Extended orthogonal space
time block codes in wireless
relay networks

This item was submitted to Loughborough University’s Institutional Repository by the/an author.


Additional Information:

• This is a conference paper [© IEEE]. It is also available at: http://ieeexplore.ieee.org/ Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.

Metadata Record: https://dspace.lboro.ac.uk/2134/5843

Version: Published

Publisher: © IEEE

Please cite the published version.
This item was submitted to Loughborough’s Institutional Repository (https://dspace.lboro.ac.uk/) by the author and is made available under the following Creative Commons Licence conditions.

For the full text of this licence, please go to: http://creativecommons.org/licenses/by-nc-nd/2.5/
This item was submitted to Loughborough’s Institutional Repository (https://dspace.lboro.ac.uk/) by the author and is made available under the following Creative Commons Licence conditions.

For the full text of this licence, please go to:
http://creativecommons.org/licenses/by-nc-nd/2.5/
EXTENDED ORTHOGONAL SPACE TIME BLOCK CODES IN WIRELESS RELAY NETWORKS

F. T. Alotaibi and J. A. Chambers

Advanced Signal Processing Group, Department of Electronic and Electrical Engineering
Loughborough University, Loughborough LE11 3TU, UK
Email: {F.Alotaibi, J. A. Chambers}@lboro.ac.uk

ABSTRACT

In this paper we propose complex extended orthogonal space-time block codes (EO-STBCs) with feedback for wireless relay networks with the assumption of quasi-static flat fading channels. Full rate in each stage and full cooperative diversity for distributed EO-STBCs (D-EO-STBCs) are achieved by providing channel state information (CSI) at certain relay nodes. Two closed-loop schemes are proposed which make use of limited feedback from the destination node to a particular number of relay nodes, not exceeding half of the total number of such relay nodes. In our simulations, we use four relay nodes. Simulation results show that these two closed-loop D-EO-STBCs achieve full cooperative diversity in addition to array gain with linear processing. In particular, the proposed D-EO-STBCs designs preserve low decoding complexity and save both transmission power and total transmit time between source and destination.

Index Terms— Extended orthogonal space-time block codes (EO-STBCs), orthogonal designs, wireless relay network, cooperative diversity, partial feedback.

1. INTRODUCTION

Space-time coding (STC) is an effective technique to exploit spatial diversity not only for multiple-input multiple-output (MIMO) point-to-point systems but also for wireless relay systems [1], thereby improving the reliability, energy usage and throughput. One of the attractive approaches for STC is space-time block codes (STBCs) because of their potential low computational complexity in decoding. They can be divided into two main classes, namely, orthogonal and non-orthogonal, which includes quasi-orthogonal codes [2]. However, orthogonal STBCs are especially promising because full diversity is achieved while a very simple symbolwise maximum-likelihood decoding algorithm can be used at the decoder. The first real/complex orthogonal space-time block code was proposed by Alamouti [3] for two transmit antennas. In [4] STBCs for more than two transmit antennas from orthogonal designs were proposed. It has been shown in [4] that a complex orthogonal design that provides full diversity and full transmission rate is not possible for more than two antennas. In fact, the Alamouti scheme is the only complex orthogonal STBC that achieves both full transmission rate and full diversity gain. A quasi-orthogonal space-time block code (QO-STBC) is designed in [5] to achieve full transmission rate at the expense of loss in diversity gain and increasing the decoding complexity. Many methods have been proposed for designing STBC schemes with a full rate and full diversity for more than two transmit antennas for example see [6], [7], [8], [9], [10], and [11]. One of these schemes is the extended orthogonal STBC based on closed-loop operation as in [6], which was proposed for three and four transmit antennas, and as in [7], for four transmit antennas. In these two schemes the transmitter exploits channel state information to achieve full transmission rate and full diversity advantage combined with transmit array gain.

In order to exploit the benefits of STBC on transmission energy efficiency and robustness, STBCs were extended into distributed wireless relay networks, which is shown in [1] to be possible by exploiting the cooperating capability of relay nodes. In [12], orthogonal and quasi-orthogonal STBCs, which were originally used in multiple-antenna systems, were proposed for distributed STBC in wireless relay networks but without achieving full rate in each stage and full diversity at the same time for more than two relay nodes.

In this paper we extend orthogonal STBCs to wireless relay networks because they can achieve both full rate in each stage and full cooperative diversity gain in addition to array gain. The remainder of the paper is organized as follows. In Section 2, EO-STBCs are described and their extension to relay networks is introduced and analyzed. In Section 3, the simulation results are presented. Finally, conclusions are drawn in Section 4.

Fig. 1. Wireless relay network with single source and destination together with a relay stage.

2. EXTENDED ORTHOGONAL STBCs

Consider a wireless relay system with one source node, one destination node, and \( R \) relay nodes, as shown in Fig. 1. Every node in the system has only one antenna. There is no direct link between the transmitter and destination receiver. Denote the fading coefficient from the source node to the \( i \)th relay as \( h_{SR_i} \), and the fading coefficient from the \( i \)th relay to the destination node as \( h_{RD_i} \). Assume the channel between any two terminals is quasi-static flat Rayleigh fading. Therefore, assume that \( h_{SR_1} \) and \( h_{RD_1} \) are independent com-
plex Gaussian random variables with zero-mean and unit-variance. To transmit the information from the source node to the destination node, they undergo two stages. From time $1$ to $T$, which is the first stage, the source node broadcasts the information $s = [s_1, ..., s_M]^T$ after modulation onto complex symbols to the relay nodes, $R_i$, where $(.)^T$ denotes vector transpose. The received signal at the $i$th relay is denoted as $y_i$, which is corrupted by both the fading coefficient $h_{SR_i}$ and the noise $n_i$. Meanwhile, the relay nodes $(R)$ receive the information where $R = 4$ in this paper. From time $T+1$ to $2T$, which is the second stage, the source node stops transmission and the relay nodes, which operate in amplify-and-forward (AF) strategy, send the received noisy signals after processing denoted $t_i$ to the destination node. We represent the received signal and noise at the destination receiver by $r$ and $w$, respectively. The noises are assumed to be i.i.d. zero-mean and unit-variance complex Gaussian random variables. Clearly \[
 y_i = \sqrt{P_i} h_{SR_i} s + n_i \quad \text{and} \quad r = \sum_{i=1}^{R} h_{R_i, D} t_i + w \quad (1) \]

where $P_i$ denotes the average transmission power at the source node. Then the mean power of the signal $y_i$ at a relay node is $P_i + 1$ due to the unit variance assumption of the additive noise $n_i$. Let $P_2$ denote the average transmission power at every relay node. The relay nodes will process and transmit the received noisy signals. The transmit signal at the $i$th relay is designed to be a linear function of its received signal and its conjugate where $(.)^*$ denotes the complex conjugation. \[
 t_i = \frac{P_2}{P_1 + 1} (A_i y_i + B_i y_i^*) \quad (2) \]

so, \[
 t_i = \frac{P_2 P_1 T}{P_1 + 1} (h_{SR_i} A_i s + h_{SR_i} B_i s^*) + \frac{P_2}{P_1 + 1} (A_i n_i + B_i n_i^*) \]

where $A_i$ and $B_i$ are the matrices used at the $i$th relays. In our system we have used four relays. These relay nodes are designed to use the following matrices: \[
 A_1 = A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_3 = A_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad 02 \]

$\quad B_1 = B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad 02 \quad B_3 = B_4 = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad 0 \quad 02 \]

The matrix $A_i = 0 \quad 2$ means that the $i$th column of the code matrix contains the conjugate of the information $s$ while the matrix $B_i = 0 \quad 2$ means that the $i$th column contains the information $s$. The matrix $0 \quad 2$ denotes the $2 \times 2$ matrix with all zeros. Define \[
 \hat{A}_i = A_i, \quad \hat{h}_{SR_i} = h_{SR_i}, \quad \hat{n}_i = n_i, \quad \hat{A}_i s = s, \quad \text{if} \quad B_i = 0 \]

\[
 \hat{A}_i = B_i, \quad \hat{h}_{SR_i} = h_{SR_i}^*, \quad \hat{n}_i = n_i^*, \quad \hat{s}^* = s^*, \quad \text{if} \quad A_i = 0 \quad 0 \]

from (2), we have \[
 t_i = \sqrt{P_2 P_1 T} \hat{h}_{SR_i} \hat{A}_i s + \sqrt{\frac{P_2}{P_1 + 1}} \hat{A}_i \hat{n}_i \]

with the second equation in (1), the received signal can be calculated to be \[
 r = \sqrt{\frac{P_2 P_1 T}{P_1 + 1}} S H + W \quad (3) \]

where \[
 S = \begin{bmatrix} \hat{A}_1 s^1 & \cdots & \hat{A}_R s^R \end{bmatrix}^T \quad H = \begin{bmatrix} h_{SR_1} h_{R_1, D} & \cdots & h_{SR_R} h_{R_R, D} \end{bmatrix}^T \quad W = \sqrt{\frac{P_2}{P_1 + 1}} \sum_{i=0}^{R} h_{R_i, D} \hat{A}_i \hat{n}_i + w \]

Therefore, without decoding, the relays generate a space-time code word $S$ distributively at the receiver. $H$ is the equivalent channel and $W$ is the equivalent noise. If $H$ is known at the receiver, the maximum-likelihood (ML) decoding is \[
 \arg \min_s \| X - \sqrt{\frac{P_2 P_1 T}{P_1 + 1}} S H \|_F \]

where $\| . \|_F$ indicates the Frobenius norm. We adopt the optimum power allocation in [13] in our proposed scheme as next described. Denote $P$ as the total transmission power in the whole scheme, so $P_1 = \frac{P}{4}$ and $P_2 = \frac{P}{4 \pi}$ where $R$ is the number of the relay nodes.

2.1. Open-Loop D-EO-STBCs

Initially, we assume there is no channel information at the relays but full channel information at the receiver. First, we define the form of the D-EO-STBC code word $S$ that we want to generate at the destination receiver. It has the following form [6] \[
 S = \begin{bmatrix} s_1 & s_1 & s_2 & s_2 \\ -s_2 & -s_2 & s_1 & s_1 \end{bmatrix} \quad (4) \]

so, we defined the information sequence which is encoded at the source node as $s = [s_1 - s_2]^T$, as in [12]. It is clear that the extended code word we select, as in [6], is built from the well known Alamouti code. Also, the extended code word has scale-free property [6]. In the case of four relays, the received signal, $r$, will be written as $r = \sum_{i=1}^{4} h_{R_i, D} t_i + w$ and since the coherence interval $T=2$, therefore \[
 r = \begin{bmatrix} r_1 & r_2 \end{bmatrix}^T, \quad t_i = \begin{bmatrix} t_{i1} & t_{i2} \end{bmatrix}^T \quad \text{and} \quad w = \begin{bmatrix} w_1 & w_2 \end{bmatrix}^T \quad (5) \]

so, the received signals $r_1$ and $r_2$ at the two independent time slots are expressed as follows \[
 r_1 = t_{11} h_{R_1, D} + t_{21} h_{R_2, D} + t_{31} h_{R_3, D} + t_{41} h_{R_4, D} + w_1 \]

$\quad r_2 = t_{12} h_{R_1, D} + t_{22} h_{R_2, D} + t_{32} h_{R_3, D} + t_{42} h_{R_4, D} + w_2 \quad (6)$

By substituting $t_i$, and taking the conjugate of $r_2$, the equivalent channel matrix corresponding to the code word in (4) used over four relay nodes is given by: \[
 H = \begin{bmatrix} h_{SR_1} h_{R_1, D} + h_{SR_2} h_{R_2, D} + h_{SR_3} h_{R_3, D} + h_{SR_4} h_{R_4, D} \\ h_{SR_1} h_{R_1, D} + h_{SR_2} h_{R_2, D} - h_{SR_3} h_{R_3, D} - h_{SR_4} h_{R_4, D} \end{bmatrix} \quad (5) \]

Applying the matched filtering at the destination receiver with the equivalent channel matrix in (8), we can obtain the Grammian matrix $G$ as follows: \[
 G = H^H H = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \quad (6) \]

where $(.)^H$ denotes Hermitian transpose and $\alpha$ represents the channel gain such that \[
 \alpha = \sum_{i=1}^{4} |h_{SR_i} h_{R_i, D}|^2 + \beta_1 + \beta_2 \]
with

\[ \beta_1 = h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D + h_{SR_2}^* h_{R_2} h_{SR_1} h_{R_1} D \]
\[ = 2 \text{Re}(h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D) \]
\[ \beta_2 = h_{SR_3}^* h_{R_3} h_{SR_4} h_{R_4} D + h_{SR_4}^* h_{R_4} h_{SR_3} h_{R_3} D \]
\[ = 2 \text{Re}(h_{SR_3}^* h_{R_3} h_{SR_4} h_{R_4} D) \]

where \( \beta_1, \beta_2 \) are the interference factors and \(|x|^2\) denotes the modulus squared of a complex number and \( \text{Re}[\cdot] \) its real part.

From (6), it is evident that the Grammian matrix of the D-EO-STBC is orthogonal, which indicates that the code can be decoded with a simple receiver. It can be shown that the signal-to-noise ratio (SNR) is

\[ SNR = \frac{\sigma^2}{\sigma_n^2} = \left( \sum_{i=1}^{4} |h_{SR_i} h_{R_i} D|^2 \right)^2 + \beta_1 + \beta_2 \frac{\sigma^2}{\sigma_n^2} \]

(7)

where \( \sigma_n^2 \) is the total transmit power of the desired signal, and \( \sigma_n^2 \) is the noise power at the receiver.

It can be seen that the \( \beta_1 \) and \( \beta_2 \) terms may reduce channel gain, and correspondingly the SNR. In order to achieve a full cooperative diversity of order four for a wireless system with four relays, we used two feedback schemes.

2.2. Closed-Loop D-EO-OSTBCs for Four Relay Nodes

We now consider that channel information at the relays. In our four relay system, partial channel information is fed into only any two relays each having different code processing which is enough to leverage the system gain to the maximum as shown in [6] and [7]. The two relays that we selected in our system are \( R_1 \) and \( R_3 \) as shown in Fig. 2. We assumed that the full channel information CSI can be estimated at the destination receiver. So, the relay transmitted signals \( t_1 \) and \( t_3 \) are multiplied by \( U_1 \) and \( U_2 \) before they are transmitted from the first and third relay nodes, respectively, while the other two are kept unchanged; we used in our simulation two feedback schemes as in [6] and [7] to determine the value of \( U_1 \) and \( U_2 \).

In [6], \( U_1 = (-1)^k \) and \( U_2 = (-1)^{k-1} \), where \( i, k = 0, 1 \), which means using \( 0 \) or \( \pi \) as the rotation angle for the signals. In [7], \( U_1 = e^{j\theta_1} \) and \( U_2 = e^{j\theta_2} \) respectively, which means using the exact angles for rotating the signals which result in maximizing \( \beta_1 \) and \( \beta_2 \). Therefore, the error performance is improved at the expense of increased feedback overhead. The phase rotation on transmitted symbols is equivalent to rotating the phases of the corresponding channel coefficients. The received signals \( r_1 \) and \( r_2 \) at two independent time intervals are expressed as follows

\[ r_1 = t_11 U_1 h_{R_1} D + t_21 U_2 h_{R_2} D + t_31 U_3 h_{R_3} D + t_41 U_4 h_{R_4} D + w_1 \]
\[ r_2 = t_12 U_1 h_{R_1} D + t_22 U_2 h_{R_2} D + t_32 U_3 h_{R_3} D + t_42 U_4 h_{R_4} D + w_2 \]

Using the same procedures as we have followed in the open loop scheme, the equivalent channel matrix \( H_e \) given by

\[ H_e = \begin{bmatrix} f_1 & f_2 \\ f_2 & -f_1 \end{bmatrix} \]

(8)

where

\[ f_1 = U_1 h_{SR_1} h_{R_1} D + h_{SR_2} h_{R_2} D \]
\[ f_2 = U_1 h_{SR_2} h_{R_2} D + h_{SR_1} h_{R_1} D \]

also,

\[ G = H_e^H H_e = \begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} \]

Fig. 2. Schematic representation of the proposed closed-loop D-EO- STBC system for four relay nodes with feedback to two nodes.

with

\[ \alpha = \sum_{i=1}^{4} |h_{SR_i} h_{R_i} D|^2 + \beta_1 + \beta_2 \]

where

\[ \beta_1 = 2 \text{Re}(U_1 h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D) \]
\[ \beta_2 = 2 \text{Re}(U_2 h_{SR_3}^* h_{R_3} h_{SR_4} h_{R_4} D) \]

taking into account that \( |U_1|^2 = |U_2|^2 = 1 \).

The feedback performance gain is \( g_f = \beta_1 + \beta_2 \). From this, it is clear that if \( g_f > 0 \), the designed closed-loop system can obtain additional performance gain, which leads to an improved whole channel gain, and correspondingly the \( SNR \) at the destination receiver.

According to the analysis in [6], \( U_1 = (-1)^{k} \) and \( U_2 = (-1)^{k-1} \). So, we can propose the following design criteria:

\[ i = \begin{cases} 0 & \text{if } h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D h_{SR_3} h_{R_3} h_{SR_4} h_{R_4} D \geq 0 \\ 1 & \text{otherwise} \end{cases} \]

(9)

\[ k = \begin{cases} 0 & \text{if } h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D h_{SR_3} h_{R_3} h_{SR_4} h_{R_4} D \geq 0 \\ 1 & \text{otherwise} \end{cases} \]

(10)

While according to the analysis in [7], \( U_1 = e^{j\theta_1} \) and \( U_2 = e^{j\theta_2} \). So, we can propose the following design criteria:

\[ \theta_1 = -\text{angle}(h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D h_{SR_3} h_{R_3} h_{SR_4} h_{R_4} D) \]

(11)

\[ \theta_2 = -\text{angle}(h_{SR_1}^* h_{R_1} h_{SR_2} h_{R_2} D h_{SR_3} h_{R_3} h_{SR_4} h_{R_4} D) \]

(12)

Similarly, we can design a closed-loop scheme for three relay nodes with one feedback link only.

3. SIMULATION RESULTS

In this section, we compare the bit error performance of the proposed extended schemes with earlier proposed schemes in quasi-static flat fading channels. The fading is constant within a frame and changes independently from frame-to-frame. Each frame consists of 64 symbols in our simulation. Only one antenna at the transmitter and the receiver was considered, but this technique can be extended to multiple transmit or receive antennas or both which result in better improvement in the performance. All schemes use QPSK modulation and have the same total power. The x-axis shows the average transmitted power from the source node in dB and the y-axis shows the
BER. In Fig. 3, we show the performance of the proposed D-EO-STBC for four relay nodes. We can see that the two closed-loop D-EO-STBCs are better than the D-EO-STBC without feedback. In particular, at a bit error probability of $10^{-3}$, the proposed D-EO-STBC scheme with feedback 0 or 1 provides approximately 3.5 dB improvement while the proposed scheme with feedback of exact phase provides approximately 4.5 dB improvement. However, the improvement increases as the average transmitted power increases. This improvement of the two closed loop schemes is because both achieve full cooperative diversity of order four combined with array gain. As shown, the closed loop scheme with exact phase as feedback provides better performance (approximately 1 dB for BER of $10^{-4}$) compared with the other closed loop scheme with 0 or 1 as a feedback. The figure also provides a comparison of the proposed schemes with the distributed orthogonal STBC (D-OSTBC) [4], using the optimal power allocation protocol for distributed STC with unequal time intervals as in [12], and the distributed Alamouti (D-Alamouti) scheme [3]. We clearly notice that the proposed scheme outperforms them. Specifically, when the BER is $10^{-3}$, our open loop scheme can achieve more than 2 dB gain over the D-Alamouti scheme and more than 14 dB gain over D-OSTBC.

As shown from the simulation, the exact phase feedback scheme presents the best performance compared to the others. The advantages of the exact phase feedback scheme can be retained by exploiting some quantization of the feedback coefficients as in [7], to yield a more practical scheme with limited feedback. In [7], very little degradation in BER performance is shown so it is not repeated in our simulations.

### 4. CONCLUSIONS

In this paper, we proposed complex extended orthogonal STBCs for wireless relay networks. Simulations showed that D-EO-STBC provides better performance than the D-Alamouti scheme and the D-OSTBC when we used both closed loop and open loop schemes. In terms of data rate, the open-loop D-EO-STBC provides full rate in each stage which is equal to the data rate of the D-Alamouti scheme and outperforms the D-OSTBC. These results are very encouraging since the total transmission power is the same or constant in addition to the information symbols being decoded separately in a very simple manner at the receiver. Our simulations also showed that with partial channel knowledge at the relays, the D-EO-STBC achieves higher gain compared with no feedback because the closed-loop D-EO-STBC achieves full diversity gain in addition to array gain. The main result we can conclude is that the extended schemes in wireless relay networks save the transmission time and the transmission power. Finally, results in this paper can be extended straightforwardly to wireless relay networks with multiple-antenna nodes.

### 5. REFERENCES


