A study on attenuating gear teeth oscillations at low engine speeds using nonlinear vibration absorbers

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Citation: FRISKNEY, B. ... et al., 2018. A study on attenuating gear teeth oscillations at low engine speeds using nonlinear vibration absorbers. SAE Technical Papers, 2018-01-1477.

Additional Information:

- This paper was presented at the 10th International Styrian Noise, Vibration & Harshness Congress: The European Automotive Noise Conference. This paper was accepted for publication in the journal SAE Technical Papers, 2018-01-1477 and the definitive published version is available at https://doi.org/10.4271/2018-01-1477

Metadata Record: https://dspace.lboro.ac.uk/2134/36169

Version: Accepted for publication

Publisher: © SAE International

Rights: This work is made available according to the conditions of the Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International (CC BY-NC-ND 4.0) licence. Full details of this licence are available at: https://creativecommons.org/licenses/by-nc-nd/4.0/

Please cite the published version.
A study on attenuating gear teeth oscillations at low engine speeds using nonlinear vibration absorbers

Abstract

Gear oscillations are one of the most common sources of Noise, Vibration and Harshness (NVH) issues manifested in automotive powertrains. These oscillations are generated mainly due to impacts of the meshing gear teeth over a broad frequency range. To mitigate NVH phenomena, automotive manufacturers traditionally couple linear tuned vibration absorbers to the driveline. Common palliatives used are clutch dampers and dual mass flywheels, which generally suppress vibrations effectively only over narrow frequency bands. Nonlinear Energy Sinks (NESs) are a class of vibration absorbers with essentially nonlinear characteristics that are designed for dissipating vibration energy over broad frequency ranges (due to the employed nonlinearity). The NES does not have a preferential natural frequency; this is rather characterized by the nonlinear stiffness. An NES functions on the principle of transferring energy between the primary system (e.g. driveline) and the absorber in two ways: (i) the NES induces a unidirectional transfer of the vibration energy excess from the primary system to the absorber and (ii) the NES induces a redistribution of the vibration energy excess in the modes of the primary structure, enhancing the energy dissipation capabilities of the primary structure. This paper presents a study on the use of NESs for reducing oscillations on gear pairs operating at low engine operating speeds. Numerical simulations were performed using a gear pair model equipped with an absorber with essentially cubic nonlinear stiffness, attached to the gear wheel. The stiffness and inertia properties of the absorber were varied with the objective of obtaining the parameter combination that induces significant attenuation of the oscillatory motion of the gear wheel. The occurring motion of the system using different sets of parameters is studied and presented.

Introduction

Recent developments in the automotive industry have led to the production of downsized turbo-charged engines. When compared to naturally aspirated engines [1], turbo-charged engines have higher torque, fuel efficiency and power output whilst producing lower emissions [2-3]. However, due to higher torque, significant torsional oscillations are able to penetrate into the drivetrain system [4]. Transmission and gear rattle are common NVH issues present in new generation powertrains with downsized engines.

Gear rattle is an audible noise that originates due to repetitive impacts of gear teeth with their counterpart pinion teeth. Although gear rattle does not result in a mechanical malfunction, it has implications on the vehicle comfort and ride quality. To overcome this drawback, manufacturers are implementing costly palliatives such as Dual Mass Flywheels (DMFs), clutch pre-dampers and centrifugal pendulum absorbers. The effectiveness of these palliatives is limited by the linear nature of these devices which confines them to operate over narrow bands of frequencies.

Past literature has examined ways in which to reduce gear rattle. De la Cruz et al. [5] performed an investigation on manual transmission drive rattle. It was concluded that the effect of engine order vibrations on rattle become significant with rising temperature. Theodossiades et al. [6] studied the interaction between the transmission gears at idle conditions by accounting for lubricated contacts. In this study it was assumed that lubricant properties affect the drag torque and inertia of idle gears, thus promoting rattle. Jadhav [7] conducted NVH analysis on the clutch and gear dynamics. It was determined that modification to the clutch damper led to a reduction in gear rattle for a given engine speed.

A different approach to reducing gear teeth oscillations due to engine harmonics proposes the use of a special class of nonlinear vibration absorber called a Nonlinear Energy Sink (NES). An NES [8] is characterized by a light mass and an essentially nonlinear stiffness. Thus, it is not tuned to any preferential frequency; instead, it is activated by the magnitude of the energy input. The NES operates on the principle of Targeted Energy Transfer (TET) whereby the vibrating energy from a primary system (driveline) is transferred in a nearly irreversible manner to a secondary system (NES) [9], where it is either redistributed in modal space or absorbed and dissipated locally.

In literature, numerous publications have been devoted to the understanding, design and optimization of translational NES absorbers. Gendelman et al. [10] and Vakakis et al. [11-12] studied the implementation of an NES in two, three and multi degree of freedom systems. Kerschen et al. [13] conducted parametric studies on understanding the dynamics of the energy pumping in NESs. McFarland et al. [17] performed investigations in structures with grounded and ungrounded non-linear attachments. However, few studies on rotational NESs are reported in literature.

The implementation of a torsional NES was examined by Viguié et al. [14] with the objective of stabilizing a rotational drilling system. Hubbard et al. [15] performed theoretical and experimental analysis for suppressing the aero-elastic torsional instabilities of a flexible swept wing using a rotational NES. Haris et al. [8] theoretically demonstrated the effectiveness of using an NES on suppressing broadband torsional vibrations in automotive powertrains connected to a turbo-charged engine. Savva et al. [16] studied the effect that NES damping has on suppressing torsional vibrations of automotive
engines operating at idling conditions. In a numerical study, Motato et al. [18] demonstrated the characteristic behaviour of an NES for attenuation of torsional vibration of a vehicle powertrain through energy re-distribution and energy dissipation. Scaglitarini et al. [19] modelled a spur gear pair under constant excitation. Bifurcation analysis and frequency response functions (FRFs) were presented for the gear pair and the system coupled with a rotary NES. The study showed that TET can be activated, causing quasiperiodic motion. As such, further promise of the application of a rotary NES to engine order vibration suppression in a gear pair is evident.

This paper presents a theoretical study on the use of an NES attached to a spur gear pair operating at harmonically varying low loads (rattling conditions of unloaded gears). Parametric studies were performed on the gear pair equipped with an NES by varying the nonlinear stiffness and inertia of the absorber. The aim is to demonstrate effective operation of the NES in attenuation of the gear’s torsional vibrations. Frequency-Energy Plots (FEPs) and Nonlinear Normal Modes (NNMs) are used to provide insight on the Gear-NES resonant behaviour.

The paper is organized as follows. In the next section, the numerical model of the spur gear pair is described. Then, the model of the spur gear pair coupled with an NES is defined. In the fourth section, the criterion and index used to determine the NES parameter combination that induces significant vibration attenuation are presented. The performance of the NES with optimum parameters is analysed next through Frequency Response Function (FRF), Frequency Energy Plots (FEP) and Poincare maps. Lastly, the conclusions and proposed future work are discussed.

Spur Gear Pair Model

The model of the spur gear pair is shown in Figure 1. The model includes a pinion with inertia $I_p$ and a gear with inertia $I_G$.

![Spur Gear Pair Model](image)

**Figure 1: Spur gear pair model**

The equations of motion describing the behaviour of the gear pair system are:

$$I_p \ddot{\theta}_p + R_p c_M x_t + R_p k_M x_t = T_p$$

(1)

$$I_G \ddot{\theta}_G - R_G c_M x_t - R_G k_M x_t = -T_G$$

(2)

where $\theta_G$, $\theta_p$ and $\theta_c$ are the angular acceleration, the angular velocity and angular position of the gear, respectively. In addition, $\theta_c$, $\theta_p$ and $\theta_t$ are respectively the angular acceleration, angular velocity and angular position of the pinion. $R_p$ is the pinion radius, $c_M$ is the damping coefficient, $R_G$ is the gear radius, $T_p$ is the pinion torque and $T_G$ is the gear torque. $k_M$ is the time-varying meshing stiffness and is a function of the angular position of the pinion and the applied torque. Structural mesh damping is assumed for the gear pair upon which this investigation is based.

$x_t$ is a piece-wise linear function describing the changing linear displacement as seen by the mesh to reflect contact on the drive and reverse teeth flanks, and the case at which there is no contact as a result of the backlash clearance. This function is dependent upon the magnitudes of the dynamic transmission error $u$ (DTE) and total backlash 2b:

$$u = R_p \dot{\theta}_p - R_G \dot{\theta}_G$$

(3)

$$x_t = \begin{cases} u - b, & u > b \\ 0, & -b < u < b \\ u + b, & u < -b \end{cases}$$

(4)

The input excitation torque to the system $(T_p)$ is a function of a pinion base torque $T_{pb}$, the harmonic frequency $\omega_{pf}$ (which corresponds to the pinion speed), the amplitude torque factor $T_A$ and time $t$. The equation describing the pinion torque is:

$$T_p = T_{pb} + T_{pb}\delta\sin(\omega_{pf}t)$$

(5)

The model parameters are shown in Table 1.

<table>
<thead>
<tr>
<th>Table 1: Model parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia of pinion [kgm$^2$]</td>
<td>0.0004</td>
</tr>
<tr>
<td>Inertia of gear [kgm$^2$]</td>
<td>0.005</td>
</tr>
<tr>
<td>Radius of pinion [m]</td>
<td>30 x 10$^{-3}$</td>
</tr>
<tr>
<td>Radius of gear [m]</td>
<td>60 x 10$^{-3}$</td>
</tr>
<tr>
<td>Pinion teeth number</td>
<td>20</td>
</tr>
<tr>
<td>Gear teeth number</td>
<td>40</td>
</tr>
<tr>
<td>Gear ratio</td>
<td>2</td>
</tr>
<tr>
<td>Pinion base torque [Nm]</td>
<td>2</td>
</tr>
<tr>
<td>Gear torque [Nm]</td>
<td>4</td>
</tr>
<tr>
<td>Backlash [m]</td>
<td>0.005</td>
</tr>
<tr>
<td>Amplitude torque factor (%)</td>
<td>50</td>
</tr>
</tbody>
</table>

The numerical model of the spur gear pair was developed in MATLAB/Simulink and was studied numerically for pinion speeds in the range of 100 rpm – 2000 rpm. The total simulation time is 2 s, but only the last 0.5 s are used for the analysis ensuring that steady state motion has been achieved. Figure 2 shows the velocity time histories for pinion speed of 800 rpm ($\omega_{pf} = 83.78 \text{ rad/s}$) and the corresponding gear speed. Figure 3 shows the velocity time history for pinion speed of 2000 rpm ($\omega_{pf} = 209.44 \text{ rad/s}$) and the corresponding gear speed.
Spur Gear Pair Model with NES

The spur gear model was modified to incorporate a cubic rotational nonlinear energy sink attached at the gear (Figure 4). The objective is to attenuate the amplitude of gear oscillations over a broad range of operating frequencies. The equations of motion governing the dynamics of the new system including the NES are:

\[ I_P \ddot{\theta}_P + R_P c_M x_t + R_P k_M x_t = T_P \]  
\[ I_G \ddot{\theta}_G - R_G c_M x_t - R_G k_M x_t + k_N (\theta_G - \theta_N)^3 \]  
\[ + c_N (\dot{\theta}_G - \dot{\theta}_N) = -T_G \]  
\[ I_N \ddot{\theta}_N - k_N (\theta_G - \theta_N)^3 - c_N (\dot{\theta}_G - \dot{\theta}_N) = 0 \]

where \( \ddot{\theta}_N \), \( \dot{\theta}_N \) and \( \theta_N \) are respectively the angular acceleration, the angular velocity and angular position of the NES. Additionally, \( k_N \) is the NES nonlinear stiffness, \( c_N \) is the NES damping coefficient and \( I_N \) is the NES inertia.

A parametric study was performed to identify the influence of varying the NES parameters (inertia and stiffness) on the suppression of gear oscillations.

NES Performance criterion and Results

The effectiveness of the NES was assessed by comparing the performance of the gear pair with locked and active NES. The locked NES refers to the system whereby the inertia of the NES is simply added to the gear inertia. The active NES refers to the system whereby the NES inertia is connected to the primary system through the essentially nonlinear stiffness. To assess how the performance of locked and active system differs, the Frequency Response Function (FRF) was used.

For the pinion speed range between 100 rpm to 2000 rpm, the steady state amplitude of gear oscillations was calculated for both locked and active systems and an FRF curve is generated. The performance index is calculated by subtracting the area under the curve of the FRF for locked system with the area under the curve of the FRF for active system (Figure 5). A positive index indicates vibration attenuation and a negative index indicates vibration increment.

The index is:

\[ \text{Index} = A_{\text{Locked}} - A_{\text{Active}} \]  
\[ A_{\text{Locked}} = \text{Area under the curve of FRF for Amplitude of oscillation (Steady State) for Locked system} \]  
\[ A_{\text{Active}} = \text{Area under the curve of FRF for Amplitude of oscillation (Steady State) for Active system} \]
The parametric study was conducted by simulating a total of 20,000 different combinations of NES inertia and stiffness for the defined pinion speed range. The range of NES parameters was varied as follows:

- NES Inertia $I_N = 1\% - 100\%$ of Pinion Inertia
- NES nonlinear stiffness $k_N = 10 - 5000$ Nm/rad$^3$
- NES damping coefficient $c_N = 0.002$ Nms/rad

The NES damping will arise from the use of any rolling elements such as bearings in the physical design, and is assumed to be small (this will be the aim for any NES physical design). For each combination the performance index of equation (9) was computed while changing the excitation harmonic frequency $\omega_{ef}$ in the range of 100 rpm – 2000 rpm. The simulation for each combination was performed until the steady-state condition has been achieved. Figure 6 shows a contour plot of the NES performance index, where the y-axis constitutes the variation of the nonlinear stiffness and the x-axis constitutes the variation of the NES inertia.

From the contour plot in Figure 6, it is found that an NES parameter combination that results in significant attenuation of gear oscillations is: $I_N = 100\%$ of pinion inertia and $k_N = 430$ Nm/rad$^3$, with the FRF of the optimum parameter presented in Figure 7.

The selected NES parameters ($I_N = 100\%$ of pinion inertia, $k_N = 430$ Nm/rad$^3$ and $c_N = 0.002$ Nms/rad), could attenuate gear oscillations for the corresponding pinion speed in the range of 700 – 1600 rpm. The NES performance for four different pinion speeds (800, 1100, 1400 and 2000 rpm) are analysed.

Figure 8a shows the gear velocity response for the locked and active systems for the corresponding pinion speed of 800 rpm, where the NES induces marginal vibration attenuation. The corresponding NES torque time history is shown in Figure 8b. The simulation was performed for 2 s but only the steady state motion is shown in these figures.
Figure 9a, shows the gear velocity time history for locked and active NES operating at 1100 rpm. The corresponding NES torque time history is shown in Figure 9b. A greater NES torque is produced for the case of 1100 rpm when compared with the case of 800 rpm.

The third working case is characterized by a dramatic reduction in gear vibration oscillations. Figure 10a, shows the gear velocity for locked and active NES for pinion speed of 1400 rpm. The corresponding NES torque time history is shown in Figure 10b. In this case the NES peak torque is 2.1 Nm (greater than in the two previous cases studied). The gear vibration reduction in this case confirms the effect of higher NES peak torques (vibration absorber action).

The NES combination selected ($I_N = 100\%$, $k_N = 430 \text{ Nm/rad}^3$) induces vibration reduction in the range of pinion speeds between 400 - 1500 rpm (gear velocities between 200 – 740 rpm). No gear vibration attenuation is achieved out of these ranges.
Frequency - Energy Plot Analysis

Frequency - Energy plots (FEPs) are produced to show that the working frequency region of vibration attenuation presented earlier is associated to 1:1 Gear-NES resonant behaviour. This is also exhibited in Poincare maps, which plot the position and velocity of the gear for one point of the periodic motion.

Linear mass-spring systems possess a single natural frequency at which a higher level of oscillation energy is achievable (an energy maximum can be transferred to the system at this single frequency). As such, the normal modes of the linear system appear as horizontal lines on an FEP. In the nonlinear system examined, the cubic nonlinearity gives rise to Nonlinear Normal Modes (NNMs), which can be considered as series of linear normal modes at different energy levels; a collection of periodic motions at varying frequencies.

The NNM computation [20] was used to calculate the NNM of the active system described in equations (6) - (8). To compute the NNM backbone curves, additional small linear (residual) stiffness connecting the NES with the gear \( k_L = 0.001 \text{ Nm/rad} \) is assumed (for computational purposes). The three linear natural frequencies are: \( \omega_1 = 0 \text{ Hz}, \omega_2 = 0.26 \text{ Hz} \) and \( \omega_3 = 5212 \text{ Hz} \). In this analysis only the NNM associated to the second resonant frequency is studied, being related to the NES. Operation of the system on the backbone curve of the FEP corresponds to 1:1 Gear-NES resonance. Physically, this means that during one cycle of the NES' motion, the NES may exhibit much larger amplitude of oscillation that that of the gear as a result of the irreversible energy transfer to the NES [9]. The motions of the two bodies may be in- or out-of-phase.

The NNM are shown in Figure 12. The NNM initiate at the second resonant frequency \( \omega_2 = 0.26 \text{ Hz} \) for low energy levels and end at the third resonant frequency \( \omega_3 = 5212 \text{ Hz} \) for higher energy levels. The NNM plot includes two internal resonance branches with the first branch occurring at 1000 Hz and the second around 1750 Hz. These branches are analogous to the behaviour of a linear system operating at its natural frequency. They represent a departure of the system dynamics from those associated with the FEP backbone curve as explained above. In such cases, 1:1 Gear-NES resonance cannot be achieved, thus TET to the NES will not occur.

Three energy levels (A, B and C) are studied in detail. The lower energy level in point A is characteristic of pinion speed of approximately 800 rpm. As shown in Figure 8 (for pinion speed of 800 rpm), marginal vibration attenuation is observed, as this point corresponds to 1:1 in-phase resonance behaviour of Gear–NES (Figure 13a). Figure 13b shows the corresponding Poincare map for the last 20 periods of the gear oscillations.

The medium energy level (point B in Figure 12) is characteristic of pinion speed of approximately 1400 rpm. As shown in Figure 10, in this case the NES induces substantial vibration reduction as a result of the greater amount of energy irreversibly transferred to the NES. Figure 14a shows the corresponding gear and NES velocities (active NES). The NES and gear velocities are out-of-phase with 1:1 resonance. This result confirms the findings in [9] where it is claimed that 1:1 resonance leads to TET. Figure 14b shows the corresponding Poincare plot.

The higher energy level (point C in Figure 12) is located at the transition region between the main branch and the internal resonance branch. This occurs at pinion speeds around 1700 rpm and is characterized by a 3:1 Gear-NES resonance behaviour. Figure 15a shows the corresponding gear and NES velocities of the active system (3:1 resonance). Figure 15b shows the Poincare plot of the gear velocity, in which three isolated points reflect the 3:1 Gear-NES resonant behaviour. Pinion speeds higher than 1700 rpm do not lead to gear vibration attenuation.
Conclusions and Future Work

This paper presented a theoretical study on the use of nonlinear vibration absorbers (NES) for reducing torsional vibrations of a spur gear pair. A numerical model of the spur gear pair was developed in MATLAB. Parametric studies were performed for different pinion operating speeds. The performance of the NES was analysed through a criterion that computes for each case the amplitude of oscillations of the gear velocity for systems with and without an active NES.

It was observed that there exists a direct relation between higher vibration reduction levels and higher NES peak torques. In addition it was possible to confirm the findings in [9] that the 1:1 Gear-NES resonance leads to TET and to vibration attenuation. It is shown through FEP and NNM analysis that the movement of the solution into the internal resonant branches, as a result of the gear and NES dynamics, causes TET to cease. This limits the working range of the NES and thus its performance at higher speeds. The transition region from working and non-working frequencies is characterized by a transient 3:1 Gear-NES resonant behaviour.

It is concluded that the NES has the potential to act as an effective passive torsional absorber, capable of attenuating vibrations over a broad range of frequencies with the optimum inertia and stiffness parameters found through the numerical study. This is unlike the linear absorbers, which are tuned to operate in a narrow frequency band. For future work and to validate these theoretical results, the design and manufacture of an NES prototype will be implemented in the examined spur gear pair.

References


Contact Information

Brett Friskney B.T.Friskney-13@student.lboro.ac.uk
Eliot Motato – e.motato@lboro.ac.uk
Ahmed Haris – a.haris@lboro.ac.uk
Mahdi Mohammad-Pour – m.mohammad-pour@lboro.ac.uk
Stephanos Theodossiades – s.theodossiades@lboro.ac.uk

Acknowledgments

The authors wish to express their gratitude to the EPSRC for the financial support extended to the “Targeted energy transfer in powertrains to reduce vibration-induced energy losses” Grant (EP/L019426/1), under which this research was carried out. Thanks are also due to Raicam clutch and Ford Motor Company for their technical support.

Definitions/Abbreviations

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DMF</td>
<td>Dual Mass Flywheel</td>
</tr>
<tr>
<td>NES</td>
<td>Nonlinear Energy Sink</td>
</tr>
<tr>
<td>TET</td>
<td>Targeted Energy Transfer</td>
</tr>
<tr>
<td>NNM</td>
<td>Nonlinear Normal Mode</td>
</tr>
<tr>
<td>NVH</td>
<td>Noise Vibration and Harshness</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>FEP</td>
<td>Frequency Energy Plot</td>
</tr>
<tr>
<td>DTE</td>
<td>Dynamic Transmission Error</td>
</tr>
</tbody>
</table>