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METHOD OF MASSES TO DETERMINE A PROJECTILE'S AERODYNAMIC COEFFICIENTS AND PERFORMANCE

by

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ABSTRACT

The thesis traces the history of missile aerodynamic prediction methods and defines the aerodynamic requirements for the subsonic free-flight projectiles configurations under consideration. Different types of trajectory model are described with the aerodynamic input requirement being analysed. Methods of generating the required aerodynamic data for the trajectory models are discussed emphasising the aerodynamic models capabilities, weaknesses and ease of use. The method of masses aerodynamic prediction method is defined, highlighting the adaptations to the method that were carried out to generate the aerodynamic stability data required for subsequent projectile trajectory analysis. An assessment of the sensitivity and accuracy of the simulated data is carried out using experimental flight trial data on different projectile configurations. Finally, using the simulation models developed in previous chapters, a parametric analysis is carried out on different projectile configurations to optimise the trajectory performance.
1.0 SUMMARY

1.1 Introduction

To determine the flight performance of a free-flight subsonic projectile requires the solution to a group of coupled differential equations which mathematically describe forces acting on the projectile. With the advent of enhanced computational power the numerical solutions to these equations can be carried out accurately and in a short time scale. However, to obtain an accurate theoretical representation of the projectile’s trajectory also requires values of the projectile’s aerodynamic coefficients to be determined. These aerodynamic coefficients are functions of the projectile’s physical configuration and flight parameters.

1.2 Methods of determining a projectile’s aerodynamic coefficients

Different techniques and methods have been developed to determine a projectile’s aerodynamic coefficients. The most commonly used technique called slender-body theory has been the foundation on which the majority of currently used aerodynamic computer prediction codes are based. However, these prediction codes have usually been developed to calculate the aerodynamic coefficients for a specific set of projectile configurations. As the codes invariably have embedded look-up-tables it means that they are limited in there output accuracy to the range of the tables which in turn was determined by the projectile configurations under investigation. It was found that for the projectile configurations and flight parameters under investigation the current aerodynamic computer prediction codes were not capable of providing the requisite aerodynamic coefficient data.

With the advent of Computational Fluid Dynamic (CFD) codes being accessible on reasonably priced computer platforms and the codes taking a momentary run time to provided accurate aerodynamic coefficient data, it was considered that using a CFD code might produce the required aerodynamic coefficient data. However, for the CFD codes investigated, it was found that it required considerable expertise to manipulate them and interpret whether the output data was correct.

S - 1
1.3 Method of masses aerodynamic prediction method

Having found that traditional aerodynamic prediction methods and CFD computer codes were not suitable for generating the requisite aerodynamic coefficient data, a known but undeveloped numerical aerodynamic prediction method called the method of masses was used as the basis of generating aerodynamic coefficient data. Using numerical integration techniques it was found that solutions to the method of masses equations was possible to provide a basis from which an accurate value of aerodynamic coefficient could be obtained for the projectiles under investigation.

Using the method of masses, all the aerodynamic coefficient data necessary to generate a trajectory for a free-flight subsonic projectile could be obtained except for drag coefficient data. Drag coefficient data cannot be obtained using the method of masses because of the method's fundamental assumption that the projectile is moving in an inviscid fluid medium. Solving the basic equations derived from the method of masses provided a coarse value of aerodynamic coefficient. However, this calculated coefficient value was not accurate enough to be used in any detailed trajectory analysis. Therefore, the basic value obtained from the method of masses equations had to be altered to take into account fin/body and body/fin interference effects, variations with angle of attack and velocity variations. In addition to these factors, variations in projectile configuration such as leading and trailing edge fin sweeps, curved fins and nose shapes also had to be taken into consideration. The numerical values due to these effects was added to the basic method of masses values. Adding these values to the basic equation values provided aerodynamic coefficient data of sufficient accuracy to carry out conceptual design work.

1.4 Limitations in using the method of masses

The method of masses was only used to determine aerodynamic coefficients for subsonic, free-flight, axisymmetric projectiles. With the configurations and flight parameters being considered, it was found that the techniques developed using the method of masses were not suitable in determining aerodynamic coefficients for the following projectile configurations:
b. Fin sweep angles in excess of 45 degrees.
c. Angles of attack in excess of 15 degrees.
d. Fin numbers not to exceed 8 in one set.

If these configuration limits are not exceeded, the method of masses including the previously mentioned additions was found to generate aerodynamic coefficient data that was consistently lower in value when compared to experimental data. The typical variation between the techniques developed and experimental data were found to be as follows:

a. Normal lift coefficient 8% lower.
b. Pitching moment coefficient 10% lower.
c. Spin damping coefficient 7% lower.
d. Spin driving coefficient 8% lower.

As previously referred to, the method of masses cannot determine drag coefficients. Therefore, drag coefficients were calculated by establishing a look-up-table of drag data from experimental data for different projectile configurations. This data was embedded into a 6 Degree of Freedom (DoF) trajectory model. Typical differences between the data generated from the drag look-up-table and experimental data was ± 0.043 for zero lift drag and ± 0.002 for lift induced drag.

With the calculated aerodynamic coefficient data directly coupled into a 6 DoF trajectory model the output data of the model was compared to three sets of experimental projectile trajectory data. It was found that there was a good degree of correlation between the experimental and simulated trajectories. This good degree of correlation indicated that the aerodynamic coefficient data generated using the method of masses was within the stipulated design tolerance limits (± 12% for $C_N$ and $C_m$ and 0.1 for $C_D$). Having developed a means of estimating a projectile trajectory, variations in the projectile's configuration and the consequential effects on the trajectory could be carried out without the need for expensive flight trials.
1.0 BACKGROUND TO WHY THE METHOD OF MASSES AERODYNAMIC PREDICTION METHOD AND PROJECTILE TRAJECTORY MODELS WERE DEVELOPED

1.1 Introduction

The aim of this thesis was to develop a method of determining the trajectory characteristics of subsonic, free-flight projectiles. Before any details concerning the numerical methods that are used and developed in this thesis are discussed, it is essential to define the type of projectile and flight parameters that are being considered. The reason for this definition is that there are potentially an enormous number of projectile configurations and flight parameters that could be considered. The numerical methods detailed in this thesis were only validated for the following projectile configurations and flight parameters:

a. Subsonic speed regime between Mach 0.1 and Mach 0.8 at sea level.
b. Man portable shoulder launched projectile.
c. Ballistic free-flight with no guidance system.
d. Range to exceed 100 metres.
e. Not powered after launch.
f. Flight times not to exceed 10 seconds (Beyond line-of-sight-range).

A diagram depicting the general outline of the projectile configuration required to satisfy the listed parameters is shown at Fig 1.1. The parameters listed above were all dictated by practical considerations. For example, the subsonic speed regime specification was due to the limitation on the mass of thrust propellant that could be carried in the launch tube. To obtain supersonic speeds required an excessive amount of launch propellant mass. It was estimated that a 300% increase in propellant mass was required to obtain supersonic velocities.

1.2 Mathematical trajectory models and aerodynamic coefficient requirements

To determine the characteristics of a projectile's trajectory requires the solution of a large number of coupled first and second order differential equations (14 coupled equations for a 6
degree of freedom trajectory model). These equations describe the forces acting on the projectile in three planes of motion. These planes are defined as X, Y and Z throughout the thesis. Fig 1.2 shows this convention. In addition, the figure also shows the projectile's inclination angles (Z plane, \( \theta \) also denoted as angle of attack, \( \alpha \). Y plane, \( \psi \) also denoted as yaw angle, \( \beta \). X plane, \( \phi \) also denoted as angular rotation, \( \omega \)). The recent advancements in computational power means the solution of these equations can be carried out using a Personal Computer (PC) in a short time frame. However, the solution of the equations by themselves does not necessarily provide an accurate projectile trajectory path. The accuracy of the projectile trajectory is a combination of the solution of the equations of motion and the data inputs to these equations. The data inputs to these equations can be divided into two categories. One category relates to the initial launch conditions (launch angle, velocity etc.) and external influences (cross-winds). These variables are defined in this thesis as flight parameters. The second category is related to the projectile's physical configuration. From the physical configuration, (Shape of nose, size of fins etc.) the projectile's aerodynamic stability coefficients can be determined. It is the magnitude and sign of these aerodynamic coefficients coupled with the flight parameters that dictate the stability of the projectile trajectory. Therefore, the magnitude and sign of these coefficients must be accurately determined if the trajectory characteristics of a particular projectile configuration are to be determined.

When designing a new projectile configuration, the equations detailing the mathematical equations for a trajectory with 3 Degrees of Freedom (3 DoF) are well documented. With reference to Fig 1.1 which is a typical projectile shape, the force in the X axis can be written as:

\[
\text{Force}(X_{\text{axis}}) = m \frac{d^2X}{dt^2} = -0.5 \rho V_x S_r C_{d0} \left( \frac{dX}{dt} - W_x \right)
\]  

(1)

Where: \( V_x = \) Projectile speed in X axis, \( m = \) Projectile mass, \( \rho = \) Density of fluid (air), \( W_x = \) Wind speed in X axis, \( S_r = \) Reference area (Body cross-sectional area), \( C_{d0} = \) Zero lift drag. The negative sign represents the projectile decelerating after launch.
From Eqn (1) the projectile and flight parameters can be extracted and categorised as follows:

**Initial launch conditions:** $V_p, \rho.$

**Projectile parameters:** $S, m.$

**Aerodynamic coefficient:** $C_{D0}.$

**External factors:** $W_x.$

From this simple equation the velocity and distance travelled by the projectile can be calculated by integrating Eqn (1) (The first integration provides velocity, the second distance). The initial launch conditions and projectile parameters are relatively straightforward to measure. However, the determination of the aerodynamic coefficient is more complex. In Eqn (1), the value of the zero lift drag aerodynamic coefficient, $C_{D0}$ determines the rate at which the velocity decreases. As a rule of thumb the value of $C_{D0}$ will vary between 0.3 for a conical nose to 1.1 for a flat face. Using this crude estimation, a trajectory for the projectile can be obtained by solving similar equations in the $Y$ and $Z$ planes (The $Z$ plane includes a gravitational term) and combining the data results to create a trajectory plot in three planes. However, these equations do not evaluate the projectile's degree of stability. Stability in this case is defined as the amount of pitch and yaw the projectile has as it flies. This motion can only be determined by using a 6 DoF trajectory model. An example of the 6 DoF equations of motion in the $X$ plane are as follows:

$$\frac{d^2X}{dt^2} = -\frac{\pi p d^2}{8m} (C_{D0} + C_{D\alpha} \alpha) V_x \frac{dX}{dt}$$  \text{Drag}$$

$$+ \frac{\pi p d^2}{8m} C_{V\alpha} V_x \frac{dX}{dt} \alpha$$  \text{Lift}$$

$$- \frac{\pi p d^3}{16m} (C_{Y_{per}} p(\alpha \cos(V_x)))$$  \text{Magnus effect}
\[
\frac{dp}{dt} = \frac{\pi \rho d^4}{16 lx} p V_x c_{lp} \quad \text{Spin damping}
\]

Where: \( V_x \) = Projectile speed. \( C_{Da} \) = Variation in drag with angle of attack (Pitch). \( \alpha \) = Angle of attack (Pitch). \( d \) = body diameter. \( C_{Na} \) = Normal lift coefficient (Varies with alpha). \( C_{typa} \) = Magnus force coefficient (varies with alpha). \( C_{ma} \) = Pitch damping coefficient (Varies with alpha). Note: In these equations the wind effect \( W_x \) has not been included.

Initial launch conditions: \( V_p, \rho \).

Projectile parameters: \( d, m \).

Aerodynamic coefficient: \( C_{Dh}, C_{Da}, C_{typa}, C_{ma} \).

As can be seen, Eqn (2) is a lot more complex that Eqn (1). Moreover, Eqn (2) only represents the projectile's linear forces and spin in one plane. For a full 6 DoF trajectory, other linear and rotational forces and moments also have to be taken into consideration (The full 6 DoF equations are provided in Chapter Three). The purpose of showing Eqn (2) at this stage is to demonstrate that there is a significant increase in the number of aerodynamic coefficients that have to be determined when considering a 6 DoF trajectory model. In addition, the magnitude and sign of the aerodynamic coefficients can vary depending on the projectile's angle of attack and velocity. The benefit of using this level of sophistication in investigating a projectile's trajectory is that the stability of the trajectory and a more accurate velocity profile for the projectile can be obtained. Having this accuracy also means that the projectile configuration can be optimised in terms of performance to obtain the greatest range with the best terminal accuracy. To achieve this degree of accuracy requires the determination of the projectile configurations aerodynamic coefficients.

Projectiles of the type described have not been the highest priority in terms of optimising their trajectories. In the past it has normally been a case of fire a lot of rounds and determine the flight characteristics from the experimental data (If it hit the target it was accurate). There is nothing wrong with this approach and it has served designers very well over many years.
nothing wrong with this approach and it has served designers very well over many years. However, with the increased cost of range time and limited research budgets an alternative to experimental methods is now required.

1.3 Numerical aerodynamic prediction methods

Numerical methods for determining aerodynamic coefficients have been primarily developed for missiles in the supersonic speed regime. These missiles are a lot more sophisticated than the projectile configurations being described in this thesis. They are guided using autopilots which means the missile's aerodynamic stability coefficients are required to design the fin actuator systems. The numerical methods and techniques used to determine a supersonic missile’s aerodynamic coefficients are not always applicable in the determination of the aerodynamic coefficients for a subsonic free-flight projectile.

As the aim of this thesis was to determine the flight trajectory of subsonic free-flight projectiles, a means of determining the aerodynamic coefficients had to be found. It was discovered from a series of technical papers written in the 1950’s that a numerical technique for determining the aerodynamic coefficients for a projectile called the method of masses was developed. This work initially carried out by Bryson [1] was commented upon by Nielson [2] in the 1960’s. It was claimed by Nielson that the method of masses was accurate for all speed regimes and numerous physical missile configurations, but very little experimental evidence has been found to substantiate these claims. The method of masses then appears to have been abandoned for two reasons. First the method of masses is extremely mathematical and requires complicated integral equations to be solved numerically. The computational power to accomplish this was beyond the reach of researchers in the 1950’s and 1960’s. Secondly, a large financial investment into aerodynamic prediction techniques was made by the United States Department of Defense. This investment manifested itself in the guise of Missile DATa COMparison (DATCOM) a computer code to determine aerodynamic coefficients. With the time and effort put into Missile DATCOM it rapidly became the commercial standard of the 1960’s until the present day. Alongside Missile DATCOM, several other aerodynamic computer codes were also developed. Most notable of these was a code developed for the Naval Warfare Centre by Frank Moore [3] called NSWC. All of these prediction codes relied
heavily on look-up-tables and numerical techniques to determine aerodynamic coefficients. Unfortunately, Missile DATCOM and the other codes were primarily designed for the sophisticated end of the missile inventory and their numerical techniques and look-up-tables were largely untried and unreliable or could not cope with the projectile configurations under investigation in this thesis.

To obtain the aerodynamic coefficients of the projectiles described in section 1.1, the method of masses was adapted to determine the required aerodynamic data. Coupled with a 6 DoF trajectory model, the trajectory of a particular projectile could be quickly determined and modified by altering the projectile’s physical characteristics (By altering the physical configuration, the aerodynamic coefficients are altered. Physical characteristics are nose shape fin shape etc.) to see if an optimum projectile shape could be attained.

1.4 Structure of thesis

Chapter Two of this thesis describes the development of numerical aerodynamic prediction methods from the beginning to current techniques. Having shown how the aerodynamic techniques have evolved, Chapter Three details the importance of the aerodynamic coefficients and shows how they are incorporated into various mathematical trajectory models. Chapter Four details a selection of current aerodynamic prediction and trajectory codes used in the United Kingdom and highlights their advantages and disadvantages. The chapter concludes with a theoretical description of the method of masses as a numerical technique for generating a projectiles aerodynamic coefficients. Following from this theoretical explanation, Chapter Five gives practical examples of how the method of masses has been adapted to cater for the projectile’s under investigation. Numerical examples are provided and the limitations of the method discussed. A trajectory analysis of three experimental projectiles compared to the theoretical data generated in Chapter Five is analysed in Chapter Six. Variations in trajectory performance with alterations to aerodynamic coefficients are investigated with the accuracy between experimental trajectory data and trajectory data generated by the method of masses being discussed. Finally, Chapter Seven details a possible projectile configuration optimisation that could be carried out using the method of masses and 6 DoF trajectory model.
2.0 BACKGROUND TO AERODYNAMIC PREDICTION THEORIES

2.1 Introduction

This chapter details the development of the numerical methods used to calculate a projectile's aerodynamic coefficients. The chapter highlights the major numerical aerodynamic prediction methods and details how they have been adapted to cater for different configurations and flight conditions. It should be noted that there are two methods of determining a projectile's aerodynamic coefficients. One method is by experiment. This requires the firing of the projectile on a firing range and by means of mathematical regression techniques calculating the configuration's aerodynamic coefficients from the experimental trajectory data. The other experimental technique is to use wind tunnels and measure the aerodynamic coefficients with force balances. Alternatively, the aerodynamic coefficients can be determined using numerical methods. It is these numerical methods that are examined in this chapter.

2.2 Aerodynamic prediction methods

The foundation of all modern missile aerodynamic prediction methods is Slender-Body theory. This theory was then adapted into the crossflow method to take non-linear effects into account, such as the variation in the lift force with large angles of attack (normally pitch angles in excess of 8°). With the rapid advance in computational power, higher methods of analysis were then made available to the missile designer by dividing the surface of the missile configuration into panels and calculating the pressure distribution around the body. This Chapter will review all the significant developments in numerical aerodynamic prediction theory. Comments upon the suitability of the methods as a means of predicting the aerodynamic coefficient for the projectile configuration outlined in Chapter One is given. Fig 2.1 is a flow diagram depicting the development of aerodynamic prediction methods from Slender-Body theory to the most sophisticated which is the solution to the Navier Stokes equations at the top of the figure.
Navier Stokes equations (Computational Fluid Dynamics)

*Thin viscous layer*  
*Neglect viscosity*

Boundary layer equations  
Euler equations

*Irrotational flow*

Generalised potential equations  
Supersonic speeds

Characteristic equations

*Small perturbations*

Linearised potential equation (Slender-Body theory)

Fig 2.1 Development of aerodynamic theories where the terms in italics detail the physical limitations placed on the method.

Slender-Body theory assumed a linear relationship between the aerodynamic coefficients and flight variables such as the angle of attack and velocity. This theory holds true for angles of attack below about $10^\circ$ and for subsonic/supersonic velocities. However, at greater angles of attack and high supersonic velocities (M$\geq$4.0) slender body theory breaks down and new theories based upon irrotational flow were developed. These theories essentially dealt with large angles of attack and high supersonic velocities. Following on from these theories, limited viscous flow was considered (Up to this point the theories considered the fluid in which the projectile moved to be inviscid) in terms of boundary layer effects. Alternatively viscous flow was still ignored and solutions to Euler equations sought. Finally, with the rapid advancement in computational power solutions to the Navier-Stokes equations were pursued using Computational Fluid Dynamics (CFD). Appendix A. provides further background details on aerodynamic prediction methods.
2.3 Background and development of Slender-Body theory

Slender-body theory or potential theory as it is sometimes referred to was first developed by Munk [4] in 1924 in his aerodynamic theory of airships and has been the foundation of all modern aerodynamic prediction theories. As the Slender-Body methodology forms the backbone of missile aerodynamic prediction theory it is crucial to have a fundamental understanding of its function. Even though there have been enormous advances in computational aerodynamics, Slender-Body theory still gives the aerodynamicist an insight into the sensitivities of a particular projectile configuration.

Slender-Body theory is a result of a first approximation of the velocity potential equation. For a steady flow this equation is:

\[
(1-\frac{\Phi^2}{c^2})\Phi_{xx} + (1-\frac{\Phi^2}{c^2})\Phi_{yy} + (1-\frac{\Phi^2}{c^2})\Phi_{zz} - 2\frac{\Phi_x}{c^2} \Phi_{xy} - 2\frac{\Phi_y}{c^2} \Phi_{yz} - 2\frac{\Phi_z}{c^2} \Phi_{zx} = 0
\]  

(1)

Where: \( c^2 = c_0^2 - \frac{\gamma-1}{2} U_0^2 \)

Where: \( c \) = Speed of sound. \( \Phi \) = Full velocity potential. \( \gamma \) = ratio of the specific heats. \( U_0 \) = Freestream velocity. \( x, y, z \) = Cartesian coordinates.

The classical approach to Slender-Body theory started with the linearisation of the potential equation. Assuming that the velocity field is composed of a uniform flow, a linear perturbation, and higher-order terms leads to the well-known Prandtl-Glauert perturbation potential equation:

\[
(1 - M^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0
\]  

(2)

Where: \( M \) = Mach number.
Slender-Body theory assumes that the longitudinal (x) derivative of the velocity potential is much smaller than the transverse (y,z) derivatives. Therefore, the term $(1 - M^2)\phi_{xx}$ in Eqn. (2) can be neglected. This therefore gives Laplace's equation in the crossflow plane:

$$\phi_{yy} + \phi_{zz} = 0 \quad (3)$$

A more up to date and formal derivation of Slender-Body theory is based upon the method of matched asymptotic expansion, Ashley and Landahl [5]. The slenderness parameter of the expansion could be the maximum width to length ratio or the maximum equivalent diameter to length ratio. The solutions to these equations can be found at Ref. [6].

The first solution to the Slender-Body equation was given by Munk [7] for axisymmetric bodies, using the method of apparent masses (This method was developed by Bryson). The foundation of this method is the relationship between the force acting on a transverse slice of a body and the change in kinetic energy contained in the matching slice of the flow field. The apparent masses aerodynamic method is explained in greater detail in Chapter Four. Calculated examples using the method are provided in Chapter Five.

Jones [8] used a twofold analysis in his study of slender wings. He used the method of apparent mass to obtain the longitudinal lift distribution. He also used a known potential, from the two dimensional theory, to obtain the load distribution, lift and induced drag. His result for the lift-curve slope, based on wing planform area, is:

$$C_{Na} = \left(\frac{\pi}{2}\right)AR \quad (4)$$

Where: $AR = \text{Aspect ratio of wing}$. $C_{Na} = \text{Normal lift coefficient with angle of attack}$.

Spreiter [9] used the Joukowski transformation to transform a plane-wing and body combination into a flat plate. Using the potential function developed by Jones, he calculated the pressure distribution, lift and pitching moment. This analysis pioneered the method of
conformal mapping for the solution of slender bodies at an angle of attack or sideslip. According to this method, actual cross-sections of the body are conformally transformed into shapes for which a solution of the potential exists. Typical transformed planes are a circle, or a flat plate. The inverse transformation provides a means of obtaining the potential in the physical plane, from which the load distribution can be calculated. For a plane-wing and body combination Spreiter obtained:

\[ C_{Na} = \frac{\pi b^2}{S_t} \left[ 1 - \left( \frac{D}{b} \right)^2 + \left( \frac{D}{b} \right)^4 \right] \]  \hspace{1cm} (5)

Where: \( D \) = Projectile body diameter. \( b \) = Fin span. \( S_t \) = Reference area (Body cross-sectional area).

Bryson [10], of which more will be discussed later, studied configurations having non-circular bodies. He conformally transformed a plane wing, elliptical-body, vertical tail configurations into a circle as a complete configuration. The benefit of this analysis is that the components are connected together. Knowing the potential function for a two-dimensional circle, he evaluated the added masses. Based on his results, the normal lift curve slope for a plane wing, elliptical-body combination was found to be:

\[ C_{Na} = 2\pi \frac{\tau - (a^4 + b')^2}{4\tau} + a'^2 \]  \hspace{1cm} (6)

where

\[ \tau = \frac{1}{2} \left( s' + \sqrt{s'^2 - a'^2 + b'^2} \right) \]

Where: \( a' \) = Semi axis of an ellipse (normal to crossflow direction). \( b' \) = Semiaxis of an ellipse (in the crossflow direction). \( s' \) = Semispan = \( b/2 \)
2.31 Aerodynamic coefficients given by slender-body theory

The aerodynamic coefficients of a body given by slender-body theory can be obtained by integrating the load distribution along the body. This distribution can be obtained from the momentum theory given by Munk [4]. Integrating this distribution over the body length gives the total forces and moments in the following equations:

\[
\frac{dF}{dx} = \frac{\partial}{\partial x}[pVS(\sin \alpha + \frac{q(x - x_{cg})}{V})] \quad (7)
\]

\[
= q_\infty \cos \alpha [2 \frac{dS}{dx} (\sin \alpha + \frac{q(x - x_{cg})}{V}) + 2S(\frac{\dot{x}}{V} + \frac{q}{V})] \quad (8)
\]

\[
\approx q_\infty [2 \frac{dS}{dx} (\alpha + \frac{q(x - x_{cg})}{V}) + 2S(\frac{\dot{x}}{V} + \frac{q}{V})] \quad (9)
\]

\[
C_H = \frac{\int_0^{lb} \frac{df}{dx} \, dx}{\frac{\partial}{\partial x} q_\infty S_r} \quad (10)
\]

\[
C_m = \frac{\int_0^{lb} \frac{df}{dx} \, (x_{cg} - x) \, dx}{\frac{\partial}{\partial x} q_\infty S_r l_r} \quad (11)
\]

Where: \( \delta f \) = Partial differential of force. \( S \) = Cross-Sectional area. \( lb \) = Length of body. \( V \) = Velocity.
For truncated bodies with non-zero frontal area, additional terms appear in the integration due to the presence of $dS/dx$ in $dfi/dx$. The aerodynamic coefficients including these terms are obtained as:

\[
C_{Na} = 2 \frac{S_b - S_o}{S_r} \quad (12)
\]

\[
C_{m_a} = 2 \frac{V_b - S_b (l_b - x_{cg}) - S_o x_{cg}}{S_r l_r} \quad (13)
\]

\[
C_{Nq} = 2 \frac{S_b (l_b - x_{cg}) - S_o x_{cg}}{S_r l_r} \quad (14)
\]

\[
C_{Na} = 2 \frac{V_b}{S_r l_r} = (C_{Nq} + C_{m_a}) \quad (15)
\]

\[
C_{m_a} = 2 \left( \frac{V_b x_{cg}}{S_r l_r^2} \right) \quad (16)
\]

\[
C_{m_q} = 2 \left[ \left( \frac{V_b x_{cg} - S_b (l_b - x_{cg})^2 + S_o x_{cg}^2}{S_r l_r^2} \right) \right] \quad (17)
\]

\[
= -C_{Na} \frac{(l_b - x_{cg})^2}{l_r} - C_{m_a} \frac{S_o x_{cg}}{S_r} \left( \frac{l_b - x_{cg}}{l_r} \right)^2 \quad (18)
\]

Where: $l_b$ = Length of body. $V_b$ = Volume of body. $S_o$ = Area of nose. $S_b$ = Area of body. $S_r$ = Reference Area. $l_r$ = Reference length (Body diameter). $x_{cg}$ = Centre of mass. $C_{Na}$ = Normal lift coefficient. $C_{m_a}$ = Pitching moment coefficient. $C_{Nq}$ = Normal lift with yaw rate coefficient. $C_{m_{dot}}$ = Normal lift with pitch rate coefficient. $C_{m_{dot}}$ = Pitching moment with pitch rate coefficient. $C_{m_{qdot}}$ = Pitching moment with yaw rate coefficient.

For pointed nose sections ($S_o = 0$), and these equations revert to the results given in Ref [5] and other standard references on aerodynamic prediction techniques. As practical missile configurations have pointed or rounded noses to reduce the drag coefficient, Eqns (12-18)
have not been developed before (Reference to them has not been found). The equations were
developed to generate aerodynamic coefficients for flat nose projectiles. They were not
developed any further as the accuracy of the data provided by them was found to be imprecise
when compared to the data generated using the method of masses techniques described in
Chapter Five (Errors of up to 30% lower than experimental data values were being
calculated).

2.4 Crossflow adaptation method

Early investigations of Slender-Bodies highlighted differences between experimental normal
force and pitching moment coefficients with those predicted by potential theory. In particular,
non linearities in normal force curves were observed at increased angles of attack. The source
of these discrepancies was identified as flow separation due to viscous effects.

The first analytical approach to the problem was provided by Allen [11] for axisymmetric
bodies. He postulated that the additional normal force could be treated independently of the
potential force. To accomplish this he related the viscous crossflow along the body to that of
a two-dimensional cylinder, impulsively started into motion with a velocity \( U = U_0 \sin \alpha \). Using
this analogy, the distance from the tip of the body is related to the time \( t \), from the start of the
motion by:

\[
x = U_0 t \cos \alpha = \frac{U_t}{\tan \alpha}
\]  

To simplify the application, the transitional nature of the crossflow was ignored and a steady-
state crossflow drag coefficient was used. This approximation reduced Allen's "impulsive-
flow analogy" to his "crossflow method". This method was validated by Allen and Perkins
[12].

Static longitudinal force and moment coefficients for Slender Bodies of circular and non-
circular cross-section with and without lifting surfaces can be predicted with algorithms from Allen’s viscous crossflow concept. This was done by adding the lift contribution from flow separation to the lift generated by potential theory.

For bodies in which the cross-sectional shape (but not necessarily the area) is constant along the longitudinal axis, the following expressions have been derived:

\[
C_n = \frac{A_b}{A_r} \sin 2\alpha \cos \alpha \left( \frac{C_n}{C_{n_0}} \right)_{SB} + \eta C_{dn} \frac{A_p}{A_r} \sin^2 \alpha \left( \frac{C_n}{C_{n_0}} \right)_{Newton} \tag{20}
\]

\[
C_m = \left\{ \left[ \frac{V - A_b(1 - x_m)}{A_r X} \right] \sin 2\alpha \cos \alpha \left( \frac{C_m}{C_{m_0}} \right)_{SB} + \left[ \frac{\eta C_{dn}}{A_r} \right] \frac{(x_m - x_c)}{X} \sin^2 \alpha \left( \frac{C_m}{C_{m_0}} \right)_{Newton} \right\} \tag{21}
\]

\[
C_A = C_{A_{\alpha=0}} \cos^2 \alpha \tag{22}
\]

\[
x_{sc} = \left( \frac{x_m}{X} \right) \frac{C_m}{C_n} \tag{23}
\]

Where:  
- \( A_b = \) Body cross-sectional area.  
- \( A_r = \) Reference area.  
- \( \eta = \) Ratio of crossflow drag for a finite-length cylinder to that of an infinite-length cylinder.  
- \( C_A = \) Axial drag.

The first term in Eqs (20-21) comes from Slender-Body theory. The second term represents the viscous crossflow.

In Eq (20), \( (C_n / C_{n_0})_{Slender-Body} \) is the ratio of the normal force coefficient for the body of non-circular cross-section to that for the equivalent body (same cross-sectional area) of circular cross-section as determined from slender-body theory. The ratio \( (C_n / C_{n_0})_{Newtonian} \) is determined from Newton impact theory (Details of this theory is provided at Appendix A). In Eqs (20-21), \( \eta C_{dn} \) is a function of both Mach number and Reynolds number. Charts depicting these values are given in Ref [13]. For bodies at subsonic Mach numbers Ref [14] suggests that \( \eta \) the crossflow drag proportionality factor can be obtained from a plot of \( \eta \) vs length-to-width-ratio (For bodies at supersonic and hypersonic Mach numbers the value \( \eta = 1 \) is used).
For a more general case of a body with lifting surfaces where the cross-sectional shape varies along the length, values of \((C_N / C_{No})_{\text{slender-body}}\) and \((C_N / C_{No})_{\text{Newtonian}}\) at stations along the horizontal axis must be used, and the terms predicted by slender-body theory and viscous theory must be written in integral form as shown in Eqns (24-25). For increasing values of body cross-section, positive \(dA/dx\) values are used.

For the projectile configurations detailed in Chapter One, it was found that crossflow effects could be avoided if the angle of attack \((\alpha)\) was kept below a certain value. This value was determined to be a function of the body length to diameter ratio. Further details concerning crossflow drag is given in Chapter Five.

\[
C_N = \frac{\sin 2\alpha \cos \alpha}{\text{Ar}} \int_0^1 \left( \frac{C_N}{C_{No}} \right)_{\text{sl}} \frac{dA}{dx} \, dx + \frac{2\eta C_{db} \sin^2 \alpha}{\text{Ar}} \int_0^1 \left( \frac{C_N}{C_{No}} \right)_{\text{Newt}} \, r \, dx 
\]

and

\[
C_m = \frac{\sin 2\alpha \cos \alpha}{\text{ArX}} \int_0^1 \left( \frac{C_N}{C_{No}} \right)_{\text{sl}} \frac{dA}{dx} (x_m - x) \, dx + \frac{2\eta C_{db} \sin^2 \alpha}{\text{ArX}} \int_0^1 \left( \frac{C_N}{C_{No}} \right)_{\text{Newt}} r(x_m - x) \, dx
\]

2.5 Component build up method

The most commonly used approach to obtaining aerodynamic coefficient data for a particular projectile configuration is to use the "component build-up method". As implied by the title the method determines the overall loading for the missile by summing the major airframe components (e.g. body, wing and tail) in isolation and then adding in the additional loads created by interference factors. The following equations detail this method:

\[
C_{NB WT} = C_{NB} + C_{NW(B)} + C_{NB(W)} 
\]

\[
C_{mB WT} = C_{mB} + C_{mw(B)} + C_{mB(w)}
\]

The term wing was used as the theory was developed for missiles which had wings to provide lift.

The body-alone terms in Eqs (26-27) are defined as loads that would act on the body if it were isolated in the freestream at the angle of incidence ($\alpha$) seen by the complete configuration. Various methods for estimating these loads can be found in Ref [15]. The $W(B)$ terms represent the loads acting on the exposed wing panels in the $Xo - Yo$ (horizontal plane). It is therefore convenient to think of the panels as halves of a wing alone. If the body diameter is very small relative to the wingspan, then two opposing exposed panels essentially act as if they constitute an isolated wing in the freestream. If the body diameter is very large relative to the wingspan, the body acts as a reflection plane for each panel and again it is appropriate to consider a wing alone composed of two exposed panels joined at their root chords. However, because of the disturbance of the freestream flowfield by the body, the angle of attack experienced by this wing alone is not equal to the body angle of attack plus the wing deflection angle. The concept of the wing alone is pivotal to the component build-up method, Ref [16].

The $B(W)$ terms can be considered as resulting from carryover to the body of the pressure field created by the wing panels. For linear conditions on a cruciform wing configuration and the nose sufficiently upstream of the wing, the $B(W)$ terms are proportional to the $W(B)$ terms, Ref [17]. For the case where vorticity is shed from the body, the effect of the wing on the development of that vorticity may have to be taken into account, Ref [18].

2.6 Method of NACA Report 1307

For more than 30 years the most popular missile aerodynamic prediction method has been that of NACA 1307 by Pitts et al. Ref [19]. Their approach was based on using linear theory or experimental results for the body and fin alone, together with the estimates for the interference
effects of the various components on each other. Their estimates for these effects were based primarily on Slender-Body theory and apply for only small angles of attack. It should be noted that a second set of fins can be used in this method. If two sets of fins are used, the first set is defined as the wing and second defined as the tail. The term wing should not be taken as to mean a large span. It is simply used as a term to distinguish between two sets of fins. It could be the case, as in double canards (used to enhance a missile’s control authority without increasing the fin span), that the fins are the same size and separated by a small axial distance.

2.6.1 Wing-body interference

If a wing is attached to an airframe, for small angles of attack and for the wing “sufficiently” far aft of the nose, the cylindrical portion of the body in the vicinity of the wing will produce little or no lift as part of the body alone, Ref [20]. Therefore, all of the lift for the section is due to the presence of the wing. The Slender-Body theory of Munk was extended to wings and wing-body combinations in supersonic flow, Ref [21]. The new theory gave the following result for the normal-force coefficient for the wing section (wings undeflected relative to the body):

\[ C_{NW(B)} + C_{NB(W)} = \frac{2\pi \alpha_s S_m^2}{S_{ref}} \left(1 - \frac{a^2}{S_{m}^2}\right)^2 \]  

(28)

Where: \(a\) = the local body radius. \(S_m\) = Fin semispan. \(\alpha_s\) = Angle of attack for complete configuration. \(S_{ref}\) = Body cross section.

Since Eqn (28) predicts no Mach number effects for slender wing-body sections, it is of little use for the direct estimation of loads. However, Ward, Morikawa, Nielsen and Kattari, independently suggested using Eqn (28) to obtain the interference between the wing and the body while improving the wing lift estimate by using linear theory or data for the wing alone. This concept was validated using experimental data. By setting \(a = 0\), the following expression was obtained for the Slender Body theory wing-alone normal force coefficient.
Dividing Eq (28) by Eq (29) and noting that $S = S_m - a$ gives

$$\frac{C_{NW(B)} + C_{NB(W)}}{C_{NW}} = \left(1 + \frac{a}{S_m}\right)^2$$  \hspace{1cm} (30)

The determination of wing-body lift using Eqn (30) together with linear theory or data for $C_{NW}$ is an essential feature of the modified slender body theory. Morikawa suggested rewriting Eqn (30) as follows:

$$C_{NW(B)} + C_{NB(W)} = (K_w + K_B) \frac{\delta C_{NW(B)}}{\delta \alpha} \alpha_c$$  \hspace{1cm} (31)

Where the wing-body interference factors are defined as

$$K_w = \frac{C_{NW(B)}}{C_{NW}}$$  \hspace{1cm} (32)

$$K_B = \frac{C_{NB(W)}}{C_{NW}}$$  \hspace{1cm} (33)
For $\alpha_e \neq 0$ and $\delta = 0$. Comparing Eqs (28 - 31), it can be seen that Slender Body theory gives:

$$K_w + K_B = (1 + \frac{a}{S_m})^2 \quad (34)$$

Eqn (34) will be used later in Chapter Five to improve the accuracy of the apparent masses prediction methodology.

2.7 Comments on slender-body theory as a means of generating the required aerodynamic coefficients

As has been stated, Slender Body theory has been used very successfully for many years to calculate a missile's aerodynamic coefficients. However, to use the theory, requires adaptations to the basic equations to cater for angles of attack and velocity variations. In addition, as can be seen from the previous sections, the research effort into slender-body theory was primarily put into determining the aerodynamic coefficient values of normal lift coefficient $C_N$ and pitching moment coefficient $C_m$ (Very little published literature detailing rotational and damping coefficients was found). As Slender Body theory assumes an inviscid fluid it cannot determine drag coefficients. The method of masses theory is based upon Slender Body theory. Therefore, a thorough understanding of the development of Slender Body theory provided the foundation to develop the aerodynamic methods described in Chapter Five. The interference effects described by Eqn (34) are pivotal in improving the accuracy of the method of masses. Further details on interference effects are provided in Chapter Five.

2.8 Spin damping coefficient

As previously stated, the previous aerodynamic methods were primarily developed to calculate a missile's normal force and pitching moment coefficients for bodies and wings in combination
and isolation. This was due to the missiles being considered having roll control autopilots and therefore not requiring spin damping coefficient data. Therefore, only a limited amount of data exists for numerically determining a projectile's aerodynamic spin damping coefficient as there was no design requirement to obtain an accurate value of spin damping coefficient.

For subsonic unguided projectile systems, the projectile's flight characteristics have been determined by firing the projectile on a calibrated range. If the trajectory appeared to meet the design specification, it was considered a satisfactory design solution. On initial inspection, this method of testing a conceptual projectile configuration appeared costly and haphazard. However, until recently, the capability to model a projectile in 6 DoF was not easily accomplished due to a lack of computational power. Consequently, in terms of financial cost and computational time it was more efficient to fire a number of different projectile configurations and measure their end point performance rather than optimise a single configuration for maximum trajectory efficiency.

The roll damping aerodynamic coefficient is required to establish a projectile's angular motion and to determine the spin rate for the projectile in flight. For an unguided ballistic projectile the magnitude of the pitch and yaw components is required to be a minimum so that the projectile's precession and nutation motion can be kept as small as possible.

2.81 The equation of rolling motion

To demonstrate why the spin damping coefficient $C_{lp}$ and the spin driving coefficient $C_{ls}$ are fundamental in the understanding of unguided projectile trajectory analysis, the equation of rolling motion needs to be defined. The equation of rolling motion about the body axis is conventionally written as:

$$ (C_{ls} \delta + C_{lp} (\frac{pd}{2V}) q_o S_{ref}) = I_{xx} \ddot{y} \quad (35) $$

$\ddot{\gamma} =$ Angular acceleration (rad/s²). $q_0 = 0.5pV^2$. $d =$ Body diameter.

The roll producing moment coefficient $C_1$ due to the fin cant angle $\delta$ is usually written as the coefficient derivative multiplied by the fin cant angle $\delta$, ($C_{1\delta}$). The opposing moment (roll damping moment) is usually written as the multiple product of the coefficient derivative, $C_{1p}$ (roll damping moment), times $(pd/2V)$:

$$\frac{C_{1\delta}}{\delta(pd/2V)} \quad (36)$$

Also $p = \dot{\gamma} =$ roll rate. The reference area, $S_{Ref}$, is based on the body reference diameter and is determined by $\pi d^2/4$. The damping moment opposing roll coefficient $C_{1p}$, is conventionally written with a negative sign to indicate a damping function.

For the steady state rolling motion, $\phi = 0$ and, therefore,

$$\frac{C_{1p}}{C_{1\delta}} = -2\left(\frac{V\delta}{p_s d}\right) \quad (37)$$

Where: $C_{1p} =$ Spin damping coefficient and $C_{1\delta}$ is the spin driving coefficient. $p_s =$ Steady state roll rate. $d =$ Body diameter.

It should be noted that if $C_{1p}/C_{1\delta}$ is to be kept constant with time, the steady state roll rate $p_s$ must vary at the same rate as the velocity along the projectile trajectory. However, for projectiles that are not powered the velocity will decrease due to the configuration's drag. Therefore, the projectile spin must decrease so as to keep the $C_{1p}/C_{1\delta}$ ratio constant. Consequently, if the projectile spin decreases then true "steady state roll" does not exist. Hence, if the ratio of $C_{1p}/C_{1\delta}$ is constant along the trajectory this will not translate to a true steady state spin value, and visa versa.
Bolz and Nicolaidas [22] found that after range testing their free-flight 'Finner' projectile in an indoor range, the ratio of $C_{\psi}/C_{18}$ appeared not to vary with velocity. However, they also called this ratio "steady state fin tip helix angle per cant angle" because their "steady state" referred to the 600-foot long range measurement where the velocity of the projectile did not significantly decrease. Therefore, they considered the projectile velocity to be constant when compared to the projectile's mid-range value. It was therefore inconclusive whether or not the ratio varied with velocity.

2.9 Fin Cant Angle and Equivalent Cant Angle

The cant angle $\delta$ for a projectile fin, refers to the angle between the body axis and the chord line of the cross-section of the whole fin panel as shown in Fig 2.2. The fin panels are deflected in one direction for all or individual panels as required. For cruciform fin settings, one panel will be deflected up while the 180 degree opposite panel will be deflected down. This combination creates the fin normal force generating the rolling moment. However, the common fin configuration that is used on the majority of all new projectile designs is to partially cant the fin. The fin cross-section chord line remains parallel to the body, with only the leading or trailing edge (or both) being chamfered at an angle to the chord line. The reason for this is twofold. First, the whole panel deflection may produce an undesirable amount of spin although a very small whole panel cant of the order of $0.1^\circ$ may be possible. However, this method is not recommended due to the manufacturing tolerances and the associated costs incurred in producing such an accurate fin configuration. Second, the complexity of manufacture and costs can be decreased by partial canting which maintains the fin chord aligned with the body axis, while only chamfering the leading and trailing edge (or both). For projectiles with only partial canting, an equivalent whole fin panel cant angle $\delta_{eq}$ must be determined and used if $C_{18}$ is to be evaluated from the measured value of $C_{18}$. 

2 - 23
2.91 The correlation of Eastman

Bolz and Nicolaidas first published their results in 1950. Adams and Dugan [23] followed in 1958 by publishing their supersonic analysis using the linearised perturbation theory for Slender Bodies. They extended a two-wing aeroplane configuration analysis to a four-fin body in the (+) formation (i.e. cruciform fins). For the cruciform configuration they obtained the following expression:

\[
\frac{C_{lp}}{C_{15}} = -0.627 \left( \frac{d}{b_o} \right) \tag{38}
\]

Where: 
- \( b_o \) = the total span of two fin panels including the centre body diameter.
- \( d \) = Body diameter.

Unfortunately, according to all published literature they did not apply this result to any configuration or compare it to any experimental data so it remained as a theoretical relationship only. Eastman [24] applied the Adams and Dugan (A and D) expression in 1965 to different projectile configurations for which data was available for both \( C_{lp} \) and \( C_{La} \), to find out if their result did have any justification for a practical configuration. From the experimental data available for \( C_{lp} \) and \( C_{La} \), Eastman empirically wrote the following relationship in a form similar to the result of the analysis of (A and D).

\[
\frac{C_{lp}}{C_{18}} = -2.15 \left( \frac{y_o}{d} \right) \tag{39}
\]

Where: \( y_o \) is the distance between the rolling body axis to the area centre of one fin panel.

Eastman showed that his correlation was valid not only for supersonic speeds as implied by the (A and D) analysis, but rather for all speed regimes. Eastman however, limited his correlation to four cruciform fin configurations, since he considered that the changes he made to the
(A and D) expression were minor and therefore the limitations of their analysis must be applied to it. It is therefore interesting to note that Eastman appears to have never applied his correlation to non-cruciform fins or to an arbitrary number of fins (i.e. other than 4 fins). The reason for this appears to be simply that there was no requirement at that time for a projectile to be designed with a different number of fin panels or curved surfaces.

2.92 Extension to an arbitrary number of fins

Eastman's empirical correlation shown at Eqn (39) does not provide the same numerical number as the (A and D) expression in Eqn (38). Although Eastman started with the (A and D) expression, he developed his own expression which, although similar in form to Eqn (38), is unrelated to the outcome of their theory or analysis. Therefore, according to Mikhail [25], Eastman's expression should not be restricted to the (A and D) limitation of being derived for 4 cruciform fins only as it has not been derived from the output of the (A and D) analysis. To validate this statement a comparison for fin numbers other than four had to be made. This work has been carried out for two configurations of six and three fins respectively by Mikhail, where he claims to have shown that Eqn (39) is valid for an arbitrary number of fin cases. However, as will be explained in more detail in Chapter Five, the more fins that are added to a projectile the damping in roll increases but at a decreasing rate. Therefore the claim made by Mikhail that there is a constant relationship between the number of fins and roll damping would appear not to be entirely correct.

2.10 Comments on methods of generating spin damping aerodynamic coefficient data

As stated in the sections on spin damping, the methods that were described appear not to be suitable to determine the rolling aerodynamic coefficients for the projectile configurations under investigation. Therefore, the fundamental equations from the method of masses were developed to calculate the projectiles spin driving and spin damping aerodynamic coefficients. Details of how the method of masses equations were modified to determine these coefficients is described in Chapter Five.
2.11 Computational Fluid Dynamics (CFD) - Panel methods

Panel methods to determine the aerodynamic coefficients of a projectile have been in existence for a long time, although for supersonic flow the number of choices are fairly limited. In general terms, panel methods can predict pressure distributions on the components of tactical missiles at a lower cost when compared to higher order CFD methods. CFD methods are based upon higher order non-linear theory, such as the full potential Euler or Navier-Stokes equations.

Panel methods can be classified into low and high order categories. Both these categories employ distributions of singularities derived from linear potential theory. The low-order panel methods provide continuity across the panel edges, and the flow tangency boundary condition is applied at the control point in each panel. The high-order panel methods incorporate quadratically varying strengths which are made continuous across the panel edges. The boundary condition includes setting the potential on the interior of the panelled component to zero. The high-order panel method can yield better results than the low-order panel methods by virtue of the smoothly varying characteristics of its singularities at the expense of longer computer running times. The modelling of surface details is also better with the high-order panel methods. Further technical details concerning panel methods can be found at Hess and Smith [26].

2.12 Higher order CFD methods

For a considerable period of time Navier-Stokes equations have been the only way to describe the motion of a fluid mathematically, Ref [27]. By assuming the fluid to be a continuum material rather than a collection of molecules, this non-linear system of partial differentiation equations provides a relationship between density, velocity and pressure that conserves mass and momentum. However, the Navier-Stokes equations cannot be solved exactly, except for very special geometries and circumstances, Ref [28]. The standard way to obtain a solution
has been to separate the flow domain into a large number of small cells and then calculate the fluid parameters on the resulting grid, Ref [29]. The generation of the grid for the region of interest is far from a trivial problem. In fact, grid generation has been cited repeatedly as being a major time consuming element in computational fluid dynamics. At present it can take orders of magnitude more man-hours to construct the grid than it does to perform and analyse the flow solution on the grid, Ref [30]. This is especially true now that flow codes of wide applicability are becoming available. The flow codes that are now being developed generally require much less esoteric expertise of the knowledgeable user than do grid-generation codes.

The construction of structured grids in complicated regions has been greatly facilitated by the use of composite-block grids in which the region is broken up into sub-regions bounded by six (four in two dimensional) curved sides. Within each side the grid is generated separately but with complete continuity across the connecting interfaces. This continuity is accomplished through the use of a surrounding layer of points outside each computational block, with grid as well as flow field values at these points set equal to those at the first layer of points in the interior of the adjacent block. This arrangement results in a two-layer overlap at each block interface, allowing block to block communication, Ref [31].

This method allows both the grid generation, and numerical solutions of the grid, to be constructed to operate in a rectangular computational region, regardless of the shape or complexity of the full physical region. The full region is treated by performing the solution operation in all of the rectangular computational blocks. With the composite framework, partial differential equation solution procedures written to operate on rectangular regions can be incorporated into a code for general configurations in a straightforward manner, since the code only needs to treat a rectangular block. The entire physical field can then be treated in a loop over all of the blocks.

The original impetus for using blocked grids was to simplify the gridding of complex configurations and to permit the solution of large problems requiring many grid points by keeping only the information required to solve one block in central memory while retaining
the information associated with the remaining blocks in secondary memory. Use of blocked grids highlighted a third reason for their use. This was that supercomputer installations generally place a high price on the use of central memory, whereas the use of secondary memory is relatively inexpensive. By blocking, the use of central memory can be controlled, not only to fit the available memory, but also to fit the available budget, Ref [32].

At high Reynolds numbers the flow becomes turbulent and the calculation can become numerically unstable, Ref [33]. This is partly due to the flow being updated through a series of floating point operations that induce round-off errors. These can eventually swamp the significant information and cause the calculation to fail. Although double-precision calculations can be used they may only delay the instability in the calculation. When instability occurs, its cause and remedy are often unknown. Despite these problems, Navier-Stokes codes have been used successfully on a wide range of missile configurations although with the reliance on elaborate turbulence models and a bewildering array of numerical tricks and approximations, Refs [34-35].

2.13 Comments on CFD methods

Without question, CFD methods are capable of generating the required aerodynamic coefficient data for the projectile configurations detailed in Chapter One. Indeed a CFD code was investigated and used to generate projectile aerodynamic coefficients. Details of this are given in Chapter Four. It was concluded that as a method of producing a projectile's aerodynamic coefficients it cannot be improved in terms of accuracy as a numerical method. But the grid generation and data interpretation, mitigates against its use (for current codes) as a rapid aerodynamic generation method. This coupled with the difficulty of porting the output data into a readily available 6 DoF trajectory model discounted CFD methods from further consideration in this thesis.
2.14 Summary of aerodynamic prediction methods

Aerodynamic prediction codes for the generation of a projectile’s aerodynamic coefficients have developed to a very high degree of sophistication since the work carried out by Munk. It is now possible using the latest CFD codes and prediction methods such as Missile DATCOM to generate all the required missile aerodynamic coefficients to a very high degree of fidelity. However, the majority of these numerical aerodynamic methods have been specifically tailored to investigate sophisticated supersonic missile systems. With the present research emphasis on high angles of attack and hypersonic velocities, it is considered in many academic quarters that the current aerodynamic methods can cater for all other missile configurations and velocity regimes. This point cannot be argued against, as with time, these methods are more than capable of generating the required aerodynamic coefficients for the projectile configurations detailed in Chapter One. Nevertheless, Chapter Three reviews the current accessible commercial and Ministry of Defence numerical aerodynamic and trajectory models and highlights the difficulties in using them to produce a trajectory for the projectiles specified in Chapter One.
3.0 TRAJECTORY ANALYSIS MODELS AND INPUT REQUIREMENTS

3.1 Introduction

To determine the theoretical performance of a projectile requires the trajectory to be mathematically modelled. This mathematical modelling is more commonly defined as trajectory analysis. The level of fidelity with which a projectile trajectory can be analysed has increased with the advancements in computer processing power. With the processing power of the desktop Personal Computer (PC), it is now possible to model a projectile trajectory with 6 Degrees of Freedom (DoF). This chapter outlines the aerodynamic coefficient data and trajectory algorithms that are required to generate a 6 DoF trajectory model. It then reviews aerodynamic and trajectory models and assesses their suitability for generating trajectory data for the projectile parameters detailed in Chapter One.

3.2 Projectile axis and notation

Before the trajectory force equations are developed, it is essential to define the frames of reference and notation that was used in this analysis. Fig (3.1) shows the axis and notation used throughout this analysis. X, Y and Z represent components of the resultant aerodynamic force along their respective axis. U, V and W are the components of the resultant linear velocity acting at the origin of the axis. L, M and N are the moment components. Where, L is the rolling moment, M is the pitching moment and N is the yawing moment. P, Q and R are the angular velocity components, where P is the rolling rate, Q is the pitching rate and R is the yawing rate. Non-dimensionalised components are represented by lowercase characters. For example, m, n and r represents the non-dimensionalised moment components. Although notation varies greatly between researchers, this notation appears to be consistent with the majority of learned texts.

3.3 Aerodynamic coefficients

The purpose of the aerodynamic prediction methods is to generate the projectile's aerodynamic coefficients for use in a trajectory model. The reason why the coefficients are
required is that throughout a projectile's flight, it is subject to various forces that may influence the projectile's motion (The aerodynamic coefficients are created as a result of non-dimensionalising the forces). The major forces acting on the projectile are lift, drag and sideforces. To determine the effect of these forces on the trajectory they are first separated into different groups. To understand this grouping and the effect each aerodynamic coefficient has on the projectile trajectory a brief description of each major aerodynamic coefficient is provided in the following sections.

3.31 The \( \alpha \) coefficients

The \( \alpha \) coefficients describe changes in forces and moments when the angle-of-attack is altered.

a. \( C_{X\alpha} \) represents the changes in the X-force with respect to changes in angle-of-attack. A positive angle-of-attack disturbance will produce a positive component of \( C_{X\alpha} \) due to the forward inclination of the lift vector and a negative component due to the rotation of the drag vector. This coefficient is important as it is the lift induced drag component. With a ballistic projectile any increase in drag is extremely detrimental to the trajectory performance (Written in this thesis as \( C_{D\alpha} \)).

b. \( C_{Y\alpha} \) is the sideforce due to the angle-of-attack. It is generally small enough to be ignored in an analysis.

c. \( C_{Z\alpha} \) is the change in the Z-force due to an angle-of-attack-change. This coefficient is one of the primary design drivers at it determines the size of the projectile's static margin. It is also defined in this thesis as the normal lift coefficient \( (C_{N\alpha}) \).

d. \( C_{L\alpha} \) is the rolling moment due to an angle of attack change. It is used to determine the spin driving force created by an offset fin configuration.
e. $C_{nx}$ is commonly known as the static stability coefficient and represents the change in pitching moment with angle-of-attack. It provides a direct measurement of the projectile's static stability, which is its ability to initially return to an equilibrium position when disturbed. It is equal to the lift curve slope multiplied by the static margin. For static stability $C_{nx}$ must be negative, thus the centre-of-gravity must be in front of the aerodynamic lift. This is another of the fundamental design coefficients.

f. $C_{nu}$ is the yawing moment due to an angle-of-attack. It is generally small enough to be ignored in the design process.

3.32 The U coefficients

The U coefficients represent the effect on forces and moments due to changes in the forward velocity component of the projectile.

a. $C_{xu}$ is the speed damping coefficient, because it gives the resistance due to a change in forward velocity.

b. $C_{zu}$ is the change in lift due to forward velocity changes. This coefficient is important and should be monitored during a trajectory to ensure the lift does not decrease to a point where instability occurs.

c. $C_{mu}$ represents the pitching moment variation with changes in the forward velocity.

3.33 The q coefficients

The q coefficients represent the effect of forces and moments due to the pitch rate of the projectile.
a. $C_{Yq}$ is the sideforce due to pitching moment. It can be ignored unless there is a high spin rate. At high spin rates there can be significant sideforce due to Magnus effects.

b. $C_{Zq}$ is the change in Z-force associated with a varying pitch rate. This coefficient has a large effect in determining change in pitch angle throughout the trajectory.

c. $C_{lq}$ is the rolling moment due to pitch rate. It is normally small and can be ignored.

d. $C_{mq}$ is the change in pitching moment due to changing pitch rate. Combined with damping in pitch coefficient it determines the total damping due to pitch.

e. $C_{nq}$ is the change yawing moment due to changing pitch rate. It can be ignored unless the projectile is subjected to high spin rates. In this case, it may make a significant contribution to a missile's dynamic modes due to Magnus effects.

3.34 The $\dot{\alpha}$ ($\alpha$dot) coefficients

The alpha dot coefficients represent the effect on forces and moments due to the rate of change of the angle-of-attack. They exist because a pressure distribution across a fin does not instantaneously adjust itself to its equilibrium value when the angle-of-attack is suddenly changed. These coefficients differ from those previously discussed because they involve unsteady flow. Therefore, they are difficult to determine using analytical methods.

a. $C_{Y\dot{\alpha}}$ is the change of sideforce due to an angle-of-attack rate. It can be ignored unless the projectile undergoes high spin rates. In this case, it may make a significant contribution to a projectile's coefficient due to Magnus effects.

b. $C_{Z\dot{\alpha}}$ represents the change in Z-force due to angle-of-attack rate. It is important in determining the coefficient of a projectile's trajectory.
c. \(C_{m\alpha}\) represents the change in pitching moment due an angle-of-attack rate. This coefficient combined with \(C_{mq}\) comprises the total projectile damping due to pitch. It is a significant coefficient in determining the stability of a projectile's trajectory.

d. \(C_{n\alpha}\) is the yawing moment due to an angle-of-attack rate. It can be neglected except for high spin rate conditions. In this case, it can make a significant contribution to a projectile's dynamic modes due to the Magnus effect.

3.35 The \(\beta\) Coefficients

The \(\beta\) coefficients represent the effect on forces and moments due to the sideslip angle. Generally, analytical techniques for determining these coefficients are not reliable due to the presence of sidewash which greatly influences the coefficients. Sidewash is caused by vortices generated from the projectile body when yawed and is difficult to accurately predict. Wind-tunnel experiments on yawed models are required to obtain accurate coefficients.

a. \(C_{y\beta}\) represents the sideforce associated with a sideslip angle (\(\beta\) is positive to the right). It is highly dependent on the sidewash but usually small enough to be ignored in the trajectory analysis.

b. \(C_{z\beta}\) is the lifting force due to sideslip angle. It can be ignored except under high spin rate conditions. In this case, it may make a significant contribution due to the Magnus effect.

c. \(C_{l\beta}\) represents the rolling moment induced by a sideslip angle. Strictly speaking, it describes a restoring moment about the X-axis due to a fin dihedral. A dihedral effectively increases the angle-of-attack on one fin, while decreasing it by the same amount on the other, resulting in a difference in lift. This difference provides the restoring rolling moment. This coefficient is normally small for the projectiles under consideration.
d. $C_{n\theta}$ is the pitching moment due to a sideslip angle. It can be neglected except when the projectile undergoes high spin rates. In this case, it may have a significant effect.

e. $C_{n\phi}$ represents the yawing moment induced by a sideslip angle. It is also called the weathercock coefficient constant because it describes the yawing restoring moment. It has an important influence on the lateral coefficient of the projectile. $C_{n\theta}$ is a measure of the directional stability of the projectile when subjected to a gusting cross-wind. It is an important coefficient in the design process.

### 3.36 The $p$ coefficients

The $p$ coefficients represent the effects on forces and moments due to a roll rate. For high spin rates, Magnus effects will occur causing significant changes to the forces and moments acting on the projectile. These coefficients, like the $\beta$ coefficients, are difficult to determine analytically due to their sensitivity to sidewash (roll-induced in this case).

a. $C_{yp}$ is the sideforce due to rolling. It is generally small enough to be ignored.

b. $C_{zp}$ is the lifting force due to the projectile rolling and it can be ignored unless the projectile is subjected to high spin rates. In this case Magnus effects must be taken into consideration.

c. $C_{ip}$ is the rolling moment associated with a roll rate and is more commonly called the damping-in-roll coefficient. It expresses the resistance of the projectile to rolling. In most cases it is only the fin that contributes to this coefficient. This coefficient is very important in determining the spin rate for a free-flight projectile.

d. $C_{mp}$ is the pitching moment due to roll. It is generally small enough to be ignored for projectile designs.

e. $C_{np}$ is the yawing moment produced by the projectile’s rolling motion. It is
generally small enough to be ignored for projectile configurations.

3.37 The r coefficients

The r coefficients represent the effects on forces and moments due to yaw rate. Yawing motion alters the velocity flow field and produces sidewashes from the body. As previously stated, sidewash effects are difficult to determine analytically and wind tunnel data is normally required to determine an accurate value for these coefficients. These coefficients are normally not required for projectile design as their values are small. Therefore, they will not be described.

3.38 The $\dot{\beta}$ (βdot coefficients)

The βdot coefficients represent the effect on forces and moments due to the rate of change of sideslip angle. They exist because a pressure distribution across the projectile body does not instantaneously adjust itself to its equilibrium value when the sideslip angle is suddenly changed. These coefficients are similar to the αdot coefficients in that they involve unsteady flow. As with the r coefficients, their values are small and considered significant in the projectile design process.

3.4 Introduction to trajectory models

There are three principal types of trajectory model. These are defined as the point mass, modified point mass and 6 Degree of Freedom (DoF) models. The simplest of these models to define mathematically is the point mass model. It only requires one aerodynamic coefficient for its input (Zero lift drag, $C_{D0}$). It is a useful model in determining outline velocity and range profiles but it cannot be used in any in depth analysis of projectile behaviour during flight (Due to the simplicity of the force equations the trajectory model cannot generate projectile stability information). This trajectory model was primarily used as a check to see if the output from the more complex models was correct (Simple velocity profiles were compared to establish if the differential equations are programmed correctly) as it is very easy to obtain extremely spurious output data from these more complex models. As projectile designs evolved using
spin stabilisation, the requirement to determine estimates of yaw, drift and Magnus effects was required. This was partially accomplished by adding axial spin to the linear force equations. This model was called the Lieske trajectory model, so named after its designer. It is extremely useful in determining a projectile's spin profile. However, the model is not capable of determining the pitch and yaw angles. The trajectory model that will provide all the flight data required for configuration optimisation is the 6 DoF model. However, to solve the mathematical equations until recently, has been beyond the scope of computing systems except for very large main frame systems. The method of determining a solution to the mathematical equations of motion is only part of the problem. In order to determine a 6 DoF trajectory the values for the majority of the aerodynamic coefficients detailed earlier are required.

Having defined the aerodynamic coefficients and trajectory model capabilities, the next stage was to define the fluid through which the projectile would fly and the equations with associated aerodynamic coefficients required to model the projectile's trajectory.

3.5 ICAO (International Civil Aviation Organisation) atmosphere

The first international standard atmosphere was adopted by the International Committee for Aerial Navigation in 1924. In 1925 this standard was merged with the U.S. National Advisory Committee on Aeronautics, to give the ICAO atmosphere, which is the most commonly used by aeroballisticians. Other standard atmospheres include the 1962 and 1976 U.S. standard, the World Meteorological Organisation Standard, the International Organisation for Standardisation and the 1942 Standard Ballistic. This 1942 standard was produced by the U.K. Ordnance Board specifically for trajectory calculations.

As the projectiles being analysed in this thesis will not fly higher than 20 m, an ICAO standard atmosphere was chosen for all calculations. This standard provides the following parameters at sea-level: Temperature = 15° C, Pressure = 1013.25 mbars and Density = 1.225 Kg m⁻³.
3.6 The point mass trajectory model

Until recently, the point mass model was the major mathematical model used to create firing tables and range safety traces (a volume of space in which the projectile would fly). The point mass model generates reasonably accurate estimates of range for stable projectiles. It can also be used to predict first-order wind disturbances. The axis system for this model is located at the position of launch at zero altitude, with the X axis pointing in the horizontal direction of launch (X-range (positive forward), Y-drift/yaw (positive to right), Z-altitude (positive up)).

The point mass model assumes that the only aerodynamic force acting on the projectile is drag. This drag force is considered to be independent of pitch angles. The drag force in the X plane is defined as:

\[ \text{Drag force (X Plane)} = m \frac{d^2 X}{dt^2} = -\frac{1}{2} \rho V_x S_r C_{D0} \frac{dX}{dt} \]  (1)

Where: \( X \) = displacement in the X plane. \( V_x \) = Projectile speed. \( m \) = Projectile mass. \( \rho \) = Density of fluid (air). \( S_r \) = Reference area (Body cross-sectional area). \( C_{D0} \) = Zero lift drag. The negative sign in the equation represents the projectile decelerating after launch.

The resultant projectile speed \( V_{Res} \) taking wind effects\(^1\) into account can be calculated as follows:

\[ V_{Res} = \sqrt{(V_x - W_x)^2 + (V_y - W_y)^2 + (V_z - W_z)^2} \]  (2)

Where: \( V_x, V_y, V_z \) are the projectile’s speed components in the three planes of motion. \( W_x, W_y, W_z \) are the wind speed components in the three planes of motion.

Ignoring earth rotation effects the projectile motion for a point mass including wind effects can be calculated as follows:

\[ \frac{d^2 X}{dt^2} = -\pi \rho d^2 C_{D0} V \left( \frac{dX}{dt} - W_x \right) \]

\( ^1 \) The inclusion of the resultant velocity from Eqn (2) in Eqn (1) did not alter the trajectory by any noticeable amount. Therefore, Eqn (2) was not used in the trajectory calculations. The wind factor was taken into account by subtracting the resolved wind speed from the projectile speed. This method produced a good correlation with test range data.
\[
\begin{align*}
\frac{d^2 Z}{dt^2} &= -g - \frac{\pi \rho d^2 C_{D0}}{8m} V_Z (\frac{dZ}{dt} - W_z) \\
\frac{d^2 Y}{dt^2} &= - \frac{\pi \rho d^2 C_{D0}}{8m} V_Y (\frac{dY}{dt} - W_y)
\end{align*}
\]

Where: \(X, Y\) and \(Z\) represent the three planes of motion (\(X =\) Down range, \(Z =\) Height, \(Y =\) Cross range).
\(W_z, W_x, W_y\) = Components of wind velocity. \(g =\) Gravitational acceleration. \(V_X, V_Y, V_Z\) = Projectile velocity in a particular plane.

3.7 The modified point mass trajectory model

There are several variants of the modified point-mass model. However, its basic format has 4 Degrees-of-Freedom (DoF). These being 3 spatial DoF plus axial spin. The modified point mass model is based on the conventional point mass model but, in addition, the instantaneous equilibrium yaw is calculated at each time step along the trajectory so as to provide estimates of yaw, drag, drift, and Magnus force effects resulting from the yaw of repose (The equilibrium yaw angle the projectile will fly at). A version of the modified point mass equations of motion is given below. These equations assume linear aerodynamics and allow for rotating earth effects. It should be noted that with the very short flight times of the projectiles being studied, the rotating earth effects have not been included in the computer simulations.

\[
\begin{align*}
\frac{d^2 U}{dt^2} &= - \frac{\pi \rho d^2}{8m} (C_{D0} + C_{D\alpha} \alpha) V_U \frac{dU}{dt} \quad \text{Drag} \\
+ \frac{\pi \rho d^2}{8m} C_{N\alpha} V_U \frac{dU}{dt} \alpha \quad \text{Lift}
\end{align*}
\]
\[
- \frac{\pi \rho d^3}{16m}(C_{vpa}p(\alpha \cos(V_u))) \quad \text{Magnus effect}
\]

\[
\frac{dp}{dt} = \frac{\pi \rho d^4}{16Ix} pVc_{ip} \quad \text{Spin damping}
\]

Where: \( U = \) Displacement in the X, Y and Z planes. Wind effects have been omitted for clarity.

The axis system is the same as the point mass model with X-axis being along the line of launch.

3.8 The six DoF trajectory model

The six DoF trajectory model is the highest level trajectory model. It allows the projectile to be studied in two yaw planes, three spatial degrees of freedom and spin. The major disadvantage with this model is its requirement for initial firing conditions and a large number of aerodynamic coefficients. If the initial conditions and aerodynamic coefficients are not available or accurate, the results from the 6DoF model may not be significantly more accurate than the modified point mass model. The fundamental six DoF equations of motion can be expressed as follows:

Linear motion:

\[
\frac{d^2U}{dt^2} = -\frac{\pi \rho d^2}{8m}(C_{d0} + C_{da}\alpha^2)V_u \frac{dU}{dt} \quad \text{Drag}
\]

\[
+ \frac{\pi \rho d^2}{8m} C_{Na} (V_u \frac{dU}{dt})\alpha \quad \text{Lift}
\]

\[
- \frac{\pi \rho d^3}{16m} C_{ypa} \frac{Iy}{Iy} pV_u \frac{dU}{dt} \quad \text{Magnus force}
\]
\[
\frac{\pi d^3}{16m} \left( CL_q + CL_a \right) \frac{1}{I_{\text{plane}}} \frac{dU}{dt} \quad \text{Damping force}
\]

Rotational motion:

\[
\frac{d^2 H}{dt^2} = \frac{\pi pd^4}{16lx} V_U C_{1p} \frac{dU}{dt} \quad \text{Spin damping}
\]

\[
+ \frac{\pi pd^3}{8ly} C_{m_x} V_U \frac{dU}{dt}
\]

Overturning moment

\[
- \frac{\pi pd^4}{16lx} C_{m_{p_y}} p V_U \frac{dU}{dt}
\]

Magnus moment

\[
+ \frac{\pi pd^4}{16ly} \left( C_{m_q} + C_{m_d} \right) \dot{\alpha} p V_U \frac{dU}{dt}
\]

Damping moment

Where: \( H \) can be taken to represent, \( P, Q, R \) depending on the rotational plane. \( U \) is the displacement in the \( X, Y, Z \) plane.

### 3.9 Coding the trajectory models

The trajectory algorithms were coded using a commercial software package on a personal computer platform. The input menu for the trajectory program was divided into two data sections. One section had all the initial flight condition data and the other all the aerodynamic coefficients. For the initial development, the aerodynamic coefficients were input manually using the output from the method of masses aerodynamic algorithms. Subsequent development of the computer simulation had the aerodynamic coefficient code being embedded within the trajectory code. Having the two codes linked meant that variations in aerodynamic coefficient data with angle of attack or velocity could be calculated at each integral time step in the trajectory model. A more representative trajectory could be generated with varying the aerodynamic coefficients with flight parameters. Details of the input data requirements for the aerodynamic and trajectory models are provided in Chapter Six.
When considering errors that can occur in the mathematical solution of the differential equations (assuming they have been coded correctly), it is the effect of the time interval used to calculate the numerical solution (A Runge Kutta 5th order numerical method was used to solve the differential equations) that is the greatest error source. It was found that if too large a time interval in the numerical integration routine was used, the projectile yawing motion was not modelled correctly. Too large a time interval introduced aliasing errors (Errors created when the sample has a greater frequency than the numerical method it is set to calculate). To determine the integration time required an estimation of the projectile pitch rate needs to be determined. This was achieved by developing the following equations for estimating the projectile pitch rate:

\[
\text{Projectile pitch rate} = t = \frac{2\pi}{\sigma} \frac{I_y}{I_x P}
\]

where:

\[
\sigma^2 = 1 - \frac{1}{s}
\]

\[
s = \frac{I_x^2 P^2}{4I_y^2 v^2 \rho d^4 C_{m\alpha}}
\]

\(I_y\) = Transverse moment of inertia. \(I_x\) = Axial moment of inertia. \(P\) = Total axial spin (rad/s). \(v\) = Projectile velocity. \(m\) = Mass of projectile. \(\rho\) = Air density. \(d\) = Diameter of projectile. \(M_{\alpha}\) = Overturning moment coefficient.

Having an estimation of the pitch rate from Eqn (3) provided an integral solution time period. (The integration time had to be set at least twice this rate - Nyquist sampling theory, to avoid aliasing errors).
3.9 Ministry of Defence (MoD) trajectory and aerodynamic prediction models

MoD is taken to encompass the Defence Evaluation Research Agency (DERA). Having detailed the mathematical trajectory and aerodynamic coefficient requirements, a review of accessible MoD trajectory and aerodynamic prediction simulation codes was carried out. Accessible is defined as simulation codes that could be used to evaluate and generate the required aerodynamic data (In addition, the source code was also available for inspection). There might be other codes in existence within the MoD organisation that could do the job more efficiently and accurately. If there are they were not found in an extensive review procedure. The benchmark for the codes were the projectile configuration parameters detailed in Chapter One, section 1.1. The simulation models detailed in the following sections were chosen as they could, with varying degrees of adaptation, be used to generate the required trajectories. Other models were investigated but disregarded due to them not being suitable for the projectile parameters under investigation. (Most codes were unsuitable due to them being designed for the wrong velocity regime). The following terms are used in the descriptions of the simulation models:

a. User friendly: The code was easy to use with the data inputs being easy to understand. The output data was easy to interpret. The code had a short learning time.

b. Aerodynamic data: The model required a source of aerodynamic coefficient data to be provided.

3.10.1 CADAC

CADAC is a modelling environment which is used in the development of 3DoF, 5DoF and 6 DoF missile models. It was originally developed by Litton industries in the 1960s. The source code is written in FORTRAN and run on a PC using Microsoft FORTRAN Power Station. To run an analysis of a trajectory, the software package “Plot it” is also required. The model is not very user friendly and takes a considerable time to be able to create a data input file. It also requires a source of aerodynamic data.
3.10.2 Hades

Hades is a generic 6 DoF simulation model. The program PV-Wave provides the graphical interface. The model uses an “object orientated” modular design, allowing sub-systems to be interchangeable, providing the interface between them remains constant. The code is written in DEC Pascal and FORTRAN and runs on a VAX-VMS platform. The model is not very user friendly and still requires a source of aerodynamic data.

3.10.3 TRAP 3.1

TRAP 3.1 is a general purpose missile fly out program which is sponsored by the United States Air Force. It claims to able to model 3DoF, 5 DoF and 6 DoF simulations. However, its aerodynamic input data is derived from the aerodynamic prediction code missile DATCOM which cannot generate rotational aerodynamic data of sufficient accuracy to justify the 6 DoF Trap claim. The code is written in FORTRAN 77, and executed on a VAX-VMS platform. It is very difficult to program and can take an excessive time (up to eight hours on a DEC Alpha workstation for a full output) to run a trajectory simulation. The aerodynamic data is extracted from DATCOM output data by means of an interface program called Postdat. Extreme care must be taken with the FORTRAN data format as any errors result in the program not running. With no error log, code checking can take a long time to generate any meaningful output data.

The 6 DoF output is in tabular form and requires a graphics package to generate graphical output. It is not possible to determine spin profiles or pitch and yawing motion from the model (Detailed explanation of missile DATCOM in Chapter Four discusses this point).

3.10.4 Damocles

Damocles is 3 DoF kinematic simulation model. No validation documentation has been produced for the model and therefore the output data is not considered very accurate. The model requires a separate source of aerodynamic input data which makes it extremely cumbersome to use.
3.10.5 "Aptec Inc." Rapid Aero-Shape Generator (RAGE)

This is an airframe optimisation tool, optimising a shape, within a set of constraints, according to some pre-determined aerodynamic strategy, such as maximised Lift/Drag ratio. The program was originally developed to help design re-entry space vehicles. The program uses shock expansion and Newtonian methods to predict the aerodynamics of airframe shapes. RAGE is written in FORTRAN and C and runs on a Silicon Graphics Indigo 2 workstation, running the IRIX operating system. The aerodynamics are only validated at speeds above Mach 3. The aerodynamic output data is quoted as only being accurate to approximately 20%.

3.10.6 Missile DATCOM

Missile DATCOM is an aerodynamic prediction code for use in missile preliminary design. DATCOM computes the aerodynamic parameters as a function of angle of attack. The source code is written in FORTRAN. A personal computer version has just been released and is going through a validation procedure. Further details concerning DATCOM are provided in Chapter Four.

3.10.7 Missile design synthesis

Missile design synthesis is a modular program, used in the conceptual design of missiles. The code was developed by British Aerospace. It uses empirical methods to predict the aerodynamics of missile components and the effects of interference between them. A drag module called Zero Incidence Predictor provides zero incidence drag data. The aerodynamic data can then be input into a 5 DoF trajectory simulation model called Kinematic Simulator. The modules were written by British Aerospace, in FORTRAN and run on a VAX-VMS system. The owning and release authority is British Aerospace through the MoD.
3.11 Computational Fluid Dynamics (CFD)

3.11.1 NEARZEUS CFD code

The NEARZEUS CFD code is written by Nielsen Engineering & Research, Inc. Ca, USA. It was designed for detailed missile aerodynamic prediction. It is valid for supersonic/hypersonic flow. In brief the codes details are; Predicts non-linear details of flow around the missile, finite-volume Godunov scheme, second order accurate Zonal Euler Solver, supersonic space marching, real gas option. External or internal flows, multiple zone semi-automatic gridding. The code runs on a workstation such as a Sun Sparc. (To obtain the code requires a US export licence which can cause problems) The code is not easy to use and appeared to be too sophisticated for the aerodynamic data that was required. Average run time was very short (20 seconds). The average time to generate the grid was about 2 hours. When the aerodynamic data was generated, it still had to be manually ported into a trajectory model.

3.12 Summary of aerodynamic and trajectory codes

It was found that a combination of the simulation codes listed might have provided the required trajectory data. It was also noted access to the source code for some of the codes was not available for inspection which meant that the theoretical methods the codes used could not be verified. For the codes in which access to the source code was possible, it was invariably found that the source code was badly documented or had been altered to accommodate experimental missile configurations. Where this had happened validation data was not available. It was therefore concluded that only one of these codes, viz missile DATCOM, might prove to be viable in generating the required aerodynamic coefficient data. This aerodynamic prediction code is analysed in more detail in Chapter Four.
4.0 AERODYNAMIC PREDICTION METHODS

4.1 Introduction

As detailed in Chapter Two, numerical aerodynamic prediction methods have developed from Slender Body theory through to Computational Fluid Dynamics (CFD) codes. Chapter Three provided a brief explanation of the available MoD aerodynamic and trajectory codes. Having performed a selection procedure and concluded that Missile DATCOM might be suitable to generate the required aerodynamic coefficients, Missile DATCOM was investigated in more detail. A second commercial aerodynamic prediction code was also investigated. This code was called RAPPIC (Unknown what RAPPIC stood for). The RAPPIC code was not accessible to the author but output data from the code was provided for different projectile configurations so that a validation procedure with other aerodynamic prediction code data could be carried out. A CFD aerodynamic code called Loftsman was also investigated to determine whether it would be suitable to provide the required aerodynamic coefficient data (The Loftsman CFD code was provided by the University of Loughborough).

This chapter details the theoretical basis of these aerodynamic codes and assesses their suitability in determining the projectile aerodynamic coefficient data. The chapter concludes with a theoretical description of the method of masses numerical method to generate projectile aerodynamic coefficient data.

To assess the suitability of the aerodynamic codes, a set of output requirements had to be established. For the projectile parameters provided in Chapter One, section 1.1, the aerodynamic prediction model output requirements to determine the projectile’s trajectory performance were defined as follows:

a. Be capable of generating all the aerodynamic stability coefficients required to generate a 6 Degree of Freedom (DoF) trajectory.

b. Be capable of generating accurate aerodynamic stability coefficient data in the subsonic speed regime.

c. Be a direct method without the need for extensive look-up-tables.
d. Link the output from the aerodynamic prediction method into the 6 DoF trajectory mathematical model.

e. Have a simple input data menu with an unambiguous method of inputting the projectile geometry.

f. Generate the required aerodynamic coefficients in a short time frame.

g. The aerodynamic and trajectory model to run on a Personal Computer platform (PC).

4.2 Loftsman CFD code

With the advent of commercial CFD codes being available on a Personal Computer (PC) and at reasonable financial cost it was considered that these codes might provide the aerodynamic data requirements. A CFD code, provided by Loughborough University, that ran on a PC was the Loftsman, Version 3.2 (1996) CFD code. This code uses a low order panel technique but it is advertised as a CFD method. Therefore, as a panel method is a class of CFD technique, the term CFD is used in this context.

4.2.1 Loftsman PC Code

At the outset it must be stated that the Loftsman code was not very easy to use. It took many hours of work to be able to obtain data output for the simplest projectile shape. Even though an output was obtained, the accuracy of the data was questionable as it was not entirely clear if the correct projectile input geometry was being analysed. This fact was reinforced by the originator of the code who in the users manual writes, “Users who do not study the documentation carefully or do not thoroughly understand what the program does, what it cannot do, and what they themselves must do, are in some danger of discovering only at the mock-up stage that what they have designed is different from what they had imagined”.

There are two main parts to the Loftsman program. First there is a classical lofting technique used to construct the required aerodynamic shapes. For a user not familiar with lofting techniques, it takes a considerable time to be able to manipulate even the simplest diagrams. A complete aerodynamic configuration is constructed by splicing different geometric objects
together (Nose, body fins etc.). Once the geometric diagram is complete, a mesh function is used so that the object can be analysed using CFD techniques. If the object is symmetrical, the mesh function is straightforward as the code assumes a mirror image. For non-symmetrical objects an additional patch function is required. For fin body interfaces, there are two methods of determining the mesh pattern. These are flow through and flow around functions. The flow around breaks the mesh at the leading edge of the fin and the panelling is carried around the root (Intersection between fin and body). The mesh is distorted but preserves proportional spacing. The user manual quotes that this type of meshing is frequently used but can produce bad neighbour relations between panels at the fin leading edge, where the inflection is large and abrupt. The flow around function leaves the panelling undistorted. This method can produce a few triangular panels adjacent to the root, but they will have a less detrimental effect on the computational analysis than the fin leading edge inflection that occurs with the flow around technique.

When the geometric object has been meshed the aerodynamic analysis can be carried out. This analysis is a three dimensional low order panel code. It is run using a file called CMARC. It was found that the execution time of this file was a direct function of the number of panels being analysed, the number of calculation time steps and the number of additional features that were included, such as numerical precision. The CMARC input files are derived from NASA’s Pmarc-12 program. They are written in FORTRAN notional format, which can cause problems if the correct convention is not adhered to. The input file calls into the CMARC program the required geometric shape from the Loftsman program. The input file also specifies the flight conditions and the dimensions for the geometric shape. It was found that a typical run time was in the order of fifteen minutes on a pentium 133 MHz machine. On completion the output data could only be read using a word processing package as the output comprised in the order of 450 pages of output data (The file is too large to be read by ‘Windows notepad’ type programs). Two example output pages are shown at Fig 4.1 As can be seen from this figure, this data it is not easy to interpret. It should also be noted that the pitching moment and normal force coefficients are considered to be linear throughout a large

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1 In written dialogue with the codes author, he considered flow through to be the most suitable option for the designs being considered.
angle of attack range. As has been previously documented this is an invalid statement for large angles of attack due to the influence of cross-flow drag.

4.2.2 Theoretical basis of CMARC program

The three-dimensional model from LOFTSMAN consists of a closed surface immersed in a field of infinite extent. The surface is subdivided into a large number of generally rectangular panels, which separate the field into inner and outer regions. Depending on the formulation of interest, either may be the subject of the analysis, the other remaining a fictitious flow. Most aerodynamic and hydrodynamic problems target the outer region. Flow is assumed to be incompressible, irrotational and inviscid. The velocity potentials in both regions are assumed to satisfy Laplace's equation which was defined in Chapter Two.

4.2.3 Computation of induced drag

CMARC computes drag using two methods. One is by integrating surface pressures. It should be noted that because of the rapid change in pressure gradient at the leading edges of lifting surfaces, a large number of chordwise panels is required to resolve the induced drag accurately.

The second method is the Treffitz plane analysis, which involves evaluating an integral in the Treffitz plane. Further details of this technique can be found at Ref [36].

4.2.4 POSTMARC

POSTMARC is a post processing program that enables data output from CMARC to be manipulated. Configuration data can be altered using POSTMARC to vary the values of forces and moments. The adjustable values are speed, density, area, fin semispan and fin chord. The aerodynamic output coefficients are referenced to the wing area, the reference chord length and the fin semispan measured from the surface of the body. Fluid density must be expressed in units consistent with the model dimensions. The same is true of the projectile velocity, which must be entered in order to obtain forces in a useful dimensional form.
4.2.5 Aerodynamic coefficients and lift curve slope

To obtain the aerodynamic coefficients for the projectile under investigation, it is necessary to run all cases (Different projectile parameters such as height, velocity configuration variation) at a minimum of two angles of attack. Since CMARC generates linear variations in force and moment coefficients with respect to angles of attack, it was found to be unnecessary to run more than four angles of attack for each case. It was recommended by the originator of the code to run different cases at up to four different angles of attack to “average out slight variations that might occur”.

Loftsman in conjunction with CMARC has the capability of generating all the aerodynamic coefficients required to generate a point mass or modified point mass projectile trajectory. However, damping derivatives or Magnus derivatives were not listed in the manuals as being obtainable from the code, which meant the full 6 DoF trajectory could not be run from the quoted Loftsman aerodynamic output.

It was therefore concluded that for outline conceptual designs and minor alterations of existing configurations, this CFD code was too complex and required too much time to program. It was also not capable of generating all the required aerodynamic coefficient data.

The limited amount of output data that was generated by CMARC is compared with other aerodynamic prediction method data in Chapter Five.

4.3 RAPPIC aerodynamic prediction code

RAPPIC is a commercial aerodynamic prediction code developed by Hunting Engineering. The code was designed to cater for the type of projectiles described in Chapter One. However, because it is a proprietary code it was not possible to examine in detail the construction of the code and the methods used to generate the output data. However, a limited number of data runs were possible using the RAPPIC code to determine the aerodynamic coefficients of different projectile shapes. This aerodynamic data was then
compared to other aerodynamic prediction methods. It was concluded that RAPPIC was capable of generating all the required aerodynamic coefficient required for a 6 DoF trajectory with the exception of drag. The projectile's drag characteristics were calculated using the British Aerospace, Zero Incidence Predictor drag prediction code. The RAPPIC code is not linked to a trajectory model and therefore trajectory comparisons could not be carried out. Several projectile configurations were analysed using the RAPPIC code. The output data from these configurations were used in the validation procedures in Chapters Five and Six.

4.4 Missile DATCOM aerodynamic prediction code

A large number of technical papers have been written about aerodynamic prediction codes describing their apparent capabilities. However, what is not readily available is the limitation and accuracy to which the code was designed for. Through discussions with the Missile DATCOM codes technical designers an understanding into how Missile DATCOM was developed, what configurations it could simulate and the expected accuracy of the output was obtained. By understanding how Missile DATCOM was developed an awareness into the limitations of aerodynamic prediction codes was obtained. The following sections detail the accuracy and design philosophy behind Missile DATCOM.

4.4.1 Missile DATCOM development

The missile prediction code, Missile DATCOM was developed by the United States Air Force and the McDonnell Douglas Corporation\(^2\). Since its inception, there has been a continuous publication of technical papers purporting to detail Missile DATCOM's development, progress and capabilities. The definitive publication that explains the aerodynamics methods selected for implementation in Missile DATCOM can be found at Ref [37].

\(^2\) It should be noted that the correct title to the code is Missile DATCOM and not the abbreviation DATCOM as DATCOM alone stands for Data Comparison and is an aircraft prediction methodology.
4.4.2 DATCOM aerodynamic method selection

For an aerodynamic prediction code to be useful, it must combine the features of rapid generation of data, applicability over a wide range of design parameters and good accuracy. Prior to Missile DATCOM, aerodynamic data generation was based upon prediction codes that were either primarily empirical, and produced a highly accurate result over a rather limited parameter space encompassed by their defining database, or alternatively, they were research-oriented with input and output requirements that were excessive in terms of the amount of data they required or generated. Fundamentally, neither could adequately address the designer's requirements. However, the aggregate of these codes contained a nucleus of well-documented accepted prediction methods. Therefore, a code based on the component "build-up" approach, relying heavily on existing methodology, was required. Component build-up codes offer all the features required in the preliminary design environment, although at the expense of the rather complex program logic required to synthesise different components and methods into a total system. Over 300 candidate methods were examined during the Missile DATCOM feasibility study. Methods applicable to tactical missiles received primary emphasis, with only secondary emphasis being given to subsonic projectiles and ballistic ordnance. This selection criteria mitigated against Missile DATCOM being used to generate the aerodynamic coefficients required for all the configurations under investigation in this thesis.

4.4.3 Missile DATCOM accuracy criteria

An important aspect in the accuracy assessments used in the development of Missile DATCOM was the criteria established by Krieger and Williams [38]. These criteria express the allowable error for a given aerodynamic parameter (e.g. axial force) as a function of the tolerances placed on vehicle performance requirements (e.g. range). Thus methods giving acceptable accuracy for early trade-off studies could be selected without unduly compromising the speed of computation. The Missile DATCOM criteria used for static longitudinal characteristics are shown in Table 1.
### Table 1. Static longitudinal characteristics and allowed errors

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Error allowed</th>
<th>Design parameter (error)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_N$</td>
<td>± 20%</td>
<td>range (± 10%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Load Factor (± 20%)</td>
</tr>
<tr>
<td>$C_m$</td>
<td>± 20% or</td>
<td>Fin Area (± 20%)</td>
</tr>
<tr>
<td></td>
<td>± 25 % L</td>
<td>Centre of Mass (± 2%L)</td>
</tr>
<tr>
<td>$C_{D0}$</td>
<td>± 10% or</td>
<td>Range (± 10%)</td>
</tr>
<tr>
<td></td>
<td>± .2 $C_D/(CA \cos \alpha)$</td>
<td>Turn Deceleration (± 20%)</td>
</tr>
</tbody>
</table>

#### 4.4.4 Missile DATCOM limitations

As previously stated, Missile DATCOM was specifically designed to predict the aerodynamic coefficients of guided missiles. The flight profile for these systems is predominantly in the supersonic velocity regime so validity of the code at subsonic speeds was not a high priority in the code validation process. Moreover, as the guided weapon systems invariably have roll autopilots, the Missile DATCOM code does not predict the roll damping coefficient for fins and body in combination as it was not considered a fundamental design requirement. Therefore, the code only predicts roll damping for the body alone. The value of the body alone damping coefficient is significantly lower than the value with a fin/body combination. This fact is not made clear in the Missile DATCOM user manuals and gave problems in the beginning of this research work, particularly in modelling spin profiles. It was only after flight trials testing that it was realised that the quoted DATCOM spin damping figures were in error.

Another limitation of the code is that it cannot model rounded projectile noses. Again the reason for this is that the noses on guided projectiles are either hemispherical, or ogival in shape and there was no requirement for other shapes to be considered. This was again due to the code being optimised to model guided supersonic systems. For a supersonic system a rounded nose will create too much drag. Therefore the requirement was not there to compute the data for this type of nose shape.
4.5 Summary of Loftsman and Missile DATCOM as methods of generating aerodynamic coefficient data

As stated in section 4.2.1 the Loftsman CFD aerodynamic code was not easy to use in terms of generating a projectile shape and originating aerodynamic coefficient data that could be used in conjunction with a trajectory model, although with time, the code might be adapted to create suitable output.

Missile DATCOM proved to be very versatile in the missile configurations it could model. However, the limitation on the rounded nose and the lack of spin damping coefficient data mitigated against its use to generate the required projectile aerodynamic data. An option that was investigated was to extract a subsonic set of Missile DATCOM algorithms and create a new aerodynamic code. It was found that the Missile DATCOM methods were so embedded with look-up-tables and numerical assumptions (Insertion of constant values to increase or decrease the output values) it was not possible to generate a practical subset of numerical algorithms. However, it was discovered that the Missile DATCOM subsonic drag methods had been rigorously validated Ref [39]. The subsonic Missile DATCOM drag data proved to be very valuable in validating the drag determination method described in Chapter Five. As there is no interface that can extract Missile DATCOM output data so that it can be used in a 6 DoF trajectory model (The program Postdat only extracts the following aerodynamic coefficients $C_{Na}$, $C_{ma}$ and $C_{De}$), it was decided not to pursue the Missile DATCOM aerodynamic program any further. However, Missile DATCOM output data proved to be very valuable in validating the aerodynamic methods developed in Chapter Five.

Having exhausted the available aerodynamic simulation codes it was decided the development of a known but neglected numerical aerodynamic technique might be applicable using modern computational techniques. This numerical method was the method of masses.
4.6 Method of masses aerodynamic prediction method

The method of masses is a direct numerical method of producing a projectile's aerodynamic coefficients if the apparent mass coefficients of the projectile cross-section are known. The method of masses to determine a projectile's aerodynamic coefficients was initially developed by Bryson in the 1950's, Refs [1,40]. It is an extremely complex mathematical method and the means of numerically solving the equations efficiently were not available at the time of its development. The method of masses appears then to have been ignored until Nielson dedicated a chapter on it in his book Projectile Aerodynamics, Ref [2]. Following the work carried out by Neilson there appears to have been no further serious exploitation carried out on this method. This lack of research into the method again appears to be due to the lack of computational power required to solve the method of masses equations. Another reason for the lack of interest in the method was the large investment being put into alternative aerodynamic methods and techniques such as Missile DATCOM.

The following sections outline the theoretical background to the method of masses. This material has been included as it provides a fundamental background into the method. It also illustrates how the method has been developed and illustrates some of its limitations and advantages. For accuracy and completeness, the work in these sections draws heavily on the work published by Bryson and Neilson. Further details of the early development work can be found at Refs[41-43].

4.7 Theoretical background to the method of masses

The general class of projectile configuration the method of masses is capable of determining the aerodynamic stability coefficients for are slender bodies with canards, rear fins or a combination of both. The projectiles detailed in Chapter One appear to be ideal candidates for having their aerodynamic coefficients calculated using this method. The physical foundation of the method assumes that the projectile moves through an infinite expanse of fluid stationary at infinity, and that the system of body axes X, Y, Z has its origin fixed at the centre of mass of the system as shown in Fig 4.2. Fins have been omitted from this diagram for clarity reasons. The method of masses is capable of determining the aerodynamic coefficients for the projectile.
configurations detailed in Chapter One. Considering a crossflow plane fixed perpendicular to the X axis, the potential in this plane only depends on the normal velocities of the projectile cross-section in the plane at the instant under consideration. Assuming \( \xi, \eta \) and \( \zeta \) are parallel to \( X, Y, \) and \( Z, \) and let \( v_1, v_2 \) be the linear velocities of the projectile cross-section in the plane along the \( \eta \) and \( \zeta \) axes, respectively. If the angular velocity of the projectile cross-section about the \( \xi \) axis is defined as \( \phi. \) The potentials due to unit values of \( v_1, v_2 \) (Velocity components of the projectile cross-section) and \( \phi \) are \( \phi_1, \phi_2, \) and \( \phi_3 \) the complete potential can be written as:

\[
\phi = v_1 \phi_1 + v_2 \phi_2 + \phi \phi_3 \quad (1)
\]

This expression ignores any influence of terms proportional to the rate of change of cross-sectional area along the projectiles body. This factor has to be taken into consideration for design purposes as the method from this basic assumption is capable of dealing with sharp discontinuities in a projectile cross-section (A result will be generated with a sharp discontinuity, but the accuracy of the result will be degraded). As the projectiles under investigation do not have large discontinuities this factor was not taken into consideration.

The kinetic energy of the fluid per unit length along \( X \) can be expressed by the integral expression:

\[
T = -\frac{1}{2} \rho \int_c \phi \frac{\partial \phi}{\partial n} ds \quad (2)
\]

Where: \( T = \) Kinetic energy of fluid, \( \rho \) is the fluid density, \( s \) is the distance measured along contour of projectile cross-section in crossflow plane.

The contour \( c \) is the periphery of the projectile cross-section in the crossflow plane, and \( n \) is the outward normal. Eqn (2) can be expanded to take into account the different velocity components. Using these components the kinetic energy can be expressed as:
\[
\frac{T}{\frac{1}{2} \rho S_R} = \frac{v_1^2}{S_R} \int_c \phi_1 \frac{\partial \phi_1}{\partial n} \, ds + \frac{v_1 v_2}{S_R} \int_c \phi_1 \frac{\partial \phi_2}{\partial n} \, ds + \frac{v_1 (\lambda p)}{S_R \lambda} \int_c \phi_1 \frac{\partial \phi_3}{\partial n} \, ds + \frac{v_1 v_2}{S_R} \int_c \phi_2 \frac{\partial \phi_1}{\partial n} \, ds + \frac{v_2^2}{S_R} \int_c \phi_2 \frac{\partial \phi_2}{\partial n} \, ds + \frac{v_2 (\lambda p)}{S_R \lambda^2} \int_c \phi_2 \frac{\partial \phi_3}{\partial n} \, ds + \frac{v_2 v_1}{S_R} \int_c \phi_3 \frac{\partial \phi_1}{\partial n} \, ds + \frac{(\lambda p)^2}{S_R \lambda^2} \int_c \phi_3 \frac{\partial \phi_2}{\partial n} \, ds + \frac{(\lambda p)^3}{S_R \lambda^3} \int_c \phi_3 \frac{\partial \phi_3}{\partial n} \, ds
\]

In the above expression the reference length \( \lambda \) (Body length) and a body reference area \( S_R \) (maximum body cross-section) have been included. The nine integrals in this expression are defined as the inertia coefficients of the cross-section. They are given the notation \( A_{ij} \) in accordance with the following matrix array:

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix} = -\begin{bmatrix}
\frac{1}{S_R} \int_c \phi_1 \frac{\partial \phi_1}{\partial n} \, ds & \frac{1}{S_R} \int_c \phi_1 \frac{\partial \phi_2}{\partial n} \, ds & \frac{1}{S_R \lambda} \int_c \phi_1 \frac{\partial \phi_3}{\partial n} \, ds \\
\frac{1}{S_R} \int_c \phi_2 \frac{\partial \phi_1}{\partial n} \, ds & \frac{1}{S_R} \int_c \phi_2 \frac{\partial \phi_2}{\partial n} \, ds & \frac{1}{S_R \lambda^2} \int_c \phi_2 \frac{\partial \phi_3}{\partial n} \, ds \\
\frac{1}{S_R \lambda} \int_c \phi_3 \frac{\partial \phi_1}{\partial n} \, ds & \frac{1}{S_R \lambda^2} \int_c \phi_3 \frac{\partial \phi_2}{\partial n} \, ds & \frac{1}{S_R \lambda^3} \int_c \phi_3 \frac{\partial \phi_3}{\partial n} \, ds
\end{bmatrix}
\]

The kinetic energy of the fluid per unit length becomes:

\[
T = \frac{1}{2} \rho S_R \left[ v_1^2 A_{11} + v_2^2 A_{22} + (\lambda p)^2 A_{33} + 2 v_1 v_2 A_{12} + 2 v_1 (\lambda p) A_{13} + 2 v_2 (\lambda p) A_{23} \right]
\]

This expression assumes the relationship \( A_{ij} = A_{ji} \). This relationship is based on Green's theorem Ref [44].

4.8 Forces acting on the body cross-section

The aerodynamic sideforce, downforce and rolling moment per unit length on the body at any given cross-section can be found by relating the velocities \( v_1 \) and \( v_2 \) to linear and angular velocities \( v, \omega, q, \) and \( r \), but with the substitution of \( \alpha \) and \( \beta \) as independent variables for \( v \) and \( \omega \). Therefore, the following expression can be obtained:
\[ v_1 = v + rX = \beta V_0 + rX \]  
\[ v_2 = \omega - qX = \alpha V_0 - qX \]  

The forces and moments \( Y, Z, L, M \) and \( N \) can be determined by differentiating the kinetic energy given in expression (5). It must be noted that the drag force \( X \) is not included as the method is not suitable for its calculation (The reason for this is the theory assumes the projectile to be moving in an inviscid fluid). The expressions for calculating the force \( dY/dX \) and \( dZ/dX \) per unit axial distance and the rolling moment \( dL/dX \) per unit axial distance are derived by Lamb Ref [45] and given here without proof.

\[
\begin{bmatrix}
Y \\
Z \\
L
\end{bmatrix} = -\frac{d}{dt}\begin{bmatrix}
\frac{\partial T}{\partial v} \\
\frac{\partial T}{\partial \omega} \\
\frac{\partial T}{\partial p}
\end{bmatrix} + p\begin{bmatrix}
\frac{\partial T}{\partial v} \\
\frac{\partial T}{\partial \omega} \\
0
\end{bmatrix} + \omega\begin{bmatrix}
0 \\
0 \\
\frac{\partial T}{\partial \omega}
\end{bmatrix} - \begin{bmatrix}
\nu \\
0 \\
0
\end{bmatrix}
\]  

(7)

The yawing moment and pitching moment per unit length are given by:

\[
\begin{bmatrix}
n \\
m
\end{bmatrix} = \begin{bmatrix}
xY \\
xZ
\end{bmatrix}
\]  

(8)

Because \( T \) is a function of \( v, \omega, p \) and the six inertia coefficients, \( T \) can change with time in two ways:

i. By changing the motion of the cross-section.

ii. By a change in shape of the cross-section.

The motion of the cross-section can also change in two ways:

4 - 59
i. By changing the linear velocity of the centre of mass of the body.

ii. By changing the angular velocity of the body.

Differentiating Eqn (5) the forces and rolling moment per unit projectile length Eqns (9-11)

\[
\frac{dY}{dX} = -\rho S_R [A_{11} \dot{v}_1 + A_{12} \dot{v}_2 + A_{13} (\lambda \dot{p})] + \rho S_R V_0 \frac{\partial}{\partial X} [A_{11} v_1 + A_{12} v_2 + A_{13} (\lambda p)] + \rho S_R p [A_{11} v_1 + A_{12} v_2 + A_{13} (\lambda p)] 
\]

(9)

\[
\frac{dZ}{dX} = -\rho S_R [A_{12} \dot{v}_1 + A_{22} \dot{v}_2 + A_{23} (\lambda \dot{p})] + \rho S_R V_0 \frac{\partial}{\partial X} [A_{12} v_1 + A_{22} v_2 + A_{23} (\lambda p)] - \rho S_R p [A_{11} v_1 + A_{12} v_2 + A_{13} (\lambda p)] 
\]

(10)

\[
\frac{dL}{dX} = -\rho S_R \lambda [A_{13} \dot{v}_1 + A_{23} \dot{v}_2 + A_{33} (\lambda \dot{p})] + \rho S_R V_0 \frac{\partial}{\partial X} [A_{13} v_1 + A_{23} v_2 + A_{33} (\lambda p)] + \rho S_R \nu_2 [A_{11} v_1 + A_{12} v_2 + A_{13} (\lambda p)] - \rho S_R \nu_1 [A_{12} v_1 + A_{22} v_2 + A_{23} (\lambda p)] 
\]

(11)

can be obtained:

Since the axial distributions of sideforce Y, normal force Z and rolling moment L are known along the body, direct integration from the projectile base to projectile nose will supply the forces Y, Z, L and moments m and n. Eqns (9-11) are non-dimensionalised by dividing the forces by \( \rho V_0^2 S_R / 2 \) and the moments by \( \rho V_0^2 S_R \lambda / 2 \). In addition, the independent variables \( \alpha, \beta, \lambda p/2V_0, \lambda q/2V_0, \) and \( \lambda r/2V_0 \) are included. With these changes the following expressions can be derived:
To obtain specific expressions for the derivatives of $C_v$, $C_z$, $C_t$, and $C_r$ by $\alpha$, $\beta$, $p\lambda/2V_0$, $\lambda q/2V_0$ and $\tau\lambda/2V_0$ (25 derivatives). From Eqn (12), and only considering the first derivative as an example the following expression can be obtained:

$$\frac{dC_v}{d(X/\lambda)} = -4A_{11}\left[\frac{\lambda \dot{V}_0}{2V_0}\right] + \frac{\lambda \dot{V}_0}{2V_0^2} + \frac{\lambda \dot{V}_0}{2V_0^2} + A_{12}\left[\frac{\lambda \dot{V}_0}{2V_0}\right] + A_{21}\left[\frac{\lambda \dot{V}_0}{2V_0}\right] + A_{31}\left[\frac{\lambda \dot{V}_0}{2V_0}\right] + A_{41}\left[\frac{\lambda \dot{V}_0}{2V_0}\right] + 2\frac{\lambda \dot{V}_0}{2V_0} + 4(\frac{\lambda \dot{V}_0}{2V_0})A_{22}$$

(15)
base are denoted by a bar as $\overline{A}_{11}, \overline{A}_{12}$ etc. $X$ integrals of the inertial coefficients $A_{ij}$ are calculated as follows:

$$B_{ij} = \int A_{ij} d\left(\frac{X}{\lambda}\right) \quad (16)$$

$$C_{ij} = \int A_{ij} \left(\frac{X}{\lambda}\right)^2 d\left(\frac{X}{\lambda}\right) \quad (17)$$

$$D_{ij} = \int A_{ij} \left(\frac{X}{\lambda}\right)^3 d\left(\frac{X}{\lambda}\right) \quad (18)$$

In terms of $A_{ij}$, $B_{ij}$, $C_{ij}$ and $D_{ij}$ the integration of Eqns (16-18) provides the projectile configurations aerodynamic force coefficients $Y$, $Z$ and the spin coefficients $I$ with respect to $\alpha$, $\beta$, $q$ and $r$. (The $X$ force component is not included as it was initially assumed that the projectile was in an inviscid fluid medium. For this medium there is no $X$ drag force). Definitions of these aerodynamic coefficients is given in Chapter Three.
\[
C_{Y_0} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{12} - 2A_{12} + 4\left(\frac{\lambda p}{2V_0}\right)B_{22}
\]
\[
C_{Y_\beta} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{11} - 2A_{11} + 4\left(\frac{\lambda p}{2V_0}\right)B_{12}
\]
\[
C_{Y_p} = -4\bar{A}_{13} + 4\alpha B_{22} + 4\beta B_{12} + 16\left(\frac{\lambda p}{2V_0}\right)B_{23} - 8\left(\frac{\lambda q}{2V_0}\right)C_{22} + 8\left(\frac{\lambda r}{2V_0}\right)C_{12}
\]
\[
C_{Y_q} = 4\bar{A}_{12}\left(\frac{X}{\lambda}\right)_b - 8\left(\frac{\lambda p}{2V_0}\right)C_{22}
\]
\[
C_{Y_r} = -4\bar{A}_{11}\left(\frac{X}{\lambda}\right)_b + 8\left(\frac{\lambda p}{2V_0}\right)C_{12}
\]
\[
C_{Z_0} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{22} - 2A_{22} - 4\left(\frac{\lambda p}{2V_0}\right)B_{12}
\]
\[
C_{Z_\beta} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{12} - 2A_{12} - 4\left(\frac{\lambda p}{2V_0}\right)B_{11}
\]
\[
C_{Z_p} = -4\bar{A}_{23} + 4\alpha B_{12} + 4\beta B_{11} + 16\left(\frac{\lambda p}{2V_0}\right)B_{13} - 8\left(\frac{\lambda q}{2V_0}\right)C_{12} - 8\left(\frac{\lambda r}{2V_0}\right)C_{11}
\]
\[
C_{Z_q} = 4\bar{A}_{22}\left(\frac{X}{\lambda}\right)_b - 8\left(\frac{\lambda p}{2V_0}\right)C_{12}
\]
\[
C_{Z_r} = -4\bar{A}_{12}\left(\frac{X}{\lambda}\right)_b + 8\left(\frac{\lambda p}{2V_0}\right)C_{11}
\]
\[
C_{k_s} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{23} - 2A_{23} + 4\alpha B_{12} + 2\beta(B_{11} - B_{22}) - 4\alpha B_{12} + 4\left(\frac{\lambda p}{2V_0}\right)B_{13} - 8\left(\frac{\lambda q}{2V_0}\right)C_{12} + 4\left(\frac{\lambda r}{2V_0}\right)(C_{11} - C_{22})
\]
\[
C_{k_p} = -4\left(\frac{\lambda V_0}{2V_0^2}\right)B_{13} - 2A_{13} + 4\alpha B_{12} + 2\alpha(B_{11} - B_{22}) - 4\beta B_{12} - 4\left(\frac{\lambda p}{2V_0}\right)B_{23} - 4\left(\frac{\lambda q}{2V_0}\right)(C_{11} - C_{22}) - 8\left(\frac{\lambda r}{2V_0}\right)C_{12}
\]
\[
C_{k_i} = -4\bar{A}_{33} + 4\alpha B_{13} - 4\beta B_{23} - 8\left(\frac{\lambda q}{2V_0}\right)C_{13} - 8\left(\frac{\lambda r}{2V_0}\right)C_{23}
\]
\[
C_{k_q} = 4\bar{A}_{23}\left(\frac{X}{\lambda}\right)_b - 8\alpha C_{12} - 4\beta(C_{11} - C_{22}) - 8\left(\frac{\lambda p}{2V_0}\right)C_{13} + 16\left(\frac{\lambda q}{2V_0}\right)D_{12} - 8\left(\frac{\lambda r}{2V_0}\right)(D_{11} - D_{22})
\]
\[
C_{k_r} = -4\bar{A}_{13}\left(\frac{X}{\lambda}\right)_b + 4\alpha(C_{11} - C_{22}) - 8\beta C_{12} - 8\left(\frac{\lambda p}{2V_0}\right)C_{23}
\]

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The pitching and yawing moment coefficients are derived by taking the moment of the $C_z$ and $C_y$ distributions about the origin of the $X$, $Y$, and $Z$ axes which was taken at the centre of mass of the projectile. These aerodynamic moment coefficients are listed as follows:

$$C_{mx} = 4\left(\frac{\lambda V_0}{2V_0^2}\right)C_{22} + 2[B_{22} + \overline{A}_{22}\left(\frac{X}{\lambda}\right)_b] + 4\left(\frac{\lambda p}{2V_0}\right)C_{11}$$

$$C_{mxy} = 4\left(\frac{\lambda V_0}{2V_0^2}\right)C_{12} + 2[B_{12} + \overline{A}_{12}\left(\frac{X}{\lambda}\right)_b] + 4\left(\frac{\lambda p}{2V_0}\right)C_{11}$$

$$C_{my} = 4[B_{23} + \overline{A}_{23}\left(\frac{X}{\lambda}\right)_b] + 4\alpha C_{12} + 4\beta C_{11} + 16\left(\frac{\lambda p}{2V_0}\right)C_{13} - 8\left(\frac{\lambda q}{2V_0}\right)D_{12} + 8\left(\frac{\lambda r}{2V_0}\right)D_{11}$$

$$C_{mq} = -4\overline{A}_{22}\left(\frac{X}{\lambda}\right)_b^2 + C_{22} - 8\left(\frac{\lambda p}{2V_0}\right)D_{12}$$

$$C_{mr} = -4\overline{A}_{12}\left(\frac{X}{\lambda}\right)_b^2 + C_{12} - 8\left(\frac{\lambda p}{2V_0}\right)D_{11}$$

$$C_{nx} = 4\left(\frac{\lambda V_0}{2V_0^2}\right)C_{12} + 2[B_{12} + \overline{A}_{12}\left(\frac{X}{\lambda}\right)_b] + 4\left(\frac{\lambda p}{2V_0}\right)C_{22}$$

$$C_{nxy} = 4\left(\frac{\lambda V_0}{2V_0^2}\right)C_{11} + 2[B_{11} + \overline{A}_{11}\left(\frac{X}{\lambda}\right)_b] + 4\left(\frac{\lambda p}{2V_0}\right)C_{12}$$

$$C_{ny} = 4[B_{13} + \overline{A}_{13}\left(\frac{X}{\lambda}\right)_b] + 4\alpha C_{22} + 4\beta C_{12} + 16\left(\frac{\lambda p}{2V_0}\right)C_{23} - 8\left(\frac{\lambda q}{2V_0}\right)D_{12}$$

$$C_{nx} = 4\overline{A}_{12}\left(\frac{X}{\lambda}\right)_b^2 + C_{12} - 8\left(\frac{\lambda p}{2V_0}\right)D_{22}$$

$$C_{nr} = 4\overline{A}_{11}\left(\frac{X}{\lambda}\right)_b^2 + C_{11} - 8\left(\frac{\lambda p}{2V_0}\right)D_{12}$$

These equations provide the basis to calculating a projectile configurations aerodynamic coefficients in terms of the inertia coefficients which can be obtained from the apparent mass coefficients. As an example, the following equations provide the apparent mass coefficients required to calculate the aerodynamic coefficients for two common projectile configurations of a body alone and a body with the addition of cruciform fins.
Circular body of radius $a$ with no fins:

\[
\begin{align*}
  m_{11} &= \pi a^2 \\
  m_{12} &= 0 \\
  m_{13} &= 0 \\
  m_{22} &= \pi a^2 \\
  m_{23} &= 0 \\
  m_{33} &= 0
\end{align*}
\]

Cruciform wing, circular body:

\[
\begin{align*}
  m_{11} &= \pi a^2 \left(1 - \frac{a^2}{s^2} + \frac{a^4}{s^4}\right) \\
  m_{12} &= 0 \\
  m_{13} &= 0 \\
  m_{22} &= \pi a^2 \left(1 - \frac{a^2}{s^2} + \frac{a^4}{s^4}\right) \\
  m_{23} &= 0 \\
  m_{33} &= \frac{2\rho a^4}{\pi} \text{ if } a = 0
\end{align*}
\]

Where: $a =$ Body radius. $s =$ Body radius + fin span. $\rho =$ air density.

4.9 Simplified method of calculating apparent masses

Having established a method of determining the apparent mass coefficients for a particular projectile shape, the next stage was to calculate inertia coefficients. There are several methods for evaluating the inertia coefficients of a projectile. They could be evaluated directly by integrating Eqn (4). However, a more powerful method exists based on the theory of residues. This requires only a knowledge of the transformation that maps the projectiles cross-section conformally onto a circle of radius $a$, with no distortion at infinity. Using this method, the inertia coefficients can be determined without difficulty except when they require summing an intractable infinite series. This method has been used by a number of authors, including Ward, Bryson, Summers and Sacks. It is not the purpose of this section to go into the mathematical rigour of Bryson's method, but to provide some simple formulas that can be
used with a practical projectile configuration. The following expressions define the apparent-mass coefficients in terms of the transformation which turns the projectile cross-section into a circle of radius $a$. In Eqn (4), the inertia coefficients have already been defined in terms of the potentials $\phi_1, \phi_2, \phi_3$ for two translations and one rotation of a given projectile cross-section. The apparent-mass coefficients can be defined as:

$$m_{ij} = m_{ij} = -\rho \phi_1 \frac{\partial \phi_j}{\partial n}$$

$$i, j = 1, 2, 3$$

(19)

The apparent-mass coefficients are usually called the "additional" apparent mass coefficients since they induce on a body in a fluid a dynamic effect in addition to that due to the mass of the body alone. The apparent mass coefficients do not have dimensions of mass, but have dimensions that are readily apparent from their relationship to the truly non-dimensional inertia coefficients.

$$A_{11} = \frac{m_{11}}{\rho S_R}$$

$$A_{12} = A_{21} = \frac{m_{12}}{\rho S_R}$$

$$A_{22} = \frac{m_{22}}{\rho S_R}$$

$$A_{13} = A_{31} = \frac{m_{13}}{\rho \lambda S_R}$$

$$A_{23} = A_{32} = \frac{m_{23}}{\rho \lambda S_R}$$

$$A_{33} = \frac{m_{33}}{\rho \lambda^2 S_R}$$

Where: The quantities $\lambda$ and $S_R$ are the reference length (projectile body length) and area (projectile body cross-section).

Once the $A_{ij}$ inertia coefficients have been calculated the projectile's $B_{ij}$ and $C_{ij}$ inertia coefficients can be calculated by integrating along the length of the projectile using Eqns (16 - 17) with the value of the integral being determined by the projectile shape at that point. For example, for the body, the integral was simply taken as a circular body. When a fin was encountered, the integral was taken to be a body and cruciform fin combination. Each integral was then summed. This summation represents the total $B_{ij}$ and $C_{ij}$ inertia coefficient for the
total projectile configuration. Having obtained the $A_{ij}$, $B_{ij}$ and $C_{ij}$ inertia coefficients, the projectile's aerodynamic coefficients could be determined. A worked example using this method is provided in Chapter Five.

4.10 Summary of current aerodynamic methods

It was concluded that even though the Loftsman CFD code was capable of generating all the projectiles aerodynamic coefficient data, it required too much time to construct the initial projectile grid. Adaptations of the configuration also took a long time as the grid had to be altered for each projectile configuration change. Finally, the CFD code was not directly linked to a trajectory model which meant the aerodynamic data had to be ported into a trajectory model which again took an inordinate amount time.

Missile DATCOM was found to be unable of modelling all the projectile shapes that were being investigated. A subset of DATCOM algorithms was investigated but abandoned due to the technical complexity of creating the look-up-tables and linked algorithms. Coupled with the DATCOM code not having an associated trajectory model, further analysis of the missile DATCOM aerodynamic code was abandoned. However, DATCOM subsonic drag data proved to be very valuable in validating a projectile drag method developed in Chapter Five.

The method of masses is a direct numerical method that appeared theoretically capable of generating the required aerodynamic coefficients needed to generate a projectile's 6 DoF trajectory. It was also discovered that if the algorithms described in previous sections were used to generate a projectiles aerodynamic coefficients, the generated aerodynamic coefficient data was found to be not of the accuracy required. Chapter Five details with examples the modifications that were made to the method of masses to create the required aerodynamic coefficient data accuracy.
5.0 MODIFICATIONS TO THE METHOD OF APPARENT MASSES

5.1 Introduction

The method of masses was selected as the method to determine the aerodynamic coefficients of the projectile configurations under investigation because it is a direct mathematical method. This means that the aerodynamic coefficients can be obtained using algorithms rather than look-up-tables. However, it was discovered that the aerodynamic coefficients generated by the method of masses were not accurate enough to be used directly to determine a particular projectile configurations trajectory. In order to generate practical aerodynamic coefficients, modifications to the method of masses algorithms had to be made. This chapter details the modifications to the method of masses that were carried out so that accurate aerodynamic coefficient data and the subsequent projectile trajectory profiles could be created.

Even though the method of masses is a direct numerical method, the accuracy of the aerodynamic coefficient data generated was the key factor. It was therefore decided that if the method of masses was worth further development, the calculated aerodynamic coefficient values had to be within a certain accuracy band when compared to experimental and other prediction code data. The accuracy criteria used was based upon the accuracy criteria of the aerodynamic prediction code Missile DATCOM\textsuperscript{1}. However, it was recognised that the velocity regime and range of projectile configurations being investigated were much more restricted than those used by DATCOM. Therefore, it was decided that the results generated by the method of masses for the projectile configurations detailed in Chapter One, should be a lot more accurate than the DATCOM design criteria. It was also found that the DATCOM data was more accurate than the figures provided in Table 1. In particular, this was found to be the case for the subsonic drag data. The reason for this appears to be due to a lot of effort having been put into the subsonic drag algorithms due to a particular design requirement. The table of accuracy criteria that was decided upon for the method of masses and with DATCOM comparisons is shown in Table 2.

\textsuperscript{1} For ease of reading Missile DATCOM is shortened to DATCOM.
Table 2. Static longitudinal characteristics and calculation tolerance errors for the method of masses and DATCOM

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>DATCOM allowed error</th>
<th>Method of Masses allowed error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_N$</td>
<td>± 20%</td>
<td>± 12%</td>
</tr>
<tr>
<td>$C_m$</td>
<td>± 20%</td>
<td>± 12%</td>
</tr>
<tr>
<td>$C_{D0}$</td>
<td>± 10%</td>
<td>Method of masses not capable of determining this coefficient. Alternate method to be determined</td>
</tr>
</tbody>
</table>

5.2 Limitations in the method of masses

As detailed in Chapter Four, the method of masses is based upon the concept that projectile shapes can be mathematically transformed to represent a circle. If this transformed circle is then moved through an inviscid fluid medium it will displace a mass of fluid that is proportional to the diameter of the transformed circle. By determining the kinetic energy that is required to move the fluid, a means of obtaining the forces and moments associated with a particular projectile shape is made available. The forces and moments are calculated by differentiating the kinetic energy of the inviscid fluid per unit length. However, the resultant equations used to calculate the aerodynamic coefficients do not take into account the following major factors:

   a. Interference effects between the body/fin and fin/body.
   b. Non-linear effects at increased angles of attack.
   c. Coefficient variation with velocity.

To take into account these factors and other influences that are specific to particular aerodynamic coefficients, the fundamental equations developed from the method of masses were modified. These modifications are described in the following sections.
5.3 Normal lift coefficient ($C_{Na}$)

The fundamental equation derived from the method of masses to determine the normal lift coefficient for a rectangular cruciform fin, circular body projectile is as follows:

$$C_{Na} = -4 \left( \frac{\lambda V_0}{2V_0^2} \right) B_{22} - 2 \left( \frac{\pi s^2}{S_R} \left( 1 - \frac{a^2}{s^2} + \frac{a^4}{s^4} \right) \right) - 4 \left( \frac{\lambda p}{2V_0} \right) B_{12}$$  \hspace{1cm} (1)

Where: $\lambda = \text{The reference length (Body diameter)}$. $V_0 = \text{Initial velocity}$. $a_0 = \text{Initial acceleration}$. $B_{22}$, $B_{12} =$ Integrals of the inertial $A_{22}$ coefficient along the projectile body length. $A_{22} = \text{Inertial coefficient (Apparent mass)} = \left( \frac{\pi s^2}{S_R} \left( 1 - \frac{a^2}{s^2} + \frac{a^4}{s^4} \right) \right)$. $p = \text{Spin rate}$. $a = \text{body radius}$. $s = \text{body radius + fin span}$. $S_R = \text{reference area (Body cross-sectional area)}$.

On initial inspection of Eqn (1) it would appear that a velocity component is used to determine the normal lift coefficient. However, the ratio of the projectile acceleration to the velocity squared was calculated for typical projectile flight profiles and found to be an insignificant value in the order of 0.000001. Therefore the acceleration and velocity components have no significant effect on the final calculated coefficient value. Also, there is no factor that takes into account different fin shapes. The value of Eqn (1) is that it provides a method of determining values of normal lift coefficient by considering different fin spans and body diameters. It provides a numerical value for the normal lift coefficient to which other factors such as interference effects can be added. It was found that the other factors such as interference effects that were added to Eqn (1) were of much smaller value. Therefore, the numerical value generated by Eqn (1) was always the dominant factor. This was found to be true of all the coefficients generated using the method of masses. Hence, the method of masses was considered to be very good in providing a rough order of magnitude as to the value of the aerodynamic coefficient. However, this statement does not always hold true for angles of attack in excess of approximately ten degrees because of increased interference and crossflow effects. These effects will be discussed in later sections of this chapter.
5.4 Interference effects

When a stabilising fin unit is added to a projectile body, there is an increase in the total projectile lift due to changes in the airflow around the projectile configuration. This means that the total projectile lift is greater than the sum of the individual fin and body lift values. By definition, the extra lift is created by interference effects between the body and fin. The change in airflow due to interference is not taken into account in the method of masses due to the mathematical transform changing the configuration shape to a circle with the assumption that there will be smooth airflow around the circle. The increase in lift with an actual projectile is created through an increase in the fin lift due to the influence of the body ($K_w$) and an increase in the body lift due to the influence of the fin ($K_b$). Interference between the fin/body and body/fin have been experimentally measured and well documented. Examples of this experimental data can be found at Ref [46]. Depending on a projectile's configuration and for large angles of attack in excess of fifteen degrees, interference effects can increase the value of the normal lift force by up to 30%. Therefore, interference effects have to be taken into consideration and added to the value of normal lift coefficient provided by Eqn (1).

It was considered that the best method of determining interference effects would be to determine the lift due to the fin alone and using this value determine the interference effects due to the fin on the body and the body on the fin. By considering the lift due to the fin alone also takes into account the fin shape that is also not considered in Eqn (1). Using the method of masses, the normal lift coefficient for a fin alone can be calculated using the following expression:

$$C_{Na} = \frac{\pi AR}{2}$$

Where $AR = \text{The fin aspect ratio} = \text{Span}^2/\text{Reference area of the fin (Taken to be the area of the fin)}$.

In Eqn (2), the fin span is taken to be the maximum value (For a swept fin). Therefore, using Eqn (2) variations in lift with fin planform can be taken into consideration. The accuracy of
using Eqn (2) to calculate the lift of different fin shapes at subsonic speeds was found to have a good correlation with experimental data. However, it was noted that with large aspect ratio (values in excess of 4) fins, Eqn (2) became unreliable due to the departure from elliptical lift distribution, Refs [47,48]. As the projectile fins under consideration have small aspect ratio fins in the order of 1.5, this factor was ignored. Having established a means of determining the normal lift coefficient due to the fin alone, the next stage was to calculate the interference effects created by the addition of the fin to the body. Using Eqn (2), the total lift force generated by four cruciform fins can be expressed as:

$$L_w = 2\pi a s q_0 (s-a)^2$$  \hspace{1cm} (3)

Where:  
- $L_w =$ Total fin lift force. 
- $q_0 =$ Free-stream dynamic pressure ($0.5\rho v^2 S_r$). 
- $\alpha =$ angle of attack. 
- $s =$ Maximum fin span + body radius. 
- $a =$ body radius.

If it is considered that the body is a circular cylinder, the increased lift on the fin due to the body can be considered to be:

$$K_w L_w = L_w(B)$$ \hspace{1cm} (4)

Where $K_w =$ Ratio of lift of fin panels in the presence of the body to lift of fin alone. $L_w(B) =$ Total lift of fin due to the body.

Neilson [2] derived a numerical expression for $K_w$ based upon empirical data. This expression is expressed as follows:

$$K_w = \frac{L_w(B)}{L_w} = \frac{1}{\pi \alpha \lambda^2 (\alpha^2 - 1)} \left[ \frac{\pi (\lambda^2 - 1)^2 + (\lambda^2 + 1)^2}{\lambda^2 + 1} \sin^{-1} \left( \frac{\lambda^2 - 1}{\lambda^2 + 1} \right) \right]$$ \hspace{1cm} (5)

Where:  
- $\lambda = \frac{s}{a}$, $s =$ Body radius + fin span, $a =$ Body radius

The lift ratio can be seen to be a function of (Fin span + body radius) / (Body radius), $(s / a)$. It was because of this relationship that Eqn (5) was chosen in the interference calculations as
Eqn (1) is also dependent on the ratio of \((s/a)\). Having the same relationship reduced the number of variables and potential errors in the calculation of the coefficient.

The increased lift on the body due to the fin, \(K_B\) was found using a similar theory. Hence \(K_B\) can be defined as follows:

\[
K_B \times L_w = L_{B(W)} \tag{6}
\]

There was another factor that has to be taken into consideration when dealing with the lift due to the body and that was the effect of the projectile nose. Eqn (1) does not consider nose effects so to combine these with the body interference solved two problems in one. It was assumed that the lift due to the nose was a function of body radius. The normal lift force due to the body alone as derived from the method of masses can be expressed as:

\[
L_B = q_0 \cdot 2 \pi \alpha a^2 \tag{7}
\]

Where: \(L_B\) = Lift due to circular body. \(a = \) body radius (Radius decreases to take into account a rounded or pointed nose). \(\alpha = \) angle of attack. \(q_0 = \) Free-stream dynamic pressure \((0.5 \rho v^2 S_p)\).

As the nose of the projectile is simply a change in body radius \((a)\), the effect of the nose on the normal lift force could be calculated by summing the integral values of different nose radii over the length of the nose section to obtain the increase in normal lift coefficient due to the projectile nose. As will be shown in later sections this method produced a good degree of correlation with empirical data and other prediction methods.

The lift on the body due to the fin can therefore be evaluated since:

\[
L_{B(W)} + L_{W(B)} + L_N = L_{\text{Combination}} \tag{8}
\]

Where: \(L_{B(W)} = \) Increased lift on the body due to the fin. \(L_{W(B)} = \) Increased lift on the fin due to the body. \(L_N = \) Lift due to the nose.

Neilson [2] also derived the value of \(K_B\) from empirical data to be:

\[5 - 73\]
\[ K_B = \left(1 + \frac{a}{s}\right)^2 - K_w \]  

(9)

Where: \( s = \) Body radius + fin span, \( a = \) Body radius.

As can be seen from Eqns (5) and (9), \( K_B \) and \( K_w \) are both functions of \((a/s)\) only. A plot of \( K_B \) and \( K_w \) against \((a/s)\) is provided at Fig 5.1. From this graph it can be seen that for \((a/s = 0)\) the value of \( K_w \) is unity due to the way it has been defined, and \( K_B \) is zero because there is no body.

Having developed a method of calculating the fin/body and body/fin interference effects, the next stage was to use these values to determine the normal lift stability coefficient for a complete projectile configuration. The new derived expression for determining the normal lift stability coefficient for a cruciform projectile configuration was as follows:

\[ C_{N_{\text{Combination}}} = C_{N_{\text{Method of Masses}}} + (K_B + K_w)C_{N_{\text{Wing}}} + C_{N_{\text{Nose}}} \]  

(10)

5.5 Pitching moment coefficient \( (C_{m_{\alpha}}) \)

The pitching moment stability coefficient \( C_{m_{\alpha}} \) was derived from the method of masses as follows:

\[ C_{m_{\alpha}} = 2[B_{1,1} + A_{1,1}\left(\frac{X}{\lambda}\right)_{\text{base}}] \]  

(11)

Where: \( B_{1,1} = \) X integral of \( A_{1,1} \) coefficient \( X \). \( X = \) Distance from projectile base to centre of mass.

\( \lambda = \) Reference length. \( A_{1,1} = \) Inertia coefficient measured from projectile base.
The pitching moment stability coefficient $C_{ma}$ is extremely important in the design of a free-flight projectile. For a finned projectile the configuration should be statically stable unless spin stabilisation is incorporated. The sign of the pitching moment should indicate whether the configuration under investigation is either statically stable or unstable. The convention used in this thesis is a negative value of pitching moment indicates static stability and a positive value indicates static instability. Having this indicator immediately showed whether a particular configuration was worthy of further investigation or if the stabilising parameters have to be altered (e.g. Increase size of fins or decrease length of nose). On inspection of Eqn (11) it can be seen that if the centre of mass is kept constant the factor that will alter the sign and size of $C_{ma}$ is the value of $B_{t,t}$ as the value of $A_{1,1}$ is determined by the ratio of the fin span to body radius only. Therefore the value of $B_{1,1}$ had to vary in relationship to fin shape, nose shape and changes in body diameter. $B_{1,1}$ is defined as follows:

$$B_{1,1} = \int_{(X/\lambda)_{base}}^{(X/\lambda)_{nose}} A_{1,1} d\left(\frac{X}{\lambda}\right)$$  \hspace{1cm} (12)

Where: $X$ is the distance from the projectile base to the projectile nose.

Note: $X$ is divided by the reference length $\lambda$ to create a non-dimensional value of length.

Investigating Eqn (12) further and with reference to Fig 5.2 which shows a slender cruciform fin. $A_{1,1}$ for a slender fin can be expressed as:

$$A_{1,1} = \frac{\pi s_m^2}{s_r} \left(\frac{X_n - X}{c}\right)^2$$  \hspace{1cm} (13)

Where: $s_m =$ Maximum fin span. $s_r =$ Reference area. $c =$ Chord length. $X_n =$ The distance from the fin apex to the reference point (Reference point can be taken as the half chord position for a fin in isolation).

When the fin is attached to a body, the reference position is taken as the projectile centre of mass. For a fin with a leading or trailing edge sweep, the fin span varies along the $X$ direction (The chord length). To determine how the $B_{1,1}$ value varies with fin sweep, a relationship can
simply be established from the fin geometry shown in Fig 5.2. Adding this fin to a circular body of radius \( a \) provides the following expression for \( B_{1,1} \):

\[
B_{1,1} = \int_{(X/nose)}^{(X/nose)} \frac{\pi}{Sr} \left[ s^2 \left( \frac{Xn - X}{c} \right)^2 - a^2 + \frac{a^4}{s^2 (Xn - X/c)^2} \right] \frac{1}{\lambda} \, dX
\]

Where: \( s = s^2(Xn - X/c)^2 \) and \( A_{1,1} \) is expressed as \( \frac{\pi s^2}{Sr} \left( 1 - \frac{a^2}{s^2} + \frac{a^4}{s^4} \right) \)

As can be seen from Eqn (14) by integrating along the projectile body from the base to the nose a value of \( B_{1,1} \) can be obtained. The value of \( B_{1,1} \) is dependent on the configuration shape. For example, if a fin is encountered a large negative value of \( B_{1,1} \) is generated. This is due to the fin being behind the centre of mass (-ve value) and the integral being large due to the fin and body combination of Eqn (14). A body alone simply consists of a radius squared integral value which is much smaller. Therefore, using Eqn (11), a means of determining the variation in pitching moment with projectile configuration had been established. To show how the value of \( B_{1,1} \) varies with configuration shape, an illustrative example is provided. Using the simple projectile configuration shown in Fig 5.3 the following \( B_{1,1} \) integrals are obtained:

\[
B_1 = \int_{-0.3}^{-0.29} 0.05^2 \frac{1}{\lambda} \, dX \quad \text{Circular body.}
\]

\[
B_2 = \int_{-21}^{-29} 0.11^2 - 0.05^2 + \frac{0.05^4}{0.11^2} \frac{1}{\lambda} \, dX \quad \text{Rectangular fins.}
\]

\[
B_3 = \int_{-21}^{0} 0.05^2 \frac{1}{\lambda} \, dX \quad \text{Circular body}
\]

\[
B_4 = \int_{0}^{0.3} 0.05^2 \frac{1}{\lambda} \, dX \quad \text{Circular body}
\]

\[
B_{1,1} \ (\text{Total}) = \frac{\pi}{S_{ref}} (B_1 + B_2 + B_3 + B_4) = 5.58 \quad (15)
\]
additional nose section which was added to the flat face of the projectile is detailed in Fig 5.3a

The leading parameters for the projectile configuration are as follows:

**Body:** Length 0.6m, Diameter = 0.05m, No nose shaping - blunt face.

Stabilising Fins: Chord = 0.08m, Span = 0.06m, Leading edge of fin = 0.51m from nose.

4 Cruciform fins. No leading or trailing edge sweep.

**Flight Conditions:** Velocity = 150 m/s, Sea level standard temperature and pressure, Centre of Mass = 0.3m from the nose.

**Additional nose:** The nose of Radius of curvature / Body diameter (rc/d) = 0.2 was added to the front of the basic projectile configuration. This has the effect of increasing the projectile length by 0.03m.

### 5.6.1 Normal lift stability coefficient for blunt face projectile.

From the dimensions in Fig 5.3, we have:

Body radius / (Body radius + fin span) = \( \frac{a}{s} = \frac{0.05}{0.11} = 0.4545 \)

Using Eqns (5) and (9) for \( K_B \) and \( K_N \), we have:

\( K_B = 0.712 \) and \( K_N = 1.404 \)

Using Eqn (2) for the normal lift coefficient due to the fin alone, we have:

\( C_{N_x(\text{Fin})} = 0.257 \) per radian

Using Eqn (1) for the normal lift coefficient for body and fin without interference, we have:

\( C_{N_x(Body+Fin)} = 8.097 \) per radian

Using Eqn (10) for the normal lift coefficient including interference effects, we have:
$C_{N_a \text{ Total}} = 8.354 \text{ per radian}$

### 5.6.2 Normal lift coefficient with rounded nose

With this configuration, the fins and body shape remain the same as the previous example except for the rounded nose. It was found that for this nose rounded shape, the normal lift coefficient increased by only 0.17 per radian. This normal lift coefficient value was calculated by determining the value of $A_{N_a}$ for three radii varying from 0.04, 0.03 and 0.02 m. To represent the rounded nose shape. These values were then added together to represent the increase in normal lift coefficient for a rounded nose. This small increase in normal lift coefficient corresponds to empirical data published on rounded nose projectiles at Ref [50]. If a conical nose had been placed on the front of the body, the same procedure as the rounded nose could be used to calculate the increase in normal lift coefficient. It was found that a larger increase in the normal lift coefficient would be obtained with a conical nose (A conical nose of length 3 calibres increase $C_{N_a}$ by 0.9 per radian). This larger increase in coefficient value was due to the longer length of the conical nose. The increase in normal lift coefficient with a conical nose also gave a good correlation with empirical data.

### 5.6.3 Pitching moment stability coefficient for blunt face

The interference values derived in the previous normal lift coefficient example are used in this calculation.

Using Eqn (11) for the pitching moment coefficient without interference effects, we have:

$$C_{m_x (\text{Body + Fin Method of masses})} = -13.129 \text{ per radian}$$

Using Eqn (17) for pitching moment effects with interference, we have:

$$C_{m_x (\text{Body + Fin + Interference})} = -13.644 \text{ per radian}$$
5.6.4 Pitching moment stability coefficient for rounded nose

With the addition of a rounded nose to the configuration, the centre of lift was found to move forwards. To determine the forward movement of the centre of lift, the $B_{1,1}$ value in Eqn (11) was increased due to the change in nose radius. This had the effect of decreasing the pitching moment and therefore moving the centre of lift forward. The following observations concerning the $B_{1,1}$ values were made from this analysis:

a. $B_{1,1}$ value for projectile with a flat face from calculation using Eqn (15) = 5.58.
b. $B_{1,1}$ value for the rounded nose from calculation using Eqn (16) = 3.509. Summing the values from Eqns (15) and (16) = 9.089. Assuming the normal lift coefficient change was insignificant with the rounded nose, the new value of pitching moment coefficient from Eqn (17) = -10.31 per radian.

5.7 Centre of pressure

The centre of pressure was calculated by dividing the pitching moment coefficient by the normal lift coefficient. For the projectile configuration with a flat face the centre of pressure was calculated as follows:

Pitching moment coefficient / Normal lift coefficient = -13.644 / 8.345 = -1.633 Calibres behind the centre of mass. This meant that the centre of pressure was 0.4633 m from the nose. This distance indicated that the configuration was statically stable which was indicated by the negative value of pitching moment.

The centre of pressure for the rounded nose configuration was calculated as follows:

Pitching moment coefficient / Normal lift coefficient = -10.31 / 8.345 = -1.235 Calibres behind the centre of mass. This meant that the centre of lift was 0.4234 m from the nose. With manufacturing tolerances of ± 1 calibre this configuration would just be deemed suitable for further investigation. This result showed that the centre of mass moved forward by 0.04m with a $(rc/d = 0.2)$ nose added to the front of the projectile.
As has been demonstrated in the previous calculated examples, the method of masses with the addition of interference effects was capable of generating values of normal lift coefficient, pitching moment coefficient and centre of mass for a complete projectile configuration.

Table 3. summarises the calculated results. The coefficient values are also given in degrees. The reason for this is that most trajectory calculations are carried out in degrees as opposed to radian measure. As it was considered that up to six degrees angle of attack the coefficients would increase linearly, the coefficient variation with angle of attack has not been included.

<table>
<thead>
<tr>
<th>Flat Face Projectile</th>
<th>Rounded Nose projectile</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{Na}$</td>
<td>$C_{ma}$</td>
</tr>
<tr>
<td>8.354/rad</td>
<td>-13.644/rad</td>
</tr>
<tr>
<td>0.145/deg</td>
<td>-0.238/deg</td>
</tr>
</tbody>
</table>

Table 3. Normal lift coefficient, pitching moment coefficient and centre of pressure for flat face and rounded nose projectiles.

5.8 Multiple number of fins

To determine the normal lift coefficient and pitching moment coefficient, for multiple finned projectiles, Eqns (19) and (20) were derived using the method of masses. For four fins these expressions revert back to Eqns (1) and (11). It was found that there was a good degree of correlation with the data generated by Eqns (19) and (20), with prediction methods and empirical data. These equations were used for projectile configurations where fin numbers other than four were used. Therefore, to obtain the total normal lift and pitching moment coefficients, the interference effects calculated at Eqns (5) and (9) were added to the values provided by Eqns (19) and (20).
\[ C_{na} = \frac{4\pi s^2}{Sr} \left\{ \frac{1 + \left( \frac{a^2}{s^2} \right)^2}{2} \right\}^s - \frac{1}{2} \left( \frac{a}{s} \right)^2 \]  

(19)

\[ C_{na} = 2[B_{2,2} + \frac{2\pi s^2}{Sr} \left\{ \frac{1 + \left( \frac{a^2}{s^2} \right)^2}{2} \right\}^s - \frac{1}{2} \left( \frac{a}{s} \right)^2 \left( \frac{X}{\lambda_{base}} \right)] \]  

(20)

Where: \( n \) = Number of fins, \( a \) = Body diameter, \( s \) = Body radius + fin span, \( Sr \) = Reference area, \( \lambda \) = The reference length.

5.9 Crossflow effects

At small angles of attack, the shape of the projectile configuration only will effect the normal lift and pitching moment stability coefficients. As the projectile angle of attack increases, the configuration is exposed to wind and a cross-flow drag component had to be added to the normal force. Allen and Perkins [11] derived an expression to determined the crossflow drag component. Further details on crossflow effects are provided in Chapter Two. This expression is as follows:

\[ \delta C_{na} = \mu C_{De} \frac{A_p}{S_{ref}} \sin^2 \alpha \]  

(18)

Where: \( \delta C_{na} \) = The increase in normal lift force due to crossflow drag, \( \mu \) = Drag proportionality factor, \( A_p \) = Area of body as seen in crossflow when \( \alpha = 90^\circ \).

In examination of Eqn (18) and using experimental data from Ref [51] it was found that the onset of viscous crossflow effects was determined by the length of the projectile body. Knowing this fact is very important to the designer as the crossflow effect can be ignored if the angle of attack is kept below a certain value for a particular length of body. A graph showing the relationship between viscous crossflow and body length for subsonic speeds is shown in Fig 5.4.
Using the data from the graph it can be seen that for a body length of six calibres, crossflow effects can be ignored up to an angle of attack of approximately $10^\circ$.

### 5.10 Velocity effects

As explained in earlier sections, the method of masses does not take into account variations of coefficient with velocity. Therefore, a relationship between projectile velocity and aerodynamic coefficients had to be established. As defined in Chapter One, the velocity regime in which the projectiles being studied were to fly was subsonic. Subsonic experimental projectile data from Ref [52] was analysed to see if a relationship between velocity and coefficients values could be established. The velocity regime that was investigated was between $M_{0.1}$ and $M_{0.8}$. It was found that for the velocity regime being investigated and for the type of projectile configuration being used there was only a very small variation in the normal lift coefficient, $C_{Na}$ ($\pm 0.1 / \text{rad}$) and the pitching moment coefficient ($\pm 0.21 / \text{rad}$) $C_{ma}$ for changes in projectile velocity between $M_{0.1}$ and $M_{0.8}$. These small variations in aerodynamic coefficient were used in the 6 Degree of Freedom (DoF) trajectory analysis program and found not to have any significant effect on the trajectory performance. Therefore, for this analysis it was concluded that the normal lift coefficient and pitching moment coefficient remained constant over the speed regime $M_{0.1}$ to $M_{0.8}$.

### 5.11 Comparison between the method of masses, commercial prediction codes and empirical data

To establish the accuracy of the methods described previously to calculate the normal lift coefficient and pitching moment coefficient a comparison had to be made against established prediction methods and, where available, empirical data. Having this comparison meant the limits of the method could be determined. To carry out this comparison, a set of basic parameters had to be established. This was considered essential as there are no definitive rules in determining what reference lengths, areas or what angular measurement is used in the calculation of aerodynamic coefficients. It was often found in analysing experimental data that a result that looked completely at odds with other data sets was in fact correct as the
originator of the data had used a fin reference area instead of the more commonly used body cross-sectional and not made mention of it. The common set of parameters against which all comparisons were made was defined as follows:

a. Subsonic speeds M0.1 - M0.8
b. Angles of attack up to 6 degrees.
c. Sea-level standard temperature and pressure.
d. All coefficients to be measured in radians.
e. Body structure to consist of a smooth finish such as polished metal or paint.
f. All projectile configurations to be slender.
g. Reference length and reference area to be the maximum body diameter and maximum body cross-section.

Having established the fundamental flight parameters and standards, the projectile configurations that would provide the maximum validation data had to be determined. The projectile configurations that were chosen for the validation were as follows:

a. Arbitrary number of fins in fin set. This would determine whether the interference algorithm for multiple fins was accurate.
b. Nose shapes. This would determine the accuracy of the method for determining different nose shapes.
c. Fin sets with leading and trailing edge sweeps. This would determine the accuracy of modelling different fin shapes.
d. Curved fins. This would determine the accuracy of method for determining coefficients for curved fin shapes.

If a good correlation between the method of masses, empirical data and established prediction codes could be established for the configurations listed above, the method of masses could be considered viable to use in conceptual designs to calculate a projectile configurations normal lift and pitching moment coefficients.
5.12 Results of validation procedure

The two aerodynamic prediction codes that were used in this validation procedure were DATCOM and RAPPIC. (RAPPIC was used to determine the aerodynamic coefficients for rounded nose projectiles and spin damping coefficient data) Further details of these codes are provided in Chapter Four. The empirical data that was used in validation was obtained from Refs [48-55]. A selection of the projectile configurations that were used for the experimental validation is provided at Appendix B. The following sections detail the observations from this validation.

5.12.1 Arbitrary number of fins in fin set

There was a good degree of correlation between the aerodynamic prediction codes for fin numbers ranging from three to eight. No noticeable difference was found in coefficient values for different subsonic velocities in the range M0.1 - M0.8. However, it was noted that the method of masses consistently provided lower values than the prediction codes. The maximum difference between the methods was 8% in $C_{Na}$ and 10% in $C_{ma}$. This maximum difference was observed with fin sets of six. With a large number of fins (Ten) very inaccurate data was recorded. This variation with a large number of fins was determined to be due to the difference in interference effects used by the prediction codes for a large number of fins. The method of masses used the same interference effect for all multiple fin numbers. This variation was not deemed serious as the projectiles under investigation would be limited to six fins. It was also observed that DATCOM values were consistently higher than RAPPIC data values. This was considered to be due to the difference in calculating interference effects. As access to the RAPPIC code was not available this statement can only remain as speculative. Empirical data was found to be in good agreement with the method of masses calculated data. Very little empirical data was found for a large number of fins (in excess of eight) due to the manufacturing difficulties involved and the diminishing trajectory benefits to be gained from a large number of fins. For what empirical data that was available a good degree of correlation was found with the method of masses technique (Differences of up to 6% lower for $C_{Na}$ and 8% lower for $C_{ma}$ were found for four and six fins). With large aspect ratio fins (Greater than 4) the data generated by the method of masses was not accurate with differences in values.
compared to empirical and prediction data of over 90% being observed. The following are the conclusions from this particular validation. Other data sources derived from the prediction codes and empirical data:

a. Fin numbers up to six. Good agreement with other data sources. (Maximum differences, 6% lower for $C_{Na}$ and 8% lower for $C_{ma}$ - empirical data).
b. Large aspect ratios above 4 are not to be used. Very inaccurate data.
c. Body length/diameter must be greater than 4 calibres or large errors will occur due to nose bluntness effects.

5.12.2 Nose shapes

DATCOM cannot model rounded noses, therefore the rounded noses were compared to RAPPIC and empirical data from Ref [50]. No noticeable difference was found in coefficient values for different subsonic velocities in the range M0.1 - M0.8. It was found that there was a good degree of correlation between the method of masses and the other data sources for rounded noses. The pitching moment was found to vary by a maximum of 8% for nose rounding ratios of $(rc / d) 0 - 0.5$ (Integral limit taken to be 0.01m). The normal lift coefficient was found to vary by 5% lower. It was found that the method of masses calculated coefficient values were always lower than the empirical data. The lower value for $C_{ma}$ was considered to be due to the course value of the integral limits. A consistently lower value of $C_{ma}$ was considered not to be a disadvantage as it provided a margin of error in favour of the designer (a lower value of $C_{ma}$ gives a smaller static margin). There was a good degree of correlation between the method of masses and other data sources for conical noses. Again it was found that the method of masses consistently gave lower values, with the maximum difference being 7% for $C_{Na}$ and 11% for $C_{ma}$ for nose lengths = 4 calibres. There was a larger difference in calculated conical nose values as compared to the rounded nose case. This larger difference was considered to be due to the coarse value being used in the integral limits for $C_{ma}$ as a longer conical nose gave a greater error. However, it was considered not worth changing the integral distance as conical noses in excess of 3 calibres would not to be used in the projectile designs as a long nose would add too much length to the body. The following are the
conclusions from this particular validation. Other data sources derived from the prediction codes and empirical data:

a. The method of masses provided differences of 5% lower for $C_{Na}$ and 8% lower for $C_{ma}$ for rounded noses of ratios $(r_c/d)$ 0-0.5.
b. Large data errors were observed with conical noses in excess of 4 calibres. For conical noses of length 3 calibres, differences of 7% for $C_{Na}$ and 11% for $C_{ma}$ are to be expected.

5.13 Fins with leading and trailing edges

There was a good degree of correlation between the method of masses, prediction codes and empirical data for coefficient values detailing fins with leading and trailing edges. No noticeable difference was found in coefficient values for leading and trailing edge sweep values for subsonic velocities in the range $M0.1 - M0.8$. If the fin sweep was kept below 45 degrees the maximum difference in values was found to be 8% lower for $C_{Na}$, and 12% lower for $C_{ma}$ for method of masses data and other sources. It was not surprising to observe the larger error in the $C_{ma}$ value as the method of masses calculation involved several integrals with coarse integral limits. A more accurate result would be obtained if smaller integral limits were used. However, for the accuracy of the data required, a lower value of $C_{ma}$ was deemed sufficient for conceptual design purposes. Leading and trailing edge sweep angles of greater than 45 degrees gave very inaccurate results of up to 90% lower than other data sources. This inaccuracy appeared due to the interference equations being in error at large sweep angles and the numerical integrals not having small enough limits. Since fin sweeps in excess of 30 degrees are not common, these factors were not considered significant. The following are the conclusions from this particular validation. Other data sources derived from the prediction codes and empirical data:

a. The method of masses method was valid in determining $C_{Na}$ and $C_{ma}$ up to fin leading and trailing edge sweep angles of 45 degrees.
b. Maximum differences of up to 8% for $C_{Na}$, and 12% for $C_{ma}$ are to be expected between prediction and empirical data for fin leading and trailing edge sweep angles up
to 45 degrees. These differences are always consistently lower than prediction codes and empirical data.

5.14 Body + Nose shapes + Multiple curved fins.

DATCOM is declared not to be capable of modelling curved fins. RAPPIC data was not available for curved fins. Therefore, the majority of the data for the validation came from experimental data at Ref [56]. Using this data no significant difference was found in normal lift coefficient or pitching moment coefficient values for different subsonic velocities in the range M0.1 - M0.8. It was found that a good degree of correlation could be obtained between the method of masses and experimental data if the curved fin was modelled as a flat fin. The method that was used was to measure the distance from a point at the maximum span with a line intersecting the body at 90 degrees. If this dimension was used as the span of a flat fin, the equivalent flat fin provided results that were a maximum of 7% lower than experimental data for $C_{Na}$ and a maximum of 10% lower for $C_{ma}$. It was interesting to note that the interference effects for an equivalent flat fin appear to provide a good relationship to a curved fin. For the limited amount of empirical data that was available the following observation concerning curved fins was made:

a. Curved fins can be modelled using the method of masses. To model the curved fin an equivalent flat fin has to be assumed. The flat fin equivalent provides a good degree of correlation with the curved fin. Maximum differences of 7% lower for $C_{Na}$ and 10% lower for $C_{ma}$ were noted.

5.15 Calculation of spin damping coefficient using the method of masses

The next aerodynamic coefficient to be examined was the spin damping coefficient. The following method describes how the method of masses can be used to calculate the spin damping coefficient $C_{p}$ of a projectile. The fundamental method of masses equation used to calculate the spin damping coefficient is given by:
\[ C_{10} = -4A_{33} + 4\alpha B_{13} - 4\beta C_{23} - 8\left(\frac{\lambda q}{2V_0}\right)C_{13} - 8\left(\frac{\lambda r}{2V_0}\right)C_{23} \]  \hspace{0.5cm} (21)

Where: \( q \) and \( r \) are the projectiles angular velocities, \( V_0 \) = Initial velocity, \( \lambda \) = Reference length, \( A_{33} \) is the inertial coefficient. \( B_{21} \) and \( C_{13} \) are the first and second integral of \( A_{33} \).

As all the projectiles considered in this section have horizontal and vertical planes of symmetry, the following inertia coefficients are zero: \( A_{12} = A_{13} = A_{23} = B_{23} = C_{13} = C_{21} = 0 \), therefore Eqn (21) reduces to:

\[ C_{10} = -4A_{33} \] \hspace{0.5cm} (22)

Eqn (22) is the fundamental method of masses equation for determining spin damping coefficient values. Different equations were derived from the basic Eqn (22) to determine the value of the inertial coefficient \( A_{33} \) for different projectile configurations (Different fin numbers and body diameter). This was done to see if a single equation could be derived that took into account different fin numbers and variations in body diameter. However, as can be seen, this was not achieved. All the following equations were found to be dependent upon either the number of fins or body radius:

\[ \overline{A}_{33} = \frac{2s^4}{\pi \lambda^2 Sr} \hspace{0.5cm} \text{For} \ a = 0 \hspace{0.5cm} \text{(Four fins)} \hspace{0.5cm} (23) \]

\[ \overline{A}_{33} = \frac{\pi s^4}{\lambda^2 Sr^8} \hspace{0.5cm} \text{For} \ a = 0 \hspace{0.5cm} \text{(Two fins)} \hspace{0.5cm} (24) \]

\[ \overline{A}_{33} = \frac{\pi s^4}{\lambda^2 Sr^8} \left\{ \left(1 + R^2\right)^2 \tan^{-1} \frac{1}{R} \right\} 2R(1 - R^2)(R^4 - 6R^2 + 1)\tan^{-1} \frac{1}{R} - \pi^2 R^4 + R^2 (1 - R^2)^2 \]

Where \( R = \frac{a}{s} \) \hspace{0.5cm} (Two fins including variations in body diameter) \hspace{0.5cm} (25)

\[ \overline{A}_{33} = \frac{\pi s^4}{2\lambda^2 Sr^2} \hspace{0.5cm} \text{For} \ a = 0 \hspace{0.5cm} n = \infty \hspace{0.5cm} (26) \]

\[ \overline{A} = \frac{0.533s^4}{\pi \lambda^2 Sr} \hspace{0.5cm} \text{For} \ a = 0 \hspace{0.5cm} n = 3 \hspace{0.5cm} (27) \]
Where: \( n \) = The number of fins, \( a \) = Body radius, \( \lambda \) = Reference length, \( s \) = Maximum fin span

As can be seen from Eqns(23) - (27), only Eqn (25) takes into account body radius. As this equation is for two fins it was not considered suitable to use as a basis to calculate the spin damping coefficient as the majority of the projectiles under investigation had four or more fins. It was therefore decided to use Eqn (23) as the basic equation from which a projectile spin damping coefficient could be calculated. As this was the fundamental equation used in the analysis it is repeated below with definitions:

\[
C_p = 4\left(\frac{2s^4}{\pi \lambda^2 Sr}\right) \quad \text{For } a = 0 \quad \text{(Four fins)} \quad (28)
\]

Where: \( C_p \) = Spin damping coefficient, \( s \) = Body radius + Maximum fin span, \( Sr \) = Reference area, \( \lambda \) = Reference length.

As can be seen from this fundamental Eqn (28), the following parameters are not accounted for:

- a. Variation in fin number and shape on the coefficient.
- b. Effect of body radius on the coefficient for a fixed fin span.
- c. Variation in coefficient with velocity.
- d. Variation in coefficient with angle of attack.
- e. The effect of curved fins on the coefficient.

Each of these parameters was analysed and their effect on the fundamental value of spin damping coefficient provided by Eqn (28) taken into account in the following sections.

5.15.1 Variation in spin damping with fin number and fin shape

On inspection of Eqn (28) it can be seen that \( C_p \) is directly proportional to the inertial coefficient \( A_{33} \). As the reference area and reference length remain constant no matter how many fins the projectile has, the following relationship could be deduced:
Where: \( n \) = Number of fins.

Numerical values for the ratio shown in Eqn (29) were calculated by using the different fin number values provided by Eqns (23) - (27). The results of this analysis which shows an increase in spin damping factor with fin number is shown in Table 2. This table of results also shows that the addition of fins to the projectile body adds to the damping in roll but at a decreasing rate.

\[
\frac{(C_y)_n}{(C_y)_2} = \frac{(\overline{A}_{33})_n}{(\overline{A}_{33})_2} \quad (29)
\]

Table 2. Variation in projectile spin damping coefficient with fin number.

<table>
<thead>
<tr>
<th>Spin damping factor</th>
<th>Number of fins</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1.7</td>
<td>4</td>
</tr>
<tr>
<td>2.27</td>
<td>6</td>
</tr>
<tr>
<td>2.44</td>
<td>8</td>
</tr>
<tr>
<td>2.66</td>
<td>10</td>
</tr>
</tbody>
</table>

The relationship between fin number and spin damping was compared to experimental data given in Refs [57-58]. It was found there was a good correlation between the experimental data and the spin damping factors shown in Table 2. However, it was observed that the accuracy of the data in Table 2 decreased as the fin numbers increased above six. With fin numbers of eight and above the difference in values was found to be up to 25 % below the experimental value. It was considered this large error was due to the interference effects between the fin not being correctly taken into consideration. For two, four and six fins the spin damping factor was found to be consistently lower than experimental values by around
9%. This value was considered to be within the limits required as it was also noted that experimental spin damping data is very difficult to measure accurately and therefore there could be an error as much as ±5% in the experimental data.

Spin damping variation with fin shape was difficult to take into account. The best correlation that could be found was to assume that the spin damping coefficient varied in a similar way to the lift coefficient. It was assumed that a rectangular fin was the standard. It was also assumed that if the lift increased due to a larger root chord then this would also increase the spin damping coefficient. Conversely, a leading or trailing edge would decrease the lift and the spin damping coefficient. It was found that there was a very good degree of correlation between the theory that spin damping coefficient reduced in a similar manner to the lift coefficient with variations in fin geometry and the experimental data at Ref [58]. In summary, to determine the change in spin damping coefficient with fin shape variation, the percentage variation in lift compared to a rectangular fin was multiplied by the fundamental value of $C_{lp}$ calculated from Eqn (28). Numerically this equates to the following expression:

$$C_{lp \, \text{Corrected}} = C_{lp} \frac{C_{No \, \text{Shaped fin}}}{C_{No \, \text{Standard rectangular fin}}}$$

(30)

5.15.2 Variation in spin damping with body radius for a fixed fin span

As in the analysis with different fin numbers, and in reference to Eqn (28). If the projectile fin span is kept constant, and the reference quantities remain constant, the body radius can be varied as follows:

$$\frac{C_{lp}}{(C_{lp})_{s=0}} = \frac{A_{33}}{(A_{33})_{s=0}}$$

(31)

To determine numerical values for the relationship shown in Eqn (31), experimental values of $C_{lp}$ for different values of body radius were obtained from the experimental work carried out
by Adams and Duggan [23]. Different values of $C_p$ for $a = 0$ were calculated using Eqns (23)-(27). It was found that the addition of a body with a value of Body radius / Fin span ($a / s$) up to a value of about 0.4 caused very little change in the damping in roll. Therefore, as the majority of the projectile configurations under investigation have fin spans greater than the body radius the variation in spin damping with body radius was considered not to be a significant factor.

5.15.2 Variation of spin damping with velocity

The method of masses Eqn (28) does not consider variations of the spin damping coefficient with velocity. On analysis of experimental data from Refs (51,53,54,55) it was found that in the subsonic velocity range and for the missile configurations detailed in Appendix B., the spin damping coefficient increases by 0.5 from $M0.1$ to $M0.9$. As the projectile moves into the transonic and supersonic velocity regimes, a discernible relationship between spin damping and velocity from flight trials data becomes very difficult to ascertain as it appeared that spin damping did not have a maximum value which coincided with a maximum velocity. The relationship therefore appeared to be very configuration dependent for velocities above $M1.0$. To calculate the spin damping coefficient value for a projectile, the speed regime in which it was flying had to be taken into account. A simple correlation to take into account the variation in spin damping with velocity was calculated from experimental data using standard correlation and regression techniques. The relationship between spin damping and velocity is given at Eqn (32).

$$C_{pMach} = 0.1 - 0.9 + (Mach\ number \times 0.5)$$  \hspace{1cm} (32)

5.16 Spin driving moment coefficient ($C_{18}$)

Having developed an expression for the spin damping coefficient the next stage was to develop an expression for the spin driving moment coefficient. It is this moment generated by the projectile fins that causes the projectile to rotate. It is interesting to note that several methods were found in published papers Refs [59-61] describing methods of how to determine the spin driving moment coefficient ($C_{18}$). It was found that none of the published methods
were direct, accurate or described the assumptions they used. Therefore, a new method employing the previously described normal lift coefficient including interference effects was developed. A cruciform fin set at an angle of attack but no deflection angle \( \delta \) will produce a normal force. Therefore, the fin normal lift coefficient including body-fin and fin-body interference effects was calculated by subtracting the body alone normal lift coefficient from the total configuration normal lift coefficient.

\[
C_{N\text{Single fin}} = (C_{N\text{ total configuration}} - C_{N\text{ Body alone}}) / 4 
\]  

(33)

The equation was divided by four to calculate the lift from one fin panel in a four fin panel combination (For other fin number combinations this value would alter accordingly). If the spinning case is now considered, with no angle of attack but with 4 fin panels canted by and angle \( \delta \), \( \delta = \alpha \), the spin driving moment coefficient \( C_{18} \) can be calculated as follows:

\[
C_{18} = \frac{(nC_{N\text{Spin}}y_c)}{\lambda} 
\]  

(34)

Where: \( C_{18} = \) The spin driving moment coefficient, \( \delta = \) The fin cant angle, \( n = \) The number of canted fins in the fin set, \( y_c = \) The moment arm of the fin panel normal force about the body axis, measured to the fin centre of area. \( \lambda = \) Reference length.

As was described in the calculation of \( C_{N\text{Max}} \), it was assumed that the angle of attack \( (\alpha) \) was kept to below six degrees. Therefore, the cant angle \( (\delta) \) must be also be kept to six degrees. This restriction will only be a limitation if a very high spin rate is required. The only uncertain value in Eqn (34) was the value of \( y_c \) which refers to the fin centre of area rather than the fin centre of pressure. As the fin semi-span was usually in the order of one body diameter, the difference between the fin centre of area and the fin centre of pressure was small at about 5%. The major advantage in using this method to determine the spin driving moment coefficient was that the normal lift coefficient had already been calculated including the interference effects. It is the interference effects that other published methods appear not to take into consideration properly or not consider at all. Comparison of spin driving moment coefficient
data generated using Eqn (34) showed a very good correlation with experimental data from Refs [54-56].

5.17 Calculated example

The following is an example of how to calculate the spin damping and spin driving moment coefficients for the basic projectile configuration shown in Fig 5.3 using the method of masses. To carry out this calculation, the fin configuration in Fig 5.3 had to be altered to create a fin spin driving force. The following changes were therefore made to the projectile configuration:

Fins: Four cruciform fins, each were canted at 4 degrees to generate a spin driving force.

5.17.1 Spin damping coefficient \( C_{ip} \)

To calculate spin damping coefficient for the projectile configuration shown at Fig 5.3, Eqn (28) was used to obtain the basic value of \( Crp \). This calculation provides:

\[
C_{ip} = -4\left(\frac{2s^2}{\pi \lambda ST}\right) \quad C_{ip} = -4.749 / \text{rad}
\]

As this equation is for four fins the damping values provided at Table 2. are included in the calculation. There is no sweep to the fins therefore this factor can also be ignored. As the projectile is travelling at \( M0.44 \) a velocity correction factor is required. The correction factor using Eqn (32) provides the following result:

\[
C_{ip} + \text{Velocity correction} = -4.749 + (-0.2) = -4.949 / \text{rad}
\]

The negative sign shows the coefficient behaves as a damping function.
5.17.2 Spin driving moment coefficient

To calculate the fin normal lift coefficient using Eqn (33) we have:

\[ C_{N\text{Body+Fin+Interference}} - C_{N\text{Body alone}} = \frac{(8.354 - 3.434)}{4} = \frac{4.92}{4} = 1.23 /\text{rad (For one fin)} \]

To calculate the spin driving moment using Eqn (34) we have:

\[ C_{18} = \frac{nC_{NF}y_c}{\lambda} \]
\[ C_{18} = 3.96 / \text{rad} \]

5.18 Comparisons with experimental flight data

The equations that were developed to calculate the spin damping coefficient and the spin driving moment coefficient were used to compare results with experimental data provided in Refs [48-57, 62,63]. It was found that there was a very good degree of correlation between the experimental data and the derived equations. The data generated from the equations was consistently lower than the experimental data by a maximum of value 8%. The lower values were to be expected as it has already been commented upon that the interference values used in these equations are slightly lower than actual experimental values. Outline details of the projectiles used to generate the experimental data are given at Appendix B.

5.19 Dynamic aerodynamic coefficients

The method of masses provides equations to determine a projectile's aerodynamic coefficients. The most important dynamic coefficient, as far as the projectile configurations under investigation are concerned, was the pitching moment coefficient due to pitch rate. This coefficient can be calculated using the following expression derived using the method of masses:
Using the configuration in Fig 5.3 the value of $C_{\text{madot}}$ can be calculated by integrating from the base of the projectile to the nose and summing the integrals for each particular part of the configuration shape. Using this configuration, the following value of $C_{\text{madot}}$ was obtained:

$$C_{\text{madot}} = -9.31 \text{ / rad}$$

The DATCOM result for $C_{\text{madot}}$ was calculated to be -9.78 / rad. As there was no suitable experimental data available for this coefficient the accuracy between the DATCOM result and the method of masses was considered to be sufficient for the $C_{\text{madot}}$ coefficient to be used in the trajectory analysis.

5.20 Determination of projectile drag characteristics

The projectile drag coefficient is one of the most important aerodynamic design coefficients for the projectile configurations under investigation. It was calculated using the 6 DoF trajectory model that an increase in the zero lift drag coefficient ($C_{D0}$) of 0.1 reduced a typical projectile's configuration range by 34m (Configuration as detailed in Chapter One). Therefore it was decided that the method used to determine the projectile drag should not be in error by more than $C_{D0} \pm 0.1$ when compared to experimental data.

The method of masses is not capable of generating a projectiles drag coefficient ($C_{D0}$). The reason for this, as discussed in Chapter Four, is that the method of masses is based upon the premise that the projectile is moving through an inviscid fluid. For an inviscid fluid there are no drag forces and therefore no drag coefficients. Therefore, an accurate method of
determining a projectile's drag had to be established. There have been numerous papers
written on how to determine the drag effects of a projectile. The most comprehensive of these
methods can be found at Refs [64,65]. However, if these techniques are followed and the
numerical methods adhered to, the computational effort becomes excessive. Therefore, an
alternative solution was sought.

To establish whether there was a key component in a projectile's overall drag figure, the
contributors to the overall drag had to be identified. Of all the projectile configuration drag
contributors, the following components were found to have by far the largest influence for the
configurations under investigation;

a. Skin friction drag.
b. Body base drag.
c. Nose wave/pressure drag.

As it was considered that the projectile designs being considered were of a similar type it was
assumed they would all have a similar values of drag. The individual drag characteristics of
the projectile configurations were calculated using the DATCOM prediction code (For the
configurations DATCOM could model). This was considered to be the most suitable method
of obtaining the drag data as the DATCOM methods used to calculate drag components have
been extensively validated against flight trial and experimental data at subsonic velocities. It
should be noted that such an extensive validation for the other coefficients has not been
carried out at subsonic velocities (DATCOM subsonic drag data was required for
sophisticated missile systems when the missile was first launched. At launch the missile is
powered and usually unguided. The drag figures were required to determine the initial
acceleration parameters at subsonic velocities). The standard projectile configuration shown
at Fig 5.3 was modelled using the missile DATCOM aerodynamic prediction code to provide
the configurations component drag data. The projectile configuration had the following
dimensions and flight parameters:

Angle of attack = 0-6 degrees. Speed = M0.41. Altitude = 0 m. Body diameter = 0.1m. Nose
- Either blunt or conical with length = 0.025m. Body length = 0.6m. Body surface - smooth
painted finish. Fins - Rectangular with root chord = 0.08m leading edge 0.52m from nose. Fins at 90 degrees to body surface.

Using DATCOM to determine the drag components at zero angle of attack ($C_{DA}$) for the projectile body alone provided the results shown in Table 3 and 4.

<table>
<thead>
<tr>
<th>Friction drag</th>
<th>Pressure/wave drag</th>
<th>Base drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08123</td>
<td>0.3844</td>
<td>0.137</td>
</tr>
</tbody>
</table>

Table 3. Drag components for body alone with conical nose.

<table>
<thead>
<tr>
<th>Friction drag</th>
<th>Pressure/wave drag</th>
<th>Base drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10196</td>
<td>0.89290</td>
<td>0.137</td>
</tr>
</tbody>
</table>

Table 4. Drag components for body alone and flat face.

As can be seen from Tables 3. and 4., the base drag was considered to remain constant. However, the base drag could be reduced by a value of 0.05 by reducing the body diameter at the rear of the projectile (Boat tail). For the projectile designs under investigation this was not considered to be viable as the fins would have to be inclined on the reducing body diameter which would make it difficult to manufacture. Again in reference to the data in Tables 3. and 4. it can be seen that the largest drag contributor was the pressure/wave drag which was a function of the nose shape. The shape of the nose and drag is considered in the next section.

5.20.1 Drag characteristics of projectile nose shapes

DATCOM is not capable of modelling rounded nose shapes and so experimental data at Refs [66-68] was used to determine if there was a large drag variation with nose shaping and Mach number. The nose shapes that were investigated are shown in Fig 5.4. From this experimental data there appeared to be little effect of projectile nose fineness ratio or bluntness ratio for Mach numbers less than 0.8. The magnitude of the variation in drag value with Mach
number was found to be in the order of 0.05. This small increase in drag with velocity was also confirmed by analysis of the experimental work detailed in Ref [69].

The next aspect of nose drag investigated was the effect of drag due to nose shape. As already recognised, the nose drag would be the largest drag contributor. Therefore, an accurate method of assessing its value had to be determined. It was obvious that the nose drag would decrease with a more pointed aerodynamic nose shape. The problem was how to accurately determine this drag decrease. Numerical methods were considered, but on investigation it was found that the computational effort was excessive for the accuracy of result achieved. The reason for the complex algorithms required to calculate the pressure/wave drag was due to the physical formation of the drag. The pressure/wave drag is formed due to the fluid in which the projectile is moving being viscous. Through the viscosity of the fluid, the skin friction drag causes a thickening in the boundary layer which results in pressure/wave drag (For a non-viscous fluid the pressures at the nose are balanced by the pressures at the base resulting in an absence of drag). Therefore, the pressure drag can be determined as not only a function of the nose shape but also a function of the body fineness ratio (length of body / diameter) and skin-friction coefficient. A larger body fineness ratio will result in a lower pressure drag.

Taking the previous factors into consideration, it was decided to use experimental data detailing the magnitude of nose pressure/wave drag and to incorporate this data into the 6 DoF trajectory program by means of a look-up-table. This was not as complex a task as first envisaged as the range of nose shapes, body diameters and body fineness ratios could be limited to a workable number through practical design considerations. These practical design considerations meant that the following nose shapes had to be incorporated into the look-up-table:

a. Rounded noses of Radius of corner of blunt nose / Body diameter (rc/d). Values 0 - 0.5. There were no significant drag benefits to be gained by increasing rc/d to a greater value. Greater values than this moved the centre of pressure too far forward and had to be compensated with larger fins.

b. Conical noses of length no more than 3 calibres. Lengths greater than this made the
projectile too long.

c. Hemispheres of nose length 3 calibres. Lengths greater than this made the projectile too long.

The optimum body fineness ratio (Body to include the length of the nose) values to give a low value of pressure/wave drag were calculated from experimental data to be between 9 and 15.

The drag look-up-table was programmed with the following nose shape drag data extracted from the experimental data at Refs [66-69]:

a. Re/d values increasing by 0.1 to a maximum of 0.5.

b. Conical nose lengths increasing by 1 calibre to a maximum of 3 calibres.

c. Hemispherical nose lengths increasing by 1 calibre to a maximum of 3 calibres.

For the projectile configurations under consideration, this range of nose shapes proved to be sufficient. If other nose shapes were required it was a simple matter of adding the requisite data to the look-up-table.

5.20.2 Drag characteristics of fins

The next factor to be considered was the drag effect of adding flat plate stabilising fins to the projectile body. DATCOM was used to model the increase in zero lift fin drag for different fin numbers and fin spans. The DATCOM fin drag results are shown in Table 5.
### Table 5. Drag increase with fin span and fin number.

<table>
<thead>
<tr>
<th>Fin semi-span</th>
<th>4 fins - total drag</th>
<th>6 fins - total drag</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04</td>
<td>0.01593</td>
<td>0.0239</td>
</tr>
<tr>
<td>0.05</td>
<td>0.01991</td>
<td>0.02987</td>
</tr>
<tr>
<td>0.06</td>
<td>0.02390</td>
<td>0.03583</td>
</tr>
<tr>
<td>0.07</td>
<td>0.028</td>
<td>0.04182</td>
</tr>
<tr>
<td>0.08</td>
<td>0.03186</td>
<td>0.04780</td>
</tr>
<tr>
<td>0.1</td>
<td>0.03983</td>
<td>0.05974</td>
</tr>
</tbody>
</table>

As can be seen from Table 5, the addition of flat plate fins did not significantly increase the projectile drag factor. Shaped fins were investigated with the result that the drag decreased by a small amount with leading and trailing edge sweep. As the fins for the configurations under investigation were considered to be flat plates, the fin drag factor was taken to be 0.03 for four fins and 0.04 for six fins.

The next stage in the drag analysis was to investigate the total variation of drag for the flat face and conical nose configurations with increases in angle of attack.

### 5.20.3 Drag characteristics with angle of attack

As can be seen from the results Table 6, the increase in drag with angle of attack is not a linear function. As the increase in drag with angle of attack is a significant factor in determining the projectile's trajectory performance, an algorithm predicting the increase in drag with angle of attack was required.
The relationship between drag and angle of attack was achieved by plotting drag values against angle of attack for different projectile shapes and determining the relationship with angle of attack using a regression technique. The resultant algorithm to determine the increase in drag with angle of attack is shown below:

$$C_{Da} = C_{Do} + (0.006\alpha + 0.008\left(\frac{1}{\alpha + 17}\right))$$  \hspace{1cm} (36)

Where $C_{Da}$ = Total drag, $C_{Do}$ = Zero lift drag, $\alpha$ = Angle of attack.

Eqn (36) can be used up to 15 degrees angle of attack with an expected error of ± 0.002 in lift induced drag being noted as compared to experimental and DATCOM data. Eqn (36) was embedded in the 6 DoF trajectory model to simulate lift induced drag.

5.20.4 Errors in calculating the drag for a complete projectile configuration

The drag for a complete projectile configuration was calculated as follows:

<table>
<thead>
<tr>
<th>Angle of attack (Alpha)</th>
<th>$C_{Do}$ Flat face + four fins</th>
<th>$C_{Do}$ Conical nose (length = 2 calibres) + four fins</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.989</td>
<td>0.125</td>
</tr>
<tr>
<td>1</td>
<td>0.996</td>
<td>0.133</td>
</tr>
<tr>
<td>2</td>
<td>1.010</td>
<td>0.147</td>
</tr>
<tr>
<td>3</td>
<td>1.029</td>
<td>0.167</td>
</tr>
<tr>
<td>4</td>
<td>1.055</td>
<td>0.194</td>
</tr>
<tr>
<td>5</td>
<td>1.087</td>
<td>0.228</td>
</tr>
<tr>
<td>6</td>
<td>1.119</td>
<td>0.262</td>
</tr>
</tbody>
</table>

Table 6. Total drag for flat face and conical nose configurations with angle of attack
Total drag = \( (C_{D_{base}}) + (C_{D_{a}}) = \) (Base drag + Skin friction drag + Pressure/wave (Nose) drag + Fin drag) + (Increase in drag with angle of attack)  \( \text{(37)} \)

Using DATCOM and experimental drag data, the following errors were observed in the drag calculations carried out using Eqn (37):

a. Base drag. Insignificant errors if large variations in body diameter were not used. (Body diameters varying between 0.03 and 0.2m were used in the validation). Value taken to be \( C_{D_{base}} = 0.14 \).

b. Skin friction drag. Value taken to be \( C_{D_{Skin friction smooth surface}} = 0.1 \pm 0.01 \), with an increase or decrease of 0.0166 for every 0.1m in body length (Smooth paint assumed as the surface covering).

c. Pressure/wave (Nose). Creates the largest drag variation. Value determined by using experimental data in a look-up-table. Error found to be \( \pm 0.03 \) in interpreting the look-up-table.

d. Fin drag. Flat plate fins have to be used. \( C_{D_{fin}} = 0.03 \pm 0.001 \), \( C_{D_{6 fin}} = 0.04 \pm 0.001 \).

e. Increase in drag with angle of attack. If Eqn (36) is used to determine \( C_{D_{a}} \) the error was found to be \( \pm 0.002 \).

If Eqn (37) is used to calculate the projectile’s total drag, the total error in the calculation was determined to be \( C_{D_{Total}} = 0.043 \) which is lower than the original stipulated 0.1 value. To put this value of error into context when considering typical trajectories. An error of \( \pm 0.043 \) in drag coefficient would give a range error of 15 m over a total range of 300m. This equates to a total range error of 5%. This percentage range error value was considered to be acceptable for type of projectile configuration under investigation.

5.21 Summary of method of masses aerodynamic coefficient method

By altering the fundamental method of masses equations for determining a projectile’s aerodynamic coefficients, it has been shown that the calculated numerical results relate favourably with experimental and predicted aerodynamic data sources. Unfortunately one of
the most important aerodynamic coefficients, drag cannot be determined using the method of masses. Therefore, as the projectile configurations under investigation had similar parameters, techniques of determining the various drag contributors were developed. Having a method of calculating a projectile’s aerodynamic coefficients, the next stage in the analysis was to determine the accuracy of the aerodynamic data by using it in a 6 DoF model and comparing the output with experimental data. This analysis was carried out and the results discussed in Chapter Six.
6.0 COMPARISON OF EXPERIMENTAL TRAJECTORY DATA WITH 6 DoF TRAJECTORY MODEL

6.1 Introduction

Chapter Three, detailed the basic algorithm requirements of three trajectory models; point mass, modified point mass and 6 Degree of Freedom (6 DoF). Chapter Five detailed the method of apparent masses to calculate a projectile's aerodynamic stability coefficients. The method of masses algorithms and drag look-up-table detailed in Chapter Five were embedded into the trajectory models which meant that a variation in the projectile configuration could be analysed as a trajectory profile variation. Before it could be determined whether an accurate trajectory profile could be generated, the trajectory model with method of masses aerodynamic coefficient data had to be validated. To accomplish this, three subsonic projectile sets of experimental range data were used to verify the output from the simulated trajectory models. The three sets of range data that were used is outlined as follows:

a. A simple projectile shape with a flat face, circular body and four flat plate fins. Defined in later sections as projectile (a).
b. A spin stabilised projectile with flat and rounded nose. Defined in later sections as projectile (b).
c. An adaptation of the projectile (a) with curved fins and a roll driving notch. Defined in later sections as projectile (c).

The range trial data for the projectile configuration (a) was used to verify that the trajectory models had been coded correctly and the aerodynamic methods described in Chapter Five were capable of generating accurate aerodynamic data. Variations in the aerodynamic coefficient on the trajectory were also analysed using this experimental data. The range trial data from the projectile configuration (b) was used to supplement the data provided by projectile (a). Projectile (b) was spun at a high rotational rate and therefore the experimental range data for (b) could be used to verify the accuracy of the rotational algorithms in the trajectory and aerodynamic models. Projectile (c) was an experimental projectile that used curved fins and a spin driving force.
If it could be verified that the trajectory and the aerodynamic models were accurate in determining the trajectories for projectiles (a) and (b), the simulation models could then be used to determine the performance of projectile (c) and subsequently improve its performance by changing the physical configuration.

6.2 Trajectory of a projectile with four flat plate fins

The projectile configuration (a) is shown in Fig 5.3 (No notable anomalies in trajectory were noted). The flight trial data for this projectile was taken from Ref [70]. The aerodynamic coefficients calculated from the methods detailed in Chapter Five and the flight parameters that were used in the trajectory models are detailed as follows:

**Projectile configuration:**
4 flat plate fins no sweep angle or offset spin driving angle. Flat nose. Centre of mass = 0.3m from nose. Moment of inertia: 
- $I_y$ (Polar) = $5.55 \times 10^6$ g.mm $^2$,
- $I_x$ (Transverse) = $1.04 \times 10^8$ g.mm $^2$.
Reference area = 0.00785 m$^2$. Reference length = 0.1m.

**Flight parameters:**
Velocity = 150 m/s. Initial launch height = 1.2m. Cross-wind = 0m/s. Initial launch angle = 3 degrees.

**Aerodynamic coefficients:**
- $C_{N\alpha \text{flat face}} = 8.354 / \text{rad}$,
- $C_{m\alpha \text{flat face}} = -13.644 / \text{radian}$,
- $C_{D\alpha \text{flat face}} = 0.989$,
- $C_{D\alpha \text{rounded nose}} = 0.366$.
- $C_{D\alpha \text{Total}} = [(0.006\alpha + 0.008(1/(\alpha+17))) + C_{D\alpha}]$,
- $C_{mp\alpha} = 12.26 / \text{rad}$,
- $\dot{C}_{mp\alpha} = 1.95 / \text{rad}$,
- $C_{modot} = -9.31 / \text{rad}$.

6.3 Comparison of experimental and theoretical trajectory data for projectile (a)

Eight projectiles were fired in this trial. Data was only published for five of the trajectories. Table 1. shows the data from one firing. All the other firings had very similar results (No notable trajectory anomalies were noted). The results from the three trajectory models for the flat face projectile configuration are detailed in Table 1. The values in brackets are the flight trial results.
<table>
<thead>
<tr>
<th>Trajectory model</th>
<th>Flight Time (s)</th>
<th>Range (m) Down range</th>
<th>Impact velocity (m/s)</th>
<th>Impact angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Mass</td>
<td>1.46 (1.5)</td>
<td>200 (200)</td>
<td>117.84 (118)</td>
<td>Unknown</td>
</tr>
<tr>
<td>Modified Point Mass</td>
<td>1.46(1.5)</td>
<td>200 (200)</td>
<td>117.84 (118)</td>
<td>Unknown</td>
</tr>
<tr>
<td>6 DoF</td>
<td>1.455(1.5)</td>
<td>200 (200)</td>
<td>116.96 (118)</td>
<td>+1.1</td>
</tr>
</tbody>
</table>

Table 1. Comparison of trajectory models for flat faced projectile configuration. Data in brackets is experimental.

As can be seen from Table 1, there is no difference in data between the point mass and modified point mass trajectory models. This is because the same equations are used in both models except where the projectile is spinning. The difference in terminal velocity and range between the 6 DoF and the other two models was due to the 6 DoF model taking into account the increase in drag due to the changes in the angle of attack (\( \alpha \)) throughout the trajectory. If the flight time was increased the differences between the output of the models was much greater with the velocity of the projectile decaying much faster with the 6 DoF model due to the increased induced drag. However, the degree of difference between the models was very much dependent on the stability of the projectile and whether it was subjected to a cross-wind. For a marginally stable projectile, the induced drag could decrease the range and velocity by up to 30m in a two second flight time. As the stability of the projectile was determined by the accuracy of pitching moment coefficient, the variation in pitching moment and its influence on the trajectory had to be examined. As stated in Chapter Five it was considered that the accuracy of the pitching moment was \( C_{ma} \pm 10\% \). This 10% variation in the pitching moment coefficient gave a variation in \( C_{ma} \) of \( \pm 1.364 /\text{rad} \). It was determined using the 6 DoF model that this change in pitching moment did not have any significant effect on the projectile trajectory for flight times up to six seconds. In terms of a physical configuration difference this variation of 10% in \( C_{ma} \) equates to a fin span change of 4 mm. As far as the other aerodynamic coefficients were concerned, they did not have any significant influence on the trajectory. This was not due to the models being inaccurate, but simply the factor of the short flight time. With a flight time of only 1.5s, there was not
time enough for the pitch and yaw components to increase and therefore destabilise the trajectory. As long as the projectile was statically stable it would fly a stable trajectory for 1.5s.

It was decided that the point mass and modified point mass trajectory models would provide no further useful information concerning the trajectory of projectile (a). Their trajectory outputs would not change with alterations in the aerodynamic coefficients, except for the zero lift drag coefficient and this was constant for all firings. The benefit of developing the point mass model was that the 6 DoF model was extremely complicated to code and errors were easily made. Having a trajectory model that could provide similar range and velocity information for short flight times enabled output errors in the 6 DoF trajectory model to be quickly corrected. Once it was established that the 6 DoF model was coded correctly, further validation was carried out using range trial data.

A second series of firings using projectile (a) were carried with the following differences in initial parameters:

Launch angle 5°, Initial velocity = 180 m/s.

In this trial a total of six projectiles were fired. For two of the firings data was not recorded. Table 2. shows one set of experimental data displayed in brackets. The other sets of experimental data demonstrated similar results. For completeness, the point mass figures are also shown in the trajectory figures listed at Table 2.

<table>
<thead>
<tr>
<th>Trajectory model</th>
<th>Flight Time (s)</th>
<th>Range (m) Down range</th>
<th>Impact velocity (m/s)</th>
<th>Impact angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Mass</td>
<td>2.66 (3)</td>
<td>400 (400)</td>
<td>103 (86)</td>
<td>Unknown</td>
</tr>
<tr>
<td>6 DoF</td>
<td>3.08 (3)</td>
<td>400 (400)</td>
<td>85 (86)</td>
<td>+ 11</td>
</tr>
</tbody>
</table>

Table 2. Comparison of trajectory models for projectile (a) at larger launch angle and initial velocity.
Table 2. clearly demonstrates the difference between using the 6 DoF and point mass trajectory models. With longer flight time the projectile’s pitch and yaw angles increased to; pitch $10^\circ$ and yaw $8^\circ$. This large pitch angle meant that the projectile was on the limit before a cross-flow drag factor would have had to be incorporated into the trajectory calculation. With the large pitch and yaw angles it was interesting to note how close the drag predictions were in the 6 DoF model. The good drag prediction also meant that the other aerodynamic coefficients were of sufficient accuracy to predict the trajectory to within 1m/s and a flight time to within 0.08s. As the results of the simulated trajectory and the flight trial were so close it was considered that the aerodynamic coefficients were within the accuracy tolerance laid down in Chapter Five. Therefore it was not considered worth carrying out a regression technique on the flight trials trajectory to determine experimental values of aerodynamic coefficients as it was considered very similar values to those calculated in Chapter Five would be obtained.

6.4 Trajectory of a spin stabilised projectile

Projectile (a) had no spin imparted to it, therefore, the aerodynamic rolling coefficients could not be fully assessed in terms of calculated accuracy. To determine the accuracy of the aerodynamic rolling coefficients generated by the method of masses and the trajectory model rolling algorithms, a comparison between the trajectory simulation and experimental data for a spin stabilised projectile was carried out. The spin was imparted to the experimental projectile by firing it from a rifled barrel launcher.

A series of flight trials were carried out using projectile (b) by Page, Refs [71,72] to determine the stability of a spin stabilised, subsonic, free-flight projectile. The following are the projectile’s aerodynamic coefficients calculated by the method of masses as detailed in Chapter Five and the trajectory flight parameters:

**Configuration:**
Cylindrical body, Length = 0.1m, Diameter = 0.037m, Mass = 0.136 Kg, Flat face, Centre of mass = 0.03m from nose. Moment of inertia: $I_y = 2.3 \times 10^{-6} \text{Kg.m}^2$. 

6 - 111
Ix = 1.24 x 10^{-4} Kg.m^2. Reference area = 0.001 m^2, Reference length = 0.037m.

**Flight parameters:**
Initial velocity = 66 m/s, Initial launch height = 1.3m, Cross-wind = 6m/s at 90° to trajectory, Initial launch angle = 7 degrees, Initial spin rate = 1631 rad / s

**Aerodynamic coefficients:**

\[ C_{Na\flat\text{face}} = 3 \text{ / rad}, \ C_{mu\flat\text{face}} = -0.016 \text{ to } +0.03 \text{ (Depending on angle of attack. -ve for 2°. +ve for 3° and above) / rad}, \ C_{D\flat\text{flat face}} = 1.1, \ C_{D\alpha} = [(0.006\alpha+0.008(1/\alpha+17)) + C_{D0}] , \ C_{mpa} = 7.2 / \text{ rad}, \ C_{mpa} = 0.16 / \text{ rad}, \ C_{\text{modot}} = -2.29 / \text{ rad}, \ C_{lp} = -.343 /\text{ rad}. \]

<table>
<thead>
<tr>
<th>Calculation Method</th>
<th>Flight Time (s)</th>
<th>Range (m)</th>
<th>Drift (m) Wind 6m/s @90°</th>
<th>Impact velocity (m/s)</th>
<th>Impact angle (deg)</th>
<th>Final spin rate (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental data</td>
<td>1.76</td>
<td>100</td>
<td>0.062</td>
<td>36.3</td>
<td>1.1</td>
<td>1104</td>
</tr>
<tr>
<td>6 DoF model</td>
<td>1.79</td>
<td>100</td>
<td>0.0518</td>
<td>35.6</td>
<td>0.823</td>
<td>1102</td>
</tr>
</tbody>
</table>

Table 2. Experimental and simulated 6 DoF trajectory data

As can be seen from the data in Table 2, the simulated and experimental data show a very good degree of correlation. Looking at the results in more detail it would appear that the spin damping calculations were correct. The velocity prediction was good considering the pitching moment changes sign and magnitude with angle of attack due to the blunt nose and short body length. For angles of attack up to 2° the pitching moment was negative. For angles of attack greater than 2° the pitching moment was positive. This change in pitching moment in the trajectory model was calculated by interpreting a value of pitch angle with associated pitching moment value every 0.01s throughout the trajectory.

A positive value of pitching moment meant that the projectile had to be spun at a high spin rate to achieve a stable trajectory. The drag figures that were used in the trajectory model appear to be
correct as the difference in velocity was 0.7 m/s and range 0.9 m. During the trajectory, the pitch angle increased from 0 to 2.5° and the yaw angle increased from 0 to 5°. The yaw angle increase being greater than the pitch angle was due to the influence of the 6 m/s cross-wind at 90° to the flight direction.

It was recognised by Page, that the flat face projectile configuration was unsuitable due to the change in pitching moment sign and the large zero lift drag due to the blunt face. Therefore, to overcome this problem, the nose of the projectile was rounded with a value of radius of rounding (rc) / Body diameter (d) = 0.2. Using the method of masses the following aerodynamic coefficients were altered to simulate the rounded nose projectile configuration:

Configuration:
Cylindrical body, Mass = 0.132 Kg, Centre of mass = 0.025m from nose.
Ix = 1.21 x 10^{-3} Kg.m². Nose rounding rc/d = 0.2

Flight parameters:
Initial velocity = 66 m/s, Initial launch height = 1.3m, Cross-wind = 6m/s at 90° to trajectory, Initial launch angle = 7 degrees, Initial spin rate = 1631 rad / s

Aerodynamic coefficients:
Cmin rounded nose (rc/d = 0.2) = 0.024 / rad, CDD (rc/d = 0.2) = 0.36.

<table>
<thead>
<tr>
<th>Calculation Method</th>
<th>Flight Time (s)</th>
<th>Range (m)</th>
<th>Drift (m)</th>
<th>Impact Velocity (m/s)</th>
<th>Impact Angle (deg)</th>
<th>Final spin rate (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental data</td>
<td>1.86</td>
<td>150</td>
<td>0.032</td>
<td>54</td>
<td>3.4</td>
<td>984</td>
</tr>
<tr>
<td>6 DoF model</td>
<td>1.98</td>
<td>150</td>
<td>0.022</td>
<td>52.8</td>
<td>3.1</td>
<td>979</td>
</tr>
</tbody>
</table>

Table 3. Trajectory comparisons for 6 DoF model and experimental data for projectile with rounded nose radius of rounding (rc) / Diameter (d) = 0.2.

As can be seen from Table 3., there is a good degree of correlation between the flight trials data and the output from the 6 DoF model. The similarity in velocity between the experimental data and the 6 DoF model indicated that the value of the drag coefficient and the variation in angle of attack
throughout the trajectory were very close. The spin rate also showed a good degree of similarity which indicated that the spin damping algorithms appeared correct. However, the problem with using this experimental data was that the accuracy with which the data was measured was not recorded. Another observation that was made when carrying out this analysis was the very short flight time. With the short flight time of around 2s, the effect of the lift induced drag was not very great as the projectile only increased in pitch by $2^\circ$ and yaw by $2.4^\circ$. In summary the close comparison between the simulated trajectory data and the flight trials data indicated that the trajectory algorithms and the aerodynamic coefficients were of sufficient accuracy to model a spin stabilised projectiles trajectory.

Comparisons with flight trials data for longer range spin stabilised projectiles was carried out. The experimental data was extracted from work carried at Ref [73]. Details of the configurations and comparison of rotational coefficient data is provided at Appendix B. From the comparisons with the experimental data the following observation concerning the method of masses aerodynamic prediction technique and trajectory model was made:

a. A variation of 10% in $C_{bp}$ equated to a spin reduction of 2 rad/s. This variation in spin rate will not create dynamic instability for a spin stabilised projectile, due to the very high spin rates that are required to spin stabilise a statically unstable projectile.

6.5 Magnus forces and moments

The Magnus force and moment coefficients were calculated using the derived method of masses equations in Chapter Five. No variation was made to the numerical values provided by the basic method of masses equations. The Values for the Magnus Force coefficient and Magnus moment coefficient using these equations was $C_{rpa} = 0.125$ / deg and $C_{mpa} = 0.0028$ / deg. It was found that using these coefficient values in the 6 DoF trajectory model did not have any significant effect on the overall projectile trajectory (It was due to this negligible trajectory variation with these coefficients that they were not researched in any great depth in Chapter Five).
6.6 Flight trials experiments for projectile with curved fins

A series of flight trials were carried out using the projectile configuration shown in Fig 6.1. This projectile configuration is defined as projectile (c). The flight trials were carried out to determine the flight characteristics for this new configuration shape. The configuration details and flight parameters for the projectile were as follows:

**Projectile configuration:**
4 curved fins with no sweep angle or offset spin driving angle, Notch in the trailing edge to generate spin. Root chord = 0.08m, Equivalent flat plate span = 0.06m Flat nose. Body Diameter = 0.1m, Body length = 0.584m, Centre of mass = 0.3m from nose. Moments of inertia: See Table 5. Reference area = 0.00785 m², Reference length = 0.1m. Mass = 3.94 Kg

**Flight parameters:**
Initial velocity = See Table 6. Initial launch height = 1.2m, Cross-wind = 0m/s, Initial launch angle = See Table 5.

**Aerodynamic coefficients:**
See Table 6.

As can be seen from Fig 6.1, this projectile configuration is fundamentally the same as that used in Chapter Five to determine the projectile’s aerodynamic stability coefficient characteristics. However, for this experimental projectile the fins were curved as the flight characteristics of curved fins were required to be known. Having curved fins made the packing of the projectile into a launch tube much easier. The curved fins were deployed using a spring mechanism when the projectile was ejected from the launch tube. In addition, a notch was placed on the trailing edge of each fin to impart spin to the projectile. Spin was given to the projectile to overcome any misalignment or shape differences caused by fin manufacturing discrepancies. In addition, to the curved fin information, data concerning the variation to the projectile trajectory created by altering the fin dimensions was also required from the experimental trial results.
With a limited financial budget, only fourteen projectiles could be manufactured and fired. With this limited number of projectiles, it was not considered viable to have large variations in fin geometry. This was due to the fact that similar fins had to be compared as the range recording equipment might have failed for a particular configuration with the subsequent loss of data, or the fin deployment might have been different for different fin configurations providing different results. Two or more sets of range data for each fin configuration had to be acquired to enable trajectories to be compared. Of more importance was that there were no accurate aerodynamic or trajectory simulation models available at the time of these trials. The method of masses as a means of generating aerodynamic coefficients had not yet been developed as neither had the trajectory simulation models. Without an accurate aerodynamic model, the estimated position of the projectile's centre of pressure ranged from 0.38m to 0.4335m from the nose depending on which rule of thumb prediction method was employed at the time. It was therefore decided that to obtain the maximum information from the limited number of firings, the projectile's centre of mass would be moved progressively backwards from the nose towards the stabilising fins. This would have the effect of making the projectile more unstable. If an unstable configuration could be obtained then the projectile would tumble. This would show that the centre of pressure for the configuration was in front of the centre of mass. Therefore, the statically stable centre of pressure position would be between this unstable position and a position of stability obtained from a centre of mass point closer to the nose. As the largest estimate for the centre of pressure was at 0.4335m from the nose, it was decided to add over a calibre to this distance and put the maximum centre of mass value at 0.446m from the nose. This should produce an unstable configuration. The centre of mass was then progressively moved forwards towards the nose to create a stable projectile configuration. It was important that all other projectile properties remained the same when the centre of mass was moved. By keeping the projectile properties the same, the different centres of mass would create the same aerodynamic effect as having different fin configurations. A smaller fin would provide less stability and a smaller static margin. This would be the same as moving the centre of mass towards the rear of the projectile.

A major problem was encountered in the movement of the centre of mass. As the centre of mass was moved the projectiles moments of inertia values were changed. If the moments of inertia for the projectiles under test were not the same, there could not be a valid comparison between their
trajectories. A method of keeping the moments of inertia constant was developed by attaching different masses to a rod running through the centre of the projectile. It was very difficult to centre the rod and subsequently the polar moments (I_y) of inertia were not all the same. It was recognised that the major influence on the trajectory would be the transverse moment of inertia (I_x) and not I_y. As can be seen from Table 4., the transverse moment of inertia was kept reasonably constant for all projectile firings. The one exception to this was the projectile with the centre of mass at 446mm from the nose. As it was expected that this projectile configuration would tumble it was not considered worth the time and expense of aligning the inertia coefficients with the other configurations.

<table>
<thead>
<tr>
<th>PROJECTILE</th>
<th>C of G mm from nose</th>
<th>STATIC MARGIN mm</th>
<th>I_x Kg mm^2</th>
<th>I_y Kg mm^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Target values</td>
<td></td>
<td></td>
<td>0.103</td>
<td>0.0055</td>
</tr>
<tr>
<td>GR/53593/A</td>
<td>327</td>
<td>149</td>
<td>0.103</td>
<td>0.00455</td>
</tr>
<tr>
<td>GR/53594/A</td>
<td>365</td>
<td>111</td>
<td>0.103</td>
<td>0.0043</td>
</tr>
<tr>
<td>GR/53595/A</td>
<td>400</td>
<td>76</td>
<td>0.103</td>
<td>0.00393</td>
</tr>
<tr>
<td>GR/53596/A</td>
<td>436</td>
<td>40</td>
<td>0.108</td>
<td>0.00358</td>
</tr>
<tr>
<td>GR/53597/A</td>
<td>446</td>
<td>30</td>
<td>0.115</td>
<td>0.00336</td>
</tr>
</tbody>
</table>

Table 4. Physical properties of trials projectiles.

Tables 5. and 6. show the projectiles initial conditions and end point accuracy as measured on the test range. The 6 DoF trajectory model showed a very good correlation with the experimental data shown in Table 6.
<table>
<thead>
<tr>
<th>ROUND</th>
<th>LAUNCH VELOCITY (M/S)</th>
<th>CENTRE OF MASS (mm from nose)</th>
<th>STATIC MARGIN (mm from C of G)</th>
<th>MASS (Kg)</th>
<th>ANGLE OF LAUNCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>143.2</td>
<td>327</td>
<td>149</td>
<td>3.903</td>
<td>3.2</td>
</tr>
<tr>
<td>2</td>
<td>144.7</td>
<td>327</td>
<td>149</td>
<td>3.862</td>
<td>2.85</td>
</tr>
<tr>
<td>3</td>
<td>131</td>
<td>327</td>
<td>149</td>
<td>3.889</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>131.7</td>
<td>327</td>
<td>149</td>
<td>3.889</td>
<td>2.87</td>
</tr>
<tr>
<td>5</td>
<td>145.4</td>
<td>400</td>
<td>76</td>
<td>3.936</td>
<td>2.85</td>
</tr>
<tr>
<td>6</td>
<td>142</td>
<td>436</td>
<td>40</td>
<td>3.94</td>
<td>2.917</td>
</tr>
<tr>
<td>7</td>
<td>137</td>
<td>400</td>
<td>76</td>
<td>3.95</td>
<td>2.917</td>
</tr>
<tr>
<td>8</td>
<td>138</td>
<td>436</td>
<td>40</td>
<td>3.955</td>
<td>2.95</td>
</tr>
<tr>
<td>9</td>
<td>141</td>
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<td>111</td>
<td>3.865</td>
<td>2.95</td>
</tr>
<tr>
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<td>139</td>
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<td>111</td>
<td>3.865</td>
<td>2.9</td>
</tr>
<tr>
<td>11</td>
<td>140</td>
<td>327</td>
<td>149</td>
<td>3.85</td>
<td>2.93</td>
</tr>
<tr>
<td>12</td>
<td>138</td>
<td>327</td>
<td>149</td>
<td>3.92</td>
<td>2.975</td>
</tr>
<tr>
<td>13</td>
<td>137</td>
<td>327</td>
<td>149</td>
<td>3.91</td>
<td>2.92</td>
</tr>
<tr>
<td>14</td>
<td>141</td>
<td>446</td>
<td>149</td>
<td>3.9</td>
<td>2.91</td>
</tr>
</tbody>
</table>

Table 5. Summary of projectile initial conditions
<table>
<thead>
<tr>
<th>Round</th>
<th>Average wind speed (m/s)</th>
<th>Average wind direction (deg)</th>
<th>Impact point X (m)</th>
<th>Impact point Y (m)</th>
<th>Centre of mass (mm from nose)</th>
<th>Motion</th>
<th>Launch velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.3</td>
<td>52</td>
<td>.4L</td>
<td>.85H</td>
<td>327</td>
<td>Stable</td>
<td>146.05</td>
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<tr>
<td>2</td>
<td>5.3</td>
<td>43</td>
<td>.41L</td>
<td>.11H</td>
<td>327</td>
<td>Stable</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>50</td>
<td>.82L</td>
<td>1.39H</td>
<td>327</td>
<td>Stable</td>
<td>144.9</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>37</td>
<td>.21L</td>
<td>.87L</td>
<td>327</td>
<td>Stable</td>
<td>131.73</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>50</td>
<td>.39L</td>
<td>.18L</td>
<td>327</td>
<td>Stable</td>
<td>139.61</td>
</tr>
<tr>
<td>6</td>
<td>2.1</td>
<td>50</td>
<td>.4L</td>
<td>.03H</td>
<td>327</td>
<td>Stable</td>
<td>137.92</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>50</td>
<td>0</td>
<td>Miss L</td>
<td>327</td>
<td>Stable</td>
<td>127.1</td>
</tr>
<tr>
<td>8</td>
<td>2.3</td>
<td>55</td>
<td>.33L</td>
<td>.66H</td>
<td>365</td>
<td>Stable</td>
<td>140.93</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>63</td>
<td>.24L</td>
<td>.11H</td>
<td>365</td>
<td>Stable</td>
<td>139.36</td>
</tr>
<tr>
<td>10</td>
<td>2.5</td>
<td>33</td>
<td>Miss L</td>
<td>Miss H</td>
<td>400</td>
<td>Miss</td>
<td>145</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>65</td>
<td>1.07R</td>
<td>.83L</td>
<td>400</td>
<td>Miss</td>
<td>137.3</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>50</td>
<td>13.6R</td>
<td>Miss L</td>
<td>436</td>
<td>Unstable</td>
<td>143</td>
</tr>
<tr>
<td>13</td>
<td>4.5</td>
<td>45</td>
<td>6.5L tumbling</td>
<td>Miss 6.1 short</td>
<td>436</td>
<td>Unstable</td>
<td>141</td>
</tr>
<tr>
<td>14</td>
<td>3</td>
<td>50</td>
<td>tumbling</td>
<td>tumbling</td>
<td>446</td>
<td>Unstable</td>
<td>139</td>
</tr>
</tbody>
</table>

Table 6. End point trajectory results

Where;  L = Left of centre of target,  R = Right of centre of target,  H = High of target,  l = low of target,  Tumbling = Unstable in the trajectory,  Miss = The projectile did not reach the target.

The experimental trial also demonstrated that the curved sprung fin assembly worked as intended. The range yaw card signature showed all four fins were erect at 47 m from the launch point. When the rounds were recovered all four fins remained attached to the projectile body. The high speed
range camera showed the fin blades first motion within 1 millisecond after shot exit from the launch tube and fully erect at around 8 milliseconds after shot exit. The rear high speed range camera showed the fin blades oscillating along their length after locking but this was soon damped out. No significant rotational motion was recorded for the first 16 m of the trajectory. As the fins were deployed in 8 milliseconds, this time was found not to effect the projectile trajectory. If the fin blades had deployed in times greater than 0.1s then an initial pitch angle of 4° was noted using the 6 DoF simulated trajectory. This initial pitch angle was found to destabilise the trajectory, particularly for configurations with small static margins in the order of 0.7 calibres.

The experimental velocity profiles for the different projectile configurations was recorded and converted to a digital format so that range velocity data could be overlaid with simulated trajectory data to determine the degree of correlation between the experimental and simulated data. The spin profile of the projectiles was also recorded at specific points along the trajectory so that a comparison between the simulated and experimental spin data could be made.

As explained in Chapter Five, the method used to determine curved fins aerodynamic coefficients was to model the curved fin as a flat plate of span - from tip chord with a straight line to root chord. For the curved fins it was calculated that the flat fin equivalent would have a span of 0.053m. This value of 0.053m for the fin span was used to calculate the aerodynamic coefficients of the projectile. Using this fin span, the method of masses method gave the projectile Centre of pressure (CP) to be 439mm from the nose. As can be seen from Table 5., the projectiles with centres of mass at 446mm and 436mm from the nose had an unstable trajectory indicating that they were unstable or on the limit of stability. The pitching moment values of $C_{ma} = +0.004$ /deg (Centre of mass = 446mm) and $C_{ma} = -0.013$ /deg (Centre of mass = 436 mm) calculated using the method of masses also gave unstable trajectories when used in the simulated 6 DoF trajectory model. The projectile with centre of mass at 400mm from the nose gave a stable trajectory - but it missed the target through excessive pitch and yaw angles (It was on the limit of stability). Therefore from the flight trial results it was concluded that the CP was between 400mm and 446mm. The method of masses predicting 439m indicated that the pitching moment and normal lift coefficient, including interference effects, appear to be in close agreement with the flight trials data. Therefore, the assumption of making the curved fin into a flat
plate appears to provide an accurate result. It is also interesting to note that the interference effects appear to be the same for the curved fin as the flat plate equivalent even though there is a greater area exposed to the fluid medium through which the projectile is flying.

Table 5. shows the two of the aerodynamic coefficients calculated for projectile (c) using the method of masses aerodynamic prediction methods detailed in Chapter Five.

<table>
<thead>
<tr>
<th>PROJECTILE</th>
<th>C of G (m) from nose</th>
<th>Static margin (m)</th>
<th>Pitching moment $C_{ma}$ /deg</th>
<th>Damping moment $C_{eq}$ /deg</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR/53593/A</td>
<td>.327</td>
<td>-.1163</td>
<td>-.192</td>
<td>-.621</td>
</tr>
<tr>
<td>GR/53594/A</td>
<td>.365</td>
<td>-.0789</td>
<td>-.13</td>
<td>-.432</td>
</tr>
<tr>
<td>GR/53595/A</td>
<td>.400</td>
<td>-.0439</td>
<td>-.072</td>
<td>-.293</td>
</tr>
<tr>
<td>GR/53596/A</td>
<td>.436</td>
<td>-.0079</td>
<td>-.013</td>
<td>-.185</td>
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<tr>
<td>GR/53597/A</td>
<td>.446</td>
<td>+.0021</td>
<td>+.004</td>
<td>-.161</td>
</tr>
</tbody>
</table>

Table 5. Aerodynamic coefficients for projectile c, calculated using the method of masses.

6.7 Rotational motion of projectile

The projectile was rotated to reduce any manufacturing differences in the fin assembly which might generate excessive pitch or yaw angles. From the flight trials data it was calculated that the projectile spin rate was increased from 0 to 7 revs/sec. Using the 6 DoF trajectory model, it was established that this spin rate would reduce the projectile velocity by about 0.02 m/s. As the accuracy of the range measuring equipment was calibrated at ± 1m/s, the decrease in velocity due to projectile rotation was ignored. From experimental data at Ref [55], it was determined that there is not a specific rotational speed to reduce fin manufacturing errors. It was concluded from the experimental data at Refs [74,75], that projectiles of the type being described should be spun at rates of between 5 and 20 revs/s to compensate for any fin imperfections. As the projectile was statically stable, the slow spin rate of 7
revs/s did not add significantly to the projectiles stability. It was calculated that a spin rate of 7 revs/s was equivalent to increasing the fin span by 3mm. The increased value of static margin due to spinning the projectile was calculated by having a statically unstable projectile and then by increasing the spin creating a dynamically stable projectile configuration. This dynamic stability figure was determined by using the 6 DoF trajectory model and noting the value of the pitch and yaw angles with an increased spin rate. Pitch and yaw angles in excess of 15° were considered to constitute an unstable configuration. The spin rate was increased by increasing the fin span (Fins were offset at a set 4° for all spans to create a spin driving force. It was not considered possible to model a notch spin driving mechanism).

6.8 Optimisation of the projectile configuration using the simulation methods

As detailed in the projectile configuration description, the projectile was spun using a notch in the trailing edge of the fin. The method of masses was not capable of modelling the spin driving force created by the notch. However, the spin driving force could be determined and an equivalent fin cant angle calculated. Knowing this equivalent cant angle meant that that the fin configuration could be redesigned to create an equivalent trajectory. The need for the redesign was required as the notch in the trailing edge of the fins was produced by machining the fin from a solid piece of aluminium. This fin unit, although providing good stability, was too heavy, too expensive to manufacture and, with the associated spring mechanism, too complex. It was subsequently requested by the customer that a fin with a leading edge sweep was required. This leading edge sweep was needed to give the projectile a better penetration capability in vegetation. As the trajectory of the experimental projectile was considered to be stable, the same trajectory performance was required but with a simpler stabilising fin assembly. Having developed the method of masses as an accurate method of determining the projectile’s aerodynamic coefficients and having this coupled to a validated 6 DoF trajectory model, it was possible to use the simulation models to design a new fin stabilising assembly.
6.9 Example of using method of masses and 6 DoF trajectory program to optimise a projectile configuration

The following section is an example of how the methods developed in Chapters Five and Six can be used to optimise the trajectory and physical design of a projectile. It was considered that projectile (c) performed a stable trajectory (With centre of mass at less than 400mm from the nose with an equivalent fin span of 0.053m). Therefore, this basic projectile shape provided the aerodynamic coefficients that could be used to generate a practical baseline trajectory. However, there were aspects in the projectile design that were not optimum and could be improved with the correct simulation methods.

As a sweep angle was required in the leading edge a new fin set was designed to include a sweep angle. The results of this analysis to provide a fin set that would produce adequate static stability and spin profile was as follows:

Six fins, spaced at 0, 60, 120, 180, 240 degrees around the projectile body. Leading edge sweep = 10°, no trailing edge sweep, root chord = 0.2 m, span = 0.04m, canted at 5° to provide spin driving force. \( C_{lp} = -5.316 \text{ rad} \), \( C_{ls} = 2.221 \text{ rad} \).

The projectile spin profile using this fin set was found to be lower than the spin profile of the experimental projectile (c) (Simulated spin rate was found to be maximum at 6.2 revs/s after 1.4 seconds). This was not considered significant as the spin rate to counter fin misalignments was between 5 and 20 revs/s (As detailed earlier).

The fin span was reduced because it was envisaged that the fin would be constructed from sprung steel. On exit from the launch tube the fin would spring into place from its wrap around position and create the required static stability. The span of 0.04m was chosen to reduce the opportunity of the fin oscillating and causing trajectory instability. It was considered that larger fin spans would oscillate too much. The reduction in the span and the sweep in the leading edge meant that the fin chord had
to be increased to 0.2m so that the fin was capable of generating the correct static stability as well as the appropriate aerodynamic spinning coefficients. This fin set gave a static margin of 1.18 calibres behind a centre of mass position of 0.3 m from the nose. This static margin was considered to be within the tolerance of manufacture without a cost penalty for more accurate assembly time. These improvements would make the fin assembly lower cost and smaller mass.

Having improved the fin assembly the next area of improvement was the shape of the nose. With a flat nose the drag coefficient was calculated to be $C_{D0} = 0.989$. If the nose was rounded by a ratio of $rc/d = 0.2$, the $C_{D0}$ value was reduced to 0.32 (body alone). The adverse side of reducing the drag was that the centre of pressure moved forward by 0.5 calibres. This movement in the centre of pressure put the static margin at 0.68m. This was considered too small with the risk of instability in the trajectory. To make the projectile more stable the following parameters could be changed:

a. Move the centre of mass forward.
b. Increase the size of the stabilising fins.
c. Increase the length of the body.

For this example, it was decided to increase the length of the projectile body to 0.9m and keep the stabilising fin unit previously described the same. A diagram of this projectile configuration is shown at Fig 6.2. The complete input parameters for this with the associated aerodynamic stability coefficients calculated from the method of masses described in Chapter Five are as follows:

**Body data:**
Total length = 0.9m, Reference diameter = 0.1m, length of rounded nose = 0.02m, Distance to the Centre of mass = 0.4m (This was calculated by considering a uniform mass distribution in the body and taking into account the mass distribution of the payload).
Fin data:
Tip chord = 0.193, Root chord = 0.2m, leading edge sweep = 10°, Mid chord sweep = 5°, semispan = 0.4m, Aspect ratio = 0.407, Wing body interference = 1.505, Body wing interference = 0.915, Fin area = 0.0157m², Reference area = 0.0079 m².

Flight parameters:
Velocity = 150 m/s, Density of air = 1.225 Kg/m³.

Aerodynamic coefficient data:
$C_{Na} = 7.39 /\text{rad}, \ C_{ma} = -39.72 /\text{rad}, \ x_{cp} = 0.537 \text{m from the nose.}$ Static margin = 1.37 m,
$C_\phi = -5.316 /\text{rad}, \ C_{ls} = 2.221, \ C_{madot} = -12.4 / \text{rad}, \ C_{D0} = 0.385 \text{(Body-smooth finish+ fins)},$
$C_{D\alpha} = C_{D0} + (0.006\alpha+0.008(1/\alpha+17)), \text{ With the slow spin rate the Magnus coefficients were not considered to be significant. (All these aerodynamic coefficients were calculated using the methods described in Chapter Five).}$

The aerodynamic data was compared to the RAPPIC aerodynamic prediction code and found to be within the tolerance limits laid down for the method of masses in Chapter Five. DATCOM was not capable of modelling this projectile configuration.

Using the aerodynamic data, the trajectory of the projectile could be analysed using the 6 DoF trajectory model. The trajectory profiles for this projectile are shown in Appendix C. As can be seen from these profiles, the performance of the projectile can be quickly determined. Also variations in the flight parameters or aerodynamic coefficients can be quickly changed and the effect on the performance determined. The input menus for the aerodynamic coefficients and the 6 DoF trajectory model are shown at Figs 6.3 and 6.4.

This analysis is shown as an illustrative example of how the performance of a projectile can be improved using the aerodynamic and trajectory methods described in this thesis.
6.10 Summary of the derived prediction models

Having the simulation models facilitates the determination of the performance of conceptual subsonic ballistic projectile designs. Before the development of the aerodynamic methods and their integration into the 6 DoF trajectory model, the design of this type of projectile was very inaccurate and took a long time. The only means of determining a representative projectile trajectory was to test fire the conceptual designs. To obtain the degree of optimisation demonstrated in the previous example would have meant the following set of trials firings:

a. A fin assembly design to determine stability and spin performance. This would have to include several fin spans, root chords and offset angles to determine the optimum configuration.
b. Different fin combinations would have to be tried to determine the optimum number.
c. Different nose shapes with the fin assembly to determine the stability performance. The centre of mass or the rounded nose would have to be determined. The range target would have to be changed as the new range with the reduced drag would not be initially known.
d. Different projectile lengths would have to be tried to determine a stable trajectory with a rounded nose and new fin assembly.

As can be seen from the this list there are a large number of firings required. Using the method of masses and the 6 DoF trajectory model, these firings can be eliminated and a configuration based upon Fig 6.2 tested straight away with the stability and range performance parameters detailed in Appendix C being used as a basis for designing the trials range in terms of target placement, camera positions and range safety traces (Where the projectile would probably land).
7.0 FURTHER DEVELOPMENTS OF THE METHOD OF MASSES AND 6 DoF TRAJECTORY MODEL

7.1 New configurations

The method of masses and 6 DoF trajectory model were developed to determine the performance of projectiles detailed in Chapter One. Chapter Six described the comparison between three experimental configurations and simulated data. It was found that there was a good degree of correlation between the simulated and experimental data. The next stage in the development of the models would be to determine the theoretical performance of conceptual projectile configurations to determine if they would be viable for developing the designs further and firing them on a test range. A design that has been investigated is shown in Fig 7.1. This design is an adaptation of the configuration shown in Fig 6.2. In this case, the stabilising fins are placed on an extending sleeve. The advantage with this design is that the projectile length is reduced when stored in the launch tube prior to launch. On launch the sleeve extends rearwards and the flip-out stabilising fins are deployed to create static stability. If static stability is not achieved dynamic stability could be achieved by imparting spin to the projectile by means of offset fins. To ascertain the physical configuration parameters required to produce a suitable trajectory, the methods detailed in Chapters Five and Six could be employed.

7.1.1 Propulsion and command guidance systems

Having developed the 6 DoF trajectory model, it would be very straightforward to incorporate a thrust profile due to a solid fuel propulsion system. The variation in the projectile's centre of mass on the stability and the required stabilising mechanism could be determined using the techniques detailed in Chapter Five if the velocity was kept in the subsonic regime. An optical command guidance system could be incorporated into the model by determining the change in lift due to fin deflection caused by a servo system. Alternatively, a thrust vectored system could be modelled by adapting the basic force equations.
7.1.2 Supersonic velocities and advanced guidance systems

The method of masses was initially declared as being a numerical method independent of velocity. As has been described in Chapter Five this is not a true statement. To determine supersonic aerodynamic coefficients would need further development of the interference effects, nose configurations and fin shapes etc. Although this appears to be a complete redesign of the methods described in Chapter Four, it has to be noted that supersonic systems are guided and, therefore, the accuracy of the aerodynamic coefficient data is not as critical as with a free-flight system. It should therefore be straightforward to calculate the required coefficients in the transonic and supersonic speed regimes. The drag look-up-tables would have to be extended to incorporate the new speed regimes. This again would not be complicated as this is how existing aerodynamic prediction models calculate drag coefficient data.

Guidance laws such as proportional navigation would not be a problem to incorporate into the trajectory model as they are an adaptation of the basic force equations.

7.2 Conclusions

It would be possible to extend the trajectory and method of masses aerodynamic model to incorporate transonic and supersonic speed regimes. However, as stated in Chapter One, there are numerous simulation models to calculate trajectories at these speeds. The methods developed in the thesis have been optimised for the subsonic speed regime and projectile configurations detailed in Chapter One. Further work using the techniques developed in the thesis is currently being carried out to determine the trajectory characteristics of the following subsonic systems:

a. Subsonic, propelled, guided air to surface missile system. Determination of range safety traces.

b. Ballistic free-flight air to surface M84 bomb. Variation in trajectory with crosswinds. Optimum launch conditions.

c. Launch parameters for laser guided bombs. Trajectory model to be incorporated
into aircraft avionics to provide range and launch conditions.

d. Range and dispersion of free-flight air to surface rockets.

e. Range of subsonic blast fragments created from detonation of missile warhead.

f. Possible optimisation of an aircraft gun system to improve the accuracy of the bullet.
Fig 1.1 General projectile configuration
Where: $X =$ Longitudinal axis, $Y =$ yaw axis, $Z =$ pitch axis, $\theta =$ Pitch angle, $\psi =$ Yaw angle, $\varphi =$ roll angle
For clarity the stabilising fins have not been included.

Fig 1.2 Co-ordinate system used to define the mathematical model
Four Fins with No Cant

Four Fins with Cant

Angle $\delta$ ($\delta$ replacing $\alpha$)

$V \sin \alpha$

Fig 2.2 Different fin cant configurations
Where: $\alpha = \text{Angle of attack in Y plane.}$
$\beta = \text{Yaw angle in Z Plane.}$
$\omega = \text{Angular rotation about the X axis}$
$U, V, W = \text{Velocities in the X,Y,Z planes.}$
$L, M, N = \text{Moments in the X,Y,Z planes.}$
$P, Q, R = \text{Angular rate in the X,Y,Z planes.}$

Non dimensionalised components are represented by lower case letters.

E.g. Non dimensionalised pitching moment coefficient.

$C_{ma} = \text{Pitching moment in Y plane due to change in pitch angle } \alpha.$

Fig 3.1 Projectile axis layout
FORCE AND MOMENT COEFFICIENTS

NOTE: If the geometry is panelled using a plane of symmetry about the Y=0.0 plane, only the total force and moment coefficients will include the contribution from the image panels.

********
WIND AXES
********

PATCH COEFFICIENTS

<table>
<thead>
<tr>
<th>PATCH NAME</th>
<th>CL</th>
<th>CD</th>
<th>CY</th>
<th>C_m</th>
<th>C_n</th>
<th>C_I</th>
<th>PATCH AREA/SREF</th>
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</thead>
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<td>1 WING (Patch 1, 10x218)</td>
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<td>0.0012</td>
<td>0.0071</td>
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<td>-0.0003</td>
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<td>0.0004</td>
<td>0.0073</td>
<td>-0.0023</td>
<td>0.0051</td>
<td>0.0018</td>
<td>0.0082</td>
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<td>3 ROOT TRANSITION FORE ST</td>
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<td>0.0004</td>
<td>0.0025</td>
<td>0.0105</td>
<td>-0.0015</td>
<td>-0.0001</td>
<td>0.3222</td>
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<td>4 ROOT LOWER STARBOARD</td>
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<td>-0.0002</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0000</td>
<td>0.1212</td>
</tr>
<tr>
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<td>0.0105</td>
<td>0.0005</td>
<td>0.0112</td>
<td>0.0037</td>
<td>-0.0021</td>
<td>-0.0001</td>
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<tr>
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<td>0.0208</td>
<td>0.0043</td>
<td>0.0146</td>
<td>0.0010</td>
<td>0.4187</td>
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COMPONENT COEFFICIENTS

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<th>C_n</th>
<th>C_I</th>
<th>2C_I</th>
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<td>0.0042</td>
<td>0.0486</td>
<td>0.0032</td>
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TOTAL COEFFICIENTS

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<td>0.2481</td>
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INDUCED DRAG COMPUTED USING TREfftZ PLANE ANALYSIS

WAKE NUMBER CDI CL

| 1 | 0.006317 | 0.241531 |

TOTAL INDUCED DRAG COEFFICIENT CDI = 0.006317
TOTAL LIFT COEFFICIENT CL = 0.241531
SPAN EFFICIENCY FACTOR = 0.734878

1

*******
BODY AXES
*******

PATCH COEFFICIENTS

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<th>CA</th>
<th>CY</th>
<th>C_m</th>
<th>C_n</th>
<th>C_I</th>
<th>PATCH AREA/SREF</th>
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<td>-0.0023</td>
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<td>0.0082</td>
</tr>
<tr>
<td>3 ROOT TRANSITION FORE ST</td>
<td>0.0059</td>
<td>0.0000</td>
<td>0.0025</td>
<td>0.0105</td>
<td>-0.0015</td>
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</tr>
<tr>
<td>4 ROOT LOWER STARBOARD</td>
<td>-0.0001</td>
<td>0.0001</td>
<td>-0.0002</td>
<td>0.0007</td>
<td>0.0006</td>
<td>0.0000</td>
<td>0.1212</td>
</tr>
<tr>
<td>5 ROOT UPPER STARBOARD</td>
<td>0.0105</td>
<td>-0.0002</td>
<td>0.0112</td>
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<tr>
<td>6 ROOT TRANSITION AFT STA</td>
<td>0.0005</td>
<td>0.0015</td>
<td>0.0208</td>
<td>0.0043</td>
<td>0.0146</td>
<td>0.0000</td>
<td>0.4187</td>
</tr>
</tbody>
</table>

COMPONENT COEFFICIENTS

Note: It was very difficult to determine the meaning of the data output.
Fig 4.1 Loftsman CFD data output.
Where: $K_b = $ Body interference factor, $K_w = $ Fin interference factor, $a = $ Body diameter, $s = $ Fin span.

Fig 5.1 Interference lift ratios for lift associated with pitch
Fig 5.2  SLENDER CRUCIFORM FIN
Fig 5.3 General projectile configuration

Additional nose rc/d = 0.2
Fig 5.3a Rounded nose - Not to scale. Dimensions of integral limits for a rounded nose. Integral limit = 0.01m
Fig 5.4 Angle of attack of a cylindrical body where crossflow drag begins to affect the normal force as a function of cylinder length for subsonic velocities.
\[ f = \text{FOREBODY LENGTH}/50 \quad b = \text{DIAMETER OF SPHERICAL BLUNTING}/50 \]

- F15: \( f = 3 \) \( b = 0 \)
- F16: \( f = 3 \) \( b = 0.3 \)
- F17: \( f = 3 \) \( b = 0.6 \)
- F18: \( f = 1.5 \) \( b = 0 \)
- F19: \( f = 1.5 \) \( b = 0.3 \)
- F20: \( f = 1.5 \) \( b = 0.6 \)
- F21: \( f = 1 \) \( b = 0 \)
- F22: \( f = 1 \) \( b = 0.6 \)
Fig 6.2 Optimised projectile using the methods described in Chapter Five.
**Representative Aerodynamic Coefficient Model Data**

**Input Menu**

Units are not defined - The user has to define the projectile units e.g. MKS. The aerodynamic coefficient data output from the model will default to the user inputs.

**Aerodynamic Model:**

**Input Flight Conditions:**
- Mach numbers: $M_0 := 0.4$
- Air static temp: $T := 288.2$
- Air density: $\rho := 1.225$
- Gas constants: $R := 287$
- Gas constant: $\gamma := 1.4$
- Speed of sound: $a_0 := \sqrt{\gamma R T}$
- Velocity: $V_0 := M a_0$
- Angle of attack: $\alpha := \frac{x}{180}$
- Sideslip angle: $\beta := 0$
- Roll rate: $p := 0$
- Pitch rate: $q := 0$
- Yaw rate:

**Projectile Input Geometry for Single Fin Set:**

- CofG position: $X_{cg} := 3.0$
- Total length of missile: $L := 4.7$
- X-distance from CG to base: $X_b := X_{cg} - L$
- X-distance from CG to nose: $X_n := X_{cg}$
- Wing semi-span: $s := \frac{1.1}{2}$
- Maximum value: $s := 1.1$
- Nose length: $n_l := 0.03$
- Nose rounding: $n_r := 0.3$
- Fin sweep angle leading edge: $l_e := 5$
- Fin sweep angle trailing edge: $l_e := 5$
- Body diameters at stations from the base: $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8$
- Fin span from the root chord measured from the base: $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8$
- Integral limits for a rounded or conical nose: $w_1, w_2, w_3$

Projectile configurations having rounded noses and shaped fins are catered for by using the methods detailed in Chapter Five. Because of the different projectile configurations that are possible only the basic outline input menu is shown in this figure. The output from this model is embedded into the 6 DoF trajectory model. A graphical comparison of the output from the model with Missile DATCOM and RAPPIC data is given in Appendix B.

Fig 6.3 Example of the input menu for the aerodynamic model using the method of masses techniques described in Chapter Five.
SIX DEGREE OF FREEDOM TRAJECTORY
MODEL DATA INPUT MENU

Units are not defined - The user has to define the trajectory units e.g MKS. The graphical output will default to the units used in the input. The aerodynamic coefficients are embedded in the trajectory code and are calculated in degree units.

- **Mass** \( m := 3.862 \)
- **Diameter** \( d := .1 \)
- **Cross-sectional area** \( A := \left( \frac{1}{2} \right)^2 \)
- **Axial Inertia** \( I_x := .09 \times 10^{-3} \)
- **Axial Inertia** \( I_x := .2 \times 10^{-2} \)
- **Zero lift drag** \( c_{do} := 1.8 \)
- **Lift induced drag** \( c_{da} := 0.03 \)
- **Overturning moment** \( c_{ma} := -.265 \)
- **Lift coefficient** \( c_{la} := .09054 \)
- **Yaw damping moment** \( c_{mq} := -.764 \)
- **Spin damping moment** \( c_{lp} := -5.31 \)
- **Magnus force coefficient** \( c_{mp} := 0 \)
- **Velocity** \( V := 143.2 \)
- **Launch angle** \( \theta := \frac{2.8 \pi}{180} \)
- **Gravity** \( g := 9.81 \)
- **Density** \( \rho := 1.225 \)
- **Magnus moment** \( c_{mp} := .028 \)
- **Wind x** \( w_x := 0 \)
- **Wind y** \( w_y := 5.9 \)
- **Wind z** \( w_z := 0 \)

The aerodynamic coefficient data shown above was generated using the methods described in Chapter Five. A demonstration of the graphical output from the trajectory model is provided at Appendix C.

Fig 6.4 Example of the input menu for the 6 DoF trajectory model
Fig 7.1 Experimental projectile with telescopic fin assembly
REFERENCES


5. Ashley, H. and Landahl, M. Aerodynamics of Wings and Bodies, Addison Wesley, Reading, MA, 1965


R-2
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<th>No.</th>
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<td>68</td>
<td>ESDU.</td>
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R-6
A1.0 Overview of Aerodynamic Prediction Theories

A2.0 Introduction

Through the requirement to determine the trajectory performance of different missile
customations, many different aerodynamic methods have been developed to generate
the required aerodynamic stability derivatives for missile configurations. The diversity of
aerodynamic prediction theories arose due to their applicability to different Mach
numbers, flow dimensionality and the shapes of the physical boundaries that need to be
analysed. The most notable of these aerodynamic prediction theories are shown in Table
1. These theories are listed in order of increasing accuracy cumulating in the Viscous
Crossflow method.

<table>
<thead>
<tr>
<th>THEORY</th>
<th>DIMENSION</th>
<th>TYPICAL SHAPE THAT CAN BE ANALYSED</th>
<th>Mach RANGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ackeret</td>
<td>2-D</td>
<td>Airfoils</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Busemann</td>
<td>2-D</td>
<td>Airfoils</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Shock Expansion</td>
<td>2-D</td>
<td>Simple shapes, airfoils</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Characteristics</td>
<td>2-D</td>
<td>Airfoils, bodies of revolution</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Strip</td>
<td>2-D</td>
<td>3-D Shapes</td>
<td>Any</td>
</tr>
<tr>
<td>Simple Strip</td>
<td>2-D</td>
<td>Swept wings and cylinders</td>
<td>Any</td>
</tr>
<tr>
<td>Supersonic Wing</td>
<td>3-D</td>
<td>Wings</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Conical Flow</td>
<td>3-D</td>
<td>Wings, cones</td>
<td>M&gt;1</td>
</tr>
<tr>
<td>Supersonic Lifting</td>
<td>3-D</td>
<td>Wings built of elliptical vortices</td>
<td>M&gt;1</td>
</tr>
</tbody>
</table>
The following sections provide a brief summary of the aerodynamic prediction theories listed in Table 1.

### A3.0 Ackeret’s Theory

Ackeret’s theory is based on the first order linearised potential equation. It is suitable only for rough estimates of pressure coefficients that are linear in the flow-deflection angle.

### A4.0 Busmann’s Theory

Busmann’s theory applies the equations of oblique shock waves to Prandtl-Meyer flow, which is expanded in a second order power series in the flow deflection direction. It is slightly more accurate than Ackeret’s theory.

### A5.0 Shock Expansion Theory

Shock expansion theory uses the equations of oblique shock waves and Prandtl-Meyer flow. Its use is limited to simple shapes such as airfoils. For simple shapes it produces very accurate results.

---

<table>
<thead>
<tr>
<th>Method of masses</th>
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<td>Viscous Crossflow</td>
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<td>Slender bodies</td>
<td>Any</td>
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Table 1. List of Aerodynamic prediction theories
A6.0 Method of Characteristics

The Method of Characteristics is a graphical method for solving two-dimensional or axially symmetric potential flows. Though it could theoretically be used in three-dimensions, the procedure is complicated and computationally inefficient to carry out.

A7.0 Strip Theory

Strip Theory consists of slicing any three-dimensional shape by a series of parallel planes. Flow in each plane is assumed to be two-dimensional with no interaction between strips. A two-dimensional theory can be used for each strip. The accuracy of the method is dependent on the two-dimensional theory that is used.

A8.0 Simple Sweep Theory

Simple Sweep theory is a method for obtaining the flow field for swept wings and cylinders. It consists of applying Strip Theory normal to the leading edge of swept wings and cylinders. Like Strip Theory, the accuracy of the method will depend upon the two-dimensional theory used.

A9.0 Supersonic Wing Theory

Supersonic Wing theory is based on the linearised potential flow equation. This involves resolving the flow equation into three partial differential equations, one relating to each independent space co-ordinate, and then solving the equations by placing boundary conditions at the plane of the wing.

A10.0 Conical Flow Theory
A10.0 Conical Flow Theory

Conical Flow Theory is a form of linearised theory applied to a line of constant flow quantities starting from the missile nose. The advantage of this theory is that a large number of wing flow fields can be constructed by superimposing conical flow fields with differing start positions, thus improving the accuracy.

A11.0 Supersonic Lifting Line Theory

Supersonic Lifting Line Theory consists of replacing the lifting surface with one or more elliptical vortices. This simplifies the flow field calculations remote from the wing. This theory is in essence Prandtl lifting-line theory for supersonic speeds with the major difference being that supersonic vortices replace subsonic vortices.

A12.0 Quasi-Cylindrical Theory

Quasi-Cylindrical theory is similar to Supersonic Wing Theory, because the same partial differential equations have to be solved. The only exception is that the boundary condition is taken along a cylindrical surface instead of the wing plane. Since a cylinder is any closed surface generated by a line moving parallel to a given line, many lifting surfaces can be generated thereby making it more applicable for cylindrical shapes than Supersonic Wing theory.

A13.0 Slender Body Theory

Slender Body theory is based upon Laplace’s equations. Laplace’s equation simplifies the mathematics and allows applications to three-dimensional bodies for many standard missile configurations to be made. The theory assumes that the body is pointed at the front end and either blunt or pointed at the aft end. The maximum radial distance of the body is assumed to be small in comparison to the overall length of the body. The method can be applied to either subsonic or supersonic flow speeds.
A14.0 Newtonian Impact Theory

Newtonian Impact theory is based on Newton's model of fluid flow. It is accurate for low supersonic speeds but as the Mach number increases to hypersonic speeds (M>5) it becomes very accurate. It is a very simple theory based upon the sine-squared law derived from Newton's second law of momentum.

A15.0 Viscous Crossflow Method

The Viscous Cross Flow Method is based on the crossflow drag coefficient, which is the drag resulting from a viscous cross force. The crossflow is considered due to the significant effect body vortices can have on the overall pressure distribution, which influences the body forces and moments. This method consists of adding the crossflow drag coefficient directly to the lift equation, which is determined from Slender Body Theory. It should be noted that the crossflow effect becomes significant at angles of attack in excess of six degrees. If this angle of attack is not exceeded Slender Body theory can be used.

A16.0 Summary of listed aerodynamic prediction theories

As can be seen from the list of prediction theories, the majority are not suitable for the projectile configurations under investigation. The methods that appear to have most promise in generating the required aerodynamic coefficients are Slender Body theory and the method of masses. A more detailed description of these methods is given in Chapters Two and Four.
APPENDIX B

B1.0 Introduction

The following sections detail the projectile configurations that were used to compare the method of masses aerodynamic coefficient data for spin damping $C_{1p}$ and spin driving coefficients $C_{1a}$. It should be noted that spin driving and spin damping coefficients were also calculated at supersonic speeds. The results at these supersonic speeds show a reasonable degree of correlation with the experimental data. However, the accuracy of the supersonic data should not be relied upon as the methods were not validated in this speed regime. The results are only included to demonstrate that the method could be used in the supersonic speed regime. The aerodynamic coefficients were generated using the methods detailed in Chapter Five.

The notation for the tables of results is: Experimental = Range trial data. Ch.5. = data generated by the methods detailed in Chapter Five.

B2.0 M 829 Kinetic Energy Projectile

The M829 kinetic energy projectile is a gun launched system, fired from a 120mm smooth-bore gun. The configuration has six fins and a reference diameter of 27mm (1.065 inches) and is shown in Fig (B1).
This configuration was first flight tested at the Ballistic Research Laboratory in about 1948. It is a configuration that was approved by the U.S. Army, Navy and Air Force Services as a test platform to verify simulation codes. The range test results were published by Eastman, Ref(24). The projectile shown in Fig (B2) has a diameter of 20mm (0.786 inches) and four flat fins. It should be noted that the data in Ref(24) was calculated using fin area and fin span as the reference area and length. This data was adjusted to be consistent with the other systems references which are body diameter and body cross sectional area.

B3.0 Basic Finner Configuration

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<th>Clδ (Experimental data)</th>
<th>Clδ Eq. (Ch.5)</th>
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</tbody>
</table>

B - 2
B.4.0 Hydra 70mm MK66 Army Missile (Curved wrap around fins)

The U.S. army 70mm (2.75 inch) Hydra projectile is shown in Fig (B3). The missile has three wraparound fins with partial fin cant. The main body with its rocket motor can be configured to function with different warheads. A combination of wind tunnel and flight testing was carried out on this projectile by Dahlke and Batiuk, Ref (54).

<table>
<thead>
<tr>
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<th>Clp (Experimental data)</th>
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<td>-</td>
<td>5.09</td>
<td>-</td>
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</tbody>
</table>

B.5 The GSRS Boeing Rocket

The modified Boeing General Support Rocket System (GSRS) is shown in Fig (B4). The system has a 230mm (9 inch) body diameter and four rectangular flat fins, all of which are wholly canted at 0.95 degrees for the projectile that was analysed. Wind tunnel and flight trials data was recorded by Monk and Phelps (63).
5.6 The Air Force 2.75-Inch Folding Fin Rocket

This rocket has a 70 mm (2.75 inch) body diameter with four fold out fins which deploy after firing. The fins are partially deployed with a 45 degree angle to the body axis. The fins are partially canted only at the tip to produce roll. Wind tunnel data about this system is provided by Uselton and Carman (46). A diagram of the projectile is shown in Fig (B.5).

<table>
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<td>2.49</td>
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</table>

B.7 Ballistic Research Projectile

The BRL projectile shown in Fig (B.6) was test fired from a 105mm rifled gun tube, using a discarded sabot. Three models with different fin configurations were tested. The first was a
quarter-ellipse (approximately) fin planform. The second was a rectangular planform, while the third was a clipped delta (trapezoidal) configuration. All the fin sets had four panels. The clipped delta was tested at both 45° and 90° offset angles. Range firing for fifteen BRL projectiles were reported, including cant angles of 2° and 0.2°. Five of the fifteen round were fired with a cant angle of 0.2°. These results have been excluded from this analysis because on discussion with the projectile manufacturers, it transpired that the accuracy to which the 0.2 ° cant angle was manufactured could have been in error by as much as 50%. With this large amount of manufacturing error it was not considered valid to analyse these five sets of results.

The clipped delta-fin with a 45° offset appeared to provide approximately 70% of the spin damping and spin driving values for the 90° case.

<table>
<thead>
<tr>
<th>Mach Number</th>
<th>Fin Planform/offset angle (4 fins)</th>
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B - 5
B.8 The Terrior-Recruit First Stage Vehicle

This projectile has a 457.2 mm (18 inches) body diameter and is about 90.23 m (27.5 feet) long. The projectile is shown in Fig (B.7). The projectile has four flat fin panels. Wind tunnel tests were carried out on this projectile by Rollstin, (49). It should be noted that spin driving and spin damping coefficients provided in Ref (49) are based upon a reference length of the length of the body rather than the body diameter. The results from this reference have therefore been adjusted to reflect a reference length of the body diameter.

<table>
<thead>
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<th>Mach Number</th>
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B9. Comparison of method of masses aerodynamic coefficient data with DATCOM aerodynamic prediction code data

Pages B8 - B13 in this appendix represent part of the graphical data that was used to for determining the accuracy of the methods used for calculating the aerodynamic coefficient data outlined in Chapter Five. Pages B14 - B16 show the experimental drag data for the projectile.
nose configurations shown at Fig 5.5 and described in Chapter Five. Using this experimental data and output drag data from DATCOM the drag look-up-tables were calculated.
Fig B1. M829 Kinetic Energy Projectile.

Fig B2. Basic Finner Projectile.
DIMENSIONS IN INCHES
Fig B4. The GSRS Boeing Rocket

Fig B5. The Air Force 2.75 - Inch Folding Fin rocket
Fig B6  Ballistic Research Projectile

Fig B7  Terrior - Recruit First Stage Vehicle
APPENDIX C

C1.0 Introduction

Pages C1-C3 in this appendix detail typical graphical output from the 6 DoF trajectory model described in Chapter Six. This graphical data was used to determine the accuracy of the trajectory simulations and associated aerodynamic coefficient data. The graphical data is automatically plotted from the numerical solutions of the trajectory equations. As the aerodynamic coefficient algorithms are embedded into the trajectory model, a variation in flight parameters or configuration will be graphically represented as a trajectory. Using this model variations in configuration can be instantly analysed with unsuitable configurations being disregarded. One of the most useful uses of the model was to note the variations in trajectory with a variation in initial conditions such as launch velocity, launch angle, initial yaw etc. With this data a range profile could be obtained for each condition. This information could be mounted on the launch tube by means of a range sighting system. Having this information would allow the operator a means of judging ranges. At the moment this is accomplished by means of a spotting round which is not very efficient.

Having a plot of the velocity profile enables the designer to estimate the time delay requirement for a tandem warhead charge initiator.
Configuration specification: Speed = M 0.41  xcg = 0.2  Nose = cone
L nose = 0.06m and 0.04m Body length = 0.6m. DAT = DATCOM  MOM = Method Of Masses
Cm = Pitching moment  CN = Normal force. xcp = Centre of pressure.
PROJECTILE BODY PLUS 4 RECTANGULAR FINS

FIN SPAN = 0.04m

FIN SPAN = 0.06m

Configuration specification:
- Speed = M 0.41
- xcg = 0.2
- Nose = cone
- L_nose = 0.06m
- Body length = 0.6m
- Fin chord = 0.08, 0.08m
- Fin span = 0.04 and 0.06m
- DAT = DATCOM
- MOM = Method Of Masses
- CM = Pitching moment
- CN = Normal force
- xcp = Centre of pressure

B9
**MULTIPLE FINS**

Configuration specification: Speed = M0.41  xcg = 0.2m  Nose = cone  Lnose = 0.06m  Length = 0.6m  Fin chord = 0.08m  Span = 0.04 and 0.06m

Span = 0.04m 6 - fins

Span = 0.06m 6 - fins

Span = 0.04m 6 - fins

Span = 0.06m 6 - fins
COMPARISON OF USING INTERFERENCE EFFECTS

Comparison between using and not using interference effects in the method of masses calculations.

Projectile configuration: Defined as projectile (a). in Chapter Six
BODY + FINS WITH 10 DEGREE LEADING EDGE SWEEP

Configuration specification: Body length = 0.6m. Nose = Cone. Length of nose = 0.1m. Speed = M0.4. Root chord = 0.08m. Span = 0.06m. 4 fins with 10 degree leading edge sweep. DAT = DATCOM. MOM = Method Of Masses. $C_m$ = Pitching moment. $C_N$ = Normal force. xcp = Centre of pressure.
BODY + FINS WITH 10 DEGREE LEADING AND TRAILING EDGE SWEEP

Configuration specification: Body length = 0.6m. Nose = Cone. Length of nose = 0.1m. Speed = M0.4. Root chord = 0.08m. Span = 0.06m. 4 fins with 10 degree leading edge sweep. DAT = DATCOM. MOM = Method Of Masses. CM = Pitching moment. CN = Normal force. xcp = Centre of pressure.
EXPERIMENTAL DRAG DATA FOR THE NOSE SHAPES SHOWN IN FIG (5.5) CHAPTER FIVE

Where: F = nose type, f = Forebody length / 50, b = Diameter of spherical blunting / 50
F18 $f=1.5 \quad b=0$

F19 $f=1.5 \quad b=0.3$

F20 $f=1.5 \quad b=0.6$
COMPARISON BETWEEN TRAJECTORY SIMULATION AND FLIGHT TRIAL DATA FOR PROJECTILE c.

Fig 1. SPIN PROFILE

Fig 2. PITCH PROFILE

Fig 3. VELOCITY PROFILE

Fig 4. DISTANCE TRAVELLED

Trajectory output for projectile (c). The trajectory model output has been overlaid with the flight trial data so that variations in the trajectory with projectile configuration alteration can be determined.
POLAR PLOT OF PROJECTILE c. PITCH AND YAW ANGLES

Extended flight time polar plot of pitch and yaw angles for projectile c. The plot was taken over six seconds. This length of time was taken to see how the instability of the projectile increased with time. For normal flight times of two seconds the maximum values of pitch and yaw would not exceed ten degrees.
6 DoF OUTPUT FOR FIN STABILISED SYSTEM.
PROJECTILE (c) GR/53593/A.

Trajectory results for projectile GR/53593/A as detailed in Chapter Six