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Metadata Record: https://dspace.lboro.ac.uk/2134/5914

Version: Published

Publisher: © IEEE

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CONVERGENCE BEHAVIOURS OF AN adaptive step-size constant modulus algorithm for DS-CDMA RECEIVERS

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ABSTRACT

A convergence analysis of the blind adaptive step-size constant modulus algorithm (AS-CMA) for direct-sequence code division multiple access (DS-CDMA) receivers is presented. Analytical results show similar convergence behaviours of the AS-CMA receiver and the adaptive step-size stochastic approximation (AS-SA) receiver. Simulations show that the blind AS-CMA algorithm performs comparably with the non-blind AS-SA in a Markovian type of nonstationary environment. The robustness of the proposed algorithm to different settings of the initial step-size is also shown.

1. INTRODUCTION

An adaptive step-size constant-modulus algorithm (AS-CMA) has been proposed for a DS-CDMA receiver operating in nonstationary environments [1, 2]. The algorithm adaptively varies the step-size in order to minimize the constant modulus (CM) criterion. In contrast to the adaptive step-size minimum output energy (AS-MOE) receiver proposed in [3], AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1]. This is because the MOE criterion is not designed to equalize the channel and the pulse shaping waveform as AS-CMA can be applied successfully in multipath fading CDMA channels [1].

In this paper, we study the convergence behaviours of the AS-CMA receiver in multipath fading nonstationary channels as compared to the adaptive step-size stochastic approximation (AS-SA) receiver [5]. Further analysis from [1] reveals similarity in convergence behaviours of the \( Y_n \) process, the derivative of the tap-weight with respect to the step-size, of AS-CMA and AS-SA.

Simulation results show that the performance in terms of signal-to-interference plus noise ratio (SINR) of AS-CMA is comparable to that of the non-blind AS-SA receiver. Insensitivity of initial step-size settings is also confirmed. The trajectories of the step-sizes of AS-CMA are shown to behave similarly to those of AS-SA while AS-CMA enjoys the advantage of not requiring a training sequence for the adaptation process.

2. SIGNAL MODEL

Consider the real signal model of an additive white Gaussian noise (AWGN) \( K \)-user synchronous DS-CDMA channel, the baseband received signal is defined as

\[
r(t) = \sum_{i=-\infty}^{\infty} \sum_{k=1}^{N} A_k b_k(i) c_k(t-iT) + w(t),
\]

where \( A_k \) represents the received amplitude of the \( k \)th user. The data bits \( b_k(i) \) are independent identically distributed (i.i.d.) and \( b_k(i) \in \{-1, +1\} \). The symbol period is denoted by \( T \). The spreading waveform of the \( k \)th user \( c_k(t) \) is \( N \)-dimensional and has unit energy property, i.e., \( \| c_k \|_2^2 = 1 \). The AWGN \( w(t) \) has power spectral density \( \sigma_w^2 \). The spreading codes \( c_k(t) \) can be modified to take into account the effect of the channel and the pulse shaping waveform as \( \bar{c}_k(t) = c_k(t) \ast \psi(t) \ast h_k(t) \), where \( \ast \) denotes convolution. \( \psi(t) \) is the pulse shaping filter and \( h_k(t) \) is the channel response of the \( k \)th user and need not be identical for different users. The continuous-time received signal \( r(t) \) is sampled to form a length-\( L_f \) received signal vector at the \( n \)th observation, where \( L_f \) is the length of a receiver with tap-weight vector \( f \),

\[
r_n = \sum_{k=1}^{K} r_n^{(k)} + w_n = \sum_{k=1}^{K} G_k b_k[n] + w_n,
\]

where \( G_k \) is the combined code-channel response matrix of the \( k \)th user and \( b_k[n] = [b_k[n + L_h - 1], \ldots, b_k[n]]^T \) with \( L_h = \left[ \frac{L_f + L_k - 1}{N} \right] \) and \( w_n = [w_nN + L_f - 1, \ldots, w_nN]^T \). Note that

\[
G_k = C_k H_k
\]

where \( C_k \) represents the block of delayed copies of the code sequence of the \( k \)th user with dimension \( L_h \times N + L_h - 1 \)

\[
\mathbf{C}_k = \begin{bmatrix} \bar{C}_k & \cdots & \bar{C}_k \end{bmatrix}
\]

and \( \mathbf{C}_k \) denotes the code matrix \( \mathbf{C}_k \) with the first \( L_h - 1 \) columns truncated. The channel response vector for the
achieved in the first user has length \( L_k \), i.e., \( h_k = \left[ h_{k}^{(1)}, \cdots, h_{k}^{(K)} \right]^T \). For an adaptive linear filter \( f \), the estimate of the transmitted data \( b[n] \) can be achieved as

\[
\hat{b}[n] = x[n] - f^T r[n] = f^T (G b[n] + w[n]),
\]

where \( G = [G_1, G_1, \cdots, G_K] \) and \( b[n] = [b_1[n], b_2[n], \cdots, b_K[n]]^T \). For brevity, we shall consider the first user as the desired user and drop the subscript \( k \) in all variables involving the first user.

### 3. Adaptive Step-Size CMA Algorithm

We re-introduce the AS-CMA algorithm as presented in [1, 2]. Consider the CM criterion for real signals,

\[
J_{CM} = \frac{1}{2} E \left( z^2 - R_z \right),
\]

where \( z_n = f^T r_n \) is the output of the receiver at time \( n \). The dispersion constant \( R_z \) is equal to unity for binary phase shift keying (BFSK) signals. By minimising \( J_{CM} \) with respect to \( f^{(1)} \) at a particular step-size \( \mu \) and taking the instantaneous gradient, the fixed step-size (fixed-\( \mu \)) CMA receiver weight update equation is given by

\[ f_{n+1} = f_n - \mu (z_n - R_z) z_n r_n, \]

which \( \mu \) is the time-varying step-size. The dispersion constant \( R_z \) is equal to unity for binary phase shift keying (BFSK) signals. The update of the step-size is given by [1]

\[
\mu_{n+1} = \frac{\mu_n - \alpha (z_n - R_z) z_n r_n}{\mu_n + \alpha},
\]

where \( [.]^{+} \) denotes truncation to the limits of the range \([\mu_{min}, \mu_{max}]\) and \( \alpha \) denotes the adaptation rate of the step-size \( \mu_{n} \) with \( \alpha > 0 \). \( Y_n \) represents the derivative \( \partial f_n / \partial \mu \big|_{\mu = \mu_n} \) as defined in [3] and, from (6), the update equation of \( Y_n \) is given by

\[
Y_{n+1} = \left[ 1 - \mu (3z_n^2 - R_z) r_n r_n^T \right] Y_n - (3z_n^2 - R_z) z_n r_n, \]

where \( I \) denotes the identity matrix with size \( I \times I \). Equations (6), (7) and (8) constitute the adaptive step-size CMA algorithm for real signals [1]. It is straightforward to extend this to the complex case, but we retain the real version for consistency with [4, 3]. For the fixed-\( \mu \) system, i.e., \( \mu_{n} = \mu \), we define \( Y_n^\ast = \partial f_n / \partial \mu \) and its update rule is given by

\[
Y_{n+1}^\ast = \left[ 1 - \mu (3z_n^2 - R_z) r_n r_n^T \right] Y_n^\ast - (3z_n^2 - R_z) z_n r_n, \]

where \( z_n^2 = (f_n^T r_n)^T r_n \).

### 4. Interpretation of \( Y_{n}^\ast \) and \( Y_n \) of AS-CMA

The behaviour of the AS-CMA algorithm is controlled by the random vector parameter \( Y_n \), defined as the derivative of the estimates of the tap-weight vector with respect to \( \mu_n \). To study the behaviour of the AS-CMA algorithm, it is essential to understand the meaning of \( Y_n \). In the algorithm development, although we do not assume that the actual system is stationary, we employ the minimisation of the expectation of a stationary error with respect to \( \mu_n \).

Thus, we study \( Y_n \) via \( Y_n^\ast \) which is introduced as a stationary sequence at \( \mu_n = \mu \) for the fixed-\( \mu \) process. Following [5], a finite difference approximation is used to study the meaning of \( Y_n^\ast \).

Due to the multimodality of the CM cost function, the local minima can be associated with a variety of mean square-error (MSE) values. However, the analysis of [6] has shown a close relationship between the "good" local minima and minimum mean-square error (MMSE) solutions. These local minima are associated with low MSE and are considered as global minima. For a meaningful interpretation of \( Y_n^\ast \), we consider the behaviour of \( Y_n^\ast \) only in the neighbourhood of local minima associated with low MSE.

Following the analysis of [5], \( Y_n^\ast \) is interpreted by considering its derivative representation. For \( \Delta > 0, \mu + \Delta \leq \mu_n \), define

\[
\delta Y_n^\ast = \frac{y_n^\ast - y_n^\ast - \Delta}{\Delta}.
\]

Therefore,

\[
\delta f_{n+1}^\ast = \frac{1}{\Delta} \left[ (y_n^\ast + \Delta - (\mu + \Delta) f_l^\ast) r_n - (f_n^T - \mu f_l^T) r_n \right],
\]

where \( f_l \) is the prediction error function of the CM cost function and \( f_l = (f_n^T + \Delta) r_n \). We consider the behaviour of the AS-CMA algorithm within the neighbourhood of an equilibrium point \( f \) in the absence of noise. By using the first-order Taylor expansion at \( f = f_0 \), we arrive at

\[
\delta f_{n+1}^\ast = \frac{1}{\Delta} \left[ (y_n^\ast + \Delta - (\mu + \Delta) f_l^\ast) r_n - (f_n^T - \mu f_l^T) r_n \right],
\]

where \( f_l \) represents the derivative of \( f_n \) with respect to \( \mu_n \). Substituting (11) and (12) in (10), we arrive at

\[
\delta Y_{n+1} = \frac{1}{\Delta} \left[ (y_n^\ast + \Delta - (\mu + \Delta) f_l^\ast) r_n - (f_n^T - \mu f_l^T) r_n \right].
\]

At \( f = f_0 \), (9) becomes

\[
Y_{n+1}^\ast = Y_n^\ast - \mu f_l^T r_n r_n^T f_n^T - \phi (z_n^2) r_n.
\]

We define \( D_n^\ast = \delta Y_n^\ast - Y_n^\ast \), therefore, at \( f = f_0 \),

\[
D_n^\ast = \frac{1}{\Delta} \left[ (y_n^\ast + \Delta - (\mu + \Delta) f_l^\ast) r_n - (f_n^T - \mu f_l^T) r_n \right],
\]

which is in a similar form to \( D_n^{(1)} \) shown in Section 3 of [5]. Similar to the moment bounds of \( Y_n \) and \( Y_n^\ast \) discussed in Appendix A of [1], the moment bound for \( D_n^\ast \) is given by

\[
\lim \sup_n E (\| D_n^\ast \|)^2 \leq O(1/\mu^2),
\]

Therefore, for the stationary process and in the neighbourhood of the low MSE local minima, \( Y_n^\ast \) retains the interpretation as a derivative in the mean square sense.
4.1. Subspace Interpretation of $Y_n$ and $Y_n^\mu$

We can study the convergence behaviours of $f_n$ in the signal subspace via the weight error vector $e_n = f_n - \bar{f}$. It is shown in [2] that mean convergence behaviour of the CMA receiver for DS-CDMA near the global minimum of the CM cost function satisfies

$$E\{e_{n+1}\} = (1 - \mu_n \sigma^2 \mathbf{G} \mathbf{B} \mathbf{G}^T) E\{e_n\}, \quad (18)$$

where $\sigma^2 = E\{b[n]^2\} = \mathbf{B}^T \mathbf{E}\{\sigma_b^2 \delta (\tau_n)b[n]\} \mathbf{B}^T[n]$. In a similar fashion of studying $Y_n$ via $Y_n^\mu$, we assume the stationary process and study $e_n$ at $\mu_n = \mu$. Defining $e_n^\mu = \mathbf{f}_n^\mu - \bar{f}$ and $e_n^{\mu+\Delta} = \mathbf{f}_n^{\mu+\Delta} - \bar{f}$, (17) can be written as

$$\lim_{\Delta \to 0} E\{ (Y_n^\mu - e_n^{\mu+\Delta} - e_n^\mu)^2 \} = 0. \quad (19)$$

Therefore, $Y_n^\mu$ also retains the interpretation as a derivative of tap error vector in the mean square sense.

By using the eigendecomposition of the Hessian matrix $\mathbf{H} \triangleq \mathbf{G} \mathbf{B} \mathbf{G}^T$, we arrive at

$$\mathbf{H} = \mathbf{U} \mathbf{A} \mathbf{U}^T , \quad (20)$$

where $\mathbf{U}$ is a unitary matrix of normalised eigenvectors and $\mathbf{A}$ is the diagonal matrix of corresponding eigenvalues. Substituting the Hessian matrix $\mathbf{H}$ defined by (20) in (18) and rearranging, we arrive at

$$E\{e_n\} = \mathbf{U} (1 - \mu \sigma^2 \hat{\mathbf{A}}) \mathbf{U}^T \epsilon_0 , \quad (21)$$

where the expectation operator on $\epsilon_0$ is removed since $\epsilon_0$ is not a considered random vector. At $\mu_n = \mu$ and $\mu + \Delta$,

$$E\{e_n^{\mu}\} = \mathbf{U} (1 - \mu \sigma^2 \hat{\mathbf{A}}) \mathbf{U}^T \epsilon_0 , \quad (22)$$

$$E\{e_n^{\mu+\Delta}\} = \mathbf{U} (1 - (\mu + \Delta) \sigma^2 \hat{\mathbf{A}}) \mathbf{U}^T \epsilon_0 , \quad (23)$$

where $e_n^{\mu+\Delta} = \mathbf{f}_n^{\mu+\Delta} = \epsilon_0$. The signal subspace eigenvectors form the column of matrix $\mathbf{U}_S$ and the noise subspace eigenvectors form the matrix $\mathbf{U}_N$, i.e., $\mathbf{U} = [\mathbf{U}_S \mathbf{U}_N]$. It is shown that the MMSE solution $\mathbf{f}_{\text{MMSE}}$ lies in the signal subspace [7]. For the binary input, it is shown in [8] that $\bar{f}$ coincides with $\mathbf{f}_{\text{MMSE}}$ implying that $\mathbf{U}_S \bar{f} = 0$. Therefore, $\bar{f}$ also lies in the signal subspace. $\mathbf{U}_S^T \epsilon_0 = 0$. Hence, from (22) and (23), we arrive respectively at

$$E\{e_n^{\mu}\} = \mathbf{U}_S (1 - \mu \sigma^2 (\hat{\mathbf{A}}_S - \sigma^2 \mathbf{I})) \mathbf{U}_S^T \epsilon_0 , \quad (24)$$

$$E\{e_n^{\mu+\Delta}\} = \mathbf{U}_S (1 - (\mu + \Delta) \sigma^2 (\hat{\mathbf{A}}_S - \sigma^2 \mathbf{I})) \mathbf{U}_S^T \epsilon_0 , \quad (25)$$

where $\hat{\mathbf{A}}_S$ denotes the diagonal matrix of nonzero eigenvalues of $\mathbf{H}$.

Setting $\mathbf{f}_n^\mu$ in the signal subspace implies faster convergence speed for both the least-mean-square (LMS) algorithm [9] and CMA [2]. From (19), (24) and (25), it is shown that $Y_n^\mu$ can be bounded within the signal subspace provided that the initialisation $\mathbf{f}_n^\mu$ is in the signal subspace. This bounding of $Y_n^\mu$ implies that the convergence speed and time constant of the algorithm can be assessed by considering only the eigenvalues associated with the eigenvectors spanning the signal subspace [2, 9].

5. NUMERICAL RESULTS

We considered a synchronous DS-CDMA system with spreading gain $31$ ($N = 31$). Without loss of generality, the first user was the desired user with unity power. The background noise was zero mean AWGN with SNR=20 dB referenced to the desired user. We considered the performance of adaptive step-size receivers in a dynamic environment where different user groups switched between three user groups according to a Markov chain. The dynamic environment was set similarly to that of [3] with minor modification. The power of the first user was set such that its SNR was 20 dB and the multiple-access interference (MAI) user groups were designated as follows. User group 1 consisted of 17 MAI users transmitting at 20 dB and two MAI users transmitting at 30 dB. User group 2 consisted of 26 MAI users transmitting at 20 dB. Finally, user group 3 consisted of eight MAI users transmitting at 20 dB and one MAI user transmitting at 30 dB. The MAI structure was constructed according to a Markov chain $\pi[n]$ with transition probability matrix

$$P = \begin{bmatrix} 0.998 & 0.01 & 0.01 \\ 0.01 & 0.998 & 0.01 \\ 0.01 & 0.01 & 0.998 \end{bmatrix} . \quad (26)$$

and state space $\{(1, 0, 0)^T, (0, 1, 0)^T, (0, 0, 1)^T\}$. The Markov chain $\pi[n]$ designates the MAI user group which affects the desired user detection at the nth symbol. The switching between the three MAI user groups occurred at every 500 symbols. Five-ray multipath fading channels were used for both the desired user and all the MAI users for all user groups. The last four rays were uniformly distributed in delay over [0, 10T_c], where $T_c$ is the chip interval, with standard deviation 0.3. Averaged SINR was used as a performance measure and all plots were averaged over 50 Monte-Carlo runs.

We compare the SINR performances of the AS-CMA receiver, the AS-MOE receiver [3] and the AS-SA receiver [5]. All the receiver lengths were chosen to be twice that of the spreading gain. For all AS algorithms, the adaptation rates $\alpha$ were $10^{-3}$ and the upper and lower step-size limits $\mu_+ - \mu_-$ were set respectively at $10^{-2}$ and $0$. The averaged SINR plots of all algorithms are compared in Fig. 1. Notice that while both the AS-SA and AS-CMA receivers have an ability to track variations of multipath channels, the AS-MOE receiver fails to operate due to mismatch. The SINR plot of AS-CMA almost coincides with that of AS-SA indicating the comparable performance of the two algorithms.

In Fig. 2, the steady-state SINR of the AS-CMA receiver for different initial settings of step-sizes $\mu_+ \in [10^{-8}, 10^{-3}]$ are compared with that of the AS-SA receiver. Despite a 100 000-fold variation in $\mu_+$, it is shown that the SINR performances of both receivers are maintained at a constant level. The AS-CMA is also shown to achieve a comparable SINR performance as the AS-SA. This result is attributed to the excellent performance of the CM criterion which has been shown to be near the Wiener receiver [10, 6]. Since the SINR results of AS-MOE are approximately at $-12$ dB throughout the setting range of $\mu_+$, AS-MOE is therefore discarded in the plot.

Fig. 3 shows the trajectories of $\mu_+$ of the AS-CMA receiver and the AS-SA at three different settings of the initial step-size $\mu_0 = \{10^{-8}, 10^{-3}, 10^{-5}\}$. The trajectories were plotted for 80 000 symbols. Sensitivity to the settings of the initial step-size for each algorithm can be observed. The adaptation behaviour of $\mu_+$
Figure 1: The SINR performance of AS-SA [5], AS-MOE [3] and AS-CMA receivers in a Markovian dynamic environment. The AS-MOE receiver fails to operate due to mismatch. AS-SA and AS-CMA are comparable in terms of convergence speed and steady-state performance.

Figure 2: SINR comparison of AS-SA and AS-CMA at different settings of initial stepsizes $p_0$. It is shown that both AS-SA and AS-CMA are not affected by various initial settings despite 100 000-fold of variation of $p_0$. Notice an almost identical performance of both algorithms.

Figure 3: Trajectories of $p_n$ for AS-SA and AS-CMA at different settings $p_0 = \{10^{-8}, 10^{-5}, 10^{-3}\}$. Insensitivity to variation in the initial step-size settings for each algorithm can be observed. Similarity in adaptation behaviours of $p_n$ of both algorithms is shown.

7. REFERENCES


