Continuous Scanning Laser Vibrometry for Measurements on Rotating Structures

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Abstract. It is readily accepted that a Laser Vibrometer measures target velocity in the direction of the incident laser beam but it is essential that, for correct measurement interpretation, the target velocity be considered in terms of the various target motion components. This paper begins with a review of the theoretical description of the velocity sensed by a dual mirror scanning Laser Vibrometer incident in an arbitrary direction on a rotating target of flexible cross-section undergoing arbitrary vibration. This comprehensive velocity sensitivity model can be applied to any Laser Vibrometer measurement configuration on any target and is sufficiently versatile to incorporate time dependent beam orientation. This is described in this paper with particular reference to continuous circular scanning Laser Doppler Vibrometry.

The velocity sensitivity model allows a detailed analysis of the form of the measurement that is obtained in actual scan configurations. For example, additional components occur in a circular scanning Laser Vibrometer measurement on a rotating target that can be shown to be due to a combination of instrument configuration and target misalignment. In this paper, the measured data obtained from a circular scanning measurement on a rotating target undergoing axial vibration is investigated as a means of demonstrating the usefulness of the comprehensive velocity sensitivity model.

Introduction

The principle of Laser Doppler Vibrometry (LDV) relies on the detection of a Doppler shift in the frequency of coherent light scattered by a moving target, from which a time-resolved measurement of the target velocity is made. The Laser Vibrometer is now well established as an effective non-contact alternative to the use of a traditional contacting vibration transducer. Laser Vibrometers are technically well suited to general application but offer special benefits where certain measurement constraints are imposed, for example by the context, which may demand high frequency operation, high spatial resolution or remote transducer operation, or by the structure itself, which may be hot, light or rotating. Measurements on such structures are often cited as important applications of LDV [1] and scanning LDV is of particular current interest.

Commercial scanning Laser Vibrometers incorporate two orthogonally aligned mirrors, and can operate point by point or in continuous scanning mode [2]–[4], a special case of which is tracking mode in which the probe laser beam remains fixed on a single point on a target such as a rotating bladed disc [5]. Throughout the remainder of this document, “scanning” LDV refers to operation in continuous scanning mode.

Recent work resulted in the extension [6],[7] of a totally general theoretical description of the velocity measured by a single laser beam incident in an arbitrary direction on a rotating target undergoing arbitrary motion [8] to the particularly challenging application of continuous scanning Laser Vibrometer measurements on rotating targets with flexible cross-sections. The original velocity sensitivity model was written in terms of laser beam orientation angles, whilst the more recent version was reformulated to make use of mirror scan angles, which is especially useful as it
is these that the user would seek to control in a real scanning system. The advanced technique of circular scanning on rotating targets was investigated as a means of illustrating the effectiveness of the model for the analysis of actual scan configurations. In particular, the origins of the additional components that occur in measured data due to instrument configuration were easily revealed using the velocity sensitivity model.

**Velocity Measured by a Dual Mirror Scanning Laser Vibrometer**

With reference to Fig. 1, a typical scanning measurement is performed by the introduction of two orthogonally aligned mirrors, separated by some distance $d_s$, into the beam path.

![Fig. 1 – The dual mirror scanning arrangement](image)

The mirror scan angles, $\theta_{sx}$ and $\theta_{sy}$, are defined as positive if in an anticlockwise sense and “zero” when the resulting laser beam direction, described by the unit vector $\hat{b}$, is along the scanning system optical axis ($z$ axis). Expressing the orientations of the mirror surfaces as unit vectors, it is possible to calculate the laser beam direction for any combination of mirror scan angles and thereby derive an expression for the velocity measured, $U_m$, in a scanning Laser Vibrometer measurement on a rotating target of flexible cross-section undergoing 6 degree-of-freedom vibration [6]:

$$
U_m = \sin 2\theta_{sx} \left[ \dot{x}_r(P_0) + \dot{x}_f(P) \right] \\
- \cos 2\theta_{sx} \sin 2\theta_{sy} \left[ \dot{y}_r(P_0) + \dot{y}_f(P) \right] \\
+ \cos 2\theta_{sx} \cos 2\theta_{sy} \left[ \dot{z}_r(P_0) + \dot{z}_f(P) \right]
$$

(1)

in which $\dot{x}_f(P)$, $\dot{y}_f(P)$, $\dot{z}_f(P)$ are the vibration velocity components in the $x$, $y$, $z$, directions due to cross-section flexibility (the point $P$ represents the instantaneous point of incidence of the laser beam on the arbitrarily deformed target) and $\dot{x}_r(P_0)$, $\dot{y}_r(P_0)$, $\dot{z}_r(P_0)$ are the resultant vibration velocity components in the $x$, $y$, $z$ directions due to rigid body vibration (the point $P_0$ represents the corresponding point on the undeformed target). These components are given by:
\[ \dot{x}_s(P_0) = \dot{x} - (\dot{\theta}_z + \Omega)(y_0 - y) + (\dot{\theta}_x - \Omega \theta_y)(z_0 - z), \quad (2a) \]
\[ \dot{y}_s(P_0) = \dot{y} + (\dot{\theta}_z + \Omega)(x_0 - d_z \tan 2\theta_{sx} - x) - (\dot{\theta}_x + \Omega \theta_y)(z_0 - z) \]
\[ \dot{z}_s(P_0) = \dot{z} + (\dot{\theta}_z + \Omega \theta_y)(y_0 - y) - (\dot{\theta}_x - \Omega \theta_y)(x_0 - d_z \tan 2\theta_{sx} - x). \quad (2c) \]

Here, \( \dot{x}, \dot{y}, \dot{z} \) and \( x, y, z \) are the translational vibration velocities and displacements, \( \dot{\theta}_x, \dot{\theta}_y, \dot{\theta}_z \) and \( \theta_x, \theta_y \) are the angular vibration velocities and displacements and \( \Omega \) is the total target rotation frequency of the target. \((x_0 - d_z \tan \theta_{dy}, y_0, z_0)\) is the position of an arbitrary known point that lies along the line of the beam. In this case the incidence point of the laser beam on the \( y \) deflection mirror is a convenient point to take \[6\].

The development of Eq. (1) is significant since it can be conveniently employed to make the analysis of complex measurement configurations more straightforward. In particular, applications in which the laser beam is scanned can be investigated by considering a time dependent known point position. Such applications are important; a circular scanning Laser Vibrometer, for example, can be used to measure the velocity profile along a predetermined path on a rotating disc in a single measurement. Post-processing of the Laser Vibrometer output signal results in a series of coefficients that describe the operational deflection shape or the mode shape \[2\] – \[4\].

**Axial Vibration Measurement using Circular Scanning LDV.**

The depth of information offered by the velocity sensitivity model will be demonstrated in this section by examining the form of the instrument output when making several circular scanning Laser Vibrometer measurements on a rotating target of rigid cross-section undergoing medium severity axial vibration. Fig. 2 shows the custom built scanning Laser Vibrometer and rotating target that were used to obtain these measurements. The arrangement used enabled independent control of the vibration amplitude and frequency and the rotation frequency.

![Fig. 2 – Custom built scanning Laser Vibrometer](image-url)
Firstly, however, the form of several more straightforward Laser Vibrometer measurements on the same target will be discussed such that the various artefacts that occur in the full measurement can be attributed to particular sources. Fig. 3 shows the velocity measured in a non-scanning Laser Vibrometer measurement on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration. This straightforward measurement constitutes a baseline for the scanning measurements, illustrating the vibration peak at 40Hz, as well as the genuine low level harmonic distortions at 80Hz and 120Hz. The underlying instrument noise floor contains peaks at 50Hz and 100Hz, which are caused by electrical interference and, as such, are present in all measured spectra presented in this section.

Fig. 3 – Velocity measured by a Laser Vibrometer on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration

Fig. 4 shows the velocity measured in a circular scanning Laser Vibrometer measurement on a non-rotating target undergoing nominally the same axial vibration. A circular scanning Laser Vibrometer measurement can be performed by deflecting the laser beam through suitable angles around two orthogonal axes simultaneously, typically by using cosine and sine functions [6],[7] of the form:

$$\theta_{sx} = -\Theta_{sx} \cos(\Omega_st + \phi_s)$$  \hspace{1cm} (3a)$$

and

$$\theta_{sy} = \Theta_{sy} \sin(\Omega_st + \phi_s)$$  \hspace{1cm} (3b)$$

where $\Theta_{sx}$ and $\Theta_{sy}$ are the $x$ and $y$ mirror scan amplitudes and $\Omega_s$ and $\phi_s$ are the scan angular frequency and initial phase.

Scanning the laser beam causes speckle pattern motions [9] during the course of the measurement that manifest themselves clearly in the spectrum at integer multiples of scan frequency [7]. The peak at 1x scan frequency is at a higher level as a result of the additional Doppler shifts from the scanning mirrors which occur due to offset between the laser beam and the mirror rotation axes in this custom built scanning Laser Vibrometer [7]. Despite these sources of additional velocity content, the spectrum represents a respectable measurement of the 40Hz axial vibration and its harmonic distortion. Worthy of note is the fact that the peaks at 40, 80 and 120Hz appear slightly broader because the vibration frequency and rotation frequency are controlled independently and, although the target motion was arranged for this measurement to be so, the
vibration frequency was not an exact integer multiple of the rotation frequency. In this measurement, these peaks contain velocity content associated with both vibration and speckle noise.

![Graph showing velocity measured by a dual mirror circular scanning Laser Vibrometer on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration.](image)

**Fig. 4** – Velocity measured by a dual mirror circular scanning Laser Vibrometer on a non-rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration ($r_S = 12.5\text{mm}$, $d_S = 50\text{mm}$, $z_0 = 1\text{m}$, $\Omega_S = 20\pi\text{ rad/s}$, arbitrary misalignment)

Fig. 5 shows a circular scanning Laser Vibrometer measurement on a rotating, non-vibrating target in which the rotation and target frequencies are exactly 10 and 12.5Hz, respectively. In this case, a speckle pattern repeat of 2.5Hz is expected and this is clearly shown in the data. Care must be taken when measurements contain noise associated with the laser speckle effect, as the resulting speckle pattern repeat frequency may not be as straightforward and data misinterpretation can easily occur.

![Graph showing velocity measured by a dual mirror circular scanning Laser Vibrometer on a rotating, non-vibrating target.](image)

**Fig. 5** – Velocity measured by a dual mirror circular scanning Laser Vibrometer on a rotating, non-vibrating target ($r_S = 50\text{mm}$, $d_S = 50\text{mm}$, $z_0 = 250\text{mm}$, $\Omega_S = 25\pi\text{ rad/s}$, $\Omega \approx 20\pi\text{ rad/s}$, arbitrary misalignment)

In the case of the full measurement, i.e. a circular scanning Laser Vibrometer measurement on a rotating target, that is illustrated in Fig. 6, the introduction of target rotation into the measurement should result in a general difference in the resulting speckle noise. For a 10Hz scan on a target rotating at 20Hz, a speckle pattern repeat of 10Hz is still expected but, since the rotation frequency is only nominally 20Hz in this measurement, the speckle pattern does not repeat perfectly at 10Hz.
Despite this, higher speckle noise levels can be seen at approximately integer multiples of 10Hz in the real measurement.

Fig. 6 – Velocity measured by a dual mirror circular scanning Laser Vibrometer on a rotating target undergoing 40Hz, 10mm/s (nominal) axial vibration ($r_S = 12.5$mm, $d_S = 50$mm, $z_0 = 1$m, $\Omega_S = 20\pi$ rad/s, $\Omega \approx 40\pi$ rad/s, arbitrary misalignment)

More importantly, Fig. 6 shows significantly higher velocity levels at 1x scan and 2x scan frequency that, without prior knowledge of the target vibration, could easily be incorrectly identified as vibration associated peaks. It is, however, possible to predict the form of the instrument output for any measurement configuration by making use of the velocity sensitivity model. Using small angle approximations and setting the flexible vibration parameters to zero and substituting the necessary rigid vibration and initial misalignment parameters in Eqs. (2a,b&c) into Eq. (1) as well as substituting for $\theta_{sx}$ and $\theta_{sy}$ using Eqs. (3a&b) enables an expression for the total velocity measured during this Laser Vibrometer configuration:

$$
U_m = \Omega \left[ \theta_{sx} x_{0m} + \theta_{sy} y_{0m} \right] \\
+ 2\Omega \theta_{sx} \left[ y_{0m} + \theta_{sx} (z_0 + d_S) \right] \cos(\Omega_S t + \phi_S) - 2\Omega \theta_{sy} \left[ x_{0m} - \theta_{sy} z_0 \right] \sin(\Omega_S t + \phi_S) \\
- 2\Omega \theta_{sx} \theta_{sy} d_S \sin 2(\Omega_S t + \phi_S) \\
+ \dot{Z} \cos(\omega_z t + \phi_z) \\
- 2\Omega \theta_{sx} \theta_{sy} \frac{\dot{Z}}{\omega_z} \left[ \sin((\omega_z + \Omega_S) y + \phi_z + \phi_S) + \sin((\omega_z - \Omega_S) y + \phi_z - \phi_S) \right] \\
- 2\Omega \theta_{sy} \theta_{sx} \frac{\dot{Z}}{\omega_z} \left[ \cos((\omega_z - \Omega_S) y + \phi_z - \phi_S) - \cos((\omega_z + \Omega_S) y + \phi_z + \phi_S) \right],
$$

(4)

where $x_{0m}$, $y_{0m}$, $\theta_{sx}$ and $\theta_{sy}$ represent the two translational and two angular misalignments, respectively, between the scanning system and target rotation axes, $\dot{Z}$, $\omega_z$, $\phi_z$ are the velocity amplitude, angular frequency and initial phase, respectively, of the axial vibration.

Fig. 6 represents a respectable measurement of the target vibration and this is confirmed by the theoretical prediction of the instrument output which is shown in Fig. 7 for a typical scanning arrangement with typical misalignment values. The harmonic distortion of the axial vibration is not included in this velocity prediction.
It is not possible to quantify the initial unknown misalignments in a real measurement but it should be possible to estimate typical expected values from the measurement configuration and therefore produce an order of magnitude prediction for the DC and 1x additional components (1st, 2nd and 3rd terms in Eq. (4)) \[7\]. The 2x additional component (4th term in Eq. (4)) can be estimated with a high degree of accuracy in a real measurement, since it is insensitive to misalignment \[6\],[\(7\)]. As shown in Fig. 7, the small sidebands (6th and 7th terms in Eq. (4)) are present in the predicted instrument output at the vibration frequency \(\pm \Omega_s\). These are caused by the misalignments and are generally very low-level components, below the real instrument’s noise floor and therefore insignificant \[7\].

In the measurement discussed in this section, the scan frequency, rotation frequency and vibration level and frequency were chosen such that the additional measurement content did not constitute a significant source of measurement ambiguity. In a real measurement such control is obviously not available and, for example, if the axial vibration level were lower or at a different frequency, then it might be less straightforward to distinguish genuine vibration peaks from additional content peaks. In such a situation, changing the scan frequency might be useful since the frequency of additional content peaks would change in sympathy whilst the frequency of the vibration peaks would remain constant. As previously mentioned, caution must be exercised as the resulting change in the speckle pattern repeat may not be straightforward.

The information presented in this paper provides the user with the ability to predict the additional components that occur in real scanning Laser Vibrometer measurements and thereby anticipate the form of the resulting spectra, of which Fig. 6 is a typical example. Such measurements can then be interpreted with confidence.

**Conclusions**

The use of Laser Vibrometers incorporating some form of manipulation of the laser beam orientation, typically using two orthogonally aligned mirrors, has become increasingly popular in recent years. This paper has reviewed a recently developed theoretical model for the prediction of the velocity sensed by a Laser Vibrometer in continuous scanning mode.

The velocity sensitivity model was presented in such a manner that it can be straightforwardly implemented to predict the Laser Vibrometer output for any system configuration and any combination of mirror scan angles. In the case of circular scanning measurements on rotating targets, the occurrence of significant additional components at DC and integer multiples of the scan frequency is predicted.
frequency is due to both the dual mirror arrangement and misalignment between the scanning system and target rotation axes.

The influence of noise generated by the laser speckle effect and the effect of offset between the laser beam and either one or both of the deflection mirror rotation axes were highlighted as sources of additional measured velocity in scanning LDV. Finally, a prediction of the Laser Vibrometer output was compared with a real circular scanning measurement on a rotating target undergoing a medium severity axial vibration. The correlation between predicted and measured data was strong and the usefulness of the model in enabling confident interpretation of the measurement was thereby confirmed.

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