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EFFECTS OF BEAM-STIFFENING ON THE SOUND POWER RADIATED BY FINITE PLATES

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1 INTRODUCTION

Beam-stiffening is a common technique used as an effort to minimise the acoustic radiation of plates, by modifying its structural properties. Although beam-stiffening has been widely used in many applications for sound and vibration control, its effects on sound radiation has not been fully studied theoretically. Hence, the use of beam-stiffening might lead to undesired acoustic results.

One of the first and most important publications regarding beam-stiffened plates is that by Maidanik\(^1\). By studying the acceleration spectrum, and its relation to the sound power, he showed that the discontinuities of the plate at the beam's location will, in general, increase the coupling between the structure and the surrounding acoustic medium. Maidanik also derived approximate expressions for the radiation efficiency of the beam-plate system as a function of frequency, by employing a statistical method, which led to the development of a then new technique called Statistical Energy Analysis.

The interaction of flexural waves, in an infinite panel, with a beam discontinuity was studied by Ungar\(^2\). Expressions for the transmission, reflection and near-field effects are presented and are related to trace matching between flexural waves in the plate with flexural and torsional waves in the beam. Lyon\(^3\) used these expressions, to investigate the radiation resistance of the infinite beam-plate system, for different configurations of the beam.

Although the study of infinite beam-plate systems, as presented in \(^2,3\), reveals the effects of an attached beam to the vibration and acoustic radiation of a plate, it does not take into account the modal behaviour of the beam. This is considered by Heck\(^4\), where a semi-infinite plate with attached beams along the finite dimension is studied. Expressions for the transmission and reflection coefficients are presented, in order to discuss the wave propagation along the complex system. The near-field effects are not taken into account as they are local and do not propagate away from the discontinuity. However, these effects are important in the acoustic radiation of structures with discontinuities.

Lin and Hayek\(^5\) give an analytical solution for the acoustic radiation from an infinite plate reinforced by a beam, when the structure is exited by a point force on the beam. Approximate expressions for the sound power are given for frequencies above and below the critical frequency. However, the point force excitation is not always a good approximation. For example, automobile floor panels are excited in much more complicated ways. Hence, a more general approach is required.

More recently, some researchers\(^7,8,9\) are focused on the numerical optimisation of the acoustic design of a structure, by taking advantage of the increasing capabilities of microprocessors. The proposed methods use optimisation algorithms, such as genetic algorithms, and numerical methods such as finite element methods and/or boundary elements method, to optimise the position and properties of a beam attached on a plate. Although, the acoustic optimisation techniques are very powerful for dealing with problems where the parameters (such as, source excitation, geometry of the structure, etc) are specific and they are known a priori, they are unable to shed light on the physical phenomena of sound radiation by beam-stiffened plates and hence to give general guidelines for designing such structures.

In this paper, the coupling of an acoustic medium and the vibration field of a semi-infinite plate with an attached finite beam is studied theoretically. The vibration field of the semi-infinite plate is...
assumed to consist of a propagating flexural wave in the infinite dimension and a specific mode-shape in the finite dimension. For this, the radiation efficiency of the structure, for a given mode number, is considered.

2 WAVE PROPAGATION ALONG THE BEAM-PLATE SYSTEM

We consider a thin plate of flexural rigidity $D$ and mass per unit area $m_p$. The transverse displacement of the plate is given by the classical equation:

$$\left(\nabla^4 - k_p^4\right)v(x, y) = \frac{F_p(x, y)}{D}$$

(1)

with $k_p = \left(\frac{m_p\omega^2}{D}\right)^{\frac{1}{4}}$ being the flexural wavenumber of the plate.

Similarly the transverse displacement of a classical beam of flexural rigidity $B$ and mass per unit length $m_b$ is given by:

$$\left(\frac{d^4}{dy^4} - k_b^4\right)\eta(y) = \frac{F_b(y)}{B}$$

(2)

with $k_b = \left(\frac{m_b\omega^2}{B}\right)^{\frac{1}{4}}$. The torsional vibration is governed by the equation:

$$\left(Jk_t^4 - \frac{d^4}{dy^4}\right)\theta = \frac{F_t(y)}{GK}$$

(3)

where $k_t = \left(\frac{\omega^2}{GK}\right)^{\frac{1}{4}}$, $GK$ is the beam torsional rigidity and $J$ is the polar mass of inertia.

In the case of beam and plate coupling, the force terms in Eqs. (1)-(3), contain the coupling forces and moments at the beam-plate interface. Hence, the two equation for the flexural and torsional vibration of the beam can be expressed in terms of plate displacement.

Assume that the beam is attached to the plate at $x = 0$ and that the plate is infinite in the x-direction and finite in the y-direction with length $y_L$. The beam is attached along the length $y_L$ (see Figure 1). The two boundaries of the semi-infinite plate, and those of the beam, are taken to be simply-supported. The displacement of the plate in the x-direction can be assumed to consist of a flexural wave travelling toward the beam in the $x_+$ direction, a reflected and transmitted wave and the near field associated with these. The displacement of the plate in the y-direction can be written as the summation of all its natural modes. In complex numbers notation this is:
Figure 1. The beam-plate system

\[
\begin{align*}
\sum_{n} a_n \sin \frac{n \pi y}{L_y} \left[ \exp(-ik_n x) + R_n \exp(i k_n x) + R'_n \exp(-k'_n x) \right] & \quad x > 0 \\
\sum_{n} a_n \sin \frac{n \pi y}{L_y} \left[ (T_n - 1) \exp(-ik_n x) + T'_n \exp(k'_n x) \right] & \quad x < 0
\end{align*}
\]

(4)

with

\[
k_n^2 = k_p^2 - \left( \frac{n \pi}{L_y} \right)^2 \quad \text{and} \quad k'_n = k_p^2 + \left( \frac{n \pi}{L_y} \right)^2
\]

The coefficients \( R_n \) and \( T_n \) are the complex reflection and transmission coefficients respectively and \( R'_n \) and \( T'_n \) are the corresponding near-field coefficients. Substituting Eq. (4) to the coupled flexural and torsional vibration equations of the beam with the plate, yields four simultaneous equations, two for each region of the plate. From the system of equations, one is able to find expressions for the four coefficients \( R_n, R'_n, T_n \) and \( T'_n \):

\[
\begin{align*}
R_n &= \frac{i \gamma \left( \frac{1}{i2Dk_n} - \frac{1}{\alpha} + \frac{k_n^2}{\beta} \right)}{k_n - ik'_n} \\
T_n &= \frac{i \gamma \left( \frac{1}{\alpha} + \frac{k_n^2}{\beta} \right)}{k_n - ik'_n} \\
R'_n &= \frac{\gamma \left( -1 \right) \left( \frac{1}{D(k_n + i k')} + \frac{i}{\alpha} - \frac{k_n k'_n}{\beta} \right)}{k_n - ik'_n} \\
T'_n &= \frac{\gamma \left( -1 \right) \left( \frac{i}{\alpha} - \frac{k_n k'_n}{\beta} \right)}{k_n - ik'_n}
\end{align*}
\]

with

\[
\begin{align*}
\alpha &= GK(n \pi)^2 + L_y \left( 2D(k_n - k'_n) + J \omega^2 \right) \\
\beta &= EI(n \pi)^4 + L_y^4 \left( 2D k_n k'_n (k_n + i k'_n) - m_p \omega^2 \right)
\end{align*}
\]

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Figure 2. Absolute squared value of transmission coefficient with respect to frequency normalised by the natural frequency of the beam

\[ \gamma = 2Dk_n (k_n + ik'_n) \]

Figure 2 shows the absolute squared value for the transmission coefficient with respect to normalised frequency. It can be seen that the transmission coefficients takes a value close to 1 at a frequency close to the natural frequency of the beam. This results are in agreement with those derived by Heckl. At this frequency, almost all of the energy of the incident wave travelling from the \( x_+ \) direction, is transmitted across the beam, to the other side of the plate. The absolute squared value of the reflection coefficient is the reciprocal of that of \( |T_n|^2 \) since, from an energy consideration, \( |R_n|^2 + |T_n|^2 = 1 \). Hence, at this frequency, there is no reflected wave, whereas, at any other frequency most of the energy of the incident wave is reflected back.

### 3 ACOUSTIC RADIATION

The acoustic power radiated by the beam plate system can be given by:

\[
P = \frac{\rho c k}{8\pi^2} \int \int |u_n(k_x, k_y)|^2 \, dk_x dk_y
\]
where, $\rho$ is the volume density of the acoustic medium, $c$ is the speed of sound, $k$ is the acoustic wavenumber and $u(k_x, k_y)$ is the velocity power spectrum of the plate, given by the double Fourier transform of the velocity distribution of the plate:

$$u_n(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} u_n(x, y) \exp(-ik_x x) \exp(-ik_y y) dx dy$$

(6)

Using Eq. (4), the velocity spectrum for the beam plate system is given by:

$$u_n(k_x, k_y) = \omega^2 a_n \left( \frac{T_n}{k_x + k_n} + \frac{R_n}{k_x - k_n} + \frac{R_n'}{k_x - ik_n'} - \frac{T_n'}{k_x + ik_n'} \right)$$

$$\times \left( \frac{2m/L_y}{k_y^2 - (\pi n/L_y)^2} \right) \sin^2 \left( \frac{k_y L_y - n\pi}{2} \right)$$

(7)

A useful index in quantifying the coupling between structural vibration and the sound radiation of the system, is the radiation efficiency, given by:

$$\sigma = \frac{P}{\rho c S \left< \bar{u}^2 \right>}$$

(8)

where $S$ is the area of the structure and $\left< \bar{u}^2 \right>$ is the space-average of the time-average squared velocity of the plate, which for harmonic waves is given by:

$$\left< \bar{u}^2 \right> = \frac{1}{S} \int_0^T \int_S u_n^2(x, y) dt dS$$

$$= \frac{1}{2S} \int_S [u_n(x, y)]^2 dS = \frac{\omega^2 L_y}{4S} |a_n|^2 |R_n - T_n|^2 = |a_n|^2 \frac{\omega^2 L_y}{4S}$$

(9)

By substituting Eqs. (5) and (9) into (8), the radiation efficiency of the beam-plate system is given by:

$$\sigma = \frac{k \int \int_{-k-k} u_n^2(k_x, k_y) dk_x dk_y}{4\pi^2 L_y}$$

(10)

The radiation efficiency of a vibrating plate, with or without discontinuities, is approaching 1 for frequencies above the critical frequency (the frequency where the structural wavenumber coincides with the acoustic wavenumber)\(^1\). Hence one is interested in the radiation efficiency below this frequency.
frequency \( k_p > k \). In that region the approximation \( k_x << k_n \) and \( k_y << n/L_y \) can be used and the radiation efficiency can be written as:

\[
\sigma_n = \frac{k^2 L_y}{\pi^2 n^2 k_p^4 \left( \frac{n \pi}{L_y} \right)^2} \left( k_p^2 + \left( \frac{n \pi}{L_y} \right)^2 \right)^2 \left( 8D^2 k_p^2 k_n^2 k_p^2 k_y^8 - 4D \epsilon k_p^2 \delta L_y^4 + \left( 13k_p^2 + 5\left( \frac{n \pi}{L_y} \right)^2 \right) \delta^2 \right)
\]

\[(11)\]

where

\[
\epsilon = k_p^4 \left( \frac{n \pi}{L_y} \right)^4
\]

\[
\delta = L_y^4 m_b \left( \omega_{b,n}^2 + \omega^2 \right)
\]

\( \omega_{b,n} \) is the \( n \)th resonant frequency of the beam.

One can observe that the radiation efficiency of the beam-plate system at low frequencies is independent of the torsional vibration of the beam, as it does not contain the beam torsional stiffness, \( Gk \), and polar mass of inertia, \( J \). Note that Eq. (11) consists of two terms. The first term is the radiation efficiency of the semi-infinite plate without the beam. The second term is the influence of the beam. This term, for most practical situations is above 1, which indicates that the beam discontinuity will increase the radiation efficiency, as it has been also shown in previous publications.

### 4 DISCUSSION AND CONCLUSIONS

The radiation efficiency of a beam plate system, including the beam’s modal behaviour has been studied theoretically. The expression for the radiation efficiency of the \( n \)th mode, Eq. (11), consists of two terms, the second being the beam influence factor. It can be seen that when the mass of the beam tends to 0 (\( \delta \rightarrow 0 \)), the beam influence factor tends to 1, which means that the influence of the beam on the acoustic radiation of the beam-plate system tends to vanish (there is no discontinuity at the location of the beam). On the other hand, when the mass of the beam tends to infinity, the beam forms a clamped edge boundary condition.

Figure 3 shows the beam influence coefficient with respect to frequency for a 1mm steel plate with a steel beam of length 300mm. It can be seen that the beam influence factor is increased as the height of the beam is increased. For the case of 2mm beam the factor is very close to one, whereas for heavier beams it takes higher values. It can be also seen that there is an anti-peak, which occurs at slightly different frequencies for the three different beams. It has been observed that this anti-peak coincides with the anti-peak of the near-field coefficients \( R_n', T_n' \). This is because the near-field evanescent wave contributes to the acoustic radiation in the far-field even below the coincidence frequency and hence it increases the coupling between the structure and the acoustic medium.
Eq. (11), provides a relatively simple expression for the change in radiation efficiency caused by the attachment of a beam on a plate. In the case of a finite plate in all dimensions, one expects to have two more factors for the two boundaries that would make the semi-infinite plate studied above to become finite. The influence of the beam, which is another boundary condition, will be given again by the second term of Eq. (11). Hence, one can predict the change in the radiation efficiency of a finite plate, which caused by the attachment of a beam.

The acoustic power radiated is proportional to the product of the radiation efficiency and the mean-squared velocity of the beam-stiffened plate (Eq. (8)). Hence, the beam influence factor indicates the minimum reduction in the mean-squared velocity of the plate that should be caused by the beam, in order for the acoustic power to be reduced.

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6 REFERENCES