High power Tesla driven miniature plasma opening switch

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High power Tesla driven miniature plasma opening switch

by

Rajesh Kumar MSc. (Physics)

A Doctoral Thesis submitted in partial fulfilment of the requirement for the award of

Doctor of Philosophy of Loughborough University, U.K

26 January 2009

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Abstract

The plasma opening switch (POS) is used in pulsed power systems where a very fast opening and high current switch is required. Plasma is injected into the switch, which carries a large conduction current, before it opens in a process that lasts for a few nanosecond and transfers the current to a parallel-connected load at a much increased voltage and with a much shorter rise time. The conduction and opening times of the switch are dependent on plasma parameters such as the distribution, speed and species, all of which are determined by the plasma source.

Most of the earlier reported work involves large dimension POSs and a correspondingly high input current (more than 100 kA) and uses carbon plasma. One main objective of the present research was to achieve a low input current (20 kA) and miniaturised POS by using hydrogen plasma rather than carbon plasma on account of its lower mass.

A cable gun was selected for producing the plasma, since although this produces both hydrogen and carbon plasma these arise different times during its operation.

For the present application a Tesla transformer was used in preference to a Marx generator to produce an initial high voltage pulse for the system, on the basis of its simpler design and cost effectiveness. This transformer together with an associated water PFL (pulse forming line) and pressurised switch was capable of producing a load current in excess of 20 kA with a rise time of 53 ns, which was fed through the POS to the final load.

Special diagnostics arrangements were necessary to measure the fast high current and voltage pulse in nonintrusive way. Faraday cups and a high speed camera were used to measure the plasma parameters.

The overall system built (i.e. including the POS) is capable of producing a 22 kA current with a rise time of 5 ns, and of generating a power of more than 10 GW.

Much of the work detailed in the thesis has already been presented in peer reviewed journals and at prestigious international conferences.
Acknowledgements

I would like to express my sincere gratitude and to acknowledge the assistance and continuous support received from my supervisor, Professor Ivor Smith. I am grateful to him for providing Departmental funding, without which it would not have been possible for me to undertake the present research work. I am also grateful to him for his reviews and advice during the writing of the thesis.

Special thanks are also due to Dr. Bucur Novac, who was a constant source of inspiration, expert advice and assistance during my research work. I am very fortunate to have been associated with him.

I am also grateful to Prof P. K. Kaw, Director, Institute for Plasma Research Gandhingar, India, Prof A. Sen, Dean, Institute for Plasma Research and Dr. Anurag Shyam, Head of the Pulsed Power Group at the Institute for Plasma Research for granting me leave for carry out my research.

I am very grateful to Mr. Charles Greenwood for his technical assistance in designing, and constructing the experimental systems and his advice related to numerous mechanical problems. Without his help none of the systems described would have been built.

I would also like to thank Dr. Partha Sarkar, Dr. Marko Istenic and Mr. Peter Senior, colleagues in the Pulsed Power Group at Loughborough University, for providing constant and friendly help.

Finally, but most importantly, I would like to thank my wife Sweta, and daughter Shatakshi and my entire family for their constant support and patience throughout my PhD studies.
## List of Principal Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c$</td>
<td>velocity of light</td>
<td>$3 \times 10^8 \text{ ms}^{-1}$</td>
</tr>
<tr>
<td>$e$</td>
<td>charge on electron</td>
<td>$1.6 \times 10^{-19} \text{ C}$</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>permittivity of free space</td>
<td>$8.85 \times 10^{-12} \text{ C V}^{-1} \text{ m}^{-1}$</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>permeability of free space</td>
<td>$4\pi \times 10^{-7} \text{ H m}^{-1}$</td>
</tr>
<tr>
<td>$m_e$</td>
<td>electron mass</td>
<td>$9.109 \times 10^{-31} \text{ kg}$</td>
</tr>
<tr>
<td>$m_p$</td>
<td>proton mass</td>
<td>$1.672 \times 10^{-27} \text{ kg}$</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann’s constant (chapter 5)</td>
<td>$1.38 \times 10^{-23} \text{ JK}^{-1}$</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Stefan Boltzmann constant</td>
<td>$5.67 \times 10^{-8} \text{ W m}^{-2} \text{K}^{-4}$</td>
</tr>
<tr>
<td>$h$</td>
<td>Planck’s constant</td>
<td>$6.63 \times 10^{-34} \text{ Js}^{-1}$</td>
</tr>
<tr>
<td>$1 \text{ eV}$</td>
<td>Energy in electron volts</td>
<td>$1.6 \times 10^{-19} \text{ Joule}$</td>
</tr>
<tr>
<td>$m_c$</td>
<td>mass of carbon ions</td>
<td>$20.071 \times 10^{-27} \text{ kg}$</td>
</tr>
<tr>
<td>$z$</td>
<td>Atomic number</td>
<td></td>
</tr>
<tr>
<td>$E$</td>
<td>Electric field intensity</td>
<td>$\text{kg m s}^{-3} \text{ A}^{-1}$</td>
</tr>
<tr>
<td>$r$</td>
<td>Orbital radius (chapter 5)</td>
<td>$\text{m}$</td>
</tr>
<tr>
<td>$v$</td>
<td>Electron orbital velocity (chapter 5)</td>
<td>$\text{m s}^{-1}$</td>
</tr>
<tr>
<td>$J$</td>
<td>Current density</td>
<td>$\text{A m}^2$</td>
</tr>
<tr>
<td>$m$</td>
<td>Electron mass (chapter 6)</td>
<td>$9.109 \times 10^{-31} \text{ kg}$</td>
</tr>
<tr>
<td>$Z_{\text{diode}}$</td>
<td>Diode impedance (chapter 6)</td>
<td>$\Omega$</td>
</tr>
</tbody>
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1 Introduction

1.1 Pulsed Power

The steady accumulation of energy followed by its rapid release can result in the delivery of a substantially increased amount of instantaneous power over a short period of time (although the total energy of course remains the same). Energy is typically stored within electrostatic fields (capacitors), magnetic fields (inductors), as mechanical energy (using large flywheels connected to special purpose high current generators), or as chemical energy (high-current lead-acid batteries, or explosives). By releasing the stored energy over a very short interval (a process that is called energy compression), a huge peak of power can thus be delivered to a load. For example, if one joule of energy is stored within a capacitor and released over one second, the peak power delivered to the load is only one watt. However, if all of the stored energy is released within one microsecond, the peak power is one megawatt, a million times greater!

1.2 What we understand as Pulsed Power

Pulsed power involves the electrical generation of very short pulses (nanoseconds to milliseconds) with the possibilities of:

- currents up to several hundreds of megamperes.
- voltages up to several megavolts.
- energy releases up to several hundreds of millions of millions of Joules per second.
- power densities of several hundreds of millions of watts per square centimetre and pressures of millions of atmospheres.
- temperatures of millions of degrees Kelvin.
1.3 History of pulsed power

The roots of pulsed power can be traced to the developments of high voltage technology and nuclear physics prior to World War II.

Pulsed power itself was first developed during World War II for use in radar. A massive development program, similar in scale to the Manhattan Project [1.1], was undertaken to develop pulsed radar, requiring very short high power pulses. After the war development continued in various other applications, leading to a generation of several highly novel pulsed power machines [1.2], [1.3].

At the beginning of the Cold War, when U.S. and Soviet scientists at Los Alamos and Arzamas 16 (today Sarov) began the race in the development of nuclear weapons, they also began to think about the production of energy by atomic fusion.

One possibility of reaching this goal was seen in the generation of super-high magnetic fields of 1000 T and more, producing immensely high magnetic pressures. To generate such high fields the so called Magnetic Flux Compression Generator was invented in the fifties independently by A. Sakharov in the Soviet union and C.M. Fowler in the U.S.

Although during the Cold War various commercial applications of pulsed power were developed, like the Lithotripter [1.4] and material shaping [1.5], most of the efforts remained concentrated in the military and nuclear sciences.

After U.S. President Reagan's proclamation of the Strategic Defence Initiative in 1983, often called "Star Wars", Pulsed Power Technology offered extremely good financial opportunities for the development of improved materials and electrical components with high energy and power densities, in both the United States of
America and the former Soviet Union. France and the United Kingdom also participated in this race, but with very low budgets compared to those of the U.S. and the Soviet Union. Topics that were investigated included electrical devices like electromagnetic guns, laser weapons and high power microwave weapons.

After the breakup of the Soviet Union in 1990, and the end of the Cold War, the financial support for pulsed power was greatly reduced. Engineers and scientists who were used to dealing with the military applications of pulsed power began to look elsewhere for new, commercial applications. They had to learn very rapidly that the introduction of pulsed power into the commercial market depended strongly on the competitiveness of their products compared with those already available. They had also to learn that the development of pulsed power products for the commercial market is a "High Risk High Pay-Off" task, and that only a few industrial partners were interested in the promotion of such a technology.

Since the security and the reliability of nuclear weapons has to be maintained without nuclear tests, one current use of pulsed power is in the simulation of the effects of fusion reactions in small volume, which has helped to push forward their civil applications.

The *International Society on Pulsed Power Applications* was founded in Gelsenkirchen Germany in 1997, specifically to support the commercial side of pulsed power applications.

### 1.4 Some applications of Pulsed Power Technology:

The applications of pulsed power can be divided into two broad categories.
1.4.1 Industrial applications

Typical industrial application that are becoming widespread include

a) Production of nano-powders [1.6]

With the advent of nano-technology there has been a great demand for a mass production process for the manufacture of nano-sized powders for various industrial applications such as cosmetics, the electrodes for MLCC (Multi Layer Ceramic Capacitors) and the production of silver fibre for anti-bacterial products. A pulsed power based electric wire explosion can be used to produce these nano particles [1.6].

The wires are exploded in a vessel that can be filled with gas at pressures between 50 and 150 kPa. Depending on the gas that is used either metal oxide, nitride, or carbide powders can be generated.

b) Food processing/pasteurization [1.7].

Pulsed electric field (PEF) processing has been shown by multiple researchers to be equivalent to pasteurization in terms of pathogen reduction for a wide range of liquid foods. For foods that are heat sensitive, it offers considerable benefits in taste, colour, and nutritional value above the other process.

c) Sludge treatment [1.7]

Sewage sludge is composed of organic materials, bacteria and mostly water. The bacteria contain digested organic materials and the objective of the sewage sludge treatment is to destroy the bacteria cell membrane. High voltage pulse power can be used to generate an arc discharge in the sewage sludge, which will ensure that this happens.
d) Medical waste treatment [1.8]

Pyrolysis is a pulsed power based process in which waste is heated in an oxygen deficit that results in the gasification of organic components (a so-called synthesis gas is formed) and the melting of mineral components.

1.4.2 Research and development

a) Inertial confinement fusion [1.9]

Inertial confinement fusion (ICF) is a process in which nuclear fusion reactions are initiated by heating and compressing a fuel target, typically in the form of a pellet that most often contains a mixture of deuterium and tritium. The heating can be achieved using laser or high X-ray radiation.

The extremely large output of pulsed power energy can be transferred into a wire array mounted in (say) the z-direction. A high current flowing in the z direction will produce a magnetic field $B(\theta)$, resulting in a $J(z) \times B(\theta)$ force in the $r$ direction. This phenomenon is called z-pinch and implodes the wire array, thereby releasing a huge bursts of X-ray radiation suitable for heating an ICF process.

b) Transmutation of nuclear waste [1.10]

Nuclear transmutation is the conversion of one chemical element or isotope into another, which occurs often through nuclear reactions. Natural transmutation occurs when radioactive elements spontaneously decay over a long period of time and transform into other more stable elements. Artificial transmutation occurs in machinery that has sufficient energy to cause changes in the nuclear structure of the elements. Pulsed power systems can be used to generate the high power needed to achieve this.
1.5 Some important requirement of pulsed power [1.11]

Some of the requirements of typical pulsed power devices are given below in Table 1.01. Pulse requirement for various applications cover a wide range, from kilo-ampere to megampere currents, kilovolts to megavolts, nanosecond to millisecond pulse widths and pulse repetition rates of 1 to 30,000 pulses per second.

<table>
<thead>
<tr>
<th>Applications</th>
<th>Voltage MV</th>
<th>Current MA</th>
<th>Rep-rate kpps</th>
<th>Load type</th>
<th>Pulsed energy MJ</th>
</tr>
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<tbody>
<tr>
<td>Advanced test accelerator</td>
<td>0.1-1</td>
<td>0.01-0.1</td>
<td>10</td>
<td>Capacitive</td>
<td>0.002-0.5</td>
</tr>
<tr>
<td>Modified betatron</td>
<td>0.5</td>
<td>0.2</td>
<td>0.001-10</td>
<td>Inductive</td>
<td>100</td>
</tr>
<tr>
<td>Lasers</td>
<td>0.01-0.1</td>
<td>0.001-0.03</td>
<td>0.01-0.1</td>
<td>Resistive/Capacitive</td>
<td>0.001-0.04</td>
</tr>
<tr>
<td>EM guns</td>
<td>0.002-0.02</td>
<td>0.1-5</td>
<td>0.001-0.05</td>
<td>Resistive and Inductive</td>
<td>5-500</td>
</tr>
<tr>
<td>Inertial fusion</td>
<td>3</td>
<td>0.1</td>
<td>0.01</td>
<td>Capacitive</td>
<td>0.1-3</td>
</tr>
<tr>
<td>EMP simulator</td>
<td>1-5</td>
<td>0.01</td>
<td>0.1</td>
<td>Capacitive</td>
<td>0.1-0.5</td>
</tr>
</tbody>
</table>

Table 1.01: Typical pulsed power requirements.

The advantages of using an energy store increase as the repetition rate decreases and in single-shot operation an energy store is almost mandatory. Energy storage and transfer system are therefore important, not only for their potential saving in overall size, weight and cost but also since the enormous peak power requirements of many applications make power supplies rated at the peak pulse power completely impractical.

1.6 Energy storage for pulsed applications [1.12]

The energy storage systems generally used are capacitive, inductive, chemical (batteries and high explosive), and inertial (rotating machines, flywheels). Table 1.02
shows the salient features used when comparing these systems, i.e. energy storage density, storages losses, charge, discharge rates (which in turn determine the minimum energy transfer time and peak power capabilities).

<table>
<thead>
<tr>
<th>Storage devices</th>
<th>Energy density MJ/m(^3)</th>
<th>Energy/weight J/kg</th>
<th>Typical transfer time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitors</td>
<td>0.01-1</td>
<td>300-500</td>
<td>µs</td>
</tr>
<tr>
<td>Inductors</td>
<td>3-40</td>
<td>10(^2)-10(^3)</td>
<td>µs- milliseconds</td>
</tr>
<tr>
<td>Batteries</td>
<td>2000</td>
<td>10(^9)</td>
<td>minutes</td>
</tr>
<tr>
<td>Explosives</td>
<td>6000</td>
<td>5x10(^9)</td>
<td>µs</td>
</tr>
<tr>
<td>Flywheels</td>
<td>400</td>
<td>10(^4)-10(^5)</td>
<td>seconds</td>
</tr>
</tbody>
</table>

Table 1.02: Comparison of energy storage methods [1.12] [1.13].

As table 1.02 shows, high explosives have the highest energy density and the shortest energy release times, but they are limited to single-shot operation and require auxiliary equipment to convert their chemical energy into electrical energy. A battery has a high energy storage density but a low power delivery capability, and requires both long charge and discharge times. Inertial storage has a high storage density and a moderate power output capability and although capacitors have the highest electrical discharge capability they have a relatively low energy storage density.

Only inductive storage systems have both a high energy and a high electrical power capability and they also have a decreasing cost per unit energy as their size increases. They can be made essentially lossless by the use of superconducting materials. Because non-superconducting or normal coils are not loss free, energy can in practice only be stored in them for less than about one \(L/R\) time constant ( where \(L\) is the inductance of the coil and \(R\) is its resistance), otherwise the coil will itself dissipate as heat much of the stored energy.
There are two major obstacles to the practical use of inductive storage in pulsed-power devices, both of which become obvious when the basic capacitive and inductive energy discharge circuits are compared as below.

In the capacitive energy discharge circuit of figure 1.01(a) the capacitor $C$ is charged through a resistor $R$ to a voltage $V$. The time constant for the subsequent self-discharge of the capacitor is $\tau_c = RC$ where $R$ is the leakage resistance which, for low inductance, high-voltage capacitors, may be of the order of tens of minutes. For such capacitors, the charging current can be kept fairly low.

![Figure 1.01: Circuits for (a) capacitive and (b) inductive discharge circuit.](image)

In a practical system the capacitor will typically be discharged into the load $Z_L$ by means of the closing switch $S_c$, which is often a spark gap. The discharge current is usually large compared to the charging current and a capacitive discharge circuit can, therefore, be considered as a current amplifier.

If $\frac{dW_C}{dt}$ is the rate of increase of stored energy in the capacitor and $\frac{dW_R}{dt}$ is the rate at which energy is dissipated in the load, then during a charging the capacitor we have
\[
\frac{dW_C}{dt} = \frac{d}{dt} \left( \frac{1}{2} CV^2 \right) = \frac{1}{RC} \frac{dV}{dt}
\]

where \( R \) is in series with \( C \) and \( V \) is the voltage across \( C \).

Suppose the capacitor \( C \) is charged by a constant current charger, such that

\[
v(t) = \frac{I}{C} t
\]

Then equation 1.01 can be written as

\[
\frac{dW_C}{dt} = \frac{T}{\tau_C}
\]

where \( T \) is the charging time of the capacitor, and slow charging is best since this gives \( T >> \tau_C \) and a low loss in the resistor \( R \) while the capacitor is charging.

In the inductive energy storage circuit figure 1.01(b), the inductor \( L \) is "charged" to a current \( I \), and the time constant for the \( L-R \) circuit is \( \tau_L = \frac{L}{R} \), where \( R \) is the combined series resistance of the current source, the switch \( S_o \) and the inductor. For inductive energy storage systems, \( \tau_L \) can be in the order of seconds, which means that inductors have to be charged in relatively short times, and high power, primary power sources are needed. The energy stored in the inductor is transferred to the load by means of an opening switch \( S_o \), which interrupts the current \( I \) in the charging circuit and a closing switch \( S_c \), which in turn connects the capacitor to the load \( Z_L \). Due to the rapid decrease of current, a high voltage of magnitude \( L(dI/dt) \) is induced across
the opening switch and load, and inductive discharge circuits can, therefore, be considered as voltage amplifiers.

If \( \frac{dW_L}{dt} \) is the rate of increase of stored energy in the inductor L and \( \frac{dW_R}{dt} \) is the rate at which energy in dissipated in the load the rate of increase of stored energy in the magnetic field of the inductor, divided by the energy dissipated in the series resistance in the same time, is

\[
\frac{dW_L}{dt} = \frac{d}{dt} \left( \frac{1}{2} L \dot{i}^2 \right) = \frac{L}{R} \frac{1}{i} \frac{di}{dt}
\]

Now \( \frac{di}{dt} = \frac{i}{T} \), where T is the charging time; and \( L/R = \tau_L \), the inductor time constant.

So for a high charging efficiency, i.e. a large \( \frac{dW_L}{dt} \), one must ensure that \( \tau_L / T \) is large, which means charging rapidly with respect to the inductor time constant.

It is evident from the above consideration that the two major technical complexities encountered in inductive energy storage systems are the charging circuit, because of the necessary fast charging of the inductor and the opening switch because of its inbuilt complexities at the currents and voltage involved.

For an inductive storage bank the charging circuit is in form of either a Marx generator or a Tesla based transformer, and using a transformer instead of a Marx generator for a pulse charging application can offer a number of significant benefits, which derive principally from two features:
1) the primary storage capacitors are not required to operate at the ultra-high output voltage

2) the complicated switching and dc charging components of the Marx circuit are not required.

Low voltage operation means that the primary capacitor bank and transformer need not be housed in a large tank of insulating oil, and the resulting system is compact, requires substantially less floor space, and is more readily accommodated in a laboratory. Also, with fewer components, the possibilities for failure are reduced and maintenance is minimized. Finally, these simplifications yield a system that costs appreciably less than its Marx counterpart to both build and operate.

### 1.7 Repetitive pulse power systems

An important current trend in pulsed power technology is the study of compact repetitive pulsed power systems, which have many important applications in defence systems, laser systems, high-energy physics, waste treatment, material processing and other industrial areas. Compact repetitive pulsed power systems can be improved by enhancing the peak and average power output, increasing the pulse repetition frequency (PRF) and reducing the equipment size to meet the demands of the increasing applications. All these possibilities are under investigation in various laboratories worldwide.

Pulsed power systems not capable of repetitive operation are commercially unusable.

Clearly, the opening switch used in an inductive storage system must be capable of repetitive operation for the system to be repetitive. Although many different types of opening switches are available an opening switch based on plasma erosion (POS) is probably the best for repetitive operation at the voltage and current levels involved.
1.8 Aim of the thesis

Planned experiments are conducted to achieve the following goals and leading to the production of a miniaturised POS as an essential first step in the development of a table top repetitive POS based machine.

1. Proof of principle of a GW rated miniaturised and repetitive POS, where volume of plasma is of the order of only tens of cm$^3$.

2. Low current operation of a POS, typically of the order of few tens of kA (20 kA to 70 kA).

This machine after successful completion will be useful in the following applications

1. EMP sources (Antenna based system) [1.14].

2. Radiation sources such as microwave generators (Virator, Milo) [1.15].

3. Electron and ion accelerators, which can be used as a source for the metallic deposition for nanotechnology [1.16].

To achieve the overall aim of this work a Tesla based transformer which charges a water pulse forming line (PFL) and operates in a resonance mode was used as a generator. A pressurized gas switch at the end of the PFL connects the stored energy in the PFL to the coaxial storage inductor, where the plasma opening switch (POS) shorts the transmission line initially. As the current grows in the POS, this switch opens and diverts current to the load with a rise time faster than the initial rise time of the current before the POS. A schematic design of the system is presented in figure 1.02.
Introduction

Figure 1.02: Schematics of the Tesla transformer based plasma opening switch.

1.9 Outline of the thesis

The thesis defines the progress of Tesla transformer based generator research, in which the water based pulsed forming line is used to increase the power level of an electrical pulse which is further enhanced by the POS. Chapter 1, which is the introduction to this thesis, has outlined the history of pulsed power, its applications, described inductor based systems and stated the overall aim of the thesis.

Chapter 2 presents details of the Tesla transformer that will be used to raise the low system charging voltage into the high voltage used to charge the PFL waterline.

Chapter 3 deals with the PFL (water line), which is charged to 300 kV and acts as a secondary capacitor for the Tesla Transformer. Details of the SF₆ pressurised output spark gap switch are also presented.

Fast nanosecond current and voltage sensors are discussed in chapter 4.

Chapter 5 outlines plasma sources for the plasma opening switch and the developments of diagnostics for the plasma. Fast camera photographs of the moving plasma are also presented.
Details of the plasma opening switch which is at the heart of this project are presented in chapter 6, where a literature survey and other most important issues regarding the existing plasma opening switch based generator are presented.

Chapter 7 summarise the findings and proposed lines of future work that will be important attraction.

A list of publication presented at international conferences and in prestigious academic journal is provided in chapter 8.
1.10 Reference

[1.1] Leslie Groves “Now it Can be Told: The Story of the Manhattan Project”


2  Tesla Transformer

2.1  A historical note

The son of a Serbian Orthodox clergyman, Nikola Tesla was born on 9 July 1856 in Croatia. After receiving his early education in Croatia, he studied engineering at a polytechnic school in Gratz, Austria. Subsequently, he enrolled at the University of Prague but dropped out after his father’s death. Tesla worked for a short time as a draftsman at the Central Telegraph Office of Hungary and as a telephone engineer in Budapest. He then took a job as an engineer with the Continental Edison Company in Paris, France.

In 1884 Tesla moved to the United States, where he worked for Edison on power plant installations in New York City. But on finding that his employer did not share his enthusiastic interest in alternating current (ac), Tesla soon resigned himself to engaging in independent research on ac systems. He began filing applications for patents during 1887 and several were issued in May 1888. In the same month he presented a classic AIEE paper entitled “A New System of Alternating Current Motors and Transformers.”

Tesla soon turned his attention to investigations of the high-frequency, high-voltage phenomena using what soon became known as the “Tesla coil,” a resonant air-core transformer with a large turns ratio between the primary and secondary windings. He gave a lecture demonstration of some of the unusual physiological and plasma discharge effects he had discovered at an AIEE meeting in May 1891.

2.2  Tesla transformer

A Tesla transformer is a device that produces very high voltages at high frequency. The voltages range, depending on the design and size of the coil, from about a few 100 kVs up to several million volts (MV).
A Tesla transformer differs significantly from a "normal" or conventional transformer; and it may be called a *resonant transformer*. Whereas in normal transformers, i.e. those designed typically for use on a 50 Hz 230 V supply, operation does not change the frequency (the input and the output frequency are always exactly the same), a Tesla coil increases both the voltage and the frequency of the input signal.

The typical Tesla transformer is composed of two circuits. The primary circuit comprises a high-voltage capacitor that is discharged through a switching device such as a spark gap, into a low inductance primary winding. The secondary winding simply features an air-wound coil with one side grounded, which forms a combination of inductance and its own stray capacitance. If the two coils are magnetically coupled, every discharge of the primary capacitor generates a magnified voltage in the secondary coil.

The working point of a Tesla transformer is influenced by the value of the capacitance and inductance of the primary and secondary windings, together with the coupling between them. Attempting to maximize the efficiency is not a trivial task, as these various parameters all have a nonlinear effect on the transformer tuning and its magnification.
Figure 2.01 Primary and secondary circuit of a Tesla transformer.

Figure 2.01 shows diagrammatically a Tesla transformer, with the resonant LC circuits \((L_pC_p \text{ and } L_sC_s)\) coupled through their mutual inductance \(M\). Losses are represented by the resistances \(R_p\) in the primary and \(R_s\) in the secondary. The open circuit resonant frequencies of the two circuits are chosen to be equal for complete energy transfer from the primary circuit to the secondary circuit \([2.1]\). The primary capacitor \(C_p\) is initially charged, and when the spark gap (S) is closed operation of the transformer is described by the following equations.

For the primary circuit

\[
R_p i_p + \frac{1}{C_p} \int i_p dt + L_p \frac{d i_p}{d t} + M \frac{d i_s}{d t} = 0
\]  \(\text{(2.01)}\)

and for the secondary circuit

\[
R_s i_s + \frac{1}{C_s} \int i_s dt + L_s \frac{d i_s}{d t} + M \frac{d i_p}{d t} = 0
\]  \(\text{(2.02)}\)

If \(q_p\) and \(q_s\) are the instantaneous charges on the capacitors \(C_p\) and \(C_s\), then

\[
i_i = \frac{dq_i}{dt} \quad i = p, s
\]  \(\text{(2.03)}\)

and equations 2.01 and 2.02 may be rewritten as
\[
R_p \frac{dq_p}{dt} + \frac{1}{C_p} q_p + L_p \frac{d^2 q_p}{dt^2} + M \frac{d^2 q_s}{dt^2} = 0 \quad (2.04)
\]
\[
R_s \frac{dq_s}{dt} + \frac{1}{C_s} q_s + L_s \frac{d^2 q_s}{dt^2} + M \frac{d^2 q_p}{dt^2} = 0 \quad (2.05)
\]

Introducing the differential operator \( D \left( = \frac{d}{dt} \right) \) and combining equation 2.04 and 2.05

[2.2] [2.3] gives auxiliary equation

\[
\left( 1 - k^2 \right) D^4 + \left( \frac{R_p}{L_p} + \frac{R_s}{L_s} \right) D^3 + \left( \omega_p^2 + \omega_s^2 + \frac{R_p R_s}{L_p L_s} \right) D^2 + \left( \frac{R_p}{L_p} \omega_p^2 + \frac{R_s}{L_s} \omega_s^2 \right) D + \omega_p^2 \omega_s^2 = 0
\]

(2.06)

where \( k = \frac{M}{\sqrt{L_p L_s}} \)

(2.07)

and \( \omega_p = \frac{1}{\sqrt{L_p C_p}} \) and \( \omega_s = \frac{1}{\sqrt{L_s C_s}} \)

(2.08)

Equation 2.06 has four complex roots \( E_i \) and its solution can be written in terms of the charge on the capacitors as shown in reference [2.3]

\[
q_s = \sum_i A_i \exp(E_i t)
\]

(2.09)

\[
q_p = \sum_i B_i \exp(E_i t)
\]

(2.10)

where \( A_i \) and \( B_i \) are constants (and \( i = 1..4 \)), which can be evaluated by using the boundary conditions at \( t = 0 \) of

\( q_s = 0, \ q_p = q \) (initial charge on \( C_p \)) and

\( dq_p = dq_s = 0, \)

Subsequently, the primary and secondary capacitor voltages can be written as

\[
V_p = \frac{q_p}{C_p} = \frac{1}{C_p} \sum_i B_i \exp(E_i t)
\]

(2.11)

\[
V_s = \frac{q_s}{C_s} = \frac{1}{C_s} \sum_i A_i \exp(E_i t)
\]

(2.12)
Solution of equations 2.11 and 2.12, when the primary and secondary resistance are neglected [2.4] gives the voltage developed across the secondary circuit capacitance $C_s$ as

$$V_s = \frac{2kV_p}{\sqrt{(1-T)^2 + 4k^2T}} \frac{L_s}{L_p} \sin \left( \frac{\omega_2 + \omega_1}{2} \right) \sin \left( \frac{\omega_2 - \omega_1}{2} \right)$$  \hspace{1cm} (2.13)

where

$$T = \left( \frac{\omega_p}{\omega_s} \right)^2 = \frac{L_s C_s}{L_p C_p}$$  \hspace{1cm} (2.14)

is one for a transformer operating in the Tesla mode and

$$\omega_1 = \omega_s \sqrt{\frac{(1+T) - \sqrt{(1-T)^2 + 4k^2T}}{2(1-k^2)}}$$  \hspace{1cm} (2.15)

$$\omega_2 = \omega_s \sqrt{\frac{(1+T) + \sqrt{(1-T)^2 + 4k^2T}}{2(1-k^2)}}$$  \hspace{1cm} (2.16)

$T$ is the tuning ratio, defined as the square of the ratio of the uncoupled resonance frequencies, while $V_p$ is the initial voltage across $C_p$. $\omega_1$ and $\omega_2$ are the resonant frequencies of the primary and secondary circuits when magnetically coupled.

### 2.2.1 Condition for maximum voltage gain

From equation 2.13, one obvious way to optimise a Tesla transformer is to obtain the maximum achievable secondary voltage at a given primary voltage. The maximum secondary voltage gain over the primary voltage can be written from equation 2.13 as

$$G = \frac{V_s(t)}{V_p} \bigg|_{\text{max}} = \frac{2k}{\sqrt{(1-T)^2 + 4k^2T}} G_L$$  \hspace{1cm} (2.17)

where
The gain $G$ from equation 2.17 can be achieved only if the sine terms in equation 2.13 are simultaneously equal to ±1, i.e.

$$\frac{\omega_2 - \omega_1}{2} = \pi t + m\pi \quad \text{and} \quad \frac{\omega_2 + \omega_1}{2} = \pi t + n\pi$$

where $n$ and $m$ are positive integers. Without losing generality, $n$ can be set to zero, thereby changing the requirement to

$$\frac{\omega_1}{\omega_2} = 1 + \frac{m}{n}$$

To make $t$ as small as possible so that the maximum voltage occurs in the shortest time from $t = 0$, we take $m = 1$, which yields the condition:

$$\omega_1 = 2\omega_2 \quad (2.19)$$

In addition, if $Z$ is the impedance operator [2.1] for the circuit shown in figure 2.02, it can be written as

$$Z = \begin{bmatrix} j\omega L_p + \frac{1}{j\omega C_p} & j\omega M \\ j\omega M & j\omega L_p + \frac{1}{j\omega C_p} \end{bmatrix}$$

and from the characteristics equation $\det Z = 0$, it follows that

$$\left\{ j\omega L_p + \frac{1}{j\omega C_p} \right\} \left( \omega L_p + \frac{1}{j\omega C_p} \right) - (j\omega M)^2 = 0 \quad (2.21)$$

introducing the substitutions $k = \frac{M}{\sqrt{L_p L_s}}$, $\omega_p = \frac{1}{\sqrt{L_p C_p}}$ and $\omega_s = \frac{1}{\sqrt{L_s C_s}}$, enables equation 2.21 to be rewritten as

$$(1 - k^2)\omega^4 - \left(\omega_p^2 + \omega_s^2\right)\omega^2 + \omega_p^2 \omega_s^2 = 0 \quad (2.22)$$
For complete energy transfer, the resonant electrical circuits must have same frequency as shown in reference [2.1], ie.

\[ \omega_p = \omega_s \]  \hspace{1cm} (2.23)

In which case equation 2.22 becomes

\[ (1-k^2)\omega^4 - 2\omega_p^2 \omega^2 + \omega_p^4 = 0 \]  \hspace{1cm} (2.24)

with roots \( \omega_1 \) and \( \omega_2 \), where

\[ \omega_{1,2}^2 = \omega_p^2 \left[ \left(1 \pm k\right)/\left(1 - k^2\right) \right] \]  \hspace{1cm} (2.25)

The ratio of the square of the frequencies is

\[ \frac{\omega_1^2}{\omega_2^2} = \frac{(1+k)}{(1-k)} \]  \hspace{1cm} (2.26)

Combining equation 2.19 and 2.26

\[ \frac{(1+k)}{(1-k)} = 4 \]

or

\[ k = \frac{3}{5} = 0.6 \]  \hspace{1cm} (2.27)

Equations 2.23 and 2.27 provide the criterion for selecting the Tesla parameters for maximum voltage gain and maximum energy transfer from the primary circuit to the secondary circuit.

### 2.3 Modelling of Tesla transformer

#### 2.3.1 Filamentary modelling [2.5]

Filamentary modelling is a simple but accurate and general purpose numerical technique that enables a full description of electromagnetic, thermal and dynamic interactions to be included in a model, while still preserving considerable mathematical simplicity. In filamentary modelling, the given assembly of conductors
is divided into an assembly of filaments in the direction of the current paths through
the conductors, once these are known. The filaments must be sufficiently small for the
current distribution in their cross sections to be regarded as uniform and, as a rule of
thumb, the dimensions will be much less than the equivalent skin depth. The number
of filaments required to describe accurately their properties under transient conditions
is obtained by calculating the parameters for a relatively small numbers of filaments,
and then repeating the calculations for a progressively increased number, until the
differences between successive calculations are less than about 1%.

Once the replacement of the solid conductors by the current filaments is complete, the
ohmic resistance of each filament is determined from its cross-sectional area, length
and temperature dependent resistivity. By assuming a uniform current distribution
across the cross section of the filaments, the self inductances can readily be calculated
from well known formulae, as can the mutual inductance between every possible pair
of filaments. Thereby the electromagnetic problem is reduced to a simple circuit
problem, in which the solid conductors are represented by an assembly of current
filaments. The currents in each branch are defined as state variables, and the circuit
equations are written as a set of linear first order differential equations that have to be
solved for the circuit currents. A typical set of equations will appear in matrix form
as:

\[
\frac{d}{dt} \begin{bmatrix}
L_1 & M_{1,2} & \ldots & M_{1,N} \\
M_{2,1} & L_2 & \ldots & M_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
M_{N,1} & M_{N,2} & \ldots & L_N
\end{bmatrix} \begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_N
\end{bmatrix} = \begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
\]  

(2.28)
where $N$ is the number of filaments, $I_i (i = 1...N)$ is the current in the $i_{th}$ filament, $V_i (i = 1...N)$ is the complete inductive voltage term in the circuit containing the $i_{th}$ filament and $M_{i,j} (i, j = 1...N)$ is the mutual inductance between the $i_{th}$ and $j_{th}$ filaments. When $i = j$, $M_{i,j}$ becomes $L_i$, the self inductance of the $i_{th}$ filament and equation 2.28 can be written in compact form as

$$\frac{d}{dt} \left( \sum_{j=1}^{N} M_{i,j} I_j \right) = V_i \ (i = 1...N) \quad (2.29)$$

The set of first order differential equations that corresponds to the filamentary circuit model can be solved using numerical ordinary differential equation solvers such as the Runge-Kutta technique, with the filamentary currents thus calculated providing the current distribution in the conductors.

Most electromagnetic devices encountered in pulsed power have rotational symmetry about one axis, which is termed axial symmetry. Because of this situation, the model here is restricted to the cylindrical co-ordinates $(r, z, \theta)$, with symmetry about the $z$-axis. Therefore, in the most general case, the effect of an arbitrary current path through the symmetrical conductors can be defined in terms of a combination of $z$ and $\theta$ currents.

In the following sections, it is shown how the self and mutual inductance associated with coils can be determined. With the turns divided into a number of filaments, their effective self and mutual inductances can be calculated assuming coaxial individual circular coils.
2.3.1.1 Mutual inductance between two parallel and coaxial circular loops

![Diagram of two loops](image)

Figure 2.02 Schematic representations of two loops.

The mutual inductance \( M \) between the two loops \( C_1 \) and \( C_2 \) shown in Figure 2.02 having radii of \( a \) and \( b \) respectively can be written as

\[
M = \frac{\mu_0}{4\pi} \int_{C_1} \int_{C_2} \frac{dS_1 \cdot dS_2}{r}
\]  

(2.30)

where \( dS_1 \) and \( dS_2 \) are the differential vectors for each loop and \( r \) is the vector shown in figure 2.02. Then \( M \) may be expressed in the form [2.5]

\[
M = \mu_0 \left\{ -\sqrt{(a+b)^2 + d^2} E\left(\frac{4ab}{(a+b)^2 + d^2}\right) + \frac{a^2 + b^2 + d^2}{\sqrt{(a+b)^2 + d^2}} K\left(\frac{4ab}{(a+b)^2 + d^2}\right) \right\}
\]  

(2.31)

where \( d \) is the axial distance between the two loops and \( E \) and \( K \) are the elliptical integrals defined as

\[
E(k^2) = \int_0^{\pi/2} \sqrt{1-k^2 \sin^2 \theta} d\theta
\]  

(2.32)

\[
K(k^2) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1-k^2 \sin^2 \theta}}
\]  

(2.33)
2.3.1.2 Self inductance of circular loops

2.3.1.2.1 Self inductance of finite ribbon cross-section conductor

![Circular ribbon](image)

Figure 2.03: Circular ribbon.

To calculate the self inductance it is assumed that a current is flowing along the circular ribbon of radius $a$ and axial length $t$ shown in figure 2.03 [2.5]. The self inductance $L$ can be calculated from the magnetic flux produced when current flows with a uniform density in the ribbon, which is equivalent to taking an average of all the mutual inductances between the current loops in the ribbon [2.5].

Then, if $t \ll a$

$$L = a\mu_0 \left(0.3862944 + 0.1730840q^2 - \ln q - 0.253863q^2 \ln q\right) \quad (2.34)$$

where

$$q = \frac{1}{2a} \quad (2.35)$$

2.3.1.2.2 Self inductance of coaxial structure

![Coaxial structure](image)

Figure 2.04: Coaxial structure.
The self inductance of a single loop with a circular cross section of finite radius $r_c$ can be obtained as [2.6]

$$L_c = \mu_0 R_c \left( \ln \frac{4\pi R_c}{2r_c} - \frac{3}{4} \right)$$

(2.36)

The results derived above will be used in the equations of section 2.3.4 to predict the inductance and mutual inductance of the transformer.

### 2.3.2 Resistance of cylindrical conductor geometry

For a hollow cylindrical conductor of length $l$, inner radius $r_i$ and outer radius $r_o$, the resistance is given [2.7] by

$$R = \eta(\rho_d, T) \frac{l}{S} = \eta(\rho_d, T) \frac{l}{\pi(r_o^2 - r_i^2)}$$

(2.37)

When it is necessary to account for the skin depth $\delta$, it is convenient to replace $r_o - r_i$ by $\delta$, giving:

$$R = \eta(\rho_d, T) \frac{l}{\pi\delta(2r_o - \delta)}$$

(2.38)

### 2.3.3 Stray capacitance between coaxial coils [2.6] [2.7] [2.8] [2.9]

The stray capacitance of a coil consists of the turn-to-turn capacitances and the turn-to-ground capacitances between the individual coil turns and a ground plane. Estimates of the effective capacitance of the secondary winding of the Tesla transformer can utilise this approach, using the representation of the secondary winding shown in figure 2.05, where the N secondary turns are treated as N independent loops (or nodes) and the ground point 0 as the single loop or node.
The stray capacitance of the secondary winding may be represented by the network of lumped node-to-node capacitive elements in figure 2.06, where nodes 1 to N compared to the N terms of the coil and node 0 corresponds to the ground plane.

Figure 2.05: Electrostatic model used in simulating capacitance matrix.

Figure 2.06: Node-to-node lumped capacitance network.
$C_{i,j}$ thus represents the capacitance between turns $i$ and $j$ and $C_{i,0}$ the total capacitance of turn $i$ to ground. The relationship between the node voltages $V$, the node charges $Q$ and the node-to-node capacitances $C$ can be written as [2.8].

\[
\begin{bmatrix}
Q_1 \\
Q_2 \\
\vdots \\
Q_N
\end{bmatrix}
= 
\begin{bmatrix}
B_{1,1} & B_{1,2} & \cdots & B_{1,N} \\
B_{2,1} & B_{2,2} & \cdots & B_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
B_{N,1} & B_{N,2} & \cdots & B_{N,N}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{bmatrix}
\]

(2.39)

where the diagonal term $B_{i,i}$ are called the Maxwell coefficients of capacitance [2.9], and the off-diagonal term are called the coefficients of induction [2.9]. Both sets of terms are related to the individual capacitance of the turns by

\[
C_{i,0} = \sum_{j=1}^{N} B_{i,j}
\]

(2.40a)

and

\[
C_{i,j} = -B_{i,j} \quad \text{and} \quad C_{j,i} = -B_{j,i}
\]

(2.40b)

Rearrangement of equation 2.39 gives

\[
\begin{bmatrix}
Q_1 \\
Q_N \\
Q_2 \\
\vdots \\
Q_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
B_{1,1} & B_{1,N} & B_{2,1} & \cdots & B_{1,N-1} \\
B_{N,1} & B_{N,N} & B_{N,2} & \cdots & B_{N,N-1} \\
B_{2,1} & B_{2,N} & B_{2,2} & \cdots & B_{2,N-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
B_{N-1,1} & B_{N-1,N} & B_{N-1,2} & \cdots & B_{N-1,N-1}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_N \\
V_2 \\
\vdots \\
V_{N-1}
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
Q_1 \\
Q_N \\
Q_2 \\
\vdots \\
Q_{N-1}
\end{bmatrix}
= 
\begin{bmatrix}
B_{1,1} & B_{1,N} & B_{1,2} & \cdots & B_{1,N-1} \\
B_{N,1} & B_{N,N} & B_{N,2} & \cdots & B_{N,N-1} \\
B_{2,1} & B_{2,N} & B_{2,2} & \cdots & B_{2,N-1} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
B_{N-1,1} & B_{N-1,N} & B_{N-1,2} & \cdots & B_{N-1,N-1}
\end{bmatrix}
\begin{bmatrix}
V_1 \\
V_N \\
V_2 \\
\vdots \\
V_{N-1}
\end{bmatrix}
\]

(2.41)

which can be rewritten as

\[
\begin{bmatrix}
Q_x \\
Q_y
\end{bmatrix}
= 
\begin{bmatrix}
B_{xx} & B_{xy} \\
B_{yx} & B_{yy}
\end{bmatrix}
\begin{bmatrix}
V_x \\
V_y
\end{bmatrix}
\]

(2.42)
where the submatrices $Q_x$, $V_x$, $Q_y$, $V_y$, $B_{xx}$, $B_{xy}$, $B_{yx}$ and $B_{yy}$ are defined as:

\[
\begin{bmatrix}
Q_x
\end{bmatrix} = \begin{bmatrix}
I_1 \\
I_N
\end{bmatrix}, \quad
\begin{bmatrix}
V_x
\end{bmatrix} = \begin{bmatrix}
V_1 \\
V_N
\end{bmatrix}
\]

\[
\begin{bmatrix}
Q_y
\end{bmatrix} = \begin{bmatrix}
Q_2 \\
\vdots \\
Q_{N-1}
\end{bmatrix},
\begin{bmatrix}
V_y
\end{bmatrix} = \begin{bmatrix}
V_2 \\
\vdots \\
V_{N-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{xx}
\end{bmatrix} = \begin{bmatrix}
B_{1,1} & B_{1,N} \\
B_{N,1} & B_{N,N}
\end{bmatrix}, \quad
\begin{bmatrix}
B_{xy}
\end{bmatrix} = \begin{bmatrix}
B_{1,2} & \cdots & B_{1,N-1} \\
B_{N,2} & \cdots & B_{N,N-1}
\end{bmatrix}
\]

\[
\begin{bmatrix}
B_{yx}
\end{bmatrix} = \begin{bmatrix}
B_{2,1} & B_{2,N} \\
\vdots & \vdots \\
B_{N-1,1} & B_{N-1,N}
\end{bmatrix}
\]

and

\[
\begin{bmatrix}
B_{yy}
\end{bmatrix} = \begin{bmatrix}
B_{2,2} & \cdots & B_{2,N-1} \\
\vdots & \ddots & \vdots \\
B_{N-1,2} & \cdots & B_{N-1,N-1}
\end{bmatrix}
\] (2.43)

Equation 2.42, can be solved as in reference [2.8], when

\[Q_y = 0\] (2.44)

it can be shown from equation 2.42 that

\[V_y = -B_{yy}^{-1}B_{yx}V_x\] (2.45)

Substitution of equation 2.45 in equation 2.42 gives

\[Q_x = (B_{xx} - B_{xy}B_{yy}^{-1}B_{yx})V_x = B_xV_x\]

or
\[ Q_x = B_x V_x \]

where

\[ B_x = B_{xx} - B_{xy} B_{yy}^{-1} B_{yx} \]  

(2.46)

Since \( Q_x \) is a 2x2 matrix and \( V_x \) is a 2x2 matrix, as sub matrices shown in equation 2.43, \( B_x \) will also be a 2x2 matrix, but with two hypothetical terms \([2.8] \), \( N', \) with capacitances such that all the other real terms (windings) from 2…N-1 have charge \( Q_2, \ldots Q_{N-1} = 0. \)

The modified equivalent circuit for the N turns secondary is given in figure 2.07.

![Equivalent circuit for secondary.](image)

Figure 2.07: Equivalent circuit for secondary.

for which the Maxwell coefficient matrix, can be written as

\[
B_x = \begin{pmatrix}
B'_{1,N} & B'_{1,N} \\
B'_{N,1} & B'_{N,N}
\end{pmatrix}
\]

(2.47)

which is related to the capacitances of figure 2.07 by equations 2.40(a) and (b) below.

\[ C'_{1,0} = B'_{1,1} + B'_{1,N} \]

\[ C'_{N,0} = B'_{N,N} + B'_{N,1} \]

together with

\[ C'_{1,N} = C'_{N,1} = -B'_{N,1} = -B'_{1,N} \]
which is the effective capacitance between turns 1 to N of the actual secondary windings.

In practice the Maxwell coefficients can be obtained by using ANSOFT-MAXWELL-2D electrostatic programme with the secondary winding model of figure 2.05. The conical secondary winding is modelled by a series of coaxial planner loops of progressively decreasing radii. Once the Maxwell coefficients are known the individual capacitance to the ground ($C_{0,i}$) of turns, turn to turn capacitance ($C_{i,j}$) and total effective capacitance of the secondary winding can be calculated as shown above.

2.3.4 Inductance calculation of transformer

2.3.4.1 The energy method [2.5] [2.7]

Once the filamentary currents of any transformer are known, the magnetic energy stored in all the filamentary self and mutual inductances can be calculated at any time during the discharge of the capacitor bank. The total energy stored in the winding can then be found directly by adding the magnetic energies associated with each filament, and the result obtained must be equal to the energy stored in the corresponding element of the lumped component model (i.e. $\frac{LI^2}{2}$). The self inductance of the single-turn primary winding can thus be written as:

$$L_p = \frac{\sum_{i=1}^{N_p} \sum_{j=1}^{N_p} M_{i,j} I_i I_j}{\left(\sum_{i=1}^{N_p} I_i\right)^2}$$

(2.48)

where $N_p$ is the total number of filaments in the primary winding.

The inductance of the multi-turn secondary winding (represented by N concentric cylinders) is given by
where \( N_s \) is the total number of filaments in the secondary winding.

Similarly the mutual inductance between the primary and secondary windings can be written as

\[
M_{p-s} = \frac{N_P N_s}{\sum_{i=1}^{N_P} \left( \frac{1}{N_s} \sum_{i=1}^{N_s} I_i \right)} \sum_{i=1}^{N_P} \sum_{j=1}^{N_s} M_{i,j} I_i I_j
\]  

Equations 2.48, 2.49 and 2.50 can be evaluated since all the turn-to-turn mutual inductances are derived using the results in section 2.3.1.

### 2.4 Requirement of present project

The present project requires a generator capable of producing a minimum current of about 20 kA rising in a minimum of 50 ns (for the low current POS operation discussed in chapter 1) into a short circuit load at an initial charging voltage \((V_p)\) of 25 kV. Since the primary capacitor used had a total capacitance of 1830 nF (with the use of 3 parallel – connected capacitors, each having capacitance of 610 nF) and able to provide a maximum current of 25 kA (a total of 75 kA). If the maximum current is as given in equation 2.51, the primary inductance should not be lower than 200 nH, for the safety of the capacitors.

\[
I_{\max} = V_p \frac{C_p}{L_p}
\]  

The primary inductance is made up of

\[
L_p = L_{pw} + L_{cap} + L_{con} + L_{sg}
\]
where $L_{pw}$ is primary winding inductance, $L_{sg}$ (60 nH) is the inductance of one spark-gap and $L_{cap}$ (20nH) is the inductance of a single capacitor and $L_{con}$ is due to the connection (10 nH). Since three capacitors and three spark-gaps are used in parallel, their respective inductance will be one-third of the individual inductance. The primary winding inductance from equation 2.52 is then $L_{pw} = 163 \text{ nH}$

Once the inductance and capacitance in the primary circuit are known the secondary inductance and capacitance must be chosen to satisfy $k = 0.6$ and $T = 1$. This process provides values of the primary and secondary transformer parameters as in table 2.01 below:

<table>
<thead>
<tr>
<th>Primary circuit capacitance</th>
<th>610 nF x3=1830nF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary circuit inductance</td>
<td>200 nH</td>
</tr>
<tr>
<td>Secondary circuit capacitance</td>
<td>7 nF</td>
</tr>
<tr>
<td>Secondary winding inductance</td>
<td>62.0 µH</td>
</tr>
</tbody>
</table>

Table 2.01: Required parameters for the Tesla transformer circuits

When in use a pulse forming line (PFL) (as discussed in chapter 1) will be attached to the secondary circuit of Tesla transformer and its capacitance will be much higher than the capacitance formed by the windings of the secondary coil ($C_{sw}$). If $C_{PFL}$ is the capacitance of the water filled PFL (order of nF) and $C_{sw}$ is the stray winding capacitance (order of pF) of the secondary windings, then

$$C_S = C_{PFL} + C_{SW}$$  \hspace{1cm} (2.53)

and since

$$C_{PFL} \gg C_{SW}$$

it follows that

$$C_S = C_{PFL}$$  \hspace{1cm} (2.54)
Although $C_{sw}$ plays no important role in the present project, due to the high capacitance of the PFL, its value will be calculated for comparison with the experimental results obtained for the secondary circuit in the absence of the PFL.

### 2.5 Manufacture of Tesla primary and secondary windings

In fabricating the Tesla transformer, an important aim is to make the primary winding inductance 163 nH and the secondary inductance 62.0 µH.

#### 2.5.1 Selection of primary circuit for Tesla transformer

The internal diameter of the primary coil should not exceed the diameter of the PFL (discussed in chapter 3) to maintain the uniformity of the system. Once the diameter of the primary is fixed the length of the primary coil must be adjusted to give the primary inductance of 163 nH (section 2.4) as calculated in Appendix–I.

A copper sheet of 100 µm width and of 236 mm length was wrapped around a plastic mandrel of diameter 240 mm to provide a single turn primary for the Tesla transformer as shown in figure 2.08 and figure 2.09 (a). The calculated value for the primary resistance for this arrangement was 100 mΩ.

![Primary and secondary windings of Tesla transformer.](image)

Figure 2.08: Primary and secondary windings of Tesla transformer.
2.5.2 Secondary winding:

2.5.2.1 Properties of tapered secondary winding

A tapered construction (with the larger diameter at the bottom) was used for the secondary coils to avoid any break-down at the high voltage end of the secondary winding. The high voltage will appear at the upper end (as the lower end is grounded) and the greater space around the high voltage terminal will prevent the possibility of any high-voltage break down.

2.5.2.1.1 Secondary winding

The secondary winding has to be placed inside the plastic mandrel used to support the primary winding. In this situation the maximum diameter and height of the secondary coil must not exceed the internal dimensions of the plastic mandrel (which not only works as a support for the primary coil but also retains the insulating oil for the secondary windings.)

Once the height and diameter for the secondary winding is fixed the reminder of the parameters for the secondary windings can be calculated as shown in Appendix–I.

The secondary winding was formed by 23 turns of commercially available copper wire 3 mm in diameter wrapped around a conical mandrel with a lower diameter of 212 mm, an upper diameter of 134 mm and height of 181 mm as shown in figure 2.09 b.

The number of turns was determined by the required secondary inductance and the secondary resistance of 2 Ω (see Appendix-I). The selection of the wire diameter is a trade off between the required low resistance of secondary windings and the higher inter-turn distance of the secondary windings to prevent any inter turn high voltage breakdown.
2.5.2.2 Tesla transformer during manufacture.

Figure 2.09: (a) Primary winding on plastic mandrel (b) secondary conical winding.

Figure 2.10: (a) secondary winding and outer grounding bar for PFL, (b) plastic pipe where secondary will be kept and base for primary winding.
2.5.3 Simulated Tesla parameters

The parameters given in this section are derived from the dimensions also given in this section. Calculation of these parameters is available in Appendix-I.

\[ L_p \text{ (primary circuit inductance)} = 200 \text{ nH} \]

\[ R_p \text{ (Primary circuit resistance)} = 100 \text{ m}\Omega \]

\[ L_s \text{ (secondary winding inductance)} = 62.69 \mu\text{H} \]

\[ C_{sw} \text{ (stray capacitance of secondary windings)} = 70 \text{ pF} \]

\[ R_s \text{ (resistance of secondary circuit)} = 2 \Omega \]

Mutual inductance between primary and secondary windings = 2.16 \mu \text{H}

2.6 Testing of Tesla parameter

2.6.1 Primary circuit testing

To confirm the calculated parameters of the primary winding of the Tesla transformer, the primary circuit capacitor was discharged, with the secondary winding open circuit. The voltage on the capacitor was measured using a commercially available North-star voltage divider with a ratio of 1:1000. The rate of change of the primary current was measured by an I-dot probe, mounted in the transmission line as shown in figure 2.13.

![Figure 2.13: Experimental setup for testing of primary in Tesla.](image)
The voltage appearing on the capacitor is given by

\[ V_p = \frac{dI_p}{dt} (L_p - L_{cap}) + (R_p - R_{cap})I_p \]  \hspace{1cm} (2.55)

Based on the voltage probe results of figure 2.14, the I-dot probe could be calibrated. Integration of the I-dot probe will give \( I_p \) \( k \) (calibration factor), and \( k \) can be obtained by solving equation 2.55. Figure 2.14 compares the primary voltage output with the calculated values using the parameters given in section 2.6.

Figure 2.14: Experimental results of primary (without connected secondary) to verify calculated primary parameters.

Figure 2.15: Experimental results of primary current (without connected secondary) compared with I-dot probe output.
The I-dot probe is located in the primary circuit to measures the \( dI/dt \) of the primary current. Integrated \( dI/dt \) signal (multiplied by the calibration factor) with the predicted current are both shown in figure 2.15.

### 2.6.2 Testing of secondary circuit

Low voltage testing of the secondary circuit was undertaken to measure the secondary parameters and the mutual inductance between the primary and secondary windings.

In these tests the Tesla transformer was not working in the Tesla transformer mode. If \( C_s \) is the combination of the stray capacitance of secondary winding and the capacitance of the PFL, then

\[
C_s = C_{sw} + C_{PFL}
\]

where \( C_{sw} \) is the stray capacitance of secondary winding and \( C_{PFL} \) is the capacitance of the PFL. Since all the calculation was performed to include the water PFL, whose capacitance is much higher than the stray capacitance of the secondary windings, these tests do not resemble the true Tesla transformer testing. Details of the water PFL will be presented in chapter 3.

![Figure 2.16: Experimental results for secondary parameter testing without connecting water PFL.](image)
The secondary voltage output without the PFL is shown in figure 2.16. Only the stray capacitance of the secondary windings is taken into account in the simulation.

Results obtained from figure 2.16 are given in table 2.02

<table>
<thead>
<tr>
<th>( \omega_p ) (Rad/sec)</th>
<th>1.373x10^6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_s ) (Rad/sec)</td>
<td>1.571x10^7</td>
</tr>
<tr>
<td>( \omega_1 ) (Rad/sec)</td>
<td>1.371x10^6</td>
</tr>
<tr>
<td>( \omega_2 ) (Rad/sec)</td>
<td>1.82x10^7</td>
</tr>
<tr>
<td>k</td>
<td>0.503</td>
</tr>
<tr>
<td>T</td>
<td>7.631x10^{-3}</td>
</tr>
</tbody>
</table>

Table 2.02: Parameters for Tesla transformer without secondary.

Results with PFL included (but without water, which considerably reduce its capacitance) are shown in figure 2.17.

Figure 2.17: Experimental results for secondary parameter testing with connecting PFL (without filling of water).
Results for figure 2.17 given in table 2.03 confirm that because the PFL is not water filled, the transformer is not working in the Tesla mode.

<table>
<thead>
<tr>
<th>$\omega_p$ (Rad/sec)</th>
<th>$1.478 \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega_s$ (Rad/sec)</td>
<td>$1.057 \times 10^7$</td>
</tr>
<tr>
<td>$\omega_1$ (Rad/sec)</td>
<td>$1.474 \times 10^6$</td>
</tr>
<tr>
<td>$\omega_2$ (Rad/sec)</td>
<td>$1.249 \times 10^7$</td>
</tr>
<tr>
<td>$k$</td>
<td>0.529</td>
</tr>
<tr>
<td>$T$</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 2.03: Parameters for Tesla transformer with PFL (not water filled).

Results obtained when the PFL was water filled PFL are shown in figure 2.18, and recorded in table 2.04. These confirm that the transformer is then working in the Tesla mode, as $\omega_2 \approx 2 \omega_1$ and $k \approx 0.60$. Details of the PFL are provided in chapter 3.

Figure 2.18: Experimental results of Tesla operation with water filled PFL.
Table 2.04: Parameters for Tesla transformer with PFL (water filled).

2.7 Conclusion

Based on the above tests the primary parameters and secondary transformer parameters can be summarised as

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Simulated</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary winding inductance</td>
<td>163.8 nH</td>
<td>165 nH</td>
</tr>
<tr>
<td>Primary circuit resistance</td>
<td>50 mΩ</td>
<td>47 mΩ</td>
</tr>
<tr>
<td>Secondary winding inductance</td>
<td>62.69 µH</td>
<td>62 µH</td>
</tr>
<tr>
<td>Stray capacitance of secondary coil</td>
<td>70 pF</td>
<td>60 pF</td>
</tr>
<tr>
<td>Resistance of secondary circuit</td>
<td>2 Ω</td>
<td>2 Ω</td>
</tr>
<tr>
<td>Mutual between primary and secondary circuit</td>
<td>2.16 µH</td>
<td>2.10 µH</td>
</tr>
</tbody>
</table>
2.8 References


3 Pulse forming line

In the present research, a high current fast rising pulse is essential as a feed to the POS. Although there are many ways to create such pulses, use of a pulse forming line (PFL) is perhaps the simplest form of generating one with the required characteristics.

Pulse forming lines essentially consists of a transmission line, usually a waterline that is charged as a lumped capacitor (which forms the secondary capacitance of the Tesla transformer) to store electrical energy and a high voltage switch to discharge this energy in the transmission line to the load.

A coaxial transmission line was selected on the basis that it could easily be made to connect to a coaxial spark-gap, while it also has a self limiting capability of suppressing electrical noise. The coaxial structure is extremely suitable for a water capacitor, as it not only provides the capacitance but will also hold the water without any additional support.

3.1 The single pulse forming line [3.1]

In most electrical circuits, the length of the wires connecting the components can be ignored. That is, the voltage on the wire at any given time can be assumed to be the same at all points along the wire. However, when the voltage changes in a time interval comparable to the time it takes for the signal to travel down the wire, the length becomes important and the connection must be treated as a transmission line. In other words the length of the wire is important when the signal includes frequency components with quarter wavelengths ($\lambda/4$) comparable to or less than the length of the wire. At this length the phase delay and the interference of any reflections on the line become important and can lead to unpredictable behavior of the system.
The charging time of a transmission line is of the order of $1-2 \mu s$ ($\lambda/4 = 40 m$) and the discharge time is of the order of $50 \text{ ns}$ ($\lambda/4 = 1.5 m$), as the PFL discharges its electric energy stored in the transmission line mode into the load.

Transmission lines are an essential part of any PFL, and they comprise two or more conductors that carry the fast electrical signals. The important feature of the transmission line is that its cross sectional configuration and the separation between the conductors must remain constant along the length of the line. If a dielectric is inserted between the lines the dielectric constant must remain constant throughout the length of the line. Although different types of line can be envisaged, the simplest and most commonly used is the coaxial line, where two coaxial cylindrical conductors with finite length work as a transmission line, as shown in figure 3.01.

![Coaxial transmission line](image.png)

Figure 3.01: coaxial transmission line.

The electrical parameters of the coaxial transmission line are uniformly distributed along its length. With reference to figure 3.01, these parameters (per unit length) for a coaxial line are:

$$C = \frac{2\pi\varepsilon_r}{\ln\left(\frac{R_{out}}{R_{in}}\right)}$$  \hspace{1cm} (3.01)
and the characteristic impedance of the line is

\[ Z_0 = 60 \sqrt{\frac{\mu_r}{\varepsilon_r}} \ln \left( \frac{R_{out}}{R_{in}} \right) \]  

(3.03)

where \( \varepsilon_r \) is the relative permittivity of the dielectric material between the conductors.

Figure 3.02 is a basic diagram of a pulse generator based on a single pulse forming line. A length \( l \) of transmission line of impedance \( Z_0 \) is charged to a potential \( V \) by the power source. Once the line is charged the switch is closed and the line allowed to discharge into the load \( Z_L \). Since the stored energy for the pulse is in the transmission line, a rectangular pulse is generated [3.1] in the load, with a duration equal to twice the transit time that a voltage step takes to propagate along the line from one end to other. The propagation velocity of such a step \( v_p \) is given by.

\[ v_p = \frac{c}{\sqrt{\varepsilon_r}} \]  

(3.04)

where \( c \) is the velocity of light. The duration of the pulse \( t_p \) that is generated is given simply by

\[ t_p = \frac{c}{v_p} = 2 \delta \]  

(3.05)

Figure 3.02: The basic pulse-forming line.
It is important that the closing and opening times of the switch are much shorter than the duration of the pulse generated, as these characteristic time dictate, in part at least, the rise and falls times of the pulse that is generated. It is also important for short duration pulses that the stray inductance $L_s$ of those parts of the circuit that connect the output end of the line to the load is minimised. If this is not the case the rise time of the pulse will be limited by this stray inductance, with a time constant $\tau_r$ given by

$$\tau_r = \frac{L_s}{Z_0 + Z_L}$$  \hspace{1cm} (3.06)

For purposes of illustration, figure 3.03 show the simplest situation, where $Z_0 = Z_L$

Figure 3.03: The potential distribution on the line at various times after switch closure.

On closure of the switch, the voltage on the load rises immediately from zero to a value determined by the initial charge voltage $V$, the characteristic impedance of the line $Z_0$ and the load impedance $Z_L$, and is given by
\[ V_I = V \frac{Z_L}{Z_L + Z_0} \]  \hspace{1cm} (3.07)

In the case where the impedance of the line and the load are equal then clearly the voltage on the load is \( V/2 \). Simultaneously a step voltage \( V_s \) propagates away from the load towards the charging end of the line with an amplitude \( V_s \) given by

\[ V_s = V \left( \frac{Z_L}{Z_L + Z_0} - 1 \right) = V \left( \frac{-Z_0}{Z_L + Z_0} \right) \]  \hspace{1cm} (3.08)

Again, in the case that \( Z_0 \) and \( Z_L \) are equal this step is simply \(-V/2\). When it reaches the charging end of the line (after a time \( \frac{1}{V_p} \)) it is reflected back with a reflection coefficient given by

\[ \rho = \frac{Z_L - Z_0}{Z_L + Z_0} \]  \hspace{1cm} (3.09)

The reflection coefficient will be \( \sigma = +1 \) because the output impedance of the charging supply is much greater than the impedance of line. Thus the potential at the charging end of the line falls to zero and a voltage step of amplitude \( V_s \) propagates back towards the load, altering as it goes the potential of the line. Again, if the line and load impedance are matched, this step has an amplitude \(-V/2\) as it propagates, and it completely discharges the line, i.e. the line potential is reduced to zero. When the step finally reaches the load, the line is completely discharged, the potential on the load falls to zero and the pulse terminates. If the impedance of the load and the line are not matched, further reflections of the propagating step occur at the load and the resulting output pulse has a more complicated shape, rather than the single rectangular pulse shown in figure 3.03.
3.2 High voltages switches [3.2]

Spark-gap switches fall into the category of closing switches. They are of simple design and low cost, and their outstanding switching characteristics display an excellent voltage withstand capacity (around a few MV) and a high charge transfer capability. Spark-gap switches have a sub nanosecond closure time.

A gas switch can be considered to be electrically closed when under a high electrical field stress the insulating gas between the electrodes becomes conducting and a plasma channel develops. The electrode geometries of many gas filled spark gaps can be described either by one of the two basic structures shown in figure 3.04 or a combination of these. The electric field between the electrodes in the sphere-sphere configuration approaches uniformity when the radius of the spheres is much larger than the sphere-sphere separation, or \( r_1, r_2 \gg d \). The behaviour of a sphere-sphere gap is polarity independent, due to its symmetry.

![Figure 3.04: Typical spark gap configuration.](image)

The point-sphere configuration can be approximated as a point plane geometry, when the gap spacing is much less than the radius of the sphere and much greater than the
radius of point (in the sub mm range), or \( r_s \gg\gg d \gg\gg r_p \). The performance of a sphere-point gap is polarity dependent.

The gap length (\( d \)) between the two spheres should be kept to a minimum to lower the inductance of the gap. A small gap can however break at an undesirable low voltage, although this can be avoided by pressurization of the gas medium. Electric breakdown dependency on both pressure and length of gap are discussed in detail in section 3.3.2.

### 3.3 Pulsed break down strength of dielectric medium

Two types of dielectrics are used in the PFL described in this thesis. Deionised water is used as the dielectric medium in the transmission line and SF\(_6\) as the dielectric medium in the high pressure switch before the load, as shown in figure 3.05. The characteristics of both of these dielectrics are discussed below.

![PFL with different dielectrics](image)

Figure 3.05: PFL with different dielectrics.

#### 3.3.1 Pulsed break-down strength of water [3.3]

Water is used as the dielectric in the transmission line, since it has very good electrical insulation properties (but only in the time range of a few microseconds) as well as a high dielectric constant (around 80). Its high dielectric constant makes it the best insulator for use when compact capacitors and high values of capacitance are required.
3.3.1.1 Basic electrical properties of water

3.3.1.1.1 Permittivity

The relative permittivity $\varepsilon_w$ and the operating pulse electric field strength $E$ are the most important properties when water is used as an insulator, since they determine the electrical energy density stored in the dielectric and given by

$$E_W = \frac{\varepsilon_w E^2}{2}$$  \hspace{1cm} (3.10)

and correspondingly, the impedance of the water filled transmission line depends on the permittivity also.

The permittivity of a substance is a complex quantity, and is dependent on the frequency of the applied electromagnetic field. According to the Debye theory [3.4],

$$\varepsilon_w = \varepsilon_{\infty} + \frac{(\varepsilon_s - \varepsilon_{\infty})}{(1 + j \omega \tau_d)}$$  \hspace{1cm} (3.11)

where $\varepsilon_{\infty}$ is the permittivity at infinitely high frequencies, $\varepsilon_s$ is the static permittivity at zero frequency and $\omega = 2\pi f$. For polar liquids, $\varepsilon_s >> \varepsilon_{\infty}$ and $\varepsilon_{\infty}$ can therefore be ignored in equation 3.08. $\tau_d$ is the Debye relaxation time (i.e. the time required to change the orientation of molecular dipoles under the action of the electric field). For water, $\tau_d \approx 10^{-11}$ s at 25 °C, therefore any dependency of $\varepsilon_w$ on frequency can be ignored for high power pulsed facilities.

3.3.1.1.2 Dependency of water permittivity on temperature of water

The permittivity of water depends on the water temperature $T$ and the frequency $f$ of the voltage changes or that equivalent to the duration of the acting pulse. Experimental data agrees well with changes in the relative water permittivity that depend on $T$ and calculated using the formula [3.5]
\[ \varepsilon_w = 78.54 \left( 1 - 4.579 \times 10^{-3} T_i \right) + 1.17 \times 10^{-5} T_i^2 - 2.8 \times 10^{-8} T_i^3 \]  

where \( T_i = T^0 - 25 \) is the Centigrade temperature. As the temperature rises water the permittivity falls slowly; from \( \varepsilon_w = 88 \) at 0°C to 78.2 at 25°C.

Although the breakdown strength (BS) of water is an important characteristic, its pulse breakdown remains unclear despite extensive experimental and theoretical studies. This is associated with the complex character of the development of the breakdown and its dependence on a combination of factors such as the duration, amplitude, and shape of the applied pulse electric field, the geometry and the material of the electrode system, the degree of water degassing, the presence and material of any mechanical particles in the water volume and at the electrode surfaces, the water resistivity \( \rho \), etc. It appears impossible to find a quantitative analytical approach for the aggregate effect of these factors, especially if there are no conventional and consistent methods for determining their physicochemical characteristics. Therefore, on the basis of experimental data, we briefly discuss below the influence of the main factors on the water breakdown strength and present empirical formulae for calculating the breakdown strength \( E_b \) for a water layer and over a surface of solid dielectric in water.

### 3.3.1.1.3 Formulae for calculating the water breakdown field strength.

Since it remains impossible to determine a priori the water BS, empirical formulae must be used to perform the calculations required when designing water-insulated facilities. To estimate the breakdown field strength of water with a resistivity \( \rho \geq 10^5 \ \Omega\text{-cm} \) for an action duration of 0.1 to 1 \( \mu s \) and a voltage increasing according to the \( 1 - \cos(\omega t) \) law in a system of coaxial electrodes with an area of \( >10^4 \ \text{cm}^2 \), J.C.
Martin proposed a formula for the electrical breakdown in water, where \( E_{br} \) is breakdown voltage and \( S \) is the surface area. This formula is given as [3.6]:

\[
E_{br}, \text{kV/cm} = K_{ef}^{-0.33} S^{-0.1}
\]

(3.13)

where the factor \( K \) is equal to 300 for positive polarity and 600 for negative polarity of the inner electrode, when the outer electrode is ground, \( t_{ef} \) is the rise time in \( \mu \text{s} \) of the field strength from 0.63\( E_b \) to \( E_{br} \); and \( S \) (cm\(^2\)) is the electrode area over which the field changes from 90 to 100\% \( E_{br} \).

Note that this formula is applicable to transformer oil, ethanol, and methanol (\( K = 500 \) for both polarities), and also to glycerine and castor oil as well (\( K = 700 \) for both polarities).

The breakdown strength of purified water was studied [3.5] as a function of the gap length \( d = 0.4–5 \) cm and the electrode surface \( S = 5–1000 \) cm\(^2\) under the action of a triangular voltage pulse with a duration \( t_{ef} = 0.1–2 \) \( \mu \text{s} \) in a field produced by Rogowsky electrodes (described in section 3.3.1.1.3.1) and in highly inhomogeneous fields of point–plane and blade–plane systems.

The relationship for the Rogowsky electrodes is [3.5]

\[
E_{brd}, \text{kV/cm} = K_d t_{ef}^{-0.25} S^{-0.1} d^{-0.1}
\]

(3.14)

where \( E_{brd} \) is electrical break down voltage in Rogowsky electrode and \( K_d \) is a factor equal to 380 and 320 for stainless steel and aluminium electrodes, respectively.

### 3.3.1.1.3.1 Rogowski electrodes

To generate a uniform electrical field between two electrodes, very large flat plates provide an ideal but somewhat difficult arrangement to realize in a laboratory of reasonable dimensions. Finite sized plates produce a uniform field near the middle of the plate, but the high field at the edges creates a problem.
Rogowski developed a technique that starts by determining a realizable field, then constructs an electrode shaped so that its surface lies on an equipotential surface.

### 3.3.1.2 Breakdown strength over the surface of solid dielectrics in water

To obtain the necessary measurements, the structural and insulating elements made of solid dielectric are placed in water or any other liquid insulator. The considerable differences in the $\varepsilon$ and $\rho$ values for water and solid dielectrics lead to difficulties in designing and using such insulators, since they may substantially modify the value and distribution of the $E$ field at the interface with the water. In order to develop a water-insulated device, it is therefore important to know how the breakdown voltage over the surface of solid insulators depends on their shape and electrical properties.

The mean break-down strength of a cylindrical insulator of length ($l$) in water is given as [3.5]

$$E_{brs, kV/cm} = k_s l^{-0.33} t_{ef}^{-0.25}$$

where $k_s = 200$ for polyethylene (used in making the switch chamber see figure 3.04)

### 3.3.2 Breakdown of gases in uniform field

When a charged particle is placed in an electric field, it will experience an accelerating force. This results in motion in the direction parallel to the electric field. However, except at very low pressures, in gases (and weakly ionized plasmas) particles collide very frequently with other particles, mainly neutral atoms/molecules. Such collisions change the direction of the moving particle sharply and in a random way, while thereby loosing its directed motion. In between such collisions the charged particles are again accelerated by the electric field. Therefore, the motion of charged particles in an electric field is a superposition of a directed motion parallel to the
electric field and a random motion. The average, directed motion as a result of an electric field is known as *drift*. The drift velocity of a particle, $v_{d_i}$, can be described by:

$$v_{d_i} = \pm \mu_i F$$

where the $\pm$ sign indicates whether the drifting particle $i$ is positively or negatively charged, $F$ is the electric field and $\mu_i$ is the mobility of particle $i$. It is clear that particles with opposite charge will drift in opposite directions. The relation between the electric field $F$ and the drift velocity is not completely linear since the mobility is a function of electric field strength. It should be noted that since electrons are much lighter than noble gas ions, the mobilities (and drift velocities) of electrons are higher.

The fundamental element of the breakdown process is the electron avalanche. Electrons drifting in an electric field towards the anode gain energy from this field and cause ionization of atoms by collisions. This results in a multiplication of electrons and ions. In the 1930s, Rether was the first to visualize an electron avalanche using a cloud chamber experiment. Electron avalanches evolve not only in time, but also in space, along the direction of the electric field. Often it is most convenient to express the rate of ionization not in terms of ionization frequency, but as an ionization coefficient $\alpha$, which is the number of ionization events by electron impact per unit length along the direction of the electric field. The coefficient $\alpha$ is also known as Townsend’s (first) ionization coefficient.

### 3.3.2.1 Townsend breakdown mechanism [3.7]

In any breakdown model a source of free electrons is required between the electrodes, and, although such source can be created by irradiation, for any volume of gas there exists a low but constant source of free electrons due to the ionising effect of cosmic particles. In the absence of an electric field the rate of electron and positive ion genera
tion is counterbalanced and a state of equilibrium exists between ions. This will however be upset by application of a sufficiently high electric field.

Townsend found that the current at first increased proportionally with the applied voltage and remained constant at a value of $I_0$, as shown in figure 3.06, despite an increase in the applied voltage. At a higher voltage the current increased above the value $I_0$ at an exponential rate.

To explain this current increase Townsend introduced his first ionisation coefficient, $\alpha$. It is the collisional ionization coefficient representing the number of new electrons produced by an electron while travelling along a 1 cm path in the direction of electric field.

The ionisation current leaving the cathode, is given by [3.7]

$$I = I_0 \exp(\alpha d)$$

(3.16)

where $d$ is the separation between the cathode and anode.

For the quantity $\alpha$, Raether [3.8] derived a relation which can be written as

$$\alpha = pf \left( \frac{E}{p} \right)$$

(3.17)

where $E$ is the electrical field strength and $p$ is the gas pressure.
This equation can be rearranged as

\[
\frac{\alpha}{p} = f\left(\frac{E}{p}\right) \tag{3.18}
\]

### 3.3.2.2 Townsend secondary ionization coefficient

Townsend observed that at higher voltages the current increases at a more rapid rate than that defined by the previous equations. This variation was due to secondary electron emission at the cathode caused by photon impact and photo-ionization in the gas itself. His modified equation that includes this effect is written as [3.7]

\[
I = I_0 \frac{\exp(\alpha d)}{1 - \gamma(\exp(\alpha d) - 1)} \tag{3.19}
\]

where \( \gamma \) is the number of electrons released from the cathode per incident positive ion. Since

\[
E = \frac{V}{d} \tag{3.20}
\]

and

\[
\frac{I}{I_0} = e^{(p.d)f\left(\frac{V}{pd}\right)} \frac{1}{1 - \gamma e^{(p.d)f\left(\frac{V}{pd}\right) - 1}} \tag{3.21}
\]

At the point where the current \( I \) becomes indeterminate and the denominator in the above equation vanishes.

\[
e^{(p.d)f\left(\frac{V}{pd}\right)} = \frac{1}{\gamma} + 1 \tag{3.22}
\]

or

\[
f\left(\frac{E}{p}\right) pd = \ln\left(1 + \frac{1}{\gamma}\right) = K \tag{3.23}
\]
For a uniform field \( V_b = Ed \), where \( V_b \) is the breakdown voltage,

\[
f \left( \frac{V_b}{pd} \right) = k
\]

or

\[
f \left( \frac{V_b}{pd} \right) = \frac{\ln(k)}{pd}
\]

which using a new function \( F \) can be written as:

\[
V_b = F(pd)
\]

which means that the breakdown voltage of a uniform field gap is a unique function of product of the pressure and electrode separation for a particular gas and electrode material. This is known as Paschen’s Law.

![Paschen’s curve](image)

Figure 3.07: Paschen’s curve.

An approximate relationship between \( \frac{\alpha}{p} \) and \( \frac{E}{p} \) is

\[
\frac{\alpha}{p} = Ae^{-Bp/E}
\]

where \( A \) and \( B \) are constant for a given gas. If ion impact is taken as the most important secondary ionisation process, experiments [3.2] have shown that \( \gamma \) is a
slowly varying function of $E/p$ over a wide range, and after using the secondary ionization equation (equation 3.16) [3.7], it follows that

$$V = \frac{Bpd}{C + \ln(pd)} \quad (3.27)$$

where

$$C = \ln \left( \frac{A}{\ln(1 + \frac{1}{\gamma})} \right) \quad (3.28)$$

This equation enables the Paschen curve shown in figure 3.07 to be generated, by using arbitrary values for A, B and \(\gamma\). It can clearly be seen that there is a minimum of the breakdown voltage, which is unique property of the gas and electrode materials, and is given by [3.7] \(V_{\text{min}} = B,\text{Exp}(1 - C)\) at \(pd = \text{Exp}(1 - C)\). There is a region to the left of the minimum where the ratio of the gap distance to the mean free path of electrons decreases, thereby reducing the probability of collisions. To maintain the ionising collision at a value sufficient to cause breakdown, higher electron energies are required and the breakdown voltage is increased. To the right of the minimum, the electron mean free path decreases, and higher electric fields are necessary to provide the electrons between collisions with sufficient energy to cause ionisation. At even higher pressures additional effects due to irregularities in the cathode surface have to be taken into account, which cause field intensification and leads to lower breakdown voltages than those given by the Paschen curve.

### 3.3.2.3 Break-down in non uniform fields

In non-uniform fields, e.g. in point-plane, sphere-plane gaps or coaxial cylinders, the field strength and hence the effective ionization coefficient \(\alpha\) vary across the gap. The electron multiplication is then governed by the integral of \(\alpha\) over the path, and at low
pressures the Townsend criterion for a breakdown shown in equation 3.19 takes the form [3.7]

\[ \gamma \exp \left( \int_{0}^{x} \alpha \gamma \, dx \right) = 1 \]  \hspace{1cm} (3.29)

where \( x \) is the spark gap length and the integration must be taken along the line of the highest field strength. The expression is also valid for higher pressure, if the field is only slightly non-uniform.

The space charge concentration in a non-uniform field will disrupt the progress of the discharge, in contrast to the uniform field conditions. The space charge will lead to the partial breakdown of an insulating gas without the formation of a complete breakdown, and the breakdown voltage will be higher than that required for a uniform field. In the case of a non-uniform field the polarity of the applied voltage plays an important role, due to the effect of field strength on the secondary ionisation mechanism at the cathode.

### 3.3.2.4 Break-down in transient fields

The breakdown theory described in the previous section is commonly known as the Townsend breakdown mechanism. It is characterized by the continuous development of successive electron avalanches between electrodes by secondary emission processes. During the last century, the Townsend theory has been very successful in explaining breakdown phenomena under various discharge conditions. For instance, the explanation of Paschen’s breakdown curves, discussed in the previous section, and effects of electronegative gases and cathode materials on the breakdown voltage.

However, in the 1930s and 40s, with the ongoing development of experimental equipment for studying the time-development of transient breakdown processes, observations of breakdown phenomena were made that did not fit in the Townsend
theory. For instance, Raether’s investigations of developing electron avalanches by cloud chamber experiments [3.9] showed breakdown features too fast to be explained by the Townsend mechanism. These observations were mainly made for discharges with high values of $pd (>10^3 \text{ Pa-m})$; that is high pressures and large gaps. Loeb [3.10] [3.11], Meek [3.12] and Raether [3.13] developed the fundamentals of a new breakdown theory that could explain the observed, high-speed breakdown features. The basis of this new theory, known as the *streamer breakdown* theory, takes into account space charge effects of a single electron avalanche. In the Townsend theory these effects are neglected. In the streamer theory, the breakdown process still starts with the development of an electron avalanche. However, the multiplication of charges in this avalanche is so large that the space charge in the avalanche head starts to modify the applied electric field, before the avalanche can reach the anode. A avalanche-to-streamer transition can be observed in the discharge gap. The space charge field in the avalanche head can be calculated, assuming complete separation between positive and negative charges. The space charge field, $E_s$, produced by a number of charges $Q$ in a sphere of radius $r$ is

$$E_s = \frac{Qe}{4\pi r^2 e_r}$$  \hfill (3.30)

where $e$ is the elementary charge. The radius, $r$, of the space charge region, by calculating the space charge formation in an electron avalanche and comparing it to the applied electric field, Meek [3.12] defined a criterion for streamer formation consequently breakdown as

$$\exp \int_0^{x<d} e dx = N_{cr}$$  \hfill (3.31)
where $N_{cr}$ is the critical electron concentration in an avalanche giving rise the to initiation of streamers, $x_c$ is the path of avalanche to reach this size and $d$ the gap length.

\[ x_c < d \]

\[ \int_0^{x_c} \alpha dx = \ln N_{cr} \approx 18 - 20 \]  \hspace{1cm} (3.32)

### 3.3.2.5 Effect of electronegative gas [3.14]

Electronegative gases such as O$_2$ and SF$_6$ have empty space in their outermost orbit, and the attachment of free electrons to electronegative gas molecules forms negative ions. SF$_6$ is used as an insulating medium in this work, since it has excellent insulating strength because of its affinity for electrons, i.e., whenever a free electron collides with a neutral gas molecule to form a negative ion, the electron is absorbed by the neutral gas molecule. The attachment of the electron with the neutral gas molecule may occur in two ways:

- $\text{SF}_6 + e \rightarrow \text{SF}_6^-$
- $\text{SF}_6 + e \rightarrow \text{SF}_5^+ + \text{F}$  \hspace{1cm} (3.33)

The negative ions formed are relatively heavy as compared to free electrons and, therefore under a given field they do not attain sufficient energy to lead to cumulative ionization in the gas. Thus this process represents an effective way of removing electrons from the space which otherwise would have contributed to forming an electron avalanche. This property therefore gives rise to a very high dielectric strength for SF$_6$, which also has the important property of fast recombination.
3.3.2.6 Spark channel development of the switch

All the different breakdown mechanisms described above result in the formation of a weakly ionised gas channel between the two electrodes. The section describes the development of this channel until it electrically closes the electrode gap.

Energy from the electric field is transferred into kinetic motion of the electrons, ions and neutral gas molecules, which results in a rapid rise of temperature of the channel to thousands of Kelvin. As a result, the weakly ionised gas channel is converted to a highly conducting narrow plasma channel. Due to the corresponding increase in the conductivity, current from the external circuit flows through the channel, dissipating more energy and giving rise to a rapid expansion of the channel and the generation of shock wave in the surrounding media. Drabkina and subsequently Braginskii proposed models for the shock wave expansion of the spark channel and provided results on good agreement with experimental data. Braginskii [3.15] proposed a formula for the channel radius $r(t)$ at time $t$ as:

$$r(t) = \left( \frac{4}{\pi^2 \rho_0 \sigma \xi} \right)^{1/6} \left[ \int_0^t I(t)^{2/3} \, dt \right]^{1/2}$$

(3.34)

where $I(t)$ is the current through the channel, $\sigma$ is the electrical conductivity of the channel, $\rho_0$ is the gas density and $\xi$ is a constant depending on the type of gas.

The switch closure process consists of various phases, each requiring a finite time, and with the total of these referred to as the switching time. One phase is called the ‘resistive phase’, in which the spark resistance decreases by many orders of magnitude as the discharge channel radius increases due to thermal ionisation. It is very difficult to quantify the resistance accurately, as it is dynamic and due to other complex processes that are statistical in nature. There are empirical formulae valid for
particular cases, but the most straightforward approach is to consider the discharge channel as having cylindrical geometry.

At time $t$ the resistance is then

$$R(t) = \frac{l}{\pi r^2(t) \sigma} \quad (3.35)$$

where $l$ is the gap length. Substituting the value of $r(t)$ from equation 3.34 gives

$$R(t) = \frac{l}{\left(\frac{4\pi \sigma^2}{\rho \xi}ight)^{1/3} \left(\int_0^t t^{2/3} dt\right)} \quad (3.36)$$

When the electrical conductivity of the spark channel becomes high, the switch behaviour is more suitably described by its inductance, with this being called the inductive phase. The value of the inductance is determined by the overall switch geometry, and it can be written as [3.2]:

$$L = 2.1 \times 10^{-7} \left(\ln \frac{2l}{r_c} - 1\right) l \quad (3.37)$$

where $r_c$ is the channel radius.

Together, the resistive and inductive phases control the rise time of a switch, with the 10-90% switch rise time $T_r$ being expressed as [3.2]:

$$T_r = 2.2 \sqrt{t_R^2 + t_L^2} \quad (3.38)$$

where $t_R$ is the time for the resistive phase, or the time from the establishment of a conducting channel to the point at which the switch resistance is equal to the inductive switch impedance. The time $t_L$ is that of the inductive phase, the portion of the subsequent rise time dominated by the switch inductance.

Another empirical formulae, used to specify the rise time of switches used in high power impulse transmitters was given by Martin (1965) as [3.16]:

---

**Pulse forming line**
where $\rho_c$ is the density of the gas in gap in gcm$^{-3}$, $Z$ is the impedance of the load circuit and $E_f$ is the breakdown field in MV/cm.

### 3.3.3 Switch recovery process: free recovery

Before a spark-gap switch can be operated again following a discharge, the conditions within the gap must recover to their pre-breakdown state. If the time between two consecutive discharges is insufficient, the switch will pre-fire at well below its self breakdown voltage level.

Initially a high temperature plasma remains in the gap even after the discharge current has ceased to flow, and the temperature and electron concentration are sufficiently high to reignite the gap on application of a reduced voltage. Such a breakdown will occur when there is thermionic emission from the electrodes and even at an applied voltage of 100 V or less. If the external voltage is not reapplied, recombination and attachment processes will deionise the plasma column in about 100 µs and only a hot neutral gas column will remain in the gap. Due to the presence of free charge carriers the gap recovery time remains reasonably constant.

Another recovery process occurs with cooling of the hot neutral gas column from several thousand Kelvin to ambient temperature. During this time the breakdown voltage depends on the gas density, and the factors affecting the rate of thermal conduction and convection are important in the gap recovery. The thermal diffusivity of the gas is the most important of these, but the gap spacing, electrode geometry, material and gas pressure are also significant. Without any assisting mechanism, the gap recovery occurs with a decrease in the gas temperature to ambient, which takes about 100 ms for most gases except hydrogen [3.17].
3.4 Design of PFL

Since this PFL acts like a secondary circuit capacitance, it must satisfy the Tesla condition for resonance. The stray capacitance of the helical winding, was taken fully into the consideration when testing the secondary parameters.

To operate a Tesla transformer in a doubly resonant mode requires a secondary capacitance much higher than the stray winding capacitance, and this will determine the capacitance required in the PFL.

The capacitance of a water filled PFL can be written by modifying equation 3.01 as

$$C_w = \frac{2\pi \varepsilon_0 k_w l}{\ln \left( \frac{R_{out}}{R_{in}} \right)}$$  \hspace{1cm} (3.40)

where $R_{out}$ and $R_{in}$ are the outer and inner radii of the PFL (see figure 3.01), $k_w$ is the dielectric constant of water and $l$ is the length of the PFL.

The outer cylinder of PFL was made using a commercially available stainless steel stock pipe of diameter of 247 mm. The length of the PFL should not exceed 1 metre for machining purposes, once length and outer diameter was fixed for the PFL, internal diameter was chosen such that its capacitance should be 7 nF using equation 3.40.

The PFL described here has an inner diameter of 140 mm, an outer diameter of 247 mm and a length of 933 mm as shown in figure 3.08. The manufactured PFL is shown in figure 3.09. De-ionised water was used as the dielectric medium between the outer and inner conductors.
3.5 Design of spark gap

The spark gap arrangement used with the PFL is shown in figure 3.10, with the two hemispherical brass electrodes (shown in figure 3.11) being 60 mm in diameter and having a separation of 5 mm. SF₆ is the insulating medium between them.
Each spherical brass electrode is supported by a polyethylene insulator, as shown in figure 3.11. Several concentric grooves were made on the surface of the plastic to increase the effective surface distance from the high voltage electrode to the base supporting aluminium ring shown at A in figure 3.12.

The spherical brass electrodes were sand blasted to prevent any possible erosion following closure of the spark gap. The different components used in the spark gap switch are shown in figure 3.12, where B is an assembled electrode with its support
and C is the aluminium spark gap chamber that enables pressurization of the gap. This chamber was tested with pressurised air up to 65 psi, and a 40 psi limit was set for safe experimental practice with the chamber.

3.6 Electrical field analysis for PFL and spark-gap

ANSOFT® Maxwell-2D (SV) electrostatic software was used to analyse the electric field in the PFL and the spark-gap, due to presence of the triple point in the PFL shown at T1 and T2 in figure 3.13. A triple point is the junction of three materials having different electrical properties, and at this the location electric field is very much higher than if the same material was used throughout. The triple point is generally regarded as the location where flashover is initiated in high voltage insulation, and as the vulnerable point at which an electrical breakdown begins.

Figure 3.13: PFL arrangement with spark-gap, T1, T2 indicate triple points where three different materials meet.
3.6.1 Electrical field in PFL

Consideration of the electrical field distribution in the PFL shown in figure 3.14 and 3.15 confirms that the operation will be breakdown free, since the electric field strength is always much less than the breakdown strength of water. The simulations established that the electric field in the regions in the vicinity of the triple points was only 70 kV/cm, well below the break-down strength of water of 193 kV/cm.
3.6.2 Electrical field in spark-gap switch

The lines of constant electric field in the spark gap are shown in figure 3.16 when one electrode is at -300 kV and other is grounded, and demonstrate that the electric field strength is constant along the axis (the line joining the centre of the electrodes) of the spark gap. Hence Paschen curve for uniform fields can be used in predicting the behaviour of the gap.

![Figure 3.16: Electric field in spark gap region, when one electrode is at -300 kV and the other is grounded.](image)

3.7 Characterization of SF₆ spark-gap

Figure 3.17 shows the measured characteristics of the spark gap for (a) 6 mm electrode separation (d₁) and (b) 5 mm electrode separation (d₂), with the breakdown voltage presented as a function of the SF₆ pressure. Both curves show a linear dependency of the breakdown voltage on the pressure in the chamber. It is evident from the figures that for operation at 300 kV the 6 mm separation requires an excessive pressure (p₁) of more than 40 psi, whereas at 5 mm separation the pressure is within the safe working pressure (p₂) of the spark gap chamber. This may be due to
p_1d_1 and p_2d_2 falling in different location in the Paschen’s curve shown in figure 3.17 c.

Figure 3.17: Breakdown characteristics for electrode separation of (a) 6 mm and (b) 5 mm and their position (c) in Paschen’s curve
3.8 Testing of the PFL

The PFL functions as the secondary capacitance for the Tesla transformer. However, since the resistance of the water in the PFL acts as a shorting resistor to the capacitor, the modified circuit shown in figure 3.18 is more appropriate than the earlier circuit of figure 2.01.

The equations for the circuit of figure 3.18 are

$$ R_p i_p + \frac{1}{C_p} \int i_p \, dt + L_p \frac{dI_p}{dt} + M \frac{dI_s}{dt} = 0 $$

(3.41)

$$ R_s i_s + \frac{1}{C_s} \int (I_s - I_w) \, dt + L_s \frac{dI_s}{dt} + M \frac{dI_p}{dt} = 0 $$

(3.42)

$$ R_w i_w + \frac{1}{C_s} \int (I_s - I_w) \, dt = 0 $$

(3.43)

which can be solved to obtained the voltage and current in the primary and secondary circuits. They can however only be used up to closure of the spark gap switch, after which the voltage and current are no longer described by equations 3.41, 3.42 and 3.43, but by the transmission line characteristics of the secondary of the Tesla transformer.
The full Tesla assembly was tested by charging the primary capacitor to 24.5 kV and with a pressure in the spark-gap switch of 28 psi the parameters listed in table 3.01 were obtained.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_p$ (primary circuit inductance)</td>
<td>230 nH</td>
</tr>
<tr>
<td>$C_p$ (Primary circuit capacitance)</td>
<td>1830 nF</td>
</tr>
<tr>
<td>$R_p$ (Primary circuit resistance)</td>
<td>47 mΩ</td>
</tr>
<tr>
<td>$L_s$ (secondary circuit inductance)</td>
<td>62 µH</td>
</tr>
<tr>
<td>$R_s$ (resistance of secondary windings)</td>
<td>2 Ω</td>
</tr>
<tr>
<td>Dielectric constant of water ($k_w$)</td>
<td>78</td>
</tr>
<tr>
<td>Water capacitor $C_w$ (Secondary capacitance)</td>
<td>$6.058.10^{-9}$ F</td>
</tr>
<tr>
<td>Mutual inductance between primary and secondary windings</td>
<td>2.10 µH</td>
</tr>
<tr>
<td>Resistivity of water in PFL</td>
<td>10 MΩ-cm.</td>
</tr>
<tr>
<td>$R_w$</td>
<td>10.69 kΩ</td>
</tr>
<tr>
<td>Electrical impedance of PFL</td>
<td>3.9 Ω</td>
</tr>
<tr>
<td>PFL transit time</td>
<td>53 ns.</td>
</tr>
</tbody>
</table>

Table 3.01: Tesla parameters.

The voltage was measured using a calibrated capacitively coupled dv/dt sensor (discussed in detail in chapter 4). After closure of the spark-gap switch, the voltage oscillations begin with a period corresponding to twice the electrical length (prorogation of voltage wave in one direction) of the PFL.
Figure 3.19: Typical secondary voltage.

Figure 3.20: Typical secondary current.

Figure 3.21: Primary current in Tesla transformer with PFL.
The secondary voltage output evident in figure 3.19, is sufficiently close to -300 kV to generate 40 kA (see figure 3.20) in the secondary circuit, which is the current the PFL is required to feed to the POS (discussed in chapter 1). The curve shows no breakdown in water due to a pulse length around 2.4 µs whereas most of the available literature suggests that the pulse length should be confined to below 2 µs for safe operation. The primary current shown in figure 3.21 confirms the safe operation of primary capacitor bank.

Simulation of the primary current and secondary voltage was performed using equation 3.41, 3.42 and 3.43. The prediction for the primary current matches closely the measured current until closure of the spark gap, as shown in figure 3.21. The secondary voltage differs from the calculated value due to the anomalous behaviour of the water resistance, as seen in figure 3.19 where at the time of closure of the spark gap the measured voltage is 280 kV, whereas the estimated value is 220 kV, giving an error of 20%.

3.9 Conclusions

The experimental results in this chapter demonstrate that the PFL when connected to the Tesla transformer is capable of producing the required characteristics of current for the POS. Important conclusions drawn from the work on the PFL are

1. The resistivity of water in the PFL must be maintained by circulating it continuously through a de-ionizer. This is the probable reason for the water breakdown only occurring at well beyond the previously reported 2 µs pulse width.

2. The electrical field strength in spark gap is uniform to within the ± 10% of axial field of $6.10^7$ V/m, which justifies the application of Paschen law in predicting self breakdown voltage of sparkgaps.
3.10 Reference


4 Fast nanosecond current and voltage sensors

An increasing number of pulsed power devices delivering short duration high intensity pulses are now being used in quite different applications. For the reliable analysis of experimental events, the noise free and accurate measurement of electrical parameters such as voltage and current is vital. Since most pulsed power systems are fast and high power, the sensors must have a high bandwidth and be located close to the system itself. The current sensors used are mainly the B-dot probe and the Rogowsky coil, and for measuring voltage the commercially available sensor is the voltage divider.

There is a frequent demand for easily-fitted probes for the non-invasive measurement of both fast-rising voltages and currents being transmitted via transmission lines, which can be simple coaxial cables or water or oil filled pulsed forming networks (PFLs). The Rogowsky coil is a sensor frequently used to measure current, but it may sometimes be difficult to install in tight geometry without modifying the actual system. Since the Rogowsky coil works on the principle of the rate of change of the magnetic field, its output may sometimes be too high for diagnostic purposes. Miniaturization of the coil is a possible solution but current measurement in a transmission line or a vacuum chamber where the geometry is fixed is not possible; similarly a voltage divider cannot be installed in a line without altering the system geometry.

Using designs similar to those developed for use with high-power water lines, capacitively-coupled voltage sensors and inductively-coupled current transducers based on an inductive groove device added on the transmission line (inductive groove) were developed. Experimental results have confirmed that these both have
rise times of about 3 ns and are suitable for use with both high and low voltage 50 Ω voltage cable as well as for transmission lines.

4.1 Current sensor

Two types of current sensor are discussed below, one, based on the inductive groove concept and other on a Rogowsky coil.

4.1.1 Inductive groove current sensor [4.1] [4.2] [4.3]

As proposal by Ekdahl [4.1] an inductive groove is incorporated into a metallic flange at the inner face of the outer conductor of a line as shown in figure 4.1. The slight increase thereby introduced into the inductance on the path of the return current flowing in the metal skin layer (of the order of 100 pH) does not appreciably modify the overall current characteristics of rise time and peak value, but it does induces an extra voltage that can be easily monitored.

As shown in figure 4.1 the basic geometry for an inductive groove is an annular channel machined in a flange added on the outer conductor of line. A vacuum tight SMA connector as shown in figure 4.1 is used for transmitting the induced voltage from groove to oscilloscope, with the main body of the connector attached to the outer conductor of the transmission line, and the central conductor connected to the inner edge of the groove. The voltage measuring method is similar to the earlier axial measurement proposal of Ekdahl [4.1] [4.2].
4.1.2 Voltage induced by the inductive groove

The groove is machined between radii $R_i$ and $R_e$ and faces the coaxial central conductor through a spacing of width $e$ (as shown in figure 4.1), allowing the penetration of any very high frequency magnetic fields generated by the passage of a current $I$ through the central conductor. This gives a rough estimate of the upper bandwidth of the inductive groove. Typically

$e \approx 1 \text{ mm}$

and a rough estimate of cut off frequency ($f_{\text{cutoff}}$) for the maximum allowed wavelength $\lambda = e$ can be given by equation 4.01, where $c$ is the speed of light in vacuum.

$$f_{\text{cutoff}} = \frac{c}{e} = 3.10^{11} \text{Hz}$$  \hspace{1cm} (4.01)

If $L$ is inductance of groove, dI/dt is rate of change of current with time, the induced voltage ($V_{\text{groove}}$) is given by

$$V_{\text{groove}} = -L \frac{dI}{dt}$$  \hspace{1cm} (4.02)
where the current passing through the central conductor, is assumed to be parallel of the groove axis, to create a $B$ field at a radius $r < R$ and to return along the groove surface. The groove inductance $L$ can be expressed in terms of the surface integral of $Bds$ over the groove part of the contour along which the current is forced to flow as

$$V_{\text{groove}} = \frac{\mu_0}{2\pi} \left( \frac{dI}{dt} \right) \left( \frac{1}{r} \right) drdz = L \frac{dI}{dt}$$  \hspace{1cm} (4.03)$$

For a groove machined between radii $R_i$ and $R_e$ the expression for the induced voltage depends on the location of the groove. In the groove where $dz = h$

$$V_{\text{groove}} = \frac{\mu_0}{2\pi} \frac{dI}{dt} h \ln \left( \frac{R_e}{R_i} \right)$$ \hspace{1cm} (4.03)

which however neglects the ohmic voltage drop along the groove. At high frequencies the main source of resistance comes from the skin depth resistance, which can be expressed as

$$\delta_{\text{skin}} = \frac{1}{\sqrt{\pi f \mu \sigma}}$$ \hspace{1cm} (4.04)

where $\mu$ is the permeability of the sensor material, $\sigma$ is the bulk conductivity and $f$ is the frequency of current pulse.

The PFL used in the present work is made of stainless steel, so that the skin depth of stainless steel will be used while calculating the skin resistance of the groove. The skin depths at different frequencies for stainless steel are given in table 4.1.

<table>
<thead>
<tr>
<th>$f$ (MHz)</th>
<th>100</th>
<th>1000</th>
<th>2000</th>
<th>6000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta_{\text{skin}}$ (µm)</td>
<td>42.1</td>
<td>13.3</td>
<td>9.4</td>
<td>5.4</td>
</tr>
</tbody>
</table>

Table 4.1: Data for stainless steel.

Equation 4.02 can be corrected for the voltage drop in the skin layer resistance $R_{sk}(f)$ around the annular groove, by

$$V = L \left( \frac{dI}{dt} \right) + IR_{sk}(f)$$  \hspace{1cm} (4.05)$$
where the skin resistance can be obtained simply by assuming a uniform current flow, which for the arrangement shown in figure 4.1 can be obtained as [4.3]

\[
R_{sk}(f) = \frac{\mu_0}{\pi\sigma} \left( \frac{2h + R_c - R_i}{(R_i + R_c)} \right) \tag{4.06}
\]

4.1.3 Cut-off frequencies of the sensor

Integration of equation 4.05 with respect to time and assuming \( R_{sk} = 0 \) gives

\[
V = j\omega LI \left( 1 - \frac{j}{\omega\tau} \right) \tag{4.07}
\]

where \( \tau = \frac{L}{R_{sk}} \) and \( f = \frac{\omega}{2\pi} \) \tag{4.08}

The low frequency limit corresponds to the condition \( \omega\tau = 1 \), or

\[
f_{lower} = \frac{R_{sk}}{2\pi L} \tag{4.09}
\]

while the high frequency limit is determined mainly by the resistance at the output of sensor to the oscilloscope, which for this situation is \( R_c \approx 50 \Omega \); thus

\[
f_{higher} = \frac{R_c}{2\pi L} \tag{4.10}
\]

This limitation is rather general. For practical purposes the geometry of the groove and dielectric (of dielectric constant \( \varepsilon_r \)) used in the transmission line play important roles, and impose an additional limitation on the high frequency limit arising from the stray capacitance \( (C_{\text{groove}}) \) and self inductance \( (L_{\text{groove}}) \) due to the grooved thickness of \( e \) and width \( s \).

Then \([4.3] [4.4]\)

\[
C_{\text{groove}} = \frac{\pi\varepsilon_r s(R_i + R)}{e} \tag{4.11}
\]

\[
L_{\text{groove}} = \left( \frac{\mu_0}{2\pi} \right) \left[ h \ln \left( \frac{R_c}{R_i} \right) + h \ln \left( \frac{R_i}{R} \right) \right]
\]

The modified higher frequency limit then becomes [4.3]
The modified higher frequency limit in equation 4.12 must be followed for a transmission line, when \( C_{\text{groove}} \) is high such as in a water filled line. Higher value of \( C_{\text{groove}} \) results in a reduction in the upper frequency limit, and making this sensor unsuitable for the measurement of fast rising current pulses.

### 4.1.4 Inductive current sensor for coaxial cable

The sensor developed for use with RG-218 coaxial cable can be straightforwardly attached without altering the central conductor and cable insulation and with only a slight modification to the ground conductor being required as shown in figure 4.2. The main parameters for the sensor are given in table 4.2 and 4.3 and figure 4.2 shows diagrammatically the prototype sensor. The total material cost for an inductive groove current sensor on the metallic flange as shown in figure 4.2 is about £25, and about 4 technician hours are required for its manufacture. Figure 4.3 (b) shows a typical result from a calibration shot of the sensor and demonstrates that the prototype unit has a rise time of less than 3 ns.

\[
 f_{\text{higher}} = \frac{1}{2\pi \sqrt{L_{\text{groove}} C_{\text{groove}}}} \tag{4.12}
\]

Figure 4.2: Current sensor (inductive groove) mounted on RG-218 coaxial cable.
Figure 4.3: Photograph of twin current sensor: (a) open and (b) closed.

<table>
<thead>
<tr>
<th>Geometrical Parameters</th>
<th>(mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
<td>9</td>
</tr>
<tr>
<td>$R_1$</td>
<td>11</td>
</tr>
<tr>
<td>$R_2$</td>
<td>16</td>
</tr>
<tr>
<td>$h$</td>
<td>4</td>
</tr>
<tr>
<td>$e$</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 4.2: Geometrical parameters for current sensor mounted on RG-218 cable.

<table>
<thead>
<tr>
<th>Electrical parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inductance</td>
<td>19.29 nH</td>
</tr>
<tr>
<td>Skin depth</td>
<td>2.8 µm @850MHz</td>
</tr>
<tr>
<td>Skin resistance</td>
<td>1.3 mΩ @850MHz</td>
</tr>
<tr>
<td>Low frequency cut-off</td>
<td>1 MHz</td>
</tr>
<tr>
<td>High frequency cut-off</td>
<td>7 GHz</td>
</tr>
</tbody>
</table>

Table 4.3 a: Electrical parameters for the current sensor mounted on a RG-218 cable.

This current sensor was calibrated against the current generated by a fast pulse generator. This generator (with an internal capacitance of 0.1 µF) was charged to 30 kV and discharged into a 58 Ω resistive load to generate a fast current pulse of 600 A, with a rise time of 5 ns. Due to the fast rise time and low current of the pulse, a commercially available current sensor was not suitable. The predicted current shown in figure 4.4 b by a dotted line was used to calibrate the current sensor. Theoretically the calibration factor for this sensor is $1/L_{\text{groove}}$ according to equation 4.03.

Calculated and experimental values of the calibration factor are given in table 4.3 b.
<table>
<thead>
<tr>
<th>Inductance of groove ($L_{\text{groove}}$)</th>
<th>Calculated calibration factor ($1/L_{\text{groove}}$)</th>
<th>Experimental calibration factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>19.29 nH</td>
<td>5.183x10$^9$</td>
<td>5.3x10$^9$</td>
</tr>
</tbody>
</table>

Table 4.3 b: Calibration factor for the sensor mounted on RG-218 coaxial cable.

Figure 4.4: Current sensor calibration. (a) $\frac{dI}{dt}$ original signal(lower) and integrated current signal(upper). (b) Integrated signal (full line) compared with theoretical prediction (dotted line).
4.2 Rogowsky coil [4.5]

Self-integrating Rogowsky coils are important tools for measuring currents in pulsed high-current circuits. The Rogowsky coil is a current transformer of deceptively simple geometry, consisting of a toroidal coil with “n” turns (minor turns), linked by the magnetic flux created by the current I to be measured.

Rogowsky coils operate on a simple principle, where an ‘air-cored’ coil is placed around the conductor in a toroidal fashion and the magnetic field produced by the current induces a voltage in the coil. The voltage output is proportional to the rate of change of current, which when integrated, thus producing an output proportional to the current.

In order to design properly a nanosecond response time Rogowsky coil, calculations must take into account the fact that the coil behaves as a delay line. The coil will have a distributed capacitance if it is placed in a conducting shield, or if it is wound back on itself as shown in figure 4.5 to prevent it from picking up signals from currents that run parallel to the plane of the coil.

The advantages of using a Rogowsky coil to measure high frequency currents are:

- no-saturation, because it is air-cored
- good linearity, due to the absence of magnetic materials
- simple circuitry and low price
- it is non-intrusive, i.e. the coil does not load the circuit carrying the current to be measured under correct conditions of impedance matching.

The simplest Rogowsky coil design is the single-layer arrangement shown in figure 4.05 a. The current $I$ passing through the centre of the coil, creates a magnetic field in the $\theta$ direction. These magnetic lines pass through the winding of the coil and the time variation of the current $I$ causes a variation in the magnetic flux $\phi$ linking the
Fast nanosecond current and voltage sensor

coil, results in an induced current $I_c$ into the Rogowsky winding depends on the inductance $L$ and resistance $R$ of winding. If a resistance $R_{CVR}$ is attached as shown in figure 4.05, this can generate a voltage $I_c R_{CVR}$ for use in the self integration mode discussed latter. The resistance $R_{CVR}$ is termed the current viewing resistor.

Figure 4.5: (a) Winding style of Rogowsky, current I is passing through the windings (b) Rogowsky with CVR in shielded case.
Figure 4.5 b shows the assembly of a typical Rogowsky coil inside the shielded enclosure together with the current viewing resistor $R_{CVR}$.

Due to the incremental pitch advancement of the helical winding along the circumferential length, any changing magnetic flux perpendicular to the loop thus created induces an error voltage into the coil output. To compensate for this undesired effect, a one-turn loop (major turn) is placed inside the helical winding in the opposite direction to the pitch advancement (shown in figure 4.5 a). As this is electrically connected in series with the coil output, a compensation voltage is induced that is equal and of opposite polarity to the voltage produced by the advancement of the coil.

It is important to differentiate between a Rogowsky coil and the familiar current transformer used in many conventional power laboratories, which consists of a wire wrapped many times around an annular iron core. The iron core is used to increase the flux linkage, even at lower currents, but at higher frequencies it can produce unpredictable behaviour leading to a false measurement of the current. So although the Rogowsky coil may be unsuitable for low current operation due to a low flux linkage capability at high and fast current, overall it represents one of the best available current measuring instrument.

### 4.2.1 Theory of Rogowsky coil [4.5]

The following terms are introduced for the typical Rogowsky coil shown in figure 4.05

- $c$ is the speed of light in vacuum
- $a$ is the minor mean radius of the coil (to centre of winding) (see figure 4.5 a)
- $r$ is the major mean radius of the coil (see figure 4.5 a)
- $d$ is the wire diameter used in the coil
\( p \) is the pitch of the coil turns
\( n \) is the number of turns in the coil
\( \rho \) is the resistivity of the conductor (wire)
\( l \) is the length of wire used in the coil winding
\( A \) is the effective cross section of the conductor where current is flowing due to skin depth
\( I_c \) is the current flowing in the Rogowsky coil
\( L \) is the self inductance of the Rogowsky coil
\( R_{CVR} \) is the resistance of the current viewing resistor

\( R \) is the total resistance of the wire used in the Rogowsky coil, and is a combination of the current viewing resistance and the skin resistance of the coil (\( R_{\text{skin}} \)), or

\[
R = R_{CVR} + R_{\text{skin}} \tag{4.13}
\]

where \( R_{\text{skin}} \) is given by

\[
R_{\text{skin}} = \rho \frac{l}{A} \tag{4.14}
\]

The length of the wire used in the coil windings is given by

\[
l = n\sqrt{p^2 + (2\pi a)^2} \tag{4.15}
\]

and the effective cross section of the wire by

\[
A = \pi d \left( \frac{\rho}{\pi f \mu_0} \right) \tag{4.16}
\]

where \( f \) is given by \([4.6]\)
where \( t_r \) is the rise time of the current pulse \( I \).

Therefore the skin effect resistance is

\[
R_{\text{skin}} = \frac{n}{\pi d} \sqrt{\rho \pi f \mu_0 (p^2 + (2\pi a)^2)}
\]  

(4.18)

and if \( \phi \) is the flux induced by the current \( I \) in the cross section of the Rogowsky coil, the circuit equation for the Rogowsky coil can be written

\[
I_c R = \frac{n d \phi}{dt} - L \frac{d I_c}{dt}
\]

(4.19)

or

\[
\frac{1}{L} \frac{n d \phi}{dt} = \frac{d I_c}{dt} + \frac{R}{L} I_c
\]

(4.20)

Consider now two particular cases:

**Case – (1)**

If \( \frac{R}{L} I_c \ll \frac{d I_c}{dt} \)

(4.21)

then

\[
\frac{1}{L} \frac{n d \phi}{dt} = \frac{d I_c}{dt}
\]

(4.22)

or, since the system is linear

\[
\frac{n \phi}{L} = I_c
\]

(4.23)
Case – (2)

If \( \frac{R}{L} I_c \gg \frac{dI_c}{dt} \) \hspace{1cm} (4.24)

\[ \frac{1}{L} \frac{nd\phi}{dt} = \frac{R}{L} I_c \] \hspace{1cm} (4.25)

or

\[ \frac{1}{R} \frac{nd\phi}{dt} = I_c \] \hspace{1cm} (4.26)

It is clear that in case (1), the current produced in the Rogowsky coil is proportional to the induced magnetic flux due to the current \( I \). and equation 4.23 is therefore the criterion for the Rogowsky coil to work in a self integrating mode.

In case (2) the current produced in the Rogowsky coil is proportional to the rate of change of the magnetic flux. The condition of equation 4.26 is therefore the criterion for a differentiating mode.

The self-integrating Rogowsky coil has several advantages over its differentiating counterpart, including the output voltage being essentially independent of frequency, and hence of the rise-time of the current being measured. It is also less sensitive to electron impact or photon induced currents and there is less cable attenuation of the signals from the coil since the output is proportional to the current rather than to its derivative and is much lower than that from a differentiating coil. A differentiating coil can generate dangerously high voltages and currents if \( \frac{dl}{dt} \) is high.

The self-integrating mode was selected for use in this project, with the voltage induced in the integrated Rogowsky coil \( V_{rogo} \) being given by

\[ V_{rogo} = I_c R_{CVR} \] \hspace{1cm} (4.27)
4.2.2 Self inductance of Rogowsky coil

The self inductance of the Rogowsky coil plays an important role in determining the self integration properties, as is evident from equation 4.20. When calculating the self inductance of the coil, the minor turns are taken into account with the major turn shorted to the housing assembly for grounding purposes.

To calculate the inductance $L$ of the minor windings, calculations are first performed for a single layer coil based on formulae for a cylindrical sheet (that is, a winding where the current flows around the axis of a cylinder in a layer of infinitesimal radial thickness on the surface of cylinder). The inductance of the winding is clearly different from that of the cylindrical current sheet, and the necessary correction is introduced by the formulae given by Grover for a solenoid [4.7] as:

$$L = L' - \Delta L$$  \hspace{1cm} (4.28)

where $L'$, the inductance of the cylindrical current sheet having $n$ number of turns in the coil of length $b$ and radius $a$, is given by [4.7]

$$L' = \frac{1}{2} \mu \pi n^2 \left(\frac{2a}{b}\right)$$  \hspace{1cm} (4.29)

and the correction factor $\Delta L$ due to the insulating space between the two adjacent windings is given by

$$\Delta L = \mu an(G + H)$$  \hspace{1cm} (4.30)

where $G$ is the winding space correction due to Rosa [4.5].

$$G = \frac{5}{4} \ln \frac{2a}{d}$$  \hspace{1cm} (4.31)

and
H is a tabulated function which can be approximated by [4.5]

\[ H = \sum_{i=0}^{3} a_i (\ln n)^i \]  

(4.32)

where the constants \( a_i \) are

\[ a_1 = 0.00070 \]
\[ a_2 = 0.17730 \]
\[ a_2 = -0.03220 \]
\[ a_3 = 0.00197 \]  

(4.33)

Combining these terms gives,

\[ L = \mu a \left[ \frac{\pi}{2} \left( \frac{2a}{b} \right) n + \ln \left( \frac{2p}{d} \right) - \frac{5}{4} - \sum_{i=0}^{3} a_i (\ln n)^{-i} \right] \]  

(4.34)

which can be re-written by substituting \( b \) by \( b = np \) and \( n \) by \( n = 2\pi r/p \) as

\[ L = \mu a \left[ \frac{2\pi \rho}{p} \right] \left[ \frac{\pi a}{p} + \ln \left( \frac{2p}{d} \right) - \frac{5}{4} - \sum_{i=0}^{3} a_i \ln \left( \frac{2\pi \rho}{p} \right)^{-i} \right] \]  

(4.35)

4.2.3 Decay constant of the coil

The ratio \( L/R \) is defined as the decay constant of the Rogowsky coil, since it determines the low frequency response and it must be sufficiently high to satisfy equation 4.20.

At high frequencies, penetration of the electromagnetic waves into the conductive (electrical) winding will be incomplete. As a result the winding exhibits a frequency dependent skin effect resistance which can be calculated using the assumption that the wire is a planar slab of width \( \pi d \) and length \( r\sqrt{\rho^2 + (2\pi r)^2} \), in which current is flowing only along a surface layer. The skin effect surface resistivity is then \( \sqrt{\pi \rho} \).
The resistance of the slab is obtained by multiplying the surface resistance by its length and dividing by its width.

The decay constant is

$$\tau = \frac{L}{R} \quad (4.36)$$

and on introducing equation 4.13, 4.17 and 4.34

$$\tau = \frac{\mu a N \left[ \pi \left( \frac{2a}{b} \right) n + \ln \left( \frac{2p}{d} \right) - \frac{5}{4} - \sum_{i=0}^{3} a_i (\ln n)^{-i} \right]}{R_{CVR} + \left( \frac{2r}{\rho} \right) \left[ \int \mu \rho \pi (p^2 + (2\pi)^2) \right]^{1/2}} \quad (4.37)$$

4.2.4 Flux linkage of current $I$ with Rogowsky coil

The Rogowsky coil output depends on the current $I_C$ which in turn depends on the flux linkage $\phi$ according to equation 4.26. The flux linkage with the Rogowsky coil is in general the flux linking the minor turn of the coil, or

$$\phi = \int B.dA \quad (4.38)$$

In the chosen geometry the total flux linkage is [4.5]

$$\phi = \frac{\mu N}{\pi} \int_{-a}^{a} \left( \frac{a^2 - x^2}{(r-x)} \right)^{1/2} dx \quad (4.39)$$

and on integrating from $\pi / 2$ to $3\pi / 2$

$$\phi = \mu N \left[ r - (r^2 - a^2)^{1/2} \right] \quad (4.40)$$

For simplicity $\phi$ can be written by expanding $(r^2 - a^2)^{1/2}$ as
\[ \phi = \mu ln \left[ \frac{1}{2} \frac{a^2}{r} - \frac{1}{8} \frac{a^4}{r^3} \right] \]  

(4.41 a)

which can be approximated by

\[ \phi = \frac{\mu na^2}{2r} \]  

(with less than a 1% error.)  

(4.41 b)

4.2.5 Parameters for the Rogowsky coil

The experimental setup on the POS (described later) requires two Rogowsky coils, one before the POS (Rogowsky 1) and another after the POS (Rogowsky 2) as shown in figure 4.6, to measure the current pulses before and after the switch. Both coils must be capable of measuring a maximum current of 70 kA with a rise time of 1 ns as required by the present POS (details of the required current were given in chapter 1). Since both coils have to be installed in the POS chamber, with a inner diameter of 65 mm, the chosen parameters were as displayed in table 4.03.

<table>
<thead>
<tr>
<th>Rogowsky coil parameters</th>
<th>Rogowsky 1</th>
<th>Rogowsky 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of turns, ( n ):</td>
<td>98</td>
<td>98</td>
</tr>
<tr>
<td>Minor mean radius of the coil, ( a )</td>
<td>3.010^{-3} m</td>
<td>3.910^{-3} m</td>
</tr>
<tr>
<td>Major mean radius of the coil, ( r )</td>
<td>5.010^{-2} m</td>
<td>4.810^{-2} m</td>
</tr>
<tr>
<td>Wire diameter (Including insulation), ( d )</td>
<td>1.010^{-3} m</td>
<td>1.210^{-3} m</td>
</tr>
<tr>
<td>Current viewing resistor (( R_{CVR} ))</td>
<td>0.15 ( \Omega )</td>
<td>0.20 ( \Omega )</td>
</tr>
<tr>
<td>Pitch, ( p )</td>
<td>3.077.10^{-3} m</td>
<td>3.077.10^{-3} m</td>
</tr>
<tr>
<td>Length of wire, ( l )</td>
<td>1.90 m</td>
<td>2.42 m</td>
</tr>
<tr>
<td>Cross sectional area of the wire</td>
<td>3.412.10^{-8} m²</td>
<td>1.461.10^{-7} m²</td>
</tr>
<tr>
<td>Total mean major circumference, ( b )</td>
<td>0.310 m</td>
<td>0.302 m</td>
</tr>
</tbody>
</table>

Table 4.3: Parameters for the two Rogowsky coils.
4.2.6 Importance of CVR in integrated Rogowsky coil

Since in theory the voltage output of an integrating Rogowsky coil is directly proportional to the current viewing resistor $R_{CVR}$, this resistance must have a very low inductance, otherwise the induced voltage in the coil will see an additional term $L_{CVR} \frac{dl}{dt}$, where $L_{CVR}$ is the inductance of the CVR.

The technique adopted to minimise the inductance of the CVR is shown in figure 4.7. Eight surface mountable 1.2 $\Omega$ resistors were located parallel to the central conductor of the BNC connector, which decreased the parasite inductance of the CVR from 9 nH (in the case of a single resistor) to 0.7 nH, thus maintaining the coaxial geometry of the BNC and also providing the requisite resistance with a very low inductance.

![Figure 4.6: Rogowsky coil installation, (a) top view (b) cable guns shown to visualise the position of the Rogowsky – 2.](image)

![Figure 4.7: Current viewing resistor, showing some of the surface mounted resistor.](image)
4.2.7 Sensitivity of Rogowski coil

The sensitivity \((k_{rog})\) of the Rogowsky coil is defined as the voltage generated by the Rogowsky coil when measuring 1 A of current. It can be calculated as

\[
k_{rog} = \frac{RCVR\phi}{LI}
\]  

(4.42 a)

Substituting for \(\phi\) from equation 4.41 b gives

\[
k_{rog} = \frac{\mu_0a^2RCVR}{2rL}
\]  

(4.43 b)

which is a constant value that depends only on the geometrical parameters of the Rogowsky coil.

4.2.8 Calibration of Rogowsky coils

Both Rogowsky coils were calibrated against a commercially available Pearson current transformer (model no. 410) arranged as shown in figure 4.8.

![Figure 4.8: Positioning of Rogowsky coils, (one before the plasma gun and one after the gun) and the Pearson current transformer.](image)
A capacitor bank of 0.22 µF charged to 3 kV was discharge through the POS chamber as shown in figure 4.8, with both the Rogowsky coils and the Pearson current transformer installed to measure the current pulse. The results obtained are shown in figure 4.9, together with those for the commercially available Pearson current transformer [4.8] the sensitivity of the two Rogowsky coils was calculated as recorded in table 4.5.

Table 4.5: Calibrated results for both Rogowsky coils.

<table>
<thead>
<tr>
<th>Current sensor</th>
<th>Sensitivity (Volts/ampere)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Theoretical</td>
</tr>
<tr>
<td>Commercial current transformer (Pearson)</td>
<td>0.05</td>
</tr>
<tr>
<td>Rogowsky-1</td>
<td>$1.283 \times 10^{-3}$</td>
</tr>
<tr>
<td>Rogowsky-2</td>
<td>$1.41 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

4.3 Voltage sensor [4.2]

The voltage sensors developed for both low and high-voltage coaxial cables and for the PFL waterline are capacitively-coupled devices, having the basic arrangement
shown in figure 4.10 (for a coaxial cable) and figure 4.11 (for a water PFL). The sensors have a coaxial structure, with a central electrode and an outer case in cylindrical geometry. The electrodes are separated by an insulator and the sensor is mounted on the coaxial cable or PFL as shown in figure 4.10 and 4.11. An equivalent electrical circuit for this sensor is presented in figure 4.12, where $C_1$ is the capacitance between the sensor and the high voltage point and $C_2$ is that between the electrodes in the sensor itself. $R_1$ is a resistor attached to the sensor (which is zero due to directly shorting of the sensor to the output terminal) and $R_2$ is the terminating resistance at the oscilloscope, which is 50 $\Omega$ in the present case.

Figure 4.10: Voltage sensor mounted on RG-218 coaxial cable.

Figure 4.11: Voltage sensor mounted on PFL.
Figure 4.12: Equivalent electrical circuit.

If \( v_1(t) \) is the input voltage and \( v_2(t) \) is the output voltage of the sensor, then it follows from figure 4.12 that

\[
C_1 \frac{d}{dt}(v_1 - v_a) = C_2 \frac{dv_a}{dt} + \frac{v_a - v_2}{R_1} \quad (4.43)
\]

\[
\frac{v_a - v_2}{R_1} = \frac{v_2}{R_2} \quad (4.44)
\]

\[
v_a = v_2 \frac{(R_1 + R_2)}{R_2} \quad (4.45)
\]

\[
C_j \frac{dv_j}{dt} = \frac{v_2}{R_2} + (C_j + C_2) \left( \frac{R_j + R_2}{R_2} \right) \frac{dv_j}{dt} \quad (4.46)
\]

If

\[
R_1 + R_2 = R \quad (4.47)
\]

and

\[
C_1 + C_2 = C \quad (4.48)
\]

then equation 4.46 can be written as

\[
\frac{R_j C_j}{RC} \frac{dv_j}{dt} = \frac{dv_2}{dt} + \frac{v_2}{RC} \quad (4.49)
\]
If $RC$ is very small, the $dv_2/dt$ term in equation 4.48 can be neglected, when the output voltage $v_2$ becomes proportional to $dv_1/dt$, which means the sensor operates in the so-called ‘V-dot’ mode

4.3.1 Criterion for the voltage sensor

To explain in more detail the criterion for a voltage sensor to work satisfactorily, figure 4.13 is an enlargement of figure 4.11 to provide a clearer view of the different parts of the sensor. The conditions that must be met are then

![Diagram of the voltage sensor](image)

**Figure 4.13: Voltage sensor installed on a transmission line.**

i. The spatial variation of the field must be such that all information concerning voltage rise time, etc reaches the sensor without distortion, i.e.

$$d \ll \frac{\tau_{\text{rise}}}{\sqrt{k_m}}$$

where $\tau_{\text{rise}}$ is the rise time of the high voltage pulse, $k_m$ is the dielectric constant of medium and $d$ is the distance between high voltage surface and the voltage probe.

ii. The conductivity of the dielectric medium can be represented as a resistance shunting the coupling capacitance $C_1$ and therefore the probe gives an accurate measurement only for a time less than the $RC$ product for the
medium, where $R$ is the effective resistance offered by the medium from sensor to high voltage point and $C = C_1$. This time alternatively can be written as $\tau_{medium} = k_m \varepsilon_0 \sigma$, where $k_m$ is the dielectric constant and $\sigma$ is the conductivity of the dielectric material.

$$iii. \quad (R_1 + R_2)(C_1 + C_2) \ll \tau_{rise}$$

**4.3.2 Parameters for voltage sensor**

The voltage sensors designed fulfil all the above conditions. The main parameters for sensors developed for both RG-218 type high-voltage coaxial cables and for low-voltage RG-58 type coaxial cables are given in Table 4.8.

<table>
<thead>
<tr>
<th>Electrical parameter</th>
<th>For RG-218 cable High voltage</th>
<th>For-RG-58 cable Low voltage</th>
<th>For water PFL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$R_2$ (Ω)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$C_1$ (pF)</td>
<td>10.2</td>
<td>12.3</td>
<td>0.908*</td>
</tr>
<tr>
<td>$C_2$ (pF)</td>
<td>2.8</td>
<td>3.2</td>
<td>10.4</td>
</tr>
</tbody>
</table>

(* it is kept low so that the output voltage from the sensor is within the range of measurement, since the PFL is charged in the range -300kV to -400 kV.)

Table 4.8: Electrical parameters of voltage sensors.
Figure 4.15: Voltage sensors on water PFL.

Figure 4.14 shows both the high-voltage sensor for RG-216 coaxial cable and the low-voltage sensors for RG-58 coaxial cable and figure 4.15 shows a voltage sensor mounted on the PFL. It is important to note that mounting the sensors is straightforward and rapid and does not require any alteration to the coaxial geometry, i.e., no significant inductance is introduced in the circuit and no additional ground connection is required. There is no increase in the risk of high-voltage breakdown either, as the original insulation is maintained. The total cost of materials for manufacturing one sensor is £20 and the required time is 4 technician hours.

### 4.3.3 Calibration of voltage sensor

The capacitively coupled voltage sensor was calibrated using the commercially available voltage dividers listed in table 4.9.

<table>
<thead>
<tr>
<th>Voltage divider</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHV 621, PMK Germany [4.9]</td>
<td>100X, Max: 2 kV(DC), Bandwidth:400 MHz</td>
</tr>
<tr>
<td>PVM-6, North Star Research [4.10]</td>
<td>2000X, Max: 100 kV(pulse), Bandwidth:90 MHz</td>
</tr>
<tr>
<td>P6015A, Tektronics [4.11]</td>
<td>1000X, Max: 40 kV(pulse), Bandwidth:100 MHz</td>
</tr>
</tbody>
</table>

Table 4.9: Electrical parameters of voltage dividers used in calibrating the voltage sensor.
Table 4.10: Fast rise time different pulsed generators.

The two different pulse generators used in calibrating the voltage probes are listed in table 4.10. These generators produce a defined voltage pulse, which was measured by both a standard voltage divider from Table 4.9 and a voltage sensor developed in the project. The outputs from the two sensors are compared to calibrate the sensors developed.

Table 4.10

<table>
<thead>
<tr>
<th>Pulse generator</th>
<th>Properties of generated pulse</th>
</tr>
</thead>
</table>
| TG-70 [4.12] (L-3 Communications) | Maximum charge voltage: 70 kV  
Output voltage into 50 Ω: 70 kV  
Output pulse polarity: Positive  
Output pulse rise time into one 50 Ω cable: $\leq$ 5 ns (10%-90%)  
Timing delay: $\sim$ 400 ns  
Timing jitter: $\leq$ 5 ns rms |
| DG-535 [4.13] (Stanford research system Inc) | Output: TTL or 32 V  
Load: 50 Ω or high impedance  
Rise time: 2 to 3 ns |

Figure 4.16: Signal from pulse generator DG-535 compared with the output from a voltage sensor mounted on a RG-218 cable.
Fast nanosecond current and voltage sensor

Figure 4.17: Output signal from a sensor mounted on RG-218 cable compared with the corresponding signal from a commercial voltage sensor PVM6.

Figure 4.18: Output signal from a sensor mounted on the PFL compared with the corresponding signal from a commercial voltage sensor P6015A.

The sensor made for the RG-218 cable was calibrated using a 3 ns rise time pulse from a standard source (DG-535) for low voltage operation with the results obtained shown in figure 4.16. The result in figure 4.16 shows that the rise time for this sensor is no more than 3 ns. The same sensor was tested for high voltage operation using a
TG-70 pulse generator as a input source. The integrated output from the sensor is compared in figure 4.17 with that of a standard voltage sensor PVM6.

The sensor installed on the PFL was compared with standard Tektronics voltage divider for low voltage (30 kV). Results of this comparison are shown in figure 4.18, which gives the calibration factor for voltage sensor for further use. This sensor was tested upto 400 kV without any problem.

4.4 Conclusion

The capacitive voltage sensor is extremely suitable for use with the waterline PFL, where other methods of measuring voltage are dangerous. The integrated Rogowsky coil provides the best method of measuring the current in fixed geometries, as it has very high bandwidth and its output voltage can be controlled by varying various parameters such as number of turns and the resistance of current viewing resistor.
4.5 References


5  Plasma sources for POS

5.1  Introduction

Plasma plays an important role in an opening switch such as the POS built for this project. It has been shown, both theoretically and experimentally, that the plasma must have sufficient density, velocity, mass, charge and spatial distribution to provide a short circuit during the conduction phase and to achieve the large bipolar threshold current needed before a POS can begin to open [5.1]. Additionally, the erosion and enhanced phases of the opening processes are also dependent on the plasma parameters, and the plasma source is thus a key component of any POS that needs optimisation to achieve its maximum performance. In the following sections the generation of plasma, various different types of plasma source and their diagnostics are all discussed.

5.2  Plasma generation

Plasma is generated in the gaseous state, when inelastic collisions occur between atoms or molecules and electrons (electron impact ionisations) or photons (photon impact ionisation) with sufficient energy to eject electrons from a neutral atom, leaving behind a positive ion in a process called ionisation. There are several methods that can supply this energy, such as heating, compression, radiation by electric or magnetic fields, and bombardment by high-energy particles, as illustrated figure 5.1. In the case of a solid, energy must first be supplied to vaporise and to form the gas that is needed.
Figure 5.1: Different ways of plasma generation.

Thermal energy is normally obtained from flames by the exothermic chemical reactions of molecules involved in the process. Compressing a gas has the capability of heating it, whilst also increasing the probability of collisions by bringing the particles closer together. Another way of enhancing the probability is to inject beams of energetic particles into the gas, forcing them to collide with the gas particles and transferring kinetic energy to them. Neutral particles are often used for this purposes in fusion applications, because they are unperturbed by the electric and magnetic fields that hold the plasma in position. Electric and magnetic fields can however also be used to heat and accelerate electrons that then collide with gas particles, transferring their kinetic energy to them. In the following sections a number of plasma
sources are described that use these principles of energy transfer. There are various types of plasma sources based on different physical principle such as:

1. Discharge plasma sources
2. Microwave discharges
3. Radio frequency discharges
4. Dielectric barrier discharges
5. Beam plasma sources
6. PIG ion sources
7. Electron cyclotron Resonance ion sources
8. MHD sources.

Most of the methods of plasma generations other than the discharge techniques are costly and complicated and require complex and extensive instrumentation. It is unsurprising therefore that the discharge plasma source is one of the frequently used method in POS application.

5.2.1 Principle of plasma generation [5.2]

The Bohr model of an atom describes a single electron of mass $m_e$ and charge $-e$ orbiting a positive nucleus with a charge $ze$ where $z$ is the atomic number.

The electrostatic force between the positive nucleus and a negative electron is opposed the effect of the centripetal acceleration holding the atom together, such that

$$\frac{m_e v^2}{r} = \frac{ze^2}{4\pi\epsilon_0 r^2}$$  \hspace{1cm} (5.01)

and this force must be overcome if electrons are to be freed from the atom. Except for the simplest of atoms, the hydrogen atom, electrons are actually found in different orbits around the nucleus, with each orbit having a different energy level. If an
electron is to move between two such levels, energy must be either supplied to the atom or emitted in the form of light or heat as shown in figure 5.2.

![Energy level diagram](image)

**Figure 5.2: Energy level diagram.**

In order to ionise an atom, the impacting electrons or photons need to have adequate energy to promote the orbiting electrons from the outermost energy level to infinity.

This is the first ionisation energy $E_1$ given by

$$E_1 = e \phi_1$$

where $\phi_1$ is the first ionisation potential. The second ionisation energy removes the next electron that is normally in the adjacent orbit closer to the nucleus, and consequently needs more energy for it to be released from the atom, and so on.

### 5.2.2 Discharge plasma sources

One of the more common methods of generating and even maintaining low temperature plasma is by applying an electric field to a neutral gas. Any gas, even the vapour emitted from the surface of a material, contains a number of free electrons and ions as a result of natural interactions with light rays and background radiation. These free charge carriers are accelerated by the electric field to velocities that are sufficient to create new charged particles when they collide with neutral gas particles.
Secondary electrons and ions arising from this process lead to further collisions, resulting in an avalanche effect that produces a dense and sustainable plasma. The electric field can be dc, ac or pulsed.

5.2.3 DC discharges [5.3]

Non thermal plasma is created inside a closed vessel by application of a dc discharge across the gas between two internal electrodes. Depending on the applied voltage and the current a variety of plasmas can be produced, from a range of discharges such as a Townsend or dark self-sustained discharge through to an arc formation as seen in figure 5.3 below.

![Figure 5.3: D.C discharge of gas.](image)

Three general regions are identified on figure 5.3, as the dark discharge region, the glow discharge, and arc discharge regions.

5.2.3.1 Dark Discharge

The region between A and E on the voltage-current characteristic is termed a dark discharge because, except for corona discharges and the breakdown itself, the discharge remains invisible to the eye.
A – B During the background ionization stage of the process the electric field applied along the axis of the discharge tube sweeps out the ions and electrons created by ionization from background radiation. Background radiation from cosmic rays, radioactive minerals, or other sources, produces a constant and measurable degree of ionization in air at atmospheric pressure. The ions and electrons migrate to the electrodes in the applied electric field producing a weak electric current. Increasing the voltage sweeps out an increasing fraction of these ions and electrons.

B – C If the voltage between the electrodes is increased sufficiently, all the available electrons and ions are eventually swept away, and the current saturates. In the saturation region, the current remain constant while the voltage is increased. This current depends linearly on the radiation source strength, a regime useful in some radiation counters.

C – E If the voltage across the discharge tube is increased beyond point C, the current will rise exponentially. The electric field is now sufficiently high for the electrons initially present in the gas to acquire enough energy before reaching the anode to ionize a neutral atom. As the electric field becomes even stronger, the secondary electron may also ionize another neutral atom leading to an avalanche of electron and ion production. The region of exponentially increasing current is called the Townsend discharge.

D – E Corona discharges occur in Townsend dark discharges in regions of high electric field near sharp points, edges, or wires in gases prior to electrical breakdown. If the coronal currents are sufficiently high, corona discharges can be technically “glow discharges”, visible to the eye. For low currents, the entire corona is dark, as appropriate for the dark discharges. Related phenomena include the silent electrical discharge, an inaudible form of filamentary discharge, and the brush discharge, a
luminous discharge in a non-uniform electric field where many corona discharges are active at the same time and form streamers through the gas.

Electrical breakdown occurs in the Townsend regime due to the addition of secondary electrons emitted from the cathode due to ion or photon impact. At the breakdown, or sparking potential ($V_B$) the current might increase by a factor of $10^4$ to $10^8$, and is usually limited only by the internal resistance of the power supply connected between the plates. If the internal resistance of the power supply is very high, the discharge tube cannot draw enough current to break down the gas, and the tube will remain in the corona regime with small corona points or brush discharges being evident on the electrodes. If the internal resistance of the power supply is relatively low, then the gas will break down at the voltage $V_B$, and move into the normal glow discharge regime.

The breakdown voltage for a particular gas and electrode material depends on the product of the pressure and the distance between the electrodes, $(pd)$, as expressed in Paschen’s law (1889).

5.2.3.2 Glow Discharge

The glow discharge regime owes its name to the fact that the plasma is luminous. The gas glows because the electron energy and density are sufficiently high to generate visible light by excitation collisions. Practical applications of a glow discharge include fluorescent lights.

After a discontinuous transition from E to F, the gas enters the normal glow region, in which the voltage is almost independent of the current over several orders of magnitude of the discharge current. The electrode current density is independent of the total current in this regime.
This means that the plasma is in contact with only a small part of the cathode surface at low currents. As the current is increased from F to G, the fraction of the cathode occupied by the plasma increases, until plasma covers the entire cathode surface at point G.

**G – H** In the *abnormal glow* regime above point G, when the voltage increases significantly with the increasing total current in order to force the cathode current density above its natural value and provide the desired current. Starting at point G and moving to the left, a form of hysteresis is observed in the voltage-current characteristic. The discharge maintains itself at considerably lower currents and current densities than at point F and only then makes a transition back to Townsend regime.

### 5.2.3.3 Arc discharges

**H – K** At point H, the electrodes become sufficiently hot for the cathode to emit electrons thermionically. If the DC power supply has a sufficiently low internal resistance, the discharge will undergo a glow-to-arc transition, H-I. The *arc regime*, from I to K is one where the discharge voltage decreases as the current increases, until large currents are achieved at point J, and after that the voltage increases only slowly as the current increases further.

### 5.2.4 Pulsed discharges

Pulsed discharges are used in plasma generation because of the advantages that offer over dc discharges:

1. the source can be operated at a higher power for the short duration of the pulse.
2. controllability of plasma parameters can be obtained by varying the voltage, current or duty cycle of the discharge.
3. a reduction in effects such as thin film deposition from variations in the neutral gas composition between the plasma boundary and centre.

5.3 Basic plasma properties [5.2]

5.3.1 Particle density

The constituents of a plasma are ions, electrons and usually un-ionised neutrals. The most commonly used plasma parameters are the plasma density and temperature. The plasma electron density is denoted by \( n_e \), the plasma ion density by \( n_i \) and the neutral density by \( n_n \). It is usual to express the density in units of particles per \( \text{cm}^3 \). If the ions are singly ionised, and since the plasma overall is charge neutral, the ion density is equal to the electron density, i.e. \( n_i = n_e \).

5.3.2 Fractional ionization

The fractional ionization or percentage ionization of the plasma is defined as the ratio of the ion density to that of the total density of ions and neutral particles.

\[
\text{Fractional ionization} = \frac{n_i}{n_i+n_e}.
\]

5.3.3 Particle energy and velocity

Plasma particle motion can be described by distribution functions. For the case when a plasma species is in thermal equilibrium, its distribution is Maxwellian [5.2] This is a function of velocity, speed and energy as shown in figure 5.4, and the various averages that can be obtain from these.

The velocity distribution function \( f(v) \) describes the number of particles in a given velocity interval. For a plasma in thermal equilibrium the Maxwellian distribution function is given by

\[
f(v) = 4\pi v^2 n \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} \exp \left( -\frac{mv^2}{2kT} \right)
\]

(5.03)

where \( m \) is the particle mass \( k \) is Boltzman’s constant and \( n \) is the particle density.
Figure 5.4: Maxwellian distribution of plasma particles.

The mean speed of a particle $\bar{v}$ in a plasma is given by [5.2]

$$\bar{v} = \sqrt{\frac{8kT}{\pi m}}$$  \hspace{1cm} (5.04)

and its equivalent rms particle speed ($v_{\text{rms}}$) by [5.2]

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$  \hspace{1cm} (5.05)

The most probable speed ($v_p$) is the speed most likely to be possessed by the particle and is given by [5.2]

$$v_p = \sqrt{\frac{2kT}{m}}$$  \hspace{1cm} (5.06)

### 5.3.4 Plasma sheath

The plasma particles (ions and electrons) are charged and therefore interact with one another at a distance via their electric and magnetic fields. When a plasma sheath forms around an electrode that is biased with a negative potential ions are attracted towards the cathode surface, forming a region of positive space charge that eventually prevents further ions from reaching the cathode. This region is termed the sheath, and
the electrons or ions must have sufficient energy to cross it before conduction can take place. In fact only ions either in the sheath or on its edge, are drawn towards the cathode, outside it the ions are shielded from the electric field and remain unaffected by the presence of the cathode. It is this effect that limits the bipolar conduction which is described in chapter 6. Almost all the voltage drop occurs across the sheath with the potential distribution being as shown in figure 5.5.

![Figure 5.5: Plasma sheath formation.](image)

### 5.3.5 The Debye length

The distance over which the sheath shields the electric fields is commonly known as the Debye length \( \lambda_D \), as shown in figure 5.5 and is given by [5.2]

\[
\lambda_D = \sqrt{\frac{e_0 kT}{e^2 n_e}}
\]  

### 5.3.6 Particle flow

Plasma particles flow like any normal gas, but because they are charged, a macroscopic current can arise. In plasma, unlike metal, it is not just electrons that give rise to current flow as the ions are also free to move. The resultant current density \( J \) is given by

\[
J = J_i - J_e = n_e e (v_i - v_e)
\]
where the drift velocities of the electrons and ions are $v_e$ and $v_i$ and it is assumed that the positive ions are singularly charged such that the ions and electrons concentrations are equal.

### 5.4 Plasma sources for the POS [5.4]

Various types of plasma sources have been used in POS experimentation. Depending upon the application and character of the POS a particular source will be selected. Most working POSs use either a flashboard or a cable plasma gun as the plasma source, both of which work on the principle of electrical discharge.

#### 5.4.1 Flashboard

The flashboard is a plasma injector based on a discharge over a dielectric surface. A flashboard, as shown in figure 5.6, has many multi-gap electrode systems located on the insulator surface, and flashover occurs along the series gap electrodes due to the geometry of the tangential electric field. The flashboard plasma has a high flow velocity (up to 10 cm/µs) and a high electron density ($10^{12}$–$10^{20}$/cm$^3$). The Hawk pulsed power generator at the Naval Research Laboratory (USA) employs 18 flashboards and has produced load powers up to 0.7 TW [5.5].

![Figure 5.6: Typical flashboard multi-gap structure.](image-url)
5.4.2 Plasma gun

Two types of plasma gun have been commonly used in the POS experiment. Most old experiments used the Mendel type of gun, but more recently the cable gun has been used. Although different designs of plasma gun exist, most of these work on the same principle of a surface discharge across a dielectric.

5.4.2.1 Mendel type plasma gun [5.5]

The Mendel gun was one of the earliest carbon plasma guns. It was a complex design, which was aimed at increasing the efficiency of plasma formation by using a conical coil for the outer conductor (anode) as shown in figure 5.7. Energy from a capacitor is translated into magnetic energy stored in the plasma gun inductance, which is initially small to allow the current to rise rapidly and ultimately to store a much increased amount of energy. The inductance of the plasma gun grows as the plasma expands outwards and the length of the coil in contact with the plasma increases.

![Figure 5.7: Mendel type plasma gun.](image)

The end of a hard insulator between the outer and inner conductors of the gun is coated with graphite based paint. Guns typically need two or three conditioning shots after each coating before the output reaches a reasonable level, but eventually the graphite paint becomes completely vapourised and the insulator is eroded. Erratic
operation ensues, with a high level of impurities appearing in the plasma. Frequent recoating is necessary if this situation is to avoided.

The benefits of the conical coil in the Mendel gun were found to be relatively small, and alternative and much simpler designs soon evolved. These also removed the need for the continual applications of graphite paint, by constructing the gun from the end of a section of coaxial cable.

5.4.2.2  **Cable plasma gun [5.6]**

A modern cable plasma gun [5.6] is also based on surface flashover and is frequently used as a plasma source for a POS because of its simplicity and high density plasma. A cable plasma gun has coaxial electrodes and an insulator separating them as shown in figure 5.8. When a high voltage is applied between the electrodes, a flashover occurs along the insulator surface and the plasma generated by Joule heating and surface ablation is pushed outside the cable plasma gun by electro-dynamic and hydrodynamic forces. The cable gun plasma has a typical flow velocity of several cm/µs and an electron density of $10^{12}$ – $10^{17}$ cm$^{-3}$. As an example, the GIT-4 pulsed power generator at the Institute of High Current Electronics (Russia) uses cable plasma guns located in two positions, which inject plasma into the plasma chamber in both radial and in axial directions, resulting in a load current rise rate of 10–12 kA/ns [5.7].

5.4.3  **Differences between cable gun and flashboard plasma [5.4]**

- The reproducibility of the cable gun was found to be better than 10% between shots and 15% between different guns.

- The cable gun has a low plasma flow velocity of around 3.3 cm/µs and an ion density around $10^{12}$-$10^{17}$ cm$^{-3}$. The flash board plasma has a high flow velocity 10 cm/µs and an ion density of $10^{15}$-$10^{16}$ cm$^{-3}$. 
Plasma sources for POS

- Cable guns are easy to make and mount.
- Cable guns produce cleaner plasma with \( z = 1 \) (initially) and can go to \( z = 2 \) for polyethylene based cable.
- The plasma parameters of a cable gun can be controlled eg ion velocity (by voltage), ion density (by charge and insulator length) and axial length of plasma (by a supplementary nozzle).

In view of these advantages, it was decided to use a plasma gun based on a cable for this project.

### 5.5 Model of a carbon plasma gun [5.8]

The zero-dimensional model of a single plasma gun by Rodriguez and Elizondo [5.8] that was adopted for the present work, has an arrangement similar to that shown in figure 5.8. The basis for their analysis of the model is the conservation of energy and momentum, with microscopic analysis of the plasma behaviour avoided by considering it to be a moving mass of charge. The following sections describe the parameters and the basic set of equations used in this model, whose geometry is shown in figure 5.8. Some of the variables defined in this section will be used in calculating the ion velocity and temperature of the plasma.

![Figure 5.8: Plasma gun diagram adopted for modelling.](image-url)
5.5.1 **Mass**

The rate of change (erosion) of mass $\dot{M}$ is assumed to be proportional to the current $I$ and given by

$$\dot{M} = KI$$

(5.09)

where $K$ is a constant of proportionality called the mass rate constant. Experimental data estimated it to be $0.4.10^{-6}\, \text{kgC}^{-1}$ for copper and $2.0.10^{-6}\, \text{kgC}^{-1}$ for graphite [5.8], so that a 440 nF capacitor charged to 30 kV would erode about 26 µg of C or nearly $10^{18}$ electron-ion pairs.

5.5.2 **Fraction ionised.**

Plasma consists of only a fraction $F_i$ of the total particles being ionised, with the remainder being neutral particles, such that

$$F_i = \exp\left(-\frac{\phi_i}{kT_n}\right)$$

(5.10)

where $\phi_i$ is the ionisation potential and $T_n$ is the initial temperature of the new material.

5.5.3 **Velocity**

The conservation of momentum can be used to determine the acceleration of the plasma mass $M$ whose velocity is $U$

$$M\dot{U} + \dot{M}U = k \left[ \frac{T}{m_i} + \frac{\mu_0}{2\pi} \ln \left( \frac{r_{gun_i}}{r_{gun}} \right) \right] U^2 - F_{ic} \left[ \frac{m_e}{e} \right] IU \frac{M}{M}$$

(5.11)

where the ionisation factor $F_{ic}$, assuming bipolar conduction, is

$$F_{ic} \geq \sqrt{\frac{m_e}{m_i}} = 0.7\% \text{ for C}$$

(5.12)

The first term on the left hand side of equation 5.11 is the net force acting on the plasma mass. The second term is the mass loading to take into account changes in the
plasma mass. The three terms on the right-hand side are respectively the hydrodynamic, magnetic and drag forces, assuming that the volume and length of the plasma are located at the centre of mass position (defined as position X in figure 5.8). The magnetic force is derived from the integral of $J \times B$ between the two electrodes, where the magnetic flux density $B$ at a radius $r$ between the two electrodes is

$$B = \frac{\mu_0 I}{2\pi r}$$

(5.13)

The drag force in equation 5.11 results from friction between the plasma and the gun walls. Other contributions such as the interaction between the plasma particles are assumed to be negligible, since almost all the plasma is moving in the same direction and collisions are infrequent.

### 5.5.4 Energy

The conservation of energy $W$ in this model is described mathematically by

$$-W = F_v \left[ \frac{m_i}{e} \right] I U^2 + \frac{\mu_0}{2\pi} \ln \left( \frac{r_{gun}}{r_{gun_i}} \right) r^2 U + \dot{M} \left( F_e \frac{e \phi_e}{m_i} + C_v T_s \right)$$

where $C_v$ is the specific heat capacity and $T_s$ is the sublimation temperature. The first term on the right hand side is the friction loss, the second is the work done by the magnetic propulsion and the third is the energy lost by the rate of mass increase. The energy used to ionise the new material is the most significant contribution to this final term.

### 5.5.5 Temperature

The plasma temperature is determined from the rate of change of thermal energy $\dot{W}_{th}$ by

$$\dot{W}_{th} = \frac{3}{2} \left( \frac{k}{m_i} \right) T_s M + \left( \frac{m_i}{e} \right) I U^2 - \left( \frac{k}{m_i} \right) T M \frac{U}{X} - \left( A \sigma_T \right) r^4$$

(5.15)
where the first term is the mass loading assuming that new ions are added at the sublimation temperature, the second is the wall friction loss and the third accounts for cooling as the plasma expands towards the end of the gun. The final term is the radiated heat from the black body whose area is \( \pi (r_{gun}^2 - r_{gun}^2) \), with the Stefan Boltzmann constant \( \sigma \) being \( 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-2} \). Solving equation 5.15 gives the temperature in terms of the thermal energy \( W_{th} \) as

\[
T = \frac{2}{3} \left( \frac{m_i}{e} \right) \frac{W_{th}}{M} \tag{5.16}
\]

Only the friction term in energy equation 5.15 heats the plasma to produce the thermal energy required to raise the temperature, equation 5.17 gives the initial temperature \( T_n \) of new material as

\[
T_n = \frac{2}{3} \frac{F_w}{k} \frac{m_i^2}{e} \frac{U^2}{K} \tag{5.17}
\]

### 5.5.6 Initial plasma velocity

If the thermal energy evolved by the friction loss is used to create new plasma, the friction and mass loading terms on the right hand side of equation 5.15 can be equated, giving an initial plasma velocity \( U_0 \) as

\[
U_0 = \sqrt{\frac{e \varphi_i}{m_i} \left( \frac{m_i}{e} + C_v T_s \right) K} \tag{5.18}
\]

Equation 5.18 gives a smaller speed than that actually measured, which is due to the acceleration produced by the neglected interaction between the magnetic field and the moving charged particles.

This model has provided extremely useful information on the processes involving in the cable plasma gun. It has demonstrated the exchange of energy between that stored
in the capacitor, the mechanical and thermal energy losses in the gun and the kinetic energy transferred to the ejected plasma.

5.5.7 Application of this model in present applications

In the present application the polyethylene insulation of the cable is used to generate the plasma for the POS. An estimate for the energy needed to ionise one unit of the chain is obtained from the ionisation energies of the atoms and the energy of covalent bonds of the polymer structure. This is compared to the energy provided from the capacitor, to estimate the quantity of material eroded in each shot. The following sections implement this model in calculating the initial velocity of the ions ejected from the cable gun and the density of the plasma.

5.5.7.1 Properties of polyethylene

Polyethylene is a polymer consisting of long chains of the monomer ethylene as shown in figure 5.9. A single mer (a mer is the smallest repetitive unit of a polymer) of polyethylene is shown in figure 5.9. Polyethylene is created through polymerization of ethane.

![Structure of polyethylene](image)

Figure 5.9: Structure of polyethylene.

Ethane gas has two carbon atoms in the chain and each of the two carbon atoms share two valence electrons with the other. If two molecules of ethane are brought together, one of the carbon bonds in each molecule can be broken and the two molecules can be joined with a carbon to carbon bond. After the two mers are joined, there are still two
free valence electrons at each end of the chain for joining to other mers. The process can continue linking more mers and polymers together until it is stopped by the addition of another chemical (a terminator), that fills the available bond at each end of the molecule. This is called a linear polymer and is the building block for thermoplastic polymers.

Ionization of polyethylene requires firstly a reversed of the polymerisation process. The total energy required for this process is given by [5.9]

\[
E(\text{CH}_2=\text{CH}_2) = 699 \text{ kJ/mole} = 7.27 \text{ ev} \quad (5.19)
\]

The remaining covalent bond energies needed to split the polymer into the individual atoms and the total energy to be supplied are given by [5.9] as

\[
E(\text{CH}_2=\text{CH}_2) = 699 \text{ kJ/mole} = 7.27 \text{ ev} \\
E(\text{CH-H}) = 448 \text{ kJ/mole} = 4.67 \text{ ev} \\
E(\text{C-H}) = 335 \text{ kJ/mole} = 3.49 \text{ ev}
\]

Total \(E = 7.27 + 2(4.67 + 3.49) = 23.59 \text{ ev} \quad (5.20)\)

Ionization of a molecule of polyethylene gives two carbon and four hydrogen ions. If the carbon ions are ionised to double positive carbon ions \(\text{C}^{++}\), the energy needed for ionisation is

\[
\phi_i(\text{H}) = 13.60 \text{ eV}, \text{ ionisation energy for H} \\
\phi_i(\text{C}) = 11.26 \text{ eV}, \text{ ionisation energy for C} \\
\phi_i(\text{C}+) = 24.38 \text{ eV}, \text{ ionisation energy for C}^+ 
\]

Total \(\phi_i(2\text{C2C}^+\text{4H}) = 125.68 \text{ eV} \quad (5.21)\)

So the total energy needed to vaporise and ionise a single unit of polyethylene is

\[
7.29 \text{ eV} + 23.59 \text{ eV} + 125.68 \text{ eV} = 156.54 \text{ eV} \text{ (or } 2.499 \times 10^{-17} \text{ J)}.
\]

This value be used to calculate maximum number of molecule which can be ionised by the available energy.
5.5.7.2 Calculation of different parameters for the plasma gun

The energy from a capacitor bank is used to create a surface discharge in the cable gun to produce plasma with details of the discharge circuit given later in 5.10.3.

The maximum number of molecule $N$ that can be ionised by the capacitive energy (Capacitance=1µF, charging voltage $V = 30 \text{kV}$) can be obtained from

\[ N = \frac{\text{Available energy}}{\text{energy required to ionise one unit of polyethylene}}, \]

\[ N = 4.207 \times 10^{18} \]

Plasma density = $N / \text{volume of plasma chamber (section 6.7)} = 1.0 \times 10^{13} \text{ cm}^{-3}$

Mass of total ionised polyethylene (assming that the plasma is totally ionised)

\[ M = N.A \] (where $A$ is Avogadro’s constant.)

Mass increase per unit charge ($K$) stored in capacitor, is

\[ K = \frac{4.207 \times 10^{18} \times 6.97 \times 10^{-26}}{1.1 \times 30 \times 10^{-3}} \]

\[ K = 4.596 \times 10^{-5} \text{ kg/C} \]

Using equation 5.18 the initial velocity ($U_0$) of carbon ions at sublimation temperature of polyethylene $473^\circ\text{K}$ is 1.862 cm/µs.

5.6 Criterion for selecting plasma gun parameters [5.7] [5.10]

Some of the criteria for a POS to work in a repetitive mode have been criterion given by Dolgachev [5.7].

1. \[ \frac{Q_{\text{conduction}}}{r_{\text{outer}}} \leq 0.5 \text{ C/m} \] where $Q_{\text{conduction}}$ is the charge transferred through the POS at the conduction phase and $r_{\text{outer}}$ is the radius of the outer electrode in the coaxial geometry.

2. Each gun must produce at least 5 mC to provide an adequate ion density.

3. The total number of guns must be sufficient to produce $Q_{\text{conduction}}$.  

5.7 Simulation for cable gun firing circuit

The firing circuit used for the plasma gun in its generation was selected on the basis of the criteria given in section 5.6.

For criterion 1

For the planned POS experiment (maximum conduction current 70 kA in 50 ns)

\[ 70 \text{ kA} \times 50 \text{ ns} / (32.5 \text{ mm}) = 0.108 \text{ C/m} \]

For criterion 2,

Total available charge on capacitor (for 1 \( \mu \text{F} \) capacitance charged to 30 kV)

\[ = 30 \text{ kV} \times 1 \mu \text{F} = 30 \text{ mC} \]

Only the first quarter of the current waveform plays a role in generating plasma, hence the resistance (\( R \)), inductance (\( L \)) and capacitance (\( C \)) of the firing circuit must be chosen such that for each gun receives at least 5 mC charge. Based on this criterion, the parameters selected for the firing circuit are given in table 5.1 a and corresponding generated plasma parameters calculated in section 5.5.7.2 are in given in table 5.1 b, where the inductance of firing circuit was adjusted to give desired charge value in its first quarter.

<table>
<thead>
<tr>
<th>Capacitance</th>
<th>Charging voltage</th>
<th>External inductance</th>
<th>Switch inductance</th>
<th>Inductance</th>
<th>External Resistance</th>
<th>Cap Resistance</th>
<th>Cable Gun resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ( \mu \text{F} )</td>
<td>30 kV</td>
<td>740 nH</td>
<td>60 nH</td>
<td>120 nH</td>
<td>0.524 ( \Omega )</td>
<td>0.1 ( \Omega )</td>
<td>0.025 ( \Omega ) for four cables in parallel</td>
</tr>
</tbody>
</table>

Table 5.1 a: Parameters for cable plasma gun firing circuit.

<table>
<thead>
<tr>
<th>No of cable guns</th>
<th>Maximum carbon plasma density (cm(^3))</th>
<th>Carbon ion velocity (cm/( \mu \text{s} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1x10(^{13})</td>
<td>1.862</td>
</tr>
</tbody>
</table>

Table 5.1 b: Parameters for the carbon plasma.
Figure 5.10 is the current pulse generated by the plasma gun, and it can be seen that experimental results coincide closely with simulated results calculated on the basis of the parameters given in Table 5.1.

Integration of the waveform plotted in Figure 5.10 from 0 to $T/4$ gives

$$\int_0^{T/4} Idt \approx 22 \text{ mC}$$

Since one gun should produce 5 mC as per criterion 2 [5.7] for an adequate density of plasma for the POS, the number of guns needed to produce a charge of 22 mC is about 4.

### 5.8 Plasma gun based POS Generator

The planned POSs are to run on 20 kA (hydrogen plasma) or/and 70 kA (carbon plasma), which are produced in a plasma gun operation at different time scales. Since most existing POS based generators work in the range of MAs, or certainly more than 100 kA and no POS working in low current has not been reported to date in any reputable report. The low current operation of POS requires low plasma density and
low ion density. To justify the selection of plasma density, number of guns and their charges, Details of some of the POS based generators are given in table 5.02.

<table>
<thead>
<tr>
<th>Machine name</th>
<th>Bank capacitance for each cable gun (µF)</th>
<th>Bank charging voltage in (kV)</th>
<th>Charge (mC)</th>
<th>Density (/cm³)</th>
<th>Ion velocity (cm/sec)</th>
<th>Guns (No.)</th>
<th>Important information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamble-I</td>
<td>0.6</td>
<td>25</td>
<td>15</td>
<td>1.5 \times 10^{13}</td>
<td>6 \times 10^6</td>
<td>3</td>
<td>Cylindrical separation from anode to inner cathode is 2.5cm. Plasma source to cathode is 10 cm. Vacuum is 5 \times 10^{-4} Torr</td>
</tr>
<tr>
<td>Japan-Syuji Miyamoto</td>
<td>0.3</td>
<td>35</td>
<td>10.5</td>
<td>1.2 \times 10^{13}</td>
<td></td>
<td>4</td>
<td>Density is at 10cm and after 4.5 µs after firing gun.</td>
</tr>
<tr>
<td>ASO-X</td>
<td>0.7 (for two cable gun)</td>
<td>25</td>
<td>8.75/gun</td>
<td>1.2 \times 10^{15}</td>
<td></td>
<td>8</td>
<td>Central electrode of plasma gun is made of copper diameter of 2.9 mm. Polyethylene is insulator. Brass nozzle diameter is 3 mm.</td>
</tr>
<tr>
<td>DECADE</td>
<td>0.6</td>
<td>25</td>
<td>15</td>
<td>1.2 \times 10^{13}</td>
<td>From 5cm of gun</td>
<td></td>
<td>1.1/4 inch semi rigid coaxial cable, Teflon as a insulator.</td>
</tr>
<tr>
<td>Gamble-I (Scaled down)</td>
<td>0.6</td>
<td>15</td>
<td>9</td>
<td>1.2 \times 10^{13}</td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>TIGR-2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>12</td>
<td>Anode cathode gap is around 12 cm.</td>
</tr>
<tr>
<td>HAWK</td>
<td>0.6</td>
<td>25</td>
<td>15</td>
<td>3 \times 10^{15}</td>
<td>3 \times 10^6</td>
<td>12</td>
<td>Cable gun substrate is Teflon. 5 cm cathode diameter. Cable gun is 9 cm from cathode. Plasma length 5 cm.</td>
</tr>
<tr>
<td>CNRS France</td>
<td>0.6</td>
<td>20</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td>Guns located 13 cm from cathode. Diameter of cathode is 10 cm. Outer diameter of anode is 23 cm, plasma length 12 cm.</td>
</tr>
<tr>
<td>Black-Jack</td>
<td>12</td>
<td>40</td>
<td>7.5</td>
<td></td>
<td></td>
<td>64</td>
<td>POS cathode diameter 5cm-10cm</td>
</tr>
</tbody>
</table>

Table 5.02: POS based pulsed power generator.
Data from the scaled down experiment on Gamble-I match closely with the present selection of plasma density, charge and number of guns given in section 5.7.

5.9 **Existing cable gun used elsewhere**

Different designs of cable gun used world wide are discussed in this section. A design based on ASO-X (section 5.9.3) is selected for use in the present work, in view of its simplicity and obvious suitability.

5.9.1 **Initial cable design**

Experiments at NRL [5.1] used between 12 and 36 cable guns to produce the high density plasma ($\sim 10^{17}$ cm$^{-3}$) needed for a long-conduction POS. These were manufactured from ¼ inch semi-rigid coaxial cable with a $60^0$ inverted cone used to direct the plasma towards the load as shown in figure 5.11. Experimentally this gun performed better than the Mendel gun, and was much simpler to make. Energy stored in a capacitor is discharged across the PTFE ($C_2F_4$) insulation between the inner conductor and the sheath. Plasma from the flashover expands outwards from the indented surface and is accelerated into the POS by magnetic fields self-generated by the current in the gun.

![Cross-sectional view of plasma source with conical tip](image)

Figure 5.11: Cross-sectional view of plasma source with conical tip.
Although this gun is simple and cheap, direction control is virtually impossible and, as the cable insulation erodes, the plasma produced is contaminated with both fluorine and chlorine ions.

### 5.9.2 Plasma gun at Ecole Polytechnique (Laboratoire de Physique et Technologie des Plasmas France)

This gun is also made of coaxial cable but fitted with a nozzle as shown in figure 5.12. Surface breakdown between the grounded nozzle and central conductor of the cable generates the plasma.

![Cable plasma gun](image)

**Figure 5.12: Cable plasma gun.**

### 5.9.3 ASO-X system, Japan

Figure 5.13 shows the cross section of the plasma gun connected to a POS chamber [5.11]. The central electrode is of copper wire 2.9 mm in diameter and is covered axially by a polyethylene insulator of outer diameter 9.7 mm. The brass nozzle has a central hole 3 mm in diameter and with a conical shape.
The design shown in figure 5.13 was adopted as the plasma gun for the current project, since it is easy to make, and the length of the insulator can be varied depending on the plasma density requirements, whereas the length of insulator used in the gun described in section 5.9.1 cannot be changed. The design used at the French Institute (section 5.9.2) has no additional nozzle to direct the plasma in the required fashion. Thus the AXO-X type cable plasma gun is most suited for this project.

5.10 Plasma diagnostics

Two types of diagnostics were to used to study the cable plasma. A Faraday cup was used to measure the velocity of the ions, their density and plasma spread and a fast optical camera to measure the velocity of plasma and its uniformity.

5.10.1 Faraday cup [5.12]

A Faraday Cup is a device for measuring the current in a beam of charged particles. In its simplest form it consists of a conducting metallic chamber or cup, which intercepts a particle beam. An electrical lead is attached which conducts the current to a measuring instrument. Detection can be as simple as an ammeter in the conducting lead to ground or a voltmeter or oscilloscope displaying the voltage developed across a resistor from the conducting lead to ground. A bias voltage applied either to the cup
itself or a repelling grid preceding the cup, or a magnetic field, is usually used to prevent secondary emission from distorting the reading.

When a beam or packet of ions hits the metal it gains a small net charge while the ions are neutralized. The metal can then be discharged to measure a small current equivalent to the number of impinging ions. Essentially the Faraday cup is part of a circuit where ions are the charge carriers in vacuum and the Faraday cup is the interface to the solid metal where electrons act as the charge carriers (as in most circuits). By measuring the electrical current (the number of electrons flowing through the circuit per second) in the metal part of the circuit the number of charges being carried by the ions in the vacuum part of the circuit can be determined. For a continuous beam of ions (each with a single charge)

\[
\frac{n_i}{t} = \frac{I}{e}
\]

(5.23)

where \(n_i\) is the number of ions observed in time \(t\), \(I\) is the measured current and \(e\) is the electron charge (about \(1.60 \times 10^{-19}\) C). Thus, a measured current of one nanoamp (\(10^{-9}\) A) corresponds to about 6 billion ions striking the Faraday cup each second.

Similarly, a Faraday cup can act as a collector for electrons in a vacuum (for instance from an electron beam). In this case electrons simply hit the metal plate/cup and a current is produced. Faraday cups are not as sensitive as electron multiplier detectors, but are highly regarded for accuracy because of the direct relation between the measured current and number of ions.
The plasma flux was characterized by the Faraday cup technique (FC) in a separately made single gun. A negative bias voltage was applied to prevent electrons in the plasma flow coming through the holes in the cathode strip of the installation as shown in figure 5.14.

Negative biasing was applied on the collector to repel electrons from the collector. The voltage was raised from a low voltage to that at which the Faraday output voltage becomes fixed despite any further increase in the biasing voltage. A still further increase in the biasing voltage will lead to a sharp increase in the Faraday output, this value of biasing voltage is defined as the dynamic breakdown voltage and is shown in figure 5.15. At this point the Faraday output becomes extremely high, shown in figure 5.15 as -210 V.

Prior knowledge of the dynamic breakdown voltage is necessary for reliable operation of any Faraday cup and the biasing voltage must be kept slightly (about 5 V) below the breakdown voltage. For the results shown in figure 5.15 the Faraday output increases with an increase of the biasing voltage, but this increase ceases at a biasing voltage of -134 V and remains so until -210 V, when the Faraday output

Figure 5.14: Electrical assembly of faraday cup.
abruptly becomes high. So a biasing voltage of -205 V was selected as the appropriate value.

![Figure 5.15: Saturation voltage for Faraday cup.](image)

For a surface hole area $S_{fc}$, the current $I_{fc}$, passing through the FC for a given charge density $\rho_{fc}$ and charge velocity $v_{ion}$ is

$$I_{fc} = \rho_{fc} v_{ion} S$$

(5.24)

The corresponding voltage $V_{fc}$ which appears across $R_{fc}$ which will be proportional to the resistance $R_{fc}$ defined by equation 5.25, or

$$I_{fc} = \frac{V_{fc}}{R_{fc}}$$

(5.25)

The ion density of the plasma is related to the charge density by

$$n_i = \frac{\rho_{fc}}{q_i}$$

(5.26)

where $q_i$ is the ionic charge. Thus

$$n_i = \frac{I_{fc}}{S_{fc} v_{ion} q_i}$$

(5.27)

For $q_i = ze$ where $z$ is the charge state of ionization, equation 5.27 can be rewritten
\[ n_i = \frac{I_{Fe}}{S_{Fe} v_{ion} z e} \]  

(5.28)

If two Faraday cups, separated by a distance \( L_{Fe} \), are used to monitor the ions and the appearance of signals from them is separated by a time \( \Delta t \), then

\[ v_{ion} = \frac{L_{Fe}}{\Delta t} \]  

(5.29)

so that the Faraday cup can be used for ion velocity measurement as well as for ion density measurement.

### 5.10.1.1 Faraday cup design

Two Faraday cups were used in this project. One cup (FC-1) had a collector 2 mm diameter and a hole of radius 1 mm and the other (FC-2) a collector diameter of 2 mm and a hole diameter also of 2 mm. Separation between the hole and collector is 10 mm as shown in figure 5.14. The collector for both the Faraday cups is a 2 mm diameter cylindrical rod, which is separated from the metal case by a ceramic tube as shown figure 5.16. The output is taken from the collector and the metal case as shown the figure 5.17. All the connecting wires are covered with a metallic shield and a plastic cover, to prevent any spurious electrical noise and also to stop ions from colliding with the wire carrying collector information.
Figure: 5.16: (a) Metal case for Faraday cup and collector (b) assembled Faraday cup.

Figure: 5.17: Different views of assembled Faraday cup with its connection and shielding.
5.10.2 Modelling of Faraday cup results

Faraday cup detectors are used extensively for the characterization of ion emission in pulsed laser deposition studies. The potential distortion of the cup signal due to space charge effects was recognized and characterized in 1970 by Green [5.13]. Due to the presence of a significant number of ions, a very large positive space charge accumulates inside the cup. If the space charge field is sufficiently strong, additional arriving ions will be repelled and the cup signal is said to be distorted by space charge effects. Green constructed an analytic model describing the behaviour of the ions inside the cup, and derived a condition to ensure that space charge distortion is avoided. His model assumes that the electron flux is stopped at the outer surface of cup, so that the only species present within the cup are ions. According to that model the ion signal used for the analytical calculations was a modified Gaussian function.

The cable plasma gun used in the present work produces mainly carbon and hydrogen ions. For modelling of the Faraday cup results it is assumed that these ions do not interact with each other.

The Gaussian function for hydrogen ions is \( fh(v_h) \) where \( v_h \) is the velocity of hydrogen ions, \( v_h \) is highest speed of hydrogen ions and \( \rho_h \) is the adjustment factor of the Gaussian distribution function for hydrogen.

\[
fh(v_h) = \frac{1}{\rho_h \sqrt{2\pi}} \exp \left( -\frac{(v_h - v_{h})^2}{2.\rho_h^2} \right) \tag{5.30}
\]

Similarly the Gaussian function for carbon ions is \( fc(v_c) \) where \( v_c \) is velocity of carbon ions, \( v_c \) is highest speed of carbon ions and \( \rho_c \) is the adjustment factor for Gaussian distribution function of carbon ions, where

\[
f_c(v_c) = \frac{1}{\rho_c \sqrt{2\pi}} \exp \left( -\frac{(v_c - v_{c})^2}{2.\rho_c^2} \right) \tag{5.31}
\]
Figure 5.18: Gaussian distribution for hydrogen ions.

Figure 5.19: Gaussian distribution for carbon ions.

Figure 5.20: Gaussian distribution for hydrogen and carbon ions with Faraday cup results at 19 cm from cable.
The Gaussian distributions for hydrogen and carbon ions are given in figure 5.18 and 5.19 respectively. The distribution functions obtained for these ions were combined to simulate the Faraday cup results. The Faraday cup results together with simulated results based on the Gaussian distribution for hydrogen and carbon ions are shown in figure 5.20. The distribution functions for plasma ions are useful in obtaining various plasma parameters such as plasma density and velocity of ion species, and this distribution function will be used to calculate the plasma density in section 5.10.3.4.5.

5.10.3 Experimental arrangements for characterising plasma using a Faraday cup

To characterise the plasma produced by the gun, only one cable from the capacitor bank was connected to the gun, with the gun located inside into a vacuum chamber evacuated to $10^{-4}$ mbar.

The arrangement shown in figure 5.21 includes cable plasma gun, firing circuit and Faraday cups. Although the cable plasma gun was outlined in previous sections, the following sections describe a few more points related to the arrangement adopted.
5.10.3.1 Cable gun

A cable plasma gun has a simple structure, with two coaxial electrodes having an insulator positioned between them. When a high voltage is applied between the electrodes, electrical flashover occurs along the insulator surface and a plasma sheath is generated by Joule heating and surface ablation, which is pushed forward by electro-dynamic and hydrodynamic forces.

Coaxial cable RG-218 was used in the design of the present cable gun as shown in figure 5.22. The inner conductor has a diameter of 6 mm and the outer conductor one of 15.5 mm, with the insulation in between being polyethylene to provide both vacuum and electrical insulation.

![Figure 5.22: Plasma Gun schematic.](image)

A conical brass nozzle with a central hole of the same diameter as the central conductor of the coaxial cable was attached as shown in figure 5.22 and 5.23. Different nozzle angle were used to study the uniformity of plasma, but for the plasma diagnostic only one nozzle angle of 60° was used, which is shown in figure 5.23b.
5.10.3.2 Firing circuit of Plasma gun

As discussed earlier a surface discharge produces plasma. This surface discharge was created by a 1 µF capacitor charged to 30 kV. The basic circuit of firing circuit of the plasma gun is given in figure 5.21.

5.10.3.3 Faraday cup arrangement for diagnostics

For characterization of the plasma only one gun was used and the Faraday cup was placed at the different locations shown in figure 5.24.

The plasma guns were operated in a vacuum chamber evacuated to $10^{-4}$ mbar. A nozzle electrode having a 60º cavity was attached at the end of the cable and a typical result from a Faraday cup is given in the following sections.
Figure 5.24: A, B, C, D, E, F, G, H and I are different locations for the Faraday cup with A, B, C and D being locations and the others being off axis.

Figure 5.25: Cable plasma gun and Faraday cup position.
5.10.3.4 Results from the Faraday cup

5.10.3.4.1 Velocity of species

Figure 5.26: Faraday cup signals for different positions (A, B, C, D) along the gun axis.

Figure 5.26 shows the signals recorded by Faraday cups aligned along the plasma gun axis (points A, B, C and D of figure 5.24) to measure the axial velocity of the emitted ions, where the time origin is taken from the start of the gun current. Since this results were based on a single Faraday cup (FC-1) an accurate velocity measurement was not possible due to uncertainty over the actual time of generation of the ions. To overcome this problem two Faraday cups (termed FC-1 12.5 cm from the gun and FC-2 at 19 cm from the gun) were installed. The output from these two cups are shown in figure 5.27, and the velocity of the carbon ion calculated using equation 5.29 is given in table 5.4. The velocity of the carbon ions was used to calculate the time of generation of ions, and it was estimated that generation of ion starts 0.25μs after current initiation in the plasma gun.
Figure 5.27: Signals from two Faraday cups FC-1 and FC-2 placed 12.5 cm and 19 cm respectively from the cable plasma gun for accurate measurement of ion velocity.

5.10.3.4.2 Velocity variation with charging voltage

The results of experiments performed to obtain the ion velocity dependency on the charging voltage of the cable gun are given in figure 5.28 and table 5.4 for the Faraday cup positioned at C (see figure 5.26)

Figure 5.28: Signals corresponding to different gun capacitor charging voltages as recorded by the Faraday cup at position C.
In calculating the velocity of the hydrogen ions, it is assumed that the initial kinetic energy of the hydrogen ions and carbon ions are equal, giving

\[ v_h = v_c \frac{m_c}{m_h} \]  

(5.32)

where \( m_c, v_c \) are the mass and velocity of the carbon ions and \( m_h, v_h \) are the mass and velocity of the hydrogen ions. The calculated velocities of ions corresponding to figure 5.28 are given in table 5.4.

<table>
<thead>
<tr>
<th>Voltage (kV)</th>
<th>Carbon ions (cm/µs)</th>
<th>Hydrogen ions (cm/µs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>2.29</td>
<td>7.04</td>
</tr>
<tr>
<td>25</td>
<td>2.19</td>
<td>6.73</td>
</tr>
<tr>
<td>20</td>
<td>1.95</td>
<td>5.99</td>
</tr>
</tbody>
</table>

Table 5.04: Variation of ion velocity with charging voltage.

### 5.10.3.4.3 Divergence of plasma density

The spread of plasma along the cathode surface is defined as the plasma length. Since the plasma density is a maximum at point A in figure 5.29 c and decreases gradually on either side of the axis along the surface of cathode. The distance between the points where the plasma density is 30% \((1/e)\) of the maximum (points \(K_1\) and \(K_2\) of figure 5.29 c) is called the plasma length (distance \(K_1K_2\)). This spread plays an important role in the bi-polar conduction limit of a POS and the off axis divergence of the plasma density is very important in selecting the POS chamber geometry as is explained in chapter 6.

Figure 5.29 a shows the off axis measurement corresponding to an on axis length of 54 mm and figure 5.29 b is the off axis measurement corresponding to an on axis
length of 72 mm from the plasma gun, since it is very difficult to measure the divergence of plasma at 25 mm from plasma gun due to very high influx of ions.

Figure 5.29(a, b): Signals from Faraday cups positioned off axis to observe plasma divergence, (c) plasma length.
Results from figure 5.29 suggests that the plasma density falls by more than 30% at 30° (position I and G) from the axial density hence maximum plasma length \( l \) for the cathode placed at 25 mm from the gun is

\[ l = 2.25 \tan(\pi/6), \] which is 30 mm.

5.10.3.4.4 Species identification in cable plasma

Figure 5.30 shows the plasma current and Faraday cup results when positioned at A and B (positions are shown in figure 5.24).

![Figure 5.30: Faraday cup results showing carbon and hydrogen peaks.](image)

The Faraday cup signals show two main peaks separated in time. It is known from modelling of the cups that the first peak of the Faraday cup signals corresponds to hydrogen ions and the second peak to carbon ions, as indicated in figure 5.30.

5.10.3.4.5 Plasma density measurement

Plasma density can be calculated as described in section 5.10.2. Equation 5.33 can be used for the density measurement from the Faraday cup signal and the carbon and hydrogen ion densities at any point can be obtained by using their respective Gaussian distribution. Thus the ion density \( n_i \) according to equation 5.28 can be written as
Plasma sources for POS

\[ n_i = \frac{I_f}{S_{fc} \cdot f_c(v_{cplat} \cdot v_{cplat} \cdot z \cdot e) } \]  

(5.33)

where \( v_{cplat} \) is the velocity corresponding to the maxima of the \( f_c(v_c) \) variations, (details are given in figure 5.19 and figure 5.20) and shown figure 5.31.

Solving equation 5.33 for the Faraday cup results at 190 mm gives the ion densities as tabulated in table 5.05.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Ion density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen ion</td>
<td>( 1.88 \times 10^{17} ) m(^{-3} )</td>
</tr>
<tr>
<td>Carbon ion</td>
<td>( 4.71 \times 10^{17} ) m(^{-3} )</td>
</tr>
</tbody>
</table>

Table 5.05: Plasma density of different species at 190 mm from the cable gun

5.10.4 Plasma gun studies using an ultrahigh speed camera

The present POS requires a four plasma gun arrangement as discussed in section 5.7.

To replicate this arrangement, four guns having identical nozzles, and with the capacitor bank charged to 30 kV, were assembled on a plastic former with a grounded stainless steel cathode placed at the centre to match closely the POS arrangement, as shown in figure 5.32. The central cathode has an outer diameter of 13 mm and the
plasma guns were placed radially 25 mm from the axis. This whole arrangement was placed in a glass bell jar evacuated to $10^{-4}$ mbar.

Three different nozzles with different nozzle angles as $30^0, 45^0$ and $60^0$ were investigated to determine the effect on the plasma uniformity when all four guns are operating. In the high speed camera arrangements shown in figure 5.33, a high-speed camera (IMACON 468) system which was capable of taking 7 pictures in a single shot was placed 1.5 m from the gun. The pictures obtained for the cable plasma are shown in figure 5.34.

Figure 5.32: Mock-up POS arrangement: 4-plasma guns positioned equally around the POS central cathode.

Figure 5.33: High speed camera and POS arrangement.
Figure 5.34: Plasma dynamics of plasma gun (in a single shot). Plasma gun starts producing 1.8 µs after the start of the gun current in the plasma (t = 0) (upper extreme left). After that second frame is for t = 2.53 µs, the third frame for t = 3.26 µs, and the fourth frame for t = 3.99 µs, when plasma has already filled in the area of the central cathode to the plasma gun surface. Lower extreme left is for 5th frame t = 4.72 µs, 6th frame for t = 6.18 µs and final frame is for t = 6.9 µs.

This picture was obtained using 30 kV charging voltage for the plasma gun and with nozzle angled 60°.

The dynamics of the plasma produced by the four guns can be observed in figure 5.34. Due to the high reproducibility of the plasma generation, a large number of frames obtained during identical shots were assembled to produce a film from which 15 frames are presented in figure 5.35. The dynamics of the plasma produced by the four guns can be observed in this figure. The time advances between frames along rows is about 100 ns from left to right and top row to bottom row sequentially, with the first frame corresponding to about 2 µs from the start of the plasma gun current. Frame analysis showed that the velocity of the luminous plasma front, is about 2.2 cm/µs, which is equal to the velocity of the carbon ions obtained during the Faraday cup study.
Figure 5.35: Plasma dynamics in the POS arrangement observed with an IMACON 468 camera (from many shots).
5.10.4.1 Nozzle angle

The conical cavity at the end of the gun provides a nozzle for the ejected plasma which helps in spreading the plasma uniformly. Three different types of conical cavity giving nozzle angles of $30^\circ$, $45^\circ$ and $60^\circ$ were studied for producing the most homogeneous ejected plasma. Figure 5.36 shows the picture of plasma produced by the gun at different nozzle angles.

![Figure 5.36: Plasma spread obtained at 4 µs after initiation of the discharge current in the plasma gun.](image)

It can be inferred from figure 5.36 b that the spread of plasma is uniform for a nozzle angle of $45^\circ$, whereas the figure 5.36 a, b clearly shows less uniformity of plasma than the figure 5.36 b. Hence a nozzle angle of $45^\circ$ is best for the production of a uniform plasma for 4-guns.

The $45^\circ$ nozzle was therefore selected for use in the present project.

5.11 Discussion

The quality of the plasma plays a major role in characterizing a POS based switch. It was noted in this work that a cable plasma gun not only produces plasma cheaply, but that it can also be controlled according to the needs of POS, such as ion velocity by
increasing the charging voltage, plasma density by the length of the cable gun, insulator and charging voltage.

The Faraday cup results shown in figure 5.20 suggest that the ejected plasma follows a Gaussian rather than a Maxwellian distribution, because the ejected plasma is not in equilibrium. The cable gun produces hydrogen plasma as well as carbon plasma at a different time, and most probably the two plasmas do not interact with each other, since hydrogen plasma is unstable and can recombine with hydrogen ions before the arrival of the carbon plasma. The carbon plasma appears at least 2-3 µs after the hydrogen plasma at a distance of 2.5 cm (cathode) from the gun, as shown in figure 5.30.

Hydrogen plasma is pure since it contains only H⁺, ions whereas carbon plasma can contain both C⁺ and C++ ions and other heavy metals eroded from the electrodes. The purity of plasma ensures the reliable operation of the POS, hence the POS operation with H⁺ plasma allows the smooth operation of the POS at low current. The POS based on carbon plasma operates at a higher current than that required for H⁺ plasma. Current research suggests that the POS operation can be realised at two different time, one for hydrogen plasma and another for carbon plasma.
5.12 Reference


6 Plasma opening switches

6.1 Introduction

Inductive energy storage in conjunction with opening switch techniques has many advantages over conventional capacitive power approaches for high-energy, high-power applications. The principal advantages of inductive storage derive from the 10-100 times higher energy storage density that is possible, which in theory makes possible compact and economical generators. In any inductive generator, the opening switch is the most critical component. One such switch is the plasma opening switch (POS). A POS is often used in pulsed conditioning, inductive energy storage systems and pulse power compression, as it provides the fastest known method of transferring currents of up to several mega-amperes to a load. It is normally used in a circuit where the rise time of the output current of a generator needs to sharpened before it is supplied to a load. The typical POS has the coaxial electrode arrangement seen in figure 6.01, with the generator at one end and the load at the other.

The generator can be a charged capacitor bank, a Tesla transformer or any other appropriate source of current, whilst at present the load is commonly a low impedance link across the electrodes. Prior to conduction of current through the switch, a source injects plasma through the anode and towards the cathode, filling the vacuum region between the load and the generator. The plasma source will typically be a plasma gun or flashboard. The generator is fired a brief time interval after firing the plasma gun, during which time the plasma has reached the necessary concentration and distribution.
POS operation is conveniently divided into the three modes of operation, conducting, opening and (finally) when fully open. During the conduction phase the plasma is in parallel with the load, short-circuiting this and taking all the current produced by the generator.
Ideally, the plasma should conduct the generator pulse for the duration required for
the current to build to a high level, whilst charging a storage inductor in vacuum
between the generator and the switch plasma, and also short-circuiting the load as
shown in figure 6.02 a. Above a critical current level a rapidly expanding gap appears
between the switch plasma and the cathode, transferring the current from the plasma to
the load in a process that provides a load current rise time significantly shorter
(around 10 times faster) than would be obtained with the generator alone. Once the
switch is fully open all the current from the generator and the storage inductor flows
through the load, as illustrated in figure 6.02 b and c.

In the majority of cases the generator applies a negative voltage to the inner electrode,
making this the cathode and the outer electrode the anode, as shown in Figure 6.01.
This is either because the intended load, such as an electron beam or inverse diode,
needs this arrangement or because attempts at a positive polarity POS have resulted in
slower opening times. It has been suggested that the slow opening is due to the
weaker cathode electric field, when it is the larger electrode, or because the plasma is
injected from outside such that it opposes the conduction current. It is now almost
universal practice to use the negative polarity arrangement, with the plasma injected
through the anode towards the central cathode.
6.2 Applications

6.2.1 Inductive energy storage

Inductive energy storage systems are widely used in high power TW nanosecond generation.

![Diagram of Inductive Energy Storage System]

Figure 6.03: Inductive energy storage system.

In the typical system shown in figure 6.03 a generator discharges a high-current $I_G$ through a storage inductor $L_1$ and through plasma in the closed POS switch. When this current reaches its peak the POS (or other opening switch) opens, becoming a high impedance and transferring the current to the load. If the load inductance $L_2$ is very small in comparison to the storage inductance $L_1$, the output pulse duration becomes $L_1/R$, and the peak voltage about $I_LR_L$. The low impedance generator can be treated as a short-circuit after the switch is opened, so that the charging current is transferred and the voltage is multiplied by the ratio of the conduction time to the output pulse duration due to the fact that while the opening occurs the current remains substantially constant but the resistance of the switch increases many fold. Unfortunately the output pulse rise-time and voltage are both restricted by the opening time of the switch and its voltage hold-off capability. A single opening switch will normally manage only about a 10-fold voltage increase.
6.2.1.1 Energy transfer efficiency in inductive storage

The energy transfer efficiency from the generator to the load side of the inductive energy storage system given in figure 6.03 is \[ E / E_0 = \left[ 1 + \frac{R_L}{R_G} + \frac{L_2}{L_1} \right]^{-1} \]  

(6.01)

where $E_0$ is the energy in the generator and $E$ is the energy transferred in the load.

6.2.2 Pulse sharpening application

This application is a form of inductive energy storage, in which the inductor $L_1$ in figure 6.03 is the stray inductance of the connection between a capacitor and the opening switch. The load is bypassed by a closed opening switch during the initial part of the capacitor discharge, and at the peak current the switch opens to force a rapid transfer of the current to the load as shown in figure 6.04. The output pulse is made much sharper than can be obtained using any form of capacitive discharge alone.

![Figure 6.04: Pulse-sharpening waveform.](image)

6.3 Types of POS

Plasma opening switches are divided into two categories depending on the conduction time. Nanosecond POSs usually conduct currents for up to 100 ns, with opening times of 10 ns, while microsecond switches have conduction times of about one
Plasma opening switches

microsecond with opening times of less than 100 ns. Both POSs are capable of operating at currents of MA levels.

Despite similarities in the performance of a POS in the two time scales, the phenomena of magnetic field diffusion, electron magneto-hydrodynamics, energy dissipation, plasma erosion and electrode-plasma formation are significantly different. Thus, experimental investigations and theoretical modelling of the POS on both time-scales are required, to improve the understanding of the fundamental phenomena, and for progress towards applications with various power levels and geometries. There are several features distinguishing the microsecond-POS from the nanosecond-POS. In order to conduct current of hundreds of kA during microsecond time scales, the plasma density is required to be $10^{15}$-$10^{16}$ cm$^{-3}$, which is nearly two orders of magnitude higher than the density needed for the nanosecond-POS due to larger current flow. Also, due to the higher density and the relatively long time, ionization and the effects of electrode plasmas can play an important role in the switch operation. Usually, microsecond-POSs have larger axial dimensions, larger losses of ions, and lower impedance when opened, as compared to the nanosecond-POS.

Due to the higher electron density the initial phase of the microsecond-POS is probably best described by an idealised magneto-hydrodynamic (MHD) theory, which predicts plasma as pushing the magnetic field. Later in time, as the density in some regions decreases due to the plasma pushing, EMHD (electro magneto hydrodynamics) effects start to play an important role, when fast magnetic field penetration may begin to dominate, in a manner similar to that of the nanosecond-POS. It should be noted that during the microsecond-POS operation, various hydrodynamic and kinetic instabilities may develop which may cause increases in the plasma resistivity, particle heating and broadening of the current channel.
### 6.4 Various POS based pulsed power generator and their developmental history of POS.

Details of various existing pulsed power generator based on POSs are given in table 6.1.

<table>
<thead>
<tr>
<th>Machine name</th>
<th>Cathode Diameter</th>
<th>Anode Diameter</th>
<th>( I_s/ \text{Volts} )</th>
<th>Plasma Density ( /\text{cm}^3 )</th>
<th>Output</th>
<th>Plasma length</th>
<th>Other important notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gamble-I</td>
<td>1.3 cm, 5 cm, 2.5 cm, 1.26 cm</td>
<td>10 cm</td>
<td>1 MV 150 kA 40 ns</td>
<td>1.5x10(^{15})</td>
<td>10 cm</td>
<td>Cylindrical separation from anode to inner cathode is 2.5 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>20 cm</td>
<td>Plasma source to cathode distance is 10 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>30 cm</td>
<td>Vacuum is 5x10(^{-4}) Torr</td>
<td></td>
</tr>
<tr>
<td>Gamble-II</td>
<td>4.4 cm</td>
<td>9 cm</td>
<td>1.7 MV 0.85 MA 70 ns</td>
<td>1.2x10(^{15})</td>
<td>4.6 MV/ 0.75 MA 20 ns</td>
<td>10 cm</td>
<td></td>
</tr>
<tr>
<td>PBFA-II</td>
<td>20 cm</td>
<td></td>
<td></td>
<td>2x10(^{13})</td>
<td>6.0 MV/ 20 MA 50 ns</td>
<td>20 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Anode cathode gap 8 cm.</td>
<td></td>
</tr>
<tr>
<td>ASO-X</td>
<td>8 cm</td>
<td>14 cm</td>
<td>30 kV 200 kA 1.3 (\mu)s.</td>
<td>1.2x10(^{15})</td>
<td>2x10(^{-10})</td>
<td>Central electrode of plasma gun is made of copper of diameter 2.9 mm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Insulator is Polyethylene.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Brass nozzle diameter is 3 mm</td>
<td></td>
</tr>
<tr>
<td>DECADE</td>
<td>8.44 cm</td>
<td>17.1 cm</td>
<td>1.2 MA 600 ns</td>
<td>1.2x10(^{15})</td>
<td>2.5 MV</td>
<td>1/4 in. semi rigid coaxial cable Teflon as a insulator</td>
<td></td>
</tr>
<tr>
<td>HAWK</td>
<td>2.54 cm</td>
<td>Variable</td>
<td>640 kV 0.5-1 (\mu)s.</td>
<td>10(^{-1})-10(^{-7})</td>
<td>1-2 MV</td>
<td>3 cm</td>
<td>separation between anode and cathode = 2 cm,</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8 cm</td>
<td>Gun and separation cathode = 12 cm</td>
</tr>
<tr>
<td>TIGR-1</td>
<td>36 cm</td>
<td>6 cm</td>
<td>0.04 MV 0.12 MA 3 (\mu)s.</td>
<td>4x10(^{15})</td>
<td>0.5 MV</td>
<td>10 mm</td>
<td>separation between anode and cathode=12 cm</td>
</tr>
<tr>
<td>HAWK</td>
<td>10 cm</td>
<td>13 cm</td>
<td>0.5 MA 0.7 MV 1 (\mu)s.</td>
<td>3x10(^{15})</td>
<td>450 kA</td>
<td>4 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.6 MV</td>
<td>5 cm</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>8 cm</td>
<td></td>
</tr>
<tr>
<td>Black-Jack</td>
<td>5 cm-10 cm</td>
<td></td>
<td>480 kV 3 MA 1 (\mu)s.</td>
<td>1.5x10(^{15})</td>
<td>3 MA</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 6.1: Parameters of various machine world-wide.
In 1983 the Naval Research Laboratories (NRL) USA [6.3] was the first Institute to use the POS to switch high power levels; prior to that it had been employed as a pre-pulse suppressor.

GAMBLE I and II accelerators employed e-beam diode drivers that delivered approximately 1 MV, 50 ns pulses at 100 kA (GAMBLE I) and 300 kA (GAMBLE II). The researchers obtained threefold voltage multiplication and 7 ns opening times after 40 ns of conduction through a POS. Experimentally, they proved that these opening characteristics were possible with the switch attached to a 5 Ω load [6.4]. The NRL machines used a cylindrical switch of 2.5 cm radius of the inner cathode and 5 cm radius of the outer anode.

Another pulsed power generator at Sandia National Laboratories PBFA II was tested with a POS under similar conditions as at NRL but was different in its voltage multiplication. This machine was operated at 3 MA current level. A voltage multiplication of more than 10 times was achieved, far greater than that obtained in the GAMBLE machines.

The opening of plasma switches at such high voltages (MV) was not understood at that time.

Weber et al. [6.5] placed magnetic probes in the GAMBLE I plasma and charted the progress of the penetrating current sheet as it moved from the generator side to the load end through the plasma. The driver pulse rose to 100 kA in 50 ns. The probes indicated that the current sheet was broad, nearly 6 cm in width and propagating across 12 to 30 cm of the plasma. There was no direct indication of a gap formation (region of lower plasma density) at either the cathode or the anode. The probe measurement was difficult and could not be duplicated on GAMBLE II because of its much higher magnetic intensities.
The results based on these investigation suggested that some anomalous mechanism was at work allowing the magnetic field to penetrate deep into the plasma. Simulations performed by Payne [6.6] taking account of anomalous collisions, matched the experimental opening times for switches on both GAMBLE I and PBFA II.

Later Cooperstein and Ottinger at NRL [6.7] concluded that electrons could be drawn from the cathode region, leaving an excess of positively charged ions and creating a strong electric field to encourage the acceleration of ions toward the cathode and their local depletion or “erosion” there. An erosion gap would open up along the cathode, and the driving magnetic field would pass though this gap to the load region, opening the switch.

Significant modifications followed from the density rise and the larger width of plasma (8 cm) of the PBFA II switch, compared to the GAMBLE I (2.5 cm) switches. At the anode there was significant field penetration coupled to a large-scale ion motion away from the anode and toward the cathode. This opened an anode plasma gap to help let the field through. The body of the plasma in the centre of the switch remained field free, shielded by diamagnetic currents running up the generator side of the filled plasma. A second and larger plasma gap formed rapidly at the cathode, opening the switch. This was correlated principally with the propagation of the larger cathode gap to the load side of the fill plasma. This leads to creation of the full cathode gap and is termed as the “conduction phase” of the POS. Although these explanations were true for fast conduction (50 ns) currents they were unable to explain the conduction time of microsecond levels.
In 1991, William Rix et al [6.8], and Bruce V. Weber et al [6.9] begin modelling of the BLACKJACK 5 and ACE-1 accelerators, where the POS on ACE 1 was conducting currents up to 2 \( \mu \text{s} \) and opened in less than 100 ns.

They explained both theoretically and experimentally that short-conduction time (50 ns) POS operation may be understood as emission-limited conduction followed by erosion opening. In many of these cases, magnetic displacement of the switch plasma during conduction was negligible. At longer conduction times (1 \( \mu \text{s} \)) the effects of magnetic forces became more apparent and may dominate both conduction and opening processes. Simple estimates of plasma displacement, combined with measurements of the current distribution in POS plasmas, impose a lower limit on the plasma mass and therefore on the plasma density. This density can be orders-of-magnitude higher in the microsecond conduction time range than that required for emission-limited conduction in the 50 ns conduction time case. The rate of gap opening is decreased in this case because of the increased density. Erosion opening is probably not the dominant mechanism in the 1 \( \mu \text{s} \) conduction time case. A combination of hydrodynamic displacement with erosion as a final step is hypothesized.

Based on these historical developments, POS opening mechanisms are described in the following sections.

6.5 Theory for working of POS [6.1]

6.5.1 POS conduction mechanisms

The conduction phase lasts for the period during which the switch isolates the load from the system by behaving as a short circuit. All the current from the generator is conducted through the switch plasma between the anode and cathode whilst charging the storage inductor. The mobility of the lighter electrons in comparison to the heavier
carbon ions means that conduction is primarily by electrons emitted from the cathode. The electron motion is influenced both by the strong electric and magnetic field present in the plasma and by collisions between the particles. At present the opening process is described by a number of conduction models that are termed bipolar, magnetic pressure and snowplough conduction.

6.5.1.1 Bipolar conduction model

In this section a bipolar conduction model of the POS is described, based on the electrostatic behaviour of electrons and positive ions by Weber et al [6.1], [6.3]. During the conduction phase a cathode sheath grows between the plasma and the cathode, restricting the flow of current. The cathode becomes a space-charge-limited electron emitter, whilst the plasma acts as space-charge-limited ion emitter. Ions from the plasma drift towards the cathode with a drift velocity \( v_d \). Electrons from the cathode cross the small space-charge cathode sheath. The emitted electron current density is about two orders of magnitude larger than the ion current density and is related to it by the bipolar ratio \( \alpha \) [6.4], where

\[
\alpha = \frac{J_e}{J_i} = \sqrt{\frac{m_i}{zm_e}}
\]  

(6.02)

in which \( J_i \) and \( J_e \) are the ion and electron current densities, \( m_i \) and \( m_e \) are the ion and electrons masses and \( z \) is the ion charge state. The ion current density is determined from their drift velocity \( v_d \) by

\[
J_i \leq n_i z ev_d
\]  

(6.03)

where \( n_i \) is the ion density and \( e \) is the charge on an electron. Using the bipolar ratio \( \alpha \) from equation 6.02 and assuming that overall neutrality is maintained limits the
electron density to the bipolar current density limit $J_{BP}$, given by

$$J_{BP} = J = n_e e v_d \sqrt{\frac{m_e}{zm_e}} \gg J_i$$  (6.04)

where $n_e$ is the electron density. Bipolar conduction is illustrated in figure 6.05 where the switch current $I_s$ equals the generator current $I_G$ and the load current $I_L$ is zero.

![Figure 6.05: Bipolar conduction.](image)

The cathode current is uniformly distributed over the cylindrical plasma length $l$ to give a bipolar current limit $I_{BP}$ of

$$I_{BP} = 2\pi r_c l J_{BP} = n_e e v_d (2\pi r_c l) \sqrt{\frac{m_e}{zm_e}}$$  (6.05)

where $r_c$ is the cathode radius. During the conduction phase the sheath expands along the length of the plasma, as in figure 6.05, until conduction ceases when it extends along the entire plasma length.

### 6.5.1.2 Magnetic pressure model [6.9]

An imbalance of current between the load and generator side of the plasma region results in a strong azimuthal ($\phi$ direction in cylindrical coordinate system) magnetic field $B_\phi$ on the generator side. This interacts with the switch current $I_s$ to cause a
magnetic pressure $B_0^2/2\mu_0$, which sweeps the plasma axially towards the load, as seen in figure 6.06.

Figure 6.06: Axial displacement of plasma by magnetic pressure.

Interaction between current flow through the plasma and the magnetic force at the centre of the plasma mass, which is a thin cylinder of inner radius $r_c$ and initial plasma length $l_0$, gives its displacement as

$$\Delta z = \frac{\mu_0 z}{8\pi^2 r_c^2 l_0 m n_e} \int I_z^2 dt$$

This displacement of plasma results in an axial current that causes radial motion of the plasma, as described below, which can eventually open the switch at a current level far below the bipolar conduction current limit $I_{BP}$ in equation 6.06.

### 6.5.1.3 Snowplough model [6.10]

A further development of the magnetic pressure model is the following one dimension snowplough model for the conduction phase proposed by Rix et al [6.10]. This uses the following assumptions:

1. the plasma develops a current-carrying channel whose width is much less than the plasma length $l$;
2. the plasma pressure is negligible;
3. the channel is roughly perpendicular to the direction of power propagation.

The snowplough movement of the plasma is represented by
\[
\frac{d}{dt}(m_u \frac{dz}{dt}) = \frac{B_o^2}{8\pi} \tag{6.07}
\]

where \( m_u \) is the mass per unit area of the plasma being displaced. Assuming this is constant, integrating equation 6.07 over a cross-section of the coaxial geometry gives [6.10]

\[
m_u \pi (r_a^2 - r_c^2) \left( \frac{d^2}{dt^2} \frac{z^2}{2} \right) = \frac{1}{100} I_s^2 \ln \left( \frac{r_a}{r_c} \right) \tag{6.08}
\]

where \( r_a \) is the anode radius. Assuming the snowplough action begins at zero displacement \((z = 0)\) and that the mass density \( m_u \) at this time is zero, then [6.10]

\[
\frac{600 \pi (r_a^2 - r_c^2)}{\ln \left( \frac{r_a}{r_c} \right)} m_u I_s^2 = 12 \int_0^{t_{ld}} \int_0^t I_s^2 (dt) dt \tag{6.09}
\]

where \( t_{ld} \) is defined as the time at which the snowplough arrives at the load end of the POS. Assuming the switch current to increase linearly with time \((I_s = I_s t)\), gives [6.10]

\[
I_s(t_{ld}) = \left( \frac{600 \pi (r_a^2 - r_c^2)}{\ln \left( \frac{r_a}{r_c} \right)} \right)^{1/2} m_u \left( 2 \int_0^{t_{ld}} \int_0^t I_s^2 (dt) dt \right)^{-1/2} \tag{6.10}
\]

showing that, to maintain the right-hand side constant, the onset of switch opening is dependent on the total charge through the switch. This is in contrast to the bipolar model, where the conduction phase ends when the generator current exceeds a current threshold.

### 6.5.1.4 Summary of the conduction mechanisms

The bipolar model is the simplest to understand and can be applied to a POS that conducts for a short duration \((<<1 \mu s)\), because the displacement of plasma towards
the load due to magnetic forces is insignificant. In a longer conduction POS this displacement becomes important, so that both the magnetic pressure and snowplough model need to be considered.

### 6.5.2 POS opening mechanisms

The switch opens when its impedance rises from zero to a high value, blocking the flow of current and transferring it to a downstream load. A common feature of all the models proposed is that the switch opens because a gap forms between the switch plasma and the cathode. The dynamics of the switch are governed by the rate of growth of the gap and it is this process that often leads to disagreement between the various models.

It is assumed in both the models that the POS is fully open when the magnetic field in the gap is sufficiently strong to deflect all the electrons towards the load end, as illustrated in figure 6.07, preventing any from traversing the gap to conduct current between the electrode. In this state the switch is described as magnetically insulated.

![Figure 6.07: Magnetic insulation gap near the cathode.](image)

This state occurs when the gap width $D$ is equal to the electron gyroradius, which leads to the critical current $I_{c1}$ [6.11]

$$I_{c1} = \frac{2\pi n_e c}{e \mu_0} \sqrt{\gamma^2 - 1} \frac{r_e}{D} \approx 8500 \sqrt{\gamma^2 - 1} \frac{r_e}{D}$$

(6.11)
where $c$ is the velocity of light in vacuum and the electron relativistic factor $\gamma$ is related to the gap voltage $V_{gap}$ by [6.11]

$$\gamma = 1 + \frac{eV_{gap}}{m_ec^2} \quad (6.12)$$

The switch voltage $V_{gap}$ is determined by the Child-Langmuir law as follows [6.23]

$$V_{gap} = \left(3.7 \times 10^4 I_{BP} \frac{D^2}{r_c I}\right)^{2/3} \quad (6.13)$$

The processes behind the growth of this gap are explained by extending the erosion, magnetic pressure and snowplough models as explained below.

### 6.5.2.1 Erosion model [6.12]

The opening stages of the erosion model are described as erosion, enhanced erosion and magnetic insulation. The conduction phase explained earlier ends when the generator current reaches the bipolar threshold current limit $I_{BP}$ of equation 6.4 and the sheath extends along the entire length of plasma. The erosion phase as shown in figure 6.08, begins when the ion current $I_i$ exceeds $I_{BP}$, at which time the gap grows because ions are collected at the cathode faster than they can be replaced by the plasma.

![Figure 6.08: Erosion phase.](image-url)

The gap grows at a rate determined by the diminishing ion density in the gap as [6.12]
\[
\frac{dD}{dt} = \frac{I_i - \alpha d_{up}}{n_1 z e (2\pi r_1)} \tag{6.14}
\]

where \(\alpha\) is the bipolar ratio from equation 6.2. After a while the electrons start to move towards the load end, although they are still able to conduct current between the two electrodes. This is the start of enhanced erosion phase, illustrated in figure 6.08, which begins when the generator current \(I_G\) reaches a second critical current level \(I_{c2}\), given by [6.11]

\[
I_{c2} = 8500k\sqrt{\gamma^2 - \frac{r^2}{D}} \tag{6.15}
\]

where \(k\) is a numerical factor that accounts for space-charge effects.

The rate of gap growth now increases dramatically, with magnetic insulation being achieved when the generator current \(I_G\) exceeds the critical current \(I_{c1}\), as shown in figure 6.09. The small fraction of the generator current that remains in the switch is due to the small ion current.

### 6.5.2.2 Magnetic pressure model [6.9]

Magnetic pressure from the generator end forces the plasma towards the load end during the conduction process, as described in section 6.5.2. An axial current \(J_z\) arises as a result of this movement of charge towards the load, and interacts with the
azimuthal magnetic field $B_\phi$ to produce a radial $J \times B$ force. This force opens a gap on the generator side of the plasma where the field is strong, as illustrated in figure 6.06. The effect diminishes towards the opposite side of the plasma where the magnetic field is zero until current begins to be transferred to the load, at which time the magnetic field opens a gap along the entire length.

The gap-opening rate is described by [6.9]

$$\frac{d}{dt} \left( m_u \frac{dD}{dt} \right) = J \times B_\phi$$

(6.16)

or in the snowplough effect by [6.9]

$$m_u \frac{d}{dt} D \frac{dD}{dt} = \beta \frac{B_\phi^2}{8\pi}$$

(6.17)

where the scale factor $\beta$ accounts for the material and the magnetic back pressure of the plasma.

### 6.5.2.3 Summary of the opening mechanisms

As mentioned previously, magnetic pressure is the opening mechanism used to describe the plasma behaviour in the long-conduction POS, where the plasma is being swept along during the conduction phase. However the opening phase of this process is too slow to explain the fast opening action that has been observed experimentally.

The prevailing idea is that once a gap forms the opening mechanism becomes predominantly enhanced erosion, with the overall mechanism being a combination of both magnetic pressure and erosion.
6.6 Requirement for the project

6.6.1 Goal of experiment

The POS required for the present project is to conduct 20 kA in hydrogen plasma and 70 kA in carbon plasma, when the conduction time for both current levels is 50 ns, and the switch should open in less than 5 ns. Since the plasma switch is conducting for 50 ns, this POS can be best described by the NRL model. Plasma is almost static for this time scale, and also the $J \times B$ force is not sufficiently high to move plasma in the direction of the load. Movement of the plasma is given by equation 6.05 which, using the parameters given in table 6.02 gives $\Delta Z = 0.031\, cm$, which is very low in comparison to the plasma length of 5 cm, justifying the selection of the NRL model for the current project.

The minimum current requirement is described by equation 6.18a for conduction through plasma (NRL model) and the conduction time ($\tau_c$) is given by equation 6.18b [6.13], where $I_{max}$ is the maximum current passing through the POS. Thus:

$$I_{BP} = zn_1ev_a(2\pi r l) \sqrt{\frac{m_i}{Zm_e}} \quad (6.18\ a)$$

$$\tau_c = \frac{\tau_f}{0.7\pi} \arcsin \left( \frac{I_{BP}}{I_{max}} \right) \quad (6.18\ b)$$

for fast opening of the POS the current passing ($I_{max}$) must be greater than the bi-polar limit ($I_{BP}$), as evident from equation 6.18b.

To satisfy this requirement, plasma must have the proper combination of density and ion velocity, which both depend on the plasma gun. The other two variables which are essential for achieving conduction of a POS at low current level are the radius of the cathode and the plasma length (which can be controlled by placing the plasma gun at proper location.). The radius of the cathode has been taken from Gamble-I, the plasma
length from HAWK and the plasma density and the ion velocity from experiments conducted on a cable gun. Based on these parameters, equation 6.18 a is used to calculate the value of \( I_{BP} \) included in table 6.02.

<table>
<thead>
<tr>
<th></th>
<th>H+</th>
<th>Carbon ions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ion density (cm(^{-3}))</td>
<td>(1.2 \times 10^{13}) [chapter 5]</td>
<td>(3.0 \times 10^{13}) [chapter 5]</td>
</tr>
<tr>
<td>Ion velocity (cm/µs)</td>
<td>6.85 [chapter 5]</td>
<td>1.97 [chapter 5]</td>
</tr>
<tr>
<td>Charge state ((z))</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Radius of cathode(cm)</td>
<td>0.65 (as in Gamble-I)</td>
<td>0.65</td>
</tr>
<tr>
<td>Plasma length(cm)</td>
<td>5 (as in HAWK)</td>
<td>5</td>
</tr>
<tr>
<td>( I_{BP} ) (kA)</td>
<td>11.7</td>
<td>27</td>
</tr>
</tbody>
</table>

Table 6.02: Various required parameters for POS operation at 2.5 cm from cathode.

### 6.6.2 Scaling of current project parameters of POS from Gamble-I

The product of ion density and velocity is one of the important parameter of the POS plasma, and a POS can work at any combination of \( n_i \) and \( v_d \) provided that their multiplication is kept constant as seen from equation 6.18 a, for any fixed geometry of cathode radius and plasma length.

Figure 6.10: Ion density and velocity combinations for Gamble-I.
Figure 6.10 shows the plasma density and ion velocity combination of Gamble –I. If the product of ion density and velocity is kept the same for the scaled down cathode radius and plasma length of the existing project, the required conduction current can be achieved, using equation 6.05, which justifies the present selection of POS parameters.

6.7 Design and manufacturing

Figure 6.11: Schematic of Tesla transformer, PFL and plasma opening switch connections.

Full details of the generator based POS are presented in figure 6.11, where the Tesla transformer is connected to the PFL which acts as the secondary capacitance of the transformer. Energy from the charged water capacitor is transferred to the POS
through a pressurised spark gap. Details of the Tesla transformer and waterline were presented in chapters 2 and 3 and the source for the plasma in chapter 5. Initially plasma shorts the transmission line, enabling charging of the inductance $L_1$ (shown in figure 6.12). When the plasma opens, the energy stored in $L_1$ is transferred to a short circuited load at a faster rate than that of the initial current rise. Details of the POS design are given in figure 6.12 and are explained below.

**6.7.1 POS chamber**

The POS chamber is coaxial with the central cathode that is made of 13 mm diameter steel as shown schematically in figure 6.12, and photographs of the actual chamber in figure 6.13 and 6.14. The outside of the chamber is of 65 mm diameter aluminium and the chamber is evacuated to $10^{-4}$ mbar. The length of the chamber is such that the total inductance ($L_1+L_2$) should produce the required rise time of 50 ns for a current of
Plasma opening switches

more than 20 kA. Plasma guns are placed at position such as $L_1 > L_2$ for maximum energy transfer from the generator to the load as described in equation 6.01. The inductances shown in figure 6.13 are as $L_1 = 35 \, nH$, $L_2 = 24 \, nH$ which are calculated for the optimised plasma length. A photograph of the full system is shown in figure 6.15.

(a)      (b)
Figure 6.13: Inner view of plasma opening switch chamber, with upper Rogowsky and plasma guns also shown.

Figure 6.14: Outer view of POS chamber and cable plasma gun.
Figure 6.15: Plasma opening switch chamber with cable plasma gun, spark-gap and PFL.
6.8 Testing of POS

6.8.1 Arrangement of POS experiment

For POS experimentation the plasma gun has to be fired before the Tesla generator to enable the plasma to act as a short for the transmission line. Since the plasma density and ion velocity both vary with time and space, synchronization of the plasma gun with the Tesla generator is of the utmost importance for proper operation of the POS. This synchronization can be achieved by setting the delay between firing the plasma gun and the Tesla generator, which can be adjusted by the multiple delay pulsed generator (MDPG). This pulse generator produces multiple 30 V pulses to fire the trigger generator TG-70, which ultimately fires the spark-gaps T-670 connecting the capacitor banks of the plasma gun to the primary circuit of the Tesla transformer. Details of this are shown in figure 6.16. Before firing the spark-gaps connecting the
capacitor banks, charger 1 delivers 25 kV to the Tesla transformer and charger 2 delivers 30 kV to the plasma gun.

The measured inbuilt delay (I.D) between the cable plasma gun and the start of the Tesla secondary current for the present generator is 2 μs, which must be taken into consideration before setting the value in MDPG.

### 6.8.1.1 Voltage and current waveform without the feeding of plasma

Voltage and current waveforms of the generator without the POS are given in figure 6.17, where charger 1 delivered 25 kV.

![Figure 6.17: Voltage (V<sub>PFL</sub>) and current (I<sub>PFL</sub>) waveform of the generator without plasma.](image)

Secondary voltage (V<sub>PFL</sub>) is 300 kV in PFL to generate current of 40 kA.
6.8.1.2 Voltage and current waveform of POS based generator

The waveforms in figure 6.18 were achieved for the generator using the parameters given in table 6.04.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage (charger-1)</td>
<td>25 kV</td>
</tr>
<tr>
<td>Voltage (charger-2)</td>
<td>30 kV</td>
</tr>
<tr>
<td>Delay set in MDPG</td>
<td>0 µs</td>
</tr>
<tr>
<td>Inbuilt delay in systems (I.D)</td>
<td>2 µs</td>
</tr>
<tr>
<td>SF6 pressure</td>
<td>30 psi</td>
</tr>
<tr>
<td>Vacuum in plasma switch chamber</td>
<td>5.10^4 mbar</td>
</tr>
<tr>
<td>Voltage in secondary</td>
<td>-240 kV</td>
</tr>
<tr>
<td>Current in PFL @-240 kV</td>
<td>24 kA</td>
</tr>
</tbody>
</table>

Table 6.04: Parameters for POS operation.

Figure 6.18: $V_{PFL}$ is voltage generated in PFL (waterline), $I_{PG}$ is plasma gun current. $I_{PFL}$ and $I_L$ are the currents before and after the plasma opening switch.
The result in figure 6.18 are expanded in figure 6.19 a to explain the compression achieved in the pulse rise time. The currents before \((I_{PFL})\) and after \((I_L)\) opening of the switch are shown in figure 6.19 a, and the reduction achieved in their rise times is recorded in table 6.05. In the absence of any plasma the generator current and load current will have almost same rise time (around 52 ns) as shown in figure 6.19 b since generator current is passing through load without interruption.
Table 6.05: Current rise times before POS and after POS.

The rise time of the output pulse is 9 ns, whereas that of the input pulse to the POS is 53 ns, which clearly illustrates the role of the POS. Different delay times in the MDPG were used to check the importance of correct synchronization between generator current and plasma current as shown by the results recorded in figures 6.20 to 6.23, whose delay details are given in table 6.06.

<table>
<thead>
<tr>
<th>Delay in MDPG</th>
<th>Describing figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 ns +I.D (inbuilt delay)</td>
<td>Figure 6.20</td>
</tr>
<tr>
<td>2 µs+I.D</td>
<td>Figure 6.21</td>
</tr>
<tr>
<td>2.5 µs+ I.D</td>
<td>Figure 6.22</td>
</tr>
<tr>
<td>3 µs+ I.D</td>
<td>Figure 6.23</td>
</tr>
</tbody>
</table>

Table 6.06: Delay settings in the MDPG.

Figure 6.20: Delay between plasma guns and Tesla generator is 100ns+I.D.
Figure 6.21: Delay between plasma guns and Tesla generator is 2 μs + I.D.

Figure 6.22: Delay between plasma guns and Tesla generator is 2.5 μs + I.D.

Figure 6.23: Delay between plasma guns and Tesla generator is 3 μs + I.D.
The results above suggest that the POS designed provides the best result with a delay of 1.0 set in the MDPG. This delay corresponds to the optimised hydrogen plasma density and ion velocity, as discussed in chapter 5. After this timing an additional delay of 100 ns in MDPG gives output current rise time of 12 ns as shown in figure 6.20, whereas the other results do not show any output current at all. The delay setting for these experiments in MDPG crosses the hydrogen plasma regime and enters into the carbon plasma regime as shown in figure 6.24, where the minimum current requires for conduction is around 30 kA much more than that required for hydrogen plasma.

![Graph showing arrival of different ions at 2.5 cm from cable plasma gun](chapter5.png)

Figure 6.24: Arrival of different ions at 2.5 cm from cable plasma gun [chapter 5].

The arrival of different ions species based on Faraday cup results at the surface of the cathode is illustrated in Figure 6.10, which is taken from chapter 5.

### 6.8.2 POS performance with dynamic load

Voltage and power multiplication can be obtained if the POS switches efficiently to a load having the proper impedance [6.4]. To the extent that the storage inductor acts as a constant current source, and the POS opens to infinite impedance, the load power and voltage increases with increasing load impedance. But in practice the load current
Plasma opening switches

decreases with increasing load impedance, due to the $L_2/R$ decay during the switch opening where $L_2$ is the small inductance between the load and the switch and $R$ is the resistance of the diode and the magnetic insulation losses between the switch and the load [6.4].

A diode load was created by altering the cathode from the previously short circuited arrangement (see figure 6.13) to that shown in figure 6.25.

Figure 6.25: Diode assembly.

The POS was tested with different anode and cathode separations $d_{ac}$. The impedance of the diode is given by equation 6.19 [6.14]

$$Z_{\text{diode}} = \frac{\mu_0 c (\gamma - 1)}{2 \pi \gamma \ln[\gamma + (\gamma^2 - 1)^{0.5}]} \frac{d_{ac}}{r_c}$$

(6.19)

where $\gamma = 1 + eV/mc^2$ $r_c$ is the radius of the cathode tip where the voltage $V$ is applied.

For the present experimental conditions, $V = 0.3$ MV, $d_{ac} = 4$ mm and $r_c = 6.5$ mm, giving a calculated impedance of 17 $\Omega$. The diode impedance is dependent on the applied voltage at the tip of cathode and knowledge of this voltage is important when
measuring the impedance. A voltage measurement was not possible, due to space constraints but a current measurement was available as shown in figure 6.26. The current measurement without plasma (figure 6.26 a) and that with plasma (figure 6.26 b) was used to measure the amplification of the current.

Figure 6.26: Generator and load current with diode (a) without POS (b) with POS.

When the POS was in use, the current in the diode load rose from 5.5 kA (rise time 95 ns) to 24 kA (rise time 5 ns), clearly showing a four times amplification of the current.

6.9 Discussion

Operation of the plasma opening switch at low current can be achieved with proper geometrical parameters [6.1] [6.3] [6.15] of the plasma chamber and conducting plasma parameters, and the use of hydrogen plasma is the best option for low current operation. It is not only light for low current operation but also pure in comparison with the other emitted plasma. Although all plasma species originate at the same time, due to their higher velocity $H^+$ ions reaches the cathode surface much earlier than all the other species, and they are therefore uncontaminated. Single species plasma avoids all the problem related to multi-species plasma described in reference [6.16].
Figure 6.21: Different points in load current, point A corresponds to 33.6 ns and current 3.6 kA, point B is having slow dI/dt from $t_1=36.8$ ns to $t_2=38.4$ ns. At point C is at 44 ns load current attains maximum current of 23.6 kA.

Results based on H$^+$ plasma in figure 6.21 shows that point B where the current is around 11.37 kA corresponds to the bi-polar conduction limit of POS. The current has a slow rise time from $t_1$ (36.8 ns, 11.37 kA) to $t_2$ (38.4 ns, 12.049 kA), which implies switching from bi-polar conduction to erosion and enhanced erosion processes. The critical current from (equation 6.18 a) $I_{sc}=17$ A, after which the magnetic insulation process is the dominant factor in the opening.

Ultimately, the load current achieves the highest current (23 kA) of the generator ($I_{PFL}$). The total time taken by POS to open is 9 ns, which makes the load current rise time 6 times shorter than that of the original current pulse.

All the calculations based on the NRL model match the experimental results obtained, which suggests that at a fast conduction time and single species of plasma this model is best to describe the POS and its parameters.

The POS performance was much improved by the installation of a diode as the load and opened in 5 ns, whereas the POS opened in 9 ns without any load with power amplification of 16.
6.10 Reference


7 Conclusion

Plasma opening switch (POS) configurations have been used for the fast switching of very high current pulses (more than 100 kA) for some time. However low current operation of a POS is difficult to achieve, due to both the plasma parameters required and the cathode geometry, and no successful low current operation has previously been reported. The main parameters of the injected plasma are the species of ion, the plasma density and the ion velocity. If the ions are light, low velocity and low plasma density can be produced with a modified cathode and a POS can be expected to work even at low current.

Because of its low mass, plasma density and ion velocity hydrogen plasma is most suitable for low current operation of a POS.

An important task at an early stage of the project was the selection of a suitable generator for the POS operation, whose rise time has subsequently to be reduced by action of the POS. A Tesla transformer was selected for this task over its Marx counterpart on the grounds of its greater simplicity. A PFL with a water dielectric formed the secondary load of the transformer. The energy in the PFL was delivered to the load through the POS and a pressurised switch.

Measurement of the high voltages and currents in the system was one of the most difficult tasks, due to the sealed geometry of the PFL and the small available space. This problem was solved by using a capacitive coupled probe and a self integrating Rogowsky coil for voltage and current monitoring respectively. The self integrating Rogowsky was difficult to make due the requirement of a low inductance of the current viewing resistor (CVR), with any substantial inductance inducing a high voltage in the output waveform due to high rate of change of the current waveform.
This was solved by using a parallel combination of surface mounting resistors in a coaxial geometry, to drastically reduce the CVR inductance.

After solving the problems relating to the generator, attention was given to the selection of the plasma source for the POS. A cable plasma gun was used for this purpose as this is capable of producing hydrogen plasma together with carbon plasma. Hydrogen plasma used with a carefully selected cathode radius made the POS operation feasible at low currents. A brass nozzle with a conical angle of 45° was used to guide the plasma into the POS chamber. Results from Faraday cup and high speed camera diagnostics confirmed the suitability of the plasma density, and ion velocity and the uniformity of the plasma produced.

A cable plasma gun produces hydrogen plasma much earlier in its operation than it does carbon plasma; about 2 µs after firing the cable gun compared with about 4 µs for the carbon plasma. To use the hydrogen plasma regime in the POS the Tesla generator was built such that the current flow from the generator to the POS began 2 µs after firing of the primary capacitor bank in the Tesla transformer. Hence the cable plasma gun and the Tesla transformer were fired at the same time, and only hydrogen plasma is present in the POS chamber when the generator current passes through it.

The plasma emitted at the required time was single species hydrogen, since other species are heavier than hydrogen and arrive at the cathode surface much later, and do not interfere with the hydrogen plasma.

The POS that was built operates at 23 kA, much lower than any figure reported previously. It opens in 8 ns and by shorting the original 53 ns pulse it enhances the output power by more than 7 times.
The complete machine was tested in a dynamic condition by the installation of an X-ray diode load, when the original rise time 53 ns was reduced 6 ns. The machine is now suitable for use in other experiments, when the load is a microwave generator such as a vircator.

Since a cable plasma gun produces hydrogen and carbon plasma on a different time scale, hydrogen plasma is suitable for low current (~20 kA) operation whereas carbon plasma is suitable for high current (~40 kA) operation. So the same machine can be tested at a higher current, using carbon plasma for the POS.

Since the present experiments have proved the feasibility of producing nanosecond POS based on hydrogen plasma, the possibility of using hydrogen plasma in a microsecond POS can be explored. If this is successful, the PFL and Tesla transformer can be replaced by a high voltage and high current compact capacitor bank since the fast rising (53 ns) generator will no longer be required, leading to a more compact generator. The overall set-up will comprise a compact generator with a miniaturised POS.

The original aims of the research were described in section 1.8. This thesis has demonstrated that these have been met. That is

1. A GW rated miniaturised POS has been developed and demonstrated, where power in excess of 10 GW was generated.

2. Low current operation of a POS has been demonstrated.
8 Publications

The following is a list of the journal and conference papers that have been published during the present studies.

8.1 Journals


8.2 Conferences


[8.2.3]. R. Kumar, B.M. Novac, I.R. Smith and C. Greenwood, “Plasma Source for a Miniature and Repetitive Plasma Opening Switch” presented at IEEE
Publications


[8.2.5]. M Istenič, B M Novac, J Luo, R Kumar and I R Smith, “Magnetically insulated pulsed transformers” IEEE pulsed power and plasma science conference June 17-22, 2007, Albuquerque Convention Center, Albuquerque, New Mexico, USA.


Appendix-I

Calculation of various parameters of the Tesla transformer

[I.1]. Capacitor matrix for secondary winding (for calculating stray capacitance $C_{sw}$)

Figure I 1: (a) actual secondary windings (b) Model used in Maxwell-2D for capacitance matrix calculation.

A model of the actual secondary windings was evaluated as in section 2.3.6 of this thesis using 2-D electrostatic axi-symmetrical finite element analysis software MAXWELL-2D. The actual windings and their adopted model are both shown in figure I 1. Since each winding is considered as a separate conductor in this analysis, it gives capacitance matrix for all 23 circular windings, which is $23\times23$ capacitance matrix as below
\[ C(23 \times 23)/10^8 = \]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
</table>

\[
Y_{v/10^8} =
\]

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
</table>
\[
Y_{XX} = \begin{bmatrix}
7.22 \times 10^{-8} & 7.241 \times 10^{-8} \\
7.241 \times 10^{-8} & 7.256 \times 10^{-8}
\end{bmatrix}
\]

\[
Y_x := Y_{xx} - Y_{xy} Y_{yy}^{-1} Y_{yx}
\]

\[
Y_x = \begin{bmatrix}
2.1 \times 10^{-11} & -70 \times 10^{-12} \\
-70 \times 10^{-11} & 9.486 \times 10^{-11}
\end{bmatrix}
\]

\(Y_x\) is the capacitance matrix of the secondary where non-diagonal term gives the stray capacitance (\(C_{sw}\)) of secondary circuit. Which is 70 pf in the existing case.

[I.2]. **Inductance and mutual of calculation of Tesla transformer**

This calculation is based on section 2.3, of the thesis where the primary and secondary windings of the transformer are divided into filaments to evaluate their self inductances and mutual inductance to be calculated.

These calculations were performed using MathCAD.
Table I 1: Dimensions used with Figure I 2 to calculate Tesla transformer

To calculate the primary and secondary inductances and their mutual inductance, these two windings are divided into the filament defined in figure I 3. The primary winding is divided into $np \times mp$ parts ($np$ rows and $mp$ columns) and each secondary winding is treated as a separate single-turn winding for simplicity.
Figure I 3: Filaments used in calculation of Tesla transformer inductances.

The primary is divided into the \( np \times mp \) filaments shown in figure 3, where \( np=100 \) and \( mp = 1 \) for simplicity.

The total no of filaments in the primary is therefore

\[
N_p = np \times mp
\]

The secondary is divided into 23 parts (each turns is as one filament), \( ns = 23 \)

Total numbers of secondary filament is

\[
N_s = ns
\]

The total no of filaments is

\[
N_t = N_p + N_s
\]

MathCAD equations for calculations

The taper ratio of the secondary is defined as

\[
p_{conic} = \frac{rsNO - rsNi}{ns - a}
\]

where \( rsNO \) and \( rsNO \) are defined in the figure I 2 and table I 1.

The radius of any secondary turn can be defined as
The following expressions are defined for the filaments positions in the primary and secondary. If the filaments are described as columns and rows, then

any filament position of column in primary is defined as
\[
\text{wchcolp}(i) := \text{floor} \left( \frac{i - 1}{np} \right) + 1
\]

any filament position of column in secondary is defined as
\[
\text{wchcols}(i) := \text{floor} \left( \frac{i - 1}{ns} \right) + 1
\]

any filament position of rows in primary is defined as
\[
\text{wchrwp}(i) := \text{mod}(i - 1, np) + 1
\]

any filament position of rows in secondary is defined as
\[
\text{wchrws}(i) := \text{mod}(i - 1, ns) + 1
\]

The distance between any two primary filaments defined as
\[
\text{distp}(i, j) := |\text{wchrwp}(i) - \text{wchrwp}(j)| \cdot hp
\]

The distance between any two secondary filaments are defined as
\[
\text{dists}(i, j) := |\text{wchrws}(i) - \text{wchrws}(j)| \cdot p
\]

The distance between any primary and any secondary filaments is defined as.
\[
\text{distps}(i, j) := \left[ \left[ \text{wchrwp}(i - 1) - hp \cdot 2 - \left( \text{wchrws}(1) - p + 1.5 \cdot 10^{-3} \right) \right] + \text{apartbelow} \right]
\]

The radius of any primary filament is given by
\[
\text{rp}(i) := \text{rpr} + \frac{\text{tp} \cdot \text{wchcolp}(i)}{2 \cdot \text{mp}}
\]

The following variables are defined as required by the equation 2.34 to get the inductance of the coil.

Values of q as in equation 2.35

For the primary
For the secondary

\[ q_s(i) := \frac{p}{2r_sN(i)} \]

Variables for the elliptical function defined as in equation 2.31

For the primary only

\[ k_p(i, j) := \sqrt{\frac{4rp(i)rp(j)}{(rp(i) + rp(j))^2 + distp(i, j)^2}} \]

For the primary and secondary (combined to provide the mutual inductance between them)

\[ k_{ps}(i, j) := \sqrt{\frac{4rp(i)rsN(j)}{(rp(i) + rsN(j))^2 + distps(i, j)^2}} \]

For the secondary only

\[ k_s(i, j) := \sqrt{\frac{4rsN(i)rsN(j)}{(rsN(i) + rsN(j))^2 + dists(i, j)^2}} \]

Inductance formed by the primary filaments as from equation 2.34

\[ L_p(i) := \mu_0 rp(i) \left( 0.3862944 + 0.1730840qp(i)^2 - \ln(qp(i)) - 0.2538683qp(i)^2 \cdot \ln(qp(i)) \right) \]

Inductance formed by the secondary filaments as from equation 2.34

\[ L_{s(j)}(i) := \mu_0 rsN(i) \left( 0.3862944 + 0.1730840qs(i)^2 - \ln(qs(i)) - 0.2538683qs(i)^2 \cdot \ln(qs(i)) \right) \]

Inductance of any secondary winding as from equation 2.36.

\[ L_s(i) := \mu_0 rsN(i) \left( \ln \left( \frac{4rsN(i) \pi}{ts} \right) - \frac{3}{4} \right) \]

Inductance of different filaments of primary and their interaction as from equation 2.31

\[ M_p(i, j) := \begin{cases} 
2\mu_0 kp(i, j)^{-1} \sqrt{p(i)rp(j)} \left( 1 - \frac{1}{2}kp(i, j)^2 \right) \text{LegendreKc}(kp(i, j)) - \text{LegendreEc}(kp(i, j)) \quad & \text{if } i \neq j \\
L_p(i) \quad & \text{otherwise}
\end{cases} \]
Inductance of different filaments of secondary and their interaction as from equation 2.31

\[ M_s(i, j) := \begin{cases} 
2 \mu_0 k_s(i, j)^{-1} \sqrt{r_sN(i) \cdot r_sN(j)} \left[ 1 - \frac{1}{2} k_s(i, j)^2 \right] \text{LegendreKc}(k_s(i, j)) - \text{LegendreEc}(k_s(i, j)) & \text{if } i \neq j \\
L_s(i) \text{ otherwise} 
\end{cases} \]

Mutual inductance between primary and secondary filaments

\[ M_p(i, j) := 2 \mu_0 k_p(i, j)^{-1} \sqrt{r_p(i) \cdot r_sN(j)} \left[ 1 - \frac{1}{2} k_p(i, j)^2 \right] \text{LegendreKc}(k_p(i, j)) - \text{LegendreEc}(k_p(i, j)) \]

Primary inductance (due to energy method) as in equation 2.46

\[ L_{\text{primary}} := \frac{\sum_{i=1}^{N_p} \left( \sum_{j=1}^{N_p} M_p(i, j) \right)}{N_p^2} \]

\[ L_{\text{primary}} = 1.638 \times 10^{-7} \text{ H} \]

Secondary inductance as in equation 2.47

\[ L_{\text{secondary}} := N_s^2 \left( \sum_{i=1}^{N_s} \left( \sum_{j=1}^{N_s} M_s(i, j) \right) \right) \]

\[ L_{\text{secondary}} = 6.269 \times 10^{-5} \text{ H} \]

Mutual inductance between primary and secondary as in equation 2.48

\[ \text{Mutual} := \frac{\sum_{i=1}^{N_p} \left( \sum_{j=1}^{N_s} M_p(i, j) \right)}{N_p} \]

\[ \text{Mutual} = 2.167 \times 10^{-6} \text{ H} \]