Closed-loop space–time block coding and resource allocation in collaborative wireless networks

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Closed-Loop Space-Time Block Coding
and Resource Allocation in Collaborative
Wireless Networks

Thesis submitted to Loughborough University in candidature for the
degree of Doctor of Philosophy.

Shakiru Kehinde Kassim

Advanced Signal Processing Group
Department of Electronic and Electrical Engineering
Loughborough University
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The focus of this thesis is to exploit closed-loop space-time block coding schemes designed for multiple antenna links composed of four transmit antennas within collaborative wireless networks. Such schemes have the potential to increase the end-to-end bit error rate performance of wireless networks as compared to established schemes which only include links composed of two transmit antennas. The two four transmit antenna closed-loop-space-time block codes (STBCs) considered are the closed-loop-quasi-orthogonal-STBC (CL-QO-STBC) and the closed-loop-extended-orthogonal-STBC (CL-EO-STBC) schemes. Both techniques benefit from linear decoding complexity and symbol-wise maximum likelihood decoding.

The theoretical capacity advantage of the CL-QO-STBC technique within a point-to-point multi-input multi-output (MIMO) system is confirmed for both ergodic and non-ergodic channels. The achievable rate region for the ergodic channel is presented in terms of ergodic capacity and that of non-ergodic channels is given as outage probability.

The CL-QO-STBC method is next applied within a two-stage wireless network with frequency flat fading links. It is confirmed that this yields increased diversity gain and end-to-end throughput as compared to the approach when the Alamouti STBC (A-STBC) is applied. The performance is improved still further by the CL-EO-STBC technique
since it is highlighted that it provides both diversity and array gain. Power management schemes are employed in all the comparative simulations to optimize resource allocation.

The CL-QO-STBC and CL-EO-STBC schemes are then utilised within a three-stage wireless network composed of frequency-selective broadband links. The broadband channels are decomposed into frequency-flat links through the application of orthogonal frequency division multiplexing. A novel intra-stage power management strategy is developed to maximise end-to-end performance. Performance improvement is confirmed in terms of both capacity and end-to-end bit error rate.

Finally, for frequency-selective fading broadband links a new adaptive space-time-frequency block encoding approach is designed to optimize link performance within a collaborative wireless network. The performance advantage is confirmed through end-to-end bit error rate simulations.
To my loving children
STATEMENT OF ORIGINALITY

The contributions of this thesis are concerned with collaborative (cooperative) relay based wireless communication systems with particular emphasis on space-time block codes (STBCs) and resource allocation. The contributions are supported by seven published conference papers and one submitted journal paper. The contributions can be summarised as follows:

In Chapter 3, the capacity and outage probability bound of a full-rate and full-diversity closed-loop quasi-orthogonal space-time block code (CL-QO-STBC) scheme are compared with those of other orthogonal space-time block codes (O-STBC) including the famous Alamouti STBC (A-STBC). This study is carried out for point-to-point multi-input multi-output systems over both ergodic and non-ergodic channels. The results have been published in:

In Chapter 4, building upon the results of Chapter 3, the capacity and outage analysis are extended to a two-stage cooperative multi-stage relay network where the end-to-end capacity and throughput are found when a full-rate and full-diversity CL-QO-STBC scheme is used at the relaying stages. Furthermore, a more practical measure of the proposed cooperative system, i.e. end-to-end bit error rate performance, is also studied when the A-STBC, the full-rate and full-diversity CL-QO-STBC and the full-rate full-diversity closed-loop extended-orthogonal STBC (CL-EO-STBC) are employed at the relaying stages. In all these, the effect of inter-stage resource allocation is also considered. The performance of the architecture is studied for frequency-flat fading channels. The results have been published in:


In Chapter 5, motivated by the diversity advantage and power efficiency of the analysed architecture in Chapter 4 over flat fading channels, a more sophisticated cooperative relay network architecture for frequency selective channels is proposed in which the Alamouti-space
frequency block code (A-SFBC), full-rate and full-diversity closed-loop quasi-orthogonal space frequency block code (CL-QO-SFBC) and the full-rate and full-diversity closed-loop extended-orthogonal space frequency block code (CL-EO-SFBC) schemes are deployed at the relaying stages. Their performance is investigated within a three-stage collaborative multi-stage relay communication network. Distributed adaptive space-frequency coding for links composed of four transmit and one/or receive antennas, i.e. exploiting quasi-orthogonal and extended-orthogonal coding schemes is also examined. Moreover, within this chapter an intra-stage power allocation strategy is proposed. The results of these and the earlier works using quasi and extended-orthogonal space-frequency coding over frequency selective channels using the orthogonal frequency multiplexing (OFDM) technique, have been presented in four conferences and submitted to a journal:


- N.M. Eltayeb, S.K. Kassim and J.A. Chambers “Adaptive Resource Allocation within Three-Stage OFDM Relay Networks ,” 42nd IEEE Asilomar Conference on Signals, Systems and Com-


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<th>Description</th>
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<tr>
<td>4G</td>
<td>Fourth Generation</td>
</tr>
<tr>
<td>A-SFBC</td>
<td>Alamouti-SFBC</td>
</tr>
<tr>
<td>A-STBC</td>
<td>Alamouti-STBC</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>BER</td>
<td>Bit Error Rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CL-EO-SFBC</td>
<td>Closed-Loop-Extended-Orthogonal-SFBC</td>
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<td>CL-EO-STBC</td>
<td>Closed-Loop-Extended-Orthogonal-STBC</td>
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<tr>
<td>CL-QO-SFBC</td>
<td>Closed-Loop-Quasi-Orthogonal-SFBC</td>
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<td>Closed-Loop-Quasi-Orthogonal-STBC</td>
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<tr>
<td>CP</td>
<td>Cyclic Prefix</td>
</tr>
<tr>
<td>CSI</td>
<td>Channel State Information</td>
</tr>
<tr>
<td>dB</td>
<td>Decibel</td>
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<tr>
<td>EGC</td>
<td>Equal Gain Combiner</td>
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<td>EO-SFBC</td>
<td>Extended-Orthogonal-SFBC</td>
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<tr>
<td>Acronyms</td>
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<td>EO-STBC</td>
<td>Extended-Orthogonal-STBC</td>
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<tr>
<td>EOBC</td>
<td>Extended-Orthogonal Block Codes</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency-Division Multiple Access</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier Transform</td>
</tr>
<tr>
<td>IEEE</td>
<td>Institute of Electrical and Electronics Engineers</td>
</tr>
<tr>
<td>IET</td>
<td>Institute of Engineering and Technology</td>
</tr>
<tr>
<td>IFFT</td>
<td>Inverse Fast Fourier Transform</td>
</tr>
<tr>
<td>IID</td>
<td>Independently and Identically Distributed</td>
</tr>
<tr>
<td>M-PSK</td>
<td>M-ary Phase Shift Keying</td>
</tr>
<tr>
<td>M-QAM</td>
<td>M-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-Input Multi-Output</td>
</tr>
<tr>
<td>MISO</td>
<td>Multi-Input Single-Output</td>
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<tr>
<td>MRC</td>
<td>Maximum Ratio Combiner</td>
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<td>O-STBC</td>
<td>Orthogonal Space Time Block Code</td>
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<td>OBC</td>
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<td>OFDMA</td>
<td>Orthogonal Frequency Division Multiple Access</td>
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<td>Orthogonal Frequency Division Multiplexing</td>
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<td>OSF</td>
<td>Orthogonal Space Frequency</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<td>QO-SFBC</td>
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<td>Quasi-orthogonal Block Codes</td>
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<td>QoS</td>
<td>Quality of Service</td>
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<td>QPSK</td>
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<td>RF</td>
<td>Radio Frequency</td>
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<td>Rx</td>
<td>Receive Antenna</td>
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<td>SC</td>
<td>Selection Combiner</td>
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<td>SER</td>
<td>Symbol Error Rate</td>
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<tr>
<td>SFBC</td>
<td>Space Frequency Block Code</td>
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<tr>
<td>SIMO</td>
<td>Single-Input Multi-Output</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-Input Single-Output</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>STBC</td>
<td>Space Time Block Code</td>
</tr>
<tr>
<td>STFBC</td>
<td>Space-Time-Frequency Block Code</td>
</tr>
<tr>
<td>TDD</td>
<td>Time-Division Duplex</td>
</tr>
<tr>
<td>TDMA</td>
<td>Time-Division Multiple Access</td>
</tr>
<tr>
<td>Tx</td>
<td>Transmit Antenna</td>
</tr>
<tr>
<td>VAA</td>
<td>Virtual Antenna Array</td>
</tr>
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</table>
MATHEMATICAL NOTATIONS

\[ E(.) \quad \text{Statistical expectation operator} \]
\[ tr(X) \quad \text{Trace of a matrix } X \]
\[ |x| \quad \text{Absolute value of } x \]
\[ ||.|| \quad \text{Euclidean norm} \]
\[ (.)^T \quad \text{Transpose operator} \]
\[ (.)^* \quad \text{Complex conjugate operator} \]
\[ (.)^H \quad \text{Hermitian transpose operator} \]
\[ I_m \quad m \times m \text{ Identity matrix} \]
\[ \mathbb{C} \quad \text{Set of complex numbers} \]
\[ \mathbb{Z} \quad \text{Set of integers} \]
\[ \Re(x) \quad \text{Real part of } x \]
\[ \angle x \quad \text{Angle of } x \]
\[ \max(a_1, \cdots, a_n) \quad \text{Maximum of } a_1, \cdots, a_n \]
$diag(a_1, \cdots, a_n)$ $n \times n$ matrix with diagonal elements $a_1, \cdots, a_n$

and zeros elsewhere

$arg(a_1, \cdots, a_n)$ Argument of $a_1, \cdots, a_n$
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Chapter 1

INTRODUCTION

The ultimate goal of wireless communications is to provide "anywhere, anytime, anymedia" wireless access at a reasonable low price [1]. How to achieve this ambitious goal with limited bandwidth and power resources at affordable complexity while adhering to various implementation constraints presents tremendous challenges to research and development engineers.

However, recent developments in multi-input multi-output (MIMO) technologies for wireless communication systems have the potential to provide solutions to some of these challenges, creating multiple data pipes between the transmitter and the receiver [2]. The spectral efficiency measured in-terms of capacity of such system can be as high as several tens of bits/s/Hz [3] [4], although this comes with the price of an increased transceiver complexity. Another major advantage of MIMO systems is the increase in link performance in a fading environment, which is due to the diversity paths between the transmitter and receiver afforded by multi-antenna technology. The use of space-time block codes (STBCs) in exploiting this diversity has been shown to be possible, resulting in a lower complexity encoder/decoder [5]. Moreover, two antenna designs have optimal maximum likelihood symbol-wise decoding [6]. As such, STBC and MIMO techniques are very attractive
approach for future wireless systems and is already being exploited in WiFi (IEEE 802.11n [7]) and WiMAX (IEEE 802.16e [8]). Extending this methodology forms part of my focus in this thesis.

The benefits offered by MIMO systems both in-terms of capacity and system design depends largely on the correlation between sub-channels that exist between the transmitting and receiving antenna elements [9]. The extra capacity and diversity potentially provided by MIMO systems is underpinned by having spatially uncorrelated sub-channels. A high correlation between the sub-channels reduces the performance of a MIMO wireless channel towards that of a single link channel hence jeopardizing its capacity and diversity advantages. The main challenge faced by a MIMO communications engineer is therefore to design an antenna array which ensures mutually uncorrelated sub-channels. Correlation between sub-channels can be caused if the mutual spacing between the antenna elements creating the MIMO sub-channels is too small causing electromagnetic coupling [10].

Therefore, MIMO systems only promise an increase in capacity and diversity if uncorrelated paths are formed between the transmit and receive antennas. Naturally, physical limitations on the size of mobile terminal will generally lead to mutually correlated sub-channels and thus jeopardizing the available MIMO capacity. However, a solution to overcome this problem as suggested in [11] would be the use of spatially distributed cooperative single antenna elements, termed Virtual Antenna Arrays (VAA).

These cooperating distributed antennas can ensure spatially uncorrelated channels between the transmitter and the receiver, and permit the deployment of conventional space-time coding
techniques in a distributed manner. This also forms an integral part of the work presented in this thesis.

“Anywhere” communications means that reliable communication should be provided irrespective of the physical location of a transceiver, therefore, with the advantage of uncorrelated link inherent in cooperative communications, researchers have been looking at methods of using some of the “idle” distributed mobile terminals [11] to work as relays thereby increasing the network capacity, thus extending the network coverage areas especially in ad-hoc/cellular networks. This has led to another flourishing research area broadly known as “Cooperative multi-stage/relaying Communications” [12].

As with other research areas, cooperative multi-stage communication technique has its own challenges [12]; some of which include (a) radio resource management [12], (b) effective coding techniques [11], (c) synchronization [13] and (d) optimal cooperation levels [13].

The overall context of this thesis is to provide a contributive step towards addressing some of these challenges, with particular focus on the use of space-time coding techniques and resource allocation within cooperative multi-stage communications system operating over frequency non-selective (flat) and frequency selective Rayleigh fading channels.

Some basic definitions relevant to the work within this thesis are next presented.
1.1 Basic Definitions

1.1.1 Relaying Methods

In relay networks, the basic idea is to introduce a relay node which forwards data from a source to the destination which is out of reach of the source, for example, in a downlink communication where relays are used to forward data to mobile terminal that is outside the coverage area of the base station.

This can generally be accomplished in two different ways [14], amplify-and-forward (non regenerative relaying) and decode-and-forward (regenerative relaying), in the first, the relay terminals re-transmit the received signal with its embedded noise, whilst in the later, the relay terminal processes its received signal to eliminate the effect of noise and interference before re-transmitting it. This extra processing thus makes decode-and-forward relaying better in-terms of capacity than amplify-and-forward relaying [15] of course, at the expense of increased complexity. Additionally, regenerative relaying also allows the deployment of MIMO capacity enhancement technologies such as optimised allocation of network resources e.g power, frequency and frame duration among all the relay terminals [16]. These are some of the reasons why regenerative relaying is considered in this thesis\footnote{Unless otherwise stated regenerative relaying is considered in this thesis}. Regenerative relaying generally allows the utilization of two access methods; time-division multiple access (TDMA) and frequency-division multiple access (FDMA) systems. In TDMA based regenerative relaying depicted in Figure 1.1 the entire frame duration is orthogonally or non-orthogonally partitioned into slots among relay stages (“Stage1-4” in the diagram), and
Figure 1.1. Principle of time-division multiple access (TDMA) based orthogonal and non-orthogonal relaying

Figure 1.2. Principle of frequency-division multiple access (FDMA) based orthogonal and non-orthogonal relaying

communications occurs over the entire bandwidth. On the other hand,
for FDMA based regenerative relaying represented in Figure 1.2, the available bandwidth is divided by orthogonal frequency-division multiple access (OFDMA) or non-orthogonally among the relay stages and communication takes place continuously over the entire frame duration. Both of these access methods are employed in the analysis of a cooperative relay network presented in this thesis. Note the $RS_i (i \in \mathbb{Z}^+)$ is the relaying stage comprising of relay node(s) and $S$ and $D$ are the source and the destination respectively.

1.1.2 Cooperative Communication and Relaying Architecture

The relaying architecture can take many forms [12] depending on application. At the heart of cooperative communication is the classical relay architecture as shown in Figure 1.3 (a), which can also be called the "three body problem". In the architecture, the source broadcasts the

![Figure 1.3.](image)
signal to both the relay (denoted by \( r \)) and destination. The relay then retransmits the information to the destination. When the link between the source and destination is too unreliable for example due to high pathloss to guarantee reliable communications [16], the architecture reduces to the case of a cascade multi-stage communication.

Other structures use this classical architecture as the basic building block for more sophisticated cooperative relay networks. Such as in Figure 1.3b, which shows a simple case of multi-branch relaying using two parallel branches of relays \( r_1 \) and \( r_2 \). In this arrangement, the relays can take the form of virtual arrays which thus mimic a MIMO system through collaboration (cooperation) and hence derive better overall performance between the source and destination.

Due to the dispersive nature of wireless channels, cooperative relaying communication techniques can be deployed over both frequency non-selective (flat) and frequency selective wireless fading channels.

### 1.1.3 Frequency Non-selective (Flat) Fading Channel

A time-varying wireless channel model with impulse response \( h(t, \tau) \) is represented as in Figure 1.4, with \( x(t) \) denoting the signal transmitted and \( y(t) \) representing the received signal which is the “faded” version of \( x(t) \). If the channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal, then the impulse response \( h(t, \tau) \) can be approximated by a delta function at \( \tau = 0 \) that may have a time varying amplitude. In other words, \( h(t, \tau) = \alpha(t)\delta(\tau) \) where \( \delta(\tau) \) is the Dirac delta function. This is a narrowband channel in which the spectral characteristics of the transmitted signal are preserved at the receiver provided \( \alpha(t) \neq 0 \).
This phenomenon is called flat fading or frequency non-selective fading, hence the channel can be referred to as a frequency non-selective or frequency flat channel.

Although this channel preserves the spectral characteristics of the transmitted signal, for other applications the bandwidth of the transmitted signal might be greater than that over which the channel gain may be assumed constant; in that case the channel behaviour varies as a function of frequency, leading to the concept of frequency selectivity.

1.1.4 Frequency Selective Fading Channel

If the channel possesses a constant gain and linear phase over a bandwidth that is smaller than the signal bandwidth, intersymbol interference (ISI) exists and the received signal is distorted [17]. A wideband channel whose response varies with frequency is called a frequency selective channel, and if this frequency selectivity is time dependent, due for example to motion between the transmitter and receiver, the channel is termed frequency selective fading.

Both of these wireless channels characteristics are considered in this thesis.
1.2 Motivation of the Proposed Research Work

The work presented in this thesis has been inspired by the contribution of [18] and [19] on space-time block codes (STBC) as applied to MIMO and by [11] on cooperative relay networks. The motivating factors in each of these works are as follows.

In the work presented in [18] by Taker, Lambotharan and Chambers, it was demonstrated that a class of STBC called the quasi-orthogonal STBC (QO-STBC) originally proposed by [20] can be used to achieve full diversity over four transmit antennas in MIMO systems, if the symbols transmitted from two out of the four transmit antennas are rotated by a phase angle obtained through a feedback from the receiver, and hence the overall operation is called closed-loop QO-STBC (CL-QO-STBC). This important achievement was made after the earlier conclusion made in [21], that full diversity and full code rate complex valued space-time block codes exist only for a dimension of two that is the Alamouti STBC (A-STBC), and has motivated much research, such as [22], in the use CL-QO-STBCs in MIMO systems.

The achievement made in [19] to use another class of STBC, extended-orthogonal-STBC (EO-STBC), originally proposed in [23], to achieve fourth order diversity by combining beamforming with the diversity gain over four transmit antennas is also exploited in this thesis. This approach is named as closed-loop extended-orthogonal-STBC because extra diversity and directivity are obtained using the same feedback techniques employed to achieve CL-QO-STBC. This code has the potential to perform better than other orthogonal-STBCs including the CL-QO-STBC because of it array gain advantage as a result of the incorporated beamforming technique.
Section 1.2. Motivation of the Proposed Research Work

Since a high correlation among transmitting antennas potentially reduces a MIMO channel to a SISO channel. The design challenges involved in the use of multiple antennas on limited sized mobile terminals and other devices, to ensure uncorrelated MIMO channels has limited the successful deployment of these codes in practical MIMO systems. Additionally, even if the deployment of multiple antennas was made possible, to deliver the promised advantages of these coding schemes over a wide coverage area in the presence of other deterministic channel effects (pathloss) still remain a great design challenge.

A solution to overcome the correlation problems in MIMO channels was suggested in [11] where an array of spatially distributed single antenna elements forms a virtual antenna array (VAA). With this arrangement, it is less likely that the different channels from these individual single antenna elements are correlated. Under the assumption of perfect cooperation among the nodes within the VAA, an uncorrelated MIMO channel can be established between a transmitter and a receiver. The resulting modified MIMO channel is generally called a distributed MIMO (D-MIMO) channel [12]. In a D-MIMO system, it is more likely that space-time codes provide the desired diversity advantage due to the cooperation existing in a VAA. Also the VAA approach can be used to overcome the range problem if these closely distributed single antenna elements are used as relays in longer range communication. This approach is what is called collaborative or cooperative multi-stage communications which is at the heart of the work presented in this thesis.

The core aim of this thesis is to transfer the advantage of CL-QO-STBC and CL-EO-STBC in point-to-point MIMO systems to
cooperative multi-stage communications, and thereby provide a framework in which the advantages of these codes are more likely to be practically realised.

An important issue in multi-stage cooperative communications is in the effective use of the available network resource among all participation nodes, this has been studied by many researchers including [11]. The framework provided in [11] has made it possible to extend the resource allocation strategies to broadband channels with higher number of relaying stage for use in future generation wireless systems. Although the work presented in [11] does not explicitly address broadband channels, the development of a VAA has motivated the study of relay networks operating in broadband channels and thus the development of effective resource allocation over collaborative multi-stage communication operating in a broadband channel in this thesis.

To accomplish this target, the following steps are taken in this thesis.

1.3 Organisation of the Thesis

An introduction and brief survey of the state-of-the-art in space-time block codes (STBCs) design, cooperative communication and resource management is presented in Chapter 1. In Chapter 2, a detailed literature survey is provided together with the necessary theoretical background for point-to-point MIMO systems and cooperative relay networks. The core research is presented in Chapters 3, 4 and 5.

With the aim of establishing a general design framework for the use of CL-QO-STBCs in MIMO systems, Chapter 3 focuses on determining the maximum achievable rate region of this code in comparison
with other orthogonal STBCs (O-STBCs) when deployed in point-to-point MIMO ergodic and non ergodic channels. Simulation results are presented in-terms of capacity and outage probability. The simulation results shown in the chapter suggest that in an ergodic channel, the capacity advantage of using CL-QO-STBC supersedes that of other O-STBCs including the A-STBC. However for non-ergodic channels, the behaviour of the coding techniques vary and as shown in Chapter 3 depend on the desired communication rate with CL-QO-STBC being more suitable for low rate transmission.

In Chapter 4 the theory exposed in Chapter 3 is extended to a two-stage multi-stage cooperative network again for both ergodic and non-ergodic channels. The maximum achievable end-to-end capacity and throughput when CL-QO-STBCs and other O-STBCs are deployed in a distributed manner at the relaying stage is studied, again simulation results show the advantage of using CL-QO-STBCs over other O-STBCs considered. Also in the chapter, a more practical measure of the performance of these coding techniques is presented in-terms of the end-to-end bit error rate (BER) and throughput as a function of the received signal-to-noise ratio (SNR) at the destination, again the CL-QO-STBC promises to the deliver a steeper BER curve by providing a diversity order of four and higher throughput than the A-STBC. At this point having established the performance advantage of CL-QO-STBC over other O-STBCs with four cooperating relay nodes, a comparison is made assuming CL-EO-STBC are deployed distributively at the relaying nodes. Simulation results show the advantage of deployment of the CL-EO-STBC over the CL-QO-STBC.

In all the simulation results presented the resource allocation strategy

With the higher data rate requirement of future generation wireless networks over a wider coverage area, Chapter 5 investigates a broadband cooperative relay communication system, extending the two stage scenario studied in Chapter 4 to a three-stage with a more sophisticated architecture. Using the orthogonal frequency division multiplexing (OFDM) technique both the CL-QO-STBC and CL-EO-STBC are implemented over frequency selective fading channels and referred to as the CL-EO-SFBC and CL-QO-SFBC respectively. A major contribution in this chapter is the development of an optimum power allocation strategy based on the OFDM sub-carrier gains, this is shown to have better end-to-end BER and capacity performance when combined with the interstage allocation presented in [11]. A comparison is made assuming equal and water-filling power allocation allocation strategies are employed, the proposed approach by simulation study shows a significant performance advantage.

Finally, a concluding summary and suggestions for future research are provided in Chapter 6.
In this chapter a literature survey and necessary theoretical background concerning cooperative communications are presented. At first, the pioneering work in multi-input multi-output (MIMO) systems is presented both from information theory and practical implementation perspectives. Then various diversity techniques are investigated leading to the discussion on space-time block codes (STBCs), orthogonal space-time block codes (O-STBCs), quasi-orthogonal space time block codes (QO-STBCs) and finally, the extended orthogonal block codes (EO-STBCs). Some landmark results which motivate much of the work in this thesis are presented.

This is followed by a review of relay communication systems and cooperative operation. The ability of a cooperative relay network to mimic the performance advantages of point-to-point MIMO is discussed both from the information-theorist point of view and practical deployment.
2.1 MIMO Communication Systems

The use of multiple antennas at the transmitter and/or receiver in a communication system can result in the creation of a number of independent fading channels between the transmitter and receiver, this extra degree of freedom brought about by these independent channels is termed spatial diversity. This characteristics of a multiple antenna system if exploited properly, can improve the error performance of the system without increasing the transmission bandwidth to induce frequency selectivity or expanding the observation window to effect time selectivity [2].

Research on MIMO systems started to flourish since the publication of the landmark papers by Telatar [3] and Foschini [4]. Their analysis shows that spatial diversity can boost the capacity (and thus data rates) of MIMO systems well beyond those available with a single antenna SISO link. More on their results on the achievable capacity of MIMO systems are discussed in Chapter 3.

The use of space-time block codes has been shown to be useful in exploiting the potential spatial diversity available within a MIMO system [5], [24] and [25]. These codes are known to induce diversity as high as the number of transmit antennas times the number of received antennas. The development trend of these technique will be reviewed in this chapter.

Spatial diversity can be categorised into two forms; transmit diversity and receive diversity. Since both of these are exploited in this thesis hence a brief review on each of them is necessary.
Figure 2.1 shows a receive diversity system with a single transmit antenna and $N_r$ receive antennas giving rise to a single-input multi-output (SIMO) channel. Each of the receive antennas received a copy of the transmitted symbol $s$ denoted by

$$ y_i = h_i s + n_i $$  \hspace{1cm} (2.1.1)

where $i$ is the receive antenna index, $h_i$ is the flat fading coefficient of the channel linking the transmit antenna to the $i^{th}$ receive antenna which is zero mean circularly symmetrical complex Gaussian (ZMC-SCG) random variable with unit variance and $n_i$ is the additive white Gaussian noise (AWGN). Intuitively, it is evident that with the same transmit power, the $N_r$ receive antennas collect $N_r$ times more signal power than does the single receive antenna. This implies an increase in the instantaneous SNR at the output of the diversity combiner that can be invoked to demodulate the symbol $s$ using a weighted superposition of the received symbols.
The output of the combiner can be expressed as

\[ \hat{s} = \sum_{i=1}^{N_r} w_i y_i = \sum_{i=1}^{N_r} w_i h_i s + \sum_{i=1}^{N_r} n_i \]  

(2.1.2)

with \( w_i \) is defined based on the combiner employed. The most common type of combiners include the maximum ratio combiner (MRC) where \( w_i = h^* \), where \( (.)^* \) denotes complex conjugate, equal gain combiner (EGC) where \( w_i = h_i^*/|h_i| \) or selective combiner (SC) where \( w_j = 1, j = \text{argmax} |h_i| \), and \( w_i = 0, \forall i \neq j \). These three combiners present different trade-offs among required channel knowledge, complexity and average error probability achieved; but they all collect the maximum receive antenna diversity even when the channel \( h_i, i = 1, \ldots, N_r \) are correlated and have different probability density functions (pdfs) [25]. This is because the pdf of the combined SNR at any of the combiner’s output possess the sum of the degrees of freedom of each individual channel’s pdf which makes it more resilient to the fading effect [25].

The receive diversity approach is well suited for uplink transmission in cellular system, because of the possibility of deploying multiple antennas at the base station. Note, for downlink transmission the multiple antennas at the base station can also be used to exploit transmit spatial diversity.

### 2.1.2 Transmit Diversity

The transmit diversity scheme in Figure 2.2 showing two transmit and one receive antennas (an example of a multi-input single output MISO topology). With the assumption of two uncorrelated flat fading channel where the two channels coefficient \( h_1 \) and \( h_2 \) are ZMCSCG random
variable with unit variance, the received symbols $y$ can be modelled as

$$y = h_1 \frac{1}{\sqrt{2}} s_1 + h_2 \frac{1}{\sqrt{2}} s_2 + n$$  \hspace{1cm} (2.1.3)$$

where $1/\sqrt{2}$ is a scale to ensure that the total transmit power is divided equally across the two transmit antennas.

Clearly, unlike the receive diversity, it can be seen that the power available for demodulation is identical to that of SISO system since the available transmit power was divided across the transmit antennas. Even if the same symbol ($s_1 = s_2$) is transmitted from both antennas, where the input-output relation is changed to

$$y = \frac{h_1 + h_2}{\sqrt{2}} s_1 + n$$  \hspace{1cm} (2.1.4)$$

since $h_1$ and $h_2$ are uncorrelated Gaussian distributed, $(h_1 + h_2)/\sqrt{2}$ is still distributed according to the same probability density function as $h_1$ or $h_2$. This is assuming no channel state information (CSI) is available at the transmitter. Hence the MISO channel still behaves like a SISO and no advantage results from the use of multiple transmit antennas.
Therefore, to be able to exploit the advantage of multiple transmit antennas, there is a need for “smart” signal design at the transmitter side. Space-time coding techniques has proven in [5] to be a suitable approach for exploiting spatial diversity from multiple transmit antennas. The next section presents some of the earlier work on the use of this coding technique.

2.2 Space-Time Block Codes (STBCs)

A space-time block code (STBC) provides proper coding jointly across spatial and time dimensions in a multiple transmit antenna system, in order to exploit the spatial diversity available to the system. In STBC, the transmitted symbols are positioned in a transmission matrix which represents the transmission both in the spatial and temporal dimensions. There are several types of STBCs, this includes the orthogonal STBC (O-STBC) [21], quasi-orthogonal STBC (QO-STBC) [20], extended-orthogonal STBC (EO-STBC) [23], differential STBC [26], [6] and unitary space-time modulation [27].

An important property of a STBC is the code rate $R$ which is defined as the ratio of the number of transmitted symbols that can be sent in one codeword to the number of durations used in transmitting the codeword.

A review of all these classes of STBCs is beyond the scope of this thesis, hence the relevant ones employed in this thesis are reviewed, namely O-STBCs, QO-STBCs and EO-STBC.
2.2.1 Orthogonal Space Time Block Code (O-STBC)

If the space-time codeword matrix containing data symbols $x_1, x_2, ..., x_s$ is given as $X$, then this coding matrix is orthogonal only if,

$$XX^H = \left( \sum_{i=1}^{s} |x_i|^2 \right) I_s$$

(2.2.1)

where $s$ is the number of symbols contained in the codeword (two for Alamouti space-time block code (A-STBC)), $(.)^H$ denotes Hermitian (complex conjugate, transpose), $|.|$ is a magnitude operator and $I$ is an identity matrix. Any deviation from this will jeopardize the orthogonality of the code and hence result in loss of diversity when used in a multiple antenna system.

Therefore, O-STBCs achieve full diversity of order that is identical to the number of transmit antennas. However, they can introduce redundancy in the temporal dimension; hence they achieve a code rate less than unity, except for the case of two antennas [21].

Some examples of O-STBC are

$$X = \begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$$

(2.2.2)

This is the famous A-STBC scheme that represents transmission of two symbols over two time slots from two antennas achieving full diversity and full code rate (unity).
Others examples include [28], [21] and [29]

\[ X = \begin{bmatrix}
    x_1 & x_2 & x_3 \\
    0 & x_1^* & -x_2^* \\
    -x_1^* & 0 & -x_2^* \\
    x_2^* & -x_3^* & 0
\end{bmatrix} \] (2.2.3)

Equation 2.2.3 represents transmission of three symbols over four time slots from three antennas, achieving full diversity and 3/4 code rate; and

\[ X = \begin{bmatrix}
    x_1 & x_2 & x_3 \\
    -x_2 & x_1 & -x_4 \\
    -x_3 & x_4 & x_1 \\
    -x_4 & -x_3 & x_2 \\
    x_1^* & x_2^* & x_3^* \\
    -x_2^* & x_1^* & -x_4^* \\
    -x_3^* & x_4^* & x_1^* \\
    -x_4^* & x_3^* & x_2^*
\end{bmatrix} \] (2.2.4)

which denotes transmission of transmit four symbols over eight time slots from three transmit antennas, achieving full diversity and 1/2 code rate; and

\[ X = \begin{bmatrix}
    x_1 & 0 & -x_2^* & x_3^* \\
    0 & x_1 & -x_3 & -x_2 \\
    x_2 & x_3^* & x_1^* & 0 \\
    -x_3 & x_2^* & 0 & x_1^*
\end{bmatrix} \] (2.2.5)

which represents transmission of three symbols over four time slots from four antennas achieving full diversity and 3/4 code rate.
Another general property of O-STBC is that the computational complexity of a maximum likelihood based receiver is linear in the number of transmit antennas. i.e. symbol-wise maximum likelihood decoding is possible.

### 2.2.2 Quasi-Orthogonal Space Time Block Code (QO-STBC)

A straightforward extension of the A-STBC is given in [30] and [20], although the matrices introduced in these publications are different, it has been shown that their properties and performance are identical [18], hence for the purpose of this thesis the code proposed in [20] will be employed to provide insight into the behaviour of QO-STBC.

Re-writing $X$ as in equation (2.2.2) $X(1, 2)$ then QO-STBC can be formed from A-STBC as

$$X_Q = \begin{bmatrix} X(1, 2) & X(2, 4) \\ -X(3, 4)^* & X(1, 2)^* \end{bmatrix}$$

(2.2.6)

which can be written explicitly as

$$X_Q = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{bmatrix}$$

(2.2.7)

Clearly, this code achieves full code rate (transmits four symbols over four time slots through four transmitting antennas) but a diversity order of two [20]. Therefore in contrast to O-STBC, while the QO-STBC achieves full code rate, it suffers from diversity loss.

To show this, suppose this scheme is transmitted through a four
transmit and one receive antenna channel with each path experiencing independent flat fading with Rayleigh distribution. The received signal at the output over four symbol intervals (after taking the complex conjugates of the symbols in the second and third intervals) can be expressed as

\[
\begin{bmatrix}
  y_1 \\
  y_2^* \\
  y_3^* \\
  y_4
\end{bmatrix} =
\begin{bmatrix}
  h_1 & h_2 & h_3 & h_4 \\
  h_2^* & -h_1^* & h_4 & -h_3^* \\
  h_3^* & h_4^* & -h_1^* & -h_2^* \\
  h_4 & -h_3 & -h_2 & h_1
\end{bmatrix}
\begin{bmatrix}
  x_1 \\
  x_2 \\
  x_3 \\
  x_4
\end{bmatrix} +
\begin{bmatrix}
  n_1 \\
  n_2^* \\
  n_3^* \\
  n_4
\end{bmatrix}
\]  
\tag{2.2.8}

\[y = Hx + n \]  
\tag{2.2.9}

where \( y \) is the received signal vector, \( H \) represents the transmission path matrix and \( n \) contains the zero-mean circularly symmetric complex valued Gaussian noise components. Applying the matrix \( HH^H \) to perform matched filtering, i.e. perfect channel state information (CSI) is assumed to be available at the receiver (this assumption is made throughout this thesis) then the output can then be expressed as

\[\hat{y} = H^H y \]  
\tag{2.2.10}

from which the estimates of the transmitted symbols becomes

\[\hat{y} = H^H y = \Delta x + H^H n \]  
\tag{2.2.11}
Section 2.2. Space-Time Block Codes (STBCs)

Figure 2.3. Baseband representation of QO-STBC showing the feedback method that set $\alpha = 0$

\[
\begin{bmatrix}
\hat{y}_1 \\
\hat{y}_2 \\
\hat{y}_3 \\
\hat{y}_4
\end{bmatrix} =
\begin{bmatrix}
\gamma & 0 & 0 & \alpha \\
0 & \gamma & -\alpha & 0 \\
0 & -\alpha & \gamma & 0 \\
\alpha & 0 & 0 & \gamma
\end{bmatrix} 
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix} + 
\begin{bmatrix}
\hat{n}_1 \\
\hat{n}_2 \\
\hat{n}_3 \\
\hat{n}_4
\end{bmatrix}
\]

(2.2.12)

where $\Delta = H^H H$.

Here it can be seen that that due to the term $\alpha$, there is a form of coupling between the received symbols. For instance, the fourth symbol interferes with the first symbols, the third symbol interferes with the second symbol, this causes the diversity loss due to the coupling of these symbols, and would increase the complexity of maximum likelihood decoding to a pair-wise operation.

Several transmit preprocessing methods have since been proposed to minimise or remove this coupling factor. One of such method is pro-
posed by Toker et al. in [18] where two feedback methods are proposed and demonstrated to set the value of $\alpha = 0$. The first one is to phase rotate the transmitted symbols from two antennas with particular angles $\theta$ and $\phi$ as shown in figure 2.3, such that

$$\theta = \cos^{-1}\left(\frac{|\lambda|}{|\kappa|} \cos(\phi + \angle \lambda) \right) - \angle \kappa$$

(2.2.13)

provided $\phi$ is in the range $\phi \in [0, 2\pi)$ if $|\lambda| < |\kappa|$, or $\phi \in [\pi - \xi - \angle \lambda, \xi - \angle \lambda] \cup [-\xi - \angle \lambda, \pi + \xi - \angle \lambda]$ where $\kappa = h_1^* h_4$, $\lambda = h_2^* h_3$ and $\xi = \cos^{-1}(|\kappa|/|\lambda|)$, with $|.|$ and $\angle$ denoting absolute value and angle respectively. However, because of practical limitations it was further shown in the same work that using a single phase angle for rotation, (keeping the feedback from the receiver to the transmitter as small as possible) yields identical performance of the system. Another method proposed in [18] was antenna weighting and selection, this involves pre-multiplying the transmitted symbols by a weighting diagonal matrix, which is determined by channel state information CSI feedback from the receiver, this helps to determine which two antennas are chosen to be weighted for higher performance. In the context of this thesis a single phase angle approach is adopted as it requires minimum feedback information while minimising the decoding complexity of maximum likelihood decoding, that is making it symbol-wise.

It should be noted that using one receive antennas the full rate and full diversity QO-STBCs provide a higher diversity order i.e. fourth order, as compared to the conventional A-STBC for two antennas, with second order diversity.

Other independent methods such as constellation rotation and par-
Section 2.2. Space-Time Block Codes (STBCs)

Potential feedback have also been provided in the literature notable of which are [31], [32] and [33]. However, all these methods either have increased decoding complexity or reduced performance and are therefore not considered in this thesis.

2.2.3 Extended-Orthogonal Space Time Block Code (EO-STBC)

Another extension of the A-STBC called the extended-orthogonal space-time block code (EO-STBC) was proposed by [23]. Slightly different from the QO-STBC, this code achieves full (unity) code rate (transmits two symbols over two time slots through four transmitting antennas) and can achieve a full transmit diversity with simpler detection. The EO-STBC is represented as

\[ X_E = \xi \begin{bmatrix} x_1 & x_1 & x_2 & x_2 \\ -x_2^* & -x_2^* & x_1^* & x_1^* \end{bmatrix} \]  \hspace{1cm} (2.2.14)

where \( \xi \) is a constant value equal to 1/2. Ignoring the constant for simplicity of representation, and assuming that this code is transmitted over a four transmit and one receive antenna channel with each path experiencing independent flat fading with a Rayleigh distribution, the received signal at the output over two symbol intervals (after taking the complex conjugates of the symbols in the second symbol interval) can be expressed as

\[
\begin{bmatrix}
y_1 \\
y_2^*
\end{bmatrix} = \begin{bmatrix}
h_1 + h_2 & h_3 + h_4 \\
h_3^* + h_4^* & -h_1^* - h_2^*
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
n_1 \\
n_2^*
\end{bmatrix} \]  \hspace{1cm} (2.2.15)

\[ y = Hx + n \]  \hspace{1cm} (2.2.16)
where \([n_1 n_2]^T\) is the additive white Gaussian noise (AWGN) vector, at the receiver.

Applying the matrix \(H^H\) to perform matched filtering, the output can then be expressed as
\[
\hat{y} = H^Hy
\]  
(2.2.17)
and can be expressed as
\[
\hat{y} = H^Hx + H^Hn
\]  
(2.2.18)
where \(H^H H\) yields
\[
\begin{bmatrix}
\alpha + \beta & 0 \\
0 & \alpha + \beta
\end{bmatrix}
\]  
(2.2.19)
with \(\alpha = \sum_{i=1}^4 |h_i|^2\), and \(\beta = 2Re(h_1 h_2^*) + 2Re(h_3 h_4^*)\).

Note here that the \(H^H H\) of EO-STBC is orthogonal (unlike that of QO-STBCs), this indicates a simpler receiver decoding (without feedback) with linear detected symbols as follows
\[
\begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} = \begin{bmatrix}
\alpha + \beta & 0 \\
0 & \alpha + \beta
\end{bmatrix}\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} + \begin{bmatrix}
(h_1^* + h_2^*)n_1 + (h_3 + h_4)n_1^* \\
(h_3^* + h_4^*)n_1 + (h_1 + h_2)n_2^*
\end{bmatrix}
\]  
(2.2.20)

However, although the decoding complexity is low, \(\beta\) may be negative, which leads to some diversity loss. Therefore, the crucial thing is to ensure that \(\beta\) is positive so as to maximise the diversity achieved by this code. Several methods have been proposed, in [23] it was proposed that the transmitted symbols at the first and third transmit antennas are multiplied by \(U_1 = (-1)^i\) and \(U_2 = (-1)^k\) where \(i, k = 0, 1\) respec-
Section 2.2. Space-Time Block Codes (STBCs)

Figure 2.4. Baseband representation of EO-STBC showing the phase rotation technique that maximised $\beta$

tively; i.e. $0$ and $\pi$ as rotated angles for the symbols. Likewise in [34], the receiver computes the “interference factor” $\beta$ and feeds back $+1$ if $\beta \geq 0$ or $-1$ if $\beta < 0$. In [19] a phase rotation technique was proposed, where the symbols from the first and third antennas are rotated by a phase-angle (phase shift); $U_1 = e^{j\theta_1}$ and $U_2 = e^{j\theta_2}$ respectively, obtained through a feedback channel from the receiver as shown in figure 2.4. Therein, the values of $\theta_1$ and $\theta_2$ that thus maximise $\beta$ are

$$\theta_1 = -\angle(h_1 h_2^*)$$
$$\theta_2 = -\angle(h_3 h_4^*)$$

(2.2.21)

Although it was shown that the phase angle will introduce time delay, as in beamforming [35], this will enable the energy to be steered in a certain direction which gives directivity and thereby chooses the direction that matches the channel.
Section 2.3. Relaying Communication System

The publication [19] also shows that by applying a common phase-angle at antennas 1 and 3, there would be an insignificant loss in performance in comparison with the two phase-angle rotation case and thus the feedback overhead will be reduced since only one angle needs to be estimated at the receiver and fed back to the transmitter. Therefore, in the context of this thesis a common phase rotation is adopted for application in a cooperative wireless relay network with nodes deploying the distributed EO-STBC scheme.

It is worth pointing out that for point-to-point MIMO systems, the EO-STBC has advantages over other O-STBCs for more than two transmit antennas. First, a single radio frequency (RF) chain can be adopted for two transmit antennas that transmit the first two columns or the last two columns of $X_E$. In addition, the full-rate is achieved with decoding delay equal to two.

2.3 Relaying Communication System

The concept of relaying was first reported by Van der Meulen in [36] dated back to 1971 and was also studied by Sato in [37]. The first rigorous information theoretical study on the relay channel was exposed by Cover in [38], with a more detailed description presented in his book [39].

In these early contributions, it was assumed that a source communicates with a destination directly and via relaying nodes composed of mobile terminals which can operate in full-duplex mode. In [38] the maximum achievable communication rate has been derived for various communication scenarios, which includes the cases with and without feedback to either source or relaying nodes, or both. The capacity of
such a relaying network was shown to exceed the capacity of a simple direct link. However, the analysis presented by these works has been performed for statistical stationary Gaussian communication channels only, which is not directly applicable to wireless fading channels.

After these early introductory theoretical studies, it was not until the late 90s that [40] presented another milestone into the study of relay networks, they suggested a low complexity protocol to exploit cooperative diversity with the aim of boosting the uplink capacity and lowering the uplink outage probability for a given transmission rate. Their designed protocol stipulates a source mobile station to transmit its data frame to the base station and to a spatially adjacent mobile terminal, which then re-transmit the frame to the base station. Such a protocol could possibly yield a higher degree of diversity because the channels from both mobile terminals to the base station can be considered uncorrelated. The simple cooperative protocol has been extended by the same authors to more sophisticated schemes, which can be found in [41] and [42].

Based on the same concept, [16] presents a mathematical extension of [40], where energy-efficient multiple access protocols are suggested based on regenerative and non-regenerative relaying technologies. It has been shown that significant diversity and outage gains are achieved by deploying the relaying protocols when compared to the direct link. Note in both [40] and [16] no distributed space-time coding has been considered. Building on [16], [14] proposed another simple but efficient protocol to reduce the uplink outage probability for a code division multiple access (CDMA) cellular system operation in both regenerative and non-regenerative relaying modes.
Furthermore, [43] derived the analytical expression for the average symbol error for a generic multi-hop network. Specific application of relaying to a cellular network was presented in [44], in this study the use of SISO relaying has enabled the extension of the coverage area of a base station, by utilizing the mobile terminals at its coverage edge as relays to provide services to mobile terminals out of its reach.

In the work mentioned above, a simple single source-relay-destination network topology have been considered (SISO relaying) i.e. the source node, relay node and the destination node are made up of one node each.

With the prospect of deploying multiple antenna elements (or multiple single antenna elements) at the source, relay and destination, the attention of researchers has shifted to exploiting the advantages of MIMO technology in the context of relay communications [12]. The next section reviews some landmark contributions in this context.

The next section presents a review of research works that lead to the deployments of STBCs in the context of relay communications.

### 2.4 Cooperative Communication Systems

The contribution of Gupta and Kumar in [45] on relaying systems deploying multiple antennas at the transmitting and receiving sides was one of the major contributions in MIMO relaying. The network topology exposed in their work is the most generic one can think of, although slightly different from the stage-by-stage topology, the maximum system capacity is achieved by allowing any mobile terminal to communicate with any other mobile terminals in the network. Their approaches are from an information theoretic perspective, they derived
an achievable communication rate region in a network of arbitrary size and topology.

The works presented by Laneman in his thesis [12] created another dimension in the study of relay communication. He demonstrated that with cooperation between spatially distributed nodes, information can be relayed from the source to the destination. Such cooperation yields full spatial diversity which allows significant power saving at the same level of outage probability for a given communication rate. Laneman's ideas were extended to more refined distributed-MIMO multi-stage communication systems by Dohler in his thesis [11] where cooperation between the spatial distributed nodes not only takes place at the relaying stage, but also at the source and destination. Dohler formally named this arrangement "Virtual Antenna Arrays" in his patent [46]. Another major contribution of Dohler's thesis is also a discussion on the fractional power allocation at every stage in the network for different communication scenarios.

The work of Laneman and Dohler has motivated many research activities in distributed MIMO multi-stage communications, including the work exposed in this thesis, both from information theoretic and practical implementation perspectives, example of which includes [47] where analysis of outage probability of a cooperative relay network with certain node distribution is presented, and it is shown that with appropriate cooperation and node topology, the effect of cooperation reduces the outage probability as compared to a non-cooperative case. Also, [48] investigates ergodic capacity and outage probability for amplify-and-forward cooperative relays with different transmission protocols. Other information theory approaches to the study of cooperative relay net-
works include [49], where the end-to-end performance of cooperative diversity networks equipped with non-regenerative relays and SC receivers was studied. The authors derived a closed form expressions for the cumulative distribution function (CDF), probability density function (PDF) and moment generating function (MFG) of the end-to-end SNR in order to determine the most appropriate channel to use on a particular link within the network or to perform optimised routing.

The concept of spatial multiplexing and beamforming in traditional MIMO configurations was extended to cooperative relay networks with the work of [50], [51] and [52]; in [50] a cooperative relay network configuration that combines spatial multiplexing and beamforming was suggested based on a two-stage non-regenerative relay protocol, in their work they derived a closed form expression for the system capacity, and under SNR maximising and capacity maximising criteria, they analysed and compared the system capacity and BER performance with that of non-cooperative systems and confirmed its performance advantage. In [51], transmit beamforming techniques at the relaying stages were proposed, whereas in the contribution of Madan et al. [52] analysis of total energy consumption for a general class of cooperative beamforming-based relay transmission schemes was investigated.

Distributed space-time coding (DSTBC) schemes were first described for use in cooperative networks in [53] in order to take advantage of the spatial diversity available in cooperative communications. This has been followed by a series of other attempts to incorporate various space-time coding techniques in cooperative networks with performance measures varying from the theoretical concepts to practical deployments. Some of such work can be found in [11] where orthogonal space-time
Section 2.4. Cooperative Communication Systems

Block codes including the A-STBC are deployed in a distributed manner for a virtual antenna array multi-stage network, other works include [54] where distributed A-STBC is used in achieving cooperative diversity in the uplink of a relay-assisted cellular network. After these early works, [55], [56] and [57] have attempted to exploit the distributed coding techniques using quasi-orthogonal space-time block codes with more cooperating nodes. Their approach is different from the one presented in this thesis in that they do not incorporate effective resource allocation in their design and because the constellation rotation method is adopted, and the computational complexity of the maximum likelihood decoding at the receiver is no longer linear in the number of cooperating relays. Other distinctions are outlined in the introductory part of Chapter 4.

Other techniques used in exploiting space-time block codes in cooperative communication can be found in [58] and [59]. In [58] a new technique for cooperative transmission is shown using a new space-time coding technique to achieve spatial diversity between the relay nodes and the destination. In [59] distributed space-time trellis codes have been designed for cooperation between two spatially adjacent mobile terminals, to achieve a lower frame error rate to one or more destination(s), where a quasi-static fading channel has been assumed. This scheme was shown to maximise the performance for the direct link from either of the mobile terminals or to the destination and the relaying link.

Another dimension to the study of cooperative communications is the development of resource allocation strategy for the network nodes. To the best of my knowledge this was pioneered by the work exposed in Dohler's thesis [11] where network resources, power, frame duration
and frequency are shared among all relay stages. In his thesis these resources are shared among various stages depending on the number of cooperating transmitting and receiving nodes, he showed from both information theoretic and practical implementation perspectives that irrespective of a given scenario this allocation strategy provides optimum end-to-end performance and better than equal resource allocation. His framework based on the virtual antenna array approach has made possible the deployment of more sophisticated space-time coding schemes in cooperative communications as is exposed in this thesis.

Other resource allocation techniques can be found in [60], [61], [62] and [63]. One common element in the allocation strategies proposed within these contributions is that they considered transmission protocols in which both the relay and the source nodes cooperate to transmit the distributed STBC to the destination, which due to the transmission protocol deployed, makes their application beyond the distributed A-STBC scheme very difficult. Moreover, they could also suffer performance loss if the link between the source and destination goes into deep fade, hence reducing the whole system to SISO relaying. Hence the virtual antenna array approach provides a better framework for optimal power distribution over the network, due to the local communication between the multiple relay nodes.

All the above work have been performed under the assumptions that the wireless transmission channels are frequency flat, however, because of the time dispersive nature of wireless channels, the application of these work are limited to low data rate applications, this is to avoid inter-symbol interference (ISI) that results in transmission of higher data rate when the channel becomes frequency selective. With the use
of the orthogonal frequency division multiplexing (OFDM) technique, to combat ISI, which is becoming a promising technique for future wireless networks, recent research effort has focused on combining the cooperative relay technique with OFDM for use in future broadband networks. Some of the reported work in this regards ranges from the extension of D-STBC to frequency selective channel to different algorithms that deal with resource allocation problems. Some of this work includes [64], [65], [66], [67] and [68]. In [64], a capacity maximising optimum resource allocation algorithm in conjunction with sub-carrier pairing for a regenerative OFDM based relay system was presented, they showed that with the availability of CSI at the relaying node, the available system capacity can be enhanced, although no D-STBC is considered in this publication, their result was presented from the information theorist point of view. In the work presented in [65], where capacity maximising power allocation algorithm was developed for a non-regenerative OFDM relay system, their strategy was accomplished in two parts, first the optimal power allocation to each stages was performed when the power of each sub-carrier is fixed and secondly an optimal allocation of power at each hop to each sub-carrier was performed. They show that these strategies improve system capacity when compared to equal power allocation. In [66], the authors studied the end-to-end resources allocation in an OFDM based multi-stage network consisting of one dimensional chain of nodes including a source, a destination and multiple relays (SISO relaying), showed the average end-to-end transmission rate can be maximised if the power allocation to each sub-carrier at each stage has the water-filling structure and the fraction of transmission time allocated to each stage is adjusted to keep
the instantaneous rate over all stages equal. In [67] the use of A-STBC in an non-regenerative asynchronous cooperative communication system over a frequency-selective channel is proposed, they showed that this scheme can achieve diversity order of two with minimum decoding complexity. However, in [68], a subcarrier grouping technique was used with A-STBC to achieve maximal spatial and multipath diversity in a non-regenerative cooperative relay system.

The limitation of the work exposed in these publications include the assumption of a direct path between the source and the destination as in [64] and [65], consideration of single cooperative relaying stage and the use of only A-STBC. These are in contrast to the work to be presented in this thesis where optimum power allocation is suggested for a multiple-stage relay communication wherein more advanced coding schemes are deployed.

Although contributions on the topic of broadband cooperative communications have begun to emerge, the amount of work undertaken is scarce in comparison to the vast amount of potential scenarios.

2.5 Summary

This chapter has introduced the reader to the research work that has been undertaking prior to the work presented in this thesis.

The first section of this chapter has focussed on the achievements made in the area of MIMO communications from the landmark work of Telatar in information theoretical concepts to the various developments in space-time coding and then the application of these background works to relay communication and cooperative communication.

In MIMO, the space-time block codes used in order to exploit the
transmit diversity offered by multiple antennas are introduced, leading to the O-STBCs, QO-STBCs and EO-STBCs. The performance advantages of each were highlighted. The chapter then proceeded to discuss the earlier attempts to expand wireless network coverage with the use of relay networks. Various theoretical and practical issues were also reviewed.

Motivated by the difficulty in deploying multiple antennas in mobile terminals, it was then highlighted that researchers have focused their attention into ways of using multiple single antenna elements to create a MIMO channel through the use of cooperation, this was later extended to relay networks and the advantages of D-STBCs was also exploited.

Note that the descriptions given above of prior contributions are not exhaustive; however, more contributions are detailed in the introduction to the respective contribution chapters.
3.1 Introduction

Capacity is a concept that relates to a large area of information theory and serves as a fundamental measure of any communication channel. The landmark work presented by Claude Shannon in [69] "A Mathematics Theory of Communication" in this regard, has served as the backbone for the now classical paradigm of digital communication.

His theory predicts the maximum achievable data transmission rate over a channel that guarantees error-free communication between a transmitter and receiver, with a given input distribution, noise power, bandwidth and transmission power and is known as "channel capacity". The analysis presented by Shannon assumes that the transceivers available to the communication system designer are of infinite complexity, which is not generally practically feasible.
With researchers and designers having to trade complexity with performance, much effort has been made to construct finite complexity transceivers operating close to the capacity bound predicted in [69]. A major achievement in this area has come with the the development of Turbo Codes [70] and their modifications in [71], [72] and [73] to further increase the capacity offered by Turbo Codes, but to date, nobody has ever managed to communicate at exactly the theoretical channel capacity.

Information theorists thought that with the discovery of Turbo Codes that the activity within their research area was reducing, but with the birth of multi-input multi-output (MIMO) communications pioneered in [4] and [3], a new dimension to the study of information theory has opened up. In MIMO systems, the additional spatial dimension is utilised to further increase the available channel capacity\footnote{These additional capacity gains are guaranteed if there exist uncorrelated subchannels between the transmitter and the receiver.}. Although to achieve maximal capacity MIMO systems also require transceivers of infinite complexity the capacity limit set in [4] and [3] serves very well as general characterisations for MIMO communication systems and have motivated many theoretical generalizations, (with derivation of different closed form expressions for MIMO capacity examples of which are presented in [74] and [75]). They have also spawned interesting research and development activities in practical methods, such as space-time block coding (STBC) strategies and Bell labs layered space time (BLAST) like systems [76]).

The interest in this chapter is in the practical use of orthogonal space-time block coding methods over point-to-point MIMO channels and thereby to leverage the spatial dimension and its associated po-
tential capacity gain. Orthogonal space-time block codes (O-STBCs) inherently reduce the MIMO channel into a single SISO channel with modified channel statistic [28], such MIMO channels are henceforth referred to here as orthogonal-MIMO channels (O-MIMO). The issue here is the investigation of the capacity bounds of O-MIMO channels using O-STBC with different transmission (code) rates. Generally, the MIMO capacity is shown to scale linearly with the minimum of the number of transmit and received antennas, but for O-MIMO channels the capacity is also a function of the transmission rate of the O-STBC involved. Therefore, for O-STBC to deliver any pay off in-terms of capacity improvement over the traditional SISO channel, the transmission rate must be preserved from the transmitter to the receiver (i.e. the O-STBC must be of full rate) and hence delivering the diversity advantage. Since according to [21], a full diversity and full rate complex value space time block code (STBC) exists only for a dimension of two e.g Alamouti-STBC (A-STBC), therefore, most earlier work in [77], [11] and [78] which presents achievable ergodic and non-ergodic capacity of O-MIMO channels were based on A-STBC over two transmit antennas and other lower rate codes over higher numbers of transmit antennas. With the extension of A-STBC into four dimensions the quasi-orthogonal space-time block codes (QO-STBCs) \(^2\) [20] which were then orthogonalised in [18] and called the closed-loop quasi-orthogonal space time block code CL-QO-STBC, then the assumption in [21] can no longer represent a generalised statement for O-STBCs. The performance of the CL-QO-STBC presented in [18] focussed on its bit error rate (BER) at different signal-to-noise ratio (SNR) regions, which is

\(^2\)Remember that this code achieves full code rate at the expense of diversity hence is not orthogonal.
different from the one presented in this chapter, the new evaluation presented here in-terms of capacity is important to overall system design such as when concatenated with capacity-achieving outer codes e.g. Turbo Codes or deployed in distributed communications. Therefore it is the aim of this chapter to present both the ergodic and non-ergodic capacity potential of O-MIMO using the CL-QO-STBC.

To this end, the remainder of this chapter is structured as follows, firstly an overview of some basic analysis presented in [69] is re-iterated for the Gaussian channel in Section 3.2. Emphasis is then placed upon Rayleigh fading in Section 3.3, which leads to the definition of capacity over ergodic and non-ergodic channels. This aids in the understanding of MIMO capacity as presented in [3] for generic MIMO systems and O-MIMO in Section 3.4. Some simulation results are presented in Section 3.5 which show the capacity potentials in using CL-QO-STBC in comparison with other O-STBCs including the popular A-STBC. Finally, in Section 3.6, a summary is presented.

### 3.2 Channel Capacity (Gaussian Case)

Consider a classical representation of a communication system presented in Figure 3.1 named in [69] as a “Schematic diagram of a general communication system”, where the information source generates messages or sequences of messages to be transmitted to the receiver through the transmitter, the transmitter then performs signal processing on the message making it suitable for transmission over the channel. The receiver, on the other hand, performs an inverse transmitter operation on the received signal to reconstruct the original message and forward it to the destination. Shannon shows that the white noise from the noise
source results in a distorted signal at the receiver. Of most interest to him was the measure of the information contained in the message received by the destination in relation to the message transmitted from information source. To do this he extended the concept of entropy originally exploited in a thermodynamic context to information theory, to measure the level of uncertainty in the transmitted message. For a continuous random variable \( x \) having a probability density function (pdf) \( p(x) \), its average entropy can be expressed as

\[
H(x) = -\int_{-\infty}^{\infty} p(x) \log_2(p(x)) \, dx \quad (3.2.1)
\]

This equation can also be regarded as the amount of information required on average to describe \( x \). If this variable is limited in variance, \( \sigma^2 \) (with any mean \( \mu \)), then \( p(x) \) which maximises equation (3.2.1) follows a Gaussian distribution, i.e. \( x \sim \mathcal{N}(\mu, \sigma^2) \), the uncertainty in \( x \) can thus be expressed as
Section 3.2. Channel Capacity (Gaussian Case)

\[ H(x) = \frac{1}{2} \log_2(2\pi e \sigma^2) \]  
(3.2.2)

where \( e \) is the natural constant with approximate value of 2.718. If \( y \) is the received message at the destination then it is also a random process, and the mutual information between \( x \) and \( y \) is given as

\[ I(x; y) = H(x) - H(x|y) \]  
(3.2.3)

which represents the uncertainty inherent in the transmitted message \( x \), minus the uncertainty in \( y \) given that \( x \) is transmitted and is called a conditional entropy of \( x \) given \( y \). If at the receiver, the conditional entropy remains i.e. \( H(x) = H(x|y) \), then the receiver will be unable to decide which realisation of \( x \) has been transmitted and the mutual information is zero; alternatively, if after receiving \( y \), the uncertainty in \( x \) is resolved partially, i.e. \( 0 < H(x|y) < H(x) \), then the receiver can determine with some level of certainty which realisation of \( x \) is transmitted, and this level of certainty depends on the properties of the channel. However, if \( H(x|y) \) completely resolves the uncertainty in \( x \) i.e. \( H(x|y) = 0 \), the receiver can determine the realisation of \( x \) with utmost certainty and \( I(x; y) = H(x) \). According to [69] theorem 16 maximum mutual information between the source and destination occurs when \( H(x|y) = 0 \). With this framework, Shannon then expressed the capacity of a channel as the maximum mutual information \( I(x; y) \) between the source and the destination for all possible distributions of \( x \) in the presence of additive white Gaussian noise (AWGN) as

\[ C = \max_{p(x)} I(x; y) \]  
(3.2.4)
The input/output relationship of the above system can be represented as,

\[ y = x + n \]  \hspace{1cm} (3.2.5)

where \( n \) is the additive white noise, which is Gaussian in nature. Hence equation (3.2.4) is maximised only if \( x \) is Gaussian too. This enabled him to define the channel capacity of an AWGN channel per channel use as

\[ C = \frac{1}{2} \log_2(1 + \rho) \]  \hspace{1cm} (3.2.6)

where \( \rho \) is the signal-to-noise ratio (SNR) defined as the ratio of average power of \( x \) (S) to the average power of \( n \) (N).

Shannon then showed that if \( x \) is limited in duration \( T(s) \) and bandwidth \( W(Hz) \) it can be represented approximately with \( 2WT \) samples. Since the capacity expression in equation (3.2.6) is the capacity per channel use, he was able to express the capacity of a band limited signal/channel as

\[ C = WT \log_2(1 + \rho) \] (bits)  \hspace{1cm} (3.2.7)

which represents the maximum number of bits that can be transmitted per channel use without error.

This can then be re-written per duration \( T \) as

\[ C \equiv \lim_{T \to \infty} \frac{WT \log_2(1 + \rho)}{T} = W \log_2(1 + \rho) \] (bits/s)  \hspace{1cm} (3.2.8)

If this expression is normalised by the bandwidth of \( x \), equation (3.2.8)
can thus be expressed per unit bandwidth as

\[ C = \frac{W \log_2(1 + \rho)}{W} = \log_2(1 + \rho) \quad \text{(bits/s/Hz)} \quad (3.2.9) \]

Equation (3.2.9) is thus referred to as the normalised capacity, defined as channel capacity per unit bandwidth. In this thesis, the normalised channel capacity is used for notational simplicity.

Finally, Shannon proved that to achieve error free communication from the source to the destination, the transmission rate must not exceed the channel capacity.

### 3.3 Channel Capacity (Fading Case)

If in Figure 3.1, the channel between the transmitter and the receiver is wireless then there generally exist different paths between the transmitter and the receiver. This will result in many delayed versions of the transmitted signal being received at the receiver, the receiver then combines these different versions (which could be different in phase and strength) together and processes the estimated received signal. Since the resulting signal is an accumulation of different received signals, the channel is no longer an additive white zero memory Gaussian noise channel [29] and its capacity cannot be explicitly defined by equation (3.2.9).

To be able to describe the capacity of such a channel, a general characteristic of wireless channels must be defined. Wireless channels generally obey large, medium and small-scale fading [29]; large scale fading results in the reduction in the received signal strength/power over a long period of time or large distance, this is a deterministic ef-
effect which is attributed to path loss or shadowing. Medium scale fading is a random effect that is observed in the spatial dimension when moved over several tens of wavelength [79] and lastly small-scale fading results in the rapid changing of the received signal power over a short period of time or distance, this is further classified in [29] into flat slow/fast fading or frequency non-selective slow/fast fading and frequency selective slow/fast fading. This random change in signal power at the receiver due to small-scale fading has resulted in use of different statistical arguments to model a wireless channel namely Rayleigh, Ricean and Nakagami fading models [17]. These statistical models define the randomness in the channel power gains $\lambda$ with their respective probability density function $p(\lambda)$. The study of all these wireless models is beyond the scope of this thesis hence analysis here concentrates only on systems operating over Rayleigh fading slow/frequency non selective channels.

If message $x$ is defined as a finite length codeword drawn from an infinite length codebook, then the capacity expression in equation (3.2.9) can be expressed in-terms of channel power gain due to the deterministic effects captured in $\gamma$ and $\lambda$ with its associated probability density function $p(\lambda)$ as follows.

### 3.3.1 Fixed Channel Realisation

If the channel gains are fixed for at least the finite duration of the codeword transmission, the channel capacity of a wireless channel with a fixed $\lambda$ can be expressed as

$$C = \log_2(1 + \lambda \gamma \rho) \quad (\text{bits/s/Hz})$$  \hspace{1cm} (3.3.1)
This expression is sometimes referred to as instantaneous capacity.

### 3.3.2 Ergodic Channel Realisation

If $\lambda$ varies through a transmitted codeword but its statistical properties (moments) are the same from codeword to codeword, then the channel can be referred to as ergodic and the capacity can be expressed as an average capacity over all possible value of $\lambda$ which is expressed in [3] as

$$C = E_\lambda\{\log_2(1 + \lambda\gamma\rho)\} \quad (\text{bits}/s/\text{Hz}) \quad (3.3.2)$$

where $E_\lambda\{\cdot\}$ is the statistical expectation operator with respect to $\lambda$ described by its probability density function. The expression in equation (3.3.2) can be referred to as the mean capacity, which can be interpreted to mean that, if the channel fades fast enough over a finite but long length of codeword, then its mean can be estimated with utmost certainty. The channel can then support a rate not exceeding the above given capacity. This expression will prove useful for the analysis presented in this chapter.

### 3.3.3 Non-ergodic Channel Realisation

If $\lambda$ is chosen randomly and kept constant for the entire codeword transmission, there exists a non-zero probability that a certain rate cannot be supported by the channel [3]. This is because for this randomly fixed channel, it might be possible that a certain rate cannot be supported, hence making it impossible to express the capacity of such a channel as maximum mutual information, rather the capacity is sometimes referred to as the outage capacity [80], and is expressed by means
of non-zero probability called the outage probability. The reliability of such a channel depends on its outage probability. This channel property will be made apparent during the discussion on MIMO capacity in the next section.

### 3.4 Capacity of a MIMO Channel

This section is focussed on the capacity achievable by a multiple antenna system over the various channel realisations discussed in the previous section and taking into consideration coding over its spatial and temporal dimensions.

#### 3.4.1 System Model

Consider a generalised MIMO transceiver model represented by Figure 3.2. The data stream $s$ from the source is fed into the transmitting block containing error control coding, signal-to-constellation Gray code mapping (multi-level quadrature amplitude modulation (M-QAM), or M-ary phase-shift keying (M-PSK)), the coding produces several separate symbol streams which are mapped to the $N_t$ transmit antennas. This mapping may be a linear weighting of antennas (as in the case of a channel with feedback) or space-time coding [81]. The resulting spatial codewords $(x_i, i \in 1, 2, \ldots N_t)$ are then transmitted through $N_t$ multiple antennas with average power $S$ to the receiver via a wireless channel. The receiver is also equipped with $N_r$ multiple antennas, receives codewords $(y_i, i \in 1, 2, \ldots N_r)$ that have been distorted by the channel and AWGN of average power $N$. The receiver performs demodulation and demapping operations to produce an estimate of the transmitted data stream $\hat{s}$ at the destination. The selection of coding and antenna
mapping can vary a great deal depending upon the application and its primarily decided by factors such as receiver and transmitter complexity, and prior channel knowledge. Again of interest in this chapter is the space-time coding process at the transmitter and the receiver to determine the achievable error-free transmission rate over this channel.

Mathematically, let the spatial transmitted codeword from all the \( N_t \) antennas at anytime instant be represented by \( x \in \mathbb{C}^{N_t \times 1} \), with the total power constraint \( S \) regardless of the numbers of transmit antennas the following holds.

\[
E\{x^H x\} = tr[E\{xx^H\}] \leq S \tag{3.4.1}
\]

where \( tr[.] \) and \( (.)^H \) defining a trace and Hermitian transpose operator respectively. Representing the general channel realisation between transmitter \( i, i \in (1, N_t) \) and receiver \( j, j \in (1, N_r) \) as \( h_{ij} \), then the entire channel realisation between the transmitter and the receiver can
be grouped into an $N_r \times N_t$ complex channel matrix $H$ expressed as

$$H = \begin{bmatrix}
    h_{1,1} & h_{1,2} & \ldots & h_{1,N_t} \\
    h_{2,1} & h_{2,2} & \ldots & h_{2,N_t} \\
    \vdots & \vdots & \ddots & \vdots \\
    h_{N_r,1} & h_{N_r,2} & \ldots & h_{N_r,N_t}
\end{bmatrix} \quad (3.4.2)$$

where $h_{i,j}$ is referred to as a sub-channel. Assuming $H$ is full rank, then there will be at least $\min(N_t, N_r)$ independent sub-channels between the transmitter and receiver. The input-output representation can thus be expressed as

$$y = Hx + n \quad (3.4.3)$$

where $y \in \mathbb{C}^{N_r \times 1}$ is the received signal vector containing the received signal at each receiver and $n \in \mathbb{C}^{N_r \times 1}$ is the noise vector containing noise samples from each receiving antenna with every dimension being a zero mean circularly symmetrical complex Gaussian (ZMC-SCG) random variable with average power $N/2$ per dimension i.e. $n \sim \mathcal{C}_c(0_{N_r}, N, I_{N_r})$, where $I_{N_r}$ and $0_{N_r}$ represent an identity matrix and an all-zero matrix respectively both with dimensions $N_r \times N_r$. In what follows, the achievable communication rate through this channel when O-STBC is employed at the transmitter is considered, taking into consideration the different channel realisations of $H$ similar to $\lambda$ in Section 3.3.
3.4.2 Fixed Channel Coefficient

If the matrix $H$ is fixed, then the channel capacity for this MIMO system due to [82] and [39] is more rigorously derived by Telatar in [3]. A lot of literature on its derivation is now available, the interested reader can find details in many books on wireless communications such as [2], [20] and [28]. For this reason only important results are summarised here.

The capacity of the system expressed by equation (3.4.3) when $x \sim \mathcal{N}_C(0, N)$, the codebook covariance matrix $Q = E\{xx^H\}$ and $tr[Q] = S$ is given in [3] as

$$C = \log_2 \det \left( I_{N_t} + \frac{HQH^H}{N} \right) \quad (\text{bits/s/Hz}) \quad (3.4.4)$$

where $\det(.)$ is the determinant operator. According to [3] $Q$ is diagonal. This implies that equation (3.4.4) can be rewritten as

$$C = \log_2 \det(I + HP^H) \quad (\text{bits/s/Hz}) \quad (3.4.5)$$

where $\Psi$ is a diagonal matrix containing fractions of the transmit power allocated to each transmitter, and as a reminder $\rho$, is the ratio of the total average transmit power $S$ from all antennas to the receiver noise power $N$

or

$$C = \sum_{i=1}^{N_t} \log_2(1 + \lambda_i \rho) \quad (\text{bits/s/Hz}) \quad (3.4.6)$$

where $\lambda_{i\in N_t}$ are the eigenvalues of $HH^H$ and $\epsilon_{i\in N_t}$ are the diagonal elements of $\Psi$ with each representing the fraction of the total power allocated to each transmit antenna i.e. $\Psi = diag(\epsilon_1, \epsilon_2, ..., \epsilon_{N_t})$, the
allocation of which depends on the presence of the channel state information (CSI) at the transmitter. The well known approach as reported in the literature that optimally distributes $\epsilon_i \in N_t$ when the transmitter has the full knowledge of the CSI is the water-filling algorithm [76], otherwise an equal proportion of the total power is allocated to each transmit antenna. Using the water filling algorithm, the optimum value of each $\epsilon_i$ is determined through Lagrangian optimisation when $S = 1$ to yield

$$\epsilon_i = (\mu - (\lambda_i \rho)^{-1})^+ \quad i \in (1, N_t) \quad (3.4.7)$$

where $\mu$ is a constant that satisfies $\sum_{i=1}^{N_t} \epsilon_i = 1$ and $(\epsilon)^+$ implies

$$\begin{align*}
(\epsilon)^+ &= \begin{cases} 
\epsilon & \text{if } \epsilon \geq 0 \\
0 & \text{if } \epsilon < 0
\end{cases}
\end{align*}$$

this ensures that a non-negative power is allocated to each antenna. Thus, the capacity expression in equation (3.4.6) can then be simplified as [11]

$$C = \sum_{i=1}^{N_t} (\log_2(\mu \lambda_i \rho))^+ \quad (bits/s/Hz) \quad (3.4.8)$$

Explicitly, equation (3.4.7) can be interpreted to mean that if the channel gains of any of the $i \in N_t$ sub-channels results in a negative power being transmitted through it, the algorithm allocates a zero fraction of the total power to its respective antenna and hence no transmission takes place through the sub-channel, therefore, the transmission power is concentrated on the sub-channels that result in a non-zero allocation, thus an optimal usage of available power. This will enhance the resulting capacity in equation (3.4.8) which is a linear combination of the capacities of the used sub-channels. However, this capacity advan-
Section 3.4. Capacity of a MIMO Channel

Figure 3.3. A simplified multi-input multi-output (MIMO) transceiver model

tage disappears at high SNRs for $N_t = N_r$ [29]. It should be noted that an explicit estimation of $\mu$ is omitted here because the method of obtaining it is well established in the literature, interested readers can check [2] and [11] for details.

As mentioned above if the CSI is not available at the transmitter, but it is known perfectly at the receiver, then equal power allocated to all transmitting antennas remains optimal [29], thus, equation (3.4.6) can then be expressed as,

$$C = \sum_{i=1}^{N_t} \log_2 \left( 1 + \frac{\lambda_i \rho}{N_t} \right) \quad (\text{bits/s/Hz}) \quad (3.4.9)$$

Transmitting an O- STBC through this MIMO results in a capacity expression not only constrained by the number of transmit and receive antennas, but also the O- STBC transmission rate. The transceiver model in Figure 3.2 is redrawn explicitly in Figure 3.3 to show the space-time coding/decoding processes at the transmitter and the re-
It is assumed that the transmitted symbols resulted from signal to constellation mapping (not shown) are mapped by the space-time encoder into a block code with spatial and temporal dimensions. The space-time block code (STBC) generated is orthogonal and could be of different code rates similar to the ones discussed in the previous chapters. To remind the reader, the code rate $R$ of O-STBC is the ratio of the transmitted number of symbols that can be sent in one codeword to the number of durations used in transmitting the codeword. The received symbols are then decoded through the space-time decoder before being passed to the signal to constellation demapping stage (not shown).

Since the use of O-STBC reduces the MIMO channel into a single SISO channel with modified channel statistics [28], equation (3.4.4) can be written as

$$C = R \log_2 \left( 1 + \frac{||H||^2_F \rho}{RN_t} \right) \quad (bits/s/Hz) \quad (3.4.10)$$

where $\rho$ is modified to mean the ratio of average transmitted symbol power $S$ to the noise power $N$ at the receiver, and $||H||^2_F$ denotes squared Frobenius norm of $H$ given as

$$||H||^2_F = \sum_{i=1}^{N_t} \sum_{j=1}^{N_r} |h_{ij}|^2 = tr[H H^H] \quad (3.4.11)$$

### 3.4.3 Ergodic Fading Channel

In contrast to the previous section, the realisation of $H$ possesses the same characteristic of the channel described in Section 3.3.2. Therefore, the capacity is now obtained by averaging equation (3.4.4) over all
realisations of $\mathbf{H}$ for a capacity maximising codebook covariance matrix $\mathbf{Q}$, this is sometimes referred to as ergodic or mean capacity. Telatar in [3] then proved that for $\mathbf{x} \sim \mathcal{N}_c(\mathbf{0}_{N_t}, \mathbf{Q})$, this ergodic capacity can be expressed for a given maximising codebook covariance matrix as

$$C = E_{\mathbf{H}} \left\{ \log_2 \det \left( \mathbf{I}_{N_r} + \mathbf{HH}^H \frac{\rho}{N_t} \right) \right\} \text{ (bits/s/Hz)} \quad (3.4.12)$$

If $n = \max(N_r, N_t)$ and $m = \min(N_r, N_t)$, the random matrix $\mathbf{HH}^H$ has a Wishart distribution with parameters $m, n$ and ordered eigenvalues with joint probability density function given in [83] as

$$p(\lambda_1, \ldots, \lambda_{N_t}) = \frac{1}{K_{m,n}} \prod_{i}^{m} \lambda_i^{n-m} e^{-\lambda_i} \prod_{i<j} (\lambda_i - \lambda_j)^2 \quad (3.4.13)$$

where $K_{m,n}$ is the normalizing factor. Any of the ordered eigenvalues have the distribution

$$p(\lambda) = \frac{1}{m} \sum_{k=0}^{m-1} \frac{K!}{(k + n - m)!} [L_{k}^{n-m}(\lambda)]^2 \lambda^{n-m} e^{-\lambda} \quad (3.4.14)$$

where $L_{k}^{n-m}(\lambda)$ is the associated Laguerre polynomial of order $k$ and it is given by

$$L_{k}^{n-m}(\lambda) = \sum_{l=0}^{k} (-1)^l \binom{k + n - m}{k - l} \binom{n - m + l}{l} \lambda^l \quad (3.4.15)$$

Then equation (3.4.12) can be expressed in-terms of the ordered eigen-
values as

\[
C = E_\lambda \{ \log_2 \text{det}(\mathbf{I}_{N_r} + \frac{\rho}{N_t} \text{diag}(\lambda_1, \ldots, \lambda_m)) \} \\
= E_\lambda \{ m \log_2 (1 + \frac{\rho}{N_t} \lambda) \} \\
= m E_\lambda \{ \log_2 (1 + \frac{\rho}{N_t} \lambda) \} \\
= \int_0^\infty \log_2 (1 + \frac{\rho}{N_t} \lambda) \sum_{k=0}^{m-1} \frac{K!}{(k + n - m)!} [L_k^{n-m}(\lambda)]^2 \lambda^{n-m} e^{-\lambda} d\lambda
\]

(3.4.16)

This is the landmark ergodic capacity expression derived by Telatar in [3] to which various explicit closed forms have been developed for simplification of its analysis and applications.

Similar to equation (3.4.10), the modified expression of equation (3.1.16) for the O-MIMO channel can be expressed as [11]

\[
C = E_\lambda \{ R \log_2 \left( 1 + \frac{\lambda \rho}{RN_t} \right) \} \\
= \int_0^\infty R \log_2 \left( 1 + \frac{\lambda \rho}{RN_t} \right) p(\lambda) d\lambda \quad \text{(bits/s/Hz)}
\]

(3.4.17)

where \( \lambda = \sum_{i=1}^q \lambda_i \) depends on the statistics of each sub-channel and \( q = N_t N_r \). Clearly \( p(\lambda) \) can be obtained through the \( q \) fold convolution with respect to the probability density functions of \( \lambda_i (p(\lambda_i)) \) i.e.

\[
p(\lambda) = p(\lambda_1) * p(\lambda_1) * p(\lambda_2) * \ldots * p(\lambda_q)
\]

(3.4.18)

where * denotes the convolution operation. Although equation (3.4.18) is analytically feasible, it has been proven it could be easily solved with the use of the moment generating function (MGF).
The MGF $\phi_\lambda(s)$ of $\lambda$ is defined as

$$
\phi_\lambda(s) \equiv \int_0^\infty p(\lambda)e^{s\lambda}d\lambda \quad (3.4.19)
$$

and when using this result equation (3.4.18) transforms into

$$
\phi_\lambda(s) = \prod_{i=1}^{q} \phi_{\lambda_i}(s) \quad (3.4.20)
$$

The probability density function of $\lambda$ is known obtained by performing inverse transformation which yields

$$
p(\lambda) = \frac{1}{2\pi j} \oint_{\sigma-j\infty}^{\sigma-\infty} \phi_\lambda(s)^{-s\lambda}ds \quad (3.4.21)
$$

where $j$ denotes the complex number $j = \sqrt{-1}$ and $\sigma$ is chosen in the region of convergence of the integral in the complex $s$ plane. Since the MGF is closely related to the Laplace transform, the operations in equations (3.4.19) and (3.4.21) are rarely performed because of the availability of large tables of either transforms e.g. [84].

With [17] and [11] equation (3.4.18) for a Rayleigh fading sub-channel can be expressed as

$$
p(\lambda) = \frac{1}{\Gamma(q)} \lambda^{q-1}e^{-\lambda} \quad (3.4.22)
$$

This probability density function has been shown in [4] to follow a $\chi^2$ distribution with $2q$ degrees of freedom and a mean of $q$ and $\Gamma(.)$ denotes an ordinary Gamma function. With this expression, equation (3.4.17) can be re-written as

$$
C = \frac{R}{\Gamma(q)} \int_0^\infty \log_2 \left(1 + \frac{\lambda \rho}{\beta N_0} \right) \lambda^{q-1}e^{-\lambda}d\lambda \quad \text{(bits/s/Hz)} \quad (3.4.23)
$$
The normalised capacity versus SNR based on this expression is shown in Figure 3.4 for different rate O-STBCs with performance compared with CL-QO-STBC. This figure will be discussed in more detail in Section 3.5.

### 3.4.4 Non-Ergodic Fading Channel

In the section, the analysis will assume a channel realisation chosen randomly according to a Rayleigh distribution at the beginning of transmission and kept constant over the codeword transmission, therefore, there is a non-zero probability that a given transmission rate $\Phi$ will not be supported by this channel [3], as mentioned earlier this probability is called the outage probability $P_{(out)}(\Phi)$. Thus the maximum mutual information is generally not equal to the channel capacity because it is not always achievable. The aim here is to express the achievable transmission rate over this channel based on the communication reliability $1 - P_{(out)}(\Phi)$.

To minimise the outage probability for a given channel, noise power, average codeword power and required transmission rate a suitable codeword $x$ must be choosing with covariance matrix $Q = E\{xx^H\}$ [3]. The outage probability can be expressed in terms of the desired rate as

$$P_{(out)}(\Phi) = \inf_{\text{tr}(Q) \leq S} \left\{ Pr\left[ \log_2 \det\left( I_{N_r} + \frac{HQH^H}{N} \right) \right] < \Phi \right\}$$

(3.4.24)

where $\inf\{\}$ and $Pr[.]$ denotes infimum and probability respectively. The choice of codewords $x$ which minimise the outage probability with given constraint $\text{tr}(Q) = S$ is not trivial and has not been solved in
Section 3.4. Capacity of a MIMO Channel

a closed form. However, Telatar in [3], provided a conjecture on the optimum form of $Q$. As he outlined, let's assume that $N_t$ transmit elements and $N_r$ receive elements are available. The conjecture states that the covariance $Q$ which minimises the outage probability of equation (3.4.24) has to be of the form

$$Q = \frac{S}{N_t} \begin{pmatrix} I_{N_t} & 0_{N_r-N_t} \\ 0_{N_r-N_t} & 0_{N_r-N_t} \end{pmatrix}$$

(3.4.25)

where $N_t = 1, ..., N_r$ is chosen such that (3.4.24) is minimised for a given rate $\Phi$ and SNR. That means that out of $N_r$ transmitters only $N_t$ are used. This allows equation (3.4.24) to be rewritten as

$$P_{\text{out}}(\Phi) = P_r \left[ \log_2 \det \left( I_{N_t} + \frac{HH^H \rho}{N_t} \right) < \Phi \right]$$

(3.4.26)

which, with reference to [3], can be expressed as

$$P_{\text{out}}(\Phi) = P_r \left[ \sum_{i=1}^{m} \log_2 \left( 1 + \frac{\lambda_i \rho}{N_t} \right) < \Phi \right]$$

(3.4.27)

This requires the calculation of an m-fold convolution of the probability density functions of $\log_2(1 + \frac{\lambda_i \rho}{N_t})$ generated by the randomness of $\lambda_i$ with pdf $p(\lambda_i)$ given by equation (3.4.14) similar to that of the ergodic channel. After some simplification and modifications [11], equation (3.4.27) can then be expressed as

$$P_{\text{out}}(\Phi) = \frac{1}{\Gamma(q)} \gamma \left( q, \frac{\rho}{R.N_t} \right)$$

(3.4.28)

where $\gamma(.,.)$ denotes the lower incomplete Gamma function.

In both equations (3.4.23) and (3.4.28) the case of SIMO and MISO
are found using the appropriate values of $N_t$ and $N_r$.

Figure 3.5 depicts the outage probability versus desired rate in (bits/s/Hz) for different rare O-STBCs and CL-QO-STBCs. Simulation studies are next undertaking to examine the expression described in equations (3.4.23) and (3.4.28).

### 3.5 Simulation Results

In complementary to previous research work on CL-QO-STBC, the results presented in this section, serve as a new generalised performance criteria for the CL-QO-STBC. Figure 3.4, shows the normalised ergodic capacity in bits/s/Hz taken from equation (3.4.23) for a multi-input single output (MISO) channels using different rate O-STBCs including the CL-QO-STBC versus SNR. Clearly, the CL-QO-STBC promises to deliver higher capacity over the ergodic channel. As can be observed the full-rate CL-QO-STBC outperforms the Alamouti-STBC by approximately 0.3dB or 0.2 bits/s/Hz and better than the SISO scheme by approximately 2dB or 0.5 bits/s/Hz. Also, the 3/4-rate schemes perform at a rate inferior to the full-rate schemes (SISO), A-STBC and CL-QO-STBC. This is as a result of the loss in transmission rate of these schemes. That means that if a transceiver deploys a channel code operating at the capacity limit, then the use of more than two transmit antennas only improves the gain for ergodic fading channels, provided the rate is maintained over all the transmit antennas. However, since the O-STBC reduces the MIMO channel to a scalar AWGN SISO channel with gain proportional to the Frobenius norm of the matrix channel, none of the capacities exceeds the capacity of a simple Gaussian SISO channel.
Figure 3.4. Normalised ergodic capacity for benchmark and space-time block coding transmission scheme versus SNR.

Although, the results only represent the case of a single receive antenna, the capacity gap between the O-MIMO channel and the equivalent Gaussian channel will vanish as the number of receive antenna increases [77]. However, the capacity of a Gaussian channel cannot be exceeded. Therefore, the simplicity of O-STBCs comes with a loss in capacity of the true MIMO channel because of it conversion to a scalar channel, hence limiting the ultimate system capacity.

Figure 3.5, demonstrates performance comparison of deploying a
Figure 3.5. Outage probability $P_{out}(\Phi)$ for benchmark and space-time block coding transmission scheme versus normalised transmission rate $\Phi$ for non-ergodic channel.

The Alamouti scheme, and other O-STBCs operating in a non-ergodic channel at an SNR of 10dB. The figure shows outage probability $P_{out}(\Phi)$ versus the desired rate $\Phi$. It can be observed that an increase in the number of transmit antennas does not always lower the outage probability. For instance, at a high rate of $\Phi = 5\text{bits/s/Hz}$, the SISO scheme provides 0.94 outage probability, that is, 6% communication reliability, the Alamouti scheme with 0.8 while other schemes including the CL-
QO-STBC provide outage probability of 1. However, at a lower rate $\Phi = 2\text{bits/s/Hz}$ the figures show that this rate is supported at 0.28 outage probability by a SISO scheme, the Alamouti scheme by 0.12 and the CL-QO-STBC by 0.2, that is a reliability of 98%. This means that the number of transmit antennas and the O-STBC employed at the transmitter should depend on the desired transmission rate.

An interesting behaviour of these curves is that they intersect at some point, which demonstrates the observation of Telatar’s conjecture in equation (3.4.25), that is, desired high rates with higher reliability should be supported by one transmit element, and desired low rates with high reliability by four elements.

As with the ergodic channel, simulation results show the case of a system with single receive antenna, a similar conclusion can be made for any number of receive antennas, with steeper outage curves when the number of received antennas increases.

It should be highlighted that the performance improvement of CL-QO-STBC is dependent upon the availability of channel state information (CSI) at the transmitter and this is assumed throughout the thesis.

3.6 Summary

This chapter has analysed capacities and associated outage probabilities of MIMO channels when O-STBCs are transmitted through it. Although the presented capacity limits can only be approached with transceivers of infinite complexity, an understanding of their behaviour is vital in designing optimum MIMO communication systems. The analyses presented will prove very useful in distributed-MIMO multi-
stage systems to be discussed in the next chapter.

To maintain a logical thread, from the introductory notes to the simulation results of O-MIMO channel capacities and associated outage probabilities, a brief introductory definition of capacity was first given, followed by Shannon's understanding and definition of capacity of an AWGN channels, this allowed for a proper understanding of the subject in later sections. Next, the fundamental difference between AWGN channel capacity and capacity of wireless channels was presented leading to fundamental differences between capacity and outage probability. Both concepts proved crucial for the remaining sections. In what follows, the traditional flat Rayleigh fading MIMO channel in its ergodic and non-ergodic realisation was dealt with from which the associated O-MIMO capacity and outage probability were presented. It was shown through simulation that O-STBCs are optimal with respect to ergodic capacity when the code is rate one and the capacity advantage grows linearly as a function of the number of transmit and receive antennas. In particular, the result shows the ergodic capacity advantage of CL-QO-STBC over the A-STBC (both rate one codes) and other lower rate codes. The CL-QO-STBC outperforms the A-STBC scheme by approximately 0.3dB or 0.2 bits/s/Hz and the 3/4-rate schemes perform inferior to both rate one schemes this is as a result of the loss in transmission rate of these schemes.

A different behaviour was experienced when this O-MIMO system operated over a non-ergodic channel. It was shown that an increase in the number of transmit antennas does not always lower the outage probability, thus deployment of CL-QO-STBC must be made taken into consideration the desired transmission rate. The simulation results
show that for a four antenna O-MIMO system deploying CL-QO-STBC, the reliability of transmitting lower rate information is higher than with higher transmission rate.

These two novel simulation results show the promised capacity and outage bound of O-MIMO channel when CL-QO-STBC are deployed at the transmitter, with perfect CSI.

In general the capacity advantage, offered by fixed, ergodic and non-ergodic MIMO channels could vanish if there exists correlated sub-channels between the transmitter and receiver and can be reduced if the relative distance between the transmitter and the receiver results in considerably high pathloss.

With further demand for increased coverage areas of mobile and other ad-hoc networks, coupled with minimal transceiver sizes, there is a need to develop strategies which preserve the theoretical capacity advantage offered by MIMO technology over a wider coverage area using transceivers of reasonable size and relatively low computational complexity. With these targets the next chapter exploits the advantage inherent to MIMO in the context of "cooperative" relay networks with particular interest in the deployment of CL-QO-STBC and the closed-loop extended orthogonal space time block code (CL-EO-STBC) presented in the previous chapter.
4.1 Introduction

Multiple antenna technologies have been shown to be a promising technology for future wireless communication systems operating in highly fading environments. With appropriate coding techniques over temporal and spatial dimensions the effect of fading can be minimised [5], [18], [21] and [20]. However, one of the limitations of using these coding schemes in traditional point-to-point multi-input multi-output (MIMO) systems is that they offer limited capacity, data throughput and can be expensive to realised due to the infrastructure required. These limitations in capacity and throughput are largely due to path loss, which causes the received signal power to decrease exponentially with distance between transmitter and receiver. Another constraint in the effective use of these coding schemes is the physical size limitation within mobile terminals and other wireless devices that prevents the successful deployment of multiple antennas which ensure existence of spatial uncorrelated channels between the source and destination terminal, a condition required to exploit fully the advantage of a multiple antenna
system. A more pragmatic approach to system design would be to allow multi-stage communication through the use of relays between the source and the destination terminals [13], this provides a flexible extension of the point-to-point system and a logarithmic increase in capacity as a function of numbers of relay stages [85].

The advantages inherent to relay networks could be further enhanced if closely spaced "idle" single antenna terminals (relay nodes) cooperate together to form a relaying stage thereby creating a virtual multi-input multi-output (MIMO) channel [11], [51] and [86]. In [11], the concept of virtual antenna arrays (VAAs) was demonstrated, where spatially distributed relays (producing relaying links which suffer uncorrelated fading paths) cooperate together to form a virtual MIMO channel that mimics the performance advantage of MIMO. This technique has been shown to provide diversity gain at the destination [14]. To fully exploit the cooperative diversity at the relay nodes, [11], [87] and [88] showed the applicability of the two antenna Alamouti code (ASTBC) as a distributed O-STBC thereby producing a diversity order of 2 for a two single antenna cooperating relay nodes. In [89] a D-STBC operating in an amplify-and-forward (AF) mode was analysed through the derivation of pairwise error probability (PEP) expressions. It was shown that the original design criteria for conventional STBC (i.e. rank and determinant criteria) still apply for the design of D-STBC schemes under the assumption that appropriate power control rules are employed at relays.

To increase the diversity order within the network, effort has been made to perform distributed coding over four antenna relaying system, [55] showed the application of a quasi-orthogonal space-time block code
Section 4.1. Introduction

(QO-STBC) with constellation rotation [32] over two relay nodes each with two antennas. Although in their work, a diversity order of four was achieved, it is very likely that as with MIMO, there is likelihood that there will be correlation between adjacent antennas at the same relay node, hence degrading the performance of this system in-terms of its loss in diversity. In a similar manner, [56] and [57] deploy similar coding techniques over relay nodes having one and two antennas each, but because of lack of cooperation between the relays, their scheme comes with additional processing complexity.

However, this chapter is aimed at exploiting the maximum relay-destination diversity when four single antenna relays cooperate together with a limited amount of total network power, similar to distributed A-STBC previously investigated. The virtual antenna array concept is exploited in this thesis to investigate the use of closed loop quasi-orthogonal space-time block codes (CL-QO-STBCs), over four antenna cooperative relay nodes. In contrast to the work presented in [55], the spacing between the four antennas ensures spatially uncorrelated channels. Additionally, unlike [56] and [57], the virtual antenna array approach provides simplified decoding operation at the destination, due to the local communication between the relay nodes.

In general, one of the problems facing relay assisted networks generally is the optimum allocation of the available network resources, in particular the transmission power among all the transmitting elements within the network. This has resulted in many suggested approaches with different performance measuring indices used in evaluating these suggestions. Notable to mention is the work presented in [60], [11], [61] and [90]. Out of all these, the virtual antenna array based resource al-
location strategies presented in [11] are employed here because of simplicity in deployment over spatially distinct relay nodes transmitting distributed space-time codes. Of particular interest is the comparison of the deployment of distributed A-STBC, CL-QO-STBC and CL-EO-STBC schemes previously deployed in the context of point-to-point communications in a wireless relay network whose relaying nodes have single antenna each.

The remaining parts of this chapter are structured as follows: Section 4.1 presents the system model of the cooperative system analysed in the chapter, followed by the end-to-end capacity and throughput evaluation of the system in Section 4.2. The analysis is split into the cases of ergodic and non-ergodic flat fading channels. The resource allocation strategy presented in [11] (previously demonstrated for a limited number of relay elements and distributed A-STBC) is extended for a higher number of elements and other distributed O-STBCs including the distributed CL-QO-STBC and are also employed in this analysis. Simulation results that show these end-to-end capacities and throughput bounds are presented in Section 4.3, these will serve as theoretical measures expected when distributed CL-QO-STBC are deployed in the context of virtual antenna array as against the distributed A-STBC. In Section 4.4 analyses are performs in-terms of end-to end BER which is a realistic measure of the performance expected from the cooperative network. Section 4.5 shows simulated end-to-end BER and throughput. Comparisons are made between CL-QO-STBC and CL-EO-STBC in the context of distributed relaying system and it is confirmed that their relative performance in point-to-point system is retained with distributed relay networks.
Finally, the chapter is concluded with a summary.

Note that in all the simulation results presented, the term optimised means resources are allocated optimally between relaying stages while non-optimised denotes a situation when resources are equally shared among the stages independent of the network topology, channel conditions and processing schemes.

### 4.1.1 System Model

In Figure 4.1, a single source and destination communicating via cooperating relay nodes is depicted. It is assumed that there is no direct path between the source and the destination, all participating nodes communicate using a single antenna configuration over narrowband flat-fading channels $h_{i,j}$, where $i \in s, d$, represent the channel from the source or destination to a particular relay node $j \in 1, 2, \ldots, N_t$ up to the maximum number of cooperating relays $N_t$. All random channel parameters $h_{i,j}$ are assumed to be zero mean circular symmetric complex Gaussian (ZMCSCG) random variables with unity variance. It is also assumed that there is full cooperation among transmitting nodes at the relaying stage, and the air interface between each node within the relaying stage is distinct from the interface used for inter-stage communication and are error free due to relatively short communication distance. Because the original signals need two stages before arriving at the destination in these system models, the model can be named a typical two-stages cooperative system.

Two indices are used here to demonstrate the performance of this network when O-STBC are deployed at the relaying stage. First, a theoretical achievable end-to-end capacity gain is presented assuming
FDMA access methodology is utilised within the network, and secondly the end-to-end BER and throughput performance are presented with the TDMA access methodology employed within the network.

In both cases, the indices suggest similar performance of the networks presented which confirms that the access method used in a relay network does not affect its capacity gain [11] and thus its potential advantages. Analogous to the point-to-point MIMO the capacity behaviour depends on the number of cooperating relaying, the channel conditions and the STBC employed.

In what follows the theoretical end-to-end capacity and throughput gains of the network are presented followed by Monte-Carlo based end-to-end BER performance and throughput evaluations for different
channel realisations.

4.2 Capacity and Throughput Evaluation

In this section, the maximum achievable capacity limit for the scenario of Figure 4.1 is investigated for various numbers of cooperating relay nodes, over both ergodic and non-ergodic channels.

4.2.1 Ergodic Channel

In this section the channel between the source and the relays and from the relays to the destination are assumed to be ergodic and the system is constrained in its total power $S$ and bandwidth $W$.

Since the notion of Shannon capacity relates to error-free communication, therefore for any relaying network, if certain capacity is to be provided from source to destination, then all the stages involved must guarantee error-free communication. Hence, the end-to-end capacity of the network will be dictated by the capacity of the weakest stage [91]. Looking at Figure 4.1, with each stage transmitting equal amount of power, it can be seen that the capacity of an individual stage are unequal, this is due to the difference in the transmitting power of the source and each relay resulting in different SNRs at the relay and the receiver. Therefore, to maximise the end-to-end capacity, it is necessary to maximise the minimum of the capacities. One way to do this is by optimally distributing the total available power and bandwidth among the two stages [11]. From equation (3.4.17) the capacity of a single $i^{th}$ stage can be expressed in term of its fractional allocated power and bandwidth
Section 4.2. Capacity and Throughput Evaluation

\[ C_i = R_i a_i \cdot \log_2 \left( 1 + \frac{1}{R_i N_i} \frac{\lambda_i}{\alpha_i} S \right) \quad \text{(bits/s)} \quad (4.2.1) \]

where \( \alpha_i, i \in 1...K \) is the fraction of the total bandwidth allocated to the \( i^{th} \) stage, \( K \) is the number of stages, and \( \beta_i \) is the fractional power allocated to the \( i^{th} \) stage with \( R_i \) denoting the O-STBC rate deployed. Note that \( \rho \) is explicitly expressed here to define the power allocation in-terms of the total transmit power \( S \), \( \beta_i S \) is the fraction of the total power allocated to the \( i^{th} \) stage.

With the approximation of \( \log_2(1 + x) \approx \sqrt{x} \) [11], the fractional resources \( \alpha_i \) and \( \beta_i \) can be decoupled from the expectation in equation (4.2.1), yielding

\[ C_i \approx \alpha_i \sqrt{\frac{\beta_i}{\alpha_i}} \sqrt{\frac{S}{N}} E\left\{ \sqrt{\frac{R_i}{N_i} \lambda_i} \right\} \quad \text{(bits/s)} \quad (4.2.2) \]

where \( E\left\{ \sqrt{\frac{R_i}{N_i} \lambda_i} \right\} \) is as defined in [11] as \( \sqrt{\frac{R_i}{N_i} \frac{\Gamma(q_i + \frac{3}{2})}{\Gamma(q_i)}} \) and \( q_i = N_{t,i} N_{r,i} \).

Finally, the ergodic capacity of the \( i^{th} \) stage can be expressed as

\[ C_i \approx \alpha_i \sqrt{\frac{\beta_i}{\alpha_i}} \sqrt{\frac{S}{N}} \sqrt{\frac{R_i}{N_i} \frac{\Gamma(q_i + \frac{3}{2})}{\Gamma(q_i)}} \quad \text{(bits/s)} \quad (4.2.3) \]

which can be re-written as

\[ C \approx \alpha_i \sqrt{\frac{\beta_i}{\alpha_i}} \sqrt{\frac{S}{N}} \Lambda(q_i, N_{t,i} R_i) \quad \text{(bits/s)} \quad (4.2.4) \]

After rigorous analysis in [11] the optimum ratio of \( \alpha_i \) and \( \beta_i \) which ensures perfect distribution of \( S \) and \( W \) is shown to be
\[ \frac{\beta_i}{\alpha_i} = K \frac{\left( \Lambda(q_j, N_{tj} R_j) \right)^{2/3}}{\sum_{k=1}^{K} \left( \Lambda(q_k, N_{ik} R_k) \right)^{2/3}} \]  

(4.2.5)

where \( \Lambda(q_j, N_{tj} R_j) = \sqrt{\frac{R_j}{N_{tj}}} \frac{1}{\Gamma(q_j)} \), \( j \) here denotes the adjacent stage.

The parameter \( \alpha_i \) is evaluated by equating equation (4.2.1) for all stages which yields

\[ \alpha_i = \frac{R_j \cdot E_{\lambda_j} \{ \log_2 \left( 1 + \frac{1}{R_j N_{tj}} \frac{\lambda_j}{\alpha_j} \frac{S}{N} \right) \}}{\sum_{k=1}^{K} R_k \cdot E_{\lambda_k} \{ \log_2 \left( 1 + \frac{1}{R_k N_{ik}} \frac{\lambda_k}{\alpha_k} \frac{S}{N} \right) \}} \]  

(4.2.6)

This intermediate result for bandwidth \( \alpha_i \) is then substituted into equation (4.2.5) to determine the optimum power allocation for the \( i^{th} \) stage. The above developed fractional bandwidth and power allocation schemes are employed in this thesis to demonstrate the advantage of optimum resource allocation on the end-to-end capacity of the relay network in Figure 4.6 when both A-STBC and CL-QO-STBC are employed at the relaying stages.

### 4.2.2 Non-ergodic Channel

As discussed earlier, the capacity behaviour of a non-ergodic channel is entirely characterized by the rate \( \Phi \) supported with probability \( 1 - P_{\text{out}}(\Phi) \). Defining the amount of information delivered error-free from source to destination as throughput \( \Theta \) for a given system, and noting that the notion of capacity defines error-free transmission, then maximising capacity will be the same as maximising the throughput through minimising the outage probability. Therefore, using \( \Theta \) alone will be appropriate to evaluate the maximum achievable rate of a non-ergodic channel. The normalised (spectral) throughput in bits/s/Hz of a non-ergodic channel can be expressed in-terms of the outage proba-
From equation (4.2.7) it can be said that there exists an optimum \( \Phi \) which maximises throughput and minimises the outage probability, however this proves intractable to determine with the outage expression in equation (3.4.27). Interestingly, in [11] the analysis is simplified by suggesting an approximation for equation (3.4.27) in the form of,

\[
P_{(\text{out})}(\Phi) = \frac{\gamma(q, \lambda)}{\Gamma(q)} \approx a \left[ (2^{\frac{S}{\lambda}} - 1) \left( \frac{S}{RN_t} \right) \right]^b
\]

where \( a \) and \( b \) are obtained numerically so as to minimise the mean error between the approximated and the exact value in the outage probability. The approximated outage probability can thus be expressed as

\[
P_{(\text{out})}(\Phi) \approx a \left[ (2^{\frac{S}{\lambda}} - 1) \left( \frac{S}{RN_t} \right) \right]^b
\]

this can be easily resolved for \( \Phi \) to yield

\[
\Phi = R \log_2 \left( 1 + b \frac{P_{(\text{out})}(\Phi)}{a} \frac{1}{RN_t} \frac{S}{N} \right)
\]

hence substituting equation (4.2.10) into (4.2.7) the normalised throughput can then be approximated as

\[
\Theta \approx (1 - P_{(\text{out})}(\Phi)). R \log_2 \left( 1 + b \frac{P_{(\text{out})}(\Phi)}{a} \frac{1}{RN_t} \frac{S}{N} \right)
\]

which after logarithmic approximation \( \log_2(1 + x) \approx \sqrt{x} \) again yields
Section 4.2. Capacity and Throughput Evaluation

\[
\Theta \approx (1 - P_{\text{out}}(\Phi)). R^{2b} \frac{P_{\text{out}}(\Phi)}{a} \sqrt{\frac{1}{R}} \sqrt{\frac{1}{N_t}} \sqrt{\frac{S}{N}} \tag{4.2.12}
\]

hence the throughput maximising outage probability can thus be obtained by differentiating equation (4.2.12) with respect to \( P_{\text{out}}(\Phi) \) and expressed in [11] as

\[
P_{\text{out}}(\Phi) \approx \frac{1}{1 + 2b} \tag{4.2.13}
\]

As with ergodic channels, end-to-end throughput in multi-stage communication through non-ergodic channels is also determined by the weakest stage, i.e. the stage with minimum throughput. Therefore to maximise end-to-end throughput, the throughput at each stage needs to be equalized and then maximised through fractional resource allocation.

Therefore, substituting equation (4.2.13) into (4.2.11) along with insertion of fractional resource allocation parameters \( \beta_i \) and \( \alpha_i \) and after simple manipulations, the throughput expression for the \( i^{th} \) stage is given in [11] as

\[
\Theta_i \approx \alpha_i R_i \left( \frac{2b}{1 + 2b} \right) . \log_2 \left( 1 + \left( \frac{1}{a(1 + 2b)} \right) \frac{1}{\alpha_i N_t} \sqrt{\frac{S}{N}} \right) \tag{4.2.14}
\]

which could be approximately expressed in term of the logarithmic approximation as

\[
\Theta_i \approx \alpha_i \sqrt{\frac{\beta_i}{\alpha_i}} \sqrt{\frac{1}{N_t}} \sqrt{\frac{S}{N}} \sqrt{R_i} \left( \frac{2b}{1 + 2b} \right) \left( \frac{1}{a(1 + 2b)} \right)^{\frac{1}{2}} \tag{4.2.15}
\]

which is similar to equation (4.2.4) and enables the adoption of fractional resource allocation strategies for ergodic channels to be used. As
with an ergodic channel the aim here is to estimate the end-to-end normalised throughput for the network in Figure 4.1 when both A-STBC and CL-QO-STBC are employed at the relaying stages.

The simulation results shown in the next section demonstrate the achievable normalised end-to-end capacity and throughput of O-MIMO channels using the distributed A-STBC and the CL-QO-STBC schemes over ergodic and non-ergodic channel versus the SNR. Also shown is the effect of optimum interstage power allocation on these capacities and throughput.

### 4.3 Simulation Results

Figure 4.2 show the normalised end-to-end ergodic capacity versus SNR when two/four relay nodes cooperate at the relaying stage. With four cooperating relays, the CL-QO-STBC scheme provides higher end-to-end capacity over the A-STBC scheme in ergodic channels independent of SNR at the receivers. Assuming an operating SNR of 10dB at all receiving nodes, the distributed CL-QO-STBC scheme delivers an normalised end-to-end capacity of approximately $1.7 \text{bits/s/Hz}$ as against $1.6 \text{bits/s/Hz}$ obtained with the use of distributed A-STBC. Notice that difference in the capacities of these two schemes widens as SNR increased, this shows that this new deployment of the distributed CL-QO-STBC in the context of virtual antenna array (single antenna relays), promises to deliver a higher capacity gain over the distributed A-STBC with increase in SNR for non-optimised case.

However, the figure also shows that with implementing fractional resource allocation (optimised case), the distributed CL-QO-STBC scheme delivers a 20% capacity improvement over A-STBC case ($2.1 \text{bits/s/Hz}$
Figure 4.2. Normalised end-to-end capacity versus SNR for a two stages ergodic channel configuration with optimum resource allocation against 1.9bits/s/Hz. The SISO result presented is just for illustration only to show the importance of cooperation among the relays.

Also for non-ergodic channels, the distributed CL-QO-STBC scheme, as illustrated in Figure 4.3, provides the higher end-to-end throughput as compared to the distributed A-STBC scheme over the entire SNR range. Again, assuming an operating SNR of 10dB the distributed CL-QO-STBC scheme delivers a throughput of approximately 1.15bits/s/Hz as against 0.9bits/s/Hz delivered by the distributed A-STBC scheme. Clearly this is 25% improvement in the end-to-end throughput for non-optimised case. Again when fractional resource al-
Section 4.3. Simulation Results

Figure 4.3. Normalised end-to-end throughput versus SNR for a two stages non-ergodic channel configuration with optimum resource allocation.

location (optimised case) is used the distributed CL-QO-STBC scheme still maintains its performance advantage over the distributed A-STBC scheme. In both cases it can be seen that the end-to-end throughput gap between distributed CL-QO-STBC and A-STBC schemes increases as SNR increased.

Therefore, for both ergodic and non-ergodic channels, using CL-QO-STBC in a distributed single antenna relay (virtual antenna array) provides greater capacity and throughput advantage as compared to the conventional A-STBC schemes.

The lower rate O-STBC in Figure 4.3 shows the loss in performance
with reduction in the rate $R$ of the O-STBC, again this is for illustration only.

Although the end-to-end capacity and throughput illustrated in Figures 4.2 and 4.3 represent the capacity and throughput delivered when distributed A-STBC and CL-QO-STBC are employed at the relays, they represent an information-theoretic perspective to system evaluation and do not represent the practical behaviour of the cooperative system under consideration. Therefore in the next section a more practical measure, end-to-end BER, is employed in the analysis of Figure 4.1

### 4.4 Link Performance

The analysis so far has dealt with the theoretical capacity and throughput for a distributed MIMO multi-stage communication network taken into account certain communication scenarios i.e. whether the channel is ergodic or non-ergodic. In all these, it is implied that transceivers having infinite complexity are available to the system designer. This is highly impractical, therefore the capacity analysis presented from the time of Shannon are just a theoretically achievable transmission rate and does not represent an accurate metric to measure the performance of a realistic communication system.

Evaluating the performance of such a system realistically is done in terms of its error-rate versus SNR. The most commonly used error-rate metric includes the bit-error-rate (BER), symbol-error-rate (SER) and frame error-rate (FER). BER and SER are important when comparing various modulation or coding schemes, whereas modern packet based systems can be gauged by FER. Since evaluation of network perfor-
formance based of different coding schemes is of interest here, SER and BER evaluation will be employed in accessing the performance of the network presented in Figure 4.1.

For any practical system the BER decreases generally asymptotically with increased SNR, therefore, finding an equivalent SNR at which the system is error-free is not unique. Hence, to guarantee an error-free link, the SNR has to approach infinity (for the given additive Gaussian noise model). That has obviously little meaning for a system designer, which is the reason why finite complexity transceivers are said to yield a virtually error-free communication when the achieved error-rate falls below a certain threshold.

Signal processing at the transmitters and receivers has proven to be an important tool in reducing the error-rate of a system at the specified SNR, therefore, this section will use this tool in showing the realistic performance of this system with TDMA access methodology across the network.

### 4.4.1 System Model

From Figure 4.1, communication between the source and the destination is accomplished as follows. In the first time frame the source node broadcasts symbol $s_k$, $(s \in Z$ and $k \in N_t)$ to the relay nodes over a number of symbol intervals $N$ which depends on the numbers of cooperating relays or the size of the O-STBC coding matrix. Assuming perfect cooperation among relay nodes, i.e. perfect synchronization and error free inter-relay communication using no channel resources (bandwidth/time), and perfect backward CSI $^1$ at the relays.

$^1$Channel state information between the source and the relays.
The distributed channel vector from source to relay $h_{sr}$ can take the form,

$$ h_{sr} = [h_{s,1}h_{s,2}...h_{s,N_t}]^T $$  \hspace{1cm} (4.4.1)

which enables the received signal at each symbol interval in the presence of Gaussian noise denoted by $n$ (i.e. ZMCSGC random variables with variance $N_0$) to be compactly represented in vector form over all the relay terminals,

$$ y_{sr} = h_{sr}s_k + n $$  \hspace{1cm} (4.4.2)

This is simple maximal-ratio-combining (MRC) which is then followed by matched filtering

$$ z = h'^T y_{sr} = ||h_{sr}||^2 s_k + h'^T n $$  \hspace{1cm} (4.4.3)

and maximum-likelihood decoding, the optimum approach for the best distributed estimation $\hat{s}_k$ of the transmitted symbol,

$$ \hat{s}_k = \arg\min_{\bar{s}_k \in S} ||\bar{s}_k - z||^2 $$  \hspace{1cm} (4.4.4)

where $||.||^2$ denotes the Euclidean distance operator, $\hat{s}_k$ represents the symbol estimate restricted to a finite set of signal constellations $S$, e.g. QPSK, and is repeated $N$ times. In the next time-frame each relay node synchronously transmits the estimated symbols in the first time-frame according to a pre-allocated column of an O-STBC matrix, here the A-STBC is first considered in comparison with the new virtual antenna array approach in deploying CL-QO-STBC to confirm the advantages of using a four antenna system with a full-rate and full diversity code, next CL-QO-STBC is compared with the CL-EO-STBC both deployed
distributively in order to compare the performance of these two four antenna coding schemes. The comparison here is different from the one presented in [19] which shows the performance of these schemes over a MIMO point-to-point system. At this stage it is assumed that the relays know their respective forward CSI. This is possible through a feedback channel between the relays and the destination. Figure 4.4, gives a clearer picture of the processing that takes places at the relaying stage. All the relays receive the transmitted symbol from the source (not shown) through their respective antennas, then decodes it before passing it onto the cooperative transceiver which combines the decoded symbols from all the antennas at each symbol interval using the MRC scheme, then accumulates them over $N$ symbol intervals. With full cooperation, the symbols are re-encoded in a distributed manner by the distributed encoder (bases on the preferred coding scheme), synchronized and re-transmitted to the destination (not shown). Next the error analysis of Figure 4.1 is presented.

4.4.2 Symbol Error Rate (SER) and Bit Error Rate (BER) Analysis

At each relay stage, the instantaneous SNR $\rho$ per symbol at the detection stage is given in [28] as

$$\rho = \frac{1}{R} \frac{\lambda S}{N_t N} = \frac{\lambda E_b}{N_t N_0} = \log_2(M) \frac{\lambda E_b}{N_t N_0} \quad (4.4.5)$$

where $M$ is the constellation size, $S$ is the average transmit signal power, $N$ is the average received noise power, $R$ is the rate, $N_t$ is the

$^2$Channel state information between the relays and the destination.
Figure 4.4. Functional block of the relaying stage.
number of transmit antennas, $N_0$ is the average receivers noise power density, $E_s$ is the average transmitted symbol energy, $E_b$ is the average transmitted bit energy and $\lambda \equiv ||\mathbf{H}||^2$. Since the instantaneous SNR ($\rho$) is random, due to the randomness of the channel realisation $\lambda$, for any given modulation scheme e.g QPSK, this results in a probability of error conditioned on $\rho$ i.e. $P(\epsilon|\rho)$. Thus the symbol error rate (SER) of O-STBC is evaluated by averaging this conditional probability over the probability density function of the instantaneous SNR per symbol as

$$P(\epsilon) = \int_0^\infty P(\epsilon|\rho) \cdot pdf_\rho(\rho) d\rho \quad (4.4.6)$$

In [92] this integral is expressed in terms of the moment generating function (MGF) $\phi_\rho(s)$ of the instantaneous SNR expressed as

$$\phi_\rho(s) = \phi_{\frac{1}{H^H X}}(s) \quad (4.4.7)$$

Hence, for coherent MPSK the probability of error (the average symbol error rate SER) $P_s(\epsilon)$ was then shown to be [92]

$$P(s) = \frac{1}{\pi} \int_0^{\pi M-1} \phi_{\frac{G_{PSK}}{\sin^2 \theta}} d\theta \quad (4.4.8)$$

where $G_{PSK} = \sin^2(\pi/M)$. Similarly, the average SER for the coherent M-QAM is given as

$$P(s) = \frac{4q}{\pi} \int_0^\frac{\pi}{2} \phi_{\frac{G_{QAM}}{\sin^2 \theta}} d\theta - \frac{4q^2}{\pi} \int_0^\frac{\pi}{2} \phi_{\frac{G_{QAM}}{\sin^2 \theta}} d\theta \quad (4.4.9)$$

where $G_{QAM} = 3/2/(M - 1)$ and $q = 1 - 1/\sqrt{M}$. The equivalent bit error rate BER $P_b(\epsilon)$ can be expressed via [17] as
for low BERs and SERs.

The probability of error expressed in equations (4.4.8), (4.4.9) and (4.4.10) above estimates the average probability of a single stage in a relaying network, however, the overall error performance of a given relaying system i.e. the end-to-end BER, is defined as the error-rate in the received signal at the destination. A bit from the source is received correctly at the destination only when at all stages the bit has been transmitted correctly, therefore the end-to-end BER can be expressed as [11]

\[
PT(\epsilon) = 1 - \prod_{i=1}^{K} (1 - P_{bi}(\epsilon))
\] (4.4.11)

where \( K \) is the number of relay stages and \( P_{bi}(\epsilon) \) represents the BER of the \( i^{th} \) stage.

Application of all the above analysis is later shown using Monte-Carlo simulations to produce the predicted end-to-end probability of error of the system in Figure 4.1, using both distributed A-STBC and CL-QO-STBC schemes.

4.4.3 Throughput and Power Allocation

As mentioned earlier, throughput is a function of information delivered from source to destination; if it is assumed that the correctness of information is determined at the receiver, then to maximise the end-to-end throughput is the same as to minimise the end-to-end BER expressed in equation (4.4.11).

If the source transmits \( B \) bits, forming \( D \) symbols defined as \( D = \)
B/\log_2 M$, to the destination over $K$ number of relay stages, the normalised end-to-end throughput $\Theta$ can be expressed as [11]

$$\Theta = \min_{i \in \{1, K\}} \left( \alpha_i R_i \log_2(M_i) \right) \cdot [(1 - PT(e))B] \quad (4.4.12)$$

where $M_i$ is the modulating index of the $i^{th}$ stage, $\alpha_i$ is the optimised frame duration of the $i^{th}$ stage constraint such that $\sum_{i=1}^{K} \alpha_i = 1$ and $PT(e)$ is the end-to-end BER. The $\min$ signifies the dependence of throughput on the weakest stage.

The end-to-end throughput is maximised by minimising the end-to-end BER through optimally assigning fractional power to each relaying stage on the assumption that the fractional frame duration is optimally allocated.

The analysis presented in [11] to achieve this firstly simplifies SER in equations (4.4.8) and (4.4.9) by invoking their upper bounds which occurs when their largest argument $\theta = \pi/2$. Therefore the upper bound of BER of the $i^{th}$ relaying stage for M-PSK is

$$P(e) \leq \sum_{i=1}^{K} \frac{M_i - 1}{M_i \log_2(M_i)} \left( 1 + \beta_i \frac{GPSK_i \gamma_i S}{N_{ti} N} \right)^{-q} \quad (4.4.13)$$

and M-QAM is

$$P(e) \leq \sum_{i=1}^{K} \frac{2g_i - 1}{\log_2(M_i)} \left( 1 + \beta_i \frac{GQAM_i \gamma_i S}{N_{ti} N} \right)^{-q} \quad (4.4.14)$$

where $GPSK_i = \sin^2(\pi/M_i)$, $GQAM = 3/2/(M_i - 1)$ and $g_i = 1 - 1/\sqrt{M_i}$, $\gamma_i$ represents the average attenuation experienced at the $i^{th}$ stage and $N_{ti}$ is the number of transmit antennas of the $i^{th}$ stage.
where $q_{\text{max}} = \arg\max(q_1, \ldots, q_k)$ and $j$ denotes the adjacent stage. $A$ and $B$ are defined for M-PSK as

$$A_i = \frac{M_i - 1}{M_i \log_2 M_i}$$

$$B_i = \frac{G_{\text{PSK}} \gamma_i S}{R_i N_i N}$$

and M-QAM as

$$A_i = \frac{2q_i}{\log_2 M_i}$$

$$B_i = \frac{G_{\text{QAM}} \gamma_i S}{R_i N_i N}$$

Using different coding schemes, the enhancement in the performance of the network with the above resource allocation is next demonstrated.

### 4.5 Simulation Results

Figures 4.5 and 4.6 show the end-to-end BER performance of the proposed relaying systems in Figure 4.1 with and without power optimisation using QPSK constellation versus SNR. Although only one antenna at the receiver is considered, this technique can be extended to multiple receive antennas. In order to show the BER performance it is assumed that the source-relays and relay destination channels are both quasi-static flat fading channels.

From the figures, it can be seen that the end-to-end BER performance of the relaying system based on MRC and distributed A-STBC
Figure 4.5. End-to-end BER performance for an MRC, A-STBC and CL-QO-STBC 2-stage relay with equal power allocation, with QPSK transmission

performs worse when compared to the end-to-end BER performance of the relaying system based on MRC and distributed CL-QO-STBC for both optimum and non-optimum cases. In Figure 4.5 for example, an improvement of $7\,\text{dB}$ is achieved with BER of $10^{-3}$ when a relaying system exploits full rate, full diversity CL-QO-STBC. Likewise in Figure 4.6 a further improvement of $2\,\text{dB}$ is gained when power allocations to the two stages are optimised. Hence it can be inferred that deploying multiple antennas within a relaying system has the capacity to produce better performance which is further enhanced if power is optimally allocated between relaying stages. These properties hence
Section 4.5. Simulation Results

Figure 4.6. End-to-end BER performance for an MRC, A-STBC and CL-QO-STBC 2-stage relay with optimum power allocation.

have the capability to increase the throughput of the entire system.

Assuming 100bits are transmitted from the source, Figure 4.7 suggests that at SNR of 10dB, when attempting to transmit QPSK constellation of these bits over the available channel, both optimum and non optimum power allocation account for loss of over 80% of the total transmitted bits with the throughput of each symbol less that 0.4 with MRC and A-STBC based relaying system. This suggests a limitation in performance of this type of configuration (as also evidenced in the BER performance). However, in Figure 4.8, the full rate, full diversity CLQO-STBC yields a throughput of 1.0, approximately, for non-optimised and 1.4, for optimised power distribution at 10dB SNR. This confirms
Figure 4.7. End-to-end throughput performance for an MRC, A-STBC 2-stage relay with optimum and non-optimum power allocation
the advantage of using more antennas at the relaying stage with appropriate O-STBC. In addition, a further performance gain results from power optimisation. In the next set of results a comparison of two four antenna relaying system is shown employing different distributed closed-loop O-STBCs.

Figure 4.9 compares the use of CL-EO-STBC and CL-QO-STBC distributed coding scheme over the relay nodes for both optimised and non-optimised scenarios, from the figure it can be seen that the performance of the CL-EO-STBC based distributed coding is always better than the that of the distributed CL-QO-STBC, at any SNR. Figure 4.9 also confirms that the difference of the slope of the non-optimised and
Section 4.5. Simulation Results

**Figure 4.8.** End-to-end throughput performance for an MRC, CL-QO-STBC 2-stage relay with optimum and non-optimum power allocation.

Optimised curves of the two schemes are increasing as the total transmit power goes higher.

A further important issue for real-world applications is what happens to the overall performance of the optimum relay network based on distributed CL-EO-STBC scheme if the power control routine fails to operate properly. There will be only 0.5 dB loss in the performance of the scheme which still performs significantly better than the optimised CL-QO-STBC; therefore this shows the robustness in the distributed CL-EO-STBC scheme in the event of power control failure.
Figure 4.9. End-to-end BER performance for an MRC, CL-QO-STBC and CL-EO-STBC 2-stage relay with and without optimum power allocation.

4.6 Summary

In this chapter the potential advantages of using distributed O-STBC in a relaying system have been presented. These advantages in relaying systems are shown to be further enhanced by appropriate resource allocation between the stages, although with additional complexity. In particular, it was shown that with appropriate coding across relays coupled with increase in the numbers of cooperating relays, results in a significant performance improvement of the entire network. This is first shown in-terms of the end-to-end capacity and throughput achievable by implementing the distributed CL-QO-STBC scheme in relay
networks as compared to the distributed A-STBC scheme.

Next, the end-to-end BER comparisons for two and four antenna relaying systems with both the distributed A-STBC and CL-QO-STBC are shown, the performance clearly shows a 7dB gain when a four antenna relaying system is deployed as compared with the two antenna relaying system. This significant improvement in performance further suggests a huge amount of power saving of a four antenna relaying system over a two antenna system.

Having established the importance of four antenna relaying systems, the chapter then compares the performance of distributed CL-QO-STBC and CL-EO-STBC schemes. The results show the performance gain of deploying the CL-EO-STBC schemes as against the CL-QO-STBC over the relay nodes. This suggest again that in a cooperative relay network, increase in the number of cooperating relays should be accompanied with appropriate coding techniques to maximise the network performance.

However, in all cases considered, the effect of optimum power allocation between the two stages was investigated, it was found that optimal power allocation between the stages further increases the performance with the effect being more pronounced in the distributed CL-QO-STBC and CL-EO-STBC relaying.

Although generally, relay networks provide a promising solution to the high data rate coverage requirements that appear for beyond 3G mobile radio systems [93] and [94], their capacity advantages only increase logarithmical with the number of relaying stage [85]. With the higher data rate applications demanded of future wireless systems, there is the need to consider relay networks which will guarantee higher
Section 4.6. Summary

data rate transmission and deliver higher end-to-end capacity. To this end the next chapter will be focussing on analyzing a relay network with higher number of relaying stages operating within a broadband channel. Also to further improves the performance of such network an intra-stage power allocation is proposed.
Chapter 5

COLLABORATIVE
BROADBAND RELAY
NETWORKS

5.1 Introduction

Cooperative communication systems have recently gained much interest due to their ability to realize the performance gains of MIMO wireless systems, with distributed single antenna terminals forming virtual antenna arrays (VAA) [11] as presented in the previous chapter. These systems can be interpreted as a distributed multiple antenna transmission system that provides "cooperative diversity" also known as "user cooperation" [41], which is an effective means of improving spectral and power efficiency of wireless networks without the additional complexity of multiple antennas [14] [89]. In order to maximize the diversity advantage in cooperative systems the use of "conventional" orthogonal space-time block coding (STBC) has been suggested in a distributed fashion in particular in [14], [95] and [96], as a mechanism to implement user cooperation. Whilst [14] investigated the use of an Alamouti space-time block code (A-STBC) to achieve a diversity order of 2, [95]
was able to achieve fourth order diversity over a relaying system based on the VAA concept using the closed-loop quasi orthogonal space-time block code (CL-QO-STBC) and [96] shows the additional array gain achieved through the use of the closed-loop extended orthogonal space-time block code (CL-EO-STBC).

Applying these distributed STBCs over frequency-selective channels has been a challenging design problem because of the dispersive nature of such channels which causes intersymbol interference (ISI) leading to unavoidable performance degradation. Since all the works mentioned above and the ones in the previous chapter have considered that the relaying networks are formed over narrowband frequency non-selective fading channels (frequency flat) their applications are only generally appropriate for low data rate transmission.

However, for higher data rate applications over broadband frequency selective fading channels (multi-path channels), there becomes a clear need for research to focus attention on extending the advantages offered by multi-stage networks to future broadband systems.

Although orthogonal frequency division multiplexing (OFDM) has become the de-facto approach to broadband transmission over frequency selective fading channels, largely because of its higher spectral efficiency and its efficient realisation based on the fast Fourier transform (FFT) in wireless broadband systems, there are still problems of power saving for extra-high data rate transmission with quality of service (QoS) constraints and its vulnerability to radio propagation and synchronisation errors [97].

One way to overcome these problems is by employing a multi-stage communication technique, which takes advantage of relay nodes to ob-
tain additional diversity gain, and is the reason it is being considered as an important component for fourth generation (4G) systems [97].

There have been only a few results reported on broadband cooperative transmission techniques for frequency-selective channels. [98] investigates the performance of a distributed OFDM-STBC scheme through a simulation study considering both AF and decode-and-forward (DF) relaying, building upon their previous work on D-STBC in [99], [100] studied the performance of a relay-assisted uplink OFDM-STBC scheme and derived an expression for symbol error probability assuming decode-and-forward with no error propagation. In [101] an OFDM cooperative diversity system was studied assuming amplify-and-forward relaying and upper bounds on the channel capacity were derived. In [102], the performance of distributed OFDM-STBC in a single-relay scenario assuming a non-fading relay-to-destination link was studied. However, it is the aim of this chapter to present a new approach to the deployment of distributed A-STBC, CL-QO-STBC and CL-EO-STBC in decode-and-forward relay system over frequency selective broadband channels.

However, to exploit fully the benefit offered by multi-stage communication over broadband channels, effective use of both radio and power resources (especially in a constrained situation) has been identified as one major issue to be addressed. This will further minimise the end-to-end bit error rate (BER), and as such has been receiving considerable research attention lately. Some of the earlier works in this area are [91], [103] and [104], but as with the narrowband application, they have limited their analysis to a two stage network, which offers limited capacity [85] and coverage area as compared to networks with higher number of stages.
The work presented in this chapter is based on the VAA concept and provides a further attempt to effectively use these "scarce" network resources for an increased QoS performance and thereby further enhance the coverage area of the previously proposed network (without jeopardizing performance) with a logarithmic increase in capacity. The advantages of different orthogonal and quasi-orthogonal coding schemes at the relaying stages are demonstrated on the performance of the network and it is shown that with effective explicit power allocation over stages and frequency, a significant improvement in the end-to-end BER performance can be achieved.

A time division multiple access (TDMA) technique is employed, where the total frame duration is divided optimally among all the relay stages, and for a high data rate application with better throughput the relays at each stage decode the received data to remove the effect of noise before forwarding to the next stage. A fixed power budget is also assumed for the entire network with a quasi-static channel fading characteristic, (a frequency selective slow fading channel).

The rest of this chapter is organized as follows. In Section 5.2 the MIMO-OFDM multi-stage system model and the general structure for rate one full diversity space frequency orthogonal, quasi-orthogonal and extended orthogonal codes are described. Section 5.3 discuss various power allocation strategies in sufficient depth, presenting three categories (1) Equal power equal sub-carrier gains (2) Un-equal power equal sub-carrier gains (3) Un-equal power un-equal sub-carrier gains. The end-to-end BER to which the power allocation strategies is applied is discussed in Section 5.4 while Section 5.5 shows the achievable end-to-end capacity of the network with different sub-carrier power allocations.
Section 5.2. System Model

Figure 5.1. Simple three-stage collaborative broadband relay architecture

Simulation results which show the end-to-end BER and capacity advantage of the proposed optimum allocation are shown in Section 5.6. Section 5.7 discusses the adaptive coding technique which modifies the coding pattern across relaying stages based on the prevailing channel conditions. Additional simulation results which demonstrate the use of this technique are shown in Section 5.8 wherein the proposed optimum power allocation strategy is applied. Final conclusions are drawn in Section 5.9.

5.2 System Model

The analysis contained in this chapter is based on a three-stages collaborative broadband relay network architecture shown in Figure 5.1. With three-stage transmission protocol, the communication between the source (S) and the destination (D) is achieved through two sets of relaying stages RS₁ and RS₂, communication between S and RS₁ is termed relay stage 1, between RS₁ and RS₂ is referred to as relay stage 2 and finally between RS₂ and D is the relay stage 3, hence the use of word three-stage protocol. Note that the word relay stage and stage will
be used interchangeably in the rest of this chapter. Each relaying stage is made up of \( k \) single antenna relaying elements where \( k \) is a positive integer but not equal to 0 (\( k \in \mathbb{Z}^+ \)). The source and the destination are each also made up of a single antenna. The nodes participating in the entire three-stage communication are denoted by each black spot in Figure 5.1. As mentioned earlier, a quasi-static frequency selective Rayleigh fading channel is assumed at each stage and the channel from the \( S \) to \( RS_1 \) is denoted as \( H_{sr} \), that from \( RS_1 \) to \( RS_2 \) as \( H_{rr} \) and from \( RS_1 \) to \( D \) as \( H_{rd} \). These channels are exactly known by the receivers of each stage but the transmitter may or may not have knowledge of the full channel state information (CSI). There is no direct communication between the source and destination as the link is highly unreliable and will not ensure meaningful information exchange, hence a multistage approach with shorter sub-links is adopted.

Let \( x_i^j \) denote the data stream transmitted within the \( i^{th} \) stage from the \( j^{th} \) relay (which would be 1 in the first stage and 1, \ldots, \( k \) in the second and third stages), transmitted over a frequency selective fading channel having \( L \) independent channel taps, where \( x_i^j = [x_i^j(0)x_i^j(1)\ldots x_i^j(N - 1)]^T \), where the \( n^{th} \) element of the vector is the \( n^{th} \) data symbol and indeed the sub-carrier index, \((.)^T\) is a transpose operator and \( N \) is the frame length.

The complex channel transfer functions of the subcarrier \( n \) from the source \( S \) to the \( j^{th} \) relay of \( RS_1 \) are denoted as \( H_j^s(n) \), from the \( m^{th} \) relay within \( RS_1 \) to the \( j^{th} \) relay within \( RS_2 \) as \( H_{mj}^2(n) \) and that of the \( m^{th} \) relay within \( RS_2 \) to the destination as \( H_j^d(n) \), note that within each stage, \( m \) is used to index the transmitting antennas whilst \( j \) indexes the receiving antennas. With these notations, \( H_j^s(n) = \sum_{l=0}^{L-1} h_j^s(l)e^{-j2\pi ln/N} \),
\[ H_{m_j}^2(n) = \sum_{l=0}^{L-1} h_{m_j}^2(l)e^{-j2\pi ln/N} \] and \[ H_{m_j}^3(n) = \sum_{l=0}^{L-1} h_{m_j}^3(l)e^{-j2\pi ln/N} , \]
where \( h_1^j, h_{m_j}^2 \) and \( h_3^j \) are their corresponding channel impulse response coefficients. The receive signal on each sub-carrier on the \( j^{th} \) receiving antenna of the \( i^{th} \) stage can be expressed as \( y_{ij}(n) = x_i^m(n)h_i^m(n) + z_{ij}(n) \), where \( h_i^m(n) = [H_i^1(n)H_i^2(n)\ldots H_i^{N_{ri}}(n)]^T \) with \( N_{ri} \) representing the total number of transmitting relays of the \( i^{th} \) stage and \( z_{ij}(n) \) is the received noise of the \( j^{th} \) receiver, which is assumed to be an independent and identically distributed (i.i.d.), zero mean and unit variance complex Gaussian noise variable at the receive antenna, and
\[ x_i^m(n) = [x_i^1(n)x_i^2(n)\ldots x_i^{N_{si}}(n)]. \]

The received signals \( y_{ij}(n) \) at the first stage (when \( i = 1 \)) are combined using a maximum ratio combiner (MRC) and distributively re-encoded (assuming full cooperation among the relays) depending on the cooperating relays and the preferred coding methods using either Alamouti space frequency block codes (A-SFBC), quasi-orthogonal space frequency block codes (QO-SFBC) and extended orthogonal space frequency block codes (EO-SFBC). The same encoding is also performed at the second relaying stage before final transmission to the destination. Note, space-frequency encoding is used to give maximum robustness to fading channels [2]

Assuming that the channel impulse response remains constant within a frame and changes from frame-to-frame (quasi static) the input-output relationship of stages two and three will be presented in the next section.
5.2.1 Space Frequency Orthogonal, Quasi-Orthogonal and Extended-Orthogonal Block Codes

In [105] an example of coding over a frequency selective channel is presented based on the Alamouti space time block code (A-STBC) where the time index is replaced by OFDM sub-carrier index. On the assumption that the channel remains constant over two consecutive sub-carriers \( n_0 \) and \( n_1 \) i.e. \( H(n_0) = H(n_1) \) (dropping the stage indices) then the receiver can detect the received symbols over two sub-carriers and the coding matrix can take the form

\[
X_{2x2} = \begin{bmatrix}
    x_1(n_0) & x_2(n_0) \\
    -x_2^*(n_1) & x_1^*(n_1)
\end{bmatrix}
\]  \hspace{1cm} (5.2.1)

where \( x_i(n), i = 1, 2 \) represent the space frequency symbols and \((.)^*\) denotes complex conjugate. Dropping the channel index \( n \) and the row and column sizes for simplicity, this codeword is proportional to a unitary matrix such that

\[
XX^H \propto \left( \sum_{i=1}^{s} |x_i|^2 \right) I_s
\]  \hspace{1cm} (5.2.2)

where \( s \) is the number of symbols contained in the codeword, two for A-SFBC, \(|.|\) is a magnitude operator and \( I \) is an identity matrix (as in equation (2.2.1)). This property ensures the absence of intersymbol interference at the receiver and transforms the decoding process into a linear symbol-wise operation. This code is rate one and provides maximum possible diversity over two transmit and \( N_r \) receive antennas because it satisfies the rank criterion as shown in [29].

However, the earlier time dimensional extension of equation (5.2.1)
presented in [20] and [23] classed as quasi-orthogonal space time block code (QO-STBC) and extended orthogonal space time block code (EO-STBC), which integrates beamforming within the code, respectively suffers diversity loss with full rate transmission over four transmit and \( N_r \) received antennas. Replacing the time index with OFDM subcarrier index and expressing these classes of codes in terms of spatial and frequency dimensions [106] [107], they can be referred to as quasi-orthogonal space frequency block code (QO-SFBC) and extended orthogonal space frequency block code (EO-SFBC) respectively. The codeword matrix for QO-SFBC is presented as

\[
X_{4 \times 4} = \begin{bmatrix}
  x_1(n_0) & x_2(n_0) & x_3(n_0) & x_4(n_0) \\
  -x_4^*(n_1) & x_1^*(n_1) & -x_2^*(n_1) & x_3^*(n_1) \\
  -x_3^*(n_2) & -x_4^*(n_2) & x_1^*(n_2) & x_2^*(n_2) \\
  x_4(n_3) & -x_3(n_3) & -x_2(n_3) & x_1(n_3)
\end{bmatrix} \tag{5.2.3}
\]

and for EO-SFBC as

\[
X_{2 \times 4} = \begin{bmatrix}
  x_1(n_0) & x_1(n_0) & x_2(n_0) & x_2(n_0) \\
  -x_2^*(n_1) & -x_1^*(n_1) & x_1^*(n_1) & x_2^*(n_1)
\end{bmatrix} \tag{5.2.4}
\]

where \( n_0, n_1, n_2 \) and \( n_3 \) denote four adjacent channel frequency indices.

These codeword matrices (whether with time or frequency index) do not satisfy equation (5.2.2), yielding coupling between the estimated symbols at the receiver and hence making decoding more complex, e.g. maximum likelihood decoding is pair-wise for QO-SFBC, as a result, there is a loss in diversity order offered by these coding schemes.

An attempt to eliminate or reduce these coupling factors was presented in [18] and [19], using a feedback approach, wherein they pre-
multiply the transmitted symbols from two antennas with a phase angle feedback from the receiver.

Similar to A-SFBC if the channel remains constant over two and four consecutive frequencies for EO-SFBC \( [H(n_0) = H(n_1)] \) and QO-SFBC \( [H(n_0) = H(n_1) = H(n_2) = H(n_3)] \), then the receiver can detect the received symbols over two/four sub-carriers and the feedback approaches developed in [18] and [19] to orthogonalize the resulting channel matrix can be exploited, although the level of feedback information can be high but the nature of the channel can be exploited to reduce this [19]. This collapses the resulting vector detection problem into a scalar detection problem and the optimum maximum likelihood detection technique is employed to extract the transmitted signal.

Using the above coding schemes the received signal at the second and the third stage on the two/four adjacent sub-carriers, at the \( j^{th} \) received antenna can be expressed as

\[
y_i^j = X_i^m h_i^m + z_i^j, i \in (2, 3)
\]  

(5.2.5)

and dropping the stage and receiver indices in the vector elements, \( y_i^j = [y(n_0)y(n_1)\ldots y(N_{f-1})]^T \), and \( z_i^j(n) = [z(n_0)z(n_1)\ldots z(N_{f-1})]^T \), where \( N_f \) is the number of sub-carriers in one block.

The vector \( h_i^m = [H^{1}(n)H^{2}(n)\ldots H^{N_f}(n)]^T \) since \( H(n) \) is quasi static over \( N_f \) sub-carriers and \( X_i^m \) is the corresponding codeword matrix in equations (5.2.1), (5.2.3) or (5.2.4) for (A-SFBC), QO-SFBC and EO-SFBC.

Note in this work the additional complexity of the grouping scheme proposed in [1] to maximally exploit both the spatial and temporal diversity of the MIMO-frequency selective channels is not included. In-
 Instead it was assumed that in a practical realisation of this scheme, convolution encoding and interleaving could be added, hence all simulations are be presented in the uncoded sense.

5.3 Optimum Power allocation Strategy

In this section, optimum power allocation within the entire network is investigated. Given a fixed power budget, an optimal inter-stage power allocation scheme presented in [11], which minimises the end-to-end bit error rate (BER) with flat fading inter-stage channels is extended to frequency selective Rayleigh fading channels. Since OFDM is a multi-carrier transmission technique, to further improve the performance of the system, an optimal intra-stage power allocation strategy over each sub-carrier is formulated in addition to the aforementioned inter-stage allocation presented in this section. It is worth noting that at this point as in the rest of the thesis, it is assumed that the channel at each stage is known to the transmitter at that stage, which is feasible in a time division duplex (TDD) system where the up and down-link channel can be assumed identical [108].

5.3.1 Equal Power Equal Sub-carrier Gains

If the total power available to transmit information from the source to destination is $P$, equal power is first allocated to each relaying stage such that each $P_i$ which is the power allocated to the $i^{th}$ stage can be expressed as

$$P_i = \frac{P}{K} \quad (5.3.1)$$
where $K$ is the number of relaying stages.

With $N_{ti}$ transmit antennas, the power allocated to each transmitting antennas of the $i$\textsuperscript{th} stage is expressed as

$$P_i[t] = \frac{P_i}{N_{ti}}, t \in (1, \ldots, N_{ti}) \quad (5.3.2)$$

where $N_{ti}$ depends on the number of cooperating transmit relay elements (where here it is either 2 or 4).

For OFDM transmission, let $\{P_{t,j}(n)\}_{n=0}^{N-1}$ and $\{\lambda_{t,j}(n)\}_{n=0}^{N-1}$ denote the transmitted power in each sub-carrier and their corresponding channel gain squared from transmitting antenna $t$ to the receiver $j$ of the $i$\textsuperscript{th} stage respectively. Hence the transmit power allocated to the $n$\textsuperscript{th} sub-carrier between the transmit antenna $t$ and any receive antenna $j$ can be expressed as

$$P_{t}(n) = \frac{P_i[t]}{N} \quad (5.3.3)$$

where $N$ is the total number of sub-carriers.

### 5.3.2 Un-equal Power Equal Sub-carrier Gains

If the total power required to deliver information from source to destination is constrained to $P$, this will be distributed among the relay stages such that

$$P = \sum_{i=1}^{K} P_i \quad (5.3.4)$$

An optimum $P_i$ that maximizes the capacity and minimizes the end-to-end bit error rate (BER) can be expressed from [11] as

$$P_i = \left[ \sum_{j=1}^{K} \alpha_j \left( \frac{q_i^{-1} A_i^{-1} B_i^q}{q_j^{-1} A_j^{-1} B_j^q} \right)^{-\frac{1}{\max + 1}} \right]^{-1} \quad (5.3.5)$$
where \( q_i = N_{ti}N_{ri} \) which is the product of the number of transmit and receive antennas of the \( i^{th} \) stage, \( q_{\text{max}} = \arg \max(q_1, \ldots, q_K) \) (note Equation (5.3.5) and (4.4.15) are the same). The parameter \( \alpha_j \) is the fractional frame duration which for the TDMA system considered satisfies \( \sum_{i=1}^{K} \alpha_i = 1 \). The \( i^{th} \) stage fractional frame duration can be expressed as

\[
\alpha_i = \frac{\prod_{j=1, j \neq i}^{K} R_j \log_2(M_j)}{\sum_{k=1}^{K} \prod_{j=1, j \neq k}^{K} R_j \log_2(M_j)}
\]  

(5.3.6)

with \( A_i \) and \( B_i \) expressed as

\[
A_i = \frac{2q_i}{\log_2(M_i)}
\]  

(5.3.7)

\[
B_i = \frac{GQAM_i \gamma_i S}{R_i \frac{N_{ti}}{N}}
\]  

(5.3.8)

where \( GQAM = 3/2/(M_i - 1) \), \( \gamma_i \) is the total channel gains of the \( i^{th} \) stage, \( q_i = 1 - 1/\sqrt{M_i} \), \( R_i \) is the code rate of the \( i^{th} \) stage while \( M_i \) is the modulating index and \( \frac{S}{N} \) is the \( i^{th} \) stage receiver signal to noise ratio. This power is allocated equally among each transmitting antenna of each stage which is further distributed equally among each sub-carrier from the \( j^{th} \) transmitting antenna, in accordance with equation (5.3.3).

### 5.3.3 Un-equal Power Un-equal Sub-carrier Gains

As shown in [11], the optimum power allocation strategy that maximises the capacity and minimises the end-to-end bit error rate (BER) is one that distributes the power based on the architecture of each stage such that the worst stage (which determines the performance of the entire network) obtains the greatest percentage of the total power to improve its performance and hence the entire network, therefore, \( P_i \)
obey equation (5.3.5).

Also as noted in [39], the well known Shannon's water filling algorithm is the optimal power allocation method for an orthogonal carrier system, but its capacity advantage disappears at larger signal-to-noise ratios (SNRs). Hence an explicit approximation of this algorithm to improve the average BER performance of the network over a larger range of SNR is proposed herein. The performance of this proposed scheme is compared with equal power allocation to sub-carriers.

If $P_i[t]$ is the total transmitted power from the $i^{th}$ transmitter of the $i^{th}$ stage, then the power allocated to the $n^{th}$ sub-carrier between the transmit antenna $t$ and any receiver antenna $j$ can be expressed as (dropping subscript $j$ for ease of representation),

$$P_t(n) = \left( \frac{\prod_{g=1}^{N} \lambda_t[g]^{-\frac{1}{2}}}{\sum_{w=1}^{N} \prod_{g=1}^{N} \lambda_t[g]^{-\frac{1}{2}}} \right) P_i[t], n = (0, 1, ..., N - 1) \quad (5.3.9)$$

If $P_i$ is distributed equally among all $N_t$ elements, then the transmitting power on each sub-carrier can be expressed as

$$P_t(n) = \left( \frac{\prod_{g=1}^{N} \lambda_t[g]^{-\frac{1}{2}}}{\sum_{w=1}^{N} \prod_{g=1}^{N} \lambda_t[g]^{-\frac{1}{2}}} \right) \frac{P_i}{N_t} \quad (5.3.10)$$

therefore

$$P_t = \sum_{t=1}^{N_t} \sum_{n=0}^{N-1} P_t(n), N_{ti} \in Z^+ \quad (5.3.11)$$

This power allocation scheme is designed for quasi-static channels.

### 5.4 End-to-End Bit Error Rate

If the $i^{th}$ relaying stage experiences independent probability of error (BER), which is denoted here as $P_{b,i\in[1,K]e}$ caused by independent
symbol error rate (SER) \( P_{s,i \in (1, K)(e)} \), the probability of error free transmission at stage \( i \) can be expressed as

\[
1 - P_{b,i \in (1, K)(e)}
\]

hence the average probability of correct end-to-end transmission \( P_{c,e2e}(e) \) can be expressed as the joint probability of correct transmission at each stage, i.e.

\[
P_{c,e2e}(e) = \prod_{i=1}^{K} (1 - P_{b,i(e)})
\]

A bit transmitted from a source terminal is received correctly at the destination only when at all the stages the bits have been transmitted correctly. Thus, the end-to-end BER \( P_{b,e2e}(e) \) can therefore be expressed as

\[
P_{b,e2e}(e) = 1 - P_{c,e2e}
\]

which at low BER at each stage can be approximated as

\[
P_{b,e2e}(e) \approx \sum_{i=1}^{K} P_{b,i(e)} \\
\approx \sum_{i=1}^{K} \frac{P_{s,i(e)}}{\log_2(M_i)}
\]

where \( M_i \) is the modulating index employed at the \( i^{th} \) stage. From equation (5.4.4), it is clear that the end-to-end BER will be dominated by the worst stage.
5.5 Capacity of Three Stage Network

This section presents the capacity analysis of the entire network assuming the power allocation strategy proposed in Section 5.3.3 is employed at each stage. Remember that the use of a cyclic prefix (CP) with application of FFT and inverse fast Fourier transform (IFFT) at the transmitters and the receivers at each stage decouples the multi-path channel (frequency selective quasi-static channel) into a number of parallel frequency non-selective fading channel. With this the normalised channel capacity in bits/s/Hz between, for instance the transmitter $t$ and the relay $j$ at the $i^{th}$ stage can be represented as , (as before dropping subscript $j$),

$$C_i[t] = \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{\lambda_t(n) P_t(n)}{\sigma(n)^2} \right)$$

(5.5.1)

where $P_t(n)$ denotes the power allocated to each sub-carrier, $\lambda_t(n)$ the channel gain of each sub-carrier and $\sigma(n)^2$ is the variance of the additive white Gaussian noise of each sub-carrier.

If the channel gains between the transmitter $t$ and the receiver $j$ on all sub-carriers of the $i^{th}$ stage are represented as $\tilde{H}_t$ i.e. $\tilde{H}_t = [\lambda_t(0), \lambda_t(1), \lambda_t(2), ..., \lambda_t(N - 1)]$, and the independent sub-carriers are treated as virtual antennas, then it is possible to analyze the capacity at each stage using the MIMO capacity concept presented in [39], [2] and [82] for a fixed channel realisation.

Let $H_i$ denote the channel matrix of all the sub-carriers from transmitter $t$ to the receiver $j$, in the $i^{th}$ stage, grouping this conveniently yields $H_i \in \mathbb{C}^{N_r \times N_t}$, where $N_r$ and $N_t$ are the total number of receive and transmit antennas of the $i^{th}$ stage.
Hence the capacity of each stage can be expressed as

$$ C_i = \alpha_i \sum_{t=1}^{d} \sum_{n=0}^{N-1} \log_2 \left( 1 + \frac{\lambda_i[n] P_i[n]}{\sigma[n]^2} \right) $$

(5.5.2)

where $d$ is the rank of matrix $H_i = min(R_i, T_i)$. Therefore the end-to-end capacity $C$ of the three stage network can be expressed as the minimal capacity between the three links i.e.

$$ C = amin(C_1, C_2, C_3) $$

(5.5.3)

The additional $\alpha$ term denotes the fact that three time slots are occupied for the transmission of one block of data from source to destination.

### 5.6 Simulation Results

The performance of the three-stage collaborative broadband relay architecture using the power allocation strategies discussed in Section 5.3 is investigated. Although, the application of these allocation strategies is been applied to the architecture presented in Figure 5.1, it can easily be extended and applied to any multi-stage configuration provided the basic assumptions are the same.

Figures 5.2, 5.3 and 5.4, show the end-to-end (average) BER performance of the proposed three stage relaying system with two and four transmit/receive antennas at the second stage when the proposed resource allocation schemes are applied. The end-to-end bit error rate (BER) is evaluated as a joint probability of the BER of each stage. For each stage the channel between the transmitter $t$ and the receiver $j$ is of length three with each coefficient a zero mean circular complex Gaussian random variable with equal variance, normalised so that
Figure 5.2. End-to-end BER OSF-OFDM as a function of SNR.

$\sum_{i=0}^{2} \sigma_i^2 = 1$. Also a 64 length OFDM symbols, a cyclic prefix of length 8 and QPSK modulation technique are employed. The received symbols at the first stage are always combined using a maximum ratio combiner (MRC).

Space frequency block coding is employed at stages two and three as it has very simple maximum likelihood decoding symbol-wise and it is straightforward. Figure 5.2 presents results for open-loop space frequency Alamouti encoding over the second and third stages. Figure 5.3 assumes channel state information (CSI) is available in the transmitters of stages two and three and exploits quasi-orthogonal space frequency
Figure 5.3. End to End BER CLQO-SFBC-OFDM as a function of SNR.

block coding, called closed-loop quasi-orthogonal space frequency block coding (CLQO-SFBC) and likewise in Figure 5.4 the closed-loop extended orthogonal space frequency block coding (CLEO-SFBC) in the second and third stages with the same assumption.

Note that in the figures, the "No-optimum and no-WF" represents equal power allocation across stages and sub-carriers, "optimised no-WF" represents optimised inter-stage allocation with equal sub-carrier power and "optimised and WF" represents optimised inter-stage and sub-carrier power allocation using the proposed strategy. Also simula-
Section 5.6. Simulation Results

Figure 5.4. End to End BER CLEO-SFBC-OFDM as a function of SNR.

tion results are parameterized by $p$ which defines each link gain with respect to the first stage, e.g. $p(0,10,5)$ means that the second link is 10 times stronger than the first one and the last link is 5 times stronger than the first one. Although this last stage has a 5 fold increase relative to the first stage, the first stage still performs better that the last stage because of its inherent array gain advantage, and with equal power distribution across stages, the radiated power $P_1$ in the first stage is $T_3$ times higher that the power radiated from individual antennas of the 3rd stage, where $T_3$ is the number of transmitting relays at the third stage.
As can be seen from the figures, the proposed resource allocation strategy in equation (5.3.10) provides the best performance when compared with equal power allocation strategy over each sub-carrier. It also shows that the performance of inter-stage power allocation over broadband multi-stage communication can further be enhanced by optimum sub-carrier resource allocation. For instance in Figure 5.2, at end-to-end BER of $10^{-2}$ the proposed optimum intra-stage allocation combined with optimal inter-stage allocation out-performs equal power allocation by approximately $3dB$ and there is a loss of approximately $1.5dB$ if not employed with optimal inter-stage allocation. However,
Section 5.6. Simulation Results

Figure 5.6. Achievable end-to-end capacity for various power allocation strategies over a broadband channel

in a system using CL-QO-SFBC, a 3.5dB gain is achieved at the same BER while for a system with CL-EO-SFBC a 3.5dB gain is achieved. However, looking at Figures 5.3 and 5.4, at 20dB the CL-EO-SFBC enjoys additional benefit of a maximum diversity gain and transmit array gain of $0.5 \times 10^{-3}$ over CLQO-SFBC, these benefits are also evidenced with both inter and intra-stage power allocation.

Figure 5.5 shows the variation of interstage power allocation over the SNR range considered. It can be seen that on the average, the third stage which is the weakest link is allocated more power in order to boost
its performance, for instance with a $1 \times 4$, $4 \times 4$ and $4 \times 1$ configuration, the power allocated to each stage at $10dB$ is 1.3612, 0.2002 and 1.4386 respectively and likewise for $1 \times 2$, $2 \times 2$ and $2 \times 1$ is 1.5007, 0.2499 and 1.2494 respectively. In both cases the total power available to the network is constrained to $K = 3$ and the interstage allocation obeys equation (5.3.4).

Figure 5.6 depicts the achievable normalised end-to-end capacity in (bits/s/Hz) versus the SNR in (dB) for the examined broadband wireless multi-stage communication with various sub-carrier power allocations. In this simulation it is assumed the channels are also quasi-static to perform power allocation.

As can be seen from Figure 5.6, the approximate power allocation strategy (AP-WF) proposed yields a near optimum water filling (WF) performance as SNR increases and better than equal power allocation (EQ).

However, looking at Table 5.1, it can be seen that the proposed allocation strategy maintains it capacity advantage over a larger range of SNR as compared to the water-filling algorithm in relation to equal power allocation. At $2dB$ the water filling algorithm has started to lose its capacity advantage over equal power allocation, while the proposed approximation of water filling algorithm maintains its capacity advantage over a larger range of SNR (up to $7dB$). It can also be seen that at SNR of $19dB$, the proposed allocation strategy begins to outperform the water-filling algorithm.
Table 5.1. Comparison of end-to-end capacity gain for using water-filling (WF) and approximated water-filling (AP-WF) algorithms with equal (EQ) power allocation strategies on each sub-carriers.

<table>
<thead>
<tr>
<th>SNR</th>
<th>AP-WF - EQ</th>
<th>WF - EQ</th>
<th>(AP-WF) - WF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.398</td>
<td>1.278</td>
<td>0.880</td>
</tr>
<tr>
<td>1</td>
<td>0.442</td>
<td>1.221</td>
<td>0.779</td>
</tr>
<tr>
<td>2</td>
<td>0.483</td>
<td>1.161</td>
<td>0.678</td>
</tr>
<tr>
<td>3</td>
<td>0.520</td>
<td>1.098</td>
<td>0.579</td>
</tr>
<tr>
<td>4</td>
<td>0.550</td>
<td>1.035</td>
<td>0.486</td>
</tr>
<tr>
<td>5</td>
<td>0.570</td>
<td>0.973</td>
<td>0.402</td>
</tr>
<tr>
<td>6</td>
<td>0.582</td>
<td>0.913</td>
<td>0.331</td>
</tr>
<tr>
<td>7</td>
<td>0.584</td>
<td>0.858</td>
<td>0.274</td>
</tr>
<tr>
<td>8</td>
<td>0.570</td>
<td>0.802</td>
<td>0.232</td>
</tr>
<tr>
<td>9</td>
<td>0.546</td>
<td>0.749</td>
<td>0.204</td>
</tr>
<tr>
<td>10</td>
<td>0.508</td>
<td>0.696</td>
<td>0.188</td>
</tr>
<tr>
<td>11</td>
<td>0.458</td>
<td>0.639</td>
<td>0.181</td>
</tr>
<tr>
<td>12</td>
<td>0.396</td>
<td>0.575</td>
<td>0.179</td>
</tr>
<tr>
<td>13</td>
<td>0.323</td>
<td>0.501</td>
<td>0.178</td>
</tr>
<tr>
<td>14</td>
<td>0.240</td>
<td>0.412</td>
<td>0.173</td>
</tr>
<tr>
<td>15</td>
<td>0.149</td>
<td>0.307</td>
<td>0.158</td>
</tr>
<tr>
<td>16</td>
<td>0.052</td>
<td>0.182</td>
<td>0.131</td>
</tr>
<tr>
<td>17</td>
<td>-0.049</td>
<td>0.038</td>
<td>0.087</td>
</tr>
<tr>
<td>18</td>
<td>-0.152</td>
<td>-0.128</td>
<td>0.024</td>
</tr>
<tr>
<td>19</td>
<td>-0.253</td>
<td>-0.314</td>
<td>-0.062</td>
</tr>
<tr>
<td>20</td>
<td>-0.352</td>
<td>-0.521</td>
<td>-0.169</td>
</tr>
</tbody>
</table>
5.7 Adaptive Space-Frequency and Space-Time-Frequency Coding

The design of optimum resource allocation algorithms for adaptive relaying transmissions in an OFDM-based relaying system is presented in this section. Performance of OFDM-based systems can be further enhanced when combined with a link adaptation technique, which adjusts link parameters of transmission on all sub-carriers, so as to improve throughput while satisfying end-to-end BER requirement [109]. Adaptation in both power and channel statistics (delay-spread) relates to the optimum fractional transmission power allocated to each stage on possible types of operational modes. Channels between relay stages in practice, may have different fading statistics, hence adaptation to channel statistics by switching the quasi-orthogonal block code (QOBC) and extended orthogonal block code (EOBC) to either space-frequency code (SFC) or space-time-frequency code (STFC) according to the channel condition, is an effective technique of further minimising the receiver error performance degradation due to the highly frequency selective nature of the channel. Quantifying the error performance degradation of orthogonal block codes (OBC) in fading channels using a fading interference metric, can be used to select the best space-diversity transmission mode for inter-stage transmission.

In what follows, these different modes are discussed.
Figure 5.7. QOBC-MIMO-OFDM (a) Time-domain spreading (STBC), (b) Frequency domain spreading (SFBC) and (c) Frequency and time domain spreading (STFBC).
Figure 5.8. EOBC-MIMO-OFDM (a) Time-domain spreading (STBC), (b) Frequency domain spreading (SFBC) and (c) Frequency and time domain spreading (STFBC).
5.7.1 Space-Time-Frequency Coding

Defining the (QOBC) and (EOBC) as given in equations (5.3.3) and (5.3.4) by dropping the sub-carrier index as

$$X = \begin{bmatrix} a & b & c & d \end{bmatrix}^T$$  \hspace{1cm} (5.7.1)

and EO-SFBC as

$$X = \begin{bmatrix} e & f \end{bmatrix}^T$$  \hspace{1cm} (5.7.2)

respectively, where vectors $a,b,c,d$ for (QO-SFBC) are defined as

$$a = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad b = \begin{bmatrix} -x_2^* \\ x_1^* \\ -x_4^* \\ -x_3 \end{bmatrix}, \quad c = \begin{bmatrix} -x_3^* \\ -x_4^* \\ x_1^* \\ x_2 \end{bmatrix}, \quad d = \begin{bmatrix} x_4 \\ -x_3 \\ -x_2 \\ x_1 \end{bmatrix}$$  \hspace{1cm} (5.7.3)

and $e,f$ for (EO-SFBC) as

$$e = \begin{bmatrix} x_1 \\ x_1 \\ x_2 \\ x_2 \end{bmatrix}, \quad f = \begin{bmatrix} -x_2^* \\ -x_2^* \\ x_1^* \\ x_1^* \end{bmatrix}$$  \hspace{1cm} (5.7.4)

Transmitting vectors $a, b, c$ and $d$ in consecutive time slots in the same sub-carrier as shown in Figure 5.7a refers to quasi-orthogonal space time coding while the transmission of $e$ and $f$ in the same manner shown in Figure 5.8a represents the extended-orthogonal space time coding. As mentioned in Section 5.2.1, if these vectors are transmitted at consecutive sub-carriers in the same time slot as shown in Figures 5.7b and 5.8b respectively, it is referred to as space frequency coding for either QOBC and EOBC.

However, if vectors $a$ and $b$ are transmitted at time $t = 1$ over two consecutive sub-carriers, and vectors $c$ and $d$ at time $t = 2$ over
the same two consecutive sub-carriers, this forms the quasi-orthogonal space-time-frequency block code (QO-STFBC) and in the same manner an extended-orthogonal space-time-frequency block code (EO-STFBC) is formed if vector $\mathbf{e}$ is transmitted at time $t = 1$ on one sub-carrier, and vector $\mathbf{f}$ at time $t = 2$ on a different sub-carrier, as shown in Figures 5.7c and 5.8c respectively. For (QO-STFBC), this is in effect transmitting an equivalent of 4 transmitter STBC/SFBC OFDM codes, but instead of using 4 adjacent time intervals as in the STBC-OFDM case or 4 adjacent sub-carriers in the SFBC-OFDM case, only 2 adjacent time intervals and sub-carriers are employed in the STFBC code.

The decoding of the STFBC is similar to that of STBC, SFBC on the assumption that the channel remain constant over 2 time interval/sub-carrier. This lead to a method of breaking down of systems employing a large numbers of transmitters when the channel remains constant over $N_t$ time intervals/sub-carriers to a system that assumes that the channel is quasi-static only over half the numbers of transmitters employed $N_t/2$. The same argument can be made for (EO-STFBC) but in this case the channel is quasi-static over a quarter of the transmitting antennas employed $N_t/4$. It will become clear later this could have a significant impact on the BER performance of the system in a fast fading channel.

5.7.2 Adapting the Space-Time-Frequency Spreading

The channel status that determines the spread of QOBC and EOBC in time or/and frequency can be categorised into the following scenarios [109]:

- If $B_c > M \times \Delta f_{sub}$ and $T_c < M \times T_{block}$ then SFBC is used.
• If $T_c < M \times T_{\text{block}}$ and $B_c < M \times \Delta f_{\text{sub}}$ then STFBC is used.

where $B_c$ and $T_c$ are coherence bandwidth and coherence time, $T_{\text{block}}$ is the total OFDM symbol duration including CP and $M$ is the number of rows of OBC design. $\Delta f_{\text{sub}}$ is the subcarrier spacing of the OFDM system.

In a highly frequency selective channel, the block code is spread over time and frequency. In order to realise the adaptive space-time-frequency spreading in the $i^{th}$ stage for instance, the transmitting relays of the stage must know the mode switching information. Thus the receiver at the stage is assumed to be equipped with a channel statistics estimator ($B_c$ and $T_c$ estimators) and based on the mode selection criteria provides the switching information to the transmitters through feedback.

5.8 Simulation Results

The performance of the adaptive SF and STF coding for extended-orthogonal and quasi-orthogonal codes is presented here. It is assumed as before that the channels across the network are spatially uncorrelated Rayleigh fading channels. Channel lengths (L) two and five are employed to represent “low and high frequency selective” channels respectively, with each normalised so that $\sum_{i=0}^{L-1} \sigma_i^2 = 1$. Also a 128 length OFDM symbols and a cyclic prefix of length 16 are employed. In order to evaluate the diversity performance of the system, the average end-to-end BER against the average SNR is plotted.

Figure 5.9 shows the performance comparison of SFBC and STFBC applied to the last link in the network in order to mitigate the error performance rate. Since the last stage is performing worse compared
to the first and second stages in this particular network architecture, the adaptive coding is best applied to this stage to further improve its BER performance, but note in a network with different architecture the switching should be based on the CSI at each stage. From the figure one can see that STFBC for extended-orthogonal coded OFDM systems are susceptible to the channel variation at high SNR. Because in the decoder the STFBC assumes that the channel responses are constant during one time slot and across one sub-carrier and SFBC assumes that

\[ N_1 = 1, N_2 = 4, M=4 \]
\[ N_1 = 4, N_2 = 4, M=4 \]
\[ N_1 = 4, N_2 = 1, M=4 \]

\[ p[0,10,5] \]

**Figure 5.9.** Performance comparison of BER for adaptive non-optimised EO-SF and EO-STF coded OFDM system for \( L = 2 \) and \( L = 5 \).
the complex channel gains between adjacent sub-carriers are approximately constant. Observe that there is about a $2dB$ degradation at $BER = 10^{-2}$ in the performance of STFBC decoder compared to the performance of the SFBC scheme. This degradation is increased with high SNR. While in Figure 5.10, STFBC for quasi-orthogonal has better performance than the SFBC scheme; i.e., SFBC scheme is susceptible to channel variation. This gain is attributed to the assumption that the channel responses are constant during two time slots and across
two frequency bins. For example, at a target $BER = 10^{-2}$ about $1dB$ gain can be obtained compared to the SFBC scheme performance.

Figures 5.11 and 5.12 shows the adaptability of the optimum power allocation strategy to each transmission matrix. Again it is compared with the system with equal power allocation discussed in Section 5.3.1 for both QOBC and EOBC respectively. It can be seen that the optimum power allocation strategy considerably preserves the power con-
Figure 5.12. Performance comparison of BER for adaptive optimised EO-STFBC for $L = 5$.

Assumption in the network for achieving the given end-to-end BER (QoS) at the destination. Those figures illustrate that using the proposed power allocation an approximate $2dB$ gain in terms of network lifetime will be obtained as compared to equal power strategy.
5.9 Summary

A three-stage relaying structure which has the aim of increasing the coverage area of a broadband communication system (without jeopardizing its performance) has been presented. The advantages of using different space frequency coding schemes over the network in order to combat the effect of multi-path propagations between the source and through various relaying stages to the destination have been shown. An extension of the inter-stage power allocation for a two-stage communication over a frequency non-selective channel presented in [11] to a three-stage communications over a frequency selective channel was presented. Simulation results showed a considerable advantage when compared to equal power allocation on each stage. An approximate water filling power allocation algorithm that provides a capacity advantage over a larger range of SNR is also developed in this chapter, and was employed to optimally distribute power across each OFDM sub-carrier at each relaying stage. Simulation results further confirm that there is a need for effective and efficient power allocation over a multi-stage communication link if the required capacity advantage, higher throughput, enhanced coverage and better QoS are to be achieved.

Finally, the chapter proposed adaptive resource allocation strategies that take both the statistical CSI and the optimum fractional power allocated to each stage into account to improve the performance of the multistage OFDM relay network. Further simulations demonstrate that the proposed power allocation strategy could considerably save the total transmitted power compared to an equal transmitted power scheme. In summary, the scheme presented here can be considered as a promising technique for high data rate wireless transmission over
Section 5.9. Summary

frequency selective multi-path fading channels for broadband wireless communication systems.
Chapter 6

CONCLUSION AND FUTURE WORK

6.1 Conclusion

The thesis uses the concept of virtual antenna arrays (VAAs) to allow possible deployment of orthogonal space-time block in particular the closed-loop quasi-orthogonal space-time block code (CL-QO-STBC) and the closed-loop extended orthogonal space-time block codes (CL-EO-STBC) in the context of collaborative multi-stage communications (Both codes have been shown to provide a maximum order diversity in MIMO systems).

It has been demonstrated that VAA deployment yields significant gains in data throughput independent of the complexity of the available transceivers and provides a better framework to achieve the multi-input multi-output (MIMO) gains in-term of capacity and bit error performance. Therefore, research into the use of diversity enhancement coding schemes originally design for point-to-point MIMO systems in the context of collaborative multi-stage communication was justified.

The contributions of the thesis are organised to give a better picture of the contribution of the research conducted.
In Chapter 3 an information theoretical approach was adopted to demonstrate the achievable capacity region of CL-QO-STBC over both ergodic and non-ergodic point-to-point MIMO channels. It was reiterated that the appropriate capacity measure for ergodic channels is the Shannon capacity, whereas for non-ergodic channels the outage probability for a given communication rate is appropriate. The simulation results shown in the chapter showed that O-STBCs are optimal with respect to capacity when the code is rate one and the capacity advantage grows linearly as a function of the number of transmit and receive antennas. Combining these two criteria, the results showed the capacity advantage of CL-QO-STBC over the A-STBC (both rate one codes) and other lower rate codes. The CL-QO-STBC outperforms the A-STBC scheme by approximately $0.5dB$ or $0.2bits/s/Hz$ and the $3/4$-rate schemes perform inferior to both rate one schemes, this is as a result of the loss in transmission rate of these schemes. However, a different behaviour was experienced when this O-MIMO system operated in a non-ergodic channel, it was shown that an increase in the number of transmit antennas does not always lower the outage probability, thus deployment of CL-QO-STBC must be taken into consideration the desired transmission rate. The simulation results provided show that deploying CL-QO-STBC must be done with a lower rate transmission for high reliability communications.

These two novel results confirms the promising capacity and outage advantages of an O-MIMO channel when CL-QO-STBC is deployed at the transmitter.

Although the techniques and benefit of CL-QO-STBC have received considerable research attention lately, these novel results set the
capacity and outage bound of O-MIMO channel deploying CL-QO-STBC over ergodic and non-ergodic flat Rayleigh fading channels.

Having established theoretical capacity and outage probability advantages of using CL-QO-STBC in an O-MIMO channel, analysis in Chapter 4 then focuses on the use of these coding schemes in distributed antenna system and over cooperative multi-stage communications. In the first analysis, the same theoretical approach used in Chapter 3 was used to determine the achievable range region of a given network topology. Similar to Chapter 3 the analysis considered both ergodic and non-ergodic channels. It was shown that maximising the end-to-end throughput over non-ergodic channel is equivalent to maximising the end-to-end capacity over an ergodic channel. The simulation results presented show that the end-to-end capacity and throughput achievable by implementing the distributed CL-QO-STBC scheme in relay networks as compared to the distributed A-STBC scheme, for instance assuming an operating SNR of 10dB at all receiving nodes, the distributed CL-QO-STBC scheme delivers normalised end-to-end capacity of approximately 1.7bit/s/Hz as against 1.6bits/s/Hz obtained with the use of distributed A-STBC, and delivers a throughput of approximately 1.15bits/s/Hz as against 0.9bits/s/Hz delivered by the distributed A-STBC scheme.

Clearly, this novel result has demonstrated the advantage of using CL-QO-STBC in virtual antenna array as against the A-STBC.

A practical measure was also used in demonstrating this performance advantage with CL-QO-STBC providing a steeper BER curve than the A-STBC ensuring a diversity order of four is achieved at the destination for the topology considered.
Thus this means that a four antenna VAA using CL-QO-STBC promises to deliver a better performance advantage compared to a two antenna VAA system deploying A-STBC.

Having established the importance of a four antenna relaying system, the chapter then compared the performance of distributed CL-QO-STBC and CL-EO-STBC schemes. The result also in Chapter 4 shows the performance gain by deploying the CL-EO-STBC schemes against the CL-QO-STBC over the relay nodes. This suggested again that in a cooperative relay network, increasing the number of cooperating relays should be accompanied with appropriate coding techniques to maximise the network performance.

However, in all cases considered, the effect of optimum power allocation between the two stages was investigated, it was found that optimal power allocation between the stages further increases the performance with the effect being more pronounced in distributed CL-QO-STBC and CL-EO-STBC relaying.

For completeness, a broadband application of cooperative multi-stage communication was studied in Chapter 5, with a three-stage relaying structure. The aim was to increase the coverage area of a broadband communication system (without jeopardizing its performance). The advantages in deploying the A-STBC, CL-QO-STBC and the CL-QO-STBC as space frequency coding schemes in order to combat the effect of multi-path propagation between the source and various relaying stages to the destination was shown in this chapter. An extension of the inter-stage power allocation for a two-stage communication over a frequency non-selective channel presented in [11] to a three-stage communications over a frequency selective channel was presented. Simula-
tion results showed a considerable advantage when compared to equal power allocation on each stage. An approximate water filling power allocation algorithm that provides a capacity advantage over a larger range of SNR was also developed in this chapter, and was employed to optimally distribute power across each OFDM sub-carrier at each relaying stage, simulation results further confirmed that there is a need for effective and efficient power allocation over a multi-stage frequency selective communication link if the required capacity advantage, higher throughput, enhanced coverage and better QoS are to be achieved.

Finally, the chapter proposed adaptive resource allocation strategies that take both the statistical CSI and the optimum fractional power allocated to each stage into account to improve the performance of the multistage OFDM relay network. Further simulations demonstrated that the proposed power allocation strategy could considerably save the total transmitted power compared to equal transmitted power scheme. In a nutshell, the scheme presented here can be considered as a promising technique for high data rate wireless transmission over frequency selective multi-path fading channels for broadband wireless communication systems.

Since the ultimate purpose of this thesis is to positively contribute to scientific knowledge through developing new strategies that could be useful in future wireless systems in particular distributed-MIMO multi-stage relaying networks, it is my opinion that the work exposed herein has opened up other challenges that may catch the imagination of future researchers.
6.2 Direction of Future Research Work

Although the thesis poses and answers to some questions relating to the understanding of cooperative relaying communication systems, as with any scientific work, it has opened up other research challenges, some of which will form the focus of my future research work highlighted below.

6.2.1 General Coding

- The potential concatenating of the coding schemes considered with outer channel codes such as Turbo codes in order to further increase the performance capacity already shown in [18] is still open research.

- The investigated scenarios can be extended to any form of coding, i.e. potentially concatenated space-time block, trellis, Golden and silver codes as well as their differential realisations.

- The framework for efficient analysis of outage and ergodic capacities of a distributed orthogonal space-time coding system where the cooperating links can be operating under different fading channels or with different SNRs will provide a more robust design since the channel at different stages could have different characteristics.

6.2.2 Cooperative Relaying

- The architecture considered in this thesis assumed a single source and destination, an extension to the multi-user scenario is a possibility which would allow one or more relaying nodes to be used by
more than one relaying chain. The issue of interference reduction also comes into play within this extension.

- It has been assumed that the CSI is perfectly known at the receiver, i.e., the relays know the channel from the source to relay and the destination knows the channel from the relays to destination. Of interest could be investigation of channel estimation at the relays and the destination nodes.

- Distributed orthogonal space-time block codes (O-STBCs) have been used to exploit cooperative diversity in this thesis with the assumption that the distributed nodes are perfectly synchronized in time. In practical systems time synchronization errors may occur which would degrade the performance. The study of the effects of timing synchronization error on the bit error rate performance of a distributed O-STBC system is another interesting research issue.

- Extending the proposed OFDM based transmission scheme to coherent, symbol asynchronous relay networks with timing errors and frequency offsets at the relay nodes is also an interesting direction for further work. This problem has been addressed in [110] for the case of two relay nodes.

- For the cooperative multi-stage relaying network studied, it is assumed that all the relay nodes cooperated perfectly this might not be possible in practice due to different relay location a study of partial cooperation among the relays is also of interest.
6.2.3 Resource Allocation

- To preserve the network resource, determining the optimum number of relaying nodes/stages will be useful in order to further increase the network lifetime.

- Finally, it may also prove useful in deriving the optimum transmit power allocation for STBCs to operate over channels with different gains.
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