Acoustic emission accompanying the onset of surface microcracks

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The acoustic emission accompanying the formation and growth of shallow cracks (microcracks) on the surface of an isotropic solid is investigated. The radiation directivity patterns of two- and three-dimensional microcracks are calculated for various conditions of their propagation.

It is a well-known fact that one of the most important modifications of acoustic emission is the emission of sound during the nucleation and growth of cracks in solids (Ref. 1). Quite a few papers have been devoted to the theoretical study of this problem in recent years. However, practically all the existing papers (see, e.g., Refs. 2-5) deal with cracks in an unbounded medium. On the other hand, cracks very often originate on the surface of a solid. This is particularly true of situations where the solid comes into contact with an aggressive surrounding medium, which lowers its safety factor considerably. In the present article the emission of sound by shallow cracks (microcracks) formed on the surface of an isotropic solid is analyzed theoretically. The application of the perturbation method in the calculations imposes a restriction on the dimensions (specifically the depth) of the cracks. We note in this connection that the restriction is insignificant, because real cracks formed on a surface usually have a depth much smaller than the characteristic wavelengths $\lambda$ of the emission spectrum, i.e., they can be regarded entirely as microcracks.

We assume that a crack of depth $h$, oriented along the $y$ axis, is present on the surface $z = 0$ of a solid. Tensile stresses $\sigma_{yz} = \sigma(t)$ are applied to the body in the direction of the $x$ axis, causing the crack either to spread (without any increase in its length $2l$ or depth $h$) if $\sigma(t)$ does not exceed certain critical values $\sigma_{cr}$, or to propagate at finite rates $v$ if $\sigma(t)$ exceeds $\sigma_{cr}$ (Ref. 6). Both of these cases can be treated within the framework of a unified approach based on the application of Rayleigh's principle for solids (Ref. 7). The displacement field of the emitted acoustic waves for a crack of constant length, i.e., a spreading crack, can be written as follows in this case:

$$u_n(r, \theta) = \int \left[ \left( n, s_{nl}(w, r')G^{m'}(w, r, r') \right) + n, s_{nl}(w, r')G^{m'}(w, r, r') \right] \exp(-iuat) \, dw \, dS,$$

where the observation point $r$ is situated inside a contour $S$ running along the free surface of the solid, the edges of the crack, and a semicircle of infinite radius in the depth of the solid, $G^{m'}(w, r, r')$ is the surface Green's tensor, which satisfies the condition of zero normal stresses on the free surface $z = 0$ of the half-space, $n$ is the outward normal to the contour $S$, and $s_{nl}$ denotes the stiffness constants. Since the transfer $G^{m'}$ satisfies null boundary conditions at $z = 0$, the integration in Eq. (1) is carried out only along the edges of the crack (the integral along the semicircle vanishes by virtue of the radiation extinction condition). The stresses $\sigma_{ij}(w, r')$ in the integral (1) are the spectral densities of the elastic stresses in the equivalent problem for a nondeformable solid, in which the edges of the crack are not free, but are subjected to external "repulsive" stresses $\sigma(t)$ (Refs. 3-5). In this case $\sigma_{ij}(w, r') = \sigma_{w}(w, r') = -\sigma(t).$ The quantities $u_{ij}(w, r')$, whose values are not known in general, describe the law of motion of the edges of the crack under the action of the applied stresses.

The calculation of the emitted radiation field for cracks of arbitrary dimensions poses an extremely complex problem. In the first place, the displacements $u_{ij}(w, r')$ of the edges of the crack must first be determined from the specified stresses $\sigma_{ij}(w, r')$, requiring the point $r$ to tend to $r'$ in Eq. (1), and then the resulting integral equation must be solved for $u_{ij}(w, r')$ (Ref. 7). In the second place, the surface Green's tensor has an exceedingly cumbersome form in the general case, and even if the displacements are determined in some way, the calculations cannot be carried out analytically.

However, the problem can be simplified considerably for shallow cracks (microcracks) by the application of a perturbation procedure, which entails "transferring" the boundary conditions from the edges of the crack to the unperturbed surface of the solid; this approach is used extensively in the theory of wave scattering by rough surfaces (see, e.g., Refs. 8-11). According to this procedure, the stresses at the edges of a shallow crack can be replaced by equivalent stresses applied to the surface $z = 0$. If we restrict the analysis to terms that do not contain the second or higher powers of $f(x)$ and $f'(x) = df/dx$, where the function $f(x)$ describes the initial (before spreading) profile of the crack (Fig. 1) and satisfies the condition $f(0)/\lambda < 1$, the boundary conditions at $z = 0$ will have the following form in the given situation:

$$\sigma_{zz} = 0, \quad \sigma_{rr} = 0, \quad \sigma_{zz} = -f'(x)\sigma(x) + f(x)\sigma_{zz}.$$  

(2)

We note that the transition to the boundary conditions (2) also implies that the integration in Eq. (1) is not carried out over the domain of the projection of the crack onto the surface $z = 0$. Since the Green's tensor satisfies the free surface condition at $z = 0$, the second term in the integral (1) vanishes, and so it is no longer necessary to determine the displacements of the edges of the crack. Taking the smallness of $f(x)$ into account, we can find the value of $\partial \sigma_{zz} / \partial z$ by the perturbation method, using the condition $\sigma_{xz}(x)\big|_{z=0} = -f'(x)\sigma(x)$ as the first approximation and solving the corresponding boundary-value problem. Inasmuch as the characteristic width $2d$ of the initial crack (see Fig. 1) is much smaller than the emitted wavelengths, this problem can be solved in the static approximation, whence it follows that

$$\left. \left( \partial \sigma_{zz} / \partial z \right) \right|_{z=0} = -\left( \sigma(x)/n \right) \int f'(x') (x-x')^{-1} \, dx'.$$

(3)
Now, substituting Eqs. (2) in the integral (1) with allowance for Eq. (3), expanding \( G_{\text{mm}}(\omega, r, r') \) in a Taylor series in the neighborhood of \( r' = 0 \), and then passing to the limit \( d \to 0 \), we find that the investigated surface crack as an acoustic emitter is equivalent to a point source of the type "double force without moment."\(^{13}\) The integral (1), which is conveniently rewritten entirely in the spectral representation, now acquires the following form in the two-dimensional case:

\[
\sigma_{z}(t, r, t') = \frac{1}{2\pi} \int d\omega \sum_{\Delta k} \frac{G_{\text{mm}}(\omega, \kappa, z)}{\mu(k)} \exp(ikz-\omega t) \exp(ik\Delta r) \exp(ikz' \Delta t) \exp(ik\Delta r'),
\]

where \( \sigma(\omega) = -\int \sigma_{z}(\omega, z) dx = h^{2} \sigma(\omega) \), and the components of the Green's tensor are expressed in the space of tangential wave numbers \( k \). In writing the expression for \( \sigma(\omega) \), we allow for the fact that the contribution of the term \( \omega x = F^{\prime}(x)\sigma(\omega) \) tends to zero as \( \omega \to \infty \). We note that the passage to the limit \( d \to 0 \) in the given situation is not completely rigorous, because the slope of the edges of the crack approaches the vertical (the quantity \( F^{\prime}(x) \) tends to a delta function) and the conditions on \( F^{\prime}(x) \) that allow us to use the first nonvanishing approximation (2) in transferring the boundary conditions to the unperturbed surface are violated. It can also be verified that the higher approximations result in divergence of the integral (1) (e.g., in the integration of the square of the delta function). All the same, it has been shown in an analysis of the scattering of Rayleigh waves by obstacles with vertical walls\(^{16}\) that the application of equations of the type (2) yields a correct result, which is in good agreement with numerical calculations and with experiment.\(^{11}\) If the condition \( |F^{\prime}(x)|/\lambda \ll 1 \) remains valid. This is true in spite of the fact that the higher approximations here, as before, are divergent quantities.

The components \( G_{\text{xx}}^{0}(\omega, \mathbf{r}, \mathbf{r} \prime) \) and \( G_{\text{xx}}^{0}(\omega, \mathbf{r}, \mathbf{r} \prime) \) used in the integral (4) have the form

\[
G_{\text{mn}}(\omega, \mathbf{r}, \mathbf{r} \prime) = \frac{2ik\nu_{1}}{\mu(k)} \exp(-r\nu_{1}-\nu_{2}) \exp(-r\nu_{1}+\nu_{2}),
\]

where \( \nu_{1} = \omega/\sqrt{\mu} \) and \( \nu_{2} = \omega/\sqrt{\lambda} \) are the Rayleigh determinant, and \( k_{1} = \omega/\sqrt{\mu} \) and \( k_{2} = \omega/\sqrt{\lambda} \) are the longitudinal and transverse wave numbers, \( \mu \) is the shear constant, and \( \nu_{1}^{2} = k_{1}^{2} - k_{2}^{2} \), \( \nu_{2}^{2} = k_{2}^{2} - k_{1}^{2} \)

We use the integral (4) to calculate the radiation field of a spreading microcrack. We carry out the corresponding calculations for the case of instantaneous application of tensile stresses at the time \( t = 0 \). In this case \( \sigma(t) = \sigma_{0} \delta(t) \), where \( \sigma(t) \) is the Heaviside step function (the given relation models an instantaneously formed crack). We assume for definiteness that the crack spreads without any increase in depth \( h \) if the depth \( h \) increases at a finite rate from zero to some value \( h_{0} \) at \( \sigma(t) = \text{const} \), the quantity \( h^{2} \sigma(\omega) \) must be inserted in the integrand of (4); now all the results obtained for the case \( h = \text{const} \) will remain valid. Substituting the values of \( \sigma_{0} = \omega_{0}^{2}/2\mu \) and \( G_{\text{xx}}^{0}(\omega, \mathbf{r}, \mathbf{r} \prime) \) in the integral (4), we compute it with respect to \( \kappa \) in the complex plane by the standard procedure (we omit the integration with respect to \( \omega \) and confine the discussion to the temporal emission spectrum).\(^{17}\) Going from the variables \( x, z \) to polar coordinates \( R, \theta \) and applying the stationary-phase method, we obtain the following expressions for the spectral components of the longitudinal and transverse waves \( u_{\theta}(\omega, R, \theta, t) \), \( u_{z}(\omega, R, \theta, t) \), which are valid in the far field:

\[
u_{\theta}(\omega, R, \theta, t) = -\frac{k_{1}^{2} G_{\theta\theta}(\omega, k_{1}^{2} R, \nu_{1}, \nu_{2})}{2\pi \nu_{1} R^{3}},
\]

\[
u_{z}(\omega, R, \theta, t) = \frac{k_{1}^{2} G_{\theta\theta}(\omega, k_{1}^{2} R, \nu_{1}, \nu_{2})}{2\pi \nu_{1} R^{3}} \sin 4\theta \exp(ik_{1}R/\nu_{1} R^{3}).
\]

The residues at the poles of the functions \( G_{\theta\theta}(\omega, \kappa, \nu_{1}, \nu_{2}) \) (5) given the spectral components of the Rayleigh-wave displacements, which have the following form at \( z = 0 \):

\[
u_{\theta}(\omega, x, z) = \frac{-ik_{1}^{2} G_{\theta\theta}(\omega, k_{1}^{2} R, \nu_{1}, \nu_{2})}{2\pi \nu_{1} R^{3}} k_{1}^{2} \sin 4\theta \sin 4\phi \exp(ik_{1}z),
\]

\[
u_{z}(\omega, x, z) = \frac{-ik_{1}^{2} G_{\theta\theta}(\omega, k_{1}^{2} R, \nu_{1}, \nu_{2})}{2\pi \nu_{1} R^{3}} k_{1}^{2} \sin 4\theta \sin 4\phi \exp(ik_{1}z).
\]

The directivity functions normalized to the corresponding maxima for the spectral components \( u_{\theta}(\omega, \theta, t) \) and \( u_{z}(\omega, \theta, t) \) are shown in Fig. 2 for a Poisson ratio \( \nu = 0.25 \). It is seen that neither longitudinal nor transverse waves are emitted in the directions of the normal and the tangent to the surface. Only the parts of the directivity patterns in the interval of angles from 0 to \( \pi/2 \) are shown in Fig. 2, because of their symmetry about the \( z \) axis.

We now consider the three-dimensional problem. We assume that a microcrack of finite length \( 2l \) is oriented along the \( y \) axis on the surface of an elastic half-space. The conditions on the crack depth and the applied stresses are the same as before. Now the dispersion field of the acoustic waves emitted by the crack for \( h = \text{const} \) can be written as follows in application to the given problem:

\[
u_{l}(t, r) = \frac{1}{h^{4}/4\pi^{2}} \int d\omega \sum_{\Delta k} \sigma(\omega) G_{\text{mm}}(\omega, k, z) \exp(-\omega t+ikp) \exp(ikz) \exp(ik\Delta r) \exp(ik\Delta r'),
\]

where \( k = [k_{1}, k_{2}] \) is the surface wave vector, \( \rho = \delta \rho_{y} \) is the surface radius vector, \( G_{\text{mm}}^{0}(\omega, k, z) \) is the three-dimensional surface Green's tensor, which satisfies the condition of zero stresses at \( z = 0 \), and \( \sigma(\omega, k) \) is the spectral
component of the tensile stresses in the region of the crack. To determine the function \( G_{\omega,k}^{(2)} \) we invoke the Lamé potentials \( \phi \) and \( \psi \), which are related to the displacements \( u \) by the equation \( u = \text{grad} \phi + \text{curl} \psi \). The following expressions for the spectral components \( G_{\omega,k}^{(2)} \) in the given three-dimensional problem are readily derived by a procedure similar to that used in finding the two-dimensional Green's functions:

\[
G_{\omega,k}^{(2)}(w,k,z) = \frac{2ik_\omega k_z}{\mu F(k)} \exp(-v_z) - \frac{ik_\omega}{\mu F(k) k_z} \left(k_y^2 F(k) - v_z^2 F(k) - 2bk_z^2\right) \exp(-v_z),
\]

\[
G_{\omega,k}^{(2)}(w,k,z) = \frac{-2ik_\omega k_z v_z}{\mu F(k)} \exp(-v_z) + \frac{ik_\omega}{\mu F(k) k_z} \left(F(k) - 2v_z^2\right) \exp(-v_z),
\]

\[
G_{\omega,k}^{(2)}(w,k,z) = \frac{-2k_z^2 v_z}{\mu F(k)} \exp(-v_z) - \frac{k_z^2}{\mu F(k) k_z} \left(F(k) - 2v_z^2\right) \exp(-v_z).
\]

Here \( k = k_x + ik_y, b = k_y^2 v_z - 2v_z k_z \).

The integration with respect to \( k \) in Eq. (8) can be carried out in polar coordinates, \( k, \chi \) in the complex planes of \( k \) and \( \chi \). Here the residue at the point \( k = k_R \), where \( k_R \) is the Rayleigh wave number, and the contribution of the saddle points in Eq. (9) in integration with respect to the angle \( \chi \) yield the far field of the Rayleigh waves emitted by the crack. The combined contributions of the saddle points in the \( k \) and \( \chi \) planes yield the far field of longitudinal and transverse bulk waves. We carry out the calculations for three possible crack-growth regimes: 1) spreading of the crack without any variation of its length \( 2l \); 2) symmetrical (two-way) propagation of the tips of the crack along the \( y \) axis at the rates \( v \); 3) one-way (asymmetrical) propagation of one of the tips of the crack at the rate \( v \). We note that the crack propagation rate \( v \) cannot exceed the Rayleigh wave velocity for the type of stresses \( \sigma(t) \) considered in the present study. We give the results of the calculations for all three cases.

The spectral density \( \sigma(\omega, k) \) for spreading of the crack without any change of length has the form \( \sigma(\omega, k) = \sigma(\omega, k) = (\hat{\text{u}}(\omega, k) \text{sin}(\theta)) / k_\theta \), and the spectral components of the Rayleigh wave \( u(R)(\omega, \rho, \varphi) \) and \( u(R)(\omega, \varphi) \) at \( z = 0 \) and also of the transverse \( u_{\theta}(\omega, R, \theta, \varphi) \) and longitudinal \( u_{p}(\omega, R, \theta, \varphi) \) bulk waves are as follows (respectively):

\[
u_{\theta}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y v_z (k_\theta - b_\omega)}{\mu F(k) k_\theta^2 \omega} \sin(k_\theta + \sin^2 \varphi) \exp(i k_\varphi - i \varphi),
\]

\[
u_{p}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y k_z v_z (k_\theta - b_\omega)}{\mu F(k) k_\theta^2 \omega} \cos^2 \varphi \exp(i k_\varphi - i \varphi).
\]

For two-way (asymmetrical) propagation of the tips of the crack at a rate \( v < \sqrt{R} \), where \( \sqrt{R} \) is the Rayleigh wave velocity, the spectral component \( \sigma(\omega, k) \) depends on \( \omega \) and \( k \) as follows: \( \sigma(\omega, k) = -\alpha \sqrt{\omega} \sqrt{\omega} (\omega - k^2 \varphi) \), and the expressions for the surface- and bulk-wave fields acquire the form

\[
u_{\theta}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y v_z (k_\theta - b_\omega) v_s \cos^2 \varphi}{\mu F(k) k_\theta^2 \omega} \exp(i k_\varphi - i \varphi),
\]

\[
u_{p}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y k_z v_z (k_\theta - b_\omega) v_s}{\mu F(k) k_\theta^2 \omega} \cos^2 \varphi \exp(i k_\varphi - i \varphi),
\]

\[
u_{\theta}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y v_z (k_\theta - b_\omega)}{\mu F(k) k_\theta^2 \omega} \exp(i k_\varphi - i \varphi),
\]

\[
u_{p}(\omega, k) = \left( \frac{2\pi}{k_\theta} \right)^n \frac{\text{h}^2 k_y k_z v_z (k_\theta - b_\omega)}{\mu F(k) k_\theta^2 \omega} \exp(i k_\varphi - i \varphi).
\]
2\pi \theta (\omega - k_{pv}). The amplitude functions of the surface and bulk waves in this case do not differ from the preceding case, but the angular distribution of the field does not differ in that the maxima of the directivity pattern are shifted in the direction of propagation of the crack:

\[ u_s^{(n)} = \frac{2\pi}{k_{sp}} \left( \frac{\sin^2 (k_{sp} \sin \theta)}{\sin^2 (k_{sp} \sin \theta)} \right) \exp (ik_{sp} \theta - \pi/4), \]

\[ u_s^{(n)} = \frac{2\pi}{k_{sp}} \left( \frac{\sin^2 (k_{sp} \sin \theta)}{\sin^2 (k_{sp} \sin \theta)} \right) \exp (ik_{sp} \theta - \pi/4), \]

\[ u_s = \frac{\hbar^{n} \sin (k_{sp} \sin \theta)}{\sin (k_{sp} \sin \theta)} \exp (ik_{sp} \theta), \]

Typical directivity patterns of the spectral components of the radiated Rayleigh waves in the three cases are shown in Figs. 3 and 4. The principal maxima of the directivity patterns in the cases of a nonpropagating crack and a symmetrically growing crack lie in the direction of the normal to the crack (for the ratio of the parameters used in the calculations, \( k_{pl} = 3 \) or \( k_{pq} \approx 3.3 \), the directivity pattern for a crack of constant length has two additional weak side lobes). The principal maxima for an asymmetrically propagating crack (for greater clarity, the curves in Fig. 4 are not normalized) lie in the direction of angles \( \theta_{\text{max}} = \arcsin (\sqrt{2} \eta) \). The directivity patterns of the radiation of longitudinal and transverse bulk waves are shown in Figs. 5 and 6 for the cases of nonpropagating and propagating cracks. Without launching a detailed discussion of the results [see Eqs. (10)-(12)], we call attention to the fact that neither longitudinal nor transverse bulk waves are emitted in the direction of the normal to the surface in any of the cases considered here.

This fact must be taken into consideration in determining the placement of acoustic-emission sensors, because the probable sites of the inception of surface cracks (zones of elevated stress concentrations, contact with an aggressive medium, etc.) can often be foreseen. Clearly, the sensors must not be placed directly below cracks. The same considerations pertain to surface waves, which are not emitted in any directions in the plane of the crack.

\[ \theta_{\text{max}} = \arcsin (\sqrt{2} \eta) \]

1Reference 7 contains an error in the representation of the boundary condition (5). This condition should have the form \( G_{x=0} = 0, G_{x=x^*} = 0 \).

2It is advisable to calculate the temporal spectrum of the signal, because narrowband acoustic-emission sensors are customarily used in practice.

Translated by J. S. Wood