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Metadata Record: https://dspace.lboro.ac.uk/2134/4118

Version: Published

Publisher: © IEEE

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ON INTUITIONISTIC FUZZY NEGATIONS AND INTUITIONISTIC FUZZY EXTENDED MODAL OPERATORS. Part 2.

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Abstract—Some relations between intuitionistic fuzzy negations and intuitionistic fuzzy extended modal operations $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$ are studied.

I. On some previous results

The concept of the Intuitionistic Fuzzy Set (IFS, see [1]) was introduced in 1983 as an extension of Zadeh’s fuzzy set. All operations, defined over fuzzy sets were transformed for the IFS case. One of them - operation “negation” now there is 27 different forms (see [2]). In [1] the relations between the “classical” negation and the two standard modal operators “necassity” and “possibility” are given. Here, we shall study the relations between the intuitionistic fuzzy negations and the intuitionistic fuzzy extended modal operations $F_{\alpha,\beta}$ and $G_{\alpha,\beta}$.

In some definitions we shall use functions $sg$ and $\overline{sg}$:

$$sg(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \end{cases}$$

$$\overline{sg}(x) = \begin{cases} 0 & \text{if } x > 0 \\ 1 & \text{if } x \leq 0 \end{cases}$$

For any two IFSs $A$ and $B$ the following relations are valid:

$A \subseteq B$ iff $(\forall x \in E)(\mu_A(x) \leq \mu_B(x))\nu_A(x) \geq \nu_B(x)$,

$A \supseteq B$ iff $B \subseteq A$,

$A = B$ iff $(\forall x \in E)(\mu_A(x) = \mu_B(x))\&\nu_A(x) = \nu_B(x)$.

Let $A$ be a fixed IFS. In [1] definitions of standard modal operators are given:

$$\Box A = \{\langle x, \mu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\Diamond A = \{\langle x, 1 - \nu_A(x), \nu_A(x)\rangle | x \in E\}.$$  

The first extended modal operator is

$$D_0(A) = \{\langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + (1 - \alpha) \cdot \pi_A(x)\rangle | x \in E\},$$

where $\alpha \in [0, 1]$. It is extended to

$$F_{\alpha,\beta}(A) = \{\langle x, \mu_A(x) + \alpha \cdot \pi_A(x), \nu_A(x) + \beta \cdot \pi_A(x)\rangle | x \in E\},$$

where $\alpha, \beta \in [0, 1]$ and $\alpha + \beta \leq 1$. Another non-standard modal operator is

$$G_{\alpha,\beta}(A) = \{\langle x, \alpha \cdot \mu_A(x), \beta \cdot \nu_A(x)\rangle | x \in E\},$$

where $\alpha, \beta \in [0, 1]$.

Obviously,

$$\Box A = D_0(A) = F_{0,1}(A),$$

$$\Diamond A = D_1(A) = F_{1,0}(A),$$

$$D_0(A) = F_{\alpha,1-\alpha}(A).$$

In [2], [3], [4], [5], [6] the following 27 different negations are described.

$$\neg_1 A = \{\langle \nu_A(x), \mu_A(x)\rangle | x \in E\},$$

$$\neg_2 A = \{\langle \overline{sg}(\mu_A(x)), \overline{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_3 A = \{\langle \nu_A(x), \mu_A(x)\rangle, \nu_A(x) + \mu_A(x)^2 | x \in E\},$$

$$\neg_4 A = \{\langle \nu_A(x), 1 - \mu_A(x)\rangle | x \in E\},$$

$$\neg_5 A = \{\langle \overline{sg}(1 - \nu_A(x)), \overline{sg}(1 - \nu_A(x))\rangle | x \in E\},$$

$$\neg_6 A = \{\langle \nu_A(x), \overline{sg}(\mu_A(x))\rangle | x \in E\},$$

$$\neg_7 A = \{\langle \nu_A(x), \mu_A(x) \rangle | x \in E\},$$

$$\neg_8 A = \{\langle (1 - \nu_A(x)), \mu_A(x) \rangle | x \in E\},$$

$$\neg_9 A = \{\langle \overline{sg}(\mu_A(x)), \mu_A(x) \rangle | x \in E\},$$

$$\neg_{10} A = \{\langle \overline{sg}(1 - \nu_A(x)), 1 - \nu_A(x) \rangle | x \in E\},$$

$$\neg_{11} A = \{\langle \overline{sg}(\nu_A(x)), \overline{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{12} A = \{\langle \mu_A(x), \mu_A(x) + \nu_A(x)\rangle,$$

$$\mu_A(x), (\mu_A(x) + \nu_A(x))^2 | x \in E\},$$

$$\neg_{13} A = \{\langle (1 - \overline{sg}(1 - \mu_A(x)), \overline{sg}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{14} A = \{\langle \overline{sg}(\nu_A(x)), \overline{sg}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{15} A = \{\langle \overline{sg}(1 - \nu_A(x)), \overline{sg}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{16} A = \{\langle \overline{sg}(\mu_A(x)), \overline{sg}(1 - \mu_A(x))\rangle | x \in E\},$$

$$\neg_{17} A = \{\langle (1 - \nu_A(x)), \overline{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{18} A = \{\langle \nu_A(x) \cdot \nu_A(x), \mu_A(x), \overline{sg}(\nu_A(x))\rangle | x \in E\},$$

$$\neg_{19} A = \{\langle \nu_A(x), \nu_A(x) \cdot \overline{sg}(\mu_A(x))\rangle, 0 | x \in E\},$$

$$\neg_{20} A = \{\langle \nu_A(x), 0 \rangle | x \in E\},$$

$$\neg_{21} A = \{\langle \nu_A(x), \nu_A(x), \nu_A(x) + \mu_A(x)^n \rangle | x \in E\},$$

where real number $n \in [2, \infty)$,

$$\neg_{22} A = \{\langle x, \nu_A(x)\rangle,$$
Theorem 1: For every IFS shall prove following assertions.

\( \mu_A(x) = \nu_A(x) + \max(0, \mu_A(x) - \nu_A(x)) \) \( \forall x \in E \),

\( -\gamma_3 A = \{ (x, 1 - \mu_A(x)) : \nu_A(x) \} \) \( \forall x \in E \),

\( \mu_A(x) = \max(0, \mu_A(x) - \nu_A(x)) \) \( \forall x \in E \),

\( \mu_A(x) = \nu_A(x) + \max(0, \mu_A(x) - \nu_A(x)) \) \( \forall x \in E \),

\( \gamma_2 A = \{ (x, \mu_A(x)) : \nu_A(x) \} \) \( \forall x \in E \),

\( \lambda \subseteq \alpha \beta \),

\( \alpha \beta \subseteq \lambda \).

Theorem 2: For every IFS \( \alpha, \beta \in [0, 1] \) the following properties are valid:

(1) \( \gamma_1 A \subseteq \alpha \beta \) \( \forall x \in E \),

(2) \( \gamma_2 A \subseteq \alpha \beta \) \( \forall x \in E \),

(3) \( \gamma_3 A \subseteq \alpha \beta \) \( \forall x \in E \),

(4) \( \gamma_4 A \subseteq \alpha \beta \) \( \forall x \in E \),

(5) \( \gamma_5 A \subseteq \alpha \beta \) \( \forall x \in E \),

(6) \( \gamma_6 A \subseteq \alpha \beta \) \( \forall x \in E \).

Proof: Let \( \alpha, \beta \in [0, 1] \) be given so that \( \alpha + \beta \leq 1 \), and let \( A \) be an IFS. Then we obtain directly that:

\( \gamma_1 \alpha \beta (A) = \gamma_1 \alpha \beta A(\gamma_1 A) \)

(7) \( \gamma_2 \alpha \beta \subseteq \alpha \beta \) \( \forall x \in E \),

(8) \( \gamma_3 \alpha \beta \subseteq \alpha \beta \) \( \forall x \in E \),

(9) \( \gamma_4 \alpha \beta \subseteq \alpha \beta \) \( \forall x \in E \),

(10) \( \gamma_5 \alpha \beta \subseteq \alpha \beta \) \( \forall x \in E \),

(11) \( \gamma_6 \alpha \beta \subseteq \alpha \beta \) \( \forall x \in E \).

Now, we see easily that:

\( 1 - \mu_A(x) - (1 - \mu_A(x) - \alpha \beta_A(x)) = \alpha \beta_A(x) \geq 0 \)

and:

\( \mu_A(x) + \alpha \beta_A(x) - \mu_A(x) \geq 0 \).

Therefore inclusion (5) is valid.

Theorem 3: For every IFS \( \alpha, \beta \in [0, 1] \) the following properties are valid:

(1) \( \gamma_1 \alpha \beta A \subseteq \gamma_1 \alpha \beta A \) \( \forall x \in E \),

(2) \( \gamma_2 \alpha \beta A \subseteq \gamma_2 \alpha \beta A \) \( \forall x \in E \),

(3) \( \gamma_3 \alpha \beta A \subseteq \gamma_3 \alpha \beta A \) \( \forall x \in E \),

(4) \( \gamma_4 \alpha \beta A \subseteq \gamma_4 \alpha \beta A \) \( \forall x \in E \),

(5) \( \gamma_5 \alpha \beta A \subseteq \gamma_5 \alpha \beta A \) \( \forall x \in E \),

(6) \( \gamma_6 \alpha \beta A \subseteq \gamma_6 \alpha \beta A \) \( \forall x \in E \).

II. Main results

Now, following and extending the idea from [7], [8] we shall prove following assertions.

Theorem 1: For every IFS \( A \) and for every \( \alpha, \beta \in [0, 1] \) so that \( \alpha + \beta \leq 1 \), the following properties are valid:

(1) \( \gamma_1 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(2) \( \gamma_2 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(3) \( \gamma_3 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(4) \( \gamma_4 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(5) \( \gamma_5 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(6) \( \gamma_6 A \subseteq \alpha \beta A \) \( \forall x \in E \).

Proof: Let \( \alpha, \beta \in [0, 1] \) be given so that \( \alpha + \beta \leq 1 \), and let \( A \) be an IFS. Then we obtain directly that:

\( \gamma_1 A \subseteq \alpha \beta A \) \( \forall x \in E \),

\( \gamma_2 A \subseteq \alpha \beta A \) \( \forall x \in E \),

\( \gamma_3 A \subseteq \alpha \beta A \) \( \forall x \in E \),

\( \gamma_4 A \subseteq \alpha \beta A \) \( \forall x \in E \),

\( \gamma_5 A \subseteq \alpha \beta A \) \( \forall x \in E \),

\( \gamma_6 A \subseteq \alpha \beta A \) \( \forall x \in E \).

Theorem 3: For every IFS \( A \) and for every \( \alpha, \beta \in [0, 1] \) the following properties are valid:

(1) \( \gamma_1 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(2) \( \gamma_2 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(3) \( \gamma_3 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(4) \( \gamma_4 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(5) \( \gamma_5 A \subseteq \alpha \beta A \) \( \forall x \in E \),

(6) \( \gamma_6 A \subseteq \alpha \beta A \) \( \forall x \in E \).

References


