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A Polynomial QR Decomposition Based Turbo Equalization Technique for Frequency Selective MIMO Channels

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A. Choice of Notation

Throughout this paper, matrices are denoted by upper case bold characters and vectors by lower case bold characters. Regular upper or lower case characters denote scalar quantities. \( [\cdot]_{kl} \) denotes the \((k,l)\)-th element of the matrix in the square brackets. The superscripts \( \ast, T, \) and \( H \) denote the complex conjugate, matrix transpose and Hermitian conjugate, respectively. \( I_p \) is used to denote the \( p \times p \) identity matrix.

Polynomial matrices and vectors are denoted by underscored bold upper and lower case characters, respectively. The use of an underscore with scalar quantities denotes a polynomial (matrix, vector, or scalar) with the qualifier \( z \). Any polynomial (matrix, vector, or scalar) with the qualifier \( (\cdot) \) denotes a polynomial in the indeterminate variable \( z^{-1} \). The \( *, T, \) and \( H \) denote complex conjugation of the coefficients in a polynomial matrix or vector. The use of \( \ast \) above a polynomial matrix or vector denotes the paraconjugate, i.e. for a given polynomial matrix \( \mathbf{A}(z) \), \( \mathbf{A}(z) = \mathbf{A}^T(z^{-1}) \). \( \| \cdot \|_F \) will be used to denote the Frobenius norm (F-norm) of a polynomial matrix, which is simply the square root of sum of the squared F-norms for all coefficient matrices.

II. POLYNOMIAL MATRIX QR DECOMPOSITION

The input to the PMQRD algorithm is a \( p \times q \) complex polynomial matrix, \( \mathbf{A}(z) \).

\[
\mathbf{A}(z) = \sum_{i=t_1}^{t_2} \mathbf{A}_i z^{-i}
\]

Where \( \mathbf{A}_i \in \mathbb{C}^{p \times q} \) is the \( i \)-th matrix tap of the polynomial matrix, \( i \in \mathbb{Z}, t_1 < t_2 \) and \( z^i \) is the unit delay operator.
The polynomial QR decomposition of $A(z)$ is shown in (3) where $Q(z)$ is a paraunitary polynomial matrix, such that $Q(z)Q(z) = I_p$ and $R(z)$ is an upper triangular polynomial matrix.

$$A(z) = Q(z)R(z) \tag{3}$$

The operation of the PMQRD algorithm is outside of the scope of the paper and is available in [3], [4].

### III. APPLICATION OF PMQRD TO MIMO CHANNEL EQUALIZATION

Without loss of generality we consider a frequency selective MIMO system of equal number transmit and receive antennas, i.e. $M_r = M_t$. The PMQRD of the channel $H(z)$ is shown in (4).

$$H(z) = Q(z)R(z) \tag{4}$$

A set of source signals of length $N$, $s(n) \in \mathbb{C}^{M_t \times 1}$ for $n \in \{0, 1, \ldots, N - 1\}$ are propagated through the MIMO wireless channel, $H(z)$, received and filtered with $\tilde{Q}(z)$, as shown in Figure 1.

$$y = \tilde{Q}(z)H(z)s + \tilde{Q}(z)n \tag{5}$$

where $n$ denotes an additive Gaussian noise process with variance $\sigma^2 I_{M_r}$. The convolutive mixing model can be rewritten as (6), where $n' = \tilde{Q}(z)n$.

$$y = R(z)s + n' \tag{6}$$

#### A. Iterative Interference Cancellation

The MIMO channel problem can now be transformed into a set of $M_r$ equalization problems using back substitution. The $M_t^{th}$ source signal is expressed as (7) which is a single channel equalization problem, which is solved using a minimum mean squared error (MMSE) equalizer [5].

$$y_{M_r} = \sum_{j=1}^{M_t} r_{M_r,j}(z) s_j + n'_{M_r} \tag{7}$$

Once $s_{M_r}$ is retrieved we use it to cancel its contribution to $y_{M_r-1}$ as follows

$$y_{M_r-1} - \tilde{r}_{M_r-1,M_r}(z)s_{M_r} = \tilde{r}_{M_r-1,M_r-1}(z)s_{M_r-1} + n'_{M_r-1} \tag{8}$$

which again is a single channel equalization problem. Therefore the $i^{th}$ single channel equalization problem can be formulated as

$$y_i - \sum_{j=1}^{M_r} r_{ij}(z) s_j = \tilde{r}_{ii}(z) s_i + n'_i \tag{9}$$

providing the streams $s_{i+1} \ldots s_{M_r}$ have been previously recovered.

### IV. CHANNEL MODEL

In our simulations, we consider a MIMO system with three antennas at the transmitter and receiver. The temporal length, $L$ of the channel between each transmitter and receiver is five. The channel has a constant power delay profile with equal average gain for each tap.

### V. DIAGONAL ENCODING (D-BLAST)

D-BLAST [6] is an encoding architecture that combines the simplicity of horizontal BLAST (H-BLAST) encoding with the performance benefits of vertical BLAST (V-BLAST) encoding [7]. The data stream is first demultiplexed into three substreams, $s_1, s_2, s_3$. Each substream is then independently convolutionally encoded and interleaved. We have used the code formatting polynomials in (10) as per the global system for mobile (GSM) CS1-CS3 [8].

$$G0 = 1 + D^3 + D^4$$

$$G1 = 1 + D^3 + D^3 + D^4 \tag{10}$$

To ensure that errors appear random and to avoid long error bursts, an interleaver is used to randomize the encoded bits prior to transmission. We have used an S-Random interleaver with a depth of 28 bits to gain maximum performance [9]. The data streams are then rotated, so that the bit stream-antenna association is periodically cycled [2]. This allows a diversity gain of $M_r M_t$ while maintaining a low computational complexity at the receiver.

### VI. RECEIVER DESIGN

The received signals are filtered with $\tilde{Q}(z)$ as shown in Figure 1, yielding the received substreams, $y_1, y_2, y_3$. Each substream is turbo equalized prior to the application of the iterative interference cancellation scheme previously described.

Turbo equalization [10] is an iterative equalization and decoding technique that can achieve impressive performance
gains for coded data transmission over intersymbol interference (ISI) channels. Repeating the equalization and decoding tasks on the same set of data and incorporating soft feedback from the decoder into the equalization process generally yields significant improvements in the BER [10]. We assume the channel coefficients of the \(i^{th}\) stream, \(r_{i}(z)\) are known to the receiver and do not vary in time within each block. Figure 3 shows the turbo equalization structure.

\[
\begin{array}{c}
\text{MMSE Equalizer} \\
\downarrow \quad \text{Deinterleaver} \\
\text{MAP Decoder} \\
\downarrow \quad \text{Interleaver} \\
y_n \rightarrow \hat{x}_n \rightarrow L_E(\hat{x}_n) \rightarrow L_c(\hat{x}_n) \rightarrow \text{data estimate}
\end{array}
\]

Fig. 3. Turbo equalization structure

The \(L\) operator is applied to quantities \(x \in \{+1, -1\}\) and is given by
\[
L(x) = \ln(P(x = +1)/P(x = -1))
\] (11)
i.e., the log likelihood ratio (LLR).

A. MMSE Equalizer

The MMSE equalizer computes estimates \(\hat{x}_n\) of the transmitted symbols \(x_n\) from the received symbols \(y_n\) by minimizing the cost function \(E \{(x_n - \hat{x}_n)^2\}\) [5] where \(\hat{x}_n\) represents the soft output from the MMSE equalizer, and \(E \{\cdot\}\) denotes the statistical expectation operator.

For a single channel \(r_{i}(z)\) the channel is denoted as \(h_0 + h_1z^{-1}, \ldots, h_{L-1}z^{-(L-1)}\) where \(L\) represents the channel length. We only consider taps of \(r_{i}(z)\) that are greater than \(\sigma^2/(M_iM_i)\) where \(\sigma^2\) represents the noise variance. We set the length of the equalizer, \(M\), to capture these taps of interest. The channel convolution matrix, \(H\), of dimensions \(M \times (M + L - 1)\) takes the form:
\[
H = \begin{bmatrix}
    h_0 & h_1 & \ldots & h_{L-1} & 0 & \ldots & 0 \\
    0 & h_0 & h_1 & \ldots & h_{L-1} & 0 & \ldots \\
    \vdots & \ddots & \ddots & & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & h_0 & h_1 & \ldots & h_{L-1}
\end{bmatrix}
\]
(12)

Let \(H_{CM}\) denote the column of \(H\) containing the most energy. Assuming symbols are temporally uncorrelated, we write \(E \{(x_n - \hat{x}_n)(x_n - \hat{x}_n)^H\}\) as a diagonal matrix \(\text{diag}(v)\) where the \(n^{th}\) element of \(v\) is \(v_n = 1 - \hat{x}_n^2\) and \(\hat{x}_n\) denotes the interleaved soft estimates of the transmitted symbol from the MAP decoder output. The MMSE weight vector, \(w_n\) is given by [10] [5]:
\[
w_n = (Hc \times \text{diag}(v_n) \times Hc^H + \sigma^2I)^{-1}HcM
\]
(13)

The MMSE equalizer output \(\hat{x}_n\) is used to obtain the difference between the posteriori and a prior LLR as follows:
\[
\Phi_E(\hat{x}_n) = \ln \frac{p\{x_n = +1|\hat{x}_n\}}{p\{x_n = -1|\hat{x}_n\}} - \ln \frac{p\{x_n = +1\}}{p\{x_n = -1\}}
\]
(14)

where \(\Phi\{\hat{x}_n\}\) denotes the real component of the quantity \(\{\hat{x}_n\}\).

B. MAP Decoder

The maximum a posteriori (MAP) algorithm [11] computes the posterior probability of symbols from Markov sources transmitted through discrete memoryless channels. Since the output of a convolutional coder passed through the equalized frequency selective channel forms a Markov source the MAP algorithm can be used for maximum a posteriori probability decoding of convolutionally encoded code [12]. For each transmitted symbol it generates a hard estimate (using thresholding) and soft outputs, \(L_D(c_n), \hat{x}_n\) in the form of the a posteriori probability of the received sequence [13].

C. Optimal Detection Ordering

The performance of the iterative interference cancellation scheme is affected by the order in which the components of \(x\) are detected. An optimal detection ordering scheme (ODO) scheme can significantly improve system performance. This is achieved by swapping the columns of \(H(z)\) and performing the PMQRD. A permutation of the columns of \(H(z)\) exists such that \(\|r_{i,3}\|_F\) is maximal. Wolniansky et al have shown [7] that the column permutation of \(H(z)\) which maximising \(\|r_{i,3}\|_F\) given that \(\|r_{i,1}\|_F\) is already maximal yields the order of optimum detection.

VII. RESULTS

We consider a wide sense quasi stationary (WSQS) situation where the channel coefficients have been assumed to be unchanged within each data block, but allowed to change between data blocks according to a zero mean complex circular Gaussian distribution. WSQS implies that the second-order time statistics of the channel are stationary and is justified in mobile channels over short periods [2]. The bit error rate has been computed for 1000 Monte Carlo simulations. The modulation scheme used is BPSK for evaluation purposes but extension to large constellations is straightforward. The number of time slots of the channel, \(N = 2048\). Initially we assume the receiver has perfect channel knowledge.

We have used a MIMO-OFDM QR scheme as a benchmark. MIMO-OFDM is a DFT based technique that decomposes the otherwise frequency selective channel of bandwidth \(B\) into \(N\) orthogonal frequency flat MIMO channels, each with a bandwidth \(B/N\) [2]. The data stream undergoes the same encoding process as the PMQRD based scheme. Prior to transmission the transmitter performs an inverse fast Fourier transform (IFFT) operation on the signal to be transmitted.
from each individual transmit antenna. A cyclic prefix (CP) of length \((L - 1)\) is then added prior to transmission. At the receiver the cyclic prefix is stripped off and an FFT is applied to the signal received at each antenna. The standard QR decomposition is then applied within each narrowband tone. The iterative cancellation within the receiver is performed on each tone individually and Viterbi algorithm is used in the error correcting decoder. OFDM transmission incurs on account of the cyclic prefix. If \(N \gg L\), this loss is negligible [2] so this has not been considered.

\[
H_t = \begin{bmatrix}
h_{11}(0) & \cdots & h_{M1}(0) \\
h_{11}(1) & \cdots & h_{M1}(1) \\
\vdots & \ddots & \vdots \\
h_{1M_r}(0) & \cdots & h_{M_M_r}(0) \\
h_{1M_r}(1) & \cdots & h_{M_M_r}(1) \\
\vdots & \ddots & \vdots \\
h_{1M_r}(L - 1) & \cdots & h_{M_M_r}(L - 1)
\end{bmatrix}
\]  

(16)

Minimizing the cost function \(\| y_t - S_t H_t \|^2 \) where \(H_t\) denotes the estimate of the channel matrix is now given by the least squares estimator:

\[
H_t = (S_t^H S_t)^{-1} S_t^H y_t
\]  

(17)

Figure 4 shows the average relative error for 1000 Monte Carlo simulations. Even with a relative low SNR it is possible to obtain a reasonably accurate estimation of the channel, for example a relative error of 0.27 at an SNR of 2dB.

\[
E_{\text{rel}} = \frac{\| H_t - \hat{H}_t \|_F}{\| H_t \|_F}
\]  

(18)

Minimizing the cost function \(\| y_t - S_t H_t \|^2 \) where \(H_t\) denotes the estimate of the channel matrix is now given by the least squares estimator:

\[
H_t = (S_t^H S_t)^{-1} S_t^H y_t
\]  

(17)

Figure 4 directly compares the proposed PMQRD and MIMO-OFDM QR schemes for both standard and ODO implementations. BER performance of the PMQRD is far superior to the MIMO-OFDM QR scheme, for example a 5db gain in SNR is observed at BER \(10^{-3}\) when ODO is applied.

A. Channel Estimation Errors

We now consider the scenario where the receiver has imperfect channel knowledge. A training sequence of length \(N_t\), \(s_t(n) \in \mathbb{C}^{M_t \times 1}\) for \(n \in \{0, 1, \ldots, N_t - 1\}\) is propagated through the MIMO wireless channel, \(H(z)\). It is assumed the receiver has prior knowledge of the training sequence. Expressing the received training sequence, \(y_t(n) \in \mathbb{C}^{N_t + L - 1 \times M_r}\) as

\[
y_t = S_t H_t + n
\]  

(15)

where \(S_t\) is an \(N_t + L - 1 \times M_t(L)\) matrix comprised of \(N_t + L - 1 \times (L)\) subblocks, with each subblock representing the convolution matrix of the training sequence transmitted from a given transmit antenna and possessing the well-known Toeplitz form and \(H_t\) is an \(M_t(L) \times M_r\) matrix containing the channel coefficients \(H(z)\) in column vector form, i.e.

\[
E_{\text{rel}} = \frac{\| H_t - \hat{H}_t \|_F}{\| H_t \|_F}
\]  

(18)

Figure 5 shows the average relative error for 1000 Monte Carlo simulations. Even with a relative low SNR it is possible to obtain a reasonably accurate estimation of the channel, for example a relative error of 0.27 at an SNR of 2dB.

\[
E_{\text{rel}} = \frac{\| H_t - \hat{H}_t \|_F}{\| H_t \|_F}
\]  

(18)

\[
E_{\text{rel}} = \frac{\| H_t - \hat{H}_t \|_F}{\| H_t \|_F}
\]  

(18)

Figure 6 shows the impact of channel estimation error on BER performance. Using an identical scheme as previously described we have computed the bit error rate for 1000 Monte Carlo simulations. The modulation scheme used is BPSK. The number of time slots of the channel, \(N = 2048\). The length
of the training sequence, $N_t = 50$. Simulations have been performed for both PMQRD ODO and MIMO-OFDM ODO schemes.

![Graph](image_url)

Fig. 6. Average uncoded BER results for D-BLAST PMQRD and MIMO-OFDM schemes for a 3x3 MIMO channel, $L = 5$, with constant power profile with channel estimation error. All schemes utilize ODO.

Figure 6 again clearly shows the BER performance benefits of the PMQRD ODO scheme. For example a 3.5dB gain in SNR is observed at BER $10^{-3}$.

VIII. CONCLUSION

We have proposed a PMQRD technique for MIMO systems with frequency selective channels and implemented a D-BLAST architecture based communications system.

For the MIMO-OFDM D-BLAST scheme, the information in each individual symbol, is constrained to a single narrow-band tone. Individual tones may have poor gain due to the frequency selective nature of the MIMO channel, resulting in degraded system performance. However in the PMQRD D-BLAST based system the information in each symbol is spread across the entire frequency bandwidth, making the system more robust to frequency selectivity, resulting in superior performance. This makes PMQRD highly suitable for MIMO-QR based applications where the transmitter has no prior channel knowledge, for example digital video broadcasting.

For modulation schemes with constant transmit energy per symbol, the peak-to-average power ratio (PAR) will be unity for PMQRD based schemes. For an identical OFDM based scheme, the IFFT operation at each transmit antenna results in each individual symbol, is constrained to a single narrow-band tone. However in the PMQRD D-BLAST based system the information in each symbol is spread across the entire frequency bandwidth, making the system more robust to frequency selectivity, resulting in superior performance. This makes PMQRD highly suitable for MIMO-QR based applications where the transmitter has no prior channel knowledge, for example digital video broadcasting.

For modulation schemes with variable transmit energy per symbol, the PAR of PMQRD system will still be significantly less than that of OFDM, reducing transmitter complexity and cost.

In addition, signal components originating from bins other than the considered one give rise to interbin interference (IBI) [16]. In the presence of channel estimation error IBI is significantly increased, leading to degraded performance of an MIMO-OFDM based scheme. Although the PMQRD scheme is also susceptible to channel estimation error, its BER performance is still significantly greater, demonstrating the robustness of the proposed PMQRD scheme.

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