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Prediction of Friction in EHL Contacts for Drivetrain Applications

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Abstract

Prediction of frictional losses in elastohydrodynamic lubricated contacts is of particular importance from the viewpoint of energy efficiency, thus reduced levels of emissions. There are increasingly stringent government regulations. Thus, the industry strives to identify potential losses and improve energy efficiency. Most losses occur in a large number of load bearing conjunctions in all forms of mechanisms and machines. These losses are affected by the operating conditions, such as applied load, contact kinematics and generated temperature. Prediction of prevailing conditions, such as generated pressures, film thickness is the prelude to evaluation of frictional power losses. Many contact conjunctions in vehicular drivetrains are subjected to elastohydrodynamic conditions, the fundamental aspects of which are still evolving. In particular, effective prediction of performance of elastohydrodynamic lubricated (EHL) contacts is subject to inclusion of realistic contact conditions, particularly with respect to inlet and outlet boundary conditions as well as kinematics of contact. This paper demonstrates the importance of boundary conditions on predictions of prevailing situations.

Keywords—Elastohydrodynamic Lubrication (EHL); Variable entrainment velocity distribution; Elliptical point contacts
1-Introduction

Accurate prediction of friction is important in order to take appropriate measures to mitigate its effect on power losses and energy efficiency. With transport contributing to nearly 70% of global energy usage [1], coupled with stricter government regulations, automotive manufacturers seek to identify and resolve potential sources of power loss. Additionally, passenger and commercial vehicles’ production grows year-on-year [1]. Thus, a greater effort is progressively expended upon powertrain efficiency.

There is ample evidence of the effect of powertrain efficiency upon the overall power losses for most vehicles [2-4]. For some vehicles, majority of power losses stem from the drivetrain and axle drives. It has been shown that 5 to 10% of all power losses can be attributed to the axle alone [4]. Hence, the current investigation focuses on the frictional losses associated with drivetrain components such as those of the axles. This paper is concerned with the untoward effects of friction upon drivetrain efficiency, therefore, energy (fuel) efficiency of vehicles. However, it should be noted that a certain level of friction is required to enable transmission of tractive power as well as act as an energy sink for excess transmitted engine power. Otherwise, the excess energy induces a plethora of noise, vibration and harshness (NVH) issues, affecting the vehicle refinement. There are many such phenomena, particularly transmission rattle [5-7], driveline elasto-acoustic response [8-10] and differential and axle vibration such as whine [11, 12]. Therefore, a link exists between generation friction, powertrain efficiency and NVH refinement which is not considered in the current paper, see [13]. Nevertheless, accurate prediction of friction is the primary interlude in any study of powertrain efficiency and refinement.

The differential at the core of the axle usually comprises hypoid or bevel gears and significant losses are attributed to the sliding of the gear teeth [4, 13]. The contacts between the meshing teeth are usually separated by a thin layer of lubricant, usually subjected to elastohydrodynamic lubrication (EHL) [14-16]. Therefore, it is necessary to accurately model and predict the contact conditions, such as the generated contact pressures, lubricant film thickness, thus the shear stress, friction and the power loss from the contact.

For the case of transmission gearing the contact footprint is either of elliptical or a narrow rectangular strip. Therefore, EHL solutions have been provided for these conditions, where
suitable boundary conditions and contact kinematics are essential in order to obtain realistic solutions, including the effect of inlet shear heating and starvation which are prevalent in gearing applications. Mohammadpour et al [17] show that lubricant entrainment occurs at an angle to the elliptical contact footprint for hypoid and bevel gears. Fundamental EHL solution for these conditions have been reported by Chittenden et al [18] and Jalali-Vahid et al [19], the latter showing good agreement with optical interferometric studies.

The inlet boundary condition is affected by the lubricant availability and flow into the contact by the relative speed of contiguous contacting surfaces as show by Tipei [20]. His numerical approach was used for circular point contact [21], with the results agreeing remarkably well with experimental measurements [21, 22]. Some of the inlet flow of lubricant counter-flow out of the contact whilst some other proportion of the flow swirls. Therefore, only some of the lubricant available at the inlet meniscus eventually flows into the contact, where no further reverse flow occurs. The boundary demarcated zero reverse flow is known as the stagnation point, which usually provides a much reduced inlet distance to the centre of the contact, resulting in contact starvation. This approach is also used for the case of hypoid gears, showing reduced film thickness and increased friction [23]. The inlet shear heating caused by the pressure gradient at the inlet converging gap also leads to shear thinning and starvation [24-26] with similar outcomes as the result of reduced contact film thickness. Therefore, it is essential to determine the inlet conditions, which are not only affected by the relative sliding motion of the contiguous surfaces, but also by spatial variation of entraining velocity along the inlet meniscus to the contact as shown later in figure 3. In fact, Tipei [20] shows that the inlet distance to the contact is not the same for all the contact footprint, but of curvilinear geometry. For the case of transmission gearing the speed of entraining motion of the free surface lubricant varies according to the location on the flank. This is the realistic inlet kinematics considered in this paper.
2-Theory

A numerical model with varied lubricant inlet entrainment velocity is developed, based on Reynolds equation. The Reynolds equation is solved simultaneously with lubricant viscosity variation with pressure [27] and that of lubricant density [28], together with the elastic film shape comprising contact deflection.

2.1 – Contacting solids

The contact of two solids such as gear teeth is reduced to an equivalent ellipsoidal rigid solid in contact with an elastic semi-infinite half-space rigid plate, as shown in figure 1 [28, 29].

![Figure 1: Equivalent ellipsoid and resulting contact footprint with geometrical references](image)

$R_x$ and $R_y$ are the radii of curvature along the $zx$ and $zy$ planes of contact of the equivalent ellipsoidal solid, whilst $a$ and $b$ are known as the semi-major and semi-minor axes of the elliptical contact footprint.

The reduced Young’s modulus of elasticity of the semi-infinite elastic half-space $E'$ becomes:

$$
\frac{1}{E'} = \frac{1}{2} \left( \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2} \right)
$$

(1)

where, $E_1$, $E_2$, $\nu_1$ and $\nu_2$ are the Young’s moduli and Poisson’s ratios for each contacting surfaces respectively.
The semi-major and semi-minor axes length can be approximated by the following relationships [28, 30]:

\[
\begin{align*}
a &= \left( \frac{6k^2\varepsilon WR'}{\pi E'} \right)^{\frac{1}{3}} \\
b &= \left( \frac{6\varepsilon WR'}{\pi kE'} \right)^{\frac{1}{3}}
\end{align*}
\]  

(2)  

(3)

where, \(W\) is the applied contact load and the equivalent (or reduced) radius \(R'\), the elliptical \(k\) parameter and \(\varepsilon\) are:

\[
\frac{1}{R'} = \frac{1}{R_x} + \frac{1}{R_y}
\]

(4)

\[
\bar{k} = 1.0339 \left( \frac{R_y}{R_x} \right)^{0.636}
\]

(5)

\[
\bar{\varepsilon} = 1.0003 + 0.5968 \left( \frac{R_x}{R_y} \right)
\]

(6)

The ellipticity of the contact can be considered as the ratio of semi-major to semi-minor axes as [30]:

\[
\bar{k} = \frac{a}{b}
\]

(7)

Thus, \(\bar{k} = 1\) depicts a circular point contact, while \(\bar{k} > 1\) depicts an elliptical contact.

2.2 - Reynolds equation

Reynolds equation in 2 dimensions can be stated as:

\[
\frac{\partial}{\partial x} \left[ \rho h^3 \left( \frac{\partial p}{\partial x} \right) \right] + \frac{\partial}{\partial y} \left[ \rho h^3 \left( \frac{\partial p}{\partial y} \right) \right] = \frac{\partial (\rho h U)}{\partial x} + \frac{\partial (\rho h V)}{\partial y} + \frac{\partial (2\rho h)}{\partial t}
\]

(8)

where \(\rho\), \(\eta\) and \(p\) are the density, viscosity and generated lubricant pressure, while \(h\) describes the film shape and the combined velocity of surfaces in the direction of entraining; \(x\), and side leakage; \(y\), given as \(U\) and \(V\) respectively.
2.3 – Elastic film shape

The film shape is given as:

\[ h = h_0 + s(x, y) + \delta(x, y) \]  

where, \( h_0 \) is the undeformed gap between the two surfaces, \( s(x, y) \) is the profile of the ellipsoidal solid and \( \delta(x, y) \) is the localised deflection [30]:

\[ s(x, y) = \frac{x^2}{2R_x} + \frac{y^2}{2R_y} \]  

\[ \delta(x, y) = \frac{2}{\pi E} \int \frac{p(x', y')}{\sqrt{(x-x')^2 + (y-y')^2}} d\chi' d\psi' \]  

2.4 – Lubricant rheology

The lubricant pressure dependent viscosity (piezo-viscosity) is obtained as [31]:

\[ \eta = \eta_0 e^{\alpha^* p} \]  

where, the resting viscosity, \( \eta_0 \), is taken at ambient temperature and pressure and modified pressure viscosity coefficient, \( \alpha^* \), is given by[27]:

\[ \alpha^* = \frac{1}{p} \left[ \ln(\eta_0) + 9.67 \right] \left( 1 + \frac{p}{1.98 \times 10^8} \right)^Z - 1 \]  

in which the desired and ambient temperatures are \( \theta \) and \( \theta_0 \) respectively, while \( Z \) and \( S_0 \) can be found by:

\[ Z = \frac{\alpha_0}{(5.1 \times 10^{-6} \ln(\eta_0) + 9.67)} \]  

\[ S_0 = \frac{\beta_0 (\theta_0 - 138)}{(\ln(\eta_0) + 9.67)} \]  

where, \( \alpha_0 \) is the pressure-viscosity coefficient and \( \beta \) is the thermo-viscosity coefficient. For isothermal cases if \( \theta \) and \( \theta_0 \) are equal, then the Houpert’s equation [27] is reduced to that of Roelands [31].
2.5 – Dimensionless parameters

For the purpose of comparison, the dimensionless groups, given below are used [30]:

\[ G^* = E' \alpha_0 \] (16)

\[ U^* = \frac{\eta_0 U}{2E' R_x} \] (17)

\[ W^* = \frac{W}{E' R_x^2} \] (18)

All film thickness and pressure results are normally presented using the following non-dimensional groups respectively:

\[ H = \frac{h}{R_x} \] (19)

\[ P = \frac{p}{p_h} \] (20)

where, \( p_h \) is the maximum Hertzian pressure [30]:

\[ p_h = \frac{3W}{2\pi ab} \] (21)

2.6 – Computational domain

Figure 2 shows the computational domain. A rectangular domain is chosen with a total length being the sum of the inlet distance, \( f_i \), and the exit distance (point of lubricant film rupture), \( f_o \), while the total width of the domain is the sum of the two side lengths, \( f_{s1} \) and \( f_{s2} \). Flow direction is set to go along the minor axis of the contact footprint. Although, it should be noted that in some gearing applications such as hypoid gears the contact ellipse precesses about the contact normal as shown by Mohammadpour et al [15, 17, 23].
Table 1 lists the chosen parameters for the current study. Direction of entrainment velocity, $U$, is along the semi-minor (i.e. $X$ axis) direction. The side leakage occurs is along the semi-major (i.e. $Y$ axis) axis.

**Table 1: Domain size description**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_i$</td>
<td>Inlet distance</td>
<td>3.5$a$</td>
</tr>
<tr>
<td>$f_o$</td>
<td>Exit boundary</td>
<td>1.5$a$</td>
</tr>
<tr>
<td>$f_{s1}, f_{s2}$</td>
<td>Side Length</td>
<td>1.5$a$</td>
</tr>
<tr>
<td>$N_x$</td>
<td>Number of elements along the $X$-axis</td>
<td>80</td>
</tr>
<tr>
<td>$N_y$</td>
<td>Number of elements along the $Y$-axis</td>
<td>80</td>
</tr>
</tbody>
</table>

**2.7 – Variable inlet velocity distribution**

It is assumed, that the velocity across the contact inlet, $U(y)$, varies according to the following relationship (see Figure 3):
\[ U(y) = \omega r(y) \]  

where, \( \omega \) represents a constant representative angular velocity and \( r(y) \) is the distance from centre of rotation and the inlet along the y axis.

\[ \mathbf{U}(y) = \omega r(y) \]  

Figure 3: Inlet velocity profile

3-Results and Discussion

To observe the effect of the inlet entrainment velocity distribution, isothermal elastohydrodynamic solution is compared with case of uniform inlet velocity. Input conditions from Lubrecht et al [24] were selected for this study as listed in Table 2.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Value</th>
<th>( R_x ) (m)</th>
<th>( \omega ) (s(^{-1}))</th>
<th>( \eta_0 ) (Pas)</th>
<th>( \alpha_0 ) (Pa(^{-1}))</th>
<th>( G^* )</th>
<th>( U^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td></td>
<td>0.02</td>
<td>12.35</td>
<td>0.041</td>
<td>2\times10^{-8}</td>
<td>4520</td>
<td>1.68\times10^{-12}</td>
</tr>
</tbody>
</table>

The results with constant uniform inlet velocity is shown for the lubricant film thickness contour plots in Figure 4 which agree with those reported by Lubrecht et al [24]. Furthermore, the numerical results show good agreement with the results obtained using Hamrock and Dowson extrapolated regression equation for lubricant film thickness [32] as listed in Tables 3 and 4 as well as in Figures 5 and 6. It is worth noting that for low ellipticity ratio of \( \tilde{k} \leq 2.88 \), the location
of the minimum film thickness $H_m$ tends to the sides of the contact, whilst for higher ratios cases
the minimum film thickness is located at the rear of the contact.

![Location of $H_m$](image)

(a)

![Location of $H_m$](image)

(b)

Figure 4: Comparison of film thickness contour plots of (a) constant and (b) variable entrainment velocity
for $k=1, 1.61, 2.88$ and $12.48$
Table 3: Comparison of non-dimensional central film thickness values

<table>
<thead>
<tr>
<th>( \frac{R_y}{R_x} )</th>
<th>( \bar{k} )</th>
<th>( W^* )</th>
<th>( H_c ) (Hamrock and Dowson [32])</th>
<th>( H_c ) (Lubrecht et al [24])</th>
<th>( H_c ) (Constant U)</th>
<th>( H_c ) (Variable U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.11E-07</td>
<td>6.27E-06</td>
<td>6.03E-06</td>
<td>6.04E-06</td>
<td>6.04E-06</td>
</tr>
<tr>
<td>2</td>
<td>1.61</td>
<td>2.13E-07</td>
<td>6.83E-06</td>
<td>6.53E-06</td>
<td>6.67E-06</td>
<td>6.67E-06</td>
</tr>
<tr>
<td>5</td>
<td>2.88</td>
<td>4.48E-07</td>
<td>7.41E-06</td>
<td>6.61E-06</td>
<td>7.03E-06</td>
<td>7.03E-06</td>
</tr>
<tr>
<td>10</td>
<td>4.48</td>
<td>7.45E-07</td>
<td>7.56E-06</td>
<td>6.43E-06</td>
<td>7.14E-06</td>
<td>7.14E-06</td>
</tr>
<tr>
<td>20</td>
<td>6.96</td>
<td>1.19E-06</td>
<td>7.47E-06</td>
<td>6.42E-06</td>
<td>7.18E-06</td>
<td>7.18E-06</td>
</tr>
<tr>
<td>50</td>
<td>12.48</td>
<td>2.13E-06</td>
<td>7.21E-06</td>
<td>6.41E-06</td>
<td>7.19E-06</td>
<td>7.19E-06</td>
</tr>
<tr>
<td>100</td>
<td>19.40</td>
<td>3.25E-06</td>
<td>7.01E-06</td>
<td>6.4E-06</td>
<td>7.17E-06</td>
<td>7.17E-06</td>
</tr>
</tbody>
</table>

Table 4: Comparison of non-dimensional minimum film thickness values

<table>
<thead>
<tr>
<th>( \frac{R_y}{R_x} )</th>
<th>( \bar{k} )</th>
<th>( W^* )</th>
<th>( H_m ) (Hamrock and Dowson [32])</th>
<th>( H_m ) (Lubrecht et al [24])</th>
<th>( H_m ) (Constant U)</th>
<th>( H_m ) (Variable U)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1.11E-07</td>
<td>3.59E-06</td>
<td>3.66E-06</td>
<td>3.38E-06</td>
<td>3.37E-06</td>
</tr>
<tr>
<td>2</td>
<td>1.61</td>
<td>2.13E-07</td>
<td>4.51E-06</td>
<td>4.33E-06</td>
<td>4.21E-06</td>
<td>4.18E-06</td>
</tr>
<tr>
<td>5</td>
<td>2.88</td>
<td>4.48E-07</td>
<td>5.52E-06</td>
<td>5.19E-06</td>
<td>5.19E-06</td>
<td>5.12E-06</td>
</tr>
<tr>
<td>10</td>
<td>4.48</td>
<td>7.45E-07</td>
<td>5.89E-06</td>
<td>5.39E-06</td>
<td>5.57E-06</td>
<td>5.53E-06</td>
</tr>
<tr>
<td>20</td>
<td>6.96</td>
<td>1.19E-06</td>
<td>5.93E-06</td>
<td>5.38E-06</td>
<td>5.6E-06</td>
<td>5.55E-06</td>
</tr>
<tr>
<td>50</td>
<td>12.48</td>
<td>2.13E-06</td>
<td>5.73E-06</td>
<td>5.37E-06</td>
<td>5.59E-06</td>
<td>5.49E-06</td>
</tr>
<tr>
<td>100</td>
<td>19.40</td>
<td>3.25E-06</td>
<td>5.56E-06</td>
<td>5.37E-06</td>
<td>5.54E-06</td>
<td>5.36E-06</td>
</tr>
</tbody>
</table>

As expected Figure 4 shows that for lower ellipticity ratios including for circular point contact, varying inlet lubricant entrainment velocity does not alter the overall film thickness distribution. However, for \( \bar{k} \geq 4.48 \) the influence of \( U(y) \) becomes significant. The minimum film thickness location appears to have shifted towards the lower velocity side of the contact (i.e. clearly a reduced flow rate with an assumed fully flooded contact reduces the lubricant film thickness). Hence, the overall film thickness distribution becomes asymmetric. Interestingly for this case, varied velocity appears not to significantly change the absolute central film thickness, despite the change in the entire distribution, as shown in Figure 5.
Figure 5: Comparison of dimensionless central film thickness values between predictions from extrapolated equations and numerical predictions, for a range of ellipticity ratios

Figure 6: Comparison of dimensionless minimum film thickness values between predictions from extrapolated equations and numerical predictions for a range of ellipticity ratios

Figures 7 and 8 show the differences in generated pressures and the corresponding film thickness between assumed uniform inlet velocity and a varied distribution. These differences are...
negligible along the centreline of the contact. Along the side leakage direction the asymmetric film thickness profile is as the result of varied lubricant entrainment at the inlet meniscus.

Figure 7: Comparison of centreline dimensionless film thickness and pressure for $R_y/R_x = 100$ ($\bar{k} = 19.40$)

Figure 8: Comparison of dimensionless film thickness and pressure for $R_y/R_x = 100$ ($\bar{k} = 19.40$) along the side leakage axis
4. Conclusions

The effect of varied inlet entrainment velocity distribution on the lubricant film thickness and generated elastohydrodynamic pressures is investigated. Varied inlet velocity distribution has a significant effect on the overall pressure distribution and film thickness, especially in the side leakage direction for elliptical point contacts with higher ellipticity ratios. The results also show that the film thickness at the part of the contact with lower localised inlet velocity is reduced which can lead to starvation and mixed regime of lubrication, thus increased friction. These points need further investigation.

Acknowledgement

The supports of EPSRC under the CDT-EI are acknowledged.

Nomenclature

\(a\) Semi-major axis half-width
\(b\) Semi-minor axis half-width
\(E'\) Effective (reduced) Young’s modulus of elasticity of elastic half-space
\(G^*\) Dimensionless Materials’ Parameter
\(h\) Film thickness
\(H\) Dimensionless film thickness \( (h/R_x) \)
\(H_c\) Dimensionless central film thickness
\(H_m\) Dimensionless minimum film thickness
\(\bar{k}\) Ellipticity ratio
\(p\) Pressure
Maximum Hertzian Pressure

Dimensionless Pressure ($p/p_h$)

Distance from centre of rotation

Principal radii of curvature of contact in the $zx$ and $zy$ planes of contact

Sliding Velocity

Dimensionless Velocity (rolling viscosity) parameter

Applied contact load

Dimensionless load parameter

\begin{itemize}
  \item $p_h$ \quad Maximum Hertzian Pressure
  \item $P$ \quad Dimensionless Pressure ($p/p_h$)
  \item $r(y)$ \quad Distance from centre of rotation
  \item $R_x, R_y$ \quad Principal radii of curvature of contact in the $zx$ and $zy$ planes of contact
  \item $U$ \quad Sliding Velocity
  \item $U^*$ \quad Dimensionless Velocity (rolling viscosity) parameter
  \item $W$ \quad Applied contact load
  \item $W^*$ \quad Dimensionless load parameter
\end{itemize}

\textbf{Greek Symbols}

\begin{itemize}
  \item $\alpha_0$ \quad Pressure-viscosity coefficient
  \item $\beta$ \quad Thermal viscosity coefficient
  \item $\delta$ \quad Localised elastic deformation
  \item $\epsilon$ \quad Simplified elliptical integral parameter
  \item $\eta$ \quad Lubricant’s dynamic viscosity
  \item $\eta_0$ \quad Lubricant’s atmospheric dynamic viscosity
  \item $\theta_0$ \quad Reference temperature
  \item $\theta$ \quad Contact Temperature
  \item $\rho$ \quad Lubricant density
\end{itemize}
References:


[23]- Mohammadpour, M., Theodossiades, S. and Rahnejat, H., “Transient mixed non-Newtonian thermo-elastohydrodynamics of vehicle differential hypoid gears with starved partial counter-


