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Distinctive characteristics of guided surface-wave propagation in complex topographic structures

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The specific characteristics of wave motion in smooth topographic waveguides of complex configuration are investigated on the basis of the asymptotic theory of Rayleigh wave propagation on smooth surfaces of arbitrary shape.

It is common knowledge that a great many solid-state structures can be used as waveguides for surface acoustic waves.1 If the waveguide properties are wholly attributable to the influence of the surface geometry, the corresponding waveguides are usually called topographic waveguides.1,2 Waveguides of this type are of special interest for acoustoelectronic signal-processing devices. The need to take into account the waveguide properties of a surface is also encountered in seismological and ultrasonic flaw-detection problems. The investigated topographic structures are treated by numerical methods in the majority of cases.1,2 Only in a few cases is it possible to formulate an approximate analytical solution.3-5 Among those situations, in particular, are smooth topographic waveguides, for which the minimum radius of the curvature of the surface is greater than the surface wavelength. Such structures were first analyzed by Wilson et al. (Refs. 6 and 8), who investigated the cases of waveguides in the form of a smooth wedge, a smooth projection, a beam of elliptical cross section, and an elliptical cavity. A solution of the problem was obtained directly in the form of waveguide modes of the corresponding structures, as a result of which the analysis was quite cumbersome. A simpler method has been proposed8 for the analysis of smooth topographic waveguides on the basis of an asymptotic expression derived in the same paper for the local velocity of a plane Rayleigh wave propagating along a smooth surface of arbitrary shape, with subsequent allowance for the nonuniform curvature of the surface in the transverse direction. Numerical methods, which are also applicable to the low-frequency case, have been used in an analysis10-11 of the waveguide behavior of cavities of various configurations, specifically an elliptical cavity and cavities in the form of a partially closed figure-eight and a rounded square.

The objective of the present study is to summarize certain results obtained in relation to the theory of smooth topographic waveguides of convex configuration by the method proposed in Ref. 9 and on the basis of the fundamental concepts of the theory of coupled waveguides. It will be shown below that this combined approach can be used to obtain all the known results in the high-frequency range by a far simpler procedure and in a physically transparent form. The same simplicity and clarity also make it possible to use the indicated approach to analyze a whole series of new and very intricate waveguide structures that defy investigation by other methods.

We proceed from the equation for the local Rayleigh wave phase velocity c on a smooth curved surface of arbitrary shape,9 writing it in the form

$$c = c_0 (1 + a/k \rho_0 + b/k \rho_1),$$

(1)

where $c_0$ is the Rayleigh wave velocity on a plane surface, $k = \omega/c_0$ is the Rayleigh wave number, $\rho_0(u, v)$ and $\rho_1(u, v)$ are the variable radii of curvature of the surface in the direction of wave propagation (in the direction of the wave normal) and in the direction perpendicular to it, respectively, and $u$ and $v$ are the surface coordinates in the same directions, and $a$ and $b$ are coefficients of the order of unity, which depend on the Poisson ratio of the medium. We omit the expressions for $a$ and $b$ in the interest of brevity. We also disregard the expressions for the displacements in the Rayleigh wave, which closely differ in the given situation ($k \rho_0, v \gg 1$) from the corresponding expressions for a plane surface. We advise the reader who is interested in the specific notation of the coefficients $a$ and $b$ that the expressions corresponding to $a$ and $b$ in Ref. 9 are nonrigorous, because the author discarded certain terms of the same order ($\nu, 1/k \rho_0, v$) as the retained terms in order to simplify the derivation of the parabolic equations used for the analysis in Ref. 9. Exact expressions for the coefficients $a$ and $b$ may be found in Ref. 12.

We now use the above-presented relation (1) to analyze specific topographic waveguide structures. We consider a topographic waveguide in the form of a rounded wedge as the simplest example (Fig. 1a). We first apply Eq. (1) to the case of Rayleigh wave...
where
\[ c = c_0 \left( 1 + \frac{1}{k_R} \left[ (a-b) \sin^2 \phi + b \right] \right), \]
(2)
where \( R \) is the radius of the cylindrical tip zone and \( \phi \) is the angle between the wave normal and the generatrix of the cylinder. The dependence on the angle \( \phi \) in Eq. (2) characterizes the anisotropy associated with the curvature of the surface. Taking into account that the Rayleigh wave velocity at the edges of the wedge (i.e., in the regions of zero curvature) represents the velocity \( c_0 \), we can write the following equation for the normal-mode propagation in an arbitrary direction in the vicinity of the rounded tip, using the Euler equations:

\[ \frac{\varepsilon}{2} (k_o(\Phi) - \gamma)^2 = m - \frac{\pi}{2} + \text{arc} \left[ \frac{\sqrt{\gamma - k_o^2}}{k_o(\Phi) - \gamma} \right]^b. \]  
(3)

Here \( \gamma \) is the normal-mode propagation constant in the waveguide, \( m \) is the order of the middle layer (clearly, \( m = (\varepsilon - e)R \) is the given situation), and \( k_o(\Phi) \) is the Rayleigh wave number in the rounded-tip zone. Making use of Eq. (2) and restricting the problem to small values of the angle \( \phi \), we write the equation for the propagation constant in the form

\[ k_o(\Phi) - k_o(1 + (1/k_R)(a \Phi^2 + b)), \]
(4)
where \( a = -2(a - b) \), \( b = -2b \). We determine the first terms of the asymptotic expression of \( \gamma/(1/k_R) \) for the zeroth mode. To do so, we allow for the fact that \( \gamma \approx k_o \cos \Phi = k_o(1 - \varepsilon^2/2) \) and substitute Eq. (4) in (3). After simple transformations,

\[ \text{tg} \left( \frac{\varepsilon - e}{2} R \right) = \left( \frac{b}{k_0 R \varepsilon^2 - 1} \right)^b. \]
(5)

Linearizing the left-hand side of Eq. (5), we reduce it to a biquadratic equation in \( \gamma \) and solve it to obtain \( \gamma^2 = \frac{2b}{(\varepsilon - e)}(k_o R)^{3/2} \). We therefore obtain the propagation constant \( \gamma \) in the form

\[ \gamma \approx k_o \left[ 1 + \frac{b}{2k_R} - \frac{b^2}{\pi - \varepsilon (k_R)^2} \right]. \]
(6)

Equation (6) has the same form as the first terms of the expansion in Refs. 6 and 8, but its derivation here is far simpler. The law governing the amplitude decay of the investigated normal mode at a distance \( d \) from the boundary of the rounded-tip zone is characterized by the factor \( \exp[-\gamma (1 + \varepsilon/2)] \), where

We emphasize that the waveguide effect is possible in the given topographic structure by virtue of the fact that the Rayleigh wave velocity along the generatrix of a convex circular cylinder \( R > 0 \) is always smaller than the Rayleigh wave velocity on the plane. This follows from the fact that the coefficient \( b \) is negative in Eqs. (1) and (2) or, equivalently, from the fact that the coefficient \( B \) is positive in Eqs. (4)-(6). It is interesting to note that the propagation constant \( \gamma \) in the given approximation is independent of the coefficient \( \varepsilon \), which characterizes the velocity anisotropy caused by the curvature of the surface.

The guided-wave propagation of a Rayleigh wave along concave surfaces in the form of channels or troughs can be analyzed similarly (note that structures of this type have not been investigated before). In order for guided-wave propagation to be possible in this case, the Rayleigh wave velocity on the bottom of the trough must be smaller than the velocity along its sloping sides. The simplest structure for this situation consists of a flat bottom with walls in the form of concave circular cylinders of radius \( R \) (Fig. 1b). We assume formally for simplicity that the width of the circular walls is infinite. Since the field in a waveguide mode decays rapidly in the transverse direction, this idealization is entirely acceptable. For illustration, we once again use the dispersion relation (3) for symmetrical modes in a three-layer system, but now we interchange the quantities \( k_o \) and \( k_o^2 \), and \( k_o^2(\Phi) \) acquires the form (minus sign is attached to the radius of curvature \( R \) for a concave surface)

\[ k_o^2(\Phi) = k_o^2[1 - (1/k_R)(a \Phi^2 + b)]. \]
(7)

Substituting Eq. (7) in (3) with allowance for the fact that \( \gamma = k_o \cos \Phi = k_o(1 - \varepsilon^2/2) \), and carrying out transformations similar to those above, we obtain the following for the fundamental symmetrical mode:

\[ \gamma \approx k_o \left[ 1 - \frac{b}{2k_R} - \frac{b^2}{(k_R)^2} \right]. \]
(8)
The law governing the amplitude decay of the field at a distance $s$ from the boundary of the channeling wave zone is also determined by the factor $\exp[-\nu(\ell+s/2)]$ in this case, where

$$\nu = (\gamma^2 - k^2)^{1/2} = k_0 \left[ \frac{\beta}{k \delta} - \frac{2\beta}{k_0^2} \frac{1}{k \delta} \right]^{1/2}.$$ 

Thus, the field decays in the transverse direction according to the same law as in the case of a rounded wedge.

We now proceed to the analysis of more complex structures having two or more zones for the possible channeling of surface waves. One of the simplest structures of this kind is a bar with an elliptical cross section. Such a structure has been investigated previously\(^a\) by the direct approach. We now show that it can be analyzed by an almost elementary procedure relying on the principles of coupled-waveguide theory. To simplify matters, we analyze a bar whose cross section is not described by the equation of an ellipse as in Refs. 6-8, but in the form of a strip whose edges are joined by two circular cylinders of radius $R$ (Fig. 2a). This simplification, of course, is not fundamental and is employed for the sole purpose of being able to make use of the results obtained above for simple waveguide structures with elements in the form of circular cylinders.

We recall the fundamental facts of coupled-waveguide theory that will be needed below. The field of coupled identical modes in the most interesting case of a system of $N$ identical planar waveguides with centers separated by a distance $d$ can be described by the system of equations\(^b\):

$$\frac{\partial u_n}{\partial z} - i\nu_n + ik_n(u_{n+1} + u_{n-1}) = 0.$$ 

(9)

Here $u_n$ is an amplitude factor, which characterizes the field in the $n$-th waveguide $(n = 1, 2, \ldots, N)$, $\gamma$ is the propagation constant of the investigated mode in the solitary unperturbed waveguide (to be specific, we assume below that the fields of the unperturbed waveguides represent symmetrical modes), $z$ is the coordinate measured along the waveguide axes (the time factor $\exp(-i\omega t)$ is tacit), and $k_n$ is the coupling coefficient of the waveguides (the coupling is assumed to be weak, and only interaction between contiguous waveguides is taken into account). The equation for the coupling coefficient $\kappa$ in the case of interest (three-layer structures) can be written in the form\(^c\):

$$\kappa = \frac{\mu^4}{\gamma(1 + \nu s/2)(\mu^4 + \nu^4)} \exp[-\nu(d - s)],$$ 

(10)

where $\mu = (k_s^2 - \gamma^2)^{1/2}$, $\nu = (\gamma^2 - k_p^2)^{1/2}$, $k_s$ and $k_p$ are the wave numbers of plane surface waves propagating in the channeling regions (regions with a lower phase velocity) and in the surrounding medium (regions with a higher phase velocity), and $s$ is the width of the layer with a lower velocity. It is seen at once that the coupling coefficient (10) is determined decisively by the factor $\exp[-\nu(d - s)]$, i.e., by the decay law of the field outside the waveguide (see above). If the end waveguides of the coupled system, which are numbered $n = 1$ and $n = N$, are practically noninteracting, it is required to set $u_0 = u_{N+1} = 0$ formally in Eqs. (9). When interaction takes place between these waveguides and is also characterized by the coupling coefficient $\kappa$ (cyclic system of waveguides), it is required to impose cyclic boundary conditions $u_n = u_{N+n}$ on the system, so that $u_0 = u_N$, $u_{N+1} = u_1$.

The subsequent considerations regarding the application of these facts to the investigated topographic structure (Fig. 2a) are rather trivial. Such a structure is obviously a coupled cyclic system of two waveguides described by the equations

$$\begin{align*}
\frac{\partial u_1}{\partial z} - (\gamma_1 + i2\omega \epsilon_1)u_1 &= 0, \\
\frac{\partial u_0}{\partial z} - (\gamma_0 + i2\omega \epsilon_0)u_0 &= 0,
\end{align*}$$

(11)

which differ from the well-known equations for two coupled waveguides on the plane only in the doubling of the coupling coefficient $2\kappa$ as a result of the cyclic property. The standard procedure of seeking a solution of Eqs. (11) in the form of normal modes of the coupled system

$$u_n = A_n \exp(ikz).$$

(12)

where $A_n = \text{const}$, gives the values of the mismatch $\Delta \gamma = \kappa - \gamma$ between the propagation constants of the symmetrical and antisymmetrical modes of the coupled system relative to the propagation constant of the unperturbed waveguide: $\Delta \gamma = \pm 2\kappa$. A positive value of the mismatch $\Delta \gamma = 2\kappa$ corresponds to an antisymmetrical mode of the coupled system ($A_1 = -A_2$), and a negative value $\Delta \gamma = -2\kappa$ corresponds to a symmetrical mode ($A_1 = A_2$). Returning to Eq. (10) for the coupling coefficient $\kappa$ and bearing in mind the relation given above for the coefficient $\nu$ in the case of a rounded-wedge waveguide (the angle $\epsilon$ must be set equal to zero in this case), we readily infer (omitting the details) that the values of $\Delta \gamma = \gamma(d)$ are exponentially small, in perfect agreement with the direct calculations of Refs. 6-8. The fields in this case are concentrated in the rounded-corner zones, and the periodic transfer of energy to the other edge is possible in the excitation of only one rounded edge, as in any coupled waveguide system.

A topographic structure in the form of a slot with rounded edges (Fig. 2b) can be analyzed similarly (The case of an elliptical-cylindrical cavity has been investigated previously\(^d\)). All the argu-

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We consider propagation by the waveguide in the form of a solid bar. As the roundoff radii $R$ of the slot tend to zero (with the understanding that the previously used asymptotic description of the field in the rounding zone is no longer applicable in this case), the investigated structure goes over into a through crack. The guided-wave propagation of surface waves along the rims of such a crack was first investigated by Freund and later by Burden.  

Cyclic structures (solid bars and open cavities) containing three, four, or more guided-wave channels (Figs. 2c and 2d) can be calculated in exactly the same way. For example, the solution of the system (9) in the case of the three-sided bar in Fig. 2c gives a mismatch $\Delta \gamma = -2 \kappa$, which corresponds to a symmetrical mode ($A_1 = A_2 = A_3$), and the doubly degenerate value $\Delta \gamma = k$, which can be associated with two independent modes: $A_1 = A_2 = A_3 = 0$ and $A_2 = -A_1$, $A_3 = 0$.

The advantage of the method described here for the analysis of complex topographic waveguides is particularly conspicuous in the case of structures containing a large number of channels (see Fig. 3). The usual procedure of substituting Eq. (12) in the equations (9) describing such a system and then determining $\Delta \gamma$ from the characteristic equation, which is obtained when the determinants of the homogeneous system of algebraic equations in $A_n$ is equal to zero, can be simplified considerably for large $N$. It is useful in this case to resort to a method based on the analogy between a system of coupled waveguides and a one-dimensional atomic lattice, and to assume that the investigated structure is continuous in the $z$ direction and is discrete in the perpendicular direction. The method essentially entails, first, the investigation of a system of infinitely many coupled waveguides. The solution of Eqs. (9) in this case is sought in the form of so-called "plane" coupling waves propagating in the investigated quasidiscrete medium at an angle relative to the $z$ axis:

$$w = A \exp(ikz + iqd), \quad (13)$$

where $q$ is the wave number in the direction perpendicular to the $z$ axis, $A$ is a constant, and $n$ is an integer. The substitution of Eq. (13) in (9) shows that the quantities $\Delta \gamma = k - \gamma$ and $q$ are related by the equation

$$\Delta \gamma = -2k \cos qd, \quad (14)$$

which characterizes the anisotropy of the "plane" waves (13) propagating in the infinite coupled system. Since $\Delta \gamma$ is a periodic function of $qd$ in Eq. (14), it is sufficient to consider values of $qd$ in the interval $|qd| \leq \pi$. As an illustration, we formulate one of the particular solutions of the given problem, namely the solution corresponding to the entry of a signal into the investigated infinite system through one of the waveguides. The field in the $n$-th waveguide in this case can be represented by a superposition of "plane" waves (13), which are normal modes of the unbounded coupled system:

$$w_n = \int B(qd) \exp(iqd - i2\kappa \cos qd + i\gamma) d(qd), \quad (15)$$

where $B(qd)$ is the spectral density, which depends on the boundary conditions. If we assume that the signal enters the waveguide numbered $n = 0$ and if we assume without loss of generality, that at $z = 0$

$$w_0 = 1, \quad w_n = 0, \quad (16)$$

it then follows from Eq. (15) in accordance with the stated boundary conditions (16) that $B(qd) = 1/2$ and, according to the well-known properties of the Bessel integral, the fields in the system acquire the form

$$w_n = (-i)^nJ_n(2\kappa z) \exp(i\gamma z). \quad (17)$$

where $J_n$ is the Bessel function of order $n$. Equation (17) coincides with the familiar result of Ref. 15, which was obtained by an alternative method and was confirmed experimentally, attesting to the validity of the proposed approach.

We now undertake the description of real topographic structures in the form of bounded systems consisting of a large number of coupled waveguides. We first consider a cyclic system (Fig. 3b). In this case the cyclic boundary conditions of the Born-von Karman type must be satisfied in any cross section of the structure; according to Eq. (12), we take the form $\exp(iqd)z = 1$, from which it follows that

$$qd = 2\pi m/N, \quad (18)$$

where $m$ is an integer. The quantization conditions (18) in conjunction with Eq. (14) and with allowance for the inequality $|qd| \leq \pi$ determine the values of the mismatch $\Delta \gamma$, i.e., the normal-mode velocities of the investigated structure. We emphasize that the values obtained for $\Delta \gamma$ are exact and hold for any number $N$ of waveguide channels in the system. This fact is easily verified by comparing, e.g., the values of $\Delta \gamma$ deduced from Eqs. (18) and (14) for cyclic structures for $N = 2$ and $N = 3$ with the corresponding values of $\gamma$ calculated above directly from the solution of the system (9).

The situation is somewhat more complicated in the case of an open system of $N$ waveguide channels (Fig. 3a). Here we can use so-called periodic boundary conditions of the Born-von Karman type, which require coincidence or anticoincidence of the fields in the two outermost waveguides when allowance is made for the fact that the investigated structure always has symmetrical and antisymmetrical solutions. These conditions have the form $\exp[iqd(N - 1)] = \pm 1$, so that

$$qd = m\pi/(N - 1), \quad (19)$$

where $m$ is an integer. In contrast with condition (18), however, Eq. (19) is approximate due to the approximation of using the stated boundary conditions for the given structure. The situation here is exactly the same as in the case where conditions of the Born-von Karman type are used for the vibrations of a bounded crystal lattice. As in the lattice problem, the accuracy of the solutions obtained for $\Delta \gamma$ by means of Eqs. (19) and (14) increases with the number $N$. The error of determination of $\Delta \gamma$ is already less than or equal to 20% for $N = 6$.

We conclude with a discussion of an interesting consequence of Eqs. (18) and (19). It follows from Eq. (18), which refers to cyclic structures, that
when \( N \) is a multiple of four, the coupled-waveguide system has modes for which \( \omega d = \pi/2 \) and, hence, \( \Delta \gamma_1, 2 = 0, \Delta \gamma_3, 4 = \pm \Delta k \). With a certain amount of effort, this result can also be verified directly by solving Eqs. (9). In the case of open structures, which are characterized by Eq. (19), that \( \Delta \gamma \) always has zero values for odd \( N \) beginning with \( N = 3 \). Despite the imprecision of Eq. (19) for small \( N \), it can be proved by means of Eqs. (9) that the stated conclusion is in fact true for all the indicated values of \( N \).

We have thus shown that elements of the theory of waves on curvilinear surfaces and the theory of coupled waveguides can be used to analyze the high-frequency behavior of a set of complex topographic waveguide structures. Of course, the indicated specific features of the propagation of surface waves in such structures and the selected examples themselves do not exhaust all the interesting situations amenable to the given analysis, the number of which can be expanded considerably. For example, if the coupling coefficients in Eqs. (9) are assumed to be weakly varying quantities \( \kappa = \kappa(z) \), it is a simple matter to analyze the problems of wave reflection and transmission in individual waveguide channels from the vertices of pyramid-type structures with rounded edges and vertex (Figs. 4a and 4b). The wave processes in such structures should obviously be similar to the coupled-mode excitation and propagation processes accompanying the reflection of a plane wave from the edge of an acute-angle wedge (Ref. 19). Consequently, oscillating dependences of the wave reflection and transmission coefficients in the individual channels on the angles characterizing the rate of the change of the coupling coefficient (the angles \( \varphi \) and \( \theta \) in Figs. 4a and 4b) should be expected by analogy with the wedge case.

The detailed analysis of such structures, like other cases of interest for particular applications, is beyond the scope of the present article.


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