Optimised control of an advanced hybrid powertrain using combined criteria for energy efficiency and driveline vibrations

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OPTIMISED CONTROL OF AN ADVANCED HYBRID POWERTRAIN USING COMBINED CRITERIA FOR ENERGY EFFICIENCY AND DRIVELINE VIBRATIONS

by

Ashley Kells

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

June 2002

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Abstract

This thesis discusses a general approach to hybrid powertrain control based on optimisation and optimal control techniques. A typical strategy comprises a high level non-linear control for optimised energy efficiency, and a lower level Linear Quadratic Regulator (LQR) to track the high level demand signals and minimise the first torsional vibration mode. The approach is demonstrated in simulation using a model of the Toyota Prius hybrid vehicle, and comparisons are made with a simpler control system which uses proportional/integral (PI) control at the lower level.

The powertrain of the Toyota Prius has a parallel configuration, comprising a motor, engine and generator connected via an epicyclic gear train. High level control is determined by a Power Efficient Controller (PEC) which dynamically varies the operating demands for the motor, engine and generator. The PEC is an integrated non-linear controller based on an iterative downhill search strategy for optimising energy efficiency and battery state of charge criteria, and fully accounts for the non-linear nature of the various efficiency maps. The PEC demand signals are passed onto the LQR controller where a cost function balances the importance of deviations from these demands against an additional criterion relating to the amplitude of driveline vibrations. System non-linearity is again accounted for at the lower level through gain scheduling of the LQR controller.

Controller performance is assessed in simulation, the results being compared with a reference system that uses simple PI action to deliver low-level control. Consideration is also given to assessing performance against that of a more general, fully non-linear dynamic optimal controller.
Acknowledgements

This work was conducted in the Department of Aeronautical and Automotive Engineering, Loughborough University and was funded by Ricardo Midlands Technical Centre, Leamington Spa.

The author wishes to thank Ricardo MTC for their financial support and guidance throughout the project, and in particular Mike Savage, Dave Kelly and Jon Wheals.

The research was supervised at Loughborough by Dr. M. C. Best and Prof. T. J. Gordon, and I would like to extend my thanks to them for their exceptional supervision and guidance throughout the project.

I also wish to thank my colleagues in the Department of AAE, and in particular my office compatriots Andrew Fairgrieve (Muppet), Ayao Komatsu (Nippon) and Simon Tuplin (The Best) for providing a tireless source of entertainment and amusement.

Finally, I would like to thank my close family and friends for their support and understanding throughout my studies.
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Nomenclature

A Vehicle frontal area (m$^2$)
$A_d$ Constant term in rolling resistance (N)
$A_i$ Initial Amp-hours used by the battery (Ah)
$A_m$ Maximum battery capacity (Ah)
$A_u$ Total Amp-hours used by the battery (Ah)
b Constant in calculation of component operation cost element
B Term in Pacejka tyre model determined from input parameters
$B_c$ Fuel consumption (kg/h)
$B_d$ Velocity dependant term in rolling resistance (kg/s)
b$_s$ Shaft damping (Nms/rad)
b$_t$ Tyre damping (Nms/rad)
b$_t^*$ Tyre damping parameter (Nm)
C Term in Pacejka tyre model determined from input parameters
c$_{1-4}$ Cost function weighting parameters
$C_d$ Aerodynamic drag coefficient
d Drivecycle distance (km)
D Term in Pacejka tyre model determined from input parameters
E Term in Pacejka tyre model determined from input parameters
f Fuel use (g)
$F_A$ Aerodynamic drag force (N)
$F_{ac}$ Force required to accelerate the vehicle (N)
$f_c$ Fuel use (g/kWh)
$f_d$ Fuel consumption (l/km)
$F_d$ Total drag force on the vehicle (N)
f$_{pr}$ Contact force between planet and annulus gears (N)
f$_s$ Fuel use (g/s)
f$_{SOC}$ Fuel use correction based on end SOC
f$_{sp}$ Contact force between sun and planet gears (N)
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<td>$I$</td>
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<td>$P_{bm}$</td>
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<td>$P_g$</td>
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<td>$P_{in}$</td>
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<td>$P_{lb}$</td>
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<td>$P_{le}$</td>
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<td>$P_{mr}$</td>
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<td>$P_{net}$</td>
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<td>$P_{out}$</td>
<td>Electrical power out of the battery (kW)</td>
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<td>$P_{sys}$</td>
<td>Total power required by the powertrain (W)</td>
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<td>$P_u$</td>
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<td>$R_s$</td>
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<td>$R_{sa}$</td>
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<td>$S$</td>
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<td>$T'_2$</td>
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<td>$T_{an}$</td>
<td>Torque required at the annulus (Nm)</td>
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<td>$T_B$</td>
<td>Brake torque (Nm)</td>
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<td>$T_c$</td>
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<tr>
<td>$T_D$</td>
<td>Driveshaft torque (Nm)</td>
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</table>
**Nomenclature**

- $T_{dg}$: Total drag torque on the vehicle (Nm)
- $T_e$: Engine torque (Nm)
- $T_{emax}$: Maximum engine torque (Nm)
- $T_{emin}$: Minimum engine torque (Nm)
- $t_f$: Final time value (s)
- $T_{lim}$: Motor torque limit (Nm)
- $T_m$: Motor torque (Nm)
- $T_{max}$: Maximum motor torque (Nm)
- $T_R$: Tyre rolling resistance torque (Nm)
- $T_{rl}$: Road load torque (Nm)
- $t_s$: Number of teeth on sun gear of epicyclic
- $T_s$: Shaft torque (Nm)
- $T_{SOC}$: Table of SOC correction
- $t_{sp}$: Time step within the drivecycle profile (s)
- $T_{sun}$: Torque from the sun element of the planetary gearset (Nm)
- $T_T$: Torque transmitted through the tyre (Nm)
- $u$: Matrix of controls
- $u_1$: Motor torque control input (Nm)
- $u_2$: Engine torque control input (Nm)
- $u_3$: Generator torque control input (Nm)
- $u_{lim}$: Upper component boundary
- $V$: Voltage (V)
- $v_0$: Constant term in tyre torque-slip relationship
- $v_1$: Demand velocity from the drivecycle (m/s)
- $V_{bus}$: Battery bus voltage (V)
- $V_{max}$: Maximum voltage of the battery (V)
- $V_{oc}$: Open circuit battery voltage (V)
- $v_v$: Vehicle velocity (m/s)
- $x_i$: System state
- $y$: Uncontrolled system input
- $\alpha$: Rate of change of power loss with respect to engine speed (kWs/rad)
- $\alpha_e$: Engine rotational acceleration (rad/s²)
<table>
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<tr>
<td>$\alpha_m$</td>
<td>Motor rotational acceleration (rad/s²)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Rate of change of power loss with respect to engine torque (kW/Nm)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Coefficients for calculation of steady-state vehicle acceleration as a function of the torque inputs</td>
</tr>
<tr>
<td>$\Delta\theta$</td>
<td>Relative shaft displacement (rad)</td>
</tr>
<tr>
<td>$\Delta\omega$</td>
<td>Relative shaft rotational velocity (rad/s)</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Error matrix</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Gain for conversion of mechanical power to battery charge (kW)</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Power gain factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Cost associated with control input use</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
</tr>
<tr>
<td>$\eta_c$</td>
<td>Coulombic efficiency of the battery</td>
</tr>
<tr>
<td>$\eta_e$</td>
<td>Engine efficiency</td>
</tr>
<tr>
<td>$\eta_{fd}$</td>
<td>Efficiency of the final drive</td>
</tr>
<tr>
<td>$\eta_g$</td>
<td>Generator efficiency</td>
</tr>
<tr>
<td>$\eta_m$</td>
<td>Motor efficiency</td>
</tr>
<tr>
<td>$\eta_{me}$</td>
<td>Electrical efficiency</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Battery power gain factor</td>
</tr>
<tr>
<td>$\Theta$</td>
<td>Angle of road with respect to horizontal (rad)</td>
</tr>
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<td>$\theta_T$</td>
<td>Tyre displacement (rad)</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Wheel displacement (rad)</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Density of air (kg/m³)</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Fuel density (kg/m³)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Speed (rad/s)</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Shuffle mode eigenvector</td>
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<td>$\omega_a$</td>
<td>Annulus rotational velocity (rad/s)</td>
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<tr>
<td>$\omega_{an}$</td>
<td>Rotational velocity required at the annulus (rad/s)</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Base speed of motor (rad/s)</td>
</tr>
<tr>
<td>$\omega_c$</td>
<td>Carrier speed (rad/s)</td>
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<tr>
<td>$\omega_e$</td>
<td>Engine speed (rad/s)</td>
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<tr>
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<td>Description</td>
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<tr>
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<td>-------------</td>
</tr>
<tr>
<td>$\omega_m$</td>
<td>Motor speed (rad/s)</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>Rotational speed of planet gear (rad/s)</td>
</tr>
<tr>
<td>$\omega_s$</td>
<td>Generator speed (rad/s)</td>
</tr>
<tr>
<td>$\omega_T$</td>
<td>Tyre rotational velocity (rad/s)</td>
</tr>
<tr>
<td>$\omega_w$</td>
<td>Wheel rotational velocity (rad/s)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Combined cost associated with component limitations</td>
</tr>
<tr>
<td>$\psi$</td>
<td>State of charge weighting parameter</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Battery SOC</td>
</tr>
<tr>
<td>$\xi_i$</td>
<td>Initial battery SOC</td>
</tr>
<tr>
<td>$\zeta_{opt}$</td>
<td>Desired (optimum) battery SOC</td>
</tr>
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</table>
This thesis considers the development of optimised control strategies for future vehicle powertrain systems, and in particular, their application to hybrid electric vehicles (HEV's). The major emphasis is placed on the development and application of a hierarchical supervisory control system applied in simulation of a commercially available hybrid electric vehicle. The supervisory control system consists of a high level controller whose demands are realised through use of low-level controllers; the optimisation of both controller levels is discussed.

In this chapter, the background to the development of HEV's is discussed and their various forms summarised. The use of a well designed control strategy is identified as a means by which significant improvement in a HEV's performance, particularly in terms of overall fuel consumption, can be made. The first torsional vibration mode of a vehicle's powertrain is also discussed. This mode is often termed 'shuffle' and can have a significant effect on a vehicle's driveability. A number of published techniques that aim to regulate shuffle exist, although these are generally applied to conventional internal combustion engined vehicles. As such, application for hybrid electric vehicles is investigated here.

1.1 Background

Reducing the air pollution from motor vehicles has become an international issue, particularly over the course of the last two decades. Carbon dioxide (CO\textsubscript{2}) emissions are largely responsible for current concerns over global warming, and motor vehicles are blamed for being a large contributor to the problem of smog that plagues many cities around the world. Even as conventional internal combustion engined motor vehicles become more environmentally friendly there are still problems due to the sheer volume
of traffic on the roads, especially in congested inner cities. Such problems are being addressed by some city councils who are continually developing transport management schemes such as pedestrianisation of inner cities and the use of ‘park and ride’ schemes in order to try and reduce the concentration of traffic, and hence air pollution. A car sharing scheme has been investigated by Yamada et al. (2000) whereby bookings can be made from a fleet of vehicles for particular journeys by members of the scheme. The scheme aims to promote the use of vehicles when they would otherwise be redundant, for example during the day at a place of work where another employee may require the vehicle for an off-site business meeting. However, one problem inherent with the scheme is that it’s difficult to guarantee vehicles will be available at the required locations at times required by the members.

Transport management schemes in general are plagued by many political problems with perhaps the most notable being that they tend to drive business, and hence valuable revenue, out of the cities thereby creating what some would say is a no win situation. This forces extra pressure on the motor industry to develop more environmentally friendly vehicles. There is also the additional impetus to develop more economical vehicles given the rising cost of fossil-based fuels. This factor becomes more of an issue when vehicles that travel large distances (e.g. heavy goods vehicles) or with low fuel economy (e.g. off-highway) are concerned.

With these environmental and economic pressures in mind, developments have been made to conventional internal combustion engines in order to reduce the amount of air pollution associated with them and increase their fuel economy. Examples of such developments are the introduction of unleaded fuel and closed loop catalytic converters, and as discussed by Teratini et al. (2000), the development of idle stop systems. Even with these improvements many people believe that powertrains of a different configuration to the standard internal combustion engined (ICE) variety may influence the future of motor vehicle transport.

Alternative powertrain systems currently undergoing research with regards to automotive applications include fuel cell electric vehicles (see for example Kawatsu, 2000; Chizek,
2001; McCraw, 2001) and pure electric vehicles (EV) as exemplified by the commercially available Th!nk Mobility city car (Think Mobility, 2000). Research is also being conducted into the use of compressed natural gas (CNG) as discussed by Tamura and Kata (2001), and hybrid vehicles have been the focus of much previous work (see for example Kawai, 2000; Tamura and Kato, 2001; Kawatsu, 2000). Although fuel cell, EV and CNG vehicles have been demonstrated, it is believed that the technology to implement such systems is not mature enough for mass-market production. There is also concern that EV's rather than solving pollution problems simply move the location to the powerstations. However, Adcock et al. (1995) found this not to be the case primarily because it will always be easier to control and monitor the emissions from a fixed generating plant than from a large number of mobile sources. Additional problems also arise through the need for revised infrastructure requirements (e.g. recharging stations, hydrogen/methanol distribution centres). However, hybrid vehicles enable current and future technologies to be synthesised into an acceptable package for immediate application.

1.2 Hybrid vehicles

A hybrid road vehicle is one in which propulsion energy, during specified operational missions, is available from two or more types of energy sources, or converters. According to Wouk (1995), a hybrid electric vehicle (HEV) is a hybrid vehicle in which at least one of the energy sources or converters can deliver electrical energy. Many different configurations of hybrid vehicle are possible, although their designs generally fit into one of two categories, namely series and parallel. A series HEV is essentially an electric vehicle with an additional power source and generator set which is used to supply electricity as and when required. The motor is the only energy converter that can provide propulsion power to the road wheels. A parallel type HEV differs from a series type in that both power sources can deliver propulsion power to the wheels, either independently or combined depending on the vehicle and mission requirements. Schematic examples of series and parallel HEV layouts are shown in Figure 1.1.
One of the advantages of a series hybrid system is that the engine is disconnected from the road wheels, and as such can be operated at a fixed point on the torque/speed map to minimise fuel consumption or emissions. Also, the choice of engine is not restricted to a conventional internal combustion engine based on the Otto Cycle. This allows the use of engines such as the gas turbine as suggested by Davis et al. (1995), or the Stirling engine (see Poulton, 1994 for further detail) to be considered in the design process. The mechanical energy generated by the engine is converted into electrical energy by the generator and then back to mechanical energy via the motor. Inherent in this process are losses which result from energy conversion, in addition to losses associated with any battery charging/discharging. For a series hybrid to achieve its maximum speed for a sustained period, the battery essentially becomes redundant, and hence the motor, generator and engine all need to be able to deliver the maximum power required by the vehicle. Because of these high power requirements, series hybrid vehicles are unsuitable for the vast majority of passenger car applications. Series hybrids do however lend themselves well to applications where the demands of the drivecycle dictate frequent stop/start modes of operation (e.g. city buses). In this case the engine and generator are
very small in relation to the motor because most of the driving energy is delivered from the battery which is charged via a combination of regenerative braking and from the engine/generator.

As a parallel hybrid vehicle can deliver power to the road wheels through a combination of the engine and the motor, the components can be specified significantly smaller than those of a series arrangement to achieve the same maximum power requirements. Losses can also be minimised during sustained steady-state periods as mechanical energy from the engine is passed directly to the road wheels. Because of the flexibility in the arrangement of the two (or more) power sources, parallel hybrid vehicles also have the advantage that they can take on a number of forms (see for example Fischer, 1996; Welke et al., 1997). A disadvantage of the parallel configuration is that unless a device such as a continuously variable transmission (CVT) is used, the engine cannot run continuously at its optimum efficiency.

Dual hybrid vehicles are strictly a subset of the parallel classification, and attempt to combine the merits of both series and parallel configurations. By allowing the engine to operate independently of the road wheels it can be optimised (for example) for either fuel use or emissions, and maintaining a direct connection to the road wheels minimises powertrain losses during sustained steady state periods. This has been demonstrated using a combination of a motor, generator and engine with either a clutch and manual gearbox as shown by Stridsberg (1998a; 1998b) or a planetary gearbox as with the Equos Research vehicle (see West, 1997 for further detail) and the Toyota Hybrid System, as outlined by Oi and Ogiso (2000). Given the additional freedom within the system when compared to conventional parallel hybrid configurations there is a clear potential for increased performance through choice of a well designed control strategy. This coupled with their relative complexity and increased performance potential has lead to dual hybrids and the Toyota Hybrid System (THS) in particular to be used as a base for the research presented in this thesis.
1.3 Toyota Prius

The Toyota Prius (Figure 1.2) was the world's first mass-produced hybrid electric vehicle when released in Japan in 1997, and was derived from a concept vehicle shown at the 1995 Tokyo Motor Show.

![Figure 1.2: Toyota Prius](image)

The powertrain of the Prius comprises an internal combustion engine, two motor/generators, a battery pack and a planetary gearset, referred to collectively as the Toyota Hybrid System (THS). The electrical machines are generally referred to as the motor and generator according to their primary mode of operation. The engine is connected to the planetary carrier, the motor is connected to the annulus and the generator is connected to the sun gear. The THS falls into the dual type of HEV in that propulsion power can be supplied to the road wheels from the engine, the motor or a combination of the two. The planetary gearset system also allows the engine to run independently from the road wheels and pass power to the generator in order to charge the battery. A schematic of the THS is shown in Figure 1.3:
The configuration also allows the battery to be charged via the motor through use of regenerative braking. Here, rather than dissipating the vehicle’s energy as heat during braking as with conventional mechanical systems, energy is used to recharge the battery. The engine used by Toyota in the THS is a purpose built 1.5 litre all-alloy unit operating with an Atkinson, or high expansion ratio cycle. Using this cycle, the engine is designed for maximum efficiency rather than maximum power, and delivers a maximum power of 43kW and a maximum torque of 102Nm at the engine’s upper limit of 4000rpm. With this limit imposed, the weight of the engine can be reduced without compromising its overall durability, and smaller friction-reducing main, big end and little end engine bearings can be used.

The use of the Atkinson Cycle design makes full use of the combustion energy by keeping the exhaust valves shut until the end of the expansion stroke as discussed by Heywood (1988). The expansion stroke is thus increased when compared to an engine using a conventional Otto Cycle, with the exhaust valves left closed until virtually all of the expansion pressure has been dissipated, therefore converting more of the combustion energy into torque on the crankshaft. This cycle is achieved by using an engine with a long-stroke design, an offset crankshaft, direct injection and variable valve timing. Figure 1.4 shows a pressure-volume diagram for the Atkinson cycle.
With reference to Figure 1.4, in a conventional four stroke (Otto cycle) engine, the compression stroke (1-2) volume and expansion stroke (3-4') volume are practically identical, and hence the compression and expansion ratios are identical. In general, any attempt to increase the expansion ratio of an engine results in an increase in its compression ratio which leads to a greater likelihood of engine knock, or pre-ignition. An engine using an Atkinson Cycle overcomes this problem by delaying the closure of the intake valves until the compression stroke has begun (position 4). This effectively delays the start of compression and hence reduces the effective compression ratio. As the closing of the intake valves is delayed in this manner, there is a small portion of the intake air that has been drawn into the cylinder returned to the inlet manifold. This small amount of back-flow into the intake manifold has an advantage in partial load conditions as it allows for an increase in throttle valve opening, thereby reducing intake manifold vacuum and hence reducing intake pumping losses. Compared to the Otto cycle the area $14'451$ has been added to the P-V diagram thereby increasing the efficiency of the engine. For a true Atkinson cycle expansion occurs until the gas pressure within the cylinder reaches atmospheric ($P_{atm}$ at position $5^*$). However, in practise this is not feasible as the work done to eject the gasses from the cylinder outweighs the benefit through further increase of the expansion stroke. Also, dimensional constraints of the engine become a greater issue as a much longer piston stroke is required. A disadvantage
with this cycle is that the indicated mean effective pressure and power density decrease significantly because only part of the total displaced volume is filled with fresh charge.

The THS uses a permanent magnet AC synchronous type of configuration for both the motor and the generator (Figure 1.3). The motor has a maximum power of 30kW from 940 to 2000rpm and a maximum torque of 305Nm from 0 to 940 rpm. The power from the motor is combined with power from the engine to ensure smooth starts and responsive acceleration. The motor is also used to convert the kinetic energy of the vehicle into electrical energy for storage in the battery during regenerative braking.

The generator has a maximum power of 21kW and is used to run the electric motor and charge the battery. In addition, the generator is used by the THS control computer to control the speeds and hence the power distribution within the planetary gear set. This is achieved by controlling the amount of electricity the generator produces, and hence the angular velocity of the sun gear in the planetary gear set. The generator also serves as a starter motor for the petrol engine.

The battery module used within the THS is a sealed nickel-metal hydride (Ni-MH) type. The battery is composed of 240 modules of 1.2 volts each in a series arrangement to form a total voltage of 288V. The capacity of the battery is 6.5 Amp-hour. The THS control computer ensures that the state of charge (SOC) of the battery stays between a narrow band to eliminate the need for external charging and preserve the life of the battery. This is achieved using a combination of generator control and the regenerative braking system.

A technical specification of the Toyota Prius is detailed in Appendix A.

1.4 Hybrid vehicle control strategies

The operating conditions of the various power sources within a hybrid powertrain system are prescribed by the vehicle's control strategy. The aim of this control strategy is to achieve the demands of the drivecycle whilst running the vehicle as economically as
possible in terms of fuel consumption. Attention should also be given to ensure that the life of components is not compromised (e.g. by excessive battery cycling). Much improvement to the performance of a particular powertrain can be made through use of a well designed control strategy.

For a series hybrid vehicle there are historically two main types of control strategy; power assist and range extension. A power assist hybrid uses the load-levelling device (e.g. battery, flywheel) to manage the power output from the auxiliary power unit (e.g. internal combustion engine, fuel cell). The auxiliary power unit must be able to supply the vehicle’s maximum sustained power requirements, with assistance given from the load-levelling device during transient events. As such, the storage capacity of the load-levelling device is relatively small. Conversely, a range extender hybrid uses a relatively small auxiliary power unit and large load-levelling device. Here the auxiliary power unit is operated when the load levelling device falls below a set charge threshold, and switched off once the charge is greater than a set upper threshold. The main advantage of this configuration is that the auxiliary power unit may be operated at a set operating condition, optimised for say fuel efficiency or emissions. This mode of operation is often referred to as a thermostat strategy.

With a parallel hybrid vehicle, a mechanical connection exists between the auxiliary power unit and the road wheels, and as such the auxiliary power unit must be able to achieve the steady state demands of the drivecycle. Load-levelling device cycling is minimised as are the losses associated with charging and discharging. This type of strategy is often termed a load-following charge-sustaining strategy.

The choice of a particular strategy should preferably be made using a holistic approach during the design stage of the vehicle, as discussed in many prior works (see for example Ohyama, 1997a; Moore, 1996; Anderson and Pettit, 1995).

A supervisory control architecture was used by Hubbard and Youcef-Toumi (1997) in the design of a control strategy for a hypothetical parallel hybrid bus. The control structure comprises two elements – a high level supervisor which sets demands to be
Introduction

achieved by independent low-level controllers for the transmission and each power source. An adaptive identifier is used by the supervisory controller to determine the vehicle’s road load so that an inference of the driver’s acceleration demand may be made.

A similar approach was taken by Saeks and Cox (1999). Here a hierarchical control architecture is implemented on a hypothetical series hybrid electric vehicle. In this application separate electric motor/generators are used at each wheel, with electric power supplied by a fuel cell and flywheel arrangement. The hierarchical system comprises three elements; a vehicle control system, an energy control system and a vehicle management system. The vehicle control system translates the driver’s inputs into corresponding steering angle and motor speed commands whilst the energy control system controls the power flow between the fuel cell, flywheel and the motor/generators. The vehicle management system provides a performance measure to the energy control system based on current and anticipated drivecycle requirements. A neural adaptive control algorithm is used to implement the vehicle control system, and an adaptive critic algorithm is then used to implement the energy control system. With this arrangement energy use is optimised at an upper level whilst the demands of the drivecycle are met at a lower level. These adaptive algorithms perform satisfactorily at continually adapting to changes in tyre slip with minimal a priori knowledge of the HEV system dynamics and tyre slip function.

It is likely that the optimal strategy for a particular application will fall between the two broad categories of load-following charge sustaining and range extension, with such strategies being realised through use of vehicle configurations comprising continuously variable transmissions. One such application of this concept was proposed by Ohyama (1997b). The system comprises two 3-cylinder engines and an electric machine with each connected via individual clutches and a planetary gearset to a wide ratio range CVT. The system then operates with a combination of the three power sources depending on drivecycle requirements and vehicle parameters. The planetary gearset allows smooth switching between the power sources with any power surplus not delivered to the road wheels absorbed by the electric machine.
Introduction

A number of optimisation techniques have been applied to HEV’s with one off-line application of interest being contributed by Kleimaier and Schröder (2000). Here a direct correlation method for the numerical solution of optimal control problems is used to determine optimised control inputs to a system defined in state space. A prototype HEV is considered in the study and is of a parallel arrangement comprising an electric motor and combustion engine connected to the road wheels via a CVT. Steady state maps are used to represent the efficiencies of the powertrain components.

The optimisation technique was applied to two drive cycles with the results being used to provide an insight into desirable characteristics for the design of an on-line strategy. More specifically, the tool showed that at low speeds in city traffic the engine should only operate during high road-load requirements, whilst for higher sustained vehicle speeds the engine should supply the necessary power to the road wheels with the CVT ensuring that the engine is operating at its most efficient condition.

Prior knowledge of a vehicle’s drivecycle provides useful information for a hybrid vehicle and allows the application of optimised control strategies. A method was investigated by Quigley and Ball (1998) which sought to determine the duration and distance of a vehicle’s drivecycle using departure time information. Fuzzy models were used on data collected from eight vehicles over a period of one month with three types of performance exhibited. The first type performed well at predicting journeys on a week day morning, but not so well for afternoon journeys. The second and third types performed poorly either because of inconsistent departure times or different journeys occurring at the same departure time. It is hypothesised that performance improvements could be made using the addition of place of departure information. This could be provided by a GPS receiver which could feasibly be incorporated into the design of a HEV.

For the Toyota Prius, a supervisory control architecture is implemented as detailed by Sasaki et al. (1997). Here a Vehicle Control ECU sets engine output, motor torque and generator speed demands for the respective low-level controllers based on accelerator angle, shift position, brake pedal effort and battery condition (Figure 1.5).
The Engine ECU demands a throttle opening angle from the throttle actuator based on the engine power command from the Vehicle Control ECU. The proportion of the total torque required at the axle that is not met by the direct torque path of the engine is then achieved via a command to the motor inverter from the Motor ECU. The target engine speed is dictated by the Vehicle ECU and is achieved using the Generator ECU to realise a generator speed demand calculated from the planetary gearset constraint equations. During operation of the engine, fuel use is minimised using stored data of the engine’s efficiency. This usually dictates that the engine operates at wide-open-throttle (WOT). Because engines are less efficient in the low-load region the Vehicle ECU dictates that by default the engine should be stopped with the vehicle continuing on battery power alone during low driving power requirements. The Brake ECU commands the pressure control valve to provide any excess brake force not achieved by regenerative braking. The vehicle has demonstrated improved performance in terms of both fuel use and exhaust gasses when compared to a conventional internal combustion engined vehicle with the same cylinder displacement. These advantages are more prominent during stop-start situations as the hybrid powertrain configuration is able to minimise engine operation during these inefficient periods.

An optimised strategy applicable to the Toyota Prius architecture has been proposed by Seiler and Schröder (1998). The basis for the operating strategy is the minimisation of efficiency loss for the entire vehicle whilst also ensuring that the vehicle is independent-driven, i.e. does not require charging from an external electrical source. An off-line
calculation of total efficiency loss for any given road speed and torque is used to calculate a look-up table of engine operating point for any driving condition. A sensitivity analysis is used to account for battery charge requirements thus providing a complete operating strategy.

1.5 Low-level control

Using a hierarchical control strategy, the principal requirements of the low-level control system are to achieve the demands set at a higher level. However, through choice of a suitable control system additional benefits may be sought. Such a system is exemplified by a concept which has recently been implemented by UK based Zytek Electric Vehicles Ltd, who in conjunction with DaimlerChrysler developed a parallel hybrid powertrain for the MCC smart city coupé as detailed by Scarlett (2001). The conventional smart powertrain comprises a six speed automated-manual transmission which (in keeping with similar systems) has a particular failing in that a smoothed jerk is felt as the gearbox changes up. A competent manual gearbox driver knows how to avoid this, but conventional automated-manual gearbox systems cannot. However, with the hybrid arrangement the motor is able to sustain drive during the gear change gap thereby smoothing the power delivery during acceleration. To illustrate further use of low-level control here, it is proposed to investigate the design of a controller which attempts to minimise the first torsional vibration mode (shuffle) of a vehicle.

Shuffle has a significant effect on a vehicle’s driveability and has been the focus of much previous research (see for example Best, 1998; Farshidianfar et al., 2001; Hwang et al., 1998; Krenz, 1985; Laschet, 1994; Mo et al., 1996; Pettersson and Nielsen, 1997; Rabeih and Crolla, 1996; Streib and Hubert, 1996). Shuffle is excited primarily through aggressive throttle manoeuvres and is felt as a longitudinal acceleration fluctuation of the whole vehicle at 2 – 5 Hz. Various approaches have been postulated for the automatic control of shuffle through modulation of the engine torque response (see for example Mo et al., 1996; Pettersson and Nielsen, 1997; Streib and Hubert, 1996; Wang et al., 2000). This is commonly achieved through computer control of spark timing, engine fuelling or
via a drive-by-wire throttle system. However, a source of limitation in system performance can be attributed to engine combustion delays. Improvements could therefore be expected from a powertrain layout containing an electrical power source with a direct connection to the road wheels and an inherently smaller actuation delay, as present on hybrid electric vehicles.

1.6 Objectives of this Thesis

Given the desire to strive for more fuel efficient road vehicles, many studies in the literature are concerned with the development of hybrid electric powertrain systems. Much improvement to a vehicle’s overall performance can be achieved through use of a well designed control strategy. Dual hybrids allow the relative merits of both series and parallel configurations to be exploited, with a practical application demonstrated by Toyota with its commercially available Prius HEV. Given the increased complexity of dual hybrids when compared to conventional parallel or series hybrids, they have been used as a focus for much of the work completed here.

The control strategy implemented on the standard Prius performs favourably when compared to a comparative vehicle fitted with a conventional internal combustion engine (see for example Table 3.2). In an effort to further improve the overall efficiency loss from the powertrain a strategy was devised by Seiler and Schröder. The strategy was developed off-line using component efficiency data and implemented in simulation, although little data detailing its performance is available in the literature.

Although a significant amount of research has been completed on the control of hybrid vehicles, the topic is broad and allows work to be undertaken at a variety of levels, from simple switching logic to highly complex controllers which consider the fast transient dynamics of the system. It is proposed that the work here should bridge these categories, and be carried out in a systematic approach using model based control and data available from published sources. A major aim is to develop a generic methodology which requires minimal ad hoc tuning and pre-calculations. This will then enable the strategy to
be implemented and adapted easily for various vehicle configurations and sizes and form a basis upon which further research could be made.

A major advantage of an on-line controller when compared to an off-line method is that errors caused by unexpected events are corrected. Such a problem may occur if the operational capability of a component changes, with no compensation made by the strategy using an off-line approach. Changes in component operating conditions (e.g. ambient temperature) can also be easily accounted for. Also, a strategy using for example on-line efficiency data allows powertrain components to be changed easily in a plug and play manner by simply substituting the required efficiency data; off-line pre-calculations thus become superfluous. The use of gradient information to drive a system towards a minimum in terms of some predefined criteria is a sensible means of improving overall system performance. Given the availability of efficiency data for the powertrain components considered here and the desirable benefit of an on-line technique, an on-line strategy using gradient information obtained from the powertrain components has been investigated.

Studies have also been completed which use the low-level control of a hierarchical system to achieve some criteria in addition to achieving the demands set at a higher level. Here it is proposed to implement a technique which attempts to minimise the first torsional vibration mode of a powertrain. Previous work on shuffle control is based on conventional internal combustion engined vehicles using a variety of techniques such as computer control of spark timing, engine fuelling or via a drive-by-wire throttle system. However, applications for hybrid electric vehicles have not yet been considered. Although shuffle is historically not a particular problem for hybrid vehicles, benefits could be realised given the inclusion of fast-acting electrical machines within the powertrain thus permitting higher performance hybrid vehicles to be developed. Also, recent advances in 42V systems have led to the development of powertrains encompassing an electrical machine fitted to the output shaft of an internal combustion engine (see for example Streater, 2001). A controller developed to minimise shuffle for a HEV could therefore also be adapted for use on these powertrain configurations.
A dual-level control architecture is employed on the Toyota Prius, therefore in order to develop a system which could feasibly be implemented on the real vehicle the same structure has been maintained here. This also has the additional benefit in that the techniques of each level can be applied (either in part or full) on other vehicle configurations.

The objective of this thesis may therefore be summarised as:

*To investigate the on-line control of an advanced hybrid electric vehicle using a dual-level controller with combined criteria for energy efficiency and driveline vibrations.*

1.7 Overview of Thesis

Chapter 2 details the development of the hybrid electric model used as a basis on which subsequent work is implemented. The two main simulation types commonly used (forward and reverse calculation) are discussed, along with details of the major subsystems modelled.

Chapter 3 describes the implementation of a Baseline control strategy representative of the production Japanese specification Toyota Prius using the model developed in Chapter 2. The strategy is hierarchical with a high level Supervisor setting demands to be achieved at a lower level. The various (continuous) operating modes of the Prius are described and a comparison is made with a commercially available software tool which purports to perform similar simulations.

In Chapter 4 a Power Efficient Controller (PEC) is developed as a supervisor which seeks to minimise the overall efficiency loss of the system whilst maintaining acceptable battery charge/discharge cycling. The technique uses an on-line calculation of the rate of change of two system states (in this case engine speed and torque) to drive the system towards the most efficient steady state. The strategy is applied and analysed in simulation with comparisons made with the Baseline controller implemented in Chapter
3. Parameters within the PEC are optimised using two published techniques and their robustness to changes in drivecycle is assessed.

Chapter 5 considers the development of the low-level controllers. Here the single-input single-output (SISO) controllers used to track the demands set by the Supervisor are replaced using a multi-input multi-output (MIMO) controller developed using linear quadratic regulator (LQR) theory. The use of the LQR controller facilitates the implementation of additional terms within the cost function definition. Here the minimisation of the first torsional vibration mode (shuffle) is considered.

Chapter 6 investigates the use of non-linear optimal control theory to provide benchmarks for the controllers developed in Chapters 4 and 5. The technique employed is that proposed by Marsh (1992), whereby the numerical solution of a two point boundary value problem is sought to obtain the optimal control inputs for specified system and initial conditions. Solutions to two separate problems are sought. The first finds a solution to the problem of energy management whilst the second is focused on shuffle regulation. Comparisons can then be made with results obtained using the controllers described in Chapters 4 and 5.

Chapter 7 concludes the Thesis with a discussion of the work presented and possible areas of future research.
A model of the Toyota Prius hybrid electric vehicle has been developed using the
dynamic simulation package EASY5 (see The Boeing Company, 1997 for further detail)
and data available from published sources. EASY5 is a graphical user interface based
software tool used to model, analyse and design dynamic systems. Models are assembled
using a combination of primitive function blocks (e.g. integrators, gains, summers), pre­
defined application specific components (e.g. gears, shafts, inertias) or user defined
blocks of FORTRAN code. Specialised component libraries are available, with the
Ricardo Powertrain Library being of particular use in this application. EASY5 may also
be linked to other software tools such as MATLAB/Simulink developed by The

The model consists of a driver, hierarchical control system and vehicle components. In
this chapter, a description of the driver and vehicle components is given with material
relating to the control system considered in the following chapter.

2.1 Model Fundamentals

Although a number of vehicle simulation tools suitable for hybrid electric powertrain
modelling are commercially available (see for example Butler et al., 1997; NREL, 2001;
Rousseau et al., 2001; Swann, 1998), it is often preferred to develop a bespoke system
tailored for specific modelling requirements. Factors affecting this decision may include
the desired software language, previous work, and the financial implications associated
with a commercial product. Perhaps the most important factor however is whether there
is a tool available which is capable of achieving the results required.
There is currently one tool commercially available that is capable of modelling the Toyota Prius architecture – Advisor, as discussed by Evans and Stone (2000). However, Advisor is based on a reverse calculation (or backward-facing) simulation technique and as such is unsuitable for this study. In this context, a reverse calculation technique assumes that the vehicle meets the prescribed vehicle speed demand of the drivecycle at each prescribed time step of the simulation. The operating condition of each component of the vehicle is then calculated working backwards from the vehicle component (Figure 2.1a). If a particular component can achieve the demand requested by its neighbour, then the request is implemented at the start of the next time step. If however the request can not be achieved, the best that can be achieved is implemented with appropriate changes made to the operation of any affected components. The calculations continue at each time step until the simulation is complete. This technique is well suited to the design of control logic, although how the component implements the requirements are beyond its purview. As such, a reverse calculation technique is considered unsuitable given the objectives of this thesis whereby the design of continuous controllers that could be applied to a real vehicle are desired. In addition, a reverse calculation technique is unable to perform the analyses required for the simulation of driveline vibrations.

![Diagram of Forward and Reverse Calculation Techniques](image)

**Figure 2.1: Forward and Reverse Calculation Techniques**
Here, a *forward calculation* (or forward-facing) simulation technique has been employed (Figure 2.1b). This approach simulates the dynamic behaviour of vehicle and control systems and enlists control loops to set and correct the behaviour of the system. A driver model is included that seeks to modulate throttle and brake commands to follow the requirements of the drivecycle. The throttle signal is converted into a torque, which is passed down the driveline and converted to a force which ultimately moves the vehicle.

During simulation, equations of motion are solved at a particular point in time in order to calculate the acceleration of all moving components. Time-based integration is then used to calculate the velocity and position of these components at the start of the next time-step (Figure 2.2).

![Figure 2.2: Integration process during a simulation](image)

The time step may be fixed or variable depending on simulation requirements. Fixed-step methods, whilst requiring fewer calls to a model for a given time step are restricted to the same time step (usually small to maintain accuracy) for the entire simulation. Fixed-step methods may therefore require a larger number of evaluations of the model equations of motion for an entire simulation (including transient events) than required by a variable-step method. Various integration methods are available (e.g. Runge-Kutta, Huen, Euler) with a variable-step Stiff Gear method employed here as detailed by The Boeing Company (1997). This method is well suited to 'stiff' systems in which there is a wide range of eigenvalues. Here, connections between mechanical components are initially made using the representation of relatively stiff shafts in order to reduce simulation times when the emphasis is on power management, although analyses employing flexible
driveshafts are discussed in later chapters. The Stiff Gear integrator ignores the high frequencies of these shafts that would otherwise force the use of very small time steps, thus optimising the time to complete a simulation.

### 2.2 Toyota Prius model

A model of the Toyota Prius component architecture has been developed based on data available from published sources, with a specification given by Hodkinson and Fenton (2001) reproduced in Appendix A. A schematic of the Prius based on Sasaki (1998) and Sasaki et al. (1997) is shown in Figure 2.3.

![Figure 2.3: Toyota Prius architecture](image)

The model consists of a drivecycle which provides a vehicle velocity versus time demand to the driver model. The driver then interprets this demand as accelerator and brake inputs which are then fed to the supervisor element of the hierarchical control
system. The supervisor then sets component demands based on drivecycle requirements and component conditions (e.g. battery state of charge). Engine, generator and motor demands are then met through use of independently tuned single-input, single-output (SISO) low-level controllers. Here, proportional-integral (PI) control is applied. Full detail of the ‘baseline’ supervisor and low-level control system implemented is given in Chapter 3, and the remainder of this chapter concentrates on the representation of the vehicle hardware.

2.3 Component modelling

The system components are modelled individually and joined via appropriate connections. Mechanical connections between the components are made using torque inputs and velocity outputs, whilst electrical connections are in terms of requested and available power. Shafts are represented as torque producing components which use the relative velocity and displacement between each end of the shaft to develop a wind-up torque. I.e.

\[
T_s = K_s \Delta \theta + C_s \Delta \omega
\]  

(2.1)

The \( C_s \) term is incorporated so as to provide a convenient source of damping due to other elements not explicitly modelled (e.g. bearings).

The reduction gear is represented using standard single gear components available within EASY5, and accounts for backlash windup during operation.

2.3.1 Electrical machines

Both the motor and generator used in the Prius are permanent magnet synchronous electrical machines, hence only analysis of the motor shall be considered here. Advantages of permanent magnet machines for vehicle powertrain applications are their relatively high power density and efficiency when compared to alternative electric propulsion sources (e.g. brushed direct current machines, induction motors etc.). As at the time of modelling the electrical machines such components were unavailable as
standard within EASY5, the models were developed and implemented as user-defined components.

The base speed of a motor is the magnitude of the maximum rotor speed before core saturation occurs. Control of permanent magnet machines below this base speed has been available through use of published techniques for some time (see for example Krause and Wasynczuk, 1989). In order to extend the operating range past its base speed, the control task is complicated by the need to advance the phase of the applied three-phase sinusoidal currents. The relationship between torque output, angular velocity and current phase advance is multi-variable and non-linear in this extended region. As such, control in this region requires high frequency (of the order of 1 kHz) stator position information and a technique such as vector control in order to analyse the stator current (see for example Stewart and Kadirkamanathan, 1998). Given the high computational requirements of this control technique and the fact that such detailed component analysis is beyond the purview of this study, a more computationally efficient representation of the electrical machines has been employed here. Also, the time constants of controlled motors are of the order of 1 ms (see for example Krause and Wasynczuk, 1989). Given this relatively small influence on the torque response of the motor, these dynamics have been excluded here.

Considering positive motor speeds and torque’s only, between stator rotational velocities of zero and base speed, \( \omega_b \), the maximum torque available from the motor is a constant value, \( T_{\text{max}} \). For motor velocities above the base speed the torque output is limited by the rated power of the machine as shown in Figure 2.4.

For the Prius, the base speed of the motor is 98.44 rad/s (940 rev/min) and the maximum torque, \( T_{\text{max}} \), is 305 Nm. The motor is rated at 30 kW, hence the maximum torque for a particular motor speed, \( \omega_m \), can be obtained from the following equations:

\[
\begin{align*}
\omega_m &\leq \omega_b \quad T_m = T_{\text{max}} \\
\omega_m &> \omega_b \quad T_m = \frac{P_{\text{rat}}}{\omega_m}
\end{align*}
\]  

(2.2)  

(2.3)
In simulation, the motor component receives a torque demand from the low-level controller. If the torque request is within the allowable range as defined by Equations 2.2 and 2.3, the required torque demand is supplied, otherwise the maximum available is used. The torque, $T_m$, is then applied to an inertial representation of the motor ($I_m = I_g = 0.005 \text{ kgm}^2$) with the equation of motion described as follows:

$$\alpha_m = \frac{T_m - T_{mm}}{I_m}$$

(2.4)

The efficiency of the motor, $\eta_m$, has been tabulated as a function of speed ($\omega_m$) and torque ($T_m$) according to data published by NREL (2001), as shown for the positive speed and torque quadrant in Figure 2.5.
If a convention of positive power flow out of the battery is adopted (i.e. power is positive when motoring), the electrical power, $P_e$, of the component when motoring can be expressed as:

$$P_e = \frac{T_m \omega_m}{\eta_m} \quad (2.5)$$

and when generating, the power is calculated from:

$$P_e = T_m \omega_m \eta_m \quad (2.6)$$

It is assumed that the electrical machines operate equally for negative torque and speed regions, hence the analysis above can be applied to the other three quadrants of the machine thus replicating full motor and generator operation.
2.3.2 Battery

The battery was modelled as a combination of a charge reservoir and an equivalent circuit comprising an open circuit voltage source \( (V_{oc}) \) in series with an effective internal resistance \( (R_i) \) subjected to a load voltage, \( V_{bus} \) (Figure 2.6). This is again developed as a user-defined component within the EASY5 environment.

\[ V_{bus} \]

\[ V_{oc} \]

\[ + \]

\[ R_i \]

\[ I \]

\[ + \]

\[ - \]

**Figure 2.6: Equivalent battery circuit**

Interpolated look-up tables for the open circuit voltage and internal resistance are used to determine these parameters as a function of battery SOC. The parameters are given in Table 2.1 according to NREL (2001), with the internal resistance chosen according to whether the power requirement is to charge or discharge the battery. The parameters are then scaled according to the number of battery modules in series – 40 in this case.
<table>
<thead>
<tr>
<th>SOC</th>
<th>( R_i ) (charge)</th>
<th>( R_i ) (discharge)</th>
<th>( V_{oc} ) (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0235</td>
<td>0.0377</td>
<td>7.237</td>
</tr>
<tr>
<td>0.1</td>
<td>0.022</td>
<td>0.0338</td>
<td>7.4047</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0205</td>
<td>0.03</td>
<td>7.5106</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0198</td>
<td>0.028</td>
<td>7.5873</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0198</td>
<td>0.0275</td>
<td>7.6459</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0196</td>
<td>0.0268</td>
<td>7.6909</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0197</td>
<td>0.0269</td>
<td>7.7294</td>
</tr>
<tr>
<td>0.7</td>
<td>0.0197</td>
<td>0.0273</td>
<td>7.7666</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0203</td>
<td>0.0283</td>
<td>7.8078</td>
</tr>
<tr>
<td>0.9</td>
<td>0.0204</td>
<td>0.0298</td>
<td>7.9143</td>
</tr>
<tr>
<td>1.0</td>
<td>0.0204</td>
<td>0.0312</td>
<td>8.3645</td>
</tr>
</tbody>
</table>

Table 2.1: Battery internal resistance and open-circuit voltage per module

The total power that the battery can deliver to the motor and generator is limited according to the SOC, equivalent circuit parameters and the motor controller’s minimum allowable voltage. If the SOC is zero, the battery is unable to supply power. Also, \( V_{bus} \) cannot drop below either the motor’s minimum voltage or the battery’s minimum voltage, both of which are constant values of 60 V and 240 V respectively. Therefore in this case, the voltage must not drop below 240V and is battery limited. If the voltage is above these prescribed limits, the maximum power available will be observed when the voltage is half the battery’s open circuit voltage, \( V_{oc} \) as follows. If \( V_{bus} \) is the larger of the minimum motor voltage, minimum battery voltage or half the open circuit voltage, the maximum power limit of the battery can be calculated as:

\[
P_{bus} = V_{bus} \times \frac{V_{oc} - V_{bus}}{R_i}
\]  

(2.7)

Differentiating Equation 2.7 with respect to \( V_{bus} \) yields;
Development of the Hybrid Electric Vehicle Model

\[
\frac{dP_{\text{bm}}}{dV_{\text{bus}}} = \frac{V_{\text{oc}} - 2V_{\text{bus}}}{R_i} \tag{2.8}
\]

Hence the maximum power available in the general case occurs when \( V_{\text{bus}} = \frac{V_{\text{oc}}}{2} \), although here \( V_{\text{bus}} \leq 240V \) as defined by the minimum allowable battery voltage.

Power is defined as

\[ P = IV \tag{2.9} \]

Kirchoff's voltage law, applied along the equivalent circuit loop requires that:

\[ V_{\text{bus}} = V_{\text{oc}} - (R_iI) \tag{2.10} \]

Combining Equations 2.9 and 2.10,

\[ \frac{P}{I} = V_{\text{oc}} - (R_iI) \tag{2.11} \]

Multiplying Equation 2.11 by \( I \) yields:

\[ P = V_{\text{oc}}I - R_iI^2 \tag{2.12} \]

Two values of \( I \) are obtained from the solution of Equation 2.12, although the larger of the two values is not considered as this would require a larger current and hence a lower voltage. Power lost to heat is calculated as:

\[ P_{\text{lb}} = I^2R_i \tag{2.13} \]

Hence power loss is minimised through choice of the smaller solution of Equation 2.12, which gives:

\[ I = \frac{V_{\text{oc}} - \sqrt{V_{\text{oc}}^2 - 4R_iP}}{2R_i} \tag{2.14} \]
During charge, the maximum voltage of the battery, $V_{\text{max}}$, must not be exceeded, hence the magnitude of the maximum charge current, $I_{\text{me}}$, is defined as:

$$ I_{\text{me}} = \frac{V_{\text{max}} - V_{\text{oc}}}{R_i} \quad (2.15) $$

The battery current, $I$, is thus the smaller magnitude of Equations 2.14 and 2.15.

The total Amp-hours used since the start of the simulation, $A_u$, is then calculated as:

$$ A_u = \int (\eta_c I) \, dt \quad (2.16) $$

where $\eta_c$ is the coulombic efficiency of the battery which is a constant value of 0.6 during charge (given the absence of a thermal model) and a value of 1 during discharge. The initial SOC is accounted for by calculating an initial Amp-hours used, $A_i$, as:

$$ A_i = (1 - \zeta_i)A_m \quad (2.17) $$

The battery state of charge, $\zeta$, can therefore be calculated as:

$$ \zeta = \frac{A_m - A_u}{A_m} \quad (2.18) $$

2.3.3 Engine

The engine is modelled in a similar manner to the motor and generator, i.e. using steady state look-up tables in conjunction with an equivalent inertia. However, the engine model is available as a standard component within EASY5. The engine’s low-level controller (see Chapter 3 for further detail) supplies a throttle demand to the engine. A linearly interpolated table of torque as a function of engine speed and throttle position is then used to determine the torque supplied by the engine. According to Krenz (1985), engine
delays after the initiation of a throttle tip-in are in the region of 50–150 ms, hence this dynamic is represented using a first order lag of time constant 0.1 s. As with the motor, the engine torque is applied to an inertial representation of the engine \((I_e = 0.5 \text{ kgm}^2)\) to allow calculation of the engine’s equation of motion:

\[
\alpha_e = \frac{T_s - T_e}{I_e} \tag{2.19}
\]

A table of fuel use (g/s) has been derived from data published by Duoba et al. (2000), and is used to determine the engine’s efficiency (Figure 2.7), power loss and power used.

The fuel use in g/kWh is calculated as:

\[
f_c = f_s \times 3600 \times \frac{1000}{T_e \omega_e} \tag{2.20}
\]

The efficiency of an engine is defined as:

---

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If a fuel calorific value of 42000 kJ/kg is assumed (see for example Bosch, 1993), the engine efficiency can be calculated as:
\[ \eta_e = \frac{1000 \times 3600}{42000 \times f_c} \approx \frac{86}{f_c} \]

The power loss \( (P_{le}) \) and power used \( (P_u) \) by the engine are then calculated as:
\[ P_{le} = T_e \times \omega_e \times \left( \frac{1}{\eta_e} - 1 \right) \]
\[ P_u = \frac{T_e \times \omega_e}{\eta_e} \]

and the overall fuel consumption for a drivecycle can then be calculated as:
\[ f_d = \int (f_e) dt \times \frac{1}{\rho_f d} \]

where \( \rho_f = 750 \text{ kg/m}^2 \).

2.3.4 Planetary gearset

The planetary gearset is available as a standard component within EASY5, although definition of the equations of motion of the system are required in order to obtain the optimal solutions in Chapters 5 and 6. Each element of the planetary gearset is modelled as an equivalent rotating inertia which is acted upon by torques from external connections (via the sun, annulus and carrier gears) as well as internal mesh connections between the individual gears in the gearset. The calculation of the equation of motion of each element then enables the accelerations of the sun and annulus gears to be determined, the detail of which is committed to Appendix C. The basic ratio of the
epicyclic, $R_{sa}$, is given by the number of teeth on the sun gear (30) with respect to the annulus gear (78). Thus

$$R_{sa} = \frac{t_s}{t_a} = \frac{30}{78}$$

(2.26)

Hence having calculated the velocities of the sun and annulus gears, the velocity of the carrier can be determined using the general speed equation of the planetary gearset:

$$\omega_c = \frac{\omega_s - R_{sa} \omega_a}{1 - R_{sa}}$$

(2.27)

2.3.5 Tyre

The tyre model relates the torque developed by the drivetrain to the tractive effort (at the tread-ground interface) which ultimately accelerates the vehicle, and is available in this form as a standard component within EASY5. The driving torque applied to the wheel is transmitted to the tread via windup in the tyre carcass thus creating an effective slip between the driveline and the vehicle. This relationship between the wheel-tread slip and the tractive force is specified using the Pacejka 'magic' tyre model as described by Pacejka and Bakker (1993). A schematic of the wheel and tyre is shown in Figure 2.8.

![Figure 2.8: Wheel and tyre representation](image_url)
Slip, $S$, is defined as:

$$ S = \frac{\omega_w - \omega_T}{\omega_T} \quad (2.28) $$

The Pacejka model empirically relates the adjusted slip to tractive force as:

$$ F_T = D \sin \{C \tan^{-1}[B S - E(B S - \tan^{-1}(B S))]\} \quad (2.29) $$

where $B = 8$, $C = 1.65$, $D = 281.25$ and $E = 0.7$ as defined by default in the On-Highway tyre component of EASY5. Tyre rolling resistance torque is calculated as:

$$ T_R = (A_d + B_d v)R \quad (2.30) $$

where $A_d = 5$ Nm, $B_d = 0.5$ Ns and $R = 0.3$m, again as defined by default in EASY5. An equation of motion for the tyre thus yields:

$$ F_T R = K_T(\theta_w - \theta_T) + C_T(\omega_w - \omega_T) \quad (2.31) $$

And a similar analysis for the wheel gives:

$$ \ddot{\theta}_w = \frac{T_D - T_B - T_R - F_T R}{J_w} \quad (2.32) $$

It is worth noting at this stage that both driven and un-driven wheels are modelled so as to account for losses due to tyre rolling resistance.

2.3.6 Vehicle

The vehicle model available within EASY5 is represented by a mass with forces acting upon it. The aerodynamic drag force of the vehicle is calculated as:
Development of the Hybrid Electric Vehicle Model

\[ F_A = \frac{1}{2} \rho_a AC_d v_v^2 \]  

(2.33)

where \( \rho_a = 1.2 \text{ kg/m}^3 \) and \( AC_d = 0.5238 \text{ m}^2 \).

If the vehicle is ascending a slope of angle \( \Theta \) with the horizontal, an equation of motion for the vehicle can be formulated as;

\[
\dot{v}_v = \frac{\sum_{i=1}^{n} F_{n_i} - F_A - m_v \sin \Theta}{m_v} 
\]  

(2.34)

where \( \sum_{i=1}^{n} F_{n_i} \) is the sum of tractive forces from \( n \) tyres.

The above component models provide a suitable representation of the Toyota Prius for use in subsequent chapters. In the next chapter, the control system required to complement the component models developed here is formulated. Simulation of the complete system is then performed.
Chapter 3

Control Strategy Implementation

This chapter discusses the methodology and implementation of a control strategy representative of the Japanese specification Toyota Prius. This enables an analysis of the operation of the Prius to be made and also provides a benchmark for controllers developed in subsequent chapters.

The strategy is based on the principle that a proportion of the power required by the road wheels be supplied by the engine, as dictated by drivecycle demands and component operating conditions. The motor then supplies any additional power required. A description of the driver model is given and a hypothetical drivecycle is used in simulation to demonstrate the strategy and identify the various modes of operation inherent in the powertrain layout.

Results from the model and control strategy are then compared with results from Advisor, a commercially available software tool used to model the Prius.

3.1 Operating modes of the Toyota Prius

Due to the configuration of the Toyota Hybrid System (THS) architecture, the powertrain can operate in a number of different (but smoothly varying) modes depending on such parameters as battery SOC and drivecycle requirements.

The engine is controlled to operate in its most efficient range, and as such is shut down for low power requirement applications such as low speed operation or coasting downhill. In this case, the motor provides propulsion power to the road wheels using energy stored in the battery (Figure 3.1).
In normal driving conditions the engine provides propulsion power to the road wheels via the planetary gearset. This is achieved with a torque reaction on the sun gear by converting mechanical into electrical power using the generator. The electricity is then used to supplement the engine’s propulsion power through the motor with any excess used to charge the battery (Figure 3.2).

If high power is required by the road wheels (e.g. during acceleration), the electricity supplied to the motor via the generator can be supplemented with power from the battery, state of charge (SOC) permitting. This mode of operation is shown in Figure 3.3.
During deceleration, some of the vehicle’s kinetic energy is converted into electricity via the motor and used to charge the battery with any additional required braking force provided using the conventional mechanical method. The engine may be started at any vehicle speed, including when the vehicle is at rest by using the generator to change the engine’s speed and varying fuel and spark as necessary.

### 3.2 Driver model

Given the use of a forward calculation simulation technique a model of a driver is required. Here the driver model uses a proportional-integral control and a proportional control on the accelerator and brake pedals respectively in order to convert a drivecycle profile into pedal inputs. This representation has proved adequate for the task required here – i.e. to closely follow a defined vehicle velocity profile.

The brake pedal input is set proportional to the vehicle speed error if the error is less than $-5\text{m/s}$. The upper limit is set so as to allow engine braking to take place for gentle decelerations. During braking, the accelerator pedal proportional gain is set to zero, and the integrator element of the PI controller is reset and held at zero to prevent integrator windup. If the vehicle speed error is greater than or equal to $-5 \text{ m/s}$, the proportional control on the brake is set to zero, and the accelerator gains are set to non-zero. Such a representation has proved sufficient for the drivecycles considered here.
3.3 Implementation of control strategy

The control strategy of the Toyota Prius is implemented using a hierarchical methodology; a supervisor sets demands to be implemented at a lower level (Figure 3.4). The dynamic interpretation of pedal inputs determines the vehicle speed demand from the driver’s pedal inputs. Here the speed demand is assumed to be proportional to the pedal travel with switching logic applied to reject any simultaneous application of the two pedals. The supervisor uses the interpreted driver pedal inputs along with knowledge of the present operating condition of the driveline components to set demands for the engine speed, engine torque and motor torque. These demands are then achieved through use of independently tuned single-input single-output (SISO) controllers. It is assumed that engine torque can be rapidly controlled by an engine management system, so this is modelled by setting the throttle through use of a known engine map with a small time delay (0.001 seconds) to avoid algebraic loop errors.

A Baseline supervisory control has been developed based on knowledge of the maximum efficiency curve for the engine. As described in Chapter 2, the drag force on the vehicle is calculated as a function of the vehicle’s velocity.
With the rolling resistance torque of each wheel, $T_R$, and the aerodynamic drag force, $F_A$, defined by Equations 2.30 and 2.33 respectively, the total drag force, $F_d$, is defined as:

$$ F_d = 4\frac{T_R}{R} + F_A \tag{3.1} $$

The force required to accelerate the vehicle to the velocity prescribed by the drive cycle can be defined as:

$$ F_{ac} = \frac{v_i - v_c}{t_sp} \times m_v \tag{3.2} $$

If $R$ is the tyre rolling radius, $\eta_{fd}$ is the efficiency of the final drive (assumed to be 0.99 here), and $g_r$ is the combined final drive and reduction gear ratio ($g_r = 0.235294$), then the total torque required at annulus of the planetary gearset can be defined as:

$$ T_{an} = \frac{R \times g_r \times (F_{ac} + F_d)}{\eta_{fd}} \tag{3.3} $$

The rotational velocity required at the annulus is then:

$$ \omega_{an} = \frac{v_i}{g_r \times R} \tag{3.4} $$

According to Kelly et al. (2002), the Prius maintains the battery pack to within a tight SOC target during operation. Here, 0.6 is used as the target SOC, and as such the total system power required is then calculated as:

$$ P_{sys} = \omega_{an} T_{an} + (\phi(0.6 - \zeta)) \tag{3.5} $$

where a suitable value for $\phi$ was found by trial and error to be 70 kW.
The optimum engine speed is then determined using a linearly interpolated table of engine speed as a function of $P_{an}$ (Table 3.1) and the engine torque is determined by wide open throttle.

<table>
<thead>
<tr>
<th>Power (kW)</th>
<th>Engine speed (RPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>1200</td>
</tr>
<tr>
<td>10</td>
<td>1263</td>
</tr>
<tr>
<td>15</td>
<td>1843</td>
</tr>
<tr>
<td>20</td>
<td>2332</td>
</tr>
<tr>
<td>36</td>
<td>4000</td>
</tr>
</tbody>
</table>

**Table 3.1: Engine speed as a function of power required at the road wheels**

At low power demands (e.g. low vehicle speed and satisfactory battery SOC) the vehicle operates using power from the battery only. As the power requirement increases so does the supplementary power required from the engine, and as such non-zero speed and torque demands are set for the engine. Excess power not supplied by the engine is delivered via the motor as determined by its low-level controller. During braking the motor acts as a generator and converts a proportion of the kinetic energy of the vehicle into electrical current. Depending on the demands from the driver, braking may be assisted using a force applied to the vehicle representative of conventional friction braking. The resulting control system is thought to be broadly representative of that implemented on the Japanese specification Toyota Prius.

### 3.4 Simulation results

Figure 3.5 shows the vehicle speed demand and that achieved for a simulated Prius incorporating the Baseline strategy on a nominal 100 second drivecycle.
Figure 3.5: Vehicle speed demand and achieved for 100s drivecycle

With reference to Figures 3.6-3.8, the engine can be seen to remain off during the first 10 seconds of the simulation when the vehicle speed demand is zero and the battery SOC is satisfactory. At 10 seconds, an increase in vehicle speed demand to 5m/s is encountered. As this is a relatively low power requirement from the powertrain the engine remains off and the vehicle is propelled purely by electric means. At 20 seconds there is an increase in speed demand to 15m/s which gives a sufficient power requirement from the powertrain to necessitate supplementary power from the engine. The engine can be seen to run at a high engine speed and torque (Figures 3.7 and 3.8) during the transient period before reducing to a power sufficient to steadily charge the battery. This is also seen during the period 45 to 75 seconds where the vehicle speed demand increases from 15 to 25m/s. During the period 75 to 82 seconds, the vehicle speed demand requires the vehicle to decelerate. This is achieved using regenerative braking with the accompanying increase in battery SOC seen in Figure 3.6. The engine throttle is closed and fuelling is postponed during braking leading to a negative torque resistance, or engine braking. As the vehicle speed demand settles, the engine power delivered is sufficient to prevent the
battery from discharging without wasting energy given that the battery SOC is at an acceptable level. Regenerative braking is again seen between 90 and 93 seconds after which the vehicle returns to rest. According to Tojima et al. (1998), the battery management sets an upper limit on the SOC. Here, a value of 0.7 is used, hence the engine continues to operate until the end of the drivecycle as the battery SOC is below this upper threshold.

![Figure 3.6: Battery SOC for 100s drivecycle](image)

*Figure 3.6: Battery SOC for 100s drivecycle*
Figure 3.7: Engine speed for 100s drivecycle

Figure 3.8: Engine torque for 100s drivecycle
3.5 Comparison with Advisor

The above control (‘EASY5’ control) will now be compared with that of Advisor (ADvanced VehIcle SimulatOR) (refer to NREL, 2001 for further detail) – a vehicle simulation package developed in the MATLAB/Simulink environment by the National Renewable Energy Laboratory, Colorado USA. Advisor uses a reverse calculation technique with a view to providing a tool for quick analysis of the performance and economy of conventional, electric and hybrid electric vehicles. Advisor enables a multitude of vehicle configurations to be modelled, and is supplied with a number of predefined vehicles and drivecycles. One such model is that of the Japanese specification Toyota Prius which has been developed using a combination of quasi-static data obtained from published sources and that collected during laboratory tests.

The ECE 15 + EUDC test cycle is performed on chassis dynamometers to assess the emissions of light duty vehicles in Europe as detailed in EEC Directive 91/441/EEC (1991). The entire test procedure consists of four ECE 15 cycles followed by one EUDC segment (Figure 3.9) with the vehicle allowed to soak for at least six hours at a test temperature of 20-30°C prior to the test.

Figure 3.9: ECE 15 + EUDC test cycle
From 2000, engine idling prior to the test has been eliminated. The ECE 15 element of the test procedure was devised to represent city driving conditions (e.g. in Paris or Rome) and is characterised by low vehicle speed, engine load and exhaust gas temperature. The EUDC (Extra Urban Driving Cycle) was added to account for more aggressive, high speed driving modes. Table 3.2 provides a summary of the characteristics for both the ECE and EUDC segments.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Unit</th>
<th>ECE 15</th>
<th>EUDC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>km</td>
<td>4x1.013 = 4.052</td>
<td>6.955</td>
</tr>
<tr>
<td>Duration</td>
<td>s</td>
<td>4x195 = 780</td>
<td>400</td>
</tr>
<tr>
<td>Average Speed</td>
<td>km/h</td>
<td>18.7 (with idling)</td>
<td>62.6</td>
</tr>
<tr>
<td>Maximum Speed</td>
<td>km/h</td>
<td>50</td>
<td>120</td>
</tr>
</tbody>
</table>

**Table 3.2: ECE+EUDC test parameters**

Results have been collated for simulations performed using Advisor and EASY5 for the Toyota Prius models on the ECE 15 + EUDC drivecycle (Table 3.3). For interest, actual combined urban and extra urban fuel consumption figures for the Prius and a 2002 Toyota Corolla equipped with a 1.4 litre engine and manual gearbox have been added. The Corolla is of similar size to the Prius with the results highlighting the potential advantage of hybrid technology.

<table>
<thead>
<tr>
<th>Simulator</th>
<th>Economy (l/100km)</th>
<th>End SOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Advisor</td>
<td>5.82</td>
<td>0.583</td>
</tr>
<tr>
<td>Easy5</td>
<td>5.02</td>
<td>0.588</td>
</tr>
<tr>
<td>Prius</td>
<td>4.90</td>
<td>Not available</td>
</tr>
<tr>
<td>Corolla</td>
<td>6.71</td>
<td>Not applicable</td>
</tr>
</tbody>
</table>

**Table 3.3: Advisor and EASY5 model comparison results**
Given the coupled nature of the powertrain elements and the fact that in both cases the drivecycle demands were met to high degree of accuracy and similarity, only results obtained for the engine and battery need be examined (Figures 3.10-3.12).

With reference to Figure 3.10, a good correlation between Advisor and EASY5 can be seen. One immediately apparent difference is due to the engine switching strategy; in Advisor the engine is switched on immediately as engine start/stop functionality is not included whereas in EASY5 conditions dictate that the engine is not required in accordance with the real vehicle. This is reflected in the overall fuel consumption figures, with Advisor predicting a fuel consumption of 5.82 l/100km compared to that of 5.02 l/100km for EASY5. Other discrepancies can be seen at higher steady state vehicle speeds (after 800 seconds) where the strategy employed by EASY5 tends to run the engine at a slightly higher speed than that of Advisor. Figure 3.11 again shows a good correlation, although the steady state engine torque from EASY5 is typically lower than the corresponding torque in Advisor. This agrees with the difference in engine speed given the same overall power requirements.

![Figure 3.10: Engine speed comparison](image)
Figure 3.11: Engine torque comparison

Figure 3.12: Battery SOC comparison
A first examination of Figure 3.12 indicates differences in the battery SOC between the two simulation algorithms even though both are modelled in a similar manner. An investigation into the sensitivity of SOC to engine speed and torque showed significant differences apparent for relatively small changes in engine speed and torque. This is largely due to the non-constant (and non-linear) effect of changes in powertrain component efficiency, and indicates that benefits could be achieved through use of an integrated holistic approach to hybrid vehicle control strategy design. The comparison is further exacerbated by the difference in regenerative braking strategies employed by the two systems. The EASY5 model allows greater energy recovery, with this being most apparent on the final deceleration phase of the drivecycle (1172 to 1206 seconds). This is reflected in the lower engine torque (i.e. more negative) during deceleration from Advisor, with less power thus available for regeneration via the motor.

3.6 Consolidation

A baseline control strategy representative of the Japanese specification production vehicle has been developed and applied to the model of the Toyota Prius vehicle architecture. The strategy was implemented in simulation and results were derived using a prescribed drivecycle. Use of the engine to regulate the battery SOC condition whilst also attempting to minimise fuel consumption was demonstrated. This is achieved through operation of the engine on its curve of maximum efficiency determined as a function of the total power requirement at the road wheels.

The model incorporating the baseline strategy was compared with results obtained using the commercially available simulation tool Advisor. The results showed a good comparison between the respective engine speed and torque values with similar overall fuel consumption figures. Minor discrepancies were seen between the equivalent battery SOC results. This is due to small variations in the engine speed and torque values which have a relatively large affect on the SOC – the battery is sensitive to changes in engine operating condition due to the varying charge and discharge rates for different battery SOC values. The discrepancies may also be attributed to the different simulation techniques employed. Here the EASY5 model uses forward calculations whereas
Advisor is based on a reverse calculation technique. However, the close correlation provides confidence in the modelling and control techniques employed.

In subsequent chapters the Baseline controller is used as a reference to assess the performance of proposed novel controllers developed both at the upper and lower level.
Chapter 4

Development of the Power Efficient Controller

This chapter discusses the development and application of a control strategy for the Supervisory element of the control system which aims to minimise the overall power loss in the powertrain by continuously adapting the operating point of each power source towards the most efficient steady state. This strategy is termed the Power Efficient Controller (PEC).

A technique for calculating the required engine operating condition in terms of minimising the efficiency loss in a hybrid vehicle was described by Seiler and Schröder (1998). As outlined in Section 1.4, this technique involves the off-line calculation of the most efficient engine operating point as a function of drive shaft speed and torque. By contrast, the method described here involves the on-line minimisation of the total power loss in the vehicle’s powertrain. The derivation of the algorithm is discussed along with an analysis of its operation through use of a prescribed drivecycle. A technique for optimising the parameters within the algorithm using two published optimisation techniques is then discussed. A comparison is then made to the Baseline controller (Chapter 3) and the chapter concludes with an assessment of the algorithm’s robustness.

4.1 Derivation

If the power consumption of the drive components is assumed known and reasonably constant, this information can readily be used to prescribe a gradient to drive the total system to a more power efficient steady state operating point (as described by Kells et al. (2000)). The seven internal states of the system are the motor torque, $T_m$, and the speeds and torque’s at the epicyclic defined as follows:
Development of the Power Efficient Controller

\( x_1 \) Motor/annulus speed (rad/s)
\( x_2 \) Torque on annulus gear of epicyclic (Nm)
\( x_3 \) Generator speed (rad/s)
\( x_4 \) Torque on sun gear of epicyclic (generator torque) (Nm)
\( x_5 \) Engine speed (rad/s)
\( x_6 \) Torque on carrier of epicyclic (engine torque) (Nm)

The road load torque in the steady state is given by \( T_r = T_m - x_2 \). With \( R_{sa} \) defined in Equation 2.26, five constraints can be imposed for the steady-state:

\[
\begin{align*}
\text{(4.1)} & \quad x_1 x_2 + x_3 x_4 + x_5 x_6 = 0 \text{ (power balance)} \\
\text{(4.2)} & \quad x_2 + x_4 + x_6 = 0 \text{ (torque balance)} \\
\text{(4.3)} & \quad x_1 - R_{sa} x_3 + (R_{sa}-1)x_5 = 0 \text{ (speed in the epicyclic)} \\
\text{(4.4)} & \quad \delta x_1 = 0 \text{ (steady state vehicle speed)} \\
\text{(4.5)} & \quad \delta T_r = 0 \text{ (steady state road load torque)}
\end{align*}
\]

Here, the total power loss in the powertrain is expressed in terms of losses in the motor, generator and engine, i.e.

\[
\tilde{P}_p(x) = P_m(x_1, T_m, \eta_m) + P_g(x_3, x_4, \eta_g) + P_e(x_5, x_6, \eta_e) \quad (4.6)
\]

If the power flow into the battery is taken into consideration, a cost function can be prescribed of the form

\[
P_{lp} = P_m + P_g + P_e + \lambda (P_{in} - P_{out}) \quad (4.7)
\]

The change in power 'cost' can then be described in terms of gradients \( \frac{\delta P_{lp}}{\delta x_1} \) which can readily be calculated from efficiency maps.

---

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\[
\delta P_p = \sum_{i=1}^{5} \left[ \frac{\partial \delta P_p}{\partial x_i} + \lambda \frac{\partial (P_{in} - P_{out})}{\partial x_i} \right] \delta x_i
\]  
(4.8)

As the system is represented by seven degrees of freedom and five constraints, the \( \delta x_i \) can be described in terms of a change in two freedoms in the system – in this case engine speed (\( \delta x_5 \)) and torque (\( \delta x_6 \)). By differentiating the first three system constraints and applying the fourth and fifth, \( \delta x_1, \delta x_2, \delta x_3 \) and \( \delta x_4 \) can be expressed in terms of \( \delta x_5 \) and \( \delta x_6 \) (see Appendix B for further detail). Therefore Equation 4.8 can be rewritten in the form

\[
\delta P_p = \alpha \delta x_5 + \beta \delta x_6
\]  
(4.9)

Supervisory control is then imposed via steepest-descents relative to the gradients \( \alpha \) and \( \beta \):

\[
\dot{x}_5 = -\varphi \alpha
\]  
(4.10)

\[
\dot{x}_6 = -\varphi \beta
\]  
(4.11)

where \( \varphi \) is an acceleration parameter. The time rate of change of power ‘cost’ is then given by:

\[
\dot{P}_p = -\varphi (\alpha^2 + \beta^2)
\]  
(4.12)

which ensures cost reductions for any positive \( \varphi \).

The algorithm is implemented here using a suitably small constant value of \( \varphi \), which ensures slow adaptation of the system’s operating point. \( \lambda \) is not constant however – the cost associated with power drain in the battery is more sensibly related to its instantaneous state of charge:

\[
\lambda = k_1 (\zeta - \zeta_{opt}) + k_2 (P_{in} - P_{out})
\]  
(4.13)
where $k_1$ and $k_2$ are constants, and $\zeta_{opt}$ is the desired operating point for the battery considered. For a particular steady state vehicle speed an amount of charge is available for recovery through use of a suitable regenerative braking strategy. As such, $\zeta_{opt}$ is variable as determined by a look-up table as a function of vehicle speed. This ensures that fuel is not wasted through overcharging the battery, although care should be taken in the selection of $\zeta_{opt}$ so as not to compromise the vehicle’s performance. Table 4.1 shows the selection of $\zeta_{opt}$ assuming an overall desired battery state of charge of 0.6 and steady state vehicle operating conditions.

<table>
<thead>
<tr>
<th>Steady state vehicle speed (m/s)</th>
<th>Target SOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 10$</td>
<td>0.6</td>
</tr>
<tr>
<td>15</td>
<td>0.598</td>
</tr>
<tr>
<td>20</td>
<td>0.596</td>
</tr>
<tr>
<td>25</td>
<td>0.595</td>
</tr>
<tr>
<td>$\geq 30$</td>
<td>0.593</td>
</tr>
</tbody>
</table>

Table 4.1: Desired battery SOC

It is clear that only small changes in SOC set point are implemented for a variation in vehicle speed. This is largely due to the requirement that the vehicle’s performance should not be compromised, although small improvements in fuel consumption are made by not unnecessarily recharging the battery.

4.2 Bicubic interpolation

To calculate the rate of change of power ‘cost’ (Equation 4.8) the PEC requires not only component efficiency values, but also the rates of change of efficiency with respect to the relevant states. The efficiencies of the individual components are tabulated as functions of speed and torque, and may or may not be equally spaced.

A linear interpolation method could be used to calculate the efficiency at any operating point, and central differencing could be used to calculate the required gradients.
However, the nature of the PEC necessitates not only continuity in the component efficiencies, but also in the first derivatives. Linear interpolation of differentials using centred differencing does not guarantee this.

Numerous published techniques can be applied to an ordered table to guarantee smoothness up to the nth differential. In this application, a bicubic interpolation method described by Press et al. (1992) is used as this computes the value and first derivatives whilst also guaranteeing smoothness. Bicubic interpolation requires data at each grid point not only for the function $y(x_1,x_2)$, but also the gradients $\frac{dy}{dx_1}$, $\frac{dy}{dx_2}$ and the cross derivative $\frac{d^2y}{dx_1dx_2}$. A cubic interpolating function is then determined with the following properties: (i) the values of the function and the derivatives are reproduced exactly on the grid points, and (ii) the values of the function and its derivatives change continuously across the table. The smoothness properties of the function are tautologically 'forced' and have nothing to do with the accuracy of the specified derivatives. The interpolation will be more accurate with more accurate derivative information, but it will be smooth no matter what derivatives are specified.

In this application each data set is reordered into an equally spaced table to enable faster determination of the required grid-square during analysis. The derivatives for each coordinate are then found using centred differencing, and the coefficients used for the linear transform are pre-computed and tabulated to enable faster on-line execution.

4.3 Analysis of PEC system operation

The PEC has been applied as a supervisory control algorithm in simulation of the Toyota Prius Hybrid vehicle model developed in Chapter 2. A value for $k_1$ was chosen as 1500 based on the calculation that a deviation in SOC of 0.1 should be of the same order as the power loss from the motor and generator so as to instigate an increase in power output from the engine. A value of 0.003 kW$^{-1}$ was selected by trial and error for the damping term, $k_2$, so as to minimise oscillations in the engine speed and torque demands, and 0.2
was selected for the acceleration parameter, $\varphi$, so as to provide a suitably rapid response to deviations is battery SOC. Through simulation of a simple vehicle drive schedule (Figure 4.1) the PEC algorithm will be analysed to assess and comprehend its operation based on the results obtained.

![Vehicle velocity profile for 100s drive cycle](image1)

**Figure 4.1:** Vehicle velocity profile for 100s drive cycle

![Engine speed and torque for 100s drive cycle](image2)

**Figure 4.2:** Engine speed and torque for 100s drive cycle
An inspection of the engine speed and torque profiles dictated by the PEC (Figures 4.2) shows similar outputs to those obtained with the baseline strategy implemented (Figures 3.7 and 3.8).

As the vehicle accelerates, the demand for supplementary power increases, and as such the engine speed and torque can be seen to increase once a sufficient condition exists to start the engine (see for example 22 < t < 25 sec). This condition is largely driven by the deviation in battery state of charge from the ideal value of 0.6 (Figure 4.3).

![Battery SOC for 100s drivecycle](image)

**Figure 4.3: Battery SOC for 100s drivecycle**

Analysis of the power losses from the engine, motor and generator (Figure 4.4) throughout the drivecycle clearly indicate that the majority of power loss for the overall system is due to the engine. As such, the rate of change of power loss in the electrical components has a small (yet significant) effect on the choice of engine speed and torque. The largest changes are instigated by changes in battery state of charge which for a depleting SOC demands an increase in power output from the engine. The electrical component power losses then fine-tune the engine's operating condition to find the minimal overall power loss.
As would be expected, the rate of change of engine power loss with respect to engine speed (delpea) and engine torque (delpeb) is positive (i.e. seeks to reduce the engine speed and torque) during all steady state periods, although brief deviations into the negative region are seen during some transient events (Figure 4.5).

Figure 4.4: Power loss for 100s drivecycle

Figure 4.5: Engine power loss gradients for 100s drivecycle
An analysis of the drivecycle between 26 and 38 seconds allows us to investigate the operation of the PEC in further detail. The motor speed, torque, power loss and rate of change of motor power loss with respect to engine speed (delpma) are shown in Figure 4.6. Throughout this period the motor torque is negative, and as such the motor is charging the battery (NB. motor velocity is positive for a forward vehicle speed). The engine is operating at maximum speed and torque (Figure 4.2) following the acceleration manoeuvre from 5 to 15 m/s until approximately 27 seconds – this corresponds to the maximum power loss from the motor.

After this point the engine reduces speed and torque, and there is a corresponding reduction in motor power loss until approximately 28.5 seconds. After this point the generator is rotating at a slower speed (Figure 4.7) than the motor (due to the corresponding reduction in engine speed), and the constraints of the planetary gearset (Equations 4.1, 4.2 and 4.3) dictate an increase in the magnitude of the motor torque, and hence a corresponding increase in motor power loss. This is reflected in the rate of change of motor power loss with respect to engine speed, delpma (Figure 4.6).

Whilst the generator is operating at a greater speed than the motor, delpma is positive and hence seeks to reduce the engine speed. This corresponds to a reduction in motor power loss whilst the engine speed is reducing, i.e. continue to reduce the engine speed to minimise motor power loss. However, when the generator speed is less than the motor speed, the torque required by the motor increases, and as such there is an increase in motor power loss. As the engine speed continues to reduce, in order to reduce motor power loss delpma is negative and hence seeks to increase engine speed.

A discontinuity can be seen in delpma as the generator speed changes from being greater to than less than the motor speed. This is caused because of the definition of the rate of change of annulus torque with respect to engine speed, which is asymptotic when motor speed and generator speed are equal as given by Equation B17 in Appendix B.
Figure 4.6: Motor speed, torque, power loss and Delpma
Figure 4.7: Motor and generator rotational velocities for 100s drivecycle

The generator speed, torque, power loss and rate of change of generator power loss with respect to engine speed (delpga) are shown in Figure 4.8.

During section A the generator is charging the battery and the generator speed is greater than that of the motor. As such, to reduce the power and power loss of the generator the engine speed should be reduced. In section B the engine speed and hence the generator speed is reducing, and the constraint equations of the planetary gearset attempt to increase the speed of the engine. In section C the generator speed is less than that of the motor and power reduction again becomes the principal issue. As such, delpga attempts to reduce the speed of the engine. In Section D the generator is motoring, and as such delpga attempts to minimise the power loss by increasing the speed of the engine to reduce the power requirement of the generator.
The rate of change of motor and generator power loss with respect to engine torque (delpmb and delpgb respectively) are shown in Figure 4.9.
The rate of change of motor and generator power loss with a change in engine torque is governed principally by the torque constraint in the planetary gearset (Equation 4.2). As both the motor and generator torques are negative during this sample period (i.e. torque is flowing out of the planetary gearset), both delpmb and delpgb strive to reduce the torque from the engine. During periods when either the motor or generator torques are positive the respective power loss gradient attempts to increase the torque from the engine.

Analysis of the rate of change of power into the battery with respect to engine speed (battpa) and engine torque (battpb) confirm that increasing engine speed and torque increases the power into the battery (Figure 4.10). Brief deviations by battpa into the negative region are once again accounted for by the constraint equations in the planetary gearset.

Figure 4.9: Rate of change of motor and generator power loss with engine torque
4.4 Optimisation of PEC parameters

The cost associated with power drain in the battery (λ) requires a suitable choice for the weighting factors $k_1$ and $k_2$, and selection of a value for the acceleration parameter ($\varphi$) is also required. For a given drive cycle, $k_1$, $k_2$ and $\varphi$ can be selected such that certain criteria are optimised through use of a number of published techniques (e.g. genetic algorithms, simulated annealing, Tabu search etc.). In this study two such techniques are applied. The first is a simplex method developed by Nelder and Mead (1965), and the second is a reinforcement learning automaton described by Frost (1998). Both techniques require no knowledge of the mechanisms of the system and provide a solution based on final values of the relevant criterion. As such, existing vehicle models are used in this study, although on-line optimisation on a real vehicle is not beyond their purview. However, due to the nature of the theory behind the techniques global minima are not guaranteed. This problem is minimised by starting the optimisations from various initial conditions and comparing the results obtained from both methods. As the parameters are
optimised for a specific drivecycle their robustness is assessed through further simulation on a variety of schedules.

4.4.1 Cost function definition

For a given drivecycle the system should be controlled such that battery life is preserved while schedule requirements are met with overall minimal fuel consumption from the engine. To this end, a cost function to be minimised has been developed based on the deviation of the battery from its optimal value throughout the simulation, the battery’s end SOC and the total fuel used. The cost function, \( L \), is thus formulated in Equation 4.14.

\[
L = f + \psi \int (\zeta - \zeta_{opt})^4 dt + f_{SOC} 
\]

The cost is represented as an overall fuel use, and as such a battery state of charge weighting factor, \( \psi \), is required. Here a suitable value for \( \psi \) was found to be 25000. This ensures that the cost associated with the integral of SOC deviation is significant when compared to fuel use in order to prevent excessive SOC deviation from the optimum value. This is required in order to ensure that practical limitations of the battery are not exceeded.

As the optimisation techniques base their decisions on end values only, the deviation of the battery SOC from its optimum condition of 0.6 is assessed as the time integral of the error between the actual and optimum battery SOC. Also, as the battery SOC at the end of a simulation may or may not be equal to the optimal condition, a table of ‘corrected fuel use’ (\( T_{SOC} \)) is used (Table 4.2). More specifically, there is an increase in cost if the end SOC is below the desired value of 0.6 (Equation 4.15) and a decreased cost if the end SOC is above this optimum condition (Equation 4.16).

\[
\begin{align*}
\zeta \leq \zeta_{opt} & \quad f_{SOC} = T_{SOC} (0.6 - \zeta) \\
\zeta > \zeta_{opt} & \quad f_{SOC} = -\eta_{ne} \times T_{SOC} (1.2 - \zeta)
\end{align*}
\]
The electrical efficiency, $\eta_{el}$, accounts for losses incurred during battery discharge and is again based on an 'average' value for the powertrain – 0.9 was used here.

The method by which this table is generated is however not trivial – a certain change in battery SOC may be achieved in a variety of ways depending on vehicle speed, engine operation and time. This freedom leads to changes in the overall system efficiency and hence fuel use to raise the battery SOC by a certain amount. Here simulations were performed with initial battery SOC's at a variety of values below that desired with the engine operating at a speed of 2000 RPM and a torque of 35 Nm in order to represent an 'average' overall powertrain efficiency. As these corrections to the fuel use 'cost' are due to the strategy not achieving the same battery SOC as at the start of the drivecycle, during the calculation of the required correction table the vehicle speed was set to zero. In implementation, this is equivalent to an additional (stationary) drivecycle period with the engine running in order to charge the battery by the required amount.

<table>
<thead>
<tr>
<th>SOC</th>
<th>Cost Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>681.15</td>
</tr>
<tr>
<td>0.1</td>
<td>574.45</td>
</tr>
<tr>
<td>0.2</td>
<td>461.25</td>
</tr>
<tr>
<td>0.3</td>
<td>347.06</td>
</tr>
<tr>
<td>0.4</td>
<td>232.02</td>
</tr>
<tr>
<td>0.5</td>
<td>116.25</td>
</tr>
<tr>
<td>0.6</td>
<td>0</td>
</tr>
</tbody>
</table>

**Table 4.2: End battery SOC cost term**

4.4.2 Nelder-Mead optimisation

The Nelder-Mead optimisation technique is a simplex method as described by Nelder and Mead (1965). A simplex consists of a pattern of at least $n+1$ points enclosing a non-zero volume in n-dimensional space, and the Nelder-Mead optimisation technique iterates to find points that are more desirable for the simplex. The current points are used
to determine the direction of the iteration, and no gradient information is required. The optimisation terminates either once a specified error tolerance has been satisfied, or the maximum number of iterations has been exceeded. In order to ensure that the global minimum has been determined the optimisation is started from various initial conditions.

Using the hypothetical drivecycle shown in Figure 3.5, the parameters $k_1$, $k_2$ and $\phi$ have been used as input parameters for the Nelder-Mead optimisation. Initial values for the optimisation were set as $k_1 = 1500$, $k_2 = 0.003$ and $\phi = 0.2$. Figures 4.11-4.14 show the change in $k_1$, $k_2$, $\phi$ and cost for each optimisation iteration.

![Figure 4.11: $k_1$ for each Nelder-Mead optimisation iteration](image-url)
Figure 4.12: $k_2$ for each Nelder-Mead optimisation iteration

Figure 4.13: $\varphi$ for each Nelder-Mead optimisation iteration
The results from the optimisation are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Parameter/value</th>
<th>Initial value</th>
<th>Final value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_1$</td>
<td>1500</td>
<td>1305.8</td>
</tr>
<tr>
<td>$k_2$</td>
<td>0.003</td>
<td>0.0014</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.2</td>
<td>0.577</td>
</tr>
<tr>
<td>Cost</td>
<td>87.96</td>
<td>78.57</td>
</tr>
<tr>
<td>Final Battery SOC</td>
<td>0.593</td>
<td>0.595</td>
</tr>
<tr>
<td>Fuel Use (g)</td>
<td>80.07</td>
<td>76.23</td>
</tr>
<tr>
<td>SOC Deviation</td>
<td>$3.89 \times 10^{-5}$</td>
<td>$1.138 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 4.3: Nelder-Mead optimisation results

With reference to Table 4.3 it can be seen that the cost is significantly reduced (by 10.7%) when the optimised parameter values are used as opposed to ‘sensible estimates’.
This is reflected with an increase in the end battery SOC and a reduction in the overall fuel use for the drivecycle. This can be largely attributed to the change in SOC deviation between the two sets of parameters – using the optimised values the deviation is significantly reduced. The optimised parameters thus allow a sufficient deviation for battery assistance during the drivecycle without excessive discharge into its more inefficient region. Excessive restriction in battery use is avoided as this leads to an increase in fuel use as the degree of hybridisation diminishes and the vehicle tends towards a purely internal combustion engined configuration.

4.4.3 CARLA optimisation

A Continuous Action Reinforcement Learning Automaton (CARLA) as described by Frost (1998) has been employed to determine optimum values for the parameters $k_1$, $k_2$ and $\varphi$ in the PEC controller for any given drivecycle. A learning automata is a general-purpose stochastic optimisation technique for solving search and optimisation problems and finds optimal solutions based on probability density within a predefined range. A reinforcement scheme is the heart of the learning automaton and is the mechanism used to adapt the probability distribution. Reinforcement learning automaton use outputs from a random or unknown environment to improve some predefined cost function, as shown for a standard case in Figure 4.15.

![Figure 4.15: Learning automaton](image)

Traditional learning automata consist of a finite number of discrete actions, with one particular problem being that there is an inherent limitation in the thoroughness of the search within the defined action space. It is possible that optima may be missed if they
lie between action points, and increasing the density of the action points significantly slows the learning process. The technique proposed by Frost aims to overcome this problem by replacing the discrete representation of the action space with a continuous probability distribution. The technique demonstrates benefits in comparison with discrete automata including faster learning and global optimum location/local optima avoidance within the defined action space.

Figures 4.16-4.18 show the iteration histories of the CARLA probability distributions for $k_1$, $k_2$ and $\varphi$ again using the prescribed drivecycle introduced in Section 3.3. The parameters were optimised to values of $k_1 = 1327.1$, $k_2 = 0.0014$ and $\varphi = 0.570$ and show a good comparison with the results obtained using the Nelder-Mead technique.

![Figure 4.16: Iteration history of a CARLA probability distribution for $k_1$](image)

Figure 4.16: Iteration history of a CARLA probability distribution for $k_1$
Figure 4.17: Iteration history of a CARLA probability distribution for $k_2$

Figure 4.18: Iteration history of a CARLA probability distribution for $\phi$
4.5 Comparison with Baseline Supervisor control

In order to assess the performance of the PEC algorithm, a comparison has been made with the Baseline developed in Chapter 3 using the prescribed 100 second drivecycle (Figure 4.19) with the battery SOC, engine speed and engine torque results shown in Figures 4.20-4.22.

![Vehicle speed demand and achieved comparison](image_url)

**Figure 4.19: Vehicle speed demand and achieved comparison**
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Figure 4.20: Battery SOC comparison for 100s drivecycle

Figure 4.21: Engine speed comparison for 100s drivecycle
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Figure 4.22: Engine torque comparison for 100s drivecycle

<table>
<thead>
<tr>
<th>Strategy</th>
<th>Fuel use (g)</th>
<th>SOC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline</td>
<td>65.71</td>
<td>0.595</td>
</tr>
<tr>
<td>PEC</td>
<td>76.23</td>
<td>0.595</td>
</tr>
</tbody>
</table>

Table 4.4: Baseline and PEC results for the 100s drivecycle

With reference to Table 4.4, it is evident that the PEC performs poorly when compared to the Baseline controller. Analysis of Figures 4.21 and 4.22 shows that the Baseline controller generally aims to minimise the engine speed and operate at WOT where the engine is most efficient. Conversely, the PEC controller tends to follow a high speed, low torque approach for the engine. This initially appears to be an odd result considering that the PEC strives for maximum efficiency, but can be explained as the selection of the parameters within the PEC that yield a reduction in fuel consumption also tend to exacerbate fluctuation in the engine torque. This is a particular problem at high vehicle speeds with the acceleration gain, $\phi$, having a strong influence. To prevent rapid battery
SOC depletion during vehicle acceleration manoeuvres a high \( \phi \) is required. However, as the electrical components switch from charging to discharging and vice versa, their influence on the rate of change of engine torque is sufficient to cause fluctuations.

As the PEC is based on theory which assumes a steady state operating condition that slowly adapts towards the most efficient operating condition, the problems associated with a high choice of the acceleration factor (\( \phi \)) are not unexpected. However, for drivecycles which have large sections of steady-state operation the PEC is seen to outperform the Baseline strategy as exemplified in Table 4.5.

<table>
<thead>
<tr>
<th>Steady Velocity (m/s)</th>
<th>Baseline</th>
<th></th>
<th></th>
<th>PEC</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fuel use (g)</td>
<td>End SOC</td>
<td>Equiv. Fuel (g)</td>
<td>Fuel use (g)</td>
<td>End SOC</td>
<td>Equiv. Fuel (g)</td>
</tr>
<tr>
<td>10</td>
<td>19.89</td>
<td>0.590</td>
<td>31.52</td>
<td>26.40</td>
<td>0.598</td>
<td>28.74</td>
</tr>
<tr>
<td>15</td>
<td>37.31</td>
<td>0.588</td>
<td>51.26</td>
<td>39.40</td>
<td>0.597</td>
<td>42.89</td>
</tr>
<tr>
<td>20</td>
<td>65.07</td>
<td>0.584</td>
<td>83.67</td>
<td>77.79</td>
<td>0.596</td>
<td>82.44</td>
</tr>
<tr>
<td>25</td>
<td>93.93</td>
<td>0.599</td>
<td>95.55</td>
<td>90.36</td>
<td>0.602</td>
<td>88.38</td>
</tr>
<tr>
<td>30</td>
<td>144.62</td>
<td>0.598</td>
<td>146.94</td>
<td>139.73</td>
<td>0.593</td>
<td>147.87</td>
</tr>
</tbody>
</table>

**Table 4.5: Steady state simulation results**

The results for the steady state simulation’s were compiled using a 100 second drivecycle with an acceleration period up to the steady-state velocity occurring between 5 and 15 seconds, and a deceleration period to zero between 92 and 98 seconds as shown in Figure 4.23.
With reference to Table 4.5 it can be seen that the Baseline controller generally outperforms the PEC in terms of outright fuel consumption, although the final battery SOC tends to be lower. The equivalent fuel consumption is calculated based on the fuel used with adjustments made according to Equations 4.15 and 4.16. A comparison of these values for all but the 30 m/s steady state velocity shows improvements from the PEC. The acceleration manoeuvres have a reduced overall effect on the drivecycle, and as such the parameter selection for the PEC is able to concentrate on the steady state rather than ensuring minimum battery discharge during acceleration events. For the 20 m/s case engine speed and torque, and battery SOC comparisons between the Baseline and PEC controllers are shown in Figures 4.24-4.26
Figure 4.24: Engine speed comparison for 20 m/s steady-state drivecycle

Figure 4.25: Engine torque comparison for 20 m/s steady-state drivecycle
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Figure 4.26: Battery SOC comparison for 20 m/s steady-state drivecycle

With reference to Figures 4.25 and 4.26 it can be seen that the PEC generally dictates a higher engine speed and lower engine torque than the Baseline to achieve a similar engine power output. Given the efficiency map of the engine (Figure 2.7), this mode of operation initially seems contrary to what would be expected. However, analysis of the overall power loss for the two controllers (Figure 4.27) shows a reduced power loss for the PEC during the steady state period even though there is a corresponding increase in the current flowing into the battery. This is largely attributed to the PEC controller dictating an engine speed such that the generator is operating in a more efficient condition thus reducing the overall power loss and increasing the current flow into the battery. This demonstrates the worth of PEC during steady-state operating conditions and the advantage of a holistically designed control strategy.
Figure 4.27: Overall power loss comparison for 20 m/s steady-state drivecycle

For the 30m/s event the Baseline slightly outperforms the PEC. Here the bias of the acceleration manoeuvre has a significant influence on the selection of the parameters within the PEC.

As shown in the following section, the PEC also provides favourable results on drivecycles with lower maximum vehicle speed requirements.

4.6 Robustness

To assess the robustness of the PEC to sub-optimal selection of the parameters $k_1$, $k_2$ and $\phi$, the optimal parameters values obtained for the 100 second drive cycle (Section 4.5) were used in simulation of a hypothetical 50 second drive cycle (Figure 4.28). This is to assess how sensitive the parameter selection is to changes in drive cycle profile, and will identify whether some form of on-line optimisation would be desirable. The results using the parameters obtained for the 100 second drive cycle are termed 'Unoptimised PEC'.

and are compared to simulation results using parameters optimised for this 50 second drivecycle (Optimised PEC). The results are shown in Figures 4.29–4.31 and Table 4.5, where results for the Baseline drivecycle are also shown for comparison.

![Diagram](image-url)

**Figure 4.28: Hypothetical 50 second drivecycle**

![Diagram](image-url)

**Figure 4.29: Engine speed responses for 50s drivecycle**
Figure 4.30: Engine torque responses for 50s drivecycle

Figure 4.31: Battery SOC comparison for 50s drivecycle
<table>
<thead>
<tr>
<th>Strategy</th>
<th>Fuel Use (g)</th>
<th>SOC</th>
<th>Equiv. Fuel use (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unoptimised PEC</td>
<td>31.52</td>
<td>0.595</td>
<td>37.33</td>
</tr>
<tr>
<td>Optimised PEC</td>
<td>24.64</td>
<td>0.595</td>
<td>30.45</td>
</tr>
<tr>
<td>Baseline</td>
<td>16.52</td>
<td>0.586</td>
<td>32.79</td>
</tr>
</tbody>
</table>

**Table 4.5: Robustness of optimised strategy**

With reference to Table 4.5 it can be seen that the PEC controller using parameters optimised for the 100 second drivecycle (Unoptimised PEC results) perform badly compared to both the optimised PEC and Baseline strategy results. The Equivalent Fuel use figure gives a measure of the overall fuel use and takes account of end battery SOC deviations from the start value of 0.6. This is calculated as the actual fuel use plus the additional fuel use according to a linear interpolation of Table 4.2 (see Section 4.5.1 for further detail). A comparison of the results for the optimised and unoptimised PEC strategies using Figures 4.29-4.31 shows little difference in the battery SOC values. However, inspection of the engine torque and speed curves indicate that to obtain virtually the same engine power output the unoptimised case tends towards a higher speed and lower torque when compared to the optimised case. The improved efficiency of the optimised case is then as might be expected given that the engine is most efficient close to WOT.

The optimised PEC also performs better than the Baseline strategy when Equivalent Fuel Use values are compared. This is due to the PEC dictating that regaining battery SOC is of high importance whereas the Baseline strategy is penalised for a low final SOC. A further advantage of the higher end SOC that occurs with the PEC is that the vehicle is better prepared for future events. For example, steep gradients or rapid vehicle acceleration demands may necessitate a high current drain from the battery which would be more limited with the Baseline strategy’s final SOC condition.

The results indicate that although the PEC is able to achieve reduced overall fuel consumption when compared to the Baseline this is largely dependant on the selection of the PEC parameters. This highlights the importance of *a priori* drivecycle knowledge or
the use of some form of on-line learning algorithm in order to achieve improved efficiency. Also, it is interesting to note that although in the 100 second 'steady-state' drivecycle (Section 4.5) overall power loss was minimised through use of a higher engine speed and lower engine torque when compared to the Baseline, this is not the case here. The 'unoptimised' PEC dictates engine operation of this type here, which is contrary to the optimised case. This leads to a clear reduction in overall system performance and again indicates that either good a priori drivecycle knowledge or on-line optimisation would be beneficial.

4.7 Conclusions

A supervisory control algorithm based on steady state efficiency criteria has been developed and applied in simulation to the hybrid vehicle model formulated in Chapter 2. The algorithm seeks to minimise the overall power loss in the powertrain by continually adapting the steady state operating point of the engine and is applied using a steepest descents technique. The technique is based on the definition of a system using constraint equations for the Prius architecture, although it could equally be applied to other vehicle configurations. A 'cost' is defined here as the overall power loss from the electrical components with an additional term relating to the state of charge of the battery. As such, additional elements (e.g. gearbox efficiency) could easily be incorporated if the relevant component data was available. The PEC thus provides a fundamentally based control algorithm which optimises the operation of the system on-line with a view to minimising the overall power loss.

The mechanism by which the PEC algorithm operates to reduce the overall power loss within the powertrain is complex, although some basic behaviour is worth clarifying. As would be expected, the majority of the power loss within the powertrain is attributed to the engine, and as such the rate of change of power loss with respect to engine speed and torque seeks to reduce the engine speed and torque respectively. The major prevention to this is due to the battery SOC requirement that requires engine power during steady state and 'low' SOC to provide a positive current into the battery. Thus a balance exists between minimising fuel use and charging the battery. The power losses due to the motor
and generator have a relatively small yet significant effect on the overall minimal power loss by fine tuning the operating point of the engine.

The PEC algorithm requires the selection of three parameters. The use of an optimisation method in this selection process was shown to have a significant effect on the performance of the PEC with a clear indication of the significance of a priori drivecycle knowledge being apparent. To gain maximum performance from the PEC the parameters should be optimised to a particular drivecycle. This could perhaps be achieved using an on-line optimisation procedure with GPS information being used to identify frequently used journeys. Also, application could be found on vehicles with well-defined operating schedules such as buses or some off-highway vehicles.

To minimise efficiency losses the battery SOC must be regulated during acceleration events, i.e. engine power should increase rapidly to prevent excessive discharge from the battery. This requires a swift reaction from the PEC to SOC deviations, and as such the parameters within the PEC must be set accordingly. The PEC is only likely to be fully optimal under slowly varying conditions, and as such performs poorly when compared to the Baseline controller on drivecycles with significant transient events. However, for steady-state operation overall power loss is reduced with the PEC as it finds a more efficient operating condition. In the example drivecycle considered this is achieved in a manner contrary to what might be expected from examination of the engine efficiency map. The most efficient operating condition is close to WOT although the PEC dictates a higher engine speed and lower engine torque to achieve approximately the same power output. In this mode of operation the generator operates more efficiently and hence the power loss from the generator is reduced and the current flow into the battery is increased.

In general, the Baseline controller performs well, although performance improvements in terms of energy efficiency have been seen from the PEC algorithm operating in the steady-state. The performance of the PEC suffers during transient events and benefit could possibly be made using a combination of the Baseline strategy for the load following requirements and the PEC for the charge sustaining requirements of a
drivecycle. The optimisation of component parameters would again be imperative to ensure maximum overall efficiency. A further benefit of the on-line approach adopted by the PEC is that a large amount of flexibility is available with changes to components easily accommodated without the need for any pre-calculations.
Chapter 5

Development of a Low-Level Controller

This chapter discusses the development of a multi-input multi-output (MIMO) controller as an alternative to the independently tuned classical low-level controllers implemented in previous chapters. The controller is based on linear optimal control theory, and more specifically is a linear quadratic regulator (LQR). LQR was chosen over other control techniques (e.g. fuzzy logic, sliding mode) as it is a convenient means of providing an optimal solution (provided a linear system model is available). Also, additional elements can easily be incorporated into the definition of the cost function.

The controller is extended to balance the demands set by the PEC with some other criteria through choice of a suitable cost function. Here, the minimisation of the first torsional vibration mode (shuffle) is considered. Although shuffle is not specifically a problem with the Toyota Prius, if higher performance hybrid vehicles were to be introduced (which is certainly feasible) it may become an issue. Also, the technique could equally be applied to vehicles fitted with 42V integrated starter/generators.

The chapter begins with the development of a state-space model of the hybrid powertrain system in a reduced form, including a correlation between this and the full non-linear (EASY5) vehicle model. Using this reduced order model, an LQR low-level controller is developed and expanded to minimise shuffle.

5.1 Development of a reduced order model

In order to develop an LQR controller a linear state-space system model is required. If the shafts connecting the motor, engine and generator are initially considered rigid, the inertias of the relevant power sources can be added to those of the planetary gearset for
the calculation of the state matrices. The reduction gear and final drive can also be considered rigid, and hence modelled simply as a gain between the planetary gearset and the driveshafts. If at this stage the driveshafts are also considered rigid and with low inertia, the wheels can be considered as an appropriate addition of inertia to the respective component of the planetary gearset (i.e. the annulus). The torque developed by the drivetrain is transmitted to the vehicle through windup in the tyre carcass. The tyre deforms as it rotates and creates an effective slip between the driveline and the vehicle, and hence the torque transmitted to the vehicle is a function of the tyre slip (defined by Equation 2.28).

The planetary gearset (and rigidly connected components) can be modelled using two states, and hence the full vehicle can be modelled in reduced form using three states, as shown in Figure 5.1.

![Figure 5.1: Reduced Model Schematic](image)

The states are defined as:

- $x_1$ Generator velocity (rad/s)
- $x_2$ Motor/wheel velocity (rad/s)
- $x_3$ Equivalent rotational vehicle velocity (rad/s)

An analysis of the equations of motion of the planetary gearset yields equations for $\dot{x}_1$ and $\dot{x}_2$, the detail of which is again committed to Appendix C.
With the rolling resistance torque of each wheel and the aerodynamic drag force defined by Equations 2.30 and 2.33 respectively, the total drag torque, \( T_d \), is defined as:

\[
T_{de} = 4 \times T_r + F_A \times R
\]  

(5.1)

Assuming the torque transmitted to the vehicle is proportional to the tyre slip, the acceleration of the vehicle is a function of tyre slip.

\[
x_3 = \frac{b^*_t}{J_v} S - \frac{T_{de}}{J_v}
\]  

(5.3)

\[
\dot{x}_3 = \frac{b^*_t}{J_v} \left( x_2 - x_3 \right) - \frac{T_{de}}{J_v}
\]  

(5.4)

\[
\ddot{x}_3 = \frac{b^*_t}{J_v} \left( x_2 - x_3 \right) - \frac{T_{de}}{J_v}
\]  

(5.5)

In the linear approximation the tyre longitudinal force is proportional to the slip velocity, yielding the following expression for the vehicle motion:

\[
\dot{x}_3 = \frac{b_t}{J_v} \left( x_2 - x_3 \right) - \frac{T_{de}}{J_v}
\]  

(5.2)

Therefore if \( b_t^* \) is a constant, the tyre damping, \( b_t = \frac{b_t^*}{x_3} \).

A comparison of the vehicle acceleration responses for the full and reduced order models for a stepped input in motor torque from 0 to 100 Nm is shown in Figure 5.2.

The responses in Figure 5.2 show a good correlation between the reduced and full models with the very small deviations being attributed to the differences (e.g. gear backlash, simplified tyre model) introduced using the simplified representation.
5.2 Application of LQR Control

As is well known, a Linear Quadratic Regulator is a method of designing an optimal solution to the problem of regulating (the system states to zero in) a linear system based on a quadratic cost function (see for example Bryson and Ho, 1975). The control system assumes that all system states are available (or can be inferred), and is a multi-input multi-output (MIMO) technique.

If $\dot{x} = f[x(t), u(t)]$ and the error, $\varepsilon$, we wish to minimise is defined as:

$$\varepsilon = Ex + Fu$$

$$\varepsilon^2 = (Ex + Fu)^T(Ex + Fu)$$

$$= (x^TE^T + u^TF^T)(Ex + Fu)$$

$$= x^T(E^TE)x + x^TE^Fu + u^TF^Ex + u^T(F^Tu)$$

$$= x^T(E^TE)x + x^TE^Tu + x^TE^Fu + u^T(F^Tu)$$

$$\varepsilon^2 = x^T(E^TE)x + x^T(2E^TF)u + u^T(F^TF)u$$

Figure 5.2: Full and reduced model acceleration response
Therefore, if the cost, \( L \), is set as the time integral of the error squared:

\[
L = \int_0^\infty (x^T E^T E) x + x^T (2E^T F) u + u^T (F^T F) u) \, dt
\]

\[
L = \int_0^\infty (x^T Q x + x^T N u + u^T R u) \, dt
\]  

(5.6)

Where,

\[
Q = E^T E
\]

\[
N = 2E^T F
\]

\[
R = F^T F + \gamma
\]

and \( \gamma \) represents the additional cost directly associated with control input use.

If the system is considered linear, an optimal gain matrix, \( K \), can be derived by solving the algebraic Riccati equation (see for example Anderson and Moore, 1971) such that:

\[
u = -Kx
\]  

(5.7)

In this application the LQR controller is required to track the demands set by the PEC, rather than act as a simple regulator. Hence ‘pseudo states’ are added to the reduced order model to represent the demands of the PEC and the vehicle’s drag torque, and thus enable a cost function to be prescribed to minimise the error between the pseudo states and the actual vehicle speed, engine speed and engine torque values. The pseudo states are represented by first order lags with time constants of \( 10^4 \) seconds, hence their time rates of change are defined by

\[
\dot{x}_i = -1 \times 10^{-4} x_i
\]  

(5.8)

Modelling the drag torque as a pseudo state rather than a conventional input allows the optimal gain matrix to be developed with knowledge of the drag torque. To verify this a cost function can be formulated as the square of the error between the vehicle speed and the vehicle speed demand:
Development of a Low-Level Controller

\[ L = \int_0^\infty [(x_3 - x_5)^2 + u^T \gamma u] dt \]  

(5.9)

Given the representation of the tyre as a variable damper, the state matrices and corresponding optimal gain matrix are derived for various vehicle speeds. In implementation the optimal gain matrix is interpolated as a function of vehicle speed. The vehicle speed and component torque responses are shown in Figures 5.3 and 5.4 respectively where the LQR controller can be seen to track the desired vehicle speed whilst the choice of \( \gamma \) has ensured that the component torques are kept within their operational boundaries. However, a consequence of this choice of \( \gamma \) is that the vehicle has slow response during the transient period. This indicates that in order to achieve some acceptable level of vehicle acceleration following a tip-in event the cost function must be defined such that component saturation is permitted.

![Vehicle speed responses](image)

Figure 5.3: Vehicle speed responses
The LQR controller can then be expanded to track a vehicle speed demand, engine speed demand and engine torque demand thereby enabling the classical low-level control system to be replaced by an LQR based system. A cost function can thus be formulated as:

$$L = \int_0^\infty \left[ c_1 (x_3 - x_5)^2 + c_2 \left( \frac{x_2 - R_{sa} x_1}{1 - R_{sa}} - x_6 \right)^2 + c_3 (u_2 - x_7)^2 + u^T \gamma u \right] dt$$  \hspace{1cm} (5.10)$$

where the following notation has been used:

- \( x_5 \)  Vehicle speed demand 'pseudo' state (rad/s)
- \( x_6 \)  Engine speed demand 'pseudo' state (rad/s)
- \( x_7 \)  Engine torque demand 'pseudo' state (Nm)

The first term in the cost function refers to the error between the vehicle speed and the demand. The second is the engine speed error (calculated according to Equation 2.27) and the third is the engine torque error. The cost function parameters are determined such
that a balance is achieved between these three elements, with the cost associated with control use ensuring that the torque inputs from the motor, engine and generator are not excessive. Here the values are set as follows:

\[
c_1 = 15 \quad c_2 = 9 \quad c_3 = 1 \quad \gamma = \begin{bmatrix} 1e^{-5} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1e^{-5} \end{bmatrix}
\]

Results for vehicle speed, engine speed and engine torque for the full EASY5 model with an LQR based low-level control system are shown in Figures 5.5 to 5.7, where it can be seen that the engine speed and engine torque closely follow their respective demands.

Figure 5.5: Vehicle Speed profile for 30s drivecycle
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Figure 5.6: Engine Speed profile for 30s drivecycle

Figure 5.7: Engine torque profile for 30s drivecycle
The notable differences shown for the engine speed are due to the restrictions imposed as the engine’s lower speed is limited to idle. The engine will stall if its speed drops below the lower ‘idle’ limit, although as the PEC is unaware of this it often dictates a lower speed during some periods of low power requirement. An element of the low-level control thus prohibits this. Some steady state offset can be seen between the vehicle speed and its demand. This is largely due to the nature of the LQR control system which contains no integral term. Given these results it is clear that the requirements of the defined cost function have been achieved through suitable choice of the cost function weighting parameters.

5.3 Case study: shuffle control

As discussed in Chapter 1, the relatively fast transient dynamics of an electrical machine coupled with a direct connection to the road wheels could offer benefits in terms of shuffle control over more conventional methods. Here the low-level MIMO controller developed in Section 5.2 will be expanded in an attempt to regulate shuffle whilst also achieving the demands set by the PEC. This is achieved through inclusion of additional terms within the defined cost function.

5.3.1 Model definition

Through development of a linear reduced order model (Figure 5.1), a vehicle model with flexible driveshafts can be derived as shown in Figure 5.8. The driveshafts are defined to be more flexible than would be expected in the Prius (and have been modelled previously) so as to exacerbate shuffle, with $k_s = 575 \, \text{Nm/rad}$ and $b_s = 12 \, \text{Nms/rad}$. This representation was shown by Farshidianfar et al. (2001) to provide an adequate means for shuffle analysis, although the identification of higher order torsional vibrations (e.g. rattle) is beyond its purview. The reduced order model again contains a non-linear tyre representation, although other non-linearity’s such as lash are ignored. A study by Best (1998) investigated the affect of non-linearities (principally lash) on control actions, and concluded that little benefit can be made from their inclusion in driveline control.
With reference to Figure 5.8, the model states are defined as follows:

- $x_1$: Generator velocity (rad/s)
- $x_2$: Motor velocity (rad/s)
- $x_3$: Spring deflection - compression positive (rad)
- $x_4$: Wheel velocity (rad/s)
- $x_5$: Vehicle velocity (rad/s)

States $x_1$ and $x_2$ are again detailed in Appendix C, with the sun, carrier and annulus inertias in the model also including the inertias of the generator, engine and motor respectively. Considering the components downstream of the planetary gearset, the driving torque is given by

$$ T'_2 = -k_s x_3 - b_s x_3 = -b_s x_2 g_r - k_s x_3 + b_s x_4 $$

$$ \therefore T_2 = -b_s x_2 g_r^2 - k_s x_3 g_r + b_s x_4 g_r $$

(5.11)

Also

$$ \dot{x}_3 = x_2' - x_4 = x_2 g_r - x_4 $$

(5.12)

The equation of motion for the wheel rotation is...
The equation of motion of the vehicle is then

$$\dot{x}_5 = \frac{1}{J_v} \left( b_x x_4 - b_t x_5 - T_{dg} \right)$$  \hspace{1cm} (5.14)

In a similar way that the reduced order rigid shaft model was used to design an LQR based low-level controller, the model with flexible shafts can be used to develop a controller capable of shuffle regulation.

5.3.2 Application of shuffle control

Three techniques were investigated to incorporate shuffle control. Given that shuffle is observed as a fluctuation in acceleration about a steady-state reference condition the first technique aims to track a vehicle acceleration calculated as a function of the control inputs. The relative motion of the states during a particular mode can be found through analysis of the system’s eigenstructure. As such, the second technique attempts to minimise vibrations as defined by the shuffle eigenvector. As with the first case, the third technique aims to regulate shuffle by tracking a defined vehicle acceleration demand, although the vehicle acceleration demand is derived from vehicle speed demand information.

The techniques were assessed through application of a step input to full throttle over 0.1 seconds with an initial vehicle velocity of 1m/s followed by a back-out when the vehicle reached 13m/s.
5.3.2.1 Calculated acceleration tracking

This technique involves the addition of a cost term to minimise the difference between the vehicle's acceleration and the steady state acceleration calculated as a function of the three component torque inputs. The steady state acceleration may be calculated based on the assumption that the connection between the planetary gearset and the vehicle is rigid. The acceleration of the vehicle can then be calculated using the equations of motion of the planetary gearset (Appendix C) with the inertia of the vehicle and wheels added to that of the annulus (with allowances made for the reduction gear). The complete cost function can thus be formulated as:

\[
L = \int_0^\infty \left[ c_1 (x_5 - x_7)^2 + c_2 \left( x_2 - \frac{R_{sa} x_1}{1 - R_{sa}} - x_8 \right)^2 + c_3 (u_2 - x_3)^2 + c_4 (x_5 - (\chi u))^2 + u^T \gamma u \right] dt
\]

The first three elements in the cost function are as described for Equation 5.10, and the fourth element seeks to minimise the error between the actual and calculated steady-state vehicle acceleration. A suitable choice of cost function parameters was found by trial and error to be:

\[
c_1 = 1 \quad c_2 = 2 \quad c_3 = 1 \quad c_4 = 0.05 \quad \gamma = \begin{bmatrix} 1 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \times 10^{-7} \end{bmatrix}
\]

Figure 5.9 shows an acceleration response comparison between a vehicle using PI low-level control and one using LQR with cost function (5.15).
It can be seen that there is little difference between the two control systems during tip in. This is due to the torque/speed limitations of the components, and the fact that the demand acceleration is a function of the motor, engine and generator torques. As the torques change (as determined by the controller) in an attempt to track the desired acceleration, the desired acceleration also changes. Changes to the cost associated with control use allow a more conservative use of the controls to be defined. However, this leads to an unsuitable result as the vehicle’s acceleration reduces and performance is compromised. A small peak in vehicle acceleration can be seen as a result of the tip-in for both cases. This may (fortuitously) have driveability benefits in that it provides the vehicle with an initial surge of acceleration before settling, thus providing the impression of increased performance.

Figure 5.9 also shows the shuffle caused by back-out, which can be seen to be worse in the uncontrolled shuffle case (PI), with the LQR controller demonstrating a significant improvement.
The component torques for the vehicle incorporating shuffle control are shown in Figure 5.10. The motor torque can be seen to saturate during tip-in. As the motor is directly connected to the vehicle's driveshafts, the LQR controller determines that this is the most efficient component to minimise shuffle. A consequence of this is that shuffle control is compromised during tip-in when compared to the back-out case where the components are well within their operational boundaries.

5.3.2.2 Minimisation of eigenvector based shuffle amplitude

In this technique an error matrix is developed which aims to regulate the amplitude associated with the shuffle eigenvector as described by the reduced-order model. The eigenstructure is calculated for the linear model and the mode associated with shuffle is determined by analysis of the damped natural frequency (approximately 3 Hz) and damping ratio (approximately 0.25) of each mode. The appropriate eigenvector is then added to the cost function for regulation as shown below:
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\[
L = \int_0^\infty \left[ c_1 (x_5 - x_7)^2 + c_2 \left( \frac{x_2 - R_{sa} x_1}{1 - R_{sa}} - x_8 \right)^2 + c_3 (u_2 - x_9)^2 + c_4 (\Omega)^2 + u^T \gamma u \right] dt \tag{5.16}
\]

where \( \Omega \) is the magnitude of the shuffle mode eigenvector normalised according to the largest element. For a vehicle speed of 15 m/s this is given by

\[
\Omega = x_1 + 0.453 x_2 + 4.62 \times 10^{-3} x_3 + 4.442 \times 10^{-3} x_4 + 8.41 \times 10^{-4} x_5 \tag{5.17}
\]

The cost function parameters are then defined as

\[
c_1 = 2.5 \quad c_2 = 2 \quad c_3 = 100 \quad c_4 = 2 \quad \gamma = \begin{bmatrix} 1 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \times 10^{-6} \end{bmatrix}
\]

Figure 5.11: Acceleration comparison with and without shuffle compensation

Figure 5.11 shows an acceleration response comparison between a vehicle using PI low-level control and one using LQR control with a cost associated with the error in regulating the shuffle amplitude to zero. The acceleration responses for systems with and
without shuffle compensation can be seen to be comparable. This is again due to the nature of the fluctuations which are caused by the motor saturating. During back-out, the controller with shuffle compensation can be seen to show an improvement when compared to the uncompensated case. Occupant perception of the back-out manoeuvre should be improved as the rate of change of acceleration (jerk) is reduced, although a similar result might be obtained by simply reducing the gains in the PI controller.

A consequence of a choice of cost weighting parameters that achieve improved shuffle control is that the engine speed demand is not followed as closely as in the PI case (where the errors are negligible), or in the case discussed in Section 5.3.2.1. The engine torque also deviates slightly from the demand. However, as the deviations only occur for relatively small durations, such deviations are probably acceptable. The engine speed and torque plots are shown in Figures 5.12 and 5.13 respectively with the deviations in achieved engine response clearly visible.

![Engine speed response using eigenvector based shuffle compensation](image)

Figure 5.12: Engine speed response using eigenvector based shuffle compensation
Figure 5.13: Engine torque response using eigenvector based shuffle compensation

5.3.2.3 Acceleration demand tracking

In this technique the LQR controller aims to track an acceleration demand (as opposed to a vehicle speed demand) along with engine speed and torque demands. Here the acceleration demand is determined as a simple high level switching strategy based on the error between the actual and desired vehicle speeds; the LQR controller aims to minimise the error between this and the actual vehicle acceleration. The acceleration demand implemented is a tip-in to maximum over 0.1 seconds followed by a sustained period and a back-out to zero so as to achieve the same vehicle speed demand as considered earlier.

A seventh 'pseudo' state is the vehicle acceleration demand, hence the overall cost function is defined as:

\[
L = \int_0^\infty \left[ c_1 (\dot{x}_5 - x_7)^2 + c_2 \left( \frac{x_2 - R_{sa} x_1 - x_8}{1 - R_{sa}} \right)^2 + c_3 (u_2 - x_9)^2 + u^T \gamma u \right] dt
\]  

(5.18)
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with parameters defined as:

\[ c_1 = 0.8 \quad c_2 = 4 \quad c_3 = 1 \quad \gamma = \begin{bmatrix} 1 \times 10^{-6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \times 10^{-2} \end{bmatrix} \]

Figure 5.14: Acceleration comparison with and without shuffle compensation

Figure 5.14 shows an acceleration response comparison between this controller and a vehicle using PI low-level control. It can be seen that shuffle is reduced considerably during back out whilst the desirable initial acceleration surge is maintained during tip-in. Analysis of the control torques (Figure 5.15) shows a rapid response from the electrical components during tip-in and back-out conditions. Also, during tip-in conditions the electrical component torques are seen to saturate which may compromise the controller’s ability to regulate shuffle. On the other hand the engine speed and torque demands are well tracked as shown in Figures 5.16 and 5.17, although this does lead to a slight reduction in acceleration after the initial surge with LQR control when compared to the PI controlled case.
Figure 5.15: Regulated LQR control torques for acceleration demand tracking

Figure 5.16: Engine speed response for acceleration demand tracking
5.4 Discussion

In this chapter, a state-space model of the hybrid system has been developed which has enabled a low-level control system to be designed using LQR optimal control. The tyre is a particularly important component to model, and has been incorporated through use of a simplified linear torque-slip relationship which has been shown to be valid for this application (Section 5.1). Using the reduced order model, a MIMO controller was developed which tracks vehicle speed, engine speed and engine torque demands, and as such provides an alternative to the classical control system used for low-level control. Given the dependence of the tyre model on the vehicle speed, optimal gain matrices are derived and linearly interpolated across a defined vehicle speed range.

Using a low-level control system based on LQR, additional elements can be added to the cost function. In this study the minimisation of shuffle has been demonstrated using a
simple vehicle model with relatively flexible shafts. Three approaches have been demonstrated and are now summarised.

The first technique attempts to minimise the error between the actual vehicle acceleration (calculated from the measured states in this application, although an accelerometer may be used in practice) and a steady state acceleration calculated as a function of the three power source torque inputs. An improvement in vehicle shuffle was shown during the back-out case where the power sources are well within their operational boundaries. However, the major limitation of this technique is that as the power sources respond to deviations from the calculated acceleration, the calculated acceleration changes. During tip-in, an acceleration peak can be seen for both the LQR and PI controlled systems. This is due to the sudden saturation of the motor, and may (fortuitously) have driveability benefits in that occupants would feel an initial surge giving the impression of increased acceleration.

The second technique attempts to minimise shuffle amplitude as determined by the corresponding shuffle eigenvector. This technique again gave the initial acceleration surge during tip-in, and showed a reduction in the torque fluctuations during back-out. In this case, the rate of change of acceleration (jerk) during back-out is reduced when compared to both the first technique and the PI controlled case. This may for example be desirable for use on a luxury vehicle where high levels of refinement take precedence over a perception of high performance. A minor consequence of the improved shuffle control is that the engine speed and torque are poorly tracked in this application during tip-in and back-out manoeuvres.

The third technique attempts to minimise shuffle by tracking a demand acceleration derived in turn from vehicle speed demand and current vehicle speed information. This technique demonstrated greatly improved shuffle control compared to a system using classical control during the back-out condition – the vehicle acceleration quickly reached the demand of zero after back-out. Using this technique the engine speed and torque demands are well tracked.
In an attempt to further improve these results, a cost function was formulated by combining the second and third techniques. Little improvement was seen using this technique when compared to tracking an acceleration demand alone.

In summary, a technique for employing LQR control at the low-level of a hybrid powertrain system has been demonstrated, and successful techniques have been identified for improving the vehicle's shuffle response should flexible driveshafts be preferred. The techniques show slightly varying responses which each have their own relative merits depending on the intended vehicle's driveability requirements. Although applied on a model of the Prius here, the technique could equally be applied to different vehicle configurations provided a state-space representation of the system is available.

In the next chapter, full non-linear solutions using an off-line optimisation technique are sought. This allows a performance assessment of the controller developed here to be made through comparison with an independent non-linear 'optimal' benchmark.
In Chapters 4 and 5 a dual-level control system was proposed consisting of an on-line minimisation of drivetrain power loss at the upper level coupled with a low-level controller developed using linear optimal control theory. To assess the performance of these elements, more general fully non-linear optimal control solutions will be sought. The technique employed is that proposed by Marsh (1992), whereby the numerical solution of a two point boundary value problem is sought to obtain the optimal control inputs for a specified system and initial conditions. The problem is too computationally expensive to be applied on-line, and as such an off-line optimisation is used to provide the required sequence of controls for the prescribed drivecycle. The technique enables solutions to be obtained either with or without a priori knowledge of the drivecycle requirements. This enables an overall benchmark of on-line controller performance to be evaluated.

The technique is first used to obtain a benchmark to assess the performance of the PEC by determining the maximum fuel efficiency for a given drivecycle, followed by an assessment of the low-level controller in minimising shuffle.

6.1 Non-linear optimal control methodology

In this section the solution of the non-linear optimal regulator problem from a given initial condition is studied.

Integrating a defined cost function, \( L[x(t),u(t)] \), over a time period \( t_0 \) to \( t_f \) yields a dynamic cost function, \( J \):
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\[ J = L_c[x(t_f)] + \int_{t_0}^{t_f} \left[ L[x(t), u(t)] \right] dt \]  \hspace{1cm} (6.1)

Adding the constraint equations to this with a vector of Lagrange multiplier functions, \( p(t) \) gives

\[ J = L_c[x(t_f)] + \int_{t_0}^{t_f} \left[ L[x(t), u(t)] + p^T(t) [\tau[x(t), u(t)] - \dot{x}(t)] \right] dt \]  \hspace{1cm} (6.2)

where \( \tau \) is determined from the state-space dynamic equations \( \dot{x} = \tau[x(t), u(t)] \). The Lagrange multiplier functions are usually termed \textit{costate} functions. The Hamiltonian function can be defined (see for example Bryson and Ho, 1975) as

\[ H = L[x(t)] + p^T(t) \tau[x(t), u(t)] \]  \hspace{1cm} (6.3)

Equation 6.2 can then be rewritten as

\[ J = L_c[x(t_f)] + \int_{t_0}^{t_f} \left[ H - p^T(t) \dot{x}(t) \right] dt \]  \hspace{1cm} (6.4)

Integrating the second term of (6.4) by parts gives

\[ J = L_c[x(t_f)] + p^T(t_0)x(t_0) - p^T(t_f)x(t_f) + \int_{t_0}^{t_f} \left[ H + p^T(t)\dot{x}(t) \right] dt \]  \hspace{1cm} (6.5)

Considering small changes \( \delta J \) in the dynamic cost caused by small changes in the controls \( \delta u(t) \) and in the states \( \delta x(t) \):

\[ \delta J = \frac{\partial L_c}{\partial x} \delta x(t_f) + p^T(t_0)\delta x(t_0) - p^T(t_f)\delta x(t_f) + \int_{t_0}^{t_f} \left[ \frac{\partial H}{\partial x} \delta x(t) + \frac{\partial H}{\partial u} \delta u(t) + p^T(t) \delta x(t) \right] dt \]
The costates can be chosen such that $\delta J$ depends only on changes in the controls by imposing the following conditions:

\[
\dot{p}(t)^T = -\frac{\partial H}{\partial x} - \frac{\partial L}{\partial x} - p^T(t) \frac{\partial \tau}{\partial x}
\]  

(6.7)

\[p^T(t_f) = \frac{\partial L_e}{\partial x}
\]  

(6.8)

hence

\[
\delta J = p^T(t_0)\delta x(t_0) + \int_{t_0}^{t_f} \left( \frac{\partial H}{\partial u} \delta u(t) \right) dt
\]  

(6.9)

The open loop series of controls which minimises the dynamic cost $J$ for initial state values is sought, hence $\delta x(t_0)=0$ which leaves

\[
\delta J = \int_{t_0}^{t_f} \left( \frac{\partial H}{\partial u} \delta u(t) \right) dt
\]  

(6.10)

To minimise the dynamic cost, Equation 6.10 must be zero for an arbitrary change in the controls, hence

\[
\frac{\partial H}{\partial u} = 0, \ \forall \ t
\]  

(6.11)

This extremum can be verified as a local minimum through analysis of the second order variations of $J$ (see for example Bryson and Ho, 1975). Equations (6.7), (6.8) and (6.11) are known as the Euler-Lagrange equations in the calculus of variations.
It is also noted from (6.9) that the change in dynamic cost due to a small change in initial state is given by

$$\delta J = p^T(t_0) \delta x(t_0)$$  \hspace{1cm} (6.12)

provided the controls are either held constant, or Equation 6.11 is satisfied. This allows each initial costate value to be interpreted as the local rate of change of dynamic cost due to a change of the corresponding initial state value.

With non-linear differential equations and a non-quadratic cost function, the solution of Equation 6.10 leads to non-linear costate equations, and hence yields a non-linear two point boundary problem over the range $t_0$ to $t_f$.

In Marsh (1992) an approximation to the continuous solution is found using a discrete sequence of controls, each held constant for a small time $\delta t$. Each segment then forms a separate element with constant controls and a cost gradient defined by

$$\frac{\partial J}{\partial u_i} = \int_{t_i}^{t_{i+1}} \frac{\partial H}{\partial u_i} dt$$  \hspace{1cm} (6.13)

An off-line gradient based optimisation algorithm can then be employed and provides a good approximation to a continuous time solution through choice of a suitably small $\delta t$. 

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6.1.1 Procedure

A schematic of the method for calculating the optimisation algorithm is shown in Figure 6.1 and summarised as follows:

a: Using initial values (which may or may not be zero) for the discretised controls, an integration forwards in time is performed using given initial conditions in order to calculate the state time histories of the system.

b: The total dynamic cost and final state values are recorded and the final costate values are calculated from (6.8).

c: The final costate values form a starting point for the backwards time integration of the costate equations using (6.7). The required cost gradients can then be found from (6.13).

d: The control sequence is updated using a line search optimisation along the steepest descent.

Steps a–d are then repeated until a suitable convergence of cost and controls is met.

The algorithm is implemented such that the system states are represented by a set of first-order ordinary differential equations. Additional functions used in the state calculations are permitted providing that the function supplies not only the value, but also any
additional partial derivatives. The Hamiltonian is calculated according to (6.3) using symbolic maths to represent the defined equations and use of the Jacobian function allows the calculation of the required derivatives with respect to the states. The differential of the Hamiltonian with respect to the states is then found (again using the Jacobian function), thus providing an auto-calculation of the required rate of change of system costates with respect to state given the definition of (6.7). Solution of (6.8) enables calculation of the costates at the end of each zero-order held control step. Given the large number of integration steps required during execution, the functions used in the calculation of the rate of change of the states with respect to time are compiled into C++ code, and a similar file is generated for the rate of change of costate.

The integration routine employed is an embedded 5\textsuperscript{th}/6\textsuperscript{th} order Runge-Kutta formula using values for the required constants as defined by Cash and Karp (for further detail see Press et al., 1992). The technique is a variable step method with the step sizes determined using a calculated error tolerance – an error estimate is calculated which determines whether fewer or greater steps are required in order to maintain a defined accuracy. Although perhaps not the fastest integration method available, the major strength of the Runge-Kutta formula (compared to say a Bulirsch-Stoer method) is that it virtually always finds a solution regardless of the smoothness of the equations.

The technique can be implemented both with and without \textit{a priori} knowledge of the required duty cycle and thus provide either an overall optimum benchmark, or a causal solution that could be expected through use of a fully non-linear control law.

To carry out the technique without \textit{a priori} knowledge of the future drivecycle, a 'rolling' optimisation is conducted. Here each predefined time step is considered in turn, and a separate optimisation is performed. Using defined initial conditions an optimisation is performed assuming the initial demand is zero-order held for a specified optimisation time, or window (Figure 6.2). Once the convergence criteria have been met a simulation using this assumed demand is performed using the optimal sequence of controls and the state values at the end of the first time step (dt) are noted. These values are then used as initial conditions for the second optimisation where the demand at time
dt is again zero order held for the specified window. This continues at intervals of dt until the full drivecycle has been performed. The complete sequence of optimal control inputs are obtained cumulatively from the control inputs applied for the first time interval (dt) of each individual optimisation.

Figure 6.2: Rolling optimisation demands

6.1.2 Algorithm verification

To verify the implementation of the algorithm, a comparison for a linear system with quadratic cost function was made with a solution obtained from LQR theory using the LQR function in MATLAB. The model used for the study is shown in Figure 6.3, and represents a simplified linear version of the hybrid powertrain with flexible driveshafts. Here a linear damper replaces the tyre.

Figure 6.3: Linear approximation of the drivetrain
The linear powertrain can be represented using five states:

\[ x_1 \] Generator velocity (rad/s)

\[ x_2 \] Motor velocity (rad/s)

\[ x_3 \] Driveshaft deflection - compression positive (rad)

\[ x_4 \] Wheel velocity (rad/s)

\[ x_5 \] Vehicle velocity (rad/s)

In the design stage of the optimal gain matrix obtained using LQR theory, a 6\(^{th}\) pseudo state was incorporated to represent a shaft windup demand (as in Chapter 5). A cost function can then be prescribed as the square of the error between the actual shaft windup \( (x_3) \) and the demand:

\[
J = \int_{t_0}^{t_f} [(x_3 - x_6)^2 + u^T u] \, dt
\]  

(6.14)

With a driveshaft windup demand set as an increase from 0 to 0.765 rads over the first 0.5 seconds of the drivecycle, results were found both using the rolling optimisation technique (Figure 6.4) with an optimisation window of 1 second and with a priori knowledge of the shaft demand (Figure 6.5). The rolling technique shows a good comparison with the LQR solution with the very small errors attributed to the convergence criteria of the algorithm. With prior knowledge of the demand the algorithm pre-empted required changes to the controls as seen by the small differences in the controls of Figure 6.5.
Figure 6.4 LQR (solid) and rolling optimisation (dashed) comparison

Figure 6.5: LQR (solid) and non-linear algorithm (dashed) comparison
6.2 Development of the non-linear model

Although the integration routine itself is able to handle discontinuities in the functions describing the rate of change of states and costates, the optimisation methodology employed does not take these into account. To illustrate this, consider a step change in cost due to say a component being driven out of its operational envelope (e.g. $T_e > T_{\text{emax}}$). Through analysis of (6.7) it is clear that at the point of the step change in cost, the rate of change of cost and hence the rate of change of costate is very high. This has the effect of instigating a step change in costate at this position. The Hamiltonian is calculated as defined by (6.3) and is a function of the costates. According to (6.13), the cost gradient for each zero-order held control step is calculated as the integral of the rate of change of the Hamiltonian with respect to control across the step. The step change in cost thus translates to an abnormally high cost gradient, or impulse, for one zero-order held control (within which the step resides). This may have the effect of (temporarily) driving the control in a direction away from the optimal solution.

This has the most significance here in that in order to apply constraints to components, continuous functions should be defined with an increased cost associated with operation outside of the limits of the component.

To determine the optimal controls required to minimise shuffle, a five state reduced order model is used as described in Section 5.3.1 and shown schematically in Figure 5.8. The tyre is again modelled assuming a linear torque/slip approximation, although an additional term is applied to ensure that torque can be transmitted at zero vehicle speed whilst avoiding discontinuities in the function. If the wheel’s rotational velocity is defined as $x_4$ and the vehicle’s as $x_5$, the torque transmitted through the tyre is defined as:

$$T_T = b_T(x_4 - x_5) \quad (6.15)$$

Then,

$$b_T^* = \frac{b_T}{x_5 + v_0 e^{(-qx_5)}} \quad (6.16)$$
Where $v_0$ and $q$ are constants and the rate of change of $b_T$ with respect to vehicle speed can be readily calculated. Figure 6.6 shows the variation of $b_T$ with $x_5$ when $b_T^* = 10,000$ Nm, $v_0 = 1$ rad/s and $q = 1$.

![Graph showing the variation of $b_T$ with vehicle speed](image)

**Figure 6.6: Variation of $B_T$ with vehicle speed**

The drag torque imposed on the vehicle is again calculated as a function of vehicle velocity as given by Equation 5.1.

To ensure that the optimisation algorithm fully utilises component limitation knowledge, functions are defined for the relevant components as listed in Table 6.1.
Non-linear Optimal Control Solutions

<table>
<thead>
<tr>
<th>Element</th>
<th>Dependant variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle speed</td>
<td>Vehicle speed</td>
</tr>
<tr>
<td>Engine torque</td>
<td>Engine torque, engine speed</td>
</tr>
<tr>
<td>Engine speed</td>
<td>Engine speed</td>
</tr>
<tr>
<td>Motor torque</td>
<td>Motor torque</td>
</tr>
<tr>
<td>Motor Power</td>
<td>Motor speed, motor torque</td>
</tr>
<tr>
<td>Generator torque</td>
<td>Generator torque</td>
</tr>
<tr>
<td>Generator power</td>
<td>Generator speed, generator torque</td>
</tr>
</tbody>
</table>

Table 6.1: Component limitations

For elements that are defined as a function of one variable only, the cost is defined as a combination of two quadratic terms – one within the operational boundary and one outside, as shown for the engine speed limitation in Figure 6.7.

![Figure 6.7: Cost associated with engine speed limitation](image)

Within the allowable range a small cost is applied in order to provide the optimisation with rate of change information. The cost should strictly be zero, although this implies
that an infinite change can be applied to a given parameter with no change in cost. Such a representation can lead to the optimisation driving the component well out of its operational region where the sudden change to an excessively large cost can cause the algorithm to fail as the required computations can not be made. The cost within the allowable range is thus chosen small enough to have negligible effect on the choice of the parameter with respect to other factors whilst also restricting excessive movement into the steep gradient region that bounds the choice of the parameter.

If the upper and lower component boundaries are represented by $u_{\text{lim}}$ and $l_{\text{lim}}$ respectively, and $m$ represents the mid-point between the two, within the acceptable range the cost is defined as:

$$L = \left( \frac{x - m}{u_{\text{lim}} - m} \right)^2$$

(6.17)

where $x$ is the current value of the required component parameter. The rate of change of cost with respect to $x$ is thus

$$\frac{dL}{dx} = \frac{2x - 2m}{(u_{\text{lim}} - m)^2}$$

(6.18)

Outside the operational region, an additional quadratic element is applied in order to increase the cost. Using the same notation and with $b$ defined as a constant, if the component parameter is greater than the upper limit, then

$$L = \left( \frac{x - m}{u_{\text{lim}} - m} \right)^2 + \left( \frac{x - u_{\text{lim}}}{b} \right)^2$$

(6.19)

and the corresponding derivative with respect to $x$ is

$$\frac{dL}{dx} = \frac{2x - 2m}{(u_{\text{lim}} - m)^2} + \frac{2x - 2u_{\text{lim}}}{b^2}$$

(6.20)
Analysis of (6.17) to (6.20) when $x = u_{\text{lim}}$ confirms that the overall function is continuous and with a continuous gradient when switching between the two elements.

A similar term can be applied if the component parameter is below the lower limit, and the derivative can be calculated analytically ensuring continuity across the range (Figure 6.8).

![Figure 6.8: Rate of change of engine speed limitation cost with respect to engine speed](image)

If the component limitation is a function of two variables, the upper and/or lower limits vary as a function of one parameter. As an example, consider the engine torque limit. If operation above idle is considered, quadratic equations can be used to provide a simple representation of the upper and lower engine torque limits as a function of engine speed as given for the Prius in Equations 6.21 and 6.22, and shown in Figure 6.9.

\[
T_{e,\text{max}} = -4e^{-4}\omega_e^2 + 0.3004\omega_e + 45 \tag{6.21}
\]

\[
T_{e,\text{min}} = -4e^{-5}\omega_e^2 + 0.0612\omega_e - 4.2143 \tag{6.22}
\]
The cost associated with engine torque is then calculated as for Equations 6.21 and 6.22 as shown in Figure 6.10.

Figure 6.10: Engine torque limit
The rate of change of cost can then be calculated as a function of the dependant variables (Figures 6.11 and 6.12).

Figure 6.11: Rate of change of cost with respect to engine speed

Figure 6.12: Rate of change of cost with respect to engine torque
To assess the performance of the PEC controller a four state reduced order model is sufficient. The vehicle is represented using two states (generator and motor speeds) as for a planetary gearset, and rigid powertrain connections are assumed as described in Section 5.1. As such, the powertrain is represented as a linear system with the wheel and vehicle inertias included in the overall inertia of the equivalent annulus element. Again, the component limitations and drag torque are represented by non-linear functions. The third and fourth states are battery SOC and fuel use defined as follows.

Although not physically sensible, during an optimisation the descent may dictate that the engine runs out of its operational range, perhaps with a negative torque value. To ensure that a fuel use value can be calculated for all regions, the fuel use for the positive speed and torque quadrants is reflected around both axes to provide an overall map (Figure 6.13). Given this coverage, the rate of change of fuel use with respect to engine speed and torque always tends to drive the engine towards zero speed and torque, and as such, no benefit in terms of fuel consumption is gained from running the engine out of its operational range. Criteria relating to the engines operational boundaries also ensure that cost is not reduced by operating in this way.

![Figure 6.13: Engine fuel use for optimisation](image-url)
As the fuel map is generated using a bicubic interpolation of tabular data, the gradients are continuous as shown in Figure 6.14 for the rate of change of fuel use with respect to engine speed.

![Figure 6.14: Rate of change of fuel use wrt engine speed](image)

The battery model developed in Section 2.3.2 is too computationally expensive to be applied here, hence a simplified model has been developed:

\[
\zeta = \int_{t_0}^{t_f} \left( r \times \frac{P_{\text{net}}}{288} \right) \, dt
\]  

(6.23)

where \( r \) represents the charge or discharge rate of the battery.

The net power flow from the battery is the sum of the net motor and generator power drawn from the battery, i.e.

\[
P_{\text{net}} = \text{motor electrical power} + \text{generator electrical power}
\]
If the torque, speed and efficiency of a component are defined as $T$, $\omega$ and $\eta$ respectively, and the electrical power is defined as $P_e$, then assuming a convention of positive power flow out of the battery, the electrical power of a component can be calculated as depicted in Figure 6.15.

![Figure 6.15: Electrical power calculation](image)

As a component switches from charging to discharging or vice versa, a discontinuity in electrical power exists. However, as the state being modelled (SOC) is an integral of the net power term, such discontinuities are acceptable by the optimisation algorithm and a method of forced smoothing is not required.

In this reduced form, the charge and discharge rates are represented by independent constant values. This simplification ignores the change in rate as a function of battery SOC with average values selected, and hence assumes that the battery’s resistance is constant. Using the 50 second drivecycle as an example (Figure 4.28), a comparison of the previously used ‘full model’ in EASY5 and this ‘reduced model’ is shown in Figure 6.16.
As for the fuel use calculation, the electrical component efficiencies are calculated using a bicubic interpolation of tabular data as described in Section 4.2.

6.3 Benchmark for PEC

Non-linear optimal solutions have been found which balance the use of the engine with that of the battery, thus providing a useful comparison for the PEC controller developed in Chapter 4. However, in this application continuous engine operation is assumed, i.e. the algorithm is unable to switch the engine on and off. Using the technique in its current form a switching strategy is difficult to achieve. A continuous cost associated with engine operation between zero and idle can be implemented, although in practice this simply restricted operation to above idle. As such the study is intended to provide an insight into ‘optimal’ component operation rather than an outright benchmark for the PEC.
Non-linear Optimal Control Solutions

6.3.1 50 Second drivecycle

Results for the 50 second drivecycle introduced in section 4.6 have been obtained with the algorithm having full \textit{a priori} knowledge of the vehicle speed demand requirements. If \( y \) is an uncontrolled input set as the drivecycle in m/s, then a cost function can be formulated as:

\[
J = \int_{t_0}^{t_f} \left( g_1 (r x_2 - y)^2 + c_1 (x_3 - 0.6)^2 + c_2 x_4 + \xi + u^T \gamma u \right) dt
\]  

(6.24)

The cost function weighting parameters are chosen such that the demands of the drivecycle are met whilst achieving minimum fuel use and maintaining battery SOC requirements with values as follows:

\[
\begin{bmatrix}
1 \times 10^{-9} & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1 \times 10^{-9}
\end{bmatrix}
\]

The profile is shown in Figure 6.17 along with the vehicle responses from the non-linear analysis and PEC controlled solutions. Through analysis of the change in vehicle speed demand at 20 seconds, it is interesting to note that the non-linear solution again pre-empts the change in vehicle speed demand, where by contrast the PEC reacts to this change in demand. This slight delay from the PEC is also apparent during the braking period. The corresponding component torques to achieve this result are shown in Figure 6.18 for the non-linear solution, with the PEC results shown for comparison.
Figure 6.17: Vehicle speed demand and achieved for 50 second drivecycle

Figure 6.18: Non-linear (solid) and PEC component torque (dashed) comparison
The optimal solution utilises the engine and motor in order to accelerate the vehicle when required as has been seen previously and would be expected. The rates of change of torque output from the components however are a lot smoother than seen previously – the controller has knowledge of impending drivecycle requirements and is able to pre-empt changes in component output as clearly seen (for example) at 25 seconds. This would have advantages in terms of component life, but would be difficult to achieve in practice without an extremely well defined operating schedule. The battery SOC obtained for the drive schedule is shown in Figure 6.19, with results for the PEC again shown for reference.

![Battery SOC for 50s drivecycle](image)

**Figure 6.19: Battery SOC for 50s drivecycle**

It can be seen that the battery SOC is held close to the desired value of 0.6 as dictated by the defined cost function. As the engine is always operational, the SOC initially rises with little discharge apparent during the first acceleration period at 10 seconds. During the second acceleration period (20 seconds), a similar amount of current drain for both controllers is observed. However, the SOC continues to fall with the optimal sequence of
controls as the algorithm calculates that SOC will be recouped during the deceleration phase (through use of regenerative braking), and also that the drivecycle does not require further high current drain from the battery. Analysis of the engine speed (Figure 6.20) also highlights the algorithm’s knowledge of the drivecycle. Here the engine speed is held close to idle as the cost function dictates that acceleration of the engine is not required for this drivecycle as this would increase fuel consumption. Conversely, the PEC accelerates the engine during the second acceleration event (20 seconds) so as to limit battery discharge and recover SOC ready for any further acceleration events that may be required.

![Engine speed comparison for 50s drivecycle](image)

**Figure 6.20: Engine speed comparison for 50s drivecycle**

The corrected fuel use for the algorithm (i.e. with consideration made to the end SOC of 0.597 as described in Section 4.4.1) is 15.94g which compares to 30.45g used with the PEC controller. This highlights the benefit of drivecycle knowledge, although it is unlikely that any non-causal controller could approach this ‘optimum’ result in real operation as this would require extremely accurate knowledge of the drivecycle and vehicle position.
The rolling optimisation technique has been used to obtain optimal controls without drivecycle knowledge as shown in Figure 6.21. The required cost function can be derived as in Equation 6.24, although an additional term relating to the cost associated with the final battery SOC at the end of each individual optimisation is required as given by Equation 6.25.

\[ L_c = 1 \times 10^5 (x_3 - 0.6)^2 \]  

(6.25)

The first ten seconds of the simulation has been omitted as the controls would simply remain constant to maintain zero vehicle speed and idle engine speed and torque. Similar results are seen when comparing the two optimisation techniques, although the corrected fuel use of 20.17 g for the rolling case is significantly higher than when the optimisation has a priori knowledge. For reference, the battery SOC at the end of the simulation is 0.595. This result again highlights the significance of prior drivecycle knowledge, although the result is much improved over that obtained using PEC control. This is largely due to the selection of the cost function weighting parameters which were specifically tailored for this drivecycle in order to obtain an 'optimal' strategy.

![Figure 6.21: Comparison of optimal control torques with prior drivecycle knowledge (dashed) and without (solid)](image)

Figure 6.21: Comparison of optimal control torques with prior drivecycle knowledge (dashed) and without (solid)
The fluctuations in control observed are due to a combination of the convergence criteria of the rolling optimisation and the underlying assumption of a zero order held demand for each optimisation. A slight decrease in tolerance is required in order to achieve acceptable optimisation times, and there is a step change in demand between each individual optimisation. This can be easily verified by applying more stringent convergence criteria and the assumption that the demand continues at its rate of change from the previous two seconds throughout the optimisation window. The resulting sequence of control torques for the period between 11 and 14 seconds is shown in Figure 6.22 where a smoother solution can clearly be seen. The results of Figure 6.21 are however only slightly compromised and the general trend can easily be seen.

![Figure 6.22: Optimal control torques with more stringent optimisation criteria](image)

6.3.2 100 Second drivecycle

In Section 4.5 a number of ‘steady-state’ drivecycles were considered in order to form a comparison between the Baseline and PEC controllers. The PEC was found to perform favourably, and detail for the 20 m/s case (shown again in Figure 6.23) was given. An
optimal solution for this case has been found using the rolling optimisation technique with a window of 5 seconds in order to provide a further comparison. The resulting sequence of control torques is shown in Figure 6.24.

Figure 6.23: Steady-state drivecycle for rolling optimisation

Figure 6.24: Rolling optimisation component torques
It is clear that the requirements of the drivecycle in terms of transient events are met through use of the motor with assistance from the engine as seen and explained in previous analyses. After the initial acceleration event, the motor torque falls close to zero. As such, the engine provides the majority of the propulsion to the road wheels with the power ‘absorbed’ by the generator in providing a sufficient reaction used to re-charge the battery (Figure 6.25). During the deceleration phase the engine torque drops to idle and the motor torque becomes negative for the regeneration process, again, as seen previously.

![Figure 6.25: Battery SOC comparison for Non-linear optimisation and PEC](image)

Figure 6.25: Battery SOC comparison for Non-linear optimisation and PEC

With reference to Figure 6.25, similar SOC values obtained for the non-linear optimal and PEC solutions are seen. However, subtle differences in terms of engine operation are observed as shown in Figures 6.26 and 6.27. The engine start dictated by the PEC is clearly visible at 5 seconds. The battery SOC for the PEC controlled system can be seen to have fallen by a significant amount before this time. As the non-linear optimisation assumes continuous engine operation, the drop in SOC is reduced when compared to that
of the PEC, although there is little difference in the lowest overall value. This is accounted for by the fact that the two techniques allow a similar deviation from the battery’s optimal value, although this is costed in different ways. The PEC allows a drop before switching on the engine, whereas the non-linear optimisation provides a lower, yet continual contribution from the engine (see Figures 6.26 and 6.27).

![Engine speed comparison for steady-state drivecycle](image)

**Figure 6.26: Engine speed comparison for steady-state drivecycle**
During the sustained steady-state period, the PEC can be seen to raise the battery SOC using the engine output, whereas the non-linear optimisation is content with a relatively small rate of increase of SOC. Analysis of the operation of the two systems during the deceleration/regenerative period can be used to explain this, as there is a significant increase in the SOC regained using the non-linear technique when compared to the PEC. This is largely due to differences in the operation of the engine during this period where the PEC drops the engine to idle speed and torque. The non-linear optimisation drops the engine torque to its lower limit in a similar manner, although by contrast the engine speed is raised to its maximum value. What seems initially to be an odd result can be explained when the configuration and constraints of the planetary gearset are considered. Power and torque are balanced in the planetary gearset as described by Equations 4.1 and 4.2. During braking the planetary gearset torque is balanced such that most braking torque is absorbed by the motor. As the vehicle speed and hence the annulus speed reduces, in order to maintain the power balance the carrier and sun (i.e. engine and generator) velocities increase. Also, the system is able to use the inertia of the engine to
decelerate the vehicle, which seems sensible given its relatively large value when compared to that of the electrical components. This has the effect of providing the battery with additional charge via the reaction at the sun gear from the generator.

Given this overall reduced use of the engine from the non-linear optimisation technique when compared to the PEC, the overall fuel use of 51.65g is a vast improvement over the 82.44g achieved by the PEC. However, in practical applications for road vehicles it is desirable to have some form of engine shut-off given the number of start/stop procedures due to traffic lights etc., hence some form of engine start criteria is required. Also, passengers may find it disconcerting for the engine to suddenly accelerate to maximum speed during a braking manoeuvre!

For this drivecycle little improvement in overall fuel use is seen through use of the optimisation technique with full a priori knowledge of the drivecycle – 48.95g of fuel is consumed. This result is sensible given the underlying assumption of the rolling optimisation technique that the drivecycle requirements stay constant for a defined window; this is largely true for the drivecycle considered here.

6.4 Application for shuffle control

To more formally assess the performance of the linear optimal control solution for shuffle regulation (Chapter 5), a more general non-linear solution has been found for the tip-in case as described by Kells et al. (2001). The LQR techniques applied in Chapter 5 provided excellent control of shuffle during back-out conditions where the components were well within their operational boundaries. However, saturation was observed from some of the components during tip-in which may compromise the controllers ability to regulate shuffle. As such, non-linear solutions for the tip-in case are considered here.

The influence of system non-linearities (principally lash) were shown by Best (1998) to have little affect on the design of an optimal solution, therefore in this study the use of a more generalised optimal control solution is principally sought to assess the benefits of the non-linear system model and more significantly the inclusion of component
Non-linear Optimal Control Solutions

limitation knowledge. If the uncontrolled input, \( y \), represents the acceleration demand of the vehicle, and \( \xi \) represents the combined cost associated with component limitations (Section 6.2), a dynamic cost function can be formulated as:

\[
J = \int_{t_0}^{t_f} \left[ (\dot{x}_5 - y)^2 + \xi + u^T \gamma u \right] dt
\]

(6.26)

where \( \gamma = \begin{bmatrix} 1 \times 10^{-5} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \times 10^{-5} \end{bmatrix} \)

With *a priori* knowledge of a step change to maximum acceleration demand between 0.5 and 0.6 seconds and component limitation information, the optimal controls provide an excellent solution to the shuffle regulation problem (Figure 6.28). Interestingly, this is achieved by progressively increasing the motor and engine torques and regulating the acceleration fluctuations through the generator (Figure 6.29). This result is sensible given the configuration of the planetary gearset where the ratios dictate an amplification of torque from the generator on the output shaft. As such, the generator’s usual role of regulating the engine speed is compromised for a short period during the shuffle event (Figure 6.30).

To accelerate the vehicle without component limitations imposed, the optimal solution prefers to decelerate the engine from its initial condition and drive it in reverse if necessary. This result is reflected in this study, as the controls increase the engine speed just prior to the increase in acceleration demand in order to allow the generator to apply shuffle control, which consequently leads to a reduction in engine speed.
Figure 6.28: Vehicle acceleration response

Figure 6.29: Optimal component torques
Through use of the rolling optimisation technique, a more sensible comparison between the performance of a fully non-linear and a linear-quadratic controller can be made. This allows the performance of the low-level controller developed in Chapter 5 to be more formally assessed for shuffle regulation. The cost function remains as for the case with full *a priori* knowledge, as defined in Equation 6.26. A comparison of the vehicle acceleration results obtained for a tip-in over 0.1 seconds (Figure 6.31) shows a significant improvement from the non-linear solution. The acceleration using the LQR controller can be seen to fall compared to that of the non-linear solution. This is a consequence of the constraints imposed by the LQR solution which balances the requirement to accelerate the vehicle with the requirement to achieve the engine speed and torque demands of the PEC controller. This is evident in a comparison of the respective component torques (Figure 6.32), where the motor and generator torques from the two controllers are closely matched whilst the engine torque can be seen to be lower with LQR control. This is due to the formulation of the non-linear solution where no cost is directly associated with tracking an engine torque demand and hence the engine torque can be increased to sustain higher vehicle acceleration. An optimum result for shuffle regulation is thus obtained.
Figure 6.31: Non-linear rolling and LQR vehicle acceleration

Figure 6.32: Non-linear rolling (solid) and LQR component torque (dashed) comparison
In order to assess the true capability of a linear solution for shuffle regulation an LQR controller has been developed with the sole criterion being to track a vehicle acceleration demand, and therefore minimise shuffle. If $x_7$ is a pseudo-state representing the vehicle’s acceleration demand, the cost is defined as:

$$J = \int_{t_0}^{t_f} \left[ (\dot{x}_5 - x_7)^2 + u^T \gamma u \right] dt$$

(6.27)

with $\gamma = \begin{bmatrix} 1 \times 10^{-5} & 0 & 0 \\ 0 & 1 \times 10^{-5} & 0 \\ 0 & 0 & 1 \times 10^{-5} \end{bmatrix}$

A comparison of the vehicle acceleration response for this controller with one obtained using the non-linear rolling solution is shown in Figure 6.33.

![Figure 6.33: Non-linear rolling and LQR vehicle acceleration](image)
The two controllers can be seen to provide similar results, both of which are excellent solutions to the problem of shuffle regulation. The non-linear controller is able to sustain a slightly higher acceleration through application of a higher engine torque (Figure 6.34). This is achieved as the engine speed is increased slightly by the non-linear solution whereas the LQR solution holds the engine speed at idle. This indicates that use of an LQR solution is valid with little performance improvement available through use of a non-linear control law when \textit{a priori} drivecycle knowledge is unavailable. This is possible as the generator is not being used to track a particular engine speed demand during the transient event as previously, and hence being free to compensate for the saturation of the motor. Such a controller would only be applicable during acceleration events, with some switching strategy from the previously described controller of Chapter 5 required.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure6_34.png}
\caption{Non-linear rolling (solid) and LQR component torque (dashed) comparison}
\end{figure}
6.5 Discussion

This chapter has considered the use of non-linear optimisation theory in order to allow a comparison with the controllers developed in Chapters 4 and 5 to be made. The technique was used to find solutions for a linear system with quadratic cost function with the results compared with those obtained using LQR. The results showed little error between the two thus indicating that the technique has been implemented successfully.

Simulations were performed with a cost set to balance the deviation in battery SOC from the prescribed desired value (0.6) with the fuel used by the engine. The results were then compared with those obtained using the Power Efficient Controller of Chapter 4. Initial results are encouraging in that given free reign on a choice of solution, the general trend in component operation is comparable between the non-linear ‘optimum’ results and those obtained with the PEC. Simulations were performed both with and without a priori drivecycle knowledge, and a clear indication of the benefit of prior drivecycle knowledge was seen.

It is interesting to note that in some instances the optimal solution allows the battery SOC to discharge during steady state periods given knowledge of future drivecycle requirements and regenerative braking information. A further interesting result was also dictated by the optimal solution during braking. Rather than reducing the engine speed and torque to idle during braking periods, the solution preferred to increase engine speed and use its inertia to decelerate the vehicle. Although a useful means of braking the vehicle, in practice such a strategy would most likely be very disconcerting to the driver!

In general, the results indicate that much improvement over the PEC (and the Baseline) can be made through use of optimal control for prescribed drivecycles. However, the solutions require large amounts of computation for any component changes and are very drivecycle specific. Further work is required in order to derive an optimal controller that could be applied on-line for arbitrary drive schedules.
Simulations were also performed in order to assess the ability of the linear based controller developed in Chapter 5 for shuffle regulation. The linear based solution provides good control of shuffle during back-out events, although the results indicate that further improvements are possible during tip-in. An optimal solution was obtained here, where the error between a demand and achieved vehicle acceleration was found. Given the non-linear nature of the algorithm employed, solutions were obtained with full knowledge of component limitations.

The optimal sequence of controls indicate that component limitation knowledge was used to good effect, as during a tip-in the motor torque (in particular) was ramped up to its maximum value and held. This provides a steady acceleration of the vehicle with shuffle then regulated by the generator. With the linear solution a requirement was made (through definition of the cost function) that dictates that the engine speed and torque demands of the PEC be followed. As such, the generator was unable to react to the saturation of the motor during tip-in hence compromising the controller's ability to regulate shuffle. As such, a linear controller was developed which dispensed with the requirement to track engine speed and torque demands. The controller was found to provide excellent control of shuffle and showed little difference when compared to the result of the causal non-linear algorithm.

In conclusion, the results have shown that little benefit is available in terms of regulation of shuffle when a non-linear solution is compared to one obtained using linear optimal control theory. However, a consequence is that a separate set of optimal gain matrices are required during tip-in events in order to obtain the best form of shuffle regulation.
Chapter 7

Discussion and Conclusions

This thesis has considered the development of optimised control strategies for future vehicle powertrain systems, and in particular, their application to hybrid electric vehicles. The major emphasis was placed on the development of a hierarchical supervisory control strategy implemented in simulation of a commercially available hybrid vehicle, namely the Japanese specification Toyota Prius. The supervisory control system consists of a high level controller whose demands are realised through use of low-level controllers; the optimisation of both levels was investigated. The controllers are fundamentally based and have been developed using a systematic approach to their design. More general fully non-linear solutions have also been found which have enabled an assessment of the controllers to be made whilst also providing an insight into the behaviour of more complex systems.

A model of the Prius was developed in the dynamic simulation package EASY5 based on data available from published sources. A Baseline controller was developed and results obtained from the model were compared with some obtained from Advisor, a commercially available software tool. A close correlation between the two was seen even though both use different simulation techniques – the EASY5 model is based on a forward calculation technique whereas that used in Advisor is a reverse method. This provides confidence in the EASY5 model and enables the Baseline to be feasibly used as a benchmark for the novel strategies developed.

The Baseline control strategy implemented on the Prius performs favourably, with much improvement in fuel consumption reported when compared to similar conventional internal combustion engined vehicles. However, the strategy implemented is largely based around optimising the efficiency of the engine only. It was hypothesised that improvement could be made through application of an integrated strategy which aims to
Discussion and Conclusions

optimise the whole hybrid powertrain for maximum fuel efficiency. To this end a Power Efficient Controller (PEC) was developed and implemented in simulation using the EASY5 model.

The PEC was developed using a fundamental representation of the vehicle’s powertrain and aims to minimise the overall power loss in the system. Using this approach the various efficiency maps of the components drive the system towards a more efficient operating condition by changing two freedoms in the system – in this case engine speed and torque.

Selection of a number of parameters within the PEC algorithm are required, and use of a priori drivecycle knowledge in this process showed a significant improvement in terms of controller performance compared to one employing PEC algorithm parameters selected using a similar drivecycle. This could prove useful where vehicle drivecycles are consistent and well documented, such as for the tasks of buses or certain off-highway vehicles. Also, recent reductions in the cost of GPS systems and their increased use in automotive applications could feasibly allow the accurate prediction of passenger car drive schedules.

The performance of the PEC algorithm was compared with results obtained using the Baseline controller. As the PEC is based on the assumption of steady-state operation, its performance suffers when compared to the Baseline controller on drivecycles with a significant number of transient events. However, during steady-state operation a reduction in overall power loss from the powertrain was demonstrated with the PEC as it found a more efficient operating condition. Inspection of the engine output revealed that this was achieved in a way which is contrary to intuition when one considers the engine efficiency map. The most efficient operating condition of the engine is close to wide-open throttle (WOT), although the PEC dictates that this methodology is not always followed. To achieve similar power output from the engine, the PEC may dictate that the engine operates at a higher speed and lower torque rather than lower speed and WOT as in the case demonstrated. In this mode of operation the generator in particular was seen
to operate more efficiently, hence power loss from the generator was reduced and current flowing into the battery was increased.

In general the Baseline controller performs well, although performance improvements in terms of energy efficiency have been seen from the PEC algorithm. The performance of the PEC suffers during transient events and benefit could possibly be made using a combination of the Baseline strategy for the load following requirements and the PEC for the charge sustaining requirements of a drivecycle. The optimisation of component parameters would again be imperative to ensure maximum overall efficiency. One further benefit of the PEC technique is that it requires no off-line precalculations – component operation is determined based on the efficiency maps of the powertrain components. As such, the strategy could easily be applied to different vehicle types employing this dual hybrid powertrain configuration by simply ‘dialling in’ the appropriate efficiency maps.

A multi-input multi-output (MIMO) low-level controller based on LQR theory was developed as an alternative to the independently tuned single-input single-output (SISO) controllers employed in the Baseline configuration. This allows the inclusion of additional terms within the cost function, and has been demonstrated here by balancing the deviation from the demands set by the Supervisor against an additional criterion relating to the amplitude of driveline vibrations. The first torsional vibration mode, or shuffle has a significant effect on the driveability of a vehicle and has been the focus of much previous research. Various approaches have been postulated for the automatic control of shuffle through modulation of the engine torque response. Although improvements have been demonstrated, a source of limitation in system performance can be attributed to the inherent delays in the engine, and as such improvements could be expected from a powertrain layout containing an electrical power source with an inherently smaller actuation delay. The Toyota Prius itself does not suffer from problems associated with shuffle, although the technique could be adapted to any hybrid or electric vehicle configuration. In addition, with the recent move towards 42V electrical systems and the inclusion of integrated starter-generators attached to the crankshaft of an internal combustion engine, the technique could find further application.
Discussion and Conclusions

A reduced order model of the hybrid powertrain was described using a state-space representation, and relatively flexible driveshafts were assumed in order to exacerbate shuffle vibrations. The tyre was found to be a particularly important component to model, although a simple linear torque-slip relationship was found sufficient for this application. Given this relationship and the variation of tyre slip with vehicle speed, optimal gain matrices were derived for a number of vehicle speeds over a defined range and linearly interpolated in operation. Results from the model were compared with some obtained from the non-linear EASY5 model and good correlation was seen.

The LQR controller developed purely as a replacement for the previously used independently tuned PI controllers at the low-level closely followed the demands of the Supervisor (as the PI controllers did). As a case study, the controller was then expanded to also incorporate shuffle control.

Three techniques were investigated with the most suitable minimising the cost associated with the error between the vehicle’s longitudinal acceleration and a defined demand. Much improvement in shuffle regulation during back-out was seen as the electrical components were well within their operational boundaries, and as such were able to react to and hence regulate shuffle when compared to a controller not costing these vibrations. However, during tip-in little improvement was seen using this technique. In this case the motor responds rapidly to a sudden increase in acceleration demand and saturates as it reaches its upper torque limit. The cost associated with tracking the engine speed demand of the PEC dictates that the generator is not used to assist in this instance.

The PEC and LQR controllers developed have shown promising results in the simulations performed. However, in order to ascertain whether their performance could be improved upon, more general fully non-linear optimal control solutions were sought both with and without a priori drivecycle knowledge. An off-line sequence of controls was calculated through solution of a two-point boundary value problem. The solution is too computationally expensive to be applied on-line, and as such an open loop series of controls was found for each defined drivecycle.
Discussion and Conclusions

The results of the optimisation with *a priori* drivecycle knowledge showed that for a given drivecycle, fuel use can be much reduced by restricting the use of the engine to an absolute minimum. As the optimisation is calculated with knowledge of the full requirements of the drivecycle, little attention is paid to regaining battery SOC after some discharge if further battery use is not required. Battery SOC is then regained through regenerative braking. The same was also apparent for simulation results obtained using controls found from optimisations without *a priori* drivecycle knowledge. Even though the optimisation had no knowledge of impending drivecycle requirements, the cost function was still formulated for each individual drivecycle in order to achieve the best possible causal result that could be calculated. In order to develop and implement a strategy such as this in practice, accurate drivecycle information would be required.

Using controls obtained without *a priori* drivecycle knowledge, engine operation during braking is contrary to what would be expected and observed with PEC control. Rather than reducing the engine to idle, the controls allowed the engine speed to increase to its upper limit thus utilising the engine as an inertia brake. The engine torque was held at idle, and as such the power of the engine was reacted by the generator and converted into additional charge for the battery. Consequently a significant increase in battery SOC recovery during braking compared to the conventional method was seen. However, such a strategy may be disconcerting for passengers as engine acceleration during braking is not as would be expected from previous experience of road vehicles!

With *a priori* knowledge of a step change in acceleration demand and component limitation information the optimal controls provided an excellent solution to the shuffle regulation problem. This was achieved by progressively increasing the motor and engine torques and regulating the acceleration fluctuations with the generator. This result is sensible given the configuration of the planetary gearset where the ratios dictate an amplification of torque from the generator on the output shaft. As such, the generator’s usual role of regulating the engine speed was compromised for a short period during the shuffle event.
Using controls obtained from the optimisation technique without \textit{a priori} knowledge, it is evident that the results obtained using the LQR controller (Chapter 5) could be improved upon. This is largely due to the fact that the LQR controller is balancing the deviation of the engine speed and torque from its demands with vehicle acceleration. As such, acceleration response and hence shuffle regulation are slightly compromised. In order to provide a direct comparison, an LQR solution with no cost associated with meeting the demands for the engine speed and torque was found. Such a controller would only be applicable during acceleration events, with some switching strategy from the previously described controller of Chapter 5 required. The use of feed-forward information perhaps using rate of change of accelerator pedal information could facilitate this in practice. When compared with the optimal solution the LQR result performed favourably with little difference evident between the two. This indicates that use of an LQR solution for shuffle control is valid with little performance improvement available through use of a non-linear control law when \textit{a priori} drivecycle knowledge is unavailable. This is due to the cost function dictating that the generator counteracts the saturation of the motor thus improving the vehicle’s response.

The following main conclusions can thus be drawn from this study:

- The use of simulation tools provides a good means of analysing the complex behaviour of hybrid electric vehicles.
- Using knowledge of component efficiency data it is possible to define an on-line control system which seeks to minimise power loss in a vehicle’s powertrain in the steady state. The strategy also allows changes to powertrain components to be made relatively easily with no off-line pre-calculations required.
- The use of a low-level MIMO controller using linear optimal control theory is a good means of minimising the first torsional vibration mode of the powertrain (shuffle) of a hybrid electric vehicle. Little benefit is available through use of a non-linear alternative, although for best performance a dedicated shuffle algorithm using feed-forward information should be employed.
- The use of non-linear optimal control theory is a good means of obtaining optimal control solutions when systems and supporting functions can be defined continuous.
Discussion and Conclusions

- Knowledge of a drivecycle obtained *a priori* enables a significant improvement in control system design to be made when compared to a causal alternative.

**Recommendations for further work**

The study has been based on the use of simulation and optimisation techniques to develop novel control strategies for the Toyota Prius architecture. As such, a useful exercise would be to validate the model and Baseline controller with data collected from the real vehicle. This would allow calibration of the individual components and also allow the worth of the modelling technique employed to be assessed further. A particular component worth further attention is the battery. With additional data obtained from the test bed, a thermal model could be developed which would improve the correlation.

The cost function of the PEC could be expanded to include additional terms such as a measure of vehicle emissions or the efficiency of the planetary gearset. Relevant maps as a function of the states defined for the PEC could again be used to provide gradient information for the application of control. For the planetary gearset the cost would simply be the power loss as a function of the torques and speeds of the planetary elements, which would ultimately be described in terms of the two free variables in the system (i.e. the engine torque and speed demand). As the cost defined in the PEC is a power loss, a suitable weighting term for particular vehicular emissions would be required. Also, given that the PEC technique uses a steepest descents method to apply control, global optimality is not guaranteed. As the efficiency maps of components are relatively smooth the technique has proved successful in its current state. However, the inclusion of emissions maps with their characteristic sharp peaks or ‘hillocks’ could lead to the PEC obtaining a local rather than global optimum. As the PEC is continually adapting the operating point of the powertrain this should only be a temporary result, and as the PEC makes decisions based on the maps it is given, should this point prove to be an issue, smoothed emissions maps could be used. In addition, tabulated data of minimum emissions as a function of vehicle speed and road load torque could be formulated. This would allow the PEC algorithm to establish whether it had achieved a global or local minimum and hence search elsewhere if necessary.
Discussion and Conclusions

A further use of the model could be to optimise the sizing of key components for a particular drivecycle. An optimisation technique such as the Nelder-Mead or CARLA techniques discussed in Chapter 4 would be suitable as would other methods such as simulated annealing, genetic algorithms, Tabu search etc. The optimiser would be wrapped around the whole model with parameters used to scale the inertia terms of the relevant components or the parameters of the battery model. Other characteristics (e.g. efficiency maps) would then be either scaled accordingly or determined from linear interpolation of entered data for a range of component sizes. Other vehicle parameters such as gear ratios could also be optimised in a similar manner.

The dual hybrid powertrain configuration and PEC controller methodology could also be applied to vehicles other than passenger cars (e.g. buses, off-highway vehicles). An optimisation technique as described above could then be used to scale the components according to some defined drivecycle profile.

The technique used to minimise shuffle could be investigated for a vehicle with an integrated starter/generator attached to the crankshaft of a conventional internal combustion engined vehicle. Initial studies would be conducted in simulation with gain scheduled optimal gain matrices obtained. The control system could then be feasibly applied to a real vehicle given the recent developments of powertrains of this configuration.

In Chapter 6 non-linear optimal solutions were found in order to provide a benchmark for the PEC. Given the computationally expensive requirements of the technique, solutions were found off-line. A non-linear feedback controller could however be derived from these results. This would provide a more suitable benchmark for the PEC in that it could be applied to the EASY5 model rather than just the simplified representation. The feedback law could be obtained using a number of different methods such as determining a non-linear analytical function or by training a neural network based on the off-line results obtained.
Discussion and Conclusions

The 'rolling' optimisation technique developed to obtain a non-linear solution without \textit{a priori} drivecycle knowledge assumes that the demand of each individual optimisation is zero order held throughout the optimisation window. Further assumptions could be investigated such as the pessimistic view that the demand increases for a set period, or the optimistic view that the demand decreases to zero within the specified window. Some form of extrapolation technique could be employed to provide an estimate of future drivecycle demands thus aiding the selection.

As a final goal, the complete control strategy could be implemented on the real vehicle thus allowing a true performance assessment to be made. Given the complex nature of the control system fitted to the real vehicle this could provide a challenge as the robustness of the system would have to be of paramount importance to ensure safe operation.
References

_How Beneficial are EV's to the Environment?_
Electric and Hybrid Vehicle Technology '95

Anderson, B. D. O. and Moore, J. B., 1971
_Linear optimal control_
Prentice-Hall, Englewood Cliffs

Anderson, C. and Pettit, E., 1995
_The Effects of APU Characteristics on the Design of Hybrid Control Strategies for Hybrid Electric Vehicles_
SAE Paper 950493

Best, M. C., 1998
_Nonlinear Optimal Control of Vehicle Driveline Vibrations_
UKACC International Conference on CONTROL '98 (IEE Conf. Publ. No.455).

Bosch, R., 1993
_Automotive handbook_,
Robert Bosch GmbH, Stuttgart

Bryson, A. E. and Ho, Y. C., 1975
_Applied optimal control: optimization, estimation, and control_
Hemisphere, New York
References

A Versatile Computer Simulation Tool for Design and Analysis of Electric and Hybrid Drive trains,
SAE Paper 970199

Chizek, P., 2001
Fuel cells in focus
Electric and Hybrid Vehicle Technology

The Gas Turbine series Hybrid Vehicle - Low Emissions mobility for the Future?
Autotech 95, Paper C498/29/110

Duoba, Michael, Ng, Henry, and Larsen, Robert, 2000
In-Situ mapping and Analysis of the Toyota Prius HEV Engine
SAE paper 2000-01-3096

Amending Directive 70/220/EEC on the approximation of the laws of the member states relating to measures to be taken against air pollution emissions from motor vehicles
Official Journal of the European Communities L242, Vol. 34

Evans, C. and Stone, R., 2000
Adaption of ADVISOR 2.0 for dual hybrid vehicle modelling
IMechE - Integrated Powertrain Systems for a Better Environment

Farshidianfar, M., Ebrahimi, M., and Barlett, H., 2001
Hybrid Modelling and Simulation of the Torsional Vibration of Vehicle Driveline Systems
References

Fischer, G., 1996
*Hybrid Drive Concepts for Emission Free City Travel - a Summary of BMW Developments*
Automotive Powertrains, C498/29/204/95, Professional Engineering

Frost, G. P., 1998
*Stochastic optimisation of vehicle suspension control systems via learning automata*
PhD Thesis, Loughborough University

Heywood, J. B., 1988
*Internal combustion engine fundamentals*
McGraw-Hill, New York

Hodkinsin, R. and Fenton, J., 2001
*Lightweight Electric/Hybrid Vehicle Design*
Butterworth-Heinemann, Oxford, UK

Hubbard, G. A. and Youcef-Toumi, K., 1997
*System Level Control of a Hybrid-Electric Vehicle Drivetrain*
Proceedings of the American Control Conference Albuquerque, New Mexico

Hwang, S., Stout, J. L., and Ling, C., 1998
*Modelling and Analysis of Powertrain Torsional Response*
SAE paper 980276

Kawai, T., 2000
*Automotive Power Sources for the 21st Century*
Toyota Technical Review, Vol. 50, No. 1, pp6-11, Toyota Motor Corporation
References

Kawatsu, S., 2000
*Fuel Cell Hybrid vehicles*
Toyota Technical Review, Vol. 50, No. 1, pp24-29

*Optimised Control of an Advanced Hybrid Powertrain Based on Energy Efficiency Criteria*
First Ricardo International Conference on Vehicle Systems Integration, pp287-296, Professional Engineering Publishing, UK

Kells, A. J., Best, M. C., and Gordon, T. J., 2001
*Control of an Advanced Hybrid Electric Vehicle Using Combined Criteria for Energy Efficiency and Driveline Vibrations*
The Institute of Mathematics and its Applications, Advanced Simulation and Control for Automotive Applications, Professional Engineering Publishing, UK

Kelly, K. J., Mihalic, M., and Zolot, M., 2002
*Battery Usage and Thermal Performance of the Toyota Prius and Honda Insight for Various Chassis Dynamometer Test Procedures*
17th Annual Battery Conference on Applications and Advances

Kleimaier, D. and Schröder, D., 2000
*Optimized Design and Control of a Hybrid Vehicle with CVT*
1st IFAC Conference on Mechatronic Systems

Krause, P. C. and Wasynczuk, O., 1989
*Electromechanical motion devices*
McGraw-Hill, New York
References

Krenz, R. A., 1985
*Vehicle Response to throttle Tip-In/Tip-Out*
SAE Paper 850967

Laschet, A., 1994
*Computer Simulation of Torsional Vibrations in Vehicle Powertrains*

Marsh, C., 1992
*A nonlinear control design methodology for computer-controlled vehicle suspension systems*
PhD Thesis, Loughborough University

Matsuo, I., Nakazawa, S, Maeda, H., and Inada, E., 1999
*Development of a Hybrid Propulsion System with CVT*
8th Aachen Colloquium "Automobile and Engine Technology"

McCraw, J., 2001
*GM's going to clean up with fuel cells*
Electric and Hybrid Vehicle Technology

*Active Control of Drivability*
SAE paper 960046

Moore, T. C., 1996
*Tools and Strategies for Hybrid-Electric Drivesystem Optimization*
1996 SAE Future Transportation Technology Conference, Vancouver, B.C.
NREL, 2001
ADVISOR
www.ctts.nrel.gov/analysis

Nelder, J. A. and Mead, R., 1965
A Simplex Method for Function Minimization
The Computer Journal, Vol. 7, No. 4, pp.308-313

Ohyama, Y., 1997a
A Fuel Efficient Hybrid Drivetrain Control System
Autotech '97 Automotive Environmental Impact and Safety

Ohyama, Y., 1997b
An Advanced Engine Drivetrain Control System
SAE Paper 970291

Oi, T. and Ogiso, S., 2000
Introduction to the New Prius
Toyota Technical Review, Vol. 50, No. 1, pp.12-17

Pacejka, H. B. and Bakker, E., 1993
Magic Formula Tyre Model
Proceedings of the 1st International Colloquium on Tyre Models for Vehicle Dynamics Analysis

Petrie, D., 1995
Moving EV's into a Mass transit Scenario
Electric and Hybrid Vehicle Technology '95
Pettersson, M. and Nielsen, L., 1997
*Driveline Modeling and RQV Control with Active Damping of Vehicle Shuffle.* SAE Paper 970536

Poulton, M. L., 1994
*Alternative engines for road vehicles*
Computational Mechanics Publications, Southampton, UK

*Numerical recipes in FORTRAN :the art of scientific computing*
Cambridge University Press, Cambridge

Quigley, C. P. and Ball, R. J., 1998
*Prediction of Journey Characteristics for the Intelligent Control of a Hybrid Electric Vehicle,*
IFAC Intelligent Components for Vehicles, Seville, Spain

*Intelligent Control of Clutch Judder and Shunt Phenomena in Vehicle Drivelines*
International Journal of Vehicle Design Volume 17, No. 3

Rousseau, A., Sharer, P., and Pasquier, M., 2001
*Validation process of a HEV system analysis model: PSAT*
SAE paper 2001-01-0953

Saeks, R. and Cox, C., 1999
*Design of an Adaptive Control System for a Hybrid Electric vehicle*
IEEE International Conference on Systems Man and Cybernetics
References

Sasaki, S., 1998
*Toyota's Newly Developed Hybrid Powertrain*
International Symposium on Power Semiconductor Devices and ICs

*Toyota's Newly developed Electric-Gasoline Engine for Hybrid Powertrain Systems*
EVS 14

Scarlett, M., 2001
*Switch to hyper drive*
Electric and Hybrid Vehicle Technology 2001

Seiler, J. and Schröder, D., 1998
*Hybrid Vehicle Operating Strategies*
EVS 15

Streater, S., 2001
*Intelligent Solution?*
Electric and Hybrid Vehicle Technology 2001

Streib, H. and Hubert, B., 1996
*Electronic Throttle Control (ETC): A Cost Effective System for Improved Emissions, Fuel Economy, and Driveability*
SAE Paper 960338

Stewart, P. and Kadirkamanathan, V., 1998
*On Steady State and Dynamic Performance of Model Reference Control for a Permanent Magnet Synchronous Motor*
UKACC International Conference on CONTROL '98 (IEE Conf. Publ. No.455).
References

Stridsberg, L., 1998a
*Dual Electric Motor Hybrid Powertrain*
EVS 15

Stridsberg, L., 1998b
*Leaner, Meaner, Cleaner*
Electric and Hybrid Vehicle Technology '98

Swann, J., 1998
*Simulation Software Tool for Advanced Powertrain Solutions*
MIRA - Technology Towards the Millenium

*Compressed Natural Gas Vehicle*
Toyota Technical Review, Vol. 50, No. 1, pp36-41

Teratini, T., Kuramochi, K., Nakao, H., Horii, K., Inayoshi, T., and Mizutani, K., 2000
*Development of a Vehicle with Idle Stop System*

The Boeing Company, 1997
*EASY5 User's Guide*

The MathWorks Inc., 1997
*MATLAB: The Language of Technical Computing*

Think Mobility, 2000
*Th!nk City Brochure*
References

Stability Analysis for Self-Excited Torsional Oscillation of Vehicle Driveline
International Journal of vehicle Design, Volume 24, No's, 2/3

Development of Battery System for Hybrid Vehicle
EVS15

Single Shaft Parallel Hybrid Drive System
SAE Paper 970286

West, J. G. W., 1997
Hybrid Vehicles
Autotech '97 - Automotive environmental impact and safety

Wouk, V., 1995
Hybrids: then and now

Yamada, M., Shibata, M., and Suda, T., 2000
Toyota's 'Crayon' car Sharing System Using an Ultra Compact EV
Appendix A – Toyota Prius Technical Specification

The following provides detail of the Japanese specification Toyota Prius as gathered from published sources.

**Vehicle**

<table>
<thead>
<tr>
<th>Type</th>
<th>Front engined, front wheel drive, 5-seater, 4 door saloon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>1250 kg</td>
</tr>
<tr>
<td>Suspension</td>
<td>F: MacPherson Strut, anti roll bar</td>
</tr>
<tr>
<td></td>
<td>R: Trailing arms</td>
</tr>
<tr>
<td>Steering</td>
<td>Power assisted rack and pinion.</td>
</tr>
<tr>
<td></td>
<td>4.1 turns lock to lock</td>
</tr>
<tr>
<td>Brakes</td>
<td>F: Regenerative with 10 x 0.9 in vented disks</td>
</tr>
<tr>
<td></td>
<td>R: 7.9 x 1.2 in cast iron drum</td>
</tr>
</tbody>
</table>

**Internal combustion engine**

<table>
<thead>
<tr>
<th>Type</th>
<th>4 cylinder in-line with aluminium block and head</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bore x stroke</td>
<td>75.0 x 84.7mm</td>
</tr>
<tr>
<td>Displacement</td>
<td>1497cc</td>
</tr>
<tr>
<td>Compression ratio</td>
<td>13.5:1</td>
</tr>
<tr>
<td>Effective Atkinson cycle</td>
<td>9.5:1</td>
</tr>
<tr>
<td>compression ratio</td>
<td></td>
</tr>
<tr>
<td>Engine control system</td>
<td>Toyota with port fuel injection</td>
</tr>
<tr>
<td>Emissions control</td>
<td>3-way catalytic converter</td>
</tr>
<tr>
<td>Valve gear</td>
<td>Chain-driven double overhead cams, 4 valves per cylinder,</td>
</tr>
<tr>
<td></td>
<td>hydraulic lifters, variable intake-valve timing</td>
</tr>
<tr>
<td>Power (SAE net)</td>
<td>57 bhp (42.5 kW) @ 4000 rpm</td>
</tr>
<tr>
<td>Torque (SAE net)</td>
<td>75 lb-ft (101.7Nm) @ 4000 rpm</td>
</tr>
</tbody>
</table>
### Electric machines

**Motor**
- **Type**: 3-phase AC permanent magnet synchronous
- **Power**: 40 bhp (29.8kW) @ 940 – 2000 rpm
- **Maximum torque**: 225 lb-ft (305 Nm) @ 0 – 940 rpm

**Generator**
- **Type**: 3-phase AC permanent magnet synchronous
- **Power**: 21 kW
- **Maximum torque**: 40.6 lb-ft (55 Nm) @ 0 – 2000 rpm

### Powertrain

- **Transmission**: Continuously variable automatic
- **Final drive ratio**: 3.93:1
- **Transmission ratio range**: Infinite

### Battery

- **Type**: Nickel-metal hydride (NiMh)
- **Number of cells**: 240
- **Cell voltage**: 1.2 V
- **Number of modules**: 40 - one module is made with six cells in series
- **Maximum capacity**: 6.5 Ah

### Additional data

- **$K_s$**: 10,000 Nm/rad
- **$C_s$**: 1000 Nms/rad
- **$K_T$**: 50,000 Nm/rad
- **$C_T$**: 2500 Nms/rad
- **$m_v$**: 1250 kg
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_T$</td>
<td>10,000 Nm</td>
</tr>
<tr>
<td>$J_v$</td>
<td>112.5 kgm²</td>
</tr>
<tr>
<td>$I_r$</td>
<td>0.0002 kgm²</td>
</tr>
<tr>
<td>$I_s$</td>
<td>0.0001 kgm²</td>
</tr>
<tr>
<td>$I_c$</td>
<td>0.0001 kgm²</td>
</tr>
<tr>
<td>$I_p$</td>
<td>0.0001 kgm²</td>
</tr>
<tr>
<td>$m_p$</td>
<td>0.01 kg</td>
</tr>
<tr>
<td>$R_c$</td>
<td>0.01 m</td>
</tr>
<tr>
<td>$t_a$</td>
<td>78</td>
</tr>
<tr>
<td>$t_s$</td>
<td>30</td>
</tr>
</tbody>
</table>
Appendix B – The Power Efficient Controller

If the power consumption of the drive components is assumed known and reasonably constant, this information can readily be used to prescribe a gradient to drive the total system to a more power efficient steady state operating point. The seven internal states of the system are the motor torque, $T_m$, and the speeds and torques at the epicyclic ($x_1$ to $x_6$) (Figure B1):

- $x_1$: Motor/annulus speed (rad/s)
- $x_2$: Torque on annulus gear of epicyclic (Nm)
- $x_3$: Generator speed (rad/s)
- $x_4$: Torque on sun gear of epicyclic (generator torque) (Nm)
- $x_5$: Engine speed (rad/s)
- $x_6$: Torque on carrier of epicyclic (engine torque) (Nm)

Figure B1: System schematic
The road load torque in the steady state is given by $T_{rl} = T_m - x_2$. For the steady state, five constraints can be imposed.

### B.1 System constraints

- **Power balance:**
  $$x_1 x_2 + x_3 x_4 + x_5 x_6 = 0 \quad (B1)$$

- **Torque balance:**
  $$x_2 + x_4 + x_6 = 0 \quad (B2)$$

- **Speed in the epicyclic:**
  $$x_1 - R_{sa} x_3 + (R_{sa} - 1)x_5 = 0 \quad (B3)$$

  where $R_{sa} = \frac{t_s}{t_a}$

- Steady state vehicle speed and road load torque:
  $$\delta x_1 = \delta T_{rl} = 0 \quad (B4)$$

  Also, noting that $x_2$ is $-ve$ when the torque is out of the epicyclic:
  $$T_n = T_m - x_2$$

  Therefore:
  $$\delta T_{rl} = \delta T_m - \delta x_2 \quad (B5)$$

  whence
  $$\delta T_m = \delta x_2 \quad (B6)$$

### B.2 Differentiating the constraints

- $x_2 \delta x_1 + x_1 \delta x_2 + x_4 \delta x_3 + x_3 \delta x_4 + x_5 \delta x_5 + x_6 \delta x_6 = 0 \quad (B7)$

- $\delta x_2 + \delta x_4 + \delta x_6 = 0 \quad (B8)$

- $\delta x_1 - R_{sa} \delta x_3 + (R_{sa} - 1)\delta x_5 = 0 \quad (B9)$
Appendix B

\[ \delta x_1 = 0 \] (B10)

\[ \delta T_{fl} = 0 \] (B11)

Aim;
Express the total power loss in the powertrain in terms of two freedoms in the system; in this case the engine speed and torque.

Therefore require \( \delta x_1, \delta x_2, \delta x_3, \delta x_4 = f(\delta x_5, \delta x_6) \)

From (B9)

\[ \delta x_3 = \frac{(R_{sa} - 1)\delta x_5}{R_{sa}} \] (B12)

(B12) into (B7)

\[ x_1\delta x_2 + x_4 \left[ \frac{(R_{sa} - 1)}{R_{sa}} \delta x_5 \right] + x_3\delta x_4 + x_6\delta x_5 + x_5\delta x_6 = 0 \]

\[ \therefore x_1\delta x_2 + x_3\delta x_4 + \left[ \frac{(R_{sa} - 1)}{R_{sa}} x_4 + x_6 \right] \delta x_5 + x_5\delta x_6 = 0 \] (B13)

(B13) + \( x_1 \)

\[ \frac{\delta x_2}{x_1} + x_3\frac{\delta x_4}{x_1} + \left[ \frac{(R_{sa} - 1)x_4}{R_{sa}x_1} + \frac{x_6}{x_1} \right] \frac{\delta x_5}{x_1} + \frac{x_5}{x_1} \frac{\delta x_6}{x_1} = 0 \] (B14)

(B14) - (B8)

\[ \left( \frac{x_3}{x_1} - 1 \right) \frac{\delta x_4}{x_1} + \left[ \frac{(R_{sa} - 1)x_4}{R_{sa}x_1} + \frac{x_6}{x_1} \right] \frac{\delta x_5}{x_1} + \left( \frac{x_5}{x_1} - 1 \right) \frac{\delta x_6}{x_1} = 0 \]

(From (B15),

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Appendix B

\[ \delta x_4 = \left\{ -\frac{\delta x_5}{x_1} \left[ \frac{(R_{sa} - 1)x_4 + x_6}{x_1} \right] - \frac{\delta x_6}{x_3 - x_1} \right\} x_1 \]

\[ \delta x_4 = -\delta x_5 \left[ \frac{(R_{sa} - 1)x_4 + x_6 R_{sa}}{R_{sa} (x_3 - x_1)} \right] - \frac{\delta x_6}{x_3 - x_1} \left( \frac{x_5 - x_1}{x_3 - x_1} \right) \]  
(B16)

(B16) into (B8)

\[ \delta x_2 = -\delta x_6 + \delta x_5 \left[ \frac{(R_{sa} - 1)x_4 + x_6 R_{sa}}{R_{sa} (x_3 - x_1)} \right] + \delta x_6 \left( \frac{x_5 - x_1}{x_3 - x_1} \right) \]

\[ \therefore \delta x_2 = \delta x_5 \left[ \frac{(R_{sa} - 1)x_4 + x_6 R_{sa}}{R_{sa} (x_3 - x_1)} \right] + \delta x_6 \left( \frac{x_5 - x_1}{x_3 - x_1} - 1 \right) \]  
(B17)

B.3 Power loss function

B.3.1 Motor

If the motor is drawing current from the battery, i.e. 'motoring';

\[ P_m = x_1 (T_n + x_2) \left( \frac{1}{\eta_m} - 1 \right) \]

\[ \delta P_m = (T_n + x_2) \left( \frac{1}{\eta_m} - 1 \right) \delta x_1 + x_1 \left( \frac{1}{\eta_m} - 1 \right) \delta x_2 + x_1 \left( \frac{1}{\eta_m} - 1 \right) \delta T_n - \]

\[ \frac{x_1 (T_n + x_2)}{\eta_m^2} \frac{\partial \delta P_m}{\partial x_1} \delta x_1 - \frac{x_1 (T_n + x_2)}{\eta_m^2} \frac{\partial \delta P_m}{\partial T_m} \delta T_m \]

\[ \delta P_m = x_1 \left( \frac{1}{\eta_m} - 1 \right) \delta x_2 - \frac{x_1 (T_n + x_2)}{\eta_m} \frac{\partial \delta P_m}{\partial T_m} \delta x_2 \]
Appendix B

\[ P_{b_m} = -\frac{1}{\eta_m} x_1 T_m = -\frac{1}{\eta_m} x_1 (T_{rl} + x_2) \] (power into battery)

\[ \delta P_{b_m} = -\frac{(T_{rl} + x_2)}{\eta_m} \delta x_1 - \frac{x_1}{\eta_m} \delta x_2 - \frac{x_1}{\eta_m} \delta T_{rl} + \frac{x_1}{\eta_m} \frac{\partial \eta_m}{\partial x_1} \delta x_1 + \]
\[ \frac{x_1 (T_{rl} + x_2)}{\eta_m^2} \frac{\partial \eta_m}{\partial T_m} \delta T_m \]

\[ \delta P_{b_m} = \left[ -\frac{x_1}{\eta_m} + \frac{x_1 (T_{rl} + x_2)}{\eta_m^2} \frac{\partial \eta_m}{\partial T_m} \right] \delta x_2 \]

If the motor is supplying the battery with current, i.e. 'generating';

\[ P_m = -T_m x_1 - \eta_m (-T_m x_1) \]

\[ P_m = T_m x_1 (\eta_m - 1) = (T_{rl} + x_2) x_1 (\eta_m - 1) \]

\[ \delta P_m = (T_{rl} + x_2)(\eta_m - 1) \delta x_1 + x_1 (\eta_m - 1) \delta x_2 + x_1 (\eta_m - 1) \delta T_m \]
\[ + (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial x_1} \delta x_1 + (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial T_m} \delta T_m \]

\[ \therefore \delta P_m = x_1 (\eta_m - 1) \delta x_2 + (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial T_m} \delta x_2 \]

\[ P_{b_m} = -\eta_m T_m x_1 = -\eta_m (T_{rl} + x_2) x_1 \]

\[ \delta P_{b_m} = -\eta_m (T_{rl} + x_2) \delta x_1 - \eta_m x_1 \delta x_2 - \eta_m x_1 \delta T_{rl} - (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial x_1} \delta x_1 - \]
\[ (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial T_m} \delta T_m - (T_{rl} + x_2) x_1 \frac{\partial \eta_m}{\partial T_m} \delta T_m \]
Appendix B

\[ \delta P_{b_m} = \left[ -\eta_m x_1 - (T_{n} + x_2) x_4 \frac{\partial \eta_m}{\partial T_m} \right] \delta x_2 \]

B.3.2 Generator

If the generator is drawing current from the battery, i.e. 'motoring';

\[ P_s = x_3 x_4 \left( \frac{1}{\eta_g} - 1 \right) \]

\[ \delta P_g = x_4 \left( \frac{1}{\eta_g} - 1 \right) \delta x_3 + x_3 \left( \frac{1}{\eta_g} - 1 \right) \delta x_4 - \frac{x_4 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_3} \delta x_3 - \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_4} \delta x_4 \]

\[ \delta P_g = \left[ x_4 \left( \frac{1}{\eta_g} - 1 \right) - \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_3} \right] \delta x_3 + \left[ x_3 \left( \frac{1}{\eta_g} - 1 \right) - \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_4} \right] \delta x_4 \]

\[ P_{b_g} = -\frac{1}{\eta_g} x_3 x_4 \]

\[ P_{b_g} = -\frac{1}{\eta_g} x_3 \delta x_3 - \frac{1}{\eta_g} x_3 \delta x_4 + \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_3} \delta x_3 + \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_4} \delta x_4 \]

\[ P_{b_g} = \left[ -\frac{1}{\eta_g} x_4 + \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_3} \right] \delta x_3 + \left[ -\frac{1}{\eta_g} x_3 + \frac{x_3 x_4}{\eta_g^2} \frac{\partial \eta_g}{\partial x_4} \right] \delta x_4 \]

If the generator is supplying the battery with current, i.e. the generator is 'generating';

\[ P_g = x_3 x_4 (\eta_g - 1) \]

\[ \delta P_g = x_4 (\eta_g - 1) \delta x_3 + x_3 (\eta_g - 1) \delta x_4 + x_3 x_4 \frac{\partial \eta_g}{\partial x_3} \delta x_3 + x_3 x_4 \frac{\partial \eta_g}{\partial x_4} \delta x_4 \]
B.3.3 Engine

\[ P_e = \frac{x_5 x_6}{\eta_e} \]

\[ \delta P_e = \left[ \frac{x_6}{\eta_e} \delta x_5 + \frac{x_5}{\eta_e} \delta x_6 - \frac{x_5 x_6}{\eta_e^2} \delta x_5 - \frac{x_5 x_6}{\eta_e} \frac{\partial \eta_e}{\partial x_5} \right] \delta x_5 + \left[ \frac{x_5}{\eta_e} \delta x_6 - \frac{x_5 x_6}{\eta_e} \frac{\partial \eta_e}{\partial x_6} \right] \delta x_6 \]

B.3.4 Overall power loss

The total power loss in the powertrain can be expressed in terms of losses in the motor, generator and engine, i.e.

\[ P_{\text{kp}}(x) = P_m(x_1, T_m, \eta_m) + P_g(x_3, x_4, \eta_g) + P_e(x_5, x_6, \eta_e) + \lambda (P_{\text{in}} - P_{\text{out}}) \]

The change in power 'cost' can then be described in terms of derivatives \( \frac{\delta P_{\text{kp}}}{\delta x_i} \) which can be readily calculated from the efficiency maps of the components:
As the system is represented by seven degrees of freedom and five constraints, the $\delta x_i$ can be described in terms of a change in two freedoms in the system – in this case engine speed and torque. Therefore:

$$\delta P_p = \alpha \delta x_s + \beta \delta x_\theta$$

Supervisory control is then imposed via steepest-descents relative to the gradients $\alpha$ and $\beta$:

$$\dot{x}_s = -\varphi \alpha$$
$$\dot{x}_\theta = -\varphi \beta$$

where $\varphi$ is an acceleration parameter. The time rate of change of power 'cost' is then given by:

$$\dot{P}_p = -\varphi (\alpha^2 + \beta^2)$$

which ensures cost reductions for any positive $\varphi$.

The algorithm is implemented here using a suitably small constant value of $\varphi$, which ensures slow adaptation of the system’s operating point. $\lambda$ is not constant however – the cost associated with power drain in the battery is more sensibly related to its instantaneous state of charge with an additional term to damp oscillations in the engine set point:

$$\lambda = k_1 (\text{SOC} - 0.6) + k_2 (P_b + P_b_m)$$

where $k_1$ and $k_2$ are constants, and $\text{SOC} = 0.6$ is the desired operating point.
Appendix C – Analysis of the Planetary Gearset

A schematic of the planetary gearset is shown in Figure C.1.

![Planetary Gearset Diagram](image)

**Figure C.1: Planetary gearset**

In order to provide a state space analysis of the system, equations of motion for the individual elements shall be considered.

A schematic of the annulus is shown in Figure C.2.

![Planetary Annulus Diagram](image)

**Figure C.2: Planetary annulus**

The equation of motion of the annulus:

\[ I_r \dot{\omega}_a = T_m + T_2 + f_{pr} R_r \]  

(C.1)
A schematic of the sun gear is shown in Figure C.3:

![Figure C.3: Planetary sun gear](image)

The equation of motion of the sun:

$$I_s \dot{\omega}_s = T_s + f_{sp} R_s$$  \hspace{1cm} (C.2)

A schematic of the carrier is shown in Figure C.4.

![Figure C.4: Planetary carrier gear](image)

Considering the equivalent inertia of the carrier:

$$I^*_c = I_c + \left( I_p + m_p R^2_e \right)$$

The equation of motion of the carrier:
Appendix C

\[ I_c^e \omega_c = T_e - f_{pr} (R_c + R_p) - f_{sp} (R_c - R_p) \]  \(\text{(C.3)}\)

A schematic of the planet gear is shown in Figure C.5.

![Planet gear schematic](image)

**Figure C.5: Planet gear**

The equation of motion of the planet:

\[ I_p \dot{\omega}_p = R_p(f_{sp} - f_{pr}) \]  \(\text{(C.4)}\)

Considering the kinematic equations of the planetary gearset:

\[ t_a \omega_a + t_s \omega_s = (t_a + t_s) \omega_c \]  \(\text{(C.5)}\)

\[ t_a \omega_a - t_s \omega_s = (t_a - t_s) \omega_p \]  \(\text{(C.6)}\)

Differentiating C.5

\[ t_a \dot{\omega}_a + t_s \dot{\omega}_s = (t_a + t_s) \dot{\omega}_c \]  \(\text{(C.7)}\)

Differentiating C.6

\[ t_a \dot{\omega}_a - t_s \dot{\omega}_s = (t_a - t_s) \dot{\omega}_p \]  \(\text{(C.8)}\)

From C.5,

\[ \dot{\omega}_p = \frac{R_p}{I_p} (f_{sp} - f_{pr}) \]

\[ \therefore \text{into C.8:} \]
Appendix C

\[ t_a \dot{\omega}_a - t_s \dot{\omega}_s = (t_a - t_s) \frac{R_p}{I_p} (f_{sp} - f_{pr}) \]

Substituting for \( \dot{\omega}_s \) and \( \dot{\omega}_a \) from C.1 and C.2:

\[
\frac{t_a}{I_r} (T_m + T_2 + f_{pr} R_r) - \frac{t_s}{I_s} (T_g + f_{sp} R_s) = \frac{R_p}{I_p} (f_{sp} - f_{pr}) (t_a - t_s)
\]

\[
\therefore f_{pr}\left(\frac{t_a R_r}{I_r} + \frac{R_p (t_a - t_s)}{I_p}\right) = f_{sp}\left(\frac{t_s R_s}{I_s} + \frac{R_p (t_a - t_s)}{I_p}\right) = \frac{T_g t_s}{I_s} - \frac{t_a (T_m + T_2)}{I_r}
\]

let

\[
\frac{t_s R_s}{I_s} + \frac{R_p (t_a - t_s)}{I_p} = \alpha
\]

let

\[
\frac{t_s R_s}{I_s} + \frac{R_p (t_a - t_s)}{I_p} = \beta
\]

let

\[
\frac{T_g t_s}{I_s} - \frac{t_s (T_m + T_2)}{I_r} = \gamma
\]

\[
\therefore \alpha f_{pr} - \beta f_{sp} = \gamma \quad (C.9)
\]

from C.3,

\[
\dot{\omega}_c = \frac{T_c}{I_c^*} - \frac{f_{pr} (R_c + R_p)}{I_c^*} - \frac{f_{sp} (R_c - R_p)}{I_c^*}
\]

Substituting for \( \dot{\omega}_s \) and \( \dot{\omega}_a \) from (C.1) and (C.2), and using (C.7):
Appendix C

\[
\frac{t_a}{I_r}(T_m + T_z + f_{pr}R_r) + \frac{t_s}{I_s}(T_g + f_{sp}R_s) = \frac{(t_a + t_s)T_e}{I_c'}
\]

\[
f_{pr}\left(\frac{R_c + R_p}{I_r'}\right)(t_a + t_s) - f_{sp}\left(\frac{R_c - R_p}{I_s'}\right)(t_a + t_s)
\]

Therefore:

\[
f_{pr}\left(\frac{t_aR_r}{I_r} + \frac{(R_c + R_p)(t_a + t_s)}{I_c'}\right) + f_{sp}\left(\frac{t_sR_s}{I_s} + \frac{(R_c - R_p)(t_a + t_s)}{I_s'}\right) = \frac{(t_a + t_s)T_e}{I_c'} - \frac{t_s(T_m + T_z)}{I_r} - \frac{t_sT_g}{I_s}
\]

let

\[
\frac{t_aR_r}{I_r} + \frac{(R_c + R_p)(t_a + t_s)}{I_c'} = \Delta
\]

let

\[
\frac{t_sR_s}{I_s} + \frac{(R_c - R_p)(t_a + t_s)}{I_s'} = \varepsilon
\]

let

\[
\frac{(t_a + t_s)T_e}{I_c'} - \frac{t_s(T_m + T_z)}{I_r} - \frac{t_sT_g}{I_s} = \eta
\]

\[
\therefore \Delta f_{pr} + \varepsilon f_{sp} = \eta
\]

(C.10)

\[
(C.9) \times \frac{\Delta}{\alpha} - (C.10):
\]

\[
\frac{a\Delta}{\alpha} f_{pr} - \frac{\beta\Delta}{\alpha} f_{sp} - \Delta f_{pr} - \varepsilon f_{sp} = \frac{\gamma\Delta}{\alpha} - \eta
\]

\[
\therefore f_{sp} = \left(\eta - \frac{\gamma\Delta}{\alpha}\right) + \left(\varepsilon + \frac{\beta\Delta}{\alpha}\right)
\]

\[
\therefore f_{sp} = \frac{(t_a + t_s)T_e}{I_c'} - \frac{t_s(T_m + T_z)}{I_r} - \frac{t_sT_g}{I_s} + \frac{T_s t_s \Delta}{I_c' \mu I_r} + \frac{t_s(T_m + T_z) \Delta}{I_c' \mu I_s \alpha \mu}
\]

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\[ \dot{\omega}_s = \frac{T_s}{I_s} + \frac{R_s}{I_s} \left[ \frac{(t_a + t_s)I_c}{I_c} - \frac{t_s (T_m + T_2)}{I_s} - \frac{t_s T_g}{I_s} - \frac{T_g t_s \Delta}{I_s \alpha \mu} \right] \]

\[ \ddot{\omega}_s = T_m \left[ \frac{-t_s R_s}{I_s \mu I_r} + \frac{t_s \Delta R_s}{I_s I_r \alpha \mu} \right] + T_2 \left[ \frac{-t_s R_s}{I_s \mu I_r} + \frac{t_s \Delta R_s}{I_s I_r \alpha \mu} \right] + \]

\[ T_c \left( \frac{t_s + t_s \Delta R_s}{I_s I_r} \right) + T_g \left[ \frac{1}{I_s} - \frac{t_s R_s}{I_s^2 \mu} - \frac{t_s \Delta R_s}{I_s^2 \alpha \mu} \right] \]

(C.11)

\[ f_{pr} = \frac{\eta}{\Delta} - \frac{\varepsilon f_{se}}{\Delta} = \frac{\varepsilon \eta}{\Delta} - \frac{\varepsilon \gamma}{\Delta} \]

\[ \therefore f_{pr} = \frac{\varepsilon \eta}{\Delta} - \frac{\varepsilon \gamma}{\alpha \mu} \]

\[ f_{pr} = \frac{(t_a + t_s)I_c}{I_c \Delta} - \frac{t_s (T_m + T_2)}{I_s \Delta} - \frac{t_s T_g}{I_s \Delta} - \frac{\varepsilon (t_a + t_s)I_c}{I_s \Delta} + \]

\[ \frac{t_s \varepsilon (T_m + T_2)}{I_s \mu \Delta} + \frac{t_s T_g \varepsilon}{I_s \mu \Delta} + \frac{t_s \varepsilon (T_m + T_2)}{I_s \alpha \mu} + \frac{t_s \varepsilon (T_m + T_2)}{I_s \alpha \mu} \]

\[ \dot{\omega}_a = \frac{T_m}{I_r} + \frac{T_2}{I_r} + \frac{(t_a + t_s)I_c R_r}{I_c \Delta I_r} - \frac{t_s R_r (T_m + T_2)}{I_s \Delta I_r} - \frac{t_s T_g R_r}{I_s \Delta I_r} - \]

\[ \frac{\varepsilon R_r (t_a + t_s)I_c}{I_c \mu \Delta I_r} + \frac{t_s \varepsilon R_r (T_m + T_2)}{I_s \mu \Delta I_r} + \frac{t_s R_r T_g \varepsilon}{I_s \mu \Delta I_r} + \frac{\varepsilon R_r T_g t_s}{I_s \alpha I_r} + \]

\[ \frac{t_s R_r \varepsilon (T_m + T_2)}{I_s^2 \alpha \mu} \]
Appendix C

\[ \dot{\omega}_a = T_m \left[ \frac{1}{I_r} - \frac{t_a}{I_r^2 \Delta} + \frac{t_a eR_r}{I_r^2 \mu \Delta} + \right] \]

\[ \text{+}T_2 \left[ \frac{1}{I_r} - \frac{t_a eR_r}{I_r^2 \Delta} + \frac{t_a eR_r}{I_r^2 \mu \Delta} + \right] \]

\[ \text{+}T_e \left[ \frac{(t_s + t_a)R_r}{I_c \Delta I_r} - \frac{\varepsilon R_r (t_s + t_a)}{I_c \mu \Delta I_r} \right] \]

\[ \text{+}T_s \left[ -\frac{t_a R_r e}{I_r \mu I_r} - \frac{\varepsilon R_r t_a}{I_r \mu I_r} + \right] \]

(C.12)