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APPLICATION OF FLUORESCENT PIV AND DIGITAL IMAGE ANALYSIS TO MEASURE TURBULENCE PROPERTIES OF SOLID-LIQUID STIRRED SUSPENSIONS

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Abstract. This study describes an experimental technique which combines Fluorescent Particle Image Velocimetry (FPIV) and digital image analysis, to quantify the hydrodynamics of a solid-liquid suspension stirred by a 45° pitched-blade turbine impeller. Soda-lime glass spheres of 1000 µm diameter were employed for the dispersed phase, with up to volumetric concentrations of 0.5 vol% in water. The magnitude of the continuous phase mean velocity did not change significantly in the impeller jet or bulk flow, with the addition of up to 0.5 vol% dispersed phase. Turbulence levels of the continuous phase, in terms of rms velocities, turbulent kinetic energy and dissipation rate decreased above particle concentrations of 0.2 vol%, and the level of turbulence suppression remained constant up to 0.5 vol%. Continuous phase integral length scales remained unchanged in the presence of solids. The locally-averaged particle concentration field showed high concentrations above and below the impeller and at the corner of the vessel base, extending up the vessel wall. Particle turbulence
levels measured at 0.5 vol% dispersed phase were lower than the corresponding continuous phase.

**Key words:** Mixing; multiphase flows; turbulence; particle image velocimetry; phase separation

1. INTRODUCTION

The study of turbulent flows in multiphase systems has presented a significant challenge to fluid dynamicists, being recognised as one of the most interesting fields of research. Investigating the dynamics of a continuous phase turbulent flow is coupled with the complexity of the response of the dispersed phase. The dispersed phase may modulate the structure of the turbulent environment, and equally the continuous phase will have a compounding impact by transferring momentum to the dispersed phase, referred to as ‘two way coupling’ (Bachalo, 1994). Previous studies of multiphase flows have been conducted mainly in pipe and jet configurations, which have reported up to 50% turbulence damping by small particles, and up to 360% increase by larger particles (Gore and Crowe, 1991). It has been suggested that this transition occurs when the particle diameter to characteristic fluid length-scale ratio is 0.1. Other theories have also been postulated to relate these effects to the particle Reynolds number and wake shedding (Hetsroni, 1989).

The focus of this paper is the study of solid-liquid stirred suspensions, which are a common unit operation in the chemical, pharmaceutical and food industries (Guiraud et al., 1997), and include steps such as solid-catalyzed reactions, dissolution and crystal growth. These processes may involve micro-mixing or mass transfer, which strongly depend on the system turbulence. Although they are widespread in industry, there is little information regarding the velocity of either or both phases in stirred vessels, mainly due to the limitations
of previously available measurement techniques. However, recent advances in laser
diagnostics and digital imagery have improved the prospects of studying two-phase flows,
and some experiments have even attempted to characterise turbulence modulation in stirred
vessel configurations. Nouri and Whitelaw (1992) applied laser-Doppler anemometry (LDA)
to quantify the mean flow and rms velocities of the dispersed phase up to 2.5 vol%. Guiraud
et al. (1997) employed phase-Doppler velocimetry to measure velocities of both phases in
stirred suspensions up to 0.5 vol%. Micheletti and Yianneskis (2004) also applied LDA for
measurements of the continuous phase flow, with solids up to 2 vol%.

In the studies mentioned above, turbulence modulation was investigated in terms of
the particles’ effect on the fluid root-mean-square velocities or turbulent kinetic energy, but
not the rate at which the turbulent kinetic energy was dissipated. The latter property is more
important in chemical processes, since it governs the degree of micromixing, and
subsequently the product selectivity, where parallel competing reactions are involved (Bourne
and Yu, 1994). Furthermore, if the key reaction is catalysed by a solid, then inclusion of the
dispersed phase may alter the fluid turbulence, and hence the product yield. The dissipation
rate is also linked to the crystal size distribution (CSD) in crystallization processes, and the
mean drop size in liquid-liquid systems (Zhou and Kresta, 1998). Hence, knowledge of the
hydrodynamics of stirred solid-liquid suspensions may enable optimization of the system
geometry as well as operating conditions, which may improve, for instance, the product yield
in a solid-catalysed reaction, or give better control of a CSD.

The lack of experimental data has also held back the validation of predictive two-
phase models, which should include inter-phase turbulence transfer terms for turbulence
modulation. In the absence of such information, most models employ simple extensions of the
standard single-phase k-ε model (Montante and Magelli, 2007). For instance, the
homogeneous $k$-$\varepsilon$ model assumes that both solid and liquid phases share the same turbulent kinetic energy and dissipation rate, when in reality this may not be the case.

Particle Image Velocimetry (PIV) techniques are traditionally used to visualise single phase flows, to determine spatial information regarding their fluid velocities and turbulence levels (Sheng et al., 2000; Sharp and Adrian, 2001; Khan et al., 2006). The opacity of most multiphase systems and increased interphase noise limits the application of PIV to the study of solid-liquid suspensions at low concentrations. However, with the inclusion of refractive index (RI) matching, optical filtering and digital post processing, some of these challenges may be overcome.

The authors are aware of only one other study which has attempted to characterise turbulence modulation in solid-liquid stirred suspensions using PIV, with up to 1.5 vol% solid concentration (Virdung and Rasmuson, 2008). The current paper presents a combination of fluorescent PIV (FPIV) and digital image analysis techniques for simultaneous velocity measurements of both phases in a stirred flow. The influence of concentration on turbulence properties such as rms velocities, turbulent kinetic energy ($k$) and dissipation rate ($\varepsilon$) will be quantified for experiments with up to 0.5 vol% of 1000 $\mu$m dispersed particles in water.

2. EXPERIMENTAL SETUP

2.1 Experimental Apparatus

Experiments were carried out in a baffled stirred tank of diameter $T = 101$ mm, equipped with a 45° pitched-blade turbine (PBT) of diameter $D = T/3$, which operated at a clearance of $C = T/4$ from the vessel base. The four baffles had a width equal to $T/10$. The tank was filled to a height $H = T$ with water. The vessel was placed inside a square tank also filled with water, to limit optical distortion at the curved surface. The vessel was made of cylindrical glass; all other components were made of acrylic. The apparatus is shown in Figure 1. The
dispersed phase particles were 1000 \( \mu \text{m} \) solid soda-lime glass beads of density 2500 kg m\(^{-3}\) and RI 1.51 at the Sodium D-line (\( \lambda = 589.3 \) nm) at 20 °C. Experiments were initiated with particles suspended in the fluid at a volumetric concentration of 0.01 vol\%, and increased incrementally until the laser light sheet extinction was too severe to enable further PIV measurements to be taken. This limit was found to be 0.5 vol\%. In all experiments the impeller speed was set at 1600 rpm, which is well above the just-suspended speed characterized by the Zwietering (1958) criterion of \(~1090\) rpm for 0.5 vol\% of 1000 \( \mu \text{m} \) particles suspended in water. This criterion ensured complete off-bottom suspension.

Fluorescent tracer particles of diameter 30 \( \mu \text{m} \) were used to seed the water. The particles were manufactured in-house from polystyrene (PS) cross-linked with divinyl benzene (DVB) and doped with pyrromethene 597-8C9 dye (exciton). The dye has an absorption peak at 527.8 nm (close to the laser excitation wavelength of 532 nm), and a fluorescence peak at 590 nm in diesel fuel. The pyrromethene dyes are not very solvochromatic, hence the wavelength changes would be small (5 to 10 nm at most) in the PS/DVB material.

### 2.2 Measurement technique

The measurement technique involved a combination of fluorescence tagging and digital image treatment. The experimental setup is depicted in Figure 2. The system employed a two-camera PIV setup. One camera was fitted with two long-pass Schott OG550 filters (UQG Ltd) with a cut-off at 550 nm, meaning that it blocked Mie scattering at the lower wavelength (532 nm) and transmitted fluorescence at higher wavelengths. In this way, the camera captured images of the fluorescent tracers without interphase noise, which could be processed to obtain velocity fields without any further treatment. PIV analysis was conducted using cross-correlation techniques on interrogation area (\( IA \)) sizes of 32×32 pixels, with 50% overlap. At
the given resolution (33 pixels per mm), the length of the interrogation area was 0.97 mm. The particle diameter (1 mm) was comparable to the IA spatial resolution. However, the majority of particles appeared as two ‘half moon’ shapes rather than filled circles in the images. They did not always occupy the entire IA and enabled enough fluorescence from the tracers to be transmitted for cross-correlation. In cases where particles overlapped or where particles saturated the IA, no additional measures were taken to avoid the problem of insufficient seeding. Instead, the cross-correlation signal peak-to-noise ratio was set relatively higher than the single phase case, so that vectors from these low seeding regions would be eliminated. The holes were later filled with interpolation of the surrounding vectors.

The second camera was equipped with a short-pass dielectric cut-off filter (Melles Griot) which also had a cut-off at 550 nm. In contrast to the first camera, this filter was used to partially block fluorescence and transmit mainly Mie scattering from the dispersed and tracer particles. Subsequently, the fluorescent tracers appeared less bright in the images, which in turn facilitated digital phase separation. This camera simultaneously captured Mie scattering images of both phases on a single frame, which were digitally post-processed to extract bright pixels corresponding to the dispersed phase. The particle velocities were obtained via cross-correlation, in a similar way to the continuous phase, using 32×32 pixel IAs with 50% overlap.

The cameras were two 8-bit TSI PIVCAM 10-30 Model 630046 cross-correlation cameras, with a resolution of 1000×1016 pixels. Both were fitted with a Nikon Micro Nikkor 105 mm lens (f 2.8-32). The system was equipped with a double-pulsed New-Wave Nd:YAG Solo III laser of 532 nm wavelength and 50 mJ pulse energy. The laser was passed through a series of cylindrical and spherical lenses (focal lengths -15 and 500 mm respectively), which shaped the resulting beam into a light sheet of approximately 1 mm thickness. The two cameras were synchronised using a TSI LASERPULSE synchroniser Model 610034. For
camera calibration, a target grid of known grid spacing was placed in the centre of the laser light sheet and both cameras were adjusted to obtain the same field of view (with an accuracy of ±2 pixels at the edges). The image acquisition rate was 15 Hz (for double pulses), with a pulse width of 6 ns and pulse delay of 263 μs. The time separation between two exposures in an image pair was 50 μs in each experiment.

Ensemble-averaged measurements were obtained in the impeller region, where the highest turbulence level was expected. The field of view was 28×28 mm² and included the blade. The baffle at the edge of the field of view was positioned just behind the laser light sheet, such that its reflections did not appear in the images. A recent study revealed that the mean flow and turbulence properties can be characterised using a minimum of 575 vector fields (Virdung and Rasmuson, 2008). Consequently, 600 double image pairs were obtained in each evaluation, and were considered to be sufficient to obtain statistics of the turbulence.

3. IMAGE ANALYSIS

After reviewing various phase separation techniques in literature, it was decided to process the multiphase Mie scattering images by the application of an algorithm based on differences in geometrical characteristics of the particles. The ‘two-parameter phase discrimination technique’ of Khalitov and Longmire (2002) was believed to be the best choice, since it considered differences in both particle size and intensity simultaneously. The algorithm comprised three main steps, which are: (i) detection of all objects within the image, followed by (ii) parameterization of the objects based on combinations of size and gray-scale intensity and finally (iii) separation of the objects into dispersed particles and tracers according to the parametric combinations. In the present study, an in-house code was developed based on these fundamental principles. However, the object detection stage of the algorithm was adapted to fit the current study, due to differences in the image patterns of the dispersed
particles observed by Khalitov and Longmire (2002), compared to the present case. The various steps of the algorithm are described to perform the phase separation on an instantaneous two-phase PIV image containing 1000 μm particles at the highest volume fraction studied (0.5 vol%). This is the worst case scenario for phase separation, as it contains overlapping particle images and excessive reflections close to the impeller.

3.1 Object detection

In their study, Khalitov and Longmire (2002) employed second order intensity gradients to identify objects within the images. This was based on the consideration that the intensity $I$ reaches a maximum in a two-dimensional image of both tracers and solid particles. In order to be able to identify each maximum however, it is necessary to capture circular image objects, and the intensity distribution within these regions must be smooth and approximately Gaussian. In the present experiments, dispersed particle clusters, broken particles, and particles at the edge of the light sheet did not appear to be circular. Moreover, the dispersed particles manifested themselves as two adjacent ‘half moon’ shapes in the images rather than filled circles, due to the refraction of light at the edges of the particles, as is shown in Figure 3. In these cases, the dispersed particles would not be picked up using second-order spatial derivatives. An alternative routine was developed and implemented in Matlab.

The background intensity of a two-phase PIV image may be variable, with darker intensities in some regions compared to others. The first stage of the algorithm was to make the background as uniform as possible, which facilitates object detection. To extract the background, a disk-shaped structuring element was created using the `strel` function, and passed over the image. Matlab uses a ‘morphological opening operation’ that removes objects which do not completely saturate the structuring element. The size of the structuring element is an input parameter, and should be larger than the objects which are to be filtered out. In the
sample PIV image shown in Figure 4(a), the maximum object size (including overlapping particles) was found to be 45 pixels. Thus, a disc filter of radius 25 pixels efficiently removed all objects. Subsequently, the background was subtracted from the original image to make it more uniform.

In order to maximize object detection, the image contrast was increased by saturating 1% of the data at both low and high intensities, and by stretching the intensity values to fill the potential gray level range of 0-255 (8-bit gray scale). The function \textit{imadjust} was used for this purpose. Next, the gray scale image was binarized into black and white form via thresholding. The binarized image enabled the application of the \textit{bwlable} function which identified all objects. The ‘pixel connectivity’ option in the object detection procedure was set such that pixels which were diagonal and touching at the corners were counted as part of the same object, as well as adjacent touching pixels. Subsequently, the \textit{regionprops} command was implemented to obtain object properties such as the ‘number of objects’ in the image and ‘object area’ (in pixels), from the binarized image. The average intensities (or brightness) of objects were also calculated from corresponding object pixels in the original image.

3.2 Parameterization

From the previous step, all detected objects were assigned the parameters of area (in pixels) and average gray-scale intensity (or brightness). Next, a size-brightness contour map from all of the object data was constructed. The map indicated the total amount of signal carried by objects with a given combination of size and brightness (Khalitov and Longmire, 2002). For the total signal density, Khalitov and Longmire (2002) found the most effective measure to be:

$$\int \int_{\text{all} \text{ objects}} IdA = A \times B \times N_{\text{obj}}$$

(1)
where \( N_{obj} \) is the total number of objects of corresponding area \( A \) and average brightness \( B \); the latter was rounded to the nearest integer. The logarithm of the total signal density was plotted versus size and brightness. The size-brightness contour map of the sample PIV image (of Figure 4(a)) is shown in Figure 5. Note that only objects from a single image were used to construct this plot, but in real operation information of objects from the entire PIV data set would be used, which would typically comprise several hundred images.

3.3 Phase separation

As may be observed in Figure 5, the size-brightness map yields two major regions containing signals carried by the tracers and dispersed particles. The tracers form a high-density region at the left-hand side of the abscissa which include small-dim particles, whereas the dispersed solids generate scattered data points closer to the top-right corner, representing larger and brighter particles. The difference in the signal densities results from the fact that a single two-phase PIV image comprises many tracers but only a few dispersed particles. As information is collected from larger PIV data sets, the dispersed particles generate stronger signals with higher density in the top-right corner, but do not form a well defined peak like the tracers, even for a few hundred images at these low volume fractions. There is also a small area of overlap between the two major regions, containing relatively low signal density. These points could arise from clusters of tracer particles or broken dispersed particles, which may be classified as ‘unidentified objects’. It is desirable to distinguish between the dispersed particles and tracers in the real images, whilst at the same time removing the ‘unidentified objects’, which would result in a false impression of the flow field if cross-correlated to produce vectors.
Separation limits (i.e. the size and brightness characterizing each phase) were obtained by defining two non-overlapping rectangles containing the strongest signals within each major region. Data which fell outside of these limits were discarded (i.e. the ‘unidentified objects’). Although the dispersed particles generated scattered data points instead of a defined peak, the majority of them could still be confined within a boundary. Subsequently, all objects cordoned by this dispersed phase separation limit could be confidently assumed to be dispersed solids. Their corresponding pixel and gray-scale intensity information was extracted from the original image and placed on a pure black background, forming ‘separated dispersed phase’ images. The dispersed phase information corresponding to the sample PIV image in Figure 4(a) has been obtained in this way and is shown in Figure 4(b). Note that reflections off the blade edge are present in the processed images, which could have been filtered out by making the phase separation limits tighter (i.e. by reducing the upper limit of object size). However, this would have been at the expense of losing overlapping particle images which also occupy a large area. Instead, the region occupied by the impeller was blanked out during the vector processing. The separated images were then processed via cross-correlation to obtain velocities of the dispersed phase.

4. THEORY

4.1 rms velocities and turbulent kinetic energy

Root-mean-square (rms) velocities provide a measure of the turbulence levels in the stirred tank. They may be obtained as the root of the mean-squared differences between the instantaneous and mean velocity components as defined below:

\[
\overline{u_i} = \sqrt{\left(u_i - \overline{u_i}\right)^2}
\]  

(2)
where \( u_i \) and \( \bar{u}_i \) are the instantaneous and mean velocities respectively, in the direction \( i \).

The studies reported here did not attempt to produce angle-resolved velocity information, although this will be the subject of some future work. In the remainder of this section, Cartesian notation is adopted, such that \( u, v \) and \( w \) represent velocities in the radial \((x)\), axial \((y)\) and tangential \((z)\) directions respectively. The overbar represents an ensemble average, i.e. the mean of the data at a particular vector position, obtained as a time-average over 600 velocity fields. Knowledge of all three rms velocity components (in the axial, radial and tangential directions) may then be used to calculate the turbulent kinetic energy, which is half of the trace of the Reynolds stress tensor:

\[
k = \frac{1}{2} \left( \bar{u}^2 + \bar{v}^2 + \bar{w}^2 \right)
\]  

(3)

The experimental setup employed in this study yields 2-D PIV data for each phase, namely the axial and radial velocity components in the \( x-y \) plane; the tangential velocity component is unknown. In this case a pseudo-isotropic assumption of the flow field may be applied to determine \( k \) (Khan et al., 2006), such that:

\[
k = \frac{1}{2} \left( \bar{u}^2 + \bar{v}^2 + \frac{1}{2} \left( \bar{u}^2 + \bar{v}^2 \right) \right) = \frac{3}{4} \left( \bar{u}^2 + \bar{v}^2 \right)
\]  

(4)

The flow isotropy may be later assessed by calculating differences between the axial and radial rms velocity components across the field of view, and subsequently validating the 2-D pseudo-isotropic approximation. It should be noted that this analysis provides no information of the tangential rms velocity component explicitly, which may deviate from the axial and radial rms velocities even if the latter two are found to be similar. Consequently, the turbulent
kinetic energy calculation may be subject to some error, which cannot be quantified in this paper. However, in a previous study, Khan et al. (2006) found that the evaluation of $k$ under the pseudo-isotropic approximation and the full evaluation using all three velocity components yielded similar results for the flow generated by a pitched-blade turbine.

4.2 Dissipation rate

The dissipation rate was calculated using the 2-D large eddy simulation (LES) analogy of Sheng et al. (2000), which was found to be the best method compared to dimensional analyses and the direct estimate. The latter requires spatial gradients of the turbulent velocity field to be resolved down to the Kolmogorov scale for an accurate determination (Ducci and Yianneskis, 2005), but this is not within the capability of PIV experiments in a stirred turbulent flow. Comparisons between the various calculation methods of $\varepsilon$ will be presented by Unadkat et al. (2009), and are not within the scope of the current paper.

In LES, the filtered Navier-Stokes (NS) equations are solved directly for the large scales, whereas the small scales are modelled via sub-grid-scale (SGS) models. The spatial resolution of PIV measurements is usually greater than the smallest, Kolmogorov eddy sizes that govern the dissipation rate. Thus, by adopting the LES analogy, it is possible to measure the resolved velocity field (analogous to solving the NS equations), and then model the unresolved scales via a SGS model. The interrogation area size is naturally the spatial filter. The Reynolds averaged SGS dissipation rate (Error! Objects cannot be created from editing field codes.) is given by:

$$\varepsilon = C_{\varepsilon} \frac{\langle \mathbf{u} \cdot \mathbf{u} \rangle}{k},$$  (5)
where the resolved scale strain rate tensor calculated from gradients of the instantaneous resolved scale velocity fields:

\[
\text{Error! Objects cannot be created from editing field codes.}
\]  

and the SGS stress tensor, modelled by the Smagorinsky model (Smagorinsky, 1963), as defined in Equation (7). Note that Sheng et al. (2000) found that both the Smagorinsky and similarity models (Lui et al., 1994) yielded consistent results for the dissipation rate.

\[
\tau_{ij} = -C_s^2 \Delta |s_{ij}| s_{ij}
\]  

In Equation (7), \( C_s \) is the Smagorinsky constant, equal to 0.17, \( \Delta \) is the filter width (or LA size) and \( |s_{ij}| \) is the characteristic filtered strain rate, defined as \( \sqrt{2s_{ij}s_{ij}} \).

The strain rate tensor consists of a total of nine components, of which five are known from PIV measurements (refer to Sheng et al., 2000). The missing terms involve gradients of the tangential velocity components, or gradients in the out-of-plane direction. Subsequently, the sum of the product of the strain rate and stress tensor was multiplied by a factor of 9/5 for a full estimate of the dissipation rate. Alternatively, expanding Equation (5) gives:
If the dissipation rate is expressed in the form of Equation (8), then the statistical isotropy assumptions of Sharp and Adrian (2001) may be employed to calculate the unknown gradients terms, rather than linear scaling. When this technique was adopted, the maximum dissipation rate estimate of the single phase fluid was found to be ~17% greater than that obtained from scaling the strain rate tensor components in the first described method, although the spatial distributions from both estimates were similar. For brevity, only the dissipation rate results from the original method of Sheng et al. (2000) will be presented here for the single and continuous phase flows, since this study is concerned primarily with turbulence modulation of the fluid after the addition of particles, rather than the calculation methods of the turbulence properties themselves.

5. RESULTS AND DISCUSSIONS

For presentation purposes, the data has been plotted on the \( r - z \) (radial-axial) plane according to polar coordinates. Also note that this point forward, in the figures, the radial mean and rms velocity is denoted \( \langle u \rangle \) and \( u_{rms} \), respectively, and similarly for the axial velocity component.
5.1 Continuous phase

The turbulence properties of the single and continuous phase stirred flows with 1000 μm dispersed particles up to 0.5 vol% are shown in this section (the effects of other particle sizes will be reported later). The velocities (both mean and rms), turbulent kinetic energy (TKE) and dissipation rate have been normalized by the impeller tip speed and impeller diameter to enable comparison between different works.

5.1.1 Mean flow

Figures 6(a) and (b) show the mean velocity vectors superimposed with the TKE contours, of the single and continuous phase with 0.5 vol% of particles, respectively. Only one third of the total number of vectors is displayed to elucidate the flow structure. First of all, the single phase flow was compared to previous works to ensure reliability of the PIV data. In Figure 6(a), the primary circulation loop characteristic of a PBT is immediately evident. The impeller generates a strong axial flow in the downwards axial direction, which reaches a maximum magnitude of $0.45V_{tip}$, at the point $z/T = 0.197$ and $r/T = 0.119$. This is in very close agreement to the value reported by Kresta and Wood (1993a). Schafer et al. (1998) also found the peak axial mean velocity to be $\sim 0.45V_{tip}$ below the impeller, where $r/T = 0.125$. As the jet reaches the vessel base, it changes direction and is deflected towards the wall at around $r/T = 0.15$, which also coincides with the strongest radial flow region. In previous studies, the change in direction has been reported to occur at around $r/T = 0.25$ (Fort, 1986; Virdung and Rasmuson 2008). Schafer et al. (1998) observed this behaviour at $Re > 1100$. The difference may be due to the reduced clearance, $C$, from the vessel base in this study ($C = T/4$) in contrast to the standard geometry used in the aforementioned studies ($C = T/3$). The jet stream then flows up the vessel wall, where the axial velocities are $\sim 0.1V_{tip}$, before returning to the top of the impeller. Note however that the field of view does not extend as far as the
vessel wall where \( r/T = 0.5 \); it reaches only \( r/T = 0.356 \). In the wall jet, the velocities can be as high as \( 0.2V_{tip} \) (Shafer et al. 1998). Since the flow was fully turbulent (Reynolds number of \( 3.08 \times 10^3 \) at 1600 rpm), this general flow pattern was found to be repeatably measured.

From Figure 6(b) it appears that the mean velocities of the continuous phase do not change significantly in the impeller suction or jet, with the incorporation of 1000 \( \mu m \) particles at 0.5 vol%. The continuous phase mean flow fields with lower particle concentrations were the same as Figure 6(b), and are not shown for brevity. This is in contrast to the findings of Guiraud et al. (1997), who studied solids suspensions of 253 \( \mu m \) particles in water at 0.5 vol%, agitated by an axial propeller. These authors reported a decrease in the impeller discharge flow rate and wall jet velocity, due to the effects of particle inertia and gravity, respectively. In a similar study of 1000 \( \mu m \) particles stirred by a PBT, Virdung and Rasmuson (2008) also reported a decrease in the axial velocity of around 46% at a solid concentration of 1.5 vol%, mainly attributed to particles resting at the vessel base and causing the impeller jet to divert towards the wall further away from the bottom. In the present case however, the particles were agitated well above the just suspended speed defined by Zweitering (1958), ensuring off-bottom suspension, so this effect was not observed.

When comparing the flow structure close to the vessel wall between Figures 6(a) and (b), it may be observed that the centre of the circulation close to the tank wall has moved 3 mm away from the wall in the flow containing 0.5 vol% particles, relative to the single phase case. The vortex centres (or zero velocity regions) are marked in Figure 6 to show this effect more clearly. This trend was observed for experiments carried out with dispersed phase volume fractions above 0.2 vol% (but not at lower volume fractions). For this reason, the axial velocity profile in Figure 7(a) depicts a change in direction of the axial flow from downwards to upwards at \( r/T = 0.32 \) in the wall jet, at particle concentrations greater than 0.2 vol%. In Figure 7(b) it can be seen that the magnitude of the mean radial velocities also
appear to be increased by 50% close to the tank wall, above 0.2 vol% dispersed phase. This result is not a real increase in the wall jet velocity, but an artefact due to the shift of the centre of the flow circulation loop.

5.1.2 rms velocities and turbulent kinetic energy

The ensemble-averaged normalized axial and radial rms velocities of the single phase flow are depicted in Figures 8(a) and 9(a) respectively. Both contour maps are plotted on the same colour scale for ease of comparison. It may be observed that the rms velocities are strongest below the impeller blade reaching ~0.3$V_{tip}$, and are an order of magnitude smaller ~0.03$V_{tip}$ in the bulk region. Differences in the spatial distributions of the axial and radial rms velocities are evident. Secondly, their maximum values are unequal; 0.27$V_{tip}$ and 0.30$V_{tip}$ respectively. The radial rms velocity is greater, which is unusual since the primary flow direction (axial in this case) is normally considered to represent the system turbulence, when for example, estimating the dissipation rate via dimensional analysis methods. However, a point-by-point comparison between the rms velocities (obtained as $\mu_{rms} - v_{rms}$) yielded differences of only 0.02$V_{tip}$ in the bulk flow, and a maximum of 0.05-0.08$V_{tip}$ underneath the blade edge and close to the centre of the vessel base (where $r/T = 0.1$). Khan (2005) reported differences in the rms velocities of a single phase flow generated by a PBT to be of the same order of magnitude. Kresta and Wood (1993b) stated that the rms velocity components in highly anisotropic flows are expected to vary up to 200%. Against this criterion, the single phase flow of the current study is considered to be isotropic, hence the application of Equation (4) is justified.

Analogous plots of the axial and radial rms velocities for the continuous phase flow with 1000 μm particles at 0.5 vol% are shown in Figures 8(b) and 9(b) respectively. From the graphs it may be observed that the particles suppress both rms velocity components beneath
the impeller. Specifically, the maximum radial rms velocity was decreased by 7%, and the maximum axial rms velocity by 12.5%; both locations were in the impeller jet. In addition to the points of maxima, the general turbulence levels in the impeller jet were also reduced in magnitude, although they remained unaffected in the bulk of the flow. In their study Guiraud et al. (1997) made different observations; the axial and radial rms velocities of the fluid were unaffected by the presence of 253 μm dispersed particles at 0.5 vol%. On the contrary, the more recent and experimentally similar study by Virdung and Rasmuson (2008) reported an increase in the rms level, also for 1000 μm particles at 0.5 vol%. However, the study of Micheletti and Yianneskis (2004) supports the current observations; the authors reported turbulence suppression in all regions of the tank by 50% when dispersing 186 μm particles at 0.5 vol%. The current experimental results depicted that the rms velocities decreased in the presence of particles at concentrations greater than 0.2 vol%, and the degree of turbulence suppression remained relatively constant up to 0.5 vol%.

The effect of particle concentration on the fluid turbulence is reflected in the TKE contour maps of the single and continuous phase flows with an increasing concentration of the dispersed phase. Figures 10(a) and (b) show the TKE of the single and continuous phase with 0.2 vol% of 1000 μm particles respectively; the magnitudes and distributions of both are comparable. However with the incorporation of 0.3 vol%, turbulence suppression in the impeller jet is immediately evident, in Figure 10(c). When the concentration is increased to 0.5 vol% in Figure 10(d), there is no noticeable further change. The results suggest that at the low volumetric fractions studied, particle-particle interactions are unimportant, and do not contribute to the observed phenomenon of turbulence suppression. Instead it is a direct consequence of the interactions between individual particles and the flow. In particulate suspensions with higher volumetric concentrations, where particle-particle interactions are
expected to be significant, the degree to which turbulence is modulated would also depend on the particle volume fraction.

Micheletti and Yianneskis (2004) found that concentrations of 0.5 and 1 vol% of 186 μm particles decreased the radial rms velocity of the fluid in the impeller disc plane by 50% and 70% respectively. On the other hand, Virdung and Rasmuson (2008) found that increasing the concentration of 1000 μm particles from 0.5 to 1.5 vol% increased the amount of turbulence augmentation of the continuous phase, as well as making their spatial distributions more homogeneous. Although these studies show opposing effects, it is suggestive that particle-particle interactions become relevant in the mechanisms which lead to turbulence modulation, above volumetric concentrations of 0.5 vol%. However, Micheletti and Yianneskis (2004) also noted that turbulence levels did not decrease further when the concentration was increased to 2 vol%, indicating that inter-particle interactions no longer governed turbulence suppression.

5.1.3 Dissipation rate

The authors do not know of any other study which has experimentally investigated the effect of particles on the dissipation rate of a continuous phase stirred flow, relative to the single phase fluid. Spatial distributions of the dissipation rate for the single and continuous phase with 1000 μm particles at 0.5 vol% are shown in Figures 11(a) and (b) respectively. From this it may be observed that the dissipation rate has decreased in the jet stream, whereas the bulk of the flow seems to be unaffected. The largest turbulence suppression occurs directly underneath the blade, where the maximum dissipation rate has decreased by ~21%. This suggests that the presence of the particles decrease the velocity gradients in the immediate vicinity of the impeller blades, as well as causing a general decrease in the fluctuating velocities throughout the jet stream.
Following the trends of the rms velocities and TKE (described in the previous section), the dissipation rate also decreased in the presence of 1000 μm particles at concentrations greater than 0.2 vol%. The turbulence suppression in this case is clearly distinguishable from the single and continuous phase experiments with lower particle concentrations, as shown by the axial and radial profiles in Figures 12(a) and (b) respectively. Note that blank area where no results are shown in Figure 12(a), (between $0.216 < z/T < 0.293$) corresponds to the region occupied by the impeller, where PIV results were eliminated. Once again, the extent of turbulence damping remains approximately constant with increasing dispersed phase concentration up to 0.5 vol%.

In a previous study, Gore and Crowe (1991) postulated that particles with a ‘diameter to fluid integral length scale ratio’ of $d_p/L < 0.1$ cause turbulence suppression, whereas those with $d_p/L > 0.1$ cause turbulence augmentation, where $L$ was identified as an integral length scale. The average 2-D integral length scale of the fluctuating axial velocity component ($\Lambda_v$) of the continuous phase flow was used to calculate this ratio for the 1000 μm particles. Details of the computation are discussed in the next section concerned with length scales. In this case $\Lambda_v$ was found to be 0.0035 m (or $0.475W$ where $W$ is the blade width), providing a ratio of $d_p/\Lambda_v = 0.285$. The particles do not enhance turbulence as predicted by Gore and Crowe’s (1991) criterion; instead they produced the opposite effect. Micheletti and Yianneskis (2004) who reported turbulent suppression found the respective ratio to be 0.15 in their study, also contradicting the much accepted theory. Although it may be argued that the ratio 0.15 is close to (or of the same order as) the transitional value of 0.1, the study of Gore and Crowe (1991) depicts $d_p/L = 0.1$ to be a clear demarcation point.
### 5.1.4 Integral length scales

In order to characterize the $d_p / L$ ratio for turbulence modulation, Gore and Crowe (1991) used the integral length scale of the single phase flow, but mainly out of necessity since data for length scales in the presence of particles were not readily available at that time. Following this comment, length scales of the continuous phase were investigated, since a change in these may in turn affect the critical value ($d_p / L = 0.1$).

The 1-D longitudinal ($\Lambda_{u_x}$) and transverse ($\Lambda_{u_y}$) integral length scales of the $u$ and $v$ fluctuating velocity components were obtained by integrating the 1-D autocorrelation functions of the respective velocity fields up to the first zero in a given direction, before taking an average over the PIV data set. Similarly, the 2-D integral length scales ($\Lambda_{u_z}$) were obtained by integrating the 2-D autocorrelation function for the volume in both directions, up to the first zeroes.

Figure 13 illustrates the integral length scales of the continuous phase with respect to dispersed phase volumetric concentration of 1000 $\mu$m particles. All length scales have been normalized by the impeller blade width $W = 0.007$ m. The general trends are that the 2-D length scales are greater than the 1-D longitudinal length scales (although for the radial velocity component they overlap at some points), which are in turn greater than the 1-D transverse length scales. So for instance in the case of the axial velocity component, $\Lambda_v > \Lambda_{vy} > \Lambda_{vx}$. For isotropic turbulence, the ratio of the longitudinal to transverse length scale is 2:1 (Pope, 2000) even when comparing different velocity components (regardless of which is considered to be the primary flow direction). In the present case, the average ratio from the radial velocity component ($\overline{\Lambda_{u_r}} / \overline{\Lambda_{u_y}}$) was found to be 1.95, but the equivalent ratio from the axial velocity ($\overline{\Lambda_{vx}} / \overline{\Lambda_{vy}}$) was 1.47, indicating some anisotropy of turbulence. Similarly, the ratios of $\overline{\Lambda_{vy}} / \overline{\Lambda_{ux}}$ and $\overline{\Lambda_{ux}} / \overline{\Lambda_{vx}}$ were found to be 2.29 and 1.26 respectively.
The 2-D length scales in the primary (axial) flow direction were always greater than the 2-D length scales in the radial direction, i.e. $\Lambda_v > \Lambda_u$.

In some cases, there are fluctuations in the magnitude of the length scales with respect to particle concentration, but the majority of these are retained within the 95% confidence intervals about the mean values of adjacent results, and there is no indication of a consistent increase or decrease. Thus, it may be concluded that the integral length scales of the continuous phase flow are unaffected by the presence of particles, and it is acceptable to estimate those of a continuous phase fluid from the single phase, at these low solids volume fractions at least.

The average 2-D length scale of the axial velocity components ($\Lambda_v$) from all continuous and single phase measurements was found to be $0.475W$. This result also supports the commonly used approximation that the characteristic integral length scale is around half the blade width ($W/2$) in the impeller discharge stream (Kresta and Wood, 1993b). However, it should be noted that the result was derived from the autocorrelation of the entire field of view, which expands from the impeller jet towards the tank wall (up to $r/T = 0.35$). It was found previously that this ratio over estimates the integral length scales within the impeller region, and under estimates them in the bulk (Cutter, 1966; Wu and Patterson, 1989; Mahouast et al., 1989; Khan, 2005). In light of these observations, it is expected that if autocorrelation was performed in sections of the field of view both close to and further away from the impeller, the measured length scales would have been different. Dividing the velocity field into smaller sections would reduce the number of instantaneous vectors available for the autocorrelation, providing less reliable results; hence this concept was not explored further. For all other measured 2-D and 1-D integral length scales, the factor $W/2$ provides an overestimate in the entire field of view.
5.2 Dispersed phase

Dispersed phase velocities (and subsequently turbulence properties) were obtained from cross-correlation techniques analogous to the continuous phase, using IA sizes of 32×32, with 50% overlap. Only particle images from experiments carried out at the highest volumetric fraction (0.5 vol%) were processed, to ensure enough particle signal in the images for cross-correlation. When calculating mean fields, areas unoccupied by particles and hence not containing velocity vectors were not included in the average. Subsequently, regions where particles never passed the field of view over the entire data set resulted in holes in the mean field. Locally-averaged particle concentration fields have also been obtained.

5.2.1 Concentration field

One of the most important aspects of solid-liquid mixing is the distribution of solid particles throughout the mixing volume, since it may affect the reactor performance, and thus efficient reactor design. The solids distribution has an effect on the course of a variety of industrial processes such as suspension polymerization, reactive crystallization and particle coating. It may be important to obtain a uniform distribution of particles in the medium such that enough particle surface area is exposed to enable heat and mass transfer between the solids and liquid. Knowledge of the particle distribution in a stirred vessel may also be useful to interpret turbulence modulation of the continuous phase.

The locally averaged particle concentration field \( \langle C_d \rangle \) was obtained for the 1000 μm particles at 0.5 vol% in water. The following procedure was adopted. The separated dispersed phase images were binarized and divided into small non-overlapping rectangular regions (analogous to the IAs in cross-correlation) of size 30×30 pixels. The number of bright pixels per region was obtained across the entire image, and for all instantaneous images. Subsequently an average was obtained in each 30×30 pixel region across all (600×2) images.
in the data set, and finally normalized by the total number of pixels in that area (900 pixels). In this way, the fraction of bright pixels per region in the 2D image was interpreted as the 3D local volume fraction within that region. The spatial distribution of the local concentration field is provided in Figure 14, where 100% on the colour scale would indicate that particles fully occupied a particular region in all image frames. However, note that the absolute value of the local concentration obtained from image analysis in the vicinity of the impeller region does not provide a reliable measure. As mentioned before, reflections from the blade were found to be strong when the impeller was captured in the field of view. Consequently, this area was blanked out during vector processing. However, the reflections off the blade also distorted images of dispersed particles surrounding the blade, and their projected image was larger than what the PIV calibration factor would suggest. Subsequently, the concentration map should only be interpreted to assess the relative distribution of particles in the field of view, but not as a quantitative measure. Notwithstanding this effect, the volume-averaged concentration of particles in the field of field (shown in Figure 14) was found to be 0.48 vol%, very close to the experimental value of 0.5 vol%.

It may be observed that the 1000 μm particles are not distributed homogeneously in the fluid. Instead, there are regions of locally high concentrations above and below the impeller, as well as a stagnation region at the bottom of the vessel (where the base meets the wall) which also extends up the vessel wall. This suggests that the observed turbulence suppression of the continuous phase is a genuine effect, and a direct consequence of the presence of particles in the discharge stream, particularly beneath the impeller.

The simulation results of Derksen (2003) for a flow generated by a Rushton disk turbine (RDT) showed high particle concentrations underneath the impeller. However unlike PBTs, RDTs generate an up-flow below the impeller, which may carry highly concentrated slurries with it. Experimental studies of RDTs conducted by Magelli et al. (1990) and
Nocentini et al. (2002) also noted locally high solids concentrations in the vicinity of the impeller, and midway between turbines for multi-impeller configurations. Interestingly, the concentration profiles of particles generated by multiple PBT impellers (Montante et al., 2001) showed trends similar to the RDT studies, despite differences in the flow patterns between the two impellers.

5.2.2 Mean flow

The ensemble-averaged mean velocity field of the 1000 μm particles (0.5 vol%) is shown in Figure 15, superimposed on contours of the 2-D TKE. It is evident that the dispersed phase velocity field is significantly noisier than the equivalent continuous phase velocity field (Figure 6(b)), since the average values have been obtained from fewer vectors (O(10) instead of O(100)). However, even at the low volumetric fractions studied, cross-correlation of the dispersed phase images was able to provide some qualitative information of the flow field, and enabled further calculation of particle rms velocities and TKE (discussed later).

The typical downward pumping flow circulation loop generated by the impeller is not obvious at first glance, but it is evident that the particle velocities are strongest underneath the impeller blade and in the discharge stream. They change direction from being predominantly axial to radial at $r/T = 0.15$ at the vessel base, analogous to the point of change in direction of the single and continuous phase. Relatively smaller upward axial velocities are attained in the wall jet, which eventually becomes radial and returns to the top of the impeller, completing the circulation loop. The maximum axial velocity of the 1000 μm particles was found to be $-0.58V_{tip}$ which occurs at the point $z/T = 0.192$ and $r/T = 0.138$, very close to the blade edge. This is 29% greater than the maximum axial velocity of the single (and continuous) phase fluid. The result seems to be quite large and may well be due to a few spurious particle vectors, which are not smoothed out in the averaging process (since the
average is only from a few vectors). However in their LDA study of a solid-liquid suspension, Pettersson and Rasmuson (1997) found that whilst the mean flow direction of both phases was the same (within a few degrees), the absolute difference in the 3D mean velocity was as much as 20% relative to the fluid. This relative difference was observed at only 0.06 vol% dispersed phase.

5.2.3 rms velocities and turbulent kinetic energy

The mean axial and radial rms velocities of the 1000 μm dispersed particles (0.5 vol%) are shown in Figures 16(a) and (b) respectively. It may be observed that the dispersed phase axial rms velocities are the greatest in the discharge stream, reaching a magnitude of \( \sim 0.17V_{\text{tip}} \) on the whole. Some infrequent spotty regions reach as high as 0.2-0.3\( V_{\text{tip}} \). By comparison, the rms velocities are small in the bulk, between 0-0.1\( V_{\text{tip}} \). It should be noted that the zero results may be an artefact of there possibly being only one dispersed phase velocity vector at that point (since the rms velocity is calculated from the difference between the instantaneous and mean velocity, and when only one vector is present, these two values are equal). The continuous phase axial rms velocity distribution was similar to the dispersed phase (Figure 8(b)), but the fluid attained higher turbulence levels directly underneath the impeller blade up to \( \sim 0.3V_{\text{tip}} \).

In Figure 16(b), the radial rms velocities of the particles are significantly lower than their axial counterparts, and the high magnitude turbulence level in the discharge stream is no longer prominent. Instead, the radial rms velocities reach up to 0.125\( V_{\text{tip}} \) only at some points in the discharge stream. The bulk rms turbulence levels are also slightly lower, between 0-0.05\( V_{\text{tip}} \). This suggests anisotropy of the dispersed phase turbulence. On the other hand, the continuous phase radial rms velocities shown in Figure 9(b) preserve the high turbulence region underneath the impeller blade and jet stream, which also reaches \( \sim 0.3V_{\text{tip}} \) as found in
the fluid axial rms velocities.

Inevitably, the dispersed phase TKE levels (Figure 15) were found to be much lower than the corresponding continuous phase fluid (shown on the same colour scale in Figure 6(b)), by 22% in the discharge stream.

The turbulence anisotropy of the dispersed and continuous phase was assessed in a similar fashion to the single phase \( \left| \frac{\mu_{\text{rms}} - \nu_{\text{rms}}}{V_{\text{tip}}} \right| \), and displayed in Figures 17(a) and (b) respectively. The turbulence of the dispersed particles is slightly anisotropic; the differences in both rms velocity components are between 0.1-0.15\( V_{\text{tip}} \) in the discharge stream. The particle turbulence in the bulk of the flow is relatively isotropic, where the difference is at most 0.05\( V_{\text{tip}} \). On the other hand, the continuous phase preserves much higher isotropy, reaching a maximum difference of 0.08\( V_{\text{tip}} \) underneath the blade edge, and close to the centre of the vessel base. This observation was very similar for the single phase, suggesting that the presence of the particles does not affect the isotropy of the flow.

6. CONCLUSIONS

This paper presents the development of an adapted phase discrimination algorithm which has been successfully applied to study stirred solid-liquid suspensions of 1000 \( \mu \)m particles with up to 0.5 vol% dispersed phase. A commonly reported result (Gore and Crowe, 1991) is that particles which have a diameter to characteristic fluid length scale ratio greater than O(0.1) enhance turbulence. Using the 2-D integral length scale of the fluctuating axial velocity component obtained from PIV, this ratio was found to be 0.285 in the current study, which would suggest turbulence augmentation according to the criterion. However, the present results show the opposite effect of turbulence suppression, hence further tests using particles of other sizes will be carried out to substantiate the observations.
Continuous phase turbulence levels in terms of rms velocities, TKE and dissipation rate were seen to decrease (relative to the single phase flow) after the addition of particles above 0.2 vol%. The level of turbulence suppression remained approximately constant when the particle concentration was gradually increased to 0.5 vol%, indicating that particle-particle interactions did not contribute to the observed phenomenon. This kind of information is valuable in the development of CFD codes for predictions of two-phase flow phenomena, as it highlights a particle volumetric concentration below which turbulence modulation effects of the dispersed phase on the continuous phase may be neglected, and thus when one-way coupling models are acceptable to implement. The greatest impact was on the dissipation rate, which showed turbulence suppression of ~21% in the presence of particles. In light of the finding that the integral length scales remained unchanged when particles were added to the flow, even simple dimensional analysis calculations methods of the dissipation rate would be able to show this effect.

Digitally phase-separated images of the 1000 μm particles at 0.5 vol% were also processed. Spatial distributions of the locally-averaged volumetric concentration field revealed three zones of locally high concentrations; one of which was the discharge stream. This supported the notion that the presence of particles was responsible for modulating the turbulence in that region. The rms velocities and TKE of the particles were found to be lower than the corresponding continuous phase fluid; the former also exhibited slight anisotropy of turbulence.

7. NOMENCLATURE

Roman letters

\[ A \quad \text{Object area in an image frame} \quad [\text{pixels}] \]
\[ B \quad \text{Average intensity of an object in an image frame} \quad [-] \]
Impeller clearance from vessel base \([\text{m}]\)  
Smagorinsky constant \([-\text{]}\)  
Locally averaged particle concentration \([\text{vol\%}]\)  
Dispersed particle diameter \([\mu\text{m}]\)  
Impeller diameter \([\text{m}]\)  
Water fill height \([\text{m}]\)  
Pixel intensity in an image frame \([-\text{]}\)  
Turbulent kinetic energy \([\text{m}^2 \text{s}^{-2}]\)  
Characteristic flow scale \([\text{m}]\)  
Number of objects in an image frame \([-\text{]}\)  
Radial, axial and tangential directions in polar coordinates \([-\text{]}\)  
Resolved scale strain rate tensor \([\text{s}^{-1}]\)  
Vessel diameter \([\text{m}]\)  
Instantaneous radial, axial and tangential velocity respectively \([\text{m s}^{-1}]\)  
Mean radial, axial and tangential velocity respectively \([\text{m s}^{-1}]\)  
Mean radial and axial velocity respectively in graphs \([\text{m s}^{-1}]\)  
Fluctuating radial, axial and tangential velocity respectively \([\text{m s}^{-1}]\)  
Rms radial, axial and tangential velocity respectively \([\text{m s}^{-1}]\)  
Rms radial and axial velocity respectively in graphs \([\text{m s}^{-1}]\)
Blade tip velocity \[ \text{[m s}^{-1}\text{]} \]

\[ W \] Blade width \[ \text{[m]} \]

\[ x, y, z \] radial, axial and tangential directions in Cartesian coordinates \[ \text{[-]} \]

**Greek symbols**

\[ \Delta \] Filter width in large eddy simulation analogy \[ \text{[m]} \]

**Error! Objects cannot be created from editing field codes.** Average turbulent kinetic energy dissipation rate \[ \text{[m}^2\text{s}^{-3}\text{]} \]

**Error! Objects cannot be created from editing field codes.** Reynolds averaged sub-grid scale dissipation rate \[ \text{[m}^2\text{s}^{-3}\text{]} \]

\[ \lambda \] Wavelength \[ \text{[nm]} \]

**Error! Objects cannot be created from editing field codes.** 1D integral length scale of velocity component **Error! Objects cannot be created from editing field codes.** in direction \[ \text{[m]} \]

**Error! Objects cannot be created from editing field codes.** 2D integral length scale of velocity component **Error! Objects cannot be created from editing field codes.** \[ \text{[m]} \]

\[ \mu \] Absolute or dynamic viscosity \[ \text{[Pa.s]} \]

**Error! Objects cannot be created from editing field codes.** Sub-grid scale stress tensor \[ \text{[N m}^{-2}\text{]} \]

**Abbreviations**

CCD Charge coupled device

CSD Crystal size distribution
DVB  Divinyl benzene
FPIV  Fluorescent Particle Image Velocimetry
IA  Interrogation area
LDA  Laser Doppler anemometry
LES  Large eddy simulation
Nd:YAG  Neodym-Yttrium-Aluminium Garnet
NS  Navier-Stokes
PBT  Pitched blade turbine
PIV  Particle Image Velocimetry
PS  Polystyrene
RDT  Rushton disc turbine
Re  Reynolds number [-]
Re\_p  Particle Reynolds number [-]
RI  Refractive index [-]
rms  Root-mean-square
rpm  Revolutions per minute [min\(^{-1}\)]
SGS  Sub-grid scale
TKE  Turbulent kinetic energy [m\(^2\)s\(^{-2}\)]
vol\%  Volumetric percentage [%]

8. REFERENCES


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