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THE TRANSIENT RESPONSE OF AUTOMOBILES TO STEERING AND WIND GUST INPUTS.

by

NICOLAS FRANK BARTER, B.Sc.

A DOCTORAL THESIS SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE AWARD OF Ph.D. OF THE LOUGHBOROUGH UNIVERSITY OF TECHNOLOGY.

October, 1975.

SUPERVISOR: PROFESSOR F.D. HALES, B.Sc.,Ph.D.,C.Eng.,M.I.Mech.E.,
F.I.M.A.,M.B.C.S.
DEPARTMENT OF TRANSPORT TECHNOLOGY.

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8. TIMERESP - COMPUTER PROGRAMME, INPUT/OUTPUT AND LISTING
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This Thesis is concerned with the problem of measuring and quantifying the transient response of automobiles to steering and side wind inputs. The overall strategy is to use a control theory approach, and describe the various responses in terms of transfer functions and the parameters which define these transfer functions. The work is a continuation of the Author’s earlier work in the same field, and the starting point is that the steer frequency response of three vehicles had been measured using a sinusoidal steer input device, but analysis of the results had not been undertaken. The historical background is reviewed briefly.

The classical three degree of freedom, linear, vehicle response model, with the addition of compliance steer and camber effects, is used to calculate steer and wind gust frequency responses from basic vehicle parameters, and the typical forms of the yaw rate, roll angle, and lateral acceleration frequency response curves are examined.

Frequency response curves produced by sinusoidal steer input tests are shown to be of the same form as those of the three degree of freedom model, and curves of this type are fitted to the experimental data.

As the sinusoidal steer input test is cumbersome and not applicable to side wind response tests, a new method of producing vehicle frequency response curves is developed, involving digital Fourier analysis of input and response for a test using a short, semi random, type of input. This is shown to give results in agreement with the sine input tests, and a digital optimisation technique (OFLS) is used to fit curves of the form of the three degree of freedom model to this data. It is thus shown that the steer responses of the vehicles involved can be described in terms of an equivalent mathematical model, whose parameters can then be used to quantify the vehicle’s response.

A similar conclusion for wind gust response could not be made as the available gust facility did not give a gust appropriate for the analysis technique used.
Finally it is shown that the fitted curves can be used to calculate the vehicle response to any specified input, and examples of these are seen to compare well with measured responses to the same inputs.
ACKNOWLEDGEMENTS.

The Author gratefully acknowledges the guidance and assistance provided by Professor Hales, both during the course of this work and throughout his earlier years at MIRA. The assistance of many members of MIRA staff has also been invaluable, with particular thanks being due to John Little for his help in the experimental programmes, and to Susan Oliver for her assistance in the computer programming and data handling work. Finally the work would not have been possible without the permission of the Director of MIRA, R.H. Macmillan, whose encouragement over a long period of time is gladly acknowledged.
1. INTRODUCTION AND HISTORICAL REVIEW.

In broad terms this work is concerned with the problem of objectively quantifying the handling behaviour of automobiles. The object of research into vehicle handling is to obtain a better understanding of the systems and processes involved so that satisfactory design criteria can be established. Handling is a complex quantity involving all aspects of the vehicle-driver-control relationship, and a complete study of handling must obviously include the driver as part of the system. However, it is necessary to have a good understanding of the vehicle dynamics before attempting to include the driver, and this work is concerned only with this stage.

The discussion of vehicle dynamics can conveniently be treated in two parts referred to as steady state and transient behaviour. As discussed below the steady state behaviour, where the control positions and external forces remain constant, has been extensively studied and is relatively well understood. However the transient part is much less well understood and is the subject of this Thesis. The general philosophy is to use a control theory approach to study the open loop transfer functions of the vehicle system. This basically involves a study of the relationship between the input and output of the system and in this case is applied to both steering and side wind inputs.

The Author has been concerned with work in this field for some time at The Motor Industry Research Association (MIRA), and before the beginning of the work for this Thesis (December 1968) had been concerned with the measurement of the frequency responses of three vehicles, using a sinusoidal steer input device. Part of this Thesis is concerned with the analysis of these results. (In vehicle handling terminology transient conditions are those where the external forces and/or control positions vary with time, and a vehicle subjected to a sinusoidally varying steer input is evidently in such a condition. Confusion can arise with control theory terminology, however, where frequency response is often referred to as a measure of steady state behaviour. Vehicle terminology is used throughout this Thesis).
In 1956 a team from what was then the Cornell Aeronautical Laboratory in Buffalo, USA, presented a series of five papers under the general heading of "Research in Automobile Stability and Control and in Tyre Performance". This series of papers represents a significant landmark in the understanding of vehicle handling and stability and as it also contains a comprehensive survey of the literature to that date is a suitable starting point for this brief historical review. In Ref. 1 Milliken and Whitcomb give the literature survey with 190 references covering vehicle response to both steer and aerodynamic inputs, and a survey of relevant work in the field of aeronautical engineering. They discuss the general approach and results of the work at Cornell, and describe how the techniques used were drawn from experience in the aeroplane field. Segel, Ref. 2, derives the now classical, linear, three degree of freedom equations of vehicle motion, and expresses them in stability derivative form following aeronautical practice. By comparing calculated and experimentally measured vehicle motion he shows that the assumption of linearity holds good up to about a lateral acceleration of 0.3g, and that in this region the equations accurately describe the vehicle behaviour. Refs. 3 and 4 provide an essential back up to this work in the measurement and description of the side force and moment characteristics of the pneumatic tyre, and finally, Whitcomb and Milliken, Ref. 5, use the simpler two degree of freedom equations of motion to examine the basic features of vehicle behaviour, and the effect of fundamental design parameters on this behaviour.

Since the publication of this work by Cornell the study of vehicle dynamics has progressed in various directions:

1) A set of standard definitions and conventions has been developed. This was pioneered at MIRA, Ref. 6, and by the SAE who have periodically produced updated versions, the latest of which is given in Ref. 7.

2) Numerous mathematical models have been introduced, some very complex and able to predict vehicle behaviour right up to the non-linear skidding situation. McHenry, Ref. 8, and Larrabee, Ref. 9, describe the work of Cornell in this non-linear area, and Mitschke, Ref. 10, gives a comprehensive literature survey of mathematical models.
As one of the objects of this Thesis is to gain a better understanding of the fundamentals of vehicle response, the simplest representative model is used. This is fully described in Section 2 and is based on a model by Hales, Ref. 11. It is a linear, three degree of freedom model, but differs from the Segel (Ref. 2) model in that it uses the concept of suspension derivatives (Ref. 12) instead of a "roll axis".

3) The measurement and description of steady state response have been studied in depth, so that this aspect of vehicle dynamics is now relatively well understood. The much used concepts of understeer and oversteer, and static margin were discussed in the Cornell work (Ref. 5) and are now well defined (Ref. 7). A detailed mathematical treatment was presented by Radt and Pacejka, Ref. 13, and studies of the effects of vehicle parameters given by Bergman, Ref. 14, Bundorf, Ref. 15, Hales and Barter, Ref. 16, and many others. A detailed theoretical discussion on the description and measurement of steady state behaviour is given by Hales, Ref. 17 and 18, and a special purpose measurement technique called Tethered Testing is described by Barter, Ref. 19, and Little and Selway-Hoskins, Ref. 20. Very recently a laboratory facility for the accurate measurement of the relevant vehicle parameters is described by Nedley and Wilson, Ref. 21.

4) The measurement and description of transient response have been tackled by various workers, but the state of the art is still relatively unadvanced, and no very firm guidelines have as yet been established. It is with this aspect of vehicle dynamics that the work of this Thesis is concerned, and so it is pertinent to examine the historical background in a little more detail.

The empirical method of studying vehicle transient response is, broadly speaking, to measure the vehicle response to various inputs and describe the results in terms of the characteristics of the particular responses. While this is satisfactory for some purposes, it tends not to give a very useful description of the vehicle dynamics from the point of view of the vehicle designer, nor to provide a very concise method of quantifying the vehicle's characteristics. In the aircraft field this problem has been tackled by using the control theory approach which was largely developed in the first instance to deal with the response and stability of electronic circuits. The
first significant application of this kind of approach to automobile response was described by Schilling, Ref. 22 (1953), who identified the difference between the "free" and "fixed" control behaviour of the vehicle and described two modes of motion citing typical natural frequencies and damping ratios. Then the Cornell work (Ref. 2) showed that the three degree of freedom model could predict yaw rate, roll rate, and lateral acceleration frequency responses to steer input, which agreed well with those measured experimentally.

In 1966 Weir et al., Ref. 23, re-examined the Cornell equations and expressed them in an operational form from which vehicle frequency response could be more readily calculated. They discussed the various response transfer functions and evaluated them theoretically for a typical American saloon car. Experimental frequency response measurements again on an American saloon car, were made by Szostak, Ref. 24, in 1967, and transfer functions of similar form were obtained. This frequency response approach was applied to a high speed bus by Kojima et al., Ref. 25 (1968), and results of a similar pattern but with lower natural frequencies are presented. Kojima also references some earlier Japanese work on passenger cars.

Versace and Forbes, Ref. 26 (1968), tackle the problem of relating the various measures of vehicle dynamics to criteria of desirability, and Milliken and Dell'Amico, Ref. 27 (1968), pursue this in greater depth citing in particular the results of work in the aircraft field (e.g. Ref. 28) relating the natural frequency and damping ratio of a particular mode of motion to pilot opinion contours.

In parallel with the work of Weir etc. described above, the Author was engaged in a programme of measurement of vehicle frequency responses at MIRA. The first part of this work was published in 1970, Ref. 29, and the relevant parts are given in Section 4 of this Thesis.

Since the beginning of this work various papers employing a frequency response approach to vehicle dynamics have been published. Kohno et al., Ref. 30, compare measured frequency responses with those predicted by a 7 degree of freedom model and obtain good agreement. Okada and Sagishima,
Ref. 31, use this approach to examine the effects of tractive force, and Fiala, Ref. 32, and Mitschke and Strackerjan, Ref. 33, for their theoretical and experimental studies of vehicle dynamics. In 1970 O'Hagan and King, Ref. 34, published a brief description of work along similar lines to this Thesis involving fitting a simple linear transfer function to an experimentally obtained lateral acceleration frequency response. They showed that up to a frequency of about 1 Hz, the measured frequency response was not dependent on the input to the vehicle, and discussed the general merits of this type of approach. Also in 1970 Bidwell, Ref. 35, provided an up to date literature survey covering all aspects of vehicle control and road holding.

So far this review has been concerned with vehicle response to steering inputs. However the response to side wind inputs is also to be examined and so a brief review of recent work in that area is also relevant.

A survey of the literature up to 1958 is given by Fosberry, Ref. 36. A large part of the work to that date and since has been concerned with the aerodynamic coefficients of vehicles and body shapes, and a lot of information in this area is provided in a series of reports on the work in the wind tunnels at MIRA, Ref. 37-49. A significant contribution to the understanding of the response of vehicles to side winds was provided by Bundorf et al, Ref. 50 (1963). They used a linear, three degree of freedom, mathematical model similar to that of Segel (Ref. 2) but including aerodynamic inputs, and confirmed its validity by carrying out tests using a hydrogen peroxide rocket motor attached to a vehicle to simulate side wind inputs.

The effect of including aerodynamic forces, on vehicle behaviour in still air, is examined theoretically by Toti, Ref. 51, using a two degree of freedom model, and more realistically by Hales, Ref. 52, using the three degree of freedom model. A general conclusion is that the effect is small up to about 70 mph. Buning and Beauvais, Ref. 53, study the transient effects as a vehicle encounters a wind gust, and conclude that up to total wind incidence angles of 15 degrees these effects may be ignored, and wind tunnel derived steady state coefficients can be used,
A frequency response approach to gust response is used by Hayashi and Furusho, Ref. 54. They measured the wind velocity at a fixed point beside a test road and the steer input and response of a vehicle driven along the road. Steer and gust response transfer functions were then calculated on the assumption that the wind input was a stationary random process. This work differs radically from that of this Thesis in the use of a statistical rather than a deterministic approach.

Braess, Ref. 55, discusses the various methods available for the study of vehicle wind gust response and gives a survey of the literature up to 1968. This was the starting date for the work of this Thesis, but since then various papers have been published. Schmid, Ref. 56, examined the effect of vehicle parameter changes on the response to a proving ground wind input, and Mitschke, Ref. 57, studied the vehicle-driver system in wind gusts using a statistical frequency response approach similar to that of Ref. 54. Hales, Ref. 52, theoretically studied the steady state response to a wind gust, and Braess, Ref. 58, examined the vehicle-driver system using several drivers and a simulated wind gust produced by an air jet fixed to a vehicle. He compared measured driver responses with those predicted theoretically using different driver models and control loop closures. Smith, Ref. 59, provided measurements of gusts occurring on British motorways arising both from natural causes and from the influence of bridges, cuttings, other vehicles, etc.
2. THE THREE DEGREE OF FREEDOM MATHEMATICAL MODEL.

A mathematical model is required for this work for two reasons. Firstly to illustrate the theoretical form of the vehicle response transfer functions, and secondly to allow frequency response curves to be computed for representative sets of vehicle data, for comparison with experimentally measured frequency responses. It is not an object of the work to produce a new or highly sophisticated model, or to attempt to accurately predict the frequency response of a particular vehicle from its basic parameters. In fact the object is to use the simplest possible representative model, so that there is a maximum chance of the transient motion described by the model being understood in physical terms, and thus being used as a method of quantifying the vehicle transient behaviour.

The simplest possible model would be the two degree of freedom system used by Whitcomb and Milliken (Ref. 5). This assumes that the vehicle has yawing and sideslip degrees of freedom only, and moves at constant forward speed. Unfortunately almost all real vehicles have a substantial amount of coupling between the above degrees of freedom and the rolling motion of the body, and so this has to be included in a representative model, particularly when transient behaviour is to be studied. The model used is thus the well established three degree of freedom vehicle derived and validated in detail by Segel (Ref. 2) and used in various forms by several authors since. The equations of motion are linear and are derived by the standard method of equating rates of change of linear and angular momentum to the applied external forces and moments. In order to keep the vehicle inertia matrix constant, an axis system fixed in the vehicle is used. In this case this has its origin at the vehicle centre of gravity, x-axis horizontal and pointing forward, y-axis horizontal and pointing to the off-side, and the z-axis pointing vertically downwards so as to form a right handed set. This is then a rotating axis system and the equations of motion can be written as:

\[
\begin{align*}
\dot{\xi}_Y &= m\dot{v} + m\dot{u}r \\
\dot{\xi}_L &= A\dot{\phi} \\
\dot{\xi}_N &= \dot{C}_r
\end{align*}
\]
where:
Y = external force along y-axis.
L = external roll moment (about x-axis).
N = external yawing moment (about z-axis).
m = vehicle mass (unsprung mass is neglected).
A = roll moment of inertia (products of inertia are neglected).
C = yaw moment of inertia.
U = vehicle forward speed.
v = side slip velocity.
φ = roll angle.
r = yaw rate of turn.
\( \dot{v}, \dot{r} \) etc. = \( dv/dt \), \( dr/dt \), etc.

Using the stability derivative approach these linear equations can be expanded to show the composition of the external forces and moments, giving:

\[
\begin{align*}
\dot{m}v + m\dot{r} &= Y_r v + \frac{\partial Y}{\partial v} v + \frac{\partial Y}{\partial \dot{v}} \dot{v} + \frac{\partial Y}{\partial \ddot{v}} \ddot{v} + Y_r v + Y_r \ddot{r} \\
A\ddot{\phi} &= L_r v + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial \dot{v}} \dot{v} + \frac{\partial L}{\partial \ddot{v}} \ddot{v} + L_r v + L_r \ddot{r} \\
C\ddot{r} &= N_r v + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial \ddot{v}} \ddot{v} + N_r v + N_r \ddot{r}
\end{align*}
\]

where \( Y_r, L_r, \) etc. = \( \partial Y/\partial v, \partial L/\partial v, \) etc. are the stability derivatives, and \( \phi \) = steer input.

These equations are now exactly in the form given by Hales (Ref. 11). Aerodynamic forces are omitted for the moment.

Implicit in the three degree of freedom concept is the assumption that the dynamics of the steering system can be ignored. For the case where the steering wheel is not held fixed (free control) this assumption is not true and the motion under these conditions was examined in detail by Sogel, Ref. 60. However, for the fixed control case the assumption is equivalent to assuming that the steering system frequencies are outside (higher than) the range of interest for the complete vehicle motion. This is not a serious limitation in most cases.

Since the intention is to examine the frequency response of the vehicle system, it is convenient to perform the Laplace transformation on equations 2.4 - 2.6, and express them in the form:
Where \( s \) is the Laplace transform operator.

The three steer response transfer functions can now be written in the form:

\[
\begin{align*}
\text{Yaw rate} & \quad r(s)/\mathcal{Y}(s) = \frac{DR}{D} \\
\text{Roll angle} & \quad \phi(s)/\mathcal{Y}(s) = \frac{DF}{D} \\
\text{Sideslip velocity} & \quad v(s)/\mathcal{Y}(s) = \frac{DV}{D}
\end{align*}
\]

Where \( D, DR, \) and \( DV \) are determinants of the relevant coefficients of equations 2.7 - 2.9. In practical terms it is not easy to measure the sideslip velocity \( v \), and the lateral acceleration (latac), \( a \), in space is of more interest. Remembering that the axis system is rotating with angular velocity equal to the yaw rate \( r \), we can write:

\[
a = \dot{v} + ru
\]

or

\[
a(s) = sv(s) + Ur(s)
\]

Hence the latac transfer function can be written as:

\[
a(s)/\mathcal{Y}(s) = (sDV + U_r DR)/D = DL/D
\]

Where this equation defines the determinant \( DL \).

These determinants can be written down direct from equations 2.7 - 2.9 to give:

\[
D = \begin{vmatrix} ms - Y_{\alpha} & -Y_{\alpha} s - Y_{\phi} & mU - Y_{\gamma} \\ -L_{\alpha} & As^2 - L_{q} s - L_{\phi} & -L_{\gamma} \\ -N_{\alpha} & -N_{q} s - N_{\phi} & Cs - N_{\gamma} \end{vmatrix}
\]

\[
DR = \begin{vmatrix} ms - Y_{\alpha} & -Y_{\alpha} s - Y_{\phi} & Y_{\gamma} \\ -L_{\alpha} & As^2 - L_{q} s - L_{\phi} & L_{\gamma} \\ -N_{\alpha} & -N_{q} s - N_{\phi} & N_{\gamma} \end{vmatrix}
\]
The expansion of these determinants is given in detail in Appendix 1 and results in polynomial expressions of the form:

(2.19)

(2.20)

(2.21)

(2.22)

Where the coefficients \( D[1], D[2], DR[0], \) etc. are functions of the vehicle mass, inertia, stability derivatives, and forward speed, and are given explicitly in Appendix 1. There are 17 of these coefficients, but because of the quotients involved the number required to specify the transfer functions can be reduced to 16, when they are expressed:

\[ r(s)/\delta(s) = C_6(s^3 + C_6 s^2 + C_7 s + C_8)/s^4 + C_1 s^3 + C_2 s^2 + C_3 s + C_4 \]  
(2.23)

\[ \phi(s)/\delta(s) = C_9(s^3 + C_9 s^2 + C_10)/s^4 + C_1 s^3 + C_2 s^2 + C_3 s + C_4 \]  
(2.24)

\[ a(s)/\phi(s) = C_{10}(s^4 + C_1 s^3 + C_7 s^2 + C_8 s + C_9)/s^4 + C_1 s^3 + C_2 s^2 + C_3 s + C_4 \]  
(2.25)

where:

\[ c_1 = D[2]/D[1], \quad c_2 = D[3]/D[1], \quad c_3 = D[4]/D[1], \quad c_4 = D[5]/D[1], \]

\[ c_5 = DR[1]/DR[0], \quad c_6 = DR[2]/DR[0], \quad c_7 = DR[3]/DR[0], \quad c_8 = DR[4]/DR[0], \]

\[ c_9 = DP[0]/DP[1], \quad c_{10} = DP[1]/DP[0], \quad c_{11} = DP[2]/DP[0], \quad c_{12} = DP[3]/DP[0], \]

\[ c_{13} = DL[1]/DL[1], \quad c_{14} = DL[2]/DL[1], \quad c_{15} = DL[3]/DL[1], \quad c_{16} = DL[4]/DL[1], \quad c_{17} = DL[5]/DL[1] \]  
(2.26)
These 16 coefficients now define the steer response of the vehicle and could be used as an objective description of this response. Alternatively, the polynomial parts of equations 2.23 - 2.25 can be factorised and the equations written in the form:

\[
\frac{r(s)}{\delta(s)} = \frac{C_r \left[ (1/\omega_r)^2 s^2 + (4\omega_r/\omega_c)s + 1 \right]}{\left[ (1/\omega_r)^2 s^2 + (4\omega_r/\omega_c)s + 1 \right]} \quad (2.27)
\]

\[
\frac{\theta(s)}{\delta(s)} = \frac{C_\theta \left[ (1/\omega_\theta)^2 s^2 + (4\omega_\theta/\omega_c)s + 1 \right]}{\left[ (1/\omega_\theta)^2 s^2 + (4\omega_\theta/\omega_c)s + 1 \right]} \quad (2.28)
\]

\[
\frac{\alpha(s)}{\delta(s)} = \frac{C_\alpha \left[ (1/\omega_\alpha)^2 s^2 + (4\omega_\alpha/\omega_c)s + 1 \right]}{\left[ (1/\omega_\alpha)^2 s^2 + (4\omega_\alpha/\omega_c)s + 1 \right]} \quad (2.29)
\]

where the \(\omega\)'s and \(\dot{\omega}\)'s are defined by these equations and are the characteristic corner frequencies and dampings of the system. The coefficients \(C_r\), \(C_\theta\), and \(C_\alpha\) are retained as these represent the steady state gain factors. These \(\omega\)'s and \(\dot{\omega}\)'s are perhaps more appropriate quantities to use as descriptors of the vehicle response as they have a more readily understandable physical significance. In particular the two factors on the denominator of the transfer functions 2.27 - 2.29 represent the two normal modes of the system, and \(\omega_r, \dot{\omega}_r, \omega_\alpha, \dot{\omega}_\alpha\) are the natural frequencies and damping ratios of these modes. (It is, of course, possible that these quadratic factors can be further factorised into real, first order factors, in which case the response is not oscillatory and the concept of natural frequency has no meaning). Physically \(\omega_r\) is the natural frequency in yaw and \(\omega_\alpha\) that in roll.

In control theory terms the transfer function numerator factors are called lead terms as they give rise to phase lead, and their roots are called the zeros of the system. The denominator factors are called lag terms and their roots the poles of the system. The stability of the system can be examined by studying the poles and zeros, and this is done by Weir et al in Ref. 23. They also examine the dependance of the \(\omega\)'s and \(\dot{\omega}\)'s on vehicle parameters and this is not done explicitly here. However, a brief look at the composition of \(\omega_r, \dot{\omega}_r, \omega_\alpha, \dot{\omega}_\alpha\) for the two degree of freedom case is given in Appendix 2.

It is interesting to note that the two degree of freedom model represents the case where the \(\omega_\alpha\) factor is cancelled by a numerator factor,
and the amount of roll coupling present in a vehicle is indicated by the difference between $\omega_L$, $J_a$, and $\omega_t$, $J_t$, for the yaw rate response, and $\omega_{lt}$, $J_{lt}$ for the lateral response.

So far the above equations have been expressed in terms of the stability derivatives $V_x$, $L_x$, etc. In order to look at the effects of physical vehicle parameters the derivatives must be evaluated in terms of these parameters. This is done by the method of Hales, Ref. 11, but with the additional inclusion of the effects of lateral force steer and camber. (The importance and magnitude of these latter effects have been brought home to the author in the course of many vehicle tests and are discussed in detail by Bergman, Ref. 61).

Following the method due to Hales, the external lateral tyre forces are assumed to be functions of their slip and camber angles $\alpha$ and $\gamma$. (In practice tyre forces are also functions of the tyre vertical load, but for a linear model the effect of any increase in load on one side of the vehicle is counteracted by the corresponding decrease on the other side and so the overall effect is zero). Any lateral tyre force can thus be written:

$$Y = K_\alpha \alpha + K_\gamma \gamma$$

(2.30)

where:

$K_\alpha =$ tyre side slip stiffness.

$K_\gamma =$ tyre camber stiffness.

The problem is now reduced to expressing $\alpha$ and $\gamma$ in terms of the motion variables $v, \dot{\phi}, \phi, r,$ and $\dot{r}$ of equations 2.4 - 2.6. This can be done by simply listing the ways in which slip and camber are generated. Slip angle is produced by:

<table>
<thead>
<tr>
<th>Effective slip angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front wheels</td>
</tr>
<tr>
<td>1) Direct steer</td>
</tr>
<tr>
<td>2) Lateral velocity</td>
</tr>
<tr>
<td>3) Yaw velocity</td>
</tr>
<tr>
<td>4) Steer due to roll</td>
</tr>
<tr>
<td>5) Lateral velocity due to roll rate</td>
</tr>
<tr>
<td>6) Steer due to lateral force</td>
</tr>
</tbody>
</table>
and camber angle by:

Effective camber angle

Front wheels  \( \frac{\partial \phi_f^i}{\partial \phi} \) \( \phi_f \)
Rear wheels  \( \frac{\partial \phi_r^i}{\partial \phi} \) \( \phi_r \)

where:

Subscripts \( F, R \) refer to front and rear.

\( x_{F,R} \) = longitudinal distance of C.G. from front and rear axles.

\( \beta/\phi \) = steer angle due to roll.

\( \psi/\phi \) = tyre lateral displacement due to roll angle. (This is analogous to the concept of roll centre height).

\( \gamma/\phi \) = steer angle due to lateral force.

\( \phi/\psi \) = tyre camber angle due to roll angle.

\( \psi/\psi \) = tyre camber angle due to lateral force.

The partial derivative notation of 4 - 8 above is that due to Hales, Ref. 12, who termed these expressions suspension derivatives. Items 6 and 8 have been added by the author. The front lateral tyre force can now be written from equation 2.30 as:

\[
Y_f = K_{sp} [v/U + (\partial \phi_f^i/\partial \phi) \cdot \dot{\phi}/U - (\partial \delta_f^i/\partial \phi) \phi + x_{F}/U - \delta] - (\partial \delta_f^i/\partial \psi) \psi_f + K_{cp} (\partial \phi_f^i/\partial \psi) \psi \]

or solving explicitly for \( Y_f \):

\[
Y_f[1 + K_{sp} (\partial \phi_f^i/\partial \psi) - K_{cp} (\partial \phi_f^i/\partial \psi)] = K_{sp} [v/U + (\partial \phi_f^i/\partial \psi) \cdot \dot{\phi}/U - (\partial \delta_f^i/\partial \psi) \phi + x_{F}/U - \delta] + K_{cp} (\partial \phi_f^i/\partial \psi) \psi \] \hspace{1cm} (2.31)

This can be written:

\[
Y_f = K_{sp} [v/U + (\partial \phi_f^i/\partial \psi) \cdot \dot{\phi}/U - (\partial \delta_f^i/\partial \psi) \phi + x_{F}/U - \delta] + K_{cp} (\partial \phi_f^i/\partial \psi) \psi \] \hspace{1cm} (2.32)

where:

\[
K_{sp} = K_{sp}/[1 + K_{sp} (\partial \phi_f^i/\partial \psi) - K_{cp} (\partial \phi_f^i/\partial \psi)] \hspace{1cm} (2.33)
K_{cp} = K_{cp}/[1 + K_{sp} (\partial \phi_f^i/\partial \psi) - K_{cp} (\partial \phi_f^i/\partial \psi)]
\]
The effect of the lateral force steer and camber can thus be thought of as modifying the tyre side slip and camber stiffnesses to $K_s$ and $K_c$. With this substitution the stability derivatives can be written down exactly as given by Hales, Ref. 11.

A similar expression to 2.32 applies to the rear tyres and the front and rear tyre force derivatives can be written down by referring to equation 2.4, as:

$\gamma_r = \frac{K_s}{U}$

$\gamma_{rf} = \frac{K_s}{U} \left( \frac{\partial Y_r}{\partial \dot{\gamma}} \right)$

$\gamma_{rf} = -K_s \left( \frac{\partial Y_r}{\partial \dot{\gamma}} \right) + K_c \left( \frac{\partial q_r}{\partial \dot{\gamma}} \right)$

$\gamma_{ff} = K_s x_c / U$

$\gamma_{f} = -K_s$

$\gamma_{ff} = -K_s$

Now given the roll stiffness $K_s$ and roll damping $K_d$, the stability derivatives can be written down by resolving laterally and taking moments about the axes. The expressions, exactly in the form given by Hales, Ref. 11, are then:

$\gamma_r = \gamma_{rf} + \gamma_{rr}$

$L_r = \left( \frac{\partial Y_r}{\partial \dot{\gamma}} \right) Y_{rf} + \left( \frac{\partial Y_r}{\partial \dot{\gamma}} \right) Y_{rr}$

$N_r = x_f Y_{rf} + x_k Y_{rr}$

$\gamma_q = \gamma_{qf} + \gamma_{qr}$

$L_q = \left( \frac{\partial Y_q}{\partial \dot{\gamma}} \right) Y_{qf} + \left( \frac{\partial Y_q}{\partial \dot{\gamma}} \right) Y_{qr} + K_d$

$N_q = x_f Y_{qf} + x_k Y_{qr}$

$\gamma_f = \gamma_{rf} + \gamma_{rr}$

$L_f = \left( \frac{\partial Y_f}{\partial \dot{\gamma}} \right) Y_{rf} + \left( \frac{\partial Y_f}{\partial \dot{\gamma}} \right) Y_{rr}$

$N_f = x_f Y_{rf} + x_k Y_{rr}$

$\gamma_f = \gamma_{ff}$

$L_f = \left( \frac{\partial Y_f}{\partial \dot{\gamma}} \right) Y_{ff}$

$N_f = x_f Y_{ff}$
By substituting these expressions in the equations for the coefficients given in Appendix 1 it is thus possible to evaluate the vehicle steer response transfer functions of 2.23 - 2.25 in terms of vehicle parameters.

So far the response to steer input has been considered. The response to a wind gust input can be derived in a very similar way by substituting the wind input terms \( Y_p, L_p, N_p \) for the steer input terms \( Y_s, L_s, N_s \) in equations 2.4 - 2.6. Where:

\[
\begin{align*}
\dot{\vartheta} & = \text{angle of incidence of the total wind.} \\
Y_p, L_p, N_p & = \text{wind gust stability derivatives } \frac{\partial Y}{\partial \vartheta}, \frac{\partial L}{\partial \vartheta}, \frac{\partial N}{\partial \vartheta}.
\end{align*}
\]

These wind gust derivatives can be evaluated from the aerodynamic coefficients measured in a wind tunnel. Using the techniques of Refs. 38 - 44 the relevant aerodynamic coefficients are:

\[
\begin{align*}
C_v & = \text{side force coefficient.} \\
C_r & = \text{rolling moment coefficient.} \\
C_y & = \text{yawing moment coefficient.}
\end{align*}
\]

These coefficients are functions of \( \vartheta \) and for any given \( \vartheta \) the forces and moments on the car are:

\[
\begin{align*}
\text{Side force} & \quad Y' = C_v A q \\
\text{Rolling moment} & \quad L' = C_r A q t \\
\text{Yawing moment} & \quad N' = C_y A q \lambda
\end{align*}
\]

(2.40)

where:

\[
\begin{align*}
A & = \text{projected frontal area of the vehicle.} \\
l & = \text{wheelbase.} \\
t & = \text{mean track.} \\
\rho & = \text{mass density of air.} \\
q & = \text{dynamic head } = (1/2) \rho V^2 \\
V & = \text{component of the relative air speed parallel to the longitudinal axis of the vehicle.}
\end{align*}
\]

Conventionally the aerodynamic coefficients are related to an axis system with origin (0) on the ground plane on the centre line of the vehicle.
midway between the axles, x-axis horizontal and pointing to the rear, y'-axis horizontal and pointing to the near side, and z'-axis pointing vertically upwards. Figure 2.1 shows the relationship between this axis and that used above for the derivation of the equations of motion. (Referred to as the C.G. system).

The co-ordinates of 0 in the C.G. system are \((x_F - L/2, 0, h)\).

Where:
- \(x_F\) = longitudinal distance from C.G. to front axle.
- \(L\) = wheelbase.
- \(h\) = C.G. height.

The forces and moments in the C.G. system can thus be expressed in terms of equations 2.40 as:

\[
\begin{align*}
Y &= -y' \\
L &= -L' + Y'h \\
N &= -N' - Y'(x_F - L/2)
\end{align*}
\] (2.41)

Examination of the data presented in Refs. 38 - 44 shows that it is a reasonable assumption to make the coefficients \(C_s\), \(C_r\) and \(C_y\) linear functions of the angle of incidence of the wind with gradients \(\partial C_s/\partial \theta\), \(\partial C_r/\partial \theta\), and \(\partial C_y/\partial \theta\). Expressing equation 2.41 in terms of a positive side wind in the C.G. system and substituting these gradients the forces and moments due to a wind at angle \(\theta\) can thus be written:
\[ Y = Aq \frac{\partial C_y}{\partial \phi} \theta = Y_p \theta \]
\[ L = Aq [t \frac{\partial C_r}{\partial \phi} - h \frac{\partial C_s}{\partial \phi}] \theta = L_p \theta \]
\[ N = Aq [2 \frac{\partial C_y}{\partial \phi} + (x_f - \ell/2) \frac{\partial C_s}{\partial \phi}] \theta = N_p \theta \]

(2.42)

where \( Y_p, L_p, \) and \( N_p \) are the required aerodynamic stability derivatives and are defined by these equations. If the longitudinal wind speed is assumed to be the same as the vehicle forward speed \( U \) these derivatives can be written finally as:

\[ Y_p = (1/2) \tau^2 A \frac{\partial C_y}{\partial \phi} \]
\[ L_p = (1/2) \tau^2 A [t \frac{\partial C_r}{\partial \phi} - h \frac{\partial C_s}{\partial \phi}] \]
\[ N_p = (1/2) \tau^2 A [2 \frac{\partial C_y}{\partial \phi} + (x_f - \ell/2) \frac{\partial C_s}{\partial \phi}] \]

(2.43)

Wind gust response transfer functions can now be written down exactly as the corresponding steer functions of equations 2.10 - 2.13 to give:

\[ \frac{\dot{\phi}(s)}{\theta(s)} = \frac{A D P}{D} \]
\[ \frac{\dot{\phi}(s)}{\theta(s)} = \frac{A D P}{D} \]
\[ \frac{v(s)}{\theta(s)} = \frac{A D V}{D} \]
\[ \frac{a(s)}{\theta(s)} = \left( s \frac{A D V + \theta A D R}{D} \right) \]

(2.44)

where the determinants \( A D R, A D P, \) and \( A D V \) are obtained from \( D R, D P, \) and \( D V \) by replacing \( Y_f, L_f, \) and \( N_f \) by \( Y_p, L_p, \) and \( N_p \) respectively. The discussion of the steer transfer functions then applies directly to these gust functions.
Although this work is not primarily concerned with steady state vehicle behaviour, it is of interest to have a measure of this for consideration in relation to the frequency responses. The steady state quantities static margin (SM), steering gain (DSTEER/DLATAC), and steer-slip gradient (DSTEER/DSLIP), can be calculated from the vehicle stability derivatives as follows:

For steady state conditions the equations of motion 2.4 - 2.6 become:

\[ m \ddot{u} = Y_q v + Y_q \dot{q} + Y_r \dot{r} + Y_f \dot{f} \]
\[ 0 = L_q v + L_q \dot{q} + L_r \dot{r} + L_f \dot{f} \]
\[ 0 = N_q v + N_q \dot{q} + N_r \dot{r} + N_f \dot{f} \]  

(2.45)

Static margin can be defined as the longitudinal distance from the vehicle centre of gravity at which the application of a side force does not produce a change in yaw rate. If we consider the vehicle travelling in a straight line and apply a force \( Y \) at the SM (distance \( x \) from the C.G.), then if there is no steer angle \( \delta = 0 \), there will be no yaw rate \( \dot{r} = 0 \) and the equations of motion become:

\[ 0 = Y_q v + Y_q \dot{q} + Y \]
\[ 0 = L_q v + L_q \dot{q} \]
\[ 0 = N_q v + N_q \dot{q} + xy \]  

(2.46)

The condition for these to have a solution is:

\[
\begin{vmatrix}
Y_q & Y_q & 1 \\
L_q & L_q & 0 \\
N_q & N_q & x
\end{vmatrix}
= 0
\]

whence:

\[ SM = x = -(L_q N_q - L_q N_r)/(Y_q L_q - Y_q L_r) \]  

(2.47)

It can be seen from equations 2.34 - 2.39 that some of the stability derivatives are functions of speed, \( U \). In order to make \( U \) appear explicitly in the equations 2.45 we can make the substitutions:

\[ Y_q = \frac{Y_q}{U} \]
\[ L_q = \frac{L_q}{U} \]
\[ N_q = \frac{N_q}{U} \]
\[ Y_r = \frac{Y_r}{U} \]
\[ L_r = \frac{L_r}{U} \]
\[ N_r = \frac{N_r}{U} \]  

(2.48)
giving:
\[
\begin{align*}
\mu_r &= Y_y (v/U) + Y_q \phi + Y_f (r/U) + Y_f \delta \\
0 &= L_y (v/U) + L_q \phi + L_f (r/U) + L_f \delta \\
0 &= N_y (v/U) + N_q \phi + N_f (r/U) + N_f \delta
\end{align*}
\]  
(2.49)

If we now consider the vehicle to be kept on a constant radius path we have:

\begin{align*}
\text{Radius} & \quad R = r/U \\
\text{Latac} & \quad Y = \mu_r \\
\text{Side-slip angle} & \quad \alpha = -v/U
\end{align*}

and equations 2.49 become:

\begin{align*}
Y - \frac{\mu_r}{R} &= Y_y \alpha + Y_q \phi + Y_f \delta \\
L_y &= -L_y \alpha + L_q \phi + L_f \delta \\
N_y &= -N_y \alpha + N_q \phi + N_f \delta
\end{align*}

Differentiating these equations w.r.t. \( Y \) gives:

\begin{align*}
1 &= -Y_y (d\alpha/dY) + Y_q (d\phi/dY) + Y_f (d\delta/dY) \\
0 &= -L_y (d\alpha/dY) + L_q (d\phi/dY) + L_f (d\delta/dY) \\
0 &= -N_y (d\alpha/dY) + N_q (d\phi/dY) + N_f (d\delta/dY)
\end{align*}

whence:

\[
\frac{d\text{DSTEER}/d\text{DLATAC}}{d\text{DY}/dY} = \frac{d\delta/dY}{} = \frac{(L_y N_q - L_q N_y) / \Delta}{d\phi/dY} 
\]  
(2.52)

\[
d\phi/dY = -(L_q N_f - L_f N_q) / \Delta
\]  
(2.53)

where:

\begin{equation}
\Delta = \begin{vmatrix}
Y_y & Y_q & Y_f \\
L_y & L_q & L_f \\
N_y & N_q & N_f
\end{vmatrix}
\]  
(2.54)

From 2.52 and 2.54 we can now write:

\[
\frac{d\text{DSTEER}/d\text{DSLIP}}{d\phi/d\alpha} = \frac{-(L_y N_q - L_q N_y) / (L_q N_f - L_f N_q)}{d\phi/d\alpha}
\]  
(2.55)
3. CALCULATION OF FREQUENCY RESPONSES FROM VEHICLE PARAMETERS.

To evaluate the steer and wind gust frequency responses from basic vehicle parameters a digital computer programme - FREQRESP - was prepared, and a listing (in Algol) is given in Appendix 3. The operation of this programme is outlined below.

The input required is the list of vehicle data given in Table 3.1. The first stage of the programme uses equations 2.33 - 2.39 and 2.43 of Section 2 to calculate the vehicle stability derivatives. These are used to calculate the steady state quantities \( \text{SM} \), \( \text{DSTEER/DLATAC} \), and \( \text{DSTEER/DSLIP} \) given by equations 2.47, 2.52, and 2.55, the 16 coefficients (4.26) defining the steer response transfer functions 2.23 - 2.25, and the 16 similar coefficients defining the gust response transfer functions 2.44. The characteristic corner frequencies and damping ratios defined by equations 2.27 - 2.28, are then calculated by extracting the roots of the quartics, cubics, and quadratics which make up the numerators and denominators of the transfer functions. The roots of the quartics and cubics are obtained using standard procedures QUARTIC and CUBIC taken from the MIRA computer library. These procedures are not included in the programme listing in Appendix 3. The roots are typically in the form:

\[ a + jb \]

where \( j = \sqrt{-1} \)

The corner frequency, which is the undamped natural frequency, is then given by:

\[ \omega_n = \sqrt{a^2 + b^2} \]

the damping ratio by:

\[ \zeta = a / \omega_n \]

and, if required, the actual damped natural frequency \( \omega_d \) by:

\[ \omega_d = b = \omega_n \sqrt{1 - \zeta^2} \]

(FREQRESP does not evaluate \( \omega_d \)).
The frequency responses of the vehicle are obtained from the transfer functions of equations 2.10 - 2.12 by replacing the Laplace operator \( s \) by \( j\omega \). Using equations 2.19 - 2.22 the frequency responses can then be written as functions of frequency, \( \omega \):

**Yaw rate**

\[
\]

**Roll angle**

\[
\]

**Lateral**

\[
\]

These are in the form:

\[
\frac{p + jq}{r + js}
\]

which can be simplified to:

\[
e + jf
\]

where:

\[
e = \frac{(pr + qs)}{(r^2 + s^2)}
\]

and \( f = \frac{(qr - ps)}{(r^2 + s^2)} \)

and FREQRESP simply evaluates the amplitude and phase for each value of frequency, \( \omega \), from expressions of the form:

\[
\text{Amplitude} = \sqrt{e^2 + f^2}
\]

\[
\text{Phase angle} = \tan^{-1}(f/e)
\]

Standard computer procedures for the evaluation of \( \tan^{-1} \) produce the result only in the range 0 - 90 deg. FREQRESP expands this to the full 0 - 360 deg. range, firstly by examining the signs of \( f \) and \( e \), and secondly by ensuring that there are no large step discontinuities in phase angle. These amplitude and phase calculations are actually done by the use of simple, specially written procedures AMP, PHASE, and FIXPHASE.

An example of the output of FREQRESP is shown in Table 3.2 and Figs. 3.1 to 3.3. The data for this Run (Run 14) is shown in Table 3.1 and is that used by Hales, Ref. 52, who largely took it from Bundorf, Ref. 50. The vehicle is a typical American saloon car.
The notation of Table 3.2, which is the computer print-out, is self explanatory. It can be seen that the vehicle is fairly understeering with a Static Margin of -8% (expressed as a percentage of the wheelbase). The yaw and roll corner frequencies are 0.65 and 1.35 Hz, as shown by the poles of the system. Hales, Ref. 52, also evaluated these poles and the fact that his values agree with these is a useful check on the programme. Listed at the foot of Table 3.2

<table>
<thead>
<tr>
<th>NOTATION OF SECTION 2</th>
<th>FREQRESP</th>
<th>UNITS</th>
<th>RUN 14</th>
<th>RUN 27</th>
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<td>KPD</td>
<td>ft, lb/rad/sec</td>
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<td>-1000</td>
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Table 3.1 Input data required for FREQRESP and values for Runs 14 and 27.
FDH/B12 50 MPH RUN 14
SM -8.009  D(STEER)/D(SLIP) .3580  D(STEER)/D(LATAC) .0892

RESPONSE COFFF STER WIND
1 1.140e+01 1.140e+01
2 1.201e+02 1.201e+02
3 4.660e+02 4.660e+02
4 1.179e+03 1.179e+03
5 2.505e+01 4.900e-03
6 6.373e+00 7.880e+00
7 6.892e+01 1.046e+02
8 1.878e+02 3.090e+02
9 -6.363e+01 6.153e-04
10 3.122e+00 1.101e+02
11 2.590e+01 -2.529e+02
12 1.198e+02 4.296e-02
13 5.688e+00 7.927e+00
14 1.151e+02 1.275e+02
15 3.190e+02 3.924e+02
16 2.865e+03 2.573e+03

POLES
REAL IMAG FREQ D KAT
1 -3.177 7.840 1.35 -0.376
2 -2.525 3.179 0.65 -0.622
3 -2.525 -3.179 0.65 -0.622
4 -3.177 -7.840 1.35 -0.376

STEER ZEROS GUST ZEROS
REAL IMAG FREQ D KAT
YAW RATE
1 -3.196 0.000 0.51 -1.000 -3.459 0.000 0.55 -1.000
2 -1.588 -7.500 1.22 -0.207 -2.211 -9.189 1.50 -0.234
3 -1.588 7.500 1.22 -0.207 -2.211 9.189 1.50 -0.234
ROLL ANGLE
1 -1.561 -4.844 0.81 -0.307 2.250 0.000 0.36 1.000
2 -1.561 4.844 0.81 -0.307 -112.4 0.000 17.89 -1.000
LATAC
1 -9.499 7.711 1.24 -0.122 -2.539 8.663 1.44 -0.281
2 -1.894 -6.627 1.10 -0.275 -1.424 5.435 0.89 -0.253
3 -1.894 -6.627 1.10 -0.275 -1.424 -5.435 0.89 -0.253
4 -9.499 -7.711 1.24 -0.122 -2.539 -8.663 1.44 -0.281

SPECIFIC STEADY STATE STEER RESPONSE
YAW RATE 3.991e+00
ROLL ANGLE 1.395e+00
LATAC 2.913e+02

SPECIFIC STEADY STATE GUST RESPONSE
YAW RATE 1.284e-03
ROLL ANGLE 1.320e-04
LATAC 9.373e-02

Table 3.2. Tabulated results corresponding to Figs. 3.1 - 3.3. Run 14.
Fig. 3.1. Steer frequency response curves for the data of Run 14.
Fig. 3.2. Gust frequency response curves (amplitude) for data of Run 14.
Fig. 3.3. Gust frequency response curves (phase angles) for data of Run 14.
are the specific steady state (zero frequency) responses. These are
given per radian of steer angle for the steer response and per degree
of wind angle for the gust response.

The amplitude and phase frequency response plots (Bode plots) of
Figs. 3.1 - 3.3 were plotted by the graph plotter available on the MIRA
computer (an ICL 4130). The amplitude curves are plotted on a decibel
(dB) scale, the value at any frequency being given by:

\[
\text{dB value} = 20 \log_{10} \left( \frac{\text{amplitude}}{\text{amplitude at zero frequency}} \right)
\]

This is done so that all the curves can be readily plotted on the same
scale and the specific steady state responses are listed separately so
that the actual values can be obtained if required.

The amplitude steer response curves show the characteristic
vehicle behaviour which will be seen throughout. The yaw rate curve has
a gentle rise from the steady state level to a low peak, followed by a
fall off at 6 dB per octave, superimposed on which is a pronounced
'kink' showing that this vehicle has considerable coupling with the
roll mode. The roll angle curve does not have the initial rise, and
begins to fall off at a much lower frequency, the final fall off rate
being 12 dB per octave. The roll mode resonance is characterised
by the typical 'kink' in the curve, which is rather more pronounced
for this vehicle than for the other vehicles examined later. The latac
curve is of particularly interesting shape with a steep fall off similar
to that of the roll angle, followed by a final rise again to what would
be a response equal to the steady state response at infinite frequency.
In mathematical terms this shape is due to the effect of the lightly
damped lead term in the transfer function, and in physical terms is
attributable to the change over from the predominance of yaw rate
generated centrifugal acceleration, to straight forward linear
acceleration along the y axis (see equation 2.13, Section 2). The
phase angle curves are similarly characteristic and tend to final
values of 90, 180, and 0 degrees for the yaw rate, roll angle, and
latac respectively. (In fact since the roll angle has opposite sign
to the yaw rate it should have an additional 180 degrees of lag. This
has been removed to ease the scaling of the graphs).
For yaw rate and lateral the gust response curves, Figs. 3.2 and 3.3, are of broadly similar form to the above, but the roll angle curve (and therefore its influence of the yaw rate curve) is quite different. This can be understood when it is appreciated that, for this model, the wind gust input is equivalent to a lateral force acting fairly well forward and about half way up on the vehicle bodywork. This bears some resemblance to the force input due to steering the front tyres from the point of view of yaw rate and lateral, but is clearly quite different for roll angle.

Although it is not an object of this work to carry out a parameter study using this model, it is helpful in interpreting the measurements examined in the following Sections, to look at some different types of response curves. It is well known that the steady state understeer or oversteer of a vehicle has a large effect on its transient response, and this can be clearly demonstrated by changing the front roll understeer of Run 14 (DDFDP = 0.32) to roll oversteer (DDFDP = -0.32). The results of this (Run 43) are shown in Table 3.3 and Figs. 3.4 and 3.5. The main points of interest in the Table are, the change in Static Margin (SM) from -8% to +10.5%, the splitting of the complex conjugate pair representing the oscillatory yaw-sideslip mode into two real roots, and the increase in steady state response by almost a factor of 10. One of the real roots, although still negative, is very small, indicating that a small increase in vehicle speed or in the level of oversteer would make the vehicle unstable. The steer frequency response curves, Fig. 3.4, are markedly different from the typically understeering curves of Fig. 3.1, and illustrate why oversteering vehicles have such a slow, heavily damped, (although large) steer response. The gust response curves, Fig. 3.5, show a similar trend, but the high frequency response is not quite so poor because of the greater influence of the roll angle in the gust response. Because the high frequency gust response is significantly better than that of the steer response it can be imagined that this vehicle would be difficult to control in gusty conditions. In this respect this vehicle is worse than that of Run 14 where the steer and gust yaw rate frequency responses were more similar.

To look at the effect of a purely aerodynamic change Run 15 is the same as Run 14 except that the vehicle has been given aerodynamic coefficients more like those of an estate car (figures taken from Ref. 52).
FDH/B12 50 MPH RUN 43

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**STEER ZEROS**

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- **LATAC**

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<td>1.24</td>
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- **SPECIFIC STEADY STATE STEER RESPONSE**

| YAW RATE | 3.788E+01 |
| ROLL ANGLE | 1.327E+01 |
| LATAC | 2.765E+03 |

- **SPECIFIC STEADY STATE GUST RESPONSE**

| YAW RATE | 4.484E+03 |
| ROLL ANGLE | 1.253E+03 |
| LATAC | 3.273E-01 |

Table 3.3. Tabulated results corresponding to Figs. 3.4 and 3.5. Run 43.
Fig. 3.4. Steer frequency response curves for an oversteer vehicle. Run 43.
Fig. 3.5. Gust frequency response curves for an oversteer vehicle, Run 43.
The parameters are therefore the same as in Table 3.1 except that:

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<tr>
<th>AERODYNAMIC COEFFICIENT</th>
<th>RUN 14 (SALOON)</th>
<th>RUN 15 (ESTATE)</th>
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<td>DCSDT (side force)</td>
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<td>DCRDT (roll moment)</td>
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<td>0.015</td>
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<tr>
<td>DCYDT (yaw moment)</td>
<td>0.01</td>
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The yawing moment coefficient has been halved at the expense of a 25% increase in side force coefficient. Table 3.4 gives the tabular output and Figs. 3.6 and 3.7 the gust response curves. The steer response is the same as for Run 14, Fig. 3.1. Superimposed on Fig. 3.6 are the gust response curves of Run 14 (dashed lines). It can be seen that the predominant effect is a large reduction in the steady state response levels, but the high frequency behaviour is also affected. Relative to the steady state response the yaw rate now has a lower high frequency response while the roll angle and lateral responses are higher. Because the yaw rate response is the most important in terms of likely deviation from the intended path the estate vehicle is clearly preferable on both steady state and transient gust response grounds.

The results looked at so far have been representative of a typical American car, which is much larger and heavier than its European counterpart. The data values headed Run 27 in Table 3.1 are intended to represent a medium sized European car and the results are given in Table 3.5 and Figs. 3.8 to 3.10. In general the picture is as for Run 14, but the significant differences are; less understeer and larger steady state response, higher yaw and roll natural frequencies and thus better high frequency response, and less roll-yaw coupling as seen by the less pronounced "kink" on the yaw rate curve. In fact it will be seen later that some of the vehicles on which measurements were made showed yaw natural frequencies significantly higher than for this Run. Two further variants of Run 27 were also run to illustrate the effects of roll-yaw coupling. The parameters which were varied are shown below and the results presented in Tables 3.6 and 3.7 and Figs. 3.11 and 3.12.
FDH/812 ESTATE RUN 15

SM -6.099 D(STEER)/D(SLIP) 0.3580 D(STEER)/D(LATAC) 0.0892

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POLES

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STEER ZEROS

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SPECIFIC STEADY STATE STEER RESPONSE

| YAW RATE | 3.991e+00 |
| ROLL ANGLE | 1.395e+00 |
| LATAC | 2.913e+02 |

SPECIFIC STEADY STATE GUST RESPONSE

| YAW RATE | 8.298e-04 |
| ROLL ANGLE | 4.508e-05 |
| LATAC | 6.058e-02 |

Table 3.4. Tabulated results corresponding to Figs. 3.6 and 3.7. Run 15.
Fig. 3.6. Gust frequency response curves for an estate type vehicle, Run 15. Curves for saloon vehicle, Run 14, superimposed dashed.
Fig. 3.7. Gust frequency response curves (phase angles) for an estate type vehicle. Run 15.
**50 MPH RUN 27**

**SM - 4.618  D(STEER)/D(SLIP) .1865  D(STEER)/D(LATAC) .0368**

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**POLES**

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**SPECIFIC STEADY STATE STEER RESPONSE**

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<td>6.316e+00</td>
<td>6.316e+00</td>
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<tr>
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<td>1.704e+00</td>
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<tr>
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**SPECIFIC STEADY STATE GUST RESPONSE**

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Table 3.5. Tabulated results corresponding to Figs. 3.8 - 3.10. Run 27.
Fig. 3.8. Steer frequency response curves for the data of Run 27.
Fig. 3.9. Gust frequency response curves (amplitude), Run 27.
Fig. 3.10. Gust frequency response curves (phase angles). Run 27.
<table>
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<th>RUN 27</th>
<th>RUN 38</th>
<th>RUN 39</th>
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<td>KSF (front tyre stiffness)</td>
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<td>-9200</td>
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<td>DDFDP (front roll centre)</td>
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<td>-2.1</td>
<td>-2.1</td>
</tr>
<tr>
<td>DDFDP (front roll steer)</td>
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<td>0</td>
<td>0</td>
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<tr>
<td>DPDFDP (front roll camber)</td>
<td>0.72</td>
<td>0</td>
<td>0</td>
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<tr>
<td>KP (roll stiffness)</td>
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<td>-25550</td>
<td>-25550</td>
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<tr>
<td>KPD (roll damping)</td>
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<td>-1500</td>
<td>-1500</td>
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The object in Run 38 was to maintain a similar level of understeer with a much reduced roll-yaw coupling, and this was primarily achieved by the removal of the roll steer and camber effects combined with the reduction of the front tyre cornering stiffness. A final adjustment was made by effectively lowering the front roll centre height and increasing the roll damping. It can be seen from Fig. 3.11 that the yaw rate response curve now shows very little evidence of the roll "kink". In mathematical terms this is illustrated by the similar values of the frequencies and damping ratios of the roll angle mode and the yaw rate zeros. When expressed as a transfer function these terms will almost cancel out leaving essentially two degree of freedom behaviour. For Run 39 the roll stiffness was increased by a factor of 100, moving the roll resonance out of the frequency range of interest and effectively giving a no roll, two degree of freedom vehicle.
### Table 3.6. Tabulated results corresponding to Fig. 3.11. Run 38.

#### 50 MPH RUN 38

**SM** - 4.781  **D(STEER)/D(SLIP)** *2110*  **D(STEER)/D(LATAC)** *0.0416*

<table>
<thead>
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<td>1.155*03</td>
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<tr>
<td>4</td>
<td>2.722*03</td>
<td>2.722*03</td>
</tr>
<tr>
<td>5</td>
<td>2.879*01</td>
<td>7.549*03</td>
</tr>
<tr>
<td>6</td>
<td>1.504*01</td>
<td>1.748*01</td>
</tr>
<tr>
<td>7</td>
<td>1.781*02</td>
<td>1.992*02</td>
</tr>
<tr>
<td>8</td>
<td>5.748*02</td>
<td>6.509*02</td>
</tr>
<tr>
<td>9</td>
<td>-1.092*02</td>
<td>1.214*02</td>
</tr>
<tr>
<td>10</td>
<td>3.911*00</td>
<td>2.392*01</td>
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<tr>
<td>11</td>
<td>5.020*01</td>
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<tr>
<td>12</td>
<td>1.242*02</td>
<td>8.038*02</td>
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<tr>
<td>13</td>
<td>1.143*01</td>
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<tr>
<td>14</td>
<td>2.447*02</td>
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<td>15</td>
<td>1.132*03</td>
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#### POLES

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#### STEER ZEROS

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#### ROLL ANGLE

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#### LATAC

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<td>1.93</td>
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#### SPECIFIC STEADY STATE STEER RESPONSE

| YAW RATE | 6.080e+00 |
| ROLL ANGLE | 2.013e+00 |
| LATAC  | 4.438e+02 |

#### SPECIFIC STEADY STATE GUST RESPONSE

| YAW RATE | 1.885e-03 |
| ROLL ANGLE | 9.847e-05 |
| LATAC  | 1.318e-01 |
Fig. 3.11. Steer frequency response curves for the data of Run 38.
50 MPH RUN 39
SM -4.781  D(STEER)/D(SLIP) -2110  D(STEER)/D(LATAC) -0416

**RESPONSE COEFFICIENTS**

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**SPECIFIC STEADY STATE STEER RESPONSE**

| YAW RATE | -6.080*+00 |
| ROLL ANGLE | 2.013*+02 |
| LATAC    | 4.438*+02 |

**SPECIFIC STEADY STATE GUST RESPONSE**

| YAW RATE | 1.805*+03 |
| ROLL ANGLE | 9.847*+07 |
| LATAC    | 1.318*+01 |

Table 3.7. Tabulated results corresponding to Fig. 3.12. Run 39.
Fig. 3.12. Steer frequency response curves for the data of Run 39.
4. THE RESPONSE OF TWO VEHICLES TO A MECHANICAL-SINUSOIDAL STEER INPUT

4.1 Introduction.

Prior to the commencement of the work associated with this Thesis the Author was involved in a programme of research at MIRA, which had at that time reached a stage where:

(1) A machine had been built and commissioned which applied a sinusoidal steer input for a range of amplitudes and frequencies. This is described in Appendix 4.

(2) Measurements had been made on three cars, the results from one of which, car A, are presented in this Section. Other than the reduction of the results to the form of amplitude and phase as functions of frequency, no analysis had been carried out.

The actual test programme was carried out by MIRA staff under the control and supervision of the Author. After beginning this Thesis measurements were made on one further vehicle, car B, and these results are also presented here. Brief descriptions of cars A and B are given in Appendix 4.

4.2 Instrumentation.

Relevant quantities measured during this test series are given in Table 4.2.1.

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<tr>
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<td>Strain gauged tube</td>
</tr>
<tr>
<td>Speed</td>
<td>AC generator on 5th wheel</td>
</tr>
<tr>
<td>Lateral acceleration</td>
<td>Accelerometer</td>
</tr>
<tr>
<td>Yaw rate</td>
<td>Rate gyroscope</td>
</tr>
<tr>
<td>Roll angle</td>
<td>Free gyroscope</td>
</tr>
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</table>

Table 4.2.1 Quantities measured and transducers used.

Measurement of road wheel angles were also attempted using various arrangements of potentiometers, but these were not very successful and
the results were not analysed in detail. The signals from these transducers were recorded on a photographic galvanometer recorder fitted with low frequency, high sensitivity galvanometers. As the accelerometer used was a mass, spring and mirror type, equivalent in operation to a galvanometer, it was mounted inside the recorder, and so this unit was mounted at the centre of gravity of the test car.

The test records were in the form of analogue traces on photographic paper. These were digitised on a trace reading machine to give the X and Y co-ordinates of the peaks (and troughs) of the sine waves, and then analysed on a digital computer in a very straightforward way to give average values of the amplitude of each trace and its phase relationship to the handwheel trace, for each of the frequencies tested.

4.3 Test Procedure.

The tests were initially attempted at MIRA, but the width of test track available proved insufficient, particularly for the combination of low frequency and high handwheel angles. To solve this problem permission was obtained to use a 150ft wide airfield runway at a neighbouring aerodrome, where there was found to be just adequate room. Throughout testing the occupants of the test vehicles wore crash helmets and safety harnesses, and all moveable equipment in the vehicles was secured.

Each set of tests was done at constant speed and with a constant amplitude of handwheel angle input. At the beginning of a set of tests the handwheel amplitude was set using a protractor attached to the rim of the wheel, and for each test the frequency was adjusted on the stepless gearbox of the machine. Before setting off along the runway a pin was inserted to lock the steering to the machine. Then, when the vehicle was travelling at the required speed the driver engaged the clutch on the machine, the observer started the recorder, and the driver switched on the machine's electric motor. At the end of the test the driver disengaged the clutch on the machine and the recorder was stopped. The aim was to record at least six cycles for each test but this was not always possible at the lowest frequencies because of the space limitation. To obtain recordings of the zero frequency responses for each handwheel input and vehicle speed, the machine was not switched on but the driver
turned the handwheel, first one way then the other, to the limits set by the machine with the pin inserted.

4.4 Analysis of Results - Manual Curve Fitting.

Since the theoretical approach to vehicle frequency response which is used here assumes linear vehicle behaviour, it is relevant to examine the behaviour of the vehicles measured in this work for linearity. This can be done for any vehicle response (for example yaw rate) by plotting amplitude ratio against handwheel angle. For linear behaviour this graph will be a horizontal straight line, and any deviation of the slope of this graph from zero will be a measure of non-linearity. Graphs of this form are presented for the yaw rate response of each vehicle for a range of frequencies.

The principal method of analysis of the frequency response results involves the assumption that the response curves can be represented by three degree of freedom transfer functions of the form derived in Section 2. The results of Section 3 show the type of curve predicted by these transfer functions, and a preliminary inspection of the measured frequency responses shown here shows that they are of similar form. In the notation of Section 2 the equations for the yaw rate, roll angle, and lateral transfer functions are:

\[
\frac{r(s)}{\gamma(s)} = \frac{C_t [ (1/\omega_\gamma) s + 1 ] (1/\omega_\nu_c) s^2 + (2/\nu_c/\nu_q) s + 1}{[(1/\omega_\gamma) s^2 + (2/\nu_c/\nu_q) s + 1] [ (1/\omega_\nu_c) s^2 + (2/\nu_c/\nu_q) s + 1]}
\]

(2.27)

\[
\frac{\phi(s)}{\delta(s)} = \frac{C_q [ (1/\omega_\phi) s^2 + (2/\nu_c/\nu_q) s + 1]}{as for r(s)}
\]

(2.28)

\[
\frac{\alpha(s)}{\delta(s)} = \frac{C_\alpha [ (1/\omega_\alpha) s^2 + (2/\nu_c/\nu_q) s + 1] [ (1/\omega_\nu_c) s^2 + (2/\nu_c/\nu_q) s + 1]}{as for r(s)}
\]

(2.29)

The \( \omega \)'s and \( \nu \)'s are the characteristic frequencies and damping ratios of the system and in as far as is possible their physical significance is discussed in Section 2. It is shown in Appendix 2 that the transfer functions of the simpler, two degree of freedom, non-rolling model can be obtained from the above equations by deleting the roll equation (2.28), and deleting the last factor of both numerator and denominator of the yaw rate and lateral expressions.
If the \( \omega \)'s, and \( j \)'s are known the frequency response of a system can be synthesised by adding together the effects of the individual factors in the numerator and denominator of the transfer function on log-log graph paper. The separate effects of these factors are well known and can be obtained from standard curves as illustrated in Figs. 4.4.1-2. The \( \omega \)'s are the corner frequencies and the \( j \)'s the damping ratios. Factors in the numerator of a transfer function are called lead terms as they produce a phase lead, and those in the denominator are called lag terms. Figs. 4.4.1 and 2 are drawn for lag terms. For lead terms the curves are similar but inverted.

In the present case the \( \omega \)'s and \( j \)'s were not known and the object was to derive them from the experimental frequency response curves. In order to do this the results were presented in the form of Bode plots. These are plots of log amplitude ratio against log frequency, where in this case amplitude ratios were peak to peak values of the quantities concerned, over peak to peak handwheel angle. The problem then was to find the set of curves corresponding to the factors of the appropriate transfer function, which when added together gave a curve which fitted the experimental data. This was done by first of all drawing asymptotes on the experimental curve to give an estimate of the \( \omega \)'s and \( j \)'s, and then gradually improving this estimate by a process of trial and error, involving actually drawing sets of curves and adding them together, until a satisfactory fit was obtained. As an example of the final result Fig. 4.4.3 shows a fitted curve and the three standard curves from which it was constructed, superimposed on a set of experimentally measured points. Curves can be fitted to the experimentally measured phase points in a similar way, to confirm that the fit obtained is good.

The quantities of principal interest are the vehicle responses, yaw rate, roll angle, and lateral acceleration (latac), and so the bulk of the results are given in these terms, although measurements were also made of steering torque and an example of this is also given. Where the experimental data were of the form predicted by linear theory and reasonable curve fits could be obtained, the fitted curves are shown superimposed on the experimental points. The combinations of standard curves from which these fitted curves were constructed are not shown in the Figures, but their various corner frequencies and dampings are presented and discussed in the following section.
Fig. 4.4.1. Standard frequency response curves for linear and quadratic factors.
Fig. 4.4.2. Phase angle curves corresponding to the curves of Fig 4.4.1.
Fig. 4.4.3. Details of a typical manual curve fit, showing a curve fitted to experimental points, and the three standard curves from which it was constructed.
Although measurements of road wheel angle were made on each vehicle, some of the measurements were not entirely satisfactory. For this reason, and also because handwheel angle is the quantity important to the driver, the results presented here are all in terms of handwheel angles. The theoretical approach, however, does not consider the dynamics of the steering system, and any effects of this will tend to cause a lack of agreement between theory and measured results. In particular at frequencies above about 2 Hz road wheel angles seemed, in general, to have a phase lag of about 20 degrees behind the handwheel angle, and this probably accounts for some of the rather high phase angles obtained at these frequencies.

4.5 Results and their Discussion.

For car A frequency response measurements were made for four handwheel angle inputs at 50 mile/h, and at three speeds at one handwheel input. Fig. 4.5.1 shows that the vehicle behaves reasonably linearly with handwheel input, and the remaining results are presented for one input only. Figs. 4.5.2 to 4.5.4 show Bode plots of amplitude ratio and phase angle for the yaw rate, roll angle, and lateral responses. Points plotted on these graphs at 0.1 Hz represent the zero frequency responses. There is generally good agreement between the shapes of these curves and those which are predicted by the transfer functions derived in Section 2.

A transfer function of the form given in equation 2.27 is fitted to the yaw rate curves of Fig. 4.5.2 giving the following frequencies and dampings:

\[ \omega_s = 0.66 \text{ Hz} \]
\[ \omega_f = 0.9 \text{ Hz}; \quad j_f = 0.7 \]
\[ \omega_t = 1.7 \text{ Hz}; \quad j_t = 0.3 \]
\[ \omega_r = 1.8 \text{ Hz}; \quad j_r = 0.3 \]

It is seen that the \( \omega_f \) and \( \omega_r \) factors almost cancel so that a two degree of freedom approximation would give quite a reasonable fit. The yaw resonance is well damped \( (j_f = 0.7) \) and the rise in the amplitude ratio curve, peaking at about 0.9 Hz, is entirely due to the lead term, as is the small amount of phase advance at around 0.2 Hz. A transfer function
Fig. 4.5.1. Linearity of yaw rate response with handwheel angle, Car A.
Fig. 4.5.2. Experimental yaw response points and fitted curves, Car A.
Fig. 4.5.3. Experimental roll response points and fitted curves, Car A.
Fig. 4.5.4. Experimental lateral response points, Car A.
of this form should result in a phase lag of 90 degrees at high frequencies but higher lags are shown in Fig. 4.5.2. In fact, above about 1.5 Hz, the phase curve deviates from the expected shape. The reason for this is probably largely due to the phase lag between handwheel angle and road wheel angle discussed in section 4.4, although it is possible that some higher order dynamics are also having an effect, arising for example from the tyres. This effect was also noticed in Ref. 24, and occurs in all the vehicles measured during this work.

A good fit can also be made for the roll angle transfer function of equation 2.27 to the curves of Fig. 4.5.3. The lead term frequency and damping are:

\[
\omega_l = 1.4 \text{ Hz} \quad ; \quad \zeta_l = 0.15
\]

and as expected the denominator terms are the same as for the yaw rate. This lightly damped lead term is responsible for the steep fall in the amplitude ratio curve above 1 Hz, and the small peak which follows is due to the roll resonance. If serious roll coupling were present then a "kink" of the sort illustrated in Fig. 4.5.3 could also appear on the yaw rate curves, as demonstrated in Section 3.

The shape of the acceleration curves of Fig. 4.5.4 can also be described by a transfer function of the form of equation 2.28, but an attempt to obtain an exact fit was not made because this measurement is affected by roll angle. However the very sharp dip at about 1.8 Hz indicates a lightly damped lead term at this frequency, and this tends to mask the effects of the higher frequency terms.

The effect of speed is illustrated by the yaw rate amplitude ratio curves of Fig. 4.5.5. Fitting two degree of freedom curves to these gives:

<table>
<thead>
<tr>
<th>Speed</th>
<th>( \omega_c )</th>
<th>( \zeta_c )</th>
<th>( \omega_r )</th>
<th>( \zeta_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 MILE/H</td>
<td>0.45</td>
<td>0.6</td>
<td>0.82</td>
<td>0.6</td>
</tr>
<tr>
<td>30 MILE/H</td>
<td>1.12</td>
<td>0.7</td>
<td>1.3</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Equation 19 of Appendix 2 indicates that \( \omega_c \) should be inversely prop-
Fig. 4.5.5. Effects of speed on yaw response. Experimental points and fitted curves, Car A.
ortional to speed, and the above results are seen to be in good agreement with this. The variation of $\omega_c$ and $J_r$ is also generally as indicated by equations 22 and 23. It is interesting to note that the higher peak of the 70 mile/h curve is almost entirely due to the reduction in $\omega_c$ rather than to a reduction in $J_r$, as might be expected.

Measurements were made on car B in three configurations.

(a) Standard, as described in Appendix 4.
(b) The roll stiffness distribution was altered by removing the front anti-roll bar and bump rubbers, and rear stabiliser bar, and fitting stiffer rear suspension torsion bars. This reduced the ratio of front to rear roll stiffness by 40%, and the total roll stiffness by 12%.
(c) As for (b) plus a camber backlash of the front wheels allowing about 0.75 degrees of positive camber (top out) on either front wheel when subjected to cornering force.

The roll stiffnesses for configurations (a) and (b) are given below.

<table>
<thead>
<tr>
<th>ROLL STIFFNESS</th>
<th>FRONT</th>
<th>REAR</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>2535</td>
<td>3000</td>
<td>5535  lb.in/deg</td>
</tr>
<tr>
<td>(b)</td>
<td>1628</td>
<td>3228</td>
<td>4856  lb.in/deg</td>
</tr>
</tbody>
</table>

Frequency response measurements were made at three handwheel angle inputs at 50 mile/h. Fig. 4.5.6 shows that the standard vehicle behaved moderately linearly, but that (b) was rather less linear and (c) significantly non-linear. The results of this are evident in the Bode plots where it was not found possible to fit curves of the form predicted by linear theory to the results from configurations (b) and (c).

Bode plots for the yaw rate, roll angle, and lateral responses are given in Figs. 4.5.7 to 4.5.9 for configuration (a) at one handwheel input. These are seen to be of similar basic shape to the corresponding curves for car A, and it was again found possible to fit curves of the form predicted theoretically. The frequencies and damping ratios obtained from these fits are:
Fig. 4.5.6. Linearity of yaw rate response with handwheel angle, Car B.
Fig. 4.5.7. Experimental yaw response points and fitted curve, Car B.
Fig. 4.5.8. Experimental roll response points and fitted curve, Car B.
Fig. 4.5.9. Experimental latac response points, Car B.
The frequencies \( \omega_q \) and \( \omega_r \) are not so close for this vehicle as for car A, with the result that a two degree of freedom fit would be less satisfactory. The general description of these curves, however, follows the same lines as that given for car A and need not be repeated here.

Fig. 4.5.10 shows a steering torque curve for this car. The shape of the curve is typical of measurements from other vehicles, with a pronounced minimum at around the yaw resonance frequency. No attempt was made in this work to analyse the dynamics of the steering system and so no further comment is made here.

Frequency response curves for the vehicle in configurations (b) and (c) at one handwheel angle are given in Figs. 4.5.11 to 4.5.13. As the phase angles associated with these curves are typically very similar to those given for configuration (a), they are not reproduced here. The amplitude ratio curves are, however, significantly different and, as mentioned above it was not found possible to fit linear curves. Configuration (c) shows the largest difference, with a much lower yaw response at low frequency, but very pronounced peak giving a similar response at around 1 Hz. The vehicle in this configuration had subjectively unpleasant handling, but it is not possible to tell whether this was due to the non-linear response to handwheel, to the peaky yaw response curve, or to some other unrecorded factor. It is probable, however, that both the former are undesirable. Work in the aircraft industry has, in fact, indicated that pilots show a preference for, and perform better when operating, linear systems (Ref. 66).

Configuration (b) was also subjectively worse than (a), but the difference was much smaller than between (c) and (a). The yaw response curve also shows a tendency to peak, but this is much less marked than for (c) because of the higher low frequency response. The effect of this peak is reflected in the roll response curves for both cases.
Fig. 4.5.10. Measured steering torque input, Car B.

Fig. 4.5.11. Experimental yaw response curves, Car B, configurations (b) and (c).
Fig. 4.5.12 Experimental roll response curves, Car B, configurations (b) and (c).
CAR 'B' H.W. 54°

LATERAL ACCELERATION, $\xi$

H.W. ANGLE, DEG.

CONFIGURATION (b)

CONFIGURATION (c)

FREQUENCY, Hz

Fig. 4.5.13  Experimental lateral response curves, Car B, configurations (b) and (c).
The general shape of the curves at higher frequencies is similar to that of the (a) curves. As with the latac results for (a) there is considerable scatter at high frequency. This is due to the low levels of acceleration being measured at these frequencies.

4.6 Conclusions.

This work confirms that the frequency responses of vehicles which have a reasonably linear response to handwheel input conform to the pattern indicated by a three degree of freedom linear model, except for rather large phase lags at the high frequency end of the response curves. It is probable that this phase lag arises from the dynamics of the steering system and the tyres but this was not fully investigated here.
5. DIGITAL FITTING OF THREE DEGREE OF FREEDOM FREQUENCY RESPONSE CURVES.

5.1 Introduction.

The fundamental assumption involved in the analysis of frequency response curves by fitting a predetermined form of curve is that the vehicle can in fact be described by a system of the form chosen, in this case the three degree of freedom system. This may seem to be an unwarranted restriction, but was adopted for this work because of the opportunity it provides for the description of the vehicle response in terms of quantities which can be theoretically and practically understood. The first attempt to apply the idea has been described in Section 4, and the success achieved despite the cumbersome manual curve fitting involved, provides the incentive to pursue more sophisticated analysis techniques.

The manual curve fitting technique, involving as it did the trial and error approach of adding three variable curves repeatedly until the best fit was obtained, was extremely cumbersome and time consuming, and probably not very repeatable between different people. A better method was therefore required and this Section describes the development of this method.

5.2 Selection of Curve Fitting Technique.

It is evident that a digital computer technique is most likely to provide the solution. The principle of fitting a polynomial function to a set of data points, by minimising the sum of the squared differences of the points from the corresponding values of the function, is well established, and in the case of a polynomial can be performed explicitly. This is usually called the "least squares" technique. The author had successfully used this approach to fit polynomial curves to vehicle steady state handling data, and so decided to try to apply it in this case.

The functions involved are the amplitude and phase angles of the yaw rate, roll angle, and lateral frequency responses, and as shown in Section 2 equations 2.23 - 2.25, are in the form of ratios of polynomials.
These complex, non-linear functions do not lend themselves to the explicit type of solution available for simple polynomials, so some kind of iterative technique is indicated. One approach would be to simulate the manual technique of forming sets of constituent curves and adding these together for successive variations of the constituent curves until a best least squares fit was obtained, but a satisfactory method of doing this is difficult to envisage. Rather than writing an iterative least squares procedure from scratch, a survey of published algorithms was made, and initially two seemed applicable to this problem.

The first, entitled "A general least squares program for fitting functions to data" by P.N.Murgatroyd, was obtained from the Institution of Electrical Engineers Computer Library, programme number CP 47. In operation the programme takes an estimated set of coefficients which define the function, and adjusts these singly, in rotation to minimise the sum of squares of differences between data values and corresponding points of the function. This seemed to be an appropriate technique and a special purpose version to fit the functions involved in this work was developed.

The second, entitled "STEEP1" by E.J.Wasscher, was found in the Communications of the ACM, Vol. 6, No. 9, September 1963, Algorithms 203 - 205. It consists of a routine for finding the minimum of a differentiable function of n variables, using the method of steepest descent, and is made applicable to the current case by using the appropriate sum of squares of differences as this function. Very broadly the technique is to form the partial derivatives of the function with respect to each of the n variables, and then to adjust all n at once by an amount dependant on the appropriate partial derivatives. During the development of the first technique it was found that rather a large number of iterations were sometimes required to obtain a satisfactory fit, and STEEP1 was tried in an attempt to speed this up. In all of the cases investigated STEEP1 was never an improvement and often produced a poorer fit. As it was also more difficult to use, involving a greater number of control parameters, investigation of this technique was abandoned in favour of the first.
At a late stage in the work, when an adequate although very time consuming curve fitting technique had been developed based on the Murgatroyd algorithm, a set of routines - OPTIMA - produced by the Numerical Optimisation Centre of the Hatfield Polytechnique, became available, Ref. 62. One of these, "OPLS - A Safeguarded Gauss-Newton Technique for Minimising Sums of Squared Terms", Ref. 63, was appropriate to the current problem and much faster in operation than the Murgatroyd technique. Preliminary trials gave excellent results and this procedure was incorporated into the final version of the curve fitting programme.

Considerable development was carried out using the Murgatroyd technique and many of the ideas continued to apply in the final system, thus although the development described below is in terms of OPLS most of the work was actually done on the Murgatroyd system and then re-checked when OPLS was incorporated.

5.3 Fitting of Individual Curves.

The first stage of the development of the fitting technique was to write a computer programme, incorporating a suitably modified version of OPLS, to fit a curve of the form given in equation 2.23, 2.24, or 2.25 to a set of either yaw rate, roll angle, or latac amplitude versus frequency data. The basic inputs to this programme were:

(1) A set of data points in the form of amplitude ratio versus frequency for either yaw rate, roll angle, or latac.

(2) A guessed set of coefficients (COEFFS[1]). These are the coefficients defined in equations 2.23 - 2.26, which describe the curve to be fitted, and it can be seen that 8 are required for yaw rate, 7 for roll angle, and 9 for latac.

(3) The control parameters required for OPLS. These and their functions are described in Section 5.5 and need not be considered here.

In essence this programme used OPLS to minimise the sum of the squared differences between each data point and the value of the curve being fitted (defined by the current set of COEFFS).
After either a specified number of iterations, or reaching a satisfactory fit, the programme gave as output a new set of COEFFS describing the fitted curve. These COEFFS were then used to calculate the characteristic frequencies and damping ratios of the system, and to plot the fitted curve alongside the original data so that the fit could be inspected. The techniques used for this calculation and plotting were similar to those described in Section 3.

As a first stage in testing the programme the theoretically derived data described as Run 14 in Section 3 was used. This had the obvious advantage that the correct values of the COEFFS were known in advance. An excellent fit to the yaw rate data was readily obtained giving accurate estimates of the COEFFS. Fig. 5.3.1 shows a typical result. (The notation on this computer drawn graph is described in Appendix 6). This result was not very sensitive to the choice of the initial values of the COEFFS provided that they were not ridiculous, but the time taken to achieve the result was longer for a poorer set. The result of Fig. 5.3.1 was obtained in 58 seconds (on an ICL 4130) from the initial COEFFS shown in Table 5.3.1. The error quoted is the final value of the sum of the squared error terms.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>INITIAL</th>
<th>FINAL</th>
<th>CORRECT</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>10</td>
<td>11.38</td>
<td>11.4</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>119.8</td>
<td>120.1</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
<td>464.6</td>
<td>466.0</td>
</tr>
<tr>
<td>4</td>
<td>1000</td>
<td>1175</td>
<td>1179</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>25.04</td>
<td>25.05</td>
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<td>6</td>
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<td>6.351</td>
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</tr>
<tr>
<td>8</td>
<td>300</td>
<td>187.3</td>
<td>187.8</td>
</tr>
</tbody>
</table>

Table 5.3.1 COEFFS for curve fitted to yaw rate data of Run 14.
Fig. 5.3.1. Curve fitted to yaw rate data of Run 14.
The above result was most encouraging, but the data of Run 14 represents a large American car and has considerable coupling between the roll and yaw motion, as illustrated by the pronounced "kink" in the yaw rate curve. It was therefore considered that data for a more typical European car should be examined and the results of Run 27 of Section 3 were selected for this purpose. The yaw rate amplitude curve for this data shows some roll coupling but considerably less than that of Run 14.

Two sets of results for this data are shown, for two initial values of COEFF 8 (C8). The initial values of the other COEFFS were the same as in the previous example (except for C5 which controls only the overall level of the curve and, as discussed later, is relatively easy to fix). The curve fits obtained are shown in Figs. 5.3.2 and 5.3.3 and the COEFFS in Table 5.3.2.

<table>
<thead>
<tr>
<th>COEFF NO</th>
<th>INITIAL (C8=300)</th>
<th>FINAL (C8=100)</th>
<th>CORRECT</th>
</tr>
</thead>
<tbody>
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<td>16.78</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>235.2</td>
<td>229.9</td>
</tr>
<tr>
<td>3</td>
<td>500</td>
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<td>1073</td>
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<td>166.3</td>
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<td>8</td>
<td>300,100</td>
<td>571.3</td>
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</tr>
<tr>
<td>ERROR</td>
<td>-</td>
<td>4.75E-7</td>
<td>3.44E-5</td>
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</tbody>
</table>

Table 5.3.2 COEFFS for curves fitted to yaw rate data of Run 27.

It can be seen that although both curve fits are visually perfect, the COEFFS in the C8 = 100 case are significantly poorer. This type of dependance on the initial COEFFS did not occur for the data of Run 14 and is almost certainly a symptom of the greater difficulty in fitting the three degree of freedom curve to data which only contains a small amount of roll information. It is interesting to note that the largest errors in the C8 = 100 case are in C4 and C3 which are both too small, and a study of the individual significance of each COEFF is helpful in showing how this can happen while the actual curve fit is still good.
Fig. 5.3.2. Curve fitted to yaw rate data of Run 27, \( \sigma_3 = 300 \).
Fig. 5.3.3. Curve fitted to yaw rate data of Run 27. C8 = 100.
The yaw rate transfer function in Laplace operator form was derived in Section 2 and is:

\[
\frac{r(s)}{\dot{\gamma}(s)} = \frac{C_r(s^3 + C_2 s^2 + C_3 s + C_4)}{(s^4 + C_1 s^3 + C_2 s^2 + C_3 s + C_4)} \quad (5.3.1)
\]

\(C_r\) is seen to control the overall gain or the general level of the curve, and the zero frequency amplitude is given by \((C_r C_2) / C_4\). In the \(C_r = 100\) example given in Table 5.3.2, the zero frequency amplitude is correct, but this is achieved with the wrong values for \(C_2\) and \(C_4\). The initial value of \(C_r\) chosen for this example was almost correct. This was done deliberately since preliminary runs had invariably moved this coefficient to near the correct value. (Further experience with the curve fitting shows that there is generally no difficulty in fixing the overall gain coefficient and that a reasonable estimate of it can usually be made from the zero frequency amplitude level).

Broadly speaking, the individual effects of the \(C\)OFFS\) are governed by the power of \(s\) to which they apply. The higher the power of \(s\), the higher the frequency at which that coefficient will have its major effect. A series of computations were carried out and frequency responses plotted, to illustrate the effect of changes in each of the \(C\)OFFS\) in turn on the shape of the frequency response curves. From these it is apparent that, as might be expected, the effects of corresponding numerator and denominator terms are roughly equal and opposite. This implies that it might be possible to obtain reasonable curve fits (subsidary minima) with pairs of numerator and denominator \(C\)OFFS\) in error by similar amounts in the same direction, and, as was illustrated above and will be seen again later, this is the type of error which occurs when difficulty in curve fitting is experienced.

As a further check of the influence of roll/yaw coupling on the effectiveness of the curve fitting procedure, some runs were carried out on the yaw rate data of Run 38 of Section 3. This is basically Run 27 modified by removing roll steer effects and adjusting roll centre heights to reduce the roll coupling even further. The results of a run using similar initial \(C\)OFFS\) to the previous examples are shown in Fig. 5.3.4 and Table 5.3.3.
Fig. 5.3.4. Curve fitted to yaw rate data of Run 38.
In this case the fit still looks excellent, although it is not in fact so good, but the COEFFS are much poorer, most of them being too low, with C4 and C8 being the worst. It is evident that the further reduction in roll coupling has made the curve fitting correspondingly more difficult.

The logical development of the above cases is to examine yaw rate data for a vehicle with no roll coupling at all. A set of such data, Run 39, was obtained by multiplying the roll stiffness of Run 38 by 100 and thus moving the roll resonance and any roll effects well out of the frequency range of interest. A curve fit from the same initial COEFFS as for the data of Run 38 is shown in Fig. 5.3.5 and the COEFFS in Table 5.3.4. For this case it is interesting to look at the "s and j's corresponding to the COEFFS and these are shown in Table 5.3.5.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>INITIAL</th>
<th>FINAL</th>
<th>CORRECT</th>
</tr>
</thead>
<tbody>
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<td>19.59</td>
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<td>238.9</td>
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</tr>
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<td>ERROR</td>
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Table 5.3.3 COEFFS for curve fitted to yaw rate data of Run 38.

<table>
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<td>30</td>
<td>28.73</td>
<td>28.79</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>8.295</td>
<td>15.04</td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td>99.87</td>
<td>14470</td>
</tr>
<tr>
<td>8</td>
<td>300</td>
<td>329.6</td>
<td>57480</td>
</tr>
<tr>
<td>ERROR</td>
<td>-</td>
<td>1.88x10^-7</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 5.3.4 COEFFS for curve fitted to yaw rate data of Run 39.
Fig. 5.3.5. Curve fitted to yaw rate data of Run 39.
The curve fit obtained is once again excellent but the COEFFS (other than C5) are seen to be hopelessly wrong. However, the yaw rate frequencies and damping ratio corresponding to these COEFFS are exactly correct, while the roll angle poles and zeros are wrong but identical so that they cancel out and have no effect.

The yaw rate data of Run 39 is, of course, equivalent to a two degree of freedom system, and in such a case it might seem more sensible to try to fit a simpler two degree of freedom curve in the first instance. This can be done using the current transfer function (equation 5.3.1) by setting C3, C4, C7, and C8 equal to zero, and a version of the programme was adjusted so that iterations could be carried out using only the non-zero COEFFS. For the data of Run 39 this technique gave correct values for $\omega_r$, $\beta_r$ and $\omega_c$ in a similar way to the 3 degree of freedom fit. As an experiment this 2 degree of freedom fit was also tried on the yaw rate data of Run 38 and 27 to see if reasonable estimates of $\omega_r$, $\beta_r$, and $\omega_c$ could be obtained despite the roll coupling. For Run 38, with a small amount of roll coupling, a moderately good fit was obtained and the $\omega_r$, $\beta_r$, and $\omega_c$ values were slightly better than for the 3 degree of freedom fit. A fit to the Run 27 data is shown in Fig. 5.3.6. The $\omega_r$, $\beta_r$, and $\omega_c$ values are seen to be not very good, although the curve obtained could probably be used to represent the vehicle's yawing motion reasonably well.

The position with regard to fitting a three degree of freedom transfer function to yaw rate frequency response data can now be summarised for three identifiable categories of data.

(1) Systems with no roll-yaw coupling (i.e. basically two degree of freedom behaviour).

<table>
<thead>
<tr>
<th>FREQ</th>
<th>DAMP</th>
<th>FINAL</th>
<th>CORRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>RATIO</td>
<td>Fig. 5.3.5</td>
<td></td>
</tr>
<tr>
<td>POLES</td>
<td>$\omega_r$</td>
<td>$\beta_r$</td>
<td>0.69 , -0.420</td>
</tr>
<tr>
<td></td>
<td>$\omega_c$</td>
<td>$\beta_c$</td>
<td>1.45 , -0.237</td>
</tr>
<tr>
<td>ZEROS</td>
<td>$\omega_z$</td>
<td>0.63</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>$\omega_{fr}$</td>
<td>$\beta_{fr}$</td>
<td>1.45 , -0.237</td>
</tr>
</tbody>
</table>

Table 5.3.5 $\omega$'s and $\beta$'s for curve fitted to yaw rate data of Run 39.
STEER RESPONSE

17 9 74 P4 DIRECT 50 MPH R27

YAW RATE
SP SS RESP 6.762 +39
POLES
0.63 -0.613 0.00 H
0.00 H 0.00 H
0.00 H 0.53 -1.000
0.63 -0.613

Fig. 5.3.6. Two degree of freedom curve fitted to yaw rate data of Run 27.
Good estimates of $\omega_r$, $f_r$ and $\nu_k$ can be made.
No information on the roll motion can be obtained.
The response coefficients cannot be established.

(2) Systems with moderate roll-yaw coupling.

Moderate estimates of the $\omega$'s and $f$'s and the response coefficients can be made depending on the amount of roll coupling. Care is required with initial values of coefficients to avoid subsidiary minima.

(3) Systems with considerable roll-yaw coupling.

All the $\omega$'s and $f$'s and the response coefficients can be found.

There are no hard boundaries between these categories, of course, the divisions simply serving to illustrate the different problems arising as roll-yaw coupling increases.

So far this section has been concerned principally with yaw rate data, but the programme was also capable of fitting curves to roll angle and latac data. Various runs were made using data from the same cases as the above. For roll angle data the behaviour was very similar to that for the heavily roll coupled yaw rate data, although there was a greater tendency towards subsidiary minima. This means that for situations where there is no, or very little, roll-yaw coupling, there is a better chance of establishing the poles of the system from the roll angle data than from the yaw rate. Curve fitting to latac data was also reasonably successful, although the subsidiary minimum problem was even greater because of the rather dominating effect of the lightly damped lead term.

5.4 Multiple Curve Fitting.

The previous Section has examined in some detail the fitting of a curve of the form of the three degree of freedom vehicle transfer function to sets of yaw rate frequency response data, and it was seen that various difficulties could arise. One of the problems arose when there was a lack of roll mode information in the yaw rate data. This suggests that some
form of combined roll angle and yaw rate curve fitting might be beneficial. Also, COEFFS 1 to 4 are common to all three transfer functions and it seems wasteful to re-calculate them in each case. A new version of the programme was therefore prepared which allowed the combined fitting of any number from 1 to 6 of the yaw rate, roll angle, and latac amplitude and phase curves. The principle of operation of this programme (the developed version of which was finally adopted for this work and which is described in detail in Section 5.5) is similar to that for the individual curve fitting. The main differences are:

1. Curves can be simultaneously fitted to up to 6 sets of data points representing the yaw rate, roll angle, and latac amplitude and phase versus frequency data,

2. The quantity minimised is the total sum of the squared differences between each set of data points and the appropriate transfer function.

3. The number of COEFFS required is basically 4, plus 4 for yaw rate, 3 for roll angle, and 5 for latac.

As the first stage in the development of this programme the combined fitting of yaw rate and roll angle amplitude was investigated. The data of Runs 14, 27, and 38 was again used and in all cases excellent fits were obtained. No difficulty was experienced with subsidiary minima and the choice of initial COEFFS had, within reason, no effect on the result, altering only the time required. Fig. 5.4.1 shows a fit to the data of Run 27 which was achieved in 2min 50s.

For this scatter free, theoretical data the combined fitting of yaw rate and roll angle amplitude curves is thus able to give excellent estimates of the relevant COEFFS and characteristic frequencies and damping ratios.

To complete the description of the three degree of freedom system the COEFFS defining the latac zeros are required and can only be obtained from latac data. The study of the individual curve fitting to latac data showed that a subsidiary minimum was frequently reached where one of the latac zeros had a positive real part although its magnitude was correct.
Fig. 5.4.1. Curves fitted to yaw rate and roll angle data of Run 27.
This same problem occurred with the combined fitting of the three sets of amplitude data and an example is shown in Fig. 5.4.2 and Table 5.4.1. (As the 11 COEFFS defining the poles of the system and the yaw rate and roll angle zeros were correct, only the latac zero COEFFS are shown in Table 5.4.1).

<table>
<thead>
<tr>
<th>COEFF</th>
<th>INITIAL</th>
<th>FINAL</th>
<th>CORRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>100</td>
<td>119.8</td>
<td>119.8</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>1.888</td>
<td>5.688</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>100.6</td>
<td>115.1</td>
</tr>
<tr>
<td>15</td>
<td>500</td>
<td>138.5</td>
<td>319.0</td>
</tr>
<tr>
<td>16</td>
<td>3000</td>
<td>2865</td>
<td>2868</td>
</tr>
</tbody>
</table>

Table 5.4.1 Latac zero COEFFS for curves fitted to amplitude data of Run 14.

It can be seen that the cause of the problem is that COEFFS 13 and 15 are much too small, but it is evident from the curve fit obtained that this solution is a valid one in the terms that the curve fitting problem is posed. However, it seems likely that this positive real part will be incompatible with the latac phase information, and so the study of the effect of including the phase curves in the fitting procedure was the logical next step.

The combined fitting of yaw rate or roll angle amplitude and phase made no difference to the conclusions already drawn, and as excellent results were obtained without the phase information its inclusion does not seem a justifiable complication. For latac data the inclusion of phase made the occurrence of the positive real part less likely, although it could sometimes still appear despite the terrible fit to the phase data it produced. The choice of initial values for the COEFFS was the governing factor in this.

The use of phase information was thus only of small help, and as the chance of obtaining a good result was, for latac data, still influenced by the choice of initial COEFFS, it seemed worthwhile investigating whether the adjustment of these could be used to good effect for the amplitude only case. For the combined fitting of all three sets of
### STEER RESPONSE

#### 21 8 74 P1 DIRECT FD/H/B12 50 MPH

<table>
<thead>
<tr>
<th>SP SS RESP</th>
<th>YAW RATE</th>
<th>PULL ANGLE</th>
<th>LATAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLES</td>
<td>ZEROS</td>
<td>ZEROS</td>
<td></td>
</tr>
<tr>
<td>1.35 -0.376</td>
<td>3.991 +00</td>
<td>1.399 +00</td>
<td>2.917 402</td>
</tr>
<tr>
<td>0.65 -0.621</td>
<td>0.51 -1.000</td>
<td>0.81 -0.306</td>
<td>1.24 0.122</td>
</tr>
<tr>
<td>0.65 -0.621</td>
<td>1.22 -0.207</td>
<td>0.81 -0.306</td>
<td>1.10 -0.275</td>
</tr>
<tr>
<td>1.35 -0.376</td>
<td>1.22 -0.207</td>
<td>1.24 0.122</td>
<td></td>
</tr>
</tbody>
</table>

#### Fig. 5.4.2

Curves fitted to amplitude data of Run 14.
amplitude data it was found possible, by using a two or three stage approach, to obtain good estimates of all the COEFFS. The first stage frequently gave the positive real part problem, but by increasing the values obtained for COEFFS 13 and 15 and then carrying out a further optimisation good results could be achieved.

Since the latac data was not necessary or particularly helpful in establishing COEFFS 1 to 4, and the use of all 16 COEFFS together made the curve fitting much more time consuming and rather less consistent, the above procedure, although largely satisfactory, did not seem particularly efficient. An alternative method was to establish COEFFS 1 to 4 from a yaw rate and roll angle fit and then to do a separate fit to the latac data iterating only over the remaining 5 COEFFS. A further fit with re-adjusted COEFFS 13 and 15 was still sometimes necessary, but this technique was nevertheless rather quicker and easier than the full three curve fit.

It might be argued that, aside from considerations of computer time and space, the most straightforward approach to the curve fitting would be to always use all 16 COEFFS and the 6 sets of amplitude and phase data. This is probably true for scatter free theoretical data where an exact curve fit is possible as the system is known to be 3 degrees of freedom, but the situation is rather different for the experimentally derived data for which the procedure is really intended. As seen in the results of Section 4, the phase angle data for real vehicles tends to deviate from the 3 degree of freedom shape above about 2 Hz and so has to be treated with some care. The measurement of latac is complicated in practice by the effect of the vehicle roll angle (if the accelerometer is mounted rigidly in the vehicle), and by the large dynamic range caused by the lightly damped lead term. Instrumentation problems must sometimes occur leaving some runs where only incomplete data is available. For these reasons it was considered worthwhile to produce a programme which could cope with some or all of the 6 sets of data, and to investigate in detail how its use was affected by the amount of data available.

This Section has demonstrated that the curve fitting procedure developed is capable of providing good estimates of the response coefficients and characteristic frequencies and dampings for theoretically
derived data, but the study of experimental data (that of Section 4) was carried out in parallel with the above, and similar conclusions were found to apply. The results of curve fits to the data of Section 4 and various other data produced by sine-input tests are shown and discussed in Section 6. A description of the final version of the fitting procedure is given in the following section and a summary of the input and output details is given in Appendix 6 along with a listing.

5.5 Computer Programme Description.

During the remainder of the work of this Thesis the curve fitting programme was incorporated into a larger programme - FITTRANS. As this programme is described in the following section and the actual fitting procedure - OPLS - is essentially a proprietary package, only the sub-routines used to prepare the data for curve fitting are described here along with the overall programme capability. These sub-routines are called FUNCTION, CALFUN, and FUNCT and Algol listings are included in the complete programme listing given in Appendix 6.

The appropriate three degree of freedom frequency response functions can be simultaneously fitted to up to 6 sets of data points selected from yaw rate, roll angle, and lateral amplitude ratio and phase angle versus frequency data. It is assumed that this data will come from one of two sources:

1) Direct. The data comes from some source external to the programme in the straightforward form of lists of amplitude and phase versus frequency.

2) By Transform. The frequency related data is obtained by Fourier transformation of time based input and response data.

So far only the direct data has been considered but the second type is examined in detail in the following sections. Normally the first point of each set of data is at zero frequency so that subsequent plotting (of amplitudes) can be done on a decibel scale relative to this point. As zero frequency off-sets are sometimes difficult to avoid, particularly for the second type of data, it is optional whether or not this point is
actually used in the curve fitting.

In addition to the basic data a guessed set of the coefficients (COEFFS) which define the transfer functions is required. These are the coefficients defined in equations 2.23 - 2.26. The first 4 COEFFS define the denominator common to all three transfer functions and are therefore always required. The number of COEFFS additionally required are 4 for yaw rate, 3 for roll angle, and 5 for lateral, making a maximum possible total of 16. Associated with each COEFFS[I] is a control parameter AKEY[I] which governs whether or not that COEFF is to be adjusted during the iterations.

For a given set of COEFFS and a data point in the form of an amplitude ratio or phase angle and the corresponding frequency, the procedure FUNCTION calculates the amplitude or phase angle of the transfer function defined by the COEFFS at that frequency and then sets:

\[
\text{FUNCTION} = \frac{(\text{transfer function amplitude} - \text{data amplitude})}{(\text{data amplitude})} \quad 5.5.1
\]

or \[
\text{FUNCTION} = \frac{(\text{transfer function phase angle} - \text{data phase angle})}{100}
\]

For amplitude data FUNCTION is normalised as shown by division by the data amplitude, so that its actual value is only dependant on the error. As the phase angle can drop to zero this type of normalisation is not appropriate and the factor of 0.01 is a compromise arrived at as follows:

Examination of typical amplitude and phase angle data and fitted curves shows that equivalent amounts of scatter are represented by 1 dB and about 10 degrees. For one amplitude data point 1 dB error means that:

\[
20 \log_{10}(\text{curve amplitude/data amplitude}) = 1
\]

whence: \[
\text{FUNCTION} = 0.122
\]

For a phase data point a 10 degree error means that:

\[
\text{FUNCTION} = (\text{curve phase} - \text{data phase}) = 10
\]
A weighting factor of 0.01 for the phase data thus seems appropriate. Various weighting factors and methods of normalisation were tried during the development of the procedure but the above were found to be the most satisfactory. If no such scaling is used the resulting curve fits are heavily biased towards the data which has the largest numerical value.

In the case of data obtained by Fourier transformation it is not necessary to store a set of frequency values corresponding to the data points as these points always come at equal frequency increments (FR), and so in this case the appropriate frequency is calculated within FUNCTION from FR and the number of the data point. For direct data FR is set equal to 999 in advance and used to indicate that the data is in the direct form.

The procedure CALFUN evaluates the function to be minimised, namely the sum of the squared values of procedure FUNCTION, for all the data points involved. FUNCT is used to calculate the derivative of FUNCTION with respect to each of the COEFFS for each data point. The number of derivatives calculated is thus the product of the number of COEFFS and the number of data points, which results in a large increase in time and computer space being involved in using the full capacity of the programme at 16 COEFFS and 6 sets of data. (Each set of data contains at least 15 points for direct data or many more for data from Fourier analysis). The control parameter DEL is used to govern the step size used in the derivative evaluation.

The basic outputs of the main procedure OPLS are a new set of COEFFS, the value of FUNCTION for these COEFFS at each data point, and the total sum of the squared values of FUNCTION. The latter two are used to judge the quality of the optimisation obtained. OPLS is an iterative procedure and the output is provided each IPRINT iterations, where IPRINT is a control parameter specified in advance. The other control parameters are IMAX, EPS, and SO and the procedure is terminated either after IMAX iterations, or when a significant decrease in the function can no longer be achieved, or when convergence defined by EPS is obtained. SO controls the size of the initial step in the iteration.
Having obtained a new set of COEFFS the programme plots the data points and the fitted curve on a graph similar to that described in Section 3, so that the fit can be visually inspected. Examples of these graphs are given earlier in this Section. The characteristic corner frequencies and damping ratios are also calculated from the COEFFS, and the values listed on the graphs under the headings of Poles and Zeros.

Guide lines for the use of the curve fitting procedure and typical values for the COEFFS and control parameters are given in Appendix 6.
6. CURVE FITS TO SINUSOIDAL STEER INPUT DATA.

6.1 Data of Section 4 - Mechanical Steer Input.

The previous Section has described the development of the curve fitting technique and demonstrated its effectiveness using theoretically derived data. This Section is concerned with using the fitting procedure for its real purpose of fitting curves to experimentally measured data, and as a first step the data described in Section 4, which was produced from a series of tests using a mechanically controlled sinusoidal steer input, will be examined. Curves were also fitted to the data by the original manual procedure and so the two sets of results can be compared.

Figs. 6.1.1 to 4 show digitally fitted curves to the yaw rate and roll angle amplitude data for the data for three speeds for Car A and one for Car B of Section 4. The procedure produced three fits quickly and easily and they are seen to be visually satisfactory. The poorest fit is probably that to the 30 mile/h data, which has the flattest yaw rate curve, and it is likely that this type of curve will always be the most difficult to deal with.

<table>
<thead>
<tr>
<th>( \omega_r, f_r )</th>
<th>70 mile/h</th>
<th>50 mile/h</th>
<th>30 mile/h</th>
<th>50 mile/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAR A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIGITAL</td>
<td>0.89, -0.58</td>
<td>1.03, -0.86</td>
<td>1.55, -0.73</td>
<td>1.37, -0.97</td>
</tr>
<tr>
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<td>0.90, -0.7</td>
<td>1.3, -0.7</td>
<td>1.1, -0.7</td>
</tr>
<tr>
<td>( \omega_d, f_d )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAR A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIGITAL</td>
<td>1.78, -0.40</td>
<td>1.54, -0.42</td>
<td>1.76, -0.33</td>
<td>1.51, -0.37</td>
</tr>
<tr>
<td>MANUAL</td>
<td>- -</td>
<td>1.7, -0.3</td>
<td>- -</td>
<td>1.8, -0.4</td>
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<tr>
<td>( \omega_c )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAR A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIGITAL</td>
<td>0.51</td>
<td>0.62</td>
<td>1.27</td>
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</tr>
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<td>MANUAL</td>
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<td>1.12</td>
<td>0.8</td>
</tr>
<tr>
<td>( \omega_{eq}, f_{eq} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAR A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DIGITAL</td>
<td>1.85, -0.34</td>
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<td>1.86, -0.32</td>
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<tr>
<td>MANUAL</td>
<td>- -</td>
<td>1.8, -0.3</td>
<td>- -</td>
<td>2.0, -0.35</td>
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</tbody>
</table>

Table 6.1.1 Characteristic frequencies and damping ratios for digital and manual curve fits to the data of Cars A and B of Section 4.
STEER RESPONSE

11 11 74 P1 DIRECT CAR A HW75 70 MPH

<table>
<thead>
<tr>
<th>SP SS RESP</th>
<th>YAW RATE</th>
<th>ROLL ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLES</td>
<td>1.839e-01</td>
<td>7.922e-02</td>
</tr>
<tr>
<td>ZEROS</td>
<td>1.68 -0.582</td>
<td>1.48 -0.109</td>
</tr>
</tbody>
</table>

Fig. 6.1.1. Curve fits to data for Car A of Section 4 at 70 mile/h.
Fig. 6.1.2. Curve fits to data for Car A of Section 4 at 50 mile/h.
96

STEER RESPONSE

11 11 74 P3 DIRECT CAR A HW75 90 MPH

<table>
<thead>
<tr>
<th>SP SS RESP</th>
<th>YAW RATE</th>
<th>ROLL ANGLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLES</td>
<td>ZEROS</td>
<td>ZEROS</td>
</tr>
<tr>
<td>1.76 -0.334</td>
<td>1.27 -1.000</td>
<td>1.65 -0.172</td>
</tr>
<tr>
<td>1.03 -0.725</td>
<td>1.77 -0.264</td>
<td>1.65 -0.172</td>
</tr>
<tr>
<td>1.03 -0.725</td>
<td>1.77 -0.264</td>
<td></td>
</tr>
<tr>
<td>1.76 -0.334</td>
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<td></td>
</tr>
</tbody>
</table>

Fig. 6.1.3. Curve fits to data for Car A of Section 4 at 30 mile/h.
STEER RESPONSE

11 11 74 P4 DIRECT CAR B (ω 50 MPH)

YAW RATE
SP SS RESP 1.622e-01
POLES 0.67 -1.000
ZEROS 1.35 -0.148
1.37 -0.965
1.37 -0.965
1.51 -0.368

ROLL ANGLE
ZEROS 7.171e-02
1.55 -0.148
1.55 -0.148
1.37 -0.315
1.37 -0.315
1.51 -0.368

Fig. 6.1.4. Curve fits to data for Car B (a) of Section 4.
Table 6.1.1 shows the ω's and j's corresponding to these fits and (where available) to the manual fits. For Car A the agreement is seen to be reasonable, but it is not quite so good for the Car B case. This is not surprising as the Car B data was difficult to deal with manually, and the digital curve looks a much better fit. As the digital technique provides a well defined and consistent curve fit these results are to be preferred.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>70 mile/h</th>
<th>50 mile/h</th>
<th>30 mile/h</th>
<th>50 mile/h</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAR A</td>
<td>CAR A</td>
<td>CAR A</td>
<td>CAR B(a)</td>
</tr>
<tr>
<td>1</td>
<td>15.3</td>
<td>19.1</td>
<td>21.5</td>
<td>23.6</td>
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<tr>
<td>2</td>
<td>214</td>
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<tr>
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<td>3.55</td>
<td>2.99</td>
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<td>5.01</td>
</tr>
<tr>
<td>10</td>
<td>2.04</td>
<td>2.15</td>
<td>3.57</td>
<td>2.89</td>
</tr>
<tr>
<td>11</td>
<td>87.9</td>
<td>84.2</td>
<td>103</td>
<td>94.5</td>
</tr>
</tbody>
</table>

Table 6.1.2 COEFFS from digital curve fits to the data of Cars A and B of Section 4.

<table>
<thead>
<tr>
<th>COEFF</th>
<th>70 mile/h</th>
<th>50 mile/h</th>
<th>30 mile/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.4</td>
<td>16.6</td>
<td>23.7</td>
</tr>
<tr>
<td>2</td>
<td>209</td>
<td>236</td>
<td>314</td>
</tr>
<tr>
<td>3</td>
<td>762</td>
<td>1100</td>
<td>1940</td>
</tr>
<tr>
<td>4</td>
<td>2130</td>
<td>3280</td>
<td>7320</td>
</tr>
<tr>
<td>5</td>
<td>36.0</td>
<td>36.0</td>
<td>36.0</td>
</tr>
<tr>
<td>6</td>
<td>10.0</td>
<td>11.7</td>
<td>15.8</td>
</tr>
<tr>
<td>7</td>
<td>160</td>
<td>167</td>
<td>183</td>
</tr>
<tr>
<td>8</td>
<td>407</td>
<td>575</td>
<td>945</td>
</tr>
<tr>
<td>9</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>10</td>
<td>2.77</td>
<td>3.90</td>
<td>6.48</td>
</tr>
<tr>
<td>11</td>
<td>55.8</td>
<td>55.8</td>
<td>55.8</td>
</tr>
</tbody>
</table>

Table 6.1.3 COEFFS for theoretical data of Run 27 of Section 3
Effect of speed.
The COEFFS corresponding to the digital fits are shown in Table 6.1.2, and as an indication of the theoretical effect of speed, Table 6.1.3 shows the COEFFS calculated at three speeds for the data of Run 27 of Section 3. The type of variation shown by the COEFFS is seen to be in good agreement, providing further justification for the fitting of this type of model to the experimental data. The $\omega$'s and $J$'s corresponding to the theoretical COEFFS naturally also show similar behaviour.

6.2 Manually Applied Sinusoidal Steer Input Data

As the installation of the sinusoidal input machine was a rather complicated procedure, and other researchers had reported some success in the use of manually applied sinusoids, it was decided to try this as part of the experimental programme involved in the later stages of this work. The experiment, which was very successful, is described in detail in Section 7, and curve fits to these sets of results are discussed below. Two additional cars are involved, designated Car E and Car F, and these are described in Appendix 7.

Yaw rate (6/3), roll angle (7/3) and latac (9/3) amplitude data and fitted curves are shown in Fig 6.2.1 for Car E. The data itself looks good with a satisfactory level of scatter. The lack of roll angle points at low frequency stems from the fact that the measurement actually made was of roll rate, which understandably gave a very low signal, and therefore poor signal to noise ratio, at these low frequencies. The roll angle points were calculated from the corresponding rate points by simple division by the frequency. The latac has been corrected for the effect of roll angle with due account being taken of the phase difference between the two signals.

Because the work of Section 5 has shown that the latac data is not particularly helpful in fitting the curves to the yaw rate and roll angle data, the curve fits of Fig. 6.2.1 were achieved by first fitting curves to the yaw rate and roll angle data only, thus establishing COEFFS 1 to 11, and then fitting to the latac data with these COEFFS fixed, to establish the remaining 5 COEFFS. The fits obtained are visually good, although the latac is perhaps slightly poorer than the other two, and once again it is seen that the three degree of freedom
Fig. 6.2.1. Curve fits to data for Car E at 50 mile/h, tyres 30F, 28R, latac fitted separately.
Table 6.2.1 COEFFS for fits to data for Car E at two sets of tyre pressures, and percentage differences between COEFFS.

The effect of fitting all three curves together is illustrated in Fig. 6.2.2, and the COEFFS corresponding to this Figure and to Fig. 6.2.1 are shown in Table 6.2.1 with the percentage difference given in parentheses. It is interesting to note that the combined fitting of all three, in general, produced larger differences in COEFFS 1 to 11 than in the specifically latac COEFFS 12 to 16. This in a sense confirms that the separate fitting is to be preferred. A further guide lies in the characteristic frequencies and damping ratios (given at the top of each Figure), where those of Fig. 6.2.2 seem rather less appropriate than the others. In particular the yaw rate corner frequency (identified by its higher damping ratio) seems rather high and is significantly larger than
STEER RESPONSE

<table>
<thead>
<tr>
<th>12 178 P1 DIRECT SINE4T30F28R</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>SP SS RESP</th>
<th>YAW RATE</th>
<th>ROLL ANGLE</th>
<th>LATAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>POLES</td>
<td>1.116×10^-01</td>
<td>6.662×10^-02</td>
<td>6.627×10^-03</td>
</tr>
<tr>
<td>ZEROS</td>
<td>0.98 -1.000</td>
<td>1.43 -0.175</td>
<td>1.76 -0.115</td>
</tr>
<tr>
<td>ZEROS</td>
<td>1.49 -0.777</td>
<td>1.22 -0.292</td>
<td>1.22 -0.292</td>
</tr>
<tr>
<td>ZEROS</td>
<td>1.49 -0.777</td>
<td>1.43 -0.175</td>
<td>1.76 -0.115</td>
</tr>
</tbody>
</table>

Fig. 6.2.2. Curve fits to data of Fig. 6.2.1, three curves fitted together.
the roll mode frequency. For this set of data these differences in the COEFFS and corner frequencies are not very large, but it will be seen from the results for Car F given later in this Section and from various results in Section 9, that large differences can occur and that the separate fitting of the latac data is to be preferred.

The tests of Fig. 6.2.1 were carried out at tyre pressures of 30F, 28R lb/in (where F and R denote front and rear respectively), and as a guide to the sensitivity of the experiment and of the frequency response curves to a small vehicle change, a second set of tests were carried out at tyre pressures of 28F, 30R. Fig. 6.2.3 shows the results with fitted curves and the curves of Fig. 6.2.1 superimposed for comparison. The changes to the roll angle and latac curves are probably not very significant, but the yaw rate curve shows a consistent increase in high frequency response for the 28F, 30R case. As this tyre pressure change would have made the car more understeering this is the type of effect that could have been expected. The COEFFS corresponding to Fig. 6.2.3 are also given in Table 6.2.1 with the percentage differences from those of Fig. 6.2.1. The differences in some of the COEFFS are unexpectedly large and some of this, in particular COEFF 10, can be attributed to the artificially high damping of the roll mode for Fig. 6.2.3, where the fitted curve does not do justice to the "kink" in the roll angle data.

Figs. 6.2.4 and 6.2.5 show the data from a manually applied sinusoidal input test on Car F, and curves fitted by the two methods discussed above. The corresponding COEFFS are given in Table 6.2.2 with the percentage differences. For this data the difference between the two types of fitting is greater, although of the same type, and the results from the separate fitting technique of Fig. 6.2.4 are much to be preferred. In fact the yaw rate corner frequency still seems rather high even for Fig. 6.2.4, and the study of results from random steer input tests to this car, given in Section 9, will suggest that this should be reduced further.
STEER RESPONSE

1275 P3 DIRECT SINE3T2FSOR

<table>
<thead>
<tr>
<th>YAW RATE</th>
<th>ROLL ANGLE</th>
<th>LATAF</th>
</tr>
</thead>
</table>

POLES

| 1.31 -0.358 | 0.72 -1.000 | 1.50 -0.217 |
| 1.37 -0.877 | 1.46 -0.457 | 1.50 -0.217 |
| 1.37 -0.877 | 1.46 -0.457 | 1.71 -0.084 |
| 1.31 -0.358 |               | 1.71 -0.084 |

ZEROS

| 1.31 -0.358 | 0.72 -1.000 | 1.50 -0.217 |
| 1.37 -0.877 | 1.46 -0.457 | 1.50 -0.217 |
| 1.37 -0.877 | 1.46 -0.457 | 1.71 -0.084 |
| 1.31 -0.358 |               | 1.71 -0.084 |

Fig. 6.2.3. Curve fits to data for Car E at 50 mile/h, tyres 28F, 30R, latac fitted separately (Curves of Fig. 6.2.1, superimposed).
Fig. 6.2.4. Curve fits to data for Car F at 50 mile/h, latac fitted separately.
Fig. 6.2.5. Curve fits to data of Fig. 6.2.4, three curves fitted together.
The results of this Section have shown that the frequency responses of the cars involved can be closely represented by three degree of freedom vehicle models. However, it has also been seen that very similar curves can be obtained with quite large differences in the COEFFS, and it may be that the curves themselves are the most useful descriptions of vehicle response and that the COEFFS simply provide an analytic description of these curves. This aspect will be considered again in Section 9.
7. MEASUREMENT OF TRANSIENT RESPONSE TO STEERING AND WIND GUST INPUTS.

7.1 Introduction.

The work of this thesis so far has shown that frequency response to steering input can be achieved by using a mechanically produced sinusoidal steer input, and that a three degree of freedom model can be fitted to the sinusoidal data, providing an analytic description of the vehicle response. However, the sinusoidal input test is time consuming and very demanding on test track space. It is also difficult to apply in the case of a wind gust input where the input is much more difficult to control than a steer input.

A potentially more elegant method of producing the frequency response curve of a system is to apply some kind of transient input to the system, measure the response, and then calculate the frequency response by Fourier analysis of input and response. The advent of the Fast Fourier Transform (FFT) technique for rapid digital calculation of Fourier transforms, means that such an approach is a practical proposition, and so it was decided to try to apply it to this vehicle response problem. A second series of tests was therefore carried out and is described in this Section.

7.2 Instrumentation.

The instrumentation system used for the tests described in Section 4 involved the recording of variables in analogue form on photographic chart. As digital analysis of the current tests was planned this was evidently not an appropriate system. It was also rather heavy and complicated, involving the interconnection of several separate modules and junction boxes. The first stage in this series of tests was thus to build or obtain a more suitable system.

After a survey of commercially available equipment and considerable consultation with the electronics department at MIRA, a system was devised based on miniature cassette tape recorders. The detailed design and construction of this system was carried out by the MIRA
electronics department from the systems concept and specification provided by the Author. Basically the system allows for the recording of up to 12 channels of information in analogue form on magnetic tape cassettes. On replay the analogue signals are digitised on an analogue to digital converter available on line to the MIRA computer, ready for whatever digital analysis is required. A description and photographs of the vehicle borne part of the system are given in Appendix 5.

The above system contains all that is required in terms of power supplies, signal conditioning, monitoring, and recording facilities. For the recording of a set of variables all that is then required are the appropriate transducers. A list of the variables measured during these tests, and of the transducers used is given in Table 7.2.1.

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>TRANSDUCER</th>
<th>LOCATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Handwheel angle</td>
<td>Rotary potentiometer</td>
<td>Gear-driven off steering column</td>
</tr>
<tr>
<td>Left and right</td>
<td>Linear potentiometers</td>
<td>Two potentiometers each side under the front suspension in parallelogram arrangement</td>
</tr>
<tr>
<td>road wheel angles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yaw rate</td>
<td>Rate gyrooscope</td>
<td>6 in. aft of the C. of G.</td>
</tr>
<tr>
<td>Roll rate</td>
<td>Rate gyrooscope</td>
<td>6 in. aft of the C. of G.</td>
</tr>
<tr>
<td>Lateral acceleration</td>
<td>Strain gauge</td>
<td>C. of G.</td>
</tr>
<tr>
<td>Speed</td>
<td>Accelerometer</td>
<td>Attached to rear bumper</td>
</tr>
<tr>
<td>Lateral &amp; longitudinal</td>
<td>A.C. generator on</td>
<td>Above the test car roof on a special mounting rack</td>
</tr>
<tr>
<td>wind speed</td>
<td>fifth wheel</td>
<td></td>
</tr>
<tr>
<td></td>
<td>High frequency</td>
<td></td>
</tr>
<tr>
<td></td>
<td>anemometers</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2.1 Quantities measured and transducers.

The use of linear potentiometers in a parallelogram arrangement for the measurement of road wheel angles, was to eliminate the effects of vertical, lateral, and longitudinal movement of the wheel, from the measurement of its steer angle relative to the vehicle body.
Although roll rate was measured here the quantity actually required is roll angle. The direct measurement of roll angle during transient manoeuvres such as those planned here requires the use of a free, position gyroscope. These are typically expensive, power consuming, and difficult to use because of drift problems, and so although the Author has used this technique in the past, it was decided to opt for the much simpler measurement of roll rate for these tests. As digital analysis of the data was planned, the integration of the rate signal to give roll angle was not seen as a problem.

As the accelerometer for the measurement of lateral acceleration was rigidly fixed to the vehicle, its signal was affected by the vehicle roll angle, and so a correction for this at the analysis stage was planned. The horizontal position of the centre of gravity was found by placing the vehicle on four scales, and its height from the ground by oscillating the whole vehicle on a specially designed compound pendulum system available at MIRA.

The tests planned for this work were all to be carried out at constant speed, and the measurement of this parameter was simply to provide a check that this was being achieved.

Lateral and longitudinal wind speed were only measured during the wind gust response tests. The anemometers used were supplied by the Electrical Research Association, and each consisted of a small perforated sphere, mounted on a vertical shaft attached to a strain gauged cantilever immersed in silicon fluid to provide damping. The two of these mounted above the roof of the test vehicle were calibrated in the MIRA Full Scale Wind Tunnel. The setting up, calibration, and development of an analysis procedure for this was a small project in itself and was not carried out by the Author. The system and techniques involved are fully described by Smith, Ref. 59.

7.3 Test Procedures.

The actual test work described here was planned and supervised by the Author but carried out by members of MIRA staff.
For all the tests the recordings of the actual tests were preceded by zeros and calibration steps, the latter being generated in the instrumentation package by networks of load resistors representing known values of angles etc. Three types of test were carried out:

7.3.1 Sinusoidal Steer Input

As indicated in Section 6 these tests were carried out using a manually generated sinusoidal steer input. The objects of this were:

(a) to examine the feasibility of using this kind of input as compared to the mechanical input used for the tests of Section 4.
(b) to provide frequency response curves for comparison with those generated by Fourier analysis of transient tests (7.3.2), for the same vehicle under similarly controlled conditions.

This type of test requires a large width of test track for the low frequency points. The only way of achieving this at MIRA is to use the full four lanes of one of the straight parts of the High Speed Circuit, and as this circuit is heavily used this is not often possible. For the car E tests arrangements were made to use the Dunlop test area based on a disused airfield at Fradley near Lichfield. Although this provided adequate space the surface was rather rougher than is desirable for this type of test. For car F where only one test was carried out the High Speed Circuit was used at a quiet time (Christmas Eve).

The test procedure was for the driver to drive the car at constant speed (50 mile/h), and to apply as near as possible to a sinusoidal steering input for several complete cycles. To provide the driver with guidance as to his frequency of input a pocket watch sized musician's metronome was attached to the dashboard in front of him. After some practice it was found that he could produce a good approximation to a sinusoidal input over the frequency range of interest. Measurements were made at a range of frequencies in about 30 steps from 0.2 to 3.5 Hz. A constant amplitude of input was used corresponding to a steady state
latac of about 0.2g at the test speed. The appropriate size for this input was found from a previously carried out steady state test on the vehicle.

7.3.2 "Random" Steer Input.

The car was driven at constant speed in a straight line. Then the driver applied a pseudo random type of steer input with the intention of putting in adequate power over the frequency range of interest. The maximum amplitude of steer input was approximately restricted to that corresponding to a steady state latac of 0.2g. Finally the car was returned to and held in the straight ahead position so that the recorded signal began and ended with zero input, and was of appropriate length for the analysis procedures. Various time histories of this type of test are shown in Section 8.

As the response characteristics of vehicles can vary with latac it was decided to carry out the above tests also about a mean latac of 0.2g rather than zero. This was achieved by performing the test while driving round the flat inner part of a bend on the MIRA High Speed Circuit. The test procedure was similar except that the steer input had a non zero mean value.

7.3.3 Wind Gust Input.

MIRA has available a system for providing an artificial wind gust. This consists of a jet engine exhaust diverted by means of a triforced pipe into a gust about 120 ft wide at wind speeds of up to 50 mile/h. A full description of this facility can be found in Ref. 64.

The test procedure was simply to approach the gust at constant speed in a straight line, fix the handwheel, and proceed through the gust allowing the vehicle to respond to the gust as the sole input. Various time histories, including the gust profile as measured by the anemometers, are shown in Section 8.
7.4 First Stage Data Reduction.

The measured data from these tests was in the form of analogue electrical signals recorded on magnetic tape cassettes. To provide flexibility in the final type of analysis to be employed, a two stage data reduction process was used. The first stage was to digitise the signals, apply all necessary calibrations, integrations, and corrections, and then store on a digital magnetic tape file directly accessible by the computer. Although there was nothing difficult about this in concept, the necessary data handling techniques were not in a developed form at MIRA at the time when the process was arranged, and so the Author spent a considerable amount of time sorting this out. The assistance of the staff of the computer department during this phase, and throughout the computer programme preparation, is gratefully acknowledged.

The digital file is arranged to contain, for each test, time histories of the following quantities:

1. Wind speed \( \text{mile/h} \)
2. Wind angle \( \text{deg} \)
3. Handwheel angle \( \text{deg} \)
4. Mean road wheel angle \( \text{deg} \)
5. Roll rate \( \text{deg/sec} \)
6. Yaw rate \( \text{deg/sec} \)
7. Roll angle \( \text{deg} \)
8. Yaw angle \( \text{deg} \)
9. Lateral acceleration(latac) \( g \)

The numbers associated with the quantities in this list are used to identify the quantities on the frequency plots shown throughout this thesis. For example 6/3 is used to mean the yaw rate/hand wheel angle response.

1 and 2 are computed from the anemometer signals using a special procedure developed separately from this work and described in Ref. 59. In simple terms the procedure carries out linear interpolation on a
chart produced from the wind tunnel calibration data for the car being used. To give consistency with the wind gust response equations of Section 2 the wind angle is subsequently used as the input to the vehicle and the longitudinal component of the wind speed is assumed to remain constant. This is a very reasonable assumption in the artificially produced side wind situation being considered here. In the testing not involving wind measurement other quantities can be inserted in these channels.

7 and 8 are normally produced by simple digital integration of 5 and 6 although the system can accept direct measurements of these if they are available. 9 is produced by correcting the accelerometer signal for the effect of roll angle, using the roll angle signal from channel 7.
8. COMPUTATION OF FREQUENCY RESPONSE FROM TRANSIENT RESPONSE.

8.1 Theory.

The transformation from the time to the frequency domain using digital computers and the theory of the discrete Fourier transform to operate on a series of data points, has had a lot of attention in a number of areas in recent years, and received considerable impetus from the advent of the Fast Fourier Transform (FFT) algorithm. Unfortunately, from the point of view of this current work is the fact that most of the published work has been concerned with random vibrations, rather than the deterministic transient responses of interest here. The basic theory for the analysis of transients is well established, however, and in terms of the current work can be expressed as follows.

The vehicle with transfer function $H(\omega)$, (expressed as a function of frequency, $\omega$) is subjected to a time input $I(t)$ and produces a corresponding response $R(t)$ as shown diagramatically in Fig. 8.1.1.

![Fig. 8.1.1.](image)

The problem is to find $H(\omega)$ from given $I(t)$ and $R(t)$ and the proposed solution is shown in Fig. 8.1.2.

![Fig. 8.1.2.](image)

This is relatively straightforward in concept, and difficulties which can arise are mostly due to the limitations imposed by the analysis.
and recording hardware, and the use of the FFT algorithm.

8.2 Practical Considerations.

In the case of digital analysis the time functions I(t) and R(t) are in the form of series of data points separated by time intervals of \( \Delta t \). The use of the FFT requires that the total number of points be an exact power of 2, say \( N = 2^n \), and gives a frequency transform consisting of \( 2^{(n-1)} \) points at frequency intervals of \( \Delta f \) where \( \Delta f = 1/(N,\Delta t) \). Table 8.2.1 illustrates the various factors involved.

<table>
<thead>
<tr>
<th>Data sampling rate, points per second</th>
<th>( 1/\Delta t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of points</td>
<td>( N = 2^n )</td>
</tr>
<tr>
<td>Total length of record, seconds</td>
<td>( T = N,\Delta t )</td>
</tr>
<tr>
<td>Frequency resolution available, rad/sec</td>
<td>( \Delta f = 1/T )</td>
</tr>
<tr>
<td>Highest frequency available, rad/sec</td>
<td>( F = 1/(2,\Delta t) )</td>
</tr>
</tbody>
</table>

Table 8.2.1 Control factors in Fourier transformation by FFT.

For this work the highest frequency of interest is about 3 Hz, suggesting a minimum sampling rate of 6 points per second. To provide a measure of breathing space it was decided to set a minimum rate of 10 points per second, and to confirm that this was adequate in practice a series of test transformations were carried out on real data with a range of sampling rates. For rates of 10 and above the shape of the transform in the frequency range of interest was found not to vary. The actual sampling rates available in this case are restricted by the 40 Hz synchronising signal used in the recording system, to 40 divided by any integer, and so the range of reasonable values for the control factors can be seen to be those shown in Table 8.2.2.

In terms of carrying out the vehicle tests the relevant factor in this Table is the time \( T \), for which there are nominally 7 possibilities. In fact the frequency resolution provided by \( T = 6.4 \) seconds is rather poor, and test track space tends to limit the maximum time possible, so that the useful times reduce to 9.6, 12.8, 19.2, or 25.6 seconds. Where possible for this work a time of 25.6 seconds has been used to give the best frequency resolution.
<table>
<thead>
<tr>
<th>Sampling Rate</th>
<th>No of Points</th>
<th>Total Time</th>
<th>Frequency Resolution</th>
<th>Max Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{\Delta t} ) (1/sec)</td>
<td>N</td>
<td>T (sec)</td>
<td>( \Delta f ) (Hz)</td>
<td>F (Hz)</td>
</tr>
<tr>
<td>10</td>
<td>64</td>
<td>6.4</td>
<td>0.156</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>128</td>
<td>12.8</td>
<td>0.078</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>256</td>
<td>25.6</td>
<td>0.039</td>
<td>5</td>
</tr>
<tr>
<td>10</td>
<td>512</td>
<td>51.2</td>
<td>0.02</td>
<td>5</td>
</tr>
<tr>
<td>13.33</td>
<td>128</td>
<td>9.6</td>
<td>0.104</td>
<td>6.66</td>
</tr>
<tr>
<td>13.33</td>
<td>256</td>
<td>19.2</td>
<td>0.052</td>
<td>6.66</td>
</tr>
<tr>
<td>13.33</td>
<td>512</td>
<td>38.4</td>
<td>0.026</td>
<td>6.66</td>
</tr>
<tr>
<td>20</td>
<td>128</td>
<td>6.4</td>
<td>0.156</td>
<td>10</td>
</tr>
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<td>20</td>
<td>256</td>
<td>12.8</td>
<td>0.078</td>
<td>10</td>
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<tr>
<td>20</td>
<td>512</td>
<td>25.6</td>
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<td>10</td>
</tr>
<tr>
<td>20</td>
<td>1024</td>
<td>51.2</td>
<td>0.02</td>
<td>10</td>
</tr>
<tr>
<td>40</td>
<td>256</td>
<td>6.4</td>
<td>0.156</td>
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<td>40</td>
<td>512</td>
<td>12.8</td>
<td>0.078</td>
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</tr>
<tr>
<td>40</td>
<td>1024</td>
<td>25.6</td>
<td>0.039</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 8.2.2 Range of control factors appropriate to this work.

Classical Fourier transform theory requires that the infinite integral of the time function be finite, which is best achieved in practice by arranging that the function begins and ends at zero. In fact the use of the discrete Fourier transform in the digital analysis means that violation of this condition is not critical, as its effect is simply to give an amplitude offset at zero frequency. Since a time input beginning and ending at zero is a neat way of defining a test, and is not normally difficult to achieve in practice, and the zero frequency offset can be rather inconvenient, an input of this form was selected for these tests. As will be seen later the exact level of the zero frequency amplitude still turns out to be rather difficult to establish and in most cases this point is not used.

The use of the discrete Fourier transform also means that the continuous Fourier transform of the time function is approximated by a series of frequency points, and the transient time function is assumed to be one cycle of a periodic function. These frequency points require smoothing to give the continuous transform and this is achieved here without loss of frequency resolution using the 'Hamming' coefficients as follows:
Say that the frequency points are: \( H(0), H(1), \ldots, H(N) \)

After Hamming these points become:

\[
\begin{align*}
H(0) &= 0.54H(0) + 0.46H(1) \\
H(n) &= 0.23H(n-1) + 0.54H(n) + 0.23H(n+1) \quad \text{for } n = 1 \text{ to } (N-1) \\
H(N) &= 0.46H(N-1) + 0.54H(N)
\end{align*}
\]

Although the analysis illustrated in Fig. 8.1.2 places no particular theoretical restriction on the form of the Fourier transform of the input signal, \( I(\omega) \), in practice this evidently has to contain adequate power over the frequency range of interest. The factors governing this are the inherent dynamic range of the instrumentation used for recording and the analogue to digital conversion, and the noise level of the tests themselves. For this work it was found that a range of about 20 dB for the amplitude of \( I(\omega) \) was about the most that could be tolerated, and so this was checked for each test before proceeding with the calculation of the transfer functions required. Care was also taken that the full ranges of the recording system and ADC were always employed.

The division shown in Fig. 8.1.2 is in fact complex, but is accomplished here by resolving the Fourier coefficients obtained from the FFT at each frequency into amplitude and phase, and then simply dividing the amplitudes and subtracting the phases. FITTRANS, the computer programme written to carry out the transformation and various data sorting required for all the above is described in detail in Appendix 6.

8.3 Steer Response Results.

Results from two cars, designated car E and car F, are presented here. Outline descriptions of these cars and details of their steady state behaviour are given in Appendix 7. The results are principally presented in the form of computer drawn graphs which are largely self-explanatory, but details of the various formats are given in Appendix 6. The position of frequency response graphs on their gain scales has frequently been adjusted to avoid confusion due to overlapping curves, and so should not be considered significant unless otherwise stated. Where comparisons of curves are involved the correct relative positions are, of course, used.
8.3.1 Car E.

Fig. 8.3.1.1 shows a time history of a test carried out at 50 mile/h on the same day and test track and with the same vehicle conditions as the sinusoidal steer input test whose results are shown in Fig. 6.2.1. This test track had rather a rough surface and it can be seen that the roll rate in particular, and to a lesser extent the latac, show a significant response even with zero steer input. As will be seen this is reflected in the frequency response graphs. The yaw and roll angle traces are produced by digital integration of the rates and tend to suffer from drift if the latter signals have any DC content. As the latac measurement is 'corrected' by this roll angle signal some drift can also be seen on the latac trace. Rather than carrying out any filtering or trend removal at this stage the effect of any drift can conveniently be removed by ignoring the lowest frequency points in the subsequent Fourier analysis.

Fig. 8.3.1.2 shows the frequency transform of the handwheel input, which can be seen to contain adequate power over the range of interest. The transfer functions for yaw rate (6/3), roll angle (5/3), and latac (9/3), relative to handwheel angle are shown on Fig. 8.3.1.3. The superimposed smooth curves are those fitted to the sinusoidal input data of Fig. 6.2.1. For yaw rate the noise on the curve is reasonable and the agreement excellent, indicating that the complete test, measurement, and analysis procedure is operating satisfactorily. The roll angle curve is calculated from the roll rate, and as anticipated from the noise on the time signal, shows considerably more noise than the yaw rate curve. Within this noise, however, the agreement with the sinusoidal input curve is seen to be good. Up to about 1.5 Hz the latac curve is also satisfactory, but in the range 1.5 – 2.5 Hz there is considerable noise. As this in the range where, because of the shape of the transfer function, the latac signal is smallest, any noise on the time signal will be likely to have a large effect. At the highest frequencies the situation is better, and the agreement with the sinusoidal input data is satisfactory.

A similar set of Figs. corresponding to the same vehicle with tyre pressures at 28 F (front), 30 R (rear) (lb/in²) instead of 30 F, 28 R, and for comparison with the sinusoidal input data of Fig. 6.2.3, are
Fig. 8.3.1.1. Time history for a test on Car E at 50 miles/h, tyers 30F, 28R.
Fig. 8.3.1.2. Frequency transform of handwheel input of Fig. 8.3.1.1.
Fig. 8.3.1.3. Transfer functions from data of Fig.8.3.1.1. and superimposed sine input curves.
shown in Figs. 8.3.1.4 - 6. The data is rather poorer than the previous set and the agreement with the sinusoidal data is not so good for the roll angle curve. The noise on the yaw rate curve is slightly greater but the agreement is again good. For the yaw rate curves the effect of the tyre pressure change was readily detectable from the sinusoidal input data, and since there is good agreement between this data and the current set it is tempting to conclude that this method of testing is also able to detect the difference. However, because of the noise on the curves it is probably more realistic to say that this difference is about the limit of what can be detected.

Individual front wheel steer angles were measured in addition to handwheel angle during these tests and time histories of the average of the two are shown in Figs. 8.3.1.1 and 4. As the steady state measurements on this car showed a large amount of effective loss in the steering system (see Appendix 7), it is interesting to examine the transfer functions relative to the road wheel input. Fig. 8.3.1.7 shows the yaw rate relative to road wheel (6/4) and, to illustrate the behaviour of the overall steering system, the road wheel relative to handwheel (4/3), transfer functions. The yaw rate curve is seen to be quite a different shape from the corresponding curve of Fig. 8.3.1.3, showing more heavily damped behaviour corresponding to a more oversteering car. This is in general agreement with the steady state results. The road wheel relative to handwheel curve is interesting in that it shows the greatest steering loss at low frequencies. Simple reasoning might have tended to suggest that the reverse would have been the case, and indeed if the loss had been due to the dynamics of the steering system as such, this would have occurred. As will be the case for many cars the losses shown are thus probably due to roll and compliance steer effects which fall off with frequency as do the roll angle and lateral responses. That this sort of analysis can readily be carried out is an illustration of the power of the overall measurement and analysis technique.

A final set of tests on this car were carried out about a mean lateral of 0.2g and a set of results is shown in Figs. 8.3.1.8 - 10. The vehicle condition corresponds to that of Fig. 8.3.1.1 except that the speed was slightly slower at 46 mile/h. The noise on the time traces and corresponding scatter on the transfer functions is much reduced compared to the previous
Fig. 8.3.1.4. Time history for a test on Car E at 50 mile/h, tyres 28F,30R.
Fig. 8.3.1.5. Frequency transform of handwheel input of Fig. 8.3.1.4.
Fig. 8.3.1.6. Transfer functions from data of Fig. 8.3.1.4. and superimposed sine input curves.
Fig. 8.3.1.7. Transfer functions from data of Fig. 8.3.1.1.
Fig. 8.3.1.8. Time history for a test on Car E with non-zero mean latac.
Fig. 8.3.1.9. Frequency transform of handwheel input of Fig. 8.3.1.8.
Fig. 8.3.1.10. Transfer functions from data of Fig. 8.3.1.9.
results. This is probably largely due to the smoother test track involved, but may also be influenced by the vehicle itself being 'taught' with a non-zero mean side force. The roll angle and lateral transfer functions (5/3 and 9/3) now clearly show the characteristic shapes of the three degree of freedom model. Unfortunately no sinusoidal input data is available for comparison with these results but the similarity with the results of Fig. 6.4.1 is clear. As might be expected from the steady state results the yaw rate transfer function is significantly flatter than that of Fig. 8.3.1.3. (The lower speed will have contributed to this but could not account for all the difference). These results are most encouraging and suggest a method for dealing with non-linear vehicle behaviour.

8.3.2 Car F.

The results for car E have illustrated the basic viability of the test and analysis methods, although the noise on the transfer functions produced from the 'random' steer input tests was rather large. It is likely that this noise was largely due to the roughness of the test track used, and it would have been interesting to carry out some tests on this car on a smooth surface. The vehicle was only available for a short period and so this was not possible. However, it was considered important to show that the technique would also work for a second car, and from the lessons learned with car E the series of tests on this car (car F) were carried out on the smooth test track surfaces at MIRA. As discussed in Section 7 it was difficult to fit in the lowest frequency points for the sinusoidal input test, but the 'random' input tests were readily accomplished on a surface two lanes wide.

Figs. 8.3.2.1 - 3 show time histories, handwheel frequency content, and the three transfer functions for a test carried out at 50 mile/h on car F fitted with cross-ply tyres at 22 lb/in² all round. The smooth curves superimposed on the transfer functions of Fig. 8.3.2.3 are the fitted curves of the sinusoidal input test shown in Fig. 6.2.4. The agreement is seen to be excellent and the noise level on all the curves is acceptable. The smooth test track thus solves the noise problem encountered with car E and, in fact, inspection of the time histories of Fig. 8.3.2.1 and 8.3.1.1 shows the improvement quite clearly. As an
Fig. 8.3.2.1. Time history for a test on Car F at 50 mile/h, cross-ply tyres.
Fig. 8.3.2.2. Frequency transform of handwheel input of Fig. 8.3.2.1.
Transfer functions from data of Fig. 8.3.2.1, and superimposed sine input curves.
Fig. 8.3.2.4. Transfer functions and superimposed sine input curves, Car F 50 mile/h. on cross-ply tyres.
indication of the repeatability of the test Fig. 8.3.2.4 shows an exactly equivalent set of results to Fig. 8.3.2.3 produced from a separate test run.

It is thus established that steer frequency response curves can be produced from the relatively simple and rapid type of transient test shown here, and so it is now possible to examine some further vehicle conditions using this method. The results of some further tests are shown in Section 9 where curve fitting direct to these frequency responses is considered.

8.4 Wind Gust Response Results.

A series of wind gust response tests were carried out, as described in Section 7, on car F fitted with cross-ply tyres at 22 lb/in² all round. Fig. 8.4.1 shows a time history of a run at 30 mile/h. The three pronged shape of the wind angle input arising from the trifurcated ducting of the MIRA wind gust generator can be seen clearly. It is interesting to notice that although there is virtually no handwheel input during the gust, significant road wheel angles are developed. This is an illustration of the roll and compliance steer effects on this car, which are also clearly demonstrated by both the transient and steady state steer response test results. The roll rate, yaw rate, and lateral responses to the gust are clearly seen, although fairly high noise levels are present before the gust itself is reached. These noise levels are high compared to the gust responses but in absolute terms are quite small, and can be put into perspective by noting that the roll rate and yaw rate scales on Fig. 8.4.1 are about four times the corresponding scales on the steer response results shown on Fig. 8.3.2.1, for example.

The 19 second length of the record of Fig. 8.4.1 compared to the duration of the actual gust of about 3.3 seconds, has been done deliberately to provide adequate resolution in the subsequent frequency analysis. However, it is clearly not very satisfactory as the response records all show considerable signals, arising from spurious sources, in the absence of significant gust input. Fig. 8.4.2 shows a frequency transform of the wind angle input of Fig. 8.4.1, and when matched against the criterion of having a maximum dynamic range in the input of around 20 dB, is seen
Fig. 8.4.1. Time history for a test on Car F at 30 mile/h.
Fig. 3.4.2. Frequency transform of wind angle input of Fig. 3.4.1.
to be inadequate over a substantial part of the range of interest. The peak at about 1 Hz, corresponding to the three pulses of the input, helps a little, but not nearly enough. A satisfactorily frequency content for the input was shown in Fig. 8.3.2,2 for a steer input, where the dynamic range over the whole frequency band is only about 10 dB. Gust response transfer functions calculated from this data were not satisfactory.

Fig. 8.4.3 shows a time history of a run at 50 mile/h. In this case the wind angle input is smaller but the yaw rate and latac responses are larger. The roll rate response is totally lost in the background noise. The apparent noise on the handwheel and roadwheel traces is due to the large scales in this Fig. compared to those of Fig. 8.4.1. The frequency transform of the wind angle input is shown in Fig. 8.4.4 and although it is significantly better than that of Fig. 8.4.2 it is still very poor. Yaw rate (6/2) and latac (9/2) gust transfer functions calculated from this data are shown in Fig. 8.4.5. Ignoring the peaks at around 0.5 and 1.5 Hz, which correspond to the dips in the input frequency transform, and everything above about 2 Hz, these curves are just about recognisably of the form of the theoretical curves of Section 3, but clearly they are not good enough for any detailed analysis.

Figs. 8.4.6 - 8 show a corresponding set of results for a 60 mile/h test on car F. The frequency content of the input is still poor, although better than the previous sets of data, but the transfer functions are, as before, only barely recognisable and not of much value.

Other tests of this type were carried out on this car and on car B, but the quality of frequency response results obtained was similar to those shown here, and so further study was not considered to be useful.

The actual measurement of the wind gust and the vehicle response was quite good for most of these tests and the time histories shown are generally satisfactory. Nor do the poor frequency response results indicate that the analysis procedure is deficient; the problem lies principally with the inadequacy of the wind gust test facility for providing information of this type. The worst feature is the poor high frequency content of the gust, but the rather short length is also a drawback. It is almost certain that a more suitable gust facility could
Fig. 8.4.3. Time history for a test on Car F at 50 mile/h.
Fig. 8.4.4. Frequency transform of wind angle input of Fig. 8.4.3.
VEHICLE RESPONSE

Fig. 8.1.5. Transfer functions from data of Fig. 8.4.3.
Fig. 8.4.6. Time history for a test on Car F at 60 mile/h.
Fig. 8.4.7. Frequency transform of wind angle input of Fig. 8.4.6.
Fig. 8.4.8. Transfer functions from data of Fig. 8.4.6.
be constructed, consisting for example of a series of sharper gusts of varying lengths occurring at 'random' intervals over a longer distance, but there was no scope for such an exercise during this programme of work. Some attempts were made to improve the frequency content of the existing gust by interposing a sharp edged barrier between the gust generator and the vehicle path, but these were not very successful. It is possible that the problem of the gust duration could have been better overcome by adding zeros to the time data at the analysis stage, or by using the data again in a mirror image form, rather than by recording 'zero input' data where there was significant spurious signal. Because of the additional problem of the poor high frequency content this was not examined in any detail.

Although the results obtained from this gust generator have been seen to be unsatisfactory for analysis into transfer function form, it is of course possible to simply examine the time responses, and a fair amount of useful comparative work has been carried out by users of the facility. For this purpose it is important to ensure that the gust remains consistent from test to test and it would probably be better if the gust profile was of a simpler form. Theoretically one of the advantages of the frequency response technique, involving the measurement of both input and response, is that the exact form of the gust is not important. Unfortunately some of the properties of the gust are important as has been seen.

For interest Fig. 8.4.9 shows a time history of a test at 50 mile/h where the driver did his best to keep the vehicle on course through the gust and afterwards. Although the course deviation was finally very small the vehicle motion was rather more violent than that shown in Fig. 8.4.3 where no correction was made.
Fig. 8.4.9. Time history for a gust response test on Car F at 50 mile/h with steering correction.
9. CURVE FITTING TO FREQUENCY RESPONSES FROM TRANSIENT TESTS.

9.1 Introduction.

This is the most stringent test of the curve fitting procedure, with the object of producing smooth, analytically defined, sets of frequency response curves from the type of data illustrated in Section 8. Because of the rather different nature of this data from the "direct" frequency response results, it was found necessary to incorporate two special features into the fitting procedure. The first of these arises from the fact that scatter is likely at the lowest, and sometimes the highest, frequencies, and allows simply for the first and last points used in the fitting to be specified (STPT and NPTS). In using FITTRANS the frequency responses are first plotted in the form shown in Section 8 and then STPT and NPTS selected by inspection before proceeding to the curve fitting.

The second feature is due to the number of points produced by the FFT procedure. As discussed in Section 8 points are produced at equal frequency intervals, and this interval is chosen to give the best feasible resolution at low frequencies. As a result the number of points per octave increases rapidly, and with the normal resolution used here of about 0.04 Hz (corresponding to a time of 25 secs) 75 points are produced between 0 and 3 Hz. Both the number and the distribution of these points are undesirable. The number because of the computer space and time required, and the distribution because this causes the curve fit to be weighted towards the high frequency end where the density of points is highest. To overcome this the programme has the capability of using all points up to 1 Hz, every second point from 1 Hz to 2 Hz, and every fourth point from 2 Hz to 4 Hz (which is the highest frequency normally used). This simple device reduces the total number of points from 75 to 45 and was found to give good results from the curve fitting. Since the Hamming smoothing is carried out before the extra points are discarded their influence is not completely lost in this procedure. It will be seen from the results that follow that the resulting distribution of points (plotted on logarithmic scales) also seem visually reasonable.
As the wind gust response transfer functions produced in Section 8 were not very satisfactory, curve fitting is only applied to the steer response results in this section. FITTRANS is capable of carrying out fits to gust response data in an exactly equivalent manner.

9.2 Steer Response Results for Car E.

As seen in Section 8 the results for this car were not very good, with large amounts of scatter present, particularly on the roll angle data. It was nevertheless considered useful to try the curve fitting procedure to see how it coped with this sort of data.

Fig. 9.2.1 shows curve fits to the data of Fig. 8.3.1.3 with the curves fitted to the sine input data of Fig. 6.2.1 superimposed for comparison. The fit and agreement are seen to be satisfactory with the largest differences occurring, as might be expected, on the roll angle curves. The characteristic frequencies and damping ratios (also given at the top of the Figs.) and the COEFFS corresponding to fits are given in Tables 9.2.1 and 2 together with other sets of results to be discussed later. It can be seen that although the curves themselves look in good agreement, there are significant differences in the frequencies and damping ratios. In particular, the results for the sine input data show a tendency for the yaw rate mode frequency and damping \((\omega_r, \zeta_r)\) to be larger and the roll angle mode \((\omega_\phi, \zeta_\phi)\) to be smaller. This kind of effect was already noticed in Section 6 and will be examined in more detail with reference to the Car F data in Section 9.3.

An attempt was made to fit curves to the data of Fig. 8.3.1.6 which was for Car E with different tyre pressures. Due to the very poor roll angle data a satisfactory fit was not obtained although a reasonable estimate of \(\omega_r\) and \(\zeta_r\) was achieved.

Fig. 9.2.2 shows curve fits to the 'relative to road wheel' data corresponding to the data of Fig. 9.2.1. Despite the scatter the fits obtained are satisfactory, clearly illustrating the much more heavily damped nature of the response relative to the road wheels. Examination of the frequencies and dampings in Table 9.2.1 shows that the difference
<table>
<thead>
<tr>
<th>COEFF NO.</th>
<th>SINE INPUT DATA</th>
<th>RELATIVE TO ROAD WHEELS</th>
<th>.2g MEAN LATA</th>
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</thead>
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<tr>
<td>FIG. 6.2.1</td>
<td>FIG. 9.2.1</td>
<td>FIG. 9.3.2</td>
<td>FIG. 9.2.3</td>
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Table 9.2.2 COEFFS for fits to Car E.
Fig. 9.2.1. Curve fits to data for Car E, 50 mile/h., and superimposed curves from sine input data of Fig. 6.2.1.
Fig. 9.2.2. Curve fits to 'relative to road wheels' data. Car E, 50 mile/h.
Fig. 9.2.3. Curve fits to data for Car E, 0.2g mean Iatac.
in the curve shape is almost entirely accounted for by the difference in the yaw rate mode, where damping has increased such that the behaviour is no longer oscillatory and two real roots are obtained. Table 9.2.2 shows that this is achieved principally by a large reduction in COEFF 4. (The increase in COEFFS 5, 9 and 12 is due to the effective steering ratio).

The best results obtained for this car were those from the tests carried out about a mean latac of 0.2g. Although the scatter in these results is small the fact that higher latacs are involved and that the curve shapes are significantly different from the zero mean latac curves, means that the car is behaving in a non-linear way, so it is not sure that the simple 3 degree-of-freedom linear concept will still be applicable. Excellent curve fits were obtained, however, illustrating that this effective piecewise linear approach can be used. Fig. 9.2.3 shows the results. Examination of the frequencies and damping ratios in Table 9.2.1 shows that the flatter yaw rate curve is largely due to the combined effect of an increase in $\omega_r$ and $\phi_r$ and a larger increase in $\omega_L$, the lead term corner frequency.

9.3 Steer Response Results for Car F.

Figs 9.3.1 and 2 show curves fitted to the results for the two runs on Car F shown in Figs 8.3.2,3 and 4. These fits were both achieved by first fitting curves to the yaw rate and roll angle data and then fitting to the latac data using the established COEFFS 1 to 4. The curve fits thonselves and the repeatability between the two separate test runs are seen to be excellent. Superimposed on both these Figs. (dashed curves) are the curves fitted to the sinusoidal input data of Fig. 6.2.4, and the agreement between the two methods is again seen to be good. The COEFFS and frequencies and damping ratios corresponding to these fits are shown in Tables 9.3.2 and 1. Also shown in the Tables are the results obtained by fitting curves to all three sets of data together for the data of Fig. 9.3.1.

Both sets of fitted curves shown in Fig. 9.3.1 and 2 and the curves obtained by fitting to all three variables together, Fig. 9.3.3, provide satisfactory representation of the vehicle's frequency responses, and it
Table 9.3.1 Characteristic corner frequencies and damping ratios, Car F.

<table>
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<tr>
<th>COEFF NO.</th>
<th>RUN 1131 LATA Fitted SEPARATELY FIG. 9.3.1</th>
<th>RUN 1130 LATA Fitted SEPARATELY FIG. 9.3.2</th>
<th>SINE INPUT DATA FIG. 6.2.4</th>
<th>RUN 1131 3 CURVES FITTED TOGETHER FIG. 9.3.3</th>
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<td>226 (0)</td>
<td>236 (4)</td>
<td>222 (2)</td>
<td>227 (.5)</td>
</tr>
<tr>
<td>15</td>
<td>992 (0)</td>
<td>1180 (19)</td>
<td>1060 (7)</td>
<td>1190 (40)</td>
</tr>
<tr>
<td>16</td>
<td>12200 (0)</td>
<td>12300 (10)</td>
<td>9800 (13)</td>
<td>10400 (7)</td>
</tr>
</tbody>
</table>

Table 9.3.2 COEFFS for fits to Car F, and percentage differences.
Fig. 9.3.1. Curve fits to data for Car F, 50 mile/h, and curves from sine input data of Fig. 6.2.4.
Fig. 9.3.2. Curve fits to data for Car F, 50 mile/h., and curves from sine input data of Fig. 6.2.4.
Fig. 9.3.3. Curve fits to data for Car F, 50 mile/h., all three curves fitted together.
is not possible from the curves above to make a confident judgement as to which is to be preferred. However, as is seen in the previous Section, there are significant differences in the corresponding frequencies and damping ratios, and these can be considered in relation to their physical significance in vehicle terms. In particular, experience has shown that the yaw rate natural frequency ($\omega_1$) is likely to lie in the region of 1 Hz with a damping ratio of about 0.7, and that the roll angle natural frequency ($\omega_4$) is likely to be higher than this with a damping ratio of about 0.3. With this in mind the results of Fig. 9.3.3 and of the fits to the sine input data are not very realistic, and mean that the curve fitting procedure has found an unacceptable subsidiary minimum. Luckily, it is reasonably easy to spot that this has occurred, but it is clear that the results of any curve fitting must be looked at critically, and that it is desirable to have data available from more than one test. Because the function being fitted is relatively complicated and may not even be capable of fitting the data exactly, it is not unexpected that problems of this type can occur, and a theoretical investigation would probably indicate that the problem was rather ill conditioned. An a priori idea about the result required must thus be regarded as an essential part of the fitting procedure. Provided it is not abused this is an acceptable condition.

In terms of the $\omega$'s and $J$'s, the form of the subsidiary minimum discussed above, which has also been seen to occur for the other sets of data, is seen principally as a swapping round of $J_r$ and $J_\phi$ and an increase in $\omega_\phi$. In terms of COEFFS 1 to 8 (the yaw rate curve), C5 is seen to be about correct but the others are all too large, C1 and C6 being usually in error by the greatest amount. With the prior knowledge that $C_1 = 16$ was a reasonable value for this data, a curve fit to the sine input data was carried out with $C_1$ fixed at this level, and a result very similar to that of Fig. 9.3.1 was obtained. This was an interesting exercise but unfortunately not very useful in the general case where no prior information about the COEFFS is available. Although the combined fitting of all three frequency response curves has been seen to be more likely to result in a subsidiary minimum of this type, it is not inevitable that this will occur, and in fact a perfectly satisfactory result was obtained in this way from the data of Fig. 9.3.2.
It is interesting to note that for this car the sine input data gave less realistic results than the transient input data. This could be due to the manually generated input being non-sinusoidal, or one or two of the points in key positions for the curve fitting being particularly poor, and highlights the particular advantages of the transient technique where the exact form of the input is not important and the short duration of the test means that several repeat tests can be done.

So far in the examination of curve fitting only amplitude data has been considered. This has been done quite deliberately because:

1) Section 5 showed that phase angle information was not particularly helpful in curve fitting and added to the computer time and space required.
2) Section 4 showed that the agreement between the measured phase angle information and the 3 degree of freedom model was not very good at high frequencies.

However, it is important to look at the phase angle information, and Fig. 9.3.4 shows the phase angles corresponding to the yaw rate (6/3), roll angle (7/3), and latac (9/3) fitted curves of Fig. 9.3.1, with the actual phase angle data superimposed (dashed lines) for comparison. It can be seen that the latac curve agrees well, the roll angle curve is good until about 1.5 Hz, but as the frequency increases above this level the measured data shows increasingly greater phase angles, and the yaw rate curve shows the biggest discrepancy with measured data having 40 degrees more phase lag at 3 Hz. These differences are broadly similar to those found in Section 4, where they were attributed to the effects of steering system and tyre dynamics. It has already been seen that the amplitude frequency response curves relative to the road wheels can be quite different from the handwheel curves, and if the above reasoning is correct it would be expected that the phase angles from the road wheel data would be in better agreement with the 3 degree of freedom model, the only remaining source of error being the tyre dynamics.
Fig. 9.3.4. Phase angles corresponding to fits of Fig. 9.3.1, and measured phase angles. Car F, 50 mile/h.
Fig. 9.3.5 shows curves fitted to amplitude, relative to road wheel angle data from the same test run as Fig. 9.3.1, and Fig. 9.3.6 shows the corresponding phase angle curves with the actual phase angle data superimposed. Tables 9.3.3 and 4 show the frequencies, damping ratios, and COEFFS corresponding to these fits, and the differences between these and those for the handwheel angle curves of Fig. 9.3.1, are seen to be very similar to the differences found in the results for Car E given in Section 9.2. As predicted the agreement between the phase angle curves and the measured data is much better than for the handwheel case. At 3 Hz the error in the yaw rate curve is now reduced to about 20 degrees.

Phillips, Ref. 65, has published some information on the response of tyres to sinusoidal steer inputs, from which it is possible to see whether the above phase lags can be reasonably attributed to the tyres. Fig. 9.3.7 is extracted from Ref. 65 and shows the amplitude and phase responses of 5 different types of tyres. Frequency on this Fig. is given as rad/ft, as tyres are found to be distance rather than time sensitive, but this can readily be converted to Hz for any specified vehicle speed. For example, 3 Hz at 50 mile/h represents 0.25 rad/ft. Phillips states that these curves are typical of the type associated with a heavily damped second order system and gives a table of the equivalent natural frequencies and damping ratios. These Figures together with an identification of the tyre type and the equivalent frequencies in Hz at 50 mile/h are given in Table 9.3.5. The tyres fitted to Car F for the results discussed above were of type A, and the phase lag for the equivalent Phillips tyre at 0.25 rad/ft (3 Hz at 50 mile/h) is seen from Fig. 9.3.7 to be about 10 degrees. This is half the figure obtained above but is clearly of the correct order. Exact agreement is not to be expected as the tyre and its operating conditions were not the same for the two cases. To keep the phenomenon in perspective it is also worth noting that 10 degrees of phase shift at 3 Hz is equivalent to a time shift of one hundredth of a second.

These Phillips results indicate that metal braced radial ply tyres would give significantly more phase shift than the cross ply tyres whose results were examined above. As it was also of interest to see the changes in transfer function shape produced by such a change of tyres,
Table 9.3.3 Characteristic corner frequencies and damping ratios, Car F, Run 1131.

<table>
<thead>
<tr>
<th>COEFF NO.</th>
<th>RELATIVE TO HAND WHEEL</th>
<th>RELATIVE TO ROAD WHEELS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FIG. 9.3.1</td>
<td>FIG. 9.3.5</td>
</tr>
<tr>
<td>$\omega_r, p_r$</td>
<td>$1.3, -.68$</td>
<td>$1.0, -.83$</td>
</tr>
<tr>
<td>$\omega_q, p_q$</td>
<td>$1.6, -.27$</td>
<td>$1.6, -.39$</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>$0.60$</td>
<td>$0.69$</td>
</tr>
<tr>
<td>$\omega_{cr}, p_{cr}$</td>
<td>$1.6, -.26$</td>
<td>$1.6, -.37$</td>
</tr>
<tr>
<td>$\omega_{cq}, p_{cq}$</td>
<td>$1.6, -.14$</td>
<td>$1.6, -.19$</td>
</tr>
<tr>
<td>$\omega_{ce}, p_{ce}$</td>
<td>$1.7, -.086$</td>
<td>$1.8, -.097$</td>
</tr>
<tr>
<td>$\omega_{ce}, p_{ce}$</td>
<td>$1.5, -.35$</td>
<td>$1.5, -.39$</td>
</tr>
</tbody>
</table>

Table 9.3.4 COEFFS for fits to data for Car F, Run 1131, and percentage difference.
STEER RESPONSE

<table>
<thead>
<tr>
<th>SP SS RESP</th>
<th>YAW RATE</th>
<th>ROLL ANGLE</th>
<th>LATAC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.62 -0.388</td>
<td>4.934e+00</td>
<td>2.688e+00</td>
<td>2.436e-01</td>
</tr>
<tr>
<td>1.04 -0.828</td>
<td>0.69 -1.000</td>
<td>1.59 -0.189</td>
<td>1.77 -0.097</td>
</tr>
<tr>
<td>1.04 -0.828</td>
<td>1.61 -0.369</td>
<td>1.59 -0.189</td>
<td>1.48 -0.392</td>
</tr>
<tr>
<td>1.62 -0.388</td>
<td>1.61 -0.369</td>
<td>1.77 -0.097</td>
<td>1.48 -0.392</td>
</tr>
</tbody>
</table>

Fig. 9.3.5. Curve fits to "relative to road wheels" data. Car F, 50 mile/h.
Fig. 9.3.6. Phase angles corresponding to fits of Fig. 9.3.5, and measured phase angles. Car F, 50 mile/h.
Test conditions: Steering input $\beta = \beta_0 \sin \left( \frac{\omega}{v} \right)$

$\beta_0 = 2^\circ$  \hspace{0.5cm} $v = 1 \text{ ft/s}$  \hspace{0.5cm} $F_3 = 500 \text{ lbf}$  \hspace{0.5cm} $p = 25 \text{ lbf/in}^2$

Tyre A = $\Delta$  \hspace{0.5cm} Tyre B = $\circ$  \hspace{0.5cm} Tyre C = $\bigcirc$  \hspace{0.5cm} Tyre D = $\diamond$  \hspace{0.5cm} Tyre E = $\vee$

Lateraforce responses of the test tyres to sinusoidal steering input

Fig. 9.3.7. (Takon from Ref. 65).
<table>
<thead>
<tr>
<th>TYRE TYPE</th>
<th>A CROSS PLY</th>
<th>B LOW PRO-BIAS PLY</th>
<th>C BELTED PLY</th>
<th>D METAL BRACED PLY</th>
<th>E RADIAL PLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>NAT. FREQ., rad/ft</td>
<td>4.2</td>
<td>5.5</td>
<td>4.0</td>
<td>3.2</td>
<td>2.5</td>
</tr>
<tr>
<td>Hz AT 50 mile/h</td>
<td>48.8</td>
<td>63.9</td>
<td>46.5</td>
<td>37.1</td>
<td>29.0</td>
</tr>
<tr>
<td>DAMPING RATIO</td>
<td>1.7</td>
<td>2.0</td>
<td>1.9</td>
<td>1.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 9.3.5 Tyre data from Ref. 65 corresponding to Fig. 9.3.7

A further series of tests were carried out on Car F. The vehicle conditions were the same except that metal braced radial ply tyres were fitted and inflated to 24 lb/in² front, 28 lb/in² rear (the manufacturer’s recommended pressures). Fig. 9.3.8 shows curves fitted to the “relative to handwheel” frequency responses derived from a test on these tyres at 50 mile/h. The fitted curves of Fig. 9.3.1, the corresponding curves for the cross ply tyres, are superimposed for comparison. It is seen that the largest difference is probably in the steady state response, which can, of course, be more sensibly measured from a steady state test. The curves are slightly different in shape, however, with the yaw rate curve for the radial ply tyres having a flatter shape up to about 0.8 Hz followed by a rather sharper peak and then a similar high frequency response. The roll angle curve for the radial tyres also shows an interesting difference in a steeper fall off at around 1.5 Hz and the characteristic ‘kink’ occurring about 6 dB further below the steady state level than for the cross ply curve.

Fig. 9.3.9 shows the corresponding “relative to roadwheel curves”, with the flatter shape which would be expected from previous results, and Fig. 9.3.10 shows the phase angles corresponding to these fitted curves with the measured phase angle data superimposed. As predicted by the tyre information the measured phase angles are further from the curves than was the case for the cross ply tyres, Fig. 9.3.6. For the yaw rate the error at 3 Hz is now about 40 degrees compared to the 20 degrees seen in Fig. 9.3.6. This is a most interesting and encouraging result, confirming both the sensitivity of the method and the validity of the reasons postulated for the occurrence of this phase angle discrepancy.
Fig. 9.3.8. Curve fits to data for Car F, 50 mile/h, radial ply tyres, and curves of Fig. 9.3.1., cross-ply tyres.
### STEER RESPONSE

<table>
<thead>
<tr>
<th>SP/SS RESP</th>
<th>POLES</th>
<th>ZEROS</th>
<th>LATAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.17 - 0.399</td>
<td>1.87 - 1.000</td>
<td>1.70 - 0.192</td>
</tr>
<tr>
<td></td>
<td>1.58 - 0.927</td>
<td>1.10 - 0.498</td>
<td>1.70 - 0.192</td>
</tr>
<tr>
<td></td>
<td>1.58 - 0.927</td>
<td>1.10 - 0.498</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.87 - 0.399</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 9.3.9.** Curve fits to "relative to road wheels" data.
Car F, 50 mile/h., radial ply tyres.
Fig. 9.3.10. Phase angles corresponding to fits of Fig. 9.3.9, and measured phase angles. Car F, 50 mile/h, radial ply tyres.
As a final experiment tests were carried out on this car, with the radial ply tyres, at 30 and 70 mile/h. Figs. 9.3.11, 12, 13, and 9.3.14, 15, 16 show the amplitude relative to handwheel, amplitude relative to roadwheel, and phase angles relative to roadwheel (equivalent to Figs. 9.3.8, 9, 10 for the 50 mile/h case) for the 30 and 70 mile/h cases respectively. As the tyres are distance sensitive their effective natural frequency (in the Phillips model) increases with speed, and so it would be expected that the phase angle discrepancy would reduce with increasing speed. In fact the 70 mile/h results, Fig. 9.3.16 do show significantly less error than the 50 mile/h case, Fig. 9.3.10, but the 30 mile/h results, Fig. 9.3.13, do not show a corresponding increase over the 50 mile/h case. This may be influenced by the curve fit to the 30 mile/h data, which, as previously found with the sinusoidal input data for Car A in Section 6, provided the least satisfactory result. It may be that at this low speed the tyres are beginning to cause some loss of amplitude, and in fact the fall off of the yaw rate curve (6/4 Fig. 9.3.12) at the highest frequencies is rather greater than the 6 dB per octave expected for the three degree of freedom model. The fitted curve has achieved this rate of fall off by having an unrealistically high value for $\omega_0$.

Examination of the handwheel amplitude curves, Figs. 9.3.8, 11, 14, shows the usual effect of speed with the very pronounced peak on the yaw rate curve (6/3) for the 70 mile/h case. This peak is much narrower than that for the Car A results shown in Section 6.

The COEFFS corresponding to all the curve fits to the test results for the radial ply tyres are shown in Table 9.3.6. It can be seen from these and from the frequencies and damping ratios shown on the Figs. that unrealistic results have been obtained for both the 30 and 50 mile/h results. The problem is the same as has been seen previously, with very high values for $\omega_0$ and the effective swopping round of $\mu$ and $\eta$. From the 70 mile/h case, where realistic COEFFS and frequencies have been obtained, the COEFFS change in the direction which would be expected theoretically as speed is reduced, but then settle in a subsidiary minimum with the yaw rate COEFFS (for example) all too large. As has already been indicated it is evidently difficult to be confident of
Fig. 9.3.11. Curve fits to data for Car F, 30 mile/h, radial ply tyres.
STEER RESPONSE

5.975 PS DIRECT 0

SP SS RESP         YAW RATE         ROLL ANGLE         LATAC
POLES              ZEROS             ZEROS             ZEROS
2.18 -0.684        4.83 -1.000      1.78 -0.200       1.91 -0.048
1.25 -0.335        1.20 -0.345      1.78 -0.200       1.95 -0.325
1.25 -0.335        1.20 -0.345      1.78 -0.200       1.95 -0.325
2.18 -0.684

Fig. 9.3.12. Curve fits to 'relative to road wheels' data.
Car F, 30 mile/h, radial ply tyres.
Fig. 9.3.13. Phase angles corresponding to fits of Fig. 9.3.12, and measured phase angles. Car F, 30 mile/h, radial ply tyres.
Fig. 9.3.14. Curve fits to data for Car F, 70 mile/h, radial ply tyres.
Fig. 9.3.15. Curve fits to 'relative to road wheels' data.
Car F, 70 mile/h, radial ply tyres.
Fig. 9.3.16. Phase angles corresponding to fits of Fig. 9.3.15, and measured phase angles. Car F, 70 mile/h, radial ply tyres.
<table>
<thead>
<tr>
<th>COEFF NO.</th>
<th>30 mile/h REL. TO HAND WHEEL</th>
<th>30 mile/h REL. TO ROAD WHEEL</th>
<th>50 mile/h REL. TO HAND WHEEL</th>
<th>50 mile/h REL. TO ROAD WHEEL</th>
<th>70 mile/h REL. TO HAND WHEEL</th>
<th>70 mile/h REL. TO ROAD WHEEL</th>
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<tbody>
<tr>
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<td>26.4</td>
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<td>11.8</td>
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<td>6.75</td>
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<td>10300</td>
<td>9360</td>
<td>9330</td>
<td>11900</td>
<td>12200</td>
</tr>
</tbody>
</table>

Table 9.3.6 COEFS for fits to data for Car F on radial ply tyres.

fitting curves to 30 mile/h data and achieve a realistic set of frequencies and damping ratios. This is probably due to the flatter and less characteristic shape of the lower speed curves. However, this is the first set of data where it has not been reasonably easy to obtain good results for the 50 mile/h case. Of two repeat 50 mile/h tests carried out at the same time as that whose results are shown here (both of which gave good repeatability in terms of the shape of the amplitude transfer functions), one gave a very similar curve fit but the other gave a more realistic result. Unfortunately, both of these tests suffered an error in time synchronisation between data channels at the replay stage, resulting in wrong phase information, so that the results could not be reasonably used. It is probable that a further series of tests would have produced a satisfactory result.
For all the curve fits shown in this Section the fitted curves
give a good representation of the vehicle's steer frequency response,
and it will be shown in the following Section that these curves can be
used to calculate the response of the vehicle to any specified time
input. However, there can clearly be cases, particularly at low speeds,
when a good curve fit is obtained with an unrealistic set of COEFFS and
frequencies and damping ratios. Experience with a wider range of vehicles
is required before the extent of this difficulty can be assessed. Luckily,
the unrealistic results can be readily recognised and so there does not
seem to be much danger of being misled.
10. CALCULATION OF TIME RESPONSE FROM FREQUENCY RESPONSE.

10.1 Introduction.

Thus far, analytic expressions for vehicle response transfer functions have been obtained, (a) by calculation from vehicle parameters and (b) by the fitting of curves of defined form to frequency response data produced by either sinusoidal steer input or Fourier analysis of 'random' steer input tests. The object of this Section is to complete the picture by using the analytic expression for the transfer function to calculate the time response of the vehicle to any defined time input. In addition to this general capability this should enable the validity of the transfer function produced by the curve fitting to be confirmed, by re-calculating the time responses from which the frequency response data was derived.

10.2 Theory.

The problem is to calculate the vehicle time response $R(t)$ to a time input $I(t)$ using the transfer function $H(\omega)$ (expressed as a function of frequency, $\omega$). Using the same concept as discussed in Section 8, the solution to this can be represented as shown in Fig. 10.2.1.

A computer programme - TIMERESP - was written to carry out this process, and a listing (without the standard procedures), and data input and output specifications are given in Appendix 8. In outline the operation of the programme is as follows:

1) Read in time input series, $I(t)$, with $N (=2^n)$ points at time interval $\Delta t$.
(2) Produce transformed series, $I(\omega)$, (in amplitude and phase form) with $N/2$ points at frequency intervals $\Delta f = 1/T$.

(3) Read in 16 COEFFS defining the vehicle response transfer function $H(\omega)$.

(4) At each frequency $n\Delta f$ ($n = 0 - N/2$) calculate the amplitude and phase of $H(\omega)$ and multiply by and add the corresponding amplitude and phase of $I(\omega)$ to produce the series $R(\omega)$.

(5) Carry out the reverse transformation on $R(\omega)$ to give the desired time response $R(t)$.

To allow comparison with the measured time histories shown in Section 8 this time response $R(t)$ and time input $I(t)$ are then plotted in a similar format to those shown in Section 8.

10.3 Car F Results.

Fig. 10.3.1 is a repeat of Fig. 8.3.2.1 showing the time histories for a test at 50 mile/h on cross ply tyres. Fig. 10.3.2 - 6 show yaw rate, roll angle, and lateral time responses to the actual input of Fig. 10.3.1, calculated by TIMERESP using the vehicle transfer functions derived from the curve fits listed in Table 10.3.1.

<table>
<thead>
<tr>
<th>FIG. NO.</th>
<th>SOURCE CURVE FIT</th>
<th>REALISM OF CURVE FIT PARAMETERS</th>
<th>OTHER DETAILS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.3.2</td>
<td>Fig. 9.3.1, Table 9.3.2</td>
<td>Good</td>
<td>Transform from same test.</td>
</tr>
<tr>
<td>10.3.3</td>
<td>Fig. 9.3.3, Table 9.3.2</td>
<td>Poor</td>
<td>Transform from same test.</td>
</tr>
<tr>
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<td>Fig. 9.3.2, Table 9.3.2</td>
<td>Good</td>
<td>All 3 curves fitted together.</td>
</tr>
<tr>
<td>10.3.5</td>
<td>Fig. 6.2.5, Table 6.2.2</td>
<td>Poor</td>
<td>Transform from sine input data.</td>
</tr>
<tr>
<td>10.3.6</td>
<td>Fig. 9.3.5, Table 9.3.4</td>
<td>Good</td>
<td>All 3 curves fitted together.</td>
</tr>
</tbody>
</table>

Table 10.3.1

All the yaw rate and roll angle calculated time histories are virtually identical, and so nearly identical to the actual measured
Fig. 10.3.1. Time histories for a test on Car F, 50 mile/h, cross ply tires.
(As Fig. 8.3.2.1)
Fig. 10.3.2. Calculated time responses to input of Fig. 10.3.1.
(See Table 10.3.1 for details).
Fig. 10.3.3. Calculated time responses to input of Fig. 10.3.1. (See Table 10.3.1 for details)
Fig. 10.3.4. Calculated time responses to input of Fig. 10.3.1.
(See Table 10.3.1 for details)
**VEHICLE RESPONSE**

29 3 75 P1 STPT= 1 NTH= 2 E= 9 DT= 0.050 080175 RUN1131

VEHICLE CASE SINE1012 120175P4

![Graphs showing time responses](image)

**Fig. 10.3.5.** Calculated time responses to input of Fig. 10.3.1, (see Table 10.3.1 for details)
Fig. 10.3.6. Calculated time responses to input of Fig. 10.3.1.
(See Table 10.3.1 for details)
curves that superimposition of the curves is not useful. (The calculated roll angles do not have the small amount of drift which arises on the "measured" curve from the integration process). Similar remarks apply to the latac curves for the low frequency parts, but in the 10 to 15 second area where the input frequency is highest, all the latac calculated curves show a small reduction in amplitude compared to the measured curve. As the frequency in this region ranges from about 5 to 7Hz, it is well outside the range being studied, and so this discrepancy is not serious. Presumably it arises from the latac fitted curve falling below the data at these frequencies. This is not unexpected as inspection of the curve fits typically shows the latac fitted curves to be already slightly below the data at 3Hz. It is also worth noting that Figs. 10.3.1 - 6 are plotted digitally at a rate of 20 points per second, so that there are only 3 points per cycle at 7Hz and the visual representation of the waveform is not good.

Fig. 10.3.2 confirms that the concepts and techniques being used are valid, by illustrating the cyclic procedure of calculating a transfer function from time input and response data, smoothing this transfer function by curve fitting, and then re-calculating the time response from the original time input and the smoothed transfer function. Fig. 10.3.3 illustrates the same procedure carried out using a curve fit with a less realistic set of defining parameters but providing a perfectly satisfactory fit to the data, and shows that, as might well be expected, practical representation of the vehicle response requires only that the shape of the transfer function curves be correct and is not influenced by the parameters defining the equations of the curves. In fact the high frequency part of the latac response is rather better on Fig. 10.3.3 than on Fig. 10.3.2, due to the better curve fit at high frequency shown on Fig. 9.3.3 compared to Fig. 9.3.1.

Fig. 10.3.4 demonstrates the more general use of the procedure by carrying out the same process using a transfer function derived from a quite separate set of test results. (As this transfer function has already been shown to be very similar to the previous one it is not surprising that an equally good result is obtained). Fig. 10.3.5 shows the use of the transfer function produced from the sinusoidal
input results, and Fig. 10.3.6 demonstrates the cyclic procedure of
Fig. 10.3.2 this time using the 'relative to road wheels' transfer
function and the road wheel input data.

Fig. 10.3.7 shows the time histories of the 30 mile/h test on
radial ply tyres from which the transfer function of Fig. 9.3.11 was
derived. The parameters defining this transfer function were particularly
unrealistic although the curve fit was good, and the responses
calculated by TILiRESP, Fig. 10.3.8, demonstrate again that the latter
is an adequate condition for providing a good representation of the
vehicle response characteristics.

In order to carry out a more stringent test on this technique,
a series of step response tests were carried out on Car F at 50 mile/h
on the cross ply tyres, and a typical set of measured time histories is
shown in Fig. 10.3.9. Fig. 10.3.10 shows the time responses to the
input of Fig. 10.3.9 calculated using the same transfer function as
used for Fig. 10.3.2 (see Table 10.3.1). In this case the calculated
and measured responses are not quite identical and the difference can
be seen from the dashed curves superimposed on Fig. 10.3.10. The drift
has been removed from the 'measured' roll angle curve. Some of the
difference is understandably due to the effect of noise and spurious
input on the measured curves, but there is also a tendency for the
calculated responses to show a slightly higher yaw rate overshoot.
Fig. 10.3.11 shows a similar set of calculated responses using the
transfer function from the sinusoidal input tests, which was also used
for Fig. 10.3.5. These curves are very similar to those of Fig. 10.3.10
although there is a tendency for the yaw rate overshoot to be nearer to
the measured level. It is interesting that the two transfer functions
do give slightly different results despite the fact that, for the yaw
rate in particular, the frequency response curve shapes are very similar
over the frequency range examined. Unexamined differences in the curves
at higher frequencies may be responsible for this. In general terms the
discrepancies between these two sets of results and the measured results
are not very large and, without carrying out a full examination, these
results are considered to be satisfactory.
VEHICLE RESPONSE RUN 1090

Fig. 10.3.7. Time histories for a test on Car F, 30 mile/h, radial ply tires.
Fig. 10.3.8. Calculated time responses to input of Fig. 10.3.7.
Fig. 10.3.9. Time histories for measured stop response tests on Car F, 50 mile/h, cross-ply tyres.
Fig. 10.3.10. Calculated time responses to input of Fig. 10.3.9. Measured curves superimposed (dashed).
Fig. 10.3.11. Calculated time responses to input of Fig. 10.3.9. Measured curves superimposed (dashed).
10.4 Car E Results.

Fig. 10.4.1 is a repeat of Fig. 8.3.1.1 and shows the time histories for a test on Car E at 50 mile/h. As already discussed considerable noise is evident on these traces (particularly the roll rate) and the transfer function, Fig. 9.2.1, also showed a large amount of scatter. Despite this the yaw rate and lateral responses calculated from the curves fitted to the data on Fig. 9.2.1 are in excellent agreement with the measured curves. Fig. 10.4.2 shows the results with the parts of the measured curves which differ from the calculated levels shown dashed where the difference is large enough to show. As expected the roll angle curves show significant differences (although part of this is probably due to drift in the integration), due to the poor quality of the measured data.

10.5 Calculated Step Input Responses.

TIMERESP is also capable of calculating vehicle response to a simulated input, and Fig. 10.5.1 shows the response of Car F at 50 mile/h, on the cross ply tyres, to the inputs shown, which have three different rise (or fall) times. Very little difference would be seen if the 250 degree/sec curve were superimposed on the 500 degree/sec curve, but the 120 degree/sec result is significantly different. For this car at least, a steer input rate of 400 degree/sec, which was quoted in the USA Experimental Safety Vehicle standard as the minimum rate for an effective 'step' input, would seem to be quite adequate. The 'kink' in the lateral curve corresponding to the peak in the yaw rate response is interesting.

The yaw rate and roll angle responses of cars A, E, and F to the same 500 degree/sec steer input are shown on Fig. 10.5.2. A major difference lies in the steady state responses, but the transient yaw rate curves are also significantly different. Peak to steady state ratios for the three cars are shown below along with the Fig. number where the transfer function is plotted.
As expected the cars with the flatter yaw rate response functions show the smaller amounts of overshoot, but the significant factor is that the different frequency response curves do mean real differences in vehicle behaviour.
Fig. 10.4.1. Time histories for a test on Car E, 50 mile/h.
(As Fig. 8.3.1.1)
Fig. 10.4.2. Calculated time responses to input of Fig. 10.4.1. Measured curves superimposed (dashed).
Fig. 10.5.1. Calculated step input responses, Car F, 50 mile/h, cross-ply tyres.
VEHICLE RESPONSE

Fig. 10.5.2. Calculated step input responses, Car A, E, and F, 50 mile/h.
11. DISCUSSION AND CONCLUSIONS.

It was clear from a relatively early stage in this work that experimentally produced steer frequency response curves were of the same form as those calculated theoretically using a linear, three degree of freedom model. Therefore seemed a good possibility of describing the response behaviour of a real vehicle, in terms of the comparatively simple and analytic parameters of an equivalent three degree of freedom model. Albeit using a cumbersome manual curve fitting process, this was shown to be possible in Section 4, where characteristic corner frequencies and damping ratios ($\omega^c$s and $\zeta$s) were established for two vehicles, using data produced from sinusoidal steer input experiments.

With this background a much more sophisticated, digital curve fitting procedure was developed, and proved capable of giving good fits of three degree of freedom curves to a range of experimentally measured vehicle steer frequency responses. That these curves did indeed represent the vehicle response characteristics was demonstrated by using the model represented by the curves to calculate the vehicle response (in the time domain) to a defined steer input, and showing that this response was the same as that measured experimentally. Although this fitting procedure almost always gave curves which fitted the data well, and was extremely consistent in the result obtained for a given set of data fitted in a specified way, the curve fitting problem did sometimes show signs of being rather ill conditioned. The typical manifestation of this was that two sets of curves could be obtained (by, for example, fitting curves to two sets of data obtained from repeat experiments on the same car in the same conditions), which looked very similar, but which had quite large differences in their describing parameters. Fortunately one of these parameters was often not very acceptable physically and so could be rejected, but that this can occur clearly means that care is required in using a given set of parameters to define a vehicle's response.

The parameters directly used as variables in the curve fitting procedure, are the set of up to 16 coefficients (COEFFS) which are the coefficients of the numerator and denominator polynomials defining the vehicle response transfer functions. The $\omega^c$s and $\zeta$s, which are more easily interpreted in physical terms, are obtained from the COEFFS by
extracting the roots of the polynomials involved. It would be possible to re-write the fitting procedure so that the $\omega$'s and $\gamma$'s were the direct variables used in the fitting; and examination of some of the results obtained with the current method indicates that this might make the problem less ill conditioned. An advantage would be that it would be easier to define bounds for the $\omega$'s and $\gamma$'s. An investigation along these lines would be an interesting extension to this work and might result in a better curve fitting procedure. Experience with results from more cars would be useful in establishing to what extent a serious problem exists.

The conclusion that a vehicle's steer frequency response can be represented by a three degree of freedom model is only strictly valid for the amplitude part of the response (and also only for the low frequency range being considered, although this is probably rather academic), and it has been seen that higher phase lags are obtained than would be predicted by the model. This is attributable to steering system and tyre dynamics. When the steering system is eliminated by looking at frequency response relative to the road wheels, the remaining extra phase lag was seen to be appropriate to that predicted by examination of tyre frequency responses. It was most interesting to find that the expected greater phase lag with radial ply tyres rather than cross ply tyres was readily measured.

A major part of the work of this Thesis was the development of the test and analysis techniques for the calculation of frequency responses from a short transient test. With good test conditions it was shown that good measures of frequency responses could be obtained, which agreed well with data obtained by the more cumbersome but probably more accurate sine input type of test. Important factors influencing the quality of the results were the frequency content of the input and the level of noise in the form of spurious inputs to the vehicle from, for example, road surface roughness. The calculation of the frequency response data was done by the simplest possible method of Fourier analysis (using an FFT algorithm) of input and response followed by division of one by the other. A frequently used method of calculating frequency responses in the case of a random input, and particularly in the presence of noise, is to use the cross-correlation between input and output. Because of the satisfactory results obtained here by the straightforward approach this method was not investigated. Discussion with other workers who have tried
to use this approach for vehicle steer frequency response calculations, however, indicates that results of the quality shown here are not obtained, and it seems probable that care in the test procedure itself is more likely to be fruitful than more sophisticated analysis techniques. This may be because of small non-linearities in the vehicle behaviour.

The special purpose instrumentation system developed during this work proved to be an invaluable asset in obtaining measurements of a consistently good quality. The combination of appropriate instrumentation and care in establishing and carrying out the test procedures is essential in producing good results from vehicle dynamics experiments. This may seem to be a self obvious statement, but the difficulties involved in onboard measurement, the number of factors which have to be controlled, and the rather emotive nature of carrying out vehicle 'handling' tests, results in a situation where the scope for errors is very large.

Because the available wind gust generator proved to be not very suitable for the frequency response approach used in this work, it was not possible to establish whether vehicle gust response behaviour could be described by an equivalent three degree of freedom model. However, the measurement techniques used could well be appropriate for other kinds of analysis, and the computer programmes for Fourier analysis and curve fitting of gust response data are available should a more suitable wind gust source be found.

A fundamental limitation of the work of this Thesis, in terms of the complete driver-vehicle-environment situation, is that only the open loop, steering fixed, vehicle response characteristics have been studied. It is known, for example, that a vehicle can exhibit a lightly damped oscillatory behaviour in the steering free situation, involving the steering system dynamics and usually heavily influenced by the inertia of the steering wheel itself. This situation would not be detected by a steering fixed test, but would probably cause a driver to dislike the vehicle. Fortunately this type of problem is not difficult to identify by other means and is not particularly common in practice. This current work has shown that, for the vehicles tested, the steering system dynamics in the steering fixed case, does not greatly influence the vehicle behaviour (or rather does not make the behaviour significantly different from that of a three degree of freedom model).
With an understanding of the limitations involved, the type of results produced here are highly appropriate for use in human factors work, where the ability of people to control systems described by their different frequencies and damping ratios is studied. A knowledge of the frequencies and dampings involved allows parallels to be drawn with work in the aircraft industry where, for example, pilot opinion contours have been drawn on a plot of natural frequency versus damping of one of the aircraft pitching modes. From the point of view of the vehicle development engineer, the ability to objectively quantify vehicle response and thus eliminate part of the subjective nature of his task, must, as it has done in other areas, enable development time to be reduced, and the combination of this experience and studies of the above type should eventually lead to the establishment of more exact guidelines for 'good' vehicle behaviour.
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APPENDIX 1. EXPANSION OF TRANSFER FUNCTION DETERMINANTS.

These determinants and their notation are defined in Section 2. They are expanded as follows:


$$
D = \begin{vmatrix}
ms - Y_w & -Y_q s - Y_q & mU - Y_r \\
- L_w & As^2 - L_q s - L_q & - L_r \\
-N_r & -N_q s - N_q & Cs - N_r
\end{vmatrix}
$$

$$
= (ms - Y_w)[(As^2 - L_q s - L_q)(Cs - N_r) - L_r(N_q s + N_q)] \\
+(Y_q s + Y_q)[-L_w(Cs - N_r) - L_r N_r] \\
+(mU - Y_r)[L_w(N_q s + N_q) + N_r(As^2 - L_q s - L_q)]
$$

$$
= (ms - Y_w)(ACs^3 - AN_s s^2 - L_q Cs^2 + L_q N_r s - L_q Cs + L_q N_r - L_r N_q s - L_r N_q) \\
+(Y_q s + Y_q)(-L_w Cs + L_w N_r - L_r N_r) \\
+(mU - Y_r)(L_r N_q s + L_r N_q + N_r As^2 - N_r L_q s - N_r L_q)
$$

$$
= mACs^4 \\
-\{mAN_r + mCL_q + ACY_r\} s^3 \\
+\{mL_q N_r - mCL_q - mL_r N_q + AN_r Y_r + CL_q Y_r - CY_q L_r + mUAN_r - AY_r Y_r\} s^2 \\
+\{mL_q N_r - mL_r N_q - Y_r L_q N_r + CY_q L_r s + Y_q L_r N_r + Y_q L_r N_r - Y_q L_r N_r - CY_q L_r s \\
-CL_q Y_r + mU L_q N_q - mU L_r N_q + Y_r L_r N_q + Y_r N_r L_q\} s \\
+\{Y_r L_q N_q - Y_r L_q N_r + Y_q L_r N_r - Y_q L_r N_r + mL_r N_q s + mU L_r N_q - mL_q L_r s + mL_q L_r s + mL_q L_r\}
$$

$$
= mACs^4 \\
-\{mAN_r + mCL_q + ACY_r\} s^3 \\
+\{m(L_q N_r - L_r N_q) + A(N_r Y_r - Y_r N_r) + C(L_q Y_r - Y_q L_r) + mU N_r - mCL_q s^2 \\
+\{m(L_q N_r - L_r N_q) + N_r(L_r N_q - L_q N_r) + C(Y_q L_q - Y_q L_r) \\
+Y_r(L_q N_r - L_r N_r) + mL_r N_r - mL_r N_r + Y_q(N_r L_q - L_r N_q)\} s \\
+\{Y_q L_r N_q - L_r N_r + Y_q N_r - L_r N_r + mL_r N_q - mL_r N_r + Y_r(N_r L_q - N_q L_r)\}
$$

A 1.2. Yaw Rate Numerator Determinant $DR$.

$$
DR = \begin{vmatrix}
ms - Y_r & -Y_q s - Y_q & Y_q \\
-L_r & As^r - L_q s - L_q & L_q \\
-N_r & -N_q s - N_q & N_q
\end{vmatrix}
$$

$$
= (ms - Y_r)(AN_{rs}^2 - L_q N_r - L_q N_q + L_r N_q) \\
+ (Y_q s + Y_q)(-L_q N_r + N_r L_q) + Y_q(L_r N_r + L_r N_q + AN_r s^2 - N_r L_q s = N_r L_q)
$$

$$
= mAN_{rs}^3
$$

or $DR = DR[0]s^3 + DR[1]s^2 + DR[2]s + DR[3] ...$ this defining $DR[0]$ etc.

A 1.3. Roll Angle Numerator Determinant $DP$.

$$
DP = \begin{vmatrix}
ms - Y_r & Y_r & mU - Y_r \\
-L_r & L_r & -L_r \\
-N_r & N_r & Cs - N_r
\end{vmatrix}
$$

$$
= (ms - Y_r)(CL_5 s - L_r N_r - L_r N_q) - Y_q(-CL_5 s + L_r N_r - L_r N_q) \\
+ (mU - Y_r)(-L_r N_r + L_r N_q)
$$

$$
= mCL_5^2
$$

or $DP = DP[0]s^2 + DP[1]s + DP[2] ...$ this defining $DP[0]$ etc.

A 1.4. Latac Numerator Determinant $DL$.

$$
DL = s * DV + U * DR
$$

$$
DV = \begin{vmatrix}
Y_r & -Y_q s - Y_q & mU - Y_r \\
L_r & As^r - L_q s - L_q & -L_r \\
N_r & -N_q s - N_q & Cs - N_r
\end{vmatrix}
$$
\[ Y_{\gamma}(ACS) - CL_q s^2 - CL_q s - AN_{\gamma} s^2 + L_q N_{\gamma} s + L_{\gamma} N_{\gamma} s - L_{\gamma} N_{\gamma} s - L_{\gamma} N_{\gamma} s \]

\[ -LY (CY_{\gamma} s^2 - CY_{\gamma} s + Y_q N_{\gamma} s + Y_{\gamma} N_{\gamma} s + mUN_{\gamma} s + mUN_{\gamma} s + Y_{\gamma} N_{\gamma} s) \]

\[ +N_{\gamma}(Y_q L_{\gamma} s + Y_q L_{\gamma} s + mUN_{\gamma} s + mUN_{\gamma} s + mUN_{\gamma} s + Y_{\gamma} L_{\gamma} s + Y_{\gamma} L_{\gamma} s) \]

\[ = ACY_{\gamma} s^3 \]

\[ +[C(Y_q L_{\gamma} s - Y_q L_{\gamma} s) + A(Y_{\gamma} N_{\gamma} s - Y_{\gamma} N_{\gamma} s) - mUN_{\gamma} s]^2 \]

\[ +[C(Y_q L_{\gamma} s - Y_q L_{\gamma} s) + Y_q (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) + Y_q (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) \]

\[ +mU(L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) + Y_{\gamma} (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s)]s \]

\[ +[Y_q (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) + Y_q (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) + mU(L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s) + Y_{\gamma} (L_q N_{\gamma} s - L_{\gamma} N_{\gamma} s)] \]

or \( DV = DV[0] s^3 + DV[1] s^2 + DV[2] s + DV[3] \ldots \) this defining \( DV[0] \) etc.

whence \( DL = s_* DV + U_* DR \)

\[ = DV[0] s^4 \]

\[ +(DV[1] + U_* DR[0]) s^3 \]

\[ +(DV[2] + U_* DR[1]) s^2 \]

\[ +(DV[3] + U_* DR[2]) s \]

\[ +U_* DR[3] \]

APPENDIX 2. INFLUENCE OF VEHICLE PARAMETERS ON TWO DEGREE OF FREEDOM TRANSFER FUNCTIONS.

The three degree of freedom equations given in Section 2, equations 2.4 - 2.6 are:

\[ m \ddot{v} + mU \dot{r} = Y_v v + Y_r r + Y_f \dot{r} \]  
\[ A \dot{\phi} = L_v \dot{v} + L_r \dot{r} + L_f \dot{r} \]  
\[ C \dot{r} = N_v v + N_r r + N_f \dot{r} \]  

A considerable simplification of these equations is obtained if the roll degree of freedom is neglected, and the analysis which follows considers the resulting two degree of freedom (v and r) equations. These are:

\[ m \ddot{v} + mU \dot{r} = Y_v v + Y_r r + Y_f \dot{r} \]  
\[ C \dot{r} = N_v v + N_r r + N_f \dot{r} \]  

The derivatives \( Y_r, N_r, Y_f, N_f \) are functions of speed, and as it is wished to use speed as a variable parameter it is convenient to make the substitutions:

\[ Y_r = Y_r/U, \quad N_r = N_r/U, \quad Y_f = Y_f/U, \quad N_f = N_f/U \]

This gives:

\[ m \ddot{v} + mU \dot{r} = Y_v v + Y_f r + UY_f \dot{r} \]  
\[ UC \dot{r} = N_v v + N_f r + UN_f \dot{r} \]  

Making the Laplace transformation and re-arranging, these equations become:

\[ [mU \dot{v} - Y_f v] + [mU \dot{r} - Y_r r] = UY_f \dot{r} \]  
\[ -N_v v + UC \dot{r} = UN_f \dot{r} \]

where \( s \) is the Laplace transform variable.

Solving these equations for \( r(s) \) and \( v(s) \) gives:

\[ r(s) = \left( \frac{D_1}{D_2} \right) \]  
\[ v(s) = \left( \frac{D_3}{D_4} \right) \]
where: 

\[ D_1 = \begin{vmatrix} (mU_s - Y_f^r) & UY_f^r & -N_f^r & UN_f^r \\ \end{vmatrix} \]

\[ D_2 = \begin{vmatrix} UY_f^r & (mU_{T} - Y_f^r) \\ UN_f^r & (UC_s - Y_f^r) \\ \end{vmatrix} \]

and 

\[ D = \begin{vmatrix} (mU_s - Y_f^r) & (mU_{T} - Y_f^r) \\ -N_f^r & (UC_s - Y_f^r) \\ \end{vmatrix} \]

By definition \( D_1/D \) and \( D_2/D \) are the yaw rate and lateral velocity transfer functions. By evaluating the determinants and re-arranging, the yaw rate transfer function can be written:

\[ \frac{\gamma(s)}{\delta(s)} = \frac{K[(1/\omega_c)s + 1]}{K_1[(1/\omega_c)s^2 + (2\omega_c/\omega_r)s + 1]} \]  \( \text{(11)} \)

where:

\[ K = U(Y_fN_f^r - Y_r^rN_f^r) \]  \( \text{(12)} \)

\[ K_1 = mU_{T}N_f^r + Y_fN_r^r - Y_f^rN_f^r \]  \( \text{(13)} \)

\[ \omega_c = K/(mU_{T}N_f^r) \]  \( \text{(14)} \)

\[ \omega_r = K_1/(mU_{T}C) \]  \( \text{(15)} \)

\[ 2\omega_r/\omega_r = -U(CY_f + mN_f^r)/K_1 \]  \( \text{(16)} \)

\( v(s)/\delta(s) \) can be similarly expressed, but the quantity of interest is the vehicle lateral acceleration, \( a \). This is the lateral acceleration with respect to a fixed axis system and is given by:

\[ a = \dot{\nu} + Ur \]

In Laplace form this is:

\[ a(s) = sv(s) + Ur(s) \]

which can be solved and written in the form:

\[ \frac{a(s)}{\delta(s)} = \frac{K_1[(1/\omega_c)s + 1]}{[(1/\omega_c)s^2 + (2\omega_c/\omega_r)s + 1]} \]  \( \text{(17)} \)

Consider the yaw rate transfer function of equation (11). This is made up of a first order lead term characterised by a corner frequency \( \omega_c \), and a second order lag term characterised by a corner frequency \( \omega_r \).
and damping ratio $j_r$. The denominator of equation (17) is the same as that of (11), and $\omega_r$ and $j_r$ can be considered to be the characteristic natural frequency and damping of the vehicle in yaw. It is interesting to examine the dependence of these quantities on vehicle parameters.

For this two degree of freedom model the stability derivatives can be expressed in terms of the effective cornering stiffnesses of the front and rear tyres, $Y_F$ and $Y_R$.

Thus:

\[ Y_r = Y_F + Y_R \]
\[ N_r = Y_F x_F + Y_R x_R \]
\[ Y_s = -Y_F \]
\[ N_s = -Y_F x_F \]
\[ Y_r = Y_F x_F + Y_R x_R \]
\[ N_r = Y_F x_F + Y_R x_R \]

where: $x_F$, $x_R$ are the distances from the centre of gravity to the front and rear wheels.

$L = x_F - x_R$ is the wheelbase.

Substituting these values into equations (12) and (16) yields, after some manipulation:

\[ \omega_r = -\frac{Y_R L}{m x_F} \] \hspace{1cm} (19)

\[ \omega_r = \left[ \frac{m u^2 (Y_F x_F + Y_R x_R) + Y_F Y_R L^2}{U(mC)^2} \right] \] \hspace{1cm} (20)

\[ j_r = \frac{-[m(Y_F x_F^2 + Y_R x_R^2) + C(Y_F + Y_R)]}{2[m U(x_F x_F + Y_R x_R) + Y_F Y_R L^2]^2 (mC)^2} \] \hspace{1cm} (21)

$\omega_r$ is seen to be simply related to vehicle parameters, and in particular to be inversely proportional to forward speed. $\omega_r$ and $j_r$ are related in a rather more complicated manner, but it is possible to consider the effects of very high or very low speed.

At very low speed (20) and (21) reduce to:
\[
\omega_r = \frac{L \left( \frac{V_F + V_R}{mC} \right)^2}{U^2} \tag{22}
\]

\[
J_r = \frac{-m(V_F x_F^2 + V_R x_R^2) + C(V_F + V_R)}{2(m^2 x_F + m x_R)^2}
\]

and at very high speed they become:

\[
\omega_r = \frac{(V_F x_F + V_R x_R)}{C} \tag{23}
\]

\[
J_r = \frac{-m(V_F x_F^2 + V_R x_R^2) + C(V_F + V_R)}{2m(C(V_F x_F + V_R x_R))^2}
\]

\(\omega_r\) is thus independent of speed at high speeds, and reduces with speed at low speeds, whereas \(J_r\) reduces with speed at high speeds and is independent at low speeds.
APPENDIX 3.  FREQRESP - COMPUTER PROGRAM LISTING

FREQRESP STEER AND GUST RESPONSE 38 32;
"BEGIN" "COMMENT" READS BASIC VEHICLE DATA, CALCULATES STABILITY
DERIVATIVES, STATIC MARGIN, DSTEER/DSLIP, DSTEER/DLATAC, TRANSFER
FUNCTIONS (GAIN AND PHASE V. FREQUENCY), AND CHARACTERISTIC
CORNER FREQUENCIES AND DAMPINGS;
"INTEGER" "ARRAY" NAME[1:100];
"INTEGER" I, J, K, M, N, Z, STRIPR, STPLT, GPRIN, GPLT, RUN, Y, N2,
ZERO, YB;
"REAL" "ARRAY" D[1:5], ADP[0:3], DR[0:3], DP[0:2],
DL[1:5], ADL[1:5], ACO, COEFFS[1:16];
"REAL" M, A, C, XE, XR, U, KSF, KCF, KSR, KCR, DYSDFP, DYSRDP, DYSDFP,
DRDP, DYPDFP, DDPDFP, DDPDFP, DDPDFP, DDPDFP, DDPDFP, DDPDFP,
DCSDT, DCRDT, DCYDT, EP, KPD, KDSF, KDCF, KDSR, KDCR, YFP, YRP, YV,
LV, NV, YPD, LRP, NRP, XR, LR, NR, YD, LD, ND, CON, YAT, LAT,
NAT, SM, DSDS, DESL, AY, AL, AN, B, PX1, PX2, PY1, PY2, APX1, APX2, APY1,
APY2, FR, W, P, Q, R, S, SCALE, G, START;
"COMMENT" INSERT PROCEDURES CUBIC, QUARTIC; (NOT LISTED HERE)

"PROCEDURE" AMP(P, Q, R, S, A);
"REAL" P, Q, R, S, A;
"BEGIN" "REAL" E, F;
E:=(P*R+Q*S)/(R*R+S*S);
F:=(Q*R-P*S)/(R*R+S*S);
A:=SQRT(E*E+F*F);
"END";
"PROCEDURE" PHASE(P, Q, R, S, PH);
"REAL" P, Q, R, S, PH;
"BEGIN" "REAL" E, F;
CHECKS( PHASE );
E:=CHECKR((P*R+Q*S)/(R*R+S*S));
F:=CHECKR((Q*R-P*S)/(R*R+S*S));
PH:=CHECKR(ARCTAN(F/E));
"IF" CHECKB(PH>0) "THEN"
"BEGIN" "IF" CHECKB(PH<0) "THEN" PH:=PH+57.3-180 "ELSE"
PH:=PH+57.3;
"END" "ELSE"
"BEGIN" "IF" CHECKB(F>0) "THEN" PH:=180+PH*57.3
"ELSE" PH:=PH*57.3;
"END";

"END";

"PROCEDURE" DRAFRAX(SCALE);
"REAL" SCALE;
"BEGIN" "REAL" A,X;
"INTEGER" N,B;
PUNCH(5); WAY(0,4);
SEORIGIN(800,0); MOVEPEN(-500,-224);
DRAWLINE(-500,-254); DRAWLINE(-470,-254);
MOVEPEN(0,0);
A:=0.1;
"FOR" N:=1 "STEP" 1 "UNTIL" 9 "DO"
"BEGIN" X:=SCALE*LN(N);
DRAWLINE(X,0); DRAWLINE(X,16);
MOVEPEN((X-45),-44);
"PRINT" FREEPOINT(1),A;
MOVEPEN(X,0); A:=A+0.1;
"END";
B:=1;
"FOR" N:=10 "STEP" 10 "UNTIL" 50 "DO"
"BEGIN" X:=SCALE*LN(N);
DRAWLINE(X,0); DRAWLINE(X,16);
MOVEPEN((X-30),-44);
"PRINT" DIGITS(1),B;
B:=B+1;
MOVEPEN(X,0);
"END";
MOVEPEN(1560,-254);
DRAWLINE(1590,-254);
DRAWLINE(1590,-224);
MOVEPEN(630,-94);
"PRINT" "FREQUENCY HZ";
MOVEPEN(0,0);
"END";
"PROCEDURE" PLOTPHAS(DUM,N1,START,N2,SCALE,P);
"REAL" START,SCALE,P;
"INTEGER" N1,N2;
"REAL" "ARRAY" DUM;
"BEGIN" "REAL" A;
   "INTEGER" N;
   PUNCH(5); WAY(0,4);
   MOVEPEN(SCALE*LN(START),(P*DUM[N2]+1016));
   A:=START+FR*10;
   "FOR" N:=N2+1 "STEP" 1 "UNTIL" N1 "DO"
   "BEGIN" DRAWLINE(SCALE*LN(A),(P*DUM[N]+1016));
   A:=A+FR*10;
   "END";
"END";
"PROCEDURE" PLOTGAIN(DUM,N1,START,N2,SCALE,G,ZERO);
"REAL" START,SCALE,G;
"INTEGER" N1,N2,ZERO;
"REAL" "ARRAY" DUM;
"BEGIN" "REAL" A;
   "INTEGER" N;
   PUNCH(5); WAY(0,4);
   MOVEPEN(SCALE*LN(START),(G*LN(DUM[N2]/DUM[0])+ZERO));
   A:=START+FR*10;
   "FOR" N:=N2+1 "STEP" 1 "UNTIL" N1 "DO"
   "BEGIN" DRAWLINE(SCALE*LN(A),(G*LN(DUM[N]/DUM[0])+ZERO));
   A:=A+FR*10;
   "END";
"END";
"PROCEDURE" FIXPHASE(DUM,N1);
"INTEGER" N1;
"REAL" "ARRAY" DUM;
"BEGIN" "INTEGER" Z;
   "FOR" Z:=1 "STEP" 1 "UNTIL" N1 "DO"
   "BEGIN" "IF" CHECKB(DUM[Z]>0) "THEN"
      "BEGIN" "IF" CHECKB(DUM[(Z-1)]<90) "THEN"
         "BEGIN" "IF" CHECKB(DUM[(Z-1)]<-90)
            "THEN" DUM[Z]:=(DUM[Z]-360);
            "END";
         "IF" CHECKB(DUM[Z]<0) "THEN"
         "BEGIN" "IF" CHECKB(DUM[(Z-1)]>90)
         "ELSE" CHECKB(DUM[(Z-1)]<-90)
         "THEN" DUM[Z]:=(DUM[Z]+360);
         "END";
      "END";
   "END";
"END";
"PROCEDURE" PLOTGAIN(DUM,N1,START,N2,SCALE,G,ZERO);
"REAL" START,SCALE,G;
"INTEGER" N1,N2,ZERO;
"REAL" "ARRAY" DUM;
"BEGIN" "REAL" A;
   "INTEGER" N;
   PUNCH(5); WAY(0,4);
   MOVEPEN(SCALE*LN(START),(G*LN(DUM[N2]/DUM[0])+ZERO));
   A:=START+FR*10;
   "FOR" N:=N2+1 "STEP" 1 "UNTIL" N1 "DO"
   "BEGIN" DRAWLINE(SCALE*LN(A),(G*LN(DUM[N]/DUM[0])+ZERO));
   A:=A+FR*10;
   "END";
"END";
"PROCEDURE" FIXPHASE(DUM,N1);
"INTEGER" N1;
"REAL" "ARRAY" DUM;
"BEGIN" "INTEGER" Z;
   "FOR" Z:=1 "STEP" 1 "UNTIL" N1 "DO"
   "BEGIN" "IF" CHECKB(DUM[Z]>0) "THEN"
      "BEGIN" "IF" CHECKB(DUM[(Z-1)]<90) "THEN"
         "BEGIN" "IF" CHECKB(DUM[(Z-1)]<-90)
            "THEN" DUM[Z]:=(DUM[Z]-360);
            "END";
         "IF" CHECKB(DUM[Z]<0) "THEN"
         "BEGIN" "IF" CHECKB(DUM[(Z-1)]>90)
         "ELSE" CHECKB(DUM[(Z-1)]<-90)
         "THEN" DUM[Z]:=(DUM[Z]+360);
         "END";
      "END";
   "END";
"END";

"END";

"END";

MORE: I:=I; INSTRING(NAME, I); I:=1;

"READ" M, A, C, XF, XR, U, KSF, KCF, KS, KSR, KCR, DYDFDP, DYDRDP, DDFDP,
     DDRDP, DPDFDP, DPDRDP, DDRDP, DDRDP, DPDFDP, DPDRDP, RO, AR, T, H,
     DCSDT, DCRDT, DCYDT, KP, KPD, FR, N, STPRIN, STPLOT, GPRIN, GPLOT, RUN;
N1:=N-1;

"BEGIN" "REAL" "ARRAY" ADV[0:3], X, Y, LX, ALX, ALY[1:4], RX,
     RY, ARX, ARY[1:3];
KDSF:=KSF/(1+KSF*DDFDY-KCF*DPDFDY);
KDCF:=KCF/(1+KSF*DDFDY-KCF*DPDFDY);
KDSR:=KSR/(1+KSR*DDRDP-KCR*DPDRDP);
KDCR:=KCR*KDSR/KSR;
YFP:=-KDSF*DDFDP+KDCF*DPDFDP;
YRP:=-KDSR*DDRDP+KDCR*DPDRDP;
YV:=CHKR((KDSF+KDSR)/U);
LV:=CHKR((DYDFDP*KDSF+DYDRDP*KDSR)/U);
NV:=CHKR((XF*KDSF+XR*KDSR)/U);
YPD:=CHKR((KDSF*DYDFDP+KDSF*DYDRDP+KDSF*KDSR)/U);
LPD:=CHKR((DYDFDP*DYDFDP+KDSF*DYDFDP+KDSF*KDSR)/U);
NPD:=CHKR((XF*KDSF*DYDFDP+XF*KDSF*DYDRDP+XF*KDSF*KDSR)/U);
YP:=CHKR(YFP+YRP);
LP:=CHKR(DYDFDP*YFP+DYDRDP*YRP);
NP:=CHKR(XF*YFP+XRF*YRP);
YR:=CHKR((KDSF*XF+KDSF*XR)/U);
LR:=CHKR((DYDFDP*KDSF*KF+DYDRDP*KDSF*XR)/U);
NR:=CHKR((XF*KF+XF*XR)/U);
YD:=CHKR(-KDSF);
LD:=CHKR(-DYDFDP*KDSF);
ND:=CHKR(-XF*KDSF);
CON:=0.5*RO*U*U*AR;
YAT:=CHKR(CON*DCSDT);
LAT:=CHKR(CON*(T*DCRD+H*DCSDT));
NAT:=CHKR(CON*(KDF-*DCYDT+(XF*XR)*0.5*DCSDT));
SM:=(-100*(LV*NP-NV*LP)/(LV*LP-LV YP)/(XF-XR));
DSDS := -U*(LV*NP-NV*LP)/(LP*ND-NP*LD);
DSDL := (LV-NP-NV*LP)/(YD'(LV*NP-NV*LP)
+LD* (NV*YP-YV* NP)+ND* (YV*LP-LV*YP))*M;
D[1] := CHECKR(M*A*Q);
+C*(LPD*YV-YPD*LV))
+M*U*(LV*NP-NV*LP)+YR*(NV*LPD-LV*NPD));
+M*U*(LV*NP-NV*LP)+YR*(NV*NP-LP*NR));
AY := YD; AL := LD; AN := ND; J := 0;
AGAIN: ADR[0] := CHECKR(M*A*AN);
ADR[1] := CHECKR(M*(AL*NP-LPD*AN)+A*(AY NV-YV*AN));
ADR[2] := CHECKR(M*(AL*NP-LP*AN)+YV*(LPD*AN-AL*NPD)
+YPD*(AL*NP-LPD*AN)+AY*(LV*NPD-LPD*AN));
ADR[3] := CHECKR(YV*(LPD*AN-AL*NP)+YP*(AL+NP-LV*AN)
+AY*(LV*NP-LP*NV));
ADP[0] := CHECKR(M*C*AL);
ADP[1] := CHECKR(M*(LR*AN-AL*NR)+C*(AY*LV-YV*AL));
ADP[2] := CHECKR(YV*(AL*NR-LR*AN)+AY*(LR*NV-LV*AN)
+(M*U-YR)*(AL*NP-LV*AN));
ADV[0] := CHECKR(A*C*AY);
ADV[1] := CHECKR(C*(YPD*AL-AY*LPD)+A*(YR*AN-AY*NR))
-M*U*A*AN);
ADV[2] := CHECKR(C*(YP*AL-AY*LP)+AY*(LPD*NR-LR*NPD)
+YPD*(LR*AN-AL*NR)+M*U*(LPD*AN-AL*NPD)+YR*(AL*NPD-LPD*AN));
ADV[3] := CHECKR(AV*(LP*AN-AL*NP)+YP*(LR*AN-AL*NR)+M*U*
(LP*AN-AL*NP)+YR*(AL*NP-LP*AN));
ADL[1] := CHECKR(ADV[0]);
ADL[5] := CHECKR(U*ADR[3]);
"IF" J<0.1 "THEN"
"BEGIN" "FOR" K:=1 "STEP" 1 "UNTIL" 5 "DO" DL[K] := ADL[K]
"FOR" K:=0 "STEP" 1 "UNTIL" 3 "DO"
DR[K] := ADR[K];
"END";
J := J + 2;
"IF" J < 3 "THEN"
"BEGIN" AY := YAT; AL := LAT; AN := NAT;
    "GOTO" AGAIN;
"END";
QUARTIC(D, X, Y);
QUARTIC(DL, LX, LY);
QUARTIC(ADL, ALX, ALY);
CUBIC(DR, RX, RY);
CUBIC(ADR, ARX, ARY);
"IF" B < 0 "THEN"
"BEGIN" PX1 := PX2 := -DP[1] / (2 * DP[0]);
    PY1 := SQRT(-B) / (2 * DP[0]);
    PY2 := -SQRT(-B) / (2 * DP[0]);
"END" "ELSE"
"BEGIN" PX1 := PX2 := 0;
    PX1 := (-DP[1] + SQRT(B)) / (2 * DP[0]);
    PX2 := (-DP[1] - SQRT(B)) / (2 * DP[0]);
"END";
"IF" B < 0 "THEN"
"BEGIN" APX1 := APX2 := -ADP[1] / (2 * ADP[0]);
    APY1 := SQRT(-B) / (2 * ADP[0]);
    APY2 := -SQRT(-B) / (2 * ADP[0]);
"END" "ELSE"
"BEGIN" APY1 := APY2 := 0;
    APX1 := (-ADP[1] + SQRT(B)) / (2 * ADP[0]);
    APX2 := (-ADP[1] - SQRT(B)) / (2 * ADP[0]);
"END";
"FOR" I := 1 "STEP" 1 "UNTIL" 4 "DO"
"BEGIN" COEFFS[I] := D[I + 1] / D[1];
    ACO[I] := COEFFS[I];
    COEFFS[I + 1] := DL[I + 1] / DL[1];
    ACO[I + 1] := ADL[I + 1] / ADL[1];
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"BEGIN" COEFFS[5+I]:=IR[1]/IR[0];
   ACO[5+I]:=AD[1]/AD[0];
"END";
COEFFS[5]:=IR[0]/IR[1];
COEFFS[12]:=SL[1]/SL[1];
ACO[5]:=AD[0]/AD[1];
COEFFS[12]:=DP[1]/DP[0];
ACO[12]:=ADP[1]/ADP[0];
PUNCH(1); I:=1;
"PRINT" "L4``; OUTSTRING(NAME,1);
"PRINT" "L2``SM``S``SAMELINE,FREEPOINT(4),SM,``S3``,
   "D(STEER)/D(SLIP)``S2``DSDS,``S3``D(STEER)/D(LATAC)`
   "S2``DSIL*57.2958;
"PRINT" "L2`RESPONSE COEFF`S3`STEER`S7`WIND``;
"FOR" I:=1 "STEP" 1 "UNTIL" 16 "DO"
"PRINT" SAMELINE,``LS9``DIGITS(2),I,``S2``ACO[I];
"PRINT"``LS24``POLES `LS15`REAL``S3``,
   "IMAG``S4``FREQ``S2``D RAT``L``;
"FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
"PRINT" SAMELINE,FREEPOINT(4),``LS4``I,PREFIX(``S``),
   X[I],Y[I],ALIGNED(2,2),SQR(X[I]*X[I]+Y[I]*Y[I])/6.2832,
   ALIGNED(1,3),(X[I]/(X[I]*X[I]+Y[I]*Y[I]));
"PRINT"``LS321`STEER ZEROS``S20`GUST ZEROS``LS44``,
   "REAL``S3``IMAG``S4``FREQ``S2``D RAT``LS``YAW RATE ``;
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"PRINT" SAMELINE,FREEPOINT(4),``S4``I,PREFIX(``S``),
   RX[I],RY[I],ALIGNED(2,2),SQR(RX[I]*RX[I]+RY[I]*RY[I])/6.2832,
   ALIGNED(1,3),(RX[I]/(RX[I]*RX[I]+RY[I]*RY[I]));
"PRINT"``LS3``ROLL ANGLE``L``SAMELINE,``S4``I,
   PREFIX(``S``);
FREEPOINT(4), PX1, PY1, ALIGNED(2,2), SQRT(PX1*PX1+PY1*PY1)/6.2832, ALIGNED(1,3), PX1/SQRT(PX1*PX1+PY1*PY1), "S", FREEPOINT(4), APX1, APY1, ALIGNED(2,2), SQRT(APX1*APX1+APY1*APY1)/6.2832, ALIGNED(1,3), APX1/SQRT(APX1*APX1+APY1*APY1), "S", FREEPOINT(4), PX2, PY2, ALIGNED(2,2), SQRT(PX2*PX2+PY2*PY2)/6.2832, ALIGNED(1,3), PX2/SQRT(PX2*PX2+PY2*PY2), "S", FREEPOINT(4), APX2, APY2, ALIGNED(2,2), SQRT(APX2*APX2+APY2*APY2)/6.2832, ALIGNED(1,3), APX2/SQRT(APX2*APX2+APY2*APY2);
"PRINT" "L2S LATAC L ";
"FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
"PRINT" SAMELINE, FREEPOINT(4), "S4", I, PREFIX("S"), LX[I], LY[I], ALIGNED(2,2), SQRT(LX[I]*LX[I]+LY[I]*LY[I])/6.2832, ALIGNED(1,3), LX[I]/SQRT(LX[I]*LX[I]+LY[I]*LY[I]), "S", FREEPOINT(4), ALX[I], ALY[I], ALIGNED(2,2), SQRT(ALX[I]*ALX[I]+ALY[I]*ALY[I])/6.2832, ALIGNED(1,3), ALX[I]/SQRT(ALX[I]*ALX[I]+ALY[I]*ALY[I]), "L";
"END";
"BEGIN" "REAL" "ARRAY" RP, PP, LAP[0:N1];
W:=0;
"FOR" Z:=0 "STEP" 1 "UNTIL" N1 "DO"
"BEGIN" R:=D[1]*W+4-D[3]*W*W+D[5];
S:=-D[2]*W+3+D[4]*W;
P:=-DR[1]*W+W+DR[3];
Q:=-DR[0]*W+3+DR[2]*W;
PHASE(P, Q, R, S, RP[Z]);
P:=-DP[0]*W+W+DP[2];
Q:=DP[1]*W;
PHASE(P, Q, R, S, PP[Z]);
P:=DL[1]*W+4-DL[3]*W*W+DL[5];
Q:=-DL[2]*W+3+DL[4]*W;
PHASE(P, Q, R, S, LAP[Z]);
W:=W+FR*6.2832;
"END";
FIXPHASE(RP, N1); FIXPHASE(PP, N1); FIXPHASE(LAP, N1);
"FOR" Z:=1 "STEP" 1 "UNTIL" N1 "DO" PP[Z]:=PP[Z]-180;
"IF" STPRIN=1 "THEN"
BEGIN" "PRINT" "HR50L4"; I:=1; OUTSTRING(NAME,I);
"PRINT" "L2S23 STEER FREQUENCY RESPONSE L2S9";
"yaw rate" "roll angle" "latac" "freq" "phase" "s7";
"phase" "s6" "phase" "l";
"for" "i:=0" "step" 1 "until" n1 "do"
"print" "same line," "l"; aligned(1, 2), i*fr,
aligned(3, 1), "4", rp[i], "s4", rp[i], "s5", lap[i];
"end";
punch(5); way(0, 4); scale:=254/ln(2); p:=254/60;
q:=2540/3/ln(10);
"if" fr<0.1 "then"
"begin" start:=1;
n2:=0.1/fr;
"end" "else"
"begin" start:=fr*10;
n2:=1;
"end";
"if" stplot=1 "then"
"begin" drafrax(scale);
y:=0;
"for" n:=240 "step" -60 "until" 0 "do"
"begin" drawline(0, y); drawline(16, y);

movepen(-100, (y-16));
"print" digits(3), n;
movepen(0, y); y:=y+254;
"end";
movepen(-100, 338);
"print" way(1, 4), "phase lag degrees";
plothas(rp, n1, start, n2, scale, p);
"print" "yaw";
plothas(pp, n1, start, n2, scale, p);
"print" "roll";
plothas(lap, n1, start, n2, scale, p);
"print" "latac";
"end";
"end";
BEGIN "REAL" "ARRAY" RA,PA,LA[0:NI];
W:=0;
"FOR" Z:=0 "STEP" 1 "UNTIL" NI "DO"
BEGIN R:=(D[1]*W+D[3])*W+D[5];
S:=-D[2]*W+D[4]*W;
P:=-I*W+D[3];
Q:=-I*W+D[2]*W;
AMP(R,P,S,RA[Z]);
P:=-D[0]*W+D[2];
Q:=D[1]*W;
AMP(P,Q,R,PA[Z]);

P:=D[1]*W+D[5];
Q:=-D[2]*W+D[4]*W;
AMP(P,Q,R,LA[Z]);
W:=W+FR*6.2832;
END;
"IF" STPRIN=1 "THEN"
BEGIN "PRINT" PUNCH(1),"L3S9"YAW RATE"S3 ROLL ANGLE"S3"
LATAC","L511"AMP"S9"AMP"S8"AMP"L";
"FOR" I:=0 "STEP" 1 "UNTIL" NI "DO"
BEGIN R:=D[I]*W+D[I];
S:=D[I]*W+D[I];
P:=D[I]*W;
Q:=D[I]*W;
AMP(R,P,S,PA[I]);
END "ELSE"
BEGIN "PRINT" PUNCH(1),"L2"SPECIFIC STEADY STATE STEER RESPONSE"L;
YAW RATE"S4",S2",SAMELINE,RA[0],"L ROLL ANGLE"S2",PA[0],"L LATAC S7",LA[0];
"IF" STPLOT=1 "THEN"
BEGIN YB:=Y; MOVEPEN(0,(Y-254));
"FOR" N:=-24 "STEP" 6 "UNTIL" 6 "DO"
BEGIN DRAWLINE(0,YB);
DRAWLINE(16,YB); MOVEPEN(-80,(YB-16));
PRINT" DIGITS(2),N;
MOVEPEN(0,YB); YB:=YB+254;
END;
MOVEPEN(-100,(Y+438));
PRINT" WAY(1,4),'GAIN DB';
ZERO:=Y+1016;
PLOTGAIN(RA,N1,START,N2,SCALE,G,ZERO);
"PRINT" YAW``;
PLOTGAIN(PA,N1,START,N2,SCALE,G,ZERO);
"PRINT" ROLL``;
PLOTGAIN(LA,N1,START,N2,SCALE,G,ZERO);
"PRINT" LATAC``;
MOVEPEN(-500,2696); DRAWLINE(-500,2726);
DRAWLINE(-470,2726); MOVEPEN(285,2540);
"PRINT" WAY(O,8) ,́STEER RESPONSE RUŃ ,
DIGITS(4),RUN;
MOVEPEN(1560,2726); DRAWLINE(1590,2726);
DRAWLINE(1590,2696); MOVEPEN(-500,3726);

"END`;

"BEGIN" "REAL" "ARRAY" ARP,APP,ALP[0:N1];
W:=0;
"FOR" Z:=O "STEP" 1 "UNTIL" N1 "DO" 
"BEGIN" R:=D[1]*W*4-D[3]*W*W+D[5];
S:=-D[2]*W*3+D[4]*W;
P:=-ADR[1]*W*W+ADR[3];
Q:=-ADR[0]*W*3+ADR[2]*W;
PHASE(P,Q,R,S,ARP[Z]);
P:=-ADP[0]*W*W+ADP[2];
Q:=ADP[1]*W;
PHASE(P,Q,R,S,APP[Z]);
P:=AIL[1]*W*4-AIL[3]*W*W+AIL[5];
Q:=-AIL[2]*W*3+AIL[4]*W;
PHASE(P,Q,R,S,ALP[Z]);
W:=W+FR*6.2832;

"END`;

FIXPHASE(ARP,N1); FIXPHASE(APP,N1); FIXPHASE(ALP,N1);
"IF" GPRIN=1 "THEN"
"BEGIN" PUNCH(1);
"PRINT" "HRSOLÁ``;
I:=1; OUTSTRING(NAME,I);
"PRINT" "L2S24 GUST FREQUENCY RESPONSE L2S9``;
"YAW RATE S3 ROLL ANGLE S3 LATAC LS FREQ S5``,
"FOR" I:=0 "STEP" 1 "UNTIL" N1 "DO"
"PRINT" SAMELINE, 'L', ALIGNED(1,2), I*FR,
ALIGNED(3,1), 'S4', ARP[I], 'S6', APP[I],
'S5', ALP[I];
"END";
"IF" GLOT=1 "THEN"
"BEGIN" DRAWAX(SCALE);
"FOR" N:=-240 "STEP" 60 "UNTIL" 180 "DO"
"BEGIN" DRAWLINE(0,Y); DRAWLINE(16,Y);
MOVEPEN(-100,(Y-16));
"PRINT" DIGITS(3),N;
MOVEPEN(0,Y); Y:=Y+254;
"END";
P:=254/60;
MOVEPEN(-100,689);
"PRINT" WAY(1,4), 'PHASE ANGLE DEGREES';
PLOTPHAS(ARP,N1,START,N2,SCALE,P);
"PRINT" YAW';
PLOTPHAS(APP,N1,START,N2,SCALE,P);
"PRINT" ROLL';
PLOTPHAS(ALP,N1,START,N2,SCALE,P);
"PRINT" LATAC';
"END";
"END";
"BEGIN" "REAL" "ARRAY" ARA,APA,ALA[0:N1];
W:=0;
"FOR" Z:=0 "STEP" 1 "UNTIL" N1 "DO"
"BEGIN" R:=D[1]*Wt4-D[3]*W+W+D[5];
S:=-D[2]*Wt3+D[4]*W;
P:=-ADR[1]*W*W+ADR[3];
Q:=-ADR[0]*Wt3+ADR[2]*W;
AMP(F,Q,R,S,ARA[Z]);
P:=-ADP[0]*W*W+ADP[2];
Q:=ADP[1]*W;
AMP(P,Q,R,S,APA[Z]);
P:=ADL[1]*Wt4-ADL[3]*W+W+ADL[5];
Q:=ADL[2]*Wt3+ADL[4]*W;
AMP(P,Q,R,S,ALA[2]);
W:=W+FR*6.2832;

"END'';
"IF" GPRINT=1 "THEN"
"BEGIN" "PRINT" PUNCH(1), "LS39"YAW RATE"S3''
ROLL ANGLE"S3"LATAC", "LS11"AMP"S9"AMP"S8"AMP"L'';
"FOR" I:=0 "STEP" 1 "UNTIL" N1 "DO"
"PRINT" PUNCH(1), SAMELINE,ALIGNED(1,2), "L", I*FR,
SCALED(4),ARA[I],"S",APA[I],"S2",ALA[I];
"END" "ELSE"
"PRINT" PUNCH(1), "L2"SPECIFIC STEADY STATE GUST RESPONSE"L'',
"YAW RATE"S4'',SCALED(4),SAMELINE,ARA[0], "L"ROLL ANGLE"S2'',
APA[0], "L"LATAC"S7'',ALA[0];
"IF" GPLOT=1 "THEN"
"BEGIN" MOVEPEN(-500,2696);
MOVEPEN(285,2540);
"PRINT" WAY(O,S), 'GUST RESPONSE RUN',DIGITS(4),RUN;
MOVEPEN(1560,2540);
MOVEPEN(-800,3726);
DRAWLINE(SCALE);
Y:=0;
"FOR" N:=-30 "STEP" 6 "UNTIL" 30 "DO''
"BEGIN" DRAWLINE(0, Y);
MOVEPEN(-100,1200);
"PRINT" WAY(1,4), 'GAIN DB'';
ZERO:=1270;
PLOTGAIN(ARA,N1,START,N2,SCALE,G,ZERO);
"PRINT" 'YAW'';
PLOTGAIN(APA,N1,START,N2,SCALE,G,ZERO);
"PRINT" 'ROLL'';
PLOTGAIN(ALA,N1,START,N2,SCALE,G,ZERO);
"PRINT" 'LATAC'';
"END";
"END";
"GOTU" MORE;
"END";
APPENDIX 4. EQUIPMENT USED IN SECTION 4.

A 4.1 Vehicle

Relevant weights and dimensions of the two vehicles concerned are:

<table>
<thead>
<tr>
<th>Weights</th>
<th>Yaw Inertia</th>
<th>Roll Inertia</th>
<th>Wheelbase</th>
<th>( x_F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Slug</td>
<td>Slug ft.²</td>
<td>Slug ft.²</td>
<td>ft.</td>
<td>ft.</td>
</tr>
<tr>
<td>Car A</td>
<td>155.3</td>
<td>3818</td>
<td>588</td>
<td>10</td>
</tr>
<tr>
<td>Car B</td>
<td>93.5</td>
<td>1564</td>
<td>288</td>
<td>7.9</td>
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</tbody>
</table>

Car A was a 4200 c.c., front engined, rear wheel drive, saloon, with wishbone and coil spring front suspension, and independent rear suspension consisting of twin coil springs, and parallel transverse links located longitudinally by radius arms. This car was fitted with textile braced radial ply tyres maintained at 30 lb/in.², front and rear.

Car B was a 1600 c.c., rear engined, rear wheel drive estate car, with an anti-roll bar and transverse torsion bar, semi-trailing arm front suspension, and swing axle, trailing link, transverse torsion bar rear suspension. The rear suspension also incorporated a transverse stabiliser bar which increased the vertical stiffness without affecting the roll stiffness. Textile braced radial ply tyres were fitted and maintained at 20 lb/in.², front and 28 lb/in.² rear throughout the tests.
A 4.2 Sinusoidal Steer Input Machine.

The machine was mounted in the front passenger compartment and powered by a one-third horse power, 24 volt, D.C. electric motor which drove a step-less variable speed gearbox through a chain. The gearbox output shaft was connected, via a rubber coupling, to a fixed ratio worm and wheel gearbox which turned the drive through 90 degrees so that it was parallel with the steering column. A dog-clutch, operated by the driver, transmitted the drive to a crank-arm that rotated in the same plane as the steering hand-wheel. A small roller mounted on the crank-arm ran in the slot of a slider mechanism so that rotation of the arm moved the slider with simple harmonic motion across the car. A chain, fixed to the slider, transferred the simple harmonic motion to a sprocket mounted on the underside of a modified hand-wheel. This sprocket was free to rotate for normal driving, until required for a test run when a peg was inserted through mating holes, thus locking the sprocket to a fixed plate on the hand-wheel boss.

The amplitude of the hand-wheel input angle could be varied by altering the location of the roller on the crank-arm, by means of a screw and locknut, thereby changing its throw. The frequency was variable between 0.14 and 3.4 Hz by use of the variable speed gearbox in conjunction with either of two differently sized sprockets on the gearbox input shaft. The frequency could be decreased to about 0.14 Hz by reducing the motor supply voltage from 24 to 12; this could easily be changed as the supply was from two 12 volt batteries.
APPENDIX 5   THE MIRA LIGHT WEIGHT HANDLING INSTRUMENTATION SYSTEM

General Description

The system has been designed as a light-weight package incorporating all necessary power supplies, control and monitoring, signal conditioning and recording facilities for up to 12 channels of information in the frequency range 0-5 Hz. It is based on miniature cassette tape recorders which are built into recording modules, 4 of which can be plugged into a master baseboard. The complete 4 module system (without transducers) weighs approximately 40 lb. (18 kg.) and measures 24.2 in. (0.62 m) long x 8.5 in. (0.22 m) high x 8.25 in. (0.21 m). A remote control unit incorporating a microphone provides easy control and the system can be used in tests involving only the driver in the vehicle.

A photograph of the complete system with one of its modules un-plugged, and a diagram of one recorder module, are shown in Figs. A 5.1 and A 5.2.

Power Supply

The system is powered by rechargeable cells contained in the master baseboard. This provides a supply of +14 volts for the recorder modules, each of which has its own voltage stabiliser and provides power for its transducers, and a 28 volt supply suitable for 3 miniature rate gyroscopes. With all 4 modules in use the batteries are adequate for a full day's testing and can absorb a full charge overnight. Charging is from a 12V supply and is controlled by a built in current regulator. In practice the system can be connected to the car battery and switched to the charge position whenever it is not in use.
Recorder Modules

Each module has one channel for speech/synchronisation and 3 data channels. Each data channel has an input impedance of 51 Kohms, and provides a ± 5 volt d.c. supply for its transducer.

The various sub-systems contained in the module are:

Tape Recorder - Oxford Instrument Company, Medilog, type 4-2, 4 track magnetic tape recorder with 3 pulse-width modulator amplifiers and one direct record amplifier. The tape speed of 25 mm/sec gives a total record time of about 1.5 hours on a C90 cassette.

Sensitivity - Switched attenuators giving a range of 0-42 dB in 3 dB steps. Input sensitivity approximately ± 1 - ± 100 mV.

Filters - Butterworth low pass filters, 3 dB down at 13 Hz, cut off at 24 dB/octave. Signals up to 5 Hz unaffected.

Balance - Balance control with range of ± 10mV at mid-range of sensitivity.

Calibration - Master calibration switch provides calibration for all channels simultaneously. 3 levels are available. All transducers are fitted with passive calibration networks in their leads and can be connected to any data channel.

Input Level Monitors - Peak Levels of test signals are stored and displayed on a meter so that optimum gain settings can be used.

Control

Control is by a hand held remote unit fitted with on/off switch, synchronising tone control, microphone and warning lights. Prior to test runs, speech can be recorded on one track of each module. During test runs a 40 Hz synchronising tone is recorded on the same track of each module, so that the time relationship between separate cassettes can be maintained.
Two Module Baseboard

Any number of modules (up to 4) can be used with the full baseboard, but a smaller 2 module baseboard is also available for situations where 6 channels are sufficient. The use of this is as above but the gyroscope power supply is not included.

Replay

A separate, laboratory based replay unit is used.

Analysis

The first stage of the analysis procedure used at MIRA, and for which the synchronising tone system was devised, involves analogue to digital conversion of each channel of data. Because the data is contained in separate cassettes only 3 channels can be done simultaneously and synchronising is preserved by using the 40 Hz tone to trigger the sampling of the ADC. A system is available for sampling every nth pulse should a sampling rate of less than 40 per second be required.
FIG. A5.1
The NIF/IAAC Weight Handling Instrumentation Package.
FIG. A5.2 DIAGRAM OF A RECORDER MODULE (FULL-SIZE) SHOWING LAYOUT OF CONTROLS
APPENDIX 6. COMPUTER PROGRAMME FITTRANS – DESCRIPTION, USE AND LISTINGS

A 6.1. General Description.

In essence the programme has the capability of carrying out two distinct operations:

1. Using a Fast Fourier Transform algorithm, Fourier transforms of up to 9 channels of time series data can be computed, and frequency response (transfer function) of any channel relative to any other can be derived from these.

2. For vehicle response information, three degree of freedom frequency response functions can be fitted to any number from 1 to 6 of the yaw rate, roll angle, and lateral acceleration (latac) amplitude and phase frequency response data. This can either be computed in 1. above or input direct, and can be relative to steer or wind gust inputs.

The programme is written round a number of procedures (sub-routines), some fundamental to the operation and others simply concerned with manipulating the data and organising the input and output. These are listed below.

IDENTIFY - labels and thus identifies all outputs.

OPENREAD, BACKSPACE - used in locating data on a magnetic tape file.

RDD - reads in appropriate time data from magnetic tape or paper tape.

DRAFRAX, FLAX, PHAX - procedures for plotting frequency amplitude and phase data.

WINDOW - applies a window to the time data (not finally used here).

FASTFOURIER, REALTRANSFORM etc - Fast Fourier Transform procedures.

FTWH - uses the above to produce a Fourier transform and then applies Hamming smoothing.
SORTPHASE  - sorts phase information into the appropriate quadrant.

FUNCTION  - calculates the difference between the value of a frequency response data point (amplitude or phase) and the value of the three degree of freedom frequency response function being fitted.

CALFUN  - calculates the function to be minimised in the curve fitting operation, namely the sum of the squared values of FUNCTION above, for all data points involved.

FUNCT  - calculates the derivative of each value of FUNCTION with respect to each of the coefficients (COEFFS) defining the three degree of freedom frequency response function. This is required for the curve fitting procedure.

OPLS etc.  - "A safeguarded Gauss-Newton technique for minimising sums of squared terms".

TF  - calculates the three degree of freedom frequency response functions corresponding to a given set of defining COEFFS.

QUARTIC, CUBIC  - procedures for evaluating the roots of quartics and cubics.

A 6.2. Input and Output Details.

A 6.2.1. Time data.

The programme is written so that many of the instructions regarding control and the operations and outputs required from any particular run, are input in the form of answers to questions appearing on the computer control teletypewriter. Many of these are relevant only to the running of the programme, have no technical significance and so are not enumerated here. The remainder are indicated alongside the normal input data shown below.

Time data can be input either from a punched paper tape or, more usually, from a digital magnetic tape file. In either case the format is:
18 CHARACTER ALPHA/NUMERIC IDENTIFICATION

RUN - up to 4 digit, integer run number.

DATPTS - number of time data points per channel. \((2^E\) points are actually used starting at point STPT and using every NTH point. STPT, NTH, E are control teleprinter instructions).

DT - time interval between points in seconds. (Frequency resolution available in the Fourier analysis is \(1/(2^E \cdot DT \cdot NTH)\)).

NCHS - number of channels of data - maximum of 9.

MAX[I] - \(I = 1\) to NCHS - nominal maximum value of each channel. Used for scaling purposes.

Followed by NCHS channels of DATPTS points identified by the order in which they come and normally as follows:

1. Wind velocity, mile/h.
2. Wind angle, deg.
3. Handwheel angle, deg.
4. Mean road wheel angle, deg.
5. Roll rate, deg/sec.
7. Roll angle, deg.
8. Yaw angle, deg.
9. Lateral acceleration (latac), g.

If required this time data can be plotted out as shown, for example, in Fig. A6.1. The information beneath the title is, from left to right:

Date when the computer run was carried out and a computer pass number, P.

STPT, NTH, E, DT as defined above.

18 character run identifier.

The figure beneath each channel description is the MAX value described above and represents 0.5 inches on the vertical scale for each channel.
Fig. A6.1 Example of time data output plot.
A 6.2.2 Frequency data.

The programme's next operation is to carry out Fourier analysis of any of the time data channels and calculate vehicle response transfer functions as required. The outputs available are, the straight Fourier analysis of any number of channels with amplitude and phase plotted on separate graphs, and frequency responses (transfer functions) corresponding to whatever ratios of channels are specified, again plotted on separate amplitude and phase graphs. This gives 4 types of graph any one of which can be obtained separately if required. Fig. A6.2 shows a typical frequency response amplitude graph. The information directly beneath the title is as described for the time response plot (Fig. A6.1). At the end of each plot is a ratio of figures indicating the frequency response concerned, the figures corresponding to the order of the time data channels described above. For all the work of this Thesis the numbers were as above and so, for example, 6/3 represents the yaw rate response to handwheel input.

There is one exception to this rule, which arises because the programme allows for the calculation of the roll angle curve either direct from the roll angle channel (7), or by division of the corresponding roll rate curve, calculated from channel 5, by frequency. Since roll angle data was produced indirectly by integration of roll rate information for most of the results of this Thesis, the latter approach can be regarded as more direct and is used almost exclusively. To show that this has been done the roll angle curve is labelled 5/3, there being little risk of confusion with the roll rate curve because of the fundamental difference in shape.

The amplitude curves are plotted on a decibel scale, the amplitude at any frequency being given by:

\[ \text{dB value} = 20 \log_{10}(\text{amplitude/amplitude at zero frequency}) \]

The actual value of the amplitude at zero frequency is given for each curve under SS RESP at the top left of the graph. In theory the low frequency end of each of the three curves shown on Fig. A6.2 should pass approximately through 0 dB, but in practice (as discussed in the main text) the zero frequency level is difficult to establish and large errors often occur. This means that the relative positions of the curves should not be regarded as significant, and the best estimate of the correct zero
Fig. A6.2 Example of frequency response amplitude plot.
frequency level has to be obtained from the low frequency portion of the curves. Since the principal interest in this type of data lies in the shape rather than the position of these curves the above is not a serious problem. Information about the zero frequency levels is best obtained from a separate steady state test.

If the frequency data for use in the next part of the programme is not produced as above, it can be read in direct from a paper tape in the following format:

```
"ALPHA/NUMERIC TEST IDENTIFIER"

N - maximum number of data points in any channel.

HWRW - frequency response identifier; relative to wind angle = 2
          relative to handwheel angle = 3
          relative to roadwheel angle = 4

PP[1,1],PP[2,1],PP[3,1] - number of yaw rate, roll angle, latac amplitude data points.

PP[1,2],PP[2,2],PP[3,2] - number of yaw rate, roll angle, latac phase data points.

Followed by data sets in the above order for each non-zero PP, in the form:

Amplitude (or phase as appropriate)  Frequency

The zero frequency point should normally be first as the dB values for graph plotting are calculated relative to the first point.
A 6.2.3. Curve fitting to frequency data.

The final function of the programme is to fit three degree of freedom curves to frequency response data derived from either of the above sources. Various additional data is required for this as follows:

CC - number of COEFFS required to define the functions to be fitted.

CPLOT - for direct frequency data only. Normally = 0, but if = 1 the frequency response functions corresponding to the COEFFS given are plotted without any curve fitting.

STFT[1,1], [2,1] etc. - first point to be used in curve fitting for each channel.

PP[1,1], [2,1] etc. - for frequency data calculated by FFT only. Final point to be used in curve fitting for each channel.

COEFFS[1], AKEY[1] - COEFFS, AKEY[1] = 1 for normal use or = 0 for non-adjustment of that COEFF during curve fitting.

and then the fitting procedure (OPLS) control parameters:

IPRINT - print control, print out every IPRINT iterations.

IMAX - maximum number of iterations.

EPS - convergence control.

SO - initial step size control.

DEL - step size for derivatives.

When the curve fitting is complete the output is in two parts. Table A6.1 and Fig. A6.3 illustrate these parts for curves fitted to the sinusoidal steer input data of car F. The notation on the tabular output, Table A6.1, is reasonably self-explanatory, the SUBFUNCTIONS being the set of values of the procedure FUNCTION and SUM OF SQUARES being the current value of the actual function being minimised, CALFUN. A set of new COEFFS,
### Table A6.1

**Example of tabulated output from a curve-fitting program.**

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<tr>
<th>INITIAL VALUES OF COEFFS</th>
<th>POINTS AVAILABLE</th>
<th>START AT POINTS</th>
<th>IPPINT</th>
<th>INAX</th>
<th>EPS</th>
<th>SUM OF SQUARES</th>
<th>FUNCTION CALLS</th>
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<td>5.679e+00</td>
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**SUM OF SQUARES** = 9.628e-02

**190 FUNCTION CALLS**

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<th>4.011e-02</th>
<th>2.096e-02</th>
<th>-2.073e-02</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.496e-02</td>
<td>-7.491e-03</td>
<td>2.512e-02</td>
<td>1.209e-02</td>
<td>-1.769e-02</td>
<td>-3.232e-02</td>
<td>3.462e-02</td>
<td>-2.840e-02</td>
<td></td>
</tr>
<tr>
<td>2.345e-02</td>
<td>4.739e-03</td>
<td>3.178e-03</td>
<td>-1.040e-02</td>
<td>-3.904e-02</td>
<td>-2.534e-02</td>
<td>5.641e-02</td>
<td>2.734e-03</td>
<td></td>
</tr>
<tr>
<td>-6.401e-02</td>
<td>1.196e-02</td>
<td>6.661e-03</td>
<td>6.609e-02</td>
<td>2.354e-02</td>
<td>7.448e-02</td>
<td>-1.038e-01</td>
<td>1.784e-02</td>
<td></td>
</tr>
<tr>
<td>-2.253e-02</td>
<td>2.683e-02</td>
<td>2.834e-02</td>
<td>3.326e-02</td>
<td>2.064e-02</td>
<td>7.188e-02</td>
<td>5.762e-02</td>
<td>-3.978e-02</td>
<td></td>
</tr>
<tr>
<td>3.618e-03</td>
<td>-9.918e-02</td>
<td>2.624e-02</td>
<td>-1.244e-02</td>
<td>-4.265e-02</td>
<td>-1.884e-02</td>
<td>4.731e-02</td>
<td>5.302e-02</td>
<td></td>
</tr>
<tr>
<td>-4.428e-02</td>
<td>-4.014e-02</td>
<td>2.026e-02</td>
<td>8.133e-02</td>
<td>3.193e-02</td>
<td>1.774e-02</td>
<td>-2.495e-02</td>
<td>-5.606e-02</td>
<td></td>
</tr>
<tr>
<td>-8.273e-02</td>
<td>1.110e+01</td>
<td>1.085e+01</td>
<td>1.626e+02</td>
<td>1.337e+02</td>
<td>5.114e+02</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**SUM OF SQUARES** = 9.628e-02

**207 FUNCTION CALLS**

**CONVNMND**
### Table: Frequency Response Data

<table>
<thead>
<tr>
<th>Mode</th>
<th>Yaw Rate</th>
<th>Roll Angle</th>
<th>Lateral</th>
<th>Roll Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole 1</td>
<td>$1.337 	imes 10^{-1}$</td>
<td>$6.445 	imes 10^{-2}$</td>
<td>$6.141 	imes 10^{-3}$</td>
<td>$1.81 	imes 10^{-1}$</td>
</tr>
<tr>
<td>Pole 2</td>
<td>$1.20 	imes 10^{-1}$</td>
<td>$1.52 	imes 10^{-2}$</td>
<td>$1.81 	imes 10^{-1}$</td>
<td>$1.81 	imes 10^{-1}$</td>
</tr>
<tr>
<td>Pole 3</td>
<td>$1.31 	imes 10^{-1}$</td>
<td>$1.52 	imes 10^{-2}$</td>
<td>$1.81 	imes 10^{-1}$</td>
<td>$1.81 	imes 10^{-1}$</td>
</tr>
<tr>
<td>Zero 1</td>
<td>$1.31 	imes 10^{-1}$</td>
<td>$0.268$</td>
<td>$0.530$</td>
<td>$0.530$</td>
</tr>
<tr>
<td>Zero 2</td>
<td>$1.94 	imes 10^{-1}$</td>
<td>$0.530$</td>
<td>$0.530$</td>
<td>$0.530$</td>
</tr>
<tr>
<td>Zero 3</td>
<td>$1.94 	imes 10^{-1}$</td>
<td>$0.268$</td>
<td>$0.268$</td>
<td>$0.268$</td>
</tr>
</tbody>
</table>

### Diagram: Frequency Response

Fig. A6.3 Example of curves fitted to frequency response amplitude data.
the SUBFUNCTIONS and SUM OF SQUARES are output each IPRINT iterations as the fitting proceeds. In the case illustrated this occurred once before the final convergence when the final values are output. The most important information is the set of new COEFFS which define the fitted frequency response function.

Fig. A6.3 is basically of the same form as Fig. A6.2. The data being fitted are plotted as individual points and the fitted curves as continuous lines. The zero frequency levels, which are related to the fitted curves for this Fig., and are thus more useful than those of Fig. A6.2, are given as SP SS RESP for each of the three curves. The information directly beneath the title would be in the same format as that of Fig. A6.2 for frequency response data computed from a time data input, but in this case it was read in direct and the information is restricted to: date and pass number of computer run, DIRECT, and the data identifying code. Also given at the top of the Fig. are the poles and zeros corresponding to the fitted curves. These are calculated, as described in Section 3, by extracting the roots of the quartics, cubics and quadratics defined by the new set of COEFFS, and are given in the form of frequency followed by damping ratio. A root is typically of the form:

\[ a + jb \]

where \( j = \sqrt{-1} \)

and the frequency (undamped natural frequency or corner frequency) is given by:

\[ \omega = \sqrt{a^2 + b^2} \]

and the damping ratio by:

\[ \eta = a/\omega \]

A 6.3 COEFFS and Control Parameter Values for Curve Fitting.

Provided the initial values for the COEFFS are of the correct order the programme is not particularly sensitive to the values chosen, but experience has shown that the following can be used as a useful standard set.
<table>
<thead>
<tr>
<th>COEFF</th>
<th>VALUE</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1200</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>1.5</td>
<td>or approx. 5,(zero frequency yaw rate amplitude)</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>4</td>
<td>or approx. 50,(zero frequency roll angle amplitude)</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>80</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0.003</td>
<td>or approx. 0.3,(zero frequency latac amplitude)</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>5000</td>
<td></td>
</tr>
</tbody>
</table>

If yaw rate, roll angle, and latac frequency response data is available, a good procedure is to fit curves first to the yaw rate and roll angle data, thus establishing COEFFS 1 to 11, and then fit to the latac data using fixed values of 1 to 4 and iterating only over 12 to 16. Sometimes a solution is obtained giving a positive real part in the latac zeros. This is usually caused by the values of COEFFS 13 and 15 being too small and can be cured by re-running with larger initial values.

A suitable standard set of control parameters is as follows:

- `IPRINT` = 10
- `IMAX` = 30 (more than 30 iterations is not normally helpful)
- `EPS` = 0.001
- `SO` = 500
- `DEL` = 0.0001
A6.4 FITTRANS – Computer Program Listing

FITTRANS VEHICLE RESPONSE DATA ANALYSIS 38 52;
"BEGIN" "INTEGER" I,J,K,RED,BREAK,NENUF,RUN,DATPTS,NCHS,STPT,NTH,N,E,
CH,ZERO,PTRND,D1,D2,START,STARTFIT,ENUF,LASTCH,KIND,
SMOOTH;
"REAL" PTD, ZEROSH, DT, SCALE, FR, PSC, GSC;
"REAL" "ARRAY' MAX[1:9];
"INTEGER" "ARRAY' TESTIDENT[1:5], NAME[1:20];

"PROCEDURE" IDENTIFY(NAME, TESTIDENT, STPT, NTH, E, DT);
"INTEGER" STPT, NTH, E;
"REAL" DT;
"INTEGER" "ARRAY' NAME, TESTIDENT;
"BEGIN" "INTEGER" I; I:=1;
OUTSTRING(NAME, I);
I:=1;
"IF" NTH=999 "THEN"
"PRINT' SAMELINE, "S2`DIRECT`S2``", OUTSTRING(TESTIDENT, I) "ELSE"
"BEGIN" I:=1;
"S`DT=`", ALIGNED(1,3), DT*NTH, "S2``", OUTSTRING
(TESTIDENT, I);
"END' ;
"END' ;

SCALE:=254/LN(2); GSC:=2540/3/LN(10); PSC:=254/60;
DIRL:
"BEGIN" "COMMENT' SEGMENT one sort out data inputs;
"INTEGER" I;

"COMMENT' Procedure OPENREAD;
"PROCEDURE' OPENREAD (H, IDENTIFIER); "COMMENT' POSITIONS TAPE AT
POINT IMMEDIATELY AFTER STRING IDENTIFYING RECORD;
"INTEGER" H; "COMMENT" HANDLER NUMBER;
"INTEGER" "ARRAY" IDENTIFIER; "COMMENT" CONTAINS STRING USED
TO IDENTIFY RECORD; MUST BE FIRST ITEM ON RECORD
& MUST CONTAIN 18 UPPER CASE CHARACTERS;

BEGIN" "INTEGER" I;
"INTEGER" "ARRAY" TEST[1:5];
MTREWIND(H); "COMMENT" REWINDS TO BEGINNING OF TAPE;
ENDELOCK(H);

READ: I:= 1;
"READ" FILE(H), INSTRING(TEST,I);
"FOR" I:= 1 "STEP" 1 "UNTIL" 5 "DO" "IF" TEST[1] "NE"
IDENTIFIER[1] "THEN"
BEGIN" MTSEEK(H);
ENDELOCK(H);
"GOTO" READ;

"END"

"END";

"PRINT" PUNCH(3),"L\'FREQ DATA FROM PT TYPE 999 ELSE 0 \";
"READ" READER(3),FR;
"IF" FR=0 "THEN" "GOTO" MORE;
"PRINT" PUNCH(3),"L\'TYPE DATE AND PASS NO STRING \";
I:=1; "READ" READER(3),INSTRING(NAME,I);
"PRINT" PUNCH(3),"L\'LOAD PT\";
WAIT;
I:=1; INSTRING(TESTIDENT,1);
"READ" N; ENUF:=2*N-1; NTH:=999;
J:=K:=1;
"GOTO" DIR5;

MORE: PUNCH(1);READER(1);
"PRINT" PUNCH(3),"L\'TIME DATA FROM PT TYPE 999 ELSE 0 \";
"READ" READER(3),PTIND;
"PRINT" PUNCH(3),"L\'WINDOW TYPE KIND \";
"READ" READER(3),KIND;
"PRINT"PUNCH(3),"L\'TYPE SMOOTH \";
"READ"READER(3),SMOOTH;
"PRINT" PUNCH(3),"L\'TYPE DATE, PASS NO AND KIND STRING\";
I:=1; "READ" READER(3),INSTRING(NAME,I);
"IF" PTIND =999 "THEN"
"BEGIN" "PRINT" PUNCH(3),"L' LOAD PT";
   I:=1; INSTRING(TESTIDENT,I);
"READ" RUN,DATPTS,DT,NCHS;
"FOR" I:=1 "STEP" 1 "UNTIL" 9 "DO" "READ" MAX[I];
"GOTO" PONLY;
"END";
"PRINT" PUNCH(3),"L' LOAD MT HANDLER 2 WITH NFB HANDLING DATA FILE"
WAIT;
"PRINT" PUNCH(3),"L' TYPE TEST IDENTIFIER STRING";
   I:=1;
"READ" READER(3),INSTRING(TESTIDENT,I); 
OPENREAD(2,TESTIDENT);
LASTCH:=0;
"READ" FILE(2), RUN,DATPTS,DT,NCHS; 
"FOR" I:=1 "STEP" 1 "UNTIL" NCHS "DO"
   "READ" FILE(2),MAX[I];
PTONLY: 
   ,"E.(FINAL NO OF POINTS=2*E)"L";
"READ" READER(3),STPT,NTH,E; N:=41E;
"IF" N > (DATPTS-STPT+1)/NTH "THEN"
"BEGIN" "PRINT" PUNCH(3),"L' TOO LITTLE DATA";
   "GOTO"PTONLY;
"END";
FR:=10/(N*DT*NTH); ENUF:=N/2;
"IF" FR/10<0.1 "THEN" START:=ENTIER(1/FR) "ELSE" START:=1;
"FOR" I:=1 "STEP" 1 "UNTIL" N/2 "DO"
"BEGIN" K:=ENTIER(I*FR/10);
   "IF" K=4 "THEN"
      "BEGIN" ENUF:=I;
         I:=N/2+1;
      "END";
   "END";
"IF" PTIND=999 "THEN"
"BEGIN" J:=9; K:=N;
   "END" "ELSE" J:=K:=1;
"END' of segment one;
DIR5: "BEGIN" "ARRAY" PTIMDAT[1:J,1:K];
   "IF" FR=999 "THEN" "GOTO" DIR2;
   "IF" PTIND=999 "THEN"
   "BEGIN" "FOR" I:= 1 "STEP" 1 "UNTIL" NCHS "DO"
   "BEGIN" "FOR" J:= 2 "STEP" 1 "UNTIL" STPT "DO"
   "READ" PTD;
   "FOR" K:=1 "STEP" 1 "UNTIL" N "DO"
   "BEGIN" "READ" PTIMDAT[1,K];
   "FOR" J:= 1 "STEP" 1 "UNTIL" NTH-1 "DO"
   "READ" PTD;
   "END";
   "FOR" J:= 1 "STEP" 1 "UNTIL" DATPTS-(STPT-1) -N*NTH "DO"
   "READ" PTD;
   "END";
   "FOR" I:= NCHS+1 "STEP" 1 "UNTIL" 9 "DO"
   "FOR" J:= 1 "STEP" 1 "UNTIL" N "DO"
   PTIMDAT[I,J]:= 0;
   "END";

"BEGIN" "COMMENT" SEGMENT two time data plot;
"INTEGER" I, TS;
"REAL" P;
"REAL" "ARRAY" A[1:N];

"COMMENT" procedures BACKSPACE and RDD;

"PROCEDURE" BACKSPACE(H, IDENTIFIER, BLOCKS);

"COMMENT" BACKSPACES TO HEAD OF RECORD CURRENTLY BEING READ, CHECKS
TEST IDENTIFIER AND POSITIONS TAPE AT POINT IMMEDIATELY
AFTER TEST IDENTIFIER;
"INTEGER" H, BLOCKS; "COMMENT" HANDLER NUMBER, NO OF BLOCKS TO BACKSPACE;
"INTEGER" "ARRAY" IDENTIFIER; "COMMENT" CONTAINS STRING IDENTIFYING
RECORD; MUST BE FIRST ITEM ON RECORD AND MUST CONTAIN 18
UPPER CASE CHARACTERS;

"BEGIN" "INTEGER" I, REWIND;
"INTEGER" "ARRAY" TEST[1:5];
REWIND:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" BLOCKS "DO" MTBACK(H);
IP:=ENDBLOCK(H);
MTSEEK(H);
FINDREC(H);
"IF" FILECOND(H) = -2 "THEN"
"BEGIN" MTREWIND(H); "COMMENT" REWIND TO BEGINNING OF TAPE;
"IF" REWIND "N" O "THEN"
"BEGIN" "PRINT" PUNCH(3),'L2S2'MT READ FAIL';
STOP;
"END";
REWIND:=1;
"GOTO" IP;
"END" "ELSE"
"BEGIN" I:=1; 
"READ" FILE(H),INSTRING(TEST,I);
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO" "IF" TEST(I)
"NE" IDENTIFIER(I) "THEN""GOTO" IP;
"END";
"PROCEDURE" RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A,
PTIND,PTIMDAT);
"INTEGER"PTIND, DATPTS,LASTCH,STPT,NTH,N,CH;
"REAL" ZEROSH;
"INTEGER" "ARRAY" TESTIDENT;
"REAL" "ARRAY"PTIMDAT, A;
"BEGIN" "INTEGER" I,D,J,BLOCKS;
"REAL" DUM;
"IF" PTIND = 999 "THEN" "GOTO" PTLABEL;
"IF" LASTCH= 0 "THEN" "GOTO" THERE "ELSE"
"BEGIN" BLOCKS:=CHECKI(ENTIER((LASTCH*CHECKI(DATPTS)+33)/128)+10);
BACKSPACE(4,TESTIDENT,BLOCKS);
"READ" FILE(2),I,D,DUM,J;
"FOR" I:=1 "STEP" 1 "UNTIL" 9 "DO" "READ" FILE(2),DUM;
THERE: "FOR" I:=1 "STEP" 1 "UNTIL" STPT-1+(CH-1)*DATPTS "DO"
"READ" FILE(2),DUM;
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
"BEGIN" "READ" FILE(2), A[1];
   "FOR" J:=1 "STEP" 1 "UNTIL" NTH-1 "DO"
      "READ" FILE(2), DUM;
"END";
ZEROSH:=A[1];
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
   A[1]:=-A[1];
"IF" CH=7 OR CH=5 "THEN"
   "BEGIN" ZEROSH:=-ZEROSH;
      "FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
         A[1]:=-A[1];
"END";
"END";
LASTCH:=CHECKI(CH);

PTLABEL: "IF" PTIND = 999 "THEN"
   "BEGIN" "FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
      A[1]:=PTIMDAT[CH,1];
      "IF" CH=7 OR CH=5 "THEN"
         "BEGIN" "FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
            A[1]:=-A[1];
"END";
      ZEROSH:=A[1];
   "END";

"PRINT" PUNCH(3), "L\"TYPE 1 FOR TIME DATA PLOT\" ELSE 0 ";
"READ" READER(3), I;
"IF" I=1 "THEN"
   "BEGIN" "IF" ABS(N*DT*NTH/5-1) < .01 "THEN" TS:=1 "ELSE" TS:=ENTIER
      (N*DT*NTH/5)+1;
      SETORIGIN(800,0); MOVEPEN(-450,-224); DRAWLINE(-450,-254);
      DRAWLINE(-420,-254);
      MOVEPEN(0,0); PUNCH(5); WAY(0,4);
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO"
"BEGIN" DRAWLINE(300*I,0); DRAWLINE(300*I,16);
    MOVEPEN(300*I-50,-44); "PRINT" DIGITS(2),I*TS;
    MOVEPEN(300*I,0);
"END";
MOVEPEN(1610,-254); DRAWLINE(1640,-254); DRAWLINE(1640,-224);
MOVEPEN(670,-94); "PRINT" TIME SEC;
WAY(1,4); MOVEPEN(-64,77); "PRINT" LATAC ; MOVEPEN(-64,291);
"PRINT" YAW ANGLE ;
MOVEPEN(-64,535); "PRINT" ROLL ANGLE ; MOVEPEN(-64,809);
"PRINT" YAW RATE; MOVEPEN(-64,1053); "PRINT" ROLL RATE ;
MOVEPEN(-64,1317); "PRINT" RW ANGLE ; MOVEPEN(-64,1571);
"PRINT" HW ANGLE ; MOVEPEN(-64,1805); "PRINT" WIND ANGLE ;
MOVEPEN(-64,2079); "PRINT" WIND VEL ; MOVEPEN(0,2286);
"FOR" I:=17 "STEP" -2 "UNTIL" 1 "DO"
"BEGIN" DRAWLINE(0,1*127); DRAWLINE(16,1*127);
MOVEPEN(30,1*127-100); "PRINT" FREEPOINT(4),MAX[(19-1)/2];
MOVEPEN(0,1*127);
"END"; DRAWLINE(0,0);
"FOR" I:=1 "STEP" 1 "UNTIL" 9 "DO"
"BEGIN" MOVEPEN(0,1*254-127);
    CH:=10-I;
    RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A, PTIND,PTIMDAT);
    "IF" CH=7"OR" CH=5 "THEN"
    "BEGIN" ZEROSH:=-ZEROSH;
        "FOR" J:=1 "STEP" 1 "UNTIL" N "DO"
            A[J]:=-A[J];
    "END";
    "FOR" J:=0 "STEP" 1 "UNTIL" N-1 "DO"
        DRAWLINE(J*DT*NTH*300/TS,A[J+1]*127/MAX[10-I]+I*254-127);
    "FOR" J:=ENTIER((N-1)*DT*NTH*300/TS) "STEP" -100
        "UNTIL" 0 "DO"
        "BEGIN" P:=ZEROSH*127/MAX[10-I]+I*254-127;
            MOVEPEN(J,P); DRAWLINE(J-50,P);
    "END";
"END";
MOVEPEN(1610,2726); DRAWLINE(1640,2726); DRAWLINE(1640,2696);
MOVEPEN(235,2540); "PRINT" WAY(0,8), "VEHICLE RESPONSE RUN",
DIGITS(4), RUN;
MOVEPEN(50,2440); WAY(0,4); IDENTIFY(NAME, TESTIDENT, STPT,
NTH, E, DT);
MOVEPEN(-450,2696); DRAWLINE(-450,2726); DRAWLINE(-420,2726);
MOVEPEN(-800,3726);
"END';
"END' of segment two;

"BEGIN" "COMMENT" SEGMENT
three Frequency plots;
"INTEGER" DCH;
"ARRAY" A, B[0:N+1];
"REAL" Q;

"COMMENT" PROCEDURES DRAFRAX, PLAX, PHAX, WINDOW, BACKSPACE, RDD,
FASTFOURIER, FTWH, SORTPHASE (BACKSPACE, RDD, FASTFOURIER
NOT LISTED HERE);

"PROCEDURE" DRAFRAX(SCALE);
"REAL" SCALE;
"BEGIN" "REAL" A, X;
"INTEGER" N, B;
PUNCH(S);
WAY(0,4);
SETORIGIN(800,0);
DRAWLINE(-500, -254);
MOVEPEN(0,0);
A := 0.1;
"FOR" N := 1 "STEP" 1 "UNTIL" 9 "DO"
"BEGIN" X := SCALE * LN(N);
DRAWLINE(X, 0);
DRAWLINE(X, 16);
MOVEPEN((X-45), -44);
"PRINT" FREEPOINT(1), A;
MOVEPEN(X,0); A:=A+0.1;
 "END";
 B:=1;
 "FOR" N:=10 "STEP" 10 "UNTIL" 50 "DO"
 "BEGIN" X:=SCALE LN(N);
 DRAWLINE(X,0); DRAWLINE(X,16);
 MOVEPEN((X-30), -44);
 "PRINT" DIGITS(1), B;
 B:=B+1;
 MOVEPEN(X,0);
 "END";
 MOVEPEN(1560, -254); DRAWLINE(1590, -254);
 DRAWLINE(1590, -224); MOVEPEN(630, -94);
 "PRINT" "FREQUENCY HZ";
 MOVEPEN(0,0);
 "END";

"PROCEDURE" PLAX(SCALE,ZERO);
 "INTEGER" ZERO;
 "REAL" SCALE;
 "BEGIN" "INTEGER" K, I;
 DRAFRAX(SCALE); K:=0;
 "FOR" I:=-30 "STEP" 6 "UNTIL" 30 "DO"
 "BEGIN" DRAWLINE(0,K); DRAWLINE(16,K); MOVEPEN(-30,(K-16));
 "PRINT" DIGITS(2), I; MOVEPEN(0,K); K:=K+254;
 "END";
 MOVEPEN(-100,1200); "PRINT"WAY(1,4), GAIN DB"; ZERO:=1270;
 MOVEPEN(50,2440); I:=1; WAY(0,4); IDENTIFY(NAME, TESTIDENT,
 STPT, NTH, E, DT);
 MOVEPEN(-500,2696); DRAWLINE(-500,2726); DRAWLINE(-470,2726);
 MOVEPEN(1560,2726); DRAWLINE(1590,2726); DRAWLINE(1590,2696);
 "END";

"PROCEDURE" PHAX(SCALE, ZERO);
 "INTEGER" ZERO;
 "REAL" SCALE;
 "BEGIN" "INTEGER" I, J;
 DRAFRAX(SCALE); J:=0;
"FOR" I:=-300 "STEP" 60 "UNTIL" 300 "DO"
"BEGIN" DRAWLINE(0,J); DRAWLINE(16,J);
    MOVEPEN(-100,J-16); "PRINT" DIGITS(3),I;
    MOVEPEN(0,J); J:=J+254;
"END";
MOVEPEN(-100,1120); "PRINT" WAY(1,4), 'PHASE ANGLE DEG';
ZERO:=1270;
MOVEPEN(50,2440); I:=1; WAY(0,4); IDENTIFY(NAME,TESTIDENT,
STPT,NTH,E,DT);
MOVEPEN(-500,2696); DRAWLINE(-500,2726); DRAWLINE(-470,2726);
MOVEPEN(1560,2726); DRAWLINE(1590,2726); DRAWLINE(1590,2696);
"END";

"PROCEDURE" WINDOW(A,KIND,N);
"INTEGER" N,KIND;
"ARRAY" A;
"BEGIN" "INTEGER" I;
"REAL" F;
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
"BEGIN" "IF" KIND=2 "THEN"
    A[I] :=("IF" I < N/2 "THEN" 2*(I-1)/N "ELSE" 2-2*I/N)*A[I];
    "IF" KIND=3 "THEN"
    "BEGIN" "IF" I < N/4 "THEN" F:=2*(4*(I-1)/N)^3 "ELSE"
        "IF" I < N/2 "THEN" F:=1-6*(2*I/N-1)^2
            -6*(2*I/N-1)^3 "ELSE"
        "IF" I < 3*N/4 "THEN" F:=1-6*(2*I/N-1)^2+6*
            (2*I/N-1)^3 "ELSE"
        "IF" I "LE" N "THEN" F:=2*(2-2*I/N)^3;
    A[I] :=F*A[I];
    "END";
    "END";
"END";
"PROCEDURE" FTWH(A, N, M, KIND, CH, FR);
"INTEGER" N, M, KIND, CH;
"REAL" FR;
"REAL" "ARRAY" A;
"BEGIN" "INTEGER" I, D;
"REAL" "ARRAY" A1, B1[0:N/2];
"FOR" I:=0 "STEP" 1 "UNTIL" N/2-1 "DO"
"BEGIN" A1[I]:=A[I+1];
B1[I]:=A[N/2+I+1];
"END";
D:=M-1;
REALTRANSFORM(A1, B1, D, "FALSE");
"FOR" I:=0 "STEP" 1 "UNTIL" N/2 "DO"
"BEGIN" A[I]:=(A1[I]+B1[I])/2;
A[I+N/2+1]:=ARCTAN(-B1[I]/A1[I])*57.3;
"IF" A[I+N/2+1]<0 "THEN"
"BEGIN" "IF" -B1[I]<0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+180;
"END";
"IF" A[I+N/2+1]>0 "THEN"
"BEGIN" "IF" -B1[I]<0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+180;
"IF" -B1[I]>0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+360;
"END";
"IF" A[I+N/2+1]=0 "THEN"
"BEGIN" "IF" A[I]<0 "THEN" A[I+N/2+1]:=180;
"END";
"END";
"IF" KIND=1 "THEN"
"BEGIN"
A1[0]:=0.54*A[0]+0.46*A[1];
A1[N/2]:=0.54*A[N/2]+0.46*A[N/2-1];
"FOR" I:=1 "STEP" 1 "UNTIL" N/2-1 "DO"
A1[I]:=0.23*A[I-1]+0.54*A[I]+0.23*A[I+1];
"FOR" I:=0 "STEP" 1 "UNTIL" N/2 "DO"
A[I]:=A1[I];
"END";
"IF" CH=5 "THEN"
"BEGIN" "PRINT" PUNCH(3), "L" TYPE 1 FOR ROLL ANGLE FROM ROLL RATE ELSE 0
"READ" READER(3), I;
"IF" I=1 "THEN"
"BEGIN" "FOR" I:=1 "STEP" 1 "UNTIL" N/2 "DO"
   "BEGIN" A[1]:=A[1]/(FR/10*I*6.2832);
       A[I+N/2+1]:=A[I+N/2+1]-90;
   "END";
   "IF" FR/10 < .2 "THEN" I:=ENTER(2/FR)
   "ELSE" I:=1;
   A[0]:=A[I];
   A[N/2+1]:=A[N/2+1]-90;
"END";
"END";
"END";

"PROCEDURE" SORTPHASE(DUM,N);
"ARRAY" DUM;
"INTEGER" N;
"BEGIN" "INTEGER" I;
   "FOR" I:=0 "STEP" 1 "UNTIL" N "DO"
   "BEGIN" "IF" DUM[I]>90 "THEN"
       DUM[I]:=DUM[I]-360;
   "END";
"END";

"PRINT" PUNCH(3), "L" TYPE 1 FOR FREQ AMP OR PHASE PLOTS ELSE 0
"READ" READER(3), D1;
"IF" D1=0 "THEN" "GOTO" L8;
"PRINT" PUNCH(3), "L" TYPE 1 FOR FREQ TRANSF GAIN PLOTS ELSE 0
"READ" READER(3), D1;
"IF" D1=0 "THEN" "GOTO" L2;
PLAX(SCALE,ZERO); I:=1;
MOVEPEN(50,2340); "PRINT" WAY(0,4), "CHAN" S4 "SS RESP";
L1:"PRINT" PUNCH(3), "L" TYPE CHANNEL NO OF PLOT REqd ELSE 0
"READ" READER(3), CH;
"IF" CH=0 "THEN"
"BEGIN" MOVEPEN(410,2540); "PRINT" WAY(0,8), 'VEHICLE RESPONSE';

MOVEPEN(-800,3726);

"GOTO" L2;

"END";

RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A,PTIND,PTIMDAT);

WINDOW(A,KIND,N); FTWH(A,N,E,SIMOOTH,CH,FR);

MOVEPEN(SCALE*LN(FR*START),GSC*LN(A[START]/A[0])+ZERO);

"FOR" J:=START+1 "STEP" 1 "UNTIL" ENUF "DO"

DRAWLINE(SCALE*LN(FR*J),GSC*LN(A[J]/A[0])+ZERO);

"PRINT" DIGITS(1),WAY(0,4),CH;

MOVEPEN(50,2340-40*I);

"PRINT" DIGITS(1),WAY(0,4),CH," S",SCALED(4),A[0];

I:=I+1;

"GOTO" L1;

L2:"PRINT" PUNCH(3); "L" TYPE 1 FOR FREQ TRANSFORM PHASE PLOTS

ELSE 0

; "READ" READER(3),DL;

"IF" DL=0 "THEN" "GOTO" L4;

PHAX(SCALE,ZERO);

L3:"PRINT" PUNCH(3); "L" TYPE CHANNEL NO OF PLOT REQD ELSE 0

; "READ" READER(3),CH;

"IF" CH=0 "THEN" "BEGIN" MOVEPEN(410,2540); "PRINT" WAY(0,8), 'VEHICLE RESPONSE';

MOVEPEN(-800,3726);

"GOTO" L4;

"END";

RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A,PTIND,PTIMDAT);

WINDOW(A,KIND,N);

FTWH(A,N,E,SIMOOTH,CH,FR);

MOVEPEN(SCALE*LN(FR*START),PSC*A[N/2+1+START]+ZERO);

"FOR" J:=START+1 "STEP" 1 "UNTIL" ENUF "DO"

DRAWLINE(SCALE*LN(FR*J),PSC*A[N/2+1+J]+ZERO);

"PRINT" DIGITS(1),WAY(0,4),CH;

"GOTO" L3;
L4: "PRINT" PUNCH(3), "L' TYPE 1 FOR TRANSFUN GAIN PLOTS ELSE 0
    "READ" READER(3), DI;
    "IF" DI=0 "THEN" "GOTO" L6;
    PLAX(SCALE,ZERO); I:=1;
    MOVEPEN(50,2340); "PRINT" WAY(0,4), 'CH RATIO S4 SS RESP';
L5: "PRINT" PUNCH(3), "L' TYPE RATIO OF CHANNELS REQD ELSE 0 0
    "READ" READER(3), CH,DCH;
    "IF" CH=0 "OR" DCH=0 "THEN"
    "BEGIN" MOVEPEN(410,2540); "PRINT" WAY(0,8), 'VEHICLE RESPONSE';
        MOVEPEN(-800,3726);
        "GOTO" L6;
    "END";
    RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A,PTIND,PTIMDAT);
    WINDOW(A,KIND,N); FTWH(A,N,E,SMOOTH,CH,FR);
    RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,DCH,ZEROSH,B,PTIND,PTIMDAT);
    WINDOW(B,KIND,N); FTWH(B,N,E,SMOOTH,DCH,FR);
    Q:=A[0]/B[0];
    MOVEPEN(SCALE*LN(FR*START),GSC*LN(A[START]/B[START])/Q)+ZERO);
    "FOR" J:=START+1 "STEP" 1 "UNTIL" ENUF "DO"
    DRAWLINE(SCALE*LN(FR*J),GSC*LN(A[J]/B[J])/Q)+ZERO);
    "PRINT" DIGITS(1),WAY(0,4),CH,"/",DCH;
    MOVEPEN(50,2340-40*I);
    "PRINT" DIGITS(1),WAY(0,4),CH," S",DCH," S",SCALED(4),Q;I:=I+1;
    "GOTO" L5;
L6: "PRINT" PUNCH(3), "L' TYPE 1 FOR TRANSFUN PHASE PLOTS ELSE 0
    "READ" READER(3), DI;
    "IF" DI=0 "THEN" "GOTO" L8;
    PHAX(SCALE,ZERO);
L7: "PRINT" PUNCH(3), "L' TYPE RATIO OF CHANNELS REQD ELSE 0 0
    "READ" READER(3), CH,DCH;
    "IF" CH=0 "OR" DCH=0 "THEN"
    "BEGIN" MOVEPEN(410,2540); "PRINT" WAY(0,8), 'VEHICLE RESPONSE';
        MOVEPEN(-800,3726);
        "GOTO" L8;
    "END";
RDDD(TESTIDENT, DATPTS, LASTCH, STPT, NTH, N, CH, ZEROSH, A, PTIND, PTIMDAT);
WINDOW(A, KIND, N); FTWH(A, N, E, SMOOTH, CH, FR);
RDDD(TESTIDENT, DATPTS, LASTCH, STPT, NTH, N, DCH, ZEROSH, B, PTIND, PTIMDAT);
WINDOW(B, KIND, N); FTWH(B, N, E, SMOOTH, DCH, FR);
"FOR" J := 0 "STEP" 1 "UNTIL" N/2 "DO"
J := N/2; SORTPHASE(A, J);
MOVEPEN(SCALE*LN(FR*START), PSC*A(START]+ZERO);
"FOR" J := START+1 "STEP" 1 "UNTIL" ENUF "DO"
DRAWLNK(SCALE*LN(FR+J), PSC*A[J]+ZERO);
"PRINT" DIGITS(1), WAY(0,4), CH, "/", DCH;
"GOTO" L7;
L8: "END" of segment three;
"COMMENT" FITTRANS PART TWO;

DIR2:

"BEGIN"

"INTEGER" NADJ, M, PH, HRW, CC, TOT, IPRINT, IMAX, CPlot;
"INTEGER" "ARRAY" REDNO[0:ENUF], AKEY[1:16], PP, STFT[1:3, 1:2],
CHAIN[1:3];
"ARRAY" DATA[1:2, 1:3, 0:ENUF], ADJ, COEFFS, SKALE[1:16], D[1:5],
X, Y[1:4], ZVAL[1:3];
"REAL" EPS, SO, DEL, FF, DFR;
CPlot:=0;
"FOR" I:=0 "STEP" 1 "UNTIL" ENUF "DO" REDNO[1]:=I;

"IF" FR "NE" 999 "THEN" "GOTO" FOUR1;
"READ" HRW, PP[1, 1], PP[1, 1], PP[3, 1],
PP[2, 1], PP[2, 2], PP[3, 2];
"FOR" PH:=1 "STEP" 1 "UNTIL" 2 "DO"
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" I:=1 "STEP" 1 "UNTIL" PP[1, PH] "DO"
"READ" DATA[PH, M, I-1], DATA[PH, M, N+1-1];
CC:=4;
"IF" PP[1, 1] "NE" 0 "OR" PP[1, 2] "NE" 0 "THEN" CC:=CC+4;
"IF" PP[2, 1] "NE" 0 "OR" PP[2, 2] "NE" 0 "THEN" CC:=CC+3;
"IF" PP[3, 1] "NE" 0 "OR" PP[3, 2] "NE" 0 "THEN" CC:=CC+5;
DIR3: "PRINT" PUNCH(3), "L 'LOAD ADDIT DATA WITH", DIGITS(3), CC,
"S 'COEFFS";
WAIT; "READ" DI;
"IF" DI "NE" CC "THEN"
"BEGIN" "PRINT" PUNCH(3), "L 'WRONG NO OF COEFFS";
"GOTO" DIR3;
"ENDIF";
"READ" CPlot, STFT[1, 1], STFT[2, 1], STFT[3, 1], STFT[1, 2],
STFT[2, 2], STFT[3, 2];
NADJ:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" CC "DO"
"BEGIN" "READ" COEFFS[I],AK[EY][I];
"IF" AK[EY][I]=1 "THEN"
    "BEGIN" ADJ[NADJ+1]:=COEFFS[I]; NADJ:=NADJ+1;
    "ENDIF";
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" NADJ "DO"
    SKALE[I]:=1/ADJ[I];
"READ" IPRINT,IMAX,EPS,SO,DEL;
    TOT:=0;
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
    "FOR" PH:=1 "STEP" 1 "UNTIL" 2 "DO"
        TOT:=TOT+PP[M,PH]-("IF" PP[M,PH]=0 "THEN" 0 "ELSE" STFT[M,PH]-1);
    START:=1; CHAN[1]:=6; CHAN[2]:=7; CHAN[3]:=9;
"GOTO" DIR4;
FOUR1:

"BEGIN" "COMMENT" SEGMENT four Prepare curve fit data;
"ARRAY" A,B[0:N+1];

"COMMENT" PROCEDURES WINDOW,BACKSPACE,RDD FASTFOURIER ETC, FTWH,
SORTPHASE (THESE PROCEDURES NOT LISTED HERE);
PUNCH(3);READER(3);

"PRINT" "L" TYPE 1 FOR CURVE FITS ELSE 0 ";
"READ" D1;
"IF" D1=0 "THEN" "GOTO" FINAL;
"PRINT" "L" PTS AVAILABLE FOR FIT (BETWEEN 4HZ),SAMELINE,
    DIGITS(5),ENUF+1,"L" TYPE 1 FOR REDUCED DATA ELSE 0 ";
"READ" RED;
"IF" RED=1 "THEN"
"BEGIN" BREAK:=ENTIER(10/FR); NENUF:=BREAK;
    "FOR" I:=0 "STEP" 1 "UNTIL" BREAK "DO" REDNO[I]:=I;
    "FOR" I:=BREAK+2 "STEP" 2 "UNTIL" 2*BREAK "DO"
        "BEGIN" NENUF:=NENUF+1;REDNO[NENUF]:=I; "END";
"FOR" I:=2 *BREAK+4 "STEP" 4 "UNTIL" ENUF "DO"
"BEGIN" NENUF:=NENUF+1; READNO[NENUF]:=I;"END";
"PRINT" "L" PTS NOW",SAMELINE,DIGITS(5),NENUF+1;
"END";

"PRINT" "L" TYPE STFT PP ELSE O O "L" YAW", 
'R' Amp ";
"READ" STFT[1,1], PP[1,1];
"PRINT" "L" YAW RATE PHASE "; "READ" STFT[1,2], PP[1,2];
"PRINT" "L" ROLL ANGLE AMP "; "READ" STFT[2,1], PP[2,1];
"PRINT" "L" ROLL ANGLE PHASE "; "READ" STFT[2,2], PP[2,2];
"PRINT" "L" LATAC AMP "; "READ" STFT[3,1], PP[3,1];
"PRINT" "L" LATAC PHASE "; "READ" STFT[3,2], PP[3,2];
CC:=4;
"IF" PP[1,1] "NE" 0 "OR" PP[1,2] "NE" 0 "THEN" CC:=CC+4;
"IF" PP[2,1] "NE" 0 "OR" PP[2,2] "NE" 0 "THEN" CC:=CC+3;
"IF" PP[3,1] "NE" 0 "OR" PP[3,2] "NE" 0 "THEN" CC:=CC+5;
"PRINT" "L" RELATIVE TO WIND ANGLE TYPE 2, HW TYPE 3, RW TYPE 4 ";
"READ" HWWR;

FOOL: "PRINT" "L" LOAD ADDITIONAL DATA WITH",DIGITS(2),CC,"S"
' COEFFS";
WAIT; READER(1);
"READ" D1; "IF" D1 "NE" CC "THEN"
"BEGIN" "PRINT" "L" WRONG NO OF COEFFS"
"GOTO" FOOL;
"END";
NADJ:=0;
"FOR" I:=1 "STEP" 1 "UNTIL" CC "DO"
"BEGIN" "READ" COEFFS[I],AKEY[1];
"IF" AKEY[1]=1 "THEN"
    "BEGIN" ADJ[NADJ+1]:=COEFFS[1]; NADJ:=NADJ+1;
    "END";
"END";
"FOR" I:=1 "STEP" 1 "UNTIL" NADJ "DO"
SKALE[I]:= 1/ADJ[I];
"READ" IPRINT, IMAX, EPS, SO, DEL;
TOT:=O;
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
"FOR" PH:=1 "STEP" 1 "UNTIL" 2 "DO"
TOT:=TOT+PP[M,PH]-("IF" PP[M,PH]=0 "THEN" 0 "ELSE" STFT[M,PH])1;
CHAN[1]:=6;CHAN[2]:=7;CHAN[3]:=9;
RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,HWRW,ZEROSH,
A,PTIND,PTIMDAT);
WINDOW(A,KIND,N);FTWH(A,N,Z,SMOOTH,HWRW,FR);
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
"BEGIN" "IF" PP[M,1] "NE" 0 "OR" PP[M,2] "NE" 0 "THEN"
  "BEGIN"
    "IF" M=2 "THEN"
    "BEGIN" "PRINT" PUNCH(3),"L"TYPE 1 FOR
    ROLL ANGLE FROM ROLL RATE ELSE
    0 ";
    "READ" READER(3),1;
    "IF" I=1 "THEN" CHAN[2]:=5;
  "END";
  CH:=CHAN[M];
  RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,
  CH,ZEROSH,B,PTIND,PTIMDAT);
  WINDOW(B,KIND,N);
  FTWH(B,N,E,SMOOTH,CH,FR);
  "IF" PP[M,1] "NE" 0 "THEN"
  "FOR" I:=0 "STEP" 1 "UNTIL" ENUF "DO"
  DATA[1,M,1]:=B[1]/A[I];
  "IF" PP[M,2] "NE" 0 "THEN"
  "BEGIN" "FOR" I:=0 "STEP" 1 "UNTIL" ENUF "DO"
  B[I]:=B[I+N/2+1]-A[I+N/2+1];
  I:=PP[M,2]; SORTPHASE(B,I);
  "FOR" I:=0 "STEP" 1 "UNTIL"
  PP[M,2]-1 "DO"
  DATA[2,M,1]:=B[I];
  "END";
  "END";
"END";
"END";
"END";
"BEGIN" "COMMENT" SEGMENT five fit curves;
   "INTEGER" I;

"COMMENT" PROCEDURES FUNCTION, CALFUN, FUNCT, OPLS (OPLS NOT LISTED);

"REAL" "PROCEDURE" FUNCTION(I, CC, COEFFS, AKEY, ADJ, DATA, M, PH, PP, FR, N);
"VALUE" I, CC, M, PH, PP, FR;
"REAL" "ARRAY" COEFFS, ADJ, DATA;
"INTEGER" "ARRAY" PP, AKEY;
"INTEGER" I, CC, M, PH, N;
"REAL" FR;
"BEGIN" "REAL" P, Q, R, S, W, D;
   "INTEGER" NCO, K, L;
   "REAL" "PROCEDURE" PHASE(P, Q, R, S);
   "REAL" P, Q, R, S;
   "BEGIN" "REAL" E, F, G;
      E := P * R + Q * S;
      F := Q * R - P * S;
      G := ARCTAN(F/E) * 57.3;
      "IF" G > 0 "THEN"
      "BEGIN" "IF" F < 0 "THEN" G := G - 180;
      "END";
      CHECKS (FUNCTION);
      "IF" G < 0 "THEN"
      "BEGIN" "IF" F > 0 "THEN" G := G - 180;
      "END";
      "IF" G = 0 "THEN"
      "BEGIN" "IF" E < 0 "THEN" G := -180;
      "END";
      PHASE := G;
      "END";
   "END";
"If" FR=999 "then" W:=DATA[PH,M,N+1]*6.2832 "else"
W:=FR*1.62832;
K:=1;
"for" L:=1 "step" 1 "until" CC "do"
"if" AKEY[L]=1 "then"
"begin" COEFFS[L]:=ADJ[K]; K:=K+1;
"end";
R:=(W*W-COEFFS[2])*W*W+COEFFS[4];
S:=(COEFFS[3]-COEFFS[1]*W*W)*W;
NCO:=5;
"if" M=1 "then"
"begin" P:=-COEFFS[6]*W*W+COEFFS[8];
Q:=(COEFFS[7]-W*W)*W;
"end";
"if" M=2 "then"
"begin" "if" CC=8 "or" CC=11 "or" CC=13 "or" CC=16 "then"
NCO:=NCO+4;
P:=-W*W+COEFFS[NCO+2];
Q:=COEFFS[NCO+1]*W;
"end";
"if" M=3 "then"
"begin" "if" CC=8 "or" CC=11 "or" CC=13 "or" CC=16 "then"
NCO:=NCO+4;
"if" CC=7 "or" CC=11 "or" CC=12 "or" CC=16 "then"
NCO:=NCO+3;
P:=(W*W-COEFFS[NCO+2])*W*W+COEFFS[NCO+4];
Q:=(COEFFS[NCO+3]-COEFFS[NCO+4]*W*W)*W;
"end";
D:=DATA[PH,M,1];
"if" PH=1 "then"
"begin" "if" PP[M,PH]=0 "then" FUNCTION:=0 "else"
FUNCTION:=(COEFFS[NCO]*SQRT((P*P+Q*Q)/(R*R+S*S)))-D)/D;
"end" "else"
"begin" "if" PP[M,PH]=0 "then" FUNCTION:=0 "else"
FUNCTION:=(PHASE(P,Q,R,S)-D)*.01;
"end";
"end";
"PROCEDURE" CALFUN(REDNO, X, SCALE, NADJ, NMAX, STFT, PP, FF, SS, INF, DATA, CC, COEFFS, AKEY, FR);

"REAL" "ARRAY" X, SCALE, SS, DATA, COEFFS;

"INTEGER" "ARRAY" REDNO, STFT, PP, AKEY;

"REAL" FF, FR;

"INTEGER" CC, NMAX, INF, NADJ;

"BEGIN" "INTEGER" I, J, M, PH, K;

"REAL" FF1;

CHECKS("CALFUN");

INF:=O; FF:=O; J:=1;

"FOR" I:=1 "STEP" 1 "UNTIL" NADJ "DO" X[I]:=X[I]/SCALE[I];

"FOR" PH:=1 "STEP" 1 "UNTIL" 2 "DO"

"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"

"FOR" I:=STFT[M,PH]-1 "STEP" 1 "UNTIL" (PP[M,PH]-1) "DO"

"BEGIN" "IF" PP[M,PH]=0 "THEN" FF1:=0 "ELSE"

"BEGIN" "INTEGER" K:=REDNO[I];

FF1:=FUNCTION(K, CC, COEFFS, AKEY, X, DATA, M, PH, PP, FR, NMAX);

"END";

"IF" PP[M,PH] "NE" 0 "THEN"

"BEGIN" SS[J]:=FF1; J:=J+1;

"END";

FF:=FF+FF1*FF1;

"END";

"FOR" I:=1 "STEP" 1 "UNTIL" NADJ "DO" X[I]:=X[I]*SCALE[I];
"IF' MAGIC "NE" 67882 "THEN"
"BEGIN" "PRINT" PUNCH(3), "L" WRONG NUMERICAL DERIVATIVE
SURROUTINE";
STOP;
"END";
"IF' ILIN=2 "THEN' "GOTO' L10;
CALFUN(REDNO,X,SCALE,N,NMAX,STFT,PP,FF,8,INF,DATACCCOEFFS,
AKEY,FR);
IEV:=IEV+1;
"IF' INF=1 "OR" ILIN=1 "THEN' "GOTO' EXIT;
L10: "FOR' I:=1 "STEP' 1 "UNTIL" N "DO' Z[1]:=X[1];
DEL:=AMAX1(DEL,.0625*.0625*.0625*.0625);
"FOR' I:=1 "STEP' 1 "UNTIL" N "DO'
"BEGIN' Z[1]:=X[1]+DEL;
CALFUN(REDNO,Z,SCALE,N,NMAX,STFT,PP,FF2,52,INF,DATA,
CC,COEFFS,AKEY,FR);
IEV:=IEV+1;
"IF' INF=1 "THEN' "GOTO' EXIT;
"FOR' J:=1 "STEP' 1 "UNTIL" M "DO'
A[J,1]:=(S[2][J]-S[J])/DEL;
Z[1]:=X[1];
"END');
EXIT: "END";
PUNCH(4);
"PRINT' "F``; IDENTIFY(NAME,TESTIDENT,STPT,NTH,E,DT);
"PRINT' "L2" INITIAL VALUES OF COEFFS``;
"FOR' I:=1 "STEP' 1 "UNTIL" CC "DO'
"BEGIN' "IF' (I-1)/8-(I-1) "DIV" 8 < .05 "THEN' "PRINT' "L``;
"PRINT' SAMELINE,SCALED(4),"S2``, COEFFS [1];
"END';
"PRINT' "L2`` POINTS AVAILABLE``,SAMELINE,PREFIX("S2``"),DIGITS(3),
PP[1,1],PP[2,1],PP[3,1],PP[1,2],PP[2,2],PP[3,2];
"PRINT' "L2`` START AT POINTS `, SAMELINE,PREFIX("S2``"),DIGITS(3),
STFT[1,1],STFT[2,1],STFT[3,1],STFT[1,2],STFT[2,2],STFT[3,2];
"PRINT' "L2`` IPRINT``,DIGITS(4),SAMELINE,IPRINT,"S2`` IMAX `,
IMAX:"S2``EPS `,SCALED(4),EPS,"S2``SO `,SO,"S2``
DEL `,DEL; ·
"IF" CLOT =1 "THEN" "GOTO" SKIPFIT;

"FOR" I := 1 "STEP" 1 "UNTIL" NADJ "DO"
ADJ[I] := ADJ[I] * SKALE[I];
OPLS(ADJ, NADJ, TOT, EPS, IMAX, IPRINT, SO, DEL,
SKALE, N, STFT, PP, DATA, CC, COEFFS, AKEY, FR);
J := 0;
"FOR" I := 1 "STEP" 1 "UNTIL" CC "DO"
"BEGIN" "IF" AKEY[I] = 1 "THEN" J := J + 1;
"ELSE" COEFFS[I];
"END";

"END" of segment five;

SKIPFIT:

"BEGIN" "COMMENT" SEGMENT six plot curves and calculate poles and zeros;
"INTEGER" I;

"COMMENT" procedures DRAFRAX, PLAX, PHAX, TF, QUARTIC, CUBIC (ONLY TF
LISTED HERE);

"REAL" "PROCEDURE" TF(I, COEFFS, M, PH, PP, FR);
"VALUE" I, M, PH, PP, FR;
"REAL" "ARRAY" COEFFS;
"INTEGER" "ARRAY" PP;
"INTEGER" I, M, PH;
"REAL" FR;
"BEGIN" "REAL" P, Q, R, S, W;
"INTEGER" NCO;
"REAL" "PROCEDURE" AMP(P, Q, R, S);
"REAL" P, Q, R, S;
"BEGIN" AMP := SQRT((P*P + Q*Q) / (R*R + S*S));
"END";
"REAL" "PROCEDURE" PHASE(P,Q,R,S);
"REAL" P,Q,R,S;
"BEGIN" "REAL" E,F,G;
E:=P*R+Q*S;
F:=Q*R-P*S;
G:=ARCTAN(F/E)*57.3;
"IF" G > 0 "THEN"
"BEGIN" "IF" F < 0 "THEN" G:=G-180;
"END";
"IF" G<0 "THEN"
"BEGIN" "IF" F > 0 "THEN" G:=G-180;
"END";
"IF" G=0 "THEN"
"BEGIN" "IF" E<0 "THEN" G:=-180;
"END";
PHASE:=G;
"END";
W:=PR/10*I*6.2832;
R:=W^4-COEFFS[2]*W^2*W+COEFFS[4];
S:=-COEFFS[1]*W^3+COEFFS[3]*W;
NCO:=5;
"IF" M=1 "THEN"
"BEGIN" P:=-COEFFS[6]*W^2+COEFFS[8];
Q:=-W^3+COEFFS[7]*W;
"END";
"IF" M=2 "THEN"
"BEGIN" "IF" PP[1,1]"NE" 0 "OR" PP[1,2]"NE" 0 "THEN"
NCO:=NCO+4;
P:=-W^2+COEFFS[NCO+2];
Q:=COEFFS[NCO+1]*W;
"END";
"IF" M=3 "THEN"
"BEGIN" "IF" PP[1,1]"NE" 0 "OR" PP[1,2]"NE" 0 "THEN"
NCO:=NCO+4;
"IF" PP[2,1]"NE" 0 "OR" PP[2,2]"NE" 0 "THEN"
NCO:=NCO+3;
P := W^4 - COEFFS[NCO + 2]^2 * W * COEFFS[NCO + 4];

"END";
"IF" PH = 1 "THEN"
"BEGIN" "IF" PP[M, PH] = 0 "THEN" TF := 0 "ELSE"
TF := COEFFS[NCO] * AMP(P, Q, R, S);
"END" "ELSE"
"BEGIN" "IF" PP[M, PH] = 0 "THEN" TF := 0 "ELSE"
TF := PHASE(P, Q, R, S);
"END";

m := 0; di := 0;
"IF" PP[i, 1] = 0 "AND" PP[2, 1] = 0 "AND" PP[3, 1] = 0 "THEN" "GOTO" PHON;
PLAX(SCALE, ZERO);
"FOR" M := 1 "STEP" 1 "UNTIL" 3 "DO"
"IF" PP[M, 1] "NE" 0 "THEN"
"BEGIN" "FOR" K := (="IF" STFT[M, 1] - 1 > START "THEN"
STFT[M, 1] - 1 "ELSE" START)
"STEP" 1 "UNTIL" PP[M, 1] - 1 "DO"
"BEGIN" I := REDNO[K];
"IF" FR = 999 "THEN" DFR := DATA[1, M, N + 1] * 10
"ELSE" DFR := FR * I;
MOVEDPEN(SCALE * LN(DFR), GSC * LN(DATA[1, M, I] / DATA[1, M, 0]) + ZERO);
CENCHARACTER(M);
"END";
I := 0; PH := 1; DFR := 1;
ZVAL[M] := TF(1, COEFFS, M, PH, PP, DFR);
I := 1;
MOVEDPEN(0, GSC * (LN(TF(I, COEFFS, M, PH, PP, DFR))
- LN(DATA[PH, M, 0])) + ZERO);
"FOR" I := 2 "STEP" 1 "UNTIL" 40 "DO"
DRAWLINE(SCALE*LN(I),GSC*(LN(TF(I,COEFFS,M,PH,PP,DFR))-LN(DATA[PH,M,O])+ZERO);
"PRINT" DIGITS(1),CHAN[M],'/",HWRW;
"END";

THISG: "IF" D2=1 "THEN"
"BEGIN" MOVEPEN(450,2540);
   "IF" HWRW=2 "THEN" "PRINT" WAY(0,8),"GUST RESPONSE"
   "ELSE" "PRINT" WAY(0,8),"STEER RESPONSE";
   MOVEPEN(-800,3726);
   "GOTO" FINAL;
"END";

D2:=1;
"FOR" I:=2 "STEP" 1 "UNTIL" 5 "DO"
D[1]:=COEFTS[I-1]; D[1]:=1;
QUARTIC(D,X,Y);
MOVEPEN(50,2300);
"PRINT" "SP SS RESP ";
MOVEPEN(100,2260);
"PRINT" "POLES";
"FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
"BEGIN" MOVEPEN(10,(2260-40*I));
   "PRINT" ALIGNED(2,2),SQRT(X[I]*X[I]+Y[I]*Y[I])
   *Y[I]/6.2832;
   MOVEPEN(150,(2260-40*I));
   "PRINT" ALIGNED(1,3),X[I]/SQRT(X[I]*X[I]+Y[I]*Y[I]);
"END";

"IF" PP[1,1] "NE" 0 "OR" PP[1,2] "NE" 0 "THEN"
"BEGIN" "REAL" "ARRAY" DR[0:3],RX,RY[1:3];
   "FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
   DR[I]:=COEFTS[(5+I)]; DR[0]:=1;
   CUBIC(IR,RX,RY);
   MOVEPEN(430,2340); "PRINT" "YAW RATE";
   MOVEPEN(400,2300); "PRINT" SCALED(4),ZVAL[1];
   MOVEPEN(460,2260); "PRINT" "ZEROS";
"FOR" I:=1 "STEP" 1 "UNTIL" 3 "DO"
"BEGIN" MOVEPEN(360, (2260-40*1));
   "PRINT" ALIGNED(2,2), SQRT(RX[1]*RX[1]
+RY[1]*RY[1])/6.2832;
MOVEPEN(500, (2260-40*1));
   "PRINT" ALIGNED(1,3), RX[1]/SQRT(RX[1]
*RX[1]+RY[1]*RY[1]);
"END";
"END";

"IF" PP[2,1] "NE" 0 "OR" PP[2,2] "NE" 0 "THEN"
"BEGIN" "REAL" B;
   "REAL" "ARRAY" PX,PY[1:2];
   "IF" PP[1,1]=0 "AND" PP[1,2]=0 "THEN"
      I:=0 "ELSE" I:=4;
   B:=COEFFS((I+6)^2-4*COEFFS((I+7));
   "IF" B<0 "THEN"
      "BEGIN" PX[1]:=PX[2]=-COEFFS((I+6)/2;
      PY[1]:=SQRT(-B)/2;
      PY[2]=-PY[1];
   "END" "ELSE"
   "BEGIN" PY[1]:=PY[2]:=0;
      PX[1]=(-COEFFS((I+6)+SQRT(B))/2;
      PX[2]=(-COEFFS((I+6)-SQRT(B))/2;
"END";
MOVEPEN(760,2340); "PRINT" "ROLL ANGLE";
MOVEPEN(750,2300); "PRINT" SCALED(4), ZVAL[2];
MOVEPEN(810,2260); "PRINT" "ZEROS";
"FOR" I:=1 "STEP" 1 "UNTIL" 2 "DO"
"BEGIN" MOVEPEN(710, (2260-40*1));
   "PRINT" ALIGNED(2,2), SQRT(PX[1]*PX[1]
+PY[1]*PY[1])/6.2832;
MOVEPEN(850, (2260-40*1));
   "PRINT" ALIGNED(1,3), PX[1]/SQRT(PX[1]
*PX[1]+PY[1]*PY[1]);
"END";
"END";
"IF" PP[3,1] "NE" 0 "OR" PP[3,2] "NE" 0 "THEN"
"BEGIN" "REAL" "ARRAY" DL[1:5], LX, LY[1:4];
  "IF" PP[1,1]=0 "AND" PP[1,2]=0
    "THEN" K:=0 "ELSE" K:=4;
  "IF" PP[2,1]=0 "AND" PP[2,2]=0
    "THEN" J:=0 "ELSE" J:=3;
"FOR" I:=2 "STEP" 1 "UNTIL" 5 "DO"
DL[I]:=COEFFS[(4+I+K+J)];    DL[1]:=1;
QUARTIC(IL, LX, LY);
MOVEPEN(1160,2340);    "PRINT" "LAC";
MOVEPEN(1100,2300);    "PRINT" SCALED(4), ZVAL[3];
MOVEPEN(1160,2260);    "PRINT" "ZEROS";
"FOR" I:=1 "STEP" 1 "UNTIL" 4 "DO"
"BEGIN" MOVEPEN(1060, (2260-40*I));
  "PRINT" ALIGNED(2,2)*SQRT(LX[I]+LY[I])/6.2832;
  MOVEPEN(1200, (2260-40*I));
  "PRINT" ALIGNED(1,3), LX[I]/SQRT(LX[I]+LY[I]);
"END";
"END";
MOVEPEN(450,2540);
"IF" HWRW=2 "THEN" "PRINT" WAY(0,8), "GUST RESPONSE";
"ELSE" "PRINT" WAY(0,8), "STEER RESPONSE";
MOVEPEN(-800,3726);
PHON: "IF" DL=1 "THEN" "GOTO" FINAL;
  "IF" PP[1,2]=0 "AND" PP[2,2]=0 "AND" PP[3,2]=0
    "THEN" "GOTO" FINAL;
PHAX(SCALE, ZERO);
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
  "IF" PP[M,2] "NE" 0 "THEN"
"BEGIN" "FOR" K:=1 "STEP" M-1 "DO"
  "IF" STFT[M,2]=1 "START" "THEN" STFT[M,2]=1
  "ELSE" "START"
"STEP" 2 "UNTIL" PP[M,2]=1 "DO"
"BEGIN" I := REDNO[K];
  "IF" FR = 999 "THEN" DFR := DATA[2, M, N + I]
    #10 "ELSE" DFR := FR * I;
  MOVEPEN(SCALE * LN(DFR), PSC * DATA[2, M, I] + ZERO);
  CENCHARACTER(M);
"END";
I := 1; PH := 2; DFR := 1;
MOVEPEN(0, PSC * TF(I, COEFTS, M, PH, PP, DFR)
 + ZERO); I := 0;
ZVAL[M£ := TF(I, COEFTS, M, PH, PP, DFR);
"FOR" I := 2 "STEP" 1 "UNTIL" 40 "DO"
DRAWLINE(SCALE * LN(I), PSC * TF(I, COEFTS, M, PH, PP, DFR)
 + ZERO);
"PRINT" DIGITS(1), CHAN[M], "/", HWRY;
"END"; D1 := 1;
"GOTO" THISG;
"END" of segment six;
FINAL:"END";
"END";
"END";
Relevant weights and dimensions of the two vehicles are:

<table>
<thead>
<tr>
<th></th>
<th>Weight</th>
<th>Yaw Inertia</th>
<th>Roll Inertia</th>
<th>Wheelbase</th>
<th>xf</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car E</td>
<td>105.9</td>
<td>2100*</td>
<td>320*</td>
<td>8.6</td>
<td>4.6</td>
</tr>
<tr>
<td>Car F</td>
<td>74.3</td>
<td>1256</td>
<td>260</td>
<td>8.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>

* approximately

Car E was a 3500 c.c. front engined, rear wheel drive, saloon, with independent front suspension and anti-roll bar, and a De Dion rear suspension. It was fitted with textile braced, 185HR x 14, radial ply tyres. Two sets of tyre pressures were used: 28F, 30R and 30F, 28R (lb/in.²).

Car F was a 1300 c.c. front engined, rear wheel drive, saloon, with independent front suspension by Macpherson struts and anti-roll bar, and a live rear axle with semi-elliptic leaf springs. Two sets of tyres were used: 5.60 x 13 cross ply tyres set at 20 lb/in.² all round, and 145 x 13, steel braced, radial ply tyres set at 24F, 28R (lb/in.²).

Steady state response measurements were made on these cars on the 108ft. radius steering pad at MIRA. The instrumentation used was the same as for the transient work except that body sideslip angle and roll angle are measured using a two wheeled, castored trolley attached to the rear of the car. This trolley was designed and constructed at MIRA and the original version is described in Ref. 67. The steering pad test procedure consists of driving round the circle at a series of steady speeds, and recording the data for about 10 seconds at each speed. The results are digitised and processed on a digital computer. An average "steady state" value is obtained for each measured quantity for each 10 second recording. The notation used on the computer drawn result graphs is as follows:
4 graphs are presented for each test:

1. 

HW/SR, STMEAN, and SISLIP plotted against LATAC. The "Ackermann" angles (geometric angles required for pure rolling round the curve) are removed from the data prior to plotting to allow continuous curves to be drawn through the left and right turn data. This is in an attempt to provide information about the gradient of these curves at zero LATAC. The curves shown are cubic curves fitted digitally by the method of least squares and the various gradients described below are evaluated analytically from these fitted curves.

2. 

dHW/dSISLIP and dSTMEAN/SR/dSISLIP plotted against LATAC. The evaluation of the concept of d(steer)/d(sideslip) in terms of stability derivative is given in Section 2 (as is d(steer)/d(LATAC) of 3, below). It can also be shown that this quantity is a simple function of the ratio of the effective cornering stiffness of the front and rear tyres, and it has been found at MIRA to correlate well with subjective impressions of a car's understeer behaviour. According to the definition of understeer given in Ref. 7, d(steer)/d(sideslip) is positive for understeer and negative for oversteer (as is d(steer)/d(LATAC)). The STMEAN curve is multiplied by SR so that the gap between it and the HW curve gives an indication of the total losses in the steering system.
3. \( \frac{dHW}{dLATAC} \) and \( \frac{dSTEERING}{dLATAC} \) plotted against LATAC. The concept of \( \frac{d(steer)}{d(latac)} \) is a more conventional method of describing understeer, and is useful in addition to 2. as it is a function of both the overall effective cornering stiffness of the tyres and the ratio of the front to rear effective stiffness.

4. ROLL plotted against LATAC. The results for the two cars each in two conditions are given in Figs. A 7.1 to A 7.4.
Fig. A7.1(a) Steering pad test results for Car E, tyres 30P, 35R (lb/in²).
Fig. A7.1(b) Steering pad test results for Car E, tyres 30F, 28R (lb/in²).
Fig. A7.2(a) Steering pad test results for Car E, tyres 28F, 30R (lb/in²).
Fig. A7.2(b) Steering pad test results for Car E, tyres 28F, 30R (lb/in^2).

- • = \frac{\partial W}{\partial \text{LATAC}}
- • = \frac{\partial \text{STRES}/\text{LATAC}}{\partial \text{LATAC}}

- • = \text{ROLL}
Fig. A7.3(a) Steering pad test results for Car F, cross ply tyres.
Fig. A7.3(b) Steering pad test results for Car F, cross ply tyres.
Fig. A7.4(a) Steering pad test results for Car F, radial ply tyres.
Fig. A7.4(b) Steering pad test results for car F, radial ply tyres.
APPENDIX 8  COMPUTER PROGRAM TIMERESP - INPUT/OUTPUT AND LISTING

As described in Section 10 this program calculates the time response of a vehicle defined by a set of 16 COEFFS, to a specified time input. It is basically built out of some of the procedures of FITTRANS (Appendix 6) used in a different order. Two inputs are required:

1. Time Input.
   
   This can either come from a magnetic tape file, in which case it is identical to that described in Appendix 6, or from a paper tape, when the format is:
   
   'ALPHA/NUMERIC IDENTIFIER'
   RUN - up to 4 digits, integer, run number.
   CH - input identifier - 2 for wind angles, 3 for handwheel, 4 for roadwheel.
   DATPTS - number of time data points [2^E points are actually used starting at point STPT and using every NTH point. STPT, NTH,E are control teleprinter inputs].
   DT - time interval between points.
   SIZE[0] - nominal maximum value of input. Used for scaling purposes.
   SIZE[1] - nominal maximum value of yaw rate response expected
   SIZE[2] - nominal maximum value of roll angle response expected
   followed by the DATPTS time data amplitude points.

2. Vehicle defining input.
   
   This is always input from a paper tape and is in the form:
   'ALPHA/NUMERIC VEHICLE CASE IDENTIFICATION'
   followed by the 16 vehicle response coefficients (COEFFS) described in Section 2 and used throughout this thesis.

   The output from this program is a plot of the time input and the yaw rate, roll angle, and latac responses. The format is similar to that from FITTRANS (Appendix 6) and is fairly self explanatory. Examples are shown in Section 10.
A 8.2 TIMERESP - Computer Program Listing

TIMERESP ONSE OF GIVEN TF TO TIME DATA INPUT 38 70;
"BEGIN" "INTEGER" I,J,K,M,CH,MTPT,LASTCH,RUN,DATPTS,NCHS,STPT,NTH,
  E,N,KIND,SMOOTH;
"REAL" DT,PR,ZEROSH;
"ARRAY" MAX[1:9],SIZE[0:3],COEFFS[1:16];
"INTEGER" "ARRAY" TESTIDENT,NAME[1:5],CASE[1:20];

"COMMENT" PROCEDURES OPENWRITE ETC,RDD,IDENTIFY,FASTFOURIER ETC,
WINDOW,FTWH,TF;

"PROCEDURE" OPENWRITE(H); "COMMENT" POSITIONS TAPE IMMEDIATELY AFTER
   TM AT END OF LAST RECORD;
"INTEGER" H; "COMMENT" HANDLER NUMBER;
"BEGIN" MTSOURCE(H,"NBHANLDINGDATAFILE1");
  MTDEST(H,"NBHANLDINGDATAFILE1","TRUE");
NEXTTM: MTSEEK(H); "COMMENT" FIND TM;
  FINDREC(H);
  "IF" FILECOND(H) = -2 "THEN"
  "BEGIN" MTHACK(H);
    MTHACK(H); "COMMENT" BACKSPACE OVER EOF & TM;
  "END" "ELSE" "GOTO" NEXTTM;
"END";

"PROCEDURE" CLOSEWRITE(H); "COMMENT" TERMINATES LAST RECORD WITH TM,
   EOF,TM,TM;
"INTEGER" H; "COMMENT" HANDLER NUMBER;
"BEGIN" "INTEGER" I;
  ENDBLOCK(H);
  I:= FILECOND(H); "COMMENT" NUMBER OF BLOCKS TRANSFERRED TO M/T;
  MMARK(H);
  MTCLOSE(H,I); "COMMENT" WRITE EOF;
"END";
"PROCEDURE" OPENREAD (H, IDENTIFIER); "COMMENT" POSITIONS TAPE AT POINT IMMEDIATELY AFTER STRING IDENTIFYING RECORD;
"INTEGER" H; "COMMENT" HANDLER NUMBER;
"INTEGER" "ARRAY" IDENTIFIER; "COMMENT" CONTAINS STRING USED TO IDENTIFY RECORD: MUST BE FIRST ITEM ON RECORD & MUST CONTAIN 18 UPPER CASE CHARACTERS;

"BEGIN" "INTEGER" I;

"INTEGER" "ARRAY" TEST[1:5];
MTREWIND(H); "COMMENT" REWINDS TO BEGINNING OF TAPE;
ENDBLOCK(H);

READ: I:= 1;

"READ" FILE(H), INSTRING(TEST, I);
"FOR" I:= 1 "STEP" 1 "UNTIL" 5 "DO" "IF" TEST[I] "NE" IDENTIFIER[I] "THEN"
"BEGIN" MTSEEK(H);

ENDBLOCK(H);
"GOTO" READ;
"END"

"END";

"PROCEDURE" BACKSPACE(H, IDENTIFIER, BLOCKS);

"COMMENT" BACKSPACES TO HEAD OF RECORD CURRENTLY BEING READ, CHECKS TEST IDENTIFIER AND POSITIONS TAPE AT POINT IMMEDIATELY AFTER TEST IDENTIFIER;
"INTEGER" H, BLOCKS; "COMMENT" HANDLER NUMBER, NO OF BLOCKS TO BACKSPACE;
"INTEGER" "ARRAY" IDENTIFIER; "COMMENT" CONTAINS STRING IDENTIFYING RECORD: MUST BE FIRST ITEM ON RECORD AND MUST CONTAIN 18 UPPER CASE CHARACTERS;

"BEGIN" "INTEGER" I, REWIND;

"INTEGER" "ARRAY" TEST[1:5];

REWIND:= 0;
"FOR" I:= 1 "STEP" 1 "UNTIL" BLOCKS "DO" MTBACK(H);
IP: ENDEBLOCK(H);
MTSEEK(H);
FINDREC(H);
"IF" FILECOND(H) = -2 "THEN"
"BEGIN" MTREWIND(H); "COMMENT" REWIND TO BEGINNING OF TAPE;
"IF" REWIND "NE" 0 "THEN"
"BEGIN" "PRINT" PUNCH(3),"L2S2"MT READ FAIL";
STOP;
"END";
REWIND:=1;
"GOTO" IP;
"END" "ELSE"
"BEGIN" I:=1;
"READ" FILE(H),INSTRING(TEST,I);
"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO" "IF" TEST[I] "NE"
IDENTIFIER[I] "THEN"
"GOTO" IP;
"END";
"PROCEDURE" FASTFOURIER,REALTRANSFORM ETC (NOT LISTED).

"PROCEDURE" RDD(TESTIDENT,DATPTS,LASTCH,STPT,NTH,N,CH,ZEROSH,A);
"INTEGER" DATPTS,LASTCH,STPT,NTH,N,CH;
"REAL" ZEROSH;
"INTEGER" "ARRAY" TESTIDENT;
"REAL" "ARRAY" A;
"BEGIN" "INTEGER" I,D,J,BLOCKS;
"REAL" DUM;
"IF" LASTCH= 0 "THEN" "GOTO" THERE "ELSE"
"BEGIN" BLOCKS:=CHECKI(ENTIER((LASTCH*CHECKI(DATPTS)+33)/128)+10);
BACKSPACE(2,TESTIDENT,BLOCKS);
"READ" FILE(2),I,D,DUM,J;
"FOR" I:=1 "STEP" 1 "UNTIL" 9 "DO" "READ" FILE(2),DUM;
THERE: "FOR" I:=1 "STEP" 1 "UNTIL" STPT-1+(CH-1)*DATPTS "DO"
"READ" FILE(2),DUM;
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
"BEGIN" "READ" FILE(2),A[I];
   "FOR" J:=1 "STEP" 1 "UNTIL" NTH-1 "DO"
      "READ" FILE(2),DUM;
"END";
ZEROSH:=A[1];
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
   A[I]:=A[I]-ZEROSH;
"IF" CH=7 "THEN"
   "BEGIN" ZEROSH:=-ZEROSH;
      "FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
         A[I]:=A[I];
   "END";
"END";
LASTCH:=CHECKI(CH);

"PROCEDURE" FTWH(A,N,M,SMOOTH);
"INTEGER" N,M,SMOOTH;
"REAL" "ARRAY" A;
"BEGIN" "INTEGER" I,D;
   "REAL" "ARRAY" A1,B1[0:N/2];
   "FOR" I:=0 "STEP" 1 "UNTIL" N/2-1 "DO"
      "BEGIN" A1[I]:=A[I+1];
         B1[I]:=A[N/2+I+1];
   "END";
D:=M-1;
REALTRANSFORM(A1,B1,D,"FALSE");
"FOR" I:=0 "STEP" 1 "UNTIL" N/2 "DO"
   "BEGIN" A[I]:=SQRT(A1[I]*2+B1[I]*2);
      A[I+N/2+1]:=ARCTAN(-B1[I]/A1[I])*57.3;
      "IF" A[I+N/2+1]>0 "THEN"
         "BEGIN" "IF" -B1[I]<0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+180;
            "END";
         "IF" A[I+N/2+1]<0 "THEN"
            "BEGIN" "IF" -B1[I]>0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+180;
               "IF" -B1[I]<0 "THEN" A[I+N/2+1]:=A[I+N/2+1]+360;
               "END";
      "END";
"IF" A[I+N/2+1]=0 "THEN"
"BEGIN" "IF" A[I]<0 "THEN" A[I+N/2+1]:=180;
"END";

"END";

"IF" SMOOTH=1 "THEN"
"BEGIN"
A[0]:=0.54*A[0]+0.46*A[1];
A[N/2]:=0.54*A[N/2]+0.46*A[N/2-1];
"FOR" I:=1 "STEP" 1 "UNTIL" N/2-1 "DO"
A[I]:=0.23*A[I-1]+0.54*A[I]+0.23*A[I+1];
"END";

"PROCEDURE" WINDOW(A,KIND,N);
"INTEGER" N,KIND;
"ARRAY" A;
"BEGIN" "INTEGER" I;
"REAL" F;
"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"
"IF" KIND=2 "THEN"
A[I]:="IF" I < N/2 "THEN" 2*(I-1)/N "ELSE"
2-2*I/N)*A[I];

"IF" KIND=3 "THEN"
"BEGIN" "IF" I < N/4 "THEN" F:=2*(I-1)/N "ELSE"
2*I/N)*A[I];

"IF" KIND=3 "THEN"
"BEGIN" "IF" I < N/4 "THEN" F:=2*(I-1)/N "ELSE"
2*I/N)*A[I];

"END";
"PROCEDURE" IDENTIFY(NAME,TESTIDENT,STPT,NTH,E,DT);
"INTEGER" STPT,NTH,E;
"REAL" DT;
"INTEGER" "ARRAY" NAME,TESTIDENT;
"BEGIN" "INTEGER" I; I:=1;
   OUTSTRING(NAME,I);
   I:=1;
   "PRINT" SAMELINE, ",STPT=",DIGITS(4),STPT,",NTH=",DIGITS(3),
   NTH,",E=",DIGITS(4),E,",DT=",ALIGNED(1,3),DT*NTH,
   "",OUTSTRING(TESTIDENT,I);
"END";

"REAL" "PROCEDURE" TP(I,COEFFS,M,PH,PP,FR);
"VALUE" I,M,PH,PP,FR;
"REAL" "ARRAY" COEFFS;
"INTEGER" "ARRAY" PP;
"INTEGER" I,M,PH;
"REAL" FR;
"BEGIN" "REAL" P,Q,R,S,W;
   "INTEGER" NCO;
   "REAL" "PROCEDURE" AMP(P,Q,R,S);
   "REAL" P,Q,R,S;
   "BEGIN" AMP:=SQRT((P*P+Q*Q)/(R*R+S*S));
   "END";
   "REAL" "PROCEDURE" PHASE(P,Q,R,S);
   "REAL" P,Q,R,S;
   "BEGIN" "REAL" E,F,G;
      E:=P*R+Q*S;
      F:=Q*R-P*S;
      G:=ARCTAN(F/E)*57.3;
      "IF" G > 0 "THEN"
      "BEGIN" "IF" F < 0 "THEN" G:=G-180;
      "END";
      "IF" G<0 "THEN"
      "BEGIN" "IF" F > 0 "THEN" G:=G-180;
      "END";
"IF" G=0 "THEN"
"BEGIN" "IF" E<0 "THEN" G:=-180;
"END";
PHASE:=G;
"END";
W:=FRI/10*I*6.2832;
R:=W^4-COEFFS[2]*W*CINO+COEFFS[4];
S:=-COEFFS[1]*W^3+COEFFS[3]*W;
NCO:=5;
"IF" M=1 "THEN"
"BEGIN" P:=-COEFFS[6]*W*CINO+COEFFS[8];
Q:=-W^3+COEFFS[7]*W;
"END";
"IF" M=2 "THEN"
"BEGIN" "IF" PP[1,1]"NE" 0 "OR" PP[1,2]"NE" 0 "THEN" NCO:=NCO+4;
P:=-W^4-COEFFS[NCO+2];
Q:=COEFFS[NCO+1]*W;
"END";
"IF" M=3 "THEN"
"BEGIN" "IF" PP[1,1]"NE" 0 "OR" PP[1,2]"NE" 0 "THEN" NCO:=NCO+4;
"IF" PP[2,1]"NE" 0 "OR" PP[2,2]"NE" 0 "THEN" NCO:=NCO+3;
P:=W^4-COEFFS[NCO+2]*W*CINO+COEFFS[NCO+4];
Q:=-COEFFS[NCO+1]*W^3+COEFFS[NCO+3]*W;
"END";
"IF" PH=1 "THEN"
"BEGIN" "IF" PP[M,PH]=0 "THEN" TF:=0 "ELSE"
TF:=COEFFS[NCO]*AMP(P,Q,R,S);
"END" "ELSE"
"BEGIN" "IF" PP[M,PH]=0 "THEN" TF:=0 "ELSE"
TF:=PHASE(P,Q,R,S);
"END";

MORE: PUNCH(3); "PRINT" L TYPE DATE AND PASS NO STRING';
I:=1; "READ" READER(3),INSTRING(NAME,1);
"PRINT" L LOAD VEHICLE CASE STRING AND 16 COEFFS DEFINING TRANSFER FUNCTIONS';
WAIT; I:=1; INSTRING(CASE,I);
"FOR" I:=1 "STEP" 1 "UNTIL" 16 "DO"
"READ" COEFFS[I];
"PRINT" "L IDENTIFY TIME INPUT BY 2 FOR WIND ANGLE,
3 FOR HW,4 FOR RW,1;
"READ" READER(3),CH;
"PRINT" "L WINDOW TYPE KIND ;
"READ" READER(3),KIND;
"PRINT" "L TYPE SMOOTH ;
"READ" READER(3),SMOOTH;
"PRINT" "L FOR TIME INPUT FROM MT TYPE 1,PT TYPE 2 ;
"READ" READER(3),MTPT;
"IF" MTPT=2 "THEN" "GOTO" PAPT;
"PRINT" "L LOAD HANDLER 2 WITH APPROPRIATE DATA FILE ;
WAIT;
"PRINT" "L TYPE MT TEST IDENTIFIER STRING ;
I:=1; "READ" READER(3),INSTRING(TESTIDENT,I);
OPENREAD(2,TESTIDENT); LASTCH:=0;
"READ" FILE(2),RUN,DATPTS,DT,NCHS;
"FOR" I:=1 "STEP" 1 "UNTIL" NCHS "DO" "READ" FILE(2),MAX[I];
SIZE[0]:=MAX[6]; SIZE[1]:=MAX[7]; SIZE[2]:=MAX[9];
"GOTO" LABEL1;
PAPT: "PRINT" "L LOAD TIME DATA PT";
WAIT;
I:=1; INSTRING(TESTIDENT,I);
"READ" RUN,CH,DATPTS,DT;
"PRINT" "L INPUT CH",DIGITS(1),SAMELINE,CH;
"READ" SIZE[0],SIZE[1],SIZE[2],SIZE[3];
LABEL1: "PRINT" SAMELINE," L RUN",RUN," L DATPTS",DATPTS,
"L PTS PER SEC",ALIGNED(3,3),1/DT,
"L TYPE STPT,NTH,E(N=21E)"L";
"READ" READER(3),STPT,NTH,E; N:=21E;
"IF" N>(DATPTS-STPT+1)/NTH "THEN"
"BEGIN" "PRINT" "L TOO LITTLE DATA";
"GOTO" LABEL1;
"END";
\[ FR := \frac{10}{(N \cdot DT \cdot NTH)}; \]

"BEGIN" "REAL" "ARRAY" A[0:N+1];

"REAL" DUM;

"INTEGER" "ARRAY" PP[1:3,1:2];

"INTEGER" TS;

"IF" MTPT=2 "THEN" "GOTO" LABEL2;

RDD(TESTIDENT,DATPTS,LASTCH,STPT,MTH,N,CH,ZEROSH,A);

"GOTO" LABEL3;

LABEL2: "FOR" I:=1 "STEP" 1 "UNTIL" STPT-1 "DO"

"READ" DUM;

"FOR" I:=1 "STEP" 1 "UNTIL" N "DO"

"BEGIN" "READ" A[I];

"FOR" J:=1 "STEP" 1 "UNTIL" NTH-1 "DO"

"READ" DUM;

"END";

ZEROSH:=A[I];

LABEL3: "IF" ABS(N\cdot DT \cdot NTH/5-1)<0.01 "THEN" TS:=1

"ELSE" TS:=ENTER(N\cdot DT \cdot NTH/5)+1;

SETORIGIN(800,0); MOVEPEN(-450,-224); DRAWLINE(-450,-254);

DRAWLINE(-420,-254); MOVEPEN(0,0); PUNCH(5); WAY(0,4);

"FOR" I:=1 "STEP" 1 "UNTIL" 5 "DO"

"BEGIN" DRAWLINE(300*I,0); DRAWLINE(300*I,16);

MOVEPEN(300*I-50,-44);

PRINT2 Digits(2),I*TS; MOVEPEN(300*I,0);

"END";

MOVEPEN(1610,-254); DRAWLINE(1640,-254);

DRAWLINE(1640,-224);

MOVEPEN(670,-94); "PRINT" "TIME SEC";

MOVEPEN(0,0); DRAWLINE(0,123); DRAWLINE(16,123);

MOVEPEN(0,123);

DRAWLINE(0,250); DRAWLINE(16,250); MOVEPEN(-64,200);

WAY(1,4);

"PRINT" "LATA"; MOVEPEN(-20,180); FREEPOINT(4);

"PRINT" SIZE[3];

MOVEPEN(0,250); DRAWLINE(0,377); DRAWLINE(16,377);

MOVEPEN(0,377);
"FOR" I:=0 "STEP" 1 "UNTIL" N-1 "DO"
DRAWLINE(I*DT*NTH*300/TS,A[I+1]*127/SIZE[0]+1750);
MOVEPEN((N-1)*DT*NTH*300/TS,1750); DRAWLINE(0,1750);
PUNCH(1);
WINDOW(A, KIND, N);
FFWH(A, N, E, SMOOTH);
PP[1, 1] := 1; PP[1, 2] := 1; PP[2, 1] := 1; PP[2, 2] := 1;
PP[3, 1] := 1; PP[3, 2] := 1;
"FOR" M:=1 "STEP" 1 "UNTIL" 3 "DO"
"BEGIN" "ARRAY" A1, B1[0:N/2];
  "REAL" AMPL, FASE;
  "FOR" I:=0 "STEP" 1 "UNTIL" N/2 "DO"
  "BEGIN" AMPL := A[I]*TF(I, COEFS, M, 1, PP, FR);
      FASE := A[I+N/2+1]*TF(I, COEFS, M, 2, PP, FR);
      A1[I] := CHECKR(AMPL*COS(FASE/57.3));
      B1[I] := CHECKR(-AMPL*SIN(FASE/57.3));
  "END";
  I:=E-1; B1[N/2]:=0;
REALTRANSFORM(A1, B1, I, "TRUE");
"IF" M:=2 "THEN"
"BEGIN" "FOR" I:=0 "STEP" 1 "UNTIL" N/2-1 "DO"
      B1[I] := -B1[I];
  "END";
"END";
MOVEPEN(0, 1250-500*(M-1));
"FOR" I:=0 "STEP" 1 "UNTIL" N/2-1 "DO"
DRAWLINE(I*DT*NTH*300/TS, A[I]*127/SIZE[M]+1250-500*(M-1));
"FOR" I:=0 "STEP" 1 "UNTIL" N/2-1 "DO"
DRAWLINE((I+N/2)*DT*NTH*300/TS, A[I]*127/SIZE[M]+1250-500*(M-1));
MOVEPEN((N-1)*DT*NTH*300/TS, 1250-500*(M-1));
DRAWLINE(0, 1250-500*(M-1));
"END";
MOVEPEN(-800, 3726);
"END"; "GOTO" MORE;
"END";