Steady-state stability of a power system as affected by tie-line reactance

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STEADY-STATE STABILITY
OF A POWER SYSTEM
AS AFFECTED BY TIE-LINE REACTANCE

by

Siu Wai Lee, B.Sc.

A doctoral thesis submitted in partial fulfilment of the
requirements for the award of the Degree of Doctor of
Philosophy of Loughborough University of Technology.
October, 1981

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SUMMARY

Various control schemes are presented in this thesis for dealing with the problems which arise from the tie-line reactance existing between a generator and an infinite-bus system.

A simplified power system model is considered, consisting of a synchronous generator connected through a lossless tie-line to an infinite bus represented by a constant voltage and frequency source of zero internal impedance. The generator rotor and its associated turbine masses are lumped together and represented by a single inertia constant, and the reactances of the tie-line and any associated transformers are combined into a single reactance. Linearized equations for this model are written in state-space form. Voltage regulation and speed governing systems are added to the basic system, and the overall closed-loop system is modelled on a digital computer. A program is developed to compute the eigenvalues of the system matrix for stability limit considerations, as well as the time response of the system when subjected to a small disturbance. Open- and closed-loop operation of the basic system on variation of the tie-line reactance are studied and compared.

The effect of additional system damping, provided by a stabilizing signal derived from the generator output power through a suitable stabilizer, and superimposed on the normal error voltage signal is investigated. It is found that this signal cannot extend the existing range of the tie-line reactance for a stable operation. When the source of the stabilizing signal is replaced by a control
law, resulted from the application of Optimal Control Theory to the system, the possibility of a wider and freely chosen range of tie-line reactance for good and stable operation is revealed. The effects of the weighting factors in the performance index on the resultant optimal system performance, and the versatility of the optimal control for disturbances of different forms and magnitudes are also investigated.

Modal Control Theory and Optimal Control Theory with a modified performance index are further used to ensure a certain degree of stability on the system. Techniques for overcoming the problems arising from the unmeasurable states of the system are then designed and compared.

The performance of the system with two input optimal controls is investigated and compared with that of the system with a single input optimal control. Finally, direct optimal control over the field voltage and input power of a synchronous generator/tie-line/infinite-bus system is considered, and the performance of the optimal system is evaluated and compared with that of the system without control.
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<td>$\psi_d$, $\psi_q$</td>
<td>stator flux linkages of d- and q-axis circuits, respectively</td>
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<td>stator voltages in d- and q-axis circuits, respectively</td>
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<td>$r_t$</td>
<td>total resistance between generator terminals and infinite busbar</td>
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<td>$x_t$</td>
<td>total reactance between generator terminals and infinite busbar (tie-line reactance)</td>
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<td>$x_a$</td>
<td>armature leakage reactance</td>
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<td>mutual reactance between field and damper circuits</td>
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<td>$\delta$</td>
<td>load angle, radians</td>
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<tr>
<td>$\omega_0$</td>
<td>rated angular frequency, rad./s</td>
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<tr>
<td>$\phi$</td>
<td>angle between direct-axis of rotor and axis of reference phase, radians</td>
</tr>
<tr>
<td>$\dot{\theta}$</td>
<td>instantaneous angular speed of rotor, rad./s (time derivative of $\theta$)</td>
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<tr>
<td>$T_{in}$</td>
<td>torque input to rotor</td>
</tr>
<tr>
<td>$T_u$</td>
<td>air-gap torque</td>
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<tr>
<td>$T_l$</td>
<td>loss torque</td>
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H inertia constant
$K_d$ damping constant
$P_{in}$ input power to rotor
$P_s$ steam power
$P_o, Q_o$ real and reactive output powers, respectively
p.u. per-unit
Y governor valve position
y $dY/dt$
$\Delta$ prefix to denote small change

**Subscript notation**

a, b, c armature phases
f field winding circuit
d, q direct and quadrature axes
k damper winding circuit
CHAPTER 1

INTRODUCTION

An electric power system consists of three principal elements: the generating stations, the transmission lines, and the distribution systems. The transmission lines are the connecting links between the generating stations and the distribution systems. A distribution system connects all the individual loads in a given locality to the transmission lines.

The locations of hydro stations are fixed by the presence of water power, but the choice of sites for steam stations using fossil or nuclear fuels is more flexible. Steam stations using fossil fuels are often located throughout the system so that some generating plants are near the largest load centres. New nuclear plants are usually very large. Often plans for a nuclear plant specify a size equal to the entire system capability at the time the design is started. Thus, nuclear plants may require transmission of large blocks of power over fairly long distances, a hydro plant may also require long transmission lines from the plant to the load centres, and fossil-fueled steam plants usually supply loads over shorter distances.

The stability of long transmission lines depends very much on their capability to transmit large amounts of synchronizing power with corresponding high power limits under normal operating conditions. With the advent of the automatic voltage regulator and its application to synchronous condensers at
the receiving end of long transmission lines, it became possible to obtain good local voltage regulation and practical to operate systems much closer to their steady-state stability limits. Parallel with this development of long-distance transmission, the interconnection of large power systems for economic and emergency purposes led to a different form of stability problem. The operational characteristics in this case depend very largely upon the properties of the tie-line, which acts as a link between a large generating unit and a main system, which has a very small overall impedance compared with that of the generator or the tie-line, and which can be approximated to an infinite-bus system. Frequently these lines operate quite satisfactorily during steady-state conditions, but in the event of a disturbance, instability is inevitable if the tie-lines are incapable of transmitting sufficient synchronizing power. However, no difficulties have been encountered, except during severe faults, when the systems are tied solidly together through tie-lines designed to carry a large amount of load. Troubles were for the most part encountered on systems which were connected through so-called "shoe string" lines. Interconnection to reduce cost and to improve service reliability thus became a factor in the stability problem.

Many studies have been conducted to consider the stability problems presented by the tie-lines\textsuperscript{1-4}, and it has been generally concluded that the most obvious method of increasing the stability limit of a power system is to reduce the tie-line reactance between the generating unit and the infinite-bus system, as this directly increases the synchronizing power that may be interchanged between
them. Tie-line reactance reduction has been achieved by reducing the conductor spacing, using bundled conductors, additional parallel lines, reduced reactance transformers and series capacitors to compensate part of the line inductive reactance. The last method is usually the cheapest and has been extensively investigated by many authors. However, series-capacitor compensated power systems give rise to a major problem called the subsynchronous-resonance (s.s.r.) phenomenon. This phenomenon occurs, owing to series-capacitor compensation, when some of the natural frequencies of the transmission system are less than the supply frequency by an amount which coincides nearly with one or more of the natural torsional modes of individual generator dynamic systems. Overstressing of generator shafts is a possibility if conditions are particularly severe, owing principally to an absence of sufficient damping, or the system may become unstable even in otherwise stable region.

Parallel with the work on the problems of reducing the tie-line reactance, Modern Control Theory and its application to power systems for stability improvement has also been extensively investigated. The objective of this thesis is therefore to present a thorough investigation into the effect of tie-line reactance on the stability of a power system where Modern Control Theory is employed to improve the performance.

Throughout the thesis, a simplified system model has been employed, which is represented by a synchronous generator feeding into an infinite-bus system through a lossless tie-line represented by a single reactance. This simplified model has in fact been the
basis for many previous power system stability studies. In cases where the generator is supplying a local shunt load, the transmission network can be reduced to a single external impedance, which is mainly inductive, together with an equivalent bus bar voltage. System model equations are then developed based on Park's equations for a synchronous machine. The linearized forms of these equations, which are valid only for a small region of the state space surrounding a specified point, are arranged in a set of 1st-order differential equations (state-space representation) for open-loop steady-state stability studies. This is followed by closed-loop stability studies when voltage regulating and governing devices are added into this system. The evaluation of the eigenvalues of the system matrix by a digital computer is used to determine whether or not a system is stable. The role of a stabilizing signal added into the summing junction of the voltage regulator to provide additional damping is investigated. The source of this signal is either the output electrical power measured directly at the generator terminals, or generated by a state-feedback controller designed by Optimal Control Theory or by Modal Control Theory. Where the state-variables required for feedback are inaccessible, state-estimating observers and suboptimal controllers are employed. Optimal controllers for other system configurations are also designed and compared.

In Chapter 2, the general effect of the tie-line reactance on the stability of a power system is outlined. Methods of modelling this system, of analysing its stability, and of solving its set of 1st-order differential equations are described.
Under suitable assumptions, the equations for a power system and its associated voltage regulation and governing systems are derived in Chapter 3. The linearized forms of these equations, suitable for steady-state stability studies, are used to demonstrate the procedure for developing state-space equations to represent either the dynamics of individual subsystems or the dynamics of the overall closed-loop system. The equations for steady operation of a synchronous generator at synchronous speed are also derived.

For certain parameter values for the three subsystems considered in Chapter 3, Chapter 4 introduces a numerical method of verifying the validity of the state-space equations representing these three systems. The effects of the tie-line reactance on the stability limit of the system, in both open- and closed-loop operations, are investigated.

Retaining the same governing system of Chapter 4, Chapter 5 looks at another voltage regulator of more recent design and a synchronous generator of a different rating. The effects of the tie-line reactance and of some generator parameters on the stability of the system in both open- and closed-loop operations are evaluated. This overall closed-loop system, called the original closed-loop system, then forms the basic system for all further studies in the thesis.

Chapter 6 looks at the role of a stabilizing signal, when added into the summing junction of the voltage regulator. The source of this signal is the output electrical power measured directly at the generator terminals. Under the performance
criteria of good voltage regulation and good damping, a suitable stabilizer transfer function is found.

In Chapter 7, state-feedback optimal controllers for different values of tie-line reactance are designed according to a suitable performance index to generate the source of the stabilizing signal. Dramatic improvements in the system performance result, and the possibility of having various ranges of tie-line reactance for good and stable operation is realized. The effects of the weighting factors in the performance index on the resultant optimal system are presented in simplified graphical form. For disturbances of different forms and magnitudes, the versatility of an optimal controller is also revealed.

In Chapter 8, Optimal Control Theory with a modified performance index, and Modal Control Theory are applied in the single-input system, to shift its eigenvalues to prescribed regions or locations in the left-half of the complex s-plane for further system damping improvement. The required controller gain is higher than that of a conventional optimal controller, and a method for overcoming this problem, in actual implementation, is suggested.

In Chapter 9, full-observer, low-order observer and suboptimal controllers are designed to overcome the problems arising from the unmeasurable states which are required for feedback. In low-order observer design, a method for solving an algebraic matrix equation of the form $QT - TA = C$ is developed.
The characteristics of these three devices for different values of tie-line reactance are also investigated and compared.

In Chapter 10, a 2-input system, formed by adding another stabilizing signal into the summing junction of the governing system, is considered. The performance of this 2-input optimal system is compared with that of the single-input optimal system. Direct optimal control over the field voltage and input power of a synchronous generator in a power system without voltage regulation and governing systems is also considered. The advantages and disadvantages of this type of control are listed.

Finally, an overall conclusion and some suggestions for further investigations are presented in Chapter 11.
CHAPTER 2

ANALYSIS OF TIE-LINE CHARACTERISTICS AND EFFECTS

Before introducing any regulating devices into a power system to mitigate the adverse effect of a long tie-line, the general effect of the line reactance on the stability of the existing system should be thoroughly understood, so that the benefits to be gained from the control devices can be appreciated. This Chapter presents a justification for the representation of a tie-line by a single reactance, together with the likely errors involved. An outline of the effect of line reactance on the effect introduced by the saliency of a synchronous machine and on the maximum power transfer is also given. Methods of modelling a power system, of analysing the system stability and of solving the set of 1st-order differential equations which describe the system are also described.

2-1 LINE MODEL JUSTIFICATION

From a system point of view, the overall electrical characteristics of the transmission line are of primary interest. Usually, these can be expressed in terms of the following four line parameters, listed below in order of importance:

(1) Line inductance L
(2) Line shunt capacitance C
(3) Line resistance R
(4) Line shunt conductance G
The reason why R and G are of least importance is that they affect the equivalent line impedance, and thus the power transmission capacity, to a relatively small extent. They do, of course, completely determine the losses incurred in transmission, and to the extent that transmission economy is of interest, their presence must be considered.

The parallel elements, of which the capacitance C is dominant, represent a leakage path for line currents. Such currents are proportional to the line voltage, and their importance increases with the magnitude of the operating voltage. For line voltages between 300 to 500 kV and line lengths in excess of 125 km, the impact of these shunt elements is of primary concern to the system engineer. As far as the steady-state stability limit is concerned, the omission of R and C usually gives, respectively, an optimistic and a pessimistic result for the theoretical maximum power transfer through a line represented by a nominal or an equivalent \( \pi \) circuit. Often the degree of accuracy obtained by making a more exact calculation does not justify the additional complexity involved.

The inductance L of a power line is by far the most significant line parameter from the system engineer's viewpoint. For normal line designs it is the dominating impedance element, directly affecting the transmission capacity of the line. It is therefore important to give full and proper attention to this particular parameter.
**2-2 EFFECTS OF LINE REACTANCE**

For a salient-pole synchronous machine connected directly to an infinite bus, the equation for the per-unit power output when losses are neglected is

\[
P = \frac{EV}{x_d} \sin \delta + \frac{V^2(x_d - x_q)}{2x_d x_q} \sin 2\delta \tag{2-2.1}
\]

where \( E \) = per-unit field excitation voltage corresponding to the field excitation (frequently referred to as the voltage back of synchronous reactance),

\( V \) = per-unit infinite bus voltage (in this case also the machine terminal voltage),

\( x_d \) = per-unit direct-axis synchronous reactance,

\( x_q \) = per-unit quadrature-axis synchronous reactance,

and \( \delta \) = angle between the equivalent field excitation voltage and the bus voltage (often called the load angle).

Equation (2-2.1) defines the steady-state power angle characteristics of a salient-pole machine. The second term is independent of the field excitation and, since it is caused by the difference in the magnetic reluctance of the direct- and quadrature-axes, it is sometimes called the reluctance power component. For a round-rotor machine \( x_d = x_q \) and this term becomes zero. Figure (2-2.1) shows \( P \) plotted against \( \delta \) for a machine for which \( x_d = 1.0, x_q = 0.6 \) and \( V = 1.0 \), and with a field excitation such that a power factor of unity is obtained at full load. The figure also shows \( P \) plotted against \( \delta \) for the same conditions, but for a round-rotor machine for which \( x_d = x_q = 1.0 \). It will be noted that the reluctance component of power increases slightly the maximum power available and also makes
the angle at which the maximum power is obtained less than 90°. It is also apparent from the figure that the load angle for the round-rotor machine is greater than that for the salient-pole machine for a given power output in the stable region. It follows immediately that the reluctance power component is appreciable in determining both the maximum power output and the angular displacement for maximum power.

In the case of a salient-pole synchronous machine connected through an external reactance to an infinite bus, the equation for the per-unit power output with losses neglected becomes

\[
P = \frac{EV}{x_d + x_t} \sin \delta + \frac{v^2 (x_d - x_q)}{2(x_d + x_t)(x_q + x_t)} \sin 2\delta \quad (2-2.2)
\]
where $x_t$ = per-unit external reactance.

A plot of equation (2-2.2) is given in figure (2-2.2) for an infinite bus voltage of unity, a typical tie-line reactance of 0.5 p.u., and a field excitation such that unit machine terminal voltage is obtained at unit load. Also shown is the corresponding characteristic obtained when saliency is neglected. It will be noted that the error in neglecting saliency is not nearly so great now as when the machine was connected directly to the bus ($x_t = 0$). This clearly arises from the decrease in the relative magnitude of the reluctance power component consequent on the introduction of external reactance. For most practical problems in which the power limit is of importance, the presence of external reactance weakens the effect of saliency and makes valid the quite common assumption that round-rotor theory may be used.

![Figure (2-2.2)](image)

Figure (2-2.2) Power/angle characteristics of a salient-pole synchronous generator with external reactance $x_t = 0.5$. (a) With $x_d = 1.0$, $x_q = 0.6$, $V = 1.0$ and $E = 1.589$. (b) When saliency is neglected and $E = 1.615$. 

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For a round-rotor machine, equation (2-2.2) becomes
\[ P = \frac{EV}{x_d + x_t} \sin \theta \]  
(2-2.3)
and the maximum power transfer between the machine and the infinite system is
\[ P_{\text{max}} = \frac{EV}{x_d + x_t} \]  
(2-2.4)
If \( E \) and \( V \) are constant, it is obvious from equation (2-2.4) that a higher \( x_t \) means inevitably a lower steady-state stability limit. Any gain to be realised by designing low synchronous reactance machines will be reduced as the external reactance increases, because \( x_d \) then becomes a smaller portion of the total effective impedance. A higher \( x_t \) also decreases the reactive power transfer, and in order to maintain a given active and reactive power requirement at higher \( x_t \) greater field excitation is necessary.

2-3 MATHEMATICAL REPRESENTATION OF SYSTEM

The per-unit notation\(^{24,25}\) together with Park's 2-axis transformation\(^{19}\) of the machine are commonly used to derive mathematical models for the transmission network and for the synchronous generator with one damper winding on each axis.\(^{17}\) The turbines and the rotor masses are lumped together and represented by a single inertia constant, and the reactances of the tie-line and any associated transformers by a single reactance. The system equations, so derived, are however nonlinear. Their linearized forms for perturbations about an operating point, which are suitable for steady-state stability studies, may be arranged in state-space
form as

\[ x = Ax + Bu \quad (2-3.1) \]

where \( x \) is the state vector containing suitably chosen state-variables,

\( u \) is the input control vector,

and \( A \) and \( B \) are matrices containing the constant quantities of

the 1st-order differential equations of the model.

Equation (2-3.1), representing the dynamics of the linearized system model, is compact and easily manipulated and solved by a digital computer. Further advantages of this form of representation become apparent when subsystems representing the characteristics of the prime mover and the exciter are added to the basic machine representation.\(^{21}\)

2-4 METHODS OF ANALYSIS AND SOLUTION

Techniques involving the Lyapunov method, the eigenvalues of the system, or the Routh Criterion are all suitable and computationally feasible for analysing the stability of systems modelled mathematically by equation (2-3.1). However, when the order of the system is large, the eigenvalue method has been shown to be the best in terms of computational time.\(^{22}\) This method is adopted in this thesis, not only because subroutines for computing the eigenvalues of the system matrix \( A \) are readily available in the NAG (Numerical Algorithms Group) library of this University's computer facilities, but also because the eigenvalues, so computed, reveal directly whether or not the system is stable, and if it is
stable, how stable it is. These eigenvalues, representing the roots of the characteristic equation of the system and hence its dynamics, are either positive or negative. Positive eigenvalues mean that the state-variables increase with time and hence that the system is unstable. Negative eigenvalues, on the other hand, mean that the state-variables decay exponentially with time and hence that the system is stable. Furthermore, when these eigenvalues are portrayed in a 2-dimensioned complex s-plane, the further they are located towards the left of the plane, the more stable is the system.

Solutions of equation (2-3.1) can be obtained by numerical integration when the control u is arbitrarily chosen. If only step responses are required, and u thus comprises zeros and constants only, the convenient method of solution shown in Appendix I requires the eigenvalues and eigenvectors of the system matrix A to be computed at the same time. This method of solution is easily implemented on a digital computer and no difficulties have been encountered in computing the responses of systems subjected either to step or to impulse disturbances.
CHAPTER 3

DERIVATION OF MATHEMATICAL MODEL

In this Chapter, an overall closed-loop power system layout is described. Under suitable assumptions and using Park's transformations, equations in per-unit form are written down for both the generator and the tie-line. Equations for an automatic voltage regulator with two stages of magnetic amplification and a standard oil-servo type governor are also derived under certain assumptions. The linearized forms of all these equations, arranged for steady-state stability studies, are used to demonstrate the procedure for developing state-space equations to represent either the dynamics of individual subsystems or the dynamics of the overall closed-loop system. Finally, the equations for steady operation of a synchronous generator at synchronous speed are also given.

3-1 OVERALL SYSTEM MODEL

The schematic layout of the overall system studied is shown in figure (3-1.1). It consists of a synchronous generator connected to an infinite bus system through a single reactance called the tie-line reactance. Regulation of the generator terminal voltage is provided by an automatic voltage regulator which senses the terminal voltage, compares its value against a reference level, and generates an error signal which, after amplification, drives the field of an exciter. The output of the exciter is then rectified to provide
Regulation of input power to the turbine is achieved by a governing system, which detects an error signal between the rotor speed and a reference level, and amplifies this signal to adjust the opening of the governor valve through which the steam is admitted into the turbine.

3-1.1 Generator equations

The layout of the windings of a 3-phase 2-pole generator is shown diagrammatically in figure (3-1.2), with the rotor field winding and the stator armature windings represented by windings \( f, \hat{a}, b \) and \( c \) respectively. Paths for current flow in the solid steel of the rotor are represented by windings \( k \), with the additional suffices \( d \) and \( q \) designating windings on the pole and interpole axes, respectively. The sign convention used in this figure corresponds to a generator action in which the stator currents are defined as positive in the direction of positive potential. The field and damper windings currents are defined as positive in the opposite sense. These mixed definitions agree with the normal mode.
Figure (3-1.2) The windings of a synchronous generator of operation of the respective windings of a synchronous generator. However, they cause a mixed-sign situation in the forthcoming generator equations.

Three assumptions are made in deriving the basic machine equations:

1. The stator windings are sinusoidally distributed around the air gap, in so far as mutual effects between them and the rotor are concerned.

2. The stator winding self- and mutual inductances vary sinusoidally as the rotor moves.

3. Hysteresis, saturation and eddy currents are negligible.

Under these assumptions, equations may be written immediately from figure (3-1.2) for the individual circuits of the generator, including
Figure (3-1.3) d, q representation of a synchronous generator with one damper coil on each axis

their mutual effects. These equations contain fundamental and second-harmonic frequency terms, which make their solutions difficult but which may be removed by use of the axis transformation due to R. H. Park. According to the 2-axis theory, figure (3-1.2) is transformed into the simplified machine shown in figure (3-1.3). The per-unit equations, when applied to this figure, are \(21, 24, 25\):

Direct axis flux linkage:

\[
\begin{align*}
\psi_f &= -x_{af}i_d + x_{fkd}i_{kd} + x_{ff}i_f \\
\psi_d &= -x_{d}i_d + x_{akd}i_{kd} + x_{af}i_f \\
\psi_{kd} &= -x_{akd}i_d + x_{kd}i_{kd} + x_{fkd}i_f
\end{align*}
\] (3-1.1)

Quadrature axis flux linkage:

\[
\begin{align*}
\psi_q &= -x_{q}i_q + x_{aq}i_{kq} \\
\psi_{kq} &= -x_{aq}i_q + x_{kqk}i_{kq}
\end{align*}
\] (3-1.2)
Direct axis voltage:

\[ v_f = - \frac{x_{af}}{w_o} p_i_d + r_f i_f + \frac{x_{rr}}{w_o} p_i_f + \frac{x_{rkd}}{w_o} p_i_{kd} \]

\[ v_d = - r_i_d - \frac{x_d}{w_o} p_i_d + \frac{\omega}{w_o} i_q + \frac{x_{af}}{w_o} p_i_f + \frac{x_{akd}}{w_o} p_i_{kd} \]

\[ - \frac{\omega}{w_o} i_{kq} \]

\[ 0 = - \frac{x_{akd}}{w_o} p_i_d + \frac{x_{rkd}}{w_o} p_i_f + r_{kd} i_{kd} + \frac{x_{kd}}{w_o} p_i_{kd} \]

\[ (3-1.3) \]

Quadrature axis voltage:

\[ v_q = - \frac{\omega}{w_o} i_d - r_i q - \frac{x_{aq}}{w_o} p_i q + \frac{\omega}{w_o} i_f + \frac{\omega}{w_o} i_{kd} \]

\[ + \frac{x_{akq}}{w_o} p_i_{kq} \]

\[ 0 = - \frac{x_{akq}}{w_o} p_i q + r_{kq} i_{kq} + \frac{x_{kkq}}{w_o} p_i_{kq} \]

\[ (3-1.4) \]

Additional equations are necessary to complete the description of the synchronous generator, although no assumptions in addition to those given previously are required. Thus

Electrical torque at the air gap:

\[ T_u = \psi_d i_q - \psi_i d \]

\[ (3-1.5) \]

Mechanical equation of motion:

\[ T_{in} - T_u - T_l = \frac{2\pi}{w_o} \dot{\theta} + K_d \dot{i} \]

\[ (3-1.6) \]
Terminal voltage:
\[ v_t^2 = v_d^2 + v_q^2 \] \hspace{1cm} (3-1.7)

and
\[ P_{in} = \frac{\theta}{\sin \omega} \]
\[ i_a^2 = i_d^2 + i_q^2 \] \hspace{1cm} (3-1.8)
\[ \theta = w_o t + \delta \]

3-1.2 Transmission system equations

In the transmission system, lumped series inductance and resistance is used to represent the transformer and the tie-line connecting the generator to the infinite bus system. Transformer magnetising and line-charging currents are neglected on the grounds that they are small.

In a similar manner to that described for the generator, the transmission-system components are resolved along the generator direct and quadrature axes, when the following per-unit equations are obtained.

\[ v_d = v_{bd} + x_t i_d + \frac{x_t}{w_o} p_i_d - \frac{x_t}{w_o} i_q \]
\[ v_q = v_{bq} + x_t i_q + \frac{x_t}{w_o} p_i_q + \frac{x_t}{w_o} i_d \]
\[ v_{bd} = V \sin \delta \]
\[ v_{bq} = V \cos \delta \]

where \( x_t \) = sum of the transformer and tie-line reactances (for simplicity called the tie-line reactance),
$r_t = \text{sum of the transformer and tie-line resistances} = 0$

(for a lossless tie-line),

and the term $\dot{\theta}$ arises from the axis transformation process where time derivatives of the terms $\sin \theta$ and $\cos \theta$ are involved ($\frac{d}{dt} \sin \theta = \cos \theta \cdot \dot{\theta}$ and $\frac{d}{dt} \cos \theta = -\sin \theta \cdot \dot{\theta}$).

3-1.3 Automatic voltage regulator equations

Figure (3-1.4) shows a block diagram of a typical voltage regulator.\textsuperscript{17,23} Its main forward loop consists of a voltage-sensing element from which an error signal is derived to drive two stages of magnetic amplification, the second stage of which controls the field of the main exciter; there are also two subsidiary stabilizing loops.

The assumptions made in the formation of the equations for the voltage regulator are that\textsuperscript{17}:

(1) The two stages of magnetic amplification and the exciter may be represented by a simple time lag.

(2) Saturation of the magnetic amplifiers may be represented by limits on their output voltage.

(3) The characteristics of the rectifiers are linear and affect the operation of the generator only in exceptional circumstances; e.g. when the rotor current is forced to zero.

Under these assumptions, the equations (in volts, amperes and seconds) of the elements of the voltage regulator, with $K_{m1}$, $K_{m2}$ and $K_x$ as constant bias voltages\textsuperscript{23}, are:
Figure (3-1.4) Block diagram of voltage regulator

First magnetic amplifier:

\[ v_{m2} = \frac{G_{m1}}{1+pT_{m1}} (u_r + v_x + v_{ms}) + K_{m1} \]  
\[ v_{m2min} \leq v_{m2} \leq v_{m2max} \]  

Second magnetic amplifier:

\[ v_x = \frac{G_{m2}}{1+pT_{m2}} v_{m2} + K_{m2} \]  
\[ v_{xmin} \leq v_x \leq v_{xmax} \]  

Exciter and rectifiers:

\[ v_f = \frac{G_x}{1+pT_{x}} v_x + K_x \]  

Amplifier stabilizer:

\[ v_{ms} = \frac{-G_{ms}}{1+pT_{ms}} v_x \]
Exciter stabilizer:
\[ v_{xs} = \frac{-G_{xs} T_{xs} p}{1 + p T_{xs}} v_f \]  
(3-1.14)

Transformer and voltage-sensitive circuit:
\[ u_r = -G_e (C_t v_t - v_r) \]  
(3-1.15)

3-1.4 Governor and turbine equations

The governor shown in figure (3-1.5) is a standard oil-servo type, actuated by either the speed of the generator or the speeder-gear motor; no other feedback is incorporated. The assumptions made in developing the equations for the governor are:

1. There is no time lag between the main shaft and the governor sleeve movement.
2. Under steady conditions, the operation of the governor is such that the steam admitted to the turbine is a linear function of the speed of the turbine throughout the governor's working range.
3. The pilot valve, relay valve and turbine-entrained steam may each be represented by a simple time lag.
4. The boiler may be represented by a source of steam at constant pressure and temperature.
5. The efficiency of the turbine does not change over the small range of speed in which it is normally required to operate.

Under these assumptions, the equations of the elements of the governing system, with \( K_t \) as a constant bias quantity, are:
Figure (3-1.5) Block diagram of governing system

Watt governor and comparator:

\[ u_t = Y_o - G_1 P_\delta \]  \hspace{1cm} (3-1.16)

Pilot and relay valves:

\[ Y = \frac{G_2}{(1+pT_1)(1+pT_2)} u_t + K_t \]  \hspace{1cm} (3-1.17)

Governor valve:

\[ P_s = G_3 Y \]  \hspace{1cm} (3-1.18)

Turbine:

\[ P_{in} = \frac{1}{1+pT_3} P_s \]  \hspace{1cm} (3-1.19)
EQUATIONS FOR SMALL CHANGES AND SMALL OSCILLATIONS

The equations describing the performance of a single synchronous machine without associated excitation or prime-mover control, when connected to an equivalent transmission system, are generally nonlinear.

Small changes and small oscillations arise when a machine operating under steady conditions is subjected to a small disturbance. If the changes in the variables are small, so that their square or product terms can be neglected, the differential equations relating the changes are linear even when the general equations are nonlinear. Also, when the changes are restricted to a narrow band there is little variation in the flux level, and the machine inductances can be regarded as constants. The equations which can then be derived can be used for studying the steady-state stability, which depends on the effect of making a small change relative to a steady condition, or for calculating the magnitude of small oscillations which may be superimposed on a condition of steady operation.

The procedure of linearization by small departures is to set up subsidiary equations by replacing each variable (such as \( i_d \)) by a reference level plus a deviation (\( i_d + \Delta i_d \)). The equation relating deviations from a reference situation have the same form as those for the original variables except where a product of variables occurs. If an equation such as \( Z = XY \) exists and increments \( \Delta X \) and \( \Delta Y \) give rise to a resultant \( \Delta Z \), then

\[
Z + \Delta Z = (X + \Delta X) (Y + \Delta Y)
= XY + Y\Delta X + X\Delta Y + \Delta X\Delta Y
\]  

(3-2.1)
Terms at the reference levels can be eliminated from this equation, leaving a relation between the deviations with coefficients that can be considered constant over a limited region. If the deviations are very small, the last term may also be neglected, unless $X$ and $Y$ are also very small.

Employing the above approximation, the linearized equations for the generator, transmission system, automatic voltage regulator and governor are written as follows:

(a) Generator

Direct axis flux linkage:

$$\Delta \psi_f = -x_{af} \Delta i_d + x_{fkd} \Delta i_{kd} + x_{ff} \Delta i_f$$

$$\Delta \psi_d = -x_d \Delta i_d + x_{akd} \Delta i_{kd} + x_{af} \Delta i_f$$  \hfill (3-2.2)

$$\Delta \psi_{kd} = -x_{akd} \Delta i_d + x_{fkd} \Delta i_f + x_{kd} \Delta i_{kd}$$

Quadrature axis flux linkage:

$$\Delta \psi_q = -x_q \Delta i_q + x_{akq} \Delta i_{kq}$$  \hfill (3-2.3)

$$\Delta \psi_{kq} = -x_{akq} \Delta i_q + x_{kq} \Delta i_{kq}$$

Direct axis voltage:

$$\Delta v_f = -\frac{x_{af}}{w_o} p \Delta i_d + r_f \Delta i_f + \frac{x_{ff}}{w_o} p \Delta i_f + \frac{x_{fkd}}{w_o} p \Delta i_{kd}$$

$$\Delta v_d = -r \Delta i_d - \frac{x_d}{w_o} p \Delta i_d + \frac{x_q}{w_o} \Delta i_q + \frac{x_i}{w_o} q \Delta \psi + \frac{x_{af}}{w_o} p \Delta i_f$$

$$+ \frac{x_{akd}}{w_o} p \Delta i_{kd} - \frac{x_{akq}}{w_o} \Delta i_{kq} - \frac{x_{akq}}{w_o} i_{kq} \Delta \psi$$

$$0 = -\frac{x_{akd}}{w_o} p \Delta i_d + \frac{x_{fkd}}{w_o} p \Delta i_f + r_{kd} \Delta i_{kd} + \frac{x_{kd}}{w_o} p \Delta i_{kd}$$  \hfill (3-2.4)
Quadrature axis voltage:

\[
\Delta v_q = - \frac{x_d}{\omega} \Delta i_d - \frac{x_d}{\omega} i_d \Delta \delta - r \Delta i_q - \frac{x_q}{\omega} p \Delta i_q + \frac{x_{af}}{\omega} \Delta i_f + \frac{x_{akd}}{\omega} i_{kd} \Delta \delta + \frac{x_{akd}}{\omega} i_{kd} \Delta \delta + \frac{x_{akq}}{\omega} p \Delta i_kq
\]

\[0 = - \frac{x_{aka}}{\omega} p \Delta i_q + r_kq i_{kq} + \frac{x_{akq}}{\omega} p \Delta i_kq\]

(3-2.5)

Torque equations and equations of motion:

\[
\Delta T_u = i_q \Delta v_d + \psi_d \Delta i_q - i_d \Delta v_q - \psi_q \Delta i_d
\]

\[
\Delta T_{in} - \Delta T_u = \frac{2H}{\omega} \Delta \delta^* + K_d \Delta \delta\quad (\Delta T_1 = 0)
\]

(3-2.6)

\[
\Delta T_{in} = \Delta P_{in} - \frac{T_{in}}{\omega} \Delta \delta
\]

\[
\Delta \delta = \Delta \delta
\]

Terminal voltage:

\[
\Delta v_t = \frac{v_d}{v_t} \Delta v_d + \frac{v_q}{v_t} \Delta v_q
\]

(3-2.7)

(b) Transmission system

\[
\Delta v_d = \Delta v_{bd} + r_t i_d + \frac{x_t}{\omega} \Delta \delta - \frac{x_t}{\omega} i_d \Delta \delta
\]

\[
\Delta v_q = \Delta v_{bq} + r_t i_q + \frac{x_t}{\omega} \Delta \delta + \frac{x_t}{\omega} i_d \Delta \delta
\]

\[
\Delta v_{bd} = V \cos \delta \Delta \delta + \Delta V \sin \delta
\]

\[
\Delta v_{bq} = \Delta V \cos \delta - V \sin \delta \Delta \delta
\]

(3-2.8)
(c) Automatic voltage regulator

\[ \Delta v_2 = \frac{G_{m1}}{1 + p T_{m1}} \left( \Delta u_r + \Delta v_{xs} + \Delta v_{ms} \right) \quad (\Delta K_{m1} = 0) \]

\[ \Delta v_x = \frac{G_{m2}}{1 + p T_{m2}} \Delta v_{m2} \quad (\Delta K_{m2} = 0) \]

\[ \Delta v_f = \frac{G_x}{1 + p T_x} \Delta v_x \quad (\Delta K_x = 0) \]  

\[ \Delta v_{ms} = \frac{-G_{ms} p T_{ms}}{1 + p T_{ms}} \Delta v_x \]

\[ \Delta v_{xs} = \frac{-G_{xs} T_{xs} p T_{xs}}{1 + p T_{xs}} \Delta v_f \]

\[ \Delta u_r = -G_e G_t \Delta v_t = -G_v s \Delta v_t \quad (\Delta v_r = 0) \]

(a) Governor and turbine

\[ \Delta Y = \frac{G_2}{(1 + p T_1)(1 + p T_2)} \Delta u_t \quad (\Delta K_t = 0) \]

\[ \frac{d}{dt} \Delta Y = \Delta Y \]

\[ \Delta P_s = G_3 \Delta Y \]

\[ \Delta P_{in} = \frac{1}{1 + p T_3} \Delta P_s \]

\[ \Delta u_t = -G_p \Delta \delta \quad (\Delta Y_0 = 0) \]
3-3 MATRIX REPRESENTATION OF EQUATIONS

Matrix representation provides a very efficient method for arranging a system of equations in compact form. When in state-space form, the eigenvalues of the system matrix provide a direct insight into the stability of the system. For the linearized equations in the previous Section, the matrix method is convenient for the inclusion of prime-mover and regulator representations into the generator and tie-line system for the evaluation of the overall performance of the system.

3-3.1 Generator and tie-line

In order to manipulate the linearized equations into a set of 1st-order differential equations and associated algebraic relationships, suitable for direct computer solution, they are first rewritten in the matrix form

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{z}_1(t)
\end{bmatrix} = P \begin{bmatrix}
x_1(t) \\
z_1(t)
\end{bmatrix} + Q x_1(t) + R u_1(t) \quad (3-3.1)
\]

where
### MATRIX $P$ (19×19)

$$
\begin{array}{cccccccccccccccccccc}
1 & & & & & & & & & & & & & & & & & & & & \hline
\omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n \\
\hline
x_1 & x_2 & \ldots & x_9 & x_{10} & \ldots & x_{19} & \ldots & x_{19} & x_{19} & \ldots & x_{19} & \ldots & x_{19} & x_{19} & \ldots & x_{19} & \ldots & x_{19} & \ldots & x_{19} \\
\hline
\omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n \\
\hline
\end{array}
$$

### MATRIX $Q$ (19×7)

$$
\begin{array}{cccccccc}
1 & & & & & & \\
\hline
-(2H + m_0K_x) & x_1 & x_2 & \ldots & x_{19} & x_{19} & \ldots & x_{19} \\
\hline
\omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n \\
\hline
\end{array}
$$

### MATRIX $R$ (19×3)

$$
\begin{array}{cccccccc}
1 & & & & & & \\
\hline
\omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n & \omega_n \\
\hline
\end{array}
$$
and the vectors $x_1$, $u_1$ and $Z_1$ are given by

$$
x_1 = \begin{bmatrix}
\Delta i_d & \Delta i_q & \Delta i_f & \Delta i_{kd} & \Delta i_{kq}
\end{bmatrix}^T,
$$

$$
u_1 = \begin{bmatrix}
\Delta v_f & \Delta P_{in} & \Delta V
\end{bmatrix}^T,
$$

$$
Z_1 = \begin{bmatrix}
\Delta v_d & \Delta v_q & \Delta v_{bd} & \Delta v_{bq} & \Delta P_0 & \Delta v_t & \Delta T_u \\
\Delta \psi_d & \Delta \psi_q & \Delta \psi_f & \Delta \psi_{kd} & \Delta \psi_{kq}
\end{bmatrix}^T,
$$

and $x_1$ is the derivative of $x_1$ with respect to time. Equation (3.3.1) is then pre-multiplied by the inverse of matrix $P$ to give

$$
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{Z}_1(t)
\end{bmatrix} = P^{-1}Qx_1(t) + P^{-1}Ru_1(t)
$$

$$
= Fx_1(t) + Gu_1(t) \tag{3.3.2}
$$

where $F = P^{-1}Q$ and $G = P^{-1}R$ (for details of the elements of matrices $P^{-1}$, $F$ and $G$, see Appendix II). After conformable partitioning, the equations of the generator and tie-line can be written in state-space form as

$$
x_1(t) = A_1x_1(t) + B_1u_1(t) \tag{3.3.3}
$$

$$
Z_1(t) = C_1x_1(t) + D_1u_1(t) \tag{3.3.4}
$$

The set of state-variables selected for the vector $x_1$ comprises axis currents, rather than flux linkages as in [26]. This choice seems to be a natural one when machines are interconnected through a transmission network, and the performance may be more readily visualized in terms of voltages and currents than in terms of flux linkages.
The inclusion of the vector $Z_1$ in equation (3-3.1), and finally in equation (3-3.4), enables important quantities such as $\Delta P_o$ and $\Delta v_t$ to be expressed in terms of the state- and control variables. This process is necessary when a voltage regulator is employed in a closed-loop system, with $\Delta v_t$ as the feedback signal, or a stabilizing signal derived from $\Delta P_o$ is required.

3-3.2 Addition of the prime-mover

An unified approach to the steady-state stability problem for the generator and tie-line system has been shown to result from the state-space form of equation (3-3.3). Further advantages of this form of representation become apparent when subsystems representing the prime-mover or the exciter are added to the basic machine representation. For variations about a specified operating point, the equations of the prime-mover can be expressed in matrix form as

$$x_2(t) = A_2 x_2(t) + B_2 u_2(t)$$  \hspace{2cm} (3-3.5)

where
To combine equation (3-3.5), which represents the governing system, with equations (3-3.3) and (3-3.4), which represent the generator and the tie-line system, matrices $B_1$ and $D_1$ of these latter equations are partitioned so that

$$ x_1 = A_1 x_1 + \begin{bmatrix} B_{11} & B_{12} & B_{13} \end{bmatrix} \begin{bmatrix} \Delta V_f \\ \Delta P_{in} \\ \Delta V \end{bmatrix} $$  \hspace{1cm} (3-3.6)$$

$$ z_1 = C_1 x_1 + \begin{bmatrix} D_{11} & D_{12} & D_{13} \end{bmatrix} \begin{bmatrix} \Delta V_f \\ \Delta P_{in} \\ \Delta V \end{bmatrix} $$  \hspace{1cm} (3-3.7)$$

where the vector of system inputs $u_1$ has been expressed explicitly in terms of its elements. The combination of equation (3-3.5) with equations (3-3.6) and (3-3.7) gives
where $M_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$.

Equations (3-3.8) and (3-3.9) represent the generator, tie-line and prime-mover equations in state-space form, and as such constitute a basic model to which various excitation systems can be added for comparison and evaluation.

**3-3.3 Addition of regulator and exciter system**

For small variations about a specified operating condition, the linearized equations of the regulator and exciter can be written in state-space form as

$$\dot{x}_3(t) = A_3 x_3(t) + B_3 u_3(t)$$  \hspace{1cm} (3-3.10)

where
The combination of equation (3-3.10) with equations (3-3.8) and (3-3.9) gives

\[
\begin{bmatrix}
\Delta v_f & \Delta v_{m2} & \Delta v_x & \Delta v_{ms} & \Delta v_{xs}
\end{bmatrix}
\]

and \( u_3 = \Delta u_r \)

The combination of equation (3-3.10) with equations (3-3.8) and (3-3.9) gives

\[
\begin{bmatrix}
\Delta u_r \\
\Delta u_t \\
\Delta v
\end{bmatrix} = \begin{bmatrix}
A_1 & B_{12}M_1 & B_{11}M_2 \\
0 & A_2 & 0 \\
0 & 0 & A_3
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
0 & 0 & 0 \\
0 & B_{21} & 0 \\
B_3 & 0 & 0
\end{bmatrix} \begin{bmatrix}
\Delta u_r \\
\Delta u_t \\
\Delta v
\end{bmatrix} \tag{3-3.11}
\]
where $M_2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}$.

Equations (3-3.11) and (3-3.12) represent the open-loop system, except for the inclusion of the amplifier-stabilizer and exciter-stabilizer loops in the excitation system.

3-3.4 Closed-loop system

For the system considered, it may conveniently be assumed without loss of generality that no changes occur in the infinite bus voltage, i.e. $AV = 0$ for the purpose of governor and regulator adjustments.

The steady-state stability boundaries of the closed-loop system can be studied by rearranging the system representation of equations (3-3.11) and (3-3.12). First the open-loop form is written more concisely as

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = Hx + Eu$$

(3-3.13)

$$Z_1 = Cx$$

(3-3.14)

where

$$Z_1 = \begin{bmatrix} C_1 & D_{12} & M_2 & D_{11} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ D_{13} \end{bmatrix} \begin{bmatrix} \Delta u_r \\ \Delta u_t \\ \Delta V \end{bmatrix}$$

(3-3.12)
With a signal proportional to $\Delta \delta$ fed back to $\Delta u_t$ and a signal proportional to $\Delta \nu_t$ fed back to $\Delta u_x$, then, in matrix form it follows that

$$u = K \begin{bmatrix} x \\ z \end{bmatrix} = K \begin{bmatrix} I \\ C \end{bmatrix} x$$

(3-3.15)

where $K$ is given by

$$K = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & k_{1,21} & 0 & \ldots & 0 \\ 0 & k_{22} & 0 & \ldots & 0 & \ldots & 0 \end{bmatrix}$$

with $K$ an $(2 \times 27)$ matrix and $I$ an $(15 \times 15)$ unit matrix.

Substitution of equation (3-3.15) into equation (3-3.13) gives the closed-loop state-space form

$$x = \left\{ H + BK \begin{bmatrix} I \\ C \end{bmatrix} \right\} x$$

(3-3.16)

where $A = \left\{ H + BK \begin{bmatrix} I \\ C \end{bmatrix} \right\}$

and
-

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G(2,2)

F(2,2)

F(2,l)

F(2,4)

F(2,5)

F(2,6)

F( 2,7)

F(j,1)

F(J,2)

F(l,l)

F(J ,4)

F(j,5)

FO ,6)

F(l,7)

F( 4,1)

F( 4,2)

F(4,l)

F(4,4)

F(4,5)

F( 4,6)

F(4,7)

F(5,l)

F(5,2)

F(5,l)

F(5,4)

F(5,5)

F(5,6)

F(5,7)

G(5,l)

F(6,l)

F(6,2)

F(6,l)

F(6,4)

F(6,5)

F(6,6)

F(6,7)

G(6,l)

F(7 , 1 )

F(7,2)

F(7,l)

F(7,4)

F(7,5)

F(7,6)

F(7,7)

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GO ,1)

:'i

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- Tl

A =
k

G2
22 T1T2

T +T
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T)

- T1T2

1
- T1T2

1
Cx

1
x

-T
GaF( 1),1)

Ca F(ll,2)

GaF(ll,)

Ga F(ll,5)

GaF( ll,4)

Ga F(ll,6)

G F(ll,7)
a

Ga G(l),l)

Tx

Gm2
Pm2
Cms Gm2

-~
Gxs

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C

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Gm1
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1
- Tm1

Cm1
Pml

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T rn2

rn,

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Pm2
Cxs Gx

--P-

x

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rn,

--;f-

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During steady-state and dynamic stability investigations, only small deviations from the steady-state are considered, and relations between these small deviations of the variables involved (i.e. the perturbation relations) are sufficient for the derivation of stability criteria. In order to compute the stability borderline, it is necessary to choose different steady operating points and thus equations for calculating the steady-state quantities from given parameters are essential.

During normal steady operation, the speed of the machine is the constant synchronous speed \( \omega = \omega_0 \). The field voltage and current are constant, the damper currents are zero, and the armature phase voltages and currents are balanced 3-phase quantities. The axis voltages and currents are thus all constant and independent of time. When the nonlinear equations of the generator and tie-line are simplified by putting \( p = 0 \) and \( i_{kd} = i_{kq} = 0 \), the resultant steady-state equations obtained are

\[
\begin{align*}
\psi_f &= -x_{af}i_d + x_{ff}i_f \\
\psi_d &= x_d i_d + x_{af}i_f \\
\psi_{kd} &= x_{kd} i_d + x_{rkd}i_f \\
\psi_q &= x_q i_q \\
\psi_{kq} &= x_{akq} i_q \\
v_f &= r_{f}i_f \\
v_d &= -ri_d + x_q i_q \\
v_q &= -x_d i_d - ri_q + x_{af}i_f
\end{align*}
\]
A steady-state phasor diagram relating the voltages and currents of the system is shown in figure (3-4.1). From this figure,

\[ i_d = i_a \sin(\phi + \theta) \]  

(3-4.13)
and \( i_q = i_a \cos(\theta + \phi) \)  

For given values of \( i_a \) and \( \phi \), the steady-state value of \( \theta \) can be obtained by substituting equations (3-4.13) and (3-4.14) into equations (3-4.7) and (3-4.9) with the result that:

\[
\theta = \tan^{-1} \left\{ \frac{-i_a (r \sin \phi - x_q \cos \phi + r_t \sin \phi - x_t \cos \phi)}{r_i \cos \phi + x_q i_a \sin \phi + V + r_t i_a \cos \phi + x_t i_a \sin \phi} \right\} 
\]

From hereon, the equations for calculating the steady-state quantities at any operating point (i.e. at different \( i_a, \phi, x_t \) etc.) are readily written down as:

\[
i_d = i_a \sin(\theta + \phi) \\
i_q = i_a \cos(\theta + \phi) \\
v_d = -r_i \frac{i_d}{x_d} + x_q \frac{i_q}{x_q} \\
v_q = V \cos \theta + x_t \frac{i_q}{x_t} + x_t \frac{i_d}{x_d} \\
v_t = \sqrt{v_d^2 + v_q^2} \\
if = \frac{v_a + x_d \frac{i_d}{x_d} + r_i \frac{i_q}{x_q}}{x_{af}} \\
\psi_d = -x_d \frac{i_d}{x_d} + x_{af} \frac{i_f}{x_f} \\
\psi_q = -x_q \frac{i_q}{x_q} \\
T_u = \psi_d \frac{i_q}{x_d} - \psi_q \frac{i_d}{x_q} \\
T_{in} = T_u + T_l \\
i_{kd} = 0 \\
i_{kq} = 0
CHAPTER 4

STABILITY STUDIES I

This Chapter presents numerical methods of verifying the validity of the matrices representing the dynamics of the three individual systems detailed in Chapter 3. Stability borderlines as portrayed in P-Q charts (charts plotted with real against reactive output powers), tables of computed eigenvalues, together with time responses corresponding to step increases in real power, are employed to study the performances of both open-loop (i.e. a manually controlled generator and tie-line system) and closed-loop systems. A brief study on the effect of the regulator gain on the closed-loop system damping is also given.

4-1 SYSTEM MATRIX VERIFICATION

Any equations or mathematical models representing the dynamics of real systems should be verified so that confidence in the validity and accuracy of the equations or models can be gained. Equation(3-3.16), representing the closed-loop dynamics of the overall linearized system in Chapter 3, can be verified either by comparison with the behaviour of an actual experimental set or, less favourably, by comparison with theoretical results in any publications in which an identical system has been considered. However, since neither of the above methods of verification was available, a numerical method of model verification was considered, based on the fact that the eigenvalues of any system matrix are the
roots of the characteristic equation of that system.

The closed-loop system matrix $A$ in equation (3-3.16) can be converted into an open-loop matrix simply by putting $k_{11}, k_{21}, k_{22} = 0$, which effectively breaks the links between the three subsystems. Using the parameter values of the individual systems as detailed in Appendix III, the computed eigenvalues of the open-loop matrix, which consists effectively of three unrelated submatrices, are as shown in table (4-1.1). Since the governor is composed of linearly cascaded blocks, its transfer function can readily be written down from equation (3-2.10) as

$$\Delta P_{in} = \frac{-G_1 G_2 G_3}{(1+pT_1)(1+pT_2)(1+pT_3)} p \Delta \delta \quad (4-1.1)$$

The characteristic equation of equation (4-1.1) is

$$(1+pT_1)(1+pT_2)(1+pT_3) = 0 \quad (4-1.2)$$

The fact that this characteristic equation is satisfied by putting successively the computed eigenvalues of the governor in table (4-1.1) in places of the operator $p$ confirms the $(3 \times 3)$ submatrix as a correct representation of the governor.

To check the validity of the $(5 \times 5)$ submatrix representing the voltage regulator, its transfer function was first developed. Referring to equation (3-2.9), it can be shown that

$$\frac{1+pT_m}{G_m} \frac{1+pT_x}{G_x} \Delta V_f = \frac{G_m}{1+pT_m} (\Delta u_r - \frac{G_{xs} T_{xs}}{1+pT_{ms}} \Delta V_f - \frac{G_{ms} T_{ms}}{1+pT_m} \frac{1+pT_m}{G_x} \Delta V_f)$$

i.e.

$$\frac{\Delta V_f}{\Delta V_r} = \frac{G_m}{1+pT_m} \frac{(1+pT_m)}{G_m G_x} + \frac{G_m G_{xs} T_{xs} p}{(1+pT_m)(1+pT_{ms})} + \frac{G_m G_{ms} T_{ms} p (1+pT_x)}{G_x (1+pT_m)(1+pT_{ms})} \quad (4-1.3)$$
\[ (1+pT_m)(1+pT_2)(1+pT_x)(1+pT_{ms})(1+pT_{xs}) + G_{m1}G_mG_xG_{xs}T_{ms}p(1+pT_{ms}) \\
+ G_{m1}G_mG_{ms}T_{ms}p(1+pT_x)(1+pT_{xs}) = 0 \]  

(4-1.4)

The fact that this characteristic equation is satisfied by putting successively the computed eigenvalues of the regulator in table (4-1.1) in places of the operator \( p \) confirms the \((5 \times 5)\) submatrix as a correct representation of the regulator.

To verify the validity of the \((7 \times 7)\) submatrix representing
the generator and tie-line system, its characteristic equation should also be derived from its basic equations. However, the characteristic equation of a power system represented by a \((7 \times 7)\) matrix is extremely complicated.\(^{27}\) To resolve this problem, and to prevent any interactions between the mechanical and electrical systems, only the five current state-variables \((\Delta i_d, \Delta i_q, \Delta i_f, \Delta i_{kd}, \text{and} \Delta i_{kq})\) were considered initially. The resultant matrix, having dimension \((5 \times 5)\) and comprising only constant elements, is independent on the loading of the generator. Under this arrangement, the reciprocals of the real parts of the computed eigenvalues (two complex and three real) are then the short-circuit time-constants of the system, namely \(T_a, T_d', T_d''\) and \(T_q'^{27,29}\). Table(4-1.2) shows the computed values of these time-constants as well as those values calculated algebraically from the equations given in \([23]\). The close agreement that can be seen serves to verify the validity of the \((5 \times 5)\) matrix. The fact that the computed eigenvalues consist of a 50 Hz oscillatory term, which is the operating frequency of the generator, further supports this claim. The computed eigenvalues of the complete \((7 \times 7)\) submatrix, formed by inclusion of the state-variables \(\Delta \delta\) and \(\Delta \delta\), are shown in table(4-1.1). Comparison of tables(4-1.1) and (4-1.2) shows that there are significant effects on some of the short-circuit time-constants, due to interactions between the mechanical and electrical systems. The extra pair of complex eigenvalues represents rotor hunting at a frequency of 1.22 Hz, which lies within the usual hunting frequency range from 0.2 to 2.0 Hz\(^5\) for most generator and transmission systems. The validity of the \((7 \times 7)\) submatrix
<table>
<thead>
<tr>
<th>Computed eigenvalues</th>
<th>Corresponding time-constants and frequencies of oscillation (s) (Hz)</th>
<th>Calculated time-constants by the approximate equations in [23] (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.32194 ± 3314.13244</td>
<td>( T_a = 0.7564 ) 50.0</td>
<td>( T_a = 0.7557 )</td>
</tr>
<tr>
<td>-29.21945</td>
<td>( T_d'' = 0.0342 )</td>
<td>( T_d'' = 0.0357 )</td>
</tr>
<tr>
<td>-0.64869</td>
<td>( T_d' = 1.5415 )</td>
<td>( T_d' = 1.4750 )</td>
</tr>
<tr>
<td>-9.92190</td>
<td>( T_q'' = 0.1007 )</td>
<td>( T_q'' = 0.1007 )</td>
</tr>
</tbody>
</table>

Table (4-1.2) Comparisons of computed and algebraically calculated time-constants at \( x_t = 0.3 \) p.u.

representing the generator and tie-line was thus established by the combination of a few precise indications.

4-2 OPEN-LOOP STABILITY

The operating limits of synchronous generators are of general importance to power system planning and operating engineers. The limits for lagging power factors have been of particular interest in the past, when synchronous generators were operated mainly with lagging power factors, i.e. in the overexcited region.

More recent developments in power systems are, however, leading to the position where many synchronous generators have to be operated at unity power factor or even with a leading power factor, i.e. in the underexcited operating region. This arises mainly from the need for sources of negative MVAR for suitable control of power flow and voltage levels in large modern systems, as, for example, in the case of long distance transmission and
underground cable distribution systems. At unity and leading power factors the steady-state stability limit can become very critical, and it is now equally important to look at the stability of the power system in both the overexcited and the underexcited regions.

A set of plots of real against reactive output power, which effectively delineates the steady-state stability limits for the open-loop system (i.e. the system represented by equation (3-3.3)) are shown in figures (4-2.1) to (4-2.3). Figure (4-2.4) shows the time responses of the same system operating at rated output power \(0.8 + j0.6\ \text{p.u.}\), when it is subjected to a 5% step increase in real power. Together with the information given by the computed eigenvalues listed in table (4-2.1), the effects of a higher \(x_t\) are therefore to reduce system damping, to require a greater field excitation if the same active and reactive output power is to be maintained, and to give poorer voltage regulation when there are load changes in the system (except for \(x_t = 0\) when the generator is connected directly to the infinite bus of fixed voltage).

It is worth noting that instability of this manually controlled system is represented by undamped unidirectional increases of the state-variables, i.e. in the unstable operating regions of figures (4-2.1) to (4-2.3), the critical eigenvalue is real and positive.
<table>
<thead>
<tr>
<th>$x_t=0.0\ p.u.$</th>
<th>$x_t=0.1\ p.u.$</th>
<th>$x_t=0.3\ p.u.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3.58 \pm j313.96$</td>
<td>$-2.28 \pm j314.08$</td>
<td>$-1.32 \pm j314.13$</td>
</tr>
<tr>
<td>$-2.94 \pm j8.96$</td>
<td>$-2.19 \pm j8.46$</td>
<td>$-1.33 \pm j7.66$</td>
</tr>
<tr>
<td>$-36.85$</td>
<td>$-32.32$</td>
<td>$-29.10$</td>
</tr>
<tr>
<td>$-18.37$</td>
<td>$-11.89$</td>
<td>$-7.66$</td>
</tr>
<tr>
<td>$-0.569$</td>
<td>$-0.508$</td>
<td>$-0.427$</td>
</tr>
</tbody>
</table>

Table (4-2.1) Computed eigenvalues of the open-loop system at rated output power. Underlined are the critical eigenvalues for figures (4-2.1) to (4-2.3)
Figure (4-2.1) Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.0$ p.u.
Figure 4-2.2 Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.1 \text{ p.u.}$

Critical eigenvalues:

- $a = 0.0$
- $b = -0.4$
- $c = -0.6$
- $d = -0.8$
Figure(4-2.3) Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.3$ p.u..
Figure (4-2.4) Time response of the open-loop system, operating initially at rated output power, when subjected to a 5% step increase in real power.
4-3 CLOSED-LOOP STABILITY

The stability boundaries for the closed-loop controlled system represented by equation (3-3.16) are shown in figure (4-3.1). When \( x_t = 0 \), there can be no terminal voltage variation and the voltage regulator is therefore inoperative. Under this condition, the only control device in operation is the governor. However, comparison between the computed eigenvalues in the first columns of both tables (4-2.1) and (4-3.1) shows that the governor has little effect on the eigenvalues of the generator and tie-line system (the addition of the governor changes the eigenvalues of the open-loop system only from \(-2.94 \pm j8.96\) to \(-2.93 \pm j8.95\) and from \(-0.569 \) to \(-0.56\)). Due to the insignificant effect of the governor, and the ineffective voltage regulator, the stability boundary for \( x_t = 0 \) is very much the same as that of the open-loop system. The stability limits for \( x_t = 0.1 \) and 0.2 p.u. show a reduction in the stable region produced by the addition of the control devices. The frequency of oscillation in the unstable operating points along the two boundaries ranges from 1.9 to 0.2 Hz (in the direction of the arrow). Instability, therefore, is caused by the rotor hunting and finally falling out of synchronism with the infinite bus system. Further reductions in the stable operating region are demonstrated by the stability limits for \( x_t = 0.25 \) and 0.3 p.u.. These two limits consist of upper and lower boundaries, with the frequency of oscillation at the unstable operating points along the lower boundaries again ranging from 1.9 to 0.2 Hz but that along the upper boundaries being around 4.0 Hz. Referring to table (4-1.1), this latter frequency suggests that the instability in regions
above the upper boundaries is caused by the voltage regulator. At rated output power, the addition of the control devices greatly reduces the range of $x_t$ for stable operation (table(4-3.1)), but, as shown in figure(4-3.2), the improvement in the voltage regulation of the system is very dramatic. This point is also reiterated by the direct comparison of the responses of the open- and closed-loop systems given in figure(4-3.3).

<table>
<thead>
<tr>
<th>$x_t$=0.0 p.u.</th>
<th>$x_t$=0.1 p.u.</th>
<th>$x_t$=0.2 p.u.</th>
<th>$x_t$=0.25 p.u.</th>
<th>$x_t$=0.3 p.u.</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.58±1314.96</td>
<td>-2.28±1314.08</td>
<td>-1.67±1314.11</td>
<td>-1.47±1314.12</td>
<td>-1.32±1314.13</td>
</tr>
<tr>
<td>-2.93±139.95</td>
<td>-2.53±18.12</td>
<td>-1.99±18.46</td>
<td>-1.72±18.19</td>
<td>-1.49±17.97</td>
</tr>
<tr>
<td>-2.40±130.03</td>
<td>-2.29±27.37</td>
<td>-0.86±25.60</td>
<td>-0.13±25.08</td>
<td>0.50±24.74</td>
</tr>
<tr>
<td>-29.43</td>
<td>-5.12±50.37</td>
<td>-5.76±50.24</td>
<td>-5.76±50.20</td>
<td>-5.70±50.25</td>
</tr>
<tr>
<td>-11.96</td>
<td>-10.79</td>
<td>-10.04</td>
<td>-10.01</td>
<td>-9.99</td>
</tr>
<tr>
<td>-5.37</td>
<td>-2.01</td>
<td>-2.01</td>
<td>-2.01</td>
<td>-2.01</td>
</tr>
<tr>
<td>-2.05</td>
<td>-0.49</td>
<td>-0.50</td>
<td>-0.50</td>
<td>-0.50</td>
</tr>
<tr>
<td>-0.56</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-0.017</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table(4-3.1) Computed eigenvalues of the closed-loop system at rated output power and with $G_m1 = 52.0$. Underlined are the critical eigenvalues.

Figure(4-3.4) shows the stability borderline with the regulator gain ($G_m1$) plotted against $x_t$. It is obvious that an increase in $G_m1$ can greatly increase the range of $x_t$ for stable operation. Detailed studies of this figure give figure(4-3.5), which shows the loci of the critical eigenvalues as well as the eigenvalues associated with the mechanical damping for typical
values of $x_t$. It can be seen that, for all $x_t$ considered, an increase in $G_{m1}$ shifts the critical eigenvalues towards the left of the complex $s$-plane to an apparently optimal region ($130 \leq G_{m1} \leq 160$), and has a relatively small effect on the eigenvalues associated with the mechanical damping. This effect is even smaller when $x_t$ is higher. Figure(4-3.6), for $G_{m1} = 150.0$, shows that because a higher $G_{m1}$ has a relatively small effect on the mechanical damping (compare the second pairs of complex eigenvalues in tables(4-3.1) and (4-3.2)), the stability borderline for $x_t = 0.1$ p.u. is very much the same as that in figure(4-3.1). At this higher $G_{m1}$ however, the upper boundary previously present in figure(4-3.1) for the stability borderline of $x_t = 0.25$ p.u. has been removed, and there are no dramatic reductions in the stable regions for higher $x_t$ ($x_t = 0.5$ and 0.8 p.u.). Figure(4-3.7), gives the step responses of the closed-loop system with $G_{m1} = 150.0$, and shows that at $x_t = 0.25$ p.u., the 4.0 Hz oscillation previously seen in figure(4-3.2) has disappeared. However, owing to the decrease in the mechanical damping of the closed-loop system for higher $x_t$ (refer to table (4-3.2)), the system is, even when stable, quite oscillatory with a frequency of oscillation around 1.1 Hz.
<table>
<thead>
<tr>
<th>$x_t = 0.1 \text{ p.u.}$</th>
<th>$x_t = 0.25 \text{ p.u.}$</th>
<th>$x_t = 0.3 \text{ p.u.}$</th>
<th>$x_t = 0.5 \text{ p.u.}$</th>
<th>$x_t = 0.8 \text{ p.u.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2.28 \pm 314.08$</td>
<td>$-2.31 \pm 314.08$</td>
<td>$-2.16 \pm 317.26$</td>
<td>$-15.48 \pm 311.22$</td>
<td>$-5.12 \pm 30.36$</td>
</tr>
<tr>
<td>$-2.77 \pm 314.12$</td>
<td>$-1.70 \pm 314.18$</td>
<td>$-2.16 \pm 314.47$</td>
<td>$-11.62 \pm 317.90$</td>
<td>$-5.76 \pm 30.20$</td>
</tr>
<tr>
<td>$-2.92 \pm 314.13$</td>
<td>$-1.48 \pm 317.96$</td>
<td>$-11.32 \pm 319.15$</td>
<td>$-5.70 \pm 30.25$</td>
<td>$-42.02$</td>
</tr>
<tr>
<td>$-1.33 \pm 314.14$</td>
<td>$-0.92 \pm 317.29$</td>
<td>$-10.02 \pm 322.35$</td>
<td>$-5.37 \pm 30.34$</td>
<td>$-42.48$</td>
</tr>
<tr>
<td>$-0.64 \pm 314.15$</td>
<td>$-0.52 \pm 316.65$</td>
<td>$-0.02 \pm 524.81$</td>
<td>$-5.25$</td>
<td>$-5.25$</td>
</tr>
</tbody>
</table>

**Table (4-3.2)** Computed eigenvalues of the closed-loop system at rated output power and with $G_m = 150.0$. Underlined are the eigenvalues associated with mechanical damping.
Figure 4-3.1 Dynamic stability limits of the closed-loop system, plotted on real against reactive output power coordinates.
Figure 4-3.2) Time response of the closed-loop system, operating initially at rated output power, when subjected to a 5% step increase in real power.
Figure (4-3.3) Time response of the open- and closed-loop systems, operating initially at rated output power and \( x_p = 0.1 \text{ p.u.} \), when subjected to a 5% step increase in real power.
Figure 4-3.4 Effect of regulator gain $G_{m1}$ on the range of $x_t$ for stable operation of the closed-loop system operating at rated output power.
region of optimal damping

imaginary part (rad./s)

real part (s⁻¹)

eigenvalues associated with mechanical damping

\begin{align*}
\text{critical eigenvalues} & \quad \text{eigenvalues associated with mechanical damping} \\
& \begin{cases}
\text{a} & x_t = 0.2 \text{ p.u.} \\
\text{b} & x_t = 0.3 \text{ p.u.} \\
\text{c} & x_t = 0.4 \text{ p.u.} \\
\text{d} & x_t = 0.2 \text{ p.u.} \\
\text{e} & x_t = 0.3 \text{ p.u.} \\
\text{f} & x_t = 0.4 \text{ p.u.}
\end{cases}
\end{align*}

Figure (4-3.5) Tracking of the critical eigenvalues and the eigenvalues associated with mechanical damping as a function of $G_m$, for the closed-loop system operating at rated output power. Values of $G_m$ are shown along the curves.
Figure (4.3.6) Dynamic stability limits of the closed-loop system with $G_m = 150.0$, plotted on real against reactive output power coordinates.
Time response of the closed-loop system operating initially at rated output power and with $G_m = 150.0$, when subjected to a 5% step increase in real power.
CHAPTER 5

STABILITY STUDIES II

While retaining the same mathematical models for the generator, tie-line and governor system, as given in Chapter 3, this Chapter looks at an overall model which includes an automatic voltage regulator typical of a more recent design. The parameter of the governor are the same as in Chapter 4, but those for the generator correspond to a machine of a different rating (for details, see Appendix III). As before, the closed-loop equations for the overall system are constructed in state-space form. Stability borderlines as portrayed in P-Q charts, tables of computed eigenvalues, together with time responses corresponding to a step increase in real power, are all employed to study the performances of the system in both open- and closed-loop operations. The effects introduced by variations in the tie-line reactance and in some of the generator parameters on the mechanical damping and the hunting frequency of both systems are also investigated.

5-1 AUTOMATIC VOLTAGE REGULATOR (MODEL AND EQUATIONS)

Figure(5-1.1) Block diagram of a voltage regulator
Figure (5-1.1) shows a block diagram of a modern regulator which in practice differs from the older one considered in Chapters 3 and 4 in that it uses solid state amplifiers and thyristors. Because of the low inherent time-constants of these devices, very much less stabilization by feedback is required. In fact, in this modern regulator, stabilization is achieved by phase advance circuit and phase delay circuit (mixer) acting as lead-lag compensator. The equations (in volts, amperes and seconds) for the elements of this regulator are:

Transformer and filter:

\[ v_z = \frac{G_z v_z}{1+pT_z} v_t \]  \hspace{1cm} (5-1.1)

Phase advance:

\[ v_a = \frac{G_a (1+pT_{a1})}{1+pT_{a2}} u_r \]  \hspace{1cm} (5-1.2)

Mixer:

\[ v_m = \frac{G_m (1+pT_{m1})}{1+pT_{m2}} v_a \]  \hspace{1cm} (5-1.3)

Output:

\[ v_o = \frac{G_o}{1+pT_o} v_m \]  \hspace{1cm} (5-1.4)

Converter:

\[ v_c = \frac{G_c}{1+pT_c} v_o \]  \hspace{1cm} (5-1.5)

Exciter:

\[ v_f = \frac{G_x}{1+pT_x} v_c \]  \hspace{1cm} (5-1.6)
Comparator:

\[ u_r = v_r - v_z \]  \hspace{1cm} (5-1.7)

The linearized forms of these equations are:

From equation (5-1.1),

\[ \Delta v_z = - \frac{\Delta v_z}{T_z} + \frac{G_v G_z}{T_z} \Delta v_t \]  \hspace{1cm} (5-1.8)

From equation (5-1.4),

\[ \Delta v_o = - \frac{\Delta v_o}{T_o} + \frac{G_o}{T_o} \Delta v_m \]  \hspace{1cm} (5-1.9)

From equation (5-1.5),

\[ \Delta v_c = - \frac{\Delta v_c}{T_c} + \frac{G_c}{T_c} \Delta v_o \]  \hspace{1cm} (5-1.10)

From equation (5-1.6),

\[ \Delta v_x = - \frac{\Delta v_x}{T_x} + \frac{G_x}{T_x} \Delta v_c \]  \hspace{1cm} (5-1.11)

From equations (5-1.1), (5-1.2) and (5-1.7),

\[ \Delta v_a = - \frac{\Delta v_a}{T_{a2}} + \frac{G_a}{T_{a2}} \frac{T_{a1}}{T_z} - 1) \Delta v_z - \frac{G_v G_z T_{a1}}{T_{a2} T_z} \Delta v_t \]  \hspace{1cm} (5-1.12)

From equation (5-1.2) and (5-1.3),

\[ \Delta v_m = - \frac{\Delta v_m}{T_{m2}} + \frac{G_m}{T_{m2}} (1 - \frac{T_{m1}}{T_{a2}}) \Delta v_a + \frac{G_G G_z T_{m1} T_{a1}}{T_{m2} T_{a2} T_z} \Delta v_t \]  \hspace{1cm} (5-1.13)

From equation (5-1.7),

\[ \Delta u_r = - \Delta v_z \hspace{1cm} (\Delta v_r = 0) \]  \hspace{1cm} (5-1.14)
When arranged in state-space form, these linearized equations become

\[ x_3 = A_3 x_3 + B_3 u_3 \]  

(5-1.15)

where

\[
A_3 = \begin{bmatrix}
-\frac{1}{T_x} & 0 & 0 & \frac{G_x}{T_x} & 0 & 0 \\
0 & -\frac{1}{T_z} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{T_o} & 0 & 0 & \frac{G_o}{T_o} \\
0 & 0 & \frac{G_s}{T_c} & -\frac{1}{T_c} & 0 & 0 \\
0 & \frac{G_a}{T_a} (\frac{T a_1}{T a_2 T_z} - 1) & 0 & 0 & -\frac{1}{T a_2} & 0 \\
0 & \frac{G_G T}{T m_1 (T a_1 T m_2 T a_2 T_z^2)} & 0 & 0 & \frac{G_m T}{T m_2 (1 - T m_1 T a_2)} & -\frac{1}{T m_2}
\end{bmatrix}
\]

\[
B_3 = \begin{bmatrix}
0 \\
\frac{G_v G_z}{T_z} \\
0 \\
-\frac{G_a v_z a_1}{T a_2 T z} \\
-\frac{G_G G_G T}{T m_1 T a_1} \\
-\frac{G_m a v_z m_1 T}{T m_2 T a_2 T z}
\end{bmatrix}
\]
The state-space form of the equations for a generator, tie-line and governing system has already been established in equation (3-3.8). With a feedback signal proportional to $\Delta \delta$ for the governing system, the combination of equations (3-3.8) and (5-1.15) gives a closed-loop state-space equation

$$x = Ax$$  \hspace{1cm} (5-2.1)

where

$$x = \left[ \begin{array}{cccccccc}
\Delta \delta & \dot{\Delta \delta} & \Delta i_d & \Delta i_q & \Delta i_f & \Delta i_{kd} & \Delta i_{kq} & \Delta P_{in} & \Delta V & \Delta Y \\
\Delta v_f & \Delta v_z & \Delta V_o & \Delta V_c & \Delta V_a & \Delta V_m
\end{array} \right]'$$

and

$$x_3 = \left[ \begin{array}{cccccccc}
\Delta v_f & \Delta v_z & \Delta v_o & \Delta v_c & \Delta v_a & \Delta v_m
\end{array} \right]'$$

and $u_3 = \Delta v_t$. 

5-2 CLOSED-LOOP SYSTEM EQUATION

The state-space form of the equations for a generator, tie-line and governing system has already been established in equation (3-3.8). With a feedback signal proportional to $\Delta \delta$ for the governing system, the combination of equations (3-3.8) and (5-1.15) gives a closed-loop state-space equation

$$x = Ax$$  \hspace{1cm} (5-2.1)

where

$$x = \left[ \begin{array}{cccccccc}
\Delta \delta & \dot{\Delta \delta} & \Delta i_d & \Delta i_q & \Delta i_f & \Delta i_{kd} & \Delta i_{kq} & \Delta P_{in} & \Delta V & \Delta Y \\
\Delta v_f & \Delta v_z & \Delta V_o & \Delta V_c & \Delta V_a & \Delta V_m
\end{array} \right]'$$

and
\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
 & F(2,2) & F(2,3) & F(2,4) & F(2,5) & F(2,6) & F(2,7) & O(2,2) \\
\hline
F(3,1) & F(3,2) & F(3,3) & F(3,4) & F(3,5) & F(3,6) & F(3,7) & O(3,1) \\
\hline
F(4,1) & F(4,2) & F(4,3) & F(4,4) & F(4,5) & F(4,6) & F(4,7) & O(4,1) \\
\hline
F(5,1) & F(5,2) & F(5,3) & F(5,4) & F(5,5) & F(5,6) & F(5,7) & O(5,1) \\
\hline
F(6,1) & F(6,2) & F(6,3) & F(6,4) & F(6,5) & F(6,6) & F(6,7) & O(6,1) \\
\hline
F(7,1) & F(7,2) & F(7,3) & F(7,4) & F(7,5) & F(7,6) & F(7,7) & O(7,1) \\
\hline
\end{array}
\]

\[
A = \begin{pmatrix}
\frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} & \frac{1}{T_2} \\
-\frac{1}{T_2} & -\frac{1}{T_2} & -\frac{1}{T_2} & -\frac{1}{T_2} & -\frac{1}{T_2} & -\frac{1}{T_2} \\
O_0 P(13,1) & O_0 P(13,2) & O_0 P(13,3) & O_0 P(13,4) & O_0 P(13,5) & O_0 P(13,6) & O_0 P(13,7) \\
O_0 P(13,1) & O_0 P(13,2) & O_0 P(13,3) & O_0 P(13,4) & O_0 P(13,5) & O_0 P(13,6) & O_0 P(13,7) \\
O_0 P(13,1) & O_0 P(13,2) & O_0 P(13,3) & O_0 P(13,4) & O_0 P(13,5) & O_0 P(13,6) & O_0 P(13,7) \\
O_0 P(13,1) & O_0 P(13,2) & O_0 P(13,3) & O_0 P(13,4) & O_0 P(13,5) & O_0 P(13,6) & O_0 P(13,7) \\
\end{pmatrix}
\]

with \( G_0 = \frac{G_0}{T_2} \), \( G_0 = \frac{G_0 G_0 P(13,1)}{T_2^2 T_2} \) and \( G_0 = \frac{G_0 G_0 G_0 P(13,1)}{T_2^2 T_2^2 T_2} \).
This equation forms the basis of all further studies in this thesis.

5-3 OPEN-LOOP STABILITY

A set of plots of real against reactive output power, which effectively delineates the steady-state stability limits for the open-loop system (i.e. the system represented by equation (3-3.3)) are shown in figures (5-3.1) to (5-3.3). Together with the information given by the computed eigenvalues listed in table (5-3.1), the effects of a higher $x_t$ are therefore to reduce system damping, and to require a greater field excitation if the same active and reactive output power is to be maintained. For a fixed field excitation, figure (5-3.4) shows $P_o$ plotted against $t$. It is obvious that a higher $x_t$ means inevitably that a lower maximum power can be delivered by the generator to the infinite bus.

Figure (5-3.5) shows the time responses of the system operating at rated output power ($0.85 + 0.526$ p.u.), when subjected to a 5% step increase in real power. It can be seen that a higher $x_t$ also gives a poorer voltage regulation when there are load changes in the system. Figures (5-3.6) to (5-3.9) show the trackings of the complex eigenvalues associated with the mechanical damping of the system as functions of $x_t$ and of ±20% variations from the base value of a certain generator parameter. These figures reveal that generally although the open-loop system remains always stable it becomes more oscillatory as $x_t$ increases, and that apart from the case in figure (5-3.6), the effects on the mechanical damping and hunting frequency of variations of some generator parameters ($x_q$, $x_d$, and $H$) also diminish as $x_t$ increases.
<table>
<thead>
<tr>
<th>$x_1=0.0 \text{ p.u.}$</th>
<th>$x_1=0.1 \text{ p.u.}$</th>
<th>$x_1=0.2 \text{ p.u.}$</th>
<th>$x_1=0.3 \text{ p.u.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3.74+j314.08$</td>
<td>$-2.67+j314.12$</td>
<td>$-2.08+j314.13$</td>
<td>$-1.70+j314.14$</td>
</tr>
<tr>
<td>$-2.05+j10.29$</td>
<td>$-1.40+j9.22$</td>
<td>$-1.04+j8.47$</td>
<td>$-0.81+j7.92$</td>
</tr>
<tr>
<td>$-31.97$</td>
<td>$-30.61$</td>
<td>$-29.86$</td>
<td>$-29.38$</td>
</tr>
<tr>
<td>$-5.46$</td>
<td>$-4.43$</td>
<td>$-3.86$</td>
<td>$-3.48$</td>
</tr>
<tr>
<td>$-0.339$</td>
<td>$-0.307$</td>
<td>$-0.283$</td>
<td>$-0.264$</td>
</tr>
</tbody>
</table>

Table (5-3.1) Computed eigenvalues of the open-loop system at rated output power. Underlined are the eigenvalues associated with mechanical damping.
Figure (5-3.1) Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.0$ p.u.
Figure (5-3.2) Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.1 \text{ p.u.}$.
Figure (5-3.3) Steady-state stability limit of the open-loop system, plotted on real against reactive output power coordinates with $x_t = 0.3 \text{ p.u.}$.
Figure(5.3.4) Power/angle characteristics of the open-loop system with constant field excitation $i_f = 1.0$ p.u.
(a) $x_t = 0.0$ p.u.  (b) $x_t = 0.1$ p.u.
(c) $x_t = 0.3$ p.u.
Figure (5-3.5) Time response of the open-loop system, operating initially at rated output power, when subjected to a 5% step increase in real power.
a, b and c are coincident

Figure (5-3.6) Tracking of the eigenvalues associated with the mechanical damping of the open-loop system operating at rated output power.
Figure 5-3.7. Tracking of the eigenvalues associated with the mechanical damping of the open-loop system operating at rated output power.
Figure 5-3.8 Tracking of the eigenvalues associated with the mechanical damping of the open-loop system operating at rated output power.
Figure 5-3.9 Tracking of the eigenvalues associated with the mechanical damping of the open-loop system operating at rated output power.
The stability boundaries for the closed-loop controlled system represented by equation (5-2.1) are shown in figure (5-4.1). The addition of control can be seen to provide stable operating regions which are otherwise unstable for open-loop operation, but to reduce the range of $x_t$ for stable operation as well as the mechanical damping, when comparisons are made between tables (5-3.1) and (5-4.1). Figures (5-4.2) and (5-4.3) show that the step response of the closed-loop system is slightly more oscillatory than that of the open-loop system, but that the closed-loop system has a shorter settling time than that of the open-loop system, as well as a very good voltage regulation for typical $x_t$ considered. The same general conclusions as given for figures (5-3.6) to (5-3.9) apply also to figures (5-4.4) to (5-4.7), with the latter figures revealing the critical values of $x_t$ above which the closed-loop system becomes unstable.

<table>
<thead>
<tr>
<th>$x_t$ (0.1 p.u.)</th>
<th>$x_t$ (0.2 p.u.)</th>
<th>$x_t$ (0.3 p.u.)</th>
<th>$x_t$ (0.4 p.u.)</th>
<th>$x_t$ (0.5 p.u.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.67 ± 3514.12</td>
<td>-2.00 ± 3514.13</td>
<td>-1.70 ± 3514.14</td>
<td>-1.44 ± 3514.14</td>
<td>-1.25 ± 3514.15</td>
</tr>
<tr>
<td>-1.33 ± 19.04</td>
<td>-0.90 ± 18.16</td>
<td>-0.54 ± 17.50</td>
<td>-0.20 ± 17.09</td>
<td>0.017 ± 16.86</td>
</tr>
<tr>
<td>-27.03 ± 10.80</td>
<td>-27.32 ± 13.91</td>
<td>-27.48 ± 14.89</td>
<td>-27.50 ± 15.46</td>
<td>-27.66 ± 15.84</td>
</tr>
<tr>
<td>-5.65 ± 11.36</td>
<td>-4.01 ± 11.16</td>
<td>-5.69 ± 10.52</td>
<td>-2.45 ± 16.26</td>
<td>-2.32 ± 16.59</td>
</tr>
<tr>
<td>-1.67 ± 12.31</td>
<td>-2.66 ± 14.24</td>
<td>-2.53 ± 15.59</td>
<td>-533.33</td>
<td>-533.33</td>
</tr>
<tr>
<td>-333.33</td>
<td>-333.33</td>
<td>-333.33</td>
<td>-142.81</td>
<td>-142.80</td>
</tr>
<tr>
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<td>-142.82</td>
<td>-142.81</td>
<td>-9.98</td>
<td>-9.98</td>
</tr>
<tr>
<td>-5.40</td>
<td>-5.32</td>
<td>-5.32</td>
<td>-3.80</td>
<td>-3.92</td>
</tr>
<tr>
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<td>-2.01</td>
<td>-2.00</td>
<td>-5.29</td>
<td>-2.96</td>
</tr>
<tr>
<td>-0.676</td>
<td>-0.672</td>
<td>-0.670</td>
<td>-2.00</td>
<td>-0.670</td>
</tr>
</tbody>
</table>

Table (5-4.1) Computed eigenvalues of the closed-loop system at rated output power. Underlined eigenvalues are associated with mechanical damping.
Figure 5-41 Dynamic stability limits of the closed-loop system, plotted on real against reactive output power coordinates.
Figure (5-4.2) Time response of the closed-loop system, operating initially at rated output power, when subjected to a 5% step increase in real power.
Time response of the open- and closed-loop systems, operating initially at rated output power and $x_t = 0.2 \text{ p.u.}$, when subjected to a 5% step increase in real power.
Figure 5.4.4 Tracking of the eigenvalues associated with the mechanical damping of the closed-loop system operating at rated output power.

The diagram shows the real and imaginary parts of the eigenvalues for different cases:

- **Case a**: $x_d = 1.418$ p.u.
- **Case b**: Base case
- **Case c**: $x_d = 2.127$ p.u.

The tracking is depicted by curves labeled a, b, and c, indicating that a, b, and c are coincident.
Figure 5-4.5 Tracking of the eigenvalues associated with the mechanical damping of the closed-loop system operating at rated output power.
Figure 5-4.6 Tracking of the eigenvalues associated with the mechanical damping of the closed-loop system operating at rated output power.
Figure (5-4.7) Tracking of the eigenvalues associated with the mechanical damping of the closed-loop system operating at rated output power.
CHAPTER 6

STABILITY AS AFFECTED BY
ADDITIONAL STABILIZING SIGNAL

6-1 SOURCE OF STABILIZING SIGNAL

The addition of control to the generator-bus system was shown in the previous Chapter to decrease the mechanical damping of that system. In this Chapter, the effect of a stabilizing signal superimposed on the normal error voltage signal to provide additional damping of any rotor oscillations is studied. The scheme is shown diagrammatically in figure(6-1.1). In practice, the extra signal \( v_s \) may be derived from a number of sources, including the generator shaft speed\(^{30-32}\), the terminal power\(^{33-34}\) and any terminal frequency deviation.\(^{35}\) In the present study, a stabilizing signal derived from the output electrical power \( P_o \) is chosen, not only because this is relatively easy to measure, but also because electrical power oscillations always represent relative motion of the generator with respect to the rest of the system, and this is precisely what it is desired to control.\(^{34}\) Having chosen the source of the stabilizing signal, the suitability of a few stabilizer transfer functions(represented by \( K(p) \) in figure(6-1.1)) is investigated.
6-2 SUITABILITY STUDIES OF STABILIZER TRANSFER FUNCTIONS

The four stabilizer transfer functions shown in figure (6-2.1) are investigated in this Section. The suitability of each depends on whether or not its employment results in an improvement in the performance of the closed-loop system represented by equation (5-2.1) (hereafter called the original closed-loop system).

Figure (6-2.1) Stabilizer transfer functions
6-2.1 Proportional type

For this stabilizer,

\[ v_s = K_s p \]  

(6-2.1)

and at the regulator summing junction,

\[ u_r = v_r + v_s - v_z \]  

(6-2.2)

Substituting equations (6-2.1) and (6-2.2) into equation (5-1.2) and linearizing, we obtain

\[ \Delta v_a = -\frac{1}{T_{a2}} \Delta v_a + \frac{G_a T_{a1}}{T_{a2} T_z} \Delta v_z - \frac{G_a G_T T_{a1} T_z}{T_{a2} T_z} \Delta v_t \]

\[ + \frac{K G T_{a1}}{T_{a2}} \Delta P_o + \frac{K G T_{a1}}{T_{a2}} \Delta P_o \]  

(6-2.3)

from which it follows that equation (5-1.13) becomes

\[ \Delta v_m = -\frac{1}{T_{m2}} \Delta v_m + \frac{G_m}{T_{m2}} \left( 1 - \frac{T_{m1}}{T_{a2}} \right) \Delta v_a + \frac{G_m a^m m_1 T_{a1}}{T_{m2} a^2 T_z} \Delta v_z \]

\[ - \frac{G_m a^m m_1 T_{a1}}{T_{m2} a^2 T_z} \Delta v_t + \frac{K G G_T T_{a1} T_z}{T_{a2} m_2} \Delta P_o \]

\[ + \frac{K G G_T T_{a1} T_z}{T_{a2} m_2} \Delta P_o \]  

(6-2.4)

With equations (5-1.8) to (5-1.11) unaltered by the addition of the stabilizing signal, the equation of the original closed-loop system employing this stabilizer may be written as

\[ \dot{x} = A_a x \]  

(6-2.5)

where \( x \) comprises the same state-variables as those in equation (5-2.1). \( A_a \) is the (16 x 16) system matrix, the elements of which are shown in Appendix (IV-1).
Figure (6-2.2) shows that the employment of this stabilizer reduces the range of $x_t$ for stable operation, consequent upon the reduction of the mechanical damping as detailed in table (6-2.1). The step responses in figure (6-2.3) show also a worsening in the good voltage regulation of the original closed-loop system consequent upon the use of this stabilizer. A combination of the above discussion items points, therefore, to the fact that this stabilizer is unsuitable.

<table>
<thead>
<tr>
<th>$K_s=0$ (original closed-loop system)</th>
<th>$K_s = 1 \times 10^{-4}$</th>
<th>$K_s = 2 \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.08 ± j314.13</td>
<td>-2.08 ± j314.14</td>
<td>-2.08 ± j314.14</td>
</tr>
<tr>
<td>-0.90 ± j8.16</td>
<td>-0.71 ± j7.67</td>
<td>-0.39 ± j7.26</td>
</tr>
<tr>
<td>-27.32 ± j3.91</td>
<td>-27.12 ± j3.42</td>
<td>-26.9 ± j3.28</td>
</tr>
<tr>
<td>-4.01 ± j1.16</td>
<td>-3.91 ± j1.25</td>
<td>-3.79 ± j1.31</td>
</tr>
<tr>
<td>-2.66 ± j4.24</td>
<td>-3.15 ± j4.48</td>
<td>-3.80 ± j4.64</td>
</tr>
<tr>
<td>-333.33</td>
<td>-333.34</td>
<td>-333.34</td>
</tr>
<tr>
<td>-142.83</td>
<td>-142.83</td>
<td>-142.83</td>
</tr>
<tr>
<td>-5.32</td>
<td>-5.32</td>
<td>-5.32</td>
</tr>
<tr>
<td>-2.01</td>
<td>-2.01</td>
<td>-2.01</td>
</tr>
<tr>
<td>-0.67</td>
<td>-0.67</td>
<td>-0.67</td>
</tr>
</tbody>
</table>

Table (6-2.1) Computed eigenvalues of the original closed-loop system employing the Proportional stabilizer. The overall system is operating at rated output power and at $x_t = 0.2$ p.u. Underlined eigenvalues are associated with mechanical damping.
6-2.2 Derivative-with-delay type

For this stabilizer,

\[ v_s = \frac{K_p}{1 + pT_s} P_o \]  \hspace{1cm} (6-2.6)

and after linearization and rearrangement, equation (6-2.6) becomes

\[ \Delta v_s = - \frac{1}{T_s} \Delta v_s + \frac{K_s}{T_s} \Delta P_o \]  \hspace{1cm} (6-2.7)

Substituting equations (6-2.2) and (6-2.6) into equation (5-1.2) and linearizing, we obtain

\[ \Delta v_a = - \frac{1}{T_{a2}} \Delta v_a + \frac{G_a(T_{a1} - 1)}{T_{a2}} \Delta v_z - \frac{G_a G_{v2} a_1}{T_{a2} T_z} \Delta v_t \\
+ \frac{G_a (1 - T_{a1})}{T_{s}} \Delta v_z + \frac{K_s G_a a_1}{T_{a2} T_s} \Delta P_o \]  \hspace{1cm} (6-2.8)

from which it follows that equation (5-1.13) becomes

\[ \Delta v_m = - \frac{1}{T_{m2}} \Delta v_m + \frac{G_m(T_{m1} - 1)}{T_{m2}} \Delta v_a + \frac{G_m G_{v2} m_1 a_1}{T_{m2} T_z} \Delta v_m \\
- \frac{G_m G_{v2} m_1 a_1}{T_{m2} T_z} \Delta v_t + \frac{G_m G_{v2} m_1 a_1}{T_{m2} T_z} (1 - \frac{T_{a1}}{T_s}) \Delta P_o \]  \hspace{1cm} (6-2.9)

With equations (5-1.8) to (5-1.11) unaltered by the addition of the stabilizing signal, and equation (6-2.7) constituting an additional 1st-order differential equation, the closed-loop state-space equation of the original closed-loop system employing this stabilizer may be written as

\[ x = A_b x \]  \hspace{1cm} (6-2.10)
where the first 16 state-variables of $x$ are the same as those in equation (5-2.1), and its 17th state-variable is $\Delta v_s$. $A_b$ is the $(17 \times 17)$ system matrix, the elements of which are shown in Appendix (IV-2).

Figure (6-2.4) shows that the employment of this stabilizer reduces the range of $x_t$ for stable operation, consequent upon the reduction of the mechanical damping as detailed in table (6-2.2). However, because of the derivative action of this stabilizer, the stabilizing signal has a transient but no steady-state effect on the overall system. As a result, as shown in figure (6-2.5), the good voltage regulation of the original closed-loop system is retained. Nevertheless, this stabilizer is considered unsuitable on the ground that it has an adverse effect on the system damping.

<table>
<thead>
<tr>
<th>$K_s = 0$ (original closed-loop system)</th>
<th>$K_s = 1 \times 10^{-4}$</th>
<th>$K_s = 1 \times 10^{-4}$</th>
<th>$K_s = 1 \times 10^{-4}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_s = 2 \ s$</td>
<td>$T_s = 1 \ s$</td>
<td>$T_s = 0.5 \ s$</td>
<td></td>
</tr>
<tr>
<td>$-2.08 \pm j314.13$</td>
<td>$-2.08 \pm j314.14$</td>
<td>$-2.08 \pm j314.14$</td>
<td>$-2.08 \pm j314.13$</td>
</tr>
<tr>
<td>$-0.90 \pm j8.16$</td>
<td>$-0.80 \pm j7.92$</td>
<td>$-0.65 \pm j7.71$</td>
<td>$-0.26 \pm j7.46$</td>
</tr>
<tr>
<td>$-27.32 \pm j3.91$</td>
<td>$-27.22 \pm j3.67$</td>
<td>$-27.11 \pm j3.40$</td>
<td>$-26.85 \pm j3.26$</td>
</tr>
<tr>
<td>$-4.01 \pm j31.16$</td>
<td>$-3.95 \pm j31.21$</td>
<td>$-3.88 \pm j31.28$</td>
<td>$-3.62 \pm j31.55$</td>
</tr>
<tr>
<td>$-2.66 \pm j4.24$</td>
<td>$-2.91 \pm j4.35$</td>
<td>$-3.24 \pm j4.39$</td>
<td>$-4.11 \pm j4.15$</td>
</tr>
<tr>
<td>$-333.33$</td>
<td>$-333.33$</td>
<td>$-333.34$</td>
<td>$-333.33$</td>
</tr>
<tr>
<td>$-142.83$</td>
<td>$-142.83$</td>
<td>$-142.83$</td>
<td>$-142.82$</td>
</tr>
<tr>
<td>$5.32$</td>
<td>$5.32$</td>
<td>$5.32$</td>
<td>$-5.32$</td>
</tr>
<tr>
<td>$-2.01$</td>
<td>$-2.01$</td>
<td>$-2.01$</td>
<td>$-1.92$</td>
</tr>
<tr>
<td>$-0.67$</td>
<td>$-0.67$</td>
<td>$-0.67$</td>
<td>$-0.67$</td>
</tr>
<tr>
<td>$-0.49$</td>
<td>$-1.00$</td>
<td>$-2.18$</td>
<td></td>
</tr>
</tbody>
</table>

Table (6-2.2) Computed eigenvalues of the original closed-loop system employing the Derivative-with-delay stabilizer. The overall system is operating at rated output power and at $x_t = 0.2 \ p.u.$. Underlined eigenvalues are associated with mechanical damping.
6-2.3 Proportional-with-delay type

For this stabilizer,

\[ V_s = \frac{K_s}{1 + T_s P_o} \quad (6-2.11) \]

and after linearization and rearrangement, equation (6-2.11) becomes

\[ \Delta V_s = -\frac{1}{T_s} \Delta V_s + \frac{K_s}{T_s} \Delta P_o \quad (6-2.12) \]

Substituting equations (6-2.2) and (6-2.11) into equation (5-1.2) and linearizing, we obtain

\[ \Delta V_a = -\frac{1}{T_a} \Delta V_a + \frac{G_a}{T_a} \left( \frac{T_{a1}}{T_z} - 1 \right) \Delta V_z - \frac{G_a G_{a1}}{T_{a2} T_z} \Delta V_t \\
+ \frac{G_a}{T_a} \left( 1 - \frac{T_{a1}}{T_z} \right) \Delta V_s + \frac{K_g a_{a1}}{T_{a2}} \Delta P_o \quad (6-2.13) \]

from which it follows that equation (5-1.13) becomes

\[ \Delta V_m = -\frac{1}{T_{m2}} \Delta V_m + \frac{G_m}{T_m} \left( 1 - \frac{T_{m1}}{T_{a2}} \right) \Delta V_a + \frac{G_m a_{m1}}{T_{m2} T_{a2}} \left( \frac{T_{a1}}{T_z} - 1 \right) \Delta V_z \\
- \frac{G_m G_{a1} G_{m1}}{T_{m2} T_{a2} T_z} \Delta V_t + \frac{G_m G_{m1}}{T_{m2} T_{a2}} \left( 1 - \frac{T_{a1}}{T_z} \right) \Delta V_s \\
+ \frac{K_{g m} a_{m1} G_{m1}}{T_{m2} T_{a2}} \Delta P_o \quad (6-2.14) \]

With equations (5-1.8) to (5-1.11) unaltered by the addition of the stabilizing signal, and equation (6-2.12) constituting an additional 1st-order differential equation, the closed-loop state-space equation of the original closed-loop system employing this stabilizer may be written as

\[ x = A_c x \quad (6-2.15) \]
where $x$ comprises the same state-variables as those in equation (6-2.10). $A_c$ is the $(17 \times 17)$ system matrix, the elements of which are shown in Appendix (IV-3).

Figure (6-2.6) shows that the employment of this stabilizer does not extend the range of $x_t$ for stable operation, but that it increases the mechanical damping of the system within that stable region. In the case where the greater increase of damping is achieved by a higher $K_s$ (refer to figure (6-2.6C)), the range of $x_t$ for stable operation is reduced. Figure (6-2.7) shows that a higher stabilizer gain also means a greater sacrifice of the good voltage regulation originally achieved by the voltage regulator. Figure (6-2.8) shows that any further reduction of the stabilizer time-constant is inadvisable, because by then this stabilizer will approach the performance of the Proportional stabilizer which has already been considered as unsuitable.

6-2.4 Cascaded type

The stabilizer considered in the previous Section is capable of increasing the damping but at the consequence of a worsening in the voltage regulation achieved by the original closed-loop system. The Cascaded stabilizer considered here is therefore constructed to supplement the previous stabilizer in such a way that not only is the system damping improved, but that the good voltage regulation achieved by the original regulator is retained.
For this stabilizer,

\[ v_i = \frac{K_s}{1+pT_s} P_o \]  

(6-2.16)

and \[ v_s = \frac{K_f T_f P}{1+pT_f} v_i \]  

(6-2.17)

and after linearization and rearrangement, equations (6-2.16) and (6-2.17) become respectively

\[ \Delta v_i = -\frac{1}{T_s} \Delta v_i + \frac{K}{T_s} \Delta P_o \]  

(6-2.18)

and \[ \Delta v_s = -\frac{1}{T_f} \Delta v_s + K_f \Delta v_i \]  

(6-2.19)

Substituting equation (6-2.18) into equation (6-2.19) gives

\[ \Delta v_s = -\frac{1}{T_f} \Delta v_s - \frac{K_f}{T_s} \Delta v_i + \frac{K_f K_s}{T_s} \Delta P_o \]  

(6-2.20)

Substituting equations (6-2.2), (6-2.16) and (6-2.17) into equation (5-1.2) and linearizing, we obtain

\[ \Delta v_a = -\frac{1}{a_2} \Delta v_a + \frac{G_a}{a_2} \left( \frac{a_1}{a_2} - 1 \right) \Delta v_z - \frac{G_a v_a T a_1}{a_2 T z} \Delta v_t \]

\[ + \frac{G_a}{a_2} \left( 1 - \frac{T_f}{a_1} \right) \Delta v_s - \frac{K_f G_a}{a_2 T_a} \Delta v_i + \frac{K_f K_s}{a_2 s} \Delta P_o \]  

(6-2.21)

from which it follows that equation (5-1.13) becomes

\[ \Delta v_m = -\frac{1}{T_m} \Delta v_m + \frac{G_m}{T_m} \left( 1 - \frac{T_m^1}{T_m^2} \right) \Delta v_a + \frac{G_m G_m^1}{T_m^2 a_2^1} \left( \frac{T_m^2}{T_m^2} - 1 \right) \Delta v_z \]

\[ - \frac{G_m G_m G_m}{T_m^2 a_2^1} \Delta v_t + \frac{G_m a m^1}{T_m^2 a_2} \left( 1 - \frac{T_f}{T_f} \right) \Delta v_s \]

\[ - \frac{K_f G_m G_m}{T_m^2 a_2^1} \Delta v_i + \frac{K_f K_s G_m G_m^1}{T_m^2 a_2^1} \Delta P_o \]  

(6-2.22)
With equations (5-1.8) to (5-1.11) unaltered by the addition of the stabilizing signal, and equations (6-2.18) and (6-2.20) constituting two additional 1st-order differential equations, the closed-loop state-space equation of the original closed-loop system employing this stabilizer may be written as

\[ x = A_d x \]  

(6-2.23)

where the first 16 state-variables of \( x \) are the same as those in equation (5-2.1), and its 17th and 18th state-variables are \( \Delta v_i \) and \( \Delta v_s \) respectively. \( A_d \) is the \((18 \times 18)\) system matrix, the elements of which are shown in Appendix (IV-4).

By taking

\[ K_s = 2.75 \times 10^{-4} \quad \text{and} \quad T_s = 0.5 \text{ s} \]

(using these parameter values in the previous stabilizer results in improvement of mechanical damping without obvious reduction in the range of \( x_t \) for stable operation), the values of \( K_f \) and \( T_f \) are so adjusted as to improve further the performance of the overall system operating at rated output power and at \( x_t = 0.2 \text{ p.u.} \). The end results are

\[ K_f = 2.0 \quad \text{and} \quad T_f = 0.4 \text{ s} \]

with the system step response shown in figure (6-2.9). This figure, together with table (6-2.3) show that the employment of this stabilizer increases the mechanical damping over both that of the original closed-loop system and that of the original closed-loop system employing the Proportional-with-delay stabilizer, while still retaining the good voltage regulation originally achieved by the voltage regulator. However, as shown in figure
(6-2.10), the improvement of the mechanical damping over the original closed-loop system happens only between a range of $x_t$ of about $0 \leq x_t \leq 0.3$ p.u.. Above $x_t \approx 0.3$ p.u., the employment of this stabilizer actually worsens the original system damping, thus reducing the range of $x_t$ for stable operation as a consequence.

6-3 SUMMARY

Under the performance criteria of good voltage regulation and good damping, the addition of a stabilizing signal, derived from the output electrical power through an appropriate stabilizer transfer function, has been shown to be capable of increasing the damping while retaining the good voltage regulation of the original closed-loop system. However, the improvement of damping occurs only within the original range of $x_t$ for stable operation, or the additional signal cannot extend this range. In the case where the greater increase of damping is achieved, the range of $x_t$ for stable operation is reduced.
<table>
<thead>
<tr>
<th>original closed-loop system</th>
<th>original closed-loop system employing the Proportional-with-delay stabilizer</th>
<th>original closed-loop system employing the Cascaded stabilizer</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.08 ± j314.13</td>
<td>-2.08 ± j314.14</td>
<td>-2.08 ± j314.14</td>
</tr>
<tr>
<td>-0.90 ± j8.16</td>
<td>-1.26 ± j8.04</td>
<td>-1.69 ± j7.64</td>
</tr>
<tr>
<td>-27.32 ± j3.91</td>
<td>-27.38 ± j3.99</td>
<td>-27.45 ± j4.09</td>
</tr>
<tr>
<td>-4.01 ± j1.16</td>
<td>-4.07 ± j0.89</td>
<td>-2.02 ± j0.66</td>
</tr>
<tr>
<td>-2.66 ± j4.24</td>
<td>-2.23 ± j4.67</td>
<td>-1.98 ± j5.55</td>
</tr>
<tr>
<td>-333.33</td>
<td>-1.95 ± j0.12</td>
<td>-333.33</td>
</tr>
<tr>
<td>-142.83</td>
<td>-333.33</td>
<td>-142.83</td>
</tr>
<tr>
<td>-5.32</td>
<td>-9.98</td>
<td>-5.29</td>
</tr>
<tr>
<td>-2.01</td>
<td>-5.31</td>
<td>-4.56</td>
</tr>
<tr>
<td>-0.67</td>
<td>-0.67</td>
<td>-3.63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-1.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-0.67</td>
</tr>
</tbody>
</table>

Table (6-2.3) Computed eigenvalues of three closed-loop systems, operating at rated output power and at \( x_t = 0.2 \) p.u.. Underlined eigenvalues are associated with mechanical damping.
Figure (6-2.2) Tracking of the real parts of the complex eigenvalues associated with the mechanical damping of the original closed-loop system employing the Proportional stabilizer. The overall system is operating at rated output power.
Figure 6-2.3  Time response of the original closed-loop system employing the Proportional stabilizer, when subjected to a 5% step increase in real power. The overall system is operating initially at rated output power and \( x_t = 0.2 \) p.u.
Figure 6-2.4  Tracking of the real parts of the complex eigenvalues associated with the mechanical damping of the original closed-loop system employing the Derivative-with-delay stabilizer. The overall system is operating at rated output power.
Figure 6-2.5  Time response of the original closed-loop system employing the Derivative-with-delay stabilizer, when subjected to a 5% step increase in real power. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Tracking of the real parts of the complex eigenvalues associated with the mechanical damping of the original closed-loop system employing the Proportional-with-delay stabilizer. The overall system is operating at rated output power.
Figure (6-2.7) Time response of the original closed-loop system employing the Proportional-with-delay stabilizer, when subjected to a 5% step increase in real power. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Figure 6-2.8 Time response of the original closed-loop system employing the Proportional-with-delay stabilizer, when subjected to a 5% step increase in real power. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Figure (6-2.9) Time response of the original closed-loop system employing the Cascaded stabilizer, when subjected to a 5% step increase in real power. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Figure 6-2.10 Tracking of the real parts of the complex eigenvalues associated with the mechanical damping of the original closed-loop system employing either the Proportional-with-delay or the Cascaded stabilizer. The overall system is operating at rated output power.
CHAPTER 7

LINEAR OPTIMAL CONTROL

Both the extensive growth of electric power systems and the development of high-voltage long-distance transmission systems separating the generation from the load centres have accentuated the importance of increasing the dynamic and the transient stability limits of synchronous machines. In recent years, considerable attention has been given to the development of compensating control schemes to provide the required system stabilization. Chapter 6 has examined the role of a supplementary stabilizing signal derived from the output electrical power. With the well-developed Optimal Control Theory utilizing the state-space representation of a multivariable system, the present Chapter studies the problem of the optimization of synchronous machine performance, by minimizing a scalar performance index in both system state-variables and inputs.

7-1 BASIC DESIGN THEORY

When the state-space representation of a controllable, linear, time-invariant system is written as

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

(7-1.1)

\(x\) is the \((n \times 1)\) state vector, \(u\) the \((m \times 1)\) input vector and \(y\) the \((1 \times 1)\) output vector of the system. \(A\), \(B\) and \(C\) are constant.
(n x n), (n x m) and (1 x n) matrices respectively. When the linearized model of a nonlinear system is considered, x and y consist only of deviations from the initial steady-state variables.

Pontryagin's maximum principle\(^{50}\) may be applied to find the optimal control which transfers the system from an initial state to a final state and at the same time minimizes a given performance index, usually expressed in quadratic form by

\[
J = \frac{1}{2} \int_{0}^{T} (x'Dx + u'Hu)dt \quad (7-1.2)
\]

where \(D\) is an \((n x n)\) matrix and \(H\) an \((m x m)\) matrix containing the weightings on the state- and control variables respectively. Once these weightings are chosen such that \(D\) and \(H\) are nonnegative definite and positive definite matrices respectively, a unique optimal control exists\(^{37}\), which minimizes the performance index \(J\) and the error response of the system after a disturbance.

The solution of the problem begins with the introduction of a costate variable vector \(f\), together with a Hamiltonian \(H_1\) of the form

\[
H_1 = \frac{1}{2}(x'Dx) + \frac{1}{2}(u'Hu) + f'(Ax + Bu) \quad (7-1.3)
\]

For an optimal control \(u\), \(f\) and \(x\) must provide the solution of\(^{10}\)

\[
\dot{x} = \frac{\delta H_1}{\delta f} \quad (7-1.4)
\]

and

\[
f = - \frac{\delta H_1}{\delta x}
\]

which lead to

\[
\dot{x} = Ax + Bu \quad (7-1.5)
\]

and

\[
f = - Dx - A'f
\]
respectively. It is also necessary that

$$\frac{\delta H}{\delta u} = 0 \quad (7-1.6)$$

which gives

$$u = -H^{-1}B'f \quad (7-1.7)$$

By assuming a solution for \( f \)

$$f = Qx \quad (7-1.8)$$

the optimal control becomes

$$u = -H^{-1}B'Qx = Kx \quad (7-1.9)$$

where \( K = -H^{-1}B'Q \) = optimal-controller-gain matrix.

The time derivative of equation(7-1.8) follows by substitution of equation(7-1.5), giving

$$\dot{Q} = -D - A'Q - QA + QBE^{-1}B'Q \quad (7-1.10)$$

which is the well-known Matrix Ricatti Equation where matrix \( Q \) is symmetric and positive definite.\(^{37,38}\) For a time-invariant system, \( Q \) is a constant matrix\(^{10}\) and is the solution of the nonlinear algebraic matrix equation(7-1.10), which can be written for a steady-state solution (i.e. \( \dot{Q} \rightarrow 0 \) as \( T \rightarrow \infty \))\(^{38}\) as

$$D + A'Q + QA - QBE^{-1}B'Q = 0 \quad (7-1.11)$$

The system equation with optimal control becomes

$$\dot{x} = (A - BE^{-1}B'Q)x$$

$$= (A + BK)x \quad (7-1.12)$$

A method of solving equation(7-1.11), together with its computer subroutine, is shown in Appendix V.
The closed-loop system described by equation (7-1.12) can be visualized by the configuration shown in Figure (7-2.1). In this figure, a feedback controller described by a transfer function \( K \) is interposed between the system states and inputs, with the system states constituting the controller inputs. Under the assumption that all the system states are available for measurement (if this is not so, it is generally possible to construct a state estimator to produce at its output the system states, when driven by both the system inputs and outputs\(^{45}\)), the controller generates the system input \( u \) (control law) which is an instantaneous linear combination of all the states of the system.
For a single-input system, the linear controller function $K$ consists of $n$ elements (or $n$ gains) from $k_1$ to $k_n$, where $n$ is the dimension of the system. Instead of choosing different values of $k_1$ to $k_n$, by trial and error, in order that the system performance is improved, Linear Optimal Control Theory provides a systematic way of choosing $k$'s which are unique for a particular system operating condition, and the resultant optimal closed-loop system must be stable even though the initial system without control is unstable. With an appropriate choice for the matrices $D$ and $H$ in the performance index, a linear optimal controller is not only easy to implement, but is also capable of improving the system performance dramatically.

7-3 PERFORMANCE INDEX

For a synchronous generator feeding into an infinite-bus system, good regulation of the terminal voltage and turbine speed are essential. Good control over the load-angle swings and output power oscillations, which represent the relative motion of the generator with respect to the rest of the system, can certainly be beneficial to the system performance. A power system with linearized system equations will therefore require minimization of $\Delta P_o$, $\Delta V_t$, $\Delta \delta$, $\Delta q$ and $\Delta u$, when it is subjected to a disturbance. The required performance index is defined as

$$J = \frac{1}{2} \int_0^\infty (\lambda_1 \Delta P_o^2 + \lambda_2 \Delta V_t^2 + \lambda_3 \Delta \delta^2 + \lambda_4 q^2 + \lambda_5 u^2) \, dt$$

(7-3.1)
where \( \lambda_1, \lambda_2 \) and \( \lambda_3, \lambda_4 \) are the weighting factors on the system outputs and states respectively, and \( \lambda_5 \) is the weighting factor on the input control. When \( \Delta P_0 \) and \( \Delta v_t \) are expressed in terms of the system state-variables, equation (7-3.1) can be expressed as

\[
J = \frac{1}{2} \int_0^\infty (x' Dx + u' Hu) \, dt \quad (7-3.2)
\]

7-4 SYSTEM EQUATIONS AND PERFORMANCES

The original closed-loop system of Chapter 6 employing the Cascaded stabilizer is considered. When the signal source \( P_o \) of the stabilizing signal is replaced by another source called \( v_{sum} \), then the equations required are the same as those developed in Section (6-2.4), except that the symbol \( \Delta P_0 \) in equations (6-2.18) and (6-2.20) to (6-2.22) is now replaced by \( \Delta v_{sum} \) which acts as the single input to the system.

When the state-space equation of the system is written as

\[
x = Ax + Bu \quad (7-4.1)
\]
x comprises the same state-variables as those in equation (6-2.23) and \( u = \Delta v_{sum} \). \( A \) and \( B \) are respectively \((18 \times 18)\) and \((18 \times 1)\) matrices, the elements of which are shown in Appendix (IV-5).

With a performance index in the form of equation (7-3.1), and \( \Delta P_0 \) and \( \Delta v_t \) expressed in terms of the system state-variables (refer to Section (3-3.1)), the matrices \( D \) and \( H \) of equation (7-3.2) are constructed as follows:—
(The following expressions are in 1900 FORTRAN code)

```
DO 7 I=1,18
DO 7 J=1,18
7    D(I,J)=0.0
DO 8 I=1,7
DO 8 J=1,7
8    D(I,J)=LANDA(1)*F(12,I)*F(12,J)+
     LANDA(2)*F(13,I)*F(13,J)
DO 11 I=1,2
11   D(I,I)=D(I,I)+LANDA(I+2)
DO 40 I=1,7
40   D(I,11)=LANDA(1)*F(12,I)*G(12,1)+
     LANDA(2)*F(13,I)*G(13,1)
40   D(11,I)=D(11,I)
11    D(11,11)=LANDA(1)*G(12,1)*G(12,1)+
      LANDA(2)*G(13,1)*G(13,1)
```

and since only a single input is considered, the matrix $H$ is a scalar given by

$$H = L\text{ANDA}(5)$$

with $L\text{ANDA}(I) = \lambda_I (I = 1, 2, \ldots, 5)$

When Linear Optimal Control Theory was applied (the computer algorithm is shown in Appendix(VI-1)), and after a series of step-response tests on the optimally controlled system operating initially at rated output power and $x_t = 0.2$ p.u., the weighting factors of equation(7-3.1) were finally chosen as

$$\lambda_1 = 1.0 , \quad \lambda_2 = 1.0 , \quad \lambda_3 = 0.1$$

$$\lambda_4 = 0.1 \quad \text{and} \quad \lambda_5 = 0.07$$

(7-4.2)

to give the satisfactory time response shown in figure(7-4.1). It can be seen that the performance of the closed-loop system with optimal control is superior to both that of the original closed-loop system and that of the original closed-loop system with the output electrical power as the source of the stabilizing signal.
In association with this optimally controlled closed-loop system, the eigenvalues are

\[-2.08 \pm j314.14\]
\[-27.32 \pm j3.90\]
\[-1.61 \pm j0.43\]
\[-333.33 \pm j142.83\]
\[-5.32 \pm j4.50\]
\[-2.09 \pm j0.67\]
and the optimal-controller-gain matrix \( K \) is

\[
K = \begin{bmatrix}
0.682 & -0.990 & -3.792 & 1.430 & 3.805 \\
3.799 & 1.524 & -0.655 & 0.114 & 1.576 \\
-3.288 \times 10^2 & -4.115 \times 10^3 & -1.354 \times 10^3 \\
5.512 & -6.166 \times 10^2 & -1.082 \times 10^3 \\
-2.944 \times 10^{-4} & -8.328 \times 10^1
\end{bmatrix}
\]

(7-4.3)

This controller is optimal only when the system operates at rated output power and with \( x_t = 0.2 \) p.u.. For other values of \( x_t \), the controller is suboptimal and a stable system is not guaranteed. Figure (7-4.2) shows that employment of this suboptimal or fix controller can provide stable operation for values of \( x_t \) above the critical value of \( x_t \) for stable operation of the original closed-loop system. The instability at low values of \( x_t \) (\( 0 \leq x_t \leq 0.05 \) p.u.) is unimportant since practical values of \( x_t \) usually range from 0.2 to 0.8 p.u., within which employment of this controller is capable of providing stable operation. Figures (7-4.3) to (7-4.5), together with tables (7-4.1) and (7-4.2), compare the performances of the original closed-loop system, the original closed-loop system employing this suboptimal controller, and the original closed-loop system.
system employing optimal controllers designed at different values of $x_t$ but with the same weighting factors as given in equation (7-4.2). At $x_t = 0.1$ p.u., the original closed-loop system performance is so good that the employment of either controller is unnecessary. At $x_t = 0.3$ p.u., both controllers give a well-damped system. At $x_t = 0.6$ p.u., the original closed-loop system is unstable, but the employment of either controller can provide stable operation, even though it is the optimal controller which gives the better damped system. For a certain range of tie-line reactance, over which a system is required to operate stably, a fix controller can always be designed by Linear Optimal Control Theory to satisfy this requirement, even though the performance of the resultant system may not be always satisfactory throughout the entire range. The shifting of the range of $x_t$ for stable operation is shown in figures (7-4.6) and (7-4.7), where other optimal controllers are used suboptimally. However, as can be seen in table (7-4.2), the individual elements of the optimal-controller-gain matrices do not vary very much for different optimal controllers designed at different values of $x_t$. In any practical implementation therefore an optimal controller constructed for one value of $x_t$ can readily be adjusted to operate optimally at other values of $x_t$, without altering the power levels of its amplifiers.
Table (7-4.1) Computed eigenvalues of the original closed-loop system with or without constant-coefficient-feedback controller. The system is operating at rated output power.
| $x_t$(p.u.) | elements of optimal-controller-gain matrix  
| K |
| --- | --- |
| 0.1 | $\begin{bmatrix} 2.676 & -0.813 & -3.213 & -3.840 & 3.241 \\ 3.243 & 4.021 & -1.001 & 0.097 & 1.260 \\ -3.137 \times 10^2 & -3.927 \times 10^3 & -1.290 \times 10^3 \\ -5.257 & -8.942 \times 10^2 & -5.121 \times 10^2 \\ 1.654 \times 10^{-4} & -9.337 \times 10^{-3} \end{bmatrix}$ |
| 0.3 | $\begin{bmatrix} -1.476 & -1.082 & -3.797 & 0.797 & 3.774 \\ 3.780 & -0.786 & -0.028 & 0.128 & 1.909 \\ -3.443 \times 10^2 & -4.271 \times 10^3 & -1.411 \times 10^3 \\ -5.771 & -5.266 \times 10^2 & -1.318 \times 10^3 \\ 1.164 \times 10^{-5} & -7.992 \times 10^{-1} \end{bmatrix}$ |
| 0.4 | $\begin{bmatrix} -3.409 & -1.095 & -3.593 & 2.586 & 3.538 \\ 3.558 & -2.644 & 0.744 & 0.141 & 2.219 \\ -3.528 \times 10^2 & -4.347 \times 10^3 & -1.438 \times 10^3 \\ -5.911 & -4.871 \times 10^2 & -1.424 \times 10^3 \\ -1.277 \times 10^{-4} & -7.851 \times 10^{-1} \end{bmatrix}$ |
| 0.6 | $\begin{bmatrix} -6.148 & -0.967 & -3.098 & 4.750 & 3.010 \\ 3.045 & -4.899 & 2.320 & 0.156 & 2.695 \\ -3.494 \times 10^2 & -4.298 \times 10^3 & -1.418 \times 10^3 \\ -5.850 & -4.508 \times 10^2 & -1.460 \times 10^3 \\ -3.713 \times 10^{-5} & -7.657 \times 10^{-1} \end{bmatrix}$ |

Table (7-4.2) Elements of optimal-controller-gain matrices for controllers designed at rated output power and different values of $x_t$. 
Figure (7.4.1) Time response of the system operating initially at rated output power and $x_t = 0.2$ p.u., when subjected to a $5\%$ step increase in real power.
junctions where transitions of two real eigenvalues into one complex eigenvalue, or vice versa, take place.

Figure (7-4.2) Tracking of the dominant eigenvalues of the closed-loop system employing the controller designed optimally at rated output power and \( x_p = 0.2 \) p.u.. The system is operating at rated output power.
Figure (7-4.3) Time response of the system operating initially at rated output power and $x_t = 0.1 \text{ p.u.}$, when subjected to a 5% step increase in real power.
Figure (7-4.4) Time response of the system operating initially at rated output power and $x_t = 0.3$ p.u., when subjected to a 5% step increase in real power.
with controller designed optimally with $x_t = 0.2 \text{ p.u.}$

with optimal controller

Figure 7-4.5 Time response of the system operating initially at rated output power and $x_t = 0.6 \text{ p.u.}$, when subjected to a 5% step increase in real power.
junctions where transitions of two real eigenvalues into one complex eigenvalue, or vice versa, take place.

Figure (7.4.6) Tracking of the dominant eigenvalues of the closed-loop system employing the controller designed optimally at rated output power and \( x_i = 0.4 \) p.u.. The system is operating at rated output power.
junctions where transitions of two real eigenvalues into one complex eigenvalue, or vice versa, take place.

Figure (7-4.7) Tracking of the dominant eigenvalues of the closed-loop system employing the controller designed optimally at rated output power and \( x_t = 0.6 \) p.u.. The system is operating at rated output power.
7-5. **EFFECT OF WEIGHTING FACTORS**

In the process of deciding the values of the weighting factors in the performance index given by equation (7-3.1), engineering experience and a trial and error method are generally necessary. The weighting factors, so decided, will result in an optimal feedback controller to give a well-damped closed-loop system at a particular operating condition. However, when the choice of these weighting factors is inappropriate, the resultant system, even if it is stable, may not behave satisfactorily. In this Section, the effect on the optimally controlled system of changing one of the five weighting factors (given by equation (7-4.2)) while keeping the rest unchanged is investigated. The system studied is operating initially at rated output power and $x_t = 0.2$ p.u., and is subjected to a 5\% step increase in real power.

A summary of the effects on the resultant system and the characteristics of its associated optimal controller is given in figures (7-5.1) to (7-5.5), when each of the five weighting factors is varied. These figures show the tracking of the real part of the complex eigenvalue associated with the mechanical damping, the three largest elements in the optimal-controller-gain matrix ($k_{12}$ is associated with the state-variable $\Delta v_z$, $k_{13}$ with $\Delta v_o$ and $k_{16}$ with $\Delta v_m$), the ratio of the peak value to the steady-state value of $\Delta b$, and the ratio of the peak value to the steady-state value of $\Delta v_t$, as functions of the weighting factor in question. The horizontal broken lines represent the behaviour of the original closed-loop system. Figure (7-5.1) shows that a decrease in $\lambda_5$ (the weighting on the input control) increases the mechanical
damping of the system, but at the expense of a higher transient overshoot and larger elements in the controller-gain matrix. An increase in $\lambda_5$, on the other hand, decreases the elements of the controller-gain matrix and may eventually put the system into a stage as if there was no control. Figure(7-5.2) shows that there is a range of small values of $\lambda_1$ within which the system performance and its associated optimal controller are virtually unaffected. Small values of $\lambda_1$ mean a small emphasis on the term $\lambda_1 \cdot \Delta P_0^2$ in the performance index, but still result in a mechanical damping increase over that of the original closed-loop system. Higher values of $\lambda_1$, on the other hand, increase the emphasis on this term and result in a further increase in the mechanical damping as well as in the elements of the controller-gain matrix and the transient overshoot.

The same arguments also apply to figures(7-5.3) to (7-5.5). The time responses in figures(7-5.6) to (7-5.10) assist in revealing the complete picture of the effects of the weighting factors on the behaviour of the resultant system. It is generally concluded that a moderate amount of control should be imposed on the system, so that a compromise between the resultant system damping and the transient overshoot can be reached.

Since similar effects produced by the weighting factors on the resultant system with optimal control are observed for other values of tie-line reactance, only the effects of changing $\lambda_5$ are shown in figures(7-5.11) and (7-5.12) for a value of $x_t$ of 0.6 p.u. A choice of $\lambda_5 = 0.4$ in figure(7-5.12) achieves a compromise between the resultant system damping and the transient overshoot, and gives a time response which is certainly better than that obtained by the optimal controller designed at $\lambda_5 = 0.07$ (refer to figure(7-4.5)).
Figure (7-5.1) The effect of changing $\lambda_5$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_1 = 0.2$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_1$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_t = 0.2$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_2$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_b = 0.2$ p.u., and is subjected to a 5% step increase in real power.
Figure (7-5.4) The effect of changing $\lambda_3$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_1 = 0.2$ p.u., and is subjected to a 5% step increase in real power.
Figure 7-5.5 The effect of changing $\lambda_4$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_e = 0.2$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_5$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_5 = 0.2$ p.u., and is subjected to a 5\% step increase in real power.
The effect of changing $\lambda_1$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_1 = 0.2$ p.u., and is subjected to a 5\% step increase in real power.
Figure (7-5.6) The effect of changing $\lambda_2$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_+ = 0.2$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_3$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_4 = 0.2$ p.u., and is subjected to a 5\% step increase in real power.
Figure (7-5.10) The effect of changing $\lambda_4$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_t = 0.2$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_5$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_\pm = 0.6$ p.u., and is subjected to a 5% step increase in real power.
The effect of changing $\lambda_5$ on the performance of the system with optimal control. The optimal system is operating initially at rated output power and $x_t = 0.6$ p.u., and is subjected to a 5% step increase in real power.
QUANTITATIVE AND QUALITATIVE CONSIDERATIONS OF DISTURBANCES

In a highly complicated system like the power generating system considered in this thesis, disturbances of any kind and magnitude may occur from time to time during daily operation. A successful design of a feedback controller should be able to bring the system back to a steady state in a satisfactory way whenever it is subjected to such a disturbance. Two types of disturbances are considered:

1. Disturbances which transfer the system from one state to the other (represented by a step demand in the generator real output power or terminal voltage).

2. Disturbances which appear for a very short period of time (represented by an impulse-type disturbance in rotor angle or field voltage).

In case(1), we look at how the system attains its new operating point, and in case(2) at how the effect of an impulse-type disturbance (or the effect of an initial condition) on the system is to be reduced to zero.

For the system with an optimal controller designed at rated output power and $x_t = 0.2 \text{ p.u.}$ (the corresponding weighting factors are given in equation(7-4.2)), figures(7-6.1) to (7-6.2) and (7-6.3) to (7-6.4) show the system time response following disturbances of the types of (1) and (2) respectively. It can be seen that the magnitude of the disturbance affects only the magnitude of the system response, with an unchanged frequency of oscillation and settling time, and that the optimized system performs satisfactorily under the variety of disturbances considered.
When a variety of disturbances is considered, an advantage of a stabilizing signal derived from an optimal control law over that derived from $\Delta P_0$ is that the control law is a linear combination of all the system states; any kinds of disturbances will be reflected in all the states which then generate a control law of sufficient magnitude to affect the system performance. In the case where $\Delta P_0$ is measured directly at the generator terminals and used as the source of the stabilizing signal, its effect is significant when a torque disturbance is considered, but may become insignificant when other kinds of disturbances are considered. This is illustrated by the similarity between the time responses shown in figures (7-6.5) and (7-6.6) for a step change in $\Delta v_r$ (the reference voltage at the summing point of the voltage regulator). This is because a change in $\Delta v_r$, as shown in figure (7-6.5), does not cause a significant change in $\Delta P_0$. The consequence is that the signal $\Delta P_0$ is not of sufficient magnitude to drive the stabilizer into producing a significant effect on the system, which therefore behaves very similarly as the original closed-loop system where no additional stabilizing circuit is incorporated.
Figure (7-6.1) Time response of the system with optimal controller, when subjected to different magnitudes of step demand. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Figure 7.6.2) Time response of the system with optimal controller, when subjected to different magnitudes of step demand. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Time response of the system with optimal controller, when subjected to different magnitudes of initial condition. The overall system is operating initially at rated output power and $x_t = 0.2 \text{ p.u.}$.
Figure (7-6.4) Time response of the system with optimal controller, when subjected to different magnitudes of initial condition. The overall system is operating initially at rated output power and $x_t = 0.2 \text{ p.u.}$.
Figure (7-6.5) Time response of the system with $\Delta P_o$ as the source of the stabilizing signal, when subjected to different magnitudes of step demand. The overall system is operating initially at rated output power and $x_t = 0.2$ p.u.
Figure (7-6.6) Time response of the original closed-loop system, operating initially at rated output power and $x_t = 0.2$ p.u., when subjected to different magnitudes of step demand.
SUMMARY

Under the assumption that all the system state-variables are measurable, the application of Linear Optimal Control Theory to a power system has been shown to be capable of stabilizing an otherwise unstable system, and of improving its closed-loop operation dramatically. A fix controller, designed optimally at one value of tie-line reactance, is capable of achieving stable operation for a wider range of $x_t$ than that of the system without control. This range can readily be shifted to suit any predetermined range of $x_t$ for stable operation, by using a fix controller designed optimally at other suitable value of $x_t$. Due to the small variations of the elements of the optimal-controller-gain matrices between different optimal controllers designed at different values of $x_t$, an optimal controller constructed at one value of $x_t$ can, in actual implementation, be readily adjusted to operate optimally at other values of $x_t$ without changing the power levels of its amplifiers.

Unsuitably chosen weighting factors in the performance index have been shown possibly to worsen the resultant system performance. A choice of weighting factors has therefore to achieve a compromise between the resultant system damping and the transient overshoot.

Finally, a stabilizing signal derived from an optimal control law is more versatile than that derived from $\Delta P_e$, when disturbances of different forms and magnitudes are considered.
CHAPTER 8

LINEAR SYSTEM OPTIMIZATION
WITH PRESCRIBED DEGREE OF STABILITY

The roots (or the eigenvalues) of the characteristic equation of a system are obtained by solving for $\lambda$ in the equation $|A - \lambda I| = 0$, where $I$ is a unit matrix and $A$ the system matrix of a system described by $n$ 1st-order differential equations. These eigenvalues, representing the behaviour of the small-signal time response of the system, are best portrayed in a 2-dimensioned complex s-plane. Their locations in the left or the right hand half of this plane determine whether or not the system is stable. Eigenvalues located to the left also give an indication of how stable the system is, and in particular the further they are to the left, the more stable is the system.

Many methods of achieving a more stable system by shifting the eigenvalues more to the left of the complex s-plane have been investigated. The equation for a controllable, linear, time-invariant system can be written as

$$x = Ax + Bu$$

and with the control $u$ expressed as $u = Kx$, the resultant system equation becomes

$$x = (A + BK)x$$  \hspace{1cm} (8-2)

The choice of the gain matrix $K$ determines the eigenvalues (hence the performance) of the closed-loop system. However, it is not in
general possible to choose $K$ in order to minimize a quadratic performance index and, at the same time, to achieve arbitrary closed-loop poles. Moussa and Elangovan et al developed algorithms for systematically choosing the matrix $D$ (for a chosen $H$) in the performance index given by the form of equation (7-3.2), to achieve left shifting of the dominant eigenvalues and at the same time minimizing the performance index. Anderson et al., on the other hand, modified the performance index to ensure that the closed-loop eigenvalues were located within a certain region of the complex $s$-plane. Pai and Olguin-Salinas applied Modal Control Theory successfully to shift the dominant eigenvalues of a power system to their prescribed locations in the complex $s$-plane. This Chapter studies the last two methods, namely Optimal Control Theory with a modified performance index and Modal Control Theory when employed in stability considerations of the single-input system described by equation (7-4.1). An impulse-type disturbance (represented by an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.)) is considered throughout the analysis.

8-1 LINEAR SYSTEM OPTIMIZATION WITH REGIONAL RESTRICTION OF THE CLOSED-LOOP POLES IN THE COMPLEX $s$-PLANE

The choice of the gain matrix $K$ in equation (8-2) determines the eigenvalues of $(A + BK)$. The problem in this Section is not to fix precisely the eigenvalues of $(A + BK)$, but rather to ensure that they will lie within a certain region of the complex $s$-plane. Typical regions might be that sector of the left half-plane $\text{Re}[s] < 0$ bounded by straight lines extending from the origin and making
angles $\pm \theta$ with the negative real axis, or they might be that part of the left half-plane to the left of $\text{Re}[s] = -\alpha$, for some $\alpha > 0$.

It is the second type of restriction which is attempted in this Section, and which is achieved not by selecting $K$ through some modification of the procedure used for solving the pole-positioning problem, but by posing a suitable version of the regulator problem. Essentially it is a solution of the regulator problem that provides not merely an asymptotically stable closed-loop system but one with the real parts of all its eigenvalues at least less than $-\alpha$.

8.1.1 Construction of the control law

On the assumption that matrix $D$ is nonnegative definite and that the system described by equation (8-1) is observable, then all the states of the system have some effect on the performance index given by equation (7-3.2). Without these assumptions, there can be unstable modes even when $J$ is finite. Similarly, controllability ensures that the system can be stabilized, and that the positive definiteness of matrix $H$ prevents the control law from becoming unbounded. These discussions establish that an optimal closed-loop system must be asymptotically stable. From here, it turns out that any desired stability margin can be guaranteed by altering the performance index. With the above assumptions, it is clear that both $x(t)$ and $u(t)$ must decay to zero as $t \to \infty$ in order for $J$ to remain finite. If equation (7-3.2) is replaced by

$$J = \frac{1}{2} \int_{0}^{\infty} (x' Dx + u' Hu) e^{2\alpha t} dt \quad (8-1.1)$$
for some $\alpha > 0$, then $x(t)$ and $u(t)$ must decay at least as fast as $e^{-\alpha t}$, in order for $J$ to remain finite. In other words, the closed-loop system must not only be stable but have also the real parts of all its eigenvalues less than $-\alpha$.

To minimize equation (8-1.1) subject to the conditions of equation (8-1), set

$$\dot{x} = xe^{\alpha t} \quad \text{and} \quad \dot{u} = ue^{\alpha t}$$

(8-1.2)

Equation (8-1) is then equivalent to

$$\dot{x} = (A + \alpha I)\dot{x} + B\dot{u} \quad \text{and} \quad \dot{x}(t_0) = e^{\alpha t_0}x_0$$

(8-1.3)

while $(x'Dx + u'Hu)e^{2\alpha t} = \dot{x}'D\dot{x} + \dot{u}'H\dot{u}$. Thus, minimization with respect to equation (8-1) of equation (8-1.1) is equivalent to minimization with respect to equation (8-1.3) of

$$J = \frac{1}{2} \int_0^\infty (\dot{x}'D\dot{x} + \dot{u}'H\dot{u}) dt$$

(8-1.4)

in the following senses:

1. The minimum value of equation (8-1.1) (expressed in terms of $x_0$) is the same as that of equation (8-1.4) (expressed in terms of $\dot{x}(t_0)$, taking account of $\dot{x}(t_0) = e^{\alpha t_0}x_0$).

2. If $\dot{u} = f(\dot{x})$ is the optimal control for equations (8-1.3) and (8-1.4), $u = e^{-\alpha t}f(xe^{\alpha t})$ is the optimal control for equations (8-1) and (8-1.1), and conversely.

The first point is not so significant as the second. For equations (8-1.3) and (8-1.4), the optimal control is

$$\dot{u} = K_\alpha \dot{x}$$

(8-1.5)

where $K_\alpha = -H^{-1}B'Q_\alpha$
and $Q_\alpha$ is the unique nonnegative definite solution of

$$D + (A + \alpha I)'Q_\alpha + Q_\alpha (A + \alpha I) - Q_\alpha BH^{-1}B'Q_\alpha = 0$$

(8-1.7)

The second remark then yields the optimal control for equations (8-1) and (8-1.1) as

$$u = K_\alpha x$$

(8-1.8)

Thus, the construction of the desired control law is essentially no more difficult than for the case when $\alpha = 0$ (this is equivalent to the conventional optimal control problem considered in the previous Chapter).

8-1.2 System Studies

The time response of the single-input system (described by equation(7-4.1)) with optimal controllers designed at rated output power, at different tie-line reactances and for $\alpha = 0$ and 1.0 are shown in figures(8-1.1) to (8-1.3). Together with the computed eigenvalues listed in table(8-1.1), it can be seen that further improvement on the system performance is achieved for $\alpha = 1.0$ over that for $\alpha = 0$ for all three values of $\alpha_t$ considered. The elements of the optimal-controller-gain matrices are, in general, greater for $\alpha = 1.0$ than that for $\alpha = 0$ (compare tables (7-4.2) and (8-1.1)). For $\alpha = 1.0$, some of these elements seem to be prohibitively large. In actual implementation however the configuration of figure(8-1.4b) can be used instead of figure (8-1.4a), so that every element of the $K$ matrix is multiplied by a factor of $10^{-4}$. This approach reduces substantially the amplification factors required by some of the amplifiers of the controller, from a few million to only a few hundred. Similar to
the case for $\alpha = 0$ (see Section(7-4)), it can be seen from table (8-1.1) that the individual elements of the optimal-controller-gain matrices for $\alpha = 1.0$ do not vary very much for different optimal controllers designed at different values of $x_t$. Note that the same weighting factors (i.e. the same matrices $D$ and $H$ in the performance indexes given by equations(7-3.2) and (8-1.1)) as given by equation (7-4.2) have been used for both values of $\alpha$. In the case of $\alpha = 0$, the weighting factors have to be chosen by engineering experience and by trial and error until the system performance is satisfactory (i.e. until the eigenvalues of the closed-loop system matrix have satisfactorily large negative real parts). With the modified approach considered in this Section however any initial choice of inappropriate weighting factors can thus be supplemented by choosing an appropriate value for $\alpha > 0$, because the resultant closed-loop system eigenvalues all have their real parts less than $-\alpha$ anyway.
<table>
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<th>$x_t$(p.u.)</th>
<th>resultant system eigenvalues</th>
<th>optimal-controller-gain matrix $K$ for $\alpha = 1.0$</th>
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<td>-333.34 -142.81</td>
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<td>-4.51 -9.98</td>
<td>$1.145 \times 10^2 1.805 \times 10^6$</td>
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<tr>
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<td>-5.32 -2.66</td>
<td>$-3.071 \times 10^6 -1.259 \times 10^4$</td>
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<td>-2.00±30.01</td>
<td>$1.242 \times 10^3$</td>
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<td>$[-1.648 \times 10^1 -1.256$</td>
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<tr>
<td></td>
<td>-2.92±38.57</td>
<td>$-1.382 \times 10^1 1.097 \times 10^1$</td>
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<td>-27.74±j6.12</td>
<td>$1.427 \times 10^1 1.379 \times 10^1$</td>
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<td>-5.33 -2.00</td>
<td>$-3.150 \times 10^6 -1.084 \times 10^4$</td>
</tr>
<tr>
<td></td>
<td>-1.99 -2.63</td>
<td>$1.054 \times 10^3$</td>
</tr>
</tbody>
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Table 8-1.1 The eigenvalues and optimal-controller-gain matrix of the system with an optimal controller designed under a modified performance index. The overall system is operating at rated output power.
Figure (8-1.1) Time response of the system with optimal control, following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.). The overall system is operating initially at rated output power and $x_t = 0.2$ p.u..
Figure (8-1.2) Time response of the system with optimal control, following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.). The overall system is operating initially at rated output power and $x_t = 0.4$ p.u.
Figure 8-1.3) Time response of the system with optimal control, following an instantaneous change in the load angle ($\Delta \alpha = 0.04$ rad.). The overall system is operating initially at rated output power and $x_t = 0.6$ p.u.
MODAL CONTROL

The central concept of modal control is very simple; it is merely that of generating the input vector of a system by linear feedback of the state vector in such a way that prescribed eigenvalues are associated with the dynamic modes of the resultant closed-loop system.

Consider a simple continuous-time system governed by the 1st-order scalar state equation

\[ x(t) = ax(t) + bu(t) \quad (8-2.1) \]

where \( a \) and \( b \) are real constants. It is obvious that in the absence of control (i.e. when \( u = 0 \)), the state of the system at any time is given by

\[ x(t) = x(0)\exp(at) \quad (8-2.2) \]

where \( \exp(at) \) defines the single dynamic mode of the system (8-2.1).

Now, if \( a \) is positive, it is obvious from equation (8-2.2) that the uncontrolled system will be unstable; also, although the system will be asymptotically stable if \( a \) is negative, the decay \( x(t) \rightarrow 0 \) may not be sufficiently rapid. However, if linear state feedback according to the control law

\[ u(t) = kx(t) \quad (8-2.3) \]

is introduced, equation (8-2.1) clearly assumes the closed-loop form

\[ x(t) = (a + bk)x(t) \quad (8-2.4) \]

where \( k \) is an arbitrary real constant. Since the solution of equation (8-2.4) may be written as

\[ x(t) = x(0)\exp(\rho t) \quad (8-2.5) \]

where \( \rho = a + bk \), it is obvious that \( \rho \) may be assigned any arbitrary
real negative value simply by choosing $k$ in the feedback law (8-2.3) according to the formula

$$k = (\rho - a)/b$$  \hspace{1cm} (8-2.6)

The above argument can readily be extended to multi-mode systems and the theories behind this are well established. A computer algorithm (see Appendix(VI-2)) for applying Modal Control Theory to the single-input system described by equation(7-4.1) is straightforward and summarized as follows:

1. Read in system parameters to form matrices $A$ and $B$
2. Compute the eigenvalues of $A$, eigenvalues $\lambda$ and eigenvectors $V$ of $A'$.
3. Compute the mode-controllability matrix $P = V'B$ as a check for the controllability of each mode.
4. Declare the required eigenvalues $\rho$ for the closed-loop system (this includes shifting only the dominant eigenvalues to their prescribed positions in the complex $s$-plane while leaving the remaining eigenvalues untouched).
5. Compute the proportional-controller gains given by

$$QK_j = \frac{1}{n} \sum_{k=1}^{n} \frac{(\rho_k - \lambda_j)}{(P_j \sum_{k \neq j}^{n} (\lambda_k - \lambda_j))} \quad (j = 1, 2, \ldots, n)$$

where $n$ is the dimension of the system,

- $QK_j$ is the $j$-th element of the $n$-vector $QK$,
- $\rho_k$ is the $k$-th element of the $n$-vector $\rho$,
- $\lambda_j$ is the $j$-th element of the $n$-vector $\lambda$,
- and $P_j$ is the $j$-th element of the $n$-vector $P$.  

(6) Compute the controller gains given by

\[ K_j = \sum_{i=1}^{n} QK_i V_{ji} \quad (j = 1, 2, \ldots, n) \]

where \( K_j \) is the \( j \)-th element of the \( n \)-vector \( K \),
and \( V_{ji} \) is the \( j \)-\( i \)-th element of the \((n \times n)\) matrix \( V \).

(7) Form the closed-loop system

\[ x = (A + BK)x \]

and compute the eigenvalues and eigenvectors of \((A + BK)\).
Compute and plot the time response of the closed-loop system
for a nonzero initial condition.

**8-2.1 System Studies**

Figures(8-2.1) to (8-2.3) show that the controllers
designed by Modal Control Theory give a very much better performance
than that of the original closed-loop system, and are even better
than the conventional optimal controllers (for \( \alpha = 0 \)) for the
various values of \( x_t \) considered. The controller-gain matrices and
the resultant closed-loop eigenvalues are shown in table(8-2.1).
In this table, the underlined eigenvalues are those which have been
shifted to their new positions in the complex s-plane for damping
improvement, while the rest of the eigenvalues, apart from the two
additional eigenvalues constituted by the stabilizer, remain the
same as those of the original closed-loop system (compare tables
(8-2.1) and (7-4.1)). The P-Q charts in figures(8-2.4) to (8-2.6)
show that the stability region of the original closed-loop system
reduces as \( x_t \) increases, so that employment of any one of the three
types of controllers, namely the controllers designed by Optimal
Control Theory for \( \alpha = 0 \) and 1.0 and by Modal Control Theory,
becomes more and more justified.
### Table (8.2.1)
The eigenvalues and controller-gain matrix of the system with a controller designed by Modal Control Theory. The overall system is operating at rated output power.

<table>
<thead>
<tr>
<th>$x_t$ (p.u.)</th>
<th>Resultant System Eigenvalues</th>
<th>Controller-Gain Matrix K</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-2.08±j314.14</td>
<td>1.379 -1.564 -1.932 x 10⁴</td>
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<tr>
<td>-3.00±j10.0</td>
<td>-6.032 2.023 x 10¹</td>
<td></td>
</tr>
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<td>1.936 x 10¹ 6.324</td>
<td></td>
</tr>
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<td>-2.66±j4.24</td>
<td>-2.435 0.026 0.454</td>
<td></td>
</tr>
<tr>
<td>-4.01±j1.16</td>
<td>7.828 x 10³ 1.235 x 10³</td>
<td></td>
</tr>
<tr>
<td>-333.33 -142.83</td>
<td>3.415 x 10⁴ 1.307 x 10²</td>
<td></td>
</tr>
<tr>
<td>-2.21 -9.98</td>
<td>1.935 x 10⁶ -3.289 x 10⁴</td>
<td></td>
</tr>
<tr>
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<td>-1.558 x 10⁴ 1.643 x 10³</td>
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</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>0.4</td>
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<td></td>
</tr>
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<td></td>
</tr>
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</tr>
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</tr>
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<td>3.820 x 10² 5.464 x 10⁵</td>
<td></td>
</tr>
<tr>
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<td></td>
</tr>
<tr>
<td>-2.50 -2.01</td>
<td>3.662 x 10³</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
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<td></td>
</tr>
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<td>-2.105 x 10¹ 0.496</td>
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</tr>
<tr>
<td>-5.34 -2.50</td>
<td>-1.222 x 10⁷ -3.698 x 10⁴</td>
<td></td>
</tr>
<tr>
<td>-2.00±j0.0047</td>
<td>4.167 x 10³</td>
<td></td>
</tr>
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</table>
Figure 8-2.1 Time response of the system operating initially at rated output power and $x_b = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure (8.2.2) Time response of the system operating initially at rated output power and $x_+ = 0.4$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure (8-2.3) Time response of the system operating initially at rated output power and $x_+ = 0.6$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure (8-2.4) P-Q chart for the system operating at $x_t = 0.2 \text{ p.u.}$
Figure 9-2.5  P-Q chart for the system operating at $x_t = 0.4$ p.u.

a  original closed-loop system
b  with controller designed optimally at rated output power and $x_t = 0.4$ p.u. ($\alpha = 0$)
c  with controller designed optimally at rated output power and $x_t = 0.4$ p.u. ($\alpha = 1.0$)
d  with controller designed by Modal Control Theory at rated output power and $x_t = 0.4$ p.u.
2. CC

unstable → stable

Figure (8-2.6) P-Q chart for the system operating at $x_t = 0.6 \text{ p.u.}$.
SUMMARY

A modified application of Optimal Control Theory has been shown to have the effect of restricting the closed-loop eigenvalues of a system to within a freely chosen region in the complex s-plane. The computer algorithm for $\alpha > 0$ is no more difficult than that for $\alpha = 0$, and both consume about 280 s of computational time. Since for $\alpha > 0$, the closed-loop eigenvalues of a stable system all possess negative real parts less than $-\alpha$, more freedom and time saving are therefore possible in the process of deciding the weighting factors for the performance index. However, in heuristic terms, the faster a state is returned to zero after a disturbance (i.e. for higher values of $\alpha$), the greater is the control power required. This has been reflected in the requirement of higher controller gains. Therefore a practical limitation on the value of $\alpha$ is imposed by the power-handling capacity of the amplifiers available.

Modal control represents a direct way of affecting the stability of a system without any time being consumed in guessing the weighting factors as for the case of optimal control. With modal control, controllable eigenvalues of a system can be shifted to their prescribed locations instead of within a prescribed region in the complex s-plane. Those eigenvalues which have not been shifted for damping-improvement purpose will remain unchanged. However, the computational time of the algorithm at about 380 s is longer than that for optimal control.

Finally, the additional effort and expense in constructing a controller designed by any one of the three methods considered (i.e. the controllers designed by Optimal Control Theory for $\alpha = 0$ and $\alpha > 0$ and by Modal Control Theory) become more justified when longer transmission links are required.
CHAPTER 9

STATE ESTIMATION AND THE DESIGN
OF A MINIMUM NORM SUBOPTIMAL CONTROLLER

The methods of designing the controllers in Chapters 7 and 8 resulted in a situation in which the control law is a function of all the states of the system. This would be satisfactory provided that the states are either all accessible or available for measurement. However, in a practical system, this measurement is not always possible, and even if it were so it is likely to be economically unacceptable. This situation has led to many investigations into methods for overcoming the problems arising from the unmeasurable states. Thus, Luenberger has considered the design of a full-observer to construct the entire state vector when the output of the plant, which represents only part of the state vector, is available for direct measurement. This observer design has a certain degree of mathematical simplicity, as well as a degree of redundancy, when some of the states are in fact measurable. A low-order observer, on the other hand, estimates only those states which are either unmeasurable or more desirable to estimate than to measure directly. This technique requires the solution of an algebraic matrix equation of the form $QT - TA = -SC$. De Sarkar et al and Kosut considered the design of a suboptimal controller by the minimum norm nearness criterion, in which the minimum norm suboptimal control vector is merely the optimal control
vector with the terms involving the unmeasurable states deleted. However, such a controller does not guarantee stable operation of the resultant closed-loop system. Hosking\textsuperscript{47} considered a modified approach to the design of a suboptimal controller in which the control law does not depend on all the states, while Elmetwally\textsuperscript{12} et al reported experimental results on the implementation of an optimal controller with respect to a reduced system model which retains only the dominant and measurable states. This Chapter studies the first three methods, namely the full-observer, the low-order observer and the minimum norm suboptimal controller, when employed in the single-input system described by equation (7-4.1) with optimal control.

9-1 OBSERVATION OF THE ENTIRE STATE VECTOR

The state-space representation of a system has some conceptual advantages over the more conventional transfer function representation. The state vector contains sufficient information completely to summarize the past behaviour of the system, and the future behaviour is governed by simple 1st-order differential equations. It is however very likely that only the inputs and the outputs of the system can be measured, while the state-variables may have no physical meaning. It is therefore necessary to estimate the state-variables, knowing the system state equations and the system inputs and outputs. An observer, driven by both the control system inputs and outputs, is required to produce outputs (or estimated system state-variables) which converge as quickly as possible to the true values of the system state-variables, and at
the same time to have no effect on the locations of the poles of the original control system.

Suppose the control system to be observed is governed by

\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]  

(9-1.1)

where A, B and C are constant matrices, x is the system state vector, u the input vector and y the output vector. An observer, driven by both the system inputs and outputs, can be represented by

\[ \dot{h} = Qh + Ru + Sy \]  
\[ h = Tx \]  

(9-1.3)

where Q, R, S and T are constant matrices and h is the state vector of the observer. The time derivative of equation (9-1.3) gives

\[ \dot{h} = Tx \]  

(9-1.4)

which, on substitution of equations (9-1.1) to (9-1.3), becomes

\[ QTx + Ru + SCx = TAx + TBu \]  

(9-1.5)

Equating coefficients of x and u in this equation gives

\[ QT - TA = -SC \]  

(9-1.6)

and

\[ TB = R \]  

(9-1.7)

respectively. In a full-observer design, the entire state vector x is estimated. When the order of the system is n, there are n states in the h vector as well as in the x vector and therefore T is an (n x n) square matrix. For mathematical simplicity, T is chosen arbitrarily to be a unit matrix (T = I) and equations (9-1.6) and (9-1.7) become
Q - A = - SC \hspace{1cm} (9-1.8)

and \hspace{1cm} B = R \hspace{1cm} (9-1.9)

respectively. Substituting equations (9-1.8) and (9-1.9) into equation (9-1.2) gives

\[ h = (A - SC)h + Bu + Sy \] \hspace{1cm} (9-1.10)

which describes the dynamics of a full-observer.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{schematic_layout.png}
\caption{Schematic layout showing the employment of a full-observer in a control system}
\end{figure}

Figure (9-1.1) is a schematic layout showing the employment of a full-observer in a control system, in which the control law \( u \) is a linear combination of the estimated states and is written as

\[ u = Kh \] \hspace{1cm} (9-1.11)

Substitution into equations (9-1.1) and (9-1.10) gives

\[ x = Ax + BKh \] \hspace{1cm} (9-1.12)

\[ h = (A - SC)h + BKh + SCx \] \hspace{1cm} (9-1.13)

respectively, or in composite matrix form.
which represents the dynamics of an overall closed-loop system comprising a feedback controller, a full-observer and the system to be controlled.

9-1.1 Method of choosing the full-observer dynamics

It is necessary that matrix $S$ in equation (9-1.10) should be chosen such that the eigenvalues of $(A - SC)$ are large negative quantities. The more negative they are, the more rapidly $h$ will approach $x$, but inevitably the larger the elements of $S$ will become. A systematic way of choosing $S$ makes use of Optimal Control Theory for a linear system, described arbitrarily as

$$z = (A' - C'S')z \quad (9-1.15)$$

and since $(A' - C'S') = (A - SC)'$, and the eigenvalues of a matrix are the same as those of its transpose, the vector $z$ in equation (9-1.15) will approach zero rapidly if the eigenvalues of $(A - SC)$ have large negative values. Equation (9-1.15) is rewritten as

$$z = A'z + C'u_1 \quad (9-1.16)$$

where $u_1 = -S'z \quad (9-1.17)$

and a performance index is defined as

$$J = \frac{1}{2} \int_0^\infty (z'Iz + u_1'\Lambda u_1)dt \quad (9-1.18)$$

where $I$ is a unit matrix and $\Lambda$ a diagonal matrix selected by trial and error. The optimal control problem posed by equations (9-1.16)
and (9-1.18) results in a Matrix Ricatti Equation

\[ I + Q_1A' + A_1Q_1 - Q_1C'C^{-1}Q_1 = 0 \]  \hspace{1cm} (9-1.19)

and an optimal control law

\[ u_1 = -C^{-1}Q_1z \]  \hspace{1cm} (9-1.20)

Comparing equations (9-1.17) and (9-1.20) gives

\[ S = (C^{-1}Q_1)' \]  \hspace{1cm} (9-1.21)

The choice of matrix \( S \) by the above approach guarantees that 
\( (A - SC) \) has negative eigenvalues, the magnitudes of which are 
governed by the choice of matrix \( \lambda \).

9.1.2 System studies

The single-input system described by equation (7-4.1) 
employing successively three different optimal controllers
(for \( \alpha = 1.0 \)), designed at (a) rated output power and \( x_t = 0.2 \) p.u.,
(b) rated output power and \( x_t = 0.4 \) p.u. and (c) rated output power 
and \( x_t = 0.6 \) p.u., is used to demonstrate the design of a full-
observer (details of the three optimal controllers are listed in 
table (8-1.1)). Assuming that only two of the system state-variables 
\( \Delta i_{kd} \) and \( \Delta i_{kq} \) are unmeasurable, the remaining 16 state-variables 
constitute the elements of the output vector \( y \) in equation (9-1.1)
(when more state-variables are assumed to be unmeasurable less 
elements appear in the output vector, but the basic design procedure 
of a full-observer remains unchanged). \( y \) is therefore a \((16 \times 1)\) 
vector and its associated output matrix \( C \) has dimension \((16 \times 18)\), 
with elements which are given by

\[ C(i,i) = 1.0 \quad (i = 1,2,\ldots,5) \]  \hspace{1cm} (9-1.22)

\[ C(j,j+2) = 1.0 \quad (j = 6,7,\ldots,16) \]
Having chosen matrix C, all the diagonal elements of the (16 x 16) matrix A in equation(9-1.18) were finally chosen to be 0.0001 to give the satisfactory full-observer design result in figure(9-1.2) for system(a) mentioned above (the design algorithm is shown in Appendix(VI-3)). The corresponding time response is shown in figures(9-1.3a) to (9-1.3g). It can be seen that the observed state-variables converge to the true system state-variables in under 0.1 s, and that no significant discrepancies exist between this response and that shown in figure(8-1.1) where all the state-variables are assumed to be measurable. With the same matrix A, full-observer design results for systems(b) and (c) mentioned above are also shown in figures(9-1.4) and (9-1.5) respectively. From these results, it can be seen that all the eigenvalues of each full-observer have large negative real parts but with reasonable magnitudes for the elements of the matrix S. Furthermore, the full-observer has no effect on the eigenvalues of the optimal-controller system, other than to add its own eigenvalues to the overall observer-optimal-controller system.
Figure(9-1.2) Design results of a full-observer for the system with optimal control. The overall system is operating at rated output power and $x_t = 0.2$ p.u.
Figure (9-1.3a) Time response of the observer-optimal-controller system operating initially at rated output power and \( x_t = 0.2 \) p.u., following an instantaneous change in the load angle (\( \Delta \delta = 0.04 \) rad.).
Figure (9-1.3b) Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\delta = 0.04$ rad.).
Figure (9-1.3c) Time response of the observer-optimal-controller system operating initially at rated output power and \( x_t = 0.2 \) p.u., following an instantaneous change in the load angle \( (\Delta \delta = 0.04 \text{ rad.}) \).
Figure 9-1.3d. Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.).
**Figure 9.1.3e** Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure 9-1.3f Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure 9-1.3g) Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure (9-14) Design results of a full-observer for the system with optimal control. The overall system is operating at rated output power and $x_t = 0.4$ p.u.
Design results of a full-observer for the system with optimal control. The overall system is operating at rated output power and $x_t = 0.6$ p.u.
The full-observer design in the above Section achieved a certain degree of mathematical simplicity by letting $T = I$. Further examination reveals that there is a certain redundancy in the full-observer, which constructs the entire state vector, when in fact the output of the system, representing part of the state vector, is available for measurement. This redundancy however can be eliminated at the expense of implementing a matrix inversion. In fact, if the order of the system to be observed is $n$ and there are $l$ measurable system state-variables ($1 \leq l \leq n$), then a low-order observer of order $n-l$ can be constructed.

For the same system described by equation (9-1.1), a low-order observer is also represented by equations (9-1.2) to (9-1.7). Since the order of the observer is now $n-l$, there are only $n-l$ states in the observer state vector $h$. This means that $T$ is an $(n-l) \times n$ matrix and cannot be inverted. To enable an estimate $\hat{x}$ of the state vector $x$ to be obtained, a new square matrix $T_1$ given by

$$
\hat{x} = \begin{bmatrix} T & C \end{bmatrix}^{-1} \begin{bmatrix} h \\ y \end{bmatrix} = T_1^{-1} \begin{bmatrix} h \\ y \end{bmatrix} = W \begin{bmatrix} h \\ y \end{bmatrix}^{n-l} \tag{9-2.1}
$$

is introduced, where $T_1 = \begin{bmatrix} T \\ C \end{bmatrix}$ and $W = T_1^{-1}$. Figure (9-2.1) is a schematic layout showing the employment of a low-order observer in a control system, in which the control law is a linear combination of the estimated states and is written as

$$
u = K\hat{x} \tag{9-2.2}
$$

which, on substitution of equation (9-2.1), becomes
Figure (9-2.1) Schematic layout showing the employment of a low-order observer in a control system

\[ u = KW \begin{bmatrix} h \\ y \end{bmatrix} \]  

(9-2.3)

By partitioning matrix \( W \) such that \( W = \begin{bmatrix} W_1 & W_2 \\ n-l & 1 \end{bmatrix} \),

equation (9-2.3) becomes

\[ u = K \begin{bmatrix} W_1 & W_2 \end{bmatrix} \begin{bmatrix} h \\ y \end{bmatrix} = KW_1h + KW_2y \]  

(9-2.4)

Substitution into equations (9-1.1) and (9-1.2) gives

\[ \dot{x} = Ax + BKW_1h + BKW_2Cx \]

and

\[ \dot{h} = Qh + RKW_1h + RKW_2Cx + SCx \]

respectively, or in composite matrix form.
\[
\begin{pmatrix}
  \dot{x} \\
  \dot{h}
\end{pmatrix} =
\begin{cases}
  n \left\{ \begin{pmatrix}
    A + BKW_2C & BKW_1 \\
    RKW_2C + SC & Q + RKW_1
  \end{pmatrix} \right\} \\
  n-l \left\{ \begin{pmatrix}
    & n \\
    & n-l
  \end{pmatrix} \right\}
\end{cases}
\begin{pmatrix}
  x \\
  h
\end{pmatrix}
\]

(9-2.5)

which represents the dynamics of an overall closed-loop system comprising a feedback controller, a low-order observer and the system to be controlled.

With arbitrarily chosen matrices \( Q \) and \( S \) in equation (9-1.2), a method for solving for matrix \( T \) in equation (9-1.6) is required. Many methods of solution have been investigated\(^{48,49}\) but one, which makes use of the fact that the algebraic matrix equation \( QT-TA=-SC \) is equivalent to a system of \((n-1) \times n\) scalar equations in the elements of matrix \( T \), is shown in Appendix VII. This specially developed program gives a high accuracy of solution, although it consumes excessive computational time when the order of the system and the number of unmeasurable states increase. Having solved for matrix \( T \), matrix \( R \) can readily be obtained in equation (9-1.7).

9-2.1 System studies

The single-input system with the three different optimal controllers considered in Section (9-1.2) is also used here to demonstrate the design of a low-order observer. Assuming again that only the state-variables \( \Delta i_{kd} \) and \( \Delta i_{kq} \) are unmeasurable, the output matrix \( C \) in this Section is therefore the same as that given in equation (9-1.22). By a choice of

\[
Q = \begin{bmatrix}
-10 & 0 \\
0 & -20
\end{bmatrix}
\]

(i.e. eigenvalues of the low-order observer will be -10 and -20)
and \( S = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}_{16} \)

in equation (9-1.2) a satisfactory low-order observer design resulted, as given in figure (9-2.2) for system (a) (the design algorithm is shown in Appendix (VI-4)). The corresponding time response is shown in figures (9-2.3a) to (9-2.3g). It can be seen that in the case where the states are measurable, the observer outputs actually coincide with the true system states. In the case where the states are unmeasurable, the observer outputs take about 0.5 s to converge to the true system states. Furthermore, there are no significant discrepancies between this time response and that shown in figure (8-1.1) where all the states are assumed to be measurable. For the same matrices \( Q \) and \( S \), low-order observer design results for systems (b) and (c) are also shown in figures (9-2.4) and (9-2.5). From these results, it can be seen that each low-order observer only adds its own eigenvalues to the control system and the original eigenvalues are unaffected. Furthermore, a high accuracy in the solution of the algebraic matrix equation \( QT - TA = - SC \) is also evident when the difference in the numerical values of its left and right hand sides is evaluated.
Figure (9-2.2) Design results of a low-order observer for the system with optimal control. The overall system is operating at rated output power and $x_t = 0.2$ p.u.
Figure (9-23a) Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
a and b are coincident

---

**Figure 9-2.3b** Time response of the observer-optimal-controller system operating initially at rated output power and \( x_t = 0.2 \) p.u., following an instantaneous change in the load angle (\( \Delta \phi = 0.04 \) rad.).
Time response of the observer-optimal-controller system operating initially at rated output power and \( x_i = 0.2 \text{ p.u.} \), following an instantaneous change in the load angle (\( \Delta \delta = 0.04 \text{ rad.} \)).
Figure 9.2.3d. Time response of the observer-optimal-controller system operating initially at rated output power and $x_2 = 0.2 \text{ p.u.}$, following an instantaneous change in the load angle ($\Delta \delta = 0.04 \text{ rad.}$).
Figure(9-2.3e) Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.).
Figure (9-2.3f) Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta t = 0.04$ rad.).
Figure 9-2.3g Time response of the observer-optimal-controller system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Design results of a low-order observer for the system with optimal control. The overall system is operating at rated output power and \( x_t = 0.4 \text{ p.u.} \).
Design results of a low-order observer for the system with optimal control. The overall system is operating at rated output power and $x_t = 0.6 \text{ p.u.}$
9.3 **DESIGN OF A SUBOPTIMAL CONTROLLER BY MINIMUM NORM NEARNESS CRITERION**

When all the states of a linear, time-invariant system represented by

\[ x = Ax + Bu \]  \hspace{1cm} (9-3.1)

are available for feedback, the optimal control of this system, with a quadratic performance index, can be written as

\[ u = Kx \]  \hspace{1cm} (9-3.2)

where \( K \) is the \((m \times n)\) optimal-controller-gain matrix. Now, if the elements of the control vector are constrained to be a time-invariant linear combination of the measurable states of the system only, then a matrix equation may be written as

\[ z = Mx \]  \hspace{1cm} (9-3.3)

where \( z \) is a \((q \times 1)\) vector \((q \leq n)\), consisting of the measurable states of the system, and \( M \) is an \((q \times n)\) matrix termed the measurement matrix. From the above definition, it is clear that the control vector \( u \) must be a linear combination of the elements of \( z \). Thus let

\[ u = Hz \]  \hspace{1cm} (9-3.4)

where the elements of the \((m \times q)\) matrix \( H \) are the design parameters of the required controller. Combining equations (9-3.3) and (9-3.4) gives

\[ u = Fx \]  \hspace{1cm} (9-3.5)

where the \((m \times n)\) controller matrix \( F \) is given by

\[ F = HM \]  \hspace{1cm} (9-3.6)
Thus equations (9-3.5) and (9-3.6) specify the control structure constraint on the control \( u \) or on the matrix \( F \).

Using the minimum norm nearness criterion, a suboptimal control is derived from an optimal control by minimizing the norm of the difference between the optimal- and suboptimal-controller-gain matrices, and it can be shown that a minimum norm suboptimal-controller-gain matrix is given by \(^{13, 46}\)

\[
F = KM' (MM')^{-1} M
\]

(9-3.7)

Since the measurable state \( z \) is a set of the state \( x \), then \( x \) could be arranged such that the first \( q \) elements of \( x \) are the measurable state \( z \). That is let

\[
x = \begin{bmatrix} z \\ \ldots \\ x_a \end{bmatrix}
\]

(9-3.8)

where \( x_a \) contains the \( n-q \) unmeasurable states. Since

\[
z = Mx
\]

(9-3.9)

the measurement matrix \( M \) can be partitioned such that

\[
M = \begin{bmatrix} I & 0 \\ q & n-q \end{bmatrix}
\]

(9-3.10)

The arrangement of \( M \) in equation (9-3.10) leads to \( MM' = I \) and equation (9-3.7) therefore reduces to

\[
F = KM'M
\]

\[
= KM_o
\]

(9-3.11)

where

\[
M_o = M'M = \begin{bmatrix} M \\ \ldots \\ 0 \end{bmatrix}^{q}_{n-q}
\]

(9-3.12)
By substituting equation (9-3.11) into equation (9-3.5), a suboptimal control vector is obtained as

\[ u = Fx = K_0 x \]

which simply means that a minimum norm suboptimal control vector is merely the optimal control vector with the terms involving the unmeasurable states deleted. However, such a controller does not guarantee stable operation for the closed-loop suboptimal system. Therefore, when adopting this form of control, the stability of the resultant system must be tested.

9-3.1 System studies

Figures (9-3.1) to (9-3.3) show the time response of the single-input system employing the three optimal controllers considered in the previous Sections, and the correspondingly derived suboptimal controllers (by deleting those terms in the optimal-controller-gain matrices associated with the states \( \Delta i_{kd} \) and \( \Delta i_{kq} \)). It can be seen that, in general, the performance of the suboptimally controlled system is inferior to that of the optimally controlled system. However, the significance of these suboptimal controllers is that they provide stable operation for all the three values of \( x_t \) considered. At low value of \( x_t \) (\( x_t = 0.2 \) p.u.), the employment of a suboptimal controller results in a control system performance which is worse than that of the original closed-loop system (compare figures (8-2.1) and (9-3.1)). Only when the values of \( x_t \) are such that the original closed-loop system is operating close to its stability limit (\( x_t = 0.4 \) p.u.)
or even beyond it into the unstable region \( x_t = 0.6 \text{ p.u.} \), are these suboptimal controllers not only easier to construct than their corresponding optimal controllers, but are also capable of providing stable and better damped system.
Figure (3.1) Time response of the system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure 9.3.2 Time response of the system operating initially at rated output power and $x_t = 0.4$ p.u., following an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.).
Figure 9-3.3  Time response of the system operating initially at rated output power and $x_0 = 0.6$ p.u., following an instantaneous change in the load angle $(\Delta \delta = 0.04$ rad.).
2-4 SUMMARY

The construction of the state vector of a linear, time-invariant system has been shown to be possible when its inputs and outputs are accessible. The observer which performs this task is itself a linear system with arbitrary time-constants. In practice, a feedback system can first be designed, based on the assumption that all the states are measurable, followed by the incorporation of an observer to construct either the entire state vector or merely that of the unmeasurable states, without affecting the locations of the poles of the original control system. The observer simply adds its own poles to the overall observer-controller system. A full-observer, which constructs the entire state vector, has a certain redundancy when some of the states are in fact measurable. A low-order observer, on the other hand, constructs only the unmeasurable states and has some advantages over a full-observer. These are:

(1) Complete freedom in choosing its dynamics (i.e. its eigenvalues can be chosen exactly according to wish).

(2) A simpler equipment.

(3) The computational time of the design algorithm for a low-order observer is about 300 s which is considerably less than the 580 s required for the design algorithm for a full-observer.

A suboptimal controller, designed by a minimum norm nearness criterion, represents a direct way of overcoming the problems arising from the inaccessibility of some of the system
states. It is easier to construct than its corresponding optimal controller, and its employment makes all the effort and expense in the construction of an observer unnecessary. However, it does not provide good system performance when the value of the tie-line reactance is small. Only when the tie-line reactance is large will the benefits to be gained from its employment increase.
CHAPTER 10

2-INPUT OPTIMAL CONTROL SYSTEMS

10-1 ADDITIONAL CONTROL THROUGH THE GOVERNING SYSTEM

Single-input control systems have been considered in Chapters 6 to 9, where a signal source ($P_o$ or $v_{sum}$) drives a stabilizer to produce a stabilizing signal which is fed into the summing junction of the voltage regulator. In this chapter, an additional signal ($v_{g3}$) is fed into the summing junction of the governing system. This signal is derived from a signal source (called $v_{gsum}$) through a transfer function similar to that considered in Section 6-2.4 and a servo motor, represented by $K_{g3}/(1+pT_{g3})$ in Figure (10-1.1). The signal source $v_{gsum}$, together with $v_{sum}$, then constitute the two inputs to the system. Optimal Control Theory is applied to design a state feedback controller for this 2-input system.

Figure (10-1.1) Schematic layout with an additional signal in the governing system
The equations for the stabilizing circuit shown in figure (10-1.1) are

\[ v_{g1} = \frac{K_{g1}}{1+pT_{g1}} v_{gsum} \]  
\[ v_{g2} = \frac{K_{g2} p}{1+pT_{g2}} v_{g1} \]  
and \[ v_{g3} = \frac{K_{g3}}{1+pT_{g3}} v_{g2} \]  

At the summing junction,

\[ u_t = Y_o + v_{g3} - G_1 p \delta \]  

After linearization and rearrangement, equations (10-1.1) to (10-1.4) become respectively

\[ \Delta v_{g1} = -\frac{1}{T_{g1}} \Delta v_{g1} + \frac{K_{g1}}{T_{g1}} \Delta v_{gsum} \]  
\[ \Delta v_{g2} = -\frac{1}{T_{g2}} \Delta v_{g2} + \frac{K_{g2}}{T_{g2}} \Delta v_{g1} \]  
\[ \Delta v_{g3} = -\frac{1}{T_{g3}} \Delta v_{g3} + \frac{K_{g3}}{T_{g3}} \Delta v_{g2} \]  
and \[ \Delta u_t = \Delta v_{g3} - G_1 \Delta \delta \]  

Substituting equation (10-1.5) into equation (10-1.6) gives

\[ \Delta v_{g2} = -\frac{1}{T_{g2}} \Delta v_{g2} - \frac{K_{g2}}{T_{g1}} \Delta v_{g1} + \frac{K_{g1} K_{g2}}{T_{g1}} \Delta v_{gsum} \]  

Since an additional signal is fed into the summing junction of the governing system, the first equation in equation (3-2.10), which describes the linearized model of this system, needs to be modified.
This equation is firstly rewritten as
\[ \Delta Y = \frac{G_2}{(1+pT_1)(1+pT_2)} \Delta u_t \]  
(10-1.10)

which, on substitution of equation(10-1.8), becomes
\[ \Delta Y = \frac{G_2}{(1+pT_1)(1+pT_2)} \left( \Delta v_{g3} - G_4 \Delta \delta \right) \]
or
\[ \Delta y = -\frac{T_1+T_2}{T_1T_2} \Delta y - \frac{1}{T_1T_2} \Delta Y - \frac{G_4}{T_1T_2} \Delta \delta + \frac{G_2}{T_1T_2} \Delta v_{g3} \]  
(10-1.11)

where \( \Delta y = \Delta Y \). If there is no additional signal in the governing system, the last term on the right-hand side of equation(10-1.11) is deleted, leaving this equation as

\[ \Delta y = -\frac{T_1+T_2}{T_1T_2} \Delta y - \frac{1}{T_1T_2} \Delta Y - \frac{G_4}{T_1T_2} \Delta \delta \]  
(10-1.12)

Again, if the signal \( \Delta \delta \) is not fed into the governing system (i.e. the governing system is in open-loop operation), then equation (10-1.12) is further reduced to

\[ \Delta y = -\frac{T_1+T_2}{T_1T_2} \Delta y - \frac{1}{T_1T_2} \Delta Y \]  
(10-1.13)

which is the same equation as considered in equation(3-3.5).

Equation(10-1.11) can thus be regarded as a modified form of equation(10-1.13), taking into account the feedback signal \( \Delta \delta \) and the additional signal \( \Delta v_{g3} \).

Using equation(10-1.11) and adding equations(10-1.5), (10-1.7) and (10-1.9) into the single-input system described by equation(7-4.1), the resultant 2-input system has an order of 21 and in its state-space equation
\[ x = Ax + Bu \quad (10-1.14) \]

the first 18 state-variables of \( x \) are the same as those in equation (7-4.1), and the 19th, 20th and 21st state-variables are \( \Delta v_{g1}, \Delta v_{g2} \)

and \( \Delta v_{g3} \) respectively. \( A \) and \( B \) are respectively \((21 \times 21)\) and \((21 \times 2)\) matrices, the elements of which are shown in Appendix(IV-6). \( u \) is a \((2 \times 1)\) vector and \( u = \begin{bmatrix} \Delta v_{sum} \\ \Delta v_{gsam} \end{bmatrix} \).

For an optimal controller design, a quadratic performance index similar to equation(7-3.1) is taken as

\[
J = \frac{1}{2} \int_{0}^{\infty} \left( \lambda_1 \Delta P_o^2 + \lambda_2 \Delta v_t^2 + \lambda_3 \Delta \delta^2 + \lambda_4 \Delta \delta^2 + \lambda_5 \Delta u_1^2 \\
+ \lambda_6 \Delta u_2^2 \right) dt
\]

\[
= \frac{1}{2} \int_{0}^{\infty} (x'Pdx + u'Hu)dt
\]

where \( \Delta u_1 = \Delta v_{sum} \) and \( \Delta u_2 = \Delta v_{gsam} \), and matrix \( D \) is constructed in exactly the same way as in Section(7-4), except that its dimension is now \((21 \times 21)\). Because there are two inputs to the system, matrix \( H \) is not now a scalar but has dimension \((2 \times 2)\), with its elements given by \( H(1,1) = \lambda_5 \) and \( H(2,2) = \lambda_6 \).

10-1.1 System Studies

By taking

\[
K_{g1} = K_{g2} = 1.0
\]

\[
K_{g3} = 2.0
\]

and \( T_{g1} = T_{g2} = T_{g3} = 0.1 \) s,

the weighting factors of equation(10-1.15) were chosen as
\[ \lambda_1 = \lambda_2 = 1.0 \]
\[ \lambda_3 = \lambda_4 = 0.1 \]
\[ \lambda_5 = 0.07 \text{ and } \lambda_6 = 0.01 \]

to give the satisfactory time response of the 2-input system, operating at rated output power and three different values of \( x_t \), in figures(10-1.2) to (10-1.4). It can be seen that the performance of the 2-input optimal system is better than the corresponding result for the single-input optimal system. Of course, more amplifiers are necessary for the construction of the required controller, which is now represented by a \((2 \times 21)\) gain matrix. In Section(8-1), a further improvement over the single-input optimal system (for \( \alpha = 0 \)) is achieved by modifying its associated performance index. This requires higher amplification factors for the amplifiers of the resultant controller (see table(8-1.1)). Here, another option for achieving the further improvement is provided, whereby the amplification factors of the required controller amplifiers are just about the same as that of the conventional optimal controller for the single-input optimal system (compare tables(7-4.2) and (10-1.1)). However, when the optimal controllers for the 2-input system are used suboptimally, at other values of output power and tie-line reactance, an unfavourable consequence is revealed in the P-Q charts shown in figures(10-1.5) to (10-1.7), wherein the stable region available for low output power operation (i.e. area around the origin in the P-Q chart) is very much reduced.
<table>
<thead>
<tr>
<th>$x_t$(p.u.)</th>
<th>elements of optimal-controller-gain matrix $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$\begin{bmatrix} 0.312 &amp; -0.459 &amp; -1.810 &amp; -0.586 \ 1.928 \times 10^1 &amp; 1.459 &amp; -1.772 \times 10^1 &amp; -1.147 \times 10^1 \ 1.820 &amp; 1.812 &amp; 0.627 &amp; -1.009 \ 1.922 \times 10^1 &amp; 1.785 \times 10^1 &amp; 1.184 \times 10^1 &amp; -1.105 \times 10^1 \ 0.017 &amp; 0.159 &amp; -1.508 \times 10^2 &amp; -2.008 \times 10^3 \ -0.393 &amp; -7.502 &amp; 7.003 \times 10^2 &amp; 2.515 \times 10^3 \ -6.276 \times 10^2 &amp; -2.529 &amp; -2.123 \times 10^2 &amp; -6.732 \times 10^2 \ 2.029 \times 10^3 &amp; 1.150 \times 10^1 &amp; 3.755 \times 10^3 &amp; -4.895 \times 10^3 \ 2.007 \times 10^2 &amp; -7.855 \times 10^1 &amp; -0.123 &amp; 0.189 \ 3.152 \times 10^3 &amp; -4.148 \times 10^3 &amp; 0.055 &amp; -1.244 \ 0.126 \end{bmatrix}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$\begin{bmatrix} -0.962 &amp; -0.377 &amp; -1.367 &amp; 0.801 \ 2.310 \times 10^1 &amp; 0.849 &amp; -1.678 \times 10^1 &amp; -1.572 \times 10^1 \ 1.357 &amp; 1.355 &amp; -0.818 &amp; -0.419 \ 1.815 \times 10^1 &amp; 1.687 \times 10^1 &amp; 1.628 \times 10^1 &amp; -1.418 \times 10^1 \ 0.021 &amp; 0.281 &amp; -1.253 \times 10^2 &amp; -1.647 \times 10^3 \ -0.425 &amp; -8.359 &amp; 6.866 \times 10^2 &amp; 3.159 \times 10^3 \ -5.187 \times 10^2 &amp; -2.100 &amp; -1.775 \times 10^2 &amp; -5.476 \times 10^2 \ 2.058 \times 10^3 &amp; 1.129 \times 10^1 &amp; 2.817 \times 10^3 &amp; -2.994 \times 10^3 \ 1.129 \times 10^2 &amp; -5.409 \times 10^1 &amp; -0.093 &amp; 0.164 \ 2.379 \times 10^3 &amp; -3.019 \times 10^2 &amp; 0.031 &amp; -1.281 \ 0.126 \end{bmatrix}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$\begin{bmatrix} -1.455 &amp; -0.279 &amp; -1.087 &amp; 1.231 \ 2.359 \times 10^1 &amp; 0.323 &amp; -1.434 \times 10^1 &amp; -1.597 \times 10^1 \ 1.072 &amp; 1.074 &amp; -1.269 &amp; 0.019 \ 1.547 \times 10^1 &amp; 1.438 \times 10^1 &amp; 1.656 \times 10^1 &amp; -1.616 \times 10^1 \ 0.023 &amp; 0.363 &amp; -1.117 \times 10^2 &amp; -1.465 \times 10^3 \ -0.441 &amp; -8.828 &amp; 6.285 \times 10^2 &amp; 3.365 \times 10^3 \ -4.615 \times 10^2 &amp; -1.871 &amp; -1.920 \times 10^2 &amp; -4.278 \times 10^2 \ 1.938 \times 10^3 &amp; 1.035 \times 10^1 &amp; 2.178 \times 10^3 &amp; -1.831 \times 10^3 \ 6.342 \times 10^1 &amp; -4.233 \times 10^1 &amp; -0.070 &amp; 0.140 \ 1.789 \times 10^3 &amp; -2.140 \times 10^2 &amp; 0.017 &amp; -1.298 \ 0.119 \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Table 10.1.1 Elements of optimal-controller-gain matrices for controllers designed at rated output power and different values of $x_t$. 
Figure (10-1.2) Time response of the system operating initially at rated output power and $x_v = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure (10-1.3) Time response of the system operating initially at rated output power and \( x_* = 0.4 \) p.u., following an instantaneous change in the load angle (\( \Delta \delta = 0.04 \) rad.).
Time response of the system operating initially at rated output power and \( x_t = 0.6 \) p.u., following an instantaneous change in the load angle \( (\Delta \delta = 0.04 \) rad\.).
Figure (10-1.5) P–Q chart for the 2-input system with controller designed optimally at rated output power and \( x_t = 0.2 \) p.u.
Figure (10-1.6) P-Q chart for the 2-input system with controller designed optimally at rated output power and $x_t = 0.4 \text{ p.u.}$.
Figure (10-1.7) P-Q chart for the 2-input system with controller designed optimally at rated output power and $x_t = 0.6 \text{ p.u.}$.
So far, voltage regulation and governing systems have been incorporated into a synchronous generator/tie-line/infinite-bus system to regulate the terminal voltage and power. Here, only a synchronous generator/tie-line/infinite-bus system is considered, and its performance is altered by direct control over the field voltage and input power of the generator. The equations describing the above simpler system have already been established as shown in equation (3-3.3), rewritten here as

\[ \dot{x} = Ax + Bu \quad (10-2.1) \]

where \( A \) and \( B \) are respectively \((7 \times 7)\) and \((7 \times 2)\) matrices, the elements of which are shown in Appendix (IV-7). \( u \) is the \((2 \times 1)\) control vector and \( u = \begin{bmatrix} \Delta V_f & \Delta P_{in} \end{bmatrix}^T \).

Defining a performance index

\[ J = \frac{1}{2} \int_0^\infty \left( \lambda_1 \Delta P_o^2 + \lambda_2 \Delta V_t^2 + \lambda_3 \Delta \delta^2 + \lambda_4 \Delta \dot{\delta}^2 + \lambda_5 \Delta u_1^2 + \lambda_6 \Delta u_2^2 \right) dt \quad (10-2.2) \]

where \( \Delta u_1 = \Delta V_f \) and \( \Delta u_2 = \Delta P_{in} \), it can be shown that equation (10-2.2), when associated with the system described by equation (10-2.1), can be rewritten as

\[ J = \frac{1}{2} \int_0^\infty (x^T Dx + u^T Hu + u^T Vx + x^T V^T u) dt \quad (10-2.3) \]

where \( D, H \) and \( V \) are respectively \((7 \times 7), (2 \times 2)\) and \((2 \times 7)\) matrices. Applying the same method of analysis as in Section (7-1), the optimization problem posed by equations (10-2.1) and (10-2.3)
results in the Matrix Ricatti Equation

\[(D-V'H^{-1}V) + Q(A-B'V) + (A-B'V)'Q - QEH^{-1}E'Q = 0\]

(10-2.4)

and an optimal control law

\[u = -H^{-1}(V + E'Q)x = Kx\]

(10-2.5)

where \(K = -H^{-1}(V + E'Q)\).

10-2.1 System Studies

Considering the basic machine parameters used from Chapter 5 onwards, the weighting factors of equation(10-2.2) were chosen to be

\[\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0.00001\]

and \(\lambda_5 = \lambda_6 = 1.0\)

to give the satisfactory time response of the 2-input optimal system, operating at rated output power and three different values of \(x_t\), in figures(10-2.1) to (10-2.3). Comparison with the time response of the system without control (i.e. in open-loop operation) shows that the three optimal controllers are successful in providing a good system performance, but are not satisfactory, as seen in the P-Q charts in figures(10-2.4) to (10-2.6), in that they reduce dramatically the stable region available for low output power operation. Furthermore, because there is no voltage regulating device incorporated into this system, the optimal controllers designed here do not necessarily provide good voltage regulation when disturbances of different forms and magnitudes appear in the system. However, each of these controllers, now represented by a \((2 \times 7)\) gain matrix, requires amplifiers with relatively low amplification factors (see table(10-2.1)).
<table>
<thead>
<tr>
<th>$x_t$(p.u.)</th>
<th>elements of optimal-controller-gain matrix $K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>$\begin{bmatrix} -0.186 \times 10^{-1} &amp; 0.362 \times 10^{-3} &amp; 0.181 \times 10^{-1} \ 0.145 \times 10^{-3} &amp; -0.581 \times 10^{-4} &amp; -0.134 \times 10^{-3} \ 0.134 \times 10^{-1} &amp; -0.192 \times 10^{-1} &amp; -0.182 \times 10^{-1} \ -0.212 \times 10^{-3} &amp; 0.133 \times 10^{-3} &amp; 0.133 \times 10^{-3} \ -0.140 \times 10^{-1} &amp; 0.223 \times 10^{-3} \end{bmatrix}$</td>
</tr>
<tr>
<td>0.4</td>
<td>$\begin{bmatrix} -0.176 \times 10^{-1} &amp; 0.402 \times 10^{-3} &amp; 0.162 \times 10^{-1} \ 0.144 \times 10^{-3} &amp; -0.746 \times 10^{-4} &amp; -0.143 \times 10^{-3} \ 0.119 \times 10^{-1} &amp; -0.172 \times 10^{-1} &amp; -0.162 \times 10^{-1} \ -0.211 \times 10^{-3} &amp; 0.139 \times 10^{-3} &amp; 0.139 \times 10^{-3} \ -0.123 \times 10^{-1} &amp; 0.222 \times 10^{-3} \end{bmatrix}$</td>
</tr>
<tr>
<td>0.6</td>
<td>$\begin{bmatrix} -0.169 \times 10^{-1} &amp; 0.442 \times 10^{-3} &amp; 0.150 \times 10^{-1} \ 0.144 \times 10^{-3} &amp; -0.898 \times 10^{-4} &amp; -0.153 \times 10^{-3} \ 0.106 \times 10^{-1} &amp; -0.159 \times 10^{-1} &amp; -0.150 \times 10^{-1} \ -0.207 \times 10^{-3} &amp; 0.146 \times 10^{-3} &amp; 0.145 \times 10^{-3} \ -0.110 \times 10^{-1} &amp; 0.219 \times 10^{-3} \end{bmatrix}$</td>
</tr>
</tbody>
</table>

Table (10-2.1) Elements of optimal-controller-gain matrices for controllers designed at rated output power and different values of $x_t$. 
Figure (10-2.1) Time response of the system operating initially at rated output power and $x_t = 0.2$ p.u., following an instantaneous change in the load angle ($\Delta \delta = 0.04$ rad.).
Figure 10.2.2 Time response of the system operating initially at rated output power and $x_+ = 0.4$ p.u., following an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.).
Figure (10-2.3) Time response of the system operating initially at rated output power and $x_t = 0.6$ p.u., following an instantaneous change in the load angle ($\Delta \theta = 0.04$ rad.).
Figure (10.24) P-Q chart for the 2-input system with controller designed optimally at rated output power and $x_t = 0.2$ p.u.
Figure (10-2.5)  P-Q chart for the 2-input system with controller designed optimally at rated output power and $x_t = 0.4$ p.u.
Figure (10-2.6) P-Q chart for the 2-input system with controller designed optimally at rated output power and $x_t = 0.6 \text{ p.u.}$.
Further improvement over the single-input optimal system (for $\alpha = 0$) is achieved by relying more heavily on a feedback controller which generates one single control law. The consequence that has been seen is a considerable increase in the required controller gains. In this Chapter, further improvement over the single-input optimal system is achieved by adding another signal into the governing system. The amplifiers of the required controller have just about the same levels of amplification as that of the optimal controller for the single-input optimal system. However, the optimal controller, which generates two control laws, has been found unfavourable when working suboptimally, due to the reduction of the stable region on the $P-Q$ chart available for low output power operation.

Direct optimal control over the field voltage and input power of a synchronous generator in a power system without voltage regulating and governing devices is capable of dramatically improving the system performance. The controller requires relatively low-gain amplifiers. However, there is no guarantee of good voltage regulation when disturbances of different forms and magnitudes affect the system. Furthermore, when this optimal controller is working suboptimally, the reduction of the stable region on the $P-Q$ chart available for low output power operation is also unfavourable.
CONCLUSION AND SUGGESTIONS FOR FURTHER INVESTIGATIONS

11-1 CONCLUSION

For a tie-line connecting a generator to an infinite-bus system, the line reactance, among other line parameters, has been pointed out to be the most important element, directly affecting the power transmission capacity of the line as well as the stability of a power system as a whole.

Matrix representation has been shown to be a very efficient method of handling the linearized equations of a power system. When expressed in state-space form, it has been shown to allow very conveniently for the inclusion of governor and voltage regulator representations in the evaluation of the performance of an overall closed-loop system. Computation of the eigenvalues and eigenvectors of the open- and closed-loop system matrices by a digital computer provides a very efficient way of determining the stability (or degree of stability) of a system, as well as of solving the set of 1st-order differential equations which describe the system.

A mathematical model for a system is usually verified by comparison with an actual experimental set, or with the theoretical results of any publications in which an identical system was considered. Since the above methods were not available, a numerical method, based on the computed eigenvalues of the system matrix, has been used successfully to verify the validity of the state-space
equations representing the three individual subsystems of a power system, including the synchronous generator/tie-line/infinite-bus, voltage regulation and governing systems.

When a synchronous generator/tie-line/infinite-bus system is in open-loop operation, an increased tie-line reactance has been shown to reduce only slightly the stable operation region as portrayed in a P-Q chart. However, when the same active and reactive output power is to be maintained, a higher excitation is required. In particular, when the system is operating at rated output power (in the overexcited region), a higher tie-line reactance reduces the mechanical damping and increases the voltage deviation after a step increase in real power. However, the system remains stable even when very high values of tie-line reactance are considered. Only in closed-loop operation, with the addition of voltage regulating and governing devices to improve the voltage regulation and damping of the system, will the overall system become unstable. In other words, the addition of control reduces the range of tie-line reactance for stable operation, or, for a given value of tie-line reactance, the addition of control can actually cause instability.

A stabilizing signal, derived from the output electrical power through a suitable stabilizer transfer function and fed into the summing junction of the voltage regulator, has been found capable of providing greater mechanical damping. However, the improvement of damping occurs only within the original range of tie-line reactance for stable operation. In other words, the addition of this stabilizing signal cannot extend this range.
In the case where a greater increase of damping is achieved, the range of tie-line reactance for stable operation is reduced.

Optimal Control Theory has been applied successfully to design an optimal-state-feedback controller according to a suitable quadratic performance index which includes the square terms of the deviations in output power, terminal voltage, rotor angle, rotor speed and control input. This controller, which now generates the source of the stabilizing signal, was found to improve drastically the performance of the resultant optimal system. An unstable system, operating at a high tie-line reactance, has also been stabilized by this method of control. A fix controller, designed optimally at one value of tie-line reactance, was found to provide good and stable operation for a wider range of tie-line reactance than that of the original closed-loop system. This range can readily be shifted by using a fix controller designed optimally at another value of tie-line reactance. The variations of the elements of the optimal-controller-gain matrices between different optimal controllers designed at different values of tie-line reactance were found to be small. In actual implementation, therefore, an optimal controller, constructed for one value of tie-line reactance, can readily be adjusted to operate optimally for other values of tie-line reactance without changing the power levels of its amplifiers.

Optimal control applied to a power system does not necessarily improve the system performance, and was found to depend very much on a suitable choice of the weighting factors in the performance index. In fact, an unsuitable choice of the weighting factors has been shown possibly to worsen the resulting system
performance. It was revealed that a choice of the weighting factors has to achieve a compromise between the resulting system damping and the transient overshoot following the onset of a disturbance. In fact, when the weighting factors are right, the resultant optimal system was shown to have good performance under a variety of disturbances which appear in different forms and magnitudes.

Optimal Control Theory, with a modified performance index, has been applied successfully to restrict all the eigenvalues of the resultant optimal system to a prescribed region of the left-half of the complex s-plane. The required computer algorithm is no more difficult than that for the design of a conventional optimal controller. In fact, with this method of approach, more freedom and time saving are possible in the process of deciding the weighting factors, simply because the degree of stability of the resultant system has initially been approximated to in the modified performance index. However, the gain of the required controller was found to be much higher than that of a conventional optimal controller.

Modal control represents a direct way of affecting the stability of a system, without any time being consumed in guessing the weighting factors as for the case of optimal control. The dominant eigenvalues of a system can be shifted to prescribed locations in the left-half of the complex s-plane for damping improvement, without affecting the locations of the other eigenvalues. However, the computational time of the required algorithm is longer than that for optimal control. For low and
high values of tie-line reactance, the three kinds of controllers, designed by conventional Optimal Control Theory, by Optimal Control Theory with a modified performance index and by Modal Control Theory, are all capable of providing good system performance. In particular, when the performance of the original closed-loop system deteriorates as the tie-line reactance increases, the benefits to be gained by employing any one of these three controllers become more and more pronounced.

The problems arising from the unmeasurable states of the state-feedback control system have been overcome by the design of a full-observer, a low-order observer and a minimum norm suboptimal controller. A self-imposed optimal control problem has been shown to be a very systematic method for choosing the dynamics of the full-observer. As in the low-order observer design, a method which gives a very accurate solution for an algebraic matrix equation of the form $QT - TA = C$ was developed. The estimated states, or the outputs of both the full-observer and low-order observer, have been seen to converge to the true observer-controller-system states within a time much less than that required for the system to settle down after a disturbance. Furthermore, these two types of observer are themselves linear systems and only add their own eigenvalues into the overall system without affecting the original eigenvalues of the control system. As a result, there are no significant discrepancies between the time response of an observer-controller system and that of a control system where all the states are assumed to be measurable. When these two observers are compared, the low-order observer has some advantages over the full-observer.
These are:

1. Complete freedom in choosing its dynamics.
2. A simpler equipment.
3. Its design algorithm consumes lesser computational time.

A minimum norm suboptimal controller provides a direct way of overcoming the problems arising from the inaccessibility of some of the control system states. It is easier to construct than its corresponding optimal controller, but the performance of a suboptimally controlled system is inferior to that of an optimally controlled system. In fact, it was found that the employment of a minimum norm suboptimal controller actually worsens the resulting system stability when the value of the tie-line reactance is small. Only when the value of tie-line reactance is such that the original closed-loop system is operating close to its stability limit, or beyond it into the unstable region, will such a controller provide a stable and better damped system.

An optimal controller, designed for the 2-input system formed by the addition of another signal through the summing junction of the governing system, has been found to improve further the system performance over that of the single-input system with a conventional optimal controller. The required controller amplifiers have just about the same levels of amplifications as that of the conventional optimal controller. However, the optimal controller, which now generates two control laws, has been found unfavourable when working suboptimally, due to a reduction in the stable region on a P-Q chart which is available for low output power operation.
Direct optimal control over the field voltage and input power of the generator in a synchronous generator/tie-line/infinite-bus system has been found capable of improving the system performance. The required controller amplifier gains are relatively low. However, when such a controller is working suboptimally at other output powers and other values of tie-line reactance, it has been found unfavourable due to a reduction in the stable region on a P-Q chart which is available for low output power operation.

11-2 **SUGGESTIONS FOR FURTHER INVESTIGATIONS**

Since tie-line reactance has very significant effects on the hunting properties of a lumped turbines and generator rotor mass, it is believed to be advisable to have a more detailed model of the prime-mover system, including high, intermediate and low pressure turbines, so that the effect of tie-line reactance on the dynamic modes of these masses can be studied in depth.

In the matter of solving a state-space equation, numerical methods of integration may be considered and compared with the method shown in Appendix I in terms of accuracy and computational time.

The stabilizing signal, as considered in Chapter 6, can also be derived from another source, or from a combination of sources. The versatility of the resultant system to different kinds of disturbance can be studied. The consequences of where the stabilizing signal is added to the system are also worth looking at.

In the design of an optimal controller, other types of performance index can be employed and the resultant system and
optimal controller characteristics may be compared with that
already revealed in this thesis.

Apart from the types of disturbances outlined in Section
(7-6), disturbances of other forms and durations may also be
considered.

In the design of an optimal controller (for $\alpha = 1.0$) which
provides regional restriction in the complex s-plane on the
resultant closed-loop eigenvalues, other values of $\alpha$ may be used
to see if a compromise between an increase in the system damping
and in the controller gains can be achieved.

Further investigations can be carried out on how the
characteristics of the single-input system and its associated
controller, designed by Modal Control Theory, change when different
eigenvalues are shifted for damping improvement. Further work may
also be done on the application of Modal Control Theory to the
2-input systems.

In the design of a state observer, other measurable outputs
and unmeasurable states may also be assumed for studying the
differences in the design results and the overall system
performances.

Further work can be done on the minimum norm suboptimal
controller formed by deleting other optimal-controller gains
associated with other states.

In the matter of solving an algebraic matrix equation of
the form $QT - TA = 0$, other methods of solution may be used and
compared with that shown in Appendix VII in terms of computational
time and accuracy.
REFERENCES


APPENDIX I

ANALYTICAL SOLUTION OF THE STATE EQUATIONS

The state equations to be solved are of the form

\[ x = Ax + Bu \]  \hspace{1cm} (I-1)

where the control vector \( u \) may be arbitrarily chosen, \( x \) is an \((n \times 1)\) column matrix containing the states of the system, \( A \) an \((n \times n)\) square matrix, \( B \) an \((n \times m)\) rectangular matrix and \( u \) an \((m \times 1)\) column matrix with \( m \)-forcing functions. Numerical methods of integration may be used to solve these equations or they may be patched up on an analogue computer. Both methods will provide \( x(t) \) for any desired \( u(t) \). However, it is sometimes desirable in analytical work to write down the solution of the controlled system in compact form. This solution is

\[ x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau \]  \hspace{1cm} (I-2)

where \( e^{At} \) is the transition matrix.

Since digital computers are very efficient in calculating eigenvalues and eigenvectors, it would be useful if these results could be applied to the determination of solutions for equation (I-1). This can certainly be done quite simply if only step responses are required, when \( u \) comprises only zeros and constants. Let

\[ x = Pz \]  \hspace{1cm} (I-3)

where \( P \) comprises the eigenvectors of \( A \). Substitute equation (I-3)
into (I-1) to give

\[ Pz = APz + Bu \]

or \[ z = P^{-1}APz + P^{-1}Bu \]

\[ = \lambda z + P^{-1}Bu \] \hspace{1cm} (I-4)

where \( \lambda = P^{-1}AP \) and is diagonal if the eigenvalues of \( A \) are distinct.

Comparisons between equations (I-4), (I-1) and (I-2) give

\[ z(t) = e^{\lambda t}z(0) + \int_0^t e^{\lambda(t-\tau)}P^{-1}Bu(\tau)d\tau \]

\[ = e^{\lambda t}z(0) + \int_0^t e^{\lambda(t-\gamma)}d\gamma P^{-1}Bu \]

Now \( \int_0^t e^{\lambda_i(t-\gamma)}d\gamma = \frac{1}{\lambda_i} (e^{\lambda_i t} - 1) \) with \( i = 1, 2, \ldots, n \)

and the \( \lambda \)'s are the eigenvalues of \( A \). Therefore

\[ x(t) = P \left[ \begin{array}{c} \lambda_1 e^{\lambda_1 t} \\ \vdots \\ \lambda_n e^{\lambda_n t} \end{array} \right] z(0) + \frac{1}{\lambda_1} (e^{\lambda_1 t} - 1) \]

\[ = P \left[ \begin{array}{c} \frac{1}{\lambda_2} (e^{\lambda_2 t} - 1) \\ \vdots \\ \frac{1}{\lambda_n} (e^{\lambda_n t} - 1) \end{array} \right] e_i \]
where $e_i = P^{-1}Bu$

i.e.

$$x(t) = P \begin{bmatrix} (z_1(0) + \frac{e_1}{\lambda_1})e^{\lambda_1 t} - \frac{e_1}{\lambda_1} \\ \vdots \\ (z_n(0) + \frac{e_n}{\lambda_n})e^{\lambda_n t} - \frac{e_n}{\lambda_n} \end{bmatrix}$$

This result was easily implemented on the ICL 1904S digital computer of the Loughborough University of Technology.
APPENDIX II

ELEMENTS OF MATRICES PI, P AND G

In the following computer printout,

\[ \Pi = P^{-1} \],

\[ \text{FLUXQ} = \psi_q \],

\[ \text{FLUXD} = \psi_d \],

\[ D = \delta \],

\[ RR = r \],

and with

\[ A_1 = XFKD \times XFKD - XFF \times XFKD \]
\[ A_2 = XAKD \times XFKD - XAF \times XFKD \]
\[ A_3 = (XD + XT) \times (A_1 + XAF \times XAF \times XFKD - 2 \times XAF \times XAKD \times XFKD + XFF \times XAKD \times XAKD \]
\[ A_4 = XAF \times XFKD - XFF \times XAKD \]
\[ A_5 = XKKD \times (XO + XT) - XAKQ \times XAKD \]
\[ A_6 = (XO \times ID - XAKQ \times IKQ) / WQ \]
\[ A_7 = XT \times 10 / WQ \]
\[ A_8 = (XAF \times IF + XAKD \times IKD - ID \times ID) / WQ \]
\[ A_9 = XT \times ID / WQ \]
\[ A_{10} = WQ / (2 \times H) \]

Matrix \( \Pi \) (19 x 19)

\[ \Pi(1,1) = 1.0 \]
\[ \Pi(2,2) = 1.0 \]
\[ \Pi(2,14) = -A_{10} \]
\[ \Pi(2,15) = -10 \times A_{10} \]
\[ \Pi(2,16) = ID \times A_{10} \]
\[ \Pi(3,3) = WQ \times A_1 / A_3 \]
\[ \Pi(3,5) = WQ \times A_2 / A_3 \]
\[ \Pi(3,6) = WQ \times A_4 / A_3 \]
\[ \Pi(3,8) = -WQ \times A_1 / A_3 \]
\[ \Pi(3,10) = -WQ \times A_1 / A_3 \]
\[ \Pi(4,4) = XKOKD \times WQ / A_5 \]
\[ \Pi(4,7) = XAKQ \times WQ / A_5 \]
\[ \Pi(4,9) = -XKKD \times WQ / A_5 \]
\[ \Pi(4,11) = -XKKD \times WQ / A_5 \]
\[ \Pi(5,3) = WQ \times A_2 / A_3 \]
\[
\begin{align*}
PI(5, 5) &= W_0 \cdot A_2 \cdot A_2 / (A_3 \cdot A_1) - X_2 K_0 A_2 / W_0 / A_1 \\
PI(5, 6) &= W_0 \cdot X_2 K_0 A_1 + W_0 \cdot A_2 \cdot A_4 / (A_3 \cdot A_1) \\
PI(5, 7) &= -W_0 \cdot A_2 / A_3 \\
PI(5, 8) &= -W_0 \cdot A_2 / A_3 \\
PI(5, 10) &= -W_0 \cdot A_2 / A_3 \\
PI(6, 3) &= W_0 \cdot A_4 / A_3 \\
PI(6, 5) &= PI(5, 6) \\
PI(6, 6) &= W_0 \cdot A_4 \cdot A_4 / (A_3 \cdot A_1) - W_0 \cdot X_2 K / A_1 \\
PI(6, 8) &= -W_0 \cdot A_4 / A_3 \\
PI(6, 10) &= -W_0 \cdot A_4 / A_3 \\
PI(7, 4) &= W_0 \cdot X_2 K_0 / A_5 \\
PI(7, 7) &= W_0 / X_2 K_0 + W_0 \cdot X_2 K_0 \cdot X_2 K_0 / (X_2 K_0 \cdot A_5) \\
PI(7, 9) &= -W_0 \cdot X_2 K_0 / A_5 \\
PI(7, 11) &= -W_0 \cdot X_2 K_0 / A_5 \\
PI(8, 3) &= X_2 T \cdot A_1 / A_3 \\
PI(8, 5) &= X_2 T \cdot A_2 / A_3 \\
PI(8, 6) &= X_2 T \cdot A_4 / A_3 \\
PI(8, 8) &= -1 - X_2 T \cdot A_1 / A_3 \\
PI(9, 4) &= X_2 T \cdot X_2 K_0 / A_5 \\
PI(9, 7) &= X_2 T \cdot X_2 K_0 / A_5 \\
PI(9, 9) &= -1 - X_2 T \cdot X_2 K_0 / A_5 \\
PI(9, 11) &= -1 - X_2 T \cdot X_2 K_0 / A_5 \\
PI(10, 10) &= 1.0 \\
PI(11, 11) &= 1.0 \\
PI(12, 3) &= 1.0 \cdot X_2 T \cdot A_1 / A_3 \\
PI(12, 4) &= 1.0 \cdot X_2 K_0 \cdot X_2 T / A_5 \\
PI(12, 5) &= 1.0 \cdot X_2 T \cdot A_2 / A_3 \\
PI(12, 6) &= 1.0 \cdot X_2 T \cdot A_4 / A_3 \\
PI(12, 7) &= 1.0 \cdot X_2 T \cdot X_2 K_0 / A_5 \\
PI(12, 8) &= 1.0 \cdot 1.0 \cdot X_2 T \cdot A_1 / A_3 \\
PI(12, 9) &= 1.0 \cdot 1.0 \cdot X_2 K_0 \cdot X_2 T / A_5 \\
PI(12, 10) &= 1.0 \cdot 1.0 \cdot X_2 T \cdot A_1 / A_3 \\
PI(12, 11) &= 1.0 \cdot 1.0 \cdot X_2 K_0 \cdot X_2 T / A_5 \\
PI(12, 12) &= 1.0 \\
PI(13, 3) &= X_2 T \cdot (V_0 / V_T) \cdot A_1 / A_3 \\
PI(13, 4) &= X_2 T \cdot X_2 K_0 \cdot (V_0 / V_T) / A_5 \\
PI(13, 5) &= X_2 T \cdot (V_0 / V_T) \cdot A_2 / A_3 \\
PI(13, 6) &= X_2 T \cdot (V_0 / V_T) \cdot A_4 / A_3 \\
PI(13, 7) &= X_2 T \cdot (V_0 / V_T) \cdot X_2 K_0 / A_5 \\
PI(13, 8) &= (V_0 / V_T) - X_2 T \cdot (V_0 / V_T) \cdot A_1 / A_3 \\
PI(13, 9) &= (V_0 / V_T) - X_2 T \cdot X_2 K_0 \cdot (V_0 / V_T) / A_5 \\
PI(13, 10) &= (V_0 / V_T) - X_2 T \cdot (V_0 / V_T) \cdot A_1 / A_3 \\
PI(13, 11) &= (V_0 / V_T) - X_2 T \cdot X_2 K_0 \cdot (V_0 / V_T) / A_5 \\
PI(13, 13) &= 1.0 \\
PI(14, 14) &= 1.0 \\
PI(14, 15) &= 1.0 \\
PI(14, 16) &= -1.0 \\
PI(15, 15) &= 1.0 \\
PI(16, 16) &= 1.0 \\
PI(17, 17) &= 1.0 \\
PI(18, 18) &= 1.0 \\
PI(19, 19) &= 1.0 
\end{align*}
\]
Matrix F (19 x 7)

<table>
<thead>
<tr>
<th>i</th>
<th>j</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1, 2</td>
<td></td>
<td>$F(1, 2) = 1.0$</td>
</tr>
<tr>
<td>2, 2</td>
<td></td>
<td>$F(2, 2) = -(\text{TIN} + \text{H0} \times \text{KD}) / (2 \times \text{H})$</td>
</tr>
<tr>
<td>2, 3</td>
<td></td>
<td>$F(2, 3) = \text{H10} \times (\text{FLUX} + 10 \times \text{RD})$</td>
</tr>
<tr>
<td>2, 4</td>
<td></td>
<td>$F(2, 4) = -\text{H10} \times (\text{FLUX} + 10 \times \text{K0})$</td>
</tr>
<tr>
<td>2, 5</td>
<td></td>
<td>$F(2, 5) = -10 \times \text{AF} \times \text{H10}$</td>
</tr>
<tr>
<td>2, 6</td>
<td></td>
<td>$F(2, 6) = -10 \times \text{AKD} \times \text{H10}$</td>
</tr>
<tr>
<td>2, 7</td>
<td></td>
<td>$F(2, 7) = 10 \times \text{AK0} \times \text{H10}$</td>
</tr>
<tr>
<td>3, 1</td>
<td></td>
<td>$F(3, 1) = \text{PI}(3, 1) \times V \times \text{COS}(D)$</td>
</tr>
<tr>
<td>3, 2</td>
<td></td>
<td>$F(3, 2) = \text{PI}(3, 2) \times \text{H6} - \text{PI}(3, 8) \times \text{A7}$</td>
</tr>
<tr>
<td>3, 3</td>
<td></td>
<td>$F(3, 3) = -\text{PI}(3, 3) \times \text{RR} + \text{PI}(3, 8) \times \text{RT}$</td>
</tr>
<tr>
<td>3, 4</td>
<td></td>
<td>$F(3, 4) = \text{PI}(3, 3) \times \text{K0} - \text{PI}(3, 8) \times \text{RT}$</td>
</tr>
<tr>
<td>3, 5</td>
<td></td>
<td>$F(3, 5) = -\text{PI}(3, 5) \times \text{RF}$</td>
</tr>
<tr>
<td>3, 6</td>
<td></td>
<td>$F(3, 6) = -\text{PI}(3, 6) \times \text{KDO}$</td>
</tr>
<tr>
<td>3, 7</td>
<td></td>
<td>$F(3, 7) = -\text{PI}(3, 7) \times \text{AK0}$</td>
</tr>
<tr>
<td>4, 1</td>
<td></td>
<td>$F(4, 1) = -\text{PI}(4, 1) \times V \times \text{SIN}(D)$</td>
</tr>
<tr>
<td>4, 2</td>
<td></td>
<td>$F(4, 2) = \text{PI}(4, 2) \times \text{H8} + \text{PI}(4, 9) \times \text{A9}$</td>
</tr>
<tr>
<td>4, 3</td>
<td></td>
<td>$F(4, 3) = -\text{PI}(4, 4) \times \text{RD} + \text{PI}(4, 9) \times \text{RT}$</td>
</tr>
<tr>
<td>4, 4</td>
<td></td>
<td>$F(4, 4) = -\text{PI}(4, 4) \times \text{RR} + \text{PI}(4, 9) \times \text{RT}$</td>
</tr>
<tr>
<td>4, 5</td>
<td></td>
<td>$F(4, 5) = \text{PI}(4, 4) \times \text{KAF}$</td>
</tr>
<tr>
<td>4, 6</td>
<td></td>
<td>$F(4, 6) = \text{PI}(4, 4) \times \text{AKD}$</td>
</tr>
<tr>
<td>4, 7</td>
<td></td>
<td>$F(4, 7) = -\text{PI}(4, 7) \times \text{KDO}$</td>
</tr>
<tr>
<td>5, 1</td>
<td></td>
<td>$F(5, 1) = \text{PI}(5, 1) \times V \times \text{COS}(D)$</td>
</tr>
<tr>
<td>5, 2</td>
<td></td>
<td>$F(5, 2) = \text{PI}(5, 2) \times \text{H6} - \text{PI}(5, 8) \times \text{A7}$</td>
</tr>
<tr>
<td>5, 3</td>
<td></td>
<td>$F(5, 3) = -\text{PI}(5, 3) \times \text{RR} + \text{PI}(5, 8) \times \text{RT}$</td>
</tr>
<tr>
<td>5, 4</td>
<td></td>
<td>$F(5, 4) = \text{PI}(5, 3) \times \text{X0} - \text{PI}(5, 8) \times \text{XT}$</td>
</tr>
<tr>
<td>5, 5</td>
<td></td>
<td>$F(5, 5) = -\text{PI}(5, 5) \times \text{RF}$</td>
</tr>
<tr>
<td>5, 6</td>
<td></td>
<td>$F(5, 6) = -\text{PI}(5, 6) \times \text{KDO}$</td>
</tr>
<tr>
<td>5, 7</td>
<td></td>
<td>$F(5, 7) = -\text{PI}(5, 7) \times \text{AK0}$</td>
</tr>
<tr>
<td>6, 1</td>
<td></td>
<td>$F(6, 1) = \text{PI}(6, 10) \times V \times \text{COS}(D)$</td>
</tr>
<tr>
<td>6, 2</td>
<td></td>
<td>$F(6, 2) = \text{PI}(6, 2) \times \text{H6} - \text{PI}(6, 8) \times \text{A7}$</td>
</tr>
<tr>
<td>6, 3</td>
<td></td>
<td>$F(6, 3) = -\text{PI}(6, 3) \times \text{RR} + \text{PI}(6, 8) \times \text{RT}$</td>
</tr>
<tr>
<td>6, 4</td>
<td></td>
<td>$F(6, 4) = \text{PI}(6, 3) \times \text{K0} - \text{PI}(6, 8) \times \text{XT}$</td>
</tr>
<tr>
<td>6, 5</td>
<td></td>
<td>$F(6, 5) = -\text{PI}(6, 5) \times \text{RF}$</td>
</tr>
<tr>
<td>6, 6</td>
<td></td>
<td>$F(6, 6) = -\text{PI}(6, 6) \times \text{KDO}$</td>
</tr>
<tr>
<td>6, 7</td>
<td></td>
<td>$F(6, 7) = -\text{PI}(6, 7) \times \text{AK0}$</td>
</tr>
<tr>
<td>7, 1</td>
<td></td>
<td>$F(7, 1) = -\text{PI}(7, 11) \times V \times \text{SIN}(D)$</td>
</tr>
<tr>
<td>7, 2</td>
<td></td>
<td>$F(7, 2) = \text{PI}(7, 4) \times \text{H8} + \text{PI}(7, 9) \times \text{A9}$</td>
</tr>
<tr>
<td>7, 3</td>
<td></td>
<td>$F(7, 3) = -\text{PI}(7, 4) \times \text{RD} + \text{PI}(7, 9) \times \text{RT}$</td>
</tr>
<tr>
<td>7, 4</td>
<td></td>
<td>$F(7, 4) = -\text{PI}(7, 4) \times \text{RR} + \text{PI}(7, 9) \times \text{RT}$</td>
</tr>
<tr>
<td>7, 5</td>
<td></td>
<td>$F(7, 5) = \text{PI}(7, 4) \times \text{KAF}$</td>
</tr>
<tr>
<td>7, 6</td>
<td></td>
<td>$F(7, 6) = \text{PI}(7, 4) \times \text{AKD}$</td>
</tr>
<tr>
<td>7, 7</td>
<td></td>
<td>$F(7, 7) = -\text{PI}(7, 7) \times \text{KDO}$</td>
</tr>
<tr>
<td>8, 1</td>
<td></td>
<td>$F(8, 1) = \text{PI}(8, 10) \times V \times \text{COS}(D)$</td>
</tr>
<tr>
<td>8, 2</td>
<td></td>
<td>$F(8, 2) = \text{PI}(8, 2) \times \text{H6} - \text{PI}(8, 8) \times \text{A7}$</td>
</tr>
<tr>
<td>8, 3</td>
<td></td>
<td>$F(8, 3) = -\text{PI}(8, 3) \times \text{RR} + \text{PI}(8, 8) \times \text{RT}$</td>
</tr>
<tr>
<td>8, 4</td>
<td></td>
<td>$F(8, 4) = \text{PI}(8, 3) \times \text{K0} - \text{PI}(8, 8) \times \text{XT}$</td>
</tr>
<tr>
<td>8, 5</td>
<td></td>
<td>$F(8, 5) = -\text{PI}(8, 5) \times \text{RF}$</td>
</tr>
<tr>
<td>8, 6</td>
<td></td>
<td>$F(8, 6) = -\text{PI}(8, 6) \times \text{KDO}$</td>
</tr>
</tbody>
</table>
F(8, 7) = -PI(3, 3) * XAKO
F(9, 1) = PI(9, 11) * V * SIN(D)
F(9, 2) = PI(9, 4) * A8 + PI(9, 9) * G9
F(9, 3) = PI(9, 4) * X0 + PI(9, 9) * X1
F(9, 4) = PI(9, 4) * RR + PI(9, 9) * RT
F(9, 5) = PI(9, 4) * XAF
F(9, 6) = PI(9, 4) * XAKD
F(9, 7) = PI(9, 7) * RK0
F(10, 1) = V * COS(D)
F(11, 1) = -V * SIN(D)
F(11, 2) = PI(11, 10) * V * COS(D) - PI(11, 11) * V * SIN(D)
F(11, 3) = PI(11, 3) * A6 + PI(11, 4) * A8 - PI(11, 8) * A7 + PI(12, 9) * A9
F(12, 3) = -PI(12, 2) * RR - PI(12, 4) * X0 + PI(12, 8) * RT + PI(12, 9) * XT + V0
F(12, 4) = PI(12, 4) * X0 - PI(12, 4) * RR - PI(12, 8) * RT + PI(12, 9) * RT + V0
F(12, 5) = PI(12, 4) * XAF - PI(12, 5) * RF
F(12, 6) = PI(12, 4) * XAKD - PI(12, 6) * RKD
F(12, 7) = -PI(12, 2) * XAKO - PI(12, 7) * RK0
F(13, 1) = PI(13, 10) * V * COS(D) - PI(13, 11) * V * SIN(D)
F(13, 2) = PI(13, 3) * A6 + PI(13, 4) * A8 - PI(13, 8) * A7 + PI(13, 9) * A9
F(13, 3) = -PI(13, 2) * RR - PI(13, 4) * X0 + PI(13, 8) * RT + PI(13, 9) * XT
F(13, 4) = PI(13, 4) * X0 - PI(13, 4) * RR - PI(13, 8) * XT + PI(13, 9) * RT
F(13, 5) = PI(13, 4) * XAF - PI(13, 5) * RF
F(13, 6) = PI(13, 4) * XAKD - PI(13, 6) * RKD
F(13, 7) = -PI(13, 2) * XAKO - PI(13, 7) * RK0
F(14, 3) = -FLUXD - (10 * XD)
F(14, 4) = FLUXD + ID * X0
F(14, 5) = ID * XAF
F(14, 6) = ID * XAKD
F(14, 7) = -ID * XAKO
F(15, 3) = -XO
F(15, 5) = XAF
F(15, 6) = XAKD
F(16, 4) = -X0
F(16, 7) = XAKO
F(17, 3) = XAF
F(17, 5) = XFF
F(17, 6) = XFKD
F(18, 3) = -XAKD
F(18, 5) = XFKD
F(18, 6) = XDKD
F(19, 4) = -XAKO
F(19, 7) = XDKO
Matrix G (19 x 3)

\[ G(2, 2) = R10 \]
\[ G(3, 1) = PI(3, 5) \]
\[ G(3, 2) = PI(3, 10) \times SIN(D) \]
\[ G(4, 2) = PI(4, 11) \times COS(D) \]
\[ G(5, 1) = PI(5, 5) \]
\[ G(5, 2) = PI(5, 10) \times SIN(D) \]
\[ G(6, 1) = PI(6, 5) \]
\[ G(6, 2) = PI(6, 10) \times SIN(D) \]
\[ G(7, 2) = PI(7, 11) \times COS(D) \]
\[ G(8, 1) = PI(8, 5) \]
\[ G(8, 2) = PI(8, 10) \times SIN(D) \]
\[ G(9, 2) = PI(9, 11) \times COS(D) \]
\[ G(10, 3) = SIN(D) \]
\[ G(11, 3) = COS(D) \]
\[ G(12, 1) = PI(12, 5) \]
\[ G(12, 3) = PI(12, 10) \times SIN(D) + PI(12, 11) \times COS(D) \]
\[ G(13, 1) = PI(13, 5) \]
\[ G(13, 3) = PI(13, 10) \times SIN(D) + PI(13, 11) \times COS(D) \]
APPENDIX III

SYSTEM DATA

III-1 FOR STABILITY STUDIES I

Generator and transmission system

Rating

37.5 MVA ; 30 MW
11.8 kV ; 50 Hz

x_a
0.14 p.u.

x_f
0.14 p.u.

x_kd , x_kq
0.04 p.u.

x_af , x_akt , x_fkd
1.86 p.u.

x_akt
1.86 p.u.

r
0.002 p.u.

r_f
0.00107 p.u.

r_kd , r_kq
0.0125 p.u.

H
5.3 kWs/kVA

w_o
100 π rad./s

T_1
0.0

K_d
0.0

r_t
0.0

V
1.0 p.u.
Automatic voltage regulator (with magnetic amplifiers)

\[ G_{vs} = -k_{1,21} = 0.00159 \]
\[ G_{m1} = 52 \]
\[ G_{m2} = 12.2 \]
\[ G_x = 3.06 \]
\[ G_{ms} = 0.00525 \]
\[ G_{xs} = 0.0139 \]
\[ T_{m1} = 0.044 \text{ s} \]
\[ T_{m2} = 0.1 \text{ s} \]
\[ T_x = 0.2 \text{ s} \]
\[ T_{ms} = 0.1 \text{ s} \]
\[ T_{xs} = 2.0 \text{ s} \]

III-2 FOR STABILITY STUDIES II

Generator and transmission system

Rating

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_d )</td>
<td>1.773 p.u.</td>
</tr>
<tr>
<td>( x_q )</td>
<td>1.726 p.u.</td>
</tr>
<tr>
<td>( x_d' )</td>
<td>0.297 p.u.</td>
</tr>
<tr>
<td>( x_d'' )</td>
<td>0.247 p.u.</td>
</tr>
<tr>
<td>( x_q' )</td>
<td>0.257 p.u.</td>
</tr>
<tr>
<td>( T_{do}' )</td>
<td>8.46 s</td>
</tr>
<tr>
<td>( T_{do}'' )</td>
<td>0.038 s</td>
</tr>
<tr>
<td>( T_{qo}' )</td>
<td>0.75 s</td>
</tr>
<tr>
<td>( r )</td>
<td>0.003 p.u.</td>
</tr>
<tr>
<td>( H )</td>
<td>5.3 kWs/kVA</td>
</tr>
<tr>
<td>( w_0 )</td>
<td>100 ( \pi ) rad./s</td>
</tr>
<tr>
<td>( T_l )</td>
<td>0.0</td>
</tr>
<tr>
<td>( K_d )</td>
<td>0.0</td>
</tr>
<tr>
<td>( r_t )</td>
<td>0.0</td>
</tr>
<tr>
<td>( V )</td>
<td>1.0 p.u.</td>
</tr>
</tbody>
</table>
Under the assumption that the armature leakage reactance, \( x_a \), is equal to 0.2 p.u., the following data are calculated:\(^{23}\)

\[
\begin{align*}
  x_f & \quad 0.1033 \text{ p.u.} \\
  x_{kd} & \quad 0.0911 \text{ p.u.} \\
  x_{kq} & \quad 0.0592 \text{ p.u.} \\
  x_{af}, x_{akd}, x_{fkd} & \quad 1.573 \text{ p.u.} \\
  x_{akq} & \quad 1.526 \text{ p.u.} \\
  r_f & \quad 0.000631 \text{ p.u.} \\
  r_{kd} & \quad 0.015763 \text{ p.u.} \\
  r_{kq} & \quad 0.006728 \text{ p.u.}
\end{align*}
\]

**Automatic voltage regulator**

\[
\begin{align*}
  G_v &= 0.002 & T_z &= 0.007 \text{ s} \\
  G_z &= 1.0 & T_{a1} &= 0.31 \text{ s} \\
  G_a &= 0.87 & T_{a2} &= 0.078 \text{ s} \\
  G_m &= 1.0 & T_{m1} &= 1.5 \text{ s} \\
  G_o &= 1.3 & T_{m2} &= 2.55 \text{ s} \\
  G_c &= 15.0 & T_o &= 0.05 \text{ s} \\
  G_x &= 5.0 & T_x &= 0.9 \text{ s}
\end{align*}
\]

**Governor**

\[
\begin{align*}
  G_1 &= -k_{22} = k_f = 0.001888 & T_1 &= 0.1 \text{ s} \\
  G_2 &= 1.33 & T_2 &= 0.188 \text{ s} \\
  G_3 &= 1.42 & T_3 &= 0.49 \text{ s}
\end{align*}
\]

**III-3 FOR STABILITY STUDIES I AND II**
APPENDIX IV

ELEMENTS OF MATRICES

IV-1 ELEMENTS OF MATRIX $A_a$ IN EQUATION(6-2.5)

With

$G_5=GA*G_2*GV$

$G_4=GM*G_5$

$G_6=<G_5*TR1>/<(TR2*TZ)$

$G_7=<G_4*TH1*TR1>/<(TH2*TR2*TZ)$

$G_8=GV*G_2/TZ$

$G_9=K_S*GA*TR1/TR2$

$G_{10}=K_S*G_4*TH1/TH2$

$G_{12}=G_10*G*M*TH1/TH2$

$B_1=F(12,3)*F(2,1)+F(12,4)*F(4,1)+F(12,5)*F(5,1)+F(12,6)*F(6,1)+F(12,7)*F(7,1)$

$B_2=F(12,1)*F(1,2)+F(12,2)*F(2,2)+F(12,3)*F(3,2)+F(12,4)*F(4,2)+F(12,5)*F(5,2)+F(12,6)*F(6,2)+F(12,7)*F(7,2)$

$B_3=F(12,2)*F(2,3)+F(12,3)*F(3,3)+F(12,4)*F(4,3)+F(12,5)*F(5,3)+F(12,6)*F(6,3)+F(12,7)*F(7,3)$

$B_4=F(12,2)*F(2,4)+F(12,3)*F(3,4)+F(12,4)*F(4,4)+F(12,5)*F(5,4)+F(12,6)*F(6,4)+F(12,7)*F(7,4)$

$B_5=F(12,2)*F(2,5)+F(12,3)*F(3,5)+F(12,4)*F(4,5)+F(12,5)*F(5,5)+F(12,6)*F(6,5)+F(12,7)*F(7,5)$

$B_6=F(12,2)*F(2,6)+F(12,3)*F(3,6)+F(12,4)*F(4,6)+F(12,5)*F(5,6)+F(12,6)*F(6,6)+F(12,7)*F(7,6)$

$B_7=F(12,2)*F(2,7)+F(12,3)*F(3,7)+F(12,4)*F(4,7)+F(12,5)*F(5,7)+F(12,6)*F(6,7)+F(12,7)*F(7,7)$

$B_8=F(12,2)*G(2,2)$

$B_9=F(12,3)*G(3,1)+F(12,5)*G(5,1)+F(12,6)*G(6,1)+G(12,1)+(-1/TH)$

$B_{10}=G(12,1)*(6X/TH)$

MATRIX $A_a$ (16 x 16)

$A(1,2)=1.0$

$A(2,2)=F(2,2)$

$A(2,3)=F(2,3)$

$A(2,4)=F(2,4)$

$A(2,5)=F(2,5)$

$A(2,6)=F(2,6)$

$A(2,7)=F(2,7)$

$A(2,8)=G(2,2)$

$A(3,1)=F(3,1)$
\[
R(1, 2) = F(1, 2) \\
R(2, 3) = F(2, 3) \\
R(3, 4) = F(3, 4) \\
R(3, 5) = F(3, 5) \\
R(3, 6) = F(3, 6) \\
R(3, 7) = F(3, 7) \\
R(3, 11) = G(3, 1) \\
R(4, 1) = F(4, 1) \\
R(4, 2) = F(4, 2) \\
R(4, 3) = F(4, 3) \\
R(4, 4) = F(4, 4) \\
R(4, 5) = F(4, 5) \\
R(4, 6) = F(4, 6) \\
R(4, 7) = F(4, 7) \\
R(5, 1) = F(5, 1) \\
R(5, 2) = F(5, 2) \\
R(5, 3) = F(5, 3) \\
R(5, 4) = F(5, 4) \\
R(5, 5) = F(5, 5) \\
R(5, 6) = F(5, 6) \\
R(5, 7) = F(5, 7) \\
R(5, 11) = G(5, 1) \\
R(6, 1) = F(6, 1) \\
R(6, 2) = F(6, 2) \\
R(6, 3) = F(6, 3) \\
R(6, 4) = F(6, 4) \\
R(6, 5) = F(6, 5) \\
R(6, 6) = F(6, 6) \\
R(6, 7) = F(6, 7) \\
R(6, 11) = G(6, 1) \\
R(7, 1) = F(7, 1) \\
R(7, 2) = F(7, 2) \\
R(7, 3) = F(7, 3) \\
R(7, 4) = F(7, 4) \\
R(7, 5) = F(7, 5) \\
R(7, 6) = F(7, 6) \\
R(7, 7) = F(7, 7) \\
R(8, 8) = -1/T_3 \\
R(8, 10) = G_3/T_3 \\
R(9, 2) = -(K_3*K_2)/(T_1*T_2) \\
R(9, 9) = -(T_1+T_2)/(T_1*T_2) \\
R(9, 10) = -1/(T_1*T_2) \\
R(10, 9) = 1, 0 \\
R(11, 11) = -1/T_X \\
R(11, 14) = G_X/T_X \\
R(12, 1) = G_8+F(13, 1) \\
R(12, 2) = G_8+F(13, 2) \\
R(12, 3) = G_8+F(13, 3) \\
R(12, 4) = G_8+F(13, 4) \\
R(12, 5) = G_8+F(12, 5) \\
R(12, 6) = G_8+F(13, 6) \\
R(12, 7) = G_8+F(13, 7)
\begin{align*}
A(12, 11) &= G8 \cdot G(12, 1) \\
A(12, 12) &= -1/T_2 \\
A(12, 13) &= -1/T_0 \\
A(12, 16) &= G0 / T_0 \\
A(14, 13) &= G6 / T_C \\
A(14, 14) &= -1/T_C \\
A(15, 1) &= G6 \cdot F(13, 1) + G9 \cdot B_1 + G10 \cdot F(12, 1) \\
A(15, 2) &= G6 \cdot F(13, 2) + G9 \cdot B_2 + G10 \cdot F(12, 2) \\
A(15, 3) &= G6 \cdot F(13, 3) + G9 \cdot B_3 + G10 \cdot F(12, 3) \\
A(15, 4) &= G6 \cdot F(13, 4) + G9 \cdot B_4 + G10 \cdot F(12, 4) \\
A(15, 5) &= G6 \cdot F(13, 5) + G9 \cdot B_5 + G10 \cdot F(12, 5) \\
A(15, 6) &= G6 \cdot F(13, 6) + G9 \cdot B_6 + G10 \cdot F(12, 6) \\
A(15, 7) &= G6 \cdot F(13, 7) + G9 \cdot B_7 + G10 \cdot F(12, 7) \\
A(15, 8) &= G9 \cdot B_8 \\
A(15, 11) &= G6 \cdot G(13, 1) + G9 \cdot B_9 + G10 \cdot G(12, 1) \\
A(15, 12) &= (G6 / T_2) \cdot (T_1 / T_2 - 1) \\
A(15, 14) &= G9 \cdot B_10 \\
A(15, 15) &= -1 / T_1 \\
A(16, 1) &= G7 \cdot F(13, 1) + G11 \cdot B_1 + G12 \cdot F(12, 1) \\
A(16, 2) &= G7 \cdot F(13, 2) + G11 \cdot B_2 + G12 \cdot F(12, 2) \\
A(16, 3) &= G7 \cdot F(13, 3) + G11 \cdot B_3 + G12 \cdot F(12, 3) \\
A(16, 4) &= G7 \cdot F(13, 4) + G11 \cdot B_4 + G12 \cdot F(12, 4) \\
A(16, 5) &= G7 \cdot F(13, 5) + G11 \cdot B_5 + G12 \cdot F(12, 5) \\
A(16, 6) &= G7 \cdot F(13, 6) + G11 \cdot B_6 + G12 \cdot F(12, 6) \\
A(16, 7) &= G7 \cdot F(13, 7) + G11 \cdot B_7 + G12 \cdot F(12, 7) \\
A(16, 8) &= G11 \cdot B_8 \\
A(16, 11) &= G7 \cdot G(13, 1) + G11 \cdot B_9 + G12 \cdot G(12, 1) \\
A(16, 12) &= (G6 \cdot G(13, 1)) \cdot (T_1 / T_2 - 1) / (T_2 \cdot T_1) \\
A(16, 14) &= G11 \cdot B_10 \\
A(16, 15) &= (G6 / T_2) \cdot (1 - T_1 / T_2) \\
A(16, 16) &= -1 / T_2
\end{align*}
IV-2  ELEMENTS OF MATRIX $A_b$ IN EQUATION (6-2.10)

With

\[
\begin{align*}
G5 &= GA*GZ*GV \\
G4 &= GH*G5 \\
G6 &= -(G5*TA1)/(TA2*TZ) \\
G7 &= -(G4*TH1*TA1)/(TH2*TA2*TZ) \\
G8 &= (GV*GZ)/TZ \\
GD1 &= KS/TS \\
GD2 &= (GA*KS*TA1)/(TA2*T5) \\
GD3 &= (GH*TH1*GD2)/TH2 \\
B1 &= F(12, 2)*F(3, 1)+F(12, 4)*F(4, 1)+F(12, 5)*F(5, 1)+
\quad F(12, 6)*F(6, 1)+F(12, 7)*F(7, 1) \\
B2 &= F(12, 1)*F(1, 2)+F(12, 2)*F(2, 2)+F(12, 3)*F(3, 2)+F(12, 4)*F(4, 2)+
\quad F(12, 5)*F(5, 2)+F(12, 6)*F(6, 2)+F(12, 7)*F(7, 2) \\
B3 &= F(12, 2)*F(2, 3)+F(12, 3)*F(3, 3)+F(12, 4)*F(4, 3)+
\quad F(12, 5)*F(5, 3)+F(12, 6)*F(6, 3)+F(12, 7)*F(7, 3) \\
B4 &= F(12, 2)*F(2, 4)+F(12, 3)*F(3, 4)+F(12, 4)*F(4, 4)+
\quad F(12, 5)*F(5, 4)+F(12, 6)*F(6, 4)+F(12, 7)*F(7, 4) \\
B5 &= F(12, 2)*F(2, 5)+F(12, 3)*F(3, 5)+F(12, 4)*F(4, 5)+
\quad F(12, 5)*F(5, 5)+F(12, 6)*F(6, 5)+F(12, 7)*F(7, 5) \\
B6 &= F(12, 2)*F(2, 6)+F(12, 3)*F(3, 6)+F(12, 4)*F(4, 6)+
\quad F(12, 5)*F(5, 6)+F(12, 6)*F(6, 6)+F(12, 7)*F(7, 6) \\
B7 &= F(12, 2)*F(2, 7)+F(12, 3)*F(3, 7)+F(12, 4)*F(4, 7)+
\quad F(12, 5)*F(5, 7)+F(12, 6)*F(6, 7)+F(12, 7)*F(7, 7) \\
B8 &= F(12, 2)*G(2, 2) \\
B9 &= F(12, 3)*G(3, 1)+F(12, 5)*G(5, 1)+F(12, 6)*G(6, 1)+
\quad G(12, 1)*(-1/TX) \\
B10 &= G(12, 1)*(G/H/TX)
\end{align*}
\]

MATRIX $A_b$ (17 x 17)

\[
A(1, 2) = 1.0 \\
A(2, 2) = F(2, 2) \\
A(2, 3) = F(2, 3) \\
A(2, 4) = F(2, 4) \\
A(2, 5) = F(2, 5) \\
A(2, 6) = F(2, 6) \\
A(2, 7) = F(2, 7) \\
A(2, 8) = G(2, 2) \\
A(3, 1) = F(3, 1) \\
A(3, 2) = F(3, 2) \\
A(3, 3) = F(3, 3) \\
A(3, 4) = F(3, 4) \\
A(3, 5) = F(3, 5) \\
A(3, 6) = F(3, 6) \\
A(3, 7) = F(3, 7) \\
A(3, 11) = G(3, 1) \\
A(4, 1) = F(4, 1)
\]
$A_{(4, 2)} = F(4, 2)$
$A_{(4, 3)} = F(4, 3)$
$A_{(4, 4)} = F(4, 4)$
$A_{(4, 5)} = F(4, 5)$
$A_{(4, 6)} = F(4, 6)$
$A_{(4, 7)} = F(4, 7)$
$A_{(5, 1)} = F(5, 1)$
$A_{(5, 2)} = F(5, 2)$
$A_{(5, 3)} = F(5, 3)$
$A_{(5, 4)} = F(5, 4)$
$A_{(5, 5)} = F(5, 5)$
$A_{(5, 6)} = F(5, 6)$
$A_{(5, 7)} = F(5, 7)$
$A_{(5, 11)} = 6(5, 1)$
$A_{(6, 1)} = F(6, 1)$
$A_{(6, 2)} = F(6, 2)$
$A_{(6, 3)} = F(6, 3)$
$A_{(6, 4)} = F(6, 4)$
$A_{(6, 5)} = F(6, 5)$
$A_{(6, 6)} = F(6, 6)$
$A_{(6, 7)} = F(6, 7)$
$A_{(6, 11)} = 6(6, 1)$
$A_{(7, 1)} = F(7, 1)$
$A_{(7, 2)} = F(7, 2)$
$A_{(7, 3)} = F(7, 3)$
$A_{(7, 4)} = F(7, 4)$
$A_{(7, 5)} = F(7, 5)$
$A_{(7, 6)} = F(7, 6)$
$A_{(7, 7)} = F(7, 7)$
$A_{(8, 8)} = -1/T2$
$A_{(8, 10)} = G3/T3$
$A_{(9, 2)} = -(K_F*B2)/(T1*T2)$
$A_{(9, 9)} = -(T1+T2)/(T1*T2)$
$A_{(9, 10)} = -1/(T1*T2)$
$A_{(10, 9)} = 1.0$
$A_{(11, 11)} = -1/TX$
$A_{(11, 14)} = G6/TX$
$A_{(12, 1)} = G8*F(13, 1)$
$A_{(12, 2)} = G8*F(13, 2)$
$A_{(12, 3)} = G8*F(13, 3)$
$A_{(12, 4)} = G8*F(13, 4)$
$A_{(12, 5)} = G8*F(13, 5)$
$A_{(12, 6)} = G8*F(13, 6)$
$A_{(12, 7)} = G8*F(13, 7)$
$A_{(12, 11)} = G8+6(13, 1)$
$A_{(12, 12)} = -1/T2$
$A_{(13, 13)} = -1/T0$
$A_{(13, 16)} = G0/T0$
$A_{(14, 13)} = G6/T6$
$A_{(14, 14)} = -1/TC$
$A_{(15, 1)} = G6*F(13, 1) + 6D2*B1$
$A_{(15, 2)} = G6*F(13, 2) + 6D2*B2$
\[ A(15, 3) = G6 \cdot F(13, 3) + G6 > B3 \]
\[ A(15, 4) = G6 \cdot F(13, 4) + G6 > B4 \]
\[ A(15, 5) = G6 \cdot F(13, 5) + G6 > B5 \]
\[ A(15, 6) = G6 \cdot F(13, 6) + G6 > B6 \]
\[ A(15, 7) = G6 \cdot F(13, 7) + G6 > B7 \]
\[ A(15, 8) = G6 > B8 \]
\[ A(15, 11) = G6 \cdot G(13, 1) + G6 > B9 \]
\[ A(15, 12) = (G7 \cdot TR2) \cdot (TA1 / TZ - 1) \]
\[ A(15, 14) = G6 > B10 \]
\[ A(15, 15) = -1 / TR2 \]
\[ A(15, 17) = G7 \cdot G(13, 7) + G7 > B7 \]
\[ A(16, 1) = G7 \cdot F(13, 1) + G7 > B1 \]
\[ A(16, 2) = G7 \cdot F(13, 2) + G7 > B2 \]
\[ A(16, 3) = G7 \cdot F(13, 3) + G7 > B3 \]
\[ A(16, 4) = G7 \cdot F(13, 4) + G7 > B4 \]
\[ A(16, 5) = G7 \cdot F(13, 5) + G7 > B5 \]
\[ A(16, 6) = G7 \cdot F(13, 6) + G7 > B6 \]
\[ A(16, 7) = G7 \cdot F(13, 7) + G7 > B7 \]
\[ A(16, 8) = G6 > B8 \]
\[ A(16, 11) = G7 \cdot G(13, 1) + G7 > B9 \]
\[ A(16, 12) = (G7 \cdot G \cdot TH1) \cdot (TA1 / TZ - 1) \cdot (TR1 / TZ - 1) \]
\[ A(16, 14) = G6 \cdot B10 \]
\[ A(16, 15) = (G7 \cdot TH2) \cdot (1 - TH1 / TA2) \]
\[ A(16, 16) = -1 / TR2 \]
\[ A(16, 17) = (G7 \cdot G \cdot TH1) \cdot (1 - TR1 / TS) \cdot (TH2 / TA2) \]
\[ A(17, 1) = G6 \cdot B1 \]
\[ A(17, 2) = G6 \cdot B2 \]
\[ A(17, 3) = G6 \cdot B3 \]
\[ A(17, 4) = G6 \cdot B4 \]
\[ A(17, 5) = G6 \cdot B5 \]
\[ A(17, 6) = G6 \cdot B6 \]
\[ A(17, 7) = G6 \cdot B7 \]
\[ A(17, 8) = G6 \cdot B8 \]
\[ A(17, 11) = G6 \cdot B9 \]
\[ A(17, 14) = G6 \cdot B10 \]
\[ A(17, 17) = -1 / TS \]
IV-3 ELEMENTS OF MATRIX $A_c$ IN EQUATION (6-2.15)

With

\[ G_5 = G_8 + G_2 \times G_Y \]
\[ G_4 = G_M + G_5 \]
\[ G_6 = - \frac{(G_5 \times T_A 1) \div (T_A 2 \times T_Z)}{T_R^2 \times T_Z} \]
\[ G_7 = - \frac{(G_4 \times T_M 1 \times T_A 1) \div (T_M 2 \times T_A 2 \times T_Z)}{G_Y + G_2} \]
\[ G_1 2 = G_5 \times T_A 1 \]
\[ G_1 4 = (G_8 + G_2 \times G_Y) \div T_A 2 \]
\[ G_1 5 = (G_M \times T_M 1 \times G_1 2) \div T_M 2 \]

MATRIX $A_c$ (17 x 17)

\[ A(1, 2) = 1.0 \]
\[ A(2, 2) = F(2, 2) \]
\[ A(2, 3) = F(2, 3) \]
\[ A(2, 4) = F(2, 4) \]
\[ A(2, 5) = F(2, 5) \]
\[ A(2, 6) = F(2, 6) \]
\[ A(2, 7) = F(2, 7) \]
\[ A(2, 8) = G(2, 2) \]
\[ A(3, 1) = F(3, 1) \]
\[ A(3, 2) = F(3, 2) \]
\[ A(3, 3) = F(3, 3) \]
\[ A(3, 4) = F(3, 4) \]
\[ A(3, 5) = F(3, 5) \]
\[ A(3, 6) = F(3, 6) \]
\[ A(3, 7) = F(3, 7) \]
\[ A(3, 11) = G(3, 1) \]
\[ A(4, 1) = F(4, 1) \]
\[ A(4, 2) = F(4, 2) \]
\[ A(4, 3) = F(4, 3) \]
\[ A(4, 4) = F(4, 4) \]
\[ A(4, 5) = F(4, 5) \]
\[ A(4, 6) = F(4, 6) \]
\[ A(4, 7) = F(4, 7) \]
\[ A(5, 1) = F(5, 1) \]
\[ A(5, 2) = F(5, 2) \]
\[ A(5, 3) = F(5, 3) \]
\[ A(5, 4) = F(5, 4) \]
\[ A(5, 5) = F(5, 5) \]
\[ A(5, 6) = F(5, 6) \]
\[ A(5, 7) = F(5, 7) \]
\[ A(5, 11) = G(5, 1) \]
\[ A(6, 1) = F(6, 1) \]
\[ A(6, 2) = F(6, 2) \]
A(6, 3) = F(6, 3)
A(6, 4) = F(6, 4)
A(6, 5) = F(6, 5)
A(6, 6) = F(6, 6)
A(6, 7) = F(6, 7)
A(6, 11) = G(6, 1)
A(7, 1) = F(7, 1)
A(7, 2) = F(7, 2)
A(7, 3) = F(7, 3)
A(7, 4) = F(7, 4)
A(7, 5) = F(7, 5)
A(7, 6) = F(7, 6)
A(7, 7) = F(7, 7)
A(8, 8) = -1/T2
A(8, 10) = 6/3/T2
A(9, 2) = -(KF*G2)/(T1*T2)
A(9, 9) = -(T1+T2)/(T1*T2)
A(9, 10) = -1/(T1*T2)
A(10, 9) = 1.0
A(11, 11) = -1/TX
A(11, 14) = 6X/TX
A(12, 1) = G6+F(13, 1)
A(12, 2) = G6+F(13, 2)
A(12, 3) = G6+F(13, 3)
A(12, 4) = G6+F(13, 4)
A(12, 5) = G6+F(13, 5)
A(12, 6) = G6+F(13, 6)
A(12, 7) = G6+F(13, 7)
A(12, 11) = G6*G(13, 1)
A(12, 12) = -1/T2
A(13, 13) = -1/T0
A(13, 16) = G0/T0
A(14, 13) = GC/TC
A(14, 14) = -1/TC
A(15, 1) = G6+F(13, 1)+G14*F(12, 1)
A(15, 2) = G6+F(13, 2)+G14*F(12, 2)
A(15, 3) = G6+F(13, 3)+G14*F(12, 3)
A(15, 4) = G6+F(13, 4)+G14*F(12, 4)
A(15, 5) = G6+F(13, 5)+G14*F(12, 5)
A(15, 6) = G6+F(13, 6)+G14*F(12, 6)
A(15, 7) = G6+F(13, 7)+G14*F(12, 7)
A(15, 11) = G6*G(13, 1)+G14*G(12, 1)
A(15, 12) = (GA/TA2)*(TA1/TZ-1)
A(15, 15) = (-1/TA2)
A(15, 17) = (GA/TA2)*(1-TA1/TA)
A(16, 1) = G7*F(13, 1)+G15*F(12, 1)
A(16, 2) = G7*F(13, 2)+G15*F(12, 2)
A(16, 3) = G7*F(13, 3)+G15*F(12, 3)
A(16, 4) = G7*F(13, 4)+G15*F(12, 4)
A(16, 5) = G7*F(13, 5)+G15*F(12, 5)
A(16, 6) = G7*F(13, 6)+G15*F(12, 6)
A(16, 7) = G7*F(13, 7)+G15*F(12, 7)
\[ A(16, 11) = G7 \cdot G(13, 1) + G15 \cdot G(12, 1) \]
\[ A(16, 12) = (G M \cdot G A \cdot T M1) \cdot (T A1 / T Z - 1) / (T M2 \cdot T A2) \]
\[ A(16, 15) = (GM / TM2) \cdot (1 - TM1 / TA2) \]
\[ A(16, 16) = (-1 / TM2) \]
\[ A(16, 17) = (G M \cdot G A \cdot T M1) \cdot (1 - TA1 / TA2) / (T M2 \cdot T A2) \]
\[ A(17, 1) = G12 \cdot F(12, 1) \]
\[ A(17, 2) = G12 \cdot F(12, 2) \]
\[ A(17, 4) = G12 \cdot F(12, 4) \]
\[ A(17, 5) = G12 \cdot F(12, 5) \]
\[ A(17, 6) = G12 \cdot F(12, 6) \]
\[ A(17, 7) = G12 \cdot F(12, 7) \]
\[ A(17, 11) = G12 \cdot G(12, 1) \]
\[ A(17, 17) = -1 / T S \]
IV-4 ELEMENTS OF MATRIX $A_d$ IN EQUATION(6-2.23)

With

\[
\begin{align*}
G_5 &= G_A * G_Z * G_Y \\
G_4 &= G_M * G_5 \\
G_6 &= -(G_5 * T_{A1}) / (T_{A2} * T_Z) \\
G_7 &= -(G_4 * T_{M1} * T_{A1}) / (T_{M2} * T_{A2} * T_Z) \\
G_8 &= (G_Y * G_Z) / T_Z \\
G_{12} &= K_5 / T_5 \\
G_{13} &= (K_F * K_5) / T_5 \\
G_{14} &= (G_A * T_{A1} + G_{13}) / T_{A2} \\
G_{15} &= (G_M * T_{M1} + G_{14}) / T_{M2}
\end{align*}
\]

MATRIX $A_d$ (18 x 18)

\[
\begin{align*}
A(1, 2) &= 1.0 \\
A(2, 2) &= F(2, 2) \\
A(2, 3) &= F(2, 3) \\
A(2, 4) &= F(2, 4) \\
A(2, 5) &= F(2, 5) \\
A(2, 6) &= F(2, 6) \\
A(2, 7) &= F(2, 7) \\
A(3, 1) &= G(2, 2) \\
A(3, 2) &= 0 \\
A(3, 3) &= F(3, 3) \\
A(3, 4) &= F(3, 4) \\
A(3, 5) &= F(3, 5) \\
A(3, 6) &= F(3, 6) \\
A(3, 7) &= F(3, 7) \\
A(3, 11) &= 0 \\
A(4, 1) &= F(4, 1) \\
A(4, 2) &= F(4, 2) \\
A(4, 3) &= F(4, 3) \\
A(4, 4) &= F(4, 4) \\
A(4, 5) &= F(4, 5) \\
A(4, 6) &= F(4, 6) \\
A(4, 7) &= F(4, 7) \\
A(5, 1) &= F(5, 1) \\
A(5, 2) &= F(5, 2) \\
A(5, 3) &= F(5, 3) \\
A(5, 4) &= F(5, 4) \\
A(5, 5) &= F(5, 5) \\
A(5, 6) &= F(5, 6) \\
A(5, 7) &= F(5, 7) \\
A(5, 11) &= 0 \\
A(6, 1) &= F(6, 1)
\end{align*}
\]
\[ A(16, 6) = G7 \cdot F(12, 6) + G15 \cdot F(12, 6) \]
\[ A(16, 7) = G7 \cdot F(12, 7) + G15 \cdot F(12, 7) \]
\[ A(16, 11) = G7 \cdot G(13, 1) + G15 \cdot G(12, 1) \]
\[ A(16, 12) = (G_{M} \cdot G_{R} \cdot T_{N1}) \cdot (T_{A1} / T_{Z-1}) / \left( T_{M2} \cdot T_{A2} \right) \]
\[ A(16, 15) = (G_{N} / T_{H2}) \cdot (1 - T_{M1} / T_{A2}) \]
\[ A(16, 16) = (-1 / T_{H2}) \]
\[ A(16, 17) = -(G_{M} \cdot T_{N1} \cdot G_{R} \cdot T_{A1} \cdot K_{F}) \cdot (T_{M2} / T_{A2} + T_{S}) \]
\[ A(16, 18) = (G_{H} \cdot G_{R} \cdot T_{N1}) \cdot (1 - T_{A1} / T_{F}) \cdot (T_{M2} / T_{A2}) \]
\[ A(17, 1) = G12 \cdot F(12, 1) \]
\[ A(17, 2) = G12 \cdot F(12, 2) \]
\[ A(17, 3) = G12 \cdot F(12, 3) \]
\[ A(17, 4) = G12 \cdot F(12, 4) \]
\[ A(17, 5) = G12 \cdot F(12, 5) \]
\[ A(17, 6) = G12 \cdot F(12, 6) \]
\[ A(17, 7) = G12 \cdot F(12, 7) \]
\[ A(17, 11) = G12 \cdot G(12, 1) \]
\[ A(17, 17) = -1 / T_{S} \]
\[ A(18, 1) = G13 \cdot F(12, 1) \]
\[ A(18, 2) = G13 \cdot F(12, 2) \]
\[ A(18, 3) = G13 \cdot F(12, 3) \]
\[ A(18, 4) = G13 \cdot F(12, 4) \]
\[ A(18, 5) = G13 \cdot F(12, 5) \]
\[ A(18, 6) = G13 \cdot F(12, 6) \]
\[ A(18, 7) = G13 \cdot F(12, 7) \]
\[ A(18, 11) = G13 \cdot G(12, 1) \]
\[ A(18, 17) = -K_{F} / T_{S} \]
\[ A(18, 18) = -1 / T_{F} \]
IV-5  ELEMENTS OF MATRICES A AND B IN EQUATION (7-4.1)

With

\[ G_5 = G_A \times G_Z \times G_Y \]
\[ G_4 = G_H \times G_S \]
\[ G_6 = \langle G_5 \times T_A, 1 \rangle / (T_A, 2 \times T_Z) \]
\[ G_7 = \langle G_4 \times T_M, 1 \times T_A, 1 \rangle / (T_M, 2 \times T_A, 2 \times T_Z) \]
\[ G_8 = \langle G_Y \times G_Z \rangle / T_Z \]
\[ G_{12} = K_S / T_S \]
\[ G_{13} = \langle K_S \rangle / T_S \]
\[ G_{14} = \langle G_A \times T_A, 1 \times G_1 \rangle / T_A, 2 \]
\[ G_{15} = \langle G_H \times T_M, 1 \times G_1 \rangle / T_M, 2 \]

**MATRIX A (18 x 18)**

| A(1, 2) = 1, 0 |
| A(2, 2) = F(2, 2) |
| A(2, 3) = F(2, 3) |
| A(2, 4) = F(2, 4) |
| A(2, 5) = F(2, 5) |
| A(2, 6) = F(2, 6) |
| A(2, 7) = F(2, 7) |
| A(2, 8) = G(2, 2) |
| A(3, 1) = F(3, 1) |
| A(3, 2) = F(3, 2) |
| A(3, 3) = F(3, 3) |
| A(3, 4) = F(3, 4) |
| A(3, 5) = F(3, 5) |
| A(3, 6) = F(3, 6) |
| A(3, 7) = F(3, 7) |
| A(3, 11) = G(3, 1) |
| A(4, 1) = F(4, 1) |
| A(4, 2) = F(4, 2) |
| A(4, 3) = F(4, 3) |
| A(4, 4) = F(4, 4) |
| A(4, 5) = F(4, 5) |
| A(4, 6) = F(4, 6) |
| A(4, 7) = F(4, 7) |
| A(5, 1) = F(5, 1) |
| A(5, 2) = F(5, 2) |
| A(5, 3) = F(5, 3) |
| A(5, 4) = F(5, 4) |
| A(5, 5) = F(5, 5) |
| A(5, 6) = F(5, 6) |
| A(5, 7) = F(5, 7) |
| A(5, 11) = G(5, 1) |
| A(6, 1) = F(6, 1) |
| A(6, 2) = F(6, 2) |
\[ A(6, 3) = F(6, 3) \]
\[ A(6, 4) = F(6, 4) \]
\[ A(6, 5) = F(6, 5) \]
\[ A(6, 6) = F(6, 6) \]
\[ A(6, 7) = F(6, 7) \]
\[ A(6, 11) = 0(6, 1) \]
\[ A(7, 1) = F(7, 1) \]
\[ A(7, 2) = F(7, 2) \]
\[ A(7, 3) = F(7, 3) \]
\[ A(7, 4) = F(7, 4) \]
\[ A(7, 5) = F(7, 5) \]
\[ A(7, 6) = F(7, 6) \]
\[ A(7, 7) = F(7, 7) \]
\[ A(8, 8) = -1/T2 \]
\[ A(8, 10) = 03/T3 \]
\[ A(9, 2) = -(G1*G2)/(T1*T2) \]
\[ A(9, 9) = -(T1+T2)/(T1*T2) \]
\[ A(9, 10) = -1/(T1*T2) \]
\[ A(10, 9) = 1.0 \]
\[ A(11, 11) = -1/TX \]
\[ A(11, 14) = 6X/TX \]
\[ A(12, 1) = 08*F(13, 1) \]
\[ A(12, 2) = 08*F(13, 2) \]
\[ A(12, 3) = 08*F(13, 3) \]
\[ A(12, 4) = 08*F(13, 4) \]
\[ A(12, 5) = 08*F(13, 5) \]
\[ A(12, 6) = 08*F(13, 6) \]
\[ A(12, 7) = 08*F(13, 7) \]
\[ A(12, 11) = 08*G(13, 1) \]
\[ A(12, 12) = -1/TZ \]
\[ A(13, 13) = -1/T0 \]
\[ A(13, 16) = 60/T0 \]
\[ A(14, 13) = 6C/TC \]
\[ A(14, 14) = -1/TC \]
\[ A(15, 1) = 06*F(13, 1) \]
\[ A(15, 2) = 06*F(13, 2) \]
\[ A(15, 3) = 06*F(13, 3) \]
\[ A(15, 4) = 06*F(13, 4) \]
\[ A(15, 5) = 06*F(13, 5) \]
\[ A(15, 6) = 06*F(13, 6) \]
\[ A(15, 7) = 06*F(13, 7) \]
\[ A(15, 11) = 06*G(13, 1) \]
\[ A(15, 12) = (GA/TA2)*(TA1/T2-1) \]
\[ A(15, 15) = -1/TA2 \]
\[ A(15, 17) = -(GA*TA1*KF)/(TA2*TS) \]
\[ A(15, 18) = (GA/TA2)*(1-TA1/TF) \]
\[ A(16, 1) = 07*F(13, 1) \]
\[ A(16, 2) = 07*F(13, 2) \]
\[ A(16, 3) = 07*F(13, 3) \]
\[ A(16, 4) = 07*F(13, 4) \]
\[ A(16, 5) = 07*F(13, 5) \]
\[ A(16, 6) = 07*F(13, 6) \]
\[ A(16, 7) = G7*F(13, 7) \]
\[ A(16, 11) = G7*G(13, 1) \]
\[ A(16, 12) = (GH*GA*TH1)*(TA1/TZ-1)/(TM2*TA2) \]
\[ A(16, 15) = (GH/TM2)*(1-TM1/TA2) \]
\[ A(16, 16) = -1/TM2 \]
\[ A(16, 17) = -(GH*TM1*GA*TA1*KF)/(TM2*TA2*TS) \]
\[ A(16, 18) = (GH*GA*TM1)*(1-TA1/TF)/(TM2*TA2) \]
\[ A(17, 17) = -1/TS \]
\[ A(18, 17) = -KF/TS \]
\[ A(18, 18) = -1/TF \]

**MATRIX B (18 x 1)**

\[ B(15, 1) = 614 \]
\[ B(16, 1) = 615 \]
\[ B(17, 1) = 612 \]
\[ B(18, 1) = 613 \]
IV-6 ELEMENTS OF MATRICES A AND B IN EQUATION (10-1.14)

With

\[ G_5 = G_A G_Z G_V \]
\[ G_4 = G_M G_5 \]
\[ G_6 = -\frac{(G_5 T_{A1})}{(T_{A2} T_Z)} \]
\[ G_7 = -\frac{(G_4 T_{M1} T_{A1})}{(T_{M2} T_{A2} T_Z)} \]
\[ G_8 = \frac{(G_V G_Z)}{T_Z} \]
\[ G_{12} = K_S T_5 \]
\[ G_{13} = (K_F K_S)/T_S \]
\[ G_{14} = (G_A T_{A1} G_{13})/T_A 2 \]
\[ G_{15} = (G_M T_{M1} G_{13})/T_M 2 \]
\[ G_{16} = G_{11}/T_{D1} \]
\[ G_{17} = K_{G2} T_{D1} \]

MATRICE A \((21 \times 21)\)

\[ A(1, 2) = 1.0 \]
\[ A(2, 2) = F(2, 2) \]
\[ A(2, 3) = F(2, 3) \]
\[ A(2, 4) = F(2, 4) \]
\[ A(2, 5) = F(2, 5) \]
\[ A(2, 6) = F(2, 6) \]
\[ A(2, 7) = F(2, 7) \]
\[ A(2, 8) = G(2, 2) \]
\[ A(3, 1) = F(3, 1) \]
\[ A(3, 2) = F(3, 2) \]
\[ A(3, 3) = F(3, 3) \]
\[ A(3, 4) = F(3, 4) \]
\[ A(3, 5) = F(3, 5) \]
\[ A(3, 6) = F(3, 6) \]
\[ A(3, 7) = F(3, 7) \]
\[ A(3, 11) = G(3, 1) \]
\[ A(4, 1) = F(4, 1) \]
\[ A(4, 2) = F(4, 2) \]
\[ A(4, 3) = F(4, 3) \]
\[ A(4, 4) = F(4, 4) \]
\[ A(4, 5) = F(4, 5) \]
\[ A(4, 6) = F(4, 6) \]
\[ A(4, 7) = F(4, 7) \]
\[ A(5, 1) = F(5, 1) \]
\[ A(5, 2) = F(5, 2) \]
\[ A(5, 3) = F(5, 3) \]
\[ A(5, 4) = F(5, 4) \]
\[ A(5, 5) = F(5, 5) \]
\[ A(5, 6) = F(5, 6) \]
\[ A(5, 7) = F(5, 7) \]
\[ R(5, 11) = G(5, 1) \]
\[ R(5, 1) = F(6, 1) \]
\[ R(6, 2) = F(6, 2) \]
\[ R(6, 3) = F(6, 3) \]
\[ R(6, 4) = F(6, 4) \]
\[ R(6, 5) = F(6, 5) \]
\[ R(6, 6) = F(6, 6) \]
\[ R(6, 7) = F(6, 7) \]
\[ R(6, 11) = G(6, 1) \]
\[ R(7, 1) = F(7, 1) \]
\[ R(7, 2) = F(7, 2) \]
\[ R(7, 3) = F(7, 3) \]
\[ R(7, 4) = F(7, 4) \]
\[ R(7, 5) = F(7, 5) \]
\[ R(7, 6) = F(7, 6) \]
\[ R(7, 7) = F(7, 7) \]
\[ R(8, 8) = -1/T3 \]
\[ R(8, 10) = G3/T3 \]
\[ R(9, 2) = -(G1*G2)/(T1*T2) \]
\[ R(9, 3) = -(T1+T2)/(T1*T2) \]
\[ R(9, 10) = -1/(T1*T2) \]
\[ R(9, 21) = G2/(T1*T2) \]
\[ R(10, 9) = 1.0 \]
\[ R(11, 11) = -1/TX \]
\[ R(11, 14) = G6/TX \]
\[ R(12, 1) = G8*F(13, 1) \]
\[ R(12, 2) = G8*F(13, 2) \]
\[ R(12, 3) = G8*F(13, 3) \]
\[ R(12, 4) = G8*F(13, 4) \]
\[ R(12, 5) = G8*F(13, 5) \]
\[ R(12, 6) = G8*F(13, 6) \]
\[ R(12, 7) = G8*F(13, 7) \]
\[ R(12, 11) = G8*G(13, 1) \]
\[ R(12, 12) = -1/T2 \]
\[ R(13, 13) = -1/T0 \]
\[ R(13, 16) = G0/T0 \]
\[ R(14, 13) = G6/TC \]
\[ R(14, 14) = -1/TC \]
\[ R(15, 1) = G6*F(13, 1) \]
\[ R(15, 2) = G6*F(13, 2) \]
\[ R(15, 3) = G6*F(13, 3) \]
\[ R(15, 4) = G6*F(13, 4) \]
\[ R(15, 5) = G6*F(13, 5) \]
\[ R(15, 6) = G6*F(13, 6) \]
\[ R(15, 7) = G6*F(13, 7) \]
\[ R(15, 11) = G6*G(13, 1) \]
\[ R(15, 12) = (GA/TA2)*(TA1/TZ-1) \]
\[ R(15, 15) = -1/TA2 \]
\[ R(15, 17) = -(GA*TA1*KF)/(TA2*TS) \]
\[ R(15, 18) = (GA/TA2)*(1-TA1/TF) \]
\[ R(16, 1) = G7*F(13, 1) \]
\[ R(16, 2) = G7*F(13, 2) \]
\[
\begin{align*}
\mathbf{A}(16, 3) &= G7 \times F(13, 3) \\
\mathbf{A}(16, 4) &= G7 \times F(13, 4) \\
\mathbf{A}(16, 5) &= G7 \times F(13, 5) \\
\mathbf{A}(16, 6) &= G7 \times F(13, 6) \\
\mathbf{A}(16, 7) &= G7 \times F(13, 7) \\
\mathbf{A}(16, 11) &= G7 \times G(13, 1) \\
\mathbf{A}(16, 12) &= (Gm \times G \times Tm1) \times (Tal / Tz - 1) / (Tm2 \times Ta2) \\
\mathbf{A}(16, 15) &= (Gm / Tm2) \times (1 - Tm1 / Tm2) \\
\mathbf{A}(16, 16) &= -1 / Tm2 \\
\mathbf{A}(16, 17) &= -(Gm \times Tm1 \times G \times Ta1 \times Kg) / (Tm2 \times Ta2 \times Ts) \\
\mathbf{A}(16, 18) &= (Gm \times G \times Tm1) \times (1 - Ta1 / Tm2) / (Tm2 \times Ta2) \\
\mathbf{A}(17, 17) &= -1 / Ts \\
\mathbf{A}(18, 17) &= Kg / Tm2 \\
\mathbf{A}(18, 18) &= -1 / Tm2 \\
\mathbf{A}(19, 19) &= -1 / Tg1 \\
\mathbf{A}(20, 19) &= Kg2 / Tg1 \\
\mathbf{A}(20, 20) &= -1 / Tg2 \\
\mathbf{A}(21, 20) &= Kg3 / Tg3 \\
\mathbf{A}(21, 21) &= -1 / Tg3 \\

\text{MATRIX } \mathbf{B} \ (21 \times 2) \\
\begin{align*}
\mathbf{B}(15, 1) &= G14 \\
\mathbf{B}(16, 1) &= G15 \\
\mathbf{B}(17, 1) &= G12 \\
\mathbf{B}(18, 1) &= G13 \\
\mathbf{B}(19, 2) &= G16 \\
\mathbf{B}(20, 2) &= G17
\end{align*}
\]
IV-7 ELEMTENTS OF MATRICES A AND B IN EQUATION (10-2.1)

MATRIX A (7 x 7)

A(1, 2) = 1.0
A(2, 2) = F(2, 2)
A(2, 3) = F(2, 3)
A(2, 4) = F(2, 4)
A(2, 5) = F(2, 5)
A(2, 6) = F(2, 6)
A(2, 7) = F(2, 7)
A(3, 1) = F(3, 1)
A(3, 2) = F(3, 2)
A(3, 3) = F(3, 3)
A(3, 4) = F(3, 4)
A(3, 5) = F(3, 5)
A(3, 6) = F(3, 6)
A(3, 7) = F(3, 7)
A(4, 1) = F(4, 1)
A(4, 2) = F(4, 2)
A(4, 3) = F(4, 3)
A(4, 4) = F(4, 4)
A(4, 5) = F(4, 5)
A(4, 6) = F(4, 6)
A(4, 7) = F(4, 7)
A(5, 1) = F(5, 1)
A(5, 2) = F(5, 2)
A(5, 3) = F(5, 3)
A(5, 4) = F(5, 4)
A(5, 5) = F(5, 5)
A(5, 6) = F(5, 6)
A(5, 7) = F(5, 7)
A(6, 1) = F(6, 1)
A(6, 2) = F(6, 2)
A(6, 3) = F(6, 3)
A(6, 4) = F(6, 4)
A(6, 5) = F(6, 5)
A(6, 6) = F(6, 6)
A(6, 7) = F(6, 7)
A(7, 1) = F(7, 1)
A(7, 2) = F(7, 2)
A(7, 3) = F(7, 3)
A(7, 4) = F(7, 4)
A(7, 5) = F(7, 5)
A(7, 6) = F(7, 6)
A(7, 7) = F(7, 7)

MATRIX B (7 x 2)

B(2, 2) = G(2, 2)
B(3, 1) = G(3, 1)
B(5, 1) = G(5, 1)
B(6, 1) = G(6, 1)
APPENDIX V

METHOD OF SOLUTION OF
MATRIX RICATTI EQUATION

In general, the application of Optimal Control Theory to a linear, time-invariant system

\[ x = Ax + Bu \]  \hspace{1cm} (V-1)

involves the solution of a Matrix Ricatti Equation of the form

\[ D + QA + A'Q - QBH^{-1}B'Q = 0 \] \hspace{1cm} (V-2)

or

\[ D + QA + A'Q - Q(E)Q = 0 \] \hspace{1cm} (V-3)

where \( E = BH^{-1}B' \). \( D, Q, A \) and \( E \) are \((n \times n)\) matrices, and \( D \) and \( E \) are positive semidefinite matrices.

Employing the computationally feasible method of J. E. Potter 40, the solution procedure of equation(V-3) starts with the formation of an \((2n \times 2n)\) matrix AM given by

\[ AM = \begin{bmatrix} A' & D \\ E & -A \end{bmatrix} \] \hspace{1cm} (V-4)

which has the property that if \( \lambda \) is one of its eigenvalues, so is \( -\lambda \).

An \((2n \times 2n)\) matrix \( T \) is then constructed as

\[ T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \] \hspace{1cm} (V-5)
where $T_{11}$ to $T_{22}$ are all $(n \times n)$ matrices. The first $n$ columns of $T$ are made up of the eigenvectors corresponding to the positive eigenvalues of $AM$. The second set of $n$ columns of $T$, on the other hand, are made up of the eigenvectors corresponding to the negative eigenvalues of $AM$.

The positive definite, symmetric solution $Q$ of equation (V-3) can be shown to be\textsuperscript{39,40}

$$Q = T_{11}^{-1}T_{21}^{-1}$$  \hfill (V-6)

In the computer subroutine (Ricatti) that follows, $NS = n$, $AA = D$, $BRB = E$, $HI = H^{-1}$, $AU = T_{11}$ and $AL = T_{21}$. The steps for computing $Q$ are:

(1) Calculate matrix $E$.

(2) Form matrix $AM$.

(3) Compute the eigenvalues and eigenvectors of $AM$.

(4) Obtain the eigenvectors corresponding to the eigenvalues of $AM$ with positive real parts. These eigenvectors are then arranged to form the first $n$ columns of $T$ (the formation of the second set of $n$ columns of $T$ is omitted since it is not required in the solution).

(5) Invert the lower half of the first $n$ columns of $T$ and premultiply the result with the upper half to obtain the complex solution of $Q$.

(6) Obtain the real parts of $Q$ as the final solution (this is because the real part of every element of complex $Q$ is always very much greater than the imaginary part which itself is very small and can be neglected).
SUBROUTINE RICATTICR(B,RR,HI,N5,NC,Q)
COMPLEX A,AL,ALL
DIMENSION A(18,18), B(18,18), AA(18,18), Q(18,18), BHB(18,18)
DIMENSION AM(36,36), AL(18,18), DD(36), INTEGER(36)
DIMENSION WR(36), HI(36), ALL(18,18), V(36,36), WH(1,18)
DIMENSION BTR(1,18)
NN=2*NS
WRITE(2,100)

CALCULATION OF MATRIX [BHB]

DO 30 I=1,NS
BTR(I,I)=B(I,1)
30 WM(I,I)=HI*BTR(I,I)
CALL F01CKF(BHB, B, HH, NS, NS, HC, DD, 1, 1, 0)
CALL F01CPF(DD, HH, 1, 0)
WRITE(2,200)
CALL PRINTOUT1(BHB, NS)

FORM MATRIX [AM]

DO 2 I=1,NS
DO 2 J=1,NS
AM(I,J)=A(J, I)
2 AM(I+NS), J)=BHB(I, J)
-AM(I, J+NS)=AA(I, J)
2 AM(I+NS), J+NS)=-A(I, J)

CALCULATE THE EIGENVALUES AND EIGENVECTORS OF [AM]
CALL F01ATF(NN, 2, AM, NS, K, L, DD)
CALL F01AKF(NN, K, L, AM, NS, NS, V, NN)
CALL F01APF(NN, K, L, AM, NS, INTGEP)
CALL F02ARF(NN, K, L, 2.0**(-37), AM, NN, NS, NS, WH, HI, INTGEP, 0)
CALL F01ARF(NN, K, L, NS, AM, NN, NS, NS)
WRITE(2,101)
WRITE(2,102)(WH(I), HI(I), I=1, NN)

OBTAIN THOSE EIGENVECTORS CORRESPONDING TO THE EIGENVALUES
OF [AM] WITH POSITIVE REAL PARTS

I=1
K=1
10 IF(HI(I)) 0, 5
   IF(HI(I)) 0, 0
   I=I+1
GOTO 6
5 IF(HI(I)) 7, 0, 7
   DO 9 J=1,NN
8 V(J,K)=V(J, I)
   HI(K)=HI(I)
   K+K+1
GOTO 6
7 DO 9 J=1,NN
   V(J,K)=V(J, I)
9 V(J,K+1)=V(J, I+1)
I = 1
16 IF(WI(I)) 12,0,12
DO 13 J=1,NS
AU(J,I)=CMPLX(V(J,I),0,0)
13 AL(J,I)=CMPLX(V(J+NS,I),0,0)
GOTO 14
12 DO 15 J=1,NS
AU(J,I)=CMPLX(V(J,I),V(J,1+I))
AU(J,1+I)=CMPLX(V(J,I),-V(J,1+I))
AL(J,1)=CMPLX(V(J+NS,I),V(J+NS,1+I))
15 AL(J,1+I)=CMPLX(V(J+NS,I),-V(J+NS,1+I))
I=I+1
14 I=I+1
IF(NS-I) 0,16,16
C
C INVERT [AI] AND OBTAIN THE SOLUTION [Q]
DO 20 I=1,NS
DO 20 J=1,NS
ALL(I,J)=CMPLX(0,0,0)
20 ALL(I,J)=CMPLX(1,0,0)
CALL FO4ADF(AL,NS,ALL,NS,NS,ALL,NS,DD,0)
DO 22 I=1,NS
DO 22 J=1,NS
AL(I,J)=CMPLX(0,0,0)
22 O(I,J)=0.0
DO 21 I=1,NS
DO 21 J=1,NS
DO 21 K=1,NS
AL(I,J)=AL(I,J)+AU(I,K)*ALL(K,J)
21 Q(I,J)=REAL(AL(I,J))
WRITE(2,203)
CALL PRINTOUT2(AL,NS)
CALL FO1CAF(INTGER,NN,1,0)
CALL FO1CAF(DD,NN,1,0)
CALL FO1CAF(WI,NN,1,0)
CALL FO1CAF(WR,NN,1,0)
100 FORMAT(1H1,100(*'),/,'10X,'SOLUTION OF THE MATRIX RICATTI EQUATION',/,'100(*')
101 FORMAT(/'1H','EIGENVALUES OF [AM]'/)
102 FORMAT(1H,'E15.5','E15.5)
200 FORMAT(1H,'MATRIX [BHB]'/)
203 FORMAT(1H,'COMPLEX MATRIX [R]'/)
RETURN
END
APPENDIX VI

COMPUTER ALGORITHMS

VI-1 ALGORITHM FOR THE APPLICATION OF LINEAR OPTIMAL CONTROL

start

read in system parameters
of the system $x = Ax + Bu$,
read in the values of the
weighting factors $\lambda_1$ to $\lambda_5$

set the load conditions
(set the values of $P_0$ and $Q_0$)

calculate the steady-state values,
form the matrices $A$ and $B$

form the matrices $D$ and $H$
of the performance index

solve the Matrix Riccati Equation

$$D + A'Q + QA - QH^{-1}B'Q = 0$$

calculate the optimal-controller-
gain matrix $K = -H^{-1}B'Q$
and form the closed-loop system

$$x = (A + BK)x$$

set value of step disturbance and
compute the system time response

printout the results and
plot the time response

stop
VI-2 ALGORITHM FOR THE APPLICATION OF MODAL CONTROL

```
start

read in system parameters of the system \( x = Ax + Bu \)

set the load conditions (set the values of \( P_o \) and \( Q_o \))

calculate the steady-state values, form the matrices \( A \) and \( B \)

compute the matrix \( V \) and vectors \( \lambda \) and \( P \)

declare the required eigenvalues and store in the vector \( \rho \)

compute the vector \( QK \), then the controller-gain vector \( K \)

form the closed-loop system \( x = (A + BK)x \)

set the value of initial condition and compute the system time response

printout the results and plot the time response

stop```

VI-3 ALGORITHM FOR THE FULL-OBSERVER DESIGN

start

read in system parameters of
the system $x = Ax + Bu, y = Cx$,
read in the feedback gain
matrix $K$ and matrix $A$

set the load conditions
(set the values of $P_0$ and $Q_0$)

calculate the steady-state values,
form matrices $A$, $B$ and $C$

solve the Matrix Ricatti Equation
$I + Q_1 A' + A Q_1 - Q_1 C' A^{-1} C Q_1 = 0$

compute the matrix $S = (A^{-1} C Q_1)'$
and the eigenvalues of $(A - SC)$

form the overall observer-controller
closed-loop system of order $2n$

set the value of initial condition,
compute the true system
states and the observer outputs

printout the results and
plot the time response

stop
VI-4 ALGORITHM FOR THE LOW-ORDER OBSERVER DESIGN

start

read in system parameters of the system \( x = Ax + Bu, y = Cx \), read in the feedback gain matrix \( K \)

set the load conditions (set the values of \( P_o \) and \( Q_o \))

calculate the steady-state values, form the matrices \( A, B \) and \( C \)

choose matrices \( Q \) and \( S \)

solve for \( T \) and \( R \) in the equations \( QT - TA = -SC \) and \( TB = R \), check the solutions

form square matrix \( T_1 = \begin{bmatrix} T \\ C \end{bmatrix} \), compute the inversion \( T_1^{-1} = W \)

partition \( W \), form the overall observer-controller closed-loop system of order \( 2n-1 \)

set the value of initial condition, compute the true system states and the observer outputs

printout the results and plot the time response

stop
APPENDIX VII

SOLUTION FOR EQUATION $QT - TA = C$

The algebraic matrix equation

$$QT - TA = C$$  \hspace{1cm} (VII-1)

where $Q$, $A$ and $C$ are given $(p \times p)$, $(n \times n)$ and $(p \times n)$ matrices respectively and $T$ is an unknown $(p \times n)$ matrix, is equivalent to a system of $(p \times n)$ scalar equations in the elements of $T$. By expanding equation (VII-1) in terms of the elements of the matrices, a general pattern is revealed by which this equation can be rewritten as

$$YX = K$$  \hspace{1cm} (VII-2)

where

$$Y = \begin{bmatrix}
Y_{11} & Y_{12} & \cdots & \cdots & Y_{1p} \\
Y_{21} & Y_{22} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots \\
Y_{p1} & \cdots & \cdots & Y_{pp}
\end{bmatrix}
\begin{array}{c}
Y_{11} \\
Y_{21} \\
\vdots \\
Y_{p1}
\end{array}
\begin{array}{c}
Y_{12} \\
Y_{22} \\
\vdots \\
Y_{pp}
\end{array}
$$

with $Y$ an $((p \times n) \times (p \times n))$ square matrix and each submatrix of which (i.e. $Y_{11}, Y_{12}, \ldots, Y_{pp}$) of dimension $(n \times n)$, and

$$Y_{kk} = \begin{bmatrix}
-a_{11}+q_{kk} & -a_{21} & \cdots & \cdots & \cdots & -a_{n1} \\
-a_{12} & -a_{22}+q_{kk} & -a_{32} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
\vdots & \vdots & \ddots & \ddots & \ddots \\
-a_{1n} & \cdots & \cdots & -a_{n-1,n} & -a_{nn}+q_{kk}
\end{bmatrix}
$$

$(k = 1, 2, \ldots, p)$
\[
\begin{bmatrix}
q_{ij} & 0 & \cdots & \cdots & 0 \\
0 & q_{ij} & 0 & \cdots & \cdots \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
\vdots & \vdots & \ddots & 0 & \vdots \\
0 & \cdots & \cdots & 0 & q_{ij}
\end{bmatrix}
\]

\(Y_{ij} = \begin{cases} 
q_{ij} & (i = 1, 2, \ldots, p) \\
0 & (j = 1, 2, \ldots, p)
\end{cases} \)

\[
X = \begin{bmatrix}
t_{11} & t_{12} & \cdots & t_{1n} & t_{21} & \cdots & t_{2n} & \cdots & t_{p1} & \cdots & t_{pn}
\end{bmatrix}^	op
\]

and

\[
K = \begin{bmatrix}
c_{11} & c_{12} & \cdots & c_{1n} & c_{21} & \cdots & c_{2n} & \cdots & c_{p1} & \cdots & c_{pn}
\end{bmatrix}^	op
\]

The lower case letters \(a, q, t\) and \(c\), together with their associated numerical subscripts, represent the elements of the matrices \(A, Q, T\) and \(C\) respectively. The patterns of arranging \(Y\) and \(K\) follow closely the pattern of arranging the \(T\) elements in \(X\). If the pattern of \(X\) changes, so do the patterns of \(Y\) and \(K\).

Equation \((\text{VII-2})\) can easily be formed on the ICL 1904S digital computer of the Loughborough University of Technology and the NAG subroutine \(F04ATF\) can be used for the solution.