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Backpropagating Constraints Based Trajectory Tracking Control of a Quadrotor with Constrained Actuator Dynamics and Complex Unknowns

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Abstract—In this paper, a backpropagating constraints based trajectory tracking control (BCTTC) scheme is addressed for trajectory tracking of a quadrotor with complex unknowns and cascade constraints arising from constrained actuator dynamics including saturations and dead zones. The entire quadrotor system including actuator dynamics is decomposed into 5 cascade subsystems connected by intermediate saturated nonlinearities. By virtue of the cascade structure, backpropagating constraints (BC) on intermediate signals are derived from constrained actuator dynamics suffering from nonreversible rotations and nonnegative squares of rotors, and decouple subsystems with saturated connections. Combining with sliding-mode errors, BC-based virtual controls are individually designed by addressing underactuation and cascade constraints. In order to remove smoothness requirements on intermediate controls, first-order filters are employed, and thereby contributing to backstepping-like sub-controllers synthesizing in a recursive manner. Moreover, universal adaptable compensators are exclusively devised to dominate intermediate tracking residuals and complex unknowns. Eventually, the closed-loop BCTTC system stability can be ensured by the Lyapunov synthesis, and trajectory tracking errors can be made arbitrarily small. Simulation studies demonstrate the effectiveness and superiority of the proposed BCTTC scheme for a quadrotor with complex constraints and unknowns.

Index Terms—Backpropagating constraints, quadrotor, constrained actuator dynamics, cascade constraints, complex unknowns, dead zones, trajectory tracking control.

I. INTRODUCTION

RECENTLY, increasing attention has been paid to Vertical Take-Off and Landing (VTOL) unmanned aerial vehicles (UAV) pertaining to a wide area of vital applications including patrolling for forest fires, traffic monitoring and surveillance.
addition to actual thrusts generated by propellers. Furthermore, the desired attitude variables resulting from virtual control signals are taken as references to be tracked by rotational dynamics. In this context, virtual control inputs have to be reasonable since the total thrust is uniformly nonnegative. In terms of unit-quaternion, Abdessameud and Tayebi [3] created a tool for extracting the thrust and desired attitudes, whereby feasible magnitudes for intermediate signals can be ensured. Within the UF framework, underactuated dynamics are driven by one or more cascaded fully-actuated subsystems, and thereby rendering backstepping-based approaches available. Typically, inspired by Lyapunov’s direct method for underactuated ship tracking [52], Do et al. [46] developed a global tracking control scheme for a QUAV by employing bounded backstepping techniques. Within the ML structure, several quasi-cascade loops are designed by employing time-scale separation philosophy, whereby the innermost (outermost) loop has to possess the fastest (lowest) tracking error dynamics since virtual controllers can only stabilize individual subsystems. In summary, there exist the issues which are open:

- **Dealing with cascade constraints.** Using TR and UF cascade structures would inevitably ignore cascade constraints hidden within subsystems due to main facts as follows: (1) The thrust constraint is bounded by non-reversibly limited propeller rotations and has to be non-negative; (2) Together with trigonometric functions of attitudes, desired cascade inputs to translational dynamics have to be feasibly constrained; (3) Cascade inputs to attitude dynamics are directly constrained by the squares of propeller rotations; (4) Thrust torques generated by individual propellers are restricted to be nonnegative and are determined by the squares of motor rotor speeds.

- **Dealing with actuator dynamics.** As analyzed above, actuator dynamics including transient responses and control input constraints would directly affect and limit the torque inputs to propeller rotation dynamics. Clearly, treating actuator dynamics as input nonlinearities/uncertainties [53], [54], linearized dynamics [43], [55], [56] or stationary mappings [1], [57] would hardly determine feasible input torques generated by propellers, and thereby resulting in uniformly unreachable regions within the desired control efforts. Nevertheless, BLDC motors in a QUAV are not allowed to rotate reversely such that uniformly upward thrust forces can be generated. In this context, it becomes empirical and risky to design control laws for torque inputs if BLDC dynamics are omitted and torque control signals are directly fed into the electronic speed control (ESC) module which generates 3-phase AC voltages via PWM signals. Hence, incorporating actuator dynamics into the QUAV model is strongly desirable for pursuing high autonomy. However, to our best knowledge, few attention to systematically dealing with actuator dynamics including control constraints has been paid for a QUAV.

In this paper, we focus on trajectory tracking control of a QUAV including cascade constraints, constrained actuator dynamics and complex unknowns, which is unsolved in the literature. By incorporating the SMC and DSC approaches into a backstepping-like framework, a backpropagating constraints (BC) based trajectory tracking control (BCTTC) scheme is proposed by devising extraction tools for cascade constraints. In the presence of actuator dynamics, unmodeled dynamics, uncertainties, measurement noises and external disturbances, the entire QUAV dynamics are formulated in a vectorial pure-feedback form with unmatched unknowns whereby intermediate constraints and underactuated dynamics appear in a cascade mode, and make traditional backstepping-based approaches unavailable. In this context, the BCTTC framework using the SMC is realized to circumvent both cascade constraints and underactuation issues, and recursively stabilize tracking errors. The DSC technique is further deployed to facilitate the derivation of intermediate signals. Since constrained actuator dynamics are sufficiently addressed, virtual control signals pertaining to Euler angles, rotation squares, and armature voltages of nonreversible motors with input saturations and dead zones are reasonably constrained by the BC extraction. In addition, intermediate tracking discrepancies and complex unknowns are further attenuated by a family of universal adaptive compensators. Eventually, the Lyapunov approach ensures that the entire closed-loop BCTTC system is asymptotically stable, and trajectory tracking errors together with other signals are uniformly ultimately bounded.

The rest of this paper is organized as follows. In Section II, the QUAV dynamics and problem formulation are addressed. Backpropagating constraints on intermediate signals are derived in Section III. The BCTTC scheme for trajectory tracking of a QUAV and stability analysis are presented in Sections IV and V, respectively. Simulation studies are conducted in Section VI. Conclusions are drawn in Section VII.

**Nomenclature:** Throughout this paper, "||·||" denotes Euclidean vector norm or Frobenius matrix norm, respectively, and a saturation function sat(·) shown in Fig. 1 is defined by

\[
sat(x; x_0, \delta_x) = \begin{cases} x, & |x - x_0| \leq \delta_x \\ x_0 + \delta_x, & x - x_0 > \delta_x \\ x_0 - \delta_x, & x - x_0 < -\delta_x \end{cases} (1)
\]

where \( x_0 \) and \( \delta_x > 0 \) are the center point and range of the saturation, and a smooth approximation to (1) is defined by

\[
sat_{\alpha}(x; x_0, \delta_x) = x_0 + \delta_x \cdot \tanh((x - x_0)/\delta_x) (2)
\]

with hyperbolic tangent function \( \tanh(·) \). Accordingly, the saturation approximation error function is defined as follows:

\[
sat_{\epsilon}(x; x_0, \delta_x) = sat(x; x_0, \delta_x) - sat_{\alpha}(x; x_0, \delta_x) (3)
\]
which is obviously bounded.

II. QUAV DYNAMICS AND PROBLEM FORMULATION

A. QUAV Dynamics

As shown in Fig. 2, a QUAV is made up of four electric motors fixed on an X-shape frame. The earth-fixed coordinate $O_x Y_o Z_o$ and the body-fixed coordinate $O'_X Y'_Z$ are considered with the origin coinciding to the gravity center of the QUAV. In the earth-fixed frame, the $Z_o$-axis points upwards, and the QUAV position is given by a vector $[x, y, z]^T$. The QUAV orientation refers to as roll, pitch, and yaw, and is given by the vector $[\phi, \theta, \psi]^T$ which is measured with respect to the earth-fixed coordinate. Actually, the entire model of the QUAV is composed by position dynamics, Euler angles, angular velocity, propeller speed and BLDC motor dynamics.

Inspired by the faithful representation for a QUAV with complete dynamics in [58], in this paper, actuator dynamics, i.e., BLDC motor dynamics together with propeller speeds, have been comprehensively incorporated into the entire QUAV dynamics which in turn become much more practical and challenging for controller design and synthesis.

The position dynamics can be described as follows:

$$\begin{align*}
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= f_1 (x_{12}) + u_1 (x_2, T(x_4)) + d_1
\end{align*}$$

with lumped unknown nonlinearities $d_1 = [d_{11}, d_{12}, d_{13}]^T$ including model uncertainties, unmodeled dynamics and/or external disturbances which exist in position dynamics, and

$$f_1 (x_{12}) = -\frac{1}{m} \begin{bmatrix} D_x \ddot{z}^2 \\ D_y \ddot{z}^2 \\ D_z \ddot{z} + g \end{bmatrix}$$

$$u_1 (x_2, T(x_4)) = \frac{T}{m} \begin{bmatrix} \cos \phi \sin \theta \cos \psi + \sin \phi \sin \psi \\ \cos \phi \sin \theta \sin \psi - \sin \phi \cos \psi \\ \cos \phi \cos \theta \end{bmatrix}$$

where $x_{11} = [x, y, z]^T$ and $x_{12} = [\dot{x}, \dot{y}, \dot{z}]^T$ are the vectors of the positions and linear velocities in the earth-fixed frame, respectively, $D_i (i = x, y, z)$ represents the air resistance coefficient respectively, $m$ is the mass of the QUAV, $g$ is the acceleration of the gravity, $T$ is the total thrust determined by

$$T(x_4) = \sum_{i=1}^{4} bw_i^2$$

here, $b$ is the thrust factor and $x_4 = [w_1, w_2, w_3, w_4]^T$ is the vector of propeller rotation speeds, and $\chi_2 = [\phi, \theta, \psi]^T$ is the vector of Euler angles governed by

$$\dot{\chi}_2 = G_2(\chi_2) u_2(\chi_3) + d_2$$

with lumped nonlinearities $d_2 = [d_{21}, d_{22}, d_{23}]^T$ which may include measurement noises and/or external disturbances pertaining to angular velocities, and

$$G_2(\chi_2) = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & -\sin \phi \cos \theta & \sin \phi \cos \theta \\
0 & \sin \phi \cos \theta & \cos \phi \cos \theta
\end{bmatrix}$$

$$u_2(\chi_3) = \chi_3$$

where $\chi_3 = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$ is the angular velocity vector in body-fixed coordinate given by the following dynamics:

$$\dot{\chi}_3 = f_3(\chi_4) + G_3 u_3(\chi_5) + d_3$$

with the diagonal matrix $G_3 = \text{diag} (1/J_x, 1/J_y, 1/J_z)$ where $J_i (i = x, y, z)$ is the moment of inertia with respect to each axis, $d_3 = [d_{31}, d_{32}, d_{33}]^T$ are unknown nonlinearities within the input channel of angular dynamics, and

$$f_3(\chi_4) = \begin{bmatrix}
\frac{J_x - J_z}{J_z} q r \\
\frac{J_y - J_z}{J_z} p q \\
\frac{J_y - J_x}{J_x} p q
\end{bmatrix}$$

$$u_3(\chi_4) = \begin{bmatrix}
lb (-w_2^2 + w_3^2) \\
lb (-w_2^2 + w_4^2) \\
(k - w_1^2 + w_2^2 + w_3^2)
\end{bmatrix}$$

where the virtual control input $u_3(\chi_4)$ is actually constrained by the nonnegative squares, i.e., $w_2^2, w_3^2, w_4^2$ and $w_3^2$. $\tau_{\alpha}(i = 1, 2, 3)$ denotes the airframe torque, $l$ is the distance from the gravity center of QUAV to the propeller rotor, $b$ is the thrust factor, $k$ is the drag factor, and the dynamics of propeller speeds $\chi_4 = [w_1, w_2, w_3, w_4]^T$ are given by

$$\dot{\chi}_4 = f_4(\chi_4) + G_4 u_4(\chi_5) + d_4$$

with $G_4 = I_4/I_r$ where $I_4 \in R^4$ is an unity matrix, $I_r$ denotes the propeller rotor inertia, $d_4 = [d_{41}, d_{42}, d_{43}, d_{44}]^T$ are lumped unknowns for propeller rotation dynamics, and

$$f_4(\chi_4) = \frac{1}{I_r} \begin{bmatrix}
-k w_1^2 - c_1 w_1 \\
-k w_2^2 - c_2 w_2 \\
-k w_3^2 - c_3 w_3 \\
-k w_4^2 - c_4 w_4
\end{bmatrix}$$

$$u_4(\chi_5) := [r_1, r_2, r_3, r_4]^T = \frac{m T}{\eta} [w_1^2, w_2^2, w_3^2, w_4^2]^T$$

where the input signal $u_4(\chi_5)$ is actually constrained by the nonnegative squares, i.e., $w_1^2, w_2^2, w_3^2$ and $w_4^2$, $c$ is the thrust factor, $r_i (i = 1, 2, 3, 4)$ denote the thrust torques generated by individual propellers, $n$ is the damping factor, $r$ is the speed.

Fig. 1. Saturation function sat(·) and its smooth approximation sat_a(·).

Fig. 2. The configuration of a QUAV.
ratio between the motor and the propeller, $\eta$ is the transmission efficiency and the dynamics of BLDC motor rotor speeds $\chi_5 = [w_{e1}, w_{e2}, w_{e3}, w_{e4}]^T$ is given by

$$\chi_5 = f_5(\chi_5) + G_5u_5(v_5) + d_5 \tag{17}$$

with $G_5 = C_m I_1 / R_a$ where $C_m$ and $R_a$ denote the electric torque coefficient and the armature resistance of the BLDC motor, respectively. $d_5 = [d_{51}, d_{52}, d_{53}, d_{54}]^T$ are lumped unknowns pertaining to motor dynamics. $u_5(v_5) = [u_{51}(v_{51}), u_{52}(v_{52}), u_{53}(v_{53}), u_{54}(v_{54})]^T$ is the armature voltage vector of motors and is practically constrained by input nonlinearity including both saturations and dead zones due to irreversible rotation shown in Fig. 3 as follows:

$$u_{5i}(v_{5i}) = \text{sat}\left( dz (v_{5i}; 0, u_{5i}^0) / 2 \right) \tag{18}$$

where

$$dz (v_{5i}; 0, u_{5i}^0) = v_{5i} - \text{sat}(v_{5i}; 0, u_{5i}^0) \tag{19}$$

here, sat(·) is defined by (1), $u_{5i}^0$ and $u_{5i}^+$ are the dead zone and the saturation of armature voltages, respectively, and $v_{5i} = [v_{51}, v_{52}, v_{53}, v_{54}]^T$ is the ideally nominal control input, and

$$f_5(\chi_5) = \frac{1}{J_r r^2 + J_m \eta} \begin{bmatrix} -C_m \omega_{e1} - w_{e1}^3 & -C_m \omega_{e2} - w_{e2}^3 & -C_m \omega_{e3} - w_{e3}^3 & -C_m \omega_{e4} - w_{e4}^3 \\ -R_a C_m \omega_{e1} - n w_{e1} & -R_a C_m \omega_{e2} - n w_{e2} & -R_a C_m \omega_{e3} - n w_{e3} & -R_a C_m \omega_{e4} - n w_{e4} \end{bmatrix} \tag{20}$$

where $J_r$ is the moment of inertia of the motor rotor, $J_m$ is the inertia moment of the rotating element that turns to rotor of the motor, $C_e$ is the voltage coefficient of the motor.

Similar to previous works formulated by Euler angles [32], [36], constraints on Euler angles are naturally required to ensure the nonsingularity of matrix $G_2$ in (9) as follows:

**Assumption 1.** Euler angles are constrained by

$$\phi, \theta, \psi \in (-\pi/2, \pi/2) \tag{21}$$

**Remark 1.** For the entire QUAV dynamics (4), (8), (11), (14) and (17), vectorial nonlinearities $u_1(\cdot), u_2(\cdot), u_3(\cdot)$ and $u_4(\cdot)$ are taken as virtual control inputs while the signal $u_5$ is referred to as the actual control input. As a consequence, a vectorial pure-feedback nonlinear system with interconnected vectorial pure-feedback nonlinearities can be innovatively established and is ready for backstepping-like controller design.

**Remark 2.** In view of the squares of propeller and rotor speeds, i.e., $w_{e1}^2$ and $w_{e1}^2$ in (13) and (16), respectively, together with (7), virtual control signals $u_1(\cdot), u_2(\cdot)$ and $u_4(\cdot)$ in (4), (11) and (14) respectively are expected to be constrained for ensuring the positiveness and boundedness of speed squares. In addition, actuator dynamics with complex constraints arising from insensitive dead-zone voltages, bounded armature voltages and nonreversible rotations have been completely formulated in (17)–(19), and thereby leading to constraints on control input nonlinear $u_5(\cdot)$. To our best knowledge, all aforementioned concerns on backpropagating cascade constraints and complex actuator dynamics have not been addressed in the literature.

**Remark 3.** In practice, the armature voltage of a BLDC motor within the QUAV is actually limited within a reasonable range, and is usually nonnegative for unidirectional rotation. In addition, both mismatched and matched complex unknowns $d_i, i = 1, \cdots, 5$ including unmodeled dynamics, uncertainties, measurement noises and external disturbances are incorporated into the QUAV model.

**B. Problem Formulation**

In this paper, we address the trajectory tracking problem of a QUAV with backpropagating cascade constraints, complex actuator dynamics and mismatched unknowns within the entire dynamics (4), (8), (11), (14) and (17). Our objective is to design a backpropagating constraints based trajectory tracking controller (BCTTC) such that the complex QUAV can track the desired trajectories under mild conditions as follows:

**Assumption 2.** The desired trajectory $(x_{d1d} := [x_d, y_d, z_d]^T$ and $\psi_{d1})$ and its time derivatives are bounded.

**Assumption 3.** Complex unknowns $d_i$ are bounded while the upper bound is unnecessarily known, i.e.,

$$\|d_i\| \leq L_i, \quad i = 1, 2, 3, 4, 5 \tag{22}$$

where positive constants $L_i > 0$ is unknown.

In practice, BLDC motors within a QUAV are expected to rotate unidirectionally and generate uniformly upward thrust.

**Assumption 4.** BLDC motors are nonreversible, i.e., $w_{e1} \geq 0$.

In order to facilitate stability analysis of the closed-loop control system, a preliminary result is stated here.

**Lemma 1.** Consider the following system:

$$\dot{x}(t) + \lambda(t)x(t) = \sigma(t) \tag{23}$$

with $\lambda(t) > 0, \forall t$, if $\sigma(t)$ is uniformly bounded, i.e., $|\sigma(t)| \leq g, \forall t$ with a positive constant $g > 0$, then states $x(t)$ and $\dot{x}(t)$ are uniformly bounded.

**Proof:** Consider the Lyapunov function $W = \frac{1}{2}x^2$. Using (23) yields the time derivative of $W$ as follows:

$$\dot{W} = x(-\lambda x + \sigma) \leq - (\lambda - \kappa ) \cdot x^2 + \frac{\sigma^2}{4\kappa} \tag{24}$$

for any positive constant $\kappa > 0$. Since $|\sigma(t)| \leq g, \forall t$, selecting $\kappa < \lambda$, we further have

$$\dot{W} \leq -\alpha W + b \tag{25}$$

with $a = 2(\lambda - \kappa)$ and $b = g^2/(4\kappa)$. Fig. 3: The input nonlinearity including saturation and dead zone.
It implies that

\[ 0 \leq W(t) \leq W(0) e^{-at} + (1 - e^{-at}) \frac{b}{a} < \infty \]  

(26)

Clearly, \( x(t) \) is uniformly bounded (UB), i.e., \( |x(t)| \leq \bar{x}, \forall t \). From (23), we further have

\[ |\dot{x}(t)| \leq \lambda \bar{x} + g < \infty, \quad \forall t \]  

(27)

which yields \( \dot{x}(t) \) is also UB. This concludes the proof. \( \blacksquare \)

**Remark 4.** Unlike previous works, a servo motor control loop is incorporated in this paper, and renders cascade constraints on intermediate signals actually backpropagate from complex actuator dynamics in addition to mismatched unknowns. In this context, an innovative backpropagating cascade constraints based control scheme for such a complex QUAV is established in the sequel.

### III. Backpropagating Constraints

In order to facilitate our control scheme, backpropagating constraints (BC) on intermediate signals are extracted from nonreversible actuator dynamics and saturations. Key results are summarized as follows:

**Proposition 1.** The following BC-based saturations hold:

\[
\begin{align*}
\dot{w}_{ei} &= \text{sat}(u_{ei}^{u}; w_{ei}^{m}/2, w_{ei}^{m}/2) \\
\dot{w}_i &= \text{sat}(u_i^{u}; w_i^{m}/2, w_i^{m}/2) \\
T &= \text{sat}(T^{u}_i; 2bw_i^{m2}, 2bw_i^{m2}) \\
\dot{u}_{4i} &= \text{sat}(u_{4i}^{u}; nrw_{ei}^{m}/(2\eta), nrw_{ei}^{m}/(2\eta)) \\
\dot{u}_{3i} &= \text{sat}(u_{3i}^{u}; 0, u_{3i}^{m}) \\
\end{align*}
\]

where \( \text{sat}(\cdot) \) is defined in (1), \( w_{ei}^{m} \) and \( w_i^{m} \) are maximal rotation speeds of motor rotors and propellers, respectively, \( \text{sat} \) denotes the unsaturated signal of \( \text{sat} \), and saturation levels for \( u_{3i} \) are given by

\[
\begin{align*}
\dot{u}_{33} &= \frac{k}{b} \min \left\{ T, 4bw_i^{m2} - T \right\} \\
\dot{u}_{32} &= \frac{l}{2} \min \left\{ T - \frac{b}{k} u_{33}, 4bw_i^{m2} - T + \frac{b}{k} u_{33} \right\} \\
\dot{u}_{31} &= \frac{l}{2} \min \left\{ T + \frac{b}{k} u_{33}, 4bw_i^{m2} - T - \frac{b}{k} u_{33} \right\} \\
\end{align*}
\]

Proof: Rewriting actuator dynamics (17) as follows:

\[
\dot{w}_{ei} = -a_i w_{ei} + b_i 
\]

with

\[
\begin{align*}
a_i &= \frac{1}{J_i r_i^2 + J_m \eta} \left( C_m C \eta \frac{R_a}{R_a} + nrw_{ei} \right) \\
b_i &= \frac{C_m}{R_a} u_{si}(w_{si}) + d_{si} \\
\end{align*}
\]

Together with Assumptions 3–4 and (18), we have

\[
a_i \geq \frac{C_m C \eta}{R_a (J_i r_i^2 + J_m \eta)} > 0, \quad \forall t \]  

(40)

Using Lemma 1, we immediately have rotor rotation \( w_{ei} \) is UB, i.e., \( 0 \leq w_{ei} \leq w_{ei}^{m}, \forall t \). Similarly, using (14), we have propeller speed \( w_i \) is UB, i.e., \( 0 \leq w_i \leq w_i^{m}, \forall t \). Together with (7) and (16), respectively, we have constraints on \( T \) and \( u_{4i} \), i.e., \( 0 \leq T \leq 4bw_i^{m2} \) and \( 0 \leq u_{4i} \leq nrw_{ei}^{m2}/\eta \). In this context, we have (28)–(31) hold.

Together with (7) and (13), we have

\[
\begin{align*}
u_{11}^2 &= -\frac{1}{2b} u_{32} - \frac{1}{4k} u_{33} + \frac{1}{2} T \in [0, w_i^{m2}] \\
u_{12}^2 &= -\frac{1}{2b} u_{31} + \frac{1}{4k} u_{33} + \frac{1}{2} T \in [0, w_i^{m2}] \\
u_{13}^2 &= \frac{1}{2b} u_{32} - \frac{1}{4k} u_{33} + \frac{1}{2} T \in [0, w_i^{m2}] \\
u_{14}^2 &= \frac{1}{2b} u_{31} + \frac{1}{4k} u_{33} + \frac{1}{2} T \in [0, w_i^{m2}] \\
\end{align*}
\]

which yields

\[
\begin{align*}
u_{33} &\leq \frac{k}{b} \left[ -(4bw_i^{m2} - T), T \right] \cup \frac{k}{b} \left[ -T, 4bw_i^{m2} - T \right] \\
u_{32} &\leq \frac{l}{2} \left[-(T - \frac{b}{k} u_{33}), 4bw_i^{m2} - T - \frac{b}{k} u_{33} \right] \cup \frac{l}{2} \left[- \frac{b}{k} u_{33}, 4bw_i^{m2} - T + \frac{b}{k} u_{33} \right] \\
u_{31} &\leq \frac{l}{2} \left[-T - \frac{b}{k} u_{33}, 4bw_i^{m2} - T + \frac{b}{k} u_{33} \right] \\
\end{align*}
\]

It follows that saturation constraints on \( u_{3i} \), i.e., (32) and (34)–(36), hold.

Using (6) yields

\[
\begin{align*}
u_{11} &= \frac{T}{m} \cos(\psi) \sqrt{\cos^2 \phi \sin^2 \theta + \sin^2 \phi} \\
u_{12} &= \frac{T}{m} \sin(\psi) \sqrt{\cos^2 \phi \sin^2 \theta + \sin^2 \phi} \\
u_{13} &= \frac{T}{m} \cos \phi \theta \cos \theta \\
\end{align*}
\]

with \( \delta = \tan^{-1}(\tan \phi/\sin \theta) \). Together with Assumption 1, we have \( |\nu_{11}| \leq 4bw_i^{m2}/m, |\nu_{12}| \leq 4bw_i^{m2}/m \) and \( 0 \leq u_{13} \leq 4bw_i^{m2}/m \). In this context, saturations on \( u_{1} \) in (33) hold. This concludes the proof. \( \blacksquare \)

**Proposition 2.** Consider desired signals \( u_{jdi}, j = 1, 3, 4 \) of BC-based saturations \( \mathbf{u}_i \), in (31)–(33) defined as follows:

\[
\begin{align*}
u_{1jdi} &= \text{sat}(u_{1jdi}; nrw_{ei}^{m2}/(2\eta), nrw_{ei}^{m2}/(2\eta)) \\
u_{1jdi} &= \text{sat}(u_{1jdi}; 0, u_{1jdi}^{m}) \\
u_{1jdi} &= \text{sat}(u_{1jdi}; 0, 4bw_i^{m2}/m) \\
u_{1jdi} &= \text{sat}(u_{1jdi}; 2bw_i^{m2}/m, 2bw_i^{m2}/m) \\
\end{align*}
\]

where \( v_{jdi} \)’s are corresponding unsaturated signals. Then, the error \( \mathbf{e}_j := u_j - u_{jdi} \) is bounded, i.e.,

\[
||\mathbf{e}_j|| \leq \zeta_j 
\]

for an unnecessarily known constant \( \zeta_j > 0 \) depending on the saturation level.
This concludes the proof.

\textbf{Remark 5.} Proposition 1 reveals that actuator constraints in (28)–(30) backpropagate recursively to preceding intermediate signals saturated in (31)–(33), and establishes recursive saturation levels which facilitate the BC-based backstepping-like control. Proposition 2 implies that virtual control discrepancies are bounded if desired signals are governed by (49)–(51).

IV. BACKPROPAGATING CONSTRAINTS BASED TRAJECTORY TRACKING CONTROL SCHEME

In this section, the BC-based trajectory tracking control (BCTTC) scheme for a complex QUAV is elaborately established, in a recursive form, by employing an SMC-based DSC framework with universal adaptive compensators for saturation and robustness. As shown in Fig. 4, the entire BCTTC scheme consists of 5 successive controllers, whereby the preceding control effort is used as the desired signal of the succeeding inner closed-loop. Hence, a cascade backstepping-like control hierarchy is synthesized.

A. Position Virtual Controller

By Proposition 2, the desired position virtual controller (PVC) \( \mathbf{u}_{1d}(\mathbf{v}_{1d}) := \lbrack u_{11d}(v_{11d}), u_{12d}(v_{12d}), u_{13d}(v_{13d}) \rbrack^T \) is saturated as (51), where \( \mathbf{v}_{1d} := \lbrack v_{11d}, v_{12d}, v_{13d} \rbrack^T \) is the ideally desired PVC determined later.

Note that the saturated signals \( \mathbf{u}_{1d}(\mathbf{v}_{1d}) \) defined in (51) are non-smooth. In order to facilitate a differentiable PVC, a smooth function \( \mathbf{g}_{1d}(\mathbf{v}_{1d}) = \lbrack g_{11d}(v_{11d}), g_{12d}(v_{12d}), g_{13d}(v_{13d}) \rbrack^T \) is devised to approximate the saturated input \( \mathbf{u}_{1d}(\mathbf{v}_{1d}) \) as follows:

\[
\begin{align*}
    g_{11d} &= \text{sat}_a \left( v_{11d}; 0, 4bw^m/2/m \right), \quad i = 1, 2 \\
    g_{13d} &= \text{sat}_a \left( v_{13d}; 2bw^m/2/m, 2bw^m/2/m \right)
\end{align*}
\]

where \( \text{sat}_a(\cdot) \) is defined in (2).

Accordingly, the saturation approximation error \( \mathbf{w}_1 := \lbrack w_{11}, w_{12}, w_{13} \rbrack^T \) is given by

\[
\begin{align*}
    w_{1i} &= \text{sat}_a \left( v_{1id}; 0, 4bw^m/2/m \right), \quad i = 1, 2 \\
    w_{13} &= \text{sat}_a \left( v_{13d}; 2bw^m/2/m, 2bw^m/2/m \right)
\end{align*}
\]

where \( \text{sat}_a(\cdot) \) is defined in (3). Obviously, \( \mathbf{w}_1 \) is bounded, i.e.,

\[
\| \mathbf{w}_1 \| \leq \bar{w}_1
\]

here, \( \bar{w}_1 > 0 \) is unknown.

Define an intermediate tracking error as follows:

\[
\mathbf{u}_{1e} = \mathbf{u}_1 - \mathbf{g}_1
\]

Using (52) and (60), we immediately have

\[
\| \mathbf{u}_{1e} \| \leq \rho_1
\]

with an unknown upper bound \( \rho_1 = \zeta_1 + \bar{w}_1 \).

Given a reference trajectory \( \chi_{11d} := \lbrack x_d, y_d, z_d \rbrack \), combining with position dynamics (4), we define the following errors:

\[
\begin{align*}
    e_{11} &= \chi_{11} - \chi_{11d} \\
    e_{12} &= \chi_{12} - \chi_{12d} - \mu_1 \\
    y_1 &= \chi_{12d} + \mu_1 - \chi_{12d}
\end{align*}
\]

where \( \mu_1 \) is a dynamic compensator determined later. \( \chi_{12d} \) is a virtual control signal, \( \chi_{12d} \) is the filtered output of \( \chi_{12d} \) given by

\[
\epsilon_1 \hat{\chi}_{12d} + \hat{\chi}_{12d} = \chi_{12d}
\]

here, \( \epsilon_1 > 0 \) is an user-defined filtering time constant.

Design sliding surfaces as follows:

\[
\mathbf{s}_{1i}(t) = e_{1i}(t) + K_{1i} \int_0^t e_{1i}(\tau) d\tau, \quad i = 1, 2
\]

where \( K_{1i} = \text{diag}(k_{1i1}, k_{1i2}, k_{1i3}) \).

In this context, the virtual control signal \( \chi_{12d} \) can be selected as follows:

\[
\chi_{12d} = -P_{11} s_{11} + \hat{\chi}_{11d} - K_{11} e_{11} - e_{12}
\]

where \( P_{11} = \text{diag}(p_{111}, p_{112}, p_{113}) \) and an ideally desired PVC for sub-system (4) can be designed as follows:

\[
\mathbf{v}_{1d} = -P_{12} \mathbf{s}_{12} - f_1(\mathbf{v}_d) + \mathbf{u}_{1d} \\
- \mu_1 - K_1 \mathbf{e}_{12} - (\dot{L}_1 + \tilde{\rho}_1) \tanh \left( \frac{s_{12}}{\xi} \right)
\]
with a positive constant $\varepsilon > 0$, and universal adapt ive compensators (UAC) for $\mu_1$, $\hat{L}_1$ and $\hat{\rho}_1$ given by

\[
\begin{align*}
\dot{\mu}_1 &= -\mu_1 + g_1((v_{1d} - v_{1d}) = 0,
\dot{\hat{L}}_1 = -\gamma_{11}\hat{L}_1 + \kappa_{L,1} \hat{L}_{12} \tanh \left( \frac{\hat{s}_{12}}{\varepsilon} \right) \\
\dot{\hat{\rho}}_1 &= -\gamma_{12}\hat{\rho}_1 + \kappa_{\rho,1} \hat{L}_{12} \tanh \left( \frac{\hat{s}_{12}}{\varepsilon} \right)
\end{align*}
\] (70)

where $P_{12} = \text{diag}(p_{121}, p_{122}, p_{123}) > 0$, $\gamma_{11} > 0$, $\gamma_{12} > 0$, $\kappa_{L,1} > 0$, $\kappa_{\rho,1} > 0$, and $\hat{L}_1$ and $\hat{\rho}_1$ are estimates of unknown bounds $L_1$ and $\rho_1$, respectively.

In this context, the sliding error dynamics can be obtained as follows:

\[
\begin{align*}
\dot{s}_{11} &= -P_{11}\dot{s}_{11} + y_1 \\
\dot{s}_{12} &= -P_{12}\dot{s}_{12} + d_1 + \hat{u}_{1c} - \left( \hat{L}_1 + \hat{\rho}_1 \right) \tanh \left( \frac{\hat{s}_{12}}{\varepsilon} \right)
\end{align*}
\] (71)

with $\hat{u}_{1c}$ defined in (61).

B. Euler Angle Virtual Controller

Substituting (51) into the input nonlinearity (6) yields

\[
\begin{align*}
T_d \cos \phi_d \sin \theta_d \cos \psi_d + \sin \phi_d \sin \psi_d &= m u_{11d} \\
T_d \cos \phi_d \sin \theta_d \sin \phi_d - \sin \theta_d \cos \psi_d &= m u_{12d} \\
T_d \cos \phi_d \cos \theta_d &= m u_{13d}
\end{align*}
\] (73)

where $T_d = m \| u_{1d} \|$ and $u_{1d} := [u_{11d}, u_{12d}, u_{13d}]^T$.

Let $\chi_{2d} := [\hat{\phi}_{d}, \hat{\theta}_{d}, \hat{\psi}_{d}]^T$ and $\chi_{2d} := [\hat{\phi}_{d}, \hat{\theta}_{d}, \hat{\psi}_{d}]^T$ where $\chi_{2d}$ is the filtered output of $\chi_{2d}$ given by

\[
\begin{align*}
\chi_{2d} &= \hat{\chi}_{2d} + \chi_{2d} = \chi_{2d}
\end{align*}
\] (75)

here, $\epsilon_2 > 0$ is an user-defined filtering time constant.

Combining with Euler angles dynamics (8), we define the following errors:

\[
\begin{align*}
y_2 &= \chi_{2d} - \chi_{2d} \\
y_3 &= \hat{\chi}_{2d} - \chi_{2d}
\end{align*}
\] (76)

Design a sliding surface as follows:

\[
\begin{align*}
s_2(t) &= e_2(t) + K_2 \int_0^t e_2(\tau) d\tau
\end{align*}
\] (78)

where $K_2 = \text{diag}(k_{21}, k_{22}, k_{23}) > 0$.

In this context, the desired Euler angle virtual controller (EAVC) for sub-system (8) can be designed as follows:

\[
\begin{align*}
u_{2d} &= \Gamma_2^{-1}(\chi_2) \left[ \chi_{2d} - K_2 e_2 \\
&= P_{2}\dot{s}_{2} + \beta_2 y_2 - \hat{L}_2 \tanh \left( \frac{s_{2}}{\varepsilon} \right) \right]
\end{align*}
\] (79)

with an UAC $\hat{L}_2$ given by

\[
\begin{align*}
\hat{L}_2 &= -\gamma_{21}\hat{L}_2 + \kappa_{L,2} \hat{L}_{2} \tanh \left( \frac{\hat{s}_{2}}{\varepsilon} \right)
\end{align*}
\] (80)

where $P_2 = \text{diag}(p_{21}, p_{22}, p_{23}) > 0$, $\beta_2 > 0$, $\gamma_{21} > 0$, $\kappa_{L,2} > 0$, and $\hat{L}_2$ is the estimate of unknown bound $L_2$.

Hence, the sliding error dynamics can be obtained as follows:

\[
\begin{align*}
\dot{s}_2 &= -P_{2}\dot{s}_2 + d_2 + G_2 u_{2c} + \beta_2 y_2 - \hat{L}_2 \tanh \left( \frac{s_{2}}{\varepsilon} \right)
\end{align*}
\] (81)

where

\[
\begin{align*}
u_{2c} &= u - u_{2d} = \chi_{3} - \chi_{3d}
\end{align*}
\] (82)

Remark 6. The derivation of (74) from (73) can be obtained by assigning a given reference yaw angle $\psi_d$. In addition, the first equation of (74) ensures the desired total thrust $T_d$ is reasonable, whereby possible saturation can be tackled later.

C. Angular Velocity Virtual Controller

The saturated angular velocity virtual controller (AVVC) $u_{3d}(v_{3d}) := [u_{31d}(v_{31d}), u_{32d}(v_{32d}), u_{33d}(v_{33d})]^T$ is designed as (50) with saturation levels in (34)–(36), where $v_{3d} := [v_{31d}, v_{32d}, v_{33d}]^T$ is the ideally desired AVVC determined later.

Note the constrained control input $u_{3d}(v_{3d})$ defined in (50) and (34)–(36) is non-smooth. In order to facilitate a differentiable virtual control law, a smooth function $g_3(v_{3d}) := [g_{31}(v_{31d}), g_{32}(v_{32d}), g_{33}(v_{33d})]^T$ is devised to approximate the constrained input $u_{3d}(v_{3d})$ as follows:

\[
\begin{align*}
g_{3d}(v_{3d}) &= \text{sat}_{c}(v_{31d}; 0, u_{31d}^\varepsilon) \\
g_{3d}(v_{3d}) &= \text{sat}_{c}(v_{31d}; 0, u_{31d}^\varepsilon)
\end{align*}
\] (83)

where $\text{sat}_{c}(\cdot)$ is defined in (2) and $u_{31d}^\varepsilon$ is given by (34)–(36).

Accordingly, the constraint approximation error $\varpi_3 := [\varpi_{31}, \varpi_{32}, \varpi_{33}]^T$ is given by

\[
\varpi_{3d} := \text{sat}_{c}(v_{31d}; 0, u_{31d}^\varepsilon)
\] (84)

where $\text{sat}_{c}(\cdot)$ is defined in (3). Obviously, $\varpi_3$ is bounded, i.e.,

\[
\|G_3\varpi_3\| \leq \bar{\varpi}_3
\] (85)

here, $\bar{\varpi}_3 > 0$ is unknown.

Define an intermediate tracking error as follows:

\[
\begin{align*}
\hat{u}_{3d} &= u - g_3 \\
\hat{u}_{3d} &= u - g_3
\end{align*}
\] (86)

Using (52) and (85), we immediately have

\[
\|G_3\hat{u}_{3d}\| \leq \rho_3
\] (87)

with an unknown upper bound $\rho_3 = \|G_3\| \geq \bar{\varpi}_3$.

Together with angular velocity dynamics (11), we design a sliding surface as follows:

\[
\begin{align*}
s_{3}(t) &= u - u_{3d} - y_3 = e_3(t)
\end{align*}
\] (88)

where

\[
\begin{align*}
ey_2 &= \chi_{3} - \chi_{3d} - \mu_3 \\
y_3 &= \chi_{3d} + \mu_3 - \chi_{3d}
\end{align*}
\] (89)

with $\mu_3$ is determined later, $\chi_{3d} := [\hat{\phi}_{d}, \hat{\theta}_{d}, \hat{\psi}_{d}]^T$ is the filtered output of $\chi_{3d} := [\hat{\phi}_{d}, \hat{\theta}_{d}, \hat{\psi}_{d}]^T = u_{2d}$ and is given by

\[
\begin{align*}
\epsilon_{3}\hat{\chi}_{3d} + \chi_{3d} = \chi_{3d}
\end{align*}
\] (91)

here, $\epsilon_3 > 0$ is an user-defined filtering time constant.
Accordingly, an ideally desired AVVC for sub-system (11) can be governed as follows:

$$\mathbf{v}_{3d} = G_3^{-1} \left[ \hat{X}_{3d} - f_3(\chi_3) - G_7^T s_2 - P_3 s_3 - G_3 \mu_3 \right] + \beta_3 y_3 - (\hat{L}_3 + \hat{\rho}_3) \tanh \left( \frac{s_{1a}}{\varepsilon} \right) \tag{92}$$

with the UAC for $\mu_3$, $\hat{L}_3$, and $\hat{\rho}_3$ given by

$$\begin{align*}
\dot{\mu}_3 &= G_3 (g_3(v_{3d}) - v_{3d} - \mu_3) \\
\dot{\hat{L}}_3 &= -\gamma_{31} \hat{L}_3 + \kappa_L s_3^T \tanh \left( \frac{S_{1a}}{\varepsilon} \right) \\
\dot{\hat{\rho}_3} &= -\gamma_{32} \hat{\rho}_3 + \kappa_{\rho,3} s_3^T \tanh \left( \frac{S_{1a}}{\varepsilon} \right)
\end{align*} \tag{93}$$

where $P_3 = \text{diag}(p_{31}, p_{32}, p_{33}) > 0$, $\beta_3 > 0$, $\gamma_{31} > 0$, $\gamma_{32} > 0$, $\kappa_L > 0$, $\kappa_{\rho,3} > 0$, and $\hat{L}_3$ and $\hat{\rho}_3$ are estimates of unknown bounds $L_3$ and $\rho_3$, respectively.

In this context, the sliding error dynamics can be obtained as follows:

$$\dot{s}_3 = -P_3 s_3 - G_7^T s_2 + d_3 + \dot{G}_3 \dot{u}_{3e} + \beta_3 y_3 - (\hat{L}_3 + \hat{\rho}_3) \tanh \left( \frac{s_{1a}}{\varepsilon} \right) \tag{94}$$

where $u_{3e}$ is given by (86).

**Remark 7.** The saturation of $T_d$ in (74) can be transferred to constraints on $u_{3d}$ given by (50).

### D. Propeller Speed Virtual Controller

The desired propeller speed virtual controller (PSVC) $u_{4d}(v_{4d}) := [u_{41d}(v_{41d}), u_{42d}(v_{42d}), u_{43d}(v_{43d}), u_{44d}(v_{44d})]^T$ is designed as (49), where $v_{4d} := [v_{41d}, v_{42d}, v_{43d}, v_{44d}]^T$ is the ideally desired PSVC determined later.

Note the constrained control input $u_{4d}(v_{4d})$ defined in (49) is non-smooth. In order to facilitate a differentiable virtual control law, a differentiable function $g_4(v_{4d}) := [g_{41d}(v_{41d}), g_{42d}(v_{42d}), g_{43d}(v_{43d}), g_{44d}(v_{44d})]^T$ is employed to approximate the non-smooth constrained input $u_{4d}(v_{4d})$ as follows:

$$g_{4d}(v_{4d}) = \text{sat}_{e} \left[ (v_{4d}; n_r w_{e1}^2)/(2\eta), n_r w_{e2}^2)/(2\eta) \right] \tag{95}$$

where sat$_{e}$(·) is defined in (2). The constraint approximation error $\mathbf{w}_{4} = [\mathbf{w}_{41}, \mathbf{w}_{42}, \mathbf{w}_{43}, \mathbf{w}_{44}]^T$ is determined by

$$\mathbf{w}_{4} = \text{sat}_{e} \left[ (v_{4d}; n_r w_{e1}^2)/(2\eta), n_r w_{e2}^2)/(2\eta) \right] \tag{96}$$

where sat$_{e}$(·) is defined in (3). Obviously, $\mathbf{w}_{4}$ is bounded, i.e.

$$\|G_4 \mathbf{w}_{4}\| \leq \bar{w}_{4} \tag{97}$$

here, $\bar{w}_{4} > 0$ is unknown.

Define an intermediate tracking error as follows:

$$\dot{u}_{4e} = u_{4} - g_{4} \tag{98}$$

Using (52) and (97), we immediately have

$$\|G_4 \dot{u}_{4e}\| \leq \rho_4 \tag{99}$$

with an unknown upper bound $\rho_4 = \|G_4\| \bar{w}_{4} + \bar{w}_{4}$.

Note that the actually desired control law $u_{4d}$ can be derived from (49). Together with the following equations deriving from (7) and (13):

$$\begin{align*}
w_{1d} &= \left( -\frac{1}{\sqrt{\eta}} u_{32d} - \frac{1}{\eta} u_{33d} + \frac{1}{\sqrt{\eta}} T_d \right)^{1/2} \\
w_{2d} &= \left( -\frac{1}{\sqrt{\eta}} u_{31d} + \frac{1}{\eta} u_{33d} + \frac{1}{\sqrt{\eta}} T_d \right)^{1/2} \\
w_{3d} &= \left( \frac{1}{\sqrt{\eta}} u_{32d} - \frac{1}{\eta} u_{33d} + \frac{1}{\sqrt{\eta}} T_d \right)^{1/2} \\
w_{4d} &= \left( \frac{1}{\sqrt{\eta}} u_{31d} + \frac{1}{\eta} u_{33d} + \frac{1}{\sqrt{\eta}} T_d \right)^{1/2}
\end{align*} \tag{100}$$

we can obtain the reference $\chi_{4d} := [w_{1d}, w_{2d}, w_{3d}, w_{4d}]^T$, and the filtered signals $\hat{\chi}_{4d} := [\hat{w}_{1d}, \hat{w}_{2d}, \hat{w}_{3d}, \hat{w}_{4d}]^T$ given by

$$\hat{e}_4 \dot{\hat{\chi}}_{4d} + \hat{\chi}_{4d} = \chi_{4d} \tag{101}$$

where $\hat{e}_4 > 0$ is an user-defined filtering time constant.

Combining with propeller speed dynamics (14), we define

$$\begin{align*}
e_4 &= \chi_4 - \chi_{4d} - \mu_4 \\
\eta_4 &= \chi_{4d} + \mu_4 - \chi_4 \tag{102, 103}
\end{align*}$$

where $\mu_4$ is determined later.

Design a sliding surface as follows:

$$\dot{s}_{4}(t) = e_4(t) + K_4 \int_{0}^{t} e_4(\tau)d\tau \tag{104}$$

with $K_4 = \text{diag}(k_{41}, k_{42}, k_{43}, k_{44}) > 0$.

In this context, the ideally desired propeller speed control law for sub-system (14) can be designed as follows:

$$\begin{align*}
v_{4d} &= G_4^{-1} \left[ \hat{\chi}_{4d} - f_4(\chi_4) - K_4 \eta_{4d} - P_4 s_4 \
- G_4 \mu_4 + \beta_4 y_4 - (\hat{L}_4 + \hat{\rho}_4) \tanh \left( \frac{s_{1a}}{\varepsilon} \right) \right]
\end{align*} \tag{105}$$

with the UAC for $\mu_4$, $\hat{L}_4$, and $\hat{\rho}_4$ given by

$$\begin{align*}
\dot{\mu}_4 &= G_4 (g_4(v_{4d}) - v_{4d} - \mu_4) \\
\dot{\hat{L}}_4 &= -\gamma_{41} \hat{L}_4 + \kappa_4 s_4^T \tanh \left( \frac{S_{1a}}{\varepsilon} \right) \\
\dot{\hat{\rho}_4} &= -\gamma_{42} \hat{\rho}_4 + \kappa_{\rho,4} s_4^T \tanh \left( \frac{S_{1a}}{\varepsilon} \right)
\end{align*} \tag{106}$$

where $P_4 = \text{diag}(p_{41}, p_{42}, p_{43}, p_{44}) > 0$, $\beta_4 > 0$, $\gamma_{41} > 0$, $\gamma_{42} > 0$, $\kappa_L > 0$, $\kappa_{\rho,4} > 0$, and $\hat{L}_4$ and $\hat{\rho}_4$ are estimates of unknown bounds $L_4$ and $\rho_4$, respectively.

In this context, the sliding error dynamics can be obtained as follows:

$$\dot{s}_4 = -P_4 s_4 + d_4 + G_4 \dot{u}_{4e} + \beta_4 y_4 - (\hat{L}_4 + \hat{\rho}_4) \tanh \left( \frac{s_{1a}}{\varepsilon} \right) \tag{107}$$

where $u_{4e}$ is given by (98).

### E. Servo Motor Actual Controller

The actually desired signal $u_{5d}$ can be derived from (49) and (105). Using (16), we can obtain the desired vector $\chi_{5d} := [\bar{w}_{e1d}, \bar{w}_{e2d}, \bar{w}_{e3d}, \bar{w}_{e4d}]^T$, and $\hat{\chi}_{5d} := [\bar{u}_{e1d}, \bar{u}_{e2d}, \bar{u}_{e3d}, \bar{u}_{e4d}]^T$ is the filtered output given by

$$\epsilon_5 \hat{X}_{5d} + \hat{X}_{5d} = X_{5d} \tag{108}$$

here, $\epsilon_5 > 0$ is an user-defined filtering time constant.

Combining with servo motor dynamics (17) and the input nonlinearities (18) and (19), we define the following errors:

$$e_5 = \chi_5 - \chi_{5d} - \mu_5 \tag{109}$$
where $\mu_5$ is determined later, $u_5(t)$ is the nonlinear input constrained by saturation and dead zone in (18) and (19).

Design a sliding surface as follows:

$$s_5(t) = e_5(t) + K_5 \int_0^t e_5(\tau) d\tau$$  \hspace{1cm} (111)

with $K_5 = \text{diag}(k_{51}, k_{52}, k_{53}, k_{54}) > 0$.

Eventually, the nominal control law, i.e., the servo motor actual controller (SMAC) $v_5$, can be designed as follows:

$$v_5 = G_5^{-1}\left[\dot{X}_5 - f_5(X_5) - K_5 e_5 - P_5 s_5 - G_5 \dot{\mu}_5 + \beta_5 y_5 - \dot{L}_5 \tanh \left( \frac{s_5}{\epsilon} \right) \right]$$  \hspace{1cm} (112)

with the UAC for $\mu$ and $\dot{L}_5$ given by

$$\dot{\mu}_5 = G_5(u_5(v_5) - v_5 - \mu_5)$$  \hspace{1cm} (113)

where $P_5 = \text{diag}(p_{51}, p_{52}, p_{53}, p_{54}) > 0$, $\beta_5 > 0$, $\gamma_5 > 0$, $\kappa_L > 0$, and $\dot{L}_5$ is the estimate of unknown bound $L_5$.

Hence, the sliding error dynamics is obtained as follows:

$$\dot{s}_5 = -P_5 s_5 + d_5 + \beta_5 y_5 - \dot{L}_5 \tanh \left( \frac{s_5}{\epsilon} \right)$$  \hspace{1cm} (114)

**Remark 8.** Bounded intermediate errors in (61), (86) and (98) decouple sliding error dynamics (71), (72), (94) and (107), and leave only $\dot{s}_2$ in (81) be driven by the input discrepancy $u_{2e}$, which is closely related with the cascade sliding surface $s_3$. In addition, as shown in Fig. 4, the BCTTC scheme is composed by 4 successive virtual sub-controllers in (69), (79), (92) and (105), and 1 actual sub-controller in (112). In this context, each sub-controller for an individual subsystem can be designed independently by using various approaches although the SMC technique is exclusively employed in this paper. In essence, this significant advantage actually benefits from the BC-based cutting by virtue of bounded intermediate errors.

**Remark 9.** Note that the ESC module is still required to be used for generating PWM waves which drive and regulate BLDCs even though actuator dynamics have been completely addressed in the proposed BCTTC scheme. Unlike traditional ESC modules which are open-loop control systems, the closed-loop ESC can be achieved in the BCTTC scheme, and thereby enhancing its regulation accuracy and robustness.

**Remark 10.** Note that the BCTTC scheme only requires a nominal model, and even is a model-free approach if nominal dynamics $f_i, i = 1, \ldots, 5$ are completely unknown and thereby encapsulating into unknowns $d_i$. In addition, nonlinear state observers can also be designed to extend the BCTTC to an output-feedback control approach.

**Remark 11.** Filters applied to virtual signals might cause high-gain problem pertaining to filter-backstepping (i.e., DSC) or high-gain observer design [59]. In the BCTTC scheme, unexpected magnitudes and/or peaks are actually saturated by BC-based constraints. The SMC technique employed in sub-controllers is expected to enhance steady-state tracking accuracy via incorporating an integral term. Actually, if integral gains $K_i$ are chosen as zeros, sliding-mode surfaces degrade to intermediate tracking errors.

**Remark 12.** From (51), (69), (79), (50), (92), (49), (105) and (112), we can see that the computational complexity of the BCTTC scheme is similar to adaptive approximation based state-feedback approach.

**V. Stability Analysis**

A key result on stability analysis is summarized as follows:

**Theorem 1.** Consider a complex QUAV system (4), (8), (11), (14) and (17), together with the proposed BCTTC scheme (51), (69), (79), (50), (92), (49), (105) and (112) with the UAC given by (70), (80), (93), (106) and (113), tracking errors are uniformly ultimately bounded and all other signals of the closed-loop control system are bounded.

**Proof:** Consider the following Lyapunov function:

$$V = \frac{1}{2} \left[ s_{11}^T s_{11} + s_{12}^T s_{12} + \sum_{i=2}^{5} s_i^T s_i + \sum_{i=1}^{5} y_i^T y_i \right]$$

$$+ \sum_{i=1}^{5} \left( -\kappa_{\rho,i} \tilde{L}_i + \gamma_5 \kappa_{\rho,5} \tilde{L}_5 \right) \tanh \left( \frac{s_5}{\epsilon} \right)$$  \hspace{1cm} (115)

with $\tilde{L}_i = L_i - \hat{L}_i, i = 1, \ldots, 5$, $\rho_i = \rho - \hat{\rho}_i$, $\hat{\rho}_3 = \rho_3 - \hat{\rho}_3$ and $\rho = \rho_4 - \hat{\rho}_4$. Together with (71), (72), (81), (94), (107) and (114), and using (62), (87) and (99), we have the time derivative of $V$ can be derived as follows:

$$\dot{V} \leq \sum_{i=2}^{5} \left[ -s_i^T P_i s_i + \beta_i y_i^T y_i + L_i \| s_i \| - \tilde{L}_i \left( \frac{s_i}{\epsilon} \right) \tanh \left( \frac{s_i}{\epsilon} \right) \right]$$

$$+ \sum_{i=1}^{5} \left( -\kappa_{\rho,i} \tilde{L}_i + \gamma_5 \kappa_{\rho,5} \tilde{L}_5 \right) \tanh \left( \frac{s_5}{\epsilon} \right)$$

$$- \kappa_{\rho,1} \rho_1 \hat{\rho}_1 - \kappa_{\rho,3} \rho_3 \hat{\rho}_3 - \kappa_{\rho,4} \rho_4 \hat{\rho}_4 - \tilde{\rho}_i \tilde{L}_i \tanh \left( \frac{s_i}{\epsilon} \right)$$  \hspace{1cm} (116)

Note that for any positive constant $\epsilon > 0$ and $\mathbf{v} \in \mathbb{R}_n$ the following inequality holds [60]:

$$\| \mathbf{v} \| - \mathbf{u}^T \tanh \left( \frac{\mathbf{u}}{\epsilon} \right) \leq k_\epsilon$$  \hspace{1cm} (117)

where $k_\epsilon = e^{-(k+1)}$, i.e. $k_\epsilon = 0.2785$.

We further have

$$\left\{ \begin{array}{l} L_i \| s_{1i} \| \leq L_1 \left( \frac{s_{1i}}{\epsilon} \tanh \left( \frac{\mathbf{s}_{1i}}{\epsilon} \right) + k_\epsilon \right) \\ L_i \| s_{1i} \| \leq L_1 \left( \frac{s_{1i}}{\epsilon} \tanh \left( \frac{\mathbf{s}_{1i}}{\epsilon} \right) + k_\epsilon \right), i = 2, 3, 4, 5 \\ \rho_1 \| s_{12} \| \leq \rho_1 \left( \frac{s_{12}}{\epsilon} \tanh \left( \frac{\mathbf{s}_{12}}{\epsilon} \right) + k_\epsilon \right) \\ \rho_3 \| s_3 \| \leq \rho_3 \left( \frac{s_3}{\epsilon} \tanh \left( \frac{s_3}{\epsilon} \right) + k_\epsilon \right) \\ \rho_4 \| s_4 \| \leq \rho_4 \left( \frac{s_4}{\epsilon} \tanh \left( \frac{s_4}{\epsilon} \right) + k_\epsilon \right) \end{array} \right.$$  \hspace{1cm} (118)
In addition, using the Young’s inequality yields

\[
\sum_{i=2}^{5} \left[ \beta_{i} s_{i}^{T} y_{i} + y_{i}^{T} y_{i} + k_{i} \varepsilon L_{i} \right]
\]

Applying (70), (80), (93), (106) and (113) to (119), we further have

\[
\dot{V} \leq \sum_{i=2}^{5} \left[ -s_{i}^{T} P_{i} s_{i} + \frac{\gamma_{i1}}{\kappa_{iL_{i}}} \dot{L}_{i} + \beta_{i} \dot{s}_{i}^{T} y_{i} + y_{i}^{T} y_{i} + k_{i} \varepsilon L_{i} \right]
\]

From (66), (75), (91), (101) and (108), we can obtain

\[
\begin{align*}
\dot{y}_{i} &= \frac{-y_{i}}{\varepsilon} + \dot{\chi}_{12d} + \frac{\rho_{i1}}{\varepsilon} \dot{\mu}_{1}, \\
\dot{y}_{2} &= \frac{-y_{2}}{\varepsilon} + \dot{\chi}_{2d} + \frac{\rho_{21}}{\varepsilon} \dot{\mu}_{1}, \\
\dot{y}_{3} &= \frac{-y_{3}}{\varepsilon} + \dot{\chi}_{3d} + \frac{\rho_{31}}{\varepsilon} \dot{\mu}_{1},
\end{align*}
\]

In this context, we have

\[
\begin{align*}
\|\dot{y}_{1} + y_{1}/\varepsilon_{1}\| &\leq z_{1}(\dot{\chi}_{12d} + \dot{\mu}_{1}), \\
\|\dot{y}_{2} + y_{2}/\varepsilon_{2}\| &\leq z_{2}(\dot{\chi}_{2d}), \\
\|\dot{y}_{3} + y_{3}/\varepsilon_{3}\| &\leq z_{3}(\dot{\chi}_{3d} + \dot{\mu}_{1}), \\
\|\dot{y}_{4} + y_{4}/\varepsilon_{4}\| &\leq z_{4}(\dot{\chi}_{4d} + \dot{\mu}_{1}), \\
\|\dot{y}_{5} + y_{5}/\varepsilon_{5}\| &\leq z_{5}(\dot{\chi}_{5d} + \dot{\mu}_{5})
\end{align*}
\]

for continuously bounded functions \(z_{i}(\cdot)\).

Together with (122)–(126), we eventually have

\[
\dot{y}_{i}^{T} y_{i} \leq \left( \frac{1}{\varepsilon_{i}} - \frac{1}{2} \right) y_{i}^{T} y_{i} + \frac{1}{2} y_{i}^{T} y_{i}, \quad i = 1, \ldots, 5
\]

In addition, using the Young’s inequality yields

\[
\begin{align*}
\dot{L}_{i} &\leq \frac{1}{2} \dot{L}_{i}^{2} - \frac{1}{2} \dot{L}_{i}^{2}, \quad i = 1, \ldots, 5 \\
\dot{\rho}_{i1} &\leq \frac{1}{2} \dot{\rho}_{i1}^{2} - \frac{1}{2} \dot{\rho}_{i1}^{2}, \\
\dot{\rho}_{i2} &\leq \frac{1}{2} \dot{\rho}_{i2}^{2} - \frac{1}{2} \dot{\rho}_{i2}^{2}, \\
\dot{\rho}_{4} &\leq \frac{1}{2} \dot{\rho}_{4}^{2} - \frac{1}{2} \dot{\rho}_{4}^{2}
\end{align*}
\]

Applying (127) and (128) to (120) yields

\[
\dot{V} \leq -s_{11}^{T} (P_{11} - \frac{I_{1}}{2}) s_{11} - s_{12}^{T} P_{12} s_{12} - s_{2}^{T} (P_{2} - \frac{\beta_{2} I_{2}}{2}) s_{2} - \sum_{i=3}^{5} s_{i}^{T} (P_{i} - \frac{\beta_{i} I_{i}}{2}) s_{i}
\]

where \(I_{i}(i = 1, 2, 3) \in \mathbb{R}^{3}\) and \(I_{i}(i = 4, 5) \in \mathbb{R}^{4}\) are unity matrices.

Selecting user-defined parameters satisfying the following conditions:

\[
\begin{align*}
P_{11} &\geq \frac{1}{2} a I_{1}, P_{12} \geq \frac{\alpha}{2} I_{1}, P_{2} \geq \frac{\beta_{2} + \alpha + 1}{2} I_{2}, \\
P_{i} &\geq \frac{\beta_{i} + \alpha}{2} I_{i}, \quad i = 3, 4, 5
\end{align*}
\]

where \(\alpha > 0\) is any positive constant, we have

\[
\dot{V} \leq -\alpha V + C
\]

with

\[
C = \frac{5}{2} \sum_{i=1}^{3} \left[ \frac{\gamma_{i1}}{2 \kappa_{iL_{i}}} L_{i}^{2} + k_{i} \varepsilon L_{i} \right]
\]

where \(\tilde{z}_{i}\) is the upper bound of function \(z_{i}\).

Together with (115) and (131), we have

\[
0 \leq V(t) \leq V(0) e^{-\alpha t} + (1 - e^{-\alpha t}) \frac{C}{\alpha} < \infty
\]

It is clear that \(V(t)\) is bounded. Moreover, together with (115), there exist a finite time \(T > 0\) such that

\[
\begin{align*}
V &= \frac{1}{2} \sum_{i=1}^{5} s_{i}^{T} s_{i} + s_{11}^{T} s_{11} + s_{12}^{T} s_{12} + \sum_{i=1}^{5} y_{i}^{T} y_{i} \\
&\quad + \sum_{i=1}^{5} \kappa_{iL_{i}}^{-1} \dot{L}_{i}^{2} + \kappa_{i}^{-1} \dot{\rho}_{i1}^{2} + \kappa_{i}^{-1} \dot{\rho}_{i2}^{2} + \kappa_{i}^{-1} \dot{\rho}_{4}^{2}
\end{align*}
\]

In this context, we have

\[
\vartheta \leq 2 \sqrt{C/\alpha}
\]

where \(\vartheta \in \{\|s_{11}\|, \|s_{12}\|, \|s_{12}\|, \ldots, \|s_{5}\|, \|y_{i}\|, \ldots, \|y_{5}\|, \|\dot{L}_{i}\|, \ldots, \|\dot{L}_{5}\|, \|\dot{\rho}_{i1}\|, \|\dot{\rho}_{i2}\|, \|\dot{\rho}_{4}\|\}\).

Using (134) and Lemma 1, we immediately have the tracking error \(\|e_{iL}\|\) is uniformly bounded. Similarly, we can obtain that all the other signals including \(e_{i2}, e_{2}, \ldots, e_{5}, y_{1}, \ldots, y_{5}, \dot{L}_{i}, \ldots, \dot{L}_{5}, \dot{\rho}_{i1}, \dot{\rho}_{i2}\) and \(\dot{\rho}_{4}\) are ultimately uniformly bounded. Together with the filtered dynamics (66), (75), (91), (106) and (113) to (119), we further have

\[
\dot{V} \leq -s_{11}^{T} (P_{11} - \frac{I_{1}}{2}) s_{11} - s_{12}^{T} P_{12} s_{12} - s_{2}^{T} (P_{2} - \frac{\beta_{2} I_{2}}{2}) s_{2} - \sum_{i=3}^{5} s_{i}^{T} (P_{i} - \frac{\beta_{i} I_{i}}{2}) s_{i}
\]
TABLE I
MAIN PARAMETERS OF THE QUAV.

<table>
<thead>
<tr>
<th>Para.</th>
<th>Value</th>
<th>Units</th>
<th>Para.</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$</td>
<td>9.806</td>
<td>m/s²</td>
<td>$m_0$</td>
<td>0.65</td>
<td>kg</td>
</tr>
<tr>
<td>$b$</td>
<td>7.5e-7</td>
<td>—</td>
<td>$l$</td>
<td>0.232</td>
<td>m</td>
</tr>
<tr>
<td>$c$</td>
<td>1e-5</td>
<td>—</td>
<td>$n$</td>
<td>1.5e-4</td>
<td>—</td>
</tr>
<tr>
<td>$k_c$</td>
<td>3.13e-5</td>
<td>—</td>
<td>$r_s$</td>
<td>1</td>
<td>—</td>
</tr>
<tr>
<td>$C_m$</td>
<td>0.08</td>
<td>N/(m/s)²</td>
<td>$C_v$</td>
<td>0.0415</td>
<td>N/(m/s)²</td>
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<td>$R_s$</td>
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<td>Ω</td>
<td>$L_s$</td>
<td>1e-3</td>
<td>kg·m²</td>
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<tr>
<td>$D_x$</td>
<td>1e-6</td>
<td>N/(m/s)²</td>
<td>$J_x$</td>
<td>0.015</td>
<td>kg·m²</td>
</tr>
<tr>
<td>$D_y$</td>
<td>1e-6</td>
<td>N/(m/s)²</td>
<td>$J_y$</td>
<td>0.015</td>
<td>kg·m²</td>
</tr>
<tr>
<td>$D_z$</td>
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<td>N/(m/s)²</td>
<td>$J_z$</td>
<td>0.026</td>
<td>kg·m²</td>
</tr>
<tr>
<td>$w_m$</td>
<td>3000</td>
<td>rpm</td>
<td>$w_m$</td>
<td>3000</td>
<td>rpm</td>
</tr>
<tr>
<td>$u_o$</td>
<td>0.2</td>
<td>V</td>
<td>$u_o$</td>
<td>14</td>
<td>V</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.9</td>
<td>—</td>
<td>$\eta$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Fig. 5. Trajectory tracking of the BCTTC scheme.

Fig. 6. Desired and actual states $x$, $y$, $z$ and $\psi$.

Fig. 7. Trajectory tracking errors $x_e$, $y_e$, $z_e$ and $\psi_e$.

Fig. 8. Desired and actual Euler angles $\phi$ and $\theta$.

Fig. 9. Desired and actual angular velocities $p$, $q$ and $r$.

(101) and (108), and the UAC mechanism (70), (80), (93), (106) and (113), we can obtain that system signals including $\dot{x}_{1234}$, $\dot{x}_{56}$, $\cdots$, $\dot{x}_{56}$, $\dot{y}_1$, $\cdots$, $\dot{y}_5$, $L_5$, $\dot{\rho}_1$, $\dot{\rho}_3$ and $\dot{\psi}_3$ are bounded. This concludes the proof.

VI. SIMULATION STUDIES

In this section, the effectiveness and superiority of the proposed BCTTC scheme is demonstrated for trajectory tracking control of a complex QUAV with actuator dynamics and cascade constraints on both control input and states, in the presence of complex unknowns. Main parameters of the QUAV refer to [58] and are listed in Table I.

The reference trajectory is governed by $x_d = -2\sin(0.1t)$, $y_d = \cos(0.3t)$, $z_d = 2\sin(0.2t) + 3$ and $\psi_d = \frac{\pi}{4} \sin(0.2t)$, and the initial condition is as follows: $\chi_{11}(0) = [1, 0, 0]^T$, $\chi_{12}(0) = [0, 0, 0]^T$ and $\chi_{2}(0) = [0, 0, 0]^T$. For the sake of simulation studies, complex unknowns are assumed to be as follows: $d_1 = 5\sin(0.01\chi_{11}\chi_{12})$, $\cos(0.02\chi_{11}\chi_{12})$, $\sin(0.015\chi_{11}\chi_{12})$ $\cos(0.02\chi_{11}\chi_{12})^2$, $d_2 = 5\sin(\chi_2) \cos(\chi_2)$, $d_3 = 5\sin(\chi_3) \cos(\chi_3)$, $d_4 = 100\sin^2(\chi_4) \cos(\chi_4)$, and
User-defined parameters of the BCTTC scheme are as follows: 

\[ P_{11} = P_{12} = P_3 = \text{diag}(10, 10, 10), \]
\[ P_2 = \text{diag}(100, 100, 100), \]
\[ P_4 = \text{diag}(1, 1, 1, 1), \]
\[ P_5 = \text{diag}(10, 10, 10, 10), \]
\[ K_{11} = K_{12} = \text{diag}(0.01, 0.01, 0.01), \]
\[ K_4 = K_5 = \text{diag}(0.01, 0.01, 0.01, 0.01), \]
\[ \gamma_{11} = \gamma_{21} = \gamma_{31} = \gamma_{41} = \gamma_{51} = 1, \gamma_{12} = \gamma_{32} = \gamma_{42} = 2, \]
\[ \kappa_{L,1} = \kappa_{L,2} = \kappa_{L,3} = \kappa_{L,4} = \kappa_{L,5} = 1, \kappa_{p,1} = \kappa_{p,2} = \kappa_{p,3} = \kappa_{p,4} = 2, \epsilon = 1, \]
\[ \epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = \epsilon_5 = 0.01, \text{ and } \beta_2 = \beta_3 = \beta_4 = \beta_5 = 1. \]

The actual and reference trajectories in 3-D space are shown in Fig. 5, from which we can see that the BCTTC scheme can render the QUAV track the desired trajectory accurately in the presence of both mismatched and matched complex unknowns. Individual positions, i.e., \( x, y \) and \( z \), and the yaw angle \( \psi \) together with their desired targets are shown in Fig. 6, from which we can see that the QUAV using the BCTTC scheme can track the desired individual trajectories with fast response.
and high accuracy, simultaneously, whereby tracking errors are shown in Fig. 7. Intermediate tracking results for other states including Euler angles, angular velocities, propeller speeds and motor rotations are shown Figs. 8–11, respectively, which demonstrate that accurate tracking of intermediate states can still be guaranteed under the constraints on propeller speeds and motor rotations (shown in Figs. 10 and 11). Eventually, control inputs to 4 motors are shown in Fig. 12, from which we can see that nonreversible constraints and saturations have been effectively addressed.

Furthermore, in order to demonstrate the superiority of the proposed BCTTC scheme, comprehensive comparisons with a PD control scheme are conducted on previous settings. To this end, PD controllers are designed as follows:

\[
\begin{align*}
U_1 &= K_{p1}(\chi_{11} - \chi_{11d}) + K_{d1}(\chi_{12} - \chi_{12d}) \\
U_2 &= K_{p2}(\dot{\phi} - \theta_d) + K_{d2}(\dot{\theta} - \phi_{d}) \\
U_3 &= K_{p3}(\theta - \theta_d) + K_{d3}(\dot{\theta} - \theta_d) \\
U_4 &= K_{p4}(\psi - \dot{\psi}) + K_{d4}(\dot{\psi} - \psi_d)
\end{align*}
\]

where \( U_1 \) is control input of position dynamics (4), \( U_2 \) through \( U_4 \) are control inputs of attitude dynamics (11), \( \phi_d, \theta_d \) and \( T := m||U_1|| \) are derived from \( U_1 \) according to (74), and fine-tuning parameters are chosen as follows: \( K_{p1} = \text{diag}(20, 100, 150), K_{d1} = \text{diag}(10, 10, 10), \) and \( K_{p2} = 1, K_{d2} = 0.3, K_{p3} = 1, K_{d3} = 0.3, K_{p4} = 1.5, \) and \( K_{d4} = 0.5. \)

Trajectory tracking result of the PD control approach and comparisons with the BCTTC scheme are shown in Figs. 13 and 14, respectively, and illustrate that the BCTTC approach can accommodate complex unknowns, and thereby achieving nearly zero steady-state discrepancies which apparently appear in PD controllers.

In order to make intensive insight into the superiority of the BCTTC, quantitative comparisons using Integrated Absolute Error (IAE) and Integrated Time Absolute Error (ITAE) indices for tracking errors are summarized in Table II. Clearly, it can be seen that the proposed BCTTC scheme is significantly superior to the PD control approach. It should be noted that the PD control strategy cannot tackle constrained actuator dynamics. As a consequence, as shown in Fig. 15, negative squares of rotor rotations reversely deriving from PD control input torques would unreasonably occur, and thereby leading to unreachable control efforts in practice and even destroying system stability. Similarly, those methods taking rotor torques as control inputs would inevitably suffer from the aforementioned negative-square dilemma. In this context, the proposed BCTTC scheme via backpropagating constraints due to constrained actuator dynamics can definitely guarantee reasonable control signals which can be completely executed by actuators.

![Fig. 15. Rotation squares of PD control scheme.](image)

<table>
<thead>
<tr>
<th></th>
<th>BCTTC IAE</th>
<th>PD IAE</th>
<th>BCTTC ITAE</th>
<th>PD ITAE</th>
</tr>
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<tr>
<td>( x )</td>
<td>0.9773</td>
<td>0.0187E+4</td>
<td>3.8507</td>
<td>0.9520E+4</td>
</tr>
<tr>
<td>( y )</td>
<td>0.9726</td>
<td>0.0291E+4</td>
<td>3.0425</td>
<td>1.0152E+4</td>
</tr>
<tr>
<td>( z )</td>
<td>1.3784</td>
<td>0.0620E+4</td>
<td>2.7317</td>
<td>0.8251E+4</td>
</tr>
<tr>
<td>( \psi )</td>
<td>0.9928</td>
<td>0.3509E+4</td>
<td>2.1992</td>
<td>0.7405E+4</td>
</tr>
</tbody>
</table>

### VII. Conclusions

In this paper, the BCTTC scheme for trajectory tracking of a QUAV with constrained actuator dynamics and complex unknowns has been proposed. Unlike previous works, the entire QUAV system has been decomposed into 5 cascade subsystems connected by intermediate nonlinearities. In this context, SMC-based sub-controllers have been recursively designed by addressing underactuation and cascade constraints, whereby the preceding sub-controller provides desired signals for the succeeding subsystem. In addition, first-order filters have been employed to avoid the smoothness requirement and decouple the iterative design within the backstepping-like procedure. By virtue of backpropagating constraints (BC), intermediate controls have been shaped within reachable regions determined by constrained actuator dynamics including saturations and dead zones. Furthermore, universal adaptive compensators have been employed to dominate complex unknowns together with BC-based intermediate discrepancies. Using the Lyapunov approach, BCTTC tracking errors can be made arbitrarily small and all signals are bounded. Simulation studies have shown that the proposed BCTTC scheme can achieve high-accuracy tracking under constrained actuator dynamics and complex unknowns, and is remarkably superior to previous approaches without addressing actuator constraints or inner nonlinearities.
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REFERENCES


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