The multi-objective optimum design of building thermal systems

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0402661019
THE MULTI-OBJECTIVE OPTIMUM DESIGN OF BUILDING THERMAL SYSTEMS

By

Heather Anne Loosemore

A Doctoral Thesis submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy

of Loughborough University

June 2002

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Acknowledgements

Most thanks go to my family for their continuous support during the study but particularly during the writing up process. To David who has been key to the development of the written aspect, Linda endless cups of coffee and Roni the much needed sugar provider.

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Other colleagues that deserve my thanks are the whole Control room group who ensured that even during the bleakest moments that the working environment was joy to be part of.

There are many friends who have provided the much needed distraction, and support. Special mention goes to Mark for always being there, Mike for inventing the second bottle principle, Scott for encouraging me to actually finish and the Dillys for the pet therapy. Not forgetting the numerous climbing partners both old and new.
Abstract

The thermal design of buildings as a multi-criterion optimisation process since there is always a pay-off (balance) to be made between capital expenditure and the operating cost of the building. This thesis investigates an approach to solving 'whole building' optimisation problems. In particular simultaneous optimisation of the plant size for a fixed arrangement of air conditioning equipment, and the control schedule for its operation to condition the space within a discrete selection of building envelopes.

The optimisation is achieved by examining a combination of the cost of operating the plant, the capital cost of the plant and building construction, and maximum percentage people dissatisfied during the occupation of the building. More that one criterion is examined at a time by using multi-criteria optimisation methods. Therefore rather than a single optimum, a payoff between the solutions is sort. The benefit of this is that it provides a more detailed information about the characteristics of the problem and more design solutions available to the end user.

The optimisation is achieved using a modified genetic algorithm using Pareto ranking selection to provide the multi-criterion fitness selection. Specific methods for handling the high number of constraints within the problem are examined. A specific operator is designed and demonstrated to deal with the discontinuous effects of the three separate seasons, which are used for the plant selection and for the three separate control schedules.

Conclusions are made with respect to the specific application of the multi-criterion optimisation to, building services systems, their control, and the viability of 'whole building design' optimisation.
**Publication List**


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Nomenclature

\( A \)  
Surface Area \( \text{m}^2 \)

\( b_i \)  
Number of individuals in the \( i^{th} \) sub-region

\( \bar{b}_i \)  
Expected number of individuals in the \( i^{th} \) sub-region

\( c \)  
Thermal Capacity Ratio \( (\frac{C_{\text{min}}}{C_{\text{max}}}) \)

\( \bar{c}(x) \)  
Normalised constraint function

\( C \)  
Thermal Capacity Ratio \( (\frac{C_{\text{min}}}{C_{\text{max}}}) \)

\( C_h \)  
Capacitance of all massive elements in the room \( \text{kJ/kgK} \)

\( CF \)  
Crowding Factor

\( C_l \)  
Capacitance of the lower half of slab \( \text{kJ/kgK} \)

\( C_{\text{min}} \)  
The lower value of thermal capacity of the moving fluid \( \text{kW/K} \)

\( C_{\text{max}} \)  
The higher value of thermal capacity of the moving fluid \( \text{kW/K} \)

\( COP_{\text{full}} \)  
Chiller coefficient of performance at full capacity

\( C_{pa} \)  
Air Capacitance \( \text{kJ/kgK} \)

\( C_u \)  
Capacitance of the upper half of slab \( \text{kJ/kgK} \)

\( cv_n \)  
Volume of a \( n \) dimensional sphere

\( d \)  
Distance between individuals

\( E_{\text{res}} \)  
Heat loss due to respiration (dry and latent) \( \text{Wm}^{-2} \)

\( E_{sk} \)  
Heat loss by water vapour diffusion and evaporation through skin \( \text{Wm}^{-2} \)

\( f(x) \)  
Function of variable \( x \)

\( g(x) \)  
Inequality Constraint function

\( h(x) \)  
Equality Constraint function

\( h_i \)  
Smoothing parameter

\( H \)  
Heat loss through convective and radiation from the clothing surface \( \text{Wm}^{-2} \)

\( i \)  
Variable identifier

\( j \)  
Constraint identifier
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
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<tr>
<td>$k$</td>
<td>Objective function identifier</td>
<td></td>
</tr>
<tr>
<td>$K$</td>
<td>Total number of Objective</td>
<td></td>
</tr>
<tr>
<td>$K_e$</td>
<td>Epanechnikov Kernel</td>
<td></td>
</tr>
<tr>
<td>$l$</td>
<td>String Length</td>
<td>m</td>
</tr>
<tr>
<td>$L$</td>
<td>Ventilated core length</td>
<td>m</td>
</tr>
<tr>
<td>$l_x$</td>
<td>Lower bound of variable $x$</td>
<td></td>
</tr>
<tr>
<td>$n$</td>
<td>Number of variables</td>
<td></td>
</tr>
<tr>
<td>$m$</td>
<td>Minimum value of the objective</td>
<td></td>
</tr>
<tr>
<td>$M_x$</td>
<td>Maximum value of the objective</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>Metabolic Rate</td>
<td>$Wm^{-2}$</td>
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<tr>
<td>$\dot{m}_a$</td>
<td>Mass air flow rate</td>
<td>$kg/s$</td>
</tr>
<tr>
<td>$M_s$</td>
<td>Supply mass flow rate</td>
<td>$kg/s$</td>
</tr>
<tr>
<td>$M_{sp}$</td>
<td>Supply mass flow rate setpoint</td>
<td>$kg/s$</td>
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<tr>
<td>$N$</td>
<td>Number of individuals in a population</td>
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</tr>
<tr>
<td>$NTU$</td>
<td>Number of transfer units</td>
<td></td>
</tr>
<tr>
<td>$O$</td>
<td>Number of objectives</td>
<td></td>
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<tr>
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<td>Objective Number $j$</td>
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</tr>
<tr>
<td>$p$</td>
<td>number of variables/parameters</td>
<td></td>
</tr>
<tr>
<td>$P$</td>
<td>Population size</td>
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<tr>
<td>$Pareto$</td>
<td>Pareto score of individuals (when in the context of an equation)</td>
<td>%</td>
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<tr>
<td>$PLR$</td>
<td>Percentage part load ratio</td>
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<tr>
<td>$PMV$</td>
<td>Percentage mean vote</td>
<td></td>
</tr>
<tr>
<td>$PPD$</td>
<td>Percentage people dissatisfied</td>
<td></td>
</tr>
<tr>
<td>$q$</td>
<td>Number of niches</td>
<td></td>
</tr>
<tr>
<td>$q_d$</td>
<td>Desired number of optimal points</td>
<td></td>
</tr>
<tr>
<td>$Q_r$</td>
<td>Radiant heat gains</td>
<td>W</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Convective heat gains</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{full}$</td>
<td>Full design load</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{cool}$</td>
<td>Cooling Load</td>
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<tr>
<td>$\dot{Q}_i$</td>
<td>Input energy</td>
<td>W</td>
</tr>
<tr>
<td>$\dot{Q}_{max}$</td>
<td>Maximum peak load</td>
<td>W</td>
</tr>
<tr>
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<td>Description</td>
<td></td>
</tr>
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<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$R_{ai}$</td>
<td>Resistance of the slab from the node in the lower half of the slab to the ventilated air</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{au}$</td>
<td>Resistance of the slab from the node in the upper half of the slab to the ventilated air</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{sl}$</td>
<td>Resistance of the slab from the node in the lower half of the slab to the zone below the slab</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{su}$</td>
<td>Resistance of the slab from the node to the upper half of the slab to the zone above the slab</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{eb}$</td>
<td>Resistance of all massive elements in the zone from the mass temperature to inside surface point</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{eb}$</td>
<td>Resistance of all massive elements in the zone from the mass temperature point to outside surface</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{sv}$</td>
<td>Resistance to convective heat transfer</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$R_{sv}$</td>
<td>Resistance to air infiltration</td>
<td>$m^2\cdot{\circ}K\cdot W^{-1}$</td>
</tr>
<tr>
<td>$s$</td>
<td>Individual Binary String</td>
<td></td>
</tr>
<tr>
<td>$Sh$</td>
<td>Sharing parameter</td>
<td></td>
</tr>
<tr>
<td>$T_{ai}$</td>
<td>Internal temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{ai}$</td>
<td>Surface temperature of lower half of slab model</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{aa}$</td>
<td>Sol-air temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{au}$</td>
<td>Surface temperature of upper half of slab model</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$\bar{T}_{av}$</td>
<td>Mean temperature of ventilation air</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{az}$</td>
<td>Zone air temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{mb}$</td>
<td>Mass air temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{ma}$</td>
<td>Mean slab mass temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{sl}$</td>
<td>Temperature at the node in the lower half of the slab</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{su}$</td>
<td>Temperature at the node in the upper half of the slab</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_o$</td>
<td>Outside air temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$T_{sb}$</td>
<td>Inside surface temperature</td>
<td>$\circ C$</td>
</tr>
<tr>
<td>$U$</td>
<td>U-value (Transmittance)</td>
<td>W $m^{-2}\cdot{\circ}K^{-1}$</td>
</tr>
</tbody>
</table>
$u_x$ Upper bound of variable $x$

$\nu$ Constraint identifier

$w_i$ Weighting factor

$W$ Rate of work

$\dot{W}_i$ Shaft power input

$x$ Variable

$z^*$ Normalised bit difference

$\theta_o$ Outside air temperature

$\theta_s$ Supply air temperature

$\theta_{sp}$ Supply temperature setpoint

$\sigma_{share}$ Sharing function

$\sigma_i^2$ Variance of the individuals serving $i^{th}$ sub-region of the non-dominated region.

$\alpha$ Power law sharing function

$\varepsilon$ Objective constraint

$\varepsilon$ The effectiveness of the heat exchanger

$\partial$ Arbitrary small non-negative number
Chapter 1

Introduction

Throughout building design and services installations there has been a need for optimisation. This is primarily due to the need to manage the initial construction cost. More recently and due to increasing environmental concerns and rising fuel costs therefore there has been a lot of focus on reducing the cost of running the building.

There are four main ways of reducing the operational cost of maintaining the environment within a building space at a comfortable level;

1. Reduce the level of conditioning and therefore affecting the level of the occupants’ comfort.
2. Control the climatic impact on the occupied space, which is usually done at the building design stage.
3. Suitably selecting the type of environmental control. This can be done by carefully choosing the type and size of air-conditioning plant.
4. Control the plant efficiently, to make use of the building, plant response, known occupation patterns, and cheap tariffs.

Optimisation can be used to examine any of these areas to find the cheapest alternative. In optimising one of the areas of cost reduction in isolation, the global optimal solution is often overlooked. In the same manner optimising just the operational cost in isolation
can lead to the building construction being adversely effected. This thesis attempts to simultaneously optimise many of these areas of cost reduction. In optimising simultaneously the aim is to achieve a rounded solution to the problem of producing an optimum environmental design of a building space, to produce a ‘whole building design’.

Optimisation is an integral part of building design, with cost often being the primary objective. Traditionally when optimising only a single objective is used, however in life the final solution is often a trade-off between many influencing factors. A basic example of a cheap building construction and therefore initial outlay, this may reduce the thermal performance leading to a more expensive operational cost. There will be more than one design solution dependent on the importance put on either the operational or the construction cost. Leaving the decision of the importance to the end of an optimisation process would lead to multiple optimal solutions.

Multiobjective optimisation is a developing field of optimisation that has been growing most substantially in the last eight years. Most of the multiobjective algorithms have been developed from adaptation of existing algorithms that have been already tried and tested for the application with single objectives. One such algorithm is the genetic algorithm, which is modelled on the idea of selection of the fittest seen in nature. As with nature, the genetic algorithm deals with a ‘population’ of ‘individuals’. The individuals represent possibly optimal solutions and because multiobjective optimisation is looking for more than one possible solution the genetic algorithm naturally lends itself to the task of multiobjective optimisation.

Multiobjective optimisation before now has not been applied simultaneously to building design, plant selection and operation. In this thesis it is applied to an example model of a single zone in a multi-zone building. The variables available for the optimisation process are a finite selection of building constructions, air-conditioning plant sizes and setpoints to control the use of the plant. The objectives being investigated in this thesis are the operational cost, capital cost and a measure of the comfort of the occupants.

The hollow-core ventilated slab is often presented as a method of saving on the operational cost of a building. This, together with the overall interaction of building structure with plant control and its effect on both comfort and cost, are investigated with the use of the multiobjective algorithm.
1.1 Aim and Objectives

The aim of this research is to investigate the application of multiobjective optimisation in the thermal design of the building.

The objectives of this research are:

- To develop and evaluate a multiobjective optimisation algorithm suitable for thermal building optimisation by investigating the concept of multiobjective optimisation and reviewing existing approaches.

- Evaluate multiobjective optimisation using existing models of building, fabric selection, thermal control, and plant selection. For both conventional and hollow core ventilated slabs.

- Examine the use of multiobjective optimisation in aiding design decision making.

- Draw conclusions as to the effectiveness and applicability of multiobjective optimisation to the thermal design of buildings.

1.2 Thesis Structure

This thesis is divided into two main areas; the model and the optimisation algorithm. It gives a background study on these two areas, then their application, results analysis, and concludes on the effectiveness of the multiobjective methodology in the field of building thermal optimisation.

Initially the background of optimisation in building thermal design, HVAC plant and controls selection is investigated (chapter 2). The actual model being utilised to perform simultaneous building thermal optimisation is described in chapter 4.

The second main area concentrates on the development of the multiobjective algorithm. The choices of multiobjective algorithm available are described, briefly detailing their historical development, in chapter 3.
A specific multiobjective algorithm is detailed with all the modifications necessary to apply the optimisation to the building thermal optimisation model (chapter 5).

The result chapters are broken down into analysis both of the performance of the algorithm by justification of the behaviour expressed in the optimisation results (chapter 6). The second results chapter explores the application of the hollow-core ventilated slab as a method of reducing the operational cost of a building (chapter 7).

The methods available for disseminating and analysing multiobjective results are investigated in chapter 8. In the final chapter the results of the application of the algorithm to simultaneous optimisation are presented with conclusions and recommendations for future research (chapter 9).

The appendix includes the explanation of the basic genetic algorithm.
Chapter 2

Optimisation of Buildings

The multiobjective algorithm in this study is developed to facilitate the optimisation of building construction selection, HVAC plant selection and HVAC plant control. The optimisation of these elements has been previously investigated, as both capital and energy cost saving is always been a concern of the building industry. Environmental concerns have led to much research being carried out into making buildings energy efficient. The evolution of building and plant models has been both a product of the driving factors but also the development of computing power facilitating more accurate and complex simulation tools. In this chapter a brief evaluation of previous research in the areas of building thermal optimisation and HVAC optimisation is carried out.

2.1 Building Thermal Optimisation

The main purpose of a building has always been to provide an environment that will sustain the occupants needs. This is summarised by D’cruz and Radford (1987) as being the need for shelter, physical comfort, security, privacy, visual continuity and appropriate spaces in which to conduct activities. Due to the conflicting relationship between these requirements building design is a complex process. This is mirrored in the early stages by the conflict of ideas between the architect, the designer and the engineer acting on the needs of the final client. Although, for all involved, cost has always been a concern in
construction, the present environmental concerns coupled with escalating energy costs have meant that there is an overall move to optimise at the initial design stage the lifecycle cost of a building (combination of capital and operating costs).

The initial introduction of building regulations generated interest in studying the thermal properties of buildings. Simon and Michel (1977) state that energy consumption in buildings depends on five parameters, air ventilation, insulation materials and thickness, thermal bridges, building shape and window quality and surface. More specifically the paper concluded the need for optimisation of building fabric insulation type and thickness, concluding that 'energy concepts have to be considered early on'.

Most papers define the factors effecting thermal performance, although these vary in detail dependent on the specific subject area of the thermal performance the authors are concentrating on and recalculating each option without using a specific algorithm. D'cruz et al (1983) basically, but comprehensively, described the factors affecting thermal performance as being:

- Building
  - Shape
  - Massing
  - Orientation
- Window
  - Sizes
  - Glass Types
  - Shading
- Surface Finishes
- Material Properties
- Ventilation and Infiltration

Much of the previous work has been done to optimise one of these criteria in isolation. Page (1974) used differential calculus and elementary thermal models to establish geometries that minimise the heat conduction for multi-story buildings, whilst Brown (1990) investigated the effect of thermal mass of commercial buildings. Both followed the general theme of optimising an element of the building for the reduction in overall energy usage of the building, which is done by varying one component of a building and
keeping all other components constant. D'Cruz et al (1983) summarised that the designer in practise needs to optimise a number of criteria, some of these are quantifiable, others non-quantifiable. It is recognised at this point that although it is better to look at more criteria, it will not be possible to look at them in as much detail as for a single criterion. This indicates the need for an automated multi-criterion optimisation that would address the level of detail required.

A number of methods have been applied previously to the task of optimising the thermal performance of a building. This is more of a reflection on the number of optimisation methods available and the appropriateness of different methods to the contrasting approaches of building thermal optimisation. Initial work in building design was limited by the available technology (Johnson (1976)) and the optimisation techniques that were available to use.

Gero et al. (1983) approached the task as a multiobjective problem. In doing so they used the concept of Pareto optimality to deal with the multi-criteria. Initially the optimisation approach used exhaustive search techniques; because this is a numerically intensive method the number of possible solutions was limited. Rosenman and Gero (1983) investigated the use of dynamic programming with the concept of Pareto optimality. The examples they used are the multiobjective optimisation of the external wall mass against the noise transmittance, and the optimisation of material, overhead and labour costs for the selection of floor-ceiling subsystems.

In future work the concept of Pareto optimality remains as the basis for dealing with multicriteria within the problem, however the optimisation is still dealt with as single criterion optimisation problem. (Jo Hun and Gero, 1998). Work is now moving toward the uses of genetic algorithms to facilitate the optimisation of space allocation problems within building design. In doing this it was felt necessary to modify the problem to allow it to be combined with the algorithm and to make improvements in the methodology of the search algorithm (Gero and Kazarkov 1998).

D'Cruz and Radford (1987) described a problem that investigated the performance criteria of thermal performance, cost, planning efficiency and daylighting availability. To achieve these objectives the problem was split into 5 models.
CHAPTER 2: OPTIMISATION OF BUILDINGS

Thermal models: Standard analytical techniques were employed at basic levels that were described as 'thermal loads at the scheme design stage'.

The daylight models: Which affected the thermal model, and was used for the daylighting objective.

The capital cost model: Used an elementary estimating model.

The planning efficiency model: Net useable areas, set rules are used for lift space, circulation areas etc.

The building model: Collation of the variables that the other models depend on as well as a grouping of the models themselves.

In the results, the thermal performance was compared against other criteria. It was not possible to see relationships between the criteria, because of the small number of results in the final optimised solution.

Al-Homoud (1997A), (1997B) investigated the optimum thermal design of both air conditioned residential buildings and offices. The approach used to solve the problem for both building types was the same, in that the same optimisation tasks were performed in the same six climates with the same optimisation variables (working in different bounds) and the same programs were used (ENEROPT and ENERCALC). The terminology used that the underling optimisation technique was complex, however the author did not name the specific optimisation technique. General conclusions were that it proved that there are energy savings, which can be made at the design stage, and that the individual peaks (heating and cooling) can be reduced. There seems to be very little justification as to whether there is a saving on the initial cost. The conclusions also list the building structural improvements, these vary dependent on the type of building being examined. The trends demonstrated for each type of building and climate are summarised.

Brown (1990) emphasised the link between varying the thermal mass, which included internal mass not indicated in D'Cruz's list, and the overall energy usage and the performance required of the HVAC design. All comparisons that have to predict the energy use and cost have to make assumptions about the HVAC plant. Bouchlaghem (1990), however was the exception to this as he minimised discomfort, based purely on the environmental temperature calculated using the admittance method. This was done purely on a passive building, avoiding the use of HVAC systems, using a combination of simplex and non-random complex optimisation. This led to a reduction of the
environmental temperature in summer to acceptable levels and a reduction in winter to unacceptable levels.

Wilson et Templeman (1976) only considered using optimisation of fabric heat loss, albeit in great detail. The optimisation took place using specific computer programs although the optimisation process was based around geometric programming. This led to the conclusion that the more insulation used the better, as this would produce considerable cost saving over the lifetime of the building.

Coley and Schukat (2002) develop a building thermal model with the use of EXCALIBUR simulation problem. The authors optimise a large number of building criteria to do with structure and form of the building, however they reduced these extensively before they are passed to the building model. This is mainly to reduce computational time and allow an annual energy cost to be optimised. The authors however are not after a single optimum design and use a genetic algorithm to produce a population of solution, keeping the best 5% of the population. The authors propose that there are many criteria that have to be evaluated when designing a building, many of which are not quantifiable and as such not suitable for optimising simultaneous. Maintaining the best 5% allows the designer to access the reduced set of solutions for these non-quantifiable criteria, e.g aesthetic appeal. They do recognise that the solutions can be used to evaluate the effect of the building properties and the optimisation criterion of cost.

Caldas and Norford, (2002), again utilise the principle of genetic algorithms. The authors’ optimise annual energy cost by assessing the thermal and lighting performance using DOE2. This information is used to optimise the placing and size of windows within a building. The optimisation algorithm is modified to minimise the computational complexity of the problem.

Miles et al (2001) used genetic algorithms to aid the design in the early stages of building design to reduce conceptual complexity for the designer. The structural form of the building is considered in conjunction with environmental impact and integration with services strategy. Cost was not considered directly or in isolation. The author ranks parts of the design based on large clear span, cost and environmental damage. These are
weighted and combined as a search engine rather than specifically as an optimisation process with penalty functions to produce a fitness function,

Neilson (2002) identified that to assess the performance of a space the thermal environment, daylight, heating demand, electrical consumption and life cycle cost must be evaluated. Although a single score has been proposed to evaluate quality of design solutions (Hendricks et al. 2000), Neilson felt the issue to be unresolved and the optimisation was limited to the life cycle cost with the other performance criteria forming constraints.

In most previous research basic plant models were used. There has been no attempt to include an optimisation of the HVAC plant design or operation, although in most cases the number of factors that have been considered in the building fabric optimisation has meant that the programs used have been complex and time consuming.

### 2.2 HVAC Plant Optimisation

As with most optimisation procedures the ultimate aim of HVAC optimisation is to minimise the operational and installation cost of the system. Wright and Hanby (1981) identified that the optimisation procedure had three main elements (Figure 2.1).

1. The identification of a number of possible system configurations.
2. For each configuration the optimisation of the size of the system components.
3. The assessment of the systems performance by selections criteria values to be used as quantitative parameters, thus enabling the selection of an optimum system.

The optimisation process is restricted by the physical connection between the components, as is system design. The selection and availability of the components also impose constraints.

Previous HVAC optimisation research has generally concentrated equally (or more) on the modelling techniques as opposed to the optimisation process being used. Although all optimisation of building fabric or HVAC design relies on modelling of the systems involved, the more the technical nature of the HVAC components the more emphasis
needs to be given to the formation of the model, due to their complex interaction with the building fabric.

Figure 2.1: Optimised Design of HVAC Systems

2.2.1 HVAC Systems: Component Modelling and Simulation.

Component models may be regarded as mathematical statements describing the region of component operation under consideration. They can take a wide variety of forms. Hanby (1987) summarised them in terms of the basic form from which are written and the function of the model. The form of a component model is summarised in Table 2.1.
### Table 2.1 Component Models: form and function

<table>
<thead>
<tr>
<th>MODEL TYPE</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fundamental models</td>
<td>Described adequately by established theoretical principles, applied to internal components (discrete nodal systems) e.g. Describing a mixing tee with mass and enthalpy balances</td>
</tr>
<tr>
<td>Semi-empirical models</td>
<td>Component behaviour is modelled partially by fundamental equations and partly a curve fit of some element of the performance.</td>
</tr>
<tr>
<td>Empirical models</td>
<td>‘black box’ (curve fit) models are used to describe the systems performance.</td>
</tr>
<tr>
<td>Algorithmic models</td>
<td>Most documented components are of this type;</td>
</tr>
<tr>
<td>Steady state</td>
<td>Widely used, and simplistic. The justification for their use in building service modelling is that the response of the plant is much more rapid than the forcing function (weather, building), therefore dynamics of the system will be indistinguishable from the forcing function.</td>
</tr>
<tr>
<td>Dynamic</td>
<td>Early work concentrated on control loop dynamics rather than comprehensive prediction of plant performance. Linearity/nonlinearity or the lumped/distributed parameter representation indicates levels of complexity in dynamic models implemented for HVAC systems.</td>
</tr>
<tr>
<td>Numerical models</td>
<td>Models of the processes that are not easily described by the fundamental equations or curve fit, and are most easily represented by a 'lookup' table.</td>
</tr>
</tbody>
</table>

In early system simulation procedures the choice of system modelling was limited to those available on a standard menu of systems and control strategies. This menu system based selection on a limited selection of predefined 'primary' and 'secondary' HVAC
systems. The disadvantages are the inflexibility of defining systems and the wasted program data leading to greater computational times. Most users of such systems are all too aware that the actual system being modelled does not fit exactly into one of the available options. Many early building simulation programs used this approach, probably the best known example of this being the building energy analysis program DOE-2 (Hirch, 1982)

The main way of overcoming the limitations of the menu-based schemes is to use component-based simulation. This allows the user to build up a description of the system from a defined set of components. One of the first dynamic modelling and simulation programs that allowed the user to define the system component by component was TRNSYS (Klein et al, 1976). TRNSYS was originally developed for the simulation of solar energy systems, but has since been successfully used in the HVAC field. As well as allowing almost any system configuration to be defined, component based simulations lend themselves to HVAC plant optimisation since they allow capital and energy costs to be associated with each component, as well as the equation describing component performance. This was examined by Wright and Hanby (1987) who defined four sub-models associated with each component required for use in HVAC optimisation:

1. Component Performance Model: Defines the operating characteristics. (Often termed component model).
2. Component Energy Model: Where the energy consumption is defined.
4. Component Constraint Model: Defines the practical operating region.

This allowed a realistic definition of the optimisation problem for any HVAC sub-system. Wright and Hanby (1987) solved the example HVAC system optimum sizing problems using direct (heuristic) search methods. The methods implemented where able to find optimum solutions although the direct search methods had difficulty in finding the optimum, once the search hit a constraint boundary, and could fail to find the optimum as a result.

It is recognised that modelling forms a significant part of any optimisation process, in the case of building climate control the plant selection optimisation is reliant on the definition of setpoint and operating parameters. It is evident that these have to be set to
make any HVAC model comparison fair, and that the plant has to be selected to achieve specific criteria, however there will be always more than one method of control possible.

2.2.2 Supervisory Control Optimisation

There are two levels of control of HVAC systems, supervisory control and local loop control. Supervisory control includes the ON/OFF time switching of the plant operation, and the scheduling of the control setpoints. Local loop control is concerned with controlling the plant output to meet the scheduled setpoints. In this respect, it is the optimisation of the control setpoints (supervisory control) that holds the most potential for energy cost savings.

Most literature is concerned with supervisory control, for either defining an optimum setpoint schedule, or for comparing one control method with another. Generally, optimisation techniques applied for both are not a global search of the possible permutations, but a performance comparison of new against old for one or more new techniques. Knabbe and Felsmann (1999) initially compare non-linear optimisation techniques but later utilise this knowledge to compare control strategies, (Felsmann and Knabbe, 2001)

Kaya et al (1982) compared a conventional control approach with proposed new control methods to determine energy savings in the HVAC space. The conventional control was temperature control (thermostat) and humidity control (humidistat) working independently. Whilst new methods of control were the simultaneous control of temperature, humidity and air velocity in the thermal comfort region recommended by ASHRAE.

As with both the fabric and system optimisation it is necessary to model, and therefore simulate, the performance of the space. It is observed that not as much detail is required for simulation of the space as the research is often concentrated primarily on just the HVAC control methods and the application of optimal control theory. Nizel et al (1984) chose the simplest model that would be as accurate as possible, and used what they termed the simplest dynamic models. Although it is recognised that to model the effects of the control system it is necessary to ultimately have a dynamic model, not all simulations are based on dynamic models of the building fabric. Kata et al (1982) put
forward that it is only necessary to initially produce steady state values for the internal environment and system outputs since the outside air changes sufficiently slowly to be assumed to be constant for the 30 minute duration of their prediction. The dynamic effects of the control equipment (sensor, controller, actuator valve) is often neglected as the time constants are very short (Nizel et al. (1984)).

House and Smith (1995) modelled the transient effect of the building envelope assuming the plant and zone air responses to be instantaneous. The simplification of their model and therefore the governing equations was justified by stating that it was all the accuracy required to discover the merits of system-based optimal control approach over conventional control approaches. The authors summarised that there had been to that date a number of system-based optimal control strategies demonstrating the improved system responses and the overall reduction in energy and that, with multiple control variables, further improvements could be demonstrated.

Throughout, the literature the modelling of the building fabric the fabric capacitance for storing energy is evaluated. In conventional control strategies the thermal storage of the building fabric is not directly utilised. For these strategies it is assumed that thermal mass works to increase operating costs, however under proper circumstances, the use of the thermal storage for load shift can significantly reduce operational costs. In early work Braun (1990) studied dynamic control strategies specifically related it to the offsetting of peak loads by the use of the thermal capacitance properties of the space, and by implementing the concept of free cooling during the unoccupied period. Ren (1997) applied the numerical optimisation method of genetic algorithms to a dynamic building model and static plant model to investigate the control characteristics of a thermal storage system. The emphasis of the report was to develop an optimum control strategy for the system, but it also concluded the viability of using the Genetic Algorithm for solving the optimisation problem.

2.3 Discussion

There has been no optimisation of the building fabric and the design of the plant and the control strategies prior to the research project associated with this thesis (Wright et al 2002, Wright and Loosemore, 2001). The papers evaluated recognised that it is not possible to look at each of the factors in pure isolation. For example it was not possible
to assess the optimum building fabric construction in terms of cost without making a prediction of the plant operation. Braun (1990) identified that, although there was significant energy savings to be had with optimised dynamic control, the energy saving depended on the thermal capacitance of the building (building fabric optimisation) and the part-load characteristics of the plant (plant optimisation).

It is recognised that the concept of "whole building optimisation" has a large number of problem variables, which is not easily handled by 'conventional' search methods. The multiobjective approach, with the use of genetic algorithms as a search technique, permits the use of more problem variables and it also has the advantage of indicating the correlation between the objectives.

Out of the papers reviewed most optimised one objective function, which is generally always either capital or operational cost, although many identified more than one evaluative criterion, Nielsen (2002) for example. One exception to this is Gero et al. (1983) who tackled to problem of building fabric by optimising four objectives; thermal performance, cost, planning efficiency and daylighting availability. To do this they used the concept of Pareto optimality with dynamic programming.

\[
\text{Cost Function } \int_{t_i}^{t_f} \left[ \sum_{j=1}^{N_z} \left( \alpha_{T_T} (\Delta T_j)^2 + \alpha_{f_A} (\Delta f)^2 + \alpha_{PMV} (\Delta PMV)^2 \right) + \alpha_{F_C} F_C^2 + \alpha_{F_H} F_H^2 + \sum_{j=1}^{N_z} \alpha_{f_T} f_T^2 + \alpha_{f_R} (f_R^2)^n \right] dt
\]

*where:*
- \( \Delta T \) = Difference in Temperature between zone and set point (\( z_j \) = Zone Ref)
- \( \Delta f \) = Difference in Air Flow Rate between zone and setpoint (N = Number of zones)
- \( \Delta PMV \) = Difference in Comfort levels between zone and setpoint
- \( \alpha \) = Specific Cost Function for each zone and Function
- \( F \) = Fuel Function for cooling coil (cc), heating coil (hc) and VAV boxes in zones (vj)
- \( f \) = Air Flow Rate for supply fan (sf) and return fan (rf)

Cost Function (House and Smith 1995)

It can be seen from the formation of some objective functions, that the optimisation problem can lend itself to the application of the use of multiobjectives the best example...
of this is the cost function shown in equation 2.1. The equation shows how each criteria were assigned predetermined weights to give an overall criterion for optimisation.

Many aspects of building construction have been optimised (e.g. mass, orientation, and window size). In some areas multiobjective optimisation has been utilised to give a selection of solutions. Although not extensively used in building optimisation, the concept of multi-criteria or multiobjective optimisation has been used to a greater extent in this area in comparison with HVAC optimisation.

Within building optimisation, when the operational cost is assessed it is necessary to apply some assumptions about how the building space is conditioned. The modelling of HVAC systems is well developed, with a number of standard models being used. Software is available for the sizing and selection of plant and assessment of component performance.

When analysing component performance the plant is affected by the way in which it is controlled. Supervisory control procedures and plant models used in this thesis were developed and evaluated previously by (Ren, 1997)

In conclusion, although multiobjective optimisation is beginning to be used to within building fabric design there has so far been no investigation into their application to the optimisation of HVAC systems control or selection. Specifically, the application of multiobjective to the problem of optimising more than one of these areas simultaneously has not been attempted.
Chapter 3

Multiobjective Optimisation

In many real world optimisation problems there are multiple measures of performance, cost, comfort etc., which should be optimised simultaneously. It is possible to optimise each separately however this rarely gives suitable solutions to the global problem. With single objective optimisation a single 'perfect' solution is obtained, this is rarely the case with multiobjective optimisation. Instead the multiobjective problems tend to be characterised by a group of alternative solutions each of which are considered equivalent. The aim is to present the decision maker (engineer) with the selection of alternatives, permitting the decision maker to make an informed choice. The purpose of these methods is to help the engineer to make the right decision with conflicting situations.

A significant proportion of research development and application in the field of optimisation is concerned with single objective optimisation, although most real world problems involve more than one objective. As these multiple objectives are often conflicting, there is normally more than one solution.

The topic of multiobjective optimisation has been studied extensively. This chapter aims to concisely identify the categories of different multiobjective optimisation. The main algorithms are briefly described, both in use and in terms of developing concepts. The specific operators available to the multiobjective algorithm used in this research are also examined.
There are many elements and features that make an optimisation problem, regardless of what is being optimised or how it is being optimised. The difference lies only with the interpretation and application of the optimisation procedure. Before the different multiobjective techniques are examined the factors that define the characteristics of an optimisation problem, together with the construction of any multiobjective optimisation problem are outlined. Methods for solving multiobjective optimisation problems are described in section 3.4 and 3.5.

3.1 Optimisation Problem Characteristics

Conventionally, optimisation problems can be defined through five categories of problem characteristics (Haupt and Haupt, 1998).

3.1.1 Function Characteristics

An optimisation algorithm minimises or maximises one or more objective functions subject to a number of constraint functions. The nature of the objective and constraint functions can influence the choice of optimisation algorithm. Where the functions are differentiable, then the problem may lend itself to an exact solution by a calculus based optimisation algorithm. However, since the objective and constraint functions used in this research are derived from a complex simulation, the gradients of the functions are not available and as such, the optimisation problems studied here can be classified as heuristic optimisation problems.

3.1.2 Single Parameter or Multiple Parameter

The number of parameters optimised can influence the choice of optimisation algorithm. Many algorithms exist for a single parameter, however, most problems have more than one parameter. The complexity of the optimisation increases with the number of parameters. The number of parameters to be optimised in the research is in the order of 200 making the problems studied here "large scale" in terms of the parameters.
3.1.3 Static or Dynamic

Solutions to static problems are independent of any other event or solution. Solutions to dynamic problems are normally a function of time, but do not necessarily have to be. Many of the building and HVAC optimisation problems are static. However, for fabric thermal storage systems the optimisation of a sequence of control setpoints is dynamic. That is, the optimum value of a setpoint in one time period is dependent on the solution for all the setpoints (and therefore building performance) in the previous time periods (Ren, 1997).

3.1.4 Discrete or Continuous

If the number of parameter values is finite, then the problem is discrete and the optimum will consist of a certain combination of parameter values. However, in the case of problems considered in this research, both discrete and continuous parameters exist so that the problem can viewed as a "mixed-integer" optimisation problem.

3.1.5 Constrained or Unconstrained

The range of parameter values is often restricted by simple bounds or constraint functions. The constraints can be formed as either equalities or inequalities. There are numerous ways of dealing with the constraints, each particular method often depending greatly on the underlying optimisation approach adopted. The constraints can act as constraint functions to limit an operation within the problem, as well as acting as bounds on the variables. Many multiobjective problems are unconstrained, although the problems studied in this research are highly constrained.

3.2 Multiobjective Optimisation Problem Formulation

There are three elements in the formulation of a multiobjective optimisation problem:

1. the problem variables;
2. the objective function;
3. the problem constraints.
CHAPTER 3: MULTIOBJECTIVE OPTIMISATION

3.2.1 Problem Variables

The optimisation problem variables can be discrete or continuous in nature. They represent the physically realisable parameters of the optimum solution(s). In this research, they are formed to represent the building construction, the size of the air-conditioning system components and the air-conditioning system control schedule. In this thesis, the problem variables are given by:

\[ X = (x_1, x_2, \ldots, x_n)^T \]

where \( X \) is a vector of \( n \) problem variables.

3.2.2 The Objective Functions

The objective function in single objective problems gives an expression, which is a measure of the optimality of the solution. In a multiobjective problems an objective function on its own does not give an expression of optimality; only all the objectives in conjunction are able to give a measure of optimality.

The multiobjective optimisation problem is, without loss of generality, the problem of simultaneously minimising or maximising the \( n^\text{th} \) components \( f_k (k = 1, \ldots, K) \), of a possibly non-linear vector function \( F \) of a general variable vector \( X \) in a universe \( \mathbb{U} \), where

\[ F(X) = (f_1(X), \ldots, f_k(X))^T \]

3.1

The problem usually has no unique, perfect solution, but a set of non-dominated, alternative solutions, known as the Pareto-optimal set (Ben-Tal (1980)).

With most multiobjective problems all the objectives are treated as minimisation problems. Any maximisation problems are inverted within the search to effectively make the search minimisation (Equation 3.3), however all objective manipulation is done using the normalised objective \( f(x) \) (Equation 3.2). With many optimisation problems, cost is an objective.

\[ f'(x) = \frac{f(x)}{F(X)_{\max} - F(X)_{\min}} \]

3.2
The objectives that we wish to be optimised are often termed ‘soft’ objectives; this is generally because in many multiobjective techniques the constraints are treated as objectives, because the constraints’ objectives have to be achieved before the solution is feasible they are termed ‘hard’ objectives.

### 3.2.3 Problem Constraints

The solution of any practical design problem may be constrained by a number of restrictions imposed on the decision variable. Most engineering design problems have constraints and with objective problems there are 2 possible places of application of the bounds.

1. Imposing bounds on the variables.
2. Linear or Non-linear constraints imposed within the design problem.

The variables are simply bounded to limit the problem to a range of interest. To ensure that the problem is modelled realistically then it is essential to impose constraints on the actual problem. For example, it is necessary to put an upper limit on the water velocity within the coil to limit the noise. The variable \( x_i \) is restricted between the lower \( l_i \) and upper \( u_i \) bounds.

\[
l_i \leq x_i \leq u_i \quad i = 1,2,\ldots,n
\]

With multiobjective optimisation, the objectives can be constrained. As there is often a number of objectives being optimised giving a large amount of information, goal restraints are imposed on the objective to limit this information to the areas of interest. Goal restraints are described in more detail in Section 3.4.4.
Both linear and non-linear constraints take the form of any of three types of constraints:

Equality Constraints \( h_j(X) = 0 \quad j = 1,2,\ldots,J \)

Inequality Constraints

\[
\begin{align*}
g_s(X) & \leq 0 \quad s = 1,2,\ldots,S \\
g_v(X) & \geq 0 \quad v = 1,2,\ldots,V
\end{align*}
\]

Where \( g \) is a real-valued function of a variable \( x \). The inequality may also be strict (\( < \) instead of \( \leq \)).

With multiobjective optimisation constraints normally fall into one of two categories:

- **Domain constraints** express the domain of definition of the objective function.
- **Preference constraints** impose further restrictions on the solution of the problem according to knowledge at a higher level.

### 3.2.4 Pareto Optimality

With single criterion optimisation the optimum is defined as the minimum optimum feasible solution. When there is more than one criterion to be optimised the notion of an optimal solution is replaced by a more generalised idea. One powerful concept in multiobjective design optimisation is known as Pareto Optimality.

Pareto optimality is a required measure of multiobjective optimisation, independent of whether the optimisation procedure uses Pareto optimality as a method of progression in the search. The concept of Pareto optimality defines the optimum solution for any multiobjective problem.

Practical problems are often characterised by several non-commensurable and often competing measures of performance, or objectives. Assuming a minimisation problem, dominance is defined as follows (Fonseca 1995):
CHAPTER 3: MULTIOBJECTIVE OPTIMISATION

Definition 3.1: (Pareto dominance). A vector of objective function values $F^*(X) = (X^*_1, \ldots, X^*_n)$ is said to dominate vector $F(X) = (X_1, \ldots, X_n)$ if and only if $F^*(X)$ is partially less than $F(X)$: $F^*(X) < F(X)$, i.e.,

$$\forall i \in \{1, \ldots, n\}, \quad X^*_i \leq X_i \land \exists i \in \{1, \ldots, n\} : X^*_i < X_i$$

3.6

Definition 3.2 (Pareto optimality) A solution $X^* \in UN$ is said to be Pareto optimal if, and only if, there is no $X \in UN$ for which $F(X) = (X_1, \ldots, X_n)$ dominates $F(X)^* = (X^*_1, \ldots, X^*_n)$.

Pareto-optimal solutions are also called efficient, non-dominated, and non-inferior solutions. The corresponding objective vectors are simply called non-dominated. The set of all non-dominated vectors is known as the non-dominated set, or the trade-off surface, or pay-off surface of the problem.

To maintain diversity a number of sharing and mating restrictions can be implemented, the most common of these are sharing and crowding techniques.

3.2.5 Sharing and Crowding

In dealing with multimodal functions (functions with more than one solution), simple GAs converge to a single peak (Goldberg and Richardson (1987)). Figure 3.1 shows a typical response of a simple genetic algorithm to a multimodal function. This tendency to converge is termed 'genetic drift' and can be defined as the convergence of a finite population in the absence of selection pressure, due to variance in the selection process (Mahfoud (1994)). In addition, mating and mutation may be less likely to produce individuals in certain regions of the trade-off surface than others (e.g. the extremes), causing the population to cover only a small part of it.
Deb and Goldberg (1989) compares that, faced with a similar problem in nature, stable sub-populations of organisms are formed by forcing individuals to share the available resources forming groups of individuals that are termed niches.

Although a number of the more defined approaches are available more specifically for the single-objective, multimodal functions the principles behind sharing and mating restrictions are often applied to form the basis for multiobjective problems.

**Crowding Techniques**

The original scheme was proposed by De Jong (1975) and termed 'crowding'; this worked by creating separate niches by replacing existing strings according to their similarity with other strings in an overlapping sub-population. The method was defined by two parameters, generation gap (G), and crowding factor (CF). The generation gap dictated the use of overlapping population model with only a proportion (G) of the population being permitted to reproduce in each generation, date crowding factor determines the size of the sub-population.

Mahfoud (1995) improved the standard crowding by introducing competition between children and parents of identical niches. Once crossover and mutation takes place, each child replaces the nearest parent if the fitness is higher. This results in two sets of tournaments (parent 1 against child 1, parent 2 against child 2) or (parent 1 against child 2, parent 2 against child 1).
Examples of less well practised niching techniques are clearing and restricted tournament selection. Although dissimilar approaches, both still implement the concept of using search spaces. Restricted Tournament selection uses the selection process of tournament selection instead of the original roulette wheel for generating the next population. Clearing is similar to fitness sharing except that it does not share the attributes between individuals in a sub-population, it attributes them only to the best member of the sub-population.

Sharing and Niching Restrictions.

Goldberg and Richardson (1987) defined the most commonly used sharing techniques. The technique they described used a sharing function to penalise an individual’s fitness, depending on the proximity of other individuals. The goal of fitness sharing is to distribute the population over a number of different peaks in the search space, with each peak receiving a fraction of the population in proportion to the height of that peak. The distance, $d_i$, is measured between the individuals either by comparing the distance in the decoded space or the encoded space.

When the decoded parameters are compared (phenotypic sharing):

$$d_{ij} = d(x_i, x_j)$$

where $x$ is the decoded variable.

Comparing the strings directly (genotypic sharing):

$$d_{ij} = d(s_i, s_j)$$

where $s$ is the individual binary string.

Although there are many sharing functions possible, and although work since Goldberg and Richardson (1987) has given many different ways of defining the parameters within the function, the actual sharing function definition (Equation 3.7) remains constant.
\[ Sh(d) = \begin{cases} 1 - \left( \frac{d}{\sigma_{share}} \right)^\alpha & \text{if } d < \sigma_{share} \\ 0 & \text{otherwise} \end{cases} \]

\( d = \) Distance between individuals \( \sigma_{share} = \) Sharing parameter \( \alpha = \) Power law sharing function (see figure 2.7).

**Sharing Function Definition**

Once a sharing function has been calculated for a distance between two individuals the sharing functions for that individual and the rest of the population are calculated in a similar manner and summed. A new fitness for that individual is calculated by penalising the old fitness function with the total sharing function (Equation 3.8).

\[
\text{New Fitness } (f'_i) = \frac{f_i}{m_i}
\]

\[
m_i = \sum_{i=1}^{N} Sh(d_{i,il}) = \sum_{i=1}^{N} Sh(d(x_i, x_{il}))
\]

**Generation of the New Fitness Function using Sharing**

\[ Sh(d) = \begin{cases} 1 - \left( \frac{d}{\sigma_{share}} \right)^\alpha & \text{if } d < \sigma_{share} \\ 0 & \text{otherwise} \end{cases} \]

\[ d = \text{Distance between individuals} \]
\[ \sigma_{share} = \text{Sharing parameter} \]
\[ \alpha = \text{Power law sharing function (see figure 2.7)} \]

**Figure 3.2**: Power law sharing functions
Figure 3.2 shows graphically how this can be increased or decreased to vary the shape of the fitness function. In most cases $\alpha = 1$ (Sareni and Krähenbühl, 1998). This results in a sharing function referred to as the triangular sharing function, and gives a linear relationship between $Sh(d)$ and $\sigma_{share}$ (Goldberg 1989). It can be seen that when $Sh(d) = 0$, $d/\sigma_{share} = 1$ when $d$ (distance) is at its maximum. This meant initially that $\sigma_{share}$ should be set at the maximum values of $d$, which ensures that all the population has some effect on the sharing function.

Deb and Goldberg (1989) gave a more formal approach to defining the parameter $\sigma_{share}$ by defining it in relation to the number of peaks (niches, $q$) and the number of variables/parameters ($p$).
In phenotypic sharing the $c_{\text{share}}$ defined as:

$$
\sigma_{\text{share}} = \sqrt{\frac{\sum_{k=1}^{p} (x_{k,\text{max}} - x_{k,\text{min}})^2}{2q}}
$$

*Phenotypic Sharing Parameter*

3.9

In genotypic sharing the $\sigma_{\text{share}}$ defined as:

$$
\frac{1}{2^l} \sum_{n=1}^{2^l} (l) = \frac{1}{q}
$$

*Genotypic Sharing Parameter*

3.10

$$
\sigma_{\text{share}} = \frac{1}{2} (l + z^* \sqrt{l})
$$

*Approximated Genotypic Sharing Parameter*

3.11

The genotypic sharing parameter is defined by the string length ($l$). Equation 3.9 can be approximated using a normal distribution especially for larger populations by Equation 3.10 (Deb and Goldberg, 1989). The normalised bit difference ($z^*$) corresponding to the fraction $\frac{1}{q}$ may be found from a normal distribution chart.

Deb and Goldberg (1989) compared the performance of the genotypic and phenotypic sharing techniques with the traditional crowding technique (proposed by DeJong (1975)) for a function with five equally sized solution points followed with a function with five size-diminishing solutions. They concluded from these examples that sharing techniques were better than the crowding technique as they found all the peaks described by the test functions. It was also observed that the genotypic sharing could not always maintain the sub-populations at the sub-optimal peaks. The use of crossover may produce offspring that do not belong to any curve. To guard against this and to improve on the overall performance they proposed a mating restriction.

Both Fonseca and Fleming (1993) and, independently, Horn and Nafpliotis (1993) implemented Goldberg’s niching and non-dominated ranking techniques and successfully applied the resulting algorithms to difficult open problems. The main differences in the
approaches lay with the selection techniques. Fonseca and Fleming (1995) implemented the principle of Pareto dominance with the original roulette wheel selection approach, while Horn and Nafpliotis (1993) used the general principle and applied them to the tournament selection method.

These sharing schemes are primarily for single objective problems and have limitations due to the fact that setting the dissimilarity threshold $\sigma_{\text{share}}$ requires a priori knowledge of how far apart the optima are and therefore the number of niches ($q$). For all individuals $\sigma_{\text{share}}$ is the same assuming therefore that all peaks are equidistant in the domain.

The major disadvantage is that all sharing methods mentioned here try to spread a population evenly over a set number of optimum, and sub-optimum solutions. In the case where there is more than one objective then there is no overall optimum or defined number of solutions. This means that for a multiobjective problem the number of niches ($q$) is undefined, making the calculation of the sharing function distorted. Fonseca and Fleming (1993) state that for multiobjective problems the ranking scheme forces the solution toward a polynomial equation based on the solution space formed by the minimum and maximum of all the objectives (Equation 3.12).

Estimate $\sigma_{\text{share}}$ by solving the $(q-1)$ order polynomial equation 3.12.

$$\frac{\prod_{i=1}^{K}(M_i - m_i + \sigma_{\text{share}}) - \prod_{i=1}^{K}(M_i - m_i)}{N\sigma_{\text{share}}} = 0 \quad \text{[for } \sigma_{\text{share}} > 0]\]$$

where; $K =$ No. of Objectives $\quad N =$ No. of Individuals in a Population $M_{\text{max}} =$ Maximum Value of Objective $n \quad m =$ Minimum Value of Objective $i$

**Calculation of Multiobjective Sharing Parameter**

3.12

The equation describes a method of calculating the sharing function that does not involve knowing the number of niches. Once calculated, the sharing parameter is used in the standard sharing function (Equation 3.7). As the sharing function is developed from
objectives, the distance \((d)\) between the individuals is defined by the difference in the objective, not the variables, as previously used.

**Kernel Density Estimation**

The fitness sharing method used in the MOGA was originally constructed for sharing between niches in SGA. The main difficulty in its application is deciding on an appropriate niche size, i.e. how close the individuals should be for degradation to occur. Density Estimation is used by statisticians, and is calculated in the same manner as niche counts except for a constant factor. Parallels are drawn in Table 3.1, (Fonseca and Fleming, 1995)

<table>
<thead>
<tr>
<th>Fitness Sharing</th>
<th>Kernel Density Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharing Function</td>
<td>Kernel Function</td>
</tr>
<tr>
<td>Niche size ((\sigma_{\text{share}}))</td>
<td>Smoothing parameter ((h))</td>
</tr>
<tr>
<td>Niche Count</td>
<td>Density Estimate</td>
</tr>
</tbody>
</table>

**Table 3.1**

The smoothing parameter is ultimately subjective but guidelines have been developed for certain kernels such as the standard normal probability density function and the Epanechnikov Kernel \((K)\)

\[
K_{\epsilon}(d/h_{\epsilon}) = \begin{cases} 
\frac{1}{2} cv_{K}^{-1}(K+2)[1-(d/h_{\epsilon})^2] & \text{if } d/h_{\epsilon} < 1 \\
0 & \text{otherwise} 
\end{cases}
\]

3.13

Where \(K\) is the number of decision variables, \(d\) is the Euclidean distance between the individuals. \(cv_{K}\) is the volume of an \(K\) dimensional sphere e.g. \(cv_{1} = 2\), \(cv_{2} = \pi\), \(cv_{3} = 4\pi/3\).

The smoothing parameter \(h_{\epsilon}\) is defined generally by Silverman (1992) (equation 3.14). However this can be simplified for \(n=1\) to 2.40 and for \(n=2\) to 2.49.
The Epanechnikov Kernel is the most commonly used with the MOGA employed here because of its links to the specific multiobjective optimisation and its proven application to this specific algorithm (Fonseca and Fleming (1995)). Within the application of the algorithm there is the option to use the more basic phenotypic sharing or alternatively, either the variable solution space or the objective space. This allows direct manipulation of the solution space if the search seems to be having difficulty in spreading in either of these areas.

**Mating Restrictions**

In conjunction with sharing functions mating restriction are often applied to manipulate the fitness of the individuals. Mating is used to stop the GA process of crossover producing a large number of invalid offspring. Deb and Goldberg (1989) used mating restrictions in the case of single objective multimodal functions to stop the individuals from separate peaks mating and producing individual strings that do not represent any peak. In the case of multiobjective optimisation a mating restriction is often applied to stop individuals from distant parts of the trade-off curve from mating and producing individuals that are dominated by, and disassociated from, the trade-off curve.

**3.2.6 Elitism**

Elitism is used to ensure that the best solutions in each population for each generation are maintained. This is often done as a sub-population which is updated and expanded with each successive generation. Some methods integrate this sub-population into the selection process for the next generation.

Elitism is a common operator for many multiobjective algorithms and, as such, Deb (2002) uses this to breakdown a review of evolutionary algorithms onto two distinct categories; non-elitist and elitist.
3.3 Methods of classification

Previously there have been many different algorithms developed for multiobjective optimisation, and consequently there have been many different attempts at classifying these algorithms into groups. The most encompassing is described by Hwang and Masud (1979) and later reiterated by both Miettinen (1999) and Deb (2001) who set out four separate groups that are based on the point in the optimisation at which the decision maker expresses preference in the choice of solution.

A no-preference articulation. No information about the importance of the objectives, but a heuristic is used to find a single optimal solution, therefore no attempt is made to find more than a single Pareto-optimal solution.

A priori articulation of preferences. The Decision Maker expresses preferences by combining the different objectives into a scalar cost function, ultimately making the problem single-objective prior to optimisation.

A posteriori articulation of preferences. The Decision Maker is presented by the optimiser with a set of candidate non-inferior solutions before expressing any preferences. The compromise solution is chosen from that Pareto optimal set.

A progressive articulation of preferences. Interactive decision making and optimisation take place between the Decision Maker and the Optimisation Process. At each step partial preference information is supplied to the optimiser by the Decision Maker which, in turn, generates better alternatives according to the information received.

Veldhuizen and Lamont (1998) uses this method of categorisation to group a number of different algorithms, then splits these categories down further. The priori techniques are broken down into 3 categories based on how the fitness and objectives are sorted and combined. The posteriori methods are categorised into 4 separate categories (these categorisations being based more on how the fitness is assigned).

Veldhuizen and Lamont (1998) also lists some examples of progressive (or Interactive) techniques. Both priori and posteriori methods may be used to the search portion of the decision making process. Interacting with the decision maker means that some of the non-optimal problems with just priori approach, and some of the extensive computation
involved with producing an overall optimal solution group, can be avoided. Both Veldhuizen and Lamont (1998) and Miettinen (2001) give some examples of this. Miettinen (2001) gives further examples and discussion on 'No preference methods' which are rare and, consequently, often omitted from the categorisation.

The simplest categorisation applicable here is whether the optimisation procedure is evolutionary or non-evolutionary. As evolutionary algorithms lend themselves to multiobjective optimisation because they possess the ability to optimise a population of solutions simultaneously. They are the focus of increasing amounts of research. However, in comparison, evolutionary algorithms are a recent development in multiobjective optimisation, as the problem of conflicting objectives has been around for a long time. Presented here are some of the more frequently used non-evolutionary approaches. A more comprehensive selection is summarised by Deb (2001) and termed 'classical' methods. Other more comprehensive studies are also given by Veldhuizen and Lamont (1998) and Ceollo Ceollo (1999).

3.4 Non-Evolutionary Methods

Most of the non-evolutionary methods combine the objectives into a single function are termed aggregated methods, (Ceollo Ceollo, 1999). Once these methods are combined to form a single aggregated solution then these are often treated using single objective optimisation methods. Combing requires accurate scalar information about the range and to an extent the behaviour of the objectives. As the aggregating solution requires input from the decision maker before the optimisation takes place, these are quite commonly regarded as priori processes. Meittinen (1999) states that as the methods can be used in a repetitive process often varying by the preferences applied to each of the objectives, a set of Pareto optimal solutions can be attained then it become a posteriori process.

3.4.1 Weighting Method

Weighting the objectives to obtain non-inferior solutions is the oldest multiobjective solution technique. In the very basic form of the weighting method each objective is assigned a weight depending on the decision maker's preference and the judged importance of each objective. Then the objectives are combined together to form a single equation for optimisation. In doing this the user is not obtaining a truly multiobjective
solution to the problem as the objectives are restricted by the decision makers judgement. In fixing the weights, the solution of the optimisation may converge on a point that is not the true optimum. The best way to overcome this is to vary the weights assigned to the objectives in an ordered procedure, and then take the best solution.

$$\text{Minimise } \sum_{i=1}^{K} w_i f_i(X)$$

Where $w_i \geq 0$ for all $i = 1, \ldots, k$ and $\sum_{i=1}^{K} w_i = 1$

The general principle of weighting the objective values can be applied deliberately to give some degree of preference articulation of the optimisation procedure in other optimisation methods (Fonseca and Fleming (1998)). A common example of this is life cycle cost in which economic weights are used to combine the capital and operating cost.

3.4.2 ε-Constraint Method

This method was first introduced by Haimes et al. (1971) and optimises one (preferably the most important) objective and the remaining objectives are treated as constraints by bounding them to an acceptable level $\varepsilon_i$.

$$\text{Minimise } f_m(X)$$

Subject to $f_i(X) \leq \varepsilon_i$ for $i = 1, 2, \ldots, M$ and $p \neq m$

The optimisation is repeated for different values of $\varepsilon_i$. The search is stopped when the decision-maker finds the solution(s) acceptable. This method has the advantage of providing separate Pareto optimal solutions for different constraint values, however the constraint values have to be chosen carefully to lie within the min/max of the objective. If the bounds $\varepsilon_i$ are too small they have to be relaxed.

3.4.3 Weighted Metrics Method

With this method the distance between a reference point and the feasible objective region is minimised. The solution depends heavily on the value chosen for $p$. 
Minimize $\left(\sum_{i=1}^{d} w_{i} | f_{i}(X) - z_{i}^{*} |^{p}\right)^{1/p}$

3.17

For large values of $p$ it becomes a matter of minimising the largest distance to the reference point. For a value of $p = 2$ then the problem is effectively measuring the Euclidean distance from the solution in each of the objectives to the ideal reference point $z^{*}$. For $p=\infty$ the problem has a specific name of the weighted Tchebycheff problem (equation 3.18)

Minimize $\max_{i=1}^{e} w_{i} | f_{i}(X) - z_{i}^{*} |$

3.18

Deb (2001) states that the Tchebycheff metric guarantees a Pareto optimal solution when $z^{*}$ is a utopian vector. As objectives take on different magnitudes it is advised that the objectives be normalised. The ideal value of $z^{*}$ is defined by independently optimising each objective. Both Meittinen (2001) and Deb (2001) document improvements to the algorithm in efficiency and avoiding weakly coupled non-dominated solution sets.

3.4.4 Goal Programming Method

Goal programming is one of the first methods expressly developed for multiobjective optimisation. The decision maker assigns targets or goals that they wish each objective to achieve. These are then incorporated into the problem as additional constraints, selected so that they are not achievable simultaneously. There are several variants of this method; the weighted approach and the lexicographic approach as well as min-max goal programming seem to be some of the most common variations, (Deb, 2002).

3.5 Evolutionary Approaches

Evolutionary Algorithms (EA) refer to a number of search and optimisation algorithms inspired by the process of natural evolution. There are currently four recognised evolutionary approaches, which are:
CHAPTER 3: MULTIOBJECTIVE OPTIMISATION

1. Genetic Algorithms: The most common form of evolutionary algorithms, which derives its behaviour from some of the mechanisms of evolution in nature.

2. Genetic Programming: Program evolves, varying the string length, rather than just the variables.

3. Evolutionary Programming: Is similar to GA, but instead places emphasis on the behavioural linkage between Parents and their offspring.

4. Evolutionary Strategies: It employs real-coded parameters and, in its original form, it relied on mutation as the search operator and a population size of one.

Within the evolutionary algorithm there is inherently a population of solutions. This lends itself to the posteriori preference decision making where a selection of optimal solutions is presented to the decision-maker at the end of the optimisation process.

There are two primary evolutionary approaches; repetitive application of a single objective EA and the more multiobjective approach where the population is assessed as a whole and a number of solutions are worked simultaneously.

Evolutionary algorithms are generally based on the manipulation of a population of solutions. Each solution is a computer encoding of the variables that form the solution called a chromosome. To progress the search towards the optimum each solution is assessed and assigned a measure of fitness. One such way of assigning fitness is using the principle of Pareto optimality (section 3.2.4). Each progression from one population to the next is termed a generation. A number of operators are used to ensure that the search progresses and develops from the first population; the normal operators are mutation and crossover. These standard operators and basic encoding are described in the context of the simple basic genetic algorithm in Appendix A.

The type of search is characterised by the method employed to assign fitness, and how the fitness is used to progress the search.
Summarised in Table 3.2 are a number of evolutionary approaches. These are just a small sample, but represent either marked changes in algorithm or a commonly applied methodology. The following abbreviations are used.

<table>
<thead>
<tr>
<th>For Pareto,</th>
<th>NP</th>
<th>No concept of Pareto optimality is utilised</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>Pareto is utilised</td>
</tr>
<tr>
<td>For Elitism,</td>
<td>NE</td>
<td>No elitist preservation on optimal solutions is used where</td>
</tr>
<tr>
<td></td>
<td>E</td>
<td>Indicates elitism is used.</td>
</tr>
<tr>
<td>For Chromosome,</td>
<td>BC</td>
<td>Binary coding of the chromosome string</td>
</tr>
<tr>
<td></td>
<td>DC</td>
<td>Diploid coded chromosome, which has a recessive and dominant selection of the chromosome.</td>
</tr>
<tr>
<td>Name</td>
<td>Author and Date</td>
<td>Description</td>
</tr>
<tr>
<td>------------------------------------------</td>
<td>-----------------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Vector Evaluated Genetic Algorithm (VEGA)</td>
<td>Schaffer (1985)</td>
<td>GA population at every generation is split into M equal sub-populations (m=no. objectives). Each sub-population is assigned the fitness by evaluating the fitness for each objective in turn.</td>
</tr>
<tr>
<td>Vector Optimised Evolutionary Algorithm (VOEA)</td>
<td>Kursawe (1990)</td>
<td>Similar to VEGA, but a Diploid Chromosome is used therefore a solution has a dominant and recessive string, the solution is given a fitness based on a weighted sum of both the dominant and recessive objective The process in done in m steps using a user defined probabilistic vector to select objective (this may vary with each generation). A swap operator exchanges the recessive and dominant genes with a selection probability of 1/3</td>
</tr>
<tr>
<td>Multiobjective Genetic Algorithm (MOGA)</td>
<td>Fonseca and Fleming (1993)</td>
<td>This is not detailed further as it is the main emphasis in the study the MOGA is described in detail. (section 3.6)</td>
</tr>
<tr>
<td>Name</td>
<td>Author and Date</td>
<td>Description</td>
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<td>-------------------------------</td>
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</tr>
<tr>
<td>Weighted Genetic Algorithm</td>
<td>Hajela and Lin (1993)</td>
<td>Similar to weighed sum, as each objective is provided with a weight, however the each individual in the population is assigned a different weight vector. The GA string represents the decision variables as well as the associated weights</td>
</tr>
<tr>
<td>Sharing Function Approach</td>
<td></td>
<td>Sharing function is used to maintain diversity for the weight vectors</td>
</tr>
<tr>
<td>Vector Function Approach</td>
<td></td>
<td>Similar manner VEGA. The weight vectors are applied to the population, from this the best members are grouped together in a sub-population. This is associated with the weighting vector, mutation/crossover and selection are done from the sub-population (repeated for each weight).</td>
</tr>
<tr>
<td>Niched Pareto Genetic Algorithm</td>
<td>Horn et al. (1994)</td>
<td>Uses Pareto domination binary tournament selection. A comparison set of ((N_{dom})) individuals are picked random. 2 random individuals picked from pop' which are then compared to comparison set for dominance, if one is non-dominated and the other isn't then non-dominated is selected, if the same then the solution with the lowest niche count is selected</td>
</tr>
<tr>
<td>Name</td>
<td>Author and Date</td>
<td>Description</td>
</tr>
<tr>
<td>-------------------------------</td>
<td>----------------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Non-Dominated Sorting Genetic Algorithm</td>
<td>Scrinivas and Deb (1995)</td>
<td>This varies from the simple GA only in its selection operator. Population is ranked based on (Pareto) domination, all non-dominated are placed in one group, a dummy fitness is assigned based on population size. Fitness of these individuals are shared with dummy fitnesses (original in variable space, objective space is an option). This group is then removed from pop' and processes is repeated with remaining population.</td>
</tr>
<tr>
<td>Random Weighted GA</td>
<td>Murata and Ishibuchi (1995)</td>
<td>WGA, the each solution in the population is assigned a weight vector and the sum of the weighed objective forms the solutions fitness. A new population is created using proportionate selection, crossover and mutation. Before accepting pop, a random proportion of population is replaced solutions chosen from an external randomly created population.</td>
</tr>
<tr>
<td>Strength Pareto EA</td>
<td>Zitler and Theile (1998)</td>
<td>External fixed size population stores the non-dominated solutions. Newly found non-dominated solutions are compared with existing external population and resulting non-dominated are kept. A clustering technique is utilised to limit the number of solutions in the final set. The fitness of the solutions is based on how many solutions it dominates.</td>
</tr>
<tr>
<td>Name</td>
<td>Author and Date</td>
<td>Description</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-----------------</td>
<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Elitist Non Dominated Sorting GA (NSGA II)</td>
<td>Scrinivas and Deb (1994)</td>
<td>2 populations exist the parent and offspring, which are combined sorted based on domination, then the new population is filled by the lowest rank solutions, then the next rank until it is full, (all remaining solutions are deleted). The solutions in the last set used to fill the new population are chosen based on a niching strategy.</td>
</tr>
</tbody>
</table>

Table 3.2: Multiobjective Optimisation
3.6 Multiobjective Genetic Algorithms

Multiobjective Genetic algorithms (MOGA's) combine the multiobjective Pareto ranking approach for assigning fitness with the genetic algorithm. This has an ability to deal with large numbers of variables and solution sets. A genetic algorithm will inherently try and converge to one solution. In a true multiobjective problem there will be many solutions forming the 'trade-off' curve. In early GA the same problem was experienced in problems that had more than one solution, i.e. the problems were multimodal.

3.6.1 Fonseca and Fleming's MOGA

Fonseca and Fleming (1993) identify a MOGA, which in most documentation is called MOGA. From now on MOGA strictly applies to the algorithm utilised in the study, which is heavily based on the principles put forward by Fonseca and Fleming.

As MOGA has been around since 1995 and in principle one of the first to actually use the principle of Pareto optimality, much comparison of the MOGA has taken place often against the authors own algorithm which have been well documented. Within the literature the main advantages and disadvantages of the MOGA are:

Advantages

- The simple fitness assignment.
- Niching allows spread of the solution in the objective space (can also be easily applied to the variable space).

Disadvantages

- The method of fitness assignment means there can be a bias towards some solutions.
- Although diversity can be maintained in the objective space, it can not be maintained in the parameter set. However in this study a sharing parameter can be applied to either the objective space or the parameter (variable) space.
The MOGA is as good as any of the multiobjective techniques available (Velduizen and Lamont 2000). Most of the disadvantages outlined in documentation can be overcome with careful use and selection of niching operators.

Multiobjective genetic algorithms make full use of the general multiobjective algorithms operators, however in their approach they have a number of specific operations that permit the manipulation of the algorithm and aid the algorithm to progress through the search space.

3.6.2 Pareto Ranking

Multiobjective algorithms all use concept of Pareto optimality (section 3.2.4) in many ways, however there is a specific methodology for utilising the principle as part of the search algorithm.

Pareto based fitness assignment was first proposed by Goldberg (1989) as a means of assigning equal probability of reproduction to all non-dominated individuals in a population. The method proposed consisted of ranking all the solutions that were not dominated by any others, 1. This set was then removed from the ranking procedure and the next sets of non-dominated solutions were assigned the rank 2, and so on.

Fonseca and Fleming (1993) proposed a different method of ranking the individuals using Pareto where an individual's rank corresponded to the number of individuals it dominated, therefore rank 0 being ideal. Fitness is assigned inverting the Pareto score. An example of this ranking method for two objective functions is shown in Figure 3.3.
As Cohon (1978) points out, this trade-off information does not necessarily make the design decisions any easier. D'Cruz et al (1983) state that the main disadvantage is the computation burden that Pareto optimisation adds to a problem, however this statement is compared to a single objective optimisation problem and not other multiobjective techniques.

Pareto is method of combining the objectives in a fair unbiased way, however it is not an optimisation procedure and requires integrating with an optimisation method (for example dynamic programming (D'Cruz et al (1983)), or genetic algorithms to form multiobjective genetic algorithms).

### 3.6.3 Goal Attainment Method

The goal attainment method described here was first demonstrated for inclusion in multiobjective ranking by Fonseca and Fleming (1993). The notion of goals puts what is effectively constraints on the objective functions. This is not to be confused with constraints discussed previously which are general physical limitations on the variables or the systems they are being used to model. Putting limits on the objective functions allows the decision maker to concentrate the search on one area of the trade-off surface. The method suggested by Fonseca and Fleming (1993) was to degrade the Pareto values
once they fall outside the goal value. The Pareto score for any individual that falls outside of a specific goal function is the number of individuals that have a lower objective value for the specific objective value that the goal value applies to, as demonstrated in Figure 3.4.

![Diagram of Goal Attainment Method](image)

**Figure 3.4: Goal Attainment Method**

### 3.7 Discussion

Optimisation problems can be characterised by the nature of their objective and constraint functions, and by the problem variables. In turn, these characteristics necessarily dictate the form of optimisation algorithm used to solve the optimisation problem (the simpler the optimisation problem, the simpler the algorithm required to solve the problem). The multiobjective optimisation problems considered in the research can be said to be multi-parameter having mixed continuous and discrete problem variables. The objective and constraint functions are non-linear and are derived from a complex simulation of the building performance (and as such, the derivatives of the functions are not available). The highly constrained nature of the problems and the large number of problem variables means that it is difficult to obtain an initial feasible solution with which to seed the optimisation (suggesting the need for a search that can begin from a randomly generated solution). Further, the problems considered here are also dynamic.
optimisation in nature. It is shown in this research that the most suitable class of optimisation algorithm for solving these large scale, highly constrained, non-linear dynamic optimisation problems are those based on genetic, or more generally, evolutionary algorithms.

Fonseca and Fleming's MOGA was chosen for this study because it was well documented and tested. It was shown to be as good as any other multiobjective algorithm available at the time of starting the study, however it is necessary to use a number of operators to enhance the performance of the MOGA. The actual operators used and specific alterations to the basic MOGA that were required for the application to an optimisation problem are documented further in Chapter 5.
Chapter 4

Example Simultaneous Optimisation of Building Thermal Design

The optimisation process takes place on a model of a room that is air-conditioned. The model is split into two parts, the model of the thermal response and performance of the building envelope, and the model of the plant operation to condition that space.

The model being used was constructed by Ren (1997) and is a mid level office in a multi-storey building, with only one external wall. The optimisation is taking place using a discrete selection of building constructions, plant sizes, and control schedule.

This chapter details the construction of that model and characteristics it portrays.

4.1 Building Thermal Model

The room is 6m by 7m by 2.8m with a single window on the south facing facade (Figure 4.1).
4.1 The Hollow Core Ventilated Slab Model

When examining the performance of the thermal storage, it is possible to add additional hollow core slabs to the floor and ceiling to imitate a Termodek™ system.
The well validated lump-parameter model (Ren and Wright 1998) is combined with a lumped parameter slab model. The hollow core ventilated slab is modelled in two sections to evaluate the heat transfer to the zones above and below the slab (Figure 4.3). The thermal capacitance is given for the lower and upper half of the slab, as \( C_u \) and \( C_l \) respectively acting on the central node point of each section. The resistance from the node point to the average ventilated slab air temperature \( (\bar{T}_{av}) \) is given by \( R_{av} \) and \( R_{al} \). \( R_{av} \) and \( R_{al} \) are the resistance from the node to the internal to the upper and lower surface temperatures \( T_{av} \), \( T_{al} \).

An average temperature for the air in the slab is used (Equation 4.1), where the length of the ventilated core is \( L \), the mass temperature is \( T_m \), \( U \) is the transmittance per length of hollow core, and \( C_{pa} \) is the air capacitance.

\[
T_{av} = T_m + \frac{(T_{sl} - T_m)}{\gamma L} (1 - e^{-\frac{L}{\gamma L}}) \quad \dot{m}_a = \text{air mass flowrate}
\]

\[
\gamma = U / \dot{m}_a C_{pa} \quad T_m = \text{mass temperature}
\]

Both lumped parameter models are combined to give the overall building model for the hollow core ventilated slab model, additional resistances are added to reproduce radiant heat transfer between the surfaces within the space (Ren and Wright, 1998). In the overall model, the ventilated slab is applied to the ceiling and floor, the lower and upper slab temperatures of each application form the surface temperatures within the zone respectively.

Figure 4.3: The Lumped Parameter Model of the Ventilated Slab
To obtain a load suitable for a building thermal system to operate, the system is designed to meet the design load for five identical zones.

### 4.1.2 Model Parameters Values

The main building model parameter values are the construction of the internal and external walls, and the temperatures surrounding these walls. The building has three possible building constructions, Light, Medium and Heavy Weight (Figure 4.4). The medium and heavy weight constructions share the same internal wall construction (Figure 4.5), which leads to very similar zone properties (Table 4.3).

![Figure 4.4: External Wall Constructions](image)

![Figure 4.5: Internal Wall, Ceiling and Floor Constructions](image)
CHAPTER 4: EXAMPLE SIMULTANEOUS OPTIMISATION OF BUILDING THERMAL DESIGN

<table>
<thead>
<tr>
<th>Resistance (m²K/W)</th>
<th>Light Weight</th>
<th>Medium Weight</th>
<th>Heavy Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>0.2149</td>
<td>0.3273</td>
<td>0.4597</td>
</tr>
<tr>
<td>Internal Wall</td>
<td>0.3806</td>
<td>1.6206</td>
<td>1.6206</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capacitance (Wm²K⁻¹)</th>
<th>Light Weight</th>
<th>Medium Weight</th>
<th>Heavy Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>1.48E+05</td>
<td>3.11E+05</td>
<td>5.51E+05</td>
</tr>
<tr>
<td>Internal Wall</td>
<td>2.49E+04</td>
<td>2.09E+05</td>
<td>2.09E+05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Thermal Resistance (KW⁻¹)</th>
<th>Light Weight</th>
<th>Medium Weight</th>
<th>Heavy Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>3.6108</td>
<td>6.3944</td>
<td>7.7227</td>
</tr>
<tr>
<td>Internal Wall</td>
<td>12.8284</td>
<td>63.5267</td>
<td>63.5267</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total Thermal Capacitance (W/K⁻¹)</th>
<th>Light Weight</th>
<th>Medium Weight</th>
<th>Heavy Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>External Wall</td>
<td>2.48E+06</td>
<td>5.22E+06</td>
<td>9.26E+06</td>
</tr>
<tr>
<td>Internal Wall</td>
<td>9.78E+05</td>
<td>8.19E+06</td>
<td>8.19E+06</td>
</tr>
</tbody>
</table>

Table 4.1: Thermal Properties of Different Wall Weights

Table 4.1 shows the capacitance and resistance for the internal and external walls, not accounting for glazing, the ceiling and floor constructions are not included, as they remain constant. Comparing in Table 4.1 the thermal resistance and capacitance of the constructions, the similarity of the heavy weight and medium weight construction, and difference in the lightweight construction can be seen.

There are three possible areas of glazing; 10%, 20% and 30% with two types of glazing; clear or low emissivity.

The occupied internal gains are set at 15W/m² for lighting, 20W/m² equipment, 30W/10m² latent occupant gains and 70W/10m² sensible gains (CIBSE). The gains are assumed to remain constant throughout the occupied period. The occupied period is from 8:00 in the morning to 17:00.

4.2 HVAC System Model

The thermal systems are a 100% fresh air heating, ventilating and air-conditioning (HVAC) system. The system consists of a heating coil, cooling coil and supply fan, with some heat recovery taking place with the use of a regenerative air to air heat exchanger, (Figure 4.6).

The dynamic response of the plant is not required in this study therefore the components has been simulated using established steady state models. The fan model is based on a
non-dimensional polynomial curve fit based on manufacturers' data for the centrifugal fan (Wright, 1991).

The heat exchanger has a fixed effectiveness of 0.85 is used to calculate the heat recovered from the exhaust air, regardless of airflow rate.

The pressure drop due the ductwork has been modelled as a simple quadratic resistance. The quadratic resistance of the coils is a function of the number of rows. The heat recovery device has been modelled on having a fixed effectiveness.

The coil models are based on the ideal thermodynamic response of the water to air heat exchanger. The effectiveness of the coil is calculated using equations for counterflow heat exchangers. The overall conductance for the coils has been taken from Holmes, 1982. The latent heat transfer in the coils was not modelled.

The models have been simplified however they retain sufficient details and characteristics to allow sufficient evaluation of the optimisation procedure.

4.2.1 Model Parameters Values

There are four components of the HVAC plant; the heat exchanger, the fan and the cooling and heating coils.

The heater exchanger is a fixed construction, the only options available to the optimisation is to turn the heat exchanger ON or OFF.

Both of the coils can be varied in construction, the parameters available are shown in Table 4.2.

There are three fan diameters available to the model 0.38m, 0.445m and 0.508m, and most operate between fixed constraints (4.3.3).

As the plant size can be optimised then the size parameters become problem variables, these are shown in more detail in section 4.3.2.
CHAPTER 4: EXAMPLE SIMULTANEOUS OPTIMISATION OF BUILDING THERMAL DESIGN

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Discrete Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width (m)</td>
<td>0.5</td>
<td>2.5</td>
<td>0.05</td>
</tr>
<tr>
<td>Height (m)</td>
<td>0.5</td>
<td>2.5</td>
<td>0.05</td>
</tr>
<tr>
<td>Rows</td>
<td>1</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>Water Circuits</td>
<td>2</td>
<td>129</td>
<td>1</td>
</tr>
<tr>
<td>Water Flow rate (kg/s)</td>
<td>0</td>
<td>10.23</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 4.2: Coil Parameters

4.2.2 HVAC Control System Mode

The control system is an open loop to the zone, as the optimisation seeks to achieve the supply temperature in each hour so that the problem constraints are met. The controller has been chosen so that it can be applied to both ventilated floor slabs and conventional buildings. The controller has two set points one for the supply temperature setpoint (θSP) and a second for supply mass setpoint (MSP). The control supervisor determines whether the heat recovery device should be operational (C2) and whether active heating or cooling is required (C3 and C4) in order to meet the setpoint. (Figure 4.6)

The supply airflow rate is controlled through a separate fan speed controller (C1).

![Figure 4.6: Air Conditioning System Layout](image-url)
Supervisory control is achieved by optimising the airflow rate and the temperature over the entire 24 hours subject to the time varying room and ambient conditions. Most of the optimisation of the plant is concerned with minimising energy cost and meeting the supply setpoints. The setpoint schedule is based on perfect prediction of the thermal disturbances acting upon the building.

4.3 Optimisation Problem Formulation

There are three problem configurations:
1. Optimisation of the control setpoints; building and plant configuration fixed.
2. Optimisation of the control setpoints and plant size; building selection is fixed.
3. Optimisation of the control setpoints, plant size and building selection.

Throughout the different configurations the initialisation of the optimisation remains the same and the zone and plant options remain constant. To achieve the boundary conditions it is necessary to run a pre-processor to set-up the input files for the optimisation run. Re-running the model five times for each design day initialises the model. The results of this initialisation from the last run are used in the optimisation.

4.3.1 Design Days

There are 3 design days used these are 3rd March, 13th April and 14th May, these are the coldest and hottest based on average temperatures for the year 1994. The minimum number of design days that can be used is two, to enable plant size however to gain an appreciation of transient days a swing season design day is also implemented. However, a limit of three is imposed to keep the codes operation to be computationally effective and to maintain the amount of data to analysis at a reasonable level.

The external temperatures for the design days are shown in Figure 4.7. The external temperatures form the boundary temperatures for the external wall, whilst the daily temperature for the adjacent zones is given by the boundary temperature.

The solar gains on the space are modelled by accounting for the short-wave heat gain, proportioned to each surface dependant on their relative surface areas. These are formed
using the solar irradiance, global diffuse and direct (Figure 4.8, Figure 4.9, and Figure 4.10)
The three-optimisation days are chosen for the max cooling and heating loads with one swing day in-between. Therefore, it is fair to say that the winter and summer design days effect the choice of plant and building and therefore the capital cost. The swing day has little or no effect on the capital cost.

The operational cost is the summation of the cost of meeting the requirements for conditioning the space for each design day. Directly within the model there is no linkage between the days. To ensure that the performance for that day is stable it is simulated for 5 days, but those days are a repeat of the same day not five consecutive days in a year.

The loads on the plant determine the operational cost. These depend on the setpoints that have to be met by the plant the extent of these is determined by the building performance. When the building and plant are fixed then the operational cost is completely separate for all three design days, however when the building and plant optimisation are operational a link between them may be present.
The comfort is also determined by the setpoints, and in the same way is completely separate for each design day. This is investigated further when examining both the performance of the algorithm and the model itself. There is no direct link inherent in the model between the design days, any link is indirect and a consequence of plant and building optimisation.

4.3.2 Problem Variables

In order to ensure that the model is capable of controlling the system when the ventilation slab is used the system has been given 24 setpoints, one for each hour for both the supply air temperature $\theta_{sp}$ and the air mass flow rate $M_{sp}$.

The plant is kept operational during occupancy 8:00 till 17:00, as there is a requirement for minimum fresh air to the space. Out of occupancy there is no longer a minimum requirement and it is possible for the plant to be turned off. This is achieved by having another 15 control variables for ON/OFF control of the plant during unoccupied periods. This gives a total of 63 control variables for each design day. A total of three design days winter, summer and swing means that there is a total of 189 control scheduling variables.

For problems requiring plant sizing there are 11 additional variables controlling the height, width, number of rows, number of circuits (as identified in Table 4.2) and the mass flow rate for both the cooling and heating coils, and well as one variable for the fan size. The three fan sizes are identified by the integers 0, 1 and 2.

<table>
<thead>
<tr>
<th></th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Discrete Increment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Building Weight</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Glazing Type</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Glazing Area</td>
<td>0</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.3: Building Problem Variables
For problems requiring building sizing there are 3 additional variables controlling the choice of building construction, window type, and window area, which are identified in the optimisation by integers. (Table 4.3)

### 4.3.3 Design Constraints

To model the system realistically it is necessary to apply constraints to the problem. There are three discomfort constraints to ensure the comfort of the occupants, within multiobjective optimisation these often form a search objective and the construction of this objective is detailed in section 4.3.7. The other constraints are concerned with limiting the plant operation to within reasonable operating bounds.

The face velocity of the coil is restricted to be below 1.8ms⁻¹, to prevent noise. The water velocity is limited to help prevent excessive corrosion. The last constraint on the coil is to ensure that the number of coil passes results in both the water circuit entrance and exit being the same side of the coil (Table 4.4).

<table>
<thead>
<tr>
<th>Constraint Function</th>
<th>Constraint Form</th>
<th>Constraint Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Velocity (m/s)</td>
<td>≤</td>
<td>2.5</td>
</tr>
<tr>
<td>Face Velocity (m/s)</td>
<td>≤</td>
<td>1.8</td>
</tr>
<tr>
<td>Water Circuits</td>
<td>=</td>
<td>0.0</td>
</tr>
<tr>
<td>Fan Speed</td>
<td>≤</td>
<td>1.0</td>
</tr>
<tr>
<td>Fan Speed (-)</td>
<td>≥</td>
<td>0.0</td>
</tr>
<tr>
<td>Volume Flow (-)</td>
<td>≤</td>
<td>1.0</td>
</tr>
<tr>
<td>Volume Flow (-)</td>
<td>≥</td>
<td>0.0</td>
</tr>
<tr>
<td>Temperature Setpoint (-)</td>
<td>=</td>
<td>0</td>
</tr>
<tr>
<td>Flow Setpoint (-)</td>
<td>=</td>
<td>0</td>
</tr>
<tr>
<td>PPD (summer)</td>
<td>&lt;</td>
<td>10%</td>
</tr>
<tr>
<td>PPD (swing)</td>
<td>&lt;</td>
<td>10%</td>
</tr>
<tr>
<td>PPD (winter)</td>
<td>&lt;</td>
<td>10%</td>
</tr>
</tbody>
</table>

Table 4.4: Problem Constraints
The fan model has limits imposed as it generated from manufacturer's data, therefore making it unwise to extrapolate beyond this data, (Wright, 1998). To prevent this there are constraints for lower and upper boundaries of the fan speed and volume flow rate. These have been normalised so that they have to fall within the constraints of 0 to 1.

The last two constraints are imposed to ensure that the HVAC plant has enough capacity to meet the requirements dictated by the air temperature and flow rate setpoints.

There are a total of 12 possible plant constraints, 6 coil performance constraints, 4 fan envelope and 2 setpoint constraints, and when not being optimised an additional 3 constraints limiting the discomfort in each seasons operation.

4.3.4 Objective Function Formulation

Although, within the multiobjective genetic algorithm (MOGA) it is possible to have any number of constraints forming problem objectives, here three main objectives are focused on. The first is the operational cost over the three sample days, the second is a measure of the occupants comfort on the same sample days, and the last is the capital expenditure.

4.3.5 Operation Cost Objective

The plant loads are calculated for the three different days, two of which are representative of the extremes of yearly weather conditions for which plant sizes are typically sized on. The plant model gives an hourly breakdown of the loads for the cooling and heating coils and the fan power for the three individual days.

The required energy input is calculated assuming the Coefficient of Performance (COP) of 4.0 for the chiller and efficiency of 0.9 for the boiler. The part loads are calculated based on a maximum load for the three days and the part load models described by Kreider and Rabl (1994).

\[
\text{Part Load Ratio (PLR)} = \frac{\dot{Q}_{\text{part}}}{\dot{Q}_{\text{full}}}
\]
Shaft Power Input ($\dot{W}_t$) = \frac{\dot{Q}_{\text{full}}}{COP_{\text{full}}} \left[ A + B(PLR) + C(PLR)^2 \right]

COP_{\text{full}} = \text{chiller COP at full-capacity point}
A, B, C = \text{chiller-specific part-load coefficients}

The chiller specific part load coefficients are taken as $A = 0.023, B = 1.429, C = -0.471$ as stated for Reciprocating compressors (Kreider and Rabl).

Input Energy ($\dot{Q}_t$) = \dot{Q}_{\text{full}} \left[ A + B(PLR) + C(PLR)^2 \right]

Full Design Load ($\dot{Q}_{\text{full}}$) = \frac{\dot{Q}_{\text{max}} \text{ (Max peak load)}}{Efficiency_{\text{full}}}

The boiler part-load characteristics are $A = 0.1, B = 1.6$ and $C = -0.7$. (Kreider and Rabl)

After the specific hours part loads have been calculated the cost for the day is based on the energy tariffs\(^1\) for gas and electricity. The fan and the chiller are assumed to be entirely electrically supplied whilst the boiler is converted to a cost using gas.

4.3.6 Capital Cost Objective Function

The capital cost is comprised of two sections, the cost purchase and installation of the buildings conditioning system and the building fabric.

Plant Costs

When sizing the plant it is possible to have the following different fan and coil setups that effect the capital cost.

---

\(^1\) Tariff structure and value based on continuous supply rate for Loughborough university for the year 2000
Both the fan and coil costs are based on data previously used with a fixed inflation cost added to them to ensure they are comparable to today's cost. The cost of the coil is shown graphically for a coil construction with one row in Figure 4.11.

Supply Fan: 3 possible diameters 0.38, 0.445 and 0.508m
Coils (heating or cooling): 41 possible widths 0.5 to 2.5 in increments of 0.05m

The number of rows possible ranges from 1 to 10.

Building Costs

The options for building cost are between the 3 building types, 2 glazing types and the percentages of glazing. As there are fixed values associated with each component it is possible to discretely calculate the total building cost for each combination. (Table 4.5).
CHAPTER 4: EXAMPLE SIMULTANEOUS OPTIMISATION OF BUILDING THERMAL DESIGN

<table>
<thead>
<tr>
<th>Glazing (% External Surface Area and Type)</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear</td>
<td>LowE</td>
<td>Clear</td>
<td>LowE</td>
</tr>
<tr>
<td>Heavy Weight</td>
<td>5459.988</td>
<td>5557.207</td>
<td>5586.559</td>
</tr>
<tr>
<td>Medium Weight</td>
<td>5369.164</td>
<td>5498.383</td>
<td>5505.277</td>
</tr>
<tr>
<td>Light Weight</td>
<td>5924.941</td>
<td>6052.160</td>
<td>6049.114</td>
</tr>
<tr>
<td>Heavy Weight</td>
<td>5535.156</td>
<td>5662.375</td>
<td>5661.727</td>
</tr>
<tr>
<td>Medium Weight</td>
<td>5444.332</td>
<td>5571.551</td>
<td>5580.445</td>
</tr>
<tr>
<td>Light Weight</td>
<td>6000.109</td>
<td>6127.328</td>
<td>6124.282</td>
</tr>
</tbody>
</table>

Table 4.5: Building Capital Costs (£)

4.3.7 Thermal Comfort Objective Function

A level of comfort is calculated using Fangers' traditional 'Percentage Mean Vote' (PMV) and 'Percentage of People Dissatisfied' (PPD) and as described in ISO 7730

\[
PMV = (0.303e^{-0.0034M} + 0.028)[(M - W) - H - E_{vk} - E_{rem}]
\]

Where \( M \) is the metabolic rate of sedentary activity (office, dwelling school, laboratory), \( 70 \text{ Wm}^{-2} \). The rate of work \( W \) is set at \( 0 \text{ Wm}^{-2} \).

\( H = \) Dry Heat loss through convection and radiation from the clothing surface \( (\text{Wm}^{-2}) \)

\( E_{vk} = \) Heat loss by water vapour diffusion and evaporation through the skin \( (\text{Wm}^{-2}) \)

\( E_{rem} = \) Heat loss due to respiration (dry and latent) \( (\text{Wm}^{-2}) \)

The multiobjective objective is not the maximum comfort that can be achieved but the maximum discomfort the occupants should experience. This is achieved by making the maximum percentage of people dissatisfied (PPD) during the occupied period the objective.

\[
PPD = 100 - 95e^{-(0.0353PMV^4 + 0.2179PMV^2)}
\]

When the PPD is being optimised for all 3 design days then PPD for the three days is summated to form one objective function.
4.4 Utilisation of the Building of the Model

Primarily the trade-off curves examined are:

- The Operation Cost vs. Maximum Percentage People Dissatisfied.
- The Operational Cost vs. Capital Cost.

Although the other trade-offs are examined the trade-off between the maximum PPD and the operational cost are the most commonly used. The capital cost trade-off is examined for the problems involving the selection of building envelope. Although the plant selection also involves a change in capital cost, the trade-off with operational cost is very limited because, if the operational cost reduces, the load the plant is required to meet also reduces. This infers that the minimal operational cost will also be the minimal capital cost.

The building elements are examined to determine their effects on the trade-off curve. The building envelope effects the load on the building and consequently the level of discomfort and the operational cost. To get a true evaluation of the buildings performance this is done with the plant also part of the optimisation. This ensures that the limits of the plants to meet a load does not affect changes in trade-off curve caused by the changes in building envelope.

4.5 Discussion

The optimisation problem presented in this study combines the optimum design of building fabric, HVAC system selection and HVAC supervisory control.

The model was developed by Ren (1997) to evaluate the use of building fabric for thermal storage. The comparison the slab construction is used to demonstrate the effectiveness of the MOGA as an optimisation and performance assessment tool.

The problem formation provides a complex optimisation problem on which to evaluate the benefits of the multiobjective algorithm. The problem provides continuous and discontinuous solution spaces depending on which element is investigated, as well as
giving a number well defined opposing objectives on which the optimisation can take place.
Chapter 5

The Multiobjective Genetic Algorithm (MOGA)

With standard optimisation a single criterion is optimised, however in many cases it would be benefit to optimise more than one criterion simultaneously. To enable more than one criterion to be optimised multicriteria algorithms are used. These are often adaptations of single criterion algorithms. This technique allows the criteria to be optimised simultaneously and the relationships between them to be examined. Here a genetic algorithm is adapted because of its ability to deal with complex highly constrained problems, from its traditional single criterion optimisation to multicriteria. The specific method employed here is a multiobjective genetic algorithm MOGA as originally developed by (Fonseca and Fleming, 1993). This chapter sets out the (MOGA) operators necessary to move the simple genetic algorithm towards multicriteria optimisation. Through the investigation of the problem it became necessary to adapt the standard set-up to deal with the ‘whole building’ problem as well as the constraints imposed on it. To measure the performance of the MOGA it is also necessary to evaluate a number of performance measures.
5.1 MOGA Operators

The genetic algorithm (GA) used in this study is a simple binary encoded GA with "roulette wheel" selection, single point cross-over and a non-overlapping population. The most basic MOGA operator, which changes it from the basic simple genetic algorithm (SGA), is the fitness assignment. The fitness is then applied to the roulette wheel within the SGA and treated as normal (see appendix A).

5.1.1 Standard Operating parameters

The genetic algorithms utilised in this study are set up with the same standard values:

- Population = 200 individuals
- Generations = 1000
- Mutation = 0.06
- Crossover = 0.4

The crossover rate and mutation were set to keep the diversity high and where chosen on a trial and error bases throughout the research, as it became evident through test problems that there was no global ideal values for the setting up the genetic algorithm.

The population was chosen based again on trial and error however this was more a compromise between computational complexity and the information available from using a higher population size.

The high number of generations was to try and do an exhaustive search; however the benefit of allowing the algorithm to continue so long is investigated further in the following chapters.

5.1.2 Assigning Fitness

The Pareto algorithm described by Fonseca and Fleming (1993) is implemented as the basic scoring method for the solutions. Therefore the non-dominated solutions have a ranking of zero, (Chapter 3, section 3.6.2).
In its basic form, the MOGA is assigned its fitness as an inverse of the Pareto score, then it is modified by additional routines (optional) and then normalised, for entry into the roulette wheel. This can cause slow convergence; therefore, the easiest way to overcome this was to advantageously weight the Pareto zero scores.

To weight the fitness to bias the lower Pareto scored individuals (less dominated individuals), (Chapter 3, section 3.6.2) an exponential fitness weighting is applied.

The Exponential Fitness Function

Using the exponential fitness method the individuals in the population are sorted from the worst Pareto score to the best Pareto score. The fitness is applied according to their sorted position. If an individual was in position 32 then its fitness will be the exponential value of 32. The fitness for those individuals with the same Pareto score is then averaged and reassigned as the individual's fitness (Figure 5.1).

It was realised that the performance of the exponential function could be modified/improved by adding a dividing factor. This modification is applied, as a fraction of the population size, to ensure that no matter what the size of the population; the exponential curve shape remains the same. This ensures that whatever the population...
or scale rank, the minimum and maximum ranks receive the same fitness, (Figure 5.2 and Figure 5.3). This prevents over domination of the roulette wheel by individual solutions.

\[
\text{Exponential Fitness} = \exp\left[\frac{\text{Rank}}{(\text{populationsize}/17)}\right]
\]

5.1

The simplest method of comparing the performance of the function is to see how well it forms the pay-off curve. There are two measures of how well it performs; how close it gets to the trade-off curve and how much of the curve it covers.

The exponential fitness is weighted so that it performs comparably to original fitness function at a standard population of 100. Figure 5.4 demonstrates, using Fonseca and Flemings (1998) function that after 100 generations with a population of 100 individuals there is little difference in the accuracy and coverage of the curve. However at higher population sizes the weighting proportional to the population size means that the roulette wheel is able to select from a greater proportion. Figure 5.5 shows the trade-off curve produced for a greater population, the weighing allows the solution to cover a greater surface area, without the assistance of additional sharing functions.
Although the weighting proportional to the population size assists in the spread of solutions, it is not meant to replace sharing or mating functions that are generally required to overcome the genetic algorithm of the MOGA from naturally progressing the search to a point.

Figure 5.4: Exponential Fitness Comparison (population 100)

Figure 5.5: Exponential Fitness Comparison (population 300)
5.1.3 Sharing and Niching (species and mating)

Within the MOGA used here there is the option to use a number of sharing techniques. The one primarily applied in this study is the Epanechnikov kernel function method as described by Fonseca and Fleming (1995), Chapter 3, section 3.2.5.

There are many other non-parametric methods available other than the kernel method, the simplest being the histogram, however in this case we are concentrating on the multivariate Epanechnikov kernel method as this is the method that is compared directly with the sharing methods employed with the MOGA. (Fonseca and Fleming (1995)).

5.1.4 Use of Elitism

Elitism is not used as part of the search of the optimising processes, in that non-dominated elite solutions are not directly passed from one generation to the next.

The non-dominated solutions from each generation for this algorithm are kept in a sub-population. In this case, because of the large number of solutions every 10 generations and the last generation, the sub-populations of elite solutions are Pareto ranked and all the dominated solutions are removed. This is mainly to ensure the sub-population does not become too large to be assessed computationally efficiently at the end of the optimisation process.

5.2 The Infeasibility Objective for Constraint Handling

The whole building problem is highly constrained and therefore it was necessary to develop a method to allow the constraints to be applied to a multiobjective genetic algorithm. The method developed was called the infeasibility objective (Wright and Loosemore 2001\textsuperscript{a}).

The infeasibility objective is a method of combining constraint violations to give a single measure of an individual’s infeasibility. The infeasibility is then treated as an objective in the Pareto ranking of the solutions.
The infeasibility objective is used to reduce the dimensionality of a problem by representing each of the constraints as a single measure of infeasibility. Traditionally constraint handling with the MOGA is limited to treating the constraints as objectives, which is the original and preferred method for the Pareto ranked multiobjective genetic algorithm (Fonseca and Fleming (1998)).

The rationale applied to the approach used here is that the constraints are considered bounds on the problem and that the interrelationship between constraints and objectives is of no interest. Where the impact of a constraint on the objectives is considered important, then the constraint may still be represented as objective (with any remaining constraints combined to produce an infeasibility objective).

As the constraints act as bounds on the problem it is only the solutions that lie beyond the bounds and are therefore infeasible that are of concern. The infeasibility objective is constructed in three stages.

1. The inequality constraints \( g_j(X) \) are formulated such that they are negative when feasible, and the equality constraints \( h_j(X) \) are zero when feasible (Equations 5.2 and 5.3).

2. The feasible constraint values are reset as zero and infeasible values as positive (Equation 5.4).

3. Finally, the solution’s infeasibility \( c(X) \) is taken as the normalised sum of the reset constraint values (Equation 5.5). The solution’s infeasibility \( c(X) \) is subsequently referred to as the infeasibility objective, which once minimised (to zero), ensures all constraints are satisfied.

\[
g_j(X) \leq 0 \quad (j = 1,...,q) \tag{5.2}
\]

\[
h_j(X) = 0 \quad (j = q+1,...,m) \tag{5.3}
\]

\[
c_j(X) = \begin{cases} 
\max(0, g_j(X)) & \text{if } 1 \leq j \leq q \\
\left| h_j(X) \right| & \text{if } q+1 \leq j \leq m 
\end{cases} \tag{5.4}
\]
\[
\bar{c}(X) = \sum_{j=1}^{c} \frac{c_j(X)}{c_{\text{max},j}}
\]

Normalising the constraint violations (by dividing by the scaling factor \(c_{\text{max},j}\)) is necessary since large differences in the magnitude of the constraint values can lead to dominance of the infeasibility by the constraints having the highest values. In the procedure implemented here, the scaling factor for each constraint \(c_{\text{max},j}\), is taken as the maximum value of the constraint violation found in the initial randomly generated population. If no infeasible solutions are found, the scaling factor is set to equal unity. The scaling factor is static and has been taken from the initial population so that for given constraint violations, the magnitude of infeasibility objective is consistent in every generation. This allows solutions from each generation to be included in the Pareto ranking of subsequent generations without the need to re-evaluate the infeasibility objective.

Although the infeasibility measure can be used directly as an objective function in a Pareto optimisation, it is necessary to use the goal attainment method (Fonseca and Fleming (1998)), to direct the optimisation towards the feasible solutions. Since all feasible solutions have the same infeasibility objective value (zero), the infeasibility objective is excluded from the Pareto ranking when it has a value of zero. This results in the ranking for feasible solutions being a function of only the true objectives, which in turn has the effect of reducing the dimensionality of the problem and thus makes it easier to interpret the solutions.

5.2.1 Example Constrained Pareto Optimisation

The infeasibility objective approach was investigated through test problems as the ‘whole building’ problem is highly constrained and provides a problem too complex for evaluation of the constraint operators. The test problem is easily visualised and is an adaptation of an established multiobjective test problem. The infeasibility objective approach is compared to the approach, which treats all the constraints as individual objectives (Fonseca and Fleming (1998)).

A number of methods are available for performance comparison (section 5.3), which are used for comparing different algorithms for the same test problem. As this is the same
MOGA with two different constraint-handling methods, the assessment is based on the ease with which the results can be interpreted, and the extent to which the infeasibility objective approach produces Pareto optimum solutions.

5.2.2 A Four Function Test Problem

The four function test problem is an adaptation of an existing two objective test problem Equation 5.6 and 5.7 (Fonseca and Fleming (1998)). A third function (Equation 5.10 provides two inequality constraint functions (Equations 5.8 and 5.9), giving a total of four test functions ($f_1(X) ; f_2(X) ; g_1(X) ; g_2(X)$).

Minimise:

$$f_1(X) = 1 - \exp \left( - \sum_{i=1}^{n} x_i - \frac{1}{\sqrt{n}} \right)^2$$  \hspace{1cm} 5.6

$$f_2(X) = 1 - \exp \left( - \sum_{i=1}^{n} x_i + \frac{1}{\sqrt{n}} \right)^2$$  \hspace{1cm} 5.7

Subject to  

$$g_j(X) \leq 0.0 \hspace{0.5cm} \forall j$$  \hspace{1cm} 5.8

Where:  

$$g_j(X) \leq 0.0 \hspace{0.5cm} \forall j$$  \hspace{1cm} 5.9

$$f_3(X) = 1 - \exp \left\{ - \sum_{i=1}^{n} \left[ (x_i - \frac{1}{\sqrt{n}})^2 \right] \quad \text{if } i = 1,3,5... \ight\}$$

$$f_3(X) = 1 - \exp \left\{ - \sum_{i=1}^{n} \left[ (x_i + \frac{1}{\sqrt{n}})^2 \right] \quad \text{if } i = 2,4,6... \right\}$$  \hspace{1cm} 5.10

In this example, the number of variables, \((n)\) has been fixed at 2. A discrete increment of 0.05 between the variable values has been chosen, and the variable range set at -2.0 to 2.0. This results in the function values being in the range 0.0 to 1.0. Figure 5.6 illustrates the test problem surface, with the shaded area representing the constrained Pareto solution space.
It would be expected that treating both constraints as separate objectives would result in solutions covering the whole of the Pareto surface. Using a population size of 100 and optimising for 100 generations, Figure 5.7 illustrates that this is the case (with the solutions indicated by a "box").

However, since the infeasibility objective is only included in the Pareto ranking for infeasible solutions, it would be expected that this approach would produce a set of solutions that represent the constrained Pareto optimum solutions for only $f_1(X)$ and $f_2(X)$. This is represented by a line following the upper limit of the Pareto surface in Figure 5.6 (the "Pareto front"). Figure 5.8, illustrates that this is the case (the solution being indicated by a "box" and the remainder of the Pareto surface by "circles").

It can be seen from Figure 5.7 and Figure 5.8 that the solutions from the infeasibility objective approach are a subset of the solutions obtained when the constraints are treated as separate objectives. However, for more complex problems, it may not be so easy to visualise the results and determine which set of solutions are equivalent to the constrained Pareto optimum solutions for the true problem objectives.
Figure 5.7: Pareto Solutions for Constraint Function Optimisation

Figure 5.8: Pareto Solutions for the Infeasibility Objective Approach
The infeasibility objective approach is an effective means of obtaining the Pareto optimal solutions for the true objectives and can be used to reduce the dimensionality of the optimisation problem where detailed analysis of the constraint behaviour is not required.

Using the infeasibility objective in a Pareto ranking optimisation results in solutions that are equivalent to the constrained Pareto optimal solutions for only the true objective functions. This is in contrast to the solutions obtained when each constraint is treated as separate objective function, which increases the dimension of the problem and limits the extent to which the interrelationship between objectives and constraints can be interpreted. The use of the infeasibility objective does not preclude the treatment of a constraint function as an objective function if knowledge of the constraint effect is considered important. The approach allows the treatment of constraints as bounds on the problem when the only concern is that the solutions are feasible, the constraint values across the Pareto optimal solutions are still available for analysis. This method also has the advantage over approaches based on the use of penalty functions in that it does not require any additional parameters, which overcomes the need to tune it to a particular problem.

5.3 Measuring Performance

When assessing the MOGA and its improvements it is necessary to have a basis on which to compare their performance. There are five main elements that need to be qualified to determine whether a MOGA is performing well or not.

1. The size of the solution space.
   This describes the size of the problem. Generally a larger solution space is desired as this more often means that a larger amount of the trade-off space is being described, however there are problems that have a concave solution space where a larger solution space will imply that the search is further away from the solution.
2. Spread over the solution space.

It is necessary for the solution set to be spread out to ensure that it has described the entire trade-off surface to its maximum detail. Spread is also necessary to ensure the all of the surface is found as well as encouraging good problem progression within the solution space.

3. How near the population is to the optimum solution.

There needs to be a measure of how close the solution gets to the actual non-dominated surface, this can be a factor of the population size as well as the number of generations that take place.

4. The proportion of non-dominated solutions in a generation.

Although this is an important measure, the meaning of the measure can be confusing. A high number of Pareto zeros in a solution can be an indication of a greater spread, larger solution surface, or because of the current MOGA set-up it can also mean that all the Pareto zero solutions are at one point.

It is recognised however that generally the higher the number the better, as this means that the generation is actually progressing towards the ideal solution, however there will always be less than 100% to allow movement in the problem.
5. The speed of the problem solving

By speed it is meant how fast (number of generations) the MOGA achieves criteria set down in [2] [3]. Comparing the results at a given generation can negate this, however in more complex problems this can be related to the number of function calls. With the test functions this is not applicable because there is no filter to stop the simulation being called, and is not necessary due to the calculation time being negligible. The building simulation however will require a filter to stop the same simulation being run repetitively to ensure running times are tangible.

5.3.1 Methods of Comparison

Measuring performance of optimisation algorithms has to be some sort of comparison, no matter how finite a defined performance measurement figure can be. In the cases using test functions it is a comparison between a known optimum solution and what the algorithm produces. In most other cases and with ‘whole building’ optimisation, it is the comparison between the results of one specific set-up of the MOGA to another.

The simplest form of comparison, and the one most used in this study is visual comparison where one objective is plotted against another to provide the end user with a curve demonstrating the optimal solutions. When a problem has two functions then it is possible to visualise the trade-off surface directly. In this study only the comparisons which are limited to 2 objectives are presented, however the complexities of comparing more than 2 objectives are recognised.

Fonseca (1995) demonstrates the prime alternative available for visually representing three or more objectives, known as parallel co-ordinates. Figure 5.10 shows how competing consecutive objectives are demonstrated by cross lines. Although this method allows the relationships between objectives to be displayed, it does not actually provide a quantifiable way to compare different multiobjective optimisation techniques.
Throughout this thesis the trade-off information is shown primarily as a trade-off curve, however throughout the optimisation process a number of statistical performance measures are calculated and stored.

### 5.3.2 Statistical Performance Analysis

The statistical output from the version of the MOGA used in this research will take two forms; on running statistical output and a calculated output that takes place after the optimisation has taken place. When there is more than one run the main outputs detail the overall trade-off surface for the run and the statistical output file.

The outputs from each generation are:

For each objective

- Maximum and minimum value for current population.
- Maximum and minimum values for the non-dominated individuals.
- The relative Performance Measure.

Three measures for all the objectives simultaneously are:

- Performance Measure.
- Distance Performance Measure
- Number of Pareto zero values in a generation.
5.3.3 Chi-squared Measures

The performance measure that is introduced at this point measures the spread of the current solution in the current non-dominated solution space. This is done by using the method as detailed in equation 5.11 and 5.12 by both Srinivas and Deb (1995) and Sareni and Krithenbühl (1998).

\[
\text{Performance Measure } i = \sqrt{\sum_{l=1}^{p} \left( \frac{b_l - \bar{b}_l}{\sigma_l} \right)^2}
\]

5.11

\[
\sigma_l^2 = \bar{b}_l \left( 1 - \frac{\bar{b}_l}{P} \right), \quad i = 1, 2, \ldots, q,
\]

5.12

In this case the ideal number of solutions in each niche is not measured in the variable space, but in the objective space. This means that there is a separate performance measure for each objective.

In many problems, the actual objective space is not actually known, thus at the moment of the calculation the performance measure describes the spread in the current population. If the individuals are spread well but in a small solution space, it will give a better performance value to one that is spread less evenly, but in a greater solution space.

To reflect the changing solution space the performance measure is made a fraction of the solution space. The size of the solution space is cubed in this case to make the performance measure more sensitive to the size of the space. (Equation 5.13).

\[
\text{Relative Performance Measure (RPM)} = \frac{\sqrt{\sum_{l=1}^{p} \left( \frac{b_l - \bar{b}_l}{\sigma_l} \right)^2}}{\left[ \frac{\text{Max Objective}_g - \text{Min Objective}_g}{\text{Max Objective}_1 - \text{Min Objective}_1} \right]^3}
\]

5.13

The relative performance measure (RPM) as with the normal performance measure gets smaller as the solution becomes better distributed, however with the relative performance measure, the figure will increase if the solution space decreases. All the changes in the
relative performance are based on the amount of change of solution space from the initial generation.

A brief investigation is made into the effect of the number of niches on the relative performance measure (Figure 5.12). This shows that as the number of niches increases the greater the RPM, and the less detail present. This is because as the number in niches increases the larger the performance measure becomes, and therefore Equation 5.13 becomes top heavy. If the number of niches remains the same then the performance measure will remain similar no matter how large the population. This consistence is important for comparison, however keeping the number of niches the same means that the size of each niche will become larger therefore reducing the detail in the performance measure (Figure 5.12). Making the niche size proportional to the population size increases the number of niches therefore increasing the performance measure, but also the detail. For consistence in the comparisons, the number of niches will remain at a constant ten for this example (Fonseca and Fleming, 1993) and in the test problem comparisons, however with other problems the standard population and solution space may negate a different niche number.

Figure 5.11: Demonstration on How the Difference in Solution Space Effects the Relative Performance Measure (RPM)
CHAPTER 5: THE MULTIOBJECTIVE GENETIC ALGORITHM (MOGA)

To give a single measure of the relative performance measure rather than a figure for each objective is purposed just to average the figures. This should give a single figure of comparison, but still retain the features described by the performance measure although in a more diluted form.

5.3.4 Distance Performance Measure

The first performance measure gives a measure of the spread of a population in each generation, and the relative performance measure indicates the distribution in respect to the total coverage of each generation. None of these give any indication to how optimal each population is.

The second performance measure (DPM) was created to measure the distance of each population to zero, therefore giving a direct way of determining how close each generation is to the optimum.

Figure 5.12: The Effect of Varing Population, Niche Size and Number on the Performance Measure – Objective I only.
CHAPTER 5: *THE MULTIOBJECTIVE GENETIC ALGORITHM (MOGA)*

Distance Performance Measure (DRP) = 
\[ \frac{\sum_{i=1}^{P} \left( \sum_{j=1}^{O} Obj_{i}^2 \right)}{\text{No. Non-dominated}} \]

where:
- \( P \) = Population size
- \( \text{Pareto} \) = Pareto score of individual \((i)\) in population
- \( O \) = Number of objectives
- \( Obj \) = Objective Number \((j)\)

As the performance measure decreases, the closer the trade-off surface is on average to zero. Therefore if one figure is lower than another, then that trade-off surface is on average more optimal. This performance measure however takes no account of the size or distribution of the trade-off surface. This can be shown more evidently by comparing the distance performance measure (Figure 5.13) with the trade-off surface at generation 8, 27 and 49 (Figure 5.14 to Figure 5.16)

![Figure 5.13: Distance Performance Measure: Comparing the trade-off surface performance. (Direct comparisons made for generation 8, 27 and 49 see Figure 5.14 to Figure 5.16)]
Figure 5.14: Direct Comparison of Trade-off Surface for Generation Number 8

Figure 5.15: Direct Comparison of Trade-off Surface for Generation Number 27
5.3.5 The effects of Random Initial Populations

The only difference between the run numbers in Figure 5.13 to Figure 5.16 is the initial random population. This seems to indicate that this is a crucial fact in the performance of the MOGA.

To compare and assess the performance of various methods it is going to be necessary to negate the random effects on the performance. Therefore, it is suggested that a number of runs should be used, and the average performance taken.

It should be ensured for each comparison that the same random numbers are used. For example the initial population for run 2 should be the same as the initial population for run number 2 on the other method. This is to ensure that there is a fair comparison between the methods.

The effects of different random initial populations seem to deteriorate as the generations’ progress. For many of the ‘whole building’ problems, an excess population size and number of generations are used to ensure an accurate production of the solution set in the
only one run. Figure 5.17 shows the trade-off solution for three different random initial populations, for an 'whole building' solution. The solutions were obtained with an excessive population size (200 individuals) and for 1000 generations. The figure shows that the difference obtained is minimal, and there are a large number of solutions for all runs.

![Figure 5.17: The Trade-off Surface Different Initial Random Population](image)

5.4 Problem Characteristics: Seasonal Divisions
(Optimisation on Weekly Coupled Pareto Sets)

The problem is based around the optimisation of three sample days, which are designed to represent the worst two worst temperature conditions and one transient day. Within the standard MOGA the measure of maximum discomfort and the measure of operational cost is combination of the seasonal information by simple summation. The effect of doing this is examined by comparison with a basic annual cost model acting as an alternative way of combining the seasonal costs. This examination lead directly to an alternative method of combing these within problem formulation.
5.4.1 Example: An Annual Cost Model

The example annual cost model calculates each day for half the year based on the straight-line relationship demonstrated by the linked three sample days. The summation of half the year's cost is then doubled to give an approximation of the annual cost.

This method of predicting the annual cost assumes a relationship between the sample days comparable to that shown in Figure 5.18. This method assumes symmetry between one half of the year and the other about the central day (182). The assumed costs at either end being an inverse gradient of the previous interpolation between summer and swing or swing and winter sample days. As the annual operation cost (AOC) is based on the calculation for half of the year it is represented by Equation 5.15 however this can be easily simplified to equation 5.18.

$$\frac{AOC}{2} = \sum_{n=0}^{n+1} (D_{n+1} - D_n) \left[ \frac{(C_{n+1} + C_n)}{2} \right]$$

5.15

$$C_0 = (D_1 - D_0) \frac{(C_2 - C_1)}{(D_2 - D_1)} + C_2$$

5.16
\[ C_4 = (D_4 - D_3) \left( \frac{(C_3 - C_2)}{(D_3 - D_2)} \right) + C_3 \]

5.17

\[ AOC = \sum_{n=0}^{n-1} (D_{n+1} - D_n) (C_{n+1} + C_n) \]

5.18

The effect of combining the operational cost was compared by initiating the control optimisation problem. This is because the variables for the seasons and therefore the individual costs for the seasons have nothing that links them within the problem. They are all mutually exclusive therefore the individual operational cost, and its related measure of discomfort, should be unaffected by the total cost no matter how it is constructed.

The effect of combining the operation cost can be expressed by examining how the seasons relate to each other. Figure 5.19 shows the individual operational cost within the final trade-off curve.

It is impossible to say which the most correct method is as each performs the best in different comparison areas such as minimal cost and spread for both the operational cost and the PPD. It can be concluded however, that the method of summation is having a significant effect on how the algorithm performs and the extent it optimises each season. Comparing the differing sample day cost against the summation of the PPD for each model demonstrates that the summer season dominated both models for lower PPD’s, and in the case of the annual cost model the entire surface. The proportion of the winter cost remains similar for both models whilst for the annual cost model the swing sample day is a consistently lower cost than the summation model. The fact that the sample days costs are more spread for the annual cost model is more emphasised by comparing the Sample Day Cost variations against each other. It can be seen clearly that for the annual cost model the swing season is more optimal than either of the other two seasons, whilst for the summation cost model the all seasons at some point seem optimised equally. In both cases the spread varies between the seasons but is comparable for each model. What is seen is consistent with the view that the annual cost model applies different weights to each sample day and optimises to different degrees accordingly, however the summation model optimises them evenly.
Although in both cases the total PPD is beginning to be optimised, the approach for both models is consistent. That is, the total PPD is the direct summation of the three individual sample days PPD's.

Therefore the different cost models should directly drive the differences seen in the PPD. For both models the changes in PPD across the trade-off surface are primarily caused by the changes in summer PPD. It is interesting that when comparing the sample days directly against each other the two models seem to show different overall patterns, with the winter and swing days for the annual cost model being similar with then lower or higher summer PPD. The summation model in comparison having lower swing day PPDs than winter and then higher or similar PPDs for the summer than the swing day.

This has led to the development of the split fitness MOGA that deals with each season separately. Throughout the further analysis the split fitness MOGA is compared with the summation MOGA although it is still imposing preconceived ideas of the importance of the each season to each other (i.e. they are all treated equally). It is recognised that treating the seasons separately may not be advantageous when the problem has not got three separate mutually exclusive seasons. This is likely to happen when building selection and plant selection affect the choice of control schedule. This is compared further in Chapter 1.

Figure 5.19: Comparison of Seasonal Operational Cost
5.4.2 The Split the Fitness MOGA

The split fitness algorithm was conceived by a need for the operational cost and PPD for each season to be treated separately, initially in control scheduling problems, however the approach is developed further for all types of optimisation problems.

Primarily the concept behind the split fitness MOGA, is that a separate fitness is produced for each season and then applied to each section of the chromosome, that represents the information for the season. This, in effect makes the algorithm have a multi-point crossover.

Control Scheduling Problems

For Scheduling Problems with operating cost and discomfort, the variable string is separated into three sections. The crossover for each section is based on the fitness assigned to the Pareto ranking of the summer, winter and swing separate operating and discomfort objectives.

In this case:

Objective [0] = Total Operational Cost  
Objective [1] = Total Discomfort (PPD)  
Objective [2] = Constraint Objective
Breakdown [0] = Summer Cost
Breakdown [1] = Swing Cost
Breakdown [3] = Summer Discomfort
Breakdown [4] = Swing Discomfort

There are three variable groups, the progression of the solution in each set is based on the ranking of the separate seasons.

<table>
<thead>
<tr>
<th>Variable Group One</th>
<th>Variable Group Two</th>
<th>Variable Group Three</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Summer Season</strong></td>
<td><strong>Swing Season</strong></td>
<td><strong>Winter Season</strong></td>
</tr>
<tr>
<td>Pareto Rank Based on:</td>
<td>Pareto Rank Based on:</td>
<td>Pareto Rank Based on:</td>
</tr>
</tbody>
</table>

There is no advantage in running capital cost as it is driven by the choice of plant and building construction, therefore is fixed within the control scheduling problem.

**Control Scheduling and Plant Sizing Problem**

For the scheduling and Plant Sizing Problem, the variables are split as with the control scheduling, however there are additional variables on the end that define the plant size. These are not dependent on a singular season, however traditionally all design plant sizing is based on extremes of seasons therefore it depends primarily on the results as defined by the summer season for cooling plants sizing and winter for the sizing of all heating plant. Although it is recognised that these seasons are the most important in the sizing of the plant, this preference is implemented naturally by preferential weighing of these seasons in the total cost and total discomfort.
For Optimisation of Discomfort and Operational Cost

<table>
<thead>
<tr>
<th>Variable Group 1</th>
<th>Variable Group 2</th>
<th>Variable Group 3</th>
<th>Variable Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer Season</td>
<td>Swing Season</td>
<td>Winter Season</td>
<td>Plant Size</td>
</tr>
</tbody>
</table>

Same as Control Schedule Problem

Pareto Rank based on:
Objective [0] vs Objective [1] vs Objective [2]

No. Variables: 11

For Optimisation of Discomfort, Operational Cost and Capital Cost.

There are 4 objectives for this example, the breakdowns remain the same.

Objective [0] = Total Cost
Objective [1] = Capital Cost
Objective [2] = Total Discomfort
Objective [3] = Constraint Objective

<table>
<thead>
<tr>
<th>Variable Group 1</th>
<th>Variable Group 2</th>
<th>Variable Group 3</th>
<th>Variable Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summer Season</td>
<td>Swing Season</td>
<td>Winter Season</td>
<td>Plant Size</td>
</tr>
</tbody>
</table>

Pareto Rank Based on:
Objective [3]

No. Variables: 48
CHAPTER 5: *THE MULTIOBJECTIVE GENETIC ALGORITHM (MOGA)*

Control Scheduling and Building Type Selection Problem

The set-up for this problem is the same as the Control Scheduling and Plant Sizing Problem, in that the objective and breakdowns remain the same. There are four variable groups as the building selection problem is also dependent on all three seasons it is assessed in the same manner as the Plant Size Variable Group. The only difference is that the number of variables that are affected by the Pareto assignment is limited to 3.

Control Scheduling, Plant Sizing and Building Type Selection Problem

The combination of both the plant and the building sizing problems doesn’t mean any addition to the number of groups, as the both depend on the same seasonal information, the Pareto rank and therefore the fitness will remain the same, so the additional variables required are combined into one group.

The Objectives and the breakdowns stay as those described in ‘control scheduling and plants Sizing problem’. The only change to this set-up for all objective combinations is the number of variables in Variable Group 4, as demonstrated in the following table for the operational cost and discomfort objective set-up.

<table>
<thead>
<tr>
<th>Variable Group 1</th>
<th>Variable Group 2</th>
<th>Variable Group 3</th>
<th>Variable Group 4</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>Summer Season</em></td>
<td><em>Swing Season</em></td>
<td><em>Winter Season</em></td>
<td><em>Plant Size and Building Type Selection</em></td>
</tr>
<tr>
<td>Same as Control Schedule Problem</td>
<td></td>
<td></td>
<td><em>Pareto Rank based on:</em></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>*No. Variables:*14</td>
</tr>
</tbody>
</table>
Use of Goals Functions with Split Fitness MOGA

For simplicity the goals are applied to the summation of the Pareto rank (p_score) for all variable groups, and then used to drive the search to the feasible region (for both the goals applied to the constraint function and the directional goals). Each seasons’ p_score is then replaced by the total p_score which will be greatly inflated as it fails the goals, the goal function should drive the solutions in the right direction efficiently. Once the goal functions are met then the individual p_scores for each of the groups are be used again.

5.4.3 Example Problem

The main aim is to have a single model that optimises all three season sample days without unfairly biasing one season over the other solutions. It is hypothesised that combining the operational cost, and/or the measure of discomfort for each season to a singular objective effectively puts some predisposed preference weighting on the final solutions.

The other way of dealing with this for the combined seasons is to add the Pareto scores from the pairs in the other seasons together. However ranking them separately on the total cost and discomfort seemed the more clean-cut way of approaching this. The other approach however, would facility the combination of just the summer and winter seasons together for the plant size selection.

Figure 5.21 and Figure 5.22 compare the difference in trade-off curves produce from the standard MOGA and the split fitness MOGA, and the effect on the individual seasonal pay-off which make up these solutions. The problem formulations are compared for a fixed conventional medium weight building type, with a low emissivity glazing forming 20% of the area on the external surface. The comparison is made for an optimisation problem that attempts to optimise the control schedule and the plant size, although this means that the problem should not have three truly independent sections representing the seasons. It does mean that the control schedules aren’t limited by the capability of the plant size, thus giving the true optimum control schedules.
CHAPTER 5: *THE MULTIOBJECTIVE GENETIC ALGORITHM (MOGA)*

Figure 5.21: Overall Trade-off Surface Comparison

Figure 5.22: Seasonal Trade-off Comparison
The fact that the split fitness MOGA finds a more optimal curve than the standard MOGA is immediately obvious, this is verified again by looking at the season individually. Examining the seasonal trade-off curves and comparing these to the individually optimised season curves (optimised using the standard MOGA), shows that not only is the split fitness MOGA more optimal than the solutions obtained by the standard fitness, and individual fitness, but also that the seasonal solutions are not linked at all. This is demonstrated by the fact that they are comparable to the solutions obtained by the individually optimised seasons. The true optimality of the solutions obtained is investigated further in Chapter 6.

Performance Comparison

The most important comparison is the visual comparison provided by Figure 5.21 and Figure 5.22. Although there are a number of performance measures available, the only not easily described by the visual comparison is the speed of convergence. The Distance Performance measure, gives the distance of the non-dominated solutions from zero.

Figure 5.23: Speed of Convergence (using Distance Performance Measure)

Figure 5.23 shows how the split fitness MOGA converges faster, within 30 generations compared to the standard MOGA which reaches same level in 100 generations. The split
fitness in turn remains more consistent for the 1000 generations. Both algorithms converge quickly and seem to be relatively unchanged for the duration. There is very minimal improvement after the first 100 generations, the standard MOGA shows very slightly more variation. In these cases this is meant to be the absolute solution, the number of generations are excessive to allow even a slight improvement to take place.

5.5 Discussion

To summarise, there are two basic forms of the multiobjective genetic algorithm utilised in this study, the standard MOGA and the split fitness MOGA. Both are used as they have specific advantages and disadvantages when applied to different aspects of the 'whole building' optimisation.

Both of these use the Pareto fitness selection, and the standard set-up of genetic algorithm (crossover, mutation, roulette wheel selection etc.). The split fitness MOGA in general does not need or use the sharing and mating operators, however the standard MOGA does use the kernel based method of applying sharing. There is the option however, of applying basic phenotypic sharing in either the variable or objective solution spaces. Both methods use the normalised exponential fitness function.

A number of performance measures are used, when appropriate, to evaluate the final solutions position in the solution space: the spread and the distance away from true non-dominated solution(s). Although the performance measures can be used for comparison and have been in the development of the optimisation process used here, it is often more informative to evaluate the final solutions graphically based on just on the objective and variable outputs. The performance measures can be used for aiding decision-making this is shown in Chapter 8.
Chapter 6

Experimental Results and Analysis:
Conventional Building Construction

The set of solutions from a multiobjective optimisation can be visualised graphically as one or more trade-off curves (also known as attainment curves). The trade-off curves examined in this chapter are for two groups of results. The first explores the relationship between the predicted discomfort that would be expressed by the building occupants and the operational cost required to achieve this level of comfort. In these results, the building type has been fixed; however the effect of building weight on the trade-off is examined in separate trade-off curves for each building type. The relationship between the plant size and the building weight is investigated further in the second section of this chapter. This section also details the results of analysis on the relationship between the capital cost of the building and air conditioning system and the system operating cost.

Two main sources of optimal solutions that can be presented as trade-off curves:

1. the non-dominated solution set for the final generation;
2. the non-dominated set of solutions from all generations.
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The solution set from both of these should be very similar, however due to progress in the search and possible changes of search direction this may not always be the case. Throughout this study, the optimal set of solutions from all generations is always displayed (unless specifically indicated).

The combined optimisation process is formed from the separate design days, referred to as seasons (Chapter 5, section 5.4.2). It is also possible to obtain different solution sets from seasonal information inherent in the combined optimisation. There are two options available to obtain seasonal information from the optimisation process.

1. the seasonal information inherent within the final combined non-dominated solutions set;
2. the seasonal information that is obtained throughout the optimisation process.

The optimality of the seasons is evaluated as part of this optimisation process. The season non-dominated solutions are recorded whether or not these solution form part of the combined non-dominated set. Displaying all of the seasonal optimal solutions leads to a more detailed understanding of that particular optimisation process and solution. Generally because of the more detailed information available it is the seasonal breakdown information obtained throughout the optimisation process that is evaluated in this chapter.

6.1 Discomfort versus Operating Cost

Although there are many possible combinations of pairs of objectives, the commonest and arguably the most important pair is the measure of thermal discomfort against the cost of providing the level of comfort to the space. This is because these are highly influenced by the system control schedule, and both are indirectly influenced by the plant size. Before the effect of the building can be evaluated the trade-off curves need analysing and justifying.
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6.1.1 Optimality of the Trade-off Curve

All the individually optimised seasons for the discomfort operational cost trade-off have been found using the standard MOGA (Chapter 3, section 3.6) as there is no additional benefit in using the split fitness Chapter 5, section 5.4.2.

It is possible to judge the optimality of a curve by examining the extremes of the trade-off curve. The lowest Predicted Percentage of Dissatisfied (PPD, Chapter 4) possible is 5% because even under the best conditions, 5% of the people within the space will be dissatisfied (Fanger 1970). Since the solutions give the trade-off between the objectives, the lowest discomfort solution corresponds to the maximum operating cost solution. Similarly, the lowest operational cost corresponds to the worst discomfort. This occurs when no active heating or cooling takes place and the fans are operating at their minimum bounds (to supply the minimum amount of outside air).

Minimum Discomfort, Maximum Operating Cost Solution

Examining the winter season's trade-off curve for the medium weight building (20% low emissivity glazing, Chapter 4) is done using the control schedule and plant sizing optimisation problem (Chapter 4, section 4.3). This problem ensures that the selection of the optimum control schedule is not limited by the plant size. Figure 6.1 shows the trade-off curve produced when optimising the thermal discomfort (PPD) and operational cost for winter operation only; the minimum PPD of 5% is achieved. This is shown in more detail in Figure 6.2, where it can be seen that the PPD is controlled tightly during occupancy and, as occupancy finishes, the amount of discomfort increases gradually. The corresponding control schedule is illustrated in Figure 6.3. The greatest amount of heating occurs during the initial hours of occupancy and one hour of pre-heating prior to the start of occupancy (as shown by the heat output of the coil). The amount of heating drops throughout the day as the required heating load decreases due to internal loads, thermal storage and increasing external temperature. The heat recovery device is on whenever the plant is operational, minimising the additional heat required by maximising the heat recovered from the extract air. Although this solution is for the highest operational cost on the trade-off curve, it is the optimal cost for a 5% PPD.
Chapter 6: Experimental Results and Analysis: Conventional Building Construction

![Figure 6.1: Winter Operation Trade-off Curve](image)

![Figure 6.2: PPD for Minimum PPD](image)
The setpoints are meaningless when there is no air supply to the space, and they can be seen to vary randomly outside times of plant operation (signified by a zero fan control signal). The supply air and the extract air become the same when no air is supplied to the space, and both represent the room temperature. Although this is the medium weight construction the temperature does not drop a great deal throughout the night, a minimum of 18.2°C at 06:00. Comparing this possible minimum of −5.6°C (ambient temperature) and a maximum extract temperature during occupancy of 25.8°C demonstrates how little heat is actually lost throughout the night in this building. To offset the loss during the night, therefore reducing the amount of heat to be made up before occupation, an amount of pre-heating is done at 22:00 for the next days operation. Preheating during off-peak times is more cost effective than doing all the conditioning during occupancy however this does not provide the control over the space required to achieve the minimal PPD.

The fan operation keeps close to the minimum fresh air with a control signal of 0.2 during occupancy. The coil only has a control signal of 0.01 where 1.0 would mean the valve to the coil was fully open and 0.0 fully closed. The overall highest operation level of the fan is during night heating, when the electrical cost is at the lower tariff. The
heating coil is operating very close to having the valve closed, signified by the control signal being so close to zero that it is not possible to distinguish its position in Figure 6.3.

The coil sizes for this particular solution are 0.9m x 2.4m with 4 rows and 32 circuits for the cooling coil 0.5m x 2.5m with 8 rows and 4 circuits for the heating coil. For this optimisation problem there is no capital cost optimisation, therefore there is nothing within the optimisation to encourage the coil size to reduce. The only thing within this optimisation that effects the optimisations choice of coil size is the coil duties and the required fan power which in turn effects the operational cost objective. The fact that the frictional effects decrease and therefore decrease the required fan power, along with the increased effectiveness as the coil size grows, means that the optimisation is unlikely to choose smaller sizes of coil. However, ideally the solution would have only one row, which would be the lowest frictional losses, however the problem has a circuit configuration constraint limits the reduction in the number of rows.

In the case of the PPD versus operating cost, the PPD in all seasons acts as a lower bound on the plant size selection. The plant must be large enough to be able to produce the conditions specified by the operational cost and the discomfort levels. The only way to enforce an appreciable upper limit to the plant size is to try to minimise the capital cost.

Minimum Operating Cost, Maximum Discomfort Solution

The other extreme of the trade-off curve is at the lowest the operational cost and the highest PPD solution. The operating schedule for this solution is shown in Figure 6.4 where the fan is only operating very close to that required for the minimum outside air requirements (the fan control signal is close to its lower bound of 0.2), and there is no plant operation out of the occupied period. In single season operation it is possible to constrain the other seasons' permissible level of discomfort (all seasons are modelled although only one is optimised), so that they do not affect the PPD level or cost in another optimised season.

Figure 6.5 shows how the PPD drops significantly as occupancy starts, from 28% to 15% when occupancy begins. The optimisation algorithm is forcing this change as it only minimises the worst PPD during occupancy. The large change in PPD is achieved by only a $2^\circ$K change in zone temperature. The temperature change is achieved with very
minimal heating coil operation and the use of the heat recovery device allowing heat recovery. The internal loads also contribute to the heating requirement further reducing the PPD throughout occupancy until the level of discomfort experienced by the occupants' decreases to 5% by the end of the working day.

Figure 6.4: Plant Operation for Minimum Operational Cost

Figure 6.5: The PPD for Minimum Operating Operational Cost
Figure 6.4 shows again how the heat recovery device is operational throughout the system (fan) operation. It can also be seen that with little additional heat how the setpoints follow the increase in external temperature throughout occupation. The almost constant difference between the supply and extract temperature is caused by the fixed efficiency of the heat recovery device.

Examining various points in isolation shows that the curve has to be near optimal, however the fact that the coil is not off demonstrates that although the solutions are very close to what is perceived as optimal, there is room for a small amount of improvement.

6.1.2 Simultaneous Three Seasons Optimisation

The results for the simultaneous optimisation of three seasons operation were obtained using the split-fitness MOGA (Chapter 5, section 5.4.2). The clear benefits of the Split fitness algorithm over the standard fitness MOGA for control scheduling and plant sizing were given in Figure 5.21, here the optimality of the split fitness trade-off curve is compared against the individual optimised season (Figure 6.6), again by comparing the extremities of the trade-off curve. It is very difficult to get a qualitative measure of the differences between the two curves as curves are formed on numerous non-comparable individual points.

![Figure 6.6: Seasonal Breakdown of Split Fitness Compared to Individual Optimised Seasons](image-url)
Continuing the comparison for the winter season, the extreme ends of the trade-off curve are again examined here for the winter trade-off obtained by the split fitness MOGA. For the lowest discomfort, the minimum of 5% PPD is achieved again for the entire occupied period (Figure 6.7). The actual achieved PPD is 5.011% with a operational cost of £1.116. The method that the plant uses to achieve this is very similar to that in individually optimised season (Figure 6.3), the main difference being that the pre-occupancy heating of the space is achieved with lower setpoint and therefore a lower load on the coil but with an increased air flow rate (Figure 6.8). There are 2 instances of post heating of the space at 19:00 and 23:00, however in this case the heating is achieved with a lower air supply rate. In both the individually optimised, and the split-fitness seasonal trade-off, the fan operation is kept to its minimum bound (the minimum fresh air requirement) during occupancy.

Figure 6.7: PPD For Minimum PPD (Split Fitness MOGA)

The system operation, for the opposite extreme of the trade-off curve is easier to interpret, as the solution is one that minimises the operating cost, and therefore the optimum is for the fan running at the lowest fresh air supply rate. Figure 6.10 illustrates that the lowest fresh air supply rate is achieved using the split fitness MOGA. Very minimal energy is used both as coil duty and fan operation. The PPD drops throughout
the day as the external temperature increases (Figure 6.9). The worst PPD achieved at the lower operational cost of £0.022 is 14.892%.

Figure 6.8: The Plant Operation for Minimum PPD (Split Fitness MOGA)

Figure 6.9: The PPD for Minimum Operational Cost (Split Fitness MOGA)
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Figure 6.10: The Plant operation for Minimum Operational Cost (Split Fitness MOGA)

From comparing the seasonal trade-off curves it is evident, that from Figure 6.6 that the split-fitness is as good, if not marginally better, at producing the trade-off than individually optimising each season using a standard MOGA. Comparing the extremities of the curves indicates that not only are the expected solutions produced at each end of the trade-off curve, but that the split-fitness MOGA and the individually optimised solutions are very similar. The main differences being in the preheating, whether due to increased flow rate keeping the coil load low or by allowing the coil load to be increased but saving power on the fan speed. Although only the winter season is shown here, it can be seen from the trade-off curves (Figure 6.6) that all curves reach the 5% PPD and a very low operational cost. The optimality of the summer trade-off curve is examined in similar detail in Wright and Loosemore (2001).

6.1.3 The Effects of Building Weight

There are three separate building weights available within the problem to either set as variables, or fixed for the plant sizing and control scheduling problems (Chapter 4). Here, a comparison is made of the trade-off solutions for the different building weights.
for a fixed glazed area and type of glazing (Low Emissivity glass and 20% glazed area). The solutions were obtained using the split fitness MOGA (Chapter 5, section 5.4.2).

Figure 6.11 indicates that the lightweight construction has the least optimal trade-off curve than either the medium weight or the heavy weight constructions. The medium and heavy weight constructions seem in comparison to have very similar trade-off curves, with the heavy weight being marginally better.

Figure 6.11: Comparison of Trade-off Curve for Different Building Weights.
As the building weight increases the resistance and the capacitance of the external wall increases. The internal wall increases in capacitance and resistance as the structure changes from light to medium weight however crucially the internal wall is the same construction as for the medium and heavy weight constructions. The importance of this is demonstrated by calculating the thermal properties for all relative areas. Table 4.1 (Chapter 4, section 4.1.2) shows how that the dominance of the internal wall construction accentuates the similarities between the medium weight and heavy weight structures, and what now turns into a much more pronounced difference between these and lightweight construction.

In comparing, the seasonal trade-off curves for each building weight (Figure 6.12), it can be seen that the combined trade-off curves are primarily affected by the summer season, (particularly the lightweight construction). For all seasons, the heavy and medium weight constructions continue to have very similar trade-off curves. For the winter and swing seasons the lightweight construction has produced most non-dominated curve, whilst for the summer season the lightweight construction produced a curve the noticeably more dominated than the other two constructions.
The trade-off curves for the seasons can be justified by analysing the operational cost for a fixed PPD, a more non-dominated curve will have a lower operational cost for that specific PPD. Figure 6.13 shows how for winter at approximately 8% PPD, the light weight building meets these criteria with no active heating. This is a result of the high air temperature set for the surrounding zones (Chapter 4, section 4.3.1); since the conductance of the internal walls dominate the zone heat transfer (Table 4.1), the high internal temperature of the adjacent zones offsets the heat loss from the external walls. The comfort conditions are met by the increasing airflow rate to the space, however this is not enough to offset the heat gains to the space and there is a short (possibly sub-optimal) period of cooling at 14:00. The extra mass in the heavy buildings works against the optimal solution by requiring far more initial heating, however the fan operation is fixed at the minimum control signal (and the minimum outside air rate).

The heating coil duty dominates the operating cost as the fan forms a small fraction of the total power requirement. Though it should be recognised that the heating coil is supplied by gas and the fans are supplied by electricity. Electricity is just over 6 times more expensive than the gas supply in this model. This has two effects, firstly the fan can dominate the solution where the heating requirement is small, however in the case in Figure 6.13 the heating coil requirement for the heavy weight construction is still more
expensive than the fan requirement for the lightweight building). The second effect is that as soon as there is a significant cooling requirement the cost escalates quickly.

Figure 6.14 shows how both light and heavy weight buildings operation for a 12% PPD during the summer day of operation. The lightweight building requires a short period of preheating, but more importantly, it requires a great amount of cooling at the end of the occupied day. The cost of doing this is being offset by increasing the airflow rate to the space. In contrast, the heavyweight structure requires very little cooling at the end of the day, and the fan is kept at a minimum for most of the occupied day.

Both the winter and summer season seem to suggest that the lightweight building is far more responsive by the way a lot of the control in the space is achieved using airflow-rate control. A minimal amount of preheating is done before the start of occupancy, although during the summer most of the time there is a cooling requirement. This is done to give a precise comfort level throughout the occupied period.
6.1.4 The Effects of Glazing.

There are two glazing variables, the type and the percentage of glazing forming of the part of the external wall construction.

The Effect of Glazed Area

The more glazing the more effect solar radiation can have on the space and in turn the more heat can be lost or gained from the external environment, therefore, the trade-off curves become less optimal as the glazing increases.

![Figure 6.15: Comparison of Trade-off Curve for Different Glazing Areas](image)

Figure 6.15, illustrates the trade-off curves for the heavy-weight building and different glazed areas (with the low emissivity glass). Note that the trade-off values are for the sum of the three season (days) operation, so that a 15% PPD indicates each of the three days were operating at a 5% PPD. The performance during each season is illustrated in Figure 6.16. For the winter and swing seasons the glazed area makes very little difference to the trade-off curves the increase in glazing making the trade-off slightly worse).
In contrast, during the summer season, the increase in glazing greatly affects the trade-off curves, this being due to the higher solar gains causing higher cooling loads.

Figure 6.16: Break Down of Seasonal Trade-off Curves for Different Glazing Areas.

Figure 6.17: Comparison of Trade-off Curve for Different Glazing Types.
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The Effect of Glazing Type

The second factor in glazing is the type, clear or low emissivity. Compared to conventional glazing, low emissivity transports less heat (low U value), and absorbs more of the sun's energy. The low emissivity glass performs better in every aspect of glazing this is confirmed by the more Pareto optimal curve (Figure 6.17). The proportion of improvement over the clear glazing is comparable for every season (Figure 6.18). No season dominates the overall trade-off solution.

The analysis of the effect of building weight and glazing on the trade-off curves indicates that, for the example building, the differences in the solutions are driven by the summer season operation (and the need for cooling). From examination of the different elements separately, the overall optimal building choice should be heavy weight building with 10% of the external wall formed from low emissivity glazing.

Figure 6.18: Break Down of Seasonal Trade-off Curves for Different Glazing Types.
6.1.5 Simultaneous Building Envelope, Plant Size and Control Schedule Optimisation

Figure 6.19 shows the operational cost versus discomfort trade-off curve as given by the simultaneous optimisation of the plant and building selection as well as the control schedule. All the solutions forming this curve are heavy weight buildings with 10% of the external wall being formed by low emissivity glazing. As all the solutions on the curve are the same construction, the optimality is confirmed by comparison to the trade-off curve obtained for the fixed building type optimisation.

![Figure 6.19: The Control Scheduling, Plant Sizing And Simultaneous Optimisation Trade-Off, Compared to Fixed Building Optimisation](image)
CHAPTER 6: EXPERIMENTAL RESULTS AND ANALYSIS: CONVENTIONAL BUILDING CONSTRUCTION

In Figure 6.20, the discontinuity of the winter season curve is caused by the lightweight building. Previously (section 6.1.5) when examining different building weights the light weight building was shown to be the most optimal building construction for the winter season. It was also shown in that section that although the light weight construction was the most optimal for both the winter and swing seasons, the combined trade-off curve was dominated by the summer season and therefore the heavy weight construction. It is therefore expected that during the optimisation process that other construction types are considered, as the seasonal results express the non-dominated solutions for each season throughout the search. Although the seasonal curves have been cultivated during the optimisation process, they are affected by the overall optimisation process, as the building and the plant selection is based on the Pareto optimality of all three seasons.
6.2 Capital Cost versus Operational Cost

When optimising the capital cost in relation to the operational cost, the analysis is concentrated on the building selection, plant size and control schedule. To have a trade-off using capital cost, the plant and building selection have to be variables, otherwise the capital cost remains constant.

The trade-off achieved from analysing the plant-size and the control schedule will be very limited. This is because a trade-off curve requires opposing objectives, where one is minimal when the other is at a maximum. When the operational cost is at a minimum then the plant-size required to meet this cost is smaller and therefore the capital cost is also close to minimum. When optimising the building and plant-size the building cost causes coarse increments in the trade-off curve however any detail will be given by the plant capital cost.

Unlike the operational cost versus discomfort it is harder to justify what is expected to be seen in the trade-off curve, in this case the curve is produced from both logical argument, and using a specifically targeted single objective optimisation.

The comfort is not optimised, however to keep the levels of discomfort to an acceptable level each season is constrained to an maximum of 10% PPD.

6.2.1 The Ideal Trade-off Curve

The trade-off curve is initially defined by examining the two extremes of the curve, which in the case of investigating capital cost versus operational cost is lowest capital cost possible and consequently the highest operational cost, and visa versa.

With no constraints applied to the problem, the minimum capital cost is the cheapest building construction using the smallest plant size. This is when the smallest plant size, 0.5m by 0.5m heating and cooling coils with only one pair of tubes and the smallest fan dimension are used (£5608.00). This combined with the lowest building cost (£5369.16), which is for the medium weight construction with 10% clear glazing on the external façade gives the lowest possible capital cost within this model as £10977.16.
The lowest operation cost is achieved theoretically when plant size is selected to achieve the minimum amount of cooling, heating and the smallest pressure. This is achieved when the fan is run at minimum power, which is off during unoccupied periods and 0.183 $\text{kgs}^{-1}$ during occupied periods. Just running the fans for the minimal use gives a cost of £0.04. The building type that allows the lowest operation cost to take place is the heavy weight construction using 10% low emissivity glazing. This is because it is the most thermally insulated and thermally unresponsive building type. This has been confirmed by successive optimisation procedures always picking this construction as the lowest operation cost.

The capital cost axis bounds are between the lowest capital cost (medium weight 10% clear) and the capital cost with the lowest operational cost (heavy weight 10% low emissivity). The lower the operational cost the less the plant is used and therefore the smaller the plant selection possible and lower the plant capital cost. Therefore it is possible to assume the trade-off curve is going to be described primarily by the increase in building capital cost. Figure 6.21 shows that only 6 construction types fall between the lowest capital cost building (medium weight 10% clear glazing) and the lowest operational cost building (heavy weight, 10% low emissivity glazing). These are heavy
weight low emissivity 10% and 20%, medium weight low emissivity 10% and 20%, heavy weight and medium weight clear 10%.

A single objective genetic algorithm was used to minimise the operating cost of each building construction. Figure 6.22 shows that only the 10% glazing building types actually form a possible trade-off curve.

![Pareto Curve Based on Cost Ranking](image)

When the design constraints are applied, it is recognised that it may not be possible to utilise the minimal control schedule and plant size for every building type. Therefore, to find the ideal trade-off curve it is necessary to use a single genetic algorithm repetitively constrained at various building types to describe the curve. The building types still primarily shape the trade-off curve as the minimal plant size and control schedule is sought, therefore the building type was fixed and control scheduling and plant sizing optimisation was used. The building type was also optimised for a number of constraints to test the validity of the building choices.
**Chapter 6: Experimental Results and Analysis: Conventional Building Construction**

The trade-off curve are shown in Figure 6.23, it shows that only a few points describe each building type, this is because of the minimal trade-off experienced between the plant sizes capital cost and the operations cost.

The operational cost is the lowest when the load is the smallest and therefore the smallest capital cost is when the smallest plant is used to meet this load. However, in Figure 6.23 it can be seen that there is more than one solution for each building type, therefore there is a small trade-off case by the plant size selection. When there is very little or no load on the coil, a reduction in operational cost can be achieved by increasing the coil dimensions, as shown by points 1 to 4 in Table 6.1. Table 6.1 demonstrates that

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![Diagram](image-url)

**Figure 6.23: Single GA Ideal Trade-off Curve for Conventional Building**

(generated by single objective GA)

<table>
<thead>
<tr>
<th>Ref Points</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
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<th>6</th>
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</tr>
<tr>
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<td>1</td>
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<td>7.24</td>
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</tr>
</tbody>
</table>

**Table 6.1: Coil Size for all solutions on ideal trade-off curve**
optimising both the capital cost and the operational cost, drives the coil configuration close its minimum bounds. The number of circuits is allowed to increase, as these have no bearing on the capital cost, and fan energy consumption. Figure 6.25 shows how for an increasing coil face area, the operational cost decreases. As the coil area increases the frictional losses reduce and the power from the fan required for overcoming this decreases. With the increasing coil dimensions, there is an increase in capital cost (Figure 6.26). The relationship between reducing operational cost with capital cost (increasing fan size) is reflected by the heavyweight construction shown in Figure 6.24.

The trade-off curve for conventional building type forms almost a corner, rather than a curve. The low emissivity building forming the left side (capital cost axis), with the medium weight forming the corner, the spread along the running cost is created by the clear glazing types. The shape of the curve may cause a problem for any search algorithm, as there is very little difference in the capital cost at one end and little difference in the operation cost at the other. The sharpness of the 'curve' will cause problems finding a good spread of solutions.

Figure 6.24 Plant Cost (Fan size always = 380mm) for Conventional Building Ideal Trade-off Curve
CHAPTER 6: EXPERIMENTAL RESULTS AND ANALYSIS: CONVENTIONAL BUILDING CONSTRUCTION

Figure 6.25: Comparison of Operational Cost with a Changing Coil Face Area

Figure 6.26: Relationship between capital cost and increasing coil face area.
CHAPTER 6: EXPERIMENTAL RESULTS AND ANALYSIS: CONVENTIONAL BUILDING CONSTRUCTION

Figure 6.27: Winter Setpoints for Conventional Building 10% glazing (for reference numbers see Figure 6.23)

Figure 6.28: Swing Setpoints for Conventional Building 10% glazing (for reference numbers see Figure 6.23)
Examine the trade-off curve, it is true to say that the progress across the curve is caused by the building type modifying the capital cost. All the solutions formed from, or close to the smallest plant size (and therefore the cheapest).

The increase in operational cost is caused by the increasing load requirement, this is caused by both an increase in fan use and coil operation. The temperature setpoints that do not correspond to active coil operation have been removed from Figure 6.27 to Figure 6.29. As the operational cost is compared, only the setpoints that cause an increase in cost are of interest. It is reasonable to assume that if there is additional fan operation outside occupancy and there is no coil load then all fresh air is being used. The same applies to occupancy however it is also possible that the heat recovery device is being used to meet the required setpoints. The heat recovery device is always operational during occupancy except when fresh air is being used to directly cool the space. An increase in coil load is not apparent from the temperature setpoints for the winter operation. Examining this in more detail it can be seen that for all building types, the heating requirement is very minimal.
For the example building there is no pre-conditioning except in the summer for the clear glazing building types, in the case of the heavy weight building this is in the form of an additional hour of fan operation just before occupancy. In the case of the medium weight this takes the form of an addition hour of cooling at 3 am. (Figure 6.30). This is followed by a little heating in the first hour of occupancy, which seems to suggest that the trade-off curve may be slightly sub-optimal.

![Graph showing plant loads for summer for Point 9 on the trade-off curve](image)

**Figure 6.30: Plant loads for summer for Point 9 on the trade-off curve**

### 6.2.2 MOGA Generated Trade-off curve

With the optimisation of the capital cost versus the operational cost the performance of the MOGA and the split fitness MOGA concentrates on a narrow band of optimal operational cost, with any spread being formed by the capital cost.

Figure 6.31 shows the results of optimisations for both MOGA and Split fitness MOGA with different crossover rates. The crossover rate was changed to try and encourage more than building type to be found in one search. The random criteria were not increased to encourage movement as it is already set quite high, any higher then there is a risk that the optimisation will never converge towards the trade-off curve.
The split fitness algorithm again seems to outperform the MOGA, however it can be seen that both are unable to find all points on the ideal trade-off curve (Figure 6.1).

Analyzing the results from the split fitness with 0.6 crossover shows that only one building type is selected, which is heavy weight low emissivity with 10% glazing. As shown before, this solution is the dominant optimal solution. The lack of spread in the building selection is consistent with all the solution groups, with only the MOGA at a 0.4 crossover choosing something other than the heavy weight building, however it still has a solution set consisting of just the medium type.

The probable reason for the failure of the search is that changing just the building weight in a solution causes a solution generally to become infeasible. If the building type for one of the split fitness 0.6 crossover solutions were changed from heavy weight to medium weight then it would be expected for the capital cost to decrease and the operational cost to increase. However if this is done directly then the comfort constraint of 10% for winter is exceeded, therefore the solution becomes infeasible and is rejected against the large number of feasible solutions already available to the algorithm.
This jump from feasible to infeasible causes a very discontinuous search space for the capital cost. The areas of continuous search space are caused by the varying plant size, and it can be seen that the algorithm manages to define these. The spreading of the solutions due to plant cost can be seen more clearly when the search algorithm has a fixed building type, Figure 6.32 shows the results from four separate split fitness MOGA searches.

![Diagram](image)

Figure 6.32: Four sets of MOGA solutions for Fixed Building Types

Again the minimal difference between the medium weight and heavy weight building constructions is demonstrated, and the emulation of the ideal curve is caused be the glazing type along the operational cost axis. Although now the spread is forced along the operational cost axis, most of the time the algorithm is not quite finding the true optimal solution. The exception to this is the medium weight clear glazing solutions, as these actually show a very slight improve on the ideal solution.

6.3 Discussion

For the example optimisation problem, heavy weight structure with low emissivity glazing is the most optimal solution for the lowest operational cost. This has been shown
conclusively by both the capital cost versus operational cost optimality justification, as well as the results from the operational versus comfort study.

The difference between the medium weight construction and the heavyweight, is very minimal, this is mainly due to the dominance of the internal wall structure on the model. The operational cost versus discomfort shows the consistent effect of the glazing type and area on the solutions.

The seasonal comparison of the building weights demonstrates that the specific building problem is primarily a cooling problem, with the summer season dominating most of the trade-off curve for the various building options. Although it is nominally a cooling problem the loads are such that plant size has little effect on the search, often the minimum size being adequate. This is confirmed by the fact the when examining the coil signals the coil operation never becomes great enough to be visible on the graphs shown throughout this chapter. This is again confirmed by the process of finding the ideal trade-off curve for the capital cost versus operational cost analysis, with the only increase in plant size being caused by the reduced resistance and therefore reduced operational cost. This is method of plant size increase is only possible, however, when there is no load on the coil for any season.

For a relatively continuous solution space the MOGA and more specifically the Split Fitness MOGA produced near optimal trade-off curves. For the more discontinuous capital versus operating cost optimisation sub-optimal trade-off curves were generated. This was caused by the shear dominance of the heavyweight low emissivity 10% as the optimal building type, as well as limitation of the algorithm itself.

Although the solutions can be seen to be finding the near optimal solution with the continuous operational cost versus comfort curve, they have also shown that there is room, even after the extensive generations, for improvement.

These solutions made use of the cheaper night conditioning especially when the optimal comfort for the occupant is sort. The effect related to this is more distinct when incorporating a ventilated slab into the model. This is investigated further in 0.
Chapter 7

Experimental Results and Analysis: Ventilated Slab Construction

Whilst studying the different building types of the conventional building it was clear that the most optimal building type was the heavy weight, due to insensitivity to the surroundings. It also became obvious that the least optimal building type was the light weight construction with the greatest quantity of standard glazing because of its sensitivity to the external environment, and the internal loads.

This chapter compares the advantages of effectively increasing the thermal mass of the building, by the use of a model of a hollow core ventilated slab into both the ceiling and floor. The ventilated slab model is described in Chapter 4.

7.1 Optimising Discomfort versus Operational Cost

The effect of the extra thermal mass provided by the hollow core ventilated slab is investigated by analysis of the effect this has on the discomfort versus operational cost trade-off surface.
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This is done on the 2 extremes of building types, the lightweight construction with 30% standard glazing and the heavy weight construction with 10% low emissivity glazing.

7.1.1 Heavy Weight Construction with 10% low emissivity Glazing

From the analysis of the conventional building, it is suggested that additional mass in the external walls was benefiting the operational cost of the building. In this case the mass was influenced by the behaviour of the heat loss/gain to the external environment. The fact that there is some preconditioning is taking place means that the mass is effecting the setting of the control schedule it however it is not being deliberately adding storage (mass) inside the building to encourage the preconditioning of the space.

Figure 7.1 shows how the conventional heavyweight building type is much more optimal in comparison with the trade-off curve produced by optimising the ventilated slab building. In this case the extra thermal mass seems to be having no beneficial effect on the building performance. This is explored in more detail by examining the seasonal differences between the trade-off surfaces in Figure 7.2. The figure shows that the trade-off surface for the ventilated slab is worse for all of the seasons, with the smallest difference being in the summer season, the difference becomes more exaggerated as the seasonal temperature decreases. The winter season therefore has the most pronounced difference.

The results for the winter season are compared at 8% PPD, as at this percentage the largest difference in results is experienced. The conditions are met with no preconditioning of the space, the fan power is kept to a minimum just providing the minimum fresh air requirement during occupancy. (Figure 7.3).
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Figure 7.1: Trade-off Surface Comparison For Different Slab Construction For The Heavyweight Construction With 10% low emissivity Glazing.

Figure 7.2: Seasonal Trade-off Surface Comparison For Different Slab Construction For The Heavyweight Construction With 10% low emissivity Glazing.
The cost of providing the coil load and the minimum fresh air, is £0.225 compared to the ventilated slab, and for meeting the same level of discomfort the cost is £0.749 almost 4 times the operational cost. The increase in cost is caused by the initial pre-conditioning of the space. With the ventilated slab there is very minimal conditioning to the space is done during occupancy. The fan is kept at a minimum to just provide fresh air to the occupants. All the air conditioning is done the hour before occupancy, with a greatly increased fan and coil load (Figure 7.4).

The reasoning behind the use of the ventilated slab is that thermal energy can be stored within the structure, therefore minimising the requirement for air-conditioning during occupancy when the energy cost is more expensive. Using this reasoning the ventilated slab is performing as expected however, the energy required for this preconditioning of the ventilated slab is nearly 4 times greater than the conventional building. A maximum of 20kW coil for one hour load compared to the maximum of approximately 6kW over 4 hours for the conventional slab (Figure 7.4).

The main reason for this extra energy requirement is the additional thermal mass, this is confirmed by the lower thermal response during the occupied day to the external air temperature and the internal heat loads of the ventilated slab.
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Figure 7.4: Plant Operation and Temperature Setpoints for the Ventilated Slab at 8% PPD

Another contributing factor to the greater difference in performance for the lower temperature seasons is that the heating is supplied by gas, which in this case is supplied at a constant cost throughout the day. Therefore, the only benefit of preconditioning the space is the lower electricity cost of providing the fan power.

Winter Operation at the Lowest PPD.

To investigate further why the hollow core ventilated slab is out performed in almost all cases by the conventional building, both the plant operation and the temperature fluctuations are compared. As there are many points on the trade-off curve the point at which all these comparisons are made are at the low discomfort, high cost end of the curve. This end of the curve is chosen because it is the condition for which the coil is most active, the discomfort level is a known minimum bound.
Figure 7.5: Plant Operation for Conventional Building

Figure 7.6: Plant Operation for the Hollow Core Ventilated Slab
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For the convention building construction, Figure 7.5 shows that some pre-conditioning of the space is used. However to achieve almost optimum levels of comfort within the space, the space is heated at the start of occupancy, with the required amount of heating reducing as the day progressed. When comparing this to Figure 7.7, the plant outlet temperature follows the same pattern as the coil loads. The reduction in plant temperature is the inverse to the increase in both the ambient temperature and the temperatures of the walls. These temperatures work in conjunction to achieve a relatively consistent zone temperature during the occupied period after the initial start. A consistent zone temperature allows the PPD to remain constant.

The plant duty for the ventilated slab shows a large amount of preconditioning to achieve the required initial occupancy conditions, which is why in comparison to the conventional building to achieve the same level of comfort it is more costly (Figure 7.6). In Figure 7.8 the plant outlet temperature is shown to be a reflection of the plant operation. The large increase in plant outlet temperature causes a small increase on supply temperature (Figure 7.9), which is necessary to bring the zone temperature up as occupancy of the space begins. As the ambient temperature and the walls temperature increase throughout the day the supply temperature decreases to maintain a consistent

![Figure 7.7: Temperature Variations for Conventional Building](image-url)
zone temperature during occupancy. To achieve this drop in supply temperature the plant outlet temperature reduces dramatically, and even resorts to doing a little cooling. Consistently the hollow core can be seen to have a large dampening effect on the supply temperature, large variations in plant outlet temperature (slab inlet temperature) have very little effect on the supply temperature.

![Figure 7.8: Temperature Variation for the Hollow Core Ventilated Slab](image)

In comparing the slab constructions, the large increase in cost to maintain the same level of comfort is caused by having to precondition the slab as well as the zone.

The slab has to be at a temperature before occupancy to give the correct supply temperature. The slab seems to retain this temperature and to modify the supply temperature requires large changes in the plant outlet temperature.
In comparing Figure 7.7 and Figure 7.8 it appears that the ventilated slab building model is cooling the zone, as the supply is less than the zone temperature. Whereas the conventional building is heating the zone, as the supply temperature is higher than the zone temperature, this is due to the high radiant temperature in the ventilated slab building leading to a need slightly lower supply temperature.

### 7.1.2 Light Weight Construction with 30% Clear Glazing

It was concluded that the lightweight construction with 30% glazing to be the worst construction type in terms of optimality of the operational cost versus discomfort trade-off surface (Chapter 1, section 6.1.3). It is expected that with an increase in thermal mass the ventilated slab will improve on the results of the trade-off surface, however from Figure 7.10 it can be seen that as with the heavy weight construction the conventional building is still out performing the ventilated slab.

![Trade-off Surface Comparison for Different Slab Construction for the Lightweight Construction With 30% Clear Glazing.](image)
As with the heavyweight construction, Figure 7.11 shows that the main cause of the difference between the conventional slab and the ventilated slab is the winter season. In the summer season there is little difference between the two slab types, with the ventilated slab actually performing better for the low operational cost.

![Seasonal Trade-off Surface Comparison For Different Slab Construction For The Lightweight Construction With 30% Clear Glazing.](image)

Figure 7.11: Seasonal Trade-off Surface Comparison For Different Slab Construction For The Lightweight Construction With 30% Clear Glazing.
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

Figure 7.12: Plant Operation and Temperature Setpoints for the Conventional Slab at Winter 40% PPD

Figure 7.13: Plant Operation and Temperature Setpoints for the Ventilated Slab at winter 40% PPD

Comparing the temperatures and the plant power usage at 40% PPD for the winter seasons shows how even with this high level of discomfort there is a requirement for
heating throughout the day with the conventional building type. Figure 7.12 show that again no pre-conditioning is done for the conventional slab, and in comparison, Figure 7.13 shows that all the conditioning is done before occupation for the ventilated slab. The airflow rate is kept to a minimum throughout the day for the conventional building type, however with the ventilated slab more control throughout the day is achieved by varying the airflow rate, therefore varying the amount of heat removed from the slab. As with the heavyweight comparison all the coil operation is done before occupation, the increased load in the space is evident as 2 hours of preconditioning requires 25kW.

As the cost of gas is constant throughout the day, the fact that the preconditioning has the fan nearly at full power using electrical energy is driving the conditioning of the space to take place before occupation. The slab has to be at specific condition before occupation, to allow the outlet temperature of the slab and therefore zone supply temperature to be achieved.

![Graph](image)

Figure 7.14: Plant Operation and Temperature Setpoints for the Conventional Slab at Summer 80% PPD

The ventilated slab only becomes more optimal for high levels of discomfort in the summer season. Examining this in more detail in Figure 7.14 and Figure 7.15 show how
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

the conventional slab again as with previous examples does no preconditioning of the space and the ventilated slab makes much greater use of this possibility.

The lightweight construction with 30% clear glazing has a high air-conditioning load to achieve any level of comfort. Despite providing cooling for a large portion of the day, the conventional slab achieves a level of 80% of PPD.

In comparison, the ventilated slab allows a great deal preconditioning to take place, by effectively giving the lightweight construction more mass. This allows the same level of maximum discomfort to be achieved by providing no additional cooling, other than that provided by using the cooler night air to give free cooling. The use of free cooling is more cost effective than providing the cooling to the building during the day hence the ventilated slab achieves a lower operational cost.

![Graph](image)

Figure 7.15: Plant Operation and Temperature Setpoints for the Ventilated Slab at Summer 80% PPD

The ventilated slab allows pre-conditioning to take place as it forms the ceiling and floor of the room, but this does not have the same effect as actual adding thermal mass e.g. making a light weight construction, heavy weight. Actual thermal mass suppresses the
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

effects of the external and internal environments on the actual condition in the environment experienced by the occupant.

Even with the ventilated slab the heavy weight construction is more optimal. The smallest amount of glazing using low emissivity ensures that the external environment has the minimum amount of impact on the space.

**Winter Operation at the lowest PPD.**

As with the heavy weight construction further comparison is made at the low discomfort, high cost end of the curve for the winter season.

The lightweight construction is more reactive to environmental and internal conditions than the heavy weight construction. Figure 7.16 shows how for the conventional slab both heating and cooling is required throughout the day. The lowest level of discomfort is achieved with only minimal pre-conditioning of the zone.

The fact that the space requires cooling is explained when examining the wall temperature in Figure 7.18 as this rises to well above the level required to supply the space. The change in temperature in the zone is also maintained utilising a larger airflow rate than experienced by the heavy weight construction. This is to facilitate control of the space as the zone is quick to react to the plant temperature changes and has little means of retaining the temperature being provided.

Unlike with the heavy weight construction a constant supply temperature does not relate to a constant low PPD, as the internal wall temperature affects the space so greatly the supply temperature decreases to compensate.
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Figure 7.16: Plant Operation for Conventional Building

Figure 7.17: Plant Operation for the Hollow Core Ventilated Slab
Figure 7.18: Temperature Variations for Conventional Building

Figure 7.19: Temperature Variation for the Hollow Core Ventilated Slab

The hollow core ventilated slab requests very minimal cooling (Figure 7.17). As with the heavy weight construct a great deal of heating is required to bring the slab temperature up to ensure that the supply temperature at occupation is met. This is what causes the cost
variations between the conventional slab and the hollow core ventilated slab. The flow rate into the space is increased for the preconditioning, reduced for occupancy and again increasing towards the end of occupancy, as with the conventional slab the airflow rates are higher than that experienced with the heavyweight construction.

The supply temperature remains relatively constant throughout occupancy however the zone temperature increases with the wall temperature (Figure 7.19). This causes a non-optimal solution for the level of discomfort, the level of discomfort increases as the zone temperature increases (Figure 7.20)

![Figure 7.20: Measure of people dissatisfied (PPD) for the Hollow Core Ventilated Slab](image)

7.1.3 Optimising Building Envelope, Plant Size and Control Scheduling.

Optimising the building fabric as part of the overall optimisation shows again that the conventional slab provide a better trade-off surface to that of the ventilated slab (Figure 7.21). As with the conventional slab construction the only building type that forms the optimal trade-off surface is the heavy weight low emissivity construction.
When comparing the optimisation results of this algorithm with the results of the fixed heavy weight low emissivity 10% glazing construction it is possible to see that the
similarity of the curves (Figure 7.22). The curve similarity is not as distinct as that produced from the same comparison for the conventional slab. In this case the optimisation process benefits from the added freedom in the search.

7.2 Capital Cost versus Operational Cost

The capital cost versus the operational cost has been compared for the ventilated slab, as done previously with the conventional building. From the results previously discussed in this chapter, it was shown that there is a greater use of the air-conditioning plant, therefore it would be expected that there will be a greater relationship shown between the operational cost and the plant capital cost, than that shown for the conventional building type.

As the comfort is not an objective, each season is constrained at 10% PPD.

7.2.1 The Ideal Trade-off Curve

The ideal trade-off curve for the ventilated slab models was constructed using the same procedure as with the conventional construction. The main difference being the additional capital cost of the construction, however, there is a fixed additional cost and this is constantly applied to all of the building types. Figure 7.23 shows the different capital cost for each of the building types. The different constructions have exactly the same relationships with each other as with the conventional building, with just the magnitude of the capital cost changing. Now the lowest theoretical capital cost that the model can achieve including plant cost is £11052.33 for the ventilated slab construction, compared to £10977.14 for the conventional building.
As with conventional slab type, a single genetic algorithm is repetitively utilised to produce the trade-off curve, as shown in Figure 7.24. The graph shows that only a few points describe each building type, this is because of the minimal trade-off experienced between the plant sizes capital cost and the operations cost. There is however, a greater relationship between the plant size and the operational cost, seen with the heavy weight low emissivity 10% for the ventilated slab construction, giving a larger plant size for a...
lower running cost (Table 7.1). The higher plant size decreases frictional resistance in turn reducing the fan power required to achieve this and therefore the operational cost. It would be expected that the number of rows would be lower to further reduce the fractional effect, although the coil constraints may be restricting the search, preventing it from finding the minimum number of rows, it demonstrates that there is still some room for improvement.

![Ref Points](image1)

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<table>
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<td>Cost</td>
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<td>No. Circuits</td>
</tr>
<tr>
<td>Max Water Flow Rate</td>
</tr>
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Table 7.1: Hollow Core Ventilated Slab Coil Size for all solutions on ideal trade-off curve

![Figure 7.25](image2)

Figure 7.25: Plant Size (Fan size always = 380mm) for Ventilated Slab Building Ideal Trade-off Curve
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

Figure 7.25 shows that only the heavyweight construction with 10% low emissivity glazing uses the relationship between increasing coil size and reducing frictional effects. This causes a very angular trade-off surface. Examining the heating coil operational cost, the operational cost increases to meet the increasing heating requirement shown in Figure 7.26. The plant size does not increase to meet this requirement, indicating that the plant size available to the optimisation process is oversized for the load being optimised. In Figure 7.28 for the clear glazing there is a marked increase in preconditioning of the space, indicative that there is an increase in load because of the glazing. Even with the increased cooling requirement, the optimisation makes good use of the free cooling available from the ambient air. The swing season, shows the least plant operation of the three sample days, only doing a couple of hours preconditioning even with the clear glazing (Figure 7.27), as it has the least conditioning requirement of the 3 design days.

In comparison with the same results for the conventional building (Figure 6.27 to Figure 6.29), the ventilated slab is doing much more conditioning of the space, agreeing with the results shown in the operational cost versus comfort comparison.

Figure 7.26: Winter Setpoints for Ventilated Slab Building 10% glazing (for reference numbers see Figure 7.24)
Examining how the trade-off curve is constructed it is true to say that the progress across the curve is caused by the building type modifying the capital cost. All the solutions with the exception of point 1 on the ventilated slab construction (Figure 7.24) are being formed from, or close to the smallest plant size (and therefore the cheapest).

The inconsistency of point 1 (Figure 7.25) is caused by the fact the cooling coil is not being used, which enables the size to increase without effecting the performance. As investigated previously in Chapter 1, the increase in plant size reduces the frictional resistance and allows a reduction in the operational cost.
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

Figure 7.28: Summer Setpoints for Ventilated Slab Building 10% glazing (for reference numbers see Figure 7.24)

7.2.2 Multiobjective Algorithms Performance

Figure 7.29: MOGA Solutions to Capital cost versus Operational Cost for the ventilated Slab
CHAPTER 7: EXPERIMENTAL RESULTS AND ANALYSIS: VENTILATED SLAB CONSTRUCTION

When viewing Figure 7.29 the performance of the algorithms seems disappointing. The solutions seem to rarely adequately describe the ideal trade-off surface. This again is caused by the difficulty the algorithm faces moving between building types.

In the operational cost versus discomfort it was evident that ventilated slab required a great deal more conditioning to meet the same levels of discomfort as the conventional slab. In that investigation there is no constraint on the capital cost therefore the coil sizes were often large to achieve the required level of conditioning. This demonstrated that achieving the same levels of conditioning in the space is a harder problem to technically achieve in comparison to the conventional slab. For example, as the plant size is forced to reduce then the flow rate though the coil increases to achieve the discomfort constraints, this in turn increases the operational cost.

The stronger relationship between the plant size and the operational cost makes the ventilated slab optimisation effectively harder than the conventional slab as the plant loads were smaller. The fact that there is a stronger relationship doesn’t mean that there is a higher degree of trade-off.

7.3 Discussion

The effect of the ventilated slab is almost exclusively sub-optimal in comparison to the trade-off surface of the conventional slab. This is because the extra conditioning required for ventilated slabs is rarely used to save money, even when the entire preconditioning takes place during the times of the day when the fuel costs is cheapest. The difference in fuel cost between on and off peak is not great, which is working against the optimality of the ventilated slab.

The heavyweight construction is again shown to be the optimal weight of construction, this demonstrates that the reactive the space is made to both internal and external fluctuations in temperature, the more optimal the trade-off surface.

The optimality of the operational cost versus discomfort trade-off surface is not investigated again as it was examined in detail in the previous chapter. The optimality of
the capital cost versus operational cost is again disappointing. This is because of the discontinuity formed in the algorithm's search when moving between building types.

Comparing the operational cost with the discomfort showed that for the same measure of discomfort the plant is doing a lot more work in comparison to the conventional slab. When comparing the capital cost the actual minimum size is not found as frequently as with the conventional slab, this is not unexpected as the algorithm is constrained to meet 10% PPD, which is harder to achieve with the ventilated slab construction (as demonstrated by examining operational cost versus comfort).

The greater relationship with capital cost and operational cost because of the increased plant loads is not as evident as expected. Even with the increased plant loads the coils are operating near the minimum control signal and are barely visible on most control signal graphs. This demonstrates the coils are still oversized, as with the conventional building construction.

The ventilated slab during this investigation shows no marked benefit to the occupants or in energy savings.
Chapter 8

Decision Making

Throughout this thesis the 'decision maker' has been referred to as the person or persons that will evaluate the set of optimal solutions. As the algorithm presented here is a posteriori optimisation process (Chapter 3, section 3.3) the decision-maker has no bearing on the progress of the algorithm.

Presented here is a demonstration of what post optimisation processes can be applied after the main optimisation process is complete. There are 3 main areas of post-processing available to do this.

1. More detailing of the specific areas of interest (use of goal functions)
2. Evaluation of variables
3. Performance of the algorithm

The examples presented here demonstrating the information available to the decision-maker are all based on the optimisation of the control and plant size of the heavy weight construction with 10% low emissivity glazing. The trade-off surface for this is shown in Figure 8.1.
8.1 The Role of the Decision Maker

The primary role of the decision-maker is to define the importance of the objectives that are being optimised.

Within the optimisation process presented here, there are number of non-bias feasible solutions which normally take the form of a curve. If the decision maker is looking for a finite solution, once they have the solution set then this is the point at which the decision about what is more important is made (Figure 8.2).

In the case of the discomfort versus operational cost, the preferred solution will likely take the form of the lowest level of discomfort for the lowest or most acceptable level of operational cost for the decision-maker.

In the case of the operational cost versus capital cost, the most optimal building type was limited to just 2 types of building and glazing type. The decision-maker may be influenced by more cosmetic reasons than just the optimal building and glazing types.
The trade-off surface gives the decision-maker the information to make a decision based on the quantity of capital outlay that can be made to offset the future operational costs.

8.2 Goal Functions

Goal functions can be utilised at the beginning of the optimisation process to restrain the search to a specific area, or they can be utilised after the initial trade-off surface is established to refine the solution in the specific area of interest to the decision maker.

Using goal functions requires the optimisation process to be repeated (Figure 8.3), however as this is a search in a smaller solution area it is possible to limit both the population size and number of generations.

It can be argued that the goal functions just repeat the same search for a smaller solution area, however a more concentrated search can lead to better defined solutions within the
area of interest. Figure 8.4 shows the heavy weight construction with 10% low emissivity glazing with a goal function limiting the solutions to being less than the £0.50 operational cost. This leads to the algorithm performance, and therefore the trade-off surface to be improved at the lower operational cost level.

![Diagram of Goal Function Process]

Figure 8.3: Diagram of Goal Function Process

Figure 8.4 shows how the trade-off surface is improved and extended with the use of the goal function. Whilst analysis of the seasonal trade-off surface demonstrates that most of the refinement of the overall surface was achieved because algorithms greater detailing of the summer month. A slight improvement in the winter month is evident also from examining Figure 8.5, again confirming that the overall optimisation is primarily effected by the optimisation of the summer control of the space, and less so by the winter control. The swing season shows little variation as it has little influence in the direction of the search, and in turn minimal influence on the shape of the final trade-off surface. The seasonal curves are made up from the individual seasonal non-dominated solutions found throughout the optimisation, which do not necessarily form part of the combined trade-off curve. This combined with the fact that the search was concentrating on achieving the operational cost, means that the solutions at the minimum PPD solutions appear sub-optimal.
The trade-off surface shows that even with an extensive initial search there is always room for a little improvement. It also shows how, although the trade-off surface
optimality can be justified as done previously, the MOGA does not achieve true optimality. This does not necessarily mean that the algorithm has been prematurely halted, but slightly sub-optimal convergence has been achieved.

8.3 Assessing Convergence

The performance algorithm as discussed in Chapter 5, section 5.3 can give the decision maker more information on how the algorithm progressed (Figure 8.6). This allows a more informed decision about how accurate the final trade-off solution is, and whether solution convergence has been achieved.

There are 5 areas, which can require evaluating to assess the performance of the algorithm.

1. The size of the solution space.
2. Spread over the solution space.
3. How near the population is to the optimum solution.
4. The proportion of non-dominated solutions in a generation.
5. The speed of the problem solving.
CHAPTER 8: DECISION MAKING

There are 7 types of figures produced to present this information to the decision maker as described in (Chapter 5 section 5.3.2).

How the algorithm covers the solution space can be evaluated by comparing the difference between the maximum and minimum values for that objective. Figure 8.7 and Figure 8.8 show how for both objectives the actual area that the population covers represent individual solutions, reduced in size initially, but remains reasonably constant after about 80 generations. The information given here however can be misleading, as the interpretation of the graph can be either good or bad depending on the shape of the non-dominated surface. If the area reduced to a very small point, this would indicate that the population has converged on a point, generally this would not be good in a multiobjective optimisation problem, however it may be the true optimal solution. The fact that it’s not a straight line, and there is a lot of noise indicates that the solution space is moving, that the search is investigating the space well, or it can mean that the algorithm isn’t converging on a Pareto optimal solution. In comparison the first objective, the operational cost has more static than the second objective, maximum PPD. This is likely to be an indication that the algorithm finds it easier to move around the operational cost rather than the PPD.

![Figure 8.7: The Spread of Objective 1](image-url)
The space that the non-dominated individuals covers does not reduce in size to the same extent as the overall population individuals. This is what would be expected as the algorithm moves towards the Pareto surface, and the description of the Pareto surface becomes more accurate as the number of non-dominated solutions in the population increases.

The performance measure is designed to give the decision-maker an idea of how spread out the solutions are. From Figure 8.9 the spread of the individual can be seen to reduce initially, which would be expected as the solution converges on the Pareto optimal surface. However there is so much variation (noise) with this particular method of comparison that little else can be learned.
The performance measure in this thesis has been adapted to take account of the amount of space over which the spread has been measured. This is measured by what is called the relative performance measure. As the value increases it indicates how the spread of the solutions is covering more of the solution space. If the solution space decreases it is represented by a increase in value of the relative performance measure.

Examining performance measures for the 2 main objectives, and the infeasibility objective (Figure 8.10) illustrates how spread over the solution space improves. The infeasibility objective after the initial movement remains relatively constant, indicating that the solution space is small and the solutions are groups. Once the solutions are
feasible then they are all represented be the same value, therefore the results from the relative performance measure would be expected to show little spread and movement.

Comparing the 2 main objectives with the relative performance measure shows again how the algorithm is moving the solutions around but again not generally showing much change in trend after the first 100 generations. It is difficult to tell whether any additional convergence on the optimal non-dominated curve is achieved.

The distance performance measure as shown in Figure 8.11, measures the distance between each non-dominated solution and zero. This shows how the individuals in the populations are moving towards the non-dominated surface. As with the other performance measures so far there seems to be little improvement after the first 100 generations. It is important to realise that although this is a good measure of how the population is moving towards a true non-dominated solution, it does not give any appreciation of the shape of the trade-off surface; it is possible for this measure to appear to get worse but be, in effect, describing a different section of the trade-off surface.
Figure 8.12: Non dominated Solutions

Figure 8.12 shows the simplest form of performance measure, a simply count of the number of non-dominated solutions in each generation, as with all the performance measures there is little variation in the performance of the variation after the 100th generation.

Mainly because of noise, the information gained from the measures of performance is very limited. The noise is caused by the measures themselves, the high permutation rate in the algorithm to encourage movement in the solution space, and primarily by the size of the population used. The accuracy of the information can only be assessed in comparison with other optimisation runs or a known optimal solution. When examining the performance measure the actual convergence is difficult to judge however all the measures agree that there is very little variation in the solution after 100 generations.
8.4 Variable Sensitivity

Throughout the study it is obvious that a lot of information is available to the decision maker from the optimisation procedure itself. Through evaluating optimality a lot of this information has already become apparent.

In the case of the discomfort versus operational cost the control processes, which were variables within the optimisation, were evaluated to justify how well the algorithm has covered the trade-off surface.

With the capital cost versus the operational cost the variables that control the plant size were investigated to discover what plant sizes were being selected, and their contribution to the shape of the trade-off surface.

Here we explore the variable changes that takes place over the trade-off surface, for the discomfort versus operational cost. Figure 8.13 and Figure 8.14 show how the air flow and air temperature variables vary over the trade-surface. The mean value all the non-dominate solutions for each of the control variables is shown with the standard deviation on either side to give the decision maker an idea of how much the variable changes over the trade-off surface. The standard deviation isn’t shown for the hours up to 8am because only a very few variables are actually active during these hours.

Examining the mean and standard deviation of the variable, it shows that generally the supply air temperature decreases throughout the day, where the airflow rate keeps very low throughout the occupied period. The standard deviation shows that the solutions for the supply temperature initially vary over 5°C, however the results get more consistent as the day progresses. The standard deviation for the air-flow rate remains relatively constant throughout the occupancy period.
How each variable actually progresses over the trade-off surface can be examined more closely, Figure 8.15 and Figure 8.16 show the variables of the non-dominated set,
associated with 9.00am and 2.00pm vary as the operation cost increases. The straight bounding lines on these figures define the variable bounds.

At 09:00 in the morning it can be seen that over the trade-off surface the airflow rate generally keeps to the minimum setting, however there is a much bigger spread of solutions with the temperature variable at this time. The broad band of temperature variations demonstrates the sensitivity of the non-dominated set to this particular variable. The broader the band of solutions the less driving force the variable must have on the optimal solution set. In the case of 9:00am the solution set is generally increasing in temperature as the operational cost increases. This is consistent with the rational that it would cost more to provide more heat to the room. The variable reaches the upper bound relatively early in the progression across the trade-off surface and as the operational cost increases the grouping of solutions gets closer to this bound. This tells the decision-maker that this is probably limiting the optimisation, and in consequence forcing the shape of the trade-off surface.

Figure 8.15: The Temperature and Air Flow Variables for 9am
At 2:00pm the airflow rate again is sticking close to the minimum bound. The supply temperatures are much more consistent with most of them limited to a small band. At the very low operational cost the temperature doesn’t vary much from 20°C. Above £0.8 operational cost the results are mainly in a narrow band between 20°C and 25°C. This shows the decision maker that no matter what level of comfort or cost they are trying to achieve it is likely that the control of this variable will have to be maintained at around 20°C. This is because the problem optimality is sensitive to this variable but as it is varying little then it has little effect on the overall operational cost.

8.5 Discussion

Described in this chapter are a number of methods available the decision-maker to aid the understanding of the problem, and validity of the solution choice.

The goal function main disadvantage is the additional searching required, if there is a specific area of interest, it much more efficient to implement goal functions at the outset.
The more information available to the algorithm at the outset the more accurate the solution set that can be presented to the decision-maker.

Performance measures performed on the algorithm are more meaningful if read together, however even when this is done the information presented can often have conflicting meanings. The main point of all the measures of the algorithm performance is only relative to the known information. All these measures have no way of knowing how far the current non-dominated set is from the true non-dominated set.

A rough judgement on convergence can be made using the performance measures, however this may not be convergence on the optimum solution but more an indication of lack of further progression by the optimisation algorithm.

Evaluating the variables gives an indication of which inputs to the problem are instrumental in the formation of the trade-off surface. With the analysis using the MOGA it is easy to get away from the point of completing the optimisation process. At the end of the day, the decision-maker must be able to use the settings, this means be able to have access to the variables. Analysing the variables gives the decision-maker the choice, which can be fixed at a specific setting, and those that will highly influence the cost. For instance at both the hours shown previously it is going to be likely that the solution will require the air flow rate to be set at minimum, and with the 2:00pm temperature variable it could be evaluated that the best setting for this would be around 20°C to 25°C.

The main point of the MOGA is that no preconceptions hinder the search, however at some point it is necessary to for a choice to be made. It is best to make this decision as informed as possible.
Chapter 9

Conclusion

Multiobjective optimisation is a valuable tool to both the designer and optimiser. The information provided to the decision-maker allows an informed decision to be made about the importance of the problem criteria and therefore giving a greater appreciation of the problem being optimised.

A number of modifications were necessary to the basic MOGA construction in this study, the main one being the introduction of the infeasibility objective to handle the large number of constraints in the specific problem. It is apparent that although the algorithm is capable of optimising many objectives it is not always possible to interpret the results from this in a meaningful way.

The MOGA was proved to perform well when the trade-off surface was continuous however performance was limited when large discontinuities in the problem existed. The main discontinuity in the problem is that there was little interaction between three sample days, this was solved by adapting the basic MOGA to deal with the fitness’s separately within the same optimisation (split fitness MOGA). The discontinuity on the trade-off surface caused by the building types was not overcome by any of the MOGA setups available when comparing the capital and operational costs. This was primarily because of the very limited trade-off actually possibly between the building types and the fact that a feasibly solution in one building type would be a highly infeasible solution in another
building type, making it difficult to move between building types within the solution space.

The GA is, and continues to be a good optimisation technique both for single and multiple optimisation tasks. The MOGA has shown to have limited applications but along with the GA, different ways of handling the multiobjective functions in conjunction with the GA, are developing all the time.

The GA utilised in this study had a large population size and number of generations to endeavour to cover the trade-off surface extensively. It was shown that although the trade-off surface improved throughout the generations the improvement was minimal after the first 100 generations. Sharing functions were used to aid the search to cover the extremities of the trade-off surface, and although these extremities were demonstrated by close examination to be very close to optimal, it was possible to achieve an improvement in the trade-off surface if a goal functions was used to concentrate the search in the area of interest.

The heavy weight construction with 10\% low emissivity glazing was found to be by far the most energy saving construction, however the differences between this and the medium weight construction were slight because of the dominance of the internal mass. The trade-off surface for the different glazing areas demonstrated that they have a direct and proportionate relationship with the comfort and cost. Although for the purposes of evaluating the MOGA the model was adequate, the limited size of the model meant that the trade-offs in the results were limited. The understanding of the capital cost was restricted by the fact that the loads were often so small that even in the worse cases these were met by the smallest plant sizes, and little trade-off was obtained.

The hollow core ventilated slab was shown by the MOGA not to be a viable alternative because for all but a very small area of the summer seasonal trade-off curve, the conventional slab produced a better trade-off surface. The main reasoning behind this is the limited size of the model, because in most cases there was little air conditioning requirement. Having to pre-condition the hollow core slab just added to load, rather than making it possibly to shift the load to a cheaper part of the day. Another factor causing this disparity between the slab constructions was that there was very little cost saving available from preconditioning. The difference between the on and off peak energy costs were not distinct enough to offset the additional fan and coil loads required for the
hollow core slab. The hollow core slab however, did demonstrate that the space can be adequately conditioned almost entirely before the space is occupied.

The optimisation of the plant size and the controls schedules to achieve the varying optimal trade-off surfaces, has shown that this simultaneous optimisation to be both viable computationally but also beneficial to the optimisation process. Allowing the model to select the plant size permitted the optimisation process to achieve greater optimality and allowed a greater appreciation of the selection of the plant size effected the performance of the model. The inclusion of the building design into the simultaneous optimisation caused discontinuities in the problem space, which the MOGA did not negotiate well. This caused changes in the building to have a dominating effect on the optimal solutions. This may be overcome when utilising a larger model so that the effective links between plant and the building become stronger.

Overall the MOGA proved itself to be a valuable tool, providing the decision-maker not only with a selection of optimal solutions, but important information about the sensitivity of the model.

9.1 Areas for Future Development

There are two distinct areas of optimisation that have been combined in this study, building thermal optimisation and multiobjective optimisation. Both these areas independently are subject to continuing research and development.

Measuring the performance of the MOGA accurately has been an area where the current methods have proved to be misleading, and often complex to evaluate. Both measures of optimality and convergence are required that can be used reliably and simply over a broad range of problems, so that future improvement can be judged effectively.

It was evident by examining the variables across a trade-off surface that some have a much more important role than others. Development of the algorithm to pick these features up would aid convergence on the optimal curve, and possibly reduce the computational complexity of the problem.
CHAPTER 9: CONCLUSION

The optimisation of the plant size and control simultaneously was valid and a useful tool, however the model of the space limited this, a useful tool would be the development of a linked multiobjective and modelling tool. Allowing the MOGA to be used as a design tool in conjunction with computer models, would require the MOGA itself to be refined extensively to limit the number of times the model is called, otherwise extensive processing time will be required.
References


Appendix A

A Simple Genetic Algorithm

Genetic algorithms were, as the name suggests inspired from the processes observed in natural evolution. They combine the approaches of survival of the fittest with a structured, yet random exchange of information, to give a search algorithm with a degree of natural randomness.

Genetic algorithms were initially developed by John Holland, and colleges, and was first suggested in his book 'Adaptation in Natural and Artificial Systems'. Many papers and dissertations since have proven the validity, and robustness, of the technique in function optimisation and control applications. The effectiveness in an every increasing number of applications has lead to research in adaptations, for increased effectiveness and efficiency of the initial simple genetic algorithm approach.

Genetic algorithms are different from normal optimisation methods and search procedures in four ways;

1. GAs work with coding of the parameter set not the parameters themselves.
2. GAs search from a population of points, not a single point.
3. GAs use trade-off (objective function) information, not derivatives or other auxiliary knowledge.
4. GAs use probabilistic transition rules, not deterministic rules.
A.1 Terminology.

The algorithm is iterative, each attempt at finding the optimum solution is termed a *generation*. Unlike many other search techniques, which maintain a single 'current best' and try to improve on it, genetic algorithms return a set of solutions called a *population*. In each generation the new population is created from an old population. The population comprise of number of *individuals*, which formed by *strings*, each of these stores information about the variables, which make up one solution to the problem. Each population will contain a user-defined number of strings and in turn the string will be a user-defined size, depending on the number of problem variables and size. Rather than storing this information numerically the string consists of *bits* (0's and 1's). Figure A.9.1 shows a simple problem of maximising the function \( f(x) = x_1 \times x_2 \) for a population of 4 strings with 2 variables (\( x_1 \) and \( x_2 \)).

<table>
<thead>
<tr>
<th>String No.</th>
<th>String Decoded Variables</th>
<th>Decoded Variables</th>
<th>( f(x) )</th>
<th>% of Total</th>
</tr>
</thead>
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<tr>
<td>1</td>
<td>0110101000 13 9</td>
<td>117</td>
<td>10.71</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1001111011 19 27</td>
<td>513</td>
<td>46.98</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0011000111 6 7</td>
<td>42</td>
<td>3.85</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1010010101 20 21</td>
<td>420</td>
<td>38.46</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>1092</strong></td>
<td><strong>100.00</strong></td>
<td></td>
</tr>
</tbody>
</table>

where

\[
0110101000
\]

Encoded Variable \( x_1 \), Encoded Variable \( x_2 \)

Figure A.9.1: Example Simple GA Structure

To create a new population, each string forms the old population, then is decoded and the individuals *objective function* \( f(x) \) is calculated. The objective function characterises the optimisation problem, a typical example is to minimise cost, and therefore for each string the cost will be calculated. The individual suitability to be in the new generation is defined by the *fitness function*. The fitness function can be a direct measure of how the individual satisfies the objective function, however the operator can weight the solution,
or use *penalty functions* to allow the user some control of the algorithm's direction. The fitness function is strictly a measure of the individual capability to produce offspring, only and is normally achieved by maximisation. (To minimise, the fitness function is reversed)

The individual for the new population is chosen using a *roulette wheel*, each individual is given a space on the roulette wheel proportional to the fitness. Then an individual is chosen by spinning the roulette wheel, this gives a random selection method with a greater chance of success given to those with higher fitness. The roulette wheel is the basic selection process, and has been the subject of many alterations for specific problems, however at this stage the roulette wheel process is retained.

Once two individuals are chosen they can either enter the population unchanged or be *mated* or *mutated*. All manipulation procedures are done using the individual strings.

Mating is the primary method of reproduction, the strings of the two chosen individuals are mated to produce two new individuals for the new population. Mating takes place by randomly choosing a point along the strings length and swapping the ends (Figure A.9.2). To ensure that there remains an element of diversity, occasionally at random one of the new individuals will be selected to mutate, in which a bit is reversed (0 to 1 or 1 to 0).

\[
\begin{align*}
\text{Old Individuals} & \quad \begin{array}{c}
0110101000 \arrow\rightarrow \\
0011000111 \arrow\rightarrow 
\end{array} \\
\text{New Individuals} & \quad \begin{array}{c}
0110101011 \\
0011000100 
\end{array}
\end{align*}
\]

Crossover Point

*Figure A.9.2: Crossover*

At the end of this, a new population is created, which should be better than the initial population. Each time a new generation is started the new population becomes the old, and the process is repeated. The initial population is created by randomly producing sets of 0's and 1's. The process is stopped once a predetermined number of generations are reached. The process is summarised Figure A.9.3
APPENDIX A: A SIMPLE GENETIC ALGORITHM

Figure A.9.3: High Level Description of Genetic Algorithm