Measures of component contribution to the failure of phased missions

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INTRODUCTION

The operational modes of many systems mean that they operate in an overall mission which consists of consecutive phases having different functional requirements. Successful completion of the mission requires the successful completion of each of the phases. As a result, a failure to meet the functional requirements of any phase will cause mission failure. The likelihood of mission failure is termed the mission unreliability and the means used to calculate this depends on if the system is repairable or non-repairable over the duration of the mission. For repairable systems Markov methods (Smotherman & Zemoudeh 1989) are frequently employed. For non-repairable systems fault tree methods offer an efficient alternative (La Band & Andrews 2004, Esary & Ziehms 1975).

This paper deals with the failure of systems which are non-repairable for the mission duration. Once a failure event occurs that condition will be present through the remainder of the mission. Repairs can be instigated prior to the start of each mission. The causes of a phase failure can occur in the phase itself or in an earlier phase. When phase failure conditions exist on entry to the phase the system, and therefore mission, failure occurs on phase transition. Having survived earlier phases also means that certain combinations of failure events, which would have resulted in a prior phase failure, cannot have occurred. Accounting for these factors increases the complexity of the problem beyond that if a standard system reliability assessment.

The evaluation of the mission reliability will enable decisions to be made regarding the acceptability of the system performance. In a conventional system assessment importance measures (Birnbaum 1969, Andrews & Beeson 2003) would also be calculated to indicate the contribution that each component failure makes to the system failure. Using this information weaknesses in the system design can be identified and the system modified in order to remove them. In a phased mission analysis the importance contributions are time-dependent. The contribution a component failure makes to the mission failure can vary from one-phase to the next. It is possible that a component failure will have a significant contribution to the failure in some phases and no contribution at all in others. To make an assessment of the weaknesses of the system the contribution a component failure makes in the phases need to be combined to give an overall mission or system failure contribution. The contribution of a failure to the phase failure will also change as the phase progresses and a failure in one phase can be a contribution to the failure of a later mission phase.

These factors make the definition and determination of the importance measures a complex issue. This paper deals with the development of two component importance measures. In conventional systems assessment these measures are equivalent to Birnbaum’s measure of importance (also known as...
the Criticality function) and the Criticality measure of importance.

2 IMPORTANT MEASURES FOR SYSTEMS RELIABILITY ASSESSMENT

2.1 Critical System States

Before considering the application to phased missions this section gives the definitions of Birnbaum’s measure of importance and the Criticality measure of importance as used in conventional systems reliability assessment. They are based on the concept of a critical system state.

A critical system state for a component i is a state of the remaining components in the system such that when i makes a transition from the working to the failed state the system also makes a transition from the working to the failed state.

Thus the state of component i is critical in determining the state of the system.

2.2 Birnbaum’s Measure of importance

Birnbaum’s measure of importance, \( G_i \), is defined as:

The probability that the system is in a critical state for component i. This can also be expressed as:

\[
G_i(t) = Q_{SYS}(1, q(t)) - Q_{SYS}(0, q(t)) = \frac{\partial Q_{SYS}(t)}{\partial q_i(t)}
\]

where \( Q_{SYS} \) is the system failure probability and \( q_i \) is the probability of failure of component i.

2.3 The Criticality Measure of Importance

The criticality measure of importance builds on Birnbaum’s measure in recognising the contribution that a component i makes to the system failure. It is the proportion of times that the failure of component i causes the system to fail. The probability of component i causing the system to fail is the likelihood that the system is critical state for component i, \( G_i \), and i has also failed, \( q_i \). This is divided by the probability that the system fails due to any cause, \( Q_{SYS} \). i.e.:

\[
I_{CR} = \frac{G_i(t)q_i(t)}{Q_{SYS}(t)}
\]

3 MISSION UNRELIABILITY

A method to calculate the mission unreliability has been presented in La Band & Andrews (2004). The method proceeds by developing a fault tree failure logic structure for the causes of failure in each phase based on the component performance in the phase under consideration and the preceding phases. Therefore if any component A contributes to the failure causes in phase j its failure can have occurred in any phase up to and including phase j and so we can represent this by the logic equation:

\[
A = A_1 + \ldots + A_j
\]

Where \( A_j \) represents that failure of component in phase j (+ in this case is OR).

For failure to occur in phase j the system must have successfully completed phases 1…j-1. A fault tree to provide the causes of phase j failure is illustrated in figure 1. This can be analysed to get the phase prime implicants and phase failure probabilities, \( Q_j \). The mission unreliability is then the sum of the phase failure probabilities.

\[
Q_{MISS} = \sum_{j=1}^{n} Q_j
\]

Failure During Phase j
Success in Previous Phases
Failure Conditions Met During Phase j

Figure 1. Fault Tree for failure in Phase j

3.1 Phase failure modes

To determine the phase prime implicants or failure modes requires the use of a special phase algebra. Details of this can be found in La Band & Andrews
To make the algebraic expressions more concise the notation for an event failure, $A$, in phase $j$, $A_j$, can be extended to incorporate the failure over a range of phases. So $A_{i,j}$ now represents the failure of component $A$ during the period from the start of phase $i$ up to and including phase $j$. The non-failure of event $A$ during this period is denoted as $\overline{A_{i,j}}$.

Therefore:

$$A_{i,j} = A_i + A_{i+1} + \ldots + A_j$$

$$A_{i,j} = A_i\overline{A_{i+1}}\overline{\ldots}\overline{A_j} \quad (5)$$

When manipulating the logic expression for the phase failure fault trees the following rules must be used:

$$A_i.A_j = 0$$

$$A_i.A_i = A_i$$

$$A_i.A_{i,j} = A_i$$

$$A_i.A_i = 0$$

$$A_i.A_{i,j} = A_{i+1,j}$$

$$A_i\overline{A_j} = 0 \text{ if } i < j \quad (6)$$

3.2 Phase failure probability

Since the phase failure fault trees are, for all but phase 1, non-coherent the phase failure modes are more correctly referred to as prime implicants. The likelihood of failure in each phase can then be calculated using the full inclusion-exclusion expansion:

$$Q_j = \sum_{j=1}^{N_p} p(C_j) - \sum_{j=1}^{N_p} \sum_{i=1}^{j-1} p(C_j \cap C_i) + \ldots + (-1)^{N_p-1} p(C_1 \cap C_2 \cap \ldots \cap C_{N_p}) \quad (7)$$

Where $C_i$, $i=1,\ldots,N_p$ is a list of the phase prime implicants. It must be noted that due to the non-coherent nature of the phase prime implicants Henley & Inagaki’s (1980) formulation of equation 7 must be used. In this formulation the probability of the success state of component $i$ throughout phase $j$ is represented by $p_{ij}$, and the failure probability of component $i$ during phase $j$ as $q_{ij}$. (Noting that combinations with products of both $p_{ij}$ and $q_{ij}$ are deleted as they have a probability of zero). These variables are treated as independent. Alternatively for large systems with many phases the coherent approximations to the phase failure modes can be obtained and approximations for the phase failure probability used such as the minimal cut set upper bound.

4 PHASED MISSION EXAMPLE

A very simple example phased mission problem consisting of non-repairable components A, B and C will be used to demonstrate the concepts presented. The detailed phased mission reliability calculations for this problem is presented in Andrews (2006). The phase failure fault trees are illustrated in Figure 2. Phase 1 lasts from $t_0$ until $t_1$, phase 2 starts at $t_1$ and finishes at $t_2$, and the final phase is from $t_2$ until $t_3$. If failure is avoided in all three phases the mission is successful and the probability of this is the mission reliability.

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Figure 2. Simple phase failure fault tree example
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5 PHASED MISSION UNRELIABILITY

The detailed analysis of all three phases are provided in Andrews (2006). In this paper only the calculations for phase 2 will be considered to demonstrate the concepts of the method. Figure 3 shows the fault tree indicating the causes of failure in phase 2. The top event structure has the successful completion for phase 1 ANDed with the causes of the conditions of failure to be met in phase 2.

5.1 Phase failure modes/failure probabilities

Boolean reduction of the phase failure fault tree will produce the phase failure modes. Consider the fault tree illustrated in figure 3. Establishing a Boolean expression for the fault tree gives the phase 2 failure modes (the negated term represents the conditions for success in phase 1, the remainder of the expression is the failure conditions for phase 2):

$$TOP2 = \overline{A_1}B_1(A_{1,2}B_{1,2})$$

$$= A_2B_2 \quad (8)$$

Figure 3. Simple phase failure fault tree example
For phases 1 and 3:

\[
TOP1 = A_j + B_j
\]

\[
TOP3 = A_{1,2} B_1 C_{1,3} + A_j B_{1,2} C_{1,3} + A_j B_{2,3} + A_{2,3} B_3
\]  \hspace{1cm} (9)

To evaluate the probability of failure in each phase \( j \), \( Q_j \), we use equation 7. Note only phase 3 contains success states where Henley and Inagaki’s (1980) formulation applies. So:

\[
Q_1 = q_{A_1} + q_{B_1} - q_{A_1} q_{B_1}
\]  \hspace{1cm} (10)

\[
Q_2 = q_{A_2} q_{B_1}
\]  \hspace{1cm} (11)

\[
Q_3 = \left[p_{A_{1,2}} p_{B_1} q_{C_{1,3}} + p_{A_1} p_{B_{1,2}} q_{C_{1,3}} + q_{A_1} q_{B_{2,3}} + q_{A_{1,2}} q_{B_1}ight] - \left[p_{A_1} p_{B_1} q_{C_{1,3}} + q_{A_1} q_{B_{2,3}} q_{C_{1,3}}ight] + 2 q_{A_1} q_{B_1} q_{C_{1,3}} + q_{A_2,3} q_{B_1} q_{C_{1,3}} + q_{A_2,3} q_{B_1} q_{C_{1,3}} + [4 q_{A_1} q_{B_1} q_{C_{1,3}}] - \left[q_{A_1} q_{B_1} q_{C_{1,3}} \right]
\]  \hspace{1cm} (12)

The probability of failure of any component \( A \) in a phase \( j \) is given by:

\[
q_{A_j} = \int_{t_j}^{t_{j-1}} f_A(t) dt
\]  \hspace{1cm} (13)

where \( f_A \) is the density function of times to failure for component \( A \).

6 PHASED MISSION FAILURE CAUSES

For a system to fail it needs to be in a critical condition for any of the \( i \) components and also the \( (i) \)th component fails.

In the context of a phased mission we have two scenarios that can result in phase and mission failure:

1. For any phase the system can be in a critical state for a component \( i \) in phase \( j \) and component \( i \) then fails during the phase causing phase failure. (Phase importance)

2. Alternatively the failure conditions for phase \( j \) may exist prior to the mission entering phase \( j \) and phase failure occurs on transition to phase \( j \). (Transition Importance).

7 BIRNBAUM’S IMPORTANCE MEASURES

7.1 Phase criticality function

Extending the concept of the criticality function given in equation 1 to phased missions gives:

\[
G_{i,j} = \frac{\partial Q_j}{\partial q_{i_j}}
\]  \hspace{1cm} (14)

where \( G_{i,j} \) is the probability that the system is in a critical state during phase \( j \) for the failure of component \( i \) during this phase to cause phase failure.

This can be demonstrated using the simple phased mission example.

For phase 1:

\[
G_{A_1,1} = \frac{\partial Q_1}{\partial q_{A_1}} = 1 - q_{B_1} \quad G_{B_1,1} = \frac{\partial Q_1}{\partial q_{B_1}} = 1 - q_{A_1}
\]

\[
G_{C_1} = 0
\]

For phase 2:

\[
G_{A_{1,2}} = \frac{\partial Q_2}{\partial q_{A_1}} = q_{B_1} \quad G_{B_{1,2}} = \frac{\partial Q_2}{\partial q_{B_1}} = q_{A_1}
\]

\[
G_{C_{1,2}} = 0
\]

For phase 3:
\[
G_{A,3} = \frac{\partial Q_j}{\partial q_{B_j}} = q_{B_j} (1 - q_{C_{ij}})
\]
\[
G_{B,3} = \frac{\partial Q_j}{\partial q_{B_j}} = q_{A_{ij}} (1 - q_{C_{ij}})
\]
\[
G_{C,3} = \frac{\partial Q_j}{\partial q_{C_j}} = p_{A_{ij}} p_{B_{ij}} + p_{A_{ij}} p_{B_{ij}} - q_{A_{ij}} q_{B_{ij}} - q_{A_{ij}} q_{B_{ij}}
\]

(17)

7.2 Phase transition function

\( Q_j \) is the probability that the conditions for failure in phase \( j \) occur. It accounts for the probability of failure during the phase and on transition into the phase. To establish the probability of transition to the phase \( j \), \( Q^T_{ij} \), the logic equations for the causes of phase failure need to remove any of the failure events which occur in phase \( j \). This is equivalent to establishing the causes of phase failure in phase \( j \) and in the \( 1...j-1 \) phases prior to phase \( j \) and can be achieved by modifying the right hand box form the top event on the fault tree in figure 1 to be 'failure conditions met prior to phase \( j \). This gives causes of transition to phase \( j \), \( \text{TOP}^T_j \):

\[
\text{TOP}^T_2 = A_j B_j (A_j B_j) = 0 \quad \text{giving} \quad Q^T_2 = 0 \quad (18)
\]

Similarly for phase 3:

\[
\text{TOP}^T_3 = A_j B_j (A_{1,2} B_{1,2} + C_{1,2})
\]
\[
= A_{1,2} B_{1,2} C_{1,2} + A_{1,2} B_{1,2} C_{1,2}
\]

(19)

giving:

\[
Q^T_i = p_{A_{ij}} p_{B_{ij}} + p_{A_{ij}} p_{B_{ij}} - p_{A_{ij}} p_{B_{ij}} - p_{A_{ij}} p_{B_{ij}}
\]

(20)

The phase transition function is the criticality function relating to the failure on transition to a phase. The phase transition function \( G^T_{i,j,k} \) is the probability that the system is critical for transition failure to phase \( j \) for the failure of a component \( i \) in phase \( k \) prior to \( j \).

\[
G^T_{i,j,k} = \frac{\partial Q^T_i}{\partial q_{i_k}}
\]

(21)

Considering failure criticality for the transition to phase 2:

\[
G^T_{A,2,i} = \frac{\partial Q^T_i}{\partial q_{A_i}} = 0 \quad \text{and} \quad G^T_{B,2,i} = \frac{\partial Q^T_i}{\partial q_{B_i}} = 0
\]

(22)

\[
G^T_{C,2,i} = \frac{\partial Q^T_i}{\partial q_{C_i}} = 0
\]

For transition criticality for phase 3:

\[
G^T_{A,3,i} = \frac{\partial Q^T_i}{\partial q_{A_i}} = 0 \quad \text{and} \quad G^T_{B,3,i} = \frac{\partial Q^T_i}{\partial q_{B_i}} = 0
\]

\[
G^T_{C,3,i} = \frac{\partial Q^T_i}{\partial q_{C_i}} = 0
\]

(23)

8 CRITICALITY IMPORTANCE MEASURES

8.1 Phase component criticality importance for component \( i \) in phase \( j \)

\[
I^P_{i,j} = \frac{\sum_{k=1}^{l} G^T_{i,j,k} q_{i_k}}{Q_j} = \frac{\sum_{k=1}^{l} \frac{\partial Q^T_i}{\partial q_{i_k}} q_{i_k}}{Q_j}
\]

(24)

8.2 Phase transition criticality importance for component \( i \) in phase \( j \)

Phase transition failure requires that the failure conditions have occurred for phase \( j \) in some phase, \( k \), prior to phase \( j \) but these conditions have not caused previous phase failure. This importance measure \( I^T_{i,j} \) is the failure contribution that component \( i \) makes to the transition failure of phase \( j \) as a proportion of the total phase failure probability i.e.:

\[
I^T_{i,j} = \frac{\sum_{k=1}^{l} G^T_{i,j,k} q_{i_k}}{Q_j} = \frac{\sum_{k=1}^{l} \frac{\partial Q^T_i}{\partial q_{i_k}} q_{i_k}}{Q_j}
\]

(25)

9 MISSION IMPORTANCE CONTRIBUTIONS

The total importance contribution of component \( i \) failure in phase \( j \) is:
A measure to indicate the total contribution made by a component \( i \) to the whole mission failure of the system is:

\[
I_{i,j} = I_{i,j}^p + I_{i,j}^T
\]

\[1\]

\[
I_j = \sum_{all} \left[ \frac{\partial Q_j}{\partial q_{i,j}} q_{i,j} + \left( \sum_{k=1}^{i-1} \frac{\partial Q_j^T}{\partial q_{i,k}} q_{i,k} \right) \right] \frac{1}{Q_{\text{Miss}}} \]

CONCLUSIONS

1. Importance measures have been established which indicate the contributions of system components to system failure in each phase of a non-repairable phased mission. These measures account for the successful completion of earlier mission phases.
2. The importance measures account for the contribution to a phase failure of a component failure in all phases up to and including the phase under consideration.
3. The phase contributions have been collected together to give an overall contribution of each component to the overall mission failure.

REFERENCES