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Fault Tree Analysis and Binary Decision Diagrams

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SUMMARY & CONCLUSIONS

Fault tree analysis is now commonly used to assess the adequacy, in reliability terms, of industrial systems. For complex systems an analysis may produce thousands of combinations of events which can cause system failure (minimal cut sets). The determination of these minimal cut sets can be a very time consuming process even on modern high speed digital computers. Also, if the fault tree has many minimal cut sets calculating the exact top event probability will require extensive calculations. For many complex fault trees this requirement is beyond the capability of the available machines, thus approximation techniques need to be introduced resulting in loss of accuracy.

This paper describes the use of a Binary Decision Diagram for Fault Tree Analysis and some ways in which it can be efficiently implemented on a computer. The work to date shows a substantial improvement in computational effort for large, complex fault trees analysed with this method in comparison to the traditional approach. The Binary Decision Diagram method has the additional advantage that approximations are not required, exact calculations for the top event parameters can be performed.

1. INTRODUCTION

The fault tree diagram itself is an excellent way of deriving the failure logic for a system and representing it in a form which is ideal for communication to other managers/designers/operators etc. The fault tree is discussed in detail in Andrews and Moss (Ref. 1). Since the method was first conceived in the early sixties, algorithms to derive the minimal cut sets have worked directly with the fault tree diagram itself using either bottom-up, Semanderes (Ref. 2), or top-down, Fussell and Vesely (Ref. 3), approaches. Computerised methods to conduct this analysis are now so well developed that further refinement is unlikely to result in vast reductions in computer time. Tackling this problem to improve computational efficiency has been the main concern over the years for many fault tree researchers, Bennetts (Ref. 4) and Benjamin et al. (Ref. 5) have both addressed this problem. Usually by modifying the established, conventional approaches such as MOCUS (Ref. 3).

It is felt that substantial improvement in computer utilisation will only result from a completely new approach. Such an approach would involve specifying the logic equation in a form which is easier to manipulate than a fault tree. A recent paper by Rauzy (Ref. 6) has indicated that an alternative approach using a Binary Decision Diagram may provide a faster and more efficient means of analysing fault trees.

2. NOTATION

\[ P(\text{Top}) \] – Probability of Top Event of a fault tree.
\[ C_i \] – Minimal Cut Set.
\[ P_{RE}(\text{Top}) \] – Rare Event Approximation of Top Event Probability.
\[ P_{MCSUB}(\text{Top}) \] – Minimal Cut Set Upper Bound of Top Event Probability.
\[ X_i \] – Boolean Variable.
\[ f(x)/f1/f2 \] – Boolean Functions.
\[ \text{ite} \] – If-Then-Else structure for Binary Decision Diagram.
\[ \text{<op>} \] – Boolean operation (· or +).
\[ F_i \] – Nodes/Vertices in a Binary Decision Diagram.
\[ Q_{sys} \] – Probability of occurrence of top event of fault tree.
\[ W(0, t) \] – Expected number of top event occurrences.
\[ w_{sys} \] – System Unconditional Failure Intensity.
\[ G_i(q) \] – Criticality Function for component i

3. ABBREVIATIONS

s.o.p - sum of products expression.
BDD - Binary Decision Diagram.

4. FAULT TREE ANALYSIS

The analysis of the fault tree is generally undertaken in two stages: qualitative analysis and quantitative analysis. Qualitative analysis involves obtaining the various combinations of events which cause system failure (minimal cut sets) and quantification then deals with calculating the probability or frequency that system failure will occur.

4.1 Qualitative Analysis

The conventional approach to obtain the minimal cut sets is to take the Boolean logic expression for the Top Event and transform it into a sum of products (s.o.p) form. One way of doing this is to use a Bottom-Up procedure such as that of Semanderes (Ref. 2). To obtain the s.o.p form for the Top Event of the fault tree, the inputs to the lowest gates are
methods only produce the most important minirnal must first be applied to the s.0.p form to obtain the minirnal. Also the expansion procedure can make extensive demands on memory space.

To overcome these problems various techniques have been employed to reduce the number of comparisons (Ref. 7). Some methods only produce the most important minimal cut sets. One of these techniques is referred to as culling, which means that cut sets of a certain order, say 4 and above, are ignored or deleted from the expression. Rasmuson and Marshall (Ref. 8) employ this technique in their paper. The disadvantage for doing this is that cut sets of a high order tend to have a low probability of occurrence and therefore do not make a significant contribution to the Top Event probability. However the disadvantage of this is that when common cause failures are involved this method results in considerable inaccuracies. Probabilistic culling can also be applied, in this case a cut set whose probability of occurrence is below some threshold limit will again be ignored.

4.2 Quantitative Analysis

The conventional approach (see Henley and Kumamoto in Ref. 9) to obtain the exact probability of the Top Event is to use the formula:

\[ P(\text{Top}) = \sum_{i=1}^{\text{nc}} P(C_i) - \sum_{i=2}^{\text{nc}} \sum_{j=1}^{i-1} P(C_i \cap C_j) + \ldots + (-1)^{\text{nc}-1} P(C_1 \cap C_2 \cap \ldots \cap C_{\text{nc}}) \]  

(1)

Where \( C_i \), \( i=1,\ldots,\text{nc} \) are the minimal cut sets of the Top Event, i.e. product terms.

Clearly if the fault tree has many minimal cut sets calculating \( P(\text{Top}) \) will require extensive calculations to evaluate each term in the expression, for many complex fault trees the requirement is beyond the capability of the available machines. To simplify the calculation the Rare Event Approximation, \( P_{RE}(\text{Top}) \), can be used which is:

\[ P_{RE}(\text{Top}) = \sum_{i=1}^{\text{nc}} P(C_i) \]  

(2)

However a more accurate approximation is the Minimal Cut Set Upper Bound, \( P_{MCSUB}(\text{Top}) \), which is:

\[ P_{MCSUB}(\text{Top}) = 1 - \prod_{i=1}^{\text{nc}} (1 - P(C_i)) \]  

(3)

5. BINARY DECISION DIAGRAM METHOD

The Binary Decision Diagram (BDD) method, developed by Rauzy (Ref. 6), first converts the fault tree to a binary decision diagram which encodes an If-Then-Else (ite) structure. The attractive thing about the BDD method is that the ite structure derives from Shannons' formula (Ref. 10), such that if \( f(x) \) is the Boolean Function for the top event of a fault tree then the Shannon formula can be written as:

\[ X_1 \cdot f_1 + \overline{X_1} \cdot f_2 \]  

(4)

and the corresponding ite structure is ite\((X_1, f_1, f_2)\), for a detailed account of this procedure refer to Ref. 11 and Ref. 12. From this diagram both the qualitative and quantitative analysis can be achieved.

The size of the resulting BDD is determined by the ordering that has to be given to the basic events in the fault tree before the BDD is constructed. This ordering has further implications for the analysis. If the BDD is not in a minimal form, then the BDD must first undergo a minimising algorithm before the minimal cut sets can be obtained, this minimising technique is discussed in section 6. The quantitative analysis must be performed on the unminimised diagram. The reason being that the minimising procedure produces a new BDD which only encodes the minimal cut sets. However if the ordering of the basic events produces a minimal BDD then both the quantitative and qualitative analysis is straightforward. It is therefore beneficial to achieve an ordering which is optimal in terms of the resulting size of the BDD. The ordering of basic events to produce a minimal diagram is considered in (Ref. 11) and discussed in section 7.

To illustrate the method of obtaining the minimal cut sets and probability of occurrence of the top event using the BDD method refer to the example fault tree in figure 1.

\[ P_{MCSUB}(\text{Top}) = 1 - \prod_{i=1}^{\text{nc}} (1 - P(C_i)) \]  

\[ P_{RE}(\text{Top}) = \sum_{i=1}^{\text{nc}} P(C_i) \]  

\[ X_1 \cdot f_1 + \overline{X_1} \cdot f_2 \]  

(4)

Assume an ordering for the basic events which is derived by considering those events at higher levels in the tree structure first:

\[ \text{Top} \]

\[ \text{G1} \]

\[ \text{X2} \]

\[ \text{X3} \]

\[ \text{G2} \]

\[ \text{X3} \]

\[ \text{X4} \]

\[ \text{X1} \]

\[ \text{Figure 1. Example Fault Tree.} \]
To obtain the \textit{ite} structures for each gate in the fault tree the following procedures are used:

1. Taking $X<Y$:
   
   \begin{align*}
   J & = \text{ite}(X, F_1, F_2) \quad \text{and} \quad H = \text{ite}(Y, G_1, G_2) \\
   J \text{<op>} H & = \text{ite}(X, F_1 \text{<op>} H, F_2 \text{<op>} H)
   \end{align*}

2. Taking $X=Y$:
   
   \begin{align*}
   J & = \text{ite}(X, F_1, F_2) \quad \text{and} \quad H = \text{ite}(X, G_1, G_2) \\
   J \text{<op>} H & = \text{ite}(X, F_1 \text{<op>} G_1, F_2 \text{<op>} G_2)
   \end{align*}

where \text{<op> } corresponds to the Boolean operation of the logic gates in the fault tree. For an AND gate \text{<op> } will be the dot or product symbol and for an OR gate \text{<op> } will be the addition symbol.

Also it is evident that:

\begin{align*}
1 \text{<op> } H &= 1 \quad \text{if} \quad \text{<op> } \text{is an OR gate} \\
0 \text{<op> } H &= H \quad \text{if} \quad \text{<op> } \text{is an AND gate} \\
0 \text{<op> } H &= 0 \quad \text{if} \quad \text{<op> } \text{is an AND gate}
\end{align*}

Therefore the BDD calculations for the fault tree in figure 1 are the following:

\begin{align*}
G_2 & = \text{ite}(X_3, 1, 0) + \text{ite}(X_4, 1, 0) \\
   & = \text{ite}(X_3, 1, \text{ite}(X_4, 1, 0)) \\
G_1 & = \text{ite}(X_2, 1, 0) + \text{ite}(X_3, 1, 0) \\
   & = \text{ite}(X_2, 1, \text{ite}(X_3, 1, 0)) \\
\text{Top} & = G_1.G_2.X_1 \\
   & = \text{ite}(X_2, 1, \text{ite}(X_3, 1, 0)).\text{ite}(X_3, 1, \text{ite}(X_4, 1, 0)) \cdot \text{ite}(X_1, 1, 0) \\
   & = \text{ite}(X_2, \text{ite}(X_3, 1, \text{ite}(X_4, 1, 0)), \text{ite}(X_3, 1, 0)) \cdot \text{ite}(X_1, 1, 0) \\
   & = \text{ite}(X_2, \text{ite}(X_3, X_4)), \text{ite}(X_3, 1, 0)) \cdot \text{ite}(X_1, 1, 0, 0)
\end{align*}

This top event \textit{ite} structure corresponds to the BDD shown in figure 2.

![Figure 2. BDD for \textit{ite}(X_1, \textit{ite}(X_2, \textit{ite}(X_3, 1, \textit{ite}(X_4, 1, 0)), \textit{ite}(X_3, 1, 0)), 0)]
<table>
<thead>
<tr>
<th>basic event i</th>
<th>q_i</th>
<th>λ_i</th>
<th>w_i = λ(1-q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.01</td>
<td>1.0E-6</td>
<td>9.9E-7</td>
</tr>
<tr>
<td>X2</td>
<td>0.02</td>
<td>4.0E-6</td>
<td>3.92E-6</td>
</tr>
<tr>
<td>X3</td>
<td>0.03</td>
<td>2.0E-4</td>
<td>1.94E-4</td>
</tr>
<tr>
<td>X4</td>
<td>0.04</td>
<td>3.0E-5</td>
<td>2.88E-5</td>
</tr>
</tbody>
</table>

**Table 1. Basic Event Data.**

Where;

- q_i - Unavailability of component i.
- λ_i - Conditional failure intensity of component i.
- w_i - Unconditional failure intensity of component i.

Since $Q_{sys}$ can be obtained from the probability of the sum of the disjoint paths through the BDD then:

$$Q_{sys} = P(X1, X2, X3 + X1, X2, X3, X4 + X1, X2, X3)$$

$$= q_{x1} \cdot q_{x2} \cdot q_{x3} + q_{x1} \cdot q_{x2} \cdot (1 - q_{x3}) \cdot q_{x4} +$$

$$q_{x1} \cdot (1 - q_{x2}) \cdot q_{x3}$$

$$= 0.01(0.02)(0.03) + 0.01(0.02)(1 - 0.03)$$

$$+ (0.04) + 0.01(1 - 0.02)(0.03)$$

$$Q_{sys} = 3.0776E - 4$$

The algorithm used by Rauzy for calculating the probability is given in Ref. 6.

### 5.2 Unconditional System Failure Intensity

For some systems it is the unreliability which is required for the top event i.e., the probability it will not work continuously over a given time period. An upper bound for this is the Expected number of top event occurrences $W(0, t)$:

$$W(0, t) = \int_{0}^{t} w_{sys} dt$$  \hspace{1cm} (7)

$w_{sys}$ is the system unconditional failure intensity:

$$w_{sys} = \sum_{i} G_i(q) \cdot w_i$$  \hspace{1cm} (8)

where $G_i(q)$ is the criticality function for each component.

The criticality function $G_i(q)$ is defined as the probability that the system is in a critical state with respect to component i and that the failure of component i will then cause the system to go from the working to the failed state, i.e., the probability that the system fails only if component i fails. Therefore:

$$G_i(q) = Q(1, q) - Q(0, q)$$  \hspace{1cm} (9)

Where:

- $Q(1, q)$ - is the probability of system failure with $q_i = 1$.
- $Q(0, q)$ - is the probability of system failure with $q_i = 0$.

Evaluating each of the two terms $Q(1, q)$ and $Q(0, q)$ for each component could be achieved by first substituting $q_i = 1$ and then $q_i = 0$, i.e., the probability that component i equals 1 and 0 respectively, and re-running the system failure probability calculations. This would require the equivalent of 2n evaluations of the top event probability to deduce all terms required in the expression for $w_{sys}$ in eq (8).

Consider the variable Xi which occurs at two nodes in the BDD (Figure 3) then:

$$Q(1, q) = \sum_{n} (pr_{x_i}(q) \cdot po_{x_i}^1(q)) + Z(q)$$  \hspace{1cm} (10)

$$Q(0, q) = \sum_{n} (pr_{x_i}(q) \cdot po_{x_i}^0(q)) + Z(q)$$  \hspace{1cm} (11)

where:

- $pr_{x_i}(q)$ - is the probability of the path section from the root node to node xi.
- $po_{x_i}^1(q)$ - is the probability of the path section from node xi to the terminal 1 node after the 1 branch from node xi.
- $po_{x_i}^0(q)$ - is the probability of the path section from node xi to the terminal 1 node after the 0 branch from node xi.
- $Z(q)$ - is the probability of paths from the root node to the terminal 1 nodes which do not go through a node for variable xi.
- n - All nodes for variable xi on the BDD.

Therefore:

$$G_i(q) = \sum_{n} pr_{x_i}(q)[po_{x_i}^1(q) - po_{x_i}^0(q)]$$  \hspace{1cm} (12)
A more efficient way to calculate $w_{\text{sys}}$ is to make one pass of the BDD to calculate $Pr(s_i(q))$, $Pr(s_i^1(q))$ and $Pr(s_i^0(q))$ for each node. With this information each $G_i(q)$ can be easily evaluated from eq (12) and $w_{\text{sys}}$ formed.

The algorithm Probpost to calculate $Pr(s_i^1(q))$ and $Pr(s_i^0(q))$ is given in figure 4. The calculation of $Pr(s_i(q))$ can be achieved by the algorithm Probprev given in figure 5. The criticality function $G_i(q)$ for each basic event is calculated as shown in figure 6.

```
Probpost(F) =
  Do for all F, end vertices to Root Vertex
  F=ite(x, G, H)
  R=PROBTABLE(x, prob(G), prob(H))
  Q1=p(x).prob(G)
  Q2=(1-p(x)).prob(H)
  insert - in - computation - table (f<prob, F, ->, Q1+Q2)
  return R
  return Q1 and Q2
  next F
```

**Figure 4. Probpost Algorithm.**

```
Probprev(F) =
  start at Root Vertex, F!
  Probprev(F) = 1
  Add Probprev(F) to Probtable, i.e.,
  Probtable(Probpost(F), Probprev(F))
  Do for all F, Root Vertex to end vertices
  F=ite(x, H1, H2)
  if H1=0 or 1 Goto [A]
  Probprev(H1)=p(x).Probprev(F)
  Add Probprev(H1) to Probtable
  [A] if H2=0 or 1 next F
  Probprev(H2)=(1-p(x)).Probprev(F)
  Add Probprev(H2) to Probtable
  next F.
```

**Figure 5. Probprev Algorithm.**

```
Set $G(x)=0$ for all i
Do for all F
  if F=Probtable(x, q1, q2, q3)
    G(x)=G(x)+q3(q1-q2)
  insert - in - computation - table G(x)
  next F.
```

**Figure 6. Algorithm for Calculating the Criticality Function, $G_i$.**

Example

Applying these algorithms to the example BDD given in figure 2 illustrates the application of this method.

The ite table for the BDD in figure 2 is:

<table>
<thead>
<tr>
<th>Node Label</th>
<th>Variable</th>
<th>1 branch pointer</th>
<th>0 branch pointer</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>X1</td>
<td>F2</td>
<td>0</td>
</tr>
<tr>
<td>F2</td>
<td>X2</td>
<td>F3</td>
<td>F4</td>
</tr>
<tr>
<td>F3</td>
<td>X3</td>
<td>1</td>
<td>F5</td>
</tr>
<tr>
<td>F4</td>
<td>X3</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>F5</td>
<td>X4</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

Performing one pass of the BDD to evaluate $Pr(s_i^1(q))$ and $Pr(s_i^0(q))$ for each node using Probpost gives:

```
Probpost(F5)
  F5=ite(X4, 1, 0)
  R=Probtable(X4, 1, 0)
  Q1=p(X4)=0.04
  Q2=0

Probpost(F4)
  F4=ite(X3, 1, 0)
  R=Probtable(X3, 1, 0)
  Q1=p(X3)=0.03
  Q2=0

Probpost(F3)
  F3=ite(X3, 1, F5)
  R=Probtable(X3, 1, prob(F5))←(X3, 1, 0.04)
  Q1=p(X3)=0.03
  Q2=(1-p(X3))(0.04)=0.0328

Probpost(F2)
  F2=ite(X2, F3, F4)
  R=Probtable(X2, prob(F3), prob(F4))←(X2,0.0688, 0.03)
  Q1=p(X2)(0.0688)=1.376E-3
  Q2=(1-p(X2))(0.03)=0.0294

Probpost(F1)
  F1=ite(X1, F2, 0)
  R=Probtable(X1, prob(F2), 0)←(X1, 0.030776, 0)
  Q1=p(X1)(0.030776)=3.0776E-4
  Q2=0
```

In performing this one pass, the top event probability can be calculated by:

$$P(\text{Top})=Q1+Q2$$  \hspace{1cm} (13)

for the top event node.

The values of Probpost 1 branch and Probpost 0 branch for each node are entered into the node probability table, PROBTABLE (see figure 7).

Next calculating the probability of the BDD path to each node is established using Probprev and entered into the 4th column of the PROBTABLE.

```
Probprev:
  Probprev(F1)=1
  F1=ite(X1, F2, 0)
  Probprev(F2)=p(X1).Probprev(F1)
    =0.01(1)=0.01
  H2=0
  F2=ite(X2, F3, F4)
  Probprev(F3)=p(X2).Probprev(F2)
```

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=0.02(0.01)=2.0E-4

\[ P_{\text{prev}}(F_4) = (1-p(X_2)) P_{\text{prev}}(F_2) = (1-0.02)(0.01)=2.0E-4 \]

\[ F_3 = \text{ite}(X_3, 1, F_5) \]

\[ H_1 = 1 \]

\[ P_{\text{prev}}(F_5) = (1-p(X_3)) P_{\text{prev}}(F_3) = (1-0.03)(2.0E-4)=1.94E-4 \]

\[ F_4 = \text{ite}(X_3, 1, 0) \]

\[ H_1 = 1 \]

\[ H_2 = 0 \]

\[ F_5 = \text{ite}(X_4, 1, 0) \]

\[ H_1 = 1 \]

\[ H_2 = 0 \]

<table>
<thead>
<tr>
<th>Node Label</th>
<th>Variable</th>
<th>post '1'</th>
<th>post '0'</th>
<th>probprev</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>X1</td>
<td>0.030776</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>F2</td>
<td>X2</td>
<td>0.0688</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>F3</td>
<td>X3</td>
<td>1</td>
<td>0.04</td>
<td>2.0E-4</td>
</tr>
<tr>
<td>F4</td>
<td>X3</td>
<td>1</td>
<td>0</td>
<td>9.8E-3</td>
</tr>
<tr>
<td>F5</td>
<td>X4</td>
<td>1</td>
<td>0</td>
<td>1.94E-4</td>
</tr>
</tbody>
</table>

\[ \text{Probtable}(i, 1) = \text{Basic event of node } F_i \]

\[ \text{Probtable}(i, 2) = \text{Probability of post '1' branch} \]

\[ \text{Probtable}(i, 3) = \text{Probability of post '0' branch} \]

\[ \text{Probtable}(i, 4) = \text{Probability of previous} \]

**Figure 7. PROBTABLE Array.**

Calculation of the criticality function is then straightforward using the algorithm provided in figure 6.

**Criticality Algorithm:**

\[ G(X_1)=G(X_2)=G(X_3)=G(X_4)=0 \]

\[ F_1 = \text{Probtable}(X_1, 0.030776, 0, 1) \]

\[ G(X_1)=0+0+1(0.030776-0) = 0.030776 \]

\[ F_2 = \text{Probtable}(X_2, 0.0688, 0.03, 0.01) \]

\[ G(X_2)=0+0.01(0.0688-0.03) = 3.88E-4 \]

\[ F_3 = \text{Probtable}(X_3, 1, 0.04, 2.0E-4) \]

\[ G(X_3)=0+2.0E-4(1-0.04) = 1.92E-4 \]

\[ F_4 = \text{Probtable}(X_3, 1, 0, 9.8E-3) \]

\[ G(X_3)=1.92E-4+9.8E-3(1-0) = 9.92E-3 \]

\[ F_5 = \text{Probtable}(X_4, 1, 0, 1.94E-4) \]

\[ G(X_4)=1.94E-4(1-0) = 1.94E-4 \]

Since we have calculated the criticality function for each component, \( w_{\text{sys}} \) can now be evaluated using the frequency data from table 1 using eq (8).

\[ w_{\text{sys}} = G(X_1)w_{X_1} + G(X_2)w_{X_2} + G(X_3)w_{X_3} + G(X_4)w_{X_4} \]

\[ = 0.030776(9.9E-7) + 3.88E-4(3.92E-6) + 9.992E-3(1.94E-4) + 1.94E-4(2.88E-5) \]

\[ = 1.976(244)E-6 \]

Using eq (7) the expected number of top event occurrences in time, \( t \), can be obtained.

### 6. MINIMISING THE BDD

In the example fault tree (figure 1) the resulting BDD (figure 2) was not minimum as it produced a redundant cut set. To obtain only minimal cut sets the BDD must first undergo a minimising procedure. From the unminimised BDD the minimising algorithm of Rauzy (Ref. 6) creates a new BDD that symbolises only the minimal cut sets of the fault tree. If \( F=\text{ite}(x, G, H) \) then let \( \delta \) be a minimal solution of \( G \) which is not a minimal solution of \( H \), then clearly the intersection of \( \delta \) and \( x \) will be a minimal solution of \( F \). Lastly, the set \( \sigma \) of all the minimal solutions of \( F \), \( \text{sol}_{\text{min}}(F) \), will also include the minimal solutions of \( H \) so:

\[ \text{sol}_{\text{min}}(F) = \{ \sigma \} \]

where:

\[ \sigma = \left[ \{ \delta \} \cap X \right] \text{sol}_{\text{min}}(H) \]

Rauzy (Ref. 6) has defined a 'without' operator which removes from \( G_{\text{min}} \) all the paths included in a path of \( H \). Applying this algorithm to the BDD in figure 2 where each node is considered in turn:

\[ F_1 = \text{ite}(X_1, F_2, 0) \] - Here there are no solutions on the 0 branch so the paths of \( F_2 \) remain unchanged.

\[ F_2 = \text{ite}(X_2, F_3, F_4) \] - Here \( X_3 \) is included in a path on both the 1 branch (\( F_3 \)) and the 0 branch (\( F_4 \)), therefore \( X_3 \) is removed from the 1 branch by replacing the terminal 1 vertex with a 0. (Refer to figure 8)

\[ F_3 = \text{ite}(X_3, 0, F_5) \] - \( F_5 \) does not contain any paths that are included in the 1 branch as this is a terminal vertex.

\[ F_4 = \text{ite}(X_3, 1, 0) \] - The without operator does not apply as both 0 and 1 branches are terminal.

\[ F_5 = \text{ite}(X_4, 1, 0) \] - Same applies as \( F_4 \).

The minimised BDD is drawn in figure 8.
Further improvements in terms of computational efficiency can be made for the more complex fault trees by modularising the fault tree before the analysis takes place. Khoda et al. (Ref. 14) define a module of a fault tree as having no inputs which appear elsewhere in the tree and no outputs to the rest of the tree except for its output event. For example consider the fault tree in figure 9. Modules which have the properties defined above are gates G2, G3 and Top.

Figures 8, 9, and 10 illustrate the process of modularising a fault tree and then using BDD techniques to analyse each module, and combining the results to provide an efficient means of analysing the whole fault tree.

**8. MODULARISING**

Conventional top-down and bottom-up techniques can lead to many redundant cut sets and calculating exact top event probability can become impossible. To improve these analysis procedures the aim has been to represent the system failure logic in a form which lends itself to the mathematical manipulation.
Representing the Boolean failure logic equation in the form of a BDD provides an alternative technique which gives significant savings in the computational efficiency and lends itself to manipulation. Also the BDD produces exact quantified results and top event parameters such as failure probability and the system unconditional failure intensity and the expected number of occurrences can be obtained with ease.

To simplify the analysis even further the fault tree may be modularised prior to the analysis. An alternative ordering of the basic event variables has also shown itself to significantly convert the logic from the fault tree structure to the BDL quantified results and top event parameters such as failure form. However early work indicates that for large, complex systems in fault tree analysis, "Modules in Fault Trees," Engineering Applications, vol 23, No. 3, 1989; pp203-211.

The trade off for the advantages described is the effort taken to convert the logic from the fault tree structure to the BDD form. However early work indicates that for large, complex trees this can produce a substantial reduction in computational effort.

10. REFERENCES


BIOGRAPHIES

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