Optimised sensor configurations with reduced order controllers applied to an EMS system

This item was submitted to Loughborough University's Institutional Repository by the/an author.

Citation: KONSTANTINOS, M. ... et al., 2010. Optimised sensor configurations with reduced order controllers applied to an EMS system. Proceedings of the 29th Chinese Control Conference, Beijing, China, 29-31 July.

Additional Information:

- This is a conference paper. Details of the conference can be found at: http://ccc10.bit.edu.cn/shouye_en.html

Metadata Record: https://dspace.lboro.ac.uk/2134/6666

Publisher: © CAA/TCCT (Technical Committee on Control Theory, Chinese Association of Automation)

Please cite the published version.
This item was submitted to Loughborough’s Institutional Repository (https://dspace.lboro.ac.uk/) by the author and is made available under the following Creative Commons Licence conditions.

For the full text of this licence, please go to:
http://creativecommons.org/licenses/by-nc-nd/2.5/
Optimised Sensor Configurations with Reduced Order Controllers Applied to an EMS System

Konstantinos MICHAiL1, Yimin ZHOU1, Argyrios ZOLOTAS1, Roger GOODALL1, George HALIKIAS2
1. Control Systems Group, Department of Electronic and Electrical Engineering,
Loughborough University, Loughborough, LE11 3TU, UK
E-mail: kon.michael@ieee.org, [y.zhou2,a.c.zolotas,r.m.goodall]@lboro.ac.uk
2. Control Engineering Research Center, School of Engineering and Mathematical Sciences,
City University, London, EC1V 0HB, UK
E-mail: g.halikias@city.ac.uk

Abstract: In many applications, sensor selection in an optimum sense is not a trivial task, especially when a number of objectives and constraints are to be satisfied simultaneously. Moreover, for practical implementation the order of the synthesized controller could be a burden of increased cost and complexity. In this paper, a systematic framework for optimum sensor selection is considered which considers issues of controller reduction. Efficacy of the proposed framework is illustrated via simulations on an Electro-Magnetic Suspension (EMS) system.

Key Words: Sensor Optimisation, Controller Order Reduction, EMS Systems, $H_{\infty}$ Robust Control

1 INTRODUCTION

Reduced order controller structures in practical implementation of engineering system solutions is an important issue due to reasons of complexity, cost and fault tolerance considerations. In this context, a number of different approaches have been developed throughout the years i.e. reduction of order of the original model, reduction of the controller size after designing on the original model, controller reduction in a closed-loop sense, weighted and unweighted reduction etc. In fact details on the exact methodologies can be found in [12] and references within the book.

Furthermore, the selection of sensors in an optimum sense for control and in complex (usually safety critical) systems can be a cumbersome task, especially in the case of large number of sensor choices. Adding choice of controller structure, raises the difficulty in the design process.

This research work concentrates on optimum sensor selection subject to given closed-loop performance requirements and controller order. In particular, for a practical engineering system the above may be cumbersome to follow manually, due to issues of strict objectives to satisfy, large number of sensor set candidates etc. A recently proposed systematic framework for optimum sensor selection appears in [10], albeit with no consideration of controller order reduction. In this context, extension of that work by considering the effects of controller reduction in the framework is taken into account in this paper and present some preliminary design albeit useful results.

The particular application in this paper is that of an EMS system. MAGLEV technology has been successfully applied in transport industry in recent years due to the advantages over conventional wheel-on-rail systems [9]. In particular, the EMS (Electro-Magnetic Suspension) system is non-linear, unstable system with non-trivial requirements [7]. From the sensors point of view for a single degree of freedom model, 5 possible measurements are considered and from those a number of candidate sets arise subject to closed-loop requirements. The controller design is based on $H_{\infty}$ robust control [14], with weight tuning based on GAs (extensively applied in control systems [13], in particular the recently developed NSGA-II (Non-dominated Sorting Generic Algorithm II) algorithm [3]. The case of the controller order reduction is considered in the objective functions of the design framework. It is worth noting that in the case of maglev vehicles, the controllers are duplicated for each side of the suspensions thus the least possible order could be desired (subject to achieving the required performance).

The paper organisation is as follows: Section 2 describes modeling issues of the EMS system emphasising the linearisation approach for the controller design. Section 3 refers to the track input characteristics that excites the EMS system, while describes the performance requirements for the designed system together with objective functions. The design framework is presented in Section 4, while Section 5 includes the simulation results and related remarks. Conclusion and future work are given in Section 6.

2 EMS SYSTEM MODEL

The diagram of a one-degree of freedom electromagnetic suspension system is shown in Fig. 1. The suspension consists of an electromagnet with a ferromagnetic core and a coil. The coil is attracted to the rail that is made out of ferromagnetic material. The carriage mass is attached to the electromagnet. $z_r$ is the rail position and $z$ is the carriage position. The controlled air gap $(z_r - z)$ can provide an appropriate suspension performance (see Section 3.3).

The positive direction is assumed downwards and the equation of motion derived from Newton’s motion law is

$$M \frac{d^2z}{dt^2} = Mg - F$$

where $M$ is the mass of the carriage, $g$ is the gravity acceleration constant taken as $9.81 m/s^2$ and $F$ is the vertical force produced by the electromagnet to keep the carriage at the operating position. The electrical circuit of the electromagnet
where \( V_c \) is the input voltage; \( R \) is the coil’s resistance; \( L \) is the leakage inductance; \( A \) is the pole face area; \( I \) is the coil current and \( B \) is the flux density. As indicated in [8], the four important variables in the electromagnetic suspension are force \( F \), flux density \( B \), air gap \( G \) and the coil current \( I \). The relationship between these variables is shown in Fig. 2. With a constant air gap, the flux density is proportional to the coil current and at constant current is inversely proportional to the air gap. The force is proportional to the square of the flux density. The MAGLEV suspension is non-linear but the system could be linearized around the operating point thus linear controllers could be used for the control purpose. To derive the LTI state space model, linearisation is performed around the operating point (nominal values) of the coil current \( I_o \), flux \( B_o \), force \( F_o \), nominal input voltage \( V_o \) and air gap \( G_o \). Linearisation is done by considering small perturbations around the operating point [10]. Therefore, the model of the system can be written in state space form as follows

\[
\dot{x} = A_m x + B_m u + B_z \delta_t
\]

where the state vector is \([i \; z \; (z_t - z)]^T\); \( i \) is the coil current, \( z \) is the vertical velocity and \((z_t - z)\) is the air gap. Note that small case letters refer to small perturbations around the operating point. The state space matrices are given by formulas (4), (5). The possible output measurements are the current \( i \), flux density \( b \), air gap \((z_t - z)\), velocity \( z \) and acceleration \( \dot{z} \) as shown in formula (6). Different sensor combinations can be selected by defined sets. All feasible sensor sets are then given by \( N_c = 2^{N_s} - 1 \), where \( N_c \) is the total number of sensors sets and \( N_s \) is the total number of sensors (i.e., for 5 sensors \( N_c = 31 \) sensor sets). The parameter values are shown on Table 1.

![Fig. 1 EMS system for MAGLEV](image)

![Fig. 2 Relationship between the key variables describing the magnet. The straight lines show the theoretical relationships and the broken lines indicate the effects of magnetic saturation in the magnet core](image)

\[
V_c = RI + L \frac{di}{dt} + NA \frac{dB}{dt} \tag{2}
\]

\[
A_m = \begin{pmatrix}
-\frac{B}{M} & -\frac{K_{14} N_A}{M} & 0 \\
\frac{K_{14} N_A}{M} & -1 & -\frac{K_{14} K_{(24-1)}}{M} \\
0 & 0 & 1
\end{pmatrix}
\tag{4}
\]

\[
B_{u_c} = \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\tag{5}
\]

\[
C_m = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{pmatrix}
\tag{6}
\]

3 DESIGN REQUIREMENTS FOR THE EMS

3.1 Stochastic Inputs

The stochastic inputs are random variations of the rail position as the vehicle moves along the track. This is caused by the steel rail installation discrepancies due to track-laying inaccuracies and unevenness. Considering the vertical direction, the velocity variations can be approximated by a double-sided power spectrum density (PSD) expressed as

\[
S_{\dot{z}_t} = \pi A_r V_o
\]

where \( V_o \) is the vehicle speed (taken as \( 15 m/s \) in this case) and \( A_r \) represents the roughness and it is assigned a value as \( 1 \times 10^{-7} \) for high quality track. Then the corresponding autocorrelation function is given as:

\[
R(\tau) = 2\pi^2 A_r V_o \delta(\tau)
\]

Regardless a linear controller is used, the simulations are actually based on the implementation to the nonlinear model. Hence, we calculate the Root Mean Squared (RMS) values of the required quantities (acceleration, current etc) using time history data. Details of implementation of linear controller onto a nonlinear model can be found in [5].

3.2 Deterministic Input

The main deterministic input to the suspension in the vertical direction is due to the transition onto a gradient. In this work, the deterministic input (see Fig. 3) is a gradient of 5% at a vehicle speed of \( 15 m/s \), an acceleration of \( 0.5 m/s^2 \) and a jerk of \( 1 m/s^3 \).
follows: the variables involved in the objectives are summarized as results from noisy measurements are also minimised. Thus, the amplitude of the noise on the control effort \( \gamma \) The robustness margin \( \gamma \) is set as objectives to minimise the vertical acceleration \( \dot{z} \). The fusion of controller reduction. Details of the original frame- work can be found in [10]. Referring to Fig. 5 note the task of tuning the \( H_\infty \)-controller filter \( \begin{pmatrix} W_p \\ W_{p\gamma} \end{pmatrix} \) (performance, control) such that the optimum Pareto front controllers \( K(s) \) is found. Each controller should satisfy all of the constraints showed in section 3.3 for each sensor set \( (y) \) with the minimum possible order. Note that the sensor sets are selected using the output matrix \( (C_y) \), as shown in (10). There are totally 5 available measurements, \( (i, b, (z_t - z), \dot{z}, \ddot{z}) \), which results to 31 sensor sets i.e. \( i, b, i\dot{z}, b\ddot{z}..., etc. \) The genetic algorithms are merged within the framework for filter optimal tuning of each sensor set. The MAGLEV state equation in

\[
\begin{align*}
\mathbf{\dot{x}} &= A\mathbf{x} + B_2\mathbf{w} + B_3\mathbf{u} \\
\mathbf{z}_\infty &= C_{\infty}\mathbf{x} + D_{\infty1}\mathbf{w} + D_{\infty2}\mathbf{u} \\
y &= C_y\mathbf{x} + D_{y1}\mathbf{w} + D_{y2}\mathbf{u}
\end{align*}
\]

formula (3) is imposed into the generalised form:

\[
\begin{align*}
\dot{x} &= Ax + B_2w + B_3u \\
\mathbf{z}_\infty &= C_{\infty}\mathbf{x} + D_{\infty1}\mathbf{w} + D_{\infty2}\mathbf{u} \\
y &= C_y\mathbf{x} + D_{y1}\mathbf{w} + D_{y2}\mathbf{u}
\end{align*}
\]

where \( w \) are exogenous inputs (deterministic and stochastic as described in Section 3.3), \( u \) is the controller output, \( \mathbf{z}_\infty \) is the regulated output, i.e., \( u_c \) is the control effort, \( (z_t - z) \) is the air gap and \( y \) is the corresponding sensor set. The infinity norm of the closed-loop transfer function from the

\[
\begin{align*}
\phi_1 &= i_{rms}, \phi_2 = \gamma, \phi_3 = \ddot{z}_{rms}, \phi_4 = u_{n_{rms}}, \phi_5 = n_K
\end{align*}
\]

\[
\begin{align*}
\phi_1 &= i_{rms}, \phi_2 = \gamma, \phi_3 = \ddot{z}_{rms}, \phi_4 = u_{n_{rms}}, \phi_5 = n_K
\end{align*}
\]

4 THE SENSOR OPTIMISATION FRAMEWORK

The proposed framework can be summarised in the flowchart of Fig. 4. The particular points include the use of \( H_\infty \) robust control, the NSGA-II method for tuning subject to the strict objective functions for performance and the infusion of controller reduction. Details of the original frame-
controller should satisfy strict performance constraints. In this context and subject to following controller reduction (in an open loop sense for simplicity) the balance normalised coprime factorisation reduction is utilised \cite{13,15}. Computationally, the aforementioned process can be obtained via Matlab command ‘\texttt{K'_r = newrfmr(K, n_K_r)\texttt{’}}’ where \( n_{K_r} \) is the desired order of the reduced order controller \( K_r \). Moreover, the variable of controller order in the objective function \( n_{K_r} \) varies from 1 (full reduction) to order 5 (no reduction), while the framework seeks the best controller satisfying the given requirements as described in Section 3.3.

4.1 Multi-objective Optimisation With NSGA-II

NSGA-II is an evolutionary process that requires some parameters to be assigned in order to make ensure proper population convergence towards optimum Pareto front. These are mainly selected from experience rather than from a-priori knowledge of the optimisation problem. The basic NSGA-II parameters used are listed in Table 3. Details on NSGA-II can be found in \cite{2,3} for the interested reader. The crossover probability is generally selected to be large in order to have a good mix of genetic material. The mutation probability is defined as \( 1/n_u \), where \( n_u \) is the number of variables. This is appropriate in order to give a mutation probability that mutates an average of one parameter from each chromosome. The population to be used consists of 70 chromosomes and the stopping criterion is the maximum generation number set at 120. Constraint handling in genetic algorithms can be done differently \cite{1}. The penalty function approach \cite{2} is used to achieve the constraint within limits. The constraints are separated into Soft and Hard constraints. The constraint violation for each soft constraint, \( k^j \), defined in Tab. 2, is given as,

\[
\omega_j(k^j) = \max \left\{ \frac{|g_j(k^j)|}{\alpha_j}, \begin{cases} 1 & \text{if } g_j(k^j) < 0 \\ 0 & \text{otherwise} \end{cases} \right. \quad (13)
\]

Each soft constraint is normalised based on (14) for less than the predefined level.

\[
g_j = \frac{k^j}{k^j}\text{\textsubscript{des}} + 1 \geq 0 \quad (14)
\]

Where \( k^j\text{\textsubscript{des}} \) is the predefined constraint value and \( k^j \) is the measured value. The hard constraint violation is given as,

\[
\psi_k(f^j) = \max \left\{ \frac{h_k(f^j)}{\beta_k}, \begin{cases} 0 & \text{if } h_k(f^j) = 0 \\ \beta_k & \text{otherwise} \end{cases} \right. \quad (15)
\]

This is transformed into a soft constraint, allowing a small tolerance value \( \epsilon \) as follows

\[
h_k = |f^j| - \epsilon < 0 \quad (16)
\]

Additionally, the overall constraint violation formed as given in (17). Notice that the overall constraint violation

\[
\| T_zw \|_{\infty} < \gamma \quad (11)
\]

For each selected sensor set, equation (11) is solved for each random pair of weighting functions that is produced by the genetic algorithm using Linear Matrix Inequalities. This can be easily done in MATLAB environment using function ‘\texttt{hinfnorm}\texttt{’} in the robust control toolbox. The weighting filters \( W_p \) and \( W_{uc} \) are appropriate low pass filter and high pass filter (see formula (12)) to adjust the performance of the controller by varying their parameters. There is no generic procedure to select weighting functions because it varies according to the application. However, some guidelines on selecting the weights for \( H_\infty \) design of a plant are suggested in \cite{14}.

\[
W_p = \left( \frac{s + \omega_p A_p^{1/n_p}}{s + \omega_p A_p^{1/n_p}} \right)^{n_p} \quad \text{and} \quad W_{uc} = \left( \frac{\tau s + A_u^{1/n_u}}{\tau s + A_u^{1/n_u}} \right)^{n_u} \quad (12)
\]

In the performance weighting \( (W_p) \), \( M_p \) is the high frequency gain, \( A_p \) the low frequency gain and \( \omega_p \) the crossover frequency. For the control effort weight \( (W_{uc}) \), \( \tau \) determines the crossover frequency, \( A_u \) the low frequency gain and \( M_u \) is the high frequency gain. Both \( n_p \) and \( n_u \) control the roll-off rates of the filters, equal to 1 in this case i.e. first order filters. The structure of the filters is shown in Fig. 6 which is typical in such an \( H_\infty \)-control design. The controller output is fixed, as this is only the applied voltage to the MAGLEV system. The controller inputs, however, vary based upon the utilised sensors, i.e., SISO controller for 1 sensor; MISO controllers for more sensor combinations. Moreover, the order of the controller is fixed to the order of the plant plus the order of the chosen filters i.e. \( 3 + 2 = 5^{th} \) order controller. In practical applications, high order controllers are not favorite since they increase the cost and complication level of the system. In such case the controller order reduction is considered within the framework. In particular, a reduced order controller \( K_r \) is aimed that is able to perform close to its original larger order counterpart. Recall that the original controller is designed via \( H_\infty \), mixed approach (based on signal excitation). The structure of the multi-objective generalised plant configuration is shown in Fig. 5. Note that the maglev system is a nonlinear, unstable and uncertain system while the desired

![Fig. 6 Performance weights structure for \( H_\infty \) controller design](image-url)
\( \Omega(k^{(i)}, f^{(i)}) \) is zero if all of the constraints are satisfied. The overall constraint violation is going to be used as a metric for the controllers that either satisfy or not satisfy the constraints described in Section 3.3.

\[
\Omega(k^{(i)}, f^{(i)}) = \sum_{j=1}^{j} \omega_j(k^{(i)}) + \sum_{i=1}^{i} \psi_i(f^{(i)})
\]

(17)

This constraint violation is then added to each of the objective functions values

\[
\Phi_m = \phi_m + R_m \Omega(k^{(i)}, f^{(i)})
\]

where \( R_m \) is the penalty parameter and \( \phi_m \) the objective function value. In this case, a dynamically updated penalty parameter is required, where it is helpful to avoid infeasible solutions. The penalty parameter is set to be a function of the generation number \([1]\), which are finalised as follows

\[
R_{\text{rms}} = \beta \ast 1 \quad R_{\text{gamma}} = \beta \ast 1 \quad R_{\text{damping}} = \beta \ast 0.1
\]

\[
R_{\text{rms}} = \beta \ast 1 \quad R_{\text{controller}} = \beta \ast 1
\]

(19)

where \( \beta \) is the generation number.

5 SIMULATIONS AND DATA ANALYSIS

As it is noted in Fig. 4 the NSGA-II can be merged to the sensor selection framework efficiently, where it produces the optimum Pareto front of optimised controllers for each possible sensor set satisfying the requirements in Section 3.3. Initially, the NSGA-II parameters, controller selection criteria, \( f_c \), and the user’s controller selection criteria, \( f_k \), are given. The last two criteria make sure that the selected controller results in a desired closed-loop performance. At the beginning of the optimisation procedure, the first sensor set is selected and the NSGA-II optimally tunes the performance weights to recover the optimum Pareto front of controllers (which is equal to the number of population, 70 controllers). In the sequence, the controllers satisfying all constraints are selected based on the overall constraint violation function (17). If there is no sufficient controller then the controller which results to minimum (17) is selected and the optimisation proceeds to the next sensor set. Otherwise, those controllers satisfying (17) are selected.

The next step is to select those controllers that satisfy the controller selection criteria \( f_c \). Finally, the user’s controller selection criteria, \( f_k \) is used to select the controller which results in the desired closed-loop response. The optimally tuned with the minimum possible order controller is saved and the algorithm moves to the next sensor set. Under the controller selection criteria, the selected controllers can guarantee the closed-loop response is less than 0.5\( \text{m/s}^2 \). The finally selected controller will have the minimum possible order due to the user’s selection criteria. The criteria can be formally written as

\[
f_c \equiv \tilde{\alpha}_{\text{rms}} \leq 0.5 \text{m/s}^2, \quad f_k \equiv \min[n_{K_r}]
\]

(20)

The overall algorithm is tested in MATLAB 7.2 simulation environment without Java function due to large computational need (simulation based). The average simulation time per sensor set was about 6 hours and the procedure for all possible sensor sets takes around 180 hours.

From the simulation results it was found that the proposed systematic framework is able to identify controllers that satisfy the constraints for 23 out of 31 sensor sets. Tab.4 presents the number of sensor sets with the corresponding order of controllers. It can be seen that one sensor set is able to successfully supply information for a third order controller and 10 sensor sets were found to satisfy 4th order controllers. Similarly, 20 sensor sets can work with 5th order controllers (i.e. the original order). Tab.5 shows 10 selected sensor sets from the total of 31. The first column identifies each sensor set. The second column is the corresponding sensor sets under test for deterministic and stochastic disturbances. The closed-loop responses are shown in the next 4 columns for stochastic and the following 4 for deterministic. Remaining columns present the performance margin \( \gamma \), the level of noise on the driving signal \( u_{\text{rms}} \), the order of the corresponding controller \( n_{K_r} \), the overall constraint violation function \( \Omega \) and whether the controller is Stable (S) or Unstable (U). As it has been mentioned \( \Omega \) indicates whether the closed-loop response with a sensor set satisfy the constraints or not. Two sensor sets, id:1 and id:7, have constraint violations on the steady state error \( \epsilon_{\text{sas}} \) and the vertical acceleration \( \tilde{\alpha}_{\text{rms}} \). It is clear that these cannot be used for control of the suspension regardless the association of smaller order controller. With regards to the single measurements (id:2,3) it can be seen that both satisfy the constraints with id:3 having a 4th order controller. Compared with the full sensor set, id:10 is seen that a control state is diminished. Observing id:4, with two sensors, can perform equally well having a third order controller. Figure 7 shows the singular value plot of the original and reduced order controller for set id:4. Clearly the balanced truncation on normalised coprime factors degrades the integral action of the controller, however the algorithm automatically tunes the reduced order controller to accommodate this. At high frequencies the plots have similar characteristics.

Figure 8 illustrates the deterministic input response of the EMS system using sensor set id:4 (3rd order controller). Clearly, the steady state error, settling time and peak values are within the required constraints. The control effort not shown here has similar shape and relies within the constraints as described in Section 3.3. Note that the stochastic input response is also satisfactory as it can be seen from Tab.5.

### Tab. 4 Reduced order controllers recovered

<table>
<thead>
<tr>
<th>n_{K_r}</th>
<th>( \beta^{th} )</th>
<th>4th</th>
<th>5th</th>
</tr>
</thead>
<tbody>
<tr>
<td>No of sensor sets</td>
<td>1</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

6 CONCLUSIONS

Using the proposed sensor optimisation framework it is possible to identify the sensor set that results to the desired performance with minimum order of the controller which satisfies the assigned constraints. Although at its current form the procedure utilises controller reduction in an open-loop sense (for simplicity), the preliminary results are quite promising and work is continuing on assessing robustness and closed-loop controller reduction issues.
Tab. 5 Optimised sensor configurations with the minimum possible order of controllers

<table>
<thead>
<tr>
<th>id</th>
<th>Sensor set</th>
<th>Stochastic input response</th>
<th>Deterministic input response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$g_{rms}$</td>
<td>$u_{rms}$</td>
</tr>
<tr>
<td>1</td>
<td>$i$</td>
<td>1.19</td>
<td>128.7</td>
</tr>
<tr>
<td>2</td>
<td>$b$</td>
<td>1.58</td>
<td>18.5</td>
</tr>
<tr>
<td>3</td>
<td>$\bar{z}$</td>
<td>1.55</td>
<td>22.0</td>
</tr>
<tr>
<td>4</td>
<td>$i$, $(z_t - z)$</td>
<td>2.03</td>
<td>27.33</td>
</tr>
<tr>
<td>5</td>
<td>$i$, $\bar{z}$</td>
<td>1.23</td>
<td>91.69</td>
</tr>
<tr>
<td>6</td>
<td>$i$, $b$, $(z_t - z)$</td>
<td>1.22</td>
<td>107.34</td>
</tr>
<tr>
<td>7</td>
<td>$(z_t - z)$, $\bar{z}$, $\bar{\tilde{z}}$</td>
<td>1.05</td>
<td>164.72</td>
</tr>
<tr>
<td>8</td>
<td>$i$, $b$, $(z_t - z)$</td>
<td>1.22</td>
<td>102.67</td>
</tr>
<tr>
<td>9</td>
<td>$b$, $(z_t - z)$, $\bar{z}$, $\bar{\tilde{z}}$</td>
<td>1.57</td>
<td>21.28</td>
</tr>
<tr>
<td>10</td>
<td>$i$, $b$, $(z_t - z)$, $\bar{z}$, $\bar{\tilde{z}}$</td>
<td>1.71</td>
<td>21.79</td>
</tr>
</tbody>
</table>

$g_p \equiv (z_t - \bar{z})_p, g_{rms} \equiv (z_t - \bar{z})_{rms}$

Fig. 7 Singular values of $K$ and $K_r$ for id:4

Fig. 8 Deterministic input response of the air gap with id:4

REFERENCES


