Microwave group delay equalizers

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MICROWAVE GROUP-DELAY EQUALIZERS

by

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Submitted for the degree of Doctor of Philosophy
of Loughborough University of Technology.

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A survey of the present state of microwave group-delay equalizers is presented. Based on this, a number of equalizers for use at microwave and millimetric wave frequencies are investigated, both theoretically and experimentally. A microwave group-delay measuring equipment is described, and a comprehensive treatment of possible sources of error presented.
I would like to thank my project supervisors for their encouragement throughout the course of this work.

I would also like to express my appreciation for the invaluable assistance and guidance provided by many people of the Marconi Co. Ltd. during my experimental work, especially those of Microwave Research and Engineering Group who allowed such free use to be made of their equipment.

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CHAPTER 1

INTRODUCTION

1.1 The definition of group-delay
1.2 Group-delay for various media
1.3 Selection of the transfer function
1.4 Reasons for, and applications of group-delay equalizers
1.5 Disadvantages of operational equalizers
1.1 The definition of group-delay

Two conditions must be satisfied by a propagation path if a transmitted signal is to remain undistorted. Firstly, over the frequency interval of interest, which is taken here to be the band containing the signal energy, the amplitude of the transfer function of the path must be constant. Secondly, over the same frequency interval, the phase of the transfer function must be a linear function of frequency. In this report, the devices used to fulfil this second restriction will be investigated.

A convenient measure of the linearity of the phase characteristic is the group-delay, \( t_g \), defined by the following equation

\[
 t_g = \frac{-d\phi}{d\omega}
\]

where \( \phi \) is the phase of the transfer function at an angular frequency \( \omega \). From this definition, it follows that a linear phase characteristic implies a constant group-delay.

1.2 Group-delay for various media

The medium considered may be unbounded, such as the atmospheric propagation employed in microwave line-of-sight links, or bounded, such as in a conventional component. For the moment, the definition of the transfer function is not attempted, a full discussion being presented in Section 1.3.

The input and output ports in the medium are spatially fixed, and the electric field at the output, \( E(t) \), is taken as

\[
 E(t) = A \exp(j(\omega t + \delta))
\]

where \( \omega \) is the angular frequency, \( \delta \) is the phase at the output port with respect to the input port at time \( t = 0 \), and \( A \) is an arbitrary constant. Such a field is monochromatic, and
the surfaces of constant phase have a propagation time from
input to output, called the phase-delay, $t_p$, given by
\[ t_p = \phi / \omega \] (1.3)

A wave train of finite length, such as a radar pulse, cannot be expressed in the simple harmonic form of equation (1.2), but may be resolved into a continuous frequency spectrum, $\Phi(\omega)$, by means of a Fourier integral,
\[ \Phi(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(t) \exp(-j\omega t) dt \] (1.4)

There are, therefore, an infinity of simple harmonic waves of the form of equation (1.2) associated with the wave train, each of which has a phase-delay given by equation (1.3). The wave train itself, however, has no specific value of phase-delay. If the frequency spectrum of the wave train is such that, over the frequency interval occupied, $\phi$ is a linear function of $\omega$, a propagation delay, termed the group-delay, $t_g$, can be assigned to the wave train. This is defined by¹
\[ t_g = \frac{d\phi}{d\omega} \] (1.5)

For example, with a carrier which is sinusoidally amplitude modulated by a low frequency, the propagation time of a point on the modulation envelope is given by equation (1.5). If the linear phase condition is not satisfied, the group-delay of each spectral component is different, and the modulation envelope at the output is distorted.

If the envelope has become distorted, the signal is said to have suffered dispersion. There are three types of dispersive transmission, depending on the form of the $\phi - \omega$ characteristic of the medium, and these are shown in Fig. 1.1. Curve 'a' is for a non-dispersive medium, where the group and
a - non-dispersive transmission
b - normal dispersion
c - anomalous dispersion

FIG 11  Types of dispersion
phase-delays are identical. Curve 'b' is for "normal" dispersion, where the group-delay is greater than the phase delay; an example of this being in a section of lossless waveguide. Curve 'c' is for "anomalous" dispersion, where the group-delay is less than the phase-delay. The anomalous type of dispersion can occur in a lossy transmission line, or in propagation through the atmosphere if the frequency approaches one of the natural relaxation frequencies of the water molecule.

1.3 Selection of the transfer function

In this section we consider the selection of a transfer function such that the group-delay so obtained has a physical meaning.

A two-port device may be approached from a current-voltage or a wave point of view, as shown in Fig. 1.2. In the figure, \( a_1 \) and \( a_2 \) are the electric field intensities of the wave incident on ports 1 and 2, whilst \( b_1 \) and \( b_2 \) are the electric field intensities of the waves leaving the two ports. By definition we have

\[
\begin{align*}
V_1 &= (a_1 + b_1)k_1 \\
V_2 &= (a_2 + b_2)k_1 \\
I_1 &= (a_1 - b_1)k_2 \\
I_2 &= (a_2 - b_2)k_2
\end{align*}
\]  

(1.6)

where \( k_1 \) and \( k_2 \) are scaling factors.

For most applications, the quantity of interest is the phase of wave \( b_2 \) relative to that of wave \( a_1 \). These two waves are linked by means of the scattering matrix, which is defined as follows

\[
\begin{bmatrix}
b_1 \\
b_2
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} \\
S_{21} & S_{22}
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2
\end{bmatrix}
\]  

(1.7)
FIG 1.2 Characterization of a two-port device
The device is assumed to have a matched load, and so $a_2 = 0$. Then by the definition above,

$$t_g = -\frac{1}{\omega} \left[ \text{Arg}(S_{21}) \right]$$  \hspace{1cm} (1.8)

where the operator Arg indicates that the phase of the quantity in brackets is to be taken.

If the device is matched at input and output, then $b_1 = a_2 = 0$, and equation (1.6) gives

$$\frac{V_2}{V_1} = \frac{b_2}{a_1} = S_{21}$$  \hspace{1cm} (1.9)

With a perfectly matched device, therefore, the voltage transfer function may be used to compute the group-delay. This is used in the case of the ideal resonant ring discussed in Chapter 5. With an unmatched device, such as the hybrid-ring equalizer of Chapter 6, the scattering matrix element must be used as the transfer function.

Even the definition using the scattering matrix, however, is not wholly satisfactory, since in practice the output port may not be perfectly matched. Since the requirements for an equalizer are that it be a good match and have low loss, the value of $a_2$, the reflection from the load, will be small. If the device is a reasonable match, the value of $S_{22}$ will also be small, and so the product $a_2 S_{22}$ may be neglected in comparison with $a_1 S_{21}$.

1.4 Reasons for, and applications of group-delay equalizers

From the previous sections it is seen that, for a propagation path to be distortionless, the group-delay must be constant over the frequency interval of interest.

The largest contribution to a non-constant group-delay in a communications complex is from any frequency selecting
filters present. These filters normally consist of resonators coupled together to give a required insertion-loss characteristic. As the band edges are approached, the rate of change of phase shift increases, resulting in a peak in the group-delay characteristic. The majority of such filters are of the minimum-phase type, and so have a group-delay characteristic and an insertion-loss characteristic which are mutually dependent. Fig. 1.3 shows one such set of curves for a Tschebychev band-pass filter, and it is found that as the slope of the insertion-loss skirts increases, so do the maxima of the group-delay curve. If good selectivity is required, therefore, large group-delay variations will be incurred.

In a frequency modulated system, a non-constant group-delay characteristic results in the generation of intermodulation noise. Due to the complexity of the mathematics involved, the noise due to a given group-delay characteristic can be computed only for the cases of linear and parabolic curves. The group-delay specification for a device, therefore, normally consists of a linear component, a parabolic component, and a residual ripple level.

Steep skirted filters, with their attendant group-delay variations, are extensively used for channel selection in satellite relay networks. The signal processing portion of a satellite operates solely at microwave frequencies, the up-path at 6 GHz, and the down-path at 4 GHz. The group-delay of the satellite filters introduces unacceptable signal distortion, which results in a degradation of the signal to noise ratio. Ideally the filters would be equalized by a group-delay equalizer operating at microwave frequencies, which would result in the required insertion-loss and group-delay characteristics
A Tschebychev band-pass filter

FIG 1.3 A Tschebychev band-pass filter

- Insertion-loss
- Group-delay

3dB point
Centre frequency
3dB point
being achieved. Such an equalizer must have a flat insertion-loss with an acceptable level of attenuation. Due to the lack of suitable equalizers, it is customary to compensate for the satellite by pre-distorting the signal at the intermediate frequency (i.f.) stage of the transmitting ground station. This technique has a major drawback, since the frequency converter from 70 MHz to 6 GHz is not distortion free. If frequency modulation is employed, the amplitude of the signal at the i.f. is independent of time. The pre-distortion applied, however, alters the relative phases of the spectral components, and so the resultant amplitude is no longer constant, but varies with time. If the frequency converter is such that amplitude modulation on the input signal produces spurious phase modulation on the output signal, distortion is permanently introduced, or "frozen" into the signal by the converter. For this reason, it is advantageous to equalize the microwave elements of the network at microwave frequencies.

With the Intelsat IV series of synchronous satellites, the channel selecting filters have bandwidths of only 40 MHz, with very steep skirts. Due to the large delay variations involved, each filter has a microwave equalizer of the circulator-reactive-termination type, discussed in Chapter 3, incorporated with it in the satellite. With the rapid increases in the capacities of such networks, the requirement for a minimum of intermodulation noise is resulting in very stringent constraints being placed on the allowable variations in group-delay. High performance group-delay equalizers are required, therefore, at microwave frequencies, and a considerable amount of work is needed if they are to be developed.
As was mentioned previously, the insertion-loss and group-delay of a minimum-phase filter are interdependent, theoretically one is the differential with respect to frequency of the Hilbert transform of the other. The minimum-phase label is used to signify that there are no zeros of the transfer function of the filter in the right half of the complex frequency plane. If, however, zeros of the transfer function are allowed on the right half-plane, the insertion-loss and group-delay can be separately specified. Such a filter is known as a non-minimum-phase filter, and these have recently received a great deal of attention. Such filters are inherently superior to a filter-equalizer combination, since the cavities have similar unloaded Q's. As the absorptive loss of a filter is proportional to the group-delay, the non-minimum-phase filter can be designed to have both a constant absorptive loss, and a constant group-delay over much of the passband. On the debit, there is the difficulty of designing and setting up such filters, the synthesis techniques still being in a formative stage. If such filters achieve their predicted performances, the requirement for microwave equalizers is satellite networks will not be so urgent.

A second field where such equalizers may be of importance is in communication links using overmoded circular waveguides at millimetric wavelengths. Here, the use of bandwidths of the order of 500 MHz necessitates the use of intermediate frequencies in the low microwave region. Due to the dispersion of the overmoded waveguide, large repeater separations can result in an appreciable variation in group-delay over a 500 MHz bandwidth. Here, then, we have a requisite for an equalizer operating in either the millimetric band or at lower microwave frequencies. The effect of group-delay on signal deterioration
can be reduced by utilizing some form of pulse-code modulation, but if very low error rates are to be attained, equalization must be used. A millimetric wavelength equalizer is discussed theoretically in Chapter 7, but at the present time equalization at the intermediate microwave frequency looks more attractive.

1.5 Disadvantages of operational equalizers

The majority of the equalizers investigated in this report are fabricated in stripline form. A cross-section of the basic transmission line is shown in Fig. 1.4, where the dielectric is either air, or irradiated polyolefin. In later chapters it is shown that, for the equalizers studied, the insertion-loss is a function of the group-delay, increasing with an increase in group-delay, and vice-versa. This also appears to be true for all equalizers now in use, and so they depart from the ideal case in that insertion of them into a propagation path will alter the overall insertion-loss characteristic. Whether such changes are allowable depends on the application, and the system designer must decide what type of equalizer, if any, will meet his requirements.

With the lumped-constant equalizers discussed in Chapter 2, it is not too difficult to obtain a well matched input and output. With microwave equalizers, however, the situation is more complex. Although the theoretical analysis may predict a perfectly matched device, the realization of this condition may not be easily achieved. Here again, it may be that the system is such that an equalizer with a non-unity input VSWR is acceptable. If not, isolating elements such as a circulator or isolator may overcome the problem.
FIG 1.4 The construction of stripline
REFERENCES


CHAPTER 2

A REVIEW OF GROUP-DELAY EQUALIZERS

2.1 Introduction
2.2 The low frequency bridged-tee equalizer
2.3 Microwave waveguide equalizers
2.4 Microwave stripline equalizers
2.5 Conclusions
2.1 Introduction

Existing microwave equalizers can be divided into two categories: those devices which contain two independent modes of resonance, and those which do not; however, the division is sometimes difficult to apply to a device. The resonant ring discussed in Chapter 5, for example, appears at first to contain only one mode of resonance, and it is only after experimental investigation that the two orthogonal-modes are revealed.

As the following chapters will show, it is, in general, more difficult to achieve good matches with the dual-mode equalizers than with the remainder, due mainly to the latter usually being associated with non-reciprocal devices.

The low frequency bridged-tee equalizer is dealt with in some length since it is used as a performance standard for the equalizers studied in later chapters. The circuit used is the one incorporated in the majority of microwave ground links, although the use of lattice transformations can rearrange the elements into a number of alternative configurations.

2.2 The low frequency bridged-tee equalizer

The prototype lumped-constant equalizer used is the lattice section shown in Fig. 2.1. It has been shown that the circuit has a purely resistive iterative impedance at all frequencies, and consequently, if there are no losses, the insertion-loss is constant. It is, however, more convenient to have one terminal common to both input and output ports. Use of lattice transformations changes the above circuit into the one shown in Fig. 2.2a, the bridged-tee equalizer, which has terminal properties identical to those of the lattice section.

Application of network theory shows that, when terminated
FIG 2.1 The lattice equalizer

$C_a L_a = C_b L_b$

(Broken lines indicate symmetry of components)
a - The bridged-tee equalizer

b - The pole-zero plot of equalizer

\[ s = \sigma + j\Omega \]

- \( \sigma \) - zero of transfer function
- \( \sigma \) - pole of transfer function

FIG 2.2
in its iterative impedance, the voltage transfer function is given by:

\[
\frac{V_o}{V_{in}} = \frac{A (s - \sigma - j \omega_0)(s - \sigma + j \omega_0)}{(s + \sigma - j \omega_0)(s + \sigma + j \omega_0)}
\]  

(2.1)

where \( s \) is the complex frequency, \( A \) is an arbitrary constant, and both \( \sigma \) and \( \omega_0 \) are constants determined by the component values of the circuit. The pole-zero plot of the transfer function, which is, incidentally, equivalent to \( S_{21} \) for the circuit, is shown in Fig. 2.2b.

Substituting a real frequency for \( s \) in equation (2.1):

\[
s = j\omega
\]  

(2.2)

the phase of the transfer function, \( \phi \) is given by:

\[
\phi = -2 \tan^{-1}\left[\frac{(\omega - \omega_0)/\sigma}{\sigma - 2 \tan^{-1}\left[\frac{(\omega + \omega_0)/\sigma}{\sigma}\right]}ight]
\]  

(2.3)

For most equalizers,

\[
|\omega - \omega_0| \ll \omega_0 \gg \sigma
\]  

(2.4)

and so we can reduce equation (2.3) to

\[
\phi = -2 \tan^{-1}\left[\frac{(\omega - \omega_0)/\sigma}{\sigma}\right]
\]  

(2.5)

We are effectively, therefore ignoring the contribution from the lower pole-zero pair.

The group-delay, \( t_g \), is given by

\[
t_g = -\frac{d\phi}{d\omega}
\]

\[
\therefore t_g = \frac{2\sigma}{\sigma^2 + (\omega - \omega_0)^2}
\]  

(2.6)

By differentiating equation (2.6) with respect to \( \omega \), the maximum group-delay is found to occur when \( \omega = \omega_0 \), and is given by

\[
|t_g|_{\text{max}} = \frac{2}{\sigma}
\]  

(2.7)
The "bandwidth" is defined as the separation in frequency of the points where the group-delay is one half of the group-delay maximum. Insertion of this condition into equation (2.6) shows that the two frequencies are given by:

\[ \omega = \omega_0 \pm \sigma \]  

(2.8)

i.e. the bandwidth is \(2\sigma\), and the product of the maximum group-delay and the bandwidth is a constant. A group-delay \(Q\), \(Q_D\), is defined in the conventional manner as

\[ Q_D = \frac{\omega_0}{2\sigma} \]  

(2.9)

By substituting \(\omega = \omega_0\) and \(\omega = \infty\) into equation (2.5), the area under the group-delay curve, over this interval, is found to be \(2\pi\) radians, irrespective of the values of \(\omega_0\) and \(\sigma\). This property is made use of in Chapter 8, for when a given group-delay characteristic is to be synthesized by a cascade of such equalizers, the equal-areas property enables a rough estimate to be made of the number of equalizers required.

A typical set of group-delay curves is shown in Fig. 2.3. A point of importance is that the phase and group-delay characteristics of a true, lossless, all-pass network are determined solely by the pole-zero plot of the transfer function. This is of great help when dealing with cascades of such devices since only the two-parameter locations of the pole-zeros need be considered, the group-delay contribution from each section being completely specified by these two quantities.

When the parasitic resistances associated with the inductors and capacitors are taken into account, it is found that the insertion loss of the circuit increases with an increasing group-delay. A second effect, which is not so obvious, is that the inclusion of the loss mechanisms results in the group-delay being higher than its value for the lossless case. This effect is negligible for low values of \(Q_D\) and losses, but can become
FIG 2.3 A family of group-delay curves
appreciable for high values of $Q_D$ and losses.

In the transmission line equalizers to be discussed later, the losses can be incorporated in the complex propagation constants of the transmission lines. If we assume a pole-zero plot similar to that of Fig. 2.2b, the complex propagation coefficient is equivalent to shifting the imaginary frequency axis of the pole-zero plot a distance along the real axis which is dependent on the attenuation coefficient, as shown in Fig. 2.4. The operating point now is one such as $P$, and if it is assumed that the lower pole-zero can be neglected, the phase of the transfer function, $\phi$, is given approximately by

$$\phi \approx -\tan^{-1}\left[\frac{\omega_0 - \omega}{\sigma - \alpha}\right] - \tan^{-1}\left[\frac{\omega_0 - \omega}{\sigma + \alpha}\right]$$

(2.10)

Differentiating equation (2.10) with respect to $\omega$, and substituting

$$\Delta = \omega_0 - \omega$$

the group-delay is given by

$$t_g = \frac{\sigma - \alpha}{(\sigma - \alpha)^2 + \Delta^2} + \frac{\sigma + \alpha}{(\sigma + \alpha)^2 + \Delta^2}$$

(2.11)

The maximum of group-delay is found to occur when $\Delta = 0$, and is given by

$$t_g\bigg|_{\text{max}} = \frac{2\sigma}{\sigma^2 - \alpha^2}$$

(2.12)

Thus, as the losses increase, so $\alpha$ becomes greater, and the maximum of group-delay exceeds its value for no loss.

2.3 **Microwave waveguide equalizers**

Pierce was the first person to suggest using the cut-off properties of waveguide to produce a group-delay equalizer. The device, shown in Fig. 2.5, consists of a circulator with one port terminated in a length of tapered waveguide. As the frequency increases, so the wave penetrates further into the
FIG 2.4 Pole-zero plot with losses
FIG 2.5 The circulator-tapered-waveguide equalizer
taper before being totally reflected at its cut-off dimension. By producing a suitable taper, any reasonable, monotonically increasing group-delay curve can be generated.

Tang's produces an approximate synthesis procedure based on the assumption that the wave is totally reflected at the plane of its cut-off dimensions. By replacing the taper with a large number of discrete steps, a good approximation to the taper performance is obtained. The results, however, show a marked ripple when designed for a linear group-delay curve, and Tang offers no way of overcoming the discrepancies. Also, due no doubt to a printers error, all values which should be in nanoseconds in his paper have been printed as microseconds.

Woo approaches the problem from a slightly different viewpoint. He uses the fact that a metal rod displaced from the centre of a circular waveguide increases the cut-off frequency of the TE_{11} mode; the frequency increasing with the displacement from the centre. His circulator termination, therefore, consists of a circular waveguide of constant dimensions in which is supported a slanting metal rod. The position of the rod is perturbed by means of a series of strings arranged along the rod, thereby allowing a certain degree of variation to be introduced into the group-delay curve. As with Tang, the synthesis procedure of Woo rests on the assumption of complete reflection at the cut-off dimension, and, again for a linear taper, the experimental and theoretical curves have considerable ripple. Since the ripple is worst at the higher frequency end, the cause is probably due to the waves penetrating further than the cut-off dimensions, the presence of the end of the taper magnifying the effect.

Both devices are dimensionally critical, and Tang's equalizer requires the use of electroforming techniques. Similar
work to that of Tang, carried out by the G.P.O., has verified the need for dimensional tolerances of ± 0.0003 in. The manufacturing cost of such a taper, including the cost of the stainless steel mandrel on which the copper is electrodeposited, is approximately £800. Unless a given taper is required in quantity, the cost is a factor which must detract from the appeal of such an equalizer.

Waldron considers theoretically the possibility of using backward waves, in a cylindrical waveguide containing a concentric dielectric rod, to obtain a linear delay characteristic. The delay variations obtained, however, are non-linear, and of the wrong slope to be of practical interest. This, coupled with the problem of efficiently launching the necessary mode, renders the device of little interest.

We now turn to the equalizers which contain two modes of oscillation, the majority of which are described by Cohn in his patent paper.

Merlo analyses a waveguide version of the directional coupler-resonator equalizer shown in Fig. 2.6. In this device, the two modes of oscillation are contained in two separate resonators, the arrangement being such that the two waves reflected from the resonators back to the input are in anti-phase, and so cancel. Merlo shows theoretically that for a single cavity termination, the group-delay characteristic is similar to the bridged-tee equalizer. If, therefore, an arbitrary delay characteristic is to be synthesized, a separate directional coupler is required for each equalizing section - a constraint which results in heavy, bulky equalizers. Furthermore, as will be shown with the hybrid-ring equalizer, the cavities must be identical if a high input return loss is to be obtained. Merlo also considers the use of multiple
FIG 2.6 The directional-coupler-resonator equalizer
resonator terminations to obtain more general delay curves, and this problem is investigated further in Chapter 3.

The waveguide version of the orthogonal-mode resonator equalizer proposed by Cohn is investigated by Abele and Wang. This consists of a cylindrical cavity connected to the broad-wall of a waveguide section by means of a coupling hole at one end. This aperture is situated at a point on the broad wall where the TE$_{1,0}$ mode in rectangular guide has a local circularly polarized field, which excites the two orthogonal, degenerate, TE$_{1,1,1}$ modes in the resonator. The analysis performed by the authors is based on small aperture theory, however the apertures required to achieve low enough Q's are large enough to render the theory inapplicable. A large screw beneath the coupling hole to increase the coupling, and tuning screws in the resonator, produce a tunable but very narrow-band equalizer. A further discussion is given in Chapter 4, where the experimental results obtained from building such an equalizer are presented.

2.4 Microwave stripline equalizers

The first stripline equalizer to be investigated was the hybrid ring-resonator combination shown in Fig. 2.7. Ferguson and Barrett produce point-by-point measurements of the group-delay. Due to the low accuracy of the measuring equipment, however, the results give little information, and no theoretical analysis is presented. The equalizer is discussed in some detail in Chapter 6.

Jones and Bolljahn investigate theoretically the stripline device shown in Fig. 2.8a. This consists of a quarter-wave directional coupler, with two of the ports shorted together. They show that the device is an all-pass network
FIG 2.7 The hybrid-ring-resonator equalizer
a - Coupled-strip equalizer

b - Meander-line equalizer
with a non-linear phase characteristic. Steenaart\textsuperscript{12} also examines the theoretical properties of the device, and extends the analysis to include several couplers in cascade with the short on the extreme end. He shows the equalizer to have a pole-zero plot similar to that of Fig. 5.2, the group-delay curve in the vicinity of the first resonance being similar to Fig. 2.3, with the coupling, \( c \), being shown increasing. Cristal\textsuperscript{13} considers the question of synthesizing an arbitrary delay characteristic with such an equalizer, whilst Kolker\textsuperscript{14} computes the insertion-loss of the device when the losses are included. Yamamoto et al\textsuperscript{15} consider the case where the coupling varies along the coupled length; the solutions, however, are derived only for special coupling distributions.

The meander-line, first investigated as an equalizer by Dunn\textsuperscript{17}, consists of a cascade of the above coupled transmission lines in the form of a meander, as shown in Fig. 2.3b. The pole-zero plot for a single turn is that of the above coupled-strip equalizer. The delay variation per section for this device is very small, and many identical sections must be cascaded in order to obtain reasonable delay variations. Hewitt\textsuperscript{18} presents an experimental equalizer with a linear delay characteristic of 300 nsec increment over a 600 MHz bandwidth at 1 GHz. The construction used is a form of strip-line with the broad-face of the central conductor perpendicular to the ground planes, thereby increasing the maximum available coupling per turn when compared with the coplanar construction. This construction technique is used by Tu, but is of little use above about 4 GHz due to the dimensional accuracy required.

The resonant ring equalizer described by Cohn\textsuperscript{7}, is shown in Fig. 2.9. Cohn suggests that it is a stripline equivalent of the waveguide equalizer studied later by Abele and Wang.
FIG 2.9 The resonant ring equalizer.
An extensive investigation of the properties of this device is presented in Chapter 5.

2.5 Conclusions

As a result of the survey, an investigation of the following devices was undertaken:

a) the orthogonal-mode waveguide equalizer considered by Abele and Wang
b) the circulator-reactive-termination equalizer
c) the resonant ring equalizer
d) the hybrid-ring equalizer.
REFERENCES


CHAPTER 3

THE CIRCULATOR-REACTIVE-TERMINATION EQUALIZER

3.1 Introduction
3.2 The tapped-resonator termination
  3.2.1 Theoretical investigation
  3.2.2 Practical measurements
  3.2.3 Conclusions
3.3 The interdigital termination
3.1 Introduction

The basic circulator-reactive termination equalizer is shown in Fig. 3.1, where \( Z(s) \) is a purely reactive impedance in the complex frequency domain \( s \). The impedance level is assumed normalized to the characteristic impedance of the input and output of the circulator. The reflection co-efficient, \( V \), of the termination may be written:

\[
V = \frac{Z(s) - 1}{Z(s) + 1}
\]

(3.1)

Due to the non-reciprocal properties of the circulator, the reflection co-efficient of the termination is identical with the transfer function of the complete device, if the circulator is ideal.

If \( Z(s) \) is to be physically realizable, it must be the ratio of two polynomials of even and odd degrees respectively:

\[
Z(s) = \frac{q_0(s)}{q_e(s)} = \left[ q_e(s)/q_o(s) \right]
\]

(3.2)

where \( q_e(s) \) is the even polynomial

\( q_o(s) \) is the odd polynomial

The transfer function, \( T \), now becomes

\[
T = \frac{q_o(s) - q_e(s)}{q_e(s) + q_o(s)} = -\frac{Q(-s)}{Q(s)}
\]

(3.3)

where, by definition, \( Q(s) \) must be a Hurwitz polynomial, and must have, as its roots, the poles of the transfer function.

The synthesis problem is therefore quite straightforward. Firstly the locations of the poles and zeros of a cascade of simple equalizers, each contributing two conjugate poles and two conjugate zeros, which approximate the desired equalization curve are computed. The method of computation is discussed more fully in Chapter 8. The poles of the resulting pole-zero plot
FIG 3.1 The circulator-reactive-termination equalizer
are then taken as the roots of the Hurwits polynomial, and by expansion $Q(s)$ found. By separating the even and odd parts of $Q(s)$, $q_e(s)$ and $q_o(s)$ are obtained, and hence the required impedance $Z(s)$.

In principle the impedance can be realized by normal network techniques, in the form of a ladder network, and the resulting configuration approximated in co-axial form by cascaded low and high impedance sections - in a similar manner to that used in the construction of low-pass filters (see section 9.1.4a). It is found, however, that to obtain the necessary impedance values, impedance transformers which are very difficult to build must be used. The basic problem is to synthesize a general microwave impedance, and so far no acceptable procedure has been forthcoming.

In the following sections various forms of terminating impedances are considered, and their group delay characteristics computed and measured.

### 3.2 The tapped-resonator-termination

The tapped-resonator is shown, with the upper ground-plane removed, in Fig. 3.2. The device was constructed in stripline form, with an air dielectric to reduce losses. The resonator was one wavelength long, with adjustable short circuits at both ends, to enable the resonant frequency and tapping point to be changed.

#### 3.2.1 Theoretical investigation

The input impedance, $Z_{in}$ of the resonator at the tapping point is given by

$$Z_{in} = \frac{Z_R}{\coth s \frac{k}{\lambda_o} + \coth (1 - k) \frac{s}{\lambda_o}}$$  \hspace{1cm} (3.4)
FIG 32 The tapped-resonator termination
where \( c \) is the velocity of electromagnetic radiation in air

\[ k = \frac{\text{distance from end of resonator to tapping point}}{\text{total length of resonator}} \]

\( \lambda_0 \) is the resonant wavelength

\( Z_R \) is the characteristic impedance of the line used for the resonator.

The reflection coefficient, \( \nu \), is given by

\[
\nu = \frac{Z_R/Z_o - \coth s \frac{k}{\lambda_0} - \coth s (1 - k)\lambda_0}{Z_R/Z_o + \coth s \frac{k}{\lambda_0} + \coth s (1 - k)\lambda_0}
\]

where \( Z_o \) is the characteristic impedance of the input line.

Taking the numerator, the zeros of the reflection coefficient are given by the solution of

\[
\frac{Z_R}{Z_o} - \coth s \frac{k}{\lambda_0} - \coth s (1 - k)\lambda_0 = 0
\]

for the complex variable \( s \). Putting \( s = x + jy \), and expanding equation (3.6) into its real and imaginary parts, we have

\[
\frac{1}{2} \frac{Z_R}{Z_o} \left[ \cosh \frac{2\pi}{\omega_o} \cos \frac{2\pi}{\omega_o} - \cosh (1 - 2k) \frac{2\pi}{\omega_o} \cos (1 - 2k) \frac{2\pi}{\omega_o} \right] - \\
\sinh \frac{2\pi}{\omega_o} \cos \frac{2\pi}{\omega_o} = 0 \quad (3.7)
\]

\[
\frac{1}{2} \frac{Z_R}{Z_o} \left[ \sinh \frac{2\pi}{\omega_o} \sin \frac{2\pi}{\omega_o} - \sinh (1 - 2k) \frac{2\pi}{\omega_o} \sin (1 - 2k) \frac{2\pi}{\omega_o} \right] - \\
\cosh \frac{2\pi}{\omega_o} \sin \frac{2\pi}{\omega_o} = 0 \quad (3.8)
\]

where \( \omega_o \) is the resonant frequency.

Since there is no simple analytic solution for \( x \) and \( y \) from these simultaneous equations, the problem is best tackled numerically. A two-variable Newton-Raphson iteration procedure
was used, with initial values of 0 for \( x \), and \( \omega_0 \) for \( y \). Some of the results obtained for \( x \) for various values of \( k \) are shown in Fig. (3.3). It was found that for most practical values of \( k \), the deviation of \( y \) from \( \omega_0 \) was less than 0.1\%. Fig. (3.3) shows that if the value of \( x \) is to be relatively insensitive to changes in \( k \), then \( Z_R/Z_0 \) must be low. This is a reasonable conclusion, since, for a given tapping point, as \( Z_R/Z_0 \) decreases, so does the loading effect, thereby increasing the loaded \( Q \) of the resonator, and decreasing \( x \).

So far, only the one-wavelength resonance of the resonator has been considered, i.e. the frequency where the length of the resonator is one wavelength. It is interesting, however, to consider the complete pole-zero plot of the transfer function (or reflection coefficient) in order to compare it with those of equalizers considered in later chapters. The general form of the plot is best found from an intuitive argument. The electric field magnitude is shown in Fig. 3.4 for the first five resonances. The tapping point is considered fixed at a value which lightly loads the one-wavelength resonance. As the frequency increases, on the odd half-wavelength resonances the loading decreases, and on the even half-wavelength resonances the loading increases. The pole-zero plot of the transfer function is therefore similar to that shown in Fig. 3.5.

This pole-zero plot is rather complex. It is therefore of interest to see if it can be simplified in a manner similar to that used for the resonant-ring equalizer in section 5.2. The worst expected case was used in the analysis, with a tapping fraction of \( k = 0.47 \), \( Z_R/Z_0 = 1.0 \), and a full-wavelength resonance at 4 GHz. The value of \( \Sigma \) for the full-wavelength
FIG 3.3 The relationship between tapping fraction and pole position.
FIG 3.4 The resonances of a tapped resonator
FIG 3.5 The pole-zero plot for tapped resonator
zero was 0.035 and for the half-wavelength zero was 1.091, the two differing by a factor of 301. The analysis showed a worst possible delay error of 3%. As explained in section 5.2, however, this is an error in the absolute delay; errors in the delay variation across the band have been computed to be not greater than 0.38% of the above error. Thus the actual error limit expected is $3 \times 0.0038\%$ or 0.01%, a very small quantity. In conclusion, for the computation of group-delay variations over a ten-percent bandwidth, the simplified pole-zero plot consisting of only the pole-zero pair labelled 1 in Fig. 3.5 is quite adequate for most purposes.

A computer programme was written to evaluate the variations in group-delay with frequency for a given resonator impedance and tapping point. The group-delay was computed by stepping to each side of the frequency of interest by a small increment, and evaluating the phase shift through the device at the two points. The group delay was then obtained by dividing the difference in phases by the sum of the frequency increments. This form of numerical differentiation was found to be quite adequate. The effects of losses were included in the analysis, and Table 3.1 compiled for the case $\omega_0 = 4$ GHz, $R/Z_o = 1$, tapping point 0.47:

To simplify the analysis, all of the full wavelength zeros were assumed to have the same value of $\gamma$ as zero 1. Similarly, the odd wavelength zeros were assumed to have the same values of $\gamma$ as the lowest one. The details of the procedure are in Sec. 5.2.
Table 3.1

<table>
<thead>
<tr>
<th>Attenuation coefficient (nepers/m)</th>
<th>Frequency increment (MHz)</th>
<th>Group-delay at 4 GHz (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0183</td>
<td>2.78</td>
<td>14.22245</td>
</tr>
<tr>
<td>0.00303</td>
<td>2.78</td>
<td>14.16942</td>
</tr>
<tr>
<td>0.001</td>
<td>0.1</td>
<td>14.27510</td>
</tr>
<tr>
<td>0.00203</td>
<td>0.1</td>
<td>14.28672</td>
</tr>
<tr>
<td>0.00303</td>
<td>0.1</td>
<td>14.34425</td>
</tr>
<tr>
<td>0.00403</td>
<td>0.1</td>
<td>14.42531</td>
</tr>
<tr>
<td>0.006</td>
<td>0.1</td>
<td>14.65717</td>
</tr>
<tr>
<td>0.0183</td>
<td>0.1</td>
<td>19.79914</td>
</tr>
</tbody>
</table>

From this table the following conclusions were drawn:

1. The frequency increment has little effect on the group-delay. The tapping of 0.47 gives the largest Q likely to be used in practice, and at an increment of 2.78 MHz the error was only 0.1 nsec.

2. As the losses increase, so does the group-delay. This verifies the results of section 1.5. Also, the delay is a more sensitive function of loss than it is of frequency increment.

3.2.2. Practical measurements

The constructed termination is shown, with the top ground-plane removed, in Fig. 3.2. The positions of the short circuits to give a resonance at 4 GHz, and tapping values of 0.42 and 0.47, were calculated. The reactive effects of the tee-junction
were allowed for by using the recommended equivalent circuit of Franco and Oliner\textsuperscript{2} to compute the positions of the reference planes. The short circuits were positioned to within 0.001 in. by means of a travelling microscope, and the following results obtained.

With 0.45 tapping, the theoretical and measured group-delay curves are shown in Fig. 3.6. The curves agreed to within the ± 0.3 nsec accuracy of the measuring equipment. The insertion-loss curve was similar in shape to the group-delay curve, and reached a maximum of 0.6 dB. By using a microwave frequency counter, the frequency of the maximum of group-delay was measured as 3998.88 MHz. The predicted frequency, using the appropriate value of \( y \) found in section 3.2.1, was 3997.94 MHz - extremely good agreement.

By altering the positions of the short circuits, the tapping was altered to 0.47. The measured and computed group-delay curves are shown in Fig. 3.7. The agreement between them was again within the accuracy of the measuring equipment. The measured peak of insertion-loss was 1.6 dB, and the frequency of the group-delay maximum 4001.58 MHz. The computed frequency of the maximum was 3999.752 MHz.

3.2.3. Conclusions

It has been shown that the tapped resonator, when used as a terminating reactance on one port of a circulator, produces an easily adjusted single-section equalizer. The device is capable of being accurately described by a pole-zero pair near the one-wavelength resonance, and so can be used at microwave frequencies in exactly the same manner as is the bridged-tee equalizer at lower frequencies.

The major disadvantage of the device is that for each single-section a separate circulator is required. This
FIG 3.6 The group-delay of a tapped resonator equalizer with $k=0.45$
FIG 3.7 The group-delay of a tapped resonator equalizer with $k = 0.47$. 
results in a multiple section equalizer, which is formed as a cascade of such single sections, being bulky, rather expensive, and lossy.

3.3 The interdigital termination

Nicholson and Powell have shown that the termination of Fig. 3.8a has a group-delay characteristic similar to a single-section equalizer; and that for certain spacings, the termination of Fig. 3.8b behaves as two separate single-section equalizers in cascade. By considering the admittances of the digits, transformed to the input, it is found that the range of equalization possible is severely limited. A four-digit equalizer, shown in Fig. 3.9, has been built; the group-delay curves obtained, however, were unpredictable and were rapidly varying functions of the digit positions.

A slight variation of the above work yields an interdigital termination, shown in Fig. 3.10, which is similar to the interdigital filter proposed by Matthei.

The interdigital structure was investigated for use in equalizing individual band-pass interdigital filters. Such filters are extensively used due to their compact form. With one filter per channel, the majority of the group-delay variation in a satellite, for example, is due to this single filter. If a compact equalizer, such as is obtained by using a circulator with an interdigital termination, can be designed as an integral part of the filter, the amount of predistortion required at the ground station will be considerably reduced.

The synthesis techniques required for deriving an interdigital filter from its low-pass prototype have been well documented for square and round digits. Since the low-pass to band-pass transformation used in the synthesis
a - single resonator equalizer

b - double resonator equalizer

FIG 3.8 The interdigital termination
FIG 3-9 A four resonator termination
Coupling gap

Digit no.: 0 1 2 3 4

FIG 3:10
is almost linear, there is the possibility that an equalizer for the prototype filter will, after the same transformation, also equalize the band-pass filter. Since the prototype equalizer can take the form of a ladder network, terminating on one port of an ideal circulator, the filter synthesis technique can be slightly modified to yield an equalizing interdigital structure with a circulator. A design procedure for such a filter-equalizer combination can be summarized as follows.

(a) Compute the elements of the low-pass filter prototype which satisfies the required insertion-loss characteristic.

(b) Compute the group-delay of the filter prototype over the normalized frequency interval to be equalized.

(c) Using optimization techniques (Chapter 8), obtain the pole-zero plot of the required prototype equalizer.

(d) Synthesize the ladder network termination which realizes the pole-zero plot (section 3.1) using a Cauer expansion.

(e) Convert the filter and equalizer ladder networks to their equivalent interdigital structures.

(f) Analyse the final filter-equalizer combination.

A computer programme has been written to perform the above steps. The results of a typical trial problem are shown in Fig. 3.11. A nine section, 0.1 dB ripple, Tchebyschev filter was chosen, with a pass band from 3.9 GHz to 4.1 GHz. A group-delay tolerance of ±0.3 nsec was set over 0.8 of the pass band, i.e. 3.92 GHz to 4.08 GHz. The group-delay curves of the filter and filter plus equalizer are shown in Fig. 3.11. The dimensions of the filter and equalizer are summarized in Table 3.2.
FIG 3-11 Interdigital filter and equalizer
<table>
<thead>
<tr>
<th>DIGIT</th>
<th>COUPLING GAP</th>
<th>FILTER</th>
<th>EQUALIZER</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>DIAM (ins)</td>
<td>LENGTH (ins)</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>0.266</td>
<td>0.738</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>0.181</td>
<td>0.595</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>0.189</td>
<td>0.567</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>0.189</td>
<td>0.566</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>0.181</td>
<td>0.595</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>0.266</td>
<td>0.738</td>
</tr>
</tbody>
</table>

Table 3.2
Since the filters used in satellite work have bandwidths of only 40 MHz, work was done to try to equalize such filters. It was found that the above method broke down. This was due to the end digit of the equalizer being such a distance from its neighbour that the coupling capacitances could not be accurately computed. There is also the problem of setting up such equalizers. Interdigital filters are tuned up by varying the end capacitances of the digits with tuning screws until the necessary V.S.W.R. curve is obtained. Such a technique cannot be used for the equalizer structure.

3.3.1 Conclusions

Although the use of interdigital equalizers looks promising for medium bandwidth filters, the spacing between the digits precludes their use with narrow band filters.
REFERENCES


CHAPTER 4

The Orthogonal-mode waveguide equalizer

4.1 Introduction
4.2 Design of the cavity and coupling aperture
4.3 Experimental results
4.4 Conclusions
4.1 Introduction

A photograph of the orthogonal-mode equalizer, with the cavity removed, is shown in Fig. 4.1. The cavity is excited in the TE\textsubscript{111} mode by the square aperture coupling it to the waveguide. The aperture is placed in a position on the broad-wall where the waveguide has two orthogonal magnetic fields of equal magnitude, each one exciting one of the degenerate cavity resonances.

The similarity in operation between the resonant ring equalizer, described in Chapter five, and the above device, enabled the source of the poor results initially obtained for the former to be located by comparison with the results for the latter.

4.2 Design of the cavity and coupling aperture

The cavity was designed for highest unloaded $Q$, with maximum frequency separation between the wanted resonance and other resonances. The design criteria have been outlined by Wilson et al\textsuperscript{4}, and the experimentally determined mode resonances of the designed cavity are shown in Fig. 4.2a. The cavity was designed for a center frequency of 6.1 GHz, and the optimum diameter found to be 1.663 ins., with a cavity length of 1.321 ins. The waveguide used was WG 14, with internal dimensions of 1.372 ins. by 0.622 ins., and the center of the exciting aperture was computed\textsuperscript{2} to be 0.337 ins. from the side wall of the waveguide.

The presence of the two orthogonal modes for the TE\textsubscript{111} mode was shown by changing the coupling aperture from square to rectangular. This gave different loadings on the two resonances, and so removed their degeneracy by separating them in frequency. The frequency separation is shown in Fig. 4.2b.

The design of the coupling aperture is purely empirical.

* The TE\textsubscript{111} resonant frequencies differ in Fig. 4.2a,b due to the different loadings used in the two experiments.
FIG 41 The orthogonal-mode equalizer
a - modal characteristic of resonator

b - Splitting of degenerate modes

FIG 4.2
The design criteria presented by Abele and Wang\textsuperscript{2} are based on small aperture theory, but the large apertures required for realistic values of $Q_D$ due to the low magnitudes of the electric fields near the waveguide walls cannot be designed using this work.

4.3 Experimental results

The initial input V.S.W.R. of the device, with no tuning screws in the cavity, reached a peak of 2.5 at the centre frequency. It was found that by detuning one of the resonances with a perturbing screw, a V.S.W.R. plot similar to that shown in Fig. 4.3a could be obtained. This result led to the use of tuning screws to improve the performance of the resonant ring. By using a second screw to introduce coupling between the degenerate modes, the input V.S.W.R. was reduced to that shown in Fig. 4.3b.

The measured group-delay, using a waveguide version of the apparatus described in Chapter 9 is shown in Fig. 4.4. No theoretical results are available for comparison. The peak dissipation-loss coincided with the peak of group-delay, and was measured as 3.9 dB.

4.4 Conclusions

The constructed equalizer did not perform as well as the one reported on by Abele and Wang\textsuperscript{2}; the input V.S.W.R. was poorer, and the insertion-loss per nanosecond at the centre frequency was 0.039 dB/nsec, compared with a reported figure of 0.015 dB/nsec. The increase in insertion-loss was most likely due to the use of brass in the construction of the cavity.

The device is inherently very narrow band, and even
a - Detuned input VSWR characteristic

b - Tuned input VSWR characteristic
FIG 4.4 The group-delay of the orthogonal-mode equalizer.
inclusion of a screw beneath the aperture to increase the electric field intensity results in very little reduction in $Q_D$. This limits the use of the equalizer to cases where equalization over very narrow bandwidths is required, and so it is of little general use.
REFERENCES


CHAPTER 5

THE STRIPLINE RESONANT RING EQUALIZER

5.1 Theoretical analysis
5.2 Simplification of pole-zero plot
5.3 Experimental results for co-planar coupling
5.4 Theoretical analysis of effects of errors
5.5 Relative merits of co-planar and broadside coupling
5.6 Experimental results for broadside coupling
5.7 Multiple ring equalizers
5.8 Conclusions

Appendices

5A. The effects of etching tolerances on the impedances of broadside and co-planar couplers
5B. The electrical length of a stripline bend
5.1 Theoretical analysis

The resonant ring equalizer is composed of a ring, one wavelength in circumference at the centre frequency, coupled over a quarter of its circumference to a transmission line. The characteristic impedance of the uncoupled portion of the ring is identical to that of the main transmission line. By varying the coupling between the ring and line, the loading on the ring is altered, and so the overall characteristics of the device may be adjusted.

For the purpose of analysis, the resonant ring structure is split into two parts as shown in Fig. 5.1. The first step is to describe the length of coupled transmission line by a matrix equation, thus:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
I_1 \\
I_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} \\
A_{21} & & & \\
A_{31} & & & \\
A_{41} & & & \\
\end{bmatrix}
\begin{bmatrix}
V_3 \\
V_4 \\
I_3 \\
I_4
\end{bmatrix}
\]

(5.1)

where the directions of the voltages and currents are defined in Fig. 5.1. Several methods are available for the evaluation of the matrix elements, but the derivation by Steenaart is followed, since this yields the most useful form. He obtains a matrix of the 'A' coefficients of the following form:

\[
\begin{bmatrix}
\cosh \gamma L & 0 & A_3 \sinh \gamma L & A_4 \sinh \gamma L \\
0 & \cosh \gamma L & A_4 \sinh \gamma L & A_3 \sinh \gamma L \\
A_1 \sinh \gamma L & -A_2 \sinh \gamma L & \cosh \gamma L & 0 \\
-A_2 \sinh \gamma L & A_1 \sinh \gamma L & 0 & \cosh \gamma L
\end{bmatrix}
\]

(5.2)

where

\[
\begin{align*}
A_1 &= \frac{(Z_{oe} + Z_{oo})}{2Z_{oe} Z_{oo}} \\
A_2 &= \frac{(Z_{oe} - Z_{oo})}{2Z_{oe} Z_{oo}} \\
A_3 &= \frac{(Z_{oe} + Z_{oo})}{2} \\
A_4 &= \frac{(Z_{oe} - Z_{oo})}{2}
\end{align*}
\]
FIG 5.1 The resonant-ring equalizer
$Z_{0\text{e}}, \ Z_{0\text{o}}$ are, respectively, the even and odd-mode characteristic impedances of the coupled lines. These are related to a 'balanced' impedance, $Z_0$, by

$$Z_0 = \sqrt{Z_{0\text{e}} \ Z_{0\text{o}}} \quad (5.3)$$

the two lines being assumed identical in width and in spacing from the ground planes. $\gamma$ is the complex propagation constant of the lines, and $L$ is the electrical length of the lines.

The length of transmission-line completing the loop can be described by the matrix equation

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} \cosh \gamma L' & -Z_0' \sinh \gamma L' \\ -\frac{1}{Z_0'} \sinh \gamma L' & \cosh \gamma L' \end{bmatrix} \begin{bmatrix} V_4 \\ I_4 \end{bmatrix} \quad (5.4)$$

where $Z_0'$ is the characteristic impedance of the line, $\gamma'$ its propagation constant, and $L'$ its length.

The only quantities required to complete the characterisation of the device are the source and load impedances, which we leave at $Z_L$ for the moment.

For the analysis, let

$$\gamma' = \gamma \\
\gamma L = 0 \\
\gamma L' = 3\theta \\
Z_0' = Z_0$$

Then by matrix expansion and rearranging, we obtain

$$V_1 = V_3 \ \left( \cosh 3\theta + \left( \frac{Z_{0\text{e}} + Z_{0\text{o}}}{Z_L} \right) \sinh \theta \right) + I_4 \ A_4 \ \sinh \theta \quad (5.6)$$

$$I_1 = V_3 \ \left( \cosh \theta + A_1 \right) - V_4 \ A_2 \ \sinh \theta$$

From the equations obtained by expanding the matrices, expressions for $V_4/V_3$ and $I_4/V_3$ can be obtained, and substitution yields the input impedance

$$Z_{IN} = \frac{V_1}{I_1} = Z_L \ \text{if and only if} \ Z_L = \sqrt{Z_{0\text{e}} \ Z_{0\text{o}}} = Z_0 \quad (5.7)$$
If this condition holds, the device provides a perfect match at all frequencies, since the variable \( \theta \) does not occur in the equations for \( Z_{IN} \). Also, since exponential forms have been retained, the loss mechanisms of the stripline may be incorporated by using a complex propagation factor. Thus even with loss, a perfect match is maintained.

Since the device is perfectly matched, and is symmetrical, from section 2.2, the group-delay can be obtained from the voltage transmission factor, \( V_3/V_1 \). The expression is

\[
\frac{V_3}{V_1} = \frac{1}{\cosh \theta + A_1 \sinh \theta + \frac{I_4}{V_3}}
\]

where

\[
\frac{I_4}{V_3} = \frac{A_2 \sinh \theta (A_1 \sinh \theta + \sinh 3\theta) + (\cosh \theta - \cosh 3\theta) A_2 \sinh \theta}{(\cosh \theta - \cosh 3\theta)^2 - (A_1 \sinh \theta + \sinh 3\theta)(A_3 \sinh \theta + Z_0 \sinh 3\theta)}
\]

### (5.9)

Expanding the trigonometric expressions, and simplifying gives

\[
\frac{V_3}{V_1} = \left\{-8(1+A)\sinh^2 \theta - (A - 3)^2\right\}\left\{\cosh \theta[(4B^2 + 8A + 6) \sinh^2 \theta + (A + 3)^2] - \sinh \theta [4(A + 1)^2 \sinh^2 \theta + (3A + 1)(A + 3)]\right\}
\]

\[
= N(\theta)/D(\theta)
\]

### (5.10)

where

\[
A = (Z_{oo} + Z_{oo})/2Z_0
\]

\[
B = (Z_{oo} - Z_{oo})/2Z_0
\]

Consider first the numerator of equation (5.10)

\[
N(\theta) = -8(1+A)\sinh^2 \theta - (A - 3)^2
\]

### (5.11)

If we define \( p = \tanh \theta \),

\[
N(p) = -\frac{(A - 1)^2}{(1 - p^2)} \left[ p^2 - \frac{(A + 3)^2}{(A - 1)^2} \right]
\]
Similarly, we factorise the denominator of equation (5.10)

\[
D(p) = \frac{1}{(1 - p^2)^{3/2}} \left[ \frac{p + (A + 3)}{(A - 1)} \right] \left[ \frac{p + (A + 3)}{(A - 1)} \right] (1 + p)
\]

(5.13)

By combining equations (5.12), (5.13):

\[
\frac{V_1}{V_3} = \frac{8(1 + A)}{(A - 1)^2} \left[ \frac{1 - p}{1 + p} \right] \left[ \frac{p - (A + 3)/(A - 1)}{p + (A + 3)/(A - 1)} \right]
\]

(5.14)

To reconvert this equation to the complex \( \theta \) plane, let us consider the major factor in the numerator:

\[
p = \frac{(A + 3)/(A - 1)}
\]

i.e. \( \tanh \theta - C \)

where \( C \) is a constant greater than one.

Let \( C = \coth \alpha \), where \( \alpha \) is a real constant. The factor is now \( (\tanh \theta - \coth \alpha) \). Carrier, Krook, and Pearson show that the following expansions are true:

\[
\sinh Z = Z \sum_{n=1}^{\infty} \frac{1}{n} \left( 1 + Z^2/n^2 \pi^2 \right)
\]

\[
\cosh Z = \sum_{n=1}^{\infty} \frac{1}{n} \left( 1 + Z^2/(n + 1/2)^2 \pi^2 \right)
\]

(5.15)

Since \( (\tanh \theta - \coth \alpha) = -\frac{\cosh (\theta - \alpha)}{\cosh \theta \sinh \alpha} \)

substitution of equation (5.15) yields

\[
\tanh \theta - \coth \alpha = -\sum_{n=1}^{\infty} \frac{1}{n} \left( 1 + (\theta - \alpha)^2/(n + 1/2)^2 \pi^2 \right)
\]

(5.16)

\[
= \sum_{n=1}^{\infty} \frac{1 + \alpha^2/n^2 \pi^2}{1 + \theta^2/(n + 1/2)^2 \pi^2}
\]
Similarly, by transforming the denominator, equation (5.14) becomes

\[
\frac{V_3}{V_1} = \frac{8(1 + A)}{(A - 1)^2} \left[ \frac{1 - \tanh \theta}{1 + \tanh \theta} \right]^{\frac{1}{2}} \prod_{n = 1}^{\infty} \frac{(\theta - \alpha + j(n + 1/2)\pi)(\theta - \alpha - j(n + 1/2)\pi)}{(\theta + \alpha + j(n + 1/2)\pi)(\theta + \alpha - j(n + 1/2)\pi)}
\]

(5.17)

Thus the pole-zero plot of the resonant ring is obtained, as shown in Fig. 5.2.
5.2 Simplification of the pole-zero plot

Let \( \theta \) be purely imaginary,

\[
\theta = j\phi.
\]

From equation (5.17),

\[
\text{Arg}(V_2/V_1) = -2\phi - \sum_{n=1}^{\infty} \left\{ 2 \tan^{-1} \left[ \frac{(n + 1/2)\pi + \phi}{a} \right] - 2 \tan^{-1} \left[ \frac{(n + 1/2)\pi - \phi}{a} \right] \right\}
\]

(5.18)

For most microwave problems, the fractional bandwidths are small compared with the centre frequency. Let us therefore put \( \phi = \pi/2 + \delta \), i.e. a deviation of \( \delta \) radians about the centre frequency.

\[
\text{Arg}(V_2/V_1) = -\pi - 2\delta - \sum_{n=0}^{\infty} \left\{ 2 \tan^{-1} \left[ \frac{(n + 1/2)\pi + \delta}{a} \right] - 2 \tan^{-1} \left[ \frac{n\pi - \delta}{a} \right] \right\}
\]

(5.19)

Referring to Fig. 5.2, the summation to give the phase can be re-arranged as follows:

- phase contribution from zero \( A = \tan^{-1}(\delta/a) \)
- phase contributions from zeros \( B \) and \( C = \tan^{-1}\left[ \frac{2\delta/a}{1 + (\pi/2 - \delta^2)/a^2} \right] \)

If \( \delta < \pi \),

- phase contribution from zeros \( B \) and \( C = \tan^{-1}\left[ \frac{2\delta/a}{1 + (\pi/2)^2} \right] \)

Summing all such contributions, we have:

\[
\text{Arg}(V_2/V_1) = -\pi - 2\delta - 2\tan^{-1}(\delta/a) - 2 \sum_{n=1}^{\infty} \tan^{-1}\left[ \frac{2\delta/a}{1 + (n\pi/a)^2} \right]
\]

(5.20)

Differentiating equation (5.20) with respect to \( \delta \),

\[
\frac{\delta}{\delta} \left[ \text{Arg}(V_2/V_1) \right] = -2 - \frac{2a}{a^2 + \delta^2} - \frac{\delta}{a} \sum_{n=1}^{\infty} \frac{1 + (n\pi/a)^2}{a^2 \left[ 1 + (n\pi/a)^2 \right]^2} / 4 + \delta^2
\]

(5.21)
As it stands, it is difficult to deduce anything about the convergence properties of the infinite series in eqn (5.21). However, the known relative magnitudes of the parameters involved can be used to simplify the series as follows.

As the coupling of the resonant ring to the main line increases, the $Q_D$ of the resonator decreases, i.e. $\alpha$ in Fig. 5.2 increases. The maximum coupling expected to be used is when $Z_{\infty} \approx 100 \, \Omega$, $Z_{0\infty} \approx 25 \, \Omega$, and this gives

$$a_{\text{max}} = 0.0588$$

$$1 + (\pi/\alpha)^2 > 1 + (\pi/\alpha)^2 = 2.850$$

Cauchy's integral test can be stated as follows. Let $\sum_{n=1}^{\infty} a_n$ be a series of positive terms such that $a_{n+1} < a_n$. If there exists a positive decreasing function, $f(x)$, for $x > 1$ such that $f(n) = a_n$, then the given series converges if the integral

$$\int_{1}^{\infty} f(x) \, dx$$

exists; the series diverges if the integral does not exist.

Further, if the integral exists, then

$$\sum_{n=1}^{\infty} a_n < \int_{1}^{\infty} f(x) \, dx$$

Applying this to the problem, let $x = n$.

$$a f(x) = \frac{1 + (nx/\alpha)^2}{\alpha^2 [1 + (nx/\alpha)^2] /4 + \delta^2}$$

Since this function cannot be easily integrated as it stands, the inequality of equation (5.22) can be used to give

$$(nx/\alpha)^2 >> 1 \quad \text{for} \quad x > 1$$

and

$$\frac{\alpha^2}{4} \left( \frac{nx}{\alpha} \right)^4 >> \delta^2$$

since for a ten percent bandwidth, or less,
\[ \delta = \frac{\pi}{40} \]

\[ f(x) \leq \frac{4}{\pi^2 x^2} \] (5.23)

\[ \therefore \int_{1}^{\infty} f(x) \leq \frac{4}{\pi^2} \]

\[ \therefore a \sum_{n=1}^{\infty} \frac{[1 + (n\pi/a)^2]}{2 + (n\pi/a)^2} \leq \frac{4a}{\pi^2} \]

From equation (5.21), therefore

\[ \text{delay contribution from summation} \leq \frac{2}{\pi^2} (a^2 + \delta^2) \] (5.24)

Taking worst case values, equation (5.24) shows that the percentage error in neglecting the summation is less than or equal to 0.2%. Since, in equation 5.21, \( \text{Arg}(V_x/V_1) \), is the phase shift through the device, and \( \delta \) is proportional to the real frequency, the differential is proportional to the group delay. The above result shows that by neglecting all but the contributions from the pole-zero pair A, the result will differ from the exact one by less than 0.2%.

An even better accuracy is obtained by including the pole-zero pair B. Comparison with Fig. 2.2b shows that, to a good approximation, the resonant ring can be represented by a plot identical to that for the bridged-tee equalizer, with the complex s-plane linked to the complex \( \theta \)-plane by the following relationships:

\[ s = \Sigma + j\Omega \]

\[ \theta = s \frac{\lambda}{ko} \]

where \( \lambda \) is the electromagnetic wave velocity in the dielectric.

\[ \Delta s = 2\theta \frac{\omega}{\pi} \]
where $\omega_0$ is the centre frequency of the resonant ring.

It should be noted that if only the variation of group-delay with frequency is of interest, the error is much less than that given by equation (5.24). This is due to the fact that the contribution from the summation in equation (5.21) is almost constant over the bandwidth considered. Since, in the majority of cases, only the variation of group-delay with $\delta$ is of importance, it is to be expected that the error in calculating the group-delay variation using a two pole-zero pair model will be much less than when calculating the absolute delay using the same model.

An approximate value for the variation in the error term with frequency can be deduced as follows. The error term, $E$, is given by

$$E = a \sum_{n=1}^{\infty} \frac{1 + (n\pi/a)^2}{a^2 [1 + (n\pi/a)^2]^{3/4} + \delta^2}$$

Using the same approximations as before,

$$E \approx a \sum_{n=1}^{\infty} \frac{k}{a^2 [1 + (n\pi/a)^2]} \left[ 1 - \frac{k \delta^2}{a^2 [1 + (n\pi/a)^2]} \right]$$

Extracting the frequency dependent term, $E_\delta$, and simplifying,

$$E_\delta = \frac{16a \delta}{\pi^4} \sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{16a \delta^2}{90}$$

This term is zero at $\omega_0$ when $\delta = 0$, and a maximum when $\delta$ is a maximum. If only variations in group-delay are of interest, then the error term of interest is that which varies with frequency.
The relative delay error
\[ \approx \frac{4(c^2 + \delta^2)\delta^2}{45} \]

This is a maximum when \( \delta \) is largest, and substituting in the typical values used previously,

\[ \% \text{ error} \approx 0.00053 \]

The error is using only the pole-zero pairs \( A \) and \( B \) to compute the delay variations over the band is, therefore, insignificant.

Since the synthesis of group-delay characteristics, using cascades of bridged-tee equalizers, is based on the simplicity of the pole-zero plot, all existing synthesis techniques developed at low frequencies can be directly utilized in the microwave region. For the purpose of group-delay computation, the transfer function can be written as

\[
\frac{V_2}{V_1} = -\frac{8(1 + A)}{(A - 1)^2} \left[ \frac{1}{1 + \tanh \theta} \right]^{\frac{1}{2}} \frac{\left( \theta - \alpha - j\pi/2 \right) \left( \theta - \alpha + j\pi/2 \right)}{\left( \theta + \alpha - j\pi/2 \right) \left( \theta + \alpha + j\pi/2 \right)}
\]

(5.25)

Transforming equation (5.25) into the s plane, we have

\[
\frac{V_3}{V_1} = -\frac{8(1 + A)}{(A - 1)^2} \left[ \frac{1}{1 + \tanh \left( s \pi/2\omega_o \right)} \right] \times
\]

\[
\frac{\left[ (s - 2\omega_o/\pi - j\omega_o)(s - 2\omega_o/\pi + j\omega_o) \right]}{\left[ (s + 2\omega_o/\pi - j\omega_o)(s + 2\omega_o/\pi + j\omega_o) \right]}
\]

(5.25a)

which is of the same form as equation (2.1) of the bridged-tee equalizer. The group-delay of the resonant ring can therefore be completely specified by a pole-zero plot containing the two variables \( \alpha \) and \( \omega_o \).
5.3 Experimental results for co-planar coupling

The technique used for obtaining the resonant rings and also the hybrid-rings described in the next chapter, was to selectively etch, by a photo-lithographic process, the copper coating bonded to a dielectric sheet. The masks for this were obtained by photographically reducing master copies cut on an "astro-foil" machine. The master copies have a dimensional accuracy of 0.001 in., and so if a ten fold magnification is used, the reduction will shrink this to 0.0001 inch on the masks. The masks were made on a 'melnex' base, and strains in this due to the processing can produce dimensional errors of 0.001 in. A photograph of one of the master copies cut is shown in Fig. 5.3.

The first equalizer made was designed to give a group-delay variation of 9 nsec at a centre frequency of 4 GHz. The phase shifts at the corners of the ring were computed for full width mitres (Fig. 5.4a) using the analysis of Campbell and separately using that of Wier and Adams. The two answers agreed to within one degree for each corner. Since the ring is designed to be one electrical wavelength in circumference at the centre frequency, at that frequency the circumferential phase shift is 360 degrees. If, however, the phase shift contributed by each corner is only known to an accuracy of, say, ±1 degree, the phase shift at the designed centre frequency may have a value in the range 360 ± 4 degrees. The actual resonant frequency, therefore, may be up to $4 \times \frac{100}{360} \%$ of the design frequency away from the required value. For example, with a required centre frequency of 4 GHz, the above uncertainty could result in an actual centre frequency somewhere in the range $4 \pm 4 \times \frac{100}{360} = 4 \pm 0.044$ GHz.

The results quoted are expected to be optimistic values, and
FIG 5.3 An artwork master
so some form of frequency tuning must be incorporated if the
correct centre frequency is to be attained. To allow some
small adjustments to be made, the double mitred corner
suggested by Wier and Adams\(^8\) was used at each corner of the
coupled section (Fig. 5.4c), while the low reflection corner
of Campbell and Jones\(^9\) Fig. 5.4b, was used for the other two
corners. The input V.S.W.R. characteristic for this first
equalizer resembled that shown in Fig. 3.3a for the orthogonal-
mode equalizer. Cohn has suggested in his patent paper,
although no proof is given, that the stripline ring can support
two modes of resonance. Elementary trigonometry shows that
a travelling wave can also be considered as two standing waves
of the same frequency in quadrature both temporally and
spatially. Thus it is possible that two standing waves on
the ring are more fundamental in the action of the device
than a travelling wave, the latter being a special case when
the two standing waves satisfy the above conditions. Work on
the orthogonal-mode equalizer shows that the above V.S.W.R.
characteristic is characteristic of the two standing waves
having different resonant frequencies. The two resonances
can not have their reflected components at the input cancelling,
since they are at different frequencies, and so the very large
peaks in the V.S.W.R. curve result. Without the results of
the waveguide equalizer available for comparison, it is
doubtful whether the cause of the poor input V.S.W.R. would
have been appreciated, and indeed the investigation was
abandoned until the similarity between the curves was
realized.

To investigate this splitting further, an air-dielectric,
triplate stripline resonant ring was designed and built for
operation at 1 GHz. A photograph of this device is shown,
a - Full width mitre

b - Low reflection mitre

c - Double mitred corner

FIG 5.4 Various types of mitred corner
with the upper ground plane removed, in Fig. 5.5. A series of 0.010 inch diameter holes were drilled in the top plate to take an untuned electric field probe, and the electric field distribution around the ring plotted. It was found that as the ring is moved towards the main line, the two separate resonances converge, and as they merge, the input V.S.W.R. falls drastically. The field plots were taken with the two resonances well separated, one plot being taken at each of the resonant frequencies. The results of the two plots are shown in Fig. 5.6. These clearly show the existence of two independent standing waves on the ring, the nodes occurring approximately at the centres of the ring sides; the nodes of one resonance are separated by approximately a quarter wavelength from those of the other resonance. The photographs of Fig. 5.7 show these results on a swept frequency basis.

By careful adjustment of the position of the ring, the input V.S.W.R. was reduced to a peak of 1.29, as shown in Fig. 5.8. However, with the high value of $Q_n$ involved, giving a group-delay variation of 45 ns, with a maximum insertion loss of 1.9 dB, the input V.S.W.R. obtained seemed reasonable.

A series of the 4 GHz rings were etched, the directions of the acid sprays $\hat{\phi}_q$ to obtain small variations in the gap between resonator and line. For the designed device this gap was only 0.0057 inches, and so was highly dependent on etching undercut. Such a ring, mounted in the test channel, is shown in Fig. 5.9. The dimensions of a number of rings were progressively altered to examine the effects on the performances of the devices. The results showed that the frequencies of two resonances could be made to converge most easily by either including shunt stubs at the middle of the
FIG 5.5 A 1GHz resonant ring
FIG 5.6 Standing waves on the resonant-ring
INPUT VSWR OF EQUALISER

ELECTRIC FIELD, 1/2-WAY ALONG TOP OF RING

ELECTRIC FIELD, 1/2-WAY ALONG SIDE OF RING

FIG 57
FIG 5.9 Two 4GHz resonant rings
FIG 5.8 Input VSWR of L-band equalizer
ring sides, or increasing the width of one of the coupled strips. To carry out this work, a silver suspension, commercially named "silver dag", was used, and although rather lossy, its versatility was found to be very useful.

It was found that by using small tuning stubs, the input V.S.W.R. could be reduced to give a peak of less than 1.4 for all the rings, the overall shape of the curve being similar to Fig. 5.8. In one case, however, no tuning was required, even though the difference in coupling gap was only 0.0005 in. compared with that of the poor performance rings. The measured group-delay of two of the rings, with slightly different gaps, is shown in Fig. 5.10. The agreement between the theoretical and measured curves for the correct gap, 0.0057 in., is within the ± 0.3 nsec measurement accuracy. The result of increasing the coupling gap by 0.001 in. can clearly be seen as an increase in the $Q_D$.

The material used in the construction of the resonant rings was copper clad irradiated polyolefin. The etching undercut can frequently be as large as the thickness of the copper, or in this case 0.0014 in.

Ideally, a cross section of the coupled strips reveals two closely spaced rectangles. Due to the undercut, however, the sides of the strips are no longer vertical, and so we now have two trapeziums. The coupling for this case is impossible to define, and even measurement of the gap cannot be done with any degree of accuracy. For this reason, and due to the relatively poor input V.S.W.R.s obtained for a, supposedly, perfectly matched device, an investigation into the effects of design errors and etching tolerances, detailed in the next section, was undertaken.

In section 5.4 it is shown that the presence of the corners
Frequency [GHz]

- theory
- experimental (0.0057 in. gap)
- experimental (0.0067 in. gap)

FIG 5.10 Group-delay of the resonant-ring equalizer
in the ring is theoretically undesirable, unmatched corners leading to very high input V.S.W.R.'s for the whole device. As a consequence of these results, the circular resonant ring, shown in Fig. 5.9, was designed and constructed. The results were very disappointing, the input V.S.W.R. still exhibiting large peaks, and the resonant frequency being too high. The second of these results was expected, since the electrical length of the ring was computed from the mean circumference. Due to current crowding, the electrical length is less than the mean circumference; however a literature search revealed no applicable work available on this phenomenon. The investigation outlined in Appendix 5B was therefore undertaken, and the results give a fairly accurate method of computing the electrical length of a stripline bend with a small radius of curvature. The second major problem, that of characterizing the curved section of coupled line, has remained unsolved, and so no general conclusions can be drawn about this type of resonant ring.

5.4 Theoretical analysis of effects of errors

The simplification of the matrix equations (5.1) and (5.2) was due to equation (5.3). If however equation (5.3) does not hold, or if the coupling section is not a quarter wavelength long, the performance of the ring may be drastically altered.

If the assumptions are not made, the resulting equations are so complex that recourse must be made to a digital computer for their solution. It is found that small deviations from a quarter wavelength in the coupling section have a negligible effect on the group-delay, and the device remains a perfect match. Variations in the odd and even mode impedances,
however, produce quite drastic changes. The correct values for the odd and even mode impedances for the device considered were 35.714Ω and 70.000Ω respectively. It was found that a change in values to 35.5Ω and 69.2Ω respectively produced a V.S.W.R. peak of 1.4, as shown in Fig. 5.11. Such a change in impedance values requires an alteration in the width of the gap of only 0.001 in. To obtain low values of input V.S.W.R., therefore, extremely tight tolerances must be held. A further variation to 30.0Ω and 75.0Ω, respectively, showed the V.S.W.R. peak dividing, as seen experimentally.

A qualitative explanation for this behaviour is easily obtained if the coupling section is considered simply as a directional coupler. If $Z_{oc} Z_{oo} = Z_o^2$, the condition for the coupler to have infinite directivity is satisfied, and there are outputs from two of the ports only. If the condition is not satisfied, however, there is an output from the third port. Port 2 in Fig. 5.1 corresponds to one of the main outputs, whilst port 4 is ideally decoupled. If there is an output from port 4, this will couple a signal out of port 1, and as the resonance condition is approached, this signal will increase to a maximum, thereby degrading the input V.S.W.R. If the product $Z_{oc} Z_{oo}$ differs from $Z_o^2$, there will be an impedance discontinuity, or mismatch, at the outputs from the coupling section. There will therefore be a discontinuity at two corners of the ring. Fig. 5.6 shows that these corners occur where the amplitudes of the two standing waves on the ring are significant. Coupling will therefore occur between the resonances, and so their resonant frequencies will differ, the frequency difference increasing as the mismatch increases.

The equivalent tee-section for a stripline corner is shown in Fig. 5.12. By considering the open-circuit and
FIG 5.11 Input VSWR of perturbed resonant ring
FIG 5.12 Equivalent tee-section of corner
short-circuit impedance, the condition for a perfect match is found to be given by

$$Z_1(Z_1 + 2Z_2) = Z_0^2$$  \hspace{1cm} (5.26)

To determine the value of $Z_1$ from the curves of Campbell and Jones, two large, and nearly equal numbers must be subtracted. The resulting inaccuracy makes it difficult to verify whether a given mitre satisfies equation (5.26).

A computer programme was written in which the tee elements of the corners were included. By slowly varying the corner parameters in such a manner that equation (5.26) no longer held, the effect on the input V.S.W.R. of reflections at the corners could be studied. The results obtained are shown in Fig. 5.13, where the corner perturbations are represented by the equivalent V.S.W.R. of the corner, equation (5.26) giving the conditions for a perfect match.

Curve d, which is for four full-width mitres, shows the input V.S.W.R. splitting into two peaks, similar to those observed experimentally. Even for curve a, where two corners are perfectly matched and two have a V.S.W.R. of only 1.025, a V.S.W.R. peak of 1.9 is reached. All of the curves shown are for a coupled section with $Z_{cc} = 70,000$, $Z_{oo} = 35.714\Omega$.

The results presented here have shown that, not only must the coupled section be dimensionally accurate, but the four corners must have individual V.S.W.R.s very close to unity if good V.S.W.R.s are to be obtained. The problem of the corners can be avoided by using a circular ring, as shown in Fig. 5.9.
Effective input VSWR's of corners:

- a - 1, 4 (1.00); 2, 3 (1.025)
- b - 1, 2, 3, 4 (1.025)
- c - 1, 2, 3, 4 (1.047)
- d - 1, 2, 3, 4 (1.13) [full width mitre]

**FIG 5.13** Input VSWR of resonant ring with mismatched corners
5.5 Relative merits of co-planar and broadside coupling

The devices so far described have had the resonant ring and main line in one plane, parallel with the ground planes. A second possible method of construction is that suggested by Shelton with the coupled section made of broadside coupled strips separated by a layer of dielectric. Such an equalizer is shown, with the upper dielectric layer removed, in Fig. 5.14, where the lower coupled strip can be seen to slightly overlap the upper.

The analysis given in Appendix 5A shown that, for the case of $Z_{oo} = 70\Omega$, $Z_{ce} = 35.7\Omega$, a 0.0005 in. reduction in the width of each strip on their coupled sides results in a variation in $Z_{oo}$, $Z_{ce}$ for the co-planar coupling four times the value of that for broadside coupling. Thus as far as the coupling section is concerned, the broadside coupling is more insensitive to etching tolerances. Unfortunately, this relative insensitivity means that there will be considerable, and unpredictable coupling effects at the ends of the coupled section, where in order to uncouple the strips as quickly as possible the 45 degree mitres are situated. Also, the work of Campbell and Jones assumes a symmetrically located strip. Since the ring is now nearer the upper ground plane, their results can only be taken as a first approximation. The corner lengths of Wier and Adams were obtained by purely geometric considerations, and empirical factors added. Since the factors were determined using a symmetrically located ring, these results again may be in error for a displaced ring.

5.6 Experimental results for broadside coupling

Several different equalizers were constructed with broadside coupling, one of which is shown, with the upper dielectric
FIG 5.14  A broadside-coupled equalizer
sheet and ground-plane removed, in Fig. 5.14. The input
V.S.W.R. results for these devices were as poor as for the
coplanar coupling.

The corners were designed using the equations derived
for a symmetrical strip. The corners, therefore were
probably poorly matched, and the results of section 5.4 have
shown what catastrophic V.S.W.R.s can result.

5.7 Multiple ring equalizers

Since the coupling of one ring to a line produced a first
order equalizer; it seemed likely that higher order equalizers
could be constructed by coupling further rings to the original
one. The theoretical analysis of such a device is very
involved, and since no physical insight into the problem could
be gained from manipulation of the equations, they were solved
numerically.

The case for two rings, shown in Fig. 5.15, was found to
yield a perfect match if the balanced condition of equation
(5.7) was satisfied by each of the coupled sections. The
group-delay response for various coupling to the second ring
is shown in Fig. 5.16. This response is, as expected, that of
a second order equalizer.

The order of the equalizer is related to the number of
poles and zeros at each resonance. The bridged-tee equalizer
is an example of a first order equalizing section, since at
the resonant frequency there is only one pole at \( \Sigma = \sigma, \Omega = \omega_c \)
(see Fig. 2.2b). A second order equalizing section is one
which has a pair of zeros at \( \Sigma = \sigma \) and \( \Omega = \omega_c ± \Delta \) (where
\( \Delta < \omega_c \)), the pattern being repeated at the other three points.
In the case of the resonant ring, the splitting can be regarded
as the normal resonant-frequency splitting which occurs when
FIG 5.15 The double ring equalizer.
Group-delay (nsec) vs Frequency (GHz)

- Coupling to main line:
  - a: $Z_{0e} = 75.0 \, \Omega$, $Z_{00} = 33.3 \, \Omega$
  - b: $Z_{0e} = 75.0 \, \Omega$, $Z_{00} = 33.3 \, \Omega$

- Inter-ring coupling:
  - a: $Z_{0e} = 52.5 \, \Omega$, $Z_{00} = 47.62 \, \Omega$
  - b: $Z_{0e} = 55.0 \, \Omega$, $Z_{00} = 45.45 \, \Omega$

FIG 5.16 Group-delay of double resonant ring
identical resonators are coupled together.

To obtain higher order equalizers, an appropriate number of rings are simply cascaded. Due to the singular lack of success in obtaining low-V.S.W.R. single-ring equalizers, the multiple ring equalizers were not subjected to an experimental investigation.

5.8 Conclusions

It has been shown that, for all practical uses, the resonant ring equalizer is a microwave equivalent of the bridged-tee equalizer. The sensitivity of the input V.S.W.R. to an imperfect corner or coupling section, however, severely limits the application of the device.

Development of a circular ring is hindered by the lack of information on tightly curved, coupled lines, but it is felt that this approach should yield better input V.S.W.R.s than are possible with a square ring.
APPENDIX 5A

The effects of etching tolerances on the impedances of broadside and co-planar couplers

Broadside couplers (Fig. 5.17a)

From Shelton\textsuperscript{10}, for loose coupling:

\[ q = \left( \frac{s + 1}{2} \right) \frac{a + 2s/(s + 1)}{a + (s + 1)/2} \]  

- (5A.1)

\[ W_0 = \frac{1}{\pi} \left[ \frac{s}{2} \ln (q/a) + (1 - s) \ln [(1 - q)/(1 + a)] \right] \]  

- (5A.2)

\[ C_f = -2 \frac{1}{\pi} \left[ \frac{1}{1 + s} \ln \left( \frac{1 - s}{2} \right) + \frac{1}{1 - s} \ln \left( \frac{1 + s}{2} \right) \right] \]  

- (5A.3)

\[ C_{f0} = 2 \frac{1}{\pi} \left[ \frac{1}{1 + s} \ln \frac{1 + a}{a(l - q)} - \frac{1}{1 - s} \ln(q) \right] \]  

- (5A.4)

\[ C_o = \frac{4\pi W}{1 - s^2} + C_{f0} - C_f \]  

- (5A.5)

\[ Z_{oo} = 120\pi \sqrt{\frac{\varepsilon_0}{\varepsilon_r}} \]  

- (5A.6)

\[ C_{fe} = 2 \frac{1}{\pi} \left[ \frac{1}{1 + s} \ln \frac{1 + a}{a(l - q)} - \frac{1}{1 - s} \ln(q) - \ln \left( \frac{1 + aq}{aq} \right) \right] \]  

- (5A.7)

\[ C_o = \frac{4\pi W}{1 - s^2} + C_{fe} + C_f \]  

- (5A.8)

\[ Z_{oe} = 120\pi \sqrt{\frac{\varepsilon_0}{\varepsilon_r}} \]  

- (5A.8)

Referring to Fig. 5.17a, s is constant, but due to etching undercut, \( W \) and \( W_0 \) can vary by an amount \( \Delta \). From equation (5A.1).

\[ \Delta g = \frac{(s - 1)^2}{(s + 1 + 2a)^2} \]  

- (5A.9)
a - Broadside-coupled strips

b - Coplanar coupled strips

FIG 5.17 Types of stripline coupling
From equation (5A.2)

\[
\frac{\partial w}{\partial a} = \frac{1}{\pi} \left[ s \frac{a \frac{\partial q_a}{\partial a} - q}{qa} - (1 - s) \left\{ (1 + a) \left( \frac{\partial q}{\partial a} + \frac{1 - q}{(1 - q)(1 + a)} \right) \right\} \right]
\]  - (5A.10)

\[
\frac{\partial w}{\partial q} = \frac{\partial w}{\partial a} \cdot \frac{\partial a}{\partial q}
\]  - (5A.11)

From equation (5A.4)

\[
\frac{\partial f_o}{\partial w_c} = \frac{2}{\pi} \left[ \frac{1}{1 + s} \left( 1 + a \right) \frac{\partial q}{\partial w_c} - \frac{1 - s}{1 - s} \frac{1}{q} \frac{\partial q}{\partial w_c} \right]
\]  - (5A.12)

If \(W\) is reduced by \(\Delta\), and \(W_c\) by \(\Delta\): the increase in \(C_o\), \(\delta C_o\), is given by

\[
\delta C_o = -\frac{4\Delta}{1 - s^2} - \frac{\delta C_{fo} \Delta}{\partial w_c}
\]  - (5A.13)

\[
\delta z_{ce} = -\frac{120\pi}{C_o^2 \sqrt{\xi}} \delta C_o
\]  - (5A.14)

From equation (5A.7)

\[
\frac{\partial f_{ce}}{\partial w_c} = \frac{3 \frac{f_{o} \partial - 2}{\partial w_c} + 2}{\pi} \left( \frac{a \frac{\partial q_a}{\partial w_c} + q\frac{\partial a}{\partial w_c}}{aq (1 + aq)} \right)
\]  - (5A.15)

\[
\delta C_e = \frac{4\Delta}{1 - s^2} - \frac{\delta f_{ce} \Delta}{\partial w_c}
\]  - (5A.16)

\[
\delta z_{ce} = -\frac{120\pi}{C_o^2 \sqrt{\xi}} \delta C_e
\]  - (5A.17)
The values for the experimental equalizer were

\[ Z_{oc} = 70\Omega, \quad Z_{\infty} = 35.7143\Omega \]

\[ s = 0.1095 \]

\[ a = 0.022879 \]

\[ q = 0.21156 \]

\[ \Delta = 0.00365 \text{ (a 0.0005 inch variation)} \]

which resulted in \( \delta Z_{oc} = 0.18\Omega \)

\[ \delta Z_{\infty} = 0.28\Omega \]

Co-planar couplers (Fig. 5.17b)

From Gohm \(^{14}\),

\[
Z_{oc} = \frac{30\pi}{\sqrt{r}} \frac{K(k_e)}{K(k_e)}
\]

(5A.18)

where \( K \) is the complete elliptic integral of the first kind,

\[
k_e = \tanh \left( \frac{\pi}{2} \frac{W}{b} \right) \tanh \left( \frac{\pi}{2} \frac{W + s}{b} \right)
\]

(5A.19)

\[ k_e^I = (1 - k_e^2)^{1/2} \]

(5A.20)

\[
Z_{\infty} = \frac{30}{\sqrt{r}} \frac{K(k_0^I)}{K(k_0)}
\]

(5A.21)

where \( k_0 = \tanh \left( \frac{\pi}{2} \frac{W}{b} \right) \coth \left( \frac{\pi}{2} \frac{W + s}{b} \right) \)

(5A.22)

\[ k_0^I = (1 - k_0^2)^{1/2} \]

(5A.23)

From equation (5A.22):

\[
\frac{\delta k_0}{\delta W} = \frac{-\pi}{2b} \left[ \tanh \left( \frac{\pi}{2} \frac{W}{b} \right) \text{cosech}^2 \left( \frac{\pi}{2} \frac{W + s}{b} \right) + \coth \left( \frac{\pi}{2} \frac{W + s}{b} \right) \times \right.
\]

\[ \frac{1}{2b} \text{sech}^2 \left( \frac{\pi}{2} \frac{W}{b} \right) \]

(5A.24)

\[
\frac{\delta k_0}{\delta s} = \frac{-\pi}{2b} \tanh \left( \frac{\pi}{2} \frac{W}{b} \right) \text{cosech}^2 \left( \frac{\pi}{2} \frac{W + s}{b} \right)
\]

(5A.25)

increase in s = - increase in W = \( \Delta \)
Combining equations (5A.24), (5A.25) to obtain the increment in $k_0$,

$$\delta k_0 = \frac{\pi A \coth \left( \frac{\pi W + s}{2b} \right) \text{sech}^2 \left( \frac{\pi W}{2b} \right)}{2b}$$  \hspace{1cm} (5A.26)

From equation (5A.23),

$$\delta k'_0 = -\frac{1}{2} k_0 \frac{(2 k_0 - \delta k'_0)}{(1 - k_0^2)^{\frac{3}{2}}} \hspace{1cm} (5A.27)$$

To obtain the partial derivations for the elliptic integrals, the polynomial approximation to $K(m)$ was used.\(^{19}\)

$$K(m) = (a_0 + a_1 m_1 + a_2 m_1^2) + (b_0 + b_1 m_1 + b_2 m_1^2) \ln \left( \frac{1}{m_1} \right)$$

where $a_0 - a_2, b_0 - b_2$ are the polynomial coefficients, and $m_1 = 1 - m$.

$$\frac{\partial K(m)}{\partial m} = -(a_1 + 2a_2 m_1) - (b_1 + 2b_2 m_1) \ln \left( \frac{1}{m_1} \right) + (b_0 + b_1 m_1 + b_2 m_1^2) (1/m_1)$$  \hspace{1cm} (5A.28)

$$\frac{\partial}{\partial m} \left[ \frac{K'(m)}{K(m)} \right] = K(m) \frac{\partial K'(m)}{\partial m} - K'(m) \frac{\partial K(m)}{\partial m} \frac{1}{K(m)} \frac{\partial K(m)}{\partial m}$$  \hspace{1cm} (5A.29)

where $m' = (1 - m^2)^{\frac{1}{2}}$

Substituting in values, $a_0 = 1.3862944, b_0 = 0.5$

$a_1 = 0.1119723, b_1 = 0.1213478$

$a_2 = 0.0725296, b_2 = 0.0288729$

$w = 0.0806, s = 0.00575, b = 0.125$

These gave $\delta Z_{oo} = -1.23 \Omega$

Thus the broadside coupler is less sensitive to etching tolerances than the co-planar coupler, by a factor of four. For example, if we have an undercut of 0.0005 in. at each edge, the resultant variations in $Z_{oe}, Z_{oo}$ from their ideal values results in the following. For the co-planar coupling, the input V.S.W.R. at the centre frequency rises from unity to 1.4. With broadside coupling, however, the input V.S.W.R. for the same dimensional error is only about 1.1.
The electrical length of a stripline bend

**5B.1 Introduction**

When designing stripline components which contain bends, it is customary to take as the electrical length of the bend, the length, \( L \), given by the mean radius:

\[
L = \theta (R_{\text{IN}} + R_{\text{OUT}})/2
\]

where

- \( L \) = Length of curve
- \( R_{\text{IN}} \) = inner radius of strip
- \( R_{\text{OUT}} \) = outer radius of strip
- \( \theta \) = angle subtended at centre by ends of curve.

This is satisfactory provided that the radius of the curve is large compared with a wavelength at the operating frequency. With a resonant ring one wavelength in circumference, however, the arithmetic mean radius, \( R_{\text{M}} \), is given by

\[
R_{\text{M}} = \lambda/2\pi
\]

\[
= 0.16\lambda
\]

(where \( \lambda \) is the wavelength)

and for a conventional hybrid ring,

\[
R_{\text{M}} = 1.5\lambda/2\pi
\]

\[
= 0.24\lambda
\]

In both of these cases, it has been found experimentally that a design using the mean radius will result in a centre frequency which is too high.

In the following sections an approximate analysis is presented for the resonant ring.

**5B.2 Construction of a theoretical model**

The basic procedure is to convert the stripline ring to a parallel-plate ring, and then approximate this with a parallel-plate regular polygon, as shown in Fig. 5.18.
The stripline configuration is first converted to its
equivalent form with no fringing field, as shown in Fig. 5.19
a, b. This is implemented by using a Schwartz-Christoffel
transformation, the dimension D being given to a good accuracy
by
\[
D = \omega + \frac{2b}{\pi} \left( \ln 2 + \frac{1}{2} \left[ 1 - \ln \left( \frac{2}{11} \right) \right] \right)
\]  
(5B.2)

However, this procedure is only valid for cases where the
inner radius of the strip is large enough to prevent coupling
between opposite sides of the ring. Experimental work has
shown that interaction effects become noticeable when the
inner diameter of the ring approaches the dimension of the
ground planes spacing. Also, to obtain equation (2), only
one strip cross-section was considered, whereas of course
there are two strip cross-sections in any diametral plane,
and so the transformation is not rigorous.

Since the strip is symmetrically placed between the
ground planes, the fields of interest are fully represented
by the portion shown in Fig. 5.19c. Using Babinet's
equivalence principle, all magnetic walls are replaced by
electric walls and vice-versa, and all lines of E and H
are replaced by lines of H and -E respectively. The
equivalent circuit of the original configuration is now
the dual of the equivalent circuit of this new structure.

With this transformation, the ring is now a parallel-plate
ring, as shown in Fig. 5.19d.

If the ring is approximated by a regular polygon, the
polygon can be cut as shown in Fig. 5.18, and treated as a
two-port device. The current and voltage convention used
is shown in Fig. 5.20a. We have

\[
\begin{bmatrix}
  V_1 \\
  I_1
\end{bmatrix}
= \begin{bmatrix}
  A & B \\
  C & D
\end{bmatrix}
\begin{bmatrix}
  V_2 \\
  I_2
\end{bmatrix}
\]  
(5B.3)
FIG 5.19 Stripline transformations
(a) The current-voltage convention

(b) The corner reference planes

FIG 5.20
where the coefficients $A, B, C, D$ depend on the properties of the ring. Since the ring is continuous, we must have

\[
V_1 = V_2 = V
\]
\[
I_1 = I_2 = I
\]

and this yields the condition

\[
(1 - A)(1 - D) - BC = 0 \tag{5B.4}
\]

The above $ABCD$ matrix can be easily constructed once the equivalent matrices are known for each corner and each side of the polygon. All that is then involved is matrix multiplication.

The corner matrix is most easily obtained from the equivalent $pi$ circuit given by Marcuvitz. (The Babinet equivalence principle states that the equivalent circuit for the stripline corner is given by the dual of the $pi$ section, i.e., a tee section). From Marcuvitz, the normalised series impedance of the $pi$ circuit, $Z_B$, is given by

\[
Z_B = j \frac{2\pi D}{\lambda} \tan \theta/2 \tag{5B.5}
\]

and the normalised shunt impedances, $Z_A$, by

\[
Z_A = - j \frac{\lambda}{D} \frac{810}{0.11 \theta \pi} \tag{5B.6}
\]

The reference planes, and angle $\theta$, are shown in Fig. 5.20b. The distance between reference planes, $L$, is $2R_0 \tan \theta/2$, and so the $ABCD$ matrix for the length of lossless line is given by

\[
A = \cos \left( \frac{2\pi L}{\lambda} \right) = D
\]
\[
B = - j \sin \left( \frac{2\pi L}{\lambda} \right) = C \tag{5B.7}
\]

whilst for the corner,

\[
A = \frac{(Z_A + Z_B)/Z_A}{Z_A} = D
\]
\[
B = Z_B \tag{5B.8}
\]
\[
C = \frac{(Z_B + 2Z_A)/Z_A^2}{Z_A^2}
\]
The necessary computations were programmed, and a Newton-Raphson iteration procedure used, with the designed centre frequency as the initial guess, to find the frequency which satisfied equation (5B.4). This was then taken as the resonant frequency. The procedure was repeated, with the number of corners successively doubled, until the results of two doublings gave answers differing by less than 0.1 MHz. The accuracy in determining the frequency satisfying equation (5B.4) was also 0.1 MHz.

The results obtained are shown in Table 5B.1 for 
\[ b = 0.125 \text{ in.}, \ t = 0.0014 \text{ in.}, \ \varepsilon_r = 2.321. \]

<table>
<thead>
<tr>
<th>Design Frequency (GHz)</th>
<th>Strip Width (ins)</th>
<th>Computed Resonant Frequency (GHz)</th>
<th>Measured Resonant Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>0.050</td>
<td>4.0380</td>
<td>4.0365</td>
</tr>
<tr>
<td></td>
<td>0.075</td>
<td>4.0471</td>
<td>4.0472</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>4.0563</td>
<td>4.0525</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>4.0658</td>
<td>4.0663</td>
</tr>
<tr>
<td></td>
<td>0.150</td>
<td>4.0747</td>
<td>4.0700</td>
</tr>
<tr>
<td>5.0</td>
<td>0.050</td>
<td>5.0598</td>
<td>5.055</td>
</tr>
<tr>
<td></td>
<td>0.075</td>
<td>5.0741</td>
<td>5.064</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>5.0885</td>
<td>5.083</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>5.1036</td>
<td>5.099</td>
</tr>
<tr>
<td></td>
<td>0.150</td>
<td>5.1176</td>
<td>5.108</td>
</tr>
<tr>
<td>6.0</td>
<td>0.050</td>
<td>6.0865</td>
<td>6.082</td>
</tr>
<tr>
<td></td>
<td>0.075</td>
<td>6.1072</td>
<td>6.100</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>6.1282</td>
<td>6.127</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>6.1502</td>
<td>6.146</td>
</tr>
<tr>
<td></td>
<td>0.150</td>
<td>6.1707</td>
<td>6.155</td>
</tr>
<tr>
<td>7.0</td>
<td>0.050</td>
<td>7.1182</td>
<td>7.114</td>
</tr>
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<td></td>
<td>0.075</td>
<td>7.1446</td>
<td>7.139</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>7.1754</td>
<td>7.178</td>
</tr>
<tr>
<td></td>
<td>0.125</td>
<td>7.2058</td>
<td>7.188</td>
</tr>
<tr>
<td></td>
<td>0.150</td>
<td>7.2341</td>
<td>7.166</td>
</tr>
</tbody>
</table>
If we define
\[ \Delta F = \text{frequency difference between designed and actual resonant frequencies} \]
\[ F_D = \text{designed resonant frequency} \]
\[ D = \text{equivalent strip width} \]
\[ D_M = \text{arithmetic mean diameter of curve} \]
\[ D_M = \frac{R_{IN} + R_{OUT}}{2} \]

A plot of \( \Delta F / F_D \) against \( D / D_M \) yields the straight line shown in Fig. 5.21.

5B.3 Experimental results

A series of resonant rings were etched to correspond with those shown in Table 5B.1. Fig. 5.23 shows one such card full size, the rings being placed for the minimum of interaction. The coupling gap was increased until the insertion-loss \( Q \) became constant, the frequency then being taken as the unloaded resonant frequency, and measured with a frequency counter.

A 'G' clamp was used to vary the pressure exerted on the dielectric sheets by the ground planes. With a low pressure, an air gap is present at the interface between the dielectrics, and since the ring is on this plane, the resonant frequency is altered. With increasing pressure, the resonant frequency decreases, and so the above readings were taken with a pressure corresponding to the point where the frequency ceased to drop. This force was undoubtedly greater than would normally be exerted by clamping screws, and so the resonant frequencies obtained with bolt fixing would be greater than the values measured.

The results obtained are shown in Table 5B.1, and plotted in Fig. 5.22 for comparison with the theoretical results.
FIG 5.22 Theoretical and experimental results
FIG 5-23 A set of test rings
The correction formula

The agreement between the theoretical and experimental results is sufficiently good for the theoretical curve to be used for correction purposes.

The straight line is defined by

$$\frac{\Delta F}{F_D} = 0.0567 \frac{D}{D_M} - 0.00042 \quad (5B.11)$$

If $\lambda$ is the wavelength at the resonant frequency, and $\lambda_D$ that at the design frequency,

$$\frac{\Delta F}{F_D} = \frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_D}\right)}{1/\lambda_D}$$

$$= \frac{\lambda_D - \lambda}{\lambda}$$

($\lambda_D - \lambda$) is the difference in electrical wavelengths, so that $\Delta L$, the length per wavelength which must be subtracted from $\lambda_D$ the mean length to give the true electrical length, is found to be given by

$$\frac{\Delta L}{\lambda_D} = \frac{0.0567 \frac{D}{D_M} - 0.00042}{0.0567 \frac{D}{D_M} + 0.99958} \quad (5B.12)$$

5B.5 Conclusions

The use of equation (5B.12) enables the electrical length of a resonant ring to be computed to an accuracy not previously attainable. This in turn will lead to accurate designs of such components.
REFERENCES


8. WIER, W. B., ADAMS, D. K. "Wideband multiplexers using directional filters", Microwave, May 1969, p. 44.


CHAPTER 6

THE HYBRID-RING EQUALIZER

6.1 Introduction
6.2 Theoretical analysis
6.3 Effects of errors
6.4 Experimental results
6.5 Conclusions
6.1 Introduction

The basic hybrid-ring equalizer is shown in Fig. 2.4. It consists of a hybrid 3 dB coupler, with the two output arms terminated in resonators. These arms differ in length by a quarter wavelength at the centre frequency of the device. The two path lengths for signals travelling from the input up to the resonators and then back to the input, therefore, differ by one half wavelength, and so there is no reflected wave at the input if the terminations are identical. The two signals travelling to the output port from the resonators, however, arrive in phase and so reinforce each other. The total phase change from input to output is clearly governed by the nature of the terminations, and so the group-delay characteristic can be specified by choosing suitable reactive terminations. For the purpose of the following analysis, the simple tapped resonator of Chapter 3 is used.

The same production techniques were used for both the resonant ring and the hybrid-ring. A master copy was cut on an "astrofoil" cutting machine, and photographically reduced to produce the masks used in the etching process. The material used as dielectric was again the low-loss, copper clad, irradiated polyolefin.

6.2 Theoretical analysis

The method used to analyse the hybrid ring was an extension of that used by Gunston and Nicholson. Each section of transmission line was replaced by its equivalent pi network. Referring to Fig. 6.1, the equivalent elements are given by
FIG 6.1 The equivalent circuit of a section of transmission line
\[ Z_A = -j \frac{Z_o \sin (\pi \omega / 2 \omega_o)}{1 - \cos (\pi \omega / 2 \omega_o)} \quad (6.1) \]

\[ Z_B = j \frac{Z_o \sin (\pi \omega / 2 \omega_o)} {1 - \cos (\pi \omega / 2 \omega_o)} \quad (6.2) \]

where \( \omega_o \) is the frequency when the line is a quarter wavelength long, and \( Z_o \) is its characteristic impedance. The positive sign is chosen for \( Z_B \) in order that both circuits pass d.c.

By repeated application of "star-delta" transformations, the equivalent 'pi' circuit for the complete equalizer was obtained. Due to the complexity of the resulting expressions, a computer programme was written to evaluate the performance of the device. Use of the work of Franco and Oliner\(^2\) enabled the reactive effects of the tee-junctions, occurring at the ring and at the resonators, to be taken into account.

The initial work was done with the above model, but it was considered necessary to include the loss mechanisms operative in the stripline. These can be split into two parts, the copper loss and the dielectric loss. The dielectric loss is normally incorporated by using a complex dielectric constant. For use in the above model, however, it was more convenient to use the conductance per unit length between the inner conductor and the ground planes. This quantity is dependent on the electric field distribution, and so a conformal mapping was used to create an equivalent model in which the electric field lines were parallel. Using the results of Cohn\(^3\), the conductivity per unit length was found to be given by

\[ G = 8 \pi f \varepsilon_r \tan \delta \frac{K'(k)}{K(k)} \quad (6.3) \]

where \( f \) is the frequency, \( \varepsilon_r \) the dielectric constant, \( \tan \delta \)
the loss tangent, and $K$ is the complete elliptic integral of the first kind, where $k = \text{sech} \left( \frac{\pi W}{2b} \right)$, and $\tilde{k} = \left(1 - k^2\right)^{\frac{1}{2}}$; $b$ is the ground-plane spacing and $W$ the width of the strip. The copper losses were calculated from the formulae given by Cohn.

Since the attenuation per wavelength was small, the equations for the equivalent circuit could be simplified to

$$Z_A = -Z_0 \left( A \cos \left[ \frac{\pi \omega}{2\omega_0} \right] + j \sin \left[ \frac{\pi \omega}{2\omega_0} \right] \right) \quad (6.4)$$

$$Z_B = Z_0 \left( A \cos \left[ \frac{\pi \omega}{2\omega_0} \right] + j \sin \left[ \frac{\pi \omega}{2\omega_0} \right] \right) \quad (6.5)$$

where $A = \omega \alpha / 4$.

$\alpha =$ attenuation coefficient in nepers/m

$\lambda_0 =$ wavelength in meters.

The results obtained using this model, are shown in Fig. 6.2 for various tapping points. The frequency dependence of the various quarter wavelength sections is shown by the increase in V.S.W.R. away from the centre frequency. The loss is not shown, since this was found to be a replica of the delay curve.

The computer programme was extended to analyse the characteristics of up to ten such equalizers in cascade. The results for two and for three cascaded equalizers are shown in Figs. 6.3 and 6.4 respectively. These clearly show the effect of the frequency sensitivity of the hybrid-rings on the input V.S.W.R. The group-delay curve was found to be almost identical to that obtained by addition of the curves for each of the equalizers alone. It was concluded that interaction effects between the equalizers does not seriously perturb the group-delay characteristic. The insertion-loss curves are not shown since it was found
FIG 6.2 Characteristics of the hybrid ring equalizer

- a - group-delay for k=0.45
- b - " for k=0.44
- c - " for k=0.42
- d - input VSWR for k=0.44
FIG 6.3 Two hybrid rings in cascade
FIG 6.4 Three hybrid rings in cascade.
that these were replicas of the group-delay curves.

By applying the analysis of Section 3.2 to the tapped half-wavelength resonator, a set of curves relating the real part of the pole-zero position to the tapping point were obtained, as shown in Fig. 6.5.

If the hybrid-ring were a perfect 3 dB power splitter at all frequencies, and its dimensions, as fractions of the operating frequency, remained constant with frequency, the ideal case would be attained, and the group-delay curves for the hybrid-ring, and circulator-termination equalizers would be identical for identical terminations. To synthesize a given group-delay curve, a method such as those discussed in Chapter 8 could be used to obtain the pole-zero positions for each equalizing section required. If a cascade of ideal hybrid-ring equalizers was to be used, Fig. 6.5 could be used to translate from the real part of the pole position to the equivalent tapping fraction. The frequency sensitivity of the hybrid-ring will, however, alter the group delay characteristic of the tapped resonator. Fig. 6.6 shows a comparison between the delay curve for a tapped-resonator on a circulator, and a curve for the same termination on a hybrid ring. The group delay error is not more than ± 0.2 nsec, which is of sufficient accuracy for the above cascading procedure to be implemented with hybrid rings.

6.3 Effects of errors

Due to the rather poor input V.S.W.R.s of the experimental hybrid-ring equalizers, the computer programmes used in the analysis of Section 6.2, were modified to include variations in selected dimensions of the equalizer. This enabled the effects of errors, such as unequal resonators,
FIG 6.5 The relationship between tapping fraction and pole position

Tapping fraction \( k \)

\( Z_{\alpha}/Z_{\omega}: \)
- a - 1.8
- b - 1.4
- c - 1.0
- d - 0.6
- e - 0.2

\( X/\omega_{\omega} \)
FIG 6.6 Comparison between hybrid ring and tapped resonator
on the equalizer performance to be studied in some detail.

The first dimensional variation considered was in the relative lengths of the arms connecting the resonators to the ring. These ideally differ by a quarter wavelength at the centre frequency. The hybrid ring, with $k = 0.44$, was analysed with a difference in arm lengths of firstly a quarter-wavelength plus 0.004 in., and secondly a quarter-wavelength plus 0.020 in. The computer results showed that for the first case the input V.S.W.R. was slightly improved compared with the ideal model. The results for the second case showed little deviation from the correct group-delay curve, but the input V.S.W.R. was slightly degraded. At a centre frequency of 4 GHz in irradiated polyolefin, the wavelength is 4.92 cm. The above error of 0.020 in. as a percentage of a quarter wavelength is over four percent.

The conclusion drawn from these results is that slight errors in the arm lengths can be tolerated.

Attention was then turned to the resonators, and here the story was quite different. Fig. 6.7 shows the resultant input V.S.W.R. when the two resonators differ in overall length by 0.002 in. and 0.005 in. respectively. With an etching tolerance of $\pm 0.001$ in. on most dimensions, a difference in resonator lengths of 0.002 in. can easily occur. These results lead to the conclusion that the resonators must be accurately matched in their overall lengths, or, equivalently, in their resonant frequencies.

Finally, the effect of slightly altering the tapping point on only one of the resonators was investigated. One tapping point was moved along its resonator by 0.005 in. The effect on the group-delay was found to be negligible, the input V.S.W.R. being altered from 1.062 to 1.085 at
FIG 6.7 Input VSWR of perturbed hybrid ring (k=0.44)
3.9 GHz for a tapping point of 0.4. Thus it was decided that this source of error could be neglected.

6.4 Experimental Results

Due to the lack of a precision stripline load, the V.S.W.R. of a single coaxial-to-stripline transition could not be measured. Instead, the V.S.W.R. of a pair of transitions back-to-back was measured. Each of the transitions will reflect a portion of the incident wave back to the input. If it is assumed that the magnitudes and phases of the two reflection coefficients are identical over a frequency range, then at the input the two reflected waves will differ in phase, due to the length of line separating the transitions. Thus as the frequency increases, so the phase difference increases, and the two reflections will interfere, resulting in maxima and minima in the total reflected field. As the line length is increased, so more variations will occur in a given frequency increment. A plot of the input V.S.W.R. with frequency will result ideally in a sinusoidal variation with frequency, deviations being due to imperfect matching of the transitions, and an unmatched load at the second transition. Thus, to a first approximation, for small V.S.W.R.s, the V.S.W.R. of one transition can be taken as one half of the peak to peak variation in the V.S.W.R. curve. The value so obtained for the frequency range 3.7 - 4.5 GHz was 1.09, which agrees with similar results obtained from the resonant ring work. This reflection is probably due to the discontinuity capacitance of the actual transition, and it is unlikely that any improvement can be made without resorting to tuning stubs.

The theoretical and experimental group-delay curves for
single rings with tapping fractions of 0.4 and 0.42 are shown in Fig. 6.8, 6.9. The measured input V.S.W.R. for the 0.4 device is shown in Fig. 6.10.

Two rings in cascade, with centre frequencies of 4.0 GHz and 4.2 GHz, and tapping fractions of 0.42 were designed and etched. It was arranged that, by removing certain of the copper paths, each device could be measured independently. Fig. 6.11 shows such a set of boards, including the one for testing the connectors. A similar set was prepared, but with the arms terminated in open circuits instead of resonators, thus enabling the properties of the hybrid-rings alone to be measured. The input V.S.W.R.s of each ring, and the two rings cascaded, are shown in Fig. 6.12. The frequency dependence of the input V.S.W.R. of the rings is clearly shown, as is the bandwidth shrinkage when rings are cascaded. From this it seems that cascading more than about three such rings will result in poor input V.S.W.R.s over all but a narrow frequency band. The results of Fig. 6.12 should be regarded more as qualitative than quantitative, since the effects of the connectors are included in the V.S.W.R. curves.

The V.S.W.R. curves for the rings with resonators in place are shown in Fig. 6.13. The curve for the 4.0 GHz ring shows a pronounced peak, very similar to those predicted in Section 6.3 for unequal length resonators. Measurements revealed a 0.005 in. discrepancy in one of the resonators on the astrofoil artwork. A tuning screw was inserted through one ground plane in order to perturb the resonant frequency of the incorrect resonator. By tuning, the improved curve of Fig. 6.12 was obtained. The cascaded V.S.W.R. has reasonable values, although not nearly as good as the predicted curve of Fig. 6.3. The theoretical and measured
FIG 6.8 Group-delay of a single hybrid ring

- theoretical
- experimental
k = 0.4
FIG 6.9 Group-delay of a hybrid ring

- theoretical
- experimental

$k = 0.42$
FIG 6.10 Input VSWR of hybrid ring
FIG 611 Test boards for cascaded equalizers
VSWR of 4GHz ring

VSWR of 4.2GHz ring

VSWR of the two rings cascaded

FIG 6.12
FIG 6.13

VSWR OF UNTUNED 4GHz RING

VSWR OF TUNED 4GHz RING

VSWR OF 4.2GHz RING

VSWR OF TUNED RINGS IN CASCADE
group-delays of the two section equalizer are shown in Fig. 6.14. The agreement is within the measurement accuracy.

A triple ring equalizer was designed and built, and is shown in Fig. 6.15. The three rings had centre frequencies of 3.8 GHz, 4.0 GHz, and 4.2 GHz with a tapping fraction of 0.42. The theoretical and measured group-delay curves are shown in Fig. 6.16, and again the agreement is good. The V.S.W.R., however, was very poor, exhibiting peaks of 1.9, probably due to slightly unbalanced resonators.

The experimental results obtained on the first rings produced, showed a slight discrepancy between the computed and measured centre frequencies. It was concluded that the cause was the small radius of curvature of the ring. The electrical length around the ring was computed from the mean radius of the strip. Due to current crowding at the inner edge of the ring, however, the electrical length is less than this, resulting in a shift in the centre frequency to a higher value. To overcome this error, the formula derived in Appendix 5.B can be used.

6.5 Conclusions

From the results obtained, the hybrid-ring equalizer appears suitable for applications where only one or two equalizing sections are required. It has been shown that the group-delay curves can be theoretically predicted to a high accuracy. The use of the simplified pole-zero plot enables the low frequency synthesis techniques, outlined in Chapter 8, to be directly applied.

A drawback of the equalizer is the limited bandwidth of the hybrid ring. The use of branch-line couplers has
FIG 6.14 Two hybrid rings in cascade
FIG 6:15 A triple hybrid-ring equalizer
FIG 6-16 Three hybrid rings in cascade
been considered, but they appear less amenable to accurate design than the hybrid-ring.
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CHAPTER 7

A MILLIMETER -- WAVELENGTH EQUALIZER

7.1 Introduction and basic operation of equalizer.
7.2 Structure and properties of the resonators.
7.3 Analysis of mode formation in the coupler.
7.4 Comparison with exact analysis when no slab is present.

APPENDICES

7.A Mode generation by capacitive and inductive grids.
7.B Derivation of mode amplitudes from aperture distribution.
7.C Exact analysis of waveguide cross.
7.1 Introduction and basic operation of equalizer

In Chapter 1, the need for equalization in oversize-waveguide communication systems, and the reasons for preferring equalization to be carried out at the carrier frequency, were briefly discussed. In this chapter the results of a theoretical investigation into an equalizer operating at millimeter-wave frequencies are presented. At the beginning of the work it was hoped to be able to build and test such a device. It was with this in mind that the work was carried out at a wavelength of 1.2 cm (25 GHz), although the device was envisaged for operation at frequencies nearer 50 GHz. Due mainly to the very high cost of the tapers needed to convert the $H_{1,0}$ mode in normal waveguide to a pure $H_{1,0}$ mode in the oversize waveguide, it was not considered economical to embark on the experimental work until the theoretical analysis had been completed. The time scale involved has ruled out this empirical phase, but it is hoped that the results presented here will enable the work to be undertaken at some later date.

The device investigated is shown in Fig. 7.1a, and is an extension of the directional coupler-resonator equalizer suggested by Cohn. At the wavelengths envisaged for its use, for example 6 mm. at 50 GHz, the waveguide wall losses, and so the resonator losses, are increased quite drastically above their theoretical values. The principal cause of this is the surface finish attained when the waveguide is manufactured. Considerable work has been done on this problem, and the general conclusion is that if the losses are to be kept at reasonable values, the waveguide surfaces must be mechanically or electrically polished. If waveguide is used at frequencies where its dimensions are many wavelengths, the formulae for the ideal case show that the
FIG 7-1 A quasi-optical waveguide equalizer
waveguide attenuation factor is reduced still further.

If the waveguide coupler were to be manufactured in normal-sized waveguide, the coupling holes required to give a 3 dB power split would be large, and the dimensional tolerances to give an accurate 3 dB split would be costly to meet. An alternative method is to use a dielectric sheet power splitter in oversize waveguide, and also fabricate the resonators in oversize waveguide. Using this, the construction is relatively simple (although possibly no less costly), and the losses are reduced. It is possible that such equalizers could be used in conjunction with oversize waveguide band-splitting filters.

It will be shown in Section 7.3.2, that from a purely optical (ray) viewpoint, the requirements for a 3 dB power split are that the dielectric slab should be, effectively, an odd number of quarter-wavelengths thick, and that the dielectric constant should be 3.4142. Schaefer Dielectrics Limited produce a magnesia ceramic with a loss tangent of 0.0005 up to 10 GHz, the dielectric constant of which may be specified in the range 3.0 to 7.0 to an accuracy of ± 0.1%. This material is easily machined in thin sheets, and so is eminently suitable for the coupler.

Fig. 7.1b shows the basic action of the equalizer. The situation depicted is for optical propagation, which necessitates an aperture of several hundred wavelengths. In practice a balance between low mode generation and overall dimensions dictates an aperture of about six to ten wavelengths. The 90 degree phase difference between reflected and transmitted waves is a property of the slab which enables the two reflected waves at the input port to cancel, whilst those at the output port interfere constructively. As the frequency varies, so the phase shift
through the device alters, the amplitudes remaining constant. Since the splitting depends on the slab being a quarter wavelength in effective thickness, the power reflected back to the input will vary with frequency. For reasonable V.S.W.R.'s, the bandwidth for a quarter wavelength slab is anticipated to be about $10\%$, or $5\,\text{GHz}$ at $50\,\text{GHz}$. Thus for equalization over a $500\,\text{MHz}$ bandwidth, the input V.S.W.R. will be low.

7.2 Formation and properties of the resonators

7.2.1 Introduction

The resonators are simple cavities, with an integral number of half wavelengths between the input iris and the short circuit. Since the rectangular waveguide is highly overmoded, simple inductive post or iris coupling cannot be used, for these would generate higher-order modes which would degrade the resonator performance. An iris is required, therefore, which produces no higher-order propagating modes when the dominant $H_{1,0}$ mode is incident on it.

Considerable work has been done on the subject of multi-strip capacitive and inductive irises$^{5-10}$, most of which involves extremely involved complex-variable theory. In order to solve the resulting equations, a number of simplifying assumptions are made which render the answers of questionable value. It was decided, that answers of sufficient accuracy for the equivalent circuit of the multi-strip iris could be obtained by using the 'two plane-waves' equivalence of the $H_{1,0}$ mode, with the results of Marcuvitz for free space irises$^{11}$, the incident angles for the highly-overmoded case being small. The choice between inductive or capacitive multiple-strip irises favours the inductive
case. The capacitive iris is highly inhomogeneous in the electric field direction, and so considerably modifies the lines of current flow. For this reason, the power handling is lower, and, more important, the losses are higher than for the inductive iris. Furthermore, for a given degree of coupling, the gap in an inductive iris is always larger than for a capacitive iris. The inductive iris is therefore less critical as far as tolerances are concerned. It is assumed in the work presented that the iris is infinitely thin.

The thickness of an iris may alter the equivalent circuit quite drastically, and so should be born in mind when considering the manufacture of the iris.

The problem now remains of finding the number of slits which will ensure that none of the propagating higher order modes will be excited by the iris. To solve this, an analysis based on a method due to Lyapunov was developed. The details of the work is given in Appendix 7A, and there it is shown that if an $H_{n,0}$ mode is incident on a 't' slit iris, the modes generated are of the form $H_{(2nt \pm n),0}$ where $N$ is any positive integer. Thus for an incident $H_{1,0}$ mode, the first higher order mode to be excited is the $H_{(2t - 1),0}$ mode. If all modes up to $H_{11,0}$ can propagate, we therefore need at least seven slits. Conversely if there is an $H_{n,0}$ mode incident on the iris due to mode conversion in the coupling section, further mode conversion back to the $H_{1,0}$ mode is undesirable. This produces a constraint $(2t - n) > 1$ or $2t > (1 + n)$, which is the same condition as that for the mode conversion above.

If spurious, higher order propagating modes are formed at the coupler, there is a possibility that spurious resonances of the cavity in the frequency range of interest
occur for these modes. As a typical example, a cavity was designed for a resonance at 25 GHz, at which it was two half-wavelengths long. The resonant frequencies for the higher order modes were computed, taking account of the frequency sensitivity of the coupling multi-strip iris. The following results were obtained

Number of slits = 14, loaded Q = 100

<table>
<thead>
<tr>
<th>Mode number, n</th>
<th>Resonant Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(TE_{n,0})</td>
<td>(GHz)</td>
</tr>
<tr>
<td>1</td>
<td>25.040</td>
</tr>
<tr>
<td>2</td>
<td>25.304</td>
</tr>
<tr>
<td>3</td>
<td>25.739</td>
</tr>
<tr>
<td>4</td>
<td>26.336</td>
</tr>
<tr>
<td>5</td>
<td>27.085</td>
</tr>
<tr>
<td>6</td>
<td>27.974</td>
</tr>
<tr>
<td>7</td>
<td>28.991</td>
</tr>
<tr>
<td>8</td>
<td>30.013</td>
</tr>
<tr>
<td>9</td>
<td>31.367</td>
</tr>
<tr>
<td>10</td>
<td>32.707</td>
</tr>
<tr>
<td>11</td>
<td>34.142</td>
</tr>
<tr>
<td>12</td>
<td>35.694</td>
</tr>
</tbody>
</table>

With a 500 MHz bandwidth, and the centre frequency of the equalizer at the lower edge of the band, the table shows that the TE_{2,0} resonance falls within the band, but all others are outside it. It is necessary, then, that the TE_{2,0} mode amplitude generated by the coupling section be much less than the TE_{1,0} amplitude. The other modes are not so critical, but as a rule of thumb, they want to be about 20 dB below the TE_{1,0} mode. A value for the TE_{2,0} mode amplitude of 20 dB below the TE_{1,0} mode should be taken as an upper limit.
7.2.2 Characteristics of the cavity at dominant resonance

If we look in at the plane of the detuned short circuit, (where this is defined as the plane nearest the iris where the impedance is zero or a minimum off resonance, and coincides with the reference plane of the iris), the reflected wave is given by

\[ b_1 = \frac{\sqrt{1 - k^2 - k^2 \exp[-(\alpha_l + j2\phi)]}}{\sqrt{1 - k^2 \exp[-(\alpha_l + j2\phi)]}} \]

(7.1)

where \( a_1 \) is the incident wave amplitude
\( b_1 \) is the reflected wave amplitude
\( k \) is the coupling of the iris
\( \alpha_l \) is the round trip attenuation of the cavity
\( \phi \) is the electrical distance from the end wall to the interior reference plane of the iris.

Neglecting losses, and differentiating equation (7.1) with respect to frequency, the following expression for group delay is obtained,

\[ t_g = -\frac{L}{c} \frac{k^2[2\sqrt{1 - k^2 \cos 2\phi} - (2 - k^2)]}{4(1 - k^2 - 4(2 - k^2) \cos 2\phi + 4(1 - k^2) \cos 2\phi + k^4)} \]

(7.2)

where \( L \) = length of cavity from end wall to reference plane of iris
\( c \) = electromagnetic wave velocity in free space.

At resonance, \( \phi = n\pi \), where \( n \) is an integer, and the maximum group delay is obtained

\[ t_g \bigg|_{\text{max}} = -\frac{L}{c} \frac{k^2[2\sqrt{1 - k^2} - 2 + k^2]}{8(1 - k^2 - 4(2 - k^2)\sqrt{1 - k^2} + k^4)} \]

(7.3)

Since \( k \ll 1 \), an approximate expression can be derived for

\[ t_g \bigg|_{\text{max}} \]

by expanding the square roots as a power series in \( k \).

This yields
Thus for a given value of $t_g\big|_{\max}$, equation (7.4) enables the corresponding value of $k$ to be roughly computed. If a group-delay maximum of 5 nsec is required, for example, equation (7.4) gives $k = 0.409$ for $L \approx 2$ cm. Altman\textsuperscript{14} gives an expression for the external $Q$, $Q_E$, of the cavity:

$$Q_E = \frac{4\pi L \lambda}{k^2}$$

(7.5)

where $\lambda$ is the guide wavelength.

Substituting in values,

$$Q_E = 125$$

Taking an unloaded $Q$, $Q_U$, of 2000, the loaded $Q$, $Q_L$, is given by

$$Q_L = \frac{Q_E Q_U}{Q_E + Q_U} = 117.5$$

The half-power-point bandwidth at 25 GHz = 213 MHz.

The corresponding 'half-group-delay' bandwidth, $Q_D$, (from equation (2.9)) is 127 MHz.

The power absorbed in the resonator is given by \textsuperscript{14}

$$P_{\text{abs}} = P_o \frac{4Q_L^2}{Q_E Q_U}$$

where $P_o$ is the incident power.

The insertion-loss for our 5 nsec case is, therefore, given by

$$\text{insertion-loss} = -10 \log_{10} \left(1 - \frac{4Q_L^2}{Q_E Q_U}\right)$$

$$= 1.01 \text{ dB}$$

Thus for a 5 nsec variation, an insertion loss of 1 dB must be tolerated. Whether or not such a figure is acceptable can only be answered by the system designer.
7.3 Analysis of mode formation in the coupler

7.3.1 Possible methods of beam splitting

Two methods of achieving a 3 dB power split were considered. The first was that proposed by Garnham\textsuperscript{15}. The device consists of two 45 degree prisms placed as shown in Fig. 7.2. The input wave is incident on the first sloping face at an angle of incidence greater than the critical angle. The wave would normally, therefore, be totally reflected. Due to the presence of an evanescent surface wave, however, the second prism extracts power from the incident wave via the evanescent field present in the gap between prisms. If the gap is decreased, the power extracted from the incident wave increases. The discontinuities presented by the dielectric prisms can be matched out\textsuperscript{16} by means of slots one quarter wavelength deep cut in the dielectric, thereby reducing the effective dielectric constant and forming a quarter-wave transformer.

The second method is that using a quarter-wave sheet of dielectric previously mentioned. The latter was chosen as the splitter, mainly because of its simplicity and low loss. With the dielectric prisms, the waves must travel through many wavelengths in the dielectric, and so the loss must inherently be much higher than for the dielectric sheet. Also, to obtain a 3 dB split, the spacing of the prisms must be kept constant along the prism face to a high degree of accuracy. The problems in machining the dielectric slab are relatively simple.

7.3.2 Properties of the dielectric slab

A typical plane wave incident on the dielectric slab is shown in Fig. 7.3a. The electric field is perpendicular
FIG 7.2 A dielectric prism power splitter
a - The dielectric slab

b - Equivalent transmission line of dielectric slab

FIG 7-3 The power splitter
to the plane of incidence. The equivalent transmission line is shown in Fig. 7.3b, where $Z_o$ is the wave impedance in region 1 along the normal to the slab, and $Z_d$ the corresponding quantity in region 2. The effective electrical length of the slab is $\Omega$. We define

$$R_1 = \frac{Z_d - Z_o}{Z_d + Z_o} = \text{reflection coeff. from region 1 to region 2}$$

$$R_2 = \frac{Z_o - Z_d}{Z_d + Z_o} = \text{reflection coeff. from region 2 to region 1}$$

$$T_1 = \frac{2Z_d}{Z_d + Z_o} = \text{transmission coeff. from region 1 to region 2}$$

$$T_2 = \frac{2Z_o}{Z_d + Z_o} = \text{transmission coeff. from region 2 to region 1}$$

If a wave of unit amplitude is incident on the dielectric slab the wave reaching the other side is given by

$$\text{output} = T_1 T_2 \exp(-j\Omega) \sum_{n=0}^{\infty} R_2^{2n} \exp(j2n\Omega)$$

Thus we have a transmission coefficient, $T$, of

$$T = \frac{T_1 T_2 \exp(-j\Omega)}{1 - R_2^2 \exp(-j2\Omega)}$$

$$= \frac{4Z_{DN}(\cos\Omega - j\sin\Omega)}{(1 + Z_{DN})^2 - (\cos 2\Omega + j \sin 2\Omega)(1 - Z_{DN})^2} \quad (7.6)$$

where $Z_{DN} = Z_d/Z_o$

By similar reasoning, the reflection coefficient, $R$ is given by
\[ R = \frac{j(z_{DN}^2 - 1) \tan \Omega}{2z_{DN} + j(z_{DN}^2 + 1) \tan \Omega} \]

It is easily shown that

\[ Z_0 = \frac{120\pi}{\cos i} \text{ ohms} \]  
\[ Z_d = \frac{120\pi}{\sqrt{\varepsilon_r} \cos r} \text{ ohms} \]

where \( \sin r = \sin i \)

\[ \varepsilon_r \]

For the special case of an angle of incidence of 45 degrees, and \( \Omega = \pi/2 \),

\[ T = -\frac{j}{\varepsilon_r} \frac{\sqrt{2\varepsilon_r - 1}}{\sqrt{\varepsilon_r}} \]

For a 3 dB power split, \( T = 1/\sqrt{2} \), therefore

\[ \varepsilon_r = 3.4142 \]

Equation (7.11) shows that to obtain a 3 dB power split with a quarter wavelength slab, the relative dielectric constant must be 3.4142. Also, under these conditions, equations (7.6) and (7.7) reduce to

\[ R = \frac{Z_{DN}^2 - 1}{Z_{DN}^2 + 1} \]
\[ T = -\frac{j2Z_{DN}}{1 + Z_{DN}^2} \]

which shows that, as required, the reflected and transmitted waves differ in phase by 90 degrees.

The actual thickness of the slab is easily calculated by considering the propagation constant in the direction of the normal to the slab:

phase constant = \( \frac{2\pi}{\lambda} \cos r \)

\( \lambda \)

\( \pi = \pi/2 \) ensures that \( T \) and \( R \) differ by 90° irrespective of the value of \( \varepsilon_r \).
\[ \therefore 2\pi \cos r \times d = \frac{\lambda}{2} \]
\[ \therefore d = \frac{\lambda}{4 \cos r} \]

where \( r \) corresponds to an angle of incidence of 45°, and \( \lambda_d \) is the wavelength in the dielectric.

\[ \therefore d = \frac{\lambda_d}{4 \sqrt{1 - 1/2\varepsilon_r}} \quad (7.12) \]

By similar reasoning, \( \Omega \) is obtained:

\[ \Omega = \frac{\pi}{2} \left[ 1 - \frac{\sin^2 i}{\varepsilon_r} \right] \quad (7.13) \]

A further quantity required in the modal analysis is the effective phase shift from point c to point e in Fig. 7.3a. Referring to this figure, the complex transmission coefficient will give the phase of \( a \) with respect to \( c \). The phase of \( b \) will lag that of \( a \) by \( 2\pi \left( \sin i \right) \left( \tan r \right) d \). The phase of \( e \) will lag that of \( a \) by \( 2\pi \left( \sin i \right) d \left( \tan i \right) \).

The 'effective phase shift' is defined as 'effective phase shift' = \[ \frac{2\pi \, d \left( \sin i \right) \left( \tan i \right)}{\lambda_d} - \frac{2\pi}{\lambda} \frac{d}{\cos i} \]

\[ = \frac{\pi \cos i}{\sqrt{4 \varepsilon_r - 2}} \quad (7.14) \]

7.3.3 Plane-wave representation of the input

The concept of representing an aperture distribution as an angular-spectrum of plane waves was first introduced by Booker and Clemmow\textsuperscript{17}. They showed that the field at all points in front of a plane aperture of any distribution
may be regarded as arising from an aggregate of plane waves travelling in various directions. The amplitude and phase of the waves, as a function of their direction of travel, constitutes an angular spectrum, and this angular spectrum, appropriately expressed, is, without approximation, the Fourier transform of the aperture distribution.

If the origin of co-ordinate $y$ is taken as the centre of the transmitting aperture, the electric field distribution along $y$ for an $H_{1,0}$ mode is

$$E = \cos \frac{\pi y}{a} \left( -\frac{a}{2} < y < \frac{a}{2} \right)$$

$$= 0 \left( y < -\frac{a}{2}, \ y > \frac{a}{2} \right)$$

where $a$ is the width of the aperture. The Fourier transform of $E$ gives the amplitude and phase distribution of the plane waves. If the angle of a plane wave with respect to the normal to the aperture is $\theta$, define

$$S = \sin \theta$$

Then the distribution function, $P(S)$ is given by

$$P(S) = \int_{-\infty}^{\infty} E \exp (j k S y) dy$$

(7.16)

where $k = \frac{2\pi}{\lambda}$ is the propagation coefficient.

Substituting equation (7.15) into (7.16), and simplifying, we obtain

$$P(S) = \frac{2\pi a \cos \left( k S a / 2 \right)}{\left( \pi^2 - k^2 S^2 a^2 \right)}$$

(7.17)

This shows that at $y = 0$, the phases of all the planes are equal. A plot of $P(S)$ against $\theta$ is shown in Fig. 7.4, from which it is seen that, by restricting $\theta$ to the range $-30^\circ$ to $+30^\circ$, very little information is lost.
FIG 7.4 The angular spectrum

Aperture - 7.2136 cm
Wavelength - 1.2 cm
7.3.4 Aperture distribution of opposite port

If a point is taken in the aperture of the opposite port, as shown in Fig. 7.5, the resultant field at that point can be approximately obtained by considering the amplitude and phase of each of the plane waves reaching the point from the input aperture, and summing them. The approximation involved being that the value so obtained is for the case where the point is in space with no electric walls in the vicinity. The presence of the waveguide walls defining the cross will perturb the field. However, if the waveguide is sufficiently overdosed, these effects will be of secondary importance.

There are two cases to consider. The first is for points such as A, where all wave-fronts (considering only those for $\theta$ between $\pm 30^\circ$) pass through the dielectric slab. The second case is for points such as B, where, for waves with an angle less than $-\chi$, no dielectric slab is encountered, whilst for other angles the slab must be traversed.

If the wave is direct, the phase lag from the centre of the input aperture to the point concerned is given by

$$\text{phase lag} = \frac{2\pi}{\lambda} (a + y \tan \theta) \cos \theta$$  \hspace{1cm} (7.18)

where $\theta$ is the wave angle, the sign convention being shown in Fig. 7.5. For transmission through the slab, the phase lag is modified by the angle of the complex transmission coefficient, and further by the 'effective phase shift' derived in a previous section, which takes into account the phase shift through the space occupied by the slab, this space being included in equation (7.18).

Thus for a wave not passing through the slab, the amplitude and phase at the point, as functions of $\theta$, are
FIG 7-5 Construction to determine opposite port aperture distribution
given by

\begin{align*}
\text{amplitude} & = P(S) \quad \text{(from equation 7.17)} \\
\text{phase} & = -\frac{2\pi}{\lambda} (a + y \tan \theta) \cos \theta \\
& = -\frac{2\pi}{\lambda} (a \sqrt{1 - s^2} + y s) \\
& \quad \text{(7.19)}
\end{align*}

For a wave passing through the slab,

\begin{align*}
\text{amplitude} & = P(S) \times \left| T \right| \\
\text{phase} & = -\frac{2\pi}{\lambda} \left( a \sqrt{1 - s^2} + ys \right) + \pi \sqrt{1 - s^2} \\
& \quad \frac{\sqrt{4\epsilon_r - 2}}{\sqrt{4\epsilon_r - 2}} \\
& \quad + \text{Arg}(T) \quad \text{(7.20)}
\end{align*}

where \( T \) is the transmission coefficient defined in equation (7.6), and \( s = \sin \theta \). If the point is one such as B, the crossover from direct wave to slab wave occurs when

\[ -\theta = \tan^{-1} \left( \frac{a}{2} - y \right) = \psi \quad \text{(7.21)} \]

The value of \( y \) at which the direct wave starts to play a part is given by

\[ \bar{y} = \tan^{-1} \left( \frac{a}{2} - y \right) \]

a

or \( y = 0.07735 \ a \quad \text{(7.22)} \)

By integrating numerically equations (7.19), (7.20)

with respect to \( s \), with the constraints of (7.21), (7.22),

the electric field distribution over the aperture is obtained. Since the resultant values contain amplitude and phase information, a plot of real and imaginary parts yields little information. For completeness, however, the aperture distribution for the case of \( a = 7.2136 \ \text{cm (WG 10)} \) and \( \lambda = 1.2 \ \text{cm} \), is shown in Fig. 7.6. Appendix 7B shows that the modes generated in the receiving waveguide can be computed by submitting the aperture distribution to a Fourier analysis. The results for the case analysed above are shown in Table 7.1. The mode amplitudes and phases are relative to a reference which is fixed for both the
FIG 7.6 Aperture distribution of port 2

- Real component (scale x1)
- Imag. component (scale x-1)
opposite port and adjacent port cases. The results for these cases can therefore be compared directly.

<table>
<thead>
<tr>
<th>Mode number n ( (TE_{n,o}) )</th>
<th>relative E field amplitude</th>
<th>relative phase (degrees)</th>
<th>amplitude in dB down on ( TE_{1,o} ) mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.7371</td>
<td>- 51.141</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.58062</td>
<td>+ 21.164</td>
<td>22.5</td>
</tr>
<tr>
<td>3</td>
<td>1.0075</td>
<td>+ 4.825</td>
<td>17.74</td>
</tr>
<tr>
<td>4</td>
<td>0.38868</td>
<td>+ 10.568</td>
<td>26.0</td>
</tr>
<tr>
<td>5</td>
<td>0.82654</td>
<td>+ 39.654</td>
<td>19.41</td>
</tr>
<tr>
<td>6</td>
<td>0.37285</td>
<td>- 24.399</td>
<td>26.35</td>
</tr>
<tr>
<td>7</td>
<td>0.32283</td>
<td>+ 11.927</td>
<td>27.6</td>
</tr>
<tr>
<td>8</td>
<td>0.38773</td>
<td>- 47.804</td>
<td>26.0</td>
</tr>
<tr>
<td>9</td>
<td>0.24439</td>
<td>- 3.037</td>
<td>29.9</td>
</tr>
<tr>
<td>10</td>
<td>0.34860</td>
<td>- 61.835</td>
<td>26.9</td>
</tr>
<tr>
<td>11</td>
<td>0.17962</td>
<td>- 7.589</td>
<td>32.7</td>
</tr>
</tbody>
</table>

Table 7.1 Output from opposite port.

7.3.5 Aperture distribution of adjacent port

For the adjacent port, port 3 of Fig. 7.1a, the path taken by a wavefront is rather more tortuous than for the opposite-port case. Fig. 7.7 shows the path taken by the normal to a typical wavefront. In this case, account is taken of the finite thickness of the dielectric slab by reducing the aperture size by an amount \( \delta \), where \( \delta \) is related to \( d \), the thickness of the slab, by

\[
\delta = \frac{d}{\sqrt{2}}
\]

Using simple trigonometry, the phase shift along the path shown, to a point distant \( x \) from the side wall, is given by
FIG 7.7 Construction to determine the aperture distribution of port 3
phase shift = \(- \frac{2\pi}{\lambda} (L_1 + L_2 - L_3)\)
\[= - \frac{2\pi}{\lambda} \left[ a - \delta - \sin\theta \left\{ \frac{(x + a)\cos\theta +}{2} + (a - \delta)(\sin\theta - \cos\theta) \right\} \right] \] (7.24)

where the phase shift at the point of reflection is ignored.

At a given point along the x-axis, there may be values of \(\theta\) for which no reflected wave is encountered. This occurs when \(\theta\) is such that

\[x = -(a + \delta \cot\theta) \tan\theta_c\]

or \(\theta_c = -\tan^{-1} \left( \frac{x + \delta}{a} \right)\) (7.25)

For values of \(\theta\) more negative than this, there is only the direct wave, the phase shift of which is given by

\[\text{phase shift} = - \frac{2\pi}{\lambda} (x \cos\theta - a \sin\theta) \] (7.26)

If the point is such that the value of \(\theta_c\) from equation (7.25) is numerically greater than 30°, then the point has a direct and reflected wave for all values of \(\theta\). Otherwise, for values of \(\theta\). Otherwise, for values of \(\theta\) greater than \(- \theta_c\), there is both a direct and a reflected wave, whilst for \(\theta\) less than \(- \theta_c\), there is only a reflected wave.

for the direct wave,

\[
\text{amplitude} = P(s) \\
\text{phase} = -\frac{2\pi}{\lambda} (x \cos\theta - a \sin\theta) \] (7.27)

For the reflected wave,

\[
\text{amplitude} = P(s) x \left| R \right| \\
\text{phase} = -\frac{2\pi}{\lambda} \left[ a - \delta - \sin\theta \left\{ \frac{(x + a)\cos\theta +}{2} + (a - \delta)(\sin\theta - \cos\theta) \right\} \right] + \text{Arg} (R) \] (7.28)

where \(R\) is the complex reflection coefficient defined in equation (7.7).
Numerically integrating the above equations over the range of $\theta$, the aperture distribution was computed. A Fourier analysis yielded the following results.

<table>
<thead>
<tr>
<th>Mode number $n$ ($\text{TE}_{n,0}$)</th>
<th>relative E field amplitude</th>
<th>relative phase (degrees)</th>
<th>dB down on $\text{TE}_{1,0}$ mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.9968</td>
<td>+ 33.109</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.0943</td>
<td>- 40.035</td>
<td>38.568</td>
</tr>
<tr>
<td>3</td>
<td>0.8491</td>
<td>- 63.821</td>
<td>19.479</td>
</tr>
<tr>
<td>4</td>
<td>0.2237</td>
<td>- 14.379</td>
<td>31.065</td>
</tr>
<tr>
<td>5</td>
<td>1.0202</td>
<td>- 28.603</td>
<td>17.885</td>
</tr>
<tr>
<td>6</td>
<td>0.1455</td>
<td>+ 4.385</td>
<td>34.801</td>
</tr>
<tr>
<td>7</td>
<td>0.71455</td>
<td>- 28.154</td>
<td>20.977</td>
</tr>
<tr>
<td>8</td>
<td>0.022116</td>
<td>- 2.663</td>
<td>51.164</td>
</tr>
<tr>
<td>9</td>
<td>0.79713</td>
<td>- 32.400</td>
<td>20.026</td>
</tr>
<tr>
<td>10</td>
<td>0.39109</td>
<td>- 31.480</td>
<td>26.213</td>
</tr>
<tr>
<td>11</td>
<td>2.2399</td>
<td>- 46.116</td>
<td>11.054</td>
</tr>
</tbody>
</table>

**TABLE 7.2**

Output from adjacent port

### 7.3.6 Conclusions

The results for the $\text{TE}_{1,0}$ mode in Tables 7.1 and 7.2 show that the amplitudes of this mode in the opposite and adjacent ports differ by 0.28 dB, also, their phase difference is 84.2 degrees. The ideal values for these parameters are 0 dB and 90 degrees respectively. With the assumptions made in obtaining these values, it is felt that the good agreement between the above quantities justifies some confidence in the relative amplitudes of the higher order modes, the computation of which was the primary objective. In Section 7.4 a further comparison is made to test the accuracy of the
above method.

Test of the accuracy of the plane-wave method

Although the results given in the last section showed that the plane-wave approach gives reasonable results, it was thought desirable to try and ascertain the absolute accuracy of the method. What was required was a problem which resembled that of the coupler, but which was capable of exact solution as well; thereby enabling the accuracy of the approximate solution to be checked.

Such a problem is that of the overmoded waveguide cross, which is closely related to the overmoded mitred waveguide corner\(^2\). The problem is therefore that of the power splitter with the dielectric slab removed.

Using an extension of a method first published by Miles\(^2\), and later used by Campbell\(^2\) to analyse stripline corners, a solution to the mode generation in the cross can be found. The accuracy of the solution is dependent only on the number of non-propagating modes considered, and is capable of accuracies of within one percent using only ten non-propagating modes.

The exact analysis is fully described in Appendix 7C, and only the results are quoted here. The dimensions and wavelengths involved are as given in the previous sections. An analysis similar to that of Section 7.3.4 was performed to obtain the aperture distribution, and the results obtained for the modes exciting port 2 are shown in Tables 7.3 and 7.4.
### TABLE 7.3

<table>
<thead>
<tr>
<th>Order of mode ((\text{TE}_{n,0}))</th>
<th>amplitude relative to incident (\text{TE}_{1,0}(\text{dB}))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>plane-wave analysis</td>
</tr>
<tr>
<td>3</td>
<td>-23.32</td>
</tr>
<tr>
<td>5</td>
<td>-21.36</td>
</tr>
<tr>
<td>7</td>
<td>-26.75</td>
</tr>
<tr>
<td>9</td>
<td>-27.33</td>
</tr>
<tr>
<td>11</td>
<td>-25.04</td>
</tr>
</tbody>
</table>

### TABLE 7.4

<table>
<thead>
<tr>
<th>Order of mode ((\text{TE}_{n,0}))</th>
<th>Phase (degrees) (\text{(relative to input)})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>plane-wave analysis</td>
</tr>
<tr>
<td>1</td>
<td>+2.54</td>
</tr>
<tr>
<td>3</td>
<td>-109.36</td>
</tr>
<tr>
<td>5</td>
<td>-60.86</td>
</tr>
<tr>
<td>7</td>
<td>-44.94</td>
</tr>
<tr>
<td>9</td>
<td>-45.36</td>
</tr>
<tr>
<td>11</td>
<td>-44.84</td>
</tr>
</tbody>
</table>

The number of modes considered in the exact solution was a compromise between computer time and accuracy. The computer time increases roughly as the cube of the number.
of variables, whilst the accuracy increases only slowly as the number of non-propagating modes increases. For the case just considered, the value of the $\text{TE}_{1,0}$ mode amplitude was plotted as a function of the number of non-propagating modes. The results for thirty and six non-propagating modes agreed to within one percent. The number of non-propagating modes considered for the above computations was therefore fixed at eight, giving an accuracy of better than one percent.

Table 7.3 indicates an accuracy of about $\pm 2 \text{ dB}$ for the mode amplitudes. With the number of approximations involved, this seems to be a reasonable value. The phases also are in excellent agreement, apart from the result for the $\text{TE}_{7,0}$ mode, for which no reason has been found.

The conclusion of this comparison was that the relative mode amplitudes for the coupler obtained by the plane wave approach would be close to the true values.

The use of different degrees of overmoding was also investigated using the exact analysis. Taking the amplitude of the $\text{TE}_{3,0}$ mode as fairly representative of the higher mode excitation, the results of Table 7.5 were computed with the broadwall dimension of the waveguide fixed at 7.2136 cm. The results show that a wavelength of 1.2 cm gives a reasonable compromise between reasonable waveguide size and low mode generation.
Overall Conclusions

The results of Tables 7.1 and 7.2 show that the TE₂,₀ mode generation of the coupling section is below 20dB relative to the TE₁,₀ mode. Thus the use of a one-wavelength cavity in Section 7.2.1 is adequate, since although the TE₂,₀ resonance is only 300 MHz from the TE₁,₀ resonance, the fields associated with it are sufficiently small to have little effect.

The work presented in this chapter has shown that the coupler–resonator equalizer is a feasible proposition for millimeter-wavelength systems where simple equalization is required.
Mode Generation by Capacitive and Inductive Grids

1. Capacitive Grids

The multi-strip capacitive iris is shown in Fig. 7.8a. Since the electric field lines are in the y direction only, electric walls may be placed perpendicular to the plane of the iris, and parallel to the broadwall, in the centre of each slit and broad strip, without perturbing the incident field. It can be seen that, due to the homogeneity in the x direction, an incident \( H_{1,0} \) mode will not excite any components of the form \( H_{n,0} \). The placing of the electric walls is restricted by the condition that symmetry be retained (an extension of the method of images), the walls being inserted until the "unit element" which generates the symmetry pattern is reached. Clearly, the only modes which will satisfy the boundary conditions are \( H_{n,2Nt} \) modes for an incident \( H_{n,0} \) mode, where \( N \) is any positive integer, and \( t \) is the number of slits.

2. Inductive Grids

The Lorentz lemma, or Lorentz reciprocity theorem, states that when \( \vec{E}_1, \vec{H}_1 \) is the field generated in the volume \( V \), bounded by a closed surface \( S \), by a volume distribution of electric current \( \vec{J}_1 \); and \( \vec{E}_2, \vec{H}_2 \) is the field generated in the same volume by a second system \( \vec{J}_2 \), then

\[
\oint_S (\vec{E}_1 \times \vec{H}_2 - \vec{E}_2 \times \vec{H}_1) \cdot \vec{n} = \int_V (\vec{E}_2 \cdot \vec{J}_1 - \vec{E}_1 \cdot \vec{J}_2) dV \quad (7A.1)
\]

For the case of a strip inductive iris, such as that shown in Fig. 7.8b, \( \vec{J}_1 = 0 \) and \( \vec{J}_2 = \vec{J} \), the latter being the current distribution on the strips which generates \( \vec{E}_2, \vec{H}_2 \).
a - Multi-strip capacitive iris

b - Multi-strip inductive iris

FIG 7.8 Multi-strip irises
Let $E_1, H_1$ be an $H_n^m$ mode incident on the iris, and henceforth denoted by $E_{1m}^n, H_{1m}^n$; and let $E_2, H_2$, the reflected fields be

$$\sum_n A_n^1 E_{2n}^n, \quad \sum_n A_n^{2n} H_{2n}^n$$

for all modes of the $H_{n,0}$ type, which are the only ones generated if the iris is uniform in the $y$ direction.

Collins shows that

$$\oint_{\partial V} (E_{1m}^n \times H_{2n}^n - E_{2n}^n \times H_{1m}^n) \cdot \mathbf{d}a = 0$$

unless $m = n$ (7A.2)

We can thus rewrite equation (7A.1) as

$$\oint_{\partial V} (E_{1m}^n \times H_{2m}^n - E_{2m}^n \times H_{1m}^n) \cdot \mathbf{d}a = \int_V (E_{1m}^n \cdot J) dV$$

(7A.3)

Splitting the fields into their components, and denoting the unit vector in the $z$ direction by $k$, we have

$$\oint_{\partial V} (E_{1m}^x H_{2m}^y - E_{1m}^y H_{2m}^x - E_{2m}^x H_{1m}^y + E_{2m}^y H_{1m}^x) k \cdot \mathbf{d}a =$$

$$= \int_V (E_{1m}^x \cdot J) dV$$

(7A.4)

where the superscripts denote the component directions.

The surface of integration is shown as surface 1 and surface 2 in Fig. 7.9. Since the only modes present are of the form $H_{p,0}^x$, $E_{m}^y = 0$ and $H_{m}^y = 0$. Thus we have

$$\oint_{\partial V} (E_{1m}^x H_{2m}^y + E_{2m}^y H_{1m}^x) da + \oint_{\partial V} (E_{1m}^y H_{2m}^x - E_{2m}^x H_{1m}^y) da =$$

$$= - \int_V E_{1m}^y J dV$$

The difference in sign of the integrands is due in one case to having a reflected wave, and in the other a transmitted wave, as shown in Fig. 7.9.

Since the positions of surface 1 and surface 2 are
 FIG 7-9 The surface of integration
arbitrary, let us choose them to be very close, as shown in Fig. 7.9. Since the fields denoted by a subscript 1 are those existing with no current distribution present, as \( \Delta z \to 0 \), the values of the fields on surface 1 and surface 2 will converge. Thus, in the limit we have

\[
2 \int_{s_1} E_{1m} H_{2m}^* \, da = - \int_{s_1} J_{1m}^Y \, dV \quad (7A.5)
\]

The current source, \( J^Y \), will radiate equally in both the positive and negative \( z \) directions. If we assign an amplitude \( A_m \) to the \( H_{1,0} \) mode travelling in the negative \(-z\) direction from the current source, and an amplitude \( B_m \) to the one travelling in the positive \(-z\) direction, then

\[
A_m = B_m
\]

Since the whole volume contains no sources, however, energy generation within the volume must be zero. With an incident \( H_{1,0} \) mode of unit amplitude incident on surface 1, there must also, therefore, be an \( H_{1,0} \) mode of unit amplitude exiting the volume at surface 2. The reflected fields are those with amplitudes \( A_m \), and the transmitted fields are those with amplitude \( B_m \) as well as the original \( H_{1,0} \) mode. Reducing the field components to their basic forms multiplied by an amplitude constant, and assuming an incident field of unit magnitude.

\[
A_m = B_m = \int_{s_1} - E_{1m}^Y J^Y \, dV \quad (7A.6)
\]

where

\[
E_{1m}^Y = \frac{k^2}{\varepsilon_x \varepsilon_0} \sin \frac{m\pi x}{a}
\]

\[
E_{2m}^x = j\omega \mu \sin \left( \frac{m\pi x}{a} \right)
\]
\[ \gamma_m = \left[ (\pi/2)^2 - (2\pi/\lambda_o)^2 \right]^{1/2} \]

\[ A_m = \frac{\varepsilon_r \varepsilon_0}{j\omega \gamma_m^2 \kappa_{ab}} \int_{\text{lm}} \int_{\text{Y}} J dV \]  

(7A.7)

where \( a, b = \text{waveguide interior dimensions (}a > b\) \)

\( \varepsilon_r \varepsilon_0 = \text{permittivity of medium in waveguide} \)

\( k = 2\pi/\text{(free space wavelength)} \)

The mode coefficients

The amplitude of the \( m \)th mode reflected from the diaphragm is given by

\[ A_m = \frac{\varepsilon \varepsilon_0}{j\omega k^2 \gamma_{ab}} \int_{\text{lm}} \int_{\text{Y}} J dV \]

The current distribution, \( J \), is zero except on the strips. The exact distribution on the strips is not known, and the value of \( A_m \) is not stationary with respect to small variations in \( J \). However, the generation of the higher order modes is more dependent on the periodic property of the slits than on the fine structure of the current distribution. The current density away from the edges of the strip will be of the same form as the incident wave. As an edge is approached, however, the current density will increase asymptotically.

An investigation has been undertaken to find the effect on the results of adding impulse functions at the strip edges to the approximate current distribution used below. It was found that the results were unchanged. If we take the current distribution on the strips to be that which would occur if there were no gaps present,

\[ J = \sin\left(\frac{\pi x}{a}\right) \delta(z) \]  

(7A.8)

\( \delta(z) \) being a delta function.

The delta function specifies the distribution of the current in the \( z \) direction. Also,
\[ F_{lm}^y = \frac{k^2}{\varepsilon \varepsilon_0} \sin \frac{mx}{a} \exp (jy_m) \]  

(7A.9)

Substituting equations (7A.8) and (7A.9) into (7A.7):

\[
A_m = \frac{1}{j\omega \gamma_m} \left[ \frac{D_1}{a} \sin \frac{mx}{a} \sin \frac{nx}{a} \, dx + \sum_{n=1}^{t-1} \left\{ \right. 
\left. \frac{1}{\sin \frac{mx}{a} \sin \frac{nx}{a}} \left[ (2n+1)D_1 + nD \right] \right. 
+ \left. \left. \frac{1}{(2n-1)D_1 + nD} \left[ (2n-1)D_1 + nD \right] \right. 
+ \left. \left. \frac{1}{(2\sqrt{D_1} + D)t} \right. 
+ \left. \left. \frac{1}{(2t-1)D_1 + tD} \right. \right. \right. 
\right. 
\]  

(7A.10)

After evaluating the integrals, and performing some trigonometrical manipulations, equation (7A.10) becomes

\[
A_m = \frac{1}{j\omega \gamma_m} \left[ \frac{\sin \left\{ (m-1) \frac{\pi D_1}{a} \right\} \left[ \cos^2 \left\{ (m-1) \frac{\pi}{2} \right\} + \sum_{n=1}^{t-1} \cos \left\{ (m-1) \frac{n\pi}{t} \right\} \right] }{(m-1)\pi} 
- \sin \left\{ (n+1) \frac{\pi D_1}{a} \right\} \left[ \cos^2 \left\{ (m+1) \frac{\pi}{2} \right\} + \sum_{n=1}^{t-1} \cos \left\{ (m+1) \frac{n\pi}{t} \right\} \right] \right] 
\]  

(7A.11)

Now \( A_m \) is zero only if both of the inner square brackets are zero.
Consider the first bracket, called BR1.

\[
BR_1 = \cos^2 \left\{ (m-1) \frac{\pi}{2} \right\} + \sum_{n=1}^{t-1} \cos \left\{ (m-1) \frac{n\pi}{t} \right\} \]  

(7A.12)

The series \( 1 + \exp (j\theta) + \exp (j2\theta) + \exp (j3\theta) + \cdots \) can be summed to give

\[
\sum_{n=0}^{n-1} \exp (jn\theta) = \frac{1 - \exp(jn\theta)}{1 - \exp(j\theta)} \]
Equation (7A.12) can be written

$$BR_1 = \cos^2\left\{(m - 1)\frac{\pi}{2}\right\} + \Re \left\{\frac{1 - \exp\left[j(m - 1)\pi/2\right]}{1 - \exp\left[j(m - 1)\pi/2\right]}\right\} - 1 \quad (7A.13)$$

where \(\Re\) implies taking the real part of the complex expression.

Expanding,

$$BR_1 = \cos^2\left\{(m - 1)\frac{\pi}{2}\right\} + \Re\left\{\frac{1 - \cos\{(m - 1)\pi\} - j\sin\{(m - 1)\pi\}}{1 - \cos\{(m - 1)\pi/2\} - j\sin\{(m - 1)\pi/2\}}\right\} - 1$$

If \(m\) is odd,

$$BR_1 = \Re\left\{\frac{1 - \cos\{(m - 1)\pi\} - j\sin\{(m - 1)\pi\}}{1 - \cos\{(m - 1)\pi/2\} - j\sin\{(m - 1)\pi/2\}}\right\}$$

$$= 0 \text{ if } m - 1 = 2N, \text{ where } N \text{ is a positive integer}$$

$$= t \text{ if } m - 1 = 2N$$

If \(m\) is even:

$$BR_1 = -1 + \Re\left\{\frac{2}{1 - \cos\{(m - 1)\pi/2\} - j\sin\{(m - 1)\pi/2\}}\right\}$$

$$= 0$$

Similarly, for the second bracket \(BR_2\):

$$BR_2 = \cos^2\left\{(m + 1)\frac{\pi}{2}\right\} + \sum_{n=1}^{t-1} \cos\{(m + 1)n\pi/2\}$$

gives

$$BR_2 = \cos^2\left\{(m + 1)\frac{\pi}{2}\right\} - 1 + \Re\left\{\frac{1 - \cos\{(m + 1)\pi\} - j\sin\{(m + 1)\pi\}}{1 - \cos\{(m + 1)\pi/2\} - j\sin\{(m + 1)\pi/2\}}\right\}$$

and if \(m\) is odd

$$BR_2 = 0 \text{ if } m + 1 = 2N$$

$$= t \text{ if } m + 1 = 2N$$
if \( m \) is even

\[ E_{2} = 0 \]

We have, therefore, shown that only for \( m = 2Nt - 1 \) and \( m = 2Nt + 1 \) is \( A_{m} \) non-zero.
APPENDIX 7B

It can be shown\(^{16, 19}\), that the amplitude of the TE\(_{n,0}\) mode electric field, due to an aperture distribution \(k(x)\), is given by

\[
E_n = j \frac{k}{eL} \sqrt{\frac{\mu}{\varepsilon}} a \int_0^a k(x) \sin \frac{n\pi x}{a} \, dx
\]

(7B.1)

where \(k = \frac{2\pi}{\lambda}, \quad \frac{L^2}{\lambda^2} = \frac{n^2 \pi^2}{a^2} - k^2\), and \(a\) is the broadwall dimension.

If the aperture distribution is represented by

\[
k(x) = a_0 + \sum_{n=1}^{\infty} a_n \sin \frac{n\pi x}{a}
\]

(7B.2)

Substitution of equation (7B.2) into (7B.1) yields

\[
E_n = j \frac{k}{eL} \sqrt{\frac{\mu}{\varepsilon}} \frac{a}{2} \sin \left( \frac{n\pi a}{2} \right) \text{ for } n \text{ even}
\]

(7B.3)

and

\[
E_n = j \frac{k}{eL} \sqrt{\frac{\mu}{\varepsilon}} \left\{ \frac{2a_0}{n\pi} + \frac{a_n}{n\pi} \right\} \sin \left( \frac{n\pi a}{2} \right) \text{ for } n \text{ odd}
\]

For the case where \(k(x)\) is complex, the above process is carried out for the real and imaginary components of the aperture distribution separately, and the results combined vectorially for each value of \(n\).
AN EXACT SOLUTION TO THE OVERMODED WAVEGUIDE CROSS

The overmoded waveguide cross is shown with various excitations in Fig. 7.10. If each arrow represents unit electric field, superposition of the four cases results in an input of four units in magnitude at port 1, with no inputs at the other ports, i.e. the problem to be analysed. Inspection of Fig. 7.10 shows that the solutions to the four one-port boundary value problems shown in Fig. 7.11 are required.

Let us consider case (b), shown in more detail in Fig. 7.11e, with the right handed co-ordinate system used. We assume an electric field in the y direction in region 2 of the form:

\[ E_y = \sum_{n=1}^{\infty} \left( A_n \cos k_n^x \cos b_n z + B_n \cos k_n x \sin b_n z \right. \]
\[ + C_n \cos k_n^x \cos b_n z + D_n \cos b_n x \sin k_n z \]
\[ + E_n \sin k_n x \cos b_n z + F_n \sin k_n x \sin b_n z \]
\[ + G_n \sin b_n x \cos k_n z + H_n \sin b_n x \sin k_n z \]  \( (7C.1) \)

where \( A_n, B_n, C_n, D_n, E_n, F_n, G_n, H_n \) are arbitrary constants, \( k_n = n\pi/a \), and \( b_n \) is the propagation coefficient of the \( \text{TE}_{n,0} \) mode in region 1. The fields in region 1 consist of an incident \( \text{TE}_{1,0} \) mode with unit electric field, and a set of reflected \( \text{TE}_{n,0} \) modes.

It is a simple matter to show that equation \( (7C.1) \) satisfies Maxwell's equations. If \( E_y \) can be shown to satisfy the boundary conditions, then the uniqueness theorem for electromagnetic fields states that the field of equation \( (7C.1) \) is unique in region 2.

The boundary condition on wall B gives

\[ E_y = 0 \text{ when } x = z - a/2 \]

Thus for all values of \( x \) in region 2, each term must satisfy the boundary condition. This gives
The four modes of excitation of the cross

- o - positive E-field
- x - negative E-field
- ----- - magnetic wall
- ----- - electric wall

FIG 7.10 The four modes of excitation of the cross
FIG 7.11 The four boundary-value problems
0 = A_n \cos k_n (z - a/2) \cos b_n z + B_n \cos k_n (z - a/2) \sin b_n z
+ C_n \cos b_n (z - a/2) \cos k_n z + D_n \cos b_n (z - a/2) \sin k_n z
+ E_n \sin k_n (z - a/2) \cos b_n z + F_n \sin k_n (z - a/2) \sin b_n z
+ G_n \sin b_n (z - a/2) \cos k_n z + H_n \sin b_n (z - a/2) \sin k_n z \ (7C.2)

Since the expression is an identity, we can equate coefficients of like terms:

\[
\begin{align*}
A_n \cos \frac{ak_n}{2} + C_n \cos \frac{ab_n}{2} - E_n \sin \frac{ak_n}{2} - G_n \sin \frac{ab_n}{2} &= 0 \\
A_n \sin \frac{ak_n}{2} + D_n \cos \frac{ab_n}{2} + E_n \cos \frac{ak_n}{2} - H_n \sin \frac{ab_n}{2} &= 0 \\
B_n \cos \frac{ak_n}{2} + C_n \sin \frac{ab_n}{2} - F_n \sin \frac{ak_n}{2} + G_n \cos \frac{ab_n}{2} &= 0 \\
B_n \sin \frac{ak_n}{2} + D_n \sin \frac{ab_n}{2} + F_n \cos \frac{ak_n}{2} + H_n \cos \frac{ab_n}{2} &= 0
\end{align*}
\] (7C.3)

Since wall A is a magnetic wall, the absence of the tangential magnetic field there must be used. From Maxwell's equations

\[
\begin{align*}
H_x &= \frac{1}{j\omega \mu} \frac{\delta E_y}{\delta z} \\
H_z &= -\frac{1}{j\omega \mu} \frac{\delta E_x}{\delta z}
\end{align*}
\] (7C.4)

The boundary condition gives

\[H_x = H_z \text{ at } x = a/2 - z\]

Substituting equation (7C.1) into equation (7C.4), and simplifying gives

\[
\begin{align*}
A_n b_n \cos \frac{ak_n}{2} - D_n b_n \sin \frac{ab_n}{2} + E_n b_n \sin \frac{ak_n}{2} + H_n b_n \cos \frac{ab_n}{2} &= 0 \\
A_n b_n \sin \frac{ak_n}{2} + C_n b_n \cos \frac{ab_n}{2} + F_n b_n \cos \frac{ak_n}{2} + G_n b_n \sin \frac{ab_n}{2} &= 0 \\
- B_n b_n \cos \frac{ak_n}{2} - D_n b_n \cos \frac{ab_n}{2} - F_n b_n \sin \frac{ak_n}{2} - H_n b_n \sin \frac{ab_n}{2} &= 0 \\
A_n b_n \cos \frac{ak_n}{2} - C_n b_n \cos \frac{ab_n}{2} + G_n b_n \sin \frac{ab_n}{2} &= 0 \\
A_n b_n \sin \frac{ak_n}{2} - D_n b_n \sin \frac{ab_n}{2} + E_n b_n \sin \frac{ak_n}{2} + H_n b_n \cos \frac{ab_n}{2} &= 0
\end{align*}
\] (7C.5)
Equations (7C.3), (7C.5) constitute eight equations in the eight unknown constants. They are, however, difficult to solve as they stand. A further assumption is therefore made that electric field at $z = 0$ due to the $n$th component of equation (7C.1) should match the $TE_{n,0}$ mode in region 1. If this assumption is not valid, two conflicting equations will be encountered, whereas if it is valid, equations will be duplicated. The latter is found to be the case.

In region 1, the $y$ component of electric field has the form

$$E_y = (e^{-j k z} + x_1 e^{j k z}) \phi_1(x) + \sum_{n=2}^{\infty} \delta_n e^{j b_n x} \phi_n(x)$$  \hspace{1cm} (7C.6)

where $\phi_n(x) = \sqrt{a} \sin \frac{n\pi}{a} (x + a)$

$$b_n^2 = \left( \frac{2 \pi}{\lambda} \right)^2 - \left( \frac{n\pi}{a} \right)^2$$

and $\delta_n$ is the amplitude of the $H_{n,0}$ mode

At $z = 0$, $E_y = a_n \sin \frac{n\pi}{a} (x + a)$  \hspace{1cm} (7C.7)

where $a_1 = (1 + \delta_1) \sqrt{a}$

$$a_n = \delta_n \sqrt{a}$$

In region 2, from the $n$th term,

$$E_y \bigg|_{z=0} = A_n \cos k_n x + B_n \sin k_n x + C_n \cos b_n x + E_n \sin b_n x$$

$$= a_n \sin k_n x \cos \frac{n\pi}{2} + a_n \cos k_n x \sin \frac{n\pi}{2}$$

Equating coefficients, we have

$$C_n = 0, \quad G_n = 0$$

$$A_n = a_n \sin \frac{n\pi}{2}$$

$$E_n = a_n \cos \frac{n\pi}{2}$$

If $n$ is even, equations (7C.3), (7C.5), (7C.8) yield the following

$$B_n = 0, \quad A_n = 0, \quad E_n = (-1)^{n/2} a_n$$
\[ D_n \cos \frac{ab}{2} + (-1)^{n/2} E_n - H_n \sin \frac{ab}{2} = 0 \]
\[ D_n \sin \frac{ab}{2} + (-1)^{n/2} F_n + H_n \cos \frac{ab}{2} = 0 \]
\[ -D_n \sin \frac{ab}{2} + H_n \cos \frac{ab}{2} - F_n (-1)^{n/2} = 0 \]
\[ D_n \cos \frac{ab}{2} + H_n \sin \frac{ab}{2} + E_n (-1)^{n/2} = 0 \]
\[ D_n \cos \frac{ab}{2} + H_n \sin \frac{ab}{2} + E_n (-1)^{n/2} = 0 \]
\[ F_n (-1)^{n/2} + D_n \sin \frac{ab}{2} - H_n \cos \frac{ab}{2} = 0 \]

(7C.9)

Two of the equations of (7C.9) are duplicated, and the remainder yield

\[ H_n = 0 \]
\[ D_n = -\alpha_n \sec \frac{ab}{2} \]
\[ F_n = (-1)^{n/2} \alpha_n \tan \frac{ab}{2} \]

Substitution of these values into equation (7C.9) gives for the \( n \)th component:

\[ E_{y_n} \bigg|_{n \text{ even}} = -\alpha_n \sec \frac{ab}{2} \left[ \cos b_n x \sin k_n z - (-1)^{n/2} \sin kx \cos b_n (z - \frac{a}{2}) \right] \]

(7C.10)

If \( n \) is odd, a similar analysis gives

\[ E_{y_n} \bigg|_{n \text{ odd}} = \alpha_n \cosec \frac{ab}{2} \left[ \sin b_n x \sin k_n z - (-1)^{n/2} \cos kx \sin b_n (z - \frac{a}{2}) \right] \]

(7C.11)

The \( x \) component of magnetic field is now computed for regions 1 and 2 at \( z = 0 \), and the values equated to enforce continuity of magnetic field across the boundary.

In region 1,

\[ H_x \bigg|_{z = 0} = \frac{1}{j \omega \mu} \left(-j b_{11} + j b_{12} \delta_1 \right) \frac{2}{a} \sin \frac{\pi}{a} \left( x + \frac{a}{2} \right) + \sum_{n = 2}^{\infty} j b_n \delta_n \frac{2}{a} \sin \frac{n \pi}{a} \left( x + \frac{a}{2} \right) \]

(7C.12)
In region 2,

\[ \begin{align*}
    k_z &= \frac{1}{j\omega} \left( - \sum_{n=0}^{\infty} a_n \sec a b_n/2 \left[ k_n \cos b_n x + (-1)^{n/2} b_n \sin k_n x \sin b_n a/2 \right] \right. \\
    &\quad + \sum_{n \text{ odd}} a_n \csc a b_n/2 \left. \left[ k_n \sin b_n x - (-1)^{n/2} b_n \cos k_n x \cos b_n a/2 \right] \right) \\
\end{align*} \]

(76.13)
Equating equations (70.12), (70.13), multiplying both sides by 
\[ \sin \frac{\pi}{a} (x + a) \] and integrating with respect to \( x \) between the limits \( \pm a/2 \) gives:

If \( m = 1 \)

\[ j b_1 - b_1 \cot \frac{ab_1}{2} = (j b_1 + b_1 \cot \frac{ab_1}{2}) \delta_1 + \sum_{n}^{\text{even}} \frac{2}{a} \delta_n \frac{2 \pi}{a} \left( \frac{\pi^2}{a^2} - b_n^2 \right) \]

If \( m \) is odd

\[ 0 = \sum_{n}^{\text{even}} \delta_n \frac{2 \pi}{a} \left( \frac{\pi^2}{a^2} - b_n^2 \right) + b_m \delta_m (\cot a b_m / 2 + j) \]

If \( m \) is even

\[ -4 \frac{\pi^2}{a^3} \frac{1}{\left( \frac{\pi^2}{a^2} - b_1^2 \right)} = \sum_{n}^{\text{odd}} \delta_n \frac{2 \pi}{a} \left( \frac{\pi^2}{a^2} - b_n^2 \right) - b_m \delta_m (\tan a b_m / 2 - j) \]  

(70.14)

Equations (70.14) constitute a set of \( m \) simultaneous equations in the unknowns \( \delta_1 \) to \( \delta_m \). Since the number of unknowns in an overs modoed corner is large, the equations were solved numerically on a digital computer using the Crout factorization procedure.

The above technique can be applied to the other boundary value problems, with the following results:

case (a):

If \( m = 1 \),

\[ (j b_1 - 4 \pi^2 \frac{1}{a^3} \left( \frac{\pi^2}{a^2} - b_1^2 \right)) + b_1 \tan b_1 a / 2 = \sum_{n = 3}^{\infty} \delta_n \frac{4 \pi^2}{a^3} \left( \frac{\pi^2}{a} \right)^2 - b_n^2 \]

\[ - \delta_1 (b_1 \tan b_1 a / 2 - j b_1 - 4 \pi^2 \frac{1}{a^3} \left( \frac{\pi^2}{a} \right)^2 - b_1^2) \]
If \( m \neq 1 \) and odd,
\[
-4 \frac{\alpha^2}{a^3} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 = \sum_{n=1, \text{n odd}}^{\infty} \delta_n \frac{4\alpha^2}{a^3} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 \delta_n \left( b_1 \tan b_1 a/2 - j \right) - \delta_n \frac{2\alpha^2}{a^3} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 \delta_n \left( b_1 \tan b_1 a/2 - j \right).
\]

Due to the symmetry of the problem, \( \delta_m = 0 \) if \( m \) is even.

**Case (c):**

Identical to case (b) but \( \delta_n \) is replaced by \( -\delta_n \) if \( n \) is even (due to the mirror symmetry).

**Case (d):**

If \( n = 1 \),
\[
jb_1 + \frac{4\alpha^2}{a^3} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 + b_1 \tan b_1 a/2 = -\sum_{n=1, \text{n odd}}^{\infty} \delta_n \frac{\alpha^2}{a} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 + (jb_1 - b_1 \tan b_1 a/2)\delta_1.
\]

If \( m \) is odd,
\[
-4 \frac{\alpha^2}{a^3} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 = \sum_{n=1, \text{n odd}}^{\infty} \delta_n \frac{2\alpha^2}{a} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 + \delta_n \frac{2\alpha^2}{a} \left( \frac{2\alpha}{a} \right)^2 - b_1^2 \delta_n \left( b_1 \tan b_1 a/2 - j \right).
\]

Due to the symmetry of the problem \( \delta_m = 0 \) if \( m \) is even.

Let the solutions to the problems in Fig. 7.11(a), (b), (c), (d) be \( A, B, C \) and \( D \) respectively. By considering the polarity of the applied field for each port in Fig. 7.10, superposition gives the following outputs:

- **Port 1**: \( A + B + C + D \)
- **Port 2**: \( A - B - C + D \)
- **Port 3**: \( -A + D \)
- **Port 4**: \( -A + D \)


CHAPTER 8

THE SYNTHESIS OF A GIVEN GROUP-DELAY CHARACTERISTIC

8.1 Introduction

8.2 A review of synthesis techniques

8.3 The application of various optimization procedures to particular cases

8.4 Conclusions
8.1 Introduction

It was shown in Section 3.2.1 that the tapped-resonator equalizer can be adequately represented by a pole-zero distribution similar to that for the bridged-tee equalizer. Section 5.2 showed a similar result for the resonant-ring equalizer. Section 6.2 indicated that, for most practical purposes, the delay curves of the hybrid-ring are the same as for the corresponding tapped resonator.

In the cases of these three microwave equalizers, therefore, the device delay is adequately specified by a pair of conjugate poles and a pair of conjugate zeros. Although simple equalization can sometimes be accomplished with a single equalizer section, more complex curves require the cascading of several such sections. The problem is to arrange the sections in such a way that the difference between the complement of the curve to be equalized, and the curve of the equalizer is within a set limit.

During the past years, considerable effort has been expended on the above problem for the low frequency equalizers. From the above results, it seems reasonable that the techniques evolved may be applied to the microwave frequency region. Section 8.2 contains a review of the work done, and Section 8.3 considers the application of standard optimization techniques to the problem.

The bulk of the cost of a computer solution to such problems is not in the final run time, but in the time taken to ensure that such programmes are running correctly. The techniques reviewed in Section 8.2 are not generally available as standard subroutines, and so the user must generate his own—a time consuming business. In Section 8.3, therefore, a look is taken at the results obtainable by using generally available
optimization routines. Since the optimization procedures used are already well documented, no attempt is made to fully describe them, an excellent source paper on these being that of Temes and Calahan.

8.2 A Review of Synthesis Techniques

It is generally accepted that the optimum error curve for the difference between a desired and generated curve, is a ripple characteristic, with constant amplitude ripples, the number of ripples being dependent on the number of varying parameters. This is termed an error curve in the Tschebychev sense.

The first analytic procedure based on Tschebychev polynomials was that of Hellerstein. Here, the required and actual curves are expanded as two sets of Tschebychev polynomials. By matching the first few terms of each series, the resultant error is largely due to the first of the remaining terms, which, being such a polynomial, gives a ripple error curve. In this, and indeed in all the methods described here, the number of equalizer sections required, and approximate values for the positions of their poles and zeros must be determined first. Fairly adequate rules-of-thumb have been developed using equation (2.7) and the equal-areas rule of Section 2.2. If, after the synthesis procedure has been used, the error is not within the required tolerance, then the number of sections must be increased and the computation repeated. Fall evolved a similar procedure to Hellerstein, but one which was adapted for use on a digital computer. However, the above techniques suffer from two disadvantages. The first is that the frequency range being equalized must include the origin, a constraint unsuitable for the microwave region where the bandwidths are
normally only about 10% of the center frequency. The second disadvantage is that the error ripple is not of constant amplitude, the amplitude tending to decrease with increasing frequency. By suitably altering certain of the terms, however, this effect can be considerably reduced.

Whilst the method described above relies on the properties of a polynomial to generate an equi-ripple error curve, that of Gibbs forces the error curve to assume such a form. Gibbs considers the effect on the group-delay of slight perturbations in each of the variables. The positions of the maxima and minima of the error curve are found, and the corrections required to achieve an equi-ripple characteristic computed. From these two operations, a set of simultaneous linear equations in the perturbations of the variables are constructed, from which the required corrections can be extracted. The process is repeated since, due to interactions between the variables, the equi-ripple curve is not immediately found. This technique can be applied to any frequency interval, and has yielded quite good results.

Kuntermann, Pfleiderer and Unbehauen have developed the work of Bosse into a powerful technique. A set of non-linear equations in the increments required in the variables is constructed, and solved by iteration. The equations are generated to satisfy the equi-ripple criterion, but a 'least-squares' error criterion is also incorporated. This method was found to be faster than the Fletcher-Powell gradient method which is discussed in the next section.

Tu has recently reported on work done with combinations of optimisation routines, and his results substantiate those outlined in the next section.
8.3 The application of various optimization procedures to particular cases

Two curves of interest were chosen to be equalized. The first was a linear slope, rising 10 nsec, from 3 to 13 nsec, in the frequency interval 3.7 - 4.2 GHz. The second was an experimentally measured filter group-delay characteristic, the center frequency of which was at 7.3 GHz.

8.3.1 The Linear Characteristic

The first optimization procedure used was a direct search. The function to be minimised was a 'least-squares' error criterion, generated by summing the squares of the differences between the desired and actual curves at a set number of frequencies over the band. The desired characteristic was, of course, the complement of the curve initially specified. No attempt was made to enforce an equi-ripple error curve, since if an optimal solution is obtained this should be automatically satisfied. An error limit of ± 0.2 nsec was set, and first of all six equalizing sections were used. The resultant error curve is shown in Fig. 8.1. A five section equalizer produced the curve also shown in Fig. 8.1. Both of these runs were executed by a program written in KDF 9 user-code, and took about 10 sec on that computer. There are two points of interest on these curves. The first is that the solutions are not equi-ripple, and therefore not optimal. The second is that reduction of the number of equalizers to five produces a rapid degradation in the error curve.

The starting values used for these programs were very poor, but as the curves show, the optimal solution has been closely approached. It was found that if the results for the six section equalizer were fed back into the program as new input data, only a slight improvement in the error curve resulted.
FIG 8-1 Equalization by direct search

a - curve to be equalized (1 h scale)
b - 6-section equalized (rh scale)
c - 5-section equalized (rh scale)

Group-delay [nsec]

Frequency [GHz]

Error [nsec]
Some slight variations on the direct search procedure have been developed, and that due to Rosenbrock is available as a subroutine for most scientific computers.

The second optimization method used was a "simplex" procedure. This is a sophisticated direct search technique, and unlike the Fletcher-Powell procedure discussed later, no error gradients have to be computed. It was found that, using the same poor initial approximation as for the direct search, no convergence was obtained. Further work showed that this method is best used when there is a good approximation to the optimal solution as a starting point. With the results of the direct search as starting values the curves of Fig. 8.2 were obtained. Even with 400 iterations, occupying 2 min. 33 sec. of computer time, no convergent result could be obtained. The most likely explanation for this is that the shape of the error curve near the optimum point caused the simplex to expand and contract in a very slowly convergent manner.

The third optimization method used was a "Fletcher-Powell" procedure. For this, the gradient of the error function with respect to each of the variables must be computed. Again it was found that a poor starting value gave non-convergent results. The use of the direct search values, however, gave the following results.

<table>
<thead>
<tr>
<th>No. of equalizers</th>
<th>max. error (nsec)</th>
<th>time taken</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>0.06</td>
<td>1 m. 38 sec.</td>
</tr>
<tr>
<td>5</td>
<td>0.16</td>
<td>1 m. 43 sec.</td>
</tr>
<tr>
<td>4</td>
<td>0.87</td>
<td>25 sec.</td>
</tr>
</tbody>
</table>

The error curves for the five and six section equalizers are shown in Fig. 8.3. Both the curves show an almost equi-ripple characteristic.
FIG 83 Results of Fletcher-Powell optimization

- six sections
- five sections

Error [nsec]

Frequency [GHz]

3.7  3.8  3.9  4.0  4.1  4.2
8.3.2 The Filter

The filter characteristic is shown in Fig. 8.4a. An eight section equalizer was chosen, and application of the direct search procedure yielded the error curve of Fig. 8.4b. These results were then used with the "Fletcher-Powell" program, and the curve of Fig. 8.4c resulted. Total computer time: 2 min. 40 sec. The error curve, however is not equi-ripple, which suggests that the direct search has settled in a local minimum, and not in the required global minimum.

8.4 Conclusions

The work outlined in Section 8.3 has shown that normal optimization techniques are quite adequate to obtain approximately equi-ripple error curves. Such techniques are readily available, and so can be directly applied to synthesis problems in the microwave region.
REFERENCES


 CHAPTER 9
THE MEASUREMENT OF GROUP-DELAY

9.1 For the frequency range 1-12 GHz
9.1.1 Review of previous work
9.1.2 Outline of basic measuring system
9.1.3 Sources of errors
9.1.4 Design of, and measurements on the complete system
9.1.5 Waveguide version of measuring equipment
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9.2 For the frequency range 20-100 GHz
9.2.1 Review of previous work
9.2.2 Discussion of the proposed method

Appendices:
9A Development of the group-delay formula for the system
9B Analysis of errors due to spurious signals
9C Errors due to the use of a finite modulation frequency
9.1.1 Review of Previous Work

A fairly comprehensive set of references on the measurement of group-delay up to 1965 is contained in a review article by Bova. A survey of the available literature shows that four basic methods have been used to measure group-delay. These are:

1. Direct measurement of the phase shift through the device, and differentiation of the phase-frequency curve to give the group-delay.
2. Amplitude modulation of the carrier frequency
3. Frequency modulation of the carrier frequency
4. Measurement of pulse propagation times through the device.

(1) The direct measurement method is the simplest in principle, but is difficult to implement. If high accuracy is required, the phase characteristic cannot be differentiated graphically. The method used in the Hewlett-Packard "automatic network analyser" (Model 8540A) is to automatically shift the carrier frequency in known increments, measure the phase shift through the device at each frequency, and obtain the increments in phase. The group-delay is then given approximately by the ratio of the phase and the frequency increments. With a programmed computer to perform the computations and supervise the increments in frequency, the measurement accuracy is of the order of ± 0.1 nsec, and delay curves can be obtained in a short time. However, if an "on-line" computer is not available, the method becomes extremely cumbersome and slow, with very low accuracy unless extreme care is taken. In addition, the 8540A is very expensive.

(2) The amplitude modulation technique is an extension of the method suggested by Nyquist and Brand, who make the
assumption that the group-delay can be approximated by

\[ \frac{\phi_1 - \phi_2}{\omega_0} = \phi_1 - \frac{\phi_1}{\omega_1} - \frac{\phi_2}{\omega_2} \omega_0 \]  

(9.1)

where \( \phi_1 \) is the phase shift of signal \( \omega_1 \) in passing through the device

\( \phi_2 \) is the phase shift of signal \( \omega_2 \) in passing through the device

\( \omega_m \) is the median of \( \omega_1 \) and \( \omega_2 \).

The basic block diagram of an amplitude modulated system is shown in Fig. 9.1. The two signals, \( \omega_1 \) and \( \omega_2 \) are the two sidebands of a sinusoidally modulated carrier, and by comparing the phase of the modulation envelope before and after passing through the device under test, equation (9.1) can be used to obtain the group-delay.

(3) A system using frequency modulation has been developed by Turner and Lisney for measuring group-delay in the 3.7 - 4.2 GHz satellite band. A simplified block diagram of their apparatus is shown in Fig. 9.2. The modulation index is kept low enough for only the first two sidebands to be significant. The basic operation of the system is therefore the same as for the amplitude modulation case. The accuracy claimed for the system is ± 0.1 nsec for point-by-point measurements, and ± 0.2 nsec for swept frequency measurements.

(4) The measurement of pulse transit times through the device is not feasible at microwave frequencies since, for a resolution of 0.1 nsec, the pulse edge must have a rise time of this order. Such a rise time would have a broad frequency spectrum associated with it, thereby masking any fine structure in the group-delay characteristic.

This review has shown that there are two feasible systems, the amplitude modulation, and frequency modulation techniques.
FIG 9.1 Basic measuring system
1 MHz crystal oscillator

---

sweeping signal osc.

---

isolator

---

8 dB pad

---

network under test

---

8 dB pad

---

directional coupler

---

isolator

---

mixer

---

70 MHz amp.

---

attenuator

---

discriminator

---

1 MHz band-pass filter

---

calibrated phase changer

---

1 MHz phase comparator

---

FIG 9.2 A frequency-modulated measuring equipment
In the following sections, the amplitude modulation method is studied in greater detail. It is shown that this system can achieve accuracies of \( \pm 0.2 \) nsec with a reasonable amount of equipment and setting up.

9.1.2 Outline of the basic measuring system

The r.f. output from the sweep oscillator is levelled by means of an external feedback loop which ensures that the r.f. output at point A in Fig. 9.1 is flat to within 0.5 dB over the frequency band being swept. The p-i-n diode modulator is fed by a d.c. source and video oscillator with values such that a sine wave modulated carrier is obtained at point A. The phase of the modulated signal is sampled and detected before and after passage through the device under test. The phases of the two detected envelopes are compared in a vector voltmeter, and their phase difference displayed. As shown in Appendix A, the group delay, \( \tau \), of the device under test is related to the phase difference, \( \Delta \phi \), by the approximation

\[
\tau \propto \frac{\Delta \phi}{\omega_m}
\]  

(9.2)

where \( \omega_m \) is the angular frequency of the modulating signal.

Since the output from the vector voltmeter is proportional to the phase shift in degrees, eqn. (9.2) shows that, for a modulation frequency of 2.78 MHz, one degree corresponds to one nanosecond of delay.

The absolute delay of a device is found by removing the device from the circuit, noting the phase reading, and then placing it back in the circuit. The difference in phase readings being the absolute delay at that frequency.

9.1.3 Sources of errors

(a) Spurious signals
Since the definition of group-delay outlined in Section 1.3 is concerned with only the transmitted wave at the output of the device, waves which have suffered multiple reflections within the measuring system must be regarded as spurious. If the signal which has suffered no reflections produces an input voltage $E_1$ at the vector voltmeter, whilst the signal which has been reflected, and is consequently of a much smaller magnitude than the first, produces an input $E_2$ of the same frequency, the phase of the composite signal may differ from that of the wanted signal by up to

$$\frac{E_2/E_1}{[1 - (E_2/E_1)^2]^{1/2}} \text{ radians} \quad (9.3)$$

Such unwanted signals can arise from several sources. The sweep oscillators not only generate harmonics, but also other spurious signals, and these will be amplitude modulated. Since the group-delay at the frequency of the spurious signal will be different from that at the frequency of interest, the wanted and unwanted modulation envelopes will differ in phase at the output, thereby producing an error.

Since the modulator is not ideal, it will generate harmonics of the modulation frequency which will also be impressed on the r.f. signal. Since these harmonics may have a different group-delay through the device from that of the fundamental, there is a possible source of error. If, however, the output detector is a perfect square-law device, i.e. voltage output is proportional to power input, and the input power to it is of the form

$$P_{in} = P_1 \cos(\omega_m t + \Delta\phi) + P_2 \cos(\omega_n t + \Delta\theta)$$

where $p$ is the harmonic number, then the detected output will be proportional to the above power. Since the vector voltmeter
locks on to the stronger signal, and has an effective bandwidth of only a few kilohertz, the \( p \)th harmonic is rejected, and so no errors are introduced. If, however, the detector has an input-output characteristic which is not a square-law, any two of the modulation harmonics present in the input may interact to give a difference frequency term with the same frequency as the wanted signal. For example, if there is a signal at frequency \( 2\omega_m \) as well as at \( \omega_m \), there may be a difference at frequency \( (2\omega_m - \omega_m) = \omega_m \), which the vector voltmeter will also accept, thereby giving an erroneous reading.

The last, and most important, source of spurious signals are multiple reflections within the system. Since all the components have V.S.W.R.s which differ from unity, reflections must be present in the system. For example, one such reflection path is that due to the signal which is reflected from the output pad, traverses the device, is reflected from the reference coupler, and returns to the output detector via the device.

(b) Non-linear phase

If the phase characteristic of the device is not linear over a frequency interval of many times the modulation frequency, centred on the carrier frequency, the measured group-delay will not be the true group-delay. This is because the derivation of equation (9A.9) is based on an assumption which is no longer valid.

It is shown in Appendix 9C, that the error between the measured and actual group-delay at a frequency \( \omega \), is given by equation (9C.10)

\[
\text{error} = \sum_{i=0}^{n} b_i (\omega - \omega_0)^{(i - 2)} \omega_m^2 \frac{i (i - 1)}{6} (9.4)
\]

where \( b_i \) are the coefficients of the polynomial expansion of the
group-delay curve about frequency $\omega_0$, $\omega_m$ is the modulation frequency, and $\omega_o$ the measurement frequency. The expression shows that the error will increase as the square of the modulating frequency. It is therefore necessary, for high accuracy measurements, that the modulation frequency be kept as low as possible.

The analysis of Appendix 9X has, however, assumed that the polynomial of the true group-delay curve is known, and this is what we are trying to find. What is measured is the summation of the true group-delay curve and the error. The coefficients of the polynomial for the true curve can be found as follows.

The group-delay measured is given by an equation, similar to equation (90.8) with a shifted zero:

measured value

\[
\sum_{i=0}^{n} \frac{b_i}{2\omega_m(i+1)} [(\omega_c - \omega_m - \omega_o)(i+1) - (\omega_c - \omega_m - \omega_o)(i+1)]
\]

at frequency $\omega_0$. By measuring the delay at a specific number of points, equal to the number of coefficients required, spread over the frequency range of interest, a matrix is formed from which the polynomial coefficients, $b_i$, can be evaluated by digital computer.

If, however, changes in the group-delay curve are not appreciable in an interval of twice the modulation frequency, the errors will be small compared to the absolute delay. It is then adequate to compute the error from the polynomial of the measured curve, reverse its sign, and add it to the measured delay. The resultant curve will then agree with the zero modulation frequency curve with an error considerably smaller than the computed error. This technique is best achieved by processing the information, obtained from the original plot, in
a digital computer.

(c) Vector Voltmeter

The vector voltmeter used for the present investigation was a Hewlett-Packard Model No. 8405A. The two r.f. signals, which can lie in the range 1 to 1000 MHz are converted into two 20 kHz i.f. signals, which retain the same amplitudes, waveforms, and phase relationship. This is done by a similar principle to that used in the sampling oscilloscope. The r.f. signal is sampled once per period, the sampling point being progressively shifted along the waveform by means of a voltage controlled oscillator. The sample value is then stretched until the next sample is taken. The composite waveform is therefore of a quantised nature.

Before application to the phase meter, the two 20 kHz sinusoidal fundamental signals are extracted from the i.f. signals by narrowband filters. The extracted signals are then amplified and clipped, retaining only the phase difference $\phi$. The triggers generate a square wave with a mark-space ratio proportional to the time between triggers, and therefore to $\phi$. The average current through the phase meter is governed by the mark-space ratio of the square wave, and so the meter indication is proportional to the phase difference, $\phi$, between the original inputs.

Due to the sampling, there will be a certain amount of jitter in the generated triggers. This results in a phase jitter, which imposes a fundamental limit on the accuracy. The jitter quoted for the 8405A is 0.1 degrees.

If the phase difference between the input signals is a rapidly varying function of time, as would happen for example if the sweep oscillator were sweeping quickly, the meter phase output will be in error. This occurs because the sampling
section cannot obtain sufficient samples to build up a true composite waveform before the phase difference between the inputs has altered. It is essential therefore that the rate of change of input phase difference with time be small.

The insertion-loss of the device under test will reduce the input at one channel of the voltmeter relative to the other input. If the loss is high, the input voltage will be insufficient to generate stable triggering pulses, and so the phase jitter will increase. A limit can therefore be placed on the maximum insertion loss allowable, compatible with a given accuracy of measurement.

(d) Crystal detectors and frequency multiplication

The equivalent circuit of the crystal detectors used is shown in Fig. 9.3a, where \( r \) is the appropriate slope resistance of the diode. Since \( r \) is determined by the bias point, which is in turn dependent on the r.f. level, there is a change in the phase shift through the detector as the r.f. level varies.

The slope resistance is given by

\[
r = \frac{\partial V}{\partial I} = \frac{1.3 \frac{kT}{q}}{5 \times 10^{-6}} \exp \left[ -\frac{4V}{1.3 kT} \right] \quad (9.6)
\]

Substituting appropriate values for \( q \), \( k \) and \( T \):

\[
r = 6.73 \exp \left[ -29.7 \text{V} \right] \Omega \quad (9.7)
\]

From Fig. 9.3b, the phase shift through the detector, \( \varphi \), is given by

\[
\varphi = \arg \left( \frac{V_0}{V} \right) = \arg \left( \frac{1}{1 + j \omega C (R + r)} \right) = \tan^{-1} \omega C (R + r) \quad (9.8)
\]

Substituting typical values, e.g.

\[ C = 10^{-11} \text{F}, \quad r = 6.73 \text{k}\Omega, \quad R = 50\Omega, \quad \omega = 2\pi \times 2 \times 10^6 \text{ rad sec}^{-1} \]
FIG 9.3 The detector
\[ V_1 = \tan^{-1} 0.852 \]
\[ V_1 = 40^\circ 26' \]
A 1\% change in \( r \) results in
\[ V_2 = \tan^{-1} 0.8595 \]
\[ = 40^\circ 41' \]
A 1\% change in \( r \) therefore produces a phase change of 0.25 degrees.

With a 50\( \Omega \) load resistor, \( R' \), on the output, we now have
\[ V' = \tan^{-1} \frac{\omega C R'(r + R)}{r + R + R'} \]
and a 1\% change in \( r \) gives
\[ V'_1 = \tan^{-1} 0.0062372 \]
\[ = 0.3574 \text{ degrees} \]
\[ V'_2 = 0.35738 \text{ degrees}. \]
The addition of the 50\( \Omega \) load has therefore reduced the phase shift through the diode by a considerable factor. In the measuring equipment, 50\( \Omega \) loads were used on both the reference path and test path detectors.

From equation (9.8),
\[ \frac{\partial V}{\partial r} = \frac{\omega C}{1 + \omega^2 C^2 (R + r)^2} \text{ rad. ohm}^{-1} \]  
(9.9)
Thus, for \( r = 6.73k\Omega \), \( R = 50\Omega \), and \( \omega = 2\pi \times 10^7 \) (10 MHz modulation frequency), the sensitivity of \( V \) to changes in \( r \) is given by
\[ \frac{\partial V}{\partial r} = \frac{\omega C}{5.25} \]  
(9.10)
Also, for 100 kHz modulation frequency, \( \omega = 2\pi \times 10^5 \), and we have
\[ \frac{\partial V}{\partial r} = \frac{\omega C/100}{1.0425} \]  
(9.11)
The ratio of equations (9.10) and (9.11) give
\[ \frac{\text{sensitivity at 100 kHz}}{\text{sensitivity at 10 MHz}} = \frac{1}{19.9} \]
From this we deduce that if a given change in input r.f. power
produces a phase change of 0.1 degree at 10 MHz, the same power change will give a phase change of $0.1/19.9 \approx 0.005$ degrees at 100 kHz.

The above computation is of special interest when the use of a frequency multiplier between the crystal outputs and the vector voltmeter is considered. If the error due to Section 9.1.3 (b) is to be reduced, the modulation frequency must be as low as possible. There is the conflicting demand of Section 9.1.3 (c) that the modulation frequency be kept as high as possible to reduce the error due to phase jitter. These requirements can both be satisfied by using a low modulation frequency, feeding the crystal outputs to a frequency multiplier, which also multiplies the phase difference, and applying the two high frequency signals so derived to the vector voltmeter. The output from the vector voltmeter is processed exactly as if the higher frequency had been used throughout. However, the error due to phase shift through the detectors is not the same as if the higher modulation had been used throughout. If the phase change, due to an alteration in signal levels, at 11.1 MHz is 0.1 degrees, the phase change at 100 kHz for the same change is, from the above calculation, 0.005 degrees. This phase change of 0.005 degrees is, however, multiplied with the frequency, to give a phase change at the output of the frequency multiplier of $111 \times 0.005 \approx 0.5$ degrees at a frequency of 11.1 MHz. By using a low modulation frequency, and a frequency multiplier, the error due to a change in signal amplitude is five times worse than if a high modulation frequency had been used throughout!

(e) **Modulation frequency**

Since the value of the modulation frequency appears directly in the computation of group-delay, any error in this frequency
will manifest itself as a proportionate error in the group-
delay.

9.1.4 Design of, and Measurements on the Complete System

A block diagram of the complete measuring system used is shown in Fig. 9.4.

(a) Spurious signals

The microwave sweep oscillator used was a Hewlett-Packard Model No. 693D. The specification for this states harmonics to be at least 20 dB below the C.W. output, and non-harmonics at least 40 dB below the C.W. output. Measurements using a spectrum analyser gave results indicating a performance considerably better than this, with the second harmonic 30 dB below the fundamental, and non-harmonics at least 50 dB below the fundamental. To completely eliminate any errors due to harmonics and high frequency non-harmonic signals, a low-pass filter for inclusion in the output line from the generator was designed and built.

The filter used was a conventional co-axial low-pass structure such as described by Matthaei, Jones and Young. The cut-off frequency was chosen to be about 6.5 GHz, with an insertion-loss of at least 40 dB at 7.0 GHz, and no spurious responses until at least 10 GHz. Using a computer programme based on the iterative solution of the equations given in reference 4, in conjunction with subroutines to compute the fringing capacitance, the required filter design was obtained, as shown in Fig. 9.5. The theoretical response displayed 0.1 dB ripple in the pass-band, with a cut-off frequency of 6.46 GHz, and 40 dB insertion loss at 7 GHz. Measurements on the completed filter (Fig. 9.6) showed a pass band insertion loss of 0.5 dB, due mainly to the effects of finite conductivity which was not taken into account in the filter
levelling loop

- microwave sweep oscillator
- filter
- PIN diode modulator
- 10 dB directional coupler and detector
- 10 dB directional coupler and reference detector
- device under test
- 10 dB attenuator
- test detector

video oscillator and dc bias

reference channel

vector voltmeter
test channel

FIG 9-4 Final measuring circuit
FIG 9.5 Sectional view of low-pass filter

(All dimensions in inches)

to N-type connector

dielectric

inner conductor

centre of filter

Rollet No 3 tubing

0.176 dia.

0.064 dia.
FIG 9-6 The low-pass coaxial filter
analysis. The 40 dB insertion-loss point was measured at 6.95 GHz, and the response found to be spurious-free up to 12.5 GHz. The inner conductor of the filter was formed by soft soldering the large diameter shunt capacitance sections onto a length of wire which formed the high impedance sections. The capacitive elements were formed by drilling a P.T.F.E. rod to an internal diameter slightly smaller than a length of rod of the correct diameter. The P.T.F.E. was then forced onto the rod, the external dimension machined to the correct value, the hole for the wire drilled, and the required elements parted off. The whole assembly was then cooled, and inserted into the Rollet tube.

The video signal source used was a Marconi Instruments video oscillator No. T.F.885A. The d.c. bias was adjusted until the P.I.N. modulator gave a maximum depth of modulation. The measured second harmonic of the video signal source was 50 dB down on the fundamental, and the third harmonic 53 dB down. The contribution to harmonic output from the P.I.N. modulator due to the video generator was therefore neglected. By means of a spectrum analyser, the second harmonic output from the modulator, due to nonlinearities, was found to be 35 dB down on the fundamental, and 50 dB down on the carrier. The third harmonic was 40 dB down on the fundamental. Due to the narrow bandwidth of the vector voltmeter, and the relative magnitudes of the harmonics, therefore, the only error due to harmonics was due to harmonic mixing in the crystal detectors. Since care was taken to ensure that the crystals operated under approximately square-law conditions, the magnitude of the fundamental component, generated by the mixing between the fundamental and second

* The fabrication of the filter was undertaken by Mr. Pink of the Marconi research workshops, and it is considered to be due mainly to his very high quality work that such good agreement between theory and experiment was obtained.
harmonic powers due to the terms of order greater than two in the crystal law, was concluded to be negligible compared to other sources of error.

Measurements with a harmonic analyser on the output from the crystals showed third harmonic outputs 40 dB down on the fundamental, and fourth harmonic outputs 50 dB down. The third harmonic output was partially caused by mixing between fundamental and second harmonic powers incident on the crystal. The mixing properties of non-linearities imply that the component generated at fundamental frequency would be of comparable magnitude to the component generated at the third harmonic. The signal added to the fundamental was therefore at least 40 dB down on the fundamental.

The error due to multiple reflections in the system varied, depending on whether an absolute value or relative value of group-delay was required. The measurement of absolute delay with the equipment was carried out as follows. The phase reading with the device replaced by a short circuit was noted. The device was then inserted into the system, and the change in phase reading gave the group-delay. The reflections within the system for the two cases can be completely different, and so the worst cases in the error analysis must be used. With measurements of relative changes in group-delay, the device is always present, and the only error will be due to the change in the component V.S.W.R.s with frequency. These changes may be very small compared with the V.S.W.R.s themselves. Therefore, if only the variation of group-delay over a band is required, and the V.S.W.R.s of the components remain constant over the band, the measurement of variations in group-delay can be of a much higher accuracy.

Any reflections from the output of the automatic level
control (ALC) loop could be treated as additions to the primary signal, and so did not introduce error. It was preferable, however, to keep such reflections low, and this was achieved by keeping the modulator, which had a V.S.W.R. of 2 to 1, inside the ALC loop, the output from the loop presenting a good match. The measured V.S.W.R.'s of the remaining components are given, with the equivalent return loss in dBs in Table 1.

TABLE 1

<table>
<thead>
<tr>
<th>Device</th>
<th>V.S.W.R.</th>
<th>Return Loss (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 dB attenuator</td>
<td>&lt; 1.05</td>
<td>&gt; 32</td>
</tr>
<tr>
<td>10 dB coupler</td>
<td>&lt; 1.05</td>
<td>&gt; 32</td>
</tr>
<tr>
<td>detector</td>
<td>&lt; 1.2</td>
<td>&gt; 21</td>
</tr>
<tr>
<td>device</td>
<td>&lt; 2.0</td>
<td>&gt; 9.5</td>
</tr>
</tbody>
</table>

Due to the uncertainty in locating the planes at which the discontinuities in the components occurred, a worst-case computation was used for each possible reflection path by using equation (9.3).

For the signal reflected first from the 10 dB pad, and then from the device, then back to the test detector, the overall return loss was $32 + 9.5 = 41.5$ dB. Since the detector was operating in its square law region, the voltage output was proportional to the power input,

$$-10 \log_{10} \frac{E_2}{E_1} = -41.5$$

or $E_2/E_1 = 1/14125$

From equation (9.3),

$$\text{max error} \approx 1/14125 \text{ rad} \approx 0.00405 \text{ degrees}.$$  

Thus the maximum error that could occur due to this multiple reflection was $0.00405$ degrees.

By repeating the above computation for the other possible
reflection paths the following contributions were obtained for errors at the test detector.

<table>
<thead>
<tr>
<th>path</th>
<th>return loss (dB)</th>
<th>maximum error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pad+coupler</td>
<td>64</td>
<td>0.000023</td>
</tr>
<tr>
<td>device+coupler</td>
<td>41.5</td>
<td>0.00405</td>
</tr>
<tr>
<td>detector+device</td>
<td>50.5</td>
<td>0.00051</td>
</tr>
<tr>
<td>detector+coupler</td>
<td>73.0</td>
<td>2.8 x 10^-6</td>
</tr>
</tbody>
</table>

The worst possible case was where these contributions added, giving an error of 0.00863 degrees.

Due to the finite directivity of the coupler feeding the reference channel, signals reflected from the components following the coupler could appear in the reference detector, thereby creating errors. For example, with a measured directivity of 30 dB, and a coupling factor of 10 dB, the signal reflected from the device would appear in the detector attenuated by $14 + 30 = 44$ dB. However, the main signal was attenuated by 10 dB, giving

\[
10 \log_{10} \frac{E_2}{E_1} = -44 + 10 = -34
\]

\[
\frac{E_2}{E_1} = 1/2,511.9
\]

which corresponds to a maximum error of 0.02185 degrees. The contributions from other reflections are listed below:

<table>
<thead>
<tr>
<th>path</th>
<th>effective return loss (dB)</th>
<th>maximum error (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pad +</td>
<td>52</td>
<td>0.00028</td>
</tr>
<tr>
<td>detector +</td>
<td>41</td>
<td>0.00445</td>
</tr>
</tbody>
</table>

The worst error in the reference path was therefore 0.0266 degrees.

By addition, the overall maximum error due to multiple reflections was $0.0266 + 0.0086 = 0.03518$ degrees. For a modulation frequency of 1.38 MHz this corresponded to a group-delay error of $\pm 0.06$ nsec.
(b) Non-linear phase

Although the errors caused by this source could have been computed from equation (9.5), a more direct approach was taken by considering the theoretical results obtained for the tapped-resonator-circulator equalizer, described in Chapter 3. In the theoretical computation of the group-delay curve, the phase was calculated at two frequencies equally separated from the center frequency, the group-delay computed from their difference, and presented as the group-delay at the center frequency. This basic method of numerical differentiation was very similar to the process involved in the amplitude modulation technique. By varying the separation of the two sampling frequencies, the effect of the modulation frequency on the measured group-delay could be seen. A tapping fraction of 0.47 was chosen with a resonant frequency of 4 GHz, corresponding to a $Q_D$ of 100. This gave a group-delay maximum of 14 nsec, and was considered to be the highest value of $Q_D$ likely to be measured. The following results were obtained:

<table>
<thead>
<tr>
<th>frequency deviation (MHz)</th>
<th>delay at 4 GHz (nsec)</th>
<th>delay at 3.98 GHz (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>14.25146</td>
<td>8.10352</td>
</tr>
<tr>
<td>1.39</td>
<td>14.22245</td>
<td>8.11537</td>
</tr>
<tr>
<td>2.78</td>
<td>14.16942</td>
<td>8.12790</td>
</tr>
</tbody>
</table>

The frequency 3.98 GHz corresponded to the point of maximum curvature of the phase characteristic. The differences from the values for a deviation of 0.1 MHz were as shown below.

<table>
<thead>
<tr>
<th>frequency deviation (MHz)</th>
<th>difference at 4 GHz (nsec)</th>
<th>difference at 3.98GHz (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.39</td>
<td>-0.02901</td>
<td>+0.00685</td>
</tr>
<tr>
<td>2.78</td>
<td>-0.08204</td>
<td>+0.0125</td>
</tr>
</tbody>
</table>

The errors due to curvature of the phase characteristic were computed from equation (9.C.10) to be $[-0.013]$ nsec for
1.39 MHz and [- 0.054] nsec for 2.78 MHz, at a frequency of 4 GHz. The values were not expected to be accurate, since the curve-fit procedure used did not produce a good fit with the low order polynomial used. The magnitudes, and variations with modulation frequency were, however, in good agreement with the tabulated values above.

(c) Vector Voltmeter

The phase jitter in the output of the vector voltmeter was 0.1 degree peak to peak, giving a limit of ± 0.05 degree. Since the group-delay curves were plotted using an X-Y pen recorder, the inertia of the inking mechanism averaged out the jitter, so reducing inaccuracies due to the jitter. The maker's specifications for the voltmeter quote the pen recorder output as tracking the meter reading to within ± 1.5% of full scale deflection. For a full scale deflection of 6 degrees, the possible error is ± 0.09 degrees. With a full scale deflection of 18 degrees, the error is ± 0.27 degrees.

(d) Crystal Detectors

The reference channel and test channel crystal detectors were a matched pair, thereby ensuring uniformity of tracking. The phase shift of the modulation through the detectors with changes in r.f. power was measured with the apparatus shown in Fig. 9.7. It is easy to show that the resultant of the incident and reflected waves in the slotted line is a standing wave in which the amplitude of the modulation envelope varies with distance, but the phase remains constant. By moving the detector along the line, any phase shifts due to changes in the r.f. power level could be detected on the vector voltmeter.

Tests on the two detectors showed that, when working in the square law range, a variation of 10 dB in the input power produced no visible change in the phase shift. The error due
levelling loop

- microwave sweep oscillator
- low-pass filter
- PIN diode modulator
- 10dB directional coupler and detector
- 10dB directional coupler and detector
- slotted-line with probe and detector
- short circuit

- video oscillator and dc bias
- vector voltmeter
- detector

FIG 9.7 Detector test circuit
to such phase shifts was therefore concluded to be less than 0.05 degrees.

(e) Modulation Frequency

Since several harmonics of 1.38 MHz were required for use, it was decided to use a modulation source consisting of a Marconi video signal generator, and a highly stable 1.38 MHz oscillator. By means of Lissajou figures, the variable generator could then be accurately set to any desired harmonic to within a few Hertz.

The circuit diagram of the complete oscillator is shown in Fig. 9.8. The output from the oscillator was fed into a 1.38 MHz tuned amplifier, which gave sufficient output to drive the X-deflection plates of the oscilloscope. Feedback via a 'raysistor' element ensured that the amplitude of the oscillation was constant with time. By means of a voltage divider, the output could be used to drive the P.I.N. diode modulator directly if required.

Measurements with a frequency counter showed that two minutes after switching on, the frequency was correct to 3 parts in $10^6$, and after one hour, 2 parts in $10^6$.

The errors so far have reached a total of 0.035 + 0.03 + 0.09 + 0.05 = 0.2 degrees. An upper limit to the error in measuring variations in group-delay was therefore set at ± 0.4 nsec with a modulation frequency of 1.38 MHz, or ± 0.3 nsec with a modulation frequency of 2.7 MHz. It should be emphasised that the above errors are worst possible cases, and in the majority of measurements taken with the equipment, the error was probably no more than ± 0.2 nsec with a modulation frequency of 1.38 MHz.

The technique of recording a group-delay characteristic was to trace the curve on paper by means of an X-Y plotter, and then, with the device removed from the system, put on
FIG 9.8 Circuit diagram of reference oscillator
calibration lines by moving the phase zero of the vector voltmeter in equal increments. The values of group-delay at individual frequencies were then obtained by measurements on the plot. Such a method can easily introduce slight errors, the magnitude of which are dependent on the scales used, so rendering it difficult to ascribe a specific value to them. This source of error was probably the major cause of slight deviations noted in the comparison of measured and theoretical delay curves.

The tapped-resonator-circulator described in Chapter 3 was considered to be the only equalizer having an accurately known group-delay curve which could serve as a reference. The curves Fig. 3.6, 3.7 of Chapter 3 show the measured and predicted group-delay curves for two tapping points, one giving a small group delay variation, the other a large variation. The agreement between the predicted and measured curves is within ±0.2 nsec.

9.1.5 Waveguide version of measuring equipment

To enable measurements to be carried out on the waveguide equalizer described in Chapter 4, a waveguide-14 version of the measuring equipment described in Section 9.1.2 was constructed. This was basically the same as the co-axial version described, but the low pass filter was of the "waffle-iron" type. An analysis of possible errors gave an overall value of ±0.35 nsec with a modulation frequency of 1.38 MHz.

To test the accuracy of the system, the delay of a 10 ft. length of waveguide-14 was measured at 5894.5 MHz. The computed group-delay was 14.87 nsec, and the following results were obtained:
<table>
<thead>
<tr>
<th>Modulation Frequency (MHz)</th>
<th>Measured Group Delay (nsec)</th>
<th>Possible Error (nsec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.38</td>
<td>14.6</td>
<td>± 0.35</td>
</tr>
<tr>
<td>2.7</td>
<td>14.75</td>
<td>± 0.20</td>
</tr>
<tr>
<td>5.5</td>
<td>14.75</td>
<td>± 0.12</td>
</tr>
</tbody>
</table>

The results confirmed to some extent the expected accuracies, an uncertainty being present in the predicted value of delay due to waveguide tolerances.

9.1.6 Conclusions

It has been shown that with readily available components, a group-delay measuring set can be constructed with a measurement accuracy better than ± 0.4 nsec.

9.2 For the Frequency Range 20 - 100 GHz

9.2.1 Review of Previous Work

The basic methods of measurement have been outlined in Section 9.1.1, and two of them have been implemented at millimeter-wave frequencies.

An amplitude modulation system developed by Chasek\textsuperscript{6} utilises the approximation of equation (9.1). A two-tone r.f. signal $\omega_1$, $\omega_2$ is generated, and fed through the unknown network. The phase shift factor is recovered by demodulation of the two-tone signal, the difference term from the square law detector being of the form

$$\sin\left[(\omega_1 - \omega_2)t + (\phi_1 - \phi_2)\right]$$

By comparing this with the reference,

$$\sin (\omega_1 - \omega_2)t$$

an output proportional to $(\phi_1 - \phi_2)$ is obtained, and division by $(\omega_1 - \omega_2)$, a fixed quantity, gives an approximation to the group-delay. This approximation improves as $(\omega_1 - \omega_2)$ decreases.
A simplified circuit is shown in Fig. 9.9. Comparison switching alternately replaces the unknown device with an equal length of waveguide, and at the same time switches the delay standard in and out of the i.f. reference path. The envelope resulting from the two-tone signal in the unknown path is recovered by a detector, and fed into the phase comparator. The reference envelope is recovered by a second detector, and similarly fed into the phase comparator. During half of the switching cycle, the signal through the unknown is compared to the signal through the delay standard. The other half of the switching cycle is used for zeroing purposes, with permits a reference phase comparison. Voltages proportional to the two phase comparisons are sampled, and stored in two capacitors. A differential d.c. amplifier and null meter are used to indicate when the phase difference between the two readings is zero. This occurs when sufficient delay has been inserted by means of the i.f. standard to equal the delay of the unknown device. Delay in nanoseconds is then read directly on the calibrated standard.

The accuracy claimed is \pm 0.2 nsec, the main source of error being the calibration of the standard. To cover wide bandwidths, two backward wave oscillators are required, and they must be phase-locked to give the necessary stable frequency difference. The separation used by Chasek was 10 MHz, and appreciable error can occur with this separation if narrow band equalizers are being measured. Good detectors, isolators, and low reflection pads are required, and if the equipment is to be swept over a wide-band, these must be broad-band. Such components are commercially available up to 40 GHz, but not above. The resolution of the phase measuring equipment was not discussed.
FIG 9.9 A millimeter wavelength group-delay measuring equipment
Fujii, Nakagawa, and Shimba used a frequency modulation technique very similar to that of Turner and Lisney. They give no breakdown of the sources of error, and although an accuracy of 0.02 nsec is claimed, this is more likely to be a value for the resolution of the system. From the published curves comparing theoretical and practical results, an approximate error of $\pm 0.4$ nsec seems more realistic.

9.2.2 Discussion of the proposed method

The proposed method of measuring group-delay at millimeter wavelengths, is an adaptation of the amplitude modulation technique discussed in detail in Section 9.1, and shown in Fig. 9.4. Owing to the difficulty of constructing low-pass filters in the millimetric range, this component will be absent. The amplitude modulator is of the Faraday rotation type, and can modulate an RF carrier at 1.38 MHz. The bandwidth over which such a modulator can operate is typically 1 GHz at a centre frequency of 35 GHz, and so severely limits the usable frequency range. Waveguide directional couplers with directivities in excess of 40 dB, and high quality crystal detectors are available for the whole millimetric band.

The use of a klystron in such a system would reduce the system flexibility, since the electronic tuning range is limited to 100 MHz or so. Recently, however, the Sanders Division of Marconi Instruments have produced a plug-in unit which can sweep 18 - 26.5 GHz and 26.5 - 40.0 GHz, and this can be levelled in the conventional manner.

The bandwidth limitation is now set only by the modulator, and it would seem that by increasing the complexity of the matching circuits this can be improved. The accuracy of such a system, since it so closely resembles the microwave circuit is expected to be of the same order i.e. $\pm 0.2$ nsec.
The relationship between group-delay and envelope phase shift

The p-i-n diode modulator is basically a variable resistance across the transmission line as far as a microwave signal is concerned. The video signal varies this resistance, and so varies the amount of r.f. power absorbed. The modulator therefore produces a sine-wave modulated power output for a sine wave video input. Thus:

$$P_o = P [1 + M \cos \omega_m t] \quad (9.4.1)$$

where $M$ is the modulation index, and $\omega_m$ the video frequency. To obtain eqn. (9A.1) the electric field must have the form

$$E_o = E [1 + M \cos \omega_m t]^{1/2} \cos \omega_c t \quad (9A.2)$$

where $\omega_c$ is the carrier frequency, and $E$ is a constant. At the input to the device under test we have

$$E_{in} = E [1 + M \cos \omega_m t]^{1/2} \cos \omega_c t$$

Since $[1 + M \cos \omega_m t]^{1/2}$ is an even function, it can be expanded as a Fourier series to give

$$E_{in} = E [1 + \sum_n A_n \cos n \omega_m t] \cos \omega_c t \quad (9A.3)$$

Expanding the product terms, we have

$$E_{in} = E \cos \omega_c t + E \sum_n A_n [\cos (\omega_c - n \omega_m) t + \cos (\omega_c + n \omega_m) t] \quad (9A.4)$$

If the phase shift through the device at frequency $\omega_c$ is $\phi$, at frequency $(\omega_c - n \omega_m)$ is $(\phi - n \Delta\phi)$, and at frequency $(\omega_c + n \omega_m)$ is $(\phi + n \Delta\phi)$, and the amplitudes of the signals suffer no change, the output of the device is given by substitution into eqn. (9A.4).

$$E_{out} = E [1 + \sum_n A_n \cos n (\omega_m t + \Delta\phi)] \cos (\omega_c t + \phi) \quad (9A.5)$$

i.e. $$E_{out} = E [1 + M \cos (\omega_m t + \Delta\phi)]^{1/2} \cos (\omega_c t + \phi) \quad (9A.6)$$

The power at the input to the device, from eqn. (9A.2) is

$$P_{in} = P [1 + M \cos \omega_m t] \quad (9A.7)$$

and at the output is, from eqn. (9A.6)
\[ P_{\text{out}} = P \left[ 1 + N \cos (\omega_{mt} + \Delta \phi) \right] \quad (9A.8) \]

If square-law detectors are used to detect the input and output powers, the video output voltages will be proportional to \( P_{\text{in}} \) and \( P_{\text{out}} \).

The group-delay is given by the slope of the phase characteristic,

\[ t_g = \frac{\Delta \phi}{\omega_m} \quad (9A.9) \]

From equations (9A.7), (9A.8), it is seen that the vector voltmeter will produce the phase shift, \( \Delta \phi \), which is proportional to the group-delay.

The assumption of a linear phase shift over an interval of many times the modulation frequency may not be justified for large values of modulation frequency. The errors which this can lead to are discussed more fully in Section 9.1.4 (b).
APPENDIX 9B

The analysis of errors due to spurious signals

Consider two signals arriving at the output detector; the power
in the first is given by

\[ P_1 = P_A \cos (\omega_m t' + \phi) \]  

(9B.1)

and in the second by

\[ P_2 = P_B \cos (\omega_m t' + \phi + \Delta\phi) \]  

(9B.2)

where \( P_B < P_A \).

Since the detector is working in its square law region, the input to
the vector voltmeter has the form

\[ E_{in} = E_A \cos \omega_m t + E_B \cos (\omega_m t + \Delta\phi) \]

\[ = A \cos (\omega_m t + \gamma) \]  

(9B.3)

where \( \gamma = \tan^{-1} \frac{E_B \sin \Delta\phi}{E_A + E_B \cos \Delta\phi} \) = phase error  

(9B.4)

The phase error will normally be less than five degrees, which

\[ \text{corresponds to a delay error of five nanoseconds at a modulation} \]

\[ \text{frequency of 2.78 MHz. Eqn. (9B.4) therefore becomes} \]

\[ \gamma \propto \frac{E_B}{E_A \sin \Delta\phi} \text{ radians} \]  

(9B.5)

\[ \frac{1 + E_B \cos \Delta\phi}{E_A} \]

By differentiating equation (9B.5) with respect to \( \Delta\phi \), and equating
to zero, the maximum possible error is found to be

\[ \gamma_{\text{max}} \propto \frac{E_B}{E_A} \text{ radians} \]  

(9B.6)

\[ \left[ 1 - \left( \frac{E_B}{E_A} \right)^2 \right]^{\frac{1}{2}} \]

and occurs when

\[ \cos \Delta\phi = -\frac{E_B}{E_A} \]
APPENDIX 9C

Errors due to the use of a finite modulation frequency

Consider a general group-delay curve described by a polynomial

\[ t_g = \sum_{i=0}^{n} A_i \omega^i \]  \hspace{1cm} (9C.1)

where \( A_i \) are arbitrary constants, and \( \omega \) the angular frequency.

Since

\[ t_g = \frac{\partial \phi}{\partial \omega} \]

where \( \phi \) is the equivalent phase lag,

\[ \phi = \int \sum_{i=0}^{n} A_i \omega^i d\omega + \phi_0 \]  \hspace{1cm} (9C.2)

where \( \phi_0 \) is a constant.

Following the procedure of Appendix 9A, but considering only the first pair of sidebands, the phase change at frequency \( \omega_c \) through the device under test is:

\[ \phi_c = \sum_{i=0}^{n} \frac{A_i}{(i+1)} \omega_c^{i+1} + \phi_0 \]  \hspace{1cm} (9C.3)

that at frequency \( (\omega_c + \omega_m) \) is:

\[ \phi_{(c + m)} = \sum_{i=0}^{n} A_i \frac{\omega_c + \omega_m}{(i+1)}^{i+1} + \phi_0 \]  \hspace{1cm} (9C.4)

and that at frequency \( (\omega_c - \omega_m) \) is:

\[ \phi_{(c - m)} = \sum_{i=0}^{n} \frac{A_i}{(i+1)} (\omega_c - \omega_m)^{i+1} + \phi_0 \]  \hspace{1cm} (9C.5)

Combining equations (9C.3) and (9C.4) we have

\[ \phi_{(c + m)} = \phi_c + \sum_{i=0}^{n} A_i \frac{(\omega_c + \omega_m)(i+1) - \omega_c}{(i+1)} \left[ \frac{1}{(i+1)} \right] \]

\[ (9C.6) \]
Similarly:

\[
\phi(o - m) = \phi_o + \sum_{i=0}^{n} A_i \left[ (\omega_c - \omega_m)(i + 1) - \omega_c(i + 1) \right]
\]

(90.7)

Recombining the sidebands with the appropriate phase shifts incorporated, the output waveform becomes

\[
A \cos(\omega_0 t - \phi_0) + A_m \left\{ \cos \left[ (\omega_c + \omega_m)t - \phi_0 - \sum_{i=0}^{n} A_i \frac{(i + 1)}{1 + 1} \right] \right. 
\]

\[
\times \left[ ((\omega_c - \omega_m)(i + 1) - \omega_c(i + 1)] + (\omega_c - \omega_m)t - \phi_0 - \sum_{i=0}^{n} A_i \frac{(i + 1)}{1 + 1} \right] \right.
\]

\[
\times \left[ ((\omega_c + \omega_m)(i + 1) - \omega_c(i + 1)] \right) \times \cos \left[ (\omega_c + \omega_m)t - \phi_0 - \sum_{i=0}^{n} A_i \frac{(i + 1)}{1 + 1} \right] \left. \right. 
\]

\[
\times \left[ ((\omega_c - \omega_m)(i + 1) - \omega_c(i + 1)] \right) \right.
\]

which simplifies to

\[
A \left\{ 1 + \pi \cos \left[ \frac{1}{2} \sum_{i=0}^{n} A_i \frac{(i + 1)}{1 + 1} \right] \right. 
\]

\[
\times \cos (\omega_0 t - \phi_0) \right. \right. 
\]

(90.8)

Thus the error in group delay is:

\[
\sum_{i=0}^{n} A_i \left\{ \frac{1}{2\omega_m(i + 1)} \right\} \left[ (\omega_c + \omega_m)(i + 1) - (\omega_c - \omega_m)(i + 1)] - \omega_c(i + 1) \right. 
\]

Since \(\omega_m/\omega_c\) is of the order of 1/4000, all terms higher than the fourth power in \((\omega_m/\omega_c)\) can be neglected. Equation (90.8) then simplifies to give

\[
\text{error} = \sum_{i=0}^{n} A_i \omega_m^2 \omega_c^2(i - 2)\frac{i(i - 1)}{6} 
\]

(90.9)
If a reference frequency, $\omega_0$, in the frequency interval of interest, is used, and the group-delay curve expanded as a polynomial about this point, there will be a new set of coefficients, $B_i$, and the error will be given by

$$\text{error} = \sum_{i=0}^{n} B_i (\omega - \omega_0)^i - 2 \omega_0^2 \frac{i(i-1)}{6}$$

(90,10)
REFERENCES


5. IBID: p. 390.


CHAPTER 10

SUGGESTIONS FOR FURTHER WORK
10.1 Reactive termination equalizers

I have shown theoretically the potential of interdigital terminations as equalizers for medium and wide band filters. Work is needed to determine the agreement between predicted and experimental equalizers, and to find a logical method of tuning up the equalizer digits.

10.2 The resonant ring equalizer

If the circular ring equalizer is to be developed, work must be done on the properties of curved, coupled transmission lines from both theoretical and experimental aspects. With the existing work on curved transmission lines, the device could then be accurately designed.

10.3 The quasi-optical equalizer

Although theoretical work has shown that mode generation in the coupler is acceptable, much experimental work is required to verify this. The quasi-optical coupler is of great interest in its own right, due to the possibility of its use at frequencies over 100 GHz within complete systems using overmoded rectangular waveguide.