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Modelling Gas Flow Pressure Gradients in Gelcast Ceramic Foam Diesel Particulate Filters

By
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MSc. (Mech. Eng.)

A Doctoral thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

October 2005

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ABSTRACT

Internal combustion engines are significant contributors to air pollution. To meet the future legislative particulate matter (PM) emissions targets for diesel engines there is a need for aftertreatment of the exhaust gases. Previous investigations have shown that diesel particulate filters (DPFs) are a potential exhaust aftertreatment technology for the reduction of PM emissions. DPF systems generally contain two elements; one or more filters (i.e. porous media) and a means of regenerating (i.e. cleaning) the filter(s). The filter must be regenerated intermittently or continuously to prevent imposing high exhaust back pressures on the engine.

This thesis presents the study of fluid flow through gelcast ceramic foams that are a potential candidate filter material for use in DPF systems. New mathematical models that predict pressure gradients of clean and PM loaded gelcast ceramic foam filters are described. Firstly, the Ergun mathematical model used for predicting pressure drop of packed bed is adapted to model gelcast ceramic foam filters. The new model referred to as the Extended Ergun Mathematical (EEM) model can be used to predict pressure gradients of gelcast ceramic foam filters with an error of < 30%. Secondly, a new improved mathematical model referred to as Multiple Orifice Mathematical (MOM) model is developed from a conceptual model of row of cells across the filter. This latter model can be used to predict pressure gradients with an error of < 25% and required no tuning using real foam flow data. Calibration was conducted, using experimental data from physical scale model foams that were manufactured using stereolithography rapid manufacturing techniques. The mathematical models were then validated using data collected from fluid flow experiments on gelcast ceramic foam filter samples. The MOM model was adapted further to model a PM loaded foam structure for the prediction of pressure gradients of PM loaded gelcast ceramic foam filters.

The research has resulted in new validated porous media models that predict pressure gradients of gelcast ceramic foam filters. It is shown how the models can be used to design DPF systems, with optimum size and shape attributes.
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December, 2005
Emmanuel Adigio
DEDICATION

This thesis is dedicated to God Almighty for being there from the beginning to the end. To my late mother, Hannah, who was my driving force throughout my career and my father Benson Adigio for being supportive. This thesis is also dedicated to my wife Mercy, and children Margaret, Florence, Anne, Jennifer, Andrew and Ruth for their sacrifice.
I lift up my eyes to the hills. From where does my help come? My help comes from the Lord, who made heaven and earth.

Psalm 121: 1-2
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NOMENCLATURE

A, B  constants for Darcian and non-Darcian coefficients
a, b  constants
\( A_{\text{filter}} \)  filter cross sectional area (m²)
\( A_{\text{flow}} \)  cross section area of the fluid in the filter (m²)
\( A_{\text{O}} \)  equivalent tube cross sectional area (m²)
\( A_{\text{w}} \)  window cross sectional area (m²)
c  particle mass concentration (g l⁻¹)
D  filter diameter (m)
d  cell diameter (m)
dₙ  PM loaded filter cell diameter (m)
d₀  equivalent tube diameter (row of cells, in m)
\( D_p \)  granule diameter in packed column (m)
dᵢ  capillary tube diameter (m)
E  particle number collection efficiency (%)
\( E_{\text{m}} \)  particle mass collection efficiency (%)
k  ratio of window diameter to cell diameter, \( w/d \)
k₁  Darcian coefficient of permeability (m²)
k₂  non-Darcian coefficient of permeability (m²)
L  length of filter (m)
\( M \)  number of cell in a row of cells across filter
\( N \)  number of cell per unit volume of filter
n  particle number concentration
\( N_{\text{row}} \)  number of rows of cells in a cross sectional area of filter
\( N_t \)  number of equivalent capillary tubes
P  filter penetration
\( P_1, P_2, P_3, \ldots, P_n \)  gas pressures (Pa)
Q  volumetric fluid flow rate (m³ s⁻¹)
q  flow rate through row of cells (m³ s⁻¹)
\( q_{\text{w}} \)  flow rate across windows (m³ s⁻¹)
Reynolds number

total wetted surface area (m²)

surface area truncated by a neighbouring cell

specific surface area (m²)

superficial velocity (m s⁻¹)

fluid velocities in cells (m s⁻¹)

fluid velocity (m s⁻¹)

volume truncated by neighbouring cell (m³)

volume of cell (m³)

volume of filter material (m³)

PM volume in filter (m³)

volume of PM deposit per filter cell (m³)

volume of sphere cell (m³)

capillary tube velocity (m s⁻¹)

total volume of cells on a row of cells (m³)

volume of void in cube (m³)

PM loaded filter window diameter (m)

window diameter (m)

Ergun’s correction factors applied on viscous loss

kinetic correction coefficient

Ergun’s correction factor applied to kinetic loss

pressure drop (Pa)

pressure drop from kinetic energy loss (Pa)

porosity

loaded filter porosity

deposited PM porosity

filtration coefficient

viscosity of the fluid (kg m⁻¹s⁻¹)

fluid density (kg m⁻³)

PM specific volume in filter (g l⁻¹)
Abbreviations

CAD  computer assisted design
DPF  diesel particulate filter
EEM  Extended Ergun Mathematical model
EGR  exhaust gas recirculation
EPA  environmental protection agency
GCF  gelcast ceramic foam
MOM  Multiple Orifice Mathematical model
PAH  polycyclic aromatic hydrocarbon
PM  particulate matter
SEM  scanning electron microscopy (or microscope)
SOF  soluble organic fraction

Symbols

NOx  oxides of nitrogen
SiC  silicon carbide

$\psi$  angle of inclination of PM deposit in filter (radians)
CHAPTER 1 INTRODUCTION

Over the last 100 years the diesel engine has been developed to become a reliable and widely used power source for both on-road and off-road vehicles. While diesel engines have many advantages including low emission of carbon dioxide, fuel tolerance, robustness, low cost and high low-speed torque, they have the disadvantage of emitting significant amounts of particulate matter (PM) and oxides of nitrogen NOx (Seiffert and Walzer, 1991).

Diesel exhaust emissions are reported to affect human health, contribute to acid rain and reduced visibility (Moran et al, 2002, United State Environmental Protection Agency Highlights, 2003). Consequently, governments of the United States, Japan and many European countries are enforcing stringent diesel emissions standards.

Although engine manufacturers have made a substantial reduction in emissions through improved engine design, there is expected to be a need for aftertreatment of the exhaust gases to meet future emission limits. Previous investigators (Forest and Lanscape, 2000 and Mayer et al, 1998) have reported
that diesel particulate filters (DPFs) are a potential exhaust aftertreatment technology for the reduction of PM emissions. DPFs consist of a filter designed to collect the PM in the exhaust stream of the diesel engine, while allowing the exhaust gases to pass through the system. Generally, they are cleaned (or 'regenerated') continuously or intermittently to prevent high exhaust back pressure on the engine due to trapped PM.

This thesis presents a study of gelcast ceramic foam (GCF), which is a potential candidate filter material for use in DPF systems. New mathematical models for the prediction of pressure gradients of the clean foam filters and PM loaded foam filters are described. Furthermore, the use of the models for the determination of filter size and shape is demonstrated. Finally, the use of the models to aid the design of DPF system is presented.

In this first chapter, PM emissions, health concerns and legislation are discussed. The research objectives are then presented, together with an outline of the research methodology, the thesis structure, and the major contributions to knowledge arising from this work.

1.1 DIESEL EXHAUST GAS EMISSIONS

Unlike gasoline engines, which require a spark, diesel engines use compression heating of the in-cylinder air to initiate combustion. Since the air/fuel mixture is heterogeneous, they are not premixed when entering the cylinder, leading to local oxygen deficiency on the surface of the fuel droplets. Consequently, there is superheating instead of pure oxidation, which leads to "crack reactions" and to the formation of other hydrocarbons (Heywood, 1988a, Snelling et al, 1999). Part of the crack products are no longer able to
partake in the subsequent after-oxidation process and appear as particulate matter (PM) in the engine exhaust products.

Diesel PM consists mainly of solid carbonaceous material and ash, volatile organic and sulphur compounds. The volatile and soluble organic compounds (generally described as soluble organic fraction, SOF) contain polycyclic aromatic compounds made up of sulphur, nitrogen and oxygen (Farrar-Khan et al, 1992). The sulphur is normally oxidised to SO$_2$. However, a tiny fraction, which is proportional to the total fuel sulphur content, is oxidised to SO$_3$ and sulphate aerosol. The inorganic ash in the exhaust is a product of engine metal compounds in the fuel and lubrication oil.

Figure 1.1 shows the composition of diesel PM (Kittelson, 1998). The SOF in the exhaust varies with engine design and operating conditions. The percentage of SOF is higher at light loads when the exhaust temperatures are low.

![Particulate matter (PM) composition for a heavy-duty diesel engine tested in a heavy-duty transient cycle (Kittelson, 1998)](image-url)
Chapter 1  

Introduction

There are various definitions of what diesel PM is, with respect to size (Mayer et al, 1998). A European joint project targeted at reducing the exhaust gas emissions of existing diesel engines defined PM as particulates that comprise all aerosol solids in the size range of 15-500 nm. Mayer, et al (1998) reported that PM emissions from diesel engines fall in the size range of up to 100 nm and the new engines in particular emit more fine particulates at all operating conditions. Kittelson et al (1999) also suggested that studies made on modern engines showed a marked decrease in mass emissions but led to a sharp increase in number emissions of the ultra-fine particles (i.e. < 100 nm). This size is considered to be the most hazardous because of their ability to penetrate deep into human lungs.

Other major emission components from diesel engine combustion are oxides of nitrogen. Oxides of nitrogen emissions consist mainly of nitric oxide (NO) and nitrogen dioxide (NO2), commonly referred to as NOx. The formation of NOx begins with atmospheric nitrogen and oxygen in a chain of high temperature reactions (> 1600°C). NOx is either formed rapidly in the flame zone, or more slowly in the adjacent gases as a result of high temperatures generated by the combustion. In fact, high temperatures favour the formation of NOx. Hence, the reduction of NOx emission can be facilitated by reducing the combustion temperature in the engine chambers.

In summary, diesel exhaust gas generally contains PM that consists of small particles of size < 100 nm, which are in the size range for human to respire deep into the lungs, and NOx which is a precursor for the formation of ozone.
1.2 HEALTH CONCERNS OF DIESEL EXHAUSTS EMISSIONS

The health concern posed by diesel engines is significant. The key factor for the determination of the effect of diesel particulates on health is their size. Particles that are < 100 nm are invisible to the eyes but can deposit in the bronchial and pulmonary tracts of the respiratory system (Hinds, 1980). Siegmann and Siegmann (1997) reported that fine particles from combustion contain thousands of different chemicals that cannot be characterised due to their unstable condition in the atmosphere. They demonstrated that the PM absorbs the heavier polycyclic aromatic hydrocarbon (PAH), including the carcinogenic species such as benzopyrene (a yellow, crystalline, aromatic hydrocarbon, C_{20}H_{12} that is normally found in coal tar and cigarette smoke) at ambient temperature. Furthermore, Siegmann and Siegmann (1997) reported, after confirming the presence of the heavier PAH, that a dose of benzopyrene received from traffic fumes, for many people, can be equated to the smoking of a few cigarettes everyday and that the danger of exposure to exhaust gases is approaching the danger level of cigarette smoke.

Many epidemiological studies have been carried out in the past decade using advanced statistical techniques. These have shown an association between exposure to small and short term increases in particulate matter (PM) levels and increases in daily mortality and symptoms of certain illness, including cancer in humans (Garshick et al, 1988, Nikula et al, 1995, Sjogren et al, 1996, Muzyka et al, 1998, Ahlvik and Brandberg, 2000, HEI Perspectives, 2002, United State Environmental Protection Agency, 2002, World Health Organisation, 2003). According to a California Air Resources Board fact sheet (2000), diesel particulate matter is identified as a toxic contaminant based on its potential to cause cancer and other adverse health effects and that emission from diesel engines are responsible for the majority of the potential airborne risk in California. It is also reported that diesel PM can induce arthritis in mice (Yoshino and Masaru, 1999).
Oxides of nitrogen on the other hand, are the primary ingredients for the ground-level ozone, a pollutant harmful to human health. Besides producing ozone smog, they contribute to acid rain and help to form the dirty brown clouds that often hang over major cities.

In summary, diesel exhaust emissions is reported to cause acute effects including irritation of the nose and eye, lung function changes, airway inflammation, headache and fatigue (Sydbom et al, 2001).

In order to minimise these problems associated with diesel exhaust emissions (including degradation of the environment), many countries have responded by placing stringent emission limits.

1.3 DIESEL EXHAUST GAS EMISSIONS LEGISLATION

Many industrialised nations have recognised the dangers of diesel exhaust gas emissions to public health and have responded with extremely stringent standards. California in the US is leading the way in placing stringent emissions limits to increasing fleets of diesel powered vehicles. Due to the peculiar climatic conditions in California, which tend towards frequent smog formation, the US government has placed much lower emissions limit in California.

There is currently a call for advanced emission controls and near-zero diesel emission levels in the future. For example, a US Environmental Protection Agency’s (EPA) Clean Air Off-road Diesel new rule will require the reduction of emissions from diesel engines by over 90 percent, which currently accounts for 47% of diesel PM emission (www.DieselNet.com). Hence, the emissions limit for PM has been set to 0.02 g/kWh by the year 2012.
In Europe, the regulations for heavy-duty diesel engines are commonly referred to as Euro I, II, III, IV and V. The Euro I standard meant for the medium and heavy-duty engines were introduced in 1992. The Euro II regulation came to being in 1996. Later, in 1998, the European Council of Environmental Ministers reached an agreement on the final Euro III standard and also adopted Euro IV and V standards for the year 2005 and 2008 respectively.

Figure 1.2 shows the emission standards for heavy-duty diesel engines and their respective implementation dates.

![Diagram showing Euro emission standards](image)

**Figure 1.2 European Union emission standards for heavy duty diesel engines (Adapted from Emission Standards European Union regulatory Unit, 2001)**

It is expected that all new diesel powered heavy-duty vehicles will effectively be required to fit exhaust gas aftertreatment devices to meet the emission limit values of 2005 and 2008.
The off-road engine definition used by the EPA is based on the principle of mobility and portability and the above regulations are not applicable to all off-road engines (for example, locomotives, marine vessels and engines used in underground mining equipment), but are covered by separate EPA regulations. Further details and legislation on other categories of vehicles can be found at www.DieselNet.com.

In order to meet the emission standards set by EPA and the EURO for diesel engines, a number of engineering techniques have been investigated and some have been adopted for the reduction of exhaust gas PM.

1.4 DIESEL PM EMISSION CONTROL

Control of diesel engine exhaust gas emissions started with the non-road and material handling equipment in underground mining to address occupational health hazards, which was later extended to other diesel engine applications worldwide due to the stringent emission standards (Mayer et al, 1998).

Engine manufacturers have made progress in the reduction of diesel engine emissions through improved engine design, which include improving the fuel injection techniques, air management, and combustion chamber design and oil control. The most significant progress in diesel engine development is in fuel injection equipment (Fairbanks, 2001). The required amount of fuel is monitored precisely by a microprocessor control and delivered at predetermined crank angle position into the combustion chamber at a very high pressure, at about 800 to 2000 bar. This development contributes to a major reduction of NOx and mass PM. Furthermore, most of the hydrocarbons (HC) that is responsible for the diesel smell are reduced as a result of the engine combustion control and the use of ultra-low sulphur diesel fuels.
A further reduction in NOx formation is achieved by the use of exhaust gas recirculation referred to as EGR (Lapuerta et al., 2000) and turbocharging (Icingur et al., 2003). Abu-Hamdeh (2003) suggested that the use of EGR is the most effective method in reducing emissions in internal combustion engines. In addition, Abd-Alla (2002) reported that the introduction of exhaust gas recycling in diesel engines also reduces fuel consumption. In another development, it was reported that the use of carbon dioxide in the place of EGR reduces the NOx and minimises the increase in PM (Ladommatos et al., 1998, Abd-Alla et al., 2001).

Modern diesel engine combustion systems have been reported to have reduced PM emission by as much as 90% (Haney et al., 1997) through the improved engine design and as much as 30% through fuel formulation. For example, the addition of biomass-derived fuels and synthetic fuels to diesel fuel base stocks is a means of producing a cleaner burning diesel fuel (Boehman et al., 2003).

However, the consequence of improved engine design and fuel formulation is a decrease in PM mass but an increase in fine particulate number which is potentially more hazardous (Mayer et al., 1998). Hence, it is inevitable that the exhaust gas must be treated to meet the stringent emission standard.

The development of a system to minimise PM from the diesel exhaust gas is complex due to the size, shape and composition of the PM, which in turn depends on a variety of variable parameters such as the engine load, fuel injection type, fuel composition and the ambient conditions. Other variables include the forms and size of the diesel engine: car, truck, marine, generator and bus engines. The PM is assumed to be spherical with an average aerodynamic diameter, which is generally used in filtration analysis.
Chapter 1

Introduction

The diesel exhaust aftertreatment generally involves the trapping of the PM or reduction of other toxic emissions from the exhaust gas before they enter into the atmosphere. In the reduction of PM from diesel exhaust gas, a number of design concepts, discussed in Chapter 2, have been investigated. However, the DPF technologies appear to be the best option for the development of aftertreatment systems to meet the forthcoming legislation.

1.5 THE FUTURE OF DIESEL PARTICULATE FILTER AND FILTER MODELLING

The technical progress in engine design, especially with the introduction of the high pressure fuel injection systems has reduced substantially PM emissions from diesel engines. However, diesel engine exhaust gases still contain higher amount of PM mass than that of the gasoline engine.

The diesel particulate filter technology has been proven as a viable option for the effective reduction of PM from diesel engines (Forest and Landscape, 2000) and mathematical modelling is increasingly becoming an engineering tool to understand, predict and control the DPF systems. The fundamental parameters to assess the quality of the DPF are the filtration efficiency and the pressure drop of the filter. Hence, it is desirable to develop mathematical models to predict these parameters that can be used within given boundary conditions to aid the design of DPF systems.

Several researchers (Konstandopoulos and Johnson, 1989, Konstandopoulos et al, 2001) have proposed mathematical models for predicting pressure drop across wall flow filters and filtration efficiency. However, there is little literature presenting modelling of ceramic foam filters, and the gelcast foam
filters in particular. Considering the many advantages of gelcast ceramic foam (GCF) filters, this research work seeks to contribute towards optimising GCF filters used as substrate for DPFs, by providing mathematical models for understanding fluid flow through them and as tools for the purpose of filter design.

The first step to develop a mathematical model is identifying the problem. Consequently, DPF systems should ideally exhibit the following attributes:

- A filtration efficiency that meets the emission limits.
- Increase in engine backpressure imposed by the DPF due to PM accumulation such that the fuel economy penalty is minimised.
- A reliable periodic regeneration under all driving conditions.
- The filter should be able to withstand sudden increases in temperatures that could lead to filter melting or cracking.
- A low residual accumulation of PM in the filter after extended vehicle distances or engine operating hours.
- A high durability, ideally covering a large part of the vehicle or engine's useful life.

To achieve these objectives, mathematical models to predict the performance of DPF materials are valuable tools.

1.6 RESEARCH OBJECTIVES AND POTENTIAL BENEFITS

The objective of this research work is to acquire deeper knowledge of gelcast ceramic foam DPF materials by developing validated mathematical models. The models can then be used to optimise the design of DPF systems. This involves:
Chapter 1 Introduction

1. Identification of cost effective filter substrates that exhibit high filtration efficiency (over 90%), maintain a low backpressure and can withstand high thermal stress.

2. Development of explicit models for the description of the dynamic flow through clean and PM loaded filter medium. Models should be calibrated and validated with experimental data.

3. Application of models in the design of a DPF for a given engine specification.

The potential benefit of the research presented in this thesis is the development of validated mathematical models that can be used to understand fluid flow in gelcast foam filters and aiding the design of a technology for the production of relatively cheap emission control system that can be exploited by diesel engine manufacturers. It is of note that the gelcast ceramic foam DPFs can be adapted to any of the conventional regeneration processes or as catalyst coated filters.

1.7 RESEARCH METHODOLOGY

The basic research methodology followed in the present research is shown in Figure 1.3 and is now discussed.

Following a detailed literature survey, mathematical models were developed to predict the pressure gradients of clean gelcast ceramic foam filters. The new models were calibrated using data from fluid flow experiments on physical scale model foams and then validated on actual gelcast ceramic foam filter samples. Experiments were conducted on several foam samples to establish repeatability.
Figure 1.3 Chart of major tasks of the research presented in this thesis
The mathematical models for clean foam filters were further developed to incorporate the morphology of filters loaded with PM. This resulted in a mathematical model for the prediction of pressure gradients of PM loaded gelcast ceramic foam filters. Foam filter samples were loaded with PM and then tested on a fluid flow rig to provide the data for model validation.

The mathematical models for clean and PM loaded foam filters were then used for the dimensioning of a practical engine DPF system using gelcast ceramic foam. Finally, a design procedure for sizing DPF systems was proposed.

1.8 THESIS OVERVIEW

This first chapter has presented a brief description of diesel particulate matter (PM) emissions and the main reasons for treating the diesel exhaust gases, highlighting the effect of PM on the health and the legislation limiting exhaust gas emissions. Furthermore, the aim and objectives of the research has been described. Finally, the research methodology has been presented.

Chapter 2 introduces the diesel exhaust aftertreatment system with a description of different design concepts for PM emission control. A review of exhaust aftertreatment including filter regeneration and filter types are described. The basic concepts on which the work of this thesis is based are highlighted.

Chapter 3 gives a review of previous mathematical modelling of ceramic foam filters. The development of a new mathematical model for the prediction of pressure gradients of clean ceramic foam filters is described, where the Ergun model for packed columns is extended for application to gelcast ceramic foam filters. This is called the Extended Ergun Mathematical (EEM) model.
Chapter 4 describes the experimental rig and procedures to collect fluid flow data from a physical scale model cellular foam and real gelcast ceramic foam filter samples. The use of the experimental data collected for calibrating and validating the mathematical model proposed in Chapter 3 is described. The EEM model performance is described.

Chapter 5 introduces another new mathematical model for the prediction of pressure gradients of clean ceramic foam filters referred to as the ‘Multiple Orifice Mathematical’ (MOM) model. Furthermore, the calibration and validation of the model is described. The MOM model is shown to be better than the EEM model, and importantly it requires no tuning using data from real foams.

Chapter 6 describes the development of the MOM model for the prediction of pressure gradients of PM loaded ceramic foam filters. The experimental rig for the PM loading is described. Furthermore, the procedures for experimentation, collecting fluid flow data from the loaded filter is presented. Finally, the validation of the PM loaded filter mathematical model is presented.

Chapter 7 describes the application of the mathematical models to diesel particulate filter design.

Chapter 8 presents the conclusions from the present research work and areas for potential future work are identified.
1.9 CONTRIBUTIONS TO KNOWLEDGE OF THE WORK PRESENTED IN THIS THESIS

The research work presented in this thesis has led to two significant new contributions to the study and design of diesel particulate filters. These are:

i. A validated mathematical model for the prediction of pressure gradients of clean gelcast ceramic filters.

ii. A validated mathematical model for the prediction of pressure gradients of PM loaded gelcast ceramic filters.

Both of these are considered to be significant advancements to the research field.

1.10 SUMMARY AND CONCLUDING REMARKS

Diesel particulate matter (PM) has been found to consist of small particles of \(< 100 \text{ nm}\), which are in the aerosol size range that can be inhaled deep into the lungs. The PM chemical composition is essentially a carbonaceous core surrounded by organic compounds, which include unburned hydrocarbon and oxygenated hydrocarbons, and inorganic species such as sulphur dioxide, sulphates and nitrogen dioxide.

The exhaust gas emissions are reported to have adverse health effects, thus, there are legislations limiting the emission of PM and NOx. Although, engine manufacturers have gone a long way to reducing the PM and NOx emissions by improved engine design and fuel reformulation, it is not sufficient to meet the stringent standards of the future. Hence, the need for treating the exhaust gas in the exhaust system. This is referred to as aftertreatment of the exhaust
gas. It is suggested that use of diesel particulate filters are presently the most viable options to significantly reduce PM emissions.

The increasing need for using mathematical models as engineering tools to understand fluid flow through porous medium and predict pressure gradients have been highlighted. Furthermore, the aims and objective of the research have been described followed by a description of the research methodology and thesis overview.

The next chapter presents a review of diesel exhaust aftertreatment and the highlights of the basic concepts on which the work of this thesis are based. The reviews cover some filtration technologies being developed to reduce PM emissions, methods of regeneration and types of filter materials.
CHAPTER 2 REVIEW OF DIESEL EXHAUST AFTERTREATMENT TECHNOLOGIES

2.1 INTRODUCTION

In chapter 1, the importance of diesel particulate filter (DPF) for the reduction of particulate matter (PM) from diesel engines was described and the need to develop mathematical models to be used as engineering tools to design and control DPFs was identified.

This chapter introduces the diesel exhaust aftertreatment with a description of some methods of filtration, types of DPF and some methods for filter regeneration. A review of previous filter modelling research is discussed and the basic concepts on which the work of this thesis is based are highlighted.
Chapter 2  Review of diesel exhaust aftertreatment technologies

The DPF is presently considered to be the most viable technology in diesel aftertreatment and the ceramic foam is being considered as potential filter substrates due to its efficiency with the nano-particles.

2.2 FILTRATION TECHNOLOGIES

Filtration in aerosol science is the separation of particles from gas streams. Aerosols are defined as a dispersion of solid and/or liquid particles suspended in a gas. The size of filterable particles is normally defined in terms of micrometers due to their relatively small dimension.

The challenge for diesel engine manufacturers is to develop a viable separator that can further reduce the PM from the engine-out exhaust gas to meet the stringent PM emissions standards. Examples of some of the filtration technologies, including the diesel particulate filters (DPFs) that have been investigated in diesel aftertreatment are discussed in this section.

2.2.1 Cyclones

Cyclones are centrifugal particle collection devices. They rely on the inertial properties of the particles to separate them from the carrier gas stream. They are devices that increase the normal centripetal force on the particles entering the device, by causing them to swirl and separate from the gas stream. Cyclones are already used across a variety of combustion and industrial processes, for example, cyclones are primarily used as pre-cleaners in solid
fuel combustion systems such as coal burning boilers. They can be used for collection of unburned particulate for re-injection into furnace.

Cyclones are generally simple in design with few parts. The main features of cyclones are a cylindrical section connected to a conical part with a nozzle and a dust hopper. The most common cyclone type is the tangential cyclone illustrated in Figure 2.1. The exhaust gas enters the cylindrical portion through the tangential inlet duct. The fluid flow spirals downwards through the conical section and inwards towards the axis and discharges through the axial outlet duct. The PM suspended in the gas is thrown outwards by the centrifugal force arising from the rotation and is ejected through the nozzle into the hopper.

Figure 2.1 Schematic of a tangential flow cyclone
Cyclones exhibit low pressure drops. They are typically used to remove relatively large particulates in the size range of 10 - 100 \( \mu m \) and the particle removal efficiency is typically < 70%. Aldersey-Williams (1989) reported that the separation efficiency is low for particle size < 50 \( \mu m \) which is significantly larger than the diesel particulate size range of 0.001 - 0.1 \( \mu m \). The separation efficiency is low due to insufficient residence time for the particulate matter (Ludecke and Dimick, 1983). The efficiency of the cyclone can be increased by increasing the swirling speed, but this entails extra energy.

Recently investigators (Sibanda et al, 2001) using a cyclone in combination with cross flow filters, commonly used for filtering liquids, reported collection efficiencies of over 99% for particles of 5 \( \mu m \), which is still insufficient for diesel PM filtration.

### 2.2.2 Electrostatic precipitators

Electrostatic precipitators separate PM from gas streams using electric forces. This type of separator consists of high voltage electrodes (positive and negative) in a concentric pipe carrying the exhaust gasses. A good precipitator performance voltage from the electrode is approximately 25 kV (Nobrega et al 2001). An electrostatic charge is imparted onto the PM which is consequently attracted to collector electrodes. The PM collection on the electrodes in a typical electrostatic precipitator builds up in a layer until the electrode is cleaned by a mechanical rapping or vibrating system or washed with water. The resulting PM or sludge is collected in a hopper. Gibbs et al (1986) have suggested the feeding of the collected particulate back to the engine cylinder.

The electrostatic precipitator is generally suitable for particle sizes > 1 \( \mu m \) and overall trapping efficiency of 99%. The main problem is that particulate
accumulation can bridge the gap between the electrodes and the pipe wall that may lead to an electric short circuit of the system.

Figure 2.2 is a schematic of an electrostatic precipitator. Electrostatic precipitators can be used in combination with cyclone separators (Weaver et al, 1986)

![Schematic of an electrostatic precipitator](image)

**Figure 2.2 Schematic of an electrostatic precipitator (Adapted from AEA Technology, 2001)**
2.2.3 Turbulent precipitators

The turbulent precipitator, also called "open filter", was developed by Dullien (1993). The system is a design where the exhaust flows through two hydrodynamic zones, a relatively fast flow zone corresponding to an open channel and a stagnant zone. Hence, the particulates are given sufficient residence time to settle in the stagnant zone. Figure 2.3 illustrates two example geometries of the system. The first geometry has an open straight channel while the second geometry has an open channel that is not straight. These systems have a relatively low pressure drop, robust, flexible and durable. The problem with these systems is that they require fine tuning to achieve satisfactory filter efficiency for diesel PM. Nevertheless, Gulijk et al (2001) have suggested that the turbulent precipitators appear to have a promising future.

Figure 2.3 Schematic of two geometry of turbulent precipitator
2.2.4 Diesel particulate filters

The diesel particulate filter (DPF) consists of a filter positioned in the exhaust stream designed to collect a significant fraction of the PM emission while allowing the exhaust gases to pass through the device. Since diesel engines produce a significant amount of PM to fill up a reasonably sized filter over a relatively short period of time, some means of disposing of the trapped PM must be provided. The burning or oxidising of the PM in the filter is the most common means of trapped PM disposal, thus, regenerating the filter. Consequently, the DPF comprises of a filter medium, a system for cleaning (regenerating) the filter and a monitoring/control system. Figure 2.4 illustrates the installation of a DPF in a vehicle, where a muffler is replaced by the DPF canister.

![Diagram of DPF installation](image)

1. Diesel engine
2. Catalytic converter
3. Temperature sensor
4. Heating element
5. Diesel particulate filter
6. Pressure sensor
7. Electronic control unit
8. Additive tank
9. Fuel tank

Figure 2.4 DPF installation in place of the muffler (adapted from Lukewille et al, 2000)
Chapter 2

Review of diesel exhaust aftertreatment technologies

In practical application, most DPFs can reduce the amount of particulates from diesel engine exhaust gas by at least 90% by mass across the whole range of particulates sizes (Mayer, 1998, Mayer, 2002, Heeb et al, 2004). DPFs are usually mounted in the exhaust system and could also play a role in acting as an exhaust silencer (muffler).

DPFs, while in operation, accumulate PM which will lead to an increase in engine back pressure that leads to high fuel consumption. Generally, the trapped PM is eliminated continuously or in intervals, to ensure uninterrupted engine operation, by regeneration. The effectiveness of a DPF relies in a great part on the regeneration process of the system. Efficient regeneration of a DPF is one important limitation to this PM emission control option and the technique of regeneration determines some of the design criteria of DPFs, such as filter material and configuration. Hence, understanding the fundamental process of regeneration is important.

2.2.5 Summary of filtration technologies

In order to meet the stringent PM emission standards, the required exhaust gas filtration devices should exhibit a filtration efficiency of > 85% in all the size range of the diesel PM. The devices should be able to withstand temperatures of over 1000°C. It is seen that most of the technology being investigated exhibit low filtration efficiencies in the particles size range < 5 μm and may have low thermal strength. However, DPFs have been demonstrated to maintain filtration efficiencies of over 90% even for nano-particle size (up to 100 nm).

Since regeneration plays an important role in filter design, the next section describes some of the regeneration techniques available for the disposal of trapped PM in DPFs.
2.3 METHODS OF FILTER REGENERATION

Filters mounted on a conventional diesel engine, are usually clogged with PM within a few hours of engine operation, thus, requiring frequent regeneration. Regeneration is normally achieved by oxidizing the trapped PM within the DPF, initiated by raising the temperature in the exhaust system up to the ignition temperature of the PM which is from 500 - 600 °C (Heywood, 1988b, Persiko and Sher, 2001). However, the exhaust temperature of diesel engines at most operating conditions is not sufficient to regenerate the filter. Persiko and Sher (2001) demonstrated from available data that such high exhaust temperature can only be attained at high engine load and speeds. Hence, the practical option is to raise the temperature of the exhaust gas by using electric heaters, burners, etc. and by lowering the PM ignition temperature with the aid of catalysts.

There are two types of regeneration namely passive and active. The passive regeneration does not require an external or active control system to dispose of the PM from the filter. PM is collected on the filter at idle or low power operation. As the engine exhaust temperatures increases due to the back pressure, the trapped PM can be regenerated. However, results from the monitoring of temperature profiles for the application of passive DPFs to solid waste collection vehicles in California in particular suggested that such DPFs may not be used on the full population of collection vehicles without significant assistance in increasing the engine exhaust temperature (Reul-Chen et al, 2005). Attempts to increase the exhaust temperature to assist passive regeneration could be achieved by using pipe insulation or locating the DPF closer to the engine (Mayer et al, 2001). Alternatively, the regeneration temperature could be lowered with the addition of catalyst.

Unlike the passive DPFs, active DPFs regeneration is obtained by raising the temperature of the engine exhaust gas through external source of heat. Active
Chapter 2 Review of diesel exhaust aftertreatment technologies

DPFs employ a variety of regeneration techniques including the burner DPF, where a burner is placed in the inflow to the filter (Park et al, 1998a and 1998b), an electric heater DPF (Goto et al, 1992), microwave DPF (Garner, 1989, Ning, et al, 1999, Nixdorf et al, 2001) and non-thermal plasma DPF (Slone et al, 2001). In fact, a significant variety of methods have been investigated and developed to facilitate combustion of the trapped PM. These include:

1. Coating of the filter material with base or precious metal. This reduces the temperature needed to oxidise the PM.
2. Adding catalyst to the fuel to reduce the combustion temperature of the trapped PM.
3. Using catalyst to oxidise NO to NO₂ which in turn acts as a low temperature PM oxidant.
4. Throttling the air intake to one or more of the cylinders, thereby increasing the exhaust gas temperature.
5. Throttling the exhaust gas down stream of the filter unit, adding backpressure to the engine, thereby causing the temperature of the exhaust gas to rise and initiating combustion.
6. Use of periodic compressed air flowing in the opposite direction of the DPF blowing the PM into a collection bag which is then periodically discarded or burned.
7. Use of fuel burners, electrical heaters or some other active method to heat the exhaust gas and/or the PM to temperatures high enough to ensure filter regeneration.

Regeneration of DPFs using burner as a means of heating the exhaust gas or directly the trapped PM has been extensively studied by Park et al (1998a). Figure 2.5 is a schematic of a burner type diesel particulate filter.
The burner regeneration system is relatively independent of engine exhaust gas temperatures. It uses normal diesel fuel to burn and requires no fuel additives. However, it was reported that burner systems are expensive to manufacture and maintain (Frankle, 2004), requiring trained personnel. Installation of the burner is also complex (Park et al, 1998b) and due to the difficulty of controlling the peak temperatures of the burner, the durability and reliability of DPFs using such means of regeneration are not guaranteed.

Regeneration of DPFs using electric heaters to raise the temperature of the exhaust gas requires significant extra energy from the engine. They can require several kilowatt of electrical power to operate, ranging from 30 to 120 kW (Zelenka and Telford, 2003) which is not generally available as electrical energy from a vehicle battery. Consequently, there is usually an increase in fuel consumption (Mayer et al, 1998), which renders them low in the option of techniques to develop. However, the energy requirement can be reduced by using additives in the fuel to reduce the regeneration temperature of the PM (Frankle, 2004).
Regeneration of DPFs using plasma is an attractive option due to the relatively low regeneration temperature it exhibits. This process converts NO to NO$_2$ that is absorbed on the PM which substantially reduces the temperature required to regenerate the filter (Okubo et al., 2004). However, the technology in plasma regeneration is yet to be fully understood and is still being investigated.

Ranalli and Schmidt (2004) reported that regeneration using engine management to raise the exhaust temperature has some serious weakness if the unburnt hydrocarbons are added to the exhaust via the engine. However, they reported that the introduction of the hydrocarbon directly in the exhaust system is a cost efficient method for regeneration of DPFs.

Regeneration of DPFs using engine intake throttling to increase the exhaust gas temperature exhibits an interesting potential that can be applied in combination with catalytic method (Mayer et al., 2003). However, throttling reduces engine fuel economy and is noisy and regeneration is difficult to control. Also, vehicle drivability is impaired.

Since diesel engine exhaust temperatures are generally too low to regenerate DPFs on its own, using catalysts to reduce the oxidation or combustion temperatures of the trapped PM in the filter is a good development. The catalysts are either used as additives in the fuel from a dosing tank or as coatings in the filter. A large number of catalytic materials for DPF regeneration have been investigated and developed (Ciambelli et al., 1996, Badini et al., 1998, Saracco et al., 1999, Setten et al., 2002, Fino et al., 2002, Ciambelli et al., 2002, Fino et al., 2003). The most common fuel additives for DPF regeneration are iron, copper or cerium based. Other catalytic materials include platinum, radium and vanadium.

Although, fuel additives also result in the formation of secondary toxic emissions such as furans and dioxin (Persiko and Sher, 2001), most of the
advances in DPF regeneration techniques are based on catalyst assisted regeneration. Fuel additives lower the PM ignition temperature from approximately $600^\circ C$ to between $350^\circ C$ and $450^\circ C$ (Zelenka and Telford, 2003). Other limitation of this method of regeneration is the additional tank that is needed, coupled with the automatic dosage system. The additive also contributes to the accumulation of ash, thus, the filter must be cleaned at regular intervals. Sulphur in the fuel which contributes to particulate emissions (sulphates and SO$_x$) also competes for active surface area sites on the catalyst, thus, reducing the efficiency of the catalyst. However, the catalyst can return to optimum levels once it is subjected to low sulphur fuel.

From the foregoing analysis, efficient regeneration can be achieved by using an external source of heat on the DPF or engine management with assistance from catalytic materials. For example, a regeneration system has been developed by a European car manufacturer, using the added benefit of common rail fuel injection system (Salvat et al, 2000). In this system, post-injection of fuel is used with additive and catalysed DPF systems to raise the temperature in front of the filter. The fuel additive tends to reduce the PM ignition temperature while the catalytic coating on the filter promotes the PM combustion, oxidising carbon monoxide and hydrocarbon emissions (Gieshoff et al, 2001).

Most of the regeneration techniques mentioned above causes high DPF temperatures during regeneration. Hence, the task for the engine designers is to select a good filter medium that has high filtration efficiency and durability to withstand the temperature during regeneration, which can be sometimes as high as $1100^\circ C$ during the rapid exothermic oxidation process (Mayer et al, 2003).

Having surveyed the main DPF regeneration methods, the following section describes types of DPF substrate that are being investigated or developed.
Chapter 2 Review of diesel exhaust aftertreatment technologies

2.4 TYPES OF DIESEL PARTICULATE FILTER

As mentioned in Chapter 1, there are various filters, in shape and materials that have been found suitable for DPF systems, while each has its advantages and disadvantages. However, the choice of a filter material depends on many factors including filtration efficiency, pressure drop, durability and cost effectiveness. The main diesel particulate filters that have been investigated and developed are described in this section.

2.4.1 Honeycomb wall flow ceramic monolith

The honeycomb wall flow ceramic cordierite monolith is the most widely used design of DPF today due to its high filtration efficiency, which is > 90% (Clerc, 1996). This filter is made from porous ceramic material with a honeycomb structure, where the adjacent channels of approximately 0.5 to 2.0 mm square are plugged at each alternate end in order to force the exhaust flow through the porous ceramic walls (see Figure 2.6). The main advantage of the wall flow DPF is the high surface area per unit volume of over 2 m² for 2.45 litre filter (i.e. 143 mm diameter x 152 mm length). The material used for this type of DPF is usually cordierite, which is relatively cheap. The porosity of the porous cell walls is approximately 50%.

The filtration in the honeycomb DPF is generally on the filter inlet channel surface, particulate accumulates within the length of the inlet channels. During the initial stage of regeneration of a heavily loaded filter, a sudden reduction of engine load may lead to uncontrolled filter regeneration. In principle, the sudden reduction of engine load leads to a situation where there is excess oxygen and reduction in heat evacuation from the filter (Cutler and
Merkel, 2000), and hence leading to high thermal gradients that may cause melting or cracking of the filter material (Henrichsen and Popuri, 2001).

Figure 2.6 Schematic of air flow through wall-flow ceramic filter

Figure 2.7 (a and b) shows a melted and cracked cordierite wall flow monolith filters damaged during uncontrolled regeneration. Even when a crack is only localised, filtration efficiency is usually seriously impaired.

Figure 2.7 Results of high soot loading of cordierite filters damaged during uncontrolled regeneration, (a) excessive peak temperature leading to melting, (b) excessive temperature gradient leading fracture or crack (Henrichsen and Popuri, 2001)
The cordierite material generally used as the filter substrate for monolith DPFs has the disadvantage of failing occasionally as a result of localised melting or cracking due to its low thermal strength. Hence, introducing the silicone carbide (SiC) which has a higher operating temperature limits and higher thermal strength, as a substrate for the monolith filters is of potential interest in DPFs design. The SiC DPFs are beginning to be installed in new engines, for example, the Peugeot 607 is equipped with a monolithic filter made of SiC using the cerium additive based catalyst regeneration system. However, the SiC filters are relatively costly and heavy.

Figure 2.8 shows examples of wall flow filters made of (a) cordierite and (b) silicon carbide materials respectively. The cordierite DPFs are manufactured as a whole by extrusion while silicon carbide filters are usually manufactured in parts and cemented together as illustrated in Figure 2.8 (b).

![Figure 2.8 Wall flow filters of (a) cordierite and (b) silicon carbide](image)
2.4.2 Metal sintered filters

The sintered metal fibre filters have a relatively high PM holding capacity and are capable of capturing fine particles in the exhaust gas stream. They are manufactured by compressing and heating to sintering temperature uniformly sized spherical stainless steel particles. These filters are generally deep bed filters, where filtration occurs within the matrix of the filter, with high porosity (up to 85%) leading to low pressure drops. The sintered metal fibre filters are thermally conductive and thus provide good heat dissipation and the metallic elasticity prevents thermal regeneration damage to the filter. The filter is reusable after the non combustible residue is washed off (Heikkinen and Harley, 2000).

Figure 2.9 is an exploded picture of a sintered metal fibre filter module.

![Sintered metal fibre filter](image)

Figure 2.9 Sintered metal fibre filter. (Adapted from Lukewille et al, 2000)

The material properties of sintered metal fibre filters allow a wide range of different design shapes. For example, the porous plates of the sintered metal
can be assembled to cell structure like that of honeycomb and welded together. Another design is composed of cylindrical cartridges that permit sequential regeneration to reduce the amount of electrical energy required for regeneration at any one point in time. In this case, the filter medium acts as substrate to trap PM, as well as a resistive element by which an electric current can be supplied to produce the heat to oxidise the PM.

Conversely, the sintered metal fibre filters are heavy and more expensive when compared to the monolithic ceramic wall flow filters. Also, they need to be water jacketed to control the surface temperatures (Penrith, 2003).

2.4.3 Metal block filter substrates

Metal block filter substrates are made from heat-resistant stainless steel foil with high electrical resistance. The steel foil is rolled with a porous electrical insulation membrane into the filter, and then corrugated. The peaks and valleys are perforated such that the exhaust gas diffuse uniformly through the filter with its flow disrupted. The PM is trapped by the stainless steel foil which can be eventually regenerated by passing an electric current directly through the foil.

The metal block filters are compact and have PM filtration efficiency of over 80%. However, these filters are relatively heavy and the regeneration process is not yet mastered (Engine Manufacturers Association, 2002).
2.4.4 Wound ceramic fibre filters

The wound ceramic fibre filters, which are mostly cylindrical, consist of three main parts; an inner filter support, the filter media and the outer filter protector. The inner support is generally made of woven or welded steel mesh and the filter media are of high-temperature fibres. The fibres are wound in a continuous layer around the support and the outer protector, which is also a metal mesh, is wrapped around the filter media, securing it in place to keep media from sagging and leaking. Figure 2.10 shows a schematic of wound ceramic fibre filter.
The wound fibre filters exhibit high filtration efficiencies for the nanoparticles size in particular, consistent with deep bed filtration. The volume fraction of the filter material is low in fibrous filter, which aids combustion zone propagation during regeneration. They are adaptable to available vehicle space due to their design flexibility. However, the wound ceramic fibre filters may disengage the trapped PM at high back pressure. The filters are also relatively expensive.

2.4.5 Ceramic foam filters

Ceramic foams until recently were mainly used as catalyst supports (Viskanta et al, 1991) and molten metal filters (Richardson et al, 2000). However, they are now being considered for DPF applications since they exhibit some favourable attributes. Ceramic foams have good filtration in the nano-particle range (Gabathuler et al, 1991). The high porous nature of ceramic foam filters is favourable to the propagation of the combustion zone during regeneration.

A significant advantage for the ceramic foam filters is that the geometry of the foam can be made in many configurations (Dhara et al, 2002). The shape of the foam filter is limited only by the complexity of the mould in which it is produced. In addition, Russo et al (2003) reported that the ceramic foam filters provide better PM-catalyst contact and easier spontaneous filter regeneration when compared to the sintered metal filters. The manufacturing processes and the microstructure of ceramic foam are discussed in Chapter 4.
2.4.6 Summary of types of diesel particulate filters

It is obvious that the above DPFs are all potential devices for the reduction of PM from diesel exhaust gas. However, ceramic foam filters exhibit attributes that make them worthy of investigation. The ceramic foam filters exhibit good contact between the catalysts and PM. They have high filtration efficiency even in the nano-particle size.

The next section describes the basic fluid flow theory required for the development of the mathematical models of gas pressure drop.

2.5 FLUID FLOW THEORY

A great portion of the fundamentals of porous media fluid flow and fluid flow theory have been described and developed by Scheidegger (1960), Collins (1961) and Bejan et al (2004). Here, the salient points are presented and discussed pertinent to the development of the models described later in this thesis. Fluid flow through filters (porous media) involves a complex pattern of fluid travelling in and around spaces in the support materials, through channels or pores of various sizes and shapes. The presence of a solid in the filter creates resistance to the fluid flow, which is directly related to the gas flow rate and the filter structural details. The resistance on the fluid flow is usually expressed in terms of the pressure drop, which is one of the properties used in characterising porous media. The pressure drop is usually related to the fluid flow rate per unit cross sectional area to the flow and the microstructure of the porous media.
Chapter 2  Review of diesel exhaust aftertreatment technologies

2.5.1 Darcy’s law

Pressure drop theory is usually based on Darcy’s law, which states that the pressure gradient across a porous medium is proportional to the fluid flow per unit area perpendicular to the direction of the flow (Daily and Harleman, 1966). Subsequent studies have shown that the velocity is additionally inversely a function of viscosity of the fluid flow through the porous medium. For a one-dimensional fluid flow with negligible gravitational effects, this law is given in the form:

\[
\frac{dp}{dx} = \mu \frac{v}{k_l}
\]  \hspace{1cm} (2.1)

where \( \frac{dp}{dx} \) is the pressure gradient (Pa m\(^{-1}\)), \( v \) is the fluid flow per unit cross sectional area (m s\(^{-1}\)), \( \mu \) is fluid viscosity (kg m\(^{-1}\) s\(^{-1}\)) and \( k_l \) is an empirical factor known as the permeability of the medium with units of length squared. Darcy’s law is valid for porous media when a modified Reynolds number defined as

\[
Re = \frac{v d \rho}{\mu}
\]  \hspace{1cm} (2.2)

is less than unity, where \( v \) is the fluid velocity (m s\(^{-1}\)), \( \rho \) is the fluid density (kg m\(^{-3}\)), \( \mu \) is the fluid viscosity (kg m\(^{-1}\) s\(^{-1}\)) and \( d \) is the average diameter of the filter cells (m).

Based on the work of Darcy, various expressions have been developed based on the physical model concepts, for example, the Hagen-Poiseuille flow.
A variation of Darcy's law is the equation for the Hagen-Poiseuille flow, which is valid under streamline flow through circular capillary tubes, and expressed as

\[
\frac{\Delta p}{L} = \frac{32 \mu}{d_i^4} \nu_i
\]  

(2.3)

where \( d_i \) is the tube diameter (m) and \( \nu_i \) is the tube velocity (m s\(^{-1}\)). The tube velocity is defined as the ratio of the superficial velocity to the porosity \( \varepsilon \) of the filter, where the porosity is the fraction of the volume of the bed not occupied by solid material.

In order to develop expressions relating the filter parameters, a conceptual physical model that can represent the structure is required.

### 2.5.2 Conceptual model

Due to the complex structure of most porous media, mathematical models of a filter can be developed in various ways by representing the porous media with a conceptual model. For example, the filter could be regarded as a system of capillary tubes in parallel along the fluid flow. Alternatively, the medium can be regarded as a system with certain length of cylindrical obstacles, which is generally known as the single fibre representation. Other physical scale models are constricted circular tubes that are equivalent to the filter pores in successive layers of real filter and isolated spheres.

By representing the porous medium with a bundle of capillary tubes and assuming Hagen-Poiseuille flow through each tube, it is possible to derive from Equation (2.1) an expression, such that \( k_1 \) is a function of the
microstructure of the medium. Hence, according to the hydraulic radius theory of Carman-Kozeny (Dullien, 1992) the correlation of the microstructure and the permeability of a granular bed is expressed as

\[ k_1 = \frac{\varepsilon^3 D_p^2}{180(1-\varepsilon)^2} \]  

(2.4)

where \( D_p \) is the average particle size, \( \varepsilon \) is the porosity of the granular bed and 180 is an empirical constant. However, Ergun (1952) proposed 150 instead of 180 for columns of packed spheres.

When the inertial forces become more prominent as a result of higher fluid flow rates, the pressure gradient becomes a quadratic function of the flow per unit area (the fluid velocity) in the porous medium. The most common expression used in this case is the Forchheimer's equation (Daily and Harleman, 1966) i.e.

\[ \frac{dp}{dx} = \mu \frac{v}{k_1} + \rho \frac{v^2}{k_2} \]  

(2.5)

where \( \rho \) is the fluid density and \( k_2 \) the non-Darcy coefficient. The second term in Equation (2.5) is known as the pressure drop due to Forchheimer inertial losses and the first term is due to the Darcy viscous loss contribution. Burke and Plummer (1928) in their study of gas flow through packed columns suggested the following expression for the non-Darcy coefficient \( k_2 \)

\[ k_2 = \frac{\varepsilon^3 D_p}{B(1-\varepsilon)} \]  

(2.6)
where B is an empirical coefficient, suggested to be equal to 1.75 by Ergun (1952). Substituting $k_1$ and $k_2$ in Equation (2.5) leads to the following expression that is widely used by researchers (for example Mauren et al, 2001)

$$\frac{\Delta p}{L} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu \frac{\varepsilon^2}{D_p} \nu + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho}{D_p} \nu^2 \quad (2.7)$$

In summary, the development of the expression that has been widely used by significant number of researchers has been described. Hence, the initial task in this work was to understand the application of the expression by other authors and adapt the same modelling approach to the microstructure of the gelcast ceramic foam filters. In Chapter 3 the details of the application of the expression on reticulated ceramic foam filters by other authors are described, with complement from the work of Ergun and Orning (1949).

### 2.5.3 Ergun and Orning (1949)

Ergun and Orning (1949) developed a mathematical model for the flow through a packed column by assuming that the granular bed is equivalent to capillaries or cylindrical channels placed in parallel such that the total internal surface and the free internal volume were equal to the total packing surface and the void volume respectively of the packed bed. They derived an expression for the pressure gradient based on the Poiseuille's equation while adding a kinetic energy loss. Hence, the pressure gradient was a function of the superficial velocity ($u$), and the specific surface area ($S_\nu$) of the packed bed:

$$\frac{\Delta p}{L} = 2x \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu u^2 S_\nu + \frac{\gamma (1-\varepsilon)}{8 \varepsilon^3} \rho u^3 S_\nu \quad (2.8)$$
where $\alpha$ and $\gamma$ are correction factors applied to each term due to the tortuous flow path of the fluid through the packed bed and $S_v$ is the specific surface area, an important geometric characteristic, defined as the wetted surface to volume ratio of the porous media. The correction factors depend on the geometry and packing of the particles (Ergun and Orning, 1949 and Richardson et al, 2000).

A number of suggested values for $\alpha$ and $\gamma$ have been reported. Ergun and Orning (1949) reported that experiments on air flow through packed beds of lead shot with packing porosities from 37 to 42% yielded values of $\alpha$ and $\gamma$ that varied from 2.0 to 2.8 and 2.0 to 2.6 respectively. They also reported that experimental results from a wide range of particle shapes yielded values of $\alpha$ and $\gamma$ ranging from 3.6 to 20 and 0.14 to 0.70 respectively. Macdonald et al (1979) suggested that $\alpha$ is a function of the porosity $\varepsilon$ and proposed a value of $\alpha = 10.6$ in consolidated porous media such as sandstone that generally exhibit high tortuosity.

In order to determine the values of $\alpha$ and $\gamma$, the appropriate value of the specific surface area ($S_v$) of the porous medium must be known. Models for estimating the specific surface area were proposed by a number of researchers. One was proposed by Ergun and Orning (1949) based on packed columns, using the assumption made by Kozeny (Scheidegger, 1960) where the cells were treated as uniform, parallel cylindrical tubes, with a constant diameter $D_p$ which is equivalent to the average cell size $d$. They assumed that the internal surface of the cylinder represented the surface of the idealized cells, thus, the specific surface area which is defined as the ratio of the total wetted surface to the total solid volume of the porous material, given by the expression

$$S_v = \frac{N L \pi D_p}{L \pi (D^2 / 4)(1 - \varepsilon)} \quad (2.9)$$
where $N_t$ is the number of tubes, $D$ is the diameter of the cylindrical porous medium; $L$ is the length and $\varepsilon$ is the porosity. The volume of the total void $(V)$ is defined as the product of the porosity $(\varepsilon)$ and the total volume occupied by the filter ($\pi D^2 / 4$), where it can be recalled that the porosity is the fraction of the void of the filter. Therefore, the expression for the void volume can be written as

$$V = \pi \frac{D^2}{4} L \varepsilon$$

(2.10)

Similarly, the volume of the void can also be defined from the parameters of the conceptual model, i.e. parallel tubes, as

$$V = N_t \pi \frac{D_p^2}{4} L$$

(2.11)

Hence, by equating Equations (2.10) and (2.11) the number of tubes $N_t$ can be expressed as

$$N_t = \frac{D^2}{D_p^2} \varepsilon$$

(2.12)

Finally, substituting $N_t$ from Equation (2.9) gives the expression for the specific surface area as

$$S_y = \frac{4 \varepsilon}{1 - \varepsilon D_p}$$

(2.13)

In conclusion, Ergun and Orning (1949) have determined expression for the specific surface area for a packed column. A similar approach was adopted by other authors (i.e. Richardson et al, 2000) as mentioned earlier to calculate the
surface area of reticulate ceramic foam filters, presented in Chapter 3. Furthermore, it is required to determine the correction coefficients.

Foam filtration, like granular filtration in a fixed bed, is naturally unsteady, i.e. filtered material builds up in the foam over time. The accumulation of PM within the filter causes changes in the microstructure of the filter medium. Consequently, the behaviour of the fluid flow through the loaded filter and efficiency of the filter can be significantly affected. Since the performance of the filter is measured by the filtration efficiency and the pressure drop often limits the length of time for the filtration run, the ability to accurately predict the efficiencies and pressure drops during the course of filtration is important. Although, modelling the filtration efficiency of the foam filter is not within the scope or objective of this present work, it is necessary to understand or study the basic filtration theory to determine the profile of deposition of PM across the length of the foam filter.

2.6 FILTRATION THEORY

One of the factors that affect the filtration process is the filter substrates which are classified into two broad categories: surface (or cake) filters and depth (or deep bed) filters. Surface filters, as the name suggests, retain the filtered particles on the upstream face of the filter while the depth filters retains the filtrates in the entire matrix of the filter. An example of surface filters are membrane filters (Davies, 1973) which operate by sieving the particles due to their relatively small pore sizes. Although, these filters are commonly used in aerosol sampling equipment, they are seldom used in industrial aerosol control since they readily clog. Granular filters, extensively studied by Tien (1989), are depth filters consisting packed beds of grains. The prevailing filtration process of the foam filters is depth filtration (Brown, 1993). The foam filters are, therefore, categorised as ‘non-fibrous depth filters’.
The filtration efficiency of filters can be expressed either in terms of particle number or mass collection efficiency. That is:

\[ E = \frac{n_i - n_o}{n_i} \]  \hspace{1cm} (2.14)

and

\[ E_m = \frac{c_i - c_o}{c_i} \]  \hspace{1cm} (2.15)

where \( E \) is the particle number collection efficiency, \( E_m \) is the particle mass collection efficiency all in fraction and \( n \) and \( c \) refer to number and mass concentration, respectively, of particles entering and leaving the filter. In some air cleaning equipment, the performance is usually characterised in terms of penetration \( P \) which is the fraction of the particles that exit the filter, i.e.

\[ P = \frac{n_o}{n_i} = 1 - E \]  \hspace{1cm} (2.16)

\[ P_m = \frac{c_o}{c_i} = 1 - E_m \]  \hspace{1cm} (2.17)

The deep bed filters can be thought of as multiple layers of thin filters where each is having a certain probability of collecting particles of a given size. The concentration of particles observed across a spent foam filter follows an exponential profile (Acosta and Castillejos, 2000), which can be derived by integrating the following first-order expression of filtration rate that was suggested by Iwasaki (1937) and quoted by Tien (1989):

\[ \frac{\partial c}{\partial z} = -\lambda c \]  \hspace{1cm} (2.18)
where $\lambda$ is the filtration coefficient, with dimension of reciprocal length. Boundary conditions are that, $c = c_i$ at $z = 0$ and $c = c_o$ at $z = L$. The filtration coefficient depends on the amount of particles accumulating within the filter. However, this present work is focused on an initial filtration period characterised by an initial filtration coefficient, $\lambda_0$. Hence, the corresponding filtration efficiency $E_m$ after substituting the integrated expression into Equation (2.15) is given as follows:

$$E_m = [1 - \exp(-\lambda_0 L)]$$

(2.19)

which establishes the dependence of the filtration efficiency with the length of the filter $L$. However, the filtration coefficient depends on the cell diameter, geometry, the fluid flow and the particles properties.

After establishing the PM deposition profile in the foam filter, it is then possible to model the pressure drop across the filter. Despite the abundance of literature concerning the modelling of pressure drop across loaded monolithic wall-flow filters, there is dearth of reported work on ceramic foam filters. The limited literature which has been mainly on the study of metal filtration requires the modelling of some of the parameters or determination of some of the coefficients experimentally. The main examples are models of Acosta and Castillejos (2000) and Pontikakis et al, (2001).

However, the basic approach in this present research work for the modelling of the fluid flow through loaded foam filters is derived from a deposit model suggested by Tien and Payatakes (1979). They considered two types of deposit morphology corresponding to two limiting cases. The first case assumes that the accumulation of the particles is spread uniformly inside the cell walls while the second case considers a situation where the deposition is around the holes connecting the cells.
The present study establishes a rational framework for the prediction of pressure gradient of lightly loaded filters that further guides the modelling of heavily loaded filter without resorting to computationally expensive schemes.

2.8 CONCLUDING REMARKS

This chapter has described some filtration technologies and the most viable option presently available for PM emission control. Filter regeneration techniques and fluid flow modelling in porous media were also reviewed. The basic dynamic fluid flow theory required for the development of the mathematical models has been presented.

The problems in developing models that can be of general application in predicting pressure drop were also highlighted. From the foregoing sections, it is suggested that the first step in the development of mathematical models is to determine a suitable physical elementary model to represent the microstructure of the filter and then apply the characteristic conservation equations on it, after establishing a correlation between the macrostructure and microstructure. The resulting mathematical model then requires calibration and validation.

Chapter 3 presents review of ceramic foam filter modelling and the development of a new mathematical model for the prediction of pressure gradients of clean gelcast ceramic foam filters.
CHAPTER 3 NEW MATHEMATICAL MODEL FOR FOAM FILTERS

3.1 INTRODUCTION

Chapter 2 presented a review of diesel exhaust gas aftertreatment covering filtration technologies, filter regeneration methods and types of diesel particulate filters (DPFs). Furthermore, the basic concepts on which the work of this thesis is based were highlighted.

This chapter presents a review of ceramic foam filter modelling, followed by the description of the development of a new mathematical model for the prediction of pressure gradients through clean gelcast ceramic foam filters. The model is a development from the work of Ergun and Orning (1949) on packed columns, by applying their various definitions on a conceptual physical structure of gelcast ceramic foam filters.
3.2 REVIEW OF CERAMIC FOAM FILTER MODELLING

Since the honeycomb wall-flow filters are the most common type of diesel particulate filter (DPF), it is not surprising that most of the validated models on fluid flow are on such filters. Konstandopoulos and Johnson (1989) for example, modelled the pressure drop across a wall flow filter by adopting an analytical solution of a one-dimensional momentum and mass balance for a single channel flow under isothermal conditions.

Most classical fluid flow expressions are derived from simple geometries such as assemblages of cylinders, spheres or tubes with periodically repeated constrictions. Hence, in order to model fluid flow through ceramic foam filters, the complex foam structures are represented conceptually by a simple model structure. For example, in the study of fluid flow in granular filters, Tien and Payatakes (1979) represented the filter structure with constricted tubes, which is in fact similar to the fluid flow through foam filters. However, the resultant equations of their work require heavy computation in solving them.

The modelling of porous media such as the ceramic foam filters, however, has been of interest to significantly fewer researchers. A work, of interest, reported by Pontikakis et al (2001) is the development of a mathematical model for the prediction of pressure drop across reticulated foam filters. Pontikakis et al (2001) assume that the struts which form the solid frame work of foam filters can be modelled as fibre elements. Although, the reported results were considered satisfactory, the model requires the experimental determination of the permeability of the foam filter. The reticulated foam filters are produced using a polyurethane foam skeleton. Generally, the polyurethane foam is soaked in aqueous slurry of ceramic, then dried and sintered. The sintering will burn off the polyurethane material leaving behind a ceramic foam with the struts taking the form of the skeleton or the void,
depending on the viscosity of the slurry. Figure 3.1 shows an image of a reticulated foam structure with almost cylindrical struts.

![Reticulated Foam Structure](image)

**Figure 3.1 Example of reticulated foam structure showing characteristic struts and windows (Schmah J and Davidson, 1993)**

Some other reported research findings on ceramic foam modelling are based on the work reported by Ergun and Orning (1949) and Ergun (1952). For example, Innocentini et al (1999) and Richardson et al (2000) developed their models by adapting the Ergun and Orning (1949) model to the foam filter structure. The results were considered satisfactory, thus, a similar approach was considered in this present work to develop mathematical model for the prediction of pressure gradients of clean gelcast ceramic foam filters. The initial task was to establish a conceptual physical model that represented the microstructure of the foam.
Chapter 3  New mathematical model for foam filters

The following sections describe the work of Innocentini et al (1999) and Richardson et al (2000). Their results are useful for the purpose of comparison.

3.2.1 Innocentini et al (1999)

Using the work of Ergun (1952) discussed in Chapter 2 as a base, Innocentini et al (1999) derive a relationship between the cell size of the ceramic foam filter and the spherical grain of the packed columns. They assumed that the cell diameter represented a cylindrical form of the hydraulic diameter and suggested the following relationship

\[ D_p = 1.5 \frac{(1-\varepsilon)}{\varepsilon} d \]  

(3.1)

where \(d\) is the average cell diameter of the ceramic foam filter. Using this equation, an equivalent cell diameter could be obtained for the cellular structure based on its mean cell size, which is then applied to the Ergun's equation rewritten below

\[ \frac{\Delta p}{L} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu}{D_p^2} u + 1.75 \frac{(1-\varepsilon)}{\varepsilon^3} \frac{\rho}{D_p} u^2 \]  

(3.2)

Results were considered satisfactory. However, Innocentini et al (1999) did not consider any correction factor relating to the effect of the tortuosity of the foam structure or energy loss that cannot be accounted for.
3.2.2 Richardson et al (2000)

Considering the expression proposed by Ergun and Orning (1949), presented in Chapter 2:

\[
\frac{\Delta p}{L} = 2\alpha \frac{(1-\varepsilon)^2}{\varepsilon^3} \mu a S_v^2 + \frac{\gamma (1-\varepsilon)}{\varepsilon^2} \rho u^2 S_y
\]

(3.3)

where \(\Delta p\) is the pressure drop, \(L\) is the filter length, \(\mu\) is the viscosity of the fluid, \(\rho\) is the fluid density, \(u\) is the superficial velocity, \(\varepsilon\) is the filter porosity, \(\alpha\) is the Ergun correction coefficient on the viscous loss, \(\gamma\) is the Ergun correction coefficient on the kinetic loss and \(S_v\) is the specific surface area. It is observed that the coefficients \(\alpha, \gamma\) and the specific surface area of the filter are the unknown expressions that depend on the microstructure of the filter and the unaccounted energy loss. Since the specific surface area is essentially a function of the microstructure of the filter, it was appropriate to initially find its value before the determination of the correction coefficient which also compensates for the tortuosity and the energy loss that cannot be estimated.

Ergun and Orning (1949) suggested a formula to calculate the specific surface for small grains with low specific surface (<2000 cm\(^2\) g\(^{-1}\)). However, the specific surface of the ceramic foam is generally very high (i.e. >1000 m\(^2\) g\(^{-1}\), Suzuki et al, 2003), thus, Ergun and Orning (1949) reported that the formula is not suitable for such materials. Richardson et al (2000) explored other experimental method for the determination of the specific surface of ceramic foam and suggested that none was suitable for such irregular structure. Consequently, they proposed mathematical models providing a relationship between the specific surface and the filter parameters. The resulting model was compared to other models earlier proposed by other researchers, which are also described below.
Models for estimating the specific surface \((S_r)\) were proposed by a number of researchers. These include the one suggested by Ergun and Orning (1949), Chapter 2, and Underwood (1968), see Richardson et al. (2000).

Using the Ergun and Orning (1949) expression directly on the foam structure by substituting the tube diameter \(D_p\) by the average cell diameter \(d\), in the expression derived in Chapter 2 (Equation 2.13), i.e.

\[
S_r = \frac{4\varepsilon}{(1-\varepsilon)} \frac{1}{d}
\]  

(3.4)

The second model, independent of cell shape assumption, was proposed by Underwood (see Richardson et al., 2000), where he considered a random, non-specified cell surface in a cubic sample of length 1, in space. Assuming there is a differential element of surface area \(dS\) in the cube and its \(xy\)-projected elemental area is \(dA\). He considered an array of lines called test lines distributed randomly over the face of the \(xy\)-plane (i.e. the lines are parallel to the \(z\)-axis) and determined the fraction of test lines intersecting \(dS\) and the number of intersections per unit length of test line. Finally, he integrated the number of intersection per unit test line over the entire surface of the cube to obtain the following relationship for the specific surface area:

\[
S_r = \frac{4}{d(1-\varepsilon)}
\]  

(3.5)

Richardson et al. (2000) on the other hand, assumed that the cell of the reticulated ceramic foam can be conceptualised as a tetrakaidecahedron, where each cell is connected to 14 other cells with connecting windows. The struts which are the frame of the foam were assumed to have triangular cross sectional area, bordering 8 hexagonal and 6 square windows of a cell. Using geometrical relationship, established by Gibson and Ashby (1997), between
the length of the window edges and the triangular edges of the strut, they derived the expression for the specific surface area of the reticulated foam, written as

$$S_r = \frac{12.979[1 - 0.971(1 - \varepsilon)^{0.5}]}{d(1 - \varepsilon)^{0.5}} \quad (3.6)$$

where $d$ is the cell diameter and $\varepsilon$ is the porosity of the foam filter. Furthermore, Richardson et al (2000) suggested empirical relations for the correction factors which they expressed as follows:

$$\alpha = 973d^{-0.743}(1 - \varepsilon)^{-0.0982} \quad (3.7)$$

and

$$\gamma = 368d^{-0.7523}(1 - \varepsilon)^{0.07158} \quad (3.8)$$

Richardson et al (2000) reported that despite the systematic approach in developing the model, it cannot give accurate pressure gradients for a wider range of ceramic foam types. Nevertheless, the model approach is useful in developing the new mathematical model for predicting pressure gradients of clean gelcast ceramic foam filter.

### 3.2.3 Summary of previous foam filter modelling

It can be seen that the main objective in each modelling approach (except the modelling by Pontikakis et al, 2001) is to derive an expression for the specific surface area of the foam filter and determine the correction factors. The specific surface areas depended on the assumed physical model representing the foam structure. Ergun's physical model represented the filter structure with rows of tubes across the filter (Ergun and Orning, 1949), Innocentini et al
(1999) considered an equivalent cell size for a cylindrical hydraulic diameter and Richardson et al (2000) represented the cell with a tetrakaidecahedron having struts of triangular cross sectional area and windows of hexagonal and square shapes.

The reviewed models were all derived using reticulated foam structures. Hence, the initial step in developing the present model is to define a physical model and its parameters that represent gelcast ceramic foam filters which are not reticulated, but foams formed from gelcasting techniques.

### 3.3 MODELLING PRESSURE GRADIENTS OF CLEAN GELCAST CERAMIC FOAM FILTERS

In order to characterise the foam structure it is necessary to have an understanding of the morphology of the ceramic foam by observation under a microscope. Figure 3.2 shows scanning electron microscopy (SEM) pictures of typical HiPor ceramic foam at a 150X magnification presented by Peng et al (2000). The image reveals regularly shaped spherical cells and several holes (windows) connecting the cells.

Figure 3.2 A typical microstructure of high purity Alumina foam: gelcast ceramic structure (Hi-Por Ceramic Ltd), Peng et al (2000)
Chapter 3  
New mathematical model for foam filters

There are a number of conceptual models that have been suggested to be used for modelling ceramic foam filters. However, most reported work on ceramic foam filters is based on the reticulated foam structure. Some of the concepts considered for ceramic foam modelling, as earlier discussed are:

- Using equations proposed for fibrous filters to derive the expression for pressure drop across the filter. The fibrous model may be suitable for the reticulated foam with the cylindrical shape of the struts in the foam. However, it can be seen that the struts of the gelcast ceramic foam filter are far from being cylindrical. Hence, the work of Pontikakis et al (2000) may not be suitable for predicting pressure gradients of clean gelcast ceramic foam filters, since an equivalent strut diameter has not been suggested. Figure 3.3 shows struts of the gelcast Alumina foam structure.

Figure 3.3 A typical cell and window of gelcast Alumina foam highlighting the strut (Hughes et al, 2002)

- Using equations proposed for packed bed filter (spherical grains) to derive the expression for pressure drop across the filter. It is evident that the fluid flow over spherical objects is different from flow through
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gelcast ceramic foams cells. Using such modelling approach on foams requires the application of equivalent characteristic diameter.

- Using the equations proposed for constricted tube to derive the expression of pressure drop across the filter. The constricted tube is a suitable physical model to represent the foam since flow through the filter will experience diverging and converging flow with the alternating arrangement of cells and windows. However, the model requires heavy computation and is not within the scope of this research.

Therefore, the Ergun model is not suitable in its traditional form to predict pressure gradient of gelcast ceramic foam filters. The gelcast ceramic foam filters essentially consists of a geometry of spherical cells connected through windows. Acosta and Castillejos (2000) reported the use of alternate cells and window in studying aluminium filtration. However, the result demands some level of detailed modelling. In this thesis, a similar physical model is considered where the gelcast ceramic foam is represented with uniformly arranged spherical cells intersecting to form windows.

The cell arrangement adopted in the present research is taken after the work of Peng et al (2000) in the study of the microstructure of the gelcast ceramic foam filters. They observed that the cells are spherical and have a narrow size distribution, thus, the model structure considered is a face centred cubic lattice. For example, a foam cell structure with a window/cell diameter ratio of 0.37 and porosity of 88% prepared by computer simulation, shown in Figure 3.4, reveals the face centred cubic lattice with the spherical overlapping cells and windows.
Although, there was no known application of the classical equations of fluid flow on a model structure formed by an assemblage of cells, its resemblance to the foam filters made it an attractive option. Hence, the assemblage of cells is used as a model structure of the gelcast ceramic foam in this work. This option enabled analysis of the filter with basic fluid flow theory while maintaining a good physical resemblance to the microstructure of foam filters.

Using the Ergun and Orning (1949) equation as a basis to develop the present model, the expression for the specific surface area can be derived for the proposed model structure. Furthermore, a relationship between the porosity, cell diameter and window diameter was established to estimate the window
diameters of the foam samples provided for the work. The proposed new model here is referred to as the 'Extended Ergun Mathematical' EEM model.

3.4 EXTENDED ERGUN MATHEMATICAL (EEM) MODEL

This section describes how the Ergun's mathematical model used in predicting pressure drop across granular filter can be adapted to a model for predicting pressure gradients of gelcast ceramic foam filters. The expression for the model discussed previously relates the pressure gradient to the superficial velocity in the study of fluid flow through randomly packed columns (see Equation 3.3).

Generally, all attempts to use the Equation (3.3) in predicting pressure gradients of ceramic foams have not been successful without using empirical values for the two correction factors $\alpha$ and $\gamma$. In order to determine the correction factors an appropriate value of the specific surface area $S_v$ of the porous medium should be known. Hence, the first step in adapting the Ergun’s model to the foam filters is to develop an expression relating $S_v$ to the gelcast ceramic foam filter parameters.

Although, each cell has a number of openings, it is assumed that the fluid flows in and out of the cell through one inlet and one outlet window respectively. This simplification is based on previous work by Acosta et al (1995) where it was reported that there is a preferential flow direction through the cells. Figure 3.5 shows a schematic spherical cell with twelve windows.
3.4.1 Determination of Specific Surface Area $S_v$

The specific surface area $S_v$ is the wetted surface (surface exposed to the fluid flowing through the filter) per unit volume of filter material. Considering a unit volume of filter, the total wetted surface is the product of the number of cells, $N$ and the wetted surface area of a cell, $S$, i.e.

$$\text{Total Wetted surface} = NS \quad (3.9)$$

where

$$N = \frac{\varepsilon}{V_{\text{CELL}}} \quad (3.10)$$

and the volume of filter material per unit filter volume $V_{\text{mat}}$ is expressed as:

$$V_{\text{mat}} = (1 - \varepsilon) \quad (3.11)$$

Hence, solving for the specific surface area ($S_v$) from Equations (3.9) to (3.11):
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(3.12) \[ S_r = \frac{S_F}{V_{CELL}(1 - \varepsilon)} \]

From Equation (3.12), it can be seen that the relationship is reduced to determining the wetted surface area \( S \) and volume of the cell \( V_{CELL} \).

### 3.4.2 Determination of cell wetted surface area \( S \)

In this analysis, assuming that there is a preferential flow through a cell, ten out of the twelve windows of the cell do not allow fluid flow, thus, the shared surface areas are added to the total wetted surface area of a cell. Hence, the total wetted surface area of a cell is equal to the surface area of the spherical cell minus twelve truncated surface areas of neighbouring cells plus ten shared window surface areas.

The surface area of a spherical cell of diameter \( d \) is given as \( \pi d^2 \). The surface area truncated by one neighbouring cell \( S_{TR} \) is also given as \( \pi d h \), whereas geometrically in Figure 3.6, \( h \) can be expressed as

\[ h = \frac{d}{2} - \frac{\sqrt{d^2 - w^2}}{2} = \frac{d}{2} [1 - \sqrt{1 - \frac{w^2}{d^2}}] \] \hspace{1cm} (3.13)

Therefore, the surface area truncated by a neighbouring cell can be rewritten as

\[ S_{TR} = \pi \frac{d^2}{2} [1 - \sqrt{1 - k^2}] \] \hspace{1cm} (3.14)
where \( k = \frac{w}{d} \).

Hence, the total truncated surface area from the twelve neighbouring cells \( S_{TRT} \) is written as

\[
12S_{TR} = 6\pi d^2 \left[ 1 - \sqrt{1 - k^2} \right]
\]  

(3.15)

![Diagram of a sector of a cell]

Figure 3.6 Diagram of a sector of a cell

Furthermore, the ten shared surface areas bounding the windows of diameter \( w \) can be written as \( 5\pi w^2 / 2 \). Finally, the total wetted surface area \( S \) is written as

\[
S = \pi d^2 \left[ 1 - 6(1 - \sqrt{1 - k^2}) + 5k^2 / 2 \right]
\]  

(3.16)
3.4.3 Determination of cell volume $V_{\text{CELL}}$

The volume of the cell $V_{\text{CELL}}$ is equivalent to the volume of a spherical cell $V_s$ minus the volumes truncated by the twelve neighbouring cells, $V_c$. That is,

$$V_{\text{CELL}} = V_s - 12V_c$$

(3.17)

The volume of the spherical cell is given as

$$V_s = \pi \frac{d^3}{6}$$

(3.18)

Following geometry of the truncated volume (see Figure 3.6), the truncated volumes can be expressed as

$$V_c = \left[ \frac{\pi}{6} \left( 3 \frac{w^2}{4} + h^2 \right) h \right]$$

(3.19)

Substituting the value of $h$ in Equation (3.18) and simplifying yields

$$V_c = \frac{\pi d^3}{48} \left[ 3k^2 + B^2 \right]$$

(3.20)

where $B = 1 - \sqrt{1 - k^2}$. Hence, substituting $V_s$ and $V_c$ in Equation (3.17) gives the expression for the volume of the cell as
By substituting the values of $s$ and $V_{\text{CELL}}$ in Equation (3.12) the specific surface area can be written as

$$S_v = \frac{12[1-6B+5k^2/2]s}{d[2-3B(3k^2+B^2)][1-s]} \quad (3.22)$$

Finally, substituting $S_v$ in Equation (3.3) gives the EEM model equation, expressed as

$$\Delta \rho \frac{L}{L} = \left[ \frac{12(1-6B+5k^2/2)}{(2-3B(3k^2+B^2))} \right] \rho_{\text{m}} \frac{\alpha u}{d^2s} + \frac{12(1-6B+5k^2/2) \gamma \alpha u^2}{(2-3B(3k^2+B^2)) \ ds^2} \quad (3.23)$$

The porosities of the ceramic foam filters provided for the validation of the mathematical models were calculated from their densities provided by the manufacturers. Also provided are their cell size and factors (with respect to the porosity) to determine their window size. However, the porosity factor is not always provided by the manufacturers, so it was desirable to use the physical scale model adopted to express the porosity as a function of the cell and window diameters.

The next subsection, therefore, describes the development of the expression relating the porosity to the cell and window size.
3.4.4 Expression for porosity $\varepsilon$

Following the assumption that the arrangement of the cells in the foam is a face centred cubic lattice, as suggested by Peng et al (2000), each cell is connected to twelve other cells.

The structure can be defined with fourteen cells arranged, such that the centre of eight of the cells is on the corners of a cubic unit and the other six cells are on the faces as demonstrated in Figure 3.7, when the cells are only touching. Hence, the cubic unit would contain eight $\frac{1}{8}$ volumes of cells and six hemispheres. When the cells are only touching, the volume of void in the cubic unit is

$$V_{\text{void}} = \left(\frac{8}{8} + \frac{6}{2}\right)\frac{\pi d^3}{6} = \frac{2}{3}\pi d^3$$  \hspace{1cm} (3.24)

where $d$ is the cell diameter.

Figure 3.7 Drawing of a face of a cube for a face centred cubic lattice with the cells touching, where $d$ is the cell diameter
By reducing the cubic space while the cell size remains constant, the cells cut each other creating windows of diameter $w$, thus, the volume of the void will decrease by the overlapping volumes, shown in Figure 3.8.

The truncated volume (see section 3.4.3, Equation (3.20)) is written as follows:

$$\text{The volume of one truncated portion} = \frac{\pi d^3}{48}(3k^2 + B^2)B$$  \hspace{1cm} (3.25)

where $k = \frac{w}{d}$ and $B = 1 - \sqrt{1-k^2}$.

In the unit cube, the 14 cells are in contact at 21 points. Hence, the total overlapping volume is the truncated portion multiplied by 42, i.e.:
Total truncated portions = \( \frac{21mL^3}{24}(3k^2 + B^2)B \) \hspace{1cm} (3.26)

The volume of the unit cube = \( a^3 = (d\sqrt{2(1-k^2)})^3 \) \hspace{1cm} (3.27)

Porosity is the ratio of the void volume to the volume of the cube, thus:

\[ \varepsilon = \frac{\pi(16-21B(3k^2 + B^2))}{24(\sqrt{2(1-k^2)})^3} \] \hspace{1cm} (3.28)

Therefore, for a given porosity the ratio of the window to cell size can be calculated, either graphically or by iteration. Figure 3.9 shows a graph of the ratio of cell to window size versus porosity in from Equation (3.28).

Figure 3.9 Graph of window/cell diameter vs. porosity of ceramic foam filter

From Figure 3.9, if the porosity is 85% and the cell size is given as 0.25 mm, the ratio \( w/d = 0.32 \). Hence, the window size is 0.08 mm. Following same
approach the window size of the validation samples can be determined. Window diameter determined from the above method is < 2% of the value provided by the ceramic foam manufacturers.

3.3.5 Summary of the development of the EEM Model

A new mathematical model has been developed for the prediction of pressure gradient of gelcast ceramic foam filters that requires the use of experimental data from filter samples to determine the values of the correction factors $\alpha$ and $\gamma$. The method for determining the correction coefficients is described in Chapter 5. Importantly, an expression for the determination of the foam window size has also been established.

3.4 CONCLUDING REMARKS

The original Ergun mathematical model is not suitable to predict the pressure gradients of gelcast ceramic foam filters in its traditional form. Hence, a new mathematical model has been developed to predict pressure gradients of clean gelcast ceramic foam filters. It was developed by adapting the Ergun model for packed columns (Ergun and Orning, 1949) to gelcast ceramic foam filters using a simplified physical scale model structure that represents the microstructure of the ceramic foam filter.

An expression has also been developed to estimate the window diameters of the gelcast ceramic foam since the foam manufacturers do not always provide such data.
The next chapter describes the calibration and validation of the Extended Ergun Mathematical (EEM) model for the prediction of pressure gradients of clean gelcast ceramic foam filters. The experimental rigs and procedures for the collection of data for the validation of the model are also presented.
CHAPTER 4 EXPERIMENTATION AND EEM MODEL VALIDATION

4.1 INTRODUCTION

Chapter 3 described a review of foam filter modelling and the development of a new mathematical model for the prediction of pressure gradients of clean ceramic foam filters.

This chapter describes the experimental procedures in determining the characteristics of cellular foam structures to validate the Extended Ergun Mathematical (EEM) model. The chapter also describes how reliable and consistent the experiments are, by collecting and comparing data from different foam samples of essentially identical physical properties. Furthermore, the effect of temperature on the results were considered and analysed. Finally, the validation of the model is presented, where it is compared to previous models using the experimental data.
The mathematical model for the prediction of pressure gradients can be rewritten as:

$$\frac{\Delta p}{L} = Au + Bu^2$$

(4.1)

where $A$ and $B$ are the constants that are characteristics of the filter's microstructure, $u$ is the superficial velocity and $\Delta p/L$ is the pressure gradient across the filter. Consequently, the data required are the pressure gradients across the filters with change in fluid flow. The temperatures and the absolute pressure of the fluid were also considered for the determination of the density of the fluid.

The objective of the experimentation is to study fluid flow through the ceramic foam filter samples and a physical scale model generic 'foam' matrix made using rapid manufacturing techniques. An experimental rig was constructed to measure pressure drops across the filter samples and the corresponding values of flow rates, temperatures and absolute pressure of the fluid. The filter samples were cylindrical discs of various thicknesses (lengths).

The following section describes the materials that were used in the experiments.

### 4.2 MATERIALS

The material for the experiments are nine sets of gelcast ceramic foam filters of various porosity and cell diameters ranging from 0.2 mm to 0.85 mm, one piece of Alumina foam with initially unknown parameters and a set of physical scale models of a generic cellular foam structure. The development
and production of the physical scale model foams are presented in detail later in this chapter.

4.2.1 Ceramic foam filters

Many methods of processing ceramic foams have been identified and the most common of them was patented by Schwarzwalder and Somers (1963). This method involved the filling (coating) of reticulate polyurethane or organic polymers (sponges) with aqueous slurry of ceramic, wetting agents, dispersion stabilizer and viscosity modifier. After drying in air at a temperature above 1000°C, the plastic vaporises and the ceramic particles sinter, forming open cell ceramic foam with a microstructure that either fits the void of the sponge, known as an image of the sponge, or fitting the structure of the sponge material. Figure 4.1 illustrates a foam structure showing the struts.

Figure 4.1 Microstructure of sintered foams made with fine RA45E Alumina showing structure of the struts (Peng et al, 2000)
This method of ceramic foam production has a disadvantage, which is, its tendency to leave either a hole or carbonaceous residue at the centre of the connecting structure known as the struts, which may result from pyrolysis of the polymer (sponge) frame. Pyrolysis is the chemical decomposition of the polyurethane by heat.

The type of ceramic foam filter substrate being studied here was produced by Hi-Por who are part of Dytech Corporation Ltd. The method of producing these foams is known as gelcasting. Hence, the foams are referred to as 'gelcast ceramic foams' (GCFs). The main components in the production of the gelcast ceramic foams are ceramic powder, organic monomers, dispersing agent, foaming agent and water. This manufacturing process relies on the gelation of an organic monomer to stabilise the foamed structure. Gelation occurs in solutions that contain long chain molecules that interact due to thermal or chemical change. Figure 4.2 shows a flow diagram of the manufacturing process of Hi-Por gelcast ceramic foams.

![Flowchart for production of Hi-Por ceramic foams using in situ polymerisation of monomers for stabilisation](image_url)

Figure 4.2 Flowchart for production of Hi-Por ceramic foams using *in situ* polymerisation of monomers for stabilisation
Gelcasting involves the foaming, either by mechanical frothing or by injection of gases into a ceramic suspension made up from the ceramic powder, water and dispersants. An \textit{in situ} polymerisation of the organic monomer promotes the setting of the suspension. After polymerisation, the specimens are cooled to room temperature before removal from the mould, dried and sintered. In ceramics, the binders help to hold the powder particles together in a desired shape until heat treatment forms a permanent strong bond.

The gelcasting method, when compared to other foam production methods, provides a substrate with a better strength and a wider choice of macro/micro structure (Peng \textit{et al.}, 2000). There are four distinct structures within foams, namely, the cells, windows, struts and the foam microstructure. A typical microstructure of high purity alumina foam is shown in Figure 4.3 indicating the cell, windows and the struts.

![Figure 4.3 A typical micrograph of a 30\% dense alumina foam (Azom, 2005)](image_url)
Chapter 4 Experimentation and EEM model validation

The foam cells are formed from the introduction of gaseous phase in the ceramic suspension. The slurry drains from the point of contact of the cells as they touch each other towards the struts. The struts are the building blocks of the foam structure. Since the slurry drained away from the point of contact of the cells, there exists a thin film of liquid which eventually collapses during the gelling and binder burnout process leaving connecting holes known as windows. The cells and window sizes can be manipulated with respect to various factors including the amount of gas introduced.

It can be seen that gelcast ceramic foam filters consist of almost spherical cells (pores) connected by the windows. The relative density of the foam filter is defined as the ratio of the volume of the alumina to the total volume of the foam. The densities of the foam filter samples used in this study range from 12 to 20% and average cell sizes from 200 to 850 μm. The porosities are, therefore, very high and comparable to fibrous filters. The porosities of the filters are calculated from their relative densities provided by the manufacturers (porosity is equal to 1 minus the relative density). Figure 4.4 shows examples of some ceramic foam structures that can be produced.

Figure 4.4 Ceramic foam filters (adapted from Research and Production, 2003)
Two series of gelcast foam filter samples, namely 243AL and A44C, were provided for the study. The 243AI samples were in sets of two with 10 mm thickness (length) and the A44C samples were in sets of five pieces with 5 mm thickness. Although, the external dimensions of the ceramic foam samples were provided by the manufacturers, each sample was measured accurately with callipers before the experiments because of possible wear, and these dimensions were used in all the analyses.

A scanning electron microscope (SEM) was used to determine the cell and window size of the alumina foam sample. These parameters were not provided by the manufacturers. The next subsection describes the procedure.

4.2.2 Determination of cell size of ceramic alumina foam

The ceramic alumina foam with bulk density of 20%, external diameter of 49.94 mm and length of 45.18 mm with unknown cell diameter was investigated. Hence, an SEM was used to scan some pieces of the samples.

The SEM consists of a column and chamber, vacuum system and a control console. A chamber was used to generate and focus a fine beam of electrons upon the specimen mounted in the specimen chamber. The electron gun has a thermionic emitter as a source of electrons. Three electromagnetic lenses beneath the gun focus and shape the electron beam before it strikes the specimen in a scanned fashion. The energy of the electron beam is adjusted from 300 V to 30 kV in 10 V steps and the electron beam current is continuously adjusted from 1 pA to 1 μA to suit the type of examination in progress. The specimen chamber of the unit holds the specimen to be viewed in such a way that it may be freely manoeuvred during examination. The
stage may be opened for examination once the vacuum within the chamber has been released. Figure 4.5 shows the major components of a SEM.

Figure 4.5 Major components of SEM

The operation of the electron optical column and specimen scanning is dependent not only on the presence of a vacuum, but also the degree of vacuum. The system cannot be switched on any of the high voltages until an adequate vacuum is reached.

An SEM is the best known and most widely-used of the surface analytical techniques. High resolution images of surface topography, with excellent depth of field are produced using a highly-focused, scanning (primary) electron beam.
The vacuum system consists of a turbo-molecular pump, backed by a rotary pump, and is mounted on the lower face of the specimen chamber. It commences pumping when the system is being evacuated and comes to rest during system venting. The normal operating vacuum should be in the range from $10^{-4}$ to $10^{-6}$ torr.

The operation of the instrument is from a desktop computer, using the mouse and keyboard. Within the console of the microscope, the computer is used to communicate the operations and the commands into actions by the SEM. The PC environment used was Microsoft Window.

The scanned samples were 3 mm thick with average diameter of 10 mm. Figure 4.6 shows one of the images produced from the scan. The cells were measured on the scanned images to a precision of 0.5 mm.

![SEM image of alumina ceramic foam with a theoretical density of 20%](image)

**Figure 4.6** SEM image of alumina ceramic foam with a theoretical density of 20%
The real diameter is determined by multiplying the measured values with the scales indicated on the scans. Since the cells are generally spherical the mean diameter from the measurement gives a good approximation of the foam cell diameters.

However, most of the windows in the scanned images were slanting; thus, the window size used in the analysis were calculated with the relationship provided by the manufacturer \( w = 0.26d \) for a foam with a theoretical density of 20\%. The distribution represented approximately 500 cells, where the average cell diameter is 261 \( \mu m \) with a standard deviation of 108 \( \mu m \). Figure 4.7 is the cell size distribution graph for the new alumina foam sample.

![Cell size distribution graph](image-url)

**Figure 4.7** Example cell size distribution for the new alumina foam sample with a porosity of 80\%.

Table 4.1 presents the parameters of the samples used in the experiments.
<table>
<thead>
<tr>
<th>Type of filter</th>
<th>Cell size, ( d ) (mm)</th>
<th>Window size, ( w ) (mm)</th>
<th>Porosity, ( \varepsilon ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>243AL-E</td>
<td>0.75</td>
<td>0.233</td>
<td>88</td>
</tr>
<tr>
<td>243AL-F</td>
<td>0.50</td>
<td>0.152</td>
<td>87</td>
</tr>
<tr>
<td>243AL-G</td>
<td>0.20</td>
<td>0.085</td>
<td>86</td>
</tr>
<tr>
<td>A44C7</td>
<td>0.85</td>
<td>0.221</td>
<td>80</td>
</tr>
<tr>
<td>A44C11</td>
<td>0.25</td>
<td>0.065</td>
<td>80</td>
</tr>
<tr>
<td>A44C6</td>
<td>0.25</td>
<td>0.075</td>
<td>86</td>
</tr>
<tr>
<td>A44C4</td>
<td>0.35</td>
<td>0.105</td>
<td>86</td>
</tr>
<tr>
<td>A44C1</td>
<td>0.75</td>
<td>0.277</td>
<td>88</td>
</tr>
<tr>
<td>A44C10</td>
<td>0.40</td>
<td>0.092</td>
<td>80</td>
</tr>
<tr>
<td>Alumina foam</td>
<td>0.261</td>
<td>0.068</td>
<td>80</td>
</tr>
</tbody>
</table>

Table 4.1 Cellular foam filter samples and their parameters

4.3 EXPERIMENTAL FLOW RIG

The experimental flow rig consisted of a filter sample holder with its connecting pipes, a flow meter, instrumentation for temperatures and pressure measurement and an air blower. The filter holder was constructed from stainless steel with an internal diameter of 60 mm and length of 850 mm connected to the air blower via the flow meter for the measurement of the fluid flow rate (see Figure 4.8).

The holder length, which was more than ten times the internal diameter of the pipe, was chosen in order to allow fluid flow to be fully developed at the time it gets to the pressure measurement point. The holder had an opening fitted with a cover for quick mounting of the foam samples. The holder had two pressure tappings placed such that one tap was approximately 200 mm upstream of the filter opening and the second tap was 50 mm downstream of the filter sample when mounted. Digital pressure gauge (differential pressure transmitter, see specification in Appendix I) was mounted across the filter using the two taps, ranging from 0-2500 Pa to 0-10,000 Pa depending on the maximum value of pressure drop across the filter, observing that the
measurement of pressure difference can be as low as 20 Pa. The range of the pressure gauge determines the resolution of the equipment.

A T-type thermocouple connected to a microprocessor thermometer was fitted before the filter sample to measure air temperatures corresponding to each pressure drop reading. The temperatures are read from the Comark 6200 10-channel microprocessor thermometer (Appendix I). In order to calculate the density of the air flowing through the filter, it was necessary to measure the absolute pressure of the air in the pipe. Hence, the absolute pressure corresponding to each pressure drop reading was measured by disconnecting the air lead of the pressure gauge from the downstream pressure tap. Using the values of the absolute pressure and the temperature the density of the air was calculated from the expression derived from the ideal gas law, written as:

\[ \rho = \frac{P}{RT} \]  

(4.2)

where \( \rho \) is the air density in kg m\(^{-3}\), \( P \) is the pressure in Pa, \( R \) is the gas constant equal to 287 J kg\(^{-1}\) K\(^{-1}\) and \( T \) is the temperature in K. A digital
pressure gauge was also used for the measurement of the pressure drops across the orifice flow plate.

The air flow rate was measured with an orifice plate meter. Orifice plates are the most common and simplest form of flow meters. The orifice plate (a type of differential pressure flow meter) is used to measure fluid flow by introducing a constriction in the flow. The pressure drop caused by the constriction is correlated to the flow rate using Bernoulli’s theorem. The orifice plate flow meter normally comprises a plate with a bore clamped between flanges. The pressure taps are usually placed either on the flanges or on the pipe. Specifications for the production of the orifice plate flow meter used in this research work are reported in the British Standards (2003). Figure 4.9 shows a schematic of an orifice plate set up.

![Schematic diagram of orifice plate flow meter](image)

Figure 4.9 Schematic diagram of orifice plate flow meter
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Experimentation and EEM model validation

The relationship between flow rate and pressure difference is determined by the Bernoulli equation, assuming that work and heat transfer are negligible and the flow is horizontal. The relationship between $P_1$ and $P_2$ is written as

$$\frac{P_1}{\rho g} + \frac{v_1^2}{2g} = \frac{P_2}{\rho g} + \frac{v_2^2}{2g} + \sum f$$

(4.3)

where $\sum f$ is total frictional loss that is assumed negligible, $\rho$ is the fluid density in kg m$^{-3}$, $g$ is the acceleration due to gravity in m s$^{-2}$ and $v$ is the fluid velocity in m s$^{-1}$. This equation can be simplified and rearranged to give

$$\frac{\Delta P}{\rho} = \frac{1}{2}(v_2^2 - v_1^2)$$

(4.4)

Using the Continuity Principle, the mass flow rate at 1 and 2 must be the same, i.e., assuming that the fluid is incompressible (density of fluid is constant), the flow rate $Q$ is given as

$$Q = v_1 A_1 = v_2 A_2$$

(4.5)

where $A_1$ and $A_2$ are the cross sectional areas of the fluid at 1 and 2. Hence, assuming that $A_2 < A_1$, solving Equation (4.4) and (4.5) in terms of $v_1$ yields an ideal equation,

$$v_2 = \sqrt{\frac{2\Delta P}{\rho (1 - \frac{A_2^2}{A_1^2})}}$$

or

$$Q = A_2 \sqrt{\frac{2\Delta P}{\rho (1 - \frac{A_2^2}{A_1^2})}}$$

(4.6)
Since $A_2$ is complex and difficult to measure, the ideal equation can be modified with a discharge coefficient, which is a function of the orifice opening and the non dimensional parameter Reynolds Number ($Re$). Replacing the areas with the diameters, Equation (4.6) can be written as

$$\rho Q = C_D \frac{\pi}{4} D_o^2 \sqrt{\frac{2 \rho \Delta P}{(1 - \left(\frac{D_o}{D_i}\right)^4)}}$$

(4.7)

where $Q$ is the flow rate ($m^3 s^{-1}$), $C_D$ is the orifice plate discharge coefficient, $D_o$ is the orifice diameter, $\rho$ is the fluid density, $D_i$ is the pipe diameter and $\Delta P$ is the pressure drop across the orifice plate. A number of discharge coefficient equations are used in different standards. The equation used in this work is recommended by the British Standards-ISO 5167 (2003), which is the Reader-Harris/Galagher equation; see Appendix III.4. The discharge coefficient is a function of the Reynolds number while the Reynolds number is a function of the flow rate which is computed using the discharge coefficient value. The discharge coefficient was considered constant for each orifice bore since the range of velocity of fluid flowing across the foam filter samples is small (0.7 to 2.0 m s$^{-1}$) and the variation in the coefficients is small.

The fabrication and installation of the flow meter was according to the British Standard-ISO 5167 (2003). In order to measure the pressure drop across the orifice plate, two pressure taps are placed such that one is 50 mm (one pipe diameter) upstream and the second is 25 mm (half pipe diameter) downstream of the orifice plate. Initially, the bore diameter of the orifice plate installed was 20 mm. However, due to a significant reduction in flow as the pressure drop increase with filter length, a new orifice plate of 15 mm was installed. The pipe diameter is the minimum recommended size of 50 mm by the British Standards-ISO 5167 (2003). Using the Reader-Harris/Gallagher
Equation, the values of the orifice discharge coefficients ($C_D$) corresponding to the 15 mm and 20 mm plates are 0.6285 and 0.6334 respectively.

The air flow through the filter samples was generated by a "Leister Robust" blower (see specification in Appendix I). Between the blower and the flow meter was installed a flow conditioner, which straightened the swirling air flow and reduced the pulsating of pressure across the orifice plate. The distance from the conditioner to the orifice plate was more than ten times the pipe diameter to allow the full development of the fluid flow before the orifice plate.

Figure 4.10 shows the setup of the experimental rig.
Other material requirements included a gasket that was malleable so as not to damage the samples during mounting and dismantling. A GW304 silk ribbon gasket of 3 mm thick was used around the samples, while a 3M Interam™ Mat sheet (which is commonly used to mount production DPFs and honeycomb catalysts in exhaust system canisters) was used on the covers to prevent air leaks.

### 4.4 EXPERIMENTAL PROCEDURE

The experiments consisted of measuring the pressure drop across various foam samples as the flow rate is varied. Each sample being tested was mounted, packed round with ribbon gasket, making sure that the faces of the samples were not covered. The filter cover was fastened over the samples and the sealing between the cover and the casing was with the 3M Interam™ mat sheet gasket. Air leakage through the cover was checked with soap bubbles before taking the readings for each sample. The process of mounting the samples is very important because a leakage between the filter samples and the casing will give a false pressure drop. In order to allay fears of making such errors, each sample was mounted and dismantled at least three successive times to establish consistency of results.

The air flow rate was varied with a butterfly valve connected to the blower. While varying the fluid flow through the sample, the pressure drop across the orifice plate and the filter sample was measured with the differential pressure transducer (same type for the measurement of pressure drop across the filter samples, Appendix I). Simultaneously, the temperature and the absolute pressure of the air before the filter were measured, which were required for the calculation of the density of the air for a given pressure drop and flow rate.
Besides the mounting and dismantling exercise on each sample, experiments were repeated three times on each sample, including the alumina foam sample to establish repeatability (see example in Appendix II). The average temperature recorded through the experiments was 28°C. Some examples of measurements and analysis from the experiments are shown in tabular form in Appendix III.

The pressure drop across the orifice plate was used in Equation 4.7 to determine the flow rates of air through the filter samples. The densities corresponding to each flow rate were calculated from Equation 4.2, applying the temperatures and absolute pressure data.

Results from the experiments and the subsequent analysis were used to plot graphs of pressure gradient versus mass flow rate for each sample and was used for the validation of the mathematical model.

The next section describes the reliability of the experiments on the experimental rig used in the present work to ensure good results and graphs profile with the effects of temperature and cell diameter change.

4.5 RELIABILITY OF THE EXPERIMENTAL RIG AND GRAPHS PROFILE

In order to ensure repeatability, data collected from ceramic foams of same parameters at different times were compared. Furthermore, the effect of temperature variation on the fluid flow is analysed to determine the variable used for results presentation. Finally, the profile of the graphs was presented using fluid flow experimental data.
4.5.1 Reliability of experimental rig

The aim of these experiments is to establish the consistency of the experimental data on the filter samples for a given microstructure and parameters. It is well known that the pressure drop across a filter is directly proportional to the filter length, i.e., the pressure gradient for a given microstructure is constant. Hence, five pieces of filter disks of diameter 55 mm and thickness 5 mm of the same microstructure (A44C7) were arranged and tested in such combination that the lengths are 10, 15, 20 and 25 mm. The pressure drops were measured with their corresponding flow rates for each given length of ceramic foam disk.

The experiments on each filter length were repeated three times to observe consistency in the results obtained. The pressure gradients of each set were plotted against their corresponding flow rates in the same graph.

Figure 4.11 shows graph of pressure gradient versus the fluid flow rate. It can be seen that the curves generated from the different lengths lie closely together. Result demonstrates that the experiments are consistent on the experimental rig used in this present work.

The slight deviation observed from the curve plotted from the experimental data of the 10 mm test sample is due to problem encountered when fitting gaskets and the sample cover, where the sample diameter may be reduced, thus, the increase in pressure drop.
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4.5.2 Effect of temperature and graph variables

Although, the experiments were performed with temperature variation of < 10ºC it was necessary to study the effects of change in temperature on the experimental results and determine the variable(s) used in comparing the results. It is evident that temperature is inversely proportional to the density of the fluid and since the pressure gradient is predominantly a function of the kinetic energy loss, it will decrease with decrease in the fluid density. The experiments entail measurement of pressure drops and flow rates of a given filter sample within two ranges of temperatures.

The experimental data at the two temperatures were compared using the relationship of pressure gradients versus velocity on one hand and pressure...
gradients versus mass flow rate on the other hand. The material for these experiments was a gelcast ceramic foam sample of porosity 80% and cell diameter of 0.85 mm (A44C7). The 2 temperatures considered are 25°C and 128°C. The pressure drops across the filter sample were measured with their corresponding flow rates and then, the pressure gradients were plotted against the fluid flow velocity and also against the flow rate on a different graph, since the scale of the abscises are different. Figure 4.12 shows the graph of pressure gradients in kPa m⁻¹ versus fluid velocity in m s⁻¹.

![Graph of pressure gradient vs. air velocity of ceramic foam filter, A44C7, comparing curves from data collected in different temperatures](image)

Figure 4.12 Graph of pressure gradient vs. air velocity of ceramic foam filter, A44C7, comparing curves from data collected in different temperatures

It can be seen that the difference in pressure gradients with temperature is clearly shown on the graph. The graph shows that the pressure gradients decrease with an increase in temperature. In this figure, it can be seen that the only convenient method to compare different foam samples is to maintain the same temperature during the experimentation. However, since this requires
additional instrumentation the effect of the temperature could be included in the graphical presentation by applying the air densities as they vary with the temperature.

In computing the experimental data to calculate the mass flow rate of the fluid, the densities were determined from the ideal gas law, \( \rho = \frac{P}{RT} \), where \( R \) is the universal gas constant, \( T \) is the absolute temperature and \( P \) is the gas pressure. The mass flow rate is the product of the volumetric flow rate and the fluid density.

The relationship between the pressure gradient and the fluid mass flow rate is demonstrated by the graph in Figure 4.13, where it can be seen that the two graphs corresponding to the two temperatures lie on the same curve.

![Graph of pressure gradient vs. fluid flow rate of ceramic foam filter, A44C7, comparing curves from data collected from different temperatures](image)

Figure 4.13 Graph of pressure gradient vs. fluid flow rate of ceramic foam filter, A44C7, comparing curves from data collected from different temperatures
In summary, the effect of temperature on the experimental results cannot be ignored since the temperatures cannot be controlled without additional instrumentation during the period of experimentation. Hence, analysing the results using the graph of pressure gradients versus the mass flow rates accounts for the temperature variations and presents a consistent comparison of results.

4.5.3 Profile of pressure gradient vs. velocity graphs with foam cell diameter

As mentioned earlier, fluid flow pressure gradient through porous media was defined using the Darcy law which states that the fluid flow rate is directly proportional to the pressure gradient (Daily and Harleman, 1966). This definition holds so long as the velocities are low and the kinetic energy loss is not significant. In this case the profile of the graph of pressure gradient vs. fluid flow rate is linear. However, the velocities of fluid flow in diesel particulate filters are generally high. Hence, the pressure gradient will have a quadratic term as a result of the increase in kinetic energy loss.

The porous medium can be defined with its porosity and the pore size (cell size). As the cell size decreases, resistance to fluid flow increases, thus there is an increase in pressure drop. Data were collected from the experimental rig used for this work to study the profile of pressure gradient versus fluid flow velocity graphs and the effect of cell diameter on the pressure gradients.

Figure 4.14 shows the graph of pressure gradient versus air mass flow rate from 4 different cell diameters. The curves demonstrate the quadratic correlation between the pressure gradient and the fluid flow rate which can be written in the form:
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\[
\frac{dp}{L} = aQ + bQ^2
\]  

(4.8)

where \( \frac{dp}{L} \) is the pressure gradient and \( Q \) is the fluid mass flow rate, and the factors \( a \) and \( b \) are constants. It can also be seen from the graph in Figure 4.14 that the decrease in cell diameter increases the pressure gradients.

Figure 4.14 Graph of pressure gradient vs. fluid flow rate, demonstrating the quadratic profile of pressure gradients of GCF filters. Average temperature of air entering filter = 30\(^0\)C
4.5.4 Summary of reliability of the experimental rig and graphs profile

The data collection from the fluid flow rig described in this chapter was consistent and the method of comparison used in the research accounts for the effect of temperature variation during experimentation. Hence, the main variables for analysing the proposed model and experimental data are the pressure gradients and the fluid mass flow rate.

It can also be seen that the pressure gradients increase with the decrease in cell diameter. Hence, the need to determine an optimum cell diameter for the design and development of DPFs to maintain low engine exhaust back pressure.

Having established the reliability of the data collection from the experimental rig, the following section describes the calibration and validation of the proposed mathematical model for the prediction of pressure gradients of clean gelcast ceramic foam filters.

4.6 CALIBRATION AND VALIDATION OF MATHEMATICAL MODEL

Model calibration is the process by which the parameters that characterise the model's coefficients are determined. In other words, it is the first stage of tuning or testing of the model to a set of field data not used in the original development of the model. On the other hand, model validation is either comparing its prediction with another model of the same system or just
determining how well it can represent the real system. The model validation in this thesis is achieved by comparing the model to experimental data from real gelcast ceramic foam filter samples.

Every model must be calibrated and validated before it will become a useful tool for predicting the behaviour of a considered system.

The new mathematical model (EEM model) was calibrated using fluid flow data from a generic physical scale cellular foam model of a well defined cell and window structure. The physical scale model foam was a reproduction of the conceptual model used for the development of the EEM model. The cells were spherical, connected through holes (windows) made from the intersections of the cells. The physical scale model foams are such that they can be fitted into the filter holders of the experimental rig like the real ceramic foams.

The next subsection describes the development of the generic physical scale model cellular foam structure.

### 4.6.1 Development of physical scale cellular foam model

A number of design options in the development of the physical scale foam model were considered for different cell diameters and they were produced using rapid manufacturing. Rapid manufacturing refers to a class of technologies that can automatically produce physical models from computer aided design (CAD) data. The aim was to produce a physical scale model that is similar to the microstructure of the real foam with practically the same cell diameters. However, the manufacturing method adopted for these cellular
foams has some limitations in producing the cell size, which is highlighted below.

The rapid manufacturing technique used for the production of the cellular foam model was stereolithography (SL). The SL technique is based on the process of photo-polymerisation, in which a liquid resin is converted to a solid polymer on exposure to computer controlled ultraviolet laser radiation (Jacob, 1992). The photopolymer is selectively cured on a layer by layer (additive) basis where the cured area corresponds to the desired cross section of the required shaped article to be formed. Figure 4.15 is a schematic illustrating the stereolithography manufacturing process.

Figure 4.15 Schematic diagram of stereolithography
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The solidified layer is lowered by the amount of the required layer thickness and a recoating blade moves over the surface to apply a new layer of resin. The process is repeated until a model of the required shape is finished. On completion of the build, the model is post-cured under high intensity ultraviolet radiation to complete the curing process.

In 2004, stereolithography was the most widely used of all the rapid manufacturing process with high accuracy. However, this technique will not produce the fine tolerances that could be obtained from finish machining or precision grinding. The layering process produces minute steps on slanting surfaces and the smallest cross section that can be generated is equal to the diameter of the Laser beam (Gettelman, 1989). Gettelman (1989) reported that the layer from each step of reproduction measure from 0.125 mm to 0.75 mm in thickness. Consequently, production of small cells (of the order of 0.85 mm) will result to poor finishing and difficulty of clearing chaffs inside the product. Figure 4.16 illustrates the finishing of a spherical surface of a produce from SL.

![Figure 4.16 Schematic drawing finishing on spherical surface of product of SL technique](image)
Therefore, the minimum cell diameter of the cellular foam model considered for production was 10 times the maximum ceramic foam cell size being studied. As mentioned earlier, the model structure is face centred with each cell attached to 12 neighbouring cells. The porosity of the cellular model foam was calculated using the design dimensions.

Figure 4.17 is a computer aided design of the physical model scale of a cellular foam filter.

Figure 4.17 Drawing of physical scale model of cellular foam filter manufactured using stereolithography

Six cylindrical physical model foams with the same parameters were produced: external diameters of 60 mm, lengths of 25 mm, cell diameter of 7 mm and window diameter of 2.6 mm. Another sample set that was designed and produced were of dimensions; external diameter of 60 mm, length of 100
mm and four rows of connected cells across the filter with different cell/window diameters. The use of the latter sample set is described in Chapter 5.

Figure 4.18 shows some of the physical scale cellular model that was produced using the rapid manufacturing technique.

![Physical scale model cellular foam filters](image)

**Figure 4.18 Physical scale model cellular foam filters manufactured using stereolithography**

Having developed the physical scale foam models, fluid flow experiments were performed on them and the data were used to calibrate the EEM model.
4.6.2 Porosity of the physical scale model foam

In conformity with the model derivation, the porosity of the physical scale foam model was calculated, using the porosity graph developed in Chapter 3. Given that the window diameter was 2.6 mm and the cell diameter was 7.0 mm, the ratio of the window to the cell diameter was 0.37. Hence, the corresponding porosity value was 89%. However, the cells in the physical scale model foam are generated within a diameter of 53.6 mm, where the model foam diameter is 60 mm. Therefore, the effective porosity of the foam model $\varepsilon_m$ is 80%.

4.6.3 Calibration of the (EEM) model

The EEM model was calibrated using the physical scale model foam filter. The correction coefficients $\alpha$ and $\gamma$ were tuned until the curve produced with the EEM model has a close fit with the Graph plotted from the experimental data. The values of the correction coefficients corresponding to the fit are the same that calibrate the mathematical model.

Figure 4.19 shows a graph of pressure gradient versus fluid mass flow rate where the curve plotted with the mathematical model is tuned to fit the curve plotted with the experimental data from the physical scale foam model. The correction coefficients from the graphs are: viscous correction coefficient $\alpha$ is 5 and the kinetic correction coefficient $\gamma$ is 0.45.

Substituting the values of the coefficients $\alpha$ and $\gamma$, the EEM model can be rewritten as

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\[
\frac{\Delta p}{L} = \left\{ \frac{12(1-6B+5k^2/2)}{5\mu} \frac{5\mu}{d^2 e} + \frac{12\mu(1-6B+5k^2/2)}{(2-3B(3k^2+B^2))} \frac{0.45}{ds^2} \right\} \tag{4.9}
\]

4.6.3 Validation of the EEM model

Having calibrated the EEM model, it was then validated using the experimental data from the real ceramic foam filter samples. This is now described.

Figure 4.19 Graph of pressure gradient vs. fluid flow rate of a physical scale model foam filter sample, for the calibration of the EEM model, where \(\alpha = 5\) and \(\gamma = 0.45\)

4.6.3 Validation of the EEM model

The process of validation entailed comparing the results from the calibrated mathematical model to the measured data from the real ceramic foam filter samples. Figure 4.20 shows graph of pressure gradient versus air mass flow.
rate, comparing the curves from the calibrated mathematical model to the
curves from the real foam filter samples experimental data.

It can be seen that the calibrated mathematical model with the kinetic
correction coefficient of 0.45 and viscous correction coefficient of 5 is < 50 % of
the experimental data, similar to an earlier report by MacDonald et al (1979)
on tightly packed grains. They also reported that the error reduced to
±15% when their model was applied on porous medium with spherical
grains, indicating that the model cannot be generalised for more accurate
result. In this case, the calibrated EEM model is not suitable for predicting
pressure gradients of clean gelcast ceramic foam filters if more accurate
results are required.

![Graph of pressure drop gradient vs. fluid flow rate through samples of ceramic foams for different foam cell diameters (d), comparing experimental data to EEM](image)

Figure 4. 20 Graph of pressure drop gradient vs. fluid flow rate through samples of ceramic foams for different foam cell diameters (d), comparing experimental data to EEM
Since the calibrated model is not suitable for accurately predicting the pressure gradients of gelcast ceramic foam filters, as illustrated and discussed above, the model was recalibrated using gelcast ceramic foam filter samples. The correction coefficients were tuned such that the curves closely fit the curves from the experimental data of the real foams. Each curve fit corresponds to a given pair of correction coefficients.

Similarly, the models discussed in Chapter 3, Ergun’s model, Underwood model and Richardson’s model were fitted to the experimental data by tuning the correction coefficients. Table 4.2 presents the summary of the results showing coefficients from the Ergun’s model (Ergun and Orning, 1949), Underwood model (See Richardson et al, 2000) and Richardson model (Richardson et al, 2000), with variations in both the viscous and kinetic correction coefficients.

<table>
<thead>
<tr>
<th>Filter samples</th>
<th>Ergun’s model</th>
<th>Underwood model</th>
<th>Richardson model</th>
<th>Present model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\gamma$</td>
<td>$\alpha$</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>A44C7</td>
<td>5.0</td>
<td>3.1</td>
<td>3.0</td>
<td>2.5</td>
</tr>
<tr>
<td>A44C11</td>
<td>5.0</td>
<td>5.0</td>
<td>3.0</td>
<td>4.0</td>
</tr>
<tr>
<td>A44C6</td>
<td>5.0</td>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 4.2 Viscous ($\alpha$) and kinetic ($\gamma$) correction coefficients used in various models

The range of the values of the correction coefficients for the new mathematical model is narrow when compared to the other three models. The best results using fixed correction coefficients for the Ergun, Underwood and Richardson model to predict the pressure gradients of the gelcast foam filters has an accuracy of $< 62\%$. Hence, from the narrower range of values of correction coefficients for the new model, the prediction error is less.
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However, before attempting to introduce constant coefficients that could be applied in general to gelcast ceramic foam filters, it was necessary to evaluate the effect of variation of porosity or the cell diameter on the correction factors using the modelled correlation. Hence, two samples with different porosities and cell diameters (A44C7 and A44C6) were studied.

The porosities of the samples were varied after fixing the cell diameters and the corresponding correction coefficients were determined by fitting the model to the experimental data. The effect of varying the porosity on the coefficients is demonstrated by Figure 4.21 showing the graph of the viscous and inertial correction coefficients versus the porosity. The graph shows that the inertial correction coefficient is proportional to the porosity while the viscous correction coefficient is independent. This implies that the accuracy of the value of the porosity will mainly affect the inertial coefficient.

Figure 4.21 Graph of correction coefficient (viscous and inertial) vs. porosity (%), on samples A44C7 and A44C6, average cell diameter of 0.85 mm and 0.25 mm respectively
Secondly, the cell diameters of the two samples were varied after fixing the porosity and the corresponding correction coefficients were determined by fitting the model to the experimental data. The window diameters corresponding to the new cell diameters were also calculated. These results are demonstrated by Figure 4.22 which shows a graph of the viscous and inertial correction coefficients versus the cell diameter. The filters were A44C7 and A44C6 with diameters of $0.25 \pm 0.5$ and $0.85 \pm 0.5$ mm respectively.

![Graph of correction coefficients (viscous and inertial) vs. cell diameter on samples A44C7 and A44C6, average cell diameter of 0.85 mm and 0.25 mm respectively](image)

This illustrates that the two correction coefficients strongly depend on the cell diameter as the cell size diminishes. The change in the inertial correction coefficient is markedly steep across the cell size range. However, the viscous coefficient is less dependent on slight variation in cell size.
It can be, therefore, concluded that the EEM model is very sensitive to variation in the filter parameters.

The next subsection describes the recalibration of the EEM model using experimental data from the real gelcast ceramic foam filter samples.

4.6.4 Recalibration of EEM model

The EEM model was recalibrated using the experimental data from the gelcast ceramic foam filter samples. A pair of correction coefficients of the EEM are tuned until the corresponding curve closely fit the curves generated from the experimental data of the gelcast ceramic foam filter samples. In this investigation, the error is reduced to $<30\%$ when the viscous coefficient is 5 and the kinetic coefficient is 1.7. Figures 4.23, 4.24 and 4.25 illustrate the results obtained after recalibrating the EEM using the real ceramic foams. Appendix III.2 describes the error calculation.

![Graph of pressure gradient vs. fluid flow rate of samples of ceramic foams, comparing experimental data to the EEM](image)

Figure 4. 23 Graph of pressure gradient vs. fluid flow rate of samples of ceramic foams, comparing experimental data to the EEM
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New Alumina foam filter, cell diameter = 0.261 mm

A44C6, cell diameter = 0.25 mm

Figure 4.24 Graph of pressure gradient vs. fluid flow rate of samples of ceramic foams, comparing experimental data to the EEM model

243AL-F, cell diameter = 0.5 mm

243AL-E, cell diameter = 0.75 mm

Figure 4.25 Graph of pressure gradient vs. fluid flow rate through samples of ceramic foams, comparing experimental data to the EEM model
4.6.5 Summary of Extended Ergun Mathematical (EEM) model

The summary from the EEM model are as follows:

This chapter has presented the development of a new mathematical model referred to as Extended Ergun Mathematical (EEM) model, for the prediction of pressure gradients of clean gelcast ceramic foam filters, with an error of < 30%. The viscous and inertial terms of the model were multiplied by correction coefficients of 5.0 and 1.7 respectively.

However, initial effort to calibrate the model using a physical scale foam model resulted in a model with an accuracy of < 50%. The correction coefficients obtained, viscous and kinetic losses are 5 and 0.45 respectively.

The EEM, showed less variation in the correction coefficients compared to previous models. Furthermore, it was demonstrated that measurement errors on the porosity and cell size largely influence the inertial correction factor. Hence, more accurate measurement of the dimensions of the parameters may increase the accuracy of the model.

4.7 CLOSING REMARKS

This chapter has presented the calibration and validation process of the Extended Ergun Mathematical (EEM) model. The calibrated EEM can be used to predict the pressure gradients of gelcast ceramic foam with an accuracy of about 50%. It was also demonstrated that by tuning the correction coefficients of the EEM model, using real ceramic foam filter samples yielded a model that can predict the pressure gradients with an error of < 30%. It is necessary
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to further increase the accuracy of the model, however, this is as far as the EEM model can probably be developed.

Furthermore, since the correction coefficients of the EEM model require tuning on real foam samples and there is possibility of improving the accuracy for the prediction of pressure gradients of ceramic foam filters, a new modelling approach using the basic fluid flow theory was considered necessary.

Chapter 5 presents the development, calibration and validation of the new mathematical model referred to as the Multiple Orifice Mathematical (MOM) model, which seeks to overcome the limitations of the EEM model.
CHAPTER 5 MULTIPLE ORIFICE MATHEMATICAL MODEL

5.1 INTRODUCTION

Chapter 4 described the fluid flow rig and experimental procedures for collecting data used in the calibration and validation of the Extended Ergun Mathematical (EEM) model. The model calibration and validation were then described where it was demonstrated that calibrated EEM model from physical scale model foam data can predict the pressure gradients of gelcast ceramic foam with an accuracy of only 50%. However, it was further demonstrated that the EEM model can be improved by tuning the correction coefficients using real gelcast ceramic foam samples, reducing the error to < 30%.

This chapter presents the development, calibration and validation of another new model referred to as the Multiple Orifice Mathematical (MOM) model, to predict pressure gradients of gelcast ceramic foam filters with an error < 25%.
Importantly, this new model only requires calibration using experimental data from the physical scale model foam first described in Chapter 4; i.e. it does not require calibration using real foam samples.

The next section describes the development of the MOM model.

5.2 DEVELOPMENT OF THE MULTIPLE ORIFICE MATHEMATICAL (MOM) MODEL

In Chapter 3, several conceptual models to represent the cellular foam filter structure were described. The choice of conceptual model that is considered representative of the foam structure in this thesis is the face centred lattice arrangement of spherical cells. Such arrangement can also be described as rows of cells connected through the windows running across the filter length. Assuming that the fluid flow across the filter is unidirectional, a single row of cells can, therefore, be used as conceptual model of the foam filter.

The MOM model for the prediction of pressure gradients of clean gelcast foam filters was developed by analysing the fluid flow through a single row of cells across the length of the filter. The fluid flow through the filter continuously experiences contraction and expansion due to the alternating arrangement of windows and cells, like fluid flow through a series of constricted tubes or orifices. As fluid passes through the windows, the velocity and the pressure of the fluid vary. The loss across these orifices is mainly kinetic.
As previously stated, for high velocities the total pressure drop is the sum of viscous energy loss and the kinetic energy loss. That is, the total pressure drop $\Delta p$ across the filter can be expressed as

$$\Delta p = \Delta p_v + \Delta p_{\text{kin}}$$

(5.1)

where $\Delta p_v$ is the viscous pressure drop and $\Delta p_{\text{kin}}$ is the kinetic pressure drop. In order to reduce the number of unknowns it was desirable to assume that the viscous pressure drop across the filter is same that was previously derived from the Ergun’s model in Chapter 3. Hence, the objective here is to derive an expression for the kinetic loss in function of the fluid flow rate, which is then added to the known expression of the viscous loss to give the total energy loss.

Figure 5.1 shows the conceptual MOM model of the ceramic foam. $p_1, p_2, p_3$, etc., are pressure measurement points from each cell across the length of the filter and $p_2-p_1$, $p_3-p_2$, etc., are the values of pressure difference across the windows, which are equivalent to the kinetic pressure losses.

![Diagram of the MOM model of the ceramic foam filter](image)

**Figure 5.1 Conceptual Multiple Orifice Mathematical (MOM) model of the ceramic foam filter**
Using the Bernoulli equation and mass conservation law a relationship can be made between the fluid flow rate and the pressure drop across each window. Bernoulli's Principle states that for a non-viscous, incompressible fluid in a steady flow, the sum of pressure, potential and kinetic energies per unit volume is constant at any point, i.e.

\[ \frac{p}{\rho} + \frac{u^2}{2} + gz = \text{const.} \quad (5.2) \]

where \( u \) is the fluid flow speed, \( p \) is the pressure, \( \rho \) is the fluid density, \( z \) is the height and \( g \) is the acceleration due to gravity. Also, the mass continuity states that for a steady flow process, flow where the fluid flow rate do not change over time and through a control volume where the stored mass in the control volume does not change, the inflow equals the outflow, i.e.

\[ \text{mass flow rate} = \rho u A = \text{const.} \quad (5.3) \]

where \( A \) is the cross sectional area.

Using the Equations (5.2) and (5.3) on the fluid flowing across window \( a_1 \), and assuming that the variation in height is negligible and density is constant, the following relationships can be obtained,

\[ p_1 + \frac{1}{2} \rho u_1^2 = p_2 + \frac{1}{2} \rho u_2^2 \quad (5.4) \]

\[ q = u_1 A_1 = u_2 A_2 \quad (5.5) \]
where \( u_1 \) and \( u_2 \) are the velocities inside the cells, \( q \) is the flow rate in a row of cells across the filter while \( A_1 \) and \( A_2 \) are the cross sectional area of the flow through cell (1) and window (1) respectively. Simultaneously solving Equations (5.4) and (5.5) and replacing the fluid velocity by the flow rate yields

\[
\beta^2 \Delta p_{12} = \frac{\rho q^2}{2A_o^2} \left[ 1 - \frac{A_w}{A_o} \right]
\]

(5.6)

where \( A_0 \) is the equivalent tube cross sectional area, \( A_w \) is the window cross sectional area and \( \beta \) is the window orifice correction coefficient that compensates for velocity profile and internal energy losses.

The total kinetic pressure drop \( \Delta p_{kin} \) across the filter is the sum of all the pressure drops across all the windows. That is, the pressure drop due to the kinetic energy loss can be expressed as

\[
\Delta p_{kin} = \Delta p_{12} + \Delta p_{23} + \Delta p_{34} + \ldots + \Delta p_{m-1,m} = M \frac{\rho q^2}{2\beta^2 A_w^2} \left( 1 - \frac{A_w^2}{A_o^2} \right)
\]

... (5.7)

or

\[
\Delta p_{kin} = M \frac{\rho q^2}{2\beta^2 A_w^2} \left( 1 - \frac{w^4}{d_o^4} \right)
\]

(5.8)

where \( M \) is the number of orifices in the row of cells across the filter, given by

\[
M = \frac{L}{\sqrt{d^2 - w^2}}
\]

(5.9)

and \( w \) is window diameter, \( d \) is cell diameter and \( L \) is the filter length.
The following subsection now describes the determination of the fluid flow rate in the row of cells.

### 5.2.1 Determination of fluid flow rate through cell row

The fluid flow rate through a row of cells \( q \) is a fraction of the total fluid flow rate \( Q \) through the filter, which is calculated as follows. Assuming that there are \( N_{\text{row}} \) number of rows of cells in the cross section of the filter and the volume of fluid flowing through the filter in one second is \( Q \), the fluid flow in a single row of cells in one second \( q \) is expressed as

\[
q = \frac{Q}{N_{\text{row}}} \tag{5.10}
\]

The porosity of the filter is expressed as

\[
\varepsilon = \frac{A_{\text{fil}} L}{A_{\text{FILT}} L} = \frac{N_{\text{row}} \pi \frac{d_o^2}{4}}{4 A_{\text{FILT}}} \text{ or } \varepsilon A_{\text{FILT}} = N_{\text{row}} \pi \frac{d_o^2}{4} \tag{5.11}
\]

where \( A_{\text{FILT}} \) is the cross sectional area of the filter and \( d_o \) is the diameter of an equivalent tube of the row of cells.

Solving for \( N_{\text{row}} \) in Equation (5.11) and substituting \( A_{\text{FILT}} (\pi D^2/4) \) yields
where $D$ is the filter diameter. Therefore, substituting $N_{row}$ in Equation (5.10) gives the expression

$$q = \frac{d_o^2}{\varepsilon D^3}Q$$  (5.13)

### 5.2.2 Determination of equivalent cell row diameter $d_o$

The relationship between the cell diameter $d$ and the equivalent tube diameter $d_o$ is such that the volume of the tube is equal to the sum of the volume of the row of cells, i.e.

$$\pi \frac{d^2}{4} L = MV_{CELL}$$  (5.14)

where the cell volume $V_{CELL}$ is written as (see Chapter 3, Section 3.4.3)

$$V_{CELL} = \frac{\pi d^3}{12}(2 - 3B(3k^2 + B^2))$$  (5.15)

Hence, substituting $V_{CELL}$ and $M$ from Equation (5.8) and solving for $d_o$ gives the expression

$$d_o = \left[\frac{d^3(2 - 3B(3k^2 + B^2))}{3\sqrt{d^2 - w^2}}\right]^{1/3}$$  (5.16)
5.2.3 Determination of the viscous pressure loss

Since the physical model considered in analysing the MOM model is similar to the model used for analysing the EEM model in Chapter 3, the viscous pressure gradient is the same. That is, the viscous pressure gradient of gelcast foam filters is expressed as (repeating Equation 3.22 developed in Chapter 3)

\[
\frac{\Delta p_{\text{vis}}}{L} = \frac{12(1-6B+5k^2/2)}{(2-3B(3k^2+B^2))} \frac{\alpha \mu v}{d^2 \varepsilon} \tag{5.17}
\]

The velocity of fluid flowing through the rows of cell, \( v \), is expressed as:

\[
v = \frac{4q}{\pi d_o^2} = \frac{4}{\pi d_o^2} \frac{4Q}{\pi d_o^2} \frac{4Q}{4\varepsilon D^2 \pi} \tag{5.18}
\]

or

\[
v = \frac{4Q}{\varepsilon \pi D^2} \tag{5.19}
\]

Finally, by substituting the expressions for the viscous and kinetic losses in Equation (5.3), the total pressure gradient is expressed as follows

\[
\frac{\Delta p}{L} = \frac{\alpha \mu 4(1-\varepsilon)^3}{D^2 \pi e^3} S^2 Q + \frac{(1-w^4/d_o^4)}{\sqrt{d^2-w^2}} \left[\frac{d_o^2}{\beta \pi e D^2 w^2} \right]^2 8 \rho Q^2 \tag{5.20}
\]

where \( d_o = \frac{d^3(2-3B(3k^2+B^2))}{3\sqrt{d^2-w^2}} \)

\[
B = 1-\sqrt{1-k^2} \quad \text{and} \quad k = \omega/d
\]
\( \alpha \) is the viscous pressure loss correction coefficient which is chosen as 5 as suggested by MacDonaId et al (1979), and \( S_v \) is the specific surface area of the foam filter defined earlier (Chapter 3 Section 3.4.1):

\[
S_v = \frac{12(1 - 6B + 5k^2/2)e}{d(2 - 3B(3k^2 + B^2))(1 - \varepsilon)} \quad (5.21)
\]

5.3 CALIBRATING AND VALIDATING THE MOM MODEL

The calibration of the MOM model required the tuning of the two correction factors \( \alpha \) and \( \beta \) such that the curve fits that of the experimental data of the physical scale foam model. As previously mentioned, the development of the model representing the viscous loss is after the work of previous researchers such as Ergun and Orning (1949), thus, the viscous correction factor was fixed by adopting the value suggested by MacDonaId et al (1979), that is, \( \alpha = 5 \). Hence, the problem was reduced to the tuning of the kinetic correction factor \( \beta \).

Furthermore, results from the calibration using the physical scale model foam were corroborated by the determination of the kinetic correction coefficient using data from fluid flow experiments on a generic multiple orifice physical scale model structure. This is now explained.

5.3.1 Calibration of MOM model using physical scale model foam

The MOM model was calibrated using the experimental data from the physical scale model foam of length 25 mm. After fixing the viscous correction coefficient as earlier mentioned, the kinetic correction coefficient \( \beta \) was tuned
until the model curve fits the curve of the experimental data. These parameters that fit the curve became the correction coefficients of the mathematical model that was then validated with real foam samples. The model calibration was repeated using data from physical scale model foam of lengths 100 mm at different ranges of Reynolds number (see Appendix VI). The comparison was repeated to six other sets of experimental foams to confirm the repeatability and wider applicability of the model.

Figure 5.2 shows the calibration graph of pressure gradient versus air mass flow rate, where the MOM model for the prediction of the pressure gradients of gelcast ceramic foam filters is calibrated using the experimental data from the generic physical scale model foam of length 25 mm.

Figure 5.2 Graph of pressure gradient vs. fluid flow rate in a physical scale model foam sample, for the calibration of the MOM model
The average value of the kinetic correction factor \( \beta \) corresponding to the fit was equal to 2.2. Therefore, the mathematical model can be rewritten after substituting the values of \( \alpha \) and \( \beta \) as

\[
\frac{\Delta p}{L} = \frac{5 \mu^4 (1-\varepsilon)^2}{D^2 \pi \varepsilon^3} S^2 Q + \frac{(1-\omega^2/d_s^2)}{\sqrt{d^2 - w^2}} \left[ \frac{d_s^2}{2.2 \pi D^3 w^3} \right]^2 \rho Q^2
\]

Furthermore, due to the importance placed on accurate modelling, a further investigation was carried out to corroborate the result obtained from the above calibration process. Consequently, the next subsection describes alternative approach to determine the kinetic correction coefficient for the MOM model.

### 5.3.2 Determination of kinetic correction coefficient using a generic multiple orifice physical scale model structure

In order to corroborate the result obtained from the calibration of the MOM model above, data from flow experiments were collected from generic multiple orifice physical scale model structures made by rapid manufacturing stereolithography to determine the kinetic correction coefficient. A combination of 4 cell diameters and 2 window diameters were used to produce eight rows of cells to yield results that can be compared.

The multiple orifice physical scale model was designed using CAD package to be generic structures with rows of cells connected through the windows. Two generic multiple orifice structures were produced with four rows of cells each. The rows of cells were of various cell/window diameters, providing enough variation for comparison. Figure 5.3 is a drawing of the generic multiple
orifice structure showing the rows of cells. The rows vary from each other from the diameters of the cells or the windows.

Figure 5.3 Drawing of generic multiple orifice physical scale model

The objective of the experimentation was to measure the pressure drop across the windows of a given row of cells for a known fluid flow rate. Hence, each row of cells consisted of 4 sets of pressure tappings to enable the measurement of the pressure drop across the four windows. Using the Continuity Principle, the mass flow rate at the orifice plate must be the same at the window in the row of cells, i.e., assuming that the fluid is incompressible (density of fluid is constant), a relationship was developed to determine the kinetic correction coefficient at the 4 windows. The Continuity Principle can be expressed as

\[ Q = Q_{\text{row}} \]  

(5.23)

where \( Q \) is the fluid flow rate from the orifice plate flow meter and \( Q_{\text{row}} \) is the flow rate across a window. The flow rate from the orifice plate was calculated
from using Equation (4.7) while the flow rate across a window was calculated from the expression derived below. Using the Bernoulli law and Continuity Principle,

\[ Q_{\text{row}} = \beta \frac{\pi}{4} w^2 \sqrt{\frac{2 \rho \Delta p}{1 - (\frac{w}{d_0})^4}} \]  

(5.24)

where \( \beta \) is the kinetic correction coefficient, \( \Delta p \) is the pressure drop across a window \( w \) is the window diameter and \( d_0 \) is the equivalent diameter. Therefore, equating Equations (5.23) and (5.24) the kinetic correction coefficient can be expressed as

\[ \beta = \frac{4Q}{w^2 \pi} \sqrt{\frac{(1 - (\frac{w}{d_0})^4)}{2 \rho \Delta p}} \]  

(5.25)

The equivalent diameter was derived such that the volume of a tube of length \( L \) and diameter \( d_0 \) was equal to the total volume of all the cells \( V_{\text{CELL}} \) in a row of length \( L \). That is,

\[ \pi L \frac{d_0^2}{4} = V_{\text{CELL}} \]

or

\[ d_0 = 2 \sqrt{\frac{V_{\text{CELL}}}{\pi L}} \]  

(5.26)

The total volume of the cell was calculated by adding up the volumes of the entire spherical cell on a row and subtracting the overlapping volumes that created the windows. That is,

\[ V_{\text{CELL}} = MV_{\text{CELL}} \]
or \[ V_{\text{cell}} = \frac{\pi d^3 L}{12 \sqrt{d^2 - w^2}} ((2 - 3B(3k^2 + B^2))) \] (5.27)

Hence, for a known flow rate and the pressure drop across the windows the kinetic correction coefficients could be calculated using Equation (5.24) to (5.27).

5.3.2.1 Experimental procedure and data analysis

In a procedure similar to the fluid flow experiments described in Chapter 4, air was supplied from the Leister blower. The pressure drops across the windows were read from digital pressure gauges with ranges from 0 to 700 Pa. The readings were taken simultaneously with that of the flow meter. The temperatures of the air were also read simultaneously to calculate the air densities.

There were two options for the experimental set up. The first option was to mount the generic structure directly in the filter holder described in Chapter 4, with three of the rows of cells blocked. However, the change of the fluid flow diameter from 60 mm to 8-10 mm was significant and the fluid flow was not fully developed when taking the readings. The second option which was adopted was to reduce the fluid flow diameter after the orifice flow meter to 10 mm and allow the fluid to be fully developed before the row of cells. Hence, a sample holder was designed and produced such that the air flow could be directed to a given row of cells and it (i.e. the holder) could be fitted to a flange on the pipe after the orifice flow meter.

The pipe that leads to the filter was >50 times the internal diameter to allow the fluid to be fully developed before the rows of cells. Figure 5.4 is a
schematic of multiple orifice physical scale model flow rig. The reducer pipe or connecting pipe of internal diameter 4.2 mm and length 450 mm is mounted after the orifice plate, then sample was mounted on the other end such that reading can be taken from each given row of cells.

![Schematic of multiple orifice physical scale model flow rig](image)

Figure 5.4 Schematic of multiple orifice physical scale model flow rig

Steel tubes of external diameter 5 mm were inserted into the pressure tapping holes in the row of cells from which they were connected to flexible hose from the digital pressure gauges (see Figure 5.5). Soap solution was used to check for air leakage after using sealants.

![Picture of the mounted sample with the inserted steel tubes](image)

Figure 5.5 Picture of the mounted sample with the inserted steel tubes
5.3.2.2 Results and discussions

Using the experimental data from the multiple orifice physical scale model, the fluid flow rates for each measurement and the corresponding kinetic correction coefficients across the cells were calculated. The results are presented in Table 5.1.

<table>
<thead>
<tr>
<th>Cell diameter $d$ (mm)</th>
<th>Window diameter $w$ (mm)</th>
<th>Fluid flow rates $Q$ (m$^3$ s$^{-1}$)</th>
<th>Kinetic correction coefficients $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.0</td>
<td>6.0</td>
<td>1.17E-03</td>
<td>2.2, 2.2, 2.2, 2.2</td>
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<tr>
<td>10.0</td>
<td>6.0</td>
<td>1.15E-03</td>
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</tr>
<tr>
<td>8</td>
<td>6</td>
<td>1.08E-03</td>
<td>3.0, 2.1, 2.1, 2.0</td>
</tr>
<tr>
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<td>6</td>
<td>1.14E-03</td>
<td>2.8, 2.1, 2.1, 2.0</td>
</tr>
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<td>1.01E-03</td>
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<td>1.07E-03</td>
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<td>1.20E-03</td>
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<td>1.26E-03</td>
<td>2.3, 2.7, 2.3, 2.5</td>
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</tr>
<tr>
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<td>1.26E-03</td>
<td>2.1, 2.0, 2.0, 2.1</td>
</tr>
<tr>
<td>11.0</td>
<td>4.7</td>
<td>1.11E-03</td>
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</tr>
<tr>
<td>11.0</td>
<td>4.7</td>
<td>1.17E-03</td>
<td>2.9, 3.0, 2.7, 2.9</td>
</tr>
</tbody>
</table>

Table 5.1 Kinetic correction coefficients from the multiple orifice physical scale model at windows 1, 2, 3 and 4, i.e. $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$.

The kinetic correction coefficients calculated from the experimental data varied from 2.1 to 3.6. It can be seen in Table 5.1 that the modal class is between 2-2.4 with an average kinetic correction coefficient of 2.24. This value for the kinetic correction coefficient determined from the multiple orifice physical scale model corroborates the results in Section 5.3.1.
5.3.2 Validation of MOM

The calibrated MOM model was validated by comparing it to experimental results from the real gelcast ceramic foam samples including the alumina foam sample. The results are demonstrated by the curves shown in Figures 5.6, 5.7 and 5.8 which are graphs of pressure gradients versus the mass flow rates.

The MOM model was first compared with the gelcast ceramic foam comprising two series of production (A44C and 243AL series corresponding to the production batches) with known parameters, provided by the manufacturer. Furthermore, the model was compared to the alumina foam which cell diameter was determined using an SEM (see detailed description in Chapter 4, Section 4.2.2). The porosity of the alumina foam was provided by the manufacturer.

![Graph of pressure gradient vs. fluid flow rate](image)

Figure 5.6 Graphs of pressure gradient vs. fluid flow rate in samples of ceramic foams, comparing experimental data to the MOM model, A44C series of filter samples and the alumina foam filter sample.
Figure 5.7 Graphs of pressure gradient vs. fluid flow rate in samples of ceramic foams, comparing experimental data to the MOM model, A44C series of filter samples.

Figure 5.8 Graph of pressure gradient vs. fluid flow rate in samples of ceramic foams, comparing experimental data to the MOM model, 243AL series of filter samples.
All the figures (Figures 5.6, 5.7 and 5.8) illustrate that the error in predicting pressure gradients of gelcast ceramic foam filters by using the MOM model is <25%.

The results show that the proposed MOM model is a promising tool for the prediction of pressure gradients of clean gelcast ceramic foam filters where the porosity and the cell diameter are known.

5.4 CONCLUDING REMARKS

The conclusions drawn from this chapter of the work are the following:

The Multiple Orifice Mathematical model was developed from the basic fluid theory using a conceptual multiple orifice model to represent the complex structure of the gelcast ceramic foam filter.

The model was calibrated using experimental data from physical scale models of the cellular foam structures. The calibrated model was then validated and the results showed that the predicted pressure gradients of the real ceramic foams were within 25% of experimental values, without the need for calibration on real foams.

Since the MOM model was found suitable for predicting pressure drop of clean ceramic foam filters, it was desirable to extend it to predicting pressure gradients of particulate matter (PM) loaded gelcast ceramic foam filters. Consequently, the next chapter describes the development of the model for predicting the pressure gradients of PM loaded gelcast ceramic foam filters by studying the morphology of the PM deposit in the filters and expressing the
new microstructure of the filter as a result of the load to the clean microstructure of the foam.
6.1 INTRODUCTION

This chapter describes the development of a new mathematical model for predicting the pressure gradients in particulate matter (PM) loaded ceramic foam filters. The model was derived from adapting the MOM model to the changing microstructure of the ceramic foam filters in the course of its loading with PM.

Before describing the development of the model a review of previous PM loaded filter modelling research is presented.
6.2 REVIEW OF PM LOADED FILTER MODELLING

Significant progress towards the understanding of transient behaviour of deep bed filtration has been made over the years, especially, in fibrous filters (Davies, 1973 and Brown, 1993) and the granular filters (Tien, 1989). Although, there is significant published literature on models predicting pressure drop across clean and PM loaded wall flow filters based on the work of the above authors, few have been published on loaded foam filters and in particular, gelcast ceramic foam filters.

The pressure drop across a loaded filter as proposed and used by some researchers (Sorenson et al, 1994, Thomas et al, 1999, Konstandopoulos et al, 1989 and Versaevel et al, 2000) is the sum of the pressure drop across the clean filter plus the flow resistance of the trapped particles, with the assumption that the deposition of the particles in the filter is uniform. This approach is considered to be suitable for filters that are predominantly exhibiting cake filtration since the Darcy law can be applied on the cake in a similar manner to the filter itself (Konstandopoulos et al, 1989). However, these models proposed for the prediction of pressure drop across loaded filters lack accuracy without adding significant empirical tuning parameters.

Other researchers, Tien and Payatakes (1979), suggested the adoption of models for clean filters to the loaded filters by relating the parameters of the clean filter structure to the loaded filter structure. During the course of loading of a filter, deep bed filters in particular, the microstructure of the filter changes, which is mainly the reduction of the space available to fluid flow. In order to relate the parameters of the loaded filter to the clean filter in this case, it is necessary to understand the morphology of the particles deposited.
Tien and Payatakes (1979) described two types of deposit morphology corresponding to two limiting cases in the study of filtration in granular filters. In the first case, it was assumed that the deposition was uniformly spread inside the cells, called the smooth coating mode. This mode of deposition reduces the porosity and increases the grain size of the granular filter. The porosity was then determined by dividing the difference of the void volume of the clean filter and the volume of the deposit by the volume of the filter. Similarly, the new grain size can be determined evaluating the volume of deposit on a grain.

The second case was with the assumption that the deposition of the particles is around the constricted portions of the fluid flow path in the filter, referred to as a 'localised mode', which can subsequently lead to blocking of the openings. Hence, it is referred to as 'localised coating mode' in this thesis. The conceptual model used by Tien and Payatakes (1979) for representing the filter is the constricted tube and the resulting equations required the aid of heavy computation. Since their model was derived for predicting pressure drop across granular filters, it may not be suitable for a direct application on ceramic foam filters and there has been no suitable conversion of the parameters of the granular filters to equivalent parameters of foam filters. Hence, it is necessary to develop a new model that considers the spherical shape of the cells of the foam.

The modelling approach for the PM loaded filter in this research was however based on Tien and Payatakes (1979) where the morphology of the PM deposit is analysed and related to the structure of the filter. The two modes of PM deposition in the filter were discussed.

The following section describes the development of the model while comparing the two cases of particle accumulation in the filter.
6.3 MATHEMATICAL MODEL FOR LOADED FOAM FILTERS

The accumulation of PM within the filter causes changes in the microstructure of the filter medium. Consequently, the behaviour of the fluid flow through the loaded filter and efficiency of the filter can be significantly affected. Since the performance of the filter is measured by the filtration efficiency and the pressure drop often limits the length of time for the filtration run, the ability to predict the efficiencies and pressure drop accurately during the course of filtration is important.

The main change as a result of PM deposition in the matrix of foam filter involves the porosity of the filter, the cell size, the window size and the geometry of the microstructure. Quantifying these changes would account for the changes in predicting the pressure drop across a loaded foam filter.

The effect of the deposition of particles in the ceramic foam filter medium is considered to be the reduction of cell and window size and effective porosity of the porous medium. Hence, adopting the approach suggested by Tien and Payatakes (1979) in the study of granular filters, a relationship is developed between the parameters of clean foam filters and those of loaded foam filters, in terms of the particulate loading. The correlation is then applied to the MOM model presented in Chapter 5.

The SEM pictures, Figure 6.1 (a) and (b), with magnification of 500X and 200X respectively, of a loaded gelcast ceramic foam filter, illustrates the pattern of PM deposition in the microstructure of the foam filters.
Figure 6.1 SEM image of loaded gelcast ceramic foam filter with porosity of 80% and cell diameter of 0.75 mm, (a) showing the heavy loading of the upstream faces of the cells, (b) showing a localised deposition of PM in the foam microstructure.
Although, the SEM images show that most of the PM is deposited around the windows, it was necessary to start with the study of a simple structure by adopting the first limiting case suggested by Tien and Payatakes (1979).

The accumulation of PM in the filter will reduce the porosity and the cell and the window diameters of the filter since the deposit occupy some of the space that was originally void in the filter. The task in the development of the loaded filter model is, therefore, to relate the microstructure of the loaded filter to that of the clean filter. Hence, it requires a study of the morphology of the PM deposition and representation of the structure of the loaded filter with a geometric model that can be related to the parameters of clean foam filters.

Assuming that the deposition is uniformly spread inside the filter, the cells coated with the PM retain the spherical structure like the clean filters. Hence, the microstructure can be represented by spherical cells arranged as a face centred lattice.

The next subsection describes the development of the mathematical model with uniformly deposited PM, known as the 'smooth coating mode'.

6.3.1 Smooth coating mode

In this mode where it is assumed that the deposition in the cells is uniform, the porosity of the filter changes in the course of loading the filter. The trapped PM occupies part of the void in the foam, thus, for unit volume of filter the porosity $\varepsilon^u$ of the loaded filter is derived as follows:

$$\varepsilon^u = 1 - V_{\text{mat}} - V_p$$  \hspace{1cm} (6.1)
where $V_{mat}$ and $V_p$ are the volume of the filter material and volume occupied by the particulates respectively. Figure 6.2 illustrates the smooth coating mode loading of the filter cells.

![Diagram of smooth coating mode filter loading]

**Figure 6.2 Schematic of “smooth coating mode” filter loading, $d_n$ is the cell diameter of the loaded filter**

The volume occupied by particulates in terms of the specific volume $\sigma$ of PM in the filter is written as:

$$V_p = \frac{\sigma}{1-\varepsilon_p} \quad (6.2)$$

Hence, replacing $V_{mat} (= 1-\varepsilon)$ and $V_p$ in Equation (6.1) yields
where $\varepsilon_p$ and $\varepsilon$ are the deposited PM porosity and initial filter porosity respectively and $\sigma$ is the PM volume per unit filter volume. The PM porosity $\varepsilon_p$ depends on the morphology of the deposits formed and changes in the course of deposition. However, since there is no reliable data on foam filters, the same assumption made by Pontikakis et al (2001) is considered in this analysis, that is, $\varepsilon_p = 90\%$.

In the course of loading the filter the diameter of the cell reduces. With the assumption that the deposition in the cells is uniform, the reduction in volume of the cell is proportional to the reduction in porosity, i.e., for a unit volume of foam filter, the porosity is expressed as

$$\varepsilon = NV_{CELL}$$

(6.5)

where $N$ is the number of cells per unit volume of filter. Rewriting the expression for the volume of the cell $V_{CELL}$

$$V_{CELL} = d^3 \frac{\pi(2-3B(3k^2-B^2))}{12}$$

$$= d^3 k_o$$

(6.6)
Similarly, the porosity of the loaded filter $\varepsilon_n$ can be expressed for a unit volume of filter as:

$$\varepsilon_n = Nd_n^3 k'_n$$  \hspace{1cm} (6.7)

where $k'_n$ is a factor of the volume of a cell as the cell diameter $d$ is changing in the course of loading and $d_n$ is the loaded cell diameter. Assuming that the ratio of the window to cell diameter remains constant with loading, the two factors are also equal, i.e. $k_n = k'_n$. Simultaneously solving Equations (6.6) and (6.7) gives the expression:

$$d_n = (\varepsilon_n/\varepsilon)^{1/3} d$$  \hspace{1cm} (6.8)

The window diameter of the loaded filter is determined by multiplying the new cell size by the known clean filter window cell size ratio determined from the porosity cell relation established earlier. The porosity and the cell and window diameters corresponding to a given load are then applied to clean filter pressure gradient expression, the MOM model.

The model for pressure gradient can be then validated with experimental data of fluid flow through loaded samples of the gelcast ceramic foam samples. The loading of the filters are accomplished from a loading experimental rig described in the next section.
6.4 LOADING AND FLUID FLOW EXPERIMENTS

The experimental work aimed to measure the flow rate of gas and pressure drop of ceramic foam samples, before and after PM loading. The data collected were used to validate the PM loaded MOM model. The fluid flow experiments and procedures were the same as described in Chapter 4. Great precaution was taken during the flow experiments not to dislodge PM since it could be easily be blown out, thus, reducing the actual filter loading level.

6.4.1 Filter PM loading rig

The soot loading rig consisted of a filter sample holder inside a filter canister that was mounted on a diesel engine exhaust system. The filter holder was fabricated from a stainless steel material in three parts so that the foam filter samples of diameter range 55 – 60 mm could be mounted with gaskets. Figure 6.3 shows the three piece filter samples holder with a filter sample inside the clamp.

![Figure 6.3 Foam sample holder for filter loading](image)

Figure 6.3 Foam sample holder for filter loading
After carefully clamping filter sample(s) between two identical pieces with 3M Interam™ mat sheet gaskets, the set was then mounted inside the filter canister. In the downstream of the filter sample holder unit was mounted a wall-flow filter of diameter 150 mm and length of 75 mm. The wall-flow filter served as a retainer for the holder unit due to the pressure from the engine exhaust gas and could also be used for the purpose of determining the foam filter filtration efficiency (see Appendix VIII.3). The filter sample holder and the wall-flow filter were mounted with fibre ribbon gaskets to aid mounting and withdrawal.

Figure 6.4 shows the schematic drawing of an assembly of filter holder, foam filter samples, wall-flow filter and the canister.
The PM was generated by an 1100 series Perkins 4.4 litre, 4 cylinder 4-stroke, and turbocharged, after-cooled diesel engine. The construction of the exhaust system was such that a bypass valve can be used to isolate the main exhaust pipe with the filter canister or to regulate the gas flow through the filter samples. The diesel engine was connected to a Froude Consine dynamometer of type AG 400 HS. The fuel for running the Perkins engine was a low sulphur diesel (30 ppm).

Figure 6.5 shows the experimental set up for loading the filters, showing the filter holder fitted in the exhaust system.

Figure 6.5 Engine test cell experimental set up for filter PM loading
6.4.2 Experimental procedures

The main objective of this experiment was to load samples of ceramic foam filters with a known amount of PM for subsequent study of the fluid flow through PM loaded foam filters. Each sample of gelcast ceramic foam filter was carefully weighed on a Sartorius weighing machine for one thousandth of a gram before mounting on the filter holder for the PM loading. The engine load during the process was 82 Nm (bmep = 117 kPa) at a steady speed of 1500 rpm. Each loading was for a duration of 1.5 hours. The loaded foam filter sample(s) was carefully dismantled and weighed. The PM load was then the difference between the clean and loaded filter samples. The weighing of the loaded PM was performed after blowing air through the sample in order to dislodge loose particles, so that the load may not change during the fluid flow experiments on the samples.

In addition, the wall flow filter mounted at the downstream of the samples was weighed before and after each loading experiment to determine the amount of PM escaping from the samples. This information was used to determine the efficiencies of the filter samples.

The weighing of the clean and loaded foam filters for each loading experiment was carried out in the same ambient conditions to maintain a consistent variation in weight.

The procedures for the fluid flow experiments for PM loaded foam filters are the same for clean filters discussed in Chapter 4. The fluid flow experimental data from the PM loaded foam filter samples was then used to validate the mathematical model for the prediction of pressure gradients of PM loaded gelcast ceramic filters.
Chapter 6  
Modelling of loaded ceramic foam filters

6.4.3 Materials

The materials for loaded filter experiments are three sets (pieces each of length of approximately 5 mm) of A44C series of gelcast ceramic foams, alumina foam disks; 85AL-B, A103ZI and A85M. The specifications of the alumina foam disks provided by the manufacturers had a tolerance range of ±100 μm. Table 6.1 is a summary of the materials used for the experimentation.

<table>
<thead>
<tr>
<th>Filter type</th>
<th>Cell diameter (mm)</th>
<th>Window diameter (mm)</th>
<th>Porosity (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A44C6</td>
<td>0.25</td>
<td>0.075</td>
<td>86</td>
</tr>
<tr>
<td>A44C4</td>
<td>0.35</td>
<td>0.105</td>
<td>86</td>
</tr>
<tr>
<td>A44C2</td>
<td>0.65</td>
<td>0.221</td>
<td>88</td>
</tr>
<tr>
<td>A103M</td>
<td>0.27</td>
<td>0.070</td>
<td>81.8</td>
</tr>
<tr>
<td>A103Z6</td>
<td>0.29</td>
<td>0.081</td>
<td>82.6</td>
</tr>
<tr>
<td>A85M</td>
<td>0.27</td>
<td>0.07</td>
<td>80.5</td>
</tr>
<tr>
<td>A103ZI</td>
<td>0.41</td>
<td>0.111</td>
<td>83.1</td>
</tr>
</tbody>
</table>

Table 6.1 Specification of ceramic foam samples

6.4.6 Filter loading results

As earlier mentioned the PM load on each sample is the difference between the clean foam filter weight and the PM loaded filter weight. Hence, after calculating the PM loads, the specific PM mass in g litre⁻¹ was determined on each sample. The specific mass of PM is the ratio of the mass load to the volume of the filter sample. Table 6.2 is a summary of the results.
Table 6.2 The specific mass of PM of the foam filter samples

Having determined the specific mass of PM, the next section describes the validation of the loaded ceramic foam filter model.

6.5 VALIDATION OF LOADED CERAMIC FOAM MODEL

Using the PM load for a given sample, the values of the new porosity, cell and window diameters were calculated using Equations (6.4) and (6.8) and the known ratio of cell to window diameter. The values are then applied to the loaded filter model and the resultant curves are compared to the curves generated from the experimental data.

Figures 6.6, 6.7 and 6.8 show the graphs of pressure gradient versus the fluid flow rate, comparing the loaded filter mathematical model to the experimental data. Clearly, the model significantly under-predicts the pressure gradient. Therefore, the model required modification to improve its accuracy.
Figure 6.6 Graph of pressure gradient vs. fluid flow rate comparing experimental data to the PM loaded MOM model: A44C2 filter sample of load = 0.525 g l⁻¹

Figure 6.7 Graph of pressure gradient vs. fluid flow rate comparing experimental data to the PM loaded MOM model: A44C6 filter sample of load = 0.848 g l⁻¹
Figure 6.8 Graph of pressure gradient vs. fluid flow rate comparing experimental data to the PM loaded MOM model: A44C6 filter sample of load = 1.325 g l\(^{-1}\)

The results imply that the smooth mode deposition was therefore not a suitable representation of the morphology of the PM deposit. The results clearly demonstrate the inadequacy of the new model based on smooth PM deposition for the prediction of pressure gradients of PM loaded ceramic foam filter.

Consequently, it was necessary to investigate the representation of the PM deposit morphology with the localised mode, where the PM accumulates mainly at the windows. In this mode the predominant changes occur with porosity and the windows while the cell diameter is considered unchanged.

Having demonstrated that the smooth model was potentially too simplistic and not suitable for the prediction of pressure gradients of PM loaded ceramic
foam filters, an improved model based on a localised PM loading model concept was developed.

6.6 PM DEPOSITION AROUND CELL OPENINGS (LOCALISED MODE)

An improved modelling approach was based on the assumption that the PM accumulation occurs mainly at the openings (windows) connecting the cells. Consequently, the window size decrease with an increase in PM load, resulting in the increase in pressure drop across the filter. Following the path of least resistance Acosta and Castillejos (2000) demonstrated that the deposits accumulate on one window per cell at a time. Scanning electron microscope (SEM) image of the window of a PM loaded ceramic foam filter (see Figure 6.9 for example) illustrated that the deposition of the PM was at the window.

Figure 6.9 SEM image of loaded ceramic foam window
Figure 6.10 shows a conceptual model of the PM loading within the cells, the latter being shaded regions upstream of each window. The shaded surface area when rotated around the x-axis is the volume of PM deposit in the cell. The task in this approach is to develop a correlation between the initial parameters and the PM loaded parameters of the foam filter. The procedure that was considered suitable in this case was to determine the volume generated by the shaded surface around the x-axis, then, equating it to the PM volume deposited in a cell for a given PM load.

Figure 6.10 Schematic of deposition of PM in a foam cell, where deposition is lodged around the window and h is the radius of the opening of the loaded filter.
The surface of the deposit was assumed to be inclined at an angle $\psi$ to a line perpendicular to the radius passing through the original circumference of the window. The inclination of the surface of the deposit was determined by tuning the angle $\psi$ in the process of validating the PM loaded foam filter model. The line intersects the cell at $(x_1, y_1)$ and the diameter of the window at $(x_2, y_2)$, thus, the new window diameter $w_n$ is equal to $2h$. The point $(x_2, y_2)$ is considered sharp by assuming that there is no disengagement of deposited particles.

The next subsection describes the derivation of the volume generated from rotating the shaded surface area around the $x$-axis.

### 6.6.1 Calculation of volume generated by the PM

The volume of PM in the cell is the volume generated by rotating the shaded area around the $x$-axis as mentioned earlier. $L_1$ is an arbitrary line coinciding with the upstream surface of the PM deposit initially considered to be perpendicular to the radius of the cell passing through the circumference of the window and $L_1'$ is a variation of $L_1$ through an angle $\psi$ ($0 \leq \psi < \theta$).

Assuming that the point of intersection between the circle $C$ and the line $L_1$ and the lines $L_1$ and $L_2$ are $(x_1, y_1)$ and $(x_2, y_2)$ respectively, $r_1$ is the perpendicular distance from the line $L_1$ (PM surface facing air flow direction) from the centre of the cell and $h$ is the radius of the opening as PM is accumulating. Then, the volume of the PM in the cell is equal to difference in volume generated by the arc of the circle between $x_1$ and $x_2$ around the $x$-axis and the volume of the line $L_1$ within the same boundary around the $x$-axis.
The equation of the circle is

$$y = \sqrt{r^2 - x^2} \quad (6.9)$$

where \( r \) is the cell radius i.e. \( d/2 \). The volume \( V_{\text{arc}} \) generated by the arc revolving around the \( x \)-axis is determined from resolving the following integration:

$$V_{\text{arc}} = \pi \int_{x_1}^{x_2} (r^2 - x^2) \, dx \quad (6.10)$$

yielding

$$V_{\text{arc}} = \pi [r^2(x_2 - x_1) - \frac{x_2^3 - x_1^3}{3}] \quad (6.11)$$

The equation representing \( L_1 \) can be written as

$$y = g_1x + k_n \quad (6.12)$$

where \( g_1 \) and \( k_n \) are the gradient and the \( y \)-axis point of intersection respectively. The volume \( V_{\text{line}} \) generated by the line revolving around the \( x \)-axis is the solution of the following integration:

$$V_{\text{line}} = \pi \int_{x_1}^{x_2} (g_1x + k_n)^2 \, dx \quad (6.13)$$

yielding

$$V_{\text{line}} = \pi [g_1^2 \left( \frac{x_2^3 - x_1^3}{3} \right) + g_1k_n(x_2^2 - x_1^2) + k_n^2(x_2 - x_1)] \quad (6.14)$$
Hence, the equivalent PM volume, $V_{PM}$, can be written as

$$V_{PM} = \pi [r^2(x_2 - x_1) - \frac{x_2^3 - x_1^3}{3} - g_1^2(\frac{x_2^3 - x_1^3}{3}) - g_1^2 \frac{(x_2^2 - x_1^2)}{2} - k_n(x_2 - x_1)]$$

...........(6.15)

6.6.2 Determination of $g_1$, $k_n$, $x_1$, and $x_2$

The gradient of the line $L_1$ is given by

$$\tan \theta = \frac{w'}{\sqrt{r^2 - w'^2}}$$ (6.16)

and $w'$ is the radius of the clean opening, i.e. $w/2$. However, since the line $L_1$ is arbitrarily chosen, the variation is considered in the calculation of $g_1$, the gradient of the line parallel to the surface of the deposit. The tangent of line $L_1$, i.e. $g_1$ is determined as follows

$$g_1 = \tan(\theta + \frac{\pi}{2} - \psi)$$

$$= \frac{\cos \psi \cos \theta + \sin \psi \sin \theta}{\sin \psi \cos \theta - \cos \psi \sin \theta}$$
Chapter 6 Modelling of loaded ceramic foam filters

\[ g_1 = \frac{1}{\tan \theta + \tan \psi} \cdot \frac{\tan \psi}{\tan \theta - 1} = \frac{\sqrt{d^2 - w^2} + \tan \psi}{\frac{w}{\frac{\sqrt{d^2 - w^2}}{w}} \tan \psi - 1} \quad (6.17) \]

The intersection \( k_n \) between line \( L_1' \) and the y-axis, and \( \sin \theta \) is equal to \( w'/r \). In order to determine \( k_n \), the following relationship is derived from the Figure 6.10 to give

\[ \frac{k_n}{\sin(\frac{\pi}{2} + \psi)} = \frac{r_1}{\sin(\theta - \psi)} \]

or

\[ k_n = \frac{r_1}{\sin \theta - \tan \psi \cos \theta} \]

and

\[ k_n = \frac{\frac{w}{d} \cdot \frac{1}{\sqrt{d^2 - w^2} \tan \psi}}{w} \quad (6.18) \]

The value of \( x_1 \) can be determined by equating Equations (6.9) and (6.12) and solving the resulting equation for \( x \), i.e.

\[ \sqrt{r^2 - x^2} = g_1 x + k_n \quad (6.19) \]

yielding:
From Figure 6.11, the intersections of the line L₁ and the circle are the values of the Equation 6.20, i.e. $x₁$ and $x'_1$. The solution required in the analysis is $x₁$, which is the least of the solution of the equation, thus,

\[
x = \frac{-g₁kₙ \pm \sqrt{g₁²r² - kₙ² + r²}}{g₁² + 1}
\]  

(6.20)

Finally, the value of $x₂$ which is the intersection between $L₁$ and $L₂$ is determined from the intersection of $L₂$ with the $x$-axis. That is:
The value of the opening \( h \) in a loaded filter is determined by substituting the value of \( x_2 \) in the equation representing \( L_1 \), Equation (6.12), i.e.

\[
x_2 = \sqrt{r^2 - w^2} = \frac{\sqrt{d^2 - w^2}}{2}
\]  

(6.22)

Furthermore, since the PM occupies part of the void in the foam filter, the porosity is the ratio of the difference in volume of the initial void and the volume occupied by PM to the volume of the filter (see section 6.3.1), thus:

\[
\varepsilon_n = \varepsilon - \frac{\sigma}{1 - \varepsilon_p}
\]  

(6.24)

Having derived the correlation between the initial parameters and the PM loaded parameters of the ceramic foam filter, the next subsection describes the determination of the volume of the PM deposited per unit foam cell.

### 6.6.3 Calculation of volume of PM per cell

Considering a unit volume of foam filter, the number of cells \( N \) is again written as follows:
where \( V_{\text{CELL}} \) is the cell volume. The volume of PM per unit cell is the volume of PM per unit filter volume, which is the specific filter deposit \( \sigma \), divided by the number of cells in the unit filter volume, i.e.

\[
V_{PM} = \frac{\sigma}{N} \tag{6.26}
\]

Substituting \( N \) in Equation (6.26), the expression for the PM volume per cell is:

\[
V_{PM} = \frac{\sigma V_{\text{CELL}}}{\varepsilon} \tag{6.27}
\]

Finally, by equating Equations (6.15) and (6.27) the value of the window size corresponding to a given load can be calculated after determining the perpendicular distance from the surface of deposit to the centre of the cell \( r_1 \).

The determination of \( r_1 \) is by iteration shown by the flow diagram in Figure 6.12. Calculation of the variables are summarised in Appendix VIII.2.
Figure 6.12 Flow diagram of the calculation of loaded filter window diameter $w_n$
The range of values of $r_1$ is defined by:

$$\left[\frac{d}{2} - \frac{w^2}{\sqrt{d^2 - w^2}}\right] \leq r_1 \leq \frac{d}{2} \tag{6.29}$$

The calculated value of the window diameter of the loaded foam filter $w_n$ and the new porosity are applied to the clean filter MOM model (Equation 5.20 in Chapter 5) to predict the pressure gradient for a given filter load.

The next section describes the validation of the improved PM loaded MOM model.

### 6.7 VALIDATION OF IMPROVED PM LOADED FOAM FILTERS MODEL

Using the PM loads reported in Table 6.2 the new filter parameters corresponding to each sample were calculated, which are the porosity and the window diameter. The calculated values were then applied to the MOM expression used for the prediction of pressure gradients of clean foam filters. The resulting expressions could then be compared to the experimental data graphically. Figures 6.13 to 6.15 show graphs of pressure gradient versus fluid flow rate comparing the new PM loaded ceramic foam filter model to the experimental data.
Figure 6.13 Graph of pressure gradient vs. fluid flow rate through PM loaded ceramic foam filters, comparing model to experimental data

Figure 6.14 Graph of pressure gradient vs. fluid flow rate through PM loaded ceramic foam filters, comparing model to experimental data
Figure 6.15 Graph of pressure gradient vs. fluid flow rate through PM loaded ceramic foam filters, comparing model to experimental data

Results show that the error in predicting pressure gradients of PM loaded gelcast ceramic foam filters using the new mathematical model is generally better than 35%.

6.8 CONCLUDING REMARKS

The conclusions drawn from this chapter of the work are the following:

A mathematical model has been developed to predict pressure gradients of PM loaded ceramic foam filters. The development of the model is based on the work of Tien and Payatakes (1979). The mathematical model is an adaptation of MOM model to include PM loading by correlating the microstructure of the clean ceramic foam filter to that of the PM loaded ceramic foam filter.
The mathematical model was calibrated using experimental data from PM loaded gelcast ceramic foam filter samples and results were encouraging, indicating that model can be used as a tool for predicting pressure gradients of PM loaded gelcast ceramic filters and for filter design.

The research has validated the PM loaded foam filters mathematical model for lightly loaded foam filters. The mathematical model prediction of pressure gradients of highly loaded foam filters maybe higher than the practical results since, disengagement of deposited PM increases with increased PM load, which can occur more around the windows. Consequently, the upstream surface area inclination with respect to the radius of the cell passing through the edge of the window will be different from the assumption in this thesis. Hence, there is a potential need for the study of heavily loaded foam filters.

The next chapter describes the application of the mathematical models developed in this work to diesel particulate filter design.
CHAPTER 7 APPLICATION OF MATHEMATICAL MODELS

7.1 INTRODUCTION

The work presented in this thesis has sought to provide mathematical models that may be used as tools for understanding the fluid flow through gelcast ceramic foam filters that can be used for design optimisation. In Chapters 3, 4, 5 and 6, the development of new mathematical models for the predictions of pressure gradients of clean and PM loaded gelcast ceramic foam filters were presented.

There are many engineering applications in which these models could be used, which include the evaluation of the dependence of the pressure drop on the cell diameter, porosity and filter surface area for given flow rates. This chapter shows how the models could be used to calculate optimum filter lengths and surface areas which correspond to target filtration efficiencies.
Chapter 7 Application of mathematical models

From these results, a suitable geometry of gelcast foam filter for a diesel engine is calculated as an example of the models’ use.

Recalling the design objectives of the DPF:

- The filtration efficiency of the filter must be such that the diesel particulate emission meets the emission limits.
- The increase in engine backpressure imposed by the DPF due to the foam filter structure and PM accumulation should be minimised so that penalty in engine fuel economy is minimised.
- Since there is space and weight limitation for additional components in a vehicle, the size of the DPF unit should be such that it can be mounted within the space available; essentially, it should be small as possible.

The filtration efficiency of the filter is enhanced by the high internal surface area of the ceramic foam filter structure (Schmahl and Davidson, 1993). Hence, increasing the thickness of the filter or reducing the cell size will increase the internal surface area, which will consequently further increase the filtration efficiency. Conversely, decreasing the cell diameter or increasing the filter thickness will increase fluid flow restriction, thus, increase in the flow pressure drop. The size of the DPF (which includes the canister) depends on the dimensions and shape of the filter material. Therefore, the aim in this chapter is to illustrate how the models can be used to determine an optimum filter thickness and filter frontal surface area that will meet the required filtration conditions, then, choose a suitable filter configuration that can be accommodated within the available space.

The dimensions of the filter are determined using experimental data reported by other authors and the mathematical models proposed in this thesis, while
the filter configuration is by choosing a geometry that fits into a recommended volume.

Since the filter thickness (length) is a major parameter that affects both the filtration efficiency and the pressure drop of the filter, the optimum filter thickness with respect to the filtration efficiency is first determined, then, using the mathematical models, the filter frontal surface area can be determined. Finally, a suitable filter configuration can be determined using the dimensions obtained.

Figure 7.1 illustrates the steps considered for proposing a suitable filter configuration.

Figure 7.1 Flow diagram for filter design process

In this thesis, the engine used as the example for the filter dimensions calculation was an Onan heavy-duty diesel engine, however, any engine might be used in a similar way. Table 7.1 shows the main specifications and exhaust gas data of the engine (detail specification in Appendix X).
Table 7.1 Engine specification and exhaust gas data

<table>
<thead>
<tr>
<th>Engine data</th>
<th>Onan LPW3, DI diesel-fuelled, naturally aspirated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement volume (l)</td>
<td>1.4</td>
</tr>
<tr>
<td>Over speed limit (rpm)</td>
<td>2100 ± 50</td>
</tr>
<tr>
<td>Exhaust gas flow at rated load (m³ s⁻¹)</td>
<td>0.05</td>
</tr>
<tr>
<td>Exhaust Temperature (°C)</td>
<td>569</td>
</tr>
<tr>
<td>Max back pressure (kPa)</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Considering the limiting conditions, i.e. the gas pressure is 5 kPa and the temperature is 569 °C, the density of the exhaust gas $\rho$ can be calculated with the ideal gas equation, assuming that the exhaust gas is an ideal gas and the atmospheric pressure is 1 bar. Therefore,

$$\rho = \frac{5000 + 100000}{286 \times (569 + 273)}$$

$$= 0.44 \text{ kg m}^{-3}$$

Similarly, at the same limiting temperature the viscosity of the gas can be determined using a gas viscosity calculator (see www.LMNOeng.com), i.e. for a temperature of 569 °C the viscosity of the gas $\mu$ is,

$$\mu = 3.841 \times 10^{-5} \text{ kg m}^{-1} \text{ s}^{-1}$$
7.2 DETERMINATION OF OPTIMUM FILTER THICKNESS

As earlier stated, there is lack of literature on filtration efficiency modelling on ceramic foam filters and gelcast ceramic foams in particular. Although, there are some reported research work on foam filter filtration modelling (Acosta and Castillejos, 2000, Xinyun et al., 2000, Pontikakis et al., 2000, Ambrogio et al., 2001), there is presently no published mathematical model that can consistently predict filtration efficiency of gelcast ceramic foam filters.

Consequently, reports that can be considered reliably and could be applicable to ceramic foam filter design are based on experimental results. It can be recalled that in deep bed filtration the efficiency increases with the filter thickness (length), due to deep bed filtration mechanism in which the whole body of the filter acts to trap PM. Tutko et al (1984) Mizrah et al (1989) and most recently Hughes et al (2002) reported that filtration efficiency increases with filter thickness up to a point beyond which only modest gains in efficiency are noted.

Tutko et al (1984) studied experimentally reticulated ceramic foam filters and reported the influence of cell diameter and filter length for a given porosity on the filtration efficiency. A summary of their experiments for a ceramic filter of porosity 85% and cell diameter of 0.420 mm is illustrated in Figure 7.2.

These show that the filtration gains reduce as the filter thickness is increased. For example, there is reduction of filtration gain of only 5% as the length was increased from 83.8 mm to 125.7 mm when compared to 15% gain as the filter thickness is increased from 41.9 mm to 83.8 mm. Furthermore, they reported that the reduction of the cell size from 0.42 mm to 0.254 mm produced a filtration gain of over 12%.
Hughes et al (2002) work was based on experimental work on gelcast ceramic foam filters. Their results from the gelcast foams of a given cell diameter show a relatively higher filtration efficiency compared to the reticulated ceramic foam of smaller cell size, presented by Tutko et al (1984). Figure 7.3 shows a result adapted from the work of Hughes et al (2002) demonstrating the correlation between the filtration efficiency of gelcast ceramic foam filters and the filter thickness.

The graph shows that there is no significant increase in the filtration efficiency of the gelcast ceramic foam filters beyond a filter thickness of 60 mm. It can be seen that the gelcast foam filter of length 30 mm yields filtration efficiency of > 75%. However, Hughes et al (2002) reported that for a given superficial velocity, the filtration efficiency can be increased by > 30% by decreasing the porosity from 94% to 87%. Furthermore, they also demonstrated that the filtration efficiency can be increased by > 20% by reducing the cell diameter.
by 100 \( \mu \) m. By extrapolation, it can be estimated that a gelcast foam filter of length 30 mm, cell diameter of 0.25 mm and porosity of 85% has a filtration efficiency that is > 90%.

![Figure 7.3 Graph comparing filtration efficiency and filter length for foam of cell diameter 0.70 mm and porosity 90%. (Adapted from Hughes et al, 2002)](image)

Based on the results reported by Hughes et al (2002) and the analysis above, a filter thickness of 30 mm and porosity of 85% are therefore adopted and used for the models application, to determine the optimum filter frontal surface area.

Having adopted a filter thickness for the design, it is necessary to understand the effect of varying the cell diameter or the filter frontal surface area on the pressure drop across clean gelcast ceramic foam filters and also loaded foam filters. Hence, the next section describes the effect of varying the filter frontal surface area and cell diameter on clean gelcast ceramic foam filters.
Chapter 7  
Application of mathematical models

7.3 EFFECT OF SURFACE AREA AND CELL SIZE VARIATION ON FILTER PRESSURE DROP USING CLEAN FILTER MODEL

The objective in this section is to evaluate the effects of varying the cell diameter and the filter frontal surface area on the pressure drop across clean foam filters for a given porosity and filter thickness. The study of the correlation of the cell diameter and the pressure drop for given values of filter frontal area gives the limit of improving on the filter filtration efficiency in reducing the cell diameter of a clean foam filter. However, since the range of the foam filter cell diameter is known, and can be arbitrarily fixed, it is necessary to first determine the optimum filter frontal surface area, which can then be used in studying the effect of the variation of the cell diameter on the pressure drop across the filter.

7.3.1 Effect of filter frontal surface area variation on pressure drop

The MOM model for predicting pressure gradients of clean ceramic foam filters presented in Chapter 5 is

\[
\Delta p = \frac{\alpha \mu 4(1 - \varepsilon)^2}{D^2 \pi \varepsilon^3} S^2 Q + \frac{(1 - \varepsilon^2 / d^4)}{\sqrt{d^2 - w^2}} \left[ \frac{d^2}{\beta \pi \varepsilon D^2 w^3} \right]^2 \varepsilon^2 \rho Q^2
\]  (7.1)

where \( \Delta p \) is the pressure drop, \( d \) is the cell diameter, \( w \) is the cell diameter, \( Q \) is the exhaust flow rate (in m\(^3\) s\(^{-1}\)) and \( D \) is the filter diameter. The pressure drop is plotted against the filter surface area (\( \pi D^2 / 4 \)) with the exhaust gas flow rate at 5.0 \( \times \) 10\(^{-2} \) m\(^3\) s\(^{-1}\). Filter frontal surface areas are varied for cell diameters of 0.25, 0.35 and 0.40 mm.
Using the MOM model for clean foam filters the pressure drops of the foam filters (3 different cell diameters) are calculated for filter frontal surface areas from 0.01 to 0.3 m². The pressure drops considered for the graph is from 0 to 6 kPa, since the maximum back pressure of the engine is 5 kPa.

Figure 7.4 show the graph of pressure drop versus the filter frontal surface area in m². The optimum filter frontal surface area corresponding to each cell diameter is determined by projecting the maximum back pressure horizontally to intercept the curves. The points of interception are the optimum frontal surface areas.

![Graph of pressure drop vs. filter frontal surface area for cell diameters of 0.25, 0.35 and 0.4 mm](image)

**Figure 7.4** Graph of pressure drop vs. filter frontal surface area for cell diameters of 0.25, 0.35 and 0.4 mm

Table 7.2 show the optimum frontal area corresponding to each filter cell diameter.
Chapter 7

Application of mathematical models

<table>
<thead>
<tr>
<th>Cell diameter (mm)</th>
<th>Filter frontal surface area (m²)</th>
<th>Filter diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>0.029</td>
<td>192</td>
</tr>
<tr>
<td>0.35</td>
<td>0.022</td>
<td>167</td>
</tr>
<tr>
<td>0.40</td>
<td>0.020</td>
<td>160</td>
</tr>
</tbody>
</table>

Table 7.2 Optimum filter frontal surface area for three foam filters of different cell diameters

7.3.2 Effect of filter cell diameter variation on the pressure drop

Using the same MOM model, the pressure drop $\Delta p$ can be plotted against the cell diameter $d$. The exhaust gas flow rate is same as considered above. However, in this example, the cell diameter is varied for filter frontal surface areas of 0.04 to 0.1 m². Figure 7.5 shows the graph of pressure drop versus foam cell diameter. The graph shows that the pressure drop increases with decrease in foam cell diameter.

![Graph of pressure drop vs. foam cell diameter for filter frontal surface areas of 0.04 and 0.1 m²](image)

Figure 7.5 Graph of pressure drop vs. foam cell diameter for filter frontal surface areas of 0.04 and 0.1 m²
Using a cell diameter of 0.25 mm in Figure 7.5 the corresponding values of pressure drop can be found. The pressure drops can then be compared to the maximum engine back pressure. The results can then be used to determine a suitable filter frontal surface area.

Table 7.3 is a summary of the results that shows that the filter of frontal surface area of 0.02 m² and cell diameter 0.25 mm has a minimum pressure drop of 8.10 kPa, which is significantly more than the maximum engine back pressure. Hence, these dimensions will not be suitable for application in this case. However, for a filter of frontal surface area > 0.03 m² and cell diameter of 0.25 mm, the pressure drop across the filter is less than half the maximum engine back pressure.

<table>
<thead>
<tr>
<th>Filter frontal surface area (m²)</th>
<th>Foam cell diameter (mm)</th>
<th>Minimum pressure drop (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.02</td>
<td>0.25</td>
<td>8.10</td>
</tr>
<tr>
<td>0.03</td>
<td>0.25</td>
<td>2.5</td>
</tr>
<tr>
<td>0.04</td>
<td>0.25</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 7.3 Minimum pressure drop for a given filter frontal surface area corresponding to cell diameter of 0.25 mm

Having evaluated the effect of varying the cell diameter and filter frontal surface area on the pressure drop, the results are summarised below.
7.3.3 Summary of model application on clean filter

Using the MOM model, correlation between the filter frontal surface area, the foam cell diameter and the pressure drop for given exhaust gas flow rate has been demonstrated. Using these results, the correlation of the foam cell diameter and the filter frontal surface area for the maximum engine back pressure of 5 kPa can be estimated. For example, the filter frontal area for a filter of porosity 85%, thickness of 30 mm and cell diameter of 0.25 mm, should be 0.03 m² to maintain the maximum engine back pressure of 5 kPa when the exhaust gas flow rate is 0.05 m³ s⁻¹.

Figure 7.6 shows graph of filter frontal area in m² versus the foam cell diameter for a maximum engine back pressure of 5 kPa for an exhaust flow rate of 0.05 m³ s⁻¹.

![Graph of filter frontal surface area vs. foam cell diameter](image)
However, since the filter will also operate with PM loading, there is the need to understand the effect of PM load on the sizing of the filter. Thereafter, the minimum filter surface area can be recalculated for PM loaded cases.

The next section presents the effect of the PM load on the pressure drop across gelcast ceramic foam filters.

7.4 EFFECT OF PM LOADING ON FILTER SIZING

Using the PM loaded MOM model presented in Chapter 6, the filter frontal surface area suitable for the diesel engine with maximum back pressure of 5 kPa and exhaust gas flow rate of 0.05 m$^3$ s$^{-1}$ can be evaluated when loading the filter with PM. The correlation between the filter frontal surface area and the pressure is evaluated for various values of specific PM load. Although, the ceramic foam filters have high PM load capacity, the load can be limited by the maximum operating engine back pressure. The PM loading limit recommended for standard DPF substrates is quoted as approximately 7 – 8 g l$^{-1}$ (Hobfeld and Kaiser, 2003).

In order to evaluate the effect of PM loading on the filter sizing, the process starts with the correlation of the pressure drop and the filter frontal surface area for various values of PM load, ranging from 0.2 to 8.0 g l$^{-1}$. Using this relationship, a relationship can be found between the PM load and the filter frontal surface area. This final relationship can be used to determine the filter frontal area for a given PM load.
7.4.1 Correlating the pressure drop to the filter frontal surface area

Using the loaded MOM model, the pressure drop across foam filter of porosity 85% and cell diameter 0.25 mm was calculated for the frontal surface areas, ranging from 0.02 to 0.2 m² corresponding to given PM loads. The PM loads considered for example in this case were 0.2, 0.8, 1.3, 2.0, 5.0 and 8.0 g l⁻¹. Using the calculated values, the pressure drop was plotted against the filter frontal surface area as shown in Figure 7.7.

![Graph of pressure drop vs. filter frontal surface area](image)

Figure 7.7 Graph of pressure drop vs. filter frontal surface area for various value of PM load, where the porosity and cell diameter of the foam are 85% and 0.25 mm respectively

The maximum engine back pressure, 5.0 kPa, can then be projected on the curves to give the values of filter frontal surface area at the intersections. For example, the maximum PM load, 8.0 g l⁻¹, considered in this case corresponds to a frontal surface area of 0.14 m² and the minimum PM load, 0.2 g l⁻¹ corresponds to a surface area of 0.032 m².
Using the above, a relationship was found, which is illustrated in Figure 7.8 to determine the frontal surface area of the foam filter for a given PM load, where the foam porosity is 85% and the cell diameter is 0.25 mm. For example, from the graph (Figure 7.8) there is a PM load limit of 5.0 g l\(^{-1}\) if the filter frontal surface area is 0.081 m\(^2\).

![Figure 7.8 MOM loaded model graph of filter frontal surface area vs. PM load for a gelcast ceramic foam of porosity 85% and cell diameter 0.25 mm. The exhaust gas flow is 0.05 m s\(^{-1}\) for an engine maximum back pressure of 5.0 kPa.](image)

**7.4.2 Summary of effect of PM loading on filter sizing**

The PM loaded foam filter mathematical model has been used to evaluate the relationship between the pressure drop and the filter frontal surface area for various PM loads. The correlation of the filter frontal surface area and the PM load has been presented for the maximum engine back pressure of 5.0 kPa. Using the relationship, the filter frontal surface area can be determined for a given PM load.
7.5 CONCLUDING REMARKS FOR FILTER SUBSTRATE
DIMENSIONING

Using the mathematical models for the prediction of pressure gradients of
gelcast ceramic foam filters, the minimum surface area of the filter that meets
the engine back pressure limit can be determined. It has been shown that a
mathematical model for predicting the pressure drop of PM loaded filters is
required to determine the functional dimensions of the filter. Table 7.4
summarises the results of the investigation showing the dimensions and
parameters of the filter substrate for an Onan heavy-duty 1.4 l diesel engine.

<table>
<thead>
<tr>
<th>Filter porosity (%)</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Filter cell diameter (mm)</td>
<td>0.25</td>
</tr>
<tr>
<td>Filter length (mm)</td>
<td>30</td>
</tr>
<tr>
<td>Filter frontal surface area (m²)</td>
<td>0.08</td>
</tr>
<tr>
<td>Maximum PM load (g l⁻¹)</td>
<td>5.0</td>
</tr>
</tbody>
</table>

Table 7.4 Specifications for gelcast ceramic foam filter as DPF substrate for
Onan heavy-duty diesel engine of 1.4 l

For an Onan heavy-duty, naturally aspirated, 1.4 l diesel engine with an
exhaust gas flow rate of 0.05 m³ s⁻¹, the minimum filter surface area for a
maximum PM load of 5 g l⁻¹ is therefore 0.08 m². However, for a DPF unit
using a continuous regeneration system, the filter surface area can be reduced,
depending on the maximum PM load that is present in the DPF at any
particular time.

Having determined the foam filter dimensions and parameters that meet the
operational limits, the next section presents the development of the filter
shape that can be contained in an optimally sized canister.
Chapter 7 Application of mathematical models

7.6 GELCAST CERAMIC FOAM FILTER CONFIGURATION

Recalling that the filter size is limited by the available space in the vehicle, the volume of a conventional wall flow filter is \( \phi 144 \, \text{mm} \) and length 152 mm. The volume of a ceramic foam filter with cylindrical configuration calculated from the recommended frontal area of 0.08 \( \text{m}^2 \) and thickness of 30 mm is 2.4 l, which is more than that of the volume of the conventional wall flow filter at 2.32 l. Using the foam filter specification recommended for a PM load capacity of 50 g \( \text{l}^{-1} \), a cylindrical shape can be adopted where the filter diameter \( D \) is 319 mm, calculated from a frontal surface area of 0.08 \( \text{m}^2 \).

The objective in this section is to present an example of geometry of foam filter material that may optimally reduce the outer diameter of the canister.

A possible shape to configure the foam filter is the “top hat” (Mizrah et al, 1989). Figure 7.9 is the cross sectional diagram of a top hat design where the thickness of the foam is \( L \).

Figure 7.9 Example “Top Hat” cross section of ceramic foam DPF prototype
Assuming that the internal diameter of the canister can be maintained at \( \leq 5.66'' \), i.e. 144 mm, the gap \( s \) can be fixed \((0 < s < D/2 - L)\), then the length \( L_e \) can be tuned to maintain the required frontal surface area \( A_{\text{FILT}} \). The total frontal surface area of the top hat can be defined with the expression

\[
A_{\text{FILT}} \geq \frac{D^2}{4} \pi + (L_e - L)(D - 2s - 2L)\pi
\]  

(7.2)

After fixing the filter diameter \( D \) and the gap \( s \) the inner canister length \( L_e \) can be selected to be able to contain the top hat shaped foam filter. For example, if the filter diameter is 144 mm and the gap \( s \) is 10 mm, then the length \( L_e \) is found to be 347 mm for the frontal surface area of 0.08 m\(^2\). However, increasing the filter diameter to 185 mm yielded a canister length \( L_e \) of 191 mm.

In summary, recalling the attributes of durability, strength, etc, of the gelcast ceramic foam filters, they may be an acceptable material for use in DPFs production. The top hat configuration allows the filter material to be shaped according to the available space in the engine. The Onan heavy-duty 1.4 l diesel engine can be fitted with a DPF of top hat gelcast foam filter material, where the foam filter length is 191 mm, internal diameter of the canister is 185 mm, filter thickness is 30 mm and the foam cell diameter is 0.25 mm.

Other foam filter configurations were suggested by Gabathuler et al (1991). They studied the performance of a variety of reticulated ceramic foam filter configurations, employing stationary engines as well as vehicle testing methods. One configuration was the Z-flow shape (see Figure 7.10) which is not unlike a large wall-flow geometry.
Figure 7. 10 Axial cross section through a Z-flow Filter

7.7 CONCLUDING REMARKS

It was explained in earlier chapters that the gelcast ceramic foam can be shaped into almost any configuration to meet the available space in the engine. The gelcast ceramic foam filter can be made in the top hat shape or the Z-flow suggested above. If there is need to further reduce the filter volume, the PM load limit can be reduced or the filter thickness is reduced while improving the trapping efficiency of the filter internal surface. Another approach is by increasing the cell diameter, which will reduce the pressure drop. With the new models proposed in this thesis, all the parameters can be investigated when optimising a DPF using gelcast ceramic foams.
The next chapter presents the major conclusions of the research work and the suggested future work.
CHAPTER 8  CONCLUSIONS AND SUGGESTIONS FOR FUTURE WORK

This thesis has presented the development of new mathematical models for predicting flow pressure gradients in clean and PM loaded gelcast ceramic foam filters. The gelcast ceramic foam is a novel filter material that is being considered as an alternative diesel particulate filter material due to the improved strength and the characteristic deep filtration mechanism it exhibits. The research has also included design of novel rigs and experiments for the generation of data used for the calibration and validation of the models.

This final chapter presents the major findings and conclusions drawn from the study of fluid flow through gelcast ceramic foam filters. It also presents recommendations for future work in this area.
8.1 CONCLUSIONS

This research has resulted in the development of new mathematical models that contribute to the understanding of fluid flow through gelcast ceramic foam filters. The models can be used as tools for diesel particulate filter design.

The major conclusions of this research are as follows:

1. For the first time, this research has developed mathematical models for predicting pressure gradients of clean gelcast ceramic foam filters, where the correction coefficients are kept constant. Researchers have previously proposed mathematical models for the prediction of pressure gradients in ceramic foam filters, but did not succeed in maintaining constant correction coefficients. Although some of these researchers later proposed empirical expressions for the correction coefficients, they reported that the resulting models could not be generalised.

Firstly, an Extended Ergun Mathematical (EEM) model was developed by adapting a model first proposed by Ergun and Orning (1949) for the prediction of pressure drop across packed columns, to that of the gelcast ceramic foam structure. The EEM model was shown to have an error of predicting pressure gradients that is better than 30%, but it required tuning of the correction coefficients on ceramic foam samples. A new model was therefore developed using the basic concepts of fluid flow called the Multiple Orifice Mathematical (MOM) model.

The MOM model was developed by considering fluid flow through rows of cells, where the connecting windows were considered to be orifices. This model had an error in predicting pressure gradients of
gelcast ceramic foam filters that is better than 25% and importantly, did not require tuning the correction coefficients using foam samples.

2. In order to calibrate the mathematical models for the prediction of pressure gradients of clean gelcast ceramic foam filters, novel physical scale model foams were developed and used. The physical scale model foams were designed based on the conceptual model of the foam structure and manufactured using the novel application of stereolithography rapid manufacturing techniques.

The MOM model was calibrated using fluid flow experimental data from the physical scale model foams, where the correction coefficients on the model were obtained.

3. The research has also developed a novel mathematical model for predicting pressure gradients of particulate matter (PM) loaded gelcast ceramic foam filters. Previous investigators have proposed loaded filter models for ceramic foams. However, these latter models were not generalised as they required the experimental determination of the permeability of the PM loaded ceramic foam or empirical correction factors.

4. The PM loaded filter mathematical model was developed by adapting the clean foam filter MOM model to PM loaded filters by studying the morphology of PM loaded foam filters. Consequently, correlations were derived between the clean foam filter parameters and those of the PM loaded foam filter.

Initially, the study of the PM loaded foam filter morphology was based on the assumption that the PM deposition was uniformly distributed in the foam structure, referred to as 'smooth mode'. However, the
resulting mathematical model based on a smooth mode was identified to be inadequate for the prediction of pressure gradients of PM loaded gelcast ceramic foam filters. An improved model was therefore developed based on an observation from scanning electron microscope photographs that the PM deposition was mainly at the cell windows, referred to as a 'localised mode'.

The determination of the porosity of the PM loaded foam was achieved by accounting for the flow space occupied by the deposited PM. The cell diameter of the PM loaded foam filter, based on the localised mode, was assumed to be unaffected by the deposit while the window diameter decreased with increased PM loading.

5. It was demonstrated in this thesis that the mathematical models for predicting pressure gradients of clean and PM loaded gelcast ceramic foam filters developed in this research can be used to determine the optimum dimensions of the gelcast ceramic foam for diesel particulate filter design. As an example, gelcast foam filter dimensions were recommended for 1.4 litre heavy-duty diesel engine used for power generation, using the mathematical models.

8.2 RECOMMENDATIONS FOR FUTURE WORK

This work has led to a number of interesting areas where further work could be directed. These include:

1. This research has improved the understanding of fluid flow through gelcast ceramic foams from the correlation of the pressure gradients and the fluid flow rates. However, it could be possible to increase the
accuracy of the model by extending the study to the arrangement of foam cells other than the face centred lattices used in the present work.

2. This work used a novel conceptual foam model to effectively develop the mathematical models for the prediction of pressure gradients of the clean and PM loaded foam filters. Similar concepts can be used to develop mathematical models for the prediction of filtration efficiency of gelcast ceramic foam filters.

3. Foam filter pressure gradient and filtration efficiency models could be linked with filter regeneration models so that simultaneous PM loading and regeneration process can be modelled under dynamic engine cycle conditions.
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Research and “Scaling up” Productions,


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References


APPENDIX I: SPECIFICATIONS OF EQUIPMENT FOR EXPERIMENTATION
## I.1 LEISTER HIGH PRESSURE BLOWER ROBUST

![Leister high pressure blower](image)

Figure I.1 Leister high pressure blower

### Technical data Blower ROBUST

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<tr>
<td>Outside diameter air outlet</td>
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The FC0332 primarily developed for monitoring the very low differential pressures between clean rooms and corridors in the pharmaceutical industry and in microelectronic manufacture. They are equally suitable for a wide variety of other applications.

The FC0332 is ideal as a replacement for liquid manometers or mechanical gauges when upgrading to a data acquisition in order to meet new regulations. It is also suitable as a general purpose two-wire transmitter in the process industry, where the static pressure and weatherproof features of the FC0352 are not required.

- Clear 'Easy Read' Display with units of your choice
Appendices

- High Sensitivity and .25% reading accuracy
- Low Cost
- Simple to install
- Programmable
- Plug in electrical connection
- Replaces Mechanical Dial Gauges

Many of these devices are calibrated on the Furness Controls FRS4 low pressure primary standard. It has an accuracy of better than .01% of reading with a resolution of .01 Pa. (.00004 "Hg"). This primary standard is the only instrument capable of calibrating the full range of Furness Controls products.

1.3 COMARK 6200 MICROPROCESSOR THERMOMETER AND THERMOCOUPLE

Figure 1.2 Comark 6200 ten-channel Microprocessor Thermometer
Appendices

Specification

Ten Input Thermometers type 6200 6300 6400 (BS4937) 6201 6301, 6401 (DIN 43710)

General Measurement

Measurement System: Auto zero, auto calibrating and integrating under microprocessor control.

Maximum Input Range: -10 mV to 75 mV

Resolution: On temperature ranges, 0.1 up to 700°C, 1°C above
On microvolt range, 1 μV

Linearity and Zero Error: Max. error ± 10 μV ± 0.5% of reading up to 75 mV.

Thermocouple Characterising Ranges:

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Notes:

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<td>-200 to -130°C ± 0.2°C</td>
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<td>-130 to 600°C ± 0.15°C</td>
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Thermocouple accuracy:

0°C, 0°F, 0 K (Kelvin)

Scales: 0.5 second to full accuracy

Reading rate: 4 per second

Warm-up: 10 minutes to full specification

Working ambient temperature range: -5 to +40°C

Supply: 110 to 120 V ± 10% or 220 to 240 V ± 10%, 50 to 60 Hz.

Other specifications: Generally conforms to IEC 348/IEC 68.
Appendices

Overall dimensions of Comark 6200

Excluding Handle - 254 x 99 x 236 mm
Including Handle - 254 x 99 x 325 mm
Weight - 2.4 kg Approximately

I.4 TRANSDUCER READOUT FOR READING PRESSURE DROPS

Figure 1.4 Transducer readout unit, FCO70

General information for FCO70

The FCO70 unit provides +15 V and -15 V DC supply to operate transducer. Maximum total 70 mA load.
A front panel meter provides a display of signal from transducers connected to the rear panel mounted sockets. It can be scaled from 0 to 100% or in various engineering units. A front panel switch allows selection of any one signal from the six transducers connected to the sockets. This signal is also available on the front panel recorder output socket.

A zero control for each transducer adjusts the signal by up to ± 5% to eliminate any zero offset.

The fast/slow switch slow meter response when set to meter slow. The FCO70 unit should be connected to a suitable mains supply via rear panel mains plug and socket.
APPENDIX II: CONSISTENCY OF EXPERIMENTAL DATA
Appendices

Alumina foam filter sample of $D = 49.94$ mm, $L = 45.18$ mm, $d = 0.261$ mm

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215
Figure I.1 Graph of pressure gradient vs. fluid flow rate comparing different experimental data on the same ceramic foam sample (alumina foam)
APPENDIX III: TABULATION OF EXPERIMENTAL DATA AND EEM MODEL RESULTS
Appendices

III.1 Pressure gradients prediction of ceramic foam filters

Data collected in the experiments are:

- Pressure drop across the orifice ($P_{or}$) in Pa, to determine the flow rate of the air, using the orifice formula described in Chapter 4.
- Pressure drop across the samples ($P_{fil}$) in Pa, used for calibration and validation of the mathematical models.
- Temperature ($T_f$) of the fluid $\circ{C}$, used for the calculation of the fluid density.
- Gauge pressure ($P_g$) of the fluid in Pa, used for the calculation of the fluid density

The density of the fluid is calculated from the relationship below:

$$\rho = \frac{P_A}{RT}$$

(1.1)

where $P_A$ is the absolute pressure which is the sum of the gauge pressure and the atmospheric pressure $P_0$, i.e. $P_A = P_0 + P_g$. $R$ is the universal gas constant taken as 286 J kg\(^{-1}\) K and $T$ is the absolute temperature in K, equivalent to 273 + $T_f$.

The pressure gradient ($P_{grad}$) is the pressure drop per unit length ($L$) of the filter sample, i.e. $P_{grad} = P_{fil}/L$. The calculated pressure gradient ($P_{calc}$) is determined using the mathematical models.

III.2 Model accuracy

The model accuracy is the ratio of the difference in experimental and model pressure gradient to the experimental pre gradient, i.e.
Model Accuracy, \( \text{Ac.} = \frac{P_{\text{ref}} - P}{P_{\text{ref}}} \times 100 \% \)

The model accuracy is calculated for all predicted values, shown in the Tables below.

Similarly, the experimental data and the calculated values, for the validation of the Extended Ergun mathematical model EEM, are presented below.

III.3 Experimental data tabulation

1. Foam sample A44C7, cell diameter = 0.85 mm, porosity = 80%

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<thead>
<tr>
<th>( P_{\text{ref}} ) Pa</th>
<th>( P_{\text{filt}} ) Pa</th>
<th>( T ) °C</th>
<th>( P_{\text{abs}} ) Pa</th>
<th>Density kg m(^{-3} )</th>
<th>Flow rate kg s(^{-1} )</th>
<th>( P_{\text{grad}} ) kPa m(^{-1} )</th>
<th>( P_{\text{calc}} ) kPa m(^{-1} )</th>
<th>Ac %</th>
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## Appendices

### 2. Foam sample A44C6, cell diameter = 0.25 mm, porosity = 86%

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<th>Flow rate kg s$^{-1}$</th>
<th>$P_{\text{grad.}}$ kPa m$^{-1}$</th>
<th>$P_{\text{gcal.}}$ kPa m$^{-1}$</th>
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<th>Pgrad. kPa m⁻¹</th>
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### 6. Foam sample 243AL-F, cell diameter = 0.50 mm, porosity = 87%

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III.A Orifice Discharge Coefficient

The discharge coefficient, $C$, given by the Reader-Harris equation is as follows:

$$C = U + V + W - X + Z$$

where

$$U = 0.5961 + 0.0261 \beta^2 - 0.216 \beta^8 + 0.000521 \left( \frac{10^6 \beta}{Re_D} \right)^{0.07}$$

$$V = (0.0188 + 0.0063A) \beta^{3.5} \left( \frac{10^6}{Re_D} \right)^{0.3}$$

$$W = (0.43 + 0.08e^{-10L1} - 0.123e^{-7L1})(1-0.11A) \frac{\beta^4}{1-\beta^4}$$

$$X = 0.031(M-0.8M^1) \beta^{13}$$
and $\beta$ is the diameter ratio of orifice plate and the pipe respectively, $d/D$ all expressed in millimetres, $Re_D$ is the Reynolds number calculated with respect to $D$, $L_1$ is the ratio of the pressure tapping of upstream measured from upstream face of orifice to pipe diameter $h_1/D$, $M = 2L_2/(1-\beta)$, $L_2$ is the ratio of the pressure tap downstream measured upstream of orifice plate to the pipe $l_2/D$, and $A = \left(\frac{19000\beta}{Re_D}\right)^{0.8}$. The following values were applied to this work as recommended from the British Standard BS EN ISO 5167-2:2003, according to the position of the pressure taps:

$L_1 = 1$

$L_2 = 0.47$

where $\beta_1 = 20/50 = 0.4$ and $\beta_2 = 15/50 = 0.3$
APPENDIX IV: FLUID FLOW RIG TECHNICAL DRAWINGS
IV.1 Technical drawings

1. Filter sample holder
2. Filter sample cover
3. Fluid flow rig piping 1
4. Fluid flow rig piping 2
5. Adaptor connection pipe
6. Pipe connection extension 2
7. Connection pipe 1
8. Pipe connection extension 1

All parts were originally designed using a CAD package (Solid Edge V14)
A

5 mm

597 mm

984.1 mm

M3

150 mm

95 mm

5 mm

115 mm

65 mm

80 mm

4 holes, \( \phi 8 \) on diameter 71.6 mm
(Same on both ends)

External \( \phi 71 \)
Internal \( \phi 65 \)

SECTION A-A

(\( 60^\circ \) on both ends)

Material: Stainless steel

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<th>Tolerance unless otherwise stated ( \pm 0.5 )</th>
<th>Drawn by E.M. Adigio</th>
<th>Course PhD</th>
<th>7/1/04</th>
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<td>-</td>
<td>-</td>
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Title: Filter sample holder

Drawing No. PD070104-1
Material: Stainless steel

All dimensions in millimetres

4 holes, $\phi 10$ mm, spaced $90^\circ$ on diameter 86 mm

4 holes, $\phi 8$ mm, spaced $90^\circ$ on diameter 63.6 mm

$\phi 50$ mm

38.1 mm

Title: Fluid flow rig piping 1

Loughborough University
All dimensions in millimetres

4 holes, φ8 mm, spaced 90° on diameter 63.6 mm

4 holes, φ10 mm, spaced 90° on diameter 86 mm

Mat: Stainless steel

Loughborough University

Title: Fluid flowing piping 2

Drawing No. PD070104-7
4 holes, \( \phi 8 \text{ mm} \), spaced 90° on diameter 72.5 mm

4 holes, \( \phi 8 \text{ mm} \), spaced 90° on diameter 64 mm

SECTION A-A

\( \phi 38.1 \text{ mm} \)

\( \phi 48.3 \text{ mm} \)

Mat: Stainless steel

All dimensions in millimetres

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</thead>
</table>

Drawn by E.M. Adigio

Course: PhD

8/1/04

Loughborough University

Title: Adaptor connection pipe

Drawing No. PD070104-4
Mat: Stainless steel

All dimensions in millimetres

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<th>Material</th>
<th>Tolerance values</th>
</tr>
</thead>
<tbody>
<tr>
<td>stainless steel</td>
<td>± 0.05</td>
</tr>
</tbody>
</table>

Drawn by E.M. Adigio

Course: PhD

Loughborough University

Title: Connection pipe 1

Drawing No. PD070104-5

232
External $\phi 49.7$
Internal $\phi 38.1$

4 holes, $\phi 8$, spaced $90^\circ$ on diameter $63.6$ mm

Mat: Stainless steel
APPENDIX V: TABULATION OF EXPERIMENTAL DATA AND MOM RESULTS
V.1 Pressure gradients prediction of ceramic foam filters

Using the same approach of calculation for the pressure gradients and accuracy in Appendix III, the experimental data and the calculated values, for the validation of the multiple orifice model MOM, were presented.

V.2 Experimental data tabulation

1. Foam sample A44C7, cell diameter = 0.85 mm, porosity = 80%

<table>
<thead>
<tr>
<th>Porif. Pa</th>
<th>Pfilt. Pa</th>
<th>T °C</th>
<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate Kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pcalc. kPa m⁻¹</th>
<th>Ac %</th>
</tr>
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<tbody>
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<td>50.63</td>
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<tr>
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2. Foam sample A44C6, cell diameter = 0.25 mm, porosity = 86%

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<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate Kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pcalc. kPa m⁻¹</th>
<th>Ac %</th>
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<th>T (°C)</th>
<th>P_{abs} (Pa)</th>
<th>Density (kg m^{-3})</th>
<th>Flow rate (kg s^{-1})</th>
<th>P_{grad} (kPa m^{-1})</th>
<th>P_{gcal} (kPa m^{-1})</th>
<th>Ac (%)</th>
</tr>
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<td>1.75E-03</td>
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<td>2503</td>
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### 4. Foam sample A44C4, cell diameter = 0.35 mm, porosity = 86%

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<th>T (°C)</th>
<th>P_{abs} (Pa)</th>
<th>Density (kg m^{-3})</th>
<th>Flow rate (kg s^{-1})</th>
<th>P_{grad} (kPa m^{-1})</th>
<th>P_{gcal} (kPa m^{-1})</th>
<th>Ac (%)</th>
</tr>
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<td>1144</td>
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<tr>
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<td>1332</td>
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<tr>
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<td>1470</td>
<td>0.84</td>
<td>3.10E-03</td>
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### 5. Foam sample 243AL-E, cell diameter = 0.75 mm, porosity = 88%

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<th>P_{filt} (Pa)</th>
<th>T (°C)</th>
<th>P_{abs} (Pa)</th>
<th>Density (kg m^{-3})</th>
<th>Flow rate (kg s^{-1})</th>
<th>P_{grad} (kPa m^{-1})</th>
<th>P_{gcal} (kPa m^{-1})</th>
<th>Ac (%)</th>
</tr>
</thead>
<tbody>
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</table>
### 6. Foam sample 243AL-F, cell diameter = 0.50 mm, porosity = 87%

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<th>Pfill. Pa</th>
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<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Ac %</th>
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<td>272.5</td>
<td>212.75</td>
<td>21.93</td>
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</table>

### 7. Alumina foam filter, cell diameter = 0.261 mm, porosity = 80%

<table>
<thead>
<tr>
<th>Perif. Pa</th>
<th>Pfill. Pa</th>
<th>T °C</th>
<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Ac %</th>
</tr>
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<tbody>
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APPENDIX VI: CALIBRATION OF MOM MODEL
VI.1 Calibration of MOM model using physical scale model foam

The MOM model was calibrated using experimental data from physical scale model foam of length 100 mm.

Using the physical scale model foam of length 100 mm at Reynolds number from 148 to 890, the kinetic correction coefficient obtained was 2.15. Further, using the same length of physical scale model foam, the data was collected at Reynolds number from 86 to 269 and 35 to 99. The corresponding values of kinetic correction coefficient are 2.0 and 2.08 respectively. This implies that the kinetic correction coefficient for the MOM model is almost independent of the Reynolds number. Therefore, the calibrated MOM model is applicable to any gelcast ceramic foam.

The results of the experiments are illustrated below, Figures VI.1, VI.2 and VI.3.

1. Physical scale model foam, cell diameter = 7.0 mm, porosity = 80%

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<tr>
<th>$P_{\text{onf.}}$ (Pa)</th>
<th>$P_{\text{fit.}}$ (Pa)</th>
<th>$T$°C</th>
<th>$P_{\text{abs.}}$ (Pa)</th>
<th>Velocity (ms$^{-1}$)</th>
<th>Flow rate (kg s$^{-1}$)</th>
<th>$P_{\text{grad.}}$ (kPa m$^{-1}$)</th>
<th>$P_{\text{gal.}}$ (kPa m$^{-1}$)</th>
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</table>
Figure VI.1 Graph of pressure gradient vs. fluid flow rate in a physical scale model foam sample, for the calibration of the MOM model. Reynolds number is from 86 to 269

Physical scale model foam of length 100 mm, cell diameter = 7.0 mm, porosity = 80%

<table>
<thead>
<tr>
<th>( P_{\text{diff}} ) Pa</th>
<th>( P_{\text{fit}} ) Pa</th>
<th>( T ) °C</th>
<th>( P_{\text{abs}} ) Pa</th>
<th>Velocity m s(^{-1})</th>
<th>Flow rate kg s(^{-1})</th>
<th>( P_{\text{grad}} ) kPa m(^{-1})</th>
<th>( P_{\text{grad}} ) kPa m(^{-1})</th>
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<td>1.47</td>
</tr>
</tbody>
</table>
Figure VI.2 Graph of pressure gradient vs. fluid flow rate in a physical scale model foam sample, for the calibration of the MOM model. Reynolds number is from 86 to 269.
Figure VI.3 Graph of pressure gradient vs. fluid flow rate in a physical scale model foam sample, for the calibration of the MOM model. Reynolds number is from 35 to 99.
VII.1 Single row cells sample adaptor technical drawings

1. Single row filter holder

2. Single filter bridle

All parts were originally designed using CAD package, Solid Edge V14.
4 holes, \( \phi 8 \) mm, spaced 90° on diameter 65 mm

2 holes, \( \phi 8 \) mm, spaced 180° on diameter 88 mm

Detail A

Detail B

Mat: Stainless steel
Mat: Stainless steel

All dimensions in millimetres

Tolerance unless otherwise specified
\pm 0.5

Drawn by E.M. Adigio
Course: PhD
Date: 5/5/04

Loughborough University

Title: Filter sample holder

Drawing No. PD050504-10
APPENDIX VIII: TABULATION OF PM LOADED FOAM FILTER EXPERIMENTAL DATA AND MODEL RESULTS
VIII.1 Determination of PM loaded window diameter

Firstly, using the relationship derived in Chapter 6 of this thesis, the PM volume per unit cell ($V_{PM}$) was calculated from the specific PM loaded obtained from the experiments. The value was then, equated to the estimated geometrical expression of PM volume per cell, $V_{pma}$.

The distance from the upstream surface of the PM deposit in the cell, of a radius passing through the edge of the window, to the centre of the cell, $r_t$, was tuned until the difference in value of the geometrical expression and the PM volume is minimal. The optimum value of $r_t$ was used to calculate the PM loaded window diameter.

The results are tabulated below. The PM loaded window diameter is calculated from value of $r_t$ before the difference changes in sign. In the example in the tables, the decrement value for tuning $r_t$ is a thousandth of the cell radius. However, for a higher accuracy, the values used for the analysis were obtained using decrement of ten thousandth of the cell radius. The expression for $r_t$ is:

$$ (r_t)_n = \frac{d}{2} - \frac{0.001dn}{2} $$ \hspace{1cm} (V.1)

where $n = 1, 2, 3, \ldots$.

The calculated value of PM loaded window diameter was used to calculate the pressure drop across the PM loaded filters.

The following section presents the calculations for the PM loaded filter window diameter using the chart (Figure 6.12).

The analysis can be summarised as follows:
Introducing the data; \( d \) (m), \( w \) (m), \( m \) (g), \( \epsilon \), \( \epsilon_p \), \( L \) (m), \( D \) (m), \( \rho_{PM} \) (kg m\(^{-3}\))

**Initial Calculations:**

1. Specific PM deposit \( \sigma \) in m\(^3\)/m\(^3\);

\[
\sigma = \frac{4m}{\pi D^2 L \rho_{PM}}
\]  \hspace{1cm} (1)

where \( \rho_{PM} = 1800 \) kg m\(^3\)

2. PM loaded filter porosity \( \epsilon_n \);

\[
\epsilon_n = \epsilon - \frac{\sigma}{1 - \epsilon_p}
\]  \hspace{1cm} (2)

where the PM porosity \( \epsilon_p \) is 90%.

3. \( g_1 \)

\[
g_1 = \frac{\sqrt{d^2 - w^2}}{w} + \tan \psi
\]  \hspace{1cm} (3)

where \( \psi = 5/180 \) rad.

4. \( x_2 \)
5. \( V_{PM1} \)

\[
V_{PM} = \frac{\sigma V_{CELL}}{\varepsilon}
\]

where \( V_{CELL} = \frac{\pi l^2}{12} \left[2-3(3k^2+B^2)B\right] \) and \( B = 1-\sqrt{1-k^2}, \quad k = w/d \)

6. \( x_1 \)

\[
x_1 = \frac{-g_1 k_s - \sqrt{g_1^2 r^2 - k_s^2 + r^2}}{g_1^2 + 1}
\]

where \( r = \frac{d}{2} \)

7. \( k_n \)

\[
k_n = \frac{r_1}{w} \frac{\sqrt{d^2 - w^2}}{d} \tan \psi
\]

where \( \left[\frac{d}{2} - \frac{w^2}{\sqrt{d^2 - w^2}}\right] \leq r_1 \leq \frac{d}{2} \) \( r_1 \) can vary with \( d/1000 \)

8. \( V_{PM2} = V_{arc} - V_{line} \)
Where \( V_{PM} = \pi[r^2(x_2 - x_1) - \frac{x_2^3 - x_1^3}{3} - g_1^2(\frac{x_2^2 - x_1^2}{3}) - g_1k_n(\frac{x_2^2 - x_1^2}{2}) - k_n^2(x_2 - x_1)] \) and

\[ V_{line} = \pi[g_1^2(\frac{x_2^3 - x_1^3}{3}) + g_1k_n(x_2^2 - x_1^2) + k_n^2(x_2 - x_1)] \]

9. If \( V_{PM1} - V_{PM2} = \text{Min} < 0 \) then calculate \( k_n \) for \( r_n \) and the window diameter of PM loaded filter \( w_n \) \( \{w_n = 2h = 2g_1\frac{\sqrt{d^2 - w^2}}{2} + k_n\} \)
### VIII.2 PM loaded foam filter data Tabulation

1. Foam filter sample A103Z1, PM load = 1.32513 g l\(^{-1}\)

<table>
<thead>
<tr>
<th>Cell, (d), m</th>
<th>Window, (w), m</th>
<th>(V_{CELL}), (m^3)</th>
<th>(\sigma_f), g l(^{-1})</th>
<th>(V_{PM}), (m^3)</th>
<th>((r)_{pm}), m</th>
<th>(V_{pmal}), (m^3)</th>
<th>Difference, (m^3)</th>
<th>(wn), m</th>
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3. Foam filter sample 65AL-B, PM load = 0.7832 g l⁻¹

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4. Foam filter sample A44C6, PM load = 0.8488 g 1⁻¹

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<td>0.5246</td>
<td>4.62E-14</td>
<td>3.23E-04</td>
<td>2.39E-14</td>
<td>2.23E-14</td>
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<td>6.50E-04</td>
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<td>1.39E-10</td>
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<td>4.62E-14</td>
<td>3.22E-04</td>
<td>4.70E-14</td>
<td>-8.64E-16</td>
<td>1.96E-04</td>
</tr>
</tbody>
</table>
VIII.3 Fluid flow results of PM loaded filters

The tables below show experimental data and calculated pressure gradients for PM loaded ceramic foam filters. The pressure gradients were calculated using the window diameters determined in Section V.1. The method of analysis and presentation is similar to Appendix II.

1. Foam sample A44C2, PM load = 0.525 g l⁻¹ window diameter = 0.196 mm

<table>
<thead>
<tr>
<th>P_{orif.}</th>
<th>P_{filt.}</th>
<th>T °C</th>
<th>P_{abs.}</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>P_{grad.} kPa m⁻¹</th>
<th>P_{gcal.} kPa m⁻¹</th>
<th>Ac %</th>
</tr>
</thead>
<tbody>
<tr>
<td>182</td>
<td>2003</td>
<td>36.2</td>
<td>2021</td>
<td>1.12</td>
<td>3.96E-03</td>
<td>66.90</td>
<td>55.04</td>
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<td>173</td>
<td>1925</td>
<td>36</td>
<td>1940</td>
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<td>3.86E-03</td>
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<td>52.62</td>
<td>18.2</td>
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<tr>
<td>163</td>
<td>1832</td>
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<td>1846</td>
<td>1.12</td>
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<td>1741</td>
<td>35.5</td>
<td>1753</td>
<td>1.12</td>
<td>3.63E-03</td>
<td>58.16</td>
<td>47.21</td>
<td>18.8</td>
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<td>134</td>
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<td>35.5</td>
<td>1575</td>
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<td>3.12E-03</td>
<td>45.30</td>
<td>36.24</td>
<td>20.0</td>
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<td>102</td>
<td>1253</td>
<td>34.9</td>
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<td>2.96E-03</td>
<td>41.85</td>
<td>33.18</td>
<td>20.7</td>
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<td>83</td>
<td>1052</td>
<td>34.5</td>
<td>1061</td>
<td>1.12</td>
<td>2.67E-03</td>
<td>35.13</td>
<td>27.81</td>
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<td>69</td>
<td>911</td>
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<td>919</td>
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<td>23.60</td>
<td>21.8</td>
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<td>51</td>
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<td>718</td>
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<td>23.76</td>
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<td>45</td>
<td>647</td>
<td>33.9</td>
<td>652</td>
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<td>1.96E-03</td>
<td>21.60</td>
<td>16.70</td>
<td>22.7</td>
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</table>

2. Foam sample A44C6, PM load = 0.848 g l⁻¹ window diameter = 0.0588 mm

<table>
<thead>
<tr>
<th>P_{orif.}</th>
<th>P_{filt.}</th>
<th>T °C</th>
<th>P_{abs.}</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>P_{grad.} kPa m⁻¹</th>
<th>P_{gcal.} kPa m⁻¹</th>
<th>Ac %</th>
</tr>
</thead>
<tbody>
<tr>
<td>39</td>
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<td>5002</td>
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<td>25.3</td>
<td>4769</td>
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<td>1.81E-03</td>
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<td>4317</td>
<td>1.18</td>
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<td>98.06</td>
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<td>93.37</td>
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<td>1.34E-03</td>
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<td>63.86</td>
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</table>

3. Foam sample A103Z1, PM load = 1.325 g l⁻¹ window diameter = 0.0712 mm
### 4. Foam sample A103Z6, PM load = 1.108 g l⁻¹ window diameter = 0.0569 mm

<table>
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<tr>
<th>Porif. Pa</th>
<th>Pfilt. Pa</th>
<th>T ⁰C</th>
<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pgral. kPa m⁻¹</th>
<th>Ac %</th>
</tr>
</thead>
<tbody>
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<td>1.51E-03</td>
<td>151.95</td>
<td>140.49</td>
<td>7.5</td>
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<td>300.43</td>
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### 5. Foam sample A85M, PM load = 0.914 g l⁻¹ window diameter = 0.0465 mm

<table>
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<tr>
<th>Porif. Pa</th>
<th>Pfilt. Pa</th>
<th>T ⁰C</th>
<th>Pabs. Pa</th>
<th>Density kg m⁻³</th>
<th>Flow rate kg s⁻¹</th>
<th>Pgrad. kPa m⁻¹</th>
<th>Pgral. kPa m⁻¹</th>
<th>Ac %</th>
</tr>
</thead>
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<td>6569</td>
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<td>94</td>
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<tr>
<td>61</td>
<td>4515</td>
<td>28.9</td>
<td>4538</td>
<td>1.18</td>
<td>1.35E-03</td>
<td>176.64</td>
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<td>3941</td>
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<td>1.23E-03</td>
<td>153.29</td>
<td>166.99</td>
<td>-8.9</td>
</tr>
</tbody>
</table>

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The exact amount of PM trapped in the filter samples and the wall flow filter mounted downstream of the samples could also be used to estimate the filtration efficiency of the ceramic foam filter. Consequently, a model was developed to estimate the filtration efficiency of the ceramic cellular foam filters, demonstrated below.

The filtration efficiency of a clean wall flow filter is known to be >90% (Mayer et al, 2005). Assuming that the total amount of PM flowing through the filter sample is \( m_t \), and \( m_{ce} \) is PM deposited on the ceramic filter sample. The PM flowing through the wall flow filter is \( m_t - m_{ce} \). Assuming \( m_w \) is the amount of PM deposit on the wall flow filter, then,

\[
\frac{m_w}{m_t - m_{ce}} = 0.95 \tag{V.3.1}
\]

The filtration efficiency of the ceramic foam is expressed as:

\[
E_{ce} = \frac{m_{ce}}{m_t} 100 \tag{V.3.2}
\]

There, substituting \( m_t \) in Equation (V.3.1) the filtration efficiency, \( E_{ce} \) is written as:

\[
E_{ce} = \frac{0.95m_{ce}}{m_w + 0.95m_{ce}} 100 \tag{V.3.3}
\]

The engine load during each filter loading period was 80 Nm at an engine speed of 1450 rpm. The average exhaust temperature was 290 °C and the average loading time per filter sample was one hour.
APPENDIX IX: PM LOADING TECHNICAL DRAWINGS
IX.1 PM loading rig technical drawings

1. Filter sample bridle supports on engine filter canister
2. Filter sample bridle on engine filter canister

All parts were designed using a CAD package, Solid Edge V14.
Mat: Stainless steel

6 holes, $\phi$ 8 mm, spaced 60° on diameter 94 mm

All dimensions in millimetres

Loughborough University

Title: Filter sample bridle support

Drawing No. PD100304-7

262
Diameter = 8 mm

Diameter = 60 mm

Diameter = 100 mm

3 slots: 8 mm

4 holes: 8 mm

Mat: Stainless steel

Loughborough University

Title: Filter sample bridle

Drawing No. PD100304-8
Diesel Generator Set
Model DNAD 60 Hz
11.5 kW, 14.4 kVA Standby
10.4 kW, 13.0 kVA Prime

ENGINE
Onan heavy-duty diesel engines provide stable power, low fuel consumption, quiet operation, and fast response to sudden load changes.

Mechanical governing is standard. Electronic governing is available for applications requiring constant (isochronous) frequency regulation such as Uninterruptible Power Supply (UPS) systems, non-linear loads, or sensitive electronic loads. Optional coolant heaters are recommended for all emergency standby installations or for any application requiring fast load acceptance after start-up.

Specifications – Engine

<table>
<thead>
<tr>
<th>Base Engine</th>
<th>Onan LPW3, naturally aspirated, diesel-fueled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Displacement in3 (L)</td>
<td>85.1 (1.4)</td>
</tr>
<tr>
<td>Overspeed Limit, rpm</td>
<td>2100 ±50</td>
</tr>
<tr>
<td>Regenerative Power, kW</td>
<td>2.70</td>
</tr>
<tr>
<td>Cylinder Block Configuration</td>
<td>Cast iron, In-line 3 cylinder</td>
</tr>
<tr>
<td>Cranking Current</td>
<td>160 amps at ambient temperature of 32°F (0°C)</td>
</tr>
<tr>
<td>Battery Charging Alternator</td>
<td>45-amp belt driven engine mounted.</td>
</tr>
<tr>
<td>Starting Voltage</td>
<td>12-volt, negative ground</td>
</tr>
<tr>
<td>Lube Oil Filter Types</td>
<td>Single spin-On, full flow.</td>
</tr>
<tr>
<td>Standard Cooling System</td>
<td>104°F (40°C) ambient radiator cooling system</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>POWER OUTPUT</th>
<th>STANDBY</th>
<th>PRIME</th>
</tr>
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<tbody>
<tr>
<td>Gross Engine, bhp (kWm)</td>
<td>20.5 (15.3)</td>
<td>18.6 (13.9)</td>
</tr>
<tr>
<td>BMEP at Rated Load, psi (kPa)</td>
<td>93.0 (641.2)</td>
<td>85.0 (586.1)</td>
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<tr>
<td>Bore, in. (mm)</td>
<td>3.38 (85.9)</td>
<td>3.38 (85.9)</td>
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<tr>
<td>Stroke, in. (mm)</td>
<td>3.15 (80.0)</td>
<td>3.15 (80.0)</td>
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<td>Piston Speed, ft/min (m/s)</td>
<td>945.0 (4.8)</td>
<td>945.0 (4.8)</td>
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<tr>
<td>Compression Ratio</td>
<td>18.5:1</td>
<td>18.5:1</td>
</tr>
<tr>
<td>Lube Oil Capacity, qt. (l)</td>
<td>4.7 (4.4)</td>
<td>4.7 (4.4)</td>
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### FUEL FLOW

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<tr>
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<th>PRIME</th>
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<tbody>
<tr>
<td>Fuel Flow at Rated Load, US Gal/hr (l/hr)</td>
<td>1.4 (5.3)</td>
<td>1.4 (5.3)</td>
</tr>
<tr>
<td>Maximum Inlet Restriction, in. Hg (mm Hg)</td>
<td>5.0 (127.0)</td>
<td>5.0 (127.0)</td>
</tr>
<tr>
<td>Maximum Return Restriction, in. Hg (mm Hg)</td>
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<td>6.0 (152.4)</td>
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</table>

### AIR CLEANER

<table>
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<th>PRIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Air Cleaner Restriction, in. H2O (kPa)</td>
<td>10.0 (2.5)</td>
<td>10.0 (2.5)</td>
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### EXHAUST

<table>
<thead>
<tr>
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<th>PRIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exhaust Flow at Rated Load, cfm (m3/min)</td>
<td>105.0 (3.0)</td>
<td>105.0 (3.0)</td>
</tr>
<tr>
<td>Exhaust Temperature, °F (°C)</td>
<td>1020.0 (548.9)</td>
<td>1020.0 (548.9)</td>
</tr>
<tr>
<td>Max Back Pressure, in. H2O (kPa)</td>
<td>20.0 (5.0)</td>
<td>20.0 (5.0)</td>
</tr>
</tbody>
</table>

Fuel System Direct injection, number 2 diesel fuel, single fuel filter and water separator; mechanical fuel transfer pump with hand primer; 10 feet fuel lift; individual fuel injection pumps.

### FUEL CONSUMPTION

<table>
<thead>
<tr>
<th>FUEL CONSUMPTION</th>
<th>STANDBY</th>
<th>PRIME</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 Hz Ratings, kW (kVA)</td>
<td>11.5 (14.4)</td>
<td>10.4 (13.0)</td>
</tr>
<tr>
<td>Load</td>
<td>1/4</td>
<td>1/2</td>
</tr>
<tr>
<td>US Gal/hr</td>
<td>0.41</td>
<td>0.62</td>
</tr>
<tr>
<td>l/hr</td>
<td>1.6</td>
<td>2.3</td>
</tr>
</tbody>
</table>

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