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ACOUSTIC EMISSION SPECTRA ASSOCIATED WITH THE FORMATION OF BRITTLE CRACKS

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1 INTRODUCTION

Acoustic emission is a spontaneous radiation of elastic waves in solids and solid structures during some irreversible processes, such as formation and development of brittle cracks, plastic deformation, dry friction, etc. The processes of formation and growth of cracks are accompanied by the radiation of elastic waves into the solid containing the crack and into the surrounding space; the intensity of the radiation is sometimes so great as to be perceptible to the ear. The practical importance of this effect lies mainly in two aspects: it can be used for prediction of catastrophic failure of important engineering structures and also for research into solid state physics and fracture mechanics. The former aspect is especially important since radiated elastic waves provide information about developing cracks, which present the greatest danger.

In the present paper, the acoustic emission accompanying the formation of brittle cracks is investigated theoretically, using the approach earlier developed by the present author and his co-workers. This approach is based on the application of Huygens’ principle for elastic solids and on the use of suitable elastic Green's functions. In the framework of this approach, the main input information required for calculations of acoustic emission spectra is the normal displacements of the crack edges as a function of frequency and wavenumber. A precise description of this function for different situations of crack formation requires the solution of a complex problem of fracture mechanics, which is not always practical. For that reason, different approximations of this function can be used for practical calculations. A simple approximation of this kind is used in this paper for calculations of the acoustic emission spectra and directivity functions of a crack of finite length. A more refined approximation based on the model of a crack of finite size as a resonator for Rayleigh waves propagating along the crack edges and partly reflecting from the crack tips is briefly discussed. The obtained theoretical results are compared with the existing experimental measurements, where available.

2 THEORETICAL BACKGROUND

2.1 Statement of the Problem

We assume that a crack is located in the interior or on the surface of an elastically stressed solid. For example, one can consider a stressed solid to be an infinite elastic half-space. The boundary conditions of zero normal stresses must be satisfied at the edges of the crack and on the free surface of the solid.

It is convenient to represent the whole problem of acoustic radiation by a crack developing in an elastic stressed medium as a superposition of the two problems, which can be done in linear approximation: 1) the static problem for the stressed solid without a crack (this problem is of no interest for the case under consideration); 2) the problem for an unstressed solid with a crack subjected to the nonzero normal stresses \( n_0 \) applied to its edges in the absence of other sources of stresses. Here the quantities \( \sigma_{ij}^0 \) represent the stresses of the first problem calculated at the site of the crack (the stresses acting on the edges of the crack in the whole problem are equal...
to zero in this case, as expected). In what follows, we will be concerned only with the second problem, which is sufficient to describe the phenomenon of crack-induced acoustic emission.

2.2 Huygens' Principle for Elastic Media

The theoretical approach considered in this paper is based on Huygens' principle for elastic solid media\textsuperscript{10}. Applying it to the problem under consideration (for simplicity, we consider a two-dimensional case), we choose a closed contour $S$ running along the surface $z = 0$ around a cut indicated the possible path of crack propagation (this path is assumed to be known) and closed by a semicircle of infinite radius. Then the corresponding mathematical representation of Huygens' principle for time-harmonic fields, which is the analogue of the well-known Helmholtz integral theorem of classical acoustics, takes the form\textsuperscript{5}:

$$u_m(r) = \int_{S} \left[ n_{j} \sigma_{ij} G_{im}(r, r') - n_{j} c_{ijkl} u_{i}(r') G_{im,k}(r, r') \right] dS.$$ \hspace{1cm} (1)

Here the point of observation $r$ lies inside the contour $S$, $u_m$ is the displacement vector, $n_i$ is the outward unit normal to the contour, $G_{im}(r, r')$ is the dynamic Green's tensor for the unbounded elastic medium, $c_{ijkl}$ are the elastic constants, and $G_{im,k} = \partial G_{im}/\partial x_k$. As usual, the summation over repeated indexes is assumed. The integral over the infinite semicircle vanishes, so that the integration in (1) is carried out only along the edges of the cut passing through the crack and along the boundary of the half-space\textsuperscript{5}. Note that the first term in (1) vanishes in the integral along the free surface due to the stress-free boundary conditions on the surface $z = 0$. In addition, if to choose the Green's tensor that satisfies the stress-free boundary conditions on $z = 0$, then the second term in the integral over the free boundary vanishes as well, and the remaining integration should be carried out only over the crack area.

2.3 Plane Crack in an Infinite Medium

In a number of situations, e.g., in the case of crack propagation in an unbounded medium, it is convenient to choose the Green's tensor in such a way that a certain type of specific boundary conditions is satisfied on the surface located between the edges of the crack. In what follows, we limit our consideration by a two-dimensional problem for a plane crack in an infinite elastic medium (Figure 1), which is of fundamental importance.

Let us introduce a Cartesian coordinate system with the $x$ axis directed along the plane of the crack and with the $z$ axis directed along the normal to it (see Figure 1) and assume that an unbounded elastic medium is subjected to the action of normal tensile stresses $\sigma_{zz} = \sigma(t)$. 

Figure 1. Plane crack in an infinite elastic medium.
According to the previous discussion, the action of these stresses is equivalent to the action of stresses $\sigma(t)$ applied to the edges of the crack. Due to the symmetry of the problem about the x axis in the given case, it is sufficient to investigate the radiated field only in one half space, for example in the lower one. As a result, we have mixed boundary conditions on the surface $z=0$:

$$
\sigma_{zx} = -\sigma(t) \text{ for } |x| < l; \quad \sigma_{zx} = 0 \text{ for } |x| \leq \infty \quad \text{and} \quad u_z = 0 \text{ for } |x| > l.
$$

To apply Huygens' principle to this problem we use the contour S as shown in Figure 1. The general expression (1) then takes the form

$$
u_{m}(x,z) = \int_{-\infty}^{\infty} \left[ -\sigma_{zz}(x')G_{zm}(z, x - x') + c_{lzm}u_{i}(x')G_{lm,k}(z, x - x') \right] dx'.
$$

For isotropic solids, it is more convenient to operate with Lame potentials $\varphi$ and $\psi$, rather than with displacements $u_{m}$. The displacement components $u_{x}$ and $u_{z}$ of the acoustic field radiated by the developing crack can be expressed in terms of the Lame potentials $\varphi$ and $\psi$ as follows:

$$
u_{x} = \frac{\partial \varphi}{\partial x}, \quad \nu_{z} = \frac{\partial \varphi}{\partial z} + \frac{\partial \psi}{\partial x}.
$$

One can also introduce the Green's tensor for potentials $\varphi$ and $\psi$, which is related to the abovementioned Green's function for displacements, $G_{lm}$, via the expressions (4). For the problem under consideration, we assume that the Green's tensor for potentials should satisfy the usual wave equations for potentials

$$
\frac{\partial^{2} \varphi}{\partial x^{2}} + \frac{\partial^{2} \varphi}{\partial z^{2}} - \frac{1}{c_{l}^{2}} \frac{\partial^{2} \varphi}{\partial t^{2}} = 0,
$$

$$
\frac{\partial^{2} \psi}{\partial x^{2}} + \frac{\partial^{2} \psi}{\partial z^{2}} - \frac{1}{c_{l}^{2}} \frac{\partial^{2} \psi}{\partial t^{2}} = 0,
$$

and the following boundary conditions at $z = 0$:

$$
\sigma_{zx} = 0 \quad \text{and} \quad u_z = \delta(x-x') \delta(t-t'),
$$

where $c_{l}$ and $c_{t}$ are the velocities of longitudinal and shear elastic waves respectively.

Finding Green's tensor for potentials and its substitution into the integral (3), which should be represented in terms of inverse Fourier transforms over $\omega$ and $k$, gives the following expressions for the frequency spectra of the potentials $\varphi(\omega)$ and $\psi(\omega)$:

$$
\varphi(\omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_{z}(\omega,k) \frac{2k^{2} - \omega^{2} / c_{l}^{2}}{(\omega^{2} / c_{l}^{2} - k^{2})^{1/2} \omega^{2} / c_{l}^{2}} e^{i(\omega^{2} / c_{l}^{2} - k^{2})^{1/2} z + ikx} \, dk,
$$

$$
\psi(\omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_{z}(\omega,k) \frac{2k}{\omega^{2} / c_{l}^{2}} e^{i(\omega^{2} / c_{l}^{2} - k^{2})^{1/2} z + ikx} \, dk.
$$
Introducing a polar system of coordinates, $R$ and $\theta$, where angle $\theta$ is counted from the normal to the crack surface, one can express the radial and tangential components of displacements, $u_R$ and $u_\theta$, respectively, in the far field of radiation via the potentials $\phi$ and $\psi$ defined by the equations (8) and (9):

\[ u_R \approx \frac{\partial \phi}{\partial R} = (i\omega/c_1)\phi, \tag{10} \]
\[ u_\theta \approx \frac{\partial \psi}{\partial R} = -(i\omega/c_1)\psi. \tag{11} \]

Obviously, the displacements $u_R$ and $u_\theta$ in equations (10) and (11) are associated largely with longitudinal and shear elastic waves respectively. As it can be seen from equations (8) and (9), added by equations (10) and (11), the Lamé potentials and displacements of radiated longitudinal and shear waves are fully determined by the spectrum $u_z^0(\omega, k)$ of the normal displacements of the crack edges, where the presence of the wave number $k$ in $u_z^0(\omega, k)$ accounts for current geometrical dimensions of the crack, in particular for its opening as well as for its growth from the nucleate stage to its macroscopic length.

The determination of the function $u_z^0(\omega, k)$ is a complex and not completely solved problem of fracture mechanics. Therefore, simplified models of cracks opening and growth can be used, based on the available numerical or experimental data. One of the simplest models is the one where it is assumed that the crack initially grows instantaneously (at an infinite rate) as a slot to the length $2l$, and then its edges spread apart, eventually approaching their static position. This crack-opening process causes the acoustic emission from the crack, within the framework of the given model.

3 \hspace{1em} CALCULATIONS OF ACOUSTIC EMISSION SPECTRA

In this section, we consider directivity and spectra of acoustic emission signals for two simple models of crack developing, which can be described by the appropriate function $u_z^0$.

3.1 \hspace{1em} Monotonous Opening of a Crack of Constant Length

Let us consider the simplest special case of opening of a crack with tip coordinates $l$ and $-l$ under the action of uniform tensile stresses $\sigma(t) = \sigma h(t)$, where $h(t)$ is Heaviside's step function. The following simple approximation for the function $u_z^0(x, t)$ can be used:

\[ u_z^0(x, t) = \begin{cases} 
  u_z^0(t), & |x| \leq l \\
  0, & |x| > l
\end{cases} \tag{12} \]

where

\[ u_z^0(t) = \begin{cases} 
  st, & 0 \leq t \leq 2l/c_1 \\
  s2l/c_1, & t > 2l/c_1
\end{cases} \tag{13} \]

and $s = \sigma/pc_l$. The rate $s$ of opening of the crack in equation (13) is determined as the result of dividing the applied tensile stress $\sigma$ by the wave impedance $pc_l$ of the medium in respect of the application of normal stresses. The crack opens up at this rate until it reaches the level $s2l/c_1$ corresponding to the well-known steady state (Westergaard’s) solution. The real elliptical form of crack opening is disregarded in the above model.

Application of the double Fourier transform
to the expressions (12) and (13) results in the following formula:

\[
u_z^0(\omega, k) = \frac{2isl^2}{\pi\omega l} e^{\frac{\omega l}{c_l}} \sin \frac{\omega}{l} \frac{\sin kl}{kl}.\]

(15)

Note that \(u_z^0(\omega, k)\) in equation (15) can be also represented as

\[
u_z^0(\omega, k) = u_z^{01}(\omega)u_z^{02}(k),\]

(16)

where the function

\[
u_z^{01}(\omega) = \frac{1}{2\pi} \int_0^\infty t u_z^0(t)e^{\frac{\omega l}{c_l}} dt = \frac{isl}{\pi\omega l} e^{\frac{\omega l}{c_l}} \frac{\sin \omega}{l} \frac{l}{c_l},\]

(17)

describes the frequency spectrum associated with the function \(u_z^0(t)\) defined by equation (13), whereas

\[
u_z^{02}(k) = \int_{-l}^{l} e^{-ikx} dx = 2l \frac{\sin kl}{kl}.\]

(18)

accounts for the geometrical dimensions of the crack, as follows from equation (12). The absolute value of \(u_z^{01}(\omega)\) as a function of frequency \(f = \omega/2\pi\) is shown in Figure 2.

![Figure 2. Absolute value of \(u_z^{01}(\omega)\) as a function of frequency.](image-url)
The parameters used for calculations in Figure 2 were as follows: the material – steel (1020), $c_l = 5893 \text{ m/s}$, $c_t = 3240 \text{ m/s}$, $\rho = 7820 \text{ kg/m}^3$, $\sigma = 3 \times 10^8 \text{ Pa}$, $l = 0.01 \text{ m}$.

Substituting the formula (15) into the expressions (8) - (11) and applying the steepest descent method for the calculation of the integrals over $k$ in the far field of radiation, one can obtain the following expressions for the frequency spectra of the displacements $u_R$ and $u_\theta$:

\[
\begin{align*}
 u_R(R, \theta, \omega) &= \frac{i \omega l^2}{\pi^2 c_l^2} e^{c_l l} \frac{\sin(\omega l \sin \theta)}{c_l} \sin \frac{\omega l}{c_l} \left( 2 \frac{c_l^2}{c_t^2} \sin^2 \theta - 1 \right) \left( -\frac{2 \pi i}{(\omega / c_l) R} \right)^{1/2} e^{c_l l} \\
 u_\theta(R, \theta, \omega) &= -\frac{i \omega l^2}{\pi^2 c_l c_t} e^{c_l l} \frac{\sin(\omega l \sin \theta)}{c_t} \sin \frac{\omega l}{c_t} \sin 2\theta \left( -\frac{2 \pi i}{(\omega / c_l) R} \right)^{1/2} e^{c_l l}.
\end{align*}
\]

(19)

(20)

Here $R$ and $\Theta$ are the polar coordinates of the observation point: $x = R \sin \Theta$ and $z = R \cos \Theta$. The analysis of the expressions (19) and (20) shows that directivity pattern of longitudinal waves (formula (19)) has a maximum in the normal direction to the crack (at $\Theta = 0$) for all spectral components, whereas shear waves (formula (20)) are not generated in the normal direction at all, in agreement with the symmetry of the problem. The results of the numerical calculations of radiation patterns of longitudinal and shear waves by the crack under consideration, according to formulas (19) and (20), are shown in Figure 3 for some typical values of the parameters given below.

Figure 3. Directivity patterns of longitudinal waves (solid curve) and shear waves (dashed curve) radiated by an opening crack (the plane of the crack is in vertical direction).
The values of $R$ and $f$ used in the calculations of Figure 3 were chosen as 0.4 m and 70 kHz respectively. Other parameters were the same as in Figure 2.

The frequency spectra of the radiated longitudinal and shear waves calculated using equations (19) and (20) are shown in Figure 4 for the value of angle $\theta = \pi/12$. The values of other relevant parameters are the same as those used for calculations in Figure 3.

![Frequency spectra of longitudinal waves (solid curve) and shear waves (dashed curve) radiated by an opening crack of finite length $2l$.](image)

As it can be seen from Figure 4, the frequency spectra of longitudinal and shear waves are similar to each other. And there are the same zero values for both spectra defined by the equations $\sin(\omega l/c) = 0$ and $\sin[(\omega l/c)\sin\theta] = 0$.

### 3.2 Oscillating Opening of a Crack of Constant Length

A number of numerical calculations and experimental observations show that cracks of finite length are opened to its static value not monotonously, as was considered in the previous section, but with a few decaying oscillations responsible for the appearance of a noticeable maximum in the corresponding frequency spectrum of crack opening $u_0(\omega)$. The characteristic frequency $\omega_0$ of such oscillations depends on the length of the crack $2l$ and one of the elastic wave velocities. It has been proposed earlier that the physical mechanisms of such oscillations can be associated with resonance elastic phenomena taking place at the edges of a crack.

One of the possible models of such resonant behaviour is the one considering a crack as a resonator for Rayleigh surface waves propagating symmetrically along both crack’s edges and reflecting from the crack’s tips, loosing part of their energy to radiation of longitudinal and shear waves, thus contributing to the overall acoustic emission. The reflection coefficients of Rayleigh waves from the tips are rather low, and it was estimated that the resulting value of $Q$ of a crack considered as a resonator is only about 3. The value of the lowest order resonant frequency of a crack $\omega_0$ can be evaluated as $\omega_0 = (\pi c R/4l)$, where $c_R$ is Rayleigh wave velocity. The above-
mentioned resonance behaviour of a crack result in the appearance of the corresponding frequency peaks in the associated acoustic emission spectra. Not discussing this issue in detail, for the sake of shortness, it is worth to mention that the comparison of the predicted crack resonant frequencies with the results of experimental measurements of the acoustic emission spectra carried out for initiated cracks with different values of 2l shows their satisfactory agreement. This fact can be used for practical evaluations of sizes of developing cracks using the acoustic emission spectra.

4 CONCLUSIONS

In this paper, the acoustic emission spectra associated with the formation of brittle cracks of finite length have been investigated theoretically using the approach based on the application of Huygens’ principle for elastic solid media. As a result of the suitable choice of Green's tensor, the main input information required for calculations of the acoustic emission spectra from a crack developing in an unbounded elastic medium is the normal displacement of the crack edges as a function of frequency and wavenumber.

The obtained theoretical predictions have been illustrated by numerical calculations of the frequency spectra and directivity functions of acoustic emission signals for a simple model of crack opening. A more refined model of crack opening, that accounts for edge oscillations and considers a crack of finite size as a resonator for Rayleigh waves propagating along the crack edges and partly reflecting from the crack tips, has been briefly discussed and compared with the existing experimental measurements.

5 REFERENCES

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