An investigation into the problems in teaching physics at CSE level to pupils of low mathematical ability and ideas for improving standards

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Additional Information:

- A Master's Thesis. Submitted in partial fulfilment of the requirements for the award of Master of Philosophy at Loughborough University.

Metadata Record: [https://dspace.lboro.ac.uk/2134/32478](https://dspace.lboro.ac.uk/2134/32478)

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AN INVESTIGATION INTO THE PROBLEMS IN TEACHING PHYSICS AT CSE LEVEL TO PUPILS OF LOW MATHEMATICAL ABILITY; AND IDEAS FOR IMPROVING STANDARDS

By

BRIAN COLLINGWOOD  BSc(Hons) (CNAA)

A master's thesis submitted in partial fulfilment of the requirements for the award of Master of Philosophy of the Loughborough University of Technology, 1983

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An investigation into the problems in teaching Physics at CSE level to pupils of low mathematical ability; and ideas for improving standards

B Collingwood

Abstract

The dissertation begins with a review of the general feelings regarding the standard of Mathematical Education in Schools and that shown by school leavers. This is followed by an analysis of the publication "Aspects of Secondary Education" which was produced after several years of investigation by Her Majesty's Inspectors of Schools and must be regarded as a highly relevant and critical appraisal of the standard of Mathematical Education in England and Wales.

The problem Science teachers have in using pupils' Mathematical knowledge as a tool in the building and understanding of scientific fundamentals is illustrated by a detailed study of two groups of pupils who were taking a Certificate of Secondary Education course in Physics. This details, step by step, the scientific theory covered by these pupils, the mathematical requirements, specific examples of its use, and examines the mathematical inadequacies of these pupils, listing the areas which need special attention specifically for the efficient teaching of a science course.

In order to promote a dialogue between the Science and Mathematics Departments of this school a table of mathematical requirements was drawn up which indicated topics, depth of treatment and at what stages during the pupils' secondary school life they would be required by the science department.

From the initial study of the pupils on this course, ideas were formulated to attempt to overcome the poor standards of mathematics found. This was done by integrating more pure
mathematical topics into the science scheme of work and building strong foundations in the areas required, by a practical approach of applying mathematical concepts.

It had been shown early in the research, by tests of a cognitive nature carried out on a sample of 11 to 12 year olds, approximately half of which had previously been taught mathematics by a practical approach and the other half by formal theoretical rote methods of learning, that the use of practical applications in the teaching of mathematics had very significant benefits in the learning and understanding process.

As the course progressed a booklet was developed, for the pupils, of brief revision notes in basic numeracy and mathematics concepts to aid the short retention life of their memories and to overcome the poor standards of work produced by this middle to lower ability range of pupils in their exercise books.

Further booklets were developed to run as extra curricular revision courses. The aims of the booklets were to show the links between topic areas, to stimulate interest and motivation by the use of materials collected from different sources and written in a way and at a reading level to assist the lower ability pupils in performing the operations, to endeavour to make the learning process less formal and at times fun, because this vastly increased motivation.

Conclusions were drawn from this work and recommendations for the future were made.

Key Words: School Physics Course, CSE Physics, Maths for Science, Maths for Physics, Cognitive Tests, Remedial Maths Booklets, Maths/Physics Interface
ACKNOWLEDGEMENTS

I would like to thank Professor A C Bajpai of the Centre for Advancement of Mathematical Education in Technology for his encouragement, help and direction in this project.

I am indebted to various institutions for the use of their resources, particularly the University of Technology, Loughborough and Ripley Mill Hill School.

I am deeply grateful to my fellow teachers for the information given, either as written evidence or as recorded interview. My gratitude is extended to the pupils who expressed their opinions in essays and recorded interview, and who participated enthusiastically in the tests and practical work.

I would like to express my appreciation to Hutchinson & Co (Publishers) Ltd. for allowing me to use diagrams from their textbook "Physics for You" which I found very helpful in this work.

Finally, I wish to express my sincere gratitude to my wife and children for their forbearance throughout this project.
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CHAPTER 1

Introduction

Since leaving industry and entering the teaching profession in 1975 the author has been concerned that many teachers teaching physics to the 11-16 year old range have found that the lack of understanding and competence in certain basic mathematical skills shown by their pupils proved a significant obstacle in the teaching of their subject. A growing amount of evidence exists which indicates that the mathematical literacy of many secondary school pupils is below the standard that many physics teachers would like or indeed, expect.

If these pupils do not possess a good understanding of the mathematical concepts and skills, such as those required for basic numeracy and simple algebraic manipulations by the end of the 'third year' when the option choices are made, they are unlikely to want to, or be able to follow a physics course successfully and hence they will be prevented from becoming possible scientists or engineers.

Derek Duckworth and Professor Entwistle in their paper titled "The Continuing Swing" [1] which related to pupils' reluctance to study science stated

"Pupils have to make important subject choices at age 13 or 14 which allow the pupils to opt out of the demanding specialist science course at a comparatively early age" [1]

It appears to be accepted by members of the teaching profession who teach science, and those in industry dealing with apprentices, that they will have to cope, in addition to
teaching their subject or trade, respectively, with the teaching of the mathematics required. The Royal Society in 1976 reported

"... the pupils who are selected come into firms unable to do the mathematics demanded of them in the job. Industry is having to spend time and money bringing their attainment up to standard" [2]

Doe [3] in March 1980 reported in the Times Educational Supplement that mathematics was taught in isolation and this referred to evidence submitted to the Cockcroft Committee by the Royal Society when they said

"The best balance between pure and applied, traditional and modern, is one which while firing the interests of pupils, most effectively meets predetermined external needs rather than any intrinsic objectives attributable to school mathematics in isolation" [3]

Mathematics teaching is not related to the 'real world' and other curriculum areas, even when these subjects either need to apply mathematic techniques directly as in the sciences, or when they give broader illustrations of ideas which are on the syllabus.

With the advent of comprehensive education from the 1960's onwards, and rapid curriculum development, mainly in mathematics with the introduction of 'Modern Maths' and to a limited extent in science with the 'Nuffield Projects', there has become a greater awareness of a 'phase difference' between mathematics and physics teaching.

"..... it is not surprising that there should occur a degree of mismatch in various areas. One such area is the maths/physics interface in schools"

Royal Society and Institute of Physics [4]
The improvement in the motivation of school pupils is very important if understanding and eventual examination success is to be achieved. Linda Dickson [5] stated that apprentices she was studying at the London Transport Department's Training School had suffered considerably throughout their school mathematics teaching and this was due "mainly to ineffective class control and low motivation through lack of practical application of the mathematics encountered". The poor mathematical ability of these apprentices being rectified in their training centre where there was a relatively high level of motivation achieved, from previously unmotivated disruptive school pupils. The Engineering Industries Training Board [6] also stated that improved motivation could be obtained by the application of mathematical concepts.

"They did overcome the problem during the first year's training because of the high motivation of trainees once they understood the needs for arithmetical skills as a part of their jobs" [6]

Graham [23] in his work related to the mathematical needs of entrants to the engineering industry pointed out most convincingly the importance of motivation, especially in the case of older children of average ability and below. This work was carried out under the supervision of Professor Bajpai, who in conjunction with Bond produced a textbook entitled "Apprentice Maths" specifically aimed at improving motivation and interest in certain areas of basic mathematics related to craft apprentices. Furthermore Gatenby [26] in his evaluation of this book found that there was such an overwhelming enthusiasm and positive response from those people using the book as either a teaching or learning aid, that he was convinced "there was a great benefit to be gained from introducing this material into schools" [26].

The author also believes this to be very true but would not limit the material to that from the craft industries but would include applications from the real world and other school curriculum areas such as the sciences, humanities and crafts.
Later chapters attempt to show that by incorporating relevance and the use of a practical approach in the teaching, that motivation can be increased in the same way as both Gatenby [26] and Dickson [5] found, and this ultimately helps all other curriculum areas which use mathematics as a necessary tool.

Bond [27] also investigated the mathematical needs of school pupils and concluded in his work

"Motivation is a vital factor in improving standards of pupils within schools.... Pupils should be taught how to relate mathematics to everyday life and practical problems" [27]

A report in a daily newspaper, regarding a government backed report indicated teachers were turning brighter pupils off mathematics by their boring lessons and it was now one of the most unpopular subjects in Britain's classrooms. Furthermore a report to Cockcroft [7] stated

"Mathematics lessons in secondary schools are very often not about anything. You collect like terms, or learn the law of indices, with no perception of why anyone needs to do such things"

In the following work the author interviews many school pupils who confirm that they find mathematics boring and that they have no comprehension of why they are being taught many of their mathematics topics.

At the present time the GCE 'O' level and the controversial CSE are taken by 83% of teenagers although these examinations were originally designed only for the top 60% (Mark Carlisle [8]). Figures issued by the Midland Region CSE Board in their pamphlet "Report on the examination 1982" explained the grading system and they indicated that from all the pupils taking this exam, (many of whom were originally thought to be below the standard set when it was created) only approximately 3% were 'ungraded'. Some of the statistics from this pamphlet are shown in Appendix I. It, therefore, appears that the
standards must have been moderated downwards to allow some approximately 20% of the entrants to be graded, and this is very evident when comparing questions from examination papers from the beginning and end of the last decade. Nowadays the vast majority of questions in subjects where mathematics is applied are devoid of any mathematical content or those few questions that contain some mathematics are restricted to very basic algorithms.

The CSE Board in their examiners' report state quite clearly that any questions involving fractions and decimals are very poorly done, whereas multiple choice type questions and brief written answers fare much better. Even at GCE standard in Physics many examination boards appear to be removing mathematical based topics, such as the field of mechanics from the syllabus. Any questions requiring a good understanding of mathematical concepts, such as formulae and their manipulation, and a proficiency in numeracy are being phased out in favour of the more abstract, non-mathematical physics concepts.

Numerous reports have been written regarding the "falling standards" of mathematical proficiency. The IMA in their evidence to the Cockcroft Committee [7] stated, regarding the basic weakness of mathematics teaching:

"The relevance of mathematics was not understood and the introduction of each mathematical skill needs to be clearly related to and illustrated by everyday practical applications of that skill"

Times Educational Supplement [3]

The author believes that more teachers should gain industrial experience by taking advantage of the secondment schemes available. Very few teachers are able to apply their subject in a meaningful way as they have no personal experiences of the mathematics utilized in the industrial field and hence find difficulty in bringing their subject to life.
Maggs [9] stated

"If schools are to be able to motivate their pupils then teachers themselves must be aware of the applications of the mathematics they are teaching. There must be a greater link than at present between school and industry".

During the author's contact with industry and commerce he formed the opinion that they were far from happy with the results of the education system with regards to pupils of average to low ability, believing overwhelmingly that the mathematics the pupils had been taught bore little or no resemblance to their needs.

The Government's Assessment of Performance Unit reported that 15 year olds are not much better at mathematics than 11 year olds, and the survey indicated practical application showed up pupils' weaknesses in basic mathematics [10].

The author feels that the country urgently needs mathematicians who have a sound mathematics foundation and the desire to use their knowledge in the solution of practical problems. Pupils often make mistakes in not being able to assess from a practical situation what particular mathematics process is required to solve the problem. The C.S.M.S (Concepts in Secondary Mathematics and Science) study [11] carried out at Chelsea College showed that many pupils were unable to recognise from the wording of a problem what particular mathematics process was needed to solve it, and that they were unable to reverse the procedure and express a mathematical problem in words. These pupils had a very low conceptual understanding.

Teachers must place their emphasis on conceptual learning not just algorithmic learning and include in any mathematical course lots of discussion so that pupils are able to express themselves and apply their knowledge to solving problems in other subject areas or unusual situations.
An article entitled "A Manifesto for Change" [12] caused the Derbyshire Education Committee to take notice, and this resulted in a course being arranged for head teachers in which they discussed this article in detail. In the article it stated

"In a modern society, oral fluency is of ever greater importance because communication in practical affairs is increasingly oral, either face to face or by telephone. At work, accurate interpersonal communication is central to efficiency, but often falls short of the necessary level of competence" [12].

Children who are able to do mathematics in their mathematics classroom often cannot cope with the same mathematics required in the sciences and other subjects because of the different way of phrasing or symbols used or possibly due to the fact that the mathematics is hidden in amongst lots of words. This was confirmed by The Royal Society and the Institute of Physics Joint Committee for Physics Education's working party on the relationship between mathematics and physics at the pre-O-level stage in secondary education [4].

Great emphasis must be placed on the teaching of mathematics as a tool to be used for the solution of problems in other curriculum areas. Professor Matthews and Margaret Seed [13] in their paper on the "Co-existence in Schools of Mathematics and Science" which appeared in the Institute of Mathematics Journal stated:

"If only such links between mathematical theory and scientific application could be made it would surely both enliven the mathematics and stress also its importance as the language of science" [13].

Children must be encouraged to think, discuss and understand the concepts behind arithmetic operations, because skills gained without understanding are in the long run unacceptable, in that there is no foundation for building on, or preparation for future innovation.
Maggs [9] confirmed this when analysing the errors made by future teachers during their training, when he tested them on some basic arithmetic skills. He stated

"These school leavers, future teachers and present teachers were unable to succeed on these fairly elementary questions, simply because of their lack of comprehension of the concepts involved" [9].

Mathematics should be taught as an integrated subject, both internally and externally. The understanding of inter-relationship of concepts, combined with their application in the world outside is of paramount importance.

"Each concept has associated skills which gives them mathematical appeal and each of concept and skills is useless without the other"

Professor G Matthews [14]

Future mathematics teachers must appreciate this and be fully proficient in applying this in their teaching, from the start of a child's education through to the moment when they use their knowledge to some useful purpose in their employment. Again Professor Matthews and Margaret Brown [15] reporting on the European Seminar on Mathematical Education stated

"The mathematics of the world of the pupils was felt to be the first priority for the future ... again it is important to educate teachers to see mathematical possibilities in football pools, newspapers, the local factories, etc" [15]

The traditional curriculum of isolated subjects, often developed as specialisms at too early an age is inadequate for the modern world. The pupils in schools should have minds which are well informed over a wide range, used to finding and using information in the solution of real life problems. Schools should provide a broad, integrated education which can serve as a good grounding for any specialism at a later stage.
Maggs [9] indicated the need for integrated subjects in secondary education

"Children working on mathematics in integrated topics or environmental studies were very motivated and were more aware of the mathematics in its relationship to the lives we lead" [9].

It is interesting to see that Linda Dickson [5] found a total lack of practical application in mathematics lessons when she interviewed the teachers of the apprentices she was studying.

"Mathematics teachers met during the course of writing this paper displayed ignorance of the practical applications of their subject in an industrial context, which with ineffective class control resulted in low motivation and achievement of the pupils" [5].

The author feels subjects are often taught in a narrow one-dimensional form which deadens, rather than arouses zest for learning. In order to appeal to the whole mind and, therefore, to develop the whole personality, subjects should be broadened into experiences which draw out, within the same context, intellectual capacity, aesthetic appreciation and practical application. It is the lack of such enrichment and relatedness that pupils often condemn what they have to learn as irrelevant, and hence mathematics has become regarded as the most boring subject in the curriculum.

Teaching subjects in all their dimensions in no way implies a loss of rigour in studies but it does give a subject an appealing reality and prepares pupils for the complexity of adult life.

An article in the School Master and Career Teacher, January 1980 expressed the view

"The school too, of course, should operate in a dynamic relationship with the surrounding community" [12].
The isolated specialist, teaching in the isolated classroom in the isolated school cannot provide the right educational environment for young people growing up in an inter-related world.

The Royal Society's working party on "School Mathematics in Relation to Craft and Technician Apprenticeships in the Engineering Industry" summarised their findings and stated

"Motivation seems to be the key factor in the progress made in mathematics during their first year of industrial training. This may indicate that pupils often lose interest in mathematics because they find it difficult when it is not taught in a context of 'real-life' situations" [2]

They also suggested that the final year of a pupil's schooling should provide a useful period for reinforcing the work (including mathematics) which they would need in their job - for example Practical courses in mathematics which they hoped would lead to motivation and hence all-round improvement in ability and comprehension.

With the view to improving the links and dialogue between schools and industry, and to stimulate school pupils' interest in local industry, the CBI are to back schools in a research competition. The initial motivation being large prizes but it is hoped that links set up by the competition will be the foundation for a strong and friendly relationship. They stated:

"The pupils of today will be responsible for creating the nation's wealth in the future"

Derby Evening Telegraph [16]

This scheme has also got the backing of the East Midland's Industrial Information Group and the local Education Authorities who obviously see the need for, and the benefits that can come from such an association.
Public concern has been repeatedly expressed on aspects of mathematical education such as:- The low motivation of pupils, the lack of application of the mathematics being taught, the lack of ability of our mathematic teachers, the reputation of mathematics as being too difficult or boring and the failure of the education system to provide all children with their basic mathematical needs. A selection of quotations which the author feels substantiates the overwhelming feeling regarding this concern follows:

Margaret Brown [17] stated regarding this concern and criticism expressed about the low level of achievement of many children in both primary and secondary schools

"There is evidence to show these criticisms have at least some justification and there are indeed a large number of children, maybe the majority, who are without what are usually considered to be basic numerical skills" [17].

From the article "Manifesto for Change" [12] it was stated

"Syllabus reforms that have been developed from outside school have less chance of acceptance and understanding than those that originate in schools themselves, but there is no doubt that more involvement by schools in consultation with Further Education and the "world of work" would result in a greater awareness of the need for school to adopt a more outward looking approach in curriculum matters" [12].

Concern expressed in a teachers' journal stated

"Until the standards of mathematics teachers improves within the schools the mathematics standards of those recruited into teaching will certainly not improve" [18]
"Primary schools cannot expect to provide satisfactory standards in mathematics unless a specialist teacher of mathematics, knowledgeable about primary practice is available to co-ordinate the curriculum and provide general teaching advice" [18].

Margaret Brown in her work on 'Cognitive Development' stated

"Many children learn algorithms in isolation from the concepts, sometimes because the teachers make no effort to relate them" [17].

Malcolm Swan [19] in his investigations for the Shell Centre for Mathematics concludes

"... that most of the mathematics currently being taught is devoid of any practical application and is completely isolated from the rest of the school curriculum" [17]

and

"... children find it quite difficult to identify the algorithms required for solving practical problems" [19].

An article in the Times Educational Supplement, entitled "Face to Face with Failure" [3] expressed the view that

"The introduction of each mathematics skill needs to be clearly related to and illustrated by everyday practical applications of that skill" [3].

The Hampshire Education Authority [20] on the decline in basic calculating skills stated

"The problem of motivating pupils is taxing teachers of the fourth and fifth years in secondary education to an increasing extent" [20].

The Royal Society [2] expressed the view

"In some cases children lose interest in mathematics after the age of 11 if they do not see its relevance or application"
and

"Specially prepared materials should be produced such as films, tape-slides, books and example sheets, illustrating the use of mathematics in industry which may help to motivate the pupils" [2].

Finally,

"Making maths enjoyable was a contribution to the long-term improvement of all aspects of maths education" [3].

In view of all the evidence and opinions, great emphasis must be placed on motivation, enjoyment and some effort must be made to integrate mathematics with examples from other curriculum areas, as a means to improving the all-round ability of school leavers.

The author fully believes that motivation amongst school children is only to be gained by teaching mathematics in an environment which is related to external needs and in a manner which stimulates interest and enjoyment. Later chapters investigate the feasibility of this by incorporating some mathematics teaching into science lessons as an integral part of science concepts. Mathematics must be taught in conjunction with other curriculum areas, not in isolation as is the present practice in the majority of schools, to overcome pupils' inability to apply their mathematics knowledge. To this end the author has analysed a typical physics course for mathematical content and produced a booklet for his own mathematics department detailing order of mathematical requirement for the success of the science course and many examples of the uses of different mathematics concepts in science.

"Mathematics in the past has often come to the aid of science .... it is now time for science to repay its debt and save mathematics from a new fossilization just as it has escaped from the old one" [13].
The author believes the fossil is hardening rapidly and something must be done soon, he feels this should be the partial integration of mathematics and science which may ultimately benefit both curriculum areas and to this end the author has carried out the work illustrated in the forthcoming chapters. Certainly in science it is necessary to counteract what is seen as a declining interest in science amongst young people and create the correct impression of the nature of scientific endeavour.
CHAPTER 2

Methods Adopted in this Enquiry

The problems the author is faced with in teaching physics to pupils between the ages of 13 and 16 are:

(1) The majority of pupils do not understand the mathematics, no matter how basic, they are expected to use.

(2) The pupils cannot adapt to the different ways of phrasing problems, or the different terminology used in scientifically based mathematical problems.

(3) The pupils have an inherent dislike of mathematics, which results in very low motivation especially in middle to low ability children.

(4) The pupils have had unsettled mathematics education due to changes in the type of mathematics course taught and constant changing of teachers, often the use of 'supply' teachers for long periods of time to replace teachers who have moved to other schools and for whom replacements cannot be found.

(5) The mathematics taught is done in isolation, very rarely is it applied to other curriculum areas.

(6) The mathematics teaching is often out of step with the requirements of the physics course and dialogue between departments has been fruitless mainly because of the mathematicians.

(7) The eventual dislike of physics due to its mathematical content, causes the pupils to 'switch off' and opt out of the more mathematical sciences. Many of whom could have possibly gained some sort of success.
The pupils are unable to discuss problems, or estimate order of magnitude answers or realise when an answer is of a totally ridiculous magnitude.

These problems have been with us for a long time, and whether the advent of modern mathematics has made the situation worse is debatable. The result is that the mathematics required for the physics course has to be taught along with the physics theory and serves as a severe obstacle to progress.

In this research the author analyses the mathematical requirements of the East Midlands C.S.E. syllabus 1 Physics course, as this is the level which seems to cause the greatest problems, and as each section is taught notes the difficulties the pupils have in coping. At various stages where particular difficulties arise, tests were developed to discover the depth of the mathematical problem (Appendix II).

This analysis led to (i) The developing of a necessary "remedial course" of mathematics in the form of bookelets, which the pupils worked through as extra curricular work both at school and at home. (Appendix III). These were basic mathematics themes taught as if they were part of a mathematics lesson (ii) A course of experiments was developed to cover some physics theory and involve lots of similar mathematics practice. The intention was to show the pupils how the mathematics they had been taught was needed and a vital part of science, to integrate and discover if application and relevance was a means to increase motivation, enjoyment and hence the possibility of improved understanding, as was the author's belief.

A questionnaire (Appendix IV) was issued to the pupils to find out their opinions of mathematics; its application; why pupils did not choose to do physics to examination level in years 4 and 5; and various items which may have related to their mathematical proficiency; was the dislike of mathematics a major contribution to the reasonably low levels of participation on such courses and hence the loss of possible future scientists and engineers? Also as Cockcroft suggests that low mathematical
attainment could result from having several different teachers, just how many mathematics teachers these pupils had had from Year 1 of the secondary school. The author is well aware of the problem of finding mathematics teachers, which virtually affects all schools and the pupils who suffer most are the middle to lower ability range.

In order to gain a deeper insight into the pupils feelings on the topic 'mathematics' the pupils were asked to write essays and some were interviewed on tape recorder, at which stage they were asked to comment on some of the application of mathematics tried in the mathematics/physics integration. (Appendix V).

At the same time as the pupils were being investigated, many from the teaching profession and associated professions were asked about the problems they faced in teaching science, with regards mathematical ability, and how those areas could be possible overcome. This was carried out in the form of an interview, tape recordings of comments, and questionnaires and hence an overall picture of the situation arose from which suggestions for the future could be made (Appendix V).

During the appraisal of the physics course a list of mathematical requirements (in chronological order) for science teaching was drawn up and examples of their uses issued as part of an initial dialogue with the mathematics department and to improve communications which hitherto were non existent. (Chapter 7)

A 'Mathematics for Science' booklet was developed for the pupils, (Appendix VI), which started with Basic Numeracy, illustrating the uses in the area of science, and which could be expanded as the course continued, in both depth of topics covered and in the number of different examples used. This was felt important as a form of mathematics revision notes, but also to show the need for learning the different mathematical concepts for use in practical application.
Some booklets of pure revision or perhaps remedial mathematics work were developed which tried to show the links between mathematics topics. They were written with the lower/middle ability pupils' reading and comprehension age in mind, including many worked examples, and some, but not too many, questions on each topic. Also included was some of the more enjoyable, amusing aspects of these topics such as quiz type questions and puzzles which tended to fascinate the pupils.

The series of experiments involving integration of mathematics and physics would show the pupils reactions to "related mathematics", so that further work could be contemplated and recommendations for the future made. This work was carried out by the author, without the help or in conjunction with, the mathematics department because they were in a total state of change, that is, they were between new heads of department and between new mathematics schemes, changing from SMP to one of the innovation of the head of department - which was contrary to the wishes of the rest of the department. (Chapter 6 and Appendix VII).

During this work the author had a feeling or hunch that application and practical mathematics aided understanding through possibly increased motivation and interest, so he tested the conceptual understanding of the pupils in Year 1 at Ripley, Derbyshire, half of which had undergone a very practical and applied course, the other half having followed a typically traditional 'rote type learning' course to see whether this was so and therefore incorporate more of this type of work in his remedial work with his science classes (Appendix VIII).

Also a form of control group, which was composed of children from throughout the Derbyshire area and not from the author's school, the author compared the mathematical abilities of these children, aged 16, with his pupils. This was done with the help of the local college of higher education who used a "National Test" on 16 year olds who were seeking employment in the printing industry in Derby. The analysis of this test (Appendix V), indicated the same problem areas were common to both groups.
A physics course is very limited in time when one considers the lengthy syllabus to be completed and so to have to teach the mathematics required in order for the pupils to understand and complete their physics course serves only to make matters more difficult. The author has had to take this into consideration in his work and not to jeopardize the chances of his pupils gaining the best grade possible in the examination.

It was hoped to tempt other science teachers into trying a method of integrating mathematics with their science teaching but the author appreciates their reluctance due to time shortage but could not fully agree that this is solely a mathematics department problem. However there is an overwhelming amount of information within the school curriculum which could be used successfully in mathematics lessons which is being ignored at present.

The enquiry starts with a section devoted solely to the H.M.I.'s views on mathematical education in England in which they critically analyse all aspects of mathematics teaching; and how this related to the author's school. (21). This work points the way for the need for research into promoting mathematics as an interesting, enjoyable, relevant, practically based subject necessary for living and working in the world at large and to the need for integration of other curriculum areas into mathematics teaching. Mathematics should be taught, partly for use by 'CONSUMERS'.

"A child who is perfectly able to give the length of a rectangle having area 10 square metres and width 2 metres may be unable to begin if the area is given as 9.5 square metres and the width 1.7 metres ... Panic brought on by the prospect of having to calculate with decimals has clouded mathematical understanding.

"This business of working with 'realistic' numbers (such as the numbers obtained from experimental measurements) often troubles pupils when they come to do calculations in science lessons. It is not always easy for the science teacher to appreciate the difficulties which children find in applying to laboratory work the skills learnt in mathematics lessons, and the joint efforts
of scientist and mathematicians will be needed if they are to surmount these hurdles successfully" (22)
A review of "Aspects of Secondary Education"

This survey, published in December 1979 by the Department of Education and Science was the result of H M Inspectors' work over a period of four years (1975 to 1978) in which they visited a structured sample of approximately 10% (384) of secondary schools in England, with regards to Mathematics and Science teaching. It involved detailed appraisal of all aspects of mathematics teaching, not only the lessons so described on the timetable, but it was considered essential to study the mathematics occurring in Science, Workshop, Art, Craft, Geography and Social Studies.

It stated that mathematics was a compulsory subject and 80% of all pupils were following examination courses:

- GCE 22% of pupils (approximately)
- GCE/CSE 13%
- CSE 52%

The range of ability of these pupils was considered to be too wide for the design of the examinations.

Although almost every pupil takes a mathematics course of some kind, the content varies considerably. Some pupils, almost invariably those with relatively low ability, take courses in Arithmetic which may or may not lead to external examination.

This variation in content of course produces public confusion and debate. There are over 50 syllabuses available at GCE and CSE level.

The report states that over the years the differences between the approaches called Traditional and Modern has diminished and most schemes of work are a compromise between the two extremes.
Figures given:  
Traditional courses  42%  
Compromise  33%  
Modern  25%  

However more schools are turning back to Traditional courses for the less able.

The H.M.I accept the fact that there is concern regarding the standards of achievement, but that changes in syllabus or greater uniformity of syllabus could, by themselves eliminate the cause for concern. Whatever the syllabus it is ultimately the interpretation which the teacher gives to the subject matter and the teaching approach used which is crucial.

Parents and public demand examination success and schools recognising this, attach great importance to it, especially in mathematics. Teaching is therefore restricted in approach, rather than expanding on ideas embodied in the syllabus.

The survey states C.S.E. courses are intended for the middle range of ability and are examined in three different modes:

Mode 1  (75% nationally in 1976): the syllabuses are decided and the examination set by the Examination Boards.

Mode 2  (5% nationally in 1976): the syllabus is formulated by the schools and the examination set by the Board.

Mode 3  (20% nationally 1976): the schools plan their own syllabus and examine them, subject to moderation.

Many schools choose a CSE syllabus that is compatible with a GCE syllabus, which enables the decision on the examination to be taken, to be kept open as long as possible. This alleviates some administration problems and pupils with CSE Grade 1 who want to continue their studies will have followed a closely
related syllabus. However the biggest drawback is that many pupils are stretched beyond their limits.

The report stated very strongly and in the opinion of the author, very necessarily, that for the 17% of pupils following a Non Examination Course, where the concentration was on arithmetic, the lessons were less encouraging and devoid of practical work, that new courses are urgently needed. These new courses will have to have very clearly stated guidelines as none exist at present. The H.M.I's appreciated the difficulty of this problem, and matters are not made easier by schools not feeling able to deploy their better teachers to the task.

Teachers have a major responsibility in interpreting the framework provided by the examination boards. Too often the syllabus is given as the reason for ignoring the 'why' in pursuit of the 'how' or for leaving out interesting and natural developments which are not in the syllabus or for giving less emphasis to those areas which are difficult to examine or rarely tested.

Correspondingly, it was noted, that where it was a simple matter to set repetitive exercises on topics that these could be pursued to excess, although the idea was more important than the routine skill.

In almost all schools seen, very considerable time was spent on arithmetic and in some schools, where leavers lacked proficiency they had certainly been taught. All schools gave prominent place to the four rules of numbers, including work with money, decimals and fractions. In spite of this some schools were found as in former times, where the arithmetic level of competence of some of the pupils was unacceptably low. Not infrequently this was associated with difficulties arising from catchment areas which the school served. Improvement of standards involves more than teaching methods alone.

The report stated the debate on 'standards' had been complicated in recent times by changes in syllabus and changes
in social needs arising from new patterns of employment and from the availability of calculators and computers.

Work with the least able (15 to 20%) gives cause for some concern, their grasp of the basic principles and their ability to retain knowledge and draw on skills other than those immediately practiced is poor. Teaching approaches need revision and greater thought is necessary to give to nurturing understanding and confidence than to covering the syllabus.

If all schools were no worse than this, there may be less cause for concern but this last paragraph was written after visiting Grammar Schools which were by no means inferior schools of their type. The paragraph is a warning against expecting too much in matters of numeracy from schools which cater for the full range of ability.

Generally the good comments on Arithmetic Competence outweighed the adverse comments by two to one, although highly adverse comments were more common than high praise,

- 86% of pupils on GCE courses were assessed competent
- 62% of pupils on CSE courses were assessed competent
- 37% of pupils on Non Exam courses were assessed competent

Modern mathematics course received somewhat less favourable assessment in this respect.

Problems arise, the H.M.I's stated, when pupils have to apply arithmetic skills on top of the fact that some pupils have difficulty enough in performing standard calculations in isolation, especially involving fractions and decimals. Hence when involved in other subjects such as the sciences failure was frequent. On non examination courses schools were excessively preoccupied with mechanical skills and insufficient concern with the uses. Little time was spent applying these techniques to other areas of knowledge such as science and craft.

In general there is a disappointing lack of reference to the applications of mathematics throughout the teaching of the subject.
The use of realistic source material such as timetables for trains, catalogues, plans, maps etc was very limited (only seen in 20% of schools). Many pupils experienced mathematics in practical settings in Science, Craft or Technical drawing. The equipment from these subjects could be utilised profitably in mathematics lessons.

"80% of schools do not relate their mathematics"

Occasionally schools were visited that had great success in relating mathematics in topics such as Rural Studies where records were kept of animals, addition in weight against feed cost, Geography with map reading, Craft, design, European Studies, economics, and science were suggested as sources which should play a part in the day to day teaching of mathematics.

The H.M.I's suggested that there was a greater need for communication between different departments within a school.

The H.M.I's stated that the time had come for a careful reappraisal of the aims and ways of teaching.

The accommodation for teaching mathematics was considered poor. Specific teaching rooms should be allocated where displays of equipment, wall charts and services are available. 66% of schools had no specially equipped rooms, and in only 40% of schools could they find evidence of mathematics display material. The problems with teaching mathematics is increased by inappropriate accommodation. The statistical evidence of the survey supported the view that, in general, specialist rooms encouraged the better teaching of mathematics.

Textbooks were not available to all pupils and reference books were even in less demand, only 7% of schools.

The staffing situation in mathematics is a very worrying problem. In the schools surveyed the following figures were tabulated:
Total Number of Mathematics Teachers

3365

54% (1809 teachers) taught maths for more than \( \frac{3}{4} \) of the week

25% (837 teachers) taught maths for between \( \frac{1}{4} + \frac{3}{4} \) of the week

21% (719 teachers) taught maths for less than \( \frac{1}{4} \) of the week.

This gives the equivalent of 2300 teachers on a full timetable.

There were approximately 70 unfilled teaching vacancies in mathematics and the survey pointed to an actual need of nearly 260 teachers for the surveyed schools. (Nationally this would be between 2500 and 3000 teachers of mathematics).

The D.E.S survey on the shortage of mathematics teachers indicates a need for between 2500 and 4000, which was to be met partly by training new teachers and partly by inservice training. This has proven to be unsuccessful over the period since the survey began:

Approximately 50% teachers had less than 3 years experience
Approximately 20% teachers had less than 1 years experience.

27% of mathematics teachers were graduates in mathematics
28% of mathematics teachers were non graduates with mathematics as main subject
22% of mathematics teachers were graduates or otherwise had mathematics as their second subject
23% of mathematics teachers had no mathematical qualifications.

These figures indicate that a large proportion of the mathematics teaching (approaching 25%) is done by teachers without any qualifications in mathematics which is a most undesirable situation. Appendix I confirms this for the County of Derbyshire.

The H.M.I's noted that there was a widespread reluctance to relate mathematics to the world outside the classroom. However, where the mathematics teaching was related there was a DRAMATIC increase in the level of motivation. The main things contributing to the low assessment in mathematics are: the lack of
variety in the content of courses, the lack of application and the full, unimaginative approach to teaching.

They stated that teaching schemes indicate the need to relate mathematics to everyday life but in reality this was very rarely seen.

There is real concern about the lack of basic skills but effort to remedy this took the form of more and more sets of questions on fractions and decimals without identifying the basic number deficiencies that are the heart of the problem. It was not surprising that mathematical activities could not be found in other areas of the curriculum and in the sciences avoided where possible.

Through the ability range, the greatest progress was made by teachers with a personal quality to capture the interests of the pupils and convince them that success was possible. Computational practice needs to be supplemented by improved diagnosis of pupils individual difficulties, a better appreciation of the role of language and oral work, more effective use of the applications of ideas, both in the world around us and in other areas of the curriculum.

During the period of the survey public attention turned to the relationship between school mathematics and Industries' requirements and the more general mathematics of adult life. This was said to be "a substantial problem requiring further investigation" Figures quoted regarding a question posed to mathematics teachers: "Is you mathematics teaching related to any of the following subject areas" are as follows:

<table>
<thead>
<tr>
<th>Is it related to Science?</th>
<th>Type of Teaching (ABILITY RANGE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSWERS:</td>
<td>GCE</td>
</tr>
<tr>
<td>YES</td>
<td>23%</td>
</tr>
<tr>
<td>NO</td>
<td>70%</td>
</tr>
<tr>
<td>UNDECIDED</td>
<td>7%</td>
</tr>
</tbody>
</table>
Is it related to Craft?

ANSWERS: YES 7% 8% 7%  
NO 85% 84% 87%  
UNDECIDED 8% 8% 6%

Is it related to other subjects

ANSWERS: YES 11% 12% 8%  
NO 82% 80% 86%  
UNDECIDED 7% 8% 6%

These figures show that above 80% of school teaching of mathematics is unrelated to other curriculum topics which specifically use mathematics as a tool in their teaching.

Summary

The survey published figures on the acceptability of the mathematical education provision in England. It assessed the provision for three ability ranges (1) the more able (2) the average ability (3) the less able.

<table>
<thead>
<tr>
<th>LEVEL OF PROVISION</th>
<th>ABILITY GROUP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>More Able</td>
</tr>
<tr>
<td>CREDITABLE (OR BETTER)</td>
<td>17%</td>
</tr>
<tr>
<td>ACCEPTABLE</td>
<td>68%</td>
</tr>
<tr>
<td>UNSATISFACTORY</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>

Unsatisfactory provision is described as restricted courses for the less able, to routine calculations divorced from context, to no application of mathematics. These figures show a very worrying situation when approximately half of the schools surveyed provided unsatisfactory mathematics teaching for the less able and one quarter for the average ability.
The main comments in the H.M.I's report are:

1. **Mathematics was too little related to the outside world and to other areas of the curriculum,** even when these subjects either need to apply mathematics techniques directly, as in the sciences, or when they gave broader illustrations of ideas which were explicit on the syllabus. Lack of success was due to the way the syllabus was interpreted.

2. Some C.S.E courses had started to base topics on the world outside, but much remains to be done to provide suitable sources of materials and to develop the ideas in ways which capture interest and convince the pupils the ideas matter.

3. **There is a need for better interdepartment links,** ie science, crafts and mathematics.

4. **Greater discussion is needed** during mathematics lessons by both teacher and pupils.

5. There is not enough equipment for practical mathematics lessons.

6. There must be a provision of suitably qualified mathematics teachers, in the order of 2500 to 4000 and more.

7. Schemes of work in schools need development. There should be discussions with Parents /Industry/Further Education before drawing up these schemes.

8. Mathematics is a subject for which pupils aptitudes and inclinations vary greatly and the design of courses with suitable content, application and of a suitable level of difficulty for pupils of varying abilities continues to present problems.

9. The Examination system although not perfect is used by Employers and Further Education institutions for a variety of purposes. In some cases mathematics skills
are needed for particular fields of employment, either directly on the job or in order to follow training courses associated with it. In other cases it is taken as an indicator of all round ability.

(10) Schools, Industry and the Public could benefit if their respective difficulties were better understood.
CHAPTER 4

Appraisal of Physics Courses

The physics courses available for pupils in schools are fairly numerous and vary from those of a pure theoretical nature, to those of application of physics and to those which concentrate on a particular field within the overall spectrum of science attributable to physics, such as Engineering Science.

Every University Examination Board caters for physics and several for the Nuffield Project type physics courses as well. Nuffield physics was developed in response to the very specific need for improving science in schools, it drew attention to the need for curriculum development in science and for a wider study of imaginative ways of teaching scientific subjects.

The Trustees of the Nuffield Foundation considered that there were great opportunities to create a course in which the material developed would enable teachers to present science in a "lively, exciting and intelligible way" ( - Brian Young, Director of Nuffield Foundation).

This work then continued with particular reference to those children whose abilities make it likely that they will take science at 'O level' and from this they developed the Nuffield Secondary Science Course for pupils from the age of 13 to 16. The aim of this course was to produce material which was flexible and capable of adaption to a wide variety of conditions so that teachers could construct courses which meet the specific needs of their own pupils. "The criterion of the content throughout is that the work should have significance for the pupils" ( - Hilda Misselbrook, general preface Nuffield Secondary Science Project).
Although the Nuffield courses rely on an investigational flavour to its experiments and the subsequent importance of 'discovering' by the pupils, it is not devoid of mathematics and once concepts had been discovered they have to be cemented into the memory by suitable exercises, which may well involve the mathematics that other courses, which may rightly or wrongly be described as Theoretical Physics, include.

The impression the author formed when examining this course was that there was some very useful material embodied in the 'Themes' but he thought that the reading and comprehension level required would test weak 'O level' candidates and those of CSE standard, and possibly prove too much for them.

The conventional physics courses of London, Northern and Associated Examining Boards embody similar 'aims' and 'objectives' which were: That students should derive "interest, enjoyment and some sense of achievement from their study of physics" (JMB Examination Council, 1982). Whilst not specifically including aspects of applied science it was expected that candidates would have studied physics in the context of its application, particularly in engineering situations and would recognise the importance and relevance of the subject.

Standard experiments are an important part of the course but the questions set test the concepts and the acquisition and treatment of data rather than experimental details. They stress importance on the students' ability to carry out simple experiments of their own design, selection of apparatus, the choice of measurements to be made, the processing of readings and the rough estimation of the reliability of the final result.

The aim is to establish the conceptional framework of physics, to provide stimulus for effective experimental work, including the ability to understand and interpret scientific information presented verbally, mathematical or graphically and to translate such information from one form to another. The ability to select and apply known laws and principles to problems which are
unfamiliar or are presented in unusual ways, "To provide sufficient opportunity for quantitative exercises in aspects of physics (AEB. Physics - 052 syllabus at 'O level').

None of these examinations included any form of compulsory practical examination.

The 'O level' examinations are aimed at the top 15-20% of the pupil population and require an understanding of mathematics to a similarly high level. The CSE was developed for pupils of an average ability range, those between the 'O level' and the bottom 40% of the pupil population, by ability. This, however, does not occur in practice, some 83% of the pupil population take either the 'O level or CSE' examinations, which is 23% more than they were designed for. (Mark Carlisle).

The country is split into Regional Examining Boards, who develop their own syllabuses, aims and objectives. The author's school is in the East Midlands Region where the physics course, syllabus 1, aims to provide pupils with a knowledge and understanding of physics, to give training in scientific methods and to develop an awareness of the importance of science in everyday life by reference to practical situations and applications.

The objectives of this course are: to recognise and handle equipment effectively, to be able to record observations with a reasonable degree of accuracy and be aware of Scientific Terminology and fact, and to relate this to everyday life, the ability to select and apply a physics principle to solve a problem situation which is unfamiliar.

The assessment is carried out by examination, involving papers on multiple choice objective questions, medium and extended questions and a fairly extensive "Practical Examination" involving taking 20 readings from different equipment and 2 experiments lasting 30 minutes each, or as an alternative to the latter, a continuous assessment scheme.
The syllabus content includes a compulsory 'Basic Core' and then a choice of 3 'Options' from various topics such as The Physics of the Weather, The Motor Car, etc. These option topics are under constant revue, as is the whole syllabus, and changes occur regularly where options topics may be deleted completely or moved into the Basic core.

There is also a Physics Syllabus 2 which has the same aims and objectives but has been designed to follow the Nuffield Secondary Science scheme, terminating in its own examination.

The mathematics involved in the courses is of a basic nature designed to reinforce the scientific discovery of the concepts. The use of calculators is generally accepted by the examining boards in examinations only for those questions of an extended, structured nature.

Also available is a limited grade, Mode 3 'Science at Work' course designed for those pupils of a low ability. This course is practically orientated and material blends practical work and information with extensive back-up for the teacher and was developed to interest and involve pupils in those aspects of science which are likely to be part of their adult life.

At the present time all examinations are under revue, on a national and regional basis, between Government, Universities and CSE Boards, for the proposed '16+ examination system'. 
East Midland Regional Examination Board: Certificate of Secondary Education: Physics Syllabus 1 (Mode 1)

The following is a detailed analysis into how this particular course is taught, the depth of understanding required in both physics and the associated mathematics. It includes typical examples of the problems experienced by the pupils (approximately 70 in the survey), the mathematical requirements, examination questions, past and present, and an indication of the mathematics syllabus which would be suitable for the teaching of physics to lower or average ability pupils.

Tables and charts have been constructed to show:
1. The frequency of requirement of each particular mathematics topic during this course
2. The Examination results of the pupils in the survey (Physics and Mathematics)
3. The Mathematics results of all Fifth Year pupils at this school taking CSE
4. The CSE Physics Syllabus

These can be found at various stages of this chapter.

General Comments

The boys and girls involved in the survey had followed a Traditional Mathematics course in Years 1 and 2, working from a 'Scottish Mathematics Textbook'. In Years 3, 4 and 5 there was no standard textbooks, various books or worksheets were used. There was no formal 'scheme of work' developed, teachers planned their own. Examinations were not common for all sets in any particular Year Group, apart from the 'Mock CSE' which was taken during the January preceding the Final Examinations in the Spring of that year. The course followed fairly traditional
mathematical lines and involved many calculations of a repetitive nature. The policy of the school was that every pupil should take mathematics as a compulsory subject and 4 periods are time-tabled in the 4th year, with 6 periods in the 5th year: Averaging at 5-35 minute periods per week.

During these pupils' school-life the maths department had moved from one building to another and now occupies four rooms on the ground floor, of which two were joined by a moveable partition through which sound travelled very easily. There was very little evidence of teaching aids, reference books or relevant wall displays; practical lessons were non-existent.

The staffing situation was and still is very changeable, mainly due to the shortage of mathematics teachers. For long periods of time the department functioned with up to 2 supply teachers out of a total of seven full-time mathematics teachers. Of the remaining 5 teachers, one was an ex-physics teacher on the point of retiring, two were from the Management Team and can only be considered as part-time members, for whom dealing with discipline problems took precedent over attending lessons. Both remaining Senior Teachers in the department were not graduates of mathematics.

Promotion in mathematics departments from Scale 1 to Scale 2 (Burnham) is fairly easy to obtain so any newly qualified teachers joining the department would move on within a time period of 1 to 2 years. This resulted in many of the pupils in the survey having as many as 6 different teachers during their first three years in the Senior School. This constantly changing staffing situation rarely affected the higher ability pupils; but the average to lower ability pupils had to make do with whatever teachers were available. In this case, work was set by a senior member of the department, consisting of page and exercise numbers and the pupils had to work unaided by a mathematics teacher.
The homework policy was for homework to be set twice a week for a period of half an hour on each occasion, and to be collected in and marked regularly. However, problems developed where 'supply' or non mathematics teachers covered lessons. Marking tended to be a tick at the end of an exercise with the absence of any constructive comments where mistakes had been made.

The highest ability pupils generally completed homeworks, the average to lower ability pupils were not in the habit of doing homework. Homework was set and marked, if handed in, but very little effort was made to make it compulsory. It was accepted that some pupils, approximately 30% in any particular year group, would not do homework when set.

Many of the average to lower ability pupils lost their exercise books when they took them home, so their exercise books had to be kept in school to ensure that at the end of the course such pupils had as full a set of notes as possible.

Equipment, such as pens, pencils and rulers were supplied to many of the pupils, and protractors or set squares had to be supplied to every pupil.

The trend of internal truancy was increasing - due to the large school site there were many places for the pupils to hide during lessons they disliked, only to reappear at break times. Time had to be spent every lesson taking registers to try to prevent this.

Discipline had changed from a trend of Corporal Punishment to one of 'discussion', this takes up a large amount of time and tends to interfere with teachers' lesson time. The standards of discipline were low and there was a considerable amount of discontentment amongst the staff with regard to this and the general lack of respect shown by the pupils and many parents for teachers and everyone in positions of authority.
The catchment area of this school included approximately:

- 10% Rural background
- 40% Outer fringes of a city (Derby)
- 50% Inner city

There were many ethnic origins of which approximately 20% had English as a second language.

One of the four 'feeder schools' of 11 year olds was a "Social Priority" school. The total number of pupils was approximately 1500. This school was fairly typical of those in the Derby area.

During the latter stages of the 3rd year, the pupils decide which subjects they would like to take during the 4th and 5th years for examination purposes. Mathematics and English are compulsory and the remaining subjects are placed in bands which are designed to give as free a choice of combinations as possible. If a pupil wishes to he can take all three sciences, that is, Physics, Chemistry and Biology.

In a school of this type approximately 15% of the 1500 pupils would have gained entrance to a Grammar School and taken GCE 'O' levels. Thus in any year approximately 45 pupils would be of an equivalent standard to those in a Grammar School, and suitable for an 'O level' course in physics. Because of the banding system, there were two 'O level groups' and two 'CSE groups', each requiring 60 pupils to conform to the necessary class sizes and give the approximate 'contact ratio' the County Council requires.

This meant that a further 15 pupils were placed into the 'O level groups' who would normally have been better suited to a CSE course, depleting the CSE groups of the top potential and leaving mediocre CSE groups which were topped up with pupils who would have done General Science Non Examination. The mathematics results at CSE indicated that some 74% of the pupils in the survey obtained Grade 3 or less.
East Midlands Regional Certificate of Secondary Education
in Physics, Syllabus 1, 1980/81

This is a Mode 1 syllabus, that is the syllabus is set by
the Board, who also examine the subject.

For convenience the analysis is carried out section by section
throughout the syllabus and at the end of each section examples
are given of the types of physics questions covered by the pupils,
the problems they had in coping with the mathematics required
and a list of the range of possible mathematical requirement of
each section, which are included in a frequency table at the
end of the analysis. The mathematics topics headings are those
suggested by Graham (23) as required in a suitable mathematics
syllabus. They are as follows:

1  Natural Numbers
2  Number system
3  Place values
4  Directed numbers
5  Prime numbers
6  Factors
7  Multiples
8  Power and Roots
9  Irrational numbers
10 Decimals
11 Fractions
12 Fractions to Decimals
13 Percentages
14 Percentage to Fraction to Decimal
15 Reading/Interpolating scales
16 Concept of rate of change
17 Index notation/Standard form
18 Reciprocals
19 Ratios
20 Proportions
21 Averages
22 Approximations
23 Estimation/Rough checking
24 Use of reference tables
25 Squares and Roots
26 Logs
27 Calculating machines
28 Areas
29 Perimeters
30 Volume
31 Types of triangle
32 Geometric proportions of triangles
33 Pythagorus
34 The Circle
35 Polygons, sum of angles at a point
36 Angles associated with parallel lines
37 Scale drawings and constructions
38 Area/Volume proportions
39 Algebra - letters to represent numbers
40 Simple formula
41 Indices
42 Brackets
43 Solution of Equations
44 Use of fractions in Algebra
45 Trigonometry - Sine, Cosine, Tangent
46 Graphs - Co-ordinates
47 Linear and Non linear graphs
48 Representing data in graphs
49 Civic arithmetic - Cost
50 SI units - weight, length
51 Imperial units
52 Speed/distance/time calculations
53 Profit/Loss/Interest
54 Renting/Buying
55 Foreign currency
56 Insurance
57 Savings
58 Wages/Salaries
59 Taxation
60 Rates/Household accounts
SECTION 1

Syllabus: Nature of Matter

Explanation of states of matter in terms of simple kinetic theory as illustrated by surface tension, expansion, change of state, latent heat, cooling by evaporation. Diffusion, Brownian motion and dilution as evidence for particulate nature of matter. Relative speed of particles in solids, liquids and gases. Relationship between kinetic energy and temperature of particles.

Practical Work

1. Kinetic theory model
2. Simple surface tension experiments such as floating a pin
3. Melting ice in water and condensing steam in water to demonstrate latent heat
4. Evaporation of ether and methylated spirit to demonstrate cooling by evaporation
5. (i) Diffusion of nitrogen peroxide or bromine vapours in air
    (ii) Diffusion of copper sulphate solution in water
6. Smoke cell experiment to demonstrate Brownian motion.

This section requires very little mathematical knowledge. However, in discussion the order of size of a molecule is mentioned as approximately $1 \times 10^{-9}$ cm, and therefore an understanding of very small sizes, and negative powers would be useful. Also the random nature of the velocities of molecules and the relationship between molecule velocity and their temperatures requires an understanding of averages. Evaporation was explained with the aid of a 'normal distribution curve'.

Pupils' Problems

Pupils of this range of ability are not familiar with negative powers and many could not write in 'figures' a large spoken number of order of magnitude $>100\,000$ or convert it to standard form $1 \times 10^5$. 
Estimate of size needed lots of attention, very few pupils new roughly what distance to hold their hands apart for a gap between hands of 30 cm. Random velocity proved a fairly difficult abstract concept, which was to lead to average velocity being proportional to temperature.
SECTION 2
Syllabus: Length, Area, Volume

Revision of proper use of measuring tapes, rulers, calipers, vernier calipers, micrometer screw gauge, set squares, pipette, burette, measuring cylinders, avoidance of parallax errors

This involved covering the following

The units of measurement required by the examination were exclusively those of The System International: kilograms, metres and seconds.
The pupils were encouraged to use rulers, tape measures and gauges to measure accurately regular and irregular shaped objects and in doing so to avoid Parallax error.

Estimation and an idea of size is important. Pupils were asked to 'guess' the size of all objects before measuring them.

Conversion between $\text{km} \div \text{m} \div \text{cm} \div \text{mm}$ was required.

A typical practical examination question involved: find the diameter of a circular object using 2 pieces of wood as shown:

\[
\text{MEASURE } x \text{ AND } y \\
\text{DIAMETER } = \frac{x+y}{2}
\]

OR

Alternately the circumference was measured and the diameter calculated from the formula: circumference of a circle = $\pi d$. 
For more accurate measurement:

The Vernier Calliper was used, with an accuracy of 0.01 cm easily achieved, and by estimation a third decimal place could be found.

Explanation of how we can read the scale to 0.0 accuracy

Vernier scale is 0.9 cm long and split into 10 divisions each 0.09 cm \( \frac{0.9}{10} \) long.

The difference in length between A and B is:

\[
0.1 \text{ cm} - 0.09 \text{ cm} = 0.01 \text{ cm}
\]

If the Vernier scale is moved so A is in line with B the Jaw has opened 0.01 cm.

Similarly, if C moves to D the Jaw has opened 0.02 cm

and if E moves to F the Jaw has opened 0.09 cm

and if G moves to H the Jaw has opened 0.1 cm and the zero on the vernier scale is now in line with 0.1 on the Main Scale.
The Micrometer Screw Gauge

The pitch of the screw thread is 0.5 mm so the spindle moves through 0.5 mm or 0.05 cm for each complete turn of the thimble. Fractions of a turn are indicated on the thimble, which has a scale of 50 equal divisions. Each division on the thimble therefore represents an opening of 0.001 cm. The micrometer reading is illustrated above.

Which instrument do we use?

This will depend on how accurate a measurement is required for example: if it is part of an experiment in which all the other measurements can only achieve a second decimal place order of accuracy then the Vernier Calliper would be quite suitable; for greater accuracy use the Micrometer.

Measurement of Area

The pupils should be able to calculate the areas of regular shapes such as rectangles, squares, triangles, circles and composite shapes involving rectangles and triangles, by measuring lengths, or height or diameter and applying formulae. The use of the correct units is essential. Calculations often involve mixed units or decimal fractions and answers often involve negative powers of 10, for example:

\[ 1 \text{ cm}^2 = \frac{1}{10000} \text{ m}^2 = \frac{1}{10^4} \text{ m}^2 = 10^{-4} \text{ m}^2 \]

\[ 1 \text{ mm}^2 = 10^{-2} \text{ cm}^2 \text{ or } 10^{-6} \text{ m}^2 \]
Measurement of Volume

This involves the measurement of regular and irregular shapes and can be achieved either by calculation using accepted formulae or by experiment using volume measuring equipment such as measuring cylinders and displacement cans. The use of the correct units is essential. Calculations often involve mixed units or decimal fractions.

Conversion of units: \( 1 \text{ cm}^3 = \frac{1}{1,000,000} \text{ m}^3 = \frac{1}{10^6} \text{ m}^3 = 10^{-6} \text{ m}^3 \)

Other volume measuring equipment included pipette, burette and standard measuring beakers.

Units: millilitre (ml); \( 1 \text{ ml} = \frac{1}{1,000} \text{ l} = 10^{-3} \text{ litre} = 1 \text{ cubic centimetre (cc) } \)

This equipment is only accurate at the temperature at which it was calibrated.

Estimation

When dealing with the kinetic theory, Brownian motion and molecules, the actual size of a molecule was discussed and to confirm this estimation an experiment suggested by Franklin (1770) in which oil is allowed to spread over the surface of water, and Lord Rayleigh later suggested that certain oils spread over water in a film 1 molecule thick.

So if we spread a known small quantity of 'light oil', that is, 1 drop of known diameter, it forms an approximate circular layer on the water, whose diameter can be measured. (This is a cylinder whose height is 1 molecule thickness).

Hence \( \text{Volume of drop} \frac{4}{3}\pi r^3 = \pi r^2 h \) where 'h' is the 'thickness' of 1 molecule

The accuracy can be improved by making a solution of the oil and a very volatile liquid such as ether, in proportions 50:1, so the amount of oil in the drop is \( \frac{1}{50} \) th of the volume. The ether allows the solution to spread more easily and it 'flashes off' to leave only the oil. The mathematics now becomes:
Volume of oil $\frac{4}{3}\pi r^3 = \pi r^2 h$. Results give $h = 10^{-7}$ to $10^{-9}$ cm.

Estimation is encouraged and can prove very valuable in experiments and everyday life.

Problems experienced by the pupils in the survey during this section

Over 60% of the pupils were not familiar with the 'metric system'. Although some knew the units of kilogrammes and metres, they were unable to relate to the fractions of these units, that is grammes, milligrammes, centimetres and millimetres and were unable to convert between them.

There was no understanding of the actual physical size of these units and it was very noticeable that this also applied to the Imperial System. These pupils had gone through Junior School at a stage during which the education system was changing over from the Imperial to the Metric system of units, and the result was that they were 'au fait' with neither system.

Conversion from one unit to another, for example, metres(m) to millimetres(mm) proved difficult. Only 40% of the pupils could multiply by factors of ten (ie. $10, 10^2, 10^3$) and even less, some 25% were able to divide by a factor of ten. Approximately 50% carried out a multiplication problem, converting 1.5 kg to grammes as follows:

\[
\begin{array}{c}
1.5 \\
1000 \\
\hline
15000 \\
\end{array}
\times
\begin{array}{c}
00 \\
00 \\
0000 \\
\hline
15000 \\
15000.0
\end{array}
\]

They were not familiar with moving the decimal point and did not realise that they could miss out these 3 lines of the problem.

An even larger percentage of the pupils some 66%, were not sure where to place the decimal point when multiplying two or more decimal placed numbers together. Example: 1.675 kg converted to grammes or the total quantity of liquid in two and a half containers each holding 0.75 litres when full.
The pupils found reading the vernier and micrometer gauges difficult and failed to understand the principle behind the vernier scale. When reading the micrometer some pupils had difficulty deciding in which columns to place the numbers, so an examination of their understanding of the number system showed a fundamental misunderstanding. Nearly 90% of the pupils could give correctly the first four column headings, those of units, tens, hundreds and thousands, but then went on to quote the next column as millions and similarly for the decimal places, but now they were a little less sure of the values of the initial columns to the right of the decimal point. Many were not aware of the relationship between successive columns, instead had just remembered HEADINGS.

Experiments showed up the pupils weaknesses in accurate work and estimations. When pupils constructed graphs of information gained from experiments they expected every value to be perfectly correct and joined the points of the graph up in a zigzag line instead of drawing in the line which best fits the set of point. Some 20% of the pupils used bar charts instead of continuous line graphs, and approximately 15% of the pupils were unable to plot a graph of any kind. More information on graphs will be dealt with later in this chapter.

The topic of area was more familiar and 80% knew the formula: Length x breadth but only 10% could say with confidence what the units of area were. Most could manage calculating areas of rectangles with whole numbers, but only 26% succeeded with decimal numbers. Some pupils need reassurance that 12 and 12.0, or 0.910 could be written as 0.91 and asked for confirmation.

Nobody was able to work out the area of a triangle and even after being shown had difficulty in deciding which was the height and base. One pupil could not see that multiply by a half(½) was the same as dividing by 2.

Nobody was able to work out the area of a circle, only a few had heard of the term Pi(π) but even these did not know its value. When used as \( \frac{22}{7} \) it proved difficult to 80% of the group. At least two boys did not realise that \( \frac{22}{7} \) meant 22 ÷ 7 and many could not convert this to 3.142 ...
Some 60% of the pupils did not know what $r^2$ meant in the area of circle formula $\pi r^2$. Again 80% of pupils were confused by problems involving 'decimals' or fractions. Most problems set could be solved more easily by cancelling but this was not done by anyone.

55% of the pupils had difficulty in converting from cm$^2$ to m$^2$ etc, as they were not familiar with moving the decimal as a means of multiplying by multiples of ten and attempted to do it as a multiplication problem with resulting mistakes.

The formula for volume was known by only 20% of the group (length x breadth x height) however nobody knew the formula: Area of base x height and again nobody knew the units of volume (m$^3$ or cm$^3$).

Similarly, conversion from m$^3$ to cm$^3$ or mm$^3$ etc, proved beyond all but two members of the group even after it had been explained.

Some comments from the pupils went as follows: "It is bad enough doing these 'difficult' problems in maths, but to do them in science as well was !!!" and another when asked how many thousandths were in an INCH, replied, "There must be millions".

The pupils were not able to work out rough answers or realise when an answer was of a ridiculous order of magnitude.

Mathematics for these pupils was all about NUMBERS whereas in science a variable is a quantity x units which invariably was forgotten.
Mathematics topics required for the section MEASUREMENT

<table>
<thead>
<tr>
<th></th>
<th>Topic</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Natural Numbers</td>
</tr>
<tr>
<td>4</td>
<td>Directed Numbers</td>
</tr>
<tr>
<td>6/7</td>
<td>Factors - Multiples</td>
</tr>
<tr>
<td>8</td>
<td>Powers</td>
</tr>
<tr>
<td>9</td>
<td>Irrational Numbers</td>
</tr>
<tr>
<td>10</td>
<td>Decimals</td>
</tr>
<tr>
<td>11</td>
<td>Fractions</td>
</tr>
<tr>
<td>12</td>
<td>Converting Fractions + Decimals</td>
</tr>
<tr>
<td>13</td>
<td>Percentages</td>
</tr>
<tr>
<td>15</td>
<td>Reading/Interpolating Scales</td>
</tr>
<tr>
<td>17</td>
<td>Standard Form</td>
</tr>
<tr>
<td>21</td>
<td>Averages</td>
</tr>
<tr>
<td>22</td>
<td>Approximation</td>
</tr>
<tr>
<td>23</td>
<td>Estimation/Rough checking</td>
</tr>
<tr>
<td>28</td>
<td>Areas</td>
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<td>29</td>
<td>Perimeters</td>
</tr>
<tr>
<td>30</td>
<td>Volume</td>
</tr>
<tr>
<td>31</td>
<td>Triangle</td>
</tr>
<tr>
<td>34</td>
<td>Circle</td>
</tr>
<tr>
<td>39</td>
<td>Algebra</td>
</tr>
<tr>
<td>40</td>
<td>Simple Formula</td>
</tr>
<tr>
<td>43</td>
<td>Solution of Equations</td>
</tr>
<tr>
<td>50</td>
<td>S.I. Units</td>
</tr>
<tr>
<td>51</td>
<td>Imperial Units</td>
</tr>
</tbody>
</table>
SECTION 3

Syllabus: Mass/Weight

Idea of quantity of matter in a body: Kg or g; use of lever balance and top-pan balance. Weight and gravity relationship. Spring balances.

This involved covering the following:

Mass as the quantity of matter in a body remains constant.
The relationship force = mass x acceleration and hence
weight = mass x acceleration due to gravity (g)
where $g = 9.81 \text{ ms}^{-2}$ (approximately $10 \text{ ms}^{-2}$)

The measurement of mass with a laboratory balance

![Laboratory balance diagram]

and the use of the lever balance and electronic top pan balance.

A greater amount of work is done on the spring balance as this includes the topic of HOOKE'S LAW and its associated experiment with springs or elastic bands to prove this law. This involved measuring the extension of the springs or elastic for different loads and tabulating the results before drawing a graph of load against extension.
Hence: Extension is proportional to load. The slope is called the Spring Stiffness that is, the load to produce a 1 cm extension.

A family of such graphs can be constructed, using springs of different strengths.

Also problems were done using combinations of springs, as shown:

At this stage a fairly extensive mathematical treatment of Hooke's Law was carried out and some typical problems were:
In an experiment with a spring the extension was found to be 4 cm when a load of 100 gf was applied. When the load was increased to 200 gf the total length of the spring was 32 cm. What was the length of the spring before any load was applied.

**Answer**

100 gf produces 4 cm extension

200 gf produces $4 \times \frac{200}{100} = 8$ cm extension

Now

New length = Original length + extension

$32 = x + 8$

$32 - 8 = x$

$24 \text{ cm} = x$

Original length of spring was 24 cm

**Part question**

Various standard masses are hung on a spring and the following extensions produced:

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Extension(cm)</td>
<td>2.5</td>
<td>5.0</td>
<td>7.4</td>
<td>9.9</td>
<td>12.4</td>
</tr>
</tbody>
</table>

(1) Plot a graph of extension ~ mass

(2) Find the stiffness of the spring

(3) What mass would produce an extension of 6 cm when hung on the spring

The answer involves initially constructing a graph of extension against mass then taking the slope of the graph to give spring stiffness (grammes/cm) and finally reading from the graph the mass corresponding to an extension of 6 cm.
Question

Three identical springs are shown

(a) What is the extension produced by the 100 gf load?

(b) Work out the load xgf.

The solution to part (b) involves ratios
for instance: 5 cm extension produced by 100 gf
1 cm extension needs $\frac{100}{5}$ gf
7 cm extension needs $\frac{100}{5} \times 7$ gf = 140gf
(answer)

Questions

Two identical springs 10 cm long and stiffness 25 g/cm.
Calculate the new length in the following situations:

(a) Solution involves: $\text{Load} \over \text{Stiffness} = \frac{120}{25} = 4.8 \text{ cm}$

New length = original length + extension = 10 + 4.8 = 14.8 cm

(b) Each spring will extend by 4.8 cm

overall extension = 4.8 x 2 = 9.6 cm

New length = 10 + 96 = 19.6 cm

(c) Each spring supports half the weight ie: 60 gf

\[ \text{:. Extension of spring} = \frac{\text{Load}}{\text{Stiffness}} = \frac{60}{25} = 2.4 \text{ cm} \]

New length = 10 + 2.4 = 12.4 cm
Problems experienced by the pupils in the survey during this section

60% of the pupils in the survey had difficulty in converting mass into weight by multiplying it by 9.8, especially when working with large numbers. The recurring problem was: lack of fluency of 'times tables' and in placing the decimal point after multiplying. Even when using the acceleration due to gravity as \(10 \text{ m/s}^2\), some pupils failed to gain correct solutions.

The understanding of acceleration as the rate of change of velocity proved, at this stage, beyond the comprehension of approximately 80% of the class, and the units of acceleration were usually misquoted.

The experiment to verify Hooke's Law proved enjoyable for the pupils, although several pupils had not the dexterity to assemble the equipment and required assistance. It was fairly obvious to the author that these pupils had not taken part actively in much practical work, although in previous years experiments had been used as the prime means of teaching science, the classes were then 'mixed ability' and in such cases the pupils of higher ability tend to take control of the performing of the experiment and the lower ability pupils remain in the background to watch.

During experiments the pupils had difficulty in deciding what information was essential and drew up very poor tables of information/readings. The graph produced the expected confusion. Pupils were not familiar with drawing line graphs. They found the decision on 'What scale to use for the axes' difficult due to lack of experience. Comments made by those pupils who could cope with the plotting of co-ordinates indicated that in mathematics problems they had experienced the scale of the axes were always given.

After discussion on finding the gradient or slope of a graph and several worked examples, the vast majority could not perform it
either successfully or accurately. Also the word and meaning of proportionality was not within their vocabulary or spectrum of educational knowledge.

Finally due to the poor numeracy and careless working of these pupils some 68% of them were unable to achieve correct solution to these problems. The problems had to be kept as simple as possible, to avoid demoralisation and loss of interest due to repeatedly getting wrong answers.

Offers of help with the mathematics required for science was generally turned down as it involved extra lessons and at this early stage in the course the examinations were not a worrying influence.

Mathematics topic required for the section Mass/Weight

1 Natural Numbers
6 Factors
7 Multiples
8 Powers
9 Irrational Numbers
10 Decimals
11 Fractions
12 Fractions → Decimal
15 Reading/Interpolating Scales
19 Ratios
30 Algebra
40 Simple formula
43 Solution of Equations
46 Graphs - Co-ordinates
47 Graphs - Linear and Non linear
50 S.I. Units
SECTION 4

Syllabus: Density

Determination by measurement and weighing for regular solids; by displacement for irregular solids. Density of liquids and air.

Practical Work:
Density of a cube and a stone.
Density of methylated spirit by density bottle.
Density of air.

This involved covering the following

The definition of density as the mass per unit volume and the relevance of this term in industry.

The formula \( D = \frac{m}{V} \) where

\[ D = \text{Density } \frac{\text{kg}}{m^3} \]

\[ M = \text{Mass } \text{kg} \]

\[ V = \text{Volume } m^3 \]

The units of density used are \( \text{kg/m}^3 (\text{kgm}^{-3}) \) or \( \text{g/cm}^3 (\text{g.cm}^{-3}) \)

and for conversion \( 1 \text{ kg/m}^3 = \frac{1000 \text{ g}}{1000000 \text{ cm}^3} = 10^{-3} \text{ g/cm}^3 \)

This topic is mainly experimental, using balances of various kinds, either calculating or using measuring equipment to find the volume. For any particular substance a graph can be constructed of mass against volume, the slope of which is the density.

The density of liquids are found using a relative density bottle and that of air by pressurising a container and finding the weight of the extra air pumped in, then releasing this air into gas jars by displacement of water technique. The volume of the gas jars can then be calculated.
For those pupils having difficulties in remembering the different versions of the density formula the 'Triangle' was used:

The required subject is covered, for instance from

\[
V = \frac{M}{D} \quad \text{(cm}^3\text{)}
\]

\[
M = D \times V \quad \text{(g)}
\]

\[
D = \frac{M}{V} \quad \text{(g/cm}^3\text{)}
\]

This topic is now continued in a mathematical context and some typical problems are as follows:

Questions involving substitution into the formula of two of the quantities, for example:

(i) Find the density of iron whose mass is 9.8g and volume 1.1 cm\(^3\) to 1 decimal place

(ii) Find the mass of a piece of wood whose density is 0.75g/cm\(^3\) and has a volume of 1.2 m\(^3\) (Note the mixed units)

(iii) Find the volume of a piece of aluminium, density 2.6 g/cm\(^3\) whose mass is 0.52 kg (Note the mixed units)
More complex problems involve calculating the volume from given facts, or using density bottles as in the following problems:

(1) A density bottle weighs 18.00 gf when empty, 44.00 gf when full of water (density 1g/cm³) and 39.84 gf when full of a second liquid. What is the density of this liquid?

(2) A light alloy consists of 70 per cent aluminium and 30 per cent magnesium by mass. What would you expect its density to be? (Density of aluminium = 2700 kg/m³ and magnesium 1740 kg/m³)

(3) Calculate the mass of air in a room of floor dimension 10 m × 12 m and height 4 m. Density of air = 1.26 kg/m³)

(4) An empty 60 litre petrol tank weighs 10 kgf. What will it weigh when full of fuel of relative density 0.72?

For the higher ability pupils discussion of Archimedes' principle and upthrust could be found useful for the future.

Problems experienced by the pupils in the survey during this section

The use of letters to represent numbers seemed a new experience for these pupils, even though the author had checked with the mathematics department that simple formula had been covered. These pupils have 'very short memories' and work must be constantly revised.

Rearranging the simple formula \( D = \frac{m}{v} \) to have \( m \) or \( v \) as the subject proved beyond all the lower ability pupils, who preferred to attempt to remember the 'Triangle' method. This would have been perfectly satisfactory if they could remember the correct ordering of the letters in the triangle.

During the experimental work a cube, with sides 2 cm long, was used and some 33% failed to calculate the volume as 8 cm³ -
instead they quoted 6 as the answer, and the majority failed to quote the correct units or any units at all. Those who quoted units tended to use cm$^2$.

When dealing with a density problem, where the answer was a decimal number, many pupils wanted to leave the answer as a whole number plus a remainder. It was obvious to the author that some work needed to be done in this area, and also some experience in "rounding off and significant figures" which did not appear in their spectrum of knowledge.

Up to 70% of the pupils were unable to cope with division by one decimal number into another decimal number, or by a larger number into a smaller. This confirmed the suggested poor numeracy problem much quoted in the newspapers. At this stage the author thought that this sample of seventy-two pupils may be a particularly poor ability group, however subsequent CSE examination results show that they were normal in cross-section of ability. Discussion of the use of rough approximation when dividing by very large numbers showed a lack of familiarity, however some pupils endeavoured to gain a solution by writing out the required timetable, say 401 timetable for the following problem, before attempting the solution, eg: $491 \sqrt{26708.9}$.

The words 'long division' seemed to worry some pupils who instantly asked permission to do 'short division', but with large numbers, this resulted in mental arithmetic mistakes. There were many different methods, or routes to answer in evidence in the pupils work, not all ad-hoc methods were unsuccessful. For those who were fairly adequate in their ability in this topic, the technique of borrowing often caused error, especially in the following methods:

\[
\begin{array}{c}
\longdiv{168.4}{19} \\
\underline{19 \times 200.0} \\
19 \underline{180} \\
\underline{180} \\
\underline{155.0} \\
8.0 \\
\underline{1.6} \\
4
\end{array}
\]

This process of borrowing proved to be untidy and often led to mistakes.
Finally with conversion from cm\(^3\) to m\(^3\) by dividing by a factor of 10\(^6\), the majority of pupils could not work out the conversion factor by multiplying 100 by 100 by 100. Answers varied from 300 to 10 000 to 1 000 000. Some preferred to do a multiplication problem rather than move the decimal point or add zeros.

These pupils were not in the habit of using reference books to find data, for instance: when checking values of densities found from experiments.
Mathematics topics required for the section on Density

1. Natural numbers
2. Number system
3. Place values
4. Factors
5. Multiples
6. Powers
7. Irrational numbers
8. Decimal
9. Fractions
10. Fractions and Decimals
11. Scales
12. Ratios
13. Reference tables
14. Area
15. Volume
16. Algebra
17. Simple formula
18. Solution of equations
19. Graphs
20. Representing data as graphs
21. S.I. units
SECTION 5

Syllabus: Time

Work with pendulum to show effect of varying mass, length and amplitude; application to clocks. Practical use of stop watch; ticker-timer and its application for the construction of tape charts involving distance/time and speed/time relationships. Use of such charts to develop the meaning of the terms speed and acceleration.

Practical work

Demonstration of the use of measuring devices.
Simple pendulum experiments.
Ticker-time experiments.
Constructing graphs and using them to determine speed, distance and acceleration.

This involved the following:-

The various units of time and conversion between units, that is, the year, day, hour and second.

The pendulum was dealt with through a series of experiments to show its 'time keeping' ability. Experiments included varying one of the following three parameters, in turn, whilst keeping the remaining two constant. These parameters were amplitude of swing, mass of the pendulum bob and the length of the pendulum, to see what effect if any there was on the Periodic Time of the pendulum, that is the time for 1 cycle. Summary of results showed that on timing 50 complete cycles the periodic time was constant for varying mass and amplitude but varied if the length of the pendulum was changed.

For the brighter pupils it could be shown that periodic time = \(2\sqrt{\frac{L}{g}}\)

where 'g' is the acceleration due to gravity.
The stopwatch was used to record the time taken for the pendulum to complete 50 cycles. The type of stopwatch was a 30-second per revolution, with a one half minute hand.

The work progressed onto the Ticker-Timer which is a device that puts a dot on a piece of moving ticker tape every 0.02 \( \left( \frac{1}{50} \right) \) of a second and was used to study speed and acceleration relationships.

Information was gained from the tapes by measuring distances between dots, tabulated and used to draw graphs to represent the form of motion. For example, these graphs show Non-uniform velocity of a trolley initially travelling down an incline and then at some later stage travelling up an incline until it stops.
Other graphs would show: constant velocity, uniform and non-uniform acceleration, and these typical graphs are shown below, with some relevant facts obtained from the experiments:

**Uniform velocity**

![Graph of uniform velocity](image)

**Uniform acceleration**

![Graph of uniform acceleration](image)

**Non-uniform acceleration**

![Graph of non-uniform acceleration](image)

**Formulas**

- **Velocity**: $\frac{\text{Distance}}{\text{Time}} = \frac{AB}{OB}$
- **Acceleration**: Rate of change of velocity $= \frac{\text{Gradient of OP}}{t}$
- **Acceleration at P**: \( = \frac{MN}{LN} \)

Other formulae used were:

1. **Average velocity** = \( \frac{\text{distance travelled}}{\text{time taken}} \) or
   \[
   \text{Initial velocity} + \text{Final velocity} \]
   \[
   = \frac{v + u}{2}
   \]

2. \( v = u + at \) where \( v = \text{final velocity}, u = \text{initial velocity}, a = \text{acceleration and } t = \text{time} \)

3. \( a = \frac{v - u}{t} \)

4. **Distance travelled** (s) = \( ut + \frac{1}{2} at^2 \). This could be replaced by using the fact that the distance travelled was equal to the area under the curve of a velocity against time graph.

The units of velocity = metres per second (m/s)

The units of acceleration = metres per second per second (m/s²)
At this stage, whilst using the ticker-timer an experiment was performed to find the acceleration of a falling object, that is the acceleration due to gravity (g).

Some typical problems on this section are:

(1) A car travels along a level road. A number of lines are marked across the road 15 m apart and a man starts a watch as the car crosses the first line. His watch read 3.7 seconds at the 2nd line, 6.0 seconds at the 3rd line, 8.9 seconds at the 5th line and 9.7 seconds at the 6th line.

(a) Devise a table which will enable you to present the information clearly
(b) Draw a graph of the motion
(c) Describe the motion
(d) How far is the car away from the 1st line after 5 seconds?
(e) At what time does the car pass the 4th line?
(f) What is the average velocity between the 2 and 5th lines?
(g) What is the velocity of the car as it crosses the 5th line?

(2) Define speed and explain average speed. A car travels from Derby to Skegness a distance of 145 km in 3 hours 45 minutes. Find the average speed.

(3) Draw a velocity against time graph for a body which starts with an initial velocity of 3m/s and continues to move with an acceleration of 1.5m/s² for 6 seconds. Show how you would find from the graph the distance travelled during this time.
(4) A body which starts from rest and travels down an incline covers the following distances in times:

<table>
<thead>
<tr>
<th>Distance (m)</th>
<th>0</th>
<th>12.8</th>
<th>20.0</th>
<th>28.8</th>
<th>51.2</th>
<th>64.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (s)</td>
<td>0</td>
<td>0.8</td>
<td>1.0</td>
<td>1.2</td>
<td>1.6</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Draw a graph to show that the motion is uniform acceleration and calculate its value.

Problems experienced by the pupils in the survey during this section

By the time this section was dealt with we were in the second term of the 4th year. We started by conversion between units of days, hours and seconds, which all pupils were familiar with, but several pupils made careless errors in the arithmetic. A greater number of pupils (75%) were unable to work out what 5 minutes was as a fraction of an hour. Although some pupils could manage this, nobody could correctly calculate \( \frac{7\frac{1}{2}}{60} \) minutes as a fraction of an hour. The problem was not one of just forming the fraction \( \frac{7\frac{1}{2}}{60} \) but rationalising \( \frac{7\frac{1}{2} \times 2}{60 \times 2} = \frac{15}{120} \)

and cancelling to \( \frac{1}{8} \).

When working with the pendulum, the pupils appreciated that a greater accuracy could be achieved by timing 50 cycles and then dividing to find the time for 1 cycle. As the experiment was carried out to a fairly high degree of accuracy this division was done to 2 decimal places. Over 50% of the pupils were confused by the term decimal places and two pupils when faced with dividing 48 seconds by 50 said that it could not be done, because 50 was bigger than 48. The same pupils when faced with a similar problem, later in the same day, in which a small number had to be divided by a larger number eg: \( 3 \div 50 \), automatically carried out the operation as \( 3 \sqrt{50} \) because to them logically '3 would go into 50' whereas '50 would not go into 3' so the problem had to be \( 3/\sqrt{50} \).

The ticker-timer proved very interesting to all and each pupil eagerly wanted to make a ticker-tape. The accuracy of many of
the graphs plotted of distance covered against time taken was so erratic, many failing to gain a graph of any kind, that it was decided to cut up the ticker tapes into $\frac{1}{10}$th second pieces and use these to plot the graph and achieve a relationship. This is shown in the diagram entitled 'Ticker Tape - irregular motion' which appeared earlier in this section.

Less than 20% of the pupils were able to use the relevant formula and substitute values to find an unknown. Nobody could rearrange the formulae to have a different subject.

When working out the distance travelled, using the theory relating to the area under a velocity against time graph, some 85% of the pupils were not able to calculate the area of triangle. This was due mainly to poor memories on the part of the majority of the pupils as it had been covered previously.

The units of velocity and especially acceleration proved confusing and similarly problems involving retardation (negative acceleration) where the formula became $v = u + (-a)t$, and average velocity $= u + \frac{1}{2}(-a)t$.

Many CSE questions involve plotting graphs of distance or velocity against time and reading values from these graphs. The average to lower ability pupils found this very difficult, especially when the co-ordinates of a point consisted of decimals.
The mathematics topics required for the section on Time

1 Natural numbers
3 Place values
4 Directed numbers
6 Factors
7 Multiples
8 Powers
9 Irrational numbers
10 Decimals
11 Fractions
12 Fractions to decimals
15 Reading/Interpolating Scales
16 Concept of rate of change
21 Averages
22 Approximations
23 Estimations/Rough checking
25 Squares and Roots
39 Algebra - letters to represent numbers
40 Simple formula
43 Solution of equations
46 Graphs - Co-ordinates
47 Representing data by graphs
48 Linear/Non linear graphs
50 S.I. units
52 Speed-distance calculations
SECTION 6

Syllabus: Forces

(a) Effect of forces in changing the shape and size of bodies. Extension of steel springs and elastic strings, elastic limit. Use of spring balance. Unit of force - Newton (N).

(b) (i) Moving a body from rest. Idea of inertia.
(ii) Altering the velocity of a body, ie: speed and direction.


(d) Friction - basic qualitative idea, reduction of friction.

(e) Upthrust related to the displacement of fluids of different densities. Flotation - application to ships and hydrometers.

Practical work

1 Spiral spring experiment to demonstrate the relationship between the load and extension and elastic limit.

2 Use of metre rule and masses to demonstrate the principle of moments.

3 Centre of gravity of a card board.

4 Equilibrium rods to demonstrate stability. Bunsen burner or funnel may also be used.

5 Using Newton meter and 1 kg mass and different surfaces to demonstrate friction.

6 Using Newton meter, 200g brass weight, displacement can or measuring cylinder and various liquids, demonstrate upthrust

7 Construction and use of hydrometers.
This section involved the following:

The effects of force covering Newton's Laws (Force = mass x acceleration), gravity and resultant effect on weight, the acceleration due to gravity \( g \) = 9.81 m/s\(^2\). The units of force: Newton and its definition.

Experiments involving friction forces and the relationship between static friction and load or area of surface in contact. For the more able the use of the relationship \( \frac{F}{R} \) = constant and \( F = \mu R \) where \( F \) = limiting frictional force, \( R \) = normal reaction force and \( \mu \) = coefficient of static friction.

Tension in strings and the simple resolution of forces, done for the more able pupils because this involves the use of sine, cosine and tangent relationships. This continues with a detailed look at Hooke's Law and the relationship load is proportional to the extension of an elastic substance up to the elastic limit.

The turning effect of a force is dealt with experimentally through levers and the use of a metre rule and masses (see-saw) to show anticlockwise moments are equal to clockwise moments for an equilibrium situation.

Centre of gravity: within any body there is a point where the loads and their resulting moments on each side at that point are in equilibrium. It appears that all the weight acts through that point. This was done experimentally on a lamina and the pupils were required to know where the centre of gravity of some regular shaped laminas such as squares, rectangles, circles, triangles and some cuboids could be found.

Stability and its three states, stable, neutral and unstable equilibrium are dealt with experimentally.
Some typical examples on this section are as follows:

(1) A spring is loaded by stages and its length noted each time. The results are shown in the table:

<table>
<thead>
<tr>
<th>Load in grammes weight</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of spring in cm.</td>
<td>36</td>
<td>41.5</td>
<td>48.5</td>
<td>54</td>
<td>60</td>
</tr>
</tbody>
</table>

Draw the graph of the results plotting load across the page and length of spring up the page. From the graph
(i) What will be the length of the spring when 110 g is applied to it? (ii) What is the length of the unstretched spring? (iii) What load will produce an extension of 20 cm?

(2) A light rod AB 20 cm long, is supported horizontally by two exactly similar springs hanging vertically, attached to its ends. A load of 200 g.wt is hung from a point on the rod 15 cm from end A and the upper end of the spring at B has to be raised 2 cm in order to make AB horizontal again. Find the force needed to extend each of the springs by 1 cm.

(3) The diagram shows two identical springs, spring stiffness = 50 g/cm. Calculate the extension in each case.

![Diagram a) and b)](image)

(4) Find the value of \( E \)

(5) A metre rule is balanced as shown. What is the weight of the rule?
(6) The diagram shows a man pulling a heavy load. What is the horizontal force applied by the man? Work out the force tending to lift the front of the load.

The problems experienced by the pupils in this survey in the section on Force

Most pupils could see the sense in approximating the acceleration due to gravity to $10 \, \text{m/s}^2$ and much preferred multiplying and dividing by 10 rather than $9.81$. The section involving "Mechanics" is always unpopular even amongst the higher ability pupils who are not capable of applying their mathematical knowledge. The problems involving forces are usually restricted to force in line with movement as when resolving for forces not in line, the problem arises that these pupils cannot use the laws of trigonometry or tables. This type of problem could be solved by geometrical construction but experiences showed that many were not able to measure lengths or angles accurately and the work tended to be very untidy and careless. Whenever decimal fractions appeared in problems the more able appeared to cope with an added confidence, whereas the lower ability range suffered with the same problems as experienced earlier in the course.

Hooke's law was reasonably successful, but accuracy in measurement needs attention. A considerable amount of difficulty was experienced by all pupils when dealing with applied mathematics problems in which the relevant information
was placed in a written text. Their strategies of approach to such problems was very poor.

Again, throughout the whole of this section, the work of many pupils was spoilt by careless mistakes in basic arithmetic and untidy presentation. It was however noted that there was a general improvement in enthusiasm for any section which included practical work and the subsequent written work benefitted from this.

**Mathematical topics required for the section on Mechanical Forces**

1. Natural numbers

6. Factors

7. Multiples

10. Decimals

11. Fractions

19. Ratios

20. Proportions

37. Scale drawings

39. Algebra

40. Simple formula

43. Solution of equations

45. Trigonometry

46. Graphs - Co-ordinates

47. Graphs to represent data

50. S.I. units
SECTION 7

Syllabus: Pressure

Relation between forces, area and pressure. Units N/m². Everyday illustrations and applications. Pressure dependence on depth and density in a fluid. Measurement of pressure in fluids by:
(a) manometer - cm on fluids
(b) simple mercury barometer - cm mercury
(c) aneroid barometer

Practical work

Construction of a manometer and its use to measure the pressure of gas supply.
Demonstration of Fortin and aneroid barometers.
Variation of pressure with depth of liquid.

This involved covering the following:

The definition of pressure and the equation Pressure = \( \frac{\text{Force}}{\text{Area}} \)

The units of pressure Newtons/m². This was followed by some simple calculations of pressures created by various loads on various areas, and terminated with an example to show that a woman in high-heeled shoes causes more damage to the floor than an elephant would, that is indicating that very high pressure can be created by relatively small forces applied to very small areas.

\[
\text{Thrust in both cases: weight of brick} = 3 \text{ kgf} = 3 \times 9.8 \text{ Newtons}
\]

\[
\text{Pressure} = 3 \times 9.8 \frac{\text{N}}{0.06 \text{ m}^2} = 490 \text{ N/m}^2
\]

\[
\text{Pressure} = 3 \times 9.8 \frac{\text{N}}{0.09 \text{ m}^2} = 327 \text{ N/m}^2
\]
The calculations of pressure in a fluid and its relationship with depth of fluid.

A simple experiment shows that pressure increases with depth of fluid and acts in all directions.

\[
\text{Pressure (N/m}^2\) = hpg
\]

where \( h \) = head of fluid, \( p \) = density of fluid (depth), \( g \) = acceleration due to gravity

This experiment used a manometer to measure the pressure and was subsequently used to measure gas pressure. When using the manometer the pupils have to take into consideration Atmospheric Pressure + Pressure due to water column or excess pressure in gas above Atmospheric Pressure = hpg (N/m²)

This value is usually quoted in cm or mm of water for low pressures, or mercury for higher pressure. For instance, Town gas pressure was approximately 15 to 20 cm of water pressure whilst atmospheric pressure is approximately 76 cm of mercury. The relevance of this depended on an understanding of the relative density of mercury 13.6 g/cm³ and water 1 g/cm³.
The barometer was dealt with next; initially the simple form then the more accurate Fortin version. Atmospheric pressure can either be measured in centimetres of mercury, or Newtons/metre$^2$ or the bar and millibar for those more able and the option topic on the weather. A method of relating the standard atmosphere in cm of mercury to the millibar as shown on a weather map, is carried out:

Assume $g = 9.81 \text{ m/s}^2$

1 atmosphere = 760 mm of mercury

Density of mercury:

\[ p = 13.6 \text{ g/cm}^3 = 13600 \text{ kg/m}^3 \]

Pressure = $hpg$ where:

- $h$ = head of mercury
- $p$ = density of mercury

So 1 atmosphere =

\[ \frac{760 \times 13.6 \times 10^3 \times 9.81}{1000} \text{ N/m}^2 \]

= 101400 N/m$^2$

Now 1 bar = $10^5$ N/m$^2$

(definition of bar)

So 1 atmosphere = \( \frac{101400}{10^5} \) = 1.014 bars

= 1014 millibars (mb)
Some typical questions on this topic

(1)

The diagrams show a block of metal weighing 1.2N resting on a surface in three different positions:

(i) In which position will it exert the greatest pressure on the surface?

(ii) Give reasons for your answer to part (i)

(iii) Calculate the pressure exerted in each case

(2) The bottom of a rectangular tank measures 10 cm x 6.5 cm. Water is poured in to a depth of 4.5 cm. What is the pressure on the bottom? What is the thrust on the base?

(3) A rectangular block with sides 5 cm by 10 cm by 20 cm is made from a material of density 2.5 g/cm³. Find the maximum pressure it can exert when resting on one of its faces.

(4) Hydraulic pumps were dealt with in the form of a question: If any part of a liquid that completely fills a closed vessel is subjected to a pressure, that pressure is transmitted equally to all parts of the containing vessel. Hence answer the following:
(5) An open 'U' tube manometer containing water shows a difference in level of 15 cm when connected to the gas supply. Find in N/m² the excess pressure of the gas above atmospheric pressure.

Problems experienced by the pupils in the survey on this section

No new problems appeared in this section which had not surfaced earlier in the course. The depth of the problems remained the same even though these pupils had been taught mathematics for a further two terms. The main areas requiring attention for this section are: lack of competence in the four rules of whole numbers and decimals, algebra, simple formula and lack of comprehension or strategy when dealing with calculations where the question is written in essay form. Only the very simplest of wording can be understood as these pupils' command of the English language is very poor. The pupils have difficulty in applying their mathematical knowledge to other subjects because they are unfamiliar with the different approach and terminology.

It was noted that the level of motivation and interest was at a maximum whenever the theory was applied to a practical situation or had some personal relevance to the pupils.
Mathematics topics required for the section on Pressure

1 Natural numbers
2 Number system
3 Place values
8 Powers
9 Irrational numbers
10 Decimals
11 Fractions
12 Fractions ÷ Decimals
15 Reading/Interpolating scales
19 Ratios
22 Approximations
23 Estimation/Rough checking
28 Areas
30 Volume
39 Algebra
40 Simple formula
43 Solution of equations
50 S.I. units
Syllabus: Work, Energy and Power

Work – as transformation of energy. Mechanically calculated as the product of force and distance moved along the line of action of the force. Unit – joule.

Energy as ability to do work. Various forms of energy to include kinetic and potential energy: heat, light, electrical and sound energy. Transformation of energy from one form to another. Conservation of energy. Unit – joule.

Power as rate of doing work or rate of use of energy. Unit – watt

Practical Work:

Various energy transformation experiments

1. battery - electric motor, belt and pulley lifting weight
2. weight falling connected via a pulley and belt to drive a dynamo and light a lamp
3. Steam engine used to drive a pulley or dynamo as in the case of a power station
4. Water turbine and hydroelectric power

Power as a rate of doing work: experiment to determine the power created by a pupil running upstairs

This Involves the Following

Energy is the capacity to do work. Experiments and discussion on changing the form of energy by various means and the conservation of energy. A more detailed discussion of two important energy forms, those of kinetic and potential energy, followed by a mathematical treatment of the work done by a force.
Work done by a force

= Force (in Newtons) × Distance moved through by the force in the direction of application (metres)

Units of Work = Newton metres (Nm) = Joules (J)

Power is the rate of doing work, that is,

\[ \text{Power} = \frac{\text{work done}}{\text{time taken to do work}} \text{ J/s} \]

and the unit 1 J/s is equal to 1 WATT (W)

**Mechanical Energy**

1. Kinetic energy which is the energy a body has by reason of its motion

   Kinetic energy = \( \frac{1}{2} \text{mass} \times (\text{velocity})^2 = \frac{1}{2}mv^2 \)

   Units = kg × \( \left( \frac{\text{m}}{\text{s}} \right)^2 \) = kg × \( \frac{\text{m}^2}{\text{s}^2} \) = N.m = Joules

2. Potential energy is the energy an object has by reason of its position or state. Potential energy due to height = mg\( \times \)h Joules

Some Typical Questions on this Section follow

1. In an experiment which appealed to the pupils, they were asked to run up a flight of stairs and time this action with a stopwatch. Then they measured the total vertical height and their weight and performed a calculation similar to the one shown below.

   Pupil weight 60 kg. Time taken to run up the stairs 5 seconds.
   Stairs have 30 steps each 15 cm high. Given g = 9.8 m/s²
Work done = force \times distance
\hspace{2cm} = (60 \times 9.8) \times \left(\frac{30 \times 15}{100}\right)
\hspace{2cm} \text{convert kg into Newtons}
\hspace{2cm} \text{convert height of 30 steps into metres}

Power = \frac{\text{work done}}{\text{time taken}} = 60 \times 9.8 \times \frac{30 \times 15}{100} \times \frac{1}{8} \text{ watts}

Power = 529.2 \text{ watts or } 0.5292 \text{ kw} = 0.5 \text{ kw}

Some of the physically more able pupils were able to generate powers of the order of 1 kilowatt, but they realised that this higher power could only be sustained for short periods of time.

A cart weighs 500 kgf and is loaded with coke which weighs 750 kgf. If a horizontal force of 250 kgf is required to pull the loaded cart 2 km along a level road.
Calculate (i) the work done (ii) if the cart had been pulled the same distance up an incline of 1 in 100, how much work would have to be done against gravity. Take \( g = 10 \text{ m/s}^2 \)

Problems Experienced by the Pupils in the Section on Work, Energy and Power

This section proved to be highly enjoyable for the pupils, especially the experiments involving the steam engine and the running up and down the stairs. Consequently when it came to writing about the experiment and performing the relevant calculations, they were carried out with an enthusiasm rarely seen. They were very keen to know just how much power they themselves were capable of generating. Even the members of the group that complain rigorously about any form of mathematics demanded help to complete the working out correctly and quickly; confirming that motivation and enjoyment go "hand in hand" and hence the improvement of overall standards.
Mathematics Topics Required for this Section

1 Natural numbers
2 Number system
3 Place values
6 Factors
7 Multiples
8 Powers
9 Irrational numbers
10 Decimals
11 Fractions
13 Percentages
15 Reading/interpolating scales
16 Concept of rate of change
17 Index notation
21 Averages
22 Approximations
23 Estimation/rough checking
39 Algebra
40 Simple formula
43 Solution of equations
50 SI units
SECTION 9

Syllabus: Heat

(a) Temperature as an indication of the 'hotness' of a body. Distinction between heat and temperature. Ways of measuring temperature - liquid in glass thermometers to include clinical and combined maximum and minimum types. Celsius scale. Thermocouple. Bimetallic thermometer

(b) Heat as a form of energy, unit - joule. Relate heat absorption by a body to increase in kinetic energy of molecular motion. Measurement of heat energy given to a body from an electrical source using a joule meter or ammeter/voltmeter method. Heat capacity. Comparison of the heat capacities of equal masses of different substances

(c) Effect of Heat

(i) Expansion of solids, liquids and gases treated qualitatively. Everyday applications including thermostats, anomalous behaviour of water. Relationship between temperature, volume and pressure of gases (no calculations on the gas laws needed)


(d) Transference of Heat

Conduction, convection and radiation - qualitative treatment related to molecular state in the case of conduction and convection. Everyday examples of these
processes including thermal insulation of buildings, the domestic hot water system and the vacuum flask. Absorption and emission of radiant heat energy from different surfaces

**Practical Work**

Demonstration and use of mercury thermometer, clinical thermometer, maximum and minimum thermometer

Thermocouple and galvanometer experiment

Construction of thermostat

Expansion of metal strips, air and liquids

Bar breaking experiment

Expansion of bi-metal strips

Using Boyle's law and constant volume apparatus - demonstrate the relationship between the volume, temperature and pressure of air

Simple experiments on evaporation, boiling, melting and freezing

Effect of reduced pressure on the boiling point

Effect of impurities on the boiling and freezing points

Domestic hot water system model

Principle of refrigeration from chart

Simple experiments on conduction, convection and radiation

Demonstration of vacuum flask

Simple experiments on absorption by and emission of heat from different surfaces
This Involves Covering the Following:

The fundamentals of expansion and its application to everyday life. The pupils are to be aware of the relationship gained from experiment that if the temperature of a gas is raised at constant volume the pressure will also increase, and that a relationship exists between pressure and volume when the temperature is kept constant (\( pV = \text{constant} \)).

From the expansion of liquids the thermometer can be explained and hence the Celsius Scale of temperature. Other temperature scales referred to were the Fahrenheit and Kelvin Scales and the conversion factors.

In the calibration of the thermometer reference is made to the boiling and freezing points of water and the effects of pressure or impurity on these values. This was investigated by experiment and graphs of temperature against time were constructed. Whilst doing this, terms such as latent heat, heat capacity and specific heat were defined and used in calculations.

The heat transfer processes were dealt with in a qualitative manner relating to the molecular state. Finally the measurement of heat energy and its unit, the Joule, were covered. The definition of the Joule stated as: 4.2 Joules of heat are required to raise the temperature of 1 gramme of water through 1°C. This was involved in some simple calculations of "heat supplied" for example the heat per minute supplied by a bunsen burner could be approximated by heating some water for a short period of time and noting the temperature rise. The calculation is only approximate as it assumes no heat capacity for the beaker etc, and no heat losses to the surroundings.

Some typical questions were as follows:

1. What amount of heat would be needed to raise the temperature of 250 g of water from 25°C to 65°C given that the specific heat of water is 4.2 J/g °C?
2 If the pressure of a gas is doubled at constant temperature what has happened to its volume? (CSE 1980)

3 An electrical immersion heater is connected to a 10 volt supply and takes a current of 3 Amps.

(i) Calculate the rate in watts at which the immersion heater converts electrical energy into heat energy

(ii) Calculate how much heat energy, in Joules, is produced in 5 minutes

(iii) Calculate the heat capacity in J/°C of the water and its container if the temperature rises from 15°C to 25°C in 5 minutes

(CSE 1982)

Problems Experienced by the Pupils in the Survey during this Section

The mathematics content in this section is usually camouflaged by masses of words and this seemed to confuse many who would normally have been able to cope. Very often the pupils had to be told which of their mathematics skills to use as they could not decide for themselves. They all found it difficult to plan any form of strategy to gain the answer to this type of question.

The specific heat of water (4.2 J/g°C) proved an awkward number in multiplication and division problems with the added complexity of the decimal point and its position in the answer. Questions usually had to be set in consistent units as questions involving mixed units were treated without regard for the units.

Calorimetry experiments and questions were found to be too difficult mathematically for all concerned and thus not dealt with in any detail.
Where graphs were required as part of the experiment there still remained a small group who could not construct the axis scales or merely labelled each centimetre division along the axis with successive values from the table resulting in an 45° line each time. For example:

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>15</th>
<th>21</th>
<th>31</th>
<th>35</th>
<th>45</th>
<th>55</th>
<th>69</th>
<th>75</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (minutes)</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>27</td>
<td>30</td>
</tr>
</tbody>
</table>

Mathematical Topics required by the Section on Heat Energy

1 Natural numbers
2 Number systems
3 Place values
4 Factors
5 Multiples
6 Irrational numbers
7 Decimals
8 Fractions
9 Percentages
10 Reading/interpolating scales
11 Concept of rate of change
12 Ratios
13 Approximations
14 Estimation/rough checking
15 Use of reference tables
16 Algebra
17 Simple formula
43 Solution of equations
46 Graphs/co-ordinates
47 Linear/non-linear graphs
48 Representing data as a graph
50 SI units
SECTION 10

Syllabus: Wave Motion

(a) **Transverse Waves**

General properties as illustrated by the ripple tank. Reflection at plane surfaces. Refraction at plane boundaries. Interference of waves

(b) **Longitudinal Waves**

As illustrated by the general properties of sound waves. Vibrations as sources of sound - need for transmission medium, reflection, echoes, refraction and interference; speed of sound in different media and one simple means of its estimation in air

(c) **Electromagnetic Spectrum**

Broad division into gamma rays, x-rays, ultraviolet, visible light, infra-red and radio waves. Sources of visible light, reflection of light at plane surfaces; refraction at plane boundaries. Single converging lens - meaning of focal point and focal length. Simple study of the nature of the image; simple magnifying lens. Simple problems for graphical solution may be set

**Experimental Work**

1. Ripple tank experiments
2. Vibration of tuning fork
3. Resonance tube
4. Use of slinky to demonstrate transverse and longitudinal waves
Bell jar and electric bell experiment to demonstrate the need for transmission medium

Velocity of sound in air experiment

Electromagnetic spectrum (chart available)

Reflection of light rays by a plane mirror
  (i) location and properties of the image
  (ii) laws of reflection

Refraction by a rectangular glass block and water. Laws of refraction

Converging of light beam by a convex lens

Formation of images by a convex lens

This Involved Covering the Following

The properties of waves, wavelength and amplitude measurement and speed of wavefront calculations

\[
\text{Speed of wavefront} = f\lambda \text{ (m/s)}
\]

where \( f \) = frequency in waves/second
\( \lambda \) = wavelength in metres

The types of wave, that is, transverse waves such as waves in the sea or electromagnetic waves and longitudinal waves such as sound waves.
When dealing with sound various methods are available to determine the velocity of sound and all involve calculation using velocity, distance and time. Other calculation-type questions covered involve sonar and echoes. For instance:

An echo sounder on a ship, receives an echo 2.5 seconds after sending the signal. At what depth is the ocean bed if the speed of sound in sea water is 1450 m/s?

Light (transverse wave) is covered in detail, starting with the rectilinear propagation of light, then continuing with reflection of light at plane and curved surfaces. Experiments are carried out to show that the angle of incidence = the angle of reflection for a plane mirror,

\[ \theta_i = \theta_r \]

and to locate the image position of an object in a plane mirror

The eye views an object O, first from position E₁ then from E₂, and at each position locates the path along which the image must lie. On projecting these lines behind the mirror the image position is found.

Note \( IQ = QO \)

and to show lateral inversion
This was followed by a thorough treatment of refraction (the bending of light on passing across a boundary between two media of different optical densities). Experiments involved determining the refractive index \((n)\) of glass and water

\[ n = \frac{\sin i}{\sin r} = 1.5 \]

Refraction index for glass to air boundary = \(\frac{1}{n}\)

Refraction index \((n)\) also = \(\frac{\text{speed of light in air}}{\text{speed of light in substance}}\)

\[ = \frac{\text{real depth}}{\text{apparent depth}} = \frac{1}{\sin \text{critical angle}} \]

The refraction of a beam of white light by a triangular prism produced dispersion and resulted in a spectrum being cast on a screen and also led into the design of convex and concave lenses. The convex lens and its properties were investigated next and a series of constructions were used to illustrate these.
Magnification $= \frac{\text{height of image}}{\text{height of object}}$

Followed by the concave lens: formation of image
Other refraction effects were covered, such as apparent depth and mirages before going on to a brief explanation of the electromagnetic spectrum:

Some Typical Problems

1. A man hammering at regular intervals of 0.4 seconds hears echoes of the hammer blows exactly halfway between successive blows when he stops hammering he hears two further echoes. Calculate his distance (perpendicular) from the reflecting surface. Velocity of sound in air is 330 m/s.

2. Two men A and B stand in a line perpendicular to a cliff, A being nearer to it than B, and the distance between A and B is 247 metres. 'A' fires a gun and hears the echo after 1 second. When B fires a gun A hears the echo after 1.75 seconds. Find (i) the velocity of sound (ii) the distance of A away from the cliff.

3. An object 4 cm high is placed 15 cm from a convex lens of focal length 5 cm. Draw a ray diagram, on graph paper (half scale), and find the position, size and nature of the image.

4. Explain with diagrams how a $45^\circ - 45^\circ - 90^\circ$ prism can be used to reflect light through (a) $90^\circ$ (b) $180^\circ$
5 A swimming pool appears to be 3 m deep. If the refractive index of water is \( \frac{4}{3} \) what is its real depth?

6 The angle between an incident ray and a mirror is 30°
   (a) what is the angle of incidence
   (b) what is the angle of reflection
   (c) what is the total angle turned through by the ray of light

7 A witness says he saw a murder committed when the hands on the clock showed 5.12 pm, but the suspect had an alibi for this time. Later, you, as the detective in-charge, realise that the witness saw the clock in a mirror by reflection. What would you investigate next?

Problems Experienced by the Pupils in the Survey on the Section on Waves

Accurate measurement was required and all apart from a small percentage were capable of measuring to the nearest millimetre but a greater percentage had trouble with the protractor and had to be shown how to use it, starting from the positioning of it and then which of the two scales to read from.

The velocity of sound revived a lot of interest because the experiment was unusual, involving a trip to the school playing field with two dustbin lids which were subsequently banged together every second whilst the pupils observed from some considerable distance away. At approximately 340 metres away from the source of sound, the bang from the first impact of the two dustbin lids just arrived as the second impact occurred, so it had taken 1 second for the sound to travel this distance.

Further calculation-type questions were set and performed reasonably well. However if instead of asking for the velocity
(v) from the equation $V = f\lambda$ or velocity = \frac{\text{distance travelled}}{\text{time taken}}

one of the other quantities was required the manipulation of the equation caused considerable problem.

Construction questions were carried out very badly. Starting with confusion over what scale to use, to poor setting out, to careless untidy work, to the mistakes made during calculating the scaled value. The standards these pupils were willing to accept was very low and must have been tolerated by their teachers. With some coercion and personal attention their standards improved, however it was very time consuming and a teacher attempting to complete a set syllabus would not have sufficient time, also the CSE Board appear to mark knowledge of subject rather than neatness or correct English.

Reciprocals and sines proved beyond the limits of nearly everyone. They appeared not to have dealt with this topic and were not familiar with the use of the "log tables".

The wavelengths associated with the visible spectrum, such as $3 \times 10^{-7}$ m were difficult to appreciate and the majority of the pupils could not say what this number represented.

The Mathematical Topics Required by this Section on Waves

1 Natural numbers
3 Place values
4 Directed numbers
10 Decimals
11 Fractions
18 Reciprocals
19 Ratios
24 Use of tables for reference
33 Pythagorus
37 Scale drawings and constructions
39 Algebra
40 Simple formula
43 Solution of equations
45 Trigonometry - sines, cosines, tangents
50 SI units
52 Speed formula
SECTION 11

Syllabus: Magnetism

Magnetic and non-magnetic materials, properties of magnets. Use of iron filings and compass needle in plotting magnetic fields. Neutral points, methods of magnetisation and demagnetisation. Temporary magnetism in iron and hence use as core of electromagnet. Permanent magnetism in steel and use in permanent magnets. Simple molecular theory of magnetism

No mathematical ability is required for this section, but the symmetrical nature of magnetic fields can be shown
SECTION 12 (a)

Syllabus: Static Electricity

Charging by friction; positive and negative charges, attraction and repulsion; conductors and insulators. Free electrons; simple atomic structure, proton, neutron and electron.
Explanations of electrostatic phenomena may be given in terms of electrons or of charges

Practical Work

1 Rubbing rods of glass, ebonite, polythene and cellulose acetate and picking bits of paper

2 Attraction and repulsion of charged rods

3 Electroscope to identify charge; to test insulating property of a material

4 Van de Graaff generator experiments (demonstration only by the teacher)

No mathematical ability is required for this section
SECTION 12 (b)  ELECTRICITY

Syllabus: Current Electricity

Electric current as a rate of flow of electrons. E.m.f producing orderly movement of electrons; difference between P.d and e.m.f. Use of ammeter and voltmeter to measure current and p.d. current; resistors in series and parallel Ampere, volt and Ohm

Practical Work

Conductors and insulators
Series and parallel circuits
Use of ammeter and voltmeter
Resistors in series and parallel
Measuring potential drop along a resistance wire
Measuring resistance of a wire using ammeter and voltmeter
Measuring electrical power of a lamp
Current: voltage graphs
Electrical fusing and colour coding

Electromagnetics

Magnetic field due to a circuit in a straight wire, flat circular coil and solenoid. Force on a current carrying conductor in a magnetic field to include turning effect on a coil and its application to a simple D.C. motor, moving coil galvanometer
Effects of moving a magnet in or near a coil. Simple qualitative investigation of the factors governing the size and direction of the induced current. Production of an alternating current. Primary and secondary coils. Simple examples of the use of electromagnetic induction.

Practical Work

1 Finding the shapes of the fields due to
   (i) a straight wire carrying current
   (ii) a short coil
   (iii) a long coil

2 Force on a conductor carrying current
   (i) Barlow's wheel
   (ii) Kicking of wire

3 D.C. motor principle

4 Faraday and Lenzes law practical demonstration to investigate the factors governing the size and direction of the induced current

5 Demonstration of transformer's principle

Electrical Cells

Examples of chemical action producing an electric current. The construction of a dry Leclanche cell. Use of cells in series and parallel. Use and care of a secondary cell.
This Involved Covering the Following

(a) Current Electricity

This section started with a description of an electric current as a rate of flow of electrons and definitions of alternating current, direct current, electromotive force and potential difference. Followed by an experiment to show Ohm's Law 

\[ I = \frac{V}{R} \text{ (at constant temperature) or } V = IR \]

This formula was presented in the triangular form for ease in transposing into \( R = \frac{V}{I} \) and \( I = \frac{V}{R} \).

In carrying out electrical experiments units and prefixes must be learnt, for instance

<table>
<thead>
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<td>micro ((\mu))</td>
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<td>mega (M)</td>
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Experiments continued with resistances in series or parallel and equations to remember were:

For series resistances:

\[ V_{\text{TOTAL}} = V_1 + V_2 + V_3 \]
\[ R_{\text{TOTAL}} = R_1 + R_2 + R_3 \]

For parallel resistances

\[ I_{\text{TOTAL}} = I_1 + I_2 + I_3 \]
\[ R_{\text{TOTAL}} = \frac{R_1R_2R_3}{R_1R_2 + R_1R_3 + R_2R_3} \]

or \( \frac{1}{R_{\text{TOTAL}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \)
Power in an electrical current and consumption of power the kilowatt hour involved the formulae:

\[ \text{Power in watts} = \text{volts} \times \text{current} = VI = I^2R = \frac{V^2}{R} \]

and \( \text{Energy consumed} = \text{power (kw)} \times \text{time (hours)} \)

This was also proven by experiment with a 6v, 12w bulb in a circuit, by measuring voltage across the bulb and current passing through it.

The pupils were shown how to read a domestic electric metre.

![First reading 1386](image1)

![Second reading 2586](image2)

Units used 2586 - 1386 = 1200 kwh

Some typical questions follow

1. A 2 volt accumulator is connected to a wire of resistance 20Ω. What current flows in the circuit? What resistance would give twice the current flow with the same accumulator?

2. A potential difference of 4 v is applied to two resistors of 6Ω and 2Ω connected in series. Calculate
   (a) the combined resistance
   (b) the current flowing
   (c) the potential difference across the 6Ω

3. A potential difference of 6 v is applied to two resistors, one 3Ω and the other 6Ω. Calculate the combined resistance if connected in parallel, the current flowing in the main circuit and the current in the 3Ω resistor
4 For the circuit shown calculate
(a) the total resistance of the circuit
(b) the total current in the circuit
(c) the voltage lost across the parallel resistances
(d) the current in the 3Ω resistance

5 A 2.25 kw electric fire is used for 10.5 hours. What is the cost at 4.25p per kwh?

6 Convert 10 mA to amps
100 μV to volts
0.1 amps to milliamps
1755 watts to kilowatts

Problems Noted for the Current Electricity Part of this Section

This section was completed well into the pupils 5th year at school and at a stage when their mathematics course had completed all the basic core syllabus and therefore all the mathematics required by this section should have been covered. This may have been the case but it certainly was not apparent in classwork. As the Aspects of Secondary Education Report stated "their grasp of the basic principles and ability to retain knowledge and draw on skills other than those immediately practiced is poor" and in the author's experience this retention time was very small and for the lower ability range in the survey their knowledge could not be applied outside the environment of the mathematics classroom.

This work involved many calculations and graphical constructions involving linear and nonlinear graphs. The pupils had great difficulty in following the circuit diagrams in order to set
up the experiment, interest of the lower ability pupils was lost and lessons were often interrupted by discipline problems. At this stage some 90% of the pupils had no motivation, they had decided that the mathematics was too difficult for them.

To indicate the level of motivation or type of pupils involved in CSE courses, three pupils decided that even though the school was prepared to enter them for the examination they themselves did not want to make the effort and would leave school prior to the examination. This is not uncommon behaviour and has occurred regularly in the four different schools in which the author has taught. A small percentage allowed themselves to be entered but had little or no intention of turning up for the examination however it was the school's policy, any pupil finishing a course should be entered for the CSE.

The graphical work was improving but the majority of pupils did not realise why they were drawing graphs or see any relationship from the results. There was still a lot of careless work in plotting co-ordinates due to the use of inappropriate scales and invariably the title of each axis and units were missing even though they had been constantly reminded. The CSE practical examination involves drawing two graphs of information found by experiment.

Division continued to be a problem especially when dividing by a larger number or involving decimal places, as is often found when dealing with units of micro--, milli-- or kilo--found in questions involving Ohm's Law.

Some pupils had difficulty reading the meters and were unable to work out values to 1 decimal place where each graduation represented 0.2 of a unit or the meter reading had to be multiplied by some factor. On a gauge, whose face was graduated as shown only 6 pupils could say that the intermediate divisions represented 0.02 and most quoted the reading as 6.6.
Problems involving parallel resistors involved formula and fractions and without doubt proved one of the most difficult for the pupils.

A frequent error was of the kind \( \frac{1}{\frac{1}{R_T}} = \frac{1}{5} + \frac{1}{5} = \frac{2}{10} \) and many failed to invert the answer, quoting \( R_T = \frac{2}{10} \).

Even greater problems were found when dealing with resistances which were fractions for instance \( \frac{1}{\frac{1}{R_T}} = \frac{1}{4} + \frac{1}{2} \) (\( R_1 = 4\Omega \) and \( R_2 = 2\Omega \)).

Some answers resulted as \( \frac{1}{\frac{1}{R_T}} = \frac{1}{2\frac{1}{4}} \) and were left because the pupil did not know what to do next.

Similar difficulties were experienced with the power formulae Power = \( I^2R \) or \( \frac{V^2}{R} \) many simply multiplied the value by two instead of squaring it.

This section as a whole emphasised the deficiencies in basic number work skills acknowledge by the School Inspectors, Industry and the public as fact. Many pupils had expressed their opinions on this section and decided not to attempt these questions in the examination. In the author's experience even GCE 'O' level candidates have trouble with the mathematics associated with this section and is obviously a cause for concern.

(b) Electromagnetics

This topic was dealt with by experimental demonstration of magnetic fields and the force on current carrying conductors involving no mathematics apart from the section on the induction of current in the transformer. This is a device for changing the voltage level of an alternating current supply. The voltage levels depend on the ratio of the number of turns in the coils of the transformer, that is,

\[
\text{voltage output} = \frac{\text{number of turns on the output coil}}{\text{number of turns on the input coil}}
\]
Some Typical Questions

1 A step-up transformer has a turns ratio of 4. If the input voltage is 10 v a.c. what is the output voltage? If there are 480 turns on the output coil how many turns are there on the input?

2 A stepdown transformer is required to transform 240 v a.c. to 12 v a.c. for a model train. If the primary coil has 1000 turns, how many turns should the secondary have?

Problems Involved in this Section on Electromagnetism

The mathematics associated with transformers is usually restricted to whole numbers and usually multiples of ten so the pupils' only real problem was to remember the formula and how to transpose it.

(c) Electric Cells

Very little mathematics was required for this section

Mathematics Topics Required for the Section on Electricity

1 Natural numbers
2 Number system
3 Place values
4 Directed numbers
6 Factors
7 Multiples
8 Powers and roots
9 Irrational numbers
10 Decimals
11 Fractions
13 Percentages
15 Reading/interpolating scales
16 Concept of rate of change
17 Index notation
18 Reciprocals
19 Ratios
20 Proportions
22 Approximations
23 Estimation/rough checking
25 Squares and roots
39 Algebra
40 Simple formula
43 Solution of equations
46 Graphs - co-ordinates
47 Linear - non-linear
48 Representing data in graphs
50 SI units
THE OPTIONS

The CSE course gives the school/pupils a choice of topics for an in-depth study. From the topics listed below three have to be covered and the examination will consist of one question on each topic, which have to be answered in 1 hour.

Topics

A  Physics of the weather
B  Colour
C  Applied Mechanics
D  Optical instruments
E  Physics of flight
F  Sound
G  Astronomy
H  Radioactivity
I  Electronics
J  Current and voltage measuring instruments
K  Generation of and distribution of electric power
L  Physics of the motor car/cycle
M  Biophysics

Very little mathematics appear in the following:

Section A  The physics of the weather and its associated problem for the pupils has been dealt with in Section 7 (pressure) where the barometer and the units of pressure have been covered

Section D  Optical Instruments

Section E  Physics of Flight

Section F  Sound  There is no further mathematics than has already been covered in the basic core
Section G  Astronomy  The only requirement is the appreciation of large distances and periodic times

Section I  Electronics  The only mathematics is that associated with the transformer and simple Ohms Law questions

Section L  Physics of the Motor Car  This includes a simple ratio of gear teeth treatment

Section M  Biophysics involves a simple treatment of human levers

The following topics have a greater mathematics content and have been covered in more detail, by illustrating the type of problems where mathematics is required.

Section C  Applied Mechanics
This includes vector addition and resolution of forces by graphical methods, for instances: Find the resultant force

This involves the construction of a triangle of force or parallelogram of force

Question on addition of vectors: A pilot aims his plane due east at 250 m/s while the wind is blowing to the north at 20 m/s. Find the resultant velocity.

This is followed by beams (reactions at the supports)
Solution  Weight of the beams = length of beam \times weight/unit length

= 30 \times 10

= 300 \text{ g}

This acts at the centre of gravity as shown

\[ \text{A} \quad \downarrow_{100\text{gf}} \quad \text{B} \]

\[ \text{RA} \quad 15\text{cm} \quad \text{RB} \quad 15\text{cm} \]

Take moments about A: clockwise moments = anticlockwise moments

\[ 300 \times 15 = R_B \times 30 \]

\[ \frac{300 \times 15}{30} = R_B = 150 \text{ gf} \]

Equate forces: upwards = downwards

\[ R_A + R_B = 300 \]

\[ R_A + 150 = 300 \]

\[ R_A = 150 \text{ gf} \]

These problems were made more difficult by the addition of weights acting at points along the beam, either acting perpendicular or at an angle or having beams of composite cross-section or involving cantilevers etc.

This was then followed by the study of Machines: and included an extension of mechanical force dealt with in section 6. It included a study of pulley systems and simple machines from which the formula

Mechanical advantage = \frac{\text{load}}{\text{effort}}

Velocity ratio = \frac{\text{distance travelled by the effort}}{\text{distance travelled by the load}}

Efficiency = \frac{\text{mechanical advantage}}{\text{velocity ratio}} \times 100\% \text{ OR } \frac{\text{work output}}{\text{work input}} \times 100\%

and Work done = \text{Force} \times \text{distance moved through by the force in the direction of application of the force}
This section was illustrated by experiments with pulleys, incline planes and screw threads. The main experiment was to show the relationship between load and mechanical advantage, and load and efficiency, followed by discussion about the shape of the curves obtained.

In connection with this experiment, the following points should be noticed:

1. The useless load consists of the weight of the lower pulley block and the string and friction in the string and bearings. The weight of string lifted depends on the distance between the pulley blocks, but the weight of the lower block is constant. The friction varies with the load, but is small in most cases. Thus, although the useless load varies somewhat, it becomes a smaller proportion of the total load as the total load increases.

2. The efficiency also increases with load for the same reasons.

3. Owing to the work wasted in overcoming friction and raising moving parts the efficiency is less than 100 per cent. Also, since there are only four pulleys altogether, the mechanical advantage cannot exceed 4.
Questions

1 If an effort of 100 N is needed to lift a load of 200 N with the system shown, what is the efficiency?

2 Draw a diagram of a single string pulley system with a velocity ratio of 6. Calculate the efficiency if the effort of 100 N is required to raise a load of 4500 N. Find the energy wasted when a mass of 500 kg is lifted through 2 metres. Take g = 10 m/s²

3 A boy is pulling a log using a rope at 20° to the horizontal and exerting a force of 100 N. Find the horizontal and vertical components of the force exerted by the rope on the log

4 The diagram shows a trolley of weight 300 N being pulled at a steady speed up a ramp by a force of 200 N.

Calculate

(a) the mechanical advantage
(b) the velocity ratio
(c) the work done on the load
(d) the work done by the effort
(e) the efficiency of the machine

5 A man uses a crowbar 1.5 metre long to lift a rock weighing 600 Newton. If the fulcrum is 0.50 metre from the end of the bar touching the rock, how much effort must the man apply? (Draw a diagram first)
6 A gear wheel A with 20 teeth is used to drive a gear wheel B with 60 teeth. What is the velocity rate? If wheel A rotates three times every second, how many times does wheel B rotate in each second?

7 Look at these diagrams of pulley systems. For each one calculate
   (a) the MA
   (b) the VR
   (c) the work done in lifting the load 1 metre
   (d) the work done by the effort in this case
   (e) the % efficiency

The theory continued with the hydraulic press, jack and brake systems for which the mathematics is included in section 7 on pressure.

This was followed by power as the rate of doing work, which was also dealt with earlier but at this stage the problems are somewhat more difficult. For example: A crane lifts a load weighing 3000 N through a height of 5 m in 10 seconds. What is the power of the crane? What would be the effect on these values if the motor on the crane was 80% efficient?

Problems Experienced by the Pupils on this Section

The CSE Board's summary indicates that every year this section is the least popular and results in the poorest efforts as the questions are usually totally mathematical.

In the author's experience this topic is rarely chosen by teachers as an option, even though it is compatible with a
GCE 'O' level syllabus and would be highly suitable for compromise courses - O level or CSE, as it causes too many problems of a mathematical nature and even O level ability children find it difficult because they are unfamiliar with the application of mathematics.

The pupils cannot apply the mathematics they are taught to any situation outside the mathematics classroom and find reading a question, sifting through for relevant information very difficult. This level of pupil cannot plan or logically work out a problem of more than one stage.

Many still cannot accurately use a protractor and have to be shown how to read it. The accuracy and neatness of construction work is very poor. Nobody in the survey was able to use two set squares in a technique to draw a series of parallel lines, a procedure used in the polygon of forces. Many had not used a set square and nobody actually had one as part of their equipment requirements. The knowledge of simple geometry was very low in all the pupils surveyed.

The beam problems produced reasonable results, as long as the questions were repeats of the worked examples. However if it were phrased slightly differently or involved the principle applied to some different situation only 10% were able to complete the problem. The pupils had problems with understanding of the fundamentals and planning strategies.

The problems involving machines and pulleys proved overly difficult for many included a large proportion of the future 'O level group'. This section proves to be the most unpopular on both 'O' level and CSE examination papers.

Section H Radioactivity

The mathematical emphasis is placed on the random nature of radioactive emissions and the definition of radioactive half-life. This is done either experimentally with Thoron gas or
by probability with dice in which one number is associated with an atom decaying. Typical graphs are shown:

For this experiment you need a large number of wooden cubes or dice, each with one face painted red. Image each cube is an atom.

If you shake and throw all the dice-atoms, some will land with the marked face upwards.

Let us pretend that these are atoms which have disintegrated and fired out α-particles or β-particles and γ-rays.

Remove these disintegrated 'atoms' and count how many atoms have survived.

Real atoms behave in a similar way: each atom disintegrates in a random, unpredictable way. A large number of atoms gives a smooth radioactive decay curve.

The graph shows a decay curve for a radioactive substance. On the graph you can see that after 1 half-life, half the atoms have disintegrated and half have survived.

The half-life of different substances varies widely - from fractions of a second up to millions of years.
Calculations often appear on half-life:

Eg.
The radioactive isotope $^{25}_{11}$Na has a half-life of 1 minute. What fraction of $^{25}_{11}$Na remains after 3 minutes?

Section J Current and Voltage Measurement

This is mainly descriptive but the conversion of galvanometers into ammeters or voltmeters by the use of shunts and multipliers respectively requires calculation.

Eg. Suppose a galvanometer has a resistance of 100Ω and gives a full scale deflection reading when a current of 1 mA passes. To convert this meter to give a full scale deflection (FSD) for 2 mA.

We must bypass (shunt) the metre with 1 mA of current to avoid damaging the meter and so it will read FSD when 2 mA flows.

From the theory on resistances in parallel we know that in this case each path for the current to flow down must have the same resistance in order for the current to split into equal proportions. Hence R must be 100Ω and the readings on the meter are multiplied by 2.

For 5 mA maximum current, 4 mA must pass through the shunt and 1 mA through the meter so R must be $\frac{1}{4}$ of the resistance of the meter and so on.

When converting the meter to a voltmeter a resistance is placed in SERIES as shown:
When used to measure a voltage of 1 volt, the multiplier is used to cause a voltage drop of 0.9 v. Hence from Ohm's Law $R$ must be 900$\Omega$ and the scale on the meter now reads values up to a maximum of 1 volt.

Section K  Generation and Distribution of Electrical Power

The mathematics in this section is an extension of that done previously in section 12 on the transformer.

Questions included:

1  Find the power wasted as internal energy in the cable when 10 kw is transmitted through a cable of resistance of 0.5$\Omega$ (a) at 200 v  (b) 2 x 10$^5$ v

Solution:

(a) \[\text{watts} = VI\]
\[10000 = 200I\]
\[I = 50 \text{ Amps}\]

(b) \[\text{watts} = VI\]
\[1 \times 10^4 = 2 \times 10^5 I\]
\[I = 0.05 \text{ Amps}\]

In cable:

Power lost \(= I^2 R\)
\[= 50^2 \times 0.5\]
\[= 1250 \text{ watts}\]
\[= 1.25 \text{ kw}\]

Power lost \(= I R\)
\[= 0.05^2 \times 0.5\]
\[= 0.00125 \text{ watts}\]

From this question it can be seen that at a voltage level of 200 v some 12.5\% of the energy is lost in transmitting the current and at 200,000 v only a very insignificant energy loss occurs.
2 Transformer efficiency: A step-up transformer is designed to operate from a 20 v supply and deliver 250 v. If the transformer is 90% efficient, determine the current in the primary windings when the output terminals are connected to a 250 v 1000 w lamp.

Solution:

(Current in secondary) \( w = VI \)

1000 = 250 I

I = 4 Amps

Efficiency 90% = \( \frac{\text{output power}}{\text{input power}} \times 100 \)

0.9 = \( \frac{1000}{P_I} \)

\( P_I = 1111.1 \) watts

So for primary windings

\( w = VI \)

1111.1 = 20 I

I = 55.6 Amps

Problems Experienced by the Pupils

No additional problems became apparent during the option topics. The main choices of the pupils tended to avoid the mathematical topics and were the physics of the weather, optical instruments and colour.
The author noted that the overall ability of the pupils in performing the mathematics involved had improved in certain areas, such as graphical work-co-ordinates but the overall level of the number work and other areas had shown little or no improvement and these pupils were at the end of their school careers.

Mathematics Topics Required by the Options

1 Natural numbers
2 Number system
3 Place values
4 Directed numbers
5 Factors
6 Multiples
7 Powers and roots
8 Irrational numbers
9 Decimals
10 Fractions
11 Percentages
12 Concept of rate of change
13 Index notation/standard form
14 Use of reference tables
15 Squares and roots
16 Areas
17 Volumes
18 Triangles
19 Pythagorus
20 The circle
21 Scale drawing/constructions
22 Algebra
23 Simple formula
24 Indices
Analysis of the Survey Results

Throughout the survey it appeared fairly clear to the author that attention was needed in the following areas:

1. Number work
2. Fractions and percentages
3. Graphs - of tabulated data
4. Estimating/rough checks/rounding off
5. Measurement
6. Simple formula - substitution into and solution of
7. Algebra
8. Geometry and constructions
9. Volume and areas
10. Angles (trigonometry)

Tests carried out confirmed this need for attention and indicated the size of the problem. These tests were carried out at various stages throughout the final two years of the pupils' school life. A test was carried out on 45 school leavers from throughout Derbyshire, all of whom were applying for jobs in the printing industry and were tested by Derby Lonsdale College of Higher Education on behalf of the industry. The results have been analysed by the author for all the questions relevant to the mathematics needs of a physics CSE course and the success rates tabulated. This confirms the problems with mathematical education and also
the findings of the author in studying the performance and problems occurring for some 73 pupils in taking the Midland Region CSE Physics course. This indicated that the pupils in the survey were by no means unusual and fairly typical of pupils in comprehensive schools throughout the country.

### CSE Results of Pupils in Survey

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<td>Physics no.</td>
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FREQUENCY OF CSE GRADE FOR MATHEMATICS AND PHYSICS

Mathematics Results of the Whole School

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<td>13</td>
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<td>14</td>
<td>48</td>
<td>63</td>
<td>42</td>
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OVERALL TOTAL 297 PUPILS

U = ungraded
N = not entered
CSE Results of Sample Group Compared with County Averages

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<thead>
<tr>
<th></th>
<th>Sample pupils</th>
<th>County averages</th>
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<tr>
<td>CSE grade 1</td>
<td>10%</td>
<td>10%</td>
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<tr>
<td>CSE grade 1–4</td>
<td>75%</td>
<td>79%</td>
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# Frequency of Required Mathematics Topics for the 13 Sections of the Physics Syllabus

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<thead>
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<th>Mathematics Topic No.</th>
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<td>4</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

| Mathematics Topic No. | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 |
|-----------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Frequency             | 0  | 4  | 2  | 0  | 2  | 2  | 0  | 0  | 3  | 0  | 10 | 11 | 1  | 0  | 11 | 0  | 3  | 7  | 7  | 5  | 2  | 11 | 1  | 3  | 0  |

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<th>56</th>
<th>57</th>
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Summary
Analysis of Survey Results

Throughout the survey it appeared fairly clear to the author that attention was needed in the following areas: hence much of the revision work and booklets concentrated on these topics:

1. Number work
2. Fractions and percentages
3. Graphs - of tabulated data
4. Estimating/rough checks/rounding off
5. Measurement
6. Simple formula - subsitution into and solution of
7. Algebra
8. Geometry and constructions
9. Volume and areas
10. Angles (trigonometry)

Physics is a practical type of course, where fundamental principles are verified by experiment with subsequent tabulation of results, constructions, graphical work and calculation. All the areas above are essential mathematical needs in order for the course to flow smoothly and aid its completion within very tight time limits. The survey has shown that many pupils attempting such courses are placed at a disadvantage due to not being able to cope with the relevant mathematics with any degree of proficiency. This results in physics or science teachers having to teach the required mathematics in conjunction with the science syllabus.

Tests of either a general mathematics nature or related to specific mathematics topics have been carried out throughout the course in order to confirm the depth of this problem and helped the author to plan a scheme of work to try to overcome it. The test papers and results have been included (Appendix II).

In order to assess the overall ability of the pupils in the survey against a wider cross-section of school pupils' ability, the author analysed the results of a test carried out on behalf
of the local Printing Industry in which 45 future school leavers wanted employment. The courses these pupils had taken varied from GCE to CSE courses, and the test paper was one devised by a national organisation. This test paper is included (see Appendix V) with an analysis of the results, giving percentage success rates of each question.

This analysis reveals similar problem areas to those found by the author with his survey pupils and shows that these pupils were by no means unusual, being fairly typical of those in comprehensive schools throughout the country.
CHAPTER 5

Cognitive Testing of 12 Year Olds

The author decided to use an established cognitive test, previously used by Chelsea College's CSMS Study (Concepts in Secondary Mathematics and Science) so that the results of the tests on 132 eleven year olds could be compared with the CSMS published figures of Brown and Küchemann, and from this evidence to see if work of an applied nature would aid the motivation and understanding of older children by carrying out a series of mathematically integrated experiments and work of a related nature.

The aims of the tests were to investigate the depth of understanding of mathematical concepts rather than manipulative skills; do the pupils know for themselves which skill to apply or have they to be guided in the right direction; and to compare the results of children who have had differing backgrounds of mathematical teaching, one highly practical and applied, and the other "non practical" traditional in nature.

These tests were carried out on eleven year olds because there were sufficient numbers who had undergone the two modes of teaching without overlapping into either mode. The pupils were told that the school was interested in how children do problems and they were given three tests:

1. Titled "How would you solve this problem" (included in Appendix VIII). There were 9 questions, on three conceptual areas ÷, × and ±; in which a short story was told and a mathematical question asked. Each question gave eight possible methods of solution and the child had to choose the correct solution.

2. A reading age test to gain equality of pupils in the test groups (App VIII)
3  Titled "Write a short story to fit the mathematics". (App VIII)
Here there were 5 questions designed to judge whether
the child has a basic understanding of operations, and
whether the child has the confidence and ability to
express in English mathematical problems and vice versa,
which is important to other curricula areas.

The marking of the papers was carried out according to the
Marking Instructions of Activity 20 of the Open University
Project Manual 3 for the course on "Cognitive Development:
Language and Thinking from Birth to Adolescence".

The results of the tests were as follows:

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<th>Type</th>
<th>% correct answers</th>
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<tr>
<td>1</td>
<td>±</td>
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<td>2</td>
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<td>D</td>
<td>×</td>
<td>40</td>
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<tr>
<td>E</td>
<td>×</td>
<td>19</td>
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</table>
The practically taught group showed a greater all-round understanding of mathematical concepts of \( \times \) and \( \div \) than did either of the CSMS study children or the "non-practically taught" children. The latter group did not fare as well as the CSMS children, being approximately 10% below on each occasion, except question 7 on which they compared. This can be explained by possibly the fact that the CSMS study included both practically taught and non-practically taught to give results for an "average British population".

The non-practical mathematics group had problems with the concepts of \( \div \) and \( \times \), many were not able to recognise from the wording of the problem what particular skill was required.

The practically taught mathematics group showed on the whole, equal to or better results than the CSMS study figures. This was encouraging, particularly as the concept of division and multiplication seemed well understood by the large majority.

Questions A to E, where problems had to be expressed in words, showed quite clearly that the pupils with a practical background were much more able, appearing to understand the concepts as well as having manipulative skills. Both sets of children fared similarly when asked to do purely manipulative number work, showing the deficiency with the non-practically taught was basically that of understanding alone.

The practical approach to mathematics seemed to have a beneficial effect on the understanding behind the processes the children were taught, and this is essential to all teachers regardless of curriculum area.

The sample of pupils who had been taught a practical course of mathematics (SMP 7-13) received this at a junior school where the Headmaster, Mr Brighouse, is one of the authors of the Mathematics Scheme, and he had gathered a team of teachers together who were convinced that mathematics teaching had to be applied and practical/experimental in nature for the good
educational interests of the children. Mr Brighouse's staff were dedicated to the teaching of the SMP scheme and did so in an enthusiastic and interesting way, which seems to be a key to its success. Another salient point is that Mr Brighouse was always available for advice regarding the best way to approach or apply topics and to overcome non-mathematics specialists' worries on conceptual understanding and the interconnections between these concepts.

Practical based courses are doomed to failure if approached in a uninteresting or unimaginative way. The pupils must be stimulated or motivated to achieve the required understanding. The problem is not solved by a change of syllabus or the involvement of more practical topics, or the linking of problems to the outside world, it appears more deep-rooted. Teachers must place their emphasis on conceptual learning not just "algorithmic learning" and make lessons lively and interesting.

These tests showed that many children have difficulty with basic operations, that is, the process and concepts behind multiplication and division, the understanding of symbols and the translation of "wordy" questions into mathematical operations.

An example of a wrongly taught concept is as follows:

What does \( \frac{3}{10} \) mean: from homework marks the answer is 3 out of 10.

So, when the problem is \( \frac{3}{10} + \frac{5}{10} \) answer \( \frac{8}{20} \).

Explanation 3 out of 10 and 5 out of 10 is 8 out of 20.

Logical, but wrongly taught concept.
It has been the author's experience, in his science teaching, that if a pupil is given a "wordy" mathematics question that the pupil does not understand, he or she will seek out the number and guess at the process required to get an answer.

An example of this occurred in a science lesson which was to lead into the topic of density (mass/volume). The pupils had spent practical lessons using lever balances to measure different masses and on calculating or measuring volumes of regular and irregular shapes.

The lesson started with a discussion of the statement "Steel is heavier than feathers" and progressed to the understanding that if we are to compare the masses of two substances fairly and to make a correct statement then we must compare equal volumes of the substances.

The question on what particular volume we should use to compare brought a mixture of both imperial and metric, but eventually the pupils decided on the volume of 1 cm$^3$ or 1 m$^3$.

When given the problem about a lump of wood, we measured its mass on a lever balanced, it was 150 grammes and by displacement found its volume to be 300 cm$^3$.

Information: 300 cm of wood has a mass of 150 g

Question: What is the mass of 1 cm$^3$ of wood?

Answer: Many said immediately it cannot be done as 300 is bigger than 150. Some were unsure and guessed at 2 because in their minds the only problem involving division that these two numbers could be involved in was $300 \div 150 = 2$. Others offered no answer. In a class of 30 mixed ability pupils of eleven/twelve years old, nobody could answer the question.
When reminded about fractions, ie one cake to be shared by 4 people was possible and what would they do, some said "cut it up".

Question: How do we divide 300 into 150?

Answer: They were now thinking of fractions but still puzzled as how to do it. Some guessed at $\frac{1}{5}$ because 3 went into 15, 5 times. Some eventually said $\frac{1}{2}$ but most were totally unsure of the division process for getting a decimal answer, or even what the decimal column headings were.

This highlighted the problem of translating from words to mathematics, the lack of understanding of some basic concepts and that pupils sometimes guess from their impression of the numbers what the mathematical process is.

In discussion with these pupils regarding how they solved a problem, it became apparent that those with correct answers had done so by several different ways and each pupil was fairly convinced that his or her approach was the best way and perhaps this was so for them. Thus for this group a formal process of manipulative skill or standard proof would not necessarily be satisfactory in trying to improve their logical understanding. A more flexible teaching approach was required, where strategies for gaining solutions to problems are discussed.
CHAPTER 6

Development and Testing of an Integrated Form of Mathematics and Physics

The basic idea of integrating the mathematics into the science syllabus was to investigate the feasibility of running a syllabus which had no predetermined boundaries which isolate one subject from another. To determine if in this way mathematics could be made more appealing to those who have an innate dislike for the subject, who at the mere mention of the word "Maths" show verbally, with grunts and groans, their feelings of discontent, and to cover the mathematical requirements of physics at each stage of the syllabus, in order that pupils can feel at ease, enabling them to enjoy learning the fundamentals of physics in a new and different atmosphere where confidence is boosted by lots of discussion and no assumptions are made as to what they should already know.

Prior to planning that approach, the analysis of the CSE Physics Syllabus, and the mathematical test carried out on both the previous sample of CSE pupils and pupils about to start the course were used to determine the depth of revision or teaching required at each stage.

The tests included those developed by the Institute of Mathematics and its applications, the CSMS study group including ideas from work done by Piaget, and also those generated by the author (Appendix II), which take into account recent research work and are related to the skills needed to successfully perform the mathematics associated with the course.

Items testing different levels of ability were written into the tests using phrasing that would be used in both a mathematics classroom and the outside world. No time limits were given to the tests apart from being finished in a 'double lesson' of 1 hour and 10 minutes, however no test was designed to occupy anywhere near this length of time. The information required
was that of 'level of understanding', not how well these pupils could cope under examination conditions working against the clock.

Furthermore, discussions were held with certain pupils who had experienced difficulties or perhaps solved a problem by an unusual method and their comments recorded (Appendix V: Pupils' comments).

This information indicated that the general level of basic numeracy was not as poor and of such an unacceptable standard as suggested in Chapter One but still required some considerable attention. The ability range of each group was very wide and certain groups of pupils had suffered more than others in the education system. It was not uncommon for certain groups to have had as many as six different teachers over a period of two years, most of whom were not qualified mathematicians, and from discussions it appeared that no set core of content had been covered, so the author decided on a short introduction to revise some basic mathematics and this was based on class participation and blackboard work rather than extensive problem solving exercises written in their exercise books.

However, a printed booklet (Appendix VI) was given out which included examples of all the operations/skills required and this was added to as we progressed through the course.

This introductory course involved the following:
(1) **Decimal number system**

An explanation of the decimal number system, covering column headings in an endeavour to overcome the misconception that the columns are units, tens, hundreds, thousands and millions.

eg: A decimal number

\[
\begin{array}{cccc}
1 & 8 & 2 & 5 \\
\text{TENS (10)} & \text{HUNDREDS (100)} & \text{THOUSANDS (1000)} & \text{TEN THOUSANDS (10,000)} \\
\end{array}
\]

So \(10 \frac{4}{10}\) could be written 18.4

or \(15 \frac{1}{100}\) 15.01

or \(516 \frac{541}{1000}\) 516.541

(2) **Addition of whole numbers and decimals**

Stress was placed on setting out, neatly in the correct columns, how and why we 'carry' and the need for checking answers (as the majority of problems are due to untidy careless work)
(3) **Subtraction of whole numbers and decimals**

Briefly covering the method of 'borrowing' and the setting out of the problems with the decimal points in line.

(4) **Multiplication of whole numbers and decimals**

This was hampered by the lack of knowledge of the 'times tables. The work covered was multiplying by decimal numbers and the subsequent positioning of the decimal point in the answers, and the multiplying by powers of ten.

(5) **Division of whole numbers and decimals**

Division had previously proved to be the area requiring the greatest attention. Emphasis being placed on the performing of LONG DIVISION, what to do if there was a remainder and the division by a decimal number.

When dividing by very big numbers a form of approximation was covered.

It was noticed that these pupils found it difficult to retain their knowledge for any appreciable length of time and constant revision was needed.

The time allocation for this basic numeracy was approximately 3 hours, which was felt to be fairly brief, but no further time could be afforded at the expense of physics. However, a booklet was designed and written, to be given to each pupil, with examples of all the aspects of basic numeracy and mathematical requirements to be used as revision notes. As new topics are covered further sheets were written for inclusion into the booklet.

After this initial mathematics revision, any required knowledge of mathematics would be integrated into the course and involve practice which would include both mathematical and scientific knowledge acquisition. It is necessary to avoid a situation where the lessons appear to be mathematics lessons which are
carried out in a science laboratory. The emphasis would be placed on practical work.

**Measurement**

This topic is the start of the physics syllabus and enables the covering of many mathematical topics simultaneously. The emphasis is placed on accuracy of measurement and clearly stating of UNITS with each measurement.

The correct way to use a ruler, and the avoidance of parallax error.

The construction and the reading of vernier calipers and micrometers which are capable of accurate measurement (±0.001 cm). Practical work involved measuring the diameters of different bars and then calculating the difference in diameters, measuring the lengths of objects both small and large. The pupils were encouraged to estimate measurements and then check by using the appropriate instrument. Small measurements could be performed
accurately with rulers if several of the objects were measured together, for instance: the thickness of a sheet of paper or an approximate method for sizing a molecule of oil.

It was noted that only two pupils from a class of 30 had rulers which measured in both centimetres and inches and that none of these pupils used the imperial system for measuring. Their estimates using both systems were very inaccurate and as indicated in the measurement test, above 80% of the pupils could not measure in inches. This was mainly due to the fact that they could not interpret the fractions such as:

\[
\frac{1}{12}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}.\]

In order to improve their understanding of the imperial system a general discussion was held and an exercise which involved measurements taken in both centimetres and inches and the subsequent plotting of a graph was carried out.

Further measurement work included area and volume, carried out by both calculation and the use of measuring cylinders, and displacement cans for regular and irregular shapes. Again the pupils were shown how the units were derived.

The tests carried out had shown a virtual total inability of the pupils to record data and plot graphs, examples of graphs relating to various experiments, such as Hooke's Law when stretching springs and heating curves for different liquids are included in the Appendix VII.

The graph is an essential means for a scientist to express information gathered during an experiment and it was felt that
such work should be included wherever possible. Several exercises were designed to use measurement and involve graphical work such as:

The pupils were asked to draw up a table of values of lengths in both inches and centimetres. This could be done in several ways, one being to draw a line and measure it, or to give out different objects for measuring. On this occasion fourteen rulers had to be issued to one class and this is not uncommon practice, many pupils find it difficult to provide themselves with equipment other than a pen for writing. To make the graph easier to plot the pupils were advised to measure lines which were exact numbers of inches long. Prior to plotting the graph there was considerable class discussion on how to decide on the scales to be used for each axis, which depended on the maximum readings, the size of the graph paper available and the accuracy required - that is, the larger the scale the more accurate it is to plot the points and read information from the graph.

In physics the concept of 'rate of change' is very important so the method of calculating the slope (or gradient) of the graph and its units was covered, and in the case of the following graph gave a fairly accurate conversion factor between centimetres and inches:

![Graph with labels: Centimetres on the y-axis, Inches on the x-axis, slope formula: \( \text{Slope} (m) = \frac{y}{x} \text{ units} \) \( \frac{cm}{inch} \)]

Following on from the slope, the equation of the line \( y = mx \) was covered and subsequently \( y = mx + c \).
Further practice in measurement and graphical work involved:

(i) Measuring area of squares and plotting against length of side

(ii) Measuring volume of cubes and plotting against length of side

(iii) Measuring the circumference of different cans and plotting against diameter. The slope of such a graph giving the term \( \pi \)

(iv) Heating curves for liquids of differing heat capacities and differing heat inputs. Included at this stage would be the design features of various types of thermometers.

It was stressed that graphs were drawn to show trends, they imparted information to the reader, so some time was spent on the 'language of graphs' and how to interpret them. For instance, when looking at a heating curve, it was easy to see if the flame had an high oxygen content from the slope of the graph, or if the slope changed whilst heating then the oxygen or gas content had been altered or if the graph reached a plateau then the substance was changing phase. Interpolation skills are rarely taught in mathematics lessons, yet these skills are highly prized in science and other disciplines. Whenever we open a newspaper or switch on the television we see information presented in tabular or graphical form. On many occasions the 'language' is misused in order to make the product more appealing or the argument more convincing or to simply blind the observer with science. It is therefore important to know how to interpret the 'language of graphs' that we may meet in order to avoid being mislead and confused by them.
The CSMS (24) tested 459 second year pupils, 755 third year pupils and 584 fourth year pupils in secondary schools on the interpretation of a simple graph relating to a journey.

![Graph](image.png)

Question: Describe what happens in the journey.

The results were 14.7%, 17.2% and 25.2% respectively, able to answer correctly. The journey was often interpreted wrongly as going up and down hills. The graph was seen as a 'picture' of the situation rather than an abstract representation of it.

Many pupils become proficient at reading and plotting points on graphs, but there was a much lower success rate in interpreting graphs which requires understanding of features such as intervals, maxima and minima, rates of change, discontinuities and so on. There is great emphasis, and very necessary, placed on 'accurate skills', such as choosing scales so that the graph will fit the page, plotting the points and joining them up with a smooth curve, reading of isolated values but the overall meaning of the graph and its significance to external features must also be explained. This will necessitate the use of 'real world graphs' in trying to understand that a graph is a form of symbolic language describing a relationship.

Some illustrations of the types of graph used are:

![Graph](image.png)

Question: How many meals were eaten?

Is the 12 on the time axis - noon or midnight?

Why?
The pupils were asked to look more deeply into the graph, for such things as "Was anything eaten at morning break?" or "How long did dinner take?" and "How could we extract this information from the graph?" This was repeated with the experiment on heating of a liquid, in which the heat input was altered from fast to slow or even switched off temporarily.

Graphs of average temperatures against months of the year are also another suitable form of information to interpret.

Question: Which is the most realistic and explain why?
Which could apply to a country south of the equator?

Other graphs used could involve petrol consumption of a car on a long journey in which there were several stops for petrol, heavy town traffic and motorway situations or growth of plants or animals, over a period of time or the variation in pulse over a period of time during which exercise was taken, rest and perhaps sleep or an interesting problem concerning Concorde or very fast aeroplanes in which two graphs are given for interpretation:
Question: How does the skin temperature change as the aeroplane travels faster?

How does the strength of aluminium alloy vary as it gets hotter?

What is the advantage of using titanium alloy?

Concorde is made from aluminium alloy, at what speed is it safe to fly?

There are many other examples of information which would be suitable for interpretation and also of use to science, in trying to achieve a good level of graphical literacy so that the pupils may appreciate the overall value and meaning of this language.

The measurement of weight

This topic started by describing weight as a force created on an object due to the gravitational attraction of the earth on that object. This was followed by considerable discussion on different methods of recording weight and the graduation of their scales. The simplest of devices is the Spring Balance, where the spring extends when a force is applied and the amount of this extension is proportional to the load (or force) applied:

Here, it is necessary to explain the meaning of proportionality, give examples, and relate it to the straight line graph.
Practical work involved plotting graphs of extension against load for a series of different springs and showing the elastic and plastic regions of their operations.

![Graph of weight against extension]

With this graph for a particular spring, which had not been permanently stretched, the pupils were able to find the weights of different objects and answer questions which needed information gained from the graph.

A typical family of spring curves up to the elastic limit is shown:

![Family of spring curves]

The pupils found the gradients of each graph, which gives the weight force per centimetre of extension, this is called the spring stiffness. Clearly the stronger springs have steeper gradients (spring stiffness).
During this topic the opportunity to set calculation problems which would give practice in basic numeracy, graphical work and problem solving techniques was taken.

Whilst using weighing devices, both mechanically and electronically operated, the concept of density and its usefulness to scientists and engineers was covered. This involved recording weights and calculating or experimentally determining the volume of different objects of different materials of construction.

The value of the density could be found by direct substitution into the formula. Density = Mass ÷ Volume - and this was covered in detail. However, to continue with the graphical work the pupils were given several pieces of wood, some regular in shape and other irregular, (so the volume could not be found from calculation,) and they plotted a graph of mass against volume, the slope of which is the DENSITY of the wood. This procedure gives a more accurate answer as it overcomes some of the inherent errors associated with measurement and 'weighing' and is analogous to taking the average value of several readings.

Families of graphs were developed of different materials such as plastic, aluminium, wood, water and iron, and from them information was read which enabled values of either mass or volume to be obtained or even given a value of mass and volume the material could be found.
The formula for density was manipulated to give three forms:

(i) \[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} \]  
\[ \text{ie: } D = \frac{M}{V} \text{ (g/cm}^3) \]

(ii) \[ \text{Mass} = \text{Density} \times \text{Volume} \]  
\[ \text{ie: } M = D \times V \text{ (g)} \]

(iii) \[ \text{Volume} = \frac{\text{Mass}}{\text{Density}} \]  
\[ \text{ie: } V = \frac{M}{D} \text{ which has units: } \frac{g}{g/cm^3} = \frac{g \text{ cm}^3}{g} = \text{ cm}^3 \]

It is necessary to cover the mathematical concepts of forming equations (true statements) and their manipulation with subsequent substitution of values into the equation. This was carried out with scientific equations, many of which the pupils were familiar or had used in the past, and some of which would be required in the future. The depth of treatment was limited by the ability of the group concerned.

Some typical formulae used were:

(i) \[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} \]  
\[ \text{ie: } D = \frac{M}{V} \]

(ii) \[ \text{Speed} = \frac{\text{Distance}}{\text{Time}} \]  
\[ \text{ie: } S = \frac{D}{T} \]

(iii) \[ \text{Speed} = \text{Initial speed} + \text{acceleration} \times \text{time} \]  
\[ \text{ie: } S = U + at, \text{where } U = \text{initial speed}, \text{a = acceleration} \]

(iv) \[ \text{Area} = \pi r^2 \]

(v) \[ \text{Volume} = \text{Length} \times \text{breadth} \times \text{height} \]  
\[ \text{ie: } V = L \times b \times h \]
\[ = \text{Cross sectional area} \times \text{height} \]  
\[ \text{ie: } V = a \times h \]
\[ \text{where } a = \text{cross sectional area} \]

(vi) \[ ^{\circ}K = 273 + ^{\circ}C \text{ (conversion from centigrade to Kelvin)} \]

(vii) \[ \text{Time period of pendulum } T = 2\pi \sqrt{\frac{L}{g}} \]  
\[ \text{where } L = \text{length of pendulum} \]
\[ g = \text{acceleration due to gravity} \]
(viii) Pressure x Volume = Constant ie: PV = C for ideal gas at constant temperature.

(ix) \( V = IR \) (ohms law) \( V = \) voltage, \( I = \) current, \( R = \) resistance

(x) \[ \frac{P_1V_1}{T_1} = \text{constant} \] ie: \[ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \] for an ideal gas

The work included manipulation of the equations to different subjects and substitution of values. This giving further practice on number work.

It must be noted that at each stage, the discussion of topics was treated with greater emphasis than doing reams of calculations, as the author felt that understanding of the concept was the essential ingredient, and once understood it would be easier for the pupils to recall, especially after some revision or the use of the booklet 'Mathematics for Science.'

It was apparent that discussion was not encouraged in other subjects, possibly due to the fact that it is easier to control a lower ability group without discussion. These pupils lacked the confidence to speak in front of a class and so it had to be built up in an atmosphere unlikely to create embarrassment, every answer being treated with encouragement whether it was correct or not. In fact time was taken to explain some of the merits in wrong answers and further encouragement given. The pupils eventually enjoyed these sessions and seemed to gain in understanding and some showed a distinct desire to continue discussing after the lesson. The author feels that this procedure is essential in developing a good relationship between teacher and pupil and to create an environment which is conducive to learning, generate or increase motivation, and to train the mind to think in a logical enquiring manner in solving some simple problems, referred to by Swann (25) in his work on strategies.

In both science and mathematics it is sometimes important to give rough estimates for measurements and other quantities. These
rough estimates we call approximations and involve mathematical knowledge of 'rounding off' and significant figures, so the next section within the overall topic of measurement covered this work and included

![Measurement Scale Diagram]

We can see that 2.3 is nearer 2 than 3, so we can say 2.3 is roughly equal to 2 or $2.3 \approx 2$. Of course, this is not exactly true but it is sometimes only necessary to know roughly what the measurement is. It was pointed out to the pupils that this was an example of Rounding Down. Then an example of 'rounding up'

![Measurement Scale Diagram]

We can say that $12.9 \approx 13$. These problems are fairly clear, however with a measurement like $5.5$ cm which is just as near to 5 as to 6, we usually round up to 6.

Larger numbers were then covered, for instance: 11478 to the nearest hundred

![Measurement Scale Diagram]

$11478 \approx 11500$

however to the nearest thousand

![Measurement Scale Diagram]

$11478 \approx 11000$

It was stressed that after a little practice there was no need to draw the scale lines to write down the approximation and often questions asked for answer given to a certain degree of rounding off and this was usually referred to as significant figures or with decimals numbers to a certain number of decimal places.
This occurs often in experiments where measurements can only be made to a certain level of accuracy, when those values are used in a calculation it is not correct to quote the result to a very much higher degree of accuracy, so we would 'round off' to the appropriate degree of accuracy. A great deal of practical experience was gained through experiments.

In the section on measurement and throughout the practical work, the pupils regularly use protractors and refer to Angular Movement in degrees, radians or revolutions. As many pupils were unsure of how to use or read a protractor some time was spent covering this. It included how to place the protractor to take a reading, the degree and the names given to angles.

How to place the protractor to measure an angle:

*Angle CAB = 45°*

Place the protractor as shown, Line AB along the $0°$ line, read from the scale the number of degrees in line with AC.  

NOTE: there are two scales - read from the scale on which you place AB in line with $0°$, in this case the inner scale.
Other points covered:

(i) $180^\circ$ in a straight line or $\frac{1}{4}$ of a circle
(ii) $360^\circ$ in a circle
(iii) $90^\circ$ in a right angle or $\frac{1}{4}$ of a circle
(iv) Acute angle is an angle up to $90^\circ$
(v) Obtuse angle is an angle between $90^\circ$ and $180^\circ$
(vi) Reflex angle is an angle between $180^\circ$ and $360^\circ$ (in one test with 5th year pupils none measured a reflex angle correctly, instead chose to measure the acute or obtuse angle)

To give plenty of practice in this skill practicals were carried out on the properties of light, with reflection and refraction where the angles of Incidence, Reflection and Refraction have to be measured; and experiments on forces, components and triangle of forces. At this stage the definitions of vectors and scalars were given and problems involved bearings.

The final topic to be covered on the theme of measurement was that of time, this proved to be fairly straightforward on the conventional '12 hour' clock. However, as information given or displayed in public places, such as railway stations, relies on the '24 hour' clock, this was covered and proved confusing to some. Later in the course when dealing with speed, questions would be set where the time intervals of the journey would have to be found from a typical timetable.

Many of the stopwatches used in experiments are those which cover 30 seconds per revolution and this gave a greater degree of accuracy, that is to $\frac{1}{10}$th of a second. This involved a new experience for the pupils and reinforced the fact that in experiments accurate measurements should be taken.
Discussion was held on many different 'TIME' devices, from the sun dial to the water clock, however the pendulum type of clock was treated in more detail and in particular the periodic time of the swing and how this could be altered. An experiment was performed in which the periodic time of a pendulum was measured (a) with varying length (b) varying amplitude of swing (c) with varying mass of the pendulum bob. To obtain the periodic time, that is the time for one complete oscillation, the pupils discussed various schemes to achieve the greatest accuracy and eventually it was decided to time 50 cycles, thereby making the error in time, of the order of 0.01 seconds. The length of the pendulum had to be measured very accurately and was done as shown.

The pendulum bob being measured with a micrometer and the radius being deducted from length x, to give the length to the centre of mass of the bob. The information gained in the experiment was recorded in tabular form,

<table>
<thead>
<tr>
<th>Length L (m)</th>
<th>Time for 50 oscillations</th>
<th>Periodic time T (s)</th>
<th>$T^2$</th>
<th>$\frac{T^2}{L}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

and $\frac{T^2}{L}$ was found to be constant, a graph of $T^2$ against L confirmed this, being a straight line through the origin.

The experiment continued with proof that amplitude of swing or variation in mass of the bob had no affect on the time period. The formula for calculating the periodic time was quoted as $T = 2\pi\sqrt{\frac{L}{g}}$ where $L =$ length of the pendulum and $g$ is the acceleration due to gravity. From this equation we were able to manipulate the form to $g = \frac{4\pi^2L}{T^2}$ to find the acceleration due
to gravity, and this was subsequently done for one of the sets of results in the previous table.

Whilst dealing with the topic on time, it was decided to cover the 'ticker'timer' and cover the laws of motion. The ticker-timer is a device which prints out a DOT on a ticker-tape once every $\frac{1}{50}$th of a second. With the ticker-tape attached to the moving object under study, the motion could be investigated by studying the series of dots produced. The pupils measured the distance between a series of dots, 5 dots or $\frac{1}{10}$th of a second, to make measuring easier. From this information or by using the ticker-tape cut into strips, graphs were drawn of the motion.

The above example of constant velocity shows equal distances covered in equal time intervals. Alternatively a distance-time graph could have been produced to show constant velocity. This was done by placing the strips of ticker-tape as shown opposite. The result being a straight sloping line. The slope of which indicated the velocity.
An experiment to show \( \text{Force} = \text{Mass} \times \text{acceleration} \):

![Ticker-tape](image)

Velocity increasing with time (acceleration)

The study continued with Newton's second law of motion, that is the effect of force on acceleration by using an elastic band to accelerate the trolley down a slight incline (to overcome the effect of friction), then two elastic bands (twice the force) etc.

From the slopes of the graphs it can be seen that an increase in force produces an increase in acceleration and that acceleration \( \propto \text{force} \)
Similarly the effect of mass on acceleration:

![Diagram of trolleys](image)

It can be seen that doubling the mass (with a constant force) gives half the acceleration:

\[ \text{acceleration} \propto \frac{1}{\text{mass}} \]

So from these experiments:

\[ \text{acceleration} \propto \frac{\text{Force}}{\text{mass}} \]

So Force \(\propto\) mass \(\times\) acceleration

This was followed by graphical work on velocity time graphs – the slope being equal to the rate of change of velocity with time (acceleration)

![Velocity-time graphs](image)

Linear acceleration

Non linear acceleration in regions A-B, B-C and D-E
The graph of non linear acceleration includes positive rate of change of velocity with time between A and C called **acceleration** and negative rate of change of velocity with time between D and E called **retardation** (negative acceleration). The value of the acceleration being calculated from the slope of the graph at any particular instant in time.

We also covered the fact that the area under a velocity against time graph gave the distance covered during this time interval. This was achieved by looking at the units of this area, that is

\[ \text{Velocity} \times \text{time} = \frac{\text{m}}{\text{s}} \times \text{s} = \text{m} \text{ (metres)} \]

Continuing on the theme of force, we extended the range to a force causing turning, commonly known as a **moment of the force**. This was done by experiment and discussion of the SEE-SAW before entering into more complicated situations. Initial demonstrations were with a metre rule, fulcrum and two unequal weights to show the balanced or equilibrium position, and that this was possible in an infinite number of different ways. Hence the relationship:
$W \times d = w \times D$ for the situation adjacent. Hence the definition of the moment of a force: as the product of the force $\times$ distance from the fulcrum. This was taken further to show that the more accurate definition was force $\times$ perpendicular distance from the fulcrum to the line of action of the force.

### Table: Anticlockwise and Clockwise Moments

<table>
<thead>
<tr>
<th>Weight $W$ (N)</th>
<th>Distance $d$ (cm)</th>
<th>Moment $W \times d$</th>
<th>Weight $w$ (N)</th>
<th>Distance $D$ (cm)</th>
<th>Moment $w \times D$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>20</td>
<td>40</td>
<td>1</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td></td>
<td>1</td>
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</table>

At this point the physics aspect could be extended into solutions with forces at angles which would involve some trigonometry or knowledge of Pythagoras and right angled triangles.

Hence to more complicated systems of forces in equilibrium and the writing down of the equation for equilibrium.

**Anticlockwise Moments = Clockwise Moments**

and for parallel force as in the bridge type situation, which from experiment gave (i) the sum of the forces in one direction = the sum of the forces in the opposite direction (ii) the sum of the anti-clockwise moments about any point is equal to the sum of the clockwise moments about that point.
This section gave ample opportunity to practice basic number work, forming and manipulating equations. The pupils had experienced in their initial work with equations in the section on Hooke's law, a tremendous amount of difficulty and this occurred again. They had difficulty deciding on which forces caused anti-clockwise or clockwise moments and in writing out the equation with an unknown value in it and furthermore, difficulty in manipulation to find the value of the unknown. Even after some considerable discussion this proved overly difficult for more than 75% of the set. It was clear that work with equations had barely, if ever, been dealt with and would require a full teaching treatment. Also it showed the small or limited retention time of facts gained from lessons, as many could not remember the work on Hooke's law involving mathematics, some doubted that it had been done, but conceded the point when looking back in their exercise books.

The theory continued with centre of gravity (centre of mass), its positions within geometric shapes and the position of it for stability. This was done by experiment, followed by extensive discussion of its implications in everyday life and we finished with some apparent 'tricks' which used the position of the centre of gravity to make 'totally stable' objects as shown below.
TRY THESE

SUPER STABLE SITUATIONS

Timing: This part of the syllabus took approximately 45 hours of teaching plus 17 hours of homework over a period of 1½ terms of the school year.
### The Physics Topics Covered

- Measurement: Length, Area, Volume, Weight, Mass
- Force: Triangle of forces, Equilibrium, Hooke's law, Moments
- Density
- Reflection/Refraction (part)
- Time: Pendulum, Gravity, Speed, Acceleration
- Newtons laws of motion

### The Mathematics Topics Covered

- Natural numbers
- Number system
- Place value
- Directed number (temperatures below 0°C)
- Factors
- Multiples (heating different masses)
- Powers and roots
- Irrational numbers
- Decimals
- Fractions
- Fractions to decimals
- Reading and interpolating scales
- Concept of rate of change
- Ratios/proportions
- Averages (of experimental readings)
- Approximations/estimations/rough checks
- Squares and roots
- Areas/volumes
- Triangle/circle/angles
- Algebra/simple formula/solution of equations
- Indices
- Graphs: linear/non-linear/gradient
- Units: SI/Imperial
- Time, speed, rate of change

The depth of cover into the mathematics was limited by the time available. However the aim behind the research was to examine the feasibility of promoting motivation in the learning of mathematics by the use of a practical, applied approach for the lower ability pupils. This, the author felt, could be justifiably examined with a limited depth mathematical approach.
At the beginning of this form of integrated course there was a little resentment visibly shown by the pupils, because of having to do mathematics in a science lesson. This was to be expected, if the reports on pupils' attitudes to mathematics were true, but as time passed the pupils gradually became acclimatized to the idea and participated reasonably fully because they were aware of the need and relevance of the work.

One of the major problems was the range of abilities found, which varied from weak GCE O level candidates through to those needing remedial help. This large ability range was brought about by the Options System for fourth year pupils. The ranges of choice being limited to make timetabling possible, even so the teacher contact ratio in science and mathematics were higher than 0.9. The result was in the two bands of choice. Physics appeared twice in each band giving approximately 120 pupils from a total of approximately 240 pupils the opportunity to take physics. This gave 60 in each band, making 1 class GCE O level in which about 15/20% of the total pupils choosing physics would be potential grammar school intake, hence the rest of the GCE O level group would then contain the remaining pupils, of which a high proportion of the pupils would be potential middle to low grade (possible CSE grade 2 down to grade 5, very few are ever ungraded).

To overcome this wide ability range, the group must be setted and taught accordingly, having extra work for and expecting better results from some, whilst working to a lesser depth with others. This is by no means a satisfactory situation but is ever-present in modern comprehensive schools. Work-sheets must be written carefully to be within the reading and comprehension ability of all, and when experiments are to be done, the worksheet read out, then the work demonstrated and left in position for the less able to copy. However, it must be stated that the less able, who were known for their poor behaviour in many lessons, took to the practical work in a very enthusiastic manner and some showed glimpses of genuine interest.
Discussion was to be a major part of the course, and initial efforts became noisy and unproductive, due largely to the fact that these pupils were not encouraged to talk in other lessons because of discipline problems it produces and their total lack of descriptive oral ability in English. It was necessary to group the class in close proximity to the teacher's bench, so all pupils were within reach if necessary and ordinary levels of voice was sufficient for all to hear, as pupils in this situation tend to mumble and it was essential to have free flowing discussion. The enthusiasm of the pupils varied, depending upon how well they could associate with the topic being discussed. Again it was a matter of acclimatization, before discussion began, to fulfil any useful purpose. A large factor in this was that the level of discussion was aimed at a standard within their range of experience, made fairly informal, humorous, all the pupils' attempts were shown to be appreciated and praise often given.

The pupils were presented with information gained by careful organisation of experiments and easily read worksheets, which directed the work step-by-step and indicated the level of notes to be taken. The pupils were not allowed to freely make their own notes as their command of written English led to notes which were too vague and confused to be suitable for good revision at a later date. At each stage it was an essential part of the research to integrate the mathematics and this proved a fairly straightforward task, as mathematics is the language of sciences and it was thought that within the scope of a physics course a suitable mathematics course could be integrated which would give a middle to lower ability pupil the majority or perhaps all the mathematic skills he would require for 'life and work' but at present would not quite fulfil all the requirements of a CSE mathematics syllabus. The author believes that for this type of pupil the formal mathematics examination system in use at present, proves uninteresting, has no apparent relevance in the mind of the pupil, lacks application, is taught in unimaginative ways, without internal co-ordination or links and thus proves totally unmotivating.
To reinforce the experimentation and theory, films, film loops and other audio visual aids proved useful. The science department encouraged the use of calculators and proposed to incorporate the facility of several BBC type B computers. The 'chalk and talk' techniques of teaching was totally unsuitable for these pupils.

The pupils problems were initially lack of confidence - most had never been in a position where they had had to do anything original or develop strategies. Previously work had been repetitive, in nature, that is, shown a worked example and then asked to repeat it many times using different numbers. So when asked to design their own experiment or discuss how experiments could be improved, or guess at possible solutions to a problem, most felt embarrassed or apprehensive when asked to participate, preferring to remain in the background. The pupils were encouraged by setting competitions in the form of "The Great Egg Race" of recent television fame in which they could put their own ideas into practice and try to win the prize, which was a Rubic cube.

The pupils, under the right circumstances, could be encouraged to become accustomed to participation even though some had accepted their low ability and were not embarrassed by it, and were of the opinion "What's the point of all this hard work", typical comments were "I have never been able to do this", "It's too difficult for me", "I cannot do maths". These pupils need to gain confidence which would lead to motivation. It was essential to get the right kind of atmosphere during lessons, reassuring, lively, applied or practical, and seen to be relevant. Although these pupils were of lower ability many had a competitive keenness to do better than others in the group. Their eagerness to get test results and compare with their friends was evident, even though the marks at times were low for the whole group, there was still the group rank order, the particular mark was irrelevant. Effort was always rewarded, whether incorrect or not, marks out of ten for work completed would always be in the range of 4 to 10, so it was hope that they would feel it was worth trying.
In a class of 30 pupils this competitiveness could not be used to its full benefit for the whole group, but in sub-groups this competitiveness could have an overall motivating influence.

The practical aspects of lessons generated a profound increase in motivation, whenever possible the routine would be changed, that is, a practical would be arranged on the playing fields or of some dramatic nature such as electrostatic lightning or a pupil charged with several thousand volts potential being able to light a bunsen burner with sparks from his finger.

Whenever pure theory lessons were adopted, the mood and co-operation of the group fell quite considerably. However, there were those who would always be quite content to sit and copy from books because this was the practice in other lessons, and it avoided thinking. Lessons had to be well planned and taught in an imaginative, enthusiastic way.

The overall impression the author gained was that given the right environment with an encouraging stimulus, these pupils were capable of producing a reasonably high level of motivation with the subsequent gains in learning of understanding and retention of knowledge.

The scheme could be modified to accommodate the mathematics and science more efficiently if the whole of the syllabus had been used, but time prevented this, and even more so if a syllabus was designed which did not take into account any predetermined needs of examination boards, which at present restricted the work covered both in depth, content and organisation.

The author decided to run a dinner-time club called "Maths can be fun and mathematics for science" for those who wanted to improve their ability. This was intended for those pursuing the CSE physics course and invitations were restricted to those participating in this study. The numbers that attended were less than 5% of the sample and this was dependent on the weather, that is, fewer came on good days. However of those who attended concern was expressed at the forthcoming CSE examination.
The format was to cover the mathematics content of the work covered in the integrated form of science and mathematics being tested, to a greater depth. To this end, booklets were made on each topic to be studied so that the work could continue at home, if so desired by the pupils. The booklets gave the information on how to do each topic, reminded them of the skills required and gave further examples for their completion. In each section of the 'Maths for science' booklet it was attempted to give clear examples of the application of these skills, and some more lighthearted uses in games such as darts, constructions which gave patterns, use of calculators for pure mathematics and games, as a form of motivation to attend the club were included.

Although the number of people involved was small, the author was convinced that the booklets and personal teaching had helped, in boosting both knowledge and confidence, and was worth continuing in future years. Perhaps making attendance of the club a semi-compulsory requirement for those choosing to take the physics examinations, or even writing a mode 3 examination to give the pupils an added CSE qualification. If this mode 3 course is developed the author would consider it necessary to consult with local industry to get their opinions.
CHAPTER 7

Mathematics for Science

The author being aware of the need for some constructive dialogue between mathematics and science departments generated and issued a document entitled "Mathematics for Science" at his school. This document is detailed in the following pages and included a chronological ordering of the mathematical requirements of the pupils, in order to cope successfully with the science teaching for years 1 to 5, and examples of the usage of these topics in a scientific context, to aid the mathematics teachers in their preparation of suitable applied material for their lessons.

In addition to this the author felt that the pupils needed their own booklet which could be used for revision of these mathematical requirements and this is included in Appendix VI.

The author also obtained the comments of pupils and teachers to which a section of this chapter is devoted.
Mathematics for Science Year 1

COMBINED SCIENCE

I Arithmetic Operations with Whole Numbers, Fractions and Decimal Numbers (to 2 places only)

(1) To find the mass of liquid in a container:

Mass of liquid = mass of container and liquid - mass of container

Mass of container and liquid = 63.8 g
Mass of container = 21.9 g
Mass of liquid = 41.8 g

(a) Volume Area and Length

Volume: regular solid

\[ \text{Volume} = \text{length} \times \text{width} \times \text{height} \]

Given

Length = 4 m
Width = 3 m
Height = 2\frac{1}{2} m

Volume = 4 \times 3 \times 2\frac{1}{2} m^3
= 30 m^3

alternatively with mixed units

Length 1 m 50 cm = 1.5 m
Width 0 m 80 cm = 0.8 m
Height 1 m 20 cm = 1.2 m

Volume = 1.5 \times 0.8 \times 1.2 m
= 1.44 m^3

(NOTE: no understanding of index of m^3 is required at this stage)
(3) To find density of water by measuring mass and volume (by practical experiment)

\[
\text{Density} = \frac{\text{mass (g)}}{\text{volume (cm}^3\text{)}}
\]

Mass = 61.0 g
Volume = 60.0 cm\(^3\)

\[
\text{Density} = \frac{61.0}{60.0} = 1.0 \text{ g/cm}^3
\]

(Note rounding off or approximation)

II Use of and Manipulation of Units in Simple Expressions

See examples 1, 2, 3 Section I.

It is important that the pupils are conversant with everyday units and have a feeling for the size of units, i.e., what is a "m/s". How fast is it. If they have a feeling for the size of units they can say whether the answer is reasonable.

Comparisons between Imperial and Metric systems.

III Understanding the Limitation of Accuracy of Data

Appropriate accuracy in expressing results of calculations with measured data. See example 3 Section I and

Volume of a cube measured to accuracy of \(\pm 1\) mm

Length = 2.1 cm
Width = 1.9 cm
Height = 2.1 cm

Volume equals 8.379 cm\(^3\)

This answer is not consistent with the accuracy obtained during the experiment of 1 decimal place, so quote answer 8.4 cm\(^3\)
IV  Components of Whole Expressed as Percentages

Conversion of % to Decimal and Fraction

Example

Composition of air:

Oxygen 20% $\rightarrow$ 0.2 $+ \frac{2}{10} (\frac{1}{5})$

Nitrogen 78.8% $\rightarrow$ 0.8 $+ \frac{8}{10} (\frac{4}{5})$

Carbon Dioxide: 0.2% $\rightarrow$ 0.002 $+ \frac{2}{1000}$

(Note: Pupils need to appreciate that very small components can be treated as negligible: inert gases in air)

V  Histograms

Pupils are asked to find the most common interval in a distribution, the number of results within any interval and to draw qualitative conclusions from the shape of the histogram.

Example

Measurement of pulse rate for the class and expression of the result as a histogram. Pupils are asked to use the histogram to find:

(a) The most usual pulse rate for the class

(b) The number of pupils with a pulse rate of say 80 beats/min

The hump shape of the histogram denoted scarcity at extremes

Other measurements: height of pupils, hand spans, etc.
VI Graphs and Tables

Expression of Experimental Data in Tables and Graphs. Simple Interpretation of Straight Line Graphs

Example

Pupils are asked to measure the extension of a spring with increasing load of 1N (approx 10 g) and express the results in a table and use the table to produce a graph.

Table:

<table>
<thead>
<tr>
<th>Load (N)</th>
<th>Original Length of Spring A (cm)</th>
<th>New Length of Spring B (cm)</th>
<th>Extension (B - A) cm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Graph:

Pupils are to draw the 'TREND' of the graph not to join one point to the next in a zig-zag fashion.
Pupils are then asked to answer the following questions:

(a) How far will the spring extend if a load of $4\frac{1}{2}N$ is hung from it?
(b) What kind of graph could you call this?
(c) What does the straight line tell you?

Answer:

(a) Pupils read off answers from their graphs
(b) Pupils are prompted and guided verbally to note the straight line
(c) Pupils are guided to the conclusion that equal extra loads produce equal extra extensions (proportionality)

Question

Which spring is the "strongest" and which is the "weakest"?

Answer:

The pupils are guided to a limited understanding of slope of the graph (a) strongest (c) weakest

VI Understanding of Proportionality and its Application to Simple Problems

(1) $5\text{ cm}^3$ of a substance has a mass of 30 g. What mass would $1\text{ cm}^3$ of this substance have?
Answer:

The volume has reduced to a fifth of its value

Therefore the mass of 1 cm$^3 = 30 \text{ g} \div 5 = 6 \text{ g}$

(2) Experiment with springs: double the load so we have double the EXTENSION

Mathematics for Science Year 2

COMBINED SCIENCE

I Arithmetic Operations with Realistic Measured Data, up to 2 decimal places

Example

(a) An experiment to find the pressure created by different pupils standing on a pad connected to a pressure measuring device (either directly to a pressure gauge or to a manometer where pressure is proportional to difference in height of fluid in the manometers limbs)

Pressure (N/m$^2$) = $\frac{\text{force (N)}}{\text{area (m}^2\text{)}}$

Typical results: Boy weighs 52 kgf

Convert to Newtons $1 \text{ kgf} = 10 \text{ N} (9.81 \text{ N})$

Area of pad 25.5 cm by 30 cm

Pressure (N/m$^2$) = $\frac{(52 \times 10)}{(25.5 \times 30)} = \frac{6797.39 \text{ N/m}^2}{100 \times 100} \approx 6797 \text{ N/m}^2$

 conversion from cm to m
Alternatively

Pressure \( \text{N/cm}^2 \) = \( \frac{(52 \times 10)}{25.5 \times 30} \) = 0.68 \( \text{N/cm}^2 \)

(b) To find the moment of a force, multiply the force by the distance.

Experimental results may be:

\[
\begin{align*}
\text{Force} &= 5 \text{ N} \\
\text{Distance} &= 9.71 \text{ N}
\end{align*}
\]

Therefore Moment = 3.55 Nm

II  Manipulation of dimensions in expressions
See I (a) and (b)

III  Measurement of Angles and Simple Construction Techniques

Example

The laws of reflection for a plane mirror

To compare angle of incidence \( \theta_i \) with the angle of reflection \( \theta_r \) and construct the normal
Pupils are asked to experiment with two mirrors and find the angles the mirrors have to be placed at to make the periscope. The idea of the mirrors being parallel to each other.

IV Further Use of Histograms and Graphs to Derive Data in Terms of Percentages and for Comparative Studies

Example

Histograms on survey of dental decay in children in two towns are shown. The pupils are asked to use the histograms to answer the following questions:

(i) What percentage of children in town A had no fillings (caries)?

(ii) What percentage of children in town B had no fillings?

(iii) In both towns a few children had a large number of teeth affected by decay. What was the greatest number of teeth affected per child in each town?

(iv) How many children in each town had 4 fillings?

(v) In which town had the children the strongest teeth?
A similar histogram was shown for town B.

Mathematics for Science Year 3

PHYSICS/CHEMISTRY/BIOLOGY

I Calculations Within the Error Limits of Data and Correct Rounding Off of Numbers to Required Number of Significant Figures; up to 4 significant figures

Example

Using a metre rule (accurate to 0.001 m or 1 mm) to find the area of a desk top
Measurements: \( x = 1.521 \text{ m (say)} \)
\( y = 0.627 \text{ m (say)} \)

Area = \( x \cdot y = 0.953667 \text{ m}^2 \)

**Pupils must round off to 0.954 \text{ m}^2**

II **The Use of Very Large and Very Small Numbers in Standard Form**

**Examples**

(i) The mass the the earth = \( 6 \times 10^{24} \text{ kg} \)

(ii) The speed of light = \( 3 \times 10^8 \text{ m/s} \)

(iii) Approx. size of molecule = \( 2 \times 10^{-9} \text{ m (diameter)} \)

(iv) Wave lengths of visible light = \( 5 \times 10^{-7} \text{ m (average)} \)

(v) Avagardro's Number = \( 6.03 \times 10^{23} \)

III **Introduction to the Use of the SI Convention for Compound Units**

**Example**

(i) \( \text{m/s} \to \text{ms}^{-1} \)

(ii) \( \text{kg/m}^3 \to \text{kgm}^{-3} \)

IV **Area and Volume**

Area: square, triangle (right angled, isosceles, equilateral) circle

Volume: cube, cuboid, cylinder, and possibly a sphere
V Further Work on Graphs and Proportionality

Example.

Interpretation of graphs:

Mathematics for Science Years 4/5

PHYSICS/CHEMISTRY/BIOLOGY ('O' level and CSE)

I Use of Electronic Calculators

The pupils are allowed to use these instruments in the 'O' level examination and part of the CSE. Typical work using operations +, -, × and ÷. Use of "Memory" for work of a repetitive nature with experimental data.

Use of Logs and Trig. functions (sine, cosine and tangent) and inversion and square/square root.

A means of approximating the answer is necessary to check the calculation has been correctly performed on the calculator.
II Correct Order of Accuracy in Calculations

Example

(i) A voltmeter and ammeter give the following readings: 
\[ V = 2.6 \text{v}, \quad I = 1.1 \text{A} \]

Using \[ R = \frac{V}{I} \] calculate the resistance

\[ R = \frac{2.6}{1.1} = 2.364 \text{\ Ņ (ohms)} \]

The pupil has to round off this answer so that the result is of the same order of accuracy as the measurements

\[ R = 2.4 \text{\ Ņ (ohms)} \]

(ii) Two sides of a square are measured using different instruments and the following results obtained:

\[ a = 1.4 \text{ m}, \quad b = 2.62 \text{ m} \]

Calculate the area of the square to the appropriate accuracy

\[ \text{Answer} \quad \text{Area} = a \times b = 1.4 \text{ m} \times 2.62 \text{ m} \]
\[ = 3.668 \text{ m}^2 \]

The pupil has to round off to the same number of significant figures as the least accurate measured value

\[ \text{Area} = 3.7 \text{ m}^2 \]

III Conservation of Quantity and its Application to Solve Problems via the Setting up of Equations

Example

(i) Quantitative problem on conservation of energy to find Kinetic Energy of a falling body and efficiency of a lifting machine.
A man, using a pulley, applies an effort of 5N through 2 m to lift a weight of 10N by 0.8 m. Calculate the energy wanted by the machine, and its efficiency.

Answer

Work done by man \( W = \text{force} \times \text{distance} \)

\[ = 5N \times 2 \text{ m} = 10 \text{ Nm} \]

\[ = 10 \text{ Joules} \]

Potential energy gained by weight \( P = 10N \times 0.8 \text{ m} \)

\[ = 8 \text{ Joules} \]

Energy wanted by machine:

By conservation of energy principle

Work done by man = work done on load + work done in machine

So \[ 10J = 8J + \text{work done in machine} \]

Hence Work done in machine = 2J

The efficiency of the machine = \[ \frac{\text{work out on load}}{\text{total work in}} \times 100\% \]

\[ = \frac{8}{10} \times 100 \]

\[ = 80\% \]

(ii) Oil film experiment to estimate the size of an oil molecule.

A droplet of oil of volume \( \frac{4}{3}\pi r^3 \) spreads to a monomolecular film (cylinder) of volume \( \pi R^2 h \) when placed on the surface of water.

The height of the film \( h \) is a reasonable approximation of the size of an oil molecule.
Answer

Since no oil is lost in the operation

\[ \frac{4}{3} \pi r^3 = \pi R^2 h \]

change subject of equation to "h"

\[ h = \frac{4}{3} \frac{r^3}{R^2} \text{ (m)} \]

Typical values obtained for \( h \approx 10^{-6} \) to \( 10^{-9} \) m.

IV  Algebraic Manipulation of Simple Expressions and the Insertion of Numbers and Units

Examples

(i) \( \text{Power} = \frac{\text{Energy}}{\text{Time}} \)

Units \( J.s^{-1} = \frac{J}{s} \)

\[ \therefore \text{Energy} = \text{Power} \times \text{Time} \quad J = J.s^{-1} \times s \]

(ii) \( \text{Power} = \text{Voltage} \times \text{Current} \)

Units \( J.s^{-1} = Jc^{-1} \times cs^{-1} \)

\[ \therefore \text{Current} = \frac{\text{Power}}{\text{Voltage}} \quad Js^{-1} = Js^{-1} \]

etc

(iii) Electrical energy units (kwh) = Power (kw) \times Time (h)

Work out the following, when electricity costs 4.2 p/kwh

(a) The cost of running a 3 kw heater for 10 hours

(b) The cost of using a 150 w TV set for 4 hours each day for a year (365 days)

(c) The cost of using a 5 w electric clock, 24 hours a day, for a year (365 days)
(iv) Work done in lifting a weight = weight (N) x height (m)

How much work is done lifting a 10N weight by 2 m

Answer: Work done = 10N x 2 m = 20 Nm
= 20 J

V Qualitative Understanding of Normal Distribution

Example

To explain evaporation and its cooling effect on the remaining liquid.

Upon evaporation only molecules with higher velocities (shaded region) escape. Therefore, the average velocity of the remaining molecules of liquid must fall. The temperature of the liquid is related to the average velocity of its molecules and hence its temperature falls.

VI Relative Measure of a Quantity Against a Standard

Example

(i) Relative atomic mass: defined as the mass of the atom relative to $\frac{1}{12}$th of the mass of the carbon atom.

$$Ar = \frac{\text{mass of atom}}{\frac{1}{12} \text{ mass of } 12^6\text{C atom}}$$

Relative atomic mass is expressed to the nearest whole number.
(ii) Relative Density: Defined as the density of a substance relative to the density of water:

Density of Water = 1 g/cm$^3$ (g/cm$^{-3}$)
Density of Meths = 0.8 g/cm$^3$ (g/cm$^{-3}$)

\[ \text{Relative density} = \frac{0.8 \text{ g/cm}^3}{1.0 \text{ g/cm}^3} \]

\[ = 0.8 \text{ (Note no units)} \]

VII Proportions Expressed as Fractions and Percentages

Example

(i) Calculate the percentage composition by mass of ammonium nitrate: $\text{NH}_4\text{NO}_3$

Atomic masses $\text{N} = 14$, $\text{H} = 1$, $\text{O} = 16$

Formula mass $\text{NH}_4\text{NO}_3 = 14 + (4 \times 1) + 14 + (3 \times 16)$

\[ = 80 \]

Percentage of N = $\frac{28}{80} \times 100 = 35\%$

Percentage of H = $\frac{4}{80} \times 100 = 5\%$

Percentage of O = $\frac{48}{80} \times 100 = 60\%$

(ii) Empirical Formula Derivation:

Calculation of the simplest empirical formula for the compound with the following composition: lead 8.32 g, sulphur 1.28 g, oxygen 2.56 g (Atomic masses Pb = 207, S = 32, O = 16)

Answer Convert into moles by dividing by atomic masses
\[ \text{Pb} = \frac{8.32}{207} = 0.04 \text{ moles} \]
\[ S = \frac{1.28}{32} = 0.04 \text{ moles} \]
\[ O = \frac{2.56}{16} = 0.16 \text{ moles} \]

Therefore ratio of atoms = 0.04 Pb : 0.04 S : 0.16 Oxy.

Since only whole numbers of atoms are allowed, divide through by the smallest number to gain ratios in terms of lowest number

1 Pb : 1 S : 4 O

Thus the empirical formula is: \( \text{PbSO}_4 \)

VIII Further Use of Numbers in Standard Form in Calculations Involving Multiplication, Division, Squaring and Square Rooting

(i) \((a \times 10^m) \times (b \times 10^n) = a \times b \times 10^{(m+n)}\)

(ii) \(\frac{a \times 10^m}{b \times 10^n} = \frac{a}{b} \times 10^{(m-n)}\)

(iii) \((a \times 10^m)^2 = a^2 \times 10^{2m}\)

(iv) \((a \times 10^m)^{\frac{1}{2}} = a^{\frac{1}{2}} \times 10^{m/2}\)

Examples

(i) Using \( R = \frac{\rho L}{A} \) calculate \( R \) if
\[ = 2 \times 10^{-7} \quad L = 6 \quad A = 3 \times 10^{-8} \]

Answer \( R = \frac{2 \times 10^{-7} \times 6}{3 \times 10^{-8}} = 4 \times 10^{-7-(-8)} \)

\[ = 4 \times 10^{1} \]

Therefore \( R = 40 \ \Omega \)
(ii) A radar signal is sent to an aeroplane $9 \times 10^3$ m away. If the speed of the signal is $3 \times 10^8$ ms$^{-1}$ find the time it takes to reach the aeroplane. How far away is another aeroplane if a radar signal takes $2 \times 10^{-4}$ s to reach it.

**Answer**

(i) \[
\text{Time} = \frac{\text{distance}}{\text{velocity}} = \frac{9 \times 10^3}{3 \times 10^8} \quad \left[ \frac{\text{m}}{\text{ms}^{-1}} \right]
\]

\[
\text{Time} = 3 \times 10^{-5} \text{ seconds}
\]

(ii) \[
\text{Distance} = \text{velocity} \times \text{time} \quad (\text{ms}^{-1} \times \text{s})
\]

\[
= 3 \times 10^8 \times (2 \times 10^{-4})
\]

\[
= 6 \times 10^4 \text{ m}
\]

**IX Further Work on Expressing Data, Graphs and Proportionality**

**Examples**

(1) Interpretation of a straight line graph through the origin as indicating direct proportionality. Ability to determine the gradient of the line and understanding its significance as the constant of proportionality. The equation of a straight line graph \(y = mx + c\).

Example of the above: Practical:

(a) Find the weight using a Spring Balance of ten masses from 100 g to 1 kg. Draw up a table of results and use this to draw a graph of weight against mass. Call the slope "g"

(b) The graph will be a straight line through the origin. Write down the relationship between weight and mass

(c) Calculate the slope 'g' and hence write the relationship between weight and mass
**Typical results**

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Weight (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>1.0</td>
</tr>
<tr>
<td>0.2</td>
<td>2.1</td>
</tr>
<tr>
<td>0.3</td>
<td>2.9</td>
</tr>
<tr>
<td>0.4</td>
<td>4.0</td>
</tr>
<tr>
<td>0.5</td>
<td>5.0</td>
</tr>
<tr>
<td>0.6</td>
<td>5.9</td>
</tr>
<tr>
<td>0.7</td>
<td>7.0</td>
</tr>
<tr>
<td>0.8</td>
<td>8.1</td>
</tr>
<tr>
<td>0.9</td>
<td>8.9</td>
</tr>
<tr>
<td>1.0</td>
<td>9.9</td>
</tr>
</tbody>
</table>

From the graph pupils are guided to conclusions

\[ \text{Weight} = \text{Mass} \]

The slope \( g = \frac{\text{change in weight}}{\text{change in mass}} \)

and this is the constant of proportionality

\[ \therefore \text{W} = g \text{M} = 10 \text{ M} \]

(2) Experiment to verify the first law of electrolysis: The mass of any substance liberated is proportional to the quantity of electricity passed through the electrolyte.

Pupils set up the experiment, take measurements and draw up the following table:

<table>
<thead>
<tr>
<th>Mass M(g)</th>
<th>Charge ( \phi ) (coulombs)</th>
</tr>
</thead>
</table>
Pupils are asked to plot a graph of $M$ against $\phi$ and

(a) State whether the first law is confirmed by their results

(b) Find the slope of the line and hence find the amount of electricity necessary to liberate 1 g of substance in their experiment

X Inverse Proportionality and Inverse Square Law

Examples

(1) Inverse-proportionality: Boyles Law

Experiment Pupils take readings of volume for a fixed sample of gas at various pressures, and draw up a table

<table>
<thead>
<tr>
<th>Pressure $P$ (Pa)</th>
<th>Volume $V$ (m)</th>
<th>$\frac{1}{V}$ m$^{-3}$</th>
</tr>
</thead>
</table>

Pupils then plot a graph of $P$ against $\frac{1}{V}$ and are asked to state a relationship between 'P' and 'V'.

Because the graph is linear and passes through the origin

$$P = \frac{1}{V} \quad \text{or} \quad P = \frac{\text{constant}}{V} \quad \text{or} \quad PV = \text{constant}$$

where the constant is the slope of the line and hence

$$P_1 V_1 = P_2 V_2$$
XI Radioactive Decay (Exponential Decay)

Qualitative treatment only for HALF LIFE of which the definition is: the time for half the atoms in the sample to decay. Pupils must understand the meaning of exponential decay and the need for the definition of half life to the random nature of radioactive materials.

Example

(i) Radioactive sample has a mass of 16 g and a half life of 10 days. What mass of original sample remains after
(a) 10 days (b) 20 days (c) 40 days

(ii) Sketch a graph to show the variation of the mass of the sample over a period of 40 days

Answer

Half life = 10 days

(a) Mass $= \frac{16}{2} = 8 \text{ g}$
(b) Mass $= \frac{8}{2} = 4 \text{ g}$
(c) Mass after 30 days $= 2 \text{ g}$
So after 40 days $= 1 \text{ g}$
XII More Complex Formula: Change of Subject and Insertion of Numbers

Examples

Equations of motion
\[ V = u + at \]
\[ S = ut + \frac{1}{2}at^2 \]
\[ V^2 = u^2 + 2as \]

Kinetic Energy
\[ (Ke) = \frac{1}{2}mv^2 \]

Potential Energy
\[ (Pe) = mgh \]

Gas Law
\[ \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \]

Electrical Resistances in parallel
\[ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \]
\[ \frac{1}{R_T} = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1R_2R_3} \]

The following are some examples involving such formula.

(i) Equations of Motion: A projectile is fired vertically upwards and reaches a height of 82.5 m. Find the velocity of the projectile at the start of its upward motion and the time taken to return to the ground (Take 'g' as 9.81 ms\(^{-2}\))

(ii) Kinetic Energy: A weight of 50N is lifted a distance of 4 m vertically upwards and then released to fall. Using the conservation of energy and assuming air resistance is negligible, find the velocity of the weight after it has fallen 3 m.

NOTE: Pupils take a proportion \(\frac{1}{4}\) of the potential energy and equate it to gain in Kinetic Energy

\[ (\frac{1}{4})mgh = \frac{1}{2}mv^2 \]
Gas Laws: Calculate the volume of gas at 290K and 720 mm Hg pressure which occupies a litre at s.t.p (273k, 760 mm Hg)

Pupils use \( \frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \) and solve for \( V_2 \)

ie, \( V_2 = \frac{P_1T_2}{P_2T_1} V_1 \)

Electrical resistors in parallel: A cell can supply a current of 1.2A through 3 resistors connected in parallel. When they are connected in series the value of the current is 0.2A. Calculate the E.M.F and internal resistance of the cell.

Pupils use \( V = IR \) and solve for \( R_I \) by using EMFS (sum of the potential differences around the circuit)

XIII Simultaneous Equations

Solved by either elimination or substitution

Example
See question relating to Electrical Resistors in parallel, section XII.

Alternatively:
A converging lens of focal length 10 cm produces an upright virtual image 3 times the object's height when used as a magnifying glass.

Find the distance from the lens. Using real is positive/ virtual is negative sign convention the focal length \( f = +10 \) cm, the object distance \( u \) is positive, the image distance \( V \) is negative. Linear magnification = -3.

Using \( M = \frac{V}{u} \) and then \( V = 3u \) \( \quad (1) \)
and \( \frac{1}{V} + \frac{1}{u} = \frac{1}{f} \) so \( \frac{1}{-V} + \frac{1}{u} = \frac{1}{10} \)

Substitute for 'v' and solve for \( u \).
.XIV Quadratic Equations

Either by factors or formula

Example

(i) A ball is thrown vertically upwards from the ground with a velocity of \(15\ \text{ms}^{-1}\). Find the time taken to reach a height of \(10\ \text{m}\) (\(g = 10\ \text{ms}^{-2}\))

**Answer**

Using \(S = ut + \frac{1}{2}at^2\)

where \(s = 10\ \text{m},\ \ u = 15\ \text{ms}^{-1}\) and \(a = -10\ \text{ms}^{-2}\)

So \(10 = 15t - \frac{10t^2}{2}\)

\(5t^2 - 15t + 10 = 0\)

\(t^2 - 3t + 2 = 0\)

Solve by factorising for two values of \(t\) or by formula

(ii) The electrical power in a resistor \(R\) of \(2\ \Omega\) due to a current \(I\) in amperes is the same as that in a resistor of \(4\ \Omega\) when the current is \(1\ \text{A}\) less than before. Calculate \(I\).

**Answer**

\(P = I^2R\)

So equate conditions \(2I^2 = 4(I - 1)^2\)

\(2I^2 = 4(I^2 + 1 - 2I)\)

Hence \(2I^2 - 8I + 4 = 0\)

Solve for \(I\):
The following points are required

(i) Knowledge and application of the definitions for sine, cosine and tangents of angles

(ii) The use of tables for sine, cosine and tangents of angles from $0^\circ$ to $90^\circ$ (alternate use of calculator)

(iii) Pythagoras' Theorem

It is also useful if pupils know the values of the sine, cosine and tangents of $30^\circ$, $60^\circ$, $45^\circ$ and $90^\circ$ ('O' level only)

Examples

(1)  
(a)  
A rod of length 2.00 m rests on a box of height 'h'. If the rod is at an angle of $18^\circ$ to the horizontal, find 'h'

(b)  Components of forces in horizontal and vertical planes

Given a street lamp is suspended over a road by wires held at the top of vertical posts. Explain how the tension in the wires varies as the angle the wire makes to the post varies.

Calculate 'T' when the lamp weighs 500N and the angle between the wire and the vertical is $70^\circ$.
(2) Calculations involving Refractive Index 'n'

\[
n = \frac{\sin i}{\sin r}
\]

**Example**

The angle of refraction 'r' in glass from air is 30°. The refractive index of air-glass boundary is 1.5. Find the angle of incidence 'i'.

(3) **Vector addition:**

Two or more forces are applied to an object, the pupils are required to find the resultant

**Example**

(i) Find the magnitude and direction of the effective force on the object.

[Diagram of forces]

Answer: by geometry or by adding algebraically the horizontal and vertical components of the forces (Triangle of forces)

(ii) Given two identical springs 5 cm long, stiffness 10 N/cm (Ncm\(^{-1}\)). Calculate the extension when (a) the springs are connected in series (b) in parallel with a load of 25N

**NOTE:** For (a) the pupils have to understand that the extension of a spring depends (is proportional) to the length of the spring, so when in series, as above, we will get twice the extension of one of the springs.

**Answer**

[Diagram of springs]

In series
- 10N gives 2 cm extension
- 1N gives \(\frac{2}{10}\) cm extension
- 15N gives \(\frac{2}{10} \times \frac{5}{2}\) cm extension

Extension = 5 cm
This gives each spring 12\(\frac{1}{2}\)N load
10N gives 1 cm extension
12\(\frac{1}{2}\)N gives \(\frac{1 \times 12\frac{1}{2}}{10}\)
Extension 1.25 cm

XVI Use of Formula, Arithmetic Operations with Experimental Data

Example Moments of forces

Definition: Moment equals the turning force x perpendicular distance from the line of action of the force to the point of turning

\[\text{Moment} = F \times \text{initially} \times \text{found by construction but later 4th year by trigonometry} \]

Units - Newton metres

From Experiment

Anticlockwise moments = clockwise moments
Draw up table

<table>
<thead>
<tr>
<th>Anticlockwise moments</th>
<th>Clockwise moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A \times x =$ (Nm)</td>
<td>$B \times Y =$ (Nm)</td>
</tr>
</tbody>
</table>

More complex problems:

```
\begin{align*}
\text{100N} & \quad 10 \quad 60cm \quad 50N \\
N & \quad \text{FULCRUM}
\end{align*}
```

Calculate the size of force 'N'

VII Symmetry

Example Centre of mass of regular shapes

- Magnetic field patterns are a fairly unusual example of symmetry.
XVIII Trigonometry: the use of 'sine'

Example

Refraactive index \( n = \frac{\sin i}{\sin r} \)

and \( a_g = \frac{1}{g_N} \)

where \( a_g \) = refractive index air to glass and \( g_N \) = refractive index glass to air.

This is found experimentally with a block of glass

The pupils should be able to work out why the emergent ray is parallel to the incident ray and that the refractive index from air to glass is the inverse of that from glass to air.

Also \( \sin \) of critical angle = \( \frac{1}{g_N} \)
Comments of pupils and teachers

In order to present a picture of the children's mathematical understanding and the opinion of those who teach them, the author interviewed many children in a new comprehensive school, comprising of a Secondary Modern School and a Technical Grammar, who had just made their option choices at the beginning of the fourth year or who were part way through their examination courses, at both CSE and 'O' level. Approximately 70% of the sample were not taking physics. In these interviews the children were encouraged to give their opinions of mathematics and the way in which it was taught, whether they enjoyed it, and the reasons for this. Questions related to physics tried to gain an insight into the pupils problems when faced with the mathematical content of the course and the effect mathematics has on the enjoyment of such a course.

The interviews were designed to be informal, to try to make the pupils feel at ease because many of them, when faced with a tape recorder apparently lost their voices and had very little to say and comments had to be coaxed from them. If a child was fluent and had strong feelings about the topic, they were allowed to continue without interruption. The interviews of each child lasted approximately 5 to 10 minutes and transcripts are included in Appendix V.

At the same time as making the tape recordings approximately 85 of these pupils, who preferred to write their opinions did so in the form of an essay. Selected passages from these essays have been included in Appendix V and they show a certain amount of discontentment with their mathematical education.

Similarly these pupils were given a questionnaire to fill in, which tried to discover basically why they did not choose physics as an option, the level of difficulty they had in
applying their mathematical knowledge; the mathematics topics they themselves felt they had difficulty in; whether they would like to drop mathematics or had it been improved by including some practical, relevant applications; whether there had been any continuity in their mathematics education, by the number of teachers they had had during the first three/four years of their secondary education; had their impression of mathematics been influenced/biased by the fact that their parents could or could not do mathematics or could they get help at home; was the mathematics they were being taught relevant to their future lives; and did they own a calculator, as the use of these instruments was now permitted in science examinations and so they should be competent in using them, as well as using them as a help in general life. The results of the questionnaire are included in Appendix IV.

The author asked several teachers for their comments on the relationship between mathematics and science. The initial survey included heads of departments of science, administrative staff who had been heads of departments of science, teachers from all three branches of science, teachers with science experience gained in several different types of school so that their opinions would have been developed relative to education generally rather than in one situation, a teacher fresh from college whose opinions would not have been biased by loyalty to a department and whose enthusiasm and expectations had not been blunted by years of "making do with the situation", teachers who had taught both mathematics and science, a science advisor for Derbyshire; a past chairman of the ASE, members of The Royal Society, senior lecturers at a local college of further education lecturing in the craft/industrial sciences, a conference of heads of science departments and a head teacher. The author thought this would give a general spectrum of the views of those who were involved in science education.
Part of the above sample recorded their comments on tape and transcripts will be found in Appendix V, the remainder preferred to talk to the author or give written summaries of their opinions also to be found in Appendix V.

From the tape recordings and other evidence, a number of salient facts became apparent in each and so the author decided to take these points and encompass them in a questionnaire which went out to a further sample of heads of science departments, from this no replies were received which seems to indicate a lack of caring or that the timing of this questionnaire was at an inappropriate time during the academic year.
CHAPTER 8

Conclusions, Recommendations and Suggestions for the Future

Every science teacher and educationalist interviewed claimed that mathematics departments were not providing an adequate service for consumers, such as those teaching other curriculum areas in comprehensive schools. Science teachers had, without exception, to resort to teaching both the mathematics required for their subjects and the scientific theory.

"Graham (23) concluded that the mathematics being taught is not suitable for science courses"

This obviously results in a severe obstacle to progress and often it is claimed a loss in interest in the science subject by the pupils with the consequent results of opting out of the sciences altogether or underachieving in external examinations.

"Children are put off physics because by the age of 13 many don't have the basic mathematical concepts and skills" (4) Royal Society and the Institute of Physics.

Having to teach the mathematics required to cope with their science subjects, and in particular physics, which seems to suffer the most, obviously puts a strain on the relationship between the mathematics and science departments...
'Professor Matthews tells the story of a heated discussion between a scientist and a mathematician at a local meeting of teachers, in which it transpired rather late on that they were from the same school, but neither knew the other.' This is a fact in the new large campus style comprehensive schools of today, where teachers meet only infrequently, at staff meetings once or twice a year. The rest of the time they remain in their department's area of the school, in isolation. The author recommends that stimulating and constructive discussions between departments is essential and every school should make great efforts to set up strong links between departments which will be to the benefit of all concerned.

The summary of the problem areas quoted in interviews and established by the author in this research are:

(1) Although the mathematics syllabuses, both modern and traditional, cover the salient topic areas for science there appears to be a "Phase Difference" in the teaching of the mathematics and the requirements of science.

(2) There is no consultation between departments of a constructive helpful nature, either discussing requirements or explaining new syllabuses and methods of teaching.

"The most important step is for mathematics and science teachers actually to meet and discuss common problems. It should also be possible to organise the timetable so that joint action is possible" (13).

"Whilst in some schools there may be open hostilities between mathematics and science" (22).

(3) The mathematics being taught appears to lack relevance and application. Cockcroft Report (See 108)

"Preparation of materials for the classroom has also been carried out on a national scale. The School Mathematics Project (SMP) which has from the outset stressed the use of mathematics in application,
supported an Industrial Fellow for two years in 1964-66. His task was to gather examples of the uses of mathematics in various kinds of employment so that these could be incorporated into the SMP texts wherever possible. The fact that this initiative achieved only limited success again draws attention to the fact that the preparation of classroom materials related to the world of work is more difficult than might be expected" (7)

This not only applies to industry, but very much to other curriculum areas and in particular Physics.

(4) Although pupils appear to have been taught the mathematics concepts they cannot utilize them in other curriculum areas. They lack the ability or confidence to decide which mathematics concept(s) will be required to solve the problem.

(5) In mathematics lessons pupils work with contrived numbers, appropriate to the easy working out of the answer, for instance in graphical work, all the points fall exactly on the line. Pupils should be taught using "naturally" occurring numbers and data from real life situations, with the normal inbuilt errors or inexactitudes.

Cockcroft report (See 25)

"This perception of mathematics, and especially arithmetic, as something which is supposed to lead to exact answers by the use of proper methods seemed to be quite common despite the fact that the numbers which arise in everyday life very often need to be rounded or approximated in some way." (7)

(6) Pupils must be encouraged to estimate as this is important in recognising whether or not an answer is reasonable and it will be of great use in life generally. Many pupils when faced with this process in physics find it very difficult and never become efficient in its usage.

"estimation is not practiced in very many classrooms" (7)
(7) A greater awareness of units is required. In mathematics lessons the pupils just work with numbers, whilst in physics, and science in general, they work with quantities, which have magnitude and units. Most mathematics teachers fail to appreciate that the real world is a world of quantities. For instance in a laboratory the pupil will be measuring and recording quantities such as length, volume, amounts of heat, etc, so for the mathematician to concentrate solely on numerical values is wrong.

(8) Understanding is important rather than mechanical ability and to this end the author feels that discussion should be encouraged during mathematics lessons. Mental arithmetic or verbal working out practiced. The pupils have difficulty in using different terminology, for instance, ADD, CALCULATE, FIND THE SUM, EVALUATE, for the same operation. Often when pupils meet an unusual way of phrasing a problem he will not understand what is required of him, and on many occasions will just guess at what process is required from the size of the numbers.

(9) In the area of graphical work, many more examples from the real world should be used and the pupils should be taught the "Language of Graphs" so that they can easily interpret the information held in the graphs found in newspapers, science, etc.

(10) The mathematics and physics (science) courses should be realigned into an acceptable phase relationship by inter-departmental discussion. This is required, to a greater extent, for those schools teaching modern mathematics where many of the topics appear at a later stage than many Physicists would like, who would normally begin to make their courses more quantitative in the 3rd/4th years but the modern mathematics 3rd year work shows that the pupils have little experience of manipulating and working with numerical values. Many modern mathematics courses try to consolidate basic skills of numeracy and algebra into the 4th year, where traditional mathematics courses would have possibly done this in the second year.
There exists a totally unacceptable situation in England and Wales where there is a need for at least 3000 good mathematics teachers and if one considers those who are teaching mathematics in schools who are not suitably qualified then this number would be considerably higher. This problem leads to many having a large number of different mathematics teachers during their school lives and this cannot help them attain the necessary firm foundations in mathematical skills - continuity of teaching is very important for all concerned.

Appendix 1: "Work of mathematics departments in Derbyshire" highlights this problem in local terms, analysing the qualifications and teaching time of teachers at 77 schools in the county.

The terminology, concepts and methods used between mathematics and physics (science) in solving the same problems is not consistent and only serves to confuse the pupils. Consequently, the transfer of skills from mathematics lessons becomes even more unlikely.

Mathematics cannot tolerate the stigma of being the most 'boring' subject in the school curriculum. Something must be done to enliven the teaching, create interest and increase motivation in the pupils. The author suggests that this can be done by application of relevant material in both theoretical and practical forms of lessons.

Cockcroft relating to practical work at primary level (but the author feels this is also needed at secondary level) stated:

"We believe that this broadening of curriculum has had a beneficial effect both in improving children's attitudes to mathematics and also in laying the foundations of better understanding" (7) (see p 286)

The development of a suitable textbook is necessary to aid the mathematics teacher show the application of his subject in other curriculum areas, such as the sciences, geography, economics the craft subjects, etc. This has been done very satisfactorily by Professor Bajpai and R Bond in their book "Apprentice Maths" [33] for craft apprentices; perhaps this idea should be developed for schools.
(15) A greater amount of practice in arithmetic concepts or skills is required, starting in primary school and continuing through into secondary school, and "that work we do with young children from age 5 to 11 may well establish for a child either a love of mathematics upon which our colleagues in secondary school can build upon, or sadly a distaste or even fear of the subject" (33)

(16) As calculators are becoming ever more available the pupils should become familiar with their use. However, an order of magnitude calculation should be done to check that the correct buttons have been pressed. Hence the understanding of significant figures, and rounding off to within the required order of accuracy are important.

Much work of an enjoyable nature can be done with calculators to build up generalisation of the effect of operations in a much wider field of decimal numbers than their own limited experience of natural numbers would have led to, it enables them to attain a certain feeling for the kinds of answers expected, for instance the calculator has the role of giving the pupils the freedom to consider the effect of say multiplying rather than the process of it. Much work has to be done in this area rather than fearing the use of the calculator in mathematics lessons.

(17) Perhaps we expect too much from our middle/lower ability range. Much of the work expects them to have attained Piaget's formal operation stage, whereas the lowest 20% of the school population never reaches this stage in mathematics. However, in science the pupils work more in the concrete operational stage and make generalisations which could be regarded as possible formal operations.

(18) Mathematics teachers should gain industrial experience in order to enable them to show a natural relevance to their teaching rather than contrived ones. Many industrial courses such as C.N.A.A degree courses require engineers to gain industrial experiences throughout the course as well as theoretical knowledge before qualifying. This would be a useful experience for teachers undergoing training.
Pupils attitudes in the 'relaxed society' of no discipline and mass unemployment does not make the teachers job any easier. Many parents, and in particular those of the middle to lower ability pupils' range, do not care about their children's education. This is stressed in the work of Gattenby (26) who attributes the degree of motivation and attainment of a pupil to their family background.

"With a few of the more deprived children there was already expectation of a life of unemployment ... absences from the 5th year averaged 20% in 1979/80 but there were many pupils attending only rarely ..." (26)

Many of these problems have been established previously in other research in mathematical related fields, for instance, Graham (23) relating to how relevance can increase motivation in Craft Apprentices said

"Once the 16 year old goes into industry and sees the application of mathematics, however, he becomes motivated and his performance in learning increases dramatically" (23)

and Bond (27) recommended the development of ideas of Mathematical Laboratories as outlined by Baig (28) where practical applications could be demonstrated by the use of equipment. He stated:

"This method is successfully used by some schools during physics experiments" (28)

The intention of this form of work would be to reinforce classroom sessions in order to ensure that the common core topics would not be overlooked.

Bond (27) stated in reference to the report of the ETIB (29)

"Probably the most important conclusion drawn from the study was that MOTIVATION of pupils to learn whatever mathematics is presented to them is sure to be critical to mathematical attainment ..." and "I would like to see all mathematical concepts related to real life situations, or, in the case of pure mathematics developed from the basic mathematics of life and modern living". (27)
Gattenby (26) in his evaluation of the book "Apprentice Maths" which was designed to aid the acquisition of knowledge by the use of relevant applied material concluded:

"The overwhelming enthusiasm and positive response to applied maths convinced the author (Gattenby) that there is great benefit to be gained from introducing this material in schools" (26)

If this material, relevant to craft apprentices is so desirable, why not develop materials relevant to the sciences and other curriculum areas?

It is important to start early in Year 1 or before, in Primary School, showing the relevance and application of mathematics because by Year 3 it could be too late and many pupils would be lost to mathematics, science and engineering, which then must affect the Technological status of this country.

At the primary stage practical work is considered invaluable and it has been said "it is evident that mathematics and science must frequently overlap and reinforce each other. Sometimes it is difficult to distinguish between the two aspects of the curriculum" (34).

Cockcroft (7) referred to practical work of the type the author is advocating:

"Practical work is fundamental to the development of mathematics at a primary state ...It is too often assumed that the need for practical activities ceases at the secondary stage but this is not the case. Nor is it the case that practical activities are needed only for pupils whose ability is low, pupils of all levels of attainment can benefit from opportunities for appropriate practical experience" (7). (See p 247)

The Royal Society (2) also tells us that we should bear in mind the relevance of mathematics in trying to foster engineering in schools and improve its image, and Professor Geoffrey Matthews and Margaret Brown (15) stated:
"We have hitherto been too concerned with unity of mathematics and must now build up relationships with the real world. The mathematics of the world of the pupils was felt to be the first priority for the future"

The author argues that part of the pupils' future is science, any improvement in the image of physics and chemistry and the pupils' ability to cope with the mathematics will eventually foster better relations with engineering. So for all concerned it is very important that mathematics departments do make it their first priority.

In assessing the work carried out by the author in integrating mathematics and science in the form of a series of experiments he concludes that a large amount of success in increasing motivation can be achieved in this way, and with such material creating the benefits of increased attainment. The pupils in this work showed interest and enjoyment and under such circumstances discipline was very easily maintained. It was apparent that the middle to lower ability pupils loved to do things with their hands, once the enjoyment aspect was realised, then with this stimulus they could be moulded into performing the collection of data, calculations, graphs etc, quite easily and fairly effectively. Even those who still had trouble with the basic mathematics (revision would be required often) were amenable to help and made the required effort within the limits of their ability.

"The potential craft apprentice is motivated by perceiving the application" (26)

These pupils worked better and more efficiently when there was an easy-to-see reason or relevance to the work. The author feels strongly enough to continue this form of integration in his teaching of physics, and feels that with careful thought a whole series of experiments and related work could be devised, which would give a pupil of this ability range a sufficient knowledge of both mathematics and science required for life in the real world at the same time. Perhaps an added benefit for the pupils would be a form of team teaching where both mathematician and
scientist conduct the same lesson.

Under the present CSE regulations a Mode 3 type examination could be written and administered which if presented to local employers could prove acceptable to them as a form of entrance qualification. This would not take the place of a mathematics examination but could run in conjunction with a more abstract non-utilitarian course that some mathematician would advocate.

The author would suggest that the problem does not only fall at the door of the mathematics department, but also at the door of the physics department. Physicists must change also, to realise some of the needs of the mathematics departments, they should make every effort to be familiar with the concepts and potentialities of the 'new ideas' in mathematics teaching.

"The scientists, in return, have been asked to be sympathetic to the use of flow diagrams and inverse mappings' and to try to avoid using heretical suggestion like - "take it to the other side, change the sign or cross multiply or minus times minus equals plus" (30)

The pupils' comments indicated the need for this application and practical form of mathematics, to overcome the boredom and instinctive dislike of this subject and any other subject associated with it. The feeling of the comments was very clear, and often well thought out, so one could not say this was a typical malaise found in school children today. It was interesting that the vast majority of the pupils realised that mathematics was generally important but they felt that the work they were doing was too 'abstract' and that it should become more relevant to them. The application of mathematics and practical lessons carried out in this work was given overwhelming approval and the opinion was that more mathematics lessons should be like this. The pupils enjoyed this sort of work and although the sample was not prolonged enough to be able to state uncategorically that there was an improvement in attainment, the author argues that where there is enjoyment, interest and motivation, then improvement in standards will follow.
The review of a recent BBC programme aimed at adult numeracy schemes stated

"Throughout the series constant references were made to ways various jobs made use of mathematics. We saw swabs being weighed in hospital to measure a patient's blood loss during operations. We studied ratios of mixtures of ingredients like pig swill and for city dwellers, concrete" (31)

These TV programmes tried to show more generally the type of application the author has shown in his work with his physics pupils. Such programmes and similar work in science would be a major source of inspiration to mathematics teachers interested in enriching the teaching of mathematics and give those pupils who are fed-up with the artificiality of many school mathematics applications, (if the work is applied at all) a real interest in their work.

The high numbers of teachers of mathematics that these children have had, up to 8 in three years, and this is not restricted to this school but similar in many schools throughout England and Wales, can only have done harm to their education. It is very important to attract mathematics teachers to the profession in order to improve on this situation. These teachers should if possible be a new 'breed' with a sound theoretical and applied knowledge.

It was noticed that many girls opted out of physics, in the survey pupils there were only 10 girls from the total of 73 pupils taking physics and a total of approximately 120 girls in the year group. The Royal Society are aware of this problem and are at present about to publish the results of the work of a committee under the chairmanship of Professor Blin-Stoyle. The author believes, and this is substantiated by the comments of girl pupils that the majority were put off physics due to mathematical content of the course, and many thought it a qualification only for engineers who were dirt and oil-covered people (men) working under horrid conditions.
Truancy is a big problem, the average attendance at this school was 80% for 5th formers, which is approximately the national average. This does not take into account the relatively new trend of 'internal truancy' where pupils get the attendance mark but fail to turn up to lessons, just wandering about the school and hiding away. Gattenby (26) acknowledges this problem and the effects on continuity and attainment and in an article in the Daily Mail newspaper (32) which discussed a mathematics test on 8000 school leavers carried out by the Institute of Mathematics and the disastrous results, stated

"The Institute of Mathematics sense of shock was increased by the knowledge that the results would have been even worse but for truancy. For many absent children were likely to be the poorest at their work" (32)

Although at this time there was a lot of vociferous comments on the type of mathematics being taught and its lack of relevance to the real world, the author could detect no apparent changes taking place, or planned for the future. The Cockcroft report has also very firmly stressed this problem and the author feels that examining boards have more control over what is taught and how. A movement by the examining boards towards greater application and relevance would mean that teachers would ultimately have to comply. However, the way the application and relevance is covered is important, work must be done at teachers training level with courses including application and industrial experience; at classroom level with increased inservice training of the kind that establishes contact between teachers and industry, (this may have to take a compulsory nature), material of a suitable nature must be developed and published to aid teachers who have difficulty in this area; money must be made available for purchase of this material and equipment to carry out the practical nature of the application. The present type of mathematics classroom will most likely be unsuitable and preferably laboratory type classrooms should be used. It may mean that science and mathematics departments share
laboratories, thereby facilitating added contact of staff, common equipment and surroundings.

In conclusion, the author stresses the need for lively, interesting, relevant and motivating mathematical courses for all ability ranges, if this problem is not to continue its downward trend at the expense of the population and the general run down of technology.

Physics appears to suffer more from the pupil's lack of mathematical ability than other curriculum areas and this is highly regrettable because of its interlinking fundamental nature with other sciences. It has a great importance in taking up a career, not only as a physicist, but as an engineer or technologist, and also because it trains the mind in logical and systematic problem solving. Those put off taking physics at 13+ by the mathematical content of the courses are deprived of career opportunities and must be a significant contribution to the shortage of Scientists, Engineers and Technologists in Britain today.

Recommendations:

(a) Teachers must be made to illustrate the curriculum in ways which appeal and motivate, by using more person-oriented situations from the real world. To this end the LEA's should give a better 'Inservice Training' support to teachers who have not got the required wealth of experience, in the form of courses at Teachers' Centres and ultimately the appointment in every school of 'onsite' expertise.

(b) Teachers' Training courses must include more work on the use of mathematics as a relevant tool and this should be backed up with industrial experience of the kind the CNAA degree courses include, or compulsory secondment to industry for a period of time.
(c) Teachers of mathematics must contrive a continuity to their course between Primary and Secondary education, with well designed records of the progress of pupils. Promoting mathematics at an early age by application and relevance is essential.

(d) An extension of the schools' careers service, to discussions with pupils in the earlier school years, promoting mathematics and physics for employment before it is too late and the pupils have wasted valuable time up to their Option Choices, or before they realise how essential a requirement mathematics is to life's needs.

(e) The development of Mathematical Laboratories, where equipment and other resources are readily available and hence more practical work may lead to a higher level of motivation and ultimately understanding, as was observed by the author during this research.

(f) A collective recognition of the problem and a commitment to change through the co-ordination of links with other departments. Perhaps initially some curriculum development towards a broad-based mathematics/science course examined at CSE on a Mode 3 basis or included in future 16+ developments.

(g) Mathematics courses should include a lot more oral and discussion type of work from both teacher and pupils. Estimation is a valuable technique used often in life and other curriculum areas, it needs to be well practiced.

(h) New textbooks are needed which include relevant application of mathematics, experimental work of a motivating, interesting nature and most essentially written at an appropriate reading age.

(i) Greater effort must be applied to promote mathematics, science and engineering to girls who seem to suffer in the system more than boys.
(j) The extent and intellectual demand of mathematics and science courses needs reviewing. These changes must have the support of all branches of education, schools, teachers, local education authorities, and government, and also the backing of industry and commerce. There must be a determination to make changes for the better.

(k) New examination courses need developing by the examination boards to foster the need for curriculum development in this area.

(l) The LEA's should employ more suitably qualified and trained mathematics teachers to make right the appalling deficit which exist today, to allow more preparation time and smaller class sizes, similar to those in practical subjects.

Suggestions for the Future

The work carried out in this project has convinced the author that motivation can be gained through a practical or integrated form of mathematics. He feels that there is a need for a vast pool of resource information regarding application, and its compilation into new textbooks.

There is scope to investigate the possibility of an integrated mathematics teaching project, where both mathematician and scientist teach pupils together - a form of team teaching, with the syllabus split into integrated modules.

Much work is necessary in developing relationships between local employers/industry and schools, and hence through discussion formulate the most suitable curriculum and syllabuses.

Pupils awareness of mathematics in an industrial or real world context could be improved by the development of suitable audio-visual aids, ie. a series of video films showing skills gained in the classroom being used in industry, etc.
The development of techniques which give pupils a greater cognitive understanding, for instance work on the language of graphs or the feel for numbers by using calculators or computers.

There is need for work to be carried out into the way girls are taught mathematics, how they cope in co-educational establishments and how the curriculum can be illustrated in ways which appeal to the person-oriented situation. Hence promoting careers in Mathematics, Science, Engineering and Technology.

Further work could be carried out into the correlation between reading age and mathematical ability, followed by a survey of the textbooks available and their appropriateness for use with pupils of different ages and levels of ability.

It is hoped that the work carried out by the author in this Thesis will help the reader appreciate the value that application of mathematics in a useful real world situation can have on increasing motivation and subsequent understanding, not only for mathematics but for all other curriculum areas.
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APPENDIX I

ANALYSIS OF THE WORK OF MATHEMATICS DEPARTMENT IN
DERBYSHIRE SECONDARY SCHOOLS. CSE STATISTICS
AN ANALYSIS OF THE WORK OF MATHEMATICS DEPARTMENTS IN DERBYSHIRE SECONDARY SCHOOLS

A survey of the teaching of mathematics in 77 Derbyshire Schools indicated:

1) Staffing of Mathematics Departments

A) Qualifications of Mathematics Teachers:

Numbers of those teaching mathematics who have: %

A degree in Mathematics with a post-graduate certificate in Education 15

A degree in which mathematics is one of the subjects with a post-graduate certificate in Education 9

Followed a main or advanced course in Mathematics (leading to a certificate or B.Ed) at a college of Education or Education Department of a Polytechnic for teaching in:

a) Secondary School 25
b) Junior/Secondary(or middle) Schools 6
c) Junior Schools 2

A degree in Mathematics (or a recognised equivalent) 3

A degree in which Mathematics is one of the subjects (or a recognised equivalent) 3

A degree or teaching qualification in

i) Computing 1
ii) A Science 7
iii) P.E. 5
iv) Psychology, Economics, English, Art, Geography etc. 14

An O.U. Degree with some Mathematics 10

100
b) The number of Teachers, the Proportion of Time Spent Teaching Mathematics and the Scales of the Posts that the Teachers hold (not Necessarily Scale Posts for Mathematics)

<table>
<thead>
<tr>
<th>Time spent Teaching Mathematics</th>
<th>Scale 1</th>
<th>Scale 2</th>
<th>Scale 3</th>
<th>Scale 4</th>
<th>Senior Teacher</th>
<th>Deputy Head Teacher</th>
<th>Head Teacher</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>More than 75% of week</td>
<td>135</td>
<td>91</td>
<td>47</td>
<td>38</td>
<td>12</td>
<td>3</td>
<td>0</td>
<td>326</td>
</tr>
<tr>
<td>75% of the week or less</td>
<td>27</td>
<td>30</td>
<td>23</td>
<td>17</td>
<td>8</td>
<td>13</td>
<td>1</td>
<td>119</td>
</tr>
<tr>
<td>50% &quot; &quot; &quot; &quot; &quot;</td>
<td>44</td>
<td>21</td>
<td>20</td>
<td>11</td>
<td>2</td>
<td>15</td>
<td>1</td>
<td>117</td>
</tr>
<tr>
<td>25% &quot; &quot; &quot; &quot; &quot;</td>
<td>31</td>
<td>14</td>
<td>20</td>
<td>5</td>
<td>2</td>
<td>8</td>
<td>2</td>
<td>82</td>
</tr>
<tr>
<td>12.5% &quot; &quot; &quot; &quot; &quot;</td>
<td>20</td>
<td>14</td>
<td>17</td>
<td>9</td>
<td>0</td>
<td>10</td>
<td>9</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td>257</td>
<td>170</td>
<td>130</td>
<td>78</td>
<td>24</td>
<td>49</td>
<td>13</td>
<td>721</td>
</tr>
<tr>
<td>%</td>
<td>35.6</td>
<td>23.6</td>
<td>18.0</td>
<td>10.8</td>
<td>3.3</td>
<td>6.8</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>

The county's requirements, based on the total of Mathematics taught, is the equivalent of 504 full-time teachers. Of this the equivalent of 34 teachers is covered by experienced, competent teachers in senior posts, and the equivalent of 40 teachers is covered by under-qualified teachers.

Further statistics

1) Only 65% of those teaching the subject are qualified in Mathematics.
2) 24% entered the profession late.
3) 30% of the teachers have taught only in one school.
4) 52% of the teachers have more than 10 years teaching experience and only 5% have more than 30 years experience.
5) 8% were trained to teach younger children.
6) 35% of those teaching Mathematics are not qualified to do so or might be considered to lack adequate background (Derbyshire County Council Nov. '82)
7) 10% shortfall in the requirements for Mathematics teachers in Derbyshire.
8) Derbyshire County Councils' Director of Education states! "Base materials, apparatus and mathematical games, puzzles and extension materials are generally not available in schools except in remedial departments. The ability to understand and apply mathematics would be fostered by a greater amount of practical work. (John Evans, Director of Education. Matlock May '82)
**East Midlands CSE Board Statistics for 1982**

Number of examination centres: 391
Number of candidates: 56,904
Number of entries per candidate: 4.9

**Entries by Mode of Examination**

<table>
<thead>
<tr>
<th>Mode</th>
<th>Overall Enters (%)</th>
<th>Physics Enters</th>
<th>Maths Enters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>76.6% (213,396)</td>
<td>(10,275)</td>
<td>(32,941)</td>
</tr>
<tr>
<td>Mode 2</td>
<td>1.2% (3,376)</td>
<td>-</td>
<td>(1,404)</td>
</tr>
<tr>
<td>Mode 3</td>
<td>22.2% (278,909)</td>
<td>(1,412)</td>
<td>(6,691)</td>
</tr>
</tbody>
</table>

**Distribution of Grades**

<table>
<thead>
<tr>
<th>Grade</th>
<th>No of entries 1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>U</th>
<th>Mean grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall entry</td>
<td>11.9 21.1 23.0 27.3 12.7</td>
<td>4.0 3.19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physics entry</td>
<td>18.6</td>
<td>9.8 22.9 24.0 24.7 16.0</td>
<td>2.7 3.26</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maths entry</td>
<td>57.89</td>
<td>13.9 13.6 15.0 23.7 22.2 11.5 3.65</td>
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<tr>
<td>Geography entry</td>
<td>17.42</td>
<td>10.1 18.8 20.4 35.0 13.1 2.7 3.31</td>
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<td></td>
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<td></td>
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<tr>
<td>History entries</td>
<td>18.78</td>
<td>7.4 27.3 25.3 25.7 30.2 7.8 3.36</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemistry entries</td>
<td>14.63</td>
<td>13.2 23.8 22.8 27.0 12.2 1.0 3.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Biology entries</td>
<td>17.87</td>
<td>6.6 19.8 20.0 38.9 12.4 2.2 3.13</td>
<td></td>
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<td></td>
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<tr>
<td>Homecraft entries</td>
<td>13.48</td>
<td>11.3 36.2 39.4 11.6 1.0 0.1 2.55</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Art entries</td>
<td>17.62</td>
<td>16.4 23.5 22.8 24.8 11.9 0.7 2.94</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>English entries</td>
<td>42.33</td>
<td>10.8 22.3 24.3 31.6 8.5 2.3 3.15</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>French entries</td>
<td>16.85</td>
<td>10.5 27.3 28.2 24.5 8.8 0.4 2.94</td>
<td></td>
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</tbody>
</table>
APPENDIX II

GENERAL COMMENTS AND ANALYSIS RELATING TO TESTS GIVEN,

TESTS AND PUPILS' WORK
General Comments on Tests and Examinations

The following section details some of the successes and failures of the pupils in the survey when coping with the mathematics required up to the CSE standard.

With regards to basic numeracy:

1. Some pupils had not mastered the art of subtraction eg 85 - 48 or 3067 or £5 - £1.83 but these tended to be the weakest pupils whose work was often careless and untidy.

2. Only a few of the average and below pupils were able to translate large, spoken numbers into figures, for example half a million or one million and fifty five thousand and sixty seven; and to work out 4.37 \times 10 \text{ or } 4.37 \times 1000 \text{ or } 4.37 \div 10. Majority of pupils thought $3 \times 4 \times 0 = 12$

3. Estimation proved to be beyond the ability of all but the best pupils, for example a problem to be estimated to 1 significant figure 852 \div 29.2 or 438 \times 21.94. It was very clear that this topic needed a lot of practice, and also an explanation of significant figures. Estimation is essential for checking whether results taken during experiments are going to "fit" the theory.

4. Division produced many unusual methods of obtaining answers. These varied from forms of mental arithmetic or short division, to a process of repeated addition, on problems such as 24 test tubes cost £4.32, calculate the cost of 1 test tube; or potassium permanganate costs £2.16 for a bottle containing 24 grams, how much does one gram cost. One pupil, who on performing some form of mental calculation, came to the conclusion that there was a remainder in his calculation and that this must have been the cost of the bottle.
"Timetables" and the recovery of information indicated that some pupils thought there were 100 minutes in an hour and further confusion arose if the 24 hour clock was used.

"Rounding off" produced a lot of muddled thinking among the average to below pupils. For instance, the number of free electrons in 1cm$^3$ of material is 31,658 round off to the nearest hundred or round off the cost of electricity of £47.65 to the nearest pound.

Percentage questions produced a common error, that is, the pupils knew they had to make a fraction, but were not sure of the right form. An example: The repair bill for the laboratory pack totalled £40. VAT is 15% and has to be added to the bill. How much is the VAT. Answer $\frac{15}{40}$ or $\frac{40}{15}$ were frequently given, or 8% of the sample of 120 items was commonly given as $\frac{8}{120} \times 100$.

The use of algebra appeared to be more confusing to the pupils than any other section. Even the substitution of numbers into a simple formula proved beyond most. For example conversion of ounces to grams. No of grams = 28.3$x$ where $x$ = number of ounces, or conversion of Fahrenheit into Centigrade. $^\circ$C = $(^\circ$F - 32)$ \times \frac{5}{9}$. Further more inequality signs $>$ or $<$ were not understood by the majority.

Trigonometry was a stumbling block, very few were able to use "Tables" for sine, cosine or tangent values and none were able to calculate an angle from given information. For instance, in a right angled triangle XYZ, angle Z = $90^\circ$, YZ = 7cm and XY = 12cm. Find the angle X to the nearest degree. Most pupils attempted to construct this question but the accuracy was far from the given tolerance and was performed in felt tip, biro, blunt pencil, etc. No pupil possessed either a protractor or a compass.

Work with different number bases was weak, and most pupils had vague knowledge of both the metric and imperial systems. Problems include base 8 work on pints and gallons or base 16 on ounces and pounds.
Decimals proved difficult, as was expected, and powers or standard form proved beyond the ability of the average to below pupils. Even the very able pupils only had an idea of how to tackle questions like: Calculate $7 \div 0.5$ or write $2.6 \times 10^{-2}$ as a decimal number or write $400 + 5000$ as a single number in standard form. There was very little familiarity with the use of half spaces instead of commas in numbers with 4 or more digits.

Weakness with fractions was universal amongst these pupils. During discussion none could answer the question "What is a half times a half". Other questions $\frac{1}{2} + \frac{2}{5}$ or $\frac{1}{2} - \frac{2}{5}$ or conversion of a fraction to a decimal, for instance $\frac{3}{8}$ as a decimal could not be done by the majority.

Square numbers proved troublesome and the most common errors were $200^2 = 4000$ or $100^2 = 2000$ or $4^2 = 8$.

In formula work many could not substitute correctly and very few pupils were able to relate to work in a practical situation. Rearranging formula was very badly done, such as, rearrange the formulae to make 'h' the subject of each (1) $b = h + a$ (2) $V = Lbh$ (3) $V = \pi r^2 h$ (4) $A = \frac{1}{2}(a + b)h$.

Poor multiplication especially by 0 and numbers greater than 10. Some were not familiar with the term "multiply".

Questions involving co-ordinates, graphs of equations or tabulated values and gradients proved unpopular. Many could not decide on the correct scale to use on the axes and were unable to plot the points accurately. No pupil could calculate the slope (or gradient) of a given graph.

Units. Questions involving units invariably were given answers without stating units, as in area and volume calculations. In the case of mixed units, such as centimetres and metres in an area question, most pupils...
simply multiplied the two numbers together, without previously converting one of the quantities

The tests seemed to indicate the need for attention in the following areas:-

1 Number work (mainly for less able and particularly multiplication and division)

2 Fractions

3 Percentages

4 Estimation/rounding off

5 Decimals, negative powers and standard form

6 Trigonometry (angle work for less able)

7 Formulae - substitution, rearranging and solving

8 Co-ordinates/graph work/ratios or proportions

9 Algebra

10 Measurement (including time and railway timetable work)

Other topic areas such as number bases, Venn diagrams (KEYS in scientific discrimination between species)

Netts, etc, were not well-understood, but not likely to be required very often in Physics. All other areas tested were thought to be of a reasonably adequate standard for the majority.
1. \(14 + 35\)
2. \(43 + 282\)
3. \(77 - 53\)
4. \(911 - 102\)
5. \(7 \times 8\)
6. \(6 \times 79\)
7. \(24 \div 6\)
8. \(243 \div 9\)
9. \(13.3 + 2.8\)
10. \(79.3 - 8.1\)
11. \(3 \times 42.5\)
12. \(13.5 \div 5\)
13. Write \(\frac{1}{4}\) as a percentage
14. Write 40% as a decimal
15. Shade in \(\frac{2}{3}\) of this diagram
16. What fraction of this diagram is SHADED

Test results of 33 candidates:

<table>
<thead>
<tr>
<th>Question number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>% correct answer</td>
<td>100</td>
<td>89</td>
<td>89</td>
<td>75</td>
<td>84</td>
<td>75</td>
<td>92</td>
<td>82</td>
<td>80</td>
<td>70</td>
<td>63</td>
<td>52</td>
<td>48</td>
<td>30</td>
<td>62</td>
<td>52</td>
</tr>
</tbody>
</table>
1. Add the weights 1035g and 77g and 988g
2. What is the difference in weight of these items 2141g and 317g
3. $729 \times 18$
4. $10.2 + 1.901 + 0.037$
5. $2937 \div 11$
6. $1.113 \times 16$
7. $48 \div 0.4$
8. $2\frac{1}{5} + 1\frac{1}{3}$
9. $3\frac{3}{4} - 2\frac{7}{8}$
10. $3\frac{1}{8} \times 4\frac{4}{5}$
11. $4\frac{1}{2} \div 1\frac{1}{2}$
12. A circle has a diameter of 20 cm. Calculate its circumference ($\pi = 3.142$)
13. $D = \frac{M}{V}$ rearrange this formula to find "V"
14. If $D = 1.1g/cm^3$ and $M = 132$ g. What is 'V' in the above question? State the units
15. Calculate the hypotenuse of this triangle
16. Draw a graph of velocity against time for the following data
   $V$ cm/s: 0 30 55 86 116 130 140 145 150 150 150
   Time s: 0 2 5.1 10.2 20 30.2 40.3 50 60 70 80
Results of the test on 40 candidates

<table>
<thead>
<tr>
<th>Question number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>% correct answer</td>
<td>93</td>
<td>72</td>
<td>83</td>
<td>77</td>
<td>86</td>
<td>77</td>
<td>46</td>
<td>58</td>
<td>40</td>
<td>32</td>
<td>50</td>
<td>19</td>
<td>42</td>
<td>23</td>
<td>28</td>
</tr>
</tbody>
</table>
(1) Add 4532
   125
   6500
   3491
   92

(2) Subtract 3887 from 20542

(3) 651 \times 47 \times 14

(4) Add $1 \frac{3}{4}$ and $2 \frac{2}{5}$

(5) What is half of a half?

(6) $\frac{5}{6} \div \frac{2}{3}$

(7) Write $\frac{2}{5}$ as a decimal and a percentage

(8) What is 15% of 40?

(9) 24 test tubes cost £4.32. How much does 1 cost?

(10) 4.37 \times 100

(11) 4.37 \div 100

(12) What is 4\??

(13) 3 \times 4 \times 0

(14) Estimate to 1 significant figure $438 \times 21.94$

(15) Divide 4 by 50.*

Results for 21 candidates

<table>
<thead>
<tr>
<th>Question Number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
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<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
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</thead>
<tbody>
<tr>
<td>% Correct Answers</td>
<td>87</td>
<td>55</td>
<td>38</td>
<td>48</td>
<td>80</td>
<td>34</td>
<td>10</td>
<td>0</td>
<td>50</td>
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<td>24</td>
<td>80</td>
<td>50</td>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

* Question 15. 60% of candidates carried out 50 \div 4 rather than 4 \div 50.
WRITE THE FOLLOWING AS NUMBER USING DECIMAL POINTS WHERE NEEDED

1  SEVEN UNITS

2  EIGHT HUNDREDS

3  FOUR TENTHS

4  TWO HUNDREDTHS

5  FIVE THOUSAND

6  TWO AND THREE TENTHS

7  THREE AND NINE HUNDREDTHS

8  FOUR THOUSAND, SEVEN TENS, SIX UNITS AND TWO TENTHS

9  1 MILLION, TEN THOUSAND AND SIX UNITS

10 1 HUNDRED THOUSAND
Plot the following points on the graph paper below:

(20, 15)
(-14, 3)
(5, -12)
1. What are the co-ordinates of point A?

2. What is the value of x when y = 2?

3. What is the gradient of the graph?

4. Describe what is happening at stages of motion in the following graph

5. The motion of a car

At which point A, B or C does the car have the greatest velocity?

Describe what is happening at E:

Between which points is the car (i) accelerating?

(ii) decelerating?
Analysis of the Test Paper on Graphs (Co-ordinates)

Being able to construct graphs is an essential feature of any Physics course and on several occasions the negative axes are also required. To this end the test was developed to see if the pupils were capable of deciding the correct scale in order to cover the whole of the range required and with the information of the points to be plotted, whether they would position the axes in an suitable position on the graph paper.

Results indicated that these pupils were not familiar with plotting points with negative co-ordinates. The vast majority (92%) started by drawing the axes along the bottom of the page and left-hand edge. Approximately 10% realised their error and tried again somewhere in the centre of the page. This was done without any thought or reference to the maximum and minimum values to be covered.

Similarly the construction of the scale along the axes caused major problems and invariably the axes were not labelled.

When positioning the points the accuracy was very poor and various means were used to indicate the position. A small percentage joined up these points with a zig-zag line and some used the first number of the co-ordinate to go up the y-axis and then along to the x value.

Generally some 80% of these pupils, who were at the start of their CSE course could not plot points on a graph.

This test was followed up with an exercise involving the recovery of information from a graph or interpretation of the trends shown. This was done late in the first term. At this stage 25% of the pupils were able to give the co-ordinates of the point A. However the difference in scales of the x and y axes caused a major problem to the remainder. Some 60% quote the x part of the co-ordinate correctly but failed with the y part.
Question 2

Nobody got this question correct. The value of x when y = 2. The vast majority thought if y had a value then x had to have one.

Question 3

Nobody at this stage of the course was capable of calculating the gradient of the graph. The term 'gradient' proved beyond the scope of many pupils' vocabulary. When informed it was the slope some pupils guessed at a number of degrees.

Question 4

Only 15% of the pupils were able to say what the graph represented. To these pupils graphs were to do with numbers, they were not aware that they carried a message. Some pupils related distance and time to speed and therefore discussed the change in speed so section B became constant or the same speed rather than there was no motion, it was stationary.

Question 5

50% of the pupils did not realise that C and D had the greatest velocity, instead quoted B because it had the steepest slope. Many pupils did not recognise the term velocity and were much more at home with speed. Only 30% of the pupils correctly answered that the car had stopped at point E and 55% were able to indicate the correct regions for acceleration and deceleration. This might have been expected given two choices. However those who gave incorrect answers indicated during discussion on this part of the problem that A to C was going uphill and the car must be slowing down and similarly D to E was downhill so the car would be going faster.
How long is the line?

From the line below measure and fill in the table. The required lengths are named. Length AB and EH have been done for you.

<table>
<thead>
<tr>
<th>MEASURE</th>
<th>LENGTH CM</th>
<th>LENGTH INCHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>2.1</td>
<td>0.8</td>
</tr>
<tr>
<td>AC</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AD</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>DE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GH</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EH</td>
<td>5.5</td>
<td>2.1</td>
</tr>
</tbody>
</table>

Now plot a graph of centimetres (cm) against inches for the points.
ANALYSIS OF THE TEST PAPER ON MEASUREMENT.

This was designed to test the pupils ability to measure accurately and then to tabulate the information before drawing a graph of the length in Imperial units against the length in Metric Units.

The author's intention was to test the familiarity of the pupils with each system and then their ability to decide on a suitable scale for their graph and the accuracy to which they plotted the points.

Question 1

There were only 10% of the pupils who gave wrong answers and these gave the answer as 7. As in previous tests, many answers were given without stating the units.

Question 2

Metric Measurement: There were 20% of the pupils who measured one or more of the lengths inaccurately. One pupil failed on seven of the eight measurements.

Imperial Measurement: This proved more difficult - 43% of the pupils failed to measure correctly one or more of the measurements and 33% of the pupils failed to get any correct at all.

Plotting the Graph: The scale proved to be unsuitable on 23% of the attempts and the accuracy of plotting was very poor - only 10% of the pupils were successful. One pupil drew a Bar chart and another used his intelligence and cheated by drawing a straight line from the origin through the only point he actually plotted, filling in the other points at random. The overall standard of the work was poor, untidy and carelessly done.
This kind of plotting data is essential in Physics where values achieved by experiment are not going to be nice round numbers. A great deal of attention is required in this area. Further discussion with many of the pupils revealed that there was much confusion about the Imperial Scales on their rules. Many could not work out what the smaller divisions were, where they were not labelled, i.e: for $\frac{1}{8}$ ths by counting 8 parts per inch hence deducing that each part was an $\frac{1}{8}$th of an inch. Fortunately, for science examinations, only the metric system would be used but surely in their lives and at work in the future these pupils would be faced many times with Imperial quantities.
WHAT IS THE LENGTH OF LINE IN EACH EXAMPLE BELOW

A

B

ANSWER
1. mm
2. mm

WHAT ARE THE AREAS OF THE FIGURES BELOW (STATE THE UNITS)

Area =

Area =

Area =

Workout the areas shaded (STATE THE UNITS)

Area =

Area =

Area =

What is the PERIMETER of the 'L shape' above?
Analysis of the Test Paper on Measurement

The test paper was designed to cover the basic measurement tasks expected of CSE pupils especially in the practical examination. Following this with area and its units which is necessary for many sections of the syllabus.

Question 1

Some 42% gave wrong answers, many ignored the scale and just counted main divisions calling them centimetres, in actual fact each main division represented 2 cm (20 mm). A further 38% failed to get the arithmetic correct or quote the units.

Question 2

This was attempted much better as the scale was 1 division = 1 mm even though it was upside down, only 8% failed to gain correct answers.

Question 3

The rectangle - 10% of the pupils gave wrong answers and a further 15% forgot the units

The trapezium - 15% of the pupils gave wrong answers and similarly forget the units

The 'L' shape - This problem involved fractions of a square unit and proved more difficult. 30% failed to get the correct answer

Question 4

Part 1 20% failed to get the correct answer, having difficulty with the decimal place and 50% either gave the wrong units or failed to quote them

Part 2 The 'L' shape produced 50% wrong answers. Many split it into two rectangles but included the "common area" in both rectangles
Part 3  28% of the pupils failed to get the correct answer. Again approximately 50% failed to state the units.

Question 5

As many as 92% were unable to work out the perimeter because they either did not know what it meant or failed with the arithmetic. The main fault being that they did not work out the lengths of the two sides not labelled but instead assumed them to be 8 cm each.
1. What is Volume?
2. What are the units of Volume?
3. What is the volume of the cube below?

4. If we make the sides of the cube TWICE as long, what is its new Volume?

5. How many 1 centimetre cubes would fit inside this box?

Number of 1 centimetre cubes: ................................

How many small cubes like this would fit into the same box?

Number of small cubes: ................................
Analysis of test paper on Volume

This paper had been designed to find out whether the pupils could calculate volumes of cubes or boxshapes initially with small whole numbers, proceeding with the relationship between volumes of different sized cubes and finally to test their use of units as these had appeared to be problem areas.

The results indicated that there was a confusion over the meaning of the term 'Volume'. Some 26% of the pupils quoted \( L \times b \times h \) and a further 16% added a correct definition; the remainder either gave poor or incorrect definition, or misquoted the formula. Some of the definitions were: The mass inside the Area or How many things you can get into a Square or Weight inside a square or What you put on a record player to turn up the sound or the area of an object.

The units of volume were incorrectly stated by 40% of the pupils, some stating inches or centimetres or square centimetres.

Question 3
The Volume of a cube whose sides were 2 cm long; 33% of the pupils failed to get the correct answer, 8 cm\(^3\), but quoted: \( 2 \times 2 \times 2 = 6 \text{ cm} \) or 4 was another popular answer, and 46% either did not quote or incorrectly quoted the units.

Question 4
The most popular answer was to multiply the answer to question 3 by a factor of 2, 43% of the pupils gave wrong answers.

Question 5
This was done a little better, only 12% gave the wrong answer for the number of 1 cm cubes which fit into the box. However when the cubes were reduced in size to have sides \( \frac{1}{2} \) cm long, there were only 10% with correct answers. The most popular answer was: double the first answer because the cube was half as big.
OTHER TESTS INCLUDED

(A)

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<td>529 \times 10</td>
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<tr>
<td>2</td>
<td>7.46 \times 10</td>
<td>74.6</td>
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<td>3</td>
<td>7.46 \times 10^2</td>
<td>746</td>
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<td>4</td>
<td>637.15 \times 10^3</td>
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<td>5</td>
<td>0.00365 \times 10^4</td>
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<td>6</td>
<td>529 \div 10</td>
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<td>7</td>
<td>7.46 \div 1000</td>
<td>0.00746</td>
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<td>8</td>
<td>7.46 \div 10</td>
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<td>9</td>
<td>637.15 \div 10^3</td>
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<td>10</td>
<td>0.0036 \div 10^2</td>
<td>3.6E-5</td>
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(B)

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<td>1</td>
<td>9 \overline{243}</td>
<td>928</td>
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<td>2</td>
<td>16 \overline{928}</td>
<td>16928</td>
</tr>
<tr>
<td>3</td>
<td>27.95 \div 1.3</td>
<td>21.64705882</td>
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<tr>
<td>4</td>
<td>0.3 \div 1.6</td>
<td>0.1875</td>
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<tr>
<td>5</td>
<td>10 \div 3 answer to 3DP</td>
<td>3.33333333</td>
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<tr>
<td>6</td>
<td>27 \times 9</td>
<td>243</td>
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<tr>
<td>7</td>
<td>165 \times 87</td>
<td>14453.5</td>
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<td>8</td>
<td>21.4 \times 3.2</td>
<td>68.28</td>
</tr>
<tr>
<td>9</td>
<td>0.03 \times 0.004</td>
<td>0.00012</td>
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<tr>
<td>10</td>
<td>3.035 \times 1000</td>
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(C)

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<tbody>
<tr>
<td>1</td>
<td>39 + 48</td>
<td>87</td>
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<tr>
<td>2</td>
<td>278 + 185 + 68</td>
<td>531</td>
</tr>
<tr>
<td>3</td>
<td>14.516 + 13.334</td>
<td>27.85</td>
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<tr>
<td>4</td>
<td>18.3 + 14.796 + 0.806 + 27.5</td>
<td>67.402</td>
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<td>5</td>
<td>2487 + 210.89 + 9.006 + 2574 + 3.97</td>
<td>7421.86</td>
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<td>6</td>
<td>48 - 39</td>
<td>9</td>
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<td>7</td>
<td>2487 - 2389</td>
<td>1298</td>
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<td>8</td>
<td>4.3 - 2.5</td>
<td>1.8</td>
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<td>9</td>
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<td>0.3875 - 0.2496</td>
<td>0.1379</td>
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(D)

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<td>\frac{7}{8} = \frac{21}{24}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>\frac{15}{4} change to mixed number</td>
<td>3 \frac{3}{4}</td>
</tr>
<tr>
<td>3</td>
<td>Write \frac{8}{5} as an Improper Fraction</td>
<td>1 \frac{3}{5}</td>
</tr>
<tr>
<td>4</td>
<td>Simplify \frac{16}{24}</td>
<td>\frac{2}{3}</td>
</tr>
<tr>
<td>5</td>
<td>\frac{1}{3} + \frac{1}{8} =</td>
<td>\frac{11}{24}</td>
</tr>
<tr>
<td>6</td>
<td>\frac{1}{4} + 3 \frac{1}{5} =</td>
<td>\frac{17}{2}</td>
</tr>
<tr>
<td>7</td>
<td>\frac{1}{2} - \frac{5}{16}</td>
<td>\frac{7}{16}</td>
</tr>
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<td>8</td>
<td>\frac{6}{8} + 2 \frac{1}{16} - \frac{3}{32}</td>
<td>\frac{23}{32}</td>
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<tr>
<td>9</td>
<td>\frac{1}{3} \times \frac{1}{4}</td>
<td>\frac{1}{12}</td>
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<tr>
<td>10</td>
<td>\frac{1}{2} \times \frac{1}{3}</td>
<td>\frac{1}{6}</td>
</tr>
<tr>
<td>11</td>
<td>\frac{1}{2} : \frac{1}{4}</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>17 \frac{1}{2} : 3 \frac{1}{2}</td>
<td>8 \frac{1}{3}</td>
</tr>
</tbody>
</table>
1 Write the following numbers correct to the required number of significant figures

(a) 16.752 to 3SF
(b) 191.34 to 4SF
(c) 6.751 to 3SF
(d) 0.003135 to 2SF
(e) 171000 to 1SF

2 Work out approximate answers to the following by using all numbers correct to 1 SF

(a) 35.1 × 141
(b) 110 × 16
(c) 1610 × 0.013
(d) 1595 ÷ 41
(e) 9.65 ÷ 99.5
VARIOUS EXAMPLES OF MISUNDERSTOOD CONCEPTS THAT APPEARED DURING THE TESTING OF THESE PUPILS

QUESTION: 100 - 29.786 =
ANSWER: This cannot be done – there are too many numbers in 29.786

QUESTION: Write down eight thousand, six hundred
ANSWER: 8000600 (When asked if this was correct the pupil could not see the error)

QUESTION: 4200 × 20 =
ANSWER:
\[
\begin{array}{c}
4200 \\
\times 20 \\
\hline \\
84000 \\
88200
\end{array}
\]

QUESTION: \(\frac{3}{10} + \frac{5}{10}\)
ANSWER: \(\frac{8}{20}\) (Pupil explained 3 out of 10 plus 5 out of 10 equals 8 out of 20)

QUESTION: CONVERT 71.86g to KILOGRAMS
ANSWER: 71.86 \(\div 1000\) (Pupil said that it was the only way of doing this problem as 1000 was too big to go into 71.86)

QUESTION: \(\frac{1}{2} \times \frac{1}{2}\)
ANSWER: 0.5
\[
\begin{array}{c}
0.5 \\
\times 0.5 \\
\hline \\
0.25 \\
0.25
\end{array}
\]
pupil stated that this line was necessary or else the working out would be wrong

QUESTION: \(10^2\) =
ANSWER: 20 (a very large percentage of pupils quoted this answer)
EXAMPLES OF SOME GRAPHICAL WORK:

IN INVOLVING: HOOKE'S LAW, INCHES N CENTIMETERS, LENGTH OF SIDE N AREA FOR A SQUARE, LENGTH OF SIDE N VOLUME FOR A CUBE.

This should be a line graph!
SAMPLES OF PUPILS WORK FOR TEST 1

3. \( 77 - 53 = 24 \)

10. \( 79.3 - 8.1 = \frac{793}{87.4} \)

12. \( 13.5 \div 5 = 2.7 \)

13. Write \( \frac{1}{4} \) as a percentage. 25

14. Write 40 per cent as a decimal. 0.4

15. Write 40 per cent as a decimal \( \frac{2}{5} \).

16. \( 79.3 - 8.1 = 1.2 \)

17. \( 13.5 \div 5 = 2.7 \)

18. \( 3 \times 42.5 = 3.4 \)

19. Write 40 per cent as a decimal. 0.4
16. What fraction of this diagram is shaded

\[ \text{60\%} \]

17. What fraction of this diagram is shaded

\[ \frac{1}{3} \]

19. \[ \begin{array}{c|c|c} 14 & 8.7 & 16 \\ \hline 7.6 & 19.0 & 26.6 \\ \hline 2.66 & 3.0 & 4.9 \\ \hline 1 & 5.8 & 5.8 \\ \end{array} \]

20. \[ \frac{20 \times 15}{100} = 0.25 = \frac{25}{100} \]

0.40

36 \[ \begin{array}{c} 2 \times 10 \\ 32 \\ 24 \end{array} \]

12 \[ \begin{array}{c} 7.95 \\ 8.1 \end{array} \]
1. 1.035
   0.77
   0.988
   2.1009
   \[ \checkmark \]

2. \[ \frac{x}{x} \]
   1.8249
   \[ \checkmark \]
   6. 1.113
   \[ \times 16 \]

3. 299 | 13122
   13 \times 18
   2392
   2990
   5382
   \[ \checkmark \]
   6. 6678
   11130
   17808
   120
   \[ \checkmark \]
   7. 148

5. \[ \frac{1}{901} = \frac{1}{901} \\
   0.037 \\
   10.200 \\
   12.138 \]
   \[ \checkmark \]

8. \[ \frac{2\frac{1}{3}}{3} + \frac{1}{2} = 3\frac{8}{15} \]
   \[ \frac{5}{3} + \frac{3}{15} + \frac{8}{15} \]
   \[ = 3\frac{8}{15} \]
   * This is a difference (subtract) not ADD

9. \[ 3\frac{3}{4} - 2\frac{7}{8} = \frac{1}{8} \]
   \[ \checkmark \]
   1\frac{1}{8} \]
8. \( \frac{2}{5} + \frac{13}{8} = \frac{3}{\frac{7}{8}} \times \)

9. \( \frac{3}{\frac{2}{7}} + \frac{2}{\frac{8}{9}} = \frac{1}{\frac{4}{9}} \times \)

10. \( \frac{3}{\frac{1}{5}} \times \frac{\frac{4}{9}}{\frac{1}{6}} = \frac{3}{\frac{5}{10}} \times \)

11. \( 3 \cdot 142 + \frac{20}{3} = 1 \times \)

12. \( \frac{\sqrt{162}}{\sqrt{9}} \times \)

13. \( x = \frac{\sqrt{9}}{\sqrt{4}} \times \)

14. \( 3\frac{4}{12} = \frac{3}{12} \times \)

(10) \( \frac{3\frac{1}{2}}{2} \times 4\frac{1}{2} = \frac{3}{10} \times \)

11. \( 4\frac{1}{2} \div \frac{1}{2} = 4 \frac{1}{2} \div \frac{1}{2} = \frac{1}{\frac{2}{4}} \times \)

12. \( 3\frac{\frac{1}{8}}{\frac{1}{9}} \div 4\frac{1}{2} = 3\frac{\frac{1}{8}}{\frac{1}{9}} \times \)

13. \( 4\frac{1}{2} \div \frac{1}{2} = 4 \times \)

14. \( 3\frac{4}{12} = \frac{3}{12} \times \)
1. \[
\begin{align*}
1035 & \quad 3299 \div 13 \div 22 \\
7 & \quad \times 18 \\
982 & \quad 2392 \\
2100 & \quad 2900 \\
\underline{264} & \quad \frac{5382}{264} \\
\end{align*}
\]
2. \[
182.4 \checkmark \\
\frac{4.113 \times 973}{17.808} \checkmark
\]
5. \[
\begin{align*}
1.701 & \\
0.034 & \times 16 \\
10.200 & \quad 66.48 \\
12.138 & \quad 11130 \\
\underline{17808} & \quad \text{and 120}
\end{align*}
\]
8. \[
2\frac{1}{5} + 1\frac{1}{3} = 4.142
\]
9. \[
9.5 = 102\frac{1}{2}
\]
11. \[
\begin{align*}
\text{b) } 2\frac{1}{5} + 1\frac{1}{3} = \frac{11}{5} - \frac{3}{1} = 3\frac{\frac{5}{3}}{1}
\end{align*}
\]
12. \[
\begin{align*}
\text{c) } \frac{9}{214.2} = 0 \\
\text{d) } v = d \div t \\
\text{e) } x = \text{7cm}
\end{align*}
\]
11) \( 20 \times 3 = 60 \) 

12) \( \frac{20}{\sqrt{25}} = 4 \) 

13. \( \text{Volume} = 1.1 \times 132 = 145.2 \text{~cm}^3 \) 

14. \( x = 12 \text{~m}^2 \times x \)

13. \( D = \frac{M}{V} \)

13. \( D = \frac{M}{V} \times V = \frac{M}{\rho} = V \) or \( V = \frac{M}{\rho} \)

12. \( \text{Circumference} = 6.284 \)

11) \( 20 \times 3 = 60 \) 

11) 0000

11) 62640

\[ x = 5 \text{~m} \]
(4) $3^{\frac{3}{4}} - x^{\frac{5}{8}}$

$\frac{7}{8} \times \frac{15}{8} = 15_{\frac{15}{8}} = x^{\frac{5}{8}}$

10. $3\frac{1}{8} \times \frac{5}{5} \Rightarrow 3\frac{5 \times 32}{40} = 3.160 = \frac{7. x}{40}$

11. $3\frac{3}{4} - 2\frac{7}{8} = 1 \frac{6 - 71}{8} = \frac{54}{8} = 8$

10. $3\frac{1}{8} \times \frac{4}{5} = 3 \frac{5 \times 32}{40} = \frac{160}{40} = 14$

11. $4\frac{3}{4} + \frac{1}{5} = 4 \frac{2 \times 2}{4} = \frac{7}{4}$

12. $\frac{1}{\pi} \times 3.142 \times 20 = 20 \times \frac{20}{200} = 3.142 \times 200 \Rightarrow \text{Answer: 120400}$

\[317 \frac{12\,14\,00}{1902} \quad \text{ANS: 0.65711 x}

\text{This question asked for the "difference" it is a Subtraction}

7) $48 \div \frac{30}{4} = 00$ \[00 \quad \text{ANS: 1.81}

7) $0.4 \sqrt{148} = 1. x$

2) $\frac{15}{18} \quad 573 \quad 7256 \quad 13122$
Velocity after 100 ms: 60
### Examples from Test 3

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5) 0

3. 3887/5

2054.2

18328

3) base 1 hold one.

(b) \( \frac{5}{6} \div \frac{2}{3} \)

15/12

3/6

1/2

5) 0

6) \( \frac{5}{6} + \frac{2}{3} = \frac{5}{6} + \frac{2}{3} \)

1/6

2

4

6.

\( \frac{5}{6} \div \frac{2}{3} = \frac{5 \times 3}{6 \times 2} = \frac{15}{12} + \frac{18}{12} = \frac{33}{12} \)

7. \( \frac{2}{5} = \frac{2}{5} \times \frac{5}{100} = \frac{50}{100} = 50\% \)

7/5 = 2.1

\( \frac{2}{5} \times \frac{5}{100} = 10\% \)

7. \( 2.5 + 50\% \)

72.5

21/8

7. 2.5 \times 2 < 5 \times
8. 15 x 40 = 600

9. \[
\frac{2.4 \div 8.432}{2.4} \times \frac{1}{2.4} = \frac{20}{24} \div \frac{17}{24} = \frac{5}{4} \div \frac{7}{4} = \frac{5}{7}
\]

10. \[
4.37 \div 100 = 0.0437 \quad \text{answer: } 437.00
\]

11. \[
4.37 \div 100 = 0.0437
\]

12. \[
4.37 \div 100 = 0.0437
\]

13. \[
2 \times 12 = 24 \quad 13 \times 12 \times
\]

14. \[
\frac{12^2}{450} = \frac{144}{450} \quad \text{ans: } 12^2 = \frac{144}{450} \times
\]

15. \[
\frac{12}{450} \times 15 = \frac{12a^2}{150} \times
\]
1) 190mm X
2) 96mm X

3) \(2 \times \frac{3}{2} \times \frac{1}{2} = 7 \) units
4) \( \frac{3}{2} \times \frac{3}{2} \times \frac{1}{2} = \frac{3^2}{2^2} \times \frac{1}{2} = \frac{9}{4} \times \frac{1}{2} = \frac{9}{8} \) units

Area = Length \times Width

0.6 = 76cm units

5) \( 2 \times 3 \frac{1}{2} \times 2 \times 3 \frac{1}{2} = 7 \times 7 \times 2 \times 2 = 98 \times 2 = 196 \) units

6) \( 2 \times 1 \times 3 \frac{1}{2} \times 2 = 9 \times 3 \frac{1}{2} \times \frac{1}{2} = \frac{9 \times 7}{2} \times \frac{1}{2} = \frac{63}{4} \times 2 = \frac{63}{2} \times \frac{1}{2} = \frac{63}{4} \) units

What is the remainder of the "L" shape drawn?

1) 183 \( \times \) not accurate
2) 105 \( \times \) units
3) (a) 11 \( \times \) units
4) (c) 2.5 \( \times \) units

Where is your working out?
2.1. \[ L \times W = 3.2 \times 2.1 = 6.73 \text{ cm}^2 \]

Area: 6.73 \( \times \) units

3.5 \( \times \) \( \text{?} \) \[ L \times W \times H = 2.2 \times 1 \times 3.5 = 7.7 \text{ cm}^3 \]

Area: 7.7 \( \times \) units

5.6 \( \times \) \( \text{cm} \)

Area: 5.6 cm\(^2\) \( \times \) \( \text{cm} \)

8 cm

Area: 2.09 cm\(^2\) \( \times \) \( \text{?} \)

9.2 mm \( \times \) \( \text{?} \)

Area: 9.2 mm \( \times \) \( \text{?} \)

9.2 mm \( \times \) \( \text{?} \)

Area: 9.2 mm \( \times \) \( \text{?} \)

2.2 \( \times \) \( \text{?} \)

Area: 2.2 \( \times \) \( \text{?} \)

2.2 \( \times \) \( \text{?} \)

Area: 2.2 \( \times \) \( \text{?} \)

4.2 \( \times \) \( \text{?} \)

Area: 4.2 \( \times \) \( \text{?} \)
1. Volume is finding the base of an object.
2. Can't do it.
3. The volume of the cube is 16 cm³.
4. The volume would be 32 cm³.
5. You would get 81 cm in the cube.
6. Number of small cubes: 0.

- The volume is the space inside something.
- The volume of the cube: the unit of a volume is a cube.
- The volume is 8 cm³ units.
- 16 cm³ is the new volume.
- You would get 12 cm³ cube init.
- 24 cubes in it.

1. The volume is the space inside something.
2. The units of volumes is how many units there is in a cube.
3. The volume is 64 units.
4. The new volume will then be 16.
5. 12.
6. 24.
Test: Write in Standard Form:

1. \(10^2\)
2. \(10^3\)
3. \(3.7 \times 10^2\)
4. \(0.075 \times 10^4\)

Write in Standard Form:

5. 1000
6. 3000
7. 30
8. 375
9. 100000
10. 0.035

Examples of Errors:

5. \(1000 = 10 \times 100^2\) \(\checkmark\)
6. \(3000 = 3 \times 10^3\) \(\checkmark\)
7. \(30 = 1 \times 10^3\) \(\times\)
8. \(375 = 3.75 \times 10^2\) \(\times\)
9. \(100,000 = 1000 \times 100\) \(\times\)

5. \(1000 = 100^1\)
6. \(3000 = 300^1\)
7. \(30 = 1 \times 10^3\)
8. \(375 = 3.75 \times 10^2\)
9. \(100,000 = 100 \times 100\)
Test A

\[ \frac{x \cdot 5.29}{5819} \cdot \text{Think} \quad 3/ \frac{7.46}{8206} \cdot \frac{3/7.46}{x \cdot 10^2} \cdot 4/2102.595 \]

This boy could not multiply by powers of 10 and had difficulty when multiplying by 10.

1/ 5,290.

2/ 20.60 ÷ 7.460 = 2.79

6/ 52.9

This boy was only able to complete three of the questions:

8/ 7.46 ÷ 1000 = 0.00746

9/ 0.36

10/ 0.36

5/ 0.00365 ÷ 104

6/ 0.00365 ÷ 10,000

7/ 0.00265

8/ 0.00265

9/ 36.5

10/ 637.15 ÷ 1088

11/ 637.15 ÷ 1088
\[ \begin{align*}
  1. & \quad 689 \times 10 = 6890 \\
  2. & \quad 7.46 \times 10 = 74.6 \\
  3. & \quad 7.46 \times 10^2 = 746 \\
  4. & \quad 637.15 \times 10^3 = 637150 \\
  5. & \quad 0.00365 \times 10^4 = 0.00365 \\
  6. & \quad 589 \div 10 = 58.9 \\[ \checkmark \\
  7. & \quad 7.46 \div 10 = 0.746 \\[ \checkmark \\
  8. & \quad 7.46 \div 1000 = 0.00746 \\[ \checkmark \\
  9. & \quad 637.15 \div 10^3 = 0.63715 \\[ \checkmark \\
 10. & \quad 0.00365 \times 10^2 = 0.36 \quad \checkmark
\end{align*} \]

Test B

\[ \sqrt{2.7^2} = \frac{2 \times 165}{3 \times 3.2} \times \frac{46.03}{0.004} \times 100 \]

\[ \sqrt{243} \]

When asked about this paper the boy replied, "I have never been able to do maths. On a late test paper this boy just put "help although when offered help was not enthusiastic and regarded it as a punishment."

\[ \begin{align*}
  \sqrt{165} & \quad \sqrt{243} \\
  & \quad \sqrt{1095} \\
  & \quad \sqrt{15200} \\
  & \quad \sqrt{14295} \\
  & \quad \sqrt{165} \quad 14.355
\end{align*} \]
1. \(27 \times 9 = 243\)

2. \(\frac{27,9}{243}\)

3. \[
\begin{array}{c}
21.4 \\
\times 3.2 \\
\hline
42.6 \\
642.0 \\
\hline
684.6 \\
\end{array}
\]

4. \[
\frac{0.03}{0.004} \\
\hline
0.0120
\]

5. \(9.28 \div 16\)

\[
\begin{array}{r}
57.26 \\
16 \underline{9.28} \\
80 \\
128 \\
122 \\
60 \\
48 \\
120 \\
96
\end{array}
\]

\[\text{Answer: } 57.36 \text{ to 2 decimal places}\]

6. **Test C**

5. \[
\begin{array}{c}
2487 \\
210.89 \\
\hline
2697.89
\end{array}
\]

\[
\begin{array}{c}
2697.89 \\
9.006 \\
\hline
2706.896 \\
2574 \\
\hline
5270.896
\end{array}
\]

- \(5270.896\)

- \(3.37\)

= \(5274.866\)
This boy has borrowed a 'ten' from the next column but only counted as a single (1)

**TEST D**

This test provided the most trouble for the pupils as previously shown.

7. $3 \frac{1}{2} - 2 \frac{5}{16} = 14\frac{1}{14}$  
3. $6 \frac{1}{16}$

8. $6 \frac{1}{8} + 2 \frac{1}{16} - 3 \frac{1}{32} = x$

9. $\frac{1}{3} \times \frac{1}{4} = 12 \times x$

5. $\frac{1}{3} + \frac{1}{8} = \frac{8 + 3}{24} = \frac{12}{24} = \frac{1}{2}$

7. $\frac{7}{2} - \frac{37}{16} = \frac{56 - 37}{16} = \frac{19}{16} = 1 \frac{13}{16}$

9. $\frac{1}{3} \times \frac{1}{4} = \frac{4 \times 3}{12} = 9 \times \frac{1}{12}$

10. $\frac{1}{2} \times \frac{1}{3} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{2}$
(8) \( \frac{68 + 2/16 - 3}{32} \)

\[
\frac{68}{8} + \frac{33}{16} - \frac{109}{32}
\]

\[
\frac{116 + 66 - 109}{32}
\]

\( = \frac{73}{32} = (2.9)_{32} \)

(9) \( \frac{14\frac{1}{8} + 2\frac{1}{16} - 3\frac{13}{32}}{32} \)

\[
\frac{114}{8} + \frac{33}{16} - \frac{109}{32}
\]

\[
\frac{196 + 66 - 109}{32}
\]

\( = \frac{299}{32} = (9.3)_{32} \)

One pupil's effort:

\[
\begin{align*}
11 & \quad \frac{21}{24}! \\
21 & \\
31 & \\
41 & \frac{2}{3}
\end{align*}
\]
d) 0.0031 x 35 = 0.0031 x 285

= 0.0031

e) 171000 x 158

= 2

\[ \begin{array}{c}
\text{2) 35.1 x 1.41} \\
\hline
35.1 \\
141 \\
\hline
500
\end{array} \]

\[ \begin{array}{c}
c) 1610 x 0.013 \\
\hline
1610 \\
0.013 \\
\hline
2.830 \\
16.100 \\
\hline
18.930
\end{array} \] *Note the method*

d) 1595 ÷ 41

= 40

e/2 x

10) 2 x

1) 31 x

(p) x 31.
APPENDIX III

EXAMPLES OF SOME OF THE "REVISION BOOKLETS" DEVELOPED
This is a diagram of a CUBE.
It has 6 sides or square faces and each edge is 1 cm in length.

The Volume of the Cube is 1 cubic centimetre (1 cm³).
The Area of each face is 1 square centimetre.

Lay 4 centimetre cubes side by side in a row.
(a) What is the total volume of the cubes?
(b) What area of your desk would the four cubes cover?

Now place another row of 4 cubes exactly in front of the first row.
(a) What is the total volume of these cubes?
(b) What is the area of the desk covered?

Here are four rows of four cubes.

(a) What is the total volume of these cubes?
(b) What area of your desk would they cover?
(c) What shape do the base of the cubes make on your desk?
Here, another layer has been placed on top of the first layer. What is the volume of the cubes now?

Now there are three layers. What is the total volume of those cubes?

What is the total volume now?

Complete the table for diagrams 1, 2, 3 and 4.

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Area of Base</th>
<th>Height of cube</th>
<th>Area of Base x Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Remember

\[ \text{Volume} = \text{Area of Base} \times \text{Height} \]

The units are \( \text{Cm}^3 \) - don't forget them!

Rectangular Boxes are called **Cuboids** and their volume can also be worked out using this method.

\[
\begin{align*}
\text{Area of base} & = 5 \times 3 \\
& = 15 \text{cm}^2 \\
\text{Volume} & = \text{Area of base} \times \text{height} \\
& = 15 \times 2 \\
& = 30 \text{ cm}^3
\end{align*}
\]

Check by counting the number of 1 cm cubes which would fit inside this Cuboid.

**TRY THESE:** Find their volume by both methods.

![Images of various cuboids with dimensions provided for volume calculation]
1. **What are the Volumes of the Following Boxes?**
   
   a) 8 cm x 3 cm x 10 cm  
   b) 12 cm x 5 cm x 8 cm  
   c) 6 cm x 5 cm x 9 cm  
   d) 9 inches x 3 inches x 4 inches  

2. **Do Rough Estimates to Check Your Answers on the Following Questions**
   
   a) 22 cm x 17 cm x 2 cm  
   b) 14 cm x 21 cm x 8 cm  
   c) 1.4 cm x 2.1 cm x 8 cm  
   d) 18 km x 3 km x 14 km  
   e) 0.3 m x 0.7 m x 18.5 m  

3. A box is 24 cm x 16 cm x 12 cm. How many matchboxes will fit into it if they are 4 cm x 2 cm x 1 cm in size?  

4. A cylindrical measuring container has a volume of 1000 cm$^3$.
   
   Its base has a cross sectional area of 25 cm$^2$. What is the height of the measuring container? What is the height of the 300 cm$^3$ graduation?  

5. a) A Builder has to fill a hole of size 12 m x 6 m x 9 m with cubic blocks of size 50 cm x 50 cm x 50 cm. How many cubes does he need?  
   
   b) The Builder cannot buy the blocks he wants but gets blocks whose dimensions are half those he intended to use.  
   
   (i) What is their size?  
   
   (ii) How many of these blocks will he need?  
   
   (iii) Explain why your answer is not twice the amount you got for part(a).  

6. A litre of paint (1000 cm$^3$) covers 10 m$^2$ of surface. What thickness in cm is the film of paint?  

7. Motorcycle and Car Engines are quoted in cubic capacity to give an estimate of their possible power. How does a 250 cc Honda bike compare with a 750 cc Honda bike? What is the engine size of your family car?
Here is an example where they have different units.

**WE CANNOT SAY** \( \text{Volume of box} = 2 \times 4 \times 120 = 960 \)!

because there is a mixture of cm. and m. units.

**WE HAVE TO CHANGE THE DIMENSIONS SO THEY ARE ALL IN THE SAME UNITS**

in this case, 120 cm change to metres = 1.2 m.

The volume of the box is \( 2 \times 4 \times 1.2 = 9.6 \text{ m}^3 \)

**Try these:**

1. 8 cm \( \times \) 12 cm \( \times \) 1 m
2. 36 cm \( \times \) 0.5 m \( \times \) 12 cm
3. 12 m \( \times \) 80 cm \( \times \) 4 m
4. 30 feet \( \times \) 9 inches \( \times \) 2 feet
5. 3 yards \( \times \) 9 inches \( \times \) 1.5 feet
6. 8 cm \( \times \) 9 mm \( \times \) 1 m

You may use calculators to check your answers.

Don't forget to say what units you are using!
Objects with holes

"A wooden block with a square hole."

To find the volume of the wooden block
(i) Work out volume of whole block if it were solid.
(ii) Work out volume of hole and subtract it from part (i)

Volume of whole block = $6 \times 8 \times 20 = 960 \text{ cm}^3$
Volume of hole = $3 \times 2 \times 20 = 120 \text{ cm}^3$

Therefore, volume of wooden block = $960 - 120 = 840 \text{ cm}^3$

Try this one:

Square hole 2cm x 2cm through block

Do your working here.
1. In this diagram the hole only goes halfway through the block.

2. Find the total volume of this block (hint: think of them as two separate blocks and then add their volumes together).

3. Please note!
The hole in this diagram goes all through the block.
**CUBE NUMBERS**

Cube length 1 unit
Volume 1 x 1 x 1 = 1 unit^3

" 2 units
Volume 2 x 2 x 2 = 8 unit^3

" 3 units
Volume 3 x 3 x 3 = 27 unit^3

List the first ten cube numbers, starting 1, 8, 27, …

---

**Volume of shapes other than boxes (cuboids)**

We use **area of base x height**
but what is the "area of the base?" The following examples will show you.

A. **TRIANGULAR PRISMS**

A shape like this is called a PRISM
It has the same base shape all the way through.

Look at the base separately (note it is the same as the top)

We have to find the area of this triangle
**Remember**
Area of $\Delta = \frac{1}{2} \times \text{base} \times \text{height}$

So area of triangular base = $\frac{1}{2} \times 5 \times 4 = 10 \text{ cm}^2$

Volume of this Prism = Area of base x height of prism
= $10 \times 8$
= $80 \text{ cm}^3$
The area of the base = Area of rectangle + Area of triangle
= $6 \times 8 + \frac{1}{2} \times 4 \times 6$
= $48 + 12$
= $60 \text{cm}^2$

The volume = Area of base $\times$ height of prism
= $60 \times 15$
= $900 \text{cm}^3$

TRY THESE:- In each question the height of the prism is given with the diagram of the base.

1. Prism Height 9cm

2. Prism Height 5.5cm

3. Height of prism 7.5cm

4. Prism Height = 14cm

5. Prism Height = 9.9cm

6. Prism Height = 21.5cm
CIRCULAR BASE: The area of a circle is roughly $3\pi \text{(radius)}^2$.

[More accurately $\frac{22}{7} \times \text{(radius)}^2$ which is $3.141592\ldots \times \text{(radius)}^2$.]

We will use $3.1 \times \text{(radius)}^2$ for all our work.

This number is known as $\pi$ and is written as $\pi$.

So the area of a circle is $\pi r^2$ where $r$ = radius of the circle.

The volume of the cylinder = Area of base $\times$ height

$$= 3.1 \times (5 \times 5) \times 10$$

$$= 775 \text{ cm}^3$$

TRY THESE:

1) $\pi$
   $\text{Base}$
   $22\text{cm}$
   $\text{V}$

2) $\text{18cm}$
   $\leftarrow 10\text{cm} \rightarrow$
   $\leftarrow 7\text{cm} \rightarrow$
   $\text{Base of a prism 20.6cm High}$
Whenever we have solid objects it is often useful to work out the surface area, that is, the amount of card you would need to make the shape.

Example:  A cube

The area of each face $= 2 \times 2 = 4 \text{ cm}^2$

There are 6 faces altogether

Therefore total surface area

$= 6 \times 4 = 24 \text{ cm}^3$

Try THESE :-

Find the surface area of

1. 3 cm
   2 cm
   3 cm

2. 15 cm
   0.5 m
   1.5 m

   NOTE MIXED UNITS

3. 2 cm
   3 cm
   4 cm

4. 10 cm

Don't forget the top and bottom.

The circumference of a circle is $2\pi r$
EXPERIMENT

Here are 8 – one centimetre cubes in a row

What is the Surface area of this shape?

Re-arrange them in as many different ways as you can
Work out the Surface Area each time.
Which shape do you think will give the least surface area?

VOLUME AND GRAPHS

   Investigate the length of the edge of a cube and its relationship with its volume.

Fill in the table, before deciding on the scales of your graph:

<table>
<thead>
<tr>
<th>Length of edge of cube in INCHES</th>
<th>Volume of cube in CUBIC INCHES</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>Etc – to 10</td>
<td></td>
</tr>
</tbody>
</table>

This type of graph has a name (Equation) \( x = y^3 \)
Volume and Liquid

Take a measuring cylinder, and measure the volume of water that drips from a tap over a period of time.

Draw up a table of results. Think how often should you take the readings?

Estimate the amount of water lost from a leaking tap per year.

Estimate the amount of water lost from taps in Derby, where there are 100,000 houses. If only one tap in each house drips.

Discuss your results. Do you think they are realistic?
N-Z PRO-O.

"Battleships"

X marks the spot along

CO-ORDINATES
Look for the **ISLE OF MAN** on the map.
We say its **CO-ORDINATES** are **(1,3)**.

On this map the Grid Squares are large and could be divided into smaller squares, for a greater degree of accuracy.

Take **BRISTOL**. The first number of the co-ordinate is **(2, )** but the second number could be 1, or 2 because it is "more" than 1 but less than 2. To overcome this we imagine the line between 1 and 2 to be split into ten parts—like decimals. So we say the second number is **1.5**.

The co-ordinates of **BRISTOL** are **(2.0, 1.5)**.
This is more useful in work on Map Reading in Geography.
For now we will only consider places that fall on the intersection of two Grid lines.
1. Name all the places that fall on the intersection of two grid lines and give their co-ordinates.

2. Can you fill in some more places on the map. Use their approximate geographic position by placing them on intersecting grid lines? Name the places you have chosen and state their co-ordinates.

Use letters to stand for points on the map

On this map:
A is a lighthouse
B is Devil's Island
C is a cave
D is a Native camp
E is a Spring
F is a Swamp
G is a lookout point
H is buried treasure

Answer these questions:
1. Where is the Swamp?
   Can you see that the swamp (F) is at (6, 4)
2. Where is the lighthouse?
3. Where is Devil's Island, the Buried Treasure and The Spring?
4. What is at these co-ordinates (4, 6) (3, 3) (6, 4) (8, 4)
Before continuing, notice how important it is always to put brackets around the co-ordinates. How confusing it would be just to write 4,6,3,3, 6,4, 8,4!

5. Draw a grid of your own, with 0 to 10 on each side.

Use crosses and letters mark these points on it:

A at (3,2)  D at (9,9)  G at (7,4)
B at (3,9)  E at (5,4)  I at (7,7)
C at (9,2)  F at (5,7)  I at (10,10)

x and y

Now we are going to be really Mathematical and use things called EQUATIONS to give each line on the grid a name.

\[
\begin{array}{c|c|c}
& \text{Line } x=0 & \text{Line } y=5 \\
\hline
\text{Line } y=1 & \text{Line } y=3 \\
\hline
\text{Line } x=5 & \text{Line } x=11 \\
\end{array}
\]

(Note: we usually use the lines only with numbers along the x and y axes.)

Look at the grid each line has a name. Some have been filled in for you.

Can you see that point A is where the line called \(x=2\) and the line called \(y=2\) cross? (If not ask for help)

Mark B with a cross where lines \(x=4\) and \(y=2\) cross
Mark C with a cross where lines \(x=2\) and \(y=14\) cross
TRY THESE:  

Label axes, scales, long enough, and always join up points in order written.

1. \((2,2) \quad (1,2) \quad (1,1) \quad (2,1) \quad (2,4) \quad (5,5) \quad (5,2) \quad (4,2) \quad (4,3) \quad (5,3)\)

2. \((2,6) \quad (1,6) \quad (1,0) \quad (3,0) \quad (3,6) \quad (2,6) \quad (3,7\frac{1}{2}) \quad (2,9) \quad (1,7\frac{1}{2}) \quad (2,6)\)

3. \((3,7) \quad (2,7) \quad (1,6) \quad (1,3) \quad (0,2) \quad (6,2) \quad (5,3) \quad (5,6) \quad (4,7) \quad (3,7)\)
\begin{align*}
(2\frac{1}{2}, 7\frac{1}{2}) \quad (3,8) \quad (3\frac{1}{2}, 7\frac{1}{2}) \quad (3,7) \\
\end{align*}
Finish by joining up \((2\frac{1}{2}, 2) \quad (2\frac{1}{2}, 1) \quad (3\frac{1}{2}, 1) \quad (3\frac{1}{2}, 2)\) and shade in.

4. \((4,7) \quad (2,6) \quad (1,5) \quad (5,4) \quad (0,2) \quad (0,0) \quad (11,0) \quad (11,2) \quad (10\frac{1}{4}, 4) \quad (10,5) \quad (9,4) \quad (7,7) \quad (4,7) \quad (4,8) \quad (3,8) \quad (3,9) \quad (2,9) \quad (2,11) \quad (4,11) \quad (4,10) \quad (5,10) \quad (5,7) \quad (7,7) \quad (7,8) \quad (8,8) \quad (8,9) \quad (9,9) \quad (9,11) \quad (7,11) \quad (7,10) \quad (6,10) \quad (6,9) \quad (5,9)\)
Now join \((2,11)\) to \((5,8)\) and \((9,11)\) to \((5,7)\)

5. \((12,7) \quad (7\frac{1}{2}, 7) \quad (5,10) \quad (5,12) \quad (6,10) \quad (0,12) \quad (3,10) \quad (1,8) \quad (2,7) \quad (4,8) \quad (5,6) \quad (5,2) \quad (4\frac{1}{2}, 1) \quad (3\frac{1}{2}, 6) \quad (4\frac{1}{2}, 0) \quad (6,1\frac{2}{3}) \quad (6,0) \quad (11,4) \quad (12,3) \quad (12,2) \quad (11\frac{1}{2}, 1) \quad (10\frac{1}{2}, 0) \quad (11\frac{1}{2}, 0) \quad (13\frac{1}{2}, 0) \quad (13,6) \quad (13,7) \quad (13,7) \quad (14,6) \quad (14,2)\)
Join \((8,7) \quad (8,6) \quad (9,5) \quad (10,5) \quad (11,6) \quad (11,7)\)
Put cross on \((3,9)\)

You can make up your own Pictures!
SECRET CODES.

Here is a secret message:
(3,1) (5,3) (3,3) (5,1)

What does it say?
Can you decode the message?
The answer is on the next page.

TRY THESE:

1. Decode these messages:
   (a) (3,3) (1,1) (5,4) (3,2) (4,4)
   (b) (1,1) (4,1) (4,2) (5,4) (4,2) (5,3) (4,3)
   (c) (4,4) (1,5) (3,3) (4,4)
   (d) (4,1) (4,2) (3,5) (4,2) (4,1) (5,1)
   (e) (3,1) (1,1) (4,3) (5,5) (5,3) (1,5) (3,1) (5,3) (4,1) (5,1) (1,1) (3,3) (5,1) (4,4) (4,1) (1,1) (2,2) (5,1) (5,4) (5,4) (5,5) (5,3) (5,1) (3,4)
   (1,2) (5,4) (4,2) (7,1) (0,3) (4,1)

Here's another idea.

Here is the message:
I KLA Understand it

Here is another:
HANT

Try to crack them before you turn over.
1. The message is decode the message.

2. ILKA means GONE?
   HANT means HELP

The code letters are replaced by ones which are "symmetrical" with them across the x=3 line. For instance on the first line:

- A to E
- 1 to 5

Symmetrical about x=3

3. Code some of your own messages. "Swap" with a friend and decode each others.

4. Try using the y=3 as the line of symmetry.

5. Can you do it by turning 90° around the letter M?

   O would code as C

   S would become I

6. Try "Moving 2 along and up" (what about going off the edge - discuss what you should do).

   eg Code Haa 0 and I as L. why?

   Make up tricks of your own

    Continue Symmetry.

   1. Draw some symmetrical shapes or patterns e.g.

   2. Draw a symmetry line on graph paper and make a symmetrical pattern by colouring in squares.

   3. Try to do shapes with more than one line of symmetry.

   Think about shapes - squares, triangles, hexagons and how symmetrical they are.

   Why is a ballroom basin plug round?
1. On centimetre graph paper draw in the x= 0 and y= 0 axes for values of x up to x= 10 and y up to y= 10
   (a) Go over the lines x= 1 , x= 4 , y= 2 and y= 5 more heavily
   (b) Colour in the rectangle you can now see
   (c) Write next to the grid the co-ordinates of the corners of this rectangle

2. Draw a second set of axes on fresh graph paper
   (a) Go over lines x= 3 , x= 6 , y= 4 , y= 8 more heavily
   (b) Colour in the rectangle you can now see
   (c) Write next to the grid the co-ordinates of the corners of this rectangle

3. On fresh graph paper draw a rectangle with corners at
   (2,3) (2,7) (6,3) (6,7)
   (a) Next to the grid lines write which lines make edges for this oblong.

4. Repeat for the rectangle with corners at (3,3) (3,8) (7,3) (7,8)

5. Draw axes on fresh graph paper:
   We have used x and y to give "names" to the lines on the grid.
   x= 0 is the name of one axis. We call it the EQUATION of the line x= 0

Points A, B, C, D, E, F and G
are in a line

We are going to try to find a "name" — an EQUATION for this line

Look at E for example.
Its x number is 2
Its y number is 4
the points in the line.

Point | x | y
-----|---|---
A    | 0 | 6
B    | 1 | 5
C    | 2 | 4
D    | 3 | 3
E    | 4 | 2
F    | 5 | 1
G    | 6 | 0

In each case the $x$ number and $y$ number add up to 6.

Can you see that we could say $x + y = 6$.

This could be the name for the line.

Exercise

1. a) On graph paper draw in and label the axes.
   Mark the points A (0,7) B(1,6) C(2,5) D(3,4) E(4,3) F(5,2) G(6,1) H(7,0)
   (b) Do a table, like the one above, for $x$ and $y$-number of the points.
   (c) What could the name (EQUATION) of this line be?

2. Draw lines of points with these equations (names)
   (a) $x + y = 4$  (b) $x + y = 8$  (c) $x + y = 9$

3. a) Plot these points on a grid (graph)
   A(0,1) B(1,2) C(2,3) D(3,4) E(4,5) F(5,6) G(6,7) H(7,8)
   (b) Do a table of $x$ numbers and $y$ numbers for these points.
   (c) Try to explain why the name - the EQUATION - of this line is $y = x + 1$.

4. On fresh graph paper draw these lines
   (a) $y = x + 2$  (b) $y = x + 4$  (c) $y = x + 5$

5. Try to draw points in a line fitting the equation $y = x - 1$

6. Draw in the lines $y = x - 2$, $y = x - 4$, $y = x - 5$

7. Try to find other "names" (EQUATIONS) for the lines in questions 4 and 6, for instance $x =$ ___?
Games

**BATTLESHIPS**

Draw a grid 10 by 10 and label it.

You have 9 ships which include:
1. Aircraft carrier (4 units long)
2. Battle Ship (3 units long)
2. Cruiser (2 units long)
4. Submarines (1 unit long)

Place these on your grid, similar to below.

Play with your neighbour. You each have a turn at trying to blow-up each others ships. The shells strike at the co-ordinates you name. When all your neighbours ships are sunk you are the winner. (If in doubt ask!)

You will need a similar grid to record your guesses.

Try to form some logic in your guessing.
Another Game

Get a dice from your teacher
Throw it twice and use the scores as co-ordinates of a point
Eg Get 2, then a 5 put a cross at (2,5)
Throw again Plot the point and join it to the first point
Throw again Plot the point and join it to the previous one
Carry on until you cross over yourself

STOP! How many turns before you cross?
Try to set up a record!

Take turns with a friend to throw the dice. Use different colours on the grid. First one to get 3 points in a line wins
We could call this game "Connect 3"
Scale / Enlargement

The idea of a grid can be used to enlarge or scale down a drawing.

These diagrams show how to Enlarge a picture, by copying it, square by square onto a bigger grid.

How about stretched in only one direction

Try to design your own - Have Fun!
APPENDIX IV

PUPILS' QUESTIONNAIRE AND RESULTS
QUESTIONNAIRE

THOSE WHO DID NOT CHOOSE PHYSICS

1 Why did you not choose physics?
   (a) the subject material was difficult  YES/NO
   (b) the subject involved too much mathematics  YES/NO
   (c) the mathematics was too difficult  YES/NO
   (d) it was not relevant for the job you wanted  YES/NO

2 Did you find the mathematics involved in the physics questions more difficult than when you did them in maths lessons?  YES/NO

3 Which of the following topic areas do you have difficulty with (or think hard)?
   (a) add/subtracts  YES/NO
   (b) divides/multiplies  YES/NO
   (c) decimals  YES/NO
   (d) fractions  YES/NO
   (e) percentages  YES/NO
   (f) graphs  YES/NO
   (g) equations  YES/NO

4 Would you like to drop mathematics as a subject? YES/NO

5 Do you prefer doing mathematics in a practical way, for instance by doing experiments? YES/NO
   Is it more enjoyable? YES/NO

6 How many different mathematics teachers have you had since you were eleven?
   1 2 3 4 5 6 7 8 9 10
   cross out the relevant number
7 Are your parents good at mathematics? YES/NO

8 Do you think the mathematics you do is relevant to you and your future life? YES/NO

9 Have you an electronic calculator? YES/NO
Pupil's Questionnaire

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<th>% answering YES</th>
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|                | Frequency of pupils |
| 7               | 73                 |
| 8               | 56                 |
| 9               | 59                 |
APPENDIX V

PUPILS' AND TEACHERS' COMMENTS, ESSAYS AND TRANSCRIPTS

OF TAPE RECORDINGS
Discussions at Derby Lonsdale College of Higher Education, their tests carried out on candidates for employment in the Printing Industry, and an analysis of the results.
Discussion with:

Mrs B Gough-Jones (Assistant Dean and In charge of Science) and Mr I Penchion, at Derby Lonsdale College of Higher Education - Green Lane college.

The author chose this establishment because he felt that the middle ability range pupils were likely to continue their education on City and Guilds courses as part of gaining their indentures and therefore the problems that appear in schools should also be seen here. It also enabled the author to get a fairly unbiased view, as these Lecturers had not been involved in their students secondary education and they would be working at an academic level equivalent to the work in schools, but applied to particular industries. The College caters for Art Students and Craft Skills on both 'full-time' and 'day release' courses. The Industrial based courses covered Building Sciences, Painting and Decorating, Printing science, and the entry qualifications of the students varied from CSE to 'O' levels with the occasional 'A' level.

Mrs Gough-Jones stated that over the years Lecturers had accepted the fact that it would be necessary for them to teach the basic mathematics required to teach the sciences and craft skills simultaneously with the course work.

They both felt strongly about the lack of basic mathematics skills on such topics as fractions, decimals, percentages, solution of equations etc, where a very large majority of the students were just not competent. It certainly could not be taken for granted that even the simplest of mathematical skills had been learnt. Hence the progress through the science course was severely hampered and many topics had to be dealt with in a manner so as to avoid carrying out too many mathematical calculations.

Mrs Gough-Jones claimed that schools were accepting poor standards and the evolution of modern mathematics had not helped the situation. She thought that much of what was being
taught had no relevance to adult life and its needs or the requirements of Industry. Similarly the stopping of the '11 plus' had caused a situation where the Junior schools had nothing to aim for and no means of measuring the success of their efforts.

Much of the problem was attributed to what seemed a lack of discipline and poor motivation, which she admitted the colleges did not have because employers kept the students under control and a letter from the college was enough to solve any problem. However, such instances were infrequent as most students were fairly mature and glad to have a job in this era of mass unemployment.

Mrs Gough-Jones and Mr Penchion carried out Mathematics and English tests of school leavers who were candidates for employment with several local firms. The author was allowed to see the test paper and the scripts for the year 1980, for employment in the Printing industry. The mathematics test was devised by the National Foundation for Educational Research in England and Wales, and there were 45 candidates from the Derby area of which 12 were asked to an interview.

The author was allowed to scrutinise every answered script and the following statistics were achieved:
MATHEMATICS TESTS GIVEN BY DERBY LONSDALE COLLEGE OF HIGHER EDUCATION TO 45 CANDIDATES FOR EMPLOYMENT IN THE PRINTING INDUSTRY.
ILL IN THE FOLLOWING PARTICULARS:

URN ALTE (block capitals) ..................................................
THER NAME(S) .................................................. SEX

SCHOOL present/last ..................................................
(Underline whichever applies)

LASS present/last ..................................................

OLLEGEs present/intended ...........................................

OURSE present/intended ..................................................

ATE OF BIRTH ..................................................

OYAY'S DATE ..................................................

Underline the type of school you last attended:

condary modern grammar technical
mprhensive independent

EAD THE FOLLOWING CAREFULLY:

When you are told to begin, turn over the page and begin at once.

Write your answers in the spaces provided on this paper.

Work as quickly and as carefully as you can. Make any alterations in your answers CLEARLY.

If you finish before time is up, check your work.

When you are told to stop, STOP WORKING AT ONCE.

You may not ask questions once the test has begun.

DO NOT TURN OVER OR OPEN THIS BOOKLET UNTIL YOU ARE TOLD
1. How many 50 millilitre bottles can you fill from a jar holding 2 litres of water? (.................. bottles)

2. One tenth of £3.75 is 37.5p. What is one tenth of £4.75? (.................. p)

3. What is the size of angle A? (..................°)

4. $\frac{4}{5} \div \frac{2}{3} =$ (..................)

5. I buy a car for £500.00 and sell it at a loss of 20%. For how much do I sell it? (..................)

6. $a + b + c =$ (..................°)

7. What fraction of a pound is 40p? Write the fraction in its simplest form. (..................)

8. The earth rotates through 360 degrees in 24 hours. Through how many degrees does it rotate in 4 hours? (..................°)

Go straight on to the next page.
9. \(16 \times 0 = \) 

10. \(3x + 4x = 28\). What is \(x\)?

11. What is the average of 2986752 and 2986754?

12. \(26.12\) metres \(\div 4 = \)

13. \(\frac{3}{4} + \frac{5}{8} = \)

14. \(80\) centimetres \(\times 2.5 = \)

15. \(7x + 6x = 78\). What is \(x\)?

16. \(0.5 \times 0.25 = \)

17. In \(\triangle ABC\), \(AB = BC = AC\). What is the size of \(\angle ABC\)?

18. \(\frac{1}{3} + 6 = \)

---

GO STRAIGHT ON TO THE NEXT PAGE
19. Given that \( p + q = 1 \), what is the value of \( (p + q)^2 \)?

20. What is the area of the shaded part?

21. \( \frac{5(8 - 3)}{5(4 - 3)} = \)

22. \( \sqrt[3]{4^2} = \)

23. What is the simple interest on £500.00 for 3 years at 4\(^o\)\(\text{o}\)?

24. How many boxes 1 metre high and 1 metre wide and 2 metres long could you get into a box 4 metres high, 5 metres wide and 6 metres long?

GO STRAIGHT ON TO THE NEXT PAGE
25. \( \frac{1}{2} + \frac{1}{2} = \)  

26. If 250 candidates sit for an examination and 225 of them pass, what is the percentage who fail? (\( \ldots \ldots \ldots \% \))

27. \( \frac{3}{4} = \)  

28. \( 2\frac{1}{2} - 1\frac{3}{4} = \)  

29. 

\[ \text{XY is parallel to AB. There is a theorem which states that all triangles ABC, where C can be any point on XY, have areas of equal size. If AB = 4 cm and the distance between the parallel lines is 2 cm, what is this size which is common to all the triangles, ABC?} \]  

(\( \ldots \ldots \ldots \text{cm}^2 \))

30. What is the value of \( x^2 - xy + y^2 \) when \( x = 2 \) and \( y = 1? \)  

(\( \ldots \ldots \ldots \))

31. \( x + 5x = 21 \). What is \( x? \)  

(\( \ldots \ldots \ldots \))
Given that:
Area of quadrilateral ABCD = 30 cm²
Area of quadrilateral OBCD = 10 cm²
Area of triangle OBD = 3 cm²

What is the area of triangle ABD?

33. The diameter of a circle is 10 cm. What is its circumference?
   \[ \pi = 3.142 \]
   (........................ cm) 33

34. \[ x(y + 3) = \]
   (........................) 34

35. If \[ x = 5 \] and \[ y = 2 \], what is \( xy^2 \)?
   (........................) 35

36. If \[ x = 3 \] and \[ y = 6 \], what is \( (xy)^2 \)?
   (........................) 36

37. Twelve straight lines meeting at a point form equal angles. How big is each angle?
   (........................°) 37

38. \[ 1 \div (\frac{1}{3} + \frac{1}{2}) = \]
   (........................) 38

39. The figure shows part of a staircase. What is the size of the angle ABC?
   (........................°) 39

GO STRAIGHT ON TO THE NEXT PAGE
ABC and DEF are equilateral triangles. They are arranged so that:
DE is parallel to CB
DF is parallel to AB
FE is parallel to CA

How big is angle x?

ABC and DEF are equilateral triangles. They are arranged so that:
DE is parallel to CB
DF is parallel to AB
FE is parallel to CA

How big is angle x? (..........................°) 40

41. A census is taken in a village every April, and it is found that the population is increasing by $\frac{1}{10}$ every year. If the population numbers 100 in April 1973, what will it be in April 1975?

(..........................) 41

42. Add together $3x^2 - 2xy - 4y$ and $6x^2 + 6xy - y =

(..........................) 42

43. $\frac{x^2}{x^1} =

(..........................) 43

44. Given that:
Area of triangle ABC = 40 cm$^2$
Area of triangle ABD = 30 cm$^2$
Area of triangle BOC = 15 cm$^2$

What is the area of triangle AOD? (..........................cm$^2$) 44

GO STRAIGHT ON TO THE NEXT PAGE

TOTAL (5)
45. If \( \frac{5x + 4}{2} = 12 \), what is \( 2x \)?

\[ \text{(..............................)} \]

46. \( -3 \times -01 = \)

\[ \text{(..............................)} \]

47. \( (a + b)(a - b) = \)

\[ \text{(..............................)} \]

48–49.

There is a geometrical theorem which states that if \( AB \) is a diameter of any circle and \( X \) is any other point on the circumference, then \( AXB \) is always a right angle. Bearing this theorem in mind, answer the following two questions:

48. Where is the location of the centre of the circle which passes through the vertices of a right angled triangle, say \( XYZ \)?

\[ \text{(..............................)} \]

49. If I have a circular metal disc, how many triangles whose largest side is equal to the diameter of the circle can I cut from the disc?

\[ \text{(..............................)} \]

50. \( 2x + 3y = 22, \)

\( 4x - y = 2. \)

What is \( x \)?

\[ \text{(..............................)} \]

END OF TEST

LOOK BACK OVER YOUR WORK UNTIL TOLD TO STOP

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PRINTED IN GREAT BRITAIN BY THE SIDNEY PRESS LTD., BEDFORD

PUBLISHED FOR THE NATIONAL FOUNDATION FOR EDUCATIONAL RESEARCH IN ENGLAND AND WALES

BY WHE PUBLISHING COMPANY LTD.

2 JENINGS BUILDINGS, THAMES AVENUE, WINDSOR SL4 1OS, ENGLAND

ISBN 0 7087 0003 9

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Percentage Correct Answers for Each Question in the "Senior Mathematics Test"

National Foundation for Educational Research in England and Wales. Senior Mathematics Test carried out at [ noche colegio ] College on the behalf of the local printing industry.
Teachers' Comments.
Mr J Wilde (Head of Science)

Question: Do you have any problems in teaching science to pupils, in particular to those not being able to cope with the mathematics in your science courses?

Answer: I have had a look at a series of questions covering the relationship between maths and science, presented by Mr Collingwood, my colleague, and the following are comments around that:

Major problems met by the department (Science Department related to maths). In lower school work there are certain areas which are not covered. For instance, formula manipulation - it is not very well dealt with in lower school in my opinion, and the mathematicians argue that it does not fit in with their courses but from our point of view it would be useful. Graphical work tends to be an area which should be an area of co-operation; it tends to become an area of confusion, mainly because, in my opinion, the children do graphical work but they do not have the ability, or are not taught that there are different graphs, which can be used in different cases with different results which can be obtained from them. A straightforward one is the line graph in maths, which is joined from point to point whereas in science you can very often get the point which falls off the line. The children do not appreciate this and join up point to point getting a very wavy straight line.

With regards the course higher up, chemistry, I don't think suffers too much from the mathematical side. A few of the odd symbols which have not been met, such as "greater than", "less than" we deal with in the lesson. Formula manipulation is a problem which tends to be met with by the chemistry teacher as they go through it rather on a rote learning basis - that there are ways of manipulating equations and this is one, and if you do it the same every time it will work. We have not got the time to start again and go through the basis of this type of stuff.
Biology tends to meet the graphical problems - bar graphs, line graphs, continuous properties and properties which have fixed integer values. The mathematicians do not tend to have done this, they have just done graphs with number, and that is an area which we often have to spend time and go through it and the interpretation of results.

Physics is a separate problem in my opinion. We are dealing here with the application of maths, the application of mathematical knowledge and physics is simply a process where experimental data is taken, interpreted, in a school sense, normally in words and then into a mathematical form, where the mathematics is used as a means of communicating the ideas normally in a far more precise way than language.

Simple example: Gravity by experiment to show effect and come to conclusion "What goes up must come down" and various language uses to get over the idea of gravity but in the end you have to have Newton's Law of Gravity Force $= \frac{constant \ m_1 m_2}{distance^2}$.

The final thing is a much more concise way of expressing gravity than by words and the problem to me is that maths is not taught as a means of communication, it is taught as a subject in itself that has its own little boundaries and its own conventions and this applicability, this fact, that maths is simply a means of communication, to me, is not emphasised in maths lessons and I believe it causes us problems when we meet it in science, because what happens is that the physics teachers have to spend a certain percentage of their time, it may be only 5-10%, teaching certain mathematical concepts or certain applications of mathematics.

Now from the APU age 11 survey (Assessment and Performance unit) we know that one of the differences that comes between girls and boys is the applicability of science, the ability to apply knowledge and this again is going to be shown in the "age 15" survey by APU. That is the very thing that physics teachers are having to emphasize, the fact that you take the mathematics and you have then to apply it. If these results from the APU are to be correlated with the way the children choose their options, I think there is a reasonable relationship between the two, in
that because mathematics does not teach applicability, physics teaches the applicability and hence the physics teacher is having to do the difficult bits of the maths and which then in the child's mind, becomes associated with physics and hence producing a negative response towards physics in the Options System.

Obviously from my point of view, I would put more of the share of where it is going wrong on maths, rather than science, but I dare say they could argue differently, but this idea of applying mathematical knowledge rather than mathematical knowledge for its own sake is an area which I think should be more seriously developed - I think often the mathematicians have applied their knowledge. Simple example: \( \frac{1}{2} + \frac{1}{2} = 1 \), application of this: \( \frac{1}{2} \) orange + \( \frac{1}{2} \) orange is 1 whole orange.

Well it is not! You cannot take two halves of oranges and join them together to make a whole orange, and yet from a mathematical point of view many maths teachers can just reel this off and are quite happy about it.

Certainly at primary level I know this type of problem is given and I know, for instance, my lad came across an example of them involving half an elephant. You cannot have half an elephant. If you have half a thing that was an elephant it is no longer an elephant now, and this applicability that they have tried to do, I don't think it has been done particularly well. I think it has been done by mathematicians sitting down and working out what mathematicians think they should apply it to.

I think a far better approach would be to somehow ... I think it would have to be co-ordinated by the mathematics department but if they want examples from real life I think they should come to the other departments to get them. I think most departments could provide, for the mathematics department, a whole series of examples from their own subject, where maths is used, and simply, then if you are doing manipulation of formula you could do examples on titrations, density calculations, Ohm's law calculations, there is a whole spectrum covered by science and maths and to me it is more applicable than using X's and Y's and Z's and P's and Q's etc all the time. I am not saying that there
is not a place for this in a maths course but if you are going to apply it, I would rather see it done in a sense that was going to be useful to other areas of the curriculum than in its own little pocket where you are not going to get anywhere very fast.

With regard to various mathematical topics and their relationship towards science. Number work I think is reasonably good; the major danger of number work is the calculator. You ask a kid to divide one number by another and they will give you the answer 8 decimal places, simply because the calculator works to 8 decimal places, which the maths department has tended to shun - it has to be taught. Calculators are with us, calculators are useful, but you have got to know how to use them - I don't think that has been done particularly well.

Decimals I am quite happy with and fractions I prefer not to come across. I like decimal work because I think it makes more sense. The changing of decimals to the standard form tends to be done very late in maths, where as I would prefer it at a much earlier stage. If I am going to get \(6 \times 10^{23}\) atoms in a mole of element, I like them to understand what \(6 \times 10^{23}\) is and most don't. Percentages are reasonably well done, in the 4th year there is a certain lack of clarity: If you work out the % of sodium in sodium chloride then you automatically know the % of chlorine. They tend to work out both times the entire way.

Graphs/Gradients already been touched on. I think the technique for doing this work is fairly well done, the application very very tenuous. Manipulation of equations: this is the subject we tend to teach ourselves and tend to put it over reasonably but just as a technique. I prefer S.I. units, I don't like Imperial units being brought in. I think it causes confusion to a situation which is already very confused. Very few areas of the curriculum use coherent units. English for instance made an effort to talk about millimetres and centimetres but always still talks about miles, so the kids build up their units as being centimetres, metres and miles and they also, if given a length, estimate in centimetres, and if you give them a height they would estimate in feet and inches.
The mathematic use in science: the problem is that they will work out an equation mathematically but fail to put down any units. What tends to happen is that they work in blocks; they do an experiment, they get some results, they work it out, the working out is maths so they forget the units because the maths is taught as number manipulation and I think that is again, this applicability of science.

I think the major problem is not that they cannot do the maths, I think of the maths as techniques and a lot of children have the techniques to use. I think that as soon as the children try to combine two mathematical techniques it falls apart and they have a problem, and where you start to apply maths in other areas, as we tend to take not just one maths topic to solve a problem, but several, and this ability to integrate topics in their own heads causes problems and really worries them.

Liason between maths and science: there has been a few words exchanged, there has been a little bit on measurement, syllabuses exchanged and beyond that I don't think it has got anywhere. I think mathematics has had its problems in the past with changing syllabuses and certainly at the present school we have gone from Traditional, to SMP 9-13, to our own scheme and in the near future we are going to change again, and this is in the space of 3 years. With the Cockcroft report, hopefully maths courses will have a certain base to them, involving ideas from Cockcroft and maths will go through a period of stability and it is in the creation of that period of stability I would like to see more applications of maths specially with reference to science being incorporated into the maths syllabuses and a slightly less subject based approach where it is maths for maths sake. I think it has got to be more applied and the area of applicability where it is very easy to expand, certainly the information is there, is in science.
Mrs P Briggs (10 years teaching experience in both Maths and Science (Biology and Physics)
(Transcript of tape recording)

Ideas on the relationship between Maths and Science.

It is obvious when teaching science that there is an awful lot of maths involved and particularly this is so in the 4th and 5th years and this is very essential.

A particular problem I am finding at the moment with the 4th year physics is the graph work - they have no idea of the relationship between the graph work they were taught in maths and that used in physics and this seems to relate back to when I was at school. The graph work which I was taught was just putting down a set of numbers and turning them into lines and the actual meaning of this did not seem to be understood at the time and I am finding that the children will plot lines as they are taught in maths, but have no idea what they are used for and how they relate to the physics they are doing. They seem quite happy to get along with the work involved until you actually mention that a graph has to be drawn and this to them is maths and it seems to cause a sort of 'block', as the maths they do in the schools, they appear not to enjoy. So when it refers back to the work they are doing in science, they think of it as maths and start to dislike the work.

Problems seem to occur even before you get to the 4th and 5th years when there is an awful lot of maths involved, because there is a certain amount of maths involved in years 1 to 3 of the science course and as soon as the children of lower ability, who have problems in maths, think that perhaps this science they are doing involves more maths than they thought was going to be in the course (than they had had before) then they begin to wonder if science is as interesting as they had once imagined and it tends to put them off a 'choice' in the future. Perhaps if maths was made more practical and interesting then these pupils would not reject science because it involved the use of maths and continue with a subject they might have enjoyed.

I find that we get problems, when children are having difficulty in maths or not wanting to do maths, that keeping
discipline is harder. Maths to a lot of children is beyond them and if it is not beyond them, then it is the thing that they don't enjoy or wish to do and therefore you get discipline problems arriving from that. Obviously, somewhere earlier on in the school or basically down to Junior or Infants level there has to be some introduction of maths as an enjoyable, interesting subject, rather than a subject of which children are frightened, simply by the way in which it has been taught. When I was at school, maths was taught as a rote type of learning in which Tables had to be learnt - you had to sit down and just do bookwork, examples and so on, it was very tedious work, probably work from textbooks that was very repetitive, it was thought that we understood what we were doing and an awful lot of teachers, even in Junior and Infants schools were taught like that in maths and so themselves have difficulty in making the subject interesting.

For there to be a lot more enjoyment in maths, the amount of project work could be increased and a way of applying the actual maths they have learnt to different situations even perhaps at Junior school to show them that maths can be used to solve other problems, particularly scientific problems (at a very simple level) and for the children to get used to the idea that maths is not a subject on its own. That it can be used in other subjects in enjoyable ways - rather than just learning out of textbooks.

The fact that a lot of maths teachers in Junior school and in the lower side of the Secondary School, are teaching the subject as it was taught to them is causing a lot of problems, in that pupils at a very early age begin to dislike science or the science that involves any sort of maths and so eventually, careers that involve science will suffer because they have not persevered with science due to their dislike of maths.

Something has to be done at an early stage in the curriculum to deal with this lack of interest/enjoyment in maths, in that for
children to progress through secondary school to careers they have to have found something in the work that some can do, but do not want to, simply because they don't enjoy the subject and perhaps a lot more practical experience early in school life, practical work, and a lot more reason to why this practical work in maths links up with that in science could be looked at.
Discussion with:
Mr. W Turton (teaching Mathematics and Science)

This interview has not been taped at the request of Mr. Turton who felt more at ease discussing and giving added written comments on the relationship between Mathematics and Science. The comments were as follows:

I personally do not have any problems as I teach maths anyway, any uncertainties the pupils have I will explain to them. In the science I teach, which is lower school work, there is not a lot of mathematics, but basic number work, simple formula, measurement length, mass, volume, area are taught and this is duplicated in mathematics lessons. There has been some discussion on the duplication, some people feel this should be left to one department or the other, but I think it should be taught and reinforced in both lessons.

I feel that mathematics should incorporate in it other curriculum areas, such as English: the uses of many words such as Evaluate, Simplify - prove difficult for some levels of ability. Science can be very useful to incorporate into maths lessons and I believe there is a place in the curriculum for 'Scientific Maths'. We have been doing magnetic fields in science, so when faced with symmetry in a maths lesson we looked at some field patterns of combinations of magnets, which had been exposed on photographic paper to retain the pattern, and drew in the lines of symmetry. Virtually every other curriculum area could be included in a mathematics course.

I believe that a 'relevant syllabus' should be used which will cover 'useful maths' whether it be scientific, engineering, domestic or commercial etc as well as some of the more abstract or pure maths topics which can be approached as enjoyment for its own sake.

Having learnt certain principles in maths from the blackboard, I would welcome the idea of making use of the knowledge in a practical way. For example, I took a group of 5th years (low ability) out in the mini-bus and they noted times-speed-mileage
etc and these were plotted on graphs back in the classroom.

The children are more ready to recall enjoyable experiences and when I am applying a maths topic they will want to tell you that they did an experiment on that, in say physics, and it had obviously proved interesting, so they appear more content with the maths exercise because it has become 'real' to them.

The fact that I came into teaching from Industry and teach both science and maths, I can relate most topic areas to my personal experiences and try to make lessons come alive. This is how I try to teach but it is not the regular habit of other maths teachers who tend to conform to past practices.

When doing areas with my class we went outside with tape measures and clinometer to calculate the height of the school building using tangent tables and also part of the area of the school playing field using trundle wheel and the notion of a right angle from a 3, 4, 5 right angled triangle. This work was thoroughly enjoyed by the class and I find some concepts are more readily accepted and remembered in these situations. However, my Head of Department frowned upon this as it was deviating from the very rigid formal treatment he requires.

Children are definitely more receptive when faced with learning in a more interesting, applied situation. So because I teach both maths and science I incorporate both into my lessons.

Liaison between maths and other departments should be encouraged. If other departments could say what would be useful to them then it should be incorporated into the syllabus. I wrote a scheme of integrated concepts from the school curriculum and submitted it to the Head, who showed an interest but alas nothing has happened since, apart from another new maths scheme being forced on us, which most of the department thinks is unsuitable, but we are going to have to teach it no matter how unwillingly.

Maths syllabi in general do not allow for flexibility, or for showing the relevance or for applying the skills being taught to other
areas, often topics are not linked together as fully as they should be to help the solving of problems in other curriculum areas.

I try to make maths as interesting and useful as possible. This cannot always be done and some basic 'Need to know' maths can be as boring as a pianist repeating scales to obtain supple fingers.

Connections have to be made with other departments, for instance, the history of numbers and various mathematical discoveries prove interesting to many. The work has to become more related or partially integrated so that the work we are doing as maths teachers can be found useful and stimulating. I also believe rote learning has an important part to play together with this more practical approach but not to the exclusion of theory. I believe that maths and science could be taught as a unit, there being a phase difference in preference to the maths and that there are many conceptual strands throughout the curriculum which could also be included in this way. eg Concept of Time

1 In Maths: Seconds, hours, minutes, etc added/subtracted. Timetables
2 In Science: Part of MKS system, many experiments, including speed, gravity etc.
3 In Geography: Solar system, years, months, speed of light
4 In P.E.: Fast motion, slow time over 100 m etc.
5 In English: Creative stories - "Time Machine"
6 In History: Events through time
7 In Art: Time in different media
In response to your request regarding specific information about my experience and observations about the mathematical skills of classes taught by me.

Perhaps my most recent experience and evident problem to appear in this area has occurred with a 3rd year average to lower ability physics group, although I might add that it is increasingly apparent that this does apply to all the groups throughout the school. Obviously any satisfactory progress made in a scientific context, especially in the topic most concerned to me, namely physics and in particular mechanics with this group, is in my opinion bound to be influenced by poor mathematical ability. During the mechanics topic which involved drawing line graphs and simple calculations, a number of problems became evident. When asked to draw graphs a large number of the children proceeded to draw histograms, bar charts and so on, the lesson had to be stopped to give basic instructions on how to draw line graphs. Those children who did draw graphs, failed to label axes, and to plot the points accurately or to complete the work by connecting the points, scales were incorrectly worked out resulting in graphs in one corner of the paper, figures were mixed up to give maximum extension at 'No load' – the topic in question being Hooke's law. When the graphs were eventually drawn correctly, not without some difficulty I might add, the children were unable to relate them to the practical work done. They just did not know how to interpret the graphs. They found it difficult to accept the concept illustrated by the graph that extension is proportional to load.

It is difficult to decide whether or not this is in fact due to specific mathematics problems or if it is due to slow cognitive development, one must obviously realise that the majority of a third year group may not have reached Piaget formal operational development stage and as such, perhaps have difficulty accepting the abstract concept illustrated by the graph, but I would suggest that a high proportion of the problems associated with physics teaching in general is due to the poor application of maths.
When presented with calculations with decimals, problems arise, almost all of the class failed to appreciate that 0.5 corresponded to the fraction \( \frac{1}{2} \), so they could not multiply \( 60 \times 0.5 \) to get the correct result. It also seemed to me that some of the children found it difficult to accept that you could multiply numbers, in different units, Newton's and metres for instance. Maths appears to the pupils as just numbers and units are unimportant.

When some progress was made in mathematical areas in a particular lesson, the work was not, on the whole, retained by the children until the next lesson, and the graph problems supposedly rectified in the first lesson, reappeared possibly not to the same extent in the next lesson and they had to be dealt with again - a lot of revision was obviously required with such children. They also tended to mix units, sets of units, they neither used SI or Imperial, they were not able to estimate lengths or weights to any degree of accuracy.

It seems to me that maths is taught by the maths department for its own sake, it appears not to be related or applied in any way to the children's own experience. It seems that the children have an instant dislike for maths and this is evident in lessons where the motivation gained, doing experimental work and discussions and so on dies away very quickly during the maths part of the lesson.

Although in my opinion it may be that some of the children do not have the necessary ability to succeed in mathematical terms, in general I think that the problem is perhaps due to the lack of a satisfactory relationship between the science and maths departments. It seems that the children fail to relate work carried out in maths lessons to that carried out in a scientific context. In addition it seems that mathematics teachers fail to give the necessary reinforcement, and practice in basic skills, which I see as being essential for science based studies.
I have been a Head of Science in a Secondary School and at the present time I am doing supply work because it fits in with my family commitments. I have experience in administration in science and lately some experience in many different schools throughout Derbyshire.

I have been asked to comment on the relationship between maths and science and I have found that the problems which arise fall into two main areas:

The first of these is that children do not understand basic maths and cannot carry out basic skills, decimals and measurement, the difference between millimetres and centimetres provide great problems. It would seem reasonable to expect these topics to have been covered in Primary School and further practice to be given in the first two years in Secondary School to reinforce these ideas would not go amiss.

Another problem which is a bone of contention with Heads of Science is that children do not know their 'Tables'; knowledge of these is essential for children to be able to answer some of the easier science calculations.

The second area of difficulty arises in that children do not expect to have to use their mathematical knowledge in any other subject. They have great difficulty in remembering how to do a particular topic from one maths lesson to the next and when faced with a mathematical problem in science it all proves too much for them and then they give up very easily, a good example of this would be using graphs. In maths they will have done graphs, probably pie charts, line graphs and bar graphs - when faced with a problem in science that requires a graph to be drawn they will not know which type of graph to draw. Another problem arises in that not enough attention, or importance, is given to detail such as labelling the axes on a graph and stating the scale on the graph.
Because children experience problems in dealing with the mathematical content of science, when writing a science syllabus for the Middle School I choose as many topics as possible that do not involve maths. Obviously topics that involved measurement were covered but not without the difficulties already mentioned. This course of action was not something that I particularly wanted to do, but with children of such a wide range of abilities to cater for, the science facilities within the school and the time scale given over to science on the timetable, it seemed the most sensible course of action and another influencing factor in this decision was the attitude of the local High School to which the children were transferred at 13+. Our syllabuses had to be written with their requirements and approval. How they overcame the problems I am not sure, but I do believe they tended to do Physical Science at examination level as opposed to pure physics and chemistry.

With problems such as these many students will lose interest in physics and chemistry and choose other easier options in the 4th year; if they choose a science subject they will usually choose biology.

The only way to overcome these problems is for the maths departments to consult other departments about their requirements. For example, it would be a great help if the maths department could cover a given topic in the same half term that is going to be experienced in a science lesson. With even greater co-operation between departments this time scale could be narrowed down even further.

(NOTE: Mrs Savory is suggesting co-operation between departments and possibly extending to integration of maths and science. I suggest a possible way of making co-operation happen is to have a Head of Department over both the science and maths departments, and a member of the maths department in a scaled post whose responsibility was for liaison and application of maths knowledge to other curriculum areas).
Mr E Fancourt (Physics/Rural Science)

(Transcript of tape recording)

Having read one or two of the questions set, I would like to pass one or two comments about physics and maths which I have experienced over the past few years in every science lesson and not just physics, that is physics up to 'O' level or CSE. I always find we eventually get to the point where the science stops and I begin to teach maths because we cannot progress any further; the children just cannot cope with the maths involved in the science itself and this is not just 'O' level and CSE but all the way through the school. It appears they cannot use their knowledge in maths in a practical sense, in other words the maths department is teaching them some maths but it is not teaching them how to apply it to any real situation and so the maths knowledge they have gained is not actually used in the classroom in a science situation or any other situation for that matter possibly.

Also I find that science becomes a "chore" for these people because of the maths content, pupils are put off, straight away, from physics for example because they cannot cope adequately with the maths and of course the end result is as they get higher up the school they choose a non maths option, (a subject without maths) or if they have to do physics because of various pressures they lose interest in the subject and the end result is that they are not motivated to do well and the end result is a poor grade and poor performance in the subject.

Now to cope with this, I don't think it is the physics departments' job. I think it is the maths department that should really get to grips with this, because after all maths is not an isolated thing, it is a tool to be used by other departments, and therefore, the pupils should be taught how to use the maths, not just for one area. For example graphs spring to mind; it is alright getting a nice set of textbook figures and plotting them and joining them up so everything is more or less on the line that you expect. In physics, we don't get this in a practical situation, where many errors can occur and the best we can perhaps do is to
draw in the trend of the graph and then it is up to us to interpret it, because after all, graphs to us in physics are a form of language. I feel that the maths department is ignoring the fundamental reading and translating of graphs and rarely putting out what a graph can tell you. So that is just one situation where I feel that the maths department is not extending childrens' knowledge far enough. It is giving them some basic information but not teaching them how to interpret that information. It is nice having a picture book graph but it's not giving it a real practical situation and it's just this general feeling that, some, even the bright ones, know their maths, they know it inside out and probably do quite well in a maths exam but when it comes to sitting in a laboratory with some apparatus in front of them, getting facts and figures and recording them, they cannot transform those facts and figures into the maths involved. They just seem to miss out some of the practical applications somewhere, and I feel this is a real problem the maths department has got to get to grips with.

Generally speaking, perhaps the maths is being taught. They are not being taught or shown the relevance of the maths and of course this - when it comes into a physics situation the pupil is not interested - he thinks "God" maths again, and he cannot translate that maths into a practical physics situation. This is the problem we are faced with at the moment. No more real comments to make on the maths teaching. I do feel the maths department has to be much more aware of the applications that the maths is required for and not just to simply treat is in isolation. As I said earlier, it is a tool and needs to be used and used frequently in all areas, and the maths department should show the relevance of these areas —
Mr. K. Eaton (Head of Biology) (Transcript of tape recording)

Teaching 11 years: 5 as Head of Department, experience of both Grammar School and Comprehensive children.

Most of the schools I have had experience of require at least one science option to be taken, biology is usually the most popular. Not wholly due to an interest in biology, but I think rather a dislike of physics and chemistry particularly in the weaker students.

Whether this is due to a low maths content in biology or not, would be speculation on my part. Biology is different from the other sciences in that it has a low maths content. Maths topics tend to be isolated and specific with such things as the formula related to levers in the movement topic, or simply the ratios in genetics but most commonly the maths associated with biology is the construction and interpretation of graphs and this relates to a variety of topics.

The problems pupils experience are as follows: most are mechanically able to do a maths exercise, but are not able to apply their knowledge to a practical situation. To them a maths exercise is an end in itself, with no application to other subjects. Many are not able to interpret the end results of an exercise in biology that involves maths, they have difficulties in drawing conclusions and inferences. Pupils are often surprised when the collection of data does not produce ideal results, they are usually unaware of errors and how to deal with them.

Biology is a lot different from the other sciences in that it is possible for the pupils to obtain a high grade without a good maths background, however, it is noticeable that the level of interest and motivation does seem to be related to the feelings of tension or ease when handling mathematical problems. In my opinion maths teaching should involve more applied topics and the application of collection and interpretation of data in order to encourage the transfer of training to other subjects.
Mr K Stinson  (Transcript of Tape Recording)

15 years teaching, all of them in a Secondary Modern School. Initially 2 years as a Mathematics teacher, gradually moving into the Science department, as a Chemistry teacher, which was a scaled post, then into Physics (for which I had been trained) and eventually became Head of Science for some 7/8 years.

Based upon my original experience as a maths teacher, I feel that recently, particularly with the advent of S.M.P. maths, maths has become a subject far more in its own right and for its own sake, rather than as a help for other subjects and from my point of view, physics, I get the impression that some years ago when maths was taught by much more traditional methods, every day examples and problems from outside of the classroom were given far more credance and applications of the principles involved seemed far more common than they are today.

There is no doubt that physics is regarded as the "Hard Option" and tends to be chosen, only by the average or better than average pupils, which is perhaps a good thing from a Teaching point of view. Having said that physics is regarded as the Hard Option, I am not certain that it is entirely due to it being a maths based topic. I say that because up to the 3rd year when the children pick their options the amount of maths needed by the pupils in this subject is very limited. All they need to do is to be able to multiply and divide fairly accurately, measure angles and plot very simple graphs. Although that is simple to put off pupils of very low ability, most pupils should be able to manage and I feel there is a group of pupils of average and slightly less than average ability who can manage the maths involved but find the terms and units, some of the more abstract concepts used in physics difficult to understand and do not pick it for that reason. The problems experienced in the 4th/5th year physics groups due to poor maths ability tends to be more highlighted in CSE groups than an 'O' level group, and this is probably what you would expect. I am not saying that all 'O' level pupils are good at maths but rather that they are more conscientious.
and they buy a pocket calculator where the CSE pupils do not bother. For any physics class the only way to overcome the problem of low mathematical ability is to teach the maths involved at the same time as the physics. It is surprising sometimes just how basic one has to become before the pupils 'cotton on' to what is really happening and then it is a case of building back up again to what you really wanted to explain and this can obviously take a lot of time.

There are a number of areas where I find particular problems, particularly with CSE classes, working in vulgar fractions for Ohm's law calculations, resistances in series and parallel etc. This is something that I have got to teach from almost first principles, particularly for pupils below 'O' level or CSE grade 1 standard. Most CSE pupils have a fair understanding of what a percentage is but literally no idea how to convert a vulgar fraction to a percentage until shown. There seems to be no transfer of training from maths lesson. Most pupils can use a protractor reasonably well but I wonder how much has really been absorbed when mistakes like 60° for 120° are often written down.

Very simple formula work is just about acceptable provided the pupils are only asked to find the subject of the formula. Rearranging even a simple formula with a CSE class is virtually impossible. It is perhaps a good job Equations of Motion are not now on the CSE syllabus. Most pupils can use 'Maths Tables' but I doubt if $\sin \theta$ over $\sin r$ would be possible without a calculator to do the decimal division.

The problem area where I find most trouble is, and annoys me more than anything else because it should be easier to solve than many of the others, is Graph Work. Pupils are taught how to plot straight lines or curves in maths where every point is in its correct textbook position. They seem to have no idea what to do when faced by a practical situation; they seem to be able to plot the points correctly but then every point has to be religiously joined to the next giving very strange shaped graphs. I always have to teach experimental error and plotting a mean or best line through their results. This is usually new to them and I feel
the maths department is not extending the pupils knowledge to real life situations. Things are not always perfect in life – are they? Again as a follow-up to graph work, I always have to teach how to interpret the graph and obtain information from it, particularly if the graph has to be extended or projected backwards to cut an axis etc.

Being honest, I feel that some of the things I complain about 'as never having been taught by the maths department' probably have been taught but from a very narrow maths orientated point of view and applications in other subjects and wider situations are generally being ignored.

I should hope that maths is seen more as a tool to find out other things rather than a subject for its own sake and nothing else. More practical examples should be looked at so that the pupils can see the relevance of what they are doing to other subjects, to life in general. There is no doubt that a few pupils do mentally switch off when a maths based problem appears in a physics lesson. They probably feel that they have had double maths already that day and don't feel like another double.
Conference of Science Teachers attended by the author

This conference was held to discuss the future science provision in a large area of Derby, covering the catchment areas of three adjacent schools, with a pupil population of over 4000, and science teaching staff of approximately 30. (Noel Baker School, The Merrill and St. Thomas Moore School).

The discussion started on the choice of Examination Board so that all three schools could share in the 6th Form teaching, because none of the schools had a viable number of pupils to economically run a sixth form, and carried on with the question raised by the author - "How can we make the choice of science subjects more attractive?"

The consensus of opinion was that chemistry and physics were suffering more than biology. The pupils regarded these subjects as being the difficult choices and only the very best should take it. A large proportion of the blame for poor attendance on these courses was placed firmly at the door of the mathematics department. It was the opinion of nearly all the teachers that pupils were 'put off' by the mathematical content of science courses in Years 1, 2 and 3 and so some of the more able who wanted an easier time and many of the middle ability opted out of taking science courses in Years 4 and 5. It was the policy of these schools to shunt the lower ability range pupils into the less mathematical general science course.

One school in particular was very worried about the situation and had tried for a period of a couple of years, teaching a course in Years 1-3 which had no mathematical content. This proved successful and attracted a large percentage increase in science option choices but failed in so far as once the courses inevitably commenced their mathematical content many pupils lost interest and failed to meet the standard.

It was decided that the problem could not be solved by avoiding mathematics in the earlier years but must be tackled through the mathematics department. It was here that the interest in science
could be improved, by improving the general attitude of the pupils to mathematics as a whole.

The local Roman Catholic School had already started liaising with its mathematics department and supplied them with chronological list of their requirements and examples of where the mathematics teaching could be applied to science in a relevant way, but they reported that it was over two years since this information had been passed on and nothing had developed, the situation was unchanged.

The conference closed with the conclusion that more effort and discussion was necessary on this topic and this would be continued within each school.

The author visited St. Thomas Moore school and talked with the Head of Science (Mr Seebrook) and the Head of Physics (Mr Margle). These teachers indicated that the department felt this to be the major difficulty in teaching science and they had come to the conclusion that the mathematics department were not providing them with an ideal 'service'. To overcome this the Science Department had spent some considerable time looking at the order in which they teach the science, to rearrange some of it to be more in phase with the maths teaching, (but this could not be done in all cases) and supplying the maths department with examples of where the application of maths could be taught. They felt that the pupils attitude to mathematics was not desirable for successful science teaching and that the pupils had to be taught how to use their mathematical knowledge in other subjects.

Since this time, Mr Seebrook stated, there had been hardly any contact between departments, mainly because the mathematics department did not desire such meetings.

The Head of Physics placed part of the blame for the poor mathematical attainment of many pupils on the trend to build 'campus-style' schools, where there were so many children that teachers were becoming lecturers and this resulted in a teacher
not appreciating a child's personal problems.

In these large schools, many with split situations, the staff did not really know their colleagues well enough and departments rarely communicated with each other. The reduction in Educational spending by the Government had resulted in poorer pupil teacher ratios and an increase on contact time with the associated increase in marking and preparation time for teachers who were already working many extra hours at home. So when demands are made to improve inter-departmental relationships, they regard this as being of very low priority, whereas Mr Margle felt it was very important for the efficient running of the school.
Meeting of Heads of Science in Derbyshire

The author attended a weekend meeting of 42 heads of science from Derbyshire at the Conference Centre in Buxton. This was designed to improve "The Managing of Change" in science departments and related to the new examination syllabi, the use of the computer and the trend of change. At this meeting were several of the County's advisory service, a former chairman of the ASE (Association of Science Education) - Mr Heaney, and Dr P H Andrews who was associated with the future 16+ examination.

The author used his time to find the general feelings of those in attendance, with regards to the relationship between mathematics and science. The experiences the author had had whilst teaching in five comprehensive schools in Derby and the surrounding area proved to be common throughout Derbyshire. The overall feeling was of dissatisfaction with pupils' ability to apply their mathematical knowledge to experimental or real life situations. Many complained about having to be primarily a mathematics teacher and then a science teacher, instead of the science teacher making use of work done in the mathematics department.

The areas of contention were that all abilities of pupils were unable to use their mathematics knowledge with any competence. The units used in science proved unfamiliar and estimation is rarely taught. Although decimals could be managed with a fair degree of success, with the exception of division of many digit numbers, fractions and reciprocals proved to be in need of attention. Quite often there were complaints about the pupils' ability to understand some mathematical terms, words like 'share' and 'take aways' were often in use and words like 'calculate', 'evaluate', 'simplify' and 'construct' proved confusing when appearing in a problem. The ability of the pupils to read a problem and decide what was required of them was in great doubt. It was felt that both graph work and
formula work needed a lot of attention and that to a certain extent the use of 'log tables' was no longer required as most exam boards permitted the use of the calculator. Yet on this subject, not all mathematics departments taught its pupils to use them efficiently and many did not have supplies of them for use during lessons.

Certain fixed terms, such as \( \pi \), were not known or readily remembered.

Inequality needed attention for some of the chemists present and the vast majority of those present complained that the pupils could not draw conclusions from a given set of data, or graphs obtained from a practical situation.

Mr Heaney, of the Association of Science Education, said they were particularly interested in this problem of mathematical use in science and of the trend for pupils to drift away from the physical sciences and would like to set up a committee to investigate the matter.

The advisory service acknowledged that this problem had been with us for some considerable time but were not prepared to lay the blame at any particular door.
Mr B Cope - Deputy Headmaster (Ex Head of Science, Head of Physics) (Transcript of tape recording)

I have been asked to give my opinions on the Maths/Physics relationship:

I have felt for a long time that there is little or no carry over from maths lessons to other subject areas, particularly physics, which basically I think is the application of maths to a large extent. Pupils seem unable to use their maths skills in any practical situation outside the maths lesson, they have to be told what maths topics to use and to be shown how to use the maths topic in solving questions in physics.

Ratio and proportion for instance are often applied in physics in various ways and yet children seem unable to carry over the concept of ratio and proportion that they learn in maths lessons into physics lessons. They can merrily give you the answer to a problem: If 6 oranges cost 36p, how much will 10 cost? but a similar problem in physics, say Heat, if so many calories will heat up so many grammes through so many degrees and so on - they just seem to have no idea of how to solve it and problems that involve several maths topics are really very very difficult for them and I personally have to spend a lot of time explaining the links between the various topics and how they have all done them in maths, so they should be able to do them in physics.

I think maths is very often taught as a series of techniques to get the right answer rather than a useful tool that can be used throughout our ordinary private lives and our working lives. A lot of what they are taught in maths, to me, seems unrelated to what they need in everyday life. The maths departments don't seem to push this link between their subject and practically everything else. It is not as obvious as say in English. We all need English and it is a fairly obvious link. Because of this 'maths difficulty' that we have in teaching physics and in science in general (or specially chemistry and physics) becomes in the pupils' mind the HARD SUBJECT and will not choose it as an Option, and they don't seem to realise that a lot of physics is applied in mathematics.
I feel that maths courses should be looked at and also the particular reasons for teaching maths and I would like to see the maths department organise it so that what they teach is relevant to other departments and relevant to other curriculum areas.

Teaching physics - at each stage, I find from years 1 to 5 and into the 6th form we have to teach some maths before we can make a particular topic understandable to the children.

In year 1 we start with measuring, weighing and calculating densities and an awful lot of work has to be done on the decimal system, on units and on simple formulae and the manipulating of the formulae, and the children find it very difficult and yet they can do the decimal multiplication and division in their maths lessons and just cannot do it in any other, or apparently not able to do it in any other.

All through the first five years children find it difficult to estimate sizes and they use a system of units which is a mixture of metric and imperial. But even then, where some will criticize the metric system, because Mum or Grandad says the Imperial System is better, if you ask them to show you a yard they have no idea what a yard is; their hands are put anywhere between 15 to 100 cm apart. In a little test I did, I asked them to ask their parents to estimate a yard with their hands (when the decimal system was first being introduced more into schools at the expense of the Imperial System) and very few reported back that their parents got anywhere near a yard, varying from 12" upwards as it was in those days.

Graphical work I find is very poor and by Year 3 one would expect them to have some concept of graphs. What the graph can tell you; how to draw one given the information, how to choose a suitable scale to fit it onto the page, and yet I find, and I'm sure my colleagues would confirm, we have to go through this every time. Choose a scale so that covers most of the page available, draw your line, if it has got to be straight line use your ruler, then pick the best line through the points.
If it's a curve, it is to be a smooth curve because we don't usually draw graphs that go zig zig from point to point and then again we have to explain what the shape means from the graphs: straight line, curves, negative gradient, positive gradient and so on. When they get to the 4th and 5th years they should be able to do this without me having to spend so much time explaining it. In maths lessons, what it seems to be is that they are given some information and they have to plot a graph and the interpretation seems to be left completely alone except if you get a curve that it may be a quadratic or so on.

Formula work always proves difficult, even into the 6th form manipulation of formula is baffling and always gives us problems. Once you get to the mathematics part of a problem or a particular topic in physics, then a lot of the motivation and interest of the class just dies, one can almost feel it die and I have tried various approaches to this: start the maths first, leave it out completely until they have a thorough grasp of the topic or introduce it gradually as you go through the topic, and in every case you can feel the interest dying away once you get to the mathematics part of the problem. In particular reciprocal work in the 4th year, when you are doing adding up resistances in parallel, the reciprocal baffles them completely and even quite intelligent people make stupid, idiotic mistakes in changing it back. Using the lens and mirror formula uses reciprocals in the 4th/5th years has always been difficult. Nowadays, this is less needed than it was. Nevertheless, the manipulation of the figures is very very difficult to get over.

In the 6th form I find that various topics have to be taught to the class before we can do it, radian measure is not introduced very early in mathematics, we need it very early in physics - that is just one example of what we need. Integration, Summing and In the Limit is also needed very early on and the maths department don't seem to do it as early as they ought to, or perhaps as early as we would like them to.
There is still the feeling that a lot of mathematics teaching is done in compartments, in total isolation, perhaps not total but in isolation, from the rest of the school. I still think maths could be made more interesting and enjoyable if they related it more to other subjects, to other areas outside the school and perhaps make it less repetitive in the mechanical sense. They can have repetitive problems but make them more interesting and also the pupils need to know why they are studying maths - a need for the use of it. They do need it but it is difficult to persuade them that they do, particularly when their interest has been killed off in maths lessons anyway. But I am sure if pupils could be shown clearly that maths is a vital tool, not just to enable them to solve maths problems, not necessarily to enable them to do well in chemistry, physics and other maths based subjects or maths related subjects, but a useful tool just to enable them to live, then I think our task would be a lot more easy in science teaching.
Mr G A Brighouse Headmaster of a junior school

Mr Brighouse stated that up until the early 1960's only arithmetic skills were being taught, and thus there was a need to broaden the whole mathematics base. To this end he followed the work of people like Edith Biggs who had done a lot of the early research in this field.

However due to the lack of Mathematics experts at Primary/Junior level, this broadening of the Mathematics base proved difficult, if not beyond many teachers, as they were unaware of its value or its place in an ordered approach, so the whole benefit of the exercise was minimised.

Many teachers returned to repetition of the old ways, or splitting the mathematics teaching into two very distinct categories, that is, Monday and Tuesday lessons being pure number work (old system) then Wednesday to Friday could be practically based mathematics (the new system). These teachers could not see a way of combining the two categories.

They dealt with it in a totally random way, having no logical links between topics or any awareness of why they were teaching some topics, for instance Translations and how this could be used in the Arts and Crafts with regard to shapes and manufacture, so its value in isolation was worthless.

Mr Brighouse firmly believes that number skills are very important but not in isolation. It is only when these number skills are being used in practical problems that one finds out whether the children have gained a good understanding of these skills, and at this stage it may be too late to rectify any inabilities.

In present day primary/junior schools there appears to be no method of assessment or structure to courses, essentially the SMP 7-13 scheme tries to build this in as a very relevant
part. This would enable pupils to change teachers or even schools and enable everyone concerned to know exactly where this child's progress had led to.

It is a very difficult problem for schools to develop their own courses, and solve the problem of applying the Basic Skills to other fields. They do not have the ability or time, so they rely on schemes that others, possibly outside of comprehensive education, have prepared. Of the 600 to 700 schools Mr Brighouse visited as part of his preparing the SMP 7-13 scheme, he found that not one had been able to design their own scheme.

The schemes readily available gave no ordered structure, no means of diagnostic testing, were of a reading ability beyond most pupils and there seemed to be no means of providing continuity from teacher to teacher. In the SMP 7-13 scheme all the structure of development of materials, concepts and language, and the development of any particular section and its links with other sections has been made clear. Diagnostic testing has been built in to show the "short falls" in particular teaching methods. The stage of progress of a pupil is perfectly clear so a new teacher would know the child's exact achievement.

Mr Brighouse stated "This course is capable of being developed to suit the particular needs of an environment or region."

Mr Brighouse felt that on moving up to the secondary school a child needs to have all the arithmetic skills, understanding and an enjoyment in doing mathematics. In the past they had been taught the arithmetic skills but could not apply these to different, unusual situations. He believes that a child should not be educated for "need", our job he said "is to give the child a love and interest in basic education and for this to act as a 'spring board' to branch into other topics. Even those who will gain no examination success will, we hope, have an interest and enjoyment in mathematics".
Mr Brighouse felt that organisational demands on secondary schools were very great and this puts a strain on teachers who were not so familiar as teachers in primary/junior schools in teaching large mixed ability groups where children worked at vastly different rates.

There is also a danger of lack of continuity in the curriculum between two schools and this disruption causes many to lose interest in mathematics. There must be a continuity of aim, attitude and organisation which are just as important as the materials used.

There are two places we lose children's mathematics interests:

1. We must ensure a child has a sound understanding of "place value" in primary school because it is often too late when done in secondary school.

2. The transition between schools at 11 years old. This must be seen as a continuation of a sound education scheme, not the starting of a new one. The change of environment, teachers and friends is sometimes too much for some children, without the need for completely new and different teaching aims.

It is obvious that to gain continuity between primary/junior schools and secondary schools that a tremendous amount of liaison has to take place, by teachers who want to improve the system. This is not going to happen "overnight" and in many many schools there appears to be a reluctance to accept liaison; it is classed as interference.

Any new teaching scheme relies on teachers being fully committed to it, and to the building of bridges between the schools for some constructive discussion. The primary school teacher has no set aims or tasks to complete by certain ages, as does the secondary school teacher, so what does a secondary school teacher do when the assessment tests show a lack of understanding
on the part of a large proportion of his class. The answer seems to be that he continues in order to complete his syllabus and gain examination success with those understanding. This is a dilemma that primary schools have not got.

Curriculum development is the life blood of a school, and all good development comes from people with genuine feelings for trying to do better, as people, or a school, or group of schools, developing for the future. Teachers must help towards promoting skills within the profession, or give the guidelines to help others develop skills. Mathematics courses at local teachers education centres are always well-attended but there seems to be a need for more in the school base. Experts should be available to attend classes, not to teach, but to help the teacher or each other in gaining expertise in topics previously unsure of and thereby raise the standards of education. This interaction or team work relies to a great extent on the compatibility of people to work together in planning, teaching and analysis of what has to be achieved. This is made difficult by the constant movement of teachers to gain promotion or the loss of good classroom teachers to administration.

In the SMP 7-13 scheme we have tried to build skills into activities and give the resources to fulfil this. It gives those teachers who need it, a "fallback", to gain information in those areas they are unsure of.

However one significant fact previously mentioned, that of bridges and liaison between schools has not been overcome even in the design of this scheme, in that, there was no consultation between the writers for the section 7-11 years and those for 11-13 years. It appears to have been written in isolation, and there is a lack of continuity in the course but not as great as in present teaching, where this course is not being used to take children through the 11 years barrier. The scheme is being modified to stretch the gifted pupil by
broadening his understanding of topics and ability to deduce through investigation. Rather than allow him to progress through the course in primary school to be held back at the start of secondary school (until the slower ones catch up) by recovering material already done. This leads to boredom, possible stagnation and perhaps a loss to Mathematics. Hence these pupils will benefit from a widening of the topics "sideways" and building up deductive reasoning.

Mr Brighouse maintained that for the benefit of mathematics education and to approach societies needs we had to:

1. Have a course whose continuity crossed through the change of schools at 11 years

2. This course had to be designed with a very clear scheme of work indicating required depth of teaching, reasons behind its need, and practical applications of it including those outside the field of mathematics.

3. Due to the shortage of qualified mathematics teachers the course is designed to help those teachers unsure of certain topics.

4. A recent survey at Matlock College of Education showed that Mathematics was the only subject that most primary/junior school teachers feared, because of the expertise required to teach it effectively.

5. There is a need for structure, clear links between topics (integration), assessment and pupil records.

6. The course needs to be practically orientated, providing enjoyment and challenge to all ability ranges.

7. Teachers must be convinced of the need for curriculum development and strive for improvement of standards.

8. Departments and teachers in schools should utilize their expertise by helping their colleagues to obtain greater understanding of the relevance and practical aspects of the course.
9 Schools and industry should work together to further these aims

10 The required changes would not come from an edict but from caring teachers, who have a concern and deep desire to help children gain a sound mathematical education which will serve them well in the future

11 We have not to be afraid of, or resist the changes which are necessary
Discussion with:

Mr R Sendal  Head of Mathematics, 11-18 years comprehensive

Ripley Junior School, of which Mr Brighouse is headmaster, feeds children at 11 years to Ripley Mill Hill School. When the technical school became comprehensive in 1980 a new course had to be found which would be suitable for a comprehensive intake, rather than the more formal "chalk and talk" type of course associated with grammar schools.

Although the staff did not want change, it was thrust upon them, and as the feeder school was using the SMP 7-13 scheme it was felt that this would be an appropriate course for the middle school. For some considerable time before going comprehensive the scheme was being examined. Many were not sure about the depth of practical work involved in the course or about the uses of certain mathematical concepts in the outside world. The greatest factor to overcome was the one of change and venturing into the unknown.

The course has now been running for one complete year and the staff have settled into the practical aspects of this course although there are reservations on its suitability for the highest ability range and on the amount of extra work it involves during lesson times on:

1. Preparing equipment
2. On maintaining discipline in a situation where pupils had to leave their desks to go and use equipment
3. On finding sufficient time to help each child for a worthwhile amount of time
4. Becoming totally familiar with all the facets of the scheme

Mr Sendall pointed out that the success or failure of this scheme depended on the commitment of the teachers, and their ability to work together as a team. Constant discussion amongst the mathematics department's staff had led to a development of particular skills, materials and course content.
Mr Sendall commented that all ability ranges enjoyed doing the practical aspects, but the higher ability range soon lost interest when they had mastered the theory behind the practical, and tended to skip the practical and do the problem as a pure mathematics function. However for the middle and lower ability it proved to be a motivating influence, but at this stage in its use it was too soon to say whether it would be beneficial in producing a greater understanding or competence in numeracy.

The work card situation enables the differing ability ranges to work at their own pace, although some incentive or stimulus had to be applied to prevent some from working at a reduced pace. The work cards were also being used by the remedial section, who were finding them very useful, but their needs were to integrate practicals with much more rote learning techniques.

Rote learning was an essential part of any mathematics course. Mr Sendall's final comment was that he had just gained promotion and he wondered if the new head of mathematics would be as firmly committed to this course.

This question was answered when the author checked at a later date and found that during the first term of the new head of department, that this course had been dropped in favour of a more traditional course which seemed devoid of any practical application.
Mr M Smedley  Head of Physics at a very large comprehensive school in Derby

I feel unable to make any pertinent comment about the teaching of Mathematics as I have not done any since my teaching practice days, or on the effect a dislike of Maths affects options choice. Although at this school, girls in the main avoid Physics, partly because it appears a "boys" subject, partly because of a fear of the maths involved.

The most difficult areas of a Science Teacher's tasks are:

1. **Division**  ie, \( \rho = \frac{\text{mass}}{\text{volume}} \) beats all but those of above average ability until year 5

2. **Transposing Formula**  in any form throughout the school  
   \( \text{Ohms law } v = IR \)  
   \( \text{Newtons 2nd law } F = Ma \)  
   acceleration \( a = \frac{v - u}{t} \)  etc

3. **Dealing with Powers**  of numbers, for instance \( 10^{10} \times 10^{-11} \)  etc

4. **Calculators**  Some of the difficulties are overcome by the use of calculators - most pupils now seem to have one. However, this throws up further difficulties:

(i) Use of realistic number of decimal places in an answer, particularly in practical work. This effect is probably seen mostly at 'A' level

(ii) Often pupils use calculators unnecessarily, eg for those of higher ability in a recent examination the pupils were given \( R_T = Ae^T \) and were asked to find a value for \( A \) at one temperature which came to \( A = \frac{1000}{e^n} \) where \( e = 2.72 \).

Then they had to find \( R_T \) at a new temperature which gave \( R_T = \frac{1000}{e^n} \times e^{10} \) thus \( \frac{1000}{e} \).
All of the pupils instantly worked out a numerical value for A, and then used this in the second part of the question, instead of leaving the calculator until a later stage, after cancelling. I think Science or Mathematics needs to consciously teach how to use a calculator with discrimination.

(iii) The need for pupils to make ESTIMATES before using calculators

Motivation I have not noticed loss of this with the average and above - they usually make an attempt, then ask for HELP.

With below average in Physics and Engineering Science I try to avoid all but the most simple Maths.

Repetition I often feel there is insufficient repetition in Modern Maths lessons. Certainly schemes like SMP do not give pupils adequate practice at basic skills.

There is a need for 2 forms of Maths. Firstly, basic arithmetic skills for all. Secondly, more esoteric Maths for the more able. There is still a lot to be said for teaching

   Arithmetic
   Algebra
   Geometry

and then Mathematics, but I doubt if one could find the time to do it all.
Pupils' Comments.
Q. Tell me something about the Maths course that you did - did you work from a textbook or worksheets or ...?

A. When we first started it, teacher would go over it on the board and we would have to do exercises once he had done that. If any then needed help he would give it to them.

Q. Did he apply the Maths that you were being taught to everyday life situations or was it just work from the textbook?

A. He did not apply it too much. He did a bit with formula but that is all.

Q. Did you have practical applications or work with equipment?

A. No

Q. Your Maths lessons were therefore composed with working from the textbook?

A. Yes

Q. Did you find that interesting?

A. Not really - No

Q. If you could have dropped Maths, would you have done so?

A. No

Q. Because it has some relevance to your future life?

A. Yes

Q. Don't you think that being shown the relevance of the Maths would have been important?

A. No not really, I would not be interested even if he did show the relevance. The Maths at that time was not interesting
Q. Should the Maths in your Maths lessons involve examples and problems related to other subject areas, i.e.: Physics, Chemistry, Technical Drawing, rather than just X's and Y's?

A. Yes, it would make it more interesting.

Q. What do you want to be when you leave school?

A. I want to go to Catering College to study to go into Hotel Management for which I would need to know Maths to keep books and stuff.
Q. Is the Physics course made more difficult because it contains a considerable amount of Maths used in a different way to that used in a Maths lesson?
A. I find that you can be taught different ways of answering a problem and you might pick the wrong way of doing it; you are given different methods of solving a problem.

Q. Do you find that you have to be told the topic to use, or the method of solving it rather than forming your own strategy?
A. I can make my own mind up although often there are different ways of solving it. You perhaps have two different methods which give the same end but you start with different facts and work on different principles but I usually can make my own choice.

Q. In Maths lessons do you work from a textbook, doing reams of problems or do you apply your mathematics?
A. It is mainly just examples from the textbook.

Q. Did your teacher show you the relevance of the Maths you were doing and how it could be used in everyday life?
A. No, I am afraid not. You were told how to do them. It was never explained why, or how to use it.

Q. Do you think a course in Maths would be any more enjoyable if you could see where and how it would be useful and how you would use it?
A. Yes, it would develop a greater interest in it.

Q. Do you think interest in the course is important and that enjoyment would make you work better?
A. Yes, you would be willing to try if you knew what you were working towards. With an interest in it you perhaps would put a bit more effort into it, rather than treating it like a 'chores'.
Tel'y Davis:

Well, I like Maths but I find that we tend to spend too long on each subject: like we do graphs and that for ages but I think it would be better if we did graphs for 1 week, something else for 1 week - to alternate it, then we could do the same amount but on alternating weeks.

Q. Would you prefer to do several topics over the period of several weeks which were all related to each other, so you could use 4 or 5 different topics to solve one problem. Rather than learning all your topics in isolation?
A. Yes, that would be better.

Q. Do you work only from textbooks?
A. No we don't; we do some topics on the board which are not in the textbook and I find that more interesting than working out of the textbook, as he explains it.

Q. Do you find Maths interesting or boring?
A. Some topics are interesting - those are the topics applied by the teacher on the board, others are not interesting.

Q. How could you improve your motivation to do better at Maths?
A. Better split into groups that need certain types of Mathematics and we studied those topics which were necessary for your career.

Q. On topics you do not find relevant to you, you do not try as hard?
A. No, I do try the same, but I do think it would be better to do things that are going to be relevant than not relevant.
Q. Does your teacher show you where the topics could be applied in life or in work?
A. No.

Q. Does he quote examples of the uses, which might motivate you to greater efforts?
A. No.

Q. Would you like to know the relevance or applications or why you need them or the actual linkage between topics?
A. It would be better if we knew how were going to need them.

Q. Why did you not choose Physics, are you doing Chemistry and Biology?
A. No, I am just doing Biology.

Q. Why did you not choose Physics?
A. The course was a bit difficult. I was not really interested in Physics.

Q. Was it because there was a lot of Maths in it?
A. Yes.

Q. Did you find that the Maths in Physics was as easy as when you did it in Maths or .. ?
A. No because it was applied to different topics.

Q. Were you unsure of the Maths topic to use to solve your problem?
A. Yes.

Q. If your Maths was 'applied' to different subjects, you would then have been able to do your Physics a little easier. If they showed you the relevance, the application for instance, instead of just using X's and Y's in equations to use equations found in Chemistry, Physics, Metalwork etc, that had relevance in life. Then you would be able to apply your Maths in other subjects?
A. Yes, I think that would be a lot better.
Q. Which areas of Maths do you find most difficult?
A. Simultaneous equations and equations.

Q. What about your graphical work, in Maths all the points lie on a straight line, in Physics they usually do not, did that tend to confuse you?
A. Yes.

Q. What type of graphs were you used to drawing in the 3rd year when you did Physics?
A. Plotted points, drew in the mean average.

Q. Did you teacher show you how to interpret graphs, because graphs are a form of language, they tell us something or was your exercises in graphs just being able to plot points in a mechanical way or did he show you graphs and explain how to interpret them?
A. No, he showed us how to do them – not why to do them.
Jo ann Williams: (Transcript from tape recording)

What I think of Maths. The first of all, when Maths is mentioned it is always made out to be one of the dreaded subjects and that is brought about by the way it is handled when you go into lessons. For most of the lessons I have had in Maths and the teachers don't want to try and explain things because they don't seem to have the time or be bothered about it, so you just go away not really sure what it is all about, but you are sure of how you are meant to do it but not understanding it properly. The teachers I have had have not been that good and I am not very bright at Maths anyway. I never have been because I don't seem to be that interested in it at all.

The work we do in Maths is not just working from textbooks - the lowest set I have been in the more basic the Maths has been, the higher the sets the stuff does not seem to have any relevance at all to anything we might need it for, whereas in lower sets they seem to stick to basics which you might need, which I can see now but I still do not find it interesting because we do work quite a bit from textbooks, although it is varied a bit when we do work on the board, but this is just copied from the textbook and put on the board a bit differently.

The way in which I would be probably motivated to do better would be 1 pupil to 1 teacher to be able to understand everything properly and explain to me and for more practical things to be brought into it so that you can find out how it is brought about and everything. Like I say, the stuff we are doing in my set at the moment is very relevant to what we might need later on in life but it is still not done in an interesting way.

Q. You chose not to do Physics - can you tell me why you did not want to do Physics?

A. I find that I enjoy it when I solve problems and stuff like that, it makes me feel good when I can do it but most of the time the problems that have been set tend to be very complicated and also with the Maths I don't
understand anyway, and I cannot work it out properly, and it's just a load of figures and numbers put down on the paper.

Q. Was the problem with Maths one that you did not know what Maths topic was required to solve the problem, you had to decide yourself the strategy for solving it, possibly using more than one Maths topic, and you had not been shown this process in Maths?
A. No, I think the way we had been taught to do things does not fit in with the way it was to be done in Physics lessons. It may be the same thing but it does not seem to be.

Q. Are you doing Chemistry as well as Biology?
A. No.

Q. So you chose a Science that did not involve Maths?
A. No not really, I have enjoyed the Biology I have done and that is why I chose it.

Q. Is there much Maths in Biology?
A. Various equations that you have to memorise but nothing that you have to work out and they are also in word form – as you know.
I am doing CSE Maths instead of 'O' level and this is one of the reasons I did not take Physics. I did Biology instead because it has not got so much Maths in it. With doing CSE Maths, the Maths is a lot easier than I would be doing at 'O' level, but it is a lot more boring; I think due to the fact we do a lot more of easy Maths, times and stuff like that so we don't do the more interesting bits. All the work we do do we have to go over a lot of times because of the people in the class who are a lot worse at Maths than me. I think I could go a lot faster in my work than the others do but we all have to go at a slow pace.

Q. Do you find when you go at this slow pace you mess about because you have some extra time?

A. Yes and with the fact that I did an 'O' level course in my first 3 years - now that I am doing a CSE course I have done all the work already. I have not really learnt anything yet this year because I have done it all before.

Q. In your lessons up to now have you been motivated to work hard or has it been a case that you are doing CSE because you were not motivated possibly because of the subject?

A. Yes, I thought I would be better doing CSE because I have never been very good at Maths. It is a case of just sitting and waiting for the exams now because everything they are doing I have done before.

Q. Did you find there was a lot of relevance in the course for you or did your teachers show you how you could use the Maths you were being taught in a practical situation?

A. No I don't think so. We do more now in CSE. We do more - telling us what we will need it for when we get a job. But we did not in the 'O' level course, it was "Learn the work, get the work right - go on to the next topic."
Q. Do you find that the method where you are shown the relevance or how it is applied is of more interest?

A. Yes. Now that I am doing the work again, I can see now what use it will be, where as before I could not.

Q. Would you be interested in a Maths lesson that was part practical, that you were actually doing projects that would be relevant and you were handling equipment and so on . . . ?

A. Yes definitely - it does get boring when you are writing it in your books all the time and doing it from the textbook and doing sums off the board all the time, it gets boring.

Q. Why did you not choose Physics? And you do Biology Chemistry as well?

A. No I am not doing Chemistry, just Biology. I was going to do Physics but I changed my mind because there was a lot of Maths in it - I would not be very good at it, so I decided I would take Biology instead.

Q. Did you find in your Physics lessons that the Maths that you were doing, you might have been able to do in a Maths classroom, with X's and Y's and so on. That you found much more difficult in a practical situation?

A. Yes, because we had not done any practical situations before in Maths. If we had done, then I might have understood how to use them but I did not.
Sean Edwards:

Q. Can you describe your Maths lessons to me?
A. I enjoy Maths. I like doing practical work.

Q. What form do your Maths lessons take, do you do lots of problems from textbooks?
A. No we are doing Constructions at the moment.

Q. Do you like doing Constructions?
A. Yes because it's drawing, practical I like it better than problems. I prefer doing practical work it's better than doing problems, I enjoy it more.

Q. Does your teacher show you why you need to do your Maths?
A. Yes

Q. Does he explain where you would use it outside the Maths classroom?
A. Yes he sometimes explains what jobs it would be used for.

Q. When he does that do you work harder at those topics because you know why you need to learn them and it's important to you?
A. Yes. Yes.

Q. Can you tell me what an ideal Maths lesson would be?
A. Don't know.

Q. Would you like lessons which had practical projects, applications being shown using equipment ...?
A. Yes.
Q. We did some work in Physics where we tried to incorporate Maths and Physics together. Did you find that more interesting?

A. Yes, it would be more interesting if we had science in more other subjects, like a bit of science in a Maths lesson. It would be more interesting.

Q. Maths could incorporate many other subjects in its lessons, can you name me any other subjects where you use Maths?

A. We use Maths for Woodwork, when measuring sides and we have to add them together to get the length of wood. We use it in Metalwork and Technical Drawing. We use it in Chemistry sometimes.

Q. Can you explain to me the difference between Physics and Maths. Do you find that in Physics we do a lot of Maths?

A. Don't know. In some places we do a lot.

Q. Is the Maths you do in Physics a little more difficult than when you do it in Maths lessons?

A. A little bit, yes. Not much.

Q. Is it because the Maths is hidden in words in unfamiliar questions?

A. Yes.

Q. In Maths you work on topics for several weeks. So when you go into the Maths lesson you know you are going to be doing the same topic, say constructions and so you prepare yourself to do a construction. Do you find in Physics you do not really know what Maths topic you are going to need to answer the question? You have to think about it?

A. Yes, it makes it hard sometimes.
Q. Would it help you in Maths lessons if they applied the Maths and showed you how to use it rather than just showing you the mechanics of working number problems out?

A. It would help ------
Alan Ferguson:

I think Maths is boring because when we get set something teacher gives us the answers too soon and he does not give you time to work them out. When you tell him he continues giving the answers and when he marks your book you get told off.

Q. Do you work from textbooks?
A. No, we work off the board.

Q. Does he put lots of problems on the board for you to do?
A. No, he does not set us many questions.

Q. Are the questions related to life in general or applied? Do you think that being shown where they are used in life, the relevance or a practical application would benefit you?
A. Yes.

Q. Would you enjoy that more?
A. Yes.

Q. Do you find that in your Maths lessons, when you get a situation where you cannot finish the problems set that you become unmotivated. So you say to yourself "I can't finish these so why bother trying?"
A. Yes and you get all frustrated when you cannot do it and you keep going up to teacher but you still cannot 'get it.'

Q. Does your teacher walk around the room, or come to you to see how you are getting on, or does he just sit at his desk and mark?
A. He does both, really, but most of the time he just sits at his desk.
Q. How do you think we could improve our Maths lessons to make them more interesting and enjoyable?

A. To do different topics every week, not just carry on with a topic for ages because it gets boring after a bit.

Q. Do you get shown the relationships between topics. To use more than one topic to solve a question or to be shown how to solve more complex problems?

A. We get shown how to answer complex questions.
Pupil's name: PAUL HARDY  (Transcript of tape recording)

I think that Maths is good sometimes, only if we go up town and go into a bank and see how much they get that day and we work it out, and then we see how much has been given in and taken out, and then work it out.

I am not very good at fractions, and I am not very good at doing it off the board.

Q  Do you do a lot of work off the board where you do lots and lots of problems?
A  Yes we do lots of problems off the board

Q  Do you ever do anything that you would find enjoyable and fun to do, like practical work?
A  I like to do practical, in Physics we did a lot of work

Q  When we tried to do Maths and Physics together?
A  Yes

Q  Did you find it easier to do the Physics, because I taught you the Maths as well, or was it just that you were unable to do some of the Maths you had been taught in Maths lesson when applied to Physics?
A  I have used some Maths from Maths in Physics, the measurement work and the graphs we drew from it

Q  You enjoyed doing that?
A  Yes, I always like that

Q  Do you do things like that in Maths itself?
A  No, we just work off the board
Q: You would enjoy doing more practical situations in Maths?
A: Yes sir

Q: Do you think you would work harder if you did?
A: Yes

Q: Do you mess about in lessons?
A: No

Q: You don't?
A: I used to at first, but I work to get good exam results this year

Q: Would it be easier if you saw that the Maths you were doing was relevant to your future job or needed in other lessons?
A: I would enjoy it a lot better if we did practicals with it. We have done a lot of it in here (Physics) and some off the board

Q: Do you find difficulty in certain Physics topics because it involves a lot of Maths?
A: Yes. I did find the ruler (measurement/graph) a bit hard but I managed it

Q: Your work on graphs - have you done much work on graphs in your Maths lesson?
A: I find graphs easy, we did some yesterday

Q: In Maths?
A: Yes

Q: What about the graphs we did in Physics? Are you shown how to interpret them in Maths? Understand what it means or do you just plot points? A graph is a form of language it tells you something
A They show you both how to plot points and interpret it

Q And that helps you in other lessons

A Yes

Q Did you choose to do the Physics Option?

A No. I like Biology a lot better, because we go out and do practicals, like we went out and put some string down to make a 1 metre square, then we counted the different types of grass, weeds, etc, then we came in and plotted a graph and went out again.

Q So you were really doing some Maths and Biology together

A Yes

Q And you found that enjoyable?

A Yes

Q Can you tell me why you choose Biology in preference to Physics?

A I think Biology is a lot easier

Q It does not involve as many equations and things like that

A Yes
Q  Can you tell me something about your Maths course?

A  For the first few years the course was all from textbooks. We had to do lots of examples, teacher showed us how to do one on the board and then all we did was work out of books. We did not find that very interesting. After that a lot of the class paid less attention, and there was less effort put into the work that we did.

Q  Are you saying that when you had a lot of problems to do and the first half dozen you found easy, then the rest you found a waste of time doing?

A  Well if you can do one you should be able to do the rest of them.

Q  These sections you were doing were spread over several weeks rather than several lessons, you just kept on repeating the problems?

A  Yes it was the same day after day, it went over the top really.

Q  That led to a lack of effort and some indiscipline in the class, did the teacher have much problem with discipline - did you mess about?

A  Yes, obviously if you cannot do the work you cannot concentrate on it and you start doing other things than the work.

Q  Having said that it is boring, how do you think you could make your lessons more interesting?

A  The only times we found it interesting, was like when he got Pascal's Triangle. No one knew what he was on about but when he got this piece of equipment out that you had to drop marbles down and you had to decide the probability of it going into a certain box, you were able to plot a graph of the marbles that were in a box at the end.
Q You enjoyed that?

A Yes, you could see it, where if you got it from a book all you would see was a load of numbers and ...

Q You are saying because you were doing something and something was happening that the lesson became more interesting.

A Yes, you actually did it yourself, you did not have to read it in a book. You could see yourself that the probability of it doing this was — you could see

Q You would like more practical situations in maths where you could do things for yourself and you could prove what you are doing.

A Yes

Q Did you do any project work in the maths, banking, etc?

A Towards the end of the maths course I did, paper 3—commercial and domestic—was applying the mathematics to everyday life, what maths was required.

Q And you found that interesting?

A Yes, well you could see how it was going to help you in life.

Q Did the class as a whole work harder?

A Yes because they thought that in a few years I will have to be buying a house, pay and saving in a bank.

Q So you all sat up and took interest.

A Yes.

Q Did you find any other parts of the course relevant?

A Well on most of the other parts we could not see the relevance and so we put less effort into it.
Q  When did you feel you learnt more from the course?

A  When we were doing the experiment things we did. It was a pity that the rest of the course we did was not involved in experiments.

Q  You did physics to CSE, did you find the physics course hard because there was a lot of maths in it?

A  Not so much hard, because the maths was hard, but in physics there are not numbers, there are words to equations. That's where the difficulty came in with physics.

Q  In maths you were just working with numbers and on the same topic week after week, where in physics you had to find out from the question what topic to use and that proved difficult?

A  Yes.

Q  Do you think that in your maths lesson you should have some problems applied to be relevant to other subjects, do your teachers show you how to use your maths in other subjects, say physics, to show you how to use several of the maths topics integrated into a more complex problem?

A  Well it's going to help, is it not, if you can see what you have to do, especially for a subject like physics, because of this a lot of people when I was doing it considered physics as the hardest subject.

Q  As part of your physics course you were obviously doing some maths, did your teacher have to teach you some of the maths that you were required to use?

A  No, he did not have the time. If you did not understand the maths you would not understand the physics.

Q  Would you have found it a benefit if he had had time to have taught you some maths and physics at the same time?

A  Yes.
Pupil's name: DARREN SMITHHURST
(Transcript of a tape recording)

A I don't think much of maths, it's boring sometimes. I think you would find it more interesting if you did practical work

Q Why don't you like your maths?
A Because he just leaves you to do it, in maths

Q What sort of work are you doing - number problems?
A No, we do constructing of angles and if you don't find it easy he just shows you once and that is it. He does not try to explain how it is done, he just does it for you.

Q How do you think you could improve your interest in maths?
A By doing practical things

Q You like to do practical and then the maths associated with it?
A Yes

Q Does your teacher try to show you how the maths you are being taught could be used outside the maths classroom, say in a science or metalwork, or a work or in the home?
A No

Q You just do maths problems which have or appear to have no relevance outside the maths classroom
A Yes

Q Do you think it would help you to work if you thought you required a certain maths topic for use personally for employment in the future or for your life in general?
A I think it would make you work harder
Earlier in the year we did some work in our physics lessons which incorporated some of your maths teaching because you were not capable of doing the maths at that time for the science lesson, and we incorporated the maths with the physics lesson. How did you feel about that?

A I found it easier to do when we did the experiments, because it showed you how to do it....

Q That was when we were doing equations, and moments and graph work

A Yes

Q Do you think that you would enjoy doing that all the time in your maths lessons. That you would like a certain amount of practical project work, a certain amount of work related to other subjects in the school, to show you how you would actually use the maths you are learning?

A Yes, yes

Q You volunteered for some extra lessons with me. Why did you do that?

A Because I am not very good at maths

Q You felt that because you wanted to do physics, that you wanted to improve your maths that you would be required to use next year?

A Yes
Pupil's name: NIGEL BONSAR

(Transcript of tape recording)

I think maths can give you qualifications for a good job

Q What sort of maths lessons do you have?

A What do you mean by that?

Q What do you do in your maths lessons. Do you do all number work or do you work from the board or the textbook?

A We do number work from the board

Q Is that the main part of the lessons?

A Yes

Q Does your teacher ever do any practical work with you, or project work?

A No. We only did it once when we were doing area of circles. We had this piece of string and we had to wrap it around this circle we had drawn.

Q To find the circumference?

A Yes. That's all we have done

Q Did you enjoy doing that sort of work, rather than just sitting down doing sums?

A Yes

Q Would you prefer to do more of that type of work?

A Yes

Q Does your teacher ever show you where you use your maths, outside the maths classroom, where it would be needed in other subjects or how to use it in other subjects?

A No
Q You are just taught maths and not shown where you could use it at home or for a job or for your life in general
A No

Q Do you think, if your teacher said a topic was definitely needed for your physics lessons in a couple of weeks time - because say, we were doing moments and you would have to solve equations, that you would try to understand it more, because you would realise you needed it
A Yes

Q How do you think you could improve your maths lessons?
A By having more practical work in them

Q Would that make you work harder?
A Yes

Q And get you more interested?
A Yes

Q In physics we tried to incorporate some maths and physics into lessons to try to overcome your difficulties with the physics we were doing at the time. Did you enjoy doing some of those practical aspects of maths?
A Yes

Q Do you find that you need extra maths help when you are doing physics?
A Yes

Q Because you cannot use some of the maths you are being taught in the maths classroom, even though you have done the subject, when it comes up in physics you don't know how to do it applied in the physics situation?
A Yes
Pupil's name: IAN COPE
(Transcript of tape recording)

I like maths sometimes but when it gets too hard I do not like it

Q What sort of maths lessons do you have? Do you do work from textbooks doing lots of adds and subtracts, multiplies and divides?

A We do all sorts out of books and off the blackboard

Q When do you really enjoy your maths lesson, what sort of work are you doing when you enjoy it?

A I like it when we do the sums off the board

Q You like doing sums, do you get them right?

A Mostly

Q That makes you keen to do more

A Yes, sir

Q Do you like doing project work and practical sort of maths where you do experiments to prove the work you are doing?

A Yes

Q Do you do a lot of that?

A No

Q In physics we tried an experiment where we tried to incorporate maths and science together. We did quite a lot of work on graphs, co-ordinates and things like that. Did you enjoy that?

A No, I don't like doing graphs, because I usually get muddled up with them
Q Did you like the idea of doing science and maths together, or say metalwork and maths together, where you can see a relevance to the maths you are doing?
A Yes

Q Do you think it would make you work harder?
A Sometimes

Q Do you think maths is important to you?
A Yes

Q Why?
A Because it is one of the most important things at school and it will get you a better job

Q Does your teacher show you the application of maths, in say, some local industry, jobs or things of local interest?
A No

Q Do you think it would help you to know where the maths is going to be used outside the maths classroom
A Sometimes

Q Do you think on certain topics you would work harder?
A Yes

Q Do you like physics?
A No, I find it too hard

Q Why? Is it the science that's hard or is it the maths that's involved that makes it hard?
A It is when you do the maths and have to work them out
Q Have you chosen physics?
A No
Q Why didn't you choose it?
A I think it's too hard, the maths in it is too hard
PUPILS' COMMENTS

Caroline Durant:

Q. What do you think about Maths?

A. Most of the time I think it's pretty boring. The Maths in the first and second year was OK, because they tried to make it interesting for us because we were younger but then towards the end of the 3rd, beginning of the 4th year it became just working from textbooks and sitting at desks which is not, (if you are good at Maths it's fine because it comes pretty easy anyway) but if you have never been too good at Maths, the motivation is lost from the 2nd/3rd year if you are just sitting at a desk all the time. I think a lot of your ability is brought out by the teacher - if the teacher has got a good attitude towards the subject and tries to make it interesting for you, then I think it is interesting for all the class not just for the ones who are good at Maths.

In the 3rd year we had little quizzes to try to help you with the subject and that was better than working from textbooks all the time.

Q. You liked the quiz because it was challenging to you?

A. Yes, it made it more interesting - it took your mind off the fact that it was a Maths lesson. I found it more fun really; anything that gets you interested is more fun. If you asked some of the teachers what you need it for, they say "Well, if you are building a bridge you will need it", but not many of us will be building bridges. We don't really see the relevance of a lot of it for our lives. My parents never did a lot of the stuff that we have started in the fourth year, if I ask either of them to help me with it they have not got a clue; they have managed to get their jobs without all the - you know, working on all the basic subjects.
Q. Do your teachers actually show you the application of the Maths, ie: where it could be used of the relevance of it, in your lessons?
A. No - not really

Q. You work from a textbook, doing different topics for weeks on end without being shown the relationships between them ie: graphs for weeks, then equations for weeks?
A. Yes

Q. If it was applied, doing practicals and the relevance was there, do you think you would be more interested in doing it?
A. Yes, if you knew that it was going to be vital to one job when you left school, you would try harder, and the job was going to depend on it. We never get told what jobs each subject is going to be useful for.

Q. Are you unhappy with your Maths Education?
A. Yes

Q. Will you endeavour to gain a qualification or have you given up.
A. On a few topics I will sail through and do fine and then others - say a 2 or 3 month's session where I will not be able to do the work. A lot of it is your own ability to understand each subject, if you don't understand it. I think there are a lot easier ways of explaining it. If they worked with Maths all their lives, they know what they are talking about but we don't half the time.

Q. What you are basically saying is that the course in general lacks interest, and due to this you don't try as hard, and if it was applied or relevant you would work harder?
A. Yes.
Simon Marriot:

I think Maths, when taught well is a good subject, but it's very boring at times, particularly when taught from a textbook. When teacher just leaves you to read it out from the textbook I find it very boring and hard to learn on your own. Unless you have a good teacher who shows you on the board. I think if the teachers showed us more how to do these things and if they showed us how you need them in later life and how to use these sums and work we are doing in later life it might be more interesting. If we did more practical work involving Maths I think it would be a lot more interesting. If the teachers, for instance, went into Industry and came back and told us how we needed all this in later life and how we could use them or what for. They might make the subject a lot more interesting.

Q. You chose not to do Physics. Can you tell us why? You are doing Biology and Chemistry?

A. It was because I could not understand a lot of the Mathematics; the sums were not that hard, but it was working out where the stuff was in the original sum.

Q. You are saying that you can do the basic numeracy side; adds, subtracts, etc, but you did not actually know the sort of Maths, the topic you were required to use, or the several topics required to be used to solve the problem?

A. Yes

Q. In Maths you are taught in blocks; Graphs for 2/3 weeks, Equations for 2/3 weeks. When it came down to an actual problem in Physics you did not actually know which topics you were going to need to solve the problem. If you had been told which topics or the strategy you could have gone on to solve it?

A. Yes
Q. Should the Maths in your Maths lessons involve examples and problems related to other subject areas, ie: Physics, Chemistry, Technical Drawing, rather than just X's and Y's?

A. Yes, it would make it more interesting

Q. What do you want to be when you leave school?

A. I want to go to Catering College to study to go into Hotel Management for which I would need to know Maths to keep books and stuff.
1. I enjoy maths at times, but some subjects we study I find irrelevant to modern day life and towards the job I want to do, therefore I do not try as hard at it as perhaps I should.

2. I don't think maths is particularly useful to general life though I have used simple addition in life which is useful.

3. The maths lessons are mostly boring because we only work from books yet in one lesson when we have a different teacher there is more fun in the lesson. Experiments would probably help because it would give us a better idea on the subject instead of just reading or copying from a book.

4. Most of the maths that we are taught seems pointless to me. They aren't needed in future life. I think if maths was doing experiments to prove maths more people would like it and wouldn't get bored so quickly of it.

5. Maths is ok but I think we should be given an example in how it will help us in later life. All we do is exercises on each topic. If I was given a certain problem to solve without being told what would be involved to work it out. How would I know which topic would help me solve it. However if we were given ideas on how to use the maths we know
6. The teachers seem so bothered about teaching us the work they never even think to tell us what it can be used for.

7. I feel that the maths lessons at our particular school are a little old-fashioned, the maths being taught on a very strict syllabus.

8. I think maths can be an interesting subject, but it depends on what topic we’re doing within the subject. We only ought to do the topics that will help us in future years.

9. The teachers do not teach their pupils the significance of mathematics in jobs outside. I think that mathematics should be made more interesting and enjoyable for the pupil by introducing practical work into the subject. This may also make mathematics more easy to learn for slower pupils, who would find difficulties in being seated at a desk and left to get on with an exercise. This would also mean that each pupil would be more ‘involved’.

10. Maths should be taught with more insight into the job that you are doing when you leave school. I think that, instead of grouping to ability, schools should group depending on how much relevance maths will have when they leave school and get a job. Maths could be made a great deal more exciting with the aid of more practical, i.e. experiments.
I think that maths is quite boring. The things we learn in maths aren't going to help me after I leave school. The improvement that could be done is to use more practical work; this would be more enjoyable than just copying work out of a book.

It does seem a waste of time if the job you are seeking doesn't involve complicated maths.

Our teacher never tells us where our maths can be applied and I think that this is wrong, because more people might sit up and take notice if they thought that the subject they find boring may help them in their future career.
I think that maths should not be a compulsory subject in school as not everyone likes or needs to learn how to construct graphs or draw triangles.

I enjoy straightforward maths where we only do numerical sums but once we get on to algebra and geometry I am lost. I find there types of sums boring because when I leave school I don't think they will be relevant.

I find doing constructions of triangles and things and then finding out angles, very difficult. A lot of equations have to be remembered just to find out an angle of a triangle, this is very boring to do and useless.

The work we do in maths is not relevant. I don't think it will be used later on in life. Logarithms and quadratic equations are stupid things to learn as not many people need them. I think if the teachers told us what jobs need which kind of maths then we would realise why we have to learn them and not be so bored in learning them.

I think that changing teachers also makes your work worse as you have to get to know different teachers.

The topics we do are very boring and irrelevant to what you need to know when you leave school.
(2) The section rephrased to be should be shown how it is applied in life. If it is, half the people who do math would not know how to apply the math if they were given a problem out of the classroom. In other subjects where we are used such as Physics, I prefer to do the math part of the problem give rather than have the other information.

(2) Although math is one of my best subjects, I thoroughly detest it. Perhaps it could be a more enjoyable subject if we had a different teacher and it was taught in a better method. Our current teacher just tells us to read the book and to get on with an exercise.

(3) A lot of the maths we do has no apparent importance to me now or as far as I can see in my further life.

(4) It can be a good subject if it is taught well and the teacher puts effort into the teaching of maths. I only try at the subject and topics which I feel will do me any good in the long run.
of the things we're taught aren't really necessary to you for future life.

I think that a mixture of practical and theory maths would be the best answer because then one could not get bored, but not waste too much time doing experiments.

I also think that many of the subjects done will not be relevant to any career (except of course, those specifying in maths).

Maths lessons are better when you do experiments and surveys but we only did these in the second year, but they made maths lessons more interesting.

Some of the maths that we learn at school will be useful when we leave school but some of the stuff we do has got nothing to do with solving real problems which will help us. Some of the things we learn about, we will never need when we leave school and so it is just a waste of time being taught them.

Personally, I think that Maths could be made more enjoyable by incorporating practical work. This would give some indication of the application of it, because at present we are taught theory from books and this can get very monotonous. It does not teach us any of the ways in which Maths can be of use to us in our lives.
I think it's boring and most of the work we don't going to be any use when we get older unless you're a maths or physics teacher, or something while you've a lot of complicated maths. You should just taught the basic maths and then be allowed to learn extra things if you need them for the job you do. It seems a waste of time doing something you aren't going to need in the future if you don't even enjoy it. Maths should be taught in a more practical way instead of 'just going through a text book of complicated things which just seem useless'.

3) Teacher say you cannot do experiments or such like which would make maths more enjoyable because it wastes time. This is true but if we spent less time on useless, boring exams, we would be able to do none of these.

1) It must be hard though, for teachers to make a complicated and numerical lesson like maths interesting and enjoyable. That's probably the reason why so many people hate maths, the continuous problems which line up in rows each waiting for their turn to attempt to baffle ones brain. Some teachers relunct what work we are doing with a practical situation but others just go on reading everything out of the book not caring if you understand it or not.

We are often taught things which will never be used in future life such as many of the graphs. We should be taught things which will be used later on rather than things which will fill up our lessons and free time at home.
only certain sections of maths apply to life's needs but maths because employers still think it is important. I’d agree with this because maths gives you a good grounding for other subjects.

I do not enjoy fourth year physics, it is too enough we were mixed in worse reaction out between third and fourth year. I liked first, second and third year physics because it had no maths involved it was quite easy. Since I’ve been in the fourth year every subject in physics was involved maths and this takes the interest out of it completely. I do not want to know the maths sides, what does not interest me.

I feel some of the topics we do are not relevant to our futures. I find it difficult to foresee me using hyperbolic graphs, cubic graphs, the linear function etc. in any job. Some topics like trigonometry is more relevant as it can be used to in house construction.

The teachers themselves do not apply the topics to everyday examples. They don't seem to be bothered. "It is in the books that I write..."

The maths subject is taught in various topics with the time they are taught are not interrupted with the topics previously taught and this makes the subject seem disjointed. The maths lessons are taught in such a way that the practical use of maths is lost and then the subject is taught in such a way that to pass the exam and not the steps in which they may be of use in the future.
I feel having the ability to do maths well holds you in good stead for physics but physics isn't just about being able to do maths, you have to know how to apply to maths to physics. Doing all three sciences seems to help because each subject helps each other since some subjects occur in more than one subject.

As the problem try to solve practical situations but they are somewhat boring.

Much of the maths taught is relevant areas, volumes etc, but a good deal is only taught apparently for its own sake to get you through the exam.

I don't get very much enjoyment from mathematics in fact I find it quite tedious at times. I understand the most part of the syllabus, the only difficulty arising when we are taught by supplementary teachers, some which have less understanding than us in maths.

About half of the syllabus is utterly useless for our future life such as indices etc, which seem pointless, whereas areas and volumes are much more useful, for example knowing the area of carpet needed for a room. Also I cannot see that algebra and equations are at all relevant for future life outside school.
Maths is taught fairly well at school although some of the teachers, i.e. Mr Jackson, make more errors than their pupils. Some stand-in teachers, i.e. Mrs Leeson, are very bad as they do not explain things in the slightest and write information on the blackboard which is not needed and only serves to confuse those who know what they are doing, and put those who don't on the totally wrong track.

One thing I greatly dislike is having to do myriads of problems all on the same subject when two or three would suffice. The only occupation in which most maths subjects are needed is an architect which very few people are.

I enjoy some of the topics in maths lessons that involve problem solving and constructions. However, I find exercises where the same steps are done in every question very tedious and this tends to induce careless mistakes.

Most maths topics aren't relevant. The only really relevant topics are percentages and the properties of triangles, etc. to people who want to be architects or builders. The mass use of calculators has however made redundant even more of the topics that are taught in maths lessons.
The maths lesson that we do is very much varied due to the fact that we have two and sometimes three different teachers. One of them makes the maths lesson very confusing. Instead of explaining how to do a subject she just sits down at the front of the classroom, tells you to read something out of the book and get on with the exercise. Even when she asks you to explain something she just babbles on and on and confuses you even more. With another one of our teachers I find maths easier. I say this because if you don’t understand something he will explain again and again until you finally understand the topic. With some people, e.g. the ones who are behind in the work he has even offered to teach people in his spare time. With the woman teacher we have she sometimes feels the topic wrong herself. We have another teacher for one day a week. He explains everything very thoroughly and clearly and makes maths seem easy.

With our official maths teacher, maths is fairly interesting. He explains everything but not well enough I don’t think. He uses the text book sometimes but then only for theorems and examples.
I do not find maths difficult when it is explained properly and examples are given on how to use it.

In our class maths is taught in the following way. One subject or topic is taken at a time in the order they come in a text book. We read the examples and theorems in the book and the teacher talks through them and writes some notes on the board which we copy. We then go on to do certain questions from the relevant exercises in the text book. The teacher often does not explain fully and when doing the questions we often come across difficulties and so he has to explain it again more thoroughly, but I appreciate that a lot of maths is very difficult to explain to other people.

We never learn how the topics we do in maths can be practically used in life or in what situations they would come in useful. Also when we are studying a certain topic we only do questions concerned with that topic then when it is finished that's it. We never get problems or questions in which different mathematical skills have to be used together.

I cannot really say whether I enjoy maths or not; it depends on what topic we are doing. Sometimes I can pick it up quickly and I quite enjoy doing the exercises but sometimes it is more difficult to understand but we always seem to spend about the same amount of time on each topic and then leave it and it is not looked at again until the time to revise for exams. Last Christmas at exam time the teacher did not revise in class with us or let us have revision time in lessons. There was a lot to revise and I found that I had forgotten many things and time was short and so I didn't do as well as I might.
I would rather have practical maths because it is more enjoyable and you learn a lot more and we never do graphs. We only do Graphs in Biology.

I think physics is good because I am interested in most things that we do. Also it is interesting because most of the things we learn are things that are in our normal life and we are told things that we sometimes see every day of our life but didn't know what they were or what they were there for.

In the 2nd year we did some practical work. We had to throw two dice about 30 times and see which numbers occurred most then we had to plot a graph for it. I would like to do more things like that because it was good fun and it was interesting.

I would like to do more practical in maths because it would be more interesting and it wouldn't be so boring.
I would like to see practical work involved in maths. I think maths is important. You need it to get a job.

Maths can be very boring in some subjects but in other lessons it can be very interesting. Physics, for example. I would like to do more practical in maths like measuring school fields and seeing what is the worst traffic we have, but in maths the teacher goes through lots of questions maybe once or twice and then and do them. So it gets boring. It's the same every time. Look at the board, shut up and listen, copy this down, do the questions on the board, be quiet. I have some of my own work to do. Our mothers and fathers pay their tax to teach us not to be doing other work.
Mathematics can at times be extremely tedious, but conversely can be interesting in different subjects. It is not very enjoyable generally as a lesson. The teacher can make a huge difference to the level of maths turned out by the pupils. If the subject is not very well taught and is made boring by the teacher, a downfall in pupils' standards.

Also if a pupil does not think that the present topic is relevant to his needs he may not want to learn it.

Also in the sciences, maths. Physics which is a very mathematical course many problems mathematically are encountered. If maths is found hard in the actual lessons there will be no motivation to find the physics problems enjoyable. Possibly, possibly, the reduction of problems in Physics or different treatment of them might make Physics different for pupils.

Why can't you have Maths and Physics combined as one subject. As Physics and Maths, both contain mathematics.

Physics is a very boring lesson. It is just like maths.
I don't like physics because there is too much maths in it.

You have to be brainy to do physics because you have to be brainy at maths.

I feel that doing maths which is practical in the previous years of my Junior school is more exciting than just answering questions off the board. When you go into a maths lesson and find that you have got lots of lots of tests own to do. You get really bored if bored with just sitting there, but if you're practical you can move around a bit more and probably go outside. That way it is more enjoyable.
APPENDIX VI

PUPILS' BOOKLET "MATHEMATICS FOR SCIENCE"
MATHEMATICS FOR SCIENCE
You Need to Know Your Tables for everything not just Science.
The Four Rules of Whole Numbers.

Adding (+) place columns in line:

\[ 24 + 3006 + 521 = \]

\[ 3006 + \]

\[ \frac{521}{3551} \]

\[ \text{R Carry unit} \]

Always check the answer *don't neglect it*

Example from Science

Electricity: Resistances in Series

Total Resistance = \( R_1 + R_2 + R_3 \) \( \Omega \)

\[ \begin{array}{ccc}
R_1 & R_2 & R_3 \\
18\Omega & 125\Omega & 16\Omega \\
\end{array} \]

Subtraction (-) place in columns

\[ 524 \]

\[ 387 \]

\[ 137 \]

Method 1

Method 2

\[ 524 \]

\[ 387 \]

\[ 137 \]

borrow from next column when required

add to number subtracted in succeeding column

Example from Science

Measuring Volume with Measuring Cylinders:

Volume of object = 78 - 40 \( \text{mL} \)

= 38 \( \text{mL} \)
\[ 236 \times \\
\]
\[ 1.14 \]
\[ 2.4 \quad \text{Start by multiplying by 4 Units} \\
2360 \quad \text{Now by the 10's term, place '0' in the Units column and then multiply normally} \\
2360 \quad \text{Now by the 100's term, place '00' in the first 2 columns and then multiply normally} \\
26884 \quad \text{Add}
\]

Note: Be neat and have clear columns, so you don't make silly mistakes.

Examples from Science
1) An empty bedroom measures 5m \times 4m \times 3m. If the density of air is 2 kg/m\(^3\), what is the mass of air in the room?

\[ \text{Volume of room} = L \times b \times h = 5 \times 4 \times 3 = 60 \text{ m}^3 \]
\[ \text{Mass of air} = \text{Density} \times \text{Volume} = 60 \times 2 = 120 \text{ Kg} \]

2) The heat required to raise a block of steel, specific heat 9 J/g\(^\circ\)C, which has a mass of 1640 grams through 52\(^\circ\)C is

\[ \text{Heat} = \text{Mass} \times \text{Specific Heat} \times \text{Temperature rise} \]
\[ = 1640 \times 9 \times 52 \text{ Joules} \]
\[ = 767520 \text{ Joules} \]

\[ \text{LONG DIVISION}\left( \div \right) \]
\[ 64 \div 2 \]
\[ 2 \]
\[ \overline{64} \]
\[ \overline{32} \]
\[ 60 \]
\[ 4 \]
\[ 0 \]

We never leave remainders

\[ 64 \div 10 \]
\[ 10 \overline{64.0} \]
\[ 60 \]
\[ 40 \]
\[ 40 \]
\[ 0 \]

place decimal point and add some 0s.
Long Division continued

\[
\begin{array}{c}
\text{1 ÷ 8} \\
\hline
0 \cdot 1 2 5 \\
\hline
8 \big| 1 0 0 0 \\
\quad - 8 \\
\quad \underline{2 0} \\
\quad 1 6 \\
\quad - 4 0 \\
\quad \underline{4 0} \\
\quad 0
\end{array}
\]

With very big numbers you have to approximate to see roughly how many times one number will go into another.

Eg

\[
\begin{array}{c}
2 2 \big| 8 0 4 \quad \text{\textit{\(804\)}} \\
\quad \underline{1 7 6} \\
\quad \underline{1 7 6} \\
\quad \underline{0 8 8} \\
\quad \underline{8 8} \\
\quad 0
\end{array}
\]

* 20 go into 170 - 8 times
So try 8
20 go into 80 - 4 times
So try 4

A more difficult problem:

\[
6 5 3 2 8 4 \div 1 9 8 4
\]

* Approximate
1984 to 2000
When you are trying to
divide e.g. 2000 goes into 6500
3 times etc.
Examples in Science

(1) A block of lead has a mass of 44 560 kg and a volume of 4 m³. What is the density of lead?

\[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{44 560}{4} \]

\[ = 11 140 \text{ kg/m}^3 \]

An example of the combination of the four rules of numbers:
An experiment to find the pressure created by a pupil standing on a pad 26 cm by 30 cm. Pupil’s weight 52 kg.

Area of pad = 26 \times 30 \text{ cm}^2 = 780 \text{ cm}^2

Weight of pupil in Newtons = 52 \times 10 = 520 \text{ N}

Pressure = \frac{\text{Force}}{\text{Area}} = \frac{520}{780} \text{ N/cm}^2

\[ = 0.66 \text{ N/cm}^2 \]

DECIMALS

(1) A decimal number:

\[ 1 4 2 . 8 9 2 \]

HUNDREDS TENS UNITS TENTHS HUNDREDTHS THOUSANDTHS

DECIMAL POINT

WHOLE NUMBERS FRACTIONS

0.8 means \( \frac{8}{10} \)

0.89 means \( \frac{89}{100} \)

0.892 means \( \frac{892}{1000} \)

Note: Adding a figure 0 to the end of a decimal number does not alter the value of the number, for example

14 is the same as 14.0

24.1 \ldots \ldots 24.10 \text{ or } 24.100 \text{ etc.}
Decimal Places.
These are the figures to the right of the decimal point.

14.2 \( \cdot \) 892

\[
\begin{array}{l}
| \text{3rd Decimal Place} \\
| \text{2nd Decimal Place} \\
| \text{1st Decimal Place} \\
\end{array}
\]

Eg 3.94 has 2 decimal places (2 DP)
5.016 has 3 \( \ldots \ldots \ldots \) (3 DP)
4.6 has 1 \( \ldots \ldots \ldots \) (1 DP)

Some Decimal Numbers from Science

<table>
<thead>
<tr>
<th>Substance</th>
<th>Density g/cm³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>13.6</td>
</tr>
<tr>
<td>Iron</td>
<td>8.2</td>
</tr>
<tr>
<td>Water</td>
<td>1</td>
</tr>
<tr>
<td>Ice</td>
<td>0.92</td>
</tr>
<tr>
<td>Air</td>
<td>0.0013</td>
</tr>
</tbody>
</table>

DECIMALS.

Multiplying numbers by powers of 10. (ie 10, 100, 1000, etc)

Move: Decimal Point to \textbf{RIGHT} by as many places as 0's in the multiplier.

Eg \( 0.52 \times 10 \) becomes \( 0.52 = 5.2 \)
\( 0.0531 \times 100 \) becomes \( 0.0531 = 5.31 \)
\( 7.1 \times 1000 \) becomes \( 7.1000 = 7100 \)

\( \text{Add more 0's} \)

Examples from Science:

(i) \( 1.5 \text{ kW} = 1.5 \times 1000 \text{ watts} = 1500 \text{ watts} \)

(ii) \( 1.5 \text{ m}^3 = 1.5 \times 1000000 \text{ cm}^3 = 1500000 \text{ cm}^3 \)
Move decimal point to LEFT by as many places as 0's in the divisor. If there are not enough figures, place 0's in front of the number to fill up the empty spaces

\[ 71 \div 10 = 7.1 \] becomes 7.1
\[ 15.1 \div 100 = 0.151 \] becomes 0.151
\[ 5.1 \div 1000 = 0.0051 \] becomes 0.0051

**Rounding Off.**

Look at the number after the Decimal place required

IF IT IS 5 or MORE add 1 as shown below

IF IT IS LESS THAN 5 do not change

Eg Write \( 16.3758 \) correct to 3 DP = \( 16.376 \)

\[ 16.3758 \text{ correct to 2 DP} = 16.38 \]

\[ 9.0293 \text{ correct to 1 DP} = 9.0 \]

**Examples in Science**

- (1) \( 2750 \text{ watts} = 2750 \div 1000 \text{ Kw} = 2.75 \text{ Kw} \)
- (2) \( 56 \text{ Newtons} = 56 \div 10 \text{ Kg} = 5.6 \text{ Kg} \)
- (3) \( 1256 \text{ cm} = 1256 \div 100 \text{ m} = 12.56 \text{ m} \)

**Rounding off**

\( \pi = \frac{22}{7} = 3.1428 \text{ to 4 decimal places} \)

or \( 3.143 \text{ to 3 decimal places} \)

or \( 3.1 \text{ to 1 decimal place} \)
**Note:** The decimal points of the numbers are placed directly in line.

Eq. \[ 24.567 + 12.074 = 36.641 \]

\[ \text{Place DP in line} \]

**Subtraction of Decimals**

Place decimal points of the number in line.

Eq. \[ 40.670 - 20.65 = 20.02 \]

**Examples in Science:**

1. **Volume of object by displacement**

   \[
   \begin{array}{c|c|c}
   \text{ml} & 8.3 & 5.8 \\
   \hline
   5.8 & 0 & \end{array}
   \]

   \[ \text{Volume of object} = 8.3 - 5.8 \text{ ml} = 2.5 \text{ ml} \]

2. **A Burette dispensing liquid.**

   \[
   \begin{array}{c|c|c}
   \text{1st level} & 25.7 \text{ ml} \\
   \hline
   \text{2nd level} & 34.9 \text{ ml} \\
   \end{array}
   \]

   Ammount dispensed \[ 34.9 - 25.7 = 9.2 \text{ ml} \]

3. **Resistances in Series**

   \[
   \begin{array}{c|c|c|c|c|c}
   \text{1st} & M & M & M & M & M \\
   \hline
   \text{2nd} & 14 \text{ a} & 36 \text{ a} & 8.5 \text{ a} \\
   \end{array}
   \]

   Total Resistance = 14 + 36 + 8.5 = 58.5 \text{ a}
Multiplying Decimals

Ignore the decimal point, multiply as if it were two whole numbers. Then count the number of decimal places in both numbers being multiplied, there must be the same number of Decimal places in the answer.

\[
\begin{align*}
21.4 & \times 3.2 \quad \text{Two Decimal Places} \\
6420 & \\
428 & \\
68.48 & \quad \text{Two Decimal Places in Answer}
\end{align*}
\]

\[
\begin{align*}
0.002 & \times 0.003 \quad \text{5 Decimal Places} \\
0.00006 & \quad \text{5 Decimal Places in Answer}
\end{align*}
\]

Example from Science:
The volume of air in a room is 16.25 m³. What is its mass if the density of air is 1.33 kg/m³?

\[
\text{Mass} = \text{Density} \times \text{Volume} = 16.25 \times 1.33 \quad \text{Kg}
\]

\[
\begin{align*}
16.25 \\
\times 1.33 \\
\hline
4875 \\
48750 \\
162500 \\
\hline
2161.25
\end{align*}
\]

Division by Decimals

0 whole number \[ \frac{8.25}{66.00} \]

\[
\begin{align*}
8 & \div 66.00 \\
64 & \\
20 & \\
16 & \\
40 & \\
40 & \\
0 &
\end{align*}
\]

14 decimal places
2 By a Decimal Number \[ 22.88 \div 1.3 \]
Make Divisor a whole number \( 1.3^2 = 13 \)
The same must be done to \( 22.88 = 228.8 \)

Now divide \[ \frac{176}{13} \]
\[ \frac{13}{228.8} \]
\[ \frac{98}{13} \]
\[ \frac{91}{78} \]
\[ \frac{78}{78} \]
\[ \frac{0}{0} \]

Examples in Science:

i) The mass of air in a bedroom weighs 50.3 kg and the volume of the room is 46 m³. What is the density of air? correct to 1 dp.

\[ \text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{50.3}{46} \text{ Kg/m}^3 \]

\[ = 1.09 \]

\[ = 46 \sqrt{50.3} \]

\[ = 46 \cdot 7.1 \]

\[ = 321 \]

\[ = 1.0 \text{ Kg/m}^3 \text{ to 1 decimal place} \]

(11) "Lignum Vitæ"

A block of wood weighs 10 g.

What is its density? correct to 1 dp

Volume of block = \( L \times b \times h \) cm³

\[ = 2.1 \times 1.9 \times 2.1 \text{ cm}^3 \]

\[ = 8.379 \text{ cm}^3 \]

Density = \( \frac{\text{Mass}}{\text{Volume}} \)

\[ = \frac{10}{8.379} \]

\[ = 1.19 \]

\[ = 8.4 \]

\[ = 100 \]

\[ = 760 \]

\[ = 1.2 \text{ g/cm}^3 \]
Some Large and Small Numbers from Science

- The density of gold = 19000 Kg/m³
- Avogadro's number = 6 000 000 000 000 000 000
- The speed of light = 300 000 000 m/s
- The distance to the sun = 150 000 000 m
- The diameter of the Earth = 13 000 000 m

- The size of a molecule = 0.0000002 cm
- The linear expansivity of aluminium /°C = 0.000 026
- The density of air = 0.0013 g/cm³
- The wavelength of red light = 0.000007 cm
- 1 micro amp = 0.000 001 amp
- 1 nanometre = 0.000 000 001 m

Decimal Money:

Amounts of money are written as decimal numbers.

- Example: Fifteen Pounds and Eighty Seven Pence
  
  £15.87

- Example: Six Pounds and Seven Pence
  
  £6.07

  Note the 0 means no ten pence.

Example in Science:

An electric fire rated at 2 kw is used for 52.5 hours. If the cost of a unit (1 kwh) of electricity is 4.8 p. How much does it cost to use the fire?

No. of Kwh. consumed = 2 x 52.5 = 105.0

Cost of electricity = 105 x 4.8 = 504.0 p

= £5.04
Fractions

These are parts of whole ones

3 \text{ Numerator} - this tells you how many "bits" you have
4 \text{ Denominator} - this tells you how many "bits" the whole we split into

Proper Fractions \quad \text{eg. less than one, } \frac{7}{8}, \frac{2}{3}, \frac{3}{4}

Note Denominator is bigger than numerator

Improper Fractions \quad \text{eg. bigger than one, } \frac{13}{8}, \frac{5}{3}, \frac{9}{4}

Note Denominator is smaller than numerator

Mixed Number

eg. Whole Numbers and fractions \quad 1\frac{7}{8}, 2\frac{3}{5}, 5;

Changing the fraction to an equivalent.

\[
\frac{1}{2} \rightarrow \frac{2}{4} \rightarrow \frac{3}{6} \rightarrow \frac{4}{8} \rightarrow \frac{16}{32} \rightarrow \frac{50}{100} \quad \text{etc.}
\]

Note. the fraction is changed to an equivalent by multiplying both the top and bottom by the same number. The fraction is still the same size.

Simplifying (cancelling)

\[
\frac{5}{20} \quad \text{cancels to } \frac{1}{4} \quad \text{(divide top and bottom by 5)}
\]

\[
\frac{27}{81} \quad \text{cancels to } \frac{1}{3} \quad \text{(divide top and bottom by 3 several times, until no more cancelling can be done)}
\]
FRACTIONS: -- Addition

You can only add fractions together which have the same denominator. So convert the fractions

\[
eq_1 \quad \frac{5}{8} + \frac{3}{16} = \frac{10}{16} + \frac{3}{16} = \frac{13}{16}
\]

\[
\frac{2}{5} + \frac{1}{3} = \frac{6}{15} + \frac{5}{15} = \frac{11}{15}
\]

\[
\frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1 \frac{1}{12}
\]

Improper fraction take out \( \frac{12}{12} = 1 \)

Addition of Mixed Numbers

Add whole numbers together and add fractions

\[
eq_2 \quad 2\frac{1}{2} + 3\frac{5}{8} = 5 + \frac{1}{2} + \frac{5}{8} = 5 + \frac{4 + 5}{8}
\]

\[
= 5 + \frac{9}{8} = 5 + \frac{1}{8} = 6 \frac{1}{8}
\]

Improper

Subtraction of fractions

\[
eq_3 \quad \frac{1}{5} - \frac{1}{6} = \frac{6 - 5}{30} = \frac{1}{30}
\]

\[
(\text{3-1})
\]

\[
3\frac{1}{5} - 1\frac{1}{8} = 2 + \frac{1}{5} - \frac{1}{8} = 2 + \frac{8 - 5}{40} = 2 \frac{3}{40}
\]

Mixed Addition and Subtraction

\[
eq_4 \quad 2\frac{1}{4} + 5\frac{1}{2} - 1\frac{3}{8} = 6 \frac{2 + 4 - 3}{8} = 6 \frac{3}{8}
\]
Fractions: Examples of use in Science

1. Light: Lenses working formula \( \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \)

\[ \text{An object 20 cm from a convex lens of focal length 15 cm.} \]

Find the nature, position and Magnification of the Image using Real is positive Sign Convention.

\[ u = +20 \text{ cm} \quad f = +15 \text{ cm} \]

Substitute in \( \frac{1}{u} + \frac{1}{v} = \frac{1}{f} \)

\[ \frac{1}{u} + \frac{1}{20} = \frac{1}{15} \]

\[ \frac{1}{u} = \frac{1}{15} - \frac{1}{20} = \frac{u-3}{60} = \frac{1}{60} \]

Invert both sides \( u = 60 \)

\[ \text{Image distance} \]

\[ \text{Magnification} = \frac{\text{Image Distance}}{\text{Object Distance}} = \frac{60}{20} = 3 \]

2. Electricity: resistances in Parallel \( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \) ...

\[ \text{let } R_1 = 6.5 \Omega \quad R_2 = 5.2 \Omega \quad R_3 = 4.0 \Omega \]

\[ \frac{1}{R} = \frac{1}{6} + \frac{1}{5} + \frac{1}{4} \]

\[ \frac{1}{R} = \frac{10 + 12 + 15}{60} \]

\[ \frac{1}{R} = \frac{37}{60} \]

Invert \( R = 60 \)

\[ \frac{23}{37} \Omega \]

Total Resistance \( R = \frac{23}{37} \Omega \) or as a decimal = 1.62 to 2D.P.
eg \[ \frac{3}{4} \times \frac{1}{5} = \frac{3 \times 1}{4 \times 5} = \frac{3}{20} \] Simply multiply tops together and bottoms together

\[ \frac{1}{2} \] of \( \frac{1}{2} \) means \[ \frac{1}{2} \times \frac{1}{2} = \frac{1 \times 1}{2 \times 2} = \frac{1}{4} \]

**Mixed Numbers**

\[ 2\frac{1}{4} \times 1\frac{1}{8} \] first convert to improper fractions

\[ 2\frac{1}{4} \] becomes \( \frac{9}{4} \) \( \left[ \left( 2 \times \frac{8}{4} \right) + \frac{1}{4} = \frac{9}{4} \right] \)

\[ 1\frac{1}{8} \] becomes \( \frac{9}{8} \)

So we have \( \frac{9}{4} \times \frac{9}{8} = \frac{81}{32} = 2 \frac{17}{32} \)

**Cancelling before multiplying**

\[ \frac{3}{4} \times \frac{27}{32} = \frac{1}{1} \times \frac{3}{8} = \frac{3}{8} \]

**Division of fractions** Turn the fraction you are dividing by upside down and multiply

eg \[ \frac{2}{5} ÷ \frac{2}{3} \] becomes \( \frac{2}{5} \times \frac{3}{2} = \frac{6}{10} = \frac{3}{5} \)

\[ 1\frac{3}{8} ÷ 1\frac{1}{2} \] becomes \( \frac{11}{8} ÷ \frac{3}{2} \) becomes \( \frac{11}{8} \times \frac{2}{3} = \frac{11 \times 2}{4 \times 3} = \frac{11}{12} \)

**Whole Numbers as a fraction**

eg \[ 4 = \frac{4}{1} \] just put 1 under the number
Convert a Fraction into a Decimal.

A fraction, like \( \frac{3}{4} \), means \( 3 \div 4 \)

So \[ 4 \longdiv{3.00} \]

\[ 28 \]

\[ 20 \]

\[ 20 \]

\[ 0 \]

So \( \frac{3}{4} = 0.75 \)

Decimal to a Fraction:

Eg \( 0.75 = \frac{75}{100} \) cancels to \( \frac{3}{4} \) hundredths.

Example from Science:

Two cells each having an emf of 1.5v and internal resistance \( 2\Omega \) are connected (a) in series (b) in parallel. Find the current in each case when the cells are connected to a 1\( \Omega \) resistor.

**a)**

Total emf of 2 cells in series
\( = 1.5 \times 2 = 3v \)

Total resistance in circuit
\( = 2 + 2 + 1 = 5\Omega \)

Current:\[ \frac{\text{emf}}{\text{Total Resistance}} = \frac{3}{5} \]

Current = 0.6amps

**b)**

Emf of 2 cells in parallel = 1.5v

Resistance of cells in parallel:
\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{2} + \frac{1}{2} = \frac{2}{2} = 1 \Omega \]

So \( R = 1\Omega \)

Now Total resistance = \( 1 + 1 = 2\Omega \)

Current = \( \frac{\text{Emf}}{\text{Total Resistance}} \)
\( = \frac{1.5}{2} \)

Current = 0.75amps

If now the 1\( \Omega \) resistor is replaced by an 11\( \Omega \) resistor

What is the new current?

Total resistance is now \( 2 + 2 + 11 = 15\Omega \)

Current = \( \frac{3}{15} \) = 0.2amps

Total resistance is \( 1 + 11 = 12\Omega \)

Current = \( \frac{1.5}{12} \) = 0.125amps
A percentage is a fraction with a Denominator 100,

\[ \text{eq. } 15\% = \frac{15}{100} = \frac{3}{20} \]

Convert Fraction into Percent \%

\[ \text{eg. } \frac{3}{5} = \frac{3 \times 20}{5 \times 100} \% = 60\% \]

Convert Decimal into Percent \%

\[ 0.215 \times 100\% = 21.5\% \]

Convert Percent \% into Fraction: simply divide by 100

\[ 60\% = \frac{60}{100} = \frac{6}{10} = \frac{3}{5} \quad \text{and} \quad 2\frac{1}{2}\% = \frac{2\frac{1}{2}}{100} = \frac{5}{2} \times \frac{1}{100} = \frac{1}{20} \]

Convert Percentage into Decimal

\[ 13.5\% = \frac{13.5}{100} = 0.135 \]

Example from Science:

1. Find the Percentage power wasted as internal energy in a cable when 10 kW is transmitted through a cable of resistance 0.5 \( \Omega \) at 200 V.

The current in this case is given by \( \text{Amps} = \frac{\text{Watts}}{\text{Volts}} = \frac{5}{200} \)

\[ = 0.025 \text{A} \]

Therefore power loss in the cable in Watts = \( I^2R \)

\[ = (0.025 \times 0.025) \times 0.5 \text{ Watts} \]

\[ = 2500 \times 0.5 \]

\[ = 1250 \text{ Watts} \]

Percentage power wasted = \( \frac{\text{Power lost \times 100\%}}{\text{Total power}} \)

\[ = \frac{1250 \times 100\%}{10000} = 12.5\% \]

1. Pulleys: Some problems require you to work out the efficiency of a pulley system. Efficiency = \( \frac{\text{Work out}}{\text{Work in}} \times 100\% \)

or

\[ \text{Efficiency} = \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \times 100\% \]

1. Efficiency of machines: Transformer = 80\%, Jack = 40\%, Motorcar = 25\%
Proportions expressed as fractions and Percentages in Science

(1) Calculate the percentage composition by mass of Ammonium nitrate \( \text{NH}_4\text{NO}_3 \).

Atomic Masses \( N = 14 \), \( H = 1 \), \( O = 16 \)

Formula Mass \( \text{NH}_4\text{NO}_3 = 14 + (4\times1) + 14 + (3\times16) = 80 \)

- Percentage of Nitrogen \( N = \frac{7}{80} \times 100\% = 35\% \)
- Percentage of Hydrogen \( H = \frac{4}{80} \times 100\% = 5\% \)
- Percentage of Oxygen \( O = \frac{6}{80} \times 100\% = 60\% \)

(2) If an effort of 100 N is needed to lift a load of 200 N with this pulley system.

What is its efficiency?

\[ \text{Mechanical Advantage} = \frac{\text{Load}}{\text{Effort}} = \frac{200}{100} = 2 \]

\[ \text{Velocity Ratio} = \text{No. of pulleys} = 3 \]

\[ \text{Efficiency} = \frac{\text{Mechanical Advantage}}{\text{Velocity Ratio}} \times 100\% \]

\[ = \frac{2}{3} \times 100 \]

\[ = 66\frac{2}{3}\% \text{ or } 66.67\% \text{ to 2 decimal places} \]
**Coordinates**

Note the axes have positive and negative directions and may have different scales. A coordinate is the “address or directions” of a point. Z has coordinates of (3, 2) — we go along the x-axis to 3 and then up the y-axis to 2, as shown. Remember along then up or down.

W has coordinates of (-4, -2)

Remember along then up or down.
Some typical Graphs from Science

The Variation of Water Density with Temperature

Graphs of Motion: The acceleration of the object is found from the slope of the graph at that point.

A Graph of the efficiency of a pulley System against Load. Note the efficiency increases with high load. Can you explain it?
A graph is a set of coordinates plotted, which are related to each other. A smooth line is drawn through the set of points to show the trend. It is not always necessary to have the negative axes.

The table shows information to be plotted:

<table>
<thead>
<tr>
<th>INCHES</th>
<th>1</th>
<th>1.5</th>
<th>3</th>
<th>5</th>
<th>6.8</th>
<th>8.4</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>CENTIMETRES</td>
<td>2.5</td>
<td>3.8</td>
<td>7.6</td>
<td>12.7</td>
<td>17.3</td>
<td>21.8</td>
<td>22.8</td>
<td>25.4</td>
</tr>
</tbody>
</table>

A B C D E F G H

First step: Decide on scale of each axis. Look at the largest value. The x-axis has to go up to 10 inches, y-axis up to 25.4 cm.

Second step: Plot the coordinates of each point A to H, in this case.

Third step: Draw a line through the coordinates which best fits the whole set.

A graph of length in inches against centimeters.

Note: There is no need to label each point, just plot each point and draw in the line. The line can be extended by following the trend as shown by the dotted line from A back to O.
Representing Data in Science

The Pie chart

Energy Supplies of Britain at present

The Bar Chart or Histogram

Tooth Decay
500 Children from Town A

No. of teeth affected by fillings (caries)
What is Area?
Area is amount of surface covered

How do we measure it?
We find the number of squares covered

10 squares are covered by this rectangle.

What size of square should we use?
Our square must be of a standard size.

For small areas like the page of a book we use
SQUARE CENTIMETRES cm²

\[ \text{cm}^2 \]
**AREA and PERIMETER**

Area is the amount of surface covered, and Perimeter is the distance around the outside (boundary).

The **UNITS of AREA** are **SQUARE CENTIMETRES** (cm²)

### Area of a Rectangle

![Rectangle Diagram]

Area = \( a \times b \) (cm²)

Perimeter = \( 2a + 2b \) cm

### Split into Two Suitable Rectangles

![Split Rectangles Diagram]

Total Area = \( (a \times b) + (c \times d) \) cm²

Perimeter = \( a + b + (a-d) + c + d + e \) cm

### Area of a Triangle

![Triangle Diagram]

This is \( \frac{1}{2} \) of the rectangle that the triangle fits in.

Area of Triangle = \( \frac{1}{2} \) (base \times height) cm²

### Area of Circle

![Circle Diagram]

Area = \( \pi r^2 \) cm² where \( \pi = \frac{22}{7} = 3.142 \) (Pi) and \( r \) = radius

Perimeter = \( 2\pi r \) cm

---

**Note** Always state the units cm² or m²
Area in Science

1. Involved in motion graphs

The area under a Velocity vs Time graph gives the distance moved by the object in a particular time, for example.

![Velocity vs Time graph]

The area of the Trapezium OABC is the distance travelled in 40 seconds.

This area can be worked out in 3 stages – two triangles and one rectangle.

Area 1 = \( \frac{1}{2} \times \text{base} \times \text{height} \) (Triangle)
\[ = \frac{1}{2} \times 6 \times 12 \]
\[ = 36 \text{ m} \]

Area 2 = \( \text{length} \times \text{breadth} \) (Rectangle)
\[ = 30 \times 12 \]
\[ = 360 \text{ m} \]

Area 3 = \( \frac{1}{2} \times \text{base} \times \text{height} \) (Triangle)
\[ = \frac{1}{2} \times 5 \times 12 \]
\[ = 30 \text{ m} \]

Total Area = 36 + 360 + 30
\[ = 426 \text{ metres travelled} \]
Volume is the space occupied by an object. Units: Cubic Centimetre \( \text{cm}^3 \)

Box shape called Cuboid

![Diagram of a cuboid](image)

\[
\text{Volume} = \text{Length} \times \text{breadth} \times \text{height} = L \times b \times h \text{ cm}^3
\]

Alternatively

\[
\text{Volume} = \text{Area of base} \times \text{height} \text{ cm}^3
\]

Triangle shaped box (Prism)

![Diagram of a triangular prism](image)

\[
\text{Volume} = \text{Area of base} \times \text{height} \text{ cm}^3
\]

Base (Note the base is identical to the top)

Cylindrical

![Diagram of a cylinder](image)

\[
\text{Volume} = \text{Area of base} \times \text{height} = \pi r^2 h \text{ cm}^3
\]

Note: Length is measure in:

- **System International (S.I. Units)**
  - 1 metre = 100 cm = 1000 mm
  - 1 metre = 39.3 inches
  - 1 cm or 10 mm = 0.393 inches
  - 1 kilometre (Km) = 1000 metre (m)

- **Imperial Units**
  - 1 inch
  - 1 inch = 2.54 cm

You should be able to measure in both types of unit.
ANGLES

60°
Measuring with a protractor

How to place the Protractor to measure an angle.

Place Protractor as shown. Line AB along the 0° line read from the scale the number of degrees in line with AC.

Note. There are two scales read from the scale on which you place AB in line with C. In this case the inner scale.

Types of Angle

- **Acute Angle** (less than 90°)
- **Obtuse Angle** (greater than 90° but less than 180°)
- **Reflex Angle** (above 180°)
- **90° (Right Angle)**
- **180° Straight line or Half a Turn**
- **360° (Circle or 1 Turn)**
Reflex Angle = 180° + 66° = 246°

Alternate = 360° - 114° = 246°

Triangle means 3 angles

\[ \angle BAC = 45° \]
\[ \angle ABC = 55° \]
\[ \angle BCA = 80° \]

The angles of a triangle = 180°

Isosceles Triangle
(Two sides equal in length
Two angles equal.)

Equilateral Triangle
(All sides equal
All angles equal 60°)

Pythagoras:
The square on the hypotenuse equals
The sum of the squares on the other two sides:

\[ z^2 = x^2 + y^2 \]
Place the glass block with one of its longer sides along AB and one of its shorter sides along AD. Carefully mark the outline of the block ABCD.

Direct a narrow ray of light along PO. Mark the position where this ray leaves the block. Remove the block and draw a line to show the direction that this ray takes after refraction by the boundary AB. Measure the angle of incidence \( \theta_i \) and the angle the refracted ray makes with the normal \( \theta_r \).

Record these angles in the table of results.

Repeat this procedure three further times for the rays QO, RO and SO, recording the values for \( \theta_i \) and \( \theta_r \) in the table of results.
Plot a graph of the results.

From the graph, find the value of $y^\circ$ for which the value of $x^\circ$ is 90°.

Show clearly how you obtain the result.

$y^\circ =$
There are many examples, here are some:

1. Magnetic field patterns
   a) A bar magnet
   ![Diagram of a bar magnet showing magnetic field lines and axis of symmetry]

b) Combinations of bar magnets
   ![Diagram of various combinations of bar magnets showing magnetic field lines and axis of symmetry]
(2) Reflection of Light and Symmetry

![Reflection Diagram](image)

- Angle of incidence
- Angle of reflection
- Normal

Axis of Symmetry

(3) Interference of Light Waves

![Interference Diagram](image)

- Light from a single coherent source
- Interference fringes using red light

Axis of Symmetry

(4) Various Items from Science

- Moon Rocket
- Atomic Bomb "Mushroom Cloud"
- Flask
- Conical Flask

- Cooling Towers at a Power Station

- Electric Light Bulb
- Chemical Balance
Molecular Symmetry and Transformation in Chemistry.

**Methane**  
\[ \text{CH}_4 \]  
\[
\begin{array}{c}
\text{H} \\
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

**Ethane**  
\[ \text{C}_2\text{H}_6 \]  
\[
\begin{array}{c}
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

**Propane**  
\[ \text{C}_3\text{H}_8 \]  
\[
\begin{array}{c}
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

**Butane**  
\[ \text{C}_4\text{H}_{10} \]  
\[
\begin{array}{c}
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

**Isomerism:** Two substances with the same formula  
\[ \text{C}_4\text{H}_{10} \]  
1. As Butane  
\[
\begin{array}{c}
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

2.  
\[
\begin{array}{c}
\text{H} \\
\text{C} \\
\text{H} \\
\end{array}
\]

Structures of Diamond and Graphite.
Miscellaneous. Number work

Simple measurement: 30 cm² Area

Laws of Levers

Anticlockwise Moments = Clockwise Moments
10N x 5cm = x Newtons x 10cm

\[50 \text{ N.cm} = 10x \text{ N.cm}\]

Division for solution

\[\frac{50}{10} = x \text{ Newtons}\]

Harder measurements: Lengths: accurately measured with micrometer

1.676 cm

3. D.P accuracy \(\frac{1}{1000}\) of a centimetre

If this accuracy was not required round off, for instance to 1 D.P, this is 1.7 cm

Volumes: Liquid measurement in measuring cylinder

500ml = 0.5 litres \(\rightarrow \frac{500}{1000}\) of each

Thickness of a page from a book: Total number of pages

Thickness of book to nearest mm = 36mm

Thickness of page = \(\frac{36}{330}\) mm

Division
More complex use of numbers

Constant pressure law: from \( \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \). Then, \( V_2 = V_1 \frac{P_2}{P_1} \frac{T_1}{T_2} \).

If \( T_1 = 20^\circ C \) and \( T_2 = -30^\circ C \),

Then \( V_2 = V_1 \frac{273-30}{273+20} \) or \( \frac{243}{293} \) cm³

Resistivity

\( R = \frac{\rho L}{A} \) where \( \rho \) is resistivity, \( L \) length, \( A \) area of wire.

Consistent units of resistance are \( \Omega \).

Standard form and significant figures

Electricity: current measurement \( 24.78 \text{ mA} = 2.478 \times 10^{-2} \text{ A} \), divide by \( 10^3 \).

Light: wavelength \( 7.5 \times 10^{-5} \text{ cm} \) or speed of light \( 3 \times 10^8 \text{ m/s} \).

Air Pressure: atmospheric pressure \( 10^5 \text{ N/m}^2 \).

Conversion of units: volume \( 4.454 \times 10^4 \text{ cm}^3 = 4.454 \times 10^3 \text{ litres} \), divide \( \text{cm}^3 \) by 1000 to give litres.

Convert \( \text{Km/h} \) to \( \text{m/s} \): \( 3.6 \times 10^2 \text{ Km/h} = \frac{3.6 \times 10^2 \text{ m}}{3600 \text{ s}} \times \frac{1}{10^3} \).
Use of tables:

**Square Roots**: \( V_{\text{RMS}} = \frac{V_{\text{max}}}{\sqrt{2}} \) (Electricity)

**Sine**:
- Snell's Law: \( n = \frac{\sin \theta}{\sin \phi} \)
- Refractive Index
- Diffraction grating: \( n\lambda = 2d \sin \Theta \)
- Critical angle: \( \sin \theta_c = \frac{1}{n} \)
  - For glass: \( \sin \theta_c = \frac{1}{1.48} \)

**Sine/Cosine/Tangent**: Parallelogram of forces (Mechanics)

**Averages, Proportions and Percentages**:

- **Hooke's Law**: Simply if load on spring doubles then extension doubles

**Mechanics**:
- Efficiency of Pulleys, Machines, Transformers:
  
  \[
  \text{Efficiency} = \frac{\text{Mechanical Advantage} \times 100}{\text{Velocity Ratio}}
  \]

- **Kinetic Energy**: \( \frac{1}{2}mv^2 \propto V^2 \)

- **Ohm's Law**: Current \( i \propto \frac{\text{Voltage} V}{L} \)
  - Hence \( V = \text{constant} \times i \)

- **Pressure**: \( P \propto \frac{1}{\text{Volume}} \rightarrow P = \text{constant} \times \frac{1}{V} \)

**Wavelength**: \( \lambda \propto \frac{1}{\text{Frequency}} \)

**Density**: Weight \( \propto \) density
  - density \( \propto \frac{1}{\text{Volume}} \)

**Reciprocals**:
- Lens: \( \frac{1}{f} = \frac{1}{u} + \frac{1}{v} \)
- Electrically: \( 1 = 1 + 1 \)
Simple Equations, substitution of numerical values in equations
Change of the subject of equations, solving linear equations

- Moments: $20 \times M = 50 \times 2$, solve for $M$.
- Density: $\rho = \frac{\text{Mass}}{\text{Volume}}$
- Newton's law: $\text{Force} = \text{Mass} \times \text{acceleration} = M \times a$
- Velocity of wave: $\text{frequency} \times \text{wavelength} = f \lambda$
- Speed: $v = u + at$
- Distance: $\text{speed} \times \text{time} = ut$
- Complex moments: $F_1 x_1 + F_2 x_2 = F_3 x_3$
- Average Acceleration: $\frac{\text{change in velocity}}{\text{time}}$

Can you manipulate such equations?

- $V = iR$
- $P = i^2R = \frac{V^2}{R} = V \times i$
- $\frac{P_i}{V_i} = \frac{P_2V_2}{V_1}$
- Pressure: $\rho hg$
- Kinetic Energy: $\frac{1}{2}mv^2$

Often electrical problems on Ohm's Law involve simultaneous equations.

Probability and Averages

Radioactive Decay happens randomly
Some develop Half life: the time for half of the substance to decay

Kinetic Theory: average velocity of molecules
$\frac{\text{C}}{\text{to Temperature}}$

Average Acceleration: $\frac{V - u}{t}$
Graphs relationship between two variables: draw suitable axes, suitable scales, plotting and reading points.

Pressure Law Experiment:

\[ P \] vs. \[ \sigma \]

\[ P \] vs. \( \frac{1}{\sigma} \)

Ohm's Law with heat:

\[ \sigma \] vs. \[ i \]

without heat (constant temperature)

\[ \sigma \] vs. \[ i \]

Speed (ticker tape experiment):

\[ \sigma \] vs. time

Distance = Area under graph

Charles Law:

\[ \sigma \] vs. temperature

Absolute zero
Geometry: **Light: Lenses**  

**Similar triangles for magnification:**

\[
\text{Magnification} = \frac{v}{u}
\]

**Vector and Scalars:** **Parallelogram of forces**

There are many more examples; make your own notes as you see them in the course.

**Mathematics is very important for Science and Life**
APPENDIX VII

EXAMPLES OF SOME OF THE INTEGRATED MATHEMATICS/SCIENCE

PRACTICAL WORK DEVELOPED AND SOME PUPILS' WORK
PROBLEM 1 — A SAMPLE OF A COMPLETED WORKSHEET

HOW BIG IS A MOLECULE?
Can you guess (estimate) \( 0_{\ldots\ldots} \) cm
(remember it is very very small)
Now we will do an experiment which will give
an approximate value.
Oil when it spreads on water ends up with all its
molecules in one layer.
We take a small drop of oil and ether mixture,
mixed in proportions 1 part oil to 49 parts ether.
The ether helps the flow and then evaporates away
very quickly to leave just oil molecules.
The volume of oil in a drop is \( \frac{1}{50} \) of the volume of drop.

Finding the volume of a drop

**Method 1**

Measure volume of 50 drops \( 0.8 \text{ cm}^3 \)
Volume of one drop = \( \frac{0.8}{50} \text{ cm}^3 \)
(correct to 4 decimal places)
\( = 0.0160 \text{ cm}^3 \)

Volume of oil in one drop = \( \frac{0.0160}{50} \text{ cm}^3 \)
(cannot correct to 4 decimal places)
\( = 3.2 \times 10^{-4} \text{ cm}^3 \)

**Method 2** using a Mathematics formula

Volume of a sphere = \( \frac{4}{3} \pi \text{ (radius)}^3 \text{ cm}^3 \)
\( \approx \frac{4}{3} \pi \times \text{ radius} \times \text{ radius} \times \text{ radius} \text{ cm}^3 \)

Magnified by a convex lens
so you can read accurately

<table>
<thead>
<tr>
<th>Diameter</th>
<th>mm or cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>mm or cm</td>
</tr>
</tbody>
</table>
Taking \( \pi \) (pi) as \( \frac{22}{7} \) (Radius) \(^3\)

Volume of drop = \( \frac{4}{3} \times \frac{22}{7} \times \times \times \text{cm}^3 \)

(correct to 4 dec. places)

\[ = \text{cm}^3 \]

Volume of oil in drop = \( \frac{50}{50} \times \text{cm}^3 \)

Compare Volumes of oil in drop

Method 1: \( \text{cm}^3 \) In STANDARD FORM \( \text{cm}^3 \)

Method 2: \( \text{cm}^3 \)

We could simplify the volume still further by approximating

the drop to a cube shape

\[ \text{Volume of cube} = d \times d \times d \text{ cm}^3 \]

\[ = \text{cm}^3 \]

Volume of oil in drop (cube) = \( \frac{50}{50} = \text{cm}^3 \)

At the end of the experiment we will discuss which method
we may consider the best for our purposes

Now drop ONE DROP ONTO THE WATER SURFACE COVERED IN
TALCUM POWDER — SEE WHAT HAPPENS

Oil pushes Talcum back

APPROXIMATELY CIRCULAR

Measure
RADIUS OF CIRCLE
9.969444 cm

How did you
measure so accurately?
The oil patch can be regarded as a **cylinder**

Volume of a Cylinder = Area of base $\times$ height

$= (\pi r^2) \times$ height

Now the height is equal to the size of a molecule

So $\pi r^2 = 299.84002 \ldots$ Is this a realistic number?

Volume of drop = Volume of cylinder

Since the volume of the oil stays constant while it changes shape.

Now work out the size of a molecule of oil.

If you need help Ask.

$299.84002 \times 1 = 3.2 \times 10^{-4}$

$x = 1.06723 \times 10^{-5}$

$= 0.00000106723$ cm.

$= 0.00000000106723$ m

Size of molecule of oil = $1.06723 \times 10^{-5}$ cm.

Convert it to metres $= 1.06723 \times 10^{-8}$ m.

and in standard form $= 1.06723 \times 10^{-8}$ m

$= 1.07 \times 10^{-8}$ m (A)

Molecules are about one-millionth of a millimetre

Atoms are about ten times smaller than this $(10^{-10} \text{ m}) \quad \frac{1}{10 000 000 000} \text{ m}$
If we choose the accurate method of finding the volume of the drop, we should work out the area more accurately.

Try this:

![Diagram showing a trough with radial lines drawn every 10° around the circle.](image)

On the bottom of the trough draw in dark felt tip lines and with your Protractor draw in RADIAL LINES every 10° around the circle. Number them 1 to 36.

Now looking down on the water trough at the oil patch, you can measure in from the side.

<table>
<thead>
<tr>
<th>No. of Line</th>
<th>Measurement CM</th>
<th>No. of Line</th>
<th>Measurement CM</th>
<th>No. of Line</th>
<th>Measurement CM</th>
<th>No. of Line</th>
<th>Measurement CM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10.5</td>
<td>10</td>
<td>10.2</td>
<td>19</td>
<td>10</td>
<td>28</td>
<td>9</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>11</td>
<td>12</td>
<td>20</td>
<td>11.1</td>
<td>29</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>10.45</td>
<td>12</td>
<td>8</td>
<td>21</td>
<td>12.5</td>
<td>30</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>13</td>
<td>8</td>
<td>22</td>
<td>13.5</td>
<td>31</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>14</td>
<td>10.75</td>
<td>23</td>
<td>14.75</td>
<td>32</td>
<td>12</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>15</td>
<td>10.4</td>
<td>24</td>
<td>12.3</td>
<td>33</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>16</td>
<td>10.8</td>
<td>25</td>
<td>8</td>
<td>34</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>17</td>
<td>9</td>
<td>26</td>
<td>8.5</td>
<td>35</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td>18</td>
<td>10.5</td>
<td>27</td>
<td>8.5</td>
<td>36</td>
<td>10</td>
</tr>
</tbody>
</table>
On the large sheet of graph paper supplied draw the lines 1 to 36 at 10° intervals and the outline of the Trough and measure 11 in from the edge of the trough to construct the exact shape of the Oil Patch.
You can now use the Graph Paper to work out the area (Remember 1 square = 1 cm)

\[
\text{Area of Oil Patch} = 220.5 \text{ cm}^2
\]

Compare this with the area found by approximating the oil patch to a circle
\[
\text{Area of circular patch} = \pi r^2 = 220.84 \text{ cm}^2 \checkmark
\]

Was this a fair assumption?

How did your estimation compare with the approximate size of a Molecule of Oil?
hand in hand. Mathematics is a very useful tool. Here is where we used the different Mathematics topics in this practical.

<table>
<thead>
<tr>
<th>PART OF PRACTICAL</th>
<th>MATHEMATICS TOPIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measuring Volume of Solar</td>
<td>COUNTING NUMBERS</td>
</tr>
<tr>
<td>Volume of 1 Drop</td>
<td>DIVIDING ( \frac{1}{4} ), Rounding off</td>
</tr>
<tr>
<td>Volume of Oil in Drop</td>
<td>RATIO, DIVIDE, Rounding off, UNITS cm³</td>
</tr>
<tr>
<td>Volume of Sphere</td>
<td>FORMULA ( \frac{4}{3} \pi r^3 ), Substitution, UNITS cm³</td>
</tr>
<tr>
<td>Measuring Oil Drop</td>
<td>DIAMETER ( \times 2 \times \text{RADIUS} ), Conversion mm to cm ( \div 10 )</td>
</tr>
<tr>
<td>Volume of Oil in Sphere</td>
<td>DIVIDING, UNITS cm³</td>
</tr>
<tr>
<td>Comparison</td>
<td>STANDARD FORM</td>
</tr>
<tr>
<td>Simplified Volume</td>
<td>VOLUME OF A CUBE, UNITS cm³</td>
</tr>
<tr>
<td>Volume of Oil in a Cube</td>
<td>DIVIDING, UNITS cm³</td>
</tr>
<tr>
<td>Diameter of Oil Patch</td>
<td>MEASUREMENT, UNITS cm</td>
</tr>
<tr>
<td>Radius of Oil Patch</td>
<td>DIVIDING, UNITS cm</td>
</tr>
<tr>
<td>Volume of Cylinder</td>
<td>VOLUME = ( \text{Area of base} \times \text{height} ), UNITS cm³</td>
</tr>
<tr>
<td>Volume of Drop = Volume of Cylinder</td>
<td>EQUATION</td>
</tr>
<tr>
<td>Size of Molecule</td>
<td>SOLVE FOR ONLY UNKNOWN &quot;HEIGHT&quot; UNITS cm</td>
</tr>
<tr>
<td>Size of Molecule / Atom</td>
<td>CONVERSION cm to m, DIVIDING 100, STANDARD FORM</td>
</tr>
<tr>
<td>Accurate Area of Patch</td>
<td>NEGATIVE POWERS, UNITS m¹</td>
</tr>
<tr>
<td>Comparison</td>
<td>ORDER OF VALUES, LARGE AND VERY SMALL, ESTIMATION</td>
</tr>
</tbody>
</table>

You will be constantly using your Mathematics. It is worth learning it well not just for science where it is essential but for everything.
A TYPICAL ERROR WAS :-

We could simplify the volume still further by approximating the drop to a cube shape

\[ \text{Volume of cube} = d \times d \times d \text{ cm}^3 \]

\[ = 0.1 \times 0.1 \times 0.1 \text{ cm}^3 \]

\[ \text{Volume of oil in drop (cube)} = \frac{0.1}{50} = 0.002 \text{ cm}^3 \]

and Standard Form.

Finding the volume of a drop

**Method 1**

Measure Volume of 50 drops \( \frac{0.7}{50} \text{ cm}^3 \)

Volume of one drop = \( \frac{0.7}{50} \text{ cm}^3 \)

(correct to 4 dec. places)

\[ = 0.014 \text{ cm}^3 \]

Volume of oil in one drop = \( \frac{0.114}{50} \text{ cm}^3 \)

(correct to 4 dec. places)

\[ = 2.3 \times 10^{-4} \text{ cm}^3 \]

IS THIS STANDARD FORM?

and Taking \( \pi \) (pi) as \( \frac{22}{7} \)

Volume of drop = \( \frac{4}{3} \times \frac{22}{7} \times 0.05 \times 0.5 \times 0.5 \text{ cm}^3 \)

(correct to 4 dec. places)

unrealistic!

\[ = 0.0005238 \text{ cm}^3 \]

Volume of oil in drop = \( \frac{0.0005238}{50} \text{ cm}^3 \)

\[ \approx 0.000010476 \text{ cm}^3 \]

In STANDARD FORM

\[ \approx 2.3 \times 10^{-9} \text{ cm}^3 \]
Practical 2

Density

Take your block of material and measure its sides to the nearest 0.1 cm (1mm) and fill in the diagram

![Diagram of a block with measurements]

Now work out the volume in Cubic Centimetres

\[
\begin{array}{cc}
15.5 & 38.75 \\
\times 2.5 & \times 3.5 \\
31.25 & 135.625 \\
7.75 & 193.75 \\
38.75 & 193.75 \\
\end{array}
\]

Volume of block = 135.625 cm³

(Note: This should be correct to 1 decimal place)

We have no Electronic Balance to find the mass of the block, but we do have an Elastic Band and some fixed masses on a hanger. Remember Hooke’s Law: it states that Extension is proportional to Load (Extension ∝ Load) so we could calibrate our Elastic Band as follows:

![Diagram of the Elastic Band setup]

We can measure the amount the elastic stretches (extends) for each load, then draw up a table and plot a graph of load against extension.
Read your ruler carefully: to nearest 0.01 cm.

<table>
<thead>
<tr>
<th>Load on Elastic (gramm)</th>
<th>0g</th>
<th>10g</th>
<th>30g</th>
<th>50g</th>
<th>70g</th>
<th>90g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unloaded Ruler Reading (cm)</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>Reading with Load (cm)</td>
<td>14</td>
<td>15</td>
<td>16.7</td>
<td>17.5</td>
<td>18.4</td>
<td>19.5</td>
</tr>
<tr>
<td>Extension (cm)</td>
<td>0</td>
<td>1</td>
<td>2.7</td>
<td>3.5</td>
<td>4.4</td>
<td>5.5</td>
</tr>
</tbody>
</table>

Now plot your graph below of Load against Extension. (Remember when choosing a scale for the axes you want to fill most of the available space with your graph.)
Now we can use our Elastic Band and Calibration Graph to find the mass (load) of the Block of Material. Hang it onto the Elastic Band and measure the extension. Then using the graph find its mass (load) in grammes.

Mass of Block = 50 g

DENSITY = \frac{\text{MASS} (g)}{\text{Volume} (cm^3)}

Now you can work out (calculate) the Density of the Block of Material:

\[
\text{Density} = \frac{1356}{5000} \cdot 361 = \frac{1356}{4068} = \frac{3}{6} = \frac{1}{3}
\]

Density = 0.361 g/cm³ (correct to 3 dec. places)

Convert this to Kg/m³ by \( \frac{9}{\text{cm}^3} \times 1000 \) cm³

So Density = 361 Kg/m³
Here are the masses of 1 m³ of some different substances. Which substance do you think your sample is? soft wood.

How did you make your choice? Our mass is 135.6 kg. In the graph if you look at 135.6 up the side and go across it comes to soft wood.

Now take the second piece of wood and without using your elastic band weighing machine. Find its mass in grammes. You know the density and can work out its volume in m³. Density 361 kg/m³

Measure: Base of triangle 4 cm. Height of triangle 4 cm. Thickness 4.5 cm

 Decimal point?

Volume = Area of cross-section x height of prism

So Area of Triangle = 4 cm²

Volume of Triangular Prism = 360 cm³ (correct to 1 decimal place)
Now by manipulating the Equation \( D = \frac{M}{V} \) you can write it in the form \( M = V \times D \) (correct to 1 dec. place).

Substitute into this equation for \( D \) and \( V \) and work out \( M \) the mass.

Then check this with your elastic band weighing machine.

\[
\text{Mass} = \frac{360.000}{0.361} \text{ ? grams}
\]

The value found by weighing = 10 \( \checkmark \) grammes

The value found by calculating = 10 \( \checkmark \) grammes

Here is where we used the different Mathematics topics:

<table>
<thead>
<tr>
<th>PART OF PRACTICAL</th>
<th>MATHEMATICS TOPIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement of block</td>
<td>Measurement with a ruler in S.I. units, Decimals</td>
</tr>
<tr>
<td>Volume of block</td>
<td>Formula, L.S.B.H, Substitution, Multiplication S.I. units, Rounding off, Decimals</td>
</tr>
<tr>
<td>Extension of elastic</td>
<td>Measurement, Subtraction Decimals S.I. units, Thematics values</td>
</tr>
<tr>
<td>Hook's Law</td>
<td>Graph from data, Scales, Co-ordinates, S.I. units</td>
</tr>
<tr>
<td>Mass of block</td>
<td>Reading information from a graph S.I. units</td>
</tr>
<tr>
<td>Density</td>
<td>Formula, Substitution, Division, Decimals S.I. units</td>
</tr>
<tr>
<td>Conversion to kg/m³</td>
<td>Division, Multiplication, Decimals S.I. units</td>
</tr>
<tr>
<td>Density of different</td>
<td>To read information from a bar chart</td>
</tr>
<tr>
<td>substances</td>
<td></td>
</tr>
<tr>
<td>Triangular prism</td>
<td>Measurement, S.I. units, Area of triangle, Volume of triangular prism,</td>
</tr>
<tr>
<td></td>
<td>Multiplication, Division, Decimals</td>
</tr>
<tr>
<td>Mass of prism</td>
<td>Formula manipulation in different subject, Multiplication, Decimals, Rounding off, S.I. units</td>
</tr>
<tr>
<td>Checking</td>
<td>ACALCULATOR WAS AVAILABLE, ROUNDING OFF.</td>
</tr>
</tbody>
</table>
**Examples of Poles Work From Patient 2 Density**

<table>
<thead>
<tr>
<th>Unloaded</th>
<th>Load (g)</th>
<th>Extension (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>30</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>40</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>60</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>70</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>80</td>
<td>0</td>
<td>18</td>
</tr>
</tbody>
</table>

Now plot your graph below of load against extension (remember when choosing a scale for axes you want to fill most of the available space with your graph).

- Extension

- Load (g)
1

\[
\text{Density} = \frac{\text{Mass (g)}}{\text{Volume (cm}^3\text{)}}
\]

Now you can work out (calculate) the Density of the Block of Material

\[
\frac{609}{127.8} = \frac{2.13}{601127.8} = \frac{2}{601127.8}
\]

To change g to Kg \( \frac{60}{1000} \)

\[
\text{Density} = 2.13 \times 1000 \text{ g/cm}^3 \text{ (correct to 3 dec. places)}
\]

Convert this to Kg/m\(^3\) by \( \frac{9}{1000} \text{ Cm}^3 \text{ /600000} \text{ Cm}^3 \)

This is \( \frac{9}{1000} \text{ Cm}^3 \times 1000 \)

So Density = \( \frac{10.606 \text{ Kg}}{1000 \times 601127.8} \text{ Kg/m}^3 \)

\( \times \)

2

\[
\text{Height} = 3.5 \text{ cm}
\]

\[
\text{Length} = 7.3 \text{ cm}
\]

\[
\text{Width} = 3.6 \text{ cm}
\]

Now work out the volume in Cubic Centimetres

\[
\begin{align*}
7.3 \times 21.9 & = 159.5 \times 76.55 & = 25.55 \\
3.6 & \times 95 & = 34.1 \times 13 & = 4.38 \\
\hline
26.28 & \end{align*}
\]

Volume of Block = \( 9.2 \text{ cm}^3 \) (correct to 1 decimal place)
Plot your graph below of Load against Extension (remember when choosing a scale for the axes you want to fill most of the available space with your graph).
Now work out the volume in Cubic Centimetres

\[
\begin{array}{c|c}
7.3 \times 21.90 & 26.28 \\
4.38 \times 78.84 & 3.0 \\
26.28 & 0.5 \\
91.98 & \\
\end{array}
\]

\[
\text{Volume of block} = 92 \text{ cm}^3
\]
(Correct to 1 decimal place)

Also:

Now work out the volume in Cubic Centimetres

\[
\begin{array}{c}
4.1 \\
4.4 \\
\hline
16.4 \\
16.4 \times 9x \\
\hline
32.8 \\
\hline
245.2 \\
\end{array}
\]

\[
\text{Volume of block} = 25.1 \text{ cm}^3
\]
(Correct to 1 decimal place)

8. Density = \[
\frac{34}{64} = \]
You have done 34 / 64
The question is 64 / 34
Density = 1.882 \text{ g/cm}^3 (correct to 3 decimal places)
DENSITY = \frac{\text{MASS (g)}}{\text{Volume (cm}^3\text{)}}

Now you can work out (calculate) the Density of the Block of Material.

\[ \frac{3600 \text{g}}{6880.68 \text{cm}^3} = 5.24 \]

\[ \frac{1.91115}{36} \]

\[ \frac{36}{40} \]

Problem is \[ 68.8 \sqrt{36} \]

Density = 1.9115 \text{ g/cm}^3 (corrected to 3 decimal places)

Measure: Base of triangle 3.9 cm.
Height of triangle 4 cm.
Thickness 4.6 cm.

Volume = Area of cross-section \times \text{height of prism}

So Area of Triangle = 18.4 cm\(^2\)

\[ \frac{18.4 \times 4}{13.4} \]

\[ \frac{3 \times 52}{12} \]

Volume of Triangular Prism = 56.2 cm\(^3\) (corrected to 1 decimal place)

Mass = Density \times V

= 0.361 \times 360

\[ \frac{3600}{36.00} \times 0.361 \]

\[ \frac{12.51}{12.51} \]

\[ 12.9196 \]
Scale?
Label axes.
PRACTICAL 3

MEASUREMENT AND HEAT

We are going to fill a can with different quantities of water and then heat it up through a rise in temperature of 50°.

First we must find the volume of water in the can, all we have is a ruler.

As you cannot see through the sides of the can you will have to lower the ruler down the inside wall and take the reading at the top of the can when the ruler just touches the water.

So Depth of water = h - d cm

Measure L, b, h and d to the nearest 0.1 cm

Now calculate the volume of water in cm³

Volume of Water = cm³ (correct to 1 dec. place)

1 cm³ water has a mass of 1 gramme (i.e. Density of water = 1 g/cm³)

So Mass of Water = g
Place the can of water onto the tripod and note the temperature of water with the thermometer

\[\text{Temperature of water} = \quad ^\circ\text{C}\]

Light the bunsen burner and time how long it will take for it to heat the water up through 50°C rise in temperature

* Use the bunsen burner on its hottest flame.

What is the final temperature when we stop timing? \(\quad ^\circ\text{C}\)

Will the stopwatch?

How long did it take\(\quad\) Seconds (read to the nearest second)

Now repeat with four more different amounts of water

<table>
<thead>
<tr>
<th>Mass of water (g)</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to heat up 50°C (seconds)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now on the sheet of graph paper supplied, plot a graph of Mass of Water (x axis) against Time to heat up 50°C (y axis)

Remember to use a scale which will use up most of the paper, and to label the axes.
A graph of mass of fluid against time to heat through 50°
Now repeat this whole exercise with a different liquid - cooking oil. Do not leave the hot oil unattended and to save time each group should use a different mass of oil and then share results.

We will weigh the oil in a beaker on the Electronic Balance

\[
\begin{align*}
\text{Weight of beaker} & \quad 9 \\
\text{Weight of oil + beaker} & \quad 9 \\
\text{Weight of oil} & = 9 \text{ grams}
\end{align*}
\]

Now heat it up and time, exactly as before in the can.

<table>
<thead>
<tr>
<th>Mass of oil</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Time to heat oil up 50%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

On the same graph paper plot this graph - remember to label each graph line.

Does water need more or less heat per kilogram than cooking oil? ________.

If the Specific Heat of Water is 4.2 J/ g\(\circ\)C, how much heat is required to heat up the water for each of the 5 different masses used in the experiment? Remember \(Q\) (heat supplied) = M \times S \times T \text{ Joules}

where \(M\) = Mass in g, \(S\) = Specific Heat in J/ g\(\circ\)C, \(T\) = Temperature rise \(\circ\)C.
Do your working out here; remember neatness

Fill in the table:

| Mass of water (g) | | | |
|-------------------| | | |
| Heat required to raise temperature 50°C | | | |

Now plot another graph of Mass of water against Heat required to raise its temperature 50°C on the next page. The axes and scales are drawn for you.
A graph of Mass against Heat required to raise its temperature 50°C (50°C deg E)

Heat required to raise water 50°C = $10^3 \text{Joules}$

Mass of Water g

0 200 400 600 800 1000 1200
Can you use your graph!

How much heat would be required to heat up
1 Kg of water through 50°C ________ Joules

Now if a kettle produced heat 5 times as fast
How long would it take a kettle to heat up
1Kg of water through 50°C ________ Joules

Now we know from the graph, which is a Straight
line, that the heat required is proportional to the Mass

Can you work out how much heat would be needed
to boil a kettle of water. If it contained 2Kg
of water, originally at a temperature of 25°C

Answer Heat required is ________ Joules.
<table>
<thead>
<tr>
<th>Mathematics Topics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decimals, fractions, percentages, ratios, proportions, exponents, radicals, algebra, geometry, trigonometry, calculus, linear algebra, discrete mathematics, probability, statistics.</td>
</tr>
</tbody>
</table>

### Unit 1: Number Sense and Operations
- **Use of Numerical Values**
  - Specific Heat
  - REPEAT ALL OF THE ABOVE FOR COOKING, DECIMALS
- **Measurement**
  - Temperature
  - Mass of water
  - Volume of can
  - Dimensions of can
  - Temperature
  - Mass of water
  - Volume of can

### Unit 2: Algebraic Thinking
- **Algebraic Concepts**
  - Linear equations, inequalities, systems of equations, quadratic equations, polynomial functions, rational functions, exponential functions, logarithmic functions.
  - Functions, relations, inverses, compositions, transformations.

### Unit 3: Geometry and Measurement
- **Geometric Concepts**
  - Points, lines, planes, angles, triangles, quadrilaterals, circles, polygons, solid figures.
  - Area, perimeter, volume, surface area.
  - Coordinate geometry, transformational geometry, symmetry, congruence, similarity.

### Unit 4: Statistics and Probability
- **Statistical Concepts**
  - Data collection, organization, analysis, probability, random variables, distributions, inference.
  - Sampling, surveys, experiments, descriptive statistics, inferential statistics.

### Unit 5: Functions and Relations
- **Functional Concepts**
  - Domain, range, function notation, composition of functions, inverse functions.
  - Graphs of functions, transformations, modeling with functions.

### Unit 6: Calculus and Analytic Geometry
- **Calculus Concepts**
  - Limits, derivatives, integrals, continuity, differentiation, optimization, accumulation, slope fields, differential equations.
  - Conics, parametric and polar equations.

### Unit 7: Discrete Mathematics
- **Discrete Concepts**
  - Logic, sets, relations, functions, graphs, trees, networks, algorithms, counting, probability, number theory.

### Unit 8: Advanced Topics
- **Advanced Concepts**
  - Advanced algebra, trigonometry, pre-calculus, probability, statistics, linear algebra, differential equations, vector calculus, complex analysis, number theory.

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Here is where we used the different mathematics topics.
The aim of this experiment is to find the power in watts of a lighted candle, by allowing the candle to heat a quantity of water in a calorimeter and then measuring the rise in temperature. You are to follow the method given, record the readings you obtain and answer the questions in the spaces provided.

Just over half fill the calorimeter with water measuring the volume of water added.

1. Volume of water added = .................................................................

2. List below three precautions that you have taken to ensure that the above reading is accurate:
   (a) ..................................................................................................................
   (b) ..................................................................................................................
   (c) ..................................................................................................................

3. Take the temperature of the water in the calorimeter and record the value below.
   Initial temperature of water in calorimeter = .....................................................

4. List below two precautions that you have taken to ensure that the above reading is accurate:
   (a) ..................................................................................................................
   (b) ..................................................................................................................

Clamp the calorimeter containing the water above a lighted candle with the flame in contact with the base of the calorimeter. Find the time it takes for a temperature rise of 45°C ± 5°C. Record the time taken and the final temperature of the water.

5. Final temperature of the water = .................................................................

6. Time taken = .................................................................................................

Blow out the candle.

The power of the candle is given by the following formula:

$$ \text{Power} = \frac{4 \times \text{Volume of Water} \times \text{Rise in Temperature}}{\text{Time in Seconds}} $$

7. Substitute your values in the formula in the spaces below.
   $$ \text{Power} = \frac{4 \times \text{............................................} \times \text{............................................}}{\text{............................................}} $$

8. Calculate the Power to the nearest whole number.
   Power = ......................................................................................................... watts

9. Give two reasons why this value is likely to be less than the actual power of the candle.
APPENDIX VIII

COGNITIVE AND READING TESTS GIVEN TO 12 YEAR OLDS
How would you solve this problem?

1. A bar of chocolate can be broken into 12 squares. There are 3 squares in a row.
   - How do you work out how many rows there are?
   \[ 12 \div 3 = 4 \]

2. The signpost shows that it is 29 miles west to Grange and 18 miles east to Barton.
   - How do you work out how many miles it is from Grange to Barton?
   \[ 29 \times 18 = 522 \]

3. John is cycling 8 miles home from school. He stops at a sweet shop after 2 miles.
   - How do you work out how much further John has to go?
   \[ 8 \times 2 = 16 \]

4. 286 people are coming to see the school play. The chairs are arranged in rows, and there are 26 chairs in each row.
   - How do you work out how many rows are needed?
   \[ 286 \div 26 = 11 \]
How would you solve this problem? (continued)

5 A bucket holds 8 litres of water. 4 buckets of water are emptied into a bath.

How do you work out how many litres of water are in the bath?

5 \times 4 \quad 4 \div 8

8 \div 4

8 \div 4

8 \times 8

8 \div 4

6 The Green family have to drive 261 miles to get to London from Leeds. After driving 87 miles they stop for lunch.

How do you work out how far they still have to drive?

87 \times 3 \quad 261 \times 87

261 \div 87

261 \div 87

87 \div 261

87 + 174

7 A shop makes sandwiches. You can choose from 3 sorts of bread and 6 sorts of fillings.

How do you work out how many different sandwiches you could choose?

3 \times 6 \quad 6 - 3

6 \times 3 \quad 3 - 6

6 \times 3 \quad 3 + 3

8 A gardener has 391 daffodils. These are to be planted in 23 flowerbeds. Each flowerbed is to have the same number of daffodils.

How do you work out how many daffodils will be planted in each flowerbed?

391 \div 23 \quad 23 \times 391

391 \div 23 \quad 23 \times 391

391 \times 23 \quad 23 \times 23

391 \div 23

9 An oven tray used for cooking little cakes will hold 36 cakes. A baker fills 28 trays.

How do you work out how many cakes he will cook?

54 \div 28 \quad 28 \times 54

28 \div 28

54 \div 28

56 \div 56
Activity 20

Trial item for 'How would you solve this problem?'

Linda has 5 LPs and Sandra has 3 more LPs than Linda.

How do you work out how many LPs Sandra has?

<table>
<thead>
<tr>
<th>Calculation</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 + 3</td>
<td>3 + 5</td>
</tr>
<tr>
<td>8 - 5</td>
<td>3 × 5</td>
</tr>
<tr>
<td>5 + 3</td>
<td>8 - 3</td>
</tr>
<tr>
<td>5 - 3</td>
<td>4 + 4</td>
</tr>
</tbody>
</table>

Trial item for 'Make up a story to fit this sum'

9 + 3

Give your subject these trial items first of all, to make sure he understands what to do. Discuss them with him as fully as seems necessary.
Activity 20  Make up a story to fit this sum

a. $84 - 28$  Story:

b. $9 \div 3$  Story:

c. $9 \times 3$  Story:

d. $84 \div 28$  Story:

e. $84 \times 28$  Story:
1. Trains can often be seen standing in a railway (engine, driver, box, station)

2. If you write with a pen, you also need (crayons, money, help, ink)

3. Children go to school in order to (sleep, run, cry, learn)

4. A horse is an animal with four (tails, eyes, legs, ears)

5. People usually go on their holidays to (enjoy, re-imburse, spite, employ) themselves

6. Coal is usually (yellow, black, white, pink)

7. Boys often like to climb up (beans, tents, trails, trees)

8. The first meal of the day is called (dinner, breakfast, tea, supper)

9. Books are made of (patent, paper, pamper, pepper)

10. Oranges and bananas are both (fruits, fruit, poisonous, animals)

11. Grass is (blue, green, white, red)

12. Before we cut meat, it should be (swallowed, stroked, cooked, crooked)

13. Mr Smith is limping because, yesterday, whilst getting off the bus, he slipped and twisted his (armlet, neck, ankle, umbrella)

14. Shoes are usually made of (leather, lather, laces, soles)

15. A giant is a (short, tall, thin, hungry) man

16. Men's socks are usually (matter, stolen, wasted, knitted)
17 Motor-cars are driven along by petrol being exploded inside the (cabin, pump, engine, steering-wheel)

18 When we go out to a friend's house for tea, we often find that the table is already laid with cups and (visitors, sand, sausages, saucers)

19 Most houses in this country today are lit by means of (candles, oil-lamps, electricity, tapers)

20 If the road is very bumpy, a ride on a bus can be very (uncomforting, uncomfortable, uncontrolled, unconverted)

21 Liquids are usually kept in (boxes, fires, drinks, bottles)

22 A steam engine usually runs on (rails, reels, stoves, signals)

23 One of the best ways of keeping healthy is to take plenty of (examination, examples, excitement, exercise)

24 When we send a letter to a friend, it is usual to fold it and put it into an (address, appliance, affluence, envelope)

25 The season of the year when young green buds appear on the trees is called (autumn, spring, winter, October)

26 The typhoon blew so hard that three thousand houses were (destroyed, annoyed, demonstrated, burst)

27 Unless one is very experienced, rock-climbing can be (lucrative, temporary, dangerous, degenerative)

28 A place where talking films are shown is called a (theatre, cinema, gallery, house)

29 A bald man has little (feet, hair, nose, cap) on his head

30 A male child is called a (boy, girl, dwarf, nuisance)
31 A head teacher granted (permission, presentation, permutation, refusal) for the boy to be absent from school on the day of his brother's wedding

32 A prisoner usually longs for his (sentence, toleration, serenade, freedom)

33 When people are ill they are often visited by the doctor who prescribes (prevention, disease, radio, medicine) for them

34 In this country, the commonest fuel used for most house fires is (wood, oil, smoke, coal)

35 The case for the prosecution so impressed the jury that they found the prisoner (dirty, guiltless, wicked, guilty)

36 A mushroom is an edible (fugitive, fungus, parlour, fantasy)

37 We use soap to wash clothes because it helps to remove the (grease, dye, geese, shrubs) from them

38 If there is one nearby, you should always cross the road at a (pedestal, railway, channel, pedestrian) crossing

39 The visitor went to the manager's office and politely asked the secretary if he could have an (interruption, extradition, interest, interview) with the manager

40 If you want to make sure that the plants in your garden will grow well, it is good to sprinkle them with (seeds, roots, fertiliser, worms)

41 When bombs drop on an undefended city, it is almost certain that they will cause a great deal of (demonstration, suspicion, destruction, conservation)

42 A city has a bigger (popularity, population, rainfall, postulation) than a village

43 The most important female participant in a wedding is the (groomsman, bridegroom, mother, bride)
A man who translates the conversation of two people who cannot speak each other's language is called an (interpreter, interloper, annotator, explouter)

Ships sail from port to port, crossing the seas and oceans, carrying (mercenaries, mensuration, meridians, merchandise) to all parts of the world

In spring the farmer is often very busy ploughing the fields, in order to make them ready for (stewing, cattle, sowing, grazing)

When two armies are engaged in battle, one of the two (adjectives, adversaries, explosions, swords) usually ends up as the victor

The wheels on a motor-car (rotund, retreat, rotate, excavate)

When walking in the woods you must be careful not to throw down lighted matches or you may cause serious (contemplation, conflagration, stipulation, conflict)

The explorers who first reached the South Pole found that the intense cold and the fierce blizzards (receded, impressed, impeded, imposed) their progress
## Reading Test Mark Scheme

<table>
<thead>
<tr>
<th>Score</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>6.0</td>
<td>6.1</td>
<td>6.2</td>
<td>6.3</td>
<td>6.4</td>
<td>6.5</td>
<td>6.6</td>
<td>6.7</td>
<td>6.8</td>
<td>6.9</td>
</tr>
<tr>
<td>20</td>
<td>7.0</td>
<td>7.1</td>
<td>7.2</td>
<td>7.4</td>
<td>7.5</td>
<td>7.6</td>
<td>7.7</td>
<td>7.8</td>
<td>7.9</td>
<td>8.1</td>
</tr>
<tr>
<td>30</td>
<td>8.2</td>
<td>8.3</td>
<td>8.4</td>
<td>8.6</td>
<td>8.7</td>
<td>8.8</td>
<td>9.0</td>
<td>9.1</td>
<td>9.3</td>
<td>9.5</td>
</tr>
<tr>
<td>40</td>
<td>9.7</td>
<td>10.1</td>
<td>10.3</td>
<td>10.7</td>
<td>11.2</td>
<td>11.6</td>
<td>12.1</td>
<td>12.6</td>
<td>13.1</td>
<td>13.7</td>
</tr>
<tr>
<td>50</td>
<td>14+</td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Survey Group of 12 Year Old Pupils

<table>
<thead>
<tr>
<th>READING AGE</th>
<th>GROUP 1: From applied/practical mathematics background</th>
<th>GROUP 2: From traditional, rote learning type of mathematics background</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.5 and below</td>
<td>% of total sample 11</td>
<td>% of total sample 12</td>
</tr>
<tr>
<td>above 9.5 below 11</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>11 years</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>12 and above</td>
<td>17</td>
<td>17</td>
</tr>
<tr>
<td>TOTAL</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>
The groups were chosen according to reading age, as a means of obtaining children in the sample who were of equal ability. Work at Sheffield City Polytechnic (1976, Usability of Mathematics Textbooks) indicated that children's mathematical ability can be directly related to their reading ages.