The simulation of the
dynamic load characteristics
of an internal combustion engine

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THE SIMULATION OF THE DYNAMIC LOAD CHARACTERISTICS OF AN INTERNAL COMBUSTION ENGINE.

by

J. H. Scutchings.

Under the supervision of:

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Submitted for the Degree of Ph.D. of
Loughborough University of Technology.

May 1968.
SUMMARY

This thesis considers the feasibility of replacing a series of diesel engines, which are used in starter-motor endurance tests, by an electrical loading system whose torque-characteristics are similar to those of a given individual engine. The torque-characteristics of diesel engines and starter-motors are briefly examined to indicate the problems associated with their simulation. The simulating system finally evolved is based on a static-switching current-regulator and useful contributions are made towards both the physical realisation of suitable circuit-techniques and the general analysis and operation of switching-mode circuits of this kind. Special consideration is given to the problems of rapid and periodic reversal of the regulator-output-current, and a static-logic system is evolved that provides this essential facility.

Some practicable and novel methods of generating high-frequency functions are presented for the production of torque/crank-angle waveforms for a range of engine-parameters, and proposals are made for the electrical simulation of the engine-inertia. The effects of this latter function upon the system-torque-requirements and the validity of the overall simulation are then examined in detail.

It is concluded that an electrical simulation is possible by use of the techniques suggested, although some complexity is unavoidable if high accuracy and performance are to be achieved.
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List of Symbols.

\(a'_0\) Fourier-coefficient of zero-frequency component in pound-force feet.

\(a_q\) Fourier-coefficient of cosine terms in pound-force feet.

\(A_p\) Area of a single piston in square inches.

\(b_q\) Fourier-coefficient of sine terms in pound-force feet.

\(C\) Capacitance (general).

\(E_A\) Rotational e.m.f. of the auxiliary machine.

\(E_B\) E.M.F. of the bias-supply in the time-ratio control-system.

\(E_D\) E.M.F. of the supply to the time-ratio control-system.

\(E_S\) Rotational e.m.f. of the simulating machine.

\(f(t)\) Function of time, \(t\).

\(f_c\) Cut-off frequency of the load-circuit-impedance.

\(f_m\) Fundamental frequency of engine-torque-pulsations.

\(f_p\) Switching-frequency of the time-ratio control-circuit. \(f_p = 1/(t_{CL} + t_{cr})\)

\(f_{w}\) Fundamental frequency of the simulated torque-waveform prior to sampling.

\(F\) Inertial force in pounds force.

\(F_v\) Viscous friction in pound-force feet per radian per second.

\(g^+\) Gear-ratio (Number of starting-ring teeth/Number of Starter-pinion teeth).

\(G\) Control-range of a time-ratio control-system.

\(i\) Integral multiplier of \(f_m\).

\(i_o\) Instantaneous output-current of the time-ratio control-system.

\(I_{2S}\) Armature-current of the simulating machine.

\(I_{2V}\) Average value of output-current of the time-ratio control-system.

\(I_E\) (Appendix II) Emitter-current of the unijunction-transistor attenuator.

\(I_{FA}\) Field-excitation-current of the auxiliary machine.

\(I_m\) Maximum possible output-current of the time-ratio control-system.

\(I_{PK}\) Peak current of a series resonant circuit.
Lower and upper values of output-current from the time-ratio control-system.

Bessel-coefficient (1st. kind) of variable \( x \), and order \( /1/ \).

Engine inertia (rotating).

Starter-motor-inertia.

Starter-motor-inertia referred to the engine- or simulator-shaft.

Simulator-inertia.

A constant of proportionality.

Gain of displacement-transducer in volts per radian.

Overall gain of time-ratio control-system and machines in pound-force feet per volt.

Forcing-factor.

Amplitude coefficient.

Fractional time-inhibition of the pulse-delay-interlock.

Inertia-simulation-factor.

Rise-time improvement factor.

Connecting-rod-length in inches.

Inductance (general).

Modulation-factor, i.e., \( (tp_{\text{MAX}} - tp_{\text{MIN}})/(tp_{\text{MAX}} + tp_{\text{MIN}}) \).

Instantaneous internal torque of an engine.

Instantaneous internal torque due to gas-pressure of a group of \( n_c \) cylinders.

Instantaneous internal torque due to reciprocating parts of a group of \( n_c \) cylinders.

Instantaneous internal torque due to reciprocating parts of a quadrature group of four cylinders.

Instantaneous value of engine-friction-torque.

Instantaneous value of simulator-friction-torque.

Instantaneous resultant torque responsible for providing acceleration.

Instantaneous output-torque of the starter-motor.
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m'$</td>
<td>Instantaneous output-torque of the starter-motor referred to the engine- or simulator-shaft.</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Number of phases of a polyphase supply.</td>
</tr>
<tr>
<td>$m_s$</td>
<td>Instantaneous value of reaction-torque.</td>
</tr>
<tr>
<td>$m_s'$</td>
<td>Instantaneous value of simulator-torque.</td>
</tr>
<tr>
<td>$n$</td>
<td>Integral multiplier of $f_p$.</td>
</tr>
<tr>
<td>$n_c$</td>
<td>Any number of regularly-phased cylinders.</td>
</tr>
<tr>
<td>$n_f$</td>
<td>Number of complete waveforms generated during one engine-revolution.</td>
</tr>
<tr>
<td>$N_e$</td>
<td>Engine-speed in revolutions per minute.</td>
</tr>
<tr>
<td>$N_m$</td>
<td>Starter-motor-speed in revolutions per minute.</td>
</tr>
<tr>
<td>$N_s$</td>
<td>Simulator-speed in revolutions per minute.</td>
</tr>
<tr>
<td>$P$</td>
<td>Laplace-operator.</td>
</tr>
<tr>
<td>$p'$</td>
<td>Number of pole-pairs.</td>
</tr>
<tr>
<td>$p_{g}$</td>
<td>Cylinder-pressure in pounds-force per square inch.</td>
</tr>
<tr>
<td>$q$</td>
<td>Harmonic order of $f_m$ for a two-cylinder engine.</td>
</tr>
<tr>
<td>$r$</td>
<td>Crank-arm-radius in inches.</td>
</tr>
<tr>
<td>$R$</td>
<td>Resistance (general).</td>
</tr>
<tr>
<td>$R_{2A}$</td>
<td>Armature-resistance of auxiliary machine.</td>
</tr>
<tr>
<td>$R_{45}$</td>
<td>Armature-resistance of simulating machine.</td>
</tr>
<tr>
<td>$R_{EB1}$</td>
<td>Internal emitter-base-one resistance.</td>
</tr>
<tr>
<td>$R_{EB2}$</td>
<td>Internal emitter-base-two resistance.</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (general).</td>
</tr>
<tr>
<td>$t_{CL}$</td>
<td>Time for which 'switch' is closed.</td>
</tr>
<tr>
<td>$t_{off}$</td>
<td>Turn-off time of a thyristor.</td>
</tr>
<tr>
<td>$t_{op}$</td>
<td>Time for which 'switch' is open.</td>
</tr>
<tr>
<td>$t_p$</td>
<td>Duration of any pulse in seconds.</td>
</tr>
<tr>
<td>$t_{fo}$</td>
<td>Mean duration of pulses in seconds.</td>
</tr>
<tr>
<td>$T_{1}$</td>
<td>Armature-circuit-time-constant of combined simulating and auxiliary machines.</td>
</tr>
</tbody>
</table>
\[ T_2 \] Mechanical time-constant of coupled simulator and starter-motor.

\[ T_f \] Time-constant of load-circuit when current is falling.

\[ T_{FA} \] Time-constant of the auxiliary machine field-circuit.

\[ T_+ \] Time-constant of load-circuit when current is rising.

\[ v \] Oil viscosity.

\[ v_c \] Instantaneous volume of a single cylinder in cubic inches.

\[ V_{BB} \] Voltage between base-one and base-two of a unijunction transistor.

\[ V_{BE} \] Base-emitter saturation-voltage of a junction transistor.

\[ V_{BS} \] Signal-voltage applied indirectly across the bases of a unijunction transistor.

\[ V_{cc} \] Supply-voltage.

\[ V_{eB1} \] Voltage between emitter and base-one of a unijunction transistor.

\[ V_{es} \] Controlling-voltage applied indirectly to emitter of a unijunction transistor.

\[ V_T \] Output-voltage of a d.c. tacho-generator.

\[ W \] Weight of reciprocating parts per cylinder in pounds-force.

\[ Z \] Impedance (general).

\[ \alpha \] Amplitude-factor.

\[ \alpha^o \] Crankshaft-angle (in degrees), as employed in Figure 34 and Appendix 6.

\[ \beta \] Instantaneous load-current as a fraction of \( I_m \).

\[ \gamma \] Fractional index.

\[ \delta \] Small increment. (Calculus notation).

\[ \eta \] Ideal efficiency of time-ratio control-system.

\[ \Theta \] Crankshaft-angle (in degrees) measured from top-dead-centre.

\[ \mu(f_p+i.f_m) \] Relative amplitude of side-frequency component \( (f_p + i.f_m) \).

\[ \Phi_A \] Flux per pole of the auxiliary machine.

\[ \Phi_{Am} \] Maximum flux per pole of the auxiliary machine.

\[ \omega \] Angular frequency (general) in radians per second.
\[ \omega_M \] Angular speed of starter-motor in radians per second.

\[ \omega_S \] Angular speed of simulator in radians per second.

\[ \dot{\omega}_E, \dot{\omega}_S \] Angular accelerations of engine and simulator, respectively, in radians per second per second.

Units.

i. Time quantities are in seconds.

ii. Torques are in pound-force feet.

iii. Inertias are in pounds (foot)^2.

iv. Electrical quantities are in M.K.S. units.
CHAPTER 1.

Introduction.

Starter-motor tests are usually performed with the type of engine for which the unit was designed and with which it will be used in service. The purposes of the tests are to assess, firstly, the commutation-performance and heat-losses of the motor during prolonged cranking-periods, and, secondly, the ability of the pinion-teeth, bearings and locking balls to withstand the stresses caused by large engine-load-pulsations and backlash.

The inevitable inconveniences of cooling water, exhaust-gases, fuel-and air-supplies, noise, vibration, excessive maintenance and inconsistent performance of diesel engines clearly emphasise the need for an electrical system which could replace the engine and accurately reproduce the usual torque-phenomena experienced by the starter-motor. Moreover, such a system would lend itself to convenient control of engine-parameters, for example, inertia, friction, and the shape and amplitude of the gas-pressure-cycle, so that it could be used to predict performance with engines still in the design stage. The phenomenon of misfire, which is cumbersome to accurately reproduce in an actual engine, could also be simulated as and when required.

Endurance tests begin with the engagement of the starter-pinion with the stationary engine starting-ring, the engine thereafter being cranked until regular detonation takes place. At a predetermined speed, after the initial acceleration, the starter is disengaged by the action of a solenoid. In some arrangements the acceleration under firing-conditions is maintained by governor-control until a relatively high speed is attained. In such cases, however, faulty disengagement
would permit the starter-motor to be driven to excessively high speeds and the ability of the motor to withstand such speeds when accompanied by high-frequency load-pulsations becomes the primary concern of the tests.

Engine-load-fluctuations are produced principally by the alternation of the compression and expansion strokes of the pistons. The exact reproduction of these variable-frequency torque-pulsations by an electrical machine, however, poses many problems, not least of which is the fact that they are in most cases cyclically alternating. Consequently, the primary requirement of the simulator is a fast-response characteristic by which such torque-loads may be satisfactorily imposed upon the starter-motor.

The use of conventional machines would, naturally, be desirable in many respects and would lead to more general application of the principles conceived, thereby widening the practical value of this otherwise specialised study. Although the various techniques proposed are applicable to engines of almost any design, the investigation, in detail, was confined to those of four, six and eight cylinders as providing a range of characteristics and parameters likely to be encountered in practice.
Figure 1. Typical pressure/volume diagram of a compression-ignition engine.
Figure 2. Torque/crank-angle relationship of a single cylinder of a 10-litre engine.

Curve a; Cranking torque.
Curve b; Firing torque.
CHAPTER 2.

Diesel Engine Characteristics.

2.1 The gas-pressure cycle.

The general form of the pressure/volume (or indicator) diagram for a single cylinder of a compression-ignition engine is shown in Figure 1. Since the instantaneous cylinder-volume may be expressed as

\[ V_c = A_p \left\{ r (1 - \cos \theta) + l \left( 1 - \frac{r^2 \sin^2 \theta}{l^2} \right) \right\} \text{ cubic inches,} \]

the abscissa of Figure 1 may easily be converted to a scale of crankshaft-angle \( \theta \). Similarly, the ordinate of the diagram may be transformed to a scale of crankshaft-torque by the use of another familiar relationship, i.e.,

\[ m_{es1} = \frac{P_a + A_p}{12} \left\{ \sin \theta + \frac{r \sin 2 \theta}{2l} \right\} \text{ pound-force feet,} \]

in which the turning-moment \( m_{es1} \) is reduced to zero whenever the piston is at one end of its stroke, i.e., when \( \sin \theta = 0 \). The general form of the torque/crank-angle relationship that may be derived is shown in Figure 2 for both the cranking- and firing-cycles of a single cylinder of a typical 10-litre engine, although it will be clear from the above expressions that the exact shape will greatly depend upon the ratio of the connecting-rod length to the crank-arm radius.
Under cranking-conditions the engine-torque-characteristics are determined mainly by the engine-geometry, the rate of heat-transfer through the cylinder-walls, and the temperature and duration of the cranking-period, and therefore are largely predictable from a knowledge of the thermodynamic laws and processes involved.

In contrast, when the engine is firing normally, the shape of the torque-diagram is radically modified by a number of additional factors, the effects of which are somewhat more difficult to assess. By far the most influential single factor involved is the combustion-characteristic of the engine concerned and the performance of its associated injection-equipment, i.e., the spray-characteristics, timing, operating-conditions and fuel-composition. Austin and Lyn\textsuperscript{1} postulate that during the combustion period burning proceeds in three distinguishable phases. The first phase lasts for only about 5° of crank-angle and corresponds to the period of rapid cylinder-pressure-rise that results from the injection of a large quantity of fuel during the 'delay period'. In fact, the maximum rate of heat-release during this phase of combustion was observed to increase slightly with engine-speed because of the extended 'delay angle' at the higher speeds. Delay-angle is always present in the diesel engine combustion-process and is regarded by Taylor\textsuperscript{2}, and Hanain and Bolt\textsuperscript{3}, as the time between the start of injection and the appearance of a flame or measurable pressure-rise due to combustion.

The second stage of combustion occupies about 40° of crank-angle of the power-stroke, approximately 80% of the total heat having been liberated by the end of this phase. The remaining 20% is released during the so-called 'tail' or third phase which persists to the end of the power-stroke.
Lyn⁴ has calculated the effect of compression-ratio and the rate of heat-release upon the torque/crank-angle diagram, and has also estimated the influence of injection-timing and the duration of the heat-release period. The rate of heat-release determines the course of the cylinder-pressure-rise and depends largely upon the injection-characteristics, the achievement of smooth pressure-rises necessitating a relatively gradual release of heat. In general, the peak cylinder-pressure and its maximum rate-of-rise tend to increase with compression-ratio, rate of heat-release, and the reduction of the heat-release period. Lyn⁴, whilst concluding that there is little to be gained from burning the fuel in less than 40° of crank-angle, demonstrates also that a prolonged duration will result in a serious loss of efficiency. Fortunately, the ideal heat-release diagram, of triangular distribution, sloping leading-edge and about 40° crank-angle duration, is very close to that which may be realised in practice.

It is of considerable consequence that the angular period of heat-release is essentially the same for all conditions of engine-load and speed and the general, current opinion is that because turbulence is the major controlling factor in the process of diesel engine combustion, speed is therefore the most influential single factor that determines the rate of heat-release. Nevertheless, there is a slight increase of burning-time associated with high-speed operation, but this is attributable to the relative lengthening of the injection-period. In order to counteract this effect the injection is automatically advanced slightly at the higher speeds, the result being to extend the delay-angle and postpone the onset of combustion and so reduce the overall burning-time.
Figure 3. The Firing-torque of a four-cylinder engine.

Figure 4. The Firing-torque of a six-cylinder engine.
Figure 5. The firing-torque of an eight-cylinder engine.
Figure 6. The distortion effect of a misfiring cylinder on the firing-torque of an eight-cylinder engine.

Curve a; Distortion due to a misfiring cylinder.

Curve b; Normal pulsating torque.
Table 1. Comparison of engine-torque-waveforms.

<table>
<thead>
<tr>
<th>n&lt;sub&gt;c&lt;/sub&gt; cylinders</th>
<th>Average-torque, lbf ft.</th>
<th>R.M.S.-torque, lbf ft.</th>
<th>Peak-torque, lbf ft.</th>
<th>Peak-to-peak torque, lbf ft.</th>
<th>Form-factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>105</td>
<td>380</td>
<td>525 to -1,430</td>
<td>1,955</td>
<td>3.62</td>
</tr>
<tr>
<td>2</td>
<td>210</td>
<td>540</td>
<td>525 to -1,430</td>
<td>1,955</td>
<td>2.57</td>
</tr>
<tr>
<td>4</td>
<td>420</td>
<td>760</td>
<td>525 to -1,430</td>
<td>1,955</td>
<td>1.81</td>
</tr>
<tr>
<td>6</td>
<td>630</td>
<td>780</td>
<td>120 to -1,500</td>
<td>1,620</td>
<td>1.24</td>
</tr>
<tr>
<td>8</td>
<td>840</td>
<td>950</td>
<td>0 to -1,300</td>
<td>1,300</td>
<td>1.13</td>
</tr>
</tbody>
</table>
The gas-pressure-torque/crank-angle diagrams that can ultimately be constructed represent, of course, the torque of a single piston exerted on the appropriate crankpin. The total turning-moment experienced by the crankshaft is thus given by the algebraic sum of a number \( n_c \) of these diagrams which are progressively displaced by intervals of \( 4\pi/n_c \) radians. From Figures 3, 4 and 5, which illustrate the total torque-waveforms for engines of four, six and eight cylinders respectively, it is apparent that whilst the firing-torque of the four-cylinder engine is alternating, that of the six- and eight-cylinder engines are predominantly unilateral. Figure 6 represents the effect of a misfiring-cylinder on the torque-waveform of an eight-cylinder engine and illustrates the degeneration of this otherwise unilateral torque to one that alternates aperiodically.

It should be appreciated that none of these torques can be measured directly because of the energy-storage capacity of the crankshaft- and flywheel-inertias which tend to smooth the overall turning-moment of the engine.

Measurements made on the waveforms of Figures 2, 3, 4 and 5 provide a basis for comparison of the average, root-mean-square, and peak values of the developed torques for engines of similar design but with different numbers of cylinders. The comparison is presented in Table 1.
2.2. Torque due to reciprocating masses.

In addition to the torque produced by cylinder-gas-pressure the crankshaft also experiences a torque due to the weight and motion of the reciprocating parts of the engine, which becomes of particular significance at high engine-speeds. At the beginning of a stroke energy is absorbed by the accelerating pistons and connecting rods and the resultant force applied to the crankpin is correspondingly modified. Towards the end of the stroke the retarding masses return the stored energy and tend to maintain the crankshaft rotation. Thus from a knowledge of the reciprocating-weight, engine-geometry and speed, the effect of this additional torque may be calculated as a function of the crank-angle. For each cylinder this cyclic torque will be zero at four instants during each revolution since, at both top-and-bottom-dead-centre when the inertial force is large, the moment arm is zero, whereas at approximately the mid-stroke positions, although the moment-arm is large, the piston-accelerations, and hence the inertial forces, are zero.

At the commencement of a power-stroke the gas-pressure-force is opposed by the inertial force and the initial effect of the stroke is somewhat diminished. At the conclusion of the stroke, however, the two forces become collateral and the overall tendency at high speeds is to re-distribute the total torque more evenly over the entire stroke. Conversely, the reciprocating-torque tends to diminish the peak values of compression-torque, this effect being particularly noticeable in four-and six-cylinder engines in which the torque-waveforms of individual cylinders do not appreciably overlap. The phase-relationship of the overall reciprocating- and gas-pressure-torques
Figure 7. The phase-relationship of the reciprocating-torque for a four-cylinder engine.

Figure 8. The phase-relationship of the reciprocating-torque for a six-cylinder engine.
Figure 9. The phase-relationship of the reciprocating-torque for an eight-cylinder engine.
Table 2. Comparison of reciprocating-torque-amplitude and frequency for various values of $n_c$.

<table>
<thead>
<tr>
<th>Number of cylinders $n_c$</th>
<th>Amplitude of reciprocating-torque $W_t \cdot N_e^2$</th>
<th>Amplitude for typical value of $\frac{L}{\pi} = 4$</th>
<th>Number of cycles per 720° crank-shaft angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>$\frac{W_t \cdot N_e^2}{6 \times 35,240}$</td>
<td>$\frac{W_t \cdot N_e^2}{6 \times 35,240}$</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>$\frac{2W_t \cdot N_e^2}{8 \times 35,240 \cdot L}$</td>
<td>$\frac{W_t \cdot N_e^2}{10.7 \times 35,240}$</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>$\frac{W_t \cdot N_e^2}{6 \times 35,240 \cdot L^2}$</td>
<td>$\frac{W_t \cdot N_e^2}{96 \times 35,240}$</td>
<td>8</td>
</tr>
</tbody>
</table>
(curves 'a' and 'b' respectively) are shown in Figures 7, 8 and 9 where the curves 'c' represent the resultant torque. Appendix 1 demonstrates that although the reciprocating-torque-component due to a single cylinder is rich in low harmonics, the total component for the complete engine is considerably more sinusoidal and is generated at a frequency proportional to the number of cylinders and numerically equal to the power-stroke-frequency. The results of Appendix 1 are summarised in Table 2 from which it is clear that in the four-cylinder engine the total reciprocating-torque is independent of the ratio \( \frac{L}{T} \). This is because all the pistons accelerate and retard together, thereby producing a correspondingly large torque-component at the fundamental frequency. The low-amplitude harmonics, which, in fact, result from the distorting effect of \( \frac{L}{T} \) are almost negligible and the effect of one piston tends to cancel that of another. In six- or eight-cylinder engines, however, all the pistons do not move in unison, and the fundamental frequency components of the reciprocating-torque are mutually balanced to leave only the aggregate of the harmonic distortion components which are relatively small.

The reciprocating-weight \( W \) is generally regarded as the total weight of the piston, piston rings, piston pin, and approximately one-third of the connecting rod. Because of the normal tapered design of connecting rods the lower two-thirds are considered to be entirely rotating.
2.3. Friction-phenomena in diesel engines.

2.3.1 Running-friction.

Running-friction, which may be defined as the friction of an engine during rotation, depends upon whether the engine is being cranked or is running under its own power. The principal factors involved are:

i. During cranking unburnt fuel dilutes the lubricating oil, particularly that between the piston and the cylinder-wall.

ii. Different peak values of cylinder-gas-pressure produce different reactionary forces between the piston and the cylinder-wall. The forces are, naturally, also a function of the connecting rod position.

iii. Higher values of cylinder-pressure produce greater deformation of the piston and lead to boundary-lubrication-phenomena and higher bearing-loads.

iv. Cylinder-temperatures control the piston and bearing-clearances and drastically affect oil-viscosity.

v. High cylinder-pressures tend to force the top compression-ring, which is normally poorly lubricated, against the cylinder-wall which then results in dry friction.

Under normal conditions the friction of individual cylinders will fluctuate periodically due to the cyclic variation of the cylinder-pressure, but, as Kruse⁵ observes from a series of tests, the total friction of a multi-cylinder engine is almost entirely free of fluctuations.
Figure 10. The effect of temperature on running-friction.
Early attempts by Selby and Barrington et al to establish a mathematical relationship between cranking-torque, engine-speed and oil-viscosity resulted in the empirical expression:

\[ m_M \propto (N_e \cdot v)^{1/2}, \]

so that for a given viscosity

\[ N_e \propto m_M^2 \]

However, such a relationship certainly cannot be true for speeds close to zero and is thus useful only within a limited speed-range. Moreover, in practice, changes occur in the oil-viscosity due to fuel-dilution and cylinder-temperature-variations, and so, for a given speed, the friction-torque decreases with temperature-rise in the general manner shown in Figure 10. Furthermore, since the temperature-rise is also a function of the cranking-time the friction will gradually decrease as cranking proceeds and give rise to an additional increase in speed. This, in turn, will cause a marginal rise in the friction due, according to Volarovich, to the squeezing of the oil-film and the resulting tendency towards boundary-lubrication conditions.

The friction-characteristics of any particular engine are clearly much easier to investigate empirically than theoretically, for even the constant-of-proportionality implied in Equation 1 must be obtained experimentally. Unfortunately, the assessment of the true engine-friction by measurement is not by any means a simple matter, and the many attempts made by Gish et al, Kruse and others to deduce it from measurements of the cranking-power absorbed have led to inexact deductions mainly because of the inability of a 'motored'
engine to precisely represent the pumping-losses, cylinder-
temperatures and pressures, and oil-viscosity that prevail under
normal firing-conditions. Consequently, Gish et al advocate that
the estimation of friction should be made from statistically-
accurate indicator-diagrams and precision measurements of brake-
horse-power, whilst Kruse prefers to measure the friction of
individual engine-components and auxiliaries. Kruse's observations
lead also to the interesting conclusion that the cyclic pumping-
losses are practically independent of the cranking-time since the
increase of the heat-and leakage-losses that would arise through the
gradual elimination of sealing-lubricant between the pistons and the
cylinder-walls are largely offset by the decrease of the cylinder-
wall heat-losses caused by the cylinder-temperature rise.

The effect of compression-ratio on engine-friction has
already been mentioned, but in addition, for low-ratios of the order
used in petrol-engines, Gish et al observe that the friction-torque
is largely independent of the load-condition, whereas for diesel
engines, with considerably higher compression-ratios, the friction
may well increase by as much as 30% for a 100% full-load increment.

In conclusion, any tendency for engine-friction to
cyclically fluctuate will be greatly masked by the relatively
large pulsations of gas-pressure-torque. Thus it is sufficient
to regard the running-friction as a steady quantity, the magnitude
of which depends upon the mean engine-speed, temperature, time and
load. For estimation purposes the mean friction-torque of a diesel
engine may be assumed to be about 15% of the full-load, mean indicated-
torque.
2.3.2 Static friction.

Volarovich\(^8\) and Siebald\(^12\) ascribe the phenomenon of static friction to a thixotropic, or crystalline cementation, property of the lubricating oil, the initial shearing force of which is consequently relatively large. To establish rotation thus necessitates a larger initial effort than that required to maintain it. However, although Kruse\(^5\) regards the inaccurate results of several attempts to measure static friction as a sufficient basis to question its existence, even, it is conceded by most investigators that the initial rotation of a crankshaft from rest definitely does require an inordinately large torque, the real difficulty being to apportion it into that responsible for the initial engine-acceleration and that required to overcome the static friction, should it in fact exist. As a result of the controversy, the term 'breakaway torque' has arisen in the literature, and, for the purposes of this thesis, will be regarded as the peak torque necessary to initiate rotation of the crankshaft.

The magnitude of the breakaway-torque was found by Kruse\(^5,11\) to be practically independent of temperature-variations over a very wide range, this being in contrast to the characteristic of running-friction. In fact, it is influenced almost exclusively by the quiescent position of the crankshaft, which determines the moment-arm of the tangential component of the piston/cylinder frictional force, so that at top-or bottom-dead-centre no piston-static-friction-torque is exhibited at the crankpin. Measurements obtained by Kruse\(^11\) confirm this supposition and the maximum value of static friction was found to occur approximately 76 degrees before and after top-dead-centre, where the moment-arm is a maximum for most
engines. The measurements also demonstrated that compression alone exercises but little influence upon the breakaway-characteristics because it could become effective only after the crankshaft rotation had commenced.

2-4. Factors affecting the starting of diesel engines.

Temperature is by far the most significant factor that affects the starting-performance of any internal combustion engine. Low ambient temperatures tend to increase the cranking-resistance (owing to the rise in viscosity of the lubricating oil), decrease the fuel-volatility, and impair the battery-performance. In the case of the compression-ignition engine, the lower air-temperatures and greater heat-losses through the cylinder-walls also delay the attainment of the self-ignition temperature, usually about 400° C, and prolonged cranking-periods therefore become necessary. However, under low-temperature, fluid-flow conditions the oil, having a high viscosity, absorbs heat more readily and the resulting rapid decrease of its viscosity produces a significant, corresponding, initial speed-rise. Above all, the engine's ability to run continuously, subsequent to the first fire, depends upon whether the energy of combustion that remains after the subtraction of the large heat-losses, is sufficient to overcome the high rotational resistances. In a spark-ignition engine the capacity of the starting equipment exercises little influence upon the starting-performance, subsequent to the first fire, and the inertia-disengagement of the starter-motor. This fact was substantiated experimentally by Selby\textsuperscript{13} who asserts that the starting-performance is much more a function of the friction and heat-loss phenomena against which the engine alone has eventually to contend. However, in a diesel engine the potentiality
for self-ignition is a function of the combustion-delay-time and
the time for which the cylinder-pressure and temperature exceed
certain critical values, and, hence, depends on the final cranking-
speed attained, which is directly related to the capacity of the
starting equipment and the battery-condition. The cranking-time
required to achieve ignition is thus that required for the cylinder
conditions to attain the critical values pertaining to the instantaneous
cranking-speed. In this respect premature injection, apart from
directly increasing the delay-time, allows fuel to be rapidly
distributed over the cylinder-walls and results in poor heat-transfer
from air to fuel which ultimately prolongs the cranking-time. The
addition of fuel to an engine being cranked was observed by Austen
and Lyn\textsuperscript{14} to cause a considerable increase of peak cylinder-pressure
during the first few revolutions, after which only a slight, though
steady, rise was perceived. If fuel is not supplied during the
cranking-phase, however, the peak cylinder-pressures are consistently
repeated for a considerable period after the second revolution.

2.5. The starter-motor equipment.

If the energy and engine-acceleration due to the first
ignition were to cause instantaneous rejection of the starter-pinion,
continuous running of the engine would be very unlikely. Accordingly,
inertia-disengagement is not used with diesel engines. Another
reason for this is that the high pulsation-torques due to high
compression-ratios could alone produce undesirable pinion-rejection.
Thus, cranking is usually continued after the engine has fired and
starter-rejection is accomplished either manually or through an engine-
speed detection-device.
Figure 11. Characteristics of a type CA45C starter-motor.
The starter-motor is a series-connected d.c. machine which is normally energised from a low-voltage, heavy-duty battery. The characteristics of a typical motor, type C A 45 C manufactured by C.A.V. Ltd., is shown in Figure 11, which also indicates the deviation of the output-torque/speed characteristic from the theoretical reciprocal relationship that could be achieved from a lossless and unsaturated machine. A close study of the almost linear torque/current curve, also shown in Figure 11, revealed in fact that the excitation-flux was, except near the origin, constant and independent of the motor-current, thus indicating the onset of magnetic saturation in a region well below the normal operating-range of the motor. Since the torque/speed curve depends especially on the internal resistance of the battery and the cable-loss, maximum gross power is developed when the motor terminal-voltage and the supply-system voltage-drop are equal.

An essential requirement of the starter-motor is that the net stand-still torque developed must exceed the engine breakaway threshold, discussed in Section 2.3.2, which will necessitate several hundred amperes. Such a current can be sustained by the motor for only short periods, but, once the crankshaft has begun to rotate, the motor has to overcome only the running-friction and gas-pressure-loads of the engine. Whilst the running-friction is sensibly constant, the pulsating gas-pressure-load produces cyclic fluctuations of speed. However, the mean cranking-speed may be found from the intersection of the starter-motor-torque/speed characteristic with the engine-resistance/speed curve and, in fact, the amplitude of the speed-fluctuations can be deduced in a similar way.
Selby\textsuperscript{15}, in an analytical approach, assumes that within a limited range the starter-motor torque/speed relationship can be expressed as

\[ N_M \cdot m_M = K \]

Thus, if within the same limits

\[ m_M^2 \propto N_e \cdot v \]  \hspace{1cm} (See Equation 1.)

then, because \[ N_M = \varepsilon \cdot N_e \],

\[ N_M \propto v^{-1/3} \] and \[ m_M \propto v^{1/3} \]

An independent experimental investigation by Meyer, De-Carolis and Stanley\textsuperscript{16} yielded the empirical relationship \( m_M \propto v^\gamma \) for which \( \gamma \) was found to be 0.33 for a diesel engine and 0.36 for a petrol engine thereby confirming the cubic expressions deduced by Selby. The accuracy of these expressions, if applied outside the range-limit, is seriously affected by the significant reduction of motor-efficiency, which, at very low or high speeds, is impaired by the excessively high copper-losses or high friction, windage and iron-losses, respectively.
CHAPTER 3.

The Simulation of Engine-Characteristics.

3.1 Introduction.

Except for the possibility that a simulating machine possessing characteristics that inherently resemble those of a diesel engine could be constructed, any system based on the use of conventional machines must employ a form of control-equipment that can continuously process input-data and provide a suitable output-signal for the direct control of the simulator-load. The input-data may be classified into either fixed engine-parameters such as connecting-rod-length, crank-arm-radius and engine-inertia (all of which may be assimilated prior to an actual simulation) or, time-dependent variables such as engine-speed and friction which must be processed as the simulation progresses. Furthermore, because the various stresses, accelerations and backlash-phenomena experienced by a starter-motor in service, and, moreso, the inherent time-dependent parameters of the starter-motor itself are all inextricably related to 'real' time it follows that any realistic simulation must be conducted on this same natural time-scale.

Now the predominant features of an engine-torque-characteristic are the two pulsation effects described in Sections 2·1 and 2·2, both of which occur at a fundamental frequency given by

\[ f_m = \frac{N_e \cdot n_c}{120} \text{ hertz.} \]

Thus, for a given engine, the speed \( N_e \) could form an input-variable to the control-unit, \( n_c \) being constant, and would be used to determine the load pulsation-frequency.
Figure 12. Flow-diagram of engine-torque.
At this stage it is expedient to summarise the effects of the fluctuating torque for various engines and conditions, the engine-system being considered by reference to the flow-diagram of Figure 12. The algebraic sum of the instantaneous gas-pressure and reciprocating-torque components of the entire engine is termed "internal torque $m_E$" and will be alternating for four-six- and eight-cylinder engines during the cranking-period as was seen in Sections 2·1 and 2·2. It is convenient to regard all torques that tend to increase the engine-acceleration as negative quantities and, hence, the friction-torque $m_F$ and the torques due to cylinder-compression effects are assigned the positive polarity. Thus when compression effects predominate during an engine-cycle, the total load-torque applied to the starter-motor is positive and is augmented by the friction. In fact, if the magnitude of the friction-torque exceeds the negative peak-values of indicated torque, the resultant $m_T$ will always appear as a load to the starter-motor, although it will also be fluctuating. Under the maximum-torque firing-conditions of a four-cylinder engine $m_E$ will again alternate, but, because the friction will no longer exceed the greatly increased peak torques of the power-strokes, $m_T$ also will become alternating. Similarly, for a firing six-cylinder engine $m_E$ is always negative, although it fluctuates cyclically to zero. Thus, whenever the magnitude of $m_E$ falls below that of $m_F$, the starter-motor will experience a sudden change to loading-conditions from the prior assistance afforded by the engine, and $m_T$ will again become alternating.
In an eight-cylinder engine, during firing, the negative internal torque will not necessarily fluctuate cyclically to zero, as is evident from Figure 9, and, therefore, if the minimum value is not exceeded in magnitude by the friction-torque, $m_r$ will always be negative. It is more probable, however, that $m_r$ will vary in a similar way to that for a six-cylinder engine, particularly if less than the maximum torque is being developed. Thus it may be concluded that $m_r$ will be alternating for all conditions except, possibly, for the eight-cylinder engine at maximum output-torque. It is shown in Chapter 7 that if the engine-inertia is separately simulated, further modification of $m_r$ takes place and it becomes apparent that, in general, any simulation may require the provision of alternating load-torques. These ultimately must be developed by an electrical machine in response to the torque-demand-signal. In this respect, the proportionality between the simulator-speed and the torque-pulsation-frequency may be regarded as purely incidental to this particular application and need not, therefore, be an intrinsic property of the simulator itself provided the relationship can be superficially and adequately imposed.

For the simulation of an engine under firing-conditions the maximum available simulator-torque must be comparable to the corresponding engine-torque, especially if the simulator-and engine-inertias are similar. Although dissimilarity of the inertias leads to a necessary modification of the simulator-torque requirement, the difficulty of designing a machine with a sufficiently low inertia, and of the type ultimately to be employed, implies that the simulator-torque will, in practice be quite large.
Figure 13. Principle of the homopolar generator.

- a - h: Brushes.
- P: Magnet-system.
- Q: Rotating conductor-system.
- V: Voltage source.
- \( VR_a - VR_g \): Variable resistances.
3.2. Simulation by use of a homopolar machine.

The total electromagnetic force exerted on a system of several current-carrying conductors, when situated in a transverse magnetic field, may be deduced from a knowledge of the flux-distribution and the relative values and dispositions of the conductor-currents. Many configurations may be contrived wherein such a conductor-system, with a prescribed current-distribution, rotates or reciprocates in the magnetic field to produce a net force, the amplitude of which will depend upon only the relative positions. As an example, Figure 13, which diagramatically represents a homopolar machine in which the conductors rotate, will serve for an examination of the salient, innate properties of such an arrangement.

The conductors which are interconnected at one end and coupled to slip-rings at the other are energised from a d.c. supply through variable resistors. In such a machine, if the flux-distribution of each pole is identical, the number of complete cycles of torque produced during each revolution will be equal to the number of poles, so that, for a four-stroke engine-cycle only one pole is required for each pair of engine-cylinders, the torque due to each pole representing that of a single cylinder. However, the prediction of the current-distribution required to produce a specific torque-function is complicated by the incremental nature of the function and the inaccuracy that results from non-uniform flux-distribution and pole-fringing effects which occur in practice. Slot-skewing, which could be used to avoid the 'stepped' torque-waveform, would further aggravate the difficulties of calculation, and considerable adjustment would need to be made empirically. Furthermore, the variety of torque-functions
obtainable would be limited, especially in the reproduction of any abrupt discontinuities.

The net torque of such a simulator should, ideally, represent the internal engine-torque minus the frictional losses, but the variation of the latter would presuppose a controllable zero-level. Although, theoretically, this could be accomplished through the use of a coupled, auxiliary machine to separately simulate the variable frictional component, the resulting inertia of the system would be prohibitive.

It is inevitable that where a force is produced by the interaction of a current with a magnetic flux, any transverse, relative motion will result in the generation of an e.m.f. which would, in this case, modify the current-distribution and distort the torque-function. The supply of these currents from a high-voltage source via swamping resistors would provide a solution to the problem, but would clearly involve intolerable power-losses. Alternatively, a high efficiency could be preserved by the use of an automatic current-regulating-system for each conductor, but it is improbable that a practical system would respond adequately to the intensely rapid changes of e.m.f. encountered at high simulator-speeds.

In conclusion, apart from the high initial cost of such a machine/control-system, the speed-dependent torques of the reciprocating masses could not be simulated unless the machine itself operated in a reciprocating-mode. In this form, however, the asymmetrical gas-pressure-torques would not be accurately reproduced because the electromagnetic torque would be identical for both stroke-directions. Thus, even a highly developed form of homopolar machine
could not be contemplated for high-speed simulation, although for cranking-conditions, where the power involved would be comparable to that of the starter-motor, such a system would appear more feasible, since the inefficiency could possibly be tolerated.

3.3. The applicability of conventional control-methods.

As a result of the various short-comings of the homopolar machine, the applicability of conventional methods of machine-control must be investigated.

3.3.1. Polyphase rectifier-systems.

Present practice in the speed- or torque-control of high-power d.c. machines makes use of thyristors or transductors which operate in a controlled polyphase, rectifying bridge-configuration to supply the armature of a separately-excited machine. For the usual supply-frequency of 50 Hz the mean time between successive phase-control-instants is $10/\pi \mu$s milliseconds if the bridge is fully controlled. A 3-phase bridge, therefore, discretely operates at mean intervals of $3.33 \mu$s during which no continuous control of the output-variable is possible. A considerable improvement could be achieved by the increase of the supply-frequency or the number of phases, but, because either a special alternator or phase-transformer, respectively, would be necessary, the system rise-time would be seriously impaired by the large additional line-reactance. Furthermore, the control-characteristic would be a very non-linear function of the speed of the controlled machine as the instantaneous bridge-current would depend upon the difference between the machine e.m.f. and the instantaneous value of the sinusoidal supply-voltage.
3.3.2. Alternating-current machines.

The stator-current of a speed- or torque-controlled induction-machine, when operated from a fixed-frequency supply, is intimately related to the slip-speed. At low speeds a large stator-current is required to produce a relatively low-torque and a complex torque/current relationship and poor efficiency are exhibited.

Continuous control, if achieved through variation of the supply-frequency, similarly exhibits a complex control-function as the stator-current depends upon the resulting variable synchronous-speed as well as the slip. In addition, any inconstancy of the rotating stator-flux will give rise to alternating torque-components. Finally, the development of torque-variations at frequencies greater than that of the supply is clearly impossible for either of the above considerations.

Similar arguments can be construed against the use of synchronous machines for which a variable-frequency supply would be essential. Torque-control, in this case being a function of the load-angle, ultimately would require very rapid speed-control of the supply-alternator, which, because of the large inertias involved, would be utterly impracticable.

3.3.3. Separately-excited, d.c. machines.

The d.c. machine exhibits a relatively simple control-characteristic. Under constant excitation, the gross-torque is proportional to the armature-current (if armature-reaction effects
Figure 14. Basic Ward Leonard control-system.

P; Induction or synchronous machine.
A; Auxiliary d.c. machine.
S; Simulating machine.
are neglected), which, according to the method of control, may or may not depend implicitly on the armature-speed. An interesting relationship is exemplified in the well-known Ward Leonard arrangement, wherein the armature-current may conveniently be considered to be a function of only one variable, namely the difference between two armature-e.m.fs., even though these in turn depend respectively upon the motor-speed and the generator-excitation. However, this e.m.f.-difference may be controlled independently of the motor-speed by comparatively simple means and thus the possibility arises of a simulator in which torque-control may be exercised quite independently of the simulator-speed. Another important aspect of the Ward Leonard system is its ability to regenerate when the armature-current is reversed. Hence, rapid reversal of the e.m.f.-difference will result in rapid reversal of the machine-torques.

3.4. The dual-machine system.

A simplified Ward Leonard arrangement is shown in Figure 14. Conventionally, for slow-response systems, the excitation-current of the field \( F_A \) is used to provide the desired control. For stable, closed-loop torque-control the current \( I_{25} \) is measured and compared to the reference signal, the difference being amplified to provide the excitation of \( F_A \). Poor response usually results, however, because of the appreciable time-constant of the field-windings. If \( I_{25} \) is to be maintained constant against only the speed-variations of \( S \), however, such a technique is adequate, particularly if the inertia of \( S \) is large.

The response of \( I_{25} \), as a separate variable, clearly depends only on the armature-circuit-parameters so that the removal, in some way,
of the field-control-function from the direct control of \( I_{25} \) should achieve a much-improved response of this variable. Hence, \( I_{25} \) must be controlled individually whilst the difference \((E_S - E_A)\) is maintained constant.

Now, because the machine \( S \) is constantly excited and \( E_S \propto N_s \), a constant e.m.f.-difference can readily be maintained by suitable control of \( F_A \). Ideally, the time-constant of the device used to control \( I_{25} \) should be negligible in comparison to that of the armature-circuit so that the response of the armature-current will not be degenerated more than is necessary.

3.4.1. The reproduction of pulsating torque.

If the simulator were restricted to the development of only fluctuating, unilateral torques, the effective production of alternating quantities could be achieved through biasing of the simulator-output with the constant torque of a coupled, auxiliary machine. However, for a considerable proportion of each cycle, relatively large torques would then be in mutual opposition and the main simulating-machine-rating would need to be considerably increased (by about 50\%) in order that the torque-amplitudes originally required might still be attained. Unfortunately, the addition of the auxiliary machine and the resulting increase of the simulator-capacity would increase the inertia of the combination to prohibitive proportions.

Some preliminary estimations revealed the difficulties of designing even a single d.c. machine to within the inertia-limits prescribed by typical engine-specifications. In fact, the problems were such that it became necessary to consider duplex-lap arrangements in order to utilise long, small-diameter armatures to avoid excessive
Machine A; Driven at constant speed.
Machine S; Excited at constant flux.

Figure 15. Principle of armature-current-reversal.
induced voltages between commutator-segments and to reduce the armature-inertia. In such a design, however, the reduction of inertia is offset slightly by the necessary provision of front and back equalisers. This design-problem is only significant, of course, if the simulator is required to operate at the maximum torque-level of the engine being investigated.

The alternative solution would be to periodically reverse the armature-current $I_{2s}$, shown in Figure 14, but, unfortunately, direct reversal of the voltage-difference ($E_S - E_A$) by simple control of the field $F_A$ would not provide a sufficiently rapid response. The method of reversal ultimately adopted is illustrated, in a simplified form, in Figure 15 and requires $E_S$ and $E_A$ to be maintained equal. The armature-current is supplied from an external, low-reactance, direct-voltage source $E_D$, and the direction of current-flow is determined by the condition of the thyristor-bridge in which either $CR_1$ conducts with $CR_4$, or $CR_2$ conducts with $CR_3$. A particular advantage of this technique is that the voltage-source $E_D$ has only to circulate the armature-current against the circuit-impedance and, in general, need be only a fraction of the rated voltage of the machines.

When operated from a d.c. supply a thyristor requires elaborate commutation-equipment due to its inherent bistable property. The difficulties encountered in the commutation of such pairs of conducting thyristors are considered later in Section 4.6.
3.5. Continuous armature-current-control.

The arrangement of Figure 15 does not indicate any means by which the armature-current is made to vary as a continuous function of time. Control by the variation of the circuit-resistance, for example, by use of a motorised rheostat, could not provide the essential rapid response, whilst the use of semiconductor devices as linear series-regulators could not withstand the power-dissipation envisaged and would result in poor overall efficiency as well as requiring the parallel operation of several devices.

A degree of control could possibly be obtained by variation of \( E_d \), especially if the voltage were derived from an a.c. supply through a controlled, thyristor bridge-converter; but the response-limitations discussed in Section 3.3.1 are equally applicable here, and any attempt to smooth the uneven output-voltage would further impair the time-response.

A practical solution lies in the use of a high-frequency static-switch, the controlling-function of which depends upon the ratio of the period for which the switch is closed to that for which it is open. Since the most significant power-losses are incurred during the actual, rapid switching-actions, high-efficiency operation may be expected over a very wide range of output-current, and the use of a high repetition-rate will render the system-response-function essentially dependent upon only the load-circuit-parameters. Such a system, hereafter referred to as Time-Ratio Control, may be regarded in its simplest form as a voltage- or current-regulator.
CHAPTER 4.

Time-Ratio Control.

4.1. Introduction.

If a simple switch, connected in series with a non-inductive, linear resistor $R$ and a direct-voltage source, of zero internal impedance and e.m.f. $E_D$, is operated repetitively, then the voltage-waveform appearing across $R$ will be rectangular with an amplitude $E_D$ and mark/space ratio $t_{cl}/t_{op}$. Because $t_{cl} + t_{op} = 1/f_p$, the average voltage across $R$, over a relatively long period, will be $E_D \cdot t_{cl} \cdot f_p$ and the average current will be $E_D \cdot t_{cl} \cdot f_p / R$.

If, however, the load-resistor $R$ possesses inductance $L$ and a 'free wheeling' diode is provided to dissipate, through $R$, the stored magnetic energy during those periods when the switch is open, the long-term-average current will still be $E_D \cdot t_{cl} \cdot f_p / R$, as is verified in Appendix 2. The inductance thus tends to smooth the load-current so that if $f_p \gg R/L$ the form-factor approaches unity.

The current-pulses of the voltage-source, however, remain rectangular so that zero source-inductance is desirable for satisfactory operation. The addition of shunt capacitance to the source will greatly assist the supply of these current-pulses in a practical circuit, provided care is taken to avoid resonance at the repetition-frequency $f_p$. The capacitance required will depend mainly on the amplitude, duration and mark/space ratio of the pulses as well as on the voltage-regulation that may be tolerated at the source.
For an ideal switch, the losses incurred during the switching-actions would be zero. Any practical switch, however, requires a finite operating-time and the time-integral of the product of the instantaneous switch-voltage and current, for each operation, constitutes the switching-energy-loss. These losses clearly will increase proportionally with the frequency of operation.

The applicability of time-ratio control to the simulation of engine-characteristics depends upon two important properties. Firstly, the continuous variation of load-current from zero is essential if smooth and gradual load-current-reversals are to be accomplished when required, whilst, secondly, operation at a very high repetition-frequency is equally vital if accurate and frequent control of the load is to be achieved. For example, if a typical torque-waveform at 233 Hz is to be controlled at forty instants during each cycle an operational rate of nearly 10 kHz will be necessary.

4.2. Thyristors used in time-ratio control.

Because of the order of magnitude of the torques to be simulated, thyristors were considered to be the most suitable switching-elements. These four-layer, semiconductor devices are manufactured with three electrodes, namely the anode, gate and cathode, and a suitable positive trigger-pulse applied to the gate, with respect to the cathode, will cause the device to conduct anode-current if the anode-potential is maintained more positive than the cathode. Once conduction has been initiated and the anode-current exceeds a certain minimum value (i.e., the holding-current) no further control can be exercised by the gate-potential, and so recovery to the non-conducting-
Figure 16. The basic commutation circuit.
state is only possible by the effective removal of the anode/cathode-voltage. Whereas an alternating anode/cathode-voltage thus provides natural commutation at the conclusion of each half-cycle, operation from a d.c. supply necessitates additional circuit-elements to achieve this essential function artificially.

4.2.1. Consideration of classical commutation-techniques.

The object of this section is to present an appropriate analysis and assessment of two fundamental commutation-techniques in order to formulate the necessary conditions for a high-performance system.

In the commutation-circuit of Figure 16 only the thyristor \( CR_1 \) is assumed to be conducting, so that the anode \( A_1 \) is almost at earth-potential whilst the anode \( A_2 \) is at the positive supply-potential. The load-resistance \( R_1 \) thus passes a current \( E_2/R_1 \). When \( CR_2 \) is made to conduct, the anode \( A_2 \) will be effectively earthed and \( A_1 \) will be driven instantaneously below earth-potential. This action will extinguish \( CR_1 \) by forcing a reverse recovery-current through the device. The resistance \( R_2 \) will then conduct a current \( E_2/R_2 \) whilst \( R_1 \) will pass only the negligible leakage-current of \( CR_1 \).

High-frequency, alternate operation is, however, restricted in so far as each thyristor must conduct for a sufficient time to ensure, firstly, that the reverse commutating-voltage is applied to the conducting thyristor for longer than the recovery-period of the device and, secondly, that the capacitor is completely and oppositely recharged in preparation for the commutating-operation to follow. The conditions

\[
R_1 \cdot \ln(2) \geq t_{off} \\
and \quad R_2 \cdot \ln(2) \geq t_{off}
\]

\[5\]

\[6\]
satisfy the first requirement whilst the second demands minimum conduction-periods of the order $3R_2C$ and $3R_1C$ seconds alternately, from which the maximum, average load-current obtainable may be deduced as

$$\left\{ \frac{E_D}{R_1} \right\} \cdot (1 - 3 \cdot f_p \cdot R_1 \cdot C) \text{ amperes}$$

whilst the minimum repetition-period $1/f_p$ will be $3G(R_1 + R_2)$ seconds. Similarly, the minimum, average load-current, except for when $CR_1$ is never repetitively operated, will be

$$\left\{ \frac{E_D}{R_1} \right\} \cdot (3 \cdot R_2 \cdot C \cdot f_p) \text{ amperes}$$

Hence, the continuous range of load-variation $G$ is given by

$$G = \frac{1}{R_2} \left( \frac{1}{3 \cdot C \cdot f_p} - R_1 \right)$$

An acceptable operational efficiency, however, results only if $R_2 \gg R_1$, so that substitution of, for example, $R_2 = 10R_1$, in Equation 9 gives

$$G = \left( \frac{1155}{f_p} - \frac{1}{10} \right),$$

if $t_{off}$ is typically 20 $\mu$s.

This exposes the serious impracticability of the arrangement, since, when $f_p$ is only 1.05 kHz, $G$ is unity and no current-variation whatsoever is possible. This value of $f_p$ also represents the maximum frequency at which the circuit will properly function repetitively.

The analysis particularly emphasises the dependency of the circuit-performance on the value of $t_{off}$ and the ratio $R_1/R_2$, as well as its independency upon the circuit power-rating provided, of course, that the turn-off-times of the thyristors are not appreciably affected by the magnitude of the load-current itself.
Figure 17. Oscillatory commutation-circuit.
Elimination of the power-loss in \( R_2 \), together with an improvement in the frequency of operation, may be realised however if the capacitor-charge is prepared prior to each commutating operation by the oscillatory action of a low-loss, resonant circuit. Figure 17 shows a configuration which forms the basis of several, more sophisticated, circuits proposed by Jones\(^{17}\), Morgan\(^{18}\), and Aoki and Hass\(^{19}\). If \( C \) had previously charged, through \( R_A \), to the supply-voltage whilst \( CR_L \) and \( CR_C \) were non-conducting, the eventual firing of \( CR_L \) will cause \( C \) to discharge through \( CR_L \), \( L \) and \( D_2 \) for one half-cycle of oscillation. Thus \( C \) will become and remain oppositely charged due to the high reverse-resistance of \( D_1 \). The conduction of \( CR_L \) also allows load-current to flow in \( Z_L \). Subsequent conduction of \( CR_C \) will, therefore, cause \( CR_L \) to be reverse-biased and extinguished by the reversed capacitor-voltage, and, eventually, the capacitor will re-charge to the supply-voltage via \( CR_C \) and \( Z_L \), \( CR_C \) being thereby self-commutated when its own forward-current decreases below the holding-current value.

The performance of this circuit with a highly inductive load-impedance is examined in Appendix 3, which shows that the control-range may be expressed as

\[
G = \left(1 + \frac{(1 - 8 f_p C R_L)^{1/2}}{1 - (1 - 8 f_p C R_L)^{1/2}}\right)^{-1}
\]

and that the practicable, operational frequency is limited to 6.25 kHz.

Moreover, the analysis also reveals that for any given operating-frequency there will be a maximum value of average load-current that
Figure 18. Modified, oscillatory commutation-circuit.
can be obtained, and that this is governed by either the minimum fraction of each cycle for which $CR_L$ must conduct to ensure that $C$ becomes oppositely charged, or by the duration of the recharging-period of the capacitor after the commutating-action; whichever is the more significant. Indeed, a certain minimum, average load-current must be tolerated owing to the relatively large recharging current that flows every time $CR_C$ is triggered. Clearly, any attempt to reduce this current will result in the protraction of this recharging-operation.

Several attempts were made to overcome these shortcomings, principally by re-allocation of the oscillatory and load-current-conduction-functions of $CR_L$ to two separate thyristors. This enabled the oscillatory preparation of the capacitor-voltage to be completed prior to the conduction of load-current, and thus removed the minimum-conduction-period restriction on $CR_L$. This modification is illustrated in Figure 18 which includes an additional thyristor $CR_R$ in series with a resistor $R_R$. The timely operation of $CR_R$ was intended to provide an additional conduction-path through which $C$ could recharge independently of the load-circuit-components. However, load-current-variation down to zero could be obtained by this method only if the inductance $L_L$ was negligible, otherwise the energy-storage property of $L_L$, during the conduction-period of $CR_L$, tended to maintain, subsequent to commutation, an appreciable proportion of the capacitor-recharging-current through $R_L$. This occurred despite the presence of $D_2$ since, during commutation, $D_2$ is reverse-biased by almost the supply-voltage. Unhappily, the addition of $CR_R$ also constituted a potential, low-
impedance path across the supply if ever CR₀ and CR₂ conducted simultaneously. Because CR₁ in such a role, was extinguished through the exponential decay of its anode-current, it could require, possibly, as long as 100μs to completely recover. Thus an unsafe situation was always imminent and the maximum frequency of operation had to be limited to well below 10 kHz.

An endeavour to reduce the long recovery-time of CR₁ by the addition of inductance in series with R₁ to provide an oscillatory reverse-biasing-action, resulted naturally in a degradation of the control-performance through the inability of this auxiliary circuit to adequately divert the recharging current of C from the load-circuit.

The critical timing required of the thyristor triggering equipment, in contending with the load-and temperature-dependency of the thyristor switching-characteristics and the inherently poor performance and unreliability of the system, led ultimately to its rejection.

Some attempts by Morgan to achieve high performance from a similar arrangement, by the use of saturable reactors as timing-elements and coupling-transformers to boost commutation, suffered from similar frequency limitations. Even though operation at zero current was possible, the continuous variation of the output from zero was not generally successful.

Despite the fact that the fundamental inadequacy of the circuit clearly lay in the re-charging process, a practical system could not be devised in which the load-impedance could be satisfactorily isolated during this cyclic operation.
Figure 19. The high-performance commutation-circuit.
4.2.2. A high-performance system.

The underlying commutation-principle of the classical methods is the direct application of a reverse-biasing voltage to the conducting thyristor, through which a large reverse-recovery-current is thereby forced to flow. In contrast, it is proposed to achieve commutation by the oscillatory discharge of the capacitor through, not the load-elements but, an auxiliary inductor $L_2$ as shown in Figure 19. It is possible, thus, to position the load-impedance in the circuit such that any reverse-current may be deliberately blocked by $D_2$, whilst the recovery-current through $CR_L$, can be limited by a resistor $R_C$. Moreover, the capacitor-recharging-time will be reduced by the resonant action of $L_2$ and $C$, which also desirably promotes the rapid and timely recovery of $CR_c$.

If, initially, $C$ is charged to the supply-voltage, the sudden conduction of $CR_0$ will cause, by the action of the low-loss circuit formed by $D_1$, $L_1$ and $CR_0$, almost complete reversal of the charge-polarity in a period of approximately $\pi (L_1 \cdot C)^{\frac{1}{2}}$ seconds, after which the triggering of $CR_L$ will permit the flow of load-current via $D_2$, $CR_L$ and $L_2$. If $CR_c$ is triggered at any time after the triggering of $CR_L$, the latter will be immediately commutated by the rise of voltage across $L_2$ to almost $2.5$ volts. Successful commutation will result only if the cathode-voltage of $CR_L$ remains above the supply-potential for a time greater than $t_{off}$, implying the condition that $\pi (L_2 \cdot C)^{\frac{1}{2}} > 2t_{off}$. Since load-current will flow only for the conduction-period of $CR_L$, which is now reducible to zero, the range of current-control becomes infinite. The only detrimental feature of the technique is that $CR_0$
must be operated well before either CR_1 or CR_2 is triggered and, as will become apparent in Section 4.3, this restricts the choice of the operational mode of the system.

Functionally, the arrangement may be regarded as the synthesis of two distinct circuits, firstly that of the load, comprising Z_L, D_2, D_3, D_5, R_C and CR_L, and, secondly, that formed by the remaining components, which constitute a commutation-system for CR_L. The latter circuit can operate repetitively either with or without the load-circuit-components, and in this respect the overall operation is governed ultimately by the time-phase-separation between the two individual circuit-functions.

In a practical case it should be emphasised, with regard to the time-response, that the detrimental, though negligible, effect of the addition of L_2 to the load-circuit is to some extent offset by the series-resistance contributed by the semiconductor devices.

4.2.3. Operational and practical considerations.

Whilst R_S, in Figure 19, ensures the complete initial charging of the commutating-capacitor prior to normal circuit-operation, excessive accumulation of charge over several switching-cycles, due to the oscillatory behaviour of C and L_2, is prevented by the action of the diode D_4 which functions also as a free-wheeling-diode for L_2.

A thyristor, if triggered whilst connected to an inductive load, will revert to its blocking-state at the conclusion of the gate-pulse if the rising anode-current cannot acquire the 'sustaining
value' quoted for the device. For such loads erratic operation can be avoided by either the application of wide gate-pulses, or by the addition of a non-reactive resistor in parallel with the load-impedance. For maximum efficiency the former technique is superior, although both were exploited in the experimental apparatus.

The most suitable power-source for a time-ratio control-system is some form of chemical cell since, through their negligible inductance, the optimum system-response can be realised. Where extremely high powers are involved, however, other forms of supply must be considered; in this case, the thyristor bridge-converter and the d.c. generator, both of which exhibit appreciable inductance. Indeed, thyristor-converters generally employ inductive line-reactors in a deliberate attempt to limit any prospective fault-currents to easily manageable proportions. This ensures the correct operation of the rather delicate protective fusegear used with such equipment. The unsuitability of rectifying equipment is further emphasised by its inability to provide a low-ripple output-voltage without the use of external smoothing components. Nevertheless, in contrast to the simple battery, a degree of control over the voltage-regulation could be exercised, and, if desirable, the voltage could even be varied in a prescribed manner. This function is equally apposite to the d.c. generator. Ultimately, the d.c. generator offers the most practicable solution as its inductance can be greatly reduced by careful design, its output-voltage can be maintained, and it is capable of sustaining very considerable short-term current-overloads.
4.3. Operational Modes.

Consideration of Expression 4 indicates the possibility of three, basic operational states of a time-ratio control-system, namely,

i. Variable $t_{cl}$, and variable $t_{op}$.

ii. Variable frequency $f_p$, with constant $t_{cl}$ or $t_{op}$.

iii. Constant frequency $f_p$, with variable $t_{cl}$ and $t_{op}$.

Although mode (i) is operable only as a closed-loop control (a seemingly advantageous property) it also exhibits some serious disadvantages. The closed-loop operation is inevitable because the alternate switching-instants are determined directly by continuous comparison of the input-signal-level to the instantaneous output-current. When the signal-level exceeds the decaying load-current, during $t_{op}$, by a predetermined margin, the flow of source-current is re-established and increases. The circuit is subsequently commutated when the increasing load-current exceeds the signal-level by a similar margin, and in this way the operation becomes repetitive. The advantage of the resulting constant ripple-amplitude of the output is, for this application, outweighed by the unwanted frequency-variation that ensues. The detailed examination given in Appendix 4 shows, in fact, that for small and large load-currents the repetition-frequency is lower than that which results for intermediate values.

Operation, in this mode, of the high-performance system described in Section 4.2.2, is severely restricted, however, because the time-interval by which the triggering of CR$_0$ must precede that of CR$_c$ is incompatible with the uncertainty of the commutation-instants as determined by the comparator. In practice, the thyristor CR$_0$ must be triggered from the comparator, and the actual commutation-instant
must be suitably delayed to permit the correct capacitor-conditions to be established. Since in high-frequency operation this interval (40 μs was used in the experimental equipment) constitutes a large proportion of the repetition-period, it follows that considerable non-linearity of the transfer-characteristic and variation of the output-ripple would result. In fact, satisfactory operation over a wide range may be achieved only at a relatively low frequency. As such, low ripple-amplitudes could be restored only with highly inductive load-impedances which are, of course, inconsistent with the simulator-response requirements.

Operation of the high-performance system in mode (ii) is a practicable proposition; however, since the pre-conduction requirement of CR₀ is consonant with the fixed conduction-period, or blocking-period, of CRₐ. The consequent linear relationship between the load-current and the instantaneous repetition-frequency, though, is not desirable and produces relatively high ripple-amplitudes at low output-currents.

The characteristics most appropriate to an engine-simulator are those exhibited by constant-frequency, mode (iii), operation. An option, of little practical significance, exists in so far that control may be exercised by variation of either the leading- or trailing-edges of the output-current-pulses, which requires the triggering of CR₀ to precede that of either CRₐ or CRₐ, respectively. The former option offers some marginal convenience and simplicity of the control-circuits to be used.
Experimental investigation into high-frequency operation showed that where very large output-current-demands required the load-current conduction-instants to almost immediately follow the commutation-instants, the system tended to operate at only half the gate-pulse-frequency. This resulted from the prolonged overlap of the commutation-periods upon the subsequent gate-pulses to $C_R L$ early in the following switching-interval. As will be shown later in Section 4.5 and Appendix 4, the avoidance of control-ratios that approach unity is both necessary and beneficial to satisfactory system-performance, particularly if closed-loop operation is anticipated.

4.4. Selection of the repetition-frequency.

For a linear transfer-characteristic the input-signal-voltage is converted discretely by the thyristor-control-circuits to a proportional, corresponding time-interval $t_{cl}$, and the system behaves as a linear pulse-width-modulator of carrier-frequency $f_p$. The load-impedance, with the free-wheeling-diode, performs simple, linear demodulation. In the experimental apparatus the comparator actually performed pulse-phase modulation which was ultimately converted to pulse-width modulation through the inherent, bistable property of the thyristors in the time-ratio control-circuit.

Previously, in the literature, analysis of the time-ratio-control-principle has been performed by piece-wise design formulae such as were used by Matsumara$^{20}$, McMurray$^{21}$, Horvat$^{22}$ and Colclaser$^{23}$. The analysis presented in Appendix 5, however, estimates the amplitudes
of all the frequency-components that result from the modulation, using a pseudo-static method of analysis.

Clearly, the principal factors to be considered in the estimation of the fundamental switching-rate are the acceptability of the higher order distortion-effects, the exclusion of the lower side-frequencies from the demodulator pass-band, the harmonic content of the original modulating signal and the effect of the repetition-frequency on the power-losses of the time-ratio control-circuit. Appendix 6 yields a complete Fourier-analysis of a typical engine-torque-waveform and shows, for example, the amplitude of the fifth-harmonic component for a four-cylinder engine to be only about 9.8% of the amplitude of the fundamental component. Appendix 5 shows that if the cross-distortion of the sideband-frequencies that exist below the maximum value of the fifth-harmonic-frequency are to be attenuated by, say, 30 dB a repetition-frequency of 6.99 kHz will be required. Such a frequency was found to be reasonably practicable, although the toleration of greater distortion for this extreme condition would profitably permit the use of a somewhat lower rate. However, the load-impedance time-constant imposes a limitation to lower switching-rates below which it would become a source of serious transient distortion. The nature and effects of the load-time-constant are elucidated in Section 4.8, where it is shown that an abrupt discontinuity is produced in the frequency-response characteristics when the system is operated under closed-loop conditions. Modulation beyond this frequency results in a markedly inferior response, although the discontinuity, itself, depends upon the specific harmonic structure of the input-signal.
Figure 20. The relationship between $K_T$, $K_R$ and $\beta$.

Curve a; $K_R = 10$
Curve b; $K_R = 7$
Curve c; $K_R = 5$
Curve d; $K_R = 3$
An important practical aspect is that as the load-current constitutes a feedback signal to the comparator, appreciable filtration is necessary to eliminate high-frequency distortion, or ripple, in order to prevent interference within the thyristor-gate-control-circuits.

4.5. Forced response.

The improvement of rise-time by the installation of a higher system-capacity than is basically necessary is a familiar technique, and is especially applicable to time-ratio control-systems. In a closed-loop system of this type the load-current, during each load-conduction-period $t_{cl}$, will always rise towards its maximum potential value although it will be commutated upon reaching some prescribed lower value. Hence, the gain of the error-amplifier, or comparator, can in no way influence the minimum obtainable rise-time.

For an increase $K_F$ of the capacity of a time-ratio-controlled system, the minimum rise-time of the output from zero to an arbitrary current $I$ will be reduced by a factor $K_T$ which may be deduced as

$$K_T = \frac{\log (1 - \beta)}{\log (1 - \frac{\beta}{K_F})}$$

where $\beta$ represents $I$ as a fraction of the original, maximum, possible value of the output-current. Clearly, for a specific value of $K_F$, $K_T$ will increase as $\beta$ is increased. Figure 20 indicates the dependency of $K_T$ upon the value of $K_F$ for various values of $\beta$. 
Figure 21. Load current reversal circuit.
Thus, for example, the time required for the current to rise from zero to, say, 50% of its original maximum, possible value, through an inductive circuit of time-constant 9.3 ms, may be reduced to 1.0 ms if a forcing-factor of 7.0 is imposed.

However, despite this simple improvement of the rise-time, the rate of current-decay through the free-wheeling-diode, during the periods of non-conduction $t_{\text{off}}$, depends only on the load-time-constant and the value of current from which the decay commences, and is therefore entirely unaffected by $K_F$. Considerable improvement of this fall-time can be achieved by the addition of series resistance into the diode-circuit, but the degradation suffered by the system-performance in other respects requires such a modification to be applied with discretion. Appendix 4 presents an analysis of the impairment so caused to the system-transfer-characteristic and the output-ripple-amplitude, and emphasises the severity of the non-linear operation that results. Although provision was made in the gate-pulse-control-circuits for the physical compensation of this type of non-linearity, the pseudo-static analysis, employed in Appendix 5 for the estimation of the repetition-frequency, must make allowance for the increased ripple-amplitude. The particular non-linear compensation-technique employed is discussed in Appendix 7.

4.6. Periodic load-current-reversal.

The use of a thyristor bridge for the reversal of load-current was introduced in Section 3.4.1. Figure 21 incorporates the basic principle into a high-performance time-ratio-control-circuit
similar to that represented by Figure 19. The diode D5, which is operative for either direction of load-current-flow, provides, with resistor \( R_E \), an improved load-current decay-rate during the non-conducting-periods of \( CR_L \). The inclusion of resistor \( R_H \) facilitates the immediate establishment of the thyristor-sustaining-current, as mentioned in Section 4.2, and permits the use of comparatively short, high-power gate-pulses.

It is arranged that the pair of bridge-thyristors appropriate to the desired direction of load-current-flow are normally triggered at the same instant as \( CR_L \). To effect a reversal of this load-current, however, it is necessary to inhibit the gate-pulses to all of the bridge-thyristors, and \( CR_L \), to enable the conducting-pair to turn off after the current has been allowed to decay to below the holding-value. Subsequent recovery of the devices, which enables them to sustain a re-applied forward-voltage, must be completed before the opposite bridge-thyristors are made to conduct. Although to a large extent the short recovery-period is predictable, the load-current decay-time is less portendable. High initial current-decay-rates can not emanate from initially low load-currents and the protracted decay-periods severely restrict the rate at which periodic load-current-reversals may be instigated. The degree of improvement that can be effected by \( R_E \), however, is limited by other considerations. Nevertheless, the very rapid collapse of load-current is essential only immediately prior to reversal-sequences, and it is feasible that on these occasions it could be deliberately encouraged by the substitution of a high resistance in place of \( R_E \). Replace-
Figure 22. Arrangement for forced commutation and isolation of $R_H$. 
ment of D5 by a thyristor CRF and a parallel-connected resistor R_R, as shown in Figure 22, facilitated this function. Between current-reversals, CRF was triggered to perform the normal function of a free-wheeling-diode, but, when this action was inhibited, extremely rapid decay of the load-current would supervene.

Timely and successful inhibition of CRF, however, involves some complexity, since the mere obstruction of the gate-pulses will be quite inadequate should the conduction-period of CRF, during the previous free-wheeling-cycle, be unduly prolonged. The only time, in fact, during which the restoration of the blocking-state may be guaranteed is whilst CR_L and the bridge-thyrists are conducting load-current, since CRF is then reverse-biased for at least most of the period. Thus, the necessary conditions may be established by the triggering of CR_L immediately after the inhibition of the gate-pulses to CRF, followed by the normal commutation of the load-current when CRF has recovered. The rapid decay of load-current through R_R then ensures reliable recovery of the bridge-thyrists, particularly as they are reverse-biased by the forward-voltage-drop of CRF, and load-reversal may be initiated almost immediately.

The effect of R_R is, unfortunately, diminished by the low, parallel resistance of R_H which, functionally, is totally unrelated to R_R. Addition of the diodes D6 and D7, of Figure 22, was thus essential in order to isolate R_H from R_R and thereby ensure satisfactory operation. The general design-procedure and other important practical aspects pertinent to time-ratio control are considered in Appendix 8.
Terminal A : Input signal from the function-generator.
Reference should be made to the Function-Legend.

Figure 23. Reversing-logic diagram.
Function-legend for Figure 23.

a' annuls normal detection.
b' advances the triggering-signal.
c' transfers sawtooth waveform.
d' transfers sawtooth waveform.
e' initiates trigger-pulse to CRo.
f' initiates the interlock-time-delay.
g' initiates trigger-pulse to CRc 40\mu s after that to CRo.
h' informs unit M of the triggering of CRo.
i' transfers rectified input-waveform.
j' initiates trigger-pulse to CR1.
k' inhibits gate-pulses to CRl for 200\mu s.
l' initiates trigger-pulse to CR2.
m' obstructs, for 200\mu s, all inhibitions applied to CRl.
n' inhibits trigger-pulses to CR1.
o' initiates trigger-pulses to CR1, CR2, CR3 and CR4.
p' inhibits trigger-pulses to CR1 and CR4.
q', r' inputs to the protection-unit K.
s' inhibits trigger-pulses to CR1.
t' inhibits trigger-pulses to CR2 and CR3.
u' cancels 'stored' information.
v' transfers information regarding the signal-phase.
w' transfers output-signal of the 'AND' function.
x' inhibits trigger-pulses to CRf.
y' initiates 20\mu s- and 200\mu s-delays.
z' informs 'memory' unit of prospective phase-reversals (instants 'c').

a" informs unit Q of end of trigger-pulse to CRo.
b" appropriately 'SETs' or 'RESETs' unit R.
c" appropriately 'SETs' or 'RESETs' unit Q at instants 'e' and 'd'.
d" informs unit Q of 'f' instants.
e" informs unit Q of end of 200\mu s-delay.
f" inhibits trigger-pulses to CRf for 20\mu s.
4.7. Thyristor gate control.

The component-arrangement of the reversible, high-performance time-ratio-control-circuit is not amenable to the interconnection of all the thyristor-cathodes and each device must therefore be operated by isolated gate-pulses. This affords the additional advantage that the high gate-power-dissipation characteristics may be exploited to ensure rapid and dependable switching-performances.

4.7.1. Control of the non-reversible system.

Without resort to forcing-factors, the time-ratio-controlled circuit produces an output-current proportional to the control-function $t_{cL} f_p$. Primarily, proportionality between $t_{cL}$ and the input-signal-voltage was established by the use of a linear sawtooth-generator, of repetition-frequency $f_p$, which is represented by unit A of Figure 23. This diagram suffices also as a functional model of the entire control-system. The instants at which the sawtooth-generator-voltage and the signal-voltage become equivalent are detected by a Schmitt-trigger circuit unit E, which then initiates the monostable operation of unit F to instantly trigger CR_L. A similar sequence is followed by units B and C, but, in this case, as a function of a pre-set, constant voltage $V_1$. Thus pulses emanate from unit C at regular, predictable instants and are used to trigger CR_C. The duration of these pulses is approximately 40 $\mu$s, and the trailing-edges are used to trigger CR_C from the monostable-unit D. The necessary synchronism of the pulses to CR_L and CR_C can be adjusted, by variation of $V_1$, to be coincident for a zero input-voltage.
4.7.2. The preservation of pulse-synchronism.

A serious predicament, apparent from the dual-circuit-concept mentioned in Section 4.2.2, is that because, for low signal-voltages, the conduction of CR_c closely follows that of CR_L, any adverse effects of temperature-change and drift may disrupt the synchronism to the extent where the conduction sequence of CR_c and CR_L becomes reversed. Because of the destructive load-currents that may thereby be produced, such a situation must be avoided. Although careful circuit-design minimised these effects, absolute elimination of the danger was secured by the inclusion of a pulse-delay-interlock, unit G. The monostable action of this interlock was initiated by each gate-pulse to CR_c, and the timed output-pulses were used to inhibit the gate-pulses to CR_L for a proportion K_p of the repetition-period 1/f_p. Thus, if poor synchronisation caused the gate-pulses of CR_c to precede those of CR_L by less than K_p/f_p seconds, the pulses to CR_L were obstructed and the load-current was reduced ultimately to zero. The operation of the interlock, however, imposed an effective limitation on the maximum load-current obtainable, owing to the reduction of the maximum possible duration of t_cl to (1 - K_p)/f_p seconds. Consequently, the minimum, average rise-time, over several cycles of operation, was likewise depreciated by the factor (1 - K_p). Hence, the mean response to a large step-input would appear as an exponential rise to the reduced maximum value, but, with the original load-circuit-time-constant. This may be verified by substitution of appropriate values of t_cl and t_o, into the equations derived in Appendix 2, whence the response to a given step-input may be constructed.
graphically. The value chosen for $K_p$ must thus be a compromise between that required to compensate against drift and temperature effects and that which gives minimum degradation of the system-response.

4.7.3. Control of the reversible system.

The sequence of operations necessary to accomplish periodic load-current-reversals may be regarded as supplementary to the non-reversible control-system outlined in Section 4.7.1. In this respect the relatively continuous process of pulse-width modulation is entirely independent of the thyristor-bridge-condition, and is controlled, therefore, from a unilateral modulating signal which is derived through rectification of the bidirectional output-voltage of the primary-function-generator described in Chapter 6. The load-reversals, however, must be initiated by the actual polarity-changes of the pre-rectified signal, and this introduces synchronisation difficulties between these two major functions. Detailed consideration of these problems is given in Section 4.7.4.

For the reversible system, the gate-pulses to the bridge-thyristors, shown in Figure 22, are generated by the monostable units H and J, of Figure 23, which are triggered from unit F in synchronism with the gate-pulses to CR. Normally, the inhibition of the pulses of either unit H or J is determined by the polarity of the bidirectional input-signal, as identified by the signal-monitor P. Unit P, anticipates each polarity-change and instantly energises the monostable unit L, which, for the duration of its quasi-stable state (slightly in excess of $1/K_p$...
seconds) 'memorises' the response of unit P. Immediately after
the actual signal-polarity-change, unit P communicates the new
polarity to the gate-circuit of unit Q where it is retained for
later use. If \( CR_0 \) receives a gate-pulse whilst unit L is
memorising a recent polarity-change, the output of the 'AND' gate,
unit K, initiates a pulse of \( 20 \mu s \) duration from unit N which, in
turn, erases the information stored in unit L and inhibits the gate-
pulses to \( CR_F \) (which normally coincide with those to \( CR_C \)). At
the conclusion of this \( 20 \mu s \) pulse, this inhibition ceases, and two
further pulses of \( 20 \mu s \) and \( 200 \mu s \) duration are released from units
S and T respectively. The latter pulse obstructs the triggering of
\( CR_F \) and would inhibit \( CR_L \) also, except that this second function
is withheld for an initial period of \( 20 \mu s \) by the pulse from unit S
which is, itself, used to deliberately trigger \( CR_L \). At the
termination of the pulse from unit S, \( 40 \mu s \) will therefore have
eixed since \( CR_0 \) last became conducting, and \( CR_C \) will be triggered
in the normal way to commutate \( CR_L \). Thus far, the result is that
\( 20 \mu s \) after the triggering of \( CR_0 \) the deliberate conduction of load-
current through \( CR_L \) causes \( CR_F \) to be reverse-biased for \( 20 \mu s \)
during the interruption of its gate-pulse.
The inhibitions imposed by unit T, however, persist whilst \( CR_L \)
reverts to its non-conducting-state, and the load-current is thus
forcibly reduced to zero through the resistor \( R_R \). The duration
of these inhibitions is sufficient to allow almost complete decay of
the load-current, and the eventual release of the inhibitions is
communicated to the gate-circuit Q. The information stored in this
unit is finally released by a pulse from unit P, which signifies the commencement of the new signal-phase, and the appropriate condition of the bistable unit R is established. Unit R then directly inhibits gate-pulses to the appropriate pair of bridge-thyristors until the entire sequence is repeated for the next signal-polarity-change. For correct operation it is important that any inhibition of the gate-pulses to CR_L is accompanied by the inhibition of pulses to all the bridge-thyristors.

The transient-storage-property of unit L ensures that the information regarding polarity-changes is not available to the system except at the auspicious moments. This is a necessary precaution since the reversal-sequences are intended to function only when the load-current is very low, and a permanent storage-characteristic, in providing accessibility to the information at any time, could lead to untoward consequences under certain minor fault-conditions.

The 'logical AND' function of the protection-unit K produces an output-pulse if pulses are directed simultaneously to all the bridge-thyristors; a situation that could occur through the failure of certain inhibition-functions. The output-pulse, of specified duration, will immediately advance the normal commutating-sequence of CR_d and CR_c whilst cancelling the overall modulating-operation by interrupting the action of the detection-unit B, and inhibiting the gate-pulses to CR_L. Although only a transient form of protection is afforded, a permanent fault causes repetitive operation of unit K which thus restricts the load-current-magnitude to within the commutating-ability of the circuit-components.
Figure 24. Typical input-signal from the function generator.

Figure 25. Rectified input-signal.
4.7.4. The signal monitor.

The basic engine-torque/crank-angle waveform was generated by use of the techniques presented in Chapter 6, and dynamically modified, as described in Chapter 7, before being applied as an input-signal to the time-ratio control-system. The modified signal thus resembles the waveform shown in Figure 24, where the abscissa z-2 defines the instantaneous position of the variable 'zero value' of the signal.

To ensure the vital non-conduction of all the bridge-thyristors, prior to every anticipated current-reversal, a reversing-logic-sequence must be initiated just prior to the conclusion of every preceding part-cycle of the signal, that is, at the instants a and b which correspond to finite signal-levels. An actual signal-polarity-change is not manifested, however, until the instants c and d, which, unfortunately, are unpredictable. Moreover, a critical variation of the zero-level could cause the instants a, c and d to occur without b, in which case the reversing-sequence initiated at a would incorrectly determine the load-current-direction for every successive part-cycle of the waveform.

Due consideration of this situation led to a technique based on independent surveys of the waveforms of Figure 24 and its rectified counterpart, Figure 25, by which pulses formed at the instants e, corresponding to a negative, finite signal-level, were used to initiate the reversing-sequence, whilst the actual signal-polarity, subsequent to the pulses formed at $c$ and $d$, were appropriately registered in the gating-unit $Q$. Pulses produced
at \( f \) then served to transfer the stored information of \( Q \) to the bistable unit \( R \), and to re-initiate the normal modulating-operation of the system upon completion of the reversing-sequence. Although it remained possible, through variation of the signal-zero-level, for the pulses at \( e \) and \( f \) to occur in the absence of a polarity-reversal at \( c \) or \( d \), it was not possible for the instants \( c \) and \( d \) to occur without \( e \) and \( f \), which implied that, in the case of an entirely unilateral signal, the reversing-sequence could not be initiated at all. The former condition also suggests that, over a small variation \( v \) of the zero-level, the reversing-sequence, though unnecessarily initiated, would nevertheless always be properly terminated to produce the appropriate load-current-direction. The pulses at \( f \) will re-instate the normal modulating-operation only when at least \( 220 \mu s \) have elapsed from the occurrence of \( e \). This ensures complete recovery of the bridge-thyristors. Furthermore, the use of these pulses to transfer the information in \( Q \) to \( R \) enables the correct polarity to be selected at the latest possible moment. This avoids incorrect selection at very low modulating-frequencies for which the instants \( e \) will be delayed from \( f \) by considerably more than \( 220 \mu s \).

The waveform of Figure 25 was generated by the use of diode-limited, operational amplifiers for which good drift and temperature-stability were necessary to achieve satisfactory correlation between the pulses derived from both the original and the rectified waveforms. An advantage of the monitoring system used was that perfect correlation was not absolutely essential.
4.8. The operation of a closed-loop, reversible system.

For reasons that are unique to time-ratio control, particularly in its application to a simulating system, the control-sequence described in Section 4.7 is equally applicable to either closed- or open-loop operation. The performance of a simulating device will, naturally, be acceptable only within a frequency-range for which the output-variable faithfully represents the input-signal, and it will be seen later that the frequency-limitation of a closed-loop time-ratio control-system occurs somewhat abruptly, normal operation below this restraint producing negligible phase and amplitude errors. Consequently, for such operation the reversing-sequence may be initiated entirely by the reference-signal which is external to the feedback-loop, each load-reversal thus being deliberately instigated at the required instant. The alternative, which is to initiate the reversals from the error-signal, is liable to produce unreliable and sporadic performance as any oscillation of the error will tend to produce parasitic reversals which could lead to the forced commutation of rather high load-currents.

The logical function of each reversing-sequence is to terminate the normal closed-loop operation just prior to a prospective polarity-reversal, that is, as the decreasing signal-voltage approaches zero. Response to the following signal-phase is then permitted only after the appropriate conditions have been established in the thyristor-bridge. Hence, during each reversal-process the system-response is entirely dissociated from the reference-variable and the responses to consecutive signal-phases are similarly separated.
The sampling-action of time-ratio control inherently produces a time-delay between an input-variation and the resulting response of the system. The use of a high repetition-frequency, however, almost eliminates such delays, in comparison to the load-circuit-time-constant, so that between load-reversals the system behaves as though only a single time-lag were present, and exhibits a highly-stable performance even with high values of loop-gain. However, as was emphasised in Section 4.5 the use of a forcing-factor produces non-linear operation, and conventional linear analysis is applicable only during each switching-period $t_{ce}$ and $t_{or}$, individually.

The aforementioned frequency-restriction, being regarded as the modulating-rate at which reproduction of the input-signal becomes inadequate, will clearly depend not only on the harmonic content, or gradients, of the input-waveform, but also on the signal-amplitude, since the speed of the system-response degenerates exponentially, as may be deduced from Figure 20, as the output-amplitude increases. Satisfactory reproduction of a prescribed waveform would therefore result only provided that its frequency was lower than that for which its gradients at no point exceeded the corresponding maximum possible rates-of-change of the output-current. Theoretically, this is not possible for decaying load-currents, but, due to the non-linear forward-resistances of the semiconductor elements and the deliberate de-linearisation as described in Section 4.5, a satisfactory response to decreasing signals was achieved. With a large forcing-factor the rate-of-rise of the output is sensibly constant for relatively low-amplitude input-functions. Thus the performance is characterised by a discontinuity at the limiting-frequency which may be determined,
Figure 26. System-response to a triangular function.

Figure 27. System-response to a triangular function.
experimentally, by measurement of the system-response to known ramp signals. Alternatively, the limit may be computed from a knowledge of the two load-circuit-time-constants, the restrictions imposed by the pulse-delay-interlock, and the forcing-factor. The response to any other signal may thus be predicted by the comparison of corresponding gradients. In practice the performance was assessed from the response to a variable-frequency triangular function, the rates-of-change observed in the output being correlated to the input-signal-gradients as shown in Figures 26 and 27. These results represent the performance of the system when operated with a forcing-factor of 4.13, the ratio $T_r/T_f = 3.25$, and a load-circuit-time-constant of 11.5 ms at the maximum value of load-current employed. The frequencies at which the discontinuities occurred were verified by calculation despite the considerable complication introduced by the non-linear forward-resistances of the various semiconductor devices, many of which were operated well below their permissible dissipation. Measurements of the forward-characteristics, however, enabled the responses to be estimated graphically, on an incremental basis. The adoption of this method also obviated any assumption that the time-ratio control-system, when operated at a high repetition-rate, behaved as a single-time-lag component, linear or otherwise.


The pseudo-static method of analysis, as appropriated in Appendix 5 to an ideal pulse-width-modulating and de-modulating system, is unsuitable for the determination of the response to sudden input-signal-changes, simply because such an input-function implies an infinite modulating-frequency which invalidates the essential
condition that $f_p$ should greatly exceed $f_m$. For an ideal, linear system the overall transient response could be synthesised from the individual responses of the de-modulator to consecutive pulses; by which, validity of the superposition-principle is implied. In the practical case, however, the imposition of unequal rise-and fall-times of the load-circuit-demodulator wrecks this maxim and renders the system intractable to such synthesis; the transient response being, therefore, much more amenable to the graphical representation of the results of iterative computations, as suggested in Appendices 2 and 4. The transient response to a simple step-function input is clearly a series of elemental exponential changes, the calculation of which are most readily performed by a digital computer.

In pulse-width modulation an abrupt change of the input-signal causes a sudden, corresponding variation of the pulse-width. For fixed-trailing-edge modulation the initial response to a step-increase will be delayed until the commencement of the next pulse, whereas, in fixed-leading-edge modulation, no information will be transferred to the output until the instant at which the existing pulse would have terminated had the input-signal remained unaltered. A complementary result is observed for step-decreases of the input-signal. Thus, an unpredictable delay is inevitable and is attributable to the inability of a pulse-sequence to disclose the exact position of any short, isolated occurrence that is propagated between two consecutive pulses.
4.8.2. Generation of the feedback-signal.

In the experimental system the load-current was monitored by a low-value shunt-resistance, the terminal-voltage of which was filtered and amplified prior to its use as a feedback-signal. The shunt was inserted in the freewheeling-diode circuit where the flow of load-current was independent of the condition of CRl and the reversing-bridge. A low-pass filter of the five-element, Butterworth-type provided satisfactory elimination of the high-frequency distortion.
Figure 28. Reversible machine-control-system.

M; Starter motor.
CHAPTER 5.

Time-Ratio Control of the Simulator-Torque.

5.1. Introduction.

The principle of time-ratio control may, with some reservation, be applied to active, or electrical machine loads as well as to the simple, passive elements considered so far. Simple systems have recently been used with d.c. machines by Weber24 and Reimers25, and contemplated particularly for traction purposes by Heumann26, Gregg27, and Mapham and Hey28. Application of a reversible system to the machine-configuration of Figure 14, however, is considerably more complex and presents additional practical difficulties. Figure 28 shows the machine system incorporated into a high-performance, reversible control-circuit, where, if $E_A$ and $E_B$ are always equal and opposite, the machines together behave as a simple inductive-resistive load. However, in practice, this e.m.f. equality cannot be maintained under all conditions of speed and load largely because of armature-reaction phenomena and the unavoidable errors incurred by the field-control-system of machine A. The e.m.f. unbalance will therefore either assist or hinder the flow of load-current, according to the thyristor-bridge conduction-mode, and introduce an asymmetry into the overall transfer-characteristic that will depend upon the load-current-direction. Although such asymmetry is greatly minimised by closed-loop operation, a more serious defect is that, during each load-current decay-period $t_{op}$, the e.m.f. unbalance may oppose the current-decay to an extent where any forced
Figure 29. Neutralisation of e.m.f.-unbalance.
commutation of the bridge-thyristors, prior to a prospective load-reversal, could become impossible. This difficulty was overcome by the inclusion of an opposing voltage-source in series with $R_x$; as shown in Figure 29. The magnitude of $E_B$ was made greater than the maximum, probable e.m.f.-difference, the value of which was estimated from the armature-reaction characteristics of the two machines and the dynamic error of the field-control-system.

Ideally, the field-flux of machine A should be proportional to the speed of the simulating machine S. If, in fact, these machines are similar, the difference between the respective armature-reaction effects will be quite small over a wide range of operating-conditions. By the use of Greenwood's normalised reaction-characteristics it can, in fact, be shown that any operational error is largely due to the variation of the reaction-coefficients as magnetic saturation is approached. The addition of compensating windings to the machines would, of course, considerably reduce the armature-reaction, particularly if a low magnetic loading was adopted at the machine-design stage. Apart from this, compensating windings would in fact be essential to obviate the high risk of a commutator-flash-over, which may well be provoked by the rapid fluctuations of the armature-current and the resulting oscillation of the magnetic axis caused by the cross-magnetisation effects.

The field-control-system is considered in Section 5.3, where the dynamic errors are examined in relation to the linearity and acceleration-rates of the simulator.
5.2. The characteristics of a d.c.-machine load.

In order that the overall simulator-response may be predicted it is necessary to estimate the armature-circuit-time-constant of the machine-system to be used, although even approximate methods of assessment are very complex. However, Tustin\textsuperscript{30} has expressed the main air-gap component of the armature-inductance of a medium-sized, uncompensated machine as

$$\frac{2.4 \times (\text{rated e.m.f.})}{(\text{rated current}) \times (\text{rated speed}) \times p^\prime}.$$  

from which the time-constant may be derived as

$$\frac{2.4 \times (\text{rated power output})}{(\text{Copper loss}) \times (\text{rated speed}) \times p^\prime}.$$  

where e.m.f. is in volts, current is in amperes, speed is in revolutions per minute and power is in watts. However, since the copper-loss at rated-current will be about four per cent. of the rated-power-output, the time-constant is thus given by

$$\frac{60}{p^\prime \times (\text{rated speed})}$$  

which, at rated-speed, is the pole-pair-period in seconds.

Peterson\textsuperscript{31}, following a more general approach, ignores any mutual coupling between the interpole and main-pole windings and expresses the total armature-circuit-inductance of an uncompensated motor as

$$\frac{3.8 \times (\text{rated voltage})}{p^\prime \times (\text{rated current}) \times (\text{rated speed})}.$$  

whence the time-constant becomes about
which substantiates, to the same order of magnitude, the previous expression.
Although a marginal reduction of the time-constant may be obtained through the use of longer (and consequently smaller diameter) armatures as well as by the use of higher rated-speeds, graded air-gaps and greater air-gap-lengths; a more significant reduction of about sixty per cent. would result from the addition of compensating windings. Generally, it may be concluded that larger machines exhibit lower time-constants since they utilise a greater number of poles, which thus directly reduces the pole-pair-period. This improvement, however, is slightly offset by the relative reduction of the copper-losses and increased efficiency.

The time-constant is affected, naturally, by the armature-circuit-resistance, and in an investigation by Nitta and Okitsu\textsuperscript{32} it was demonstrated that this parameter was in fact very inconstant and dependent upon the speed-and load-conditions. The tendency for it to increase slightly with speed is ascribable to the rise of the brush-contact-drop whilst any load-fluctuation, above about 10 Hz, significantly increases the iron-loss of the armature-core which, according to Nitta and Okitsu, appears ultimately as additional resistance. Although the extreme variations claimed by the investigators were not observed in the experimental simulator-machines, the fact, that in the simulator the load fluctuated at a rate proportional to the simulator-speed, resulted in an overall
Figure 39. Frequency-characteristic of armature-circuit-time-constant.
reduction of the time-constant as shown in Figure 30; although accurate measurements were difficult to obtain. In conclusion, it should be remembered that the time-constants of both machines will be substantially affected by the temperature of the armature-windings and, hence, upon the level of load imposed during the simulation.

With regard to the commutation of the two machines, serious problems may arise because of the armature-current-fluctuations. Although no difficulty was experienced with the experimental machines, despite the absence of interpoles, the successful operation of large machines will require non-saturating interpoles constructed of laminated, low-loss material in order to minimise the distortion and phase-displacement of the interpole-flux at high frequencies. Commutation of the variable-speed simulating machine will be particularly critical as normally the commutation-black-band narrows as speed rises, even with constant loads, and the commutation must be a compromise for both extremes of speed. In addition, load-fluctuations tend to increase the reactance-voltage of the coils undergoing commutation, and this further aggravates the commutation difficulties at high speeds.

5.3. Control of the rotational e.m.fs.

The e.m.f.-equality between machines A and S of Figure 15 may be maintained only if \( \Phi_A \), and hence \( E_A \), are constrained to be proportional to \( N_5 \). For satisfactory operation under all speed- and transient-conditions, however, a rapid-response, low-ripple and infinitely-variable source of excitation is necessary.
Figure 31. Cranking-speed as a function of time.
The rate of engine-acceleration under starting- or firing-conditions may be estimated, as a continuous variable, from a knowledge of the starter-motor-torque-characteristics, the total system-inertia and the various instantaneous engine-torques. Calculations for an Albion E.N. 335 - engine (of only 30lbft\(^2\) rotating inertia), with a type CA 45 starter-motor and a gear-ratio of 12, yielded an unusually high maximum acceleration-rate of 10,750 rev/min/s, as is evident from Figure 31. Thus, if the maximum engine-speed of 3,500 rev/min is to correspond to the maximum excitation \(\Phi_{Am}\) of the auxiliary machine A, then the excitation must rise, initially, at the rate of 10,750 \(\Phi_{Am} / 3,500\) webers per second, which will ultimately represent the steepest ramp-input to the field-control-system. The maximum, dynamic excitation-error will thus be 10,750 \(\Phi_{Am} \cdot T_{FA} / 3,500\) webers which, for example, will result in a tolerable error of 2\% of \(\Phi_{Am}\) with a value of \(T_{FA}\) as low as 6.5 ms.

The engine-or simulator-acceleration decreases, however, as the speed rises because of the increase of the running-friction and the decrease of the starter-motor-torque. The dynamic error will thus progressively reduce as the simulation proceeds. Moreover, at the higher excitation levels, corresponding to the higher simulator-speeds, the onset of magnetic saturation reduces the field-time-constant, and the dynamic error is decreased still further.

Clearly, the primary function of the field-winding is to provide the necessary magnetising force. This may be achieved with an extremely wide variety of winding-designs so that almost any time-
Block A; D.C. amplifier.
Block B; Shaping-unit and signal-convertor.
Block C; Error-amplifier.
Block D; Time-ratio amplifier loaded by resistance $R_{FA}$.
Block E; Time-ratio amplifier for field-control of machine A.

Figure 32. Flow-diagram of a possible means of field-control for machine A.
constant may be realised in practice. Nevertheless, any improvement in the response will demand a proportional increase of excitation power, and field-systems with a very low time-constant would require a controlled supply, the power of which could well exceed the capabilities of most vacuum-tube-or transistor-regulated systems. In this case, time-ratio control clearly affords a solution to the problem and ably satisfies the other requirements mentioned earlier in this section. Moreover, it is possible, if used in an open-loop mode, to largely counteract the magnetic saturation characteristics of Fₐ by deliberate de-linearisation of the transfer-characteristic of the time-ratio control-system.

The experimental equipment was operated under open-loop conditions and over a restricted region of the machine magnetisation curve, the simulator-speed being used as the reference-signal. Although closed-loop operation offered the advantages of a forced-response characteristic, the field-non-linearity, which would be effective within the loop, would have required the simulator-e.m.f. as the reference-signal, instead. This quantity however is not measurable, physically, although it could possibly be computed from the terminal-voltage, armature-current and the appropriate machine-parameters. Under transient conditions, however, the variations of the machine-reactances would present serious difficulties and lead to approximations. Alternatively, a search-coil could be used to indicate the e.m.f., proportionally. Again, closed-loop operation would be feasible if the reference-signal were derived, by use of the magnetisation-characteristic, from the actual field-excitation-current, although this approach is truly valid for only a compensated machine. The system could then be represented as in Figure 32 where the system-output-variable is the field-current.
However, the overall accuracy of such a technique would depend upon the calibration-errors of the shaping circuit employed.

Although closed-loop control would have provided stabilisation against the temperature-dependency of the field-winding-resistances, an open-loop system was employed for the investigations. As such, the excitation of both the simulator-field and the time-ratio-controlled field of the auxiliary machine were supplied from the same d.c. source, which was, in turn, derived from the industrial a.c. supply through a smoothed, 3-phase, thyristor bridge-converter designed to provide substantial stabilisation against supply-voltage-changes. Functionally, the time-ratio control-system resembled the high-performance arrangement of Figure 19, but, as load-reversal and forced commutation were not required, the bridge-thyristors were omitted and the thyristor-gate-pulse-circuits considerably simplified. The system was designed in accordance with the general principles outlined in Appendix 8 which also enumerates other circuit details.

5.4. The effect of magnetic remanence and hysteresis.

Non-linearity caused by remanence and hysteresis phenomena of the field-system of the auxiliary machine have so far passed unmentioned. The vertical displacement of the open-circuit magnetisation-curve by the small residual e.m.f. which is exhibited when both the simulator-speed and e.m.f. are zero depends, of course, on the histogram of the field-excitation and adds to the difficulty of preserving the balance of the two rotational e.m.fs. The fact that the flux $\Phi_A$, in practice, was never required to reverse,
greatly ameliorated this difficulty however, and for small discrepancies the simple remedy provided by a slight increase of the voltage $E_a$, of Figure 29, proved satisfactory in the experimental equipment.

Whereas in many industrial applications the remanent flux of a d.c. machine can be, and often is, neutralised by the magnetising force of a separate field-coil, the unavoidable mutual-coupling effects disqualify the use of this technique where rapid flux-changes are imposed unless, as has become normal transductor practice, a second, similar mutual-coupling device or transformer is cross-connected to counterbalance the induced e.m.f.

Unfortunately, the addition of such a component, which must possess a suitable time-constant, appreciably increases the excitation-power required, apart from generating second-harmonic distortion through the inevitable inequality of the transformer and field-system magnetic loadings.

5.5. Transient error due to breakaway-phenomena.

In a simulator, at the commencement of the cranking-phase, a sudden rise of speed to a value corresponding to $N_{M1}$, shown in Figure 31, would cause transient inequality of $E_a$ and $E_s$. However, the discrepancy will be insignificant as $N_{M1}$ is usually relatively small. If, in addition, no attempt is made to simulate the actual breakaway-phenomenon, then even lower transient-errors will result since, in general, the simulator-static-friction will be lower than that of the simulated engine. This lower value of $N_{M1}$
will result from the correspondingly lower starter-motor torque necessary to overcome the reduced breakaway-threshold.

Actually, in the case of the particular engine characterised by Figure 31, backlash would occur regularly during most of the cranking-phase due to the low engine-inertia, and such a performance would not be tolerated in practice.
CHAPTER 6.

Generation of the Engine-Torque-Function.

6.1. General requirements.

This chapter is primarily concerned with the representation of the engine-torque-characteristics as a voltage-signal, particularly as a function of engine-speed and crankshaft-position. Proportionality between the signal-frequency and engine-speed may, in practice, be imposed either by rigid electrical or mechanical synchronisation, or by artificial linearisation between two appropriate variables. Whilst the former method offers no outstanding advantages, the latter features adjustability of the proportionality to an acceptable accuracy, and does not presuppose an integral relationship.

The acceleration of an engine at the start of the cranking-period was realised in Section 5.3 to be extremely high, since, at a possible rate of 10,750 rev/min/s (see Figure 31), an engine could, for example, attain a speed of 104 rev/min during a crankshaft displacement of only three degrees. Accurate reproduction of the torque-waveform at very low engine-speeds is thus completely unnecessary particularly as, during such a minute movement, the enormous driving torque of the starter-motor at low speed will unfailingly be transmitted to the crankshaft without producing backlash. In fact, even for the extreme case depicted by Figure 31, it will be appreciated that no opportunity will arise for backlash to occur until the system
Figure 33.
The gas-pressure-torque of two oppositely-phased cylinders.

Relative crankshaft-angle in degrees

Curve a; Cranking torque.
Curve b; Firing torque.
operates in the region A-B, by which time the relatively high engine-speed of 460 rev/min will have been surpassed and the cranking-phase will be well under way.

It will be apparent from Section 2.1 that the torque-waveform of any engine is determined largely by the number of cylinders it possesses because the individual cylinder-torque-diagrams differ very little between various engine-designs. Thus to provide versatility in the reproduction of the torque-waveforms it would seem logical to algebraically summate a number of separate functions, each representing, with the correct phase-displacement, the torque of a single cylinder. The summation of an additional sinusoidal function would then provide simulation of the reciprocating-torque components, the amplitude of which should be proportional to the square of the engine-speed.

6.2. Torque due to gas-pressure.

From Figure 2 it is apparent that during every complete engine-cycle the torque developed by each cylinder is effectively zero for about 470° of crankshaft-rotation. Hence, the waveforms of two oppositely-phased cylinders appear alternately, without any overlap, once during each cycle of 720°, as shown in Figure 33. Consequently, the total torque of an engine of \( n_c \) cylinders may be represented by the summation of \( n_c /2 \) identical waveforms, similar to that of Figure 33, each being mutually displaced by \( 720 \div n_c \) degrees.

In the experimental function-generator these waveforms were derived by the multiple sampling of a single function which was generated, repetitively, a large number of times \( n_p \) during each engine-revolution. This method achieved a high degree of
conformity between the several output-waveforms and necessitated the adjustment of only the original, sampled waveform. The maximum frequency at which this primary function was generated was determined from the relationship

$$f_w = \frac{D_p \times N_e}{60} \text{ hertz.}$$

Although a complex arrangement of diode-limited operational amplifiers was originally devised to generate the primary function, the inadequate high-frequency performance confined the use of the technique to relatively low frequencies. A superior method was evolved which employed a modified 'transistor-diode pump-circuit', the basic operation of which is examined in Appendix 9. In principle, the circuit was supplied with constant-amplitude pulses each of which imparted a small predetermined quantity of charge onto a large storage capacitor. For a regular pulse-sequence the capacitor-voltage thus increased relatively smoothly, and linearly, with time, and at a rate proportional to the pulse-repetition-frequency. As such, any finite voltage-increment proportionally represented the number of pulses received during a corresponding time-interval. The usefulness of this technique was enhanced when the capacitor was exposed, alternately, to groups of positive- or negative-amplitude pulses, the former causing the capacitor to charge and the latter causing it to discharge. Variation, through circuit-parameter-changes, of the respective charge or discharge rates thus provided a versatile form of function-generator, the output of which was determined by appropriate sequential selection of the circuit's various operational modes.
Figure 34. Rectilinear representation of Figure 32.

Curve a; Cranking torque.
Curve b; Firing torque.
For experimental purposes it was considered permissible to approximate the waveform of Figure 33 to the rectilinear form represented by Figure 34, in which the phase-displacements of the discontinuities A, B, C and D were measured in terms of the number of pulses supplied to the function-generator. The counting circuits employed were also based on the principle of the transistor-diode pump-circuit, and, regardless of the input-pulse-frequency, could be represented extremely accurately. The input pulses to these circuits were derived from the same source as those supplied to the function-generator module.

The desired linearity between the primary-pulse-frequency and the simulator-speed was achieved through the linear voltage-control of an astable multivibrator circuit, a detailed analysis of which is presented in Appendix 10. In essence, the discharge-rates of the transistor-intercoupling-capacitors were directly regulated by the output-voltage of a d.c. tacho-generator coupled to the simulating machine, and less than one per cent. deviation from true linearity was achieved over a frequency-range of approximately 30 to 1.

6.2.1. Waveform sampling.

During each engine-revolution the primary torque-waveform was generated \( n_p \) times, and, therefore, to provide one complete cycle of output in this time, the primary waveform was sampled \( (n_p - 1) \) times per revolution. Thus the time-separation of the output-functions from adjacent samplers was \( n_p / (\frac{1}{2} \cdot n_0) \) primary-pulse-periods.
Figure 35. Frequency-division and sampling-system for gas-pressure-torque.
and accurate phase-relationships were readily preserved through digital pulse-frequency-division. Clearly, in this technique, the value of $n_p$ must be an integral multiple of the lowest common multiple of all the values of $n_c$ of interest, and, because its value also defines the overall resolution of the sampler-outputs, it should be as high as possible. Thus, if simulations are restricted to either two-, four-, six- or eight-cylinder engines a suitable choice for $n_p$ is 48, which dictates a sampling-rate of 47 instants per revolution.

The sampling-instants must, of course, be precisely synchronised with the primary pulses supplied to the function-generator, and thus the corresponding pulse-rate of the primary multivibrator must be divisible by both $n_p$ and $n_p-1$; in this case, requiring 2,256 pulses per revolution. Figure 35 illustrates the complete sampling system, but it will be observed that the primary pulses are generated at the rate of 9,024 pulses per revolution. As such, each cycle of the primary torque-waveform is constructed of, not 47, but 188 minute increments which produces an even higher resolution.

The samplers are operated in rotational sequence from a pulse-distribution-circuit which ensures precise phase-separation and synchronism. The various pulse-frequency-dividers, shown in Figure 35, are based on the transistor-diode pump arrangement already mentioned, which provides a satisfactory and economic counting-technique. Division by 48 is achieved by successive counts of six and eight respectively whilst division by 47 is similarly obtained after the elimination of one input-pulse from each complete counting-cycle.
6.3. Representation of reciprocating-torque.

The two major considerations pertaining to the generation of the sinusoidal function were, firstly, that of ensuring precise synchronism with the gas-pressure-torque-function over the wide frequency-range, and, secondly, the problem of amplitude-attenuation as a square-law function of the simulator-speed. A pulse-construction technique, similar to that described in Section 6.2, was appropriated in this case to the synthesis of a constant-amplitude, trapezoidal waveform which was subsequently converted to a sinusoid by a simple biased-diode shaping network. The desired phase-relationship with the gas-pressure-torque-waveform, though adjustable, was readily maintained through the operation of both function-generators from the same primary pulse-source.

6.3.1. Square-law attenuation.

The rapid acceleration that ensues during the starting-phase of an engine disqualifies the use of any slow-response, electro-mechanical methods of attenuation-control such as a servo-driven rheostat. However, because the frequency of the sinusoidal function is proportional to the simulator-speed, approximate double differentiation of the waveform by a low-loss, series-resonant circuit, of capacitance C and inductance L, could theoretically offer a solution. In fact, for an input-voltage of $E \cdot \sin \omega t$, the voltage across the inductor would be given by $\left( \frac{L \cdot C \cdot \omega^2 \cdot E \cdot \sin \omega t}{\omega^2 \cdot L \cdot C - 1} \right)$ which would exhibit an almost constant phase-displacement of nearly $\pi$ radians,
relative to the input-voltage, provided $\omega^2 L C$ was very much less than unity for the entire practical range of $\omega$. Unfortunately, the output-signal-amplitude would be reduced to $\omega^2 L C E$ volts and would require considerable amplification. Moreover, unless the input-voltage was purely sinusoidal, the double differentiation of even small harmonic components would produce serious distortion of the output.

Gosling $^{33, 34}$ has investigated the characteristics of field-effect-transistors, when operated near "pinch off", and has suggested their use as voltage-controlled attenuators. For proper operation and low distortion the amplitude of the signal to be attenuated must not exceed about 200 mV. Moreover, the relationship obtained between the attenuation-ratio and the controlling voltage is far from the desired square-law. However, it was envisaged that similar behaviour might be exhibited by a unijunction transistor if the emitter-junction-characteristic could be continuously and stably controlled by regulation of the emitter-current. Some observations, related in Appendix 11, revealed that, over a considerable region of the control-characteristics, a square-law relationship obtained, and that signal-amplitudes of up to two or three volts, of either polarity, could be attenuated without introducing appreciable distortion. Simple biasing-networks were used to establish the operating-region of the device, but the d.c. level of the output-signal was observed to depend on the controlling voltage. For this application, however, the amplitude of the output is negligible at low-frequency operation and capacitive coupling to the final summing amplifier was considered entirely satisfactory. This approach to the
problem yielded a simple, inexpensive and accurate method of attenuation and obviated the use of more complex and sophisticated electronic multiplying-methods.

6.4. Operational requirements of the function-generator.

6.4.1. The simulation of misfiring-phenomena.

The digital techniques utilised in the experimental function-generator are particularly adaptable to the simulation of certain malfunctions provided the system is modified in two essential ways. Firstly, the primary generated torque-waveform should represent the gas-pressure-torque of only a single cylinder, rather than two; and thus as many samplers as there are cylinders are required. Secondly, the waveforms for both cranking-and firing-conditions must be generated separately and simultaneously, and not as alternative functions of a single generating-unit, as was in fact the case. In this way both waveforms are always available for sampling, and this facilitates representation of the engine-torque for any combination of misfiring cylinders.

6.4.2. The reproduction of secondary functions.

It is necessary, for the realistic simulation of an engine, to provide some additional control over the primary engine-torque-signal in order to reproduce its secondary functions of time, speed and temperature as described in Sections 2.3.1 and 2.4. The analysis of the controlled unijunction-transistor-attenuator, given in Appendix 11, suggests that the control-characteristics obtainable are by no
means confined to square-law relationships and it is apparent that such a device could be used, with amplification, to provide the overall control of the signal-amplitude if each of the controlling variables was represented by a direct voltage.

6.4.3. Signal distortion.

Minute voltage-increments, due to the adoption of a relatively low sampling-rate, were superimposed on the final output-signal and were especially prominent at very low-frequency operation. Because the sampling-rate exceeded the frequency-range of the generator, it was practicable to attenuate this distortion by use of a low-pass filter with an abrupt cut-off frequency, the value of which was given by the product of the sampling-rate and the torque-signal pulsation-rate at the lower end of the speed range. As such the fundamental frequency-component of the signal appeared always within the pass-band of the filter. Although entirely satisfactory at the lower frequencies, the filter could not pass the important signal-harmonics at the upper end of the operating-range. In practice, however, it was found that the poor high-frequency performances of the summing amplifiers, alone, were sufficient to eliminate the sampling-distortion, and the filter was necessary only for output-frequencies corresponding to the cranking-speed-range. Experimental observations suggested that an increase of the sampling-rate would obviate the use of a filter altogether, and, since no formidable difficulties were incurred, this proved to be a profitable solution.
6.4.4. Rectilinear representation.

The generation of the primary torque-waveform is not restricted, except in the interests of economy and simplicity, to the linear approximations of Figure 34. Theoretically, and practically, the waveform may be represented by any number of linear sections, the chief consequence being that another pulse-counter becomes necessary for the determination of each additional break-point or discontinuity.
CHAPTER 7.

The Simulation of Engine-Inertia.

7.1. Introduction.

The versatility of an engine-simulator will depend ultimately upon the extent to which the total system-inertia may be altered. Adjustment through the use of interchangeable flywheels would be cumbersome and introduce problems of dynamic unbalance, but in particular would confine all useful simulations to engines with higher rotational inertia than the simulator alone. However, it is conceivable that the overall behaviour of a simulating system, of specified inertia, could be artificially constrained to follow any desired course simply by the suitable modulation of its driving or loading torque. Indeed, if the applied torques could be augmented, even the restriction mentioned could be overcome.

The method of Bates simulated inertia by computing the desired shaft-displacement, or one of its derivatives, as a function of time and the applied torques. The computed result, in analogue form, was applied as a reference-signal to a closed-loop control-system, the driving torque thus being regulated by the error-function. For the extremely low rates of acceleration concerned, the computation of the reference-signal was performed by servo-driven rheostats which were used directly to control the machine-behaviour.

In this section a technique is evolved whereby rapid accelerations can be simulated without recourse to computation by limited-response
elements. The merits of the method are discussed and compared to the closed-loop technique already outlined, and particular attention is given to the backlash-phenomena that occur in an engine-simulator-system.

7.2. System relationships.

If torques that resist the normal rotation of the starter-motor are regarded as positive, then, if engine-friction and backlash-phenomena are ignored, the net driving torque of the combined engine/starter-system is given by

\[(m'_M + m'_E) = (J'_M + J'_E) \dot{\omega}_E\]  

Similarly, for a simulator/starter-system with the same gear-ratio, the relationship will be

\[(m'_M + m'_S) = (J'_M + J'_S) \dot{\omega}_S\]  

Hence, if \(K_s\) is defined as \(\frac{(J'_E - J'_S)}{(J'_M + J'_E)}\) and \(\dot{\omega}_S\) is equated to \(\frac{\dot{\omega}_E}{(J'_M + J'_E)}\), the result is that

\[m_s = m'_E (1-K_s) - K_s m'_M\]  

The torque \(m'_E\) is represented by the output of the signal-generator, described in Chapter 6, so that, if it is multiplied by \(1-K_s\) and added to a fraction \(K_s\) of the referred starter-motor-torque \(-m'_M\), the resultant signal will represent the torque that must be developed by the simulator to enable it to behave as though it possessed an inertia \(J_E\). Although during any particular simulation \(K_s\) is constant, the value of \(m'_M\) must be determined as a continuous function of time.
Figure 36. Starter-motor/engine engagement.
The physical interpretation of Equation 13 is that for a particular value of $K_s$, which implies a specific value of inertia to be simulated, the total loading torque of the simulator must be controlled as an overall function of both $m'_M$ and $m_E$, in accordance with the Equation, so that the net driving-torque will be modified continuously and in proportion to the ratio $J_s/J_e$. Thus the actual driving torque $-m'_M$ will be merely re-deployed against the modified electromagnetic and inertia-loads of the simulator. The stresses in the starter-motor-shaft and pinion, therefore, cannot distinguish between the effects of a real inertia and an equivalent, apparent value.

7.3. Consideration of backlash-phenomena.

It is important that the conditions under which backlash occur in the actual engine/starter-motor combination are preserved when part of the inertia is simulated. Figure 36 depicts the engagement of the simulator-starting-ring and the starter-motor-pinion, the mean diameter-ratio (and thus the gear-ratio) being $g_+ : 1$. If each inertia-component were allowed to accelerate freely under the influence of its own applied torque, the angular accelerations would be

$$\dot{\omega}_S = \frac{m_S}{J_s}, \quad 14$$

and

$$\dot{\omega}_M = \frac{m_M}{J_M}, \quad 15$$

respectively. However, physical engagement of the two components imposes equality of the circumferential accelerations at an intermediate value of

$$\left\{ \frac{g_+ m_M + m_E}{g_+ J_M + J_s} \right\}$$
Between the backlash limits, however, physical engagement cannot exist, and the angular accelerations given by Equations 14 and 15 ensue. Hence, from the diagram, teeth A and C will disengage if \( m_s \) and \( m_M \) are in the directions shown and the condition

\[
\frac{\mathcal{g} \cdot m_s}{J_s} > \frac{m_M}{J_M}
\]

applies, whereas teeth B and C will separate when the opposite condition,

\[
\frac{m_M}{J_M} > \mathcal{g} \cdot \frac{m_s}{J_s}
\]

pertains.

In general, the onset of backlash, in either direction, will occur at the instant when

\[
\frac{m_M}{J_M} = \mathcal{g} \cdot \frac{m_s}{J_s}
\]

16.

The effect of the inertia-simulation on the instants at which backlash occurs may be examined if \( m_e \) is suitably modulated such that

\[
m_e = K_m \cdot m_M \cdot f(t)
\]

17.

From Equations 13 and 16, backlash therefore occurs when

\[
\mathcal{g} \cdot m_e \cdot (1-K_s) - \mathcal{g} \cdot K_s \cdot m_M = \frac{J_s}{J_M} \cdot m_M
\]

which, by substitution of Equation 17, yields

\[
\mathcal{g} \cdot K_m \cdot f(t) = \left( \frac{J_s}{J_M} + \mathcal{g}^2 \cdot K_s \right) \cdot \left\{ \frac{1}{1-K_s} \right\}
\]

The definition of \( K_s \) then facilitates simplification to

\[
f(t) = \frac{J_s}{J_M \cdot \mathcal{g} \cdot K_m}
\]

from which it is apparent that the time-instants for which \( f(t) \) has the value

\[
\frac{J_s}{J_M \cdot \mathcal{g} \cdot K_m}
\]

are entirely independent of \( K_s \), and the moments at which backlash occur are therefore unaffected by the proportion of inertia being simulated.
m_E \rightarrow \text{Add} \rightarrow \text{Invert} \rightarrow (m_E + m_{FE})

(m_{E}(1-K_S) - K_S m'_M - m_{FS} + (1-K_S)m_{FE})

m_E \text{ is positive (loading) for compression-periods.}
m_{FS} \text{ is always negative (driving).}
m_{FS} \text{ is always positive.}
m_S, \text{ when positive, causes simulator to load the starter-motor.}

Figure 37. Flow-diagram corresponding to Equation 19.
It is necessary also to consider the impact experienced by the engaged teeth immediately after the backlash region has been traversed. If the turning-moment of the reaction-force is \( m_R \) (referred to the starter-pinion) then, after the moment of impact, the circumferential acceleration of the two rotating members must be equal. Therefore, in a simulator/starter-motor combination,

\[
\frac{m_M - m_R}{J_M} = g^+\left(\frac{m_s + g^+m_R}{J_s}\right) \quad \text{for the case where} \quad m_M J_s \geq g^+ J_M m_s,
\]

and, by substitution of Equation 13, it can be deduced that

\[
m_R = m_M \left( J_M (1-K_s) - K_s J_M \right) - J_M \left( (1-K_s) m_e - K_s m_M \right) / g^+ \left( J_M + J_M (1-K_s) - K_s J_M \right).
\]

whence

\[
m_R = \left\{ \frac{m_M J_E - m_E J_M}{g^+ (J_M + J_E)} \right\}
\]

A similar result may be obtained for the condition \( m_M J_s < g^+ J_M m_s \).

The Expression 13, being independent of \( K_s \), thus demonstrates the independency of the backlash-impact-stresses upon the degree of inertia-simulation, \( K_s \).

7.4. Operational characteristics.

Equation 13 was derived on the assumption that frictional torques were negligible, and thus implies synonymity between the developed and net torques of the simulating machine. Since crankshaft-rotation is unidirectional, allowance may be made for friction as indicated in the flow-diagram of Figure 37 which graphically interprets the modified expression.
\[ m_g = m_e \cdot (1-K_s) - K_s m_M - m_{F5} + (1-K_s) m_{FE} \tag{19} \]

which is comparable to Equation 13. The frictional component of this expression, namely \((1-K_s)m_{FE} - m_{F5}\), will generally exhibit a characteristic similar to the simulator-friction alone, especially for values of \(K_s\) that are close to unity. Furthermore, because the friction of an engine will be somewhat larger, generally, than that of an equivalently-rated electrical machine, the differential torque-component above will be designated as positive, except for very high and improbable values of \(K_s\), and any increase of the friction with speed will provide a form of stabilising feedback within the system. However, in this application a far more predominant stabilising-influence is exercised by the speed-dependency of the starter-motor torque.

Over a limited range the inverse relationship

\[ N_s \cdot m_M = K \]

may be assumed (see Section 2.5), which, when substituted into Equation 19, gives

\[ m_g = (1-K_s) m_e - \frac{K_s m_F}{N_s} - m_{F5} + (1-K_s)m_{FE} \tag{20} \]

Thus \(m_g\) may be considered to comprise two torque-components represented respectively by \((1-K_s)m_e\), the amplitude of which will not necessarily depend upon speed, and

\[ \left\{ m_{FE} - m_{F5} - \frac{K_s m_F}{N_s} \right\} \tag{21} \]

Even though \((1-K_s)m_{FE} - m_{F5}\) could increase with speed, the overall value of Expression 21 will in all practical cases decrease with speed.
This must be so since only in this way can a final, stable cranking- or running-speed be attained. Although this inherent stabilising-effect is essential to the system, its non-linear inverse characteristic renders the relative system-stability and the final accuracy dependent upon the prevailing operating-point on the speed/torque curve.

Closed-loop operation is fundamental to the simulation-technique suggested by Bates. However, in any closed-loop system the amplified error-function depends upon the system-gain and time-constants, and is thus not representative solely of the reference-variable. As such, whereas the input-signal may exactly prescribe the desired final acceleration, speed or displacement, when computed as a function of \( m \), the torque-load finally applied to the starter-motor will be determined by numerous parameters peculiar to the entire controlling system and, therefore, will become a distorted reproduction of the torque-waveform that would actually be imposed by an engine. Apart from the fact that any distortions of the starter-motor stresses are impermissible, any excessive misrepresentation could also induce unrealistic backlash effects which destroy the validity of the simulation. Moreover, because the system-gain varies according to the starter-motor torque/speed characteristic, the damping-factor, also, becomes a non-linear function of the output-variable, and even the distortion will not remain constant over the speed-range.

The method proposed, however, operates fundamentally in an open-loop mode whereby a simple analogue computation provides a precise indication of the load-torque which must be applied, by the simulator,
to ensure the correct loading of the starter-motor. Consequently, the actual simulator-acceleration, -speed and -displacement, themselves not being directly controlled, must then be purely incidental. In this respect the system behaves exactly as the comparable engine/starter combination, and, as was verified in Section 7.3, the backlash conditions will be accurately preserved.

Whilst both the open-loop and closed-loop methods depend equally upon the accuracy of the computed data, the proposed technique, through the use of simple, non-reactive attenuators, can provide greater computational precision than can the complex equipment necessary for the closed-loop method. Moreover, the negligible time-lag imposes no unnecessary frequency-limitation upon the system-response.

7.5. Determination of the driving torque.

The precision of both the techniques examined depends largely upon the accurate representation of the starter-motor-torque $m_m$. Three methods by which this may feasibly be achieved are briefly discussed in the following paragraph.

Firstly, the starter-motor torque-reaction could be measured directly by strain-gauges provided the motor could be suitably supported. Unfortunately, such a method is satisfactory only under slowly-changing torque-conditions as the type of response obtained is seriously affected by the considerable stator-inertia and the suspension-characteristics.
Secondly, from a knowledge of the armature-torque/armature-current characteristic the starter-motor-torque could be ascertained, by use of a simple shaping-network, from the continuous measurement of armature-current. Some inaccuracy would result if the motor-flux appreciably lagged the armature-current variations, but it is fortunate that, because magnetic saturation prevails over most of the operating-current-range, variations of the field-flux are almost imperceptible, and the output-torque is in phase with the current. The most serious errors encountered would, in fact, be those caused by the inclusion of a shunt measuring-resistance into the low-voltage circuit, and, since currents of several hundred amperes must be accommodated, the method presents some difficulties.

The third possibility is to compute the motor-torque, in analogue form, as a function of its speed by the use of a biased-diode function-generator. No practical difficulties are encountered, but the overall accuracy obtained rests upon the insignificance of the armature-circuit-inductance and the precision with which the true torque/speed characteristic may be functionally represented. Unhappily, this characteristic is not exclusively peculiar to the motor but depends largely upon the battery's internal resistance and its state of charge. Because these deteriorate, particularly as a function of the discharge-time, considerable errors are inevitable over prolonged cranking-periods.

Although, generally, the measurement of armature-current could be expected to furnish the most reliable and precise estimation of torque, the third technique was utilised in the experimental system since it was more easily adapted to both the physical simulation and representation on the analogue computer which was used later to verify the system-performance.
Figure 38. Physical arrangement for the verification of inertia-simulation.
Figure 39. Machine-torque / speed characteristics.
7.6. The physical simulation of engine-inertia.

The flow diagram of Figure 37 was investigated experimentally by use of the practical arrangement shown in Figure 38, in which different values of inertia could be simulated by variation of the coupled potentiometers of attenuation-factors $K_g$ and $(1-K_g)$. The torque/speed characteristic of the machine $M$, represented by curve 'a' of Figure 39, was accurately reproduced by a shaping-network as suggested in Section 7.5, because in this case the 'starter motor', being supplied from a reasonably constant voltage, exhibited a predictable characteristic and a fast response. Since the proving-tests were of a general nature, and not pertinent to any particular engine, the engine-friction-torque was regarded as zero, and only the component $m_{rS}$ of the complete friction-component $(1-K_g)m_{FS} - m_{FS}$ was represented. This, being a function of simulator-speed, was derived from a simple function-generator, the characteristic of which was adjusted to resemble curve 'b' of Figure 39.

From the continuous recordings of simulator-speed the instantaneous acceleration-rates were measured and correlated to the input-signal $m_E$ for a number of corresponding values of $K_g$. For comparison purposes the speed-responses to several step and repetitive, triangular input functions were calculated by a simple iterative process based on the curves of Figure 39. However, because of cumulative computational and graphical errors, the performance was ultimately verified by an analogue computer simulation in which the inertia-parameter was, itself, directly varied without the use of the attenuators.
Settings for curves of Figure 39.

- $a_1 = 0.80$
- $a_2 = 0.21$
- $a_3 = 0.33$
- $a_4 = 0.15$
- $a_6 = 0$
- $a_8 = 0$

Scales:

- $X$: 1 V represents 106 rev/min.
- $Y$: 1 V represents -53 rev/min.
- $Z$: 1 V represents -11.1 rad/s, i.e., 1.74 A of simulator-current.
- $W$: 1 V represents -0.5 lbf ft.
- $V$: 1 V represents -0.5 lbf ft.
- $U$: 1 V represents -0.05 lbf ft.

Figure 40. Computer-connection for verification of inertia-simulation.
Table 3. Comparison of results.

<table>
<thead>
<tr>
<th>Input-data and settings (used or implied)</th>
<th>Computer results.</th>
<th>Simulator results.</th>
<th>Calculated results.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1−Kₜ)</td>
<td>Kₛ</td>
<td>mₑ lbf ft (step-function)</td>
<td>Initial-acceleration</td>
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<td>0</td>
<td>0.81</td>
<td>19.4</td>
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<td>0.81</td>
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<td>0.33</td>
<td>0.62</td>
<td>15.7</td>
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<tr>
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<td>0.50</td>
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<tr>
<td>0.50</td>
<td>0.50</td>
<td>0.40</td>
<td>14.2</td>
</tr>
</tbody>
</table>
The analogue simulation thus portrayed the real system with a variable engine-inertia $J_E$. The common application of the curves of Figure 39 to all three investigations thus ensured a valid comparison of the performances.

Figure 40 illustrates the computer arrangement and the various scaling-factors employed in the direct simulation of the relevant system-equation (similar to Equation 11) wherein

$$m'_M + m_E - m_F = \ddot{\omega}_E (J'_M + J_E).$$

The amplifiers $A_1$ and $A_2$, together with the potentiometers $a_1$, $a_2$, $a_3$ and $a_4$, serve to represent curve 'a' of Figure 39 on the appropriate computer-scales, whilst amplifier $A_2$ also summates the friction-torque-signal from $a_6$, $a_5$ and $A_4$. Integrator $I_1$ provides the inertia-integration-constant, and $A_3$ with $a_7$ are used merely to offset the output of $I_1$ to provide a suitable speed-reference-signal for the engine-torque function-generator, described in Chapter 6. The curve 'a' of Figure 39 may be observed at node-Z as the function of a positive voltage applied at node-X when the stabilising-link L is disconnected and amplifier $A_4$ is removed. Step changes of engine-torque were applied through $a_9$ from the negative supply, whereas pulsating signals were applied, also through $a_9$, from the coupled torque-function-generator.

The initial acceleration-rates and final system-speeds, as were attained by each of the three methods, are compared in Table 3, and it is apparent that the acceleration-rates produced by the simulator were slightly lower than the computed and calculated values. This
was due mainly to errors incurred in the determination of the actual system-inertia as measured by a retardation-technique similar to that suggested by Prescott. Further minor discrepancies may be attributed to other factors, enumerated below, in the knowledge of which the comparison is remarkably conclusive.

1. Differential linearity-errors between the $K_g$ and $(1-K_g)$ attenuators.
4. Absence of the breakaway-phenomena in both the computer-simulation and the calculations.
5. The existence of a time-lag in the torque-current relationship of the starter-motor, which applied only to the actual simulator-system.
6. Cumulative errors incurred in the iterative calculations.
7. The linear approximation of the friction-characteristic applied to the analogue computer.
8. Measurement-errors incurred in the interpretation of the various recordings.


It is evident from Section 7.2 that an increase in the value of simulated inertia dictates correspondingly larger loading-, and smaller driving-, torque pulsations from the simulating machine. Since the extremes of peak torque depend on the fraction $K_S$, it is important that the rating-requirements of the simulator are ascertainable for
Figure 41. Normalised torque-relationships.

Curve a; $K_S = -2$
Curve b; $K_S = -1$
Curve c; $K_S = 0$
Curve d; $K_S = +0.5$
various conditions. Moreover, as it is theoretically and practically feasible to represent values of total inertia that are lower than the physical quantities, that is, when \( K_\delta \) is negative, the effect of all probable values of \( K_\delta \) should be examined.

If \( m_{Fe} \) is assumed to be negligible, then from Equation 19,

\[
m_s = (m_E + m_{Fe}) (1-K_\delta) - K_\delta m_M'
\]

and, because both \( m_{Fe} \) and \( m_M' \) are known functions of the engine-speed, they may readily be related to the frequency of the engine-torque-fluctuations.

It is enlightening to consider the normalised equation

\[
\frac{m_s}{m_M'} = \left(1-K_\delta\right) \left\{ \frac{m_E + m_{Fe}}{m_M'} \right\} - K_\delta
\]

in which \( m_s/m_M' \) is clearly linearly related to \( (m_E + m_{Fe})/m_M' \) provided \( K_\delta \) is maintained at a constant value. As shown in Figure 41, this result may then be used directly to derive the simulator-torque-waveform from a known engine-torque-variation for any prevailing values of \( m_M' \) and \( K_\delta \). As an example, the waveform of \( m_s/m_M' \) that results from a given triangular variation of \( (m_E + m_{Fe})/m_M' \) has been shown for a value of \( K_\delta = -1 \). The validity of this approach relies on the assumption that, during each cycle of the engine-torque-pulsation, the value of \( m_M' \) remains sensibly constant. Except at extremely low speeds, which do not prevail for any appreciable time, any variation of \( m_M' \), as a function of speed, will be negligible since the severity of the cyclic speed-fluctuations induced by the pulsating load are, in practice, greatly attenuated by the system-inertia. The tendency of the average
Figure 42. Characteristics of Equation 25.

Amplitude factor $\alpha$.

Curve a; $K_s = -2$
Curve b; $K_s = -1$
Curve c; $K_s = 0$
Curve d; $K_s = +0.25$
Curve e; $K_s = +0.50$
speed-level to rise is relatively gradual, and use of the normalised equation will thus readily indicate both the peak and average values of the quantity \( m_s/m_M \). However, the rating and design of the simulating machine will depend more specifically upon the root-mean-square value of the developed torque, and this may be found either graphically or analytically, depending on how \( (m_E + m_{FE})/m_M \) is expressed.

In any engine, operating under normal firing-conditions, the negative peak values of engine-torque exceed the positive values, and, as an analytical example, it will be supposed that the total torque of an eight-cylinder engine may be represented approximately by the relationship

\[
\left\{ \frac{m_E + m_{FE}}{m_M} \right\} = \alpha (1 + \sin \omega t).
\]

Thus the torque \( m_s = m_M \cdot (\alpha \cdot (1-K_s) \cdot (1 + \sin \omega t) - K_s) \),

the root-mean-square value of which can be shown to be

\[
m_{s\text{ rms}} = m_M \sqrt{\left( \alpha \cdot (1-K_s) - K_s \right)^2 + \frac{1}{2} \alpha^2 \cdot (1-K_s)^2}.
\]

Hence, if \( K_s \) is zero, then \( m_{s\text{ rms}} \) is identical to the root-mean-square value of \( (m_E + m_{FE}) \) and evaluates to 1.225 \( \alpha \cdot m_M \). It follows therefore that the ratio

\[
\frac{m_{s\text{ rms}}}{(m_E + m_{FE})_{\text{rms}}} = \frac{1}{1.225} \left\{ \frac{3 \cdot (1-K_s)^2 + K_s^2 - 2 \cdot K_s \cdot (1-K_s)}{\alpha^2} \right\}^{\frac{1}{2}}
\]

Figure 42 illustrates the value of this ratio as a function of \( \alpha \) for various values of \( K_s \), and demonstrates that for values of \( \alpha \) greater than about four, the relationship is largely independent of \( \alpha \) and,
consequently, of \( m_l \). Auspiciously, during a typical simulation, \( \alpha \) will increase from a value of about four, at the commencement of the cranking-period, to much higher values as the speed increases and \( m_l \) diminishes. Thus, particularly under firing-conditions, the required simulator-torque-rating will depend almost exclusively on the value of \( K_g \). It is interesting to note that for positive values of \( K_g \) below unity, the ratio given by Equation 25 is also less than unity, and inspection of Figure 41 will confirm that such values of \( K_g \) produce a vertical displacement of the waveform of \( (m_g/m_l) \) which tends to distribute the waveform more evenly about the abscissa. This, naturally, decreases the root-mean-square value in addition to the direct reduction afforded by the scale-factor \((1-K_g)\).

7.3. Breakaway simulation.

An engine-crankshaft, in coming to rest, will tend to assume a final position where any further forward rotation would be opposed by the engine compression-torque. During the final revolution, such a stable position, of zero restoring torque, may be approached in an oscillatory manner, but, because of the presence of coulomb friction, the final rest-position will not be accurately predictable. However, in a well-lubricated engine the uncertainty will generally be negligible. When the engine is subsequently cranked, forward rotation will be delayed until the starter-motor can overcome the breakaway-threshold of the engine. After this the opposing compression-torque will tend to maintain the rigid starter-engagement already established and thereby discourage any immediate backlash tendency of the system.
In a simulator, a corresponding behaviour can only be imposed artificially by the appropriate synchronisation of the engine-torque-waveform with the onset of rotation. At the instant at which breakaway occurs, however, the value of $m_M$ will greatly exceed the maximum possible value that $m_s$ can have under cranking-conditions, and, in general, $J_s$ will also exceed $J_M$. Consequently, the necessary condition for backlash to occur, as given by Equation 16, cannot possibly prevail, and it may be concluded that no parasitic backlash will result even in the absence of waveform-synchronisation.

Due to the extreme values of starter-motor-current that normally precede breakaway, the motor-time-constant is appreciably reduced by the magnetic saturation of the field-system, and the various breakaway-phenomena are accomplished very rapidly. The initially large current will therefore contribute little towards the heat-dissipation of the motor-windings, and, at zero speed, will not pose any serious commutation difficulties. Thus, even for a simulator with a marginally lower breakaway-threshold than the engine under consideration, the value of a meticulous simulation of the phenomenon is small. Nevertheless, the method by which it could be simulated offers a satisfactory solution to a far more serious practical difficulty which will be considered in the following section.

7.3.1. Initiation of the cranking-phase.

Ideally, at the commencement of cranking, the simulator should remain at rest while the starter-motor-torque rises to the appropriate breakaway value but, clearly, unless the mutually opposing
Figure 43. Block-diagram for breakaway-simulation.
simulated-friction-and starter-torques are increased simultaneously, undesirable rotation will almost certainly result. Reverse rotation, in particular, must be prevented, otherwise the e.m.f. balance between the simulating and auxiliary machines will be upset because of the non-reversibility of the auxiliary-machine field-control-system. The most palpable solution is to restrain the reverse rotation mechanically, in which case undesirable forward rotation, also, could be precluded by the prior application of a sufficiently large opposing torque from the simulator. No forward rotation could therefore take place until this reactionary torque was either overcome by the starter-motor or deliberately reduced. Hence, it is possible, in this way, to simulate any level of breakaway-torque without incurring premature rotation.

Alternatively, a closed-loop position-control-system, applied to the simulator-shaft, could be employed for the same purpose. If an arbitrary shaft-position is permanently represented by a zero reference-signal, any disturbance of the shaft from its equilibrium-position would be opposed by a reactionary torque, developed as a result of the position-error imposed. The success of such a technique depends implicitly upon the existence of an error and upon the magnitudes of the various system-time-constants, the most prevalent being those of the machines through which the reactionary torque would be applied. The existence of an error, and its time-derivatives, thus renders the validity and value of such a technique open to question.

Figure 43 represents the block-diagram of the practical arrangement for which the closed-loop transfer-function may be written as

\[
\frac{m_s (p)}{m'_m} = \frac{K_A \cdot K_B}{F_V \cdot (1 + pT_1) (1 + pT_2) + K_A \cdot K_B}
\]
In the diagram, the inherent stabilising effect of $w_M$ upon the simulator-speed has been disregarded. The hypothetical arrangement, therefore considered, offers a poorer performance than would occur in practice, although the effect is only transient and almost negligible for small speed-variations. For simplicity, the transfer-function excludes any coulomb friction components, and therefore permits the application of conventional linear analysis. Oscillatory responses must be avoided otherwise spurious backlash may be induced and periods of reverse-rotation will ensue. Thus, in general, some form of phase-compensation will be necessary.

Application of the method to the experimental equipment presented little difficulty owing to the extremely low values of $K_A$ and $K_B$ (0.05 volts per radian and 0.37 lbf ft per volt, respectively) although phase-advance compensation was necessary. The position-error was readily generated by integration of the d.c. tacho-generator-output previously made available for control of the torque-waveforms, as indicated in Section 6.2, and the field-excitation system, as described in Section 5.3. The final steady-state error was estimated for the compensated system to be about $17^\circ$ per pound-force foot of input deflecting-torque, but appreciable discrepancies were observed in practice. These errors were due mainly to the effects of coulomb friction and drift of the integrator-output. The error, though considered acceptable in this case, should be assessed with regard to the stability-performance obtainable. Generally, for a larger system, a better performance will be achieved owing to the greater predominance of $T_2$ over $T_1$, which also simplifies the application of the phase-advance compensation-network.
CHAPTER 8.

CONCLUSIONS.

The deliberations and deductions of the preceding chapters have indicated technically-feasible solutions to the numerous problems entailed in the electrical simulation of diesel engine characteristics, the final proposals having been investigated both analytically and empirically, albeit on a somewhat reduced scale. The salient features of the combined solutions are versatility, accuracy and sophistication, particularly in comparison to the performances that could be expected from hydraulic or pneumatic simulators. Either of the latter would, however, be relatively simple and less expensive.

In particular, the thesis has involved a study of several widely-applicable disciplines, perhaps the most significant being the simulation of inertia and the infinitely-variable control of relatively large currents by time-ratio control. The investigation of the former included an analysis of the effects of backlash and static-friction-phenomena, though in many applications, where these are of no interest, considerable simplification would be possible.

For the present application of the current-control-system, it was deduced that the larger the simulating machine, of the type employed, the lower would be the armature-time-constant. This property could be exploited either to yield an improved response-characteristic and permit higher-frequency operation, or to provide a relatively mediocre performance with the economic advantages of a smaller
forcing-factor. It is for the starter-motor manufacturer to specify the maximum test-speeds of the motors to be investigated, and these limits will then largely determine the ratings and parameters of the simulating system (see also Appendix 12).

By far the most severe requirement of the simulator is the necessity to periodically reverse the output-current of the time-ratio control-system when supplying an inductive machine-load. It will be apparent that, despite the fact that remarkably high reversing-rates were obtained from the experimental apparatus, this reversing-duty, in becoming more arduous with an increase of simulator-size and rating, will impose an arbitrary limitation to the reversing-rate that may be economically achieved in a full-scale system. In this respect it should be appreciated that although a higher maximum torque-pulsation-rate will necessitate a higher-rated simulating machine, which will exhibit a lower armature-circuit-time-constant, the reduction of this parameter will not necessarily be inversely proportional to the size, and a larger forcing-factor will, in general, have to be applied.

Clearly the next stage of the investigation would be to construct, according to the findings and proposals of this thesis, a prototype system of considerable size from which more exact extrapolations could be made with regard to the parameters of a full-scale simulator. For such an apparatus, the desirability of inertia-simulation is emphasised, though the simulation of the breakaway-phenomenon is considered to be of less consequence.
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APPENDIX 1.

Torque due to Reciprocating Masses.

The inertial force \( F \) of an accelerating piston may be expressed as

\[
F = \frac{W \cdot r \cdot N_e^2}{35, 240} \left\{ \cos \Theta + \frac{t}{l} \cos 2\Theta \right\} \text{ pounds-force, 26}
\]

from which the turning-moment at the crank-pin may be derived as

\[
m_{ER1} = F \cdot \frac{t}{12} \left\{ \sin \Theta + \frac{t}{l} \sin 2\Theta \right\} \text{ pound-force feet. 27}
\]

The combination of Equations 26 and 27 gives

\[
m_{ER1} = \frac{W \cdot r^2 \cdot N_e^2}{12 \times 35, 240} \left\{ \frac{1}{2} \sin 2\Theta + \frac{t^2}{4l^2} \sin 4\Theta + \frac{t}{l} \sin(2 - 3 \sin^2 \Theta) \right\} \text{ pound-force feet. 28}
\]

Thus, for a two-cylinder engine, \( \Theta \) may be replaced \((\Theta + \pi)\) to give the reciprocating-torque of the opposing piston, the total torque being obtained by summation of the two components to give

\[
m_{ER2} = \frac{W \cdot r^2 \cdot N_e^2}{12 \times 35, 240} \left\{ \sin 2\Theta + \frac{t^2}{2l^2} \sin 4\Theta \right\} \text{ pound-force feet. 29}
\]

Hence, for a four-cylinder engine, in which all the pistons accelerate and retard together, the total turning-moment becomes

\[
m_{ER4} = 2 \cdot m_{ER2} = \frac{W \cdot r^2 \cdot N_e^2}{6 \times 35, 240} \left\{ \sin 2\Theta + \frac{t^2}{2l^2} \sin 4\Theta \right\} \text{ feet. 30}
\]

In a typical, modern diesel engine design \( l/4t \) is approximately unity and the term \( \frac{t^2}{2l^2} \sin 4\Theta \) is negligible, so that

\[
m_{ER4} = \frac{W \cdot r^2 \cdot N_e^2}{6 \times 35, 240} \sin 2\Theta.
\]
If an eight-cylinder, in-line engine is regarded as comprising two quadrature groups of four cylinders with an angular displacement of \( \pi /2 \) radians, the total turning-moment may again be derived if \( \Theta \) is replaced by \( \Theta + \pi /2 \), in Equation 30, to give

\[
m'_{\text{ERS}} = \frac{W + \frac{t^2}{2} \sin 4\Theta}{\frac{8 \times 35,240}{L} \sin \frac{\pi}{3}} \text{ pound-force feet.}
\]

Equation 31, when added to Equation 30, yields

\[
m_{\text{ERS}} = \frac{W + \frac{t^2}{2} \sin 4\Theta}{\frac{6 \times 35,240}{L} \sin \frac{\pi}{3}} \text{ pound-force feet.}
\]

In addition, it may be deduced that Equation 32 is also applicable to a regular, eight-cylinder, V-type engine since, if all the crank-pin-planes and piston-stroke-axes are displaced through identical angles, the individual torque-components will remain unaltered in time-phase and magnitude.

Finally, a six-cylinder engine may be regarded as three groups of two opposing cylinders, with an angular displacement of \( 2\pi /3 \) radians, for which the total turning-moment may be computed, from Equation 28, to be

\[
m_{\text{ERS}} = \frac{3 \times W + \frac{t^2}{2} \sin 3\Theta}{\frac{8 \times 35,240}{L}} \text{ pound-force feet.}
\]

The results of this analysis are discussed in Section 2.2.
\[ t_2 - t_1 = t_{cl} \]
\[ t_3 - t_2 = t_{op} \]
\[ t_3 - t_1 = t_{op} + t_{cl} = \frac{1}{f_p}. \]

Curve 0-A has time-constant \( T_r \).

Curve A-B has time-constant \( T_f \).

Figure 44. Output of a time-ratio control-system.
APPENDIX 2.

Linearity of Time-Ratio Control with an Inductive Load.

Figure 44 illustrates the output-current-waveform of an inductance-smoothed time-ratio-control-circuit when delivering a steady, average current. If $E_d/R_L = I_m$, then, during the period $t_{CL}$,

$$i_o = I_m \left( 1 - e^{-\frac{t}{T_r}} \right)$$

and during the period $t_{OP}$

$$i_o = I_2 e^{-\frac{(t-t_2)}{T_f}} = I_m \left( 1 - e^{-\frac{t_2}{T_r}} \right) e^{-\frac{(t-t_2)}{T_f}}$$

But $i_o = I_2$ when $t = t_2$ and, thus, from Equation 33

$$I_2 = I_m \left( 1 - e^{-\frac{t_2}{T_r}} \right)$$

Also, since $I_o = I_m \left( 1 - e^{-\frac{t_1}{T_r}} \right) = I_2 e^{-\frac{t_{OP}}{T_f}}$,

the average value of $i_o$ may be deduced, from the area under each exponential curve, as

$$I_{AV} = \frac{I_m}{t_{CL} + t_{OP}} \left[ t_{CL} - (T_+ - T_f) \left\{ 1 - e^{-\frac{t_{OP}}{T_f}} \right\} \left( 1 - e^{-\frac{t_{CL}}{T_+}} \right) \right]$$

Therefore, in a simple case, where $T_+ = T_f$,

$$I_{AV} = \frac{I_m \cdot t_{CL}}{(t_{CL} + t_{OP})} = \frac{E_d \cdot f_p \cdot t_{CL}}{R_L}$$

The combination of Equations 35 and 36 gives

$$1 - e^{-\frac{t_1}{T_r}} = e^{-\frac{t_2}{T_f}} \left( 1 - e^{-\frac{t_2}{T_+}} \right)$$

and, as $t_2 = t_1 + t_{CL}$,

$$e^{-\frac{t_1}{T_r}} \left\{ 1 - e^{-\frac{t_{CL} + t_{OP}}{T_f}} \right\} = 1 - e^{-\frac{t_{OP}}{T_f}}$$
Equations 35 and 36 may be used also to express the peak-to-peak amplitude of the ripple-current as

\[(I_2 - I_1) = I_m \left\{ 1 - e^{-\frac{t_1 + t_{cl}}{T_+}} \right\} - I_m \left\{ 1 - e^{-\frac{t_1}{T_+}} \right\} \]

which, by use of Equation 39, simplifies to

\[(I_2 - I_1) = \frac{I_m \left( 1 - e^{-\frac{t_{cl}}{T_+}} \right) \left( 1 - e^{-\frac{t_{op}}{T_f}} \right)}{\left( 1 - e^{-\frac{t_{cl}}{T_+} + \frac{t_{op}}{T_f}} \right)} \]

In this analysis Equation 37 relates the average load-current to the control-ratio \(t_{cl}/(t_{cl} + t_{op})\), which is linear only if \(T_+ = T_f\). Equation 40 defines the ripple-amplitude in terms of the system-time-constants and the control-variables \(t_{cl}\) and \(t_{op}\).
APPENDIX 3.

Performance of a Basic, Oscillatory Time-Ratio-Control-System with a Large Load-Inductance.

If, in the circuit of Figure 17, $L_L$ is sufficiently large that over the operating-range of load-current the inductive energy stored greatly exceeds the commutation-energy stored by $C$, then the cyclic discharges of $C$ through $CRc$ and $L_L$ will be approximately linear during the commutation process and, in fact, until $C$ has almost recharged to the supply-voltage. Now for high-frequency, low-ripple operation the current in $L_L$ immediately prior to commutation will be given approximately by $E_D \cdot fp \cdot t_{cL} / R_L$.

However, if $t_{cL}$ is comparable to $1/fp$, $C$ discharges and recharges to the supply-voltage very rapidly. Thus, for a linear discharge at the maximum possible prevailing current of $E_D / R_L$ amperes, the time for $C$ to discharge by $E_D$ volts will be $C.R_L$ seconds which, to ensure satisfactory commutation, must exceed $t_{off}$ which is usually of the order of $20 \mu s$. Furthermore, the subsequent recharge must be completed in a time $(1/fp - t_{cL})$ seconds, so that

$$\left(\frac{1}{fp} - t_{cL}\right) = \frac{2.C.E_D}{I_L} = \frac{2.C.R_L}{fp \cdot t_{cL}}.$$

Hence, $t_{cL} = \frac{1}{2} (1 - 8.fp \cdot C.R_L)^{\frac{1}{2}}$ seconds which implies that there are both minimum and maximum values of $t_{cL}$ for which the load-current is, respectively, either just sufficient to recharge $C$ during the correspondingly long periods $t_{op}$, or that the shorter durations of $t_{op}$ are just long enough to allow $C$ to recharge by a
Operational frequency in kilohertz.

Figure 45. Control-range characteristics.
correspondingly large current. Thus, if C.R.L is about 20\(\mu s\), the control-range G is given by

\[
G = \frac{t_{cL(\text{maximum})}}{t_{cL(\text{minimum})}} = \frac{1 + (1 - 1.6 \times 10^{-4} \rho_p)^{1/3}}{(1 - (1 - 1.6 \times 10^{-4} \rho_p)^{1/3})}
\]

which is represented in Figure 45. The maximum possible operational frequency is observed, for this case, to be 6.25 kHz. It is important to note that the minimum value of \(t_{cL}\) is also governed by the necessity for C.R.L to conduct for periods greater than \(\tau(L.C.)^{1/3}\) in order to ensure completion of the oscillatory recharges of C in preparation for each commutating-sequence. Consequently, in practice, an additional narrowing of the control-range will be manifest for high-frequency operation.
Curve a; $T_r = T_f$.  
Curve b; $T_r = 2T_f$.  
Curve c; $T_r = 4T_f$.  
Curve d; $T_r = 8T_f$.  

Figure 46. Control characteristics.
Figure 47. Ripple-amplitude characteristics.

Control-ratio $t_{cL}/(t_{cL} + t_{op})$

Curve a: $T_r = 8T_f$
Curve b: $T_r = 4T_f$
Curve c: $T_r = 2T_f$
Curve d: $T_r = T_f$
Curve e: $T_r = 4T_f$
APPENDIX 4.

The De-linearisation of the Time-Ratio-Control Transfer-Characteristic.

In Equation 37 a degree of non-linearity between the output-variable and the control-ratio is introduced by the term

\[
(T_+ - T_0) \left\{ \frac{-t_{op}}{T_0} \right\} \left\{ \frac{-t_{ca}/T_0}{1 - e^{\frac{t_{ca}}{T_0}}} \right\} \left\{ \frac{1}{1 - e^{\frac{t_{ca} + t_{op}}{T_0}}} \right\}
\]

which may be approximated, by the use of Maclaurin-Series, to

\[
(T_+ - T_0) \left\{ \frac{T_+ + T_0}{t_{cl} + t_{op}} \right\} ^{-1}
\]

provided the terms \(t_{cl}/T_+\) and \(t_{op}/T_0\) are sufficiently small fractions. Equation 37 then simplifies to

\[
I_{av} = \frac{I_m}{t_{cl} + t_{op}} \left( t_{cl} - (T_+ - T_0) \left( \frac{1}{t_{cl} + t_{op}} \right) \right)
\]

The relationship between the output-current, expressed a fraction of \(I_m\), and the control-ratio \(\left( \frac{t_{cl}}{t_{cl} + t_{op}} \right)\) is shown in Figure 46, for various values of \((T_+/T_0)\), for a typical case where \(T_+ = 20\cdot (t_{cl} + t_{op})/3\)

It is apparent from this diagram that if the control-ratio is limited to below about 0.4, the linearity over the operating-range is only marginally affected by the value of \(T_0\), although the actual output-current, and hence the system-gain, is considerably reduced as \(T_0\) is decreased for any given control-ratio.

Similarly, the ripple-amplitude, as defined by Equation 40, may be computed as a function of the ratios \((T_+/T_0)\) and \((t_{cl}/(t_{cl} + t_{op})\) ) to yield the characteristics illustrated in Figure 47, from which the
Ripple-amplitude is observed to be a maximum for the condition $T_+ = T_r$, when $t_{cl} = t_{op}$. As $T_+/T_r$ is increased the ripple-component increases, particularly for high values of the control-ratio.

Curve 'e' of Figure 47 shows the reduction of the ripple-amplitude when the switching-period $(t_{cl} + t_{op})$ alone is decreased by a factor of three, for the actual condition used, where $T_+ = 4T_r$. The fact that the ripple-amplitude is reduced by approximately the same factor may be verified by examination of the equation

$$ (I_2 - I_1) = I_m \left( \frac{T_+}{t_{cl}} + \frac{T_r}{t_{op}} \right)^{-1} $$

which is an approximate form of Equation 40.

It may be concluded that direct analysis by the above method yields information concerning the system-linearity and output-ripple-amplitude for any output-level. A clear advantage over the spectrum-analysis presented in Appendix 5 is that it does not rely on the principle of superposition and is, therefore, especially applicable to time-ratio control-systems that operate non-linearly. This is so because the computations are iterative, and different values of the various parameters can thus be easily accommodated under changing conditions. The results are particularly amenable to graphical representation, and, in this respect, the computations can be simplified by the use of graphical aids.
Function \( a \): \( 1 + \cos(2\pi f_m t) \) volts.

Function \( b \): Modulated pulse-train.

Figure 48. Pulse-width modulation.
5.

Spectrum-Analysis of the Time-Ratio Control-System.

Time-ratio control, when operated at a constant repetition-frequency, may be regarded and analysed as a pulse-width modulation-system. Figure 48 illustrates the discrete transformation of an analogue input-signal, in this case \( (1 + \cos 2\pi f_m t) \), into unit-amplitude pulses, the durations of which are determined individually by the appropriate instantaneous values of the signal-voltage. For the case illustrated, the trailing-edges of the pulses occur at regular intervals of \( \frac{1}{f_p} \) seconds, whilst the leading-edges are determined by the intersections of the signal-voltage with the sawtooth waveform. Hence the duration \( t_k \) of the \( k \)th pulse, the trailing-edge of which occurs at the instant \( \frac{k}{f_p} \), is proportional to the signal-amplitude at the instant \( \frac{k}{f_p} - t_k \). This results in the transcendental relationship,

\[
t_k \propto 1 + \cos \left\{ 2\pi f_m \left( \frac{k}{f_p} - t_k \right) \right\},
\]

which may be approximated, for analytical purposes, to

\[
t_k \propto 1 + \cos \left\{ 2\pi f_m \frac{k}{f_p} \right\}
\]

By inspection of Figure 48, the maximum error in \( t_k \) will occur when \( t_k = \frac{1}{2} f_p \), (that is, at the point of greatest slope of the signal-waveform). This error will therefore be small only if \( f_p \) greatly exceeds \( f_m \); a condition which is later found to be applicable to most practical systems.
The modulated pulse-train is ultimately applied to the time-ratio-control-circuit load-impedance which, thereafter, performs the process of de-modulation through its simple filtering-action.

The Fourier-series for a regular pulse-train of pulse-duration $t_p$ and unit-amplitude may be expressed as

$$f(t) \approx f_p \sum_{n=1}^{\infty} \frac{1}{n} \sin(2\pi n t_p f_p) \cos(2\pi n t f_p)$$

$$-\frac{1}{n} \sum_{n=1}^{\infty} \frac{1}{n} \sin(2\pi n t f_p) \cos(1-\cos 2\pi n t p)$$

if the instant $t = 0$ coincides with the end of a pulse. Thus if the duration $t_p$ is modulated by, say, a cosine-function such that $t_p = t_p(1+m \cos 2\pi \cdot f_m t)$, the Fourier-series becomes

$$f(t) \approx f_p \frac{1}{n} \sum_{n=1}^{\infty} \frac{1}{n} \sin[2\pi n t_p f_p(1+2\pi \cos 2\pi f_m t)] \cos(2\pi n t f_p)$$

$$\approx \frac{1}{n} \sum_{n=1}^{\infty} \frac{1}{n} \sin(2\pi n t f_p)[1-\cos(2\pi n t_p f_p(1+2\pi \cos 2\pi f_m t))]$$

Subsequent simplification by the use of trigonometrical identities and the convergent-series-representation of Bessel-Functions of the first kind then gives,

$$f(t) \approx f_p \frac{1}{n} \sum_{n=1}^{\infty} \frac{1}{n} \sin(2\pi n t f_p)$$

$$+\frac{1}{n} \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \frac{1}{n} \left(\left.\frac{\sin(2\pi n t_p f_p + \frac{1}{2} \pi \nu j n)}{n} \right|_{l=0}\right) \cos(2\pi n f_p + 2j n \nu f_m) t \right.$$
Figure 49. Attenuation-characteristics of $\frac{2\pi m f_p t_{po}}{J_{il}(2\pi m f_p t_{po})/2\pi m f_p t_{po}}$. 

- Curve a; $|\beta| = 1$
- Curve b; $|\beta| = 2$
- Curve c; $|\beta| = 3$
- Curve d; $|\beta| = 4$
- Curve e; $|\beta| = 5$
- Curve f; $|\beta| = 6$
- Curve g; $|\beta| = 7$
- Curve h; $|\beta| = 8$
- Curve i; $|\beta| = 9$
- Curve j; $|\beta| = 10$
This result reveals the existence of an infinite number of side-frequencies which are represented by all the possible values of 
\( n.f_p \pm i.f_m \). To a varying extent every side-frequency will produce distortion, but the most serious effects will be caused by those that appear within the demodulator-pass-band and give rise to so-called cross-distortion.

Now, from the last equation, the amplitude of a side-component of frequency \((f_p + i.f_m)\), for any value of \(i\) except zero, is given by

\[
\frac{1}{\pi} \left\{ J_{li1}^2 (m2nf_p.t_p) \sin^2 (2nf_p^2t_p + i\pi) + J_{li1}^2 (m2nf_p^2t_p) \cos^2 (2nf_p^2t_p + i\pi) \right\}^{\frac{1}{2}}
\]

which simplifies to

\[
\frac{1}{\pi} J_{li1} (2\pi mf_p.t_p)
\]

The amplitude of the component of frequency \(f_m\), however, is \(f_p.m.t_p\). Hence, the amplitude of any side-component relative to that of the basic modulated output may be written as

\[
\mu(f_p + i.f_m) = \frac{J_{li1} (2\pi mf_p.t_p)}{(\pi mf_p.t_p)} = 2 \frac{J_{li1} (2\pi mf_p.t_p)}{(2\pi mf_p.t_p)}
\]

the characteristics of which are represented in Figure 49.

If the pass-band of the load-impedance extends from zero to a frequency \(f_c\), then components of frequency \((f_p - i.f_m)\) will lie outside this frequency-band if \((f_p - i.f_m) > f_c\). However, since the ultimate concern is with values of \(f_m\) that approach \(f_c\), a limiting condition, \(fp \geq f_m (1 + i)\), is implied. Now, in Equation 42 the maximum possible value of \(2\pi.m.f_p.t_p\) is \(\pi\), (if \(m = 1\), and \(t_p = 1/2f_p\)) for which value the relative amplitude of \(\mu(f_p - i.f_m)\), for example, will be diminished by almost 30 dB. Hence, if this particular side-
Figure 50. Equivalent circuit.
frequency is to be excluded from the demodulator pass-band, \( f_p \) must exceed \( 6 f_m \). Nevertheless, it remains possible that for values of \( n \) greater than unity there will be other groups of side-frequencies, given by \((n \cdot f_p - i \cdot f_m)\), which may well exist within the pass-band. However, for the condition \( f_p = 6f_m \), such will be the case only if \( i \gg (6n - 1) \) and the Bessel-coefficients associated with such high values of \( i \), as given by

\[
J_{1/2}(x) = \frac{x^{1/2}}{2^{1/2} \cdot 1!} \left( 1 - \frac{x^2}{2(2i + 2)} + \frac{x^4}{2 \cdot 4 \cdot (2i + 2) (2i + 4)} \right)
\]

demonstrate the insignificance of these components. For a practical case, therefore, where \( i = 5 \) and \( f_c \) is sufficiently high to pass, say, the fifth harmonic of a fundamental modulating-frequency of 233 Hz, the switching-frequency \( f_p \) must be \((i + 1) \cdot 5 \times 233\), (that is, 6.99 kHz) if the side-frequency of 1,165 Hz, corresponding to the highest fifth harmonic frequency, is to be attenuated by 30 dB. Since this computation applies only for \( f_m \) at its maximum value, the distortion will decrease rapidly as the fundamental modulating-frequency is lowered, provided \( f_p \) remains constant.

The modulated pulse-train, which may be considered as originating from the simple equivalent circuit 'a' of Figure 50, will produce very non-linear operation if applied to the load-circuit 'b' alone, since the current-pulses cannot decay exponentially when the switch is opened. However, the inclusion of the diode \( D_F \) enables the load-current to behave exactly as it would have had the switch been permanently closed, without \( D_F \), and the pulse-train
generated from a perfect source of zero internal-impedance. The free-wheeling diode thus linearises the operation so far as the load-current is concerned, and the applicability of the superposition-principle permits the response to each side-frequency to be considered individually.

Several important facts emerge from the foregoing analysis. Firstly, the pseudo-static approach is only valid if \( f_p \) is very much greater than \( f_m \), so that, during successive periods of \( (1/f_p) \) seconds, the modulating signal appears relatively constant. If this were not the case the pulse-duration would not be proportional to the instantaneous signal-amplitude. Secondly, \( f_p \) will not, in general, be an exact multiple of \( f_m \), particularly as the latter varies over a wide range. Thus the modulated pulse-train will never be exactly repeated during consecutive modulating-periods, and the Fourier-series-representation is not strictly applicable. Nevertheless, the actual discrepancies are negligible provided \( f_p \) is relatively high. Thirdly, for those instances when \( f_p \) and \( f_m \) are commensurate, the frequency-spectrum will be degenerated since certain groups of side-frequencies will become coincident. For example, the component of frequency \((f_p - 8f_m)\) will coincide with those represented by \((2f_p - 18f_m), (3f_p - 28f_m)\) and so on, when \( f_p = 10f_m \), and each of these components will occur at the frequency \( 2f_m \). Finally, it is apparent from Equation 41 that components of frequency \( i \cdot f_m \) are normally entirely absent from the modulation process, and it may be concluded, therefore, that if the demodulation is linear no harmonic distortion, as such, will be exhibited in the final
output-waveform. This is so because, generally, the pulse-train explores the entire modulating waveform over several of its cycles, and the averaging effect eliminates any harmonic distortion. However, when \( f_m \) is an integral fraction of \( f_p \), the output-pulse-train becomes repetitive, and the coincidence of certain side-frequencies may easily be interpreted as harmonic distortion.

For the condition where the demodulator-time-constants \( T_r \) and \( T_f \) are unequal, the linearity of the demodulation-process is impaired and harmonic distortion appears in the load-current-waveform. However, if the overall system-linearity is restored by the method suggested in Appendix 7 this distortion is removed, although harmonics will be generated in the modulated pulse-train due to deliberate de-linearisation of the sawtooth modulating-waveform. The existence of these two internal, complementary non-linearities, however, render the system generally non-analytical and the repetitive, computational technique of Appendices 2 and 4 must be applied.

Lastly, although the overall linearity is restorable, a secondary effect of the demodulator-non-linearity is the appreciable broadening of the demodulator-pass-band, particularly if a large system-forcing-factor is utilised. As a result, certain side-frequencies that were originally rejected from the pass-band will now tend to increase the output-distortion and must, therefore, be re-excluded by an appropriate increase of \( f_p \). Any distortion-components thereafter appearing within the pass-band will be associated with higher values of \( i_n \), and will thus possess correspondingly lower amplitudes.
Harmonic Structure of the Engine-Torque-Waveform.

The indicated-torque produced by a pair of oppositely phased cylinders of a typical diesel engine is rectilinearly represented by Figure 34 which forms the basis of the harmonic analysis for various values of \( n_0 \). Because the function is asymmetrical, the Fourier Series will contain the coefficients \( a_0 \), \( a_q \) and \( b_q \) where

\[
\begin{align*}
t_{\text{tot}} &= a_0 + \sum_{q=1}^{\infty} a_q \cos (q \cdot \alpha^\circ) + \sum_{q=1}^{\infty} b_q \sin (q \cdot \alpha^\circ) \\
\text{For the particular waveform considered it was shown that} \quad a_0 &= -202 \text{ lbf ft}, \\
a_q \cdot \tau \cdot q^2 &= -350 + 2450 \cos (90^\circ \cdot q) + 7120 \cos (105^\circ \cdot q) - 9854 \cos (114^\circ \cdot q) + 634 \cos (245^\circ \cdot q) \text{ pounds-force feet,} \\
\text{and} \quad b_q \cdot \tau \cdot q^2 &= 2450 \sin (90^\circ \cdot q) + 7120 \sin (105^\circ \cdot q) - 9854 \sin (114^\circ \cdot q) + 634 \sin (245^\circ \cdot q) \text{ pounds-force feet.}
\end{align*}
\]

The terms \( a_q \cos (q \cdot \alpha^\circ) + b_q \sin (q \cdot \alpha^\circ) \) may then be combined to form single sinusoidal functions of amplitude \( (a_q^2 + b_q^2)^{1/2} \) which therefore represent the appropriate harmonic components.

If \( n_0/2 \) separate, identical functions, as shown in Figure 34, are mutually displaced by \( 720/n_0 \) degrees and added algebraically, the resultant may be represented by the related Fourier series.
<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>Harmonic coefficients for $n_c = 2$</th>
<th>Harmonic coefficients for $n_c = 4$</th>
<th>Harmonic coefficients for $n_c = 6$</th>
<th>Harmonic coefficients for $n_c = 8$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$a_q$</td>
<td>$b_q$</td>
<td>$(a_q^2 + b_q^2)^{1/2}$</td>
<td>$a_q$</td>
</tr>
<tr>
<td>0</td>
<td>-202</td>
<td></td>
<td></td>
<td>-404</td>
</tr>
<tr>
<td>1</td>
<td>+4.62</td>
<td>-31</td>
<td>468</td>
<td>-4.42</td>
</tr>
<tr>
<td>2</td>
<td>-221</td>
<td>+3.0</td>
<td>407</td>
<td>+262</td>
</tr>
<tr>
<td>3</td>
<td>+1.20</td>
<td>-151</td>
<td>193$^u$</td>
<td>-1.80</td>
</tr>
<tr>
<td>4</td>
<td>+1.31</td>
<td>-35</td>
<td>156$^v$</td>
<td>+76</td>
</tr>
<tr>
<td>5</td>
<td>+10</td>
<td>+122</td>
<td>122</td>
<td>-8</td>
</tr>
<tr>
<td>6</td>
<td>-90</td>
<td>-9</td>
<td>91$^w$</td>
<td>-34</td>
</tr>
<tr>
<td>7</td>
<td>+29</td>
<td>-71</td>
<td>77</td>
<td>+38</td>
</tr>
<tr>
<td>8</td>
<td>+38</td>
<td>+42</td>
<td>57$^x$</td>
<td>-24</td>
</tr>
<tr>
<td>9</td>
<td>-4.2</td>
<td>+23</td>
<td>48$^y$</td>
<td>-1</td>
</tr>
<tr>
<td>10</td>
<td>-4</td>
<td>-40</td>
<td>4.0</td>
<td>-16</td>
</tr>
<tr>
<td>11</td>
<td>+28</td>
<td>+9</td>
<td>28</td>
<td>+14</td>
</tr>
<tr>
<td>12</td>
<td>-17</td>
<td>+22</td>
<td>28$^z$</td>
<td>-19</td>
</tr>
</tbody>
</table>

Coefficients are in pound-force feet. $u,v,w,x,y$ and $z$ indicate the correlation between coefficients.
However, this waveform repeats at \( \frac{n_c}{2} \) times the frequency of the original function \( m_{\text{EC}_2} \). Thus, for \( n_c = 8 \), the amplitude coefficients of the fundamental components are \( 4 \cdot a_4 \) and \( 4 \cdot b_4 \). Similarly, the second-harmonic coefficients are \( 4 \cdot a_8 \) and \( 4 \cdot b_8 \). The computed numerical values of \( a_q \), \( b_q \) and \( (a_q^2 + b_q^2)^{\frac{1}{2}} \), for the basic two-cylinder waveform, are given in Table 4, included in which are the derived harmonic coefficients for several other values of \( n_c \). From this table it is apparent, for example, that the fifth harmonic of an eight-cylinder engine, when operating at 3,500 rev/min, has an amplitude of only 58.100/1,450 \% of the highest peak-torque, whereas the same frequency-component of a comparable four-cylinder engine has only half this relative value. Further inspection of Table 4 will disclose that, as \( n_c \) increases, the harmonic distortion is relatively diminished, and the final output-function approaches a purely sinusoidal waveform.

Because, generally, the cyclic variations of the basic waveform are not as abrupt as depicted in Figure 34, the higher harmonic components are not so large as the computations indicate. Furthermore, in practice, the waveform will be modified, especially at high speeds,
by the reciprocating-torque-function which is generated principally at the same frequency as the total gas-pressure-torque; although this is not so for a two-cylinder engine. This introduces no particular difficulties since the simulator must, as a primary requirement, accommodate such an input-frequency from the gas-pressure-torque-fluctuations.
Figure 51. Basic relaxation-oscillator.

Figure 52. Modified relaxation-oscillator.
Figure 53. Compensated relaxation-oscillator.
APPENDIX 7.

The Compensation of Non-Linear Demodulation.

The magnitude of the control-ratio $\frac{t_{CL}}{t_f}$ will be proportional to the input-signal-level only if the sawtooth-generator, mentioned in Section 4.7.1 and Appendix 5 for modulation of the pulse-train, exhibits a linear voltage-ramp. Such a generator was based on the relaxation-circuit shown in Figure 51. As $C$ charges through $R_1$ the emitter-junction of the unijunction transistor becomes forward biased when the peak-point-emitter-voltage is exceeded. This allows $C$ to discharge almost completely through $R_{B1}$. The emitter-junction then reverts to the non-conducting-state and $C$ re-charges through $R_1$, the cycle being thereafter repeated. Hence, both the increase and decrease of $v_{out}$ are approximately exponential. The charging-characteristic, however, may be greatly linearised if $R_1$ is replaced by the constant-current generator shown in Figure 52. Complete linearisation can then be obtained by the application of positive feedback to the base of TR1, thereby dynamically modifying its collector-current. This is most readily achieved by the addition of a phase-inverter stage as shown in Figure 53. If the coupling capacitor $C_0$ is very large, compared to $C$, the additional base-current extracted from TR1 increases as $v_{out}$ and $v_1$ increase, and the undesirable degeneration of the charging-rate of $C$ is remedied by the corresponding rise of collector-current. By
this technique, with suitable adjustment of VR₁, it is possible to
considerably over-compensate the otherwise non-linear rise of \( V_{\text{out}} \),
and this affords compensation of the non-linear transfer-characteristic
of a degraded demodulator of the type discussed in Section 4.5 and
later analysed in Appendix 2. The compensation-characteristic
actually required may be easily determined, for an individual case,
by reference to Figure 46 and Equation 37. In practice it was not
possible, for severe cases, to restore exact linearity over the entire
operating-range, but, under closed-loop conditions, the remaining
deviations were significantly reduced and an adequate performance was
realised.
APPENDIX 8.

Design Considerations for High-Performance Time-Ratio-Control.

This appendix presents the essential aspects of the design procedure adopted for the time-ratio control-system illustrated in Figure 19 and described in Section 4.2.2. A similar, though simplified, technique was applied to the synthesis of the field-excitation-control-system for the auxiliary machine, as mentioned in Section 5.3.

The most important function of such a time-ratio control-circuit is the periodic commutation of the load-thyristor, for which the commutating-capacitor must be sufficiently large to maintain, during each of its discharge-cycles, the reverse-bias of the device for at least the characteristic turn-off-time $t_{\text{off}}$. The main discharge-current may be considered to flow via the inductance $L_2$—particularly if $R_O$ is relatively large. The value of $L_2$ will be a compromise, however, since it must, to achieve rapid system-response, contribute negligible inductance to the load-current-path, and yet be sufficiently large to limit the magnitude of the oscillatory discharge-current from the capacitor in order to reverse-bias the load thyristor for an adequate period.

The peak value of the oscillatory discharge-current of the capacitor from its reverse-charge-condition is given by
whilst the condition for successful commutation may be expressed as

\[ t_{\text{off}} \leq \frac{\pi}{2} \left( \frac{L_2}{C} \right)^{\frac{1}{3}} \]

Combination with Expression 43 then gives

\[ L_2 \geq \frac{4 \cdot E_p \cdot t_{\text{off}}}{\pi \cdot I_{pk}} \]

The values of \( I_{pk} \) and \( t_{\text{off}} \) will be prescribed by the load-thyristor characteristics and capabilities, and so the value of \( L_2 \) will depend mainly on the supply-voltage used. Ultimately, the value of \( C \) may be computed from Equation 44. Consideration must, however, be given to the fact that \( L_2 \) will conduct load-current continuously, and, since during the oscillatory-discharge-period it must not saturate, it should preferably be air-cored. This load-current-conduction-function modifies, of course, the initial current extracted from the capacitor at the instant \( CR_c \) is triggered, but, as this is generally insignificant, the calculations need not be complicated needlessly. In this respect the capacitor may be considered to supply, at the beginning of the turn-off period of \( CR_L \), a current equal to the load-current about to be commutated.

The oscillatory action of \( L_1 \) and \( C \) may now be formulated. If reversal of the initial charge on the capacitor is to be completed within, say, \( 40 \mu s \) (a somewhat arbitrary period),
then \[40 \times 10^{-6} = \pi (L_1 C)^{1/2}\], and, if \(t_{\text{off}}\) is assumed to be \(20 \mu s\), \(L_1\) will be similar to \(L_2\).

The current-ratings of thyristors \(CR_0\) and \(CR_c\), diode \(D_1\), \(L_1\) and \(C\) must, of course, be compatible with the oscillatory currents incurred, and \(D_1\), moreover, should exhibit a fast-recovery characteristic so as to prevent excessive loss of charge during the reverse-recovery-period of this curtailed oscillatory operation. In contrast, however, \(D_2\) and \(CR_L\) conduct load-current only during the exponential rises, whilst \(D_4\) and \(D_5\) carry only decaying pulses of current. The exact duties of the components that conduct oscillatory currents cannot be finalised, however, until the limitations and the timing of the pre-commutation cycle have been decided.

Due to the extreme susceptibility of thyristors to transient over-voltages and-currents, it was vital that an adequate system of protection should be integrated with the time-ratio control-circuit from the outset. In the experimental system, individual over-voltage protection was afforded to each device through the use of parallel-connected resistor-capacitor networks designed to absorb the transient energy. Because the protection capacity required was not accurately assessable, considerable overall protection of the system was provided by the addition of a large capacitor across the source \(E_P\). In practice this was also necessary to facilitate the extraction of large pulses of current from the source. The design of these semiconductor-protection-circuits was based on the empirical formulae employed by Gutzwiller et al\(^{37}\).
Separate protection of the free-wheeling-thyristor, later employed as shown in Figure 22, was provided by shunt-connected, non-linear transient-voltage-suppressors which exhibited a rapidly-decreasing resistance-characteristic when operated above their normal working-voltage. As such, interference of these suppressors with the normal forced-commutation process was eliminated, whilst adequate protection was afforded against faulty operation such as the initiation of commutation during high load-current-conduction. Clearly, in practice, it will be necessary to install very high dissipation suppressors or to ensure, electronically, that the fault conditions cannot arise.

It is an established routine in many thyristor applications to procure over-current protection through the use of special, fast-acting fuses. At normal power-frequencies, and provided sufficient series-inductance is present in the system to limit the rate of rise of prospective fault-currents, such precautions are usually sufficient. For high-frequency time-ratio-control, however, where the system-response-time is of paramount importance, the load-circuit inductance is by necessity very small, and generally fault-currents will not be cleared by the fuses in time to prevent destruction of the semiconductor devices. It was considered expedient, therefore, to employ a fast-response load-current-detection-circuit which would, when necessary, instigate immediate commutation of the load-current before the same could attain dangerous proportions or increase beyond the commutating-ability of the system. This simple function was readily arranged to provide either temporary or permanent termination of the system-operation.
For faultless operation of thyristors in high-frequency and high-current switching-circuits, it is essential to suppress any rapid rise of forward-bias voltage to a non-conducting thyristor. In fact, with present-day devices a sudden voltage-rise alone may be sufficient to cause unwanted conduction, particularly if the rate exceeds about 20,000 volts per second. Voltage transients of this order may inadvertently be induced by the switching-action of other circuit-components even where low supply-voltages are used. Since these transients can be largely reduced by the addition of shunt-capacitance to each circuit-switching-element, special consideration given to the design of the protection networks, previously discussed, will obviate this difficulty. Alternatively, multigate thyristors may be employed, these being more tolerant of high rates of voltage-rise owing to the lower junction-temperatures attained under comparable operating-conditions.

An important aspect of system-design is the estimation of various losses and the overall efficiency. It will be realised, however, that in a circuit of the type illustrated in Figure 19, even if all the switching components were ideal and generated no heat-loss, the efficiency of the system fundamentally depends upon the cyclic loss incurred by the periodic replenishment of the capacitor-charge from the source $E_s$. For an undamped, oscillatory discharge of $C$, when $C_{R_o}$ conducts, the peak discharge-current is given by

$I_{pk} = 2 E_s (C/L_2)^{1/2}$ provided all other discharge-paths may be discounted as far as additional charge-extraction is concerned. Thus, whilst the anode-potential of $C_{R_o}$ decreases co-sinusoidally
Figure 54. Ideal efficiency as a function of control-ratio.
to earth-potential the entire discharge-current must emanate from the source \( E_D \). The diode \( D_4 \), however, prevents further decline of this anode-voltage, and the average current for this quarter-cycle of oscillation becomes, therefore, \( 4 \cdot E_D \cdot (C/\pi L_2)^{1/3} \) amperes. Hence, the mean power supplied by the source is \( 4 \cdot E_D^2 \cdot (C/\pi L_2)^{1/3} \) watts, and the energy expended during each commutation-cycle is thus \( 2 \cdot E_D^2 \cdot C \) joules. Now the useful energy dissipated in \( R_L \) during each switching-period is given by \( (E_D \cdot t_{cl}^2) \cdot f_p/R_L \) joules, and the ideal efficiency of the system may therefore be written as

\[
\eta_I = \frac{\left(\frac{E_D^2 \cdot t_{cl}^2 \cdot f_p}{R_L}\right)}{\left(\frac{E_D^2 \cdot t_{cl}^2 \cdot f_p}{R_L} + 2 E_D^2 \cdot C\right)} \text{ per unit,}
\]

which simplifies to

\[
\eta_I = \frac{1}{1 + \left(\frac{2 \cdot C \cdot R_L \cdot f_p}{t_{cl}^2 \cdot f_p^2}\right)} \text{ per unit.}
\]

Figure 54 shows this ideal efficiency as a function of the control-ratio \( t_{cl} \cdot f_p \) for the condition where \( C = 20 \mu F \); \( R_L = 1.5 \Omega \); and \( f_p = 4 \text{ kHz} \).

In the foregoing analysis any consideration of the almost instantaneous reverse-recovery-currents, sustained by \( CR_L \) and \( CR_0 \), was precluded, and it was also assumed that the initial charge on \( C \) was completely reversed, by the operation of \( CR_0 \), without loss. Consequently, the capacitance of \( C \) must, in practice, be slightly
larger than the value deduced by Equations 43 and 44, and the ideal
efficiency can never be entirely realised. Other losses incurred
are mainly due to the finite, non-linear, forward-resistances of
the semiconductor devices and the finite switching-times of the
thyristors, during which relatively high currents may occur
simultaneously with high switching-voltages. The former losses
may be calculated from the forward-characteristics of each device,
whilst the latter, which naturally rise proportionally with switching-
frequency, may be estimated by use of the empirical formulae deduced
by Gutzwiller et al.\textsuperscript{37}

It is possible to reduce the switching-on-loss of a thyristor by
the addition of a series-saturable-reactor which opposes the growth
of current through the device (and supports the sudden change of
forward applied-voltage) until the entire gate-junction-area has
become fully conductive. Multigate thyristors may again be used
with advantage in this respect, since their faster switch-on-time
will, in general, require lower series-reactance than conventional
elements.

A significant practical difference between the dual-machine
load of Figure 15 and its equivalent, purely passive, impedance is the
presence, in the former, of considerable commutation-noise. This was
observed to consist largely of irregular, approximately-triangular
pulses of two- or three-volts amplitude and several micro-seconds
duration, and a slight, overall increase of amplitude was detected
for increasing speed and load.
Figure 55. Commutation filter.

- $R_1: 6 \, \text{k} \Omega$
- $R_2: 1 \, \text{k} \Omega$
- $R_3: 1 \, \text{k} \Omega$
- $R_4: 1 \, \text{k} \Omega$
- $C_1: 4 \mu F$
- $C_2: 0.1 \mu F$
Suppression of these transients was necessary to avoid interference with the commutating-action of the control-circuit, and was achieved ultimately by the diode-filter arrangement of Figure 55 in which $R_2$ permits $C_1$ to discharge between successive transients, of either polarity, whilst $R_3$, $R_4$ and $C_2$ provide suppression of higher-frequency components.

Whilst, at present, thyristors are available with current-ratings of up to 350 amperes it should be appreciated that short turn-off-times are not exhibited by such devices, and that the peak oscillatory currents sustained by the thyristors $CR_0$ and $CR_c$ of Figure 19 may need to be reduced, by appropriate design modifications, due to the restricted overload-capacity of these larger devices. Thus a strict limitation is imposed upon the maximum permissible frequency of operation of the time-ratio control-system, the effect of which is to limit, in turn, the minimum obtainable distortion of the load-current-waveform. The optimum conditions cannot be exactly estimated without some knowledge of the ratings of the simulating and auxiliary machines, ultimately to be employed, and an assessment of the total armature-circuit-time-constant. It is clear, therefore, that a compromise must be made between the machine-rating, and hence its applicability to high-frequency operation, and the frequency limitation imposed by the thyristor-parameters themselves. Parallel operation of thyristors would alleviate the problem, but because, in general, current-sharing reactors or even 'swamp resistors' are necessary to compensate for dissimilarities between nominally identical devices, the technique must be applied with discretion. Simple
arrangements are possible provided the devices are properly de-rated, but neither optimum efficiency nor economy will be obtainable. The use of current-sharing reactors, nevertheless, should be based on an assessment of their high-frequency performance and the deterioration of the overall system-performance caused by their contribution of inductance to the armature-circuit.

Series-connection of thyristors need not be considered for this application, since devices are available whose voltage-ratings exceed the normal terminal-voltages employed for d.c. machines. In any case, the time-ratio control-system needs to be rated only for a voltage given by

\[ K_f \text{ (Armature circuit voltage drop at full load current),} \]

which, except for inordinately large forcing-factors, will be lower than the individual machine-voltages.

As the system-capacity is increased the importance of isolation and the screening of all gate-signals and electronic equipment will increase, owing to the higher rates-of-change of current that will be produced by the thyristor-switching-operations.
P; Input-pulses from multivibrator.
Q; Output terminal.

Figure 56. Transistor-diode pump-circuit.
APPENDIX 9.

Properties of the Transistor-Diode Pump-Circuit.

Figure 56 shows the fundamental pump-circuit which is supplied with regular pulses from an astable multivibrator. When the base-potential of TR\(_1\) is decreased to zero, TR\(_1\) cuts off so that its collector-potential rises exponentially to a voltage given by \(V_{cc} \cdot \frac{R_2}{R_1 + R_2}\), whilst current flows into \(C_1\) and \(C_2\) via D. The total charge delivered is therefore

\[
\frac{C_1 \cdot C_2}{C_1 + C_2} \cdot \left( \frac{V_{cc} \cdot R_2}{R_1 + R_2} \right)
\]

if the forward-resistance of D is negligibly small. Consequently, the final voltage-increment \(\delta V\) imparted to \(C_2\) is given by

\[
\delta V = \left[ \frac{C_1}{C_1 + C_2} \right] \cdot \left[ \frac{V_{cc} \cdot R_2}{R_1 + R_2} \right]
\]

When TR\(_1\) subsequently becomes conducting, the diode will be reverse-biased and will cause TR\(_2\) to conduct. \(C_1\) will then discharge and subsequently re-charge, with the opposite polarity, to a voltage equivalent to that across \(C_2\). This condition will persist until TR\(_1\) is again cut-off, when a similar pulse of \(V_{cc} \cdot \frac{R_2}{R_1 + R_2}\) volts transfers another, similar voltage-increment to \(C_2\). Successive increments are therefore identical and independent of the actual voltage across \(C_2\).

Provided D and TR\(_2\) are low-leakage, silicon devices, and \(C_2\) is not loaded, the voltage-rise of \(C_2\) will be an extremely linear function.
of the number of input-pulses applied to TR1. If a unijunction transistor is then coupled to the output-stage, as shown, the periodic, rapid discharge of C2, that ensues whenever the peak-point-emitter-voltage is exceeded, renders the circuit-operation repetitive. Because the unijunction transistor may be easily temperature-stabilised by discreet selection of R4 and R5, the operation is, moreover, very reliable.

For application to the generation of an engine-torque-waveform, C2 must be extremely large in order that a large number of pulses will be necessary for the construction of each section of the waveform, which must, of course, be reasonably smooth. In addition, the use of low-leakage components is essential to ensure uniform operation over a very wide frequency-range, and in this respect any loading of C2 must be avoided by the use of either cascaded emitter-follower or, preferably, source-follower stages; the latter utilising field-effect-transistors. For the frequency-range investigated, ample isolation was achieved from complementary emitter-follower arrangements.

For successful operation over a wide frequency-range it is imperative that each input-pulse is completely assimilated by the circuit before C1 commences to re-charge through TR2 in preparation for the next pulse. C1 and C2 must therefore be chosen to comply with both the circuit-time-constant, which is given by

\[
\frac{R_1 R_2}{R_1 + R_2} \left( \frac{C_1 C_2}{C_1 + C_2} \right)
\]

seconds, and the charge-leakage-characteristics that predominate at
low-frequency operation. Although low values of $R_1$ and $R_2$ obviate this difficulty the circuit power-requirements increase, and high-power transistors become necessary.

It is profitable to note some additional merits of the transistor-diode pump-circuit, apart from its suitability for the generation of engine-torque-waveforms. By simple modifications it provided, for example, an economic method of counting large numbers of pulses, and enabled time-delays to be controlled linearly by a direct voltage. It is also possible, by simple amplitude modulation of the input-pulse-train, to multiply or divide two intermediate-frequency variables. A distinguishing feature of its operation as a function-generator was the ability to produce almost any waveform over a continuous and wide frequency-range.
Figure 57. Primary pulse generator.
APPENDIX 10.

The Linear, Voltage-Controlled Primary-Pulse-Generator.

The basic, astable multivibrator used to generate the pulses required for the construction of the engine-torque-waveform is shown in Figure 57, in which the base-resistors $R_b$ are coupled to a variable controlling voltage of $-V_T$ volts. In this circuit, when $TR_1$ conducts and saturates, the rise of potential of $B_2$ above earth will be limited by the forward-conduction of $D_2$, and $C_1$ will rapidly discharge, mainly, via $D_2$ and the emitter-collector junctions of $TR_1$. The forward-resistance of this discharge-path, being very non-linear, does not permit an accurate estimation of the initial discharge-characteristic. However, as this discharge nears completion, the additional extraction of charge caused by $-V_T$, through $R_b$, will predominate, though at a somewhat lower rate. If, in comparison to this effect, the initial discharge-period is very short, then $-V_T$ will become the main controlling-influence during the astable-periods.

Now $TR_2$ will conduct when its base-potential eventually decreases sufficiently to forward-bias its emitter-base-junction, and the time required for this to occur, if the initial discharge of $C_1$ is ignored, is proportional to $\ln \left(\frac{V_T}{V_T - V_{BE}}\right)$, which, by Maclaurin-series, may be approximated to

$$\frac{V_{BE}}{V_T - V_{BE}} - \frac{1}{2} \left(\frac{V_{BE}}{V_T - V_{BE}}\right)^2 + \frac{1}{3} \left(\frac{V_{BE}}{V_T - V_{BE}}\right)^3 \cdot 45$$
Figure 58. Theoretical linearity-characteristic.

Curve a; $V_{BE} = 0.3V$.  
Curve b; $V_{BE} = 0.7V$.  
Curve $s_1$; Linear asymptote.  
Curve $s_2$; Linear asymptote.
Figure 59. Practical control-characteristic. (typical)

- **Curve a**: Observed characteristic.
- **Curve S₁**: Linear asymptote.
Thus, over a range for which the controlling voltage $V_T$ greatly exceeds $V_{BE}$, the repetition-period is proportional to the ratio $V_{BE}/V_T$, which indicates a linear relationship between $-V_T$ and the operating-frequency. Figure 58 illustrates the reciprocal value of Expression 45, in relation to $V_T$, when $V_{BE}$ is ascribed the values of 0.3 and 0.7 volts for germanium and silicon devices, respectively. Figure 59 shows the observed frequency/control-voltage relationship for a practical circuit, designed for germanium transistors, and indicates a slightly inferior performance than that predicted by Figure 58. Thus, provided the low-frequency discrepancy can be disregarded, the circuit-performance is adequate over an appreciable range of operation.

The inferior linearity is mainly due to the high-frequency operation of the circuit, for which the coupling-capacitors cannot completely recharge, repetitively, to the supply-voltage. This is because the base-emitter reverse-voltages are limited by the forward-characteristics of the diodes, and, although this is quite in order, the initial capacitor-discharges will occupy a somewhat shorter period. This effect is slightly offset, however, by the non-linear increase of the discharge-path-resistance. Nevertheless, in practice, the high frequencies obtained are slightly greater than the expected values, and this tends to counteract the low-frequency errors produced by low values of $V_T$.

A minor disadvantage that arises from the incomplete capacitor-recharge at high frequencies is that a suitably biased
pulse-amplifier is necessary to retrieve the constant-amplitude pulses ultimately required. The maximum obtainable frequency is reached, however, when the increasing inability of the capacitors to completely recharge finally reduces the amplitude of the collector-waveforms to zero. This is, of course, the penalty for a high degree of linearity.

A second deficiency is that the temperature-stability of the system depends largely on the temperature-variations of the base-emitter junctions. Because $V_{BE}$ decreases as temperature rises, the frequency will correspondingly increase. For the experimental generator, the operating-range was limited by the maximum output of 30 volts from the d.c. tacho-generator when driven from the simulating machine.
Figure 60. Determination of attenuation-characteristics.
APPENDIX II.

The Unijunction Transistor as a Voltage-Controlled Attenuator.

In the unijunction transistor of Figure 60, B₁ and B₂ constitute a bar of n-type silicon upon which has been formed an emitter, or p-n junction, at some point along its length. The resistances of the two base-regions B₁ and B₂, within the device, thus form a potential-divider with the external resistor Rₑₑ₂. In the absence of injected carriers from the emitter, the hole-concentration of the junction is low, but, if the emitter-base-one junction is sufficiently forward-biased, the conductivity between B₁ and B₂ is increased by the rise of the hole-concentration and the accompanying rise of electron-concentration that occurs to maintain overall charge-neutrality. Thus, Rₑₑ₁ varies as a function of the emitter-base-one current. If, simultaneously, Vₑₑ₅ is such that it prevents holes from entering the emitter-base-two region then Rₑₑ₂, also, will remain unchanged.

In the familiar, oscillatory applications of the unijunction transistor, the negative-slope-characteristic of Rₑₑ₁ motivates a self-sustaining increase of the forward emitter-current and provides the periodic, unstable relaxation-phases. The addition of a suitable, external resistance Rₑ, however, tends to cancel this unstable resistance-characteristic and establish a predictable relationship between the emitter-current and the controlling voltage Vₑₑ₅.
Curve a: $V_{ES} = +20$ volts.
Curve b: $V_{ES} = +15$ volts.
Curve c: $V_{ES} = +10$ volts.
Curve d: $V_{ES} = +5$ volts.
Curve e: $V_{ES} = +2$ volts.
Curve f: $V_{ES} = 0$ volts.

Figure 61. Attenuation characteristics.
Figure 62. Attenuation-factor as a function of $V_{ES}$.
In a preliminary investigation $V_{es}$ was varied, and $V_{BB}$ was measured, for various values of $V_{BS}$ (of either polarity) for a type 2N1671 A unijunction transistor. Figure 61 shows the characteristics obtained, for an ambient temperature of $25^\circ C$, when both $R_{B2}$ and $R_e$ were $10\,k\Omega$; the important result being that the relationships between $V_{BB}$ and $V_{BS}$ were linear. Figure 62, which was derived from these results, illustrates the constant gradients $\frac{\delta V_{BB}}{\delta V_{BS}}$ thereby obtained for each value of $V_{es}$, and so anticipates, as a function of $V_{es}$, the attenuation-factor of an a.c. signal applied in place of $V_{BS}$. For the particular values of $R_{B2}$ and $R_e$ considered, it will be apparent that if a pseudo-origin is established at the point $P$ on Figure 62, then the relationship

$$\frac{\delta V_{BB}}{\delta V_{BS}} \propto (7.5 - V_{es})^2$$

pertains. Thus, for the attenuation of a signal that represents engine-reciprocating-torque, the voltage ordinate $(7.5 - V_{es})$ must be made proportional to the simulator-speed, or the tacho-generator-voltage $V_T$, this being readily achieved through suitable biasing-arrangements. However, when the control-function $(7.5 - V_{es})$ is reduced to zero, the corresponding output-signal will have a magnitude given by $R-P$, so that, in order to reduce the output-amplitude to zero, it is necessary to summate the output to an inverted waveform of equivalent error-magnitude $R-P$.

A particularly undesirable characteristic of the attenuator is that although virtually no harmonic distortion is generated, the
mean level of the output-signal rises as $V_{ES}$ is increased. This is apparent from the spread of intercepts on the $V_{BB}$ axis of Figure 61. This effect was cancelled by summation of the waveform of reciprocating-torque to that of the gas-pressure-torque through a capacitive coupling. For this application this proved to be quite satisfactory since the amplitude of the attenuated, sinusoidal function was negligible at low frequencies.

The linear characteristics of Figure 61 obtain only if the current $I_{B2}$ is made small with respect to $I_E$. As such, the resistance $R_{EB1}$ remains a function, mainly, of $I_E$ and is independent of $I_{B2}$. In practice, the potential of the emitter must always exceed the peak-point-emitter-voltage that corresponds to the 'intrinsic stand off ratio' of the doped, silicon bar. Thus, for satisfactory operation with very low values of $V_{ES}$, the amplitude of $V_{BS}$ must be limited to two or three volts. The operation is, however, modified slightly by the fact that when $V_{BS}$ is negative, $V_{ES}$ injects holes into the base-two region, as well as base-one, and $R_{EB2}$ is reduced. The current through $R_{EB1}$, however, will be decreased, due to the counter-flow of holes moving towards $V_{BS}$, and $R_{EB1}$ will increase, offsetting the decrease of $R_{EB2}$. Consequently, the overall attenuation-characteristic obtained will depend upon the $V_{ES}/I_E$ characteristic of the combined junction-resistance, $R_{EB1}$, $R_{EB2}$ and $R_E$.

The square-law characteristic observed will apply only to a particular combination of circuit-constants. It was verified experi-
mentally, however, that a very wide variety of relationships could be obtained by appropriate circuit-design and the positioning of the pseudo-origin.
APPENDIX 12.

The Estimation of Simulator Performance.

The purpose of this Appendix is to realise a practicable simulating system and to assess the limits of performance that could be expected from the simulation of various engine-characteristics. Due to the extremely complex inter-dependencies between the numerous operational modes and parameters of such a system, a numerical analysis will be undertaken, for which purpose simulations will be presumed to repeat at ten-second intervals.

For the reasons discussed in Chapter 7, much is to be gained by the use, when possible, of electrical inertia-simulation and the computations will therefore be directed to this end. In fact, only by electrical simulation of inertia can a possible reduction of the required peak simulator torques be exploited, (these being quite independent of the choice of the duty-cycle interval). Thus, such a technique can afford a corresponding decrease of both the control-system capacity and the machine ratings.

A smaller machine, however, would exhibit a correspondingly lower inertia and thus more inertia could be simulated. Hence, the immediate problem is to establish the condition for which the design procedure becomes stable.
An equally important aspect of the entire system is the rate at which the armature current could be repetitively reversed, and before any system-parameters can be determined this limitation, which is particularly unpredictable without the knowledge of such parameters, must, at this stage, be chosen rather arbitrarily. However, it will depend mainly on the overall armature-circuit time-constant of the simulator, the applied forcing factors, and the magnitude of the currents to be reversed. The time-constant and the forcing factors will themselves be inter-related and will, in turn, be dictated by the machine-ratings. Hence, the frequency-limit will depend ultimately on the degree of inertia-simulation.

Experimental evidence and rudimentary calculations suggest that a reversal rate of 233 Hz, for an eight-cylinder engine at 3,500 rev/min, is impracticable and it is considered more expedient, therefore, to attempt a true simulation of the four-cylinder Albion EN 335 engine, which, at 3,500 rev/min, will require a reversal rate of only half this frequency. The simulator performance for the eight-cylinder engine may then be examined against this more realistic restriction.

In order to elucidate the analysis each simulation will be considered to comprise only that period during which the engine, having started, would accelerate the starter-motor, under full torque and governor-control, to the appropriate maximum simulator speed on the assumption that the simulator loading requirement, prior to firing, would be relatively small (for an exact analysis the inclusion of other conditions would be a straight-forward matter). On this basis the most severe duties will prevail when the internal friction of the
Curve a; Firing-torque for the Albion four-cylinder engine.

Curve b; Firing-torque for the 10-litre eight-cylinder engine.

Figure 63. Variation of total gas-pressure torque with crankshaft-angle.
engine is high; since full torque would then be applied for prolonged periods.

The oil-drag-friction torque of the Albion engine, when using an SAE-30 oil at 30°F, will be taken as 110 lbf ft, whilst analysis of Figure 63, curve 'a', which illustrates the firing-torque waveform of this engine, indicates an average internal torque of 280 lbf ft. For these values and a total estimated system inertia of 30 lb ft², 2.03 seconds will therefore be required to accelerate the engine to full speed. This, however, is an underestimate of the actual time required since viscous friction, in general, increases as speed rises, in respect of which an acceleration period of four seconds would be more authentic.

It is now possible to derive a relationship, similar to Equation 25, by which the r.m.s. value of the simulator-torque, during the four-second period, may be expressed as a function of the corresponding value of engine torque, principally in terms of $K_s$.

For the waveform 'a' of Figure 63, this relationship, for large values of $\alpha$, is

$$\frac{m_{s,rms}}{(m_E + m_{FE})_{rms}} = 1 - 2.12 K_s + 1.12 K_s^2$$

and is essential for the derivation, for various values of $K_s$, of the corresponding continuous ratings of the simulating machine. Such ratings may be estimated on the principle that the main electrical losses of the machine when running either under its rated conditions or as a simulator will be equal; although this presupposes that the machine will, through suitable design, withstand the repetitive over-speed and over-load conditions essential to the task of simulation.
Figure 64. Relationship between continuous rating and $K_g$. 
Now the r.m.s. torque, as obtained from curve 'a' of Figure 63, is 550 lbf ft, and, for $K_5 = 0$, the simulator should impose an identical torque waveform for four seconds whilst its speed rises steadily to 3,500 rev/min. Hence, if $N$ is the continuously-rated speed in revolutions per minute and $M$ is the continuously-rated torque in pound-force feet, then, for each unit, respectively, of copper- and iron-loss incurred per second under rated conditions, the machine as a simulator will incur corresponding losses, averaged over the ten-second duty cycle, of

$$\frac{1}{10} \times (\frac{550}{M})^2 \text{ watts and}$$

$$\frac{1}{10} \int_0^4 \left(\frac{3,500}{4N} t\right)^{3/2} dt \text{ watts,}$$

if it is reasonably supposed that the combined eddy-current and hysteresis losses are proportional to $(speed)^{3/2}$. (The eddy-current loss within the armature winding can be disregarded as it can usually be made quite small).

Now for maximum efficiency at the continuous rating, the two principal electrical losses are equal. Therefore, the combined losses incurred during simulation duty may be equated wholly to the total losses for the rated conditions, whence, by differentiation, the product $M \cdot N$ may be minimised to give, for $K_5 = 0$, $M = 367$ lbf ft and $N = 945$ rev/min, which corresponds to a continuously-rated output of 66 hp. For a probable efficiency of 95 per cent. the electrical input power is thus 51.8 kW.

Similar calculations for other values of $K_5$, after substitution into Equation 46, therefore permit the construction of curve 'a' of Figure 64. Nevertheless, the required rating remains indeterminable until
the ratings can be related to the armature inertia, whence the appropriate actual values of $K_s$ could be computed and a second curve established to intersect curve 'a'.

Now the armature dimensions of a d.c. machine are determined almost exclusively by the work done by the armature per revolution, and are related to the power by the so-called Esson coefficient, $C_0$. Clayton and Hancock have published an empirical relationship between $C_0$ and the energy per revolution, which, by slight approximation, may be written as

$$C_0 = 2 + \frac{12P}{N}$$

(where $P$ is the armature power in kilowatts).

As a result, an armature of diameter $D$ metres and core-length $L$ metres can be related to the speed and power by

$$D^2L = \frac{P}{N.C_0} \text{ cubic metres.}$$

Now the most decisive factor in the design of d.c. machines is the magnitude of the induced e.m.f. that appears between adjacent commutator segments which, for normal single-turn coils working in a flux-density of 1.0 Wb/m², is given by

$$\frac{2\pi D.L}{60} \text{ (Maximum speed in revolutions per minute) volts,}$$

and which, for a fully-compensated machine, should not exceed 30 volts.

Therefore, by combination of the above relationships, for a maximum speed of 3,500 rev/min,

$$D = \left( \frac{P}{N + 6P} \right) \times 6.1 \text{ metres.}$$

(In commercial designs the armature dimensions are decided largely by economic considerations of the field system, and the voltage-limitation of the commutator may not always be utilised).
Once the armature proportions have been determined, the mass of the armature can be computed, from an assumed average density of 0.293 lb/in$^3$, whence the polar moment of inertia becomes

$$351 \times 10^3 \left( \frac{P}{6P + N} \right)^3 \text{ pound foot-squared.}$$

In order to assess the commutator inertia a commutator diameter of 0.7D and a brush-current density of 50 A/in$^2$ may be assumed. Moreover, as the machine, for the power envisaged, will probably comprise two pole-pairs and thus four brush-arms, the commutator-length can be deduced. Thus its inertia can be written as

$$13.1 \ D^4 \ \left( P + 9.75 \right) \ \text{ pound foot-squared}$$

if it is regarded as a solid copper drum and half an inch is allowed for the axial length of the risers. A further ten per cent. of the armature inertia, to allow for the winding-overhang, and an additional estimated 6 lb ft$^2$ for the ring-gear assembly, then gives a total inertia of

$$6 + 386 \times 10^3 \ \left( \frac{P}{6P + N} \right)^3 \cdot \left( 1 + \frac{4.3}{100} \ \left( \frac{P}{N + 6L} \right) \ (P + 9.75) \right) \ \text{ pound foot-squared}$$

Unfortunately this expression yields inertia values comparable to that of the actual engine and ring-gear, which implies that any inertia-simulation could be only marginal.

However, if a duplex armature winding were utilised the e.m.f.-per-coil limitation mentioned earlier would be doubled, whence armature diameters of $D$ with core-lengths of $4L$ become practicable, thereby reducing the overall machine inertia approximately by a factor of four. However, because of the repetitive overload and overspeed conditions imposed on the machine, the construction must be particularly robust and it is
doubtful that the full advantage of this winding could in fact be realised, especially as a somewhat elongated commutator also results. Therefore, except for the ring-gear, reductions of 50 per cent. will be assumed.

For each value of inertia so computed, the corresponding value of $K_5$ may be calculated (see Section 7.2) from the estimated engine-inertia of 25 lb $\text{ft}^2$ to yield the relationship represented by curve 'b' of Figure 64. Thus, with $K_5 = 0.58$, (corresponding to the intersection of curves 'a' and 'b') the required continuous ratings of the machine may be deduced as 27.3 hp, 945 rev/min, and 152 lbf ft. Substitution of $K_5 = 0.58$ into Equation 13 then indicates a reduction of the peak simulator torque from 1,180 lbf ft to about 495 lbf ft.

Now, according to Section 5.2 the armature-circuit time-constant of the machine will be approximately $80 \times 0.4/2N$ which evaluates to 17 ms. Unfortunately, for the Albion engine at 3,500 rev/min, the peak gas-pressure torque is attained in about 0.5 ms, so that an inordinately large forcing factor $K_F$ would be necessary for the torque pulsations to be exactly reproduced by the simulator. Clearly, therefore, at such speeds some distortion will be unavoidable and an intermediate, though realistic, value $K_F = 7$ will be considered.

Now if the same machine is used also to simulate the 10-litre engine, which has an inertia of 45.5 lb $\text{ft}^2$, and a starter-motor inertia of 24 lb $\text{ft}^2$, the corresponding value of $K_5$ will be 0.545 which therefore permits a reduction of the peak simulator torque, in this case, to $1600 (1-0.545)$ or 728 lbf ft. Hence the peak simulator torque for the Albion engine is only 68 per cent. of that for the 10-litre engine, and, by reference to Section 4.5, the fraction $\beta$ is thus 0.68. The rise-time will be lengthened however by the effect of $K_F$ due to the
pulse-delay interlock, as discussed in Section 4.7.2, so that the effective value of $K_F$, if $K_p = 0.1$, will be 7 (1-0.1) or 6.3.

This, according to Figure 20, results in a time-improvement factor of $K_I = 10$. Consequently the rise-time of the simulator torque to 495 lbf ft will be only 1.7 ms provided the time-ratio control system is designed for the peak simulator torque of 728 lbf ft. In practice the actual engine-torque rise-time also is prolonged, by the effect of the reciprocating masses, so that the distortion will be less serious than the calculations suggest.

The armature resistance of a d.c. machine can be expressed, from a realistic estimation of the copper-losses, as a function of the rated voltage and power; that is,

$$ R_{as} \propto \frac{4 (\text{Rated voltage})^2}{P \times 10^5} $$

so that if, at 3,500 rev/min, the back e.m.f. is not to exceed, say, 1,000 volts, then at 945 rev/min the rated voltage will be 270 volts. Hence, for $P = 21.5$ kW,

$$ R_{as} \propto 0.135 \text{ ohms.} $$

Now the auxiliary machine, shown in Figure 28, will exhibit a similar resistance, so that if the rated armature current of $1000 \times P/270$ amperes produces a torque of 152 lbf ft, the maximum peak armature current ever required will be, by direct ratio of torques, 382 amperes, which will necessitate, for circulation through both armatures, a driving voltage of $2 \times 0.135 \times 382$ or 103 volts. Thus the magnitude of $E_D$, the supply voltage to the control system, must be, (for $K_F = 7$) 721 volts, with the result that both the auxiliary and simulating machines must be insulated for approximately $(1,000 + \frac{721}{2})$ volts.
It is important to stress at this point that, because the rotational e.m.f.s. of the two machines are mutually cancelling, the supply to the control system is $E_B$ alone; although a slight increment will be necessary to offset the effect of $E_B$, as discussed in Sections 5.1 and 5.3. Moreover, the peak current required for the 10-litre-engine simulation is well within the short-time capabilities of present-day thyristors, so that together with the fact that the rated r.m.s. current is considerably lower still, the control system could utilise single thyristor elements and the problems of parallel or series connection should not arise.

It is now possible to predict the simulator performance for the 10-litre-engine characteristics. Since the total engine-oil-drag friction for an SAE-30 oil at $30^\circ$ F is about 232 lbf ft then, from mensuration of curve 'b', Figure 63, the net average driving torque is $(640 - 232)$ lbf ft. Hence the time to accelerate to 1,750 rev/min (the limiting armature-current reversal-rate) will be about 0.98 seconds, if the acceleration is constant. Thus for a more realistic acceleration-time of 2.0 seconds, the simulation of this engine will occupy only one-fifth of the duty-cycle period.

Now the value of $K_s$ for this engine is 0.545 so that, from Figure 42 (or Equation 25), the ratio $m_{\text{rms}}/(m_e + m_{Fe})_{\text{rms}} = 0.42$. Therefore, as the r.m.s. engine torque (computed from Figure 63, curve 'b') is 830 lbf ft, the r.m.s. simulator torque computed over the complete duty-cycle is $0.42 \times 830 \times (0.2)^{\frac{1}{2}}$ or 156 lbf ft, which, being approximately the rated value for the simulator, suggests that a reduction of the duty-cycle period may be possible. Thus, again,
if the total electrical losses incurred by the simulating machine are to be equal under both rated conditions and simulator operation, then we can write

\[
\left\{ \frac{1}{\text{Duty-cycle in seconds}} \right\} \times \left\{ 2 \times (0.42)^2 \times \left( \frac{330}{152} \right)^2 + \int_0^2 \left( \frac{1750 + t}{2 \times 945} \right)^{3/2} \, dt \right\} = 2
\]

whence a duty-cycle of 6.25 seconds could be utilised.

A comparison should now be made between the decay-rates of the engine torques, as measured from Figure 63, and the armature-circuit time-constant of the simulator. For the Albion engine the indicated torque decays almost linearly from its maximum value to zero in 100 degrees of crankshaft rotation whereas 70 degrees are required for the 10-litre engine. Hence, the initial time-constants of these decays, for the limiting torque-pulsation frequency of 116.5 Hz, are respectively 4.77 ms and 6.67 ms, and, since the armature-circuit time-constant is 17 ms, the torque-decay of the simulator must be forced by a factor of at least seven to ensure that the armature current will be sufficiently small at the appropriate instant to permit reliable armature-current reversal. This forcing is achieved by the resistor \( R_e \), shown in Figure 21, which in this case will be \((7-1) \times 0.135 \times 2\) or 1.62 \( \Omega \). Unduly-high values of \( R_e \) should be avoided because of the initial induced voltage that will appear across the bridge thyristors at the instant diode D5 becomes conducting. \( R_e \) is, of course, necessary only during high-speed operation when the torque decay-rates are high, so that the additional losses thus incurred could, if desired, be eliminated in the lower speed range.
All the above parameters have been estimated on a 'worst case' basis, that is, the engine friction was assumed to be high for the calculation of the acceleration-times, whilst it was totally ignored in the computation of the peak simulator torques. In practice, these torques will be reduced by engine friction according to Equation 19 of Section 7.4. Moreover, as will be apparent from Figures 7, 8 and 9, the peak engine torques are further diminished by the inertial forces of the reciprocating masses - at least at high engine speeds - so that the duty, as regards the simulator torque, becomes somewhat less arduous as the speed rises. This, fortunately, alleviates the problems of armature-current reversal as the frequency limit is approached.

The dimensions of the simulating machine are strictly dictated by the low inertia and low time-constant requirements, and consequently an unconventional design results. The auxiliary machine, however, need only combine low armature- and field-circuit time-constants regardless of the inertia considerations. It is imperative however that both machines should be fully compensated against the effects of the rapid armature-current pulsations and high peak overloads, particularly in the auxiliary machine where such operation sometimes occurs under extremely weak field conditions.

In conclusion, it should be noted that if $K_F = 7$ is employed only for the Albion-engine simulation, then $E_D$ need only be $721 \times 495/723$ or 490 volts. Thus the available forcing factor for the 10-litre engine will be reduced to 4.75. This should be acceptable as the rise-time of the peak torque is not so severe when the speed is limited to 1750 rev/min.
It will also be appreciated that the underlying reason for the seemingly-low continuous rating of the simulating machine is the choice of a somewhat protracted duty-cycle. Since any reduction of the duty-cycle would demand a more-than-proportional increase of the machine rating - with a corresponding increase of inertia - it would thus be advantageous, in more frequent simulations, to operate two identical simulating machines alternately from a single control unit.