Criteria for the comparative evaluation, selection and integration of methods of financing industrial capital expenditure

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CRITERIA FOR THE COMPARATIVE EVALUATION, SELECTION AND INTEGRATION OF METHODS OF FINANCING INDUSTRIAL CAPITAL EXPENDITURE.

VOLUME I
ANALYSIS & PROCEDURES.

R.A. Fawthrop

A Thesis
Submitted for the Degree of Doctor of Philosophy
of Loughborough University of Technology.

October 1962
PREFACE

The genesis of this thesis lies in my experience as chief accountant to the U.K. and European subsidiaries of one of the faster growing American corporations. In terms of sales, profits and capital employed growth in the U.K. has been equally rapid; and a significant contributor to that expansion has been the exploitation of the financial and cost leverage potential inherent in the skilful utilisation of debt finance. I hasten to add that such utilisation was, as it must be, a function of financial management rather than of accountancy. Subsequent moderately extensive reading in the theory of capital expenditure control evoked a growing conviction that insufficient regard is paid in most developments of the theory as to the manner of financing such expenditure.

Generally, either intellectual excitement over the mathematical niceties of multilateral profitability appraisal is such that no consideration is given to the basic problem of finding the money in the first place; or a somewhat sterile controversy over the potential reactions at the interface between declared capital structure and (very) hypothetical shareholder, has caused methods of financing to be categorised into the two classes of equity and debt with an implied homogeneity of class that is not valid in fact. There is no such thing in industrial finance as 'debt', whatever its validity as an analytical concept. There are many methods of procuring finance which have as common attributes that the procurement is for a specific period and that it involves servicing and repayment under specified terms. But increasingly the period and the terms are so widely diversified, and usually so capable of varying degrees of amendment in the light of economic circumstance, that to lump these methods under one common heading for the purposes of analytical treatment (or, worse still, to use one form as necessarily representative of them all) is to assume too much. An application of the variety of debt financing schemes which are commonly available inevitably leads to the realisation that their incorporation into financial planning involves significant appraisal as to their relative costs, and - equally importantly - as to their implications for future cash-flows. In a Britain ridden with economic crises, corporate liquidity becomes as important as profitability.
There are signs that the picture is changing. Information published in the 'Social Science Research Council Newsletter', the 'Journal of Business Finance', the 'Journal of Accounting Research' and the like indicate a growing volume of current research into different aspects of debt financing. Work by Chambers, Mao and Peterson (for which references are given later in this thesis) is typical of the growing concern over liquidity as a criterion of capital budgeting equal in importance to profitability. It is of considerable interest to note that these developments require an attention to accounting practices and procedures. Hopefully, one result may be that accountants will cease to follow only with such grudging scepticism the pioneering concepts of economists and mathematicians, and will start to contribute their technical realism to a joint solution of these problems. This present thesis is one such attempt, however imperfect and incomplete.

In its preparation, I must gratefully acknowledge the help and advice received from so many academic and industrial friends and colleagues. Especially am I indebted to the senior management of the various Finance Houses enumerated in the text for their courteous and patient response to my many questions; to the small number of necessarily anonymous members of the Inland Revenue who have without prejudice expressed their private opinion on various issues; to Messrs. J.R. Barber, Ram Aiyar and G. Hayhurst who over three years have endured and implemented my ineptly stated computer programming requirements; and to my wife for patience and fortitude above and beyond the call of housewifely duty in the preparation of the typescript.
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THE EVALUATION TABLES

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This thesis is concerned with establishing criteria whereby an optimum managerial choice may be made between alternative methods of financing industrial equipment. The alternative financing methods considered are: equity-financed outright purchase, debt-financed outright and deferred purchase, and non-ownership acquisition by leasing. Each method gives rise to a different pattern, in value and in time, of cash flows and consequent reinvestment opportunities. The chief sources of these variations are differences in the taxation allowances, investment grants and disposal values which ensue to the equipment itself; on which are superimposed different patterns of deposit, amortization and interest payments. Due to the element of time, practicable combinations of these parameters are most conveniently evaluated by discounting techniques; yielding a comparative basis on which criteria for the selection of financing method can rationally be based. But such selection cannot be made in isolation of the fundamental requirements of total corporate planning. In particular, there must be considered the dictates of financial strategy as expressed in capital structure and its potential impact upon investment decisions and growth criteria. What is sought to be presented here is, first, a set of evaluations upon which financing decisions can readily be based; and second, a procedure wherein such decisions can be integrated into a total capital expenditure budgeting system. Considerations are restricted to manufacturing, construction and distribution plant and machinery; buildings and data-processing equipment being specifically excluded from the study.

The major argument is presented in two parts, supplemented by an abbreviated statistical analysis. Part One analyses the conventions, calculations and uses of the Evaluation Tables - a series of tabulations wherein a wide range of assumptions as to taxation allowances, investment grants and disposal values are evaluated within the terms of alternative financing methods. The Tables allow for variations in both the earning rate at which the reinvestment potential of each method can be exploited, and the discount rate which is used for appraisal. Some of the
non-quantifiable aspects of hire-purchase and leasing are reviewed as part of the appraisal, and specific attention is paid to the different implications which obsolescence may hold for each financing method. The Tables are examined with a view of establishing orders of significance for the different parameters. Rational criteria are thus established for the selection of financing methods.

In Part Two, there are considered the relationships of financial decisions based upon the Evaluation Tables, to the broader requirements of corporate financial strategy. The criterion of economy of financial choice is considered within the potential constraint of capital structure insofar as this may reflect upon investment decisions. A rescheduling of investment decisions over time is shown to have important implications for financial decisions so far as the debt servicing-adequacy of cash flow is concerned. Using a mathematical programming approach, the framework of an accounting model is developed so as to solve simultaneously the twin criteria of optimum profitability within a selected set of projects, and maximum liquidity-generation by that set of projects so as to facilitate optimum financing selection. This is extended into a less rigorous procedural approximation, wherein conventional treasurership routines are developed which incorporate, albeit imprecisely, the criteria developed by the model into a capital expenditure budgeting programme making wide use of the Evaluation Tables.

The argument thus far establishes certain criteria of economy and flexibility upon which the financing decision in respect of industrial equipment can be made. The Evaluation Tables are set in discounted value terms, and their correct usage requires that there be selected that discount rate which is appropriate to the firm. The calculation of cost-of-capital based discount rates is a major subject in itself; but Part Three takes the form of a short statistical supplement wherein an examination is made of the possibility of making such a selection on an 'industry' basis as a matter of practical utility. The argument of this Part is not meant to be exhaustive, but relates directly to certain problems of discount rate quantification associated with the model developed in Part Two.
PART ONE

FINANCIAL DECISION MAKING
CHAPTER ONE

THE IMPLICATIONS FOR FINANCIAL MANAGEMENT

Introduction: The Financing Decision

This thesis is concerned with a fundamental management decision: how best to finance a capital expenditure project. Initially, there is an assumption that the project has been appraised by some procedure and measure, and has passed those tests. That is, the project has been shown to be consistent with company objectives, technologically sound and economically devised; and is estimated to yield an acceptable level of profit, judged by some measure of profitability. There is thus a derived assumption that the decision to proceed with the project (the investment decision) can be separated from the financing decision. This second assumption is discussed in Part Two of this thesis; where, although the convenience of the separation is conceded, the assumption itself is thrown into doubt on the grounds of logical argument. Separation is shown to be all the more illogical where the appraising measure is one based upon a discounting process. Nevertheless, the computational difficulties of integrating the two sets of decisions are great, and on these grounds alone an initial separation is conceded in a procedural approach.

A decision as to the method of financing one project will usually impinge upon decisions as to the financing of other projects which the company may wish to implement. The total investment programme sets a first figure to the overall amount of finance which is required. There may be occasions when the amount so required is acceptable to the company and is immediately available, or can easily be obtained. The logic of a discounting appraisal process, set within a managerial objective of maximising the value of the company to its shareholders, is that the company will proceed with the investment programme. More frequently the total amount of finance required will exceed that which the company is able to sustain. The 'sustainment limit' may be set by many things: but chief among them will be inability to service the amount in terms of remuneration and/or repayment, over-riding corporate strategy, limitations to managerial capacity and credit or currency restrictions imposed under national planning. It will be argued
in Part Two that the first of these is not nearly so finite a limit as is assumed in much of the literature on capital rationing situations, due principally to the growth of instalment debt (especially if this is imperfectly disclosed). It will be further argued that a major aspect often attributed to corporate strategy - that of a predetermined optimum capital structure - is distinctly suspect. But observably many companies are short of funds, and capital expenditure budgeting under conditions of capital rationing is a valid segment of management science.

Projects thus compete for more or less scarce resources of management and money. Given that there are usually several different ways in which a piece of equipment can be bought, a building project financed and development expenses under-written, it would seem axiomatic that management should be as concerned most profitably to exploit such alternatives as they are to secure the most efficient plant etc. Even where no capital rationing exists, efficiency in the finance function is as mandatory as it is for production, selling and the rest of the organisation. Capital rationing merely adds constraints to the problem. These may be stringent, but they do not alter the basic management responsibility - to be efficient, to be profit-conscious.

This then is the subject matter of Part One of this thesis: to examine broad alternatives of financing method (1) in terms of the non-financial and financial implications of each and to present the set of Evaluation Tables which, under a moderately wide range of alternative parameters, seek to present a specific comparative measure of their various costs. For the time being, the broader issues of budgeting and strategy are postponed. The question which is now sought to be answered is the unambiguous one of "is it cheaper to finance this project by outright acquisition or by an instalment plan; and is it cheaper to use the firm's own money or borrow it from someone else?" where both parts of the question are directed solely to the one project.

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(1) It is not proposed to submit a detailed array of hundreds of different financing variants, and subject each one to scrutiny. The very large majority of such variants can be closely aligned with one or another of the broad alternatives to be discussed.
Section 1. Methods of Financing

The alternative financing methods generally available to the firm, and which are considered in this thesis, may be classified as variations on a two-vector array:

<table>
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<tr>
<th>OWNERSHIP</th>
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<td>Borrowed Money</td>
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For purposes of discussion, financing methods are broken down into six broad classifications:

1. Use of own equity funds.
2. Use of borrowing.
   2.1. By issuing a formal acknowledgement of debt such as a debenture or promissory note.
   2.2. By establishing a bank overdraft, acceptance credit or similar banking instrument.
   2.3. By establishing a line of credit or revolving fund with an industrial finance house.
   2.4. By ad hoc financing contracts involving a series of period payments comprising part capital and part interest; whether there is a transference of ownership thereby or not.
   2.5. By operating rentals.

In the following discussion, Class 2.1. is referred to as 'funded debt'. Such funded debt may be publicly negotiable, or it may consist of a private placement which (though theoretically negotiable) is not expected to be so considered and which would probably enjoy only a narrow market. For reasons of convenience only, negotiable funded debt and all banking instruments (Class 2.2.) are termed 'primary borrowing' - solely because these will be the debt resources which management will first consider if only because of habit and familiarity. Private placings of funded debt and Classes 2.3. & 2.4. are termed 'secondary borrowing' - again because they represent methods of borrowing which are less immediately familiar to most managers. Class 2.5. (operating rentals) is used to cover that method of asset acquisition which is most frequently met with in service equipment - e.g., communication equipment, data processing equipment and transport facilities. The essence of this
method of financing is that it is cancellable on giving due contractual notice and (perhaps) after payment of some moderate cancellation costs. The possibility of cancellation is explicitly recognised in the contract as a major part of the terms thereof. By contrast, Class 2.4. (known henceforth as instalment debt) does not envisage cancellation. A precautionary provision for cancellation may be written into the contract, but the relatively very high penalties of one sort or another attaching to such an act clearly indicate that non-cancellable is really envisaged. Indeed, cancellation clauses may be specified as being non-operable at any penalty cost for some minimum period. (But contractual provision for replacement of obsolescent with more modern plant during the term of the contract is not ruled out). The real distinction between operating rentals and instalment debt lies in the immutability of the servicing charge. It is this which designates operating rentals as a cost of sales, compared with instalment debt servicing charges which are clearly financing costs. For similar reasons, block discounting or debt factoring schemes are classed with operating rentals. This differentiation is of course merely one which is convenient and relevant for financial analysis.

In view of the very different pattern of cash flows generated by different financing methods, the fundamental differentiation between leasing and owning is perhaps more important. Both in the U.K. and the U.S. the basic question asked by the revenue authorities is - is it intended that the lessee shall acquire all or part of the equity, without regard as to how or at what rate that acquisition takes place. Any such acquisitory intent classifies the transaction as a sale, not a lease. It is noteworthy that the U.S. authorities are the more stringent in their tests; thus it appears that whereas a lease is divided into a primary and a secondary period (the typical U.K. practice), the transaction is the more likely to be defined as a lease if (a) there is likely still to be a significant (up to 20%) but not excessive residual value in the asset at the end of the primary period; and (b) the secondary period rentals are not inordinately - i.e., having regard to (a) - reduced from the primary period rentals. Yet so long as the option quotes a figure reflecting the 'significant residual value' an option to purchase at the end of the primary period does not
necessarily disqualify the transaction from classification as a lease. In the U.K. such an option would at once cause the transaction to be defined as deferred purchase, but whether the primary period does or does not exhaust the working life is only a matter of commercial strategy rather than a Revenue requirement. (1) A study of the renewal credits accruing to equipment returned at the end of the primary period indicates that the lessors usually aim to exhaust the working life. Secondary period rentals are usually 'inordinately' reduced. Owing to the existence of investment grants, it is doubtful whether the U.K. authorities will agree to classify as a lease anything less than a "full pay-out" transaction - i.e., the primary period rentals must recover the cash cost equivalent of the asset. "Partial pay-out" plans are common in the U.S. (Care must be taken not to confuse residual value and cash cost equivalent. The former is a function of working life, irrespective of cost).

Additionally, the differentiation between operating rentals and instalment debt is deliberately extended to deferred payment schemes in this thesis, as quite moderate cancellation provisions are, increasingly, being written into deferred schemes operated by manufacturers. It is of course possible to cancel hire-purchase schemes by allowing the vendor to repossess - but the loss of payments made up to the date of repossession or of equity established by those payments is a substantial penalty.

It is not a necessary condition of instalment debt that ownership be synonymous with usage in part or in whole: and hence this classification includes a wide range of hire-purchase and leasing agreements. In what follows in terms of financial argument, primary borrowing and instalment debt are the principle objects of consideration. Apart from any commitment charge, Class 2.5. is not really assessable until it materialises as one of the other classifications. Usually, operating rentals and deferred payment schemes are too diverse in their pattern, too tailored to a given situation to permit of any treatment other than the most general. Finally: there are excluded from the following considerations property or space leasing, short period convenience leasing and full-maintenance computer rental or "third party" computer leasing.

(1) As will be seen, recent developments are testing this.
This last is excluded because of the possibility of investment grant revocation.

What is seriously considered here is that a company already in possession of sufficient funds of its own to finance a certain project, may yet find it more profitable to borrow the necessary finance and use its own funds for some other purpose. Essentially the concept is one of opportunity cost, where the opportunity cost is the earnings which can in general be expected to accrue to new assets in the company. If a given method of instalment debt finance is examined as an alternative to acquisition by using presently available equity funds:

(a) Quite different streams of cash flow will be seen to accrue, resulting largely from the complex of grants and taxation allowances and liabilities peculiar to each method.

(b) In some cases this difference in cash stream will be accentuated by a salvage or disposal value which attaches to ownership but not to rental.

(c) Compared with the total committal of funds inherent in the utilisation of own funds in outright purchase, such forms of borrowing permit a decreasing utilisation of a residual equity capital in other areas of the company.

This third point highlights a complexity of the investment decision which often appears to be overlooked in the literature of financial theory. We must also review an all too often overlooked complexity of the financial decision. Borrowing is usually seen as the raising of funds by the execution of a formal (and usually negotiable) instrument of debt acknowledgement; or as a negotiated bank credit facility with a definite term. Both forms are seen to carry a strong implicit

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(1) A comparison of the costs of outright purchase by funded debt against the costs of outright purchase by own funds is bound to reflect adversely against the former, given that (because purchase is outright) the same tax flows will accrue to both decisions. It may be argued that the investment of the (diminishing) residual fund of equity capital is a second investment decision. My argument is that it is not however a separable decision, because its size, pattern and term is determined by the instalment debt contract of the first decision. We have in fact a "composite" reinvestment decision. No such restrictions of pattern or (probably) term would attach to the alternative investment of equity funds freed by loan funds devoted to outright purchase; and it therefore is practicable to analyse the situation in terms of two independent decisions. Notice that the conditions here examined are quite different from what are usually referred to as "dependent" and "mutually independent" decisions, which are so categorised on technical or resource grounds. (c/f Bierman & Smidt, 8 Chapters 3 & 5).
utilisation in the outright acquisition of assets. But realistically this is much too restrictive. There are many other forms of borrowing, and borrowing can be used to finance further borrowing. A useful classification is by use:

A. Single lump-sum borrowing
   1. Outright single-payment purchase of an asset.
   2. As working capital, subsequently to assist in the servicing of deferred payment or instalment debt financing.

B. Period drawing facilities
   In the form of revolving funds or lines of credit, to service as they arise:
   1. Acquisition of a series of assets.
   2. Servicing of a series of deferred payment or instalment debt schemes.

C. Contractual credit schemes
   Offers of deferred payment or instalment debt facilities by the asset manufacturer or his financial agent.

Category B2 will usually be indirect - e.g., the use of line of credit to finance stock-holding, thus freeing earnings to finance a leasing contract. But this involves two separable investment decisions - e.g., to hold stocks and to lease (say) plant - and so this category is excluded. The other categories include for far more flexible systems of loan financing than those which most writers in financial theory seem prepared to specify; and, as will be discussed, tend to reduce the generality of application of some of their arguments.

Perhaps more importantly, account needs to be taken of 'pyramiding' and 'linkage', both of which are essentially forms of "borrowing in order to borrow". By 'pyramiding' is meant the application of one category of credit to finance a second and often enlarged category. Thus a bank overdraft may directly or indirectly finance a large value hire-purchase debt - a process with a history of abuse. By 'linkage' is meant the ability of the net cash flows of one project to service the financing of a second. Traditionally this is the purview of retained profits; but here a specific raising of new loan capital to promote early and/or high cash flow yielding projects is envisaged. In recent years, the rapid development in terms of both scope and type of instalment
debt has made this 'linkage' increasingly possible. To accumulate retentions until a necessary capital sum is available is a long process - and very restrictive in that utilisation of the accumulated retentions is a 'once-for-always' decision. But if cash flow as it emerges can be ploughed back into instalment payment, the pace of growth can be quickened and a wider range of appraised projects considered - for the process is conceivably a geometric progression. Capital rationing becomes much more flexible. This potential 'linked' financing is an important aspect of the way that the financing decision in respect of one project impinges on those of all other projects.

Vancil, in one (1) of the few studies which is specifically directed towards instalment debt (principally, in his case, towards leasing), uses the concept of the firm's "credit pool":

"A credit pool, then, refers to the amount of fixed obligations which a corporation has outstanding at any point in time. In talking about a corporation's credit pool, it is important to draw a distinction between the size of the pool and the extent to which it is utilised. Theoretically, it is hard to conceive of an absolute limit on the size of the credit pool in a specific instance------ It may well be that a corporation has exhausted its credit at a 6% interest rate, but additional debt may be available at 8 or 10%. If new debt is issued at 8%, this does not mean that the company's credit pool has grown; the company is just using more of the credit that was there all the time."

If the concept of "borrowing in order to borrow" is correct, it seems reasonable to claim that this is for all practical considerations a widening of the credit pool. For if the first stage borrowing is not carried out, the derived borrowing facility does not exist.

The question may be asked - what then prevents the firm from widening the credit pool to an infinitely large size? The answer

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(1) R.F.Vancil (34) Chap.2. The distinction between operating rentals and instalment debt used in this thesis - that of a cancellation provision - is derived from Vancils use of the same concept to distinguish between operating leases (rentals) and financial leases (equipment leases).
must be - nothing but there are limits to the extent to which the pool will be utilised. Factors which set those limits include as the more important ones:

1) Constitutional limits set by the Articles of Association.
2) A diminishing debt servicing-adequacy of the cash flow. (1)
3) The logical irrationality of a management objective of maximising the size rather than the value of the company to the detriment of the equity.
4) Very broad and usually uncertain limits set by institutional conventions. At times these can be rendered much more stringent by national monetary or economic policy.

These factors are discussed in detail in Part Two of this thesis. A notable omission from this list of limiting factors is:

5) A managerial financial strategy manifested in a predetermined capital structure which reflects a presumed optimal or otherwise preferred equity-debt ratio.

If this factor (i.e., a predetermined equity-debt ratio) is not so much strategic planning as plain managerial prejudice against debt, it perhaps ought to be included in the list. But the validity of a predetermined equity-debt ratio as an expression of financial planning is questioned in this thesis, and such a ratio would then cease to be a limiting factor.

However, and subject to such limiting factors; if there otherwise exists a somewhat stringent capital rationing situation, "borrowing in order to borrow" especially in the sense of a 'linkage' potential might become a criterion as important as that of minimising financial cost - i.e., in this sense liquidity might be nearly as important as profitability. But this is an aspect of the integrated total system model of Part Two. The primary criterion must be profitability if maximisation of the value of the company is to be the managerial objective. In terms of the financing decision, this means cost minimisation. This can be established only by a comparative evaluation of alternative financing methods. But quantitative evaluation cannot very well take into account certain more subjective

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(1) 'Servicing-adequacy' of cash flow - the ability of a projected net cash inflow to sustain the interest cost and redemption of debt, having due regard to all prior claims (legal and strategic) and the requirements of equity growth. The concept is discussed at some length in Part Two, Chap. Five.
considerations of undeniable importance. It is as well to establish these as a necessary environment within which a purely quantitative judgment must be subjected to critical review.

Section 2. Non-Quantifiable Considerations

These considerations are reviewed under the headings of Ownership Preference, Improved Credit Resources, Administration Economics, and Control.

The Ownership Controversy

"Profits accrue to use, not to ownership". This is a fundamental part of the argument in favour of instalment debt, especially leasing, and it figures largely in promotional advertising. (1) Undoubtedly there is an emotional bias in favour of ownership of plant to be found in U.K. manufacturing industry. (It is perhaps not so surprising to find much less bias in construction and in transport, where there has for many years been a wide-spread familiarity with the rival concept of contract-hire). It is difficult to support this statement with statistical fact. All that can be said is that conversation with non-financial executives reveals such a bias time after time; and this includes conversations with senior managers from such leading firms as British Leyland, Rolls Royce, Hawker Siddeley, Viyella, Courtaulds, Cancellloid and Dunlop - as well as upwards of thirty small private companies. By contrast, managerial reaction at B.O.C., Calor Gas, Tarmac, Margreaves Group, F.C. Construction, Amev(Oxford) and Everingham (all with large contracting or transport industries) was much more relaxed. As Mr. H.J.D. Kearns, Chairman of the Kearns Machine Tool Co. observed:

"It is difficult to sell the idea of leasing to users. Very many customers still prefer to own their machine tools; sometimes this is sheer pride of ownership, but it also means that the owner is able to do exactly what he wishes with the machine, that is, convert it to other uses.

(1) See for example that of United Leasing Corporation Ltd. Other leasing companies documentation studied in connection with non-quantifiable considerations includes that of Astley Leasing, Mercantile Credit, Forward Trust, Bowmaker, U.D.T. British Wagon, Black Arrow and Lombard Banking: together with leasing facilities offered by Staveley Industries, Alfred Herbert, Kearney Trecker, and Ford & Slater (Leyland Motors).
or alter it as he wishes."  (1)

Sometimes this emotional bias is reinforced by ignorance (the word is not used in a pejorative sense). Thus at a seminar on investment appraisal conducted by me at Northampton, nineteen out of twenty-seven managers in small and medium-sized firms were not aware that plant could be leased - and yet all of them were sufficiently 'forward looking' in their managerial approach to already be making tentative attempts to use discounted cash flow in investment decisions.

Similarly, six out of fifteen company accountants working for South London firms which use budgetary control as a routine instrument of management (and this exhibits some degree of modernity of outlook) were not aware that office equipment other than computers could be leased. Finally, a half-way emotional attitude was expressed to me by a small group of managers from the Lucas-Girling Group:

"If your plant has a short life, or your product is soon overtaken by new models, leasing might well be advantageous. But if you have long life plant and a technologically static model, you need to own your plant."

a partly rational statement in that it focusses attention upon the relationship of leasing to obsolescence. But it still obviously associates leasing with stop-gap measures.

Undoubtedly profits do accrue to use rather than ownership. This ignores any advantages accruing to salvage or trade-in values, (2) which exceptionally might be very high: thus 10-year old, 5-20 ton presses in the Midlands are currently fetching higher prices than when new! The linked problems of economic life and residual value are discussed later. This apart it seems strange that in the face of so obvious a truism as 'profits accrue to use', management should give free reign to an emotional preference for ownership. As this

(1) Quoted in 'Totalworking Production': "Is there a future for machine tool leasing?" June 1967.

(2) Slack Arrow, alone of all leasing companies queried on this point, stated positively that taking out a fresh lease on the expiration of a primary period on the old lease would automatically earn improved terms as a 'reward' for continuity of business. It is more usual to find that such favourable treatment is conditional upon the disposal proceeds raised by the sale of assets returned after the primary period, exceeding some notional base residual value. However, this notional figure may be quite low - e.g., Bowmaker quote a figure as low as £25 for some leases.
is not a thesis in industrial psychology, it is not pertinent to attempt to establish to what extent this preference is merely a convenient metamorphosis of several potentially more rational reactions. But it is useful to point out what those reactions might be, whilst acknowledging that the "old fashioned" virtues of proud independence and thrift, together with a strong sense of property, might still be very relevant!

1. Risk Aversion

Chief among these reactions may well be a dislike of debt which reflects instinctive recognition of the immutability of interest charges and capital repayments. There is in this case a fundamental distrust of the servicing-adequacy of cash flow, listed on Page 16 as a factor which limits debt. Indeed, it is argued in Part Two of this thesis that within widely set limits, servicing-adequacy of cash flow is the principle determinant of debt. An emotional preference for ownership may well mask an inability or unwillingness to forecast cash flow. Whether this arises from a lack of expertise or of confidence, or even of mental energy, the risk of debt will be avoided. This reaction is noted in a letter from the Financial Controller of a very large U.K. capital goods manufacturing firm:

"....neither scheme has been particularly successful as buyers....appear to want to own their plant.... as they are not keen to increase their bank borrowings, which affects our own scheme" (1)

The schemes referred to cover both deferred payment and leasing arrangements, the former especially making wide and profitable use of bank facilities underwritten by this firm. Insofar as a preference for ownership is then partly hidden instinctive risk-aversion, the Evaluation Tables presented here may be of some use in rationalising that aversion. Obviously, ability and confidence in forecasting the servicing-adequacy of cash flow is a matter of management education, but the Evaluation Tables must help in the subsequent decision by quantifying the comparative advantages to be gained from pledging that cash flow.

(1) Private correspondence, in which a request for anonymity was specially requested.
2. Freedom to Modify

In the quotation on Page 17, Mr. H. J. D. Kearns refers to the ability to modify or convert plant which ownership confers. It certainly is a feature of most financial leases that the consent of the lessor must be obtained before such modifications etc. are effected. This, obviously, is to protect the estimated residual value which the lessor has taken into account in setting up the leasing terms. The same is largely true of deferred payment agreements. How strictly this observance is enforced is impossible to say, and much must depend on individual cases. My own personal experience with three major U.K. leasing companies, and conversation with the management of two others, leads to the following very tentative conclusions:

1. Minor modifications to auxiliary components which do not alter the basic usage of the equipment can in fact be carried out almost without notification. An example would be a complete modification of the discharge gear of a road tanker.

2. The consent of the lessor is generally easy to obtain if the modification improves the performance or capacity of the basic usage. An example would be the application of numerical control to a machine tool.

3. Only if the modification narrows or restricts the basic usage is the lack of the right to modify or convert really onerous. Where the modification or conversion is that of a relatively low-capacity general purpose machine to some high-capacity but specialised usage, the lessor's consent may be difficult to obtain. An example would be the replacement of steam heated drying units by infra-red units on textile finishing machinery which is to be used exclusively on synthetic fabrics. Lack of ownership is then a definite disadvantage. In the case of a deferred payment scheme, even in this case, consent would be readily forthcoming if the contract were in the later stages of its life.

It is impossible to quantify for decision making purposes what are the costs of having to seek consent, or of the risk that consent might be withheld. Generally that cost would not seem realistically to be great, but (as in many other matters) management will act on what they believe to be the truth rather than on what is the truth.
One indisputable usage-restriction must be noted. This is the effect of increased leasing charges for double or treble shift working. From a lessors point of view, this is fair as his equipment is subject to a reduction in economic life and possibly of residual value. But it is an undeniable deterrent to the user; who does not gain the same cost reduction as would accrue to a wider spread of depreciation, and who does not enjoy any less costs than under ownership.

The Improved Credit Resources Controversy

These two underlying motivations for the ownership preference; that is, aversion of the risk accruing to debt, and restriction on modification rights; can be described as fears of a loss of flexibility in subsequent decision making. They are largely non-quantifiable, although a measure of the gain from risk acceptance is afforded by the Evaluation Tables. Yet another unquantifiable feature of instalment debt can be described as a gain in flexibility. This is the increase afforded thereby to the debt capital sources open to a company - a widening of Vancil's "credit pool". There seems to be little point in following his semantic argument as to whether it is the pool which is widened or the exploitation thereof improved. So far as the financial manager is concerned the question is - does the use of instalment debt improve the flow of capital available to him? Within limits already noted, I think that it does. This is not so much because of "clean balance sheets" or any other non-disclosure of current debt; although this may be of some assistance. In this country, there is no statutory obligation to disclose in published accounts any more than a bald statement of charges for the hire of plant and machinery, which presumably includes leasing charges. There is certainly no obligation to disclose either as part of the Balance Sheet or by way of a note annexed thereto the capital value (however calculated) of leased plant: and it is by no means universal practice to disclose any balance of the capital value of plant acquired under hire-purchase or deferred terms insofar as that balance is not yet part of the equity of the company. In the U.S. the recommendations of the

(1) See the quotation on Page 15.
(2) 1967 Companies Act, Schedule 2 (12).
(3) The alleged justification for non-disclosure of outstanding balances of capital value being that "ownership is necessary before an item can be classified as an asset".
Institute of Certified Public Accountants have since 1949 been that leasing rentals should be disclosed as part of the accounts or by way of a note thereon where they are 'long term' and 'material': and a research study of the same body recommended in 1962 that the user rights under a lease be classified in the Balance Sheet as a form of asset, offset by a compensating rental suspense liability.\(^{(1)}\)

However, the 1963 Survey "Another Look at Leasing"\(^{(2)}\) indicates that of 186 U.S. companies using leased equipment, only one-third found their commitments sufficiently material to warrant disclosure as to rentals, and none in fact disclosed any sort of capitalised value of leasing rights.

But does non-disclosure in fact expand credit facilities?

The 1963 Survey quotes one financial executive:

"You cannot cover lease obligations with Balance Sheet ratios. Ability to pay either rent or debt service is the important test, since it takes dollars to cover either of these obligations".

This opinion seems to be confirmation of that secured earlier by the Vancil and Anthony survey of 1959\(^{(3)}\) where 16% of corporate respondents opined that leases were equivalent to debt, requiring adequate cash flow as the primary consideration and that creditors reacted to leasing commitments accordingly. However 21% of financial analysts responding to this survey said they did not regard leases as debts, and 24% implied that they were so wedded to conventional Balance Sheet ratios as to allow non-disclosure of lease commitments to exclude such types of debt from their analysis. Incredible though this may seem, a follow-up enquiry elicited the remarkable fact that out of the 75% balance of analysts who (presumably) did regard leasing as equivalent debt-finance, only about a half could demonstrate formal procedures which would automatically test for such equivalent

\(^{(1)}\) J.H. Nyers (79).
\(^{(2)}\) H.G. Hamel & C. Thompon (63).
debt. The Hamel-Thompson survey alleges that by 1963 there was "a growing recognition that leasing is primarily a financing device, and the inclination of bankers, accountants, credit analysts and institutional investors to regard all long-term leasing as equivalent to long-term debt has produced a gradual trend towards making some disclosure of lease obligation". No evidence is preferred in support of this cautious statement.

There may be some truth in the claims that instalment-debt (especially leasing) permits some extension of credit by reason of non-disclosure of such commitments. A more likely-sounding explanation of this phenomenon lies not so much in non-disclosure as in the rarity with such debt capital is made subject to the restrictive covenants which typify funded debt. (But a form of 'Reverse Covenant' may exist. Thus, Astley Leasing requires a specific waiver from debenture holders before concluding any leasing contract). (1) As a result, a company is free to "go the rounds" of finance houses and leasing manufacturers - where the competition to sell may be such that not too many questions are asked concerning existing commitments. For the same reason, of lack of restrictive covenants, a company can reasonably hope to escape from the limits set by the widespread reluctance of primary and secondary lenders to become over-involved in the financing of any one company. Nevertheless, it is postulated that the real expansion of credit facilities afforded by instalment debt arises out of the two aspects of "borrowing in order to borrow", with especial emphasis on "financial linkage". If the servicing-adequacy of cash flow is insufficient, disclosure or non-disclosure, covenants or no covenants will cease really to matter to lender or borrower alike. (2)

(1) Information provided to the author by Astley Leasing Co.Ltd., and substantiated by sight of actual contracts containing this condition. Copies of typical instalment-debt contracts provided by U.D.T. contain the same condition from time to time.

(2) A special extension of this in the field of lease financing may lie in the claim that 'leasing permits 100% financing'. This is duly recorded in the Hamel-Thompson 1963 Survey. However, a relatively brief review of the deferred payment terms now offered by several U.K. machine tool manufacturers leads to the unavoidable conclusion that the early payments under such schemes are so small that the same claim can realistically be made for this form of debt finance too.
The Packaged Cost Controversy

It is claimed on behalf of leasing and hire-purchase that the user is spared the expense and inconvenience of various operations incidental to the ownership of plant. Without doubt the supplier incurs the expense instead, and transmits this to the user as part of the leasing charge. Nevertheless, the user might thereby enjoy the cost minimisation accruing to the supplier's specialised skill and larger scale of operating in industrial finance. To the smaller firm especially this 'package service cost' might represent a very significant saving.

These incidental expenses will include:

Legal and Professional Costs: On bank overdrafts, the form of debt most commonly used by small companies, these costs are restricted to loss of management time in arranging the overdraft facilities. The only small advantage then offered by leasing or hire-purchase is that negotiations may be conducted in his own office and may be less prolonged. But if a private placing of equity or of funded debt is the alternative, then leasing or hire-purchase may offer a very considerable saving in time and money. Conversely, as third-party(1) borrowing charges automatically include for this service, the user who never in fact would utilise funded debt is paying something of an excessive charge. Presumably, for the large company with fully staffed secretarial and accounting functions, the legal and professional costs of raising new debt or equity finance must be regarded as costs marginal to an already large fixed overhead. Within those marginal costs, and on the realistic assumption that the amount involved is large (else why even consider such formal capital?); it is improbable that the cost in highly-paid managerial time is any less in raising a large line of (say) leasing credit than it would be in negotiating a new equity issue or a placement of funded debt.(2) Brokerage or investment banker

(1) i.e., leasing or hire-purchasing by way of a finance corporation such as Astley or Bowmaker, rather than direct from the equipment manufacturer.

(2) Many senior financial managers seem shocked at the thought of gearing up by the use of leasing rather than funded debt. Rejection seems instinctive. It is quickly rationalised as "leasing is more expensive", but close questioning reveals that this has never been really quantified.
etc. charges may be counter-balanced by commitment fees and (for the less than blue-chip company) an increment to that fee or to the leasing premiums, reflecting the fact that if third-party leasing is being resorted to (as almost certainly it will in these circumstances) then the leasing company is lending its credit status in support of the lessee's. There must however be considerable savings of stamp duty and the like if leasing is substituted for equity.

It may be argued that these are all quantifiable items which can be added to the result of an Evaluation Table comparison. This is so; but there are various reasons for not too closely evaluating comparative legal and professional costs. Firstly, there is the issue of convenience, which is rather more than a straight cost of management time: it is more an opportunity cost - difficult to measure, always changing but very real in a shortage of managerial expertise. Secondly, it is true to say that capital raising expenses ensue from one raising operation to another. Within the firm, the learning curve is very steep: outside the firm, all participants are better acquainted with the firm's strengths and weaknesses. Thirdly, as one raising operation is seldom exactly the same as the previous one, it is not easy to predict just what the costs of professional advice and administration, especially, are going to be. There is a case for regarding legal and professional costs as to an extent one of the imponderables. Some of the more obvious and fixed comparative costs - e.g., stamp duty, brokerage fees, - must be quantified.

Accounting and Recording Costs: On the assumption (not universally realistic) that the user normally keeps a plant register and more-or-less detailed fixed asset accounts, it is argued that leasing or hire-purchasing reduces such costs. As this implies that the user does not care what happens to plant when he does not own it, the argument is quite unacceptable. If "profits accrue to use, not to ownership", the user must be concerned to achieve the efficient promotion of an investment, regardless as to how it is financed. Data of non-utilisation time, repair costs etc. are just as important a contribution to the post-completion audit and to subsequent investment decisions. Indeed, if
contractual obligations as to maintenance, repair and insurance are to be scrupulously discharged, such records are even more necessary. And if the user habitually never keeps such records in respect of owned plant, this aspect of instalment-debt is a distinct cost - for the supplier or financing company will have to keep detailed locational and financial records of all outstanding contracts; an administration cost which will be passed on in the rental. The user himself must keep adequate financial records of rentals paid.

One area of administrative saving accruing to leasing lies in the simplified taxation accounting routines, to which must be added some consequent saving in audit fees. (Leasing accounts and agreements are easily audited). A further specific saving accrues as between new equity financing and instalment-debt - if the new equity attracts new shareholders then there will be additional costs now and forever in the registrars department. Otherwise, this issue of accounting and recording costs seems to be largely dependant on the extent to which such records were previously thought unnecessary and now have to be kept. For or against, the costs involved are not likely to be of sufficient materiality to warrant the cost of identification.

The Question of Control
One unquantifiable but distinct advantage of instalment debt seems seldom to be mentioned either in the analytical writings or in the advertisement literature. This is the issue of control. Where the amount involved is relatively small, the issue is immaterial. Where the amount is relatively large, the issue can be of considerable importance. Financing out of reserves should not affect control, so long as it does not evoke a shareholder revolution. The effect of a new equity issue on control can be nil - the new issue being a rights issue (1) or confined to the existing shareholding of a private company; or it can be traumatic - the new issue being wholly privately placed. A private placing of debt will almost certainly have the same effect. I am not here referring to formal restrictive covenants in the indentures, but to the effect on control ranging

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(1) Assuming each shareholder exercises his rights.
from the informal influence of a major institutional loan stockholding to the formal appointment of a debenture holders' "watchdog" to the Board. Even a bank overdraft carries overtones of such control, as it may carry an obligation to submit period financial statements for review and comment; and it is becoming commonplace for a bank to require personal guarantees from major shareholders as additional collateral for especially large facilities to small companies.

Generally, once the contract is signed, third-party and manufacturing lessors and hire-purchase agents are remarkably unconcerned to attempt any supervision of their investment. Their attitude is that of secured creditors, especially if debenture holder waivers have been executed. Only in the event of rental default do they appear to wish to be closely concerned, and even then their attitude is one of minimum commitment. Conversely, they are all the more ready to remove their assets without very much sympathetic collaboration with a management trying to ride out a crisis.

Conclusion

In fact there is no conclusion to this section. Various considerations which apply strongly to leasing, but which also are relevant to many forms of hire-purchase and deferred payment, have been separately examined because they do not lend themselves to a uniform quantitation analysis. These considerations are subject to controversy as to their very existence. Certainly their materiality must differ from time to time and circumstance to circumstance. Their evaluation must largely be subjective, as by a process of taking a cold look at each time and circumstance and coming to strategically orientated decisions within the context of the firm as a whole. Only a few aspects, such as those of certain legal costs, are likely to be capable or worthy of closer quantification.

Section 3. Economic Life, Obsolescence and Disposal Value

These three linked concepts have underlain much of the discussion of the previous Section. The issues are twofold:
1. To what extent do different financing methods afford protection against the risk of obsolescence?

Obsolescence is taken to manifest itself as:

**Equipment Obsolescence:** unduly high and avoidable costs of operating, plus a 'catastrophic'\(^1\) loss of disposal value (where by disposal value is meant re-sale or trade-in value net of salvage costs). Under obsolescence, disposal value approximates to scrap value.

**Product Obsolescence:** a catastrophic loss of disposal value, only. Operating costs are irrelevant in this situation.

2. To what extent do different financing methods imply a loss to the user of any potential disposal value in circumstances where obsolescence is not important?

In practice, it is not really possible to regard these two questions separately. A sharp loss of disposal value is possible without the existence of obsolescence where this is measured in terms of unduly high operating costs. (A typical example of this is to be found in heavy goods vehicles on the recent introduction of the new "plating" regulations.\(^2\) Hauliers demanded new "plated" vehicles which offered no real operating cost savings over existing "non-plated" vehicles, the disposal value of which dropped sharply; yet the regulations specifically provided for the "plating" of existing vehicles.\(^3\)

Perhaps obsolescence should also be defined so as to include capacity obsolescence - the need for the user to be able to increase the range

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\(^1\) This somewhat dramatic term is used solely to distinguish between a very rapid fall in disposal values and the more-or-less gradual fall normally associated with increasing asset life.

\(^2\) Whereby there must be affixed to the chassis of a vehicle a permanent plate specifying the unladen vehicle weight on each axle.

\(^3\) General vehicle distribution trade opinion is that the "panic" reflected the widespread breaching of M.O.T. weight regulations. Obviously existing vehicles would be "plated" at their true weight (often greater than new vehicles) which would give the haulier no excuse for not reducing his loads to legal limits. There is therefore strong pressure by users upon manufacturers to become extremely conscious of the weight of the unladen vehicle; but this is no new thing to the manufacturers.
or rate of output.

This is really a special category of unduly high cost operating, as the extra capacity may well be sought by excessively high rates of machine speed and/or feed, by uneconomical overtime working and the like. This form of obsolescence is particular to the firm, and may very well not be accompanied by any catastrophic loss of disposal value - although excessive operating speeds or feeds may have so reduced the mechanical efficiency of the asset that there is in fact a reduction in disposal value also. But capacity obsolescence is sufficiently important to warrant separate mention in the ensuing discussion. Other obsolescence of equipment or product will be termed technological obsolescence.

In that discussion, no account is taken of asset appreciation, either in the form of unduly low costs of operating or of increasing disposal value. Circumstances where such appreciation is material are infrequent, and are usually the result of exceptional and non-recurrent causes. The obvious exceptions to this are land and buildings. The degree of rational analysis that is required even in this area can be illustrated:

"Land appreciates in value and equipment depreciates. We buy our buildings as a hedge against inflation, keeping working capital in a state of liquidity...... Leasing is a practical means of procuring equipment without eroding capital" (1)

It is to be hoped that a comparison is made between the value of that hedge and any additional financing costs of equipment leasing over equity-financed outright purchase. Minimisation of a potential loss situation may be more worth-while than maximisation of some other potential profit situation.

Outright purchase, however financed, exposes the purchaser to the full risk of obsolescence and of declining disposal value. Conversely, an owner has full equity in whatever disposal value there may be. In technological obsolescence this will probably be small - this is the case of the "catastrophic" loss (2); But where capacity obsolescence supervenes, the disposal equity may be valuable; especially in the form of trade-in value for mobile assets. The same applies to the situation where obsolescence is not important.

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(1) Aircraft industry executive, quoted H. Schustor "The Plant Engineer and Equipment Leasing" (Plant Engineering', May 1967).

(2) For example, the I.C.I. anhydrites plant at Billingham which cost £7 million to build and was closed down as an obsolete process within nine months of opening in 1965.
The impact of balancing allowances or charges does no more than modify these conclusions somewhat. For although variations in disposal proceeds give rise to equal and opposite variations in the balancing allowance(1) it is to be remembered that:

firstly: the disposal proceeds have a 100% value, whereas the balancing allowance has a value equal only to the operative rate of corporation tax.

secondly: the disposal proceeds do not depend upon a tax liability to have effective value. The balancing allowance does.

thirdly: the balancing allowance is received nine to eighteen months after the disposal, so that in an appraisal its present worth is of less weight accordingly.

This situation is exacerbated by the Inland Revenue practice of disallowing the writing-down allowance in the year of disposal, and increasing the subsequent balancing allowance accordingly.

Nevertheless balancing allowances do somewhat mitigate the change in the cost of ownership(2) from year to year, assuming that (at whatever rate of change) disposal value is a decreasing function of asset life. Such mitigation may be increased if, as is used and explained later in this thesis, the earning opportunities of the various arising cash flows are brought into the appraisal study. The exact situation cannot be predicted without a range of residual values, one for each year of equipment life: together with some specification of the supervision of technological obsolescence, and assuming minimum standards of mechanical efficiency of the equipment at each year of life. Uncorrelated and random replacement studies(3) of resale values for machine tools and heavy goods vehicles permit the somewhat unsure generalisation that disposal values for such equipment fall

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(1) For the remainder of this thesis, the term "Balancing allowance" is taken to include for "balancing charges": which is only a negative value of the allowance.

(2) Excluding for any avoidable high costs of operating assets. These can be regarded as measured by the difference in ownership costs between high disposal value conditions and low disposal value conditions. A low disposal value condition in part reflects unduly high operating costs.

(3) Ad hoc studies carried out in the Department of Industrial Engineering & Management, Loughborough University of Technology.
sharply in the early years of life, drift slowly downwards in middle life and fall away sharply in later years. As the curve of writing-down allowances is asymptotically convex to origin, variations in the cost of ownership will obviously be uneven, with a variety of discontinuities at the time of disposal and some twelve months thereafter. A further complication is caused by the revocability of investment grants if disposal is within three years of the receipt of that grant.

I conclude that ownership provides no real protection against technological obsolescence but that capacity obsolescence costs are to an extent compensated for by disposal equity. In circumstances where obsolescence is not a significant factor, ownership enjoys the disposal equity, but the loci of writing-down allowances and of normal disposal values are too dissimilar to permit generalised predictions. Balancing allowances modify but do not alter these conclusions.

As an illustration consider the costs of ownership of an asset purchased for £1000: which, purchased in mid-June:

a) receives a 20% investment grant twelve months later.

b) ranks for a 20% writing-down allowance. As the asset was purchased mid-June; and given that the accounting year ends 31st December, the year of assessment will be the following year, and the first allowance 'received' at the end of Year 2 (actually due 1st January, Year 3). The company pays tax when due. (1)

c) has an estimated normal disposal value of £400 at the end of Year 5, £250 at the end of Year 6 and £100 at the end of Year 7; unless technological obsolescence supervenes, in which case disposal values will approximate to £100, £50 and £10 respectively.

d) will be maintained throughout to a high standard of mechanical efficiency.

(1) This highlights a minor problem in DCF appraisal studies. Firstly, it is the date of the end of the basis accounting year within which the asset is acquired, which determines the due date of receipt of allowances, not the date of asset acquisition (more accurately, asset outlay). Secondly, as allowances are received by way of a diminution of other corporation tax liability; it is when the company pays its tax, that matters, not the due date. Many companies are habitually late in their tax payments. These two aspects are often ignored in writings on DCF.
The date of disposal will in each case be the 30th December, so that any balancing allowance will be due twelve months later. Disposal will be for spot cash. The Inland Revenue can be expected to disallow the final writing-down allowance and adjust the balancing allowance accordingly. Other important assumptions are:

e) the earning opportunity of the various arising cash flows will be introduced into the appraisal study. This practice is defended in the next Chapter. (1)

f) earnings will be calculated on a year-end basis, rather than continuously. Neither is strictly defensible; but here the convenience of annual rests is utilised.

g) on average, assets employed in the company earn at 12½ per annum, net of corporation tax. By 'earn' is meant a true internal rate of return.

h) the cost of capital to the company is set at 10½, net of corporation tax.

i) as earnings are potentially, to infinity; a 9-year study period is utilised for a 5-year life, and so on. This means that all earnings are compounded forward up to 9 years, and tax thereon up to the 10th. The use of a 9-year study period permits the introduction of a nominal two years loss of earnings capacity accruing to the taxation of earnings.

j) the earning and discount rates (2) are unchanged over the study period.

Samples of the detailed calculations involved are included in the appendix to this Chapter. The significant final values of those calculations are listed in the following Exhibit I.

(1) The cash flows are: (1) investment grant (2) writing-down and balancing allowances (3) disposal proceeds. Each cash flow generates an earnings stream, which is taxed (twelve months lag) with a loss of earnings accruing to all taxation flows. See Chapter Three.

(2) That these two rates are normally not the same is discussed in Part Two, Chapter Six (Section 5).
EXHIBIT I

THE COSTS OF OWNERSHIP - GRANT QUALIFYING ASSET

(all amounts in terms of £, discounted at 10%)

<table>
<thead>
<tr>
<th>Sale at end of</th>
<th>OBsolescence</th>
<th>NON-OBsolescence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5yrs.</td>
<td>6yrs.</td>
</tr>
<tr>
<td>Disposal value</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Cash Flows: Grant</td>
<td>174</td>
<td>174</td>
</tr>
<tr>
<td>W.D. All'ces</td>
<td>138</td>
<td>153</td>
</tr>
<tr>
<td>Bal. All'ces</td>
<td>51</td>
<td>44</td>
</tr>
<tr>
<td>Disposal</td>
<td>62</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>425</td>
<td>399</td>
</tr>
<tr>
<td>Earnings: Grant</td>
<td>81</td>
<td>90</td>
</tr>
<tr>
<td>W.D. All'ces</td>
<td>53</td>
<td>67</td>
</tr>
<tr>
<td>Bal. All'ces</td>
<td>11</td>
<td>9</td>
</tr>
<tr>
<td>Disposal</td>
<td>16</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>161</td>
<td>173</td>
</tr>
<tr>
<td>TOTAL INFLOW</td>
<td>586</td>
<td>572</td>
</tr>
<tr>
<td>Net Cost ex £1000 (1)</td>
<td>414</td>
<td>428</td>
</tr>
</tbody>
</table>

(The comparative results are based upon a moving study period, whereby all earnings streams are calculated for four periods in extension beyond the disposal period and tax streams to one period further yet. Thus, although the investment grant is a sum common to all asset lives, the earnings stream accruing thereto increases as asset life is prolonged).

The impact of obsolescence is quite clear, although modified somewhat by the discounting process. The greater balancing allowances are insufficient, even including accruing earnings, totally to eliminate this impact; but the amelioration therefrom is quite substantial, being of the order of 40% of the obsolescence cost which otherwise would accrue to disposal at 5 and 6 years. At Year 7, normal (i.e., non-obsolescence) disposal value is sufficiently low for obsolescence not to count for very much: on the other hand, unduly high operating costs might by that time be really large, given that so 'old' an asset may be relatively mechanically inefficient anyway. What does emerge from

(1) Strictly, as the asset was purchased half way through Period 1, inflows should be compared against a 6-month discounted outlay of £1000, i.e., £954. However, this "error" is allowed to stand as an offset against a similar "error" in the leasing appraisal later to be compared with the present appraisal. In any event the argument here is concerned with changes, rather than in levels, of the costs of ownership as economic life varies.
a study of the obsolescence situation is that ownership costs in this later age range are rather insensitive to a sudden supervision of obsolescence: which is an obvious enough deduction.

In the non-obsolescence situation, asset life is seen to be highly important, due to the strong impact on disposal value and the earnings accruing thereto. The writing-down and balancing allowances (plus their respective earnings) can do comparatively little to modify this effect; in this illustration, the loci of the allowances and of disposal value are too dissimilar. Here the disposal values are falling off sharply, and where this is the case there would seem to be a strong argument for replacement studies at the end of the 5th year.(1) If replacement economics indicate a longer optimum asset life, there must alternatively be a review of the financing decision vis-a-vis this asset. An important aspect of ownership is that if the owner is thus exposed to the impact of loss of disposal value, he has the freedom to react by replacement or re-financing: albeit this second alternative may be most difficult to achieve.

To what extent are these tentative conclusions modified by the use of funded or instalment debt to achieve ownership? There will be imposed upon these considerations a pattern of after-tax interest costs, adjusted for recovery of potential earning from the tax shield. So far as funded debt is concerned, that imposed pattern (being uniform(2)) will not affect the conclusions at all. So far as hire-purchase is concerned, the principal concern must be the extent to which the contract is determinable during its period. The contract period will automatically provide for the recovery by the financing organisation of the full cost of the asset: it may or may not exhaust the economic life of the asset. On the other hand, the purchaser can within acceptable limits stipulate both the initial deposit and the period of the contract. Yet again, in many cases a surrendered(3) asset is so returned under fairly rigid physical contractual conditions which exclude trade-in negotiated by the purchaser. Under these circumstances, disposal value is something akin to distress

(1) This is NOT to say that replacement must take place. Replacement decision must take account of earnings, operating costs and replacement costs as well. The basic investment decision must also be re-appraised.

(2) Assuming unchanged interest rates.

(3) I use 'surrendered' rather than 'repossessed' to emphasise that cancellation by the purchaser is voluntary rather than involuntary.
sale value: and as the contract will invariably give the financing organisations a prior claim to some minimum resale proceeds, the purchaser may suffer a total loss of disposal value during the life of the contract. Finally, cancellation of such contracts is invariably accompanied by a liability to pay a compensatory lump sum which (so far as I can see) is little if at all mitigated by an intention to enter into a subsequent contract. It may be mitigated by the disposal proceeds ultimately realised by the financing organisation, but we have seen that these may be minimal. (1)

The first tentative conclusion must be that acquisition by hire-purchase will is anything expose the owner even more to the risks both of obsolescence and long-term loss in resale value. In the obsolescence situation, what small disposal value may still accrue

(1) The method used to calculate the compensatory lump sum appears to differ as between financing organisations. A typical clause reads:

"...that in the event of your exercising your right to terminate the agreement by surrender, there will remain a liability on your part to account to the XYZ Financing Company for any amount by which the proceeds of sale... (after removal and disposal costs) fall short of the balance outstanding on the agreement at the time of such termination".

Clearly disposal value (such as it may be) mitigates the compensatory lump sum; but just what is "the balance outstanding"? This depends entirely upon the extent to which service costs are regarded as recovered by time. For example: a £1600 contract, 25\% deposit and interest added at 5\% per annum for 4 years (monthly instalments) implies service costs of £240 and monthly instalments of £30. A common method is apply the "rule of 78" whereby the service costs for each period form a descending arithmetic progression based upon a "sum-of-digits" write off of the total service cost. The "sum-of-digits" for 48 periods amount to 1,224; therefore, interest in month 1. would be taken as \( \frac{48}{1224} \times £240 = £9.41 \) - in period 2 as \( \frac{47}{1224} \times £240 = £9.21 \) - etc.

The "balance outstanding" is thus:

<table>
<thead>
<tr>
<th>End Period</th>
<th>Service Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>£1200</td>
</tr>
<tr>
<td>1</td>
<td>£1200 - (30-9.4) = £1179.4</td>
</tr>
<tr>
<td>2</td>
<td>£1179.4 - (30-9.2) = £1158.6</td>
</tr>
</tbody>
</table>

which approximates closely to the 'true' internal interest cost given by:

\[
\frac{(1+1)^n - 1}{1(1+1)^n} \]

where \( R = £30 \) and \( i = \) true monthly interest rate

Another and popular method is to assume that service costs accrue uniformly - in this example at £8 per month.
to relatively early surrender is subject to even further diminution on the grounds stated. Such a surrender will at least cut off unduly high costs of operating, but (especially if the 'rule of 78' or the true interest method are used as a basis for determining the 'amount outstanding') the relatively lower capital write-off in the earlier years of the contract will mean a relatively larger compensatory lump sum penalty. The somewhat insensitive reaction of ownership costs to asset life which characterises the obsolescence situation would urge that the contract be allowed to run its term but a prolonged period of unduly high costs of operating would then accrue. The only effective defence lies in a shrewd forecasting of obsolescence and the negotiation of a short contract term in the first place. There will be a saving in interest costs as a 'bonus'; but the amount of finance acquired is correspondingly less, effectively, in the sense that the amount outstanding on average over a given number of years will be less. Where there is a more gradual loss of resale value, the penalty of surrendering may be somewhat less, but (as we have seen) the ownership cost of not-surrendering is greater. Any diminution of the disposal value accruing to the user is an unwelcome hindrance to the one effective defence the owner has - the ability to dispose of the asset in a timely fashion. Again, the initial term negotiation is seen to be critical. Fortunately, most hire-purchase agreements for industrial fixed assets are, at the instance of the financing organisation, of relatively short term.

The pattern of payments under deferred payments schemes are so devised as to preclude any safe generalisations. As these are in the main the subject of direct negotiation between user and manufacturer, the physical conditions of surrender are even more stringent. Trade-in is with the financing organisation itself, which clearly is not to be expected to be overly enjoyed at the prospect of disposing of a large number of second-hand machines of its own manufacture, thus spoiling the market for its own new products. (1)

(1) Manufacturers' attitudes vary. Thus, Alfred Herbert are quite active in re-leasing surrendered machine tools which have been renovated; and so offer quite good surrender terms. NCR and IBM, on the other hand, will give but very poor trade-in terms for data processing equipment other than computers; and follow a practice of "putting under the hammer" surrendered machines. It is convenient to stress here that the arguments of this Section are not meant to relate to computers.
Schemes are usually for a fixed period, so that a defensive tailoring of the contract term is not possible. On the other hand, there are a sufficient number of different manufacturers' schemes in most equipment fields to allow some flexibility in terms so long as the choice of machine make is not critical. A review of half a dozen schemes operated by machine tool and other metal working machine manufacturers indicates that a typical payment pattern involves the user in acquiring some 20/25% of the equity in the asset in the first two years and the balance over two or three years. The interest content of each instalment seems universally to be calculated as a true rate on the balance of capital outstanding; consequently as the majority of schemes feature a cancellation provision, the concomitant liability for the outstanding balance is large in terms of capital but there is a significant avoidance of interest cost. Generally the impression is that deferred payment schemes if anything are a little more rigid than hire-purchase schemes, and, because they introduce an even larger likelihood that the user will in fact forfeit much of residual value, offer even less defence against obsolescence and resale depreciation.

For both hire-purchase and deferred payment schemes it may be argued that the user is protected from loss of capital where technological obsolescence occurs during the period of the contract. All such contracts are by definition 'full pay-out' contracts - i.e., they recover the cost of the asset plus the financing organisations charges. They assume a NIL or near-NIL disposal value at the end of the contract term. If in fact disposal value drops to NIL during the contract, the user's instalments are unaffected: but as they contain an element of capital repayment, to acquire something which is now 'worthless', their continuation involves the user in a loss of capital. But what may be to the point is that that loss of capital is spread over a longer period: the depreciation charge to profits is less. Conversely, however, interest is being paid to little purpose; and the unduly high costs of obsolescent operating (which presumably are quite significantly inversely correlated with loss in disposal value) have to be born during the remaining term of the contract. Finally, the financing organisations are as aware of this situation as in the user. and it is noteworthy that

(1) Probably even more so, as they may be expected closely to question the equipment manufacturer concerning his plans for developing new models, before agreeing to set up standard financing terms for his present models.
manufacturers' deferred payment schemes especially are for very short periods. For these, a 5-year term is a long-term scheme.

The relationship of obsolescence and disposal values to leasing is, again, complex. There can be little doubt that a 'cancellable' lease - i.e. operating rental - affords as near complete protection against obsolescence as it is possible to have. Such rentals are determinable on giving the requisite period of notice. Indeed, many rental agreements specifically provide 'break clauses' when newer models may be substituted, albeit at an increased rental. To the extent that this increase is material, of course, some part of obsolescence costs are probably being transferred from the rentier to the user; but the ability to determine one rental agreement and to 'shop around' for the best terms on which new models can be rented, affords a valuable hedge to the user. \(^1\) This facility is even more valuable if the obsolescence is that of the product rather than of the asset, so that the user has no further need of any model of the relevant asset. There is however almost invariably a total loss to the user of any disposal value. To some extent this is a matter which the user can control. If the loss in disposal value is sudden, or expectably constitutes a steep gradient, the user enjoys the benefit of the hedge. If the loss is relatively gradual, the user can prolong his rental until the present value of the opportunity cost of prolongation (measured by the avoidable increase in operating costs which presumably accrues as assets age) is equal to the present value of the lost disposal value. \(^2\) This decision - to prolong an existing rental - is relevant only in the context of an existing decision to rent, however. Vancil \(^3\) presents a reasoned case for assuming that the rental prolongation decision is pre-empted by the financing.

\(^1\) Data-processing equipment and light vehicle hire are two good examples of this. It is impossible in practice to split higher rentals for newer models into that part which is attributable to the (usually) higher cash cost of new developments and a residual which is effectively a transfer of obsolescence cost of the old model.

\(^2\) More exactly, until the present value of the marginal opportunity costs equals the marginal lost disposal value. Especially where rental covers repair and maintenance costs, the avoidable increase in operating costs relates less to measurable costs of labour, material and overhead; and more to assumed costs of loss of operating capacity during breakdown or due to slower operating times; frequently plus subjectively assessed costs of loss of prestige and status. Annual car fleet hire renewal must surely be the prime example of this last.

\(^3\) Vancil (54) Chap.6. His argument contains some confusion in that his treatment of this problem is one in financial decision making; yet earlier he specifies operation-rental as essentially a matter of investment-decision making.
decision — which he restricts as one which initially is between outright purchase and operating rental. Essentially, his solution is to establish for each successive year of asset life, that disposal value which must necessarily obtain if the outright purchase and the operating rental financing methods are to break even. If, for a given life, the 'necessary' disposal value is thought not likely to accrue; then the decision must be to rent — and, by definition, the rental must be for that life-period. Presumably, Vancil would agree that a change in the pattern of disposal values or of renting terms would necessitate a re-appraisal along the same lines, comparing outright purchase with the best available rental. The hedge against sudden obsolescence is automatically built into this appraisal technique, being reflected in a low probability of the 'necessary' disposal value obtaining. What most strongly emerges, however, is the realisation that financing method is very much conditioned by estimates of asset life, different estimates giving different financial method optima. In such conditions, variations in the cost of ownership over time will prove doubly significant. Vancil concludes that under U.S. tax regulations, the cost of ownership tends to stabilise with time; yet the cost of operating rental continues to climb, albeit at a reducing rate due to the discounting effect. We have partially examined this conclusion in the light of U.K. investment grants and writing-down allowances: and in any case Vancil does not take into account the earning opportunities of capital released by rental. It is convenient to postpone further discussion on this point to a later time.

Having stipulated earlier that operating rental is not a prime concern of this thesis, I turn instead to a consideration of the relationships of obsolescence and economic life to financial leasing. Firstly, as to obsolescence. Because financial leases are by definition non-cancellable, they would appear to offer no shield against obsolescence. As the restriction usually relates to the primary period, this would appear especially to be the case where obsolescence suddenly and unexpectedly supervenes in the primary period. The lessee apparently is 'stuck' with the costs of obsolescence until the primary period is up. But just how realistic is this?
1. In fact it is sometimes not too difficult to substitute an improved model of the asset during the primary period. (I have personally seen this effected). Obviously there is an upward revision of the premiums; and the situation is identical with that of operating rental - some of the increase reflects a genuine addition to asset cost and some is transference of obsolescence cost to the lessee. As such substitutions are usually the subject of ad hoc negotiations, during which the question of responsibility for any loss on disposal is raised; the extent of the transference can be made explicit. It is in fact probably substantial, and the ability to substitute avoids only the penalty of the excessive operating costs of obsolescent equipment. There is often, however, one mitigating circumstance where the lease is of the "third-party" type. When the substitute is an improved model of the same asset, the lessor is usually willing to allow the lessee to negotiate 'trade-in' values for the discarded asset. The recovery of what disposal value may remain - and it may be substantial if there is a secondary market for the discarded asset - may be considerable, and the upward revision of leasing premiums mitigated accordingly.

2. Even more than with hire-purchase contracts, the lessee is able to set his own primary period. Especially where capacity obsolescence is anticipated, lessors appear to prefer the setting of a short primary period to the possibility of later ad hoc negotiation of some compensatory lump sum on substitution. (1) The same onus rests on the lessee so far as technological obsolescence can be foreseen. In this way, at least the unduly high operating costs of obsolescence can be reduced to a minimum. If technological obsolescence is sudden in its onset, the lessee is indeed unable to avoid those high operating costs.

These two points serve at best to indicate where, under non-cancellable leases, there is some protection against obsolescence - a possibility of maximising whatever disposal value remains and of its ensuring

(1) This assertion is based upon conversations with leasing company managers.
to the lessee; and of avoiding obsolescent operating costs. But this
does not avoid the catastrophic loss of disposal value (even where
it is foreseen) any more than does ownership - and indeed the lessee
has to be a shrewder forecaster of obsolescence than the owner.
This conclusion is based upon an assumption: that, long or short, the
primary period of a lease will recover the original cost of the asset
plus the lessors' servicing charge,\(^1\) less any disposal value. This
is largely true of equipment leasing in the U.K., for the reasons
discussed on Page 12.

The lease premiums thus directly reflect the estimate of the
lessor as to what the disposal value of the asset will be at the end
of the primary period, which is when the first contractual opportunity
for cancellation accrues. Under typical U.K. "full pay-out" leases,
the lessor will expect the disposal value to be but nominal at the
end of the primary period; i.e., the lessor assumes an identity of
pay-out life and economic life. If cancellation is effected because
the lessee, solely within the context of his own operations, is
suffering from capacity obsolescence: then (and especially because
he can probably negotiate a good sale price or 'trade-in' terms) the
lessee can expect to receive all of the disposal proceeds in excess
of the lessors' nominal figure. The same is true of cancellation
during the secondary leasing period which normally is renewed on a
year-to-year basis at the lessees' option. Here the situation is
identical with that of operating rental; with the vital exception
that any excess disposal value over the lessors' nominal value will
emerge to the lessee.

It follows that where equipment is not normally subject to
technological obsolescence, a leasing contract based upon a
relatively short primary period normally offers just about as good
a protection against capacity obsolescence, or more-or-less gradual
reduction in disposal value, as does ownership. This is not to say
that leasing is as good a financing method as purchase (outright or
instalment debt), which is a matter for total appraisal. It is to
say that leasing is neither worse nor better so far as the impact
of asset life on production needs or disposal value is concerned -
short of technological obsolescence.

\(^1\) i.e., costs of administration and finance, plus the lessors'profit.
The same is largely true if technological obsolescence supervenes after the primary period has expired. The lessee will not renew his lease and (as the property is not his) may even avoid the costs of disposal of an asset whereas the disposal value may be only as scrap. If such obsolescence supervenes during the primary period, then:

(a) Successful substitution negotiations will minimise unduly high operating costs but will not protect the lessee from the catastrophic loss in disposal value.

(b) Otherwise the lessee must undergo those high operating costs, but to him the leasing premium remains unchanged. The lessor's assumption of an identity of pay-out life and economic life then protects him to some extent from loss of capital when economic life is found to be less than pay-out life. But the lessor will be equally alert to the possibility of this, and the would-be lessee may very well find that his choice of primary period is distinctly limited in situations where technological obsolescence is frequently observed.

Finally, what of economic life and leasing? It is useful to examine this in the context of the financial leasing of the same asset which was utilised to examine ownership costs and economic life.

The following assumptions are made in this appraisal of leasing costs:

(a) As with the ownership illustration (Exhibit I.), the asset was purchased at end-June and qualifies for a 20% investment grant received by the lessor twelve months later,

(b) Unlike the user under hire-purchase and deferred payment schemes, the lessee is acquiring no rights in the asset. Because the asset is less efficient (and because leasing is effectively a transfer from fixed to working capital outlays) there is some loss of working capital: but in many cases, even 'obsolete' assets are capable of some contribution to operating capacity so long as they are in situ. They would have no value at all elsewhere, and may cost more to dispose of than they would realise. This argument — i.e., some contribution — may be quite invalid in a situation of product obsolescence.
plus 20% writing-down allowances.

(b) Lease premiums are taken as annual amounts for the sake of simplicity, although in practice they would be payable monthly. Premiums are taken as payable in advance, and for a 5-year primary period amount to £256 in Year 1. and £216 in Years 2 through 5. The decreases is due to receipt of the investment grant by the lessor, and is assumed to be transferred at once to the lessee by way of a cash refund of the excess Year 1. premium. All premiums are assumed to qualify as fully tax deductible, from which it follows that the grant refund must also be subject to tax. Secondary period premiums are £10 p.a. renewable annually at the lessees' option.

(c) Disposal values are as used in Exhibit I. These are assumed to ensure to the lessee, less a £50 nominal disposal value reserved to the lessor. The net receipts are subjected to tax as chargeable gains.

(d) Maintenance and other operating expenses are born by the lessee, and the asset is maintained to a high standard of mechanical efficiency.

(e-j) Exactly as for the ownership illustration (Exhibit I.) (Page 32).

(k) As a first step, there is introduced the flow of 'residual capital earnings' - based upon the concept that if the lessee leases as an alternative rather than as a necessity, he will have available a residual capital, decreasing as premiums are paid, which will be able to earn elsewhere during the study period. At this point, and pending the discussion on 'residual capital earnings' in the next Chapter, "earn elsewhere" may be most conveniently thought of as external investment in other companies - but this is only convenience, not essential necessity.

(1) This is a common but not universal leasing practice in the U.K. It follows that the refund would be received mid-way through a calendar year, and it has been set to earn accordingly. The same is true of the investment grant in Exhibit I.
EXHIBIT II.

THE COSTS OF LEASING - GRANT QUALIFYING ASSET
(all amounts in terms of £, discounted at 10%)

<table>
<thead>
<tr>
<th>Sale at end of</th>
<th>OBSOLETE</th>
<th>NON-OBSOLETE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposal value</td>
<td>100</td>
<td>50</td>
</tr>
<tr>
<td>Cash Flows:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grant Refund</td>
<td>23</td>
<td>23</td>
</tr>
<tr>
<td>Tax Shield</td>
<td>297</td>
<td>299</td>
</tr>
<tr>
<td>Disposal Refund</td>
<td>20</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>340</td>
<td>322</td>
</tr>
<tr>
<td>Earnings:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual Capital</td>
<td>168</td>
<td>167</td>
</tr>
<tr>
<td>Grant Refund</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Tax Shield</td>
<td>94</td>
<td>110</td>
</tr>
<tr>
<td>Disposal Refund</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>274</td>
<td>286</td>
</tr>
<tr>
<td>TOTAL INFLOW</td>
<td>614</td>
<td>608</td>
</tr>
<tr>
<td>Total Premium Cost</td>
<td>931</td>
<td>947</td>
</tr>
<tr>
<td>Net Cost</td>
<td>317</td>
<td>339</td>
</tr>
</tbody>
</table>

(There is the same small study period - induced distortion as was noted in the ownership study. Also, taking the premiums as being annual rather than monthly amounts results in a slightly high present value of premiums - e.g., on an annual basis, £256 in Period 1, payable in advance has a present value of £256; whereas on a monthly basis the present value would be £245. The overstatement of the present value of leasing premiums in this study is approximately £40, corresponding closely to the over-stated present value of ownership outlays - see the footnote to Page 33. At the same time, because premiums are deemed to be paid in advance, the use of annual amounts exaggerates the reduction in residual capital. Hence discounted earnings on residual capital are understated by the very small amount of £15 approximately).

The leasing study indicates the following:

1. As the discounting of lease premiums at a cost of capital-based
factor of 10\% results in a present value of less than £1000, leasing on these terms is a relatively cheap source of capital to the company.

2. As in the ownership study, the costs of leasing are less volatile in the 'obsolescence' situation, when a catastrophic loss of value in the early years leaves little room for disposal value variability in subsequent years. In this study, leasing costs are slightly more volatile than ownership costs in the obsolescence situation, and slightly less volatile in the non-obsolescence situation.

3. To assess the comparative impact of asset life extension on leasing and on ownership costs requires a detailed study of the various cash flows and accruing earnings streams. They will be seen to be contradictory in their influences.

a) Grants and Tax Allowances.

These are the principal cash flows - investment grant and writing-down allowances accruing to ownership, and a grant refund and a tax-deductability of premiums ("tax shield") accruing to leasing. (1)

<table>
<thead>
<tr>
<th>Asset Life</th>
<th>Cash Flows: (£, discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5-years</td>
</tr>
<tr>
<td>Grant plus W-D. All'ces</td>
<td>312</td>
</tr>
<tr>
<td>Refund plus Tax-Shield</td>
<td>320</td>
</tr>
<tr>
<td>Difference:</td>
<td>(8)</td>
</tr>
</tbody>
</table>

Ownership cash flows respond positively to an extension of asset life - more allowances are received. Leasing cash flows are indifferent to asset life - the secondary period premiums are so very small and late that their discounted tax-deduction value is negligible. If asset life is so far prolonged that writing-down allowances (especially when discounted over the now longer period) become insignificant, then the ownership allowances cash flow will also become indifferent to

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(1) Balancing allowances are taken as part of disposal proceeds. As, in this model, the writing-down allowance in the year of disposal is incorporated into the balancing allowance; ownership cash flows in the "grants and allowances" category are here under-stated. So however are leasing flows - which ought to include the value of the reductions of premiums which accrue as soon as the lessor receives the grant.
asset life. Up to that point, extension of asset life reduces ownership costs more than it reduces leasing costs.\(^1\) So far as concerns earnings accruing to this category of cash flow, the superiority of ownership cash flows will attract higher earnings, and the increase in that superiority as asset life is extended means that earnings will improve even more, relative to leasing cash flow earnings. Further, as earnings are calculated at compound rates, this dual benefit will be compounded with time — although of course discounting will also have an increasing effect with time.

b) **Disposal Proceeds.**

Gross disposal proceeds accruing to ownership include the full disposal value plus the balancing allowance. As asset life is prolonged and more writing-down allowances received, the value of the balancing allowance is a direct reflection of the extent to which the decrease in disposal value parallels the decrease in writing-down allowances. Given an asymptotic curve of writing-down allowances, and the typical pattern of disposal values of a sharp fall during early life, a less steep fall in middle life and a sharp fall away in late life; the situation may be that over the middle life of the asset balancing allowances will typically be of low value. This is the situation depicted in this model. So far as concerns leasing, there is received only an excess of disposal value above some minimum stipulated in the leasing contract.

<table>
<thead>
<tr>
<th>Asset Life (years)</th>
<th>Cash Flows (£, discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
</tr>
<tr>
<td>Disposal Value (£)</td>
<td>100</td>
</tr>
<tr>
<td>Disp.Proc.plus Ball.</td>
<td>113</td>
</tr>
<tr>
<td>Allce.</td>
<td></td>
</tr>
<tr>
<td>Excess Disp.Value</td>
<td>20</td>
</tr>
<tr>
<td>Difference:</td>
<td>93</td>
</tr>
</tbody>
</table>

\(^1\) It is important to realise that this statement makes no comment as to whether, even so, leasing is absolutely cheaper or dearer than ownership. It simply says that with a longer asset life and from the point of view of these cash flows only, ownership becomes cheaper but leasing is not very much affected.
Although the absolute superiority of ownership cash flows is clear, it seems that in situations where there is no violent change in disposal value (either the asset is 'middle-aged' in a non-obsolescence situation, or obsolescence has already supervened so that no significant value remains to be changed), a moderate extension of asset life reduces that superiority. This is largely because of the relative imperturbability of leasing disposal cash flows. Uncushioned by balancing allowances, excess disposal value soon reaches an irreducible minimum. This "tendency to equality at nil disposal value" will be very mildly reinforced by the earnings stream which, however, starts too late in economic life and so is too heavily discounted to count for very much. Over a long extension of life; where disposal value, writing-down allowances and balancing allowances alike tend to become very small; ownership cash flows too will tend to become indifferent to asset life. Generally, and subject entirely to the rate of change in disposal value, extension of asset life does not increase leasing costs as much as it does ownership costs. (1)

c) Residual Capital Earnings.

This flow accrues to leasing finance only. The validity of this item is a matter of controversy, but discussion is postponed until the next Chapter. Suffice it to note here that, providing that the earnings rate and the discount rate are not too dissimilar; and provided that the proposed disposal is in any event after the primary period; (2) this stream is impervious to asset life, and so need not concern the present discussion.

The impact of asset life extension is therefore contradictory

(1) In interpreting this observation, the reader must remember that a reduction in a cash flow is an increase in net financing cost.

(2) Firstly, this is contractually obligatory. Secondly, this means that there is caused no inter-period variability due to varying premiums. Thirdly, this means that low secondary period premiums will apply. Lastly, this means that those low premiums will be comparatively heavily discounted and so affect residual capital but little.
as between the "grant plus allowances" flow and the "disposal value" flow; and this contradiction is itself inverted if the extension is substantial. Using as it does two sets of disposal values - one of high values, one of low values - the present model indicates that on balance the marked susceptibility of ownership cash flows to declining disposal values means that moderate extension of economic life will be of less concern to lease financing than to ownership financing; there will be a less risk of unfavourable financing cost trends setting in. For a capital-hungry company, for which leasing already provides a welcome source of additional capital, this conclusion must be heartening in that it foretells greater flexibility of replacement planning and consequently fewer imperatives on straightened cash resources.

One outstanding feature that is common to both methods of financing is the relative stability over time of all cash flows other than disposal value, and of the net earnings stream. Patently, disposal value is of great importance in evaluating the correlation of financing costs to changes in asset life; and such changes by themselves are of not too great importance in terms of financing cost, unless disposal value changes sharply with asset life extension.

No account is taken here of changes in earning rates or in the discount rate - this is discussed in Chapter Three. Obviously such changes would be most important for the tentative conclusions so far reached. The model data imply that:

a) For a given discount rate and a 'high' earnings rate: as comparison of Exhibits I & II reveals the superiority of leasing earnings streams (accepting the validity of residual capital earnings); extension of economic life would be of greater benefit to leasing costs than to ownership costs.

b) Conversely, and for the same reason, for a given earnings rate and a 'high' discount rate, extension of economic life would be to the detriment of leasing costs. But at the same time, the 'disadvantageous' susceptibility of ownership costs to changes in disposal value over time will be ameliorated; extension of economic life will be less to the detriment of ownership costs.
Primary cash flows - the "grants and allowances" category - would be affected about equally as between the two financing methods.

These are very imprecise conclusions which must await the discussion of Chapter Three for greater clarification. But, generally, (and especially in the higher-earning rate category) financing costs for grant-qualifying assets increase less to leasing than to ownership as asset life is extended - there is a relative advantage to leasing as a method of financing long-life assets. Is this still true in the non-grant qualifying situation?

The illustrative model is revised as follows:

For ownership: a 30% initial allowance is substituted for the investment grant. This allowance is deemed to be received at the same time as the first writing-down allowance, which is decreased by the initial allowance. Subsequent writing-down allowances and the balancing allowance are amended accordingly.

For leasing: the grant refund is eliminated, and primary period annual premiums are increased by £42 p.a. (gross of tax) above the grant-reduced figure. The 'tax shield' and residual capital earnings are affected.

The new situation is summarised in the following Exhibits:

**EXHIBIT III**

**THE COSTS OF OWNERSHIP - NON-GRANT QUALIFYING ASSET**

(All amounts in terms of £, discounted at 10%)

<table>
<thead>
<tr>
<th>Sale at end of</th>
<th>5 yrs.</th>
<th>6 yrs.</th>
<th>7 yrs.</th>
<th>5 yrs.</th>
<th>6 yrs.</th>
<th>7 yrs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposal Value</td>
<td>100</td>
<td>50</td>
<td>10</td>
<td>400</td>
<td>250</td>
<td>100</td>
</tr>
<tr>
<td>Cash Flows:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial All'ce</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
<td>99</td>
</tr>
<tr>
<td>W-D. All'ce</td>
<td>134</td>
<td>145</td>
<td>154</td>
<td>134</td>
<td>145</td>
<td>154</td>
</tr>
<tr>
<td>Bal. All'ce</td>
<td>35</td>
<td>31</td>
<td>29</td>
<td>(33)</td>
<td>(9)</td>
<td>12</td>
</tr>
<tr>
<td>Disposal</td>
<td>62</td>
<td>28</td>
<td>5</td>
<td>248</td>
<td>141</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>330</td>
<td>303</td>
<td>287</td>
<td>448</td>
<td>376</td>
<td>316</td>
</tr>
<tr>
<td>Earnings:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial All'ce</td>
<td>44</td>
<td>49</td>
<td>54</td>
<td>44</td>
<td>49</td>
<td>54</td>
</tr>
<tr>
<td>W-D. All'ce</td>
<td>53</td>
<td>63</td>
<td>73</td>
<td>53</td>
<td>63</td>
<td>73</td>
</tr>
<tr>
<td>Bal. All'ce</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>(7)</td>
<td>(2)</td>
<td>3</td>
</tr>
<tr>
<td>Disposal</td>
<td>16</td>
<td>7</td>
<td>1</td>
<td>64</td>
<td>37</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td>120</td>
<td>125</td>
<td>134</td>
<td>154</td>
<td>147</td>
<td>143</td>
</tr>
<tr>
<td>Total Inflow:</td>
<td>450</td>
<td>428</td>
<td>421</td>
<td>602</td>
<td>523</td>
<td>459</td>
</tr>
<tr>
<td>Net Cost Ex £1000</td>
<td>550</td>
<td>572</td>
<td>579</td>
<td>398</td>
<td>477</td>
<td>541</td>
</tr>
</tbody>
</table>
In comparison with ownership costs for the grant qualifying asset (Exhibit I):

Costs are increased by £140-£150 generally. The principal cause is inevitably the substitution of a 30% tax allowance (12½ value at 40% tax) for a 20% investment grant; with a correspondingly reduced earning stream. Writing-down allowances are not overly affected, but the balancing allowance is significantly reduced. Earnings to balancing allowances accrue too late in the study period to be greatly altered in terms of discounted values. Cost-volatility is again the greatest in the 'non-obsolescence' situation and to a very small degree, costs generally are more sensitive to asset life extension than in the grant qualifying situation. For the range of disposal values studied, the impact of obsolescence on costs is well-nigh identical with that accruing to the grant qualifying asset. Once again, disposal value is seen to be the principal element in cost-sensitivity to changes in asset life.

EXHIBIT IV.

THE COSTS OF LEASING - NON-GRAINT QUALIFYING ASSET
(all amounts in terms of £, discounted at 10%)

<table>
<thead>
<tr>
<th>Sale at end of</th>
<th>OBsolescence</th>
<th>Non-obsolescence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disposal Value</td>
<td>100 50 10</td>
<td>400 250 100</td>
</tr>
<tr>
<td>Cash Flows:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tax Shield</td>
<td>337 339 340</td>
<td>337 339 340</td>
</tr>
<tr>
<td>Disposal Refund</td>
<td>20 0 0</td>
<td>138 72 17</td>
</tr>
<tr>
<td>Disposal values</td>
<td>357 339 340</td>
<td>475 411 357</td>
</tr>
<tr>
<td>Earnings:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Residual Capital</td>
<td>158 156 155</td>
<td>158 156 155</td>
</tr>
<tr>
<td>Tax Shield</td>
<td>107 125 146</td>
<td>107 125 146</td>
</tr>
<tr>
<td>Disposal Refund</td>
<td>4 0 0</td>
<td>33 17 4</td>
</tr>
<tr>
<td>Total Inflow</td>
<td>626 620 641</td>
<td>773 709 662</td>
</tr>
<tr>
<td>Total Premium Cost</td>
<td>1042 1058 1064</td>
<td>1042 1058 1064</td>
</tr>
<tr>
<td>Net Cost</td>
<td>416 433 423</td>
<td>269 349 402</td>
</tr>
</tbody>
</table>

In comparison with the appraisal of the grant qualifying asset (Exhibit II):

Generally, leasing costs are increased by approximately £100 over all lives. This is due entirely to the increase in the discounted value
of the premiums. Inflow is actually improved, a small improvement in the cash flow and earnings from the tax shield more than offsetting the loss of the grant refund and a weakening of residual capital earnings. Cost volatility is unchanged in extent or in pattern - costs are impervious to life extension in the obsolescence situation, but are very sensitive in the non-obsolescence situation. As with the grant qualifying asset, this is due entirely to the change in disposal cash flow; other cash flows being impervious to life, and net earnings streams being equally stable. Where the increase in cost is so attributable to a change in discounted premium costs, it must follow once again that a change in the discount rate is probably of greater significance than a change in asset life.

A comparison of the impact of asset life extension on ownership costs as against leasing costs points to the same tentative conclusions as were established in the grant qualifying situation; viz:

<table>
<thead>
<tr>
<th>TAX FLOWS:</th>
<th>CASH FLOWS (£, discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Life:</td>
<td>5 yrs.</td>
</tr>
<tr>
<td>Initial &amp; W-D. All'ce</td>
<td>233</td>
</tr>
<tr>
<td>Tax Shield</td>
<td>337</td>
</tr>
<tr>
<td>(104)</td>
<td>(95)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DISPOSAL FLOWS:</th>
<th>CASH FLOWS (£, discounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset Life:</td>
<td>5 yrs.</td>
</tr>
<tr>
<td>Disposal Value (£)</td>
<td>100</td>
</tr>
<tr>
<td>Disposal Proceeds plus Bal. All'ce</td>
<td>97</td>
</tr>
<tr>
<td>Excess Disposal Value</td>
<td>20</td>
</tr>
<tr>
<td>77</td>
<td>59</td>
</tr>
</tbody>
</table>

The clear superiority of leasing cash flows from the tax shield over the basic capital allowances is seen gradually to diminish with asset life. The superiority of total disposal cash flows from ownership likewise diminishes with time, and at a faster rate than accrues to the diminution of the advantage of leasing tax flows. The net result must be to shift the cost advantage more in favour of leasing as life is prolonged - a shift which will be reinforced by a greater rate of growth of accruing earnings. And, even more than in the grant
qualifying situation, earnings streams accruing to leasing are greater than those accruing to ownership - thus providing a larger compounding base for the greater rate of growth already noted. Discarding the concept of residual capital earnings will not significantly alter this conclusion.

Because of the higher leasing premiums, an increase in the discount rate will be less severe on net leasing costs than is the case for grant qualifying assets; but because of the relatively more important earning streams, an increase in the earning rate will increase the comparative advantage of leasing vis-a-vis the grant qualifying situation.

There are thus prima facie grounds for concluding that as the life of an asset which does not qualify for investment grant is prolonged, leasing financing offers the advantage that leasing financial costs increase at a lower rate than ownership financial costs. This same advantage was found to apply to the case of grant qualifying assets, but was somewhat less pronounced than in the present case.

This still is not to say that leasing is absolutely more advantageous than ownership (or vice versa). In this example it appears to be; but that presupposes an as yet unsubstantiated acceptance of certain implicit assumptions, as well as certain leasing terms, and disposal values which may not be untypical but which certainly are not universal. The issue here is to observe the relationship of economic life to each of two alternative financing methods. On evidence of this appraisal study, a small balance of advantage rests with leasing. This is certainly not great enough categorically to overcome any hesitation over acquisition without ownership which might accrue from the non-quantifiable considerations discussed earlier; and would add but little reinforcement if those same considerations were felt to be in favour of leasing.

The trend of disposal values is seen to be of the greatest significance to this conclusion. Ownership costs over increasing asset life are less at a relative disadvantage, the 'flatter' is the descending curve of disposal value measured against increasing asset
life. For a given set of disposal values and a given asset earning rate; ownership costs over increasing asset life are less at a relative disadvantage, the higher is the discount rate - but this result is modified the higher are the leasing premiums; as for instance, in the case of assets which do not qualify for investment grants. For a given set of disposal values and a given discount rate; ownership costs over increasing asset life are at a greater relative disadvantage, the higher is the asset earning rate. This last should occasion no surprise, as it is but another variant of cost leverage.

But these assertions concerning the earnings rate and the discount rate again involve acceptance of such basic assumptions as the validity of the earnings concept and the commonality of the discount rate; and a measure of just how significant are the results of changes in these two parameters involves a formidable array of comparisons. It becomes necessary to examine and to explain the Evaluation Tables, which incorporate both the assumptions and the array.
APPENDIX TO CHAPTER ONE.

APPRAISAL STUDY CALCULATIONS.

Demonstrated in this appendix are abbreviated samples of the calculations involved in producing Exhibits I through IV. in Chapter One. As well as indicating the basis of the data thus exhibited, this appendix serves as a necessary preliminary to a discussion of the Evaluation Tables to be carried out in the next two chapters. Apart from certain modification, which will be enumerated (and which were facilitated by the computer routines used to develop those Tables - the Exhibits being manually developed): the Tables are compiled on the same principles as were the Exhibits. It has not been thought necessary to develop every illustrative table in respect of every asset life, nor in respect of every period within one asset life. Neither is every cash flow or earnings stream illustrated, as these are at times identical in their arithmetic.

1. OWNERSHIP

1.1. Allowances and disposal values (£ undiscounted)

<table>
<thead>
<tr>
<th>Arising at end of period</th>
<th>Written down value</th>
<th>Investment grant</th>
<th>Writing down all'ces.</th>
<th>Disposal value</th>
<th>Balancing all'ces.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(20%)</td>
<td>(20%)</td>
<td>(i)</td>
<td>(ii)</td>
</tr>
<tr>
<td>0</td>
<td>1000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 1/2</td>
<td>800</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>640</td>
<td>160</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>512</td>
<td>128</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>410</td>
<td>102</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>328</td>
<td>82</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>262</td>
<td>66</td>
<td>100</td>
<td>400</td>
<td>228 (72)</td>
</tr>
<tr>
<td>7</td>
<td>210</td>
<td>52</td>
<td>50</td>
<td>250</td>
<td>212 12</td>
</tr>
<tr>
<td>8</td>
<td>168</td>
<td>42</td>
<td>10</td>
<td>100</td>
<td>200 110</td>
</tr>
</tbody>
</table>

Notes

(a) Balancing allowance calculated on a basis of disregarding the writing-down allowance in the year of sale - e.g., (year 5) (72) = 328 - 400

(b) A bracketed balancing allowance is a balancing charge.

Column (i) indicates the obsolescence situation, column (ii) indicates the non-obsolescence situation.
(c) The grant is timed as being received one year after purchase. As purchase is set at mid-year for an accounting year ending 31st December; e.g., purchased June 1968, so the year ending 31st December 1968 is the end of period 1. Writing-down allowances are deemed to be received at 12-monthly subsequent intervals, i.e., commencing from the end of period 2, using period 1 as a basis period for time purposes. As explained in the text, discounted outlay cost is not adjusted for the 6-month interval between 't = 0' and outlay time. In the Evaluation Tables, 't_0' = outlay time.

(d) The substitution of a 30% initial allowance for the investment grant, where the initial allowance is received at the same time as the first writing-down allowance (to which it is added for the purpose of calculating subsequent writing-down allowances) yields the following:

\[ \begin{array}{cccc}
\text{Arising} & \text{Initial} & \text{Writing} & \text{Balancing} \\
\text{at end} & \text{allowance} & \text{allowance} & \text{allowance} \\
\text{of period} & \text{(i)} & \text{(ii)} & \\
2 & 300 & 200 & \\
3 & 100 & & \\
4 & 80 & & \\
5 & 64 & & \\
6 & 51 & 156 (144) & \\
7 & 41 & 155 (45) & \\
8 & 33 & 154 64 & \\
\end{array} \]

(e) As allowances are allowances against corporation tax; then for appraisal purposes they assume a value of:

"allowance x tax rate": e.g., (year 5) 82 x .40 = 33 assuming a 40% tax rate. It is this tax-adjusted "value of allowance" which is used in all subsequent calculations.

1.2. Earnings Streams

These are set at a compound 12% per annum rate, calculated on yearly rests. Earnings are taxed at 40%, deemed to be paid one year in arrears. Such tax payments are regarded as creating opportunity costs (forfeited earning potential) at the same
12% rate. The effect is to create a net-of-tax earnings stream, but by using this "gross" approach the all-important issue of cash flow timings is satisfactorily dealt with. For reasons discussed briefly in the text and in greater detail in the next Chapter, earnings accruing and forfeited are calculated through period 9 for a 5-year asset life, period 10 for a 6-year life, and period 11 for a 7-year life. Taxation on earnings accrued is taken through one more period in each case.

Abbreviated illustrations of the calculations involved are:

<table>
<thead>
<tr>
<th>Earnings Generated by the Investment Grant (£, undiscounted)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Received at end of period</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>1/2</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

**Notes**

(a) Earnings to end of period 2 is for 6 months only
(b) Tax in period 9 refers to earnings of period 8 (not shown)
### Earnings Generated by Writing-Down Allowances. (Grant Qualifying)

(£. undiscounted)

<table>
<thead>
<tr>
<th>Received at end</th>
<th>Writing-Down All'ees.</th>
<th>12% Earnings on Forfeited Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>64</td>
<td>7.7</td>
</tr>
<tr>
<td>3</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>41</td>
<td>14.7</td>
</tr>
<tr>
<td>5</td>
<td>33</td>
<td>5.9</td>
</tr>
<tr>
<td>6</td>
<td>26</td>
<td>1.3</td>
</tr>
<tr>
<td>etc.</td>
<td>etc., etc., etc.</td>
<td>etc., etc.</td>
</tr>
</tbody>
</table>

#### Notes.

(a) After the end of period 4, calculations become somewhat intricate. Disposal at the end of period 5 means that:

1. One more writing-down allowance is received (£33) and the earnings stream (calculated thereafter up to and through period 9) is calculated at compound rates on a fixed base of this accumulated sum of four writing-down allowances.

2. The fifth writing-down allowance (£26) is merged with the balancing allowance.

(b) Disposal at the end of period 6 means that the fifth writing-down allowance (£26) is now added to the fixed base on which the earnings are calculated. The sixth allowance is merged with the new balancing allowance; and so on. Algebraically:

\[
\sum_{i=t}^{t+s} E_i = \left[ \sum_{j=n}^{t-1} A_j (1 + r)^{t+s-j} \right] - \sum_{j=n}^{t-1} A_j
\]

for \(1 \leq t \leq n\) : that is, this equation is valid only for the computation of \(E\) at some point in time at or after disposal. Computation before that time would appear not to have any particular relevance, as it would involve financial decision making without consideration of disposal values.

(n.b.) \(E\) = total accrued earnings at period 'i'.

\(A_j\) = the writing-down allowance received in the 'jth' year, assuming that this allowance is first received 'n' periods after the base time.

t = the period at the end of which the asset is disposed of.
s = the number of periods after period 't' to which
the study period is extended.

r = the earning rate.)

Further expressions could be developed for the taxation and
forfeited earnings streams, but would serve no immediate
purpose.

The earnings streams generated by the balancing allowances
and the disposal values are identical in pattern to those
generated by the investment grants. Their timings are of
course very different. Those for the balancing allowances
start at the end of period 't+1' (i.e., the beginning of
't+2') and proceed to the end of 't+4'. Those for the
disposal proceeds start at the end of 't' and proceed to the
end to 't+4'. In both cases the taxation calculations extend
one further period. As the appraisal involves two disposal
values for each of the three alternative economic lives, there
are in all six sets of calculations each for the earnings
streams generated firstly by the balancing allowances and
secondly by the disposal values.

1.3. Discounting

Using discrete, end-of-year factors, each separate calculation
is discounted back to a common base time 't=0'. Because it is
desired to review separately the cash flows and the generated
earnings streams, each cash flow (i.e., each year in each set
of calculations) is discounted separately. Some small economy
of effort in manual development is gained by annually summing
the earnings, tax and forfeited earnings in each set of
calculations; and discounting each annual sum. As an example:

<table>
<thead>
<tr>
<th>Balancing Allowance (Disposal end period 6, disposal value £50)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arising at end of period</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>Total:</td>
</tr>
</tbody>
</table>
Notes.

(a) The negative figure in period 11 is the tax on earnings in period 10.

(b) Total: see the appropriate data in Exhibit I. It may be instructive to observe that the summed net earnings of period 10 consist of:

£12.8 compound earnings in the third year after receipt of the balancing allowance less £4.6, being 40% tax on the £11.4 compound earnings in the second year; all less £0.5, being 12% forfeited earnings on £4.1, which is 40% tax on the £10.2 compound earnings in the first year. See Page 32 assumption (1).

2. LEASING.

Calculation of the cash flows and related earnings streams accruing to leasing is complicated by the convenience use of annual calculations and discounting, because essentially the cash flows consist of monthly increments. In the Evaluation Tables, the use of computer techniques facilitates monthly time intervals, and the approximations inherent in the textual model are avoided. The framework of calculation is the same. In the model:

1. Allowance for Premiums (Tax Shield).

As an illustration (disposal at the end of 5 years)

<table>
<thead>
<tr>
<th>Period</th>
<th>Premiums Paid</th>
<th>40% Tax Shield</th>
<th>12% Earning</th>
<th>40% Tax on Earning</th>
<th>12% Forfeited Earnings</th>
<th>Summed Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>128</td>
<td>51.2</td>
<td>6.1</td>
<td></td>
<td></td>
<td>6.1</td>
</tr>
<tr>
<td>2</td>
<td>236</td>
<td>94.4</td>
<td>18.2</td>
<td>2.4</td>
<td></td>
<td>15.8</td>
</tr>
<tr>
<td>3</td>
<td>216</td>
<td>86.4</td>
<td>30.8</td>
<td>7.3</td>
<td>0.3</td>
<td>23.2</td>
</tr>
<tr>
<td>4</td>
<td>216</td>
<td>86.4</td>
<td>44.8</td>
<td>12.3</td>
<td>1.2</td>
<td>31.3</td>
</tr>
<tr>
<td>5</td>
<td>216</td>
<td>43.2</td>
<td>60.6</td>
<td>17.9</td>
<td>2.8</td>
<td>39.9</td>
</tr>
<tr>
<td>6</td>
<td>108</td>
<td>73.0</td>
<td>24.2</td>
<td>5.9</td>
<td></td>
<td>42.9</td>
</tr>
<tr>
<td>7</td>
<td>81.8</td>
<td>32.7</td>
<td></td>
<td></td>
<td></td>
<td>(32.7)</td>
</tr>
<tr>
<td>8</td>
<td>81.8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Notes

1. This table presents both cash flows and the generated earnings stream.
2. The premiums are payable in advance, commencing half-way through period 1. The amounts are:
   a) £21.4 per month for the first 12 months, when (in line with the ownership calculations) the lessor is deemed to receive the investment grant which is at once passed on to the lessee by way of a premium reduction and a refund.
   b) £18.0 per month for the four remaining years of the primary period.
   c) £10 per annum thereafter during the secondary period.
3. It follows that there fall into:
   period 1 ... 6 premiums of £21.4
   period 2 ... 6 premiums of £21.4 plus 6 premiums of £18.0
   periods 3, 4 & 5 ... 12 premiums of £18.0
   period 6 ... 6 premiums of £18.0
   (n.b. for a 6-year life, the premiums in period 6 would be £118.0; i.e., £108.0 plus a first £10 secondary period premium. For a 7-year life, the premium in period 7 is £10.0 only)
4. The tax shield is allowed in the year of assessment next following the basis period, upon the total premium cost incurred in that basis period.
5. The earnings stream pattern is identical to that defined for the earnings streams accruing to ownership.
6. The alteration or continuation of premiums payable in year 6 and beyond for lives in excess of 5 years introduces an intricacy of calculation similar to that experienced in connection with the earnings streams accruing to writing-down allowances.
   As an example, for a 6-year life:
At end of period | Premiums Paid | 40% Tax Shield | 12% Earnings | 40% Tax on Earnings | 12% Forfeited Earnings | Summed net Earnings
--- | --- | --- | --- | --- | --- | ---
1-5 | Identical with calculations for a 5-year life
6 | 118 | 86.4 | 44.8 | 12.3 | 1.2 | 31.3
7 | 47.2 | 60.6 | 17.9 | 2.8 | 39.9
8 | 73.5 | 24.2 | 5.9 | 43.4
9 | 82.3 | 29.4 | 9.6 | 43.4
10 | 92.2 | 32.9 | 14.1 | 45.2
11 | 36.9 | (36.9)

Notice:

a) The prolongation of the study period is identical to that of the ownership calculations.
b) The change of premium in period 6; of the tax shield in period 7; and of the earnings stream in period 8 as a result thereof.

Modifying the equation for earnings to writing-down allowances to include for receipt of the tax shield in its own form in the year of disposal: we may stipulate (e.g.,) for the "12% earnings" stream:

\[
\sum_{i=t}^{t+s} E_i = \left[ \sum_{j=n}^{s-j} D_j (1 + r)^{t+s-j} \right] - \sum_{j=n}^{s-j} D_j
\]

for i < t < n

using the symbols as before, except that D (tax deduction) is substituted for A (tax allowance).

2. Grant Refund

The source of this cash flow is specified in Note(2) of the previous Table. The amount of the refund is the sum of the excess premiums paid up to the day of the refund. It is of course to be realised that all leasing contracts in respect of grant-qualifying equipment are not drawn up in this form. It is equally common to find a constant premium charged throughout the primary period. The monthly rate then reflects the lessors certainty of receiving the grant, plus an annual average adjustment for under-recovery of the due proportion of asset cost.
(plus the lessor's expenses and profit) during the period which elapses before the grant is due to be received. (In short, the value of the otherwise refunded excess is, in this method, averaged out over all primary period premiums. In this illustration the present value of 12 months excess monthly premiums of £3.4 each, payable in advance, less the present value of a £40 refund at the end of twelve months; where that twelve months period commences six months after the base time; is £1.7 approximately. This fractional advantage to the lessor must be averaged out as a surcharge over the flat-rate monthly premiums alternative, in such wise that the present value of the surcharge series is equivalent to £1.7. This presupposes certainty of receiving the grant on the due date. In fact the flat-rate premium charge method is usually one or two shillings per month per £1000 above the adjustment-on-receipt-of-grant method. It is difficult to be exact about this, as leasing companies tend to use one method or the other, rather than both. Competitive quoting between companies then makes comparisons of the methods imprecise. Finally, it is noteworthy that not all leasing companies are as mathematically precise in their actuarial calculations as are the leaders in the field).

As the excess premiums have been taken as tax-deductible, it follows that the refund is subject to tax. The pattern of cash flow and earnings stream (12% earnings less 40% tax thereon and less 12% forfeited earnings on that tax) is identical with that already established. It is worth noting that calculation of the grant refund cash flow and accruing earnings stream requires the following elements of calculation:

<table>
<thead>
<tr>
<th>Period</th>
<th>Grant Refund</th>
<th>Tax</th>
<th>Forfeited Earnings on Tax</th>
<th>Summed Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1.2</td>
<td>39.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>2.4</td>
<td>5.1</td>
</tr>
<tr>
<td>3</td>
<td>16.0</td>
<td></td>
<td>5.1</td>
<td>20.1</td>
</tr>
<tr>
<td>4</td>
<td>1.9</td>
<td></td>
<td>5.7</td>
<td>2.0</td>
</tr>
<tr>
<td>5</td>
<td>2.2</td>
<td></td>
<td>6.4</td>
<td>2.3</td>
</tr>
<tr>
<td>etc.</td>
<td>etc.</td>
<td></td>
<td>etc.</td>
<td>etc.</td>
</tr>
</tbody>
</table>
Notes

1. In the Exhibits in the text, the refund cash flow is shown net of tax.

2. As in all other cases, the earnings streams are taken to periods 9, 10 and 11 for 5, 6 and 7-year lives respectively; with taxation calculations extended for one year.

3. The basis year for the refund is period 2, and the relevant assessment is raised in the following period.

3. Disposal Proceeds

The cash flows and accruing earnings streams are identical to those calculated for the ownership study - save that a flat £50 "minimum disposal value" is deemed to have been set in the leasing contract.

As an example:

(£, undiscounted) (5-year Life; Disposal Value £400; 12% Earnings; 40% Tax)

<table>
<thead>
<tr>
<th>At end of period</th>
<th>Disposal Proceeds</th>
<th>Tax Forfeited</th>
<th>Earnings on Proceeds</th>
<th>That Tax</th>
<th>Forfeited Earnings</th>
<th>Sumsed Earnings on that Tax Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>350</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>140</td>
<td>42.0</td>
<td></td>
<td></td>
<td></td>
<td>42.0</td>
</tr>
<tr>
<td>7</td>
<td>16.8</td>
<td>47.0</td>
<td>16.8</td>
<td></td>
<td></td>
<td>13.4</td>
</tr>
<tr>
<td>8</td>
<td>18.3</td>
<td>52.0</td>
<td>18.8</td>
<td></td>
<td></td>
<td>13.1</td>
</tr>
<tr>
<td>9</td>
<td>21.1</td>
<td>59.0</td>
<td>21.1</td>
<td></td>
<td></td>
<td>12.3</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>23.6</td>
<td></td>
<td>(23.6)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes

1. The proceeds are taxed as chargeable receipts, being pure profit accruing to the disposal of an asset; the cost of which (premiums) has been wholly deducted from previous tax liability. In a sense, this is a balancing charge.

2. Earnings streams will be extended as heretofore for longer asset lives.

3. There are in all six such disposal proceeds calculations.

4. Residual Capital Earnings

(The following is descriptive of a calculation method used. It does not raise the validity of the concept).

When using annual calculations as a short-cut method, the principal difficulty is that of deciding the representative annual
residual capital. Thus in the first twelve month period of the present illustration, residual capital varies from £979 (first monthly premium payable in advance) down to £872; and from £851 down to £744 from the beginning to half-way through the second period (the half-way point being the end of the first full year's premiums) and thence to £618 at the beginning of the third period. For the purposes of the appraisal study only (the Evaluation Tables using monthly calculations), the representative annual residual capital is set at the mid-point of each premium year - i.e., at the end of each 't' period less one monthly premium (to take into account payment in advance). Effectively therefore, the representative residual capital for each of the five periods is that remaining respectively after the payment of 7, 19, 31, 43 & 55 monthly premiums. The result is somewhat to understate the accruing earnings stream, as the representative is biased towards understatement. Some compensation is afforded by:

a) Earnings are then calculated as accruing initially for a twelve-month first period, whereas they should be set to accrue for only six months as the contract does not start until mid-year.

b) When in period 5 residual capital becomes negative, it is a lower negative than that which ultimately obtains under monthly calculations. The consequent 'forfeited earnings' stream is understated.

As indicated in the text, the net effect is a very small understatement of residual capital earnings, amounting to approximately £15 over a 5-year life. Using such a life as an example:

(Continued overleaf)
<table>
<thead>
<tr>
<th>At end of period</th>
<th>Residual Capital</th>
<th>Earnings on Residual Capital</th>
<th>Tax on Earnings</th>
<th>Forfeited Earnings</th>
<th>Summed net Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>851</td>
<td>102.1</td>
<td></td>
<td></td>
<td>102.1</td>
</tr>
<tr>
<td>2</td>
<td>618</td>
<td>86.4</td>
<td>40.8</td>
<td>9.9</td>
<td>45.6</td>
</tr>
<tr>
<td>3</td>
<td>402</td>
<td>70.9</td>
<td>34.6</td>
<td>4.9</td>
<td>31.4</td>
</tr>
<tr>
<td>4</td>
<td>186</td>
<td>53.4</td>
<td>28.4</td>
<td>9.6</td>
<td>15.4</td>
</tr>
<tr>
<td>5</td>
<td>(49)</td>
<td>31.8</td>
<td>21.4</td>
<td>14.2</td>
<td>(3.8)</td>
</tr>
<tr>
<td>6</td>
<td>41.3</td>
<td>12.7</td>
<td>18.5</td>
<td></td>
<td>10.1</td>
</tr>
<tr>
<td>7</td>
<td>46.3</td>
<td>16.5</td>
<td>22.2</td>
<td></td>
<td>7.6</td>
</tr>
<tr>
<td>8</td>
<td>51.9</td>
<td>18.3</td>
<td>26.9</td>
<td></td>
<td>6.5</td>
</tr>
<tr>
<td>9</td>
<td>58.9</td>
<td>20.8</td>
<td>32.3</td>
<td></td>
<td>5.8</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>23.2</td>
<td></td>
<td></td>
<td>(23.2)</td>
</tr>
</tbody>
</table>

5. Discounting

Discrete end-of-year factors are used to discount separately each year of cash flow and/or accrued earnings stream. As an example:

**Allowance for Premiums ("Tax Shield") - discounted.**

(5-year life; 10% discount)

<table>
<thead>
<tr>
<th>At end of period</th>
<th>Tax Shield (40%)</th>
<th>Summed net Earnings</th>
<th>Discount Factor</th>
<th>Discounted Tax Shield</th>
<th>Discounted net Earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>51.2</td>
<td>68.6</td>
<td>.826</td>
<td>42.3</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>94.4</td>
<td>6.1</td>
<td>.751</td>
<td>70.9</td>
<td>4.6</td>
</tr>
<tr>
<td>4</td>
<td>86.4</td>
<td>15.8</td>
<td>.683</td>
<td>59.0</td>
<td>10.7</td>
</tr>
<tr>
<td>5</td>
<td>86.4</td>
<td>23.2</td>
<td>.621</td>
<td>53.6</td>
<td>14.4</td>
</tr>
<tr>
<td>6</td>
<td>86.4</td>
<td>31.3</td>
<td>.564</td>
<td>48.6</td>
<td>17.7</td>
</tr>
<tr>
<td>7</td>
<td>43.2</td>
<td>39.9</td>
<td>.513</td>
<td>22.6</td>
<td>20.5</td>
</tr>
<tr>
<td>8</td>
<td>42.9</td>
<td>2.9</td>
<td>.466</td>
<td>20.0</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>43.0</td>
<td>42.4</td>
<td>.385</td>
<td>18.2</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>(32.7)</td>
<td></td>
<td></td>
<td>(12.6)</td>
<td></td>
</tr>
</tbody>
</table>

Totals (see Exhibit II) 297.0 93.5

All other cash flows and earnings streams are similarly discounted.

6. **Leasing Non-Qualifying Equipment.**

Where the asset to be leased does not qualify for an investment grant but does qualify for an initial allowance (standardised at 30%), the monthly premium typically becomes a uniform figure...
of £21.5 p.m. The "tax shield" and residual capital earnings are as a result increased and reduced respectively; but the pattern and the timing of calculations remain unchanged. The grant refund cash flow and its accruing earnings stream cease to apply. Disposal proceeds and accruing earnings streams remain unchanged.

The calculations examined and illustrated in this Appendix are laborious and intricate rather than complex or difficult. The timing of an element of cash flow or earnings stream is the critical characteristic, as this vitally affects the discounting process. If timing were to be ignored the calculations would be simplified, the various percentages of percentages being 'nested' inside each other to reveal that, typically, an earnings stream is a compounding sum of 6.624% of each of an irregular series of elements in a generating cash flow. But timing can not be ignored.

It has been said earlier that it is possible to cast each cash flow and accruing earnings stream into algebraic form. This has been demonstrated in respect of the undiscounted earnings (untaxed) accruing to writing-down allowances. As an alternative it is possible to demonstrate that the undiscounted, untaxed earnings stream derived from, say, an investment grant 'G' received at the end of period 't' is, for an earning rate 'e':

\[
\begin{align*}
& \text{at } t_2 \ldots \ Ge \\
& \text{at } t_3 \ldots \ Ge + Ge^2 \\
& \text{at } t_4 \ldots \ Ge + 2Ge^2 + Ge^3 \\
& \text{at } t_5 \ldots \ Ge + 3Ge^2 + 3Ge^3 + Ge^4 \\
& \text{and so on. From this it can further be demonstrated that the discounted, taxed stream at (e.g.,) } t_4 \text{ is, for a tax rate 'o' and a discount rate 'r':}
\end{align*}
\]

\[
\frac{Ge + 2Ge^2 + Ge^3 - o(Ge + Ge^2) - cGe^2}{(1 + r)^4}
\]

which simplifies to:

\[
\frac{[(1 - o)(Ge + 2Ge^2)] + Ge^3}{(1 + r)^4}
\]
and for the stream derived from the first three writing-down allowances is, for a capital sum ‘K’ and allowance at ‘ae’:

$$\frac{Kae(7 + a + 2e^2 + e^3 - 3a^2 - a^2) - Kc^2ae(3 + e - a) + Kc^2ae}{(1 + r)^5}$$

Three difficulties will be observed. Firstly, the staggering of tax and the further staggering of forfeited earnings force the expression for each period into a long and clumsy expansion. Secondly, the basic undiscounted, untaxed earnings stream is a binomial expansion in the shape of "Fascal's triangle"; which, for 'tn', where 'n' is comparatively large, is again a long and ponderous expansion. Thirdly, the application of a different discount factor to each separate period's taxed earnings precludes a separate compounding of gross earnings, taxation thereon and forfeited earnings thereon; summing the compounded amounts; and discounting in total - which would facilitate a more generalised algebraic expression. The extension of the taxation study period beyond the forfeited earnings study period adds a small but awkward term to such an expression. Finally, although many cash flows (and hence their earnings streams) are derived from other cash flows - e.g., balancing allowances are a function of disposal values and writing-down allowances, and each writing-down allowance is itself a function of the previous one - the different timings of the cash flows make any attempt to derive combined generalised expressions extremely complicated.

For these reasons, no generalised formulae or mathematical models are presented either for the textual illustration or for the various Evaluation Tables. It would be possible to do so, but the result would be of little, if any, practical value.

..................
CHAPTER TWO  
THE METHODOLOGY OF THE EVALUATION TABLES.

Introduction: Relevant Questions and Exclusions.

The previous Chapter reviewed some preliminary conclusions as to the effect of changes in the discount factor upon the comparative advantages of ownership and lease financing in conditions of changed economic life. The next Chapter will continue that discussion in terms of absolute advantage in conditions of constant economic life.

In this present Chapter there are discussed the implications and methodology of the Evaluation Tables. These seek to answer, in purely quantifiable terms, the questions: what is the most economical method of financing some item of capital expenditure; and by how much is that method the most economical. It is not claimed that such an answer then constitutes a categorical imperative - only that the various problems of a non-quantifiable nature can then be put into the decision scales in the form of a subjective evaluation of the quantitative answer. The final decision is thereby better informed, as the cost of exercising some emotively preferred choice of financing can be measured. The Evaluation Tables are in this sense seen as being as much an instrument of guidance towards a decision, as the decision itself.

Suppose, however, that there is no discernible realistic advantage in terms of non-quantified considerations to be gained from the use of one financing method rather than another. It would appear to follow that the Evaluation Tables would then provide the final answer. That would be to say that use of any financing method shown by the Tables to be the most economical could be effected without fear of subsequent repercussions of any sort. The firm which makes no use of budgeting or other planning techniques, which takes each decision on a 'spot' basis, might think this to be the case; and if that firm were wealthy enough and managed by its owners (i.e., set to satisfy the requirements of management rather than shareholders), this indeed might be so. Even in the widely owned, shareholder-return motivated firm using sophisticated planning routines; relatively small projects and emergency decisions of less than the greatest
magnitude might use the Tables as the ultimate arbiter of financing method. But in a more general sense, financing decisions affect capital mix: and notwithstanding whatever financing method appears from the Tables to be the optimum, the final decision must take into account the repercussive effects of changes in capital mix. For these effects may include variations in the cost of capital and the discount factor.

So important is this issue that it is made a principal element of the theoretical argument developed in Part Two of this thesis. For the moment, the practical applications of the Tables will be considered, subsuming either that problems of capital mix are not significant or that they can be handled in some procedural manner not necessarily related to the Tables themselves.

Even this requires consideration of four separable issues:

1. A justification of certain assumptions basic to the calculation of the Tables - especially of the study period techniques and of the earnings concept.
2. A justification of the use of a common discount rate to discount all alternatives.
3. A decision as to whether it is equitable to compare all methods of financing, each against all others.
4. A revision of the arithmetic of the calculations.

Only then can a review of the Tables be essayed, commenting upon the salient trends revealed by comparisons between financial methods and explaining the manner in which the Tables are to be used.

It is regrettable that the very many variants of deferred payment schemes in existence (as opposed to pure hire-purchase) preclude the presentation of Evaluation Tables for this method of finance. Thus, for example, deferred payment schemes for metal machining and forming plant from five leading suppliers have the following patterns:

<table>
<thead>
<tr>
<th>% of Cash Cost paid by User</th>
<th>Staveley</th>
<th>Herbert</th>
<th>T.I.</th>
<th>Cincinatti</th>
<th>Churchill</th>
</tr>
</thead>
<tbody>
<tr>
<td>By the end of:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 1</td>
<td>25.0</td>
<td>7.5</td>
<td>3.5</td>
<td>1.0</td>
<td>2.5</td>
</tr>
<tr>
<td>2</td>
<td>32.5</td>
<td>15.0</td>
<td>20.0</td>
<td>53.0</td>
<td>3.5</td>
</tr>
<tr>
<td>3</td>
<td>62.5</td>
<td>22.5</td>
<td>55.0</td>
<td>105.0</td>
<td>58.7</td>
</tr>
<tr>
<td>4</td>
<td>92.5</td>
<td>72.5</td>
<td>90.0</td>
<td>-</td>
<td>113.0</td>
</tr>
<tr>
<td>5</td>
<td>122.5</td>
<td>122.5</td>
<td>125.0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
with each scheme requiring a different initial payment in advance as part of the first year payments; and the investment grant being transferred to the user either as a lump sum or by way of a general reduction in all payments. It is manifestly impossible to produce standardised Evaluation Tables in the face of such diversity. The only generalised conclusion that can be drawn is that the true interest cost of such schemes is around 7.5\% and that 5-7 year periods (with some emphasis on the shorter terms) are typical.

For the rest, there seems to be no alternative but to evaluate each scheme on an ad hoc basis, using the methodology employed in the Evaluation Tables. It is especially noteworthy that many deferred schemes (such as those quoted above) utilise low early payments; residual capital earnings are high in the early years, and are not heavily reduced by the discounting procedure. The larger part of the tax shield however is received late, and enjoys a much reduced value accordingly; and the same is true of the writing-down allowances which for the majority of schemes appear to be received only pro-rata to the capital content of the instalments. The advantages claimed for deferred payments schemes are, significantly, couched more in "pay as you earn" terms rather than in terms of competitive financing costs. They are addressed to liquidity - conscious rather than profit-conscious management, and, as such, must be evaluated as a potential break-out from the restrictions of capital budgeting.

Something of the same problem attaches to the evaluation of lease financing. However, such is the current competition in this field, that monthly premium figures are not widely different for similar categories of equipment, and so exemplifying Tables are presented. By categories is meant:

1. The equipment is, in a general sense, production or transport equipment. Specifically, it is not communication or data processing equipment.

2. The equipment qualifies for either a standard investment grant or a standard initial allowance.

A difficulty does arise in connection with the different treatment accorded to the investment grant by different lessors. Some use it to reduce generally all primary period premiums; others refund
it to the lessee in a lump sum. This has necessitated the provision of short supplementary Tables which must be applied by way of adjustment to the main Tables. These may themselves require to be used as the basis for an adjusting calculation where premiums are significantly different from the range of examples given. It is also to be remembered that the Evaluation Tables for leasing are restricted to the primary period, and that “heavy” primary premiums may receive some relief by reduced secondary premiums—though the low size range of secondary premiums and the low discounted value of their earnings streams does not offer much scope in this direction.

Finally it must be stressed that all evaluations are carried out on the assumption of a liability to Corporation Tax. Such is the potential difference in the timing of the respective cash flows, and in the applicable rates of tax; that the Tables are not suitable for appraisal under a Schedule D liability.

Section 1. Basic Assumptions

The first assumption which is basic to the calculation and use of the Evaluation Tables is that of the 5-years asset life. It will be recalled that in fact this involves a considerably longer period if the various cash flows and earnings streams are all to be brought into the calculations. Thus, in ownership flows, the earnings lost on the tax entailing to the earnings accruing to the balancing allowance do not eventuate until the 9th year, after disposal at the end of the 5th year. Consequently all earnings streams are taken up to and including the 9th year—which means that the arising tax liabilities extend through to the beginning of the 11th year. This standard termination pattern—a full nine years for earnings and up to eleven years for taxation—is imposed on all the Evaluation Tables, regardless of when the individual cash flows and the earnings streams which they generate, begin; but the termination pattern is dictated by the choice of a 5-year asset life.

This length of asset life was chosen firstly because 5 years appears to be the most frequent period of financial planning; evidence in support of this statement is given in Part Two, (1) where a 5-year budgeting model is examined. Secondly, 5 years represents a fair indication of the maximum period which most

(1) See footnote (1) to Page 174, & Page 237
financing companies will consider for the larger majority of instalment debt contracts. Hire-purchase contracts especially are very frequently drawn up for lesser periods - three years is a common period: whilst the larger leasing companies are willing to enter into ten year primary periods for standard leases on run-of-the-mill metal forming plant such as small presses and guillotines. Nevertheless, many contracts of both types are drawn up for 5 years, which thus constitutes a good representation. It is relevant to note that most finance company advertising literature features 5-year examples. Thirdly, the onset of numerically controlled or other automated plant in engineering, chemical and textiles is tending to reduce the average life of plant in these industries, as potentially obsolescence becomes more important. Industries as diverse as motor manufacturing, electrical engineering, pharmaceuticals, plastic extrusion and hosiery are actively considering depreciation rates higher than those calculated on an asset life of 10 years. At the same time, 5 years is becoming a commonplace life expectancy for heavy trucks, passenger transport and civil engineering plant.\(^1\) 5-10 years thus appears to be reasonably representative of intention or practice. Finally, any estimate of disposal value more than 3-5 years after acquisition appears to be an exercise of pure imagination, in many cases. All in all, therefore, a 5-year asset life represents an average of sorts (in what is generally recognised to be an area of considerable statistical ignorance) between financing preferences on the one hand and industrial practice on the other. The discussion of the previous Chapter affords some evidence for believing that an extension of this period would not materially alter the order of financial advantage, although it might alter the extent of that advantage.

The second major assumption in the calculation of the Evaluation Tables is that there can be validity in the concept of earnings streams. This assumption is the more vital in that it affects not...

(1) Information gleaned in conversation with management of, inter area, the following firms: BMC-Leyland, Ford, AEI, Electrical & Industrial, Boots, Courtaulds, Cascelloid, Viyella, John Beale Associates, York Trailer, Hargreaves Group, Calor Gas, Port of London Authority, Tarmac etc. In many cases, cost inflation in wages and materials is tending to induce hesitancy in adding yet a further burden on profits, even where the potential need for a faster write-off is accepted.
only the extent but also the order of financial advantage between the methods. Thus, in the appraisal study, a refutation of the residual capital earnings concept alone would transform the evident advantage of lease financing in the particular circumstances depicted. Indeed, this point highlights the fact that the problem of earnings streams must be considered in two parts: firstly, is the concept itself valid, and secondly, is each earnings stream therefore as valid as all the other streams?

The rate at which earnings streams accumulate may be temporarily defined as that which the firm can normally expect to accrue (gross of tax) on its employment of assets over the long period. The earnings streams themselves are a simple recognition that a net cash inflow into a firm will normally either be disbursed to shareholders, remitted as taxation or re-invested in the assets of the firm. There may be some short period increase in cash balances, but any extension beyond this is sufficiently unusual or dangerous - given the contemporary conjunction of cash shortages and take-over psychosis - to be ignored. Many of the cash flows considered in the evaluation Tables have a dual characteristic:

a) They are negative outflows rather than positive inflows - they reduce tax liabilities rather than appear as increments of cash. What with transfers to deferred taxation or tax equalization accounts and the U.K. preference for a stable dividend rather than a constant pay-out ratio; the larger part of such flows are in fact re-invested. Grants are treated in like fashion: usually being placed wholly to capital reserve or distributed (by way of a reduction in depreciation, in reported tax liability or as an overt subsidy to P.A.L) over the life of the relevant asset, the balance being in fact retained and re-invested.

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(1) A theoretical analysis of the earning rate, and a more exact specification are to be found in Section 5 of Chapter Six (Part Two). The analysis considers the relationship of the earnings rate to the discounting rate. It would be tautological to rehearse the argument here, but it is there concluded that the two rates may reasonably be dissimilar. This is a requirement of the Gordon-Shapiro model, and is necessary to explain the observed difference between the asset value and the market value of most public companies. See Durand (53) and Gordon (62).

(2) A preference enforced by the current Treasury control of dividend increases.
Disposal proceeds, however, stand in greater likelihood of being distributed (especially if comparatively small) as 'exceptional' or 'non-recurring' etc. receipts; and so, in the Evaluation Tables, are not set to general earnings streams.

b) They constitute for those many projects which have a comparatively slow gestation period the principal sources of cash flow in the early, uncertain project years. At this stage prudent management will be loath to initiate substantial distributions. They will prefer to wait until the project is firmly established as earning an operating profit. Once re-invested, they must ipso facto earn; if the firm is at all profitable. And indeed, as is argued in Appendix II: if the original project investment appraisal was by a discounting method, they must then earn at the internal rate of the project if that appraisal is to be justified - the more especially as there is no distribution. To compare financing methods without considering the accruing earnings streams is to neglect a vital implication of financing decisions. The concept of earnings streams may be seen as an alternative to dual-rate discounting; which is itself a belated recognition that, where the project is financed by equity or funded debt, disbursement of the arising cash flows by way of a repayment of the capital is antithetical to business practice and company law - so that reinvestment is inevitable and yet is unlikely to be at the internal rate of the project.

Calculation of the Evaluation Tables presupposes a total reinvestment of all cash flows arising from the financing decision, except for disposal proceeds. Strictly, this is excessive in that

(1) The approach used in this thesis is in a sense alternative to that used by Vancil (34) Chap.5: who by "washing out" the borrowing facilities afforded by financial leasing and then discounting at an earnings opportunity rate is by implication using a not dissimilar argument.

(2) The concept of earnings streams thus presupposes disbursement of arising cash flows outside the generating project and reinvestment into some 'average' asset situation. See Chapter Six, Section 4 for further discussion on this point.

(3) See also Pearson Hunt (16) Chap.2,4 for an alternative proposal for two-rate analysis, which shares a great deal of the same motivation, albeit expressed in an inverted form.
some distribution is not impossible - but the excess must be reckoned to be common to all financing methods in that there is no reason to suppose that the cash inflows of one method are inherently more subject to distribution than those of another. It must however be admitted from this that the somewhat greater importance of earnings streams in the total inflows of leasing finance (compared with ownership inflows) may mean that there is some risk of under-statement of the cost of leasing, especially at higher rates of earning.

A second presupposition is that specified above - that the reinvested cash flows must earn. It is not impossible to conceive of a company which is so satiated with cash flows that no further internal investment opportunities exist. In the short period this is quite realistic, especially where accompanied by disinvestment in a shrinking or in a technologically unsatisfactory market. The Distillers Company and C.P.A. are good examples of this; as are Powell Duffryn (in 1940) and more recently C.K.N., where disinvestment was compulsory (nationalisation). But it is noticeable that this phase does not last as liquidity is deployed into new external investment\(^1\) - voluntarily as with these companies, or involuntarily as a result of absorption in a merger. This is not to say that the earning rate must be high; that is, in excess of the discount rate. Given a tenacious management and a static technology such a situation may well be prolonged - witness large areas of the U.K. ship-building industry. Typically, the market value of the firm will then fall below its asset value. In the long run, reorganisation, take-over or closure must eventuate. Impossibility or near-impossibility of reinvestment earnings is essentially a short-term or exceptional situation, where either a superfluity of cash renders financing decisions irrelevant or the economic future is so bleak that there are few if any investment decisions to be financed.

Nevertheless certain conceptual problems remain. They arise out of a common paradox: the reinvestment within or without the firm of cash flows arising from financing decisions is itself a financing decision - and it is a decision to finance by equity in the form of retained funds. The concept of financing a series of successive investments each as it were out of the finance and operating cash

\(^1\) External, that is, to the firm as at present constituted. Incorporation within the firm usually accompanies such investment today.
flows of the previous investment; and of so re-ordering the series as to maximise the total investment complex within the constraint of maximising the present value of the firm; is treated at length in Part Two of this thesis, where it is found to be complex, imprecise and potentially unstable. I am concerned here to examine briefly certain implications of the paradox.

Postulate that there exists a situation where a very high earning rate affords such cost leverage that some sort of debt financing (say leasing) is always more economic than ownership. Each financing decision would then be in favour of debt. (1) How then could the arising cash flows be reinvested? There is some temptation to modify the postulate, and observe that a diminishing return to investment (the earning rate) might reverse the advantage in favour of equity financing. This might substantially be countered by permitting the firm to invest widely in external opportunities, purely on a portfolio basis to reduce limitations of management capacity. But the firm is then beginning to take on the appearance of an industrial banker, which as well as exposing management to charges of 'ultra vires' would appear to run into those limits to debt financing which are discussed in Part Two. Within those limits it does seem feasible that there may be sectors of investment profitability where internally generated funds (largely tax remissions and grants) are left at worst to lie comparatively idle and at best to be automatically added to working capital, rather than be used to finance new fixed assets: which is carried out by new debt finance. But at once the earnings streams of such debt finance are reduced, as they in their turn are shown to be non-investable; and immediately the postulate of more or less permanent high earnings is shown to be untenable and 100% debt financing to be most unlikely to be justified. What emerges as most important from this discussion is that the earnings rate must be that which can be expected to accrue over the long term to the employment of all assets - net current as well as fixed. And in view of the short-term exceptions which have been admitted against the general argument, the importance of long term expectations becomes clear.

(1) The implications for capital mix and the cost of capital are considered in Part Two, and are ignored at this stage.

(2) "...a moving average rate developed retrospectively from recent experience will suffice" Pearson Hunt, op. cit. Page 20.
The concept of earnings streams is thus defended on grounds of practical relevance. Its principal arithmetical significance lies in the fact that because the generating cash flows are, as between alternative financing methods, different in their timings and their frequencies over time; very different earnings potentials exist in terms both of size and of duration. When in addition time-adjusted discount factors are applied, the results of comparing alternative financing methods using (discounted) cash flows plus earnings streams may be completely at variance with the results gained by comparing (discounted) cash flows only. A warning note must at once be sounded in this context. Strictly, it is incorrect to speak of discounting earnings streams, unless these can with safety be regarded as close cash equivalents. As the element of non-equivalence is probably the same for all earnings streams, this (as with the assumption of total reinvestment) does not invalidate the procedure, but it may result in some error in the reported cost of any financing method (typically, leasing) while earnings streams are the more important fraction of total inflows.

There is one earnings stream which is sufficiently different from all others as to deserve some separate mention. This is the earnings to residual capital, which is different in that it is generated not so much by a cash inflow as the deferment of a cash outflow. Such a deferment is afforded by instalment debt. The argument postulates that the firm has capital available, but in insufficient quantity for the set of projects available for profitable investment - there is a situation of capital rationing. The serial repayment by instalments releases a dwindling ('residual') capital available for investment elsewhere; and the earnings to this can properly be applied by way of a reduction of the cost of the instalment financing. Almost by definition, a situation of capital rationing is required; otherwise the "elsewhere" investment doesn't exist, and the residual capital lies idle - there are no residual capital earnings.(1)

(1) Even so, it might in certain circumstances, be more profitable to employ instalment debt. Such circumstances would include low writing-down allowances, high asset-earning rates, a high tax liability and significant falls in disposal value as asset life is prolonged. A strong contributory factor would be the possibility of high yielding external portfolio investment of the residual capital. To the extent that this is always possible, it can be argued that there is no such thing as a situation where capital rationing does not apply.
In the simplest case the residual capital can be conceived of as invested at month-to-month deposit rates. The earnings of residual capital would then have to be set at some specific rate rather than at the average asset-earning rate envisaged so far: which is not impossible, but presents a minor inconvenience of separate calculation. More sophisticationly, the released capital is wholly reinvested within the firm, presumably earning at the average asset-earning rate. The serial repayments are made out of general revenue cash flow, which is increasingly and permanently reduced thereby and which results in an increasing loss of potential reinvestment earnings. The two opposed streams of earnings, both at average rates, are conveniently and accurately measured as a stream of earnings accruing to a diminishing residual capital. This substitution of revenue financing for capital expenditure is a feature of instalment debt advertising; but the recognition of the increasing loss of reinvestment potential demonstrates the greater truth that it is a substitution of working capital expenditure for fixed capital expenditure. The residual capital is, strictly, retained (retainable) earnings.

The argument of 'revenue financing' does however throw a useful light on one important aspect. Instalment debt avoids the acquisition of an increase in the overt formal capital structures of the firm, and its piecemeal nature tends to obscure the effective impact on retained earnings. A substantial use of this type of finance has often been associated with an unwillingness or an inability to raise further equity or funded debt. It is strange that this association should have overtones of financial embarrassment, even delinquency: for the decision not to expand permanent capital may be perfectly rational and in the best interests of present shareholders. It may also indicate a welcome managerial competence in forecasting future cash flows.

Instalment debt is, then, no more than an alternative financing

(1) It is essential to realise that these are operation earnings, not earnings from the reinvestment of cash flows arising from financial decisions.

(2) Some aspects of the non-recognition of instalment debt in capital gearing are considered in Part Two.

(3) See, for example, Hazel & Thompson ( ).

method. It is still very much debt capital. But it is a peculiarly informal capital, both in the case with which it can be acquired and in its relative lack of repercussive effect upon shareholders and existing creditors alike. It generates immediate acquisitory power - there is no accumulation period, as with retained earnings. It is short-lived and self-cancelling; unlike new equity or funded debt, which are permanent unless or until redeemed by a formal transaction. Current account financing permits such formal capital as there is now free to be immediately invested elsewhere in the operations of the company - i.e., permits residual capital earning - without permanently increasing the formal capital structure of the company. It is this unique threefold attribute of instalment debt (informality, immediacy, impermanency) which so distinguishes this form of capital that residual capital earnings are ascribed solely to it.

Further emphasis is given to this argument by the much greater tendency of instalment debt to be specific in the assets which it finances: most new equity or funded debt issues being blanket acquisitions of funds. Thus the use of instalment debt to acquire certain specific assets (typically, fixed manufacturing plant) for which short-term banking instruments are often not available, may most usefully free equity resources to acquire other assets (typically land) which most frequently either only can be acquired by equity - because of the inherent risk involved in such assets, e.g., mineral-bearing land; or which strategically ought to be so acquired - because of a high appreciation potential in such assets. Residual capital earnings is a very realistic concept in such circumstances.

But the two basic requirements of the concept must not be overlooked. First - and most probably the more easily satisfied - is the condition that free resources must be able to earn elsewhere.

(1) There must be some limit to the lack of repercussion effect. Contrast the opinions gleaned from two companies: "leasing makes no difference to our borrowing powers" (Scottish and Newcastle Breweries, who use substantial leasing finance) but "in some of our overseas projects, equipment leasing is regarded by our bankers as an appropriation of the total credit facility" (RTZ). The issue is discussed at length in Part Two.
Given the facility to invest outside the company this should present little difficulty. Second: resources must exist to be freed. If instalment debt is considered as a means to expand from an already totally committed equity base, for example; then residual capital earnings take on a different, albeit still valid, meaning. As between two competing instalment debt plans, residual capital earnings are a useful imputed valuation of the different opportunities offered by the different patterns of cash flows implicit in the two plans. But in these circumstances; when comparing between new instalment debt and other incremental methods of finance, residual capital earnings should be excluded from the comparison evaluation because ex hypothesi they cannot exist.

The description of residual capital earnings as an "imputed valuation" in these circumstances opportunely permits a short reflection upon the nature of all the Evaluation Tables. These are internally consistent data which may be used to select a financing method by comparison of their respective values. They utilise a specific scale of measurement, based upon a specific calculation methodology, which is unique to the purpose and which is not necessarily relevant to any other financial quantity or intent. Thus, the Evaluation Tables are COST-RELATIVES; they are not absolute COSTS. Certainly, they could not be incorporated into formal accounts; and as decision criteria are much more safely to be used in judging between alternatives rather than in 'accept-reject' situations.

Finally, it is necessary to be clear that residual capital earnings are not a reciprocal of a cost of equity. In the present Evaluation Tables, the discounting process effectively informs the user that, under the given set of conditions as to grant, allowance, disposal value etc; the various cash and earnings inflows "fails to recover the £1000 capital cost over 5 years at 'n'\% by a sum with the present value of £----". Clearly, 'n'\% (the discount rate) is inclusive of all the cost of equity that is required.
Section 2. The Validity of Comparisons.

The discussion of residual capital earnings leads automatically to a consideration of the extent to which evaluationary comparisons between different financing methods are valid. An evaluation of the costs of instalment debt, whether inclusive of residual capital earnings or not, and a comparison of those costs with the costs of equity in particular; cannot go unchallenged. It has been noted (Page 78) that recourse to instalment debt might imply unwillingness or inability to raise new equity or permanent debt. Is a comparison of the two meaningful?

It is argued(1) that it is invalid to compare leasing with equity financing. The argument is grounded generally on the premise that leasing is a form of borrowing; and so (Gant) the realistic comparison is between equity and loan capital at large, with leasing merely to be compared with other forms of borrowing. There can be no argument but that leasing is a form of debt finance, and that the principal feature contributing to such a classification is the immutability of the monthly or other period premiums. But against that immutability may be set the potentiality of residual capital earnings, albeit these are not of the same quality in terms of probability. Gant's whole argument postulates that because the possible consequences of the two different sources of capital are different, then the capitals themselves are different; i.e., borrowing requires unfailling servicing, but dividends can be foregone. Therefore they are incompatible. And yet if the servicing-adequacy of cash flow is insufficient (and this is what lays behind the gloom of such ominous terms as 'immutable' and 'irretrievable') it does not matter how attractive are the preferred leasing terms, vis-a-vis ownership costs. There is not much point in contemplating leasing or any other form of instalment debt unless forecasted cash flow is adequate; and if it is, then straight cost comparisons with any form of financing appear to be perfectly valid. The 'unfailing servicing' criterion ceases to be of major importance.

Vancil's argument is much more complex, appearing at times rather to shift its ground. It is possible to separate out two strands in his discussion, which are set in juxtaposition rather than integrated.

(1) Gant (57,58): Vancil (34,86): hereinafter referred to as 'Gant' and 'Vancil'.
Firstly there is the categorical statement that leasing\(^{(1)}\) is not part of the investment decision, and so is part of the financing decision only; thus clearly implying a separation of the two decisions. In the Introduction to Chapter One this separation was accepted as a useful convention, subject to the conceptual criticism of the argument contained in Part Two. But the repudiation of leasing as a part of the investment decision is made on distinctly odd grounds - it is our old friend the "irrevocable commitment to make the required payments". Thus:

"It is because of the contractual nature of financial lease payments that we may say that when a lessee enters into a financial lease he has in effect purchased the equipment" 
(Vancil, p.93)

i.e., the investment decision is pre-empted. The investment decision is to be made on the basis of purchasing the equipment. This of course does not involve actually purchasing the equipment, but working out the balance of present worth if purchase were to be undertaken. Presumably there is something about financial leasing that precludes such an exercise - only to think of it is to be committed "irrevocably" to it. The problem of "irrevocability" of leasing premiums in contradistinction to the 'voluntary' nature of dividends has already been dealt with. There may be some advantage of calculation in carrying out the investment decision analysis in terms of the tax flows accruing to ownership if, as Vancil alleges, it is not likely that an appraisal which fails under those conditions will succeed under the cash flows accruing to leasing.\(^{(2)}\) But with the investment decision thus based giving a favourable reply Vancil then proceeds to compare leasing with other forms of financing as part of the financing decision, implying that leasing may prove to be a cheaper form of finance! It at once becomes feasible to conceive of an investment decision which fails under ownership cash flows but which succeeds

\(^{(1)}\) That is to say financial leasing rather than operating rental. Vancil uses the concept of non-cancellability to distinguish between financial leasing and operating leasing. Very late information coming into the hands of the writer indicates that this is not necessarily a valid distinction in U.K. tax law.

\(^{(2)}\) Perhaps fortuitously, Vancil side-steps his own "irrevocability" problem by using an investment opportunity based appraisal factor. Had he used a cost-of-capital based factor, calculated on some weighted average of the cost of equity and of debt, an awkward inconsistency would have arisen - for funded debt is just as "irrevocable".
under leasing cash flows. Finally Vancil stands the argument on its head, as it were, by recommending that in the situation where an investment appraisal using tax flows generated by operating rental financing indicates acquisition, then there should be carried out the additional precautionary exercise of matching acquisition by purchase against acquisition by rental. This he describes as a 'routine investment decision' - but it is plainly just as much a financing decision. In other words, rental is seen to be not restricted to the investment decision, which at least would follow as a natural corollary to the restriction of leasing to the financing decision. Nor can Vancil introduce "irrevocability" into the argument: for the recommended matching is of rental and ownership, which 'ownership' may well be financed by funded debt (or even banking instrument as a bridging operation).

The whole argument is of course superfluous. If in fact a company uses or is willing to use instalment debt, then the investment decision must be carried out in terms either of an overall investment opportunity discount rate (which avoids the problem) or a cost-of-capital based discount rate which acknowledges the existence of instalment debt as part, but only as part, of the total capital mix. To appraise certain projects in the total set of available projects wholly in terms of leasing cash flows and using a leasing-cost-of-capital based discount factor is to be guilty of discriminatory financing evaluation. (1) To appraise all potential projects wholly in terms of ownership tax cash flows yet using a discount rate which acknowledges that some of those projects will in fact be financed by leasing is inconsistent. To assess each project on a fixed ratio of ownership tax flows and leasing tax ratios is not only numerically clumsy: it presupposes a predetermined capital structure, a concept which this thesis rejects. The problem is discussed in Part Two, but for the present only a convention of carrying out the investment decision wholly in terms of ownership tax flows can be accepted as a necessary convenience, without the need for spurious arguments concerning "immutability".

Vancil repeatedly and correctly stresses the analogy between financial leasing and more overt instruments of debt capital.

(1) See Chapter Four, Section 1.
Thus:
"The lessees's obligations under a net financial lease are, therefore, similar to the obligations incurred under a debt instrument. Because of this similarity, management should not enter into a financial lease unless that action is consistent with the company's previous decisions concerning the capital structure, and a desirability of raising new capital at this time ....... and that this money should be raised by the issuance of fixed-obligation securities". (p.92).

They key phrase here is "decisions concerning the capital structure". Strangely, Vancil does not use this as an argument to support the implied non-compatability of equity and lease finance. If such a decision did exist then those two would indeed not be entirely to be compared - for implementation of a predetermined capital-mix decision might make use of one or the other mandatory, irrespective of comparative financial costs for a particular project. The oddity of such a situation (for who would care to substitute the shadowy gains of capital cost minimisation for the much more certain tax gains of instalment debt, where such tax gains exist) is a further indictment of the practical validity of the capital-mix decision concept in anything except the most general, long-term sense.

The probable reason for this omission by Vancil lies in his confusion between ownership (which is a legal state) and equity financing (which is a monetary vehicle). The reader is often confused as to which Vancil really means. Thus Chapter Four is termed the "Lease or Buy Decision": which is strictly a comparison of operating rental with the tax flows of ownership - which presumably, but never specifically, is financed by equity. Chapter Five is termed the "Lease or Borrow Decision" which introduces overt debt financing plans without ever specifying very clearly whether and at what rate the equity in the asset is transferred to the user; but as the plans are referred to as "conditional sales contracts", presumably they are either deferred payment plans or hire-purchase plans, which implies a transfer of ownership to the user (with the rate of transfer unspecified). But the heading "Lease or Borrow Decision" clearly conforms to the argument that
leasing is a form of debt by implying that this is the correct comparison; in which Vancil follows Gant.

It is all the more strange that Vancil does not compare, in his various numerical illustrations, leasing with debt financing. Instead the comparison is with ownership - and an ownership very clearly financed by equity at that. Indeed the validity of a comparison between leasing and borrowing plans can often\(^{(1)}\) be repudiated because the different pattern of instalments means that the amount of credit offered by the user is different in amount and timing as between those plans. Such a situation is intolerable in Vancil's model, which specifies the constraint that the amount of debt outstanding in a period is fixed immutably as a consequence of the predetermined capital-mix decision:

"The decision as to the amount of debt to carry each year should be a conscious one on the part of management" (p.100).\(^{(2)}\)

I have much more sympathy for his earlier argument that often such plans cannot be compared because:

"---plan 'x' makes more funds available to the company and since our calculations assume that available funds can be invested to earn 10\% after taxes, these additional funds constitute an important advantage". (p.99).

It is possible to read into this statement an awareness of the reinvestment assumption implicit in many DCF calculations, which Vancil of course is using. But it is difficult to interpret in any realistic sense at first. The competing 'plans' which Vancil argues cannot be compared are all alternative methods of financing the acquisition of the same asset(s). As such, they all have an identical earning potential. But their different servicing patterns effectively

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(1) But not always. If the financing capability is identical in size and timing, comparison is valid.

(2) Just how management is to select one mix out of the many available from all alternative financing plans for a multiplicity of projects, which best satisfies this criterion, Vancil does not specify. Some of the problems created by such a criterion, at the margin of servicing-adequacy of cash flow, are discussed in Part Two.
varies the working capital left in the hands of the borrower, net of the opposing benefit of the tax shield. Vancil can thus only mean residual capital earnings. If these are specifically recognised in each comparison, then it is quite permissable to compare different schemes of instalment debt. Such recognition is afforded in the Evaluation Tables.

However, it is the constraint of fixed outstanding indebtedness plans which really lead Vancil to reject the automatic validity of comparisons between instalment debt plans which offer different financing facilities. It is one question as to whether that constraint is realistic (1) or not. It is quite another as to whether Vancil ought therefore, in dispute of his own Chapter heading, to compare instead leasing with equity financed ownership; but this he does - thereby quite happily discarding the "leasing is a form of debt and so must only be compared with debt" argument; which, as noted above, he earlier advances on the grounds of the 'nature of the beast' and which is nearly as equally implicit in a predetermination of capital mix as in the obligation of a managerially conscious decision as to the amount of outstanding debt to be tolerated at any point in time.

He effects the comparison between leasing and equity financed ownership by imputing, as part of the leasing premium, a nominal or typical interest charge on the outstanding residual capital. Only the balance of the premium is then taken as a tax deduction, representing an 'equivalent depreciation' on the 'capital repayment' part of the premium. (2) The leasing plan cost is thus loaded with the reciprocal of an interest cost tax deduction in respect of the additional finance which it affords; and as no account is taken of the subsequent ameliorating 'tax shield' earnings flow, is thus placed at a permanent disadvantage vis-a-vis ownership by equity financing. In view of the fact that this substantial and increasing

(1) The reader will have gathered that I do not think it is. See also the argument in Chapter Four, Section 1.

(2) It is not without interest that the evaluation of U.K. hire-purchase contracts must proceed in this way, in view of the different treatment given to the investment grant, capital allowances and interest charges by the Inland Revenue. This procedure is applied in the Evaluation Tables, using the contractual added interest rather than a nominal rate.
earning opportunity is an advantage, to so handicap leasing finance appears distinctly illogical. Finally, as (for the reasons already stated) Vancil does not generally compare leasing with other deferred payment or instalment debt plans, his method seems more than a little inadequate for practical purposes. If the concept of a predetermined capital-mix is rejected, then (except at the limits discussed in the next Chapter) differences in the 'natures' of equity and different types of debt cease to be important, and inter-method comparisons are conceptually valid. The residual capital earnings procedures quite adequately accounts for the indisputably different amounts of additional finance which each instalment debt method offers.

Different financial methods are thus seen to be capable of inter-comparison. This statement presupposes that the decision under review is a 'financial' one, to be made in respect of a project already appraised as part of an investment decision - a supposition which for the moment is taken to be acceptable within the context of an advanced capital budgeting system. It must be observed that the very concept of a preliminary investment decision process followed by a secondary financing decision process requires that (assuming the investment decision to make use of discounting methods of appraisal) a discount factor can be and is determined independently of the financing decision. Assuming that discount factor is a function of the cost of capital: (1) which must itself be some function of the mix of capitals used: this appears to be impossible. Further, the matching of financial methods via the Evaluation Tables, and the selection criterion of the lowest-cost method: implies an easy switching from equity to debt, and between various alternative forms of debt. This contravenes that part of capital budgeting doctrine which proceeds upon the axiom of a predetermined (optimum) capital mix. Even if this complication is shrugged off, the evidently simultaneous

(1) Net present worth analysis is clearly recommended rather than internal rate of return. The discount factor may be equal to the cost of capital or in excess thereof as a threshold 'investment opportunity' rate: but in this second case the cost of capital must set a floor. In the Evaluation Tables both rates are used. The matter is discussed in considerable detail in Part Two, including consideration of the use of a common discount rate to evaluate alternative financial decisions which by their very nature imply different capital mixes.
nature of the two sets of decisions cannot be ignored. Any procedure which uses less than simultaneous solution techniques risks instability of the discount factor. Constant reiteration of the investment decision routine becomes necessary, with each recycle setting up a new financing requirement - which might involve a new capital mix, cost of capital, and hence discount factor - which would initiate the decision reiteration. Superimposed upon this quite sufficiently complex problem is a second: the same variation of the discount factor would involve an additional reiteration of the financing decision itself if that were based upon the Evaluation Tables (which use the discounting process).

Evidently, then, the procedure within an advanced capital budgeting system is by no means as simple as at first appears. This is the motivation for Part Two of this thesis, where these and accruing secondary problems are examined in the context both of a theoretical investment model and of a more practical procedural approach set within the capacity of an accounting of planning function possessing computer facilities.

Where the control functions of the firm are not so advanced as to include capital budgeting, (1) comparison of financing methods through the Evaluation Tables may still be valid (more accurately, as valid as capital expenditure decisions taken in isolation from a planned budget ever can be.) The principal problem might be thought to be that of discriminatory financing - appraising the alternatives of a financial project (e.g., "buy or lease") each in terms of its own financing method: i.e., utilising the discount factor which relates to the individual cost of the particular type of capital used. A moments reflection shows that this is not so. As the Evaluation Tables utilise discounting procedures, it is fair to hope that the investment decision (to acquire or not to acquire) might utilise the same technique: although in the non-budgeting firm this will involve the selection of an arbitrary discount rate. (2)

(1) See footnote (1) to Page 174
(2) Which probably happens in the advanced budgeting firm also. (And how many times will it be 10%?). See Chapter Six, Section 1. (Step 1).
This must be set at the firms' best estimate of its average cost of capital. In a firm of this type, there can be no expectation of project ranking, each project being regarded as a "one-off" isolable decision. It follows that each project can be discounted back to the net present worth of its streams of operating cash out-flows and inflows; and that this net present worth can conveniently matched against each alternative financing method, using the Evaluation Tables (adjusted as necessary for residual capital earnings streams) at the appropriate discount rate which because it is common to all financial methods avoids the error of discriminatory financing. As the Tables incorporate the grant and/or taxation cash flows and accruing earnings streams which relate to the individual financing method, investment and financing decisions are simultaneous, being reduced to noting that financing method which offers the lowest present financial cost against the net present worth of the project.

It is easy to conceive of potential subsequent developments in managerial control. The experience of comparing the various alternative financial matchings, combined with the frustration experienced when seemingly viable projects cannot be proceeded with because of a shortage of funds; will lead directly to some first attempts to rank presently known projects in terms of descending profitability - probably measured by the net present worth of the gap between the net present worth of the project and the net present cost of the 'cheapest' financing method. (It would not take much longer to come to the stage of requiring probably informal indications of anticipated future projects). But this process implies that the firm is willing to use instalment debt, if that is the lowest cost financing - an implication which may very well catch management as it were unaware that possibly here is a means to break out of the straight-jacket of a shortage of funds. It is very probable that innate managerial caution will lead to a quick appreciation of the implications, too, for future cash flow; whether that appreciation is expressed in an overt manner or takes the form of the 'pride of ownership' syndrome. (2) In short, the firm finds itself face-to-face with capital budgeting

(1) i.e., operating sales minus incremental operating costs.
(2) See Chapter One, Section 2.
in terms of investment decisions and financial decisions. The solution adopted may be to apply somewhat crude and arbitrary constraints based on reactions of ignorance or emotion, or of rational cost-benefit assessments of proceeding to further levels of sophistication. Thus the discount factor may hold unchanged not because effectively it ought to be (and this may well be the case for all meaningful calculations) but because the possible impact of a change in capital mix upon the cost of capital is not understood; instalment debt may be rigidly minimised because of an emotional dislike of such financing; project cash flows, future capital allowances, disposal values - these and other cash flow estimates may be so uncertain that it just is not worth while incurring the undeniably high administration costs of complex capital budgeting routines. Some of these considerations find their place in the procedural approach discussion of Chapter Six, where some of the theoretically essential but practically unquantifiable constraints of the finance-investment linkage model of Chapter Five are relaxed.

Thus it is claimed that use of the Evaluation Tables not only can facilitate the capital expenditure decision in the firm where no planning or budgeting is effected. The same use may very well lead to a heightened appreciation of the need and value of such planning or budgeting and so to some however incomplete procedures of financial control. Further managerial probing will lead to the less translucent depths of full scale capital budgeting and capital financial planning, where the cost of the necessary administration and computational resources may begin to outrun the advantages to be gained.
Section 3. An Arithmetical Revision.

The general principles upon which the calculations are founded were set out in the Appendix to the previous Chapter. They consist of an array of cash flows and tax-adjusted compound earnings streams accruing to those cash flows. The justification of the earnings concept has already been discussed in Section 1. of this present Chapter, as have the reasons for the standard study period over which the calculations are made. There remain a number of specific points, of varying degrees of importance, which must be reviewed if the limitations of the Evaluation Tables are to be appreciated.

All earnings streams are calculated on monthly rests, using one-twelfth of the nominal annual earning and discounting rates; i.e.,

for earnings: \[ A = P \left( 1 + \frac{i}{12} \right)^t \]

for discounting: \[ P = \frac{A}{\left( 1 + \frac{i}{12} \right)^t} \]

The effect is that both true annual rates are fractionally higher than the nominal annual rates; but the distortion is very minor and tends to be self-cancelling to the extent that the earning rate and the discount rate are not widely dissimilar. This method has been used for two reasons:

a) The distortion is a small price to pay for the familiar simplicity of one-twelfth of an integer annual earning rate in the minds of most managers. This point is valid if, as is hoped, the Evaluation Tables are to find a practical use.

b) Monthly rests are used rather than continuous rests to correspond with the familiar practice of monthly management accounts. Perhaps more importantly, it really is not valid to argue that all assets earn continuously. Working capital investment in stocks and net trade credit realises nothing until it is liquidated and implemented into a further round of production and sales; the earning frequency is thus
a function of working capital turnover - which may be rapid or slow. Monthly rests therefore represent an unspecific compromise between the earning frequencies of different types of asset.

In these Tables, there are certain specific departures from the methodology of the appraisal study of the previous Chapter. Chief among these is the simulated timing of the expenditures and the subsequent cash flows and earnings streams.

**Purchase (Outright or Hire-Purchase).**

This is deemed to take place at such a time that the investment grant or initial allowance, and the first writing-down allowance, are received twelve months after the actual expenditure. Reasons were advanced in the appraisal study for doubting the realism of this; but as the possible alternatives are innumerable, with no more reason for using one than any other, this convention is adopted in the Evaluation Tables on no better grounds than that it is familiar from common usage and offers certain economies of calculation effort. This means that outlay is made either on the last day of an accounting year ending on 31st December (assuming prompt payment of assessments on 1st January) for a company trading prior to 1st April, 1965; or 3 months prior to the last day of an accounting year (ending at any date) for a company starting to trade after 1st April 1965. If the old-established company should change its accounting year-end after 1st April 1965, then the timing of the Evaluation Tables implies outlay on the last day of that accounting period, whenever it now ends again assuming prompt payment of tax. This also assumes liability to Corporation Tax rather than Schedule D. **IF IN FACT EXPENDITURE IS EARLIER THAN THE INDICATED DATE, IT WILL BE NECESSARY FURTHER TO DISCOUNT THE EVALUATED DATA BY A FACTOR REPRESENTING THE ELAPSED TIME BETWEEN THE DATE OF ACTUAL EXPENDITURE AND THE APPROPRIATE CONVENTION DATE, TO ARRIVE AT AN ABSOLUTE MEASURE OF FINANCIAL COST.** But in comparison of alternative financing methods this will be unnecessary as all methods evaluated commonly either use or build upon this convention.

The hire-purchase evaluations follow the same convention so far as the grant and/or capital allowances are concerned. It is most
carefully to be noted that, following Inland Revenue preference, the grant is deemed to be received pro-rata to the capital content of each instalment; but the capital allowances are deemed to accrue in full with the first instalment. However, as monthly instalments (payable on the last day of the month and in arrears) are envisaged, the tax shield on the interest content of the instalments will be receivable in respect only of the annual total of that content. Accordingly, the first deduction against tax for interest is deemed to accrue at the start of the third-year - i.e., two years after the first instalment and one year after the twelfth: the second deduction two years after the thirteenth instalment and one year after the twenty-fourth: and so on.

Disposal proceeds are presumed to be received immediately on disposal of the asset at the end of the standard 5-year study period. No earnings are deemed to accrue to the disposal proceeds, in the Evaluation Tables themselves. (1) A separate Table gives discounted values for disposal values - but if the user wishes to employ these for any purpose he should note that these are the *taxed* proceeds of leasing disposals.

Writing-down allowances in the year of disposal are *not* aggregated to the balancing allowances which, again, is deemed to be received twelve months after disposal, - as a similar convention to that determining the timing of writing-down allowances. This non-aggregation is contrary to Inland Revenue practice, but it does not affect the discounted evaluation and it does permit ready adjustment of the discounted evaluation if the rates of either allowance should change.

A footnote on the structure of the hire-purchase contracts used in the Tables is to be found on Page 101.

**LEASING**

Premises are deemed to be payable on the first day monthly, in advance. As with hire-purchase interest, the tax deduction will operate only in respect of an annual total of premiums and so the timing of the tax shield is the same - i.e., the first deduction in respect of the

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(1) See Page 74 - disposal proceeds are the cash flow most likely to be distributed rather than reinvested.
first twelve\(^{(1)}\) premiums is received at the start of the third year, and so on. It follows that the leasing premium tax shield is received one year later than the corresponding writing-down allowance, with a consequent effect upon discounted evaluations.

Two refinements in the leasing Evaluation Tables compared with the appraisal study in the text, are:

**Earnings on Residual Capital:** the negative residual capital which accrues when the total of premiums is paid (which exceeds the cash cost of the asset) is continuously charged against the accumulated earnings on residual capital to-date in arriving at the compounding base for each subsequent period's earnings. There is thus automatically deducted the loss of those earnings which would potentially have accrued to the excess cost if this had not had to be paid.

**Earnings on Grant Refund:** as in the appraisal study, the refund is taxed; but in addition net-of-tax lost earnings on that tax is now allowed for. Calculations relating to the grant refund are displayed in a supplementary table, as not every leasing plan makes use of this device.

As with ownership Tables, no earnings are accredited to disposal proceeds. If the (common) adjusting Table of the disposal value earnings stream is used, care must be taken to select only the excess value over the minimum stipulated in the leasing contract. **No basic disposal values are built into the Evaluation Tables** as they are for ownership and hire-purchase. This omission is to cater for leasing contracts with no disposal proceeds clause.

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\( (1) \) Twelve premiums - this follows from the convention that leasing follows the same time conventions as were established for purchasing.
Section 4. The Application of Corporation Tax.

As might be expected, changes in this rate are generally more reactive on instalment debt financing than on grant qualifying ownership finance. If the appraisal study of the last Chapter is revised for a change in Corporation Tax to 50%:

a) Ownership: grant qualifying

Writing-down allowances and balancing allowances have an increased value, but earnings streams on those allowances are not greatly changed - the increase in the calculation base being offset by the higher taxation of earnings and loss of earnings on that increased taxation. Earnings on the grant and on disposal proceeds are more heavily taxed. Given the relatively low importance of earnings streams in this financing method, the increase in the value of capital allowances is the important feature, with the obsolescence situation benefiting the more because of the larger substitution of balancing allowances for disposal value. On average the 10-point increase in the rate of tax reduces ownership costs for grant qualifying assets by approximately 7% in the obsolescence situation and approximately 4% in the non-obsolescence situation. Generally these reductions improve as asset life is extended, especially in the non-obsolescence situation.

b) Ownership: with initial allowance

The change in tax rate reacts sharply upon the value of the initial allowance; and, as a greater proportion of earnings streams are based upon allowances which are now increased in value, there is a greater tendency for increases in tax rates to reduce ownership costs under these circumstances. At the same time there is a dampening effect due to the heavier taxation of earnings streams which are relatively more important than in the grant qualifying situation. Nevertheless, ownership costs are reduced by some 14% in the obsolescence situation, and approximately 10% in the non-obsolescence situation. Again, the reduction is improved as asset life is extended,
more definitely so in the non-obsolescence situation.

c) Leasing

The impact of changes in the rate of Corporation Tax is not greatly different whether the asset qualifies for a grant or an initial allowance. The value of the tax shield is proportionately enhanced, and so is the accruing gross earnings stream. But the value of residual capital earnings is reduced by the greater taxation; and the enlarged earnings to the tax shield are also taxed more heavily, so that on balance they are slightly reduced. The net effect is a reduction of some 5% in the costs of leasing, a figure which tends to be somewhat lower in the non-obsolescence situation and is not overly affected by extensions in asset life.

If residual capital earnings are not applicable, the reduction in leasing costs consequent upon an increase in tax rates is much improved, becoming rather better than 10%.

It must be stressed that these results are peculiar to the illustration. In particular they can not be regarded as typifying the affect of changes in the rate of Corporation Tax upon the data of the Evaluation Tables - although after due allowance is made for the specified differences in parameters and methodological assumptions the results presented here are not inconsistent. A more generalised study is to be found in Section 3 of the next Chapter; suffice it at this moment to say that the impact of tax changes is apparently great enough and, between methods, different enough to warrant re-appraisal of the financing decision.

Conclusion

It must be remarked, however ruefully, that the methodology of the Evaluation Tables is nowhere held so simple and comprehensive as compiler and user alike would prefer. It is felt that the basic framework constraints - a 5-year asset life, the earnings concept, monthly rests and the range of earnings, tax allowances and disposal rates - are representative enough. But the impact of changes in tax rates, and the complexities of residual capital earnings and
of "grant versus initial allowance" situations; mean that considerable care and due respect for detail must enter into any appreciation of the Tables. A closer scrutiny of the Tables and their utilisation is now appropriate.
CHAPTER THREE

THE EVALUATION TABLES: INTERPRETATION AND CHARACTERISTICS

Introduction: The Scope of the Tables

In this Chapter there is discussed, firstly, the manner in which the Evaluation Tables are set out; and secondly, some of the characteristics and cost trends to be discerned in them. Discussion of such characteristics and trends must be eclectic. The wide variety in the patterns of hire-purchase and leasing contracts, and the numerous combinations of grants, writing-down allowances and disposal values; each example or combination being evaluated against a range of eleven earning rates, the individual results of which are then in turn subjected to a discounting procedure at four alternative rates of discount; yields in effect a multi-dimensional matrix of financing costs. To report on each and every cost trend which is thus revealed could result only in total stupefaction on the part of writer and reader alike. As it is, the reader is forewarned that the interplay of the several parameters is such as to involve even cursory analysis of cost trends in quite complicated explanations. It is possible to deduce certain generalisations as to financing method preferences under given parameters of earning potential and/or choice of discount rate. But all too often these turn out to be so sensitive to changes in taxation allowance, disposal value or whatever that either their use is dangerous or involves a mental check list of such dimension that it is easier and as quick to refer to the individual Tables.

It is also important to recollect the relative nature of the tabulated data. If therefore the term "cost trend" is used, it must be appreciated that this does not mean a directional movement of accounting data, but of units of evaluation specially constructed as a unique measuring device. The various assumptions, methodologies, limitations and discrepancies built into the Tables have been discussed in the previous Chapter and, generally, will not be referred to again in the present discussion.

The Evaluation Tables can conveniently be regarded as being of two kinds - principal and supplementary - although they are not formally designated as such. The principal Tables are set out in
terms of the present value of the net financial cost of acquiring an asset or assets of a cash cost of £1000. The supplementary Tables are set out in terms of the present value of a given cash flow together with its accruing earnings streams, which may be applied by way of adjustments to the data in the principal Tables so as to vary the assumptions upon which these are constructed.

As a late adjustment, rendered necessary by the somewhat rapid changes in the rate of Corporation Tax over the last few years, the Tables are presented at two rates of tax - 40% and 45%, with footnotes indicating to what extent extrapolation can be used for other rates of Corporation Tax. Generally, linear extrapolation does not lead to significant error over the wider range of 35% – 50%, but is not always safe outside these limits.

Where writing-down allowances apply they have been taken into the calculations at 15%, 20% and 25%, subject to normal tax calculations in respect of equipment - i.e., on a reducing balance basis and on a cost net of grant or (with the exception of the first year's allowance) net of initial allowance, over a 5-year asset life. Balancing allowances (charges) are calculated on the varying assumptions of 10%, 15% and 20% disposal proceeds. Data is not presented in respect of straight-line allowance calculations, nor in respect of 'free depreciation' allowances. Calculations based on 20% investment grant or 30% initial allowance are tabulated, together with supplementary data to permit variation of the grant to 40% or to 0% (which last implies 0% initial allowance also).

With the exception of disposal proceeds, all cash flows are set to earn at from 5% to 15% p.a. inclusive, in steps of 1%. The methodology and period of earnings calculations is in all cases as explained in the previous Chapter. Each earnings step is discounted in full at the alternative rates of 5%, 7½%, 10% and 12½% per annum, using discrete annual or monthly rates as appropriate.

The Tables consist of:

Table A. Net financing costs of outright purchase using available equity, in respect of assets attracting 20% investment grant:

A1 on a basis of 40% Corporation Tax
A2 on a basis of 45% Corporation Tax
A3 supplementary data to vary Tables A1 and A2 for 0% and 40% investment grant.

Table B.
Net financing costs of outright purchase using available equity, in respect of assets attracting 30% initial allowance:

B1 on a basis of 40% Corporation Tax
B2 on a basis of 45% Corporation Tax

Table C.
Net financing costs of purchase using hire-purchase facilities, in respect of assets attracting 20% investment grant:

C1 on a basis of 40% Corporation Tax
C2 on a basis of 45% Corporation Tax
C3 supplementary data to vary Tables C1 and C2 for 0% and 40% investment grant.

Table D.
Net financing costs of purchase using hire-purchase facilities, in respect of assets attracting 30% initial allowance:

D1 on a basis of 40% Corporation Tax
D2 on a basis of 45% Corporation Tax

Table E.
Net financing costs of leasing:

E1 on a basis of 40% Corporation Tax
E2 on a basis of 45% Corporation Tax
E3 supplementary data to vary Tables E1 and E2 where disposal proceeds accrue to the lessee.

Table F.
Supplementary data to vary all Tables E for the inclusion of one years excess premiums and a subsequent refund.

Table G.
Supplementary data to vary Tables A and B for outright purchase using short-term loan capital.

Table H.
Supplementary data of the residual capital earnings content of Tables C, D and E.
A reasonable degree of care needs to be exercised if the supplementary Tables are to be applied correctly.

All data are calculated using an I.C.L. 1905 computer operating on FORTRAN 2 programs, with card input and print output. Programs are designed to permit single card substitution to vary data for parameters (tax rate, discount rate, earning rate etc.) other than those utilised in this presentation.

Section 1: Purchasing Tables

It is recommended that entry into Tables A and B should follow the order: (1) Earning Rate  
(2) Writing-down Allowance  
(3) Disposal Value  
(4) Discount Factor

Additional parameters relating to hire-purchase (Tables C and D) are:  
(5) 'Interest added' rates of 5%, 6%, 7% and 8%.

Evaluations for rates of interest added in excess of 8% may be obtained by linear extrapolation as appropriate to the relevant rates of earning, writing-down allowance, disposal and discount.

(6) Initial deposits of 15%, 20% and 25% of the cash cost.

The typical hire-purchase contract evaluated in those Tables has the following form:

<table>
<thead>
<tr>
<th>5-Year Contract</th>
<th>£</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash Cost:</td>
<td>1000</td>
</tr>
<tr>
<td>Deposit 15%:</td>
<td>150</td>
</tr>
<tr>
<td>6% added interest, 5 years:</td>
<td>255</td>
</tr>
<tr>
<td>Total serial payments:</td>
<td>£1105</td>
</tr>
</tbody>
</table>

giving rise to 60 equal monthly payments of £18.417, payable at the end of the month and consisting of £14.167 capital repayment and £4.250 interest. This simple pro-rata basis of the division of the monthly payment into its capital and revenue components is as adopted by the Inland Revenue and the Board of Trade for the purposes (1) Alternative methods of deriving the interest content of each instalment are discussed at a later point in this Chapter.
of calculating the relevant tax allowances and grant, which are: initial and/or writing-down allowances based upon the total cash cost ab initio, but payment of grant based upon the initial deposit with subsequent serial payments based on the capital repayment content of quarterly payments (e.g., 20% of £14,167, per quarter). Late evidence has come to hand that there is a slowly growing practice of permitting grant to be claimed on the total cash cost ab initio. Interest is taken to be wholly chargeable against Corporation Tax liability. It is recommended that entry to Tables C and D should be in the order of steps (1) - (5) as enumerated on the previous page. These Tables are presented in terms of a 15% deposit rate, with standard increments to convert to 20% or 25% deposit. Except for high discount rates combined with high asset earning rates, the deposit rate conversion factors are not relatively large in amount.

Tables A and C envisage the applicability of a 20% investment grant received after the expiration of twelve months from the date of purchase, deposit or quarterly instalment (or from the date of the last of the three monthly payments in each quarterly sub-total). Provision is made in Tables A3 and C3 for variation of the rate of grant from 20% to nil or to 40%; fractions of these 20% variations being calculable at need by linear interpolation. It will be observed that both Tables A3 and C3 are indifferent to changes in the disposal rate, and that C3 is indifferent to changes in interest added. Entry into these two Tables should be modified accordingly. Tables B and D envisage the applicability of a 30% initial allowance, received twelve months after the date of purchase. No supplementary data is provided to vary this rate, except that which is implicit by the application of Tables A3 and C3 for the 'nil grant' circumstance - which also yields a 'nil initial allowance' circumstance. Care must be taken to adjust for variation in grant in the right direction - thus an adjustment to 40% grant is a deduction from the relevant principal Table, as it constitutes a reduction in net financing cost.

Tables C, D and E include for earnings to residual capital. If this circumstance is inapplicable, recourse must be had to Table H for factors to be added to C, D and E accordingly. In passing, it may be noted that the impact of an initial deposit in the standard
A hire-purchase contract is, in general, severely to reduce the residual capital earnings potential compared with that inherent in (say) a leasing contract.\(^1\) In a sense conversely, Tables A and B can be varied by reference to Table G, if outright purchase is financed by short-term loan. Table G lists the net interest costs and capital repayment values (including all accruing 5-15% earnings opportunity streams foregone) of borrowing £1000 at a standard 8.33% p.a., variation from this rate being directly proportional. The Table envisages that this interest is compounded on monthly rests (closely equivalent to £100 p.a. or 10% on annual rests), and that the accrued interest is paid to the creditor(s) in annual instalments, thereby creating an annual tax shield; the capital being repaid in a lump sum at the end of the 5-year term. Capital recovery forms of short-term finance (typified by a fixed annual repayment consisting of a reducing interest cost plus an increasing capital repayment) are not presented, in that acquisition of equipment assets by this mortgage type of finance is unusual except insofar as it is in the form of a lease.\(^2\)

Evaluation of the cost of such debt finance would best be served by a major Table similar in some respects to Table C or D: embodying a full range of separate calculations of capital allowances and tax shield over a range of mortgage rates (rather than added interest rates) and asset disposal proceeds. In

\(^{(1)}\) This is a fair example of the sometimes dangerous type of generalisation which can be deduced from the Tables. One ought to add: "except where asset earnings rates are low, or discount rates are high, especially when combined with a low level of initial deposit and/or a low added interest rate" - a series of qualifications, which except in the unusual circumstances of extreme and hence conflicting values of each of these parameters, renders the generalisation so vague as to be uncertain. However, in this particular case, the differential in residual capital earnings in all circumstances covered by these Tables is so great that the generalisation is safe enough.

\(^{(2)}\) It is perfectly valid to derive the true interest cost of a lease in terms of capital recovery or amortisation factors (c.f. Bennet Grant & Parker (6) Chap.4) in that the premiums represent to the lessor a recovery of capital outlay on the leased asset, as well as interest thereon plus a service charge and profit (Vancil (34) Chap.5.).
principle, the Table might incorporate the relevant residual capital earnings; but in practice this method of financing industrial equipment is incompatible with the possession of adequate equity, and is in conflict with the alleged banking principle that overdraft facilities are not available for fixed capital expenditures. In addition, the concept of residual capital earnings is based upon the decisional choice of immediate or instalment lay-out of existing 'own funds'.

However, where outright purchase is financed by short-term loans of the type indicated in Table G, the adjustment to Tables A and B will be of the form

$$A \text{ (or } B) \times (G-1000)$$

where A, B and G are the data of those Tables at the relevant earning and discount rates, care being taken to adjust for both interest and capital repayment data of Table G. The adjustment is of this form because the data of Tables A and B are the net costs ex £1000 own equity.

Section 2: Leasing Tables

The data in Tables E1 and E2 are based upon eight not untypical patterns of premium over a 5-year primary period contract; the patterns being listed on an introductory "Title Sheet". Patterns 1 through 7 are composed of a uniform series of payments over the primary period, and it is noted that the net costs of similarly uniform patterns in different amounts can be determined by linear extrapolation. A footnoted table provides the necessary coefficients for £20 p.a. variations in premiums over the 5-15% earning range. Table F provides data for the adjustment of E1 and E2 where there is payable an excess premium payable (typically) over the first year until the investment grant or initial allowance is received by the lessor, whereupon the excess is refunded. Pattern 8 is a descending premium over a 5-year primary period, and is not untypical of the pattern employed by a major leasing company operating in the office furniture and equipment field.

All leasing data in Tables E1 and E2 presume monthly premiums payable at the start of each month, and totally tax-deductable in
terms of liability to Corporation Tax. A second assumption is that no disposal value will accrue to the lessee, although this assumption can be abandoned by the application of Table E3 to E1 or E2. Thus, contracts are deemed to be "whole-life", with no equity passing (as an option or as an implication) to the lessee during or at the end of the contract. The competition between leasing companies is such that usually there is not too much scope left in varying primary period premiums; the principal weapons of selling being variations in the non-financial clauses of the lease such as the lessees right to update equipment or to vary or cancel the contract; or in the pattern of secondary period premiums. A great variety of patterns are met with here, according to whether the primary period premiums are calculated on a 'full' or 'partial pay-out' basis. Thus a third instrument of selling lease finance is to write in a clause granting the lessee disposal proceeds in excess of a minimum sum, which may be nominal or significant. Other tactics are to grant preferential terms on leasing renewal or expansion and - lately, and occasioning a potential future argument with the Inland Revenue as to whether the contract is not really a deferred sales contract - to grant an option to purchase at the end of the primary period. (1)

Thus there are many variations in leasing contracts as a 'package deal', and the variety is growing as leasing becomes more sophisticated. Nevertheless the type of contract represented in Tables E1 and E2 seems still to be that most commonly utilised in equipment leasing, other than in computer or communications equipment. Table E3 permits variation of E1 and E2 to cover the situation where disposal proceeds accrue to the lessee on termination of the lease at the end of the primary period. No tabular provision is made for the evaluation of secondary period premiums in view of the potentially wide variety in terms of dimension, periodicity and life. An adequate, if not wholly

(1) Quotations supplied by City Leasing Company, who state that this arrangement "is achieved without any detrimental effect on the position of the Inland Revenue" (1). The lease is terminated and the equipment is then sold to the erstwhile lessee at a predetermined price "close to the tax written down values", presumably as second-hand assets. I am informed, unofficially, that the "Commissioners are watching the position closely".
accurate, estimate of the costs of secondary period premiums can if necessary be obtained by determining the present worth of a net-of-tax uniform series of premiums over a period predetermined by the individual user. The principal error in such a calculation lies in the failure to adjust for the different timings of premiums compared with their accruing tax relief. A suggested compromise - in the sense of improved accuracy whilst still reducing the computational load involved in the procedure of the full Evaluation Tables - is given by:

\[ S = \sum_{t=n}^{j} S_t \left( \frac{P_t \left( 1 - \frac{c}{1+i} \right)}{(1+i)^t} - \frac{(cP_t e (1+e)^j - t)}{(1+i)^t + 2} \right) \]

where the secondary period premiums \( P \) extend for a period commencing in year \( n \) and ending in year \( j \). \( S \) is then the present value, at discount rate \( i \), of the series:

\( P_n, P_{n+1}, P_{n+2}, \ldots, P_{j-1}, P_j \) which attracts tax relief at a rate \( c \); where all accruing cash flows earn at a rate \( e \). As tax is payable in arrears, \( c \) is deflated by a standard \( \frac{1}{1+i} \), and as earnings on tax shield accrue over the year following receipt of each installment of relief, \( cP_t e \) is deflated by a standard \( \frac{1}{(1+i)^{t+2}} \).

Compensation for not calculating tax liability (and consequent earnings forfeited) on \( cP_t e \) is offered by compounding to a point \( (1+e)^{j-t} \) only, rather than \( (1+e)^{j+2-t} \); i.e., only as far as the end of the series of premiums only.

Evaluation of the formula is not so formidable as it might seem to be when it is realised that \( (1 - \frac{c}{1+i}) \), \( P_t \) and \( cP_t e \) are constants in any given situation and that values for the sums of the various compounding or discounting series are to obtain from any set of interest tables. Because secondary period premiums are usually paid on an annual basis, and because of the small value involved, monthly rests are not recommended. A margin of error in the evaluation of the net cost of secondary period premiums is tolerable because of the
relatively remote period over which the series is discounted. Thus, assuming $P = £10$, $n = 5$, $j = 9$, the net present worth cost of the secondary premiums under the formula is:

<table>
<thead>
<tr>
<th>Earning Rate</th>
<th>Discount Rate</th>
<th>Net Present Worth Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>10%</td>
<td>£24</td>
</tr>
<tr>
<td>12%</td>
<td>10%</td>
<td>£23</td>
</tr>
<tr>
<td>12%</td>
<td>5%</td>
<td>£26</td>
</tr>
</tbody>
</table>

A check against fully worked-out evaluations in the manner of the Tables indicates an error of less than 10% (the formula tending slightly to over-value at high earning rates or low discount rates), which for the amounts involved and the uncertainty inevitably attaching to estimates of the length of the secondary period, is scarcely significant.

Tables E1 and E2 are calculated on a basis which assumes residual capital earnings. If this circumstance does not apply, recourse must be had to Table H, the net cost data therein being added to that in E1 and E2. The introductory "Title Sheet" to the E Tables notes for each leasing pattern the true rate of interest implicit in the pattern (i.e., that rate of interest which discounts the primary period premium series into equivalence with the asset cash cost of £1000). It is of interest to compare patterns 3 and 8, which in this sense of interest cost are very similar to each other. The absolute premium cost of pattern 8 (diminishing payments) is a little less - £1236 compared with £1250 for pattern 3; but reference to Tables E1 and E2 indicate that the net cost of the diminishing premiums pattern is nevertheless consistently higher than that of the (comparable) uniform premium pattern, and this differential increases with the earning rate - albeit that this cost disadvantage decreases somewhat with higher discount rate and higher Corporation Tax. The decreasing premium pattern really represents something of a variation of the basic leasing principle towards hire-purchase, in that the higher earlier (non-refunded) premiums to some extent simulate a deposit.

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(1) i.e., (as we are using discrete end-of-year factors, the first secondary premium is paid at the end of year 5 (beginning of year 6) and the final premium at the end of year 9 (beginning of year 10).
Section 3: The Effect of Changes in Corporation Tax Rates

From the point of view of rapid comparisons between financing methods or of discerning the various cost trends implicit within them, it is unfortunate that changes in tax rates (which are potentially substantial and yet unpredictable) should have an impact which is generally significant in terms of the change in net financing costs. The magnitude and direction of the effect of changes in tax rates upon tax allowance cash flows is fairly obvious, and represents a constant ratio. But the impact on earnings streams is self-contradictory. Insofar as they are generated by tax allowances their value will alter in the same direction as the change in tax rates; whatever their basis of generation, they will bear greater or less taxation as tax rates vary, and so their value will alter in the opposite direction to changes in tax rates. Prima facie, it appears that the higher the earning rate the greater will be the tendency for the net effect upon their value to be in the same direction as the change in Corporation Tax. But the pattern at which the generating base accrues is also of considerable importance. If the pattern is an increasing one - e.g., the leasing tax shield for leases starting at mid-period - the improvement in the base is relatively late in arriving, so that the improvement in earnings is discounted the more heavily, yet the increased taxation of earnings starts at once. Whereas if the generating base conforms to a decreasing pattern - e.g., a combination of initial allowance and writing-down allowance - the increase in earnings is experienced at an early point in time and with no more than an average earning rate there is on balance an improvement in the discounted value of the earnings stream. Given the relatively greater importance of earnings streams in the evaluation of leasing costs, it is clear that it is virtually impossible to provide rule-of-thumb prediction as to what the effect of changes in Corporation Tax on those costs will be. The analysis of Section 4 of the last Chapter introduced brief considerations of the impact of such changes on different asset lives. It is opportune to consider the impact within different earning and discount rates.

Acquisition by Purchase

In the case of outright purchase of 20% grant-aided assets, an increase in tax rate provides a reduction in net financing cost which
however approaches insignificance when a high earning rate is combined with a low discount rate.\(^{(1)}\) It is of interest to note that at low earning rates, the reduction in net financing cost decreases as discount rate increases; but at high earning rates, the cost reduction is smaller but is a positive function \(^{(2)}\) of the discount rate. The magnitude of the reduction in cost is not very sensitive to the size of the writing-down allowance (fractionally larger at larger rates of allowance) and is little more sensitive to the size of the estimated disposal proceeds (smaller at higher disposal values). Plainly it is unsafe to generalise in terms of a 'representative' reduction in cost, the range of which is \(1 - 7\%\).

As grant is increased, the cost reduction gained from an increase in the tax rate weakens; becoming somewhat indifferent to the discount rate for low earning rates, but emphasising the previous trend at higher earnings rates by becoming a sharply negative amount when combined with low discount rates. Conversely, at the nil grant (includes nil initial allowance) situation; an improved cost reduction accrues with the increase in the rate of tax, becoming a shallowly negative function of discount rate at all levels of earning rate (even less so at higher rates) and becoming fairly static in absolute size irrespective of earning rate, for a given rate of discount.

Finally, it is worthy of note that the ultimate appearance of

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\(^{(1)}\) There is here a clear implication that the two rates are not the same in their nature. The concept of the earnings rate was discussed in Section 1 of Chapter Two, and the relationship of the earning rate to the discount rate is discussed in Section 4 of Chapter Six. A further relationship is implicitly discussed in Appendix I, where the reinvestment assumption is discussed in the context of the Gordon-Shapiro cost of capital model. That model clearly sees the two rates as separate entities - indeed, it argues that an increase in earning rates ipso facto will lead to a reduction in the cost of capital.

\(^{(2)}\) In the following discussion of the Evaluation Tables, there is meant by the expression "a positive function" that changes in net financing cost move in the same direction as the independent variable, and vice versa. The rate of change (as opposed to the direction of change) is not specified unless it is relatively substantial; as to do so in all cases would involve an excessive, even intolerable preoccupation with the minutiae of financing costs.
significantly negative values of net cost at 45% tax is very much a result of the combination of high earning rates with low discount rates rather than of an increase in the rate of tax as such. The appearance of a net subsidy to development area investment in plant and machinery by firms with a high profitability expectation (noting that, as the subsidy as calculated accrues only to assets enjoying higher writing-down allowances, it will accrue all the more in a 'free depreciation' situation) presumably is in accord with Government economic thinking, but it makes it all the more necessary to appreciate the 'value-relative' nature of the term 'cost' in this context.

The reduction in cost is much more constant (in absolute terms) in respect of the outright purchase of assets attracting initial allowance at 30%. As a percentage, the range is from 6% to 10% (higher than for 20% grant-aided assets, as was noted in respect of the textual illustration in Section 4 of the previous Chapter), but as an absolute value the reduction is remarkably steady at £30 - £40 regardless of earning rate, discount rate, writing-down allowance or disposal proceeds. Some faint trends are discernible, perhaps the most interesting being that of the transition of the cost reduction from a shallow negative to a shallow positive function of the discount rate as earnings rate increases. There is a slight tendency for the cost reduction to diminish at higher rates of writing-down allowances, in company with a decreased sensitivity to discount rate.

The extra parameters involved in hire-purchase evaluations (interest added and initial deposit) inevitably complicate the considerations of the effects of changes in tax rates. However, so far as concerns contracts relating to 20% grant-aided assets, these effects are generally similar to those accruing to outright purchase. An increase in tax rate occasions a reduction in net financing cost which is significant at low earning rates, where it (the reduction) is only moderately sensitive to the discount rate for a low interest added figure but becomes increasingly sensitive (as a negative function of discount rate) as interest added increases. (1) But at higher earning rates the cost reduction approaches insufficiency, so that when these

---

(1) This is due to the residual capital earnings content of the evaluations scheduled in Tables C1 and C2, D1 and D2. An increase (footnote continued on next page)
are combined with low discount rates and the lower rates of interest added, the reduction in cost is converted into a significant increase (an intensification of the outright purchase trend). Again, the level of writing-down allowance is not critical, and an increase in the level of disposal value evokes only a small (positive) reaction in the size of the cost reduction. Initial deposit is not a critical parameter. As with outright purchase, the earnings rate is easily the principal determinant of the effect of a change in tax rate: but in hire-purchase contracts, the level of interest added is the next critical factor, and the size of the discount rate only third in importance. (1) At 40% grant rate, the cost reduction accruing to a change in tax rate is diminished - which is expectable - to an extent such that significant increases accrue at low levels of earning and at high rates of discount. At nil grant (nil initial allowance) the cost reduction is intensified and applies virtually over the whole range of circumstances. (Conversely, the impact of a change in grant rate is somewhat more pronounced at lower levels of tax, a trend which is more pronounced at higher earning rates but which is insensitive to interest added). Again expectably, at lower levels of grant the otherwise only moderate sensitivity of the tax-induced cost reduction to changes in the discount rate increases somewhat for all (but especially for low)

(continued from previous page) in the rate of interest added manifests itself inter alia as a reduction in residual capital and so in the earnings accruing thereto. For a given discount rate (earnings rate held constant), the lower residual capital earnings offer a less 'target' for the reducing effect of an increase in tax rate, so that the 'beneficial' effects of that increase in tax rate on capital allowances is subject to a less 'contra'. Hence the reduction in tax rate gives a greater cost reduction at a higher interest added. If the discount rate is increased, the basis differential between residual capital earnings at different interests added is reduced so that the scope for diminution of the impact of the increase in tax rate is itself reduced. This whole complex effect is itself intensified as the earning rate is increased, so increasing the basis differential between residual capital earnings at different interests added.

(1) The obvious complexity of the effect of increases in tax rates on the net financing costs of hire-purchase is the cause of the non-linearity indicated on the "title sheets" to Tables C and D by an approximating adjustment factor. The pure linear extrapolatory error is greater - gives too low a result - at higher earnings rates, lower interest added, and lower discount rate: but is not rendered critical by changes in writing-down allowances, disposal proceeds or initial deposit.
levels of interest added where earning rates are low; but at higher rates of earning this does not apply, cost reduction reverting to a comparative insensitivity to changes in the discount rate.

For assets attracting initial allowance there is a pronounced difference in the tax rate induced cost reductions on hire-purchase financing as compared with outright purchase. A feature of the latter was the stability of reduction (in absolute terms); whereas under hire-purchase there is a reversion to significant levels of cost reduction at low earning rates tailing off to insignificance at high earning rates and even converting to a cost increase if high earning rates coincide with low discount rates. There is the same complex interaction of rates of added interest, earning rates and discount rates as was evinced in the previous paragraph. At low earning rates, the cost reduction is a negative function of discount rate, a condition which is emphasised at higher rates of interest added; but at high interest rates, the cost reduction increases with the discount rate especially at low rates of interest added. It is noteworthy that the general level of cost reduction (and of the occasional increase) occasioned by a change in tax rate is higher at 30% initial allowance than at nil allowance; and that the sensitivity of the cost reduction to changes in discount rate is greater at a nil allowance level when earning rate is low, but greater at a 30% allowance when earning rate is high (still maintaining a negative sign in the first case, and positive in the second).

From this plethora of observations of cost trend, the general conclusions that may be deduced are:

a) That an increase in tax rate is conducive to a reduction in financing cost for all forms of acquisition via purchase.
b) This is especially true at low earning rates, but less true at high earning rates - becoming erroneous if in the latter case a low discount rate is also used.
c) Within these conclusions, the levels of writing-down allowance, disposal proceeds or initial deposit are not very critical.
d) At lower earning rates, the reduction in cost is only moderately affected by variations in the discount rate;
but at higher earning rates a greater sensitivity accrues. For hire-purchase financing, the levels of the rate of interest added tends to introduce a third function into this relationship, inversely correlated with the earning rate for all levels of discount and with the discount rate for all (but especially the lower) levels of earning.

e) For all grant-attracting assets these conclusions are the stronger, the lower is the rate of grant. But for assets attracting initial allowance, the higher is the rate of that allowance the more generally correct are these conclusions. The improved cost reduction from a given change in the rate of tax to be obtained from an increase in initial allowance is not, however, as much as that to be obtained from the same (percentage) decrease in the rate of grant.

f) These conclusions are at their least defensible when the method of financing is outright purchase of assets attracting initial allowance. Cost reduction accruing to an increase in tax rate is here much more uniform in absolute terms, regardless of earning rate of discount rate (conclusions (a), (b) and (d). Conclusion (c) - insensitivity to writing-down allowance and disposal proceeds holds good: but there is very little difference between the cost reductions accruing to a given tax change for a 30% or for a nil initial allowance, the absolute value of the reduction still remaining reasonably stable. The variation in reduction is ipso facto very considerably less than that accruing to a decrease in grant of the same order (conclusion (e)).

Acquisition by Leasing

The impact of a change in tax rate on the net cost of financing by

(1) Given an assumption of secular increases in Corporation Tax, the appeal by 'non-manufacturing' industry for an end to 'fiscal' discrimination by a universal application of grant is from this point of view not well founded. The cost reduction from a given increase in tax rate is greater under 30% initial allowance than it is under 20% grant (but of course the original cost is much higher).
leasing is not very dissimilar from that which obtains under hire-purchase. For lower rates of earning, the net effect is a reduction in cost; but this becomes insignificant, as the earning rate increases and converts into an increase in cost (at about the same conversion rate as for hire-purchase; break-even is around 13% earning rate) when combined with a premium pattern which implies a relatively low true rate of interest. Cost reduction correlates negatively with discount rate in all circumstances (when there is a cost increase, correlation is still negative). This relationship is relatively insensitive for low premium patterns at low earning rates and high premium patterns at high earning rates, and relatively sensitive for low premium patterns at high earning rates and high premium patterns at low earning rates. Reading premium pattern as a substitute for interest added, these trends are in the same direction as those accruing to hire-purchase.

Generally, however, the overall level of cost reduction is higher (and of cost increase is smaller) in leasing (especially at lower earning rates) than in hire-purchase. This is due to the one hundred percent tax deductibility of leasing premiums, but (as is illustrated in the next Section) the greater (positive) sensitivity of leasing residual capital earnings to upward revisions of the tax rate, especially at higher earning rates, means that much of the improved cost reduction in leasing as a whole is lost at higher earning rates. For descending-premium patterns, the cost reduction is a little higher than for comparable uniform-premium patterns; the advantage increasing as earning rate and discount rate are increased.

For leasing patterns, tax-induced reductions in financing cost are not a linear function of the change in tax rate. (Residual capital earnings are). This non-linearity is a negative function of the discount rate and a positive function of the earning rate and, especially, the size of premium. For the range of patterns examined the curvature is nowhere very steep, and at need, (i.e., for tax rates outside the range 40-45%) it would seem safe to use a linear extrapolation as a rough and ready approximation which is known to give an over or under-statement of net financing cost equal to around .3 Pe where P = annual premium and e is the appropriate earning rate.
Strictly, it would be preferable to recalculate net financing costs for tax rates other than in the range 40-45%.

The effect of an additional premium in the first year depends upon whether there is a subsequent refund. If so, the effect is negligible. If there is no refund, the existence of a first years premium is to modify the existing conclusions in favour of increasing the cost reduction - the improvement being a positive function of the earning rate (and thus somewhat reversing the earlier findings) and a decreasing but less significant function of the discount rate (intensifying the earlier findings). Generally, the existence of an excess premium serves only to reduce the slope of the trend of cost reductions induced by changes in tax rates. The existence of disposal proceeds intensifies the discerned trend, their value being a negative function of the discount rate and unrelated to earning rates.

Section 4: Residual Capital Earnings

It is convenient to introduce a study of residual capital earnings at this juncture, inasmuch as a potential complexity is thereby removed from the discussion on cost trends. Also, such a study can conveniently encompass both tax rate effects and cost trends, and thereby a bridge is effected between the consideration of tax rate effects and cost trends in the principal Tables.

Residual capital earnings - henceforward to be abbreviated in this section to r.c.e. - were defined, and explained as a concept, in Chapter Two. They are a feature of hire-purchase and of leasing, and the present review is conducted in terms of the comparative significance of r.c.e. in the evaluation of those methods of financing. This means that the basis on which contracts of hire-purchase and leasing are deemed to be comparable with each other must be defined - and for the moment that basis is taken to be the true (internal) annual rate of interest implied by the relationship of the monthly payments to the principal involved (assuming discrete monthly compoundings). For hire-purchase contracts, the principal involved is the amount 'cash cost less initial deposit' and the true rate of interest is
calculated as that which satisfies the formula:

\[
R = P \left( \frac{i(l+i)^t}{(1+i)^t-1} \right)
\]

where \(R\) is the monthly payment, \(P\) is the principal involved and \(t\) is the length of the contract. The same evaluation is applied to leasing where the principal involved is the cash cost equivalent of the leased asset. In both cases \(i\) is then expressed as an annual rate.

At a later point in this Section an alternative basis of comparison - that of cash outflow - between hire-purchase and leasing is suggested; but on the present basis of cost comparison it will be clear that the true rate of interest implicit in hire-purchase contracts is a direct function of the rate of interest added. (It is independent of the initial deposit because the monthly instalment is reduced proportionately to the increase in deposit - there is a constant relationship between principal involved and monthly instalment for any given rate of interest added). For added interest rates of 5%, 6%, 7% and 8% the corresponding true interest rates are 9.2%, 10.9%, 12.5% and 14.2%. As such they are on this basis roughly comparable with leasing patterns 3, 4, 4/5 and 5/6, in that order; where by '4/5' is meant, a pattern mid-way between patterns 4 and 5.

R.o.e. are of course independent of the grant or tax allowance structure appertaining to the asset in consideration, and also of the disposal value. It follows that in hire-purchase evaluation, constant r.o.e. for a given earning rate constitutes a wide range of percentages of net financing cost and are more significant in some cases than in others purely as a reflection of the reduction effects of

(1) See footnote to page 35. The formula now used is the inverse of that which appears in the footnote. This is not the same basis as that which is commonly used - i.e., the relationship of a uniform annual interest charge to the average capital outstanding over the life of the contract; a basis which yields a rate of interest fractionally less than twice the rate of interest added. The algorithm in common use by the management of many finance houses is

\[
i = \frac{2rt}{(t+1)}
\]

where 'i' is the approximate true rate of interest, \(r\) is the rate of interest added and \(t\) is the number of payments in the contract. Several managers quoted this algorithm in conversation.
improved grant or tax allowances on total net cost. For purposes of discussion, the following r.o.e. are illustrated:

TABLE 1
Residual Capital Earnings (1)

<table>
<thead>
<tr>
<th>Hire-Purchase</th>
<th>5% Int.added</th>
<th>8% Int.added</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td>5%</td>
<td>12 1/2%</td>
</tr>
<tr>
<td>Tax Rate:</td>
<td>40%</td>
<td>45%</td>
</tr>
<tr>
<td>15% Deposit:</td>
<td>5% Earn.</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>149</td>
<td>161</td>
</tr>
<tr>
<td>25% Deposit:</td>
<td>5% Earn.</td>
<td>132</td>
</tr>
<tr>
<td>132</td>
<td>93</td>
<td>141</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leasing</th>
<th>£250 p.a.</th>
<th>£275 p.a.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate</td>
<td>5%</td>
<td>12 1/2%</td>
</tr>
<tr>
<td>Tax Rate:</td>
<td>40%</td>
<td>45%</td>
</tr>
<tr>
<td>5% Earn.</td>
<td>40</td>
<td>46</td>
</tr>
<tr>
<td>15% Earn.</td>
<td>172</td>
<td>186</td>
</tr>
</tbody>
</table>

Ignoring for the moment differences between the methods, r.o.e. generally can be seen to:

a) Reduce in value with increases in tax, especially where earning rates are high, discount rates are low and/or annual or monthly payments are low.

b) Increase in value as earning rates increase - which is obvious; especially where discount rates are high and payments levels and tax rates are low.

c) Increase in value as discount rates increase, especially

(1) Source: Table H (£275 p.a. interpolated). The true rates of interest are:

H.P: 9.2% and 14.2%
Leasing: 9.7% and 14.3%

and the annual H.P. instalments are:

15% Deposit: £212 at 5% and £238 at 5% int. added.
25% Deposit: £187 at 5% and £210 at 5% int. added.

so that it may fairly be argued that the leasing position is on the whole understated.
where earning rates and payment levels are high. Generally, the increase is magnified at higher levels of tax.

d) Reduce in value with increases in the level of period payments - again, an obvious point. This is more pronounced at lower levels of tax, higher rates of earning and lower discount rates.

Expectably, the rate of earning is all important, with the level of period payments the next significant parameter. Discount rate becomes really significant only where higher levels of period payments apply, and changes in the tax rates really significant only at higher earnings levels. None of these results are contrary to expectations - the increase in r.c.e. values as discount rate increases being a reflection of the more-than-proportionately greater 'slimming effect' upon relatively late tax outflows; a tendency which high tax rates inevitably strengthen.

As between financing methods,

a) Leasing r.c.e. are higher - and at the upper earning level, considerably higher - than hire-purchase r.c.e. If the basis of comparison were altered to size of period cash outflow, the difference is even greater. For example, comparing pattern 1 (£220 per annum) with 25% deposit, 8% added hire-purchase (£210 per annum); at 15% earning, leasing r.c.e. are £200 or 400% greater than hire-purchase r.c.e., whereas at pattern 5/6, they are only some 50% greater.

The difference is attributable to the larger base of residual capital which accrues to leasing, where there is no initial deposit. The loss of residual capital in the first, relatively discount-indifferent, earning year is a considerable 'setback' to hire-purchase financing costs, and one which is perpetuated throughout the nine years on which gross r.c.e. are calculated. Within the spectrum of contracts considered, the range in the size of hire-purchase instalments is not great for a given rate of interest added (£210 - £238 p.a. at 8%) and so the variation in r.c.e. value is not all that wide. But the change in annual payments between leasing patterns 1 and 5/6 is significant; especially when there is considered the continuation of the loss of r.c.e. as residual capital turns negative,(1) which 'doubles' the

(1) See Appendix to Chapter One, Pages 63 et seq.
r.c.e., differential.

b) Leasing r.c.e. is more sensitive to changes in tax rate, especially where the reduction in the value of r.c.e. is generally large - typically, at high earning rates. This comparison is made in absolute terms. In percentage terms, both sets of r.c.e. are about equally sensitive to changes in tax rate.

The greater absolute reduction in the value of leasing r.c.e. is no more than a reflection of the greater base of calculation, which manifests itself in other directions, viz:

c) In absolute terms, leasing r.c.e. is more sensitive to:
   (i) Changes in discount rate
   (ii) Changes in earning rate especially when the comparison is carried out against high-deposit hire-purchase contracts.

There is at 45% tax an indication that at very low earning rates, a reduction in the discount rate will evoke an increase in the value of hire-purchase r.c.e.

The confusion which can be evoked by consideration of changes in r.c.e. values as the various parameters change is illustrated by the following table, which expresses r.c.e. as a percentage of net financing costs prior to their deduction.

**TABLE 2**

| % Decrease in Net Financing Cost due to Residual Capital Earnings |
| (assets attracting 20% writing-down allowances and 10% disposal proceeds) |

(1) Sources: Tables C1 and C2, D1 and D2, and E1 and E2; all adjusted by Table H. The percentages in respect of hire-purchase:

a) increase with higher writing-down allowance

b) increase with higher disposal proceeds

and in respect of leasing:

a) increase with lower annual premiums (an increasing function)

b) increase if disposal value ensues to the lessee.
<table>
<thead>
<tr>
<th>Interest Added:</th>
<th>40% TAX</th>
<th>45% TAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Rate:</td>
<td>5% 12% 8% 12% 5% 12% 8% 12%</td>
<td></td>
</tr>
<tr>
<td>with 20% Grant:</td>
<td>5% earn 8 9 2 5 7 9 2 5 15% earn 63 54 24 30 5 48 14 25</td>
<td></td>
</tr>
<tr>
<td>15% Deposit:</td>
<td>5% earn 7 8 2 2 6 8 5 4 15% earn 50 47 22 27 46 41 12 22</td>
<td></td>
</tr>
<tr>
<td>25% Deposit:</td>
<td>5% earn 5 6 2 1 5 6 4 3 15% earn 32 32 13 19 26 28 8 16</td>
<td></td>
</tr>
<tr>
<td>with NIL Grant:</td>
<td>5% earn 6 7 2 4 6 7 2 4 15% earn 37 38 15 22 29 34 8 19</td>
<td></td>
</tr>
<tr>
<td>15% Deposit:</td>
<td>5% earn 5 6 2 1 5 7 4 4 15% earn 38 38 16 22 31 35 9 19</td>
<td></td>
</tr>
<tr>
<td>25% Deposit:</td>
<td>5% earn 32 32 13 19 26 28 8 16</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LEASING</th>
<th>40% TAX</th>
<th>45% TAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Annual Premium:</td>
<td>£250</td>
<td>£275</td>
</tr>
<tr>
<td>Discount Rate:</td>
<td>5% 12% 5% 12% 5% 12% 5% 12%</td>
<td></td>
</tr>
<tr>
<td>5% earn 6 8 3 5 6 8 2 5</td>
<td>15% earn 37 39 18 26 29 35 11 22</td>
<td></td>
</tr>
</tbody>
</table>
Prima facie, r.c.e. generally are not proportionately much more important in the evaluation of one method of instalment-debt financing than in another, although they do figure with rather more importance where grant accrues - and would be of major importance at a 40\% grant rate. At low earning rates, r.c.e. are of insignificant importance. But it is most important to appreciate that a change in these percentages as much reflects a change in the base financing cost as it does a change in the absolute value of r.c.e. The normally greater absolute value of such earnings in the leasing evaluation has already been noted, so that any parity in percentage terms is a clear indicator of the much higher leasing cost gross of such earnings. (1)

The existence of a reasonable similarity where these earnings are compared in proportional terms but of a significant disparity when examined in absolute terms is of considerable importance in any comparison (based upon true interest cost) between the net financing cost of leasing and those of hire-purchase, where the relevant earning rate is other than low - say 8% and above. The effect is that the taking of residual capital earnings into account may alter the order of preference (lowest cost preferred) from that established gross of such earnings or it may very well seriously alter the magnitude if not the order of preference. As a by no means extreme example:

**HIRE- PURCHASE:** Nil grant; 15\% deposit; 5\% interest added;
15\% earning; 12\% discount; 10\% disposal
gross of r.c.e.: £427
r.c.e. as \% of gross: 38\%
net of r.c.e.: £266

(1) If the hire-purchase/leasing comparison is affected in terms of annual payment rather than true interest cost, the relevant data for £220 p.a. premium (c/f 5\% added = £213 at 15\% deposit, and 8\% added = £210 at 25\% deposit) are:

<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>40% TAX</th>
<th>45% TAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>5% earn</td>
<td>12</td>
<td>13</td>
</tr>
<tr>
<td>15% earn</td>
<td>65</td>
<td>61</td>
</tr>
<tr>
<td>(no disposal proceeds attributed).</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this case, as the base cost is relatively low (comparing favourably with the base cost at NIL grant or 30\% initial allowance), the sharp increase in percentage does reflect a considerable increase in the absolute value of residual capital earnings.
Here the order of preference is unchanged but the magnitude of preference is more than halved. Given the important but unquantifiable considerations which were discussed in Chapter One as entering into the financing decisions; and the slightly improved cushion which leasing was shown in Chapter Two to offer against sudden obsolescence or the effects of a prolongation of asset life; it is not impossible that the introduction of these earnings will cause a switch in the financing decision.

A second and vital point relates to the basis of comparison. Hitherto comparisons in this Section have been based upon parity of true interest cost as reflecting a commonly used measure of the cost of a financing method. But it will be seen to be a basic hypothesis of Part II of this thesis that liquidity (servicing-adequacy of cash flow realised through project linkage) is as important an optimising objective as is profit (cost of financing), within an integrated capital budgeting routine. Accepting the hypothesis for the moment, (and after all it is the rate of cash outflow which determines the size of residual capital) it can strongly be argued that the true comparison basis is that of annual cash outflow in absolute terms. Thus, as a non-extreme example:

**HIRE-PURCHASE:** 20% grant: 15% deposit: 5% added: (giving £213 outflow p.a.) 15% earning: 5% discount: 10% disposal:
gross of r.o.e.: £237
r.o.e. as % of gross: 65%
net of r.o.e.: £285

**LEASING:** £220 per annum: 15% earning: 5% discount:
no disposal value:
gross of r.o.e.: £414
r.o.e. as % of gross: 65%
net of r.o.e.: £146
The magnitude of preference is drastically reduced by the introduction of residual capital earnings, and if a 10% disposal value (i.e., excess above some minimum value) is attributed to the leasing contract then even without unquantifiable considerations being brought into account the order of preference becomes very marginal indeed.\(^{(1)}\) This possibility is strongly to be borne in mind in any application of the procedure outlined in Chapter Six, where the matching of financing alternatives on a basis of the exhaustion of uncommitted cash flows is predicated.

It is clear that any evaluation of the financing decision which involves consideration of hire-purchase and/or of leasing must be carried out only after a clear perception of the applicability or otherwise of the residual capital concept. Insofar as this complicates an already none too straightforward a procedure it is to be regretted, and indeed the problem of r.c.e. might be ignored at earning rates below 9% without causing significant error in the very large majority of comparisons. But at higher earning rates, the concept is of considerable significance to net financing cost and to ignore it is to exclude a major opportunity cost.

Section 5: Cost Trends and Comparisons

The discussion so far presented indicates amply the intricacy of comparative analysis in financing decision-making. What now follows is a comparative review of the trends implicit in the net costs of the different financing methods, shorn as much as possible of the amending factors of tax rate and residual capital earnings. This review is carried out in terms of a 45.5% tax rate and on the assumption of the applicability of residual capital earnings. Even so it is selective, as was predicted in the introduction to this Chapter. It is pertinent

\(^{(1)}\) It must however be admitted that the greater sensitivity of leasing residual capital earnings to changes in earning rates (perhaps the most volatile parameter) and to tax rates (certainly the most arbitrary parameter) might constitute a second order of somewhat unquantifiable (in the sense that they are not deterministic) considerations. A use of a realistic earnings rate and an observation that tax rates seldom move downwards ought however substantially to ameliorate the effects of such uncertainty.
further to observe that inasmuch as assets do or do not qualify for grant, enjoy only one rate of writing-down allowance, are quoted in terms of a fairly narrow set of leasing alternatives, and so on; many comparative analyses of cost trend are numerically feasible but practically invalid.

Outright Purchase

The following primary cost relationships generally obtain:

1. Cost decreases as earning rate increases.
2. Cost increases as discount rate increases.
3. Cost decreases as writing-down allowance increases.
4. Cost decreases as disposal value increases.
5. As grant increases, (1) and (2) are emphasised, (3) is modified and (4) is unchanged. The adjustments are nowhere large in value.

But none of these trends should occasion surprise, and all conceal more subtle trends. For example:

6. The decrease in (1) accrues at an increasing rate as earning rate increases; and, by way of complication, this trend is a positive function of writing-down allowance - a function which however decreases with increases in the discount rate.
7. The increase in (2) accrues at a decreasing rate as discount rate increases; and, by way of complication, the decreasing rate intensifies slightly as earning rate increases.
8. The decrease in (3) accrues at a decreasing rate as writing-down allowance increases; the decreasing rate being a positive function of earning rate and a slightly positive function of discount rate.
9. The decrease in (4) is a constant for all disposal values, but accrues at a decreasing rate as earning rate increases and as discount rate increases.

Perhaps some sort of an ultimate is reached in the following:

10. For a given disposal value, the diminishing decrease in cost as the discount rate increases accrues at an increasing rate as earning rate increases; the whole being
emphasised as writing-down allowance increases, but being generally independent of the particular disposal value selected. (1)

a statement which can be shown to be true but which seems hardly worth while. The meaningful question to be answered is: which are the important parameters, and what is the order of their importance? A necessary and generally true assumption in the answer is: cost increases or decreases with respect to a given parameter may not be linear functions of that parameter, but may safely be taken as such so far as grants, capital allowances and disposal values are concerned. Thus, only the first parts of relationships (6) and (7) on the previous page need to be taken into account individually and reciprocally; for the rest, (3), (4) and (5) may be allowed to stand.

The following data is of assistance in answering the question:

<table>
<thead>
<tr>
<th>TABLE 3:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Net Financing Costs of Outright Purchase</strong> (2)</td>
</tr>
<tr>
<td>45% Corporation Tax</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Discount Rate:</th>
<th>7½%</th>
<th>10½%</th>
<th>15½%</th>
<th>5½%</th>
<th>10½%</th>
<th>15½%</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Earning Rate: 5½%</strong></td>
<td>10%</td>
<td>15%</td>
<td>5½%</td>
<td>10%</td>
<td>15%</td>
<td></td>
</tr>
<tr>
<td>10% W-D.A:</td>
<td>10% Disp.</td>
<td>581</td>
<td>532</td>
<td>475</td>
<td>651</td>
<td>613</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>543</td>
<td>496</td>
<td>443</td>
<td>619</td>
<td>583</td>
<td>541</td>
</tr>
<tr>
<td>15% W-D.A:</td>
<td>10% Disp.</td>
<td>550</td>
<td>489</td>
<td>417</td>
<td>613</td>
<td>565</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>512</td>
<td>454</td>
<td>386</td>
<td>581</td>
<td>536</td>
<td>482</td>
</tr>
<tr>
<td>20% Grant</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% W-D.A:</td>
<td>10% Disp.</td>
<td>435</td>
<td>354</td>
<td>258</td>
<td>507</td>
<td>442</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>397</td>
<td>318</td>
<td>226</td>
<td>475</td>
<td>412</td>
<td>338</td>
</tr>
<tr>
<td>25% W-D.A:</td>
<td>10% Disp.</td>
<td>410</td>
<td>319</td>
<td>212</td>
<td>477</td>
<td>403</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>372</td>
<td>284</td>
<td>181</td>
<td>445</td>
<td>374</td>
<td>290</td>
</tr>
<tr>
<td>30% Init.All'ce.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% W-D.A:</td>
<td>10% Disp.</td>
<td>539</td>
<td>474</td>
<td>399</td>
<td>598</td>
<td>546</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>501</td>
<td>439</td>
<td>366</td>
<td>567</td>
<td>517</td>
<td>450</td>
</tr>
<tr>
<td>25% W-D.A:</td>
<td>10% Disp.</td>
<td>515</td>
<td>441</td>
<td>354</td>
<td>568</td>
<td>509</td>
</tr>
<tr>
<td>20% Disp.</td>
<td>477</td>
<td>406</td>
<td>322</td>
<td>537</td>
<td>479</td>
<td>411</td>
</tr>
</tbody>
</table>

(Footnotes (1) and (2) on next page)
From this data the following comparison can be drawn up:

**TABLE 4 (a):**

Increase in Net Financing Cost of Outright Purchase

<table>
<thead>
<tr>
<th>Corporation Tax</th>
<th>from 7½% Disc. 10% Earning</th>
<th>to 12½% Disc. 10% Earning</th>
</tr>
</thead>
<tbody>
<tr>
<td>45%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>45%</td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>15% W-D.A: Nil Grant</td>
<td>81</td>
<td>87</td>
</tr>
<tr>
<td>20% Grant</td>
<td>88</td>
<td>94</td>
</tr>
<tr>
<td>30% Init.Alloc.</td>
<td>72</td>
<td>78</td>
</tr>
<tr>
<td>25% W-D.A: Nil Grant</td>
<td>76</td>
<td>82</td>
</tr>
<tr>
<td>20% Grant</td>
<td>84</td>
<td>90</td>
</tr>
<tr>
<td>30% Init.Alloc.</td>
<td>68</td>
<td>73</td>
</tr>
</tbody>
</table>

Here there are compared the absolute changes in cost (increases) accruing to a 5% increase in the discount rate as against that accruing to a 5% decrease in the earning rate; where the result of the change is a convergence on to a common discount rate/earning rate complex.\(^{(1)}\) It may be observed that:

11. In the nil grant (nil initial allowance) situation, the effect of a change in the discount rate is much the stronger influence; and this is emphasised principally by a low value of writing-down allowance and secondly by a high

---

\(^{(1)}\) A necessary condition of equitable comparison, in view of the non-linear (and differently non-linear) cost functions of the two rates. This non-linearity is clearly seen in the data of Table 3. It will also be observed that the various discount rate/earning rate complexes in comparison evoke not too dissimilar net financing costs in each pair comparison: for example at 15% W-D.A. nil grant the convergence is from £532 to £613, and from £568 to £613 respectively.

---

\(^{(2)}\) Source: Tables A2, A3 and E2. In this and subsequent tables, a rounding of difference figures occasionally yields fractionally different data from that derived exactly from the Evaluation Tables.
value of disposal proceeds.

12. In the 20% grant situation, the same effect generally is to be observed: but at high values of writing-down allowances, the discount rate and the earnings rate are just about equally important, moving from a small bias in favour of the earning rate to a small bias in favour of the discount rate as disposal proceeds improve.

13. In the 30% initial allowance situation, modifying effect is continued. The two rates are just about equally important at high values of writing-down allowance.

14. As a general conclusion: the discount rate is on the whole more important than the earning rate - i.e., net financing costs are more sensitive to changes in the discount rate.

As general conclusions:

15. Relationship (14) is stronger at low values of writing-down allowance (principally) and high values of disposal proceeds (secondarily); and vice-versa.

16. Writing-down allowance is on the whole a more critical determinant of net financing costs under outright purchase than is the value of disposal proceeds, especially as reflected against changes in the discount rate.

17. Relationship (16) applies a little more closely to assets attracting nil investment grant, and its significance is a negative function of the size of grant or initial allowance.

But there may be devised other pair-comparisons of discount rate/earning rate which satisfy the condition of convergence on to a common complex. Within the ranges of values accorded to this study to the two parameters there may be distinguished (calling that pair comparison just discussed by the reference letter A, and describing the common convergence point as one of 'high discount, medium earning');

(1) The 'description' labels relate to the convergence situation, and are best regarded as representing the present position of a hypothetical firm, from which a movement to either of the two points of origin is contemplated, voluntarily or involuntarily. There is thus effected the comparison: which is more advantageous to the firm, a reduction in the discount rate or an equivalent improvement in the earning rate? (Or relatively disadvantageous, if the shift in the two rates are reversed).
3. (i) 7½% discount, 5% earning
(ii) 12½% discount, 10% earning
    converging on 12½% discount, 5% earning and
described as a "high discount, low earning"
situation.

<table>
<thead>
<tr>
<th>TABLE 4 (b); (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Convergence Differentials</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C(i)</th>
<th>C(ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>15% W-D.A: Nil Grant</td>
<td>70</td>
<td>76</td>
</tr>
<tr>
<td>20% Grant</td>
<td>72</td>
<td>78</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>59</td>
<td>66</td>
</tr>
<tr>
<td>25% W-D.A: Nil Grant</td>
<td>65</td>
<td>69</td>
</tr>
<tr>
<td>20% Grant</td>
<td>67</td>
<td>73</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>53</td>
<td>60</td>
</tr>
</tbody>
</table>

This pair comparison indicates clear agreement with relationships (11) through (17).

C. (i) 5% discount, 10% earning
(ii) 10% discount, 15% earning
    converging on 5% discount, 15% earning and
described as a "low discount, high earning"
situation.

<table>
<thead>
<tr>
<th>TABLE 4 (c);</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Convergence Differentials</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>C(i)</th>
<th>C(ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10%</td>
<td>20%</td>
</tr>
<tr>
<td>15% W-D.A: Nil Grant</td>
<td>100</td>
<td>105</td>
</tr>
<tr>
<td>20% Grant</td>
<td>120</td>
<td>125</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>98</td>
<td>103</td>
</tr>
<tr>
<td>25% W-D.A: Nil Grant</td>
<td>95</td>
<td>100</td>
</tr>
<tr>
<td>20% Grant</td>
<td>116</td>
<td>122</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>94</td>
<td>99</td>
</tr>
</tbody>
</table>

(1) All subsequent Tables 4 are deemed to be in a format identical with
that of Table 4 (a). Thus columns (i) demonstrate the change in net
financing cost consequent upon a change in the discount rate, and
columns (ii) the change consequent upon a like but opposite change
in the earning rate.
As with E, this pair comparison agrees with all relationships (11) through (17).

D. (i) 10% discount, 10% earning
(ii) 5% discount, 5% earning

converging on 5% discount, 10% earning and
described as a "low discount, medium earning"
situation.

TABLE 4 (a):
Convergence Differentials

<table>
<thead>
<tr>
<th>D(i)</th>
<th>10% Disp.</th>
<th>20% Disp.</th>
<th>D(ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15% W-D.A: Nil Grant</td>
<td>93</td>
<td>98</td>
<td>56</td>
</tr>
<tr>
<td>20% Grant</td>
<td>99</td>
<td>105</td>
<td>91</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>81</td>
<td>87</td>
<td>72</td>
</tr>
<tr>
<td>25% W-D.A: Nil Grant</td>
<td>85</td>
<td>91</td>
<td>67</td>
</tr>
<tr>
<td>20% Grant</td>
<td>93</td>
<td>99</td>
<td>100</td>
</tr>
<tr>
<td>30% Init. All'ce.</td>
<td>75</td>
<td>81</td>
<td>82</td>
</tr>
</tbody>
</table>

Exactly the same conclusions can be drawn as in comparisons B and C.

Further pair comparisons; "medium discount, high/medium/low earning"(1) situations E, F and G; can be built up in a like manner. All give conclusions which agree closely with relationships (11) through (17).

It can therefore be advanced as a demonstrable hypothesis that the net financing costs of outright purchase reflect firstly the discount rate, secondly the earning rate, thirdly the value of writing-down allowance and fourthly the disposal proceeds; and that this is generally true irrespective of value of grant or initial allowance for which the asset may qualify.

It is further suggested that the following proposition is valid:

and that it is valid whatever the grant or initial allowance situation may be. If by sensitivity to change in the convergence earning/discount complex is meant, the absolute (or percentage - either is applicable)(2)

(1) These are the 'duals' of the convergences of A, C and D respectively. Thus, 12½% discount/15% earnings; 7½% discount/10% earning converges at A (10½% discount/10% earning) or at B (7½% discount/15% earning).

(2) It is possible to derive the same ranking by having regard to the ranges of change in net financing cost over the three grant-initial allowance situations in illustration, and it is noticeable that the difference in ranges between the first and the second, the second and the third etc. complexes is largest between the first and the second, next largest between the second and the third etc. complexes, and so on.
change in net financing cost for a given common change in the complex, then the ranking of sensitivity is (greatest sensitivity first):

1. The low discount, high earning situation (C)
2. (equal) The low discount, medium earning situation (D)
3. (equal) The medium discount, high earning situation (E)
4. The medium discount, medium earning situation (F)
5. (equal) The high discount, medium earning situation (A)
6. (equal) The medium discount, low earning situation (G)
7. The high discount, low earning situation (B)

**NOTE:** the ranking of D, F and E; and of A and G; is somewhat arbitrary as the differences between the complexes in each group are very small.

If for the sake of the argument we make two assumptions - firstly that there will be a high correlation between earnings per share and the earning rate; and secondly that earnings per share are ultimately a significant (inverse) factor in the cost of capital - then there is some significance to be found in this proposition. For of all the parameters used in the Evaluation Tables, the earning rate is potentially the most volatile and the discount rate - if the two assumptions are correct - potentially the second most volatile (although disposal proceeds are not usually noted for constancy). Thus the discount/earning rate complex may change. But for a company in situation A, a 5-point change in the complex still will not greatly (comparatively speaking) reduce financing costs, which must to say the least be a source of managerial frustration. Whereas a company in situation C will find that a small further improvement in earnings will still induce substantial (comparatively speaking) economics of financing cost. That such success breeds success is not of course an unusual business phenomenon, and the other side of the coin must not be overlooked. An A-type company is nowhere near so exposed, in terms of asset financing cost by outright purchase, to secular increases in market discount rates or to short period or cyclical earnings depressions, as is a C-type company.

It is also to be remarked that though this ordering of sensitivities is true over all grant-initial allowance situations, the magnitude of the differences as between grant and initial allowance situations is not constant for all complexes. Thus it is true that
a C-type complex is more sensitive than an E-type complex at nil grant, at 20% grant and at 30% initial allowance; and an E-type complex is in the same way more sensitive than a B-type complex. But for 20% grant and 30% initial allowance, C is about as sensitive over E as E is over B; but for nil grants, C is only about one-third more sensitive over E as E is over B. There is a wide variety between the magnitude of these increases in sensitivity of individual grant or initial allowance situations. Thus the general proposition of increased sensitivity is subject, not so much to error as to reduction in its practical utility.

A study of the actual convergence differentials for all types of complexes reveals that, in all cases, the net financing costs of assets attracting initial allowance are less sensitive to changes in the discount rate than those of assets attracting no allowance or attracting grant; that, in all cases, the net financing costs of assets attracting no allowance are less sensitive to changes in the earning rate than those of assets attracting initial allowance or grant; and that, in all cases, the net financing costs of assets attracting grant are the most sensitive to both changes in the discount rate and the earning rate - the increased sensitivity being a positive function of the size of grant.

Finally, there may be noted from Table 3 the rather surprisingly small advantage (in terms of the net financing cost of outright purchase) which a 30% initial allowance offers over a nil allowance (nil grant) situation. The cause of this lies in the sharp reduction in the writing-down allowance in the second and third years of asset life, which accrues to the initial allowance situation. In these periods, the 'slimming' effect of discounting is significant, but not yet as it were overwhelming. Expectably, the net financing costs of the grant situation are the lowest. This advantage, of course, is primarily a positive function of the earning rate and secondarily a negative function of the discount rate (the magnitude of the order is not great); and is somewhat emphasised at higher levels of writing-down allowance but is indifferent to the level of disposal proceeds.

HIRE-PURCHASE

The cost relationships under hire-purchase are sometimes similar to and sometimes non-comparable with those of outright purchase. The
following simple relationships can be observed:

18. Cost decreases as earning rate increases.
19. Cost decreases as writing-down allowance increases.
20. Cost decreases as disposal value increases.

which are expectable, and (differences in scale of cost variation apart)
are in conformity with relationships 1, 3 and 4. Specific to
hire-purchase are:

21. Cost increases as interest added increases, and this
generally to a noticeable extent.
22. Cost increases as initial deposit increases, but the size
of the cost variation ranges only from zero to relatively
small amounts.
23. As grant increases, relationship (18) is emphasised, (19)
is modified (20) is unchanged, (21) is unchanged and (22)
is modified.

The effect of variations in the discount rate is a noticeable absentee
in this catalogue; and this is because this cost relationship is, in
the case of hire-purchase, not simple. In the following Table 5 there
are exemplified the nil grant, 20% grant and 30% initial allowance
situations; each illustrated for three earning rates - 5%, 10% and 15%.
Table 5 lists non-numerical indicators of the change in the net
financing cost as the discount rate is increased over the range
5% - 12½%. The indicators may be interpreted as:

a) A noticeable decrease in net financing cost.
b) A slight decrease in net financing cost.
c) Little or no change in net financing cost.
d) A slight increase in net financing cost.
e) A noticeable increase in net financing cost.

and for each earning rate there are considered five states, each
representing a set of parameters which are altered in turn:

(1) A base state of 15% writing-down allowance, 10% disposal
proceeds, 15% deposit and 5% interest added.
(2) 25% rather than 15% writing-down allowance, other
parameters held constant.
(3) 20% rather than 10% disposal, other parameters held
at base values.
(4) 25% rather than 15% initial deposit, other parameters held at base values.

(5) 2% rather than 5% interest added, other parameters held at base values.

The indicators are 'measured' in each case as between the base state and the change under consideration - i.e., the indicators do not have a cumulative value.

**TABLE 5:**

<table>
<thead>
<tr>
<th>State (1)</th>
<th>State (2)</th>
<th>State (3)</th>
<th>State (4)</th>
<th>State (5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nil Grant</td>
<td>5% Earn</td>
<td>a</td>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>10% Earn</td>
<td>b</td>
<td>b</td>
<td>c</td>
<td>d</td>
</tr>
<tr>
<td>15% Earn</td>
<td>d</td>
<td>c</td>
<td>d</td>
<td>d/e</td>
</tr>
<tr>
<td>20% Grant</td>
<td>5% Earn</td>
<td>c</td>
<td>b</td>
<td>d</td>
</tr>
<tr>
<td>10% Earn</td>
<td>d</td>
<td>d</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>15% Earn</td>
<td>e</td>
<td>e</td>
<td>e</td>
<td>e</td>
</tr>
<tr>
<td>30% In.Al.5% Earn</td>
<td>a</td>
<td>a</td>
<td>a</td>
<td>a</td>
</tr>
<tr>
<td>10% Earn</td>
<td>b</td>
<td>a</td>
<td>b</td>
<td>b</td>
</tr>
<tr>
<td>15% Earn</td>
<td>b</td>
<td>c</td>
<td>d</td>
<td>b</td>
</tr>
</tbody>
</table>

A few broad generalisations may be made:

24 (i) Generally, net financing cost decreases with increases in the discount rate in the nil grant or 30% initial allowance situation, but net cost increases with increases in the discount rate at 20% grant (and even more so at 40% grant).

(ii) But as earning rate increases reductions in cost become less or even convert to small increases, while increase in cost are emphasised.

(iii) These cost trends are more likely to be in the nature of an increase (or an emphasising thereof, at high earning rates) if the disposal and/or initial deposit rates are high.

The term 'noticeable' is intended to be read as being expressed in

(1) Source: Tables C2, 63 and D2.
absolute values, yet even so is nowhere a 'large' number (say, in excess of £20). The cost trends thus labelled by the indicators are not linear, being a diminishing function of the increase in discount rate in all cases. In no case does the change in the net cost caused by a standard 2½% upward step in the discount rate exceed 5% of the 'starting' level of net cost. Compared with outright purchase (where the same 2½% variation in discount rate induces up to a 25% variation in cost), the net financing costs of hire-purchase are insensitive to the discount rate.

So far as concerns writing-down allowance and disposal value: the absolute values of variations in cost are identical, for given variations of either parameter, with those accruing to outright purchase. This is expectable, as neither parameter affects those elements of financing cost which differentiate outright and hire purchase - namely, the after-tax cost of interest added and the after-tax value of residual capital earnings. But in contradiction of relationship 16, disposal value is on the whole the slightly more critical determinant of net financing cost in hire-purchase, in that cost is over the larger (lower) part of the range of earning rates more sensitive to changes in disposal proceeds. The rate of conversion from primacy of cost sensitivity to disposal value, to primacy of cost sensitivity of writing-down allowance is a negative function of the size of grant or initial allowance. Writing-down allowance is on the whole a more critical determinant of net cost than is the discount factor, especially in the nil grant and 30% initial allowance situation. But relationship 24 points out the different effect of increases of the discount rate upon net financing cost in the various grant and initial allowance situations, which complicates comparison of the writing-down allowance and the discount rate. (Sensitivity ignores signs). Thus cost is more sensitive to writing-down allowance, except:

(a) At low earning rates, in the 30% initial allowance situation; especially if these coincide with low disposal rates and a high rate of interest added.

(b) At high earning rates, in the 20% grant situation; especially if these coincide with high disposal rates and a low rate of interest added.

The nil grant situation tends to be similar to that for the 30% initial
allowance, but with a reduced superiority of writing-down allowance at high earning rates. A similar pattern emerges from a comparison of disposal proceeds with the discount rate, save that here the superiority (in terms of cost sensitivity) of disposal proceeds is rather less marked and the areas of inferiority are emphasised. If this reduced superiority appears to conflict with the earlier conclusion that on the whole cost is more sensitive to disposal proceeds than to writing-down allowances, it is to be realised that changes in cost due to changes in the discount rate over a range of disposal values are greater than those accruing to the same changes in the discount rate over the same range of writing-down allowances.

Inspection of the relevant Evaluation Tables indicates that those parameters wherein changes most significantly affect net financing cost are the earning rate and the rate of interest added. The technique of pair-comparisons and convergence differentials can be used to assess the relative sensitivities of cost to these two influences. Thus the comparison to be effected is; which is the more advantageous to the firm, an increase in the earning rate or an equivalent decrease in the rate of interest added? (Or disadvantageous, if the shifts in the two rates are reversed). In the tables which follow, all columns (i) demonstrate the change in net costs due to a change in the earning rate and all columns (ii) the change due to a like but opposite change in the rate of interest added. The descriptions 'low' and 'high' as applied to rates of interest added is very much a relative term, as the range of comparison is only 5% - 8%. If the pair-comparisons are to be valid, variations in the earning rate must be restricted to the same range, wherefore two pairs of earning rates are tested - 5% & 8%, and 11% & 14% - respectively termed 'low', 'low-medium', 'high-medium' and 'high'.

M. (i) 5% earning, 5% interest added
(ii) 8% earning, 8% interest added

converging of 8% earning, 5% interest added and which must be described as a "low-medium earning, low interest added" situation.
### TABLE 5 (M): Convergence Differentials

<table>
<thead>
<tr>
<th>N-D.A. Deposit</th>
<th>Discount</th>
<th>10% Disp.</th>
<th>20% Disp.</th>
<th>10% &amp; 20% Disp.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Nil Grant</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% 15% 5%</td>
<td>61</td>
<td>59</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 15% 5%</td>
<td>53</td>
<td>51</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>25% 5%</td>
<td>58</td>
<td>56</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>49</td>
<td>48</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>25% 15% 5%</td>
<td>69</td>
<td>77</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>60</td>
<td>59</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>25% 5%</td>
<td>66</td>
<td>76</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>56</td>
<td>64</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td><strong>20% Grant</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% 15% 5%</td>
<td>73</td>
<td>71</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 15% 5%</td>
<td>60</td>
<td>59</td>
<td>75</td>
<td></td>
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<tr>
<td>25% 5%</td>
<td>71</td>
<td>69</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
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<td>57</td>
<td>66</td>
<td></td>
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<tr>
<td>25% 15% 5%</td>
<td>79</td>
<td>77</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>65</td>
<td>64</td>
<td>75</td>
<td></td>
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<tr>
<td>25% 5%</td>
<td>76</td>
<td>75</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>62</td>
<td>62</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td><strong>30% Initial Allowance</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15% 15% 5%</td>
<td>71</td>
<td>69</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 15% 5%</td>
<td>61</td>
<td>59</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>25% 5%</td>
<td>68</td>
<td>66</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>57</td>
<td>56</td>
<td>66</td>
<td></td>
</tr>
<tr>
<td>25% 15% 5%</td>
<td>77</td>
<td>75</td>
<td>88</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>65</td>
<td>64</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>25% 5%</td>
<td>73</td>
<td>72</td>
<td>78</td>
<td></td>
</tr>
<tr>
<td>12½% 5%</td>
<td>62</td>
<td>60</td>
<td>66</td>
<td></td>
</tr>
</tbody>
</table>

N. (i) 5% earning, 8% interest added
(ii) 8% earning, 5% interest added
converging on 8% earning, 8% interest added.
This is the 'reversal' of M and might be described
as a "low-medium earning, high interest added" situation.

<table>
<thead>
<tr>
<th>TABLE 5 (N): Convergence Differentials</th>
</tr>
</thead>
</table>

<table>
<thead>
<tr>
<th>N(i)</th>
<th>N(ii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% Disp.</td>
<td>20% Disp.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>W-D.A. Deposit, Discount.</th>
<th>15%</th>
<th>15%</th>
<th>5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
<td>52</td>
<td>50</td>
<td>88</td>
</tr>
<tr>
<td>12½%</td>
<td>46</td>
<td>44</td>
<td>75</td>
</tr>
<tr>
<td>25%</td>
<td>50</td>
<td>48</td>
<td>78</td>
</tr>
<tr>
<td>12½%</td>
<td>44</td>
<td>42</td>
<td>66</td>
</tr>
<tr>
<td>25%</td>
<td>59</td>
<td>58</td>
<td>88</td>
</tr>
<tr>
<td>12½%</td>
<td>54</td>
<td>52</td>
<td>75</td>
</tr>
<tr>
<td>25%</td>
<td>57</td>
<td>55</td>
<td>78</td>
</tr>
<tr>
<td>12½%</td>
<td>50</td>
<td>50</td>
<td>66</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20% Grant</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
</tr>
<tr>
<td>15%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>30% Initial Allowance</th>
</tr>
</thead>
<tbody>
<tr>
<td>15%</td>
</tr>
<tr>
<td>15%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
<tr>
<td>25%</td>
</tr>
<tr>
<td>12½%</td>
</tr>
</tbody>
</table>
Tables 5 (i) and 5 (ii) demonstrate clearly the generally lower degree of sensitivity to changes in the earning rate of the nil grant situation, when compared with the 20% grant situation; and the slightly lower sensitivity at low discount rates of the 30% initial allowance situation, when compared with the 20% grant situation. These differentials exist for all earning rates, and are slightly accentuated when the convergence is between higher levels of earning rate. But as this accentuation is slight, and as the sensitivity to changes in the interest added rate is indifferent as to the grant or initial allowance situation, the remainder of convergence differential analysis for hire-purchase will be carried out in terms of the 20% grant situation only.

0. (i) 5% earning, 5% interest added
   (ii) 8% earning, 8% interest added
    converging on 5% earning, 8% interest added. This is the dual of N, and might be described as a "low earning, high interest added" situation.

| TABLE 5 (0) |
| Convergence Differentials |

<table>
<thead>
<tr>
<th></th>
<th>10% Disp.</th>
<th>20% Disp.</th>
<th>10% &amp; 20% Disp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Grant.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>15%</td>
<td>5%</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td></td>
<td>123/4%</td>
<td>54</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>62</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>123/4%</td>
<td>52</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
<td>5%</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>123/4%</td>
<td>59</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>68</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td></td>
<td>123/4%</td>
<td>56</td>
</tr>
</tbody>
</table>

P. (i) 5% earning, 8% interest added
   (ii) 8% earning, 5% interest added
    converging on 5% earning, 5% interest added. This is the dual of N, and might be described as a
"low earning, low interest added" situation.

**TABLE 5 (P):**

**Convergence Differentials**

<table>
<thead>
<tr>
<th>W-D.A.</th>
<th>Deposit</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>20% Grant</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>12½%</td>
<td>60</td>
<td>59</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>71</td>
</tr>
<tr>
<td>12½%</td>
<td>58</td>
<td>57</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>12½%</td>
<td>65</td>
<td>64</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>76</td>
</tr>
<tr>
<td>12½%</td>
<td>62</td>
<td>62</td>
</tr>
</tbody>
</table>

As an example of the effect of higher earning rates:

(i) 11% earning, 5% interest added

(ii) 14% earning, 5% interest added

converging on 14% earning, 5% interest added and which might be described as a "high earning, low interest added" situation.

**TABLE 5 (Q):**

**Convergence Differentials**

<table>
<thead>
<tr>
<th>W-D.A.</th>
<th>Deposit</th>
<th>Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>20% Grant</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>12½%</td>
<td>73</td>
<td>71</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>82</td>
</tr>
<tr>
<td>12½%</td>
<td>70</td>
<td>69</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
<td>5%</td>
</tr>
<tr>
<td>12½%</td>
<td>79</td>
<td>77</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>90</td>
</tr>
<tr>
<td>12½%</td>
<td>76</td>
<td>75</td>
</tr>
</tbody>
</table>
It seems unnecessary to detail the data for the reversal of Q (i.e., 11% earning, 5% interest added - a "high-medium, low interest added" situation which may be labelled R) or the 'duals' of these last two, which may be labelled S and T respectively: for this sub-set of situations can be proved to exactly correspond with the previous sub-set M, N, O and P. The following propositions appear on this evidence to be valid:

25. The net financing cost of hire-purchase is more sensitive to changes in interest added than to asset earning rate. Interest added is the primary determinant of cost.

26. This greater sensitivity to changes in interest added can be significantly affected (reduced) by increases in the rate of initial deposit. This is true of all grant or initial allowance situations, but is the more true the higher is the rate of grant or initial allowance. At nil grant (only), the size of the writing-down allowance is equally significant.

27. This greater sensitivity to changes in interest added is emphasised when the pair-comparison is effected at higher levels of earning rate. Again this is true of all grant and initial allowance situations, but is the more true the lower is the rate of grant or initial allowance.

28. The greater sensitivity to changes in interest added is not appreciably affected by changes in the discount rate, and is not materially affected by changes in disposal proceeds.

In relationship (22), the low sensitivity of cost to changes in initial deposit was remarked upon. Purely in order to permit a ranking of parameter-sensitivities, this was analysed in terms of convergence differentials against the discount factor, where an inferior sensitivity to variations in deposit rate was quite clear. It is therefore advanced as a demonstrable hypothesis that the net financing costs of hire-purchase reflect firstly the rate of interest added, secondly the earning rate, thirdly the disposal rate, fourthly the size of writing-down allowance, fifthly the discount rate and lastly the rate of initial deposit. For given circumstances of low or high earning rates, the ranking of disposal proceeds, writing-down
allowance and discount rate may be varied or even reversed. Nevertheless, this ranking is generally in sharp contradiction to that which obtains for outright purchase, especially with respect to the discount rate. It is noteworthy that the 'relegation' of the discount factor from a position of prime to one of secondary importance is due to a sharp reduction in the numerical value of changes in net cost induced by changes in the discount rate, rather than to a more than proportionate increase in the values of changes in net cost attributable to changes in the other parameters. This 'relegation' of the discount factor is of course simply a reflection of the reduction in the present value of the serial instalment payments - a reduction which considerably offsets the reduction in the present value of the various cash inflows and accruing earnings streams.

It is possible to rank the various earning rate/interest added complexes in order of sensitivity to change in the complex. In order of descending sensitivity, the ranking is:

1. The low interest added/high earning rate situation.  (Q)
2. (equal) The low interest added/upper-medium earning rate situation.  (R)
2. (equal) The high interest added/high earning rate situation.  (T)
4. The high interest added/upper-medium earning rate situation.  (S)
5. The low interest added/ lower-medium earning rate situation.  (M)
6. (equal) The high interest added/lower-medium earning rate situation.  (N)
6. (equal) The low interest added/low earning rate situation.  (P)
8. The high interest added/low earning rate situation  (O)

(NOTE: Situations R, T & S are not illustrated in the preceding text, but have elsewhere been calculated on exactly the same basis as for Q, P, N & O).

The ranking is valid only in terms of quite small differentials between the various situations, and not too much importance can be attached to this table. Nevertheless, the ranking quite closely corresponds with
that established for earning/discount rate complexes in outright purchase, with the order of cost sensitivity correlating well with reductions in the level of the earning rate and correlating roughly with increases in the rate of interest added (vide the discount rate in outright purchase).

Any causative relationship between earning rate and interest added is probably tenuous. Rates of interest added are usually a function of the general structure of interest rates in the economy as a whole. But evidence exists that these provide a floor-rate and that individually negotiated contracts pay some attention to the financial situation of the borrower. This is usually reflected in the size of initial deposit required; but to the extent that it is also reflected in the rate of interest added, an O-type company will suffer from the same frustration - but also enjoy the same protection against short-term earnings fluctuations or secular increases in market interest rates - as an A-type company under outright purchase. (More accurately; and because the ranking differentials are here more blurred; will suffer increments of frustration or enjoy accretions of protection in like manner if not in like quantity).

Finally, as with outright purchase, the general validity of the ranking over all grant and initial allowance situations should ideally not be allowed to obscure the fact that in some of these situations this validity is "more valid" than in others. Generally, the validity is more suspect in the nil grant situation; but this modification for size of grant is less pronounced than in outright purchase, and can be ignored for practical purposes.

The comparatively small advantage (in terms of net financing cost) which was noted as accruing to a 30% initial allowance situation in the discussion of net costs under outright purchase, is repeated in hire-purchase - predictably so, as the structure of capital allowances

(1) In the form of photocopies of upwards of a dozen specifically negotiated hire-purchase type contracts for substantial capital expenditures, kindly provided by United Dominions Trust Ltd.; and of correspondence with F.C. Finance Ltd. indicating specific regard to the liquidity prospects of the purchaser. The re-sale potential of the asset in the event of a re-possession is also a primary determinant of the size of initial deposit.
is unaffected by the method of purchase. But because under hire-purchase, grant is deemed to be received in quarterly instalments according to the agreed capital content of the relevant three monthly payments, thereby reducing the calculation basis for the accruing earnings stream at any point in the study period; and thereby exposing both grant and earnings stream to a greater degree of 'slimming' from the discounting procedure; the advantage enjoyed by the accrual of grant is much reduced under hire-purchase. The point is most easily seen by extracts from Tables A3 and C3:

TABLE 6:
REDUCTIONS in Net Financing Costs
arising from a 20% Investment Grant
(45% Corporation Tax)

<table>
<thead>
<tr>
<th></th>
<th>5% Discount</th>
<th>12.5% Discount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Writing-down Allowance:</td>
<td>15% 25%</td>
<td>15% 25%</td>
</tr>
<tr>
<td>Outright Purchase:</td>
<td>5% Earn 146 141</td>
<td>144 136</td>
</tr>
<tr>
<td></td>
<td>15% Earn 223 213</td>
<td>203 192</td>
</tr>
<tr>
<td>Hire-Purchase:</td>
<td>5% Earn 114 109</td>
<td>92  86</td>
</tr>
<tr>
<td></td>
<td>15% Earn 158 147</td>
<td>123 111</td>
</tr>
<tr>
<td>25% Deposit:</td>
<td>5% Earn 118 113</td>
<td>98  92</td>
</tr>
<tr>
<td></td>
<td>15% Earn 165 154</td>
<td>132 120</td>
</tr>
</tbody>
</table>

The diminution under hire-purchase of the reduction in net cost evoked by a 20% grant is of the order of one-third. The cost-reduction impact of increases in the earning rate is sharply cut, and the increased 'slimming' effect of increases in the discount rate is plain to see. As is to be expected, there is no difference in the effect of increases in writing-down allowance, and the increase in deposit has an expectably small effect.

The effect of residual capital earning.

It was laid down as a criterion of this discussion that the existence of these earnings would be assumed. Nevertheless it is relevant to examine to what extent the principal cost sensitivities - those of interest added and earning rate - are affected by residual capital earnings inasmuch as both of these parameters are major
determinants of those earnings. For this purpose two pair-comparisons, Q and M, are recapitulated net of residual capital earnings:

Q. (i) 11% earning, 5% interest added
(ii) 14% earning, 8% interest added
converging on 14% earning, 5% interest added: the
"high earning, low interest added" complex.

TABLE 6 (a):
Convergence Differentials (Net of R.C.E.)

<table>
<thead>
<tr>
<th>W-D.A. Deposit</th>
<th>Discount</th>
<th>10% Disp.</th>
<th>20% Disp.</th>
<th>10% &amp; 20% Disp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Grant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>15%</td>
<td>5%</td>
<td>63</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>12.5%</td>
<td></td>
<td>43</td>
<td>41</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td></td>
<td>61</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>12.5%</td>
<td></td>
<td>44</td>
<td>43</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
<td>5%</td>
<td>69</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>12.5%</td>
<td></td>
<td>49</td>
<td>47</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td></td>
<td>69</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>12.5%</td>
<td></td>
<td>50</td>
<td>49</td>
</tr>
</tbody>
</table>

M. (i) 5% earning, 5% interest added
(ii) 8% earning, 8% interest added
converging on 8% earning, 5% interest added: the
"low-medium earning, low interest added" complex.

TABLE 6 (b): reproduced overleaf

(1) As residual capital earnings are indifferent to grant or initial allowance, the discussion is conducted in terms of a 20% grant situation. All previous issues concerning differences in net financing cost of hire-purchase arising out of different grant or initial allowance situations will apply with equal force in the present discussion.
### TABLE 6 (b):
Convergence Differentials (Net of R.C.E.)

<table>
<thead>
<tr>
<th>W-D.A. Deposit</th>
<th>Discount</th>
<th>H(1) 10% Disp.</th>
<th>20% Disp.</th>
<th>H(11) 10% &amp; 20% Disp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>20% Grant</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>15%</td>
<td>5%</td>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>12½%</td>
<td></td>
<td>34</td>
<td>33</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>5%</td>
<td>52</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>12½%</td>
<td></td>
<td>35</td>
<td>34</td>
</tr>
<tr>
<td>25%</td>
<td>15%</td>
<td>5%</td>
<td>58</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>12½%</td>
<td></td>
<td>39</td>
<td>38</td>
</tr>
<tr>
<td>25%</td>
<td>5%</td>
<td>5%</td>
<td>57</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>12½%</td>
<td></td>
<td>39</td>
<td>39</td>
</tr>
</tbody>
</table>

The effect of residual capital earnings upon cost sensitivity is clear. In the absence of such earnings:

29. The net financing cost of hire-purchase is more sensitive to the earning rate than to interest added (a reversal of relationship (25) where the earning rate is high; but tends to become roughly equally sensitive to these two parameters as the earning rate is reduced to a low level, with primacy passing to interest added where this coincides with a high discount rate.

30. The considerable influences previously exercised in this pair-comparison by initial deposit (relationship 26) largely disappears in the absence of residual capital earnings (which is expectable, in view of the importance of initial deposit in the determination of residual capital); and is replaced by a reintroduction of the discount factor as a principal influencing element. As was noted in relationship 29, the discount factor can reverse the primacy of two parameters in terms of cost sensitivity; but it must be noted that an increase in the deposit rate can still achieve the same effect, albeit less decisively.

This reintroduction of the discount factor as an element of some significance raises the question: does this mean that, in the absence
of residual capital earnings, the discount factor assumes the same importance in the overall determination of net financing cost under hire-purchase as it exercises under outright purchase? The answer is no. Its resumption of a position of importance in the foregoing pair comparison is on a basis of a maximum 7½-point shift. When this shift is reduced to a size comparable with other parameters - e.g., a 5-point shift to compare with disposal values, or a 3-point (2½-point) shift to compare with interest added, the parameter-ranking hypothesis on page 140 is unchanged save for the reversal of the first and second parameters at high earning rates, and at low earning rates subject to the coincidence of a low discount rate and a high initial deposit.

**LEASING**

Leasing cost relationships, as might be expected, show substantial resemblance to those of hire-purchase, insofar as the parameters involved in the two financing methods are comparable. Simple relationships, excluding any consideration of excess premiums, refunds or disposal values, are:

31. Cost decreases as earning rate increases.
32. Cost increases as size of premium increases. In Section 4 of this Chapter, there was discussed the equivalencing of the size of primacy period annual premiums to an internal or 'true' rate of return. This rate of return was compared with the hire-purchase internal rate, which was shown to be a function of the rate of interest added. Thus relationship 32 is reasonably compatible with relationship 21, differences in size apart - although the net financial cost of leasing certainly also increases to a 'noticeable' extent as this parameter increases.

Manifestly the argument advanced in Section 2 that size of annual cash outflows is perhaps a better comparative measure of instalment debt than is the implicit true rate of return, is not applicable here. The only common unit of measurement between annual premium, earning rate and discount rate is a percentage. Inasmuch as what is sought is an analysis of the relative importances of the determinants of net
financing cost, rather than the determinants of the financing decision itself, this is no detriment.

33. Cost decreases as the rate of discount increases. Generally, this trend is much less complex than the comparable trend in hire-purchase, but it is possible to discern certain modifications of the general statement as was found necessary in formulating relationship 24. Thus, this present relationship is also only true at lower rates of earning or higher true rates of interest (higher premium values). The same influence of earning rate was found in hire-purchase.

If these simple cost relationships are expanded to take multiple effects into account:

34. The decrease in cost as earning rate increases is an increasing function of the earning rate itself, though the rate of increase is nowhere great. The rate of increase is higher at higher discount rates, but lower at higher true rates of interest. Indeed, for low discount rates combined with high true rates, the decrease in cost as earning rate increases accrues at a decreasing rate at high earning rates.

35. The increase in cost as the size of premium increases is a constant function of that parameter, but (as a constant increase in the size of premium implies a slightly declining increase in the true rate of interest), the increase in cost as the true rate increases is a slightly increasing function of the true rate, especially as higher earning rates. The increase in cost as the true rate increases is slightly less for each successive level of the discount rate.

36. At each level of earning rate, the decrease in cost is approximately constant for each successive and equal increase in the discount rate; but this constant decrease is a decreasing function of the size of the earning rate, and an increasing function of the true rate (annual premium).
At high earning rates the effect of the discount rate on cost tends to be insignificant - and, indeed, is nil at low true rates of interest. Thus, at 15% earning rate, Table E2 indicates that the net financing cost under Pattern 1 (3.3% true rate) is identical for all levels of discount; and that even under Pattern 7 (19.0% true rate) the decrease in cost is less than 5% for each 2½-point step in the discount rate.

But inspection of Table E2 demonstrates that a 2½-point step in the discount rate nowhere decreases cost by an amount in excess of 5½; as in hire-purchase, the discount rate is scarcely a parameter of great significance. Earning rate and size of annual premium, in the form of the implicit true rate of interest, can be ranked in terms of their importance in determining cost by the usual pair-comparison tests of convergence differentials. Double comparisons are applied to both parameters, to observe if the ranking is significantly affected as either the premium or the earning rate moves from the lower towards the higher limits of the range of values quantified in the leasing Evaluation Tables. The tests are reported on the following parametrical values:

- **annual premiums:** £220 p.a. (3.3% true): a 'low' rate.  
  £242 p.a. (8.3% true): a 'low-medium' rate.  
  £250 p.a. (9.7% true): a 'high-medium' rate.  
  £277 p.a. (14.7% true): a 'high' rate.  
  (values obtained by interpolation within Table E2)

- **earning rate:** 5% p.a.: a 'low' rate.  
  10% p.a.: a 'medium' rate.  
  15% p.a.: a 'high' rate.

There are thus made possible two comparison levels of annual premium (3.3% versus 8.3%; and 9.7% versus 14.7%), and two comparison levels of earning rate (5% versus 10%; and 10% versus 15%). By measuring the convergence differential for each pair-comparison at the 5% and 12½% discount levels, the subsidiary effect of this parameter may be noted. The values of the differentials are listed in Table 7 (a) on the next page, where by 'int' is meant 'true interest rate' (a proxy for annual premium); by 'earn', asset earning rate; and by 'disc', the discount rate.
### TABLE 7 (a): Convergence Differentials - Leasing

<table>
<thead>
<tr>
<th>Pair-comparison</th>
<th>Converging on Situation</th>
<th>Differentials</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3.3% 5%</td>
<td>8.8% 5%</td>
</tr>
<tr>
<td></td>
<td>8.8% 10%</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>&quot; &quot;</td>
<td>3.3% 10%</td>
</tr>
<tr>
<td></td>
<td>&quot; &quot;</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>3.3% 10%</td>
<td>3.3% 5%</td>
</tr>
<tr>
<td></td>
<td>8.8% 5%</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>&quot; &quot;</td>
<td>8.8% 10%</td>
</tr>
<tr>
<td></td>
<td>&quot; &quot;</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>9.7% 5%</td>
<td>14.7% 5%</td>
</tr>
<tr>
<td></td>
<td>14.7% 10%</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>&quot; &quot;</td>
<td>9.7% 10%</td>
</tr>
<tr>
<td></td>
<td>&quot; &quot;</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>14.7% 5%</td>
<td>9.7% 5%</td>
</tr>
<tr>
<td></td>
<td>9.7% 10%</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>3.3% 10%</td>
<td>3.3% 15%</td>
</tr>
<tr>
<td></td>
<td>8.8% 15%</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>9.7% 10%</td>
<td>14.7% 10%</td>
</tr>
<tr>
<td></td>
<td>14.7% 15%</td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>9.7% 15%</td>
<td>14.7% 15%</td>
</tr>
<tr>
<td></td>
<td>14.7% 10%</td>
<td></td>
</tr>
</tbody>
</table>

There are omitted from this Table, pair-comparisons H (the dual of G), I (of which J is the dual), K and L (the reversals of I and J), N (the dual of M), and O (of which P is the dual). These have been calculated elsewhere.

Inspection of this data indicates that the conversion differentials for variations of the earning rate are consistently greater than those for variations of the true interest rate except in the case of P at low discount rates. The following relationships can be established:

37. Generally, the net financing cost of leasing is more sensitive to changes in the earning rate than it is to
equivalent changes in the annual premium where the premium is measured by the implicit true rate of interest.

38. This greater sensitivity is a diminishing function of both the true rate and the earning rate, and indeed where both are at a high level, relationship 37 can be reversed and the true rate (annual premium) become the greater influence.

39. Relationship 38 is the more emphatic at low rates of discount; and the reversal of 37 accurs the sooner, the lower is the rate of discount.

40. The diminishing functions noted in 38 have the following ranking. At low true rates (annual premium) the greater cost-sensitivity to earning rate declines more rapidly as the true rate is shifted by a given amount than as the earning rate is so shifted; but at higher true rates, the decline in that same greater cost-sensitivity is more rapid if the earning rate is shifted by the given amount than if the true rate is so shifted.

Random pair-comparison tests between the discount rate and the true rate indicate a markedly greater sensitivity of cost to the latter, with a pattern of changes in that sensitivity closely akin to those summarised in relationships 38 and 40 (reading true rate for earning rate and discount rate for true rate). The greater influence of the true rate on cost, compared with the influence of the discount rate, is an increasing function of the earning rate. The order of importance to the net financing cost of leasing, within the ranges specified by the Evaluation Tables, is thus firstly, earning rate; secondly, true rate (annual premium); thirdly, discount rate.

The various true rate - earning rate complexes can be ranked in descending order in the usual terms of cost sensitivity, in the form:

1. The low true rate, low/medium/high earning rate (in that order) situation.
2. The medium true rate, low/medium/high earning rate situation.
3. The high true rate, low/medium/high earning rate situation.

with some distortion of this even pattern as, at the higher-medium and high true rates, relationship 40 becomes effective and shifts in the earning rate become preponderant. It is of interest to note that net
costs are most sensitive to changes in the low true rate, low earning rate complex; and least sensitive to changes in the high true rate, high earning rate complex. This is of course somewhat different from the conclusions reached in connection with hire-purchase. However, inasmuch as there is certainly no significant correlation between the earning rate enjoyed by a firm and the size of the annual premiums of the leasing contracts (which per se tend to be non-negotiable) entered into by the company, it is not clear that this ranking in order of cost sensitivity is particularly meaningful. Perhaps there is here a certain advantage to leasing vis-a-vis hire-purchase, in that where a low premium level coincides with a low earning rate, there is every encouragement to the firm to improve the earning rate; but there is also every reason to fear a general increase in market premium rates. The converse holds good where high earning rates coincide with a high premium structure. To this extent the financial implications of leasing might be said to be more volatile than those of purchase, outright or by instalment, at low asset earning rates; and to be less volatile at higher earning rates. On the assumption that the low earning rate situation is indicative of an inability of the firm easily to weather adverse variations in financing costs, the volatility of lease financing costs in that situation must mean that there is added to the financing decision yet another significant unquantifiable aspect - what is the probability that leasing premiums in general will go up? Anything short of a balance of probability against such an increase must tilt the decision scales against leasing; unless of course, the firm enjoys a high earning rate now and in the foreseeable future. The application of a first-year excess premium plus subsequent refund does not alter these conclusions. If anything, the predominance of the earning rate is slightly enhanced. Cost sensitivity to changes in the discount rate is also increased, but by insignificant amounts. As disposal values are potentially common to all contracts, the inclusion of this parameter in no way alters the convergence differentials of true rate - earning rate comparisons: and, although cost sensitivity to changes in the discount rate is increased a little, it is still quite insufficient to predominate over the true rate.

Residual Capital Earnings

As in the analysis of hire-purchase financing costs, it is
pertinent to examine the extent to which the foregoing conclusions are dependent upon the assumption of the existence of residual capital earnings.

41. Relationship 31 - cost decreases as the earning rate increases - is largely still true but the rate of decrease is considerably reduced. The previously observed tendency of the discount rate to have an insignificant effect upon cost at high earning rates (and especially in conjunction with low true rates) is now sufficiently exaggerated for cost to increase as the earning rate increases to high levels. Within the range set by the Evaluation Tables, the increase is nowhere significant.

42. Relationship 32 - cost increases as annual premium increases - still holds good, but again at a much reduced rate.

43. The rather complex relationships of the discount rate to net cost, discussed in 33 and 36, still hold good. Except for a noticeable but (in terms of total cost) barely significant reduction in the size of the effects as the true rate increases, exclusion of residual capital earnings does not very much affect the influence of the discount rate on cost. The relative unimportance of the discount rate except for high levels of period payments was noted in discussion on residual capital earnings in Section 4.

However, it is in the pair-comparisons of earning rate and true interest rate that the impact of residual capital earnings is most noticeable.

TABLE 7 (b): continued overleaf
TABLE 7 (b):

**Convergence Differentials - Leasing (Net of R.C.E.)**

<table>
<thead>
<tr>
<th>Pair-comparison</th>
<th>Converging on Situation</th>
<th>Differentials</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Index</strong></td>
<td><strong>Int. Earn.</strong></td>
<td><strong>Int. Earn.</strong></td>
</tr>
<tr>
<td>A</td>
<td>3.3% 5%</td>
<td>8.3% 5%</td>
</tr>
<tr>
<td>B</td>
<td>&quot; &quot;</td>
<td>3.3% 10%</td>
</tr>
<tr>
<td>C</td>
<td>3.3% 10%</td>
<td>3.3% 5%</td>
</tr>
<tr>
<td>G</td>
<td>14.7% 5%</td>
<td>9.7% 5%</td>
</tr>
<tr>
<td>M</td>
<td>9.7% 10%</td>
<td>14.7% 10%</td>
</tr>
<tr>
<td>P</td>
<td>9.7% 15%</td>
<td>14.7% 15%</td>
</tr>
<tr>
<td></td>
<td>14.7% 10%</td>
<td>12% 5%</td>
</tr>
</tbody>
</table>

From this abbreviated table it is easily seen how, at the higher levels of discount, the previously noted superior cost-sensitivity accruing to changes in the earning rate sometimes switches, albeit marginally, to changes in the true rate (vide comparisons A, B, C & G). At other times the order of cost sensitivity is not changed (vide comparison M). On the other hand, where (as in comparison P) a superior cost-sensitivity to changes in the true rate was noted in conditions where residual capital earnings accrues - at low levels of the discount rate - there is now a substantial switch to a superior cost-sensitivity to changes in the earning rate. In most cases (specifically, in all cases except those of a high true rate situation) the size of the superiority in cost sensitivity is much reduced, in whatever direction that superiority may lie; and in the same cases, any superiority at the 12% discount rate is insignificant. As may be expected the ranking of cost sensitivity is much changed - in fact it is not too much of an exaggeration to say that it is reversed. In the preceding discussion, it was noted that in the case of leasing, the potential non-correlation of the two parameters detracted from the ascription of any meaningful significance in the sensitivity ranking. To this argument there must
in the present case be added the relatively small sizes of the various sensitivity superiorities, such that at high levels of discount ranking is virtually impossible.

**Section 6. Net Financing Cost Comparisons**

So far, the question "which is the least costly financing method" has been studiously avoided; but now it must be faced. The preceding discussions of this Chapter must clearly have demonstrated that no categorical answer is to be expected. Specifically, the answer depends upon:

a) The weight awarded to unquantifiable considerations such as those discussed in Chapter One.
b) Estimates of the probability and timing of the various types of obsolescence, and the extent to which ad hoc negotiation of instalment debt contracts can hedge against these.
c) The particular pattern of grant, capital allowances and disposal value attaching to the asset under appraisal.
d) The particular terms (deposit, interest added, annual premium, excess premium etc.) of the available alternative instalment debt contracts.
e) The existence or otherwise of residual capital earnings.
f) The level of the earning rate and of the discount rate.

Obviously, no single answer is possible. But a small number of specific illustrations may serve to underline some very general conclusions - not the least of which will be confirmation of a warning which has been given from time to time earlier in this thesis: that the quantifiable differences in the net costs of the alternative financing methods are often not so great that unquantifiable considerations might not easily over-rule the particular decision indicated by the Evaluation Tables.

The following illustrations are based upon quotations advertised or privately provided by various finance houses. In each case, costs are calculated for 5%, 10% and 15% earning rates and for 5% and 12½% discount rates: at 45% Corporation Tax. Data are drawn from the appropriate Evaluation Tables.
1. Net Financing costs for non-specialised metal forming plant attracting 20% grant and 15% writing-down allowances and with an estimated 10% disposal value at the end of 5 years.

**OUTRIGHT PURCHASE**

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net financing cost:</td>
<td>392</td>
<td>301</td>
<td>195</td>
<td>507</td>
<td>442</td>
<td>365</td>
</tr>
</tbody>
</table>

**HIRE-PURCHASE**

(20% deposit, 7% interest added over 5 years). (1)

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net financing cost:</td>
<td>442</td>
<td>329</td>
<td>201</td>
<td>441</td>
<td>347</td>
<td>236</td>
</tr>
</tbody>
</table>

**LEASING**

(£260 p.a. premium until receipt by the lessor of the investment grant - one year delay assumed - then £220 p.a. for the balance of the 5 year primary period preceded by a cash refund of the one year excess premiums. Disposal proceeds in excess of 5% of asset cash cost are transferable to the lessee). (1)

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Basic cost:</td>
<td>453</td>
<td>316</td>
<td>165</td>
<td>414</td>
<td>299</td>
<td>164</td>
</tr>
<tr>
<td>add excess &amp; refund (netted)</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>less part disposal proceeds</td>
<td>22</td>
<td>22</td>
<td>22</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Net financing cost:</td>
<td>433</td>
<td>297</td>
<td>147</td>
<td>402</td>
<td>287</td>
<td>154</td>
</tr>
</tbody>
</table>

In these circumstances; outright purchase is seen to be the cheaper method of finance at low discount and low earning rates, with the cost advantage shifting to leasing as these are increased. Both forms of instalment debt are cheaper than outright purchase at high levels of discount, increasingly so as the earning rate is increased. Finally,

(1) Bowmaker Ltd.
it must be remarked that in some cases the differences in cost are not substantial, especially as between the two instalment debt schemes. It is not thought that this necessarily follows from the fact that the quotations have a common source. The size of the leasing premium will be seen to be fairly typical. It is reasonable to expect that competition between different forms of instalment debt will often cause broadly similar costs to accrue.

This matter of small difference in cost at once evokes a query as to the extent of the influence of residual capital earnings. It the data are recast to exclude such earnings we have:

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td>5%</td>
<td>10%</td>
<td>15%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>Net financing costs:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outright purchase</td>
<td>392</td>
<td>301</td>
<td>195</td>
<td>507</td>
<td>442</td>
</tr>
<tr>
<td>Hire-purchase</td>
<td>457</td>
<td>364</td>
<td>255</td>
<td>465</td>
<td>402</td>
</tr>
<tr>
<td>Leasing</td>
<td>491</td>
<td>425</td>
<td>352</td>
<td>460</td>
<td>417</td>
</tr>
</tbody>
</table>

Differences in cost now tend to be more marked at low discount rates. For the data now in illustration, it is doubtful whether the decision at high discount rates indicated by the Evaluation Tables is sufficiently emphatic to overcome any non-quantifiable considerations except perhaps as between outright purchase and instalment debt in general at low interest rates. Outright purchase enjoys a clear and significant cost advantage at low discount rates; which (for what it is worth) passes to hire-purchase rather than to leasing as the discount rate is increased, (1) except perhaps at low earning rates.

It is sensible to ask, what are the influences of changes in grant, capital allowance, disposal value and so on? Firstly, the influence of grant and capital allowances:

2. Net financing costs for merchandising equipment attracting 25% writing-down allowance only and estimated to have 10% disposal value at the end of 5 years. Hire-purchase terms: 25% deposit, 7% interest added. Leasing terms: £255 p.a. (2) with no disposal value enuring to the lessee.

---

(1) See pages 121 & 122.
(2) Mercantile Leasing.
<table>
<thead>
<tr>
<th></th>
<th>Discounted at:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2%</td>
<td>12%</td>
<td></td>
</tr>
<tr>
<td><strong>Earning rate:</strong></td>
<td>5% 10% 15% 5% 10% 15%</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Net financing costs:</strong></td>
<td>with residual capital earnings:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% 10% 15% 5% 10% 15%</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>without residual capital earnings:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5% 10% 15% 5% 10% 15%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outright purchase</td>
<td>512 445 365</td>
<td>613 565 509</td>
<td></td>
</tr>
<tr>
<td>Hire-purchase</td>
<td>531 429 312</td>
<td>508 418 316</td>
<td></td>
</tr>
<tr>
<td>Leasing</td>
<td>561 446 318</td>
<td>509 410 296</td>
<td></td>
</tr>
</tbody>
</table>

Where residual capital earnings accrue, the previously noted conclusions apply except that hire-purchase is an equal beneficiary with leasing as discount and earning rates accrue, and that the size of the cost differentials between outright purchase and instalment debt then becomes significant at all earning levels. (Indeed the non-competitiveness of outright purchase where high discount rates coincide with a nil grant/nil initial allowance situation is striking, irrespective of residual capital earnings). At low rates of discount there is little to choose between the financing methods unless residual capital earnings are excluded, when leasing clearly becomes disadvantageous. Exclusion of residual capital earnings does not much affect the implicit decision at high rates of interest, except that the emphasis of the decision is reduced. Hire-purchase becomes marginally more advantageous than leasing, whereas with residual capital earnings there was an (even slighter) advantage to leasing.

Any increase in the estimated disposal proceeds or annual premium will normally reduce the competitive posture of leasing insofar as the increase does not accrue to the lessee. Similarly, any increase in the rate of interest added will reduce the competitive posture of hire-purchase, especially at low discount rates and/or high earning rates. As has been noted in the discussion of cost trends in hire-purchase, deposit rates are not a very important element of hire-purchase costs except at high rates of both earning and discount rates.

The impact on relative competitiveness of changes in capital allowances
alone (i.e., grant remaining unchanged) is not easy to judge in view of the lack of statistical evidence of the impact of such changes on leasing premiums. On the not unlikely assumption that a reduction in such allowances would (because it increases the amount of the lessor's capital outstanding at a given point in time) induce an increase in premium, illustration (2) is modified to give the following hypothetical case:

3. Net financing costs as in illustration (2), but assuming 15% writing-down allowance and leasing premiums of £270 p.a.

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th>5%</th>
<th>12½%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td>5%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Net financing costs:

<table>
<thead>
<tr>
<th></th>
<th>with residual capital earnings:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outright purchase</td>
</tr>
<tr>
<td>Discounted at:</td>
<td>5%</td>
</tr>
<tr>
<td>Earning rate:</td>
<td>5%</td>
</tr>
</tbody>
</table>

There accrues a pattern of costs which is still very similar to that of illustration (2). (Not too much should be made of the increased disadvantage of leasing at low discount rates in view of the hypothetical nature of the data). Evidently the rate of writing-down allowance is not a particularly important element in financing decisions.

4. Net financing costs for vehicles attracting 30% initial allowance, 25% writing-down allowance and with 15% estimated disposal proceeds at the end of 5 years. Hire-purchase terms: 15% deposit, 8% interest added. Leasing terms: (1) £260 p.a. full disposal proceeds ensuring to the lessee.

<table>
<thead>
<tr>
<th>Discounted at:</th>
<th>5%</th>
<th>12½%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earning rate:</td>
<td>5%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Net financing costs:

<table>
<thead>
<tr>
<th></th>
<th>with residual capital earnings:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outright purchase</td>
</tr>
<tr>
<td>Discounted at:</td>
<td>5%</td>
</tr>
<tr>
<td>Earning rate:</td>
<td>5%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>without residual capital earnings:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Outright purchase</td>
</tr>
<tr>
<td>Discounted at:</td>
<td>5%</td>
</tr>
<tr>
<td>Earning rate:</td>
<td>5%</td>
</tr>
</tbody>
</table>

(1) Freight Transport Leasing Ltd.
Where the asset attracts initial allowance, there is the same trend towards cost advantages accruing to instalment debt at higher discount rates, but hire-purchase is quite noticeably cheaper than leasing. Moreover, instalment debt is marginally cheaper at low discount rates if a high earning rate accrues. If residual capital earnings are excluded, hire-purchase is still distinctly cheaper than outright purchase at high discount rates, but the same advantage accrues to leasing to a significantly less extent which reduces as the earning rate increases. Given the same exclusion, outright purchase is cheaper at low discount rates, but only marginally so compared with hire-purchase at high earning rates. In passing, it is of interest to note that the rather unusual ensuring of full disposal proceeds to the lessee roughly compensates for the cost reduction above 15% writing-down allowances which ensure to the two purchasing methods from the 25% allowance.

These examples are few in number, but it is argued that they clearly demonstrate the immediacy of comparison afforded by the Evaluation Tables, within the bounds of the set parameters. They demonstrate with strong unanimity that, as the discount rate (and to a less extent the earning rate) increases, cost advantages accrue to the use of instalment debt rather than outright purchase; the choice between hire-purchase and leasing then being determined by the particular terms of the alternative contract. Especially, much depends upon the extent to which leasing premiums reflect different writing-down allowances and the extent to which disposal proceeds ensure to the lessees. It is also clear that a great deal depends upon the existence of otherwise of residual capital earnings. Especially at high earning rates, leasing ceases to be competitive if such earnings are absent.

Using the data of Table G; if the firm enjoys earning rates in excess of 10%, then predictably (as a situation of financial cost leverage applies) a supplementary cost advantage accrues to borrowing the outright cash cost of the asset and using own funds on a residual capital earning basis to discharge annual interest costs and the end-of-period capital repayment. At low discount rates, this supplement is substantial - but we have seen that at low discount rates outright purchase is cheaper than instalment in almost any event. At high discount rates, the supplement is seldom sufficient to overcome the
advantage accruing to instalment debt under the same conditions of residual capital earnings.

Section 7. Conclusion

This prolonged review of the Evaluation Tables has elicited some moderately surprising aspects of the net costs of financing. It would be tedious in the extreme to present a full recapitulation; but special mention may perhaps be made of one or two items. The reduction in the cost of financing which accrues to increases in the level of corporate taxation is clearly to be seen. This is a matter which does not appear always to be recognised by non-financial management, who tend to regard such increases as totally iniquitous. The relatively low importance of the size of writing-down allowances (which always figure prominently in discussions on capital expenditure) is even less generally appreciated. This is so even when the earnings potential generated by the receipt of such allowances is taken into account - as it usually is not. The quite low influence of the discount factor on the various costs of instalment debt is to be noted, especially when compared with the importance of this factor in outright purchase. Nevertheless, the over-riding importance of the discount factor in choosing between financing methods was clearly demonstrated in the last Section. The generally low relative importance in hire-purchase evaluation of the initial deposit, compared with the rate of interest added, also deserves to be stressed. The concept of residual capital earnings is seen to be of vital importance to the financing decision, the more especially when due weight is given to the rate at which those earnings accrue. Thus, in leasing finance, this matter is more important than equivalent variations in the size of annual premiums. With comparatively few exceptions, \(^{(1)}\) little or no use is made in the literature of capital budgeting of a specific reinvestment rate, but the review and the Tables themselves demonstrate forcibly just how important are the implications of the reinvestment assumption to the financing decision. Taken in conjunction with the discount rate, reinvestment earnings opportunities are the primary cause \(^{(2)}\) of the

\(\text{(footnote continued on next page)}\)

(1) e.g., Pearson Hunt (16).

(2) If the earning rate is set to zero, a re-tabulation of the four
clear recommendation that, other things being equal, a combination of a high earning rate and a high discount rate can be advantageously exploited by the utilisation of instalment debt financing. Given that it is reasonable to expect, via some growth inspired price-earnings causative effect on the cost of ordinary capital, an inverse correlation of earning rate and discount rate; this recommendation is less applicable to the generality of firms than might at first be thought. But the cost advantage passes to instalment debt before the incidence of the highest earning rate of the highest discount rate - and a (say) 10% earning rate and a (say) 10% discount rate are neither incompatible nor beyond the capacity of very many firms. Given that quantification of the earning rate is not a difficult task, selection of a correct discount rate becomes all the more important.

"Other things being equal" - but they are not necessarily so, especially as concerns this selection. In particular the argument has so far proceeded on the assumption that the changes in the capital structure of the firm which are involved in basing financial decisions on pure cost advantages have no reciprocal implications for the quantification of the discount rate, or consequential implications for the other segment of the capital budgeting procedure - the investment decision. The assumption of a feasible separation of the investment and the financing decisions must now be more critically examined; and consideration of the choice of discount rate, and some suggestions as to a closer integration of the two segments, form the burden of Part Two of this thesis.

(continued from previous page) Illustrations of the previous Section reveals that at 12 1/2% discount the net financing costs of instalment debt are still below those of outright purchase except in the case of leasing costs in the 20% grant situation (illustration 1). In every case, the lowest financing cost is that of hire-purchase. As the earning rate is zero, the question of the inclusion of residual capital earnings is irrelevant.
PART TWO

FINANCE-INVESTMENT LINKED BUDGETING
CHAPTER FOUR
THE NATURE OF THE DISCOUNT FACTOR

Introduction: The Discount Factor

The review in Chapter Three of the salient features of the Evaluation Tables indicated the importance of the discount function in their general usage. The question at once arises: which discount factor should the practical user of the tables select in his evaluation? The choice of the wrong discount factor could lead to:

a) The outright selection of a wrong financing method or,
b) The secondary selection of a wrong financial method.

As was earlier observed, the discounted quantitative advantage of one method over another is, in many cases, very marginal - sometimes sufficiently so for other more subjective considerations to overcome such a small advantage. In such cases, the wrong choice of discount factor could easily either prevent such a contradiction being properly exercised or could permit the contradiction erroneously to succeed. A particular critical decision area in this connection is that of long-term planning of the capital mix: when the financing decision indicated by the Tables might seem to be in conflict with any such plan. So very much of current writing in the field of decision analysis is from the stand-point - implicit or explicit - of a predetermined long-term capital mix.\(^{(1)}\) In this there are considerable advantages for developing capital expenditure budgeting principles and procedures, as will be seen; and, judging solely from that favourable evidence which is quoted, some at any rate of the leading U.S. companies consciously adopt a certain capital mix posture; see for example the quotation from Standard Oil of Ohio on Page 284. Just how strong such a posture really is could be ascertained only by an audit of the additional gearing provided by leasing, uncancelled inter-company debt and short-term debt especially at times when balance sheets are not published: but a suspicion of dogmatic convenience and of uncritical acceptance of large company financial public relations statements attaches to the proposition that company

\(^{(1)}\) See Vancil (34), Chap.5. Also Kerrett & Sykes (19), Chap.4.4, although here there is ambiguity as to whether current or planned weighting is implied. See also Brigham & Smith (47).
practice is, or ought to be, to take up such a posture and embody it in long-term planning.

"The determination by management of the appropriate blend of fixed and contingent obligations is frequently referred to as leverage, and one can find examples in American industry that cover practically the full spectrum of possible combinations". (1)

To what extent, it may be asked, is that so-called 'determination' no more than an ex-post rationalisation of substantially uncontrolled accidents of financing? Especially in a 'stop-go' economy, with its attendant credit squeezes and expansions, a likely answer must be very much so. In such circumstances, 'ought to' so determine becomes an unrealistic proposition.

This proposition, then, may or may not be correct in principle (and it will be later contended that realistically it is true only in terms of a limiting debt capacity); but nevertheless, any departures from the argument for long-term capital structure plans must be based upon firm grounds of principle and material advantage. (2)

It must be obvious that there is a strong possibility of a 'circular relationship' in capital expenditure budgeting procedures. A series of decisions as to financing method may cumulatively have an effect upon the discount factor itself, necessitating a review of the underlying investment decisions. The case is discussed later in the form of a model of financial linkage in the budgeting of capital expenditure projects. This model is shown to be most complex and demanding at the computational level. Accordingly, it is supplanted by a sub-optimal procedural approach which, hopefully, is both capable of application at the practical accounting level and of sufficient accuracy to represent an improvement in capital budgeting practice.

(1) Vancil (34) p.25.
(2) Even if such plans are to be granted, they are not sacrosanct. Marginal infractions are inevitable, given a developing complex of credit facilities as both customer and financial institution improve the sophistication of their monetary affairs in the light of evolving economic policy and environment. See also Lerner & Carleton (67)

"...no single capital budget will be optimal under diverse external market circumstances."

An excellent resume of the 'circular relationship' is also given by Lerner & Carleton (66), Section 1.
Selection of the discount factor by the individual firm must first answer two questions:

1) What is the discount factor measured by?
2) How best can it be quantified?

The first question will be briefly answered now. The second question is one on which much debate has been generated, and there is considerable disbelief in the practical realism of that debate itself. There seem to be two main sets of problems involved:

2a) What are the separate components of the discount factor, and how is each to be quantified?
2b) Is there a reaction between the component costs as the proportion of each component within the total is varied?

It would be sterile to recapitulate the various stances adopted - and abandoned - by the several participants in this debate. But a future section of this thesis is occupied with a theoretical model relating to the investment-finance linkage referred to on the previous page. In the accompanying discussion, suggestions (not necessarily original) are advanced concerning Question 2a. Both in an easy transference between equity and debt; and in the use of the Evaluation Tables to assess that transfer; it is assumed that there is no such reaction as is hypothesised in Question 2b. (Although, as a result, the averaged overall discount factor must vary as the component proportions vary). Such an assumption is shown to be utterly basic both to the model and the subsequent procedural approach, and must therefore be justified in advance. This question is accordingly examined next after a consideration of the nature of the discount factor. Finally, in deference to the lack of real assurance which attaches to measures of the discount factor at the individual firm level; and to the fact that in an economy where some 80% of firms have no public quotation or reliable share-pricing mechanism, such measures are most frequently doubly approximate; a brief analysis is appended at the end of Part Two to see if any guidance can be given at industry rather than at the individual firm level.
Section 1. The Marginal Cost of Capital

"Although there may be disagreement about methods of calculating a firm's cost of capital, there is substantial agreement that the cost of capital is the rate at which a firm should discount future cash flows in order to determine their present value." (1)

About the only opposed viewpoint to this is that of Roberts (2), who prefers an external investment opportunity rate, or 'lending' rate. But as this can be shown to equate to the equity earnings yield of the lending company, in a perfect market (and Roberts himself so equates it), the supposed difference boils down to methods of calculating the cost of capital. A subsidiary argument of this thesis puts forward (3) what might be termed a reciprocal of Roberts' postulate - namely, that for certain types of basic investment which can wholly be financed by self-generating debt, appraisal by the appropriate borrowing rate is valid. Again this is a question of the calculation rather than the nature of the discount factor.

The basic argument for the use of cost of capital as the discount factor is somewhat platitudinous one of - a company has no money of its own; therefore it has to obtain this from investors and/or creditors; these 'people' will want a return on their money; therefore investments must be appraised in a manner which reveals their ability to yield that return as a minimum. (4) As this is not a text on the theory and practice of project appraisal, further discussion of the point seems unnecessary. Suffice it to say that, in the proposed calculation of the cost of equity capital, shareholder expectations (which must involve their investment opportunities) are involved. And in the financial decision process, the 'earnings rate' is taken to be a managerial investment opportunity rate.

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(1) Lorie & Savage (69)
(2) H.V. Roberts (80) - see also further discussion on Page 303 of this thesis.
(3) See Page 200 et seq.
(4) The cost of capital is thus that return which will persuade investors to leave their money with the firm; equivalent to saying that it is "...a discount rate with the property that an investment with a rate of profit above this rate will raise the value of the firm". Gordon (59), p.218
Strictly speaking, capital budgeting theory demands that the relevant discount rate is set by the marginal cost of capital—that the incremental project be assessed against the related incremental financing cost. This seems often to be ignored in the literature. Sometimes this is explicitly so:

"...it seems likely that for large firms the average cost function is quite flat, so that the effort of estimating the marginal cost may not be worthwhile." (1)

Solomon is equally specific (2) in considering changes in the marginal cost of capital due to marginal changes in gearing. In effect, his recommendation is to ignore the marginal cost curve, and use the average cost of capital.

Nevertheless, it is possible and necessary to conceive of a marginal cost of capital which is separate from the average cost. An attempt to illustrate such a concept is that of Weston. (3) It would be tedious to recapitulate the whole of his argument, which seeks to measure an incremental cost "reflecting the changes in capital structure introduced by the new issues of securities". Summarising his argument: two weighted average costs of capital are calculated:

1) the current cost; based on the current financial mix,

2) the prospective cost; based on the prospective financial mix, and in both cases the weights are the market values of the various capitals. As an example in the context of a 40% Corporation Tax:

(1) R.W. Johnson (65) p.16

(2) Solomon (30) p.88. This point is incorporated into the procedural approach by proposing a review of the appraisal rate, but generally without any re-appraisal process in view of the various uncertainties of the appraisal and budgeting processes.

(3) J.P. Weston (93)
## CURRENT

<table>
<thead>
<tr>
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<th>Book Value</th>
<th>Market Value</th>
<th>Yield</th>
<th>Cost</th>
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</thead>
<tbody>
<tr>
<td>8% Instalment Debt</td>
<td>£ 6,000</td>
<td>£ 6,000</td>
<td>4.8%</td>
<td>£ 256</td>
</tr>
<tr>
<td>6.5% 10-year Loan Stock</td>
<td>15,000</td>
<td>13,000</td>
<td>4.6%</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>£70,000</td>
<td>£142,000</td>
<td>6.2%</td>
<td>£ 8,719</td>
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## PROSPECTIVE

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<th>Book Value</th>
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<th>Yield</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>8% Instalment Debt</td>
<td>12,000</td>
<td>12,000</td>
<td>4.8%</td>
<td>384</td>
</tr>
<tr>
<td>6.5% 10-year Loan Stock</td>
<td>15,000</td>
<td>13,800</td>
<td>4.4%</td>
<td>607</td>
</tr>
<tr>
<td>5.5% Convert. Debentures</td>
<td>10,000</td>
<td>10,300</td>
<td>3.1%</td>
<td>314</td>
</tr>
<tr>
<td>Ordinary Share Capital</td>
<td>20,000</td>
<td>141,000</td>
<td>7.5%</td>
<td>10,575</td>
</tr>
<tr>
<td>Free Reserves</td>
<td>33,000</td>
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<td></td>
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<tr>
<td></td>
<td>£30,000</td>
<td>£177,100</td>
<td>6.7%</td>
<td>£11,880</td>
</tr>
</tbody>
</table>

(Note: the following suppositions are deemed to have been made by management:

1) The yield on instalment debt is the internal rate of return implicit in such contracts.
2) The Loan Stock yield is the redemption yield. A prospective easing of the structure of interest rates is foreseen.
3) The prospective convertible will command a slight premium because of anticipated growth in equity earnings.
4) Present earnings to book equity are 15% with a prospective 5% growth rate. Distributed earnings are thought to average 2 of earnings to equity. Prospective earnings are 20%, with an unchanged pay-out ratio and 20% capital gains tax adding 1% to the expected 5% gross earnings yield now currently accruing to the share of this class of company. Market yields generally are expected to improve as the present effects of a shortage of stock and the somewhat euphoric market conditions become less pronounced in the future. Retained earnings are assumed to have the same cost as new equity. A further important point is that the sharp increase in gearing (book value terms) from 52% to 70% of total equity...
is not in itself assumed to affect the cost of equity capital. (1)

Weston argues that the marginal cost of capital is:

<table>
<thead>
<tr>
<th></th>
<th>Market Value</th>
<th>Cost</th>
<th>% Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Projected:</td>
<td>£177,100</td>
<td>£11,680</td>
<td>6.7%</td>
</tr>
<tr>
<td>Current:</td>
<td>£142,000</td>
<td>£8,719</td>
<td>6.2%</td>
</tr>
<tr>
<td>Marginal:</td>
<td>£35,100</td>
<td>£3,161</td>
<td>9.4%</td>
</tr>
</tbody>
</table>

net of Corporation Tax.

The use of market values to establish the marginal cost is of course mathematically consistent. But is it meaningful? The extra investable capital is £20,000; not £35,100. The problem would not arise if book values were used as weights. But what is the relevance of market yield to book value? Book yields would have to be instituted; but if the objective is maximisation of shareholder wealth, book values are irrelevant. The argument can be put thus - to raise £20,000 in the manner indicated will 'cost' £3,161 per annum if the market value of the firm is not to be lower than that which now obtains plus adjustment for anticipated growth. On this basis, the marginal cost of capital is 15.8%.

There is yet another problem. At various points in this thesis, reference is made to the issue of 'discriminatory appraisal'; i.e., appraising a project at the cost of that particular segment of capital which will finance that project. Pending discussion of the matter, let it be postulated (as it generally is) that this is an erroneous procedure. Let us set against this Weston's dictum (2), which neatly epitomises the marginal approach: "For evaluating current opportunities, current cost (of capital) must be used". To what extent are these two

(1) This development of a prospective yield on ordinary is much more sophisticated than that actually presented by Weston, but it does seem to be what is required to render his generalisation effective at a practical level. 'Prospective' thus refers to a 5-year study period (anticipating some later assumptions of this thesis). The assumptions embodied in this development are discussed in some detail in a later part of the thesis, as is the assumption regarding the cost of retained earnings. Free reserves exclude all taxation reserves and debt redemption reserves. The independence of the cost of equity from changes in gearing is also discussed at a later stage.

(2) Weston, op.cit., p.83.
precepts in conflict? A simple model helps in the analysis.

\[ E_0 \] = the quantity of equity capital in use at the start of a budget period.
\[ e_0 \] = the cost thereof, however defined.

\[ E_1 \] = the incremental quantity of equity capital, determined by the budget in some way.
\[ e_1 \] = the cost of equity as a result of the budget and of managerial assumptions on the impact of that budget on shareholders expectations.

\[ L_0 \] = the quantity of all kind of loan capital at the start of the budget period.
\[ i_0 \] = the average servicing cost of that quantity.

\[ L_1 \] = the incremental quantity of loan capital, determined by the budget in some way. Also influenced by \( L_0 \), but not by \( E_0 \) and/or \( E_1 \).
\[ i_1 \] = the anticipated average servicing cost of that incremental quantity.

Notice that \( e_1 \) and \( i_1 \) are very different. Any change in shareholder expectations will attach to all equity - i.e.,

\[ (E_0 \cdot e_0 + E_1 \cdot e_1) \] is a nonsense. It must be \( (E_0 + E_1) e_1 \).

Thus the marginal cost of equity is:

\[
\frac{(E_0 + E_1) e_1 - E_0 \cdot e_0}{E_1}
\]

which is a 'current cost' and, because of the inclusion of \( E_0 \) in both elements of the numerator, is non-discriminatory.

But because the servicing costs of debt are the subject of specific contractual obligation for each tranche of debt,\(^{(1)}\) it is obligatory to write:

\[ L_0 \cdot i_0 + L_1 \cdot i_1 \]

\(^{(1)}\) This presupposes that the yield on existing debt is not affected by the yield on new debt. (It also begs the question as to how restrictive covenants on new debt - e.g., subordination - are quantified into the yield). The assumption is valid enough over that greater part of funded debt which is made up of private placings and banking instruments (see footnote (2) Page 174 ).

Again there is a reflection upon the unrealistic assumption made in so much of the literature of financial theory; that debt capital always is publicly traded. It is not - and so is often isolable in cost.
and the marginal cost of debt capital is:
\[
\frac{(L_0 \cdot i_0 + L_1 \cdot i_1)}{L_1} - L_0 \cdot i_0 = i_1
\]
which is discriminatory.

If there is assumed a predetermined fixed relationship, \( L_1 = f(E_1) \); and if the budget is for a number of projects; the discriminatory effect disappears as between this period and all other periods, and as between those projects. If there is no such fixed relationship - and doubts have been and will be expressed on this point - there is inter-budget period discrimination. (Thus in one period, all new projects may be appraised entirely at 'i': in another, entirely at 'e'; and so on). If there is no such relationship and only one project, then that project will be appraised discriminatorily.

Thus there are three problem areas:

1. Is the marginal cost of capital worth calculation?
2. How is it best calculated?
3. If it is worthy and capable of calculation, does its use imply a risk of erroneous procedures?

all of which of course presuppose that 'e' and 'i' can reasonably be quantified.

In the development of a procedural approach to a finance-investment linked budget, the marginal cost of capital is abandoned on grounds (1) and (2). As a proxy measure there is substituted a two-stage average cost, viz:

Stage 1: \( E_0 \cdot i_0 + L_0 \cdot i_0 \), subject to review and modification, \( E_0 + L_0 \)
to Stage 2: \( (E_0 + E_1) \cdot e_1 + L_0 \cdot i_0 + L_1 \cdot i_1 \)
\( E_0 + E_1 + L_2 + L_1 \)

The description of this as a 'proxy' measure is an admission that a marginal cost is theoretically preferable. This I believe to be the case, holding that the imperatives of marginal analysis must take priority over the possibility of discriminatory appraisal. That possibility exists because the relationship \( L_1 = f(E_1) \) is denied.
but it seems safe to say that the possibility is at best a remote one, so far as inter-project discrimination is concerned. So far as inter-period discrimination is concerned, this must surely be a very minor obstacle in view of the imperfect substitutability of periods - due to changes in economic climate, technology etc. - which even theoretical analysis must recognise.

In addition, dissatisfaction with the basic assumption that money in the future is worth less than in the present (the fundamental principle of the discounting process used in the models about to be examined); in view of the preference for future cash at some specific future time (the basic objective of cash budgeting); leads to speculation that liquidity is as important a managerial objective as profitability. If this is true - and the case is argued in the next Chapter(1) - then total company strategy replaces individual project profitability as the issue to be optimised, and the incremental project ceases to be all important. An average basis for appraisal seems, intuitively, to be a correct approach.

In the finance-investment linkage model, a marginal cost is used because this model is couched in terms of cash flow, not balance sheet values. The model is concerned to 'evaluate current opportunities in terms of current costs', to match cash outlays and inflows. The marginal cost used is:

$$\frac{(E_0 + E_1)\delta_1 + L_0 \cdot i_o + L_1 \cdot i_1}{E_1 + L_1} = \frac{E_0 \cdot \delta_o + L_0 \cdot i_o}{E_1 + L_1}$$

However, at a few points in the introduction to the model; and in the discussion of the constraints to the model: where insistence on the use of the (rather more complex) marginal concept would obscure the matter then in issue; an average cost (Stage 2) as above is used as a simple proxy.

(1) See Gordon (61) who demonstrates a good correlation of investment with liquidity as measured by funds flow, and a poor correlation of investment with profits in his sample drawn from the U.S. chemical industry. Gordon quotes Meyer & Kuh: "The Investment Decision: an Empirical Study" Harvard 1957, as reaching comparable conclusions. Gordon's argument is derived from the thesis that, as a managerial criterion for investment decisions, security is as important as profit. This would agree with the present concept of the servicing-adequacy of cash flow, but might disagree with the total exploitation thereof envisaged by the model developed in Chapter Five hereof.
Section 2. The Optimality of the Discount Factor.

Any discussion of the choice of the discount factor which is to be used in the application of the Tables must clearly bear in mind the purpose of the Tables. That is, that they are concerned with a FINANCIAL decision: they assume that a validly separable INVESTMENT decision - the decision to acquire the asset - is already and correctly made. "Correctly" must imply that the net operating earnings of the proposed asset exceed the cost of the asset by some 'adequate amount'. Hopefully, that 'adequate amount' will be a positive net present worth of sufficient size to compensate for any unduly high risk attached to the employment of the asset; either explicitly so, or implicitly in the form of a smaller positive net present worth derived after the application of sensitivity analysis to the expected cash inflows. (1)

In this connection, it is relevant to draw attention once more to the somewhat unusual nature of the Tables: namely, that these represent the discounted net capital cost of various financial alternatives. That is, cash flows arising from grants, capital allowances and disposal values; or from tax allowances and charges arising out of initial or subsequent revenue flow; need not (save in respect of operating COSTS) be taken into account when computing cash flows estimated to accrue to the proposed investment. A more or less simple discounted net cash inflow - the result of an offset of operating cost outlays against operating sales inflows; or of computed cost outlay reductions in a replacement situation; both adjusted as necessary for Corporation Tax - can directly be compared with a net discounted capital cost-cum-revenue implications complex. (2)

The cost of the investment is therefore the cash cost which the seller must receive from the buyer, either as a lump sum or in__

(1) Due allowance having been made for any reinvestment implications. See Appendix (11).
(2) A present worth, rather than a rate of comparison is clearly preferable in view of the fact that the capital cost and the derived revenue implications are not instantaneous but are themselves a stream of cash flows over a period of time. Further, it is reasonable to suppose that managerial attention will tend to be focussed upon instalment-debt finance mostly in periods of capital rationing; and it is now fairly well accepted that in this situation present worth offers significant advantages over rate of return.
instalments; directly or by the agency of an intermediate financial institution. If the investment seems profitable enough to proceed with, a second decision determines the financing method. At first sight, some of the implications of this statement could appear as heretical in the light of much of the currently-received body of opinion on capital budgeting.

As was noted earlier, the statement seemingly predicates an easy movement between debt and equity financing which is the very antithesis of long-term gearing plans. If this is so, I have earlier remarked upon both the short-term and long-term lack of reality in the presumption that there exists within the typical firm tightly considered plans as to what the gearing of the company's capital shall be. I confess to considerable scepticism on this point, both from experience and observation. Top financial management has to shift with the times, which tend to reflect a bewildering change in the inter-active complexity of reported profits, dividends and economic climate on share prices and equity yields, on interest rates and (last but not least) on individually tailored monetary facilities. There possibly are, for different circumstances, different broad relationships between debt and equity financing which are, prima facie, 'acceptable to the market' and others which are 'not acceptable to the market' in those circumstances; but the 'market' is many people and institutions in many places, and 'prima facie' is not very important if a powerful company or a strong-minded Chairman is determined to raise finance in a certain way. (2) (This does not underestimate the limitations implied.

(1) See for example the "Accountants Magazine" May 1968: article "Long-term Planning as a Management Tool" where 52% respondents to a questionnaire addressed to major companies on the use of budgets confessed neither to using them nor intending to use them; and out of the 48% users or intending users, 13% used only specific occasional budgets not including budgets of finance. Evidently 10% also of the 35% respondents now preparing regular financial budgets used specialised staff - so how many 'financial budgets' were mere forecasts of revenue cash flows rather than funds-raising plans?

(2) In the period 1953-64: 78% of all new equity and loan issues in the U.S. was composed of loan capital, ranging from a minimum of 72% in 1961 to 89% in 1963. Of these loan issues, nearly half (48%) were direct - i.e., private placings (minimum 34% maximum 67%). (Securities and Exchange Commission data, quoted Federal Reserve Bank of Cleveland 'Economic Review' March 1965). And of course, all instalment debt is essentially a private placing. So far as equity issues in the U.K. are concerned, see Herrett et al (21) Table 4.9 p.84: some 45% of issues for new money were by placings. The average size of such issues was much smaller than for public issues etc. and it is impossible to say whether the choice of placing as the issuing method was the suggestion of issuing house or client. Finally, it is hard to believe that the market conventions of New York or London are particularly relevant to the gigantic and individualistic fund-raising operations on the Euro-Dollar market: all of which are essentially direct placings.
in the Companies Acts or specified in many Articles of Association upon the borrowing powers of Directors). Under the pressure of Corporation Tax, the London market has been induced to accept an enormous increase in gearing ratios in the last three years, without so much as a murmur. It would be convenient for financial theory if one could with conviction argue that the relative conservatism of pre-1965 gearing ratios was a reflection of inadequate appreciation by U.K. shareholders of the principles of leverage (see footnote to Page 194); that the Corporation Tax-induced switch to loan stock has coincided with a new awareness of those principles—an awareness due in no small measure to the growing professionalism of shareholders; and that 1968 saw some reaction against all but the very best loan issues (and perhaps not even then) to the slight discount on the Rolls Royce 8½% debenture issue in June 1968 principally because this more professional market is now reaching a self-imposed limit of risk acceptance where by risk is meant financial risk evidenced by capital gearing.

This thesis is attractive, but not capable of proof. Traditional cost-of-capital analysis posits a U-shaped average cost-of-capital curve, with the 'upturn' of the curve principally the result of an increase in

(1) Approximately 25% for quoted companies (B.of T. Statistics in "Financial Statistics", H.M.S.O. March 1968) based on a sample of 1,720 quoted companies in manufacturing and distribution industries. See Page 179 of this present text for a definition of gearing. Using a measure of gearing based, effectively, on prior charges cover, Singh & Whittington (29) produce statistics which average out at 8.4% gearing for 1948-60 in respect of 354 companies in engineering, food, clothing and tobacco. In respect of the low geared companies used as Sample A. in this present text (pp. 176 et seq.); the ratio of gearing as defined in this text to gearing defined by prior charges cover is approximately 1.6 to 1.0. Applied to Singh & Whittington's data, this gives a gearing of 13.5% for their sample to compare with the 25% in the B.of T. sample.

(2) More accurately, to the growing investment importance of professional management of institutional funds: estimated at anywhere between 25% and 60% of typical company equity investment in the chemical and engineering industries. C.f. the estimate of 32% of all quoted industrial equities at the end of 1966, quoted in the Times, 18th. September 1968. As institutional investment tends to be selective in favour of growth companies, the % investment in companies in which such investment is to be found would exceed 32% on average.

(3) Loan capital issues in the first quarter of 1968 were the lowest in any quarter for the past four years; and for the fourth quarter of 1967 were the lowest for that quarter for the past three years. But it is debatable whether this does not more reflect a reluctance by industry to offer, rather than by the market to accept, new loan capital. Some recovery is discernible in the Midland Bank figures of new money issues in the second quarter of 1968 but it is noticeable that that same quarter saw a sharp upturn in new equity issues.
the equity yield as risk-aversive shareholders move out of an overly geared situation. Any casual observation of the London stock market over the last two years indicates that an economic freeze, a shortage of new issues and a shortage of current equity induced by capital gains tax; and more latterly take-over speculation; will in their different ways far outweigh such subtle considerations as the risk implications of capital structure. (1)

Section 3. A Statistical Test of Optimality

Something of this sort of overwhelming reaction to such cruder influences is moderately discernible in the following data. The argument is based upon the assumption that aversion to financial risk - as evidenced by shareholder reaction against immoderate gearing - is most to be expected in situations of some considerable business risk. While discussion of the consequent effect upon the cost of capital to the firm is usually conducted in terms of variations within different 'industries' - where the classification is almost always in terms of degree of business risk subsumed in technology or in market served - at one point of time; there seems to be no reason why the usual arguments advanced would not be valid for variations across different times for one set of 'industries'. (2)(3).

With the benefit of hindsight, we can express any one or any few years as containing more business risk than some other year or years. The increase in business risk will, generally, be some function of the change in economic climate between the relevant periods. We are not

(1) Gordon (61) (IV- Conclusions) accepts that one interpretation of a lack of significant negative correlation between his 'excess security variable' (a devised measure of gearing in excess of a norm) and investment is that "corporations do not behave as if they wish to maintain or restore some level of security considered satisfactory" (p.617).

(2) The classical presentation of this fundamental tenet of traditional cost-of-capital analysis is to be found in Schwartz (82). A useful summary containing specific argument on this point is to be found in Solomon (30) Chap.8. The relationship of the present argument to Modigliani & Miller, 'Proposition II' (74) is discussed later in this Section.

(3) A recent and not too dissimilar piece of empirical research implicitly using the same argument as is used above arrives at much the same sort of conclusions. C/f Samuels (81).
here concerned to argue whether or not different 'industries' will react more or less sharply to such changes; we simply hypothesize that, in a year of deflation, nearly all 'industries' are at greater business risk than in a year of reflation. 1964 was a year of transition, but with Bank Rate at 5% until the end of November, can reasonably be termed a low risk year, with industry still optimistic after the 1962 reflation policies. 1966-67 was plainly a difficult year for large sectors of industry - it seems superfluous to quote supportive data - and so can fairly be termed a high risk year. It is of considerable relevance to the ensuing argument that 1964 was the last year before the inducement for the use of loan capital was promoted by Corporation Tax, if we assume that about six months would be required before the lesson sank in and the necessary steps could be taken. By comparison 1966-67 was probably the year when managerial reaction to that inducement came into, as it were, full flower. Capital issues by quoted companies (industrial and commercial) were:

<table>
<thead>
<tr>
<th>Year</th>
<th>Ordy Shares</th>
<th>Pref. Shares</th>
<th>Loan Capital</th>
<th>Total</th>
<th>Loan as % of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1963</td>
<td>118.3</td>
<td>10.0</td>
<td>206.3</td>
<td>334.6</td>
<td>62%</td>
</tr>
<tr>
<td>1964</td>
<td>157.6</td>
<td>7.5</td>
<td>247.1</td>
<td>412.2</td>
<td>60%</td>
</tr>
<tr>
<td>1965</td>
<td>62.7</td>
<td>(14.3)</td>
<td>359.1</td>
<td>407.5</td>
<td>88%</td>
</tr>
<tr>
<td>1966</td>
<td>123.4</td>
<td>24.0</td>
<td>427.4</td>
<td>574.8</td>
<td>75%</td>
</tr>
<tr>
<td>1967</td>
<td>61.1</td>
<td>(2.4)</td>
<td>352.1</td>
<td>410.8</td>
<td>86%</td>
</tr>
</tbody>
</table>


It would seem appropriate to examine 1964 and 1966-67 data to discover shareholder reaction, as evidenced by ordinary share price movements, to the change in 'business risk' in these two years, for different capital structures. More specifically as the yield on ordinary shares is commonly accepted as at any rate one of the elements of cost of capital, it might be instructive to look more closely at changes in the earnings yield.

(The argument is not concerned with measuring changes in costs of capital as a function of different gearings. It is concerned with discovering if movements in one important element of the cost of
capital are sufficiently consistent with any hypothesis of that function to sustain belief that some more-or-less subtle cost of capital/gearing relationship does exist, of whatever nature. It therefore seems a mere sophistry to argue whether dividend yield or earnings yield should be used. This reminder of the quite imprecise purpose and nature of the analysis now to be carried out, is necessary in face of some of the more exciting conclusions it will seem - fallaciously - to permit to be drawn).

Two samples, each of 25 publicly-quoted companies (1), were drawn in respect of the financial year ending in 1964. (To the extent that for some of the companies, the accounting year ended variously between March and September; the periods could perhaps be better described as 1963-4 and 1966-7. A simple average gives a sample year end of August 1964 for sample A and January 1967 for sample B). (2)

Sample A consisted of 25 companies which were 'low-g geared' in 1964 but which had increased the gearing of their capital structure by December 1966. Sample B consisted of 25 companies which were already 'moderately' or 'heavily' geared in 1964 but which nevertheless had increased their gearing yet further by December 1966. In summary, each Sample contained:

<table>
<thead>
<tr>
<th>TABLE 2.</th>
<th>No. of Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample A.</td>
</tr>
<tr>
<td>Construction and Construction Supplies</td>
<td>5</td>
</tr>
<tr>
<td>Electrical and/or Mechanical Engineering</td>
<td>11</td>
</tr>
<tr>
<td>Rubber, Chemicals and Plastics</td>
<td>3</td>
</tr>
<tr>
<td>Paper and Textiles</td>
<td>1</td>
</tr>
<tr>
<td>Shoes and Clothing</td>
<td>-</td>
</tr>
<tr>
<td>Food, Drink and Tobacco</td>
<td>4</td>
</tr>
<tr>
<td>Services and Distribution</td>
<td>1</td>
</tr>
</tbody>
</table>

(*) Includes one company - Rank Organisation - with substantial engineering interests.

(1) Source: Moodies Investors Handbook. Details of the firms are given in Appendix IV.
(2) '1964': accounting years ending March-December 1964. Thus, some companies include 9 months of 1963 and three of 1964; etc. '1966': accounting years ending September 1966 - September 1967. The same delayed time effect accrues. This period was chosen because the Index of Manufacturing Production began to fall away in the last quarter of 1966. The Times Industrial Share Index was at a four-year low in November 1966, and had improved but little at February 1967.
'Gearing' was measured by the % of preference capital plus long-term loan stock to 'equity' - i.e., ordinary capital plus capital and free revenue reserves. Bank overdrafts; and taxation reserves or deferred liabilities; were not included. The frequency of scrip issues in the period indicated that distributable reserves could be regarded as tantamount to ordinary shares, and so must be included in the denominator of the gearing ratio. 'Low geared' was taken as a capital structure containing 20% or less gearing as defined.

'% Earning to Ordinary' was adjusted for scrip or other issues as necessary. Share prices (an average of the years low and high prices) and 'net assets per ordinary share' were adjusted so as to be capable of expression in terms of a standard 10/- share; this was done so that comparable % increases or decreases could be deduced, and so as to be able to compare 'earning yields'. The 'per firm' averages are simple, unweighted data. (1) It is not claimed that the Samples used are random - indeed, the selection criteria involved some rather laborious searches; but it is claimed that the twin criteria of a specific gearing range in 1964 and a significant increase therein by December 1966 so reduced the population of acceptable companies that the Samples are quite representative for the restricted purposes on hand. Upwards of 500 companies were examined. About 100 satisfied the requirements of category A, and about 80 the requirements of category B. The balance were largely 'low-g geared' companies in both periods - having typically between 10% and 20% gearing; with relatively few 'moderately' or 'high' geared companies which either remained unchanged or in some few cases reduced their gearing between the two periods. It was most discernible that companies with substantial gearing in the earlier period apparently found it easier or more internally acceptable to increase that gearing in the second period.

(1) That is, data for each firm is given equal weighting regardless of the size of the firm, as a conscious decision of analytical method. The reminder on Page 177 of the purpose of the exercise is recalled. We are concerned to observe a result which will indirectly reflect the number of times a thing happens, rather than by how much it happens. Nevertheless, some bias in the comparisons is still possible; given that there is an inverse and significant correlation between size of firm and share price variability. See Haskel Benishay (41).
The results of the analysis are given in the following Tables:

**TABLE 3a**

<table>
<thead>
<tr>
<th>Sample</th>
<th>Equity</th>
<th>Prior Capital</th>
<th>Total</th>
<th>Gearing</th>
<th>Equity</th>
<th>Prior Capital</th>
<th>Total</th>
<th>Gearing</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(25 firms)</td>
<td>£40.3</td>
<td>£4.3</td>
<td>£44.6</td>
<td>9.7%</td>
<td>£46.7</td>
<td>£14.0</td>
<td>£60.7</td>
<td>34.5%</td>
</tr>
<tr>
<td>B(25 firms)</td>
<td>29.3</td>
<td>11.4</td>
<td>40.7</td>
<td>42.1%</td>
<td>35.2</td>
<td>22.7</td>
<td>57.9</td>
<td>69.2%</td>
</tr>
<tr>
<td>B adj.</td>
<td>30.2</td>
<td>11.9</td>
<td>42.1</td>
<td>46.6%</td>
<td>36.2</td>
<td>23.5</td>
<td>59.7</td>
<td>70.2%</td>
</tr>
</tbody>
</table>

Notes:
1. Sample B contained 2 firms - Butlins and Thorn Electric - whose per share earnings were very significantly in excess of other companies in the Sample, and whose share prices commanded exceptionally high premiums over the underlying asset values. An adjustment, excluding these two firms, is therefore made to Sample B.
2. £ = millions of £'s.
3. Gearing is an unweighted average of the individual gearing percentage of each firm in the Sample.
4. Fortuitously, the financial size of the average firm in all Samples is much the same; and, contrary to what might be expected, the dispersions of the two principal Samples are not very different. The coefficients of variation are, 
   \[ \frac{S_a}{\bar{x}_a} \quad \frac{S_b}{\bar{x}_b} \]

<table>
<thead>
<tr>
<th></th>
<th>1964</th>
<th>1966/67</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample A</td>
<td>1.04</td>
<td>0.96</td>
</tr>
<tr>
<td>Sample B</td>
<td>1.15</td>
<td>1.04</td>
</tr>
</tbody>
</table>

(The adjustment to Sample B does not materially alter the coefficients).
Only in respect of comparative 1966/7 gearings are the dispersions significantly different. (F-tests on size for both years and on gearing for 1964 are significant only beyond the 10% level). The significant difference in the 1964 gearing dispersion is not unexpected. It is a function of the sharp contraction in Sample B in the 1966/7 gearing, which induces speculation as to whether there is not some (very high) upper level beyond which firms are either not willing or not able to proceed; such as the point fixed by regulation of directors' borrowing powers. This point becomes fundamental in the later discussion.

Comment

The increase in the financial size in both Samples is around £16-17 million; composed of some £6 million equity and £10-11 million prior capital, very largely long-term debt. The proportion of the increase in total capital made up of long-term debt (62%) is somewhat less than that for industrial and commercial quoted companies as a whole (72% of the increase in 1966/7 - See Page 177). It is still something of the same order, and so seems reasonably representative. The average increase in gearing in both Samples (24% in Sample A and 27% in Sample B) is satisfyingly similar. The extent of the increase in gearing when taken in conjunction with the smallness of the increase in the equity capital base is surprising; not so much from the point of view of the companies involved as from the evident unperturbed acquiescence of the shareholders.

(1) Four companies in Sample A - Carrington & Dewhurst; Ross Group; Albright & Wilson; Delta Metal - increased significantly their preference capital. In both Samples, two-thirds of the increase in equity consisted of increases in retained profits.
## TABLE 3b

**SHARE PRICES, EARNINGS & ASSET SUPPORT, 1964 & 1966/67.**

*Average of 50 U.K. Industrial Companies.*

<table>
<thead>
<tr>
<th>Sample</th>
<th>Share Price</th>
<th>Earnings to Ord.</th>
<th>Asset Val. of Share</th>
<th>Price/Asset Ratio</th>
<th>Share Price</th>
<th>Earnings to Ord.</th>
<th>Asset Val. of Share</th>
<th>Price/Asset Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>29/5½d</td>
<td>20.0%</td>
<td>17/8d</td>
<td>1.7</td>
<td>26/2½d</td>
<td>20.7%</td>
<td>20/9d</td>
<td>1.3</td>
</tr>
<tr>
<td>B</td>
<td>42/11½</td>
<td>27.5%</td>
<td>22/5½</td>
<td>1.9*</td>
<td>38/2</td>
<td>22.3%</td>
<td>25/1</td>
<td>1.5</td>
</tr>
<tr>
<td>Badj</td>
<td>34/4</td>
<td>24.6%</td>
<td>21/10</td>
<td>1.6</td>
<td>31/11</td>
<td>18.3%</td>
<td>24/-½</td>
<td>1.3</td>
</tr>
</tbody>
</table>

(* c/f note (1) to Table 3a*)

**Notes.**

1. Prices and asset values are in terms of a standardised share of 10/-d. par value.
2. Asset values are at balance sheet values, after full satisfaction of prior claims and are accordingly subject to accounting valuation conventions.

**Comment**

The average fall in share prices (Oct.1966/Sept.1967 compared with Jan./Dec.1964) is reported as:

- F.T. - Actuaries '500 Industrial': a fall of 6%
- F.T. - Index of Ordinary Shares: a fall of 4.5%
- 'Times' Index of Industrial Ordinary Shares - (Large coys.): a fall of 7.3%

which may be averagely expressed as a fall of 6%. The fall in the price of shares in Sample A at 11.5% is almost twice this 'national average' and that for Sample B (unadjusted) at 15% is similarly high; but for Sample B (adjusted) the fall of 7% is quite close to this 'average'.

In some sense, the fall of 6% in share prices can be said to reflect in considerable measure the sharp reaction in shareholder
expectations to the freeze of the Autumn of 1966. This reaction was soon overtaken (from Sept. 1967) by the part-euphoric, part-stock-shortage induced price rise which has continued since that latter date. It would therefore seem reasonable to ascribe this amount of fall in share price to factors quite unconnected with the individual firms which make up Samples A. and B. In the next Table, these are corrected accordingly.

If Sample B is compared with Sample A, there is some mild evidence of a leverage effect in the data for 'Earnings to Ordinary'. This is positive for 1964, and (as would be expected in a difficult year) negative for 1966/7 - negative, that is, in the sense of a 15% reduction in earnings to ordinary for Sample B (25% reduction for Sample B. adj.); compared with a small (3%) increase for Sample A. This appearance of financial risk is opportune for this argument.

The price-asset ratios for Sample A and B. adj. are in accordance with general expectations for U.K. industry; being somewhat low for 1966/7 as a result of a general price depression noted above. The absolute increase in underlying asset values is strikingly similar across all Samples, but is in excess of the average growth of assets per company indicated by the B.O.T. samples of around 1900 companies.

The price-asset ratios for Sample A and B. adj. are in accordance with general expectations for U.K. industry; being somewhat low for 1966/7 as a result of a general price depression noted above. The absolute increase in underlying asset values is strikingly similar across all Samples, but is in excess of the average growth of assets per company indicated by the B.O.T. samples of around 1900 companies.

TABLE 4.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sample A.</td>
<td>Sample B.</td>
</tr>
<tr>
<td></td>
<td>1964</td>
<td>1966/7</td>
</tr>
<tr>
<td>Gearing</td>
<td>9.7%</td>
<td>34.5%</td>
</tr>
<tr>
<td>Share Price</td>
<td>29/5½d</td>
<td>27/9d*</td>
</tr>
<tr>
<td>Earnings Yield</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Actual</td>
<td>6.8%</td>
<td>7.9%</td>
</tr>
<tr>
<td>-Adjusted</td>
<td>7.5%</td>
<td>7.5%</td>
</tr>
</tbody>
</table>

(* 1966/7 Share prices adjusted by 6% general market reaction factor).

(1) The F.T.- Actuaries '500 Industrial' Index reveals a 1½% increase in the earnings yield on Ordinary shares in the same period. The F.T. Index of Ordinary Industrial shares gives a 1½% increase in the same yield for that period. At any rate, the price adjustment was excessive in terms of earnings results, and must have reflected a basic market readjustment to the national economic situation.
The change in earnings yield as adjusted for the reaction factor was:

Sample A: an increase of approximately 10%.
Sample B: a decrease of approximately 14%.
Sample B': a decrease of approximately 25%.

all three results being against absolute increases in gearing of approximately the same extent.

(Over the same period, and making the same market reaction adjustment, the 'national' earnings yield varied between a 7% increase (F.T. - Actuaries 500 Index) and a 5% decrease (F.T. Ordinary Share Prices Index): a very inconclusive situation. The nature and size of the Sample firms lends intuitive support to a belief that the F.T. - Actuaries 500 is here the more comparable index, but no immediate indication of the change, if any, in gearing of the firms comprised in the index is available. Some rather specious manipulation of the B.O.T. data published in the Tables "Balance Sheet Summaries of Quoted Industrial Companies", "Appropriation Accounts of Quoted Industrial Companies" and "Capital Issues" in the January 1967 and April 1969 issues of 'Financial Statistics' indicates that by the end of 1966, gearing had increased 'nationally' by something like 8-11%. This of course allows for the very large number of industrial companies which still are not significantly, if at all, geared: and is measured at too early a date for the full increase in gearing - as the Balance Sheets of many companies would still be outstanding for the purposes of these statistics).

Inasmuch as both Samples A and B are mixed in terms of 'Industries' (however defined); it may be pertinent to ask whether the heterogeneity of the one is so vastly different from that of the other, that in this lies the cause of the difference in the movements of earnings yields. Intuition and formal theory unite in ascribing any such causation to one principal factor - risk.(1)

The 'industry' composition of the Samples is set out in Table 2. Casual inspection reveals that the only real differences between them lie in the greater 'engineering' content of Sample A and the greater 'shoes and clothing' and 'services and distribution' content of Sample B.

(1) Size of firm is ruled out by footnote 4 to Table 3a.
Examination of the individual firms in each Sample throws up the presence in Sample B of three 'engineering' firms which might be said to be exceptionally exposed to the vagaries of the capital goods market - AEI, Cammell Laird and Sheepbridge Engineering - compared with one such firm (Hawker Siddeley) in Sample A. But are EHI and Thorn (Sample B) more or less risk-free than Parkinson-Cowan and Plessey (Sample A)? The generally lower rate of return (earnings before interest and taxation) on net assets discovered by Singh & Whittington (op.cit., Table 240) for the 'clothing and footwear' industry 1954-60: together with the wider dispersion of those results; both compared with several other industries and a general industrial average, indicates a tentative conclusion that a slightly higher risk element might attach to Sample B accordingly. (But Table 2.13b in the same work implies that the market did not necessarily agree in 1960 as the market value/book value 'valuation ratio' has increased relative to the 'all industry' figure). Yet, taking into account the probable variability of earnings of such 'service' firms as Butlins and Rank, there is still a predisposition to believe that Sample B is slightly the more risky.

Some further support - albeit not too reliable - to the conclusions so far derived is given by the application of 'risk factors' developed by Econtel Ltd. (1) Based on nearly 600 U.K. quoted companies, these are calculated from:

let $r_j$ = average annual GROWTH in 'earnings yield to Ordinary' for a sample of firms in industry 'j' (defined principally by product or market) realised in 1960-65.

let $\bar{r}$ = the same, but for the "total population" of 600 firms.

let $q_j$ = average annual LEVEL of 'earnings yield' for a sample of firms within industry 'j' realised in 1955-60.

let $\bar{q}$ = the same, but for the "total population" of 600 firms.

let $Q_j$ = the required average annual LEVEL of 'earnings yield' for 1955-60 in industry 'j'; where:

$$Q_j \cdot r_j = \bar{q} \cdot \bar{r}$$

(1) "Risk Factors in British Industry" (Factual Series No. 3); ECONTEL RESEARCH LTD. London 1967. The factors range from 0.5 - 2.5 approximately; with a mean of 1.0.
i.e., the **required** yield is that which, with the benefit of hindsight of **realised** growth in yields in the industry during 1960-65, ought to have applied during 1955-60 if 'overall 600 average performance' were to be achieved in 1960-65. The explicit use of this '600 average' as a touch-stone is that 'very safe' and 'very risky' industries will thus tend to cancel out somewhat, so that the average is representative of a sort of norm.

then $Z_j$ = the 'risk factor' for industry 'j' given by

\[
\frac{q_j}{q^*_j} = \frac{\text{actual 1955-60 yield}}{\text{required 1955-60 yield}}
\]

If $Z > 1.0$, then in 1955-60, either estimates of 1960-65 growth rates were too low ('error') or, if there was no 'error' were discounted by some risk factor attached peculiarly to the industry. Essentially, for such industries, ordinary shares were *on average* under-valued in 1955-60. An opposite and symmetrical interpretation applies if $Z < 1.0$. It is argued that wrong estimates or 'error' could apply only to the individual firm and that by restricting the risk factors to the average of several firms in an industry, 'errors' will tend to cancel out, leaving $Z$ as a fair measure of industrial risk. (Econtel recognises in several instances that this just does not appear to have accrued. They also point out that $Z$ will have different values if calculated as between other periods - so that the present application to 1964 and 1966-67 data is suspect).

However imprecise this measure may be, it somewhat uniquely attempts to measure the extent to which industrial risk modifies expectations of earnings, and so is clearly within the context of the present argument. Accordingly, it is applied to Samples A and B. in the following:
### TABLE 5.

**INTER-SAMPLE RISK COMPARISON.**

<table>
<thead>
<tr>
<th>Industry</th>
<th>'Econtel' Risk Factor</th>
<th>Sample A</th>
<th>Sample B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construction &amp; Construction Supplies</td>
<td>1.20</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Food</td>
<td>1.11</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Engineering</td>
<td>0.90</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>Rubber</td>
<td>1.25</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Chemicals</td>
<td>0.75</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Paper</td>
<td>0.76</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Textiles</td>
<td>1.37</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Footwear</td>
<td>0.94</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Entertainment - Cinema</td>
<td>1.48</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Entertainment - General</td>
<td>2.13</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Transport</td>
<td>1.07</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Stores</td>
<td>2.49</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(Classification of firms to an 'industry' is identical with the classification used by 'Econtel' for the same firms).

**AVERAGE 'RISK':** 0.98 1.06

**STANDARD DEVIATION of Samples:** 0.194 0.255

est. POPULATION STD.DEV.

\[
\frac{S_a(N_a - 1) + S_b(N_b - 1)}{(N_a + N_b - 2)} = 0.224
\]

**SIGNIFICANCE**

\[
t = \frac{A - B}{\sqrt{\frac{1}{N_a} + \frac{1}{N_b}}}
\]

\[
t = 0.08 = 5.56
\]

\[
0.0144
\]

which is significant at the 0.1% level.
VARIANCE

\[ F = \frac{S_B^2}{S_a^2} \]

\[ F = 0.065 \times \frac{1.75}{0.037} \]

which is significant at the 10% level only

On average, Sample B is the more risky; and from the 't' - test, very significantly so. Yet 'B' is the Sample for which yield reduces on an increase in gearing. There is a fair presumption that the observed movements in earnings yield are not caused by differences in risk between the Samples.

In detail, these movements are: an increase in earnings yield when gearing increases from a small to a moderate level, and a decrease in yield when gearing moves from a moderate to a very high level. Does this conform to any established theory of the affect of gearing on the cost of capital?

Section 4. The "Equivalent Risk Class" Assumption

It certainly does not conform to the 'traditional' theory of a U-shaped equity yield curve. The observed curve is T-shaped and as such conforms to the logical implications, at higher levels of gearing, to the Modigliani-Miller "Proposition II", viz:

"The expected yield of a share of stock is equal to the appropriate capitalisation rate 'k' for a pure equity stream in the relevant class, plus a premium related to financial risk equal to the debt-to-equity ratio times the spread between 'k' and the rate of interest on loan stock" (1)

This can be expressed as:

\[ e = k + (1-t)(k-r) \frac{D}{E} \]

(1) Modigliani & Miller (74) Subsequent argument is based also on the same authors, (77) and (78).
$e = \text{earnings yield to equity}$

$k = \text{a capitalisation rate of a mixed uncertain equity earning stream plus a certain debt earning stream}$

$t = \text{rate of company taxation}$

$r = \text{capitalisation rate of a certain debt earning stream}$

$L = \text{market value of the company's loan capital}$

$E = \text{market value of the company's equity}$

(A brief commentary on this expression, relating especially to 'k', appears in Appendix I).

Several writers\(^{(1)}\) have drawn attention to the possibility that, according to this formulation it is possible for the earnings yield first to increase as gearing increases and then to decrease as gearing becomes so great that a correspondingly high rate of interest payable on such a large quantity of loan stock turns the fraction $(k-r)$ into a negative quantity. Robichek & Myers contend that in practice this is not possible, as it would involve a marginal earnings yield on debt in excess of the marginal earnings yield on equity: which they conceive to be impossible given that debt, in addition, will even at the margin enjoy the security of prior rights over equity. But even given the trade-off between future capital gain and current income that a high level of personal taxation engenders, a reverse yield of this nature is not at all impossible. The mean redemption yield on industrial equities has on several occasions during the frenetic increase in share prices in 1968-69 fallen below that of first-class industrial debentures.\(^{(2)}\)

On the other hand, it is not at all clear that in fact interest rates do increase as additional amounts of loan capital are sought.

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\(^{(1)}\) e.g., Solomon, "Leverage and the Cost of Capital" Journal of Finance, May 1963; Robichek & Myers (25); Bargas (4).

\(^{(2)}\) e.g., On 10th December 1968, the Times Index reported a 4.68% earnings yield on ordinary shares and a 7.12% flat yield on debentures. Given 30% capital gains tax, this implies a prospective 75% increase in the general level of share prices before the combined income yield and capital gain on ordinary is equal to the flat yield on debentures. It is significant that the "Sunday Times" of the 15th December was able to report a sample of 15 convertible loan stocks which offered a cheaper way into equities than by open share purchases.
Any empirical investigation would have to make some quantifying estimate of the effect on 'true' interest costs of alterations in priority between different tranches of loan capital, of restrictions on yet further capital-raising operations specifically attached to the present new instalment etc. But this apart, a casual examination of nominal interest rates attaching to the several incremental loan capitals raised by a dozen highly-gearied major U.K. companies between 1962-1967 indicated that the increase in coupon rates for nine of those companies was less (and in several cases considerably less) than the increase in the market yield on 20-year bonds in the same period. Insofar as that yield reflects medium term interest rate changes in the economy as a whole, unrelated to any changing risk pattern in an individual company; for these companies, 'i' was not an increasing function of 'L'.

It is more to the point that, as 'k' is an average capitalisation rate, one of whose constituents is 'i'; then there must arise the paradox of a sharp increase in the value of 'i' accruing to a small enough base of 'L' such that 'k' is not much increased: for

\[ k = (w,e,w,i); \]
\[ w = (i,L). \]

Nevertheless it must be asked whether the foregoing Sample analysis does not in fact demonstrate that the equity earnings yield can reduce, as an adjusted-for-tax Proposition II might seem to indicate. The answer must surely be - not in terms of that Proposition. In developing the argument which leads to Proposition II, Miller & Modigliani say(1): "We shall assume that firms can be divided into equivalent return classes, such that the return on the shares issued by any firm in any given class is proportional to the return on the shares issued by any other firm in the same class. By taking the ratio of the return to the expected return, the probability distribution of that ratio is identical for all shares in the class"

---

The "equivalent class" concept is thus basic to Proposition II and is defined in terms of profits, not in terms of technology, of products or of markets. An "equivalent class" is one in which all the constituent firms enjoy in common (1) the same probability distribution of values for the coefficient:

\[
\text{Achieved return for total assets} = \text{Expected return from total assets}
\]

In the nature of market preference for the larger company (2) some modification in terms of the size of the return is necessary; the more so in that the expected return is the mean of a probability distribution, and that the uncertainty implicit in the relationship of an actual parameter value to a narrow range of probable values of that parameter is very different from the situation where the range of probable values is wide - even though the distribution means be identical.

It is thus possible to borrow from Barges (3) the concept of a coefficient:

\[
\text{standard deviation of achieved from average expected return to total assets} = \text{average expected return to total assets}
\]

and stipulate an "equivalent class" as in one in which all constituent firms enjoy a probability distribution of values for this coefficient of variability of earnings in common. (4) But as this measure defies

(1) And, as expectations change, presumably both at any one moment and for all time, I confess to scepticism as to the existence of such a degree of continuing uniformity in mass money market psychology; but on the other hand this is one of several points on which Miller & Modigliani are less than chrysalid clear. C/f their "reply" ( 75 ) p.285.

(2) Benishay ( 41 ). The point here is that the larger company is expected to have the more stable income in terms of dispersion about a mean.

(3) Barges ( 47 ) Chap.2.

(4) Barges further argues that, as actual equity earnings are exposed to greater variability about a mean in a highly geared situation, the "equivalent class" must employ a fairly high homogeneity of capital structure in its member firms. But in the paragraph prior to that quoted above, Miller & Modigliani say:

"...the uncertainty (of expectation) attaches to the mean value over time of the stream of profits and should not be confused with variability over time of the successive elements of that stream.... the elements of a stream can be variable even though known with certainty."

Once again the authors are less than clear; but it seems to be implied that they regard this capital-structure effect as one of quantifiable risk within the total range of uncertain total asset income - "the effect of variability......is at best a second-order one".
really meaningful quantification so long as the only method of evaluating expectations is to extrapolate (however subject to qualification) from those same achieved results to which they are to be related; a proxy measure must be sought. Given an emphasis on total asset return and on the uncertainty which must attach to forward projections of that return; (and to the equity earnings derivable from that return, which Miller & Modigliani seem to see as risk capable of being mentally quantified by the shareholder as secondary derivatives from his primary uncertain expectations of total asset earnings); the most approximate measure must be that of 'industry', as reflecting the business environmental risk which must be a large element in profit uncertainties. The industry classification used by Miller & Modigliani, and of others after them is seldom really clear as between product, technology or market - but this is perhaps endemic in so nebulous a concept. My own preference would be in terms of market, inasmuch as this must be the ultimate determinant of just what risk a business faces.

This translation of the definition of "equivalent class" from profit terms to industry terms has been accepted, generally, in a remarkably uncritical way. One of the features of empirical work in this field is the disappointing lack of clear cut, statistically highly significant support or repudiation of the various arguments. The a priori conclusion that in the main investors 'buy' individual companies and managements rather than economic agglomerations often seems inescapable; and lends support to the concept of a class of firms correlated by profit potential with little or no regard as to what they make, how they make it, or to whom they sell it. More sophisticatedly, the substitution of industrial for profit-potential classification is defended in terms of the supposed common variability of earnings (total or equity) to be found within an industry.

(1) The point is well illustrated in the previous footnote.
(2) See, for example, R.W. Johnson (65). For one hesitant moment in mid-argument Johnson queries his own usage. By contrast, J.F. Weston is well aware of the limitations implicit in this assumption - see (92).
Thus, Wipern (1):

"The assumption that firms within the same industry are subject to equivalent degrees of basic business uncertainty is widely employed in financial theory and underlies many of the approaches to valuation in the security analysis literature. The principal area in which this assumption is used, and is of major importance, is the study of the effects of capital structure on the cost of capital.... A measure of the cyclical variability of past earnings is the most widely-used basis from which influences are drawn regarding the uncertainty of the receipt of future earnings."

Over fairly small samples and using analysis of variance techniques, Wipern does not find statistically significant different earnings variability as between the eight industries sampled by him; industries being classified on a product basis. He concludes:

".....industry groups do not provide an adequate basis on which to insure (sic) homogeneity of basic business uncertainty".

The samples used in this present text certainly do not enjoy a Barges-type homogeneity of capital structure. The coefficients of variation of gearing on Page 180, while not excessive, are surely high enough to demonstrate that even for Sample 3 in 1966-67 this condition is not sufficiently well met. And examination, albeit by inspection only, of earning variability indicates that the Samples are not "equivalent classes" on that basis. There remains the question: are the Samples "equivalent classes" by the test of the Bonteel risk factors?

It can be seen from Table 5 that the 't'-test indicates that the Samples are not from the same 'risk populations'; they are not all from...

1) R.F.Wipern (95): who lists Hirschleifer & Markowitz in support of his contention concerning earnings variability. To these could be added Gordon(59,60) and Benishay (41). It is of interest to note that immediately after the passage quoted, Wipern comments that the degree of total earnings variability is often a constraint upon the amount of gearing a firm can safely undertake; and that a principal undesirable effect of financial leverage is that it increases the potential variability of equity earnings. This is essentially Barges point in note (4) Page 191 - and indeed one suspects Wipern of having Barges in mind. But, if the cost of capital/capital gearing relationship is a function of "equivalent (industrial classes"; yet those same classes are a function of gearing: this argument is circular.
one common "equivalent class". The 'F'-test indicates however that the two estimates of variance are not significantly different, so that bias from range of risk does not exist.

It seems reasonable to conclude that the samples, not being of the same class, can not be said to support or be explained by "Proposition II". In the context of this argument it would be illogical if they could, for the fundamental mechanism of "Proposition II" is stockholder arbitrage in response to changes in financial risk. It is precisely this sensitivity which is now in doubt. The U.K. stock market does not seem sufficiently sophisticated so far as non-institutional investors are concerned: (1) and the potentially greater awareness of the professional institutional investor is swamped by the cruder impact of the prices and incomes policy, devaluation, capital gains tax and a shortage of stock.


Section 5. The Limits to Debt Financing.

At any rate for the present, longer-term plans of capital structure evidently need to be but little influenced by considerations of the effect of that structure on the cost of capital. It follows that switching between debt and equity financing of investments (as indicated to be optimal be the Tables), will not offend on that score. But - assuming for the sake of the argument, a prolonged series of debt financings - does longer-term capital planning not so much prohibit as limit the easy switching from equity to debt? And is that limit imposed, not so much because of its impact on the cost of capital, but because of institutional constraints which are a function of security considerations?

(1) Else why two articles on the effect of gearing on equity earnings - 'Investors Chronicle', 16th. February and 1st. March 1968? And the recent controversy between Sir Frank Kearton and the Chairman of the British Insurance Association (over the part played by institutional investors in the Courtauld merger activities of recent months) indicates scepticism in some quarters concerning the efficiency and awareness of even professional investment managers.

* mid-1968
The principal constitutional restraint is embodied in many sets of Articles of Association, consisting of restrictions on the borrowing powers of Directors to (some) multiple of the total of issued capital plus free reserves. (Failing such a specific restraint, the 1948 Companies Act imposes a limitation of equality of borrowing papers and issued capital plus reserves). It is noteworthy that the inclusion of this restraint often seems quite automatic, with no real appreciation of the potential benefits as well as of the potential dangers of gearing.

Following Donaldson, (1) we can also note the constraint of the capacity to bear incremental cash flows; regarding the servicing and redemption of debt as an immutable incremental negative cash flow. This capacity, expressed in a more or less formally assessed cash flow projection, constitutes an ultimate limit to debt financing, if not pre-empted by the constitutional constraint. Rule of thumb or conventional gearing ratios; and prior-charges cover ratios; may become increasingly inadequate to set limits to the use of hire-purchase or leasing financing alternatives - the more so as the relatively unemphasised way in which these are dealt with in financial accounts may tend to diminish managerial awareness of their existence. (2) Without doubt this is the reason for the requirement of their partial disclosure under the 1967 Companies Act. One secondary implication of the impact of the use of the Tables on medium and long-term financial planning may be to heighten the need for the preparation of flexible cash budgets and to spell out the relative priorities of alternative discretionary outlays such as dividends, & sales promotion - and capital expenditure. I would submit that this is what is really meant by "market-acceptable levels" of debt: the question, not of total insolvency, but of adequate solvency - both absolutely, and comparatively against the competing claims of other discretionary outlays. Certainly it has been my experience that applications for overdraft facilities have been the more sympathetically received when accompanied by a cash flow budget which set out clearly such priorities. In any event, a formal appreciation of existing immutable cash flow commitments, and the margin of safety accruing thereto, is an indispensable adjunct to the consideration of further debt financing.

(1) G. Donaldson (11).

(2) Gant (57); Hamel & Thompson (63); Kancil & Anthony (89). But see Donaldson, op. cit (50) p.177 for an unenthusiastic attitude towards formal lease capitalisation.
as a measure both of the capacity to take on further debt commitments and the need to do so because of inadequate equity resources.

In this sense, the easy movement from equity to debt financing induced by the use of the Tables may have to be curtailed. But planning the financing of capital expenditure projects is as much a part of capital expenditure budgeting as is the appraising of those same projects. Setting the limits of instalment and other forms of debt is no more than a part of setting the shares of all appropriations of the net cash flow of the firm. The critical concept is that of the adequacy of that cash flow to sustain immutable commitments. For periods of time which are significant in terms of investment decisions, those limits can be regarded as a financial parameter. Given that no more than £x can be assigned to servicing debt, then the problem is one of ranking projects in descending order of profitability; establishing the lowest-cost method of financing from the Tables; and matching projects against debt capacity so as to maximise aggregate profitability and yet minimise aggregate cost of financing. Much of the subsequent argument of this thesis is taken up with a model synthesising this dual requirement of profitability and financial costs followed by a suggested procedure to implement these requirements in a practical way.

It is first necessary to make due acknowledgement of other, more general restrictions on the use of debt finance. Analysis reveals that these are really no more than a somewhat specialised application of the concept of servicing-adequacy of cash flow; but they are of sufficient importance to warrant some separate comment.

There may well be an institutional reaction against an excess of funded loan capital - by which is meant loan capital represented by a formal, more or less negotiable long-dated instrument of debt acknowledgement. In terms of market operations this reaction is almost certainly no more than a surplus of fixed-interest stock in institutional portfolios. In terms of ad hoc fund-raising operations at the individual firm level, a supportive equity base is a normal pre-condition of lending. It will be argued that this is not in any

(1) The concept of the 'servicing-adequacy of cash flow' will be found to be central to much of the ensuing argument.
way an oblique form of capital gearing requirement, but that it is a highly condensed appreciation of the logic of limited liability. For the moment, the requirement is taken at face value. One version of this requirement is to be found in the growing clearing-house banking practice of demanding personal guarantees by major shareholders as additional collateral, in the case of smaller firms. (This seems to me to be a most remarkable infringement of the principal of limited liability). Another version is implied in the appraisal of new funded debt issues in terms of 'asset cover' (1) by the fixed-interest departments of stockbroking firms. It is relevant to the present argument that this appraisal by creditors is of their security of repayment; their security of income from the loan is appraised in terms of interest cover, i.e., servicing-adequacy.

To this perhaps not inconsiderable extent, the requirements of long-term capital structure planning might have to be observed. It is thus conceivable that the advantages indicated by the Tables in one form of financing over all others, might have to be over-rulled by those requirements. In the case of small scale expenditures, especially of a non-budgeted emergency nature (intuitively, the unplanned but vital replacement decision falls into this category), even such requirements might be breached with impunity. Financial management just is not that precise. But in the more important respect of large-scale planned expenditure, there can be envisaged a programme of selection of the most advantageous financing method for each project, followed by an aggregation of the values of each so chosen method so as to compare the equity-debt structure thus derived against a long-term capital budget based upon market conventions and requirements. Any gross disparity between the two might necessitate a switch in the chosen financial method for those projects where the advantages of the first choice was least impressive. It must be admitted that any such revision is seen only as occurring in extreme cases and in terms of large aggregates of capital.

Mention must also be made of the restraint upon the use of debt capital which can be imposed by strategic considerations. A typical example of this would be consideration of the affect of high gearing

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upon Trustee status - although preliminary studies carried out in the Department of Industrial Engineering and Management at Loughborough University of Technology have not found any significant correlation between Trustee status and low gearing levels. A second example is the consideration of currency regulations which the financial management of the U.K. subsidiaries of overseas corporations must have in mind - e.g., the greater degree of automatic consent which is given to servicing foreign loan capital than to foreign equity capital. Finally, there may well be circumstances where the immediate use of debt capital is precluded by plans for a future major debt financing operation - which, it is thought, might be prejudiced by present action. This restraint appears to operate most strongly where debt capital is to be raised upon international markets - e.g., the Euro-Dollar market.

It is opportune to observe that all such constraints on the use of debt capital require that the true debt finance nature of instalment debt is fully recognised. Other than the imprecise requirements of para.12 (1) g.b. of the 1967 Companies Act, requiring disclosure of hire charges: there is no statutory obligation upon companies to disclose the capital value of leasing finance. Such disclosure, at least by way of a note appended to the Balance Sheet, is required by the usance of good auditing practice in the U.S. (although the requirement does not seem to be assiduously prosecuted); but other than by the endeavours of individual auditors, U.K. practice lags behind. (1)

The fixed commitments on cash flow are none the less real, and the cover for the future servicing of overt debt none the less reduced. In addition, there seems to be considerable uncertainty as to the extent of which limitations on the borrowing powers of Directors, imposed by Articles of Association, do or do not include the capitalised value of instalment debt.

One other consequence arising out of the repudiation of a pre-planned structure must be mentioned. Acceptance of the 'traditional' analysis of the interaction of increasing debt upon equity yields leads to the concept of an 'optimal gearing structure'.

(2) c.f. references - "unpublished dissertations" - R.H.Barnes.
such that the cost of capital is minimised.\(^{(1)}\) It is usually postulated from such empirical work as has been carried out in this field that the curvature of the average cost curve for equity capital is very gradual, so that over a wide range of gearing that average cost is but insignificantly changed. Nevertheless, it is possible to conceive of some capital structure such that the cost of capital is at a minimum for all levels or amounts of total capital. Thence it can be argued that shareholder wealth will be maximised, as the consequent minimisation of the appraisal rate will permit maximum project exploitation. But, realistically, that which permits maximum project exploitation is the sufficiency of finance to do so. Such a sufficiency may very well not be forthcoming if an overly fixed obsession with a certain capital mix precludes exploitation of servicing-adequacy because of some restriction of equity - or, indeed, vice versa. Even if a departure from that mix causes the cost of capital to increase for given total amounts of capital; but permits the financing of a number of projects which otherwise must be cancelled; so long as incremental projects still yield a positive net present worth at that increased cost, shareholder wealth may be promoted by their exploitation. In this light, the 'optimal financial mix' concept is seen to be something of a rationalisation or justification of the convenience for capital budgeting theory of the pre-assumption of some given capital mix.\(^{(2)}\)

Not that this concept is necessarily inconsistent with the present thesis. To anticipate, in order that the matter may for the moment be conveniently disposed of: the retention portion of the master cash flow may be set such that the debt-capacity of the balance

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\(^{(1)}\) The argument as presented here is greatly abbreviated. The concept is also acceptable under the later, tax-corrected, analysis of Modigliani & Miller. They draw the same conclusions as to the gradualness of the slope of the average cost curve for equity capital. See for example, Bower "Leverage and the Cost of Capital" (Eng.Econ. vol.10 No.2). (44).

\(^{(2)}\) See Brigham & Smith (47) p.6. "The relative weights for the weighted average calculation (of the firm's cost of capital) are typically based on the book value of the firm's different capital sources. This assumes (1) that the firm is operating somewhere near it's optimal capital structure,...and (2) that in acquiring new funds, the firm will maintain it's relative proportions of the different types of capital". Fundamentally, in this thesis maximisation of profits is preferred to maximisation of the rate of return. - c/f Amey (2).
may result in a capital structure which in fact minimises the cost of capital. Two difficulties arise. Firstly, the result may still be to impede full project exploitation, because the cheapest financing method consonant with total system optimisation may not then be available. Some project or projects which could more optimally be financed by debt must now be financed by equity, or vice versa. As a result finance will not be available for additional projects.(1) Secondly, if capital cost curves are indeed so gradual in their curvature, it seems doubtful if the concept of an 'optimal financial mix' is really very practical. It can be postulated that within the argument advanced generally in this thesis as to the effect of gearing on the cost of capital, and the absence of capital structure planning: and as a consequence of the finance-investment linkage model and procedure to be advanced: 'optimal financial mix' as part of financial planning is relegated to a secondary role. With it, of course, goes any doctrine that financial management ought to pre-plan the financial mix, even if in fact they do not.

Section 6. Discriminatory Appraisal Approaches

The concept of the successive matching of evaluated projects against alternative financing methods may appear to constitute a second cardinal infringement of accepted capital budgeting theory, in that if the separateness of the investment from the financial decision is not appreciated, it appears to postulate that a given project can be appraised in terms of a selected type of finance. The theory holds that the ability of certain projects to attract cheaper types of finance that would be denied to other projects does not mean that the first set of projects can be appraised by a lower-cost formula than the second.

(1) As stated, this is to anticipate the argument of the model and the procedural approach. Therefore, this argument may presently be obscure, and reference back at a later point in time may become advantageous.
Thus Vancil (op.cit.):

"The attractiveness of a new investment should not be influenced by the method in which the funds for that investment are to be provided. Assuming that the money can be borrowed at a low rate does not make the investment more attractive than if the money were to be provided by the sale of common stock." (P.5).

Alternatively, Weston(1):

"It is inaccurate and inappropriate to consider the cost of the particular type of financing required at a particular time as the cost of capital for financing a particular segment of expansion." (P.81).

But Herrett & Sykes(2) came very close to advocating discriminatory appraisal rates:

"The weights appropriate to calculating this average (cost of capital) are the proportions in which the two types of capital will be used to finance the actual projects under consideration .... The lower return required.... (on) retained earnings is irrelevant, since if any projects under consideration are shown to be unacceptable, the reduction in capital requirements will be made in the relatively expensive new share issue." (P.33).

These authors came this close because essentially they are advocating a marginal cost of capital approach. Further, it is convenient to note at this point that Vancil uses the postulate of non-discriminatory appraisal rates as the basis for the separation of the investment from the financing decision.

Yet a moment's reflection brings the realisation that the use of a common appraisal rate of discount over all alternative financing methods, means that there is no more discrimination involved in the selection of one of those methods than there is involved in

(1) F.Weston (93).
(2) Herrett & Sykes (19).
preferring for one project that one set of technological specifications and/or phasing of capital outlays among several possibilities, which maximises the present worth of the arising net cash flows from that project. Chemical plant especially is capable of construction alternatives in that the same capacity can be achieved by one large or several smaller linked units. The capital cost is thus a single sum or a series of instalments, and it is basic to capital expenditure appraisal that both alternatives be reviewed and the more profitable alternative selected.

Nevertheless, I confess to some impatience over the implicit totality of application which is awarded to the precept of non-discriminatory appraisal rates. For its application, the precept requires perfect substitutability of capital between projects. This is by no means universally to be found. Capital can be as imperfectly competitive, within the firm, as the next factor of production. Postulate for example, high cost, advanced technology equipment such as chemical plant. This is a field of industry in which rapidly growing and capital-hungry firms are often to be found. The construction or acquisition of such high-cost plant by such a firm is with increasing frequency financed by debenture loan capital specifically secured on that plant. A guarantee that such debenture finance will be forthcoming on completion of the plant is often used as collateral for a construction bridging loan. Such transactions are much sought after by many institutions - e.g., insurance companies - because of the relatively high rate of interest which the debenture carries, the excellent security of up-to-date plant and processes and the access which the arrangement gives to the (profitable) financing of the newer fields of technology. The loan would not be available to the company for any other asset: the plant AND THE SUBSEQUENT EARNING STREAM would be unavailable but for the loan. Such plant is, frequently, uncertain as to its cash flow in the early, D.C.F. - important years, because of its technological complexity. Such firms, because of their capital hunger, typically have an otherwise high cost of capital. The combination of a necessarily severe sensitivity-analysis and a high cost of capital will invariably result in a modest or even negative net present worth. Yet such plant in such circumstances is often absolutely vital if the firm is to
progress or even to continue. Capital expenditure theory typically enters a caveat in such circumstances by making reference to non-financial strategic criteria. But this is most clearly a case of acute financial judgement, and to have recourse to other criteria as the ultimate determinant seems to indicate a significant discrepancy in an area of management science. A strong case exists for the assessment of such cases to be in terms of their own individual financing.(1)

It seems reasonable to conclude that the procedure of matching evaluated projects against alternative financing methods does not conflict with generally received capital expenditure budgeting theory either because of the implicit conflict with long-term capital structure planning assumptions or because of a superficial discrimination in appraisal. It has been seen that the fundamental servicing-adequacy of cash flow which must lay at the root of the concept of long-term capital structure plans may impose limits to the matching process. This limiting process may well be reinforced at the limit by constitutional limits to borrowing powers and may be further influenced by especially strong observancies of the basic cash flow problem taking the form of conventions adopted by lending institutions. Donaldson (op.cit.) remarks that lending institutions are not more likely to be pessimistic about solvency constraints than are the more responsible applicants for debt finance. After all, the lending institutions enjoy the advantage of the principle of average loss, but to the manager an error in judgement can threaten total failure.

The servicing-adequacy of cash flow is thus seen to be a central part of any realistic capital expenditure budgeting process. In as

(1) See also R.S. Bower (43); Bower demonstrates the error of using an average cost of capital to appraise projects which by their nature can give rise to further, additional debt financing facilities. Essentially, his technique is to set up the cash flows arising from the debt finance facility as a credit within the main project flows and to appraise the net incremental flows by a pure equity discount rate. (Vide Solomon "Measuring a Company's Cost of Capital" (83).) Bower concludes; "Only when the link is 100% (i.e., the project is 100% self debt financing) "and the form of the loan is such that the cumulative flows to owners are positive" (exactly the two conditions typifying heavy complex process plant erected under long term construct-and-operate contracts) "will the comparison of the debt interest rate and the project rate of return provide a correct decision". This solution is discussed further on Page 229, et seq.
much as the acceptance of projects itself is a generator of further cash flows, there is evidently a strong looped inter-relationship between the decision to invest and the decision to finance. This inter-relationship or linkage is discussed at some length in the next Chapter.
CHAPTER FIVE

A FINANCE-INVESTMENT LINKAGE MODEL

Introduction: The Cost of Debt

A strong case can be made for thinking that the principle of servicing-adequacy is not at all clear cut in its effects. Earlier comment has been made upon the ability of Chairmen to secure additional capital under the most seemingly adverse circumstances, and of the ability of large companies not to raise the cost (in real terms) of debt against themselves. This lack of clarity is intensified, as was noted on Page 198, where instalment debt is used but, due to unemphasised accounting routines, is not immediately evident in all its implications. Nevertheless, ultimate managerial doubt about the servicing-adequacy of cash flow in effect causes the curve of the cost of debt capital to rise vertically at some point. Institutional conventions, reflecting more or less specifically the same doubt, will cause the same sort of effect. As the size of the firm increases presumably its aggregate cash flow shows potential increase. There must be a corresponding increase in debt capacity. Thus, the cost function of debt capital to a firm must, diagrammatically, consist of a non-continuous curve where of each segment will consist of a horizontal part followed by a (probably rapidly) increasing part and an ultimately vertical part. If there is indeed a negative correlation between size of firm and cost of debt capital, the overall position may well be as illustrated in Figure 1 below.

FIGURE 1.
Conclusions similar to those depicted in Figure 1. are reached by Brigham & Smith, (1) supported by evidence drawn from the reports of the Federal Reserve Board. These authors also note the discontinuity in the curve as a firm grows from size 1 to size 'n'. Some relevant, albeit negative, evidence of the general sense of the argument seems to be afforded by the recent cases concerning Davies Investment, and Pinnock Finance: where attempts unreasonably to defer the vertical upturn of the interest cost curve ended in financial disasters. No U.K. statistical data appears to be readily available to support that evidenced by Brigham & Smith in the U.S., but there is no great reason to believe that circumstances are greatly different as between the two economies (except in respect of the greater emphasis on the use of gearing by the small firm in the U.S.). In a sense, the very existence of the I.C.F.C. is proof of this, in that its raison d'etre is the provision of loan capital to small companies to whom such capital would not otherwise be available, however deserving the firm may be.

The curvature between horizontal and vertical in Figure 1. will be more or less gradual, according to such factors as:

1. The real cost accruing to restrictive covenants which each successive debt tranche may place on further capital raising activities.

2. The degree of recourse to secondary lending institutions which give evidence of being less restrictive in their appraisal conventions, but more overtly expensive accordingly. (2)

3. The delaying effect on cost increases which will accrue to the spreading of largely fixed fund-raising expenses over larger tranches. This favourable effect will tend to be more pronounced for the larger firm, with a larger average size of tranche.

(1) Brigham & Smith (47).

(2) A variant of this factor may lie in the reduced negotiability of privately placed debt, for which no wide public market may exist. This will be even more pronounced if ad hoc instalment debt is arranged on a triangular basis between supplier, financing institution and buyer. See J. Dundas Hamilton (op. cit.).
4. The extent to which constitutional limitations on Directors' borrowing powers can be voided by instalment debt.

Almost certainly the net effect is ultimately to raise the average cost of capital. For while we have seen that there is no significant reaction of equity yield to substantial injections of funded debt capital (and it is the more probable that there will be none such to injections of the less conspicuous instalment debt); that point on the curve where doubts over the servicing-adequacy of cash flow introduces a sharp upward turn will surely represent the point when debt servicing will infringe upon the priority given to cash flow utilised for future capital expenditure. Further investment opportunities will have to be forgone because of shortage of cash flow, earnings growth rates will decline and the equity yield will start to rise also. (1)

Under standard capital budgeting theory, the optimal limit of capital investment is thus set at the intersection of the rising marginal cost curve to be derived from these effects and the diminishing marginal net revenue curve of investment opportunities. It is in this field of capital expenditure budgeting that the appraisal of financial decisions afforded by the Tables can play a part, in examining the hitherto somewhat neglected side of such budgeting theory - i.e., the supply of capital (as opposed to the demand for capital) implicit in the investment decision. The ensuing discussion turns to the topic of capital expenditure budgeting, accordingly. It must be realised that this is an extension of the application of the Tables beyond that afforded by the need to appraise the financing of emergency or other unplanned investment decisions.

(1) For a similar conclusion, see Robichek & Myers (25) P.41. The essential point is that if share prices are a function of average estimated future equity earnings, which mathematically implies - if those earnings are estimated to be constant or nearly so - an estimate of the future managerial investment strategy (or, by the Gordon-Shapiro model, can be true for any pattern of future expectations); any change in that strategy, induced by illiquidity, must change share prices and yields. Just conceivably the change could be for the better e.g., shareholder distrust of continued investment in a declining industry.
Section 1. A Numerical Illustration of Financial Linkages

The ranking of investment projects which normally is considered to take place under capital budgeting procedures establishes the relative desirabilities of those projects in terms of profitability. But it cannot be claimed that any priority ranking of the order in which projects shall be implemented in time, is established thereby. Urgent re-equipment projects, may rank lower in profitability appraisal than the new factory in a development area; but often they cannot hang fire for the two years or so that this will require for planning, negotiation and completion. The timing of projects, and the consequent starting of their arising cash flows is vital to a forecast of total company cash flow from all sources. This affects the servicing-adequacy and hence the 'shape' of the cost curve for debt capital of all sorts - with the concomitant effect upon the cost of capital.

Consider a project just beyond the margin of acceptance in terms of profit appraisal against some predetermined cost of capital. Let that project be one wherein the yearly cash flow is markedly uneven (most probably strongly positive in the early years), albeit over the life of the project it is not so favourable - hence its rejection. Yet those few timely positive cash flows, expressed as part of a medium-term overall company cash flow forecast, might indicate a servicing-adequacy of sufficient size to induce substantial incremental instalment debt capital. Thereby the cost of capital might possibly be sufficiently reduced to a point where that same project becomes acceptable.

This very real problem seems to be endemic in the use of discounted cash flow for capital budgeting routines. The insistence on converting cash flows to a present value tends to ignore the fact that for the servicing of capital, cash flow is "wanted when it is wanted"(1), which may very well not be the present. Project appraisal is usually discussed in terms - whether evaluated or arbitrarily chosen -

(1) C/f the very apposite introductory remarks to the specific model developed by D.Chambers (49): "---it will be important that funds should become available when they will be needed" Chamber's relation of this observation to his context of a firm using mainly internally generated finance appears unduly restrictive.
of a bench-mark rate of interest; either as a positive net present worth after discounting at a predetermined rate of interest or as an internal rate of return whereof the acceptability is assessed, again, against a predetermined rate. This predetermined rate, it has been noted, is derived from some supposedly optimal or near-optimal capital mix which will minimise the capital cost function for all amounts of total capital: the minimised capital cost becoming the rate in question. But if we now accept that:

1. The real limitation to debt capacity is the servicing-adequacy of cash flow;

2. Debt capacity as thus measured includes instalment debt financing; which at one and the same time must be taken into account in any realistic cash flow forecast and yet tends in its nature to be less conspicuously 'objectionable' under rule of thumb, conventional limitations. This is true of institutional, constitutional or restrictive covenant aspects;

3. There is no real reason to believe that even formal or funded debt very significantly affects equity yields in an opposing or countervailing direction;

4. Apart from cash flows forecasted to arise from the continuation of current operations, any total company cash flow forecast is a function of the order in which new projects are commenced and start to yield their own additional cash flows;

then this 'optimal financing mix' is not so much a single determinable absolute as a formidable schedule of alternative possibilities, each a function of:

(a) what projects are appraised as acceptable; and
(b) in what order those projects are commenced.

Constraint (a) is a function ultimately of the bench-mark rate of interest. This is itself a function of the cost of capital curve. It has been demonstrated that this curve is, in its turn, a function of the servicing-adequacy of cash flow. But that cash flow depends upon the order of project commencement. This order cannot be determined until it is known what projects are acceptable. The
circular or looped nature of this procedure might conceivably be solved either by a system of simultaneous equations or by iteration routines, given the necessary constraints of project innovation and managerial capacity together with those relating to servicing-adequacy which were previously outlined on Page 206. Even so, an intricate and voluminous computational routine would be necessary to achieve an optimal solution. Certainly, it would be beyond the capacity of the average company treasurer’s department to cope with such a related set of problems without very advanced computer facilities, even supposing that the necessary mathematical programming were to be complete.

A highly simplified numerical illustration of this argument appears to be opportune; but it is necessary first to clarify certain basic assumptions which will be implicit in all that follows concerning the finance-investment linkage model and the subsequent procedural approach. The first of these concerns the definition of a 'project' which is taken as:

(a) proposing the timed commitment of a specific quantity of financial resources to achieve a stipulated objective which is in accord with general company objectives; and

(b) being capable of evaluation in terms of a comparison of timed expected monetary inflows (however adjusted for uncertainty) against that commitment. The uncertainty adjustment is such as to reduce the expectations of one project to the same degree of probability as those of all other projects under review at the base date; and

(c) requiring a specific senior management decision before the proposal can be implemented.

Thus major replacement, income-generation and cost-reduction projects aimed at maintaining or increasing profits are included; but routine revenue commitments and minor capital expenditures are not. Projects include those expenditures which for normal accounting or taxation purposes would be entitled 'capital expenditure' and also exceptional, non-recurring revenue expenditures of substantial amount such as major repairs, R & D expenditures, advertising expenditures etc. Excluded however are major capital or revenue expenditures of a non-profit nature, whether motivated as such (e.g., social and purely administrative projects) or incapable of being quantified as such

(1) See Charnes, Cooper & Miller (49).
(e.g., welfare and political projects). Such projects may be taken into account by adding a constant increment to the threshold appraisal rate. (For a discussion on this issue see Presanis, "Corporate Planning in Industry", Business Publications, 1968).

The second assumption relates to the timing of project implementation. The procedure for ranking projects by descending net present worth can of course take into account the different starting-times of projects so long as these are predicated. Thus ranking can to a limited extent take into account the technological or supply imperatives of business life. But the procedure cannot easily (that is to say, without complete recalculation) take into account shifts or reschedulings of project starting times. It is assumed that such reschedulings are feasible with considerable flexibility. Realistically, certain inevitable delays will mean that a project cannot be shifted forward in time - but it can be postponed. In what follows, this one-way mobility of certain project starting dates is subsumed, and is not again specifically referred to.

Suppose, then, a company to have under consideration the following schedule of projects at a certain time. The projects are taken to be technologically independent but not mutually exclusive.

**TABLE 6**

*Schedule of Projects*

*(All amounts in terms of £'n.)*

<table>
<thead>
<tr>
<th>Project</th>
<th>Outlay (t=0)</th>
<th>Cash Flows (positive) arising in period:-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(t_0-t_1)</td>
<td>(t_1-t_2)</td>
</tr>
<tr>
<td>A</td>
<td>800</td>
<td>200</td>
</tr>
<tr>
<td>B</td>
<td>1130</td>
<td>400</td>
</tr>
<tr>
<td>C</td>
<td>1030</td>
<td>300</td>
</tr>
<tr>
<td>D</td>
<td>850</td>
<td>300</td>
</tr>
<tr>
<td>E</td>
<td>700</td>
<td>80</td>
</tr>
<tr>
<td>J</td>
<td>200</td>
<td>30</td>
</tr>
<tr>
<td>J_1 etc</td>
<td>etc</td>
<td>etc</td>
</tr>
<tr>
<td>(\ldots)</td>
<td>etc</td>
<td>etc</td>
</tr>
<tr>
<td>P etc</td>
<td>etc</td>
<td>etc</td>
</tr>
</tbody>
</table>

where the time intervals are constant (e.g., years) and refer to time intervals of project life.
Let it further be supposed that:

(1) Project appraisal in this company is restricted to a 5-year study period. This assumption is defended later in this discussion.

(2) All cash flows accrue evenly during the various periods.

(3) That the finance presently available to the company in respect of these new projects is:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>£2,540</td>
<td>67%</td>
</tr>
<tr>
<td>Debt</td>
<td>£1,270</td>
<td>33%</td>
</tr>
<tr>
<td></td>
<td>£3,810</td>
<td>100%</td>
</tr>
</tbody>
</table>

(All amounts in terms of £'n.)

Prima facie, this appears to management to be:

(4) An acceptable capital structure in that it does not transgress the Articles of Association, and so far as they can see does not increase their present estimate of the cost of capital.

(5) The maximum finance available in so far as no further share issue is planned and the projected cash flow is inadequate either to accumulate higher retentions or to service more debt.

Finally,

(6) Due to problems of supervisory capacity, projects can on average be expected to be started only at 6-monthly intervals.

Using a 10% discount rate (management's present estimate of the cost of capital), the projects can be appraised and ranked as follows, without assuming at this stage any project start-up schedule:
TABLE 7.

PROJECT RANKING TABLE

(All amounts in terms of £'s.)

<table>
<thead>
<tr>
<th>Project</th>
<th>Outlay</th>
<th>Net Present Worth ( @ 10% Discount)</th>
<th>Cumulative Outlay</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1130</td>
<td>210</td>
<td>1130</td>
</tr>
<tr>
<td>C</td>
<td>1030</td>
<td>165</td>
<td>2160</td>
</tr>
<tr>
<td>A</td>
<td>800</td>
<td>115</td>
<td>2960</td>
</tr>
<tr>
<td>D</td>
<td>850</td>
<td>60</td>
<td>3810</td>
</tr>
<tr>
<td>J</td>
<td>200</td>
<td>25</td>
<td>4010</td>
</tr>
<tr>
<td>E</td>
<td>700</td>
<td>(10) neg.</td>
<td>4710</td>
</tr>
</tbody>
</table>

and so on, up to and including project P

(Note: ranking is by simple NPW. Calculation of and ranking by some such profitability index as 'NPW per £ outlay' may be superior, but is not necessary for the present purpose; which is simply to set up an initial order of projects).

On this basis, projects B, C, A, and D are acceptable and are to be proceeded with in the stipulated order of start-up. Project J is acceptable but cannot be financed. Project E is neither acceptable nor can it be financed. This is the typical situation depicted by capital expenditure budgetary control under DCF routines, as commonly to be found in the literature on the subject.

The evident cash flow generated by this acceptance and start-up ranking appears to be (on a 6-monthly basis; and ignoring now and for the rest of this immediate discussion cash flows from existing operations):
TABLE 8A.

PROJECT CASH INFLOW SCHEDULE
(all amounts in terms of £’s)

<table>
<thead>
<tr>
<th>Project</th>
<th>$t_0 - t_1$</th>
<th>$t_1 - t_2$</th>
<th>$t_2 - t_3$</th>
<th>$t_3 - t_4$</th>
<th>$t_4 - t_5$</th>
<th>$t_5 - t_6$</th>
<th>$t_6 - t_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>B</td>
<td>200</td>
<td>200</td>
<td>175</td>
<td>175</td>
<td>150</td>
<td>150</td>
<td>100</td>
</tr>
<tr>
<td>C</td>
<td>150</td>
<td>175</td>
<td>200</td>
<td>150</td>
<td>150</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>150</td>
<td>150</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Total</td>
<td>625</td>
<td>650</td>
<td>625</td>
<td>600</td>
<td>550</td>
<td>500</td>
<td>450</td>
</tr>
</tbody>
</table>

where (a) and (b) are the first and second halves of each time interval, respectively. As the first three periods and the last three periods especially are incomplete as to cash flows accruing to projects not now under budgetary consideration; only the middle seven periods are summed as relevant to the discussion. (1)

It is as well to note at once that there is revealed an immediate and potent contradiction. The indicated staggering of project timing will at once affect the present value of delayed projects, especially by the impact on the initial outlay or on large cash flows which are late in the project life. Thus, in this simple example, the ranking priority of C over A nearly disappears as a result of the staggering. This is a sobering thought – that a revision of the technological or strategic priority given to a project may so bias the pattern of its cash flow in and over time that the project assumes a totally different profitability ranking. As it is, account of this change in profitability must be taken in any model built to represent finance-investment linkage.

But a different pattern of cash flows will accrue if the same four accepted projects are started in a different order. For example:

(1) This procedure is defended at a later point. It will be noted that Table 8A also ignores the effect of rescheduled cash outflows. This omission is a temporary convenience.
Two linked but distinguishable sets of observations follow:

(a) the cash flow pattern D,B,A,C is better than that arising from B,C,A,D; in that it is less variable as between time-intervals. As such it is very likely to represent less risk to a supplier of loan capital, and presumably will accordingly represent an improved servicing-adequacy compared with a cash flow pattern of similar magnitude but greater variability, or even with a cash flow of somewhat larger magnitude but pronounced variability.

(b) the minimum two-period flow (t₄₋t₅, (a) and (b)) of B,C,A,D is improved by £100n per period in D,B,A,C. In this second investment pattern, one of the same two periods represents an equal minimum with any other period. This is an improvement of some 20% in the 'weakest link' of the chain of cash flow servicing-adequacy. This 'weakest link' - the minimum cash flow accruing in any time interval during the study period - is the measure of servicing-adequacy; in as much as no lender will advance loan capital in excess of that which can be serviced under the least advantageous cash flow circumstances. Exactly the same sort of reasoning must apply in terms of self-financing; new issues apart, the generation of equity capital by retentions must be conditioned by the size of the cash
flow in any one time-interval. (1)
If then the crucial element is the size and (probably to a less extent) the variability of cash flow, the question to be asked is - which cash flow?

Section 2: Project Amending and Rescheduling

Let us suppose that the limitation of debt capital to £1270 (see Page 212) is based upon the cash flow projection of the investment pattern B, C, A, D; that is, upon an acceptability ranking of projects. Let us further suppose that management adopts a policy of 'total exploitation' of the (debt) servicing-adequacy of cash flow. (2) Necessary assumptions are that this is not expected to offend constitutionally; and that there will be no retentions. (All these assumptions are a necessary but temporary convenience if the argument is to be pursued). An implementation of pattern D, B, A, C would facilitate up to £250 (20%) more debt capital and thus relax the severity of capital rationing (assuming no increase in debt servicing costs).

Under these conditions, management would now have the funds to implement project J. This in its turn would create an alteration in the cash flow pattern, which might permit further debt facilities and so on. There might also be an effect upon the cost of capital, possibly in a downward direction. As a result, project E (see Page 211) might assume a positive net present worth at the lower discount rate, although of course implementation would still depend upon cash flow. This possibility is discussed later.

Very evidently, there must be some ordering of project

(1) It is essential to an appreciation of the argument constantly to remember that the various quantities discussed are in cash terms, not profitability or accounting terms. Thus 'retentions' are not non-distributed profits; they are non-distributions (i.e., there are no dividend warrants) of the reinvestable cash flow of the company.

(2) "An ideal capital structure....is an ordinary share base with the maximum loan stock in issue, if possible without any conversion rights or subscription rights" (Chairman, Slater Walker Securities - Stock Exchange Gazette, December 1968).
implementations in time (subject to technological, supply and strategy imperatives)\(^{(1)}\) which will optimise the critical minimum cash flow accruing to any one time-interval in the study period of a capital expenditure budget. The process of determining that optimal ordering can be described only in somewhat laborious terms. A number or SCHEDULE of projects, capable of being financed from present resources, are ordered (as to the timing of their implementation) in a certain way. This will generate a certain pattern of cash flows which can be used to finance further projects; which will give incremental cash flows which can be used to finance yet further projects; and so on. This first adding of subsequent projects will be termed an AMENDING process. A second SCHEDULE of projects might now be selected, which will yield a different cash flow pattern and hence a different series of AMENDMENTS. This alternative SCHEDULE together with its attendant AMENDMENTS will be called a RESCHEDULING process. It must also be realised that each amending stage of a given schedule is itself a series of alternatives. As an illustration:

**SCHEDULE A, B, C**, finances the additional AMENDING projects **D** and **E**; which will finance project **R**. An alternative AMENDMENT would let SCHEDULE A, B, C finance AMENDING projects **R** and **S**; which would finance **D** and **T**; or would finance project **E** only, etc.etc. **RESCHEDULE B, R, E** finances the AMENDING projects **A** and **S** etc.etc.

The OPTIMAL SCHEDULE is that schedule which permits the maximum series of amendments, where each successive amendment maximises the capacity for the greatest number of amendments consequently to be financed. The solution of this series appears to be a problem in iteration or dynamic scheduling. In what follows, rescheduling is taken (as a convenient shorthand) to include what may essentially be a process of optimal reamendment of a given (optimal or non-optimal)

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\(^{(1)}\) The discussions of 'Starter' projects on Page 230 and the sub-set X, on Page 238 is relevant to these terms. Jointly, these two discussions imply that there is a certain unalterable minimum content and order of the project schedule which may unavoidably lead to a sub-optimal solution.
So far only the variations in project cash inflow patterns have been considered. Re-timing project implementation will also affect the pattern of cash outlays. As a simplifying assumption, these are taken to consist of single-period outlays so far as concerns the present discussion. A relaxation of this assumption might well seriously add to the range of significant variations in the net cash flow pattern, but it should not affect the principle of the solution. It will certainly add to the complexity of solution. It is also convenient at this time to point out that consideration must be given to all intervals affected by the quinquennial study period. Whatever order of implementation is proposed, the opening and subsequent periods will be affected by a fixed amount enuring from the implemented projects of previous study periods; and the closing periods by a fixed range of alternatives as implementation matrices are studied for future quinquennia. It follows that the cash flows generated within a time-interval by projects implemented within the study period of which that time-interval forms part are only one component in the determining basis of servicing-adequacy. What were described on Page 209 as 'cash flows from current operations', now seen essentially to consist of enuring cash flows from the implementation programmes of previous study periods; plus programmes yet to be formulated in the late intervals of the present study periods are of equal, possibly greater, importance in this respect. But these two factors may be taken as parameters for any one time-interval in the present study period, unchanged for all the alternative cash flows which may accrue to that interval from project rescheduling within this study period. It is the ability of such rescheduling to optimise interval cash flows which is so important to capital expenditure budgeting under conditions of capital rationing, and which at the analytical discussion level permits exclusion of such parametrical values from that discussion. But one issue does arise from this last observation. An optimal implementation programme, in terms of maximising the minimum net cash flow in any one time-interval during the study period for one quinquennium, might leave a "tail" of enuring cash flows into the next quinquennium which preclude a really satisfactory solution of the
scheduling problem in that next quinquennium - satisfactory, that is, in that optimality for that quinquennium would desirably be at a high level of net cash flows. (1)

The problem could be serious. For example, if one study period contains only or largely heavy capital-outlay projects; but there inures to it from the previous study period an inheritance of very weak inflows in the initial time-intervals of the present period; the flexibility of implementation programming could be much reduced. If the 'inheritance' were to be very uneven, flexibility would be enhanced but the complexity of programme optimisation much increased. There seems to exist here a special variation of the 'knapsack' problem in mathematical programming. (2) However, it is possible that the multiplicity of alternative combinations of schedules and amendments and the possibility of non-negative coefficients (outlays exceeding inflows (3) - which I take to be a precondition of the 'knapsack' solution) rule this out. A formal approach is suggested later; for the moment, it is immediately clear that some sub-optimal solution must be accepted. Such a solution may be termed 'satisficing'.

The issue of interest payments enuring from one study period to the next must also be clarified, for an element of conflict is to be observed in this item of cash out-flow. Establishing the servicing-adequacy of cash flow must relate to net cash flow; including net of interest out-flows accruing to the study period either as a result of debt financing commencing within the period or as an 'inheritance' enuring from previous study periods in the form of unexpired financing commitments. Yet, when the present worth of the finally-programmed set of projects is calculated, this must be upon the basis of net cash flows gross of those same interest out-flows. These are

(1) This represents an interesting variation of the Lorie & Savage problem of a spill-over of expenditures between periods. See J.H.Lorie & L.J.Savage ( 69 ).

(2) See J.K.Weingartner ( 90 ).

(3) I am indebted to Mr.G.Hayhurst for this observation. His own research into the problem of optimal batch production scheduling within machine capacity constraints has lead him to considerations not totally dissimilar from the above - considerations which he has entitled "Combinatorial Scheduling Problems".
of course subsumed in the appraising discount rate.

A simple numerical example will serve to illustrate these issues to some degree. In the illustration, four possible implementation schedules are listed:

1) The ORIGINAL schedule B,C,A,D - based upon some original profitability ranking.

2) The OPTIMUM schedule D,B,A,C - taking as optimising cash inflow in terms of maximising the minimum cash flow in any one time-interval in the study period; and minimising inter-interval cash flow variability.

3) The CHALLENGER schedule A,D,B,C - a schedule developed by simple iterative programming which maximises minimum cash flow as efficiently as the OPTIMUM; but which does not minimise cash flow variability.

4) any AMENDED schedule D,B,A,C,J₁ - this schedule amends the OPTIMUM schedule by adding project J₁. It will be recalled that this is the project deemed to be capable of being implemented if the improved servicing-adequacy of the OPTIMUM over the ORIGINAL schedule is taken up.

(Certain simplifying assumptions have been made in the AMENDED schedule. These are:

a) Debt capability for J₁ is exploited in the same interval as that in which the outlay for J₁ is made; and this debt finance is redeemed at the end of the study period. These are convenient but not essential assumptions.
b) Exactly the same amount of capability is exploited as is required for the outlay on $J_1$. This is a relaxation of the assumption of total managerial exploitation of servicing-adequacy if $J_1$ by itself or in combination with other projects $J_2 \ldots P$ does not exactly exhaust that capability.

c) $J_1$ is implemented last in time. This is counter to the concept that the inclusion of each new project, as amendment proceeds, involves a complete rescheduling of all projects thus far selected. Purely as a simplification, it is assumed that even so $J_1$ would be started last.

Table 9 lists non-discounted cash flows arising from each of these four schedules. In respect of the AMENDED schedule, it incorporates the new increment of debt and its ultimate repayment, together with an after-tax interest payment on a semi-annual basis. An appendix indicates the present worth of those cash flows in total, utilising an unchanged 10% discount rate. Obviously, this is wrong so far as the AMENDED schedule is concerned if the inclusion of the finance for $J_1$ alters the cost of capital. However, maintaining the same discount rate does permit immediate inter-schedule comparison. The present value of the AMENDED schedule is of course calculated on cash flows gross of the interest payments.

(Note: TABLE 9 IS TO BE FOUND ON THE NEXT PAGE).
### TABLE 9.

**SUMMARY OF PROJECT SCHEDULE CASH FLOWS.**

(All amount in terms of £'n.)

<table>
<thead>
<tr>
<th>PROJECT SCHEDULE</th>
<th>CASH FLOWS</th>
<th>$t_0$</th>
<th>$t_1$</th>
<th>$t_2$</th>
<th>$t_3$</th>
<th>$t_4$</th>
<th>$t_5$</th>
<th>$t_6$</th>
<th>$t_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ORIGINAL</strong></td>
<td>Out</td>
<td>1130</td>
<td>1030</td>
<td>800</td>
<td>850</td>
<td>625</td>
<td>650</td>
<td>600</td>
<td>650</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>200</td>
<td>350</td>
<td>450</td>
<td>625</td>
<td>650</td>
<td>600</td>
<td>550</td>
<td>500</td>
</tr>
<tr>
<td></td>
<td>Net</td>
<td>(1130)</td>
<td>(830)</td>
<td>(450)</td>
<td>(400)</td>
<td>(625)</td>
<td>(650)</td>
<td>(600)</td>
<td>(550)</td>
</tr>
<tr>
<td><strong>OPTIMUM</strong></td>
<td>Out</td>
<td>850</td>
<td>1130</td>
<td>800</td>
<td>1030</td>
<td>550</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>150</td>
<td>350</td>
<td>400</td>
<td>550</td>
<td>600</td>
<td>600</td>
<td>600</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>Net</td>
<td>(850)</td>
<td>(980)</td>
<td>(450)</td>
<td>(630)</td>
<td>(550)</td>
<td>(600)</td>
<td>(600)</td>
<td>(600)</td>
</tr>
<tr>
<td><strong>CHALLENGER</strong></td>
<td>Out</td>
<td>800</td>
<td>850</td>
<td>1130</td>
<td>1030</td>
<td>625</td>
<td>575</td>
<td>525</td>
<td>600</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>100</td>
<td>250</td>
<td>500</td>
<td>625</td>
<td>575</td>
<td>525</td>
<td>600</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>Net</td>
<td>(800)</td>
<td>(750)</td>
<td>(880)</td>
<td>(530)</td>
<td>(600)</td>
<td>(625)</td>
<td>(550)</td>
<td>(575)</td>
</tr>
<tr>
<td><strong>AMENDED</strong></td>
<td>Out</td>
<td>850</td>
<td>1130</td>
<td>800</td>
<td>1030</td>
<td>200</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>In</td>
<td>150</td>
<td>350</td>
<td>400</td>
<td>750</td>
<td>615</td>
<td>630</td>
<td>630</td>
<td>630</td>
</tr>
</tbody>
</table>

**NOTE:** INTERVAL $t_{1a}$ IS THE FIRST HALF OF INTERVAL $t_0 - t_1$; etc.

(Bracketed amounts are net out-flows).

### TABLE 9: APPENDIX

(All amounts in terms of £'n).

**Total Net Cash Flows, Discounted Semi-Annually at a Constant 10% Rate.**

<table>
<thead>
<tr>
<th>SCHEDULE</th>
<th>OUTFLOW</th>
<th>INFLOW</th>
<th>NET CASH FLOW</th>
</tr>
</thead>
<tbody>
<tr>
<td>ORIGINAL</td>
<td>£ 3576</td>
<td>£ 4203</td>
<td>£ 627</td>
</tr>
<tr>
<td>OPTIMUM</td>
<td>3548</td>
<td>4155</td>
<td>607</td>
</tr>
<tr>
<td>CHALLENGER</td>
<td>3512</td>
<td>4169</td>
<td>657</td>
</tr>
<tr>
<td>AMENDED</td>
<td>3870</td>
<td>4490</td>
<td>620</td>
</tr>
</tbody>
</table>
The following observations can be made:

A. **Sub-optimal Satisficing**

If the inheritance of net inflows ensuing from the previous quinquennium is not:

1. for period $t_o \geq £1400$
2. for period $t_{1a} \geq £1530$
3. for period $t_{2a} \geq £1160$

then in any of those periods the OPTIMUM schedule will yield a minimum period cash flow of less than the present £550 minimum (see Table 8A). The slightly sub-optimal CHALLENGER schedule could then with advantage be substituted as the final schedule so long as the cash flow

4. for period $t_{1b} \geq £1430$ (to achieve an interval minimum of £550; see Table 8A) if this constraint were not met, then the even less optimal ORIGINAL schedule could with advantage be substituted so long as the cash flow

5. for period $t_o \geq £1580$ (to achieve an interval minimum of £450; see Table 8A)

6. for period $t_{1a} \geq £1280$ (to achieve an interval minimum of £450; see Table 8A)

Given for example, a failure to meet constraints 3, 4 and 5; then a further enquiry must be made as to which sub-optimal solution nevertheless affords the best satisficing function. (1)

(1) In comparing CHALLENGER v OPTIMUM, interval $t_{1b}$ is larger for CHALLENGER, and therefore sets no primary constraint for OPTIMUM. In comparing ORIGINAL v CHALLENGER, intervals $t_o$ and $t_{1a}$ are larger for ORIGINAL, and therefore set no primary constraint for CHALLENGER.

In both comparisons, it has (for the sake of brevity of exposition) been assumed that a successfully challenging schedule is able to satisfy its own minimum cash flow requirements in those intervals where the defending schedule failed. These comparisons, of course, are by no means exhaustive.
B. Net Present Value

The Appendix to Table 9 reveals a dual conflict of objectives. Currently, the most frequently proposed managerial objective is that of maximisation of shareholder wealth - more specifically, of the present worth of shareholder wealth. (1) The Appendix to Table 9 demonstrates that the present worth of a particular project implementation schedule will not necessarily be maximised by optimising, as nearly as possible, the crucial interval cash flow: thus that schedule which is OPTIMUM in terms of cash flows is the least attractive in terms of aggregate net present worth - liquidity requirements are in conflict with profitability requirements. (2) The great probability must be that this will very often be the case. For by definition, 'maximisation of present worth of shareholders wealth' predicates the capitalised values of cash flows at a common base time. Whereas 'maximising the servicing-adequacy of cash flow' predicates the contemporary importance of a series of liquidity positions over a range of time-intervals. Aggregation now is all-important to the first objective; continuity over time is all-important to the second. Only at a NIL discount rate will the two automatically reconcile, so long as the totality of project values is the same for all implementation schedules. The larger the discount rate, the greater the potential conflict.

Everything will depend upon the cash flow time patterns in each schedule. For example, the present worth superiority of the CHALLENGER schedule can be shown to be more a result of the delay in the outlays on projects B and C than of any improvement in the time pattern of inflows. A present worth criterion will always be better satisfied by projects with cash flow patterns of delayed outlays and accelerated inflows: whereas a servicing-adequacy criterion will prefer a closer offsetting in time of the two flows.

A more complex conflict in objectives can be discerned. Optimising

(1) In one form or another, most eminent writers in the field of financial theory have stipulated this objective. It would be tedious to illustrate this. By 'wealth', I mean both income and capital gain subsuming generalised patterns of individual shareholder tax liability and time-preferences: cf Bobicheck & Myers (25). It is thus possible to speak of the maximising approach as being sufficiently correlated to an optimising objective. See Amey (2), p.14.

(2) Ignoring for the moment the present value of enabled amending projects.
of servicing-adequacy is desirable because it generates debt capability, which is needed to maximise project implementation finance. This exposition as so far advanced generally assumes full exploitation of the improved servicing-adequacy created by implementation rescheduling. But full exploitation means that no cash flows ever accrue to equity either as dividends or as retentions. All optimised cash flows are envisaged as promptly being pledged to creditors either as interest on or repayment of funded debt; or as instalment debt rentals or premiums. Any non-operating inflows such as writing-down allowances, investment grants or disinvestment proceeds will be similarly pledged. There is no equity in leased assets, and on the not unreasonable assumption that most funded debt will tend to be secured there will be no real equity in assets financed by this means. Even unsecured debt ranks prior to share capital, so that even assets financed in this manner are strictly not unencumbered. Any cash flow leverage would be utilised for yet further debt capacity. Any unexpired asset value remaining after debt redemption (and this is implicit in a present worth calculation but will be exposed only to the extent that conventional asset valuations are equal to or below the then present value of the remaining income-earning capacity of the asset) will be used to generate further debt. Only when all loans are redeemed out of total asset disposal on a winding-up, will there be a sudden increment to shareholder wealth: presuming that leased assets are surrendered without any penalty. Truly a case of "jam tomorrow".

The market value of a share might reflect the estimated surplus on a winding-up. A necessary condition for any such surplus is that realisation prices of unencumbered assets should be in excess of the book value of the equity. This is a notoriously difficult area of estimation. Any such estimate must be subjective both in quantitative content and as to when it is expected to accrue. The only thing that can be even moderately certain is that the event will be somewhat

---

(1) Insofar as such allowances appear rather as a reduction in tax liabilities rather than as positive cash inflows; their reinvestment assumes an explicit managerial restriction of working capital to a given level unless investment projects in stock-holding or in customer-credit policy is included in these considerations.
distant, in the large majority of cases. If the value of a share is taken to be a function of the present worth of the series of future cash flows expected to accrue to ownership of that share: the increment in value arising out of a distant surplus of considerable uncertainty is not likely to be very great.

So long as shareholder expectations are that management strategy will continue to be one of full exploitation of servicing-adequacy, there can be no expectations of dividends, and no expectation of retentions. Accordingly there can be no expectations of capital gain in the form of an increase in share price based on either growth expectations or potential scrip issues. (Indeed, any scrip issues can only be regarded as a dilution of the present equity interest; causing a pro-rata fall in the share price). Although there are cases of share prices improving in a nil-dividend situation - e.g., Ling-Temco in the U.S. and Crown Cork in the U.K.; there are in both cases strong growth expectations within the spectrum of shareholder time-horizons. One interesting possibility remains: a total exploitation policy must largely be reflected in the profit and loss account in the form of leasing premiums, interest charges and similar financial charges to profit. Taxable profit will be at a minimum and unused capital allowances would accumulate to a useful sum. Share prices might then show a gain in anticipation of take-over moves. The possibility would again be heavily discounted, however. Generally, a total exploitation policy offers nothing to shareholders, and a steady fall away in share prices seems inevitable. The result may appear to demonstrate that the primary managerial objective in such a policy is more the maximisation of long-term creditor wealth than that of shareholder wealth. The assumption of total exploitation of servicing-adequacy must obviously be relaxed as being inconsistent with the stipulated objective of maximising the present worth of shareholder wealth.

The Discount Factor

On Page 216 reference was made to the possibility that the exploitation of servicing-adequacy might have repercussions upon the cost of capital. It is necessary to repeat that this reaction arises solely because the cost of capital is seen as a weighted average of the equity yield and the yield or interest cost of loan capital, (1) i.e., ultimately distributable retentions.
funded or instalment. It was argued further on Page 206 that the interest cost on debt will ultimately be an increasing function of the amount of debt exploited by the company. It has been contended that in the current state of the U.K. economy there is no reaction in the equity yield to the use of a very wide range of capital gearings. Finally it has been argued that instalment debt is in its nature, and in the manner in which it is presented in financial statements, somewhat inconspicuous and therefore somewhat less likely to exercise an upward influence upon the cost of debt capital. From all this it follows that any reaction upon the cost of capital which might arise from the exploitation of servicing-adequacy is likely to be solely through the agency of an increase in the cost of debt capital, and that any such increase will be accelerated to the extent that instalment debt is used in the capital structure.

Suppose the company now in illustration were to be at that stage in its loan capital operations where interest rates can be expected soon to turn against it. Let it be further supposed that the debt servicing-adequacy of cash flow can be measured by a parametrical coefficient with a value of 2.5. That is to say, the schedule of debt capability and servicing cost which faces this company is such that, for the present level of debt employed, £1 of cash flow is adequate to service £2.5 new debt capital. It must be remembered that 'service' includes an estimate of the ability to redeem that debt as well as cover the interest payments thereon, either as separate considerations for funded debt or as a series of combined premiums for instalment debt. For the remainder of this whole discussion of finance-investment models and the subsequent procedural approach, the following symbols will be used:

'Sₜ' : the cash flow accruing in a time-interval 't', where such cash flow is net of all operating out-flows, capital expenditure and other unavoidable or priority discretionary outlays. It is inclusive of both operating and new project inflows, plus all non-operating items such as grants, capital allowances and asset disposal proceeds.

'Sₜ*': the maximised value of Sₜ, where Sₜ is the lowest
value in the series $S_1, S_2, S_3, \ldots, S_n$ accruing to
the 'n' time-intervals of a given study period. The
maximisation of $S_t$ is achieved by project
implementation rescheduling.

\[ f(S) \text{ or } f(S^*) \text{ : the loan capital generating capacity - the servicing-adequacy of the cash flows } S \text{ or } S*. \]

Thus for this company:

\[ f(S^*) = 2.5 \]

and Table 10 indicates the situation which might arise. It is based on optimising inflows only, which avoids rescheduling with each new project addition; and also ignores sub-optimal satisficing constraints. It is therefore a much less than complete treatment. As the Table is illustrative only, small variations which realistically might be ignored are here taken to be important.

**TABLE 10.**

<table>
<thead>
<tr>
<th>Schedule</th>
<th>Min. Interval Cash Flow (Intervals $t_{2b} - t_{5a}$)</th>
<th>Capital Mix</th>
<th>Cost of Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity $M_1$</td>
<td>Loan $M_2$</td>
<td>Equity $'a'$</td>
</tr>
<tr>
<td>1. Original</td>
<td>450</td>
<td>2540</td>
<td>1270</td>
</tr>
<tr>
<td>2. Optimal</td>
<td>550</td>
<td>2540</td>
<td>1520</td>
</tr>
<tr>
<td>3. Amended 1.</td>
<td>575</td>
<td>2540</td>
<td>1582</td>
</tr>
<tr>
<td>4. Amended 2.</td>
<td>575 + $a_{jt}$ min</td>
<td>2540</td>
<td>1582 + $f(S)a_{jt}$ min</td>
</tr>
</tbody>
</table>

* An approximate function of £1270 @ 6%, £250 @ 8% and £62 @ 12%.

At the first amended stage the cost of capital is sufficiently reduced to make project E (see Table 7) yield a positive net present worth; but the debt capability of £62n. is inadequate to finance E (£700n.) Presumably some project 'j' exists with capital outlay $b_{jo}$ and inflows $a_{j1}, a_{j2}, \ldots, a_{jn}$; for which

\[ b_{jo} \leq £62n \]

and

\[ b_{jo} \leq \left[ \frac{n}{t=1} \left( \frac{a_{jt}}{(1+k_{4} t)^t} \right) \right] \text{ where } 'k_{4}' = 9.8\% \]
Patently this process is capable of reiteration until there are available for implementation no more projects whereof the capital cost is no more than the last potential increment of loan capital and whereof the net present worth at the last average cost of capital is positive. Equally patently, as companies observably do not proceed to abnormally high levels of funded or instalment debt — although they evidently proceed to very high levels — there must be some limiting factor to the process. One such factor has already been reviewed; an upper limit to loan capital set by a melange of institutional cost and constitutional factors — significantly weakened though these may be by recourse to instalment debt. Other factors will be seen in due course to lie in \( f(S) \).

Section 3. The Implication of Financial Linkage

For the moment it is convenient to review certain other complexities attaching to the discount factor. It will be observed that the process as far described is a generalised version of what Bower\(^{(1)}\) has termed "FINANCIAL LINKAGE". It will be recalled that Bower suggests that financially linked projects be netted of their debt-capability and assessed by a pure equity discount rate; except for \( 100\% \) self-debt-financed projects. I have noted earlier an acceptance of the second part of this theory, but the first part — the use of a pure equity rate — is rejected.\(^{(2)}\) This rejection is based upon an insight into the maelstrom of reaction and inter-reaction created by the influence of general financial linkage of projects upon the accepted objective of maximisation of the present worth of shareholders equity. For this objective requires that the aggregate of the present worths of projects be maximised. Let us call this aggregate \( \Pi \). Let the projects involved \( (A,B,C,D,E...J...P) \) be

\[(1) \text{ Bower } (43).\]

\[(2) \text{ Although, as Bower's theory also involves deducting interest costs from net cash flows, it would very conveniently resolve the different requirements of } f(S) - \text{ net of interest } - \text{ and the discounting process as noted on Page 219.}\]
comprised within the set 'X' (which will later be argued as finite). Using the symbols as so far developed, a simple model of the system might be:

Maximise

$$\Pi = \sum_{j=1}^{P} \left[ \sum_{t=1}^{n_j} \left( \frac{a_{it} - b_{it}}{(1 + k)^t} \right) \cdot x_j \right]$$

subject to as yet unspecified constraints and where, for the illustration used, n = 14 time-intervals. 'a', 'b' and 'k' defined as in the last Section. However, one constraint has been defined - that the capital cost of incremental projects must not exceed the incremental instalment of capital. Let M1 represent a necessary minimum equity and M2 total generated loan capital. Then:

$$\sum_{j=1}^{P} \left[ \sum_{t=1}^{14} \left( b_{jt} \cdot x_j \right) \right] \leq \left[ M1_t + M2_t \right]$$

where M1 and M2 are essentially measures of liquidity rather than balance sheet values. But M2 is increased by optimising implementation schedules such that S is optimised to S* for the operation of f(S*).

As indicated in Table 10 the effect well may be to vary k. It has been seen that, ex hypothesi, there is a conflict situation between optimising S and (for a given X and k) maximising \( \Pi \). Dogmatically,

1) as M2 varies, X will increase;
2) as X increases, \( \Pi \) will increase;
3) as M2 increases, k will vary;
4) as k varies, \( \Pi \) will vary;
5) to increase M2, X must be re-ordered;
6) as X is re-ordered, \( \Pi \) will vary for (k=constant);

Two matters arise:

1) Bower's theory becomes very attractive, because it advocates discounting by the cost of equity capital e. And e is so far held conveniently constant.

But in the situation predicated here, X may be in any order. One or more projects will have to be 'starters' to create the first opportunities for S* and hence f(S*). These

(1) Otherwise what is there to be maximised? But note that except for extreme values M2 \( \neq f(M1) \).
'starters' will ultimately have to be appraised if $\Pi$ is to be computed. This appraisal involves a valuation of $k$ which has been seen to be a weighted function of $e$ and $i$. The weights are of course $M_1$ and $M_2$. But $M_2$ (loan capital) is within limits a function of servicing-adequacy $f(S)$ - more specifically and in the optimal solution, of $f(S^*)$. Yet $f(S)$ is at any rate a function of the projects undertaken, $X$, and in the particular an optimal rescheduling of $X$ to optimise $f(S^*)$. How can it be known in advance that the 'starters' in $X$ will be such as to yield the optimal scheduling to give $f(S^*)$? And yet pending some ordering of $X$ how can one know what $f(S)$ and hence $M_2$ and hence $k'$ will be, and so how is $\Pi$ to be computed? But if $\Pi$ is not computed (and computed at a maximum) how can one know that the set $X$ is an optimal set? There is a presupposition of an unstable model.

In addition, Bower's theory would use the existence of $f(S)$ or $f(S^*)$ to justify netting all projects of their debt-capability and discounting by a pure equity rate. That is, the negative cash flows of interest payments emuring to a project must be deducted from the linked positive cash flows of debt finance arising out of the increment to $f(S)$ created by that project. In short, projects would be appraised net of interest costs and by an equity based discount rate. But the argument of the previous paragraph casts doubt upon the possibility of quantifying $M_2$. This involves potential variations in the interest rate 'i'. Interest cost is thus unknown and cannot be deducted in advance of a solution of the total model. Under Bower's theory each project $X_A, X_B, \ldots, X_p$ would have a different value on each rescheduling of project implementation (re-ordering of $X$). In short, until $\Pi$ is maximised it can not be determined what project will be financed by what means, and $k$ can have one common value for all projects appraised within this model only for one re-ordering of $X$ at a time. (Very large, self-debt-financing projects would seem to be external to the model: by definition, they are not financially linked.
forward. They might however be regarded hypothetically as 'starters' - but their certain size and complexity makes this technologically improbable).

2) More importantly an unstable model appears to have been envisaged. A priori, it is not possible to say whether relationships (2), (4) and (6) above - all involving \( \Theta \) - are additive or opposed. The circular insolubility discerned in the penultimate paragraph is disturbing. It becomes necessary to develop further constraints to the model in an endeavour to reach a stable and soluble version.

(The 'financial-investment linkage' model).

The method of presentation of the model will be:

1) To 'set down the model and its constraints.

2) To discuss each constraint in turn.

Fundamental assumptions to the model are:

1) A limited liability company with share capital and a wide enough formal or informal market for those shares to permit a fair market price for the shares to exist.

2) An objective of maximisation of the present value of shareholders wealth.

3) A managerial willingness to use leverage, subject to the limitations specified in the constraints.

4) Unimpeded recourse to all segments of the loan capital market up to the limit of borrowing powers \((L)\) imposed by the Articles of Association; admitting (deliberately) uncertainty as to the limits imposed thereby to the use of instalment debt, but supposing that the limitation is less ambiguous to auditors than to management. Hence, \((? L)\) will in fact be used, \('?\) indicating the uncertainty. By 'primary' lenders will be meant first-class financial institutions.
dealing mainly in funded issues which will be the subject of an offer for sale; and bank overdraft facilities other than mere short period bridging finance. By 'secondary' lenders will be meant smaller institutions, private placings and lines of credit, and instalment debt. The classification is consistent with that adopted in Chapter One.

5. Shareholders of a sufficient number and/or sufficient means such that the company can sensibly consider the raising of new equity capital, at however infrequent intervals.

The model as developed here is without doubt inelegant and incomplete. This is in large measure due to a lack of expertise in the field of mathematical programming. But the use of a formal framework, without specifying the forms of the functions involved, permits useful insights into the increasingly complex layers of inter-relationships implicit in such a model as this. Hopefully, a sufficient perception of the elements and relationships involved will be achieved such that some analyst more competent in mathematical programming may build on the suggestions presented here, and adde the necessary functions to permit a closer examination of the purely computational requirements of the model. Hence this model is couched in deterministic rather than probabilistic terms; except that as was specified in definition (b) on Page 210, projects are envisaged as being of a common quality. The present purpose will be satisfied if the major determinants can be identified, their natures exposed and their inter-relationships investigated. As it is, the model is derived largely from Baumol & Quandt; and Weingartner.\(^1\) It cannot be too strongly emphasised that the model is in terms of cash flows and not in terms of balance sheet entities.

(1) Baumol & Quandt (40); Weingartner (36, 90). The elegant model developed in full detail by Chambers (48) was brought to my attention by Mr. R. Adelson after this present text was written. Chamber's model is designed to satisfy constraints of balance sheet conventions, and utilises only one source of finance - retained profits. It is nevertheless of seminal value in "linkage analysis" (my term), and its methodology and conclusions are not so very different from those now presented. See also T.C. Mao (70), Charnes Cooper & Miller (48), Bierman et al (9), Fabricky & Tongersen (33) and Miller & Orr (reported in Robichek (26) Chap.6).
AN INVESTMENT – FINANCE LINKAGE MODEL

MAXIMISE \( \pi \) FOR

\[
\pi = \sum_{k=1}^{n} \left[ \frac{p}{j=1} \left( \frac{a_{it} - b_{jt}}{(1+k)^t} \right) x_j \right]
\]

for \( x = x_A + x_B + \ldots + x_p \)

and \( t = 0, 1, 2 \ldots n \).

1. PROJECT CONSTRAINTS

1.1. ALL \( x_j \) ARE INTEGERS AND NON-NEGATIVE

1.2. \( x_A \neq x_B \neq \ldots \neq x_p \)

1.3. \( t_{bA} > t_{BB} > \ldots > t_{bp} \)

1.4. \( t_{aj} < (t+1)_{aj} < \ldots < (t+n)_{aj} \)
2. **Flexible Capital Rationing Constraints**

2.1. \( \sum_{j=1}^{P} \left[ x_{jt} \cdot b_{jt} \right] \leq \left[ m_{1t} + m_{2t} \right] \)

2.2. \( m_{1t} = R(S_{t}) + m_{1N} > 0 \), for \( 0 \leq R \leq 1 \)

2.3. \( 0 \leq m_{2t} = \left[ f(S_{t}) \right]_{t} \leq (zL) \)

2.4. \( S_{t} \equiv S_{t} = \sum_{j=1}^{P} \left[ a_{j} \cdot b_{jt} \cdot x_{j} \right] \), maximised for \( a_{j} \cdot x_{j} \), when \( x^{*} \) is the optimum project set \( X \)

2.5. \( f(S) \equiv f(S^{*}) = \left[ R, m_{2t-1}, i \right] \)

3. **Cost of Capital Constraints**

3.1. \( x_{t} = w(s_{t}, i_{t}) \)

3.2. \( i_{t} = z[m_{1t}, m_{2t-1}, f(S)] \)

3.3. \( e \) is, for the moment, an independently determined parameter.
In addition to those previously specified the following symbols have been used:

- **MlN**: new issues of equity which raise new money: i.e., excluding scrip issues or substitution for vendors' equity, and excluding funding of existing short-term debt.
- **y**: cash flows ensuing from previous periods - 'operating cash flows'. (This symbol is not used until Page 244)
- **R**: a retention coefficient. Note that this is a retention of post-dividend cash flows, the other part of which is used to finance new loan capital.
- **a**: cash inflow, net of interest costs and planned shareholder distributions. This is discussed at a later stage of the argument.
- **w**: a weighted average function.
- **Z**: a discontinuous cost function.

Other symbols will be developed as the need arises.

The looped or simultaneous relationships giving rise to insolubility are clear; see for example constraints 2.3 and 2.5. Faced with this sort of problem, it is common practice to have recourse to determinants external to the model: typically, to introduce a predetermined consumption-investment utility function and solve in one way or another by isoquant analysis. (1) This way out is not available here, for there is a specific commitment to invest - otherwise increases in \( \Delta Z \) would be irrelevant. (Theoretically, management could increase debt capital and squander, consume or hoard it; rationally so, if thereby a positive net present 'utility' worth accrued to them in/capacity of major shareholders. Factually, loan capital suppliers tend to be less flexible in their attitudes - especially when further increases are sought. A "lender's displeasure coefficient" could be applied to \( f(S) \), but it seems hardly worth serious consideration). In a sense the \( f(S) \) function is a utility function, maximising time-interval liquidity instead of

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(1) See for example Hirschleifer (64). However Baumol & Quandt (op.cit.), Weingartner (op.cit.) and Lorie & Savage (69) ignore the problem by assuming some given value for 'k'.
subjective utility.

One external determinant which reasonably could be introduced is managerial capacity. This is essentially a short-period model, and in the short period limitations of managerial capacity are likely to be pretty inflexible. The effect is to be expected as setting lower and upper limits to the effective or meaningful ranges of values to be attributed to certain coefficients and functions within one study period. Recruitment, a wider experience, education of all sorts, technological innovation; these will combine to set different limits and values as between study periods. This is a shifting pattern in the epic manner and the present model is capable only of one-reel moving pictures. Further study of the model constraints will reveal inter alia what additional external determinants can with safety be introduced.

A second external determinant which can however be introduced from the outset is the arbitrary setting of $t$ to a value of $n$ time-intervals. In this model, the use of a five-year study period and of one 6-month time-interval delay between each project implementation would give $n=14$ for five projects, (i.e., $X = A, B, C, D, J$). Baumol & Quandt (op.cit.) observe: "Unfortunately the literature offers no satisfactory method for solution of the horizon period". In the article referred to on footnote (1) to Page 174, five years appears to represent the practical limit to forecasting periods for firms other than the large international corporations. This convention has been adopted in this model, which is after all part of the general corporate planning model. Obviously, $n$ would vary even within a given study period according to the number of projects and the length of time-interval lagging. It is not necessary that $t$ be a constant dimension, but it obviously is most convenient to the solution of the model. Indeed, in corporate profit planning, it is common practice to express plans quarterly for the proximate years, semi-annually for the mid-distant years and annually thereafter. But if inter-implementation time lags and time-intervals are both constant:

$$n = (X - 1)\lambda + SP.T.$$

where $\lambda$ is that time lag in terms of time intervals, $SP$ is the study period and $T$ the number of time intervals per time unit of study.
period. (X is used here to denote the finite numerical size of the set. In the present model, T = 2, as the time unit of study period is a year and cash flows are measured at 6-monthly intervals). Another alternative to determine the study period would be to restrict this to that period within which no new major share issue could be made. This is not utterly impractical, and would be most useful in quantifying $M_t$. Some problems would arise if the study period was made too short by frequent rights issues - e.g., Royston Industries, Power Securities.

I turn now to a study of the constraints to the model.

1) Project Constraints

The set $X$ comprises the relevant projects $A...J...P$. FIRSTLY: this set is deemed to be finite in any one study period. This accords with limited managerial capacity both in terms of innovation (often in short supply) and competent project supervision. Generally speaking, $X$ is probably fairly small numerically so far as really significant changes in the value of $S$ are concerned. Iterative routines required to establish $S^*$ may turn out to be quite simple. In a sense, $X$ consists of two sub-sets:

$X_0$: termed 'starters' on Page 230. If such projects are denoted as '$j_0$', the sub-constraint could be introduced:

$$2.1.1. \sum_{j=A}^{P} \left[ X_{j0} \cdot b_{j0} \right] = \left[ M_t + M_2_t \right]$$

$X_1$: the balance of projects, such that $X = (X_0 + X_1)$

FIRSTLY: a priori, it is impossible to say very much about these sub-sets. In the illustration so far used in this discussion, $X_0 = X_A', X_B', X_C'$, and $X_D'$. It is possible that managerial strategy

(1) "...... in practice there are reasons other than the availability of cheap capital which place distinct limits on investment, reasons which may be perfectly valid and rational from an overall management point of view; such as the limits of managerial or organisational capacity...... I accept as part of reality that non-financial restraints on investment decisions are often dominant in many of our leading business corporations." Donaldson (1969), p.199.
and the imperatives of technology will establish non-financial priorities to certain projects. Otherwise the programme calls for scanning each possible combination of projects satisfying the model and sub-constraint 2.1.1.

SECONDLY: due to limits on managerial capacity, projects can be started in any sequential order with specific time intervals 't' between each (constraints 1.2 and 1.3)(1); remembering that in this model projects are indivisible (constraint 1.1) and that when started, cash inflows are non-discontinuous (constraint 1.4). Staggered implementation is consistent with limited managerial capacity, but a more sophisticated model might usefully vary the staggering of implementation as between (say) pairs of products. 'n' would then have to be determined by separate calculation. Equally, the assumption of indivisible outlays is one of convenience rather than necessity: but sequences of outlays for one project would certainly add considerably to the computational complexities of the model.

THIRDLY: constraint 2.1 sets an upper limit to the value dimension of X, but it does not necessarily set an upper limit to the numerical dimension of X. This will be so where \((M_1 t + M_2 t)\) represents an effective capital rationing situation - i.e., where both (but especially \(M_2 t\)) are at maximum values. X may still be finite but in excess of \((M_1 t + M_2 t)\), in which case project schedules and/or amendments will have to reject projects as E was rejected in Table 10. Rescheduling, or more especially re-amending, might consider several small projects in competition with a few large projects, whereas \(X\) would potentially be of different numerical dimension at each rescheduling. 'n' would then vary also. What is more; if, as is realistically probable, inter-implementation lagging were to be in different time interval dimensions as between large and small, large and large, small and small projects, (small projects being freer of the constraint of managerial capacity); calculation of 'n' might become most complex. A predetermined study period would become 'n'-dimensionally

---

(1) Following Weingartner; 1.2 indicates the reversible sequential chain nature of scheduling, with the financially-linked contingency of one project on its predecessors. 1.3 indicates the potentially free initial ordering of the chain.
elast, for each schedule. This would vitiate inter-schedule comparisons, so that some quite arbitrary cut-offs might have to be applied to the value of 'n' for certain rescheduling iterations - thereby weakening the optimising potential of the model.

It becomes apparent that two of the seemingly more simple elements of the model - 'n' and 't' - are in fact quite complex and not easily susceptible of arbitrary pre-quantification as potential external determinants.

Also, it seems almost superfluous to point out that \( \Pi \) will vary the more for each iterative implementation rescheduling, the more different are the variations in the size and the time patterns of each project's cash flows. Given closely similar time patterns and closely similar sizes of cash flow across those patterns, \( \Pi \) will not vary much - subject always to the size of the change in 'k'. Finally, it is necessary to note that the requirement that all model values shall be non-negative, requires that disinvestments be represented as 'a' flows rather than 'b'; and loan capital redemptions be termed 'b' flows rather than '-a' flows. Other occasional elements will doubtless prove to require similar expression.

2. Capital Rationing Constraints

Baumol & Quandt assume a rigid capital rationing condition. The present model is one of partially flexible capital rationing - only partially because limiting maximum values of \( M_{1t} \) and \( M_{2t} \) are envisaged; but flexible because different amounts of \( M_{1t} \) and \( M_{2t} \) are envisaged; and yet capital rationing because each amendment is rationed to the debt capability of the previous rescheduling or reamendment. (1)

Constraints 2.1 and 2.1.1 are the basic capital rationing

(1) The maximisation of \( M_{2t} \) in the form of an infinitely high rate of interest is termed by Hirschleifer (64) a "Scitovsky situation"; and this he describes as "empirically significant only in the short term". In the present context this is an attractive idea, in view of the different \( S^* \) and \( f(S^*) \) of different (short-term) study periods. It does however under-emphasise at least one long-termed circumstance where this will apply; i.e., the constitutional restriction on \( M_{2t} \) (that is, (? L)) where \( M_{1t} \) is static or very nearly so.
expressions. The 'problem areas' of exposition lie in the elements $M_1, M_2$ and $f(S)$. Certain minimal characteristics of these elements can be discerned.

(i) $M_1$ must be positive; the model is of a profit-making limited liability company maximising the value of shareholders' wealth. Also, suppliers of loan capital will ordinarily expect to see some equity commitment somewhere in the firm. (1)

(ii) Within one study period, $M_t$ probably varies from $M_{t-1}$ principally due to the retention coefficient $R$. The possibility of a study period of that duration which normally would preclude new or rights issues has been mentioned. Yet in any event, such issues are seldom tied to individual projects other than those very large one-off projects which because of their fundamental nature tend to lie outside the scope of normal capital expenditure budgeting (see Page 231). The more usual purposes seem to be either to reduce outstanding redeemable debt commitments, to fund short term debt (most typically bank overdraft), or generally to improve the liquidity of the organisation as a whole (i.e., for working capital as well as fixed capital projects). Often the two aspects are joined in one issue. Only the third aspect affects the capital rationing system, although all three may affect 'e' and hence the project appraisal rate 'k'. If the first two operations be termed 'redemption' (MNC) out of total new issues (MNH) the following additional constraints can be specified:

\[ M_t = MN_t - MNC_t + R(S_t) - RC_t \]  

(2)

(1) This expectation is too specific to the firm and to the occasion to constitute any significant reverse debt-equity ratio convention. Its existence, and the impact of its satisfaction on the terms of a loan (especially from a bank), however; account for the inclusion of $M_t$ in constraint 3.2. The phrase 'equity base' is sometimes met, implying some minimum commitment or debt-equity ratio. However, it seems clear that what is meant is rather a lenders' minimum than a managerial optimum ratio. Leverage does require an equity commitment to make basic sense.

(2) More than in any other instance, this expression indicates the marginal cash flow nature of the model. Invested equity inherited from prior study periods (say, $M_{t-n}$) is irrelevant. Obviously ($M_{t-n} + M_t$) would represent a balance sheet figure at time 't'.

...
(It must be noted that a similar coefficient RC will exist if redemption is from existing cash balances or from retained cash flows generated by budgeted projects).

2.2.2. \( M_2^t + RC^t = c(M_2^t, U) \)

a predetermined redemption function to be applied whenever the model generates a value of \( M_2^t \) above some level 'U'. This would not reflect an 'optimal ratio'. It would reflect a redemption liability, leading to:

2.3.1. \( 0 \leq M_2^t = (f(S^t), 1) - c(M_2^t, U) \leq (7L) \)

if redemption were to be of a revolving fund of short term debt facility, e.g., an acceptance credit. (Constraint 2.3.1. is a modification of constraint 2.3).

It is not likely that the \( c \) function would be large, especially if instalment debt (which is period self-liquidating) were the larger part of \( M_2 \). Similarly, undated funded stock in \( M_2 \) would reduce this function. Finally, the \( c \) function would probably permit a re-issue of \( M_2 \) in the next interval especially if temporary non-funded finance such as bank overdraft was redeemed. In short, constraint 2.2.2. is seen as something of a fail-safe device in the model and therefore (subject to due concern over the effect on \( e \)) could be quantified at a fairly arbitrary level.

\( \text{MIN} \) or \( \text{MIN} - \text{MIC} \) can be introduced only as an arbitrary independently determined factor. In some sense, it is quantified only by the failure of the set \( X \) as measured by constraint 2.1. to include all potential projects with a positive net present worth now available to the company. Limitations, in managerial capacity have been shown to be fundamental to the derivation of \( X \), so that the existence of a 'project gap' of this sort is still no guarantee that \( \text{MIN} \) would be activated. There are further the extraneous but important matters of timing new issues and their associated administration costs. For each study period, the opening \( M_2 \) value is inherited from the last study period (in the form of uncommitted funds generated in that study period) and all that can be envisaged at this stage.
in respect of MIN in the new period is an arbitrary instruction to provide for some given value for this, after an inspection of total project availability and, so far as can be discerned without evaluation of the model, a rough estimate of both financial capacity and redemption commitments (other than for self-liquidating debt).

(iii) The introduction of the retention coefficient $R$ into constraint 2.2. is capable of producing yet another instability factor into the model. Firstly it is necessary to clarify $R$. Capital allowances, investment grants and other non-discretionary generatives are subsumed in $a_{jt}$. $R$ therefore refers to the retention from the net cash inflow $a'_{jt}$ and would be a function of:

1. The rate of distribution, if any, of investment grants by way of subsidy to depreciation or taxation provisions. (1)
2. Transfers to and from tax equalisation or other deferred taxation accounts.
3. Appropriations to revenue reserve from disposable after-tax cash flows.
4. Transfers to capital reserve of non-operating income, other than for wholly non-cash gains such as asset revaluation.

It is important to stress that $R$ is a cash relationship. These four determinants require pre-quantification: yet items (1) and (3) derive directly from the model itself. Grants are obviously a function of project implementation and timing - i.e., of the optimum schedule. Appropriations (or conversely dividends) relate directly to $\Pi$ insofar as shareholders prefer income or capital gain. This last problem might be overcome by an independent decision establishing a reference pay-out ratio. As well as facilitating a quantification of $R$, this would much simplify the problem of quantifying $\gamma$ if it were thought that this might be attempted via a Gordon-Shapiro

(1) Thereby increasing distributable surplus.
approach. It is also tempting to regard (1) as not very important; but in fact this seems increasingly to matter in earnings distributions. (2)

(1) A constant or reference pay-out ratio does not seem a realistic assumption in the context of U.K. industry, where the preference seems to be toward a constant dividend - c.f. the short earned dividends of I.C.I., Albright & Wilson and (nearly) Dunlop in recent years. The situation is additionally confused by the existence of voluntary or compulsory dividend restraints imposed by the Government prices and incomes policy. A basic preference for a stable dividend rather than a stable pay-out ratio is shown by Benishay (41) to be a significant factor in supporting share prices, and so is a quite justifiable policy. BUT IT IS ESSENTIAL TO RECLOAD THAT IN THE FINANCE-INVESTMENT LINKAGE MODEL, THE DIFFERENCE BETWEEN CASH-FLOW AND RETENTIONS IS NOT DIVIDEND. RETENTIONS ARE CAREFULLY DEFINED SUCH THAT DISPOSABLE CASH FLOW IS THE SUM OF RETENTIONS, DIVIDENDS AND LOAN FACILITY FINANCING. The following constraints or expressions have been or will be defined as essential to the model:

\[
S = f(a' - b)X
\]

\[
R = S - f(S)
\]

\[
a' = a - i - qM + y : \text{where } i = \text{loan interest}
\]

and q is a dividend coefficient.

Substituting 2.4.1. into 2.4., and thence into 2.2.4,

\[
R = f(a-b)X + y - i - qM
\]

\[
= f(aX) + y - (f(bX) + i) - qM - f(S)
\]

or

\[
R + qM + f(S) = f(aX) + y - (f(bX) + i)
\]

and as the second half of this expression is interpretable as cash inflow from budgeted projects plus cash inflow enuring from previous study periods (cash from operations); less cash outlays on budgeted projects and less loan interest; it is in fact disposable cash flow within the study period. Remembering that all related tax flows are subsumed in \(a, b, q, i\) and \(y\).

(2) An informal survey of some 85 larger companies publishing accounts for financial years ending January-December 1967 indicates that retained grants constituted 2/8% (average 4%) of net capital employed; and this after so short a life of the scheme. Rough extrapolation indicates that after five years, and assuming no major change in the size of grants or in the conditions under which they are made, investment grants alone might represent as much as 13% of net capital employed. If locational grants are added, by no manner of means can grants be dismissed as unimportant.
Accordingly it would be necessary to specify a constraint of the general form, of:

$$2.2.3. \quad R = \left( g \sum_{j=A}^{P} (x_j b_j) \right) + (1 - r) (S_t - G_t)$$

using $g$ as a coefficient of grant retention, $G$ as a measure of grants received (not due) and '$r$' as the basic reference pay-out ratio. But this would at once render constraint 2.1. insoluble. In the end, $R$ has to become an independently determined coefficient - probably a simple fraction of disposable cash flow held constant over all time-intervals within a study period. This is a distinct weakness; but it is necessary for another reason. Loosely, a maximum value of servicing-adequacy of cash flow is:

$$2.2.4. \quad S_t = R(S_t) + f(S_t') \text{ or } f(S_t') = S_t - R(S_t')$$

although $f(S_t')$ is in fact subject to additional influences, as will be seen. If $R$ is, through constraint 2.2.3. made unstable; $f(S_t')$ would become unstable. As this would be an opposing instability; and as $R$ is a determinant of $M_l$ and $f(S_t')$ of $M_2$: the two instabilities might still result in the same total of available finance $M$. However, the relationships are all too tenuous for the analyst to be sure of this with an acceptable degree of probability. Even an independently-determined value of $R$ will introduce enough distortion to the problem of establishing optimal amendments to each rescheduling iteration.

The predetermination of a value for $R$ is in a sense equivalent to establishing a planned rate of growth of the equity. This is consistent with:

1. A limited but not fixed managerial capacity.
2. The reinvestment assumption implicit in discounting techniques in the context of factual business finance and accounting procedures. (See Appendix III.)

This consideration of $M_l$ ignores for the time being any inter-relationships arising with the cost of equity $e$. It is convenient, however, to note that the model is not directed towards optimising $e$. 
(iv) It must be acknowledged that a weakness of constraint 2.4. is that in concentrating upon maximising minimum time-interval cash flow, it completely neglects the additional desirability of minimising inter-interval variability of cash flow. If such variability is a measure of risk to the lender some impact on \( f(S) \) is likely, either directly or through \( i \) in constraint 2.5.

(v) \( M2 \) is determined largely by the intersection of the loci of \( i \) and \( f(S) \). It also has its own limiting value, which can be expressed as an infinite value of \( i \). The principal causes of this limiting value were set out on Pages 205/6. Of these causes, the most effective might be that set by constitutional limits, albeit weakened by ambiguity concerning instalment debt. The second cause listed on Page 206 (recourse to secondary lenders) can be expressed as a derived value of \( f(S) \). Hence:

\[
2.3.2.1. \quad M2 \leq \left[ v = (i, f(S)) \right] \\
2.3.2.2. \quad M2 > \left[ v = (?L, f(S), et al.) \right]
\]

where \( v \) is the value of \( M2 \) at the start of the upturn in the curve (Figure 1, Page 205) and 'et al' signifies causes (1) and (5) on Page 206. The somewhat ambiguous quantification of 'et al' might with advantage offset the ambiguity of (?L) to give for the second half of the expression:

\[
2.3.2.2. \quad M2 \geq \left[ v = (L, f(S)) \right]
\]

with \( L \) the more dominant the further above \( v \) is \( M2 \).

Usefully, the omission of \( i \) in 2.3.2.2. reflects the indifference to the cost of loans which characterises management who are desperate for funds in a crisis. In an extreme liquidity crisis, \( L \) is ignored (assumes an infinite value). If the firm is basically sound, \( f(S) \) becomes random; if the firm is not sound \( f(S) \) assumes a nil value. A minor constraint is that \( L = f(M1) \); see above, Para.(i) and footnote (1') on Page 240. Indirectly this gives \( M2 = f(M1) \) for \( M2 > v \) - which is an accepted constitutional limit to gearing having nothing
whatever to do with optimising debt-equity ratios.

(It is to be noted that the c function should be inserted in all constraints for M2; but as c is expected not usually to be large, it is omitted in order to simplify the exposition).

(vi) $f(S)$ is therefore the effective demand schedule for M2.

It is important to realise the $f(S)$ is a purely monetary function - the assessed ability to pay interest and otherwise service loan capital. Non-monetary considerations such as restrictive or consultative conditions attached by existing debt-holders to further tranches of debt are part of the self-determining values of M2 and appear as a tendency for $i$ to approach infinity. As management exploits the servicing-adequacy of cash flow, it is probable that the locus of $f(S)$ will initially be horizontal. At a later stage, suppliers of loan capital (especially of funded debt) will require a more ample cover; directly or by imposition of restrictions of some sort. The curve of $f(S)$ can be expected to become concave to origin. Ultimately, recourse will be had to secondary lenders, at which point $f(S)$ becomes discontinuous. At first thought, it may be assumed that the curve will shift to the left.

However such scanty evidence as exists tends to leave a suspicion that it might equally well shift to the right.

(In my own experience, private placings of debentures with the larger U.K., Canadian or American insurance companies are not cheap; but they are moderately easy to arrange, especially if accompanied by reciprocity in substantial company insurance matters, and very especially if supported by long-term supply contracts with major customers. In a sense, these companies are substituting direct or indirect interest charges for cover. Evidently, industrial banks in the U.K. tend also to do this, if the recent cases of Davies Investments and of Pimcock Finance are to be taken as reliable evidence). Where the substitution is overt, $f(S)$ becomes partially determined by $i$, as cash flow must be adequate to service higher interest costs (see constraint 2.5.). Thereafter $f(S)$
may resume a new concave path.

\[ f(S) \] is thus seen to be cause and effect. It is without doubt a determinant of \( M_2 \), and yet is influenced itself by \( M_{2-1} \), especially at the point of transition between primary and secondary lenders. At the same time, as it must be sufficient both to cover interest and to service redemption, \( f(S) \) is influenced by changes in interest rates. These concepts are reflected in constraint 2.5. The phrase 'et al' refers to a non-quantifiable aspect - Donaldson's important point that a wrong estimate of \( f(S) \) is a greater threat to management (to whom it may spell disaster), than it is to institutional lenders (to whom the principal of average loss will apply). Secondly, as \( f(S) \) is deemed to follow a generally concave path; but the administration etc. costs of raising capital contain a large fixed element; small and hence uneconomic increments of debt may be expected to develop. It would therefore seem necessary to apply a fail-safe device to \( f(S) \) in the shape of an increasing negative 'capacity and cost' factor which would reduce \( f(S) \) to a nil value somewhat earlier than would otherwise be the case. It is essential to note that none of these various determinants can fairly be termed an optimal equity-debt ratio in disguise.\(^2\)

\(^1\) G. Donaldson (11,50).
\(^2\) Compare G.F. Weston (intro. to 'Weston & Woods' (94) on Modigliani & Miller "Some Estimates of the Cost of Capital to the Electric Utility Industry" (A.E.R. June 1966):

"since they do not admit of a rising cost of capital function at some high level of leverage, they suggest that firms would be induced...... to use the maximum amount of debt. But as firms obviously do not use extremely high debt ratios, H.M. resort to the concept of a target debt ratio to explain the limit on increasing leverage. But a target debt ratio signifies some policy on the financing mix of debt and equity. But this is what business finance has argued and H.M. have denied. They state that:

'The determination of the optimal value of ...... the firms' target ratio involves many difficult issues

(Note (2) is continued on next page.)
(vii) The other principal determinant up to the value v (constraint 2.3.2.2.) is stated to be i. The locus of this element is discussed on Pages 205 & 206.

Suffice it to say here that the substitution of i for f(S) by secondary lenders might be expressed as an upward discontinuity at the point of transition from primary to secondary lenders.

(2) continued

for which no completely well-worked out theoretical analysis is yet available'. They imply that (this ratio) is related, but uniformly, to the "maximum permitted by lenders". This is an imprecise way of stating that the cost of capital rises, at some point, sharply with leverage. This in turn suggests an optimal debt to equity ratio'.

In order of the points made by Weston:

1. My own analysis unwittingly parallels M.M's so far as maximum limits to loan capital can be defined. The phrase "maximum permitted by lenders" can as easily mean "in terms of servicing-adequacy of cash flow" as "in terms of some conventional debt-equity ratio".

2. Some firms, anyway, obviously do use extremely high debt ratios. My argument (and presumably that of M.M.) is that these are those with enlightened financial management. Even Sample A firms in 1966-67 average over the 30% gearing often implied by American writers (e.g., Donaldson) as the limit of market acceptability. See also Note 2, to Page 174.

3. A 'lender permitted' maximum does not at all suggest an optimal debt-equity ratio; unless we suddenly substitute creditor security as the optimising objective in place of maximising shareholders present wealth. The maximum permitted by lenders is as likely to be above as below the optimum from shareholders point of view.

Weston is still arguing in terms of an advanced market risk-sensitivity which almost certainly does not exist; and still assumes that all loan capital is necessarily freely negotiable and enjoys a formal market.
(vii) continued.

\( f(S) \) acts as an indicator of this in constraint \( 3.2 \); where \( M_1 \) also indicates the minimum equity commitment. (See Page 241 footnote (1). Also \( M_{2-t-1} \) indicates the tendency of \( i \) to increase with \( M_2 \). The general principle is illustrated in Figure 2.

**FIGURE 2**

\[
\begin{align*}
\text{Debt Capital (M2)} & \quad \% i \\
0 & \quad Y \\
V & \quad D \\
E & \quad f(S) \\
\end{align*}
\]

where \( M_2 = OY \) is the discontinuity of transition between classes of lenders. At \( M_2 > OV \), \( i \) begins to become infinite. At \( M_2 = OL \), \( i \) becomes infinite. \( f(S) \) ceases to determine \( M_2 \) shortly after \( v \). Figure 2. is of course illustrative, not deterministic.

Deductive and empirical research into the nature and determination of loan capital financing is long overdue. Donaldson and Vancil apart, the complexities and controversies in the received texts on the cost of equity have tended to induce a cavalier treatment of debt.\(^{(1)}\) An increasing interest cost-loan capital schedule is always presumed; some reasons for doubting this have been put forward in this present text. The influence of instalment debt on capital budgeting and on the cost of capital seems to be overdue for consideration and analysis.

\(^{(1)}\) e.g., Bierman & Smidt (8) devote 15 pages to equity cost, and effectively one half-page to debt. Their treatment of instalment debt is purely mechanistic rather than analytical.

The use of a weighted average cost of capital, where the weights are the respective incremental cash flows derived from each source of capital, to define the discount factor $k_i$ was discussed in Section 1. One point of inconsistency arises in the use of a constant discount factor over the study period, assuming optimality of the model. The change in the project set $X$ and subsequently of $k$ in the next study period, to which some at least of the cash flows of the present study period will ensue, means that projects might in fact be assessed at two or even more somewhat unrelated discount rates. This risk seems unavoidable given the use of study periods of less than project lives, especially such that project outlays may be spread between two or more study periods; but in this model such risk is minimised because by definition all outlays are taken to be instantaneous. At the same time, project assessment within a study period assumes a constant range of values for $e$, $i$ and $k$ in that period - i.e., assumes no bodily shifts to the left or to the right of the various determining schedules. However, this problem is not novel, nor is it restricted only to this model. On the (momentary) assumption of an optimising solution of the model, such that in fact $N_2$ is quantified at any rate for one study period; $k$ is constant to the extent that $i$ is constant. But what of the other determinant of $k$ - viz. $e$?

Constraint 3.3. simply specifies a value for $e$ which is temporarily taken as externally and independently determined. Evidence has been adduced to support an independence of $e$ from $N_2$; but whether $e$ is independent of the components of $N_1$ as given in:

$$2.2.1. \quad M_t = N_t + M_t + M_{1, t} + R(S_t) - RC_t$$

is an issue demanding further consideration.

The two issues on which discussion can not be avoided are: $R$ - the effect of retentions; and $M_t$ - the effect of redemption operations.

**RETENTIONS.**

Utilisation of a constant value coefficient for $R$ was proposed so as to avoid the introduction of a complex instability, and was shown to be consistent with the behavioural assumptions of the model. On Page 243, attention is drawn to the facility that this would
accord to a Gordon-Shapiro cost of capital model as a means of quantifying e. Nor does the finance-investment linkage model, as developed, prima facie introduce any preconditions as to base or exit values of shares or as to dividends which are inconsistent with the Gordon-Shapiro approach. However, there still remain very substantial objections within the finance-investment linkage model to the use of this approach. Briefly, these may be listed as:

(i) The G-S approach assumes an expectation of a constant pay-out ratio. It has been noted that this is not consistent with U.K. practice(2)

   a) U.K. companies appear to prefer a constant after-tax dividend.
   b) Dividend limitation within Government prices and incomes policies precludes constant pay-out ratios in situations of earnings growth.

This is not inconsistent with the finance-investment linkage model. Footnote (1) to Page 244 outlines the relationships between retentions and equity dividend distributions. It is quite conceivable that the dividend distribution qM_t should be for a fixed sum or for a fixed proportion of equity capital as defined in that Footnote. (In this instance 'equity capital' must be taken as the aggregate \( \sum_{t=1}^{n} M_t \) for all study periods). However, given fluctuations in M_t, a fixed value for qM_t is illogical and inconsistent with the behavioral assumptions of the model. If q is set as a fixed proportion of equity capital, another potential instability factor is introduced to the model; thus

\[
S^*_t = \sum_{j=A}^{P} (a'_{jt}) \text{ etc., etc.}
\]

(1) See Appendix 2 for a review of some of the implications of the Gordon-Shapiro approach - referred to in the rest of this discussion as 'G-S'.

(2) There is some evidence that the reluctance to alter dividend rates is more powerful in a downwards than in an upwards direction; see also Walter (35) p.55. As the G-S approach is based upon a growth situation, a rejection of the concept of a reference or a target pay-out ratio is perhaps not so well grounded.
2.4.1. \( a'_{jt} = a_{jt} - i_t - qM_{t-1} + y_t \)

where \( y \) is the cash flow from existing operations as distinct from that from new projects, and enures from previous study periods. Now essentially:

\[
a_{jt} = f\left(\sum_{j=A}^{P} (x_j)\right) \text{ yet}
\]

per 2.1. \( \sum_{j=A}^{P} (x_j) = f(M_1, M_2) \)

and 2.2. \( M_t = R(S_t) \) etc., etc., accepting that \( S_t^* = S_t \).

There is a potential instability arising out of the presence of \( S_t \) function on both sides of the equation which results from the various substitutions implicit in this series of functions.

A proxy measure of \( qM \) would be to use a constant proportion of cash flow; i.e.,

\[
2.4.1. \ a'_{jt} = a_{jt} - i_t - qa_{jt}
\]

This would present no difficulty in the solution of the model. Interestingly, \( qa_{jt} \) would also represent a proximate measure of a constant pay-out ratio, thus conforming to the G-S assumption. But how good a substitute of \( qM \) is \( qa_{jt} \)? By the very conditions of the model, not a very good one. For if \( a_{jt} \) increases, such that \( f(S) \) increases; the repudiation of some preferred equity-debt ratio means that \( M_2 \) is as likely to increase as \( M_1 \). Equally, if \( a_{jt} \) increases such that \( f(S) \) increases; \( M_1 \) is as likely to increase by \( R \) - see constraint 2.2.1. - as by \( M_1 \) - yet no dividend is payable on \( R \). Consequently \( qa_{jt} \) is no real measure of \( qM \), and this otherwise attractive idea is not really acceptable.(1)

The objection to the G-S assumption is not overcome.

---

(1) One advantage which does attach to \( qa_{jt} \) is that it is a very clear measure of liquidity back-up to reported leverage. This is most true where an increasing series of \( a_{jt} \) \( F(S_t) \) and \( M_2_t \) leads to an increase in \( X_t+1 \). \( T_t+1 \) may be increased, but \( qa_{jt+1} \) is a sharp test of servicing-adequacy of \( M_2_t - 1 \).
(ii) The second implication of the G-S approach is a constant reinvestment rate together with a constant return on that level of reinvestment.

Thus: (using the original G-S symbols and noting the lack of specification as to whether the approach is expressed in profit or cash flow terms): if 'b' is the retention fraction of earnings Y which is reinvested, and 'r' is the return on that fraction;

\[ Y_t = (Y_{t-1} + rbY_{t-1}) = Y_{t-1} (1 + rb) \]

whence

\[ Y_{t+1} = Y_t (1 + rb) = Y_{t-1} (1 + rb)^2 \]

This does not preclude that \( r \) is a decreasing function of \( b \); but

"It does exclude the case where for a given b, r is expected to take different values over time." (1)

Gordon accepts the sweeping nature of this assumption of a constant return, and it can be shown that it is directly opposed to some of the basic assumptions of the finance-investment linkage model. Here the reinvestment rate is essentially derived from an aggregation of \( R \) and \( f(S^*) \).

The relevant G-S time periods \( t, t+1, \ldots, t+n \) would appear in the case of the model to be the entire study periods; thus, in the illustration used in this text, consideration of the G-S assumptions relates to time periods, for example, of \( (t_0 - t_1 = 5) \) years. The assumption of a constant return for each level of reinvestment is not likely to be met by the model, for the following reasons:

(a) Although \( R \) by definition is a constant function of cash flow (assuming cash flow to be an acceptably proximate measure of earnings), the reinvestment effect of \( f(S^*) \) is not likely to be constant as between study periods. The \( f(S^*) \) curve of Figure 2, is to be conceived of as being volatile in terms of bodily shifts to left or to right according to changes in economic climate, financial and fiscal legislation,

(1) Gordon (59) p. 438

(2) In fact, being a cash item it is probably a more acceptable concept than an earnings retention, in what is essentially a DCF concept.
and the degree of financial sophistication in the capital market. The finance-investment linkage model, as developed, very much looks forward to varying relationships between S and f(S*) at different periods of time. It is therefore most unlikely either that f(S*) will represent a constant retention fraction of total cash flow over a sequence of study periods; or that it will represent anything like a constant level of reinvestment.

(b) The rescheduling and readjusting of projects, implicit in the optimisation of f(S), almost certainly mean that the aggregate average rate of return of each potential project set will be much different from that of any other set. In a sense, ranking in this model is two-dimensional; involving ranking in terms of f(S) as well as "in terms of k." While the relatively inelastic treatment accorded to each study period more or less meets Bodenhome's requirement that:

"the rate of return on retained earnings in any period is independent of the volume of retained earnings in any other period: i.e., depends only on the volume of retained earnings in the period when the earnings are retained." (1)

so that k is clearly the prior criterion which must be met as a minimum; a whole range of average rates of return r* on different project sets of the same aggregate reinvestment value can be envisaged.

In other words, 'r' is not necessarily a unique value for a given 'b', even though by coincidence 'b' is held pretty well constant over two or more periods - a coincidence which has already been noted as not at all likely to accrue. (2)

---

(1) On this issue, and the relevance of Bodenhome's requirement see J.W. Bennett, J. McE. Grant and R.H. Parker (6) Chap.3.

(2) e.g.,

\[ Y_t = Y_{t-1} (1 + rb) \]

but \[ Y_{t+1} = Y_t (1 + r'b) + Y_{t-1} (1 + rb) \]

where b is a constant, but \( r \neq r' \). In addition, of course

\[ \frac{dr}{db} < 1 \text{ and } \frac{dr'}{db} < 1 \text{ but } \frac{dr}{db} \geq \frac{dr'}{db} \]
(iii) The G-S approach makes two assumptions regarding financial mix:

(a) That there is no new equity financing. This is not totally incompatible with the finance-investment linkage model if the alternative definition of a study period is adopted - i.e., one in which no new issues are made. But this could contravene the utilisation of the entire study period as the relevant time intervals of a G-S application. For new issues between one study period and the next are by no means improbable under the model.

(b) That there is a fixed expected equity-debt ratio. Under the finance-investment linkage model even a broad expectation of such a fixed rate is ex hypothesi impossible.

(iv) The G-S approach is an investor orientated evaluation process. While Gordon (op.cit.) claims that the approach is valid as an investment model

"to establish whether or not the investment of the corporation is determined by the objective of maximising its value"

evertheless the variables are by definition shareholder expectations. It is consistent with investment appraisal studies that management should seek to utilise such expectations in planning investment strategy. But in the finance-investment linkage model the second (albeit secondary) criterion of project selection - that of optimising f(S) - does exist. This is and can only be a purely managerial activity. Intuitively it seems inconsistent that the one criterion should be based upon shareholder estimates and the other on management estimates; indeed, under such management-shareholder conflict situations as those recently instanced

(1) The original G-S model postulated all-equity financing. Gordon (59) modifies this: "We assume that investors estimate a retention rate 'b' and a debt-equity rate 'q', that they expect the corporation to maintain for the indefinite future." (59) p. 438.
by Garda Trust; Cunard; British Printing Corporation; or Crown Cork Company; the bases might be very inconsistent indeed. (Dollar Land indicates a conflict in the other direction, with assumptional preferences between management and shareholders reversed.)

The G-S approach assumes a growth in share values exactly equal to the growth in earnings and (given a reference pay-out ratio) in dividends. The study by Samuès\(^1\) finds that on average share prices do in fact approximate to this; but goes on to say ".... a number of (particular) companies show significant differences between....dividend yield plus a factor for growth in dividends, and dividend yield plus a share price growth factor". The Econtel study 'Risk Factors in British Industry',\(^2\) estimates errors of market valuation of individual company share prices in the light of experienced earnings growth rates in those companies. The range of error is enormous - i.e., expectations were very often very wrong. Yet share prices in fact often relate pretty quickly to divulged earnings, thus bearing little or no relation to expectations. Finally, many 1968/69 share prices bear little or no relation to earnings performances, estimated or realised. The market has developed a rationale of its own, for reasons already discussed.\(^3\) Even the broadest approximation of \(e\) cannot be based on such an incorrect assumption. A further very valid objection refers to the sometimes very severe distortions arising in the narrow markets for the shares of individual smaller private companies.

For these reasons, a G-S type approach to quantify \(e\) does not appear sufficiently consistent with the finance-investment linkage model to encourage its use here. Some other alternative must be found.

---

One such might be the general U.K. average industrial equity costs

(1) Samuès, ( 81 )
(2) See Page 185
(3) A similar situation; and relevant comment upon the inapplicability of theoretical financial models as so far developed to the resulting share prices; is described in 'Fortune' November 1961, "Why the Stock Market acts that way".
developed by Merrett & Sykes. (1) A most useful extension is the range of values for this concept which is given in their 'Figure 4.1.', allowing for different levels of retentions. Two disadvantages are:

1. Their estimates of the costs of capital are averages. Samuels (op.cit.) points out the significant differences between average and individual company capital costs. Certainly it would not seem to be very valid for companies to use a common cost coefficient if they are divergent in capital intensity of technology, in rate of technological development and in economic type of market supplied. On the whole (and excepting the legal etc. costs of new issues) such evidence as is available does not point conclusively to any correlation (presumably inverse) of size of firm with cost of equity. (2)

2. The Merrett & Sykes data are essentially based upon the G-S approach. The objections to this in the context of the finance-investment linkage model have already been rehearsed. Nevertheless, an average of this nature, modified for the selected level of R, might provide a most useful and reasonable proxy valuation for e. The distortions in the required growth of share prices are smoothed by the joint averaging effect of the large number of firms and the long study period used by Merrett & Sykes. Any company which feels on grounds of technology or of market that its shareholders are exposed to some exceptional degree of risk can make use of the 'Figure 4.1.' provided by the authors. Finally, the authors' calculations are specifically based upon U.K. taxation procedures and levels. Certain other aspects of their contentions are however worthy of brief consideration if their values for e are to be used in solving the finance-investment linkage model.

(1) Merrett & Sykes ( 19 ) Chap.4.
(2) See references previously provided on Samuels and on Brigham & Smith. Some contradiction is provided by Benishay ( 41 ) who finds significant negative correlation between equity yields and size of firm. Durand ( 53 ) argues "it would be far easier, and make far more sense, to estimate a rough average cost for the ...... industry than to attempt individual estimates", basing this plea for averages on the probably large margin of error in most estimates of future cash flows.
Firstly, their categorical repudiation (their P.33) of the relevance of the cost of retained earnings in any weighted average value of k if new equity is to be used at all; on the grounds that the relatively more expensive new equity will prove to be the truly marginal capital. This has obvious implications if the model study period is determined by that period within which no new equity is issued. Within the study period, new financing would be exclusively through R and f(S). For under such a case, the logic of the Herrett & Sykes argument is that e would be determined entirely by the cost of relatively more expensive retained earnings as this would then be the marginal finance. This would be true in each of two contiguous periods which ex hypothesi would be separated by a new issue - perhaps at a very different cost; which nevertheless presumably is irrelevant under this argument. This situation is nevertheless a highly artificial one and perhaps should be ignored. The real confusion lies in the failure of the authors to follow the logic of the theoretical rationale to their calculations. Appendix II to this thesis demonstrates the inseparable paralleling of share prices to earnings growth which is fundamental to the G-S approach - i.e., earnings on formal equity and retentions alike. In other words, the price of equity shares on the market embodies the earnings to retentions. Consequently, the cost of new issues cannot be separated from the cost of present retentions. This of course is no more than an inverted way of saying that the cost of retained earnings cannot as a minimum be less than the cost of new equity when either can be invested outside the firm in identical risk situations.\(^{(1)}\)

Secondly, Herrett & Sykes postulate that k shall be determined by planned rather than current weights of equity and loan capital. Interestingly, they do not proceed to postulate that those plans shall proceed to some point of minimum value of k. Such an extension would of course run counter to the basic thinking of this thesis; but (more importantly) the mechanics of the finance-investment linkage model would modify the concept of 'planned' weights to one of 'emerging' weights. The Herrett & Sykes postulate envisages an independently determined long-term capital mix. This is a popular concept; but the finance-investment linkage model develops the capital mix as an integral part of the total decision system analysis.

\(^{(1)}\) See Solomon ( \(30\) ) p.53-54.
It is seen as opportunistic in its nature - a matter of managerial tactics rather than strategy. In short, for constraint 3.1.

\[ k = w (e_t, \ i_t) \]

\( w \) emerges from the project rescheduling process and is thus a continuously evolving function. It is also to be noted that \( e \) and \( i \) are contemporary costs of capital; i.e., current costs only. They are in no way evaluated by reference to the historical costs of the existing capital mix.

Finally, and not least importantly; the emergent weights would be, not balance sheet values, but flow of funds figures. This is important in any model of corporate financial planning as clearly distinguishing between retained investible funds and book transfers to reserve. The concept of depreciation as a source of investible funds is thus consigned to the limbo to which it properly belongs.

Weston, (1) in developing the model of marginal cost of capital which was discussed in Section 1., uses firstly the current earnings yield and secondly an anticipated earnings yield:

"It is argued that the current market price...reflects not only current earnings but prospective earnings as well.... What we are seeking is the measure of the cost of capital that reflects the earnings expectations of the investor related to the current price of the stock. This represents the level of earnings the firm will have to achieve to maintain the price of the current common stock."

Appendix II establishes that under a G-S model; \( k, P_0 \) and \( P_t \) (the base and exit prices of the share, respectively) will be in equilibrium only if \( P_t = (1 + rb)^t \) for an earnings-to-assets rate \( r \) and a constant retention fraction \( b \). It is further demonstrated that the effect of such an exit price is to establish an indenticality of earnings-price ratios:

\[
\frac{E_1}{P_0} \ldots \ldots \frac{E_6}{P_5} \ldots \ldots \frac{E_n}{P_{n-1}} = \frac{E_a}{P_0}
\]

using \( a \) for anticipations and \( o \) for current. This is exactly the relationship which Weston seeks; but the underlying assumptions as to

(1) Weston (93).
a constant retention fraction and asset-earning rate have been shown to be inconsistent with the present model.

In addition to these implicational problems, there are two conceptual problems. The basic assumptions upon which the finance-investment linkage model was based included one specifying the existence of a formal or informal market price. It is relevant to note that an informal price, however, is probably the subject of a most imperfect market and so could not necessarily be expected to conform to the necessary criteria involved in the evaluation of a outlined above; i.e., in respect of the exit share price. It is too facile to have uncritical alternative recourse to some such representative as 'a quoted company in the same industry' or 'the average for the same industry' when what should be meant is 'in the equivalent risk class'. A comparison of earnings variability as a percentage of the average return on assets employed with that for the 'industry' might be a reasonable proxy measure of equivalence; but this is not an easy calculation - and what happens if the comparison shows a wide variation?\(^{(1)}\) Barges' correction for different gearing structures must be noted, for it is likely that if the company is a private unquoted one, gearing may well be low compared with the 'industry' sample. To the extent that any gearing in the firm is convertible whereas that in the sample is not; or vice versa; there will be further distortions built into the comparison. Other and more general disadvantages in using some industrial average were noted earlier.

Secondly, the approaches so far discussed seem notably to fail to clarify the nature of the quantities used. The G-S approach itself uses what is essentially a discounting procedure. This operates, strictly speaking, on cash flow but the quantities used by G-S are never defined in such accounting terms as clearly to permit their identification as cash flows. Almost certainly shareholders do not use cash flow, and yet it is their expectations which form the basis of this valuation approach. How accurate these are:

\((1)\) i.e., If 'industry' and 'equivalent risk class' are not found to be coterminous. Does the principle of 'equivalent risk class' take precedence, and so one compares a private engineering company with the dairy industry?
expectations must be, if the approach is to be correct, is very clearly shown in the valuation requirement of the exit share price. For the U.K., Samuels (op.cit.) finds that on average the market gets pretty near this requirement, but is much less likely to do so in respect of the individual firm. Anthony remarks:(1)

"The cost of (equity) capital is extraordinarily difficult to estimate. One starts with the market price of the stock, and then tries to estimate what fraction of that market price is attributable to the markets' judgement of the profits on the present equity, and what fraction is attributable to the markets' judgement of future profits that will be earned on retained earnings. Few people will have much confidence in the result of such estimates."

I conclude that given the constraints of the finance-investment linkage model it is not possible to quantify e on the basis of a specific valuation model utilising H. It must be externally and independently determined, and in view of the poor degree of confidence attaching to any very sophisticated approach, it might as well enjoy the convenience of simplicity.

Firstly:
It will be based upon earnings. This avoids problems of reference pay-out ratios. By implication, it does not attribute much importance to dividends. It is a matter for speculation as to what extent the strong preference of most American writers(2) for dividend-based models is a result of the dividend practices of American companies. Walter(3) cites the increasing practice in America of paying quarterly dividends and argues that this so reduces share price fluctuations (because the information content is so much greater with that increased frequency of occurrence) that capital gain potential is much reduced. He further argues (p.p.12,13)

(1) Quoted Durand ( 53 ), who openly opts for industrial averages accordingly. See Page 258 for an apposite remark at a very practical level.

(2) But see : Friend & Puckett ( 56 ).

"Annual payments...might result in more attention being paid to tax differentials"

- i.e., to capital gain as opposed to income. This of course would switch some attention from dividends to earnings. He refers to:

".....sustainable dividends that reflect managerial expectations...........and (extra) dividends that are simply distributions of current earnings.......(and which) are declared annually in the 4th. quarter".

Apparently these "extra" dividends are used as indications of future increased sustainable dividends, on some future consolidation of the growth potential. Compare the almost universally staid U.K. practice of repeating interim dividends - a practice which has an absolute minimum of informational content. Analysis is forced back upon earnings considerations, of which the quotation on the next page is a typical example. Any casual perusal of financial journalism today will show the constant attempt to translate interim earnings' statements into share price expectations, and jobbers seem increasingly to follow the same reasoning.(1) Ultramar and Vickers are cases in point, where increased interim earnings in 1968 were still below expectations, so that prices fell - although the interim dividend was maintained. During recent private discussions with London stockbrokers, the several partners present were unanimous that "it is price-earnings that count now. Only older shareholders give priority consideration to dividends, and they are increasingly concerned with dividend cover".

Secondly,

It will attribute a common cost to new equity and to free retentions, following Solomon(2) in arguing that the external investment opportunity cost is common.

The cost of equity cash flow will therefore be defined as the relationship of anticipated equity earnings to the anticipated share

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(1) The absence of jobbers on U.S. stock exchanges, and their presence on U.K. exchanges, would be worth investigating as a cause of differences in valuation approaches. To what extent does a jobbers' "marking-up (or down) prices" exaggerate, stifle or confirm shareholder expectations?

(2) Solomon (30) p.p. 53-54. See also footnote to Page 259
where time 'a' is at the end of the study period. The need for anticipated share price is set by two considerations.

a) To allow for general secular movements in share prices. This involves taking a view of share yields as a whole. For example, it has been recently calculated that if earnings per share fail to rise on average by some 50% in the next three years then share prices generally will have to ease to give that target 9% yield on which the calculation is founded. As in fact the cumulative effect of devaluation and expected productivity increases are together not expected to add more than 25% to earnings in 1968/69, plus the assumed underlying 4½% p.a. growth rate: share prices in general can be expected to ease by some 10% from current levels. (1) A more general point in the same argument is that earnings yields have exceeded 9½% in all market down-turns since 1960. This would be a relevant consideration if a study period was expected to end coincidentally with a cyclical down-turn.

b) To allow for such situations as are evidenced in the following extract:

"....at 20/6d, the shares yield 3.6% with a prospective P/E ratio of 24.4 - a rating which assumes considerably more than a modest gain on Viyella's average profits for the last four years. The target has been stated...

(1) The 'Times' 12th October, 1968. The actual argument used therein is not the clearest, and the above text serves merely to demonstrate the need to consider secular trends. Using the 'Times' data and applying it to a G-S model, earnings will have nearly to double by 1970 to give a true 9½% return. The 'Times' return appears to be a less sophisticated earnings yield. The target 9½% is adapted from the argument of Professor Harold Iose that equities have historically yielded better than 9½% and in addition must command some premium over good quality debenture stocks - which today* yield better than 8½% to maturity.

(* late - 1968)
publicly... namely profits of £8 million per annum before tax, which would reduce the prospective P/E ratio to 15.6. The snag is... that it might be well into the next decade before the target becomes reality, and share price could be left high and dry."
(The 'Times' 12th September 1968).

The situation typified is one of substantial divergence between management planning and shareholder expectations. A study of the company since it first came into prominence in 1965 gives no indication of either a reference pay-out ratio or a planned gearing ratio. Both ratios fluctuate quite sharply. (1)

The target mentioned in the extract represents a future expansion of equity earnings of about 14% per annum. Share price, having hovered at about a 5% earnings yield for three years, dropped sharply in response to the poor 1967 results; but has since risen meteorically to 20/6d. Taking 1972 price as the exit $P_e$; planned equity earnings would justify a price of around 26/-d. for a P/E ratio of 20 (which is rather better than average for the industry). Discounting this back at 14% to 1968 would give a current price of around 17/6. Obviously 20/6d. already heavily discounts future growth, and to use this price as a yield basis would cause under-estimation of $e$.

The exit $P_e$ would then be around 39/6d., giving an earnings yield of 3 1/2% for a P/E figure in excess of 30. The average 1968 price to-date is 15/-d. Equally, this is an unreliable indicator - being overly depressed by the poor 1967 results. Where shareholder expectations give rise to excessively high or low current share prices, management plans must be substituted to derive an earnings-based cost of capital.

This sort of situation is most likely to be present in conditions of pronounced growth; but it is in those conditions...

(1) A G-S approach is not therefore very applicable. But for illustration only:

using 3% dividend yield and a reference pay-out ratio of .85 (short-earned in 1967); $e$ would evaluate at 5.2%; for a 14% growth rate. A price of 20/6d. would imply shareholder expectations of about a 21% growth rate, giving $e = 6%$. 
that management expectations are most explicit.

Even stronger reservations must be entertained concerning the price established by an informal market. Despite the previously expressed doubts on the validity of comparison with other, quoted companies or with industrial averages; the management of non-quoted firms will be forced to have some regard to those criteria. Their attitude would have to be based on Samuels' findings - that on balance the market is averagely correct in its growth expectations. If the rate of earnings growth for a particular firm is in excess of the average, a proportionately higher current informal share price is acceptable as a basis of calculation, for conceivably it is impossible to estimate a future price in a narrow market. Similar considerations might have to obtain for a quoted company where growth has been and is expected to be secure solely on the basis of acquisitions of subsidiary companies.

If, generally, management expects that the average equity earnings return on the finally selected project schedule will not be vastly different from that which presently accrues to the equity, then there exists an expansion rather than a growth situation. (See Appendix II). It has been noted in the discussion of the retention coefficient \( R \) that this is not necessarily a particularly good proxy for a reference pay-out factor, even though it is a constant fraction of cash flow \( a \). But in default of any other measure, it may have to suffice. Taken into conjunction with the present earning rate, this will give the growth rate necessary to calculate \( P_a = P_o \left( 1 + rb \right)^t \). As \( E_o \) would expand pari passu, the question must be asked: why not use \( \frac{E_o}{P_o} \)?

The answer is as already given - that in a non-growth normal market situation, it could be; but that a calculation of \( P_a \) forces management to take a view as to its sensibility in \( P/E \) ratio terms as well as to give some thought to secular share price trends. A further and vital point is that this will permit a rough comparison of the underlying asset value of the share on completion of the budget. This practical aspect of share valuation seems frequently to have been overlooked in writings on financial theory.\( (1) \) If we assume (as we must) a

\( (1) \) See Durand (53) on Solomon (30); which leads to an acceptable definition of the minimum cost of capital - the rate which over the long run equates share price to disposable asset value (as a going concern). Just how that rate could ever be quantified is another matter.
positive net present worth for all projects, book value evaluated in terms of historical outlay will almost certainly be lower than the income earning capacity upon which \( P_a \) and \( P_n \) are postulated. Therefore if the ratio \( \frac{P_a}{P_o} < \frac{AV}{AV_o} \) (using \( AV \) to represent underlying asset value), there is a justifiable suspicion that something is amiss. In so complex a procedure, it is not certain that there must be, especially if important projects have prolonged gestation periods and come into fruition only after the end of the study period. But a valuable review opportunity exists which is of considerable importance to a prominent quoted company, especially if the market is generally bearish.

**REDEMPTION**

The second factor which was thought to be relevant to this discussion of \( e \) was that of redemption cash flows - \( HIC_e \) and \( RC_e \). There is much speculation as to the effect of changes in gearing on \( e \), but very little exploration seems to have been carried out as to the effects of redemption, convertibility and scrip issues on \( e \). As developed, the finance-investment linkage model is not competent to account for conversion operations and scrip issues. For these are not cash flows, and so lay outside the model. And yet they affect the optimal solution because indisputably they effect \( e \), albeit as externally determined parametrical values. Conversion operations also affect \( i \) and \( f(S) \) as they represent a shift back along the two converging functions. And of course the offering of conversion rights has an important initial affect upon the shape of the cost function.

Redemption however is as cash flow transaction and is capable of assimilation into the model. Nevertheless it has been suggested that it is either often small in amount or automatically provided for in net cash flows (in the case of instalment debt redemption). Where it is large, it will tend to be of sufficient non-recurring importance to warrant extra-model consideration (the more so if the redemption is the purpose of a new issue) and so would be capable of introduction into the solution as an externally evaluated variable.
The three operations of redemption, convertibility and scrip issues are therefore best regarded as significantly affecting the optimal solution only through their independent (to the model) reactions upon $e$ and (for the first two) $i$. Strictly, they are not quite so independent as they seem, for it lies within the capacity of financial management substantially to enforce redemption and/or conversion, within the terms of the loan agreement, at times which are opportune to the firm. The instruments of enforcement are of course the market prices of loan stock and shares, as relevant. Redemption or conversion rights can be expanded or contracted, within limits sufficiently wide to create considerable inducement or discussion. Yet share prices especially are evidently capable of reflecting capital expenditure policies and finances in terms of ensuing growth rates, as we have seen. Even if the model precludes any really satisfactory measurement of that reflection, an indirect circular relationship is discernible. Redemption affects cash flow; cash flow affects financing; financing affects capital expenditure; capital expenditure affects earnings, especially their growth; earnings growth affects share prices; share prices are part of redemption strategy. This process will but little affect loan stock prices, so that redemption rights are not much to be altered. But the altered level of share prices could greatly affect the cost of a new share issue made to finance redemption. Conversion and scrip issues are each more subtle in their relationships, for their effect is upon the appraisal and acceptance of projects. The potential effect on share prices is the same as for redemption, and thus these two operations are seen to be capable of influencing their own costs.

But, in practical terms, only the more immediate considerations can receive attention here. A decision, e.g., to effect a substantial loan stock redemption, will be viewed as it affects cash flow and appraisal rates. Subsequent rescheduling to minimise the cost of redemption is not envisaged. The size and timing of such an operation is seen as an independent financial decision, taken without much thought as to its affect upon capital expenditure appraisal or financing. Some rescheduling might be considered if the affect on cash flow in a time-interval is unusually traumatic or can be unusually conveniently accommodated; but if the affect is about
as bad in one time-interval as the next, the redemption operation will be an independent variable in the system.

Especially if there has been positive leverage, redemption and conversion are bound to depress anticipated share prices - the more so the nearer the operation appears to be. As however $M_2$ is reduced thereby, $f(S^*)$, may well be correspondingly increased with no corresponding increase in $i$. If leverage continues to be anticipated as positive, the restoration of $M_2$ may re-create a growth situation. Obviously, so far as $e$ is concerned, everything depends upon whether the increase in $M_1$ is thought to dilute the equity. This consideration would apply to scrip issues. If the improvement in $f(S)$ is not exploited, the sharp reduction in overt gearing might create conformity with an otherwise breached institutional convention and so improve access to institutional funds, with a consequent improvement in $e$. This could be very true of a company seeking trustee status (1) to widen its capital market.

It remains a matter for conjecture as to what extent $Pa$ can be expected reasonably to predict such reactions. Redemption, conversion and share issues are normally of sufficient importance to the financial well-being of a company to warrant close study and the taking of expert advice. The closer the current share price is to rational expectations the easier the task will be, albeit the improvement is only one of degree. The current price may already fully discount the forthcoming operations, so that any further adjustment of $Pa$ is superfluous. Insofar as $Pa$ is this mean of a more or less formal subjective probability distribution in the minds of financial management, the impact of such operations as redemption upon $e$ could be reflected by shifting a greater weight towards the more pessimistic range of possible $Pa$ values: for generally such operations will tend to be price-depressive. Empirical research into recent redemption etc. operations might seek to set up a cross function between $e$ and $C$ in some such form as:

(1) Although unpublished research by R.H. Barnes in the Department of Industrial Engineering & Management at Loughborough University of Technology does not reveal any significant difference between the earnings yields of trustee and non-trustee companies.
when $g_t$ is a measure of pre-redemption gearing and the third term is a measure of subsequent replenishment of debt. Given an experience and expectation of positive gearing, parameter $a_1$ will be negative; $a_2$ will be positive. If $a_0$ represents the mean market reaction to redemption (all values of variables having been normalised) it will probably be positive in anticipation of dilution of the equity. The measure will be very crude, so that a measure based over all industry will be quite adequate.

On all counts, therefore, $e$ is seen as pre-determined independently of the finance-investment linkage model. So far as the averaging of $e$ and $i$ into $k$ is concerned; the important weights are those of the incremental capital mix. That is, they will emerge naturally in the shape of cash flows $M1$ and $M2$ as the model is evolved, without recourse to any spurious pre-determined capital mix. It is especially to be noticed that the resulting cost-of-capital $k$ is a marginal cost and that the weights are flow-of-funds values, not balance sheet values. As $M1$ and $M2$ are finalised only after final selection of the project schedule, which cannot be until $k$ is known; the solution is a complex problem of simultaneous equations demanding advanced computational facilities.

**CONCLUSION**

This review of the project constraints gives some indication of those key elements for which some independently determined values might be said to lie within the competence of management. Given such values, the optimal solution ($\Pi = \text{maximum}$) of the finance-investment linkage model appears to be a problem in linear or dynamic programming. By 'independent' values it is most probable that 'proxy' values are meant. The relevant values are those for:

$$M1_t \text{ (part), } i, e, \text{ and } f(s)$$

with $R$ and $q$ as constants, and $n$ and $y_t$ as given data common to all alternative values of $\Pi$. (1) A necessary condition to a solution

(1) The possibility of varying $n$ at each rescheduling is not taken seriously. It ought to be, as it is practicably possible; but it would seem to make a solution of the model almost indeterminate. Secondly, the indivisibility of many of the variables indicates that an integer programming approach might be more apposite.
is that \( X \) is finite. This is assumed to be so, and the assumption has been defended. It is specifically set out in the series of constraints for a known number of projects which are to be implemented over a pre-determined number of time-intervals.

The discussion of constraints 2.2.1. and 2.2.2. on Pages 241 et seq laid the foundation for assuming an independent parametric value to \( MLN_t \), the new issues fraction of \( M_t \). The fail-safe \( c \) function would require senior financial managerial definition in accordance with the terms of revolving credit-funds; and the same level of decision would apply to major redemption operations. The size and timing of these would be independently determined, and fed into the model at constraint 2.2.1. Conversion and scrip decisions would be divorced from this model except through their induced influence on \( e \).

The need for independently determined values for \( i \) and \( f(S) \) arises in connection with the supply of loan capital \( M_2 \). This is a vital and ubiquitous element in the finance-investment linkage model. A review of the expressions and functions developed for the model reveals that \( M_2 \) appears as a dependent or independent variable in no less than seven constraints - which is to be expected as \( M_2 \) is the principal element of flexibility in the capital rationing environment of the model. The basic constraints affecting \( M_2 \) are:

\[
\begin{align*}
2.3. & \quad 0 \leq M_2^t = (f(S^*),i) \leq (?L) \\
2.3.1. & \quad 0 \leq M_2^t = (f(S^*),i) - MLN_t \leq (?L) \\
2.3.2.1. & \quad M_2^t \leq v = (i,f(S)) \\
2.3.2.2. & \quad M_2^t > v = ((?L),f(S) \text{ et al}) = \text{say},(L,f(S))
\end{align*}
\]

These can now be seen as a series of elaborating or refining amendments to the basic \( M_2 \) function. Given that there exists a firm managerial attitude towards instalment debt such that \( L \) is finite and known; and excluding the near liquidity crisis situation of 2.3.2.2. (when it is doubtful if any rational analysis other than for the very short term is valid anyway); \( M_2^t \) is shown to be a function of the servicing-adequacy of cash flow and the cost of debt capital. Yet:
by 2.5. \( f(S)_t = (R_t^m M_{2-t-1} \text{ et al}) \)

reinforced by 2.2.4. \( S_t = R_t = f(S)_t \) a qualified constraint.

But by 3.2. \( i_t = Z(M_{1-t} M_{2-t-1} f(S)_t) \)

all of which represents a set of constraints invoking an insoluble circularity. Only by the completely arbitrary award of a value or set of values to \( i \) and \( f(S) \) can they be solved.

To award an arbitrary value to \( f(S) \) is not impossible. Restrictions of time and/or opportunity will probably preclude the fullest transition from \( f(S) \) to \( f(S^*) \) - i.e., the arbitrary value will be sub-optimal. In addition, the value will be determined by examination of the instalment debt schemes presently available and by exploratory discussion with prospective suppliers of debt capital.\(^{(1)}\) A good example of both processes would be the setting up of line-of-credit commitments with major leasing companies. A good example of the second process would be the negotiation of bank overdraft facilities, or of more formal medium-term credit facilities by merchant bank underwriting contracts such as acceptance credits. Both processes require some fore-knowledge of cash flow as a necessary basis of discussion. Even the routine utilisation of standard leasing or hire-purchase terms should occupy the attention of a liquidity-conscious management. Prior to the solution of the model, the most that can be known is the cash flow accruing to existing operations plus those projects in the sub-set '\( x_0 \)' which so easily clear the present threshold appraisal rate that no reasonably foreseeable change in that rate would occasion their rejection. Conceivably this is a most emasculated forecast of cash flow, and for this additional reason, \( f(S^*) \) will be even more sub-optimal; but this would be no great disadvantage given the almost unpredictable downward slope of \( f(S) \).

The same examination of standard instalment debt programmes and the same exploratory discussions will permit the award of an arbitrary range of values to \( i \). This is bound to be extremely crude, especially as the real cost of the subordinating or otherwise

\(^{(1)}\) In distributed seminarian material on the topic of 'financial contingency planning' Donaldson specifically advocates the same approach, a dilation of which is to be a chapter topic in his forthcoming book.
restrictive conditions attaching to increasing debt commitments are just about unforeseeable as well as unquantifiable. Yet some notion of the cost of short-, medium- or long-term debt is available. Thus, for the first half of 1968 estimates of 6\%\%, 7\%\% and 8\% in pre-tax money terms would not be too far out; with a 3\% reduction for medium or long-term convertibility options. How much such estimates need to be adjusted for increasing debt commitments is not so easily quantified; but as by assumption the model is set in terms of total debt commitment of a less than v quantity, a following assumption of "no change" might not be too invalid.

Finally the discussion on Page 251 et seq outlines a method for an independent, external determination of e. Given e and i set at constant or near constant values, a proxy value of k is determinable albeit once more at a value which may or may not be optimal. An approximate solution of the finance-investment linkage model becomes feasible for a finite project set X.

To what extent is this approximation sufficient and acceptable? Given the uncertainty and even outright error which must attach both to the size and to the timing of cash flow expectations; given the political and macro-economic imponderables which bedevil all planning at the level of the firm; and, last but not least, given the error attaching to the numerical expressions which must replace the normative functions of the present model; it would seem that some inexactitude in the values awarded to the various key elements is a matter of relatively minor importance. But the analyst cannot easily be sure that the resulting distortion is minor; nor can he be sure that the errors are not additive, leading to a major total error in his solution. If the requisite computational facilities are to hand, presumably it would not be impossible to set ranges of values for M, e, i and f(s); and by solving the model for each permutation of these values to set up some sort of schedule of results which could be subjected to sensitivity analysis or to tests of confidence levels. Obviously such a procedure is beyond the resources of all but the very large organisation - which will may have the greatest need for such preplanning of its manifold complexities of capital expenditure budgeting. Other firms must adopt some system of approximation upon
which they can rely.

Section 5. Financial Linkage in Practice.

This is more fully discussed in the next Chapter. It may be relevant to recount at this juncture, personally observed implicit or intuitive applications of finance-investment linkages such as are (sought to be) formalised in the foregoing model. Especially in smaller companies, X is not large; and because of managerial limitations of capacity and facilities, rescheduling and amending processes must be very limited. A typical application covered five projects, regarded by management as having the following 'average return' profit ranking:

Project A: a re-tooling project for milling machines.
" B: application of numerical control to a group of capstan lathes.
" C: improvement of material handling and storage - especially of steel rod, bar and sheet.
" D: replacement of a section of single spindle vertical drillers by three multi-spindle machines.
" E: disposal of product - obsolescent specialist cylinder grinders and re-occupation of the space by enlarged turning facilities.

each of major importance and capital commitment for a firm of the size under review and in total beyond present capital resources. Project supervision was not on this occasion an especially limiting factor owing to the general similarity in type of projects A and D, and of C and B. The technological imperatives of precision engineering implied that project A and (to a less extent) D were 'starters' or \( X_0 \) type projects. Management were introduced to standard machine tool leasing programmes, to the facilities of ICFC and to the Board of Trade machine-tool hire-purchase scheme for export production. There was an inevitable confrontation over the two principal aspects of such finance - the inherent cost of the schemes and the insistence placed upon the servicing-adequacy of projected cash flows; but the idea that cash flow might be optimised by a rescheduling of projects (end project supervision strengthened) so that all projects would be implemented
to the ultimate improvement in the benefit to shareholders, was quickly grasped. Project A, as an imperative, and project C, as most quickly freeing working capital and hence cash flow, were implemented from current financial resources. The improved cash flow facilitated an ICFC loan for project B - a very major project to this company. The completion of the 'disposal' part of projects D and E facilitated initiation of a standard machine tool instalment debt scheme for the new vertical drillers, subsequently servicing-adequacy to be derived from project B. It is noteworthy that the commitment of ICFC to project B was a major feature in successful negotiation of this lease with the machine tool manufacturers, at the most economic standard tariff. The re-siting part of section part E was to be financed, admittedly after some delay, from improved retained earnings expected to accrue to the other projects.

(It is of interest to note that in all cases observed, management instinct has been to evaluate projects on the basis of netting out the fresh funds generated by the project from the cash equivalent capital cost and assessing profitability calculated thereon against the interest cost of the debt finance. Regrettably, in no case was the calculation of profitability carried out by discounting procedures - 'average return' on one basis or another being uniformly used).


Before concluding this discussion, it seems opportune to re-emphasise two fundamentals of the finance-investment linkage model. The first of these is the central importance of the 't' - intervals within the study period; more particularly, the spread of project implementation across those intervals. Just how management could predetermine at what rate and in what combinations within that rate, a quantity (possibly a large quantity) of projects of greater or less technical complexity could be implemented; is not at all clear. Yet common sense; and practical observation of scarcities of engineering skill, of prolonged delivery dates for essential parts, of delays in securing planning permissions or the agreement of unions to redeployment or retraining - all these indicate that project
implementation has to be staggered as a result of conscious decision or of external influences. In a sense, then, the introduction of a delay factor - however arbitrarily decided upon - is some recognition of this fact. Much could be written upon this issue: (1) but as the determination thereof is not immediately central to the main burden of this thesis, I am content to have raised the issue.

Secondly, the purpose served by the introduction of the retention factor R must be emphasised. Detailed reiteration would be wearisome. Suffice it to say that it is an essential condition of the basic economic assumption of the model - maximisation of shareholders wealth - which conceivably is in conflict with a primary behavioural assumption; namely, the willingness of management to exploit the servicing-adequacy of cash flow to the fullest extent. (2)

The model was introduced as inelegant and incomplete. Its purpose is to demonstrate that in the reality of investment decision making, the financial decision is not really separable from the investment decision; although there are indisputable advantages of comprehension and orderly procedure in assuming such a separation. This point is taken up in the next Chapter. Nor does the model really seek to set up a complex numerical solution of the full capital budgeting problem. Rather, it seeks to establish a loose normative framework within which the various looped relationships of that problem can usefully be explored. Hopefully, the discussion of some of the major elements and expressions of the model will also clarify some of the implications behind the financing decision, whether separated or not.

As it not uncommon with financial models a para-mathematical presentation might seem to lend a spurious air of accuracy to the model. At the same time, the over-simplification of representative symbolism tends to obscure the practicalities of cash flow and the requirements of accounting practice. A mathematical type of

(1) The literature of capital budgeting is conspicuously silent upon the point apart from the occasional wry reflection, some examples of which have been reported in earlier footnotes.

(2) This potential conflict is an alternative statement of the manager-shareholder conflict touched upon by Solomon (30) P.24: where managerial status correlated to the size rather than the ownership of assets becomes a competing objective.
presentation and the use of representative symbols does permit a logic and clarity of analysis which is of inestimable use in obtaining a close insight into the requirements and limitations inherent in a finance-investment linked capital budgeting procedure. But a less rarified atmosphere must now be sought, and a more immediately practical procedural approach developed.
CHAPTER SIX

A PROCEDURAL APPROACH TO CAPITAL EXPENDITURE BUDGETING USING FINANCIAL LINKAGE.


At the earlier stages of this discussion, a distinction was made between the investment and the financial decision, with the clear implication that the two are separable. The implication is well rooted in financial theory - e.g., Vancil:

"The attractiveness of a new investment should not be influenced by the method in which the funds for the investment are to be provided...... Thus the money provided by raising new capital should be used in the most efficient manner. Thus, the concept has arisen of "separating" the investment decision from the financing decision and modern investment decision theory rests on this concept." (1)

or, alternatively, Solomon:

"......there is a clear cut conceptual parallelism between the task of optimal investment selection and that of optimal financing." (2)

(Note: but parallels never meet).

The model developed in the previous section endeavoured to show that such a separation is not necessarily valid. The two quotations have been chosen out of context because they lead, each in its own way, to disclosure of the reasons for this apparent contradiction. Vancil is proceeding on a different set of assumptions, Solomon is reflecting on computational differences.

'Modern investment decision theory' leads to a separation of the investment and financing decisions because it presupposes:

(a) An externally predetermined capital mix representing at least a medium term financial budget.

(b) Projects are financially independent.

(1) Vancil (34) P.5.
(2) Solomon (30) P.152
Projects are implemented in the order in which they are ranked. (1)

Just how financial managers are to develop gearing plans without a forecast of cash flows; and just how cash flows are to be forecast without some indication of what projects are to be put in hand and at what intervals of time; is not too clear. But if financing plans can be developed as if they were in isolation, then obviously optimal solutions in financing and investment can separately be developed. Of course, modern investment decision theory does not really claim any such thing. In defining the three basic tasks of the finance function:

(1) what specific assets should an enterprise acquire;
(2) what total funds should an enterprise commit;
(3) how should the funds required be financed;

Solomon observes:

"These three questions are closely inter-related... (they) are really three facets of a single underlying question and in practice they must be solved simultaneously". (2)

Yet even so, Robichek & Myers quote the same three basic tasks and say:

"The first two questions will be referred to jointly as the investment decision, the third as the financing decision". (3)

and then go on to repeat Solomon's observation. A separation of treatment is evidently regarded as permissible. The inter-relationship of the two decisions is envisaged as consisting of:

(1) a schedule of investment determining the finance to be raised; and
(2) an externally determined mix of different sources of finance to establish the threshold and ranking factors necessary to set up the schedule of projects.

(1) Or implemented in the order implicit in the order in which they are ranked.
(2) Solomon ( 30 ) P.8.
(3) Robichek & Myers ( 25 ) P.2.
Each is deemed to be a finite, specific sum. A derivation of the concept considers methods of rationing (2) if (1) is restricted. Apart from these two points of contact (important though they are) the two sets of decisions can be examined in mutual isolation.

The two salient points of objection to this simplification appear to have one common origin - the growth in importance and sophistication of instalment debt finance as a source of capital. The vital importance to capital budgeting theory of this development lies in one aspect of such capital - the facility it affords to invest 'on margin'. A comparatively small change in suitably risk-assessed expected revenue cash flow can now initiate a major capital expenditure. It follows that:

1. An optimal ordering of cash flow is feasible. Project ordering by time as well as profitability becomes important.
2. Project can 'breed' project, accordingly; yet the birth rate of 'amending projects' is a function of the optimisation programme of (1).
3. Because of (a) changed time patterns of cash flow and (b) changed patterns of financial mix the profitability ordering of projects can alter, expand or even contract.

Solomons' later separation of the two decision-series is on grounds of empirical practicability in the operational sense. The two are in the same plane of management science; but while a sufficiently valid operational process of investment decision making has been developed, financial decision making is not so advanced:

"The operational restatement of a financing decision, i.e., the decision to incur one kind of 'liability' as opposed to another, involves categories for which we have very little accumulated evidence or experience as far as capital numerical values are concerned..... it requires a level of knowledge about investor and lender reaction which we do not as yet possess" (1)

which is still largely true today. To this problem can be added (1) Solomon (30) P.153.
the computational problems shown in the previous section to accrue to the instalment debt induced integration of the two sets of decisions.

This then is the reason for these reflections on some of the more obvious aspects of current capital budgeting theory. In this thesis, the investment and financing decisions will now be separated not because they ought to be; in fact to do so is inherently wrong in any evaluation of instalment debt financing. They are separated because they have to be; for the computational problems involved are beyond the facilities of the average firm, and the arbitrarily awarded values of key elements necessary to solve the simultaneous equations of the integrated system, even by iteration, result in an approximate optimal solution where the degree of error is an unknown quantity.

Section 1. The Assumptions of the Procedural Approach.

One such two-tier approach is now presented. Such novelty as it possesses is directed towards a more nearly approximate optimalisation of the financing part of the operation. Presentation is kept as simple as possible\(^{(1)}\) - no attempt is made to discuss every doubtless relevant quantification or refinement. Certain basic postulates are made which are in line with the general tenor of this thesis. They are that:

a) Equity share prices are independent of gearing over a very wide range of gearing.

b) There is a planned growth rate in the asset value of the equity interest. It is a convenient but not essential sub-assumption that this planned growth rate is more or less faithfully mirrored in a growth of share prices - which means that the market assumes increased rates of profit will accrue to the new assets, or that continuation of the old rates will benefit the equity through increased gearing.\(^{(2)}\)

(1) And is non-mathematical in approach; because, as will be seen, the procedure is inexact, and mathematical treatment would imply a spurious exactitude unless backed up by a wealth of textual quantification.

(2) See Appendix II.
c) Management can ignore, or there do not exist, any lending-institution conventions which specify any firm equity base to debt financing operations, other than a loose requirement that there shall be some (probably varying from time to time and from case to case) 'shareholder commitment' in growth or expansion situations.

d) There are no irrational managerial prejudices which preclude management from exploiting every kind of financial source open to them. A useful secondary expression of this assumption would be that management will exploit debt potential up to a total of all variants of debt not exceeding limits imposed by the Articles of Association on directors' borrowing powers. Although ever-present, this limit will not further be specifically referred to in the ensuing discussion.

e) By diligent investigation of standard instalment debt facilities offered by finance houses, equipment manufacturers and distributors, and specialised Government or para-Government agencies; and by recent exploratory discussions concerning funded debt or overdraft facilities with the relevant financial institutions: management has a reasonably accurate view of the servicing-adequacy of its cash flow and some idea of the probable cost function (i.e., interest on debt capital). (1) In the context, there must be included in the view of servicing-adequacy not only what value of capital goods can be acquired but also what type of asset can be financed by what type of capital.

f) Irrespective of the amount of debt capital, moderately firm plans exist as to the raising of fresh equity

---

(1) This seems to be a reasonable requirement of any efficient treasurer function; even when carried out by the secretary, accountant, or office manager of the medium sized firm and the auditor of the small firm. The smaller the firm, the smaller the debt capital - (assumption (d)) and so the less onerous is this duty: for the range of sources is restricted thereby, and anyway one or two sources will suffice to handle the lot.
capital, including a clear idea as to what extent that operation represents new money and to what extent a redemption operation. This assumption seems reasonable in the light of the degree of preparation necessary to execute such plans. Assumption (a) must be linked to such plans.

A finite set of projects (as defined on Page 210) together with a fair idea of implementation dates. Such dates will be set, either as categorical imperatives for certain projects due to technological, personnel or other non-financial criteria (which will be reflected in the timing of expected cash flows); or arising out of a standard time-lag between the implementations of successive projects. This standard time-lag also will be determined by non-financial criteria. To the extent that any projects are technologically or strategically chained or similarly dependent, a third implementation sequence cutting across the other two can be envisaged. It seems likely that the first group will be the most important in terms of capital commitments. The standard inter-project implementation time-lag may be the veriest guess, or even set equal to zero.

Section 2. The Procedural Approach.

In step form, the approach over a given budgeting study period is as follows:

Step 1.
Calculate a proxy discount factor using a present balance sheet value weighted average of capital costs. Note that this is not a marginal cost. A marginal cost can not be calculated

(1) For a defence of the proxy measure, see Solomon (30) p.88. Capital costs - especially equity cost - to be calculated as discussed on page 251 et seq of this thesis.
without some predetermination of capital structure, which is
denied ex hypothesi. Furthermore, a suspicion of discriminatory
financing has been seen to attach to a marginal cost of capital
concept. But if it is insisted that some weight be given to
quite generalised market conventions the following report from
Standard Oil of Ohio is indicative of an alternative method of
establishing a proxy cost of capital. (The order of the
quotation has been slightly changed):

"In the case of equity funds, it is an earnings rate on the
equity . . . . we have used the actual received rate of return
over a recent period of years, usually not less than
5 years. This is a rate of return on book value of
capital and excludes any unusual items . . . . (this) includes
a rate of return on retained earnings of prior years just
the same as though they represented new equity investment
from outside sources . . . . (On structure) we use our own
experience and the pattern of our industry, with more
weight given to the latter than to the former. We do not
use our current capital structure at any point in time
since this is almost certain to change. Our attempt is
to use a relationship which reflects our expectation of a
normal capital structure in our industry". (1)

I know from private discussions that Easo Petroleum (U.K.) use
6% "because all our funds are borrowed from the U.S. parent
company": Biruid use 15% "because after corporation tax that
is just a little better than the average for industry"
(evidently a reference to the national average return on
capital employed of 13%); the Co-operative Wholesale Society Ltd.
uses 6% "because we have got to get this thing off the ground
again, and then we shall use 10%": a leading firm of brewers
use 7½% "because it looks about right (1)"; and a medium-sized
plastics firm uses 20% "because that is about what we get, after
tax, on our better lines" - yet one of this firm's parents
(Union Carbide) reported to Cohen & Robbins:

"We don't measure the cost of capital because we don't know

(1) Quoted Cohen & Robbins "The Financial Manager" (Harper
International) 1966.
The determination of proxy costs of capital evidently holds out great hopes for some future researcher into industrial mythology. Durand would be delighted!

**Step 2.**

Plot every potential project, in terms of detailed phased outflows and inflows, on a time chart. The outlay (i.e., implementation) date will be specific for 'categorical' projects (see assumption (g)), and for the second or later of 'chained' projects, given the date of the first. 'Non-categorical' projects will be plotted at random subject to the predetermined standard inter-project implementation time-lag.

**Step 3.**

Appraise each project (net present worth) to a common base time. Review randomly dated projects to take advantage of the more obvious improvements in aggregate net present worth if these dates are rescheduled. Discard all projects with negative net worth.

**Step 4.**

If the review of 'non-categorical' projects indicates date rescheduling, replot all retained projects as in Step 2. In any event, sum cash inflows of retained projects, time-interval by time-interval.

**Step 5.**

Enter on the plot, in due time-interval order, the forecasted net cash inflow from operations. Sum to new project inflows. Deduct, in due time-interval order:

1. Review by inspection. The errors and uncertainties attaching to project estimates; the mistakes that will occur in implementation and operation; and non-financial considerations (not, however, imperatives); will make a more complex review something of a waste of time.
2. The 'tail' of net inflows inherited from previous periods. See Page 214 et seq., and especially, footnote (1) to Page 219.
(a) Existing current or provisional financial charges, including an estimate of taxation, on current operations.
(b) Cash cost of projected dividends on present share capital.

The resulting cash flows correspond to the series $a_j^t$ (for $t = 0, 1, 2 \ldots n; x = A \ldots J \ldots P$) of constraint 2.4.1.

**Step 6.**
Deduct from these $a_j^t$ cash flows the standard retention fraction. (1)
Add to these the opening cash balance, in order to obtain what might loosely to be termed "gross internally generated cash flow" - i.e., gross of new capital expenditure. To this must be added that part of the proceeds of new equity issues which is not utilised in debt redemption operations, and there must be deducted any redemption of funded debt out of existing resources.

The resulting cash flow is equivalent to the equity cash flow $M_1$ for each time-interval, in the form of an accumulating sum over the period $t_0 - t_n$. The total of the $a_j^t$ cash flow is now equal to the cash flow denoted effectively by:

$$S_t + \sum_{j=A}^{P} (b_j^t \cdot x_j) = R_t$$

for each successive interval $t$ over the period $t_0 - t_n$.

These are the servicing-adequacy cash flows - the basis of $f(s)$.

**Step 7.**
The set of ranked projects must now be separated into two sub-sets:

- $x_{m1}$ being capable of equity financing only - "compulsory $x_{m1}$ projects".
- $x_{m2}$ being capable of various forms of financing.

The first sub-set must be deducted from the equivalent cash flow $M_1$. (2) Any balance therefrom is available as alternative financing for $x_{m2}$ projects. Any deficiency of $M_1$ implies either

- a) a cut back to a low enough total of such projects;
- or b) a re-assessment of $M_1$ in its various components.

---

(1) The cash nature of $R$ was stressed on Page 216.
(2) As will appear in Step 9, this selection is not final.
Which step is taken must depend upon the net present worths of $x_{m1}$ projects compared with $x_{m2}$ projects. Normally one would not expect to find many $x_{m1}$ projects, so that the necessary comparison procedure should not be overly complex. But an echo of the formal model is to be heard here, in that any increase of (say) R to accommodate $x_{m1}$ projects results not only in the potential exclusion of an $x_{m2}$ project; but also in a potential reduction of subsequent cash flows which may result in yet a further decrease in $x_{m2}$ projects. Unless the necessary computational facilities are to hand to carry out the fairly simple but distinctly laborious iteration routines required at this point, a sub-optimal solution of the procedure is clearly possible.

**Step 8.**

Projects of the $x_{m2}$ type must now be scheduled in descending present net worth order; each project having entered against it its discounted capital cost under each alternative financing possibility - the cost being derived from the Evaluation Tables. For example:

**Table 11.**

<table>
<thead>
<tr>
<th>Project</th>
<th>PURCHASE</th>
<th>INSTALLMENT DEBT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity</td>
<td>Loan</td>
</tr>
<tr>
<td>B</td>
<td>$q_{b1}$</td>
<td>$q_{b2}$</td>
</tr>
<tr>
<td>C</td>
<td>$q_{c1}$</td>
<td>$q_{c2}$</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P</td>
<td>$q_{p1}$</td>
<td>$q_{p2}$</td>
</tr>
</tbody>
</table>

(1) On page 211, it is suggested that the integration of non-profit earning or unquantifiable projects should be effected by adding a supplement to $k$. It can now be seen that the supplementation ought to be to $e$, because such projects may very well be $x_{m1}$ type projects. If objection is taken to what is really a subsidising of such projects by direct profit-earning projects, or if $e$ is too much distorted thereby; then such projects could be regarded as a first or otherwise stipulated charge on the M1 cash flow. It is known that this is the approach favoured by I.C.I.
where $Q_j$ = discounted capital cost for any project 

\[ j = A,B,...P; \text{ and where } Q_{j1} \leq Q_{j2} \leq Q_{j3} \leq Q_{j4} \]

In using the Evaluation Tables to compute the discounted capital cost $Q$:

a) The discount factor would be that established in Step 1, using linear interpolation as necessary.

b) Regard would be had to the hire-purchase or leasing payments programme, and interest cost thereon, which basic assumption (a) indicated to be relevant to the particular project under computation. There may be no such instalment debt programme available, the alternatives being to finance out of equity, funded loan capital or overdraft facility only. The 'loan' column in the Comparative Project Financing Table might need to be sub-divided into several types.

c) The earnings factor would represent that rate of earnings which the company would normally expect to accrue to its employment of assets over the long period. A fuller discussion of the relationship of this rate to the discount factor of Step 1 is postponed until Page 302. The 'convenient sub-assumption' of postulate (b) is also relevant.

d) It was observed in Chapter Two, Section 1: that where the use of instalment debt is to expand an already totally committed equity base, residual capital earnings to instalment debt were irrelevant. Thus if $n_1$ projects totally exhaust available equity funds, Table II should be expressed in terms net of residual capital earnings. The purpose of this step is to arrive at a minimal financing cost of

\[
\sum_{j=A}^{P} \left( b_j \cdot x_j \right) \quad (1)
\]

Comparative Project Financing Table must be broken down into a second schedule, for which a suitable title might

\[ (1) \text{ Implementation date is an irrelevant consideration at this stage, as appraisal is of competing financing methods for any one project at a common time.} \]
be 'Minimum Project Financing Method Schedule'. This will schedule projects, still in descending net present worth order, specifying the undiscounted capital costs of each under the heading of its minimum cost financing method. For example:

Table 12.

<table>
<thead>
<tr>
<th>Project</th>
<th>PURCHASE</th>
<th>INSTALLMENT DEBT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Equity</td>
<td>Loan</td>
</tr>
<tr>
<td>B</td>
<td>1150</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>800</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
</tr>
<tr>
<td>etc.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(The hypothetical projects and their capital costs, and their order of ranking, are taken from the original numerical example on Page 211. The allocation to financing methods is, of course, made arbitrarily for the purpose of illustration).

The total of each financing method column will give the total amount of that type of capital preferred as a first, i.e., lowest cost, choice. For the sake of brevity, each such total will be termed the 'preferred' amount in the ensuing discussion.

Step 2.

The crucial task is to match the preferred amounts against the available amounts - which, it will be recalled, are equity less x_{ml} projects plus a period-by-period series of servicing-adequacy cash flows.

IT MUST BE ADMITTED AT THE OUTSET THAT THIS MATCHING WILL BE SUB-OPTIMAL. FIRSTLY, IT IS NUMERICALLY INEXACT, AS IT MUST BE. SECONDLY, NO PROVISION IS MADE FOR RESCHEDULING TO OPTIMISE INCRENT SERVICING-ADEQUACY OF CASH FLOW. However, it represents a few steps closer to the fully optimal solution.
The first and perhaps the easiest step is to match the preferred amount of equity against the available equity. Postulate that, as is possible, there is an excess or a deficiency of availability. Any excess must be transferred back to servicing-adequacy cash flow. If a deficiency accrues, projects will be selected by net present worth ranking, and discarded projects will be fitted at their appropriate rank into the next preferred method column, and the total adjusted accordingly. What is essential to appreciate is that this selection is not final.

If servicing-adequacy availability proves insufficient, such that preferred \( x_{m2} \) projects are discarded, then there must be compared, the difference between the net present worth of each such discarded project and the net present worth of preferred \( x_{m1} \) projects of equal capital cost (or of groups of selected \( x_{m1} \) projects of equal aggregate capital cost) against the increase in preferred capital cost if the discarded \( x_{m2} \) project is financed by equity. For it may be that the excess net present worth of the \( x_{m2} \) project over the \( x_{m1} \) project(s) is greater than the increase in financing cost due to in fact financing the \( x_{m2} \) project by second-choice equity funds. If the compared \( x_{m2} \) project fails in this test, it is indeed discarded - the capital rationing takes effect. If the compared preferred \( x_{m1} \) project(s) fail, they must then be compared against compulsory \( x_{m1} \) projects in the same way. (They cannot succeed against any other \( x_{m2} \) project, as by definition it was the weakest \( x_{m2} \) projects which were originally discarded.) The failure projects in the inter-\( x_{m1} \) project comparisons are finally rejected - again the capital rationing takes effect.

(One problem remains: that of the project - a challenging \( x_{m2} \) project is a critical illustration - parts of which can be

---

(1) So long as the original appraisal rate was (reasonably) correct, it must pay to utilise this excess in promoting qualifying projects rather than in distributing it to shareholders.

(2) i.e., not (yet) \( x_{m1} \) projects compulsorily financed by equity. This is a convenient opportunity, also, to acknowledge the use of M.A.P.I terminology in this Step.

(Footnote (3) on next Page.)
financed by different methods. No real solution is presented in this thesis. Clearly, to allow for such projects in the model would be to drop the indivisibility constraint 1.1 and the introduction of linear programming into what is largely an integer programming approach. Hopefully, such projects will be very large ones - a complete new works for example. It is possible that such projects will be so central to the firm's continuity that unquantified considerations of managerial strategy will boost an otherwise low net present worth, creating in effect an artificially high ranking for project selection purposes.

Theoretically, this is bad practice. Otherwise, such a project could be 'matched' in its separate components, each labelled to indicate that final failure by one component is a failure by all components - except that I feel sure that further consideration of this approach would reveal the need for a joint profitability approach: i.e., the defender project would have to succeed against the challenge of the total project.)

The matching of \( x_{m2} \) projects against servicing-adequacy in the first place (where either projects or servicing-adequacy may be enhanced by a transfer from the \( x_{m1} \) -equity matching process): requires for its execution an exploitation of the basic postulate (e) that management in a review of debt facilities ought to gain an idea of the cost function for debt which faces the firm.

A priori, there are no quantitative grounds for preferring one

(Footnote (3) from previous Page.)

(3) If, in the two stages of comparison, exact capital cost equality cannot be secured: then the net present worth of the challenging project must be reduced by the loss of earnings (at the appraisal rate) on the idle excess of defender capital cost over challenger capital cost. Obviously, there can be no excess challenger cost, or the challenger would not be financed. Generally the literature of capital expenditure budgeting under capital rationing pays little or no attention to inequalities between capital supply and demand, assuming continuous curves for both - which is by no means always true. New issues, whether of equity or loan stock, create distinct discrete steps in the supply curve. It can be shown that rate of return ranking of projects can lead to wrong project selection in cases of capital rationing, through this problem: which can further be shown to be an extension of the reinvestment rate problem. (See Solomon ( '84), also J.C.T. Mao "The Internal Rate of Return as a Ranking Criterion" (Eng.Econ. 71) for a mathematical statement of the point).
form of debt capital to another, except perhaps the 'package' facilities claimed for leasing. (Some consideration of other various strategic advantages which might be deemed to accrue to one financing method rather than another, were discussed in Chapter One). Management might well take a view of these, and adopt a posture on one form or another; so specifying a strategic bias. One of the advantages offered by the Evaluation Tables is that the cost of such a specification can be quantified. It is then up to management to say whether the loss of capital cost minimisation plus the loss of the net present worth of any project which has to be discarded because the bias restricts the size of M2, is compensated for by strategic gain. But, to repeat, a priori there is no means of deciding whether servicing-adequacy should be applied to funded debt or to overdraft or to instalment debt or whatever. The cost function for debt is seen as a quite inexactly-weighted average of all the servicing costs of the different types of debt. A review of the different preferred amounts of debt capitals from the Minimum Project Financing Method Schedule will give some indication (ignoring the chance of discards) of the mix of debt capitals and if the servicing costs (largely administration, legal and interest costs) of different forms of debt are quite different, then the weightings can be based on that schedule. Certainly this will be a necessary precaution if Government credit policy artificially restricts one or more sources of debt while leaving others less constrained - for it is usually the more expensive forms of debt which remain the less constrained. But because discards cannot at this stage be identified, and yet may accrue unevenly across preferred capitals, the inexactitude of this weighting must be acknowledged. The application of the weighted average servicing cost function to the minimum period servicing-adequacy cash flow will at once reveal the availability of debt capital M2 in total. (1)

One other sub-optimality of the procedure is at once revealed.

(1) This presupposes that the actualities of raising debt capital will correspond to the forecasts of exploratory discussions. There is a temptation to talk of contingency allowances. This is resisted, firstly because it is illogical; secondly because the degree of error in the forecasts of operational and project cash inflows which are the basis of servicing-adequacy f(S) are probably so great that to introduce contingency allowances in f(S) is like building on sand.
Because there is no rescheduling of projects, the critical minimum period servicing-adequacy cash flow is not maximised. Inter-period variability of cash flow also remains a problem, as in the theoretical model. Suppose the critical minimum cash flow is exceptionally depressed, vis-à-vis the cash flow of other periods however; it seems obvious that a re-timing of the contributory cash flows must be considered. The most flexible source might well be operation cash flows in the form of accelerated debt collection or arrangements for a special period of high trade credit. This is in fact equivalent to securing special short-period debt capital facilities.

Projects whose preferred capital source is debt capital, plus transferred equity-preferring projects, will be matched against the derived $I_2$ in descending order of net present worth, up to the point of exhaustion of $I_2$. Subsequent projects will be submitted to the comparison process against equity financed projects, as outlined earlier.

A somewhat complex reaction can now accrue. At the end of the matching and comparing processes, there will be a 'pile' of discarded projects. But all projects were scheduled (Steps 2 - 6) on the 'master' cash flow plot from which is derived firstly $I_1$ and secondly the servicing-adequacy cash flow which, via $f(S)$, yields $I_2$. If any project is discarded, the 'master' cash flows are reduced accordingly. Both $I_1$ and $I_2$ would be so reduced. Marginally profitable projects would have to be discarded, reducing the 'master' cash flows - and so on. In the end, the master cash flow plot would consist of net inflows from operations only - yet these would have an $I_1$ and $I_2$ potential, however reduced. Projects could be financed, but their retention would increase the 'master' cash flow,... and so on, upwards. The apparent result is a wildly oscillating instability. This should not occasion surprise, as the unstable looped relationships of the theoretical model were stressed in the
It is possible to depict a long term situation of complete project cut-back. The following example is a miniature of the process, but to enlarge the illustration all that is necessary is to imagine a cut-back process prior to any project $j$. For the sake of brevity only one general type of finance is considered, and the study-period is restricted to 4 time-intervals, which is to be taken as only a part of expected project life:

Table 13.

<table>
<thead>
<tr>
<th>Time (in terms of $t_n$)</th>
<th>Capital Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>t_1</td>
<td>50</td>
</tr>
<tr>
<td>t_2</td>
<td>100</td>
</tr>
<tr>
<td>t_3</td>
<td>200</td>
</tr>
<tr>
<td>t_4</td>
<td>150</td>
</tr>
</tbody>
</table>

If $f(S) = 10; 0$, $M_2 = 1500$ (minimum time-interval servicing-adequacy cash flow = 150). Neither project can be financed. Even a rescheduling - e.g., bringing forward project B to start in $t_1$ and postponing project A to $t_2$ (or even not doing this if it can be avoided) - is unavailing. In fact it is doubtful if a management aware of growth possibilities through capital expenditure projects would tolerate such a situation. Exceptional measures would be taken, as briefly suggested above, to improve operating cash flow or to secure deferment of initial servicings of debt. The situation, obviously, is unrealistically extreme and there are some fairly obvious limits to the project cut-back process.

The practical considerations most probably will be (and they

(1) Secondary considerations reinforce the argument. If the concept of servicing-adequacy is correct, theoretically an infinite number of projects can be exponentially financed. The imposed limits are an upturn in 'i' putting bounds to $f(S)$; a legal limit $L_i$; a finite set of projects; and limitations to managerial capacity. The reverse process - a theoretical tendency towards zero projects - must be equally possible, and imposed limits would have to be applied in this direction.
will remove the oscillation from realistic analysis):

1. Except in the very short period, management will not tolerate 'nil' retentions. (1) It will not be allowed to reduce to 'nil', but will have some floor value. The comparison process is essentially a seeking of discarded $x_{m2}$ projects to transfer as $x_{m1}$ projects. As they fail and are finally discarded, 'master' cash flow is cut back until minimum $R$ is reached. The resulting minimum $H$ will just finance those $x_{m1}$ projects (original or successfully transferred) which, with operations cash flow, generate sufficient 'master' cash flow to yield minimum $R$. In other words, there will be no debt financing - all growth will be financed out of retentions. The resulting all-equity firm is at the opposite end of the spectrum to the geared firm which has been able to exploit servicing-adequacy of cash flow up to the final legal limit $L$ (basic postulate (b)).

2. The critical servicing-adequacy of cash flow is the minimum in any one time-interval during the study period. Sooner or later, as projects are discarded, there will come the time when the last discarded project does not seriously affect the minimum time-interval cash flow set in the last round of discards - i.e., $M2$ stabilises at a minimum.

3. $f(S)$ is defined as being a decreasing function of $M2$. In a reverse application, a given constant reduction of 'master' cash flow will not result in a constant reduction in $M2$. This can have only an ameliorating effect on project cut-back, as the function always has a positive value.

4. Sooner or later, the discarded project will consist of a large capital cost associated with relatively small cash

---

(1) The implied cessation of growth would cause shareholder wealth to diminish as share prices fell away. As well as being tolerable in the short period, of course, management may have to accept this permanently in the insolvency situation: but by then there are no effective growth prospects.
flows during those time-intervals of its life which fall within the present study period. The reduction in capital requirements caused by the discarding of this project will equal or exceed the reduction in $I2$ which follows the reduction in the minimum time-interval cash flow. Any resulting excess of remaining servicing-adequacy would then have to be applied to the next ranking project which although discarded at the last review, can now be financed from that excess.

5. Even in the extreme situation depicted in Table 13 cash flow from operating will still accrue - if it does not, the situation is one of irrelevant insolvency. Given no current project expenditure, such a cash flow becomes an enforced retention; and would by accumulation permit a slow increase in purely equity financed capital expenditure. This presupposes unchanged dividend plans. The situation is quite realistic and illustrates quite accurately the slow growth of the typical small company with a conservative, anti-debt management. It is a "we must wait until we can afford it" situation. As soon as projects can be afforded, cash flow will improve and servicing-adequacy can be exploited if management so wish.

6. A shorter and more decisive route to the same end would be the raising of new equity to exploit at any rate those projects with a high net present worth. The subsequent improvement in the 'master' cash flow would finance yet further projects. (1)

The cut-back process represents the true 'external capital rationing' situation. (2) None of the above reasons for thinking

(1) This is the true meaning of the phrase "necessary equity base". Quite clearly, no optimal gearing ratio is implied.

(2) The phrase is that of Bierman & Smidt (8) pp 182 et seq. In their subsequent discussion, these authors illustrate various such situations and from time to time use the phrase "...it would be profitable to borrow an amount...". Profitable, perhaps; but would it be feasible? This facile assumption of borrowing capacity, without considering the necessary basis of servicing-adequacy of cash flow, is a frequent weakness in the literature.
that a limit will apply to the cut-back process are wholly satisfactory in the short period, in that however they decry the probability of total cut-back, they cannot deny the possibility of it. But they do, especially if taken more or less together, deny even the possibility in the long term.\(^{(1)}\)

Reason (1) does imply a certain bias towards equity financing in the overall procedure, but the realism of this is attested to in many company reports\(^{(2)}\) and is explained for some cases in reasons (5) and (6). After all, this is simply one more application of the fundamental tenet of business finance that equity is the real risk capital.

In this way, by what might fairly be described as a process of successive approximation, forecasted capital demand ranked by profitability is brought into reasonable reconciliation with forecasted capital supply. It will be noted that the approximate nature of this process is enhanced by sundry assumptions which are IMPLICIT in this process. Among them are:

1. That capital - especially debt capital - will be forthcoming in the amount required at the forecasted cost pretty well on spot demand as projects are implemented. That is, there is an assumption of unchanged interest rates over time. The assumption of spot demand may not be invalid if the interest cost function is shifted upwards by the extra 1\(\%\) or 1\(\%\) usually payable as the commitment fee required in return for a guarantee of future loan capital being made available on request.

2. That changes in the variability of the 'master' cash flow and its derivatives will not alter servicing-adequacy. As projects are discarded, compared.

\(^{(1)}\) Compare the identification of capital rationing as "empirically significant in the short term only" by Hirschleifer - Page 240 note (1).

\(^{(2)}\) Thus, in a recent television interview, the Financial Director of I.C.I. stated that the next multi-million quinquennial capital expenditure budget would be wholly self-financed.
re-discarded and re-compared, cash flows will change in their inter-period variability as well as in their period minima. Yet the same interest cost function is applied. This might be a source of significant error.

3. That as the same processes of project discard and substitution are worked out; the changing complex of preferred amounts of different sorts of debt capital do not substantially change the interest-cost function. This point was discussed earlier; it is sufficiently important to be repeated here, with the caveat that if the resulting change in servicing-adequacy is significant, then this ought to be taken into account at each discard, comparison or substitution stage. Without computer facilities, a most laborious task is inevitable.

4. Perhaps more importantly, as the $x_{m1}/x_{m2}$ comparison and substitution proceeds, the weightings of equity cost and debt cost in the overall cost of capital project appraisal rate will change. This is of sufficient importance to warrant consideration in a separate step of the procedure.

**Step 10.**

It is manifestly a necessary precaution to check as to whether the result of such changes in weighting result in a significant change in the appraisal rate. From a theoretical point of view it is essential, as the data of preferred amounts of different capital will now permit the calculation of a study period marginal cost of capital by the Weston method. It will be recollected that the discussion of the theoretical model developed such a marginal cost. The appraisal rate used in Step 1. is an average cost, based on historical rather than forecasted weights. Even the marginal cost as now calculated is the 'average marginal' cost over the study period. To calculate a project-by-project marginal cost, with due regard to discards, substitutions and so forth, would be a theoretically correct but practically untenable recommendation.
Care must be taken to be sure that the preferred amounts and the historical capital mix are in the same terms - viz cash flows or balance sheet entities.

If the resultant cost of capital is very different from the proxy average rate used in Step 1, project re-appraisal may be necessary. Strictly, the whole procedure now may have to be re-cycled, and possibly again thereafter; if the re-appraisal drastically alters project acceptance or ranking. By 'very different' must be meant 'capable of drastically altering acceptance and/or ranking'. If projects have, on the whole, a fairly standardised time-pattern of cash flows; a 50% change in the appraisal rate is not going to alter rankings much but it almost certainly will alter acceptance. If projects have very different cash flow patterns, a 5% change might not alter acceptance but could very well alter rankings. Relevant considerations will be:

(a) How certain are the estimates of project cash flow?
(b) How certain are the estimates of servicing-adequacy (which are basic to the weightings)?
(c) How certain are the estimates of the interest cost function? This will affect both servicing-adequacy and the cost of debt capital used in the calculation of a new discount factor.
(d) How clear cut are the preferences for one form of financing over another? This is important for two separate reasons:

i) If they are very clear cut, a minor change in the appraisal rate might drastically alter the preferred finance method - hence induce a second change in the appraisal rate. This is not thought to be very likely, but a few such cases will arise.

ii) If the preference is not very clear cut, opportunism at the actual time of project implementation (or for that matter the incidence of unexpected but necessary projects - e.g.,
replacement projects) may suggest an acceptable change in financing method and the subsequent weightings actually accruing.

Some or all of these considerations may make any iteration not worth while. A suggested procedure prior to any full iteration would be to select a small number of selected projects which are included in the final budget and which are either at the margin of such acceptance; or which have a cash flow pattern which could be susceptible to a change in the appraisal rate - although the project itself is well within the budget. Such projects could be reviewed for acceptance and ranking (Step 3.). A change of less than \( x \% \) in net present worth; or a ranking change of less than 'y' places would indicate that re-iteration was not worth while. Suggested determinants of 'x%' are: the % error in estimates of profitability revealed by post-completion audits; the setting of confidence limits of say 95% probability, by using the changes in \( \text{NPW} \) as standard deviations from the original \( \text{NPW} \) as a mean. A suggested determinant of 'y' places is: to derive the mean rankings of the original set of projects and of the finally budgeted set and to use the difference in means as the indicator of 'y'. These determinants are at best a crude approximation of the lack of need for an iterative re-processing.

Intuitively, there arises a suspicion that the cost and time incurred in such re-processing might be equally important determinants; much depending on the computational facilities available. And it is probably true to say that changes in the appraisal rate themselves will become very small, with no perceptible affect on \( \Pi \), as \( f(S) \) nears exhaustion and changes in \( M \) become smaller. It is impossible even to generalise. What must be recognised is, again, the sub-optimality of the procedure.\(^{(1)}\)

\(^{(1)}\) But see Vancil (34) p.73 in another context: "An error in estimating the (appraisal) rate may change the project rankings, but the reshuffling of, the list (of projects) will not be so radical as it would be if the cost estimate for one of the projects was erroneous". Pearson Hunt (16) p.20 clearly shows how relatively insensitive is the relationship of present worth and cash flows to quite substantial changes in the discount rate.
Section 3. A Summary of the Procedural Approach.

The purpose of this procedure was stated as being to lay down a framework for a closer integration of the investment and the financial decisions in capital expenditure budgeting. It will be seen that, essentially, the two sets of decisions are still separately considered:

Steps 1 - 3: The investment decision, but recognising the effect of inter-project implementation time lags.

Steps 4 - 8: The financing decision, but recognising that this is not final.

Steps 9 & 10: The integration process, matching the two sets of decisions and recognising their interdependence. Some peremptory limits are suggested to the looped nature of the reactions arising from that interdependence.

The process of matching capital supply and demand is not, it is clear, a tidy, orderly, numerically exact procedure. It involves possible recourse to exceptional debt finance, which is outside the definitions and assumptions of the total procedure. It involves some rather scrappy, disjointed comparisons between projects, and even recourse back to projects which have seemingly been finally discarded. It admits of the under-utilisation of cash flows in the short term. It may even reduce to NIL in the short term, though this is not really likely. It is sub-optimal in that cash flow is not maximised by project rescheduling. It is long-winded. Finally, it would be difficult at the end to draw up a neat work-sheet demonstrating a proof that nothing has been overlooked. But capital budgeting is inexact, despite the clinical perfection of diagram or mathematical model. It is a complex exercise in resource allocation, where the demands are no more than probabilities; and where the resources are flexible, with varying degrees of flexibility according to what steps are taken. Save within the allocation of funds to a division of the firm (and not even there, really) capital rationing is not absolute. It may be convenient for an exposition in financial theory to assume exactitude of estimates, immobility of gearing and absolute rationing; but save at the extreme,
such assumptions are so unrealistic as to render almost useless a potentially valuable instrument of management science, for all practical purposes. By recognising the financial linkage of projects, and by attempting to permit financing of projects at minimum cost while still retaining the selection and ranking of projects by present net worth, the matching process as described can be claimed to take a few albeit uncertain steps towards integrating the financial and investment decisions in an optimal fashion. Finally - and by no means least - the process is capable of practical implementation by the accounting or treasury function of a firm, and corresponds to the realities of cash flow.

One additional benefit accrues to the procedure. There is no need to predetermine a cut-off rate or level of expenditure. Such a cut-off arises naturally out of the procedure. And it can fairly be claimed that it is one which arises out of the realities of managerial strategy, technological imperatives and supervisory capacity as well as out of the financial inexorability of a capital supply correlated to best estimates of the growth of equity and the servicing-adequacy of the cash flow accruing up to that cut-off.

(Some difficulty in determining cut-off, and so completing the capital budget, may arise where competing projects are identical in profitability. Financial criteria can add nothing to the solution of this problem if all the relevant expectations enjoy the same degree of certainty. Probably the choice will be made on more emotional grounds of managerial preference. This is not to deny that those same emotional grounds will not enjoy a somewhat general application, anyway.)

Section 4. The Earning Rate.

The final item of discussion concerns the earning rate. This was temporarily defined as "that rate of earnings which the company would normally expect to accrue to its employment of assets over the long period". Some rather more specific definition must now be attempted. The two related questions are; firstly - what is meant by the definition given; and secondly - what is the relationship, if any, of
the earning rate to the discounting rate.

Essentially, the earning rate measures opportunity cost. It specifies the proceeds which might be expected to accrue to the cash flows generated or released by a given method of financing. It might be thought sufficient simply to specify and discount the cash flows themselves - which is the normal analytical method applied to (e.g.,) lease financing.\(^1\) This takes into account that the cash flows generated by different financing methods differ in timing; but raises problems as to whether or not there is capital repayment - by the definition of financial linkage there would not be; and is overly facile in assuming reinvestment at the same rate of interest as that used in the discounting process.

(Not only are the cash flow timings different. The cash flow frequencies are different. The smooth flow of residual earnings under a monthly-premium leasing plan is more nearly continuous than is the discontinuous stream of earnings accruing to capital allowances, as these are supplemented annually. The result must be an increase/decrease in the inter-time-interval variability, with a potential effect on \(f(S)\). This is not brought out in either the theoretical model or the procedural approach. One way of allowing for this might be to weight the Evaluation Tables for non-variability of net cash flows but without more specific knowledge of \(f(S)\), such weighting would be utterly unreliable).

The earnings rate is therefore an explicit investment opportunity rate. As cash flows evolve from a particular financing method, they could be made explicit by externally investing a like amount of cash. The earning rate on such external investment is frequently termed a 'lending rate' in writings on capital budgeting. In terms which echo Roberts: Bierman & Smidt state:

"... lending means acquiring a portfolio of securities that have approximately the same average risk characteristics as the assets presently owned by the firm".\(^2\)

\(^1\) See, for example, Vancil (34); Bennett et al (6); Herrett & Sykes (19, 20); Bierman & Smidt (8); Bower et al (45); McEncharon (73).

\(^2\) H.V. Roberts (80); Bierman & Smidt (8) p.161 et seq.
(They then define an excess of the borrowing rate over the lending rate as an essential condition of "external capital rationing". The choice of terms seems unfortunate as a subsequent analysis is extended to include inter alia the case where internally generated funds are in excess of the capital requirements of the set of projects under budgeting consideration — scarcely a rationing situation. The usefulness of this differentiation between lending and borrowing rates (investment opportunity and borrowing opportunity rates, to use Vancil's terminology (1) is the accompanying comment that any such difference is a function of capital market imperfections. For in a perfect market, and given that "the same average risk" refers both to business and to financial risk, the firm and the "portfolio of securities" are in the same equivalent risk class. If then the firm's earning rate on internal investment is lower than the lending rate, the price of its equity shares must fall until the yield on its shares is in equilibrium with the rest of its class. Its cost of capital will rise accordingly; i.e., the borrowing rate will equate with the lending rate. But market imperfections do exist, and so differentiation is possible. It is difficult to see that it can be very great however — because a higher borrowing rate in one firm means (given inter-company investment) a higher earning rate to another, and any such advantage in a wide sector of the market would be exploited to the full for anything more than a short period or a marginal advantage (2).

On this basis, it appears that there cannot be too much difference between the cost of capital (which is what the borrowing rate is) — i.e., the discount rate, and the earning rate. Vancil perhaps unwittingly, reinforces this point. He says on adjacent pages:

p.72 - "The (determining) question is "What will the rate of return on the least attractive investment that

(1) Vancil (34, 86) p.34 - 39, 72 - 75.
(2) The modern equivalent of this is to say that if the market value of a firm is consistently below its asset value, and yet some profitability expectations exist; investment in the form of a total or partial take-over bid will be made. A good example, is Hoyston Industries and its 'black-box' division.
we will make during the next few years, be?"

p.73 - "As a minimum, the investment opportunity rate should not be less than the average cost of capital for the firm" and then proceeds to argue in clear terms that the "least attractive investment" will earn at that rate - otherwise it would be better for the firm not to invest at all but return the money to its owners.

Yet it is possible to disagree with all this. In the first place; external investment is irrelevant to the firm - it might as well go into liquidation unless it is a financial holding or investment company. External investment might be rational in short periods of excess liquidity to ensure liquid funds for redemption purposes; or for reasons of corporate defence strategy not applicable to the individual shareholder; but generally speaking it is a contradiction in terms. Realistically, the earning rate (or lending rate, or investment opportunity rate, or whatever) must refer to internal investment.

Secondly, and in despite of Vancil, a criterion of 'least attractive investment' is not valid on practical grounds. Uncertainty is inherent in all project estimates, and no investment would take place if such estimates as were made consistently indicated a return little or no better than financing costs. Some projects will yield very high net present worths, others will not yield so generously. But management will be looking (in whatever appraisal terms are used) for a mix of projects which will yield a positive average net present worth which is sufficiently large to satisfy their unquantified fears and aspirations. Indeed, a management acutely conscious of growth will have quite specific standards of profitability which will be well above a nil net present worth. The cash flows accruing to any financing decision will be reinvested at random in the project complex and so can be expected to earn in excess of the minimum acceptable rate. Such cash flows can be applied anywhere along the curve of marginal efficiency of capital, which over its range lies above that of the marginal cost of capital.

(1) To be fair to Vancil, it later appears that an average of sorts rather than a 'least attractive investment' is his real criterion.

(2) For example see Gordon (61) on security as an investment criterion (not (1) on page 172 also refers)."
Thirdly, the cut-off arising from the matching of capital supply and demand as proposed in this thesis, is seen to arise from servicing-adequacy of cash flow - supported strongly by constitutional limits and less certainly by institutional conventions. The cut-off has been likened to a sharp upturn in the financing cost curve. But is this what happens in fact? What is more realistic is that lenders just refuse to lend any more (the institutional conventions); managers may not borrow any more (the constitutional limit); or \( f(S) \) becomes so weak that managers, faced with unequal quantities of marginal supply and demand, do not invest right up to the cut-off point. Contributory features to this short-fall are: the labour and cost of exploiting the last 'ounces' of \( f(S) \), near-exhaustion of project supervisory capacity, an emotional opposition to certain forms of secondary lending, anticipatory fear of steeply increasing servicing charges. The converging lines of diminishing marginal efficiency of capital and increasing marginal cost of capital very often do not in fact meet, and it is as well to recognise this. The investment opportunity rate stays above the borrowing rate, even at the 'marginal project'.

Note has already been taken of the reinforcement of this effect which is occasioned by the non-linearity of the two curves, as neither the supply of capital or the project outlays are infinitely divisible.

Thus the earning rate should be taken as being different from the appraisal rate, and normally will be set at that rate which on average is expected from project investment. In a static situation - technologically and market-wise - the recent past will prove a fair guide. In a changing situation, some expectation must be specified. The change may be environmental or induced. The earning rate underlying the ordinary share price might act as a starting indicator but will

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(1) "The traditional writers have generally failed to appreciate this operational distinction between the cut-off rate and the cost of capital" Durand (53) p.6). By 'operational', Durand means "in practice given managerial infirmity of estimation, control and purpose".

(2) So that if the firm were wholly equity financed; and if shareholder expectations exactly coincided with those of managers; cost of capital and earning rate would tend towards equality. Also: if shareholder expectations in such a firm are outrageous, but management fails to realise this and blindly accepts earnings yield as an element of cost of capital, while setting the earning rate at more realistic levels, appraisal rate could exceed earnings rate - but not for very long! But see footnote (2) to page 76.
be much modified by management plans, especially if the induced change is activated by growth-hungry management. However, the growth situation reveals quite clearly the complex, indirect but no less real connection between appraisal rate and earnings rate.\(^{(1)}\)

One aspect of this connection is worthy of discussion. It leads to the previously established conclusion that the cost of capital (appraisal rate) sets a floor to the efficiency of capital (earnings rate), and in that sense takes the argument no further. But it seems to stress the realism that must attach to estimates of the earning rate. For if these are wrong:

(a) The sums to be discounted are wrong; and
(b) The discount rate itself is wrong.

Appendix II demonstrates the reinvestment assumption lying behind the discount process if project cash flows cannot be disbursed from the company. A decision to finance by one method in preference to another is a project, and subject to the same set of assumptions. The cash flows arising from a finance project are partly implicit, partly explicit. Residual capital changes\(^{(2)}\) and tax allowances are implicit - i.e., there is no actual receipt of cash; grants and disposal proceeds are explicit. Implicit cash flows are the more easily conceived of as being non-disbursed; but it is general practice to put grants wholly and permanently to capital reserve, or partially and temporarily to revenue reserve 'over the life of the related assets'. Treatment of disposal proceeds is much less predictable, but at any rate the larger part of cash flows are not disbursed.

If, then, discounting by a certain rate (say A\(^\%\)) is to be meaningful the earnings accruing to the non-disbursed majority cash flow must accrue at least at A\(^\%\). Excluding for disposal proceeds, the earnings rate must be equivalent to the discounting rate (which is the project appraisal rate); and they must be realistically so - they really must accrue at that rate or better - otherwise the whole fabric of the

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\(^{(1)}\) Mention has already been made of the explicit difference between earning rate and discount rate contained in the G-S model - which is essentially based on a growth situation.

\(^{(2)}\) Including the initial change from pre-decision zero to the initial maximum available to the decision. Obviously, this is a conceptual flow; hence the term implicit. The initial residual capital is essentially an "opportunity capital", foregone if the decision is one of outright purchase.
investment-finance linkage approach is indefensible.

But if the discounting process implies reinvestment of non-disbursed cash flows at a rate equal to the discount rate; is not the setting of those flows to earn at a specific rate (as in the Evaluation Tables) an error of double-counting? If the data in the Tables were to be regarded as accounting data (specifically, capital cost data) the accusation might be deemed a fair one. But, as was stressed in Chapters Two and Three, the data are cost-relatives; transformed values with validity in no absolute, only in a comparative, sense. They permit the comparison of alternative financing decisions in respect of a given single project, regarded for the purposes of that decision in isolation (although the constraints upon the decision are multilateral). It is logical to conceive of the cash flows accruing to each alternative as being disbursed from the project so far as it, in its insular condition, is concerned; so that, within that decision, no reinvestment assumption accrues. Mention was made in Chapter Two that the earnings stream concept is in some respects an alternative to dual-rate discounting, but in this context it is an improvement on dual-rate discounting - which explicitly allows for refunding the capital outside the company. This is valid enough for instalment debt, but not for equity or funded debt; and even for instalment debt would imply debt servicing wholly out of financing decision cash flows rather than from subsequent operation cash flows. It must however be admitted that in the full investment-linkage model where ex hypothesi the financing decision is not taken in isolation, a suspicion of double-counting is somewhat more tenable. However, the application of the earnings stream concept to all financing methods means that the error (if it is one; and it is by no means clear that it is an error) is common to all alternatives. Some bias may however be created in favour of those alternatives which enjoy exceptionally wide use of the earnings stream concept.

Section 5. Conclusion

This concludes the discussion of an approach to investment-finance linked budgeting. The approach is seen to be inexact and sub-optimal, but it is claimed that a framework for further exploitation of the
concept of linkage has been laid at both theoretical and procedural levels. Even so, substantial intricacies of computation are involved, and it seems fairly certain that an application of the procedure here laid down would involve the financial planning function in a distinctly laborious series of calculations. However, in so complex and involved an operation as the appraisal and financing of capital expenditure, a large volume of accounting work is to be expected if the set of proposed projects is at all extensive. If to this there is added a situation of capital-rationing, and more particularly, a determination to minimise that rationing by the exploitation of all available sources of funds; then inevitably the task is one of Herculean proportions.

The contribution of the Evaluation Tables is that of a set of 'rapid comparison' tables as between alternative financing methods. It well may be that their use in the field of 'spot' or emergency decisions, unrelated to any formal budgeting procedure, is of equal value.

What remains to be done on the wider front is to establish more exactly the nature of the relationships existing between the parameters in the various functions of the model; and thence to derive the coefficients permitting a specific solution of the optimising process. There still will exist the differences and non-comparabilities of present-worth valuations and quantities measured by accounting conventions; and the imprecision attaching to measurement of the cost of capital will continue to cast a shadow on the completeness of the final answer. The problems of the smaller private company in this area, and in respect of the capacity and expense of the computational requirements of the model or the procedural approach also remain as formidable obstacles to a widespread acceptance of "linkage analysis" in capital budgeting. But if this present work can be regarded as contributing usefully towards the necessary provision of an extra dimension to capital budgeting, I shall be well content.
APPENDIX I
A NOTE ON THE M.M. FORMULATIONS

It is not the purpose of this note to add anything new to the controversy as to the effect of gearing upon the cost of capital (1) which was initiated by the original article by Modigliani & Miller (hereinafter referred to as M.M.). In any case, that controversy is now worn threadbare, and has been shown by writers such as Bower, and Robichek & Myers, (2) to be largely ephemeral. But as the M.M. 'Proposition II' is discussed in the light of the empirical analysis of Pages 176 et.seq., in this thesis; it seems appropriate to note in passing one area where that Proposition seems to be less than consistent in its argument.

The basic M.M. position is established in the following set of equations, where (in respect of a company of a given risk class):

\[ Y = \text{expected future earnings before interest and taxation}, \]
\[ I = \text{interest on debt capital, where } i \text{ is the yield on debt capital of market value } L. \text{ } I \text{ is deemed to be more or less certain.} \]
\[ t = \text{the rate of corporate taxation.} \]
\[ D = \text{disposable earnings; i.e., earnings after deducting interest and taxation, accruing to shareholder's interest. For the sake of simplicity, shareholders are taken to be ordinary shareholders.} \]

whence \[ X = \text{total corporate income after corporate tax, i.e., } X = I + D \]
and \[ V = \text{the market value of the firm} = S + L, \text{ where } S \text{ is the market value of the equity.} \]
also \[ e = \text{the equity yield at market value.} \]
and \[ k = \text{the capitalisation rate such that:} \]

(1) Modigliani & Miller (74).
(2) Bower (44); Robichek & Myers (25) Chapters 2 & 3.
\[ V = \frac{X}{k} \] 

but by definition \[ X = D + I \] 

and by definition \[ D = (Y - I)(1 - t) \] 

so, by substitution in Equation (1) 

\[ V = \frac{Y(1 - t) + tI}{k} \] 

or 

\[ V = \frac{Y(1 - t) + tI}{k} \] 

but, from Equations (2a) and (2b) 

\[ Y(1 - t) = (X - tI) \] 

substituting in Equation (3a) 

\[ V = \frac{(X - tI) + tI}{k} \] 

which is, of course, tautological - being equal to Equation (1); but this expansion permits a certain scrutiny of the M.M. reasoning. 

In the context of U.K. corporation tax, \( k \) is thus far an after-tax capitalisation rate; operating as it does on earnings (before interest and taxation) reduced by taxation which is, albeit indirectly, related to those earnings. But it is also a "bastard" capitalisation rate, evaluating a mixed stream of uncertain equity earnings and more-or-less certain debt earnings. M.M. are influenced by this quality of debt earnings to argue in their "Correction" (1) article that they should be capitalised at a specific rate which, being comparatively risk-free, will be different from (lower than?) the general capitalisation rate: 

amending Equation (3a) 

\[ V = \frac{Y(1 - t) + tI}{k} \] 

Notice that the use of \( i \) - the yield on this firm's debt stock - as the capitalisation rate implies that \( L \) is an equilibrium market value of that stock. This implies a happy reconciliation of servicing-adequacy assessments by lender, management and the general capital market. As the servicing-adequacy supportiveness of cash flow is emphatically a function of income; the M.M. assumptions imply 

some generally-held opinions concerning Y. But this is the 'source' of equity capitalisation calculations. Are we then to assume that debt stockholders and shareholders are so sundered, never to meet; that the one group enjoys a happy confidence in, but the other is most indecisive about, the same projected stream of earnings accruing to the same firm? (And this is to take place within the perfect market situation, implying perfect knowledge and communication, hypothesized by M.M.) Admittedly, there is (especially for higher-levered firms) a potentially substantial difference between estimating the adequacy of an income stream in contradistinction to the level of that stream. Nevertheless, there is implied in the argument a commonality of estimation that M.M. seem to have ignored.

Applying Equation (5) to Equation (3b), we have:

\[
V = \frac{(X - tI)}{(k_i)} + \frac{tI}{i} \quad \quad \quad \quad \quad \quad \quad \quad (5a)
\]

or, maybe

\[
V = \frac{(X - tI)}{(k_i)} + \frac{tI}{i} \quad \quad \quad \quad \quad \quad \quad \quad (5b)
\]

The bracketed term is clearly seen to be itself a mixture of debt and equity income.

I feel that much of the subsequent M.M. analysis is at fault because of an over-hasty assumption that:

\[
Y(1 - t) = (X - tI)
\]

is an all-equity income stream. It is not. It is part equity income, part debt income. As such, it makes Equation (5a) illogical: the term \( tI \) is capitalised at two different rates. It is also immediately derived from 'Y', a total corporate income estimate, of which any part is presumably as uncertain as the whole - for one cannot know or be somewhat 'more' certain that income will be sufficient to cover debt servicing, any more than any other capital-servicing liability, fixed or contingent. But Equation (5b) implies exactly this.

The point can be illustrated by a simple example:

\[
\begin{align*}
\text{let} & \quad Y = 100 \\
I & = 10 \\
t & = .40 = \frac{36}{90} \\
D & = 24
\end{align*}
\]
whence \( X = D + I = 64 \)
and \( D = (Y - I)(1 - t) = 90 \times 0.6 = 54 \)

let \( k = 0.12 \)
let \( i = 0.08 \), noting that \( i < k \)
so that \( L = 10 = 125 \)

then; Equation (1) \[ V = \frac{64}{0.12} = 533.3 \]

Equation (3) \[ V = (100 \times 0.6) + (4 \times 10) = 533.3 \]

and as by definition, \[ V = S + L \]
then \[ 533.3 = S + 125 \]
whence \[ S = 408.3 \]

but, from Equation (4)
\[ (100 \times 0.6) - (64 - (4 \times 10)) = 60 \]

substituting in Equation (5)
\[ 533.3 = \left( \frac{64}{0.12} - \frac{4}{0.12} \right) + \left( \frac{4 \times 10}{0.12} \right) \]
\[ = 500.0 + 33.3 \]

We see at once that the term:
\[ \left( \frac{X}{k} - \frac{ti}{k} \right) = \left( \frac{64}{0.12} - \frac{4}{0.12} \right) = 500.0 \]
is greater than the value of the equity:
\[ S = 408.3 \]
so that this term is evidently more than an equity earnings stream. By the same token, the second term:
\[ \frac{4}{0.12} = 33.3 \]
is less than the value \( L = 125 \) and so is less than any certain debt stream. On both counts, the M.M. preferred Equation (5a) is not valid, in fact, that Equation would yield:
\[ V = \left( \frac{64}{0.12} - \frac{4}{0.12} \right) + \frac{4}{0.08} = 550 \neq \text{Equation (1)} \]
and the illogicality of the dual capitalisation rate on 
$tI = 4$ is very plain.

Equation 5(b) yields:

$$V = \left( \frac{64}{0.12} - \frac{4}{0.08} \right) + \frac{4}{0.08} = 533.3$$

which is at least consistent with Equation (1). In fact the first, 
mixed-income term is satisfactorily capitalised by a mixed rate; but 
the arbitrary capitalisation of the second, sub-certain term by a 
certain rate is still open to question.

These observations have value because of the consequent arguments 
based by M.M. on, essentially, Equation (5a). These are:

- a statement of the average pre-tax cost of capital, viz...

$$k' = \frac{V}{V} = \frac{k}{(1-t)} \left( 1 - \frac{tJ}{V} \right) \quad \text{(6)}$$

- a statement of the effect of gearing on that cost, viz..

$$k' = \frac{V}{V} = k - t \left( k - i \right) \frac{L}{V} \quad \text{(7)}$$

together with a statement of the effect of gearing on 
after-tax (corporate) equity yields, viz...

$$e = k + (1-t)(k - i) \frac{L}{S} \quad \text{(8)}$$

Equation (6) is true if, and only if, Equation (5a) is accepted; with 
one exception. If the contention that the mixed nature of both terms 
in the basic Equation (3) demands capitalisation of both by the mixed 
rate $k$ is correct; then the earlier (1) M.M. average cost of capital 
formulation:

$$k' = \frac{Y}{V} = \frac{k}{(1-t)} \left( 1 - \frac{t}{kV} \right) \quad \text{(9)}$$

is valid, but for $tI$ we may substitute $tJ$; and if we accept that 
for capitalisation of mixed stream purposes the rate $i$ used by M.M. 
is really not distinguishable from $k$ (that is, the capitalisation 
rate $i$ is not the debt yield $i$; but $i$ (cap.) $= k$) then Equation (9) 
becomes Equation (6). Under such circumstances of course, $(k - i) = 0$, 
so that:

(1) i.e., in the 1958 article (74) (p. 156).
Equation (7) reduces to \( \frac{X}{V} = k = \text{Equation (1)} \)

and Equation (8) reduces to \( e = k \)

This last is clearly ridiculous. The only way out of the impasse is to accept that \( i \) in Equation (8) is a comparatively risk-free (i.e., a more or less certain) debt-income stream, typical of the sort of industry of which this firm is part, and above which \( e \) commands a further risk premium. The empirical argument of Pages 176 et seq. certainly occasions doubt that, in the U.K. of the '60s anyway, this premium is a function of \( L \) as Equation (8) would indicate. But if these definitions of \( i \) and \( e \) are accepted; (and they are definitions establishing a difference of degree rather than of kind - which is compatible with the argument advanced above immediately after Equation (5b); noting again, the essential pre-condition that \( L \) is an equilibrium market value): Equation (8) has strong intuitive appeal.

Unfortunately Equation (8) is correct if and only if Equation (5a) is correct. Further, and specifically in the context of U.K. corporation tax, it suffers from the inclusion of a comparison term \( (k - i) \) where \( k \) is an after-tax rate but \( i \) is a pre-tax rate.

A possible solution is as follows:

by definition

\[ D = (Y - I)(1 - t) \]

Substituting \( iL = I \), we can write

\[ D = Y (1 - t) - iL(1 - t) \]

but

\[ e = \frac{D}{S} \; ; \; \text{hence} \]

\[ e = \frac{Y(1 - t) - iL(1 - t)}{S} \] \hspace{1cm} (10)

By Equation (9),

\[ k' = \frac{Y}{V} \; \text{or} \; Y = k' \cdot V \]

but as by definition

\[ V = S + L \]

then

\[ Y = k' \cdot S + k' \cdot L \] \hspace{1cm} (11)
Substituting in Equation (10), we have:

\[ e = \left( k' \cdot S + k' \cdot L \right) \left( 1 - t \right) - iL \left( 1 - t \right) \]

which reduces to:

\[ e = (1-t) \left( k' + (k' - i) \frac{L}{S} \right) \]

which is the original "Proposition II" modified for the tax factor. It is of course consistent with Equation (5) so long as \( k' \) is defined by Equation (9); both of which presuppose (as I think, correctly) that all the components of the mixed income stream are capitalised by a common, mixed factor. The problem of delineating a 'risk' class still remains untouched.

..........................
APPENDIX II

THE ARITHMETIC OF INCOME MEASUREMENT

The purpose of this Appendix is to develop an analytical method which will expose clearly the arithmetical assumptions lying behind the process of discounting future cash flows as a method of investment appraisal. Because these assumptions are well known, there can be no originality in this aim; but a recapitulation via a certain form of presentation can throw a useful light on the process and its various manifestations. In particular, it permits a close scrutiny of the Gordon-Shapiro model of the cost of equity capital and consequently of the applicability of that model to the finance-investment linkage model developed in this thesis. Logically, some further thought must be given to the reinvestment assumptions clarified by this analysis.

The Analytical Method

Let \( V \) = the net present value of a discrete cash stream

\[
F_0, F_1, \ldots, F_t \text{ accruing at the end of}
\]

constant time intervals \( 0, 1, \ldots, t \); using

an interest rate \( i \)

i.e., \( V_0 = \frac{-F_0}{(1+i)^0} + \frac{F_1}{(1+i)^1} + \frac{F_2}{(1+i)^2} + \ldots + \frac{F_t}{(1+i)^t} \) \( \cdots \cdots \cdots \cdots (1) \)

where \( -F_0 \) is the initial outlay at \( t_0 \) necessary to obtain the subsequent cash inflows:

and \( V_1 = \frac{F_1}{(1+i)^1} + \frac{F_2}{(1+i)^2} + \ldots + \frac{F_t}{(1+i)^t} \) \( \cdots \cdots \cdots \cdots (2) \)

Let \( V \) = the gross present value of the same stream of cash inflows under the same circumstances.

i.e., \( V_0 = \frac{F_1}{(1+i)^1} + \frac{F_2}{(1+i)^2} + \ldots + \frac{F_t}{(1+i)^t} \) \( \cdots \cdots \cdots \cdots (3) \)

(1) See Jaedicke & Sprouse (18) and Bobichek & Myers (26) for analogous expositions. Some of the symbols used here are those of J.A.M. Essentially it is concern over the implicit assumptions which has led to the development of dual-rate discounting and the two-rate analysis of Pearson Hunt (16).
Multiply Equation (3) by \((1+i)\)

\[
V_0 + iV_0 = F_1 + \frac{F_2}{(1+i)} + \ldots + \frac{F_t}{(1+i)^{t-1}} \quad \text{..........................(3a)}
\]

Substitute Equation (3a) into Equation (2)

\[
W_1 = V_0 + iV_0 \quad \text{or, more generally}
\]

\[
W_t = V_{t-1} + iV_{t-1} \quad \text{.................................(4)}
\]

Subtracting Equation (3) from Equation (1), and ignoring the sign of \(F_0\)

\[
W_0 = V_0 = F_0 \quad \text{or, more generally}
\]

\[
W_t = V_t + F_t \quad \text{.................................(5)}
\]

(where in the special case \(t = 0\), \(F_t\) is negative).

Substituting Equation (5) into Equation (4)

\[
V_t + F_t - V_{t-1} = iV_{t-1} \quad \text{or,}
\]

\[
iV_{t-1} = F_t + (V_t - V_{t-1}) \quad \text{.................................(6)}
\]

Now \((V_t - V_{t-1})\) is the inter-period change in the present value of the cash inflow; i.e., it is a measure of depreciation (if a negative quantity) or appreciation (if a positive quantity). Using \(D\) with an appropriate sign \((- = \text{depreciation})\) as a measure of this change:

\[
iV_{t-1} = F_t + (-D_t) \quad \text{.................................(7)\, (\text{a})}
\]

But \(iV_{t-1}\) is a measure of the income accruing in period \(t\) to an investment of value \(V_{t-1}\); i.e., to the unexpired value of the investment at the end of period \(t-1\) or the start of period \(t\).

Writing \(Y_t\) for the income in period \(t\), and rearranging Equation(7),

\[
F_t = Y_t - (-D_t) = Y_t + D_t \quad \text{.................................(7a)}
\]

or

\[
F_t = Y_t - (+D_t) = Y_t - D_t \quad \text{.................................(7b)}
\]

for investment depreciation or appreciation, respectively.
The arithmetic of discounting thus specifically states that each increment of the subject cash flow is part income and part change in the capital value, where the income fraction is equal to the project rate of return \( r \) on the capital value of the project at the end of the previous period \( V_{t-1} \). Assuming for expository purposes that \( D_t \) is negative i.e.,

\[
V_t < V_{t-1} \quad \ldots \ldots < V_0
\]

and that the cash inflows \( P_1, \ldots \) are disbursed from the project; then as \( D_t \) is disbursed - i.e., there is no reinvestment assumption - \( Y_t \) is a constant return \( r \) to the diminishing capital value of the project.

(it is of course in this sense that such writers as Merrett & Sykes, and Alfred & Evans, deny the existence of a reinvestment assumption in the DCP process).

But the key phrase in this argument is "disbursed from the project" - a concept possibly of satisfaction to a management concerned to appraise each project on its own; but of little use to the shareholder to whom disbursement must mean "out of the Company" if he is to be aware of it.

Suppose the fraction \( D_{t-1} \) be reinvested as it arises at the end of period \( t-1 \) at the rate \( r \) during period \( t \). Then (using \( V' \) and \( Y' \) for the aggregate values):

\[
V'_t = V_t + D_{t-1}
\]

and

\[
Y'_t = Y_t + rD_{t-1}
\]

Assuming capital depreciation, such that \( D_{t-1} \) is a negative quantity, equal to \( (V_t - V_{t-1}) \):

\[
V'_t = V_t - Y_t + V_{t-1}
\]

and

\[
Y'_t = Y_t - rV_t + rV_{t-1}
\]

or, more generally and for a series of such reinvestments for periods \( 0, 1, \ldots, t \):

(1) See Edwards & Bell (13) for an incorporation of this, specifically, into an accounting model; and Amey (2) Chap.6 for the consequent impact upon asset accounting.

\[ V'_t = V_t \cdot V_{t-1} + V_{t-1} + \ldots + V_0 \quad \cdots \quad (8) \]

and
\[ Y'_t = Y_t \cdot Y_{t-1} + Y_{t-1} + \ldots + Y_0 \quad \cdots \quad (9) \]
simplifying to:
\[ V'_t = V_0 \quad \cdots \quad (8a) \]
and (as \( Y_t = iV_{t-1} \) by definition)
\[ Y'_t = iV_0 \quad \cdots \quad (8b) \]

The result of the reinvestment of \( D_t \) at rate \( i \) is to yield a constant percentage return to a constant investment value. Hence there is a constant absolute income value.

**The Gordon-Shapiro Model**

It is interesting to apply this otherwise obvious analysis to a modified G-S illustration, looking at the process from the point of view of the individual shareholder.

Postulate an initial share price of £100, i.e.,
\[ P_o = P_o = 100, \text{ where } P_o \text{ is negative} \]
and postulate a basic earnings rate of 10%. A constant 40% retention is able to be exponentially invested in additional projects at 20%. \(^{(1)}\)

A 5-year study period is demonstrated with sale of the investment at the end of that period. For the sake of simplicity, taxation is ignored. Then, the pattern of reinvestment and dividends is \(^{(2)}\):

<table>
<thead>
<tr>
<th>Period</th>
<th>Cumulative Reinvestment</th>
<th>20% Reinvestment</th>
<th>10% Basic Earnings</th>
<th>Total Earnings</th>
<th>Retentions</th>
<th>Dividends</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>10.00</td>
<td>10.00</td>
<td>4.00</td>
<td>6.00</td>
</tr>
<tr>
<td>2</td>
<td>4.00</td>
<td>0.80</td>
<td>10.00</td>
<td>10.80</td>
<td>4.32</td>
<td>6.48</td>
</tr>
<tr>
<td>3</td>
<td>8.32</td>
<td>1.66</td>
<td>10.00</td>
<td>11.66</td>
<td>4.66</td>
<td>7.00</td>
</tr>
<tr>
<td>4</td>
<td>12.98</td>
<td>2.60</td>
<td>10.00</td>
<td>12.60</td>
<td>5.04</td>
<td>7.56</td>
</tr>
<tr>
<td>5</td>
<td>16.02</td>
<td>3.6</td>
<td>10.00</td>
<td>13.60</td>
<td>5.44*</td>
<td>8.16</td>
</tr>
</tbody>
</table>

*Though the investment is liquidated, the shareholder will not receive any larger gross dividend than 60% of his share of total earnings."

\((1)\) Solomon (\(30\)) p.62 et seq, would categorise this as a dynamic 'true-growth' model, as opposed to an 'expansion' model.

\((2)\) The rate of growth of total earnings retention and dividend (8.%) is exactly as predicted by Merrett & Sykes (\(19\)), while it albeit with something less of an air of mystery.
and the arising cash flows are:

<table>
<thead>
<tr>
<th>$P_0$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
<th>$P_4$</th>
<th>$P_5$</th>
<th>Disposal Proceeds ($P_5$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100</td>
<td>+6.00</td>
<td>+6.48</td>
<td>+7.00</td>
<td>+7.56</td>
<td>+8.16</td>
<td>+146.92</td>
</tr>
</tbody>
</table>

(All amounts received at end of period.)

where the exit share price $P_5$ is assumed to have appreciated pari passu with the earnings and dividend growth rate - i.e., the initial and exit anticipated price-earnings ratios are identical. (1)

The internal rate of return $k$ of this cash stream is 14%, thus satisfying the basic G-S model:

$$k = \frac{P_0}{P_0} + rb$$

or

$$0.14 = \frac{6}{100} + (0.4 \times 0.2)$$

Before proceeding further, several observations must be made:

1. The values are those necessary to establish equilibrium. In particular:
   a) $P_0$ must be set equal to £100. If $P_0 = £50$, yet anticipated exit share price remained unchanged, (a situation of irrationally - i.e., unrelated to underlying profit trends - high growth such as characterised many U.K. share prices in 1968); $k$ would increase sharply - presumably in excess of shareholder requirements, so inducing a bull market in those shares and inducing $k$ to "return" to 14% as $P_0$, $P_1$ etc. increased.
   b) Therefore $P_1$, the exit share price, must be set

(1) i.e.,

$$\frac{P_0}{E_1} \cdot \frac{100}{10} = \frac{P_5}{E_6} = \frac{146.92}{E_6}$$

By extension of the above table, anticipated $E_6 = 14.69$. Note the resemblance to Solomon's (1968), "$k = \frac{E_6}{P}$".

(2) $P_1$ etc.: allowing for a time lag beyond $P_0$ and thus a short-period disequilibrium.
equal to earnings or dividend growth over the study period; i.e.,

\[ P_t = P_0 \ (1+rb)^t \]

where \( r \) = the rate of growth of incremental earnings and \( b \) = a retained portion of those earnings, which portion is anticipated to be a constant percentage:

if \( k, P_0 \) and \( P_t \) are to be in equilibrium. (1) If the exit share price is in fact materially different from \( P_0 \ (1+rb)^t \) - as in this particular case - there is a situation of disequilibrium and the G-S model is non-applicable.

2. If in fact, \( P_0 = 50 \) and \( P_t = P_0 \ (1+rb)^t = 73.46 \) for \( t > 0(\text{ jsx}) \) then the internal rate of return is 20%, obviously enough.

a) This represents the true expansion model, for current as well as additional earnings are accruing at a common rate of 20%. Either the growth or the expansion model is consistent with the finance-investment linkage model. Given the concept of a sub-set \( x_0 \) of "starter" projects, the growth model has a spurious affinity; but no assumptions are made in the model as to the relative profitabilities of \( x_0 \) and \( x_1 \).

b) It can be seen that \( k = r = \frac{E_{t+1}}{P_t} \) (2), the anticipated earnings-price ratio, or earnings yield. This is of course an extreme case of the basic supposition

---

(1) The reciprocal of Gordon's specific assumption "...the price at any future date is expected to be the discounted value of the subsequent dividends" (14, 59) as, ex hypothesi, \( D_0, D_t \) etc. will grow at the same rate \( rb \). Again, this is identical with "Rule 2" in Herrett & Sykes (19).

(2) See Gordon (19, 59) p.439: "The price of a share is independent of the corporation's retention and investment, if the rate of return which the corporation can earn on investment is the same as the rate of return which shareholders require".
of the G-S model that the rate of profit required by investors when placing a value on a dividend expectation is an increasing function of the rate of growth: \( r \) in the expansion model is on average higher than \( r \) in the rational growth model.

The analytical method developed in the first part of this Appendix will, when applied to the numerical illustration, yield the following data:

| \( t \) | \( V_t \) (value 'cum div') | \( V_t \) (value 'ex div') | \( F_t \) | \( Y_t \) | \( D_t \) *
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>100.00</td>
<td>-100.00</td>
<td>Mill (100.00)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>114.00</td>
<td>108.00</td>
<td>6.00</td>
<td>14.00 (8.00)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>123.12</td>
<td>116.64</td>
<td>6.48</td>
<td>15.12 (8.64)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>132.97</td>
<td>125.97</td>
<td>7.00</td>
<td>16.33 (9.33)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>143.51</td>
<td>136.05</td>
<td>7.56</td>
<td>17.64 (10.08)</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>155.08</td>
<td>0</td>
<td>155.08</td>
<td>19.03 -136.05</td>
<td></td>
</tr>
</tbody>
</table>

(* As all values of \( D \) are deducted from the values for \( Y \) in the same period, i.e.,

\[ F_t = Y_t - D_t \] (Equation 7b)

\( D \) is appreciation in this Table. An appreciation of £100 in \( t = 0 \) merely recognises that the original commitment of £100 is now sunk capital and irrelevant to further appraisal studies. At \( t = 5 \), \( D = 136.05 \) is actually composed of 10.87 appreciation and 146.93 disposal proceeds on disinvestment.

For the year \( t = 0 \), depreciation of £100 \((Y_0 = 0)\) is a mathematical cypher, and can be ignored as being meaningless in reality. As \( F_t = Y_t - D_t \) then plainly \( D_t \) (in this case, appreciation) is reinvested; the G-S model implies reinvestment of the capital appreciation.

(This apparent contradiction to the proofs of Equations 8a and 8b lies in the appreciation, as opposed to depreciation, of capital which is embodied in this illustration and indeed in the G-S model).

Under these circumstances - i.e., the distribution of only an (increasing) income derived from an increasing capital - the G-S
model must by very definition claim that dividends are the determinant of the cost of equity capital. The assumption of an exit share price which has grown at the same rate as earnings is an introduction of an earnings determinant reflected in capital gain, But as that gain is a function of the retention rate, or alternatively, of the pay-out ratio, it is fundamentally still a dividend determined value.

The Reinvestment Assumption

Intuitively, the model makes a second appeal in terms of business practice. Although the model implies a 'negative depreciation', nevertheless that depreciation is reinvested - which is in line with business practice (abstracting from difficulties arising out of differences between depreciation as measured here and as measured by accounting convention). Earlier in this Appendix the phrase "disbursed from the project" was used to define a non-reinvestment assumption in the context of managerial decision making. But whether the objective chosen be that of maximisation of the value of the firm or of the present value of shareholder's equity (two not incompatible but not identical criteria); management must be concerned with the disposition of funds released by maturing investment cash flows in the typical reality context of capital which neither legally nor contractually can be repaid during the project life. Given a typical capital mix as envisaged in the finance-investment linkage model, a significant part of project capital cannot be disbursed as part of $F_t$ outside the company.

Insofar as a project can be looked at in isolation, the disposition of its accruing cash flows other than $F_0$ are a matter of indifference to the appraisal of that project. By definition, this dispensation can not be extended to mutually exclusive projects in any capital situation, nor to independent projects in a capital rationing situation where some form of ranking is inevitably necessary. The finance-investment model developed in this thesis is within a capital rationing situation, and a reinvestment assumption is relevant.
Even in the context of self-liquidating capital such as instalment debt, the problem of reinvestment of cash flow accruing to the project after the debt finance is totally redeemed must be faced. This of course assumes a positive present net worth for any such projects. That net worth is only valid if, after extinction of the capital debt, subsequent inflows can continue to be reinvested at the project rate. It is the realisation of this implication which has lead to the later developments in dual-discount rate procedures.

As accountant rather than economist, I find the problem to be realistically more serious than the literature of investment appraisal appears to appreciate, for this reason. A larger and significant part of accruing cash inflows consists of tax allowances which actually will accrue as a reduction in corporation tax liability rather than as a positive cash benefit. For many companies in a capital rationing situation, the result will be a welcome relief of the strain on existing working capital.\(^1\)

Almost by default rather than by conscious decision, the situation thus created will be allowed to remain without any corresponding transfer from working capital to investible funds. In short, accruing cash flows to a significant extent will be invested in working capital.

In a special way, the present finance-investment model is bound up in this concern with the reinvestment assumption. The whole concept of 'servicing-adequacy' of cash flow \(f(s*)\) means reinvestment of accruing cash flows. Additionally, a retention coefficient \(R\) is specifically introduced into the model as a source of investment capital. It is to be noted that the reinvestment problem of this model is not only to ensure reinvestment at the minimum rate of return \(k\) but within that constraint to reinvest in that set of projects which will maximise \(f(s*)\) in as nearly an exponential manner as possible. A more or less unconscious, permanent investment in working capital (as envisaged in the

\(^1\) Working capital strains in expansion or growth situations are common experience. Management seems frequently to underestimate the working capital implications of replacement decisions in the context of rapidly developing production technologies - perhaps due in part to the rigid manner in which their financial advisors separate out working from capital. Accounting period conventions are much to blame here - see Park & Gladson 'Working Capital' (MacMillan). (23)
preceding paragraph) may very well assist in maximising $f(S*)$.
Externally raised finance may then be devoted to fixed asset investment.
Instalment-debt is particularly apt to this situation: in the first
place, it tends to be exclusively financed out of working capital -
now seen to be strengthened; in the second place, in its leasing
form the "danger" of such unconscious investment is lessened because
fewer tax allowances accrue to this sort of finance. (1) The "danger"
referred to is the tendency of working capital investments typically
to enjoy lower returns than those accruing to fixed asset investments. (2)
Leasing apart (and maybe not even then) a potential conflict within
the model is discernible if reinvestment in working capital tends to
improve $f(Sf)$ but also tends to earn at below $k$. The earning criterion
must take precedence; expansion for the sake of expansion, growth for
the sake of growth, are irrelevant to the stipulated objective of the
model. If there is any real truth in the allegations that generally,
British industry tends to have excessive stocks and debtors and too
low a working capital turnover; one of the advantages of the finance-
investment model may be that its concentration on financial linkage
and cash flow reinvestment must make for a keen managerial awareness
of just what are the relative magnitudes of the firms' different
capital uses.

(1) This of course is not strictly true insofar as lease premiums are
reduced by the lessor passing on his tax concessions. So long as
premiums are reduced evenly over the primary leasing period, the
"danger" is more evenly spread and conceivably less critical
in its impact.

PART THREE

"INDUSTRY" DISCOUNT FACTORS
CHAPTER SEVEN

"INDUSTRY" DISCOUNT FACTORS: A STATISTICAL EXAMINATION.

Introduction: The Purpose of the Analysis

During the discussion on "cost of capital constraints" in Chapter Five, fairly frequent mention was made of the use of an "industrial average" or similar representative statistic as a means of quantifying the cost of equity capital, e., when calculating the discounting factor k for use in evaluation of the investment-finance linkage model. It was previously observed in Chapter Four that in a great deal of the literature of financial theory there seems to be implied a somewhat uncritical substitution of "industry" for the rather nebulous concept of "risk class" which is basic to argument concerning the cost of capital. All too frequently, the category "industry" remains unspecified, (1) with a most particular uncertainty as to whether what is meant is "industry" in terms of technology or of product/market. In what follows, "industry" is defined in the latter sense. The purpose of this analysis is to examine, albeit in a perfunctory manner, whether in the U.K. "industry" in this sense is a meaningful concept so far as concerns the cost of equity capital. (2)

It is important to note that there is made no attempt to measure that cost, for the firm or the industry. But it seems a necessary first step towards such measurement to ask if an approach lies through the use of industry as a proxy for risk class. Much of the argument is to be found in Section 4, Chapter Five; but it may not be inappropriate to summarise it here. The relevant steps are:

1. The cost of equity capital is measured by that rate of return

(1) Durand (53) is a perfect example of this - despite, as was pointed out in footnote (2) to Page 258, specifically opting for an "industrial" statistic.

(2) This analysis was devised and substantially completed before Samuels (81) published his findings. Although his methodology and approach (i.e., the use of the individual firm rather than industry) is dissimilar, Samuels' conclusions are not inconsistent with those of the present analysis. The abandonment of quotation marks around the term "industry" from this point on in the present text, is not to be taken as a relaxation of a critical attitude towards the term.
which discounts the aggregate of future expectations of earnings accruing to possession of the equity into equality with the present market price of that same equity.

2. Those expectations are in fact the means of probability density functions projected by the holders of the equity.

3. Two at least of the practical determinants of those projections are:

(a) the past yield performance of the equity in question. (1)
(b) the extent to which the equity in question can be identified as belonging to some "class" of share, whose characteristics are seemingly predictable to the holders of the equity.

This summary is very abbreviated, but two things clearly emerge. Firstly, that there are such things as distinguishable "classes" of share (and, in this analysis, there is sought to be tested the use of industry as a proxy for class). Secondly, the projection of future expectations from a basis of historic actuality permits the observation that the more unsettled has been that history, the more uncertain must be the expectations. Reference is made to the work of Wippern(2) in this connection; and, somewhat in extension of his work, the present analysis consists of a statistical examination of the affect of earnings variability upon share price on an industry basis. The statistical technique used is different, as is the approach. Further, the environment is that of the U.K. rather than that of the U.S.

Nevertheless, the statistical conclusions reached are very similar and are capable of the same sort of interpretation.

It may seem defeatist to speculate that that which is not yet begun will prove to be inconclusive. But such have been the results of many much more sophisticated statistical forays into the field of cost of capital. Much of the reason for this must be attributed to the statistically "impure" nature of a great deal of financial data.

(1) Benishay (41) especially, demonstrates how much more complex and widely drawn than this are the total relationships of expectations and achievements.

(2) Wippern (95); and Page 193 of this text.
The design of statistical experiments implies that the physical nature of the data which is to be manipulated is fully known to the observer, and is capable of being exactly measured; and that the number of replications can be controlled by him. The measurement of financial data is imprecise, and usually inconsistent. (1) Derived from the reports of others, its precise replication is quite uncontrollable by the observer; who often must resort to expedients and approximations to render the data sufficiently malleable for his purpose. Thus the statistical instruments which he uses are ill adapted for his raw material, and all conclusions are subject to qualification and seemingly endless refinement.

It seems opportune to examine the data of the present experiment from this point of view, and this is carried out in Section 1 of this Chapter. In the following Section, the approach and methodology of the analysis is presented and discussed; the processing of the data and interpretation of the results being presented in Section 3. The limitations of the conclusions thus reached are discussed in Section 4, together with some suggestions as to potentially useful expansions of this present approach.

Section 1: A review of the Data

The statistical techniques used consist of the submission of reported equity earnings and share prices, 1956-64, for different industries; to analysis of variance and regression analysis. In this context, industry is defined on a "product market" basis, on the grounds that the market is the principal generator of business uncertainty - which is basically what the exercise is concerned to

(1) And often statistically dubious. What, for example, is the "population" of share prices? Does it exist? The use of sampling techniques implies that it does; but it then seems to be so all-inclusive as to repudiate meaningful conclusions, or so small and fragmented as to defy useful conclusions. Nor must it be overlooked that most statistical sampling theory presupposes a normal distribution of the data - a requirement by no means certain to exist in reported financial data.
examine. The industries thus utilised are:

- Chemicals
- Textiles
- Engineering Supplies and Components
- Mechanical Engineering
- Construction
- Heavy Engineering
- Construction Supplies & Components
- Food Processing.

As an indication, the samples are comprised of firms producing, inter alia:

1. Chemicals - industrial chemicals and gasses; carbon-based products, fertilisers and pharmaceuticals; but excluding the operations of companies generally known as "oil companies".

2. Textiles - wool, cotton and synthetic textile manufactures, including clothing and hosiery.

3. Engineering Supplies and Components - engineering consumable stores; castings, bearings and silicates; small components for electrical and mechanical engineering; steel stockholding, but not the operations of companies generally known as "the iron and steel industry", other than G.K.N.

4. Mechanical Engineering - light and medium non-electrical engineering manufactures such as vehicles, textile machinery, manufacturers plant, contractors plant, machine tools etc. Essentially, these products are finished machines or spare parts therefor.

5. Construction - the output of building and civil engineering; plant hire; heating and ventilation engineering.

(1) A list of the companies comprising each industry, together with details of equity earnings and share price variability over the period of the analysis, is to be found in Appendix III. Originally two additional industries - 'Consumer Durable Supplies' and 'Services' - were processed; but the heterogeneity of companies falling into these categories was sufficiently great to cast overwhelming doubt upon the validity of results utilising these samples.
6. Heavy Engineering - one-off or small-batch large monolithic products for power generation and distribution; steel works and other heavy industry, or complete chemical plant,

7. Construction Supplies and Components - raw materials (timber, stone); part processed components (joinery work, cement, bricks); and finished components - electrical equipment, sanitary equipment, paint etc.

8. Food Processing - food and drink, excluding alcoholic products; whether part processed (bacon, fish, tea etc.) or fully processed (confectionery, mineral waters etc.).

With the growing trend towards multi-product firms, a considerable amount of artificiality is inevitable in ascribing companies to particular industries. For example, Newton Chambers is included in the 'Heavy Engineering' sample - yet in the later years of this analysis, some 30% of earnings derived from pharmaceutical products. Throughout the period 30% of Tarmac earnings derived from the sale of construction supplies (tarmacadam) to users other than its own civil engineering functions; yet this company is included in 'Construction'. And, given its enormous reliance on the textile market, is I.C.I. correctly ascribed to 'Chemicals'? There is of course no satisfactory answer to this problem, and the analyst must in the end fall back upon a combination of the majority activity of the company in question and due regard for the technology used - which is of course inconsistent with the present definition. One advantage of that requirement of the 1967 Companies Act which requires attribution of sales and profits to different activities,\(^{(1)}\) is that some allocation of multi-product companies over industries might become possible.

Further difficulties in this area arise even if the problem of multi-production were to be avoided by careful selection of representative companies. In practice this would be difficult if samples are to be of a respectable size; but even so, what steps can be taken to reconcile differences between companies which, producing

\(^{(1)}\) See Amey (2) p.39.
the same sort of product, nevertheless are at different extremes of a price spectrum; are different in size and hence in reputation; or sell in geographically distinct markets - e.g., export and domestic; or are guided by totally different "management philosophies"? This last differentiation might be the most important, especially where financial policy is concerned. Thus two otherwise acutely competitive companies might have so divergent policies that they cannot be regarded as parts of a common group. (1)

In the present analysis, industries are represented by samples of 15 companies per industry. These are anything but random samples - despite the implications for the significance tests described in Section 3 - in an endeavour to eliminate some of the foregoing problems. All companies are large, well-known (at least to professional and institutional investors) companies, relative to the modal size of firm in the particular industry. (2) As well as forming a prerequisite for the necessary continuity and detail of data, such selection avoids to some extent problems arising out of any positive correlation of share price (irrespective of earnings performance) with size or reputation. (3)

Another selection criterion relates to gearing: all sample firms utilising less than 20% gearing (as defined in Chapter Four) over the period of the exercise. (Many have increased it since). The two

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(1) Further areas of analysis difficulty are to be found in Barges (4) Chapter 2; and Fisher (55).

(2) i.e., "large" in terms of book value of assets employed; "well-known" in that all companies are quoted on the London Stock Exchange, where their quoted securities are the subject of daily (or thereabouts) transactions: "relative" in that it is doubtful if any textile company will be as "large" and "well-known" as I.C.I.

(3) On size see Benishay (41) who finds a significant positive correlation between size (asset value) and share price stability. The present analysis does not altogether eliminate this effect, as on average, the sample (e.g.,) textile firms are much smaller than the sample construction or heavy engineering firms. On reputation - which is probably another interpretation of size - see Barges (4); who finds a significant positive correlation between share price and "over-the-counter-trading" status: but see the footnote to page 269 on the issue of trustee status in U.K. firms. 95% of sample firms enjoy trustee status. Fisher (55) is concerned to draw up sufficiently large samples on an "industrial" basis which conform to a marketability (of shares) criterion. See also Amey (2) Chap.V, on the difficulties of determining inter-firm compatibility.
advantages of this criterion consist of:

(a) minimising insuperable incompatibilities of capital structure between companies which otherwise could reasonably be regarded as being parts of a common group.

(b) permitting satisfaction of the basic purpose of the whole exercise - i.e., examination of business risk - with a minimal 'random noise' effect derived from inter-company differences in financial risk.

Finally, conducting the analysis in terms of equity earnings rather than equity dividends eliminates another inter-company differentiation; that of different dividend policies, with relative effects on share prices different pay-out ratios may have. Such different effects may, again, occasion investment differentiation between companies which otherwise would be regarded as of the same group. The problems of multi-production, geographical market areas and product price-spectra remain basically unsolved.

The choice of the study-period on which to base the statistical analysis reflects two considerations:

(a) The period must be long enough to ensure some degree of statistical validity, and yet recent enough for conclusions based upon it to have relevance to the present. A relevant consideration was that 1965 saw the beginning of the current wave of take-overs etc. - since when it has been much more difficult to find enough well-known companies with adequate continuity of identity and activity.

(b) The peculiar problems associated with the introduction of Corporation Tax in 1964/65 and Capital Gains Tax in 1965

(1) I have in mind both the overt problem of the relationship of dividends to share price (e.g., the G-S model) - for pay-out ratios are a function of earnings, so that such models are fundamentally a product of two tiers of financial parameters; and the covert problem of the information content of pay-out ratios for investor expectations - see Benishay (41); who - comparing projections with realisations - finds an excessively positive correlation of market rates of return with pay-out ratios; explicable as an extrapolatory error on the part of the investor. The interpretation of the effects of pay-out ratios in correlation analysis is difficult. On the controversy as to whether earnings or dividends are the better determinant of share price, see Friend & Puckett (56).
must be avoided if possible.

These "peculiar problems" are well enough known not to require more than a brief relation to the present context. Potential distortion of distribution policies, often but not universally in the direction of increasing pay-out ratios to maintain shareholder disposable income, would introduce the type of within-group differentiation previously referred to, and on a potentially larger scale: and might well result in statistically abnormal earnings-price ratios over the short period. The focussing of attention on share price at the base date of April 1965 (for capital gains purposes) would tend to introduce a systematic bias into subsequent share price data. The subsequent artificial shortage of equity stock would also introduce bias into later data.

(In the event, of course, the impact of those and other factors - principally the post-1965 rapid development of institutional investment, as unit trusts and the like grew in popularity and as pension and trustee funds switched into equities as a hedge against inflation - together with the effect of inflation itself upon company reported profits; caused over 1967/68 so fundamental a variation from the previous general levels of share prices and earnings yields that it is doubtful if the criterion of relevance to present-day conditions is met. However, it is too early to be overly pessimistic on this account, especially in the face of a firm conviction that the 'Times' ordinary share index will come back 20 - 30 points during 1969). (1)

A further practical issue is the difficulty of linking pre-Corporation Tax financial statistics with later data, except on a rough-and-ready basis. The transitional relief provisions and the substitution of grants for allowances render the re-calculation of after-tax equity earnings either excessively complex and time-consuming, or subject to a wide margin of error. Just to what extent changes in post-Corporation Tax share prices, when compared with pre-Corporation Tax prices, are a reflection of performance variations is impossible to say - not the less so because the performances themselves are nearly incompatible in after-tax terms. Straight proportioning of post-1965 prices to adjustments in after-tax earnings made on the hypothesis of a continuation of income-tax plus profits tax, just will not do - for

(1) A conviction held with effect from mid-year 1968.
the price to be proportioned contains an unknown premium (or a discount) which reflects the very existence of a changed tax structure.

For these reasons the study-period 1956-64 was chosen as avoiding the complications associated with changes in the bases of company and shareholder taxation and yet being of an acceptable time-span in terms of length and contemporaneity. Admittedly, investor and market behaviour was in this period somewhat different from that of today. Less exposed to capital gains tax, the market contained a smaller element of institution-management investors. As a result, prices tended to be more flexible due to a greater willingness to take profits and a tendency for transactions to be more personal and fragmented. At the same time, the greater volume of daily transactions in readily available securities tended towards the price-smoothing effect of a technically broad market. Certainly the market was more truly representative of company performance, even though investors were probably on balance less sophisticated - preferring over the period as a whole dividends to capital gain, save for the sharper minority who saw the undeniable advantage of long term tax-free capital gain over short-period surtax liability. (1)

To some extent therefore, a statistical analysis based upon this period and using earnings data rather than dividend data as a primary determinant of share price, is subject to error. However, the principal advantage of earnings data - avoidance of the difficult interpretation of the informational content of pay-out ratios - has already been stipulated; and during the period growing familiarity with the simpler concepts of return on capital employed was beginning to focus equal attention upon earnings. Another aspect of the market psychology of the time was a lingering suspicion of gearing, with no tax-induced leverage potential as yet available. A third reason for the sample selection of low-gearied companies is to avoid any systematic bias in share price which that suspicion might otherwise induce.

Further clarification of the term '1956-64' is needed. So far as concerns earnings:

'1956' (e.g.,) means the company financial year ending during the calendar year 1956. Thus, in fact, the actual period of trading covered may include (say) nine months of 1955 and three months of 1956; and (say) nine months of 1963

(1) See G.R.Fisher "Some Factors Influencing Share Prices" Econ.Journal March 1961: A statistical examination of share price variability over four industries, 1949-56 concludes that dividends are perhaps twice as important as earnings. (Ref.55). Friend & Puckett (56) report opposed findings.
and only three months of 1964. The overall average company year end for all sample companies is August.

So far as concerns share price:

'1956' (e.g.) means the average of the highest and lowest prices recorded on the London Stock Exchange during the calendar year 1956.

The effect of using these two slightly discordant time series is that on average prices are related to the results of a preceding period. On average, this is advantageous: for if price is reactive to earnings performance, then price variation must wait upon the announcement of those earnings - which is itself subject to audit delay. There are however various unknowns in the situation:

(a) To what extent is performance estimated and price adjusted in advance of any actual announcement? Presently, one would have thought - very much so: especially given the growth of interim reports (as opposed to dividends - see Page 262). However, this tendency was very probably not so well developed during the study-period, especially in the earlier half.

(b) Where the financial year ends on December 31st, any delay in price reaction would throw that reaction into the next calendar year. Should such a reaction be emphatic and yet short-lived the result might be an uncorrelated earnings variation in the opening year and an uncorrelated price variation in the following year. In this particular analysis, the regression of earnings or price on time is not of great importance in itself, so that lack of correlation during the period is of secondary importance.

(c) Much more importantly, the determination of the share price in any year as an average of recorded high and low prices might mean that changes in share price in response to earnings variability are nevertheless unrecorded. If the high and the low price are reactions to some external event; e.g., the Budget; which swamp performance reactions, then the analysis is correlating items of no inter-causation.
The only solution is time-consuming. It is to construct a time-weighted average share price based upon a larger number of recordings during the year - and one which, ideally, would ignore all variations not induced by performance (if these could be identified). But such variations are often more truly the result of investor expectations than reactions to performance variations. It is evidently a serious fault of this type of analysis (by proxy) that when the "genuine article" is available, it requires removal because it introduces distortion!

The summary effect of these general unknowns must be to introduce a random bias into the data. There are in addition two further sources contributing to random bias:

(d) Several writers in the field of statistical analysis of financial data have commented upon the bias introduced by the differences between real and reported earnings which must accrue to the use of certain accounting conventions. Depreciation is a principal factor in this connection. Thus Walter(1) observes that ".....the relatively higher growth of depreciation charges" (compared with the growth in pre-depreciation profits) "may well mean ..... that changes in the relative importance of non-cash changes have reduced the meaningfulness of intertemporal comparisons". An increasing awareness of the need to make adequate provision for the replacement of fixed assets at inflated prices was something of a feature of accounting controversy in the late '50's and early '60's. To what extent this is reflected by increased depreciation reserves made by the sample companies in the study-period is a matter of mere conjecture - yet the possibility remains. Probably more potent in introducing bias was the widely different treatment accorded by the companies concerned to capital allowances. Substantial transfers to or from deferred taxation accounts and taxation equalisation accounts (which were in a more or less

(1) Walter ( ) pp 9/11. A relevant commentary on the problem of reported earnings is that of Solomon & Loya "Measurements of Company Profitability - some Systematic Errors in the Accounting Rate of Return", reported Robichek ( 26 ) Chap.7.
inexact way a substitute for replacement reserves) were a more common feature of the period than today, when the greater "current liability" nature of Corporation Tax tends to reduce the scope for such items, except to the extent that they are substituted for by wholly or partially retained investment grants. And especially in the "construction" and "heavy engineering" industry samples, some rather varied practices in the valuation of work-in-progress, progress payments, retentions and the like are capable of introducing some very considerable distortions in reported earnings. (1)

(2) A further cause of random bias lies in the method of measuring equity earnings. These are expressed for the purpose of this analysis as a "percentage to issued equity capital", the source being Moodies Investors Handbook. The base "issued equity capital" is that in issue at the end of the study period. There is thus introduced a random "damping" or "exaggerating" effect on the measure of earnings variability utilised in this exercise, for the "percentage to issued equity capital" is, in Moodies, adjusted for each new issue by a simple proportional calculation. Inasmuch as the last equity capital is then by definition the largest, the range of adjusted percentages must ipso facto be reduced - yet the mean of those percentages may be reduced by a greater or a less proportion. All measures of dispersion are thereby subjected to a random distortion. An illustration may clarify the point. This is based upon the reported data for Wimpey, included in the "construction" industry sample. The number of stock splits - three - in the equity of this company during the period is not greatly atypical of companies in the various samples.

(1) Thus, for example, in the period Taylor Woodrow included finished but unbilled work at "valuation less retentions" as part of work-in-progress: whereas (so far as can be ascertained) Wimpey included such work at "cost less provision for losses". Given the old civil engineering maxim that "profit lies in the extras" (which are usually finally agreed on contract completion) the difference between 'valuation' and 'cost' could be very substantial indeed.
<table>
<thead>
<tr>
<th>Average Share Price</th>
<th>E to 0</th>
<th>Year (as reported in Moody's)</th>
<th>Stock Split</th>
<th>Issued Equity</th>
<th>'True' Share Price</th>
<th>E to 0 'Own Year' Basis</th>
</tr>
</thead>
<tbody>
<tr>
<td>£/A.</td>
<td>£</td>
<td>1956</td>
<td>1 for 1</td>
<td>£/mm</td>
<td>£/A.</td>
<td>£</td>
</tr>
<tr>
<td>3/8</td>
<td>23.9</td>
<td>1956</td>
<td>1 for 1</td>
<td>1.0</td>
<td>29/4</td>
<td>191</td>
</tr>
<tr>
<td>3/9</td>
<td>25.0</td>
<td>7</td>
<td>1 for 1</td>
<td>2.0</td>
<td>15/0</td>
<td>100</td>
</tr>
<tr>
<td>4/1</td>
<td>29.0</td>
<td>8</td>
<td>2.0</td>
<td>2.0</td>
<td>16/4</td>
<td>116</td>
</tr>
<tr>
<td>7/8</td>
<td>29.8</td>
<td>9</td>
<td>2.0</td>
<td>2.0</td>
<td>30/8</td>
<td>119</td>
</tr>
<tr>
<td>9/8</td>
<td>37.6</td>
<td>1960</td>
<td>2.0</td>
<td>2.0</td>
<td>38/8</td>
<td>150</td>
</tr>
<tr>
<td>16/1</td>
<td>35.4</td>
<td>1</td>
<td>4.0</td>
<td>4.0</td>
<td>32/2</td>
<td>71</td>
</tr>
<tr>
<td>20/1</td>
<td>34.1</td>
<td>2</td>
<td>4.0</td>
<td>4.0</td>
<td>40/2</td>
<td>68</td>
</tr>
<tr>
<td>25/9</td>
<td>53.0</td>
<td>3</td>
<td>8.0</td>
<td>8.0</td>
<td>25/9</td>
<td>53</td>
</tr>
<tr>
<td>30/10</td>
<td>80.6</td>
<td>4</td>
<td>8.0</td>
<td>8.0</td>
<td>30/10</td>
<td>81</td>
</tr>
</tbody>
</table>

Notes:

"True Share Price" Reported share prices are "true" (actual) only in respect of 1963 and 1964. Earlier prices have, in Moody's, been calculated by taking the "true" price for any year and reducing it pro-rata to the number of shares then in issue compared to those in issue at the end of 1964. This process is reversed in the column 'True' Share Price in the above table.

"E to 0 'Own Year' Basis" A parallel process is applied to 'Earnings to Ordinary'. The phrase "Own Year' Basis" is meant to indicate "calculated on the basis of the equity in issue at the end of the year in question".

The potential random bias in subsequent statistical processing can be seen by taking as examples three sets of earnings circumstances:

1. 'Own Year' Basis - i.e., assuming there had been no stock splits;

2. 'Latest Year' Basis - assuming 1 stock split only, in 1961, and recalculating the above table on an issued equity capital of £4 million:
3. 'Latest Year' Basis - accepting the 3 stock splits which actually occurred, and basing on an issued equity capital of £8 million:

The results are:

<table>
<thead>
<tr>
<th></th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean Earning %</td>
<td>105.4%</td>
<td>54.7%</td>
<td>38.7%</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>42%</td>
<td>15%</td>
<td>17%</td>
</tr>
<tr>
<td>Coefficient of Variation</td>
<td>0.40</td>
<td>0.75</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Tests on randomly selected other companies included in the various samples indicate that the change in the coefficient of variation for equity earnings cannot be expressed as a simple function of the frequency or timing ("early" or "late") of stock splits. There is some indication that on balance the effect of this "Moodie Treatment" is to increase the value of the variation coefficient if earnings percentages decrease after the stock split, and to decrease the value of the coefficient if earnings percentages increase after the split - but Wimpey is a good example of the net unpredictability of several splits. There is no reason to believe that stock splits are any more frequent or occur any later in one industry than another; and so as well as a randomness of direction of bias, there is a randomness of size of bias as between the industry samples. The same unpredictability accrues to the treatment of share prices, but here there is an additional factor to be considered. The "Moodie Treatment" in fact assumes that share prices move exactly in inverse proportion to stock splits, whereas it is a common observation that this is not always so. A good example of this is to be found in the case of Johnson Mathey (not a sample company). Moodies Handbooks for 1966 and for 1967 give the following data. (There was a 1 for 1 stock split in late 1965):
Examination is confused by the introduction of Corporation Tax in 1964/65; but if 1965 and 1966 reported post-tax equity earnings are roughly adjusted to the old basis:

Earnings to Ordinary: Johnson Mathey

1965: £1.75m = 48.9% on increased 1965 equity
1966: £1.90m = 53.3% on unchanged 1966 equity

If there had in fact been no stock split until 1967; and having rough-and-ready regard for the disinflationary effect of the 1966 Budget by examining the effect on share prices of several companies reporting moderately improved earnings in 1966 over 1965; then the projected 1966 share price might stand pretty close to the 81/6d. of 1965 - perhaps a little less. The effect of "Moodie Treatment" of a subsequent stock split would be to reduce that price to 40/9d. or a little less. In fact it was 44/7d. - shareholder expectations were more buoyant than a mathematical ratio would imply. Thus, where stock splits occur very late on in the study-period, and prior to the full recording of after-stock-split prices, the "Moodie Treatment" may well understate 'true' share price in the last couple of years, reducing the mean price and almost certainly exaggerating the value of the variation coefficient. And, as was noted earlier, there still arises the possibility that the annual high and low on which the average price is based will exceed the split-reaction price anyway: so that this double random bias will be eliminated.

(There is of course nothing to be gained by using 'reversed' data. This analysis is not concerned to
establish as a result a regression of earnings or share price on time; but to correlate earnings and price variabilities. As the "Hoodie Treatment" is arithmetically identical for both sets of data, 'reversal' is irrelevant in that context; is, as we have seen just as capable of introducing random bias; and is just as likely to be superseded by annual high and low prices reactive to external stimulants).

An important potential source of systematic bias in the analysis may be derived from growth trends in either earnings or share prices. Benishay(1) found it necessary to introduce these two trends as corrective coefficients in his multiple correlation equations. Thus, for example, if share price seemed to be in excess of that explicable by the current revealed earnings performance of the firm (shareholder expectation buoyancy); Benishay attributes this to a somewhat euphoric extrapolation by investors of recent earnings growth trends, ignoring the current performance. He therefore develops a coefficient to measure such trends, and introduces this as a correction factor into his correlation equations. This is a valiant effort to overcome the inherent disadvantage of analysing expectations by the proxy of historical data referred to above. Barges(2) also notes the same source of bias, and expresses the opinion that it will be all the stronger in growth industries. But this is not, fortunately, an issue of great importance to the present analysis, which is more concerned to observe variations around a trend line - and so automatically is compelled to derive the location of such a line. It cannot be too strongly stressed that, for the present purpose, that trend line itself is of no significance except as a point of departure from which to measure variations - but this is a matter of methodology; to an exploration of which it is now opportune to turn after so long a preamble. Yet the deficiencies of the data cannot be ignored, and any ultimate conclusions must be reached in their light.

(1) Benishay (41). As this writer operates in terms of earnings-price ratios instead of price-earnings ratios it is necessary for the present purpose to invert his conclusions: a positive correlation in his work becoming negative in the present context, and vice-versa.

(2) Barges (4).
Section 2: Approach and Methodology

In the Introduction to this Chapter, the present analysis was defined as consisting of a statistical examination of the affect of earnings variability upon share price on an industry basis. More precisely: it is concerned to examine whether, as between industries, share prices in the one are more reactive to variations in earnings performance, than they are in the others. If share prices are a function of expectations; if expectations are a reflection of uncertainty; and if uncertainty is positively correlated with an observed tendency for past earnings performance to fluctuate significantly; then any concept of industry costs of capital - which must take ordinary share price and yields into account as a major component of that cost - must enquire whether that correlation is stronger in one industry than another.

For if it is, then it follows that the cost of capital for the industry with a stronger correlation will be greater\(^{(1)}\) than that for an industry with a weaker correlation, regardless of the fact that present equity yields in the two industries may be identical, and that equity earnings in one have a history of variability no greater than equity earnings have in the other. If there is no difference in the correlation at the industry level, then uncertainty over the future either is not a function of historical earnings variability - a hypothesis which can be defended but which intuitively seems overly exclusive\(^{(2)}\); or, in any case cannot or does not distinguish between industries in forming projections about future returns. In this case, if there is such a thing as a cost of capital which is peculiar to one industry rather than another, it must arise as a function of present circumstances, unaffected by considerations of the future. (Unaffected, that is, relative to other industry projections). This conclusion seems sufficiently at odds with such generally accepted concepts as "growth industries", "export based industries" and so on as to infer the alternative conclusion that there

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\(^{(1)}\) To argue that the cost of capital will be greater is to argue that investors as a class are risk-aversive. Consequently any greater degree of uncertainty in forming projections about the future will be reflected in higher probabilities being assigned to relatively pessimistic projections.

\(^{(2)}\) Benishay (41) finds a moderately positive correlation between price and earnings stability, without specifying as between industries.
is in fact no such thing as a cost of capital which is peculiar to one industry - an inference already partially anticipated earlier in this thesis.

Two assumptions underlie this whole argument. These are:

a) That it is possible to distinguish, not only between the sensitivity of prices to earnings variability as between industries, but also between those earnings variabilities themselves. Obviously, it would be possible to have a greater price sensitivity to a common variability of earnings in one industry than in another; but, analytically, it is necessary to test whether such price sensitivity differentials are likely to be accentuated because of underlying earnings variability differentials. This is the lesser of the two assumptions, and is tested in this analysis.

b) That share price is wholly a function of earnings. This is very probably not valid. Benishay finds a strong positive correlation between share price and size of firm, and (strangely) a negative correlation with the past price history of the share in question. Walters finds a moderately positive correlation between share price and frequency of reporting. The need to correct for growth trends has already been commented upon; and this must doubly be true where there are strong national economic trends. Then there are the unquantifiable effects of shareholder ignorance, apathy and loyalty. Nevertheless, and except in the very short period, it is generally accepted that earnings are at least a significant determinant of share price. (But see the footnote to Pages 134 & 136).

The parallelism of this analysis with that of Wippen (2) has already been mentioned. Wippen is concerned to answer the first of the two preceding questions. Using analysis of variance, he seeks to distinguish between industry earnings variability; and, using industry as a proxy for risk class, to establish the statistical validity of this latter concept. To adjust for variability caused by growth, his measure of variability

(1) See for example Pages 258 and 261.
(2) Wippen (95 ), See also Page 193.
consists of the standard error of estimate around a linear regression fitted by the 'least squares' method. In fact, he finds little or no significant differentiation between his "industry" samples.

Wippern's attention is directed to this work by the same considerations as listed above - namely the functional relationships of projection uncertainty, share price and earnings variability. It can be argued, however, that a statistical analysis of earnings variability between industries is, on its own, a somewhat sterile exercise. For it is not so much a question of statistical truth which is involved, as what investors believe - whether that belief be well or ill-founded. The cost of capital is a market phenomenon, and analyses adduced to its elucidation must surely reflect that fact. For this reason, share price variability is introduced into the present analysis, as an indicator of the extent to which investors perceive - or think they perceive - differences in earnings variability within industry. The inadequacy of this approach is already acknowledged, and the statistical techniques employed could doubtless be improved upon; but it is a first step.

In an endeavour to concentrate upon business risk as a determinant of earnings variability; i.e., to eliminate the random (or systematic, according to the school of thought) 'noise effect' of financial risk, Wippern uses total earnings before interest and taxation. Hopefully, he implies that such earnings are a fair proxy for equity earnings in the context of U.S. financial structures. A closer concern with the impact upon equity share prices, and an apprehension of the vagaries of U.K. corporate taxation during the study-period, compel the use of equity earnings in the present analysis. The problem of 'random noise' from financial risk is minimised by the conviction stipulated in this thesis that generally U.K. investors neither were nor are particularly appreciative of this form of risk; and by the selection criterion of using only low-gearied companies in the various samples.

Section 3: Statistical Procedures and Interpretations
The steps in the present statistical analysis are as follows:
1. Select for each of the eight stipulated "industries", samples of 15 companies conforming to the criteria of relatively large size, of trustee status as far as possible (as a measure of reputation) and of relatively low gearing.

2. Schedule for each sample company over the period 1956-64, data of annual average share prices and annual after-tax earnings to issued equity capital; both adjusted for stock splits to a 1964 issued capital basis.

3. Using a 'least squares' technique, fit both to earnings data and to share price data, a third-order (cubic) regression curve on time. The purpose of this is to remove growth trends from the measures of variability of the two sets of data. Choice of a third order curve of the type:

\[ y = a_0 + a_1x + a_2x^2 + a_3x^3 \]

was decided upon after calculating for a random sub-sample of 10 over all selected companies, regression equations for the first to fifth order polynomials; and successively testing (1) the reductions in the root mean squares of the deviations from the regressions for significance. In seven cases out of ten, the reduction in the root mean square ceased to be significant at the 5% level after the third order polynomial. On this basis, the third-order curve was accepted as the growth curve. It is usually wave-shaped for these companies in this period.

4. Calculate the standard error of estimate (root mean square) for each selected company, both for earnings and for share price; modify by Bessel's correction for small samples; and express the standard error as a percentage of the average of earnings or of share prices as the case may be. The resulting percentage is regarded as a coefficient of variation,

\[ \bar{y} = \frac{C_{y,x1}}{C_{y,x2}} \text{ for } (n_1 + n_2 - 2) \text{ degrees of freedom} \]

and where \[ 1 > 2 \]

\[ C_{y,x1} = \left( \frac{n_1}{n_1 - 1} \right) s_{y,x1}^2 \]

\[ C_{y,x2} = \frac{C_{y,x1}}{C_{y,x2}} \]

(1) Using
measuring for each company the variability during the study-period of earnings or of share price about the trend over the period.

5. List company coefficients of variation of earnings (and secondly of share prices) in industrial groups, and carry out multi-group analysis of variances supported by 'least significant difference' matrix comparison. The purpose of this is to establish whether and to what extent significant inter-industry differences exist between variability of earnings and of share prices as measured by the coefficients.

6. For each industry in turn, correlate company coefficients of share price variability with coefficients of earnings variability: assuming for simplicity a linear relationship. Test the coefficients of correlation for significance. The purpose of this is to establish whether there is any prima facie statistical evidence that share price is sensitive to earnings variability; but the other determinants of share price are not taken into account. They are in effect assumed to be common to all "industries". This is a distinct weakness, but to bring them into the reckoning would require the calculation of representative coefficients and the use of multiple correlation.

7. Using Fisher's Z Transformation, pair-test the correlation coefficients for significant differences. The purpose of this is to establish whether there is any significant difference in the degree of share price sensitivity to earnings variability, as between industries; even though that sensitivity itself may or may not be very pronounced.

8. Purely by way of confirmation of the tests performed on the correlation coefficients: from the coefficient of correlation calculate the standard error of estimate and express as a coefficient of variation as in (4) above; and compare the coefficients of variation.

The principal calculations and results of each step are given below; steps 1 - 3 inclusive being taken as read.
4. **Coefficients of Variation**

\[ \hat{S}_{y,x} = \sqrt{\frac{(y - y_{est})^2}{N}} \cdot \sqrt{\frac{N}{(N-2)}} \]

and \( C \) of \( V = \frac{\hat{S}_{y,x}}{\bar{y}} \) for each sample company.

**INDUSTRY**

<table>
<thead>
<tr>
<th>Industry</th>
<th>Variability of Earnings</th>
<th>Variability of Share Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>Mean 14.65%</td>
<td>Mean 15.27%</td>
</tr>
<tr>
<td>Textiles</td>
<td>Mean 19.15%</td>
<td>Mean 10.52%</td>
</tr>
<tr>
<td>Engineering Supplies</td>
<td>Mean 15.50%</td>
<td>Mean 11.27%</td>
</tr>
<tr>
<td>Mechanical Engineering</td>
<td>Mean 16.07%</td>
<td>Mean 13.13%</td>
</tr>
<tr>
<td>Construction</td>
<td>Mean 22.26%</td>
<td>Mean 14.47%</td>
</tr>
<tr>
<td>Heavy Engineering</td>
<td>Mean 15.03%</td>
<td>Mean 15.59%</td>
</tr>
<tr>
<td>Construction Supplies</td>
<td>Mean 16.73%</td>
<td>Mean 11.01%</td>
</tr>
<tr>
<td>Food Processing</td>
<td>Mean 9.84%</td>
<td>Mean 11.69%</td>
</tr>
</tbody>
</table>

Inspection reveals the high earnings variability, and the high dispersion of that variability, in construction especially when compared with food. It is noteworthy that the same amount of differentiation is not apparent in share price variability. It must further be remarked that the construction industry data is much distorted by the high earnings variability and share price variability of one company in the sample.

5. **ANALYSIS OF VARIANCE**

Using for the best estimate of population variances:

\[ \sigma_x^2 = \frac{N}{n-1} \left( \frac{s_x^2}{\bar{X}^2} \right) \]

where \( s_x^2 = \frac{\sum (X - \bar{X})^2}{n} \left( \frac{1}{(n-1)} \right) \)

for (n-1) degrees of freedom; where \( n \) = no. of samples, and \( N \) = no. in sample.

The table of variances for earnings variability and share price variability (together) is as follows:
VARIABILITY OF EARNINGS

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Total</td>
<td>119</td>
<td>13905.7</td>
<td>116.8</td>
</tr>
<tr>
<td>(b) Sample Means</td>
<td>7</td>
<td>1270.9</td>
<td>182.7</td>
</tr>
<tr>
<td>(c) Samples Pooled</td>
<td>112</td>
<td>12626.8</td>
<td>112.7</td>
</tr>
</tbody>
</table>

F-ratio 1.62

Entering the tables for the variance ratio with 7 and 112 degrees of freedom, F = 2.10 at the 5% level. Evidently there is no significant difference, in general, between the samples either in terms of earnings variability or share price variability. Using:

'least significant difference between sample means' as a more detailed test of sample differentiation:

\[
LSD = t_{0.05} \sqrt{\frac{MS_{pooled}}{N}},
\]

and f = degrees of freedom for the pooled samples estimate of variance.

and *05 = for 95% confidence limits.

whence \( LSD(\text{earnings variability}) = 1.981 \sqrt{112.7 \times 2/15} \)

= 7.31

and \( LSD(\text{share price variability}) = 1.981 \sqrt{35.5 \times 2/15} \)

= 4.31

Matrices of the differences in sample means are:

**EARNINGS VARIABILITY**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chem.</td>
<td>*</td>
<td>-3.50</td>
<td>-0.85</td>
<td>-1.42</td>
<td>-7.61</td>
<td>-0.38</td>
<td>-2.08</td>
<td>4.81</td>
</tr>
<tr>
<td>Text.</td>
<td>*</td>
<td>2.65</td>
<td>2.08</td>
<td>-4.11</td>
<td>3.12</td>
<td>1.42</td>
<td>8.31</td>
<td></td>
</tr>
<tr>
<td>B.Sup.</td>
<td></td>
<td>-0.57</td>
<td>-6.66</td>
<td>0.47</td>
<td>-1.23</td>
<td>5.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H.Eng.</td>
<td></td>
<td>-6.19</td>
<td>1.04</td>
<td>-0.67</td>
<td>6.23</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cons.</td>
<td></td>
<td></td>
<td></td>
<td>7.22</td>
<td>5.53</td>
<td>12.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>H.Eng.</td>
<td>*</td>
<td></td>
<td></td>
<td>-1.70</td>
<td>5.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.Sup.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.89</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Food.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
from which it can be deduced with 95% confidence that earnings variability in the construction industry is significantly different from that in chemicals, heavy engineering (which is surprising) and food processing, and is significantly different in textiles compared with food processing. In imprecise terms, earnings variability in food processing and in construction is in general at least moderately distinguishable from that of the other industries, and from that of each other. A second somewhat surprising feature is that earnings variability in construction supplies is so nearly significantly different from that of its customer industry.

<table>
<thead>
<tr>
<th>SHARE PRICE VARIABILITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chem.</td>
</tr>
<tr>
<td>Text.</td>
</tr>
<tr>
<td>E.Sup.</td>
</tr>
<tr>
<td>M.Eng.</td>
</tr>
<tr>
<td>Cons.</td>
</tr>
<tr>
<td>H.Eng.</td>
</tr>
<tr>
<td>C.Sup.</td>
</tr>
<tr>
<td>Food</td>
</tr>
</tbody>
</table>

from which, still with 95% confidence, it can be deduced that in terms of share price variability, the chemical industry is significantly different from textiles, and very nearly so from the construction supply and components industry; that textiles and heavy engineering are significantly diverse, and textiles and the construction industry nearly so; that, surprisingly enough, the engineering supplies and components industry is significantly different from one of its major customers, heavy engineering - which latter industry is again significantly different from the construction supply and components industry, and nearly so from food processing. No general conclusions can be drawn, except that in terms of share price variability, both supply industries are fairly "different" from their customer industries. It is wryly typical of statistical analysis of financial data that significant and nearly-significant differences between the share price variabilities of these industries are too sparse to permit useful generalisations and yet too numerous to be dismissed out of hand. The "spread" of individual sample companies across industries may act as a smoothing factor on share price
variability differentiation.

6. CORRELATION OF EARNINGS VARIABILITY & SHARE PRICE VARIABILITY

The correlation coefficient is calculated from:

\[ r = \frac{\sum XY - \bar{X} \bar{Y}}{s_x s_y} \]

using as a check

\[ r = \frac{N \sum XY - (\sum X)(\sum Y)}{\sqrt{(N \sum X^2 - (\sum X)^2)(N \sum Y^2 - (\sum Y)^2)}} \]

and testing \( r \) for significance by

\[ t = r \sqrt{\frac{N-2}{1-r^2}} \]

giving the following results:

<table>
<thead>
<tr>
<th>Industry</th>
<th>'r' Value of Correlation Coefficient</th>
<th>Level of Significance for 14F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>.429</td>
<td>nearly 10%</td>
</tr>
<tr>
<td>Textiles</td>
<td>.395</td>
<td>25%</td>
</tr>
<tr>
<td>Engineering Supplies</td>
<td>.629</td>
<td>1%</td>
</tr>
<tr>
<td>Mach. Engineering</td>
<td>-.017</td>
<td>negligible</td>
</tr>
<tr>
<td>Construction</td>
<td>.376</td>
<td>25%</td>
</tr>
<tr>
<td>Heavy Engineering</td>
<td>-.092</td>
<td>below 25%</td>
</tr>
<tr>
<td>Construction Supplies</td>
<td>.427</td>
<td>nearly 10%</td>
</tr>
<tr>
<td>Food Processing</td>
<td>.315</td>
<td>25%</td>
</tr>
</tbody>
</table>

On this evidence, only so far as concerns the engineering supplies and components industry is share price significantly sensitive to earnings variability. The chemicals industry and the construction supplies and components industry also evidence some small degree of meaningful share price sensitivity. As the previous analysis failed to disclose much more than occasional differentiation in share price or earnings variability as between industries; apparently only in engineering supplies is there any significant share price reaction to a fairly common degree of earnings variability. Given the suspect natures of much of the data analysed, so isolated a result must be viewed with considerable scepticism.

7. COMPARISON OF CORRELATION COEFFICIENTS

Inter-industry comparison is effected by the Z transformation of the
coefficients: (1)

\[ Z = \frac{1}{2} \log_e \left( \frac{1+r}{1-r} \right) = 1.1513 \log \left( \frac{1+r}{1-r} \right) \]

and setting up at 95\% confidence levels the null hypothesis that the transformed coefficients are from the same population with average \( M \)

i.e., \( M_{z1} = M_{z2} \)

whence under this hypothesis

\[ z = \frac{z_1 - z_2 - (M_{z1} - M_{z2})}{\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \]

where \( \sqrt{\frac{1}{N_1} + \frac{1}{N_2}} \)

giving the following results:

| Chemicals   | .458 |
| Textiles    | .418 |
| Engineering Supplies | .740 |
| Mechanical Engineering | -.017 |
| Construction | .395 |
| Heavy Engineering | -.092 |
| Construction Supplies | .456 |
| Food Processing | .326 |

Pair comparison of these transformation values is effected in the following simple matrix of z-values, calculated as in the equation (*) given above:

(1) Spiegel (31), Chap. 14. Transformed values from "Cambridge Statistical Tables". The statistical assumption here is that the sample values of \( Z \) are randomly distributed throughout a normal population of \( Z \) values; so that expressed as a percentage of the difference between the sample standard deviation (which, given the null hypothesis of a common population, must reduce to a function of the sample sizes), the usual percentage points of the normal distribution can be used to establish significance tests. Comment has already been made as to the reality of a population, and as to the non-random nature of the sample.
From the normal distribution function, and at the 5% level of probability; the null hypothesis would be rejected only for values of \( z \) greater than +1.96 or less than -1.96. The transformation values for engineering supplies and heavy engineering are different at the 4% level of significance; and between engineering supplies and mechanical engineering the difference is significant at the 6% level. The individuality in this respect of the engineering supplies industry from its customer industries is clear; but otherwise the null hypothesis cannot be rejected - with this one exception, the reaction of share price to earnings variability is not noticeably different between the industries sampled. Given that variability itself cannot be easily differentiated, this is an expectable result seeming to imply that shareholders themselves do not think with any great conviction that they can distinguish different earnings variabilities. (1) There are however some faint indications, albeit not at meaningful significance levels, of a general differentiation between the engineering (both mechanical and heavy) industries and the others. Strangely, there is no such even faint general indication.

(1) Or, if they think they can so distinguish, still do not react to their beliefs. This is not impossible if expectations are based only in part on observed earnings performance; and if the other determinants have higher and opposing values for their coefficients. Given that the data processed in this exercise is in terms of trend-removed variabilities, however, it is rather difficult to see what these other determinants might be which would have the necessary large values. These determinants would have to be common to all the samples processed - which would eliminate otherwise potential determinants such as size of firm. (It has been pointed out that all sample companies are large in relation to the typical size of firm in their industries; it is not claimed that all firms enjoy the size and reputation of I.C.I. or G.K.N.).
vis-a-vis the construction industry, which is plainly very individualistic in its technology and its market.

The final confirmatory operation of deriving the standard errors of estimation of share price variability from the correlation coefficients and expressing these as comparable coefficients of variation, is calculated from:

\[
C_{of \, V} = \frac{S_{y.x}}{Y} = \frac{S_y \sqrt{(1-r^2)}}{Y}
\]

and yields the following results:

<table>
<thead>
<tr>
<th>Industry</th>
<th>(S_{y.x})</th>
<th>(C_{of , V})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>3.49</td>
<td>.23</td>
</tr>
<tr>
<td>Textiles</td>
<td>4.35</td>
<td>.41</td>
</tr>
<tr>
<td>Engineering Supplies</td>
<td>3.66</td>
<td>.34</td>
</tr>
<tr>
<td>Mechanical Engineering</td>
<td>6.31</td>
<td>.48</td>
</tr>
<tr>
<td>Construction</td>
<td>6.58</td>
<td>.47</td>
</tr>
<tr>
<td>Heavy Engineering</td>
<td>8.39</td>
<td>.54</td>
</tr>
<tr>
<td>Construction Supplies</td>
<td>3.79</td>
<td>.34</td>
</tr>
<tr>
<td>Food Processing</td>
<td>4.62</td>
<td>.40</td>
</tr>
</tbody>
</table>

Evidently, and with the possible exception of chemicals, the assumption of a linear correlation between share price variability and earnings variability is not a particularly good one. It is possible that a non-linear correlation equation would give a better fit, improving the significance of the correlation coefficients and yielding greater degrees of significance in the correlation differences between industries. Such a refinement would not necessarily improve the lack of differentiation between variability of share prices or earnings as between industries.

Comment has already been made upon the distortion in the results for the construction industry sample caused by the extreme variability in the earnings record of one company - W.C. French. A somewhat smaller, but still significant distortion is caused by the earnings variability of Midland Electric to the construction supplies and components results. If these exceptional items are removed, and the statistical processes repeated (in respect of earnings only), the following results accrue:
"INDUSTRY"  

<table>
<thead>
<tr>
<th>&quot;INDUSTRY&quot;</th>
<th>VARIABILITY OF EARNINGS</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>14.65%</td>
<td>5.64%</td>
<td></td>
</tr>
<tr>
<td>Textiles</td>
<td>18.15%</td>
<td>9.56%</td>
<td></td>
</tr>
<tr>
<td>Engineering Supplies</td>
<td>15.50%</td>
<td>6.54%</td>
<td></td>
</tr>
<tr>
<td>Mechanical Engineering</td>
<td>16.07%</td>
<td>7.59%</td>
<td></td>
</tr>
<tr>
<td>Construction</td>
<td>17.67%</td>
<td>6.09%</td>
<td></td>
</tr>
<tr>
<td>Heavy Engineering</td>
<td>15.03%</td>
<td>6.57%</td>
<td></td>
</tr>
<tr>
<td>Construction Supplies</td>
<td>13.61%</td>
<td>8.95%</td>
<td></td>
</tr>
<tr>
<td>Food Processing</td>
<td>9.84%</td>
<td>5.92%</td>
<td></td>
</tr>
</tbody>
</table>

The construction industry mean is reduced by 4% points, and the standard deviation is reduced to a third of the previous figure. The construction supplies industry mean is reduced by 3 points, and the standard deviation is reduced to two-thirds of the previous figure.

The effect on the analysis of variance is not pronounced, (1) the F-ratio value becoming 1.714 as opposed to 1.621, and is still not significant at the 5% level. But there is a pronounced rationalising effect upon the "least significant difference test".

EARNINGS VARIABILITY

<table>
<thead>
<tr>
<th>Difference in Sample Means</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chem.</td>
</tr>
<tr>
<td>Text.</td>
</tr>
<tr>
<td>E.Sup.</td>
</tr>
<tr>
<td>H.Eng.</td>
</tr>
<tr>
<td>Cons.</td>
</tr>
<tr>
<td>H.Eng.</td>
</tr>
<tr>
<td>C.Sup.</td>
</tr>
<tr>
<td>Food</td>
</tr>
</tbody>
</table>

For LSD = $t_{0.05} \cdot s \sqrt{\frac{2}{N}}$

\[
= 1.981 \cdot \sqrt{3.90} \cdot \sqrt{\frac{2}{15}} = 5.55
\]

The significant difference in earnings variability in the food industry at

(1) The variance table is reproduced in Appendix III
once becomes a prominent feature. This is consistent with the visual impression created by the low industry mean earnings variability. The less than significant difference of the construction supplies industry compared with construction is to be contrasted with the earlier results, when this same comparison nearly reached a significant value. The earlier overall conclusion of a generally distinguishable earnings variability in the construction industry is also now no longer valid.

Turning to the correlation of earnings and price variabilities for the two affected industries: the adjustment to the construction industry sample if anything reduces the significance of any such correlation (from 0.376 - a very slightly significant value: to 0.20 (negative) - which indicates a totally insignificant correlation). But for the construction supplies industry, the adjustment improves the correlation to a 5% level of significance. The hitherto isolated case of a significant value of correlation in engineering supplies now receives reinforcement from a second industry - and from a component and supplies industry at that.

Prima facie, there is nothing about the economics of these two industries which would lead the observer to expect such a relatively high shareholder sensitivity in the components industries rather than in their principal customer industries. The samples for both components industries include large and well-known companies, several of whom have integrated forward into user activities on a substantial scale. It is tempting to speculate that earnings variability in the engineering components industry is but a mirror of that of the two user industries - and certainly the difference in the adjusted sample means is small enough: and that share prices in the components sector are really sensitive to earnings variability in the user industries. But the correlation of components share price variability and user earnings variability is of mixed quality: (1) and why should user industry share prices remain so indifferent to user industry earnings variability?

And between the construction user and components industries, earnings

(1) For mechanical engineering earnings and engineering supplies share prices, $r = 0.119$, which is insignificant; but for heavy engineering earnings and engineering supplies share prices, $r = 0.741$, which is significant at the 1½ level. (Such an extremely high correlation significance must be suspect as a statistical freak).
Variabilities are very different.

The situation is not greatly clarified by a comparison of the correlation coefficients:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Chem.</td>
<td>*</td>
<td>0.10</td>
<td>0.69</td>
<td>1.16</td>
<td>1.65</td>
<td>1.35</td>
<td>0.40</td>
<td>0.32</td>
</tr>
<tr>
<td>Text.</td>
<td>*</td>
<td>0.79</td>
<td>1.07</td>
<td>1.51</td>
<td>1.25</td>
<td>0.49</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td>E.Sup.</td>
<td>*</td>
<td></td>
<td>1.86</td>
<td>2.39</td>
<td>2.04</td>
<td>0.30</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>M.Eng.</td>
<td>*</td>
<td>0.77</td>
<td>0.18</td>
<td>1.56</td>
<td></td>
<td>0.82</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cons.</td>
<td>*</td>
<td>0.38</td>
<td></td>
<td>2.01</td>
<td></td>
<td>1.26</td>
<td>1.02</td>
<td></td>
</tr>
<tr>
<td>H.Eng.</td>
<td>*</td>
<td></td>
<td></td>
<td>1.75</td>
<td></td>
<td>0.72</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C.Sup.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>Food</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Compared with the non-adjusted matrix of \( z \) - values, the effect upon differentiation of the construction industry is very marked; and yet in terms of significant differentiation (5% level) between industries, is restricted to the two supplies and components industries compared with their user industries. This was to be expected from the observations concerning the correlation coefficients themselves. The significance of differences in non-adjusted correlation coefficients is reinforced as to that between engineering supplies and engineering manufacturing by that now accruing between construction supplies and construction itself. The significant difference which is now observed to accrue between engineering supplies and construction can best be understood as a negative cross-linkage; supported by the evident close similarity as between the two components industries (the \( z \) - value is low at 0.30) and the reverse differentiation (\( z = 1.75 \), nearly significant at 8%) between heavy engineering and construction supplies.

These results, however, do not do much more than confirm what was already to be suspected. In the following "conclusions" therefore, not too much emphasis is placed upon the modifying effects of the adjustments to the samples - and this for four reasons:

1. High variability of earnings is a fair reflection of the technical unpredictabilities of the construction industry, subject as it is to the vagaries of geological structure.
and of climate.

2. The construction industry, and its suppliers, are particularly exposed to drastic changes in national economic planning, especially as these affect local authority expenditure. (The Minister of Public Works recently estimated that as much as 80% of the value of civil engineering work is local and national authority sponsored).

3. The mid-contract valuation of work-in-progress for financial year-end purposes exposes the earnings of construction companies to an unusual potential variability. Profit does not accrue evenly throughout the contract, especially when these are large scale involving a wide range of construction activities. The uncertain pace of developments in system building is a smaller contributory factor of the same nature.

4. Brick, aggregates, cement and timber especially have been subject to fluctuating degrees of surplus capacity over the last decade. The construction supplies industry has, after all, one of the highest rates of company failure in the U.K.

Thus, "distortion" in these industries from occasionally exceptional variabilities in earnings performance is to be expected. Finally, enough comment has already been made upon the non-random nature of the samples to wish to add an indefensible criterion of "no extreme values" to the list. Only when the sample adjustments effect very pronounced alterations in the results, are they referred to.

Section 4. Conclusions

Within the constraints already specified as to the suspect nature of the data used in this exercise, and as to the limitations of the implied methodological assumptions and statistical techniques employed; the only general conclusion that can be reached is one of "not proven". There appears to be inadequate ground for a general assumption of greatly significantly different degrees of variability of equity earnings or of ordinary share prices, between the industries sampled. The deliberate inclusion among the samples of two which are consumer orientated - textiles and food processing; two which are 'contracts' orientated -
construction and heavy engineering; two which are component industries; and two general manufacturing industries; has not resulted in the disclosure of any supplementary "type of industry" differentiation at this stage. What variability differentiation there is tends to be between pairs or trios of industries - and as often as not is within "type" pairs; e.g., there is significant differentiation in earnings variability as between the two consumer industries and as between the two "contracts" industries: the 'sample adjustment' modifications yielding a very rational and significant differentiation of the one wholly consumer-orientated industry - food processing. There is some slight evidence that the component industries can be differentiated, in terms of share price variability, from their customer industries. In earnings variability terms, construction and food processing are generally distinguishable; and in share price variability terms, heavy engineering is generally distinguishable. But the general picture is one of seemingly unrelated occasional pair differences, and the suspicion cannot be avoided that different representative samples might well yield different results.

Correlation analysis of the two variabilities yields little that is conclusive except for a sharp differentiation in share price sensitivity between the two component industries on the one hand and the three relevant customer industries on the other. The correlation coefficients in the component industries are themselves uniquely significant in value, among all the industries sampled. Just why this should be so is not clear. These results in respect of the construction supplies industry become evident only after the samples of that industry and its customer industry are modified for extreme earnings variability - a statistical process which, in an industry where risk is endemic, is sufficiently suspect to weaken the validity of the modified conclusions. Further analysis of pairs of component and customer industries (e.g., a split of textiles and clothing, of food processing and food distribution) is necessary to elucidate this issue with any finality.

On the evidence presented, a full proof is still to be sought that there is any really substantial statistical validity in the concept of industry as a proxy for "equivalent risk class" in cost of capital calculations. Nor is there any real demonstration of a close link
between earnings expectations based upon extrapolations from history, and share price; so far as that link is a function of uncertainty as measured by earnings variability in the past. Whether such a link exists, but tends to be obscured, even negated, by other share price determinants is a task for sophisticated multiple correlation analysis of the type pioneered by Benishay. Whether investors do indeed extrapolate with any precision; or can do so; is a subject of recent controversy too large and complex for the space now available in this thesis. A reading of financial journalism inclines one to the partial belief that if uncertainty about the future is a determinant of share prices and yields, then it is as much an uncertainty concerned with economic and technological change yet to come as with performance in the past. "History is bunk" said Mr. Ford; and in the changing climate of industrial management, technology and organisation now prevalent in the U.K., maybe British investors agree with him.

But my own earlier work in analysis of variance of industry price-earnings ratios (not otherwise relevant to this thesis) (1) leads me to

(1) An analysis of variance of samples of price-earnings ratios over the period 1963-67 (inclusive) for the "Financial Times" industrial categories: 'breweries', 'chemicals', 'construction', 'electrical and radio' and 'auto and aero components'. The definition of 'industry' is not fully in accord with that adopted for this thesis. The 'least significant difference' matrix yielded:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Brew.</td>
<td>*</td>
<td>0.28</td>
<td>2.63</td>
<td>1.92</td>
<td>3.24</td>
</tr>
<tr>
<td>Chem.</td>
<td>*</td>
<td>*</td>
<td>2.25</td>
<td>1.54</td>
<td>2.58</td>
</tr>
<tr>
<td>Cons.</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>-0.71</td>
<td>0.71</td>
</tr>
<tr>
<td>E.&amp;.R.</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>1.42</td>
</tr>
<tr>
<td>A.&amp;.A. Comp.</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

for LSD = 1.97 at 95% probability

from which it is possible to deduce that the one customer-orientated industry (breweries) is significantly different from construction, electrical and radio, and auto and aero components; and that chemicals 'differ' from construction and the components industries - the latter two industries having some capital-goods based affinity. Because of the somewhat non-selective bases used by the "Financial Times" in defining industry groups, these results cannot be interpreted in any strongly specific sense. For example, 'electrical and radio' includes both manufacturing and distribution.

It is of some interest to note that a hierarchically designed analysis of variance test performed on the same data but as between years rather than as between industries indicated a much more significant differentiation than in the above table. One interpretation of this would be in support of the "random walk" theory of share prices.
believe that for financial theory, the concept of "industry" has little real validity except on a very broad and "opposite end of the spectrum" basis. There is nothing in the present analysis to repudiate that belief - rather to the contrary. A more powerful statistical technique may modify this conclusion. Certainly one would like to increase the size of sample, and expand the number of industries examined - perhaps applying a tighter specification of "market" to the definition of "industry". Much closer attention needs to be paid to the problem of representative share prices and to the time lag between accounting year-ends and price reactions to earnings disclosures. A desirable refinement would be to use multiple correlation of share price variability on earnings variability, paying particular attention to size and pay-out ratios; and testing these other partial coefficients for significance before proceeding further. It would also be most interesting to establish to what extent the results of this and similar analysis are inconclusive because of failure to correct for non-normality in the distribution of the data; intuitively I suspect a great deal.

But these are tasks for a separate thesis on their own. For the present, and subject to all these criticisms, the case for "industry" classifications and hence for "industry" costs-of-capital remains not proven and suspect. Failing such guidance for the smaller or non-quoted company, determination of a valid discount rate may remain the biggest single obstacle to the general acceptance of financial decision models and procedures such as are advocated in this thesis.
APPENDIX III

SAMPLE DATA FOR THE ANALYSIS OF VARIANCE AND CORRELATION

Data of equity earnings percentages and annual average share prices were abstracted from Moody's "Investors Handbook" (1965 Series) in respect of the sample companies listed below. All share prices were ratioed to a common 5/-d. nominal value to facilitate aggregation within groups; and both share prices and earnings percentages were expressed for the whole period 1956-64 in terms of the number and denomination of shares in issue at the end of the 1964 accounting period. Calculation of the cubic regression of (successively) earnings and prices on time, so as to derive and hence eliminate growth trends, whether positive or negative; and of the resulting standard error of estimate (root mean square of error); was by a standard library programme for an ICT 1905 Computer developed in the Computer Centre at Loughborough University of Technology. In the following tabulations, company root mean squares are expressed as percentages of company means over the period, to yield coefficients of variation; so as to compensate for the often very substantial inter-company differences in the size of the means of earnings. Otherwise a random bias would attach to the calculations in that a given absolute variation in earnings could be suspected as conducive to a variation in the share price of one company but not to another, purely because that absolute variation would be a significant departure from typical earnings values in one case but not in the other. By the same process statistical exaggeration of the importance of a given absolute share price reaction is avoided. Coefficients are corrected for small sample size by applying \( \frac{N}{N-2} \) to the calculated standard error. Data of the cubic regression of earnings or of share price on time is not listed below, partly for reasons of space but principally because the regressions themselves are not a prime objective of the analysis: serving purely as a datum line from which non-trend variability can be calculated.
### CHEMICALS

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Albright &amp; Wilson</td>
<td>8.5</td>
<td>16.7</td>
</tr>
<tr>
<td>Anchor Chemicals</td>
<td>9.8</td>
<td>15.0</td>
</tr>
<tr>
<td>Berk</td>
<td>12.5</td>
<td>16.9</td>
</tr>
<tr>
<td>Boots Pure Drug</td>
<td>8.4</td>
<td>12.3</td>
</tr>
<tr>
<td>Borax (deferred ordinary)</td>
<td>24.6</td>
<td>14.4</td>
</tr>
<tr>
<td>Calor Gas</td>
<td>17.4</td>
<td>14.0</td>
</tr>
<tr>
<td>Coalite &amp; Chemical</td>
<td>15.8</td>
<td>11.7</td>
</tr>
<tr>
<td>Glaxo</td>
<td>10.0</td>
<td>11.8</td>
</tr>
<tr>
<td>Greaff</td>
<td>18.3</td>
<td>14.4</td>
</tr>
<tr>
<td>Hickson &amp; Welch</td>
<td>20.0</td>
<td>19.5</td>
</tr>
<tr>
<td>I.C.I.</td>
<td>9.1</td>
<td>13.5</td>
</tr>
<tr>
<td>Laporte</td>
<td>20.8</td>
<td>15.3</td>
</tr>
<tr>
<td>Laves Chemical</td>
<td>16.2</td>
<td>12.5</td>
</tr>
<tr>
<td>Reichhold</td>
<td>22.2</td>
<td>27.5</td>
</tr>
<tr>
<td>Yorkshire Dyewares</td>
<td>6.2</td>
<td>13.6</td>
</tr>
</tbody>
</table>

**Sample Totals**

| 219.8 | 229.1 |

| Sample Means          | 14.65   | 15.27   |
| Sample Standard Deviations | 5.64   | 3.87    |
| Correlation Coefficient (r) | 0.429  |
| z transformation of (r)  | 0.458   |

### TEXTILES & CLOTHING

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aristoo</td>
<td>11.6</td>
<td>13.1</td>
</tr>
<tr>
<td>Barber Textiles</td>
<td>42.8</td>
<td>22.7</td>
</tr>
<tr>
<td>John Bright</td>
<td>23.0</td>
<td>3.0</td>
</tr>
<tr>
<td>Bulmer &amp; Lamb</td>
<td>14.8</td>
<td>8.0</td>
</tr>
<tr>
<td>Carrington &amp; Dewhurst</td>
<td>25.2</td>
<td>5.4</td>
</tr>
<tr>
<td>M Corah</td>
<td>14.9</td>
<td>13.6</td>
</tr>
<tr>
<td>John Collett</td>
<td>5.3</td>
<td>3.5</td>
</tr>
<tr>
<td>Ellis &amp; Goldstein</td>
<td>12.2</td>
<td>11.5</td>
</tr>
<tr>
<td>A S Henry</td>
<td>10.5</td>
<td>16.8</td>
</tr>
<tr>
<td>Nottingham Manufacturing</td>
<td>9.9</td>
<td>8.3</td>
</tr>
<tr>
<td>Parkland Manufacturing</td>
<td>31.0</td>
<td>8.5</td>
</tr>
<tr>
<td>Porritt &amp; Spencer</td>
<td>19.2</td>
<td>13.1</td>
</tr>
<tr>
<td>Low &amp; Bonar</td>
<td>8.2</td>
<td>7.6</td>
</tr>
<tr>
<td>West Riding Woosted</td>
<td>19.9</td>
<td>9.2</td>
</tr>
<tr>
<td>Woolcombers</td>
<td>25.7</td>
<td>10.5</td>
</tr>
</tbody>
</table>

**Sample Totals**

| 272.2 | 157.8 |

| Sample Means          | 18.15   | 10.52   |
| Sample Standard Deviations | 9.56   | 4.73    |
| Correlation Coefficient (r) | 0.395  |
| z transformation of (r)  | 0.418   |
### Engineering Supplies 

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barton &amp; Sons</td>
<td>7.8</td>
<td>10.3</td>
</tr>
<tr>
<td>Bells Asbestos</td>
<td>11.1</td>
<td>5.6</td>
</tr>
<tr>
<td>Birmid</td>
<td>16.9</td>
<td>9.2</td>
</tr>
<tr>
<td>British Thermostat</td>
<td>10.0</td>
<td>5.2</td>
</tr>
<tr>
<td>Clayton Dewandre</td>
<td>25.2</td>
<td>12.0</td>
</tr>
<tr>
<td>Dowty</td>
<td>9.5</td>
<td>12.5</td>
</tr>
<tr>
<td>G.K.N. (period 1955-63)</td>
<td>13.5</td>
<td>10.8</td>
</tr>
<tr>
<td>Joseph Lucas</td>
<td>7.4</td>
<td>11.5</td>
</tr>
<tr>
<td>Engineering Components Ltd.</td>
<td>15.5</td>
<td>12.3</td>
</tr>
<tr>
<td>Morgan Crucible</td>
<td>13.9</td>
<td>12.2</td>
</tr>
<tr>
<td>Renold Chain</td>
<td>21.5</td>
<td>9.1</td>
</tr>
<tr>
<td>Simms Motor &amp; Electric</td>
<td>32.2</td>
<td>25.6</td>
</tr>
<tr>
<td>Hamsome &amp; Marles</td>
<td>10.9</td>
<td>13.6</td>
</tr>
<tr>
<td>Tap &amp; Dye Corporation</td>
<td>13.8</td>
<td>3.6</td>
</tr>
<tr>
<td>Universal Grinding Wheel</td>
<td>15.2</td>
<td>12.6</td>
</tr>
<tr>
<td><strong>Sample Totals</strong></td>
<td><strong>232.5</strong></td>
<td><strong>169.1</strong></td>
</tr>
</tbody>
</table>

| Sample Means                           | 15.50        | 11.27     |
| Sample Standard Deviations             | 6.54         | 4.98      |
| Correlation Coefficient (r)            |              | 0.629     |
| z transformation of (r)                |              | 0.740     |

### Mechanical Engineering

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.S.A.</td>
<td>20.7</td>
<td>27.0</td>
</tr>
<tr>
<td>J. Brockhouse</td>
<td>15.2</td>
<td>4.2</td>
</tr>
<tr>
<td>Brook Motors</td>
<td>26.8</td>
<td>6.0</td>
</tr>
<tr>
<td>Broom &amp; Wade</td>
<td>19.7</td>
<td>21.3</td>
</tr>
<tr>
<td>Crofts Engineers</td>
<td>7.9</td>
<td>8.9</td>
</tr>
<tr>
<td>Alfred Herbert</td>
<td>9.4</td>
<td>5.2</td>
</tr>
<tr>
<td>Nick Hargreaves</td>
<td>9.4</td>
<td>11.7</td>
</tr>
<tr>
<td>Leyland Motors</td>
<td>32.8</td>
<td>12.1</td>
</tr>
<tr>
<td>Nather &amp; Platt</td>
<td>16.7</td>
<td>9.3</td>
</tr>
<tr>
<td>Hansome, Sims &amp; Jeffrey</td>
<td>17.8</td>
<td>15.8</td>
</tr>
<tr>
<td>Ruston &amp; Hornaby</td>
<td>14.0</td>
<td>10.1</td>
</tr>
<tr>
<td>Serck</td>
<td>7.8</td>
<td>21.2</td>
</tr>
<tr>
<td>Sheepbridge Engineering (period 1957-65)</td>
<td>9.6</td>
<td>13.6</td>
</tr>
<tr>
<td>Stothert &amp; Pitt</td>
<td>7.3</td>
<td>18.3</td>
</tr>
<tr>
<td>G.&amp;.J. Weir</td>
<td>25.5</td>
<td>12.3</td>
</tr>
<tr>
<td><strong>Sample Totals</strong></td>
<td><strong>241.1</strong></td>
<td><strong>197.0</strong></td>
</tr>
</tbody>
</table>

| Sample Means                           | 16.07        | 13.13     |
| Sample Standard Deviations             | 7.59         | 6.32      |
| Correlation Coefficient (r)            |              | -0.017    |
| z transformation of (r)                |              | 0.017     |
## CONSTRUCTION

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bannister Walton</td>
<td>16.0</td>
<td>12.3</td>
</tr>
<tr>
<td>Matthew Hall</td>
<td>18.0</td>
<td>19.8</td>
</tr>
<tr>
<td>W.C. French</td>
<td>86.5</td>
<td>26.6</td>
</tr>
<tr>
<td>Taylor Woodrow (1964 gearing exceeds 20%)</td>
<td>11.4</td>
<td>18.7</td>
</tr>
<tr>
<td>G.N. Hadwen</td>
<td>13.7</td>
<td>22.9</td>
</tr>
<tr>
<td>John Laing</td>
<td>6.0</td>
<td>15.0</td>
</tr>
<tr>
<td>Limmer &amp; Trinidad</td>
<td>18.8</td>
<td>5.3</td>
</tr>
<tr>
<td>Marchwiel Holding</td>
<td>29.2</td>
<td>14.6</td>
</tr>
<tr>
<td>A. Monk</td>
<td>18.1</td>
<td>24.8</td>
</tr>
<tr>
<td>John Howden</td>
<td>25.0</td>
<td>15.1</td>
</tr>
<tr>
<td>Power Securities</td>
<td>18.0</td>
<td>14.6</td>
</tr>
<tr>
<td>Val de Travers</td>
<td>23.0</td>
<td>3.2</td>
</tr>
<tr>
<td>Blackwood Hodge</td>
<td>24.8</td>
<td>6.1</td>
</tr>
<tr>
<td>Tarmac</td>
<td>14.7</td>
<td>13.5</td>
</tr>
<tr>
<td>Wimpey</td>
<td>10.7</td>
<td>4.6</td>
</tr>
</tbody>
</table>

**Sample Totals**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Means</td>
<td>22.26</td>
</tr>
<tr>
<td>Sample Standard Deviations</td>
<td>18.15</td>
</tr>
<tr>
<td>Correlation Coefficient (r)</td>
<td>0.376</td>
</tr>
<tr>
<td>z transformation of (r)</td>
<td>0.395</td>
</tr>
</tbody>
</table>

(N.B. Exclusion of 'French' gives:
\[ \bar{X} = 17.67 \text{ with } S_x = 6.09; \text{ and } \bar{Y} = 13.61 \text{ with } S_y = 6.53; \]
with \( r = 0.20. \)
and \( z = 0.198. \))

## HEAVY ENGINEERING

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.E.I.</td>
<td>8.9</td>
<td>12.0</td>
</tr>
<tr>
<td>B.I.C.C.</td>
<td>5.0</td>
<td>5.2</td>
</tr>
<tr>
<td>Davy-Ashmore</td>
<td>16.9</td>
<td>29.3</td>
</tr>
<tr>
<td>Wessoe</td>
<td>14.5</td>
<td>15.6</td>
</tr>
<tr>
<td>G.E.C.</td>
<td>21.9</td>
<td>11.4</td>
</tr>
<tr>
<td>Hadfields</td>
<td>20.7</td>
<td>13.5</td>
</tr>
<tr>
<td>International Combustion</td>
<td>13.1</td>
<td>10.3</td>
</tr>
<tr>
<td>Newton Chambers</td>
<td>16.2</td>
<td>23.7</td>
</tr>
<tr>
<td>Parsons</td>
<td>20.9</td>
<td>8.2</td>
</tr>
<tr>
<td>Reyrolle</td>
<td>10.6</td>
<td>4.3</td>
</tr>
<tr>
<td>Head Wrightson</td>
<td>4.4</td>
<td>16.2</td>
</tr>
<tr>
<td>John Thompson</td>
<td>30.6</td>
<td>7.1</td>
</tr>
<tr>
<td>T.W.Ward</td>
<td>14.5</td>
<td>32.4</td>
</tr>
<tr>
<td>Wellman Engineering</td>
<td>9.9</td>
<td>26.2</td>
</tr>
<tr>
<td>Woodhall-Duckham</td>
<td>16.5</td>
<td>17.7</td>
</tr>
</tbody>
</table>

**Sample Totals**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample Means</td>
<td>15.03</td>
</tr>
<tr>
<td>Sample Standard Deviations</td>
<td>6.57</td>
</tr>
<tr>
<td>Correlation Coefficient (r)</td>
<td>-0.092</td>
</tr>
<tr>
<td>z transformation of (r)</td>
<td>0.032</td>
</tr>
</tbody>
</table>
### CONSTRUCTION SUPPLIES & COMPONENTS

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Associated Portland Cement</td>
<td>4.2</td>
<td>8.6</td>
</tr>
<tr>
<td>Derbyshire Stone</td>
<td>16.2</td>
<td>5.5</td>
</tr>
<tr>
<td>Expanded Metal</td>
<td>7.6</td>
<td>16.4</td>
</tr>
<tr>
<td>S.Glikstein</td>
<td>30.8</td>
<td>15.5</td>
</tr>
<tr>
<td>Glymed Tubes</td>
<td>10.9</td>
<td>4.9</td>
</tr>
<tr>
<td>H &amp; R Johnson</td>
<td>11.4</td>
<td>14.0</td>
</tr>
<tr>
<td>London Brick</td>
<td>9.9</td>
<td>7.3</td>
</tr>
<tr>
<td>Donald McPherson</td>
<td>5.1</td>
<td>9.8</td>
</tr>
<tr>
<td>Neston Valley Brick</td>
<td>18.7</td>
<td>17.0</td>
</tr>
<tr>
<td>Montague L. Meyer</td>
<td>5.4</td>
<td>8.5</td>
</tr>
<tr>
<td>Midland Electric</td>
<td>60.3</td>
<td>13.0</td>
</tr>
<tr>
<td>Pernamite</td>
<td>6.7</td>
<td>9.8</td>
</tr>
<tr>
<td>Rugby Portland</td>
<td>7.7</td>
<td>4.8</td>
</tr>
<tr>
<td>Universal Asbestos Mfg.</td>
<td>29.1</td>
<td>14.6</td>
</tr>
<tr>
<td>Twyford Sanitary</td>
<td>27.0</td>
<td>15.4</td>
</tr>
<tr>
<td><strong>Sample Totals</strong></td>
<td><strong>250.2</strong></td>
<td><strong>165.1</strong></td>
</tr>
</tbody>
</table>

| Sample Means                              | 16.73        | 11.01     |
| Sample Standard Deviations                | 14.50        | 4.20      |
| Correlation Coefficient \( (r) \)        | 0.427        |           |
| \( z \) transformation of \( (r) \)      | 0.456        |           |

\( \text{N.B. Exclusion of 'Midland Electric' gives:} \)

\[ \bar{X} = 13.61 \text{ with } S_x = 3.95; \text{ and } \bar{Y} = 10.86 \text{ with } S_y = 4.33; \]

\[ \text{with } r = 0.550, \text{ which is significant at the 5% level; and } \]

\[ z = 0.619. \]

### FOOD PROCESSING

<table>
<thead>
<tr>
<th>Sample Companies</th>
<th>Earnings (X)</th>
<th>Price (Y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basset</td>
<td>12.8</td>
<td>8.7</td>
</tr>
<tr>
<td>Haynard</td>
<td>5.4</td>
<td>10.1</td>
</tr>
<tr>
<td>Cerebos Salt</td>
<td>6.6</td>
<td>5.1</td>
</tr>
<tr>
<td>Danish Bacon</td>
<td>12.0</td>
<td>10.0</td>
</tr>
<tr>
<td>Express Dairies</td>
<td>8.2</td>
<td>15.6</td>
</tr>
<tr>
<td>Gardner</td>
<td>10.9</td>
<td>16.1</td>
</tr>
<tr>
<td>H.P.Sauce</td>
<td>4.0</td>
<td>8.3</td>
</tr>
<tr>
<td>National Canning</td>
<td>26.8</td>
<td>13.0</td>
</tr>
<tr>
<td>Ross Group</td>
<td>19.1</td>
<td>17.9</td>
</tr>
<tr>
<td>Schweppes</td>
<td>9.7</td>
<td>15.7</td>
</tr>
<tr>
<td>Spillers (Share Price: 1957-65)</td>
<td>5.7</td>
<td>8.4</td>
</tr>
<tr>
<td>Tate &amp; Lyle</td>
<td>9.1</td>
<td>7.6</td>
</tr>
<tr>
<td>Typhoo Tea</td>
<td>5.8</td>
<td>11.3</td>
</tr>
<tr>
<td>Tesco</td>
<td>7.0</td>
<td>22.7</td>
</tr>
<tr>
<td>United Biscuit</td>
<td>4.2</td>
<td>4.6</td>
</tr>
<tr>
<td><strong>Sample Totals</strong></td>
<td><strong>147.6</strong></td>
<td><strong>175.3</strong></td>
</tr>
</tbody>
</table>

| Sample Means                              | 9.04         | 11.69     |
| Sample Standard Deviations                | 5.92         | 4.87      |
| Correlation Coefficient \( (r) \)        | 0.315        |           |
| \( z \) transformation of \( (r) \)      | 0.326        |           |
Details of the analyses of variance are:

**ANALYSIS OF VARIANCE OF EARNINGS (X)**

<table>
<thead>
<tr>
<th>Industry</th>
<th>$\Sigma X$</th>
<th>$\bar{X}$</th>
<th>$\Sigma (X^2)$</th>
<th>$(\Sigma X)^2/n$</th>
<th>Group S.S.</th>
<th>$\Sigma^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>219.8</td>
<td>14.65</td>
<td>3696.72</td>
<td>2220.80</td>
<td>475.92</td>
<td>214.62</td>
</tr>
<tr>
<td>Textiles</td>
<td>272.2</td>
<td>18.15</td>
<td>6313.26</td>
<td>4939.50</td>
<td>1373.76</td>
<td>329.42</td>
</tr>
<tr>
<td>Eng. Sup.</td>
<td>232.5</td>
<td>15.50</td>
<td>4246.25</td>
<td>3603.70</td>
<td>642.55</td>
<td>240.25</td>
</tr>
<tr>
<td>Mech. Eng.</td>
<td>241.1</td>
<td>16.07</td>
<td>4738.75</td>
<td>3375.30</td>
<td>863.45</td>
<td>258.24</td>
</tr>
<tr>
<td>Cons.</td>
<td>333.9</td>
<td>22.26</td>
<td>12373.21</td>
<td>7432.60</td>
<td>4940.61</td>
<td>495.51</td>
</tr>
<tr>
<td>H. Eng.</td>
<td>225.4</td>
<td>15.03</td>
<td>4036.26</td>
<td>3397.00</td>
<td>649.26</td>
<td>225.90</td>
</tr>
<tr>
<td>Cons. Sup.</td>
<td>250.9</td>
<td>16.73</td>
<td>7352.23</td>
<td>4196.70</td>
<td>3155.53</td>
<td>279.89</td>
</tr>
<tr>
<td>Food Proc.</td>
<td>147.6</td>
<td>9.84</td>
<td>12977.94</td>
<td>1452.40</td>
<td>525.54</td>
<td>96.83</td>
</tr>
<tr>
<td>Totals</td>
<td>1923.4</td>
<td>128.23</td>
<td>44734.62</td>
<td>30828.90</td>
<td>13905.72</td>
<td>2140.66</td>
</tr>
</tbody>
</table>

using $n = \text{no. of samples}$, and $N = \text{no. of observations in each sample}$;
and where the total for $(\Sigma X)^2/n = (1923.4)^2/N$

The variance of the sample means $S_{\bar{X}}^2$ is given by:

$$S_{\bar{X}}^2 = \frac{\Sigma (X^2) - (\Sigma X)^2}{n} \cdot \frac{1}{n-1}$$

= 12.18

and an estimate of the variance of the population $\sigma_x^2$ is given by:

$$\sigma_x^2 = N \cdot S_{\bar{X}}^2 = 182.7$$

whence, using $(n-1)$ degrees of freedom, the estimate derived from the sample means of the population sum of squares is:

$$(n-1) \cdot \sigma_x^2 = 1278.90$$

yielding the following table of variances:

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>119</td>
<td>13905.72</td>
<td>116.85</td>
<td></td>
</tr>
<tr>
<td>Sample Means</td>
<td>7</td>
<td>1278.90</td>
<td>182.70</td>
<td>F=1.621</td>
</tr>
<tr>
<td>Samples Pooled</td>
<td>112</td>
<td>12626.82</td>
<td>112.74</td>
<td></td>
</tr>
</tbody>
</table>

which, for entry into the table for 5% points of the $F$-distribution with $v_1 = 7$ and $v_2 = 112$, is below the significance value 2.10.

Modification of the sample for construction and for construction supplies in respect of the distortion caused by observations of extreme earnings...
variability; yields the following results:

\[
\begin{array}{l}
\Sigma X \quad \bar{X} \quad \Sigma (x^2) \quad (\Sigma X)^2/n \quad \text{Group S.S.} \quad \bar{X}^2 \\
\text{Totals} \quad 1776.6 \quad 120.52 \quad 33616.23 \quad 26302.56 \quad 7303.72 \quad 1862.72
\end{array}
\]

yielding:

\[\hat{s}_{x}^2 = 6.73\] (nearly 50% reduction)

\[\hat{c}_{x}^2 = 100.95\]

and \((n-1)\hat{c}_{x}^2 = 706.65\)

to give the following table of variances:

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>119</td>
<td>7303.72</td>
<td>61.38</td>
<td></td>
</tr>
<tr>
<td>Sample Means</td>
<td>7</td>
<td>706.65</td>
<td>100.95</td>
<td>1.714</td>
</tr>
<tr>
<td>Samples Pooled</td>
<td>112</td>
<td>6597.07</td>
<td>58.90</td>
<td></td>
</tr>
</tbody>
</table>

which is a non-significant value of \(F\). The 'least significant difference' coefficient is (for the 5% level, \(t_5,05 = 1.981\))

\[\text{IND} = 1.981 \times \sqrt{58.90} \times \sqrt{2/15} = 5.55\]

ANALYSIS OF VARIANCE OF SHARE PRICES (Y)

<table>
<thead>
<tr>
<th>Industry</th>
<th>(\Sigma X)</th>
<th>(\bar{Y})</th>
<th>(\Sigma (Y^2))</th>
<th>((\Sigma Y)^2/n)</th>
<th>Group S.S.</th>
<th>(\bar{Y}^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chemicals</td>
<td>229.1</td>
<td>15.27</td>
<td>3721.69</td>
<td>3499.1</td>
<td>222.59</td>
<td>233.17</td>
</tr>
<tr>
<td>Textiles</td>
<td>157.8</td>
<td>10.52</td>
<td>1996.16</td>
<td>1660.1</td>
<td>336.06</td>
<td>110.67</td>
</tr>
<tr>
<td>Eng,Sup.</td>
<td>169.1</td>
<td>11.27</td>
<td>2277.05</td>
<td>1906.3</td>
<td>371.55</td>
<td>127.01</td>
</tr>
<tr>
<td>Mech.Eng.</td>
<td>197.0</td>
<td>13.13</td>
<td>3184.60</td>
<td>2587.3</td>
<td>597.30</td>
<td>172.40</td>
</tr>
<tr>
<td>Cons.</td>
<td>217.1</td>
<td>14.47</td>
<td>3899.31</td>
<td>3142.1</td>
<td>756.21</td>
<td>209.38</td>
</tr>
<tr>
<td>H.Eng.</td>
<td>233.9</td>
<td>15.59</td>
<td>4712.39</td>
<td>3647.3</td>
<td>1065.09</td>
<td>243.05</td>
</tr>
<tr>
<td>Cons.Sup.</td>
<td>165.1</td>
<td>11.01</td>
<td>2082.41</td>
<td>1817.2</td>
<td>265.21</td>
<td>121.22</td>
</tr>
<tr>
<td>Food Proc.</td>
<td>175.3</td>
<td>11.69</td>
<td>2405.41</td>
<td>2048.7</td>
<td>356.71</td>
<td>136.66</td>
</tr>
<tr>
<td>Totals</td>
<td>1544.4</td>
<td>102.95</td>
<td>24278.82</td>
<td>19876.43</td>
<td>4402.39</td>
<td>1353.56</td>
</tr>
</tbody>
</table>

which yields \(\hat{s}_{x}^2 = 4.10\)

and \(\hat{c}_{x}^2 = 61.50\)

whence \((1-n)\hat{c}_{x}^2 = 430.5\)
to give the following table of variances:

<table>
<thead>
<tr>
<th>Source</th>
<th>Degrees of Freedom</th>
<th>Sum of Squares</th>
<th>Mean Square</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>119</td>
<td>4402.39</td>
<td>36.99</td>
<td></td>
</tr>
<tr>
<td>Sample Means</td>
<td>7</td>
<td>430.50</td>
<td>61.50</td>
<td>1.734</td>
</tr>
<tr>
<td>Samples Pooled</td>
<td>112</td>
<td>3971.89</td>
<td>35.46</td>
<td></td>
</tr>
</tbody>
</table>

which is below the 2.10 significant value of the variance ratio at the 5% level.

Correlation Analysis

Coefficients of correlation - assumed to be linear - are calculated from

\[ r = \frac{\sum XY - \overline{X}\overline{Y}}{S_x S_y} \]  

(1)

using as a check

\[ r = \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{(n\sum X^2 - (\sum X)^2)(n\sum Y^2 - (\sum Y)^2)}} \]  

(2)

From (1):

Chemicals \[ r = \frac{233.08 - 223.71}{5.64 \times 3.87} = 0.429 \]

Textiles \[ r = \frac{208.77 - 192.93}{9.56 \times 4.73} = 0.395 \]

Engineering Supplies \[ r = \frac{195.18 - 174.62}{6.54 \times 4.98} = 0.629 \]

Mechanical Engineering \[ r = \frac{210.19 - 211.00}{7.59 \times 6.32} = -0.017 \]

Construction \[ r = \frac{370.58 - 322.10}{18.15 \times 7.11} = 0.376 \]

or adjusted \[ r = \frac{232.70 - 249.49}{6.09 \times 6.53} = -0.196 \]

Heavy Engineering \[ r = \frac{229.22 - 234.32}{6.57 \times 8.43} = -0.092 \]

Construction Supplies \[ r = \frac{210.18 - 184.20}{14.50 \times 4.20} = 0.427 \]

or adjusted \[ r = \frac{169.21 - 147.80}{8.95 \times 4.33} = 0.550 \]

Food Processing \[ r = \frac{124.12 - 115.03}{5.92 \times 4.87} = 0.315 \]
These coefficients of correlation are tested for significance, on the assumption that they are normally distributed, by:

\[ t = \frac{r \sqrt{n-2}}{\sqrt{1-r^2}} \]

the results being scheduled in the text. The assumption of linear correlation between earnings variability and share price variability is tested in some measure by establishing the standard error of estimation of share price variability, and expressing this as a function of average variability within the industry. The resulting coefficient is an approximate measure of the efficiency of the estimation afforded by the correlation coefficient.

In most industries this efficiency is little better than 50% (C of V approximates to .50); the exceptions being the two components industries and chemicals. Thus any conclusion as to differentiations in the two components industries is suspect in that, by a statistical accident, the assumption of linear correlation is more valid than in most of the other industries.
APPENDIX IV.

SAMPLE DATA FOR THE ANALYSIS OF CAPITAL STRUCTURE AND EQUITY EARNINGS YIELD

In this Appendix are listed the sample companies for this analysis, together with details of their individual capital structures, equity earnings and book asset values. The source is Moodies Investment Handbook, 1965 and 1967/8 series. Details of the selection criteria are set out in the text of Chapter Four, and so are not repeated here.

CAPITAL STRUCTURE

By the term 'equity' in the following table is meant, the value at the end of the financial year ending during 1964; and during 1966 or the first quarter of 1967; of issued ordinary capital plus free capital and revenue reserves. By "free" is meant, exclusive of taxation reserves and other reserves for deferred or future liabilities. No adjustment is made in respect of any fraction of such free reserves which might, under the Articles of Association of a particular company, enure to other than ordinary shareholders. By 'prior capital' is meant, issued capital other than ordinary capital plus funded loan capital. Bank overdraft and inter-company current accounts are excluded. The average gearing % is the arithmetic mean of the individual company gearing.

<table>
<thead>
<tr>
<th>Sample &quot;A&quot; Companies</th>
<th>1964</th>
<th>1966/7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>Prior Capital</td>
<td>Gearing %</td>
</tr>
<tr>
<td>£ 22.3</td>
<td>£ 0.3</td>
<td>1.3</td>
</tr>
<tr>
<td>16.0</td>
<td>0.8</td>
<td>5.0</td>
</tr>
<tr>
<td>87.5</td>
<td>12.4</td>
<td>14.2</td>
</tr>
<tr>
<td>64.4</td>
<td>9.9</td>
<td>15.4</td>
</tr>
<tr>
<td>11.2</td>
<td>0.5</td>
<td>4.5</td>
</tr>
<tr>
<td>6.0</td>
<td>0.5</td>
<td>8.3</td>
</tr>
<tr>
<td>187.1</td>
<td>8.1</td>
<td>4.3</td>
</tr>
<tr>
<td>5.7</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td>66.8</td>
<td>10.7</td>
<td>12.3</td>
</tr>
<tr>
<td>13.8</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(table continued on next page)
(table continued from previous page)

<table>
<thead>
<tr>
<th>1964</th>
<th>SAMPLE &quot;A&quot; COMPANIES</th>
<th>1966/7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>Prior</td>
<td>Gearing</td>
</tr>
<tr>
<td>£ 11.6</td>
<td>£ 1.9</td>
<td>16.4</td>
</tr>
<tr>
<td>21.8</td>
<td>1.3</td>
<td>6.0</td>
</tr>
<tr>
<td>32.1</td>
<td>6.1</td>
<td>19.0</td>
</tr>
<tr>
<td>49.2</td>
<td>9.3</td>
<td>18.9</td>
</tr>
<tr>
<td>6.3</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>113.5</td>
<td>20.2</td>
<td>17.8</td>
</tr>
<tr>
<td>2.7</td>
<td>0.4</td>
<td>14.8</td>
</tr>
<tr>
<td>61.1</td>
<td>12.2</td>
<td>20.0</td>
</tr>
<tr>
<td>3.5</td>
<td>0.5</td>
<td>14.3</td>
</tr>
<tr>
<td>102.2</td>
<td>9.2</td>
<td>9.0</td>
</tr>
<tr>
<td>8.4</td>
<td>1.0</td>
<td>11.9</td>
</tr>
<tr>
<td>19.6</td>
<td>0.1</td>
<td>0.5</td>
</tr>
<tr>
<td>45.3</td>
<td>2.0</td>
<td>4.4</td>
</tr>
<tr>
<td>4.2</td>
<td>0.3</td>
<td>7.1</td>
</tr>
<tr>
<td>16.0</td>
<td>0.6</td>
<td>3.0</td>
</tr>
<tr>
<td>£ 49.3</td>
<td>£ 4.3</td>
<td>2.7% AVERAGE</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1964</th>
<th>SAMPLE &quot;B&quot; COMPANIES</th>
<th>1966/7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>Prior</td>
<td>Gearing</td>
</tr>
<tr>
<td>£ 4.4</td>
<td>£ 2.2</td>
<td>50.0</td>
</tr>
<tr>
<td>4.9</td>
<td>1.7</td>
<td>34.7</td>
</tr>
<tr>
<td>36.5</td>
<td>17.3</td>
<td>47.4</td>
</tr>
<tr>
<td>10.3</td>
<td>3.5</td>
<td>34.0</td>
</tr>
<tr>
<td>9.9</td>
<td>3.2</td>
<td>32.3</td>
</tr>
<tr>
<td>9.9</td>
<td>4.8</td>
<td>48.5</td>
</tr>
<tr>
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EARNINGS AND ASSET VALUES

By "earnings %" in the following table is meant, the percentage of which earnings (net of exceptional items) bore to the values/issued ordinary capital; both earnings and capital being in respect of the financial years 1964 and 1966/7 as defined above. The figure of 'share price' is the average of the highest and lowest prices during those same periods, adjusted pro-rata to a common par value of 10/0d. per share. Actual par values are quoted in brackets after the name of the firm in the following tables. "Asset value" is the total of all assets at net book value at the end of those same periods, after payment in full of all prior claims, divided by the number of ordinary shares in issue at the end of those same periods. Goodwill is included.

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AVERAGE 20.7% 26/21 20/9

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AVERAGE 20.7% 26/21 20/9

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The following books and articles were consulted during the preparation of this thesis. Where specific reference is made to any one of these in the text, the appropriate reference number is quoted.

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Where an article has been consulted in one of those collections, the
appropriate index (A), (B), (C), or (D) appears at the end of the reference, and the page numbers are those of that collection.


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Elementary Statistical Tables.

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"Unpublished Bachelors Dissertation"
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