Reduction of sound radiation from automotive-type panels

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Reduction of Sound Radiation from Automotive-Type Panels

by

Andreas Rousounelos

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Abstract

The problem of the effect of structural modifications on sound radiation from a structure is very important with many applications in the design of a variety of products. In practise, for the design of acoustically optimum structures simplified theoretical models are used which can lead to unexpected behaviour of the real structures. Alternatively, measurements or numerical simulation can be used. The disadvantage of these two methods is that the study of alternative structures (structures with different properties) is not easy since for every modification a new structure must be manufactured or a new numerical model must be created. Moreover, these two methods do not easily give an insight into the physical mechanisms of sound radiation from modified structures.

In the first part of this thesis the problem of vibration and sound radiation from a plate with an attached beam stiffener is studied theoretically. This extends the current theories of vibroacoustic behaviour of infinite plates with an infinitely long beam discontinuity and infinite plates with an infinite number of equidistant beam discontinuities forming a periodic structure. Firstly, the propagation of flexural waves and the subsequent sound radiation from an infinitely long plate strip is considered. The scattering of plate flexural waves by a finite beam across its width is considered. Changes to the mean square velocity, sound power and radiation efficiency of the plate strip due to the introduction of the beam stiffener are identified. Simplified approximate analytical expressions for the low-frequency range, well below the critical frequency, are also presented. This model is extended for the case of a finite rectangular plate by incorporating wave reflection from the two additional boundaries of the plate. Expressions for the radiation efficiency
and the mean squared velocity of the stiffened plate are derived. The results are compared with results derived using well established numerical methods.

For structures with more complicated modifications the derivation of analytical expressions is not easy. For this reason numerical optimisation is often used. In this thesis numerical optimisation is used to optimise the modes of a structure in order to radiate acoustic energy weakly into the acoustic medium. The effectiveness of point mass and line stiffener modification to create acoustically optimum modeshapes in a flat simply supported plate is firstly studied. The results show significant reduction in the sound power radiated by the optimised structural modes. These results are also verified experimentally.

This optimisation method has certain advantages when it is used on automotive panels. The main advantage is that the panel under consideration can be isolated from the rest of the automotive structure and it can be optimised alone. This drastically reduces the time required and makes the optimisation practically applicable. A simplified car model is used and the proposed optimisation is applied to one of its floor panels. Firstly, point masses and line stiffeners are used as structural modifications to create weakly radiating modeshapes. Then more commonly used geometrical modifications are used on the floor panel. Two such modifications are studied; swages and domes. The results show a significant reduction in the radiation efficiency and sound power radiated by the optimised panel.

A MATLAB programme with a Graphical User Interface (GUI) has been developed for panel optimisation using the methods described in this thesis. A description of the GUI can be found in Appendix B. The MATLAB programme and all the necessary codes can be found on the attached CD.
Acknowledgement

The financial support of Jaguar and Land Rover Research for this project is gratefully appreciated. Especially, I would like to thank Nigel Taylor and Libin Wang for their interest in the project, their ideas and assistance.

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Nomenclature

NVH  Noise Vibration and Harshness
NGM  Noise Generation Mechanism
CAE  Computer Aided Engineering
NPA  Noise Path Analysis
TPA  Transfer Path Analysis
FEM  Finite Element Method
BEM  Boundary Element Method
SEA  Statistical Energy Analysis
FFT  Fast Fourier Transform
GA   Genetic Algorithm
DOF  Degrees of Freedom
GUI  Graphical User Interface
Notations

If not explicitly stated otherwise:

- $D$: flexural rigidity (Nm)
- $E$: Young’s modulus (N/m$^2$)
- $f$: frequency (Hz)
- $f_n$: cut-on frequency of the $n^{th}$ mode of a stiffened plate (Hz)
- $G$: shear modulus (N/m$^2$)
- $G_{p1p2}$: cross spectrum of signals $p_1$, $p_2$ ()
- $h_p$: plate thickness (m)
- $I$: second moment of inertia ($m^4$)
- $I_n$: normal intensity (W/m$^2$)
- $K$: torsional stiffness (Nm/rad)
- $k_b$: wavenumber of flexural waves in a beam (1/m)
- $k_n$: wavenumber of flexural waves in a stiffened plate (1/m)
- $k'_n$: wavenumber of evanescent waves in a stiffened plate (1/m)
- $k_p$: wavenumber of flexural waves in a plate (1/m)
- $L_x$: $x$ dimension of a plate (m)
- $L_y$: $y$ dimension of a plate (m)
- $m$: mass (kg)
- $N$: total number of modes in a modal superposition ()
- $P$: sound power (W)
$R_n$ reflection coefficient of flexural waves ()
$R'_n$ reflection coefficient of evanescent waves ()
$T_n$ transmission coefficient of flexural waves ()
$T'_n$ transmission coefficient of evanescent waves ()
$t$ time (s)
$u$ velocity $(m/s)$
$\bar{U}$ Fourier transform of velocity ()
$w$ displacement $(m)$
$w_0$ displacement amplitude constant $(m)$
$w_{inc}$ displacement of incident flexural wave $(m)$
$w_s$ displacement of scattered flexural waves $(m)$
$a_n$ modal expansion coefficient for the $n^{th}$ mode ()
$\nu$ Poisson’s ratio ()
$\rho$ mass density $(kg/m^3)$
$\sigma$ radiation efficiency ()
Chapter 1

Introduction

Reducing the amount of sound radiated by machines has become a main concern over the last decades. This is mainly because societies are becoming more aware of the impact that prolonged exposure to noise has to human health. As a result standards and regulations regarding noise are becoming more and more strict. Moreover, it has become apparent to manufacturers that many aspects of their products related to sound contribute to the overall quality of the products. The term sound quality has been used for all these aspects.

All of the above suggest the study of sound radiation from vibrating structures and ways to modify it in a desirable way. In many applications the targets of noise radiation and sound quality are conflicting with other product targets such as final cost of the product, reduced weight etc. Hence, in the design process it is desirable to take into account most of these aspects.

In this first chapter of the thesis an introduction will be presented of acoustic engineering as it is applied to the automotive industry. This is usually termed Noise Vibration and Harshness (NVH) engineering. In this section, the domain of the problem with which this thesis is concerned will be also discussed. Moreover, a review of the most relevant published work related
to the problem studied will be presented. In the final section of this chapter
the organisation of the remainder of the thesis will be presented.

1.1 Automotive NVH

Noise Vibration and Harshness is the study and modification of noise and vibration characteristics of vehicles. As the name suggests, this comprises of the study and modification of objective quantities such as noise and vibration levels, as well as subjective quantities such as harshness which in general is the subjective impression of annoyance or pleasantness of different aspects of a product related to its sound.

The purpose of NVH engineering, according to Harrison [1], includes the following:

- **Legislation** - The permitted noise levels emitted by vehicles are specified by national and international directives and standards. The sale of non-conforming vehicles is prohibited.

- **Marketing to new customers** - Good sound quality can be used to distinguish a vehicle from its otherwise similar competitors.

- **Customer expectation** - Customers have come to expect continuous improvement in new vehicles that they buy. They expect their new purchase to be better equipped, more comfortable, and perform better than the vehicle they traded in.

- **Marketing to existing customers** - Since the modern car industry needs turnover to survive people need to be encouraged to trade in regularly. Hence, improving the NVH performance of a new model compared to an existing one encourages customers to do so.
Since NVH is an important part of product development it must be included in the design process, as described in the textbook edited by Crocker [2], the ISO technical report for general machineries [3], and textbooks specifically for automotive development [4, 5]. According to the ISO technical report [3] there are four stages in the design process:

1. **Clarification of Task** - At this stage the functional economic and safety requirements of the future product have to be specified, including noise specifications. These specifications, as also mentioned above, need to make reference to legislation, request from clients, products of competitors, sales arguments, and so forth.

2. **Conceptual Design** - Here potential solution principles have to be compared and the most appropriate ones are selected. In automotive engineering this stage will produce the vehicle platform which specifies features such as cockpit design, package variables, suspension and steering, drivetrain, etc. At this stage knowledge of the final product is still limited. Judgement of the noise behaviour is made using first principles of acoustic engineering, existing products and former experience.

3. **Detailed Design** - Here the general layout of the vehicle, basic materials and individual components are defined. At this point it is easier to make reasonable estimates of the expected noise emission and to compare different design options. Computer Aided Engineering (CAE) tools are used extensively to model, predict and optimise the acoustic and vibration behaviour of the future product. It is very important for the engineers to be able to predict any problem at this stage since they can be fixed in a shorter time with less cost.

4. **Prototyping** - At this stage the first prototype is built. Noise and vibration measurements can be performed to check the overall behaviour. Problems that have been identified at this stage need to be solved by redesigning the vehicle. For an effective developing process it is important to reduce the number of prototypes built.
A common methodology of dealing with noise and vibrations in cars and other complex machineries is adopting a system analysis approach. In this, one or more excitations produce one or more responses through the transfer function of the system. In vehicles, the excitations are all the possible noise and vibration sources (or Noise Generation Mechanisms, NGM) such as engine vibrations, exhaust system acoustic radiation, etc. Usually, engineers are interested in the response of these sources at certain points inside the car compartment, such as driver’s and/or passengers’ ears locations. Hence, the transfer function in the system approach is the transfer function from the noise or vibration source to the receiver and this can be vibrational, acoustic or a combination of both.

In cases of many noise and vibration sources a cascade approach is usually used to categorise them. A basic distinction of noise sources is structure-borne and air-borne sources. For the case of vehicle NVH structure-borne noise sources include engine vibrations, vibration due to tyre/road interaction, vibrations of the exhaust, etc. Air-borne sources include acoustic radiation from the engine block, aerodynamic noise, exhaust and intake noise, etc. An alternative to this is to consider all possible sources at the first level of the cascade model and at the second level to consider the transmission paths that the sources follow, categorised as acoustic and structural paths [2].

The cascade analysis can help in the development process described above. At the first stage, targets can be set for each individual noise path. In the detailed design stage these paths can be studied individually and different designs can be implemented to achieve the targets. In the prototyping stage a design can be verified experimentally.

A very powerful tool that has been extensively used in the automotive industry for the analysis of different noise paths is the so called Noise (or Transfer) Path Analysis (NPA or TPA) [1]. Using this, the transfer functions of the paths that have been defined in the cascade model can be measured. Also
operational data of all different sources can be determined. Hence, a complete description of noise characteristics of a vehicle can be achieved with a good insight into its causes. Moreover, for a more detailed description of a noise path, it can be subdivided into different components. For example, for noise generated by the engine vibrations as it is heard by the driver, the path consists of the car body and the acoustic cavity in the car compartment. The structural path (car body) can be subdivided into different panels, such as dash board, roof panel, floor panels, etc. The contribution of each panel to the sound pressure at the driver’s location can then be determined by performing a panel contribution analysis [6].

From the above discussion it is clear that in order to minimise noise to achieve predefined targets for a vehicle, one needs to either modify the noise generated by different sources (or the way these are coupled to the vehicle structure) if possible, or modify the paths that noise follows. This project deals with the latter. Specifically, the remainder of the thesis considers the problem of panel modifications for acoustic radiation reduction, when this panel has been previously identified to contribute to unwanted vibro-acoustic phenomena (e.g. excess noise levels compared to predefined targets).

1.2 Literature Review

Methods for the study of vibro-acoustic problems can be categorised into analytical and numerical methods. The advantages of analytical methods are that they can predict acoustic and/or vibration results without many computations required in a short time and usually without frequency limitations. They can also shed light to physical mechanisms involved in certain phenomena. The main disadvantage of analytical methods is that they can be used only for simple problems (e.g. simple structures). Analytical methods can be used to quickly compare different design alternatives without the need for considerable analysis effort, by roughly seeing how different parameters
influence quantities such as radiated sound pressure or sound power. This can be very useful in the conceptual stage.

On the other hand numerical methods are very powerful in accurately modeling real life situations. They can be used to predict vibration and sound radiation from complex structures. The most widely used numerical methods are the Finite Element Method (FEM) and the Boundary Element Method (BEM). Both of them can be used for structural and acoustic analysis but due to certain advantages and disadvantages FEM is mainly used for structural problems whereas BEM is used for acoustical problems. In contrast with analytical methods, numerical methods require a great number of computations and usually take a long time to be performed. Moreover, they need considerable manual effort to design detailed models. They are also only applicable for low-frequency analysis. Numerical methods are used extensively in the detailed design stage. Many commercial computer software can perform FEM and BEM analyses. For the purpose of minimisation of sound radiation by structures, numerical methods have been used in conjunction with numerical optimisation algorithms to achieve optimum designs.

In the following sections the most relevant published work for analytical methods and numerical vibro-acoustic optimisation is presented. These serve as the basis for the subsequent analysis in this thesis.

### 1.2.1 Analytical Methods

The literature on analytical methods in acoustic radiation is vast and chronologically ranges from the late 19\textsuperscript{th} century with the work of Lord Rayleigh [7], essentially defining the science of acoustics. Inherently the problem of acoustic radiation includes the problem of vibrations of the structures under consideration. The most important theories and analytical methods can be found in the classical acoustic textbooks [8, 9, 10]. Fundamental problems, such as acoustic radiation by simple sources or flexural waves, are presented.
1.2 Literature Review

Important acoustic phenomena, such as the hydrodynamic short-circuiting, critical frequency for sound radiation, etc. are described.

In this project, the problem of structural modifications and how these influence vibration of and acoustic radiation from panels is considered. A very common structural modification of panels is the attachment of stiffeners that can lead to improvement in the static and dynamic behaviour of the structure.

Vibration of Stiffened Plates  The simplest case is that of a single stiffener of infinite length attached on a infinite plate. This case has been first studied by Ungar [11]. In his paper he derived mathematical expressions for the motion of a stiffening beam when it is excited by flexural waves in an attached plate. Shear deformation is neglected for both the plate and the beam. By assuming an incident wave to the beam Ungar was able to calculate the amplitude coefficients of a reflected and a transmitted wave together with two coefficients for the near-field in both sides of the beam. Hence, in Ungar’s analysis, the beam-stiffener is modeled as a discontinuity in the medium carrying flexural waves and, as every discontinuity, it scatters the energy of the flexural waves. Ungar found that for certain frequencies and angles incidence there is maximum or minimum transmission of flexural waves across the beam. This was interpreted in terms of trace matching between flexural waves in the plate with flexural and torsional wave in the beam. Ungar’s work also discusses the concept of stress and strain concentrations factors for fatigue analysis.

Similar analysis has been carried out by Fahy and Lindqvist [12] for an infinite plate with a single beam stiffener and two parallel beam stiffeners forming a frame. The wavenumbers of free propagation in the waveguide consisting of the two parallel beams are calculated. The numerical results show that for damped plates, the frames greatly reduce the effectiveness of damping near the cut-off frequency of the waveguide. Application of damping to the beams
was found not to suppress this behaviour.

Another publication concerned with transmission of panel flexural waves across a single stiffener is that by Maidanik and his co-researchers [13]. In it a different approach is adopted. The surface impulse response function of the structure is used to calculate the transmission and reflection coefficients. This method can lead to more complicated analysis compared with the analysis mentioned above. In this paper the panel is considered to have a membrane behaviour that simplifies but also limits the analysis.

More recently, Arruda and his co-researchers [14] have proposed a method for experimentally obtaining the reflection and transmission coefficients of propagating flexural waves in effectively infinite thin plates caused by reinforcing beams. This method has potential use in modeling the vibration behaviour of a structure using ray-tracing; a method that is currently under development.

Kessissoglou and Pan [15] have used the same methodology as Ungar [11] to calculate reflection and transmission of flexural waves through a reinforcing beam on plate. As it was also mentioned in Ungar’s analysis the reinforcing beam is very effective in attenuating wave transmission except at two frequencies. This was explained in terms of trace matching of waves in the plate and the beam. Kessissoglou and Pan used an active control method to improve the attenuation of wave transmission at these frequencies. Significant reduction was achieved by using an array of point forces and point moments.

Grice and Pinnington [16] presented a method for the vibration analysis of structures made up by large beams and flexible plates. By taking advantage of the usually great difference in the wavelengths between the long waves in the stiff beams and the short waves in the plates, the proposed method suggests the analysis of these two types of waves separately. For the total response of the structure the two responses are combined. The exchange of energy between the long and short waves acts as a damping mechanism.
Moreover, in this paper a discussion is presented on the better applicability of analytical methods for solving the problem under consideration, compared with FEM and Statistical Energy Analysis (SEA) methods. Due to the short waves in the plates a great number of elements is required to obtain adequate accuracy of a FEM model of such a structure. The resulting model may be too large to analyse in an acceptable time. Alternatively, the long waves in the stiff beams may have so low modal density that SEA is inappropriate.

The application of the method of considering a stiffening beam on a plate as a discontinuity that scatters flexural waves has been also used for the analysis of wave reflection and transmission at structural junctions [17].

The published work presented so far deals with infinite beam and plate structures. Most of the publications aim for the analysis of large structures such as ship, aircraft or aerospace structures where the assumption of infinite structures may be reasonable. For smaller structures the assumption is not valid because of the presence of strong reflections from the boundaries that create a modal behaviour of the structure.

Heckl [18] was the first to analyse a beam-plate strip system, finite in one dimension and infinite in the other. His analysis starts with a single finite beam lying along the finite dimension of the plate. The main difference with the previously published work on infinite structures is that here the modal behaviour of the beam discontinuity is taken into account. Propagation of flexural waves in the plate is considered in the infinite dimension. The plate in this dimension is separated by the beam into two sections, \((\infty,0)\) and \((0,\infty)\). In the finite dimension the vibrational velocity is given as a modal superposition by an infinite trigonometrical series. The two boundaries of the plate are simply supported. Heckl, using the expressions derived by Ungar [11] for the plate/beam interface, gives expressions for the reflection and transmission coefficient of flexural waves in the plate caused by the beam. The near-field coefficients are neglected. Evaluation of the transmission coefficient shows that the beam prevents much of the energy to be transmitted.
across it except at a frequency very close, but not at, the resonant frequencies of the beam. The method is extended for the case of more identical beams at equal distances (periodic structure). Formulas for the force impedance and the mean squared velocity of the beam-plate system are also given.

Kessissoglou [19] used the same methodology to control the energy transmission in a finite-width beam-reinforced plate by active means. Significant attenuation of the resonant peaks in the flexural wave transmission due to flexural and torsional resonances in the beam is achieved.

In the publications above the main consideration is plates with a single beam discontinuity. Another theoretically very important category of structures is that of periodic structures.

A review of the contribution from University of Southampton to the research on periodic structures is presented by Mead [20]. Many important phenomena, such as pass- and stop-bands in the wave propagation of flexural waves in periodic structures, have been predicted theoretically. Several methods of analysis are presented.

Rumerman [21] analysed the vibration and wave propagation in periodically stiffened (ribbed) plates. A general solution was obtained for the forced vibration of an infinite thin plate. The analysis is carried out in the wavenumber domain. The classical approach of idealising the ribs as parallel line attachments that exert line forces and moments on the plate is followed.

Azimi and his co-researchers [22] have used the receptance method to calculate the free vibrations of finite rectangular periodic plates with simply supported or clamped end conditions. Gupta [23] used the knowledge of natural flexural waves and the propagation constant to predict free vibrations of plates and beams with periodic supports. Takahashi and Chishaki [24] presented a method for the analysis of free vibrations of plates with intermediate frames. This method takes into account the behaviour of the beams in the frames which is incorporated in terms of restraining forces acting on the
plate. Zarutskii and Prokopenko [25] studied the influence of discrete ribs on the wavenumber of harmonic propagating waves by using a variational approach and the Galerkin method. They also investigated the effects of the stiffness of the ribs on the natural frequencies and modeshapes of a cross-ribbed plate. Kelkel [26] used Green’s function and the receptance method for the analysis of structures consisting of beams and plates. Nicholson [27] used Green’s functions to form integral equations for the free vibration of stiffened rectangular plates. All the above publications present the most common analytical techniques for the analysis of stiffened plates.

The effect of fluid loading on the vibration of stiffened plates has been considered by many researchers. Mace [28, 29] considered in two papers the response of periodically stiffened plates for different types of excitations. He found that the fluid loading results in acoustically damped propagating waves. Similar analysis was carried out by Eatwell and Butler [30], and Maidanik and Dickey [31]. In the latter publication it is concluded that the effect of fluid loading on the spectral characteristic of the vibrations of a regularly stiffened panel is minimal when the line and line moment impedance of the ribs are not unusually high. Eatwell [32] also studied the free-wave propagation in plates, with or without fluid loading, for regularly (periodic) and irregularly stiffening.

Most of the published work presented above deals with idealised structures, such as, infinite or semi-infinite plates or periodic stiffening. It was only recently that the study of vibration of finite non-periodic stiffened plates was studied by Lin and Pan [33] and a closed form solution was found. The assumption of infinitesimally narrow stiffeners is used. The equations of flexural and torsionally vibrations of the stiffening beam are combined with the equation for the flexural vibration of the plate. Results are given in the form of modal superposition for different types of excitation.
1.2 Literature Review

Sound Radiation from Stiffened Plates  One of the first and most important publications regarding sound radiation from stiffened plates is that of Maidanik [34]. By studying the acceleration spectrum of infinite and finite structures, and its relation to the sound power he showed that the discontinuities of the plate at the beam’s location will, in general, increase the coupling between the structure and the surrounding acoustic medium. Maidanik also derived approximate expressions for the radiation efficiency of the beam-plate system as a function of frequency by employing a statistical method, which led to the development of a then new technique later called Statistical Energy Analysis.

Lyon [35] studied the sound radiation from a beam when it is attached to a plate. He used the expressions previously derived by Ungar [11] for an infinite beam-plate system. Using the velocity spectrum of the vibrational field of the structure consisting of an incident wave, a reflected and transmitted wave and two evanescent waves (one at each side of the beam), he derived formulas for the radiation resistance of this structure. Explicit expressions are given for the case where the beam is modeled as a simply supported, clamped or edge supported line.

Seren and Hayek [36] dealt with the development of analytical models for the prediction of acoustic radiation from an infinite plate with a single line discontinuity due to an incident plane acoustic wave. They firstly formed the solution in the form of a Fourier integral. They then evaluated the integral using three methods; the steepest descent path method, conformal transformation and the modified saddle point method. They concluded that the modified saddle point asymptotic series solution gave numerically identical results.

Lin and Hayek [37] researched the problem of acoustic radiation from an infinite plate with a single stiffener excited by a point force acting on the stiffener. Their solution shows a new coincidence angle, where the sound pressure peaks in the directivity function, which depends on the relative
stiffness and mass of the attached beam. Approximate expressions were also derived for the radiated sound power.

Many researchers have also focused on the problem of sound radiation from periodic structures. Evseen [38] investigated the sound radiation from an infinite fluid-loaded plate with equidistant beams excited by a harmonic mechanical force. He used the Fourier transform to derive a solution in closed form. Numerical evaluations for two cases of driving forces show a significant increase in the radiated sound pressure for certain frequencies. Garrelick and Lin [39] presented further simplifications in the expressions derived by Evseen by neglecting the effect of fluid loading. They also presented examples of the radiated sound pressure by an infinite plate with two, four and an infinite number of stiffeners. Their results, as those obtained by Evseen, show increase in the sound pressure for certain frequencies.

Romanov [40], using a similar method to that of Evseen, derived an expression for the sound pressure radiation in the far-field and the radiated sound power for an infinite plate with two reinforcing beams exited by a field of random forces between the beams. His results show an increase in the radiation at low frequencies. Belinskii [41] studied the sound radiation from an infinite periodically stiffened plate with projecting stiffeners by taking into account short wave diffraction. This phenomenon is important for large ratios between the height of the stiffeners and the spacing between them.

Cray [42] studied the acoustic radiation from an infinite plate fluid-loaded on one side with two sets of stiffeners. The stiffeners of each set are identical and equidistant. The far-field radiated pressure was obtained for the case of a line driving force using the method of stationary phase. Similar analysis has been previously presented by Mace [43]. In his work one set of stiffeners represented the bulkheads and another set the intermediate frames.

**Vibration and Sound Radiation from Plates with Other Types of Discontinuities** Extensive research has been carried out for the vibration
and acoustic radiation from plates with other types of modifications. Berry and Nicolas [44] used the Rayleigh-Ritz method to analyse structural vibrations of a finite panel with added point masses and vertical line stiffeners. The panel at the boundaries was supported by translational and rotational springs to simulate general elastic boundary conditions. The Rayleigh integral was used to calculate acoustic radiation from the baffled panel.

Lebedev [45] presented a work on the influence that general mass or stiffness discontinuities have on the sound radiation from structures. He used a modal superposition method and a perturbation method to derive expressions for the case of discontinuities with small input impedance compared to that of the characteristic impedance of the structure. Lebedev, based on his analysis, concluded that one of the characteristics of the influence of discontinuities on sound radiation is the transfer of energy of high order modes, with greater radiation resistance, into low order modes. Another characteristic is that, unlike the case of a uniform structure where sound radiation below the critical frequency is generated by a system of local sources, such as excitation point and structural boundaries, in the presence of discontinuities the structure radiates sound from its entire surface. Several conclusions were also drawn for the specific example of sound radiation by a cylindrical shell with a point mass. The results of this analysis were later confirmed experimentally by Ekimov and Lebedev [46].

Howe and Heckl [47] adopted a different approach to study the effect of density and stiffness discontinuity on the sound radiation by plates. The fluctuation on these two parameters in the material of the plate were modeled as random functions of position of the plate. The objective of this study was to consider the effect of different parameters that are not taken into account when a structure under consideration is modeled using idealised theories.

Weissenburger [48] presented a method for analysing the effect of local modifications, such as additional point masses or springs, on the vibration of any linear system. The method has its basis in an eigenfunction expansion of the
solution of the modified system in terms of the eigenfunctions of the unmodified system. The solution is applicable even for large modifications. More recently, Wu and Luo [49] presented a method for the analysis of the same problem, but in their work the system of equations resulting from Weissenburger’s method was treated numerically. An eigenvalue problem was formed which can be easily solved using computers.

Cuschieri and Feit considered a similar problem to that of sound radiation, which is the sound scattering from a plate with distributed mass and stiffness inhomogeneity. This paper is presented in this section since similar methods are used for these two problems. The method used by Cuschieri and Feit was that of transforming the governing equations into the wavenumber domain. The transformation in the presence of inhomogeneities takes the form of a Fredholm integral equation of the second kind with a singular kernel. To obtain a solution a singularity substraction technique is used in conjunction with the Nystrom approximation.

Many researchers have considered the effect of only inertia discontinuities on the vibration and sound radiation of structures. Boay [50] used the Rayleigh’s method for the free vibration analysis of thin rectangular plates with a concentrated mass. The analysis of Nicholson and Bergman [27] extents for thick plates with a number of concentrated masses as well. More recently, Kopmaz and Telli [51] presented a method for free vibration analysis of thin rectangular plates carrying a distributed mass.

A different approach was adopted by Steinberg and McCoy [52] for the analysis of vibrations of plates with mass inhomogeneities. This was based on the theory of multiresolution decomposition. Using this, the solution of a dynamical system with microscale inhomogeneities is reduced to two coupled formulations on different scales.

Li and Li [53] recently considered the effect of distributed masses on the sound radiation from a plate. They presented numerical results of their analysis for few specific cases of loading for acoustic radiation in air and water.
1.2.2 Numerical Optimisation

Optimisation has been used extensively in engineering to consider complex problems. In an optimisation procedure the objective function(s) and the design variable(s) need to be defined. An objective function is the quantity that is needed to be minimised or maximised. This is achieved by optimising the design variables, which are variables of the objective function, usually within some predefined limits (constraints). In vibroacoustic optimisation the objective function is often a quantity such as vibration level or acoustic pressure and the design variables are properties of the structure such as material, thickness or geometry. An optimisation algorithm is required to search for optimum values for the design variables that minimise (or maximise) the objective function.

For structural-acoustic optimisation an extensive review on the concept, methods and the most important work done has been published by Marburg [54], who himself has contributed significantly to the field. Structural-acoustic problems can be categorised into two main categories; interior problems, in which the acoustic radiation is in a closed domain (e.g. acoustic radiation of a panel in a car compartment), and exterior problems, in which the acoustic radiation is in an unbounded domain (e.g. acoustic radiation of a panel outside a car). For interior problems a common choice for the objective function is the sound pressure at one or more points of interest. For acoustic radiation in a car compartment a common objective function is the sound pressure at the approximate location of the driver’s ears. Since usually low frequency analysis methods are used, the sound field inside the car compartment for this frequency range is insensitive to small variations of the receiver’s location. On the other hand, for the exterior problems, where usually there are no specific points of interest, as for example with the case of car pass-by noise, sound power is used as an objective function. Other objective functions such as mean squared vibration velocity, which defines an upper limit for the sound power, and modal sound power have been reported. The latter will be discussed later in this section.
1.2 Literature Review

The design variables in a vibroacoustic optimisation problem are linked to the structural properties. Since the Finite Element Method is the most common method for structural analysis, the design variables are certain local properties of the structure at the model’s nodes or elements. Lamancusa [55] optimised the thickness of the structure at nodal points for different objective functions. Naghshineh and his co-researchers [56] used the Young’s modulus and material density of each element in their structural model as design variables. These types of modifications can be very effective in designing structures with optimum behaviour but are difficult to manufacture and frequently lead to structures with increased weight which can be an undesirable factor for many applications. The increase of weight can be avoided by setting constraints in the optimisation algorithm of the maximum allowed total or added mass of the structure.

Another choice of design variable, instead of changing locally the material properties or the thickness of the structure, is to change its geometry. A common such modification that has been used extensively for automotive structures is applying panel swages, beads or stamped ribs. Using these techniques the stiffness of the structure is increased locally without changing its mass. Marburg [57] presented an optimisation procedure where several keypoints are defined in the function of the geometry of the structure. The position of these keypoints is then optimised to reduce the level of a noise transfer function. In Part II of his paper [58] this method is applied to a vehicle dashboard. The main advantage of the geometry based modification is that it can generate a great variety of new designs that are relatively easy to manufacture and that it gives great flexibility to the optimisation. On the other hand it is very difficult to create and parameterise global geometry based models for complex structures such as a vehicle body. Moreover, for complex geometries many keypoints need to be defined which result in a large number of design variables that need to be optimised. This makes the search space of the optimisation algorithm more complex and reduces its efficiency. The same problem of increased number of design variables arises when the properties of each node or element are used as design variables for
To overcome these problems Marburg [59] proposed a design modification procedure based on the concept of modification functions. In this method the modifications are applied directly to the finite element mesh without need for a geometry model. In the modification procedure proposed by Marburg modification domains are defined. These are the areas on the structural model under consideration that the modifications will be applied to. The type of modifications that will be applied are defined by modification functions. A modification function determines how the geometry (the coordinates) of a node that falls inside the modification area will be changed. The value of the modification function may depend on where the node lies in the modification area (for example, distance from its centre or boundaries). This can result in swages and other types of beads, of which the advantages were discussed above. The main advantage of the method is that there is no need for a design variable for each node in the model. Instead, the design variables are the parameters required for defining the modification function and the domains of modification. Hence, a smaller number of design variables is used and still all nodes in the model can be modified. Marburg and Hardtke [60] used the concept of modification functions for the structural-acoustic optimisation of a sedan floor panel. Another application of the method is that by Fritze et al [61].

Different types of optimisation algorithms have been used for vibroacoustic optimisation. In earlier published work usually gradient based methods were used. The advantage of the gradient based optimisers is that they are very effective without the need for many iterations of the computationally expensive vibroacoustic analysis. Gradient based optimisers require information about the derivatives of the objective function with respect to each design variable. This procedure is known as sensitivity analysis. Pritchard and his co-researchers [62] used sensitivity analysis for the optimisation of the location of nodal points of the modeshape of a structure in order to reduce its vibration levels. Sensitivity analysis was performed using the finite differ-
1.2 Literature Review

ence method. A similar approach was used by Hambric [63] for the acoustic optimisation of a ribbed cylindrical shell. Structural analysis was performed using FEM. For the acoustic analysis a low-order approximation of the expression for the sound radiation was used. Finite difference method was used for the sensitivity analysis. A program for global acoustic design sensitivity analysis was developed by Wang and Lee [64] using a FEM and BEM formulation of the problem. Results were presented for the optimisation of a half scale automobile cavity.

A gradient based optimiser that has been used in many structural-acoustic applications is the one developed by Shanno and Phua called CONMIN [65]. Lamancusa [55] and Patil and Crocker [66] have used CONMIN for vibroacoustic optimisation. In both papers different objective functions are used and compared including mean squared velocity, modal or frequency average sound power and radiation efficiency.

The main disadvantage of the gradient based optimisers is that they are local optimisers and they are inappropriate for highly nonlinear problems, such as sound radiation, since they generally expect the design spaces to behave in a manner consistent with locally measured design sensitivities [67]. Moreover, gradient based optimisers require this design sensitivity which for realistic structures can only be approximately performed using low order finite difference schemes.

Another type of optimisation algorithms that do not have the disadvantages of gradient based optimisers and they have been used successfully for vibroacoustic optimisation is the genetic algorithms (GA) [68]. These algorithms make use of the principle of natural selection for dealing with complex nonlinear multidimensional optimisation problems. GA are blind algorithms in the sense that they do not require any knowledge of the function and its derivatives other than function values at selected points. Genetic algorithms have outperformed classical optimisation methods in several cases [69].

Ratle and Berry [70] have used a genetic algorithm for the vibroacoustic
optimisation of a plate carrying point-masses. The position of masses is optimised for different optimisation criteria. Zhang and his co-researchers optimised the position of masses on a Distributed-Mode Loudspeaker for optimal sound pressure response. Genetic algorithms have also been used for automotive NVH applications. Xiaolong et al considered the vibroacoustic optimisation of one [71] and two [72] stamped ribs in a panel. Moreover, Kropp [73] presented a work on full vehicle body NVH optimisation.

The main disadvantage of genetic algorithms, as well as all other heuristic methods, is that they require a large number of function evaluations. In the case of structural-acoustic analysis using FEM and BEM, which are the most common methods used for vibroacoustic analysis, each function evaluation requires a large number of computations to be performed. This makes their use impractical for large complex structures. For this reason simplified, approximate analysis tools have been developed to reduce the computational time for every function evaluation, and hence the time required for the optimisation. Acoustic analysis is usually computationally more expensive since, following the BEM formulation, it produces in general full, non-symmetric, frequency dependent matrices. Habrict [67] derived expressions for the sound radiation of a structure based on a Taylor series expansion of the Helmholtz Integral Equation. A different approach for the simplification of the acoustic analysis is the use of radiation modes as described by Cunefare and Currey [74] for a general 3D structure and Currey and Cunefare [75] for a baffled panel. Radiation modes can be used for the reconstruction of the impedance matrix and the determination of sound power. The calculation of radiation modes can be performed before the optimisation procedure only once for each frequency of interest. The results can then be stored and retrieved for use in the optimisation. Marburg et al [76] proposed the concept of acoustic influence coefficients. Using this the sound pressure at any point in the domain of the problem can be determined by solving simple algebraic expressions that involve the nodal displacement or velocity of a structure and the influence coefficients. Similar to the above mentioned radiation modes approach, acoustic influence coefficients are evaluated in the first step of the
optimisation and then are repeatedly used in the entire optimization process.

In vehicle NVH it is sometimes desirable to optimise a single panel that has been identified to contribute to an unwanted acoustic phenomenon. A direct approach would be to compute the vibroacoustic response of the whole vehicle structure to the operational forces, which are not necessarily applied on the panel under consideration, and modify the panel for an optimum response. A different approach is the use of super-elements for the part of the structure that is not modified during the optimisation [58] to reduce the computational time. More recently, a wave-based substructuring approach have been developed by Donders et al [77] for the same purpose.

The method proposed in this thesis and will be presented in Chapters 5 and 6, suggests the isolation of the panel under consideration to avoid the analysis of the rest of the structure. One problem with isolating the panel from the rest of the structure is that appropriate boundary conditions need to be applied to the isolated panel in order to simulate the behaviour of the panel in the structure. Translational and rotational stiffness is used as elastic boundary conditions for which values need to be appropriately determined. Since the vibration response of the panel to any excitation can be given as a summation of its structural modes, these structural modes are optimised in order to minimise the sound radiated by them, hence creating weakly radiating structural modes. Apart from reducing the computational time and therewith making the optimisation applicable to practical applications, this method also has the advantage of designing a panel with optimum acoustic radiation characteristics for any excitation that will excite the optimised structural modes. Hence, the optimised panel is insensitive to redesigns of the rest of the structure or of the sources. A more detailed discussion on the use of this optimisation method for automotive-type panels is presented in Chapter 5.

Koopmann and his co-researcher [78, 79, 80] were the first to consider the concept of weakly radiating structural modes. As it is described in the book
by Koopmann and Fahnline [78], this method tries to impose to the structure modeshapes that radiate acoustic energy weakly. These are modeshapes that create strong acoustic cancellations in the vicinity of the structure and hence prevent acoustic energy from propagating into the far-field. The method presented in the book and in references [79, 80] uses point masses and gradient based optimisation to create up to three weakly radiating modeshapes. The results are verified numerically and experimentally. Later in the thesis this method is extended for other types of structural modification and optimisation method.

1.3 Organisation of the thesis

In the next chapter an analytical study on the effect of a single stiffener in the vibration and acoustic radiation of a plate strip is presented. For this, exact and approximate expressions are presented. The study is later extended for the case of a finite plate with a single stiffener. The results are compared with results from numerical simulation.

In Chapter 3 a methodology for the numerical optimisation for the minimisation of acoustic radiation from plates is presented. Structural modes that radiate acoustic energy weakly are imposed to the plates by placing point masses and line stiffeners with optimum properties at optimum locations. Principles of different optimisation algorithms are presented and the development of a genetic algorithm is discussed. The work presented in this chapter extends the previously published work on the field. A theoretical analysis is presented for the vibration of plates with point masses and line stiffeners at arbitrary locations and orientations.

In Chapter 4 the results for the optimised plates described in Chapter 3 are validated experimentally. The experimental procedure of measuring sound power in an anechoic chamber using the intensity method is described in
details. The experimental results show reduction in the sound power radiated by the optimised modes.

In Chapter 5 the methodology of Chapter 3 is extended for the case of automotive-type panels. A simplified car model is used and one of its floor panels is optimised for sound radiation minimisation by changing the location and properties of a number of point masses and line stiffeners. An optimisation algorithm is designed to use Nastran for the FEM structural analysis of the modified panel. A boundary element formulation is used for the calculation of the sound power radiated by the structure.

In Chapter 6 geometrical panel modifications are used as design variables for the optimisation. For this, the concept of modification functions is used. The design of swages and domes, two panel modifications commonly used in automotive panels, is optimised for the purpose of sound power minimisation.

In the last chapter of the thesis a summary of all of the analyses and results of the thesis is presented along with suggested future work.
Chapter 2

Investigation of the influence of stiffeners on vibration and acoustic radiation of plates

In this chapter the effect of a beam stiffener on the vibration and sound radiation of a finite width beam-plate strip and a finite rectangular beam-plate system is considered. From the review of literature presented in the previous chapter it can be seen that the effect of a single or a small number of (not necessarily equidistant) stiffeners on the acoustic radiation of plates has not been studied theoretically before. Merely analytical models for the sound radiation from stiffened plates excited by simplified types of forces, such as point or line forces exist. In automotive applications the excitation mechanisms are complex and simplified models are not applicable. For example automotive panels are usually excited by the transfer of vibrational energy from neighbouring panels. In this chapter by considering wave propagation in a stiffened plate strip and a stiffened rectangular plate a more general approach is presented compared to the specific excitation mechanisms considered in previous publications.
In the first section of this chapter an infinitely long beam-stiffened plate strip is considered. Expressions for the reflection and transmission coefficients, as well as the near-field close to the beam discontinuity, are developed. Expressions are also derived for the radiation efficiency of the scattered flexural waves caused by the beam. Numerical results calculated using the derived equations are compared with results calculated using previously published expressions for an infinite beam-plate system. The comparison shows that the exclusion of the modal behaviour of the beam-plate system can significantly underestimate the radiation efficiency of the structure at frequencies below its critical frequency. In Section 2 this work is extended for a finite rectangular plate with a beam-stiffener in the middle of its length. An expression for the mean squared vibrational velocity of the structure is derived in closed form. Results for the radiation efficiency are compared with numerical calculation carried out using finite element and boundary element software.

2.1 Beam-stiffened plate strip

2.1.1 Theory

In this section, the sound radiation and the vibrational field of a plate strip infinite in one dimension with an attached beam across the finite width is studied theoretically. The vibrational field of the beam-plate system is assumed to consist of propagating flexural waves in the infinite $x$ dimension and specific mode-shapes in the finite $y$ dimension.

In an approach similar to that presented by Lyon [35] and Heckl [18], the displacement of the structure is represented as the summation of an incident flexural wave of unit amplitude traveling from $-\infty$ to 0 in the $x$-direction together with a reflected propagating wave with complex amplitude $R_n$ and a transmitted propagating wave with complex amplitude $T_n$, as well as the
2.1 Beam-stiffened plate strip

Figure 2.1: Illustration of the beam stiffened infinite plate strip of width $L_y$ with an incident, reflected and transmitted propagating wave, and reflected and transmitted evanescent waves.

Reflected and transmitted evanescent waves in the vicinity of the beam located at $x = 0$ with complex amplitudes $R_n'$ and $T_n'$ respectively. This is illustrated in Figure 2.1. Using complex exponential notation the wave field in the $x$-direction for $x < 0$ is $\exp(-ik_n x) + R_n \exp(i k_n x) + R_n' \exp(k_n' x)$. Equivalently, the wave field in the $x$-direction for $x > 0$ is $T_n \exp(-ik_n x) + T_n' \exp(-k_n' x)$.

In the $y$-direction the displacement is given as a modal superposition across the width, $L_y$, of the plate strip. Both the boundaries of the plate strip are assumed to be simply supported. Since the beam is attached to the plate, the boundary conditions for the beam are simply supported as well. For simplicity, the effect of fluid loading has been neglected, which limits the analysis to cases of light fluid loading. In order to identify the influence of the beam discontinuity on the wave propagation in the plate strip, the displacement field, $w(x, y)$, can be written as the summation of an incident wave field $w_{inc}(x, y)$ and a scattered wave field $w_s(x, y)$ which arises due to the beam discontinuity:

$$ w(x, y) = w_{inc}(x, y) + w_s(x, y) \quad (2.1) $$

where the incident wave field in the $x$-direction is a propagating wave trav-
eling undisturbed by the beam discontinuity from $-\infty$ to $\infty$, and it is given as:

$$w_{inc}(x, y) = w_0 \sum_n a_n \sin \frac{n\pi y}{L_y} \exp(-ik_n x)$$ (2.2)

The scattered wave field due to the beam discontinuity is given as:

$$w_s(x, y) = w_0 \left\{ \begin{array}{ll}
\sum_n a_n \sin \frac{n\pi y}{L_y} \left[ R_n \exp(ik_n x) + R'_n \exp(k'_n x) \right] & x < 0 \\
\sum_n a_n \sin \frac{n\pi y}{L_y} \left[ (T_n - 1) \exp(-ik_n x) + T'_n \exp(-k'_n x) \right] & x > 0 
\end{array} \right.$$ (2.3)

with $w_0$ being the displacement amplitude constant and the wavenumbers of the propagating and evanescent waves are given by

$$k_n^2 = k_p^2 - \left( \frac{n\pi}{L_y} \right)^2$$ and $$k'_n = k_p^2 + \left( \frac{n\pi}{L_y} \right)^2$$

respectively where $k_p$ is the free plate wavenumber. The coefficient $a_n$ is the modal expansion coefficient of the $n^{th}$ mode and its value depends on the excitation of the beam-plate system.

In all the above expressions the $n$ subscript indicates that the given quantity is defined for the $n^{th}$ mode in the $y$ dimension of the infinite beam-plate strip. Moreover, a harmonic wave process is assumed and the time harmonic term $\exp(i\omega t)$ is implied.

The wavenumber of the propagating flexural waves, $k_n$, imposes a low frequency limit to the waves that the structure can support. In this case $k_n$ is
2.1 Beam-stiffened plate strip

a real quantity only when

\[ k_p^2 > \left( \frac{n\pi}{L_y} \right)^2. \]  

(2.4)

Thus, the cut-on frequency for the \( n \)-th mode, \( f_n \), which is the minimum frequency for the plate to support propagating waves, is given as [14], [19]:

\[ f_n = \left( \frac{1}{2\pi} \right) \left( \frac{n\pi}{L_y} \right)^2 \sqrt{\frac{D}{\rho h_p}}. \]  

(2.5)

In Equation (2.5), \( D = \frac{E_p h^3}{12(1-\nu)} \) is the flexural rigidity of the plate, \( h_p \) is the thickness of the plate and \( E_p, \nu \) and \( \rho \) are the Young’s modulus, Poisson ratio and mass density of the material of the plate respectively.

In his paper [11] Ungar gave the differential equations for the flexural and torsional vibration of a beam excited by flexural waves in an attached plate. Classical theory for the plate and the beam was used, hence the effect of shear deformation was not taken into account. In Ungar’s study a rotary inertia term for the beam arises. However, since it was shown to contribute little to the vibration of the structure, it has been neglected from the following analysis in this chapter. Using Equation (2.1) and assuming continuity of displacement and its first spatial derivative at the beam-plate interface, four equations are derived, two for each side of the beam. From these four equations the four unknown coefficients \( R_n, R'_n, T_n \) and \( T'_n \) can be derived as:

\[
\begin{align*}
R_n &= \frac{(ik'_n - k_n)}{d_n} \left[ \beta (\alpha + 2A k_n^2 k'_n) - 2C k'_n \alpha \right] \\
T_n &= \frac{2ik_n (k_n - ik'_n) [C (\alpha + 2A k'_n (k_n^2 + k_n^2)) - Ak'_n^2 \beta]}{d_n} \\
R'_n &= \frac{2k_n [\beta (\alpha + Ak_n k'_n (k_n - ik'_n)) - C (k'_n + ik_n) \alpha]}{d_n} \\
T'_n &= \frac{2k_n (ik'_n - k_n) (Ak_n k'_n \beta + iC \alpha)}{d_n}
\end{align*}
\]  

(2.6)
where \( d_n = (k_n + ik'_n) [\alpha + 2Ak_nk'_n (k_n - ik'_n)] [2C (k'_n + ik_n) - \beta] \), \( \alpha = k_b^4 - \left( \frac{n\pi}{2\gamma} \right)^2 \) and \( \beta = k_t^2 - \left( \frac{n\pi}{2\gamma} \right)^2 \). Moreover, the coefficient \( A \) represents the ratio of the flexural rigidity of the plate, \( D \), to the flexural rigidity of the beam, \( B \), where \( B = E_bI \), \( E_b \) is the Young’s modulus of the material of the beam and \( I \) is the second moment of area of the beam. The coefficient \( C \) represents the ratio of the flexural rigidity of the plate \( D \) to the torsional stiffness of the beam \( Gk \). The wavenumbers \( k_b \) and \( k_t \) are the flexural and torsional wavenumbers of the beam respectively.

The expressions in Equation (2.6) have been also derived, in different forms, by Heckl [18] and Kessissoglou [19]. Note the relationship between the absolute value squared of the reflection and transmission coefficients \( |R_n|^2 + |T_n|^2 = 1 \) which comes from an energy consideration of all the waves in the structure [11]. As was previously discussed by Heckl [18] and Kessissoglou [19] \( |T_n|^2 \) has a peak \( (|T_n|^2 = 1) \) at a frequency close to the \( n \)th resonant frequency of the beam.

The sound power, \( P \), of a planar radiator is given as [8]:

\[
P = \frac{\rho_0ck}{8\pi^2} \int_{-k}^{k} \int_{-k}^{k} \frac{|\tilde{U}(k_x, k_y)|^2}{\sqrt{k^2 - k_x^2 - k_y^2}} dk_x dk_y
\]

(2.7)

where \( \rho_0c \) is the characteristic impedance of the acoustic medium, \( k \) is the acoustic wave number and \( \tilde{U}(k_x, k_y) \) is the spatial Fourier transform of the velocity field, \( u(x, y) \), of the plate defined as:

\[
\tilde{U}(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u(x, y) e^{-i(\xi k_x + \eta k_y)} dx dy.
\]

(2.8)

The radiation efficiency of flexural waves in plates, with or without discontinuities, is well known to be close to unity for frequencies above the critical frequency of the plate [34]. Thus, for the remainder of the analysis in this
chapter only sound radiation below the critical frequency is considered. For this frequency range sound radiation from the incident wave, $w_{inc}(x, y)$, in Equation (2.1) is only due to the standing waves in the $y$ dimension for which mathematical expressions are well-known [8]. Here, sound radiation from the scattered field, $w_s(x, y)$, due to the beam discontinuity is considered.

The scattered displacement field, $w_s(x, y)$, given by Equations (2.3), can be converted to velocity, $u(x, y)$, required by Equation (2.8), by multiplication with $i\omega$, as all the waves are assumed to be harmonic.

Substituting the velocity field, $u(x, y)$, into Equation (2.8) for a given value of $n$, and evaluating the integral gives the spatial Fourier transform of the velocity field as:

$$\tilde{U}_n(k_x, k_y) = \omega u_n \left( \frac{T_n - 1}{k_x + k_n} + \frac{R_n}{k_x + ik_n} - \frac{T_n'}{k_x - ik_n} \right) \left( 1 - \left( \frac{n\pi}{L_y} \right)^2 - k_y^2 \right) \frac{1}{L_y}$$

where $u_n = w_0 a_n$ is the velocity amplitude constant of the $n^{th}$ mode. The expression in the first bracket is due to the waves, propagating and evanescent, in the $x$-direction, which is equivalent to the expression for the spatial Fourier transform of the scattered field derived by Lyon [35]. The expression in the second bracket is due to the standing wave in the $y$-direction [8].

Letting the complex reflection and transmission coefficients be written in amplitude and phase form as $R_n = |R_n|e^{i\phi_R}$ and $T_n = |T_n|e^{i\phi_T}$ respectively, the absolute squared value of the spatial Fourier transform of Equation (2.9), can be written as:
\[ \left| \tilde{U}_n(k_x, k_y) \right|^2 = \omega^2 \left| u_n \right|^2 \left| \tilde{U}_s(k_x) \right|^2 \left| \tilde{U}_y(k_y) \right|^2 \] (2.10)

where \( \left| \tilde{U}_s(k_x) \right|^2 \) is the modulus squared of the Fourier transform of the scattered propagating waves in the \( x \)-direction, which by excluding the near-field effect is given as:

\[
\left| \tilde{U}_s(k_x) \right|^2 = \frac{(1 + |T_n|^2) (k_x - k_n)^2 + |R_n|^2 (k_x + k_n)^2 + 2 (k_x^2 - k_n^2) |R_n| \cos(\phi_R)}{(k_n^2 - k_x^2)^2} \\
- \frac{2 (k_x - k_n)^2 |T_n| \cos(\phi_T) + 2 (k_x^2 - k_n^2) |T_n||R_n| \cos(\phi_T - \phi_R)}{(k_n^2 - k_x^2)^2} \] (2.11)

It can be proven that \( \cos(\phi_T - \phi_R) = 0 \) (see the Appendix A), hence the above expression can be further simplified as:

\[
\left| \tilde{U}_s(k_x) \right|^2 = \frac{(1 + |T_n|^2) (k_x - k_n)^2 + |R_n|^2 (k_x + k_n)^2 + 2 (k_x^2 - k_n^2) |R_n| \cos(\phi_R)}{(k_n^2 - k_x^2)^2} \\
- \frac{2 (k_x - k_n)^2 |T_n| \cos(\phi_T)}{(k_n^2 - k_x^2)^2} \] (2.12)

\( \left| \tilde{U}_y(k_y) \right|^2 \) is the modulus squared of the Fourier transform of the standing wave in the \( y \)-direction given by Cremer et al [8] as:

\[
\left| \tilde{U}_y(k_y) \right|^2 = \left( \frac{2\pi n L_y}{k_y^2 L_y^2 - n^2 \pi^2} \right)^2 \sin^2 \frac{k_y L_y - n\pi}{2} \] (2.13)
The mean squared velocity of the scattered field per unit length of the structure for the $n^{th}$ mode, neglecting the near-field, is:

$$\langle \bar{u}_n^2 \rangle = \frac{1}{S} \left( \frac{1}{T} \int_0^T u_n^2(x, y) dt \right) dS = \frac{|u_n|^2 \omega^2}{2} \left[ 1 - |T_n| \cos(\phi_T) \right]$$ (2.14)

where $T$ in the limits of the inner integral is the wave period and $u_n(x, y) = i\omega w_s(x, y)$ for a given mode $n$. When the transmission coefficient $T_n$ has a real value of 1, and hence there is no discontinuity at the location of the beam, both $\left| \tilde{U}_s(k_x) \right|^2$ and $\langle \bar{u}_n^2 \rangle$ vanish.

The radiation efficiency of a structure is defined as the ratio of the acoustic power radiated by the structure to the acoustic power radiated by a piston with the same area. For a given mode $n$ it is given by:

$$\sigma_n = \frac{\bar{P}_n}{\rho_0 c S \langle \bar{u}_n^2 \rangle}$$ (2.15)

which can be calculated using Equations (2.10), (2.7) and (2.14). The total (frequency-averaged) radiation efficiency is then given by [8]:

$$\sigma = \frac{\sum_n \sigma_n \langle \bar{u}_n^2 \rangle}{\sum_n \langle \bar{u}_n^2 \rangle}$$ (2.16)

The double integral of Equation (2.7), by substituting Equation (2.10), does not have a known solution. For frequencies well below the critical frequency though, when $k_p \gg k$, the following approximation can be made: $k_x \ll k_n$ and $k_y \ll n\pi/L_y$. Equations (2.12) and (2.13), which constitute the integrant of Equation (2.7) can be simplified by neglecting $k_x$ and $k_y$. The sound power per unit length radiated by the scattered vibrational field at the $n^{th}$ mode can then be approximated by:
\[ \bar{P}_n \simeq \frac{|u_n|^2 \rho_0 c k_n^2}{\pi k_n^2} \left( \frac{L_y}{n \pi} \right)^2 \left[ 1 - |R_n| \cos(\phi_R) - |T_n| \cos(\phi_T) \right] \] (2.17)

and its radiation efficiency per unit length is given by:

\[ \bar{\sigma}_n \simeq \frac{2k^2}{\pi k_n^2 L_y} \left( \frac{L_y}{n \pi} \right)^2 \left[ 1 + \frac{|R_n| \cos(\phi_R)}{|T_n| \cos(\phi_T) - 1} \right] \] (2.18)

Equations (2.17) and (2.18) are valid only at frequencies well below the critical frequency of the plate strip and they do not include the sound radiation from the vibrational near-field caused by the beam discontinuity.

From Equation (2.17) it can be seen that the sound power radiated by the scattered field of the beam-plate strip system is the sound power radiated by the \( n^{th} \) mode, as it is given by Cremer et al [8] for frequencies well below the critical frequency, multiplied by the factor \( \frac{2}{\pi k_n^2} (1 - |R_n| \cos(\phi_R) - |T_n| \cos(\phi_T)) \) where the term in the brackets is the difference of the real part of the amplitudes of the propagating waves in the scattered field, \( w_s(x, y) \). When the transmission coefficient \( T_n \) is real and equal to 1, hence when there is no discontinuity, the sound power becomes 0. When the reflection coefficient \( R_n = e^{i\pi} \) and the transmission coefficient \( T_n = 0 \), meaning that the beam discontinuity forms a simply supported boundary condition, the sound power radiated by the scattered field of the beam-plate strip system is equal to the sound power radiated by the \( n^{th} \) mode multiplied by the factor \( \frac{4}{\pi k_n^2} \).

Similarly for the radiation efficiency given by Equation (2.18) it can be seen that it is equal to the radiation efficiency of the \( n^{th} \) mode, as it is given in reference [8], multiplied by the factor \( \frac{k}{\pi k_n^2} \left( 1 + \frac{|R_n| \cos(\phi_R)}{|T_n| \cos(\phi_T) - 1} \right) \) where the fraction inside the brackets is the ratio of the real part of the amplitudes of the propagating waves in the scattered field, \( w_s(x, y) \). For \( T_n = 1 \) where the sound power and mean squared velocity of the scattered field are both equal to 0, Equation (2.18) takes an indeterminate form. For \( R_n = e^{i\pi} \) and \( T_n = 0 \)
2.1 Beam-stiffened plate strip

Figure 2.2: Comparison of the radiation efficiency of the scattered vibrational field caused by a rectangular stiffening beam with dimensions 7 × 7 mm for a beam-plate strip system (—) and an infinite beam-plate system (−−).

the radiation efficiency of the beam-plate system is equal to the radiation efficiency of the $n^{th}$ mode multiplied by the factor $2k/\pi k_n^2$. Note that the expressions of the radiation efficiency in Equation (2.18) is independent of the velocity amplitude constant $|u_n|$.

2.1.2 Results

Figure 2.2 shows the comparison of the radiation efficiency of a plate strip with a rectangular stiffening beam with dimensions 7 × 7 mm with the radiation efficiency of an infinite beam-plate system with the same stiffening beam calculated using the expressions given by Lyon [35]. The comparison shows that the radiation efficiency of the infinite beam-plate system, which disregards the modal behaviour of the plate and the beam, can significantly underestimate the radiation efficiency of the infinite beam-plate strip system below the critical frequency.
2.1 Beam-stiffened plate strip

Figure 2.3: Radiation efficiencies of an infinite plate strip with rectangular stiffening beams across the finite dimension with cross sections: $7 \times 3.5$ mm (—), $7 \times 7$ mm (—), $5 \times 5$ mm (—). The radiation efficiency of the plate strip in Figure 2.2 was calculated by using Equations (2.7), (2.9), (2.14) and (2.15). The double integral in Equation (2.7) was calculated using the \textit{dblquad} MATLAB function which makes use of the adaptive Lobatto quadrature. For the results a steel plate was used with $E_p = 200$ GPa, $\rho = 7872$ kg/m$^3$ and thickness $h_p = 3.5$ mm. The beam is of the same material. The width of the plate, and hence the length of the beam is $L_y = 0.208$ m. The radiation efficiencies in Figure 2.2 were calculated from the cut-on frequency of the 1$\text{st}$ mode of the plate strip, $f_1 = 194$ Hz, up to the critical frequency of the plate strip, $f_c = 3507$ Hz. For the total radiation efficiency the first 4 modes were used in the summation of Equation (2.16) as the cut-on frequency of the 5$\text{th}$ mode is at 4845 Hz which is higher than the critical frequency of the plate.

It is known that the double integral of Equation (2.7) has singularities at $|k_x| = |k_y| = k$. In order to avoid the singularities and make the numerical evaluation of the integral possible mechanical damping is introduced by
making the Young’s modulus of the plate and the beam material complex in the form \( E_d = E(1 + i\eta) \), where \( \eta \) is the loss factor. By doing so the singularities are shifted away from the real axis and the integration path is free of singularities. A loss factor of \( \eta = 0.05 \) was used for both the plate and the beam for all the results in this chapter.

Figure 2.3 shows the radiation efficiency of plate strip with beams of various rectangular cross sections. A low value for the radiation efficiency can be seen at the 1st resonant frequency of the stiffening beam for the results shown. The 1st resonant frequency of the beams with cross section 7 × 3.5 mm (—) and 7 × 7 mm (—), where the first dimension is the height of the beam and the second dimension is the width of the beam, is at 370 Hz. The 1st resonant frequency of the beam with cross section 5 × 5 mm (---) is at 264 Hz. Comparing the first two structures with beams with the same height and resonant frequencies, it can be also seen that the value of the radiation efficiency at the 1st resonant frequency of the beam depends on the width of the beam where the beam-plate system with a stiffening beam with larger width has a lower radiation efficiency than that with a stiffening beam with smaller width. The dependance of the radiation efficiency to the properties of the stiffening beam becomes unclear when the frequency approaches the critical frequency at which the radiation efficiency is independent of the beam properties.

2.2 Finite beam-stiffened plate

2.2.1 Theory

In the previous section wave propagation and sound radiation from a plate strip with an attached beam was considered. In this section the analysis is extended for finite rectangular plates simply supported at all edges with an attached vertical beam. In the \( x \)-direction, the results of the previous sec-
tion for the reflection and transmission of flexural waves and the evanescent waves at the beam discontinuity are used. Moreover, an infinite number of reflections from the two boundaries of the plate are now introduced. In the $y$-direction, the vibration field is considered to be one of the modes shapes of the simply supported plate.

In the $x$ dimension the length of the plate is $L_x$, with boundaries at $-\frac{L_x}{2}$ and $\frac{L_x}{2}$. As in the previous section, the analysis starts with an incident wave in the $x$-direction traveling from $x = -\infty$ to $x = 0$, which is the location of the beam, $\exp(-i\left[k_n\left(x + \frac{L_x}{2}\right) + \frac{\pi}{2}\right])$. The real part of the incident wave is 0 at the boundary at $x = -\frac{L_x}{2}$ which complies with the simply supported boundary condition. This incident wave, when it reaches the beam, will create a reflected and transmitted wave as:

$$\bar{R}_n \exp\left(i\left[k_n\left(x + \frac{L_x}{2}\right) - \frac{\pi}{2}\right]\right) x < 0 \quad (2.19)$$

$$\bar{T}_n \exp\left(-i\left[k_n\left(x - \frac{L_x}{2}\right) + \frac{\pi}{2}\right]\right) x > 0 \quad (2.20)$$

and two evanescent waves, same as in Equation (2.3). The reflected and transmitted propagating waves in Equations (2.19) and (2.20) have phases adjusted so they comply with the boundary conditions at $x = -\frac{L_x}{2}$ and $x = \frac{L_x}{2}$ respectively. This phase difference between the incident and the reflected and transmitted waves needs to be incorporated in $R_n, T_n, R'_n$ and $T'_n$. Following the same procedure to obtain the values for these coefficients as in the previous section, the coefficients for the finite stiffened plate can be written in terms of the coefficient of the beam-plate strip system as:
\[
\begin{align*}
\bar{R}_n &= \exp(-ik_nL_x) R_n \\
\bar{T}_n &= \exp(-ik_nL_x) T_n \\
\bar{R}'_n &= -i \exp(-ik_nL_x/2) R'_n \\
\bar{T}'_n &= -i \exp(-ik_nL_x/2) T'_n
\end{align*}
\] (2.21)

In the analysis of the beam-plate strip system in the previous section, the reflected propagating wave in the \(x < 0\) section of the plate was assumed to travel from 0 to \(-\infty\). In the case of a finite plate the reflected wave will again be reflected from the boundary of the plate at \(x = -\frac{L_x}{2}\) with a reflection coefficient of \(R_b = -1\) for a simply supported boundary condition [81]. The resulting propagating wave will travel towards the beam at \(x = 0\) where it will be reflected once again and the same procedure will be repeated. Extending this procedure for an infinite number of reflections from the beam and the boundary of the plate and adding all the waves in each direction will result in two waves traveling in the \(x^+\) and \(x^-\) directions in the \(-\frac{L_x}{2} \leq x < 0\) section of the plate. The amplitudes of the waves are given by the following infinite mathematical series:

\[
w^+ = \sum_{m=0}^{\infty} (-1)^m \bar{R}_n^m = \frac{1}{1 + \bar{R}_n}
\] (2.22)

and

\[
w^- = \sum_{m=1}^{\infty} (-1)^m \bar{R}_n^m = -\frac{\bar{R}_n}{1 + \bar{R}_n}
\] (2.23)

for the \(x^+\) and \(x^-\) directions respectively. Note that \(w^+\) includes the incident wave. The wave traveling in the \(x^+\) direction with amplitude \(w^+\) will be transmitted across the beam to the \(0 < x \leq \frac{L_x}{2}\) section of the plate with an amplitude \(\bar{T}_n w^+\). This will start the procedure described above again. Assuming again that the boundary of the plate at \(x = \frac{L_x}{2}\) is simply
supported will result in two propagating waves one with amplitude $\bar{T}_n w^2_+$ in the $x_+$ direction and one with amplitude $-\bar{T}_n w^2_+$ in the $x_-$ direction. The latter will again be transmitted across the beam to the $-\frac{L_x}{2} \leq x < 0$ section of the plate and the procedure will start over. Taking an infinite number of transmissions from one section of the plate to the other and adding up all the waves in each direction, results in four waves, two in each direction for each section of the plate. Their amplitudes given by an infinite mathematical series that converges to:

$$
\begin{align*}
  w_{1+} &= \frac{1 + \bar{R}_n}{(1 + \bar{R}_n)^2 - \bar{T}_n^2} \quad w_{2+} = -\frac{\bar{T}_n}{\bar{T}_n^2 - (1 + \bar{R}_n)^2} \\
  w_{1-} &= 1 - w_{1+} \quad w_{2-} = -w_{2+}.
\end{align*}
$$

The above expressions are valid when $(1 + \bar{R}_n)^2 - \bar{T}_n^2 \neq 0$ and $\bar{T}_n^2 - (1 + \bar{R}_n)^2 \neq 0$. Hence, the above analysis is valid only in the presence of a stiffening beam.

Using the amplitudes $w_{1+}, w_{1-}, w_{2+}$ and $w_{2-}$ with the waves

$$\exp\left(-i \left[k_n (x + \frac{L_x}{2}) - \frac{\pi}{2}\right]\right) \text{ and } \exp\left(i \left[k_n (x + \frac{L_x}{2}) - \frac{\pi}{2}\right]\right)$$

for the $x_+$ and $x_-$ directions respectively, the displacement in the $x$ dimension of the structure after making use of trigonometrical identities can be written in the following form:

$$w(x) = w_0 \begin{cases} 
  (1 - 2w_{1+}) \sin \left(k_n (\frac{L_x}{2} + x)\right) + (w_{1+} \bar{R}_n - w_{2+} \bar{T}_n) \exp(k'_n x) & -\frac{L_x}{2} \leq x < 0 \\
  2w_{2+} \sin \left(k_n (\frac{L_x}{2} - x)\right) + (w_{1+} \bar{T}_n - w_{2+} \bar{R}_n) \exp(-k'_n x) & 0 < x \leq \frac{L_x}{2}
\end{cases}
$$

The displacement in the $y$ dimension, as in the case of the beam-plate strip system, is given as the summation of sine functions. Converting the displace-
2.2 Finite beam-stiffened plate

ment of the plate into velocity, its Fourier transform can be calculated from Equation (2.8) for the \( n \)th mode:

\[
\tilde{U}_n(k_x, k_y) = u_n \tilde{U}_x(k_x) \left( 1 - (1)^n e^{-ik_y L_y} n\pi \right) \frac{1}{(\frac{n\pi}{L_y})^2 - k_y^2} L_y \tag{2.25}
\]

where

\[
\tilde{U}_x(k_x) = (1 - 2w_{1+}) - \exp \left( \frac{ik_x L_x}{2} \right) k_n + k_n \cos \left( \frac{k_n L_x}{2} \right) + ik_n \sin \left( \frac{k_n L_x}{2} \right) +
\]

\[
+ 2w_{2+} \exp \left( \frac{ik_x L_x}{2} \right) k_n - k_n \cos \left( \frac{k_n L_x}{2} \right) + ik_n \sin \left( \frac{k_n L_x}{2} \right) +
\]

\[
+ (w_{1+} \tilde{R}_{n'} - w_{2+} \tilde{T}_{n}) \frac{1 - \exp \left( -\frac{L_x(k_n' - ik_x)}{2} \right)}{k_n' - ik_n} +
\]

\[
+ (w_{1+} \tilde{T}_{n'} - w_{2+} \tilde{R}_{n'}) \frac{1 - \exp \left( -\frac{L_x(k_n' + ik_x)}{2} \right)}{k_n' + ik_n} \tag{2.26}
\]

Equation (2.25) can be used to calculate the radiated sound power by numerically solving the double integral in Equation (2.7).

The mean squared velocity of the finite structure, by neglecting the near-field, can be given in closed form as:

\[
\langle \tilde{u}_n^2 \rangle = |u_n|^2 (4 |w_{2+}|^2 + |1 - 2w_{1+}|^2) \left( \frac{k_n L_x - \sin (k_n L_x)}{4k_n} \right) \frac{L_y}{4S} \tag{2.27}
\]
2.2 Finite beam-stiffened plate

2.2.2 Results

Radiation Efficiency

To check the validity of the model described above, the radiation efficiency calculated using the above analysis is compared with the radiation efficiency calculated by FEM/BEM numerical methods, using Nastran/Patran and LMS Virtual Lab software. The comparison is shown in Figure 2.4. The material and the thickness of the plate is the same as given in the previous section. The dimensions of the plate are $L_x = 0.307$ m and $L_y = 0.208$ m. The beam has a rectangular cross section with dimensions $7 \text{ mm} \times 7 \text{ mm}$. In Figure 2.4 the plate in the case of the FEM/BEM calculation was excited by a unit point force at a node close to the corner of the plate in order to excite all modes in the frequency range of the calculation. Note that from Equations (2.25) and (2.27) it can be seen that the radiation efficiency is independent of the velocity amplitude coefficient $u_n$, and hence independent of the relative amplitude $a_n$ of each mode. 400 quadrilateral elements were used in both the FEM and BEM model of the stiffened plate.

Figure 2.5 compares the radiation efficiency of the plate with properties as described above and beams with cross section of different dimensions. Unlike the results for the beam-plate strip system with beams with the same properties as in Figure 2.5, the radiation efficiency of the finite stiffened plate does not change significantly for the different beam properties. This is not the case for all beam stiffened plates. Figure 2.6 shows the radiation efficiency of the same plate with beams of different dimensions. The changes in the radiation efficiency of the structure due to the changes of the beam dimensions can be seen.
2.2 Finite beam-stiffened plate

Figure 2.4: Comparison of the predicted radiation efficiency calculated using FEM/BEM (—) and the wave propagation model (——) for a finite steel plate with a rectangular steel beam stiffener 7 mm high and 7 mm wide.

Figure 2.5: Radiation efficiency of a finite rectangular beam-stiffened plate with beams with rectangular cross section with different dimensions: 7 × 3.5 mm (——), 7 × 7 mm (—), 5 × 5 mm (——).
2.2 Finite beam-stiffened plate

Figure 2.6: Radiation efficiency of a finite rectangular beam-stiffened plate with beams with rectangular cross section with different dimensions: $7 \times 7$ mm (—), $1 \times 1$ mm (—), $11 \times 11$ mm (—).

Mean squared velocity

The validation of Equation (2.27) for the mean squared velocity of the finite stiffened plate requires the knowledge of the frequency and mode dependent coefficient $u_n$. Unfortunately, there is no known method for the determination of this coefficient. Figure 2.7 shows the comparison of the mean squared velocity of a $1.2$ mm thick, steel plate with a stiffening beam with rectangular cross section $7 \times 7$ mm calculated using FEM and Equation (2.27). A constant value $|u_n|^2 = 5 \cdot 10^{-4}$ for the absolute squared value for the velocity amplitude coefficient was used for the comparison. For the FEM calculation the stiffened plate was excited by a unit point force at $(L_x/4, L_y/4)$ which was $(0.077 \text{ mm}, 0.052 \text{ mm})$. From the comparison of Figure 2.7 it can be seen that although the amplitudes of the two curves do not exactly coincide, since the value for the velocity amplitude coefficient has been set arbitrarily, the wave propagation model can predict the peaks at the resonant frequencies of the stiffened plate.
A useful observation can be made about the shift of the resonant frequencies of the plate due to the stiffening beam when Equation (2.27) is evaluated for a single mode \( n \) in the \( y \) dimension of the plate. Figure 2.8 shows the comparison of the mean squared velocity of a plate with a rectangular beam stiffener with cross section \( 5 \times 5 \) mm and a plate without a stiffener for \( n = 1 \). The results for the plate without the stiffener were calculated using Equation (2.27) for a beam stiffener with a rectangular cross section with both its dimensions set close to 0. From the comparison it can be seen that the resonant frequencies of the plate for a given mode \( n \) in the \( y \) dimension that are below the \( n^{th} \) resonant frequency of the beam, \( f_{bn} \), are shifted higher in frequency after the attachment of the beam. For example, the first resonant frequency of the plate without the stiffener for \( n = 1 \) (mode (1,1)) is at 98 Hz, which is lower than the first resonance frequency of the beam at 212 Hz indicated by a vertical line in Figure 2.8. Hence, this resonant frequency is shifted higher in frequency due to the attachment of the beam. The opposite effect is observed for resonant frequencies of the plate that are higher in frequency than the

Figure 2.7: Comparison of the mean squared velocity of a steel plate using FEM (—) and the wave propagation model for a rectangular beam with cross section \( 7 \times 7 \) mm (—).
2.2 Finite beam-stiffened plate

Figure 2.8: Calculated results of the mean squared velocity of a 1.2 mm thick steel plate without a stiffener (—) and a stiffened plate with a beam stiffener of rectangular cross section with dimensions $5 \times 5$ mm(—−).

resonant frequency of the beam. For example, the fifth resonant frequency of the plate without the stiffener for $n = 1$ in the $y$ dimension of the plate (mode (5,1)) is higher than the first resonant frequency of the beam, hence, it is shifted lower in frequency after the attachment of the beam. In other words, the resonant frequencies of the plate are shifted towards the resonant frequency of the stiffening beam with same mode number as that in the $y$ dimension of the plate. The second and fourth modes of the plate in Figure 2.8 (modes (2,1) and (4,1)) are affected slightly by the beam because both these modes have a nodal line at the location of the beam. The same effect regarding the shift of the resonant frequencies has been previously predicted by Soedel [82] for stiffened cylindrical shells using the receptance method.

The effect of a stiffener on the resonant frequencies of a plate is also verified experimentally. Figure 2.9 shows the input mobility of a plate with and without a stiffener. The properties of the plate and the stiffener are the same as those used for the comparison of Figure 2.8. The plate was placed inside
2.3 Conclusions

In this chapter the effect of an attached beam on the vibration and sound radiation from an infinitely long plate strip and a finite rectangular plate has been studied theoretically.

Figure 2.9: Experimental measurements of the input mobility of a 1.2mm thick steel plate without a stiffener (—) and a stiffened plate with a beam stiffener of rectangular cross section with dimensions $5 \times 5$ mm (−−).

...
For the infinitely long beam stiffened plate strip the scattering of an incident propagation flexural wave in the unbounded $x$ dimension due to a beam discontinuity in the bound $y$ dimension is considered. Expressions for the reflection, transmission and near-field coefficients are presented. Based on these coefficient, expressions for the sound power, radiation efficiency and mean squared velocity of the beam-plate strip system are developed. Simplified approximate analytical expressions for the low-frequency range, well below the critical frequency, are also presented. From numerical calculation it was shown that the radiation efficiency of the plate strip has a low value at the resonant frequencies of the beam at frequencies below the critical frequency of the plate. The exact value of the radiation efficiency depends on the properties of the stiffening beam. For a beam with a rectangular cross section, the radiation efficiency is decreased when the width of the beam is increased.

The beam-plate strip model is then extended for finite rectangular plates. Wave propagation in the finite structure and the scattering of the flexural waves due to the beam are considered. An expression for the displacement field of the structure is developed by taking into account an infinite number of wave reflections from the boundaries of the plate. Moreover, expressions for the sound power, mean squared velocity and radiation efficiency of the stiffened plate are derived. The model is validated by comparing it to calculations carried out using FEM and BEM methods. Numerical results for the radiation efficiency of a plate with attached beams of different dimensions are presented. The effect of a stiffener on the resonant frequencies of the plate is also discussed. It was observed that the resonant frequencies of the plate are shifted toward the resonant frequency of the beam with the same mode number as that in the $y$ dimension of the plate. This observation is in agreement with previously published prediction for a stiffened cylindrical shell.
Chapter 3

Optimisation of structural modes of simply supported plates by using line stiffeners and point masses

In this chapter a numerical optimisation approach is adopted for the minimisation of sound radiation from plates. The acoustic optimisation of structural modes of simply supported plates is considered. This optimisation method was developed by Koopmann and his co-researchers [79, 80]. The optimisation approach, as discussed in details in the book by Koopmann and Fahnlne [78], is based on the design of a structure in order to have structural modes that radiate sound weakly. The optimised modes are termed weak radiators. This method has the advantage that the optimisation does not depend on the way the structure is excited, and once weak radiating modes have been imposed to the structure, they will radiate weakly for any input forces that will excite them. This means that once a plate of certain geometrical and physical properties has been designed to have weak radiating modes in a certain frequency range, it can be used for any application with-
out having to take into account the possibly complex excitation mechanisms. The main property of weakly radiating structural modes is that they create strong acoustic cancellations in the vicinity of the structure, hence preventing acoustic energy from radiating into the far-field. This effect is known as hydrodynamic short-circuit [8]. This is possible only at frequencies below the critical frequency of the structure, which gives a large frequency range of applicability for automotive panels which are usually very thin with high critical frequencies.

In a previous publication Wodtke and Koopmann [80] used the optimisation method described above. The Rayleigh-Ritz method was used for structural analysis of a clamped rectangular plate with added point masses to optimise one of its modes. For the acoustic analysis the volume velocity of the structure was used as a low-frequency sound power approximation. The design variables were the weights of the added masses whereas their location was fixed. The results showed great reduction for the optimised mode. The predicted results were also verified experimentally. Pierre and Koopmann [79] presented results for the optimisation of 1 and 3 modes of a clamped plate using 4 and 9 masses at fixed locations. In the work done by Pierre [79] a more accurate vibroacoustic analysis was used compared with that of Wodtke and Koopmann [80] by using FEM for structural analysis and full calculation of the sound power.

In this chapter the method used by Pierre [79] is extended in two ways. Firstly, stiffeners in addition to masses are used to design weakly radiating plate modes and secondly, the locations of the added masses and stiffeners, in addition to their properties, are optimised. Also a genetic algorithm is used for the optimisation instead of gradient-based minimisers, which are local optimisation methods and hence are inappropriate for this problem. Another advantage of genetic algorithms over gradient-based optimisation is that the usually tedious structural sensitivity analysis is not required.

For the implementation of the optimisation procedure a structural model for
the analysis of the plate with the added masses and stiffeners is required. In the approach adopted here the response of the modified plate is given in terms of the eigenfunctions of the unmodified plate as discussed in the classical publication by Weissenburger [48]. Wu and Luo [49] used this principle for the case of a rectangular plate with attached point masses and springs. The analysis leads to an eigenvalue problem which is solved numerically to give the natural frequencies and mode shapes of the modified plate. In this chapter this analysis is extended in order to include the effects of added stiffeners, instead of springs, in arbitrary locations and orientations on the plate. For the case of weak discontinuities (when the impedance of the added masses and stiffeners is not much greater than the characteristic impedance of the plate) only a small number of modes of the unmodified plate is required to obtain an accurate solution [48]. This makes the structural computation faster compared to other methods such as FEM, which is an important requirement since the analysis needs to be carried out several times during the optimisation procedure.

3.1 Structural model of a plate with added masses and stiffeners

3.1.1 Theory

The equation of motion for the free vibrations of an unconstrained plate of uniform thickness, neglecting the effects of shear deformation and rotary inertia, is given by:

\[ D\nabla^4 w(x, y, t) + \rho_p h_p \ddot{w}(x, y, t) = 0 \quad (3.1) \]

where \( w \) is the transverse deflection of the plate, \( \ddot{w} \) imply second derivative of
w with respect to time, \( \rho_p \) is the material density and \( D \) the flexural rigidity of the plate.

The effect of constraints, such as attached masses and stiffeners, can be considered by adding forcing terms in the right side of Equation (3.1) which represent the constraint forces. Thus, for point masses and line beam-stiffener constraints Equation (3.1) becomes:

\[
D \nabla^4 w(x, y, t) + \rho_p h_p \ddot{w}(x, y, t) = -\sum_l m_l \ddot{w}(x, y, t)M_l(x, y) - \sum_s [EI_s \nabla^4 w(x, y, t) + \rho_{s,b} h_{s,b} \ddot{w}(x, y, t)]S_s(x, y) \tag{3.2}
\]

where \( m_l \) is the weight of the \( l^{th} \) mass, in kg, and \( EI_s \) and \( \rho_{s,b} h_{s,b} \) are the bending stiffness and surface density of the \( s^{th} \) beam-stiffener respectively. \( M_l(x, y) \) and \( S_s(x, y) \) are functions of the location of the masses and stiffeners respectively. For point masses and line stiffeners these functions are given by:

\[
M_l(x, y) = \delta(x - x_l)\delta(y - y_l)
\]

and

\[
S_s(x, y) = \begin{cases} 
\delta(x - f_s(y)) & y_{1s} < y < y_{1s} \\
0 & \text{otherwise}
\end{cases}
\]

where \( \delta(x) \) is the Dirac function, \( x_l, y_l \) are the coordinates of the \( l^{th} \) mass and \( f_s(y) \) is the function of the line of the \( s^{th} \) stiffener starting at coordinates.
Figure 3.1: Schematic diagram of the coordinates of the $l^{th}$ point mass and the $s^{th}$ line stiffener. 

$(x_{1s}, y_{1s})$ and finishing at coordinates $(x_{2s}, y_{2s})$. $f_s(y)$ is given by:

$$f_s(y) = \frac{y - \beta_s}{\alpha_s}$$

with $\alpha_s$ and $\beta_s$ determined by solving the equation of a line using the starting point and ending point coordinates:

$$\alpha_s = \frac{y_{2s} - y_{1s}}{x_{2s} - x_{1s}}, \quad \beta_s = y_{1s} - \frac{(y_{2s} - y_{1s})}{x_{2s} - x_{1s}} x_{1s}$$

In Equation (3.2) the influence of the rotation of the beam-stiffener has been neglected. The inhomogeneous partial differential equation, Equation (3.2), can be solved by means of the solution of the homogeneous Equation (3.1) which gives the natural modes of the unconstrained plate (plate without added masses or stiffeners). Using modal superposition for a simply supported rectangular plate the solution of Equation (3.2) is given in the form:
3.1 Structural model of a plate with added masses and stiffeners

\[ w(x, y, t) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} W_{ij}(x, y)q_{ij}(t) \]  \hspace{1cm} (3.4)

with

\[ W_{ij}(x, y) = \sin \frac{i\pi x}{L_x} \sin \frac{j\pi y}{L_y} \]  \hspace{1cm} (3.5)

where in Equation (3.4) \( q_{ij} \) is the modal expansion coefficient of the \((i, j)\) mode of the plate and \( L_x, L_y \) in Equation (3.4) are the dimensions of the plate in the \( x \) and \( y \) dimensions respectively. Substituting Equation (3.4) into (3.2), multiplying each term by \( W_{rb}(x, y) \), integrating each term over the surface of the plate and dividing by \( \rho_p h_p L_x L_y / 4 \) gives:

\[ \omega_{rb}^2 q_{rb} + \ddot{q}_{rb} + \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} A_{ij,rb} \ddot{q}_{ij} + \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} B_{ij,rb} q_{ij} + \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} C_{ij,rb} \ddot{q}_{ij} = 0 \]  \hspace{1cm} (3.6)

where

\[ \omega_{rb} = \sqrt{\left( \frac{r\pi}{L_x} \right)^2 + \left( \frac{b\pi}{L_y} \right)^2} \sqrt{\frac{D}{\rho_p h}} \]

are the natural frequencies in radians per second of the unconstrained plate. The above methodology that makes use of the orthogonality of the mode-shapes, \( W_{ij} \), of the plate was previously used in references [49] and [51] in a similar analysis.

In Equation (3.6) \( A_{ij,rb} \) is a function of the point masses, \( B_{ij,rb} \) is a function of the stiffness of the line stiffeners and \( C_{ij,rb} \) is a function of the mass of the
3.1 Structural model of a plate with added masses and stiffeners

These are defined as:

\[
A_{ij,rb} = \sum_l \frac{4m_l}{L_x L_y \rho_p h_p} W_{ij}(x_l, y_l) W_{rb}(x_l, y_l) \tag{3.7}
\]

\[
B_{ij,rb} = \sum_s \frac{4EI_s}{L_x L_y \rho_p h_p} \int_{y_1}^{y_2} d^4 \frac{dy_1}{dy_4} W_{ij}(f_s(y), y) W_{rb}(f_s(y), y) dy \tag{3.8}
\]

\[
C_{ij,rb} = \sum_s \frac{4\rho_{s,b} h_{s,b}}{L_x L_y \rho_p h_p} \int_{y_1}^{y_2} W_{ij}(f_s(y), y) W_{rb}(f_s(y), y) dy \tag{3.9}
\]

To reduce the number of indices in Equation (3.6), new indices \(m\) and \(n\) are introduced as [51]:

\[
m = N_y (r - 1) + b
\]

\[
n = N_y (i - 1) + j
\]

for \(i, r = 1, \ldots, N_x\) and \(j, b = 1, \ldots, N_y\). Hence, Equation (3.6) becomes:

\[
\omega_{rb}^2 q_m + \ddot{q}_m + \sum_{n=1}^{N} A_{n,m} \ddot{q}_n + \sum_{n=1}^{N} B_{n,m} q_n + \sum_{n=1}^{N} C_{n,m} \ddot{q}_n = 0, \quad m = 1, \ldots, N \tag{3.10}
\]

where \(N = N_x N_y\). For harmonic excitation, Equation (3.10) can be further simplified by substituting \(\ddot{q}_n = -\omega^2 q_n\). This yields

\[
\omega_{rb}^2 q_m + \sum_{n=1}^{N} B_{n,m} q_n - \sum_{n=1}^{N} (A_{n,m} + C_{n,m} + \delta_{n,m}) \omega^2 q_n = 0, \quad m = 1, \ldots, N \tag{3.11}
\]
3.1 Structural model of a plate with added masses and stiffeners

or in matrix form:

\[(\omega^2 + [B])\{q\} = \omega^2([A] + [C] + [I])\{q\}\]  \hspace{1cm} (3.12)

or in the more familiar form:

\[[K]\{q\} = \omega^2[M]\{q\}\]  \hspace{1cm} (3.13)

with \([K] = [\omega^2] + [B]\), \([M] = [A] + [C] + [I]\) both \(N \times N\) matrices, \([\omega^2]\) a diagonal matrix with the resonant frequencies of the unconstrained plate and \([A]\), \([B]\) and \([C]\) \(N \times N\) matrices given by Equations (3.7), (3.8) and (3.9) respectively. The integrals of Equations (3.8) and (3.9) do not have a known analytical solution and hence must be solved numerically. The differentiation in Equation (3.8) can be solved analytically using Leibniz identity defined as:

\[
\frac{d^4}{dy^4}W_{ij}(f_s(y), y)W_{rb}(f_s(y), y) = \sum_{k=0}^{4} \binom{4}{k} \frac{d^{4-k}}{dy^{4-k}}W_{ij}(f_s(y), y) \frac{d^k}{dy^k}W_{rb}(f_s(y), y)
\]  \hspace{1cm} (3.14)

Using Equations (3.5) and (3.3) this yields:
3.1 Structural model of a plate with added masses and stiffeners

\[ \frac{d^4}{dy^4} W_{ij}(f_s(y), y) W_{rb}(f_s(y), y) = \]
\[ \left[ \frac{2i\pi^2}{L_x L_y \alpha} \cos \frac{i\pi f_s(x)}{L_x} \cos \frac{j\pi y}{L_y} - \left( \left( \frac{i\pi}{\alpha L_x} \right)^2 + \left( \frac{j\pi}{L_y} \right)^2 \right) \sin \frac{i\pi f_s(x)}{L_x} \sin \frac{j\pi y}{L_y} \right] \]
\[ \cdot \left[ \frac{2r\pi^2}{L_x L_y \alpha} \cos \frac{r\pi f_s(x)}{L_x} \cos \frac{b\pi y}{L_y} - \left( \left( \frac{r\pi}{\alpha L_x} \right)^2 + \left( \frac{b\pi}{L_y} \right)^2 \right) \sin \frac{r\pi f_s(x)}{L_x} \sin \frac{b\pi y}{L_y} \right] \]

Equation (3.13) forms an eigenvalue problem that can be solved numerically. The eigenvalues \( \omega \) are the natural frequencies of the modified structure and the corresponding eigenvectors \( \{ q \} \) can be used in Equation (3.4) to obtain the mode shapes of the structure.

3.1.2 Validation of the structural model of the constrained plate

In order to validate the above structural model, results for the resonant frequencies of a plate with added masses and stiffeners are compared to FEM results calculated using MSC Nastran/Patran. The results of the modal superposition model presented in the previous section were obtained by solving the eigenvalue problem of Equation (3.13) in MATLAB with the use of the function \( \text{eig} \).

The material of the plate used for the comparison is steel with material properties as given in Chapter 2. The dimensions of the plate are \( L_x = 0.307 \text{m} \) and \( L_y = 0.208 \text{m} \). The geometry of the plate with the added masses and stiffeners used for the comparison is shown in Figure 3.2 where the red triangles show the location of the point masses and the blue lines show the location of the line stiffeners. The location of the four masses is at
Table 3.1: Comparison of natural frequencies calculated by the analytical model and FEM for relatively small discontinuities.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Analytical (Hz)</th>
<th>FEM (Hz)</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>63.9</td>
<td>63.8</td>
<td>0.2</td>
</tr>
<tr>
<td>2nd</td>
<td>105.7</td>
<td>104.9</td>
<td>0.8</td>
</tr>
<tr>
<td>3rd</td>
<td>150.2</td>
<td>154.9</td>
<td>3.0</td>
</tr>
<tr>
<td>4th</td>
<td>161.5</td>
<td>168.6</td>
<td>4.2</td>
</tr>
<tr>
<td>5th</td>
<td>387.8</td>
<td>384.3</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The accuracy of the results of the modal superposition model depends on the total number of modes used in the calculations as well as the location and properties of the mass and stiffener constraints. Table 3.1 shows the comparison of the resonant frequencies of the plate with constraints as shown in Figure 3.2 calculated using the modal superposition model with $N = 225$ modes and FEM using 400 elements. It can be seen that the modal superposition model, with matrices with significantly less elements than the FEM model, has a relative average error of only 1.8% for the first 5 modes compared to the FEM model.

However, when heavier masses and/or stiffeners are used (stronger discontinuities), the results become less accurate for the same number of modes. Table 3.2 shows the comparison of the resonant frequencies of the same plate but this time using 0.4kg masses and 0.005m wide and 0.01m thick stiffeners. For stronger discontinuities a larger number of modes needs to be used which consequently increases the calculation time.
3.1 Structural model of a plate with added masses and stiffeners

Figure 3.2: Geometry of the plate with added masses and stiffeners used for the validation of the structural model.

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Analytical (Hz)</th>
<th>FEM (Hz)</th>
<th>Error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1&lt;sup&gt;st&lt;/sup&gt;</td>
<td>50.0</td>
<td>54.4</td>
<td>8.1</td>
</tr>
<tr>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>73.5</td>
<td>81.4</td>
<td>9.7</td>
</tr>
<tr>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>103.7</td>
<td>119.7</td>
<td>13.3</td>
</tr>
<tr>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>121.8</td>
<td>138.2</td>
<td>11.9</td>
</tr>
<tr>
<td>5&lt;sup&gt;th&lt;/sup&gt;</td>
<td>381.3</td>
<td>346.1</td>
<td>10.2</td>
</tr>
</tbody>
</table>

Table 3.2: Comparison of natural frequencies calculated by the analytical model and FEM for relatively large discontinuities.
3.2 Inverse problem

The aim of the optimisation procedure as mentioned earlier is to impose on a plate structural modeshapes that radiate acoustic energy weakly. A method for calculating such acoustically optimum modeshapes for a beam structure is presented by Naghshineh and his co-researchers [56]. A quadratic expression for the sound power radiated by the beam was derived in terms of a vector with structural velocities at discrete points on the beam and the radiation impedance matrix. The same analysis can be extended for other structures such as plates. The quadratic expression for the sound power was then optimised using the Lagrange multiplier theorem which led to the following eigenvalue problem:

\[ [D]\{v\} = \lambda[E]\{v\} \]  

(3.16)

where matrix \([D]\) is the radiation impedance matrix. Matrix \([E]\) is a set of basis functions. After solving the eigenvalue problem in Equation (3.16) the optimum modeshapes are constructed by the multiplication \(v^T[E]\). The basis functions in matrix \([E]\) need to satisfy the structural boundary conditions.

The results of the Lagrange multiplier optimisation are modeshapes that give local minima or local maxima of the quadratic expression for the sound power. For the task of sound power minimisation only the modeshapes for the local minima need to be selected and this can be done by using the eigenvalues of Equation (3.16), \(\lambda\), which are directly related to the radiation efficiency of each mode shape. Since the radiation impedance matrix \([D]\) is frequency dependent, the same holds for the optimum modeshapes.

The optimisation presented above is in the acoustic domain only, without any reference to any properties of the structure apart from its geometry. Hence, a further step is required where the optimum modeshapes are imposed to a specific structure. By looking at the eigenvalue problems of Equations (3.13)
and (3.16) it can be seen that if the basis functions of the acoustic optimisation problem are the same with the modes of the unmodified panel $W_{ij}$ of Equation (3.4). Then in order to impose the optimised modes to the structure the coefficients of the optimised modes in vector $\{v\}$ need to be equalised with the coefficients of the modes of the modified structure $\{q\}$. Hence, one needs to solve the inverse eigenvalue problem, that is, given the eigenvector $\{q\}$ (which is derived from vector $\{v\}$ of the acoustic optimisation), and the eigenvalue $\omega$ (which is the frequency for which the radiation impedance matrix $[D]$ in the acoustic optimisation was calculated), one needs to find the appropriate matrices $[K]$ and $[M]$ in Equation (3.13). Some information for $[K]$ and $[M]$ matrices can be given a priori, for example the mechanical properties of the plate and the types of modifications (point masses, line stiffeners, etc.). The unknown parameters in these two matrices can be related to the location and properties of the modifications and can be determined from the inverse eigenvalue problem. The inverse eigenvalue problem has been previously discussed by the authors in the following references [83, 84, 85]. Its main application is in the identification of structural properties or the construction of structural analysis models, such as FEM, from measured eigendata.

One obvious advantage of using the inverse optimisation can be seen in the above analysis. This advantage is that the modes that are determined using the acoustical optimisation are the weakest structural radiators for the given frequency and geometry of the structure.

However, there are certain disadvantages of using the inverse optimisation approach. The main disadvantage is that the determination of the unknown parameters in matrices $[K]$ and $[M]$, in general, leads to a system of nonlinear equations. Such a system of equations does not have a unique solution (in this case there are many possible modifications that can lead to the desired optimum modes of the structure) and needs to be solved using numerical optimisation, least mean square methods or similar approximation techniques. The modes that will result from the approximate solution of the inverse prob-
lem will be, in general, different from the optimum modes, especially when many optimum modes are imposed to the structure at the same time. The solution of the inverse problem hence does not guarantee that the actual modes that will be imposed to the structure will radiate acoustic energy weakly.

Another disadvantage is that the acoustic optimisation returns a number of weakly radiating structural modes for the chosen frequency. However, only one of these modes has to be imposed to the structure at this frequency. The choice is very important since not all the modes can be practically implemented on a structure at a specific frequency. Naghshineh et al [56], who used the inverse optimisation for a one-dimensional beam, selected the modes that can be practically imposed to the structure according to the number of their nodal points. For example, for the second vibrational mode of the simply supported beam they used the optimum modeshape which had one nodal point. For two- or three- dimensional structures the choice of the appropriate modeshapes is not straight-forward because of the variety of optimum modeshapes with the same number of nodal points or lines.

3.3 Direct problem

The direct problem is defined as follows: find the optimum parameters related to some predefined modifications (location and weight of point masses, location and height of line stiffeners, etc.) in order to minimise the average sound power radiated by a number of modes. This leads to the following objective function that needs to be minimised as it is given by Koopmann and Fahnline [78]:

\[ \bar{P} = \frac{1}{M} \sum_{\mu=1}^{M} \bar{P}_\mu \]  

(3.17)
where \( \bar{P}_\mu \) is the sound power radiated by the \( \mu^{th} \) mode and \( M \) is the number of modes in the optimisation.

The sound power radiated by a mode can be calculated using the radiation matrix approach [9]. For an accurate prediction of the sound power a sufficient number of points of the velocity profile of the plate needs to be taken into account. However, the computation cost of calculating the radiation matrix increases dramatically when the number of these sampling points is increased. Since the sound power must be calculated for every modeshape in every iteration of the optimisation procedure, a more efficient calculation method is required. One such method is to use the velocity spectrum of the modeshape as follows [9]:

\[
\bar{P}_\mu = \frac{\rho_0 c k}{8\pi^2} \int \int_{-\infty}^{\infty} \text{Re} \left[ \frac{|U_\mu(k_x, k_y)|^2}{k_\mu - k_x^2 - k_y^2} \right] dk_x dk_y
\]

(3.18)

where \( U_\mu(k_x, k_y) \) is the velocity spectrum (wavenumber transform) of the velocity of the \( \mu^{th} \) modeshape. The velocity of the \( \mu^{th} \) modeshape is derived by firstly calculating its displacement by solving the eigenvalue problem in Equation (3.13) and using the eigenvectors with Equation (3.4). Then, since the process is harmonic, the velocity is found by multiplying the displacement by \( j \omega_\mu \) where \( \omega_\mu \) is the \( \mu^{th} \) circular natural frequency.

The velocity spectrum can be calculated efficiently using the Fast Fourier Transform (FFT). Because only the real part of the integrant contributes to the integral, the limits of both the integrals become finite, with limits \(-k_\mu\) to \(k_\mu\) as shown in Equation (2.7), hence numerical evaluation of the integral is possible. Thus, the FFT of the modeshapes must contain wavenumber components \( k_x \) and \( k_y \) ranging at least from \(-k_\mu\) to \(k_\mu\), where \( k_\mu = \omega_\mu / c \) is the acoustic wavenumber for the \( \mu^{th} \) natural frequency. A condition is not necessary for whether wavenumber components outside this range are contained in the FFT spectrum since they will result in an imaginary integrant and will not contribute to the integral. The maximum and minimum wavenumber
components contained in the FFT spectrum $U_\mu(k_x, k_y)$ when the function of the velocity of the modeshape $u_\mu(x, y)$ has been sampled uniformly, are given by [86]:

$$k_x = \pm \frac{n_x \pi}{L_x}$$

(3.19)

$$k_y = \pm \frac{n_y \pi}{L_y}$$

(3.20)

where $n_x$ and $n_y$ are the number of sampling points in the $x$ and $y$ dimension of the plate respectively. Hence, the number of sampling points must be chosen so $|k_x| > k_\mu$ and $|k_y| > k_\mu$. This will ensure that enough wavenumber components are contained in the velocity spectrum $U_\mu(k_x, k_y)$ for the accurate calculation of the sound power in Equation (3.18).

### 3.4 Optimisation

In the previous section a direct optimisation procedure was described and an objective function, Equation (3.17), was developed. Thus an optimisation algorithm is required in order to find the best values for various given parameters of the objective function in order to minimise it. For the application discussed in this chapter the parameters (or design variables) are the coordinates and weights of a number of masses and the coordinates and dimensions of the (rectangular) cross sections of a number of beams. All these parameters are described in Section 3.1.

Many types of numerical optimisation algorithms exist [87, 88], including gradient-based methods and linear and non-linear programming. It has been shown however [67] that there are certain disadvantages in using any one of these optimisation methods for vibroacoustic problems. Instead, a method that has been used successfully and is shown to outperform classical optimi-
Optimisation methods in several cases [69] is the genetic algorithm (GA) [68].

Genetic algorithms are categorized as global search heuristic methods. Global search means that there is no restriction in the problem domain in which the algorithm is searching for an optimum solution. This is opposed to local search methods where the search domain is limited and it is more likely to return local minima of the objective function. Heuristics is a type of algorithms that can produce acceptable solutions but for which there is no formal proof of their correctness.

A genetic algorithm mimics the process of evolution that occurs in biology, based on Darwin’s theory of the survival of the fittest. In a genetic algorithm an initial population of individuals is randomly formed. Each individual consist of genes; a set of values which are the design variables. For the application considered in this chapter each individual in the population is a set of 8 values which are the parameters of the objective function that needs to be optimised. These parameters are related to the mass and stiffness modifications of the plate as described above (location and properties of point masses and line stiffeners). Each individual has a fitness value, which in this application is the sound power calculated from Equation (3.17) of a plate with modifications as prescribed by the genes (parameters) of that individual.

Each iteration of the algorithm is equivalent to time passing in a biological system. Hence, in each iteration a new generation of individuals is produced by combining (breeding) previous design variables and the old population dies off. Offspring are produced by pairs of parents breeding with genes that are composites of the genes of their parents. This procedure is termed crossover. For the algorithm to move towards an optimum solution that will minimise the sound power radiated by a number of structural modes, the parents that are chosen to reproduce are chosen randomly with the most fit being more likely to reproduce. Moreover, in order for the algorithm to look outside the parents’ population for a better solution there is a chance that
some of the parents’ genes will be mutated. This means that the genes of the offspring are not exact copies of their parents but they are randomly (and usually slightly) altered. In mathematical terms this helps the algorithm to avoid finding local minima. Care must be taken for the mutated genes not to exceed some predefined limiting values (e.g., coordinates of the constraints should not exceed the dimensions of the plate). In a GA the number of individuals in the population must remain constant. Hence, a number of individuals must die off, which will equal the number of those that are born. The selection again is random with the least fit being more likely to die off. In the MATLAB implementation of the genetic algorithms for this application the calculation of the probabilities of those individuals who breed and those who die is made based on the description by Cox [89].

The iterative process described above is terminated when the fitness value of all individuals is equal to or their difference is below some predefined threshold. A flow chart of the algorithm is illustrated in Figure 3.3.
3.5 Implementation and Results

In this section optimisation results for a steel rectangular plate with dimensions $0.307m \times 0.208m$ and thickness $0.0012m$ are presented. The structural model described in Section 3.1 and the genetic algorithm described in Section 3.4 were implemented in MATLAB.

The population size used in the genetic algorithm was 40. For the evaluation of the sound power in Equation (3.18), required for the objective function of Equation (3.17), a 512 points 2D FFT was used. For the numerical solution of the eigenvalue problem of Equation (3.13) the eig MATLAB function was used. This function, as any algorithm for the solution of eigenvalue problems, produces modeshapes of arbitrary amplitude, which would affect the calculated sound power. For a fair comparison of different designs all the modeshapes were normalised in order to have a unit mean squared velocity before their sound power was calculated. Hence, for a vector $\{u\}$ with the vibrational velocities of the plate at certain points, the normalised vector $\{u_{\text{norm}}\}$ is given by:

$$\{u_{\text{norm}}\} = \frac{\{u\}}{\sqrt{\text{mean}(\{|u|^2\})}}$$

(3.21)

where the function $\text{mean()}$ returns the mean value of all the entities in vector $\{u\}$.

The aim of the optimisation, as mentioned earlier, is to create structural modeshapes that take advantage of the hydrodynamic short-circuit effect below the critical frequency of the structure in order to create strong acoustic cancellations in the vicinity of the plate. This is achieved when acoustic pressures of equal magnitude and opposite sings (with zero acoustic pressure being the atmospheric pressure) create a zero resultant pressure when they exist one next to the other. One case where such conditions in the vicinity
of a structure are met is when the structural velocity profile is symmetric and hence so is the near-field acoustic pressure. One known velocity profile that does not radiate acoustic energy in the far field because of acoustic cancellation in the near-field is an infinite sine wave [8].

Hence, in order to create symmetric modeshapes, symmetric modifications need to be used. Indeed, the use of symmetric modifications significantly improved the results and convergence time of the optimisation. In the results presented in the next sections, pairs of constraints with same properties placed symmetrically have been used. Hence, if one of the coordinates of a given constraint, in normalised form ranging from 0 to 1, is \((\bar{x}, \bar{y})\) its pair constraint with same properties will have coordinates \((1 - \bar{x}, 1 - \bar{y})\).

Another advantage of using pairs of constraints is that the number of total design variables is reduced. For a pair of masses the design variables are the two coordinates of one of the masses and one value for the weight of both masses. This is a total of three design variables for two point masses. In the case where no symmetry was to be used the number of design variables would be doubled. For stiffener constraints four coordinates are required to define a line from which the coordinates of a line symmetric to the first one can be found. The number of design variables associated with the properties of the stiffeners might vary. In the results given in the next sections the stiffeners are beams of the same material as that of the plate and a rectangular cross section. Hence, two variables to define the dimensions of the rectangular cross section are required. In the examples below the width of the cross section is kept constant at 5mm for manufacturing purposes and in order to reduce the number of design variables. The total number of design variables for a pair of stiffeners is then five.

Limiting values have also been used for the design variables. For the coordinates of the modifications the limits are the dimensions of the plate. For the weight of the masses the limits are 10g to 400g. For the height of the beam stiffeners the limits are 1mm to 10mm.
3.5 Implementation and Results

3.5.1 Added masses

As a first step the optimisation of two pairs of masses is considered. The simplest case is that of optimising one structural mode of the plate in order to radiate acoustic energy inefficiently. It must be noted that because the 1st structural mode does not have any nodal lines, and hence its whole surface radiates sound, it cannot be turned into a weak-radiator.

By implementing the genetic algorithm described in Section 3.4 and the structural and acoustic models described in Sections 3.1 and 3.3 the 2nd structural mode of a simply supported steel plate with dimensions 0.307m × 0.208m and thickness 0.0012m is optimised. The material data used are $E = 200$ GPa and $\rho_p = 7872$ kg/m$^3$. The location and weight of the masses on the plate, as resulted from the optimisation algorithm, are shown in Figure 3.4. The weights of the masses are significantly lower than those published by Pierre and Koopmann [79] for constant mass locations. The number of modes used in the superposition in Equation (3.4) was $N_x = 8$ and $N_y = 8$ which resulted in matrices with dimensions 64 × 64. The optimisation of the plate with the mass constraints, which did not include matrices $B$ and $C$ in Equation (3.12), took about 30 min for 150 iterations on regular PC. Note that for the MATLAB implementation of the structural model, all the necessary matrices were constructed in nested for loops. The time taken for the optimisation can be significantly reduced by implementing the for loops in a lower level programming language such as C, C++ or FORTRAN [90]. The cost of this would be the increase in the complexity of the code. Throughout this thesis simplicity in the implementation is preferred to improved calculation time.

In order to check the results of the optimisation the acoustic behaviour of the plate with the optimum constraints is simulated using FEM for the calculation of structural displacements at predefined nodal points and BEM for the calculation of the sound power radiated by the modified plate using the previously calculated nodal displacements as boundary conditions. For the FEM analysis the commercial software NASTRAN/PATRAN is used with
3.5 Implementation and Results

Figure 3.4: Optimum location and weight of 2 pairs of masses for the 2\textsuperscript{nd} mode.

quadrilateral thin plate elements (CQUAD4). A uniform mesh was used that divided the plate into 20 elements in each direction (a total of 400 elements). In order to ensure that the assumption of point masses in the theoretical model is valid for real masses, in the finite element model distributed masses were used with area equal to the area of one element of the FE model and total weight equal to the weight of the point masses (Figure 3.4). For the acoustic analysis the commercial software LMS Virtual Lab was used to predict the sound power radiated by the modified structure using the direct BEM. The mesh used for the acoustic analysis was the same as that for the structural analysis. For all the results presented in this thesis the reference value for sound power levels expressed in dB is $10^{-12} \text{W}$. Similarly for vibrational velocity levels expressed in dB the reference value is $10^{-9} \text{m/sec}$.

The optimised plate was excited by a point force at the bottom left corner node (not counting the nodes at the boundaries where simply supported boundary conditions have been applied). The choice of the type and location of the force is that all the structural modes in the frequency range of the simulation need to be excited. Hence, by placing a point force close to the corner of the plate one avoids the nodal points of at least the lower order modes.

Figure 3.5 shows the sound power and mean squared velocity of the plate with the two pairs of masses optimised for the 2\textsuperscript{nd} structural mode. It can
be seen that even though there is a peak in the mean squared velocity graph at the frequency of the 2\textsuperscript{nd} mode (around 100Hz), this peak does not appear in the sound power graph. This means that even though the 2\textsuperscript{nd} structural mode carries vibrational energy, this energy is not radiated into the acoustic medium. Because, instead of altering the input energy to the structure, the way the structure vibrates at its 2\textsuperscript{nd} mode was altered, the same phenomenon can be observed for any excitation force that excites that mode. In Figure 3.5 it can be also seen that the radiation efficiency of the 3\textsuperscript{rd} structural mode has been reduced even though the structure was optimised only for its 2\textsuperscript{nd} mode. Another interesting phenomenon is that the given set of constraints has created strong acoustic cancellations in a frequency slightly below 300Hz where there is no structural mode.

For comparison with the optimised results, Figure 3.6 shows the sound power and mean squared velocity of the unmodified plate excited at the same location. In this case all peaks in the mean squared velocity graph appear in the sound power graph as well. By comparing Figure 3.5 with 3.6 it can be seen that the overall sound power is higher for the optimised plate. This apparent paradox is due to the choice of the objective function which intends to reduce the radiation efficiency of only the 2\textsuperscript{nd} structural mode of the plate. Hence, the purpose of the numerical example presented here is solely to demonstrate the behaviour of a weakly radiating structural mode. More practical applications are presented in the next chapters.

The optimisation is also extended for more than one mode. Figures 3.7 and 3.8 show the sound power and mean squared velocity of the same plate optimised for the 2\textsuperscript{nd} and 3\textsuperscript{rd}, and 2\textsuperscript{nd} and 5\textsuperscript{th} structural modes respectively using again 2 pairs of masses. It can be seen that the given modes have been optimised. However, the optimisation was not as efficient as in the case of one structural mode. As in the previous case it can be seen that other modes apart from those considered in the optimisation also have weak radiating characteristics.
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.5: Optimisation of the 2nd structural mode using 2 pairs of masses.
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.6: Unmodified plate without optimised modes.
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.7: Optimisation of the 2\textsuperscript{nd} and 3\textsuperscript{rd} structural mode using 2 pairs of masses.
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.8: Optimisation of the $2^{nd}$ and $5^{th}$ structural mode using 2 pairs of masses.
The results of Figures 3.7 and 3.8 suggest that when the number of modes to be optimised is increased, the complexity (number of constraints) of the structure needs to be increased as well. Figure 3.9 shows the sound power and mean squared velocity of the same plate optimised for the 2nd, 4th and 5th structural modes that was achieved by using 4 pairs of masses.

When a large number of modes needs to be optimised, a large number of added masses needs to be used. This will increase the total weight of the structure, which is undesirable for many applications. For this reason combination of added masses and stiffeners can be used as constraints to the structure.

### 3.5.2 Added masses and stiffeners

In this section firstly the simple case of optimising the 2nd structural mode of the plate using only a pair of line stiffeners is examined. Beams with a rectangular cross section were used with 5mm width. The location and height of the beams were optimised. The limiting values for the height of the beam were 1mm to 7mm. Figure 3.11 shows the sound power and mean squared velocity of the optimised plate. It can be seen that line stiffener constraints are not effective in constructing weakly radiating structural modes compared to mass constraints. The results did not improve when 2 sets of line stiffeners were used or when the limits of the constraints were changed. This suggests that point masses can change the modeshapes of a structure to some desired modeshapes more effectively than line stiffeners.

However, the combination of masses and stiffeners showed to be very effective in optimising a number of structural modes compared to masses or stiffeners alone. Figure 3.12 shows the sound power and mean squared velocity of the plate optimised for the 2nd, 3rd, 4th and 5th structural modes using 2 pairs of masses and a pair of stiffeners placed symmetrically as in the case of masses only (Figure 3.13). The excitation point for the results of Figure 3.12 was
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.9: Optimisation of the 2\textsuperscript{nd}, 4\textsuperscript{th} and 5\textsuperscript{th} structural mode using 4 pairs of masses.

Figure 3.10: Optimum location and weight of 4 pairs of masses for the 2\textsuperscript{nd}, 4\textsuperscript{th} and 5\textsuperscript{th} mode.
3.5 Implementation and Results

(a) Sound Power

(b) Mean Squared Velocity

Figure 3.11: Optimisation of the 2\textsuperscript{nd} structural mode using 1 pair of stiffeners.
placed at the point with coordinates \((0.187m, 0.03m)\). This point was chosen by observing the modeshapes of the modified structure in order to avoid nodal points of all the modes in the frequency range 10-500Hz.

In Figure 3.12 it can be seen that the addition of stiffener constraints significantly improved the results. The radiation efficiency of all the modes included in the optimisation has been significantly reduced. As a result the sound power curve appears "smoother" without large peaks at the resonance frequencies, even though the mean squared velocity curve contains those peaks. The effect of the optimisation of the structural constraints in the sound power is similar to the effect that damping has in the structural velocity of the plate. The advantage of the structural mode optimisation approach is that damping treatment would have minimal effect for this low frequency range.

The drawback of adding the effect of stiffeners in the optimisation procedure is that the integrals of matrices \(B\) and \(C\) in Equation (3.12) need to be computed numerically in every iteration, which increases the computation time. Moreover the convergence of the series in Equation 3.4 becomes slower for stiffeners in arbitrary orientations hence more modeshapes of the unmodified plate need to be included. The number of modes used in the superposition for this case was \(N_x = 16\) and \(N_y = 16\) (a total of 256 modes). For the evaluation of the integrals the Legendre-Gauss Quadrature method was used. For the calculation of the 30 points of integration along the line of the stiffener and the corresponding weights a code written by Winckel [91] was used. For the results of Figure 3.12 the optimisation took around 4 hours, which is much longer than the time required in the case where only added masses where considered, but still not prohibited. Again, the time taken would be considerably shorter if the code was written in a lower level programming language.
Figure 3.12: Optimisation of the $2^{nd}$, $3^{rd}$, $4^{th}$ and $5^{th}$ structural mode using 2 pairs of masses and a pair of stiffeners.
3.6 Conclusions

In this chapter, the optimisation of modes of a plate in order to radiate acoustic energy weakly is considered. This is achieved by optimising the position of added masses and stiffeners on the plate. The main advantage of this optimisation method is that the optimum design does not depend on the possibly complicated excitation mechanism of the structure and hence once a plate has been designed it can be used for any application. Also this method is very effective in reducing sound radiation at low frequencies where the reduction of the vibrational levels using damping treatment is inefficient. The work presented in this chapter extends the previously published work on the subject by considering line stiffener modifications, in addition to point masses, optimising their position as well as their properties and using a more appropriate optimisation algorithm (GA).

Firstly, a mathematical model for the structural analysis of a plate with added point masses and line stiffeners at arbitrary locations and orientations is developed. Structural analysis using this model can be significantly faster.
than other commonly used methods, such as FEM, which is a significant requirement if this is a part of an iterative optimisation algorithm. Two possible optimisation approaches are discussed; a direct and an indirect approach. In the indirect optimisation the optimum modeshapes for a given geometry of the structure and frequency are calculated. Then the optimum modeshapes are imposed to the structure by the optimisation of certain structural constraints. In the direct approach no information is known a priori about the optimum modeshapes. Certain structural constraints are again optimised, this time in order to minimise the sound power radiated by a number of modes of the structure. For reasons discussed earlier in this chapter the latter approach is used. For the optimisation a genetic algorithm is used. The functioning of the algorithm is also discussed.

The method is firstly applied for the optimisation of the 2\textsuperscript{nd} structural mode of a plate using two sets of symmetrical masses. The results show that even though the optimised mode carry vibrational energy, this energy is not radiated into the acoustic medium. The 2\textsuperscript{nd} mode of the plate is hence termed a weak radiator. It was observed that in order to increase the number of modes included in the optimisation the number of constraints must be increased. Results for the optimisation of more structural modes using two and four pairs of masses are presented.

Line stiffeners did not prove to be as efficient as masses in creating weakly radiating structural modes. The best results were obtained when a combination of two sets of masses and a set of stiffeners was used.
Chapter 4

Experimental verification

In the previous chapter the concept of optimising a number of structural modes in order to radiate acoustic energy inefficiently was discussed. Several examples were presented where the sound power of the optimised structures was calculated using FEM and BEM simulation methods. In this chapter these results are verified experimentally.

There are several standardised methods for measuring the sound power radiated by a structure. Some of them are based on sound pressure measurements [92, 93, 94] whereas others are based on sound intensity [95, 96, 97]. Methods based on sound pressure require the measurement of sound pressure at many different points in the acoustic field of the sound source (20 for the case of a baffled source in an anechoic environment). The measurement procedure can be very tedious and is sensitive to human error especially in the determination of the measuring points. For the validation of the results of the previous chapter the sound intensity measurement method is used.

For the determination of sound intensity, which requires the measurement of sound pressure and particle velocity at the same point, a p-p intensity probe was used (Figure 4.1). The sound intensity can be calculated from the cross-spectrum of the sound pressure signals of the two microphones from
the following expression given by Fahy [98]:

\[ I_n(\omega) = \frac{1}{\rho_0 \omega d} \text{Im} [G_{p2p1}(\omega)] \] (4.1)

where the subscript \( n \) in \( I_n \) signifies normal intensity (as defined by the orientation of the intensity probe), \( \rho_0 \) is the density of the acoustic medium, \( d \) is the space between the two microphones and \( \text{Im} [G_{p2p1}(\omega)] \) is the imaginary part of the cross-spectrum.

## 4.1 Measurement apparatus

The results of the previous chapter have been calculated for a baffled simply supported plate. The measurement apparatus used (Figure 4.2) has been
4.1 Measurement apparatus

designed in order to approximate these conditions. It consists of a concrete
baffle with dimensions $1.48m \times 1.28m$ and thickness 0.1m. At the centre of
the baffle there is a steel frame in which a plate with dimensions $0.307m \times
0.208m$ can be placed. The frame has two parts, one fixed on the baffle as it
can be seen in Figure 4.2 and one identical removable part that can be screwed
on the fixed part with 14 screws (Figure 4.3). The plate is placed between the
two frames. In order to simulate the simply supported boundary conditions
(zero displacement and bending moment along the boundaries) inside the
two steel frames two smaller curved wooden frames are placed as shown in
Figure 4.3. All edges of the plate are also sharpened in order to have a 1mm
long V shape. In this way, if the plate has the appropriate dimensions, the
wooden frame restricts the displacement of the plate boundaries but allows
their rotation. In other words, the bending moments at the boundaries of
the plate are minimised.

In order to evaluate the performance of the apparatus the input mobility
of a 1.2mm thick steel plate is measured. The results are compared to the
4.2 Measuring equipment

For the acquisition and analysis of the results a 4 channel Photon+ portable analyser was used in conjunction with the RT Pro signal analysis software. In the measurements presented below an LDS shaker driven by the output of the analyser was used to excite the plate and its input mobility was measured as well as its sound power via sound intensity measurements. For the input mobility measurement a B&K 8200 force transducer was attached to the end of the shaker (between the shaker and the plate) and a B&K 4334 theoretical input mobility of a simply supported plate as given in [99]. The comparison is shown in Figure 4.4. It can be seen that only a few modes are close in frequency which suggests that the boundary conditions are not identical to the theoretical simply supported boundary conditions. This is considered as a challenge to the optimised plate of the previous chapter in non-idealised, real-life situations.

Figure 4.3: Frame where the plate is placed.
4.2 Measuring equipment

Figure 4.4: Comparison of the measured input mobility in the measurement apparatus to the theoretical input mobility of the same plate.

An accelerometer was attached at the same point on the other side of the plate. Both transducers were chosen based on their frequency response (flat between 50Hz and 10kHz) and weight. The weight of the transducers is an important factor since it can alter the results when attached on a thin light plate. The weight of the force transducer is 17g and that of the accelerometer is only 2.4g. Both transducers were connected to two B&K 2635 charge amplifiers where their charge sensitivity was adjusted according to the specifications given by the manufacturer. The output of the charge amplifiers was connected to the 1st and 2nd channel of the Photon+ analyser. The intensity measurements were carried out using a B&K 3548 intensity probe with two B&K 4182 preamplified condenser microphones. The 200V polarisation that is required for the microphones was provided by a G.R.A.S 12AA power module. The two outputs of the power module, which are the preamplified outputs of the two microphones, were connected to the two remaining channels of the Photon+ analyser. Before the measurements both microphones were calibrated using a G.R.A.S 42AB calibrator. The connectivity is shown schematically in Figure 4.5.

In the RT Pro analysis software FFT was performed on the time signals
4.3 Sound intensity measurement methodology

As mentioned earlier the sound power measurement was performed using the scanning sound intensity method as described in BS EN ISO 9614:2002 [97]. The first step in this standardised methodology is the determination of the scanning surfaces that enclose the source. For the measurements presented below, the surfaces were chosen to be the surfaces of a cuboid consisting of six rectangular surfaces of which one was the surface of the source. The remaining five surfaces that enclosed the source were the scanning partial surfaces. The dimensions of the cuboid are shown in Figure 4.6.

The normal intensity of each partial surface was measured by placing the two microphones of the intensity probe perpendicular to each surface. The
4.4 Results

Using the method and equipment described above the results of the previous chapter are validated experimentally. For all plates measured the same applied force was used controlled by the RT Pro software. The excitation signal was a swept sine for 1Hz to 800Hz with swept time of 1s. All measurements were carried out at the anechoic chamber of Loughborough University.

Firstly, the sound power of a plate with dimensions and properties as described in the previous chapter without any modifications is measured (the calculated results can be seen in Figure 3.6(a)). Figures 4.7 and 4.8 show the measured sound power and input mobility respectively. It can be seen that the structural modes of the plate that can be identified by the peaks in
the input mobility measurement create peaks in the sound power measurement, which indicates that these modes are "strong-radiators". The same behaviour was predicted theoretically in the previous chapter. Figures 4.9 and 4.10 show the repeatability and non-uniformity indicators with respect to frequency. The first indicator is a measure of the repeatability of the two measures performed on each partial surface. BS EN ISO 9614:2002 states that for a good repeatability this indicator should be less than 2 in the given frequency range for each octave band. In Figure 4.9 the repeatability indicator is less than 2 everywhere apart from very narrow peaks. The non-uniformity indicator is a measure of the non-uniformity of the acoustic field. The same limits are specified for the values of this indicator as with the repeatability indicator. Again, as can be seen in Figure 4.10 the non-uniformity indicator is between the limits for all frequencies apart from narrow peaks. Moreover, the average value of the coherence function between the force transducer and the accelerometer and the force transducer and the microphones is shown in Figures 4.11 and 4.12. This is an average of all the measurements on all partial surfaces (for the case of microphones it is also the average of both microphones). The coherence in both cases is poor for low frequencies which is due to the limitations of the transducers used. Moreover, for the case of the acoustic signals, reflections from the walls might have caused poor coherence at low frequencies since the anechoic chamber where the measurements took place only provides a reflection-free field at 100Hz and above. This can explain the low frequency noise in the measurements of Figures 4.7 and 4.8.

The next plate measured is the one optimised for its 2nd mode using 2 pairs of masses (Figure 3.4 in the previous chapter). The plate with the attached masses fitted in the measurement apparatus is shown in Figure 4.26 (displayed towards the end of this chapter). In this figure the location of the accelerometer used to measure the input mobility of the modified plate can also be seen. On the other side of the plate at the same location the plate is excited by the shaker. The optimum masses are glued to the plate. Figures 4.13 and 4.14 show the measured sound power and input mobility of this plate respectively. It can be seen from the input mobility measurement
that the 2\textsuperscript{nd} mode of the structure is around 120Hz which is the 2\textsuperscript{nd} peak of the graph. As in the results of the numerical simulation, this 2\textsuperscript{nd} peak does not appear so strong in the sound power graph unlike the 1\textsuperscript{st} (around 75Hz) and 3\textsuperscript{rd} (around 160Hz) peaks. Hence, even though the 2\textsuperscript{nd} mode carries vibrational energy, this energy is not radiated into the acoustic medium. By looking at the average coherence functions for this measurement (Figures 4.17 and 4.18) it can be seen that their values at around the 2\textsuperscript{nd} mode are not significantly lower than those for the 3\textsuperscript{rd} mode and are higher than those of the 1\textsuperscript{st} mode, which clearly appears in the sound power graph (Figure 4.13). This provides evidence that the reason why the 2\textsuperscript{nd} mode does not create a strong peak in the measured sound power is not due to an error in the measurement.

The last plate measured is the one optimised for the 2\textsuperscript{nd} to 5\textsuperscript{th} structural modes using 2 pairs of masses and a pair of stiffeners (Figure 3.13 in the previous chapter). The plate with the optimum modifications in the measurement apparatus can be seen in Figure 4.27. The excitation point in this case is not at the corner of the plate but at the point with coordinates \((0.187m, 0.03m)\) in order to excite all the modes of interest as discussed in the previous chapter. Figures 4.19 and 4.20 show the measured sound power and input mobility respectively. It can be seen once again that with the exception of the 2\textsuperscript{nd} mode, modes 3-5 have weak acoustic radiation characteristics. The validity of the measurement can be verified from Figures 4.21 to 4.24.

Figure 4.25 shows the comparison between the unmodified plate and the plate optimised for its 2\textsuperscript{nd} to 5\textsuperscript{th} modes.
4.4 Results

Figure 4.7: Measured sound power of the unmodified plate.

Figure 4.8: Measured input mobility of the unmodified plate.
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Figure 4.9: Repeatability indicator for the sound power measurement of the unmodified plate.

Figure 4.10: Non-uniformity indicator for the sound power measurement of the unmodified plate.
4.4 Results

Figure 4.11: Average coherence function between the force transducer and the accelerometer for all measurements of the unmodified plate.

Figure 4.12: Average coherence function between the force transducer and a microphone for all measurements of the unmodified plate.
4.4 Results

Figure 4.13: Measured sound power of the plate optimised for the 2\textsuperscript{nd} mode.

Figure 4.14: Measured input mobility of the plate optimised for the 2\textsuperscript{nd} mode.
4.4 Results

Figure 4.15: Repeatability indicator for the sound power measurement of the plate optimised for the 2\textsuperscript{nd} mode.

Figure 4.16: Non-uniformity indicator for the sound power measurement of the plate optimised for the 2\textsuperscript{nd} mode.
4.4 Results

Figure 4.17: Average coherence function between the force transducer and the accelerometer for all measurements of the plate optimised for the 2\textsuperscript{nd} mode.

Figure 4.18: Average coherence function between the force transducer and a microphone for all measurements of the plate optimised for the 2\textsuperscript{nd} mode.
Figure 4.19: Measured sound power of the plate optimised for modes 2 to 5.

Figure 4.20: Measured input mobility of the plate optimised for modes 2 to 5.
4.4 Results

Figure 4.21: Repeatability indicator for the sound power measurement of the plate optimised for modes 2 to 5.

Figure 4.22: Non-uniformity indicator for the sound power measurement of the plate optimised for modes 2 to 5.
4.4 Results

Figure 4.23: Average coherence function between the force transducer and the accelerometer for all measurements of the plate optimised for modes 2 to 5.

Figure 4.24: Average coherence function between the force transducer and a microphone for all measurements of the plate optimised for modes 2 to 5.
4.5 Conclusions

This chapter is concerned with the experimental validation of the optimisation procedure described in the previous chapter. The measurement apparatus and equipment used is described. The procedure of measuring the sound power of the structures under consideration using sound intensity according to BS EN ISO 9614:2002 is also discussed.

The measurements carried out under non-idealised conditions strongly confirm the (idealised) numerical results of the previous chapter. As it is mentioned earlier in this chapter the boundary conditions of the plate in the measurement apparatus were not identical to simply supported boundary conditions. Even though the optimisation of the modification had been carried out using simply supported boundary conditions, the same modifications created weakly radiating structural modes of a plate with different boundary conditions. Another non-idealisation is the presence of solid bodies (masses and stiffeners) in the acoustic field of the measured plates. In the results of the previous chapter the masses and stiffeners did not interfere with the

Figure 4.25: Comparison of the sound power radiated by the unmodified and optimised plates for modes 2-5.

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Figure 4.26: Plate with attached masses optimised for the 2\textsuperscript{nd} structural mode.

Figure 4.27: Plate with attached masses and stiffeners optimised for the structural mode 2 to 5.
acoustic field since they were considered as a part of the flat plate. In reality it is likely that the attached bodies created acoustic scattering of the energy radiated by the plate. Also, for the plate optimised for its 2nd to 5th mode, in the previous chapter idealised point masses and line stiffeners were used for the modifications. The experimental results with real masses and stiffeners provide evidence that these modifications can create weakly radiating structural modes.
Chapter 5

Optimisation of structural modes of automotive-type panels by using line stiffeners and point masses

In Chapter 3 an optimisation procedure in which a number of modes of a structure is optimised in order to radiate acoustic energy weakly was developed and discussed. In this chapter this method is applied to automotive-type panels.

There are certain advantages of using this method for vehicle body panels compared with traditional optimisation methods. In NVH engineering it is common to set targets for different noise paths. When there is an excess level above the predefined targets for a noise path, it is usually possible to identify the part of the vehicle structure that contributes most by performing panel contribution analysis (experimental or simulated). When the panel has been identified its vibroacoustic behaviour needs to be optimised. Using a conventional optimisation method for every iteration of the optimisation
algorithm used the vibrational response of the whole vehicle structure as well as the acoustic response at a given point needs to be calculated for a given noise path. The most widely used tools for calculating vibroacoustic responses are the FEM and a combination of FEM and BEM methods. For typical vehicle models consisting of millions of degrees of freedom (DOFs) the time required for the optimisation can be prohibitively high. Using the method described in Chapter 3 the panel can be optimised without reference to the excitation or the rest of the structure. This drastically reduces the time required for the optimisation process. Since the response of a structure can be given as a superposition of its structural modes, if these are turned into weak radiators, then the structure will radiate acoustic energy weakly for any excitation. Using this optimisation method the panel under consideration is effectively isolated from the rest of the structure. When the panel is isolated appropriate boundary conditions need to be applied to the boundaries of the panel in order to simulate its behaviour when it is a part of the whole structure. A method for identifying the appropriate boundary conditions is discussed later in this chapter.

Another advantage of using the proposed optimisation method, apart from the significant reduction in the optimisation time, is that the optimisation of the panel does not depend on the way it is excited or the properties of the rest of the structure. During the vehicle development, the vehicle structure and different vehicle parts are likely to be changed in order to improve certain (not necessarily acoustic) characteristics. If a panel has been optimised using a traditional optimisation then it is likely that it will need to be re-optimised for the new design of the vehicle. When the panel has been optimised to have weakly radiating structural modes it will keep these characteristics even when the rest of the vehicle structure or the noise sources have been changed.

The proposed method can also be used effectively for low frequency modes. Low frequency structural modes can create high acoustic levels especially when they are close to the frequencies of the acoustic modes of the cavity of the car compartment. In this case an acoustic ‘booming’ is created. Damping
treatment is usually inefficient for low frequency modes. Using the proposed optimisation method these modes can be turned into weak radiators.

Since this method does not reduce the acoustic radiation of a structure by controlling its input energy, it can be used for further improvement when all measures have been taken to prevent energy from getting into the structure. Using this method the coupling of structural vibration and acoustic radiation can be minimised.

In this chapter the proposed optimisation method is applied to a simplified car model shown in Figure 5.1. This simplified model has been used to demonstrate how the principle of weakly radiating structural modes can be applied for the optimisation of some of its panels. The same method can be used for more detailed, realistic car models.

The structure is excited by two point forces in its front part simulating forces from the engine at the engine mounts. The panel to be optimised is the left-front floor panel (can be identified by the local coordinate system symbol shown in Figure 5.1). The floor panel consists of four identical panels separated by 5cm thick rectangular beams. Each sub-panel has dimensions 0.75m × 0.55m. The material used for all the panels and the beams is steel with properties specified in the previous chapters. The thickness of the panels is 2.5mm.

The floor panel under consideration is excited by neighbouring panels in a complex way. It is difficult to identify these forcing mechanisms and form an optimisation problem for the panel when it is exited by them. Moreover, changes in the panel will cause changes in the excitation forces. Hence, using a traditional optimisation approach, the vibroacoustic response of the whole car model needs to be calculated for every iteration of the optimisation algorithm. Using the proposed method the panel will be isolated and its structural modes will be optimised.
5.1 Identification of boundary conditions

The first step in the proposed methodology is the identification of the appropriate boundary conditions that need to be applied on the parts of the isolated panel previously connected to the rest of the structure. Practically, isolating the panel means that from the simplified car FEM model only the nodes and elements of the panel under consideration need to be considered whereas all other nodes and elements can be disregarded. Then boundary conditions need to be applied to the nodes previously connected to the disregarded elements. Hence, translational and rotational stiffness is imposed to these boundary nodes for which the values need to be specified in order for the isolated panel to have the same vibroacoustic behaviour to that of the panel connected to the rest of the car structure. The isolated panel can then be optimised and retain the optimum characteristics when it is reattached to the structure.

Several methods have been proposed for the identification of the boundary

Figure 5.1: Simplified car model used for the application of the proposed optimisation to automotive-type panels.
conditions of a structure. The main application of these methods is the improvement of numerical models of a structure or combination of structures by providing more accurate values for the unknown parameters. Ahmadian and his co-researchers [100] proposed a boundary condition identification method based on a reduced characteristic equation of the structure. They applied this method for the determination of the translational and rotational stiffness of the boundaries of a beam. Pabst and Hagedorn [101] proposed a method that uses measured modal data to identify the properties of the boundary conditions of a structure. A boundary identification method using neural networks was proposed by Takahashi [24]. These are only a few of the relevant publications. The method used for the application discussed in this chapter is similar to that used by Zhu and Huang [102]. In their publication the vibrational response at a specific point of the structure of which the boundary properties need to be specified is calculated (or measured if a real structure exists) for a given excitation force and excitation location. This data is then used to identify the appropriate boundary conditions.

For the case of identification of the properties of the boundaries of the floor panel of the simplified car model, the input mobility of the panel when attached to the vehicle structure is calculated using FEM. The input mobility is calculated at a node close to the corner of the panel but not at any of the boundary nodes. The reason for this choice of the location of the input mobility calculation is to avoid nodal points and hence excite as many of the lowest order modes of the floor panel as possible. The input mobility of the isolated panel is then calculated at the same location and compared to that of the floor panel for different values of translational and rotational stiffness at the boundary nodes. In Nastran the stiffness at the boundaries is applied by using the CBUSH element. Figure 5.2 shows the input mobility of the floor panel for the frequency range 10Hz-200Hz as well as the input mobilities of the isolated panel with two different sets of values for the elastic boundary conditions. The input mobility of the floor panel contains peaks at the resonant frequencies of the panel as well as at some of the resonant frequencies of the whole structure in the given frequency range. To identify the resonant
5.1 Identification of boundary conditions

Figure 5.2: Input mobility of the floor panel and isolated panel of two different sets of stiffness values.

frequencies of the panel, its displacement profile at the frequencies of the peaks is observed. Figure 5.3(a) shows the displacement profile of the panel at 58Hz which can be identified as the 1st mode of the panel. Figure 5.3(b) shows the displacement profile at 80Hz where the input mobility has a large peak. It can be seen that this is not a displacement profile of a modeshape of the panel, since it does not have any of the characteristics of a modeshape, such as orthogonality [82]. Using this approach all the modes of the panel in the given frequency range have been identified and are shown by arrows in Figure 5.2.

The values used for the 'Panel elastic BC1' curve in Figure 5.2 are $10^{10}$ N/m for the translational stiffness and 1000 Nm/rad for the rotational stiffness. The peaks of this input mobility and the peaks of the modes of the panel in the floor panel input mobility do not exactly match. Better matching is observed with 'Panel elastic BC2' curve. The value for the translational stiffness used for this curve is $10^{10}$ N/m and for the rotational stiffness 5000 Nm/rad which results in boundary conditions very close to the theoretically clamped boundary conditions. These values are identified as the appropriate values and are used later for the optimisation of the panel.
5.2 Implementation of the optimisation procedure

Alternatively to this methodology, once the frequencies of the modes of the floor panel have been identified from the calculated input mobility, eigenanalysis can be performed on the isolated panel using FEM for different elastic boundary conditions. The resulting modal frequencies can then be compared to the modal frequencies of the floor panel previously identified.

Several assumptions are made during the methodology presented above. The first one is that the boundary conditions that need to be identified are linear. The assumption is valid for small displacements of the panel (which is also an assumption for the structural analysis using classical FEM). Another assumption is that since the frequency range of interest is relatively narrow, the elastic properties of the boundaries are frequency independent. The last assumption is that the boundary conditions are the same for the whole perimeter of the panel. The last two assumptions mentioned are shown to be valid from the good agreement of the input mobilities of the floor panel and the panel with the identified boundary conditions. The method presented above can certainly be extended for more general and complex structures as discussed at the end of the next chapter.

5.2 Implementation of the optimisation procedure

When the appropriate boundary conditions have been applied to the isolated panel, its structural modes can be optimised. The optimisation algorithm used is the same genetic algorithm described in Section 3.4. For every iteration of the optimisation algorithm a routine is executed. A block diagram of the main routine is shown in Figure 5.4. The code for the routine is written in MATLAB. Firstly, the design variables (the variables related to the modification of the panel) are provided by the genetic algorithm. Based on these variables a triangular mesh is generated for the whole area of the panel. Using the mesh and the known properties of the panel (material, thickness
Figure 5.3: Displacement profile of the panel at two different frequencies.
5.2 Implementation of the optimisation procedure

Figure 5.4: Main routine executed for every iteration of the optimisation.

and boundary conditions) a Nastran .bdf file is generated. Nastran is then called through MATLAB and the modes of the model in the .bdf file are calculated. This procedure is described in more details later in Section 5.2.1. The calculated nodal displacements for all modes included in the optimisation are read from the Nastran output file and are used to calculate the sound power radiated by those modes which is the objective function that needs to be minimised. The calculation of the sound power from nodal displacements is described in Section 5.2.2.

5.2.1 Structural analysis

In Chapter 3 a structural model for the theoretical modal analysis of a modified plate based on the modes of the unmodified plate was developed and used in the optimisation procedure. In this chapter FEM is used to calculate the modeshapes and resonant frequencies of the modified panel using Nastran, a commercial FEM solver. There are two reasons why FEM is used. The first reason is that in the analytical model of Chapter 3 the number of modes of the unmodified plate used, which is related exponentially to the
5.2 Implementation of the optimisation procedure

time required for the calculations, needs to be increased for 'stronger' discontinuities (heavier masses and beams). Hence, for a given number of modes the analytical model is limited to small discontinuities. FEM does not suffer from this problem, and its only limitation is that it can be used only for low frequencies (certain number of elements per structural wavelength). Another reason is that FEM can be easily extended to incorporate structures of general geometry as discussed at the end of the next chapter. Hence, a mesh of any structure (not necessarily a flat plate) can be imported and optimised. This mesh of the structure can be a part of an existing model, for example a floor panel of a real vehicle model.

The design variables are the same as those used in Chapter 3; two sets of coordinates defining a line for a line stiffener, the dimensions of the rectangular cross section of the stiffener, one set of coordinates for each point mass and the weight of the mass. The material used for the beam stiffeners is the same as that of the plate. As it is discussed later, symmetric modifications are used (as in Chapter 3) and hence one set of design variables can be used to define more than one set of line stiffeners and masses.

In order to apply the modifications prescribed by the design variables, the mesh of the panel needs to be generated with nodes at the points of the modifications. This means that the mesh should have nodes at the coordinates defined by the coordinates of the point masses. Moreover, it should have nodes along the lines defining the line stiffeners distributed with a specific distance. With the aim of creating a well formed mesh this distance was defined by the maximum distance of the elements used during the mesh generation.

The freely distributed code DistMesh was used for the triangular mesh generation [103]. This code was chosen because a number of fixed nodes can be defined in the geometry to be meshed. A description of the DistMesh code can be found in Appendix C. The input arguments of the main function of DistMesh (distmesh2d) are the geometry of the panel (in this case defined
by the drectangle function provided by the package), the maximum length of the elements, two coordinates defining the minimum and maximum values for the coordinates (in this case the two opposite corners of the panel) and a vector with the coordinates of the fixed nodes. The output of the distmesh2d function is a vector with the coordinates of all the nodes and the connectivity matrix defining the nodes of each triangular element. Figure 5.5 shows a panel mesh with two point masses and two line stiffeners. Lines and circles have been added to the figure to indicate the position of the stiffeners and masses respectively.

The data for the panel mesh along with the material data, the thickness of the plate, the dimensions of the cross section of the stiffeners, the weight of the masses and the translational and rotational stiffness at the boundaries are used to generate a .bdf Nastran file. A description of the format of a .bdf file is presented in Appendix D. A SOL 103 analysis is used for the calculation of the normal modes in Nastran. The properties of the panel are defined by a PSHELL card in the .bdf file. The properties of the stiffeners are defined by a PBARL card. The nodes are defined by a GRID card and the
triangular elements for the panel, the line elements for the stiffeners and the point elements for the masses are defined by CTRIA3, CBAR and CONM2 respectively. The stiffness at the boundary nodes is defined by using PBUSH and CBUSH cards. The material for the panel and the stiffeners is defined by MAT1. Once the .bdf file has been generated Nastran is called from MATLAB using the dos() function in order to calculate the modeshapes and the resonant frequencies of the panel.

5.2.2 Acoustic analysis

Once Nastran has calculated the modeshapes and the resonant frequencies of the panel the nodal displacements of each mode are read from Nastran .f06 output file. The nodal displacements are converted to nodal velocities by multiplying each value by \( j\omega \) in order to calculate the sound power of each mode. As stated in Chapter 3 the objective function that needs to be minimised is the summation of the sound power radiated by a number of modes of the panel.

The sound power for each mode is calculated based on the method described by Cunefare and Koopmann [104]. For this method an impedance matrix is required that relates nodal velocities to the nodal sound pressures through

\[
p = Zu
\]  

(5.1)

where \( p \) and \( u \) are vectors with nodal pressures and normal velocities respectively and \( Z \) is the impedance matrix. The impedance matrix can be derived from the influence matrices in a BEM analysis. In the case of a flat baffled panel the analysis can be greatly simplified by using a Rayleigh based BEM formulation. In that case the impedance matrix, for long acoustic wavelengths compared to the elements size, is given by [105]:
5.2 Implementation of the optimisation procedure

\[ Z_{ij} = \frac{j\omega\rho_0 \exp(-jkR_{ij})}{2\pi R_{ij}} \]  \hspace{1cm} (5.2)

where \( R_{ij} \) is the distance between the \( i \) and \( j \) nodes. The diagonal elements of the impedance matrix are given by \( Z_{ii} = \frac{\omega k}{2\pi} \). Note that in reference [105] each component of the matrix is multiplied by the area of each element. In the analysis presented here this is carried out later in Equation (5.5).

Based on the analysis of Cunefare and Koopmann [104] the sound power of the \( n^{th} \) mode is given as:

\[ P_n = u^T B u^* \]  \hspace{1cm} (5.3)

where superscripts \( T \) and \( * \) indicate vector transpose and complex conjugate respectively, and

\[ B = \frac{1}{4} (A + A^H) \]  \hspace{1cm} (5.4)

where the matrix \( A \) can be constructed from the submatrices \( A_j \). Assuming the same interpolation functions for both the structural and acoustic elements, \( A_j \) is given by:

\[ A_j = Z_j^T \left( \int_{S_j} N N^T dS_j \right) S_j \]  \hspace{1cm} (5.5)

where \( N \) is the vector of interpolation functions of the element \( j \), \( S_j \) is the area of the same element and \( Z_j \) is the submatrix of the radiation impedance matrix that yields the pressure at the nodes of element \( j \).

In order to validate the implementation of the sound power calculation pre-
5.3 Results

In this section results are presented for the optimisation of the floor panel of the simplified car model of Figure 5.1. In order to ensure the validity of the FEM and sound power calculation the upper frequency of interest is 200Hz. The unmodified panel has five modes in the frequency range of interest.

Two line stiffeners and four point masses have been used to optimise the 2nd to 6th modes of the panel. The upper limit for each point mass in the optimisation was 0.5kg whereas the upper limit for the dimensions of the

Figure 5.6: Radiation efficiency of the (3,1) mode of a simply supported plate with respect to frequency normalised by the critical frequency of the plate calculated using the sound power calculation method presented in this section and an analytical formula.

Presented above the radiation efficiency of a given mode of a simply supported plate is calculated for which an analytical expression exist [106]. Figure 5.6 shows the comparison of the radiation efficiency for the (3,1) mode of a simply supported plate calculated analytically and with the above method. It can be seen that the two curves are identical.
rectangular cross section of each stiffener was 0.02m. Once the optimum parameters for the modification of the isolated panel have been found (locations and properties of the stiffeners and masses), they are applied to the floor panel of the simplified car model. The whole structure is then excited by two point forces at the front part of the vehicle as discussed previously. Its structural response is calculated using Nastran. The sound power and radiation efficiency of only the floor panel is then calculated using LMS Virtual Lab and the Indirect BEM.

Figure 5.8 shows the sound power of the unmodified floor panel and the sound power of the panel with optimum modifications. No symmetry was used for the modifications, hence all stiffeners and masses were defined independently. The geometry of the modifications on the panel can be seen in Figure 5.7. The total added mass to the panel from the point masses is 1.5kg, which is a 42% increase in its total weight. The 2nd to 6th modes of the isolated modified panel were calculated to be at 78Hz, 103Hz, 122Hz, 131Hz and 155Hz. From Figure 5.8 it can be seen that reduction in the sound power has been achieved for frequencies at or around the optimised modes of the panel. The peaks that do not correspond to the resonances of the panel are due to global modes of the car structure and their frequencies do not change for both curves of Figure 5.8. In order to validate that the reduction in sound power is due to weakly radiating structural modes, the radiation efficiency of the panels has also been calculated and is shown, in logarithmic scale, in Figure 5.9. It can be seen that significant reduction has been achieved for frequencies at and around the optimised modes. However, the difference between the linear dB average of the sound power of the modified and unmodified panels over the frequency range 10Hz-200Hz shows a small increase of 1.1dB for the modified panel. A small decrease of 0.3dB was observed for the average radiation efficiency for the same frequency range. Note that in this frequency range there are also modes of the panel that were not included in the optimisation. This can also explain the relative increase in the radiation efficiency above 155Hz, which is the frequency of the highest optimised mode.
5.3 Results

Figure 5.7: Geometry of the optimum modifications (two stiffeners and four masses) of the panel using no symmetry.

Figure 5.8: Sound power of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using no symmetry.
5.3 Results

Figure 5.9: Radiation efficiency of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using no symmetry.

As discussed in Chapter 3 significant improvement of the results was observed when symmetric modifications were used. The same optimisation was performed but this time symmetry was imposed to the modifications. In this case the coordinates and parameters of the modifications are linked to each other. Four identical masses were used and their coordinates were related to each other so if \((\bar{x}, \bar{y})\) were the normalised coordinates of one mass ranging from 0 to 1, the coordinates of the other masses would be \((1 - \bar{x}, \bar{y})\), \((\bar{x}, 1 - \bar{y})\) and \((1 - \bar{x}, 1 - \bar{y})\). Hence, only the coordinates and weight of one mass were used as design variables and the others were determined from these. Similarly four points were defined for two identical stiffeners. If the normalised coordinates of the starting point of a stiffener are \((\bar{x}, \bar{y})\) then the normalised coordinates of the end point of the same stiffener are \((1 - \bar{x}, 1 - \bar{y})\). The normalised coordinates of the starting and ending points of the second stiffener would then be \((1 - \bar{x}, \bar{y})\) and \((\bar{x}, 1 - \bar{y})\). Again there is some reduction in the design variables defining the stiffeners since only one point and the properties of one stiffener need to be specified. The result of using the symmetry described above is cross stiffeners of which the crossing point is always at the centre of the panel. The resulting panel with the four masses and two stiffeners is symmetric in both the \(x\) and \(y\) dimensions. Figure 5.10
5.3 Results

Figure 5.10: Geometry of the optimum modifications (two stiffeners and four masses) of the panel using symmetry around $x$ and $y$.

shows the geometry of the optimum modifications of the floor panel using the symmetry described above. The comparison of the sound power radiated by the unmodified and optimally modified panels is shown in Figure 5.11. Unlike the results using no symmetry, the average sound power over the specified frequency range for the modified panel is 1.9dB lower than that of the unmodified panel. The reduction was achieved again by creating weakly radiating structural modes as seen in Figure 5.12 which compares the radiation efficiency of the two panels. The reduction in the radiation efficiency is 2.4dB. The weight of all the added masses used is 1.2kg which is an increase of 34% in the total weight of the panel. The height of the optimum stiffeners is 0.003m and the width is 0.015m.

Another type of symmetry used for the optimisation of the floor panel was to define the starting and ending points of the line stiffeners with symmetry only in one dimension. Hence, if $(\bar{x}_1, \bar{y}_1)$ and $(\bar{x}_2, \bar{y}_2)$ are the coordinates of the starting and ending points of one stiffener, the same points for its symmetric stiffener will be $(1 - \bar{x}_1, \bar{y}_1)$ and $(1 - \bar{x}_2, \bar{y}_2)$. This type of symmetry showed to give some extra flexibility to the modifications in order to create weakly radiating structural modes. Figure 5.13 shows the geometry of the optimum
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Figure 5.11: Sound power of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using symmetry around $x$ and $y$.

Figure 5.12: Radiation efficiency of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using symmetry around $x$ and $y$. 

5.3 Results

Figure 5.13: Geometry of the optimum modifications (two stiffeners and four masses) of the panel using symmetry around $x$.

modifications using symmetry only in the $x$ coordinates. The comparison of the sound power radiated by the unmodified and optimally modified panel is shown in Figure 5.14. The average reduction achieved for the sound power is 4.5dB whereas for the radiation efficiency (Figure 5.15) is 3.1dB. The results show further reduction compared to the previous optimisation results with less mass. The weight of all the added masses is 0.8kg (22% increase in the total weight). The height of the optimum stiffeners is 0.002m and the width is 0.013m. Similar results were achieved when symmetry of the $y$ coordinates was used. In this case the reduction in the sound power was 4.2dB and the reduction in the radiation efficiency was 2.9dB.

In order to ensure that the structural mode optimisation can include more modes, hence it can be used to optimise the vibroacoustic behaviour of panels for a broader frequency range, the floor panel of the simplified car model was optimised for the 2$^{nd}$ to 10$^{th}$ modes. The best results again were achieved by using symmetry only in one of the coordinates. Figure 5.16 shows the comparison of the sound power of the modified and optimised floor panels. The average reduction achieved in the frequency range 10Hz-250Hz (that included all the optimised modes) is 11.1dB. The reduction in the radiation
5.3 Results

Figure 5.14: Sound power of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using symmetry around $x$.

Figure 5.15: Radiation efficiency of the unmodified floor panel and the optimised floor panel with two stiffeners and four masses using symmetry around $x$. 
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Figure 5.16: Sound power of the unmodified floor panel and the optimised floor panel for modes 2-10.

Figure 5.17: Radiation efficiency of the unmodified floor panel and the optimised floor panel for modes 2-10.
efficiency (Figure 5.17) is 3.2dB. Major reduction in the sound power and radiation efficiency is observed around 150Hz which is the 4th mode of the optimised panel. The optimum added mass is 0.95kg (an increase of 26% in the total weight of the panel).

5.4 Conclusions

In this chapter the optimisation procedure discussed in Chapter 3, in which weakly radiating modes are imposed on a structure, is applied to automotive-type panels. The main advantage that this optimisation procedure offers is a significant reduction in the time required to optimise different parts of a vehicle structure, since the parts under consideration can be isolated and optimised independently. Another advantage of the method is that the optimisation is independent of the rest of the structure and the applied forces. Hence, when the vehicle structure or its parts are redesigned, which is common in the concept stage of vehicle development, the optimised panels retain their optimum acoustic characteristics. Moreover, the method is very effective in the reduction of sound radiation from low frequency modes where damping treatment is usually inefficient.

The proposed optimisation method is applied to a simplified car model. A floor panel of the car model is isolated and its structural modes are optimised. The first step is to identify the appropriate boundary conditions that need to be applied to the isolated panel in order to simulate the behaviour of the floor panel attached to the vehicle structure. For this, a simple method based on calculations of the input mobilities of the floor panel and the isolated panel is used in order to determine the values of translational and rotational stiffness that need to be applied to the isolated panel.

For every iteration of the genetic algorithm optimiser a routine is executed where a mesh for the panel under consideration is generated based on the de-
sign variables, which are related to the location and properties of the modifications. A Nastran .bdf file is generated based on the mesh and the properties of the panel and modifications. The modeshapes and resonant frequencies of the panel are then calculated using Nastran. A simplified BEM formulation based on the Rayleigh integral is used to calculate the sound power radiated by each structural mode, which is then used as the objective function of the optimiser.

This optimisation is used to optimise the location and properties of four masses and two stiffeners on the floor panel of the simplified car model. Linking the design variables by defining symmetrical modifications improved the results of the optimisation. The sound power calculated for the floor panel with the optimum modifications was reduced compared to the unmodified panel. This reduction is mainly due to the reduction in the radiation efficiency of the panel achieved by converting structural modes into weak radiators.
Chapter 6

Optimisation of structural modes of automotive-type panels by geometrical modifications

In the previous chapter point masses and line stiffeners were used to optimise the vibroacoustic behaviour of a floor panel of a simplified vehicle structure model. In automotive engineering, however, geometrical modifications are usually used on panels to improve their static and dynamic characteristics. For example, press-outs on a flat panel are common structural modifications. Figure 6.1 shows several geometrical modifications on panels in the back seat of a car. The advantage of applying geometrical modifications is mainly that the stiffness of lightweight panels can be increased without an increase in the weight of the structure, which is a crucial factor in automotive body design. In this chapter the optimisation of structural modes of automotive-type panels using geometrical modifications will be studied by extending the work done in the previous chapter.
The optimisation of the floor panel of the simplified car model described in the previous chapter will be considered. In the previous chapter all panels of the car model had a thickness of 2.5mm. In this chapter the thickness of all the panels is changed to 0.8mm, which is a typical thickness used for steel automotive panels [107]. The properties of the beams separating the floor in the simplified car model are the same. In order to confirm that the values of the translational and rotational stiffness of the boundaries of the panel are the same as those identified in the previous chapter, the input mobilities of the floor panel and the isolated panel with $10^{10}$N/m translational stiffness and 1000Nm/rad rotational stiffness at the boundaries are compared. The results are shown in Figure 6.2. It can be seen that these values can be used on the isolated panel in order to simulate the behaviour of the 0.8mm floor panel. In contrast with the results in Figure 5.2 for the input mobility of the floor panel in the previous chapter, the input mobility of the floor panel for the model with 0.8mm thick panels does not have peaks due to global modes of the whole vehicle structure. This suggests that the decrease of the thickness of the floor panel relative to the thickness of the separating beams at the boundaries of the panel prevents energy from being transferred to the rest of the vehicle structure.
6.1 Modification functions

In this section two common geometrical panel modifications used in automotive engineering are discussed. These are swages, which are modifications of a rectangular area on the panel, and domes, which are modifications of an elliptical area on the panel. The modifications applied on the panel need to be parameterised and the parameters related to the modifications can be used as design variables in the numerical optimisation procedure. An effective method of parameterising the geometrical modifications is the use of modification functions as described by Marburg and his co-researchers [57, 59, 61]. Using this method, firstly the area in which the modification will take place is determined. For example, for swages as mentioned before the modification area is a rectangular area defined inside the area of the panel whereas for domes the modification area is an elliptical area. When the modification area has been specified the way in which the geometry of the panel inside the modification area will change needs to be determined. For example, for swages on a flat panel lying in the $x$-$y$ plane the value of the $z$ coordinate of all the nodes inside the rectangular modification area is equally increased to create press-outs on the panel (Figure 6.3(a)). For domes on the same panel
6.1 Modification functions

the value of the \( z \) coordinate of all the nodes inside the elliptical modification area is increased. The value with which the \( z \) coordinate of each node is increased in the case of a dome modification depends on the distance of this node to the centre of the ellipse that defines the modification area (Figure 6.3(b)). Hence, the increase of the \( z \) coordinate of a node at the centre of the ellipse has a predefined maximum value whereas for a node at the perimeter of the ellipse the increase is 0.

6.1.1 Swages

The geometrical domain of modification for a single swage is a rectangular area. There are several ways to define a rectangular area. Here, a line with coordinates \((x_1, y_1)\) and \((x_2, y_2)\) together with a value \(w\) that represents the width of the area are used as shown in Figure 6.4. The four sets of coordinates at the corners of the rectangular modification area \((x_{1,1}, y_{1,1})\), \((x_{1,2}, y_{1,2})\), \((x_{2,1}, y_{2,1})\) and \((x_{2,2}, y_{2,2})\) can be calculated using trigonometry. The coordinates \((x_{1,1}, y_{1,1})\) and \((x_{1,2}, y_{1,2})\) can be calculated by considering the two right triangles with hypotenuse \(\frac{w}{2}\) as shown in Figure 6.5. These two
6.1 Modification functions

Figure 6.4: Geometry of a swage on a panel.

Figure 6.5: Definition of the coordinates of the modification area for a swage.
sets of coordinates can be given in relation to the coordinates \((x_1, y_1)\) and the angle \(\theta\) between the line defined by the coordinates \((x_1, y_1)\) and \((x_2, y_2)\) and the \(x\) axis as:

\[
x_{1,1} = x_1 - \frac{w}{2} \sin(\theta)
\]

\[
y_{1,1} = y_1 + \frac{w}{2} \cos(\theta)
\]

\[
x_{1,2} = x_1 + \frac{w}{2} \sin(\theta)
\]

\[
y_{1,2} = y_1 - \frac{w}{2} \cos(\theta)
\]

The angle \(\theta\) can be calculated for \(x_2 > x_1\) as:

\[
\theta = \sin^{-1}\left(\frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\right)
\]

and for \(x_2 < x_1\) as:

\[
\theta = \sin^{-1}\left(-\frac{y_2 - y_1}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}\right)
\]

Similarly the coordinates \((x_{2,1}, y_{2,1})\) and \((x_{2,2}, y_{2,2})\) at the corner of the rectangular area can be calculated by:
\[ x_{2,1} = x_2 + \frac{w}{2} \sin(\theta) \]  
\[ y_{2,1} = y_2 - \frac{w}{2} \cos(\theta) \]  
\[ x_{2,2} = x_2 - \frac{w}{2} \sin(\theta) \]  
\[ y_{2,2} = y_2 + \frac{w}{2} \cos(\theta) \]

Using the calculations described above the rectangular modification domain for a swage can be determined using only five design variables: \((x_1, y_1)\), \((x_2, y_2)\) and \(w\). One more design variable is required to define the height of the swage, that is, the increase in the value of the \(z\) coordinate of the nodes inside the modification area. Hence, a total of six design variables is required to describe one swage. However, the design variables for two different swages can be linked by using symmetric modifications, as discussed in the previous chapter and shown to improve the results. Thus, six design variables can be used to define two symmetric swages on a panel. In practice, less design variables means less dimensions in the search space of the optimization algorithm and hence better convergence.

As in the previous chapter where masses and stiffeners were used as structural modifications, the design variables are used for the meshing of the panel. The DistMesh MATLAB mesh generator is again used (a description of DistMesh can be found in Appendix C). The meshing of the panels with swages is done in two parts. Firstly, an area is created by subtracting the area of the swages from the total area of the panel and a mesh is generated for the resulting area. Fixed nodes are defined on the boundaries of the panel (nodes that were previously connected to the rest of the structure). Moreover, fixed nodes are imposed on the mesh along the interface of the panel and the swages. Figure 6.6 shows the mesh of the area which resulted from subtracting the area of a swage from the area of the panel. The fixed nodes along the panel/swage interface are indicated by a black circle. The fixed nodes are equidistant with
distance equal to the general size of the elements in the mesh, a constant used for the mesh generation (input to distmesh2d). A minimum of three nodes for each side of the rectangular modification area of the swage is used to ensure adequate meshing even for small modification areas.

The second part in the meshing of a swaged panel is the meshing of the area of the swages, defined by the design variables. The same fixed nodes used before at the panel/swage interface are now specified at the boundaries of the area of the swages. A predefined height is added to the $z$ coordinate of all the nodes of the mesh of the swages in order to create a press out. The mesh of the panel without the area of the swages and the mesh of the area of the swages are then combined. Practically this means that a new matrix is created containing the coordinates of all the nodes by combining the two node coordinates matrices of the two different meshes. The nodes of the area of the swages are appended at the bottom of the matrix with the nodes of the panel and their node number is changed appropriately. A similar procedure is followed for the connectivity matrix. The two meshes have identical nodes at the panel/swage interface. These nodes are equalised by deleting the fixed nodes at the panel/swage interface of the mesh of the panel and by replacing
them with their identical nodes in the mesh of the swages in the connectivity matrix. The result is a flat panel with the nodes inside the modification area being pressed out as shown in Figure 6.3(a).

6.1.2 Domes

The geometrical domain of modification of a single dome is an elliptical area. An elliptical area, like the one shown in Figure 6.7, can be defined by using five parameters: the coordinates of the centre of the ellipse \((x_0, y_0)\), the two radii of the ellipse \(a, b\) and the angle \(\theta\) of the ellipse to the \(x\) axis. The value of the \(z\) coordinate of the nodes inside the modification area of the dome is increased according to the distance of the node from the centre of the ellipse. For a node located at the centre of the ellipse with coordinates \((x_0, y_0)\) the increase in the value of the \(z\) coordinate is a predefined maximum value \(h_d\) whereas for nodes at the perimeter of the elliptical modification area the increase is 0. This means that the changes in the values of the \(z\) coordinate
of neighbouring nodes in the mesh of the panel is more smooth compared to those in the swaged panels. Hence, for the panel with domes the whole area of the panel is meshed and the modifications that create domes are applied on this mesh. Once the whole panel area has been meshed it is checked for every node of the mesh whether it lies inside the elliptical modification area (or modification areas if more than one dome is imposed on the panel) defined by the design variables shown in Figure 6.7. This can be checked by using the equation of an ellipse rotated by an angle $\theta$:

$$c_e(a, b, \theta) = \frac{(x_d \cos(\theta) + y \sin(\theta))^2}{a^2} + \frac{(y_d \cos(\theta) - x \sin(\theta))^2}{b^2}$$  \hspace{1cm} (6.11)

where $x_d = x_n - x_0$, $y_d = y_n - y_0$ are the distances of the $n^{th}$ node to the centre of the ellipse in the $x$ and $y$ dimensions respectively. For a node with coordinates $(x_n, y_n)$ lying inside the modification area the above equation takes a value less than 1 whereas for a node outside the modification area it takes a value greater than 1. The exact value of Equation (6.11) depends on the distance of the node to the centre of the ellipse given by $x_d$ and $y_d$. Hence, Equation (6.11) can be used to determine the increase of the value of the $z$ coordinate, $z_d$, of the nodes lying inside the modification area:

$$z_d = [1 - c_e(a, b, \theta)] h_d$$  \hspace{1cm} (6.12)

For a node at $(x_0, y_0)$ $c_e(a, b, \theta)$ becomes 0 and hence $z_d$ in Equation (6.12) becomes $h_d$. When $(x_n, y_n)$ defines a point at the perimeter of the ellipse $c_e(a, b, \theta)$ becomes 1 and hence $z_d = 0$. By applying the geometrical modification described above a panel with a dome as shown in Figure 6.3(b) can be created.

Since the panel to be optimised has been isolated from a structure as described in the previous chapter, it is important that the boundary nodes of the isolated panel are not modified during the optimisation since these nodes
will be later connected back to the rest of the structure. This means that the whole modification area of the domes must lie inside the area of the panel. Hence, during the optimisation procedure, before a modification is applied to the panel it is checked whether the whole modification area lies inside the panel area by using the central point and the 2 radii of the elliptical modification area. If the modification area is extended outside the panel area, the coordinates \((x_0, y_0)\) of the centre of the ellipse are shifted appropriately.

The total design variables required for a dome are six. Five design variables are used to define the modification area and one to define the maximum height of the dome. Symmetry can be used to link the design variables of different domes and hence with six design variables more than one node can be created as it was also discussed in the case of swages.

6.2 Acoustic analysis

In the previous chapter when stiffeners and masses were used for the optimisation of the floor panel the modes to be optimised were specified. Usually in practical engineering problems it is not known which modes of a panel cause excess noise radiation. Hence, here a more convenient way to define the optimisation problem is adopted. Instead of specifying individual modes to be optimised a frequency range is selected and all the modes inside this frequency range are optimised. The calculation of modes inside a frequency range can be carried out in Nastran by using the command

\[
\text{EIGRL 1 } f_1 \quad f_2
\]

where \(f_1\) and \(f_2\) are the lower and upper frequencies in the frequency range respectively.

As previously mentioned, in an optimisation procedure it is important to reduce the time required to evaluate the objective function for a given set of
6.2 Acoustic analysis

Figure 6.8: Comparison of the sound power of a panel with 15mm high swages calculated using a simplified BEM and full BEM.

design variables. Optimisation procedures are iterative and require a great number of evaluations of the objective function. For this reason in the acoustic analysis of the geometrically modified panels described in the previous section the assumption that the panels are flat is made. The assumption is valid as long as the out-of-plane modifications of the panel are small compared to the acoustic wavelength. Figure 6.8 shows the comparison of the sound power radiated by a panel with two cross swages using the simplified BEM formulation for the flat panel discussed in the previous chapter and a full BEM formulation. The swages are 15mm high which is the upper limit for the height of the swages used later in the optimisation of the floor panel. The full BEM formulation for the sound power makes use of the Direct BEM method [108, 109] with a symmetry plane at the plane of the panel (baffle). Using this method the influence matrices can be calculated and from these the radiation impedance matrix is derived which can be used with Equations 5.1-5.5 in the previous chapter for the calculation of the sound power radiated by the panel.

The comparison in Figure 6.8 shows that there is no significant difference in the sound power calculated using the two methods for frequencies up to
6.3 Results

In this section results are presented for the optimisation of the floor panel of the simplified car model, which was presented in Chapter 5, using swages and domes. As in the case of line stiffeners and point masses, the isolated panel with the appropriate boundary conditions, as discussed at the beginning of this chapter, is optimised. The optimum panel is then placed in the FEM model of the simplified car replacing the unmodified floor panel. The car structure is excited by point forces at the front part as described in the previous chapter and the sound power and radiation efficiency of the optimised floor panel are calculated using the commercial software LMS Virtual Lab.

The frequency range used for the optimisation is 10-200Hz. The challenge for the optimisation of the 0.8mm thick steel panel is that in this frequency range, the unmodified panel has 23 modes compared to just 5 modes of the 2.5mm thick panel of the previous chapter. The optimisation code for all the results presented here was ran overnight on a PC with 3.2GHz CPU and 2GB RAM and the time required was 12-15 hours for each case.

Figure 6.9 shows the floor panel with the optimum modifications using two swages as derived from the structural modes optimisation procedure described in the previous chapters and the geometrical modifications described in Section 6.1.1. The upper limit for the height of the swages used in the optimisation was 15mm. The height of the optimum swages in Figure 6.9 is 2.5mm for both swages. As it was discussed in the previous chapter im-
posing symmetry to the modifications around one dimension improved the results of the optimisation. Hence, for the optimisation of the swages in the floor panel symmetry was used in the definition of the modification areas. Assuming that the design variables defining the modification area of the first swage are \((x_1, y_1), (x_2, y_2)\) and \(w\), the design variables for the modification area of the second swage are \((L_x - x_1, y_1), (L_x - x_2, y_2)\) and \(w\) where \(L_x\) is the \(x\) dimension of the panel.

The optimised panel has 17 modes in the frequency range 10-200Hz and they are shown in Table 6.1. Figure 6.10 shows the comparison between the sound power of the unmodified panel and the modified panel with the optimum modifications shown in Figure 6.9. The average reduction achieved using the optimum modifications is 5.2dB. Unlike the results presented in the previous chapter where line stiffeners and point masses were used, the reduction using geometrical modifications was achieved without an increase in the total weight of the panel. Figure 6.11 shows the comparison of the radiation efficiency of the unmodified and the optimally modified panel. The average reduction in the radiation efficiency achieved by creating weakly radiating structural modes on the panel in the given range is 4.5dB. A significant reduction can be seen around modes 16 and 17 in both sound power and radiation efficiency. Moreover, some reduction can be seen around modes 3, 5, 6 and 14. An increase in the sound power and radiation efficiency of the floor panel can be observed around the 1\(^{st}\) mode which does not have any nodal lines and hence, it cannot be turned into a weak radiator. An increase can also be seen in the frequency range 100-120Hz where the optimum panel does not have any modes.

Figure 6.12 shows the floor panel with the optimum modifications using, this time, two pairs of symmetric swages. The optimum swages are two long, crossing swages together with two shorter, non-crossing swages located very close to the first ones. The height of all four swages is 1.3mm. The optimum panel has 20 modes in the frequency range 10-200Hz which are shown in Table 6.2. The reduction achieved for this panel is 6.1dB for the sound
Figure 6.9: The optimum floor panel using one pair of symmetric swages.

<table>
<thead>
<tr>
<th>Mode Num.</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26.3</td>
</tr>
<tr>
<td>2</td>
<td>48.9</td>
</tr>
<tr>
<td>3</td>
<td>49.5</td>
</tr>
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<td>4</td>
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<td>8</td>
<td>101.3</td>
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<td>11</td>
<td>131.8</td>
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<td>12</td>
<td>149.0</td>
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<tr>
<td>16</td>
<td>175.2</td>
</tr>
<tr>
<td>17</td>
<td>191.4</td>
</tr>
</tbody>
</table>

Table 6.1: Modal frequencies of the optimum floor panel with one pair of swages.
6.3 Results

Figure 6.10: Sound power of the unmodified floor panel and the optimised floor panel with one pair of symmetric swages.

Figure 6.11: Radiation efficiency of the unmodified floor panel and the optimised floor panel with one pair of symmetric swages.
Another modification function used for the optimisation of the floor panel of the simplified car model is that of domes as discussed in Sections 6.1.2. Figure 6.15 shows the floor panel with four domes as derived from the optimisation of its structural modes in the frequency range 10-200Hz. The coordinates of the centre of the modification areas of the four domes are linked in the following way: assuming that the coordinates of the centre of the first dome are \((x_0, y_0)\), the coordinates of the centre of the other three domes are \((L_x - x_0, y_0)\), \((x_0, L_y - y_0)\) and \((L_x - x_0, L_y - y_0)\). Similarly, assuming that the angle of...
Table 6.2: Modal frequencies of the optimum floor panel with two pairs of swages.
6.3 Results

Figure 6.13: Sound power of the unmodified floor panel and the optimised floor panel with two pairs of symmetric swages.

Figure 6.14: Radiation efficiency of the unmodified floor panel and the optimised floor panel with two pairs of symmetric swages.
the first dome is $\theta$ the angle of the other three domes are $-\theta$, $-\theta$ and $\theta$ respectively. The maximum height of all the domes is the same. For the optimum panel of Figure 6.15 the maximum height is 2mm. By linking the parameters of the domes in the way described above only the 6 design variables of the first dome are required in the optimisation. The average reduction in the sound power (Figure 6.16) achieved using the optimum dome modification on the panel is 5.7dB. The average reduction in the radiation efficiency (Figure 6.17) is 3.5dB and can be observed mainly around the modes of the modified panel shown in Table 6.3.
### 6.3 Results

<table>
<thead>
<tr>
<th>Mode Num.</th>
<th>Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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</tr>
<tr>
<td>2</td>
<td>32.5</td>
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<td>3</td>
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<td>64.3</td>
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<td>80.4</td>
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<td>8</td>
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</tr>
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<tr>
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<td>149.4</td>
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<td>170.9</td>
</tr>
<tr>
<td>19</td>
<td>180.8</td>
</tr>
<tr>
<td>20</td>
<td>187.4</td>
</tr>
</tbody>
</table>

Table 6.3: Modal frequencies of the optimum floor panel with four symmetric domes.
Figure 6.16: Sound power of the unmodified floor panel and the optimised floor panel with four symmetric domes.

Figure 6.17: Radiation efficiency of the unmodified floor panel and the optimised floor panel with four symmetric domes.
Due to the interest of Jaguar and Land Rover Research, which sponsored this project, in the optimisation code and results presented above it was decided to extend the optimisation code in order for it to be used during the vehicle development and refinement. The improvements required were the ability to optimise any given structure, not necessarily a flat rectangular panel, and the automatic identification of the values of the elastic boundary conditions of the structure. The first improvement also requires the ability to calculate the sound power of any three-dimensional structure of open or closed geometry. The improvement that was decided to be implemented as a part of this PhD is that of importing a general structure into the optimisation code and applying geometrical modifications to it. The calculation of the sound power radiated by this structure and the automatic identification of its boundary properties are discussed in the next chapter as future work.

In the optimisation procedure presented in this and the previous chapter the panel to be optimised was a rectangular panel which was defined by the length of its two sides. The area of this panel was meshed during the optimisation and modifications were applied. In the extension of the optimisation code a mesh of a structure, which has been previously created using an external software, is used. The mesh of the structure to be optimised can be a part of the whole vehicle structure. For example, Figure 6.18 shows a panel from the lower part of the boot (spare wheel housing) of a vehicle extracted from the whole vehicle structure model. In order to optimise this panel the nodes and the connectivity matrix of its mesh are read from a Nastran .bdf file.

After the mesh has been imported modifications need to be applied on the panel in order to optimise its structural modes. Here, dome modifications are considered, however the same method can be used for any modification function. The main difference between the implementation of the dome
modifications on a flat rectangular panel as discussed above and an arbitrary three-dimensional structure is that a modification volume instead of a modification area needs to be defined. Hence, for the case of a dome the modification volume is defined as the volume of an ellipsoid as:

\[
\frac{(x_d \cos(\theta) + y \sin(\theta))^2}{a^2} + \frac{(y_d \cos(\theta) - x \sin(\theta))^2}{b^2} + \frac{(z_d \cos(\theta) - x \sin(\theta))^2}{c^2}
\]

where \((x_d, y_d, z_d)\) are the coordinates of the centre of the ellipsoid and \(a, b, c\) are its three radii. Equation 6.13 is the extension of Equation 6.11 and gives the modification area of a dome on a flat panel in three dimensions. As in the two-dimensional case all nodes that lie inside the modification volume are modified to create a dome. In the case of a flat rectangular panel the modification applied to the nodes was to shift them towards the positive \(z\) direction, which is the direction of the normal vector of each node. For an
arbitrary, three-dimensional panel, such as the one shown in Figure 6.18, the same method is applied but this time the normal vector of each node needs to be calculated. The normal vector of each node is calculated by adding the normal vectors of all the elements adjacent to this node. Figure 6.19 shows an example of the calculation of a node connecting two line elements with normal vectors $N_1$ and $N_2$. The normal vector of each node inside the modification volume is calculated and normalised to derive the unit normal vector for each node. The translation (modification) of each node is then calculated by multiplying the unit normal vector of each node by its scalar distance to the centre of the ellipsoid $(x_d, y_d, z_d)$. An example of a dome modification on the boot panel of Figure 6.18 is shown in Figure 6.20. Since the panel already had some geometrical modifications, the dome created using the above method is indicated by a red circle.

In the extended optimisation procedure, part of which is the analysis presented here, the structural modes of the modified structure can be calculated using Nastran and the sound power radiated by each mode can be calculated (using for example an indirect BEM code) to evaluate the objective function for each structure with different modifications. The calculation of the sound power of the arbitrary structure has not been implemented and it is suggested for future work, hence results for the optimisation of an arbitrary structure are not presented here.
In this chapter geometrical modifications are used in order to impose weakly radiating structural modes on a panel. Specifically two types of geometrical modifications are considered: swages and domes. The concept of modification functions is used to create geometrical modifications on the panel. Using this, a modification area is firstly defined. For the case of swages the modification area is a rectangular area whereas for the case of domes that modification area is an elliptical area. All the nodes of the mesh of the panel that fall inside one of the modification areas are modified appropriately to create geometrical modifications (press-outs) on the panel. A simplified BEM formulation is used to calculate the sound power radiated by a number of structural modes of the modified panel based on the formulation of a flat, unmodified panel.

The results of the optimisation of the floor panel of the simplified car model introduced in the previous chapter show significant reduction in the sound
power and radiation efficiency of the panel. All the modes of the floor panel in the frequency range of 10-200Hz are optimised. In the results presented in this chapter the optimised panels had 17-20 structural modes in this frequency range. Comparing this with the results of the previous chapter where 6-10 modes of the floor panel were optimised, it shows that the proposed optimisation procedure can be efficient even when the number of modes to be optimised is increased. The best results were achieved when two pairs of symmetric swages were used. The average reduction for the sound power in the given frequency range was 6.1dB and for the radiation efficiency was 5.7dB.

In the last section of this chapter some work and discussion towards the extension of the optimisation code is presented. In this, the modes of an arbitrary, three-dimensional structure can be optimised by modifying its geometry.
Chapter 7

General conclusions and future work

This thesis is concerned with the study and minimisation of sound radiation of panels with structural modifications. In Chapter 2 of this thesis the problem of vibration and sound radiation from a plate with an attached beam stiffener is studied theoretically. This extends the current theories of vibroacoustic behaviour of infinite plates with an infinitely long beam discontinuity.

Firstly, the effect of an attached beam on the vibration and sound radiation from an infinitely long plate strip and a finite rectangular plate was studied theoretically. For the infinitely long beam stiffened plate strip the scattering of an incident propagation flexural wave in the unbound $x$ dimension due to a beam discontinuity in the bound $y$ dimension is considered. Expressions for the reflection, transmission and near-field coefficients are presented. Based on these coefficient, expressions for the sound power, radiation efficiency and mean squared velocity of the beam-plate strip system are developed. Simplified approximate analytical expressions for the low-frequency range, well below the critical frequency, are also presented. From the numerical calculations it was shown that the radiation efficiency of the plate strip has a low
value at the resonant frequencies of the beam when these are below the critical frequency of the plate. The exact value of the radiation efficiency depends on the properties of the stiffening beam. For a beam with a rectangular cross section, the radiation efficiency is decreased when the width of the beam is increased.

The beam-plate strip model is then extended for finite rectangular plates. Wave propagation in the finite structure and the scattering of the flexural waves due to the beam are considered. An expression for the displacement field of the structure is developed by taking into account an infinite number of wave reflections from the boundaries of the plate. Moreover, expressions for the sound power, mean squared velocity and radiation efficiency of the stiffened plate are derived. The model is validated by comparing it to calculations carried out using FEM and BEM. Numerical results for the radiation efficiency of a plate with attached beams of different dimensions are presented. The effect of a stiffener on the resonant frequencies of the plate is also discussed. It was observed that the resonant frequencies of the plate are shifted toward the resonant frequency of the beam with the same mode number as that in the $y$ dimension of the plate. This observation is in agreement with previously published predictions for a stiffened cylindrical shell.

For more complex and realistic structural modifications than a single beam stiffener numerical optimisation is used. The optimisation strategy adopted in this thesis is that of optimising the modeshapes of a structure in order to radiate acoustic energy inefficiently. The main advantage of this optimisation method is that the optimum design does not depend on the possibly complicated excitation mechanism of the structure and hence once a plate has been designed it can be used for any application. Also this method is very effective in reducing sound radiation at low frequencies where the reduction of the vibrational levels using damping treatment is inefficient.

In Chapter 3 the previously published work on the acoustic optimisation of structural modes is extended by considering line stiffener modifications, in
addition to point masses, optimising their position as well as their properties and using a more appropriate optimisation algorithm (GA).

Firstly, a mathematical model for the structural analysis of a plate with added point masses and line stiffeners at arbitrary locations and orientations is developed. Structural analysis using this model can be significantly faster than other commonly used methods, such as FEM, which is a significant requirement if this is a part of an iterative optimisation algorithm. Two possible optimisation approaches are discussed; a direct and an indirect approach. In the indirect optimisation the optimum modeshapes for a given geometry of the structure and frequency are calculated. Then the optimum modeshapes are imposed to the structure by optimisation of certain structural constraints. In the direct approach no information is known a priori about the optimum modeshapes. Certain structural constraints are again optimised, this time in order to minimise the sound power radiated by a number of modes of the structure. For reasons discussed in Chapter 3 the latter approach is used. For the optimisation a genetic algorithm is used.

The method is firstly applied for the optimisation of the 2nd structural mode of a plate using two sets of symmetrical masses. The results show that even though the optimised mode carries vibrational energy, this energy is not radiated into the acoustic medium. The 2nd mode of the plate is hence termed a weak radiator. It was observed that in order to increase the number of modes included in the optimisation the number of constraints had to be increased. Results for the optimisation of more structural modes using two and four pairs of masses are presented.

Line stiffeners did not prove to be as efficient as masses in creating weakly radiating structural modes. The best results were obtained when a combination of two sets of masses and a set of stiffeners was used.

In Chapter 4 the results of Chapter 3 are verified experimentally. The measurement apparatus and equipment used is described. Also the procedure of measuring the sound power of the structures under consideration using sound
intensity according to BS EN ISO 9614:2002 is discussed. The experimental results that lack the idealisation of the theoretical analysis in Chapter 3 are in good agreement with the results of the numerical simulation and provide evidence that the optimum modifications derived from the optimisation procedure can create weakly radiating structural modes.

In Chapter 5 the proposed optimisation method is applied to automotive-type panels. The main advantage that this optimisation procedure offers is a significant reduction in the time required to optimise different parts of a vehicle structure since the parts under consideration can be isolated and optimised independently. Another advantage of the method is that the optimisation is independent of the rest of the structure and the applied forces. Hence, when the vehicle structure or its parts are redesigned, which is common in the concept stage of vehicle development, the optimised panels retain their optimum acoustic characteristics. Moreover, as mentioned earlier the method is very effective in the reduction of sound radiation from low frequency modes which are responsible for the acoustic 'booming' and where damping treatment is usually inefficient.

The optimisation method is applied to a simplified car model. A floor panel of the car model is isolated and its structural modes are optimised. The first step is to identify the appropriate boundary conditions that need to be applied to the isolated panel in order to simulate the behaviour of the floor panel attached to the vehicle structure. For this, a simple method based on calculations of the input mobilities of the floor panel and the isolated panel is used in order to determine the value of the translational and rotational stiffness that need to be applied to the isolated panel.

For every iteration of the genetic algorithm optimiser a routine is executed where a mesh for the panel under consideration is generated based on the design variables, which are related to the location and properties of the modifications. A Nastran .bdf file is generated based on the mesh and the properties of the panel and modifications. The modeshapes and resonant frequencies
of the panel are then calculated using Nastran. A simplified BEM formulation based on the Rayleigh integral is used to calculate the sound power radiated by each structural mode which is used as the objective function of the optimiser.

This optimisation is used to optimise the location and properties of four masses and two stiffeners on the floor panel of the simplified car model. Linking the design variables by defining symmetrical modifications improved the results of the optimisation. The sound power calculated for the floor panel with the optimum modifications was reduced compared to the unmodified panel. This reduction is mainly due to the reduction in the radiation efficiency of the panel achieved by converting structural modes into weak radiators.

In Chapter 6 geometrical modifications are used in order to impose weakly radiating structural modes on the floor panel of the simplified car model. Specifically two types of geometrical modifications are considered: swages and domes. The concept of modification functions is used to create geometrical modifications on the panel. Using this, a modification area is firstly defined. For the case of swages the modification area is a rectangular area whereas for the case of domes that modification area is an elliptical area. All the nodes of the mesh of the panel that fall inside one of the modification areas are modified appropriately to create geometrical modifications (press-outs) on the panel. A simplified BEM formulation is used to calculate the sound power radiated by a number of structural modes of the modified panel based on the formulation of a flat, unmodified panel.

The results of the optimisation of the floor panel show significant reduction in the sound power and radiation efficiency of the panel. All the modes of the floor panel in the frequency range of 10-200Hz are optimised. In the results presented in this chapter the optimised panels had 17-20 structural modes in this frequency range. Comparing this with the results in Chapter 5 where 6-10 modes of the floor panel were optimised, it shows that the proposed
optimisation procedure can be efficient even when the number of modes to be optimised is increased. The best results were achieved when two pairs of symmetric swages were used. The average reduction for the sound power in the given frequency range was 6.1dB and for the radiation efficiency was 5.7dB.

Due to the limited time available for this research some extension of the current work could not be carried out and is discussed here as future work. More precisely, for the analytical work on the vibration and sound radiation from beam stiffened plates in Chapter 2 a possible extension could be the introduction of more stiffeners in the analysis. Hence, following the analysis in Chapter 2 a flexural wave in the plate is reflected and transmitted by a stiffener. The portion of the flexural wave that is transmitted across the stiffener is then again reflected and transmitted by another stiffener at a certain distance from the first one. The analysis can be continued for a number of stiffeners. Similar to the analysis in Section 2.2 an infinite number of reflections and transmissions from all the stiffeners and the boundaries of the plate needs to be considered to determine the vibrational field of the stiffened plate. This analysis can help to reveal the interaction of two or more stiffeners and its influence on the vibration and sound radiation from a stiffened plate.

In Section 2.2 the stiffener attached to the finite plate is placed in the middle of the plate. Hence, another extension of this work can be to make the distance of the location of the stiffener to the location of one of the boundaries of the plate variable. This can reveal the influence of the location of the stiffener to the vibration and sound radiation of the plate. Moreover, instead of simply supported boundary conditions, other types of boundary conditions can be implemented in the analysis. For the boundaries at $x = -L_x/2$ and $x = L_x/2$ different reflection characteristics can be introduced instead of the reflection coefficient $R = -1$ that has been used for the simply supported boundary conditions in this thesis. For the boundaries at $y = 0$ and $y = L_y$ different modeshapes can be used in the modal summation. This con-
fines the analysis to boundary conditions that produce well-known analytical
modeshapes.

A possible extension of the numerical optimisation code is described in Sec-
tion 6.4. In this, the modes of an arbitrary, three-dimensional structure, not
necessarily an initially flat rectangular panel, can be optimised by modifying
its geometry. This will provide a very useful tool for the acoustic optimisation
of automotive panels during vehicle development and refinement.

One part of the extension of the optimisation code was implemented as part
of this project. This is the application of geometrical modifications (domes)
on an arbitrary, three-dimensional structural FEM model as discussed in
Section 6.4. Another very important part of the optimisation, which was
not implemented during this project, is the calculation of the sound power
radiated by a number of modes of the structure. Since the geometry of the
structures to be optimised is arbitrary, an Indirect BEM approach is required
instead of the simplified BEM that was used for the applications described
in this thesis or a Direct BEM that can only deal with closed geometries
[109]. Another improvement of the optimisation would be to use two different
meshes for the FEM and BEM analysis since in a BEM analysis the mesh
used does not need to be as dense as in a FEM analysis. This will require the
assignment of nodal displacements (or velocities) on the coarse BEM mesh
based on the nodal displacements (or velocities) calculated by FEM on the
dense FEM mesh. This will reduce the time required for the optimisation,
especially for larger panels, without reducing the quality of the analysis if
the coarse mesh meets the requirements of the BEM analysis (approximately
6 elements per wavelength [109]).

Finally, another improvement that is suggested is the automatic identification
of the boundary conditions of the panel to be optimised when attached to the
rest of the structure. This can be done by using an optimisation procedure
that will optimise the values of the translational and rotational stiffness of the
boundary nodes of the FE model of the panel in order for the isolated panel
to match the behaviour of the panel which is attached to the structure. A measure of the vibrational behaviour of the panel can be its input mobility at a certain node as used in Chapters 5 and 6. To reduce the number of variables to be optimised the boundary nodes can be grouped, depending on the application, so one value for the translational and rotational stiffness of each group of nodes needs to be optimised. Moreover, substructuring techniques such as the Component Mode Synthesis (CMS) [110] and the Wave-Based Substructuring (WBS) [77] methods can be used to isolate the part of the structure that needs to be optimised.
References


REFERENCES


REFERENCES


[95] BS EN ISO 9614. Acoustics - determination of sound power levels of noise sources using sound intensity - part 1 measurement at discrete points. CEN, 1993.


Appendix A

Proof of \( \cos(\phi_T - \phi_R) = 0 \)

In order to prove that \( \cos(\phi_T - \phi_R) = 0 \), where \( \phi_T \) and \( \phi_R \) are the phase of the complex transmission and reflection coefficients respectively, it is useful to use the expressions for \( T_n \) and \( R_n \) in the form given by Heckl [18] which are equivalent to the expressions in Equation (2.6) in Chapter 2. The transmission and reflection coefficients are given in the form:

\[
T_n = \frac{i\delta}{\beta + i\gamma} \quad (A.1)
\]
\[
R_n = \frac{\alpha}{\beta + i\gamma} \quad (A.2)
\]

where \( \alpha, \beta, \gamma \) and \( \delta \) are real numbers that can be found by comparing the above equations with Equations (6) and (7) in reference [18].

The proof of \( \cos(\phi_T - \phi_R) = 0 \) requires that \( |\phi_T - \phi_R| = \frac{\pi}{2} \). From Equations (A.1) and (A.2) it can be seen that:

\[
T_n = iCR_n \quad (A.3)
\]
where $C$ is a real number. Equation (A.4) is equivalent to:

$$T_n = CR_n e^{i\frac{\pi}{2}}$$  \hspace{1cm} (A.4)

which proves that the phase difference between the complex numbers $T_n$ and $R_n$ is $\pm \frac{\pi}{2}$, where the sign depends on the sign of the real number $C$. 
Appendix B

Panel Optimisation GUI and code

In this appendix the Graphical User Interface (GUI) developed for the application of panel optimisation presented in Chapter 6 is described. All the files containing the code necessary to execute the application can be found on the CD attached with this thesis.

B.1 MATLAB GUI Installation

In order to execute the application it is recommended to copy the files contained on the attached CD to a folder on a hard disk drive. This folder then must be added to the MATLAB path. This can be done by choosing 'File→Set Path...' from the MATLAB menu and pressing the button 'Add with Subfolders...'. From the menu that follows the folder to which the files from the CD were copied to needs to be selected. Once the files have been copied and the MATLAB path has been changed appropriately several lines in the code files need to be customised according to the system that the
code will be run on. Firstly, in GUI\_main.m file the path of the working
directory needs to be changed to the path where the code files from the CD
were copied to. This can be done in line 134. The working directory is the
directory where the NASTRAN file of the final optimised panel (and all the
intermediate files) is stored after the optimisation has finished. Moreover, the
path in which NASTRAN is installed on the system running the optimisation
needs to be set in the files:

\begin{verbatim}
SoundPowerPlateDomesFEM.m - line 174
SoundPowerPlateDomesMassFEM.m - line 193
SoundPowerPlateSwagesFEM.m - line 249
SoundPowerPlateSwagesMassFEM.m - line 279
\end{verbatim}

It is necessary to have NASTRAN installed on the system running the opti-
misation.

\section*{B.2 \textit{Description of the GUI}}

In order to start the panel optimisation GUI the file GUI\_main.m needs to
be run. The form that appears after executing this file is shown in Figure
B.1. From this form the type of panel modification (design variables) can be
chosen. The available options are swages, swages and masses, domes, and
domes and masses as discussed in Chapter 6. The number of modifications
and the symmetry used between them is fixed and it can be changed by
modifying the source code appropriately.

After choosing the type of the panel modification another form appears in
which the properties of the panel as well as the limiting values for the design
variables can be specified. Figure B.2 shows the form that appears if swages
and masses modifications have been chosen in the form of Figure B.1. In
this form the dimensions of the panel can be specified as well as its ma-
B.3 Description of the MATLAB code

Figur B.1: The first form of the panel optimisation GUI.

Apart from the MATLAB files containing the GUI forms (their filename starts with the prefix GUI) there are also 4 files that contain the genetic algorithm optimisation code (their filename starts with the prefix GA) one for each of the 4 predefined panel modifications. The genetic algorithm used
is discussed in Section 3.4 and it shown in a flow chart in Figure 3.3. The implementation of the code is based on the code developed by Cox [89].

Each of these 4 files containing the genetic algorithm code calls the appropriate function, depending on the panel modification used, which calculates the sound power of a number of modes of the modified panel which is the objective function of the optimisation. The filenames of the files that contain the functions for the evaluation of the sound power start with the prefix SoundPowerPlate. The first step in these 4 functions is the construction of the mesh of the panel with the appropriate modifications using DistMesh as discussed in Section 6.1. Appropriate comments have been added to the code for this and the subsequent steps in order to be easily understood. Using the data for the mesh of the panel a NASTRAN .bdf file is written using the functions writebdf for panels without mass modifications, and writebdfmass for panels with mass modifications. The code of the function can be found in the files with the same names as the functions. After the .bdf file has
been created (with filename testfile.bdf) NASTRAN is called through MATLAB using the dos() function in order to calculate the structural modes of the panel in the given frequency range. Once NASTRAN has finished calculating the modes of the panel the results are stored in a NASTRAN .f06 results file. The number of modes in the specified frequency range can be read from the NASTRAN results file using the function xreadnummode. A for loop is then used to iterate through all the structural modes calculated from NASTRAN and to calculate their sound power. As a first step in the iteration the nodal displacements of each modeshape of the panel is read from the NASTRAN results file using xreadmodesh function. Then nodal displacements are used as input to the CalcSoundPower function which calculates the sound power radiated by each modeshape after firstly normalising all the modeshapes to have a unit mean squared velocity. The theory for the calculation of the sound power implemented in CalcSoundPower.m MATLAB file is described in Section 5.2.2.
Appendix C

DistMesh - MATLAB mesh generator

DistMesh is a collection of freely distributed MATLAB functions for unstructured 2D triangular mesh generation created by Persson and Strang [103]. DistMesh is a free software under the terms of the GNU General Public License as published by the Free Software Foundation. In this appendix the functions used for the applications discussed in Chapters 5 and 6 are presented.

**distmesh2d**  This is the main function of the mesh generator. This function generates a 2D triangular mesh using distance functions. The syntax of the function is:

\[
[P,T]=\text{DISTMESH2D}(\text{FD},\text{FH},\text{H0},\text{BBOX},\text{PFIX},\text{FPARAMS})
\]

- **FD** is a MATLAB distance function (some distance functions are described later). This describes the geometry to be meshed.
- **FH** is the desired edge length given as a function of \(x\) and \(y\), refereing to
coordinates inside the meshing area. FH can be used to refine the mesh at locations in the meshed area wherever is required. In the applications of Chapters 5 and 6 a uniform function (FH = @huniform) is used.

- H0 is the distance in the initial distribution of the nodes. H0 also defines the maximum length of the edges of each element in the final mesh.

- BBOX is the bounding box for the region to be meshed and it is given in an array (bbox=[xmin, ymin; xmax, ymax]).

- PFIX is an array with the fixed node positions.

- FPARAMS is used to pass additional parameters to the FD and FH functions.

- The output argument P is a $N \times 2$ array that contains the $x$, $y$ coordinates for each of the $N$ nodes of the mesh.

- The output argument T is the connectivity matrix that describes the way nodes are connected to form elements. The size of the matrix is $M \times 2$, where $M$ is the number of elements in the mesh. Each raw, which is associated with one triangular element, has 3 integer entries that specify the node numbers of the element.

drectangle This is one of the distance functions that can be used as an input argument to distmesh2d function. drectangle returns the signed distance of one or more points to a rectangle. The input arguments of this function is an array with the coordinates of one or more points and 4 number that are the coordinates of 2 opposite corners of a rectangular area. If the point(s) described by the first argument falls inside the rectangular area drectangle returns a negative value whereas if the point(s) is outside the rectangular area it returns a positive value. For points at the boundaries of the rectangular area drectangle returns 0.
**dpoly**  This function is equivalent to drectangle for a polygonal area.

**dintersect**  This function returns a distance function created by the intersection of 2 distance functions. dintersect was used for the meshing of swaged panels in Chapter 6 to create a distance function by combining the areas of the swages.

**ddiff**  This function returns a distance function created by the substraction of 2 distance functions. ddiff was used for the meshing of swaged panels in Chapter 6 to create an area of the panel to be meshed by cutting out the areas of the swages.
Appendix D

NASTRAN .bdf file format

In this appendix the NASTRAN .bdf file format is described and the entries used for the applications described in Chapters 5 and 6 are discussed. A full reference of all the NASTRAN commands can be found in the NASTRAN Quick Reference Guide [111].

A .bdf file is a text file that contains NASTRAN commands which describe a FE model and give instructions to NASTRAN on the type of the analysis. Hence, the first line in a .bdf file is a SOL command that specifies the type of the analysis. In the application of Chapters 5 and 6 a SOL 103 command has been used that instructs NASTRAN to calculate the normal modes and frequencies of the FE model described later in the same .bdf file. Other types of analysis include frequency response (which can be based on direct calculations or modal superposition) static analysis, transient analysis and nonlinear analysis, among others.

After the type of the solution has been specified a CEND command can be used to insert direct text input for execution control. This can include a TITLE command that needs to be placed after CEND to specify the title of the job to be executed. After the direct text input for execution control direct text input for a subcase can be inserted using the SUBCASE command.
This can be again a title for the subcase and the quantities that need to be calculated (this includes nodal forces, displacements, velocities, etc).

The main part of a .bdf file is the bulk section which begins with a BEGIN BULK command. In this section the FE model is specified. There are two possible types of formats that can be used to write the commands in this section: the long format and the short format. In the bulk section each line is separated into fields which are 16 characters long if the long format is used and 8 characters long if the short format is used. The default option is the short format which is the format that has been used in the applications described in Chapters 5 and 6.

If SOL 103 has been used in the beginning of the file an EIGRL command is required in this section. The syntax of this command is:

\[
\text{EIGRL 1 F1 F2 ND}
\]

where F1 and F2 specify the lower and upper frequencies for the calculated normal modes and ND specifies the maximum number of modes to be calculated. It must be noted that all the syntaxes of the commands presented in this appendix are specifically for the applications developed in Chapters 5 and 6. A full description of all NASTRAN commands can be found in [111].

A command is required to define the type of elements to be defined later in the bulk section. More than one command can be used if more than one type of elements is required. For the applications in Chapters 5 and 6 triangular shell elements were used. The PSHELL command can be used to define such elements. The syntax of the command is:

\[
\text{PSHELL PID MID1 T MID2 MID3}
\]

where PID is the identification number of the element type that is used later to associate elements with this type. MID1, MID2 and MID3 are the material identification numbers for the membrane, bending and transverse
shear properties of the element. $T$ is the thickness of the shell element.

A MAT1 command can be used to specify one or more materials. The syntax of this command is:

```
MAT1 MID E G NU RHO
```

where MID is the material identification number (that can be used as an input for MID1, MID2 and MID3 entries in PSHELL). $E$, $G$, $NU$ and RHO define the Young’s modulus, shear modulus, Poisson ratio and mass density respectively.

Apart from shell elements defined by PSHELL, beam elements were also used in the application of Chapter 5. The PBARL command is used to define a beam element type. The syntax of this command is:

```
PBARL PID MID TYPE DIM1 DIM2
```

where PID is the property identification number for the beam elements and MID is the material identification number of the material of the beam. TYPE is an alphanarithmetic input that specifies the type of the beam. Some of the allowed types are ROD, TUBE, I and T. For beams with rectangular cross section, like the ones used in Chapter 5, the type BAR is used. For a PBARL property of type BAR 2 extra arguments are required to specify the dimensions of its cross section. These arguments are DIM1 and DIM2. For other types of beams more that 2 arguments can be used to define the cross section.

The last type of element used in the applications described in Chapters 5 and 6 is the bush type used to impose translational and rotational stiffness at the boundaries of a panel. The bush element type is defined by PBUSH command as follows:

```
PBUSH PID "K" K1 K2 K3 K4 K5 K6
```
where PID is the property identification number, the character K indicates that the 6 following values are stiffness values, K1-K3 are the values of the translational stiffness in $x$, $y$ and $z$ directions respectively and K4-K6 are the values of the rotational stiffness around $x$, $y$ and $z$ axis respectively.

Once the properties of the elements have been specified the elements of the model need to be defined. For triangular shell elements the CTRIA3 command is used with the following syntax:

```
CTRIA3  EID  PID  G1  G2  G3
```

where EID is a unique for each element identification number, PID is a property identification number of a PSHELL or another element type specifier and G1-G3 are 3 grid point identification numbers which define the three nodes of the triangular element. One CTRIA3 command should be written in the bulk section of the .bdf file for each triangular element in the model. The grid points in the model are defined using the GRID command as follows:

```
GRID  ID  X1  X2  X3
```

where ID is the identification number of the grid point and X1-X3 are its coordinates. One such command is required for each grid point in the model. The grid points that are associated to elements, using for example the CTRIA3 command, become the nodes of the FE model.

Equivalent to CTRIA3 command is the CBAR command for defining beam elements. The syntax of the command is:

```
CBAR  EID  PID  GA  GB
```

where EID is a unique element identification number, PID is the property identification number of a PBARL and GA and GB are the grid point identification numbers of the nodes of the line element.

Apart from beam elements concentrated mass elements were also used in the
application in Chapter 5. These elements are defined by a CONM2 command as follows:

CONM2 EID G CID M

where EID is the element identification number, G is the identification number of the grid point where the concentrated mass is placed, CID is the coordinate system identification number (the default value is 0 which is a coordinate system with cartesian coordinates with origin at 0,0,0) and M is the mass in kg.

Lastly, the bush element is defined by the command:

CBUSH EID PID GA

where EID is the element identification number, PID is the property identification number of a PBUSH and GA is the identification number of a grid point that gives the location of the bush element.