Highly redundant and fault tolerant actuator system: control, condition monitoring and experimental validation

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Highly-redundant and fault tolerant actuator system: control; condition monitoring; and experimental validation

by

Hasmawati Ponding @ Antong

A Doctoral Thesis Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

May 2017

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Abstract

This thesis is concerned with developing, a control and condition monitoring system for a class of fault tolerant actuators with high levels of redundancy. The High Redundancy Actuator (HRA) is a concept inspired by biomimetics that aims to provide fault tolerance using relatively large numbers of actuation elements which are assembled in parallel and series configurations to form a single actuator. Each actuation element provides a small contribution to the overall force and displacement of the system. Since the capability of each actuation element is small, the effect of faults within an individual element of the overall system is also small. Hence, the HRA will gracefully degrade instead of going from fully functional to total failure in the presence of faults.

Previous research on an HRA using electromechanical technology has focused on a relatively low number of actuation elements (i.e. 4 elements), which were controlled with multiple loop control methods. The objective of this thesis is to expand upon this, by considering an HRA with a larger number of actuation elements (i.e. 12 elements).

First, a mathematical model of a general n-by-m HRA is derived from first principles. This model can be used to represent any size of electromechanical HRA with actuation elements arranged in a matrix form. Then, a mathematical model of a 4-by-3 HRA is obtained from the general n-by-m model and verified experimentally using the HRA test rig. This actuator model is then used as a foundation for the controller design and condition monitoring development.

For control design, two classical control method-based controllers are compared with an \( \mathcal{H}_\infty \) approach. The objective for the control design is to
make the HRA track a position demand signal in both healthy and faulty conditions. For the classical PI control design, the first approach uses twelve local controllers (1 per actuator) and the second uses only a single global controller. For the $\mathcal{H}_\infty$ control design, a mixed sensitivity method that utilizes the primary, control and complementary sensitivity functions is used to obtain good tracking performance and robustness to modelling uncertainties. Both of these methods demonstrate good tracking performance, with a slower response in the presence of faults. As expected, the $\mathcal{H}_\infty$ control method’s robustness to modelling uncertainties, results in a smaller performance degradation in the presence of faults, compared with the classical designs.

Unlike previous work, the thesis also makes a novel contribution to condition monitoring of HRA. The proposed algorithm does not require the use of multiple sensors. The condition monitoring scheme is based on least-squares parameter estimation and fuzzy logic inference. The least-squares parameter estimation estimates the physical parameters of the electromechanical actuator based on input-output data collected from real-time experiments, while the fuzzy logic inference determines the health condition of the actuator based on the estimated physical parameters.

Hence, overall, a new approach to both control and monitoring of an HRA is proposed and demonstrated on a twelve elements HRA test rig.
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Chapter 1

Introduction

1.1 Problem Statement

Modern engineering technologies depend strongly on complex control systems to meet their performance aims/goals. Over the past few decades, control systems have become widely used in manufacturing, automotive and aerospace industries, and critical infrastructures. In the aerospace industry, for example, Flight Control System (FCS) that consists of sensors, actuators and digital flight control computers (as the system’s core) has long been used to provide aircraft control and envelope protection in pitch, roll, and yaw axes [1,2]. This control system makes a key contribution to determining the efficiency and safety of the aircraft operation. In a safety critical system like this, unexpected faults in components can be catastrophic not only to the system itself but also potentially to humans within its vicinity. In a non-safety critical system, faults can have a profound economic impact, increasing down-time and life-cycle costs. In response to this, engineers are working on designing fault-tolerant systems that can withstand component malfunctions to maintain desirable performance and stability properties [3]. Hence, fault-tolerant system design has developed into a major area of research in the past three decades.

Sensors and actuators are, of course, vital components in any control system. Sensors and actuators provide a mean to measure and then act upon the system, thereby allowing control of the system. Sensors perform a
CHAPTER 1. INTRODUCTION

wide variety of functions, including measuring temperature, motion, force, acceleration and many other aspects of the physical world in real time. Actuators, on the other hand, convert energy into motion. In most applications, sensors will send feedback to the system and cause the actuators to perform desired tasks. For example, a sensor could provide input to the controller which drives an actuator that causes a robotic arm to perform a spot welding task on a vehicle in an automobile assembly plant [4].

Since actuators and sensors are key to the operation of a control system, it is important to keep the actuators and sensors at the required reliability to ensure the system can achieve the required performance. A common method of achieving the required reliability of these instruments has been to employ some redundancy. Examples of components with multiple redundancies are known in aircraft [5, 6] and nuclear power system [7].

In [5], it was mentioned that modern aircraft’s primary flight controls adopted quadruplex flight control computers and quadruplex actuators to ensure high reliability of the fly-by-wire systems. Similarly, according to [6], the Airbus fly-by-wire systems adopted five computers that are simultaneously active to compute the control law. In the event of faults, it is still possible to fly the aircraft safely with one single computer active.

In a nuclear power plant, the nuclear reaction inside the reactor is controlled by the insertion and removal of control rods into and from the reactor. According to [7], the insertion and removal of the control rods are achieved by a control system called control rods control system (CRCS), and it is crucial to maintain high reliability of the CRCS since its malfunction can cause an unexpected shutdown of the nuclear power plant. Therefore, the authors introduced hardware redundancy to the nuclear power system by building a duplex-CRCS to ensure high reliability of the control system.

Both sensor and actuator redundancy is achieved by connecting two or more of these devices in parallel as shown in Figure 1.1, and their output is combined through voting (for sensor redundancy) or mechanical consolidation (for actuator redundancy). Each actuator or sensor is capable of providing the necessary function if faults occur within one of the devices. This method works well for sensors, however, for actuators, over-actuation
1.2. HIGH REDUNDANCY ACTUATOR

increases size and cost and thus reduces the efficiency of the system. Parallel actuation redundancy is also problematic because it will be rendered useless in the presence of a lock-up fault that can effectively seize the whole actuation assembly. While the need for sensor redundancy can be reduced through analytical replication to provide fault tolerance, this strategy is not applicable to actuators because, whereas sensors deal with information, actuators involve energy conversion. Consequently, actuator redundancy is essential to achieve fault tolerance in the presence of actuator faults [8]. Hence, this research concentrates on demonstrating the concept of high redundancy actuator (HRA) to allow the system to gracefully degrade instead of going from fully functional to total failure in the presence of actuator faults.

Figure 1.1: Traditional sensor and actuator redundancy.

1.2 High Redundancy Actuator

The high redundancy concept, inspired by biomimetics, is an approach to fault tolerant actuation that aims to provide fault tolerance using a relatively large numbers of actuation elements as shown in Figure 1.2. The elements are arranged both in series and parallel to form a single actuator, and each element provides only a small contribution to the required force and travel of the system. Since the capability of each element is small, the effect of faults within individual elements on the overall system is also small, and faults in elements can be intrinsically accommodated.

An HRA reduces the problems suffered by parallel redundancy actuation by reducing the need for over-engineering. For example, an HRA may em-
ploy 100 actuation elements, but the specified operation may only require 70. Thus, the high redundancy actuator, in this case, is only 30% over-engineered in comparison to a triplex or quadruplex configuration which is 200% and 300% over-engineered.

Issues with a lock-up fault that can make the parallel redundancy actuator useless are resolved with the use of serial actuation elements. Due to the relatively small contribution of individual actuation element on the overall performance of the system, the HRA will gracefully degrade when one or more of the elements are faulty, unlike the parallel redundancy actuator where the system may change from fully functional to total failure in a short period.

Figure 1.2: Example of high redundancy actuator.

1.3 Research Motivation

The concept of HRA was proposed previously by colleagues in the Control Systems Research Group at Loughborough University [9–30].

The work in [9] investigates the feasibility of the HRA through a relatively low number of actuation elements (4 elements) which are controlled through a passive fault tolerant control methodology. It aimed to demonstrate the concept of HRA using four electromechanical actuation elements as well as designing and testing two types of control method: classical control; and Linear Quadratic Gaussian (LQG) control in the healthy and faulty conditions.

Both simulation and experimental results show that both the classical and LQG control method can be used to control the HRA with different
1.3. RESEARCH MOTIVATION

tolerant capability on the types of faults injected. There are three types of actuator faults considered in the study: open-circuit, short-circuit and lock-up fault [11].

Other work on HRA is described in [14]. The research work modelled, controlled and monitored an HRA based on electromagnetic actuators with 16 actuation elements. The element model for a moving coil actuator is derived from first principles, verified experimentally and then used to form higher-order, non-linear HRA models for simulation.

For control design purposes, a reduced order representation of the model is used. Controller design involves passive fault tolerant control (PFTC) design, and multi-agent system inspired active fault tolerant control (AFTC) design for the HRA to achieve near-nominal performance under fault conditions.

Fault detection and health monitoring of the HRA is also explored in [14] to ensure that the HRA can be repaired as the degraded performance get close to critical capability level [15]. Two methods of fault detection and identification (FDI) methods were discussed in this work: a rule-based approach for use in the AFTC; and an interacting multiple-model method for health monitoring.

Based on the review done on the previous works related to HRA, it is found that further improvements can be made. HRA using electromechanical actuation technology has been realised but with a relatively low number of actuation elements (4 elements) [9]. The work in [14] realised an HRA using electromagnetic actuation technology with a higher number of actuation elements (16 elements) and designed an active FTC as well as condition monitoring scheme for the HRA. However, the condition monitoring schemes required feedback from each actuation elements, and this increased the need for sensors as the number of actuation elements increased. Also, the control and condition monitoring scheme studied and designed in [14] were not tested experimentally.

Therefore, the founding motivation of the work in this thesis is to expand upon the work in [9] by considering a 12 elements HRA based on electromechanical actuation and to explore different condition monitoring
methods that have been introduced in [14]. The work within this thesis also aims to explore a simpler control scheme than the one used in both of the previous works. The focus of the work discussed in this thesis is to show that we can build an HRA with a complex mechanical structure and optimised its performance using simpler control structure. Mean time between failure and reliability are not the focus of the work in this thesis but there has been some work related to them by other researchers in the Control Systems Research Group such as in [31–33].

The novel contributions of the work presented in this thesis are:

1. **Larger number of actuation elements**
   This work designed the HRA using 12 actuation elements based on electromechanical actuation technology. The work in [9] is based on the same actuation technology but with a relatively small number of actuation elements (i.e. 4 elements).

2. **Condition monitoring scheme**
   The condition monitoring scheme that is considered in this work does not need position/velocity feedback from each actuation element. A condition monitoring scheme that requires one position sensor and one current sensor to measure the overall position and current of the HRA is developed. Its efficacy is tested in simulation and on the experimental test rig. The work in [14] used condition monitoring scheme that required position feedback from each actuation elements. This increase the complexity of the system because the number of sensors needed increased as the number of actuation elements increased.

3. **Controller design**
   Classical controller and $H_\infty$ controller are designed to optimise the HRA performance under healthy and faulty conditions. The classical control scheme considered in this work is far simpler than the one used in [9] (i.e. no inner current loop is considered). $H_\infty$ control method has not been tested with HRA in either of the previous works, so the work within this thesis intends to test this control method. Performance of the two control scheme is compared.
1.4 Research Aim and Objectives

This research aims to demonstrate the concept of an HRA, control scheme and condition monitoring through a 12-elements actuation system based on the electromechanical actuator (EMA). The specific objectives of this research are:

1. to design, develop and test an HRA experimental rig that consists of 12 actuation elements.

2. to model the HRA in Simulink and to validate the model against the experimental rig; the model will then be used for controller and condition monitoring design.

3. to design and apply classical and $\mathcal{H}_\infty$ control scheme and compare their performance in both simulation and experimental environment.

4. to explore possible methods of condition monitoring that does not require multiple sensors and to develop, test and validate an approach for doing this.

1.5 Publications

The research work leading to this thesis has generated several papers as listed below:


### 1.6 Thesis Outline

As described in the previous section, this thesis presents the modelling of an HRA, the design of classical and $H_\infty$ controllers and the development of condition monitoring algorithm for the HRA. This section explains how the thesis is organised.

Chapter 2 gives an introduction to relevant key concepts such as fault tolerance, faults and failure, and parallel redundancy. Previous works on HRA concept is also summarised. An introduction to classical control and $H_\infty$ control design as well as the concept of condition monitoring are also discussed.

Chapter 3 describes the development of the HRA experimental rig used to conduct real-time experiments. The electrical components, mechanical components and data acquisition and hardware network are discussed in detail which includes the design of signal conditioning circuits for the posi-
tion and current sensors, and assembly of the actuation elements. The work presented in this chapter addresses the first research objective.

Chapter 4 is concerned with the derivation of a mathematical model for a single element actuator, multiple element actuators and the full HRA assembly. The element models are derived from first principles and verified experimentally. The model was then used as the foundation on which to design both the control and condition monitoring schemes. The second research objective is addressed in this chapter.

Chapter 5 discusses the classical controller design. Two types of control structures: local and global were designed and tested. For the local control structure, each actuation element has its controller that requires position feedback from each of the element, while the global control structure employs only one controller that takes total displacement as the feedback to control the whole assembly. Results of driving a single actuation element, 3 actuation elements in series and the full HRA assembly in the healthy and faulty conditions are presented and discussed. This chapter contributes half of the third research objective.

Chapter 6 contributes the second half of the third research objective. It explains the $\mathcal{H}_\infty$ controller design. The mixed sensitivity method using three sensitivity functions: primary sensitivity; control sensitivity; and complementary sensitivity was used to design the $\mathcal{H}_\infty$ controller. The primary sensitivity function was adjusted for good tracking performance. The control sensitivity is to limit the control action in the system while the complementary sensitivity function was designed so that the system is robust to modelling uncertainty. Robustness to modelling uncertainty is necessary because, as the health condition of the system changes from healthy to faulty the dynamics of the system will also change. Results of applying the designed $\mathcal{H}_\infty$ controller to 3 actuation elements in series and the full HRA assembly in the healthy and faulty conditions are presented and discussed.

Chapter 7 covers the development, implementation and test of a least-squares parameter estimation and fuzzy logic inference based condition monitoring algorithm. The least-squares parameter estimation combined discrete time parameter estimation with the key physical parameters (in-
ductance, resistance, back-emf constant, motor torque constant, inertia and friction) of the electromechanical actuator. The fuzzy logic inference uses the estimated physical parameters to determine the HRA’s health condition. Results of evaluating the condition monitoring algorithm, as the HRA’s health condition changes from healthy to faulty, are discussed. The fourth research objective is addressed in this chapter.

Chapter 8 provides a conclusion to the work described in this thesis, by reviewing the key findings from each chapter. It then outlines suggestions for possible extensions and improvements to the current state of the techniques.

The appendices contain material which is not central to the main thesis: appendix A contains nomenclature that consists of list of abbreviations and symbols used throughout the thesis; appendix B shows some parameter calculation; appendix C contains sample of MATLAB code used for $\mathcal{H}_\infty$ controller design; data sheets for the DC motor, linear actuator and sensors used in this work can be found in Appendix D.
Chapter 2

Literature Review

2.1 Introduction

The key elements of the work presented in this thesis are fault tolerance, HRA, controller design and condition monitoring design. Therefore, the literature review starts with an overview of the broader research publications related to fault tolerant systems, actuator redundancy, system identification, condition monitoring, classical controller design and $\mathcal{H}_\infty$ controller design. After that, previous work related to HRA is examined to form an understanding of what has been done before and develop some ideas for the experimental rig setup, controller design and condition monitoring scheme design that come later in this thesis.

2.2 Faults and Failure

A fault may be defined as an unpredictable defect that occurs in the system structure or parameter that eventually leads to degradation in closed-loop system performance. The literature presents various definitions for faults and failure. For example, in [34], a fault is defined as ‘...a deviation in the system structure or the system parameters from a nominal situation...’.

In [35], a fault is defined as ‘...a non-permitted deviation of a characteristic property (feature) of the system from the acceptable, usual, standard condition...’. Based on these definitions, a fault can be concluded as something
that causes the system to deviate from its usual or standard behaviour.

Failure, on the other hand, is defined as ‘...A failure is a permanent interruption of a system’s ability to perform a required function under specified operation conditions...’ [35]. Which means, a system fails if the faults cause the system to be unable to complete an expected task. The consequences of failures could include damage to the plant, its environment, or people in the vicinity of the plant [36]. According to [8], the term fault is used to denote malfunction rather than a catastrophe while the term failure is used to suggests complete breakdown.

The concept of fault and failure can be explained as shown in Figure 2.1 [14]. The nominal behaviour $b_{\text{nominal}}$ lies within a region of acceptable system behaviour where the system is operational. The presence of faults will change the behaviour of the system, and thus it will behave differently from the nominal behaviour. The faulty system behaviour can be located anywhere within the acceptable region or outside the acceptable region. If faulty system behaviour is within the acceptable region, the system is considered to be tolerant to these particular faults, represented by $b_{\text{faulty}}$. However, if faulty system behaviour is located outside the acceptable region, the system will be unable to complete its required tasks and the fault has led to system failure ($b_{\text{failure}}$).

Figure 2.1: Concept of nominal system and faulty system [14].
2.2. FAULTS AND FAILURE

2.2.1 Classification of Faults

Faults can be classified based on several criteria, such as the time characteristics of the faults, physical locations in the system and the effect of faults on system performance [37].

When classified according to their physical locations, three main faults can be defined: plant/process faults, sensor faults and actuator faults as shown in Figure 2.2.

![Diagram of Types of Faults](image-url)

Figure 2.2: Types of faults based on their physical locations.

1. **Plant/Process Faults**
   This type of fault directly affects the physical parameters of the system and thus changes the dynamic input/output properties of the system [37]. Plant/process faults are also known as component faults, arising as a variations from the structure of parameters used during the system design, and therefore cover a wide variety of possible fault such as components that are slowly degrading over time due to wear and tear, ageing or environmental effects [38].

2. **Sensor Faults**
   Sensors refer to any instrument that function to take measurements or readings from a system such as: potentiometers, load cells, current sensors, etc. Sensor faults mean that incorrect readings or measurements are taken from the real dynamic systems [38].

   Sensor faults can occur in any of the sensor network components such as the sensing devices, transducers, signal processors or data acquisition equipment [39] for a number of reasons like power failure, physical damage, joint or connection failure, and miscalibration [40].
The majority of fault tolerant control research to date has concentrated on sensor faults. Advances in fault tolerance have been made in this area which is mainly due to the nature of sensors that deal with information, and measurements may be processed or replicated analytically to provide fault tolerance.

3. **Actuator Faults**

Actuator faults can cause severe degradation (partial actuator faults) or complete loss of control (complete actuator faults) of the system [41]. Partial actuator faults refer to the case where an actuator has a slower response or becomes less effective and hence provides the plant with part of the nominal actuation signal, while complete actuator faults mean that the actuator produces no actuation signal regardless of the input to it [38]. Partial or complete actuator faults can occur due to a malfunction or equipment ageing leading to performance degradation.

Actuator faults are the focus of the studies presented in this thesis, particularly those related to EMA, which is the actuation element being considered for the HRA in this thesis. EMA fault diagnosis poses an interesting research problem as it is composed of electrical and mechanical subsystems, which results in intricate failure modes and effects. However, two major fault modes are of interest to the current research as described in Figure 2.3.

- **Lock-up faults:** refer to a condition where the actuation elements cannot move at all (it becomes still or jam in place) [14, 17]. It might occur due to excessive wear of the ball screw that creates mechanical interference within the mechanism and thus causes the actuator to jam in place. Under this condition, the actuator becomes very stiff and hence results in loss of travel capability of the actuator.

- **Short-circuit faults:** refer to a condition where an external voltage cannot be applied to the actuation elements due to faults in the wiring [9]. However, the DC motor electrical circuit is fine. This condition means
that the DC motor cannot generate torque to move the actuator forward but, a reverse torque can be generated when a sufficient external force is applied to back drive the actuator.

The strategies for fault tolerance that have been applied to sensor faults are not applicable for actuator faults because, unlike sensors that deal with information, actuators involve energy conversion. Thus, actuator redundancy is fundamental if fault tolerance is to be achieved in the presence of actuator faults. Actuation force will always be required to keep the system in control and bring it to the desired state [8]. No approach can avoid this requirement.

2.3 Fault Tolerance

Fault tolerance is a characteristic of a system that allows it to continue operating in the event that one or more of its components experience a fault [42]. The goal is to prevent a component fault from becoming a system failure [36]. If the system’s performance decreases, this will be proportional to the severity of the fault, instead of total breakdown even with small fault. This attribute makes fault tolerance a highly desirable feature in a safety-critical application such as aircraft, nuclear reactors and chemical plants where a malfunction can lead to a catastrophe.

The basic ingredient behind providing a fault tolerance capability is to provide the system with extra (redundant) resources, which includes information redundancy, hardware redundancy, software redundancy and time.
redundancy [43, 44]. The focus of this thesis is on hardware redundancy which can be achieved by incorporating extra hardware into the design to detect or override the effects of failed components [45]. The HRA incorporates extra actuation elements to increase the travel and force capability of the system above the capability of an individual element, and thus, provides inherent fault tolerance where if one or more of the elements fail, the capabilities of the actuator may be reduced, but it does not become dysfunctional [16].

2.4 Overview of Redundancy

The word redundancy in common usage is often associated with a negative implication of something excessive or over-abundance, or in some cases it might mean something useless and meaningless. However, redundancy can be a powerful device to suppress errors [46]. For example, by introducing a simple repetition (redundancy) in a conversation might reduce the possibility of misunderstanding (error).

In engineering, the term redundancy refers to the duplication or replication of critical components of a system (usually a safety-critical system) with the intention to increase the reliability of the system. In modern engineering, redundancy is associated with safety and integrity. The principle of redundancy is simple: an element is redundant if another element exists (a backup) to do the required task if the first fails. By extension, a redundant system is a system that contains redundant elements [47]. Thus, a redundant system could be a system with several elements that work simultaneously but also capable of performing the task individually if required (such as the engine of civil aircraft) [48]. It can also refer to a system with elements that are not in use (in idle condition) until the system needs them (such as a backup power supply in a computer) [49]. Redundancy has served as a central principle of high-reliability engineering for over five decades, especially in the design of civil aircraft which depend heavily on redundancy for their safety.

Redundancy can be applied to any components in a system; sensor,
actuator, control computer, etc. and parallel redundancy or also known as direct redundancy which refers to multiple independent elements that operate in parallel to form a system [3], has been widely used in engineering design. This type of redundancy is well documented in the aircraft industry but due to some limitations, researchers and engineers are looking for an alternative to the conventional parallel redundancy especially for actuators and this generated a concept called high redundancy actuators.

2.4.1 Parallel Actuation Redundancy

Actuators are important components in many safety-critical systems such as aircraft, nuclear power plants and railway, where an undetected fault can lead to serious consequences. The conventional way of providing fault tolerant actuator especially in these safety-critical systems is through parallel replications as illustrated in Figure 1.1. In this configuration, the individual actuators that are capable of providing the required control actions are connected in triplex or quadruplex (see Figure 2.4) with some form of consolidation to sum their output. If one or more of the actuators become faulty, the remaining healthy actuators would be able to meet the control requirements, avoiding an actuation system failure.

While redundancy seeks to increase the reliability of a system, it does increase the initial procurement cost, as well as maintenance cost [50] and weight [14] and these, could be a limitation in certain applications. In aerospace applications, for example, they have a very strict constraint on weight and volume, where multiple redundant systems come with a consid-

![Figure 2.4: Quadruplex electrohydraulic actuator for aerospace applications. (Courtesy of Moog, Inc.)](image)
erable cost and weight penalties [51].

Another disadvantage of a purely parallel configuration is that it will be rendered useless in the presence of lock-up faults [14, 16] because parallel elements push/pull the same load causing the available force to increase with the number of actuation elements, but the travel remains constant. Therefore, if one element locks up, travel becomes zero, and the whole assembly fails.

In response to the issues faced by traditional parallel redundancy, the concept of a high redundancy actuator (HRA) has been introduced for actuator fault tolerance.

2.4.2 High Redundancy Actuator

The HRA is a concept for actuator fault tolerance that is inspired by human musculatures [18–20]. There are similarities between human muscles and an actuator in the sense that both convert energy into motion. Human muscles convert energy from consumed food into complex motions such as walking, running or dancing while electromechanical actuators convert electrical energy into mechanical motion. Human musculature works in an ingenious way because each muscle cell provides only a minute contribution to the travel and force of the overall muscle system making the muscle highly resilient to damage of individual cells.

By adopting the same cooperative principles, the HRA concept focuses on delivering a fault tolerant actuator that comprises a relatively large number of actuation elements that work together to form a single actuator as shown in Figure 1.2 [15, 21]. Actuation elements are connected in parallel and series configuration to improve their reliability and availability, and at the same time reduce the need for over-sizing. Parallel configurations increase the generated force and improve loose fault tolerance, while the series configurations increase travel and improve lock-up fault tolerance [22, 24]. According to [14] and [15], significant advantages of the HRA are:

- *Increased reliability* by allowing the system to do its required task and warn maintainers when it starts to approach a critical level of faults.
2.4. OVERVIEW OF REDUNDANCY

- **Reduced over-dimensioning and weight** by using a greater number of smaller actuation elements instead of using triplex or quadruplex structure.

- **Intrinsic accommodation of faults** because each actuation element provides only a small contribution to the required force and travel of the actuator and thus the effect of a fault in an individual element on the overall actuation system is also small.

- **Graceful degradation** because the capability of the system will not immediately go to zero in case of faulty.

During normal operation, it is possible that some of these actuation elements are operational and some are faulty. In this situation, the HRA may still work, but with a reduced performance. So the reliability of the HRA depends on the required performance of the application. In this thesis, the HRA is designed with application to an aircraft aileron actuation system in mind for a study case; but it can of course be used for other safety critical systems.

Faults within the actuator will affect the maximum capability of the actuator and responding to faults in the HRA is a challenge because the serial elements are making the system more complex [24,25]. However, with a properly designed controller, it should be possible to provide an actuator that gracefully degrades and allows an operator to take corrective action after a fault is detected, such as preventive maintenance or a safe shutdown. Moreover, through the appropriate integration of condition monitoring algorithm, it should be possible for the actuator to identify that it is getting closer to its performance limit to allow a proper action to be taken before the element fault causes a system failure.

For the HRA described in this thesis, the intended application is as ailerons actuation system. However, the HRA, in general, can be used in any applications. The 12 elements HRA described in this thesis is designed to meet the aileron specification of an unmanned aerial vehicle (UAV) developed by the British company BAE Systems called HERTI which stands for “High Endurance Rapid Technology Insertion”. According to [52], HERTI
is a medium-altitude long-endurance UAV developed for the UK’s defence forces to perform intelligence, surveillance, reconnaissance and target acquisition operations. It is the first UAV in the UK to be accredited by United Kingdom Civil Aviation Authority certification.

The following subsection will discuss the HRA as an ailerons actuation system.

**HRA As Ailerons Actuation System**

The operation of the ailerons is described in Figure 2.5. Its main function is to control the roll of the aircraft. The ailerons usually work in the opposite direction. Meaning, as the left aileron is deflected upward, the right is deflected downward and vice versa.

Traditionally, aileron actuation systems have employed 2-3 identical actuators in order to provide redundancy in the event the primary actuator developing a fault. This approach increases the complexity of the design not to mention the weight of the vehicle. HRA may replace the traditional three redundancy actuator setup with an ideal 1-actuator setup.

The requirements in terms of functional characteristics are given in Table 2.1. This details the BAE System requirements for the HERTI UAV application [26] and the 12 elements HRA discussed in this thesis (as well as some of the controller design requirements) is designed based on this requirements.

![Figure 2.5: Schematic showing the operation of aileron.](image-url)
Table 2.1: Functional characteristic requirements of aileron actuator for HERTI

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>HERTI requirements</th>
</tr>
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<tbody>
<tr>
<td>General design</td>
<td>Electro-mechanical</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>5kg</td>
</tr>
<tr>
<td>Working stroke (mm)</td>
<td>100mm</td>
</tr>
<tr>
<td>Overshoot</td>
<td>10%</td>
</tr>
<tr>
<td>Force requirement</td>
<td>80N</td>
</tr>
</tbody>
</table>

2.5 Controller Design

As mentioned before, the HRA discussed in this thesis is intended for flight control system and according to [53], some of the popular techniques used to design flight control systems are; classical (P, PD, PI, PID) controller [54–56], robust ($\mathcal{H}_\infty$, LQ, LQG, LTR) controller [57–59], dynamic inversion [60–62], quantitative feedback theory [63, 64], linear parameter varying (LPV) control, [65, 66], model predictive control [58, 67], backstepping [68–70], neural networks [71, 72], adaptive control [73, 74], model following [55, 60, 61], sliding mode control [75–77], fuzzy logic [58, 78], and eigenstructure assignment [79, 80].

In this thesis, a classical controller and an $\mathcal{H}_\infty$ controller have been chosen to be tested with the HRA. The classical control method was chosen due to simplicity in the design process while the $\mathcal{H}_\infty$ control method was chosen due to its capability to deal with uncertainties in a system.

The HRA employs multiple actuation elements and all of these elements are assumed to be identical during the modelling process. However, in reality, there will be slight differences in the parameters of every individual element and these introduce parameter uncertainties to the system. Moreover, when faults are injected to one or more of the elements, the dynamics of the system will change which causes time-varying uncertainties.

Though classical control methods can provide good tracking performance, it has been suggested that it has a limitation when dealing with uncertainties in a system [81]. An $\mathcal{H}_\infty$ control scheme not only has the ability to provide better robustness to inherent uncertainty in the system.
model [82–84] but is also less complex in implementation compared to adaptive and nonlinear techniques [85].

2.6 Condition Monitoring

It is desirable for the HRA to continue to operate within an acceptable region even when one or more of its elements are faulty. However, after several faults, the capability of the system will eventually fall below that required by the application. At this point, maintenance is required to replace the faulty elements and thus, a form of condition monitoring is needed to provide an indication of the HRA capability in order to schedule maintenance or halt operation. Early detection of the condition, while the system is still operating in the acceptable region, can help to avoid a situation where the HRA can no longer meet requirements and reduce productivity losses which in turn can help avoid major system breakdowns and catastrophes. The concept of condition monitoring is discussed in this section.

2.6.1 Overview of Condition Monitoring

Condition monitoring or health monitoring, refers to the continuous evaluation of a plant or equipment throughout its serviceable life [86] that can either be done online or offline [87]. Online condition monitoring means the system is monitored continuously during operation while offline monitoring refers to periodic inspection that requires the machine to be shut down and an operator to be present. Condition monitoring is considered to be related to Fault Detection and Isolation (FDI) schemes, but with a greater view towards maintenance.

Developmental work can be traced back as far as World War II [88]. Before then, the emphasis was on repairing failed machines instead of preventing failure from occurring. During World War II, a reduction in the skilled labour and the demand for higher production quantities combined with the need to investigate the cause of failure in engines of the warplanes led to a demand for some type of monitoring systems. The objective of the
monitoring system is to provide warning of a failure so that actions could be taken to repair the damage.

In [89], maintenance schemes are classified into three types: 1) reactive maintenance that is also known as breakdown maintenance. It is the earliest maintenance that takes place due to plant failure; 2) preventive maintenance which also known as planned maintenance. It is performed on a fixed-time basis regardless of the health status of the system. This type of maintenance helps reduce unplanned breakdown and thus lead to fewer catastrophic failure that results in expensive secondary damage; 3) predictive maintenance or condition based maintenance, that aims to prevent the propagation of incipient faults and component faults from causing system failure.

2.6.2 Condition Monitoring Scheme

The goal of condition-based maintenance (CBM) is to reduce the number of breakdowns by monitoring the dynamic system’s performance, aiming to detect faults at an early stage so that corrective actions can be taken to avoid the fault from causing failure and, as such, is regarded as the most advanced maintenance strategy [90]. The tool for achieving this is widely known in the literature as condition or health monitoring which involves FDI schemes to detect and identify fault.

According to [91], condition monitoring (CM) scheme can be grouped into two major categories: model-based condition monitoring; and knowledge-based condition monitoring. The former relies on a priori information represented by the sytem’s mathematical model while the latter is used where mathematical models may not be available due to for example poor modelling of the physics of the system or significant uncertainty about the models and potential parameters.

A paper that was written by Liu et al. [92] and a review article by Isermann [93] mentioned that model-based CM includes: parameters estimation; state or output observer; parity equation; and signal analysis. Knowledge-based CM, on the other hand, includes methods that are based on an expert system such as fuzzy logic and neural network.
Applications of model-based CM can be found in nuclear fusion cryogenic vacuum system [94], electromagnetic positioning system [95,96], wind turbine gearboxes [97], aircraft fuel systems [98] and railway wheel-rail adhesion monitoring [99].

In [94], Wright et al. developed a model-based CM for a neutral beam injector cryopumping system. The CM scheme used a residual generation approach where a bank of Kalman filters was used to produce the estimate of the process variables from real-time measured data. After that, a residual evaluator was used to produce diagnostic information using the difference between the estimated and the measured process variables.

Dixon [96] and Dixon et al. [95] discussed the development of a model-based CM for an electromechanical actuator intended to be applied on civil aircraft. The CM scheme involves the use of the simplified refined instrumental variable (SRIV) method to obtain the estimate of the key physical parameters of the electromechanical actuator from two discrete transfer function (TF) models (i.e. voltage to current and speed). Subsequently, fault symptoms were then formulated by monitoring the changes to models parameters over time using fuzzy logic sets.

In [97], Garlick et al. explained the development of a model-based CM for wind turbine bearings. The CM scheme used a least squares parameter estimation method to obtain the estimate of the parameters of a discrete time TF model relating the wind turbine generator temperature and the bearing temperature. The discrete time TF model was obtained by applying system identification to raw (unfiltered) Supervisory Control and Data Acquisition (SCADA) data.

The work described by Bennet et al. in [98] concerned the development of a model-based CM scheme that used a Kalman filter to detect a leak fault on the Advanced Diagnostic Test-bed (ADT), which is a representation of a modern aircraft fuel system. A mathematical model of the ADT was derived from first principles using equations for fluid height in the tanks, pressure and flow rate. The derived model was validated using real data obtained from the ADT for a series of tests.

Development of model-based CM for detection of low adhesion on the
2.6. CONDITION MONITORING

Wheel-rail contact is presented in [99]. The CM scheme employed a Kalman-Bucy filter to estimate the total lateral force and yaw moment at the wheel-rail interface which then interpreted into adhesion levels.

Examples of knowledge-based CM can be found in [100, 101]. In [100], fuzzy logic based CM to detect faults in induction motor by monitoring the motor’s stator current was discussed. Stator current amplitude was chosen as the fuzzy input variable while the stator condition was chosen as the fuzzy output variable. Three fuzzy membership functions were derived to represent the fuzzy output: good (G); damaged (D); and seriously damaged (SD) while four membership functions were derived to represent the input: zero (Z); small (S); medium (M); and big (B). From these fuzzy input-output membership functions, 14 if-then rules were derived to map the input functions to the output functions.

The work presented in [101] described the development of Artificial Neural Network (ANN) based CM for fault diagnosis of rolling element bearings (REBs) in a machine. The CM scheme used vibration signal analysis method to monitor the condition of REBs where raw vibration signals that were obtained from normal and defective bearings were used to extract features such as standard deviation, skewness, kurtosis etc. These features were then used as input to the ANN classifier for fault diagnosis.

Condition Monitoring Scheme for the 12-elements HRA

The mathematical model of the actuation elements (i.e. electromechanical actuator) used for the HRA studied within this thesis can easily be derived from first principles. Therefore, it was decided that a model-based CM scheme is the most suitable scheme to monitor the health condition of the full HRA assembly. After reviewing some model-based CM scheme available in the literature, it was concluded that the combination of parameter estimation for fault detection and fuzzy logic inference for fault diagnosis will be developed as the CM scheme for the HRA presented in this thesis.

Building on the ideas presented in Dixon and Dixon et al. described in [95,96], parameter estimation based algorithm will be used to obtain the estimates of the key physical parameters of the electromechanical actuator
in the healthy and faulty conditions. The fuzzy logic inference will be used to
give an indication of the actuator’s health condition based on the estimated
parameters.

2.7 Conclusion

A literature review pertaining to actuator redundancy, classical and \( \mathcal{H}_\infty \)
controller design, and condition monitoring has been presented in this sec-
tion. From the information gathered, an HRA with 12 actuation elements
based on electromechanical actuators will be realised in this project. An
experimental rig will be built, and mathematical model of the actuation
elements will be derived from first principles.

The previous work in [9] on HRA based on electromechanical actuation
technology used only four actuation elements. Note that the work in this
thesis is not simply an expansion of the work in [9] because the derivation
of the mathematical model of the actuation elements (presented in Chapter
4) discussed in this thesis is different from the one discussed in [9]. For the
work described in [9], a gearbox was used in between the DC motor and the
linear actuator. Therefore, a parameter called motor stiffness was included
in the actuator model. However, the work described in this thesis has a
simpler mechanical structure because no gearbox was used in between the
DC motor and the linear actuator hence the motor stiffness is not included
in the modelling of the actuation element. Also, in this thesis, the stiffness
between the DC motor shaft and the ball screw is assumed to be very high.
Therefore, the motor stiffness can be ignored in the modelling.

Also, in this thesis, a general n-by-m model of the HRA was derived
which can be used to model any number of electromechanical-based actua-
tion elements connected in matrix form, such as 20-by-30 or 100-by-100.

The control structure designed for the HRA in this thesis is also different
from the one designed in [9]. In this thesis, no multiple loop control structure
was considered. Only a single loop control structure is considered which
uses displacement as the feedback. Both the designed classical and \( \mathcal{H}_\infty \)
control method will be tested in the healthy and faulty conditions and will
be validated using the experimental test rig.

Condition monitoring scheme based on least squares parameter estimation and fuzzy logic inference will be designed to give an indication on the health condition of the HRA (e.g. healthy, one element fail, 2 elements fail and so on). Unlike the previous work in [14] that require position feedback from every actuation elements, the condition monitoring designed in this project modelled the HRA as a single actuator, and as such, eliminate the need to use a large number of position sensors.
Chapter 3

Experimental Setup

3.1 Introduction

In order to demonstrate the concept of HRA, an experimental test rig was designed and built. The experimental rig will be used for: model validation; and conducting experiments to test the designed control structure as well as the developed condition monitoring algorithm.

This chapter explains the equipment and setup used to collect the data presented in this thesis. The experimental setup is shown in Figure 3.1. It consists of three main parts: the mechanical components, the electrical components and the data acquisition and control hardware components.

A schematic of the HRA experimental setup is illustrated in Figure 3.2. It can be seen that the mechanical parts are connected to the electrical parts through wiring while the data acquisition parts are connected to the electrical parts through connector cables and break-out board. Control signals were sent to the actuation elements through the signal conditioning circuits and motor drivers to move the elements. As the actuation elements move forward and backwards, position and current signals were sent to the input/output (I/O) cards for data analysis.

The data acquisition and control hardware part is based on MATLAB’s xPC Target system and consists of a host PC and a target PC. All Simulink models are built and compiled in the host PC hence MATLAB/Simulink with xPC Target toolbox is installed in it. The models are then downloaded
to the target PC to carry out real-time experiments. I/O cards are plugged-in in the target computer to allow it to interact with the analogue world outside the PCs. The I/O cards have an analogue to digital converter (ADC) channels with an input range of $\pm 10V$, $\pm 5V$, $\pm 1V$ and $\pm 0.2V$, and digital to analogue converter (DAC) channels with an input range of $\pm 10V$.

In this chapter, the first section discusses in detail the mechanical parts which include types of actuation elements and linear guide used. The electrical components such as the types of sensor used to read the position and current signals are discussed in detail in Section 3.2. Section 3.3 explains the data acquisition and control hardware setup which includes the target PC, host PC and types of I/O cards used. This chapter ends with a conclusion in Section 3.4.

Figure 3.1: HRA experimental setup.
Figure 3.2: Schematic of the HRA experimental setup.
3.2 Mechanical Part

A plan photograph view of the mechanical part can be seen in Figure 3.3 while Figure 3.4 shows the schematic diagram of the mechanical components. Figure 3.5 shows the detail interconnection between the actuation elements. This part was assembled with the help of technicians from the mechanical workshop. It consists of the actuation elements, external load, linear guide and carriage, connecting plate and rod, and aluminum case to house the actuation elements. The experimental rig has a total length of 190cm (each actuation element has a length of 30cm), a width of 17.5cm and a height of 25cm. Each component is explained in detail in the following subsections.

![Figure 3.3: Plan view of the mechanical components.](image)

![Figure 3.4: Schematic diagram of the mechanical components.](image)
3.2. MECHANICAL PART

Figure 3.5: Schematic diagram of the interconnection between actuation elements.

3.2.1 Actuation elements

Twelve THK VLACT35 linear actuators are used as the actuation elements with six elements mounted on each side of the aluminum case. Each actuation element has a maximum stroke of 50mm, which means the whole assembly has a maximum stroke of 150mm. Recall from Table 2.1 that the HERTI requirement for working stroke is 100mm, which means the HRA designed in this thesis have 50mm ‘redundant’ stroke for fault tolerant purpose. A Bühler DC motor is used to drive each linear actuator. Details of the technical specifications of the actuator and the DC motor are shown in Table 3.1.

In Figure 3.3, the top two serial branches (i.e. actuation elements 1,2,3 and actuation elements 10,11,12) can be seen. The other two serial branches (i.e. actuation elements 4,5,6 and actuation elements 7,8,9) are in parallel below the top serial branches.

The end part of element 1 is connected to the DC motor flange of element 2 using a nut (refer to Figure 3.6 for the detail). Therefore, when element 1 is moving forward/backwards, it will push/pull element 2.
Table 3.1: Actuator Specifications

<table>
<thead>
<tr>
<th>Item</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ball screw mechanism</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated speed</td>
<td>300</td>
<td>mm/s</td>
</tr>
<tr>
<td>Rated thrust</td>
<td>80</td>
<td>N</td>
</tr>
<tr>
<td>Maximum thrust</td>
<td>240</td>
<td>N</td>
</tr>
<tr>
<td>Ball screw lead</td>
<td>6</td>
<td>mm</td>
</tr>
<tr>
<td>Ball screw shaft diameter</td>
<td>8</td>
<td>mm</td>
</tr>
<tr>
<td>Stroke</td>
<td>50</td>
<td>mm</td>
</tr>
<tr>
<td><strong>DC motor</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rated voltage</td>
<td>12</td>
<td>V</td>
</tr>
<tr>
<td>Rated power</td>
<td>90</td>
<td>W</td>
</tr>
<tr>
<td>Rated torque</td>
<td>30</td>
<td>Ncm</td>
</tr>
<tr>
<td>Rated speed</td>
<td>3000</td>
<td>rpm</td>
</tr>
<tr>
<td>Rated current</td>
<td>12</td>
<td>A</td>
</tr>
</tbody>
</table>

Figure 3.6: Detail of the ball screw and DC motor flange connection.

3.2.2 External load

A THK SKR33 linear actuator with Maxon DC motor (148867) is used as the external load. The linear actuator has a maximum stroke of 200mm which is sufficient because the HRA has a maximum stroke of 150mm. This actuator will be used to represent the external dynamic loads against which the HRA will need to operate.
3.2.3 Other components

Some of the actuation elements have a fixed base (e.g. element 1 and 10 in Figure 3.3) while some elements have a moving base, such as elements 2, 3, 11 and 12 in Figure 3.3. NSK PAU15ALS linear guide and carriage (as shown in Figure 3.7) is used to provide the ‘moving base’, where the actuation element is attached to the linear carriage to allow it to move along the rail.

A connecting plate and output rod as shown in Figure 3.8 is used to connect the HRA and the external load. Four actuation elements are connected to the plate using nuts so the HRA can push/pull the load.

![Figure 3.7: Detail of the linear guide and carriage.](image-url)
Figure 3.8: Detail of the connecting plate and output rod.
3.3  Electrical Components

The photograph plan view of the electrical components is shown in Figure 3.9. The electrical components consist of position sensors, current sensors, motor drivers, signal conditioning circuits, power supplies, break-out boards and shielded connector cables to connect the electrical part with the data acquisition part.

3.3.1  Current sensors

Thirteen LEM HAIS 50-P current sensors were used: 12 are used to measure the actuation element’s current; and 1 to measure the load’s current. The sensors employ hall-effect measuring principles. Each sensor has a measuring range of $\pm 150$A and requires a supply voltage of 5V. The output is configured to measure a current of the range $\pm 10$A by looping a wire (5 loops) through the current sensor. In this case, 1V is equal to 2A. More technical specifications of the current sensors are given in Appendix C.
3.3.2 Position sensors

There are thirteen MK30-P Micro-Epsilon position sensors used: 12 to measure the actuation element’s displacement while another one to measure the load’s (total) displacement. The position sensor is a draw-wire displacement sensor with a measuring range of 150mm. Figure 3.10 shows how the position sensors that are used to measure the actuation element’s displacement is mounted. As the actuator moves forward and backwards, the wire extends and retracts accordingly, giving an electrical position reading via the internal potentiometer.

A 5V supply voltage is applied across the device, and after signal conditioning, 1V of output corresponds to 90mm of displacement.

![Arrangement of position sensor on actuation elements.](image)

3.3.3 Motor drivers

Seven Sabertooth 2x5 motor drivers were used in this project. Each motor driver has two channels and is therefore capable of driving two DC motors. So, six motor drivers were used to drive the actuation elements and one to drive the external load. Each channel can provide up to 5A continuous current but limited to 10A per channel for a few seconds. A 12V supply voltage is used to power the motor driver.

The motor driver accepts a voltage of the range 0V to 5V as a control input to control the motor movement with 2.5V corresponds to no movement.
3.3. ELECTRICAL COMPONENTS

Signals above 2.5V will command a forward motion while signals below 2.5V will command a backwards motion. So signal conditioning circuits were designed to provide an offset of 2.5V so that when the computer is turned off, it will not cause the actuators to moved backwards. More technical specifications of the Sabertooth motor drivers can be found in [102].

3.3.4 Other components

The signal conditioning circuits are used to: scale the potentiometer output voltage to meet the input voltage requirement of the ADC and DAC channels of the xPC target; and to provide offset for the motor driver.

The break-out boards (NI CB-68LPR) together with the shielded connector cables are used to connect the electrical part and the xPC Target for data acquisition to allows signals between the sensors and the target/host PC to be exchanged. Current and position signals were sent to the target PC to be plotted or saved, and control signals from the host PC were sent to the motor drivers to move the actuation elements.
3.4 Data Acquisition and Control Hardware

A close-up view of the data acquisition and control hardware is shown in Figure 3.11. The data acquisition system is based on MATLAB’s xPC Target system [103] that requires two desktop computers to run; one working as a host PC while the other working as a target PC. I/O cards are also required as interface devices for taking measurements from and sending control signals to the analogue world outside the PCs. All of the components are explained in detail in the following subsections.

![Figure 3.11: Close-up view of the data acquisition and control hardware.](image)

3.4.1 Target and host PC

A standard Windows operating system (OS), MATLAB/Simulink with xPC Target toolbox and a C compiler are installed in the host computer. All Simulink models were built in the host computer. Two types of I/O cards (PCI-6229 and PCI-6704) for data acquisition are plugged-in in the target computer.

The target computer runs all of the real-time control algorithms. It is booted using a boot disk that is created using the host computer (hence, no Windows OS is installed in the target computer). Both computers are connected through an unshielded twisted pair (UTP) cable. The target computer is connected to the electrical part of the experimental setup through the connector cables and break-out boards shown in Figure 3.9.
3.5. **CONCLUSION**

### 3.4.2 I/O card

In this project, two I/O cards are used for data acquisition; PCI-6229 and PCI-6704 from National Instruments. The PCI-6229 has 32 analogue-to-digital (A/D) channels and 4 digital-to-analogue (D/A) channels. The A/D channels of this card are used to save sensor signals to the target PC.

The PCI-6704 card is a purely D/A card (no A/D channels). It has 32 D/A channels: 16 for voltage output; and 16 for current output. In this project, the PCI-6704 card is used to send voltage control signals to the actuation elements through the motor drivers.

### 3.4.3 Other components

A MATLAB compatible network card is installed on both the target and host computers for TCP/IP communications. Refer to [103] for more detail on the supported network card.

### 3.5 Conclusion

This chapter has explained in detail the experimental test rig used to collect experimental data presented in this thesis. The components of all three main parts: the mechanical part; the electrical part and the data acquisition part have been discussed in detail. Real-time experiments start with studying the behaviour of all actuation elements separately in open-loop to make sure that all components are connected properly and also to validate the derived mathematical model of the actuator. The open-loop experimental results were discussed in Chapter 4.

The HRA experimental rig is also used to implement and test the classical and $\mathcal{H}_\infty$ controller in the healthy and faulty conditions. The results were discussed in Chapter 5 and 6, respectively. Finally, the experimental rig is used to collect data for the model-based condition monitoring algorithms, and the results were discussed in Chapter 7.
Chapter 4

Actuator Modelling

4.1 Introduction

A good mathematical model of the system is essential in order to design an effective closed-loop control system. Such a model can also be helpful for understanding the behaviour of a system and in predicting the effect of different components on the behaviour of a system. This section is dedicated to the mathematical modelling of the electromechanical actuator (EMA) used in this research. The approach taken is to model one actuation element, then build on this by combining elements until finally, a model of the 4-by-3 actuator can be constructed.

This section starts with a brief overview of the EMA used. Then, a mathematical model and open-loop study of a single actuator is presented in Section 4.3. In Section 4.4 and 4.5, the mathematical model and open-loop study of a purely serial and parallel configuration that consists of 2 actuation elements is discussed. In Section 4.6, the derived model of a general n-by-m HRA is explained. Then, the mathematical model and open-loop study of the HRA with 12 actuation elements that are arranged in a 4-by-3 matrix configuration is presented in Section 4.7. In Section 4.8, the model validation is described by directly comparing the model output to the actual system output obtained experimentally. This chapter ends with a conclusion in Section 4.9.
4.2 Electromechanical Actuator

The HRA employs a linear EMA as shown in Figure 4.1 as the actuation element. Basically, it consists of a DC motor and a ball screw mechanism. Table 3.1 shows the actuator specifications [104,105] while Table 4.1 shows the actuator parameters used in the simulation.

Figure 4.1: VLACT linear electromechanical actuator from THK Co., LTD.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R$</td>
<td>Armature resistance</td>
<td>0.4</td>
<td>Ω</td>
</tr>
<tr>
<td>$L$</td>
<td>Armature inductance</td>
<td>8</td>
<td>mH</td>
</tr>
<tr>
<td>$J^1$</td>
<td>Motor and ball screw inertia</td>
<td>4.92x10^{-5}</td>
<td>km²</td>
</tr>
<tr>
<td>$D$</td>
<td>Motor and ball screw viscous friction</td>
<td>9.549x10^{-8}</td>
<td>Nm/rads^{-1}</td>
</tr>
<tr>
<td>$K_e$</td>
<td>Motor back emf constant</td>
<td>0.027</td>
<td>V/rads^{-1}</td>
</tr>
<tr>
<td>$K_T$</td>
<td>Motor torque constant</td>
<td>0.027</td>
<td>Nm/A</td>
</tr>
<tr>
<td>$l$</td>
<td>Ball screw lead</td>
<td>9.5493x10^{-4}</td>
<td>m/rad</td>
</tr>
<tr>
<td>$c$</td>
<td>End of screw damping coefficient</td>
<td>26.90x10^{4}</td>
<td>Ns/m</td>
</tr>
<tr>
<td>$k$</td>
<td>End of screw stiffness</td>
<td>201.06x10^{6}</td>
<td>N/m</td>
</tr>
<tr>
<td>$M$</td>
<td>Actuator mass$^2$</td>
<td>2</td>
<td>kg</td>
</tr>
</tbody>
</table>

1 Calculated using formula from [106]
2 Mass of DC motor plus ball screw

Figure 4.2 shows the equivalent schematic representation of the linear EMA. The DC motor drives the ball screw mechanism by generating torque. The ball screw then translates the motor torque into axial force to move any load connected to the actuator. The actuator can be controlled to achieve the desired force and speed by changing the voltage supplied to the motor. If a load is attached to the actuator, the ball screw will drive the load forward and backwards depending on the polarity of the voltage. When power is removed, the actuator will hold its position but can be back driven when sufficient force is applied.
4.3 Modelling of A Single Actuation Element

4.3.1 Mathematical Model

In order to model the EMA, it is necessary to develop the equations related to the DC motor and the ball screw. Since no gearbox is used to couple the motor and the ball screw, they will have the same rotational speed. Given the motor angular speed (rad/s) as $\dot{\theta}_m$ and the ball screw angular speed (rad/s) as $\dot{\theta}_s$, Equation (4.1) is obtained.

$$\dot{\theta}_s = \dot{\theta}_m \tag{4.1}$$

The ball screw mechanism then translates this rotational motion to linear motion. Given the ball screw lead (m) as $l$, the ball screw linear speed (m/s), $\dot{X}_s$ can be expressed as shown in Equation (4.2).

$$\dot{X}_s = \frac{l}{2\pi} \dot{\theta}_s = \frac{l}{2\pi} \dot{\theta}_m \tag{4.2}$$

If a load is attached to the EMA in Figure 4.2, a schematic diagram as shown in Figure 4.3 can be drawn where $k$ and $c$ refer to the stiffness and damping coefficient of the end of the screw (the part that is connected to the load), $X_a$ is the actuator displacement and $X_L$ is the load displacement. Generally, the EMA can be divided into two subsystems (see Figure 4.4): an electrical subsystem that consists of the motor armature current; and a mechanical subsystem that consists of the mechanical loading of the motor.

Based on these, the following equations are derived:

1. Motor armature current
CHAPTER 4. ACTUATOR MODELLING

Figure 4.3: Schematic diagram of the EMA with a load attached.

Figure 4.4: Electrical and mechanical subsystems of the EMA.

\[ V_s - RI - L\dot{I} - V_b = 0 \] (4.3)

where \( V_s \) is the supply voltage, \( I \) is the armature current, \( R \) is the armature resistance and \( L \) is the armature inductance. \( V_b \) is the back electromotive force (emf) and can be related to the motor angular displacement as \( V_b = K_e \dot{\theta}_m \), where \( K_e \) is the motor back emf constant. So, Equation (4.3) can be re-written as:

\[ \dot{I} = \frac{1}{L}[V_s - RI - K_e \dot{\theta}_m] \] (4.4)

2. Motor mechanical loading

\[ T - J\ddot{\theta}_m - D\dot{\theta}_m - T_L = 0 \] (4.5)

where \( J \) is the motor and ball screw inertia and \( D \) is the motor and ball screw friction. \( T \) is the electrical torque and can be related to the armature current as \( T = K_T I \), where \( K_T \) is the motor torque...
4.3. MODELLING OF A SINGLE ACTUATION ELEMENT

constant. $T_L$ is the torque applied to the rotor by the external load and can be expressed as:

$$T_L = k(X_a - X_L) \left[ \frac{l}{2\pi} \right] + c(X_a - X_L) \left[ \frac{l}{2\pi} \right]$$

where $X_a$ is the actuator displacement and $X_L$ is the load displacement. The constant $\frac{l}{2\pi}$ is used to convert force to torque. Note that the actuator displacement is equal to the screw displacement and therefore can be expressed as $X_a = \frac{l}{2\pi} \theta_m$. So, Equation (4.5) can then be re-written as:

$$K_T I - J \ddot{\theta}_m - D \dot{\theta}_m - \frac{kl}{2\pi} \left( \frac{l}{2\pi} \theta_m - X_L \right) - \frac{cl}{2\pi} \left( \frac{l}{2\pi} \dot{\theta}_m - \dot{X}_L \right) = 0$$

where $h = \frac{l}{2\pi}$,

$$K_T I - J \ddot{\theta}_m - D \dot{\theta}_m - kh (h\theta_m - X_L) - ch (h\dot{\theta}_m - \dot{X}_L) = 0$$

$$\ddot{\theta}_m = \frac{1}{J} \left[ K_T I - kh^2 \theta_m - (D + c \dot{h}) \dot{\theta}_m + kh X_L + ch \dot{X}_L \right] \tag{4.6}$$

3. Load

Figure 4.5 illustrates the free-body diagram (FBD) of the load attached to the actuator in Figure 4.3. Based on this diagram, the following mathematical equation is derived:

$$M_L \ddot{X}_L - k \left( X_a - X_L \right) - c \left( \dot{X}_a - \dot{X}_L \right) = 0$$

$$M_L \ddot{X}_L = k \left( \frac{l}{2\pi} \theta_m - X_L \right) + c \left( \frac{l}{2\pi} \dot{\theta}_m - \dot{X}_L \right)$$
\[ \ddot{X}_L = \frac{1}{M_L} \left[ k h \dot{\theta}_m + c h \dot{\theta}_m - k X_L - c \dot{X}_L \right] \] (4.7)

Figure 4.5: Free-body diagram of the applied load.

Equations (4.4), (4.6) and (4.7) are used to construct the Simulink model of the single actuator. In the Simulink model, an end-stop is provided by setting the saturation limit of the integrator block that is used to integrate actuator velocity to position. The upper saturation limit was set to 50mm which is the maximum stroke of the linear actuator while the lower saturation limit is set to 0mm to indicate that the actuator is fully retracted.

### 4.3.2 Open-loop study using a single actuation element

Figure 4.6 shows the open-loop results of a single actuation element with external load of 1kg and 20kg. The supply voltage is a step signal with amplitude of 5V and applied to the system for 0.2s. The actuator and the external load have the same displacement, which is 30.51mm with a 1kg load and 28.84mm with a 20kg load.

The peak output force is approximately 5.55N with a 1kg load and around 84.72N with a 20kg load. The peak output current is approximately 10.78A with a 1kg load and approximately 11.06A with a 20kg load.

The results show that the peak current only increase slightly when external load is increased from 1kg to 20kg. This is because, the load is relatively small compared to the referred mass of the actuator’s inertia, which is 53.92kg as shown in Equation (4.8).
Refered mass of actuator’s inertia = $Jl^2$  

$$= (4.92 \times 10^{-5} \text{kgm}^2) \left( \frac{1 \text{rad}}{9.5493 \times 10^{-4} \text{m}} \right)^2$$

$$= 53.92 \text{kg}$$

Figure 4.6: Open-loop simulation results using single-element actuator with external load of 1kg and 20kg.

4.4 Modelling of Serial Actuation Elements

Figure 4.7 shows 2 actuation elements connected in series with a load attached to the second actuator. Note that actuator 1 has its base connected to a fixed structure and therefore, the base of actuator 1 is treated as non-moving while actuator 2 has its base connected to the end of actuator 1 which means that the base of actuator 2 will move due to the force exerted by actuator 1. Therefore, the total displacement of actuator 2 is the sum of the displacement of its base and its screw ($X_2 = X_{b2} + X_{s2}$) where $X_2$ refers to the displacement of actuator 2, $X_{b2}$ is the displacement of actuator
2’s base and $X_{b2}$ is the displacement of actuator 2’s screw, which means that, when modelling actuator 2, an extra equation is needed to account for position of the actuator base. The models are developed below for both sub-actuators (actuator 1 and 2). These models and the load model can be used to model any number of actuator in series.

![Figure 4.7: Schematic diagram of an actuation system with 2 actuation elements connected in series.](image)

### 4.4.1 Mathematical Model of Actuation Element 1

1. **Armature Current**
   
   Equation for the armature current is:
   
   $$\dot{I}_1 = \frac{1}{L} \left[ V_s - RI_1 - K_e \dot{\theta}_1 \right]$$
   
   (4.9)

   where $I_1$ refers to actuation element 1 armature current and $\dot{\theta}_1$ refers to actuation element 1 angular speed.

2. **Mechanical loading**

   Equation for the mechanical loading can be obtained by replacing the load with actuation element 2 in Equation (4.6), as:

   $$\ddot{\dot{\theta}}_1 = \frac{1}{J} \left[ K_T I_1 - kh^2 \dot{\theta}_1 - (D + ch^2) \dot{\theta}_1 + khX_{b2} + ch\dot{X}_{b2} \right]$$

   (4.10)

   where $X_{b2}$ and $\dot{X}_{b2}$ refers to the displacement and velocity of the base of actuation element 2.
4.4. MODELLING OF SERIAL ACTUATION ELEMENTS

4.4.2 Mathematical model of Actuation Element 2

1. **Armature current**
   
   Equation for the armature current is:
   
   \[
   \dot{I}_2 = \frac{1}{L} \left[ V_s - RI_2 - K_e \dot{\theta}_2 \right]
   \]  
   (4.11)

   where \( I_2 \) refers to the armature current of actuation element 2 and \( \dot{\theta}_2 \) refers to its angular speed.

2. **Mechanical loading**
   
   Equation for the mechanical loading is given as:
   
   
   \[
   K_T I_2 - J \ddot{\theta}_2 - D \dot{\theta}_2 - \frac{kl}{2\pi} (X_2 - X_L) - \frac{cl}{2\pi} \left( \dot{X}_2 - \dot{X}_L \right) = 0
   \]

   substitute \( X_2 \) with \( X_{b2} + X_{s2} \) (while \( X_{s2} = \frac{l_2}{2\pi} \theta_2 \)) results the following equation:

   \[
   \ddot{\theta}_2 = \frac{1}{J} \left[ K_T I_2 - khX_{b2} - ch\dot{X}_{b2} - kh^2 \theta_2 - (D + ch^2) \dot{\theta}_2 + khX_L + ch\dot{X}_L \right]
   \]  
   (4.12)

   where \( X_L \) and \( \dot{X}_L \) refers to the displacement and speed of the load.

3. **Base**
   
   Equation for the base of actuation element 2 is derived from Figure 4.8 as:

   Figure 4.8: Free-body diagram of actuation element 2.
\[ M_2 \ddot{X}_{b2} - k (X_1 - X_{b2}) - c (\dot{X}_1 - \dot{X}_{b2}) + k (X_2 - X_L) + c (\dot{X}_2 - \dot{X}_L) = 0 \]

\( M_2 \) is the mass of actuation element 2. Knowing that \( X_2 = X_{b2} + X_{s2} \), the following equation is obtained:

\[ M_2 \ddot{X}_{b2} = k (X_1 - X_{b2}) + c (\dot{X}_1 - \dot{X}_{b2}) - k (X_{b2} + X_{s2} - X_L) - c (\dot{X}_{b2} + \dot{X}_{s2} - \dot{X}_L) \]

Substitute \( X_1 = \frac{l_2}{2\pi} \theta_1 \) and \( X_{s2} = \frac{l_2}{2\pi} \theta_2 \), the following equation is obtained for the base of actuation element 2:

\[ \ddot{X}_{b2} = \frac{1}{M_2} \left[ kh\theta_1 + ch\dot{\theta}_1 - 2kX_{b2} - 2c\dot{X}_{b2} - kh\theta_2 - ch\dot{\theta}_2 + kX_L + c\dot{X}_L \right] \quad (4.13) \]

### 4.4.3 Mathematical model of the load

Equation for the load is derived from Figure 4.9 as:

\[ M_L \ddot{X}_L + k (X_L - X_2) + c (\dot{X}_L - \dot{X}_2) = 0 \]

\[ M_L \ddot{X}_L = k (X_{b2} + h\theta_2 - X_L) + c (\dot{X}_{b2} + h\dot{\theta}_2 - \dot{X}_L) \]

\[ \ddot{X}_L = \frac{1}{M_L} \left[ kX_{b2} + c\dot{X}_{b2} + kh\theta_2 - kX_L - c\dot{X}_L \right] \quad (4.14) \]

where \( M_L \) is the mass of the load.
4.4. MODELLING OF SERIAL ACTUATION ELEMENTS

4.4.4 Travel and Force Capability

Travel and force capability of the series actuation system is expressed in multiples of the travel and force of the individual elements [107]. For a purely serial configuration, travel capability of the actuation system can be calculated as:

\[ C_{ts} = E_t n \]  

(4.15)

where \( C_{ts} \) is the travel capability of the serial assembly, \( E_t \) is the travel of the individual element and \( n \) is number of elements in series. Since there are two elements connected in series, the travel capability of the system is expected to be twice of the travel capability of the individual element.

The force capability of a purely serial configuration, on the other hand is equal to the force of the individual element.

\[ C_{fs} = E_f \]  

(4.16)

where \( C_{fs} \) is the force capability of the assembly and \( E_f \) is the force of the individual element.
4.4.5 Open-loop study using serial configuration

1. Healthy Condition

Figure 4.10 shows the simulation results of the 2-in-series actuator assembly under healthy condition with applied load of 1kg and 20kg. X1 and X2 refer to the displacement of actuation element 1 and 2, respectively while XL refers to the load’s displacement. F1 and I1 refers to the output force and current of actuation element 1 while F2 and I2 for actuation element 2.

It can be seen that when a 1kg load is attached to the actuator, X1=30.24mm while X2=30.42mm and the total displacement is 60.66mm. When a 20kg load is attached, X1=26.98mm while X2=27.15mm and the total displacement is 54.13mm. These results are as expected because the total displacement of the whole actuation assembly is the sum of the displacement of both actuation elements.

Peak output force of F1=21.31N and F2=10.74N are generated when a 1kg load is attached to the system. F1=141.66N while F2=135.02N when a 20kg load is applied. For peak current, I1=10.83A and I2=10.80A with a 1kg load while I1=11.28A and I2=11.26A with a 20kg load. As mentioned earlier, even though the force at the load changes substantially, the current does not change as much as most of the motor torque is used to accelerate the inertia of the motor and ball screw.
4.4. MODELLING OF SERIAL ACTUATION ELEMENTS

2. Effect of Lock-up Fault

Lock-up fault refers to a condition where the actuation element is jammed in place (it cannot move either forward or backwards) and therefore, a loss in travel capability is to be expected. In the Simulink model, a lock-up fault is introduced to the element by multiplying its speed with zero. Since there are 2 elements connected in series, when either one of the element experience lock-up fault, the travel capability of the system will be equal to the travel of the healthy element.

Figure 4.11 shows the simulation results when either one of the actuation elements experience lock-up fault. When A1 is locked, X1=0 while X2 and XL equal to 30.5mm. The peak output force, F1=F2=5.5N. Both forces are the same because when A1 is locked, A2 will have a fixed base and, as such, the force exerted to the base of A2 by A1 is called a static force which will be equal to the force exerted by A2 to the load [107].

In the case that A2 is locked, X2=0 while X1 and XL equal to 30.3mm. For the peak output force, F1=16.1N while F2=5.4N. This is because A1 must apply a force to accelerate A2 and the load, whilst A2 merely transmits...
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Figure 4.11: Simulation results of the 2-in-series configuration under lock-up faults.

a force to the load.

In both cases, the peak current of the healthy element is approximately 10.6A while the locked element draws a maximum current as it drives against the lock.

3. Effect of Short-circuit Fault

Short-circuit fault refers to a condition where the external input voltage cannot be supplied to the actuation elements. This might be due to faults in the wiring. In the Simulink model, a short-circuit fault is introduced to the actuation elements by making its supply voltage equal to zero.

Figure 4.12 shows the simulation results when either A1 or A2 experience short-circuit fault. From the graphs, it can be seen that the faulty element has zero displacement and current while the healthy element and the load have approximately 30.6mm displacement.

For the peak force, F1=F2=5.4N when A1 is short-circuit while F1=16N and F2=5.2N when A2 is short-circuit. Again, the results are as expected from the fact that A1 moves A2 and the load. (Note: In this case, neither
A1 nor A2 are back driven because the output force is not large enough to overcome the friction in the ball screw and the DC motor.

In both cases, the peak current of the healthy element is around 10.8A while the faulty element has zero current because the external supply voltage cannot be applied to it.

![Simulation results of the 2-in-series configuration under short-circuit faults.](image)

Figure 4.12: Simulation results of the 2-in-series configuration under short-circuit faults.
4.5 Modelling of Parallel Actuation Elements

Figure 4.13 shows 2 actuation elements connected in parallel. In this configuration, both elements will push/pull the same load and both elements have a fixed base. This means that only equation for the load needs to be modified. Equation for the armature current of both actuation elements will be similar to Equation (4.4) and equation for the mechanical loading of both actuation elements will be similar to Equation (4.6).

Figure 4.13: Schematic diagram of an actuation system with 2 actuation elements connected in parallel.

Equation for the load is derived from the FBD of the load as given in Figure 4.14:

\[
M_L\ddot{X}_L = k(h\theta_1 - X_L) + c(h\dot{\theta}_1 - \dot{X}_L) + k(h\theta_2 - X_L) + c(h\dot{\theta}_2 - \dot{X}_L)
\]

\[
\ddot{X}_L = \frac{1}{M_L} \left[ kh\theta_1 + ch\dot{\theta}_1 + kh\theta_2 + ch\dot{\theta}_2 - 2kX_L - 2c\ddot{X}_L \right] \tag{4.17}
\]
4.5. MODELLING OF PARALLEL ACTUATION ELEMENTS

4.5.1 Travel and Force Capability

For a purely parallel configuration, the travel capability of the assembly is equal to the travel of the individual element:

\[ C_{tp} = E_t \]  \hspace{1cm} (4.18)

where \( C_{tp} \) is the travel capability of the parallel assembly and \( E_t \) is the travel capability of an individual element.

The force capability, on the other hand is equal to the sum of the force of the individual element which can be expressed as:

\[ C_{fp} = E_f n \]  \hspace{1cm} (4.19)

where \( C_{fp} \) is the force capability of the parallel assembly, \( E_f \) is the force of the individual element and \( n \) is number of elements in parallel.

4.5.2 Open-loop study using parallel configuration

1. Healthy Condition

Figure 4.15 shows the simulation results of the 2-in-parallel assembly under healthy condition with an external load of 1kg and 20kg. In the graph, X1 refers to the displacement of A1, X2 for A2 and XL for the external load.
F1 and F2 refers to the output force of A1 and A2, respectively while FT refers to the total force exerted on the external load.

From the graph, it can be seen that XL=X1=X2=30.5mm (satisfy Equation (4.18)). For the peak output force, when load of 1kg is attached, F1=F2=2.80N while FT=5.60N. When a 20kg load is attached to the system, F1=F2=48.35N while FT=96.70N. In both cases, total peak force exerted on the load is twice the peak force generated by the individual actuation element (satisfy Equation (4.19)).

The peak output current in both cases is approximately 10.8A for both A1 and A2.

![Figure 4.15: Simulation results of the 2-in-parallel configuration under healthy condition with external load of 1kg and 20kg.](image)

**2. Effect of Lock-up Fault**

Figure 4.16 shows the simulation results when either A1 or A2 experience lock-up fault. It can be seen that when either one of the actuation element is locked, the whole assembly is rendered immobile because displacement of both actuation elements as well as the external load is zero. As discussed in Chapter 2, lock-up fault is one of the main disadvantage of the traditional
parallel actuation that is widely employed in flight control system.

The output force of both actuation elements are the same, which is approximately 339.29N, but in different directions. This is because, as the healthy element generate force to push the external load, it is pushing the faulty element at the same time and causes the faulty element to generate the same amount of reaction force on the opposite direction. Oscillation can be observed in the output force signal which is due to the stiffness of the ball screw.

Output current is 12A, which is the rated current of the DC motor because the motor is applying maximum torque in order to try to move the locked ball screw.

3. Effect of Short-circuit Fault

Figure 4.17 shows the simulation results when either one of the actuation elements experiences a short-circuit fault. The results show that the actuation system has the ability to tolerate a short-circuit fault because the whole assembly is not rendered immobile in the presence of the short-circuit fault. This is possible because the actuation elements are back drivable, so the healthy element can move the faulty element. However, the movement of the faulty element will be limited due to the screw stiffness.

The results show that in both short-circuit cases, the actuation elements and the external load’s displacement is approximately 15.27mm. The healthy element generates larger output force (181.49N) compared to the reaction force that the faulty element generates (-101.40N) to produce a total force of 80.09N. Slight oscillation can be observed in the output force graphs which was due to the stiffness of the ball screw.

For the output current, the faulty element has zero current because there is no voltage supplied to it. The healthy element, on the other hand, generates approximately 11.39A peak current.
Figure 4.16: Simulation results of the 2-in-parallel configuration under lock-up faults.

Figure 4.17: Simulation results of the 2-in-parallel configuration under short-circuit faults.
4.6 Modelling of n-by-m HRA

The full HRA assembly which is the focus of the work in this thesis is arranged in a matrix form as shown in Figure 4.18. This figure shows an n-by-m HRA where n refers to the number of rows and m refers to the number of columns in the matrix assembly. The letter j represents the columns (i.e. j=1 to m) while the letter i represents the rows (i.e. i=1 to n).

Based on this diagram and the knowledge from modelling the previous actuator assemblies, the following mathematical equations are obtained to represent a general n-by-m HRA.

1. Armature circuit of all actuation elements

\[ \dot{I}_{ij} = \frac{1}{L} \left[ V_s - RI_{ij} - K_a \dot{\theta}_{ij} \right] \] (4.20)

2. Mechanical loading of actuation elements in the first column (i.e. j=1)

\[ \ddot{\theta}_{ij} = \frac{1}{J} \left[ K_T I_{ij} - kh^2 \theta_{ij} - (D + ch^2) \dot{\theta}_{ij} + khX_{bi(j+1)} + chX_{bi(j+1)} \right] \] (4.21)

Note: the subscript b means base.

3. Mechanical loading of actuation elements in the second column up to the second last column (i.e. j=2 to j=m-1)
\[
\ddot{\theta}_{ij} = \frac{1}{J} \left[ K_T I_{ij} - k h \dot{X}_{bij} - c h \dot{X}_{bij} - k h^2 \theta_{ij} \\
- (D + c h^2) \dot{\theta}_{ij} + k h X_{b(i+1)} + c h \dot{X}_L \right] 
\] (4.22)

4. Mechanical loading of actuation elements in the last column (i.e. \(j=m\))

\[
\ddot{\theta}_{ij} = \frac{1}{J} \left[ K_T I_{ij} - k h X_{bij} - c h \dot{X}_{bij} - k h^2 \theta_{ij} \\
- (D + c h^2) \dot{\theta}_{ij} + k h X_L + c h \dot{X}_L \right] 
\] (4.23)

5. Base of actuation elements in \(j=2\) up to \(j=m-1\)

\[
\ddot{X}_{bij} = \frac{1}{M_{ij}} \left[ k h \theta_{i(j-1)} + c h \dot{\theta}_{i(j-1)} - 2 k X_{bij} - 2 c \dot{X}_{bij} \\
- k h \theta_{ij} - c h \dot{\theta}_{ij} + k X_{b(i+1)} + c \dot{X}_{b(i+1)} \right] 
\] (4.24)

6. Base of actuation elements in \(j=m\)

\[
\ddot{X}_{bij} = \frac{1}{M_{ij}} \left[ k h \theta_{i(j-1)} + c h \dot{X}_{i(j-1)} - 2 k X_{bij} - 2 c \dot{X}_{bij} \\
- k h \theta_{ij} - c h \dot{X}_{ij} + k X_L + c \dot{X}_L \right] 
\] (4.25)
7. Load

\[ \ddot{X}_L = \frac{1}{M_L} \left[ \sum_{i=1}^{n} kX_{bim} + c\dot{X}_{bim} + k\dot{\theta}_{im} + c\dot{\theta}_{im} \right] - 4kX_L - 4c\ddot{X}_L \quad (4.26) \]

Note: in this equation, \( j = m \) because the load is attached to the elements in the last column.

4.6.1 Travel and Force Capability

In the n-by-m HRA, there are m number of elements connected in series. Therefore, the travel capability of the n-by-m HRA under nominal condition is:

\[ C_t = mE_t \quad (4.27) \]

where \( C_t \) is the travel capability of the HRA, \( E_t \) is the travel capability of an individual element and m represents the number of elements connected in series.

Recall that in the HRA assembly, there are n number of elements that are pushing/pulling the load. Therefore, the force capability of the n-by-m HRA under nominal condition is given as:

\[ C_f = nE_f \quad (4.28) \]

where \( C_f \) is the force capability of the HRA, \( E_f \) is the force capability of an individual element that push/pull the load and n refers to the number of elements pushing/pulling the load which also means there are n parallel branches.
4.7 Modelling of 4-by-3 HRA

Figure 4.19 shows the schematic diagram of the HRA assembly used in the work presented in this thesis. Three actuation elements are connected in series and four of these series arrangements are connected in parallel.

The mathematical equations that represent the 4-by-3 HRA assembly is derived from the equations of the general n-by-m HRA and given as follow:

1. Armature circuit of all actuation elements
   Similar to Equation (4.20) with \( i = 1 \rightarrow 4 \) and \( j = 1 \rightarrow 3 \).

2. Mechanical loading of actuation elements in \( j=1 \)
   Similar to Equation (4.21).

3. Mechanical loading of actuation elements in \( j=2 \)
   Similar to Equation (4.22).
4.7. MODELLING OF 4-BY-3 HRA

4. Mechanical loading of actuation elements in j=3
   Similar to Equation (4.23).

5. Base of actuation elements in j=2
   Similar to Equation (4.24).

6. Base of actuation elements in j=3
   Similar to Equation (4.25).

7. Load
   Similar to Equation (4.26) with \( i = 1 \rightarrow 4 \) and \( m = 3 \).

4.7.1 Open-loop Study Using The Full HRA Assembly

1. Healthy Condition

Figure 4.20 shows the simulation results of the HRA under healthy condition with external load of 1kg and 20kg. Recall from Figure 4.19 that \( X_{11} \), \( X_{12} \) and \( X_{13} \) are the displacement of actuation elements \( A_{11} \), \( A_{12} \) and \( A_{13} \), respectively. \( X_L \) is the load displacement. \( F_{11} \) is the force exerted by element \( A_{11} \) to the base of element \( A_{12} \) while \( F_{12} \) is the force exerted by element \( A_{12} \) to the base of element \( A_{13} \). \( F_{13} \) is the force exerted by element \( A_{13} \) to the load. \( F_T \) is the total force acting on the load and from Figure 4.19, \( F_T = F_{13} + F_{23} + F_{33} + F_{43} \). Since \( F_{13} = F_{23} = F_{33} = F_{43} \) when no fault present, then \( F_T \) can also be written as \( F_T = 4F_{13} \) (similar to Equation (4.28)). \( I_{11} \), \( I_{12} \) and \( I_{13} \) are the current of actuation element \( A_{11} \), \( A_{12} \) and \( A_{13} \), respectively.

The results show that \( X_{11} = 30.00\text{mm} \), \( X_{12} = 30.18\text{mm} \) and \( X_{13} = 30.53\text{mm} \) while \( X_L = 90.71\text{mm} \) when 1kg load is attached to the HRA. In the case that a 20kg load is used, \( X_{11} = 28.76\text{mm} \), \( X_{12} = 28.93\text{mm} \), \( X_{13} = 29.28\text{mm} \) and \( X_L = 86.97\text{mm} \). In both cases, the displacement of the load is equal to the sum of the displacement of the elements that are connected in series.
For peak output force, when a 1kg load is attached to the system, $F_{11} = 34.67 \text{N}$, $F_{12} = 24.55 \text{N}$, $F_{13} = 3.97 \text{N}$ and $F_T = 15.88 \text{N}$. When a 20kg load is used, $F_{11} = 90.10 \text{N}$, $F_{12} = 81.81 \text{N}$, $F_{13} = 64.95 \text{N}$ and $F_T = 259.80 \text{N}$. In both cases, the total force acting on the load is four times the output force of actuation element $A_{13}$, which satisfy Equation (4.28).

These results are consistent with the expected results based on engineering judgment which are: 1) displacement of the external load is equal to the sum of the displacement of the elements connected in series; and 2) total force acting on the external load is equal to the sum of the output force of the elements pushing/pulling the load.

In Figure 4.20, only results of actuation element $A_{11}$, $A_{12}$ and $A_{13}$ are shown for simplicity. However, actuation elements $A_{21}$, $A_{31}$ and $A_{41}$ have similar results as actuation element $A_{11}$. Actuation element $A_{12}$ represents actuation elements $A_{22}$, $A_{32}$ and $A_{42}$ while actuation elements $A_{23}$, $A_{33}$ and $A_{43}$ have similar results as actuation element $A_{13}$.

Figure 4.20: Simulation results of the HRA under healthy condition with external load of 1kg and 20kg.
2. Effect of Lock-up Fault

The number of lock-up faults tolerable by the 4-by-3 HRA depends on the location of the faults. The worst scenario is that the HRA will fail when all of the elements that are connected in series (e.g. $A_{11}A_{12}A_{13}$) experience lock-up fault simultaneously. On the other extreme, the best outcome is that the HRA can move the load even with 8 elements locked. However, displacement capability will be reduced.

Figure 4.21 shows the simulation results of the HRA when its elements experience lock-up faults. For simplicity, only $X_L$ (load displacement) and $F_T$ (total force acting on the load) are shown.

In Figure 4.21a, 3 actuation elements that are connected in series are subjected to lock-up fault as shown in the diagram. It can be seen that in this condition, the HRA failed to move the external load. The oscillation observed in the output force result is due to the stiffness of the faulty elements.

In Figure 4.21b, 8 actuation elements are subjected to lock-up fault as shown in the diagram. The graph shows that the external load has a displacement of 30.51mm, which is the same as the displacement of a single actuation element. These results show that the HRA can tolerate between 2 and 8 locked elements.

The simulation results again confirm the expectation based on engineering understanding.
(a) 3 elements under lock-up fault.

(b) 8 elements under lock-up fault.

Figure 4.21: Simulation results of the HRA under lock-up faults.
3. Effect of Short-circuit Fault

Short-circuit faults do not depend on the location of the faults the way lock-up faults do. Instead, it depends on the number of actuation elements that fail. As more actuation elements experience short-circuit faults, displacement and force capability are reduced further. In the most optimistic case, up to 9 short-circuit elements can be tolerated with the actuator still able to offer force and displacement (albeit with reduced capability) as shown in Figure 4.22.

In Figure 4.22a, 4 actuation elements (one from each parallel branch) are subjected to short-circuit fault as shown in the diagram. It can be seen that in this condition, both total displacement and total peak force reduced to 49.3mm and 8.56N, respectively.

In Figure 4.22b, 9 actuation elements are subjected to short-circuit faults as shown in the diagram. The results show that total displacement and total peak force reduced further to 22.75mm and 4.02N as more actuation elements experienced short-circuit fault.

These results once again confirm the expectation based on engineering understanding.
CHAPTER 4. ACTUATOR MODELLING

(a) 4 elements under short-circuit fault.

(b) 9 elements under short-circuit fault.

Figure 4.22: Simulation results of the HRA under short-circuit faults.
4.8 Model Validation

In this section, the derived model of a single actuator and the complete 4-by-3 HRA are validated. The validation consists of comparing the model against the actual system output. The validation set-up is shown in Figure 4.23. The validation is conducted in open-loop by applying a square wave input of 4V amplitude to both the system and the model. Displacement readings are taken with the actuator moving forward and backwards without reaching the end stops. Results are shown for validation of one actuation element and for the complete 12-elements HRA system.

![Model validation setup](image)

The results shown in Figure 4.24 show the actual and model output of a single actuation element obtained using parameter values given previously in Table 4.1. The actual output of the actuator was compared against the simulation models output using the well-known coefficient of determination (CoD) function, $R^2_T$. The coefficient is calculated as:
\[ R_T^2 = 1 - \frac{\sigma^2}{\sigma_y^2} \]  

(4.29)

where \(\sigma^2\) is the sampled variance of the model residuals \(e(k)\) and \(\sigma_y^2\) is the sampled variance of the measured output \(y(k)\) about its mean value [108, 109]. A ‘perfect’ fit results in a unity output (or 100%), with anything below zero being uncorrelated.

From Figure 4.24 it can be seen that for the displacement signal, a good fit \((R_T^2 = 90.52\%)\) is obtained between the model and actual output. However, for the current signal, there is a significant difference between the model values and the actual values \((R_T^2 = 23.01\%)\)

![Figure 4.24: Actual and model output of a single actuation element.](image)

Therefore, MATLAB Design Optimization Toolbox was used to obtain better estimates of some of the parameters in order to improve the current fit. The software estimates model parameters by comparing measured data with simulation data generated from Simulink model. It uses least-squares method to compute the error between the measured data and the simulated data. During optimisation, the least-squares method minimizes the error
between the measured and simulated data by changing the parameter values.

The parameters being estimated are: viscous friction, $D$; torque constant, $K_T$; and back emf constant, $K_e$. Armature inductance, $L$ and armature resistance, $R$ are being fixed because their value can be checked by measurement while the inertia, $J$ is fixed because its value is available in the data sheet.

The parameters were being estimated due to the following reasons:

- The value of $D$ is not available in the DC motor and ball screw data sheet so it was calculated using formula from [110] (calculation can be found in Appendix B). Therefore, it was decided to adjust $D$ to cross-check the calculated value.

- The value of $K_T$ is given in the DC motor data sheet, but this value is a general value for the Bühler DC motor. Therefore, it was decided to adjust $K_T$ in order to get a more accurate value for the DC motor model that is used in this work.

- $K_e$ was assumed to have the same value as $K_T$ but in reality, this is not quite the case. So, $K_e$ ought also to be adjusted.

Figure 4.25 shows the measured data used for the parameter optimisation. The top plot is voltage, which is input signal. The middle and bottom plots are displacement and current respectively, which are output signals.

Figure 4.26 shows the optimisation results. Only current and displacement signals were used for the optimization process. The graph shows a good match between the measured and simulated responses.

Table 4.2 lists the parameter values before and after estimation using the design optimisation toolbox. From this table, it can be seen that $D$ has varied by 13.1%, while $K_e$ and $K_T$ have varied by 36.5% and 14.4%, respectively. These results are reasonable as data for motors electrical parameters is often inaccurate [111].
Figure 4.25: Measured data used for design optimisation.

Figure 4.26: Results of parameter optimisation.
### 4.8. Model Validation

Table 4.2: Value of actuator parameters used in the simulation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Unit</th>
<th>Before optimization</th>
<th>After optimization</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Armature resistance, $R$</td>
<td>Ω</td>
<td>0.4</td>
<td>0.4</td>
<td>NO</td>
</tr>
<tr>
<td>Armature inductance, $L$</td>
<td>mH</td>
<td>8</td>
<td>8</td>
<td>NO</td>
</tr>
<tr>
<td>Inertia, $J$</td>
<td>kgm$^2$</td>
<td>8.92x10$^{-5}$</td>
<td>3.686x10$^{-5}$</td>
<td>NO</td>
</tr>
<tr>
<td>Viscous friction, $D$</td>
<td>Nm/rads$^{-1}$</td>
<td>9.54x10$^{-1}$</td>
<td>8.298x10$^{-1}$</td>
<td>NO</td>
</tr>
<tr>
<td>Motor back emf constant, $K_e$</td>
<td>Nm/A</td>
<td>9.54x10$^{-1}$</td>
<td>8.298x10$^{-1}$</td>
<td>YES</td>
</tr>
<tr>
<td>Motor torque constant, $K_T$</td>
<td>Nm/A</td>
<td>2.7x10$^{-2}$</td>
<td>3.089x10$^{-2}$</td>
<td>YES</td>
</tr>
<tr>
<td>Ball screw lead, $l$</td>
<td>mm/rev</td>
<td>6</td>
<td>6</td>
<td>NO</td>
</tr>
<tr>
<td>Screw damping coefficient, $c$</td>
<td>Ns/m</td>
<td>26.30x10$^3$</td>
<td>201.06x10$^6$</td>
<td>NO</td>
</tr>
<tr>
<td>Screw stiffness, $k$</td>
<td>N/m</td>
<td>201.06x10$^6$</td>
<td>201.06x10$^6$</td>
<td>NO</td>
</tr>
<tr>
<td>Actuator and motor mass, $M$</td>
<td>kg</td>
<td>2</td>
<td>2</td>
<td>NO</td>
</tr>
</tbody>
</table>
Having found the modified set of parameters for the model, the validation experiment is repeated using the full 4-by-3 HRA assembly. Figure 4.27 shows the results and it is clear that the model gives an excellent match to the real system in terms of both current and position. The current fit is 90.13% while the displacement fit is 98.73%.

4.9 Conclusion

In this chapter, mathematical equations that represent a single-element was first derived. After that, the modelling was extended to the development of series and parallel architecture that eventually allow the construction of n-by-m models. Then, open loop-studies were conducted in simulation environment in the healthy and faulty conditions to validate the model qualitatively. The simulation results show that series configuration increases travel capability while parallel configuration increases force capability above the capability of an individual element and the results are consistent with the expected results based on engineering judgement.
4.9. **CONCLUSION**

After that, the derived model was validated quantitatively by directly comparing it to the actual system (i.e. comparing simulation and experimental responses). First, the actual and model results of a single actuation element were compared and it was found that there is a significant difference between the actual and model current output results. So, MATLAB parameter optimisation was used to estimated 3 of the 10 actuator parameters. After optimisation, validation using both the single actuation element and the 4-by-3 HRA (to represent the n-by-m model) shows a good match between the actual and model results.

The element models and the 4-by-3 model will be used as the foundation of the control and condition monitoring studies in this thesis. The n-by-m model allows future work by other researchers to study any case of EMA based HRA.
Chapter 5

Classical Controller Design

5.1 Introduction

Classical control is a conventional control methodology using Lead/Lag, proportional (P), proportional-integral (PI) or proportional-integral-derivative (PID) controller. These type of controllers are widely used in industrial processes not only due to their functional simplicity (they can be operated in a simple, straightforward manner), but also due to their good performance in a wide range of operating conditions [112,113].

Figure 5.1 shows the classic single-input single-output (SISO) closed-loop system. The plant, $G(s)$ could be any physical system, but in this case is the HRA. The plant has an input $u$ and an output $y$. Again, in the case of the HRA of interest, the input $u$ is voltage and the output $y$ is position. The command signal, $r$ is the position demand. So, the objective of the control system is to make the actual output as close as possible to the command signal, $y \approx r$. In the case of the HRA, the objective is to make the actual position follow the position demand.

The actual output is compared to the demand signal to obtain the error signal, $e = r - y$ that will be fed to the controller to tell it how far apart these two numbers are. The main concern of the controller design process is to select the controller, $K(s)$ that will determine the amount of control action, $u$, required to make $y \approx r$.

The classical controller design usually starts with the designer defining
the control design requirements: time response (i.e. rise time, overshoot, settling time and steady-state error); and frequency response (i.e. phase margins and gain margins). The controller gain is then tuned so that the system’s time and frequency response meet these design requirements. Controller tuning is the process of obtaining the controller parameters or gain, such as the proportional gain, $K_p$ or the integral gain, $K_i$ to meet the predefined control design requirements.

The simplest classical control methodology is the proportional (P) controller that has only one control parameter, the proportional gain, $K_p$, that changes the output according to the error [114]. P controller will reduce rise time and steady-state error [115]. If the P controller is not sufficient to meet the design requirements, than a more complex controller should be used which includes: proportional-integral (PI), proportional-derivative (PD), and proportional-integral-derivative (PID) controller.

The mathematical model of the HRA, presented previously in Chapter 4, is used for controller design and validated on the experimental rig. In this chapter, the design of a classical controller to study the performance of the HRA is discussed.

Using classical control, there are three controllers designed under two different structures; local and global position feedback structures as summarised in Table 5.1.

Table 5.1: Classical controller

<table>
<thead>
<tr>
<th>Control Structure</th>
<th>Controller</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local</td>
<td>Proportional</td>
<td>Local P</td>
</tr>
<tr>
<td></td>
<td>Proportional plus Integral</td>
<td>Local PI</td>
</tr>
<tr>
<td>Global</td>
<td>Proportional plus Integral</td>
<td>Global PI</td>
</tr>
</tbody>
</table>

Figure 5.1: Classic closed-loop system.
This chapter is structured as follow: Section 5.2 discusses in detail the local and global control structure. Section 5.3 explains the time and frequency response requirements of the controller design. Section 5.4 explains the travel and force capability of the full 4-by-3 HRA assembly. In section 5.5, the HRA setup for controller design is explained. Controller design for a single actuation element, 3 actuation elements in series and the full HRA assembly is presented in Section 5.6, 5.7 and 5.8, respectively. This chapter ends with a conclusion in section 5.9.

5.2 Control Structure Block Diagram

Figures 5.2 and 5.3 depict the block diagram of the local and global position feedback control structure, respectively. The solid lines represent the mechanical connection between the actuation elements and between the actuation element and the external load, while the dashed lines represent the control system connection.

$A_{11}, A_{12},...,A_{43}$ are the actuation elements, and $X_{11}, X_{12},...,X_{43}$ are their measured positions. $e_{11}, e_{12},...,e_{43}$ are the errors between the measured position and the demand position, $X_d$ while $V_{11}, V_{12},...,V_{43}$ are the input voltage of the actuation elements. $X_L$ refers to the position of the external load.

For the local control structure shown in Figure 5.2, each actuation element has its own controller, $k$, that takes the actuation element’s position as the feedback. Also note that the demand position, $X_d$ is multiplied by 1/3 because there are 3 elements connected in series and total displacement will be the sum of the displacement of these serial elements. This control structure is more complex because there will be 12 controllers required to drive the full HRA assembly.

Conversely, the global control structure shown in Figure 5.3 is much simpler because the structure employs a single controller, $k$, that takes the total displacement, $X_L$ as the feedback to control the whole assembly.
Figure 5.2: Block diagram of the local position feedback control structure.
Figure 5.3: Block diagram of the global position feedback control structure.
CHAPTER 5. CLASSICAL CONTROLLER DESIGN

5.3 Control Requirements

In a classical control design, there are two categories of requirements that need to be specified prior to any design work: time response and frequency response requirements. The time response requirements include settling time (ST), rise time (RT), overshoot (OS) and steady-state error (SSE) while the frequency response includes gain margin (GM) and phase margin (PM).

As mentioned in Chapter 2, the work in this thesis is intended to be applied as an aircraft aileron actuation system. Ailerons are small hinged sections on the outboard portion of the left and right wings of an aircraft. The ailerons are used to bank the aircraft, which means to cause one wing tip to move up and the other wing tip to move down [116,117]. The banking that is created by the ailerons caused the aircraft to turn.

In Table 2.1, it can be seen that aileron actuator requires <10% overshoot. Furthermore, it is desirable for the actuator to move to the expected position (zero steady-state error). According to the capability of the actuators given in the data sheet, both settling time and rise time are chosen to be less than 0.5s and less than 0.2s, respectively.

According to [118], it is a rule of thumb to use phase margin of 30° to 60° in order to obtain a good closed-loop transient response with gain margin of 6 to 15dB. However, for aircraft flight control, it is a common practice to use phase margin of 45° with gain margin of 6dB due to a pilot usually being ‘in the loop’ [118, 119]. Based on all these assessments, the control requirements can be formulated as shown in Table 5.2.

This is a set of requirements chosen to demonstrate the applicability of the HRA to the aileron application and should not be seen as limitations of the HRA when considered in a wider industrial context.

5.4 Travel and Force Capability

Based on the configuration of the HRA, it is possible to deduce the nominal capabilities of the HRA.

Each actuation element has a travel capability of 50mm. Since there are
Table 5.2: Control requirements for the classical controller design

<table>
<thead>
<tr>
<th>Control Requirements</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Response</strong></td>
<td></td>
</tr>
<tr>
<td>Settling Time (ST)</td>
<td>&lt;0.5s</td>
</tr>
<tr>
<td>Rise Time (RT)</td>
<td>&lt;0.2s</td>
</tr>
<tr>
<td>Overshoot (OS)</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>Steady-state error (SSE)</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency Response</strong></td>
<td></td>
</tr>
<tr>
<td>Gain Margin (GM)</td>
<td>&gt;6dB</td>
</tr>
<tr>
<td>Phase Margin (PM)</td>
<td>&gt;45°</td>
</tr>
</tbody>
</table>

3 elements connected in series, the HRA has a travel capability of 150mm. However, the position demand is set to 100mm (based on the aileron actuator requirements in Table 2.1) which means the HRA has 50% more travel capability than the application requirement.

In terms of force capability, each actuation element has a force capability of 80N. Since there are 4 parallel branch, the HRA has a force capability of 320N. The aileron actuator force requirement is 80N (refer to Table 2.1), which means the HRA has 4 times greater force capability than the application requirement to account for unforseen loads.

### 5.5 HRA Setup for Controller Design

Ailerons usually work in opposite directions; when the right aileron is deflected upward, the left is deflected downward, and vice versa which also means that when the right aileron actuator moves forward, the left aileron actuator moves backward at the same time. Therefore, for the purpose of controller design in this thesis, the individual actuation elements are positioned at mid-point, which is 25mm (load initial position is 75mm) as illustrated in Figure 5.4. This setup will allow the HRA to move forward to demonstrate upward deflection and move backward for downward deflection of the aileron. For simplicity, upward and downward deflections are assumed to be the same range.

The goal is to drive the load from its initial position of 75mm to a final
position of 100mm (25mm displacement) in the positive direction, and from 75mm to 50mm final position in the negative direction.

Figure 5.4: HRA setup for controller design.

5.6 Controller Design Using Single Element

This section deals with designing a P and PI controller to drive a single actuation element. In both cases, heuristic tuning is used to determine the suitable proportional gain, $K_p$, and integral gain, $K_i$, to drive the actuation element without causing saturation in the current signal. The design process is explained in detail in the following subsections.

5.6.1 Proportional Controller

The proportional (P) controller is designed by selecting the proportional gain ($K_p$) to give appropriate time and frequency responses to meet the control requirements. Initially, $K_p$ is set to 1V/m and the Nichols chart is plotted as shown in Figure 5.5. The plot shows the gain to be -59.7dB at -135° which is corresponding to 45° PM. Therefore, the maximum $K_p$ that can be used to drive the single actuation element is calculated as:

$$K_{p(max)} = 10^{-\frac{\text{Gain}}{20}} = 10^{-\frac{(-59.7\,\text{dB})}{20}} = 966\,\text{V/m} \quad (5.1)$$

Then, a P controller with $K_p = 966\,\text{V/m}$ is used to drive the single
actuation element in simulation setting and the result is shown in Figure 5.6. From this figure, it can be seen that when the maximum value of $K_p$ is used, the peak output current is saturated at 10A. This result is not practical because the maximum current that the motor driver (Sabertooth motor driver) can supply to the DC motor is 10A and it can only supply this peak current for a short period of time. So, further tuning was done to find the value of $K_p$ suitable to drive the single actuation element without causing current saturation.

This was done by reducing the value of $K_p$ until a satisfactory time response results was obtained as shown in Figure 5.7. This figure shows the simulation and experimental results of the single actuation element driven using a P controller with $K_p = 130\text{V/m}$. It can be seen that the peak output current is 10A but no saturation is observed. However, when a P controller is used, the actuation element cannot reach the expected position (SSE of 1.33mm is recorded). Therefore, integral action was introduced to improve the SSE performance of the single actuation element.
Figure 5.5: Nichols plot with $K_p=1\text{V/m}$. 
5.6. CONTROLLER DESIGN USING SINGLE ELEMENT

Figure 5.6: Simulation results showing time response of the single actuation element driven using a P controller with $K_p = 966 \text{V/m}$.

Figure 5.7: Simulation and experimental results showing time response of the single actuation element using P controller with $K_p = 130 \text{V/m}$. 
5.6.2 Proportional-Integral Controller

As shown in the previous results, the P gain alone was not enough to fulfil the design requirements. Therefore, integral action requires adding. The integral gain in the proportional-integral (PI) controller has the effect of removing steady-state errors [115]. However, it has the disadvantage of adding phase lag into the system that can cause instability [120]. The controller can be expressed as shown in Equation 5.2:

\[
K(s) = \frac{K_{pi}(1+s\tau_i)}{s\tau_i}
\]  

(5.2)

where \( K_{pi} \) is the overall controller gain, \( \tau_i \) is the integrator time constant and \( s \) is the Laplace variable.

The integral gain addition was performed whilst leaving the P gain at its current level of \( K_p = 130V/m \). At this value the crossover frequency of the system (i.e when the gain crosses below 0 dB) is approximately 10.8 rad/s as shown in Figure 5.8. The break frequency of the PI controller requires selecting at a frequency much lower than this and shown here is a sample calculation to determine the PI controller transfer function.

1. Let \( K_{pi} = 130V/m \)

2. Choose \( \frac{1}{\tau_i} \) that is much lower than \( \omega_i \)

\[
\tau_i = \frac{10}{\omega_i} = \frac{10}{10.8 \text{ rad/s}} = 0.925925925s \approx 0.926s
\]

3. Substitute \( K_{pi} \) and \( \tau_i \) into Equation 5.2

\[
K(s) = \frac{K_{pi}(1+s\tau_i)}{s\tau_i} = \frac{130(1+0.926s)}{0.926s} = \frac{120.38s+130}{0.926s}
\]

The PI controller was tested in both simulation and real-time experimental setup. Further tuning was done and a PI controller with \( \tau_i = 5/\omega_i \) gives the best results as shown in Figure 5.9.

This graph shows the time response of the single actuation element driven using a PI controller with transfer function of \( K(s) = \frac{60.19s+130}{0.463s} \). It can be seen that when a PI controller is used, the system can reach the expected position. The results also show a good correlation (displacement’s \( R_T^2 \) is approximately 99% while current’s \( R_T^2 \) is approximately 98%) between the actual and the model output. Which further validate the model
5.6. CONTROLLER DESIGN USING SINGLE ELEMENT

Figure 5.8: Bode plot.

and gives confidence in its use to design controller and predict the closed loop responses.

Figure 5.10 shows the frequency response of the single actuation element model using a P and PI controller while Table 5.3 lists both the time and frequency responses of the single actuation element as compared to the control design requirements. It can be seen that with a PI controller, all of the control design requirements are met.

Note that the PM and GM is very large (for both P and PI controller) compared to the design requirements. However, these values cannot be reduced further because it will make the input voltage and output current saturated at $\pm 12V$ and $\pm 10A$, respectively.
Figure 5.9: Time response of the single actuation element using PI controller.

Figure 5.10: Frequency response of the single actuation element model using P and PI controller.
### Table 5.3: Time and frequency responses of the single actuation element using P and PI controller

<table>
<thead>
<tr>
<th>Responses</th>
<th>P controller</th>
<th>PI controller</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Response</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.2</td>
<td>0.2</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.09</td>
<td>0.1</td>
<td>&lt;0.2</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>&lt;10%</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>1.33</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency Response</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>35.1</td>
<td>34.7</td>
<td>&gt;6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>81.0</td>
<td>69.7</td>
<td>&gt;45</td>
</tr>
</tbody>
</table>

*The time response values were obtained from experimental data while the frequency response values were obtained from simulation data using MATLAB’s Linear Analysis Toolbox. Refer to MATLAB’s documentation for details on how to perform frequency-domain analysis using the Linear Analysis Toolbox.*
CHAPTER 5. CLASSICAL CONTROLLER DESIGN

5.7 Controller Design Using 3 Elements in Series

The second step is to design a controller to drive three actuation elements connected in series as shown in Figure 5.11. Based on the results of the single actuation element where steady state error was present, a P controller will not be used to drive the series elements. Rather, a PI controller will be designed under two different structures; local and global position feedback structures as described in Section 5.2.

The local and global PI controller will be used to study the actuators performance under both healthy and faulty conditions.

Figure 5.11: Schematic diagram of 3 actuation elements connected in series.

5.7.1 Local PI Controller

In the local control structure, each actuation element has its own controller that uses the element’s displacement as feedback. Since the objective of the controller is to drive the assembly to a total displacement of 25mm, each actuation element will have a position demand of 8.33mm.

Healthy Condition

Figure 5.12 shows the actuation elements time response in the healthy condition (all elements are working) when local PI controller with transfer function of $K(s) = \frac{60.19s+130}{0.463s}$ (note that this is the same controller that is used for the single actuation element) is used while Figure 5.13 shows the total displacement. It can be observed from Figure 5.12 that each actuation element has displacement of approximately 8.33mm to produce a total
displacement of 25mm.

Peak input voltage of each actuation element is ±5V while the peak output current is ±10A depending on the direction of movement. As expected, the results show slight difference between the actual and model results which is due to modelling error.

Figure 5.14 shows the frequency response of the actuation elements while Table 5.4 summarizes the time and frequency responses of the actuation elements under healthy condition. It can be seen that all of the control requirements are satisfied.

Figure 5.12: Time response of 3-elements-in-series using local PI controller under healthy condition.
CHAPTER 5. CLASSICAL CONTROLLER DESIGN

Figure 5.13: Total displacement of 3-elements-in-series using local PI controller under healthy condition.

Figure 5.14: Frequency response of 3-elements-in-series model using local PI controller under healthy condition.
Table 5.4: Time and frequency responses of 3-elements-in-series using local PI controller under healthy condition

<table>
<thead>
<tr>
<th>Responses</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.20</td>
<td>0.20</td>
<td>0.35</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.10</td>
<td>0.13</td>
<td>0.14</td>
<td>&lt;0.2</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>&lt;10</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency Response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>33.3</td>
<td>33.2</td>
<td>33.4</td>
<td>&gt;6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>69.8</td>
<td>71.3</td>
<td>70.4</td>
<td>&gt;45</td>
</tr>
</tbody>
</table>
Faulty Conditions

In this thesis, two types of faults are studied: lock-up fault and short-circuit fault. In the xPC Target/Simulink model that is used to run the real-time experiments, lock-up fault is demonstrated by setting the position demand of the faulty element/s to be 25mm at all time (so that it is ‘locked’ at mid-point). The short-circuit fault, on the other hand, is demonstrated by multiplying the input voltage of the faulty element/s with zero to demonstrate a fault with the electrical connection.

Figures 5.15 and 5.16 shows the actual and model output when one or more of the actuation elements are subjected to lock-up and short-circuit faults, respectively. For simplicity, only total displacement is shown. From both graphs, it can be seen that when a local PI controller is used to drive the system (3 elements in series), the tracking performance of the system is reduced if one or more of its elements are faulty.

This is due to the structural limitation of the local controller. Even though the actuation elements are capable of moving by 25mm, their movement is limited to 8.33mm due to the position demand setting of the local controller. Therefore, when one or more elements are subjected to lock-up or short-circuit fault, the healthy element’s displacement is limited and as such, reduced the tracking performance of the system.

Table 5.5 summarizes the time and frequency responses of the actuator assembly under healthy and faulty conditions and it can be seen that under faulty conditions, the SSE is not zero. To overcome the limitation of the local controller, the global control structure that uses total displacement as feedback is introduced.
5.7. CONTROLLER DESIGN USING 3 ELEMENTS IN SERIES

Figure 5.15: Graph showing effect of lock-up faults to system with local PI controller.

Figure 5.16: Graph showing effect of short-circuit faults to system with local PI controller.
Table 5.5: Time and frequency responses of 3-elements-in-series using local PI controller under healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Lock-up fault</th>
<th>Loose fault</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 element</td>
<td>2 elements</td>
<td>1 element</td>
</tr>
<tr>
<td></td>
<td></td>
<td>locked</td>
<td>locked</td>
<td>short-circuit</td>
</tr>
<tr>
<td>Time response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.12</td>
<td>0.15</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>8.6</td>
<td>16.1</td>
<td>8.6</td>
</tr>
<tr>
<td>Frequency response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>33.3</td>
<td>33.2</td>
<td>33.4</td>
<td>33.2</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>69.8</td>
<td>71.3</td>
<td>70.4</td>
<td>71.3</td>
</tr>
</tbody>
</table>

*These are the time and frequency responses of A1.
5.7. CONTROLLER DESIGN USING 3 ELEMENTS IN SERIES

5.7.2 Global PI Controller

In the global control structure, all three elements share the same controller that takes the total displacement as feedback. Therefore, the position demand is set to 25mm which means each actuation element has the potential to move by 25mm.

Healthy Condition

Following the same design procedure introduced in Section 5.6.2, a global PI controller with transfer function of \( K(s) = \frac{47.47s + 40}{1.187s} \) is used. Figure 5.17 shows the actuation elements time response in the healthy condition while Figure 5.18 shows the total displacement.

From Figure 5.17, it can be seen that both A2 (dotted line) and A3 (dash-dotted line) have approximately the same displacement which is 7.2mm, while A1 (dashed line) have a displacement of approximately 10.6mm to generate total displacement of 25mm as shown in Figure 5.18. Slight difference between the actual and model results can be observed, which occur due to modelling error.

The peak input voltage of all the actuation elements are ±5V while the peak output current is ±10A.

Figure 5.19 shows the frequency response of the 3-elements-in-series actuation system using global PI controller. The time and frequency responses of the system with global PI controller under healthy condition is summarized in Table 5.6 and it can be seen that all of the control requirements are met.
Figure 5.17: Time response of 3-elements-in-series using global PI controller under healthy condition.

Figure 5.18: Total displacement of 3-elements-in-series using global PI controller under healthy condition.
Table 5.6: Time and frequency responses of 3-elements-in-series using global PI controller under healthy condition

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time Response</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.20</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.12</td>
<td>&lt;0.2</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>&lt;10</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>Frequency Response</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.1</td>
<td>&gt;6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>71.8</td>
<td>&gt;45</td>
</tr>
</tbody>
</table>

Figure 5.19: Frequency response of 3-elements-in-series using global PI controller under healthy condition.
Faulty Conditions

Figures 5.20 and 5.21 show the total displacement (both actual and model) when one or more of the actuation elements are subjected to lock-up and short-circuit faults, respectively. From both figures, it can be seen that when a global controller is used to drive the actuation elements, the system can go to the expected position even under faulty condition. However, the performance is slower compared to when the system is in the healthy condition.

For example, if one element is locked, the SSE is zero but ST increases from 0.2s to 0.5s while RT increases from 0.12s to 0.2s. Table 5.7 lists the time and frequency responses of the system under healthy and faulty conditions. Overall it can be seen that the frequency response of the system satisfied the control design requirements. However, in the faulty condition, ST and RT fail to satisfy the control design requirements.

Note that the results of lock-up faults and short-circuit faults are very similar because the load did not produce sufficient force to back drive the actuation elements in the case of short-circuit faults.

Figure 5.20: Time response of 3-elements-in-series using global PI controller under lock-up faults.
Figure 5.21: Time response of 3-elements-in-series using global PI controller under short-circuit faults.
Table 5.7: Time and frequency responses of 3-elements-in-series using global PI controller under healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Lock-up fault</th>
<th>Loose fault</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1 element locked</td>
<td>2 elements locked</td>
<td>1 element short-circuit</td>
<td>2 elements short-circuit</td>
</tr>
<tr>
<td>Time response</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>ST, (s)</td>
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<td>0.50</td>
<td>1.00</td>
<td>0.60</td>
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<tr>
<td>RT, (s)</td>
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<td>0.20</td>
<td>0.49</td>
<td>0.24</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>Frequency response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.1</td>
<td>37.5</td>
<td>43.6</td>
<td>37.6</td>
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<tr>
<td>PM, (°)</td>
<td>71.8</td>
<td>70.9</td>
<td>62.6</td>
<td>70.7</td>
</tr>
</tbody>
</table>
5.8 Controller Design Using HRA

The final test of the classical approach is to design a controller to drive the full HRA assembly. Based on the results obtained in Section 5.7, only global PI controller is used to study the HRA performance under both healthy and faulty conditions.

Healthy Condition

Following the same design procedure introduced in Section 5.6.2, a global PI controller with transfer function of $K(s) = \frac{94.876s+40}{2.3719s}$ is used to drive the full HRA assembly. Figure 5.22 shows the time response of three of the HRA’s elements that are connected in series, which are elements 1, 2 and 3 ($A_{11}$, $A_{12}$ and $A_{13}$). From the figure, it can be seen that $A_{12}$ (dotted line) and $A_{13}$ (dash-dotted line) have similar displacement, which is approximately 7.8mm while $A_{11}$ (dashed line) have a displacement of approximately 9.2mm. The other actuation elements have similar response as shown in Figure 5.22.

Peak input voltage is in the range of ±5V and peak output current is in the range of ±10A for all of the actuation elements. Figure 5.23 shows the total displacement of the HRA. Again, a slight difference which is due to modelling error can be observed between the actual and model results.

Figure 5.24 shows the frequency response of the HRA with global PI controller while Table 5.8 summarizes the time and frequency responses of the HRA under healthy condition. It can be seen that all of the control design requirements are met.
CHAPTER 5. CLASSICAL CONTROLLER DESIGN

Figure 5.22: Time response of the full HRA using global PI controller under healthy condition.

Figure 5.23: Total displacement of the full HRA using global PI controller under healthy condition.
Table 5.8: Time and frequency responses of the full HRA using global PI controller under healthy condition

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Response</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
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<td>&lt;0.5</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.15</td>
<td>&lt;0.2</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>&lt;10</td>
</tr>
<tr>
<td>SSE, (mm)</td>
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<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency Response</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.0</td>
<td>&gt;6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>68.2</td>
<td>&gt;45</td>
</tr>
</tbody>
</table>

Figure 5.24: Frequency response of the full HRA using global PI controller under healthy condition.
Faulty Conditions

Figure 5.25 shows the HRA performance when its elements are subjected to lock-up faults. In Figure 5.25a, 3 elements are locked as shown in the diagram and the results show that this condition causes the whole assembly to be locked which is a common problem in the traditional purely parallel redundancy actuation system.

In Figure 5.25b, 8 elements are locked as shown in the diagram and the results show that the HRA can move to the expected position but slower compared to when it was in the healthy condition.

Figure 5.26 shows the HRA performance when its elements are subjected to short-circuit faults. In Figure 5.26a, 4 elements experienced short-circuit fault while in Figure 5.26b, 9 elements experience short-circuit as shown in the diagrams. From both graphs, it can be seen that when 4 of the HRA’s elements are subjected to short-circuit fault, its performance is almost the same as the healthy condition. When 9 elements are subjected to short-circuit fault, the HRA’s performance is slower compared to the healthy condition.

Table 5.9 summarizes the responses of the HRA under both healthy and faulty conditions. Overall it can be seen that in all cases, the frequency response of the system satisfied the control design requirements. In the faulty conditions, ST and RT are larger than the required values. Also, when 3 elements that are connected in series are subjected to lock-up fault, SSE is not zero because the whole assembly is locked.
5.8. CONTROLLER DESIGN USING HRA

(a) 3 elements under lock-up fault.

(b) 8 elements under lock-up fault.

Figure 5.25: Time response of the full HRA using global PI controller under lock-up faults.
(a) 4 elements under short-circuit fault.

(b) 9 elements under short-circuit fault.

Figure 5.26: Time response of the full HRA using global PI controller under short-circuit faults.
Table 5.9: Time and frequency responses of the full HRA using global PI controller under healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Lock-up fault</th>
<th>Loose fault</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3 elements locked</td>
<td>8 elements locked</td>
<td>4 element short-circuit</td>
<td>9 elements short-circuit</td>
</tr>
<tr>
<td><strong>Time response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.40</td>
<td>-</td>
<td>0.70</td>
<td>0.50</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.15</td>
<td>-</td>
<td>0.39</td>
<td>0.23</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>25</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency response</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.0</td>
<td>42.2</td>
<td>43.6</td>
<td>37.5</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>68.2</td>
<td>90.0</td>
<td>55.4</td>
<td>65.3</td>
</tr>
</tbody>
</table>
5.9 Conclusion

In this chapter, a study of the application of closed loop classical control to the HRA has been conducted. Two types of controllers; P and PI under two different structures: local and global control structures were designed. These controllers were then used to study the performance of three different actuator assemblies: a single element, 3-elements-in-series and the full HRA under healthy and faulty conditions. Results of simulation and real time experiments were recorded and compared.

The P controller that was tested with the single actuation element is not sufficient to drive the element to the desired position. This is because, the proportional gain cannot be increased further without causing the current to be saturated at ±10A (recall that the motor driver can supply maximum of ±10A current to the DC motor). On the other hand, the PI controller is able to drive the single actuation element to the desired position and satisfies all of the control design requirements.

The performance of the 3-elements-in-series was studied using local PI and global PI controller. Although the results show that under healthy condition, both controllers were able to satisfy all of the control design requirements, the local PI controller was not sufficient to drive the assembly to the desired position under lock-up and short-circuit faults. This is due to the structural limitation of the local control structure where the actuation element’s displacement were restricted to only 8.33mm (due to the position demand setting) even though they are physically capable of giving a displacement of 25mm.

Of course, this could be overcome if exact knowledge of faults was available and reconfiguration of the controller structure was permitted. In the absence of such a reconfiguration scheme, fixed local PI control cannot adequately handle faults. In contrast, the structurally simpler global PI controller is found to satisfy all the requirements when healthy and to be capable of providing continued operation with zero steady state error after a fault has occurred.

For the complete (12 elements) HRA, the results show that the assembly has the ability to tolerate up to 8 locked and 9 short-circuit elements. How-
ever, the HRA failed when 3 of its elements that are connected in series were subjected to lock-up fault. This failure is due to the mechanical structure of the HRA where the other parallel branch cannot push/pull the faulty branch. This is a common problem with the purely parallel redundancy actuator.

In all cases, the results show a good match between the actual output obtained from real time experiments and the model output obtained from simulation using the mathematical model derived in Chapter 4. This gives confidence in the use of this model for control design analysis for both failure designs in future and possibly for simulation of other structures with greater number of elements.
Chapter 6

$\mathcal{H}_\infty$ Controller Design

6.1 Introduction

$\mathcal{H}_\infty$ control theory was introduced by Zames [121] in the early 1980s. This technique combines both the frequency- and time-domain approaches and influenced the trend of control system development in the 1980s and 1990s. The $\mathcal{H}_\infty$ control method has the ability to provide better robustness to inherent uncertainty in the system [82-84] which is advantageous for the HRA.

The HRA employs multiple actuation elements and all of these elements are assumed to be identical during the modelling process. However, in reality, there will be slight differences in the parameters of every individual element and these introduce parameter uncertainties to the system. Moreover, when faults are injected to one or more of the elements, the dynamics of the system will change which causes time varying uncertainties.

Therefore, the aim of this chapter is to investigate the performance of the full HRA assembly using $\mathcal{H}_\infty$ controller in the healthy and faulty conditions. In this chapter, $\mathcal{H}_\infty$ controller design procedure implemented for the HRA is explained. Then, results of testing the $\mathcal{H}_\infty$ controller to 3-elements-in-series and the full HRA assembly in the healthy and faulty conditions are presented and discussed.

This chapter is structured as follow: Section 6.2 discusses in detail the concept of sensitivity and weighting functions. Section 6.3 explains about
the parametric uncertainty that arise due to the difference in the linear EMA’s physical parameters when it is in healthy and faulty conditions. Section 6.4 explains the procedure (a six-step procedure) used in the $\mathcal{H}_\infty$ controller design. Results of the $\mathcal{H}_\infty$ control design applied to 3-elements-in-series configuration are presented in Section 6.5. In Section 6.6, performance of the full HRA assembly with $\mathcal{H}_\infty$ controller is discussed. The chapter ends with conclusion in Section 6.7.

## 6.2 Sensitivity and weighting functions

The $\mathcal{H}_\infty$ paradigm formulated an optimization design process where the uncertainty of the plant was considered explicitly in the design process and the key component in the $\mathcal{H}_\infty$ control is the sensitivities. These are transmission paths of the extended feedback structure (Figure 6.1) and are weighted with user selected functions. Minimisation of weighted sensitivities to ensure the peak transmission is dissipative, is the core of the $\mathcal{H}_\infty$ paradigm.

Consider the extended output feedback system shown in Figure 6.1. This system could represent either a single-input single-output (SISO) system or a multiple-input multiple-output (MIMO) system. In this figure, $G(s)$ represents the plant dynamics, $K(s)$ denotes the controller dynamics while $s$ is the Laplace variable. The inputs to the system are: $r(s)$, reference command signals; $d(s)$, disturbance inputs; and $n(s)$, measurement sensor noise. The output variables are given as $y(s)$ while $e(s)$ and $u(s)$ represent the error signals and control signals, respectively.

From Figure 6.1, six unique closed-loop input-output transmission paths can be derived. Each is known as a sensitivity function because it is ‘sensitive’ to a particular error signal. The frequency response of each of these sensitivity functions can be shaped with a suitable weighting function and will have impact on the controller behaviour. The most commonly utilised sensitivity functions are the primary, control and complementary sensitivities. However, the number of sensitivities chosen usually depend on the application and its characteristic requirements.

The three main sensitivity functions are presented next and are described
for SISO system because the full HRA assembly is described as a SISO (voltage as input signal and linear displacement as output signal) for the controller design purpose.

Figure 6.1: Extended disturbance system [100].
6.2.1 Primary sensitivity function

The primary sensitivity function describes the output $y(s)$, as a function of the disturbance input, $d(s)$. It also defines the response of the tracking error $e(s)$ to the reference input $r(s)$. Considering the closed loop transfer function from the reference signal $r(s)$, to the tracking error $e(s)$, the transfer function is given by:

$$ e(s) = \frac{1}{1 + K(s)G(s)} r(s) \quad (6.1) $$

where the primary sensitivity function is then defined by:

$$ S(s) = \frac{e(s)}{r(s)} = \frac{1}{1 + K(s)G(s)} \quad (6.2) $$

This function is considered to mainly drive the transient performance of the system, and is required when designing for regulation and tracking as it is the transmission between $r(s)$ and $e(s)$. For good command following and disturbance rejection, it is desirable to keep $S(s)$ small [122].

6.2.2 Control sensitivity function

The control sensitivity is considered as the closed loop transmission from the command or disturbance signal $r(s)$, to the controller output $u(s)$, with the transfer function formed as:

$$ u(s) = \frac{K(s)}{1 + K(s)G(s)} r(s) \quad (6.3) $$

where the control sensitivity function is then defined as:

$$ R(s) = \frac{u(s)}{r(s)} = \frac{K(s)}{1 + K(s)G(s)} \quad (6.4) $$

This function is generally seen as a limit to the amount and rate of control action used. It is desirable to keep the control signals small, as to not saturate the servomechanism [122].

The corresponding weighting function can be chosen as a linear gain across the entire frequency rate to ensure control limits are not breached at
any point in the frequency range, by scaling the output of the system to a usable level [123].

6.2.3 Complementary sensitivity function

The complementary sensitivity relates the output of the system, $y(s)$ to the input reference signal, $r(s)$. It also specifies how the output $y(s)$ is affected by the noise, $n(s)$. Considering the closed-loop transfer function from the reference signal $r(s)$ to the output $y(s)$, the transfer function is given by:

$$y(s) = \frac{K(s)G(s)}{1 + K(s)G(s)} r(s) \quad (6.5)$$

where the complementary sensitivity function is then defined as:

$$T(s) = \frac{y(s)}{r(s)} = \frac{K(s)G(s)}{1 + K(s)G(s)} \quad (6.6)$$

The complementary sensitivity is chosen so that the system is robust to known and predicted uncertainty in the model chosen for the design of the controller [123]. To make the system robust to unmodeled dynamics as well as for noise attenuation, it is desirable to keep $T(s)$ small [122]. Of course, for good tracking one would want $T(s) = 1$ (at the required frequencies).

6.2.4 Trade off between $S(s)$ and $T(s)$

Overall, it is desirable to keep both $S(s)$ and $T(s)$ small for small tracking error and good robustness. However, it is very difficult to keep both small over the whole frequency range because of the $S + T = 1$ constraint. Therefore, the frequency demand of one sensitivity must be traded off against those of another. Meaning, the primary sensitivity must be traded off against the demand of the complementary sensitivity.

It is well known and confirmed by [122] that, reference signals and disturbances usually happen at low frequencies, while sensor noise and modelling errors mostly occur at high frequencies. So the trade off is to make $|S(j\omega)|$ small at low frequencies and $|T(j\omega)|$ small at high frequencies.
6.2.5 $\mathcal{H}_\infty$ Control Formulation

$\mathcal{H}_\infty$ control is a design technique with a state-space computational solution that utilizes frequency-dependent weighting functions to tune the controller’s performance and robustness characteristics. A standard $\mathcal{H}_\infty$ control block formation is shown in Figure 6.2 where $P(s)$ is the augmented plant transfer function matrix and $K(s)$ represents a linear transfer function matrix of the controller. The input $w$ denotes the exogenous inputs (e.g. reference commands, disturbances and noise) while the input $u$ represents the control inputs. The output $z$ denotes the regulated performance output variables (e.g. tracking errors, performance variables, and actuator signals) while the output $y$ is the measured output variables.

![Controller block formation.](image)

Details of the augmented plant with weighted sensitivities of $Z_1(s)$, $Z_2(s)$ and $Z_3(s)$; and weighting functions of $W_1(s)$, $W_2(s)$ and $W_3(s)$ is shown in Figure 6.3. The signal spaces for each of the weighted sensitivities are:

\[
Z_1(s) = W_1(s)\gamma_1 S(s)
\]
\[
Z_2(s) = W_2(s)\gamma_2 R(s)
\]
\[
Z_3(s) = W_3(s)\gamma_3 T(s)
\]

where $Z_1(s)$, $Z_2(s)$ and $Z_3(s)$ are the primary, control and complementary weighted sensitivities respectively. $W_1(s)$, $W_2(s)$ and $W_3(s)$ are the primary, control and complementary weighting functions respectively.

The transfer function matrix for the system with all three weights is:
6.2. SENSITIVITY AND WEIGHTING FUNCTIONS

Figure 6.3: Augmented system [101].

\[
\begin{bmatrix}
  z_1 \\
  z_2 \\
  z_3 \\
  y
\end{bmatrix} =
\begin{bmatrix}
  W_1 & -GW_1 \\
  0 & W_2 \\
  0 & GW_3 \\
  1 & -G
\end{bmatrix}
\begin{bmatrix}
  w \\
  u
\end{bmatrix}
\]  \hspace{1cm} (6.7)

Assuming that the simplified augmented system is represented by:

\[
\begin{bmatrix}
  z \\
  y
\end{bmatrix} =
\begin{bmatrix}
  P_{11}(s) & P_{12}(s) \\
  P_{21}(s) & P_{22}(s)
\end{bmatrix}
\begin{bmatrix}
  w \\
  u
\end{bmatrix}
\]  \hspace{1cm} (6.8)

where \( z = [z_1 z_2 z_3]^T \)

and

\[ y = Ku \]  \hspace{1cm} (6.9)

the transformation (or mapping) from the input \( w \) to the output \( z \), \( T_{zw} \) is called the lower linear fractional transformation, \( F_l(P,K) \) and can be expressed as:

\[ T_{zw} = F_l(P,K) = P_{11} + P_{12}K(1 - P_{22}K)^{-1}P_{21} \]  \hspace{1cm} (6.10)
The $\mathcal{H}_\infty$ controller is then found by minimizing the infinity norm of the input-output map $T_{zw}$ or $\|T_{zw}\|_\infty < 1$. The challenge in the design process is how to choose the appropriate weighting functions for the control design. There are no specific rules for this and the final choice varies from one application to another. A few suggested guidelines are presented below based on ideas in [122], [123] and [124].

6.2.6 Primary weighting function

According to [122], the primary sensitivity function should have a low gain at low frequencies for good tracking performance and high gain at high frequencies to limit overshoot. This can be accomplished by selecting a weighting function, $W_1$ such that $W_1^{-1}$ reflects the desired shape of the sensitivity function. So, a low pass weight can be used and according to [124], a first order function of the form shown in Equation (6.11) is often chosen.

$$W_1(s) = \frac{s + b_1}{s + a_1} \gamma_1, \quad b_1 > a_1 \tag{6.11}$$

$\gamma_1$ is used to tune the overall gain of the system.

6.2.7 Control weighting function

For the control weighting function, a D.C. gain of the form shown in Equation (6.12) can be chosen [124].

$$W_2(s) = a_2 \gamma_2 \tag{6.12}$$

This restrict the level of control action across the whole frequency range. Alternatively, a transfer function could be used for example to limit the control action more at low frequency than high frequency. Again, $\gamma_2$ is used to tune the overall gain of the system.
6.2.8 Complementary weighting function

For noise attenuation and insensitivity to unmodelled dynamics which usually occur at high frequencies, the complementary sensitivity function should have a high gain at low frequencies and low gain at high frequencies [122]. So a high pass weight can be used as $W_3$ to reflect the desired shape of $T(s)$ and a first order transfer function of the form shown in Equation (6.13) often chosen as $W_3$ [124].

$$W_3(s) = \frac{s + b_3}{s + a_3 \gamma_3}, \quad b_3 < a_3$$

(6.13)

with $\gamma_3$ is used to tune the overall gain of the system.

6.3 Parametric Uncertainty

It is well known and documented in [125] that parametric uncertainty is a type of plant model uncertainty that was caused by poor knowledge of parameters in the plant. However, the structure of the model (including order) is known. The parametric uncertainty can be quantified by assuming that each uncertain parameter is bounded within some region.

In the work described in this thesis, the actuator model structure and the physical parameters are known as shown in Chapter 4 (derived from first principles). However, the actuator’s physical parameters are changing as the actuator health condition changes from healthy to faulty conditions. The variation in the actuator’s physical parameters is treated as parametric uncertainty and must be considered when designing the $H_\infty$ controller.

To include parametric uncertainty in the $H_\infty$ controller design, it is necessary to calculate relative errors as given in Equation (6.14) [125].

$$l_I(\omega) = \left| \frac{G_p(j\omega) - G(j\omega)}{G(j\omega)} \right|$$

(6.14)

where $l_I(\omega)$ is the relative error, $G(j\omega)$ is the plant nominal model and $G_p(j\omega)$ is the plant model with different parameter values. For example, $G(j\omega)$ could be a delay-free model while $G_p(j\omega)$ could be the plant model with delay and different parameter values. In [125], the relative errors Bode
magnitude plot is then used as a guidance to select the complementary weighting functions $W_3(s)$ and it must be greater than the relative errors over the entire range of frequencies.

In this thesis, plant model in the healthy condition is considered as $G(j\omega)$ while plant model in the faulty conditions are considered as $G_p(j\omega)$. Also, $W_3(s)$ is chosen based on Equation (6.13) and then compared to the relative errors to check that it is larger than the relative errors over the entire range of frequencies. The $\mathcal{H}_\infty$ controller design procedure is explained in the next section.

6.4 $\mathcal{H}_\infty$ Controller Design Procedure

For the HRA controller to be developed here, it is desirable to design a controller so that the system has a good tracking performance and is robust to modelling uncertainties. Therefore, the mixed sensitivity method that takes into account all of the three sensitivity functions is employed in the $\mathcal{H}_\infty$ control design within this thesis.

The $\mathcal{H}_\infty$ controller design relies on the following six-steps procedure:

1. Define the plant dynamics $G(s)$.
2. Calculate relative errors.
3. Select suitable weighting functions.
4. Perform controller design.
5. Evaluate the designed controller.
6. Implement the controller to Simulink model and experimental rig to evaluate the time and frequency responses of the system.

Each step of this procedure is discussed in the following subsections.
6.4. $\mathcal{H}_\infty$ CONTROLLER DESIGN PROCEDURE

6.4.1 Define Plant Dynamics

The plant is a linear electromechanical actuator (EMA) that consists of a DC motor and a ball screw mechanism. The mathematical model of the EMA can be derived from first principles as shown in Chapter 4. The equation for the motor armature current and motor mechanical loading are shown in Equation (6.15) and (6.16), respectively:

$$V_s(s) = RI(s) + LsI(s) + K_e s \theta_m(s)$$  \hspace{1cm} (6.15)

$$K_T I(s) = J s^2 \theta_m(s) + D s \theta_m(s)$$  \hspace{1cm} (6.16)

where $s$ is the Laplace operator, $V_s(s)$ is the input voltage, $I(s)$ is the motor armature current, $\theta_m$ is the motor angular displacement, $R$ is the armature resistance, $L$ is the armature inductance, $K_e$ is the back-emf constant, $K_T$ is the motor torque constant, $J$ is the system’s inertia and $D$ is the system’s damping.

As mentioned previously, the HRA is treated as a SISO system with voltage as input signal and linear displacement as output signal. So, Equation (6.16) is modified as shown below:

$$I(s) = \frac{J s^2 \theta_m(s) + D s \theta_m(s)}{K_T}$$  \hspace{1cm} (6.17)

Equation (6.17) is then substituted into Equation (6.15) to get the voltage to angular displacement transfer function as shown below:

$$G(s) = \frac{\theta_m(s)}{V_s(s)} = \frac{K_T}{(JL)s^3 + (JR + DL)s^2 + (DR + K_T)} \left[\frac{\text{rad}}{V}\right]$$  \hspace{1cm} (6.18)

A model of the EMA linear displacement can then be obtained by multiplying Equation (6.18) with $\frac{L_e}{2\pi}$ (recall that $L_e$ is the ball screw lead) as shown below:
\[ G(s) = \frac{X(s)}{V_s(s)} = \left[ \frac{K_T}{(JL)s^3 + (JR + DL)s^2 + (DR + K_t K_T)s} \right] \left[ \frac{Le}{2\pi} \right] \left[ \frac{m}{V} \right] \]

\[ (6.19) \]

### 6.4.2 Calculate Relative Errors

As mentioned before, the actuators physical parameters (i.e. \( R, L, K_e, K_t, J \) and \( D \)) vary as the actuator health condition changes from healthy to faulty and creates parametric uncertainty. The range of possible plants resulting from this parametric uncertainty must be considered in the \( \mathcal{H}_\infty \) controller design.

The values of the actuator physical parameters in the healthy and faulty conditions are obtained from experimental data using least-squares parameter estimation that is designed for the condition monitoring algorithm (which will be explained later in Chapter 7). The estimation procedure combined discrete time parameter estimation with the physical model of the system to facilitate the estimation of the key physical parameters of the actuator \(^1\).

To calculate the relative errors, the nominal plant, \( G(s) \) is obtained by substituting the estimated parameters in the healthy condition into Equation (6.19) while the plant with faulty conditions, \( G_p(s) \) is obtained by substituting the estimated parameters in the faulty conditions into the same equation. The relative errors are then calculated using Equation (6.14).

### 6.4.3 Weighting Functions Selection and Controller Design

The most difficult part in the \( \mathcal{H}_\infty \) controller design is the selection of weighting functions. As discussed previously, a low pass weight (refer to Equation (6.11)) is used as \( W_1(s) \), static gain is used as \( W_2(s) \) while high pass weight (refer to Equation (6.13)) is used as \( W_3(s) \).

\(^1\)Full detail of the parameter estimation process is explained later in Chapter 7.
The weighting functions selection and controller design was carried out as follow:

1. Tune \( W_1(s) \), \( W_2(s) \) and \( W_3(s) \) by choosing a value for \( a_1, b_1, \gamma_1, a_2, \gamma_2, a_3, b_3 \) and \( \gamma_3 \).

2. Create an augmented system, \( P(s) \) that consists of the plant, \( G(s) \) and all of the weighting functions using the MATLAB command \texttt{augw}.

3. Using \( P(s) \) created in step 2, the controller \( K(s) \) is designed using MATLAB command \texttt{hinfsyn}.

Step 1 to 3 are repeated a few times and the value of \( a_1, b_1, \gamma_1, a_2, \gamma_2, a_3, b_3 \) and \( \gamma_3 \) are manually increased or decreased until a controller \( K(s) \) that satisfies all of the \( \mathcal{H}_\infty \) paradigm is obtained. The complete MATLAB code is given in Appendix B.

### 6.4.4 Evaluate Design

The weighting functions chosen from the previous step must produce a controller \( K(s) \) that satisfies the following \( \mathcal{H}_\infty \) paradigm requirements:

1. The transmission from \( w \) to \( z \) must be smaller than 1 or 0dB over the entire range of frequencies.

   \[
   \|T_{zw}\|_\infty < 1 \quad (6.20)
   \]

   Recall that \( T_{zw} \) can be calculated using the lower linear fractional transformation technique given in Equation (6.10).

2. The Bode magnitude plot of \( S(s) \), \( R(s) \) and \( T(s) \) must be smaller than the Bode magnitude plot of \( W_1(s)^{-1} \), \( W_2(s) \) and \( W_3(s)^{-1} \) over the entire range of frequencies.

   \[
   \|S(s)\| \leq \|W_1(s)^{-1}\|, \forall \omega > 0
   \]

   \[
   \|R(s)\| \leq \|W_2(s)^{-1}\|, \forall \omega > 0
   \]
\[ \|T(s)\| \leq \|W_3(s)^{-1}\|, \forall \omega > 0 \] (6.21)

3. \(W_3(s)\) must be larger than the relative errors over the entire range of frequencies.

### 6.4.5 Apply \(K(s)\) to Simulink Model and Experimental Rig

Once a controller \(K(s)\) that satisfies all of the \(H_\infty\) paradigm requirements is obtained, it is applied to the Simulink model and experimental rig to assess the time and frequency responses of the system in the healthy and faulty conditions. The Simulink model that is used to assess the time and frequency responses of the system in simulation environment is illustrated in Figure 6.4. The plant is the EMA (3-elements-in-series or 4-by-3 HRA) and the controller \(K(s)\) is the \(H_\infty\) controller obtained from subsection 6.4.3.

The step input is a position demand from an initial position of 75mm to a final position of 100mm.

![Figure 6.4: Block diagram shows the implementation of the controller \(K(s)\) in Simulink model.](image)

The controller must be able to satisfy the time and frequency responses requirements given in Table 6.1 (Note that this is similar to the design requirements of the classical control design).

Results of implementing the \(H_\infty\) controller design procedure to the 3-elements-in-series configuration and the full 4-by-3 HRA assembly are discussed in the following sections.
Table 6.1: Control requirements for the $\mathcal{H}_\infty$ controller design

<table>
<thead>
<tr>
<th>Control Requirements</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time Response</strong></td>
<td></td>
</tr>
<tr>
<td>Settling Time (ST)</td>
<td>$&lt;0.5s$</td>
</tr>
<tr>
<td>Rise Time (RT)</td>
<td>$&lt;0.2s$</td>
</tr>
<tr>
<td>Overshoot (OS)</td>
<td>$&lt;10%$</td>
</tr>
<tr>
<td>Steady-state error (SSE)</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency Response</strong></td>
<td></td>
</tr>
<tr>
<td>Gain Margin (GM)</td>
<td>$&gt;6dB$</td>
</tr>
<tr>
<td>Phase Margin (PM)</td>
<td>$&gt;45^\circ$</td>
</tr>
</tbody>
</table>

6.5 Controller Design with 3-elements-in-series

In this section, the results of applying $\mathcal{H}_\infty$ controller to the 3-elements-in-series configuration are presented and discussed.

6.5.1 Controller Design

The first step in the design procedure is to define the plant model. Even though there are 3 DC motors and 3 ball screws in the system, they are modelled as a single DC motor and a single ball screw. This is based on the findings by Du in [9] that shows a low-order model (similar to a single actuator model) shows a very similar performance (in time and frequency domain) as a higher-order model. So, Equation (6.19) can be used to represent the 3-elements-in-series configuration.

The second step is to calculate the relative errors. Table 6.2 lists the estimated parameters in the healthy and faulty conditions that are used to calculate the relative errors for the 3-elements-in-series configuration. In the case of 3 actuation elements in series, four failure modes are considered: 1 element locked; 2 elements locked; 1 element loose; and 2 elements loose.

Figure 6.5 shows the Bode plot of the nominal plant model, $G(s)$ and all of the four failure modes model, $G_p(s)$. Figure 6.6 shows the Bode magnitude plot of the relative errors. When selecting the complementary weighting function $W_3$, it is desirable for $W_3(j\omega)$ to be greater than all of the relative errors over the entire frequencies.
Table 6.2: Actuator’s estimated parameters used to calculate relative error of the 3-elements-in-series configuration

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy</td>
<td>2.668</td>
<td>0.976987</td>
<td>0.007124</td>
<td>2.714326x10$^{-6}$</td>
<td>6.710936x10$^{-5}$</td>
</tr>
<tr>
<td>1 element locked</td>
<td>3.908</td>
<td>1.512175</td>
<td>0.009896</td>
<td>4.610142x10$^{-6}$</td>
<td>1.328251x10$^{-5}$</td>
</tr>
<tr>
<td>2 elements locked</td>
<td>5.783</td>
<td>2.643569</td>
<td>0.015116</td>
<td>3.254789x10$^{-6}$</td>
<td>8.316589x10$^{-5}$</td>
</tr>
<tr>
<td>1 element loose</td>
<td>3.920</td>
<td>1.533475</td>
<td>0.009485</td>
<td>4.614568x10$^{-6}$</td>
<td>1.337134x10$^{-5}$</td>
</tr>
<tr>
<td>2 elements loose</td>
<td>5.514</td>
<td>2.601142</td>
<td>0.014644</td>
<td>3.226044x10$^{-6}$</td>
<td>8.747519x10$^{-5}$</td>
</tr>
</tbody>
</table>

*$K = K_e = K_T$ because in the least squares parameter estimation algorithm, it is assumed that $K_e = K_T$.

Figure 6.5: Bode plot of $G(s)$ and all of the four $G_p(s)$s.
The next step is to select suitable weighting functions, design a controller $K(s)$ and perform evaluation to make sure that the designed controller satisfies the $H_\infty$ paradigm and parametric uncertainty requirements.

To start the weighting function selection and controller design process, the following parameter values were chosen:

$$
\begin{align*}
    a_1 &= 1 \\
    b_1 &= 100 \\
    \sigma_1 &= 1 \\
    a_2 &= 1 \\
    \sigma_2 &= 0.5 \\
    a_3 &= 20 \\
    b_3 &= 1 \\
    \sigma_3 &= 10
\end{align*}
$$  \hspace{1cm} (6.22)

which produce the following weighting functions:

$$
\begin{align*}
    W_1(s) &= \frac{s + 100}{s + 1} \\
    W_2(s) &= 0.5 \\
    W_3(s) &= \frac{10s + 10}{s + 20}
\end{align*}
$$  \hspace{1cm} (6.23)
The augmented plant, $P(s)$ is given in state-space matrices as:

$$A = \begin{bmatrix}
-1 & 0 & 0 & 0 & -29.36 \\
0 & -20 & 0 & 0 & 58.71 \\
0 & 0 & -390.9 & -125.5 & 0 \\
0 & 0 & 128 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}$$

$$B = \begin{bmatrix}
8 \\
0 \\
0 \\
2 \\
0 \\
0
\end{bmatrix}$$

$$C = \begin{bmatrix}
12.38 & 0 & 0 & 0 & -3.67 \\
0 & 0 & 0 & 0 \\
0 & -11.88 & 0 & 0 & 36.7 \\
0 & 0 & 0 & 0 & -3.67
\end{bmatrix}$$

$$D = \begin{bmatrix}
1 \\
0 \\
0.05 \\
0 \\
1
\end{bmatrix}$$

The controller $K(s)$ state-space matrices are:

$$A = \begin{bmatrix}
-1 & -2.632 \times 10^{-18} & 0 & 0 & -2.936 \times 10^{-5} \\
-1.693 \times 10^{-16} & -20 & 0 & 0 & 58.71 \\
2.063 \times 10^{5} & -643.2 & -1.622 \times 10^{4} & -4.884 \times 10^{4} & -2.137 \times 10^{6} \\
8.102 \times 10^{-33} & 3.887 \times 10^{-29} & 128 & 5.821 \times 10^{-11} & 1.624 \times 10^{-26} \\
-5.766 \times 10^{-17} & -2.766 \times 10^{-13} & 0 & 1 & -5.739 \times 10^{-11}
\end{bmatrix}$$
The results shown in Figures 6.7 to 6.12 are obtained from the initial design steps with the weightings as per Equation (6.22).

Figure 6.7 shows the Bode magnitude plot of $T_{z1w1}$, $T_{z2w2}$ and $T_{z3w3}$. From the figure, it can be seen that only $T_{z3w3}$ is below 0dB. This does not satisfy the $\mathcal{H}_\infty$ norm requirements.

Figure 6.8 and 6.9 show the primary sensitivity function with respect to $W_1(s)^{-1}$ and the control sensitivity functions with respect to $W_2(s)^{-1}$, respectively. From both graphs, it can be seen that both $S(s)$ and $R(s)$ are greater than $W_1(s)^{-1}$ and $W_2(s)^{-1}$ over the entire frequency range. Again, this does not satisfy the $\mathcal{H}_\infty$ norm requirements.

Figure 6.10 shows the Bode magnitude plot of the complementary sensitivity function with respect to $W_3(s)^{-1}$ and it can be seen that $T(s)$ is smaller than $W_3(s)^{-1}$ over the entire frequency range. This satisfies the $\mathcal{H}_\infty$ norm requirements.

Figure 6.11 shows the Bode magnitude plot of $W_3(s)$ with respect to the relative errors. From the figure, it can be seen that $W_3(s)$ is greater than all of the relative errors over the entire frequency range. This satisfies the parametric uncertainty requirements.

Figure 6.12 shows the time response of the system (i.e. the Simulink model time response) driven using the designed controller $K(s)$. It can be seen that the system failed to track the demand position (SSE of 1.69mm is recorded). Other than that, the current is saturated at 10A. As explained in Chapter 5, this is not practical because the Sabertooth motor driver can only supply a maximum of 10A current to the DC motor. The system’s rise and settling time are 0.17s and 0.4s, respectively. These values satisfy the
time response design requirements.

The results obtained show that some of the $\mathcal{H}_\infty$ norm and time response requirements are satisfied while some are not. Therefore, the weighting functions selection and controller design process is repeated using different values of $a_1$, $b_1$, $\sigma_1$, $a_2$, $\sigma_2$, $a_3$, $b_3$ and $\sigma_3$. Their values are manually increased or decreased until a controller $K(s)$ that satisfies all of the $\mathcal{H}_\infty$ norm and time response requirements are found.

However, by examining the frequency plots and the time response, two things are clear:

1. $a_1$ must be decreased. This is because the slope of $W_1(s)$ is the integral action and determines how much offset the output will have at steady state. So, decreasing $a_1$ will move the break frequency lower and causes an increase in the integral action so that the steady-state error can be reduced to zero.

2. $W_2(s)$ must be decreased to avoid current saturation.

![Figure 6.7: Bode magnitude plot of $T_{z_1w_1}$, $T_{z_2w_2}$ and $T_{z_3w_3}$ for the 3-elements-in-series configuration.](image)
Figure 6.8: Bode plot of $S(s)$ versus $W_1^{-1}$ for the 3-elements-in-series configuration.

Figure 6.9: Bode plot of $R(s)$ versus $W_2^{-1}$ for the 3-elements-in-series configuration.
Figure 6.10: Bode plot of $T(s)$ versus $W_3^{-1}$ for the 3-elements-in-series configuration.

Figure 6.11: Bode plot of relative errors and $W_3(s)$ for the 3-elements-in-series configuration.
After a few trials, the following values are found to produce controller $K(s)$ that satisfies all of the requirements.

$$
\begin{align*}
    a_1 &= 0.01 & b_1 &= 10 & \sigma_1 &= 0.033 \\
    a_2 &= 2.7 & \sigma_2 &= 0.063 \\
    a_3 &= 30 & b_3 &= 1 & \sigma_3 &= 26.85
\end{align*}
$$

The resulting weighting functions are:

$$
\begin{align*}
    W_1(s) &= \frac{0.033s + 0.33}{s + 0.01} \\
    W_2(s) &= 0.02333 \\
    W_3(s) &= \frac{26.85s + 26.85}{s + 30}
\end{align*}
$$

The augmented plant $P(s)$ state-space matrices are:
The controller $K(s)$ state-space matrices are:

$$A = \begin{bmatrix}
-0.01 & 0 & 0 & 0 & -1.835 \\
0 & -30 & 0 & 0 & 117.4 \\
0 & 0 & -390.9 & -125.5 & 0 \\
0 & 0 & 128 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}$$

$$B = \begin{bmatrix}
0.5 & 0 \\
0 & 0 \\
0 & 2 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}$$

$$C = \begin{bmatrix}
12.38 & 0 & 0 & 0 & -3.67 \\
0 & 0 & 0 & 0 & 0 \\
0 & -11.88 & 0 & 0 & 36.7 \\
0 & 0 & 0 & 0 & -3.67 \\
\end{bmatrix}$$

$$D = \begin{bmatrix}
0.033 & 0 \\
0 & 0.02333 \\
0 & 0 \\
1 & 0 \\
\end{bmatrix}$$

$$A = \begin{bmatrix}
-9.995 \times 10^{-3} & 3.672 \times 10^{-12} & 0 & 0 & -1.835 \times 10^{-6} \\
-2.441 \times 10^{-14} & -30 & 0 & 0 & 117.4 \\
2312 & 434.9 & -428.8 & -246 & -6350 \\
1.869 \times 10^{-29} & 1.87 \times 10^{-23} & 128 & 5.821 \times 10^{-11} & -7.284 \times 10^{-23} \\
-6.236 \times 10^{-15} & -6.238 \times 10^{-9} & 0 & 1 & 2.436 \times 10^{-8} \\
\end{bmatrix}$$
6.5. CONTROLLER DESIGN WITH 3-ELEMENTS-IN-SERIES

\[ B = \begin{bmatrix}
0.5989 \\
-1.224 \times 10^{-9} \\
-1.563 \times 10^{-24} \\
9.377 \times 10^{-25} \\
-3.128 \times 10^{-10}
\end{bmatrix} \]

\[ C = \begin{bmatrix}
965 & 181.5 & -15.81 & -50.29 & -2651
\end{bmatrix} \]

\[ D = [0] \]

The transmissions Bode magnitude plot is shown in Figure 6.13. From the figure, it can be seen that all three lines are below 0dB over the entire range of frequencies. Therefore, the first requirement is satisfied.

Figures 6.14, 6.15 and 6.16 show the Bode magnitude plot of the sensitivity functions with respect to their weighting functions. From the figures, it can be seen that magnitude of the sensitivity functions are smaller than the magnitude of the inverse of their respective weighting functions. Therefore, the second requirement are also met.

Figure 6.17 shows the Bode magnitude plot of the relative errors and \( W_3(s) \). Again, it can be seen that the third requirement is satisfied because \( W_3(s) \) is larger than the relative errors over the entire range of frequencies.
Figure 6.13: Bode magnitude plot of $T_{zw1}$, $T_{zw2}$ and $T_{zw3}$ for the 3-elements-in-series configuration.

Figure 6.14: Bode plot of $S(s)$ versus $W_1^{-1}$ for the 3-elements-in-series configuration.
Figure 6.15: Bode plot of $R(s)$ versus $W_2^{-1}$ for the 3-elements-in-series configuration.

Figure 6.16: Bode plot of $T(s)$ versus $W_3^{-1}$ for the 3-elements-in-series configuration.
In Figures 6.14, 6.15 and 6.16 it can be seen that the weighting functions can be pushed closer to the sensitivity functions. However, this is the best results that can be obtained without causing either: oscillation in the system’s time response; or hinfsyn command unable to compute the $\mathcal{H}_\infty$ controller.

The last step in the design process is to apply the controller $K(s)$ to the Simulink model and experimental rig to evaluate the time and frequency responses of the system in the healthy and faulty conditions.

### 6.5.2 Healthy Condition

Figure 6.18 shows the actuation elements time response in the healthy condition when the controller $K(s)$ is used to drive the system. In the figure, A1 (dashed line) have a displacement of 10.1mm, A2 (dotted line) and A3 (dash-dotted line) moves 7.45mm, respectively to generate a total displacement of 25mm as shown in Figure 6.19.

The peak input voltage of all actuation elements are $\pm 5V$ while the peak output current is $\pm 8A$. Slight difference between the actual and model
results can be observed, which occur due to modelling error.

Figure 6.19 shows the total displacement of the system, obtained by summing the individual actuation element’s displacement shown in Figure 6.18. It can be seen that both the actual and model output is capable of tracking the demand position with slight difference between the actual and model output due to modelling error.

Figure 6.20 shows the Nichols plot of the Simulink model in the healthy condition. The time and frequency responses of the system is listed in Table 6.3 and it can be seen that all of the control design requirements are satisfied.

Figure 6.18: Time response of the 3-elements-in-series using $H_\infty$ controller under healthy condition.
CHAPTER 6. $\mathcal{H}_\infty$ CONTROLLER DESIGN

Figure 6.19: Total displacement of the 3-elements-in-series using $\mathcal{H}_\infty$ in the healthy condition.

Figure 6.20: Frequency response of the Simulink model of the 3-elements-in-series using $\mathcal{H}_\infty$ in the healthy condition.
6.5.3 Faulty Condition

Figures 6.21 and 6.22 show the total displacement when one and two actuation elements are subjected to lock-up and short-circuit faults, respectively. From both figures, it can be seen that the system remains capable of moving to the expected position even under faulty conditions albeit with slower performance compared to the healthy condition.

It can also be seen that when one actuation element is locked or short-circuit, the system’s performance is very similar to that in the healthy condition. Again, slight differences between the actual and model results can be observed which were due to modelling error.

The system’s time and frequency responses in the healthy and faulty conditions are listed in Table 6.3. From the table, it can be seen that the ST and RT of the system increased as more actuation elements fail. However, the other control design requirements are satisfied.

By examining Figure 6.21, Figure 6.22 and Table 6.3, it can be deduced that the $\mathcal{H}_\infty$ controller exhibits robust stability because it is capable of stabilising all of the plants (i.e. plant with fully healthy elements and plant with faulty element/s).

In terms of performance, in the case of 1 faulty element, the system is still capable of satisfying the control design requirements. Moreover, there are only small changes in the system’s performance between when it is in the healthy and faulty condition. Therefore, it can be deduced that the $\mathcal{H}_\infty$ controller exhibits robust stability and robust performance in the case of 1 actuation element locked or loose.

In the case of 2 faulty elements, both settling and rise time are higher than the control design requirements when the elements are subjected to lock-up faults with percentage change of 28.6% for the settling time and 20% for the rise time.

If both faulty elements are subjected to loose faults, the settling time satisfied the control design requirements while the rise time is higher than the design requirements by 9%.

This means the $\mathcal{H}_\infty$ controller exhibits robust stability but not robust performance in the case of 2 actuation elements locked or loose.
The results match with the theory of robust stability and robust performance which states that there must be a trade-off between robust stability and robust performance. Improving robust stability will reduce robust performance and vice versa. In the case of the highly redundant actuator for fault tolerance, it is desirable for the actuator to have a good stability regardless of its health condition. However, performance reduction is acceptable as the actuator health condition changes from healthy to faulty.
6.5. CONTROLLER DESIGN WITH 3-ELEMENTS-IN-SERIES

Figure 6.21: Total displacement of the 3-elements-in-series using $H_\infty$ controller under lock-up fault.

Figure 6.22: Total displacement of the 3-elements-in-series using $H_\infty$ controller under short-circuit fault.
Table 6.3: Time and frequency responses of 3-elements-in-series using $\mathcal{H}_\infty$ controller under healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Lock-up fault</th>
<th>Loose fault</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 element locked</td>
<td>2 elements locked</td>
<td>1 element short-circuit</td>
</tr>
<tr>
<td><strong>Time response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.19</td>
<td>0.23</td>
<td>0.70</td>
<td>0.24</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.10</td>
<td>0.12</td>
<td>0.25</td>
<td>0.13</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td><strong>Frequency response</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>23.1</td>
<td>26.5</td>
<td>32.7</td>
<td>26.6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>72.8</td>
<td>77.8</td>
<td>83.3</td>
<td>77.6</td>
</tr>
</tbody>
</table>
6.5.4 Comparing Global $\mathcal{H}_\infty$ and PI controller for 3-elements-in-series

Table 6.5 shows the comparison between the system’s (3-elements-in-series) performance with the global $\mathcal{H}_\infty$ controller and the global PI controller described in Section 5.7.2. From the table, it can be seen that the system has a faster response (i.e. smaller settling and rise time) both in the healthy and faulty conditions with the global $\mathcal{H}_\infty$ controller.

However, performance degradation can be observed in both cases ($\mathcal{H}_\infty$ and PI) as more actuation elements are subjected to lock-up or short-circuit faults. Table 6.4 shows the percentage increment in ST and RT as the system’s health condition changes from healthy to faulty for both the $\mathcal{H}_\infty$ and the PI controller.

Table 6.4: Percentage increment in settling time and rise time for $\mathcal{H}_\infty$ controller and PI controller for 3-elements-in-series in the faulty condition

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Controller</th>
<th>ST</th>
<th>RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 element locked</td>
<td>$\mathcal{H}_\infty$</td>
<td>21%</td>
<td>20%</td>
</tr>
<tr>
<td>1 element short-circuit</td>
<td>$\mathcal{H}_\infty$</td>
<td>26.3%</td>
<td>30%</td>
</tr>
<tr>
<td>2 elements locked</td>
<td>$\mathcal{H}_\infty$</td>
<td>268%</td>
<td>120%</td>
</tr>
<tr>
<td>2 elements short-circuit</td>
<td>$\mathcal{H}_\infty$</td>
<td>163%</td>
<td>150%</td>
</tr>
</tbody>
</table>

It can be seen that the global $\mathcal{H}_\infty$ controller produces smaller increment in the settling and rise time and thus, smaller degradation in the system’s performance as the system’s health condition changes from healthy to faulty.

This shows that the global $\mathcal{H}_\infty$ controller exhibits a better robustness to parametric uncertainty as oppose to the global PI controller.

Overall, in Section 6.5.1, it has been shown that the $\mathcal{H}_\infty$ design approach yields a controller which meets the design requirements and is able to continue effective control of the HRA after faults with good performance in simulation and real world test. In Section 6.5.4, it has been shown that the $\mathcal{H}_\infty$ controller exhibits better robustness to parametric uncertainty com-
pared to the global PI controller discussed in Section 5.7.2.

The next subsection will discuss about designing $\mathcal{H}_\infty$ controller for the full HRA assembly.
Table 6.5: Comparison of the time and frequency responses of the 3-elements-in-series using global $H_\infty$ and PI controller in the healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>1 elements locked</th>
<th>2 elements locked</th>
<th>1 elements short-circuit</th>
<th>2 elements short-circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Global PI</td>
<td>$H_\infty$</td>
<td>Global PI</td>
<td>$H_\infty$</td>
<td>Global PI</td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.20</td>
<td>0.19</td>
<td>0.50</td>
<td>0.23</td>
<td>1.00</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.12</td>
<td>0.10</td>
<td>0.20</td>
<td>0.12</td>
<td>0.49</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.1</td>
<td>23.1</td>
<td>37.5</td>
<td>26.5</td>
<td>43.6</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>71.8</td>
<td>72.8</td>
<td>70.9</td>
<td>77.8</td>
<td>62.6</td>
</tr>
</tbody>
</table>
6.6 Controller Design with Full HRA Assembly

In this section, the results of applying $\mathcal{H}_\infty$ controller to the full HRA assembly are presented and discussed.

6.6.1 Controller Design

The transfer function given in Equation (6.19) can also be used to represent the full HRA assembly because even though there are 12 actuation elements, they are treated as a single actuator. Meaning, 12 DC motors are modelled as a single equivalent motor and 12 ball screws are modelled as a single equivalent ball screw.

To calculate the relative errors between the dynamics of the HRA in the healthy and faulty conditions, again estimated parameters obtained using least-squares parameter estimation presented in Chapter 7 are used. As explained in Chapter 7, for the 4-by-3 HRA, four failure modes are considered: 1 element fail; 2 elements fail; 3 elements fail; and 4 elements fail. In the case of 1 element fail, 12 different failure combinations (i.e. $A_{11}$ fail, $A_{12}$ fail and so on) are tested. For 2 elements fail, 68 different combinations were tested to obtain the range of estimated parameters while more than 100 combinations were tested for both 3 and 4 elements fail. Figure 6.23 shows some of the relative errors calculated using the estimated parameters obtained under different failure conditions.

The next step is to select weighting functions and design the controller using the procedure explained in Section 6.4.3. After a few trials, the weighting functions given in Equation (6.26) are found to produce $K(s)$ that satisfied the $\mathcal{H}_\infty$ paradigm and parametric uncertainty requirements as shown in Figures 6.24, 6.25, 6.26, 6.27 and 6.28.
$W_1(s) = \frac{0.028s + 0.56}{s + 0.001}$

$W_1(s) = 0.17875$

$W_3(s) = \frac{26.85s + 26.85}{s + 30}$  \hspace{1cm} (6.26)

Figure 6.23: Relative errors of the full HRA assembly.
Figure 6.24: Bode magnitude plot of $T_{z_1w_1}$, $T_{z_2w_2}$ and $T_{z_3w_3}$ for the full HRA assembly where $T_{zu}$ is smaller than 0dB over the entire frequencies.

Figure 6.25: Bode magnitude plot of $S(s)$ versus $W_1(s)^{-1}$ for the full HRA assembly where $S(s)$ is smaller than $W_1(s)^{-1}$ over the entire frequencies.
6.6. CONTROLLER DESIGN WITH FULL HRA ASSEMBLY

Figure 6.26: Bode magnitude plot of $R(s)$ versus $W_2(s)^{-1}$ for the full HRA assembly where $R(s)$ is smaller than $W_2(s)^{-1}$ over the entire frequencies.

Figure 6.27: Bode magnitude plot of $T(s)$ versus $W_3(s)^{-1}$ for the full HRA assembly where $T(s)$ is smaller than $W_3(s)^{-1}$ over the entire frequencies.
Figure 6.28: Bode magnitude plot of relative errors and $W_3(s)$ where $W_3(s)$ is greater than the relative errors over the entire frequencies.

After that, the controller $K(s)$ with the following state-space matrices are applied to the HRA Simulink model and experimental rig to assess the time and frequency responses of the system in the healthy and faulty conditions.

$$A = \begin{bmatrix} -8.794 \times 10^{-4} & 1.933 \times 10^{-11} & 0 & 0 & -1.072 \times 10^{-6} \\ -7.595 \times 10^{-1} & -30 & 0 & 0 & 34.29 \\ 3585 & -72.39 & -436.2 & -241.8 & -5170 \\ -7.47 \times 10^{-29} & -4.562 \times 10^{-24} & 128 & 5.821 \times 10^{-1} & 5.084 \times 10^{-24} \\ -6.645 \times 10^{-14} & -4.059 \times 10^{-9} & 0 & 1 & 4.581 \times 10^{-9} \end{bmatrix}$$

$$B = \begin{bmatrix} 0.7926 \\ -2.336 \times 10^{-10} \\ -2.156 \times 10^{-25} \\ -2.298 \times 10^{-25} \\ -2.044 \times 10^{-10} \end{bmatrix}$$

$$C = \begin{bmatrix} 2262 & -45.67 & -25.59 & -79.95 & -3261 \end{bmatrix}$$

$$D = [0]$$
6.6. CONTROLLER DESIGN WITH FULL HRA ASSEMBLY

6.6.2 Healthy Condition

Figure 6.29 shows the time response of three of the HRA’s elements that are connected in series, which are $A_{11}$, $A_{12}$ and $A_{13}$ (refer to Figure 5.4 for the actuation element’s configuration). From the figure, it can be seen that $A_{12}$ (dotted line) and $A_{13}$ (dash-dotted line) have similar displacement, which is approximately 7.8mm while $A_{11}$ (dashed line) have a displacement of approximately 9.2mm. The other actuation elements have similar response as shown in Figure 6.29.

Peak input voltage is $\pm 4V$ for the actual system and $\pm 4.4V$ for the model. Peak output current is $\pm 10A$ for all of the actuation elements.

Figure 6.30 shows the total displacement of the HRA obtained by summing the displacement of the elements that are connected in series (e.g. $X_{11}$, $X_{12}$ and $X_{13}$). It can be seen that both the actual and model output are capable of tracking the demand position with slight differences between the actual and model output due to modelling error.

Figure 6.31 shows the frequency response of the Simulink model of the HRA. Summary of the time and frequency responses of the HRA in the healthy condition can be found in Table 6.6 and it can be seen that all control design requirements are satisfied.
Figure 6.29: Time response of the full HRA using $\mathcal{H}_\infty$ controller under healthy condition.
6.6. CONTROLLER DESIGN WITH FULL HRA ASSEMBLY

Figure 6.30: Total displacement of the full HRA using $\mathcal{H}_\infty$ controller under healthy condition.

Figure 6.31: Frequency response of the Simulink model of the HRA using $\mathcal{H}_\infty$ controller under healthy condition.
6.6.3 Faulty Conditions

Figure 6.32 shows the HRA performance when its elements are subjected to lock-up faults. In Figure 6.32a, 3 elements are locked as shown in the diagram and the results show that this condition causes the whole assembly to be locked which is a common problem in the traditional purely parallel redundancy actuation system.

In Figure 6.32b, 8 elements are locked as shown in the diagram and the results show that the HRA can move to the expected position but slower (73% and 92.3% increment in the ST and RT, respectively) compared to when it was in the healthy condition.

These two cases were tested because: 1) to prove that when 3 elements that are connected in series are locked, the whole assembly will be rendered immobile; and 2) 8 is the maximum number of actuation elements locked that can be tolerated by the HRA.

Figure 6.33 shows the HRA performance when its elements are subjected to short-circuit faults. In Figure 6.33a, 4 elements are subjected to short-circuit faults while in Figure 6.33b, 9 elements experience short-circuit faults as shown in the diagrams. From both figures, it can be seen that when 4 of the HRA’s elements are short-circuited, its performance is almost the same as the healthy condition. When 9 elements are short-circuited, the HRA’s performance is slower (83.3% and 53.8% increment in the ST and RT, respectively) compared to that in the healthy condition.

These two cases were tested to show that performance decrease further as more actuation elements are subjected to short-circuit faults. From the real time experiments that were conducted, it was found that 9 is the maximum number of short-circuit elements that can be tolerated by the HRA.

Table 6.6 summarizes the responses of the HRA under both healthy and faulty conditions. Overall it can be seen that in all cases, the frequency response of the system satisfied the control design requirements.

By examining Figure 6.32, Figure 6.33 and Table 6.6, it can be deduced that the $H_\infty$ controller exhibits robust stability because it is capable of stabilizing all of the plants (i.e. healthy, short-circuit and lock-up).

In terms of performance, in the case of 3 actuation elements are sub-
jected to lock-up fault, the whole assembly is rendered immobile (which is a common problem in the current purely parallel redundancy). In the case that 8 elements are locked, both settling time and rise time are higher than the control design requirements with percentage change 3.8% for the settling time and 20% for the rise time.

In the case of 4 actuation elements subjected to short-circuit faults, all control design requirement are satisfied. However, for the 9 short-circuit elements, percentage change of 9% is recorded for the settling time.

Again, the $\mathcal{H}_\infty$ controller exhibits robust stability but not robust performance because control design requirements are not satisfied in some cases. However, the deviation from the required values are not very high.
CHAPTER 6. $\mathcal{H}_\infty$ CONTROLLER DESIGN

(a) 3 elements under lock-up fault.

(b) 8 elements under lock-up fault.

Figure 6.32: Time response of the full HRA using $\mathcal{H}_\infty$ controller under lock-up faults.
(a) 4 elements under short-circuit fault.

(b) 9 elements under short-circuit fault.

Figure 6.33: Time response of the full HRA using $\mathcal{H}_\infty$ controller under short-circuit faults.
Table 6.6: Time and frequency responses of the full HRA using \( H_\infty \) controller under healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>Lock-up fault</th>
<th>Loose fault</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>3 elements</td>
<td>8 elements</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>locked</td>
<td>locked</td>
<td></td>
</tr>
<tr>
<td>Time response</td>
<td></td>
<td>4 element</td>
<td>9 elements</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>short-circuit</td>
<td>short-circuit</td>
<td></td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.30</td>
<td>-</td>
<td>0.52</td>
<td>0.32</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.13</td>
<td>-</td>
<td>0.25</td>
<td>0.14</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>25</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>Frequency response</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>19.2</td>
<td>27.5</td>
<td>28.9</td>
<td>22.7</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>71.3</td>
<td>87.6</td>
<td>83.8</td>
<td>77.3</td>
</tr>
</tbody>
</table>
6.6.4 Comparing Global $\mathcal{H}_\infty$ and PI Controller for the full 4-by-3 HRA

Table 6.8 shows the comparison between the HRA’s performance using the global $\mathcal{H}_\infty$ controller and the global PI controller described in Section 5.7.2. From the table, it can be seen that with the $\mathcal{H}_\infty$ controller, the HRA has a faster response (i.e. smaller settling and rise time) both in the healthy and faulty conditions.

However, performance degradation can be observed in both cases (PI and $\mathcal{H}_\infty$) when some actuation elements are subjected to faults. Table 6.7 shows the percentage increment in ST and RT as the full HRA’s actuation elements are subjected to lock-up and short-circuit faults with both $\mathcal{H}_\infty$ and PI controller.

Table 6.7: Percentage increment in settling time and rise time for $\mathcal{H}_\infty$ controller and PI controller for the full HRA assembly in the faulty condition

<table>
<thead>
<tr>
<th>Failure mode</th>
<th>Controller</th>
<th>ST</th>
<th>RT</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 elements locked</td>
<td>$\mathcal{H}_\infty$</td>
<td>73.3%</td>
<td>92.3%</td>
</tr>
<tr>
<td></td>
<td>PI</td>
<td>94.4%</td>
<td>160%</td>
</tr>
<tr>
<td>4 elements short-circuit</td>
<td>$\mathcal{H}_\infty$</td>
<td>6.7%</td>
<td>7.7%</td>
</tr>
<tr>
<td></td>
<td>PI</td>
<td>38.9%</td>
<td>53.3%</td>
</tr>
<tr>
<td>9 elements short-circuit</td>
<td>$\mathcal{H}_\infty$</td>
<td>83.3%</td>
<td>53.8%</td>
</tr>
<tr>
<td></td>
<td>PI</td>
<td>177.8%</td>
<td>206%</td>
</tr>
</tbody>
</table>

It can be seen that the global $\mathcal{H}_\infty$ controller produced smaller increment in the settling time and rise time and thus, smaller degradation in the system’s performance as the system’s health condition changes from healthy to faulty.

This shows that the global $\mathcal{H}_\infty$ controller exhibits a better robustness to parametric uncertainty compared to the global PI controller.
Table 6.8: Comparison of the time and frequency responses of the full HRA using global PI and $\mathcal{H}_\infty$ controller in the healthy and faulty conditions

<table>
<thead>
<tr>
<th>Responses</th>
<th>Healthy</th>
<th>3 elements locked</th>
<th>8 elements locked</th>
<th>4 elements short-circuit</th>
<th>9 elements short-circuit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Global PI</td>
<td>$\mathcal{H}_\infty$</td>
<td>Global PI</td>
<td>$\mathcal{H}_\infty$</td>
<td>Global PI</td>
</tr>
<tr>
<td>ST, (s)</td>
<td>0.36</td>
<td>0.30</td>
<td>-</td>
<td>0.70</td>
<td>0.52</td>
</tr>
<tr>
<td>RT, (s)</td>
<td>0.15</td>
<td>0.13</td>
<td>-</td>
<td>0.39</td>
<td>0.25</td>
</tr>
<tr>
<td>OS, (%)</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SSE, (mm)</td>
<td>Zero</td>
<td>Zero</td>
<td>25</td>
<td>Zero</td>
<td>Zero</td>
</tr>
<tr>
<td>GM, (dB)</td>
<td>34.0</td>
<td>19.2</td>
<td>42.2</td>
<td>43.6</td>
<td>28.9</td>
</tr>
<tr>
<td>PM, (°)</td>
<td>68.2</td>
<td>71.3</td>
<td>90</td>
<td>55.4</td>
<td>83.8</td>
</tr>
</tbody>
</table>
6.7 Conclusion

This chapter discussed a global $\mathcal{H}_\infty$ controller design using the mixed sensitivity method where three sensitivity functions: primary, control and complementary are used in order for the system to have a good tracking performance and robust to modelling uncertainties. The essence of the $\mathcal{H}_\infty$ controller design lies in the process of selecting weighting functions that lead to the design of a controller that satisfies the $\mathcal{H}_\infty$ paradigm requirements.

In this thesis, the $\mathcal{H}_\infty$ controller design was applied to two actuator configurations: 3 actuation elements in series and the full HRA assembly to study their performance in the healthy and faulty conditions. In both cases, the weighting functions were tuned manually to obtain a controller that satisfies the requirements. The controller was then applied to the Simulink model and experimental rig to evaluate the system’s time and frequency responses. Results of simulation and real time experiments were recorded and compared.

In both cases (3 elements in series and complete HRA), the $\mathcal{H}_\infty$ controller is found to satisfy all the control design requirements when healthy and to be capable of providing continued tracking after a fault has occurred. For the 12 elements HRA, results show that it has the capability to tolerate between 2 and 8 locked elements (with exception of all elements that are connected in series are locked). In the case of short-circuit faults, the most optimistic case is 9 short-circuit elements can be tolerated with the actuator still can track the position demand albeit with slower performance.

The recorded results also show a good match between the actual output obtained from real time experiments and the model output obtained from simulation using model derived from first principle.

By comparing the 3-elements-in-series and the HRA’s performance in the healthy and faulty conditions, it was found that the global $\mathcal{H}_\infty$ controller exhibits robust stability regardless of the system’s health condition but exhibits reduced robust performance in the faulty condition. This is because, a trade-off between robust stability and robust performance must be made (due to the relation between primary and complementary sensitivity
functions) and in the case of the high redundancy actuator studied within this thesis, it is desirable for the actuator to have a good stability in the healthy and faulty conditions but performance degradation is expected in the faulty condition.

Comparison between the 3-elements-in-series and the HRA’s performance using the global $H_\infty$ and the global PI controller presented in Section 5.7.2 were also made. Results show that the global $H_\infty$ controller have a better robustness to parametric uncertainty compared to the global PI controller because smaller performance degradation was recorded with the global $H_\infty$ controller.

In terms of complexity, the $H_\infty$ controller design is more complex because there are 8 parameters to be tuned (i.e. $a_1$, $b_1$, $\sigma_1$, $a_2$, $\sigma_2$, $a_3$, $b_3$, and $\sigma_3$) compared to only two (i.e. $K_{pi}$ and $\tau_i$) for the global PI controller. Other than that, there are many $H_\infty$ norm requirements need to be satisfied. In the case of the global PI controller, only time and frequency responses need to be satisfied.

Overall, it can be concluded that the $H_\infty$ control design process is more complicated than the global PI control design process but it shows better performance in the healthy and faulty conditions and thus proven to be beneficial to the HRA.
Chapter 7

Condition Monitoring
Development

7.1 Introduction

It is desirable for the HRA system to continue to operate with an acceptable performance even when one or more of its components are faulty. However, after several faults, the capability of the system will eventually fall below that required by the application. At this point, maintenance is required to replace the faulty elements or more likely, replace the entire HRA and send it to the base to be repaired. Thus, a form of condition monitoring is needed to provide an indication of the HRA’s performance in order to schedule maintenance or halt operation. Early detection and diagnosis of faults while the system is still operating in the acceptable region can help to avoid unwanted event progression and reduce the chance of the HRA being unable to meet its requirements. This in turn can help avoid major system breakdowns and catastrophes.

Therefore, the aim of this chapter is to develop a model-based condition monitoring algorithm for the full HRA assembly. A flow chart of the proposed algorithm is illustrated in Figure 7.1.

The condition monitoring algorithm consists of two main parts: least-squares parameter estimation that combines discrete time parameter estimation with the physical model of the system to facilitate the estimation
of the key physical parameters (inductance, resistance, back-emf constant, torque constant, inertia and friction) of the electromechanical actuator; and fuzzy logic reasoning to determine the actuator’s condition (i.e. probability of healthy and faulty) based on the estimated parameters.

This chapter is structured as follows: Section 7.2 discusses in detail the six-step procedure implemented to the development of the condition monitoring scheme that involves least-squares parameter estimation and fuzzy logic inference. Results of applying the least-squares parameter estimation to a single actuation element, 3 actuation elements connected in series and the full 4-by-3 HRA is presented and discussed in Section 7.3. Results of the fuzzy logic inference is discussed in Section 7.4. This chapter ends with conclusion in Section 7.5.

7.2 Parameter Estimation Procedure

Estimation of the physical parameters of the actuator relies on the following six-step procedure:

1. Perturbation of the system with a suitable input signal and recording of the resulting input-output data (i.e. input voltage, output current and displacement).

2. Averaging and down-sampling of the recorded input-output data to remove the noise in the signal.
7.2. PARAMETER ESTIMATION PROCEDURE

3. Estimate parameters of the equivalent discrete linear model of the system.

4. Model assessment (using $R^2_T \times 100\%$) to ensure it is sufficiently accurate (or if not to repeat the experiment in step 1).

5. Transformation of the discrete model to a continuous time model.

6. Algebraic transformation of the continuous time model parameters to the key physical parameters of the system.

This design procedure is applied to three actuator configurations: a single actuation element; 3 actuation elements connected in series and the full 4-by-3 HRA assembly. For the 3-elements-in-series and the full HRA assembly, the model is of a single equivalent model based on similar reasoning to that discussed in Chapter 6. Each step of this procedure are discussed in the following subsections.

7.2.1 Input Signal Selection

The purpose of an input signal is to excite the system so that the collected output data contains the relevant frequency components necessary to generate a useful model. For simplicity, input signal selection was performed using a single actuation element. Once the suitable input signal to drive the system is obtained, it is used to run experiments using 3 elements in series and the full HRA assembly.

Two types of input signals were tested: sine wave with band limited noise added to it, which is similar to the input signal used in [95] and [96]; and pseudo-random binary sequence (PRBS) signal which is widely used for system identification because it can excite a wide band of frequencies.

Note that both of these signals are not a normal operating cycle for an aircraft system. Therefore, the condition monitoring algorithm would be a part of a self-diagnosis test that would take place pre- or post-flight.
7.2.2 Averaging and down-sampling of the recorded input-output data

The averaging and down-sampling process is graphically illustrated in Figure 7.2. The input-output data from experiments was recorded at 1kHz. The output data, which are displacement and current were then averaged over 5 samples.

After that, the input-output signals were down-sampled to 200Hz. This step is necessary to reduce the noise in the input-output data recorded from real-time experiments especially for displacement signal that will be differentiated to obtain angular speed.

Figure 7.2: Block diagram showing the averaging and down-sampling process.

7.2.3 Estimation of the parameters of identified discrete linear model of the system

A suitable model of the electromechanical actuator can be obtained from physical laws and can be expressed as:

\[
\begin{bmatrix}
    I(s) \\
    \dot{\Theta}(s)
\end{bmatrix} = \frac{\begin{bmatrix}
    Js + D \\
    K_T
\end{bmatrix}}{LJs^2 + [LD + R]s + [RD + K_eK_T]} V(s) \quad (7.1)
\]

Equation (7.1) is a transfer function (TF) matrix form of a single-input two-output linear model of the system [95] where \( s \) is the Laplace operator, \( I(s) \) is the motor winding current, \( \dot{\Theta}(s) \) is the motor shaft speed, \( V(s) \) is the voltage applied to the stator windings, \( J \) is the system inertia, \( D \) is the system viscous friction, \( K_e \) is the motor back-emf constant, \( K_T \) is the motor
7.2. PARAMETER ESTIMATION PROCEDURE

torque constant, \( L \) is the motor winding inductance and \( R \) is the winding resistance.

Equation (7.1) can also be expressed in state-space form as:

\[
\begin{bmatrix}
\dot{I}(s) \\
\dot{\Theta}(s)
\end{bmatrix} =
\begin{bmatrix}
\frac{-R}{L} & \frac{-K_T}{L} \\
K_T & \frac{-D}{J}
\end{bmatrix}
\begin{bmatrix}
I(s) \\
\dot{\Theta}(s)
\end{bmatrix} +
\begin{bmatrix}
\frac{1}{L} \\
0
\end{bmatrix} V(s)
\] (7.2)

The continuous time TF shown in Equation (7.1) can be transformed into discrete time as shown in Equation (7.3) assuming a zero order hold transformation.

\[
\begin{bmatrix}
I(z^{-1}) \\
\omega(z^{-1})
\end{bmatrix} =
\begin{bmatrix}
b_{11}z^{-1} + b_{12}z^{-2} \\
b_{w1}z^{-1} + b_{w2}z^{-2}
\end{bmatrix}
\frac{1}{1 + a_1z^{-1} + a_2z^{-2}} V(z^{-1})
\] (7.3)

Since the model structure of the system is known, it is quite straightforward to estimate the parameters of the equivalent discrete time TF model using least squares estimation method.

7.2.4 Model assessment

The model output obtained using least-squares estimation method is evaluated by visual inspection and by statistical measures using the well known Coefficient of Determination (CoD), \( R_T^2 \) given by:

\[
R_T^2 = 1 - \frac{\sigma^2}{\sigma_y^2}
\] (7.4)

where \( \sigma^2 \) is the sampled variance of the model error and \( \sigma_y^2 \) is the sample variance of the measured system output about its mean value [108, 126]. Value of \( R_T^2 \) tends to unity as the correlation between the actual data and the model data improves. In this work the model correlation is described in percentage (i.e., \( R_T^2 \times 100\% \)) and the minimum permissible value is set to 80\%. This value is chosen because after so many experiments conducted, it was found that it can be achieved by both the current and displacement signals.
CHAPTER 7. CONDITION MONITORING DEVELOPMENT

7.2.5 Transformation of the discrete time model to a continuous time model

The discrete time model (in state-space form) obtained from the least squares estimation is transformed to continuous time using MATLAB command \texttt{d2c} assuming a zero-order hold on the input. The continuous time state-space model takes the form:

\[
\begin{bmatrix}
\dot{I}(s) \\
\dot{\Theta}(s)
\end{bmatrix} =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\begin{bmatrix}
I(s) \\
\dot{\Theta}(s)
\end{bmatrix} +
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} V(s) \tag{7.5}
\]

Once the parameters \(a_{11}, a_{12}, a_{21}, a_{22}, b_1\) and \(b_2\) are obtained, the next step is to calculate the actuator physical parameters from those coefficients.

7.2.6 Parameter calculation

Equation (7.5) is used to map the actuator physical parameters to those in Equation (7.2) and the following equations are obtained which are used to calculate the estimate of the physical parameters from the estimated model.

\[
\hat{L} = \frac{1}{b_1} \quad \hat{R} = -a_{11}L \\
\hat{K}_e = -a_{12}L \quad \hat{K}_T = K_e \\
\hat{J} = \frac{K_T}{a_{21}} \quad \hat{D} = -a_{22}J \tag{7.6}
\]

Results of implementing the six-steps procedure to a single actuation elements, 3-elements-in-series and the full HRA are discussed in the following sections.
7.3 Parameter Estimation Results

In this section, the results of applying the previously explained six-step procedure of the least squares parameter estimation method to three different actuator configurations are presented and discussed. The configurations considered are: single actuation element; 3 actuation elements connected in series; and the full 4-by-3 HRA.

7.3.1 Single Actuation Element

Firstly, a suitable input signal to excite the system so that the experimentally obtained input-output data can be used by the system identification algorithm to generate a model that is a good representation of the actual system is selected.

The first input signal tested was a sine wave with band limited noise as shown in Figure 7.3 (upper plot). A series of experiments conducted show that sine-waves with an amplitude of 0.3 and frequency of $3 \text{rad/s}$ with band limited noise of amplitude $8 \times 10^{-4}$ and sampling time of 0.006s give a good match between the actual and model output current (CoD above 80%) as shown in Figure 7.4a. However, a good match between the actual and model angular speed cannot be achieved (CoD is less that the minimum permissible value) as shown in Figure 7.4b.
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Figure 7.3: Graph showing input (voltage) and output (displacement and current) signal when sine wave with band limited noise is used as input signal.

Figure 7.4: Graph showing the correlation between actual and model output when sine-wave with band limited noise is used as input signal.
The second input signal tested was a PRBS signal as shown in Figure 7.5 (upper plot). Also shown in this figure are the output signals: displacement (middle plot) and current (lower plot). The displacement signal was then differentiated to obtain angular speed (this is because Equation (7.1) requires current and angular speed information).

Experiments conducted show that with PRBS as the input signal, a good match between the actual and model signals for both the current and angular speed can be achieved as shown in Figure 7.6 with CoD of 93.7% and 99.5% for current and angular speed, respectively. Therefore, it was concluded that with PRBS as input signal, the estimated model is a good representation of the actual system.

![Figure 7.5: Graph showing input (voltage) and output (displacement and current) signal when PRBS is used as the input signal.](image)

The above mentioned experiment was repeated 20 times and the estimated key physical parameters of the actuator were plotted as normal distribution curve as shown in Figure 7.7. This figure shows the range of each estimated parameter. For example, the range of $\hat{L}$ is between 2.3$mH$ to 2.5$mH$.

The parameter values obtained using the least-squares parameter estimation method are repeatable (good precision) as shown in Figure 7.7.
Based on these reasons (and the fact that the CoD of the current and angular speed are above the minimum permissible value), it can be deduced that the estimated parameter values shown in Figure 7.7 are within reasonable range.

Results of implementing the parameter estimation method to 3-elements-in-series configuration is presented in the following subsection.
7.3. PARAMETER ESTIMATION RESULTS

(a) Estimated values of $L$

(b) Estimated values of $R$

(c) Estimated values of $K_e$ and $K_T$

(d) Estimated values of $J$

(e) Estimated values of $D$

Figure 7.7: Estimated parameters of a single actuation element.
7.3.2 3-Elements-In-Series

The least-squares parameter estimation method has been tested using a single actuation element and results show that the designed algorithm works as expected with PRBS as input signal. The next step is to test the algorithm using a more complex structure, which is 3 actuation elements connected in series. This step is necessary before testing the algorithm using the full HRA assembly to ensure that the algorithm works perfectly under healthy and faulty conditions.

Healthy Condition

Figure 7.8 shows the input-output data when 3 actuation elements that are connected in series are driven using PRBS with all elements under healthy condition.

![Figure 7.8: Experimental results showing actuation element’s voltage, displacement and current for the 3-elements-in-series with all elements under healthy condition.](image)

The individual voltage, displacement and current is summed up to get total voltage, displacement and current. The averaged and down-sampled total input-output signals are shown in Figure 7.9.

Figure 7.10 shows the actual current and angular speed with their estimated model. The upper left and right plots show the actual and model signals in full scale while the lower left and right plots show the signals in zoomed scale (from 0.5s to 1s). It can be seen that the CoD of both signals (i.e. current and angular speed) are more than the minimum permissible values with 87.16% and 99.52% for current and velocity, respectively. This means the estimated model is a good representation of the actual system.

Figure 7.9: Graph showing total voltage, displacement and current for the 3-elements-in-series.
CHAPTER 7. CONDITION MONITORING DEVELOPMENT

Faulty Conditions

The same process is repeated with one or more actuation elements subjected to lock-up and short-circuit faults and Figure 7.11 shows the measured individual input-output under faulty conditions. The upper left plot shows the input-output signal when one actuation element (i.e. A1) is subjected to lock-up fault while the upper right plot shows result when two actuation elements (i.e. A1 and A3) are locked. The lower left plot shows the input-output signal when A3 is subjected to short-circuit fault while the lower right plot shows result when A2 and A3 are subjected to short-circuit fault.

Figure 7.12 shows the actual and model output. It can be seen that a good match between the actual and model output can be obtained even in the faulty conditions.

The experiment is repeated for 20 times (for both the healthy and faulty conditions) and the range of estimated parameters for the healthy and faulty conditions are shown in Figure 7.13.

From this figure, it can be seen that the estimated parameter values are approximately the same regardless of whether the actuation elements are subjected to short-circuit or lock-up faults. Also, the value of $J$ and $D$ are very similar regardless of whether the actuator is in the healthy or faulty conditions.

The reason for this is because the effect of short-circuit fault is very
similar to the effect of lock-up fault. In both cases, loss of travel is to be expected because the faulty elements loose their capability to move. The only difference is that in the case of short-circuit faults, the faulty elements can be back-driven if sufficient force is applied while in the case of lock-up faults, the faulty elements are 'jammed' in a place.

To confirm this theory, a frequency response analysis was conducted.

Figure 7.11: Experimental results showing the input-output data of the 3-elements-in-series under lock-up and short-circuit faults.
CHAPTER 7. CONDITION MONITORING DEVELOPMENT

Figure 7.12: Graphs showing actual and model output of the 3-elements-in-series under lock-up and short-circuit faults.

(a) One actuation element locked

(b) Two actuation elements locked

(c) One actuation element short-circuit

(d) Two actuation elements short-circuit
7.3. PARAMETER ESTIMATION RESULTS

(a) Estimated values of $L$

(b) Estimated values of $R$

(c) Estimated values of $K_e$ and $K_T$

(d) Estimated values of $J$

(e) Estimated values of $D$

Figure 7.13: Estimated parameters of the 3 elements in series.
Frequency Response Analysis

A frequency analysis was conducted to confirm the theory that the loose faults that were demonstrated in the experimental rig by making the supply voltage to the faulty element/s equal to zero appear like a lock-up fault.

The frequency analysis is based on the following steps:

1. Run open loop experiments by moving the actuator forward and backward using sine wave with different frequencies as input signal.

2. Record the input (voltage) and output (current and displacement) to calculate gain and phase.

3. Plot Bode diagram of current, displacement and angular speed based on the calculated gain and phase.

Figure 7.14 shows the Bode plot of current, displacement and angular speed when one actuation element is subjected to lock-up and short-circuit faults (2 experiments were conducted for each failure case). From this figure, it can be seen that the behaviour of the system are very similar regardless of whether the actuation elements are subjected to lock-up or short-circuit faults which confirms the theory that the effect of short-circuit fault is similar to lock-up fault.
7.3. PARAMETER ESTIMATION RESULTS

(a) Bode plot of current.

(b) Bode plot of displacement.

(c) Bode plot of angular speed.

Figure 7.14: Bode plot of current, displacement and angular speed when one actuation element is subjected to short-circuit and lock-up fault.
7.3.3 Full HRA Assembly

In this section, the parameter estimation is tested using the full HRA assembly. First, the HRA was driven forward and backward with all actuation elements functional and the physical parameters of the actuator are estimated.

Figure 7.15 shows the total input-output signals. Total voltage and current were obtained by summing all twelve individual voltages and currents while total displacement was obtained by summing the displacement of three elements that are connected in series (e.g. $X_T = X_{11} + X_{12} + X_{13}$). This effectively the same as just having one position and one current sensor to measure the total displacement and current of the whole assembly.

The actual and model output are shown in Figure 7.16 and it can be seen that there is a good match between the actual and model output with CoD of 86.57% and 99.32% for the current and angular speed, respectively.

Again, the experiment was repeated for 20 times and after that, the key physical parameters of the actuator with all actuation elements functional were estimated and the results are shown in Figure 7.17. This figure shows the estimated parameters range. For example, the range of $\hat{L}$ is between 2.35mH to 2.7mH.

Then, the same procedure was repeated with different combinations of 1 element fail, 2 elements fail, 3 elements fail and 4 elements fail. Twelve combinations of one short-circuit element were tested (i.e $A_{11}$ short-circuit, $A_{12}$ short-circuit and so on). For 2 short-circuit elements, 68 different combinations were tested to obtain the range of estimated parameters while more than 100 combinations were tested for both 3 and 4 short-circuit elements.

Tests were done with up to 4 actuation elements failed due to the exponential increase in complexity to do simulations and experiments because there will be more than 200 different failure combinations for more than 4 faulty elements.

The range of estimated parameters under different health conditions is given in Table 7.1. These values will be used to design the fuzzy logic input membership function which is discussed in the next section.
7.3. PARAMETER ESTIMATION RESULTS

Figure 7.15: Total input-output signal of the full HRA assembly under healthy condition.

Figure 7.16: Current and velocity estimation of the full HRA assembly under healthy condition.
Figure 7.17: Estimated parameters of the full HRA assembly under healthy condition.
7.4. FUZZY LOGIC

The second part of the condition monitoring algorithm is fuzzy logic inference to give an indication of the actuator performance (i.e. healthy or faulty condition). The concept of fuzzy logic was first introduced by Dr. Lotfi Zadeh in the 1960s [127]. Fuzzy logic is a form of probabilistic logic that maps an input space to an output space using a set of if-then rules [128].

Since then, fuzzy logic has been widely used in different fields including control, failure mode analysis and condition monitoring. For example, [129] uses fuzzy logic to diagnose fault in power transformer while [130] employs fuzzy logic for failure mode, effects and criticality analysis in automobiles. Fuzzy logic based condition monitoring for induction motor and wind turbine is discussed in [100] and [131,132], respectively.

In this thesis, fuzzy logic is used to analyse the actuator performance based on the changes in the actuator physical parameters as the actuator health condition changes from healthy to faulty. Fuzzy logic method is favoured because it provides a tool to directly working with the linguistic terms such as high, low, healthy, 1 element fail or 2 elements fail in making assessment.

Generally, fuzzy logic design involves three main steps as discussed in the following subsection.
7.4.1 Fuzzy Logic Design Process

As explained in [133] and [130], the fuzzy logic design process can generally be divided into three steps:

**Step 1: Fuzzification of the input data**

This step refers to the process of assigning a membership function (MF) to the fuzzy input data. A MF can be defined as “...a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1...” [133]. If the value 1 represents a point entirely within the input space and the value 0 represents a point entirely outside the input space, then any point partially in the input space will have a value between 0 and 1.

The MATLAB fuzzy logic toolbox includes eleven MFs. The simplest being the triangular MF, \( \text{trimf} \) and the trapezoidal MF, \( \text{trapmf} \). Other MFs are: Gaussian distribution based MFs (\( \text{gaussmf}, \text{gauss2mf} \)); generalized bell MF (\( \text{gbellmf} \)); Sigmoidal MFs (\( \text{sigmf}, \text{psigmf}, \text{dsigmf} \)) and polynomial based MFs (\( \text{zmf}, \text{pimf}, \text{smf} \)). Figure 7.18 shows the general shape of all eleven MFs.

![Membership functions in the MATLAB fuzzy logic toolbox.](image)
7.4. FUZZY LOGIC

Step 2: Derivation of the fuzzy rules and output functions

The fuzzy rule base which often expressed as *if-then* rules, relates a current values in the input variable to the entire fuzzy set in the output variable. It is usually expressed as a conditional statement of the form:

if $x$ is A, then $y$ is B

where $x$ and $y$ are linguistic variables (e.g. service, height or temperature), while A and B are linguistic values (e.g. good, average or hot) determined by fuzzy sets on the ranges of X and Y, respectively.

The fuzzy rule can be divided into two parts: antecedant or premise (the if-part); and consequent or conclusion (the then-part). Both the antecedant and consequent part can have multiple variables which are calculated simultaneously and resolved to a single number using either the fuzzy AND operator or the fuzzy OR operator.

Output functions were then built using the fuzzy rule’s conclusion.

Step 3: Defuzzification of the fuzzy output

Finally, the output is defuzzified to obtain a single number to represent the output set in order to decipher the meaning of the fuzzy conclusion and their membership values. The output could be riskiness ranking, a warning level, or a weather condition.

The MATLAB fuzzy logic toolbox supports five built-in defuzzification methods: centroid, bisector, middle of maximum (mom), largest of maximum (lom), and smallest of maximum (som). Refer to [133] for detail explanation of each method.

7.4.2 Implementation of The Fuzzy Logic Design to The 4-by-3 HRA Assembly

Step 1: Fuzzification of the input and output data

For the work described within this thesis, the fuzzy input functions are the estimated parameters of the actuator under different health conditions.
Recall from Section 7.4 that there are five parameters estimated using the least squares parameter estimation: inductance, \( L \); resistance, \( R \); motor torque constant, \( K_t \); back emf constant, \( K_e \); inertia, \( J \); and friction, \( D \). However, only 3 parameters: \( L \), \( R \) and \( K_e \) were used in the fuzzy logic design. This is because \( K_t \) was assumed to be equal to \( K_e \) in the parameter estimation process while \( J \) and \( D \) were not considered because there are no significant difference in their estimated values regardless of the actuator’s health condition (refer to Figure 7.13). In other words, the faults overlap and become unidentifiable from \( J \) and \( D \).

Figure 7.19 shows the fuzzified parameters built using the parameter range listed in Table 7.1. At first, three MFs were considered to represent the fuzzy input functions: trapezoidal, generalised bell and Gaussian (\( gauss2mf \)). In the end, trapezoidal MF was chosen to represent the fuzzy input functions (as shown in Figure 7.19) because it is easier to be implemented compared to the generalised bell MF and the \( gauss2mf \).

**Step 2: Derivation of the fuzzy rules**

For the 4-by-3 HRA condition monitoring, five fuzzy rules were derived to map the input set to the output set.

**Rule 1:**
if \( (L \) is healthy) and \( (R \) is healthy) and \( (K_e \) is healthy) then (output is healthy)

**Rule 2:**
if \( (L \) is 1Fail) and \( (R \) is 1Fail) and \( (K_e \) is 1Fail) then (output is 1Fail)

**Rule 3:**
if \( (L \) is 2Fail) and \( (R \) is 2Fail) and \( (K_e \) is 2Fail) then (output is 2Fail)

**Rule 4:**
if \( (L \) is 3Fail) and \( (R \) is 3Fail) and \( (K_e \) is 3Fail) then (output is 3Fail)

**Rule 5:**
if \( (L \) is 4Fail) and \( (R \) is 4Fail) and \( (K_e \) is 4Fail) then (output is 4Fail)

The MFs for the fuzzy output derived from the fuzzy rules are shown in Figure 7.20.
7.4. FUZZY LOGIC

(a) Membership function of $L$.

(b) Membership function of $R$.

(c) Membership function of $Ke$.

Figure 7.19: Membership functions for the fuzzy input.
Figure 7.20: Membership functions for the fuzzy output.

Step 3: Defuzzification of the fuzzy output

In this thesis, the middle of maximum defuzzification method is employed to defuzzy the output. With this method, the crisp output value is the mean of the maximum value assumed by the aggregate membership functions.
7.4.3 Testing The Fuzzy Logic Inference with The 4-by-3 HRA Assembly

To test the designed fuzzy logic inference, the HRA was tested with the PRBS signal explained earlier. Then, faults were injected to some of the elements at a 5s interval. For example, at $t=5s$, one element ($A_{13}$) is subjected to loose fault. Five seconds later, another element ($A_{31}$) is loose and a third element ($A_{32}$) is subjected to loose fault at $t=15s$. Lastly, $A_{42}$ is subjected to loose fault at $t=20s$.

Now, the estimation process is done at every sample interval using moving window method, which is different from the estimation process explained in the previous subsections where a single estimation is done for the whole 5s data.

Figure 7.21 shows the HRA condition as it changes from healthy to faulty condition in probability form as an output of the fuzzy inference system. For example, at $t=0s$ until $t=5s$, the healthy condition probability is equal to 1 because the HRA is in healthy condition. However, at $t=5s$, one element started to fail. Therefore, the healthy condition probability starts to decrease and the 1 element fail probability starts to increase and finally becomes 1 after a few seconds. The one element fail probability remains 1 until $t=10s$ because at this time, another element is failing.

At $t=10s$, two elements are failing (i.e. $A_{13}$ and $A_{31}$) so the two elements fail probability started to increase and finally becomes 1 at $t=11.8s$. Similarly, at $t=15s$, the probability of three elements fail increased and becomes 1 at approximately $t=16s$ (at this time, three elements are failing which are $A_{13}$, $A_{31}$ and $A_{33}$). Finally, at $t=20s$ four fail elements are recorded and the four elements fail probability started to increase and becomes 1 at $t=21s$.

These results show that the designed fuzzy logic is capable of detecting changes in the actuator condition. This result could be used to plan maintenance predictively because as more element fail, the actuator capability will drop below that required by the application.
7.5 Conclusion

In this chapter, a condition monitoring algorithm based on least-squares parameter estimation and fuzzy logic inference to monitor the health condition of the HRA as it changes from healthy to faulty has been discussed. The estimation method combined discrete time parameter estimation with the physical model of the system to facilitate the estimation of the physical parameters of the electromechanical actuator.

The least-squares estimation method was tested using a single actuation element, 3 actuation elements in series and the full HRA assembly. Using the designed algorithm, a good correlation between the actual and model output ($R^2_T$ of current output is above 85% and $R^2_T$ of angular speed is above 95%) can be obtained both in the healthy and faulty conditions.

Results also show that the estimated parameter values are approximately at the same range regardless of whether the actuation elements are subjected to lock-up or loose faults. This is because, the short-circuit faults have similar effect as the lock-up fault which was proven through the frequency response analysis.

The estimated parameter values are then used to design the membership function of the fuzzy logic input. Five Mamdani rules and fuzzy output membership functions were derived based on the actuator health condition (i.e healthy, 1 element fail, 2 elements fail, 3 elements fail and 4 elements
fail). The fuzzy inference was then tested using the full HRA assembly and results show that the fuzzy logic is capable of detecting changes in the actuator condition and gave indication on how many actuation elements are fail based on the estimated physical parameters.

As more actuation elements fail, the actuator capability will drop below that required by the application. This condition monitoring algorithm can be developed and used to inform the maintenance/repair/replacement plan in order to avoid any catastrophic failures.
Chapter 8

Conclusions and Future Work

8.1 Conclusions

In this thesis, research related to highly redundant and fault tolerant actuator has been presented. The HRA is an approach to fault tolerant actuation that aims to provide fault tolerance using a relatively large number of small actuation elements. The actuation elements are assembled in matrix (i.e. in parallel and series) configuration to form a single actuator that has intrinsic fault tolerance.

Firstly, background study on the previous works related to HRA done by colleagues in the Control Systems Research Group at Loughborough University was done. It was concluded that the previous work that was based on electromechanical actuation technology with relatively low number of actuation elements (4 elements) could be expanded by employing a larger number of actuation elements. Following the other HRA-related work that was based on electromagnetic actuation technology, different condition monitoring scheme that does not require the use of multiple sensors was explored and developed.

The objectives of the thesis were:

1. to develop a HRA experimental rig that consists of 12 actuation elements

2. to model the HRA in Simulink and to validate experimentally the
model, which will then be used as the foundation of the controller and condition monitoring design

3. to design and implement classical and $H_{\infty}$ control scheme and compare their performance

4. to explore possible method of condition monitoring that does not require multiple sensors and to develop and validate an approach for doing this

The main content of the thesis began in Chapter 3 where the development of the experimental test rig with 12 actuation elements was presented and discussed. Detail explanation of the mechanical, electrical and data acquisition and control hardware were given. The test rig was used to do real-time experiments to validate the actuator’s mathematical model as well as to implement the controller and the condition monitoring algorithm studied within this thesis.

Chapter 4 presented the modelling of: a single element actuator; multiple elements actuator (i.e. two elements in series and two elements in parallel); and a general n-by-m HRA assembly that was derived from first principles. The model of the 4-by-3 HRA was obtained from the general n-by-m model. The general n-by-m model allows future work by other researchers to study any case of EMA based HRA with matrix configuration. The derived models were validated qualitatively in simulation environment in the healthy and faulty conditions and it was found that series configuration increases travel capability while parallel configuration increases force capability above the capability of an individual element. Quantitative validation was also done by comparing simulation and experimental results. A good match between the simulation and experimental results was achieved after adjusting some of the not well defined (i.e. not available in the data sheet) actuator parameters using MATLAB design optimization toolbox. The derived models were then used as the foundation of the control design and condition monitoring development.

Chapter 5 considered a classical control strategy for: a single actuation element; 3 elements connected in series; and the full HRA assembly. Two
control structures: local and global with two different controllers: P controller and PI controller were designed to study the actuator’s performance in the healthy and faulty conditions. The resultant performance showed that with the local P controller, the actuators cannot track the demand position unless the gain was increased to the point that it causes saturation in the input voltage and output current. Similarly, poor tracking performance was recorded in the faulty conditions when the local PI controller was used which was due to the structural limitation of the controller. Whereas, with the global PI controller, the actuator demonstrated a good tracking performance even in the faulty condition, albeit slower response compared to when the actuators were in the healthy condition.

$\mathcal{H}_\infty$ control strategy that was implemented in 3 actuation elements in series and the full HRA assembly was discussed in Chapter 6. The aim was to investigate whether the claimed “robustness to inherent uncertainties” properties of the $\mathcal{H}_\infty$ control method would lead to a global controller that would better tolerate faults in the subactuators. Mixed sensitivity method that considered the primary, control and complementary sensitivity functions was used to design the $\mathcal{H}_\infty$ controller. The actuator’s performance was examined in the healthy and faulty conditions. The parametric uncertainty that arose due to the changes in the plant dynamics as the actuator health condition changes from healthy to faulty was also taken into consideration by calculating the relative errors between the plant’s dynamics in both conditions. The resultant performance showed that the global $\mathcal{H}_\infty$ controller exhibits robust stability regardless of the system’s health condition but exhibits reduced robust performance on the faulty conditions. However, this results are acceptable because for the HRA studied within this thesis, good stability is to be expected in the healthy and faulty conditions but performance degradation in the presence of faults are to be expected. Other than that, smaller performance degradation was recorded with the global $\mathcal{H}_\infty$ controller as opposed to the global PI controller. This proves that the $\mathcal{H}_\infty$ controller demonstrates better robustness to parametric uncertainty and is beneficial to the HRA.

Finally, the development of a condition monitoring scheme was pre-
sented in Chapter 7. The condition monitoring algorithm would be a part of a self-diagnosis test that would probably take place pre- or post-flight and its purpose is to provide an indication of the actuator’s health condition to schedule maintenance or more likely, replacement. The developed condition monitoring scheme was based on least-squares parameter estimation and fuzzy logic inference. The least-squares parameter estimation used experimentally obtained input-output of the system to estimate the actuator’s key physical parameters as the actuator health condition changes from healthy to faulty. Afterwards, the fuzzy logic inference used the estimated parameter range to give information on the actuator’s health condition. The developed condition monitoring scheme was tested using a single actuation element, 3 elements in series and the full 4-by-3 HRA assembly. The recorded results showed that the condition monitoring is capable of giving information on the number of faulty actuation elements in the 4-by-3 HRA setup.

8.2 Suggestions for Future Work

The work presented in this thesis has provided significant contribution to the HRA research. However, the author (as well as other researchers involved with the project) is aware that it is still a long way (and more works are required) before the HRA concept can be implemented in real industry.

The experimental test rig that was fabricated for the work described within this thesis was aimed for laboratory testing. Therefore, it is very big to be implemented as an aircraft aileron actuator. Therefore, in the future, researchers that will be doing the HRA research should aim to built an experimental rig that can be implemented in the real industry. For example, smaller actuation elements such as microactuator that is based on micro-electromechanical system (MEMs) or piezoelectric actuator should be considered so that HRA with more actuation elements (e.g. a 10-by-10 HRA) can be built with reduced size.

The previous HRA-related works considered a complicated control structure that involves multiple-loop classical controller and active fault tolerant
control. In this thesis, it has been shown that a good tracking performance in the healthy and faulty conditions can be achieved with simpler control structure. However, the control method was designed with application in aircraft in mind. Therefore, other control method, perhaps for other critical safety applications could be researched and investigated.

The condition monitoring is another area to be considered for future work. There is scope for extension of the least-squares parameter estimation based condition monitoring. For example, the condition monitoring algorithm is capable of giving information on the number of faulty elements but it is not capable of giving information on which actuation element is failing. It was felt that this information is important to avoid system failure due to all of the elements that are connected in series experience lock-up fault.

Finally, the formulation of design synthesis methodologies to develop tools and standard procedures for moving from system requirement to HRA realisation is also an area that requires investigation. Factors such as required reliabilities, capabilities and dimensioning should be considered to aid and promote industrial implementation.
Bibliography


[105] THK, THK Low Price Actuator: VLACT35 Data Sheet. THK Co., LTD.


Appendix A

Nomenclature

A.1 Abbreviations

ADC  Analogue-to-Digital converter
ADT  Advanced Diagnostic Test-bed
AFTC  Active Fault Tolerant Control
ANN  Artificial Neural Network
CBM  Condition based maintenance
CM  Condition monitoring
CoD  Coefficient of determination
CRCS  Control rod control systems
DAC  Digital-to-Analogue converter
EMA  Electromechanical actuator
emf  Electromotive force
FCS  Flight control system
FDI  Fault detection and identification/isolation
GM  Gain margin
HERTI  High Endurance Rapid Technology Insertion
HRA  High Redundancy Actuator
MIMO  Multiple input multiple output
OS  Overshoot
P  Proportional
PFTC  Passive fault tolerant control
PI Proportional-integral
PID Proportional-integral-derivative
PM Phase margin
REB Rolling element bearings
RT Rise time
SISO Single input single output
SRIV Simplified refined instrumental variable
SSE Steady-state error
ST Settling time
TF Transfer function
UAV Unmanned aerial vehicle

A.2 Symbols

$K_p$ Proportional gain
$K_i$ Integral gain
$K_{pi}$ Overall PI gain
$\tau_i$ Integrator time constant
$\omega_i$ Cross over frequency
$V_s$ Input voltage
$R$ DC motor armature resistance
$L$ DC motor armature inductance
$K_e$ DC motor back-emf constant
$K_T$ DC motor torque constant
$J$ DC motor inertia
$D$ DC motor viscous damping
$\theta_m$ DC motor angular displacement
$I(s)$ DC motor armature current
$Le$ Ball screw lead
Appendix B

Parameter Calculation

B.1 Viscous friction calculation

The DC motor viscous friction can be calculated using the following formula:

\[ T = D \omega \]  \hspace{1cm} (B.1)

where \( T \) is torque in Nm, \( D \) is viscous friction in Nms/rad, \( \omega \) is angular speed in rad/s.

From the DC motor data sheet, \( T = 0.3 Nm \) and \( \omega = 314.159 rad/s \). So, DC motor viscous friction can be calculated as follow:

\[ D = \frac{T}{\omega} = \frac{0.3}{314.159} = 9.549 \times 10^{-4} Nms/rad \]

B.2 End of actuator’s screw stiffness

The screw stiffness is calculated using Hooke’s Law and Young’s Modulus formula:

1. Hooke’s Law states that:

\[ F = k \Delta X, \]

So,
\[ k = \frac{F}{\Delta X} \tag{B.2} \]

2. Young’s Modulus states that:

\[ E = \frac{\sigma}{\epsilon} = \frac{F}{\frac{A}{X}} \]

So,

\[ \Delta X = \frac{FX}{AE} \tag{B.3} \]

where \( E \) is Young’s Modulus. Assuming material is steel, \( E = 200 \times 10^9 \text{N/m}^2 \).

Combining Equation B.2 and B.3 yields the following equation that is used to calculate the screw stiffness, \( k \):

\[ k = \frac{AE}{X} = \frac{\pi R^2 E}{X} = \frac{\pi (4 \times 10^{-3})^2 (200 \times 10^9)}{50 \times 10^{-3}} = 201.06 \times 10^6 \text{N/m} \]

### B.3 End of actuator’s screw damping coefficient

The screw damping is determined using the transfer function of a spring-damper-mass as follow:

1. Transfer function of a simple spring-damper-mass in Laplace form is:

\[ H(s) = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + Cs + K} = \frac{\frac{1}{M}}{s^2 + \frac{C}{M}s + \frac{K}{M}} \tag{B.4} \]

2. The transfer function can be re-written in terms of its pole location as:

\[ H(s) = \frac{\frac{1}{M}}{s^2 + 2\xi \omega_n s + \omega_n^2} \tag{B.5} \]

Comparing Equations B.4 and B.5 results the following:
B.3. END OF ACTUATOR’S SCREW DAMPING COEFFICIENT

\[ 2\xi \sqrt{\frac{K}{M}} = \frac{C}{M} \]

where

\( \xi = \) damping coefficient (is assumed to be 1 which is the case of a critical damping) \( C = \) screw damping coefficient \( K = \) spring stiffness \( M = \) spring mass

Knowing the screw stiffness and mass, the screw damping coefficient can be calculated as:

\[ C = 2\xi M \sqrt{\frac{K}{M}} = (2)(1)(0.9)\sqrt{\frac{201.06 \times 10^6}{0.9}} = 26.9 \times 10^3 Ns/m \]
APPENDIX B. PARAMETER CALCULATION
Appendix C

Sample Matlab Code for $\mathcal{H}_\infty$ Controller Design

1
2 %define the parameter values
3 L = 0.002668;
4 R = 0.976987;
5 Kt = 0.007124;
6 Ke = 0.007124;
7 J = 2.714326e 6;
8 D = 6.710936e 5;
9 Le = 6e 3;
10
11 %changing bode plots properties
12 opts = bodeoptions;
13 opts.Title.FontSize = 16;
14 opts.XLabel.FontSize = 16;
15 opts.YLabel.FontSize = 16;
16 opts.TickLabel.FontSize = 16;
17 opts.Title.FontWeight = 'bold';
18 opts.XLabel.FontWeight = 'bold';
19 opts.YLabel.FontWeight = 'bold';
20 opts.TickLabel.FontWeight = 'bold';
APPENDIX C. SAMPLE MATLAB CODE FOR $\infty$ CONTROLLER DESIGN

% calculate numerator of transfer function
num = Kt*Le;

% calculate denominator of transfer function
den1 = J*L*2*pi;
den2 = ((J*R)+(D*L))*2*pi;
den3 = ((D*R)+(Ke*Kt))*2*pi;

% the single actuator's transfer function (continuous time)
G = tf([num],[den1 den2 den3 0]);

figure(1);
set(gcf, 'color', 'w');
bode(G, opts); grid
title('Bode plot of plant');
h = findobj(gcf, 'type', 'line');
set(h, 'linewidth', 3);

% DEFINE PRIMARY SENSITIVITY WEIGHTING FUNCTION
% W1's transfer function, which is a low pass filter
W1 = tf([1 10],[1 0.01]);

% multiply W1 with gamma
gamma_1 = 0.033;
W1 = W1*gamma_1;

% DEFINE CONTROL SENSITIVITY WEIGHTING FUNCTION
% W2's transfer function, which is a static gain
W2 = tf([1],[2.7]);
\begin{verbatim}
53 \%W2 = 1;
54 \%multiply W2 with gamma
55 gamma_2 = 0.063;
56 W2 = W2*gamma_2;
57
58 \%DEFINE COMPLEMENTARY SENSITIVITY WEIGHTING FUNCTION
59 \% W3's transfer function, which is a high pass filter
60 W3 = tf([1 1],[1 30]);
61
62 \%multiply W2 with gamma
63 gamma_3 = 26.85;
64 W3 = W3*gamma_3;
65
66 \%CREATE AUGMENTED SYSTEM
67 P = augw(G,W1,W2,W3)
68
69 \%make the system perform the H_infinity design
70 [K,CL,GAM,INFO] = hinfsyn(P)
71
72 \%perform the lower fractional transformation to get
73 \the transmission
74 G11 = W1;
75 G12 = G*W1;
76 G21 = 0;
77 G22 = W2;
78 G31 = 0;
79 G32 = G*W3;
80 G41 = 1;
81 G42 = G;
82
83 G_wz1 = G11 + G12*K*(1 G42*K)^1*G41;
\end{verbatim}
APPENDIX C. SAMPLE MATLAB CODE FOR $\infty$ CONTROLLER DESIGN

85 $G_{wz2} = G21 + G22*K*(1 + G42*K) \times 1*G41;$
86 $G_{wz3} = G31 + G32*K*(1 + G42*K) \times 1*G41;$
87
88 \%plot the bode diagram of the closed loop model.
89 $w = \{0.1,10\};$
90
91 \textbf{figure}(2);
92 \textbf{set}(\textbf{gcf},'color','w');
93 \textbf{bodemag}(G_{wz1},G_{wz2},G_{wz3},\textbf{opts},w); \textbf{grid}
94 \textbf{legend}('T_{\{z_1w_1\}}', 'T_{\{z_2w_2\}}', 'T_{\{z_3w_3\}}')
95 \%title('Bode diagram of Gwz');
96 $h = \textbf{findobj}(\textbf{gcf},'type','line');$
97 \textbf{set}(h,'linewidth',3);
98
99 \%compare Bode magnitude plot of primary sensitivity function and the
100 \%weighting functions.
101 \%weighting functions.
102 $G = \textbf{ss}(G);$  
103 $S_{primary} = 1/(1 + (K*G));$
104 $S_{weight} = 1/W1;$
105
106 $w=\{0.1,10\};$
107
108 \textbf{figure}(3);
109 \textbf{set}(\textbf{gcf},'color','w');
110 \textbf{bodemag}(S_{primary},S_{weight},\textbf{opts},w); \textbf{grid}
111 \textbf{legend}('S(s)', 'W_{1}\{1\}');
112 $h = \textbf{findobj}(\textbf{gcf},'type','line');$
113 \textbf{set}(h,'linewidth',3);
114
115 $R_{control} = K/(1 + (K*G));$
116 $R_{weight} = 1/W2;$
figure(4);
set(gcf,'color','w');
bodemag(R_control,R_weight,opts,w);grid
legend('R(s)','W_2^\{1\}');
h = findobj(gcf,'type','line');
set(h,'linewidth',3);

T_comp = K*G / (1 + (K*G));
T_weight = 1/W3;

figure(5);
set(gcf,'color','w');
bodemag(T_comp,T_weight,opts,w);grid
legend('T(s)','W_3^\{1\}');
h = findobj(gcf,'type','line');
set(h,'linewidth',3);

%%% extract the controller state space
A = K.a;
B = K.b;
C = K.c;
D = K.d;
APPENDIX C. SAMPLE MATLAB CODE FOR $\mathcal{H}_\infty$ CONTROLLER DESIGN
Appendix D

Data Sheets
<table>
<thead>
<tr>
<th>Characteristics*</th>
<th>Nenndaten*</th>
<th>Rated voltage Nennspannung U/V V</th>
<th>12</th>
<th>24</th>
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<tr>
<td>Rated power Nennleistung PN W</td>
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<td>Rated torque Nenndrehmoment T/MN Ncm</td>
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<td>30</td>
<td></td>
<td></td>
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<tr>
<td>Rated speed Nenndrehzahl nN rpm/min⁻¹</td>
<td>3000</td>
<td>3000</td>
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<tr>
<td>Rated current Nennstrom IN A</td>
<td>12</td>
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<table>
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<th>No load characteristics*</th>
<th>Leerlaufdaten*</th>
<th>No load speed Leerlaufdrehzahl nO rpm/min⁻¹</th>
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<th>3700</th>
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<td>No load current Leerlaufstrom IO A</td>
<td>1.3</td>
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<tr>
<th>Stall characteristics*</th>
<th>Anlaufdaten*</th>
<th>Stall torque Anlaufmoment T/Ma Ncm</th>
<th>130</th>
<th>140</th>
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<tr>
<td>Stall current Anlaufstrom IS/IH A</td>
<td>48</td>
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<table>
<thead>
<tr>
<th>Performance characteristics*</th>
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<th>140</th>
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<td>max. Constant torque max. Dauerdrehmoment Tmax/Mmax Ncm</td>
<td>20</td>
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<table>
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<tr>
<th>Motor parameters*</th>
<th>Motorparameter*</th>
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<th>1100</th>
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<tr>
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<td>gcm²</td>
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<td>Terminal resistance Anschlusswiderstand R</td>
<td>Ohm</td>
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<td>Inductance Induktivität L</td>
<td>mH</td>
<td>0.8</td>
<td>1.9</td>
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<td>Mech. time constant Mech. Zeitkonstante τm</td>
<td>ms</td>
<td>15</td>
<td>15</td>
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<td>Electr. time constant Elektr. Zeitkonstante τe</td>
<td>ms</td>
<td>3.5</td>
<td>2.0</td>
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<tr>
<td>Speed regulation constant Drehzahregelkonstante Rreg</td>
<td>rpm/Ncm</td>
<td>29</td>
<td>26</td>
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<td>Torque constant Drehmomentkonstante k/kw</td>
<td>Ncm/A</td>
<td>2.7</td>
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<td>Thermal resistance Thermischer Widerstand Rth</td>
<td>K/W</td>
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<tr>
<td>Thermal time constant Thermische Zeitkonstante τth</td>
<td>min</td>
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<tr>
<td>Axial play Axialspiel</td>
<td>mm</td>
<td>&lt; 0.01</td>
<td>&lt; 0.01</td>
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<tr>
<td>Direction of rotation Drehrichtung</td>
<td>bidirectional / bidirektional</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Design

- **Commutator**: Copper/12-segments
- **RFI Protection**: 2 chokes
- **Insulation class**: Winding H, otherwise A
- **Protection class**: IP40
- **Commutation**: Carbon brushes
- **Armature**: Straight slot
- **Magnet system**: Permanent magnets, 2-pole
- **Bearings**: 2 preloaded ball bearings
- **Housing**: Steel, corrosion protected
- **End shields**: Zinc die-cast on both sides
- **Life expectancy**: up to 4000 h

### Operational conditions*

- **Temperature range***: T °C -10 - +70
- **Axial force**: $F_a$ N 50
- **Radial force, 15 mm from mounting surface**: $F_r$ N 200

* at 25 °C
** depending on the operating conditions
*** extended temperature range on request

### Aufbau

- **Kollektor**: Kupfer/12-teilig
- **Grundentstörung**: 2 Drosseln
- **Isolierstoffklasse**: Wicklung H, ansonsten A
- **Schutzart**: IP 40
- **Kommutierung**: Kohlenbürsten
- **Anker**: gerade Nut
- **Magnetsystem**: Permanentmagnete, 2-polig
- **Motorlager**: 2 vorgespannte Kugellager
- **Gehäuse**: Stahl, korrosionsgeschützt
- **Lagerschilde**: beidseitig Zinkdruckguss
- **Lebensdauer**: bis 4000 h

### Operational conditions*

- **Temperature range***: T °C -10 - +70
- **Axial force**: $F_a$ N 50
- **Radial force, 15 mm from mounting surface**: $F_r$ N 200

* at 25 °C
** depending on the operating conditions
*** extended temperature range on request

---

**Graph:**
- **n [rpm]** vs. **T [Ncm]**
- **I [A]** vs. **η [%]**
- **U = 12 V**
- **U = 24 V**
Current Transducer HAIS 50..400-P and HAIS 50..100-TP

For the electronic measurement of currents: DC, AC, pulsed..., with a galvanic isolation between the primary circuit (high power) and the secondary circuit (electronic circuit).

### Electrical data

<table>
<thead>
<tr>
<th>Primary nominal current rms</th>
<th>Primary current measuring range</th>
<th>Type</th>
<th>RoHS since date code</th>
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</thead>
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<td>I_{PM} (A)</td>
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</tr>
<tr>
<td>50</td>
<td>± 150</td>
<td>HAIS 50-P, HAIS 50-TP (^1)</td>
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<tr>
<td>100</td>
<td>± 300</td>
<td>HAIS 100-P, HAIS 100-TP (^1)</td>
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<tr>
<td>150</td>
<td>± 450</td>
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<tr>
<td>200</td>
<td>± 600</td>
<td>HAIS 200-P</td>
<td>45231</td>
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<tr>
<td>400</td>
<td>± 600</td>
<td>HAIS 400-P</td>
<td>47096</td>
</tr>
</tbody>
</table>

### V_{OUT} Output voltage (Analog) @ I_{P}

- \( V_{OE} \) ± (0.625 \cdot (I_{P} / I_{PN})) \, V

### G_{TH} Theoretical sensitivity

- \( 0.625 \cdot V_{OE} / I_{PN} \) mV/K

### V_{REF} Reference voltage

- \( 2.5 \pm 0.025 \) V

### V_{REF} Load impedance

- typ. 200 \, \Omega

### R_{L} Load resistance

- \( \geq 2 \) k\, \Omega

### R_{OUT} Output internal resistance

- \(< 5 \) k\, \Omega

### C_{L} Capacitive loading (± 20 %)

- =4.7 nF

### I_{C} Current consumption @ \( V_{C} = 5 \) V

- 19 mA

### Accuracy - Dynamic performance data

- X: Accuracy \(^6\) @ I_{PN}, T_{A} = 25°C
- \( \leq 1 \) % of I_{PN}
- E_{L}: Linearity error 0..I_{PN}
- \( \leq 0.5 \) % of I_{PN}
- TCV_{OE}: Temperature coefficient of V_{OE}
- \( \leq 0.3 \) mV/K
- TCV_{REF}: Temperature coefficient of V_{REF}, +25°C..+85°C -40°C..+25°C
- \leq 0.015 \%/K
- TCV_{REF}/V_{REF}: Temperature coefficient of V_{OE} / V_{REF}
- \leq 0.2 mV/K
- TCG: Temperature coefficient of G
- \( \leq 0.05\% \) of reading/K
- V_{OE}: Electrical offset voltage @ I_{PN} = 0, T_{A} = 25°C
- \( V_{OE} \leq 0.025 \) V
- V_{CM}: Magnetic offset voltage @ I_{PN} = 0, after an overload of I_{PM}
- HAIS 50-(T)P
- \( \leq 0.5 \) % of I_{PN}
- HAIS 100-(T)P..400-P
- \( \leq 0.4 \) % of I_{PN}
- t_{r}: Reaction time @ 10 % of I_{PN}
- \( < 3 \) \mu s
- t_{r}: Response time to 90 % of I_{PN} step
- \( < 5 \) \mu s
- di/dt: di/dt accurately followed
- \( > 100 \) A/\mu s
- V_{no}: Output voltage noise (DC ..10 kHz)
- \( < 15 \) mVpp
- (DC .. 1 MHz)
- \( < 40 \) mVpp
- BW: Frequency bandwidth (-3 dB) \(^5\)
- DC .. 50 kHz

Notes:

- \(^1\) TP version is equipped with a primary bus bar.
- \(^2\) It is possible to overdrive \( V_{REF} \) with an external reference voltage between 1.5 - 2.8 V providing its ability to sink or source approximately 5 mA.
- \(^3\) Maximum supply voltage (not operating) < 6.5 V
- \(^4\) Excluding Offset and Magnetic offset voltage.
- \(^5\) Small signal only to avoid excessive heatings of the magnetic core.

LEM reserves the right to carry out modifications on its transducers, in order to improve them, without prior notice.
Current Transducer HAIS 50..400-P and HAIS 50..100-TP

General data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient operating temperature</td>
<td>-40 .. + 85 °C</td>
</tr>
<tr>
<td>Ambient storage temperature</td>
<td>-40 .. + 85 °C</td>
</tr>
<tr>
<td>Mass (in brackets: TP version)</td>
<td>20 (30) g</td>
</tr>
<tr>
<td>Standards</td>
<td>EN 50178: 1997</td>
</tr>
</tbody>
</table>

Isolation characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated isolation voltage rms with EN50178, IEC61010-1 standards</td>
<td>at following conditions</td>
</tr>
<tr>
<td>- Over voltage category III</td>
<td></td>
</tr>
<tr>
<td>- Pollution degree 2</td>
<td></td>
</tr>
<tr>
<td>- Heterogeneous field</td>
<td></td>
</tr>
<tr>
<td>Single insulation</td>
<td>1000 V EN50178</td>
</tr>
<tr>
<td>Reinforced insulation</td>
<td>1000 V IEC61010-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rms voltage for AC isolation test, 50 Hz, 1 min</td>
<td>2.5 kV</td>
</tr>
<tr>
<td>Partial discharge extinction voltage rms @ 10pC</td>
<td></td>
</tr>
<tr>
<td>HAIS 50..400-P</td>
<td>&gt; 1 kV</td>
</tr>
<tr>
<td>HAIS 50..100-TP</td>
<td>&gt; 1.4 kV</td>
</tr>
<tr>
<td>Impulse withstand voltage 1.2/50 μs</td>
<td>8 kV</td>
</tr>
<tr>
<td>Creepage distance</td>
<td>&gt; 8 mm</td>
</tr>
<tr>
<td>Clearance distance</td>
<td>&gt; 8 mm</td>
</tr>
<tr>
<td>Comparative tracking index (Group I)</td>
<td>&gt; 600</td>
</tr>
</tbody>
</table>

If insulated cable is used for the primary circuit, the voltage category could be improved with the following table:

<table>
<thead>
<tr>
<th>Cable insulation (primary)</th>
<th>Category</th>
</tr>
</thead>
<tbody>
<tr>
<td>HAR 03</td>
<td>450V CAT III</td>
</tr>
<tr>
<td>HAR 05</td>
<td>550V CAT III</td>
</tr>
<tr>
<td>HAR 07</td>
<td>650V CAT III</td>
</tr>
</tbody>
</table>

Safety

This transducer must be used in electric/electronic equipment with respect to applicable standards and safety requirements in accordance with the manufacturer’s operating instructions.

Caution, risk of electrical shock

When operating the transducer, certain parts of the module can carry hazardous voltage (eg. primary busbar, power supply). Ignoring this warning can lead to injury and/or cause serious damage.

This transducer is a built-in device, whose conducting parts must be inaccessible after installation. A protective housing or additional shield could be used. Main supply must be able to be disconnected.
Dimensions HAIS 50..400-P and HAIS 50..100-TP (in mm)

**HAIS 50..400-P**

**HAIS 50..100-TP**

**Terminal Pin Identification**

1...+5V  
2...0V  
3...OUTPUT  
4...Vref. (IN/OUT)  
5...Core Earth (*)  
6...NC.

**Recommended PCB hole**

Pin 1-4 : 0.7 ±0.1mm  
Pin 5-6 : 1.5 ±0.1mm  
Primary bus bar : 2.3 ±0.1mm  
General tolerance : ±0.2mm

(*) should be connected to 0V of Power Supply for better dv/dt immunity.  
Arrow indicates positive current direction.

**Operation Principle**

**Required Connection Circuit**

- 1...+5V  
- 2...0V  
- 3...OUTPUT  
- 4...Vref. (IN/OUT)  
- 5...Core Earth (*)  
- 6...NC.

LEM reserves the right to carry out modifications on its transducers, in order to improve them, without prior notice.

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