A novel re-initialization technique for CMA in the presence of channel noise

This item was submitted to Loughborough University's Institutional Repository by the author.


Additional Information:

- This is a conference paper [© IEEE]. It is also available at: http://ieeexplore.ieee.org/ Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or Redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

Metadata Record: https://dspace.lboro.ac.uk/2134/5814

Version: Published

Publisher: © IEEE

Please cite the published version.
This item was submitted to Loughborough’s Institutional Repository (https://dspace.lboro.ac.uk/) by the author and is made available under the following Creative Commons Licence conditions.

You are free:

- to copy, distribute, display, and perform the work

Under the following conditions:

**Attribution**. You must attribute the work in the manner specified by the author or licensor.

**Noncommercial**. You may not use this work for commercial purposes.

**No Derivative Works**. You may not alter, transform, or build upon this work.

For any reuse or distribution, you must make clear to others the license terms of this work.

Any of these conditions can be waived if you get permission from the copyright holder.

Your fair use and other rights are in no way affected by the above.

This is a human-readable summary of the Legal Code (the full license).

[Disclaimer](#)

For the full text of this licence, please go to:

http://creativecommons.org/licenses/by-nc-nd/2.5/
A NOVEL RE-INITIALIZATION TECHNIQUE FOR CMA IN THE PRESENCE OF CHANNEL NOISE

S. Lambotharan, A.G. Constantinides, J.A. Chambers and K. Skourratanamont

Signal Processing and Digital Systems Section
Department of Electrical and Electronic Engineering
Imperial College, London, SW7 2BT
E-mail: s.lambotharan@ic.ac.uk

ABSTRACT
The error surface associated with the Constant Modulus Algorithm (CMA) is multimodal. Therefore, an equalizer adapted with CMA has the potential to converge to a local minimum that has large noise amplification. A technique is, therefore, proposed based upon a solution of linear and quadratic equations to re-initialise the equalizer parameters. Linear equations are obtained from the fact that the retrieved transmitted signal with various delays which correspond to different minima are uncorrelated. The quadratic equation is obtained from the knowledge of the transmitted signal power. Simulations are included to demonstrate the robustness of this new scheme.

1. INTRODUCTION
The Fractionally Spaced Constant Modulus Algorithm (FS-CMA) does not posses local minima for an equalizer which models an inverse of a channel completely and in the absence of noise. All minima associated with this FS-CMA are of equal depth and they correspond to the retrieval of the transmitted signal with various delays and sign. However, when there is channel noise, as this noise is amplified by an amount which is equal to the squared $l_2$ norm of the equalizer impulse response, various minima behave differently and the minima with smallest equalizer norm are optimal. We provide a re-initialisation technique to obtain convergence to the global minima for CMA.

The first work in this context appeared in [8], where a channel surfing and re-initialisation scheme was proposed. However, this method requires inversion of a matrix. Another method is a Gram-Schmidt orthogonalisation based adaptive algorithm for which a bank of at least two parallel equalizers is required [7]. A reference equalizer is adapted with CMA while the second equalizer parameters are adapted according to a Gram-Schmidt orthogonalisation procedure, so that the second equalizer output does not have any contribution from the delayed transmitted signal that is being retrieved by the first equalizer. The Cross-Correlation and Constant Modulus Algorithm (CC-CMA) that was proposed for the simultaneous retrieval of multi-user transmitted signals, [5] and [6], was recommended for the optimum delay selection in [1]. A reference equalizer is adapted with the CMA algorithm and an another equalizer is adapted with the CC-CMA algorithm. The crosscorrelation term ensures that the second equalizer retrieves the transmitted signal with a particular delay that is different from the delay introduced by the reference equalizer. The first equalizer is initialised with the second one, if the second equalizer performs better than the first. The searching continues until the equalizer parameter with optimum delay is found. One other
technique has been proposed based upon a fact that the equalizer parameter norm associated with the global minimum is small, and the following cost (for BPSK) is proposed to obtain global convergence, [2], [3] and [4].

\[ J = E\{(y^2(k) - 1)^2\} + \kappa w^T w \]  

(1)

where \( w \) is the equalizer impulse response vector and \( \kappa \) is a mixing parameter. The algorithm which aims to minimise (1) is a leaky constant modulus algorithm and this method does not require any searching procedure nor more than one equaliser, but the drawback is the bias in the position of the minima.

The technique introduced in this paper is a block based method to provide equalizers which retrieve the transmitted signal with various delays which are different from the delay that is introduced by the reference equalizer. A matrix inversion as in [8] is required, but unlike as in [8], this scheme is also applicable to sub-equalizers of length 1, e.g. a beamformer.

2. RE-INITIALISATION SCHEME

The equalizer structure for the re-initialisation scheme is depicted in Figure 1. Let the impulse response vector of the channel, equalizer-1 and equalizer-2 respectively be \( c \), \( w_1 \) and \( w_2 \). The equalizer-1 is adapted with CMA. Assume that the equalizer-1 retrieves the transmitted signal with delay \( d \), i.e. \( y_1(k) \) is approximately equal to \( s(k-d) \). Let \( x_k \) be the equalizer regressor vector. We desire to initialise equalizer-2 such that its output does not have any contribution from \( s(k-d) \). Suppose we want \( y_2(k) \) to be approximately equal to \( s(k-d+\tau) \), then

\[ E\{y_2(k)y_1(k+i)\} = \begin{cases} 0 & i \neq \tau \\ \sigma_s^2 & i = \tau \end{cases} \]  

(2)

\[ w_2^T R_1 w_1 = \begin{cases} 0 & i \neq \tau \\ \sigma_s^2 & i = \tau \end{cases} \]  

(3)

where \( R_1 = E\{x_k x_{k+i}\} \) and \( \sigma_s^2 \) is the power of the transmitted signal. For a channel of order \( M \) and an equalizer of order \( N \), \( 0 \leq d \leq (M+N) \). Therefore, in order to consider all possible delays, the range for \( i \) should be \(- (M+N) \leq i \leq (M+N) \). Write \( R_1 w_1 = u_1 \) which is also equal to \( E\{x_k y_1(k+i)\} \). From eqn.(3), we have \( u_1^T w_2 = 0 \). \( i \neq \tau \) and \( u_1^T w_2 = \sigma_s^2 \). \( i = \tau \). In addition to this set of equations, we have the power normalisation which yields

\[ w_2^T w_2 = \sigma_s^2 \]  

(4)

Solving eqn.(3) and eqn.(4) means finding the intersection of a set of hyper-planes with a hyper-ellipsoid. Write \( p = M+N \). \( U = [u_{-p} \cdots u_{-1} u_0 u_{r+1} \cdots u_p]^T \). The Least Square (LS) solution is

\[ w_2 = (U^T U)^{-1} U^T e_v \]  

(5)

where \( e_v \) denotes a vector with 1 in the \( v \)th position and zeros elsewhere. e.g., \( e_2 = [0 0 1 0 \cdots 0]^T \) and \( v-p = \tau \). Considering the power normalisation of eqn.(4), \( w_2 \) is written as

\[ w_2 = \frac{\sigma_s^2}{w_2^T R_1 w_2} \]  

(6)

and the second equaliser is initialised at \( w_2 \). The square error associated with the LS solution of (5) is

\[ \sigma_v^2 = 1 - e_v^T U(U^T U)^{-1} U^T e_v \]  

(7)

Various initialisations are obtained for different values of \( v \) except for \( v = p \). Those initialisations such that \( \sigma_v^2 \) is approximately unity are discarded as \( \sigma_v^2 \approx 1 \) indicates that the corresponding delay is outside the possible range of delay that the channel and the second equalizer can span, i.e. \( d - \tau \notin (0, M+N) \).
3. SIMULATIONS

Three simulations were performed to verify the potential of this method. First, a fractionally spaced channel was simulated as sub-channel-1 $= [0.8944 \quad 0.4472]$ and sub-channel-2 $= [0.8944 \quad -0.4472]$. The noise power at each sensor is 0.1, i.e., SNR = 10 dB. There are two local minima and two global minima. The equalizer is initialised at $[0.5 \quad -0.5]$ and adapted with CMA, and the equalizer converges to a local minimum where the Mean Square Error (MSE) is measured to be 0.2155. The equalizer is then re-initialised which is marked by $\star$ (Figure 2), and this initialisation is very close to the global minimum where the MSE is measured to be 0.0699.

The second example considers a baud spaced channel $[0.8944 \quad 0.4472]$ and an equalizer of length 2. The equalizer is initialised at $[0 \quad 1]$ and adapted with CMA. Convergence to a local minimum is observed. Figure 3. SNR is 10 dB. An initialisation point which is marked by $\star$ is obtained with our proposed method which leads CMA to a global minimum. The MSE at the local minimum and the global minimum are 0.2661 and 0.1759.

The final example considers a fractionally spaced channel of length 4, and an equalizer of length 3. Therefore, there are 6 possible delays. Suppose, we know the channel impulse response, a Minimum Mean Square Error (MMSE) equalizer for a desired delay $d$, can be designed as follows

$$
\mathbf{w} = (\mathbf{C}^T\mathbf{C} + \frac{\sigma^2}{\sigma^2_s})^{-1}\mathbf{C}^T\mathbf{e}_d
$$

where $\mathbf{C}$ is the channel convolution matrix, $\sigma^2$ and $\sigma^2_s$ are respectively the signal and the noise power. The MSE at this delay $d$ is given by

$$
\sigma^2_e = \sigma^2(1 - e^T \mathbf{C}(\mathbf{C}^T\mathbf{C} + \frac{\sigma^2}{\sigma^2_s})^{-1}\mathbf{C}^T\mathbf{e}_d)\sigma^2
$$

For a sub-channel-1 $= [0.7782 \quad 0.0642 \quad -0.2502 \quad -0.5725]$ and sub-channel-2 $= [-0.1499 \quad 0.7902 \quad 0.4599 \quad -0.3798]$. MSEs of the MMSE equalizers which correspond to 6 different delays were obtained. Figure 4. The optimum delay is 1. The equalizer is initialised at $[-0.0374 \quad -0.1951 \quad -1.0311 \quad 0.2777 \quad 0.6906 \quad -0.3385]$ which is the MMSE equalizer that corresponds to the worst delay, i.e., 5, and adapted with CMA. This equalizer converges to a local minimum where MSE is large (0.2923). The re-initialization parameters were obtained according to eqn.(5). Eqn.(7) indicates that the obtained initial CM solution is for delay 5, and five other solutions can be obtained from the first five columns of $(\mathbf{U}^T\mathbf{U})^{-1}\mathbf{U}^T$. A bank
of five parallel equalizers was initialized with these parameters, and adapted with CMA. The MSE at those minima are depicted in Figure 4. The MSEs at the global minimum is 0.1242. A sequential search can also be performed instead of employing these five parallel equalisers, and in this case, the parameters that correspond to the smallest element of $\sigma_k^2$ can be used as an initial guess.

5. REFERENCES


4. CONCLUSION

We proposed a crosscorrelation based re-initialisation scheme for constant modulus algorithms to obtain convergence to a global minimum. This method involves estimation of the autocorrelation matrix of different delays, and performing an inverse of a matrix; hence, it has a computational complexity that is similar to [8], but provides re-initialization parameters for all possible delays simultaneously. Unlike [8], this scheme is also capable of providing solutions for FSE of length (sub-equalizer) one, i.e., a beam former. We demonstrated that these methods can be successfully applied to an FSE to obtain global convergence where the noise amplification is small. Moreover, this method can also be applied to obtain possible global convergence for a baud spaced equalizer.