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ADVANCED BRAKING CONTROL
STRATEGIES FOR TRAINS

by

Mustafa R. Abuzeid, M.Sc, B.Sc.

A Doctoral Thesis
Submitted in partial fulfilment of the requirements
for the award of the degree of
Doctor of Philosophy
of the Loughborough University

September 1996

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IN THE NAME OF GOD MOST GRACIOUS MOST MERCIFUL
To my
parents,
my wife
Mobarka Mansoor,
my children
Elias,
Malak,
Mohamed
and Maahlim.
ACKNOWLEDGEMENTS

I wish to express my sincere gratitude to both supervisors Professor Roger Goodall and Mr Bill Gabb for their indispensable help, valuable guidance and advice throughout this work.

I gratefully acknowledge my colleagues in the "control and systems" group for useful discussions and assistance, especially: Abdulla El-Abbar, Malcolm Fraser, Peter Holme, Mike Oliver, Jonathon Paddison, Dipesh Patel, John Pearson, Ian Pratt, Gang Shen.

Last but not least, my great thanks to my brothers and sisters for their encouragement and help.
ABSTRACT

The thesis describes modelling methods that are being developed to support the design and evaluation of intelligent railway braking control systems. A particular feature is that the models include higher order vehicle and train dynamics, the effects of which are expected to become important as the performance of braking systems improve. The thesis describes mathematical techniques for modelling braking systems starting with braking of a single wheelset on its own, then a single braked wheelset in a bogie, followed by a single braked wheelset in a complete vehicle and finally four wheelsets braked in a complete vehicle. The mathematical model for the braking system combines the non-linear creep laws governing the braking forces generated between wheel and rail with the suspension dynamics of a typical high speed railway vehicle.

The train model used in the simulation study consists of eight coupled vehicles each having two bogies, and includes the effects of braking on the vertical displacement and pitch of each vehicle in the train. The model is based on a British Rail MK III vehicle for which data is readily available, but it is envisaged that this model will be adapted to include newer vehicle designs once it has been successfully applied to the MK III vehicle. The simulation can readily be extended to a full train of typically eight vehicles or more once the preliminary studies have been completed.

Simulations have been carried out to assess the effect of interaction between the braking action of a number of wheelsets, and these have included both conventional control laws in which the braking action is switched on or off to avoid wheelslide under braking and control laws with proportional braking action. The thesis assesses the importance of including the dynamic interaction between the various wheelsets caused by changing vertical wheel load, considering a single bogie, a single vehicle, and extending up to a train of eight coupled vehicles. Both linear and nonlinear dynamic models were used to design the braking control system. The thesis also considers the
Abstract

effect of changing adhesion levels between the wheel and rail.
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<thead>
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<tr>
<td>a</td>
<td>bogie wheel base</td>
</tr>
<tr>
<td>A</td>
<td>state matrix</td>
</tr>
<tr>
<td>B</td>
<td>control input matrix</td>
</tr>
<tr>
<td>C</td>
<td>output matrix</td>
</tr>
<tr>
<td>c</td>
<td>damper rate (Ns/m)</td>
</tr>
<tr>
<td>D</td>
<td>control output matrix</td>
</tr>
<tr>
<td>C_e</td>
<td>slope of the graph for small value of α</td>
</tr>
<tr>
<td>C_p</td>
<td>primary damping coefficient / axle (Ns/m)</td>
</tr>
<tr>
<td>C_s</td>
<td>secondary damping coefficient / bogie (Ns/m)</td>
</tr>
<tr>
<td>[c]</td>
<td>damping matrix</td>
</tr>
<tr>
<td>Fa</td>
<td>vertical force created at pin joint between vehicle and bogie (leading end)</td>
</tr>
<tr>
<td>Fb</td>
<td>vertical force created at pin joint between vehicle and bogie (rear end)</td>
</tr>
<tr>
<td>FB</td>
<td>brake hanger force</td>
</tr>
<tr>
<td>Fp</td>
<td>deceleration force of the leading bogie</td>
</tr>
<tr>
<td>Fq</td>
<td>deceleration force of the rear bogie</td>
</tr>
<tr>
<td>FR</td>
<td>longitudinal braking force (kN)</td>
</tr>
<tr>
<td>FRn</td>
<td>longitudinal braking force (kN) at wheel n (where n = 1, 2, ... )</td>
</tr>
<tr>
<td>g</td>
<td>acceleration due to gravity (m/s²)</td>
</tr>
<tr>
<td>h</td>
<td>height of the ramp in the track</td>
</tr>
<tr>
<td>I</td>
<td>vehicle body pitch inertia</td>
</tr>
<tr>
<td>I_{ba}</td>
<td>bogie pitch inertia n(where n = 1, 2, ... )</td>
</tr>
<tr>
<td>I_c</td>
<td>integral controller gain constant</td>
</tr>
<tr>
<td>IW</td>
<td>wheelset inertia</td>
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### List Of Principal Symbols

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<thead>
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<tr>
<td>J</td>
<td>motor inertia</td>
</tr>
<tr>
<td>K</td>
<td>spring stiffness (N/m)</td>
</tr>
<tr>
<td>[k]</td>
<td>stiffness matrix</td>
</tr>
<tr>
<td>( K_s )</td>
<td>proportional controller gain constant for the electro-mechanical actuator</td>
</tr>
<tr>
<td>( K_d )</td>
<td>viscous drag coefficient</td>
</tr>
<tr>
<td>( K_L )</td>
<td>leading stiffness</td>
</tr>
<tr>
<td>( K_r )</td>
<td>reservoir stiffness / bogie (N/m)</td>
</tr>
<tr>
<td>( K_s )</td>
<td>secondary spring stiffness / bogie (N/m)</td>
</tr>
<tr>
<td>( K_t )</td>
<td>torque constant</td>
</tr>
<tr>
<td>( K_p )</td>
<td>primary stiffness / axle (N/m)</td>
</tr>
<tr>
<td>( K_{pc} )</td>
<td>proportional controller gain constant</td>
</tr>
<tr>
<td>( KV, K_\alpha )</td>
<td>parameters in linearised creep equation</td>
</tr>
<tr>
<td>( K_v )</td>
<td>back e.m.f constant</td>
</tr>
<tr>
<td>( L_A )</td>
<td>armature inductance</td>
</tr>
<tr>
<td>1</td>
<td>length of vehicle (m)</td>
</tr>
<tr>
<td>11</td>
<td>swinging arm distance (m)</td>
</tr>
<tr>
<td>12</td>
<td>swinging arm distance (m)</td>
</tr>
<tr>
<td>13</td>
<td>swinging arm distance (m)</td>
</tr>
<tr>
<td>( l_c )</td>
<td>semi vehicle length (m)</td>
</tr>
<tr>
<td>M</td>
<td>Body mass</td>
</tr>
<tr>
<td>[m]</td>
<td>mass matrix</td>
</tr>
<tr>
<td>( M_{Bn} )</td>
<td>bogie mass n (where n=1,2,...)</td>
</tr>
<tr>
<td>( M_v )</td>
<td>mass of the vehicle (kg)</td>
</tr>
<tr>
<td>n</td>
<td>rotary / linear gear ratio</td>
</tr>
<tr>
<td>R</td>
<td>vehicle reaction force (kN)</td>
</tr>
<tr>
<td>Ra</td>
<td>reaction force at the pin joint between the vehicle body and the bogie</td>
</tr>
<tr>
<td>( R_A )</td>
<td>armature resistance</td>
</tr>
</tbody>
</table>
List Of Principal Symbols

\begin{itemize}
  \item \( R_b \) reaction force at the pin joint between the vehicle body and the bogie
  \item \( R_B \) distance from the centre of the wheel to the brake hanger
  \item \( R_W \) radius of the wheel
  \item \( T_1 \) vehicle body torque
  \item \( T_2 \) leading bogie torque
  \item \( T_3 \) rear bogie torque
  \item \( t_r \) time
  \item \( T_{a_i} \) integral time constant for the electro-mechanical actuator
  \item \( T_i \) integral time constant
  \item \( \tau \) actuator time constant
  \item \( \mu \) coefficient of friction
  \item \( u \) control vector
  \item \( V \) forward speed of the train (m/s)
  \item \( x \) state vector
  \item \( \alpha \) creep
  \item \( Z \) vehicle body displacement (m)
  \item \( Z_{n_b} \) bogie displacement where \( n (n=1,2,..) \) (m)
  \item \( Z_{s_n} \) small mass displacement where \( n (n=1,2,..) \) (m)
  \item \( Z_{m_n} \) track displacement where \( n (n=1,2,..) \) (m)
  \item \( \Phi \) vehicle pitch displacement (rad)
  \item \( \Phi_{b_n} \) bogie pitch displacement (rad)
  \item \( \omega \) angular velocity of the wheel (rad/sec.)
\end{itemize}

All other symbols are defined as they appear
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CHAPTER 1
INTRODUCTION

This introductory chapter gives a brief review of the historical development of railway braking systems. It explains the field of study of this work, the definition of a braking system, and also the current braking systems that are used by British Rail. The goals of the research and the methods of approach are introduced. Improvements were gradually introduced to take into consideration not only the factors related to the vehicles, but to include the effects of the environment, such as the various adhesion conditions on the track.

The main requirement of the braking system is that the train has to stop within a minimum "emergency" braking distance under all conditions. In the early days only the locomotive and a brake van at the end of the train had brakes. As train speeds rose increasingly effective braking systems were required. The continued trend towards higher speed demanded increased performance and new approaches to the problems associated with stopping a train.

The braking ability of vehicles is one of the most important features which influence the safety and reliability of traffic. The maximum applicable deceleration enabling fast forward speed reduction depends on a great number of factors, the physical manifestations of which are not yet fully understood.

The primary factor is wheel / rail adhesion. This is in turn determined by wheel profile, rail-head profile and surface roughness variations. Since there is a limit to the amount of available adhesion, the logical follow-on to the demand for more adhesion
than is available, is to provide a protection system; or more specifically, to have more knowledge of which slide conditions are acceptable.

Adhesion. The adhesion represents the force of attachment between two contacting objects. The importance of wheel / rail adhesion to the braking action needs no emphasis. The traction and braking forces are limited by the availability of friction between the wheel and rail. The detection and correction of wheelslide are very important in the braking of the wheel, because the adhesion between the wheel and the rail is probably the most significant single factor influencing the braking performance that can be achieved by a rail vehicle. The ever increasing performance required from rail vehicles in terms of increased rates of deceleration lead inevitably to the available adhesion being exceeded at some stage. This results in wheel slide, or in severe cases, wheel skid, i.e the locking up of one or more wheelsets.

Creep is the fractional difference between the peripheral speed of the wheel and the speed of the train.

Slide occurs when too much braking force has been applied to the wheel, resulting in the slowing down or stopping of the wheel in relation to the rail, and therefore the creep limit has been exceeded.

Slip occurs when too much drive torque has been applied to the wheelset. In this case the wheelset rotates faster in relation to the rail and once again the creep limit has been exceeded. (This happens during acceleration but this is not the main issue of the thesis).

Variations in adhesion provide the most problematic aspect when decelerating as they can be quite unpredictable. If poor adhesion occurs, there is a chance that a wheel or wheels may slide which could result in damage to the rails and to the wheels. Many factors govern the usable adhesion, such as irregularities on the track, oxide layers, surface contamination.

There are two generally accepted objectives in the use of wheelslide protection:-
The developments in railway braking, as train speeds increase, implies that brake performance needs to be improved in terms of both technology and economy.

The advantages of having brakes on all the vehicles of a train operating in a centralised manner were soon recognised, and many suggestions for obtaining this have been made. It was clear that manual operation would be quite impractical to operate all the brakes on a train simultaneously, and that some form of power brake would therefore be necessary.

Many schemes have been proposed including:
- Steam
- Hydraulic.
- Pneumatic (either compressed air or vacuum)
- Electrical
- Mechanical (e.g shaft, rod, chain etc.)

The steam brake has never been used in a centralised braking system, mainly because of its slow action due to condensation. Hydraulic brakes were experimented with but only appeared in practical applications at the end of 19th century. Air and vacuum brakes were regarded as being interchangeable as the operation seemed possible by either means. Mechanical brakes were applied in practice in the mid 18th century. Note that vacuum brakes are restricted to one atmosphere of pressure, whereas compressed air can achieve several atmosphere [Moor - 1975 and Power - 1978].

The brakes should provide the following functions:
1- The driver or the operator should be capable of stopping the train through the application of the brakes.
2- The effect of the brakes should be virtually instantaneous if necessary, but also capable of gradual application in a controlled manner.
3- Minimal force must be required to apply the brakes.
4- The brakes must apply automatically if a part of the train breaks away.
5- The brakes must be able to work independently.
6- The brakes must not be affected by the failure of one vehicle or more.
7- The brakes must be simple and easy to maintain.

1.1 HISTORICAL OVERVIEW OF BRAKING SYSTEMS

From the earliest days of railways, it has been necessary to provide a means of braking the vehicles. The heavier loads, even on the early tramways using horse traction, demanded some form of brake control, particularly where gradients were involved. An excellent illustration of this is shown in Figure 1.1 where the wheels were braked by blocks. Technical developments over the years gave rise to vehicles having higher performance. The current standard for railway vehicle is the MK III design which was developed by British Rail. A typical MK III vehicle is illustrated in Figure 1.2.

A brake is defined as a mechanism for applying a force to a moving surface to slow it down or bring it to rest in a controlled manner. In doing so, a friction brake converts the kinetic energy of motion into heat which is dissipated into the atmosphere. Brakes are used in motor vehicles, trains, aeroplanes, elevators, and other machines. Most brakes are of a friction type in which a fixed surface is brought into contact with a moving part that is to be slowed or stopped.

The early wagons were provided with brake blocks mounted on either side of a brake lever, pivoting between the blocks and extending upwards to form a handle. The brakes of a railway vehicle were fitted to the engine tenders and to certain vehicles, and were applied by the fireman and the guards. An attempt was also made to employ an over-run brake, where buffer compression was used to force brake blocks onto the wheels. This proved most unsatisfactory because of the reactions caused between the
vehicles, and was soon abandoned. Steam brakes were later introduced which consisted of a small steam cylinder operating a beam connected to toggles carrying the brake blocks. However the frames and crank axles of the period were too weak to withstand the extra strain of power braking and, by and large, locomotives continued to be manually braked. At the locomotives a few were fitted with sledge brakes acting directly on the rail.

Until about the end of the 19\textsuperscript{th} century therefore, braking in general was performed by hand throughout most of the world. These were mainly screw brakes, usually operating brake blocks on all wheels of a vehicle, but sometimes sledge brakes directly into the rail were used. The sledge brakes were used on tenders and braked vehicles, and also on certain goods vehicles. In Britain lever brakes applied by a man on the ground remained the standard for goods vehicles, often operating on one wheel only.

Braking systems used on railways in Britain have been continually improved since their inception in early the 19\textsuperscript{th} century. This started from the mechanical system, and went on to hydraulic, power, and then to the introduction of disc brake systems. A more recent improvement in braking was the introduction of automatic anti-lock system [Newcomb and Spurr - 1967].

Anti-wheelslide systems have existed since the recognition of wheelslide. The earliest method which attempted to prevent the wheelsets from sliding involved the use of sand by the driver. This first attempt to improve the situation relied on the driver appreciating slide was occurring. This method is difficult, if not impossible, in a multi-vehicle system consisting of, say, an eight vehicle multiple unit where, for example, the last vehicle is sliding.

Wheelslide systems were improved and started to use mechanical equipment in their operation. Such systems used a valve connected to the brake cylinder which operated pneumatically. The valve was also connected to the axle by a flexible connection. Any rate of deceleration which exceeded the threshold limit would caused the valve to
operate and hence release the brake. This would allow the wheelset to return to the normal vehicle speed before the brakes were reapplied and this prevented the wheelset from sliding. Simple slip / slide braking mechanisms were introduced in the 1930's in Electric Multiple Units.

Since the early days, improvements in reliability and efficiency were the main targets for the designer. Historically railway braking systems may be subdivided into the following categories:-

- Pneumatically driven brakes
- Electrical brakes
- Electrical Pneumatic brakes

Each of the above three designs had their own unique problems associated with them.

The improvement of braking systems is not just to make the train decelerate or stop more rapidly. Although that is the primary reason, another important factor is concerned with ride quality requirements. Braking systems have now reached a level which satisfies the requirements of current high speed trains [Andersson - 1984]. The research will investigate advanced braking control strategies which can further improve the performance so that they can be applied to faster, more advanced trains.

1.2 OBJECTIVES OF THE WORK

A simulation model for the development of an advanced braking system is a very important subject in railway engineering because it enables various techniques for improving braking performance to be tested [Murtaza - 1993 and Kumagai - 1993].

Train simulation involves the mathematical modelling of train dynamics. There are a large numbers of parameters to be considered and it is therefore a somewhat complex process. It is now possible to simulate various train systems and track characteristics
and deduce their interaction. In general, a vehicle is subjected to coupler forces due to the track and rolling and braking forces. If the vehicle is a locomotive then the tractive effort or dynamic braking effort will also contribute to the dynamics [Oldrich and Jaroslav - 1986].

For the purposes of this research a train consisting of British Rail Mark III vehicles [Williams - 1986], has been considered, and the vehicle's vertical suspension and the interconnections and couplers have been represented in a simulation model. The model may readily be adapted for different vehicles if required. The purpose of this work has been to study the MK III vehicle passive vertical suspension system and its interaction with the braking system, with the objective of designing an advanced braking control system for the train.

Modelling methods that are being developed for the evaluation of railway braking control systems are also described. These methods are to be used in the development of high performance braking control strategies. Vehicle and train dynamics are expected to be important factors governing the improvements in the performance of braking systems.

The work started with the modelling of a single vehicle and this was incrementally built up to a multi-vehicle train. After the single vehicle had been modelled and tested the brakes were then included in the single vehicle, starting with a single wheelset braking and then subsequently applying the four wheelsets braking to the vehicle; that is, the single vehicle is under full braking control.

Two methods were used to simplify the single vehicle MK III vertical suspension model; time domain sensitivity analysis and frequency domain sensitivity analysis. During recent years there has been interest in the advanced vehicle train braking control system, due to the technical and financial benefits which may be derived from
Chapter 1 Introduction

an improved control braking system. This thesis targets the following:-
To design the tool to enable improved braking system for MK III passive vertical suspension vehicles to be designed and to test the new improved system on the computer model. A MK III vehicle was chosen for the base as there is extensive documented data that will allow verification of the simulation. The system can be adapted to other vehicles at a later date.

As the braking system improves, this leads to the following:-
A- The stopping time can be minimized as well as the stopping distance.
B- The wear of the wheelsets, brakes and track can be reduced.

1.3 OVERVIEW OF THESIS

The thesis is concerned with the design of advanced braking control strategies that can be used in trains. Mathematical modelling and simulation of single vehicle and eight coupled vehicles train provided a good understanding and a perception of the behaviour of such a system. To achieve this task the thesis is organised as follows:-

Chapter 1 Introduction

This chapter discusses the aim of the project and it describes the scope and organisation of the remainder of the thesis. Also the main contribution of the thesis is presented.

Chapter 2 Railway Braking System Overview

This chapter provides the background necessary to the investigation. It also gives a full description of the differences between the past work and this research. In this chapter some possible control system approaches are discussed.
Chapter 3 Wheel Rail Modelling

This chapter started with a discussion of modelling and simulation requirements. The chapter also introduces the theory of wheel rail interaction and presents a model of a single wheel brakes.

Chapter 4 Vehicle Modelling

This chapter provides the necessary modelling of a single vehicle, and the inclusion of the brakes. The model has been successfully tested, by both the reaction forces response of the single vehicle model and by the response of the different state behaviours. Simulation results for the braking of a single wheelset in a single bogie, braking of a single wheelset in a single vehicle and braking of four wheelsets in a single vehicle are included. It also describes the construction of a single vehicle and the principles of operation of the brake system.

Chapter 5 Train Model

This chapter presents the two different methods that were applied to simplify the single vehicle model. A full description of the coupling of eight vehicles is presented. It also shows the effect of a single wheelset braking without the inclusion of adhesion and the effect of all wheelsets braking in eight coupled vehicles.

Chapter 6 Model Linearisation

This chapter gives a full description of how the model had been linearised over three different regions in the coefficient of friction versus creep curve.
Chapter 7 Control Design

This chapter describes the controller design for the linear system model using Nichols charts, it also describes how the linear control system design parameters were implemented in the nonlinear model.

Chapter 8 Conclusion

This chapter gives the general conclusions and comments which have arisen throughout the work. Some recommendations are then made to show how the present work may be continued and enhanced.

1.4 CONTRIBUTION OF THE THESIS

The main contributions of this work are as follows:

1- A comprehensive multi-vehicle model which includes the dynamic interaction between the suspension and the brakes.
2- Modelling of existing wheelslide protection control system.
3- Modelling of a creep controlled braking system in which interactions between the wheelsets have been studied.

The models were validated against known data.
Figure 1.1 The early tramway brake.
Figure 1.2 The MK III vehicle of British Rail.
CHAPTER 2

RAILWAY BRAKING SYSTEM OVERVIEW

The primary aim of this research is to assess the performance and identify the advantages of new advanced braking control systems in railway vehicles. This chapter introduces the problem, and outlines the work performed towards solving the problem, and identifies some of the control system approaches of current braking systems.

2.1 DESCRIPTION OF A MODERN BRAKING SYSTEM

The basic system considered here is that of a typical passenger railway vehicle. The vehicle model is based on a British Rail MK III vehicle. The development of the MK III vehicle by BR is highly advanced in many aspects. Although the MK III vehicle entered service in the mid 1970s, the MK III continues to provide the benchmark for passenger vehicles for British Rail a two decades later [Eur - 1994]. In addition to the reasons mentioned in the previous chapter there are two factors which account for this performance. Firstly, the steel-built shell is structurally stiff. Secondly, the bogie design was extremely advanced design for its time.

This section also analyses the feasibility of an antilock brake control system. Over the last couple of decades, antilock brake systems have made their debut in several types of vehicles.

Some state of the art antilock brake systems employ hydraulic valve control to regulate the brake pressure during antilock operation. Brake pressure in this type of system is determined mainly by wheel deceleration during braking where the brake pressure is controlled to increase, decrease or hold according to each specific condition.
The analysis of wheelslide braking control system is focused on the closed-loop control of wheel slide. There has been an interactive wheel slide system employed since the earliest trains, using the driver's reactions to feed back and modify the braking by detecting wheel slide. However, an individual driver's reactions may tend to over correct or compensate for variable conditions. The early multiple unit braking systems were pneumatically controlled and thus, experienced a long reaction time. This encouraged low braking rates to prevent wheel slide. This problem was alleviated by subsequent electronic control. The use of electronics to control the air brakes opened up a vast opportunity to improve all aspects of the control. The driver's (or Automatic Train Control) (ATC) brake demand could be made infinitely variable.

The vehicles of the Glasgow Underground system which replaced the ageing 43 vehicles dating from the early 1890s, were fitted with an 'energize to apply' electro-pneumatic (EP) brake capable of continuous blending with the dynamic rheostatic brake, with a fully shadowing automatic air brake. The automatic brake provided a fail-safe back-up in the event of any EP brake failure and allowed the trains to be driven manually with full brake control. This avoided the more complex recovery procedures required with an "energize to release" EP brake and also allowed a locomotive equipped with automatic air brake control to haul the vehicles in a braked mode. This was the first new fleet to be fitted with this brake system which has been used very successfully in several other locations around the world.

2.2 SURVEY OF RELEVANT LITERATURE

A great number of authors have attempted to solve the problem of the contacting bodies, some of them solved the friction problem, while others attempted to solve the problem of wheel lockup. A survey of the relevant literature concerning braking control systems is presented in the following sections, along with some fundamental
Carter's theory [Kalker - 1991]. This is the rolling contact theory developed early in 1926. His theory is concerned with the action of a railway wheel. The theory depends on Coulomb's law, which shows the effect of the braking until the Coulomb maximal value is reached. Higher values of the force are not attainable, which is insufficient for the purposes of vehicle motion simulation.

Hertz theory [Kalker - 1991]. This theory, which replaced Carter's theory, considers the motion in rolling direction with the inclusion of creepage. About 1956, De Pater and Johnson actively investigated this area and modified Hertz theory as they found that the Hertz theory can be used to solve the problem of the contact area, but could not solve the problem completely, because of the underlying assumptions of the theory.

De Pater and Johnson [Kalker - 1991]. As mentioned above the Hertz theory still required some developments to solve the problem of the contact area. De Pater and Johnson studied the motion of the wheel which considers the creepage, but they used the generalization of Carter's motion of creepage, which as was later pointed out by Kalker is a rigid body motion in the contact area between the wheel and rail. The velocity which corresponds to the wheel motion is a translation and a rotation about the centre of the contact area.

Murray (1979). Studied the adhesion and wheel slide protection. One approach to considering the adhesion requirements of braking, is to look at the adhesion demands of various railway administrations based on existing vehicles and signalling distances. It is further complicated by the possibility that information on adhesion may be so specific to the test vehicle as to be not transferable in a general sense. There was some discussion as to where the peak of such a characteristic would lie in order that optimal
performance might be achieved. This would seem to be an area where some degree of consideration of the real situation is worth making, in that wheel slide would not normally be detected until the linear creepage region of the graph had been exceeded. This includes the torque response function, wheelset inertia, and the axle-load function over the period of interest.

Oldrich and Jaroslav (1986). In their study of the brakes, the basic variables relevant to this task are considered to be random processes with analytically simple properties. With this approach the influence on braking times and braking distances could be studied.

Braking effects on vehicles are predominantly judged from the point of view of safety and reliability of their operation. In order to improve the effectiveness of braking processes, they suggested antiskid devices with wheel brakes.

Murtaza and Garg (1989). They presented brake modelling as used in a train simulation study. Their model is based on the analysis of a pneumatic train circuit through which braking demand is impressed on each vehicle of the train and analysis of the vehicle pneumatic circuit which senses braking demand and corresponding brake cylinder pressure. The results are in good agreement with the experimental data. It is found that detailed and exhaustive modelling requires large amounts of computing time.

Their study concluded that the effectiveness of the brake system depends upon the pneumatic transmission of braking demand along the train length, and the kinematics of the valve controlling the supply of air to the individual brake cylinder. The fact that the pneumatic braking systems are slow to act limits the performance that can be obtained.

Lin, Dobner and Fruechte (1993). They studied the robustness of the linear controller which was analysed by a parametric study of the system's root locus. They
found that the linear controller presents difficulty in solving the sensitivity problem. They later analysed nonlinear feedback control. A formulation of sliding-mode control for the brake system is derived, and a simple solution to the brake control command is also proposed.

Their approach has shown effective control in preventing wheel lock on slippery surfaces, but it displays significant pressure and wheel slip excursions.

To enhance the brake control quality during antilock operation, an innovative approach to antilock brake control was conceived by General Motors NAO Research and Development Centre. This approach uses an electric motor as a prime mover for the brake modulator to achieve fine control of brake pressure and wheel slip. However, even though motor control is well-developed technology, questions arise on the effect of motor inertia on the overall system performance.

Non of the previous attempts have solved the current problem, because they have not taken into account the effect of creep and how it affects the interaction between the wheelsets when the brakes are in action.

2.3 CONTROL SYSTEM APPROACHES

This section aims to describe in detail the control approaches which could be used for advanced braking systems that are being developed for the evaluation of railway braking systems. This section also presents the control system designs that have taken into account the linear model parameters.

Existing methods for braking systems use open loop control of braking force and wheel slide protection. These use a simple control system consisting of an on-off control system, where the brakes are released when a certain level of creepage occurs. The objective of this research is to move towards having feedback control of the brakes. The measuring of braking force is not very helpful in low adhesion conditions which
might demand more than available, hence the use of control wheelslide or creep, as described in the next chapter by which optimum braking can be achieved. The creep force characteristic is highly non-linear and there is considerable interaction between wheelsets through the suspension system. Therefore a high quality simulation model is needed. The control of wheelslide/creep can be achieved using individual wheelset control. The control system design must, however, accommodate the non-linearities of the system.

The control system approach used in this research involves classical control system analysis based mainly upon frequency responses techniques. The control system must satisfy all the requirements to achieve the best braking system design for the MK III vehicle. Classical control design produces simple controllers for simple systems, in this case a single loop braking control system.

An alternative was to consider modern control system design. Such a control design technique has not been followed for the following reasons:

1- A classically designed controller met all the requirements of the MK III vehicle braking control system.
2- Optimal control designs can be difficult and complex.
3- Practical implementation is difficult, because often all of the states cannot be measured and this produces complex controllers.

Initially a single vehicle was simulated. From this the simulation progressed quite readily to look at coupled carriages, typically eight vehicles, once the preliminary studies were completed. The control design for the MK III vehicle started with the existing simple control system described earlier.

Subsequently, classical proportional control schemes and also proportional plus integral control schemes were used with the simulation models. The models of single wheelset
braking, single vehicle with single wheelset braking and single vehicle with four wheelsets braking, were all tested with proportional controllers and also with proportional plus integral controllers. The actuator was represented by a first order filter lag. Also a full design and modelling of an electro-mechanical actuator is presented.

In order to make the system more robust to environmental variations, the control parameters derived for the linear model have been tested with the nonlinear model. This nonlinear model controller design takes the same approach used in the wheelset slide control but in this case the change in the adhesion level between the wheel and rail is the main effect on the longitudinal braking force. It has been proven to be an effective control for the nonlinear model robust design.

Other possibilities for control strategies could be:-

• Multivariable control of the four wheelsets on a vehicle to overcome the interactions.

• Formal robust control methods such as $H_\infty$ to cope with non-linearity.

• Feed forward or preview control from one wheelset to the next.
CHAPTER 3
WHEEL RAIL MODELLING

Modelling means many things so it is necessary to begin by defining what it means with regard to this research. The aim of the research is to investigate advanced braking strategies. Modelling and simulation are part of the analysis and design process in most engineering projects and are essential in all but the simplest system. The difference between modelling and simulation is that a model is a simplified representation of a system, and simulation is a model adapted for simulation on a computer, that is mathematical or logical relations and operational rules built in to the computer program, which together are known as a computer simulation model or simply simulation model.

Simulation is similar to the laboratory experiments conducted by physical scientists to gain insight into the existing theories or to develop and validate new theories. Studying the behaviour of the system by this indirect method (i.e. by modelling and simulation) becomes a necessity in several situations where no other alternative (e.g. observation, analysis, experimentation, non-destructive testing, etc.) is possible or the alternatives available are not efficient or too costly.

This chapter presents a model to illustrate the interaction due to braking between the wheel of a railway vehicle and the rail. It also shows the derivation of the equations of motion for a single wheelset of the railway vehicle. These established equations have been used now for several years for the solution of railway dynamics problems.

The effects of the wheel rail modelling will be demonstrated when the brakes are applied to the wheel. Both the linear and nonlinear situations will be examined, as will the effects that they introduce to the vehicle dynamics.
Chapter 3 Wheel Rail Modelling

In the early days, tread brakes were used which acted on the rim of the wheel. Nowadays disc brakes are used where the disc is fitted either to the wheel itself, or to the axle which connects them, and in some cases both. The wheel brakes of a railway vehicle are the type of brakes which remain the most widely used throughout the world today, although the technology has greatly changed and improved since its early days.

This chapter also describes methods that are currently being developed for the evaluation of railway braking control systems. These are being developed for high performance control strategies.

A fundamental point is that the maximum tractive (or brake) effort which can be transmitted is the product of the prevailing adhesion factor and the vertical reaction force. The limit on the axle loading depends on the maximum speed and unsprung mass and, therefore, the maximum tractive or brake effort that could be transmitted. For wheel/steel rail contact, the surface conditions, both in terms of smoothness and cleanliness, affects adhesion considerably. The adhesion factor also varies as a function of the slide speed between the wheel and the rail. The breakaway to a low friction condition is sharp as shown in Figure 3.1, which shows the coefficient of friction versus creep. This fact, coupled with high inertias inherent in railway wheelsets, means that the majority of slip or slide systems concentrate on detecting the slip or slide as quickly as possible, and taking corrective action.

Mathematical models are being developed for the design and the assessment of advanced high performance railway braking control strategies. The modelling includes both linear dynamics and the nonlinear braking forces generated at the wheel/rail interface. The equations of motion for a single wheel and the effect of single wheel braking are reviewed in the next section. Results from a progression of models of increasing complexity are then presented. Since an understanding of the dynamic behaviour of a wheel is fundamental to the study of the dynamics of a railway vehicle an isolated wheel moving
Chapter 3 Wheel Rail Modelling

along level track is considered first. These equations are extended to the case of a rigid frame supported on two wheels, representing a single bogie. This model is further extended to represent a single vehicle, and finally to an eight vehicle train.

3.1- INTERACTION BETWEEN WHEEL AND RAIL

The railway vehicle wheelset is in the form of two wheels mounted rigidly on a common axle. In a superficial view, the behaviour of a railway vehicle wheelset is determined by purely geometrical effects.

The pure rolling motion is modified by the action of forces tangential to the wheel/rail contact point. These forces induce "creepage" or "creep". This important aspect of rolling contact behaviour is described below [Kalker - 1980, 1986, 1991, 1994].

The mechanism of the braking system works according to the following sequence. Brake pads are pushed onto a disc fitted to either the axle or the wheel. The force generated, $F_B$, is reacted by a brake hanger connected to the frame of the bogie. This opposes the rotation of the wheel and creates an elastic deformation of the contact patch between the wheel and rail. As a consequence of this a longitudinal braking force, $F_R$, is developed. The theory of this mechanism relies upon the creepage or fractional difference between the peripheral speed of the wheel and the speed of the train. The relationship between the coefficient of friction, $\mu$, and the creep, $\alpha$, depends upon the materials, but for steel wheels and rails, is typically as shown in Figure 3.1, which also indicates the adhesion limit [Andrews - 1986, Burton and Whitman - 1980].

The geometrical features of interest relate to the behaviour of the contact point on each wheel where the creep is created. A formal definition of creepage based upon the variables shown in Figure 3.2 is given in Equation (3.2). It can be seen that the longitudinal creepage is defined in terms of velocity at which the rail and wheel material
would pass through the contact zone, and is expressed as the difference between the components in the longitudinal direction of these rigid body velocities divided by the forward speed of the wheel. The creepage is produced by a tangential force and a moment about the normal axis. The tangential force is usually also resolved into its longitudinal and lateral components, although this thesis is only concerned with creep and force in the longitudinal direction. Conversely the creepage can be considered to be specified and the force and moment calculated from it. In theoretical work this is generally the case.

3.1.1- Wheel Equation Of Motion

The dynamic model of the wheel should be as simple as possible. For purposes of simulation and analysis it should also contain all the important parameters of the particular properties which are being investigated on the dynamic system of the whole vehicle. Figure 3.2 shows the forces acting on a single wheelset under braking. For the motion of a wheelset with small velocity differences relative to the track on which it is running, the creep, $\alpha$, of a single wheelset is given by the equation:

$$\alpha = \frac{(RW \cdot \omega - V)}{V} \quad (3.1)$$

in which RW is the radius of the wheel, $\omega$ is the angular speed and $V$ is the forward speed of the wheel. For braking, this results in negative values of $\alpha$, a convention which is used throughout the thesis.

When the speed difference is a significant proportion of the wheel or track speed, it is more appropriate to use the following equation, in which the difference is divided by the average of the speeds:
Equation (3.2) was used because, although it results in some increase in computational complexity, we were expecting the simulation to reach high levels of creep under some circumstances and so the more accurate equation should be used.

The dynamic equation of the wheel is therefore as follows:

\[ IW \omega = FR \cdot RW - FB \cdot RB \] (3.3)

in which \( IW \) is the moment of inertia of the wheelset, and where:

\[ FR = \mu(\alpha) \cdot R \]

The coefficient of friction \( \mu(\alpha) \) is a non-linear relationship, typically shown by Figure 3.1 for dry rail where the same characteristic applies for the negative values of creep encountered under braking. The equation can be considered to be substantially linear for a small braking force (for which adhesion is approximately proportional to creep), but it is obviously highly non-linear once the limit of the adhesion is approached.
The equation for the linear longitudinal braking force, FR, is:

\[
FR = \frac{R \left( RW \cdot \omega - V \right) C_c}{V}
\]  \hspace{1cm} (3.4)

in which \( R \) is the reaction force and \( C_c \) is the slope of the graph for small value of \( \alpha \).

The longitudinal braking force FR also depends upon the reaction force R. However, this force will change as a consequence of the redistribution of wheel loads under braking, an effect which also depends upon the dynamic properties of the vehicle suspension. It is this effect which creates the interaction between the braking systems of the various wheelsets in a train, and which must be properly understood and modelled to allow the maximum potential of high performance braking systems to be determined.

The following subsections apply those equations to single wheelset braking using parameters based upon British Rail's Mark III coach.

3.1.2- Braking Of A Single Wheelset

There are several parameters which are used to model the braking behaviour of a single wheelset. These are the radius of the wheel, \( RW = 0.5 \) m, the distance from the centre of the wheel to the brake hanger, \( RB = 0.25 \) m, and a quarter of the mass of the train, \( R = 7853.5 \) kgf or 77042.8 N. For all the preliminary studies a starting train speed of \( V = 20 \) ms\(^{-1}\) was used, although at a later stage the results were also assessed at higher and lower speeds. For a single wheelset the dynamics are simply represented by:

\[
M \ddot{V} = - FR
\]  \hspace{1cm} (3.5)
To illustrate the effect of the brakes on the velocity, creep, and angular velocity of a single wheelset running on a track, a brake application was simulated. In this case a linear adhesion - creep characteristic was used, (just limiting FB so that the adhesion limit was not exceeded). The simulation results show that as the brakes are applied, the velocity of the vehicle and the angular velocity of the wheelset start to fall, the rate of falling depending upon the braking force. The simulation was stopped when the speed of the wheelset reached zero. Figure 3.3 shows the linear speed of the wheelset and the peripheral speed of the wheel, while Figure 3.4 shows the longitudinal braking force.

Results for a steadily increasing applied force, FB, which creates longitudinal braking in excess of the adhesion limit, are shown in Figure 3.5. The upper graph shows the train speed and the peripheral speed of the wheel. The difference is small because of the low levels of creep which exist for steel on steel contact. The lower graph indicates the longitudinal braking force, FR, which is generated at the wheel/rail contact point. At this stage the braking force FB = 60 kN and the longitudinal braking FR is as shown in Figure 3.5, but all other parameters are the same as before. The simulation under development includes a simple control law representative of current practice, which switches off the braking effort if the creep exceeds a value of 0.03, and reapplies it when the wheel is no longer sliding. The resulting cyclic effect is shown in the lower graph of Figure 3.5. From this, it is clear that the average braking force achieved is significantly less than the peak which is available. In order to take full advantage of the adhesion available, it is necessary to achieve stable control of the creep at a value corresponding to the peak of the adhesion/creep curve shown in Figure 3.1, (typically around 3% although some discussion of variation in this characteristic is included later in the thesis). Note that for this case an ideal brake actuator has been assumed, i.e. no dynamic effects. Some allowance for brake actuator dynamics is included in a later chapter.

The design of control laws to enable this kind of performance to be achieved require more complex models which include the dynamic interaction between wheelsets described in the
previous section. The following chapters outline the development of such models and demonstrate the kind of interaction which can be expected.

3.2- WHEEL SLIDE CONTROLLER

Wheeled vehicles are usually braked by way of their wheels and, although there are various types of auxiliary braking systems in use, friction brakes are normally used as the most popular means of decelerating or stopping the vehicle [Gabb and Leigh - 1978]. The friction force is developed between the wheel and the rail at the time when the wheel is braked. The braking force is the output force available that will be used to decelerate or stop the train. Figure 3.6 shows the diagram of the braking scheme, including a first order filter to represent the braking system dynamics. The input is a creep signal and if the creep value is below the maximum value permitted, then that will allow the brake to be applied. If the input signal has equalled or exceeded the maximum value permitted then the brake will switched off. This force could have any profile, such as a step or a ramp and in these studies a braking ramp was used. The time constant of the actuator dynamics is typical of a pneumatic braking system.

The adhesion between the wheel and rail is the limiting factor in the braking performance on railways. Figure 3.7 shows the diagram of the wheel slide control design of a single wheelset. If the creep limit is exceeded once the brakes have applied, then they are immediately switched off. When the wheelset recovers from sliding the brakes are reapplied and this cycle is repeated. This method of wheel slide controller was implemented for Figure 3.5 results. Figure 3.8 shows the diagram of the four wheelsets slide control design. The same slide control system employed in the single wheelset was employed in the four wheelsets system, where every wheelset has an independent control system.
3.3- SUMMARY

The effect of interaction between the braking action of individual wheelset was analysed. The assessments have included the effect of simple control laws for avoiding wheel slide under braking.

The next stage in the research programme is to use these dynamic models in order to design braking controllers which will control the braking effort being applied very effectively in order to maximise the utilisation of the available adhesion. It is expected that each vehicle will have a single loop controller for each wheelset, the design of which will take account of the interactive nature of the four braking control loops. The study will also take account of changing adhesion levels, for which it is expected that connections between controllers on different vehicles will become important.
Chapter 3 Wheel Rail Modelling

Figure 3.1 The coefficients of $\mu$ versus $\alpha$ in dry conditions.
Figure 3.2 The single wheel with all forces included.
Chapter 3 Wheel Rail Modelling

Figure 3.3 The braking affect in a single wheelset.

Figure 3.4 The longitudinal braking force of a single wheelset.
Figure 3.5 The braking of a single wheelset, with simple wheel slide control.
Figure 3.6 The diagram of a controlled single wheel braking.
Figure 3.7 The diagram of a single wheelset slide control design.
Figure 3.8 The diagram of four wheelsets slide control design.
CHAPTER 4
VEHICLE MODELLING

This chapter describes the development of a sideview vehicle model which includes all the main braking and suspension components. The model was built up incrementally with extensive tests being undertaken at each stage of development to ensure that the vehicle was correctly modelled. The complexity of the model is determined to a large extent by the number of degrees of freedom. Various assumptions are made in developing the mathematical models and the applicability of a model depends on these assumptions. A complete derivation of the equations of motion is presented in Appendix A. Once the characteristics of the basic vehicle model had been verified, a model of the braking system was added to operate on the first wheelset only. This allowed the behaviour of the vehicle to be analysed under various braking conditions.

This chapter also shows the characteristic performance of a single bogie when the leading wheelset brakes. It also demonstrates the difference between the single bogie under one wheelset braking, a single vehicle under the leading wheelset braking, and a single vehicle under four wheelsets braking.

This chapter is split into three parts. The first part, sections 4.1 to 4.4, covers a range of general considerations involved in building a vehicle model [Ingemar and Mats - 1986], and looks in detail at the modelling of the secondary vertical suspension. The second part, sections 4.5 and 4.6 describes the inclusion of the braking system in the vehicle models. Finally the third part, sections 4.7 to 4.9, details the control techniques used in the braking system.
Chapter 4 Vehicle Modelling

4.1- DEVELOPMENT OF VEHICLE MODEL

1. A real railway vehicle consists of many components, most of which need not be modelled in full detail. The task of building a mathematical model for the vehicle involves identifying those parts of the vehicle which are important to its dynamic behaviour, and generating an accurate mathematical representation of these elements.

2. The secondary suspension of the MK III vehicle is provided by air springs. A secondary suspension spring carries the vertical load of the vehicle body. A full mathematical analysis of this is presented in appendix A.

3. The degree of complexity needed for modelling the primary suspension depends on the suspension arrangement. The MK III vehicle has a trailing arm which is lumped in with the wheelset in the dynamic analysis. This is equivalent to assuming that the wheel bearings are rigid, and that the centre of mass of the arm is at the bearing, which is a reasonable approximation at the suspension resonant frequency. The primary suspension will also influence other aspects of the vehicle behaviour, such as stability, ride and track forces.

4.2- FORMULATION OF THE EQUATIONS OF MOTION

The mathematical expressions defining the dynamic displacements are called the equations of motion of the system [Koffman - 1969, Koffman and Batchelor - 1963]. The solution of these equations of motion provides the required displacement history and the formulation of the equations for a dynamic system is possibly the most important phase of the entire analysis procedure.

The equations of a linear mechanical system are written by first constructing a model of the system containing interconnected linear elements and then by applying Newton's Law
Chapter 4 Vehicle Modelling

of motion to the free-body diagram. For translational motion, the Newtonian mechanics applied to a number of interconnected masses can be combined into a single matrix equation of motion and may be expressed as:

\[
\begin{bmatrix} m \\ c \\ k \end{bmatrix} \ddot{z} + \begin{bmatrix} m \\ c \\ k \end{bmatrix} \dot{z} + \begin{bmatrix} m \\ c \\ k \end{bmatrix} z = E
\] (4.1)

Where \([m], [c] and [k]\) are the system mass, damping and stiffness matrices respectively, \(\ddot{z}, \dot{z} and z\) are the vectors of accelerations, velocities and displacements respectively of the masses of interest and \(E\) is the vector of externally applied forces [Wickens and Gilchrist - 1977, Wickens et al 1969].

4.3- DEVELOPMENT OF STATE SPACE MODEL

The suspension scheme is shown in Figure 4.1. The matrices \([m], [c] and [k]\) are defined in appendix A. The parameters of the model are presented in Appendix F. The state vector for the system can be defined as:

\[
x = \begin{bmatrix} z \\ \dot{z} \end{bmatrix}
\]

which for this system consists of the displacements and rotations of the body and the bogies, along with their first derivatives and are as follows:

\[
x^T = [ Z \Phi Z_{s1} Z_{s2} Z_{b1} Z_{b2} \Phi_{b1} \Phi_{b2}
\]

\[
\begin{bmatrix} \dddot{z} \\ \ddot{z} \\ \dot{z} \end{bmatrix} = - \begin{bmatrix} m \end{bmatrix}^{-1} \begin{bmatrix} c \end{bmatrix} \begin{bmatrix} \dddot{z} \\ \ddot{z} \\ \dot{z} \end{bmatrix} - \begin{bmatrix} m \end{bmatrix}^{-1} \begin{bmatrix} k \end{bmatrix} \begin{bmatrix} m \end{bmatrix}^{-1} \begin{bmatrix} m \end{bmatrix} \begin{bmatrix} m \end{bmatrix} \begin{bmatrix} m \end{bmatrix}^{-1} E
\] (4.3)
The state space model can then be formulated as:

\[
\begin{bmatrix}
\dot{z}
\
\dot{\dot{z}}
\end{bmatrix} =
\begin{bmatrix}
0 & I

- m^{-1}k & - m^{-1}c
\end{bmatrix}
\begin{bmatrix}
z
\
\dot{z}
\end{bmatrix} +
\begin{bmatrix}
0

m^{-1}
\end{bmatrix} F
\] (4.4)

4.4- THE EQUATIONS OF MOTION

The addition of the four track inputs which are \( z_{11}, z_{22}, z_{33} \) and \( z_{44} \) to the vehicle passive vertical suspension model described previously, means that the equations of motion need to be extended. Although the main interest is in a track position input, the presence of primary dampers mean that the velocity must also be specified. Consequently track velocity is also used as an input, and track position is included as an extra state in the representation, derived by integrating the track velocity input. Figure 4.2 shows the free body diagram for the front bogie B1 which is taken to have a vertical displacement \( Z_{B1} \) and angular rotation \( \Phi_{B1} \). The figure also shows the primary suspension. The adjusted equations of motion for the MK III vehicle passive vertical suspension model are detailed in Appendix B. The eigenvalues, frequencies and damping of the adjusted equations of motion for the MK III vehicle passive vertical suspension model after the inclusion of the track inputs are listed in Table 4.1. Figures 4.3 A and B shows the free body diagram for the swinging arm of the front bogie B1 and the track inputs.
<table>
<thead>
<tr>
<th>Eigenvalues</th>
<th>Frequency (Hz)</th>
<th>Damping</th>
<th>Associated with</th>
</tr>
</thead>
<tbody>
<tr>
<td>-179.45</td>
<td>28.56</td>
<td>100%</td>
<td>small masses</td>
</tr>
<tr>
<td>-177.50</td>
<td>28.57</td>
<td>100%</td>
<td>small masses</td>
</tr>
<tr>
<td>-22.91</td>
<td>3.65</td>
<td>100%</td>
<td>small masses</td>
</tr>
<tr>
<td>-21.69</td>
<td>3.45</td>
<td>100%</td>
<td>small masses</td>
</tr>
<tr>
<td>-0.72 ± 4.67i</td>
<td>0.75</td>
<td>15.27%</td>
<td>body bounce</td>
</tr>
<tr>
<td>-15.22 ± 29.22i</td>
<td>5.24</td>
<td>46.19%</td>
<td>bogie bounce</td>
</tr>
<tr>
<td>-15.23 ± 29.87i</td>
<td>5.34</td>
<td>45.42%</td>
<td>bogie bounce</td>
</tr>
<tr>
<td>-1.29 ± 6.26i</td>
<td>1.01</td>
<td>20.17%</td>
<td>body pitch</td>
</tr>
<tr>
<td>-7.23 ± 25.29i</td>
<td>4.19</td>
<td>27.46%</td>
<td>bogie pitch</td>
</tr>
<tr>
<td>-7.21 ± 25.29i</td>
<td>4.19</td>
<td>27.39%</td>
<td>bogie pitch</td>
</tr>
</tbody>
</table>

Table 4.1 The eigenvalues, the frequencies and the damping of the MK III vehicle.
From the previous analysis it was found that the vehicle body vertical displacement mode fundamental frequency was 0.75 Hz. The two small masses, S1 and S2, which have been included in the model for facilitating the modeling procedure, have bounce modes at 28.57 Hz, 28.56 Hz, 3.65 Hz, and 3.45 Hz and the two bogies B1 and B2 have a vertical displacement mode at 5.24 Hz and 5.34 Hz.

4.4.1- The Inclusion of Track Input to The Vehicle Model.

A railway vehicle travelling along the track is subjected to the same track input to each wheel, only delayed in time. Thus a four axle vehicle is subjected to four vertical inputs ($z_1$, $z_2$, $z_3$ and $z_4$) with a time delay between them. Including the track input motion to the vehicle passive vertical suspension model, modifies both the equations of motion by including degrees of freedom representing the track input motions. The equations of motion for the vehicle vertical suspension model are then expressed in the standard form for the dynamic and output equations in state-space [Singiresu - 1990 and Steidel - 1989].

4.4.2- Governing Equations In State-Space Notation

The state space model defined by equation (4.3) is now extended to include track inputs. The track displacement can be generated from the track velocities by integration. Therefore, the state vector is extended to include track displacements and then track velocities only are required as inputs. The new set of equations of motion for the vehicle model with the track inputs is detailed in Appendix B [Murray - 1979, Newland and Cassidy - 1974].
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The extended state-space representation is:

\[
\begin{bmatrix}
\dot{z} \\
\dot{\xi}
\end{bmatrix} =
\begin{bmatrix}
0 & I & 0 \\
- m^{-1}k_{11} - m^{-1}c_{11} - m^{-1}k_{12} & 0
\end{bmatrix}
\begin{bmatrix}
z \\
\xi
\end{bmatrix} +
\begin{bmatrix}
0 \\
0
\end{bmatrix} F +
\begin{bmatrix}
0 \\
0
\end{bmatrix} u
\]

(4.5)

where

\[ u_t^T = [ z_{1t} \ z_{12} \ z_{13} \ z_{14} ] \]

and

\[ F^T = [ FR1 \ FR2 \ FR3 \ FR4 ] \]

represents the vector of longitudinal braking forces. Equation (4.5) enables the response of the body and bogie motions due to the known track input motion, \( u_t \), to be calculated.

The outputs required are the state vector and the reaction forces.

The reaction forces are defined by:-
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\[ R = \begin{bmatrix} k_{21} \\ c_{21} \end{bmatrix} z + \begin{bmatrix} c_{22} \end{bmatrix} u_t + \begin{bmatrix} k_{22} \end{bmatrix} u_t \]  

(4.6)

The output vector is therefore:-

\[ y_z = \begin{bmatrix} z \\ \dot{z} \\ R \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ k_{21} & c_{21} & k_{22} \end{bmatrix} \begin{bmatrix} z \\ \dot{z} \\ u_t \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ c_{22} \end{bmatrix} u_t \]

Which can be rewritten as:-

\[ y_z = Cx_z + Du_t \]  

(4.7)

4.4.3- Track Input Motion

To analyse the response of the vehicle dynamics a ramp was employed as a trial track motion input. A suitable delay dependent upon the train speed \( V \), being introduced to simulate the axle and bogie spacing and generate the track input at the four wheels. The four track inputs \( z_{u1}-z_{u4} \) are illustrated in Figure 4.4.

The ramp function for the first wheel \( z_{u1} \) is \( r(t) \) and was defined as follows:-

\[
\begin{align*}
  r(t) &= 0 \text{ m/sec.} & t < t_i \\
  r(t) &= h \text{ m/sec.} & t_i \leq t \leq t_f \\
  r(t) &= 0 \text{ m/sec.} & t > t_f 
\end{align*}
\]
The parameters \( t_i \), \( t_f \) and \( h \) are chosen to give the desired slope and length of ramp. The input to the wheels is a time-delayed version of the basic ramp input, and therefore the corresponding ramp functions for the other wheels will have different \( t_i \) and \( t_f \) values. For example, \( t_i \) is assumed to be 0.5 seconds and the speed \( V \) of the vehicle is 40 m/s, then the four wheels inputs \( z_i \), \( z_2 \), \( z_3 \) and \( z_4 \) hit the ramp at four different times. These times can be calculated to be \( t_1 = 0.5 \) sec., \( t_2 = 0.565 \) sec., \( t_3 = 0.90 \) sec. and \( t_4 = 0.965 \) sec. respectively. Figure 4.4 also illustrates \( t_i \) and \( t_f \) for the first wheelset.

The simulation response of the system when the track input is applied is described as follows:-

1- When the vehicle first hits the ramp

a- The vehicle body has a vertical displacement which settles to a steady state value as the vehicle passes the ramp and the same thing happens to the two small masses \( S_1 \) and \( S_2 \) and the two bogies \( B_1 \) and \( B_2 \). Figure 4.5 shows the vertical displacement response of the vehicle body. Figure 4.7 and Figure 4.8 show the vertical displacement response of the two small masses \( S_1 \) and \( S_2 \) respectively. Figure 4.9 and Figure 4.11 show the vertical displacement of the two bogies \( B_1 \) & \( B_2 \) respectively.

b- The vehicle body pitches to the rear and then returns to zero as the vehicle passes the ramp as shown in Figure 4.6. The pitch response of the two bogies \( B_1 \) and \( B_2 \) are shown in Figure 4.10 and Figure 4.12 respectively, which clearly indicates the effect of the ramp on every bogie.

c- The resultant imposed force on the bogie due to the track input rises as the vehicle hits the ramp input before moving to a steady state. Figure 4.13, Figure 4.14, Figure 4.15 and Figure 4.16 show the response of the resultant forces \( R_1 \), \( R_2 \), \( R_3 \) and \( R_4 \) respectively, which clearly indicate the effect of the ramp on the vehicle reaction forces.
Note, the steady state load is not shown for clarity, i.e. only the dynamic forces are displayed.

4.5- EFFECT OF LONGITUDINAL BRAKING FORCES

The model developed so far already possesses degrees of freedom to cater for the vertical vehicle-weight and reaction forces of the previous section. This section considers the horizontal braking forces on the bogies and railway vehicle body and their effect on the vehicle suspension system [Wickens - 1975]. In keeping with the current model assumptions, the railway vehicle is assumed to be practically rigid in the longitudinal direction. [Esveld - 1989].

4.5.1- External Forces

Figure 4.17 shows a railway MK III vehicle together with the direction of travel and the external forces of interest acting on it during braking. These forces are the weight of the vehicle body $M$, and the two bogies $M_{b1}$ and $M_{b2}$ respectively, where $M_{b1} = M_{b2}$. [Borgeaud - 1963].

The equation for the horizontal motion of the vehicle is: 

$$(M + 2M_b) \ddot{V} = -(FR1 + FR2 + FR3 + FR4) \tag{4-8}$$

Although they are not external forces Figure 4.17 also indicates the horizontal and vertical reaction forces imposed upon the body and the bogies at the connection points $Fa$ and $Ra$ for the leading bogie, $Fb$ and $Rb$ for the trailing bogie.

The equation for the horizontal motion of the railway vehicle body is: 

$$M \ddot{V} = -(Fa + Fb) \tag{4-9}$$
4.5.2- Incorporating The Longitudinal Forces

The braking forces FR1, FR2, FR3 and FR4 may be replaced by their equivalent translational and moment effects at the centre-of-mass of their respective bogie, as shown in Figure 4.17 and Figure 4.18.

For the front bogie B1

\[ F_p = FR_1 + FR_2 - Fa - M_g \ddot{V} \]  \hspace{1cm} (4-10)

\[ T_1 = (FR_1 + FR_2) \cdot h_1 + Fa \cdot h_3 \]  \hspace{1cm} (4-11)

For the rear bogie B2

\[ F_q = FR_3 + FR_4 - Fb - M_g \ddot{V} \]  \hspace{1cm} (4-12)

\[ T_2 = (FR_3 + FR_4) \cdot h_1 + Fb \cdot h_3 \]  \hspace{1cm} (4-13)

The vehicle body retarding forces Fa and Fb, may also be replaced by their equivalent translation and moment effects, as shown in Figure 4.17.

\[ F_S = (Fa + Fb) = - M \dot{V} \]  \hspace{1cm} (4-14)

\[ = FR_1 + FR_2 + FR_3 + FR_4 - (F_p + F_q) \]

\[ T_3 = (Fa + Fb) \cdot h_2 \]  \hspace{1cm} (4-15)
Thus the equation of motion for the railway vehicle is given, as before. The equations are partitioned as:

\[ \bar{x} = \begin{bmatrix} x \\ u_t \end{bmatrix} \]

Equation (4.10) describes the longitudinal braking dynamics of the leading bogie. It relates the total longitudinal force to the straight line acceleration. Equation (4.11) describes the torques acting on the leading bogie and relates this to its pitching dynamics. Similar equations exist for the dynamics of the rear bogie. The straight line forces acting on the body are related to its acceleration by equation (4.14), the body pitching dynamics are related to the external forces in equation (4.15). The input matrix is now arranged to contain the torque effects and the masses of the vehicle body and the two bogies. The input matrix is now arranged as follows:

\[
F = \begin{bmatrix} -Mg \\ T3 \\ 0 \\ 0 \\ -M_B g \\ T1 \\ -M_B g \\ T2 \end{bmatrix}
\]

The effect of the self-weight is arranged as follows.
and $x_{(0)}$ can be employed as the initial condition for the braking study analysis.

4.6- BRAKING OF A BOGIE

The next step is to incorporate two wheelsets into a bogie. A side view bogie representation is shown in Figure 4.19, consisting of the two wheelsets with primary suspension, in which each wheelset possesses a parallel spring and damper. When a bogie is running on a track, it is influenced by any disturbances introduced to the wheels such [Wickens 1965] as irregularities in the track or a braking force. The irregularities of the track were ignored in the simulation as they were not of interest. The research here is concerned with the brakes and their effects on the vehicle dynamics. The load on both wheels is the main factor in determining the value of the maximum braking force per wheelset. This loading is affected by the distribution of the normal reaction forces, which in turn determines the value of the maximum braking force [Kalousek and Johnson - 1992].

For this analysis a single bogie running along level track is considered. The model includes a steady vertical force applied to the centre of the bogie, with a value equal to half the vehicle body weight. In this way the reaction forces are of a representative level,
although any dynamic interaction through the secondary stage of suspension is excluded. The two brake hanger forces are reacted on the bogie frame, and this effect must be included. The simulation is based on state space analysis technique, where the state, output, and input variables relate to system displacement \( Z_0 \) and pitch angle, \( \Phi_B \). The parameters of the bogie are arranged in the state space \([A], [B], [C] \) and \([D]\) matrices as shown in the earlier section of this chapter. The variables of the wheel are also arranged into a state space formulation [Vijay and Rao - 1984].

The equations of the wheelset variables, which are the acceleration and the angular velocity of the wheel, were arranged as follows:

\[
\begin{bmatrix}
\ddot{\theta} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\theta \\
V
\end{bmatrix} +
\begin{bmatrix}
RW/IW - RB/IW \\
0 \\
-1/MV
\end{bmatrix}
\begin{bmatrix}
FR \\
FB
\end{bmatrix}
\] (4.16)

The relevant outputs are:

\[
\begin{bmatrix}
Y_1 \\
Y_2 \\
V \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
RW & 0 & -1 \\
\frac{RW}{2} & 0 & 1 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{\theta} \\
\theta \\
V
\end{bmatrix}
\] (4.17)

where \( \ddot{\theta} = \omega \) which is the angular acceleration and \( \dot{\theta} = \omega \) which is the angular speed. By inspection of equation (4.17) \( Y_1 \) is the difference between the speed of the wheel and the speed of the train, whilst \( Y_2 \) is the average between the two speeds. The quotient \( Y_1/Y_2 \) gives the creep value, where \( V \) is the velocity of the wheelset and \( \theta \) is defined as before.
The dynamics of the creep mechanism and the kinematic dynamics of the suspension are incorporated in a single model. The brakes of the first wheelset were applied in this case without any wheel slide control, and the behaviour of the bogie was analysed. The characteristic of the reaction forces is shown in Figure 4.20. The braking creates a small pitch to the front, with a corresponding increase in the reaction force on the front wheelset and a decrease on the rear wheelset. The change in the reaction forces is noticeable and this will affect the longitudinal braking force, FR, although at this stage the interaction is not large. The damped oscillations are the result of the dynamic modes of the bogie suspension.

4.7- BRAKING OF A SINGLE VEHICLE

In order to assess the effect of body dynamics on the reaction forces, the next stage in the model development is the full vehicle system [Misum - 1990], as shown in Figure 4.21. The full vehicle model has 16 states. Four states account for the vertical displacement and pitch mode of each of the three masses, and an extra four states are associated with the secondary suspension dynamics. A similar technique is used here as the one used in the bogie simulation, where the variables of a single vehicle are arranged in the form of a state space analysis. The same technique followed to couple the vehicle dynamics with the wheelset dynamics in one MATLAB program. Figure 4.22 shows the program flow chart of a single vehicle with front wheelset braking. The dynamic response of a complete vehicle under leading wheel braking is again analysed. Applying the brake on the first wheelset causes the vehicle body to pitch forward and increase the load on the front wheel. The pitch of the vehicle body is shown in Figure 4.23, but there is no vertical displacement. The S1 and S2 vertical displacement are as shown in Figure 4.24 and Figure 4.25 respectively. The front bogie pitches forward and the rear bogie pitches to the rear. The vertical displacement and pitch of the front bogie are as shown in Figure 4.26 and Figure 4.27 respectively. Also, the vertical displacement and pitch of the rear bogie are shown in Figure 4.28 and Figure 4.29 respectively. Figure 4.30 shows the
reaction forces at the four wheels, and demonstrates the extra dynamic interaction shown both in the increased variation in forces and in the lower frequency variations corresponding to the dynamic modes of the body [Abuzeid et al - 1994].

4.8- CONTROLLED BRAKING OF A SINGLE VEHICLE

In the previous section the effects of dynamic interaction between the wheelsets has been demonstrated through the application of braking effort to just one of the wheelsets. In practice, however, all wheelsets will be braked together [Sauvage and Pascal - 1990, Stepan - 1991], and this section demonstrates what happens to the four wheelsets on a single vehicle when the adhesion limit is exceeded. The simple control law used earlier in chapter 3 has been included for each wheelset. Although the simulation model includes the interactions, the controllers themselves are completely independent of each other.

Figure 4.31 shows the braking signals for the four wheels to indicate whether the brakes are on or off. The simulation used in this section follows the same technical steps used in the previous section with the vehicle dynamic simulation. However, in the wheelset dynamic simulation, the only difference here is that the four wheelsets perform the braking instead of one wheelset. The wheelset variables in the state space equation follow a similar format to the single wheelset model. Clearly, the equations will be extended to accommodate all four wheelsets. The characteristic of the controlled vehicle is analysed and Figure 4.32 shows the corresponding reaction forces at each wheel under a steadily increasing applied braking effort. The redistribution of wheel loads is clearly shown, and the order in which the wheels slide as the adhesion limit is exceeded is a consequence of this redistribution; wheelset number four slides first, then wheelset number two, followed by wheel number three and finally wheel number one. The wheel slide control applied to each wheelset creates the kind of cyclic on-off braking pattern which was seen for a single wheel (the lower graph of Figure 3.5) but now this involves all the wheelsets in a more complex pattern, a consequence of the interaction through the vehicle dynamic system.
4.9- CONTROL BRAKING TECHNIQUE

The braking control system used to stop or decelerate the railway vehicle should be a very advanced system. The braking control system aims to use the maximum braking force effort available while avoiding wheel slide. Wheel slide is a major problem in railway braking [Theodor and Aurel - 1982].

4.9.1- Weight Transfer

As described earlier, the railway vehicle body is generally linked to the bogies by secondary suspension. Clearance is provided by the suspension and this permits a certain displacement. The starting point of braking system design is the static weight distribution [Kalker and Piotrowski - 1989]. Under normal operating conditions, when a vehicle stands freely on the track, its weight is distributed evenly over its axles. Measurements show the different reaction forces of the wheels due to the application of the braking effort. When the brakes are applied the weight on the leading axle of a two bogie vehicle is increased by primary weight transfer. The leading bogie also receives, in addition, an increase from the secondary body weight transfer. There is weight transfer due to the way in which the braking force is applied to the vehicle. The individual braking forces act at the rail surface, whereas the mass of the vehicle is considered as being concentrated at the centre of gravity and the distance between the wheel/rail contact point and the centre of gravity above the railway surface. A reaction to the braking force, of the same magnitude will therefore act at this distance. The vehicle will tend to rotate in such a way that the loading of the front axle will increase and that on the rear axle decrease. This is clearly shown in the reaction forces illustration (see Figure 4.32). The weight transfer to the front causes the last wheel of the vehicle to slide first. As the wheel slides the brakes are turned off. This results in less weight transfer since this is in relation to the slide which simply means that the sliding wheel has a less weight acting on it. The braking on - off cycle is shown in Figure 4.31.
4.10- ESTABLISHED STRATEGIES

In a diesel or electric locomotive, where under normal operating conditions the traction equipment allows either traction or braking effort to be applied without slipping, a pair of wheels can slip under power without the driver being aware of it. Unless means are incorporated to prevent it, excessive slipping can quickly cause very serious damage to the wheel and rail. When slip happens in one pair of wheels then the control system needs to take appropriate action by reducing the power. In the case of braking, the brakes have to be switched off until the wheelsets recover from sliding and regain the original speed. One of the most important factors in the avoidance of wheel slip has been found to be the smooth application of power and the absence of sudden increases in traction effort. An important contributory cause of slipping, where power axles are not coupled, is the transference of weight from the leading axles to the trailing. This is particularly noticeable at starting with standard forms of bogie suspension. Draw gear reaction (the connection between vehicles) causes the leading wheels to be unloaded during acceleration and the trailing wheels during braking, with a resultant reduction in achievable traction and braking force.

4.11- SUMMARY

A side view vertical model of a railway vehicle has been developed and validated by assessing the eigenvalues and examining its response to track inputs. This model has been adapted to have braking forces at the wheels as inputs, and also to generate the corresponding reaction forces. This enables the dynamic interaction to be examined in response to braking forces and a number of results have been assessed to ensure that the model is properly representative.

The studies have indicated the importance of including the dynamic interaction between the various wheelsets caused by the changing vertical wheel load. The results have shown
that both bogie dynamics and body dynamics are important.
Figure 4.1 The MK III vehicle vertical suspension model.
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\[ K (Z_{s1} - Z_{b1}) + C \dot{Z}_{s1} - \dot{Z}_{b1} \]

Figure 4.2 The free body diagram of the front bogie B1 displacements.
Figure 4.3 The free body diagram of the swinging arm of the front bogie B1 and track inputs.
Figure 4.4 The four track inputs $z_{i1} - z_{i4}$. 
Figure 4.5 The vehicle body displacement response to a ramp.

Figure 4.6 The vehicle body pitch response to a ramp.
Figure 4.7 The S1 displacement response to a ramp.

Figure 4.8 The S2 displacement response to a ramp.
Figure 4.9 The front bogie displacement response to a ramp.

Figure 4.10 The front bogie pitch response to a ramp.
Figure 4.11 The rear bogie displacement response to a ramp.

Figure 4.12 The rear bogie pitch response to a ramp.
Figure 4.13 The dynamic reaction force of the first wheelset response to a ramp.

Figure 4.14 The dynamic reaction force of the second wheelset response to a ramp.
Figure 4.15 The dynamic reaction force of the third wheelset response to a ramp.

Figure 4.16 The dynamic reaction force of the fourth wheelset response to a ramp.
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height from rail vehicle - bogie coupling to vehicle centre of mass.

$V_h$ horizontal acceleration of vehicle.

height, bogie (cm), of pinned coupling between rail vehicle and bogie.

height from braking force line of action to centre of mass of bogie.

Figure 4.17 The vehicle with all the external forces acting on it during braking.
Figure 4.18 The equivalent translation and moment effect.
Figure 4.19 The side view bogie representation.
Figure 4.20 The bogie reaction forces, with uncontrolled braking
Figure 4.21 The side view MK III vehicle suspension model.
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Figure 4.22 The program flow chart of a single vehicle with front wheelset braking.
Figure 4.23 The vehicle body pitch response to first wheelset braking with braking force (FB = 60000 N).
Figure 4.24 The S1 vertical displacement response to first wheelset braking.

Figure 4.25 The S2 vertical displacement response to first wheelset braking.
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Figure 4.26 The front bogie B1 vertical displacement response to first wheelset braking.

Figure 4.27 The front bogie B1 pitch response to first wheelset braking.
Figure 4.28 The rear bogie B2 vertical displacement response to first wheelset braking.

Figure 4.29 The rear bogie B2 pitch response to first wheelset braking.
Figure 4.30 The reaction forces of a single vehicle with one wheelset braking, uncontrolled braking.

Figure 4.31 The on-off braking signal.
Figure 4.32 The reaction forces of a controlled single vehicle four wheelset braking, using the nonlinear model of wheel/rail contact.
CHAPTER 5
TRAIN MODEL

This chapter describes the methods adopted to model a full train and the studies undertaken to simplify that model. In order to investigate possibilities for limiting the complexity of the multi-vehicle model, a sensitivity analysis was carried out on the time and frequency domain responses of the single vehicle model. This was to identify areas where simplification might be possible. The analysis involved making small changes to the values of a number of key parameters in the primary and secondary suspension, and determining the corresponding changes to the responses.

The reaction forces are one of the main factors affecting the longitudinal braking forces. The effects of the changes were quantified by calculating the r.m.s variations in the time responses of the four reaction forces compared with the initial responses, and combining these variations into a sensitivity matrix. The frequency response analysis was used to give a deeper understanding of these sensitivity figures by indicating the band or bands of frequency most affected by changing a particular parameter.

The two types of analysis technique used to investigate the possibility of simplifying the vehicle passive vertical suspension model were:

1- Time domain sensitivity analysis.
2- Frequency domain sensitivity analysis.

An analysis was then developed of eight coupled vehicles with the leading wheelset braking. Finally the modelling of eight coupled vehicles with all wheelsets braking was considered.
The sensitivity analysis in both time and frequency domain were carried out, using the MATLAB package, from a state-space representation of the model.

5.1- TIME DOMAIN SENSITIVITY ANALYSIS

The idea of the time domain sensitivity analysis is to calculate the change in the time response to a particular input when various parameters are altered compared with the time response using nominal values for the parameters. The relative size of the change enables the sensitivity of the parameters to be assessed.

The vehicle was simulated running along a level track at constant velocity and the brakes of the first wheelset were applied. No control system was incorporated at this stage.

The response of the vehicle model to the application of brakes on the leading wheelset of the front bogie with the nominal system parameters is shown in Figure 5.1. Each parameter of the MK III vehicle model was changed in turn by 20%, and the responses to such changes are shown in Figures 5.2, 5.3, 5.4, 5.5 and 5.6. The responses of the vehicle model with its nominal system parameters and the responses when the parameters of $K_u$, $K_r$, $K_p$, $C_r$ and $C_p$ were independently changed by 20% respectively are then compared.

In order to quantify the difference caused by varying the parameters the r.m.s error between the nominal and altered responses is calculated.

The reaction forces of the single vehicle, when the brakes were applied to the first wheelset, are the nominal reaction forces which can be used as a reference datum. The value of a single parameter of the model was changed by 20% and a comparison was made between the adjusted and the nominal reaction forces. Further comparisons were
made by systematically adjusting each parameter of the model in turn. The r.m.s difference was calculated between the nominal and changed parameters.

The RMS error is

$$E_{RMS} = \sqrt{\frac{1}{T} \int_0^T [R - R']^2 \, dt} \quad (5.1)$$

Here the set of the nominal reaction forces, $R$ is given by

$$R = [ R_1 \ R_2 \ R_3 \ R_4 ] \quad (5.2)$$

and the set of reaction forces when there is change to the system parameters, $R'$ is

$$R' = [ R'_1 \ R'_2 \ R'_3 \ R'_4 ] \quad (5.3)$$

The importance of this test is not the absolute value, but the difference between the nominal and changed reaction forces and none of the values were large enough to give justification for simplifying the model. If one value or more was small enough than the others, the simplification would have been possible. Table 5.1 shows the different values of the r.m.s error of the four reaction forces when all parameters of the system are increased by 20%.

From Table 5.1 the following points can be observed:-

1- The change in the reaction forces due to variations in primary suspension parameters is larger than the changes due to variations in the secondary suspension parameters. This is primarily due to the effect caused by the braking force being applied to the leading wheelset of the single vehicle model.

2- The changes in the reaction forces of the leading bogie due to variations in the
primary suspension is larger than the change in the reaction forces of the trailing bogie, and that is once again due to the braking force been applied to the leading wheelset of the single vehicle.
Table 5.1 The R.M.S change in reaction forces (N) for 20 % variation in suspension parameters.

<table>
<thead>
<tr>
<th>Changed parameter</th>
<th>R1</th>
<th>R2</th>
<th>R3</th>
<th>R4</th>
</tr>
</thead>
<tbody>
<tr>
<td>K_s</td>
<td>48.3</td>
<td>48.3</td>
<td>48.3</td>
<td>48.3</td>
</tr>
<tr>
<td>K_r</td>
<td>100.7</td>
<td>100.7</td>
<td>100.7</td>
<td>100.7</td>
</tr>
<tr>
<td>K_p</td>
<td>116.4</td>
<td>122.6</td>
<td>42.4</td>
<td>42.8</td>
</tr>
<tr>
<td>C_t</td>
<td>60.0</td>
<td>60.0</td>
<td>60.0</td>
<td>60.0</td>
</tr>
<tr>
<td>C_p</td>
<td>147.1</td>
<td>149.7</td>
<td>12.6</td>
<td>13.4</td>
</tr>
</tbody>
</table>
5.2- FREQUENCY DOMAIN SENSITIVITY ANALYSIS

The frequency-domain analysis is derived from the well known state space equation of the system given by

\[
\dot{x} = Ax + Bu \quad (5.4)
\]

\[
y = Cx + Du \quad (5.5)
\]

Thus the Laplace transform of equation (5.4) is

\[
sX(s) = AX(s) + BU(s) \quad (5.6)
\]

\[
(sI - A)X(s) = BU(s) \quad (5.7)
\]

\[
X(s) = (sI - A)^{-1}BU(s) \quad (5.8)
\]

This can be formulated in terms of \(Y(s)\) which represents the wheel reaction forces, and \(U(s)\) which represents the wheel braking forces.

The Laplace transform of equation (5.5) is

\[
Y(s) = CX(s) + DU(s)
\]

Applying the same technique to the system dynamics of equation (4.1) results in
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\[ s^2 \ [m] Z(s) + s \ [c] Z(s) + [k] Z(s) = F(s) \]  

(5-9)

or

\[(s^2 \ [m] + s \ [c] + [k]) Z(s) = F(s)\]  

(5-10)

By setting

\[ R(s) = (s^2 \ [m] + s \ [c] + [k]) \]  

(5-11)

then

\[ Z(s) = R^{-1}(s) F(s) \]  

(5-12)

The frequency response is given in the normal way using

\[ Z(j\omega) = R^{-1}(j\omega) F(j\omega) \]  

(5-13)

The frequency domain response is represented by the magnitude and the phase of the vehicle model system outputs. Nominal and changed responses were taken in two steps as follows:

1- When there is no change in the system parameters, the nominal response is shown in Figure 5.7.

2- When there is an increase of 20% to each of the system parameters \( K_s, K_r, K_p, C_r \), and \( C_p \), in turn the responses are shown in Figures 5.8, 5.9, 5.10, 5.11 and 5.12 respectively. These show the responses of the front wheelset reaction
force with respect to the change of the model parameters, and the nominal responses.

While the responses were certainly affected more by changing some parameters than others, none of the variations was sufficiently small to be able to justify a significant simplification of the vehicle model. It had, for instance, been hoped that it would be possible to neglect the primary suspension dynamics, which would have created a considerable reduction in complexity. The results make it clear, however, that this would not have been appropriate, and so the multi-vehicle model was created by full replication of the single vehicle model.

5.3- INTER-VEHICLE COUPLING EFFECTS

The next stage in the modelling process is to link together a number of individual vehicles to form a complete train, in this case containing eight vehicles which is a typical train length.

The developed equations of motion necessary to the interaction of vehicles are as shown in Appendix C.

There are a number of effects caused by the coupling of vehicles during braking. These are:-

1- Due to the longitudinal coupling bar between vehicles. This is the main contribution.

2- Due to the gangway. This is meant to be soft, so that any effect caused by it is negligible. Therefore the stiffness (and damping) of the inter-vehicle gangway connection was not included in the dynamic model.

3- Due to the reaction force change created on all vehicles by deceleration from any braked wheel.
A linear dynamics model is shown in Figure 5.13 which is used for an eight-vehicle coupled train.

Each coupled vehicle has the same dimensions and parameters as the MK III single vehicle model. The eight-vehicle model has a simplified restrictive force in which the coupling bar is modelled simply in the vertical direction as a spring and longitudinally as a rigid bar. Clearly this coupling will effect the pitch and bounce modes of the connected vehicles [Sharp - 1982 and Lohmeier - 1993]. The coupling between the eight vehicles is represented by a stiffness $K_c$, where the couplers connect the vehicles together. Since the couplers are not connected along the vehicle centre lines they have an effect in bounce and pitch modes.

The vertical displacement analysis of the coupling of the eight-vehicles also allows for braking in the current model of the eight-vehicles of the train. The analysis considers the following:

1- The braking of a front wheelset in an eight-vehicle train (uncontrolled braking for an eight vehicle train).

2- The braking of all wheelsets in an eight-vehicle train (controlled braking of an eight vehicle train).

In all cases the train is assumed to be travelling on straight level track. The external forces acting on the eight-vehicles are the effects of the braking forces at the wheel/rail contact points. The link forces and the reaction forces at the latter contact point are incorporated in to the train vehicle model.

### 5.4- UNCONTROLLED BRAKING FOR AN EIGHT VEHICLE TRAIN

The effect of braking results in dynamic interactions between vehicles in the train. The longitudinal force in the vehicle coupling generated by these interactions plays an important role in the vertical analysis of the whole train [Garg and Dukkipati- 1984]. Figures 5.14, 5.15 and 5.16 show the reaction forces of the front, second and rear vehicles when the brakes are applied to the first wheelset of the front vehicle. As the
brakes are applied the body of each vehicle has a vertical displacement due to the forces in the inter-vehicle connections, and they all pitch due to the braking forces [Keizer - 1986]. The reaction forces for the thirty two wheelsets in the eight vehicle train are all different. However the results clearly show the effect of the coupler when the reaction forces of the leading vehicle are compared with the reaction forces of other vehicles in the train. The results also show that the additional interaction compared with a single vehicle is relatively small.

5.5- CONTROLLED BRAKING OF AN EIGHT VEHICLE TRAIN

This section represents the most realistic results for the eight coupled vehicle train as all the thirty two wheelsets are subject to braking [Murtaza and Garg - 1989 and 1992]. This is the most complex model as the simulation involves the coupling of eight single vehicles. Each vehicle is assigned an equal braking force following the same strategy as detailed in chapter four. Figure 5.17 shows the reaction forces of the four wheelsets of the leading vehicle, Figure 5.18 shows the four reaction forces of the second vehicle and Figure 5.19 shows the reaction forces of the four wheelsets of the rear vehicle. Figure 5.20 shows the braking signals for the four wheelsets of the leading vehicle to indicate whether the brakes are on or off following the same control principle as in the controlled single vehicle.

When comparing the controlled braking of an eight vehicle train and a single vehicle the results showed that there was no significant difference between the responses of the reaction forces. The on-off control signal of the leading vehicle of the eight coupled train and the single vehicle are following the same pattern. This similarity allows concentration on the responses of a single vehicle as the resulting simpler simulation still produces results that can be applied to multiple vehicle trains.
5.6- SUMMARY

From the results of both time domain and frequency domain analysis the following points were observed:-

1- The three methods used to simplify the model known as the RMS error, the time domain and the frequency domain analysis agreed that the model can not be simplified any further.

2- Both the time domain analysis and the frequency domain analysis indicated that any change in the secondary parameters of the model would not cause a significant difference in the reaction forces of the model.

3- Both the time domain analysis and the frequency domain analysis agreed that any change in the primary parameters of the model would not cause a significant difference in the reaction forces of the model.

As mentioned earlier the longitudinal braking forces are affected by the reaction forces, and it is found that any change in the primary or secondary suspension will not cause a significant difference in the reaction forces. This in turn will not make a significant effect in the longitudinal braking force, which indicate that the simplification is not possible. The output responses show that neither the primary nor the secondary suspension can be eliminated.

The analyses confirmed that the original model cannot be simplified any further.

From the results it was also found that there were no significant changes between the response of the reaction forces of a single vehicle model in isolation and the response of the reaction forces of the eight interconnected vehicle model. The results indicated that the on-off braking signal in the single vehicle and in the leading vehicle of the eight coupled train follow similar patterns. This indicated that any further study could be carried out on the single vehicle model.
Figure 5.1 The reaction force of the first wheelset with no change in parameters.

Figure 5.2 The combined reaction force of the nominal and changed response of the first wheelset when $K_i$ is increased by 20%.
Figure 5.3 The combined reaction force of the nominal and changed response of the first wheelset when $K_r$ is increased by 20%.

Figure 5.4 The combined reaction force of the nominal and changed response of the first wheelset when $K_r$ is increased by 20%.
Figure 5.5 The combined reaction force of the nominal and changed response of the first wheelset when $C_r$ is increased by 20%.

Figure 5.6 The combined reaction force of the nominal and changed response of the first wheelset when $C_p$ is increased by 20%.
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Figure 5.7 The reaction force of the first wheelset with no change in parameters.

Figure 5.8 The combined reaction forces of the nominal and changed response of the first wheelset when $K_1$ is increased by 20%.
Figure 5.9 The combined reaction forces of the nominal and changed response of the first wheelset when $K_r$ is increased by 20%.

Figure 5.10 The combined reaction forces of the nominal and changed response of the first wheelset when $K_r$ is increased by 20%.
Figure 5.11 The combined reaction forces of the nominal and changed response of the first wheelset when $C_1$ is increased by 20%.

Figure 5.12 The combined reaction forces of the nominal and changed response of the first wheelset when $C_2$ is increased by 20%.
Figure 5.13 The side view model of eight coupled MK III vehicles.
Figure 5.14 The reaction forces of the leading vehicle uncontrolled braking.

Figure 5.15 The reaction forces of the second vehicle uncontrolled braking.
Figure 5.16 The reaction forces of the rear vehicle uncontrolled braking.

Figure 5.17 The reaction forces of the leading vehicle controlled braking.
Figure 5.18 The reaction forces of the second vehicle controlled braking.

Figure 5.19 The reaction forces of the rear vehicle controlled braking.
Figure 5.20 The on-off braking control signal for the leading vehicle.
A common technique in nonlinear system analysis and design is to create a linearised representation of the system [Bharath et al - 1990]. This allows frequency response methods to be used in the design of a control system and greatly simplifies the design and analysis processes. Linearisation is the procedure in which a set of nonlinear differential equations is approximated by a linear set [Lathi - 1974].

This chapter explains the methods that have been used for the linearisation of the nonlinear vehicle model in order to prepare the model for the design and application of a control system. These methods were applied to obtain linear models for single wheelset braking, single vehicle with single wheelset braking and single vehicle with four wheelsets braking. This chapter explains the linearisation methods that will be used to generate linear models for the design of single loop control systems and this is described in the next chapter [Athans and Falb - 1966]. The actual linearisation was carried out by linearising the wheel rail interface dynamics.

The ultimate aim of an advanced braking control system is to control the longitudinal braking force in response to a braking command. For an individual wheel this is a function of the normal reaction force, angular velocity of the wheel, and longitudinal wheel velocity. Therefore by linearising the system with respect to these variables it is possible to design a control system that can control the longitudinal braking force.

6.1- LINEARISATION APPROACH

There are two fundamental non-linearities; the creep equation, which governs the
wheel/rail interface (a function of angular velocity, \( \omega \) and the train speed, \( V \)) and the force equation (a function of the creep (\( \alpha \)) and wheel/rail reaction force (\( R \))):

The creep equation is

\[
\alpha(\omega, V) = \frac{2 \cdot (RW \cdot \omega - V)}{(RW \cdot \omega \cdot V)}
\]

and the force equation is

\[
FR(\alpha, R) = R \cdot \mu(\alpha)
\]

The model is linearised by choosing an operating point and partially differentiating the nonlinear equations with respect to each of the independent variables. Three different points on the coefficient of friction versus creep curve have been selected. They are:

A- On the rising slope of the coefficient of friction versus creep curve.
B- On the peak of the coefficient of friction versus creep curve.
C- On the falling slope of the coefficient of friction versus creep curve.

The model is linearised at each of these chosen points. The aim will be to operate at the peak of the curve to give optimum braking, but it is necessary to ensure that the controller is designed to be stable and operate effectively for deviations away from this point.

The values of creep at the three different operating points are as follows:

1- 2.5%
2- 3%
3- 3.5%

These chosen points respectively correspond to the points 0.025, 0.03 and 0.035, as
shown in Figure 6.1. The three different points represent a stable region, a neutral region and an unstable region respectively. The coefficient of friction versus creep curve parameters of the model need to be linearised at each of the chosen operating points. The output of the linear model represents the longitudinal braking force. The block diagram in Figure 6.2 shows the linear model for a single vehicle. The gains $K_\omega$, $KV$, $K_\alpha$ and $KR$ represent the variation of $\omega$, $V$, $\alpha$ and $R$ respectively at a given operating point. After the linearisation has been performed, frequency response methods can be used to design the control system.

The following equations give the expressions for the linear gains:-

$K_\omega$ is given by

$$K_\omega = \frac{\partial \alpha (\omega, V)}{\partial \omega} \tag{6.3}$$

$KV$ is given by

$$KV = \frac{\partial \alpha (\omega, V)}{\partial V} \tag{6.4}$$

$K_\alpha$ is given by

$$K_\alpha = \frac{\partial \mu (\alpha)}{\partial \alpha} \tag{6.5}$$

and $KR$ is given by
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\[ KR = \frac{\partial FR}{\partial R} \]  

6.2- MODEL LINEARISATION

The linearisation needs to be performed at the three operating points in order to observe how the model behaves at these points. In the first region the wheel under the action of a braking force should perform without any slide, in the second region the wheel under braking force will be at an optimum point and the third region is where the wheel is under the action of a braking force and is expected to slide. The linear model of a single vehicle is shown in Figure 6.3, which clearly shows the different parts of the whole system and how they are interlinked to indicate the required optimum braking force. A description of the linear single vehicle model is presented at the end of this chapter.

6.2.1- Linearising The Model At Stable Point (0.025)

The first point at which the model was linearised was chosen to be at a very stable point on the coefficient of friction versus creep curve. The value of creep in this very stable region was chosen to occur just before the peak of the curve at a value of 0.025. From the main creep equation (6.1) the angular velocity of the wheel was then calculated. After the angular velocity of the wheel had been calculated then the values of \( K_\omega \), \( KV \), \( K_\alpha \) and \( KR \) were determined, detailed calculation is presented in later sections.

6.2.2- Linearising The Model At Neutrally Stable Point (0.03)

The second point was chosen to be at the neutral region (peak of the curve); this is the
ultimate operating point of the braking force. This point has a creep value of 0.03. The angular velocity of the wheel ($\omega$) was calculated from the main creep equation (6.1). After the angular velocity of the wheel had been calculated then the values of $K_\omega$, $K_V$, $K_\alpha$ and $K_R$ were determined as described in later sections.

6.2.3- Linearising The Model At An Unstable Point (0.035)

The third point was chosen to be at an unstable region. The creep at this unstable region was chosen to be at the point which occurs after the peak of the curve where the wheelset is expected to slide; a creep value of (0.035) was selected. The values of $K_\omega$, $K_V$, $K_\alpha$ and $K_R$ were determined by the same method used for the other operating points.

6.3- CALCULATING THE LINEAR PARAMETERS.

This section describes the calculation of the mentioned gains. To calculate the $K_\omega$, equation (6.1) should be differentiated with respect to $\omega$, which produces

$$
K_\omega \frac{\partial \alpha}{\partial \omega} = \frac{2 \cdot RW \cdot [RW \cdot \omega + V] - [RW \cdot \omega - V] \cdot 2 \cdot RW}{(RW \cdot \omega + V)^2}
$$

(6.7)

Thus further simplifies to

$$
K_\omega = \frac{4 \cdot (RW) \cdot V}{(RW \cdot \omega + V)^2}
$$

(6.8)
In order to calculate $K\omega$, $\omega$ should be computed first at the three chosen points on the coefficient of friction versus creep curve. At the stable region $\omega$ is calculated as follows:

The creep at the stable region = - 0.025
From the coefficient of friction versus creep curve the corresponding $\mu(\alpha)$ value to the above creep = 0.22
The other parameters are:-
$RW = 0.5$ m
$V = 10$ m/s, 20 m/s and 40 m/s
Substituting the above parameters with $V = 10$ m/s in equation (6.1) gives:

\[-0.025 = \frac{2 \times 0.5 \times \omega - 20}{0.5 \times \omega + 10} \]

and therefore $\omega = 19.506$ rad/sec.

The same technique was followed to find $\omega$ at the other two chosen points in the coefficient of friction versus creep curve at speeds ($V = 10$ m/s, 20 m/s and 40 m/s), which are at the neutral point (peak of the curve) and at the unstable region. Table 6.1 shows all the calculated values of $\omega$ for the three speeds and the three creep values.
Table 6.1 The calculated values of $\omega$ at the three different regions, with $V = 10\text{ m/s, } 20\text{ m/s and } 40\text{ m/s}$.

Once $\omega$ has been calculated for each of the three operating points at speed ($V = 10\text{ m/s, } 20\text{ m/s and } 40\text{ m/s}$), then $K\omega$ can be calculated using equation (6.8).

The first value of $K\omega$ is calculated as follows

$$K\omega = \frac{4 \cdot (0.5) \cdot 10}{(0.5 \cdot 19.506 + 20)^2}$$

giving a value for $K\omega$ of 0.05128

The same method was used to establish the values of $K\omega$ at the other two operating points, and was repeated for the other two speeds. All the values of $K\omega$ are shown in Table 6.2.
Table 6.2 The calculated values of $K_w$ at the three different regions, with speed ($V = 10$ m/s, $20$ m/s and $40$ m/s).

The second parameter is the variation of the speed $V$, which is represented by $K_V$. To calculate the values of $K_V$, equation (6.1) should be differentiated with respect to $V$ as follows:

$$K_V = \frac{\partial \alpha}{\partial V} = - \frac{2 \cdot [RW \cdot \omega + V] - [2 \cdot RW \cdot \omega - 2 \cdot V]}{(RW \cdot \omega + V)^2}$$

This simplifies to

$$K_V = \frac{-4 \cdot (RW) \cdot \omega}{(RW \cdot \omega + V)^2}$$

Substituting the corresponding parameters in equation (6.10) with speeds ($V = 10$ m/s) gives
\[ KV = \frac{-4 \times (0.5) \times 19.506}{(0.5 \times 19.506 + 10)^2} \]

such that \( KV = -0.0999843 \)

The same method was used to establish all the other values of \( KV \) with speed \( V = 10\text{m/s, } 20 \text{ m/s and } 40 \text{ m/s} \), and was carried out for all the operating points. The all values of \( KV \) are shown in Table 6.3.

<table>
<thead>
<tr>
<th>Condition</th>
<th>KV at ( V = 10 \text{ m/s} )</th>
<th>KV at ( V = 20 \text{ m/s} )</th>
<th>KV at ( V = 40 \text{ m/s} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>-0.09999</td>
<td>-0.04999</td>
<td>-0.02499</td>
</tr>
<tr>
<td>0.03</td>
<td>-0.09998</td>
<td>-0.0499986</td>
<td>-0.0249988</td>
</tr>
<tr>
<td>0.035</td>
<td>-0.099978</td>
<td>-0.04998</td>
<td>-0.02499</td>
</tr>
</tbody>
</table>

Table 6.3 The calculated values of \( KV \) at the three different regions, with speed \( V = 10 \text{ m/s, } 20 \text{ m/s and } 40 \text{ m/s} \).

The third linearised parameter required in the model is the slope of \( \mu(\alpha) \), which is represented by the linear gain \( K\alpha \). The calculation of \( K\alpha \) is evaluated at the three operating points. \( K\alpha \) is the gradient of the \( \mu/\alpha \) curve at each of these three different points. The three values for \( K\alpha \) are as shown in Table 6.4.
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Table 6.4 The calculated values of $K_x$ at the three different regions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K_x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>2.6192</td>
</tr>
<tr>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>0.035</td>
<td>-2.7054</td>
</tr>
</tbody>
</table>

Since the reaction force $R$ is one of the main parameters which affects the braking force it is necessary to identify the linear value (gain) of $R$, which is represented by $KR$ at the three chosen operating points in the coefficient of friction versus creep curve. However, $KR$ depends on the coefficient of friction which is a function of creep. Therefore, the coefficient of friction in the wheel braking model has to be established at the three operating points at which the linearisation is to be performed. The three values of $KR$ are shown in Table 6.5.

Table 6.5 The calculated values of $KR$ at the three different regions.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$KR$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.225</td>
</tr>
<tr>
<td>0.03</td>
<td>0.235</td>
</tr>
<tr>
<td>0.035</td>
<td>0.225</td>
</tr>
</tbody>
</table>

Tables 6.6 - 6.8 shows all the values required for the linear model, whilst Figure 6.3 shows the block diagram of the linear vehicle model. From the block diagram in Figure 6.3 the reaction forces are the output product of the vehicle dynamic model. The reaction force is used as one of the inputs to the linearised model with the other two inputs being the angular velocity and the speed of the wheel. The longitudinal
Chapter 6 Model Linearisation

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K_\omega$</th>
<th>KV</th>
<th>KR</th>
<th>Slopes of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.05126</td>
<td>-0.09999</td>
<td>0.225</td>
<td>2.6192</td>
</tr>
<tr>
<td>0.03</td>
<td>0.05151</td>
<td>-0.09998</td>
<td>0.235</td>
<td>0.01</td>
</tr>
<tr>
<td>0.035</td>
<td>0.05177</td>
<td>-0.099978</td>
<td>0.225</td>
<td>-2.7054</td>
</tr>
</tbody>
</table>

Table 6.6 The gains required for the linear models at speed ($V = 10$ m/s).

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K_\omega$</th>
<th>KV</th>
<th>KR</th>
<th>Slopes of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.02563</td>
<td>-0.04999</td>
<td>0.225</td>
<td>2.6192</td>
</tr>
<tr>
<td>0.03</td>
<td>0.02576</td>
<td>-0.0499986</td>
<td>0.235</td>
<td>0.01</td>
</tr>
<tr>
<td>0.035</td>
<td>0.02588</td>
<td>-0.04998</td>
<td>0.225</td>
<td>-2.7054</td>
</tr>
</tbody>
</table>

Table 6.7 The gains required for the linear models at speed ($V = 20$ m/s).

<table>
<thead>
<tr>
<th>Condition</th>
<th>$K_\omega$</th>
<th>KV</th>
<th>KR</th>
<th>Slopes of $\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.025</td>
<td>0.01281</td>
<td>-0.02499</td>
<td>0.225</td>
<td>2.6192</td>
</tr>
<tr>
<td>0.03</td>
<td>0.01288</td>
<td>-0.0249988</td>
<td>0.235</td>
<td>0.01</td>
</tr>
<tr>
<td>0.035</td>
<td>0.01294</td>
<td>-0.0249988</td>
<td>0.225</td>
<td>-2.7054</td>
</tr>
</tbody>
</table>

Table 6.8 The gains required for the linear models at speed ($V = 40$ m/s).

Braking force is the output of the linearised model which is used as the input to the wheel dynamics model and the vehicle dynamics model. The deceleration of the wheel...
was carried out in the wheel dynamics model. Both the speed \( V \) and the angular velocity of the wheel \( \omega \) form the outputs of the wheel dynamics model. When this is combined with the vehicle dynamics these outputs become the inputs of the linearised model. The new longitudinal braking force is an output which will be the new input to the vehicle dynamics model and the wheel dynamics model. The new reaction force from the vehicle dynamics model, the new angular velocity and the speed of the wheel are then calculated. This pattern repeats until the vehicle stops.

6.4- SUMMARY

This chapter has shown the mathematical methods that were used to linearise the model at three different points on the coefficient of friction versus creep curve. Because the vehicle model has now been linearised, linear control system design techniques can be applied.
Figure 6.1 The coefficients of friction versus creep in the dry condition.
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Figure 6.2 The block diagram represents the system used for determining the linear longitudinal braking force.
Figure 6.3 The linear model of a single vehicle.
This chapter begins with an explanation of the design of control systems suitable for
the braking control of a single wheelset in isolation, a single vehicle with one wheelset
braking and a single vehicle with four wheelsets braking. It explains how a controller
can be designed to provide adequate closed-loop control of the creep, where the
braking force controlled by the output of creep. This control system allow the
maximum braking force without wheel slide. This is followed by a description of the
different types of control strategies that can be used in the controller in the above
mentioned vehicle models. These control strategies include a proportional controller
and a proportional plus integral controller. Actuator dynamics are an important
consideration in the control system. Therefore, a description of the modelling of the
actuator and how it is integrated into the main vehicle model is included. This chapter
also includes a discussion and the implementation of the control system design of a
linear model on a nonlinear system. The performance of the nonlinear system in terms
of creep output under the proportional plus integral controller and the inclusion of the
actuator with different ramp input commands is presented. Finally this chapter presents
a description of how the nonlinear model was tested under different conditions of
adhesion to demonstrate how the control system behaved in cases where track
conditions change.

The overall design methodology used classical control in the form of a proportional
controller and a proportional plus integral controller. The design was done at the
nominal operating point for the single wheelset braking. Then the design was checked
around the nominal operating point at creep inputs ($\alpha = 0.025$, and 0.035). This
control design was checked with different initial operating speeds to make sure that the
different speeds do not cause a problem. This controller was applied to a full vehicle to check for undesirable interactions.

7.1- CONTROL DESIGN FOR THE LINEAR SYSTEM

This section describes some of the tools and techniques used for tackling linear control system problems. The frequency response of a system is the fundamental information required in the design of classical control systems [Kuo - 1987]. There are many ways of presenting a frequency response one of which, the Nichol Chart will be considered. The Nichol Chart has its own individual method of illustrating the frequency response. It is a well-established means of illustrating the effect of open loop phase changes on the frequency response and is therefore a useful tool for designing the controller [Albert and Coggan - 1992]. Such frequency response techniques were used to carry out classical control system design for the creep control. Figure 7.1 shows the block diagram of the closed loop system, in which C(s) is the compensator transfer function.

7.2- PROPORTIONAL CONTROLLER

The proportional control is simply an amplifier with constant gain. This type of control action is formally known as the proportional control, since the actuating signal at the output of the controller is simply related to the input of the controller by a proportional constant. For a controller with proportional control action, the relationship between the input of the controlled model, which for the train model is braking force demand, and the input error signal, E(t) is

\[ \text{Force demand} = K_{PC} \cdot E(t) \]  (7.1)
7.3- INITIAL ACTUATOR REPRESENTATION

This section describes how a simplified representation of the actuator dynamics has been included as part of the overall vehicle model. (A latter section will develop a proper actuator model, rather than this simplified representation.) A first order lag is used to model the actuator to a first-order approximation. This is then integrated in the models for single wheelset braking, single vehicle single wheelset braking and single vehicle four wheelsets braking. The transfer function used to represent the actuator is as follows

\[ X_{ACT}(s) = \frac{1}{(1 + s\tau)} \]  

(7.2)

The value of \( \tau \) used in the initial actuator representation was 0.016 which gives an actuator time constant which would represent a high performance braking actuator of the type needed for advanced railway braking systems (e.g. electro - mechanical rather than the normal pneumatic actuators). The electro - mechanical actuator as its name suggests utilises electrical and mechanical components to provide linear movement which may be used to apply a load or force. Using the electro - mechanical actuator, the response of the system will become faster than with pneumatic actuators.

Full explanation for the design and modelling of the electro-mechanical actuator, which represents the practical situation, along with the response of this actuator when included in the single wheelset, is presented later in Section 7.6.
7.3.1- Proportional Controller Applied To a Single Wheelset Braking With Simple Actuator Representation.

The matrices for the state space model of the linearised single wheelset were developed in the previous chapter, and are written as follows

\[
A_w = \begin{bmatrix}
-KV \cdot Ka \cdot g & -K\omega \cdot Ka \cdot g \\
KV \cdot Ka \cdot M \cdot g \cdot RW & K\omega \cdot Ka \cdot M \cdot g \cdot RW
\end{bmatrix}
\begin{bmatrix}
V \\
\dot{w}
\end{bmatrix}
\]

\[
B_w = \begin{bmatrix}
0 \\
RB
\end{bmatrix}
\]

\[
C_w = [KV \quad K\omega]
\]

The states of the single wheelset dynamics are the forward speed and the angular velocity of the wheel, and the output is the creep. Then the PI controller, the simple actuator representation and the model of the wheelset are incorporated in one MATLAB program.

The preliminary value for proportional controller gain was chosen using the following equation

\[
K_{Pc} = \frac{100 \cdot \text{Output span}}{\% PB \cdot \text{Input span}}
\]

(7.3)
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Where the % PB is the percentage proportional band, the output span is the maximum braking force (60000 N) and the input span is the maximum creep (0.865). Using a proportional band of 5%, which is a typical value for a practical control system, the proportional gain works out to be 138,000 (up to three significant figures).

The MATLAB package was used to calculate the frequency response of the single wheelset model. The Nichols Chart response for the single wheelset and the initial actuator representation with the proportional controller added, are shown in Figure 7.2. This Nichols Chart response is for the single wheelset under the proportional controller and at initial operating speed (V = 20 m/s) and creep input commands (α = 0.025, 0.03 and 0.035). From this figure it can be observed that there is an excess of stability. The open-loop responses showed that at creep inputs (α = 0.025 and 0.035) the system has low frequency gains. Therefore integral control has to be added to the system in order to increase the low frequency gains and to ensure zero steady-state error.

Proportional control action was also analysed with single wheelset and four wheelsets braking applied to a complete vehicle but apart from changes in gain level, the differences were not particularly significant.

7.4- PROPORTIONAL PLUS INTEGRAL CONTROLLER

The models of a single wheelset braking, single vehicle single wheelset braking, and single vehicle four wheelsets braking gave responses under proportional control that were excessively stable. To increase the low frequency gain of the system and to improve its sluggish performance, proportional plus Integral (P + I controller) was implemented on the three models. P+I control also provides the facility to counteract steady-state errors. The action of a proportional plus integral controller is defined by
the following transfer function

\[ C(s) = K_{PC} \left[ \frac{1 + sT_i}{sT_i^2} \right] \]  

(7.4)

where \( K_{PC} \) is the proportional gain and \( T_i \) is integral time constant. The value of \( T_i \) used in the model was 0.0176, and the value of \( K_{PC} \) used in the model was 138000 (up to three significant figures). Both \( K_{PC} \) and \( T_i \) can be adjusted if necessary. The integral time adjusts the integral control action, whereas a change in the value of \( K_{PC} \) affects both the proportional and integral parts of the control action. Appendix D shows in full detail how the P + I controller was designed.

7.4.1- PI Controller Applied To a Single Wheelset Braking With Simplified Actuator Representation

As mentioned in the previous section, and for all those reasons the proportional controller is not enough as a control system for the braking of the mentioned models, therefore a use of a PI controller is required. The actuator has been included in the single wheelset braking model and the simulation now contains a representation of all the components of the real wheelset braking system. Applying the PI controller to the model of a single wheelset braking gives frequency responses which are shown in the Nichols Charts of Figures 7.3 - 7.5 for the three different creep input commands (0.025, 0.03 and 0.035) and at three different operating initial speeds (\( V = 10 \text{ m/s}, \ 20 \text{ m/s and } 40 \text{ m/s} \)) respectively. Some frequencies are indicated on the responses in order to assist interpretation. As mentioned earlier, the aim is to use one condition (\( \alpha = 0.03 \)) at any operating speed, then the model has to be checked with other conditions of \( \alpha \) and \( V \). By inspection of the phase margins of the system, these results show that
the model is stable under proportional plus integral control for the three different input commands and the three different initial operating speeds of the system. The response verified that the model gives a stable response at creep condition = 0.03 even though the system is stable under the other two conditions. At the creep input command = 0.025 the output response shows that the system response is over damped and at creep input command = 0.035 the response of the system is the most critical condition, it is also conditionally stable. There are small differences between the responses, but they are not particularly significant variations between the responses when the initial operating speed is changed from ( V = 10 m/s, 20 m/s and 40 m/s), also the frequencies are different in the three different mentioned speeds.

7.4.2- PI Controller Applied To A Single Vehicle Single Wheelset Braking With Simplified Actuator Representation

The simulation model is now extended to show the effect of single wheelset braking on the whole vehicle. Figures 7.6 - 7.8 show the Nichols Charts response of the system at creep command inputs (α = 0.025, 0.03 and 0.035) and at different initial operating speeds (V = 10 m/s, 20 m/s and 40 m/s) which indicate once again that the system is stable and also there are small differences between these responses, but they are not particularly significant.

7.4.3- PI Controller Applied To A Single Vehicle Four Wheelset Braking With Simplified Actuator Representation

This section represents the most complex stage of modelling and consists of a single vehicle with four wheelsets braking, the PI controller and the initial actuator representation. The Nichols Charts of the system are shown in Figures 7.9 - 7.11 for the three different creep input commands (0.025, 0.03 and 0.035) and at different
initial operating speeds \((V = 10 \text{ m/s}, 20 \text{ m/s} \text{ and } 40 \text{ m/s})\) respectively. The results confirmed once again that the model response is stable at creep input command \((\alpha = 0.03 \text{ and } V = 20 \text{ m/s})\), and the system is stable under the other points but \((\alpha = 0.035)\) is the critical condition (i.e. close to instability). These results confirmed that the model is stable even under the complete model of a single vehicle four wheelsets braking and at the three different creep input commands and at the three different initial operating speeds. Also from the responses of the system the variations were not particularly significant for the three different speeds. By inspection of the system responses the PI controller satisfies all the test conditions that were required for the design of the braking control system of the vehicle train.

The three system responses known as (single wheelset braking, single vehicle single wheelset braking and single vehicle four wheelsets braking) were combined in one plot as shown in Figure 7.12. This response of these systems is plotted when the creep input command is \((\alpha = 0.03)\) and initial operating speed \((V = 20 \text{ m/s})\). This gives a good illustration of the stability of the three mentioned models at the ultimate operating point of the control system. Also it gives a good view for comparing the response of the three mentioned models as it showed that there is small differences between the responses, but the differences are not significant between the three responses of the three different models.

7.5- PROPORTIONAL PLUS INTEGRAL CONTROLLER WITH THE NONLINEAR MODEL

Once the design of the control system was completed for the linear models, which include the single wheelset braking, the single vehicle with single wheelset braking and the single vehicle with four wheelsets braking, and the results had confirmed that the control system for these linear models gave satisfactory results, then the next stage was
to apply the control strategies to the respective non-linear model. Three operating points were chosen for the nonlinear control system; a very stable operating point with creep value of 0.025, a neutral operating point with creep value of 0.03 and finally an open-loop unstable operating point with creep value of 0.035 and at different initial operating speeds (V = 10 m/s, 20 m/s and 40 m/s). By using one set point at a time for the three different speeds and using the same controller parameters that were used in the linear system model, the performance of the controller could be assessed. Figure 6.1 in the previous chapter shows the curve of the coefficient of friction versus creep with the three chosen operating points identified. The non-linear model should accurately represent the practical situation and the response of the system should be stable at the three different points [Nagrath - 1978] and at three different initial operating speeds.

7.6- ELECTRO - MECHANICAL ACTUATOR MODELLING

In the early stages of this chapter the electro - mechanical actuator has been represented by a simple representation of the actuator dynamics. This section describes the design and modelling of the complete electro - mechanical actuator and Figure 7.13 shows the block diagram of the electro - mechanical actuator system. The electro - mechanical actuator consisted of a D.C motor to provide a rotary motion and a gear box which converts the produced rotary motion in to linear motion. This linear motion will push the brake pad producing a braking force which will be used to stop the wheelsets. The information from manufacturers has been exploited to derive data for the model [Goodall and Whitfield - 1985]. The D.C. motor was found to be rated at about 600 W and this was calculated as follows:-

The braking force FB = 60000 N.
Coefficient of friction between pad and disc = 0.4.
Therefore the normal force = 150000 N.

Estimated distance of the spring compressed = 1 mm (This spring is connected between the brake pad from one side and the gear box of the actuator from the other side).

Actuation time = 0.25 s.

Therefore the speed = 4 mm/s.

Then the motor rating = 0.004 * 150000 = 600 W.

n is the rotary/linear gear ratio where n was calculated and found to be 1.273 * 10^5 m/rad.

The maximum power that the motor needs to give is 600 W. However the motor that is selected here has a power rating of 200 - 300 W which is specified by the manufacturer. (Printed motors limited). The ratio of the maximum power rating to the continuous power rating strictly depends upon the duty cycle, but a figure 2:1 is reasonable.

The parameters used for the electro-mechanical actuator model are as follows:

- Back e.m.f constant $K_v = 0.00183 \text{ V/rads}^{-1}$
- Torque constant $K_T = 0.108 \text{ Nm/A}$
- Armature resistance $R_A = 0.95 \Omega$
- Armature inductance $L_A = 0.0001 \text{ H}$
- Viscous drag coefficient $K_d = 0.01215 \text{ Nm/rads}^{-1}$
- Motor inertia $J = 0.0012 \text{ kg m}^2$
- Lead-screw stiffness $K_L = 150000000 \text{ Nm/rad}$
- Rotary/linear gear ratio $n = 1.273 \times 10^5 \text{ m/rad}$

The electro-mechanical actuator model was arranged in state-space form as follows:
The states of the electro-mechanical actuator model are the motor torque, the angle and the angular velocity of the motor, where the output is the force.

Once the electro-mechanical actuator has been modelled, the next step is to add close-loop feedback in order to make the actuator a force control. This is achieved by feeding back the force output. The design of the compensator for the actuator follows the same technique which was used in designing the PI controller as shown in

\[
A = \begin{bmatrix}
-\frac{R_A}{L_A} & 0 & -\frac{K_T}{L_A} \\
0 & 0 & 1 \\
\frac{1}{J} & -\frac{K_L}{J} & -\frac{K_d}{J}
\end{bmatrix}
\begin{bmatrix}
T \\
\delta \\
\delta
\end{bmatrix}
\]

\[
B = \begin{bmatrix}
\frac{K_T}{L_A} \\
0 \\
0
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 & \frac{K_L}{n} & 0
\end{bmatrix}
\]
Appendix D. Where the compensator transfer function is

\[ C_d(s) = K_c \left( \frac{1 + sT_a}{sT_a} \right) \]

Where the value of \( K_c \) is 71,000 and the value of \( T_a \) is 0.07.

Then the controller, the electro-mechanical actuator with the compensator and the nonlinear model of the single wheelset braking combined in one MATLAB program. The whole system block diagram is shown in Figure 7.14. The creep output and the longitudinal braking force of the system is shown in Figures 7.15 - 7.16 respectively when the creep input command is 0.025. The creep output and the longitudinal braking force of the system is shown in Figures 7.17 - 7.18 respectively when the creep input command is 0.03. The creep output and the longitudinal braking force of the system is shown in Figures 7.19 - 7.20 respectively when the creep input command is 0.035, all these responses were carried out for an initial operating speed \( (V = 20 \text{ m/s}) \). From the response of the system it was clear that the electro-mechanical actuator used in this program to represent the practical situation is more appropriate than the initial representation as the results now represent the realistic situation. The results have confirmed that the designed control system works perfectly under any stated condition.

7.7- PI CONTROLLER AND SIMPLE ACTUATOR REPRESENTATION WITH THE NONLINEAR MODELLING

The single wheelset braking model with the PI controller and the electro-mechanical actuator has been simulated in the previous section. This section introduces the
modelling of the single wheelset braking, the single vehicle single wheelset braking and the single vehicle four wheelsets braking with the initial actuator representation and compensated using PI control.

7.7.1- Single Wheelset Braking

The single wheelset system dynamics equations were arranged in state space form in the form of A, B, C and D matrices. The MATLAB package was used to simulate the equations.

The four matrices for the wheelset dynamics model are as shown in the previous chapters. The PI controller combined with the simple actuator representation dynamics and the nonlinear single wheelset dynamics take the following state-space form

\[
A_w = \begin{bmatrix}
0 & 0 & 0 & 0 \\
\frac{T_i}{\tau} & -\frac{1}{\tau} & 0 & 0 \\
0 & \frac{RB/IW}{\tau} & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\alpha_e \\
FB \\
w \\
V
\end{bmatrix}
\]

\[
B_w = \begin{bmatrix}
1 & -1 & 0 \\
-\frac{K_{PC}}{\tau} & \frac{K_{PC}}{\tau} & 0 \\
0 & 0 & (RW)/IW \\
0 & 0 & -1/MV
\end{bmatrix}
\begin{bmatrix}
\alpha_m \\
\alpha_{out} \\
FR
\end{bmatrix}
\]
The states of the system are the error signal, the braking force, the angular velocity of the wheel and the forward speed. The output of the system is creep.

The creep output and the longitudinal braking force of the system are shown in Figures 7.21 and 7.22 respectively for creep input command values of \( \alpha = 0.025, 0.03 \) and \( 0.035 \), where all the creep inputs ramp from zero to \( \alpha \) over a two second period. These graphs represent the result for an initial operating speed \( V = 10 \text{ m/s} \). The creep output and the longitudinal braking force of the system are shown in Figures 7.23 and 7.24 respectively for creep input command values of \( \alpha = 0.025, 0.03 \) and \( 0.035 \). These graphs represent the result for an initial operating speed \( V = 20 \text{ m/s} \). The creep output and the longitudinal braking force of the system are shown in Figures 7.25 and 7.26 respectively for creep input command values of \( \alpha = 0.025, 0.03 \) and \( 0.035 \). These graphs represent the result for an initial operating speed \( V = 40 \text{ m/s} \).

The output responses of the system confirmed that the controller which was designed for the linear single wheelset performed adequately in the nonlinear system for the three different initial operating speeds. The responses show that there is no significant change between the different input commands at the three different initial operating speeds. Also the results had confirmed that the ultimate designed point gives the best longitudinal braking force.

7.7.2- Single Vehicle With Single Wheelset Braking
In the single vehicle with the front wheelset braking, the vehicle dynamics and the wheelset dynamics are integrated to form one simulation routine in MATLAB. The complete vehicle with the front wheelset braking was simulated for input command values of 0.025, 0.03 and 0.035 and at three different initial operating speeds (V = 10 m/s, 20 m/s and 40 m/s) but the results are not illustrated as there are no significant changes from the single vehicle with four wheelsets braking.

7.7.3- Single Vehicle With Four Wheelsets Braking

With all the wheelsets braked the command input is changed from 0.025 to 0.03 and to 0.035. The output responses show that the system is stable at the three different creep input command. Figures 7.27 and 7.28 show the creep output and longitudinal braking force for a creep input command of 0.025 and initial operating speed V = 20 m/s. Figures 7.29 and 7.30 show the creep output and longitudinal braking force for a creep input command of 0.03 and initial operating speed V = 20 m/s. Figures 7.31 and 7.32 show the creep output and longitudinal braking force for a creep input command of 0.035 and initial operating speed V = 20 m/s. The results clearly show that the difference between the single wheelset braking and the single vehicle four wheelsets braking is that in the single wheelset braking the system settles at the command input point without any oscillation. In the single vehicle four wheelsets braking the output settles to the value of the command input after transient oscillation. This is due to the inclusion of the vehicle dynamics and the interaction between the bogies and the vehicle body.

7.8- CHANGING THE LEVEL OF ADHESION

The changing of adhesion levels affect the braking force required to stop the wheel or
to decelerate it. The change of the adhesion level is dependent upon the track conditions. In practical situations, for dry conditions the adhesion comes from steel-to-steel contact, whereas for changing rail conditions, there is an additional factor, normally the presence of a substance between the wheel and the rail. Under typical braking conditions the longitudinal braking force produced by the wheel will vary with slide conditions. Figure 7.33 shows the variation of $\mu$ versus $\alpha$ as it is representing the dry and new changed rail conditions. Under the dry rail condition the thesis shows in detail all the circumstances related to the braking and the maximum braking force is established under dry conditions. At higher values of slide the creep coefficient reaches its lowest value, and this affects the braking force [Louam et al - 1988]. The single vehicle with four wheelsets braking was tested when the condition of the track was changed from dry to wet. Figures 7.34 and 7.35 show the creep output and the longitudinal braking force for an input command of 0.03. After the first five seconds, the conditions of the rail are changed. The results show clearly that the braking force reduces, but the creep remains in control despite the large change in wheel-to-rail conditions. The results also clearly indicate that as the first wheelset reached the new condition the brakes released immediately and regulate to make the wheelset avoid slide. Also as a consequence of this the magnitude of the creep increases transiently, then it recovered to the settling point. The other wheelsets follow the same behaviour. The time delay between the wheelsets is clearly shown in the response.

7.9- SUMMARY

This chapter started with the design of a controller for the linear system commencing with a proportional controller and leading to proportional plus integral controllers. The results show that the three different models known as single wheelset braking, single vehicle single wheelset braking and single vehicle four wheelsets braking with the inclusion of the actuator dynamics in the three mentioned models were stable for the
three input commands and for the three different initial speeds; the difference between the responses were not significant.

The simulation moved to the second stage which consisted of the implementation of the controller on the nonlinear system, based upon the control parameters of the linear system. The simulation results showed that the three different models operating under the three input commands and the three different speeds had similar stable responses.

The level of adhesion has been changed in the nonlinear model to represent the situation where there is wet rail and the results showed that the maximum longitudinal braking force was reduced at the time the wheelset reached the new condition, and regained so that the slide was avoided. This affected the creep which reached its lowest value and recovered to the stable value. It has been shown that linearised models covering a range of conditions can be used to design controllers which work with the non-linear system.

The classical control which was designed in the form of the proportional controller and proportional plus integral controller has been shown to operate very effectively and efficiently in the three mentioned models. The responses of the three different systems indicated that the three systems were stable under the above mentioned classical control. This results concluded that classical control in the form of PI controller is found to be an ideal control system for braking the vehicle train as it achieves a good performance under this controller. The mentioned classical control therefore satisfies all the requirements for achieving the best braking force. This means that classical control is adequate at this stage for the railway braking and therefore there is no need to consider any further advances in the control system.
Figure 7.1 The block diagram of the closed loop system.
Figure 7.2 The output response of a single wheelset braking under proportional controller only with creep input commands (\( \alpha = 0.025, 0.03 \) and 0.035) and initial operating speed (\( V = 20 \text{ m/s} \))
Figure 7.3 The output response of a single wheelset braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 10$ m/s.
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Figure 7.4 The output response of a single wheelset braking under PI controller, creep input commands (0.025, 0.03, and 0.035) and speed $V = 20$ m/s.
Figure 7.5 The output response of a single wheelset braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 40$ m/s.
Figure 7.6 The output response of a single vehicle single wheelset braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 10$ m/s.
Figure 7.7 output response of a single vehicle single wheelset braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed \( V = 20 \text{ m/s} \).
Figure 7.8 The output response of a single vehicle single wheelset braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 40$ m/s.
Figure 7.9 The creep output response of the first wheelset in a single vehicle four wheelsets braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 10 \text{ m/s}$.
Figure 7.10 The creep output response of the first wheelset in a single vehicle four wheelsets braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 20$ m/s.
Figure 7.11 The creep output response of the first wheelset in a single vehicle four wheelsets braking under PI controller, creep input commands (0.025, 0.03 and 0.035) and speed $V = 40$ m/s.
Figure 7.12 The output response of the single wheelset braking, single vehicle single wheelset braking and single vehicle four wheelsets braking under PI controller and at creep input command ($\alpha = 0.03$) and initial operating speed ($V = 20$ m/s).
Figure 7.13 The electro-mechanical actuator system
Figure 7.14 The block diagram of the controller, the electro-mechanical actuator with the inner force control loop and the single wheelset braking model.
Figure 7.15 The creep output of the single wheelset braking at creep input command $=0.025$ and speed $V = 20$ m/s, with an electro-mechanical actuator.

Figure 7.16 The longitudinal braking force of the single wheelset braking at creep input command $=0.025$ and speed $V = 20$ m/s, with an electro-mechanical actuator.
Figure 7.17 The creep output of the single wheelset braking at creep input command \( =0.03 \) and speed \( V = 20 \text{ m/s} \), with an electro-mechanical actuator.

Figure 7.18 The longitudinal braking force of the single wheelset braking at creep input command \( =0.03 \) and speed \( V = 20 \text{ m/s} \), with an electro-mechanical actuator.
Figure 7.19 The creep output of the single wheelset braking at creep input command = 0.035 and speed $V = 20$ m/s, with an electro-mechanical actuator.

Figure 7.20 The longitudinal braking force of the single wheelset braking at creep input command = 0.035 and speed $V = 20$ m/s, with an electro-mechanical actuator.
Figure 7.21 The creep output of a single wheelset braking at creep input commands \( \alpha = 0.025, 0.03 \) and \( 0.035 \) and initial operating speed \( V = 10 \text{ m/s} \), the creep input ramp from zero to \( \alpha \) over two second.
Figure 7.22 The longitudinal braking force of a single wheelset braking at creep input commands ($\alpha = 0.025$, 0.03 and 0.035) and initial operating speed ($V = 10$ m/s), the creep input ramp from zero to $\alpha$ over two seconds.
Figure 7.23 The creep output of a single wheelset braking at creep input commands (\( \alpha = 0.025, 0.03 \) and 0.035) and initial operating speed (\( V = 20 \text{ m/s} \)), the creep input ramp from zero to \( \alpha \) over two second.
Figure 7.24 The longitudinal braking force of a single wheelset braking at creep input commands ($\alpha = 0.025$, 0.03 and 0.035) and initial operating speed ($V = 20$ m/s), the creep input ramp from zero to $\alpha$ over two second.
Figure 7.25 The creep output of a single wheelset braking at creep input commands ($\alpha = 0.025, 0.03$ and $0.035$) and initial operating speed ($V = 40$ m/s), the creep input ramp from zero to $\alpha$ over two second.
Figure 7.26 The longitudinal braking force of a single wheelset braking at creep input commands ($\alpha = 0.025$, 0.03 and 0.035) and initial operating speed ($V = 40$ m/s), the creep input ramp from zero to $\alpha$ over two second.
Figure 7.27 The creep output of the leading wheelset of a single vehicle four wheelset braking at creep input command (0.025) and initial operating speed $V = 20 \text{ m/s}$.

Figure 7.28 The longitudinal braking force of the leading wheelset of a single vehicle four wheelset braking at creep input command (0.025) and initial operating speed $V = 20 \text{ m/s}$
Figure 7.29 The creep output of the leading wheelset of a single vehicle four wheelset braking at creep input command of (0.03) and initial operating speed $V = 20$ m/s.

Figure 7.30 The longitudinal braking force of the leading wheelset of a single vehicle four wheelset braking at creep input command of (0.03) initial operating speed $V = 20$ m/s.
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Figure 7.31 The creep output of the leading wheelset of a single vehicle four wheelset braking at creep input command (0.035) and operating speed $V = 20 \text{ m/s}$.

Figure 7.32 The longitudinal braking force of the leading wheelset of a single vehicle four wheelset braking at creep input command (0.035) and operating speed $V = 20 \text{ m/s}$.
Figure 7.33 The coefficients of friction versus $\alpha$ in the dry and wet conditions.
Figure 7.34 The creep output of a single vehicle with four wheelsets braking at creep input command (0.03) and initial operating speed $V = 20$ m/s.

Figure 7.35 The longitudinal braking forces for the single vehicle four wheelsets braking at creep input command (0.03) and initial operating speed $V = 20$ m/s.
CHAPTER 8

CONCLUSION AND FUTURE WORK

This chapter presents an overall conclusion on the work described in the thesis, together with a number of suggestions for further work aimed at using the simulation software to improve braking system performance.

8.1- OVERVIEW OF THE WORK

The research investigation, which forms the subject of this discussion, was directed at designing a simulation to enable braking control system for use with modern railway vehicles to be evaluated. Braking ability is one of the most important features which influence the safety and reliability of traffic and railway vehicles as speeds have reached a very high level where the braking has to operate effectively to meet requirements of safety, stopping distance and ride quality. The braking systems developed from the research can be applied to either single vehicles or to complete trains. The braking control evaluation software was designed with a dry rail model. The robustness of the control system was tested by using different conditions, such as a wet rail condition. The model, in both circumstances, illustrated the performance of control strategies for the braking systems of railway vehicles. The results demonstrated that the controller operated successfully for all rail conditions and confirmed the validity of the simulation software.

The effect of interactions between the braking action of individual wheelsets was analysed using single bogie single wheelset braking, single vehicle single wheelset braking, single vehicle four wheelset braking, eight coupled vehicle single wheelset braking and eight coupled vehicle thirty two wheelsets braking. The assessment included the effect of simple control laws for avoiding wheel slide under braking. The
Chapter 8 Conclusion and Future Work

studies have indicated the importance of including the dynamic interaction between the various wheelsets caused by the changing vertical wheel load. The results have shown that both bogie dynamics and body dynamics are important, but that the additional interaction between vehicles is relatively small.

The simulation work of the research programme developed dynamic models in order to design and evaluate braking controllers which will control the braking effort being applied very effectively in order to maximise the utilisation of the available adhesion. The research has provided the tools with which to evaluate whole train braking control systems. Each vehicle had a single loop controller, the design of which took into account the interactive nature of the four braking control loops.

The side view vertical model of a single railway vehicle was developed and a number of results have been assessed to ensure that the model is properly representative.

The results of both the time domain analysis and the frequency domain analysis which had been applied to the single vehicle model confirmed that the original model could not be simplified any further, and therefore subsequent vehicle analysis was carried out using this model.

From the results it was also found that there were no significant changes between the response of the reaction forces of a single vehicle model and the response of the reaction forces of the eight interconnected vehicle model. The results indicated that the on-off braking signal in the single vehicle and in the leading vehicle of the eight coupled train followed similar patterns. This indicated that any further study could be carried out on the single vehicle model.

The model was linearized in order that the linear control system can be designed and was linearized at three different operating points. The full description of the
mathematical analyses which makes the model ready for frequency response analysis was presented. These linear models allow the system to be analysed in the frequency domain by calculating the frequency response at each of these operating points.

The design of a controller for the linear system was presented. Classical control techniques, such as proportional control and proportional plus integral control were applied to the three different models. The results showed that the three different models known as single wheelset braking, single vehicle single wheelset braking and single vehicle four wheelsets braking with the inclusion of the actuator dynamics were modelled to represent the practical braking system of a railway vehicle. The results showed that the systems were stable and this was confirmed by inspection of the phase margin of the Nichols plots. The controller parameters were then implemented and applied to the nonlinear vehicle model, (which was based upon the parameters of the linear system). The simulation results showed that the three different models operating under the three different input commands and the three different initial operating speeds were stable and performed as expected.

The robustness of the controller operating on the nonlinear vehicle model was assessed by varying the level of adhesion between the wheel and the rail. The results showed that the controller proved robust to changes in adhesion, such as travelling from dry rail conditions to wet rail conditions.

8.2- SIGNIFICANT CONTRIBUTIONS OF THE RESEARCH

The work has contributed engineering knowledge in the following areas:-

- Sophisticated vehicle and multi - vehicle simulation showing the braking induced interaction.
• Application and appraisal of classically formulated control laws for these systems.

• The type of actuator required for the braking system has been outlined.

The classical control designed for the mentioned models was found to be very satisfactory and efficient for the braking of a railway vehicle. This classical control approach satisfies all the requirements of the braking system, therefore there was no need for modern control strategies to be applied at this stage.

8.3- DIRECTIONS FOR THE WORK

This research has achieved many objectives. However, the limited time available has meant that many ideas have yet to be researched. The following sets out briefly some suggestions for further work.

• More advanced control design methods.

The thesis has considered SISO classical control strategies which have proved satisfactory in achieving an acceptable braking system, but an obvious extension of the work would be the design of a multi-variable controller and the simulation software described allows this comparison to be made.

• Intelligent adaptive braking systems with links throughout the train.

The control systems for vehicles at the rear use knowledge/information provided by the control systems at the front of the train, to improve the overall performance of the complete train braking.
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APPENDIX A

MK III EQUATIONS OF MOTION

THE BASIC EQUATIONS OF MOTION FOR THE MK III VEHICLE
PASSIVE VERTICAL SUSPENSION MODEL

\[ M \ddot{Z} + 2K_z Z - K_s Z_{s1} - K_s Z_{s2} = 0 \] \hspace{1cm} (A1)

\[ 1 \ddot{\phi} + 2 I_i K_s \phi - I K_s Z_{s1} - I K_s Z_{s2} = 0 \] \hspace{1cm} (A2)

\[ M_1 \ddot{Z}_{s1} - K_s Z \cdot K_s Z_{s1} + K_r Z_{s1} - K_r Z_{B1} + C_r \dot{Z}_{s1} - C_r \dot{Z}_{B1} = 0 \] \hspace{1cm} (A3)

\[ M_2 \ddot{Z}_{s2} - K_s Z \cdot K_s Z_{s2} + K_r Z_{s2} - K_r Z_{B2} + C_r \dot{Z}_{s2} - C_r \dot{Z}_{B2} = 0 \] \hspace{1cm} (A4)

\[ M_B \ddot{Z}_{B1} - K_r Z_{s1} + K_r Z_{B1} + 2 K_p Z_{B1} + C_r \dot{Z}_{s1} + C_r \dot{Z}_{B1} + 2 C_p Z_{B1} = 0 \] \hspace{1cm} (A5)

\[ I_B \ddot{\phi}_{B1} + 2 a K_p \Phi_{B1} + 2 a^2 C_p \Phi_{B1} = 0 \] \hspace{1cm} (A6)

\[ M_B \ddot{Z}_{B2} - K_r Z_{s2} - K_r Z_{B2} + 2 K_p Z_{B2} - C_r \dot{Z}_{s2} + C_r \dot{Z}_{B2} + 2 C_p \dot{Z}_{B2} = 0 \] \hspace{1cm} (A7)

\[ I_B \ddot{\phi}_{B2} + 2 a K_p \Phi_{B2} + 2 a^2 C_p \Phi_{B2} = 0 \] \hspace{1cm} (A8)
Appendix A

The [m], [c] and [k] matrices of the MK III vehicle passive vertical suspension model.

\[
\mathbf{m} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0
0 & 0 & M1 & 0 & 0 & 0
0 & 0 & 0 & M2 & 0 & 0
0 & 0 & 0 & 0 & M_B & 0
0 & 0 & 0 & 0 & 0 & I_B
0 & 0 & 0 & 0 & 0 & M_B
0 & 0 & 0 & 0 & 0 & I_B
\end{bmatrix}
\]

(A9)

\[
\mathbf{c} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & C_r & 0 & - C_r & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & C_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & (2a^2C_p) & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & (C_r+2aC_p) & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & (2a^2C_p) & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & (2a^2C_p)
\end{bmatrix}
\]

(A10)

\[
\mathbf{k} = \begin{bmatrix}
2aK_s & 0 & -K_s & -K_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
(2a^2K_p) & (lK_s) & (-lK_s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
- K_s & 1aK_s & (K_s+K_p) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
- K_s & 1aK_s & (K_s+K_p) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & -K_r & 0 & (K_r+2aK_p) & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & (2a^2K_p) & 0 & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & (K_r+2aK_p) & 0 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & (2a^2K_p)
\end{bmatrix}
\]

(A11)
APPENDIX B
MK III IMPROVED EQUATIONS OF MOTION

THE IMPROVED EQUATIONS OF MOTION FOR THE MK III VEHICLE
PASSIVE VERTICAL SUSPENSION MODEL.

A- INCLUSION OF TRACK INPUT MOTION.

1- The equations of motion for the vehicle body and the two small masses are the
same as those in appendix A.

2- The new equations of motion for the two bogies are as follows:

\[ M_B \ddot{z}_{bl} = K_r (z_{bl} - \dot{z}_{bl}) + C_r (\dot{z}_{bl} - \ddot{z}_{bl}) + F'_4 + F_4 \]  \hspace{2cm} (B1)

\[ I_B \dddot{\phi}_{bl} = (F'_4 - F_4) a \]  \hspace{2cm} (B2)

The forces \( F_4 \) and \( F'_4 \) are the resultant imposed forces on the bogie due to the track
inputs as shown below in equations (B3c) and (B3d).

For moment equilibrium, a moment about \( B \) and \( B' \) as shown in Figure (4.2), to
obtain

\[ F_1 * l_1 = (F_2 * l_2) + (F_3 * l_3) \]  \hspace{2cm} (B3a)

\[ F'_1 * l_1 = (F'_2 * l_2) + (F'_3 * l_3) \]  \hspace{2cm} (B3b)

For vertical forces equilibrium:

\[ F_4 = F_1 + F_2 + F_3 \]  \hspace{2cm} (B3c)

\[ F'_4 = F'_1 + F'_2 + F'_3 \]  \hspace{2cm} (B3d)
The forces in the spring and dampers are given by:

\[ F_2 = K_p (Z_c - Z_p) \]  
\[ F'_2 = K_p (Z_c' - Z_p') \]  
\[ F_3 = C_p (\dot{Z}_D - \dot{Z}_p) \]

\[ F'_3 = C_p (\dot{Z}_{p'} - \dot{Z}_{p'}) \]

Where the notation \( Z_p \) denotes the vertical displacement at point P.

Equation (B3c) gives

\[ F_4 = F_1 + F_2 + F_3 \]

but from (B3a) is obtained the equation

\[ F_1 = \frac{12}{11} F_2 + \frac{13}{11} F_3 \]

Substituting this in (B3c) results in the following equation:

\[ F_4 = \frac{12}{11} F_2 + \frac{13}{11} F_3 + F_2 + F_3 \]

i.e.

\[ F_4 = (1 + \frac{12}{11}) F_2 + (1 + \frac{13}{11}) F_3 \]

Appendix B
Substituting the values for $F_2$ and $F_3$ from (B3e) and (B3g) into the expression above gives

\[ F_4 - (1 + \frac{12}{H}) K_F (Z_C - Z_P) - (1 + \frac{13}{H}) C_F (\dot{Z}_D - \dot{Z}_E) \]  

(B4a)

Following a similar argument, $F'_4$ can be found

\[ F'_4 - (1 + \frac{12}{H}) K_F (Z_C' - Z_P') - (1 + \frac{13}{H}) C_F (\dot{Z}_D' - \dot{Z}_E') \]  

(B4b)

\[ F''_4 - F'_4 - (1 + \frac{12}{H}) K_F (Z_C'' - Z_P'') - (1 + \frac{13}{H}) C_F (\dot{Z}_D'' - \dot{Z}_E'') \]  

(B5a)

\[ F''''_4 - F'_4 - (1 + \frac{12}{H}) K_F (Z_C'''' - Z_P''') - (1 + \frac{13}{H}) C_F (\dot{Z}_D'''' - \dot{Z}_E'''') \]  

(B5b)

It remains to determine the displacements $Z_C$, $Z_P$, $Z_C'$, $Z_P'$, and the velocities $Z_D$, $Z_E$, $Z_D'$, and $Z_E'$. Referring to Figure (4.2) and employing the notation $Z_P$, and $\theta_P$ to denote vertical displacement and rotation respectively, at point P, for $(Z_C - Z_C')$ and $(Z_C - Z_C')$.

\[ Z_C - Z_A - (H + 12) \theta_A \]

Since

\[ Z_A - Z_B - (a - H) \Phi_B, Z_C - Z_B - (a - H) \Phi_B - (H + 12) \theta_A \]

and

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$Z_P = Z_{B1} - (a + l2) \Phi_{B1}$

then

$Z_C = Z_P - (l1 + l2) \theta_A \Phi_{B1} - (l1 + l2) (\theta_A \Phi_{B1})$ \hspace{1cm} (B6)

Now

$$\theta_A = \frac{Z_P - Z_A}{ll} - \frac{Z_{B1} - (a - ll) \Phi_{B1}}{ll} - \frac{Z_B - Z_{B1} \Phi_{B1}}{ll}$$ \hspace{1cm} (B7)

$$\theta_A \Phi_{B1} = \frac{Z_B - Z_{B1} \Phi_{B1}}{ll} - \frac{Z_{B1} \Phi_{B1}}{ll} - \frac{Z_{B} \Phi_{B1}}{ll}$$

This result is substituted into equation (B6) to yield

$Z_C = Z_P - (l1 + l2) (Z_{B1} - Z_{B1} \Phi_{B1})$ \hspace{1cm} (B8)

Similarly

$Z\delta = Z\delta - (l1 + l2) \theta\delta$

As

$Z\delta = Z\delta - (a - l) \Phi_{B1}$

$Z\delta - Z_{B1} \Phi_{B1} - (l1 + l2) \theta\delta$
Since

\[ Z_D' - Z_{B1} \cdot (a \cdot I2) \Phi_{B1} \]

\[ Z_C' - Z_D' \cdot (I1 + I2) \delta_A' - (I1 + I2)(\delta_A' - \Phi_{B1}) \]  \hspace{1cm} (B9)

As

\[ \Theta_{A'} = \frac{Z_{D'} - Z_D'}{Z_{A'}} = \frac{Z_{A'}[Z_{B1} - (a - I1) \Phi_{B1}]}{Z_{A'}[Z_{B1} - (a - I1) \Phi_{B1}]} \]  \hspace{1cm} (B10)

Thus equation (B9)

\[ Z_C' - Z_D' \cdot (I1 + I2) (\Theta_{A'} - \Phi_{B1}) \]

\[ \times I1 \cdot I2 \frac{Z_{A'} - Z_{B1} - (a - I1) \Phi_{B1} - I \Phi_{B1}}{I1} \]

i.e.

\[ Z_C' - Z_D' \cdot (I1 + I2) \frac{Z_{A'} - Z_{B1} - a \Phi_{B1}}{I1} \]  \hspace{1cm} (B11)

Consider now

\[ (Z_D' - Z_{B1}) \text{ and } (Z_B' - Z_{B1}') \]

\[ Z_D' = Z_A' \cdot (I1 + I2) \delta_A \]
Substituting for $\theta_A$ gives:

\[ Z_D = Z_A - (I - I_3) \left( \frac{2 \theta_A^2}{I} \right) \]

\[ - Z_d[1 - (1 + \frac{I_3}{I})] + (1 + \frac{I_3}{I}) \theta_B \]

\[ - Z_A^2 - (1 + \frac{I_3}{I}) \theta_B \]

i.e.

\[ Z_D^2 - \frac{I_3}{I} \theta_B^2 (a - I_1) + (1 + \frac{I_3}{I}) \theta_B \]

and

\[ Z_B = a + I_3 \theta_B \]

Hence

\[ Z_D - Z_B^2 - (I_3 - I_1) \theta_B + (1 + \frac{I_3}{I}) \theta_B \]

\[ -(1 + \frac{I_3}{I}) \theta_B^2 (a - I_1) + (1 + \frac{I_3}{I}) \theta_B \]

\[ -(1 + \frac{I_3}{I}) (Z_B - \theta_B) + \frac{I_3}{I} (a - I_3) \theta_B \]

i.e.

\[ = \theta_B^2 - a + I_3 \theta_B \]
\[
\hat{z}_D - \hat{z}_K = \left(1 + \frac{13}{H}\right)(\hat{a}_{11} - \hat{z}_{B1} + a \ \Phi_{B1})
\]  

(B12)

Repeating the argument

\[
\hat{z}_{D'} - \hat{z}_{A'} = (H + 13) \delta_{A'}
\]

\[
-\hat{z}_{A'}(1 - (1 + \frac{13}{H})) \cdot (1 + \frac{13}{H}) \hat{z}_{D'}
\]

\[
-\frac{13}{H} \hat{z}_{A'} \cdot (1 + \frac{13}{H}) \hat{z}_{D'}
\]

Namely

\[
\hat{z}_{D'} = -\frac{13}{H} [(\hat{z}_{B1} + (a - H) \ \Phi_{B1})] \cdot (1 + \frac{13}{H}) \hat{z}_{D'}
\]

But as

\[
\hat{z}_{D'} - \hat{z}_{B1} \cdot (a + 13) \ \Phi_{B1}
\]

\[
\hat{z}_{D'} - \frac{13}{H} [(\hat{z}_{B1} + (a - H) \ \Phi_{B1})] \cdot (1 + \frac{13}{H}) \hat{z}_{D'} - \hat{z}_{B1} - (a + 13) \ \Phi_{B1}
\]
It is now possible to evaluate the resultant imposed track input forces $F_4$ and $F'_4$ together with their sum ($F_4 + F'_4$) and difference ($F_4 - F'_4$).

From equation (4 a)

$$F_4 = (1 \cdot \frac{13}{II}) K_p (Z_c - Z_p)$$

$$+ (1 \cdot \frac{13}{II}) C_p (\dot{Z}_d - \dot{Z}_b)$$

Substituting for $(Z_c - Z_p)$ equation (B8) and $(Z_d - Z_b)$ from equation (B12) into the above expression result in

$$F_4 = (1 \cdot \frac{13}{II})^2 K_p (Z_d - Z_{bl} + a \Phi_{bl})$$

$$+ (1 \cdot \frac{13}{II})^2 C_p (\dot{Z}_d - \dot{Z}_{bl} + a \Phi_{bl})$$

i.e.

$$F_4 = KU (Z_d - Z_{bl} + a \Phi_{bl}) + CU (\dot{Z}_d - \dot{Z}_{bl} + a \Phi_{bl}) \quad (B14)$$
Appendix B

Where

\[ KU = (1 \cdot \frac{H}{L})^2 K_p \]

and

\[ CU = (1 \cdot \frac{H}{L})^2 C_p \]

Similarly, from equation (B4b)

\[ F_4' = (1 \cdot \frac{H}{L}) K_p (Z_c' - Z_p') \cdot (1 \cdot \frac{H}{L}) C_p (\dot{Z}_D' - \dot{Z}_D) \]

Substituting \((Z_c - Z_p)\) from equation (B11) and \((Z_c' - Z_p')\) and equation (B13) into this produces

\[ F_4' = (1 \cdot \frac{H}{L}) K_p (Z_a - Z_b) \cdot (1 \cdot \frac{H}{L}) C_p (\dot{Z}_a - \dot{Z}_b) \]

i.e.

\[ F_4' = KU (Z_a - Z_b) \cdot CU (\dot{Z}_a - \dot{Z}_b) \]

From equation (B5 a)

\[ F_4' \cdot F_4 = (1 \cdot \frac{H}{L}) K_p (Z_c' - Z_p' + Z_c - Z_p) \cdot (1 \cdot \frac{H}{L}) C_p (\dot{Z}_D' - \dot{Z}_D) \]
It can be written

\[ F'_4 \cdot F_4 \cdot (1 \cdot \frac{12}{H})^2 K_p (Z_a - Z_{Bl} - a \Phi_{Bl} \cdot Z_d - Z_{Bl} \cdot a \Phi_{Bl}) \cdot \]

\[ (1 \cdot \frac{13}{H})^2 C_p (\dot{Z}_d - \dot{Z}_{Bl} \cdot a \Phi_{Bl} \cdot \dot{Z}_d - \dot{Z}_{Bl} \cdot a \Phi_{Bl}) \]

i.e.

\[ F'_4 \cdot F_4 \cdot KU (Z_a - Z_{Bl} - 2 Z_{Bl}) \cdot CU (\ddot{Z}_d + \dot{Z}_d - 2 \dot{Z}_{Bl}) \]  
(B16)

From equation (B5b)

\[ F'_4 - F_4 \cdot (1 \cdot \frac{12}{H})^2 K_p (Z_a' - Z_d' - (Z_C - Z_P)) \]

\[ \cdot (1 \cdot \frac{13}{H})^2 C_p (\dot{Z}_d' - \dot{Z}_d' - (\dot{Z}_d - \dot{Z}_d')) \]

It can be written

\[ F'_4 - F_4 \cdot (1 \cdot \frac{12}{H})^3 K_p (Z_a - Z_d - 2 a \Phi_{Bl}) \]

\[ \cdot (1 \cdot \frac{13}{H})^3 C_p (\dot{Z}_d - \dot{Z}_d - 2 a \Phi_{Bl}) \]

i.e.

\[ F'_4 - F_4 \cdot KU (Z_a - Z_d - 2 a \Phi_{Bl}) \]

\[ \cdot CU (\dot{Z}_d - \dot{Z}_d - 2 a \Phi_{Bl}) \]  
(B17)

1.1- EQUATIONS OF MOTION FOR BOGIES

The equations of motion for the front bogie B1 are repeated here from equation (B1)
and equation (B2)

\[
M_B \ddot{Z}_{B1} - K_e (Z_{sl} - Z_{bl}) \cdot C_r (\dot{Z}_{sl} - \dot{Z}_{bl}) \cdot F_4' \cdot F_4
\]

\[
I_B \ddot{\Phi}_{B1} = (F_4' - F_4)a
\]

Substituting for \((F_4' + F_4)\) from equation (B16) and \((F_4' - F_4)\) from equation (B17) into the appropriate equations above yields

\[
M_B \ddot{Z}_{B1} - K_e (Z_{sl} - Z_{bl}) \cdot C_r (\dot{Z}_{sl} - \dot{Z}_{bl}) \cdot K_e (Z_{al} - 2 Z_{bl}) \cdot \dot{Z}_{al} - 2 \dot{Z}_{bl})
\]

\[
I_B \ddot{\Phi}_{B1} = a KU (Z_{al} \cdot Z_{al} - 2 a \Phi_{B1}) \cdot a CU (\dot{Z}_{al} - \dot{Z}_{al} - 2 a \Phi_{B1})
\]

Writing these in standard form, the equations of motion for the front bogie B1 are

\[
M_B \ddot{Z}_{B1} - C_r \cdot 2 \cdot CU \cdot \dot{Z}_{B1} - CU \cdot \dot{Z}_{al} - CU \cdot K_e \cdot Z_{B1}
\]

\[
\cdot (K_e + 2 KU) \cdot Z_{B1} - KU \cdot Z_{al} - KU \cdot Z_{al} = 0
\]

(B18)

\[
I_B \ddot{\Phi}_{B1} = 2 a^2 CU \Phi_{B1} \cdot a CU \dot{Z}_{al} - a CU \dot{Z}_{al}
\]

\[
\cdot 2 a^2 KU \Phi_{B1} - a KU Z_{al} - a KU Z_{al} = 0
\]

(B19)

The corresponding equations for the rear bogie B2 are obtained from equation (B18) and equation (B19) and by substituting \(Z_{B2}, \Phi_{B2}, Z_{sl}, Z_{al}\), and \(Z_{al}\) for \(Z_{B1}, \Phi_{B1}, Z_{al}, Z_{sl}\), \(Z_{al}\) and \(Z_{al}\) respectively and hence the equations of motion for the second bogie B2

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are

\[ M_B \ddot{z}_{Bl} - C_r \dot{z}_{Bl} \cdot (C_r + 2 \, C_U) \dot{z}_{Bl} - C_U \dot{z}_{ul} - C_U \dot{z}_{ul} - K_r \, Z_{s2} \]
\[ - (K_r + 2 \, K_U) \, Z_{s2} - K_U \, Z_{ul} - K_U \, Z_{ul} = 0 \]  
(B20)

\[ I_B \, \ddot{\phi}_{Bl} + 2 \, a^2 \, C_U \, \phi_{Bl} \cdot a \, C_U \, \dot{z}_{ul} - a \, C_U \, \dot{z}_{ul} \]
\[ - 2 \, a^2 \, K_U \, \phi_{Bl} \cdot a \, K_U \, Z_{ul} - a \, K_U \, Z_{ul} = 0 \]  
(B21)

### 1.2- EQUATIONS OF MOTION FOR THE TRACK INPUT FREEDOM

The resultant vertical force input at the \( Z_{ul} \) and \( Z_{ul} \) freedoms of the front bogie B1 are given by the forces \( F_4 \) and \( F'_4 \) respectively.

The equations of motion at these two freedoms are given by equations (B3c) and (B3d) and which have been evaluated as equations (B14) and (B15) respectively.

Rewriting these later equations, give for the \( Z_{ul} \) freedom, (see equation (B14)).

\[ - C_U \, \dot{z}_{Bl} \cdot a \, C_U \, \ddot{\phi}_{Bl} \cdot C_U \, \dot{z}_{ul} \cdot K_U \, Z_{Bl} \cdot a \, K_U \, \phi_{Bl} \cdot K_U \, Z_{ul} = F_{Z_{ul}} \]  
(B22)

and for the \( Z_{ul} \) freedom, (as shown in equation (B15))

\[ - C_U \, \dot{z}_{Bl} \cdot a \, C_U \, \ddot{\phi}_{Bl} \cdot C_U \, \dot{z}_{ul} \cdot K_U \, Z_{Bl} \cdot a \, K_U \, \phi_{Bl} \cdot K_U \, Z_{ul} = F_{Z_{ul}} \]  
(B23)

To obtain the equations of motion for the \( Z_{ul} \) and \( Z_{ul} \) freedoms \( Z_{s2}, \phi_{s2}, Z_{ul} \) and \( Z_{ul} \) can be substituted for \( Z_{Bl}, \phi_{Bl}, Z_{ul} \), and \( Z_{ul} \) respectively, to obtain:-

For freedom \( Z_{ul} \)
-CU $\dot{z}_{b2} \cdot a$ CU $\Phi_{b2} \cdot CU Z_{13} \cdot KU Z_{b2} \cdot a$ KU $\Phi_{b2} \cdot KU Z_{13} \cdot F_{z1}$

(B24)

For freedom $Z_{a4}$

-CU $\dot{z}_{b2} \cdot a$ CU $\Phi_{b2} \cdot CU Z_{14} \cdot KU Z_{b2} \cdot a$ KU $\Phi_{b2} \cdot KU Z_{a4} \cdot F_{z4}$

(B25)
APPENDIX C

COUPLED VEHICLES EQUATION OF MOTION

THE EQUATION OF MOTION FOR COUPLING OF EIGHT VEHICLES

1- For vehicle number one

\[ M\ddot{Z}_1 (2K_{e}K_{e}Z_1 - K_{e} \Phi_{1} - K_{Zz1} Z_1 Z_2 - K_{Z} \Phi_{2} = 0 \]  
(C1)

\[ I_{e} \ddot{Z}_1 (2K_{e}K_{e}Z_1^2 - K_{e} \Phi_{1} = 0 \]  
(C2)

\[ M_{Z1} \ddot{Z}_1 K_{e} \Phi_{1} = 0 \]  
(C3)

\[ M_{Z1} \ddot{Z}_1 K_{e} \Phi_{1} = 0 \]  
(C4)

\[ M_{Z1} \ddot{Z}_1 K_{e} \Phi_{1} = 0 \]  
(C5)

\[ I_{BL} \ddot{Z}_{BL} (2a^2 K_{e} \Phi_{BL} = 0 \]  
(C6)
\[ M_{2-7} \ddot{Z}_{(n-1)} - K_{2} \dot{Z}_{(n-1)} + C_{p} \dot{Z}_{(n)} = 0 \]  

(C7)

\[ I_{2-7} \dddot{\Phi}_{(n-1)} + 2 \dot{Z}_{(n)} \Phi_{(n)} + 2 \dot{Z}_{(n)} \Phi_{(n)} = 0 \]  

(C8)

2- For vehicles number two, three, four, five, six and seven

\[ M_{2-7} \dddot{Z}_{(n-1)} - K_{2} \dot{Z}_{(n-1)} + C_{p} \dot{Z}_{(n)} = 0 \]  

(C9)

\[ I_{2-7} \dddot{\Phi}_{(n-1)} + 2 \dot{Z}_{(n)} \Phi_{(n)} + 2 \dot{Z}_{(n)} \Phi_{(n)} = 0 \]  

(C10)

\[ M_{n-1} \dddot{Z}_{(n-1)} - K_{n} \dot{Z}_{(n-1)} - C_{p} \dot{Z}_{(n)} = 0 \]  

(C11)

\[ M_{n-1} \dddot{Z}_{(n-1)} - K_{n} \dot{Z}_{(n-1)} - C_{p} \dot{Z}_{(n)} = 0 \]  

(C12)
where \( n \) is the number of vehicle.

3- For vehicle number eight,

\[
M \ddot{Z}_8 - K Z_8 \dot{Z}_8 + 2 \cdot K \dot{Z}_8 \dot{Z}_8 = 0
\]

\[
J \ddot{\phi} + 2 \cdot a^2 K \Phi + 2 \cdot a^2 C \dot{\phi} = 0
\]
Appendix C

\[ I_{18}^2 K J Z K J^2 \Phi \tau K Z e(2 \cdot K J)^2. \]

\[ K J Z^2 \Phi \tau K J Z_{18} - K J Z_{16} = 0 \]  \hspace{1cm} (C18)

\[ M_{18}^2 - K^2 K Z - K \Phi \tau K Z_{18} K Z_{18} K Z_{18}^2 C \tau_{18}^2 C \tau_{18} 0 \]  \hspace{1cm} (C19)

\[ M_{18}^2 - K Z_{16} K \Phi \tau K Z_{16} K Z_{16} \]

\[ - K Z_{16} C \tau_{16} C \tau_{16} = 0 \]  \hspace{1cm} (C20)

\[ M_{18}^2 - K Z_{16}^2 K Z_{16} K \Phi \tau K Z_{16}^2 K Z_{16}^2 C \tau_{16} C \tau_{16} 2 \cdot C \tau_{16}^2 0 \]  \hspace{1cm} (C21)

\[ I_{18}^2 - B_{18}^2 - a^2 K \Phi B_{18}^2 - 2 \cdot a^2 C \Phi B_{18} = 0 \]  \hspace{1cm} (C22)

\[ M_{18}^2 - B_{18}^2 - K Z_{16}^2 K Z_{16}^2 K Z_{16}^2 K \Phi B_{18}^2 C \tau_{16} C \tau_{16}^2 C \tau_{16}^2 0 \]  \hspace{1cm} (C23)

\[ I_{18}^2 - B_{18}^2 - 2 \cdot a^2 K \Phi B_{18}^2 - 2 \cdot a^2 C \Phi B_{18} = 0 \]  \hspace{1cm} (C24)
This Appendix highlights the design of the P + I controller parameters.

Let’s choose $\omega = 10$ rad/sec. by inspection of the Bode plot.

Suppose point P is translated to point R as shown in Figure below. The required gain is therefore 118dB.

$M_{db} = 118$ dB

Conversion of $M_{db}$ to a linear quantity M gives:-

$$M = 10^{(M_{db} / 20)}$$

$$= 10^{(118 / 20)}$$

$$= 794000$$

The required phase shift, also from the Figure below, is $\phi = -130^\circ - (-50^\circ)$

$\phi = 80^\circ$

So

$\theta = 90^\circ - \phi$

$\theta = 10^\circ$
The time constant of the PI controller, $T_c$, is calculated using

$$T_c = \tan \theta / \omega$$

$$= \tan 10^\circ / 10$$

$$= 0.0176$$

The gain factor of the controller may be calculated using

$$K = M \sin 10^\circ$$

$$= 794000 \times \sin 10^\circ$$

$$= 138000$$ (to 3 significant figures)

By substituting the PI controller parameters in the following transfer function:

$$C(s) = K \left[ \frac{1 + sT_c}{sT_c} \right]$$

The transfer function of the resulting controller is:

$$C(s) = \frac{138000 [1 + 0.0176]}{0.0176}$$
The Nichols chart of the open loop model of the single wheelset braking.
% This m file is to test the single vehicle model while its running on track using a ramp on the rail.
% File name ( single vehicle dynamic test).
% Initialise vehicle variables and constants.

m = 26000;
mb = 2707;
m1 = 260;
m2 = 260;
i = 927500;
ib = 1970;
ks = 1160000;
kr = 488000;
cr = 50000;
kp = 360000;
cp = 8400;
l = 8;
a = 1.3;
l1 = 0.535;
l2 = 0.265;
l3 = 0.505;
cu = ((1 + l3/l1)^2) * cp;
k = ((1 + l2/l1)^2) * kp;

% Form vehicle model.

m = diag([m i m1 m2 mb ib mb ib 0 0 0 0]);
\[ e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ k = \begin{bmatrix} (2*ks) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \]

\[ m_{ll} = m(1:8, 1:8); \]

\[ k_{ll} = k(1:8, 1:8); \]

\[ e_{ll} = e(1:8, 1:8); \]
k12=k(1:8,9:12);
c12=c(1:8,9:12);
k21=k(9:12,1:8);
c21=c(9:12,1:8);
k22=k(9:12,9:12);
c22=c(9:12,9:12);

A=[zeros(m11) eye(m11)
-m11\k11 -m11\c11];

step;
ud = ud';

B1=[zeros(8,4)
-m11\k12];

B2=[zeros(8,4)
-m11\c12];

A1=[A B1
zeros(4,16) zeros(4,4)];

B=[B2
eye(4,4)];
CA=[eye(8,8) zeros(8,8) zeros(8,4)
zeros(8,8) eye(8,8) zeros(8,4)
k21 c21 k22];

D=[zeros(8,4)
zeros(8,4)
c22];

% perform vehicle dynamics simulation.

y=lsim(A1,B,CA,D,ud,t);

% Show plots.

plot(t,y)
pause
clg
subplot(211),plot(t,y(:,1))
xlabel('TIME (SECS.)')
ylabel('COACH BODY BOUNCE (m)')
grid
subplot(212),plot(t,y(:,2))
xlabel('TIME (SECS.)')
ylabel('COACH BODY PITCH (rad.)')
grid
pause
clg

subplot(211),plot(t,y(:,3))
```matlab
xlabel('TIME (SECS.)')
ylabel('ZS1 BOUNCE (m)')
grid
subplot(212), plot(t,y(:,4))
xlabel('TIME (SECS.)')
ylabel('ZS2 BOUNCE (m)')
grid
pause
clg

subplot(211), plot(t,y(:,5))
xlabel('TIME (SECS.)')
ylabel('BOGIE B1 BOUNCE (m)')
grid
subplot(212), plot(t,y(:,6))
xlabel('TIME (SECS.)')
ylabel('BOGIE B1 PITCH (rad)')
grid
pause
clg

subplot(211), plot(t,y(:,7))
xlabel('TIME (SECS.)')
ylabel('BOGIE B2 BOUNCE (m)')
grid
subplot(212), plot(t,y(:,8))
xlabel('TIME (SECS.)')
ylabel('BOGIE B2 PITCH (rad)')
grid
```
pause
clg

subplot(211), plot(t,y(:, 17))
xlabel('TIME (SECS.)')
ylabel('REACTION FORCE R1 (N)')
grid
subplot(212), plot(t,y(:, 18))
xlabel('TIME (SECS.)')
ylabel('REACTION FORCE R2 (N)')
grid
pause
clg

subplot(211), plot(t,y(:, 19))
xlabel('TIME (SECS.)')
ylabel('REACTION FORCE R3 (N)')
grid
subplot(212), plot(t,y(:, 20))
xlabel('TIME (SECS.)')
ylabel('REACTION FORCE R4 IN (N)')
grid
pause
clg
clg
%End.
% This m file is representing the four inputs to the wheelsets of the single vehicle (ramp in the track).
% This m file called (step).
t=0:1/100:10;
step=0.04*ones(1,101);
ud(1,:)=[zeros(1,50) step zeros(1,850)];
ud(2,:)=[zeros(1,57) step zeros(1,843)];
ud(3,:)=[zeros(1,90) step zeros(1,810)];
ud(4,:)=[zeros(1,97) step zeros(1,803)];
subplot(221),plot(t,ud)
xlabel('Time (secs.)')
ylabel('Height (m)')
%title ('Figure 4.4 a ramp in the track')
% End.
% Program to simulate a single wheelset braking nonlinear model with PI controller (the
% parameters which were designed for the linear model.
% This m file called (nonlinear single wheelset braking)
% with changing adhesion level
% Clear work.
clear

% Define constants
P = 138000; % proportional gain
I = 138000/0.0176; % integral gain
tau = 0.016; % brake first order lag time constant (seconds)

RW = 0.5; % wheel radius (m)
RB = 0.25; % braking radius (m)
IW = 700; % wheel inertia (kg*m^2)
MW = (26000+2*2707)/4; % quarter mass of the vehicle (kg)
g = 9.81; % gravity (N*s^2/m)
N = MW*g; % normal reaction force (N)
deltat = 0.005; % simulation time step
tfinal = 10; % simulation final time
vinitial = 20; % initial vehicle speed (m/s)
winitial = vinitial/RW; % wheel angular velocity (rad/s)

% Define look-up mu/creep table
xx=[-4.0;-3.0;-2.0;-1.0;-0.40;-0.35;-0.30;-0.25;-0.20;-0.18;-0.16;-0.14
-0.12;-0.10;-0.08;-0.075;-0.07;-0.065;-0.06;-0.05;-0.04;-0.03;-0.02;-0.01;0.0;0.01;0.02;0.03;
0.04;0.05;0.06;0.065;0.07;0.075;0.08;0.10;0.12;0.14;0.16;0.18;0.20;0.25;0.30;0.35;0.40;1.0]
2.0;3.0;4.0];

yy=[-0.125;-0.130;-0.135;-0.140;-0.146;-0.149;-0.152;-0.155;-0.158;-0.162
-0.166;-0.172;-0.182;-0.20;-0.215;-0.22;-0.225;-0.228;-0.23;-0.22;-0.20;-0.15;-0.10;-0.05
0.0;0.05;0.10;0.15;0.20;0.22;0.23;0.228;0.225;0.22;0.215;0.20;0.182;0.172;0.166;0.162
0.158;0.155;0.152;0.149;0.146;0.140;0.135;0.130;0.125];
tabd=[xx yy];

xxw=0.5*[ -4.0; -3.0; -2.0; -1.0; -0.40; -0.35; -0.30; -0.25; -0.20; -0.18; -0.16; -0.14
-0.12; -0.10; -0.08; -0.075; -0.07; -0.065; -0.06; -0.05; -0.04; -0.03; -0.02; -0.01; 0.0; 0.01; 0.02; 0.03;
0.04; 0.05; 0.06; 0.065; 0.07; 0.075; 0.08; 0.10; 0.12; 0.14; 0.16; 0.18; 0.20; 0.25; 0.30; 0.35; 0.40; 1.0
2.0;3.0;4.0];

yyw=0.5*[ -0.125; -0.130; -0.135; -0.140; -0.146; -0.149; -0.152; -0.155; -0.158; -0.162
-0.166; -0.172; -0.182; -0.20; -0.215; -0.22; -0.225; -0.228; -0.23; -0.22; -0.20; -0.15; -0.10; -0.05
0.0; 0.05; 0.10; 0.15; 0.20; 0.22; 0.23; 0.228; 0.225; 0.22; 0.215; 0.20; 0.182; 0.172; 0.166; 0.162
0.158; 0.155; 0.152; 0.149; 0.146; 0.140; 0.135; 0.130; 0.125];
tabw=[xxw yyw];

% Initialise system variables
% Define initial state
X = [ 0;0;w initial;v initial ];
cr = 2*(R W*w initial-v initial)/(R W*w initial+v initial);

mu=table1(tabd,cr);
FR = -N*mu;
ttotal = [];
crtotal = [];
wtotal = [];

% Initialis e system variables
vtotal = [];  
FRwtotal = [];  
crintotal = [];  
FBtotal = [];  
Etotal = [];  
w=winitial;  
v=vinitial;  

crin = -0.03;  

% Define state-space model  

a = [ zeros(1,4)  
      I/tau -1/tau 0 0  
      0 RB/IW 0 0  
      zeros(1,4) ];  

b = [ 1 -1 0  
      P/tau -P/tau 0  
      0 0 RW/IW  
      0 0 -1/MW ];  

c = [ 0 0 1 0  
      0 0 0 1 ];  

d = zeros(2,3);  

% Define creep command profile  

\[ \text{ramptab, } t = \text{ramp}(0.1, 2, 10, 0, -0.03); \]
\[ \text{ramptab} = [t' \text{ ramptab}']; \]

\% perform time simulation loop
\[ \text{disp}(''); \]
\[ \text{disp('***** PERFORMING TIME SIMULATION LOOP *****');} \]
\[ \text{disp('');} \]

\text{for time = 0:delta\text{t}:tfinal,} \]

\% Report on simulation status
\[ \text{disp('***** TIME SIMULATION NOW AT: ',num2str(time),' SECONDS *****');} \]
\[ \text{disp('');} \]

\% changing to wet condition
\[ \text{if time} >= 5, \]
\[ \text{tabd = tabw;} \]
\[ \text{end; } \% \text{if} \]

\[ \mu = \text{table1}(\text{tabd}, \text{cr}); \]

\% Calculate FR
\[ \text{FR} = -N\times\mu; \]
\[ \text{crin} = \text{table1}(\text{ramptab, time}); \]

\% Simulate over a time step
\[ \text{deltaX} = a\times X + b\times [\text{crin}; \text{cr}; \text{FR}]; \]
\[ X = X + \text{deltaX}\times\text{deltat}; \]
% Creep evaluation
w = X(3,1);
v = X(4,1);
FB = X(2,1);
cr = 2*(RW*w-v)/(RW*w+v);
E = cr - crin;

% Accumulate variables
ttotal = [ ttotal time ];
crtotal = [ crtotal cr ];
crintotal = [ crintotal crin ];
wtotal = [ wtotal w ];
vtotal = [ vtotal v ];
FRtotal = [ FRtotal FR ];
FBtotal = [ FBtotal FB ];
Etotal = [ Etotal E ];

% Break out of loop if v <= 0.15;
if (abs(v) <= 0.15),
  break;
end; % if

end

% Ploting routine
%figure(1);
% Plot creep response
subplot(221);
plot(ttotal,crtotal);
title('Creep');
xlabel('Time (seconds)');
ylabel('Creep');
grid

% Plot wheel angular velocity response
subplot(222);
plot(ttotal,wtotal);
title('Angular speed (wheel)');
xlabel('Time (seconds)');
ylabel('rad/s');
grid

% Plot velocity response
subplot(223);
plot(ttotal,vtotal);
title('Travelling speed');
xlabel('Time (seconds)');
ylabel('m/s');
grid

% Plot longitudinal braking force response
subplot(224);
plot(ttotal,FRtotal);
title('Logitudinal force');
xlabel('Time (seconds)');
ylabel('Newtons');
grid
% End.
## APPENDIX F

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body Mass</td>
<td>M</td>
<td>26000 kg</td>
</tr>
<tr>
<td>Bogie Mass</td>
<td>$M_B$</td>
<td>2707 kg</td>
</tr>
<tr>
<td>Body Pitch Inertia</td>
<td>$I$</td>
<td>927500 kgm$^2$</td>
</tr>
<tr>
<td>Bogie Pitch Inertia</td>
<td>$I_B$</td>
<td>1970 kgm$^2$</td>
</tr>
<tr>
<td>Secondary Spring Stiffness/Bogie</td>
<td>$K_s$</td>
<td>1.160 MN/m</td>
</tr>
<tr>
<td>Reservoir Stiffness/Bogie</td>
<td>$K_r$</td>
<td>0.488 MN/m</td>
</tr>
<tr>
<td>Secondary Damping/Bogie</td>
<td>$C_r$</td>
<td>50 KNs/m</td>
</tr>
<tr>
<td>Primary Stiffness/Axle</td>
<td>$K_p$</td>
<td>0.360 MN/m</td>
</tr>
<tr>
<td>Primary Damping/Axle</td>
<td>$C_p$</td>
<td>8.4 KNs/m</td>
</tr>
<tr>
<td>Primary Damper End Stiffness</td>
<td>$K_e$</td>
<td>5.0 MN/m</td>
</tr>
<tr>
<td>Bogie Centre Spacing</td>
<td>$2l$</td>
<td>16 m</td>
</tr>
</tbody>
</table>
### Appendix F

<table>
<thead>
<tr>
<th></th>
<th>2a</th>
<th>2.6 m</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bogie Wheel Base</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Swinging Arm Geometry</strong></td>
<td>11</td>
<td>0.535 m</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.265 m</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0.505 m</td>
</tr>
</tbody>
</table>