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Predicting Corporate Bankruptcy: Whither do We Stand?

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Abstract

Following triggered corporate bankruptcies, an increasing number of prediction models have emerged since 1960s. This study provides a critical analysis of methodologies and empirical findings of applications of these models across 10 different countries. The study’s empirical exercise finds that predictive accuracies of different corporate bankruptcy prediction models are, generally, comparable. Artificially Intelligent Expert System (AIES) models perform marginally better than statistical and theoretical models. Overall, use of Multiple Discriminant Analysis (MDA) dominates the research followed by logit models. Study deduces useful observations and recommendations for future research in this field.

JEL classification: G33; C49; C88

Keywords: Corporate bankruptcy; Prediction models; Distress diagnosis; Corporate financial distress; Financial ratio analysis

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1. Introduction and Research Motive

Prediction of corporate bankruptcy is a phenomenon of increasing interest to investors/creditors, borrowing firms, and governments alike. Timely identification of firms’ impending failure is indeed desirable. One major focus of Basel II regulations is also to minimize credit risk. Global economies have become cautious of risks involved in corporations’ liabilities, especially after the demise of giant organizations like WorldCom and Enron.

It is vital to develop means and ways to identify potentially bankrupt firms. The models used to predict corporate bankruptcy are based on univariate analysis, multiple discriminant analysis (MDA), linear probability analysis, logit analysis, probit analysis, cumulative sums (CUSUM) methodology, partial adjustment process, recursively partitioned decision trees, case-based reasoning, neural networks, genetic algorithms, rough sets, entropy theory, gambler’s ruin theory, cash management theory, credit risk theories, and some other techniques. These methods of corporate bankruptcy prediction have their own strengths and weaknesses and, hence, choosing a particular model may not be straightforward. This study aims at providing a one-window shopping facility to potential users of bankruptcy prediction models. It presents a critical comparison of models’ methodologies and their empirical applications with a view to improve future research in this area. To that end, this study divides corporate bankruptcy prediction models into three broad categories: statistical models, Artificially Intelligent Expert System (AIES) models, and theoretic models.

Previous such attempts, albeit useful in many respects, are still deficient in a number of ways. Scott (1981), for example, is out of date and limited in its coverage, although it presents an admirable review of both statistical and theoretical models of
corporate bankruptcy prediction. Zavgren (1983) is limited to statistical models only without any reference to theoretic approaches to bankruptcy prediction. Altman (1984) offers an interesting appraisal of business failure prediction models attempted outside USA, which included ten countries. The study, however, focuses mainly on one particular type of statistical models. Jones (1987) summarises recent research on corporate bankruptcy prediction with an aim to provide a comprehensive guide to prediction methodologies. The study fails, however, to adequately discuss theoretic models. Keasy and Watson (1991) indicate managerial uses and limitations of bankruptcy prediction models in the context of decision usefulness. Narrow in focus, the study discusses very limited types of statistical models. Dimitras et al. (1996) successfully review various methods of constructing bankruptcy prediction models with a particular emphasis to include more recent models. More comprehensive than previous offerings, their study ignores theoretic models altogether. Moreover, the review framework could possibly be improved further. Overall, Morris (1998) provides the most comprehensive review of to date bankruptcy prediction models.

The book offers a very useful discussion on many important prediction techniques and their empirical use. It, however, lacks a deserved discussion on some important artificially intelligent expert system models. A few important theoretic developments emerged afterwards have not been included, too. Focus of Zhang et al. (1999) is to present a general framework for understanding the role of neural networks in bankruptcy prediction. Authors comprehensively review empirical applications of neural networks in the domain of bankruptcy prediction. The study, however, makes no reference to other types of models. Crouhy et al. (2000) exceptionally reviews the current credit risk models. Covering the most important theoretic models of bankruptcy prediction, the study does not discuss other type of models. Given the
exclusive objective of comparative analysis of current credit risk models, one may not look forward to have other models discussed in this study.

Thus, past surveys lack comprehensiveness. Particularly, no study provides a comprehensive critical comparison of different approaches towards bankruptcy prediction. This study critically analyses basic methodologies of different models of corporate bankruptcy prediction and notes that all three approaches are comparable in terms of their predictive powers. The study maintains this hypothesis following an empirical verification procedure in which it provides an ample comparison of empirical findings and common attributes of past prediction studies. For the purpose of methodological and empirical understanding of corporate bankruptcy literature, this study consults more than 180 sources. Major proportion of these sources comes from the journal articles followed by textbooks, and some web references. To analyse empirical applications of corporate bankruptcy prediction models, this work benefits from 89 studies.

The paper is organized as follows. Section 2 presents a brief understanding of methodological underpinning of these models. Section 3 provides a critical appraisal of the models discussed. Applications of corporate bankruptcy prediction are analysed and discussed in section 4. Study concludes in section 5 with some recommendations for further research.

2. **A methodological briefing on corporate bankruptcy prediction models**

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1 Some studies employ more than one prediction techniques. We count such a study for as many numbers of times as the techniques used in it, towards a total of 89 [This is also the approach followed by Dimitras et al. (1996)]. This study is based on a sample of 89 empirical studies in the field. Only these and some other studies quoted in the paper are listed in the reference section to save the space. A complete list of references is available from authors upon request.
To date attempts of corporate bankruptcy prediction have primarily used balance sheet information as likely symptoms of firm failure. Others have constructed models by looking at the causes of failure of corporations, the qualitative explanations of bankruptcy. Both strands of research have resulted into a number of prediction methods and models. In fact, almost all the models aim to predict corporate bankruptcy (the dependent variable) in a multivariate fashion. The only exceptions are the models constructed in the era before late 1960s, which did the job in a univariate manner.

This section briefly discusses methodologies of more commonly used prediction models, which are loosely classified into three broad categories: statistical models, artificially intelligent expert system models, and theoretic models. The first two categories look at the symptoms of failure, while the last considers causes of failure only.

2.1 Statistical models

The statistical models include univariate and multivariate analyses of which the latter dominates and uses multiple discriminant, linear probability, logit, and probit models.

2.1.1 Univariate analysis

Univariate analysis is a traditional method of interpreting financial statements using firms’ financial ratios. These ratios serve as explanatory variables or the bankruptcy predictors, which are likely to exhibit significant differences across the failing and non-failing firms. The nature of analysis is, however, univariate in the sense that the variables are observed and examined one after the other. There is no allowance for an analysis capturing an integrated effect of any two or more variables.
together on financial health of the firm. After a careful analysis of these ratios, researchers would provide certain inferences about firms’ financial health.\textsuperscript{2}

2.1.2 Multiple Discriminant Analysis (MDA)

The discriminant analysis is a type of multivariate technique that allows to differentiate between two or more groups of objects with respect to several variables simultaneously. MDA is used to classify an observation (the firm here) into one of several a priori groupings (the bankrupt and non-bankrupt, in our case) dependent upon the observation’s individual characteristics.

Under usual assumptions of regression analysis\textsuperscript{3}, the MDA model is a linear combination of the discriminatory variables of the following form:

\[
Z = \alpha + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_n X_n \]

where \(Z\) is a transformed value (score) of \([A]\) used to classify the object, \(\alpha\) is a constant, \(\beta_i\) are discriminant coefficients, and \(X_i\) are values of independent discriminatory variables.

Due to the nature of \(Z\) that is actually a resultant score of linear combination of X variables in \([A]\), estimates of discriminant coefficients are obtained following a specialized discriminant model estimation procedure. The classification typically involves defining some notion of distance between the case and each group centroids with the case being classified into the closest group. The results are, usually, presented in a classification matrix (also called accuracy matrix), which is often used to test the accuracy of the classification procedure too. The percentage of the known cases, which are correctly classified, is an additional measure of group differences. As

\textsuperscript{2} To further understand univariate analysis, see Altman (1993) and Morris (1998)

\textsuperscript{3} (1) The assumption of full rank, (2) Equality of variance, and (3) Normal distribution.
a direct measure of predictive accuracy, this percentage is the most intuitive measure of discrimination and can be used to test the power of classification procedure.

As with any inferential technique based on sample data, the percent correct prediction overestimates the power of the classification procedure. A remedy is to use a hold out sample. One can validate the classification procedure by randomly splitting the sample into two subsets. One subset is used to derive the function and the other to test the classification.4

2.1.3 Linear Probability Model (LPM)

To fix the idea, let us start by considering the following model:

\[
Y_i = \beta_1 + \beta_2 X_i + \mu_i \tag{B}
\]

Where,

\[X_i: \text{ the explanatory variable(s)}\]

\[Y_i = 1 \quad \text{if the event occurs (say firm fails)}\]

\[Y_i = 0 \quad \text{if the event does not occur (say the firm does not fail)}\]

Models like [B], which express the dichotomous \(Y_i\) as a linear function of the explanatory variable(s) \(X_i\), are called LPM because the conditional expectation of \(Y_i\) given \(X_i\), can be interpreted as the conditional probability that the event will occur given \(X_i\); that is, \(P( Y_i = 1 \mid X_i)\). Such a model can be estimated by using OLS technique, whereas variable \(Y_i\) follows a probability distribution in which probability must lie between 0 (when event does not occurs) and 1 (when event occurs). So, LPM models require that the conditional probability must lie between 0 and 1.

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4 For an enhanced discussion on MDA, see Klecka (1981), Altman (1993) and Morris (1998)
In application of LPM to bankruptcy prediction, a boundary value has to be found that will distinguish between those failing and non-failing firms in the population. Minimising the classification errors does this. LPM coefficients are used to construct performance scores for firms. Alternatively, the LPM scores may be interpreted as probabilities of failure.\(^5\)

2.1.4 Logit model

Under logit, the dichotomous dependent variable is simply the logarithm of the odds that a particular event (fail/non-fail) will occur. That is, here modelling of the ‘log odds’ of belonging to a group is pursued, rather than modelling the group membership itself.

Although it would be possible to model the odds, it is simpler to model the log (natural log, \(\ln\)) of the odds \([\ln(\text{odds}) = \ln(P / 1-P)]\). This transformation into natural log, allows the dependent variable to take any value between negative infinity and positive infinity. In this way, the dependent variable becomes continuous too, rather than discrete. Now, \(B\) can be written in the logistic regression functional form as:

\[
\ln(P/(1-P)) = \beta_1 + \beta_2 X_i + \mu_i \tag{C}
\]

Hence, the probability that an event may occur, failure of firm in this case, is given by:

\[
P = \frac{1}{1 + e^{-(\beta_1 + \beta_2 X_i)}} \tag{D}
\]

[D] is estimated using Maximum Likelihood method. Assuming that 0 indicates bankruptcy, the greater the resulting decimal fraction is above 0.5 (which implies an

equal chance of a company being a failure or non-failure), the less chance there is of the subject firm going bankrupt.\(^6\)

2.1.5 Probit model

In principal, one could substitute the normal cumulative distribution function in place of logistic into \([D]\) and get the resulting probit model to be estimated by Maximum Likelihood method. Rest of the interpretations remain the same as in case of logit.\(^7\)

2.1.6 Cumulative Sums (CUSUM) procedure

CUSUM procedures are among the most powerful tools for detecting a shift from a good quality distribution to a bad quality distribution. They are a set of sequential procedures based on likelihood ratios for detecting a shift in a process. For many common distributions, the CUSUM procedure reduces to calculating cumulative sums, hence the name CUSUM.

A CUSUM model determines, in an optimal manner, the starting point of the shift and provides a signal of the firm’s deteriorating state as early as possible soon after the shift occurs. A time series behaviour of the attribute variables for each of the failed and non-failed firm is described by a finite order VAR model. Based on sequential probability ratio tests and the theory of optimal stopping rules, the CUSUM model provides a signal of the firm’s deteriorating condition.

According to the CUSUM model, the overall performance of a given point in time is assessed by the cumulative (dynamic) time-series performance score of a firm. For as long as the firm’s annual (static) time-series performance scores are positive

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\(^6\) For further details on logit, see Maddala (1983), Theodossiou (1991), Gujarati (1998), and Morris (1998).

\(^7\) For further details on probit, see Maddala (1983), Theodossiou (1991), Gujarati (1998), and Morris (1998).
and greater than a specific sensitivity parameter, the CUSUM score is set to zero indicating no change in the firm’s financial condition. Converse signals for the firm’s changed condition.\textsuperscript{8}

2.1.7 Partial adjustment process

Partial adjustment models are a theoretic rationale of famous Koyck approach to estimate distributed-lag models. Application of partial adjustment model in bankruptcy prediction can best be explained by using cash management behaviour of the firms as an example.

According to Laitinen and Laitinen (1998), cash management refers to the management of cash from the time it starts its transit to the firm until it leaves the firm in payments. Failure of the cash management can be defined as an imbalance between cash inflows and outflows. This leads to failure usually defined as the inability of the firm to pay its financial obligations as they mature.

Traditionally, cash management behaviour of a firm is described by different models of demand for money, e.g., the quantity theory of demand for money, which assumes that the demand for money does not differ from the demand for any funds in the firm. The most popular and simple approach to the demand for money in this framework is that followed by the inventory cash management approach, where demand for money by a firm is assumed to depend on the volume of transactions. The idea may be summarised as follows.

The actual cash balance of a firm in period $t$ is a multiplicative function of $S$ and $i$ as follows:

\[
\ln(M(t)) = \ln(D) + e_i \ln(S(t)) + e_i \ln(i(t)) + u(t) \]

\[E\]

\textsuperscript{8}For more insight on CUSUM, see Page (1954), Healy (1987), and Kahya & Theodossiou (1999).
Where;

\[ \ln: \text{ natural logarithm} \]

\[ M(t): \text{ actual cash balance in period t} \]

\[ D: \text{ a scale constant} \]

\[ S(t): \text{ the volume of transactions} \]

\[ i(t): \text{ the opportunity cost} \]

\[ e_c: \text{ the elasticity of cash balance with respect to } S \]

\[ e_i: \text{ the elasticity of cash balance with respect to } i \]

\[ u(t): \text{ a random error variable with standard autoregressive property} \]

Equation [E] is static in nature whose dynamic version presented in partial adjustment form is as below:

\[ \ln M(t) = y\{\ln D + e_c \ln S(t) + e_i \ln i(t) + u(t)\} + (1 - y)M(t - 1) + yu(t) \cdots \cdots \{F\} \]

where \( y \) and \( (1-y) \) are the weights representing adjustment rate.

The overall classification and prediction process, in this particular example of partial adjustment model, follows the following criterion:

- For a failing firm, absolute values of the elasticities of cash balance with respect to the motive factors (volume of transactions and the opportunity cost here) will be smaller than for a similar healthy firm
- For a failing firm, the rate of adjustment \( y \) may be even greater than unity and will certainly exceed the rate for healthy firm
- Validity of the results can be tested by any appropriate technique like Lachenbruch procedure\(^9\)

2.2 *Artificially Intelligent Expert System (AIES) models*

Initially considered numeric machines, it was later realized that computers can also process symbols to exhibit the intelligent behaviour of humans’ cognitive activities like problem solving. This realization triggered a search for programs that could emulate human cognitive skills in an acceptable way. Hence, a body of knowledge dealing with designing and implementation of such programs started to emerge since 1950s. Since this ‘intelligence’ of computers is contained in machines, and not in human brains, their exhibited behaviour is known as ‘Artificial Intelligence’ (AI).

Humans use their intelligence to solve problems by applying reasoning based on the knowledge possessed in their brains. Hence, knowledge plays the pivotal role in human intelligence. AI, in order to be as competitive as human intelligence or at least comparable, should benefit from similar knowledge in application of its reasoning to the problem posed. Expert systems (ES) were developed to serve this purpose for AI.

An ES initiates from the process of transferring knowledge, which is considered to be ‘the bottleneck problem’ of ES. Two automation processes have dominated research in the field of knowledge acquisition: ‘machine teaching’ and ‘machine learning’, of which latter has assumed more significance than former.

‘Learning’ may be considered as a system capable of improving its performance on a problem as a function of previous experience. A machine may learn under strict or no supervision, yet moderate supervision is observed more in practice.
Subsequent research resulted into a variety of supervised machine learning methods, which proved quite successful in solving problems for different domains, including bankruptcy prediction. Following discussion provides a basic understanding of most commonly used techniques and their application in bankruptcy prediction.

2.2.1 Recursively partitioned decision trees (Inductive learning model)

One form of supervised learning is inductive learning. An inductive learning program is able to learn from examples by a process of generalization. Many human experts also learn in this way. Decision trees are one way of inductive learning. A decision tree partitions a training data set into sub-classes. Procedure then proceeds to recursively replace each of the subset with a decision tree, resulting into a final decision tree for the initial training set.

Friedman (1977) first introduced recursive partitioning decision rule for nonparametric classification. As suggested by Pompe and Feelders (1997), ‘the basic idea of recursive partitioning is to fit a tree to the training sample by successively splitting it into increasingly homogeneous subsets until the leaf nodes contain only cases from a single class or some other reasonable stopping criterion applies’ (pp. 270).

In bankruptcy classification, the decision tree is constructed by recursively partitioning the training sample until the final nodes of tree contain firms of only one type: bankrupt or healthy. Any new object (firm) is then classified according to the place of final node it falls in the tree. This node identifies the firm’s group membership and associated probability.

2.2.2 Case-Based Reasoning (CBR) model

Like human experts, CBR solves a new classification problem with the help of previously solved cases in the same domain of knowledge. A case, in the context of
CBR, would consist of a contextual knowledge that represented an experience. Usually, a CBR process of knowledge acquisition would pass through four stages: (1) identification, acceptance and representation of a new problem, (2) retrieval of old similar cases from the case library, (3) adapting the cases retrieved in step 2 in a way that they fit to the new situation and provide an appropriate solution to it, and (4) evaluation of the suggested solution and finally storing the evaluated solution in the case library for future use.

In the context of corporate bankruptcy prediction, a CBR program would first develop a case library of previously solved prediction problems. It would, then identify, accept, and represent any new prediction problem. Next, it would adapt a similar case retrieved from the case library to appropriately fit the new problem and provide prediction result. Before storing this solution in the case library, a CBR program would also evaluate the suggested prediction result.\footnote{See Kolodner (1993) for deeper understanding of CBR.}

2.2.3 Neural Networks (NN)

Although capable of outperforming human brain in basic arithmetic calculations, computers are certainly inferior when it comes to tasks involving symbolic recognition like signs of bankruptcy in a firm. Neural networks are enthused by biological works related to brain and its nervous system to triumph over this lack of computational efficiency in computers. Neural networks perform the classification task, in response to impending signals of financial health of a firm, in the way a brain would do for example in deciding whether the food is salty or sweet by its taste signal.

Human brain is made up of certain types of neurons (nerve cells), which is the base of neuroscience. Neurons, in neural networks, are called ‘processing elements’ or
‘nodes’. Like real neurons, these nodes are connected to each other through ‘weighted interconnections’ (synapses in neuroscience terms). Nodes are organized in layers. Each node takes delivery of, joins, and converts input signals into a single output signal via weighted interconnections. This output signal is accepted as the classifying decision if it satisfies the researcher; otherwise it is transmitted again as an input signal to many other nodes (possibly including itself). Process keeps going until satisfaction is gained from researchers’ point of view.

Perhaps the major task of any neural network is to determine appropriate weights to interconnections of different nodes. Neural networks perform this task by a training process in which knowledge about the relationship between input and output signals is learned following certain principle. This knowledge produces a distinct structure of nodes (in one of the network layers called ‘hidden layer’) and connection weights, which correctly classifies the objects into their respective known groups. Technically, this process of mapping is termed as ‘convergence’. Following a mathematical theorem, the network is always able to converge.

While predicting corporate bankruptcy, NN would take information on explanatory variables at input nodes via input layer. The hidden layer nodes, connected to input nodes through weighted interconnections, collect and process this information to suggest a probability of a firm getting failed or succeeded.11

2.2.4 Genetic Algorithms (GA)

Based on the idea of genetic inheritance and Darwinian theory of natural evolution (survival of the fittest), GAs work as a stochastic search technique. GAs

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11 For further information on NN, see Salchenberger et al. (1992), Coats & Fant (1993), and Yang et al. 1999).
perform their search for optimal solution to the problem posed from a large and complicated space of solutions.

GAs are usually explained with the help of vocabulary, inevitably, borrowed from natural genetics. Each individual potential candidate solution to the problem is represented by a ‘string’ (also called ‘chromosome’, ‘genotype’ or ‘structure’). These ‘strings’ are made of ‘units’ (also called ‘genes’, ‘features’, ‘characters’, or ‘decoders’). Under GAs, an evolution process is run on a population of ‘strings’ that corresponds to a search through a space of potential solutions.

GAs execute this search process in three phases: genetic representation & initialisation, selection, and genetic operation (crossover and mutation). Genetic representation that is normally in binary alphabet (0 and 1) creates an initial population of solutions. After the initialisation, each string is evaluated with the help of a user-defined fitness function. Over time, such a selection process is likely to result into best performing strings only. Straightforward reproduction of selected strings entails no benefit in terms of exploration of solution space, as this will only reproduce the identical off springs from the parent strings. Genetic operations of Crossover and Mutation are introduced for this purpose. The process continues until the actual population converges towards increasingly homogeneous strings. In general, the process is stopped when we are satisfied with a certain level of homogeneity.

In order to solve a classification problem like bankruptcy, researchers extract a set of rules or conditions using GAs. These conditions are associated with certain cut off points. Based on these conditions, the model would predict whether or not a firm is likely to go bankrupt.12

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2.2.5 Rough sets models

The central quandary of rough sets theory is classification. Theory aims at complete classification of objects to a specified category with the help of information on these objects that is factually inadequate. Hence, this indiscernible or imprecise information about the objects to be classified, is the mathematical basis of rough sets theory. A set of all indiscernible objects is labelled ‘elementary set’, which is the universe of objects. A set of objects consisting of elements that are union of some elementary sets is called crisp (or precise). Otherwise the set is known as rough (or imprecise) set.

In a rough set model, inadequate knowledge about the objects is presented in the form of an information table. Rows, columns, and entries of the table are respectively called ‘objects’, ‘attributes’, and ‘attribute values’. This information table can also be considered a decision table containing sets of condition and decision attributes. The decision table is used to derive the decision rules of the model. These rules are derived on the basis of inductive learning principles and are the end result of rough sets model. Every new object is classified by matching their characteristics with the set of derived rules.

In its application to the case of corporate bankruptcy prediction, a rough set model collects and presents the available information on firms to be classified as bankrupt or healthy in an information table. Following inductive learning principle, the model generates a set of rules that help determine the actual group membership of the firms.13

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13 For more details on rough sets, see Pawlak (1982), Ziarko (1993) and Dimitras et al. (1999).
2.3 Theoretic models

Focus of statistical and AIES models is on firms’ symptoms of failure, rather than causes. These models are able to predict bankruptcy by looking at distress conditions present in the firms. However, another way of approaching this problem is to look at the factors that force corporations to go bankrupt. Under this approach, prediction models are constructed based on some theoretic arguments. Quite a few attempts have been made in this respect and are briefly described in this section.

2.3.1 Balance Sheet Decomposition Measure (BSDM) / Entropy theory

One way of identifying firms’ financial distress could be a careful look at the changes occurring in their balance sheets. Following this procedure, the argument would tag along this guideline: “like any enterprise, firms would tend to maintain a state of equilibrium that ensures sustaining existing firms’ structure”. If a firm’s financial statements reflect significant changes in their balance sheet composition of assets and liabilities over a reasonable period of time, it is more likely that the firms are incapable of maintaining the equilibrium state. Since these changes are likely to become uncontrollable in future, one can foresee financial distress in these firms. This economic rationale of firms’ likely failure is the argument of BSDM or entropy theory.14

2.3.2 Gambler’s Ruin theory

The basic idea of this theory relates with the game of a gambler, who plays with an arbitrary sum of money. Gambler would play with some probabilities of gain and loss. Game would continue until the gambler loses all his money. Theory would also talk about gambler’s ultimate ruin and expected duration of the game.

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14 For further understanding of BSDM, refer to Theil (1969), Lev (1973), and Booth (1983).
In context of the firm’s failure, firm would take the place of a gambler. Firm would continue to operate until its net worth goes to zero, point where it would go bankrupt. The theory assumes that firm has got some given amount of capital in cash, which would keep entering or exiting the firm on random basis depending on firm’s operations. In any given period, the firm would experience either positive or negative cash flow. Over a run of periods, there is one possible composite probability that cash flow will be always negative. Such a situation would lead the firm to declare bankruptcy, as it has gone out of cash. Hence, under this approach, the firm remains solvent as long as its net worth is greater than zero. This net worth is calculated from the liquidation value of stockholders’ equity.15

2.3.3 Cash management theory

Short-term management of corporate cash balances is a major concern of every firm. Cash or funds flow statements of the firms report this cash management function of corporations, particularly from 1980s. An imbalance between cash inflows and outflows would mean failure of cash management function of the firm. Persistence of such an imbalance may cause financial distress to the firm and, hence, bankruptcy.

2.3.4 Credit risk theories

Credit risk theories, closely related to Basel I and Basel II accords, mostly refer to the financial firm. The proposed Basel II framework consists of three pillars: (1) minimum capital requirements, currently set equal to 8%, according to a purposely-defined capital ratio, (2) supervisory review of an institution’s internal assessment process and capital adequacy, (3) effective use of public disclosure to strengthen market discipline as a complement to supervisory efforts.

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15 See Scott (1981) and Morris (1998) for more details on gambler’s ruin theory.
The current Basel II Accord utilises concept of a capital ratio that is calculated dividing bank’s capital amount by a measure of risk faced by it (referred to risk-weighted assets). There is a wide variety of risks faced by banks and other financial institutions these days including credit risk, market risk, operational risk, investment risk, interest rate risk, exchange rate risk, concentration risk and country transfer risk. Basel II focuses mainly on the first three of these with a view that other risks are implicitly covered. Basel II framework adequately treats both market risk (that results due to trading activities) and the operational risk (defined as the risk of losses due to inadequate or failed internal processes, people and systems, or external events). However, the Accord clearly recognises that, for most banks, it is the credit risk that matters more. Focus of our study is also limited to credit risk only, for it is related to counterparty failure (the borrowing firm).

As noted by Westgaard and Wijst (2001), credit risk is the risk that a borrower/counterparty will default, i.e., fail to repay an amount owed to the bank. Credit risk includes all of the counterparties and reasons for which they may default on their obligations to repay. Following Basel II guidelines, in the last few years, a number of attempts have been made to develop internal assessment models to measure credit risk. A few of them have gained more respect than others including JP Morgan’s CreditMetrics, Moody’s KMV model, CSFP’s CreditRisk+ and McKinsey’s CreditPortfolio View. More importantly, with one or two exceptions, these models and risk predictions thereof have been based on either microeconomic or macroeconomic theories of corporate finance. Collectively these models may be referred as credit risk theories.

The most famous microeconomic theory is related to the theory of option pricing as suggested by Black and Scholes (1973) and later developed by Merton
An option is a security that gives the holder a right to execute a transaction (to buy or sell an asset) in future at a price determined today. Options are of two types: a call option gives the right to buy, whereas the put option means the right to sell. Options are used in many instances including speculation, hedging a borrowing, capital preservation, covered call etc. A simple example is a call option on a common stock, in which the payout on the call is determined solely by the value of the stock. Excess of stock price over the strike price determines the payout to holder who will exercise the call. In the opposite case, payout will be zero and the holder will not exercise his right. Right pricing or valuation of the options is important. Black and Scholes presented a complete general equilibrium theory of option pricing that constructed a valuation formula, which is based on observable variables. Both Black & Scholes and Merton recognize that their approach could be applied in developing a pricing theory for corporate liabilities in general. They determine the option value as the solution of a partial differential equation to which the price of any option must conform, subject to boundary conditions given by the form of the payout. Under this asset value option pricing approach, firms’ default process is endogenously related to its capital structure. Firm would default on its obligations to the bank, if the value of its assets falls below certain critical level determined by the respective credit risk model. Option pricing theory is also the base of JP Morgan’s CreditMetrics and Moody’s KMV models.

An example of macroeconomic theory is the one that relates to credit portfolio risk measurement that was introduced by Wilson (1997a, 1997b, 1998). The theory states that credit cycles follow business cycles closely, i.e., a worsening economy would be followed by downgrades and defaults increase. Here default probability of a firm is a function of macroeconomic variables like unemployment rate, interest rates,
growth rate, government expenses, foreign exchange rates, and aggregate savings etc. This theory also serves as the base for McKinsey’s CreditPortfolio View model.

3. A critical analysis of corporate bankruptcy prediction models

3.1 A critique to statistical models

Univariate analysis of financial ratios was, initially, the approach followed by researchers like Beaver (1966). One critical assumption of this approach is that there exists a proportionate relationship between the variables in numerator and denominator of the ratio being calculated. However, as noted by Whittington (1980) and Keasey & Watson (1991), this assumption is very likely to violate on two grounds: (1) the relationship between the two variables may be non-linear resulting into non-proportionate outcome, (2) a constant term may also play some role in the relationship between two variables of the ratio under study, which will prevent proportionality to exist. Moreover, univariate analysis emphasises on individual signals of firms’ impending distress and hence classification can take place for only one ratio at a time. As noted by Zavgren (1983) and Altman (1993), ratio analysis in such a univariate fashion is susceptible to faulty interpretation and is potentially confusing. Of course, financial status of a firm depends on multidimensional factors, and no single ratio may be capable to depict all these together.

Flawed with such limitations, univariate analysis was later replaced by multivariate analysis. Of these multivariate techniques, multiple discriminant analysis (MDA) has been on use quite extensively, starting from Altman (1968). MDA is neither a flawless model. It works on the assumptions that the group dispersion (variance-covariance) matrices are equal for failed and non-failed firms, and the population must be distributed in a multivariate fashion. Many studies, including Karles and Prakash (1987) have shown that these assumptions are often violated by
the data set under study. Non-random sampling of distressed and non-distressed firms also creates biasness in results [Lin and Piesse (2001)]. In all, MDA works on very demanding assumptions, some of which are often violated in practice.

In search for a bankruptcy prediction model with lesser demanding assumptions, researchers suggested use of condition probability models like LPM, logit, and probit. LPM rests upon a number of assumptions that are usually not met. For example, error term is not normally distributed and is heteroskedastic. Further, it will generally produce lower measures of goodness of fit and there remains a possibility of value of dependent variable lying outside the 0-1 ranges [Gujarati (1998)].

Problems, with which LPM is beset by, can be overcome by selecting a probability function that follows cumulative distribution like that of logit or probit. Many have preferred to use logistic over probit merely for practical ease. Both logit and probit perform best when the sample size is large. Unfortunately, number of bankrupt firms is usually not large enough to make these models an optimum choice. Small sample size usually restricts use of logit or probit models in practice [Stone and Rasp (1991)]. Their results are also affected when the number of predictors is very large and the variables are continuous [Morris (1998)]. Moreover, Logit and probit models are comparatively difficult in computational terms than MDA.

There have been some attempts to employ time series framework under CUSUM and partial adjustment models. The major problem faced by these models is to employ a reasonable length of time series. These models might be subject to econometric limitations like very short length of available time series in case of bankruptcy data. Additionally, these have failed to get an encouraging response from academicians and practitioners so far.
3.2 A critique to AIES models

Artificially Intelligent Expert System (AIES) models are also subject to certain limitations. For example, Inductive Learning model (recursively partitioned decision trees) is a forward selection method that is liable to reconsidering a currently analysed variable at some later stage too. It is also exposed to the problem of over fitting [Dimitras et al. (1996)].

Some AIES models, like case based reasoning (CBR), are still at the stage of infancy in their life. Such models require a lot of improvements. For example, CBR lacks a convincing methodology of interviewing human experts and collecting cases. Index selection in CBR is still a problem to be addressed. Solutions provided by the CBR are built-in with the help of previously solved problems. However, deriving truly creative solutions requires studying further the process of brainstorming in human experts. Optimal size of cases to be represented, accommodating continues case situations, and their connectivity also counts towards CBR limitations [Kolodner (1993)].

Despite a number of studies advocating usefulness of Neural Networks (NN), there are flaws in these models too. As noted by Shin and Lee (2002), finding an appropriate NN model to reflect problem characteristics is not an easy job. It is because there are a number of network topologies, learning methods and parameters. Most importantly, NNs are characterized as ‘black boxes’ due to inability of the users to readily comprehend the final rules acquired by NNs to solve the problem. Additionally, Altman and Varetto (1994) note that long processing time to complete the NN training stage, requirement of having a large number of tests to identify appropriate NN structure, and the problem of over fitting can considerably limit the use of NNs.
Genetic Algorithm (GA) models are also in the process of development. Major problem of GAs, identified by Shapiro (2002), is that they are difficult to tune and have no convergence criteria. Another important shortcoming of GAs is that there is no pre-defined way of including constraints into GAs [Aickelin and Dowsland (2003)]. This particular problem does not make GAs readily amenable to most real world optimisation problems.

Finally, Rough set models don’t perform well with numeric data set. Theory requires conversion of numeric data into non-numeric form before it can be used [Mak and Munakata (2002)]. Basic disadvantages of rough sets, as noted by Yasdi (1995), are: high noise sensitivity, multimodality, and lack of performance-oriented fitting to task requirements.

3.3 A critique to theoretic models

Both statistical and AIES models were built without any theoretical base. Predicting corporate bankruptcy using a model without a theoretic support has long been questioned. Researchers have, therefore, tried to explain the failure process of firms with the help of some theories as discussed in previous section. This section presents a brief discussion on limitations of such theories and models constructed thereof.

Balance Sheet Decomposition Measure (BSDM) or entropy theory is characterized with a major flaw in it: it focuses only on the change in balance sheet structure not caring for the direction of this change. This fact limits the theory to distinguish between a firm whose balance sheet changes are not due to failure but due to growth. Booth and Hutchinson (1989) have also found this limitation in an empirical work. Moreover, some researchers, including Moyer (1977), concluded from their studies that BSDM is not a useful predictor of bankruptcy.
The simplest version of gambler’s ruin model assumes that firm has no access to external capital in order to finance its losses. However, as noted by Scott (1981), attempts to apply this model have been disappointing. Obviously, firms do have at least an imperfect access to external capital market as suggested by Scott (1981). Although model suggested by Scott overcomes the flaw present in simple gambler’s ruin model, no one has attempted to use this method in practice.

Cash management theories do provide a reasonable explanation of firm failure, yet this is not the only cause of distress. Many other significant predictors may still remain un-captured, if only cash flow variables are assumed to be significant. Particularly, firm’s stock and equity may have some important role to play as suggested by credit risk theories.

Study discusses four models representing credit risk theories. KMV has strongly criticised the use of transition probabilities by CreditMetrics, which is based on average historical frequencies of defaults and credit migration. As observed by Crouhy et al. (2000), KMV objects on the two critical assumptions of CreditMetrics: (1) all firms within the same rating class have the same default rate, and (2) actual default rate is equal to the historical average default rate. KMV considers this cannot be true since default rates are continuous, while ratings are adjusted in discrete manner. KMV has proved, through a simulation exercise, that the historical average default rate and transition probabilities can deviate significantly from the actual rates. Moreover, Derviz and Kadlecakova (2001) observe that assumption of default free deterministic interest rates makes the model insensitive to market risk and underlying changes in economic environment. They also note that the model proxies asset returns correlations by equity return correlations, and this might lead to an imprecise estimation.
On the other hand, KMV model is considered to be a too capital simplistic structure of the firm, as noted by Derviz and Kadlcakova (2001). They also consider that the assumption of high diversification may not necessarily meet in real world, and this may misrepresent the need of economic capital. Finally, they note that the relationship between Distance to Default and EDF is based on US data, and their derivation is not thoroughly explained. Therefore, straightforward implementation of the model, outside USA, might be questionable. Crouhy et al. (2000) observe that KMV assumes no market risk and fails to deal with non-linear products like foreign currency swaps.

Major drawbacks of CreditRisk+, as observed by Crouhy et al. (2000), are assumption of no market risk and inability to deal with non-linear products. Derviz and Kadlcakova (2001) state another limitation of the model that relates to the specification of default rates for individual obligors. Specification of these default rates is quite ambiguous, despite the fact they enter the model as basic input.

Crouhy (2001) consider that CreditPortfolioView model necessitates reliable default data for each country, and possibly for each industry sector within each country. This is, obviously, not an easy job to do. They also criticise the ad-hoc procedure to adjust the migration matrix. Derviz and Kadlcakova (2001) view the dependence of default on macroeconomic factors, as an assumption too strong. After all, microeconomic factors do play a role in default and credit quality migration too.

3.4 Authors’ note

A careful critical analysis of different methods and models of corporate bankruptcy prediction leaves an impression that, in effect, these models are not much different from each other. Historically, researchers first suggested the use of statistical models. Availability of computers and technological advancements, particularly since
1980s, motivated some to invent technology-oriented models. Artificially Intelligent Expert System (AIES) models, for example, emerged as an alternative to classical statistical models in use for long. They were the result of technological advancement used to transform human intelligence in computers. Since human intelligence was initially inspired by conventional statistical techniques, AIES models employed the characteristics of both univariate and multivariate methodologies. Hence, broadly speaking, AIES models may be considered an automated offspring of statistical approach. They, however, appear to be more sophisticated. Models built on theoretic grounds do not necessarily look at the modelling technique first. Rather, they would try to model the argument usually by employing an appropriate available statistical technique. So, even theoretic models seem to have benefited from statistical techniques at large. Therefore, role of statistical models within theoretic approach cannot be ignored either.

The fact that statistical techniques stand somewhere within all types of corporate bankruptcy prediction models and are in use for decades now, one may expect to see use of statistical models more often in applications to the case of bankruptcy prediction. Within this category, however, we are indifferent between MDA, logit and probit models as regards to their predictive performance. MDA should remain comparable, despite very demanding assumptions, as logit and probit continue to face the problem of small sample size within bankruptcy domain. LPM is unlikely to be of great use due to its unrealistic assumptions. Time series models like CUSUM and partial adjustment are improbable to produce encouraging results, as the data set is unlikely to be large enough.

AIES models may also prove useful and comparable to statistical models. It is not surprising, as they are developed using human intelligence that had learnt problem
solving mainly with the help of statistical techniques. Although neural network approach seems more appealing, an empirical exercise only can provide a definite answer as to which AIES model might do the prediction job better.

Theoretic models have a completely different route to follow. Their predictive use may remain limited, as the theory under consideration might have overlooked some other possible causes of firm failure. However, this also necessitates a quantitative verification.

Owing to the verity that almost all the models of corporate bankruptcy prediction are more or less dependent on statistical approach, we expect that their predictive accuracies should broadly remain comparable. Being in use for the longest period, it is also reasonable to assume that previous applications of prediction models to the case of bankruptcy prediction should have benefited more from statistical approach. An equally important concern would be to know which particular individual model provides best prediction results. All these questions invite for an empirical investigation. Undertaking an empirical work to answer these and other similar questions of interest is definitely a challenging task. There have been a large number of empirical applications of these models to the case of corporate bankruptcy prediction. This paper accepts the challenge and provides an empirical analysis of such a widespread literature. Major goal of the next section is to provide quantitative answers to these and many other attention-grabbing questions.
5. Applications of corporate bankruptcy prediction models

To undertake the empirical exercise, study benefits from a total of 46 major applications of prediction models to the case of corporate bankruptcies (43 journal articles, 1 technical report, 1 discussion paper, and 1 departmental document). Table 1 reports almost all the critical information from these studies. It refers to the models used in previous research, which happen to be 89 in 46 studies. Table reports only best predictive accuracy rates of the models, one year before failure, to keep the analysis consistent and simple. The abbreviations used in Table 1 are explained in Appendix given in the end.

A careful look at the attributes presented in Table 1 reveals quite interesting results. For example, a large number of journals seem interested in this area of research. However, ‘Journal of Business Finance and Accounting’ takes a lead by publishing roughly 16% of analysed papers. ‘European Journal of Operational Research’ stands second by publishing 8% of the studies. ‘Financial Management’ and ‘Expert Systems with Applications’ follow next. Future research may take this finding as a loose index to locate the journals in this area of research.

Predictive results of any empirical work value more in the presence of a holdout sample. However, only 46% of the total studies used a holdout or test sample of firms to verify their predictive claims. Such a weakness in past research warns future research in this area to recognize the importance of holdout sample.

Problem of small sample size has always been a predestined limitation of application of these models to the case of corporate bankruptcy prediction. Table 1 confirms this too. Although the estimation sample size in these studies ranges from 32 to 35287 numbers of firms, about 42% studies worked with a sample of only less than
100 firms. This inevitable constraint suggests that future research may not be criticised much on this particular account.

Conventionally, bankruptcy prediction studies have used financial ratios to predict failure in firms. This fact is also evident from Table 1, where more than 60% studies use only financial ratios as explanatory variables. About 7% studies work with cash flow information. Remaining studies employ a mix of financial ratios and other variables. These studies happen to use a wide range of financial ratios including the ones measuring liquidity, solvency, leverage, profitability, asset composition, firm size, growth etc. Other variables of interest include information on macroeconomic, industry specific, location or spatial, and firm specific variables. These findings re-emphasize the importance of information on company accounts. However, we would suggest using a mix of variables possibly in proportion to their use in past studies.

Bankruptcy being more common in public firms and relatively easy access to the required data, almost all the studies work on data sets of public limited companies. Further, most researchers tend to work on a sample of mix industry firms. Around 43% studies construct their empirical analysis on the data of mix industries. Manufacturing sector ranks second with 25% share, which includes occasional enclosure of retail or mining industry. Limitation of small sample size and finding of the study in favour of mix industry, it may prove useful for future research to work with mix industry sample.
Table 1
(Summary of Previous Research Attributes and Findings)

<table>
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<th>No.</th>
<th>Author &amp; Year</th>
<th>Model</th>
<th>OPA (%)</th>
<th>TypeI (%)</th>
<th>TypeII (%)</th>
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One major focus of this section is to trace past trend of bankruptcy prediction studies with respect to methodological approaches being followed. Figure 1 is obtained from the information contained in Table 1. Figure indicates that 64% studies prefer to use statistical models, followed by AIES and theoretic models with respective shares of 25% and 11%. This is in line with study’s expectations, as use of AIES models for bankruptcy prediction is relatively new. Moreover, historically, most practitioners find it useful to predict bankruptcies by looking at only the symptoms of firm failure. Lesser inclination of practitioners towards theoretic models explains its lower representation in past studies.

It may also prove useful to see which of the individual models is used more frequently. Figure 2 (constructed from Table 1) shows that more than 30% studies use MDA model for bankruptcy prediction, while another 21% prefer logit model. Both models belong to statistical models group and make up 77% share of the statistical models. This fact suggests that other type of statistical models could not attract the attention of many researchers. Within AIES models, neural networks ranks first with 9% share followed by recursive portioning. Of theoretic models, entropy (BSDM)
theory is employed the most with a share of 4.5%. These results suggest that, to many classical researchers, MDA may still remain a preferred model in future.

Figure 2 (Proportion of models employed by past studies)

Figure 3 summarises the average overall predictive accuracies of these models, one year before actual bankruptcy. Almost all the models are capable of successfully predicting firm’s financial health achieving a collective average of more than 85% predictive accuracy rate. Individually, the gambler’s ruin theory seems to perform better with an accuracy rate of 94%. Yet, it is important to recall from Figure 2 that gambler’s ruin constitute only 1.12% of total studies. Higher predictive accuracy rates of rough sets, credit risk models, probit and genetic algorithms may also be questionable on similar account. These models need further applications to establish their apparent rankings. On this ground, performance of MDA and Logit is more notable as their accuracies are 86% and 87%, respectively. Apparently, these results support the use of MDA and logit models in applications of bankruptcy predictions.
Do notable predictive accuracies of MDA and logit, as individual models, suggest that statistical approach is the preferred one? Figure 4 provides a ‘No’ answer to this question. Evidently, this is the AIES approach that provides overall best predictive accuracy rates of 88%. Surprisingly, even theoretic approach seems to perform slightly better than statistical approach. Given that theoretic models constitute only 11% of the total studies analysed here, their performance may rather be considered comparable to other two approaches. These results indicate that future research may benefit more from AIES models, should the approach overcome its major weaknesses.
Sample of the study covers data set of 10 countries in all. Of these, USA dominates clearly as more than 47% studies work on US data set. UK follows next with 18% studies. Collectively, European data set constitutes 42.7% share of the studies analysed. Asian data set makes up the remaining 10.1%. This particular finding indicates that US and European data sets are more accommodative for research on corporate bankruptcy prediction (see Figure 5).

Many researchers would assess a model’s predictive power by the misclassification rates committed by the models. Type I error is the one that measures number of failed firms that are classified as non-failed. This type of misclassification is considered to be very costly to the lenders. Figure 6 presents average Type I error rates of studies analysed in present research. Cash management model seems to have committed the most errors, which is 26%. Both MDA and logit, the most frequently used models, report 15% error rates. However, NN model also remains comparable by committing 17% error. The least errors are observed in case of LPM. Yet, it constitutes only 3.4% of analysed studies. Same argument applies to lesser error rates of GA and RS. If type I error is taken as the criterion, one may feel more comfortable with either the statistical (MDA or logit) or AIES (NN) models.
On the other hand, classifying non-failed firms as failed is a Type II error. Less dangerous than Type I, it is still important to classify healthy firms as healthy. Figure 7 reports Type II errors of these models. Again cash management model shows an average of 35% misclassifications, the largest of all. MDA and Logit are still comparable with 12 and 10% error rates, respectively. NN performs much better than MDA and logit with an error rate of only 6%. No misclassification in case of LPM and low rates of credit, GA and RS may better be assessed in terms of their insignificant contribution towards the analytical sample of study. Type II error rates continue to support the use of MDA, logit and NN models.
As a final observation, table reflects that the studies cover a publication period of 1968 to 2003. Despite a dedicated effort of more than 35 years, research community still tends to disagree as to what particular approach or model is more useful for the case of corporate bankruptcy prediction. Major analytical finding of this section, that all the approaches are broadly comparable, may help reduce this tension and bridge-up the gap.

5. **Conclusion and recommendations**

Corporate bankruptcy is certainly not desirable and an early detection of impending distress in a corporation is always enviable. Identification of financially distressed firms and taking corrective measures is better than protection under bankruptcy law.

Realizing the significance of prediction of corporate bankruptcy, a wide variety of models have been developed and empirically tested. These methods and models are based on statistical techniques, artificial intelligence, or theoretic arguments. In their own domain of model development, they have all done the job well. However, there remain substantial disagreements on underlying methodologies of these broad groups of models. Within each category, one can observe variable degrees of differences among alternative models.

Despite the availability of so many prediction models, search for best distress prediction models is still in progress. This study provides a critical analysis of more commonly used corporate bankruptcy prediction models under three approaches: statistical, AIES, and theoretic. The study notes that all the models seem comparable in terms of their predictive powers. It maintains this hypothesis following an
empirical analysis of past applications of these models to the case of bankruptcy prediction. Major findings of the study are:

- Study indicates that past attempts of corporate bankruptcy prediction have primarily used statistical models, particularly MDA and logit. Individually, MDA has been used most frequently, followed by logit. These findings are not surprising, as statistical models are in use for a longer time period. AIES approach is relatively new, whereas practitioners do not seem interested much in theoretic approach.

- Overall, AIES approach reflects marginally better predictive accuracies than statistical or theoretic approaches. This is desirable, as AIES models are an automated development over classical statistical models. However, superiority of this approach becomes questionable when it comes to predictive powers of individual models. On this account, MDA and logit models (statistical approach) provide consistently better predictive accuracies. Reported low average Type I & II error rates also advocate using MDA, logit or NN models in future research.

- It is still not common, to almost half the researchers discussed in present study, to use a holdout sample for validation of their results. This trend should not be encouraged in future research, as prediction results value more in presence of a holdout sample.

- Past research on bankruptcy prediction has, usually, employed a relatively small sample size. The fact that it is nearly inevitable to work with small sample size, imminent research may not be over criticised on this basis.

- Past studies have largely worked with financial ratios as explanatory variables. Information on cash flow and other variables had played a relatively little role
in prediction task. We would suggest using a mix of these variables, possibly in proportion to their representation in past studies.

- Study finds that past research, at large, has been working on mix industry. In presence of finding of the study and problem of small sample size, future research might be more useful under mix industry data set.

- Another understanding of the study is that US and European data sets are generally more accommodative for research in this area. Hence, future work may continue to benefit mainly from similar countries’ data sets.

- Finally, paper finds that a large number of journals publish research on this area with ‘Journal of Business Finance & Accounting’ taking the lead. Study’s finding on the frequency of different publishing journals may serve as a crude base, when journal ranking in this area is the target objective.

Based on the observations of this study, it seems logical to admit that almost all models of corporate bankruptcy prediction are capable of doing their job. Usefulness of a particular model depends on the particular research objective. Observations and recommendations presented in this study are likely to play a guiding role, while using these models in future research.

It is also hoped that the brief introduction to methodological details of these models, presented in this paper, would be of great use to those with recent interest in this field. It may also serve as a quick refresher to those who are already engaged in this area of research. For some, it may still bring a few new methodologies in light.
References


## Appendix
(List of Abbreviations used in the Study)

<table>
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<tr>
<th>Abbreviation</th>
<th>Description</th>
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<tr>
<td>BSDM</td>
<td>Balance Sheet Decomposition Measure (Entropy Theory)</td>
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<tr>
<td>Cash</td>
<td>Cash Management Theory</td>
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<tr>
<td>CBR</td>
<td>Case-Based Reasoning</td>
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<td>Cash Flow</td>
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<td>Const.</td>
<td>Construction</td>
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<td>Credit</td>
<td>Credit Risk Theories (including ‘Option Pricing’ and ‘Macroeconomic’ theories</td>
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<td>Recursive Partitioning (Decision Tree) Analysis</td>
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