Closely coupled metallodielectric electromagnetic band gap structures

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Closely Coupled Metallodielectric Electromagnetic Band Gap Structures

by

George Apostolopoulos, MSc

A Doctoral Thesis

Submitted in Partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy by Loughborough University

June 2006

Department of Electronic and Electrical Engineering
Loughborough University
United Kingdom

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"It is not the position that you stand, but the direction in which you look at."

To my Father, my Mother and my Sister.
Abstract

The last few years, much research is aimed at using light as an information carrier in systems. Photonic crystals are materials with varying dielectric properties, designed to interact with photons. If these crystals are arranged in a periodic structure they can control the propagation of electromagnetic waves through the structure. Photonic Band Gap (PBG) crystal is a periodic structure that prohibits propagation of all electromagnetic waves within a particular frequency band. Original PBG research was done in the optical region, but PBG properties are scaleable and applicable to a wide range of frequencies. In recent years, there has been increasing interest in microwave and millimeter-wave applications of PBG structures. Currently, research has also extended to Metallodielectric Electromagnetic Band Gap structures (MEBG), which are replacing the photonic crystals. MEBG structures are composed of periodic metallic elements usually printed in a dielectric region.

The research effort in this thesis concentrates on the analysis, modelling and practical implementation of a novel concept called CCMEBG. Closely coupled Metallodielectric electromagnetic band gap structured materials are a class of artificial periodic metamaterials that prohibit propagation of electromagnetic waves within a particular frequency band. They are formed by two arrays printed on either side of a thin dielectric sheet, closely spaced to each other and shifted appropriately in order to produce high element coupling. Due to the close proximity and the strong coupling between the shifted elements, the effective electrical length of the element increases, resulting in a significant decrease of the array resonant frequency. This property is attractive for miniaturization purposes, since by maintaining the overall size of the unit cell the same or even reducing it, a much lower cut off can be achieved.

Closely coupled MEBG structures can be used for surface wave suppression and antenna performance enhancement, in microwave circuit components, as filters and in a series of active and passive devices.

Keywords- Closely Coupled Metallodielectric Electromagnetic Band Gap (CCMEBG), Complementary MEBG (CMEBG), Periodic Structures, Surface waves, miniaturisation.
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Publications From this Research


# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>i</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>ii</td>
</tr>
<tr>
<td>Publications from this Research</td>
<td>iii</td>
</tr>
<tr>
<td>Contents</td>
<td>v</td>
</tr>
</tbody>
</table>

## CHAPTER 1

### INTRODUCTION TO PHOTONIC BAND GAP CRYSTAL

1.1 Photonic Band Gap Crystals .............................................. 1

1.2 Metallodielectric Electromagnetic Band Gap Structures .......... 3

1.3 Organisation of this Thesis ........................................... 5

1.4 Organisation of this Thesis ........................................... 8

References.................................................................................. 10

## CHAPTER 2

### THEORY OF ANALYSIS

2.1 Introduction ........................................................................... 14

2.2 Modal Analysis of Electromagnetic Band Gap Surfaces.......... 15
Table of Contents

2.3 Modal field Representation ................................................. 17

2.4 Formulation of Scattering for CCMEBG Array ......................... 23
  2.4.1 Fields at different interfaces ...................................... 23
  2.4.2 Application of Boundary Conditions .............................. 26
  2.4.3 Electric Field Coupled Integral Equations ....................... 40

2.5 Method of Moments ........................................................ 42

2.6 Plane Wave Propagation .................................................. 49

2.7 Complementary Bang Gap Structures Analysis ......................... 53

2.8 Irreducible Brillouin zone and the array element .................... 60

2.9 Conclusions ............................................................... 62

References .............................................................................. 64

CHAPTER 3

MEBG & CCMEBG DIPOLE AND TRIPOLE ARRAYS

3.1 Introduction ..................................................................... 67

3.2 Dipole Analysis .................................................................. 69

3.3 Simulations for MEBG and CCMEBG Dipole Arrays ................. 70

3.4 Dispersion Curve Diagrams for MEBG and CCMEBG Dipoles .......... 74

3.5 Array Manufacturing ...................................................... 79

3.6 Antennas and Measurement Setup ..................................... 80
Table of Contents

3.7  Measurements for MEBG and CCMEBG Dipole Arrays........ 82
    3.7.1  Plane Wave Measurements................................. 82
    3.7.2  Surface Wave Measurements............................... 84

3.8  Tripole Analysis..................................................... 89

3.9  Simulations for MEBG and CCMEBG Tripole Arrays.......... 91

3.10 Dispersion Curve Diagrams for MEBG and CCMEBG Tripoles. 94

3.11 Measurements for MEBG and CCMEBG Tripole Arrays......... 97
    3.11.1 Plane Wave Measurements................................. 97
    3.11.2 Surface Wave Measurements............................... 100

3.12 Conclusion.................................................................. 104

References........................................................................ 106

CHAPTER 4

APERTURE AND COMPLEMENTARY DIPOLE ARRAYS

4.1  Introduction............................................................. 110

4.2  Aperture Theory and Geometry ................................... 112

4.3  Dispersion Characteristics........................................ 113

4.4  Aperture Simulations............................................... 114

4.5  Aperture Measurements............................................. 117
CHAPTER 5

COMPLEX GEOMETRIES AND CCMEBG STRUCTURES COMPRISING COMPLEX ELEMENTS

5.1 Introduction .................................................................. 135
5.2 Measurement Setup .................................................. 136
5.3 Measurements of Complex Elements ............................ 137
5.4 Measurements of CCMEBG Complex Arrays .................. 139
5.5 Conclusion .................................................................. 143

References ...................................................................... 144

CHAPTER 6

CONCLUSIONS .................................................................. 145
CHAPTER 1

Introduction

1.1 Photonic Band Gap Crystals

Since the suggestion in 1987 [1], that creating a periodicity in dielectric materials could prevent the propagation of electromagnetic waves at certain frequencies, there has been much work, both theoretical and experimental, in the field of Photonic Crystals to create a so called ‘photonic bandgap’ (PBG) material [2].

PBG crystals are periodic dielectric structures with significantly different dielectric constants of their consisting materials. Whenever light encounters a change in refractive index (or dielectric constant), some light is transmitted while the rest is reflected. In PBG materials, the periods of index modulations are set such that reflections from consecutive periods add in phase. This produces a strong overall reflection in a narrow frequency range, called the “stop band”. When light, whose frequencies are in the stop band, is incident on a PBG material the transmitted light decays exponentially and propagation is not permitted. This is called a photonic bandgap[3,4].

Photonic band-gap materials can reflect light in more than one spatial dimension. Photonic crystals of two and three dimensions are being intensively investigated [5]. Whereas a 2-D photonic crystal is easier to fabricate, it possesses periodicity only in the $x$-$y$ plane and is finite in the $z$ direction. Figure 1.1 [14] shows an example of a 2-D square lattice photonic crystal, surrounded by air or a low dielectric material (not shown). Frequencies in its absolute photonic bandgap region will be prohibited for any in-plane propagation, for any polarization and any direction along the $x$-$y$ plane. Propagation in the $z$ direction will not see any bandgap since there is no dielectric variation in the $z$ direction [6–10].
For the three dimensional photonic crystal, the dielectric variation will have periodicity equal to that of its lattice. Such a system can have a complete photonic band-gap. Thus, photons with frequencies within the region of the absolute photonic band-gap are forbidden to propagate in any direction. Figure 1.2 [14] illustrates an example of a 3-D square lattice photonic crystal, surrounded by air or a low dielectric material (not shown). Fabrication of such a photonic crystal lattice is still a challenge at present.
1.2 Metallo-dielectric Electromagnetic Band Gap Structures

Original PBG research was done in the optical region, but PBG properties are scaleable and applicable to a wide range of frequencies. In recent years, there has been an increasing interest in microwave and millimeter-wave applications of PBG structures. That interest led the scientific community in the expansion of the research into Metallo-dielectric Electromagnetic Band Gap or MEBG structures. MEBG structures have emerged as direct analogues to PBG's, where the periodic high dielectric constant cavities are replaced with periodic planar metallic elements. Microwave and infrared measurements have shown that MEBG exhibit much larger electromagnetic stop bands than the photonic crystals [11,12]. Initially the terminology adopted caused some controversy amongst the microwave community and different opinions were expressed [13]. Nowadays the term Electromagnetic Band Gap is widely acceptable in the circle of the microwave community and thus it will be used throughout this thesis. Some typical element geometries employed for the formulation of an MEBG array, are illustrated in Figure 1.3 [14].

![Element geometries of an MEBG array on a square lattice, with periodicity D](image)

Figure 1.3: Element geometries of an MEBG array on a square lattice, with periodicity D
The research presented in this thesis investigates the 1D and 2D dimensional band gap properties of the dipole and tripole elements, respectively.

The 2-D planar version of an MEBG structure is in effect a type of Frequency Selective Surface (FSS). Frequency Selective Surfaces [14, 15] are two-dimensional periodic array structures consisting of thin conducting elements, often printed on a dielectric substrate for support. Frequently these arrays take the form of periodic apertures in a conducting plane. When FSSs are excited by an incident electromagnetic wave at an arbitrary angle, they behave as passive electromagnetic filters, having stopband and passband characteristics. If the direction of propagation is along the plane of the array, the FSS can be regarded as a planar 2D MEBG structure and it will exhibit band gap properties in the plane of the array.

Electromagnetic Band Gap structures can be used in a variety of applications, such as substrates to improve the performance of microwave antennas (high impedance surfaces) [10]. They can also be used to suppress surface waves in microstrip antennas and improve their directivity [17,18]. A typical example of an EBG material used for antenna applications is an optimised dipole antenna on an EBG structure. By fabricating the antenna on the EBG material with the driving frequency in the stop band, no power should be transmitted into the forbidden frequency region, thus all the power should be radiated in the desired direction [19,20].

Conventional dielectric substrates act as waveguides and a large percentage of a microstrip antenna's input power can be lost in the substrate as surface waves and dielectric losses. The substrate can be thinned to minimise the wave guiding loss, but the rigidity of the substrate will be compromised. Creating a periodic pattern in a substrate can create a 2-D EBG structure. Two-dimensional EBG structures with a band gap in the microwave spectrum are used as substrates to suppress surface waves and increase the radiating efficiency of microstrip antennas. EBG materials have also been investigated for microstrip circuit applications and have exhibited very high suppression in the stopband [21,22]. Another application utilises the EBG to act as a band reject filter within a waveguide [23]. Moreover, EBG materials can also act as spatial filters, as a result of joining the grating properties with the resonance [14].
Using the transition region between the transmission and stop band and a suitable feeding mechanism, EBG materials can behave as leaky wave structures [15].

MEBG structures exhibit a bandgap frequency, which is directly related to the electrical length of the element. The resonant length is about half to quarter wavelength for common elements such as dipoles, tripoles and loops. Hence the arrays have rather large dimensions, which make them unsuitable for integration on compact microwave devices, particularly in applications of a few GHz, such as the mobile communication operating bands. Miniaturisation of microwave components and antennas has become increasingly important in recent years. Modern wireless communication terminals require small microwave elements which are pertinent to high level integration into compact light-weight systems. In this context, miniaturisation of EBG structures is a desirable feature, with application to a variety of active and passive devices.

1.3 Closely Coupled MEBG Structures

In the last few years research interest has risen in miniaturisation of frequency selective surfaces, by a technique comprising cell size reduction with the use of complex elements[24]. Recently miniaturisation of EBG structures was introduced, by a study incorporating complex conducting elements shorted to a continuous ground plane with via pins[25].

In this thesis, the concept of closely coupled metallodielectric electromagnetic band gap structures is introduced and rigorously studied. The configuration of a CCMEBG structure comprises two arrays, closely spaced to each other and shifted appropriately in order to produce high element coupling. Based on that novel concept, dipole and tripoles elements are employed as representative geometries capable of producing 1D and 2D band gap performances. The two arrays used for the implementation of the CCMEBG structure, are printed on either side of a thin dielectric sheet. Due to the close proximity and the strong coupling between the shifted elements, the effective
electrical length of the element increases, resulting in a significant decrease of the array resonant frequency. This property is attractive for miniaturization purposes, since by maintaining the overall size of the unit cell the same or even reducing it, a much lower cut off can be achieved. A representative example of closely coupled arrays is illustrated below, in the form of tripoles.

![Diagram of closely coupled tripoles](image-url)

**Figure 1.4: Arrangement of double layer tripole CCMEBG array and geometry of a tripole array unit cell**

The black lines represent the first tripole array, while the grey lines represent the second tripole array which is rotated by 60° and shifted appropriately with respect to the first one. Both arrays are printed on either side of a thin dielectric (not shown here), in order to produce maximum coupling.

A series of studies regarding the dimensions of the elements, the relative shifting of the closely coupled arrays and the separation distance between them was conducted. For that purpose a modal analysis based on a coupled electric field integral equation was developed. The analysis was employed in order to produce a series of dispersion curves, giving an insight on the behaviour of the infinite array. The reflection and transmission properties of various arrays for plane waves of various incident angles, were also studied. For comparison purposes the same procedure was followed with single layer arrays.
Measurements were produced for both array elements, for plane wave as well as surface wave incidence, validating the predictions of the simulations. Minaturisation has been achieved, along with superior bandwidth performance and exceptional angular stability, for different incident angles. [26,27]

The concept of closely coupled electromagnetic band gap structures is not restricted to conducting elements. It can be extended to a configuration that combines a conducting array with an aperture array, as was previously shown in the context of FSS [28,29]. The latest case is examined thoroughly in the present thesis, with the use of dipole elements, and the study is significantly extended in order to introduce the concept of Complementary MEBG (CMEBG) structures.

A lot of research has been carried out over the last few years on defected ground structures (apertures or slots) comprised of apertures in a ground plane. These structures provide a band gap (stop band) performance for a large number of applications, such as antennas, passive and active microwave circuits. In this thesis, a layer of conducting elements was placed in close proximity to the apertures, forming a Complementary Metallodielectric Electromagnetic Band Gap structure. Using a single aperture array as reference, a modal analysis based on coupled electric and magnetic field integral equations was initially employed. Dispersion curves were implemented for complementary linear dipole elements (i.e. conducting dipoles and aperture dipoles in a conducting screen). A commercial software package was used to simulate the transmission responses of finite CMEBG structures. Parametric studies were carried out for different widths and lengths of conducting elements, periodicities and number of elements. Measurements validated the predicted results and illustrated the advantages of the proposed structures.

An alternative technique for reducing the physical dimension of MEBG structures, is based on the use of complex elements geometries within the unit cell [30,31]. The last session of the thesis illustrates the miniaturisation produced from such geometries, by means of comparative examples. The complex elements used for that purpose are based on tripoles. Combining that technique with the closely coupled concept, we form a CCMEBG structure consisting of an array of simple tripoles and an array of complex elements.
1.4 Organisation of the Thesis

This thesis is comprised of six chapters with the main thrust of the work described in Chapters 2, 3, 4 and 5.

Chapter 1 acts as an introduction to the work carried out in the body of the thesis. Brief background comments are made regarding the overall subject of electromagnetic band gap structures and the origins which are linking those materials with the photonic band gap crystals. The concept of closely coupled metallodielectric electromagnetic band gap arrays is briefly presented, including the main principles involved and the aims under which the research is motivated.

Chapter 2 underlines the theoretical approach used in order to model the behaviour of the electromagnetic band gap arrays accommodated in the research. A modal analysis is applied in order to expand the total fields from the infinite periodic array into a set of floquet modes. By the application of boundary conditions at the interface of the multiple dielectrics, the electromagnetic fields are matched, deriving a set of coupled integral equations. These can be solved with the method of moments. The key attribute of this method is that it is computationally efficient and exhibits a good relative convergence for small separation distances between the arrays. The mathematical interpretation of this method is illustrated, with a series of equations. The analytical approach is extended to closely coupled conducting arrays illuminated by an incident field at an arbitrary angle, along with a surface wave propagation. Furthermore a mathematical study is done for the case in which a complementary structure is employed.

Chapter 3 focuses in the introduction of the CCMEBG concept, using as references dipole and tripole elements. Two different sections are presented. The first one is referred to dipole arrays, which can only produce 1D band gap. A single array of dipoles is initially used for comparison purposes. Simulations and measurements are illustrated. Then an array of closely coupled dipoles with the same dimensions is presented. Parametric studies, simulations and measurements along with dispersion
curves illustrate the advantages of the CCMEBG concept in comparison with a single layer dipole array. Superior angular stability for plane wave illumination, vast reduction of the cut-off frequency, increase of the bandwidth, wider band gap areas and miniaturisation capabilities are a few of the key points thoroughly examined and proven.

The second section is dedicated to tripole arrays which, as more symmetrical 2D elements, can produce an absolute band gap in the plane of the array for surface wave propagation. In addition, a tripole array under plane wave incidence exhibits a similar band gap for any polarisation of the incident field. Again, predicted and measured results are illustrated verifying the series of conclusions made in section 1.

Chapter 4 covers the Complementary Metallodielectric Electromagnetic Band Gap structures, which can considered as a hybrid of CCMEBG arrays. A layer of conducting dipoles and a layer of aperture dipoles are etched either side of a dielectric substrate and shifted appropriately, producing very strong fields in the separation region. The use of complementary structures for the suppression of surface waves is a new concept, used for its miniaturization capabilities.

Chapter 5 introduces a series of complex elements which are based on the tripoles illustrated in chapter 3. Geometries such as fractals, convoluted, interdigital and periodically loaded tripoles are researched for their capabilities to produce a band gap in lower frequency regions, than the simpler tripole. Furthermore the concept of closely coupled arrays with the combination of a tripole and a complex element array is investigated. Measurements are presented.

Finally, Chapter 6 draws conclusions from the work presented in the thesis and analyses the results as a whole. Considerations are made as to future developments and improvements that could be implemented.
REFERENCES


Chapter 2

Theory of Analysis

2.1 Introduction

The main aspect of the present chapter is to introduce the concept of the Modal analysis for an infinite closely coupled EBG (Electromagnetic Band Gap) array, placed in a multiple dielectric substrate. The analysis is based on the total fields from the periodic array, where the tangential field TE (transverse electric) and TM (transverse magnetic) components can be expanded in terms of Floquet modes[1]. Based on Floquet's theorem the fields can be described in terms of a complete, orthogonal set of modes (Floquet modes) in the vicinity of each array element.

This modal analysis method [2] was first applied by Chen [3] for induced current on the conducting plates of a 2D array in free space. Montgomery [4] included a dielectric substrate on which the periodic array of thin conductors were printed.

The aim of the analysis is to describe the behavior of the fields in a surface wave propagation Figure 2.1, $\theta=90^\circ$ [10], so the determination of the stop-band (band-gap) characteristics of the array can be exploited. The array, with periodicity in two dimensions, exhibits a band-gap area in the plane of the double periodicity. In order to achieve an absolute bang-gap, it is essential to evaluate the propagation mode in all the propagation directions, within the two dimensional plane. The key elements examined as periodic array components are the dipole and the tripole.

Based on the symmetry lines of the upper elements, the idea of the first and irreducible Brillouin zone [5,6,7] is presented. By examining different geometrical lattices (hexagonal, square), a relationship between the angle of symmetry of the first Brillouin zone and the line of symmetry of the array elements, can be obtained. That relationship defines the orientation of the irreducible first Brillouin zone.

The modal analysis extends in the general case, where the polarized field is incident on the array, at an arbitrary direction, with an angle $\theta$ with respect to the z axis.
The structure acts as a frequency selective surface (FSS), oriented in the x-y plane. Finally, the concept of closely coupled surfaces is extended a step further by the introduction of complementary structures. An array of conducting elements is placed in very close proximity with an array of apertures, with a rotation between them of 90°. The modal analysis for surface wave propagation has been done, highlighting the main differences with the closely coupled conducting array.

### 2.2 Modal Analysis of Electromagnetic Band Gap Surface

The CCMEBG array used for the modal analysis is assumed to be infinite. Each element is located in a unit cell, allocated in a periodic configuration. The elements are assumed to be infinitely thin and perfectly conducting. The array elements are printed on either side of a dielectric substrate in close proximity to each other and shifted properly (chapter 3), in order to produce maximum coupling.

Due to the periodicity and the infinite structure of the conducting array, the Floquet theorem can be used. The fields can be expanded in terms of Transverse Electric (TE$^{(0)}$) and Transverse Magnetic (TM$^{(0)}$) Floquet modes, taking into account the periodic boundary conditions at the interface between the different layers of the structure. The behaviour of the Floquet modes in the multi-layered structure can be modelled with a series of equivalent transmission lines. In this way, equivalent voltages and currents can be used to describe the variation of (TE$^{(2)}$) and (TM$^{(2)}$) Floquet modes in the z stratification direction.

By applying the standard electromagnetic boundary conditions[8],[9] at the interfaces between the different dielectrics, the Electromagnetic fields are matched, to derive a set of Coupled Integral Equations (CIE). These equations are solved in terms of the unknown currents on the conducting elements, of the different arrays[10]. The (CIE) are developed assuming the same number of Floquet modes for the unit cell on each array. In that way the same Floquet indices $p$ and $q$ denote the same order Floquet mode, in each unit cell on the two arrays.
Using the Method of Moments [11,12], the Coupled Integral Equations can be reduced to a linear system of simultaneous equations. The unknown currents are expressed as a series of basis functions. With a Numerical routine which utilises Crout’s factorisation, the unknown coefficients of the basis functions are obtained. The current coefficients allow one to obtain the reflected and transmitted field amplitudes. Thus, from the total reflected and transmitted field, the reflection and transmission coefficients are calculated.

Other approaches to the analysis of multi-layer array structures have been reported, whereby each layer is considered as a separate block and cascaded with others by means of either multiple port circuit techniques [13, 14] or a generalised scattering matrix formulation based on a multimodal equivalent circuit [15]. However, computational constraints are posed when closely spaced arrays are considered in these methods, because they involve manipulation of matrices whose dimensions depend on the Floquet mode number. The coupled integral equation (CIE) method on the other hand is based on the solution of a system of equations with intrinsic higher orders of interaction and is amenable to predicting responses of closely coupled surfaces without excessive computational demands. The key attribute of this method is that it is computationally efficient and exhibits a good relative convergence for small separation distances between the arrays.

Figure 2.1: Geometry of a periodic array on a square lattice

---

```
Theory of analysis
Chapter 2

(a)
(b)
```

16
2.3 Modal Field Representation

Due to the periodicity of the structures, the transverse Electric and Magnetic fields can be expanded using Floquet modes.

\[
E(x, y, z) = \sum_{pq} a_{pq} e^{\pm j \beta_{pq} z} e^{-j k_{pq} \mathbf{r}}
\]  
(2.1)

\[
H(x, y, z) = \sum_{pq} a_{pq} e^{\pm j \beta_{pq} z} e^{-j k_{pq} \mathbf{r}}
\]  
(2.2)

Where  \( p \rightarrow \) Index for the harmonic floquet modes in the 1\textsuperscript{st} periodicity direction  
(0, ±1, ±2, ±3, ...)

\[ q \rightarrow \) Index for the harmonic floquet modes in the 2\textsuperscript{nd} periodicity direction  
(0, ±1, ±2, ±3, ...)

Each Floquet mode is an evanescent component which can propagate in the inner dimensional structure:

\[
\Xi_{pq}(x, y, z) = \Psi_{pq}(x, y) e^{\pm j \beta_{pq} z}
\]  
(2.3)

The behaviour of the Floquet mode in the transverse x-y plane is given by:

\[
\Psi_{pq}(x, y) = e^{-j k_{pq} \mathbf{r}}
\]  
(Floquet modes for transverse periodicity)

(2.4)

Defining the transverse vector \( \mathbf{r} \) as:  
\( \mathbf{r} = x \hat{x} + y \hat{y} \)

\[
\Xi_{pq}(x, z) = e^{-j k_{pq} \mathbf{r}} e^{\pm j \beta_{pq} z}
\]  
(2.5)

\[
k_{pq} = k_{pqx} \hat{x} + k_{pqy} \hat{y}
\]
\[ k_{ipq} = k_{00} + pk_1 + qk_2 \]  

(2.6)

\[ k_{foo} = k_{00x} + k_{00y} \]

\[ k_{foo} = k_0 \sin \theta \cos \phi + k_0 \sin \theta \cos \phi \]

For surface wave propagation, the angle \( \theta \) is 90 degrees.

Where \( k_{foo} \rightarrow \) is the transverse propagation vector for the main harmonic – also known as block wave number

\[ k_1 = -\frac{2\pi}{A} \hat{z} \times D_v \]

\[ k_2 = \frac{2\pi}{A} \hat{z} \times D_u \]

\[ A = |D_u \times D_v| \]

where \( A \) - Periodic unit cell area and \( D_u, D_v \) are direction vectors defined in fig 2.2

![Figure 2.2: CCMEBG Array on an arbitrary lattice](image-url)
The longitudinal propagation (z direction) constant is given from the formula:

\[
\beta_{zpq} = \sqrt{k_o^2 \varepsilon_r - (k_{x_ypq}^2 + k_{y_zpq}^2)} = \sqrt{k_o^2 \varepsilon_r - k_{ypq}^2 \cdot k_{tpq}} = \sqrt{k_o^2 \varepsilon_r - \left| k_{tpq} \right|^2}
\]

\[
\beta_{zpq}^2 = k_o^2 \varepsilon_r - \left| k_{tpq} \right|^2, \quad \text{where } k_o = \frac{2\pi}{\lambda_o}, \quad \lambda_o = \frac{c_o}{f}
\] (2.7)

Based on the upper equation for the propagation constant and knowing that the outcome of the square root depends on \( f \) and \( \varepsilon_r \), there are several possible mathematical solutions. It is interesting to divide these solutions in terms of the vacuum and dielectric slab regions, since they express different physical phenomena.

\[
\beta_{zpq}^2
\]

\[
\beta_{zpq}^2 > 0
\]

\( B_{zpq} \) is Real.
The wave is propagating in the dielectric slab

\[
\beta_{zpq}^2 < 0
\]

The wave is propagating in the dielectric slab, but not in the vacuum.
(Surface Wave-Evanescent)

\[
\beta_{zpq}^2 \leq 0
\]

\( B_{zpq} \) is Imaginary.
There is no wave propagation

\[
\beta_{zpq}^2 = 0
\]

The wave is propagating in the dielectric slab, and in the vacuum.
(Leaky Wave)

\[
\beta_{zpq}^2 < 0
\]

The wave is not propagating in the dielectric slab nor in the vacuum.
(No Wave)
The present work is focused in the study of surface wave propagation, but taking into account the existence of leaky wave modes.

The Floquet modes can be obtained by using common wave guide analysis. By choosing the z-direction as the longitudinal propagation direction, the TE\(^z\) and TM\(^z\) vector functions can be expressed as follows:

**TM modes** – The transverse component of the TM modes

\[
\begin{align*}
E_{\text{tpq}} &= \frac{k_{\text{tpq}}}{k_{\text{tpq}}} \Psi_{pq} = \kappa_{pq} \Psi_{pq} \quad (2.8) \\
H_{\text{tpq}} &= \eta_{\text{tpq}} \nabla \times \kappa_{\text{tpq}} \Psi_{pq} \quad (2.9)
\end{align*}
\]

Where \( \eta_{\text{tpq}} = \frac{\kappa \eta}{\beta_{\text{tpq}}} \) with \( \kappa = \omega \sqrt{\mu \varepsilon} \) and \( \eta = \sqrt{\frac{\varepsilon}{\mu}} \), \( \varepsilon \) and \( \mu \) are the permittivity and the permeability of the medium.

The Transverse Magnetic vector having its magnetic component parallel to the plane of the array. \((H_{\text{tpq}} = 0)\)

**TE modes** – The transverse component of the TE modes

\[
\begin{align*}
E_{\text{tpq}} &= \kappa_{2pq} \Psi_{pq} \quad (2.10) \\
H_{\text{tpq}} &= \eta_{2pq} \nabla \times \kappa_{2pq} \Psi_{pq} \quad (2.11)
\end{align*}
\]

The transverse electric vector has its electric component parallel to the plane of the array \((E_{\text{tpq}} = 0)\).
Where \( \eta_{pq} = \frac{\beta_{pq} \eta}{\kappa} \) with \( \kappa = \omega \sqrt{\mu \varepsilon} \) and \( \eta = \sqrt{\frac{\varepsilon}{\mu}} \), \( \varepsilon \) and \( \mu \) are the permittivity and the permeability of the medium.

The tangential electromagnetic field can now be described as a summation of TE and TM Floquet modes.

For any Electric field of any possible modal solution of the given array we can write:

**Electric field**

\[
\bar{E}(r,z) = \sum_{pq} (TM^z \text{ Floquet \cdot Modes} + TE^z \text{ Floquet \cdot Modes})
\]

\[
\bar{E}(r,z) = \sum_{pq} (T_{1pq}^{TM} \Psi_{pq}(r) \kappa_{1pq} \cdot e^{+j\beta_{pq}z} + R_{1pq}^{-TM} \Psi_{pq}(r) \kappa_{1pq} \cdot e^{-j\beta_{pq}z} + T_{2pq}^{TE} \Psi_{pq}(r) \kappa_{2pq} \cdot e^{+j\beta_{pq}z} + R_{2pq}^{-TM} \Psi_{pq}(r) \kappa_{2pq} \cdot e^{-j\beta_{pq}z})
\] (2.12)

It needs to be mentioned that the subscript has the value 1 and 2 denoting TM and TE modes, respectively.

The factors \( T_{1pq}^{TM}, R_{1pq}^{-TM} \) and \( T_{2pq}^{TE}, R_{2pq}^{-TM} \) are the amplitudes of the TM and TE Modes.

Similarly for the magnetic field:
Magnetic field

\[ H(r,z) = \sum_{pq} \left( T_{1pq}^{TM} \cdot \eta_{1pq} \cdot \Psi_{pq}(r) \cdot \hat{z} \cdot e^{j\beta_{pq}z} + R_{1pq}^{TM} \cdot \eta_{1pq} \cdot \Psi_{pq}(r) \cdot \hat{z} \cdot e^{-j\beta_{pq}z} \right) + \sum_{pq} \left( T_{2pq}^{TE} \cdot \eta_{2pq} \cdot \Psi_{pq}(r) \cdot \hat{z} \cdot e^{j\beta_{pq}z} + R_{2pq}^{TE} \cdot \eta_{2pq} \cdot \Psi_{pq}(r) \cdot \hat{z} \cdot e^{-j\beta_{pq}z} \right) \]

Again the subscript takes the value 1 or 2 and denotes the TM and TE modes, respectively.

In order to examine and formulate the modes of the conducting array, there are two important factors that need to be found:

1. The Block transverse Propagation constant \( (K_{100}) \)
2. The amplitude of each TE and TM floquet mode \( (T_{mpq}, R_{mpq}) \)

Each possible modal solution of the periodic structure is defined by its block-wave number value \( (K_{100}) \) and its corresponding field pattern, which is determined by the fields spectral coefficients \( (T_{mpq}, R_{mpq}) \).
2.4 Formulation of Scattering for a CCMEBG Array

2.4.1 Fields at different interfaces.

The following figure illustrates the cross-sectional view of a Closely Coupled Electromagnetic Band Gap array, embedded in three layers of dielectric substrate, surrounded by air. The electromagnetic wave travelling in any layer, can be described as a combination of a $+$ $\hat{z}$ propagating Floquet waves ($T^{(n)}$) and a $-$ $\hat{z}$ propagating Floquet waves ($R^{(n)}$). The thickness of each substrate is denoted as $z(n)$, $T^{(n)}$, $R^{(n)}$ are the transmitted and reflected field amplitudes of the forward and backward travelling waves, respectively.

Figure 2.3: Cross-Sectional view of a CCMEBG Array embedded in different dielectric substrates
The next step is to define the modal tangential electromagnetic field for each region. It is important to understand that the Electromagnetic field can be represented as a summation of Floquet modes and equivalent voltages and currents. The Floquet modes describe the behaviour of the EM field in the transverse periodicity plane (x-y), while the equivalent Voltages (\( V_{mpq}^{(e)} \)) and equivalent currents (\( I_{mpq}^{(i)} \)) can model the behaviour of the EM field in the longitudinal multi-layered z direction.

**Region 0**

For \( z \leq z^{(0)} \)

\[
E^{(0)}(r, z) = \sum_{mpq} \Psi_{pq}(r) K_{mpq} \cdot V_{mpq}^{(0)}(z) \\
= \sum_{mpq} \Psi_{pq}(r) K_{mpq} \left( T_{mpq}^{(0)} \cdot e^{-j \beta_{pq}^{(0)}(z-z^{(0)})} + R_{mpq}^{(0)} \cdot e^{+j \beta_{pq}^{(0)}(z-z^{(0)})} \right) \tag{2.14}
\]

But for region 0 since there is no \( T_{mpq}^{(0)} \) the equation 2.14 becomes:

\[
E^{(0)}(r, z) = \sum_{mpq} \Psi_{pq}(r) K_{mpq} \left( R_{mpq}^{(0)} \cdot e^{+j \beta_{pq}^{(0)}(z-z^{(0)})} \right) \tag{2.15}
\]

Equally, and again taking into account that there is no \( T_{mpq}^{(0)} \) the magnetic field can be written as:

\[
H^{(0)}(r, z) = \sum_{mpq} \Psi_{pq}(r) \hat{z} \times K_{mpq} \cdot I_{mpq}^{(0)}(z) \\
= \sum_{mpq} \Psi_{pq}(r) \hat{z} \times K_{mpq} \left( -\eta_{mpq}^{(0)} R_{mpq}^{(0)} \cdot e^{+j \beta_{pq}^{(0)}(z-z^{(0)})} \right) \\
= -\sum_{mpq} \Psi_{pq}(r) \hat{z} \times K_{mpq} \left( \eta_{mpq}^{(0)} R_{mpq}^{(0)} \cdot e^{+j \beta_{pq}^{(0)}(z-z^{(0)})} \right) \tag{2.16}
\]
To express the equivalent voltage and current standing wave functions ($V_{mpq}^{(n)}$ and $I_{mpq}^{(n)}$), the point $z=z^{(0)}$ has been used as the reference point in the exponential functions.

**Region n**

For $z^{(n-1)} \leq z \leq z^{(n)}$, where $n = 1, 2, 3$

$$E^{(n)}(r, z) = \sum_{mpq} \Psi_{pq}(r) K_{mpq} \cdot V_{mpq}^{(n)}(z)$$

$$= \sum_{mpq} \Psi_{pq}(r) K_{mpq} \left( T_{mpq}^{(n)} \cdot e^{-j\beta_{pq}^{(n)}(z-z^{(n)})} + R_{mpq}^{(n)} \cdot e^{+j\beta_{pq}^{(n)}(z-z^{(n)})} \right)$$

(2.17)

and for the magnetic field:

$$H^{(n)}(r, z) = \sum_{mpq} \Psi_{pq}(r) \hat{z} \times K_{mpq} \cdot I_{mpq}^{(n)}(z)$$

$$= \sum_{mpq} \Psi_{pq}(r) \hat{z} \times K_{mpq} \left( \eta_{mpq}^{n} T_{mpq}^{(n)} \cdot e^{-j\beta_{pq}^{(n)}(z-z^{(n)})} - \eta_{mpq}^{n} R_{mpq}^{(n)} \cdot e^{+j\beta_{pq}^{(n)}(z-z^{(n)})} \right)$$

(2.18)

As it was done with region 0, the upper point $z=z^{(n)}$ is used as the reference in the exponentials, for the regions 1, 2, 3.

**Region 4**

In this region, the point $z=z^{(3)}$ is used as a reference in the exponentials, since point $z=z^{(4)}$ does not exist (it extends to infinity).
For $z \geq z^{(3)}$

\[
\mathbf{E}^{(4)}(r, z) = \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \cdot V^{(4)}_{mpq}(z) \\
= \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \left( T_{mpq}^{(4)} \cdot e^{-j\beta_{pq}^{(4)}(z-z^{(3)})} + R_{mpq}^{(4)} \cdot e^{+j\beta_{pq}^{(4)}(z-z^{(3)})} \right) \quad (2.19)
\]

But for region 4 since there is no $R_{mpq}^{(4)}$ (semi infinite region) the upper equation will be:

\[
\mathbf{E}^{(4)}(r, z) = \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \cdot V^{(4)}_{mpq}(z) \\
= \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \left( T_{mpq}^{(4)} \cdot e^{-j\beta_{pq}^{(4)}(z-z^{(3)})} \right) \quad (2.20)
\]

Equally, and taking into account that there is no $R_{mpq}^{(4)}$ the magnetic field can be written as:

\[
\mathbf{H}^{(4)}(r, z) = \sum_{mpq} \Psi_{pq}(r) \Sigma_{pq}^{(4)} \cdot I_{mpq}^{(4)}(z) \\
= \sum_{mpq} \Psi_{pq}(r) \Sigma_{pq}^{(4)} \left( T_{mpq}^{(4)} \cdot e^{-j\beta_{pq}^{(4)}(z-z^{(3)})} - \eta_{mpq}^{(4)} R_{mpq}^{(4)} \cdot e^{+j\beta_{pq}^{(4)}(z-z^{(3)})} \right) \\
= -\sum_{mpq} \Psi_{pq}(r) \Sigma_{pq}^{(4)} \left( T_{mpq}^{(4)} \cdot e^{-j\beta_{pq}^{(4)}(z-z^{(3)})} \right) \quad (2.21)
\]

### 2.4.2 Application of Boundary Conditions

Having defined the modal tangential electromagnetic fields for each dielectric layer, the next step is to apply boundary conditions to match the fields between 2 layers at their common boundary[8]. The main aspect is to compute the reflected and transmitted wave amplitude coefficients ($T_{mpq}^{(n)}$ and $R_{mpq}^{(n)}$), by obtaining a relationship that will
relate them. In order to do so, the study has to begin from the upper region and continue towards the lower regions. In the regions where there is no array, the field amplitudes can be formed by using continuity and the orthogonality properties of Floquet modes.

**Interface between Regions 4-3 at the point z=z(3)**

Applying the continuity theorem for the Electric and Magnetic fields for the regions 3 and 4, at the point z=z(3):

**Electric field**

\[
E^{(3)}(r, z^{(3)}) = E^{(4)}(r, z^{(3)}) \iff \quad \text{(Continuity)}
\]

\[
\sum_{mpq} \Psi_{pq}(r) \chi_{mpq} V_{mpq}^{(3)}(z^{(3)}) = \sum_{mpq} \Psi_{pq}(r) \chi_{mpq} V_{mpq}^{(4)}(z^{(3)})
\]

(2.22)

Using the orthogonality properties of the Floquet functions, the summations in the equation vanish, simplifying it into:

\[
V_{mpq}^{(3)}(z^{(3)}) = V_{mpq}^{(4)}(z^{(3)}) \iff \quad \text{(Orthogonality)}
\]

\[
T_{mpq}^{(3)} e^{-j\beta_{pq}^{(3)}(z^{(3)})} + R_{mpq}^{(3)} e^{+j\beta_{pq}^{(3)}(z^{(3)})} \bigg|_{z=z^{(3)}} = T_{mpq}^{(4)} e^{-j\beta_{pq}^{(4)}(z^{(3)})} + R_{mpq}^{(4)} e^{+j\beta_{pq}^{(4)}(z^{(3)})} \bigg|_{z=z^{(3)}}
\]

(2.23)

But at the interface z=z(3) the exponential terms of the reflective and transmitted amplitude coefficients get the value of 1, therefore:

\[
T_{mpq}^{(3)} + R_{mpq}^{(3)} = T_{mpq}^{(4)} + R_{mpq}^{(4)}
\]

(2.24)
but at region 4 there is no $R_{mpq}^{(4)}$, therefore:

$$T_{mpq}^{(3)} + R_{mpq}^{(3)} = T_{mpq}^{(4)}$$  \hfill (2.25)

**Magnetic field**

$$\vec{H}^{(3)}(r, z^{(3)}) = \vec{H}^{(4)}(r, z^{(3)}) \iff \quad (\text{Continuity})$$

$$\sum_{pq} \vec{\Psi}_{pq}^{(3)}(r) \vec{z} \times \vec{K}_{mpq} \cdot J^{(3)}_{mpq}(z^{(3)}) = \sum_{pq} \vec{\Psi}_{pq}^{(4)}(r) \vec{z} \times \vec{K}_{mpq} \cdot J^{(4)}_{mpq}(z^{(3)})$$  \hfill (2.26)

$$J^{(3)}_{mpq}(z^{(3)}) = J^{(4)}_{mpq}(z^{(3)}) \iff \quad (\text{Orthogonality})$$

According to the equation 2.27:

$$\eta_{mpq}^{(3)}(T_{mpq}^{(3)} - R_{mpq}^{(3)}) = \eta_{mpq}^{(4)} \cdot T_{mpq}^{(4)}$$

$$T_{mpq}^{(4)} = \frac{\eta_{mpq}^{(3)} \cdot T_{mpq}^{(3)} - R_{mpq}^{(3)}}{\eta_{mpq}^{(4)} \cdot 1}$$  \hfill (2.28)

Substituting $T_{mpq}^{(4)}$ in the equation (2.25):  

$$T_{mpq}^{(3)} + R_{mpq}^{(3)} = \frac{\eta_{mpq}^{(3)} \cdot T_{mpq}^{(3)} - \eta_{mpq}^{(3)} \cdot R_{mpq}^{(3)}}{\eta_{mpq}^{(4)}}$$  \hfill (2.29)
Finally,

\[ R_{mpq}^{(3)} = T_{mpq}^{(3)} \frac{\eta_{mpq}^{(3)} - \eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)} + \eta_{mpq}^{(4)}} \]  

But it is observed that the second part of the product in the upper equation is the equivalent reflection coefficient seen by the equivalent waves which propagate along region 3, at the interface between the regions 3 and 4, from the equivalent propagating waves. Therefore:

\[ P_{mpq}^{(3)} = \frac{\eta_{mpq}^{(3)} - \eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)} + \eta_{mpq}^{(4)}} \]  

Substituting (2.32) in (2.31),

\[ R_{mpq}^{(3)} = T_{mpq}^{(3)} \cdot P_{mpq}^{(3)} \]  

**Interface between Regions 3-2 at the point \( z=z^{(2)} \)**

Moving to the regions 3 and 2 it is clear that the presence of the array in the interface between them, requires a different approach in the methodology. To begin with, the total electric field is continuous at the interface of the array.

\[ \bar{E}^{(2)}(r, z^{(2)}) = \bar{E}^{(3)}(r, z^{(2)}) \]
In contrast there is no continuity of the magnetic field due to the total current in the array.

\[ \mathbf{H}^{(2)}(r, z^{(2)}) - \mathbf{H}^{(3)}(r, z^{(2)}) = J^{(u)}(r, z^{(2)}), \]  

(2.35)

where \( J^{(u)} \) is the current surface density of the upper array.

The equations for the electric and magnetic fields can be expressed using Floquet mode expansion, taking into account that the total current in the upper periodic array can also be expanded as a set of Floquet modes, due to the periodicity of the currents.

\[ \mathbf{J}^{(u)}(r, z) = \sum_{mpq} j_{mpq}^{(u)} \cdot \delta(z - z^{(2)}) \cdot \mathbf{\Psi}_{pq}(r) \times \mathbf{K}_{mpq} \]  

(2.36)

Where \( j_{mpq}^{(u)} \) stands for the current spectral coefficients of the upper array.

Considering the Electric and Magnetic field in turn:

**Electric field**

\[ \mathbf{E}^{(2)}(r, z^{(2)}) = \mathbf{E}^{(3)}(r, z^{(2)}) \iff \]  

(Continuity)

\[ \sum_{mpq} \mathbf{\Psi}_{pq}(r) \mathbf{K}_{mpq} \mathbf{V}^{(2)}_{mpq}(z^{(2)}) = \sum_{mpq} \mathbf{\Psi}_{pq}(r) \mathbf{K}_{mpq} \mathbf{V}^{(3)}_{mpq}(z^{(2)}) \]  

(2.37)

Using the orthogonality properties of the Floquet functions:

\[ \mathbf{V}^{(2)}_{mpq}(z^{(2)}) = \mathbf{V}^{(3)}_{mpq}(z^{(2)}) \iff \]  

(Orthogonality)

\[ T_{mpq}^{(2)} e^{-j\beta_m^{(2)}(s-z^{(2)})} + R_{mpq}^{(2)} e^{+j\beta_m^{(2)}(s-z^{(2)})} = T_{mpq}^{(3)} e^{-j\beta_n^{(3)}(s-z^{(2)})} + R_{mpq}^{(3)} e^{+j\beta_n^{(3)}(s-z^{(3)})} \]  

(2.38)
But at \( z=z^{(2)} \) the exponential terms of the reflective and transmitted amplitude coefficients, at the first part of the equation, get the value of 1 therefore:

\[
T_{mpq}^{(2)} + R_{mpq}^{(2)} = T_{mpq}^{(3)} \cdot e^{-j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})} + R_{mpq}^{(3)} \cdot e^{+j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})}
\]  

(2.39)

**Magnetic field**

\[
\vec{H}^{(2)} (r, z^{(2)}) - \vec{H}^{(3)} (r, z^{(2)}) = \vec{J}^{(u)} (r, z^{(2)}) \quad \text{(Discontinuity)}
\]

\[
\sum_{mpq} \Psi_{pq} (r) z \times \kappa_{mpq} \cdot J^{(2)}_{mpq} (z^{(2)}) - \sum_{mpq} \Psi_{pq} (r) z \times \kappa_{mpq} \cdot J^{(3)}_{mpq} (z^{(2)}) =
\]

\[
\sum_{mpq} j^{(u)}_{mpq} \cdot \delta (z - z^{(2)}) \cdot \Psi_{pq} (r) z \times \kappa_{mpq}
\]  

(2.40)

Using the orthogonality properties of the Floquet functions:

\[
I_{mpq}^{(2)} (z^{(2)}) - I_{mpq}^{(3)} (z^{(2)}) = j^{(u)}_{mpq} \Leftrightarrow
\]

\[
j^{(2)}_{mpq} (r^{(2)}) e^{-j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})} - j^{(2)}_{mpq} (r^{(2)}) e^{+j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})} \\
- j^{(3)}_{mpq} (r^{(3)}) e^{-j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})} + j^{(3)}_{mpq} (r^{(3)}) e^{+j \phi_{pq}^{(3)} (z^{(2)} - z^{(3)})}
\]  

(2.41)

But at \( z=z^{(2)} \) the exponential terms of the reflective and transmitted amplitude coefficients, at the first part of the equation, get the value of 1 therefore:
Having as reference the equation (2.33), we substitute $R_{mpq}^{(3)}$ and then solve the upper equation for $T_{mpq}^{(3)}$

$$T_{mpq}^{(3)} = \frac{\eta_{mpq}^{(2)} (T_{mpq}^{(2)} - R_{mpq}^{(2)}) - j_{mpq}^{(u)}}{\eta_{mpq}^{(3)} (e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} - P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})})} \tag{2.43}$$

By substituting (2.33),(2.43) in the last mathematical formula of Electric field section (2.39):

$$T_{mpq}^{(2)} + R_{mpq}^{(2)} = (e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} + P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})}) \cdot T_{mpq}^{(3)} \Rightarrow$$

$$T_{mpq}^{(2)} + R_{mpq}^{(2)} = \frac{[e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} + P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})}] \cdot \eta_{mpq}^{(3)} (T_{mpq}^{(2)} - R_{mpq}^{(2)}) - j_{mpq}^{(u)}}{[e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} - P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})}] \cdot \eta_{mpq}^{(3)}} \tag{2.44}$$

For the sake of simplicity in mathematical calculations, a dimensionless factor $\omega^{(3)}$ is introduced. The index 3 refers to the interface $z=z^{(3)}$.

$$\omega^{(3)} = \frac{[e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} + P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})}]}{[e^{-j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})} - P_{mpq}^{(3)} \cdot e^{+j\beta_{pq}^{(3)}(z^{(2)}-z^{(3)})}]} \tag{2.45}$$

So equation (2.44) can be re-written as:

$$T_{mpq}^{(2)} + R_{mpq}^{(2)} = \omega^{(3)} \left[ \frac{\eta_{mpq}^{(2)} (T_{mpq}^{(2)} + R_{mpq}^{(2)}) - j_{mpq}^{(u)}}{\eta_{mpq}^{(3)}} \right] \tag{2.46}$$

With a series of mathematical calculus:
\[
R_{mpq}^{(2)} = \frac{\omega^{(3)} I_{mpq}^{(2)} - \eta_{mpq}^{(3)}}{\omega^{(3)} I_{mpq}^{(2)} + \eta_{mpq}^{(3)}} T_{mpq}^{(2)} - \frac{\omega^{(3)}}{\omega^{(3)} I_{mpq}^{(2)} + \eta_{mpq}^{(3)}} J_{mpq}^{(u)}
\]

(2.47)

For the sake of simplicity, the terms \( \frac{\omega^{(3)} I_{mpq}^{(2)} - \eta_{mpq}^{(3)}}{\omega^{(3)} I_{mpq}^{(2)} + \eta_{mpq}^{(3)}} \) and \( \frac{\omega^{(3)}}{\omega^{(3)} I_{mpq}^{(2)} + \eta_{mpq}^{(3)}} \) can be represented with the dimensionless factors \( \tau^{(3)} \) and \( \phi^{(3)} \) respectively. Therefore the upper equation can be re-written as:

\[
R_{mpq}^{(2)} = \tau^{(2)} \cdot T_{mpq}^{(2)} - \phi^{(2)} J_{mpq}^{(u)}
\]

(2.48)

**Interface between Regions 2-1 at the point \( z=z^{(1)} \)**

Having acquired the relationship for the amplitude transmission and reflection coefficients in the two upper interfaces, the next step is to apply the boundary conditions at the interface \( z=z^{(1)} \), located between regions 1 and 2. As with the previous case, the presence of an array indicates continuity of an electric filed, but discontinuity of the magnetic field.

\[
\bar{E}^{(1)}(r,z^{(1)}) = \bar{E}^{(2)}(r,z^{(1)})
\]

(2.49)

\[
\bar{H}^{(1)}(r,z^{(1)}) - \bar{H}^{(2)}(r,z^{(1)}) = \bar{J}^{(1)}(r,z^{(1)}),
\]

(2.50)

where \( \bar{J}^{(1)} \) is the current surface density of the lower array.

The equations for the electric and magnetic fields can be expressed using Floquet mode expansion, taking into account that the total current in the lower periodic array can also be expanded as a set of Floquet modes, due to the periodicity of the currents.

\[
\bar{J}^{(1)}(r,z) = \sum_{mpq} j_{mpq}^{(1)} \cdot \delta(z-z^{(1)}) \cdot \Psi_{pq}(r) \hat{z} \times \mathbf{k}_{mpq}
\]

(2.51)
Where $J_{mpq}^{(l)}$ stands for the current spectral coefficients of the lower array.

Considering the Electric and magnetic fields in turn:

**Electric field**

\[
\vec{E}^{(1)}(r,z^{(1)}) = \vec{E}^{(2)}(r,z^{(1)}) \Leftrightarrow \quad \text{(Continuity)}
\]

\[
\sum_{mpq} \Psi_{pq}^{(1)}(r) \kappa_{mpq} V_{mpq}^{(1)}(z^{(1)}) = \sum_{mpq} \Psi_{pq}^{(2)}(r) \kappa_{mpq} V_{mpq}^{(2)}(z^{(1)}) \quad \text{(2.52)}
\]

Using the orthogonality properties of the Floquet functions:

\[
V_{mpq}^{(1)}(z^{(1)}) = V_{mpq}^{(2)}(z^{(1)}) \Leftrightarrow \quad \text{(Orthogonality)}
\]

\[
T_{mpq}^{(1)} e^{-j\beta_{pq}^{(1)}(z^{(1)})} + R_{mpq}^{(1)} e^{+j\beta_{pq}^{(1)}(z^{(1)})} \bigg|_{z=0} = T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} + R_{mpq}^{(2)} e^{+j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} \bigg|_{z=0} \quad \text{(2.53)}
\]

But at $z=z^{(1)}$ the exponential terms of the reflective and transmitted amplitude coefficients, at the first part of the equation, get the value of 1 therefore:

\[
T_{mpq}^{(1)} + R_{mpq}^{(1)} = T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} + R_{mpq}^{(2)} e^{+j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} \quad \text{(2.54)}
\]

**Magnetic field**

\[
\vec{H}^{(1)}(r,z^{(1)}) - \vec{H}^{(2)}(r,z^{(1)}) = \vec{J}^{(l)}(r,z^{(1)}) \quad \text{(Discontinuity)}
\]
\[
\sum_{mpq} \Psi_{pq}(r) z \mathbf{k}_{mpq} \cdot I_{mpq}^{(1)}(z^{(1)}) - \sum_{mpq} \Psi_{pq}(r) z \mathbf{k}_{mpq} \cdot I_{mpq}^{(2)}(z^{(1)}) = \\
\sum_{mpq} j_{mpq}^{(u)} \delta(z - z^{(1)}) \Psi_{pq}(r) z \mathbf{k}_{mpq}
\] (2.55)

Using the orthogonality properties of the Floquet functions:

\[
I_{mpq}^{(1)}(z^{(1)}) - I_{mpq}^{(2)}(z^{(1)}) = j_{mpq}^{(l)} \iff \\
\eta_{mpq}^{(l)}(T_{mpq}^{(1)} e^{-j\beta_{pq}^{(1)}(z^{(1)}-z^{(0)})} - R_{mpq}^{(1)} e^{+j\beta_{pq}^{(1)}(z^{(0)}-z^{(1)})}) - \eta_{mpq}^{(2)}(T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} - R_{mpq}^{(2)} e^{+j\beta_{pq}^{(2)}(z^{(0)}-z^{(2)})}) = j_{mpq}^{(l)}
\] (2.56)

But at \( z = z^{(1)} \) the exponential terms of the reflective and transmitted amplitude coefficients, at the first part of the equation, get the value of 1 therefore:

\[
\eta_{mpq}^{(1)}(T_{mpq}^{(1)} - R_{mpq}^{(1)}) = \eta_{mpq}^{(2)}(T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} - R_{mpq}^{(2)} e^{+j\beta_{pq}^{(2)}(z^{(0)}-z^{(2)})}) + j_{mpq}^{(l)}
\] (2.57)

Using the relationship obtained in the upper region, which relates \( T_{mpq}^{(2)} \) and \( R_{mpq}^{(2)} \) (2.48), the equation (2.57) can be written as:

\[
\eta_{mpq}^{(l)}(T_{mpq}^{(1)} - R_{mpq}^{(1)}) = \eta_{mpq}^{(2)}[T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} - (T_{mpq}^{(2)} - \beta_{pq}^{(2)} R_{mpq}^{(2)} T_{mpq}^{(2)} - \beta_{pq}^{(2)} J_{mpq}^{(u)} + \beta_{pq}^{(2)} J_{mpq}^{(u)} e^{+j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})}]
\] + j_{mpq}^{(l)}

\[
= \eta_{mpq}^{(2)} T_{mpq}^{(2)} e^{-j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})} - \eta_{mpq}^{(2)} T_{mpq}^{(2)} + \eta_{mpq}^{(2)} \beta_{pq}^{(2)} J_{mpq}^{(u)} e^{+j\beta_{pq}^{(2)}(z^{(1)}-z^{(2)})}
\] + j_{mpq}^{(1)}
\] (2.58)

35
By solving equation (2.58) in terms of $T_{mpq}^{(2)}$ and substituting multiple terms in order to produce simpler representations,

$$T_{mpq}^{(2)} = \omega_{mpq}^{(1)} T_{mpq}^{(1)} - \omega_{mpq}^{(1)} R_{mpq}^{(1)} - \omega_{mpq}^{(1)} R_{mpq}^{(1)} j_{mpq}^{(1)} + \frac{\omega_{mpq}^{(1)} j_{mpq}^{(1)}}{\eta_{mpq}^{(1)}} j_{mpq}^{(1)}$$  

(2.59)

Where:

$$\omega_{mpq}^{(1)} = \frac{\eta_{mpq}^{(1)}}{\eta_{mpq}^{(2)} e^{-j\beta^{(2)}(z^{(1)} - z^{(2)})}}$$  

and

(2.60)

$$\tau_{mpq}^{(1)} = \frac{\eta_{mpq}^{(2)} e^{+j\beta^{(2)}(z^{(1)} - z^{(2)})}}{\eta_{mpq}^{(1)}}$$  

(2.60a)

By introducing $T_{mpq}^{(2)}$ in the Electric field equation (2.54),

$$R_{mpq}^{(1)} = \frac{1 - \phi_{mpq}^{(1)} T_{mpq}^{(1)} + \phi_{mpq}^{(1)} j_{mpq}^{(1)}}{1 + \phi_{mpq}^{(1)}} R_{mpq}^{(1)} + \frac{\phi_{mpq}^{(1)} T_{mpq}^{(1)} + \phi_{mpq}^{(1)} e^{+j\beta^{(2)}(z^{(1)} - z^{(2)})}}{1 + \phi_{mpq}^{(1)}} j_{mpq}^{(1)}$$  

(2.61)

$$R_{mpq}^{(1)} = A_{mpq} T_{mpq}^{(1)} + A_{mpq} j_{mpq}^{(1)}$$  

(2.62)

where,

$$A_{mpq}^{(1)} = \frac{1 - \phi_{mpq}^{(1)} T_{mpq}^{(1)} + \phi_{mpq}^{(1)} j_{mpq}^{(1)}}{1 + \phi_{mpq}^{(1)}}$$

$$A_{mpq}^{(2)} = \frac{\phi_{mpq}^{(1)} T_{mpq}^{(1)} + \phi_{mpq}^{(1)} e^{+j\beta^{(2)}(z^{(1)} - z^{(2)})}}{1 + \phi_{mpq}^{(1)}}$$

As it was done with all the upper regions, a relationship between the reflection ($R_{mpq}^{(1)}$) and transmission ($T_{mpq}^{(1)}$) amplitude coefficient was obtained.
Interface between Regions 1-0 at the point $z = z^{(0)}$

Applying the continuity theorem for the Electric and Magnetic fields for the regions 1 and 0, at the interface $z = z^{(0)}$:

Electric field

\[ E^{(0)}(r, z^{(0)}) = E^{(1)}(r, z^{(0)}) \iff (\text{Continuity}) \]

\[ \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} V^{(0)}_{mpq}(z^{(0)}) = \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} V^{(1)}_{mpq}(z^{(0)}) \]

Using the orthogonality properties of the Floquet functions, the summations in the equation vanish, simplifying it into:

\[ V^{(0)}_{mpq}(z^{(0)}) = V^{(1)}_{mpq}(z^{(0)}) \iff (\text{Orthogonality}) \]

Then

\[ T^{(0)}_{mpq} e^{-j \beta^{(0)}(z-z^{(0)})} + R^{(0)}_{mpq} e^{+j \beta^{(0)}(z-z^{(0)})} \bigg|_{z=z^{(0)}} = T^{(0)}_{mpq} e^{-j \beta^{(0)}(z-z^{(0)})} + R^{(0)}_{mpq} e^{+j \beta^{(0)}(z-z^{(0)})} \bigg|_{z=z^{(0)}} \]

But at $z = z^{(0)}$ the exponential terms of the reflective and transmitted amplitude coefficients get the value of 1, therefore:

\[ T^{(0)}_{mpq} + R^{(0)}_{mpq} = T^{(1)}_{mpq} + R^{(1)}_{mpq} \]

But at region 4 there is no $T^{(0)}_{mpq}$, therefore

\[ R^{(0)}_{mpq} = T^{(1)}_{mpq} + R^{(1)}_{mpq} \]
Magnetic field

\[ \mathbf{H}^{(0)}(r, z^{(0)}) = \mathbf{H}^{(1)}(r, z^{(0)}) \quad \Rightarrow \quad (\text{Continuity}) \]

\[
\sum_{mpq} \Psi_{pq}(r) \hat{z} \times \mathbf{K}_{mpq} \cdot I_{mpq}^{(0)}(z^{(0)}) = \sum_{mpq} \Psi_{pq}(r) \hat{z} \times \mathbf{K}_{mpq} \cdot I_{mpq}^{(1)}(z^{(0)}) \quad (2.67)
\]

\[ I_{mpq}^{(0)}(z^{(0)}) = I_{mpq}^{(1)}(z^{(0)}) \quad \Rightarrow \quad (\text{Orthogonality}) \]

\[
\eta_{mpq}^{(0)}(T_{mpq}^{(0)} - R_{mpq}^{(0)}) = \eta_{mpq}^{(0)} \cdot R_{mpq}^{(0)} \quad \Rightarrow \quad (2.69)
\]

Combining equations (2.62),(2.66)

\[
\eta_{mpq}^{(0)} \cdot R_{mpq}^{(0)} = \eta_{mpq}^{(0)} \cdot T_{mpq}^{(0)} - \eta_{mpq}^{(0)} \cdot A_{mpq}^{(1)} \cdot T_{mpq}^{(1)} - \eta_{mpq}^{(0)} \cdot A_{mpq}^{(1)} \cdot A_{mpq}^{(2)} \cdot J_{mpq}^{(1)} + \eta_{mpq}^{(0)} \cdot A_{mpq}^{(2)} \cdot j^{(u)} \quad (2.70)
\]

Solving (2.70) in terms of \( T_{mpq}^{(1)} \):

\[
T_{mpq}^{(1)} = \frac{\eta_{mpq}^{(0)} \cdot R_{mpq}^{(0)} + \eta_{mpq}^{(1)} \cdot A_{mpq}^{(1)} \cdot j^{(l)} - \eta_{mpq}^{(1)} \cdot A_{mpq}^{(2)} \cdot j^{(u)}}{\eta_{mpq}^{(0)} (1 - A_{mpq}^{(1)})} \quad (2.71)
\]
Finally substituting $T_{mpq}^{(1)}$ and $R_{mpq}^{(0)}$ in the Electric field equation and solving it in terms of $R_{mpq}^{(0)}$:

$$R_{mpq}^{(0)} = T_{mpq}^{(1)} + R_{mpq}^{(1)} = T_{mpq}^{(1)} + A_{mpq} \cdot T_{mpq}^{(1)} + A_{mpq}^{(1)} j_{mpq}^{(1)} - A_{mpq}^{(2)} j_{mpq}^{(u)} \Leftrightarrow$$

$$R_{mpq}^{(0)} = T_{mpq}^{(1)} (1 + A_{mpq}) + A_{mpq}^{(1)} j_{mpq}^{(1)} - A_{mpq}^{(2)} j_{mpq}^{(u)} \Leftrightarrow$$

$$R_{mpq}^{(0)} = \frac{\eta_{mpq}^{(0)} \cdot R_{mpq}^{(0)} + \eta_{mpq}^{(1)} A_{mpq}^{(1)} j_{mpq}^{(1)} - \eta_{mpq}^{(1)} A_{mpq}^{(2)} j_{mpq}^{(u)}}{\eta_{mpq}^{(0)} (1 - A)} (1 + A_{mpq}) + A_{mpq}^{(1)} j_{mpq}^{(1)} - A_{mpq}^{(2)} j_{mpq}^{(u)}$$

(2.72a)

Setting $X_{mpq}^{(1)} = \frac{(1 + A_{mpq})}{\eta_{mpq}^{(0)} (1 - A)}$

$$R_{mpq}^{(0)} = (\eta_{mpq}^{(0)} \cdot R_{mpq}^{(0)} + \eta_{mpq}^{(1)} A_{mpq}^{(1)} j_{mpq}^{(1)} - \eta_{mpq}^{(1)} A_{mpq}^{(2)} j_{mpq}^{(u)}) X + A_{mpq}^{(1)} j_{mpq}^{(1)} - A_{mpq}^{(2)} j_{mpq}^{(u)}$$

(2.72b)

Solving the equation 2.73 in terms of $R_{mpq}^{(0)}$,

$$R_{mpq}^{(0)} = \frac{(\eta_{mpq}^{(1)} A_{mpq}^{(1)} X_{mpq}^{(1)} + A_{mpq}^{(1)} j_{mpq}^{(1)} - (\eta_{mpq}^{(1)} A_{mpq}^{(2)} X_{mpq}^{(1)} + A_{mpq}^{(2)} j_{mpq}^{(u)}))/(1 - \eta_{mpq}^{(0)} \cdot X_{mpq}^{(1)}(0)}$$

(2.74)

Finally a relationship between the reflected amplitude coefficient $(R_{mpq}^{(0)})$ and the unknown spectral currents coefficients $(j_{mpq}^{(1)}, j_{mpq}^{(u)})$ has been obtained.
2.4.3 Electric Field Coupled Integral Equations.

Once the relationship between the equivalent wave amplitude coefficients \( R_{mpq}^{(e)}, T_{mpq}^{(e)} \) and the spectral current coefficients \( j_{mpq}^{(I)}, j_{mpq}^{(e)} \) has been obtained, the electric field integral equations must be applied, for the two perfect conducting arrays.

The Electric field must vanish in the perfect conductors of the lower and upper arrays, at \( z=z^{(1)} \) and \( z=z^{(2)} \):

\[
\begin{align*}
E(r, z^{(2)}) &= 0 \quad \text{Upper Array} \quad (2.75) \\
E(r, z^{(1)}) &= 0 \quad \text{Lower Array} \quad (2.76)
\end{align*}
\]

Beginning with the lower array, the electric field can be expressed using Floquet mode expansions.

\[
\sum_{mpq} (T_{mpq}^{(I)} \cdot e^{-j \beta_{mpq}^{(I)} z^{(1)}} + R_{mpq}^{(I)} \cdot e^{+j \beta_{mpq}^{(I)} z^{(1)}}) \Psi_{pq}(r) \kappa_{mpq} = 0
\] \quad (2.77)

But according to equation (2.62), \( R_{mpq}^{(I)} \) can be written in terms of \( T_{mpq}^{(0)} \). Therefore the upper equation can be expressed only in terms of the transmitted amplitude coefficient.

Continuing, it was proven (in the region 1-0 at the interface \( z=z^{(0)} \)), that \( T_{mpq}^{(0)} \) is directly related to \( R_{mpq}^{(0)} \) (equation (2.71)) and that the amplitude reflection coefficient for that interface, depends only in the unknown spectral currents(2.74).

So the equation (2.77) can be written only in terms of the unknown spectral current coefficients \( j_{mpq}^{(I)}, j_{mpq}^{(e)} \) and it will have the form:
\[
\sum_{mpq} (T_{mpq}^{(1)} \cdot e^{-j\beta_{pq}^{(1)} z^{(1)}} + R_{mpq}^{(1)} \cdot e^{+j\beta_{pq}^{(1)} z^{(1)}}) \Psi_{pq}(r) \kappa_{mpq} = \\
\sum_{mpq} (D_{mpq}^{(u)} j_{mpq}^{(u)} + D_{mpq}^{(l)} j_{mpq}^{(l)}) \Psi_{pq}(r) \kappa_{mpq} = 0
\]

(2.78)

Where:

\(D_{mpq}^{(u)}\) are the electric field spectral coefficients in the lower array, due to the upper array currents \(j_{mpq}^{(u)}\)

\(D_{mpq}^{(l)}\) are the electric field spectral coefficients in the lower array, due to the lower array currents \(j_{mpq}^{(l)}\)

Similarly for the upper array, the electric field can be expanded in terms of Floquet modes expansions.

\[
\overline{E}(r, z^{(2)}) = 0 \iff \left( \begin{array}{c}
\sum_{mpq} (T_{mpq}^{(2)} \cdot e^{-j\beta_{pq}^{(2)} z^{(2)}} + R_{mpq}^{(2)} \cdot e^{+j\beta_{pq}^{(2)} z^{(2)}}) \Psi_{pq}(r) \kappa_{mpq} = 0
\end{array} \right)
\]

(2.79)

(2.80)

As was done with the electric field integral equation for the lower array, the target is to relate the amplitude reflected and transmitted coefficients with the spectral currents coefficients \((j_{mpq}^{(l)}, j_{mpq}^{(u)})\) of the arrays. From the equation:

\[
R_{mpq}^{(2)} = e^{(2)} \cdot T_{mpq}^{(2)} \cdot \phi_{mpq}^{(u)} \quad (2.80)
\]

(2.80) can be expressed in terms of \(T_{mpq}^{(2)}\) only. But it was proven that the transmitted amplitude coefficient, is related to \(T_{mpq}^{(1)}\) and \(R_{mpq}^{(1)}\) from the relationship:
\[
T_{mpq}^{(2)} = \omega_{mpq}^{(1)} T_{mpq}^{(1)} - \omega_{mpq}^{(1)} R_{mpq} - \omega_{mpq}^{(1)} T_{mpq}^{(1)} j_{mpq}^{(u)} + \frac{\omega_{mpq}^{(1)}}{\gamma_{mpq}^{(1)}} j_{mpq}^{(l)}
\] (2.81)

So equation (2.80) can be written in terms of \( T_{mpq}^{(1)} \) and \( R_{mpq}^{(1)} \). But from the analysis of the lower array, it was proven that \( T_{mpq}^{(1)} \) and \( R_{mpq}^{(1)} \) can be related only to the spectral currents of the arrays. Therefore (2.80) can be written in terms of \( j_{mpq}^{(u)} \) and \( j_{mpq}^{(l)} \) only and it will have the form:

\[
\sum_{mpq} \left( T_{mpq}^{(2)} \cdot e^{-j\beta_{pq}^{(2)} z^{(2)}} + R_{mpq}^{(2)} \cdot e^{+j\beta_{pq}^{(2)} z^{(2)}} \right) \Psi_{pq}(r) \kappa_{mpq} = 0
\] (2.82)

\[
\sum_{mpq} \left( C_{mpq}^{(u)} j_{mpq}^{(u)} + C_{mpq}^{(l)} j_{mpq}^{(l)} \right) \Psi_{pq}(r) \kappa_{mpq} = 0
\] (2.83)

Where:

\( C_{mpq}^{(u)} \) are the electric field spectral coefficients in the upper array, due to the upper array currents \( j_{mpq}^{(u)} \)

\( C_{mpq}^{(l)} \) are the electric field spectral coefficients in the upper array, due to the lower array currents \( j_{mpq}^{(l)} \)

### 2.5 Method of Moments

In order to solve the two Coupled Electrical Field Integral Equations, it is essential to relate the total currents distribution functions \( j_{mpq}^{(u)}(r) \) , \( j_{mpq}^{(l)}(r) \) with the spectral coefficients \( j_{mpq}^{(u)} \) , \( j_{mpq}^{(l)} \). For this purpose the total currents in both arrays are expressed in terms of basis functions:
Lower array: \[ J^{(l)}(r, z^{(l)}) = \sum_{n} l_n \cdot f^{(l)}_n(r) \cdot e^{-j k_{\text{so}} \xi} \quad n=0,1,2\ldots N \] (2.84)

where \( l_n \) is the basis function current coefficient of the lower array.

Upper array: \[ J^{(u)}(r, z^{(2)}) = \sum_{n} u_n \cdot f^{(u)}_n(r) \cdot e^{-j k_{\text{so}} \xi} \quad n=0,1,2\ldots N \] (2.85)

where \( u_n \) is the basis function current coefficient of the upper array and \( N \) is the highest limit for the currents coefficients and basis functions index \( n \).

The total current distributions for both arrays, are expanded in terms of basis functions \( f_n(r) \) as illustrated above. The aim is to relate the spectral current coefficients to the basis functions coefficients. For that purpose it is important to obtain the electric field created by an elementary current, since then the electric field created by any current distributions in the arrays can be obtained.

For an arbitrary elemental periodic \( x \)-directed current at \((r', z')\):

\[ J_{\text{elemental}}(r', z') = \delta(r - r') \cdot \delta(z - z') \hat{x} \cdot e^{-j k_{\text{so}} \xi} \] (2.86)

The block wave term \( e^{-j k_{\text{so}} \xi} \) has been used, since the currents must be coherent with the periodic fields.

This elemental \( x \)-directed periodic current can also be expanded in terms of Floquet modes

\[ J_{\text{elemental}}(r', z') = \sum_{m\rho q} j^{\text{elem}}_{m\rho q} \cdot \delta(z - z') \Psi_{\rho q}(r) \cdot \hat{x} \cdot K_{m\rho q} \] (2.87)

where \( j^{\text{elem}}_{m\rho q} \) are the spectral coefficients associated to this elemental current.

By using Floquet mode orthogonality, the summation is eliminated, leading to:

\[ \delta(r - r') \cdot \delta(z - z') \hat{x} \cdot e^{-j k_{\text{so}} \xi} = j^{\text{elem}}_{m\rho q} \delta(z - z') = F(r', z') \] (2.88)
So knowing the spectral coefficients $j_{mpq}^{elem}$ for any elementary current $\hat{x}$ or $\hat{y}$ directed, located at any transverse point $r'$ and also located at any longitudinal point $z'$, it is easy to find the electric field produced at any observation point $(r, z)$ by this current:

$$\vec{E}_{(r,z|z',z')}^{elem} = \sum_{mpq} V_{mpq}^{z} \Psi_{mpq}(r) \cdot \vec{K}_{mpq} \cdot \vec{E}(r',z')$$ (2.89)

The upper equation is also known as Green’s function [17,18], allowing us to obtain the fields created by any elementary current, as a combination of the Floquet modes ($\Psi_{mpq}(r)$), the elementary current spectral coefficients $F(r',z')$ and the equivalent transmission line functions $V_{mpq}^{z}$.

Knowing the field created by any elementary current, the field created by a current distribution can also be known:

$$\vec{E}_{(r,z)} = \int \sum_{mpq} V_{mpq}^{z}(z) \cdot F(r',z')\Psi_{mpq}(r) \cdot \vec{K}_{mpq} \cdot J(r')\delta(z-z')dr'$$ (2.90)

So taking into account the analysis above, the two Electric Field Integral Equations can be written as:

$$\vec{E}(r,z^{(1)}) = \sum_{mpq} (D_{mpq}^{(u)} \Psi_{pq}(r) \vec{K}_{mpq}) \cdot \int \vec{F}^{u}(r') \cdot J^{u}(r')dr' +$$

$$\sum_{mpq} (D_{mpq}^{(l)} \Psi_{pq}(r) \vec{K}_{mpq}) \cdot \int \vec{F}^{l}(r') \cdot J^{l}(r')dr' = 0$$ (2.91)

$$\vec{E}(r,z^{(2)}) = \sum_{mpq} (C_{mpq}^{(u)} \Psi_{pq}(r) \vec{K}_{mpq}) \cdot \int \vec{F}^{u}(r') \cdot J^{u}(r')dr' +$$

$$\sum_{mpq} (C_{mpq}^{(l)} \Psi_{pq}(r) \vec{K}_{mpq}) \cdot \int \vec{F}^{l}(r') \cdot J^{l}(r')dr' = 0$$ (2.92)

where the $D_{mpq}^{(u)} D_{mpq}^{(l)} C_{mpq}^{(u)} C_{mpq}^{(l)}$ are the field spectral coefficients.
By expanding the currents (Method of Moments) of the upper equations:

\[
\mathbf{J}^u(t', z^{(0)}) = \sum_n \mu_n f_n^{(u)}(t')
\]

\[
\mathbf{J}^l(t', z^{(2)}) = \sum_n \lambda_n f_n^{(l)}(t')
\]

(2.93)  

(2.94)

Where \(\mu_n, \lambda_n\) are the basis coefficients and \(f_n^{(u)}(t'), f_n^{(l)}(t')\) are the basis functions.

The total electric field integral equations then can be expressed as:

\[
\mathbf{E}(r, z^{(l)}) = \sum_n u_n \sum_{mpq} D_{mpq}^{(u)} \Psi_{pq}(r) \mathbf{K}_{mpq} \cdot \int f_n^{(u)}(t') \cdot \mathbf{E}^{(u)}(t') \, dt'
\]

\[
- \sum_n \lambda_n \sum_{mpq} D_{mpq}^{(l)} \Psi_{pq}(r) \mathbf{K}_{mpq} \cdot \int f_n^{(l)}(t') \cdot \mathbf{E}^{(l)}(t') \, dt' = 0
\]

(2.95)

\[
\mathbf{E}(r, z^{(2)}) = \sum_n u_n \sum_{mpq} C_{mpq}^{(u)} \Psi_{pq}(r) \mathbf{K}_{mpq} \cdot \int f_n^{(u)}(t') \cdot \mathbf{E}^{(u)}(t') \, dt'
\]

\[
- \sum_n \lambda_n \sum_{mpq} C_{mpq}^{(l)} \Psi_{pq}(r) \mathbf{K}_{mpq} \cdot \int f_n^{(l)}(t') \cdot \mathbf{E}^{(l)}(t') \, dt' = 0
\]

(2.96)

Where \(\mathbf{E}^{(l)}(t'), \mathbf{E}^{(u)}(t')\) are the Green's functions and \(f_n^{(u)}(t'), f_n^{(l)}(t')\) are the basis functions.

By applying the Gallerkin Method [19] and introducing Test functions \(\mathbf{f}(r)\) in order to discretize the two Electric Field Integral equations we obtain:

For the lower array:
\[\sum u_n \sum_{mpq} D_{mpq}^{(u)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^u}(r) \, dr \cdot \int f_n^{(u)}(r') \cdot F^{(u)}(r') \, dr' +
\sum l_n \sum_{mpq} D_{mpq}^{(l)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^l}(r) \, dr \cdot \int f_n^{(l)}(r') \cdot F^{(l)}(r') \, dr' = 0 \quad (2.97)\]

For the upper array:

\[\sum u_n \sum_{mpq} C_{mpq}^{(u)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^u}(r) \, dr \cdot \int f_n^{(u)}(r') \cdot F^{(u)}(r') \, dr' +
\sum l_n \sum_{mpq} C_{mpq}^{(l)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^l}(r) \, dr \cdot \int f_n^{(l)}(r') \cdot F^{(l)}(r') \, dr' = 0 \quad (2.98)\]

If we express:

\[Z_{u}^{u}(\gamma, n) = \sum_{mpq} D_{mpq}^{(u)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^u}(r) \, dr \cdot \int f_n^{(u)}(r') \cdot F^{(u)}(r') \, dr' \quad (2.99a)\]

\[Z_{u}^{u}(\gamma, n) = \sum_{mpq} D_{mpq}^{(l)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^l}(r) \, dr \cdot \int f_n^{(l)}(r') \cdot F^{(l)}(r') \, dr' \quad (2.99b)\]

\[Z_{u}^{u}(\gamma, n) = \sum_{mpq} C_{mpq}^{(l)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^u}(r) \, dr \cdot \int f_n^{(u)}(r') \cdot F^{(u)}(r') \, dr' \quad (2.99c)\]

\[Z_{u}^{u}(\gamma, n) = \sum_{mpq} C_{mpq}^{(u)} \int \Psi_{pq}(r) \kappa_{mpq} \cdot \overline{f^u}(r) \, dr \cdot \int f_n^{(u)}(r') \cdot F^{(u)}(r') \, dr' \quad (2.99d)\]

we obtain for the lower array:

\[\sum l_n Z_{u}^{u}(\gamma, n) + \sum u_n Z_{u}^{u}(\gamma, n) = 0 \quad \text{with } \gamma=0,1..N \quad (2.100)\]

and for the upper array:

\[\sum l_n Z_{u}^{u}(\gamma, n) + \sum u_n Z_{u}^{u}(\gamma, n) = 0 \quad \text{with } \gamma=0,1..N \quad (2.101)\]

where \( N \) is the highest limit for the equations and test functions index \( \gamma \).

This set of equations can be written now in matricial form:
The method of Moments matrix consists of sub-matrices which depend on the currents coefficients index \( n \) and the equations coefficient \( \gamma \), as follows:

\[
\begin{bmatrix}
Z^{ld}(\gamma,n) & Z^{lu}(\gamma,n) \\
Z^{ul}(\gamma,n) & Z^{uu}(\gamma,n)
\end{bmatrix}
\begin{bmatrix}
l_n \\
u_n
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

In the same way all the sub-matrices \([Z^{lu}(\gamma,n)], [Z^{uu}(\gamma,n)]\) can be expanded. Each sub-matrix has \( N+1 \) rows and \( N+1 \) columns, therefore having dimensions \((N+1)\times(N+1)\).

This is a set of linear equations which form an homogeneous system. There are two possible solutions. The first is a trivial one, in which the coefficients of the Coefficients vector Matrix are zero. As a result the currents in the upper and lower array are zero and the fields are zero (not applicable in the present study-no wave propagation).
In order to have non-trivial solutions, the determinant of the matrix \([Z]\) must be zero. This is known as the characteristic determinant of \([Z]\), which depends on the unknown block wave number.

\[
\det(Z) = 0 \iff \Delta(K_{\text{too}}) = 0 \quad \text{where } K_{\text{too}} \text{ is the block wave number}
\]

The matrix \([Z]\) is calculated with a Numeric algorithm routine, which utilises Crout's factorisation to obtain the characteristic determinant. By varying the propagation constant \(\beta\) from 0 to the boundary of the irreducible Brillouin zone, all the corresponding characteristic determinants of \([Z]\) are computed out for each value of \(\beta\). From the characteristic determinant plot as a function of the block wave number, all the true set minima are obtained, corresponding each minima to a given individual propagation mode.

![Figure 2.4: Propagation mode determined from the plot of characteristic determinant](image1)

With the use of a single bisection method (which evaluates the characteristic determinant values before and after each point) all the propagation constants associated to each of the minima are obtained.
By exploring the whole 2D irreducible Brillouin zone, all the possible modes that exist on the x-y plane could be found. The range of frequencies where there is an absence of any propagation mode, that range of frequencies is considered to be a band-gap (stopband).

The solution of the matricial equations, will also give the values of the basis function spectral coefficients \(d_n, u_n\). Using equations (2.84) and (2.85) the total currents for the lower and upper arrays can be obtained. By knowing the total currents, using the Green's function the total electric field can be described.

### 2.6 Plane Wave Propagation

The plane wave analysis of a periodic array can be described based on the modal analysis presented in the above sections, taking into account an important additional factor. There is an introduction of an incidence electric field at an arbitrary angle \(\theta\), with \(\theta \neq 90\) (surface wave propagation) – Figure 1. The methodology used remains the same, considering some changes produced by the incident field. The present section illustrates the main differences in contrast with the surface wave propagation, without though expanding too much the analysis, since the intention of this chapter is not that.

The first difference is notified in the definition of the block wave number. It is observed that \(k_{lo0}\) is now dependent on the angle of incidence \(\theta\)

\[
k_{lo0} = k_0 \sin \theta \cos \phi + k_0 \sin \theta \cos \phi
\]

With \(\theta \neq 90\)
The next difference is mentioned in the definition of the modal tangential electromagnetic fields for the region 0. In this region the incident field is introduced, therefore it should be taken into account.

**Region 0**

\[
E^{(0)}(r, z) = E^{inc} + \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \cdot \mathcal{V}^{(0)}_{mpq}(z)
\]

\[
= E^{inc} + \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} R^{(0)}_{mpq} \cdot e^{+j\beta^{(0)}_{pq}(z-z^{(0)})} \tag{2.102}
\]

\[
H^{(0)}(r, z) = H^{inc} + \sum_{mpq} \Psi_{pq}(r) \hat{z} \times \kappa_{mpq} \cdot \mathcal{I}^{(0)}_{mpq}(z)
\]

\[
= H^{inc} - \sum_{mpq} \Psi_{pq}(r) \hat{z} \times \kappa_{mpq} \eta^{(0)}_{mpq} R^{(0)}_{mpq} \cdot e^{+j\beta^{(0)}_{pq}(z-z^{(0)})} \tag{2.103}
\]

The incident field can be expressed in terms of the zeroth order Floquet mode (p,q=0) as:

\[
E^{inc}(r, z) = \sum_{m=1} T^{inc}_{m00} e^{-j\beta_{00}(z-z^{(0)})} \mathcal{V}_{00}(r) \kappa_{m00} \tag{2.104}
\]

\[
H^{inc}(r, z) = \sum_{m=1} \eta_{m00} T^{inc}_{m00} e^{-j\beta_{00}(z-z^{(0)})} \mathcal{I}_{00}(r) \hat{z} \cdot \kappa_{m00} \tag{2.105}
\]

The introduction of the incident field will alter the calculation of the boundary conditions for the Interface between Regions 1-0 at the point \(z=z^{(0)}\)
Interface between Regions 1-0 at the point $z=z^{(0)}$

\[ \vec{E}^{(0)}(r, z^{(0)}) = \vec{E}^{(1)}(r, z^{(0)}) \iff \]

\[ \vec{E}_{\text{inc}}^{(0)}(r, z^{(0)}) + \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \cdot V_{mpq}^{(0)}(z^{(0)}) = \sum_{mpq} \Psi_{pq}(r) \kappa_{mpq} \cdot V_{mpq}^{(1)}(z^{(0)}) \quad (2.106) \]

\[ H^{(0)}(r, z^{(0)}) = H^{(1)}(r, z^{(0)}) \iff \]

\[ \vec{H}_{\text{inc}}^{(0)}(r, z^{(0)}) + \sum_{mpq} \Psi_{pq}(r) \hat{z} \times \kappa_{mpq} \cdot I_{mpq}^{(0)}(z^{(0)}) = \sum_{mpq} \Psi_{pq}(r) \hat{z} \times \kappa_{mpq} \cdot I_{mpq}^{(1)}(z^{(0)}) \quad (2.107) \]

By using orthogonality the relationship between equivalent amplitude reflection and transmission coefficients can be obtained, taking into account the influence of the incident field. As was produced with the analysis for the surface wave propagation, a relationship between the amplitude reflection coefficient $R_{mpq}^{(0)}$ and the current spectral coefficients $D_{mpq}^{(0)}$, $D_{mpq}^{(1)}$, $C_{mpq}^{(0)}$, $C_{mpq}^{(1)}$ can be obtained at the end, but with a difference.

Now the coefficients $D_{mpq}^{\text{inc}}$, $C_{mpq}^{\text{inc}}$ due to the incident field must be taken into account.

The last difference can be notified in the formulation of the Electric Field Integral Equations. In order to reach to the matricial form, it is essential to separate the part which depends on the incident field from the part which depends on the currents. So the two EFIE will be:

For the lower array

\[ \sum_{mpq} (D_{mpq}^{(u)} - D_{mpq}^{(1)} \cdot \Psi_{pq}(r) \kappa_{mpq} = \sum_{m=1}^{2} D_{m00}^{\text{inc}} \Psi_{00}(r) \kappa_{m00} \quad (2.108) \]
For the upper array

\[ \sum_{mpq} (C_{mpq}^{(u)} j_{mpq}^{(u)} + C_{mpq}^{(l)} j_{mpq}^{(l)}) \Psi_{pq} (r) k_{mpq} = \sum_{m=1,2} C_{m00}^{inc} \Psi_{00} (r) k_{m00} \]  

(2.109)

Where \( D_{m00}^{inc}, C_{m00}^{inc} \) are the spectral coefficients due to the incident field.

Finally applying the Method of Moments the matricial form can be written as:

\[
\begin{bmatrix}
Z^{ll} (\gamma, n) \\
Z^{lu} (\gamma, n)
\end{bmatrix}
\begin{bmatrix}
Z^{lu} (\gamma, n) \\
Z^{uu} (\gamma, n)
\end{bmatrix}
\begin{bmatrix}
l_n \\
u_n
\end{bmatrix}
= \begin{bmatrix}
E^t \\
E^u
\end{bmatrix}
\]

Where \( E^t, E^u \) are the excitation vector coefficients.

In this case the system of equations is not homogeneous, but the excitation vector is known. Therefore the system can be solved for the induced current coefficients and using the Green's functions the fields can be obtained, as was done before.

The solution for the unknown current coefficients is produced by the inversion of the Method of Moment matrix \( Z \), which is performed using Grout's factorisation in a NAG routine.
2.7 Complementary Band Gap Structures Analysis

The present section of the thesis illustrates the modal analysis used for the design and implementation of complementary structures, studied in Chapter 4. The concept of complementary structures follows the same principles of the CCMEBG arrays, apart from an important parameter. Instead of having two arrays of conducting elements closely spaced to each other, the one array has been replaced with an array of apertures (slots). The apertures are rotated 90° with respect to the conductors, so that both elements will be polarized with the electric field as depicted and will therefore be resonant. The actual designs, results and conclusions are presented analytically in a forthcoming chapter. The following analysis outlines the main differences produced from that used for the closely coupled conducting arrays. Using as a reference point Figure 3, the region 1 is assumed to be air and therefore is not taken under consideration so $R_{npq}^{(0)}, T_{npq}^{(0)}$ are equal to 0. The conducting array is placed at $Z=Z_0$ and the aperture array is placed at $Z=Z_2$.

**Interface between Regions 1-0 at the point $z=z^{(1)}$**

The tangential Electric field is continuous and the tangential Magnetic field is discontinuous.

\[
\begin{align*}
\overrightarrow{E}^{(1)}(r, z^{(1)}) &= \overrightarrow{E}^{(2)}(r, z^{(1)}) \\
\overrightarrow{H}^{(1)}(r, z^{(1)}) - \overrightarrow{H}^{(2)}(r, z^{(1)}) &= \overrightarrow{J}^{(c)}(r, z^{(1)})
\end{align*}
\]

Expanding them in terms of Floquet Modes and using orthogonality:
Theory of analysis

Chapter 2

Electric field

\[ V_{mpq}^{(1)}(z^{(1)}) = V_{mpq}^{(2)}(z^{(1)}) \iff \]
\[ T_{mpq}^{(1)} e^{-j\beta_{pq}(z^{(1)})} + R_{mpq}^{(1)} e^{+j\beta_{pq}(z^{(1)})} \bigg|_{z=z^0} = T_{mpq}^{(2)} e^{-j\beta_{pq}(z^{(2)})} + R_{mpq}^{(2)} e^{+j\beta_{pq}(z^{(2)})} \bigg|_{z=z^0} \] (2.112)

\[ R_{mpq}^{(1)} = T_{mpq}^{(2)} e^{-j\beta_{pq}(z^{(1)}-z^{(2)})} + R_{mpq}^{(2)} e^{+j\beta_{pq}(z^{(1)}-z^{(2)})} \bigg|_{z=z^0} \] (2.113)

Magnetic field

\[ I_{mpq}^{(1)}(z^{(1)}) - I_{mpq}^{(2)}(z^{(1)}) = \widetilde{J}_{mpq}^{(c)} \iff \]
\[ \eta_{mpq}^{(1)}(T_{mpq}^{(1)} - R_{mpq}^{(1)}) = \eta_{mpq}^{(2)}(T_{mpq}^{(2)} e^{-j\beta_{pq}(z^{(1)}-z^{(2)})} - R_{mpq}^{(2)} e^{+j\beta_{pq}(z^{(1)}-z^{(2)})}) + \widetilde{J}_{mpq}^{(c)} \]
\[ R_{mpq}^{(1)} = \eta_{mpq}^{(2)} \left[ R_{mpq}^{(2)} e^{+j\beta_{pq}(z^{(1)}-z^{(2)})} - T_{mpq}^{(2)} e^{-j\beta_{pq}(z^{(1)}-z^{(2)})} \right] \frac{\widetilde{J}_{mpq}^{(c)}}{\eta_{mpq}^{(1)}} \] (2.115)

Combining equations (2.113) and (2.115),

\[ T_{mpq}^{(2)} \left( 1 + \frac{\eta_{mpq}^{(2)}}{\eta_{mpq}^{(1)}} \right) = R_{mpq}^{(2)} \left( \frac{\eta_{mpq}^{(2)}}{\eta_{mpq}^{(1)}} - 1 \right) \frac{\widetilde{J}_{mpq}^{(c)}}{\eta_{mpq}^{(1)}} \] (2.116)

Setting: \( G_{mpq} = \left( 1 + \frac{\eta_{mpq}^{(2)}}{\eta_{mpq}^{(1)}} \right) \) and \( N_{mpq} = \left( \frac{\eta_{mpq}^{(2)}}{\eta_{mpq}^{(1)}} - 1 \right) \)
\[ T_{mpq}^{(2)} = R_{mpq}^{(2)} \left( \frac{N_{mpq}}{G_{mpq}} - \frac{J_{mpq}^{(c)}}{n_{mpq}^{(1)}} \right) \cdot \frac{1}{G_{mpq}} \]  

(2.117)

**Interface between Regions 2-3 at the point \( z=z^{(2)} \)**

At the interface \( z=z^{(2)} \) the tangential electric and magnetic fields are continuous. Furthermore the tangential electric field is equal to the magnetic currents.

\[ \vec{E}^{(2)}(r, z^{(2)}) = \vec{E}^{(2)}(r, z^{(2)}) = \vec{M}_{mpq} \]

(2.118)

Where \( \vec{M}_{mpq} \) are the spectral coefficients for the Magnetic currents.

After field and current expansion and after applying orthogonality, the equation 2.118 can be written as:

\[ T_{mpq}^{(2)} e^{-j\beta_{mpq}(z^{(2)}-z^{(3)})} + R_{mpq}^{(2)} e^{+j\beta_{mpq}(z^{(3)}-z^{(2)})} \bigg|_{z^{(2)}} = T_{mpq}^{(3)} e^{-j\beta_{mpq}(z^{(3)}-z^{(2)})} + R_{mpq}^{(3)} e^{+j\beta_{mpq}(z^{(2)}-z^{(3)})} \bigg|_{z^{(3)}} = \vec{M}_{mpq} \]

But at the interface \( z=z^{(2)} \) the exponential terms of the reflective and transmitted amplitude coefficients, at the first part of the equation, get the value of 1 therefore:

\[ T_{mpq}^{(2)} + R_{mpq}^{(2)} = T_{mpq}^{(3)} e^{-j\beta_{mpq}(z^{(2)}-z^{(3)})} + R_{mpq}^{(3)} e^{+j\beta_{mpq}(z^{(3)}-z^{(2)})} \bigg|_{z^{(2)}} = \vec{M}_{mpq} \]

(2.119)

Also

\[ T_{mpq}^{(2)} + R_{mpq}^{(2)} = \vec{M}_{mpq} \]

(2.119a)

Combining (2.117) and (2.119a) gives

\[ R_{mpq}^{(2)} = \frac{\vec{M}_{mpq} \cdot G_{mpq} - \frac{J_{mpq}^{(c)}}{n_{mpq}^{(1)}}}{N_{mpq} + G_{mpq}} \]

(2.120)

\[ T_{mpq}^{(2)} = \frac{\vec{M}_{mpq} \cdot N_{mpq} + \frac{J_{mpq}^{(c)}}{n_{mpq}^{(1)}}}{N_{mpq} + G_{mpq}} \]

(2.121)
Interface between Regions 4-3 at the point $z=z^{(3)}$

Applying the continuity theorem for the Electric and Magnetic fields for the regions 3 and 4, at the point $z=z^{(3)}$:

**Electric field**

$$E^{(3)}(r, z^{(3)}) = E^{(4)}(r, z^{(3)})$$  \hspace{1cm} (2.122)

By using the orthogonality properties of the Floquet functions and also taking into account that in region 4 there is no $R_{mpq}^{(4)}$:

$$T_{mpq}^{(3)} + R_{mpq}^{(3)} = T_{mpq}^{(4)}$$  \hspace{1cm} (2.123)

**Magnetic field**

$$H^{(3)}(r, z^{(3)}) = H^{(4)}(r, z^{(3)})$$  \hspace{1cm} (2.124)

By using the orthogonality properties of the Floquet functions and also taking into account that in region 4 there is no $R_{mpq}^{(4)}$:

$$T_{mpq}^{(3)} - R_{mpq}^{(3)} = T_{mpq}^{(4)} \frac{\eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)}}$$  \hspace{1cm} (2.125)

By adding and subtracting (2.123) and (2.125):
Theory of analysis

\[
T_{mpq}^{(3)} = T_{mpq}^{(4)} \frac{1 + \eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)}} \quad \text{and} \quad R_{mpq}^{(3)} = T_{mpq}^{(4)} \frac{1 - \eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)}}
\]

(2.126) and (2.127)

In order to express \( T_{mpq}^{(4)} \) in terms of \( \tilde{M}_{mpq} \), we combine the upper equations for \( T_{mpq}^{(3)}, R_{mpq}^{(3)} \) with equation (2.119)

\[
T_{mpq}^{(4)} = \frac{2\tilde{M}_{mpq}}{\left(1 + \frac{\eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)}} \right) e^{-\mu_{pq}^{(3)}(z^{(3)} - z^{(1)})} + \left(1 - \frac{\eta_{mpq}^{(4)}}{\eta_{mpq}^{(3)}} \right) e^{+\mu_{pq}^{(3)}(z^{(3)} - z^{(1)})}}
\]

(2.128)

Setting the denominator to be equal with \( H_{mpq} \), the upper equation can be simplified to

\[
T_{mpq}^{(4)} = \frac{2\tilde{M}_{mpq}}{H_{mpq}}
\]

(2.129)

Now we can express the transmitted and reflected amplitude coefficients \( T_{mpq}^{(3)}, R_{mpq}^{(3)} \) in terms of the spectral magnetic currents coefficient \( \tilde{M}_{mpq} \).

Combining (2.126),(2.127) with (2.129),

\[
T_{mpq}^{(3)} = \frac{\tilde{M}_{mpq}}{H_{mpq}} \quad \Leftrightarrow \quad T_{mpq}^{(3)} = \tilde{M}_{mpq} \Omega_{mpq}
\]

(2.130)

\[
R_{mpq}^{(3)} = \frac{\tilde{M}_{mpq}}{H_{mpq}} \quad \Leftrightarrow \quad R_{mpq}^{(3)} = \tilde{M}_{mpq} \Pi_{mpq}
\]

(2.131)
Theory of Analysis

\[ \Omega_{mpq} = \frac{1 + \eta^{(4)}_{mpq}}{H_{mpq}} \quad \text{and} \quad \Pi_{mpq} = \frac{1 - \eta^{(4)}_{mpq}}{H_{mpq}} \]

Electric and Magnetic Field Coupled Integral Equations.

By applying the Electric Field Integral Equation at the interface \( z = z^{(1)} \) the electric field must vanish in the array of the perfect conductors.

\[ \mathbf{E}(r, z^{(1)}) = 0 \]  \hspace{1cm} (2.131)

The electric field can be expressed using Floquet mode expansions:

\[ \sum_{mpq} (T_{mpq}^{(2)} \cdot e^{-j\beta_{pq}^{(2)} (z^{(1)} - z^{(2)})} + R_{mpq}^{(2)} \cdot e^{+j\beta_{pq}^{(2)} (z^{(1)} - z^{(2)})}) \Psi_{pq}(r) k_{mpq} = 0 \]  \hspace{1cm} (2.132)

By substituting \( T_{mpq}^{(2)} \) and \( R_{mpq}^{(2)} \) in the above equation, with their given expressions (2.120) and (2.121):

\[ \sum_{mpq} \left( \frac{\tilde{M}_{mpq} \cdot N_{mpq} + \tilde{J}_{mpq}}{N_{mpq} + G_{mpq}} \cdot e^{-j\beta_{pq}^{(2)} (z^{(1)} - z^{(2)})} \right) + \]

\[ \sum_{mpq} \left( \frac{\tilde{M}_{mpq} \cdot G_{mpq} - \tilde{J}_{mpq}}{N_{mpq} + G_{mpq}} \cdot e^{+j\beta_{pq}^{(2)} (z^{(1)} - z^{(2)})} \right) \Psi_{pq}(r) k_{mpq} = 0 \]  \hspace{1cm} (2.133)

With a series of mathematical expansions, calculations and settings the upper equation can be written as:

58
\[ \sum_{mpq} (E_{mpq} \tilde{J}_{mpq}^{(c)} + \Phi_{mpq} \tilde{M}_{mpq}) \Psi_{pq}(r) \kappa_{mpq} = 0 \]  \hspace{1cm} (2.134)

Where \( E_{mpq} \) and \( \Phi_{mpq} \) are the field spectral coefficients.

So the Electric Field Integral Equation can be expressed only in terms of \( \tilde{J}_{mpq}^{(c)} \) and \( \tilde{M}_{mpq} \).

The Magnetic Field Integral Equation (MFIE) at the interface \( z = z^{(2)} \) is obtained by applying the condition that the magnetic field is continuous across the apertures:

\[ \hat{H}^{(2)}(r, z^{(2)}) = \hat{H}^{(3)}(r, z^{(2)}) \]  \hspace{1cm} (2.135)

The magnetic field can be expressed using Floquet mode expansions.

\[ \sum_{mpq} (T^{(2)}_{mpq} e^{-j\beta^{(2)}_{pq}(z^{(1)}-z^{(2)})} + R^{(2)}_{mpq} e^{+j\beta^{(2)}_{pq}(z^{(1)}-z^{(2)})}) \eta^{(2)}_{pq} \Psi_{pq} \cdot \hat{z}(r) \kappa_{mpq} = \]

\[ \sum_{mpq} (T^{(3)}_{mpq} e^{-j\beta^{(3)}_{pq}(z-z^{(2)})} + R^{(3)}_{mpq} e^{+j\beta^{(3)}_{pq}(z-z^{(2)})}) \eta^{(3)}_{pq} \Psi_{pq} \cdot \hat{z}(r) \kappa_{mpq} \]  \hspace{1cm} (2.136)

Substituting \( T^{(2)}_{mpq}, R^{(2)}_{mpq}, T^{(3)}_{mpq}, R^{(3)}_{mpq} \) in the (MFIE) and following the same procedure with the Electric Field Integral Equation, the Magnetic Field Integral Equation can be expressed in terms of \( \tilde{J}_{mpq}^{(c)} \) and \( \tilde{M}_{mpq} \).

\[ \sum_{mpq} (H_{mpq} \tilde{J}_{mpq}^{(c)} - Y_{mpq} \tilde{M}_{mpq}) \Psi_{pq}(r) \hat{z} \kappa_{mpq} = 0 \]

The MFIE and the EFIE can now be coupled into a single matrix equation, which follows the same principle as that described in the above analysis. In this case it is the spectral coefficient of \( \tilde{J}_{mpq}^{(c)} \) in the EFIE and \( \tilde{M}_{mpq} \) in the MFIE which describe the coupling between the conductors and apertures on the different layers.
2.8 Irreducible Brillouin zone and the array element

The irreducible Brillouin zone will depend on the circularly symmetrical nature of the array element. The angles between lines of symmetry for the array element and the first Brillouin zone of the lattice must be taken into the consideration. The larger angle of the two symmetry properties is chosen for the irreducible Brillouin zone, provided that the smaller angle is a multiple factor of it. If not, the next larger angle is chosen as the irreducible Brillouin zone which is a multiple factor of the two angles [21].

The irreducible Brillouin zone will be determined by the angle between the lines of symmetry of its first Brillouin zone. Due to the symmetric and periodic properties within the first Brillouin zone, the smallest region (the shaded portion in Figure 2.5) of the first Brillouin zone, is irreducible. Thus it will be sufficient to consider only the irreducible zone as the rest are just mirror images of it.

In Figure 2.5, for a triangular lattice, the angle between the lines of symmetry of its first Brillouin zone is 30°. For the square lattice in Figure 2.6 the angle will be 45°.

\[ \text{Irreducible Brillouin zone} \]

\[ \text{Figure 2.5: Lines of symmetry in the first Brillouin zone of a triangular lattice} \]

\[ \text{Irreducible Brillouin zone} \]

\[ \text{Figure 2.6: Lines of symmetry in the first Brillouin zone of a square lattice} \]

In the case of a dipole as the array element, there are only two lines of symmetry (Figure 2.7a). Thus the dipole is only quarterly circular symmetric (90°). Whereas the
angle of symmetry for first Brillouin zone of a square and triangular lattice is 45° and 30° (Figure 2.6 a,b) respectively. Therefore the irreducible Brillouin zone must cover at least 90° of the first Brillouin zone.

![Irreducible Brillouin zones](image)

Figure 2.7 · (a) Line of symmetry of the dipole, (b) Irreducible first Brillouin zone of a dipole in a square reciprocal lattice (c) Irreducible first Brillouin zone of a dipole in a hexagonal reciprocal lattice.

For the tripole array element, the response depends on the contribution of the current in each of the three legs of the tripole. The response would be the same in the direction of A and A', B and B' and C and C'. Thus the angle of symmetry for the tripole array is 30°, coincidentally for the first Brillouin zone of the triangular lattice it is also 30° (Figure 2.8). So the irreducible Brillouin zone will have an angle of symmetry of 30°.

![Lines of symmetry](image)

Figure 2.8 · Lines of symmetry for a tripole and the first Brillouin zone of a triangular lattice.
For a tripole on a square lattice, the angle of symmetry for its first brillouin zone is $45^\circ$ and the angle of symmetry for the tripole array is $30^\circ$. The angle of $45^\circ$ cannot be chosen as the irreducible Brillouin zone because $30^\circ$ is not a factor of it, thus the next higher angle is chosen. In this case it is $90^\circ$ which both angles are a factor of.

![Figure 2.9 - The irreducible Brillouin zone of a tripole in a square lattice.](image)

2.9 Conclusion

The present chapter focused in the introduction of a modal analysis method, in order to simulate the response of an infinite closely coupled double layer periodic array. The array was placed in a multiple dielectric substrate, with each element located in a unit shell. The array elements were printed on either side of a dielectric substrate, in close proximity to each other, in order to produce maximum coupling.

The first stage of the analysis was based on a CCMEBG array of conducting elements, excited by a surface wave at an angle of $\theta = 90^\circ$ (Fig 2.1). Due to the periodicity and the infinite structure of the conducting array, the Floquet theorem was employed to simplify the analysis. This was achieved by expanding the fields in terms of Transverse Electric and Transverse Magnetic Floquet modes. The application of the standard electromagnetic boundary conditions, at the interfaces between the different dielectrics, matched the electromagnetic fields producing a set of Coupled Integral Equations (CIE). These equations were solved in terms of the unknown currents on the conducting element, of the different arrays.
Using the Method of Moments, the Coupled Integral Equations could be reduced to a homogeneous system of linear equations, written in matricial form. The unknown currents were expressed as a series of basis functions. Solving the matricial equations, for the non trivial solution, the basis function spectral coefficients were obtained. Then the total currents for the lower and upper conducting arrays could be found. By knowing the total currents, using the Green’s function, the total electric field could be described.

The second part of the modal analysis was focused on a CCMEBG array of conducting elements, excited by an incident electric field at an arbitrary angle $\theta$, with $\theta \neq 90^\circ$ (Figure 2.1). The methodology used is the same, considering some changes produced by the incident field. The most important differences can be noted in the formulation of the electric field Coupled Integral Equations and the Method of Moments. In the first case it was essential to separate the part which depends on the incident field from the part which depends on the currents. In the Method of Moments, the system of equations was not homogeneous. The excitation vector though was known. Therefore the system could be solved for the induced current coefficients and using the Green’s functions the fields could be obtained, as was done before.

The third part of the modal analysis presented in this chapter, was concentrated in the study of complementary structures. The placement of a conducting array with an aperture array in a very close proximity, with a rotation between them of $90^\circ$, was the main difference with the CCMEBG array. The Coupled Integral Equation produced consisted of an Electric Field Integral equation from the conducting array and a Magnetic Field Integral Equation from the aperture array. The MFIE and the EFIE were coupled into a single matrix equation, following the same principle as that described in the CCMEBG analysis.

The second chapter concluded with a brief introduction of the first Brillouin zone and the irreducible Brillouin zone of dipoles and tripoles, on different geometrical lattices. Taking into consideration the angles of symmetry of each array element and the angles of symmetry of the first Brillouin zone on a specific lattice, the irreducible Brillouin zone could be defined.
REFERENCES


CHAPTER 3

MEBG & CCMEBG Dipole and Tripole Arrays

3.1 Introduction

Closely Coupled Metallodielectric Electromagnetic Band Gap (CCMEBG) structured materials are a class of artificial periodic metamaterials that prohibit propagation of electromagnetic waves within a particular frequency band[1-3]. Closely coupled MEBG structures are formed by two arrays (in this case the arrays consist of conducting elements) printed on either side of a thin dielectric sheet, closely spaced to each other and shifted appropriately in order to produce high element coupling. Due to the close proximity and the strong coupling between the shifted elements, the effective electrical length of the element increases, resulting in a significant decrease of the array resonant frequency. The shifting of the arrays and the separation distance between them are the key attributes in the formulation of such structures.

Lowering the resonant frequency enhances the bandgap stability, by virtue of avoiding the grating responses. Methods such as capacity loading of dipole and cross dipole elements and super dense surfaces have been presented [4, 5], but substantial lowering of the resonance could be achieved by close coupling and relative shifting of a double layer MEBG structure. This will potentially have a beneficial impact on the design of high performance EBGs, because it deals with a size reduction process effectively. The miniaturization capabilities of a CCMEBG structure are accompanied by an exceptional angular stability for plane wave illumination, along with a superior bandwidth in comparison with a single MEBG array consisting of the same elements. CCMEBG structures find applications in surface waves suppression [6,7] and antenna performance enhancement [8-11]

Closely coupled arrays have been presented using conducting dipole and slot arrays, as well as a combination of complementary elements [12-14]. In an equivalent circuit approach, the band gap of a CCMEBG structure emerges by virtue of the resonance of an equivalent LC circuit. The inductance emerges from the currents induced on the
conducting elements and the capacitance is formed in the interelement space. Therefore, reduced frequency can be achieved by increasing the capacitance value. This can be achieved by careful coupling of two arrays, leading to closely coupled arrays. This property is attractive for reducing the size of the structure whilst maintaining superior bandwidth and angular stability. It is investigated here in the context of MEBG structures, leading to Closely Coupled MEBG (CCMEBG) structures.

The concept of CCMEBG structures is general and can be applied to various element geometries. For the purpose of the present research two designs with different characteristics were chosen: The dipole and the tripole. Qualitatively, the difference between a dipole and a tripole array is the type of band gap they can produce. The dipole, as a single polarisation element can only produce one dimensional band gaps. The tripole as a more symmetrical element can produce an absolute 2D band gap.

This chapter presents a thorough study on the two different geometries. Beginning with the dipole array, a description of the actual element, along with the formulation of the analysis of an MEBG and CCMEBG dipole array is done. Plane wave simulations and parametric studies are carried out to predict the best possible outcome, with respect to measurements. Then an introduction in the fabrication process of the arrays and the measurement set-up gives the practical aspects of the project. Measurements for plane wave and surface wave propagation are produced, verifying the simulations and highlighting the differences between an MEBG and CCMEBG dipole arrays. Finally, dispersion curves are formed for both single and double layer dipoles to describe the behaviour of the TE modes on the surface wave propagation.

The second section of the chapter is focused on tripole geometries. A tripole description and the formulation of MEBG and CCMEBG tripole arrays are introduced. Two different element orientations are examined. Plane wave and surface wave simulations and measurements are produced, for both single and double layer tripole arrays and for both orientations. Dispersion curves for surface wave propagation are presented. Finally, closing remarks and a discussion of the results are presented in the conclusion.
3.2 Dipole analysis

Dipole arrays are initially utilised to implement an MEBG and CCMEBG design. The thin linear dipole is the simplest among a variety of elements and it can be used to obtain one dimensional EBG performance. The dipole is a single polarisation element, thus it cannot produce a common 2D band gap when used in a two-dimensional EBG design. The stop band appears due to the element resonance when the electric field orientation is parallel to the dipole and diminishes as the electric field becomes perpendicular to the dipole (no resonance).

The geometry of the unit cell of a dipole array is shown in Fig.(3.1) The dipoles are arranged in a square lattice and have the following dimensions: Length: $L = 15.5$ mm, Width: $W = 0.5$ mm, Periodicity on the X-axis: $D_x = 5$ mm, Periodicity on the Y-axis $D_y = 17.5$ mm. The thickness of the metallic elements is considered to be infinitesimal.

![Figure 3.1: Single layer dipole array and cross sectional view of the unit cell](image)

The single dipole array is printed on a thin dielectric sheet, with a dielectric constant of $\varepsilon_r=3$ and thickness equal to $s=0.071$ mm. A dielectric substrate of thickness $S_{sub}=1.13$ mm and permittivity of 2.2 is used to support the thin array surface.
A double layer structure is formed by combining two identical dipole arrays. The array elements are etched on either side of a thin dielectric substrate. The second layer is shifted along the y-direction with respect to the first one by $D_y/2$ (Fig. 3.2). This displacement has been reported [12] to produce maximum coupling between the two layers. Due to the close proximity and the strong coupling between the shifted elements, the effective electrical length of the element increases. A maximum shift to the array resonance towards lower frequencies is accordingly produced. The two layers are separated by a thin dielectric layer with permittivity $\varepsilon_r = 3$. A dielectric substrate of thickness $S_{\text{sub}} = 1.13$ mm is used to support the arrays, as shown in Fig 3.2.

![Figure 3.2: Cross sectional view of a CCMEBG dipole array](image)

### 3.3 Simulations For MEBG and CCMEBG Dipole Arrays

A series of parametric studies have been carried out in order to study the performance of the Closely Coupled Electromagnetic Band Gap dipole Array. The parameters examined were the separation distance between the two dipole arrays and the relative shifting between the arrays and their elements. The plane wave modal analysis [15,16] discussed in Chapter 2 is implemented, in order to simulate the transmission response of double as well as single layer dipole arrays. For the purpose of simulations 5 basis functions were used and 17 Floquet modes. Beginning with the MEBG array, four different angles of incidence are simulated (0, 30, 60, 85 deg). At normal incidence ($\theta = 0^\circ$), with the electric field parallel to the dipoles, the array resonates at 10.7 GHz.
Grating lobes appear at higher frequencies (not presented here) due to the array y-periodicity. At higher angles of incidence the resonance moves towards lower frequency values (10.2 GHz for $\theta = 85^\circ$) and is affected by grating responses, which start appearing at frequencies near the resonance. The -10 dB fractional bandwidth at normal incidence is calculated to be 32.7% or 3.5 GHz absolute bandwidth. Figure 3.3 illustrates the simulations for the different angles of incidence.

![Figure 3.3: Plane wave simulation of MEBG dipole array, for different angles of incidence](image)

The double layer structure introduced in this chapter is formed by combining two identical dipole arrays. The second layer is shifted along the y-direction (Fig. 3.2), with respect to the first one. The relative shifting between the two arrays will produce a maximum coupling effect. When there is no shifting between the arrays, the CCMEBG structure acts like an MEBG array, producing a resonance at 10.7 GHz. As the second dipole array is shifted, with respect to the first one, the resonance moves down in frequency. There is an optimum point at which the displacement between the two layers produces maximum coupling, and thus introduces a maximum shift to the total array resonance. According to the simulations, that optimum point is equal to $D_y/2$ or 8.75 mm displacement for the two layers.

Figure (3.4) illustrates the different shifting distances and the effect produced in the resonance frequency.
Figure 3.4: Parametric study on the relative shifting of identical dipole arrays to form a CCMEG structure.

In order to simulate the plane wave incidence for the CCMEBG array, a parametric study on the separation distance between the two dipole arrays is produced. Simulations have shown that as the array separation distance $S$ is reduced, the coupling between the two layers becomes stronger, resulting in a decrease of the resonant frequency.

For $S = 0.1$ mm the resonance decreases dramatically and appears at 2.4 GHz, which corresponds to more than 4:1 frequency shift. Thus, an electrical dipole length which produces a $\lambda/2$ resonance is formed due to the interlayer coupling, whereas the physical dipole length in the array is of the order of $\lambda/8$. Smaller separation distances with values approaching zero would result in a further shift of the resonant frequency, however they become impractical in terms of fabrication and are not presented here. For $S = 0$ mm, the dipoles of the two arrays would collapse with each other and an inductive grid of infinitely long metallic strips would be formed. The double layer array with $S = 0.071$ mm will be referred to throughout the paper as dipole CCMEBG.

Figure (3.5 ) illustrates the different separation distances which they were simulated between the two conducting arrays.
Having defined the separation distance between the identical dipole arrays (0.071 mm) and the relative displacement amongst them (8.75 mm), a simulation can be obtained to describe the transmission response of the CCMEBG array for different angles of incidence.

The plane wave simulation for the CCMEBG array illustrates a radical decrease of the resonant frequency, in comparison with the MEBG array. The strong coupling between the two dipole layers caused the shifting of the resonance, which now appears at 2.4 GHz. That corresponds to more than 4:1 frequency shift. The $-10$ dB fractional bandwidth at normal incidence is calculated to be 204% or 4.9 GHz.
absolute bandwidth, in contrast with 32.7% or 3.5 GHz for the MEBG array. Finally an exceptional angular stability is observed for the CCMEBG array.

3.4 Dispersion Curve Diagrams for MEBG and CCMEBG Dipoles

One of the most important concepts in examining electromagnetic band gaps, are the phase and group velocity that will determine the band gap areas. The speed of light in a medium is the velocity at which a plane wave is propagating in that medium, while the phase velocity is the speed at which a constant phase point travels. For a TEM plane wave, these two velocities are identical, but for other types of guided wave propagation, the phase velocity may be greater or less than the speed of light. If the phase velocity and the attenuation of a line or guide are constants with frequency, then the phase of a signal that contains more than one frequency component will not be distorted. If the phase velocity is different for different frequencies, then the individual frequency components will not be able to keep their original phase relationships as they propagate along the transmission line or waveguide, and a signal distortion will occur. The phenomenon is called dispersion, since different phase velocities allow the ‘faster’ waves to lead in phase, relative to the ‘slower’ waves. The original phase relationships will gradually be dispersed as the signal propagates along the line [21].

It is useful to plot the propagation constant, $\beta$, versus the propagation constant of the unloaded line, $k$(or $\omega$), in order to clearly define the pass band and stop band characteristics of a periodic structure. Such a graph is called a $k$-$\beta$ diagram, or Brillouin diagram (after L. Brillouin, a physicist who studied wave propagation in periodic crystal structures). For instance, consider the dispersion relation for a waveguide mode: $\beta = \sqrt{k^2 - k^2_e}$ where $k_e$ is the cut-off wave number of the mode, $k$ is the free space wave number, and $\beta$ is the propagation constant of the mode. The relation is plotted in a $k$-$\beta$ diagram of Figure 3.7 [21]. For values of $k < k_e$, there is
no real solution for $\beta$, so the mode cannot propagate. For $k > k_c$, the mode can propagate, and $k$ approaches $\beta$ for large values of $\beta$ (TEM propagation).

![Dispersion diagram of waveguide modes](image)

Figure 3.7: Dispersion diagram of waveguide modes

The $k - \beta$ diagram is very useful in interpreting the various wave velocities associated with a dispersive structure. The phase velocity is $v_p = \frac{\omega}{\beta} = c \frac{\kappa}{\beta}$, which is seen to be equal to $c$ (speed of light) times the slope of the line from the origin to the operating point on the $k - \beta$ diagram. The group velocity is $v_g = \frac{d\omega}{d\beta} = c \frac{d\kappa}{d\beta}$, which is the slope of the $k - \beta$ curve at the operating point. Thus, from the figure, it can be observed that the phase velocity for a propagating waveguide mode is infinite at cut-off and approaches $c$ (from above) as $\kappa$ increases. The group velocity, however, is zero at cut-off and approaches $c$ (from below) as $\kappa$ increases. Actually a $k - \beta$ diagram can be used to study the dispersion characteristics of many types of microwave components and transmission lines.

Dispersion diagrams have been produced for both single and double layer dipoles, in order to examine qualitatively the modes propagating and the band gaps occurring. The MEBG and CCMEBG dipole arrays have their elements spaced out periodically on two axes, separated by angle $\alpha = 90^\circ$ and placed on a square lattice (Figure 3.8). Due to the symmetric and periodic properties of the first Brillouin zone and the dipole element (Section 2), the shaded region is determined as the irreducible Brillouin zone.
Propagation in this region is the same as the other 3 quadrants. The maximum phase constant ($\beta_x$ and $\beta_y$) in the direction of $x$ and $y$-axis within the irreducible Brillouin zone is $\frac{\pi}{a}$ (the square lattice has its element spaced out periodically on two axes separated by an angle $a$).

The vector Floquet modal analysis described in chapter 2 is employed to model the periodic arrays. A set of coupled integral equations (CIE) is derived and solved using the method of moments. A homogeneous equation is produced and is used for predicting the propagation constant. The procedure in the predictions requires the scanning of the phase constant at each frequency point and obtain the roots of the determinant of the matrix in the characteristic equation. The key attribute of this method is that it is computationally efficient and exhibits a good relative convergence for very small separation distances between the arrays, as opposed to other methods (e.g. the Generalised Scaterring Method).

Based on the geometry of Figure 3.2, the dipoles only impose a boundary condition on electric field components along $y$-axis and that is due to their very small width. Among the surface wave modes supported by the dielectric slab, only those with an electric field component parallel to the $y$-axis can excite currents on the dipoles. The effect of the periodic dipole array (i.e. retardation of the waves and electromagnetic band gap) is hence evident only on those modes, at the vicinity of the dipole resonance. Those are TE modes for propagation along the $x$-axis ($\Gamma X$ part of dispersion curve) Figures 3.8 and 3.9, TM modes for propagation along the $y$-axis ($\Gamma Y$) part and hybrid TE/TM modes for oblique direction of propagation ($X \rightarrow M \rightarrow Y$ in the dispersion curve).

In the following figures, the horizontal axis represent the phase constant of the propagation mode in various directions, and the vertical axis illustrates the frequency respectively.

Beginning with the MEBG dipole array, the first TE mode (surface wave) starts at 0.3 GHz. In the direction ($\Gamma - X$) where the plane of propagation is parallel to the dipole array, the surface wave propagates until the frequency of 7.4 GHz where it ceases. At this frequency, the half free-space wavelength is 20.43mm. This suggests a $\lambda/2$.  

76
resonance for the single layer dipole array, with some variation attributed to the permittivity of the substrate, the mutual impedances between the dipole elements and the fringing capacitances at the edges of each dipole. This is the beginning of the band gap along the x-direction, which extends for 7.6 GHz reaching the upper band gap at 15 GHz. As the propagation direction moves from x towards the y-axis (XM and MY), the resonant mode that excites the dipoles and produces the bandgap is a hybrid TE/TM mode. The band gap gradually narrows until the end of MY area and the frequency of 8.4 GHz. The transition from the end of MY and the beginning of YT area (y-direction), reveals the end of the band gap and the beginning of full wave propagation. As it was expected and verified from the modelling, the dipole array cannot produce an absolute band gap. The predicted band gap for the dipole array, for the different planar directions, coincide well with the measurement illustrated in Figure(3.17)

![Dispersion diagram](image)

**Figure 3.8**: Dispersion diagram for the first TE mode of the MEBG dipole array, (a) direct lattice, (b) reciprocal lattice and its first Brillouin zone
For the CCMEBG dipole array, the surface wave propagation starts at 0.4 GHz (Figure 3.9). In the direction where the dipoles are aligned parallel to the electric field (Γ - X) the propagation continues up to 2.2 GHz. This is the beginning of the band gap along the x-direction. It extends for 8.6 GHz, reaching the upper band gap edge at 11 GHz. This corresponds to a frequency shift in the range of 3.4:1. The physical length of the dipole is about $\lambda_0/8$, where $\lambda_0$ is the free space resonant wavelength. The fractional band gap bandwidth is 110%, much wider than the corresponding 65% of the single layer array. As the propagation direction moves from x towards the y-axis (XM), the band gap gradually narrows. In the (MY) direction the band gap becomes extremely narrow, until it reaches the (YT) area (y-direction), where it diminishes. Full wave propagation occurs from this point. As with the case of the MEBG dipole array, the double layer dipole array cannot produce an absolute band gap.

Figure 3.9 - Dispersion diagram of the first TE mode of the CCMEBG dipole array, (a) direct lattice, (b) reciprocal lattice and its first Brillouin zone
3.5 Array Manufacturing

One of the most important parameters in the realisation of this thesis are the arrays themselves. The manufacturing of the arrays is an issue quite complicated and sensitive, since extreme accuracy and detailed procedures are required for a satisfactory result. Considering the limitations of the process in terms of element definition and alignment between two layers (CCMEBG), it is clear that a small manufacturing mistake can make a noticeable difference in the measured results.

The MEBG and CCMEBG arrays considered throughout this thesis are manufactured in the same way using a chemical etching technique. The initial material is a laminate of copper, adhesive and dielectric, which can be either polyester or Kapton (which has a lower loss tangent). The copper is 1/4oz, having a thickness of 9 microns and the dielectric comes in a variety of different thicknesses from 21 microns to 100 microns for the single copper and 71 microns for the double sided copper laminate.

For the MEBG array the laminate must be mounted on a rigid board using a polymide tape backed by a silicon thermosetting adhesive. It must be fixed copper side up and sealed on all four sides to prevent water and chemicals leaking under. The surface of the copper is cleaned using a scrub cleaner (a process that removes any oxide deposits and creates slight abrasion to promote improved adhesion for the dry film photo resist. When dry, the laminate is placed in a pre-heated oven at 80°C with the photo resist. Then the copper is laminated with a negative dry film resist, using a hot roll laminator. A photographic artwork of the desired array design is then placed on top of the photo resist and exposed to ultra violet light. The artwork is removed and the photo resist can be developed in a spray developer using a 1% solution of sodium carbonate. The final process involves the etching away of the unwanted copper in a spray etching machine, using a solution of ammonium persulphate and the removal of the remaining dry film resist, in a dry film stripping solution.

For the manufacture of the CCMEBG array the same process is followed, except that the two photographic artworks are produced and pre-aligned with the appropriate displacements. Great care is needed during this alignment procedure to ensure the double layer array will perform as expected.


3.6 Antennas and Measurement Setup

In order to carry out plane wave and surface wave measurements over a wide range of frequencies two wideband end-fire antipodal Vivaldi antennas have been used [17,18]. The antennas are printed on the same dielectric material which is used as a substrate for the CCMEBG. The Vivaldi antenna on the dielectric slab is a slow wave structure. The wave generated by the Vivaldi will be bound to the surface of the slab until a discontinuity or non-uniformities appear where subsequently it will be radiated. The feeding of the antenna is by a smooth transition from a microstrip line to a parallel strip line and then to a symmetrically flared out slot line, which is the antenna element itself. The radiating part is the inner edge of the antenna which is exponentially curved out. Therefore, different locations of the antenna inner edge radiate at different frequencies. Due to the small dielectric thickness the polarisation of the radiated field is mainly parallel to the plane of the antenna/dielectric (TE mode). The cross polar component (TM mode) is low at low frequencies and increases at higher frequencies due to the inclination of the radiated electric field.

Figure 3.10: (a) Vivaldi antenna on RT-Duriod slab, thickness(ε) = 1.125mm (b) Vivaldi antenna radiating at a low frequency, (c) Vivaldi antenna radiating at a higher frequency

80
It can be seen from Figure 3.10, that the E field component is skewed with respect to the alignment of the antenna. With a small dielectric slab thickness, at low frequencies the skew will be small (Figure 3.10b). But at high frequency, the skew angle will increase and give high cross-polarisation. The Vivaldi antenna in this case is mainly a TE mode slow leaky end-fire travelling wave antenna. Figure 3.11 illustrates the top and bottom view of two vivaldi antennas, used for plane wave measurements.

Figure 3.11: Top and bottom view (photograph) of balanced Vivaldi antenna used for Plane Wave Measurements

Figure 3.12 presents the top and bottom view of two vivaldi antennas printed on the same dielectric slab, used for surface wave measurements.

Figure 3.12: Top and bottom view (photograph) of balanced Vivaldi antenna used for Surface Wave Measurements
3.7 Measurements For MEBG and CCMEBG Dipole Arrays

3.7.1 Plane Wave Measurements

For the plane wave measurements (normal and oblique incidence), the dipole CCMEBG surface has been placed in an open window of the same dimensions within an absorbing screen. The Vivaldi antennas used as transmitter and receiver, were placed on either side of the CCMEBG array, at a distance such that the array was within their far field at frequencies under consideration. The measurement set-up consist of an HP 8757D Scalar Network Analyser with a dynamic range up to 83 dB and the ability to measure insertion loss, gain, return loss and SWR. It is combined with an HP 83650L Series Synthesized Sweep/CW Generator with a frequency range 10MHz to 50 GHz.

By rotating the CCMEBG plane we have measured the transmission response of the structure for various angles of TE incidence. For comparison purposes, the same set of measurements was carried out for a single layer MEBG with exactly the same array dimensions[22,23]. Figure 3.13 illustrates the setup used for the plane wave measurements.

Figure 3.13: Measurements set up for plane wave measurements, using Vivaldi antennas as transmitter and receiver
The limited gain of the Vivaldi antennas, which was measured between 6dB and 3dB over the operating frequency range, restricted the dynamic range of the measurements. In addition, at low frequencies the size of the CCMEBG was relatively small (2λ at 3 GHz) which altered the transmission response of the array compared to the one predicted from the infinite array analysis in the simulation section.

Three different angles of incident field are measured, for both MEBG and CCMEBG arrays (0, 30, 60 degrees). Due to the set up limitations during the measurements, the 85 degrees incidence measurement is not presented. For the MEBG array the resonant frequency moves from 11 GHz (0 degrees) to 10.4 GHz (60 degrees). The -10dB fractional bandwidth at normal incidence is measured 42% (4.6GHz absolute bandwidth). The ripple observed in the measurements is due to standing waves between the array and the vivaldi antennas. The fluctuations observed before and after the resonance are attributed to the small size of the array. Simulation and measurements for the MEBG array are in good agreement.

![Graph](image)

Figure 3.14: Plane wave measurement of MEBG dipole array, for different angles of incidence

The CCMEBG dipole array produces a resonant frequency at 3.5 GHz for normal incidence, with a minor shift at 3.3 GHz for the 60 degrees measurement. Due to the fact that the CCMEBG resonant frequency is well below the grating lobe frequencies, the CCMEBG response is less sensitive to the angle of incidence, than the single layer MEBG. The appearance of the resonant frequency at 3.5 GHz, for the normal incidence, corresponds to more than 3:1 shift in comparison with the MEBG dipole.
array (11 GHz). Furthermore the CCMEBG exhibits a wider band gap. The $-10$ dB fractional bandwidth at normal incidence is $98\%$ (5.4 GHz absolute bandwidth) compared to $42\%$ (4.6 GHz absolute bandwidth) for the MEBG dipole array. The small discrepancies occurred between simulation and measurements for the CCMEBG array at low frequencies, are due to the small size of the printed arrays. The ripple in the measurements is due to standing waves between the array and the vivaldi antennas. Figure 3.15 illustrates the plane wave measurements of the CCMEBG array for different angles of incidence.

![Figure 3.15](image)

Figure 3.15: Plane wave measurement of CCMEBG dipole array, for different angles of incidence

### 3.7.2 Surface Wave Measurements

To examine the surface wave propagation of the MEBG and the CCMEBG array on a dielectric slab, the two Vivaldi antennas are printed directly opposite each other on the RT-Duriod dielectric. The transmitting antenna concentrates most of its field as surface waves bound on the dielectric slab, while the receiving Vivaldi antenna printed directly at the opposite end measures the surface waves propagated on the slab. In these measurements the forward surface wave transmission response is taken on the plane of and normalised with respect to the Vivaldi antennas alone. This could be considered equivalent to a $90^\circ$ TE incidence but the wave is now bound to the
surface. Given that Vivaldi antenna is mainly a TE mode travelling wave antenna, it will transmit and detect mainly TE modes.

In order to investigate the surface wave propagation mode properties of the 2-D conducting elements, the arrays can be printed on the dielectric slab between the two antennas.

However, this technique only enables the study of the Band-Gap properties along one direction from the transmitting to the receiving antenna with respect to the 2-D conductor array. To study the Band-gap properties in all directions along the surface, either the receiving antenna must scan in all directions, by moving all around the array, or the array must be rotated on the slab while fixed to the receiving antenna. The first method is not feasible as it is desirable to keep the receiving antenna at a fixed distance from the transmitter. Furthermore it has been observed that the received power is different at different angle positions, even if the distance is kept the same. The latter method would be a better method as the distance and positions of the antenna are fixed.

The second method mentioned, is achieved by printing the array on a very thin dielectric sheet, which is then placed on the slab for measurement. To study the propagation in all directions with respect to the array, the dielectric sheet on which the array is printed is simply rotated. The dielectric sheet used has a thickness of 0.071mm and a dielectric constant of 3. The modelling program that was written to accommodate different multiple dielectric layers can readily include this. It is also noticed that the dielectric sheet has very little effect on the band gap properties of the slab as it has a dielectric constant close to the RT-Duroid used, and its thickness is small compared to the thickness of the slab. This is also a cost effective method as it not only enables the study of different propagation directions by rotating the array on the dielectric sheet, but it can also provide measurements for different arrays just by printing them on different dielectric sheets.

Following the second method, the dipole MEBG and CCMEBG responses for surface wave propagation along the array are measured. Two different array set ups are examined. In the first configuration, the electric field is parallel to the element
orientation, yielding a band gap (Figure 3.16). In the second configuration the dipole array is aligned along the propagation direction, causing full propagation.

![Figure 3.16: Photograph of surface wave (SW) measurements configuration with dipole CCMEBG.](image)

In these measurements the forward surface wave transmission is taken on the plane of and normalized to the vivaldi antennas alone. The configuration used for this kind of measurement is illustrated above (Figure 3.16). This set up excites mainly TE modes along the array, however the cross polar components of the vivaldi antennas cause a very weak excitation of TM modes.

Beginning the measurement analysis for the MEBG dipole array, the first interesting feature observed is that the surface wave response yields a gain up to 8 dB before the cut-off (-10 dB), at the pass band frequencies (Figure 3.17). This is because the dipoles of the array are behaving as guiding slow wave elements for the transmitting vivaldi antenna. In the pass band frequencies, it concentrates the fields on the dielectric slab in the direction of the receiving antenna. Naturally, in the y-direction, where the dipole is aligned along the propagation direction, this gain diminishes. Furthermore there is a tightening of the radiating fields to the surface. A certain type of surface wave can also exist below and above the band gap area [19]. When the dipole array is aligned parallel to the electric field ($0^0$), the cut-off emerges at 7.4 GHz. The roll-off
between pass band and stop band is substantially steep. Due to the steep roll-off, very low transmission values are obtained at the beginning of the stop band. The end of the upper band gap appears at 14.2 GHz, producing an absolute bandwidth of 6.8 GHz. Shifting the dipole array orientation in order to be aligned with the propagation direction ($90^0$), causes full propagation (black line in Figure 3.17).

![Figure 3.17: Surface wave measurement of MEBG dipole array for 0 and 90 degrees orientation](image)

In accordance with the MEBG surface wave measurement, the CCMEBG surface wave response yields a gain up to 8 dB before the cut-off (Figure 3.18). The roll-off between the pass band and stop band is equally steep, leading to low transmission values at the beginning of the stop band. When the dipole array is aligned parallel to the electric field ($0^0$), the lower band gap edge emerges at 2.4 GHz, which corresponds to 3:1 frequency shift in comparison with the MEBG dipole array. This configuration produces an ultra wide bandwidth, with the upper band gap extending...
up to 18 GHz. When the CCMEBG dipole array is placed with a $90^\circ$ orientation between the vivaldi antennas, no band gap emerges, allowing full propagation (black line in Figure 3.18). Figure 3.15 illustrates the measured surface wave response for the CCMEBG dipole array.

Figure 3.15: Surface wave measurement of CCMEBG dipole array for 0 and 90 degrees orientation
3.8 Tripole Analysis

The dipole element is the simplest of all elements in terms of research and study of the band gap properties of an array. It is a single polarisation element, thus it cannot produce a common band gap when used in a two-dimensional (2D) EBG design. A more desirable element for applications requiring more than 1 dimensional Band Gap would be the tripole. The tripole element is composed of three dipoles arranged at an angle of 120° with each other and connected at one end (Fig 3.19). As with the modeling of the dipole element, the width of the arms of the tripole is assumed to be small and current only flows along the arms [23,24,25].

As a more symmetrical 2D element the tripole can be used in Electromagnetic Band Gap designs which exhibit common band gap in the plane of the array for surface wave propagation. In addition, a tripole array under plane wave incidence exhibits a similar band gap for any polarisation of the incident field [14]. The tripole element response depends on the contribution of the current on each of the three tripole arms.
Thus the angle of symmetry for the tripole element is $30^\circ$ which coincides with the angle of the first Brillouin zone of the hexagonal lattice [20]. Therefore, the irreducible Brillouin zone for a tripole array arranged on a hexagonal lattice is $30^\circ$ (Fig 3.19).

![Diagram of tripole array]

**Figure 3.20: Tripole CCMEBG arrangement**

Tripole arrays in single layer MEBG and double layer CCMEBG configurations are investigated in this section for their band gap properties when excited by plane as well as surface waves. The tripole array geometry used has the following dimensions: $L = 5\text{mm}$, $W = 0.5\text{mm}$, $D_u = D_v = D = 12\text{mm}$. The array is printed on a thin dielectric layer of $\varepsilon_r = 3$ and is then mounted on a flat dielectric board, similar to the one used with the dipole arrays, in order to produce the single layer tripole MEBG. A double layer array is formed by printing the tripoles on either side of the thin dielectric layer. The thickness of the thin layer was actually measured as $0.071\text{mm}$, as in the case of the dipole CCMEBG. Following the dipole CCMEBG design, in order to
produce maximum coupling between the two layers, the points of maximum current of the second array need to be placed in the inter-element space of the first one. To achieve that, the tripole elements of the second layer are rotated by 60° with respect to the elements of the first array and then shifted in the y-direction by $D \tan 30° = 6.928$ mm. This arrangement is illustrated in (Fig 3.20) for 0° element orientation.

### 3.9 Simulations For MEBG and CCMEBG Tripole Arrays

The plane wave modal analysis discussed in chapter 2 is implemented, in order to simulate the transmission response of single as well as double layer tripole arrays. Seventeen floquet modes are used for simulation purposes Two different tripole orientations (0 and 30 degrees) and four different angles of incidence (0, 30, 60, 85 degrees) are simulated, producing similar although not exact same responses for both MEBG and CCMEBG tripole arrays.

Beginning the investigation of the MEBG tripoles with 0° orientation, the array resonance occurs at 11.9 GHz for normal incidence ($\theta = 0°$). At higher angles of incidence the resonant shifts to 11.6 GHz ($\theta = 60°$). The following figure illustrates the simulations produced for the different angles of incidence for the 0° orientation.

![Figure 3.21: Plane wave simulations of MEBG tripole array (0° orientation), for different angles of incidence](image.png)
The response is affected by grating lobes due to array periodicity, particularly at high incident angles. The -10 dB fractional bandwidth at normal incidence is calculated to be 8.4% or 1 GHz of absolute bandwidth.

The simulations for the 30° orientation produce similar results to the ones for the 0° orientation. For normal incidence (θ = 0°) the resonance occurs at 11.7 GHz and the -10 fractional bandwidth is also 8.4% or 1 GHz of absolute bandwidth. The noticeable difference between the two orientations is the exceptional angular stability observed in the simulations for the 30° orientation.

![Figure 3.21: Plane wave simulations of MEBG tripole array (30° orientation), for different angles of incidence](image)

The simulations for the CCMEBG tripole array for both orientations, reveal a massive shift of the resonant frequency towards lower frequencies. For the 0° orientation, the resonant frequency has moved from 11.9 GHz to 4.1 GHz (normal incidence). That corresponds almost to a relative shift of 3:1. Grating responses at high angles of incidence appear as nulls in the transmission response at higher frequencies. The tripole CCMEBG design yields approximately a λ/15 resonance, with regards to the length of the tripole arm. The -10 dB fractional bandwidth is predicted to be 39% or
1.7 GHz absolute bandwidth, which is considerably wider than the one predicted for the MEBG array (8.4 % or 1 GHz absolute bandwidth). The CCMEBG tripole array exhibits also an extremely stable resonance with respect to the angle of incidence, in contrast with the MEBG array. Figure 3.22 presents the simulations produced for the CCMEBG tripole array with 0° orientation.

Figure 3.22: Plane wave simulations of CCMEBG tripole array (0° orientation), for different angles of incidence

The simulations for the CCMEBG tripole array with 30° orientation are very similar to the ones for the 0° orientation. The resonant frequency occurs at 4.3 GHz from 11.7 GHz (MEBG array – normal incidence). That corresponds to a relative shift of 2.8:1. The -10dB fractional bandwidth is calculated 41% or 1.8 GHz absolute bandwidth, which is a much wider band gap compared to the equivalent MEBG array (8.4 % or 1 GHz absolute bandwidth). As in the case of 0° orientation, grating responses at high angles of incidence appear as nulls in the transmission response at higher frequencies. The CCMEBG tripole array exhibits an exceptional angular stability, with respect to the incidence polarization. Figure 3.23 illustrates the simulations produced for the CCMEBG tripole array with 30° orientation.
Figure 3.23: Plane wave simulations of CCMEBG tripoles array (30° orientation), for different angles of incidence

3.10 Dispersion Curve diagrams for MEBG and CCMEBG Tripoles

Following the case of MEBG and CCMEBG dipole arrays, dispersion diagrams have been produced for both single and double layer tripoles arrays. As it was mentioned, when a tripoles array is arranged on a hexagonal lattice, the irreducible Brillouin zone of the reciprocal lattice is much smaller, compared to the one with the dipole arrays.

For the MEBG tripoles array, the first TE mode propagates from 0.2 GHz up to 10 GHz for the 0° propagation direction (Γ-M) and gradually decreases to 9.6 GHz for the 30° propagation direction (Γ-X). The lower band gap edge does not vary over a large range with the propagation direction. The upper band gap edge occurs at 18
GHz for the (Γ-M) direction and 18 GHz for the (Γ-X) direction. This produces a common band gap of 7 GHz, marked with the gray shaded area in the following dispersion diagram. A second TE mode starts propagating at 19GHz (not shown in the measurements due to the frequency range of the vivaldi antennas) for the 0° and 30° propagation direction.

Figure 3.24 illustrates the dispersion diagram for the MEBG tripole array.

![Dispersion Diagram](image)

Propagation constant (normalised $\frac{D}{2\pi}$), where $D$ is the Periodicity

Figure 3.24: Dispersion diagram of the MEBG tripole array. (a) direct lattice (b) reciprocal lattice, (c) irreducible Brillouin zone

The dispersion diagram of the double layer tripole CCMEBG is presented in Figure (3.25)
The first TE mode propagates from 0.2 GHz up to 3.7 GHz for the (Γ-M) direction and decreases to 3.5 GHz in (Γ-X) direction. Again the lower band gap does not vary over a large range with the propagation direction. The upper band gap in the 0° propagation direction occurs at 10 GHz and decreases to 6 GHz at 30° direction. The modeling results show a common band gap of 2.3 GHz (lower shaded area, Figure 3.25). The next common band gap emerges at 10 GHz extending to up 17.4 GHz [26]. Both band gaps are illustrated with a gray shaded area in (Fig.3.25). Dispersion diagrams and measured results are in good agreement.
3.11 Measurements For MEBG and CCMEBG Tripole Arrays

3.11.1 Plane Wave Measurements

The tripole MEBG and CCMEBG structures have been fabricated as 20 cm square sheets, attached to a flat dielectric board of the same size. Plane wave measurements have been carried out for normal and oblique TE incidence using the Vivaldi antennas, described in section 3.6. Both MEBG and CCMEBG tripole arrays have been placed in an open window of the same dimensions, within an absorbing screen. The Vivaldi antennas used as transmitter and receiver, were placed on either side of the CCMEBG array at a distance such that the array was within their far field at frequencies under consideration.

Three different angles of incident field are measured, for both MEBG and CCMEBG arrays (0, 30, 60 degrees). Figure 3.26 illustrates the plane wave measurements of MEBG tripoles for different angles of incidence.

![Figure 3.24: Plane wave measurements of MEBG tripole array (0° orientation), for different angles of incidence](image)

As with the plane wave measurements for the dipole arrays, the limited gain of the Vivaldi antennas, which was measured between 6dB and 3dB over the operating frequency range, restricted the dynamic range of the measurements. In addition, at low
frequencies the size of the CCMEBG was relatively small. That altered the transmission response of the array compared to the one predicted from the infinite array analysis, in the simulation section.

Due to the set up limitations during the measurements, the 85 degrees incidence measurement is not presented. For the MEBG array with 0 deg orientation, the resonant frequency moves from 11.8 GHz (0 degrees) to 11.2 GHz (60 degrees) – Figure 3.26. The -10dB fractional bandwidth at normal incidence is measured 8.4% (1GHz absolute bandwidth). The ripple observed in the measurements is due to standing waves. The simulation and measurements for the MEBG array are in good agreement.

In the CCMEBG design for 0° orientation, the resonance at normal incidence has moved to 5.2 GHz, which corresponds to more than 2:1 frequency shift (Figure 3.27). This is approximately a λ/12 resonance. Moreover, a very good angular stability of the resonance is observed with the CCMEBG array. Furthermore the CCMEBG exhibits a wider band gap. The -10 dB fractional bandwidth at normal incidence is 19.2 % (1 GHz absolute bandwidth) compared to 8.4 % for the MEBG dipole array. The small discrepancies occurred between simulation and measurements for the CCMEBG array at low frequencies, are due to the small size of the printed arrays.

Figure 3.27: Plane wave measurements of CCMEBG tripole array (0° orientation), for different angles of incidence
In the case of the MEBG tripoles with $30^\circ$ orientation, the measured results are similar with the ones for the $0^\circ$ orientation. For normal incidence ($\theta = 0^\circ$) the resonance occurs at 11.6 GHz and moves up to 11.5 GHz for ($\theta = 60^\circ$) – Figure 3.28. As it was predicted from the equivalent simulations, there is a very good angular stability for the MEBG tripole array with $30^\circ$ orientation. The -10 dB fractional bandwidth at normal incidence is calculated to be 10.3 % or 1.2 GHz of absolute bandwidth. This is a slightly wider stop band, than the one produced with the array of $0^\circ$ orientation (8.4 %). The following graph illustrates the measured transmission responses of MEBG tripoles for TE plane wave incidences at $30^\circ$ orientation.

![Figure 3.28: Plane wave measurements of MEBG tripole array ($30^\circ$ orientation), for different angles of incidence](image)

In the CCMEBG design for $30^\circ$ orientation, the resonance at normal incidence has moved to 5.0 GHz (Figure 3.29), which corresponds to 2.3:1 frequency shift, compared to the equivalent MEBG design. The -10 dB fractional bandwidth at normal incidence is calculated to be 26 % or 1.3 GHz of absolute bandwidth. That is a wider stop band than the CCMEBG tripole array with $0^\circ$ orientation (19.2 %). The following graph presents the measured transmission responses of CCMEBG tripoles for TE plane wave incidences at $30^\circ$ orientation.
Figure 3.29: Plane wave measurements of CCMEBG tripole array (30° orientation), for different angles of incidence

3.11.2 Surface Wave Measurements

Using the same technique applied for the surface wave measurements of the dipole arrays, two vivaldi antennas are printed directly opposite each other on the RT-Duriod dielectric. The tripole array is printed on a very thin dielectric sheet, which is then placed in between the vivaldi antennas for measurement. To study the propagation in all directions with respect to the array, the dielectric sheet on which the array is printed is simply rotated. The transmitting antenna will concentrate most of its field as surface waves will bound on the dielectric slab, while the receiving Vivaldi antenna printed directly at the opposite end will measure the surface waves propagated on the slab.

In these measurements the forward surface wave transmission is taken on the plane of and normalized to the vivaldi antennas alone. The configuration used for this kind measurement is illustrated above in Figure 3.30.
Beginning the measurement analysis for the MEBG tripole arrays for 0 and 30 degrees orientation, there are some common observations. The first interesting feature noticed, is that both designs exhibit a relative gain up to 8 dB before the cut-off (at the pass band frequencies) –Figure 3.31. Likewise the dipole arrays examined in section 3.6.2, the tripole elements have also become guiding elements for the transmitting vivaldi antenna. Due to the array being very closed to the Vivaldi antennas, the bandwidth of the measurement could be slightly affected. In the pass band frequencies the fields are concentrated in the direction of the antenna, onto the dielectric slab, thus the directivity is increased. Another common characteristic of both orientations is the very steep transition from the pass band to the stop band frequencies.

The TE band gap emerges at 10.1 GHz for the 0° orientation and extends up to 14.2 GHz (Figure 3.31). The –10 dB absolute bandwidth is calculated to be 4.1 GHz. The band gap for the 30° orientation emerges a little bit lower, at 9.7 GHz and extends up to 18 GHz. The surface wave response of the MEBG tripoles with 30° orientation shows lower transmission values in the stop band, when compared to the response of the 0° orientated tripoles, for a wide range of frequencies.
In accordance with the MEBG surface wave measurements, the CCMEBG surface wave responses yield a gain up to 8 dB before the cut-off (Figure 3.32). The roll-off between the pass band and stop band is equally steep, leading to low transmission values at the beginning of the stop band, for both orientations. For the 0° oriented CCMEBG tripoles, the lower band gap emerges at 4 GHz and extends up to 6.3 GHz, producing 2.3 GHz of absolute bandwidth. This corresponds to 2.5:1 shift of the band gap towards the lower frequencies, in comparison with the equivalent MEBG array. For the 30° orientation, the band gap emerges a little bit lower, at 3.8 GHz and extends up to 6.6 GHz. The -10 dB absolute bandwidth is measured as 2.8 GHz. As with the case of plane wave measurements, the tripoles with 30° orientation are producing slightly wider bandwidth. The following graph illustrates the measured transmission response of a CCMEBG tripole array, for TE surface wave incidence at 0° and 30° orientation.
Figure 3.32: Measured transmission response of CCMEBG tripole array for TE surface wave incidence at (a) 0° and (b) 30° orientation
3.12 Conclusion

The present chapter focused in the introduction of MEBG and CCMEBG structures and studied their transmission characteristics for plane wave as well as surface wave excitation. Dipoles has been used initially to implement a CCMEBG design. A significant lowering of the band gap frequency (4:1) together with a superior angular stability and bandwidth have been predicted using a plane wave modal analysis of the structure. The trends shown in the simulation results have been confirmed with measurements carried out for plane wave as well as surface wave excitation. Despite the discrepancies at low frequencies due to the small size of the arrays, the simulations provided useful design guidelines upon which the construction of the CCMEBG structures was based. For plane wave measurements, a frequency shift of more than 3:1 has been obtained with the dipole CCMEBG in comparison with the MEBG dipole array. Furthermore, improved angular stability of the resonant frequency was observed. The fractional bandwidth of the stop band also increased by 56% using the CCMEBG design. A similar shift in the stop band has been obtained during the surface wave measurements. An interesting characteristic of the surface wave response, for both single MEBG and double layer CCMEBG arrays, is that a relative gain of up to 8 dB is observed in the pass band. The transition to the stop band is very steep. Furthermore, the bandwidth of the stop band is wider compared to the one obtained in the plane wave measurements. In addition to simulation and measurements, dispersion curves have been produced in order to examine qualitatively the modes propagating and the band gaps occurred. As it was illustrated dipoles could not produce an absolute band gap due to symmetry.

The second section of this chapter focused in tripole geometries. Tripoles have been used to form CCMEBG structures, which are less dependant on the polarisation of the incident field. Tripole elements arranged on a hexagonal lattice exhibited common band gap for any incident polarisation. Two different orientations were simulated and measured. The tripole CCMEBG design exhibits in general a 2:1 frequency shift with respect to the single layer MEBG. The angular stability of the stop band was improved and the bandwidth was also increased. The 30° element orientation yielded a wider bandwidth on both plane wave and surface wave measurements.
The frequency shift property of CCMEBGs makes these structures very attractive for designs of compact EGB materials, with superior angular stability and bandwidth performance. Symmetrical 2D elements can be used in order to obtain common band gaps. Conformal designs of CCMEBGs could be used as filters for both plane and surface wave propagation, as filters for micro strip patch antennas and printed circuit applications.
REFERENCES


CHAPTER 4
Aperture & Complementary Dipoles

4.1 Introduction

In this chapter aspects of complementary metallodielectric electromagnetic band gap surface are examined. As its name suggests, a CMEBG is, effectively, a hybrid of the closely coupled electromagnetic band gap surface (CCMEBG) [1] investigated in Chapter 3. The significant difference is that instead of having two layers of conducting elements etched either side of a supporting dielectric sheet, a layer of aperture elements and a layer of conducting elements are etched either side of a supporting dielectric substrate. The CMEBG takes advantage of the interaction between the different elements, to produce very strong fields in the separation region and a resonant frequency that is much lower than that of a single layer array (aperture or conductor).

In the last few years intense research has been done in the field of aperture arrays or defected ground structures as they also called, with applications in certain areas such as: millimetre wave integrated circuits, high performance microwave circuits (dividers, couplers, low pass filters), amplifiers (for the tuning of the different harmonics) and waveguides [2-6]. They have also been investigated for their wide stop band properties, suppression of surface waves in patch antennas, their slow wave effects and for their use in a series of active devices [7-9]

The use of complementary structures for the suppression of surface waves [10] is a fairly new concept, used for its miniaturization capabilities [11,12]. By applying an array of conducting elements in very close proximity to the existing aperture array, the overall structural size and the size of the unit cell remains the same, while the band gap is shifted towards lower frequency regions. The extended capabilities of this feature can be applicable to passive and active devices and to most of the applications mentioned for the apertures, with a great reduction in overall size. The structure of the chapter is divided into two main sections.
The first one introduces the concept of aperture [13] dipoles as opposed to conducting dipole arrays discussed in chapter 3. A circuit analysis describes qualitatively the behaviour of the slots in terms of capacitance. Dispersion curves [14] based on Floquet modal analysis [15] are produced to study the modes supported by a single layer dipole aperture array. Representative simulation and measurements verify the predictions and form the basis for the design of the CMEBG.

The second section is dedicated to the detailed explanation of the CMEBG concept. The combination of the aperture and dipole array and their placement in very close proximity increases the capacitance of the structure, triggering the shift of the resonant frequency towards lower values. It will be shown that for polarised elements such as the linear dipole, the conductors and apertures must be rotated by $90^\circ$ with respect to each other, in order for both to be resonant. The physical interpretation of such a structure is described qualitatively with a circuit analysis in section 4.7.

Dispersion curves are produced to describe the modes supported from the array and to give a first estimate on the band gap produced. The Floquet modal analysis described in chapter 2 is used for that purpose. It is interesting to mention that when the electromagnetic boundary conditions are applied at each interface, this gives rise to two integral equations but not of the same nature as the ones introduced for the CCMEBG array. In this case a MFIE (Magnetic Field Integral Equation) is formed for the fields within the apertures and an EFIE (Electric Field Integral Equation) for the currents in the conductors.

A series of simulations and parametric studies are performed using a commercial software package for the design of a finite CMEBG array. The structures under investigation can be used as compact EBG ground planes for various microwave circuits and components. Therefore in this section full wave simulations and measurements are performed for compact, finite EBG structured ground planes, excited by a microstrip line. Different periodicities and conducting lengths are examined, taking into account the significance of the overall structural size. Measurements for three different structures with the same size and number of elements but with different conducting widths are produced. Simulation and measurements are in very good agreement, validating the theoretical analysis.
4.2 Aperture Theory and Geometry

An aperture [16] element array, at normal incidence, is the exact complementary structure of a conducting dipole array. It resonates at the same frequency as a conducting dipole array (assuming no dielectric substrates). Due to the duality principal it exhibits a pass-band at resonance, instead of a stop-band (conducting elements). A distinct band gap is obtained when the electric field vector is perpendicular to the aperture dipoles (slots).

When illuminated by a field at an incidence angle, dipole arrays perform as parallel LC type resonators in a parallel configuration [17]. They produce a band-pass for all angles of incidence at resonance and are highly inductive. The FSS array is highly transparent at the resonant frequency.
For Surface wave propagation along the array (horizontal incidence), the aperture array acts as a network of parallel LC resonators cascaded in series. Instead of exhibiting a pass-band as for the plane wave incidence, the network exhibits a stop-band at the resonance. Surface waves can propagate far from the resonance (as in a dielectric slab over ground plane). This model will be validated by the results in the following sections.

4.3 Dispersion Characteristics

A full wave modal analysis of an infinite aperture array is applied, in order to get a first indication on the band gap properties of the structure and the modes supported by the array.
In the Figure 4.4 the horizontal axis represents the phase constant of the propagation mode in various directions, and the vertical axis illustrates the frequency respectively. The black dashed line indicates the light line. The first TM mode (surface wave) starts at 0 GHz. For propagation along y (Γ - Y) direction, TE mode contains the \(E_x\) component, while TM mode contains \(E_y\) and \(E_z\) components. From those electric components, what excites the apertures is the \(E_y\), therefore the mode appearing in the dispersion curve is a TM.

\[
\rightarrow E_x \quad \rightarrow E_y \quad \overrightarrow{E_z} = \text{TE Mode} \quad \overrightarrow{E_z}, \overrightarrow{E_z} = \text{TM Mode}
\]

In the direction (Γ - Y) where the electric field is perpendicular to the apertures, the surface wave propagates until the frequency of 6.8 GHz where it ceases. This is the beginning of the band gap which extends up to 15 GHz. As the propagation direction moves from y towards the x-axis (YM and MX), the band gap gradually narrows until the end of MX area and the frequency of 8.6 GHz. The transition from the end of MX and the beginning of XΓ area (x-direction), reveals the end of the band gap and the beginning of full wave propagation. As was expected and verified from the modelling, the dipole aperture array cannot produce an absolute band gap. The application of full wave modal analysis for an infinite aperture array gave an indication of the cut-off frequency the extension of the band gap and the behaviour of the first TM mode.

4.4 Aperture simulations

Realising the concept of aperture arrays and taking the study a step further, a finite array is designed and simulated. For the computation of transmission responses, a commercial software package (Microwave Office) [18] is used, enabling the design
of electromagnetic (EM) structures and generating layout representations of these designs. All layouts are illustrated in the frequency domain. Simulations are performed using a 3D-planar EM simulator (EM Sight), which applies Maxwell's equations to calculate the response of a structure from its physical geometry. For the analysis of microstrip, stripline, and coplanar structures the Galerkin Method of Moments (MoM) in the spectral domain, is used in this structure.[19]

The study of the aperture dipoles is based on the configuration shown in figures (4.5, 4.6). A square board with dimensions $35 \times 35 \text{ mm}^2$ is used to accommodate the finite array, consisting of 5 unit cells. A 50 $\Omega$ microstrip transmission line with dimensions $3.5 \times 35 \text{ mm}^2$ is printed on a thin polyester dielectric sheet of ($\varepsilon_r = 3$) and thickness 0.071 mm. The thin dielectric is glued at the top of a grounded RT-Duron substrate with ($\varepsilon_r = 2.2$) and thickness 1.125 mm. At the bottom of the substrate the aperture array is etched. Figure 4.5 illustrates a side view of the aperture structure.

![Side view of the aperture array arrangement.](image)

The transmission line is printed in the middle of the dielectric sheet and de-embedded from the edge of the substrate by 2mm, in order to allow the pin connectors to be attached. The aperture dipoles have a length of $L = 15.5 \text{ mm}$, a width of $W = 0.5 \text{ mm}$ and a $y$ periodicity of $D_y = 5 \text{ mm}$. The aperture array is designed in such a way, that the centre of each element is set to be in the middle of the transmission line. The Figure 4.6 illustrates a plan view of the design explained.
In order to conform to the specifications described above, a series of parametric studies were performed, to establish the best possible design. Having as a reference point the length and the width of the apertures (15.5 mm and 0.5 mm – the same as the conductors examined in previous chapters), the periodicity of the elements on the y-axis, and the number of elements in the lattice were investigated. The first parametric study performed was related to the different periodicities of the aperture elements. Figure 4.7 shows simulated transmission response for the aperture array with periodicities of (2 mm, 3 mm, 4 mm, 5 mm and 6 mm) a decision on the appropriate element spacing was taken.
As the periodicity of the elements on the y-axis is increased, the cut-off frequency is shifted towards lower frequency regions, while the bandwidth is increased. There is a point between 4mm and 6mm in which the dynamic range of the band gap is decreasing. In order to coincide with the dimensions of the conducting elements in chapter 3 and since there is no actual difference, the periodicity is chosen to be 5mm.

The second parametric study was concerned with the actual number of elements used in the square lattice.

![Graph showing different number of aperture elements](image)

**Figure 4.8: Different number of aperture elements**

It is observed in Figure 4.8 that when more unit cells are used for the formulation of the aperture structure, better bandwidth is achieved, the cut-off frequency emerges at slightly lower frequencies and the dynamic range of the simulation is bigger. There is a point in which the addition of extra unit cells causes only the increase of the upper band gap. Therefore in order to keep the overall size of the structure as small as possible, five aperture elements are chosen.

### 4.5 Aperture measurements

Based on the simulation predictions, the aperture dipoles were fabricated. A Taconic board with dielectric constant of \((\varepsilon_r = 2.2)\) and thickness of 1.125mm was used. The thin dielectric upon which the transmission line was printed has a thickness of
0.071 mm and \( \varepsilon_r = 3 \). Five aperture elements with dimensions \( L = 15.5 \text{ mm} \), \( W = 0.5 \text{ mm} \) and \( D_y = 5 \text{ mm} \), were used. Photographs in fig.4.9(a) and (b) are illustrating the top and bottom view of the fabricated dipole aperture array excited by a microstrip line.

![Figure 4.9: (a) Top and (b) bottom view of the fabricated aperture dipole array.](image)

By using the same measurement equipment introduced in section 3.6.1 the transmission response of the upper structure was measured. With a cut-off point taken at \(-10 \text{ dB}\), the lower band gap emerges at 6.7 GHz and extends up to 12 GHz (upper band gap). This produces a bandwidth of 5.3 GHz. The simulation and measurements are in very good agreement, fig.4.10

![Figure 4.10: Simulation, Measurement and Schematic of aperture dipoles](image)
4.6 Complementary Theory and Geometry.

Complementary Electromagnetic Band - Gap structures (CMEBG) [20,21] are a hybrid of CCMEBG, whereby a layer of periodic aperture array and a layer of periodic conducting array, are etched either side of a supporting dielectric substrate. The array elements need not necessarily be the same, either side of the substrate. In the present case though the simplest of all elements, the dipole, was chosen for both conductors and apertures. By the interaction between the different elements, very strong fields are produced in the separation region. The CMEBG creates electrically large elements from physically small ones. The dipole apertures are rotated 90° with respect to the conductors, so that both elements will be polarized with the electric field as depicted in fig 4.11 and will therefore be resonant.

The key property of the proposed Complementary MEBG is the coupling of the evanescent fields within the dielectric region. This is achieved by having the elements closely spaced to each other, so there is a field interaction, due to each other. It is important to appreciate the significance of the aperture rotation by 90° with respect to the conductors, so that both elements will be polarized with the electric field. Figure 4.11 illustrates the array geometry and element orientation with respect to the incident electric field.

![Figure 4.11: Schematic of Complementary Dipole array](image-url)
4.7 Qualitative Circuitry Analysis.

In terms of an equivalent circuit description, the design of complementary dipoles can be defined as a network of parallel LC resonators, cascaded in series. Each resonator represents the unit cell of an aperture dipole array also used in the modelling of FSS. The response of such a network for surface wave propagation will produce a stop band at the resonant frequency.

\[ \omega_0 = \frac{1}{\sqrt{LC}} \] [22]  
where \( L \) is the inductance and \( C \) is the capacitance of the equivalent circuit. The capacitance is relevant to the width of the conductors and can be given from the formula:

\[ C = \frac{\epsilon A}{d} \]  
where \( \epsilon \) is the dielectric constant, \( A \) is the overlapping area between conductors and apertures and \( d \) is the separation distance between the two layers. The separation distance remains exactly the same for all the geometries studied throughout this research. As the width of the conductors is increased, that results in capacitance getting higher values. The increase of the capacitance leads to the decrease of the resonant frequency. So by enlarging the width of the conductors we shift the band gap to lower frequencies.

The bandwidth can be calculated from the formula given below and corresponds to the width of the surface wave band gap.

\[ BW = \frac{\sqrt{L/C}}{\nabla} = \frac{Z_0}{n} \]  
where \( Z_0 \) is a characteristic impedance of the surface and \( n \) is the impedance of free space. As the width of the conductor is increased and as a result the capacitance increases, the bandwidth is decreased. This explains the
reduction of fractional bandwidth occurring when the width of the conductors is increased as will be shown in section 4.9. Due to close proximity and high order modes, it is not possible to produce quantitatively results with great accuracy, from the circuitry approach. However a useful insight of how the structure works is gained.

4.8 Dispersion Curves.

Dispersion curve diagrams are produced in order to examine whether the predictions for the behaviour of an infinite CMEBG array are similar to the ones for the finite CMEBG array. The vector Floquet modal analysis is employed to model the periodic arrays. By the application of electromagnetic boundary conditions at each interface, two coupled Integral Equations are derived. A Magnetic Field Integral Equation (MFIE) is formed for the fields within the apertures and an Electric Field Integral Equation (EFIE) for the currents in the conductors. These can be solved with the Method of Moments for the unknown fields and currents. A more detailed description of the method is presented in chapter 2 of the thesis.

Two different parametric studies, regarding the length and the width of the conductors are conducted with the use of dispersion diagrams. For simplicity purposes, only the part of the dispersion diagram where the cut-off of the band gap emerges is introduced. The primary target in this study is the validation of the concept that the increase in the overlapping area will decrease the resonant frequency. Therefore the behaviour of the lower band gap in comparison with different conducting lengths and widths is the main concern.

The first study is concerned with the alteration of the conducting lengths from 7.5 mm to 15.5 mm and as a result with the periodicity. In Figure 4.13 (below), the horizontal axis represents the phase constant of the propagation mode, for the first part of the irreducible Brillouin zone (FY), while the vertical axis include the frequency range where the true minima occur. The dashed line is the light line indicating the boundary between the leaky wave and surface wave modes above and below it. For the lengthvalue of 7.5 mm the stop band emerges at 3.4 GHz. As the length of the
conductor is increased, the cut-off shifts to lower frequencies. For longer conductors (10.5mm) the band gap emerges at 2.8 GHz and drops to 2.4 GHz when the conductor’s length takes its maximum value (15.5mm).

The second parametric study examines the alterations caused in the lower band gap of the CMEBG structure, due to the different conducting widths. In figure 4.14, the horizontal axis represents the phase constant of the propagation mode, for the first part of the irreducible Brillouin zone (\(\Gamma Y\)), while the vertical axis represents the frequency range where the true minima occur. The dashed line is the light line indicating the leaky wave and surface wave modes above and below it. For the thinnest conductor, the band gap emerges at 4.3 GHz matching the simulated result. When the width of the conductor increases to 2.5 mm, the cut-off lowers to 3.3 GHz verifying the circuitry analysis and the accuracy of the simulations. Finally when the width of the conductor gets the same value as the length of the slot, the band gap emerges at 2.9 GHz being in a perfect agreement with the simulations (section 4.9). Figure 4.14 illustrates the results produced for the different widths.

![Figure 4.13: Dispersion curve diagram for different conducting dipole lengths](image-url)
The study of the CMEBG Band-Gap is based on the configuration shown in fig. 4.15. A square board with dimensions \((35 \times 35 \text{ mm}^2)\) is used to accommodate the finite array, consisting of 5 unit cells. A 50 \(\Omega\) microstrip transmission line with dimensions \((3.5 \times 35 \text{ mm}^2)\) is printed on a thin dielectric sheet \((\varepsilon_r = 3)\) and thickness 0.071 mm. The thin dielectric is glued at the top of a grounded RT-Duron substrate with \((\varepsilon_r = 2.2)\) and thickness of 1.125 mm. The aperture array is etched at the bottom of the substrate. Another thin dielectric with \((\varepsilon_r = 3)\) and thickness 0.071 mm is placed in very close proximity, where the conductors are printed. The finite array is consist of 5 unit cells. Measuring the transmission response of the microstrip line, the band-gap is directly obtained.
Simulated and measured transmission responses ($|S_{21}|$) have been obtained for the complementary dipole structure with the dimensions mentioned above. The cut-off frequency of the band-gap is defined at $|S_{21}| = -10$ dB throughout the research. The simulated $S_{21}$ responses of finite structures are obtained using a MoM based software package, introduced in section 4.3. A series of parametric studies were performed. The first parametric study is concerned with the number of complementary elements and the actual size of the array in each case, in order to produce a desirable band gap from a relatively small structure. Figure 4.16 illustrates the results for the different number of elements used.

![Figure 4.15: Side view of the CMEBG array arrangement.](image)

![Figure 4.16: Parametric study on different number of CMEBG elements](image)
It is observed that when the array contains between one and three elements, the cut-off shifts towards lower frequencies. Above five elements the cut-off frequency remains stable and only the upper band gap increases a little. The increase in the number of elements, will cause an increase in the overall size of the structure. Since there is no noticeable difference when more than five elements are used (apart from the slight increase of the fractional bandwidth and a better dynamic range), five complementary elements were chosen for the design of the CMEBG structure.

Using five unit cells, the next parametric study is concerned with the effect on the alteration of the conductor lengths. Four different lengths and periodicities are simulated. The smaller length is chosen to be 4.5 mm with a $O_y$ element periodicity of 5 mm. The width is chosen to be 0.5 mm for all the simulations. Figure 4.17 presents the different simulations produced for the alteration of the conducting lengths.

![Figure 4.17: Parametric study on conducting lengths of CMEBG elements](image)

When the length of the conductors has the smaller value (4.5 mm) the lower cut-off emerges at 4.3 GHz and the upper band gap occurs at 7.3 GHz. When the length of the conductors is increased to 7.5 mm and the $O_y$ periodicity to 8 mm, the band gap shifts towards lower frequencies and the cut-off emerges at 3.4 GHz. The absolute bandwidth is decreased from 3 GHz, to 1.4 GHz. When the length of the conductors is increased to 15.5 mm (maximum length - equal to the width of the apertures), the cut-
off is emerging at its lowest value at 2.4 GHz. The bandwidth has been drastically decreased in comparison with the initial simulation. There is a perfect agreement in the emergence of the cut-off, between the simulation of the finite array and the dispersion curve of the infinite array.

The last parametric study is concerned with the alteration of the conductor width. The main advantage in comparison with the previous study is that in this case the overall size of the structure is kept the same for all different simulations. The shift of the band gap is caused due to the increase of the overlapping area between the apertures and the conductors. This is directly proportional to the increase of the overall capacitance as introduced in section 4.7. For the purpose of this study, the width, length and periodicity of the apertures are the same with the ones used for the manufacture and measurement of the aperture array (section 4.5). The length of the conductors is chosen to be 4.5 mm and the overall size of the structure (35 x 35 mm²). Figure 4.18 presents the different simulations produced for the alteration of the conducting widths.

![Figure 4.18: Parametric study on conducting widths of CMEBG elements](image)

When the width of the conducting dipoles is 0.5 mm the band gap emerges at 4.3 GHz extending up to 7.3 GHz, producing an absolute bandwidth of 3 GHz. The increase of the width from 0.5 mm to 2.5 mm causes an increase in the capacitance and as a result the cut-off emerges lower, at 3.3 GHz. The upper band gap extends up to 5.4 GHz, giving an absolute bandwidth of 2.1 GHz. Finally when the width of the conductors
has the same value as the length of the apertures (15.5 mm), the band gap emerges at 2.9 GHz and extends up to 3.4 GHz. The bandwidth is decreased as the capacitance is increased. The absolute bandwidth is now measured to be 500 MHz. There is a perfect agreement between the theoretical predictions for an infinite and a finite CMEBG dipole array.

4.10 Measurements for Complementary dipoles

Based on the simulations and using the design and the materials mentioned in section 4.6, the CMEBG array was fabricated. Three different conducting arrays were produced for the different complementary structures. The unit cells of each array are identical being on an orthogonal lattice, with periodicities $D_x = 17.5$ mm and $D_y = 5$ mm. The element dimensions are: for the conductors $L_1 = 4.5$ mm, $W_1 = 0.5$ mm, $W_3 = 2.5$ mm, $W_4 = 15.5$ mm and for the apertures $L_2 = 15.5$ mm, $W_2 = 0.5$ mm. The thickness of the dielectric substrate is 1.125 mm and the dielectric constant is 2.2. The thickness of the thin dielectrics is 0.071 mm and $\varepsilon_r = 3$. The transmission coefficients of the three different structures were measured, using the same equipment with the measurements performed throughout this research.

By placing the layer of conducting elements in close proximity with the apertures, the resonant frequency shifts lower (Fig 4.19). The width of the conductors is considered to be $W_1 = 0.5$ mm.

![Figure 4.19: Simulation, measurement and schematic of CMEBG dipoles with $W_1 = 0.5$mm](image.png)
The cut-off now emerges at 4.3 GHz instead of 6.8 GHz (apertures) and extends up to 7.2 GHz. This corresponds to 50% fractional bandwidth or 2.9 GHz or absolute bandwidth and comes in direct agreement with the dispersion curve presented in section (4.8). The structure is miniaturised by 63% compared to single apertures. Figures 4.19 (a) and (b) illustrate the top and bottom view of the complementary structure used for measurement purposes.

![Complementary Dipole Structure](image)

**Figure 4.19**: (a) Top and (b) bottom view of the fabricated complementary dipoles with conducting width $W_1 = 0.5$ mm

As the width of the conductors is increased from 0.5 mm to 2.5 mm (Figure 4.20), the band gap shifts even lower. The lower cut-off emerges at 3.3 GHz and extends up to 5.4 GHz giving a fractional bandwidth of 48%. The structure is miniaturised by 48%.

![Bandwidth Comparison](image)

**Figure 4.20**: Simulation, measurement and schematic of CMEBG dipoles with $W_1 = 2.5$ mm

128
Increasing the width of the conducting dipoles from 2.5 mm to $W_4 = 15.5$ mm (the conducting width equals to the length of the apertures) the shift of the band gap obtains its lower possible value Fig 4.21. There is a 43% decrease of the stop band centre frequency in comparison with the geometry in figure 4.10. The band gap emerges at 2.9 GHz and extends up to 3.4 GHz giving a bandwidth of 500 MHz. The fractional bandwidth is 15.9%. The measured and simulated results are in very good agreement.

![Graph showing band gap](image)

**Figure 4.21**: Simulation, measurement and schematic of CMEBG dipoles with $W_4 = 15.5$ mm

**Figure 4.22**: Top and bottom view of the fabricated complementary dipoles with conducting width $W_1 = 15.5$ mm
4.11 Conclusion

The present chapter focused on the introduction of aperture and complementary electromagnetic band gap structures and studied their transmission characteristics for surface wave excitation. The simplest of elements, the dipole, has been employed for the implementation of both geometries.

The first section of the chapter presented the aperture array dipoles and defined the differences between an array of conducting dipoles. Furthermore, it stated the condition under which an aperture array could produce a distinct band gap, clarifying that the electric field vector should be perpendicular to the slots. A qualitative circuitry analysis was then employed to give an overview of the aperture behaviour in terms of capacitance and inductance. For surface wave propagation along the array, the aperture array acts as a network of parallel LC resonators cascaded in series. The next step was to produce dispersion curves by applying a full wave modal analysis of an infinite aperture array. That gave a first indication on the band gap properties of the structure and the modes supported by the array.

A finite aperture array then was designed and simulated in order to realise the aperture array concept. Different parametric studies concerning the y-periodicity and the total number of elements were produced. Based on the simulation results an aperture array was fabricated and measured. The simulation and measurements were in excellent agreement, while the dispersion curve provided fairly good indications.

The second section of the chapter focused in the formulation of complementary dipoles. A layer of aperture and conducting dipoles were placed in close proximity with each other, to produce field interaction and increase of the capacitance. The coupling of the evanescent fields within the dielectric region was the key property for the realisation of the CMEBG structure. An essential requirement was the aperture rotation by 90° with respect to the conductors, so that both elements would be polarized with the electric field. A qualitative circuitry analysis was then used, for the understanding of the effect produced by the alteration of the capacitance. In terms of equivalent circuitry, complementary dipoles were defined as a network of parallel LC
resonators, cascaded in series. A dispersion curve analysis was then produced to study the modes supported by the infinite array.

A finite complementary array was designed and different parametric studies concerning the y-periodicity and the number of elements of the complementary structure were performed. Simulations for different conducting widths and lengths were produced. Three complementary designs with different conducting widths were manufactured and measured. Simulations and measurements were in very good agreement. The increase of the electrical length of the array elements, due to strong coupling between the two layers, yield maximum miniaturisation in the order of 2.3:1 compared to apertures. A substantial decrease of the band gap frequency was obtained, as the overlapping area of the conducting dipoles was increased.
REFERENCES


CHAPTER 5

Complex Geometries and CCMEBG structures comprising Complex Elements

5.1 Introduction

The concept of closely coupled metallodielectric electromagnetic band gap surfaces is expandable and applicable to geometries which are not consisting of the same element structure. It can incorporate arrays with different element consistency between them. Based on that principle, the present chapter introduces a novel concept which combines two different miniaturisation schemes. The first technique is based on the use of complex elements in the unit cell. Complex element geometries can increase the resonant current path, packing more electrical length in a fixed physical space[1-4]. The second technique is sustained on the principle introduced in chapters 3 and 4 and refers to the placing of two arrays in close proximity, to produce maximum coupling effect. The combination of these two approaches, can yield maximum miniaturisation.

The structure of the chapter is divided into two sections. The first one introduces the complex elements used for the research. Based on a simple tripole element, four different geometries are designed. Fractal, convoluted, interdigital and loaded tripoles. Figure 1 presents an overview of the four different geometries.

![Figure 5.1: Convoluted, fractal, interdigital and periodically loaded tripoles, respectively](image)

The second section focuses on the combination of the complex element arrays, with an array of tripoles. The target is to produce a structure that could incorporate complex elements, appropriately shifted and placed in close proximity to the array of
Complex Geometries and CCMEBG structures comprising complex elements

Chapter 4

tripoles, for maximum coupling effect. It is important to mention that due to the complexity of the array elements and the limited computational resources, dispersion curves could not be produced, neither very accurate simulations. Therefore measurements are primarily the comparative factor.

5.2 Measurement Setup

In order to carry out surface wave measurements two wide band, double-ridged, horn antennas have been used as transmitter and receiver. They have operating frequency range of 1 -18 GHz and they are positioned on either side of a tunnel formed by absorbers, where the MEBG array lies supported by foam (Fig5.2). By rotating the horns appropriately, TE surface wave polarisation is obtained. The measurement set-up also consists of an HP 8757D Scalar Network Analyser with a dynamic range up to 83 dB and the ability to measure insertion loss, gain, return loss and SWR. It is combined with an HP 8365 0L Series Synthesized Sweep/CW Generator with a frequency range 10MHz to 50 GHz.

Figure 5.2: Measurement Setup of Complex arrays for TE surface wave polarisation
5.3 Measurements of Complex Elements

The design and fabrication of the complex elements is based on a simple tripole. The length of the tripole arms is 3mm and the width of the loadings is 0.2mm. The unit cell is hexagonal with diameter of 6.2 mm. The Complex element arrays are printed on a dielectric sheet with thickness of 0.1 mm and dielectric constant of 2.2. The following figures are illustrating the element geometries used for the formulation of the complex arrays.

Figure 5.3: Periodically Loaded tripole array

Figure 5.4: Fractal tripole array

Figure 5.5: Convoluted tripole array

Figure 5.6: Interdigital tripole array
The transmission responses of the complex element arrays have been measured and superimposed in the Figure 5.7. For comparison purposes the electromagnetic band gap response of the simple unloaded tripole is also illustrated.

Figure 5.7: Transmission responses of tripole, fractal, convoluted, interdigital Periodically loaded tripole arrays

The measurements presented are not normalised with respect to the horn antennas. For the simple tripole array, the resonant frequency emerges at 14.3 GHz and the value of the fractional bandwidth is 5.6%. The first complex geometry examined is the periodically loaded tripole array. It produces a resonant frequency at 9 GHz with a band gap area of 600 MHz. This corresponds to miniaturisation factor of 1.59:1. For the fractal tripoles the cut-off emerges at 8.5 GHz and the band gap produced is 700 MHz or 8% fractional bandwidth. The interdigital tripoles produce a cut-off at 5.1 GHz with a band gap area of 300 MHz. The miniaturisation factor is 2:72:1. Finally the convoluted tripoles have a cut-off emerging at 4.6 GHz, producing a bandwidth of 100 MHz. The miniaturisation factor in comparison to simple tripoles is 3:1. The following table summarises the results produced by the transmission responses of the complex element arrays.
Complex Geometries and CCMEBG structures comprising complex elements

Chapter 4

Table 1: Summary of all the transmission response results of complex elements

<table>
<thead>
<tr>
<th></th>
<th>Tripoles</th>
<th>Peridically Loaded Tripoles</th>
<th>Fractal Tripoles</th>
<th>Interdigital Tripoles</th>
<th>Convolute Tripoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut-off Frequency (GHz)</td>
<td>13.9 GHz</td>
<td>8.75 GHz</td>
<td>8.5 GHz</td>
<td>5.1 GHz</td>
<td>4.6 GHz</td>
</tr>
<tr>
<td>Bandwidth (GHz)</td>
<td>0.8 GHz</td>
<td>0.6 GHz</td>
<td>0.7 GHz</td>
<td>0.3 GHz</td>
<td>0.1 GHz</td>
</tr>
<tr>
<td>Miniaturisation Factor</td>
<td>---------</td>
<td>1.59:1</td>
<td>1.62:1</td>
<td>2.72:1</td>
<td>3:1</td>
</tr>
</tbody>
</table>

5.4 Measurements of CCMEBG Complex Arrays

The formulation of the closely coupled structures consists of a layer of complex elements and a layer of simple tripoles, placed in very small proximity to produce maximum coupling effect. The dimensions of the elements employed in the arrays, are exactly the same as the ones used for the MEBG complex surfaces. The dielectric, on either side of which the arrays are going to be printed, has a thickness of 0.1 mm and relative dielectric constant of 2.2. The measurement setup is the same as the one used for the complex structures. Four different CCMEBG arrays were designed and manufactured. A closely coupled tripole array, a closely coupled periodically loaded array, a closely coupled fractal array and a closely coupled interdigital array. Although the closely coupled convoluted array was designed, it was not possible to be manufactured. The difficulty in the correct alignment of the two arrays was a restrictive factor in the fabrication process. The transmission responses of the CCMEBG arrays are measured and superimposed with the equivalent MEBG complex array results for comparative purposes are illustrated in Figs. 5.8-5.11 and Table 2.

Beginning with the closely coupled tripole array, the cut-off emerges at 6.3 GHz and the upper band gap extends up to 6.7 GHz. The bandwidth is measured 400 MHz. The miniaturisation in comparison with the single MEBG tripole array is 2.05:1. Figure 5.8 Illustrates the measured transmission responses for the CCMEBG and MEBG...
The next measurement is referred to CCMEBG periodically loaded tripoles. The cut-off emerges in this case at 5.4 GHz and the upper band gap extends up to 5.6 GHz. The bandwidth produced is 200 MHz. The Miniaturisation factor in comparison with the equivalent MEBG array is of the order 1.56:1. Figure 5.9 presents the transmission responses of the CCMEBG and MEBG periodically loaded tripole arrays.

![Figure 5.8: Measured Transmission responses of CCMEBG and MEBG tripole arrays](image)

![Figure 5.9: Measured Transmission responses of CCMEBG and MEBG periodically loaded tripole arrays](image)
The third measurement provides a comparison between the CC fractal tripole array and the single layer fractal tripole array. For the closely coupled fractals, the cut-off appears at 4.5 GHz. The bandwidth is measured to be 200 MHz and the miniaturisation factor is of the order 1.93:1.

![Graph]

Figure 5.10: Measured Transmission responses of CCMEBG and MEBG Fractal tripole arrays

Finally the last measurement produced, is related to the transmission response of the closely coupled interdigital tripole array. In this case the cut-off emerges at 4.6 GHz and the upper band gap extends up to 4.8 GHz. The bandwidth is 200 MHz and the miniaturisation factor is 1.1:1. Figure 5.11 presents the transmission responses of the CCMEBG and MEBG interdigital tripole.
The following table summarises the outcome of the measurements produced. The comparative values used in the tables, such as the miniaturisation factor, are related to the equivalent MEBG arrays.

<table>
<thead>
<tr>
<th></th>
<th>CC Tripoles</th>
<th>CC Peridically Loaded Tripoles</th>
<th>CC Fractal Tripoles</th>
<th>CC Interdigital Tripole</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cut-off Frequency (GHz)</strong></td>
<td>6.3 GHz</td>
<td>5.4 GHz</td>
<td>4.5 GHz</td>
<td>4.6</td>
</tr>
<tr>
<td><strong>Bandwidth (GHz)</strong></td>
<td>0.4 GHz</td>
<td>0.2 GHz</td>
<td>0.2 GHz</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Miniaturisation Factor</strong></td>
<td>2.05:1</td>
<td>1.56:1</td>
<td>1.93:1</td>
<td>1.1:1</td>
</tr>
</tbody>
</table>

Table 2: Summary of all the transmission response results of CCMEBG complex elements
5.5 Conclusion

The present chapter focused on the introduction of a miniaturisation technique, which combined the concept of closely coupled metallodielectric electromagnetic band gap structures and complex elements in the unit cell. A simple tripole was used for the formulation of four different complex geometries. Convoluted, fractal, interdigital and periodically loaded tripoles, were designed and fabricated. Comparative measurements of the transmission responses were produced, verifying the accuracy of the principle, that complex element geometries could increase the resonant current path, packing more electrical length in a fixed physical space. A miniaturisation in the order of 3:1 was achieved with the use of convoluted tripoles.

The four complex arrays, subsequently were combined with an array of simple tripoles, for the formulation of closely coupled structures. The larger electrical length accommodated in the unit cell, in combination with the strong coupling between the elements due to small separation distance, yielded even bigger miniaturisation. Closely coupled fractal tripoles managed to achieve a miniaturisation of almost 2:1.

The large increase in the demand for small compact microwave devices and antennas, along with the requirement of microwave elements, which are pertinent to high level integration into compact light-weight systems, makes the structures studied prime candidates for a series of applications.
REFERENCES


CHAPTER 6

Conclusion

The present thesis has researched and described a study into the theory and practical implementation of closely coupled metallodielectric electromagnetic band gap structures. CCMEBG arrays are a class of artificial metamaterials that could prohibit propagation of electromagnetic waves, within certain frequency regions. Three different CCMEBG configurations were employed and thoroughly examined for their miniaturisation capabilities. Conducting arrays of the same element design, conducting arrays with different element design and complementary structures, where a layer of aperture dipoles and a layer of conducting dipoles were etched on either side of a supportive dielectric slab.

Initially, a modal analysis method was developed and used in order to simulate the response of an infinite closely coupled periodic array, consisting of dipole or tripole conducting elements. The array was placed in a multiple dielectric substrate, with each element located in a unit shell. The array elements were printed on either side of a thin dielectric, in close proximity to each other, in order to produce maximum coupling. Two different types of excitation were examined. A surface wave and a plane wave illumination. Due to the periodicity and the infinite structure of the conducting array, the Floquet theorem was employed to simplify the analysis. This was achieved by expanding the fields in terms of Transverse Electric and Transverse Magnetic Floquet modes. The application of the standard electromagnetic boundary conditions, at the interfaces between the different dielectrics, matched the electromagnetic fields producing a set of Coupled Integral Equations (CIE). These equations were solved in terms of the unknown currents on the conducting element, of the different arrays, with the use of the Method of moments.

Dispersion diagrams were produced for both MEBG and CCMEBG conducting dipole and tripole arrays, in order to examine the modes propagating and the band gaps
occurring. For the dipole elements, in both MEBG and CCMEBG cases, excitation of the currents was achieved only from the modes supported by the dielectric slab, with an electric field component parallel to the array. The effect of the periodic dipole array (i.e. retardation of the waves and electromagnetic band gap) was hence evident only on those modes, at the vicinity of the dipole resonance. Those were TE modes predominantly. The dipole elements produced a one dimensional band gap. Tripole elements being more rotationally symmetrical than dipoles, produced a two dimensional common band gap. In both element cases, dispersion diagrams illustrated the vast reduction of the resonant frequency produced by a CCMEBG array in comparison with an MEBG one.

Dipoles were used initially to implement a CCMEBG design. A significant lowering of the band gap frequency (4:1) together with a superior angular stability and bandwidth were predicted using a plane wave modal analysis of the structure. The trends shown in the simulation results were confirmed with measurements carried out for plane wave as well as surface wave excitation. Despite the discrepancies at low frequencies due to the small size of the arrays, the simulations provided useful design guidelines upon which the construction of the CCMEBG structures was based. For plane wave measurements, a frequency shift of more than 3:1 was obtained with the dipole CCMEBG in comparison with the MEBG dipole array. Furthermore, improved angular stability of the resonant frequency was observed. The fractional bandwidth of the stop band also increased by 56% using the CCMEBG design. A similar shift in the stop band has been obtained during the surface wave measurements. An interesting characteristic of the surface wave response, for both single MEBG and double layer CCMEBG arrays, was that a relative gain of up to 8 dB was observed in the pass band. The transition to the stop band was very steep. Furthermore, the bandwidth of the stop band was wider compared to the one obtained in the plane wave measurements.

Tripoles were used to form CCMEBG structures, which were less dependant on the polarisation of the incident field. Tripole elements arranged on a hexagonal lattice exhibited a common band gap for any incident polarisation. Two different orientations (0° and 30°) were simulated and measured. The tripole CCMEBG design exhibited in general a 2:1 frequency shift with respect to the single layer MEBG. The angular
stability of the stop band was improved and the bandwidth was also increased. The 30° element orientation yielded a wider bandwidth on both plane wave and surface wave measurements.

The research was also extended to the new concept of Complementary metallodielectric electromagnetic band gap structures, used for its miniaturization capabilities. A layer of periodic dipole aperture array and a layer of periodic conducting dipole array, were etched either side of a supporting dielectric substrate. The dipole apertures were rotated 90° with respect to the conductors, so that both elements would be polarized appropriately with the electric field, and therefore would be resonant. By the interaction between the different elements, very strong fields were produced in the separation region. The CMEBG created electrically large elements from physically small ones. The key attribute of the proposed Complementary MEBG was the coupling of the evanescent fields within the dielectric region.

Using a single dipole aperture array as reference, a modal analysis based on coupled electric and magnetic field integral equations was initially employed. Dispersion curves were implemented for complementary linear dipole elements (i.e. conducting dipoles and aperture dipoles in a conducting screen). A commercial software package was used to simulate the transmission responses of finite CMEBG structures. Parametric studies were carried out for different widths and lengths of conducting elements, periodicities and number of elements. A qualitative circuit analysis was employed to give an insight on the physical interpretation of the CMEBG capacitance. Measurements for CCMEBG's with different conducting widths were performed, validating the predicted results and illustrating the advantages of the proposed structures. The increase of the electrical length of the array elements, due to strong coupling between the two layers, yield miniaturisation in the orders of 1.55:1, 2.05:1 and a maximum value of 2.3:1 in comparison with the single aperture array.

Finally an alternative way for reducing the physical dimension of MEBG structures was introduced, based on the use of complex element geometries within the unit cell. Complex element geometries could increase the resonant current path, packing more electrical length in a fixed physical space. The tripole was used as a reference element
in terms of design and comparison. Representative measurements were illustrated in which interdigital tripoles produced a miniaturisation factor of 1.61:1, while convoluted tripoles were proven to be more compact with a 3:1 factor.

Based on that principle, a closely coupled arrangement was presented. The arrays placed in close proximity were comprised of tripoles and tripole based complex elements. The combination of these two different techniques yielded a further 10% miniaturisation. Representative results were produced.

Being arrays of resonant elements, MEBG structures exhibit a bandgap frequency, which is directly related to the electrical length of the element. The resonant length is about half to quarter wavelength for common elements such as dipoles, tripoles and loops. Hence the arrays have rather large dimensions, which make them unsuitable for integration on compact microwave devices, particularly in applications of a few GHz, such as the mobile communication operating bands. Miniaturisation of microwave components and antennas has become increasingly important in recent years. Modern wireless communication terminals require small microwave elements which are pertinent to high level integration into compact light-weight systems. In this context, miniaturisation of EBG structures is an important consideration for the microwave engineer. Conformal designs of CCMEBGs are potent candidates for a series of applications due to their miniaturisation capabilities. They could be used as filters for both plane and surface wave propagation or as filters for micro strip patch antennas increasing at the same time the radiating efficiency. They could be used in active and passive devices and printed circuit applications. They could find applications in waveguides acting as band reject filters. One of the most attractive applications of CCMEBG arrays is their use in mobile handsets. Holding a terminal handset with a human hand affects the antenna performance in at least two ways, namely antenna detuning and efficiency degradation. The coupling of the user’s hand to the antenna, occurs via the electromagnetic fields that the antenna excites at the spatial location of the user’s hand. Dressing the mobile handset with a conformal miniaturised MEBG array suppresses the EM fields at the locality of the user’s hand. The performance degradation of the handset can be reduced. The isolation leads to reduced antenna detuning and increased efficiency.
The concept of closely coupled metallodielectric electromagnetic band gap structures could be further developed. More geometries could be studied for complementary surfaces, acting as ground planes for antennas and microwave circuits. The application of two complex element arrays placed in close proximity, could produce a very promising miniaturisation scheme. A research interest could also rise in the application of CCMEBG structures in artificial magnetic conductor surfaces. When printed on a grounded dielectric substrate, MEBG arrays fully reflect incident plane waves. Furthermore, within a frequency range, in grounded MEBG the reflection occurs with zero or near-zero degrees phase. This behaviour is the dual of an electric conductor (AMC). The miniaturisation concepts presented in this thesis could also be investigated for the miniaturisation of AMC surfaces. Another interesting application would be the use of CCMEBG structures for left handed materials (metamaterials).