Elastic constants of elastomers and their effect on engineering applications

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ELASTIC CONSTANTS OF ELASTOMERS AND
THEIR EFFECT ON ENGINEERING APPLICATIONS

Submission for the Degree of Doctor of Philosophy
of the
Loughborough University of Technology

by
B.P. Holownia Dip.Tech (1st Class Honours)
M.Sc. (Birmingham)
C.Eng. M I Mech E
December 1985
FOREWORD

The author's long standing interest in mechanical properties of elastomers, in particular Poisson's ratio and bulk modulus, is reflected in a number of published papers on aspects of theoretical and experimental work associated with these elastic constants.

This compilation brings together the results of ten particular studies which have been central to the author's work on mechanical properties of elastomers. The presentation includes original technical reports in a sequence which reflects the logical development of the subject matter.

Some of the original publications have been enlarged to A4 size in order to make the binding of this thesis easier.
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1. INTRODUCTION

The author's interest in the field of elastomers stemmed initially from his experience in the general field of stress analysis of metals.

One of the significant differences of elastomers, when compared with other materials, is that the value of Poisson's ratio $v$ is close to 0.5. This causes difficulties in the analysis because the general stress-strain equations of elasticity contain terms such as $\frac{vE}{(1+v)(1-2v)}$ which approaches infinity when $v$ approaches a value of 0.5. Pure natural gum has a value of 0.49989. The addition of fillers will lower this value slightly.

The published work included in this thesis starts with a fundamental approach of stress distribution in bonded blocks for ideally incompressible materials. This was followed by finite difference analysis of cylindrical rubber blocks bonded to metal plates, and the effect of varying $v$ on the stress distribution within the blocks. This was carried out for thin blocks with $D/h > 9$ and thick blocks with $D/h < 6$. The results show big changes in stress distribution even when the value of $v$ is only altered in a third significant figure.

Following the theoretical analysis which showed a significant effect of $v$ on the stress distribution within bonded rubber blocks, an experimental technique was devised to measure $v$ to an accuracy of $\pm 0.0001$. This was done indirectly by measuring Young's modulus $E$ and bulk modulus $K$, a value of $v$ was calculated using $E = 3K (1-2v)$. The measurement of these elastic constants was carried out on different types of rubbers by varying both the amounts of filler and the particle size of the carbon black.
Attention was also turned to an engineering application, making use of the high bulk modulus of rubbers in the field of metal powder compaction prior to sintering. A provisional patent was taken out on this original idea which is described in Chapter 4.

An interesting engineering application was the investigation of temperature build-up in cylindrical bonded rubber blocks subjected to cyclic loading. Good agreement was achieved between the theoretical and experimental results. The value of Poisson's ratio used in the theoretical treatment was an important factor in accurate prediction of temperature build-up.

Another engineering application was the theoretical prediction of stiffness on cylindrical rubber blocks with varying coefficients of friction on the faces of the rubber.

Chapter 8 compares Finite Difference technique used by the author in previous papers with the well-known Finite Element method. The paper considers a cylindrical bonded rubber block with various values of $v$, and points out the large error [$\approx 100\%$] obtained using Finite Element for an axisymmetric problem when $v$ approaches 0.5.

Finally, an experimental technique using holographic interferometry was developed to measure the dynamic bulk modulus of elastomers up to a frequency of 200 Hz. Subsequently further development of this method was sponsored by the Ministry of Defence and, with further refinement, the frequency range was extended up to 1000 Hz.
The author through his publications gained world-wide recognition in the field of elastomers and recently had an invitation (Appendix I) to be a speaker at an International conference taking place in India in November 1986.
Other papers on related topics of holographic measurements in engineering application

by

B P HOLOWNIA


2. COMPRESSION BONDED RUBBER BLOCKS

Journal of Strain Analysis Vol 6 No 2 1971 p 121

A simplified theoretical treatment for an incompressible material presented by Lindley* in the booklet Engineering design with natural rubber, and further developed by Gent and Meinecke reference (4) for very small deflections, was extended further by the author.

Private communication with research workers reveal that a thin bonded rubber block subjected to tension will develop internal rupture when the strains are bigger than those predicted by Gent and Meinecke, as they have neglected the significant 'eh' term which has been included by the author. The 'eh' term has a large effect on the level of stress inside the rubber block, particularly for strains above 5%.

A closer agreement with experimental results was obtained with the present analysis.

* Engineering Design with Natural Rubber
N R Technical Bulletin M.R.P.R.A
by P B Lindley
Following the classical analysis for a completely incompressible material, the next paper considers real materials with Poisson's Ratio $\nu$ varying between 0.450 and 0.4999.

A numerical method of dynamic relaxation was employed to predict the stress distribution within bonded rubber blocks subjected to compression loading. There are numerous applications of bonded rubber blocks in engineering, i.e., bridge bearings, engine mountings, flexible drives, etc.

For the ease of comparison with other workers only axisymmetric bonded rubber blocks were examined in detail. The results reveal a big variation of stress within a rubber block and in particular attention was focussed on a great variation of stress with a slight change of Poisson's Ratio.

Up until now the term thick and thin rubber block was very loosely defined. A strict definition of thin block is when the stress distribution within the block is hydrostatic (i.e., equal stress in three directions).

In real bonded blocks the stress within a block is never perfectly hydrostatic. The author defines thin blocks as those with the stress variation between the normal stresses at any point in three directions to be within 10%, and this occurs with cylindrical blocks when $D/h > 6$. 

Dynamic relaxation is an iterative method and the computer time increases rapidly when $v$ approaches 0.5. The highest value of $v$, taken was 0.4999.

The stress distribution with increasing value of $v$ approaches that given by the author in his previous paper.
DISCUSSION OF PAPER (3a) BY DR V.E. GOUGH


The previous paper (3a) had a favourable review as a Communication in the Journal of Strain Analysis by Dr Gough who had a considerable practical and theoretical background in the general field of rubbers.

His comments on internal rupture and fatigue are confirmed by the theoretical results obtained by the author.
Following the theoretical analysis demonstrating a great change in stress value for a very small change of $\nu$, experimental work was undertaken to measure $\nu$ to an accuracy of four decimal places.

This paper describes an original approach in determining the value of Poisson's Ratio by measuring the static bulk modulus and Young's modulus for a given elastomer. The report covers extensive experimental work on four different elastomers with varying amounts of carbon black. It shows that even with a heavy filled rubber up to 120 parts of Carbon black per 100 parts of rubber, only a small change in $\nu$ occurs.

Judging by the amount of correspondence with the author, this publication caused a considerable interest in research and industrial establishments.

Twelve months later Rubber Chemistry and Technology asked the author for permission to reprint the paper in full in their Journal. Both papers are included here.
4b EFFECT OF CARBON BLACK ON POISSON'S RATIO OF ELASTOMERS


The following is the reprint of the previous paper which was printed twelve months later in the Journal of Rubber Chemistry and Technology.
Having measured experimentally the bulk modulus of rubbers, the author was inspired by the incompressibility of rubbers to investigate the use of this property in the field of metal powder compaction.

After developing the original idea described below, a provisional patent was taken out and a small amount of money was received from SRC to further improve the technique.

Many components are made by compacting fine particles of metal powder in a shaped mould. The fragile compacted articles are then placed in a high temperature oven where fusion of the metal particles produces a solid metal component with good surface finish and dimensional accuracy.

No machining is necessary for most components. Typical components produced in this way are simple bushes, gears, shafts and numerous other small articles.

There are a number of advantages in using powder compaction in producing engineering products.

(i) No waste material
(ii) No machining
(iii) Simple material handling.

The aim of the exercise was to reduce the ejection forces following the high compaction forces necessary to produce a 25 mm diameter cylindrical billets. The ejection force in a normal die was in the region of 5 tons.
With the use of the elastomer to balance the compressive force as shown in Figure 1 the ejection force was reduced to almost zero, i.e. the component was pushed out with a finger.

Several firms were approached with this idea, but after enthusiastic welcome of the method, they declined to put up the necessary funds to develop this idea for commercial use.
FIGURE I. Metal powder compaction of 25mm diameter cylindrical billets
Having established the variation of elastic constants with different amounts of carbon black for a number of different elastomers, an obvious next step was to measure the elastic constants using different types of carbon blacks.

The same method as described in previous paper was employed to determine the values of $E$, $K$ and $\nu$.

Four different types of carbon blacks were considered using Natural Rubber RSS 1 (NR) as base with varying the amounts of carbon black content between 0 and 120 parts per 100 parts of rubber. This meant that a large number of specimens had to be prepared and tested which took a long time, and therefore this paper was not published until 1980.
6. TEMPERATURE BUILDUP IN BONDED RUBBER BLOCKS DUE TO Hysteresis


There are numerous engineering applications where a rubber block undergoes a cyclic loading resulting in a temperature rise within the block due to hysteresis.

Temperature rise in rubber tyres has always been a problem. Other devices such as rubber suspension units, anti-vibration devices and flexible couplings may have the same problem depending on the application.

The build-up of temperature depends on the type of rubber, shape of rubber, frequency and intensity of stress cycling. In most isolation devices there is no problem of excessive rise in temperature.

In flexible drive systems such as propeller shaft on a ship, the design has to be carefully studied so that some damping is present in the coupling to isolate the propeller vibrations from the engine, but too much damping will cause undesirable temperature rise in the coupling, resulting in the breakdown of rubber structure.

At first the results of temperature distribution within the rubber block were different to what was expected until a close look was taken at the strain energy equation.

Poisson's Ratio $\nu$ again has played a very big part in the heat generation, particularly in thin rubber blocks. It was found that, although for thin blocks the stress levels were very high, leading to an expectation of high heat generation, the stresses were almost
hydrostatic, and hence the normal stress terms in equation (4) almost cancelled each other when $v \approx 0.5$. The strain energy was therefore mainly dependent on the shear stress distribution being much higher towards the outer bonded edge.

The graphs Figure 4.5 in the paper verify the above, showing higher temperature towards the outer edge than in the centre in a thin rubber block. This discovery plays an important part in the design of flexible couplings.
7. RUBBER HYSTERESIS IN BIAXIAL AND TRIAXIAL LOADINGS


Theoretical and experimental work was undertaken into the viscoelastic behaviour of rubber under biaxial and triaxial loading. This work was presented as a paper at a Bob Payne Symposium held at Loughborough University in 1978 and subsequently published in a book form by Applied Science Publishers entitled "Elastomers: Criteria for Engineering Design".

An interesting outcome of this work was that the percentage hysteresis loss seem to depend on the deformed shape of the rubber specimen rather than the stress levels.

The experimental results show that the biaxial and uniaxial stress loading gives the same percentage hysteresis loss within the experimental accuracy.

The triaxial stress loading showed that for all rubbers tested varying in carbon black content between 0 and 120 pph the hysteresis loss was the same at about 5% level. This small amount was attributed to the backlash and friction in the equipment used rather than a true hysteresis loss.
8. STIFFNESS OF RUBBER BLOCKS IN CONTACT WITH DIFFERENT SURFACES

All the work up to this time dealt with cylindrical rubber blocks bonded to metal plates. However there are a number of engineering applications where the rubber blocks are placed between two surfaces and used in compression without any bonding on the surfaces.

The resulting stress distribution depends on the friction coefficient between the rubber and the surface in contact.

Theoretical study was undertaken to examine the stress distribution within the cylindrical rubber blocks subjected to compression and in contact with surfaces having different friction coefficients.

The resulting stress distribution was similar to a bonded rubber cylinder but the magnitude of stress varied depending on the friction factor used on the surfaces in contact with rubber. The stiffness of the rubber blocks was obtained from the stress distribution within the block.
9. COMPARISON BETWEEN FINITE ELEMENT AND FINITE DIFFERENCE METHODS FOR STRESS ANALYSIS OF ELASTOMERS

Plastics and Rubber Processing and Application (1985)

In the last 20 years or so Finite Element method has developed into a very comprehensive and sophisticated tool for solving stress/strain problems. It developed in the Civil Engineering field dealing with frame works and then was extended to continuous systems, for the solution of two and three dimensional problems.

The amount of man-hours spent on developing the Finite Element packages is enormous, however there still exists a problem with Finite Element method when the material has a Poisson's Ratio close to 0.5. Most of the Finite Element packages advise not to use the package for $v > 0.45$.

Having used Finite difference technique on many different problems, it was decided to compare Finite Difference with Finite Element for a simple axisymmetric bonded rubber cylinder for $v$ varying between 0.45 to 0.4999.

Pafec 75 Finite Element package available at Loughborough University of Technology was used and compared with the Finite Difference method called "Dynamic Relaxation" which was used by the author on many different problems.

The paper shows that for $v$ up to 0.4990 both methods give similar results, but for higher values of $v$ the Finite Element gives increasingly higher stress over-estimate such that for $v = 0.4999$ the stress over-estimate is in the order of 100%.
Having established a reliable experimental method for the measurement of elastic constants of elastomers under static conditions, the author moved on to the measurement of these properties under dynamic conditions.

The dynamic value of Young’s modulus has been measured by many authors to determine the dynamic stiffness of rubber blocks, however very few workers have attempted to measure the dynamic bulk modulus, and most of them have used indirect method of determining the bulk modulus of elastomers.

The authors interest in holographic application in engineering measurements has inspired the idea of using this to measure a volume change of a rubber sphere under pressure. Since elastomers are nearly incompressible in bulk, the method required a very small measurement of the contraction of rubber sphere under pressure, which made the holographic interferometry technique ideal for this purpose.

While the author was developing this idea, he was approached by the Ministry of Defence Department (M O D) who were interested in the measurement of dynamic bulk modulus of elastomers. A small research grant was obtained to look at the feasibility of the method. Subsequently, with encouraging results, the grant was extended for another year and the method was gradually improved. However, due to the limits of the apparatus used the frequency was limited to 200 Hz.
The resulting paper was published in the Rubber Chemistry and Technology journal with MOD permission. The results were so encouraging that a bigger grant for another two years was obtained from MOD in order to extend the frequency range up to 1000 Hz.

To the author's knowledge no-one else has tried to use holographic interferometry technique for the measurement of bulk modulus of elastomers.
11. MEASUREMENT OF DYNAMIC BULK MODULUS AND PHASE ANGLE USING ESPI

Accepted for publication in the journal of Rubber Chemistry and Technology in 1986.

The bigger research grant for another two years has allowed for a research worker (A Rowland) to improve the technique further under the author's supervision.

It was decided from the start that Electronic Speckle Pattern Interferometry (ESPI) system should be used which has a big advantage over holography giving instantaneous live fringes. The method was further developed using cylindrical rubber specimens and an attempt was made to measure the phase change in bulk compression. The success of this is reported in the following paper.

The engineering application of the bulk modulus of elastomers in the underwater research by MOD is confidential and the author has dealt only with the fundamental aspects of measurement.

A letter from the Editor of Rubber Chemistry and Technology together with referees' comments is included in Appendix II as proof of acceptance for publication.
APPENDIX I
It is our great pleasure to inform you that the Indian Institute of Technology, Kharagpur in collaboration with different industries and research institutes will organise an International Rubber Conference on 7-8 November, 1986 at Jamshedpur, an industrial town, about 200 km away from Calcutta.

On behalf of the Organising Committee, I would like to extend an invitation to you to deliver an invited lecture. We would consider it a great honour if you are able to accept our invitation.

Invited lectures are exempt from payment of registration fees and they will be provided with free local hospitality at Jamshedpur. Unfortunately, due to very tight budgetary restrictions, we are unable to pay for your travel.

The lecture is scheduled as a special lecture of 45 minutes duration which includes 5 minutes for discussion. The topic on which we would like you to speak may cover any of the following general areas: (a) Thermoplastic elastomers; (b) Adhesion, composites and failure; (c) Rubber-rubber and rubber-plastic blends; (d) Engineering applications of rubber and rubber-like materials.

We propose that the first circular, due in May, 1985, and the successive second & third circulars will contain names of the invited speakers.

We realise that you must have an extremely busy schedule but we would appreciate it if you could inform your acceptance by April 20, 1985, in order to assist us in formulating our programme.

Awaiting your early reply,

Yours sincerely,

(S.K. De)
Secretary
Organising Committee

To
Dr. B. P. Holownia
Dept. of Mechanical Engg.
Loughborough University of Technology
Loughborough, Leicestershire
England
APPENDIX II
September 25, 1985

Mr. B. P. Holownia
Department of Mechanical Engineering
University of Natal
King George V Avenue
Durban 4001
Republic of South Africa

Dear Mr. Holownia:

Enclosed are the comments of the reviewer(s) who have read your manuscript, "Measurement of Dynamic Bulk Modulus and Phase Angle Using E.S.P.I." (250-85). Please make the minor revisions indicated and submit two copies of the modified manuscript to my attention. With these changes, your paper will be accepted for publication in RUBBER CHEMISTRY AND TECHNOLOGY. Additionally, if you have not done so, please include the original drawings of your figures.

Sincerely,

Gary R. Hamed
Editor

Note: Figures are unsuitable and must be redrawn before publication.
B. P. Holownia, A. C. Rowland

Measurement of Dynamic Bulk Modulus and Phase Angle Using ESIPI

This is an excellent paper. It is built on a solid background of previous work in the field. It is based on original and innovative technique, with successful results. There is a good review of possible source of error, and the magnitude of error to be expected. The authors also have suggestions for further improving the technique. I have noted some grammatical and typing errors in the text. Otherwise, I would recommend the article without changes.

In my opinion, this manuscript should be published:____ without revision ______ with major revision
____ with minor revision ______ not at all

[Signature]
B B Holownia & A C Reward

Measurement of Dynamic Bulk Modulus and Phase Angles Using ESPR

The paper presents a technique for measurement of the dynamic modulus of elastomers using electronic specific pattern interferometry to measure the volume contraction of rubber. It offers advantages over the previous technique which used holographic interferometry. The cylindrical sample offers advantages over the spherical sample used in an earlier publication by the author.

I feel that the paper would appeal to readers of R&BT interested in bulk modulus measurement. The phase angle measurements are inconclusive.

A few typos were noticed on the manuscript.

Please return this manuscript for publication as follows:

_____ with minor revision  _____ with major revision
_____ without revision  _____ not at all