The thermomechanical properties of 224-carbon steel at high strain rates

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THE THERMOMECHANICAL PROPERTIES OF 224-CARBON STEEL
AT HIGH STRAIN RATES

by

Philip R Dixon

A Thesis submitted in partial fulfilment of the requirements for the award of the degree of Doctor of Philosophy of the Loughborough University of Technology

February 1990

Supervisor: Dr D J Parry

Department of Physics

© by P R Dixon
"Work on, Hope on, Self Help is noble schooling, 
Do your best and leave the rest to God Almighty's ruling"

(Some anonymous words found on a plate belonging to my mother)
Dedicated to the memory of those loved ones who passed away during the course of this project: Nein, Auntie Doris and Grandma

* * * * *
I profoundly and sincerely thank my supervisor, Dr D J Parry for being a steadfast source of encouragement and advice throughout the course of this project.

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CHAPTER 1

INTRODUCTION

1.1. MATERIALS MECHANICS

Ever since prehistoric times, man has been aware of the differing nature of materials and throughout the ages has exploited their mechanical properties in making tools, weapons and other implements to meet a required purpose. Nowadays there is widespread development of new materials and the study of their mechanical properties is of major importance. Scientists involved in this area of research are very often concerned with the relationship between the macroscopic mechanics of the material and its microstructure and how the knowledge of this relationship can be used to design structures able to withstand prescribed strain and stress levels.

A century ago, the strengths of metals were commonly evaluated by applying successive dead weights to the end of thin wires and recording the extension in the wire produced by each addition. In this manner the yield stresses and ultimate tensile strengths of metals could be determined. However, it was soon realised that the observed mechanical strength in such tests depended on the time allowed for the wire to respond to the change in applied load. In other words mechanical strength depends on load rate and thus strain-rate. Hence, during this present century, numerous machines have been developed to test materials at constant and continuous load and strain-rates over a range of strain-rates.

In many real instances such as in transport crashes, earthquakes, missile impact etc, the deformation of matter occurs rapidly i.e. at a high strain-rate. Consequently, it is often necessary in practical
design problems to have a good understanding of the high strain-rate properties of the materials concerned. The design of flasks for the transportation of nuclear waste is a good example where this is true in order to ensure that in the event of an accidental impact none of the contained radioactivity is released.

Over the years 1986 to 1989 in the Physics Department at Loughborough University, the dynamic mechanical behaviour of type 224 carbon manganese steel (BS 1503 grade 430), which is used to fabricate nuclear waste and fuel containers in the UK, has been thoroughly investigated and the study forms the subject of this thesis. The mechanical behaviour of type 224 steel, like all other materials, depends critically on the imposed temperature and strain-rate. In experiments in materials mechanics these two parameters should always be specified.

1.2 TEMPERATURE, T

Temperature is a key parameter in all materials mechanics since it determines the type or types of deformation mechanisms which operate. Many materials have a critical temperature at which there is a ductile-brittle transition in mechanical behaviour. Under certain conditions in metals, thermal energy may assist in the ability of dislocations to overcome obstacles to their motion and for such conditions the observed flow stress is reduced.

Generally, most materials show a negative thermal sensitivity in that increasing the test temperature produces a decrease in the material strength or observed flow stress. There are, however, instances in which the opposite is true such as at temperatures which allow the diffusion of solute atoms in metals.
1.3 STRAIN-RATE, \( \dot{e}_p \)

1.3.1 The Meaning of High Strain-Rate
Strain-rate is usually defined as the strain caused by an applied stress divided by the time taken to produce that strain. It is more difficult, however, to define the classification of high strain-rate and low strain rate since this depends on the nature of the material (Cottrell, 1957). A high rate of strain does not merely mean one which is beyond the scope of conventional Instron or hydraulic testing apparatus but rather implies that the rate at which deformation occurs is comparable to the rate at which molecular processes take place within the material itself. In a substance such as glass for example, no matter how slowly it is bent or stretched to fracture, the rate of strain remains 'high' in the sense that the deformation occurs too quickly to allow plastic flow to occur.

Table 1.1 shows a classification of four different strain-rate regimes which apply generally to the deformation and testing of metals. Within each regime a particular type of deformation mechanism operates. The range of strain-rates covered by the work described in this thesis is \( 10^{-4} \ s^{-1} \) to \( 5 \times 10^{3} \ s^{-1} \). At low strain-rates below about \( 10 \ s^{-1} \), mechanical tests may be termed quasi-static since these tests are slow enough to allow a true equilibrium of applied stresses in the sample throughout the whole test. In tests at strain-rates above about \( 100 \ s^{-1} \) the deformation of sample material is determined by the propagation of stress waves which may be elastic or plastic.

1.3.2 Elastic Waves
Elastic stress waves travel through a metal without causing any permanent disturbance to the location of the atoms within the metallic structure. These waves propagate by the transmission of elastic energy from atom to atom, that is through distances of \( \sim 3 \ \text{Å} \) in a time \( \sim 10^{-13} \text{s} \). The speed of elastic waves, \( C_0 \), in a long rod, is given by
Strain-rate Regime | Method of Testing | Wave Propagation Effects
--- | --- | ---
1. Low rate \( \dot{\varepsilon} < 0.1 \text{ s}^{-1} \) | Standard mechanical testing procedures e.g. Instron machine | Not important
2. Medium rate \( 0.1 \text{ s}^{-1} \leq \dot{\varepsilon} \leq 200 \text{ s}^{-1} \) | Servo hydraulic frames cam plastometer drop weight test | Influences load and strain measurement stress waves may cause 'ringing' of load cells
3. High rate \( 200 \text{ s}^{-1} \leq \dot{\varepsilon} \leq 10^5 \text{ s}^{-1} \) | Split Hopkinson pressure bar (SHPB), rod impact (Taylor) | Affects uniform stress approxima-
ation wave analysis required for the interpretation of results
4. Very high rate \( \dot{\varepsilon} > 10^5 \text{ s}^{-1} \) | Flyer plate impact | Critical

**TABLE 1.1: CLASSIFICATION OF STRAIN RATE REGIMES IN METALS TESTING**

\[
C_0 = \sqrt{\frac{E}{\rho}} \tag{1.1}
\]

where \( E \) is the Young's modulus and \( \rho \) the density of the material.

For type 224 steel at room temperature in which \( E = 200 \text{ GPa} \) and \( \rho = 7850 \text{ kg m}^{-3} \), \( C_0 \) is \( 5048 \text{ ms}^{-1} \).

It can be shown (e.g. Cottrell, 1957) that the strain rate, \( \dot{\varepsilon}_e \), produced by the passage of an elastic wave through a material of thickness, \( L \) is

\[
\dot{\varepsilon}_e = \frac{\sigma C_0}{\rho L} \tag{1.2}
\]

where \( \sigma \) is the stress amplitude of the wave.
The fact that the elastic strain-rate, $\dot{\varepsilon}_e$ is proportional to the reciprocal of the Young's modulus, $E$, means that the observed elastic strain-rates in steels are often low when compared with strain-rates produced by plastic waves.

1.3.3 Plastic Waves

The propagation of plastic waves is more complicated than that of elastic waves since atoms no longer remain in their original cells but make permanent changes of position. Plastic waves therefore involve the shifting of a larger mass of material than elastic waves and consequently for the same applied force they travel with a lower velocity than the elastic wave speed, $C_0$.

Taylor (1946) has derived the following equation for the velocity, $V(\varepsilon)$, of an element of plastic wave in which the plastic strain is $\varepsilon$:

$$V(\varepsilon) = (1 + \varepsilon) \sqrt{\frac{\partial \sigma / \partial \varepsilon}{E}}$$

(1.3)

where $\partial \sigma / \partial \varepsilon$ is the slope of the true stress-strain curve. It is worth noting that as $\varepsilon \rightarrow 0$ and $\partial \sigma / \partial \varepsilon \rightarrow E$, equation (1.3) will reduce to equation (1.1).

For most metals in the plastic region $\partial \sigma / \partial \varepsilon$ diminishes with increasing strain. In accordance with equation (1.3), this leads to a dispersion of a plastic wave as the rear parts of the wave move more slowly than the leading parts. Type 224 carbon steel is an interesting material in that for many test conditions it shows a definite yield point in which $\partial \sigma / \partial \varepsilon$ becomes zero and usually negative. The consequence of this is that plastic shock fronts are created as the rear parts of the wave catch up with the leading front. These static fronts can be formed even in slow experiments and are the well known 'Luders bands' (to be discussed in Section 2.5).
The behaviour of elastic and plastic stress waves is a vast subject on its own and the more important aspects have been recorded in the books by Kolsky (1953) and Johnson (1972).

1.3.4 Thermal Softening at High Strain-Rates

Often in high strain-rate tests there is insufficient time for the thermal energy generated during the deformation to escape to the surroundings so that a significant temperature rise in the material may result. When this occurs the flow stress is often substantially reduced - adiabatic softening. When the rate of softening is equal to or greater than the rate of work-hardening an instability condition arises (Rogers, 1979) in which the deformation becomes increasingly more concentrated in one localised region of the specimen. The formation of the 'cone' in 'cup and cone' fracture of tensile tests pieces is a good example of this.

1.4 METHODS OF MATERIALS TESTING

Around 365 BC Aristotle postulated that in this world 'we are just as much the players as the spectators'. What this means in experimental physics is that it is not possible to measure anything without affecting the thing being measured and thereby influencing the outcome of the measurement. The only way of dealing with this dilemma is to take such precautions as to minimise the errors due to the method by which the measurement is made.

This principle is especially true in the mechanical testing of materials where the sample geometry, the testing machine and the lubrication used can have a profound effect on the results obtained. Ideally we require that the mechanical properties recorded should be independent of these factors so that accurate predictions about larger structures of the same material may be made. The project described in
this thesis has covered a range of strain-rates from $10^{-4}$ s$^{-1}$ to 5000 s$^{-1}$. Consequently it has been necessary to use three different pieces of apparatus: an Instron machine plus an ESH hydraulic machine for $\dot{\varepsilon} < 10$ s$^{-1}$ (described in Chapter 3) and a split Hopkinson pressure bar for $\dot{\varepsilon} > 80$ s$^{-1}$ (described in Chapter 4).

For each of the three methods used a critical analysis has been made of the validity of the method, the sample geometry, and lubrication to ensure that the results presented here have been obtained with maximum precision.

Few materials testing machines exist which may be used over a range of strain-rates greater than any one of the regimes presented in Table 1.1. This can prove quite irksome when experimental data is required over a wide strain-rate range since several techniques must then be used. However this fact has led to the development of a considerable number of different methods for materials testing within each strain-rate regime. The good thing is that these methods can be compared and validated against each other as well as the results obtained from the same material by different methods.

1.5 THE APPLICATION IN NUCLEAR TRANSPORTATION FLASK DESIGN

Impact properties and high strain-rate phenomena in materials such as concrete, steel, wood and soil are of particular interest to the nuclear industry because of the need to construct high integrity plant. This subject has been recently reviewed by Miles (1989) where it was noted that there is still a surprising sparsity of well researched and reliable data on these relevant materials available through the technical literature. Steel is the most researched material of the four listed above and has been reviewed with regard to high strain-rate applications in the nuclear industry by Albertini and Montagnini (1977) and Tinkler (1986).
FIG1.1 AGR IRRADIATED FUEL FLASK
The project described in this thesis has been sponsored entirely by the United Kingdom Atomic Energy Authority (UKAEA) based at Winfrith in Dorset*. Their specific interest in the dynamic mechanical properties of type 224 carbon steel arises from their work in the designing and testing of containers for the safe transportation of nuclear fuel and spent fuel.

The main objective of the Winfrith safety programme is the production of a consistent and tested method for the design of any spent fuel transportation container to survive a specified impact. To do this means that the factors contributing to the ultimate performance of the flask (plus contents) must be well understood.

Designing a flask to withstand a severe impact requires:

a) a stress analysis of the components of the flask to establish the maximum 'effective' stress (or strain) and location for a number of imposed decelerations at various orientations; and

b) a knowledge of the material properties to set the maximum allowable stress (or strain) to prevent failure.

There are two general types of flask (cylindrical and cuboid) used for the transportation of nuclear fuel by road or rail in the UK today. They are illustrated in the 'exploded' views of Figures 1.1 and 1.2 (Neilson, 1984 and 1985).

* Now called AEA Technology (Winfrith)
FIG1.2 EXCELLOX FLASK FOR TRANSPORT OF IRRADIATED OXIDE FUEL
Flasks used for the transportation of AGR fuel are of the cuboid type with side lengths 2.2 x 2.3 x 2.6m and laden weights of nearly 50 tonne (see Figure 1.1). The fuel load carried in such a flask is only 1 tonne. Twenty fuel elements are held in the fuel skip surrounded by a composite stainless steel/lead liner within a finned steel vessel. The fuel is immersed in water with a small ullage volume (container not completely filled) and ports in the liner and fuel skip to allow heat generated by radioactive decay to be transferred to the flask vessel by thermal syphon. The flask lid, made mostly of 224 steel, is bolted to the body and the seal is formed by a compressed elastomer gasket. The fins on the flask body provide a form of shock absorber as well as assisting with the transfer of heat to the surroundings.

Figure 1.2 shows the Excellox cylindrical flask which is used for the transportation of light water reactor (LWR) fuels. A number of fuel elements are carried in a "bottle" surrounded by a composite stainless steel/lead liner. The 1.5m diameter x 5m cylindrical body of the flask is finned to assist in the rejection of decay heat, internal heat being transferred to the outer case via a water thermosyphon. The flask lid (224 steel) is attached to the body by a number of bolts and the seal is effected by elastomer gaskets. The laden weight of the Excellox flask is typically 70 Mg.

Balsa wood is often incorporated as a shock absorber in the lids of transportation flasks since it displays remarkable energy absorbing capacity for its mass.

Figure 1.3 illustrates the way in which the cuboid AGR type flask is transported by rail on a flatroll carriage.

The impact testing of full size and scale model flasks is a crucial aspect of safety work in the nuclear industry. The most memorable example of such tests was the public demonstration test in 1984 where
FIG 1.3 Flask vehicle (flatrol)  
(after Miles, 1989)

FIG 1.4 Drum impact attitudes (after Miles, 1989)
a 140 tonne Type 46 locomotive plus six coaches were crashed into a derailed flask/flatroll (Figure 1.3) at a speed of 45 ms\(^{-1}\) (\(\sim 112\) mph) (Miles, 1989). 700 ms after impact the flask and locomotive plus carriages attained a common velocity of 33.5 ms\(^{-1}\) (\(\sim 83\) mph). The flask detached itself from the flatroll and finally came to rest at a distance of 60m down the track from the impact point. The mechanical damage to the flask was insignificant (Miles et al, 1985 and Tomlinson et al, 1985).

At Winfrith there are two main impact test facilities available for the testing of full size and scale model flasks. The drop test facility is capable of lifting masses up to 90 tonnes to a height of 30m. The surface onto which the flask is dropped is formed from one or more armoured-steel plates grouted on to a massive reinforced concrete backing structure. The recently developed horizontal impact facility is capable of launching 2 tonne model flasks at speeds up to 45 ms\(^{-1}\) at a large reinforced concrete structure.

Drop tests are the easiest and most commonly performed type of impact test and allow a variety of impact orientations to be investigated (see Figure 1.4 from Miles, 1989). Often the impact attitude chosen is the one which is most likely to cause the maximum damage to the flask or model.

Neilson (1985) has reviewed the variety of impact tests carried out on nuclear material containers in Germany, USA, France and Japan as well as the UK. These studies have demonstrated that the damage or permanent deformation (strain) produced in a flask during an impact test is proportional to the square of the impact velocity. Furthermore it has been found that impact tests on models of flasks generate data which accurately describes the behaviour of the full sized flask provided that the dimensions of the model are not less than 1/4 those of the actual flask.
FIG. 5 Detail of deformed finite element mesh at base of drum

FIG. 6 Comparison of computer model of flask impact with experiment
Current regulations for licensing transportation flasks require that the flask should be capable of surviving a 9m drop onto an armoured steel plate/reinforced concrete surface. Hart et al (1985) have estimated that the probability of a nuclear fuel flask suffering an impact as severe as the one in the regulatory drop test during normal transportation is $10^{-8}$ per year.

A vital aspect of the Winfrith safety programme is the computer analysis and modelling of flask behaviour in impact. Prediction of the strains generated in a flask from a specified impact is usually done with the aid of finite element analysis (illustrated in Figure 1.5). By this method the computer constructs the flask as a complex mesh of elements to each of which the material properties are assigned. From an initial specification of the impact conditions the computer calculates the deformation caused by solving the boundary conditions between each of the elements. For an acceptable level of accuracy in the calculation a large number of finite elements are required and hence the problem can only be solved using very large computer systems.

At Winfrith three main finite element codes are employed: DYNA 3D, HONDO-II and ABAQUS. The relative merits and performance of each code have been discussed with regard to nuclear fuel flasks by Neilson (1984), Cooper et al (1988) and Neilson et al (1989).

The real test of these computer codes comes of course when the results they produce are compared with experimental measurements made during an actual flask impact test. Such comparisons can be readily made when strain gauges are fixed to the flask at points which correspond to certain nodes in the equivalent finite element mesh.
Figure 1.6 shows a comparison between computer calculations of hoop strain using the HONDO-II finite element code and the actual hoop strain measured by strain gauges mounted on the impacted flask. Importantly this graph shows the better agreement between experiment and theoretical calculation obtained when strain-rate effects are incorporated in the computer analysis.

This last observation reveals the purpose of the project described here; to establish a thorough knowledge of the mechanical behaviour of 224 carbon steel over a wide range of strain-rates so that impact deformations produced in flasks made of this metal can be predicted and modelled by computer with greater accuracy. It is hoped that the work presented in this thesis will ultimately lead to the safer design of containers for the transportation of radioactive materials.

1.6 THE OBJECTIVES OF THIS PROJECT

The initial principal aims of the work described in this thesis are outlined below:

a) To record the mechanical properties of 224 carbon steel over a strain-rate range from $10^{-4} \text{s}^{-1}$ to $5000 \text{s}^{-1}$ and at temperatures of $-40^\circ\text{C}$, room temperature ($20^\circ\text{C}$) and $150^\circ\text{C}$. In actual fact this original temperature range was expanded to include tests at $-110^\circ\text{C}$ and $+300^\circ\text{C}$ (results presented in Chapter 9).

b) To compare the tensile and compressive properties of the steel (Chapters 6, 7 and 9).

c) To assess two (Instron and an ESH hydraulic) materials testing machines at Loughborough for tests at low and intermediate strain-rates and to compare the results on 224 steel from each machine (Chapter 3).
d) To assess and validate the SHPB facility in the Physics Department at Loughborough University for both compression and tensile tests at high strain-rates (Chapters 4, 5 and 7).

e) To assess the importance of specimen geometry and lubrication in the above mechanical testing systems and to establish optimum specifications for each (Chapters 3, 4, 6, 7 and 9).

f) To attempt to measure and quantify the effects of adiabatic heating on the flow stress in compression samples at high strain rates (Chapter 8).

g) To establish a theoretical model which relates the observed macroscopic mechanical properties of 224 carbon steel to the mechanisms which control the motion of dislocations (Chapter 10, thermal activation).

h) To examine the microstructure of the steel and relate this to the observed mechanical properties (Chapter 11).
CHAPTER 2
MECHANICAL PROPERTIES OF CARBON STEEL

2.1 INTRODUCTION

Steels are by far the most widely used metallic materials around the World, accounting for about 80% by weight of alloys in general industrial use today. Multifarious steels have now been manufactured, which provide an extensive range of mechanical properties from moderate yield strengths of 200 to 300 MPa with excellent ductility and toughness, to very high strengths in the region of 2000 MPa with adequate ductility.

Today it is possible to design a steel to have a particular strength and ductility by careful control of the quantities of its alloying elements and heat treatment. Due to the large number of steels existing there is inevitably, much duplication of mechanical properties and so research into the mechanics of steel is important in order to establish the optimum and most cost-effective design of a steel for a given application (Baird and Preston, 1973).

The work described here is concerned with 224 carbon manganese steel (BS 1503-224-430 grade) which is used in the fabrication of large containers for nuclear waste transportation and, therefore, should be expected to display good impact properties i.e. high yield strength and ductility at high strain rates.

Due to the large number of variables involved in steel making, both in terms of chemical additions and thermal processes, the relationship between structure and mechanical properties is very complex. This literature review begins by discussing the fundamental features of pure iron and then considers the basic mechanisms which strengthen
FIGURE 2.1 Temperature dependence of the mean volume per atom in iron crystals (Hume-Rothery, *The Structure of Alloys of Iron*, Pergamon, 1966)

FIGURE 2.2 (a) Tetrahedral and (b) octahedral sites in the α-iron lattice. (Hume-Rothery, 1966)
iron when alloying elements are added to form steel. For a complete understanding of these strengthening mechanisms a knowledge of dislocation theory is required. Here the more important historical developments in the theory are covered and the essential significance of strain rate (\(\dot{\varepsilon}\)) and temperature (\(T\)) is described. Finally high strain rate effects are noted and a review of other work on the dynamic mechanics of carbon steel is made.

2.2 IRON AND ITS SOLID SOLUTIONS

2.2.1 Iron
To understand how the mechanical properties of steel are related to its microscopic structure it is necessary to have some preliminary knowledge of the crystallography of iron and to then consider how the iron lattice is affected by the addition of solute atoms. This is the approach used here and through the following sections.

Pure iron has two crystal forms, one body centred cubic (BCC) known as \(\alpha\)-iron or FERRITE which is stable from low temperatures up to 910°C (the \(A_3\) point) when it converts to a stable face-centred cubic (FCC) form known as \(\gamma\)-iron or AUSTENITE. At 760°C, the \(A_2\) point, iron becomes non-magnetic until 1390°C, the \(A_4\) point when it reverts back to the BCC form now termed \(\delta\)-iron. This third form is magnetic and remains stable up to the melting point at 1536°C.

Figure 2.1 shows the phase transformation in a plot of the mean volume per atom of iron versus temperature. It may be deduced from the graph that the \(\gamma\rightarrow\alpha\) phase conversion is accompanied by an atomic volume change of roughly 1% which leads to the generation of internal stresses. This reflects the fact that FCC is a more closely packed structure than BCC.
The five most important mechanisms by which iron can be strengthened are:

1. Solid solution strengthening by interstitial atoms
2. Solid solution strengthening by substitutional atoms
3. Dispersion strengthening
4. Refinement of grain size
5. Work hardening.

In plain carbon steels, it is the first of these which is of most consequence since it is the interstitial solutes, carbon and nitrogen which make the largest contribution to the strength of the iron lattice. They play a key role in interacting with dislocations and in combining with some of the metallic alloying elements found in steels.

2.2.2 Carbon and Nitrogen in Solid Solution with Iron

The geometry of unit cells of α and γ-iron has a particular bearing on the solubility and diffusivity of carbon and nitrogen and so ultimately determines the general behaviour of plastic deformation. Figure 2.2 illustrates the two possible sites in the α-iron lattice which the interstitials, carbon and nitrogen may occupy. They are named tetrahedral and octahedral sites after the configuration of surrounding atoms and there are equivalent octahedral and tetrahedral sites in the FCC γ-iron.

Table 2.1 shows the largest diameters of spheres which will fit into the interstices of BCC and FCC iron. When one compares the available space with the atomic sizes of C and N given in Table 2.2, it is clear that there must be some lattice distortion when these atoms are present in iron. In fact, it is found that C and N atoms tend to occupy the smaller octahedral sites in α-iron displacing only two nearest neighbour atoms rather than the larger tetrahedral locations.
which would involve the displacement of four iron atoms and would therefore require more strain energy.

<table>
<thead>
<tr>
<th></th>
<th>Diameter</th>
<th>Diameter in iron (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCC tetrahedral</td>
<td>0.29r</td>
<td>0.36</td>
</tr>
<tr>
<td>octahedral</td>
<td>0.15r</td>
<td>0.19</td>
</tr>
<tr>
<td>FCC tetrahedral</td>
<td>0.23r</td>
<td>0.28</td>
</tr>
<tr>
<td>octahedral</td>
<td>0.41r</td>
<td>0.51</td>
</tr>
</tbody>
</table>

\( r = \) atomic radius of \( \alpha \)-iron.

**TABLE 2.1: SIZE OF LARGEST SPHERES FITTING INTERSTICES IN BCC AND FCC STRUCTURES**

<table>
<thead>
<tr>
<th>Element</th>
<th>Atomic Radius, ( r ) (Å)</th>
<th>( r/r_{Fe} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )-Fe</td>
<td>1.28</td>
<td>1.00</td>
</tr>
<tr>
<td>C</td>
<td>0.77</td>
<td>0.60</td>
</tr>
<tr>
<td>N</td>
<td>0.72</td>
<td>0.57</td>
</tr>
</tbody>
</table>

**TABLE 2.2: ATOMIC SIZES OF CARBON AND NITROGEN IN IRON**

<table>
<thead>
<tr>
<th>Temperature (°C)</th>
<th>Solubility wt%</th>
<th>at %</th>
</tr>
</thead>
<tbody>
<tr>
<td>C in ( \gamma )-iron</td>
<td>1150</td>
<td>2.04</td>
</tr>
<tr>
<td>723</td>
<td>0.80</td>
<td>3.6</td>
</tr>
<tr>
<td>C in ( \alpha )-iron</td>
<td>723</td>
<td>0.02</td>
</tr>
<tr>
<td>20</td>
<td>&lt;0.00005</td>
<td>&lt;0.00012</td>
</tr>
<tr>
<td>N in ( \gamma )-iron</td>
<td>650</td>
<td>2.8</td>
</tr>
<tr>
<td>590</td>
<td>2.35</td>
<td>8.75</td>
</tr>
<tr>
<td>N in ( \alpha )-iron</td>
<td>590</td>
<td>0.10</td>
</tr>
<tr>
<td>20</td>
<td>&lt;0.0001</td>
<td>&lt;0.0004</td>
</tr>
</tbody>
</table>

**TABLE 2.3: SOLUBILITIES OF CARBON AND NITROGEN IN \( \gamma \)- AND \( \alpha \)-IRON**

(Honeycombe, 1981)
FIG2.3a The iron-carbon diagram (after Hansen. Constitution of Binary Alloys. 2nd ed, McGraw Hill, 1958)

FIG2.3b—Iron-carbon constitution diagram.
Table 2.3 illustrates how the solubilities of carbon and nitrogen are greater in austenite than in ferrite due to the larger interstitial spaces existing in austenite. It also shows how the solubilities increase with increasing temperature for the range of temperatures presented, a principle which is exploited in the heat treatment of steel.

The solubility trends of Table 2.3 reveal themselves in the iron-carbon equilibrium diagram which forms a fundamental foundation for understanding the nature of all steels (Figure 2.3a). The diagram itself should really be regarded as representing the metastable equilibrium between iron and iron carbide, Fe₃C, known as CEMENTITE since Graphite occurs rarely on its own in an equilibrium phase in steel.

The increased solubility of carbon in austenite is demonstrated by its large phase field compared to that of ferrite. At point E the carbon solubility reaches a maximum of 2% wt at 1147°C. The bottom left hand corner of Figure 2.3a has been expanded in Figure 2.3b. It shows how severely restricted the α-iron phase field is with a maximum carbon solubility of only 0.02 wt% at 723°C (point P).

The vast difference in carbon solubility between γ- and α-iron leads to the rejection of carbon in the form of iron carbide at the boundaries of the γ-phase field. The γ→α -iron transformation occurs via a eutectoid reaction at a temperature of 723°C where the eutectoid composition contains 0.8% C (Point S). Thus upon slowly cooling alloys which contain less than 0.8% C from temperatures above A₃ = 910°C, hypo-eutectoid ferrite is formed from austenite throughout the range 910 to 723°C. Consequently, the residual austenite is steadily enriched in carbon until at 723°C, when its carbon content reaches 0.8%, it transforms to PEARLITE, a lamellar mixture of ferrite and iron carbon (cementite).
Figure 2.4. Fe–N phase diagrams (from Keh and Leslie, 1963)

Figure 2.5 The Burgers circuit in, A, edge and, B, screw dislocations. (After Friedel, 1964, Dislocations, Pergamon Press, Oxford.)
This is what takes place in the manufacture and subsequent cooling of 224 carbon steel whose carbon content is 0.18% wt (see Table 2.4). Hence, the two phases ferrite, and pearlite are the prime constituents of the microstructure of this type of steel and naturally influence its mechanical properties.

As indicated earlier, nitrogen can also play an important part in the structure of steel and this element too has various phase equilibria with iron (see Figure 2.4). However in the case of the 224 carbon steel under investigation here, the concentration of nitrogen is very small and most of it will be combined with aluminium anyway. Thus the predominant mechanical effects are produced by the 0.18% wt carbon.

<table>
<thead>
<tr>
<th>C</th>
<th>Si</th>
<th>Mn</th>
<th>P</th>
<th>S</th>
<th>Cr</th>
<th>Mo</th>
<th>Ni</th>
<th>V</th>
<th>Al</th>
<th>Cu</th>
<th>Sn</th>
<th>H2</th>
</tr>
</thead>
<tbody>
<tr>
<td>.18</td>
<td>.34</td>
<td>1.21</td>
<td>.009</td>
<td>.005</td>
<td>.12</td>
<td>.03</td>
<td>.15</td>
<td>.01</td>
<td>.016</td>
<td>.22</td>
<td>.015</td>
<td>2.9</td>
</tr>
</tbody>
</table>

TABLE 2.4: MANUFACTURERS ANALYSIS OF 224 CARBON MANGANESE STEEL BS 1503-430 GRADE
(From Forgenasters Steel Ltd, Sheffield)

2.2.3 Substitutional Solid Solution in Iron

Apart from carbon and hydrogen, all the constituent elements of Table 2.4 occupy substitutional sites within the α-iron lattice, that is they replace actual Fe atoms in the BCC structure. The effects of these additional alloying elements on the iron-carbon equilibrium diagram (Figure 2.3) are remarkably complicated. Wever (1928) showed that alloying elements could influence the Fe-C phase diagram in one of two ways:
by expanding the $\gamma$-field and thereby encouraging the formation of austenite over wider compositional limits and temperatures. The elements which produce this effect are termed 'γ-stabilisers' and include Ni, Mn, Co, Ra, Rh, Ph, Os, Ir and Pt. Both Ni and Mn depress the phase transformation $\gamma \rightarrow \alpha$-iron to lower temperatures i.e. $A_1$ and $A_3$ are lowered.

by contracting the $\gamma$-field and enhancing the formation of ferrite. 'α-stabilisers' include Si, Al Be, Pt, W, V, Mo, Cr, B and Ta.

In the first half of this century, before the mechanisms of solid solution strengthening and dislocation dynamics in iron were fully understood, Edwards (1952) carried out a comprehensive range of investigations into the effects of alloying elements on the mechanical and chemical properties of mild steel. His observations on the more important elements of Table 2.4 are summarised qualitatively below.

SILICON acts as a deoxidising agent but if added in too large a quantity it will create cavities within the steel.

SULPHUR is an unwanted impurity which causes weakness and cracks. It is neutralised by manganese in the formation of MnS.

PHOSPHORUS is always present in steels. It tends to make the metal harder but more brittle and so should be <0.04% in a high quality steel.

MANGANESE is very often added to steel as a cleansing agent since it combines with a good proportion of the dissolved oxygen in liquid steel and neutralises sulphur. The distribution of Mn is more complex than that of the other elements since as well as taking up
substitutional sites in the iron lattice it forms sulphides and carbides which combine with cementite. Increasing the Mn content generally hardens the steel - a trend to be explained later.

ALUMINIUM is used as a deoxidising agent and is normally added to the ladle before tapping. It forms no carbides but does combine with the dissolved nitrogen.

The presence of NICKEL in steel is inevitable since the element is indigenous to iron ore. Increasing Ni in steel tends to increase the yield stress and ultimate tensile stress without decreasing the elongation at fracture. It does this by lowering the A₃ point which in turn leads to grain refinement.

The presence of more than 0.1% COPPER in steel is undesirable for many applications where high ductility is essential but it does have the favourable feature of increasing the steel's resistance to corrosion. Like Ni, copper does not form carbides.

CHROMIUM and TITANIUM are only found in steel if added intentionally to raise the metal's resistance to corrosion. If sufficient quantities of Cr and Ti are added the steel is decarburised by the formation of TiC, Cr₃C, Fe₃CrC etc and consequently no distinguishable yield point appears in the stress-strain curves for such steels. The reason for this will be made clear in the following sections.

2.3 HEAT TREATMENT OF STEEL

There are essentially two main heat treatments commonly carried out on steels, normalising and annealing. Both are rather vague and variable processes and are usually adjusted to fit the requirements of the steel.
NORMALISING is a process in which the steel is reheated to between 50°C and 100°C above the A₃ temperature to form austenite, followed by air cooling through the γ→ α transformation. This recrystallisation process results in the refinement of the austenite and ferrite grain sizes and achieves a fine pearlite structure. The rate of cooling will depend on the dimensions of the steel although forced air cooling is sometimes used on larger billets and ingots. The 224 carbon steel received for the investigation described in this thesis had been normalised from 880°C. This seemingly low normalisation temperature was possible because of the lower A₃ point for 224 steel due to the relatively high Mn content.

ANNEALING covers a wide range of heat treatments. It is generally defined as heating to a temperature which is suitable for the removal of any hot or cold-rolling stress or 'work-hardening' which may have been put upon the material. Usually the cooling is carried out slowly in a furnace giving a coarse pearlite thereby providing good machineability.

2.4 DISLOCATIONS

The mechanical properties of iron and steel have been studied for several hundred years but the mechanisms which control their behaviour in plastic deformation have not been understood until recently. In the 1920's it was realised that the slipping of planes of atoms over each other occurred as a result of a large force being applied to a crystal. However, the stress required was several orders of magnitude lower than the theoretical shear stress calculated by assuming a periodic variation of shearing stress with atomic displacement. The discrepancy could only be accounted for by the existence of imperfections or DISLOCATIONS in crystal lattices and thus the subject of dislocation theory was developed in the middle of this century.
An outstanding book was written in 1953 by A.H. Cottrell which describes the early dislocation theory in relation to plastic flow. Much of what is presented there still holds true today.

The emergence of the electron microscope in the 1960's led to increased knowledge of the behaviour and structure of dislocations by direct observation of them within crystals. Brandon and Nutting (1960) were two of the first to use this invaluable research tool to study dislocations in α-iron for a variety of annealing and deformation conditions. Since then numerous electron microscope investigations have been carried out on iron and steel. The remarkable feature of the theory is how well the type and mobility of crystal defects can be defined, so that at the present time we have achieved a very precise understanding of yield and plastic flow. This does not mean, however, that our knowledge is complete.

Since the activity of dislocations is at the root of the mechanical behaviour of steel it is worth noting here some of their essential features. There are two basic types of dislocations termed EDGE and SCREW dislocations, the form and structure of which are best appreciated by studying ball and wire models (Bilby, 1950). Figure 2.5 shows schematic diagrams of the two types of dislocation.

The edge dislocation is characterised by the introduction of an extra half plane of atoms into the crystal. The half plane may be above the slip plane in which case the edge dislocation is termed POSITIVE or below it in which case the dislocation is NEGATIVE. For a given shear stress positive and negative dislocations will move in opposite directions.

Edge dislocations are defined precisely by their Burgers vector, \( \mathbf{b} \) which describes their displacement direction.
The Burgers vector is defined by means of a Burgers circuit, shown in Figure 2.5, which is an atomic path involving two lattice directions normal to each other which is then retraced using vectors of the same strength but opposite sign. In the lower half of Figure 2.5a a Burgers circuit is applied to a perfect part of the lattice and the circuit is a closed rectangle. However in the upper half of this diagram a similar circuit encloses an edge dislocation and there is a resulting 'closure failure', the magnitude and direction of which defines \( b \) for the dislocation.

The Burgers vector, \( b \) is defined in a similar way for a screw dislocation as shown in Figure 2.5b. As with edge dislocations, screw dislocations also have a sense, they can be either LEFT or RIGHT HANDED and the two types will move in opposite directions under a given shear stress.

As a result of the presence of the extra half plane of atoms, edge dislocations are much more restricted in their mobility than screw dislocations. Both types of dislocation are able to move on their own slip planes without diffusion of atoms - a CONSERVATIVE motion known as GLIDE. However edge dislocations can only move into parallel adjacent slip planes by shedding or gaining atoms - NON CONSERVATIVE motion known as CLIMB which is thermally activated. This type of movement does not apply to screw dislocations. Glide of many dislocations results in SLIP.

Dislocation lines cannot begin or end in the middle of a crystal structure, they must do so at a crystal face or grain boundary or form loops.

The force, \( F \), on a dislocation line or loop, acting at right angles to it, is given by
where \( \mathbf{F} \) is the shear stress acting on the slip plane. In BCC iron and steel the favoured slip planes are \((1,1,0)\), \((1,1,2)\) and \((1,2,3)\) and slip direction, \(<1,1,1>\). The magnitude of shear stress required to move a dislocation was first determined by Peierls (1940) and Nabarro (1947) and thus is commonly known as the PEIERLS-NABARRO STRESS.

Since even well-annealed steel contains "grown in" dislocations, it follows that every slip plane will be threaded by dislocations so that a dislocation gliding in one particular slip plane will have to intersect with the dislocations crossing that plane. These native dislocations are known as FOREST DISLOCATIONS. During plastic deformation when dislocations belonging to different slip systems interact they create JOGS and thereafter their motion is inhibited. Joggs may be viewed as steps on the dislocation which move it from one atomic slip plane to another. Similarly steps in dislocations occurring on the same slip plane are called KINKS.

The above has presented some of the fundamental features of dislocations which will be expanded upon in describing yield point and plastic flow phenomena in carbon steel. The book by Hull and Bacon (1984) is an excellent introductory guide to dislocations.

2.5 THE YIELD POINT IN CARBON STEEL

2.5.1 General Description of the Yield Point
The mechanical behaviour of carbon steel is different from most other metals in that deformation of the steel beyond its elastic limit is characterised by a very definite yield point in which the stress, and thus load, on the test piece are seen to drop whilst the strain continues to increase (Owen, 1963). Figure 2.6 shows the two types of
A) Uniform yield

B) Non-uniform yield

FIG2.6 Schematic examples of yield

FIGURE 2.7 (a) Elastic elements of a tensile machine. (b) Effect of the spring constant $K$ on the stress–strain curve
engineering stress-strain curves which are commonly obtained from standard tensile tests on metals. Figure 2.6b illustrates the type of yield point usually observed in carbon steels, whereas Figure 2.6a displays the type of uniform yielding common to many other metals and alloys, for example aluminium, copper or stainless steel.

Uniform or continuous yield is so named, since all the elements of the test piece yield and deform simultaneously and strain is uniformly distributed along the gauge length. There is a gradual transition from the elastic to plastic region as shown, so that defining the point at which the onset of plastic deformation begins can be quite arbitrary.

In the second type of yield, Figure 2.6b, concomitant to the deformation of carbon steel, the distribution of strain is not uniform throughout the specimen. Normal linear elastic extension occurs with increasing stress (AB) up to a level known as the upper yield stress, \( \sigma_{UYP} \). Thereafter, deformation proceeds at a decreased stress level, termed the lower yield stress, \( \sigma_{LYP} \) but the deformation is not homogeneous at this stage. The specimen is divided into regions known as Luders bands [after their discoverer Luders (1860)] where the strain is of the value, \( \varepsilon_L \) (Figure 2.6b) and undeformed regions of zero strain known as Hartmann lines (Hartmann, 1896).

The Luders strain, \( \varepsilon_L \) is dependent on the grain size and may be as high as 5%. Luders bands can often be observed optically by critical illumination (see for example, Winlock, 1953). \( \sigma_{UYP} \) can be regarded as the nucleation stress for Luders band formation, while \( \sigma_{LYP} \) is the stress at which they grow until they have spread along the whole length of the specimen (point D). Luders bands form with their fronts approximately 45° to the tension axis (Hall, 1951).
Beyond point D the deformation becomes homogeneous once more until the ultimate tensile stress $\sigma_{\text{UTS}}$ is reached at point E. Thereafter 'necking' develops leading eventually to ductile fracture at F.

One of the consequences of the formation of Luders bands is that it takes some small time for the formation of effective nuclei. Thus, if a load is impulsively applied to a specimen there will be a finite and measurable delay before the specimen yields. Russell et al (1961) have studied the question of 'delayed yield' in low carbon steels.

The whole subject of yield point phenomena in steel and other metals is excellently reviewed in the book by Hall (1970).

2.5.2 Effect of Testing Machine and Specimen Shape on the Yield Point

Both the type of machine and the shape of test specimen used have a crucial bearing on the magnitude and shape of the yield point in carbon steel. In the last century, stress-strain relationships were determined by noting the relative extensions in wires caused by the addition of dead weights. Obviously, by this method it would be impossible to record a lower yield point. Since the early part of this century screw driven Instron type machines and more recently hydraulic machines have been used to conduct tensile experiments on metals.

These two types of machine are capable of detecting and recording upper and lower yield points if they are stiff enough to respond to the sudden drop in load. The important factor in this kind of low strain rate testing is the relative elastic stiffness of the machine as compared to that of the specimen. Very often in the tensile testing of steels the specimen may have a larger elastic modulus than the machine so that the displacement of the gripping arms of the machine may totally swamp out any yield effects.
Figure 2.7 illustrates how the elasticity of the testing machine can affect the resulting yield point in the stress-strain record. Figure 2.7a shows how the testing machine can be regarded as an elastic spring coupled to the specimen. The machine displacement is then the product of the load and the elastic spring constant, K. Figure 2.7b shows how less marked the yield point becomes as K decreases (i.e. the machine becomes softer) and the difference between $\sigma_{\text{UYP}}$ and $\sigma_{\text{LYP}}$ is depressed. Johnston (1962) has paid particular attention to the question of the role of machine stiffness in yielding of LiF which shows marked upper and lower yield points. His work proved that an increase in machine 'hardness' or 'stiffness' by a factor of 25 could double the difference in stress between upper and lower yield points (i.e. $\sigma_{\text{UYP}} - \sigma_{\text{LYP}}$).

In tensile testing the shape of the specimen has a marked effect on the yield point. As pointed out earlier the upper yield stress, $\sigma_{\text{UYP}}$ is the stress which causes the onset of nucleation of Luders bands which will occur at the point of maximum stress concentration usually at the fillets of a test piece (Sylewewstrowicz and Hall, 1951). Thus by the use of specimens which consist of fillets having large radii of curvature and therefore a more gradual change in cross section from the gripping head to the central parallel region, it is possible to record very large values of $\sigma_{\text{UYP}}$ (Edwards et al, 1943). For these reasons, Crussard (1963) concludes that $\sigma_{\text{LYP}}$ is a fairly reliable measure of the mechanical strength of the steel unlike $\sigma_{\text{UYP}}$ which is very sensitive to specimen shape and testing technique.

2.5.3 Effect of Strain-Rate and Temperature on Yield Point
Most of the foregoing discussion referring to low or intermediate strain rates ($< 1 \text{ s}^{-1}$) applies at higher strain rates. In the dynamic testing of steel, specimen shape and the relative 'stiffness' of the testing apparatus are still very important.
Although the subject will be discussed at greater length in the following sections, it is worth noting here that strain-rate and temperature have reciprocal effects on the yield point of iron and steel. Thus increasing the temperature of the test tends to lower the level of the yield point which also occurs when the strain rate is decreased and vice-versa. Kenyon and Burns (1934) observed that in low carbon steel the yield point disappeared above a testing temperature of about 300°C.

Much work has been carried out in the study of the direct effect of temperature on the yield point of iron (Brown and Ekvall, 1962 and Altschuler and Christian, 1967).

### 2.5.4 Strain Ageing

An important aspect of the mechanical behaviour of carbon steels is strain ageing (Baird, 1963) which has very important consequences in the forming of sheet steel and deep drawing operations. If after stressing a steel sample beyond its yield point it is unloaded and then restrained immediately the yield point does not return. If however the sample is rested for a long time at room temperature or a shorter time at an elevated temperature between 100 and 200°C then the yield point reappears on subsequent restraining. This is the phenomenon which is known as strain ageing.

In elevated temperature tensile or compression tests on iron and steel the sharp yield point is gradually eliminated as the test temperature is raised. At a certain critical temperature the stress-strain curve becomes serrated or, in other words, composed of a large number of small yield points as shown schematically in Figure 2.8. The effect was first reported by Portevin and Le Chatelier (1923) and may be described as dynamic strain ageing. It is also known as blue brittleness (Kenyon and Burns, 1934) as blue is the interference
FIG2.8 The Portevin–Le Chatelier effect as observed in iron (schematic)

FIG2.9 Interstitial atoms in the vicinity of an edge dislocation: a. random atmosphere; b. condensed

FIG2.10 Factors contributing to the strength of C-Mn steels (Irvine et al., JISI, 1962, 200, 821)
colour of the surface of steel when oxidised at around 300°C. Lobe et al (1982) have made a detailed study of the effect in cast iron.

2.5.5 Cottrell-Bilby Theory of the Yield Point

The observed yield point in carbon steel cannot be regarded as an intrinsic quality of iron itself since the sharp upper and lower yield point can be obviated by annealing the metal in an atmosphere of wet hydrogen which reduces the carbon and nitrogen content to a very low level (Edwards et al, 1943). Hence the existence of such pronounced yield points must be directly due to the presence of the interstitial atoms of carbon and nitrogen.

Cottrell and Bilby (1949) first showed that these interstitial atoms would interact strongly with the strain fields of dislocations. The interstitial atoms themselves have associated strain fields so that when they move to sites within the dislocation strain fields their combined total strain energy is reduced. This results in the formation of atmospheres of interstitial atoms in the vicinity of dislocations. When such atmospheres form along the cores of dislocations they are said to be condensed.

Figure 2.9 displays the interstitial atmospheres which may arise around an edge dislocation. In the condensed atmosphere Figure 2.9b where a carbon atom takes up a position where there is maximum dilation of the lattice the binding energy is about 0.5 eV. Consequently dislocations can be locked in position by the atmospheres of carbon or nitrogen atoms which form around them. The upper yield stress, according to this model, may be regarded as the stress required to 'unlock' dislocations from their interstitial atmospheres. As more and more dislocations are unlocked and so become free to move less stress is required for the deformation to proceed and hence a lower yield point develops.
One attractive feature of the theory is that a very small concentration of interstitial carbon or nitrogen is needed to produce the locking of a whole length of all the dislocations in annealed iron. For a typical dislocation density of $10^8$ lines cm$^{-2}$ in annealed iron only $10^{-6}$ wt % carbon is sufficient to 'lock' all of them. Hence yield points are commonly observed in nearly pure iron (Hutchinson, 1963).

The Cottrell-Bilby theory also provides an explanation for the effects of Strain Ageing and Dynamic Strain Ageing described in the preceding section. The fact that both carbon and nitrogen atoms diffuse very readily at temperatures between 20°C and 150°C to create atmospheres around dislocations accounts for the re-emergence of the yield point in ordinary strain ageing. In Dynamic Strain Ageing at elevated temperatures the diffusion of interstitial atoms to lock dislocations is in direct competition with the applied stress which is setting them free and as a result numerous localised yield points develop. Dynamic Strain Ageing is seen to occur at those particular strain rates and temperatures where there is a balance between the rate of diffusion of interstitials and the rate of unlocking of dislocations.

By similar mechanisms, dislocations can be locked by substitutional atoms present in the steel (Suzuki, 1963) or by precipitates (Hale and McLean, 1963).

2.5.6 Multiplication Theory of the Yield Point

Sylwestrowicz and Hall (1951) argued against the Bilby-Cottrell locking theory on the basis that the introduction of free dislocations into test pieces by either scratching or making cuts did not destroy the upper yield point. Thence the nature of the yield point in mild steel became a subject of some controversy in the 1950s. A second theory was developed which made the assumption that once condensed
atmospheres form in iron the dislocations remain locked and the yield point arises from the generation of new dislocations. Johnston and Gilman (1959) successfully derived the theory for LiF crystals. Later Hahn (1962) applied the theory to iron and low carbon steel and Cottrell (1963) modified his earlier locking theory to incorporate dislocation multiplication.

From direct measurements of dislocations in LiF by etch pit techniques Johnston and Gilman (1959) found that for small deformations where $\varepsilon < 0.1$ the multiplication of dislocations took the form

$$\rho = \rho_0 + C \varepsilon^a$$  \hspace{1cm} (2.2)

where $\rho$ = dislocation density
$\rho_0$ = initial dislocation density before yield
$C$ = constant $\approx 10^8$ cm$^{-2}$
$a$ = constant $\approx 1 \pm 0.5$.

Recently Mikkola (1984) has studied equation (2.2) above under conditions of shock loading ($\dot{\varepsilon} > 10^5$ s$^{-1}$) and found it to be valid.

There are many mechanisms by which the dislocations can multiply (Hall and Bacon, 1984a), the most influential being Frank-Read Sources (Frank and Read, 1950) or by cross-slip (described by Conrad, 1961).

Stein and Low (1960) empirically found that the velocity, $V$ of dislocations had a stress dependence of the form:

$$V = \left(\frac{\sigma}{\sigma_r}\right)^M$$  \hspace{1cm} (2.3)
where $\sigma_r$ is a reference stress i.e. the stress required to give a
dislocation velocity of 1 cm s$^{-1}$, $\sigma$ is the yield stress and $M$ is an
index which is characteristic of the material. For low carbon steel $M$
varies between 20 and 40 (Hahn, 1962). In recent times equations
(2.2) and (2.3) have been confirmed for mild steel by thin film
electron microscopy.

The shear strain-rate, $\dot{\varepsilon}$ can be defined in terms of the movement of
dislocations as:

$$\dot{\varepsilon} = \rho \bar{V} b \quad (2.4)$$

where $\rho$ is the number of mobile dislocations per unit area, $\bar{V}$ is their
average velocity and $b$ is the Burgers vector. Thence the strain rates
at the upper yield point ($\dot{\varepsilon}_u$) and the lower yield point may be written
as

$$\dot{\varepsilon}_u = \rho_{UYP} V_{UYP} b \quad (2.5)$$

and

$$\dot{\varepsilon}_L = \rho_{LYP} V_{LYP} b \quad (2.6)$$

where $\rho_{UYP}$ and $\rho_{LYP}$ are the mobile dislocation densities at the upper
and lower yield points respectively. Then equating (2.5) and (2.6)
gives

$$\frac{V_{UYP}}{V_{LYP}} = \frac{\rho_{LYP}}{\rho_{UYP}} \quad (2.7)$$

and employing equation (2.3)

$$\frac{\sigma_{UYP}}{\sigma_{LYP}} = \left(\frac{\rho_{LYP}}{\rho_{UYP}}\right)^{1/M} \quad (2.8)$$

From equation (2.8) above it is clear that if $M$ is small (i.e. $\leq 20$
i.e. if the velocity of propagation of the dislocations is not too
large, then the ratio $\sigma_{UYP}/\sigma_{LYP}$ will be large and hence a large sharp
yield drop will be observed. Also if $\rho_{UYP}$ is low, perhaps as a result
of solute atom locking, then again a large yield drop can be expected
if there is a sizeable generation of new dislocations.
To summarise, the yield point observed in low carbon steels may be explained by a combination of the Cottrell-Bilby theory (Section 2.5.5) and the multiplication theory described above. The occurrence of a sharp yield point is brought about by the sudden increase in the number of mobile dislocations. If the pinning or locking of dislocations by carbon atmospheres is weak then the yield point arises as a result of unlocking but if the locking is strong then the yield point will be a result of the generation of new dislocations.

In conditions of dynamic strain ageing where atmospheres of carbon diffusively condense on newly generated dislocations, a higher density of dislocations is required to complete the deformation as most of the newly created ones become locked. This effect has been verified by electron microscopy. It accounts for the fact that increased work hardening rates have been observed in the blue brittle region as compared to room temperature tests (e.g. Kenyon and Burn, 1934; Manjoine, 1944).

2.6 WORK HARDENING

Once the Luders bands have covered the entire gauge length of the specimen so that it has completely yielded (i.e. point D on the schematic stress - strain curve of Figure 2.6b), increasing stress is required to produce further strain. The reason for this is that the dislocations generated or unlocked during the yield start to interact with each other so that the resistance to their motion increases. As plastic deformation proceeds barriers to dislocation movement are continually created so that a progressively larger stress is required to produce increasing strain.

The phenomenon is known as work hardening or strain hardening and applies to the section of curve D→E in Figure 2.6b. In the
The manufacture of steels work hardening is an important strengthening mechanism employed in cold forming operations.

For carbon steel and most other metals the region of work hardening may be described approximately by the simple empirical power law:

\[ \sigma_T = K\varepsilon_T^n \]

where \( n \) is the 'strain-hardening exponent' which varies between 0.1 and 1 and is typically about 0.25. \( K \) is the 'strength' coefficient. Kocks (1982) has discussed equation (2.9) above and has reviewed some alternative theoretical and empirical relationships which can more accurately represent the work hardening of some materials. In his paper, Kocks points out that the work hardening may be strain-rate dependent so that it is better described by:

\[ \sigma_T = K_0\varepsilon_T^m \]

where \( m \) is the 'strain-rate hardening exponent'.

Gladman et al (1970) have investigated the effect of microstructure and composition on the work hardening of 80 low carbon steels (mean carbon content = 0.094% wt) and concluded that equation (2.9) provided an inadequate fit to their experimental data: They found that the work hardening behaviour of the carbon steel could be more closely represented by an expression of the form:

\[ \sigma_T = a + b \ln\varepsilon + c\varepsilon \]

where \( a, b \) and \( c \) are constants.

In most stress-strain work, however, equation (2.9) may be applied as a first approximation.
2.7 SIGNIFICANCE OF GRAIN SIZE

So far in the discussion no distinction has been made between single crystals and polycrystalline structures. Although the theory presented above works well for single crystals of iron and steel most ferrous solids are polycrystalline consisting of a large number of randomly orientated grains separated by grain boundaries. These have an important effect on the mechanical behaviour of carbon steels since they present barriers to the motion of dislocations.

Each grain is a single crystal, the boundary of which is no more than a couple of atoms thick. When the misorientation between grains is small, the boundary comprises an array of dislocations termed a low angle boundary. For large misorientations at the boundary the atomic arrangement is more complex and will vary with the angle of misorientation.

Hall (1951) and Petch (1953) were the first to empirically establish the equation relating lower yield stress, \( \sigma_{\text{LYP}} \) to the grain size in iron and mild steel

\[
\sigma_{\text{LYP}} = \sigma_1 + K_y d^{-1/2}
\]  

(2.12)

where \( \sigma_1 \) is a friction stress which opposes dislocation motion, \( K_y \) is a constant related to the difficulty in spreading yielding from grain to grain and \( d \) is the mean grain diameter.

The remarkable feature of the Hall-Petch equation is that although it was established entirely empirically, it has been applied successfully to a wide range of metals including FCC and HCP (Armstrong, 1983) as well as ferrous alloys (Gladman and Pickering, 1983). The equation is most often used in the form of (2.12) above for lower yield stress,
\( \sigma_{UYP} \), but has been slightly modified to define upper yield stress, 
\( \sigma_{UYP} \) (Petch, 1964) and flow stress, \( \sigma_f \) after yield (Conrad and 
Schoeck, 1960) as linear functions of \( d^{-1/2} \).

The mechanistic model for equation (2.12) assumes that a dislocation 
source operates within a grain to produce a dislocation pile-up at the 
grain boundary. The pile-up then causes a stress to be generated 
within a neighbouring grain which, when it achieves a critical 
intensity, activates a new dislocation source within that grain. The 
process is self repeating and in this manner yielding propagates from 
grain to grain and is observed macroscopically by the passage of 
Luders bands. The grain size determines the number of dislocations in 
the pile-up and hence the stress generated. Clearly then, the coarser 
the grain size the more dislocations are present in the pile-ups which 
leads to a greater stress intensification and so yielding propagates 
at a lower applied stress.

The main criticism of the model described above is that dislocation 
pile-ups are not frequently observed in practice. It has been argued 
that the form of equation (2.12) merely reflects the obstacles imposed 
by the grain boundaries to the propagation of slip and the motion of 
dislocations (Gladman and Pickering, 1983).

The gradient, term, \( K_y \) in the Hall-Petch expression has been proved to 
be relatively insensitive to strain-rate and temperature (Harding, 
1969), whereas \( \sigma_1 \) is markedly sensitive (Heslop and Petch, 1958). 
Armstrong (1962) has pointed out that it is the grain size, \( d \), which 
in fact determines the strain-rate in deformation tests.

The constant \( \sigma_1 \) in equation (2.12) is known as the friction stress. It 
represents the yield stress of a single crystal (\( d^{-1/2} = 0 \)). Cracknell 
and Petch (1955) first resolved \( \sigma_1 \) into two parts, an athermal
component $\sigma_1$ dependent on the structure (i.e. presence of interstitial atoms and precipitates) and a thermal component $\sigma^*$, highly sensitive to strain-rate and temperature. The significance of the friction stress $\sigma_1$ will be discussed in Section 2.10.

The effect of grain size as a strengthening mechanism is always considered in the fabrication of steel. Very often small amounts of particular grain refining elements such as niobium (Mackenzie, 1963) are added to the hot ladle of mild steels in order to dramatically improve their final mechanical properties (Baird and Preston, 1973).

2.8 SOLID SOLUTION STRENGTHENING AND DISPERSION STRENGTHENING

The presence of both interstitial and substitutional alloying elements in carbon steel makes an analysis of the individual contributions to material strength from each type of element very complex. The important role carbon plays in locking dislocations was outlined in Section 2.5, however it further influences the strength of the steel by the formation of carbides of certain elements. The dispersion of such carbide precipitates within a steel presents obstacles to the movement of dislocations thereby increasing its resistance to deformation (Orowan, 1948). Furthermore, the precipitation of carbides around dislocations may lock them in a similar manner to carbon and nitrogen atoms alone.

Probably the most influential carbide in carbon steels is cementite (Fe$_3$C). This is usually found in the lamellar mixture with ferrite known as pearlite, mentioned in Section 2.2.2. The strength of pearlite itself depends on the interlamellar spacing, increasing as the spacing is decreased.
Since pearlite work hardens at a greater rate than pure ferrite, the tensile strength of some carbon steels is very sensitive to the pearlite concentration. However, in steels of carbon content < 0.3% (such as 224) the yield stress is found to be insensitive to pearlite concentration as most of the deformation is taken up by the ferrite matrix (Preston, 1983).

Irvine et al (1962) identified the three main strengthening mechanisms in carbon-manganese steels as being solid solution hardening, dispersion strengthening from lamellar pearlite and grain size (discussed in the previous section). The relative contribution of each of these to the total tensile strength of normalised carbon- manganese steel is illustrated in Figure 2.10. Variation of the carbon at constant manganese level causes a substantial increase in strength which is almost entirely due to the increased proportion of pearlite in the steel. When manganese is varied at constant carbon content the situation is more complicated since all three mechanisms are affected. Mn causes the eutectoid composition to occur at lower carbon contents so that the relative proportion of pearlite is increased within the microstructure. Also Mn is an effective solid solution strengthener as well as having a grain refining effect.

Pickering and Gladman (1963) expanded on the work of Irvine in carrying out investigations on the strength of 60 experimental casts of plain carbon-manganese steels containing up to 0.25% C and 1.5% Mn in order to isolate the key strength determining factors. Their work produced a useful set of regression equations which defined lower yield stress, $\sigma_{\text{LYS}}$, ultimate tensile stress UTS and percentage reduction of area in terms of $\%\text{ Mn}$, $\%$ pearlite and $d^{-1/2}$. 

FIG 2.12 Deformation map for carbon steel (after Rosenfield & Hahn, 1966)
2.9 TWINNING

Most plastic deformation of iron and steel occurs by the motion of dislocations along specific slip planes. However, at very high strain rates or at low temperatures the metal may undergo sudden localized shear processes known as twinning. The crystallography of twinning in iron and steel is the same for all BCC metals and occurs on the $\{112\} \langle 111 \rangle$ system.

Figure 2.11 is a two dimensional schematic illustration of the twinning process along a plane $K_1$ in the twinning direction $n_1$. As shown, twinning is a cooperative movement of atoms in which individual atoms move only a fraction of their interatomic spacing but the final result is a macroscopic shear where the twinning lattice becomes the mirror image of the parent lattice.

Twinning is usually a highly localized shear mechanism. Regions of twinned material can often be viewed microscopically after deformation and frequently appear as narrow parallel sided bands (Neumann bands) seldom thicker than 5 μm (Kelly, 1953).

In the constitutive modelling of the mechanical behaviour of BCC metals at high strain rates the twinning process has to be considered carefully before accurate resemblance can be achieved (Armstrong et al, 1989). The precise contribution of twinning to plastic deformation and strain hardening is, at the present time, poorly understood. It is possible that accommodation of strain by twinning could lead to a lower stored dislocation density thereby producing a lower strain hardening rate. Conversely, if twin boundaries act as obstacles to dislocation movement then yield and flow stress should increase. The argument has recently been discussed by Follansbee (1989).
2.10 EFFECT OF STRAIN-RATE AND TEMPERATURE (THERMAL ACTIVATION THEORY)

Research into the mechanical properties of carbon steel has been conducted for well over a century. At a very early stage it was realised that the observed mechanical behaviour was dependent on the speed of testing and the temperature (Le Chatelier, 1909). Obviously over the years a vast quantity of literature has built up on the subject and hence a large amount of data has been produced which relates the mechanical properties of iron and steel to their microstructure and the conditions of testing (Harding, 1987 and Nojima, 1986).

Still by far the most influential variables in carbon steel mechanics are temperature and strain rate. Many attempts have been made to develop constitutive relationships or mechanical equations of state of the form

\[ \sigma = f (\varepsilon_p, \dot{\varepsilon}_p, T) \]  

relating flow stress, \( \sigma \) to current values of strain, \( \varepsilon_p \), strain rate, \( \dot{\varepsilon}_p \) and temperature, \( T \) in a deforming carbon steel. However such a function can never be a true state function i.e. at a given strain-rate and temperature the stress will not be uniquely defined by the strain since the mechanical characteristics of the steel will depend on its thermal and strain history. Furthermore, the work hardening exponent, \( n \) for most carbon steels is itself strain-rate dependent (Vinh et al, 1979).

Nevertheless, a number of good models have evolved which describe strain-rate and temperature effects with a reasonable degree of accuracy. Harding 1981 and 1989 has reviewed the best of these models which apply at high strain rates.
2.10.1 The Deformation Map

In order to describe the plastic deformation of steel in the form of a constitutive equation or set of equations, it is necessary to understand the type of physical mechanism which controls the deformation. The nature of the dislocation dynamics involved will depend upon the strain rate and temperature. For a given set of conditions more than one kind of dislocation mechanism may operate, thus complicating the analysis. Klahn et al (1970) and Bodner (1968) have discussed the various types of mechanisms which play a role in the deformation of steel according to the temperature and strain-rate.

As a useful aid to the presentation of such information deformation maps are often used (Lindholm, 1978; Frost and Ashby, 1982). Usually these maps have axes of stress versus temperature or stress versus strain rate and are divided up into definite regions in which particular dislocation mechanisms operate. For each region a constitutive relation may be formulated either theoretically or from experimental data. Inevitably at the regional boundaries, there are areas of overlap in which two or more mechanisms may act.

Rosenfield and Hahn (1966) drew up a map for low carbon steels which is particularly pertinent to the work of this thesis and has been reproduced in Figure 2.12. The map covers temperature from 0 to 300K and strain rates from $10^{-5}$ to $10^{5}$ s$^{-1}$ and illustrates four distinct regions which reflect four different mechanisms of yielding and dislocation dynamics.

REGION I which includes standard low strain-rate testing at room temperature is characterised by a yield stress which is insensitive to strain rate and temperature. In this region the flow is predominantly controlled by the edge dislocation mobility (Stein and Low, 1960) and their long range interaction with precipitates. The early pioneering
FIG 2.13
Variation of lower yield stress with strain rate, at constant temperature.
(After Campbell & Ferguson, 1970)

FIG 2.14 Schematic graph showing temperature dependence of friction stress at constant strainrate
work of Manjoine (1944) and Winlock (1953) uncovered the existence of this region.

In REGION II yield stress and flow stress are both markedly sensitive to temperature and strain rate. In this region the motion of dislocations is determined by thermally activated processes, the theory for which is described in the next section.

REGION III is characterised by a reduced sensitivity of stress to strain-rate and temperature. Here twinning plays a dominant role in the mechanical deformation process.

REGION IV which commences at strain-rates above $10^3 \text{s}^{-1}$ is a region of extreme strain-rate sensitivity, the principal dislocation mechanism being one of viscous drag (Regazzoni et al, 1987). It is worth noting that the boundary between regions II and IV is independent of temperature.

The area inside the rectangular box drawn in Figure 2.12 indicates part of the range of testing conditions used in the experimental work described in this thesis. The complete range includes elevated temperature tests up to a temperature of 573K. Hence the deformation tests carried out here should involve primarily those dislocation mechanisms associated with regions I and II.

Figure 2.13 shows a set of results from shear tests carried out on an annealed mild steel (0.12% C and 0.62% Mn) by Campbell and Ferguson (1970). The graph delineates the relative strain rate sensitivities for each of the regions I, II and IV. In region II the yield stress is linearly proportional to the logarithm of the strain-rate.
2.10.2 Thermal Activation Theory (Region II)

Zener and Hollcman (1944) first proposed that the effects of strain-rate, $\dot{\varepsilon}_p$ and temperature, $T$ could be condensed into a single parameter, $Z$, so that if equation (2.13) were written as

$$\sigma = f(\varepsilon_p, Z)$$  \hspace{1cm} (2.13a)

then

$$Z = \dot{\varepsilon}_p \exp \left( \frac{U}{kT} \right)$$  \hspace{1cm} (2.14)

where $U$ is an activation energy, $k$ is Boltzmann's constant and $Z$ is constant at a given stress and strain.

A slight refinement of equation (2.14) leads to the following Arrhenius type equation:

$$\dot{\varepsilon}_p = \dot{\varepsilon}_o \exp \left\{ -\frac{\Delta G (\sigma^*)}{kT} \right\}$$  \hspace{1cm} (2.15)

where $\dot{\varepsilon}_o$ is a frequency factor (or nominal limiting strain rate) which depends on the mobile dislocation density and therefore on the structural state of the material. $\Delta G$, which is the Gibbs free energy of activation, and is a function of the local thermal stress $\sigma^*$ and the absolute temperature $T$, and can be expressed as:

$$\Delta G = \Delta G_o - V \sigma^*$$  \hspace{1cm} (2.16)

where $V$ is the activation volume for the process, $\Delta G_o$ is the total energy (free activation energy in the absence of stress), required for dislocations to overcome obstacles to their motion and $V \sigma^*$ is the contribution to this required energy provided in the form of mechanical work done by the applied load.
Equations (2.15) and (2.16) are nowadays generally accepted and are frequently used in constitutive modelling [Seeger (1956); Conrad (1961); Bennett and Sinclair (1966) and Perzyna (1974)].

Figure 2.14 illustrates the relationship between the thermal component of stress, \( \sigma^* \), in equation (2.16) and the friction stress \( \sigma_i \) which was introduced in Section 2.7 (Equation (2.12)). The work of Cracknell and Petch (1955) showed that the friction stress, \( \sigma_i \), could be represented by

\[
\sigma_i = \sigma_i' + \sigma^* \tag{2.17}
\]

where \( \sigma_i' \) is the athermal component of friction stress which is virtually independent of temperature apart from the small variation of shear modulus, \( G \) with temperature.

From Figure 2.14 it can be seen that \( \sigma^* \) decreases from \( \sigma^*(0) \) to zero as \( T \) increases from \( 0K \) to \( T_c \). This is explained by the fact that at low temperatures \( <T_c \), the motion of dislocations is mainly opposed by short-range barriers (such as dislocation-dislocation interaction) which can be overcome by thermal activation, i.e. the thermal vibrations of the lattice can assist in overcoming these barriers. However at temperatures \( >T_c \) the main obstacle to dislocation motion comes from long range forces such as those due to the presence of interstitial atoms and precipitates. In this case the long range forces present barriers which are too large for thermal activation to be significant and the athermal stress component \( \sigma_i' \) dominates.

Combining equations (2.17), (2.16), (2.15) and (2.12) gives the following idealised constitutive expression:

\[
\sigma = \sigma_i' + \Delta G/V + (kT/V) \ln \left( \frac{\dot{\epsilon} / \dot{\epsilon}_0}{1 + (K_i a^{-1/2})} \right) \tag{2.18}
\]
Although equation (2.18) gives a reasonably good description, Harding (1981) has found greater accuracy for BCC carbon steels in the semi-empirical relationship

$$\sigma = \sigma_1' + (\sigma_0 - \sigma_0') \left( \frac{\dot{\varepsilon}}{\varepsilon_0} \right)^{\frac{1}{3}} \frac{kT}{V(\sigma - \sigma')} \quad (2.19)$$

where $\sigma_0$ is the applied stress in the absence of thermal energy (0 K), indicated in Figure 2.14.

In recent times many authors (Follansbee et al. (1985); Johnson and Cook (1983); Armstrong and Zerilli (1988); Klepaczko (1984 and 1987)) have refined the basic activation theory described above to describe more accurately the behaviour of BCC steels at high strain-rates. The best of the models have been reviewed by Harding (1989). The disadvantageous feature common to all advanced constitutive theories is that the equations can become very complex especially when such effects as twinning and viscous drag (region IV) are taken into account. The extra terms and parameters incorporated usually means that a large amount of material data is required before they can be used. Nevertheless they have been found to give accurate descriptions of the behaviour of certain BCC metals at high strain rates.

2.11 A REVIEW OF THE DYNAMIC TESTING OF LOW CARBON STEEL

In 1944 Zener and Holloman and, independently Manjoine reported results from the first dynamic tensile tests on low carbon steel at various temperatures (730 to 873 K) and strain-rates (10^{-1} s^{-1} to 10^{3} s^{-1}). This early work was significant since it represented an advance in the method of dynamic tensile testing and also provided empirical evidence of a thermally active deformation mechanism. Campbell (1953) reviewed this early work on mild steel and compared the results with thermal activation theory based on the ideas of Cottrell and Bilby (1949).
Over the last fifty years an immense amount of experimental work has been performed on the dynamic mechanics of carbon steel which reflects the number of applications for which this material is chosen for its impact strength. Recently Nojima (1986) and Wagasugi (1985) have reviewed experimental developments in this area while Tinkler (1986) has made a literature survey of high strain rate properties of steel with particular regard to the electricity generating industry. Table 2.5 cites the more important papers in the field which have appeared since 1944.

The table quotes the carbon and manganese contents of the steels investigated as well as their heat treatment prior to testing. As was explained above, these three features play a key part in determining the microstructure and hence mechanical behaviour of steel (Pickering and Gladman, 1963). Comparing these contents with those laid down by the specifications in British Standards No 1501 (British Standards Institute, 1980) 224 carbon steel (grade 430) it is found that only the steel tested by Hashmi (1980, reference 29) meets these specifications. However a number of authors (references 4, 6, 10, 11, 13, 21, 25, 27 and 31 in Table 2.5) tested steels whose carbon content (0.16 to 0.2%) matched the 224 specification. It should be remembered that direct comparisons are difficult to make due to the huge number of variables which influence the final structure of a carbon steel (Honeycombe, 1981).

Table 2.5 reveals that the majority of studies have used dynamic compression rather than tension since the former is easier to perform accurately and suffers fewer problems from bending waves and inertial effects. Campbell and Harding in Oxford emerge as the main contributors to the subject over the last 40 years (references 5, 7, 8, 9, 12, 17, 19, 23 and 25). Most workers have carried out quasistatic tests to compare with observed material strength at higher strain rates.
About half the dynamic work cited in the table was carried out at room temperature which limits its usefulness in testing thermal activation theory. However, three papers (references 14, 23 and 28) describe work specifically undertaken to prove thermal activation theory and analyse their results accordingly. Generally it is found that most of the results from the investigations listed in Table 2.5 comply with the thermal activation theory presented in Section 2.10.2.
<table>
<thead>
<tr>
<th>AUTHOR(S)</th>
<th>YEAR</th>
<th>COMPOSITION OF STEEL</th>
<th>HEAT TREATMENT BEFORE TESTING</th>
<th>TENSION (T) OR COMPRESSION (C)</th>
<th>STRAIN RATE RANGE (s⁻¹)</th>
<th>TEMPERATURE RANGE (K)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zener &amp; Holloman</td>
<td>1944</td>
<td>0.25% C</td>
<td>Quenched and tempered</td>
<td>T</td>
<td>10⁻⁶ to 10³</td>
<td>73 to RT</td>
<td>Early pioneering work which not only produced the first successful tensile tests at high strain rates but also experimentally verified thermal activation</td>
</tr>
<tr>
<td>Manjoine</td>
<td>1944</td>
<td>'Low carbon open hearth steel'</td>
<td>Annealed</td>
<td>T</td>
<td>110⁻⁶ to 10³</td>
<td>RT, 473, 673, 873</td>
<td></td>
</tr>
<tr>
<td>Winlock</td>
<td>1953</td>
<td>5 sheet steels of C - content: 0.06, 0.21, 0.34, 0.48 and 1.03%</td>
<td>Normalised</td>
<td>T</td>
<td>3x10⁻⁵ to 3</td>
<td>RT</td>
<td></td>
</tr>
<tr>
<td>Krafft, Sullivan &amp; Tipper</td>
<td>1954</td>
<td>0.19% C, 0.51% Mn &amp; Iron (&lt;0.01%C)</td>
<td>Normalised</td>
<td>C</td>
<td>2000 s⁻¹</td>
<td>78 to 373</td>
<td>Investigation of point at which twinning occurs</td>
</tr>
<tr>
<td>Campbell</td>
<td>1954</td>
<td>0.14% C, 0.76% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>20 (elastic)</td>
<td>RT</td>
<td>Development of drop weight machine</td>
</tr>
<tr>
<td>Alder &amp; Phillips</td>
<td>1954</td>
<td>0.17% C, 0.62% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>1 to 40</td>
<td>1200 to 1473</td>
<td>Assessment of hot rolling condition Glass lubricants used</td>
</tr>
<tr>
<td>Campbell &amp; Duby</td>
<td>1956</td>
<td>0.24% C, 0.72% Mn</td>
<td>Normalised &amp; Annealed</td>
<td>C</td>
<td>10⁻³ and 500</td>
<td>RT</td>
<td></td>
</tr>
</tbody>
</table>
### Table 2.5 (continued)

<table>
<thead>
<tr>
<th>AUTHOR(S)</th>
<th>YEAR</th>
<th>COMPOSITION OF STEEL</th>
<th>HEAT TREATMENT BEFORE TESTING</th>
<th>TENSION (T) OR COMPRESSION (C)</th>
<th>STRAIN RATE RANGE (s⁻¹)</th>
<th>TEMPERATURE RANGE (K)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 Campbell &amp; Maiden</td>
<td>1957</td>
<td>0.32%C, 0.62% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>10⁻³</td>
<td>189 and RT</td>
<td>Effect of impact on static strength</td>
</tr>
<tr>
<td>9 Campbell &amp; Duby</td>
<td>1957</td>
<td>0.32%C, 0.62% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>10⁻³ and 500</td>
<td>RT</td>
<td>Measurement of time to yield after application</td>
</tr>
</tbody>
</table>
| 10 Cook              | 1957 | Various steels: 0.15 to 1% C  
0.17 to 0.68% Mn | Normalised                     | C                              | 1 to 100                 | 1173 to 1473          | Hot rolling applications                                                |
<p>| 11 Taylor            | 1957 | En 2V3: 0.19%C . 0.54%Mn | Normalised                     | T                              | 1 to 8                  | RT                    |                                                                          |
| 12 Campbell &amp; Harding| 1960 | 0.21% C, &lt;0.01% Mn   | Annealed                       | T                              | 10⁻³, 960 &amp; 2600        | RT                    | Observations of effects of grain size (d⁻¹/²) and neutron irradiation   |
| 13 Taylor &amp; Malvern  | 1960 | 0.19% C, 0.54% Mn     | Normalised                     | T                              | 1 to 5                  | 188 to 373            |                                                                          |
| 14 Conrad &amp; Frederick| 1962 | Iron (0.014% C)       | Annealed                       | T                              | 10⁻⁴ to 10⁻³            | 90 to 523             | Thermal activation analysis                                              |</p>
<table>
<thead>
<tr>
<th>AUTHOR(S)</th>
<th>YEAR</th>
<th>COMPOSITION OF STEEL</th>
<th>HEAT TREATMENT BEFORE TESTING</th>
<th>TENSION (T) OR COMPRESSION (C)</th>
<th>STRAIN RATE RANGE (s⁻¹)</th>
<th>TEMPERATURE RANGE (K)</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Krafft</td>
<td>1962</td>
<td>0.11% C, 0.62% Mn</td>
<td>Annealed &amp; Normalised</td>
<td>C</td>
<td>0.1 to 10³</td>
<td>RT</td>
<td></td>
</tr>
<tr>
<td>Krafft &amp; Sullivan</td>
<td>1962</td>
<td>Various steels:</td>
<td>Normalised</td>
<td>C</td>
<td>1 to 100</td>
<td>RT</td>
<td>Investigates effects of C and Mn plus grain size (d⁻¹/²)</td>
</tr>
<tr>
<td>Campbell &amp; Cooper</td>
<td>1966</td>
<td>En 2A: 0.045% C, 0.45% Mn</td>
<td>Annealed</td>
<td>T</td>
<td>10⁻³ to 10²</td>
<td>RT</td>
<td>Observes Luders bands and delay time for yield</td>
</tr>
<tr>
<td>Fearnshough</td>
<td>1966</td>
<td>12 Carbon steels:</td>
<td>Normalised</td>
<td>T</td>
<td></td>
<td>RT to 600</td>
<td>Employs modified Charpy Impact test. Irradiation effects studied</td>
</tr>
<tr>
<td>Cooper &amp; Campbell</td>
<td>1967</td>
<td>En2A: 0.045% C, 0.45% Mn</td>
<td>Normalised</td>
<td>C</td>
<td>10⁻¹ to 10²</td>
<td>RT</td>
<td>Constant load compression in soft hydraulic machine</td>
</tr>
<tr>
<td>Hawkyard, Eaton &amp; Johnson</td>
<td>1968</td>
<td>En1A: 0.11% C, 1.24% Mn. En2: 0.13%, 0.5% Mn. BS 970</td>
<td>Annealed and drawn bright</td>
<td>C</td>
<td>5 x 10³</td>
<td>RT to 973</td>
<td></td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>COMPOSITION OF STEEL</td>
<td>HEAT TREATMENT BEFORE TESTING</td>
<td>TENSION (T) OR COMPRESSION (C)</td>
<td>STRAIN RATE RANGE (s⁻¹)</td>
<td>TEMPERATURE RANGE (K)</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>--------------------</td>
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<td>------------------------------------------------------------------------------------</td>
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<td>--------------------------------</td>
<td>-------------------------</td>
<td>-----------------------</td>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>21 Samanta</td>
<td>1968</td>
<td>5 carbon steels: C: 0.1 to 0.86% Mn: 0.34 to 0.79%</td>
<td>Annealed</td>
<td>C</td>
<td>6.6 x 10⁻² and 350 to 550</td>
<td>RT to 1328</td>
<td>Drop hammer system</td>
</tr>
<tr>
<td>22 Slater, Johnson &amp; Aku</td>
<td>1968</td>
<td>0.55% C</td>
<td>Annealed</td>
<td>C</td>
<td>10⁻³ and 500</td>
<td>RT</td>
<td></td>
</tr>
<tr>
<td>23 Campbell &amp; Ferguson</td>
<td>1970</td>
<td>En38: (0.12% C, 0.62% Mn)</td>
<td>Annealed</td>
<td>Shear</td>
<td>10⁻³ to 10⁴</td>
<td>195 to 713</td>
<td>Detailed thermal activation analysis</td>
</tr>
<tr>
<td>24 Slater, Aku &amp; Johnson</td>
<td>1971</td>
<td>0.55% C</td>
<td>Annealed</td>
<td>C</td>
<td>218 to 480</td>
<td>RT</td>
<td>1473</td>
</tr>
<tr>
<td>25 Harding</td>
<td>1977</td>
<td>BS 968: 0.2% C, 1.36% Mn</td>
<td>Normalised</td>
<td>T</td>
<td>10⁻³ to 2500</td>
<td>77 and 225</td>
<td>By Instron, hydraulic machine and drop hammer</td>
</tr>
<tr>
<td>26 Wulf</td>
<td>1978</td>
<td>1023: 0.21% C, 0.63% Mn, 4130: 0.34% C, 0.45% Mn</td>
<td>Hot-rolled and Tempered</td>
<td>C</td>
<td>3 x 10³ to 3 x 10⁴</td>
<td>RT</td>
<td>Observation of thermal softening at high strain rate</td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>COMPOSITION OF STEEL</td>
<td>HEAT TREATMENT BEFORE TESTING</td>
<td>TENSION (T) OR COMPRESSION (C)</td>
<td>STRAIN RATE RANGE (s⁻¹)</td>
<td>TEMPERATURE RANGE (K)</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>-----------</td>
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<td>--------------------------------</td>
<td>-------------------------------</td>
<td>--------------------------</td>
<td>------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Costin et al</td>
<td>1979</td>
<td>1018 CRS: 0.18%C, 0.71% Mn, 1020 HRS: 0.26%, 0.5% Mn</td>
<td>Normalised</td>
<td>Shear</td>
<td>5 x 10⁻⁴, 500 and 1000</td>
<td>116 to 394</td>
<td>Temperature rise in specimens measured using an infrared detector</td>
</tr>
<tr>
<td>Tanaka &amp; Nojima</td>
<td>1979</td>
<td>2 steels: 0.02%C, 0.31% Mn, 0.45%C, 0.64% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>10⁻⁴ to 10³</td>
<td>78 to 290</td>
<td>Thermal activation analysis</td>
</tr>
<tr>
<td>Hashmi</td>
<td>1980</td>
<td>0.2%C, 1.02% Mn</td>
<td>Annealed</td>
<td>C</td>
<td>10⁵ (max)</td>
<td>RT</td>
<td></td>
</tr>
<tr>
<td>Fitzpatrick &amp; Pratt</td>
<td>1982</td>
<td>En2A: 0.054%C, 0.5% Mn</td>
<td>Annealed/Normalised</td>
<td>C</td>
<td>10⁻² and 8 x 10⁻³</td>
<td>RT</td>
<td></td>
</tr>
<tr>
<td>Shirakashi, Maekawa &amp; Usui</td>
<td>1983</td>
<td>0.18%C, 0.48% Mn</td>
<td>Annealed</td>
<td>C &amp; T</td>
<td>200 to 2 x 10³</td>
<td>293 to 1000</td>
<td>ε and T varied during test to study &quot;the brittleness&quot;</td>
</tr>
<tr>
<td>Haque &amp; Hashmi</td>
<td>1985</td>
<td>En8: 0.41%C, 0.78% Mn</td>
<td>Normalised</td>
<td>C</td>
<td>10³ to 10⁵</td>
<td>243 to 508</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER 3
COMPRESSION TESTS AT LOW AND INTERMEDIATE STRAIN-RATES
$(3.8 \times 10^{-4} \text{ to } 4 \text{ s}^{-1})$

3.1 INTRODUCTION

Compression tests at low and intermediate strain-rates were carried out using a screw driven Instron machine and an ESH (Edwards, Solloman and Hill Ltd) hydraulic testing machine respectively. Machines of these types have now become well established in materials testing laboratories and are well documented (Davis et al, 1982). This chapter describes how the two machines were adapted to perform compression tests over the strain-rate range $3.8 \times 10^{-4}$ to $4 \text{ s}^{-1}$ and at temperatures between $-40^\circ \text{C}$ and $300^\circ \text{C}$. Before the machine records could be interpreted correctly it was necessary to characterise the compliance of each machine.

3.2 SPECIMENS

Figure 3.1 shows the general arrangement for the compression of type 224 steel specimens by both ESH and Instron machines. The specimens themselves were solid cylinders of original dimensions: height, $H_0 = 5.0 \pm 0.1 \text{ mm}$ and diameter, $D_0 = 10.0 \pm 0.2 \text{ mm}$, as used in the Split Hopkinson Pressure Bar tests (following Chapter). In some of the Instron tests slightly smaller samples ($H_0 = 4 \pm 0.1 \text{ mm}$, $D_0 = 8 \pm 0.02 \text{ mm}$) were used in order to achieve more strain at maximum load.

It was found that this change in size did not influence the shape of the resulting true stress versus true strain curves. The specimens were cut from a billet of type 224 steel supplied by the UKAEA.
FIG3.1: Schematic diagram of homogeneous compression of a specimen.
A) x200 magnification

B) x50 magnification showing non-parallelism

FIG3.2 Talystep profile of a compression sample
(Winfrith) and their end faces were finely ground, to produce smooth flat parallel surfaces.

An assessment of the quality of the finished samples was made using a diamond tipped talystep machine which is capable of measuring the variation in height of a surface to ±10 μm. Figure 3.2 shows two records from this machine which illustrate the end face surface profiles of two samples typical of a batch of randomly selected 10 x 5 mm and 8 x 4 mm specimens. Figure 3.2A shows that the roughness of the sample faces does not vary by more than ±250 μm. Figure 3.2B shows the relative variation in distance between the two end faces of a sample over a distance of 2 mm. The particular trace presented represents a worst case and it can be seen that the total variation over this distance is only 1 μm. Hence over the whole diameter, 10 mm, we should not expect the total variation to be more than 5 μm which indicates a high degree of parallelism between the sample faces.

3.3 INSTRON TESTING

Figure 3.3 is a simplified diagram of the 5000 kg (50 kN) capacity Instron machine used for low strain rate testing. Compression of specimens is caused by the downwards motion of the screw-driven crosshead towards the fixed lower platen. A load cell below this platen measures force acting on it by means of a resistive network of strain gauges responding elastically to the load. The output from this load cell is amplified and fed to a chart recorder incorporated in the machine's control panel. The crosshead speed could be preselected between 0.05 mm/min and 50 mm/min, so by fixing the paper speed of the chart recorder, details of total crosshead displacement versus load could be obtained. For tensile tests (Chapter 5) the crosshead moves in the opposite direction.
FIG 3.3 Schematic diagram of Instron machine

FIG 3.4 Static compression tests arrangement (Instron)
Before use, the Instron machine was always calibrated using a set of dead weights. All compression tests were carried out using a crosshead speed of 0.5 mm/min which produced an engineering strain rate in the 10 x 5 mm specimens of approximately $10^{-3}$ s$^{-1}$. Tests were terminated as close as possible to the maximum load i.e. 5000 kg.

Figure 3.4 illustrates how the samples were actually compressed between the Instron platens. They were sandwiched between two 1/2" diameter 431 stainless steel adaptors which produced a testing geometry similar to that met with in the Split Hopkinson Pressure Bar (SHPB) dynamic tests (Chapter 4).

### 3.4 MACHINES USED FOR COMPRESSION TESTS AT INTERMEDIATE STRAIN-RATES

(0.1 s$^{-1}$ to 100 s$^{-1}$)

The three most common machines for materials testing at intermediate strain rates (0-1 s$^{-1}$ to 100 s$^{-1}$) are the servo-hydraulic compressor, the cam plastometer and the drop weight tower. These three techniques are comprehensively reviewed in the 9th Edition of the ASM Metals Handbook (1987) by Follansbee, Hockett and Dudder respectively. The first of these, the servo-hydraulic test frame has been used for the work undertaken here and will be described in the following section (3.4).

The cam plastometer was first used for compression testing by Orowan (1950). It comprises two platens between which the sample is compressed when one of the platens is driven by a logarithmically shaped cam while the other platen remains stationary. The main advantage of this type of machine is that the strain-rate is constant throughout the test due to the logarithmic shape of the cam. Usually permanent compressive strains in excess of 50% are achieved.
FIG 3.5 Schematic diagram of ESH system
The drop weight tower is a mechanically simple device which produces uniaxial compression by the free fall of a heavy weight onto a cylindrical specimen. Such machines can deliver much larger loads than either servo-hydraulic test frames or cam plastometer, e.g. up to approximately 900 kN within times between 0.1 and 20 ms. However, since the weight decelerates naturally upon impact with the specimen, both the displacement rate and load rate vary continually throughout the test, which is a major disadvantage of the method.

3.5 COMPRESSIVE TESTING OF INTERMEDIATE STRAIN RATES USING AN ESH HYDRAULIC MACHINE ($3.8 \times 10^{-4} \text{ s}^{-1}$ to $4 \text{ s}^{-1}$)

Figure 3.5 illustrates the layout of the ESH hydraulic machine used for compression tests at intermediate strain rates. This machine had two key advantages over the Instron machine used previously for low strain rate testing. Firstly it was capable of sustaining a much greater load (300 kN maximum) on the sample and hence produced more plastic deformation and secondly it was capable of a maximum displacement speed of $50 \text{ mm s}^{-1}$. The screw driven Instron machine was capable of a maximum load of 50 kN and $0.8 \text{ mm s}^{-1}$ maximum crosshead speed.

Testing of specimens on the ESH machine was almost identical to Instron testing, except that the samples were placed between two hardened steel cylindrical platens (diameter = 25.4 mm, length = 25.4 mm) which were sandwiched in turn between the jaws of the ESH. The adaptors shown in Figure 3.4 could not be used here as the high ram speeds (up to $20 \text{ mm s}^{-1}$) may have caused hazardous buckling to occur.

From Figure 3.5 it can be seen that the load is transmitted from a piston which is acted upon by pressurised fluid. Variation in position of the piston is made possible by a servo valve which
controls the flow of fluid from one side of the piston to the other. The piston moves within the 'hydraulic actuator' and its movement is measured by a linear variable differential transformer (LVDT).

The LVDT transducer is an electromechanical device which produces an output voltage proportional to displacement. It comprises a transformer with a primary and two secondary coils wound on one cylindrical former and a ferrite core aligned axially within the hollow body of the transformer. The ferrite core is attached to the piston. The secondaries are connected in series opposition so that for a particular location of the core (the null position) the induced emf's are equal and cancel each other so that the net output is zero. If the core is displaced upwards from this position the mutual inductance between the primary and one of the secondaries increases while the other decreases so that an overall positive voltage is produced. If the core moves downwards the opposite changes occur and the output voltage becomes more negative. The signal from the LVDT is amplified before being fed to readout instrumentation.

The load is monitored by a resistive load cell similar to that used in Instron testing and the signal from it is amplified before being sent to recording instrumentation. Both the force and displacement channel outputs are d.c. voltages varying with linear proportionality between -10V and +10V. For most of the tests carried out in this work the load channel was set to give 0.1V/kN and for the displacement channel 5 V/mm. The voltage resolution on both channels is 20 mV (corresponding to ±200N and ±4 μm respectively).

As Figure 3.5 delineates, the ESH machine is controlled by an electronic closed loop system. The outputs from the force and displacement channels are simultaneously fed to a feedback selector as well as the readout recording device. The feedback selector allows
the user to choose whether the machine should be placed under 'load control' or 'displacement control' i.e. whether the test should be carried out at a constant loading rate or a constant displacement rate. For all tests described in this thesis 'displacement control' was used. The controller continuously compares the selected feedback variable with a preselected command signal and then transmits a correction signal to the servo valve according to the level of disparity.

In this experiment, the command signals were linear ramped voltages which, since they were followed by the displacement channel, determined the actual displacement rates. The action of the hydraulic piston or ram was superbly smooth and could be halted instantly by setting 'trip' voltages at a preselected load or displacement on the 'command' panel.

At the lower displacement rates, 0.002 mm s\(^{-1}\), 0.02 mm s\(^{-1}\) and 0.2 mm s\(^{-1}\) the signals from each channel were fed directly into the usual X-Y plotter and load versus displacement curves obtained. At the higher ram speeds of 2 mm s\(^{-1}\) and 20 mm s\(^{-1}\), where the total test time was only 1s and 0.1s respectively, the plotter pen was unable to follow the changing signals accurately, especially around the yield region. It was then necessary to use a Datalab DL902 transient recorder to first store and then plot out load versus displacement after the test.

Calibration of the ESH machine is not as straightforward as that for the Instron machine where a set of dead weights are used. It would be an arduous task to calibrate a machine having a load capacity of 300 kN using standard weights. Instead the ESH machine described above is calibrated about once a year by a service engineer who uses calibration rings. These are large steel rings containing a
deflection indicator and are compressed or stretched elastically in the machine, whereby the true load is determined. The temperature of the calibration ring is important and is always noted. Half way through the ESH testing programme major maintenance repairs were made to the machine and the calibration just described was performed.

3.6 MACHINE STIFFNESS

In Section 2.5.2 it was noted that the compliance of the testing machine plus grips can have a significant effect on the shape of the true stress versus true strain curve especially for materials which show a definite yield point. Ideally, the testing machine should be considerably stiffer than the material being tested if the true behaviour is to be observed.

Welter (1945) carried out a sequence of experiments using a variety of testing machines to study the effect of machine stiffness on observed yielding phenomena. His work demonstrated that the size of yield drop in mild steel, $\sigma_{YUP} - \sigma_{LYP}$, could be reduced if a soft machine was used, and also that false upper and lower yield points could be generated by a very rigid machine.

Figure 3.6 is a schematic illustration of a compressive deformation machine. Part of the crosshead motion is taken up by the elastic compliance of the machine plus platens and this is represented by an imaginary spring. The elastic displacement of machine and platens due to an applied load, $F$ is

$$\Delta x_{el} = \frac{F}{K}$$

(3.1)

where $K$ is the effective spring constant or 'machine stiffness'.

FIG 3.6 Schematic view of a deforming apparatus

\[ \frac{\Delta x}{\Delta t} = \dot{S} = \text{constant displacement speed} \]

\[ \Delta H = \text{sample displacement} \]

FIG 3.7 Illustrative Instron record
If $\Delta H$ is the total amount of sample deformation then the total crosshead displacement may be written as:

$$\Delta x = S_c t = \Delta x_{e1} + \Delta H$$  \hspace{1cm} (3.2)

where $t$ is time. Engineering strain in the specimen is

$$\varepsilon = \frac{\Delta H}{H_0} = \frac{(S_c t - F/K)}{H_0}$$  \hspace{1cm} (3.3)

So that the engineering strain rate is:

$$\dot{\varepsilon} = \frac{(S_c - K^{-1} \frac{dF}{dt})}{H_0}$$  \hspace{1cm} (3.4)

Although the above has been derived for compressive testing the argument applies equally well to tensile testing on such machines.

For hard machines where $K$ is large equation (3.4) tends to:

$$\dot{\varepsilon} = \frac{S_c}{H_0}$$  \hspace{1cm} (3.5)

Equation (3.5) has been used quite generally to define quasistatic strain rates by two previous workers in this laboratory, Ellwood (1983) and Walker (1987), but it may not be entirely accurate if the elasticity of the machine is comparable to that of the specimen.

Johnston (1962) has used equation (3.4) in his study of yield points in LiF crystals. Hockett and Gillis (1971) have undertaken an empirically based analysis of equation (3.4) and suggest that the situation is further complicated by a variation in $K$ throughout the test and from test to test.
In determining engineering stress versus strain from load-displacement curves the compliance of the machine cannot be ignored. It is usual to subtract the elastic component of machine displacement from the overall load-displacement curve before calculating the strain in the specimen.

3.7 A COMPARISON BETWEEN THE STIFFNESS OF THE INSTRON AND ESH MACHINES

3.7.1 Instron Compliance

Figure 3.7 delineates a typical load-displacement record from an Instron test on a 10 x 5 mm specimen. Also shown is the machine compliance curve which was determined from the mean of several compressive runs with no sample between the 431 adaptors. It therefore represents the elastic component of crosshead displacement $\Delta x_{el}$ due to the machine plus adaptors.

The figure demonstrates that a large proportion ($\sim 74\%$) of the total crosshead displacement in a test taken up to 5000 kg, is due to the machine compliance itself. From equation (3.1) we can calculate the 'stiffness' (spring constant) $K_{INS}$ for this particular Instron machine. Hence,

$$K_{INS} = 36 \text{ kN mm}^{-1}$$

which represents a fairly 'soft' machine.

The determination of specimen displacement alone requires the subtraction of the machine compliance from the total crosshead displacement curve at given loads. Hence it was possible to determine the true strain rate in the sample from the total test time.
FIG3.9 Examples of ESH records at 0.02mm/s & 273K

FIG3.8 Compliance curves for the ESH machine.
3.7.2 ESH Compliance

Figure 3.8 shows a group of compliance curves for the ESH machine, at three different crosshead displacement speeds. All three are more or less parallel and display the same elastic displacement, $\Delta x_{el}$ for a given load. By equation (3.1) again, the machine stiffness $K_{ESH}$ is:

$$K_{ESH} = 287 \text{ kN mm}^{-1}$$

It is instructive to compare this result with that attained for the Instron. The ratio of the two is

$$\frac{K_{ESH}}{K_{INS}} \approx 8$$

Hence the ESH machine gives a marked improvement on the Instron machine in terms of stiffness alone and one should expect that the actual material behaviour would be more closely followed in ESH testing.

Figure 3.9 shows a typical set of three curves from ESH compression tests on three samples under the same conditions (i.e. room temperature, displacement rate: 0.02 mms$^{-1}$). Clearly the consistency between tests is very good, the average variation within each set of this kind being only 2%. At a load of 75 kN the mean total displacement is 1.4 mm of which only 0.26 mm (19%) is due to machine compliance (Figure 3.8). Compare this with the 74% for the Instron compliance at a load of 50 kN (Section 3.6.1). The ESH machine may be described as hard.

At the higher crosshead speeds of 2 mms$^{-1}$ and 20 mms$^{-1}$ where the DL902 transient recorder was required the compliance curves were found to agree with those presented in Figure 3.8 thus indicating that the machine compliance is insensitive to testing speed. This is described in Section 3.8.
In an identical manner to the Instron records, the specimen displacement in ESH testing was calculated by subtraction of the compliance curve from the total displacement-load curves.

3.8 CALCULATION OF TRUE STRESS AND TRUE STRAIN

Before each test on either the Instron or ESH machines the specimen dimensions were measured using a Moore and Wright digital micrometer (accuracy: ±1 μm) and were noted. Thus after establishing the total specimen displacements at a number of loads, F in a test, it was an easy matter to calculate engineering stress, \( \sigma_\circ \) (= \( 4F/\pi D_0^2 \)) and engineering strain, \( \varepsilon_\circ \) (= \( \Delta H/H_0 \); see Figure 3.1 for symbolism). These were converted in turn to give true stress, \( \sigma \) and true strain, \( \varepsilon \) via the standard equations:

\[
\varepsilon = -\ln (1 - \varepsilon_\circ) \tag{3.6}
\]

and

\[
\sigma = \sigma_\circ (1 - \varepsilon_\circ) \tag{3.7}
\]

\( \varepsilon, \varepsilon_\circ, \sigma \) and \( \sigma_\circ \) are assumed positive in compression.

The derivation of equations (3.6) and (3.7) can be found in many texts on plastic deformation, for example, Johnson (1972).

3.9 VERIFICATION OF THE ESH METHOD AT ROOM TEMPERATURE

As mentioned in Section 3.4 two types of readout instrumentation were used; a normal X-Y plotter for low testing speeds (0.002 mms\(^{-1}\) to 0.2 mms\(^{-1}\)) and a Datalab DL902 transient recorder (storage capacity = 2 kbytes per channel) at the higher crosshead speeds (2 mms\(^{-1}\) and 20 mms\(^{-1}\)). Obviously the results recorded should not be dependent on the type of instrument used. To verify that this actually was the
FIG 3.10 ESH results at 293K & $3.8 \times 10^{-3}$ s$^{-1}$ (X-Y plotter)

FIG 3.11 ESH results at 293K & $3.8 \times 10^{-3}$ s$^{-1}$ using a transient recorder
case two sets of three tests were carried out at the same crosshead speed of 0.02 mms⁻¹. In one set the recording device was the X-Y plotter while in the other set the transient recorder was used on a long time base setting at 100s full scale (digitising rate: 1 sample per 50 ms).

Figure 3.10 shows the three curves of true stress versus true strain calculated by equations (3.6) and (3.7) from load-displacement curves recorded by the X-Y plotter while Figure 3.11 shows the set for which the transient recorder was used. In both cases the samples used were 10 x 5 mm of 224 steel. The first important feature to note which is common to both graphs is the remarkable consistency in the curves belonging to each set. This small amount of variation in the results was typical of most of the tests performed under different strain rate and temperature conditions. The mean true strain rate for the six shots depicted in Figures 3.10 and 3.11 was calculated as 3.8 x 10⁻³ s⁻¹ from equation (3.4).

The mean curves for the results presented in Figures 3.10 and 3.11 are compared with each other in Figure 3.12. The higher of the two curves, (a), represents the mean of the X-Y plotter results (Figure 3.10) while the other curve (b), represents the mean of the transient recorder results. There appears to be only a small difference between curves (a) and (b), indicating that the replacement of the X-Y plotter by the DL902 transient recorder should not influence the nature of the results. At a strain level of 28% the difference between the two curves is 15 MPa which is within the experimental error for the two methods.

In Section 3.4 it was mentioned that the ESH machine was calibrated half way through the testing programme. The tests which yielded the results curves of Figures 3.10 and 3.11 were carried out in February
(a) = XY plotter, (b) = transient recorder, X = ESH results (March '88), O = Instron curve

FIG. 3.12 Comparison of mean stress versus strain curves from FIGS. 3.11 & 3.10 (10x5mm sample)

FIG. 3.13 ESH results at 293K & 4s⁻¹
1989 shortly after the calibration and extensive machine repairs had been completed. However, a considerable amount of room temperature ESH data was collected nearly a year earlier (March to April, 1988). It was felt that a possible drift in the machine's behaviour may have meant that the earlier results were in error. The crosses plotted in Figure 3.12 which represent mean true stress - true strain values for three room temperature tests at 0.02 mm/s carried out amongst the first batch of ESH tests. They show good agreement with the more recent curves of (a) and (b) up to a strain of 12% above which they fall below the two curves. At 28% strain the discrepancy between the old ESH data and curve (b) amounts to 25 MPa and between the old data and (a), 40 MPa. These figures are larger than might be expected from experimental error alone and suggest a change in the ESH machine's performance after the calibration had been carried out.

Figure 3.13 compares pre- and post-calibration ESH results recorded at a machine speed of 20 mm/s. Again the agreement is satisfactory at low strains below 10%, but above this level the old pre-calibration curve rises above the set of three post-calibration curves. At 28% strain the mean discrepancy is 30 MPa.

In the light of all this the post-calibration data was considered to be a more trustworthy indication of the actual material behaviour at higher strains (> 10%) and so was used in further analysis (Chapter 9).

Also plotted in Figure 3.12 are circles which represent the mean of room temperature results from the Instron tests on 10 x 5 mm samples, for a mean strain-rate of 1 x 10^{-3} s^{-1} (by equation 3.4). Since this strain-rate is comparable with the 3.8 x 10^{-3} for the ESH solid curves in Figure 3.12 the closeness of the Instron points to the curves implies that the observed material mechanical behaviour is the same by both methods. However, one important difference was noticed.
FIG3.14 Photograph of the cylindrical cooling jacket
In all the ESH records, upper and lower yield points were clearly visible whereas in the Instron chart records they were seen infrequently. This discrepancy is a result of the relative hardness of the two machines (see Section 3.6.2).

The Instron points do not proceed beyond a strain of 10% since at this point the load applied by the machine has reached maximum capacity i.e. 50 kN.

3.10 COMPRESSIVE TESTS AT -40°C

3.10.1 Instron Tests at -40°C
In order to attain compressive testing temperatures of -40°C a hollow cylindrical copper cooling jacket was made (see photograph in Figure 3.14). The hollow core of the jacket was designed to fit snugly around the 1" cylindrical platens plus specimen used in ESH testing (see Figure 3.5) and so these platens were used in all -40°C tests.

Cooling was achieved by filling the jacket with liquid nitrogen through a funnel and the whole arrangement was thermally lagged with fibre glass wool. Frequent refilling of the jacket was necessary since conduction of heat through the platens was considerable. The temperature inside the core of the jacket was monitored using a chromel-alumel thermocouple (K-type) which was affixed to the upper platen at a point as close as possible to the specimen itself.

Initially, a compliance run was made using the 1" platens and no sample at -40°C. At the beginning of the actual tests themselves the temperature was seen to rise markedly by as much as 10 or 15°C. Later, this was believed to be caused by the enhanced conduction of heat through the top platen due to the improved thermal contact between this platen and the crosshead once the load was applied. It
presented a major source of possible error in the resulting stress-strain analysis.

3.10.2 ESH Tests at -40°C
Exactly the same cooling arrangement was used with the ESH machine, but with the experience gained from the Instron tests, a small load of about 500N was continuously maintained on the platens and specimen during the cooling process so that at the start of each compression test the amount of heat conduction through the top platen and the driving ram head did not change. Consequently no dramatic shifts in temperature were observed during the tests.

A new compliance curve was produced for -40°C which was then used in the subsequent analysis.

3.11 COMPRRESSIVE TESTS AT ELEVATED TEMPERATURES (150°C and 300°C)

3.11.1 Instron Tests at 150°C and 300°C
In the elevated temperature Instron tests the 431 adaptors were used as in the room temperature tests (Figure 3.4). Heat was generated through a simple Nichrome wire coil heater wrapped around a 75 mm long ceramic tube (of internal diameter 15 mm) which accommodated the adaptors plus specimen. The coil was lagged with a 2" thickness of glass fibre tape. Figure 3.15 illustrates how the heater was powered by an a.c. current.

It was found that a current of 2A supported a temperature at the specimen of 150°C monitored by a K-type thermocouple fixed to the upper adaptor as close as possible to the specimen. A current of 4A had to be supplied to the coil in order to maintain a temperature of 300°C.
3.11.2 ESH Tests at 150°C and 300°C

The ESH machine is equipped with its own cylindrical furnace and temperature controller. However, it is designed to operate at temperatures higher than 250°C and thus could not control at the modest 150°C. Nevertheless, initial attempts were made to use the furnace by sensing the temperature as close as possible to the specimen using a K-type thermocouple and controlling the desired temperature (i.e., 150°C) manually by periodically switching on and off the current to the furnace. A few tests were executed under these conditions.

However, it was found that since the ESH furnace embodied a large mass of cylindrical ceramic, the initial heating up was very slow, taking between 15 and 30 minutes to reach 150°C. Furthermore, it was very difficult to manually control the 'overshoot' once the desired temperature had been arrived at and cooling of the furnace was twice as slow as heating.

In the light of these problems, it was decided to build a second resistive coil heater similar to the one depicted in Figure 3.15, the only difference being a change in geometry to accommodate the 1" platens plus specimen. This second simple heating coil, like the first, was found to work very satisfactorily. Initial heating was very rapid, taking less than 5 minutes to reach 300°C and once the designated temperature had been attained, it was found that at constant current, natural conductive and convective heat balance produced a remarkable thermal stability throughout the tests.

All the elevated temperature tests plus the -40°C tests carried out on the ESH machine took place after the machine's calibration in January 1989.
FIG3.15 Arrangement used to heat samples in elevated temperature tests

FIG3.16 Barrelling when the friction coefficient is 1 at the specimen/platen faces

FIG3.17 Zone formation in barrelling
3.12 BARRELLING

At quasistatic and intermediate strain-rates the true stress-strain curves for most metals in uniaxial compression correspond closely with those in uniaxial tension. However, tensile tests are limited by the onset of 'necking' where the deformation becomes non-homogeneous at moderate strains. In compression tests, on the other hand, far greater permanent strains may be attained. However, the disadvantage of compression testing is the presence of friction between the specimen and platen faces which can affect the shape of the final true stress-strain curve. This probably accounts for why the tensile test is most often favoured at these strain-rates.

Friction between the cylindrical specimen and the platens produces 'barrelling;' as illustrated in Figure 3.16. Points which were originally on the cylindrical surface of the sample at the start of the test are 'rolled over' to the flat end faces as shown. When this happens the calculation of engineering strain by the reduction of height $\Delta H/H_0$ (equation 3.3) is no longer valid since effectively, $H_0$, the gauge length, is decreasing as the test proceeds.

The movement of material through right angles as demonstrated by points A, B and C in Figure 3.16 is caused by a markedly inhomogeneous deformation. In the severest cases of barrelling conical zones in contact with the platens are created in which little plastic deformation takes place. These zones are often referred to as dead metal zones (DMZ) and are depicted in Figure 3.17. In all three distinguishable zones may be identified in a barrellled specimen (Figure 3.17). Zone 1 represents the dead metal zones which have only slight movement relative to the platens. Zone 3 is an annular region which mainly moves radially outwards and Zone 2 is the region where plastic deformation occurs as material moves from the interfaces with Zone 1 into Zone 2 and then into Zone 3.
Mascall et al (1983) have used a finite element model to confirm this pattern of behaviour in a barrelled sample.

To minimise the effects of frictional resistance so that conditions of homogeneous deformation are more closely achieved it is important to lubricate the specimen-platen interfaces before testing commences. However, even with the most efficient lubricant, there will always be a degree of frictional constraint between the specimen and the platens. Over the years many attempts have been made to establish the ideal homogeneous compressive deformation from actual tests in which barrelling takes place. The more important of these have been reviewed and criticised by Hsu (1969) and recently by Gunasekera et al (1982).

Read et al (1943) have derived a correction factor which allows one to calculate true homogeneous stress from the actual observed stress according to the amount of barrelling produced in the sample. However this correction formula only works for cylindrical specimens whose original height ($H_o$) to diameter ($D_o$) is 1 or greater.

Latham et al (1968) tested a wide variety of metals in compression and developed a method of correcting true stress-strain results from barrelled specimens based on the ratio of sample height and radius of curvature of the barrelling. Their work showed that in a well lubricated test, the departure of actual compression of steel specimens from the true homogeneous compression was insignificant below 25% sometimes 30% strain.

It appears that the ideal cylindrical compression specimen is one which has such a height that end effects are negligible. However, in practice, the ratio $H_o/D_o$ should be less than 2 if buckling is to be avoided. Cook and Larke (1945) tackled the friction problem by
testing a number of specimens having different initial $H_0/D_0$ ratios but identical lubrication. They then extrapolated their data to produce the desired stress-strain curve for a specimen of infinite $H_0/D_0$ ratio.

Slater (1977) ensured homogeneous deformation by a method of incremental compression. Starting with a sample of initial $H_0/D_0$ ratio of 1, he stopped the test at every 10% reduction in height in order to remachine the sample so that $H/D$ could be returned to unity. At each interval the platens and specimen were cleaned and relubricated. In this manner, Slater was able to achieve true homogeneous compressive strains up to 200% for a variety of metals.

Polakowski (1949) carried out a series of compression tests on carbon steel cylinders (C contents: 0.07% to 0.44%) and found that provided the specimens were well lubricated to begin with, there was no appreciable difference between idealized and actual compression results for strains below 25%. In a similar way to Slater, he stopped the tests every 25% to remachine the specimens and thereby achieved final true strains close to 100%.

As yet, no reliable correction factor has been developed to correct the true stress versus true strain results from a test in which significant barrelling has occurred. Indeed there is some controversy in the literature on this subject. For example, Cook and Larke (1945) state that friction increases the resistance to compression while Polakowski (1949) observes that it decreases the resistance. Hsu (1969) has concluded that the only sensible policy is to ensure that the compression remains homogeneous throughout the test. Gunasekera et al (1982) endorse this view and observe that in a compression test where barrelling has occurred the true stress will be over estimated if a normal reduction of height type analysis is
applied. The effect of barrelling will be discussed in the light of the results of Chapter 7.

3.13 SPECIMEN LUBRICATION

In view of the deleterious effects of friction as described in the preceding section (3.10), the platens and specimens were well lubricated in all compression tests carried out in this project. Due to the range of test temperatures, three different lubricants were employed. At \(-40^\circ C\) PTFE spray was used. At room temperature and \(150^\circ C\) silicone grease, and at \(300^\circ C\) molybdenum disulphide ('Molyslip') were used. These same lubricants were also used at the stated temperatures in the SHPB tests on compression pieces (Chapter 4).
CHAPTER 4

COMPRESSION TESTS AT HIGH STRAIN RATES

4.1 INTRODUCTION

The last Chapter described compression tests on type 224 mild steel at low and intermediate strain-rates ($10^{-4}$ to $4s^{-1}$). This Chapter is concerned with compression at high strain-rates ($> 100s^{-1}$) where the effects of wave propagation and inertia cannot be ignored. As a consequence of this, the analysis and derivation of true stress versus true strain in such tests is naturally more complicated.

A large number of methods have been developed to test materials at high strain-rates and some comprehensive review papers have been written on the subject [Bitans and Whitton (1972), Lindholm (1974), Holzer (1979) and Lataillade (1989)]. At very high strain rates ($> 10^4s^{-1}$), the main two techniques which are used are rod impact tests [Taylor (1948), reviewed by Erlich (1987)] and expanding ring or cylinder experiments [reviewed by Ahmed (1988)].

In the strain-rate range $100s^{-1}$ to $10^4s^{-1}$ the literature on testing techniques clearly shows that the split Hopkinson pressure bar (SHPB) has become the dominant method since the pioneering work of Kolsky (1949). The following passages describe the Loughborough SHPB system which was the main research tool used in this project to determine the properties of type 224 steel at high strain-rates.
4.2 THE SPLIT HOPKINSON PRESSURE BAR (SHPB) METHOD

4.2.1 Historical Perspective
Several authors have recounted the historical development of the SHPB method of materials testing at high strain-rates [e.g. Lindholm (1964), Ellwood (1983), Follansbee (1987b) and Walker (1987)], thus only a brief summary is required here.

The technique takes its name from the work of Bertram Hopkinson (1914) who was the first to investigate the propagation of impulsive stress waves in a long elastic metal bar. The stress waves were initially generated by the detonation of explosives or impact of bullets at one end of the bar. At the other end of the bar a 'time-piece' (a short length of bar) was loosely fixed which would fly free once the initial stress wave had passed through it. By using a series of time pieces of different lengths and measuring their final momenta the details of the original stress disturbance in the bar could be reconstructed assuming the wave had propagated with no distortion or attenuation.

In 1948, Davies made a theoretical and experimental study of the Hopkinson pressure bar. He replaced the time piece with a condenser to monitor displacements in the bar.

Kolsky (1949) introduced a split in the bar in which a small cylindrical specimen could be sandwiched. He showed how the stress and strain within the deforming specimen could be related to the displacements in the split bars, termed incident and transmitter bars. Figure 4.1 is a schematic representation of the SHPB as used by Kolsky.
FIG4.1 Basic SHPB apparatus (Kolsky, 1949)

FIG4.2 Schematic view of strain pulses incident at a specimen
The diagram shows the form of SHPB generally used for compression testing. A stress pulse is initiated in the incident bar either by impact of a projectile or detonation of an explosive charge. If the amplitude of this initial stress pulse is restricted to within the elastic limit of the bar material, and if the length of the bar is greater than at least five bar diameters, then the pulse will propagate without distortion.

Upon reaching the specimen, part of the wave will be reflected and part of it will be transmitted through the specimen into the transmitter bar. Kolsky (1949) used capacitive displacement transducers at points A and C to measure these reflected and transmitted components. Nowadays most workers use strain-gauges attached to the bars at equidistant points (A and B) from the sample as shown in Figure 4.1.

Figure 4.1 is representative of most SHPB systems currently used in research, although in some cases the projectile may itself be the incident bar [Wulf (1974) and Gorham et al (1984)]. Due to the major contribution from Kolsky to the technique the SHPB is often referred to as the Kolsky bar.

The use of strain gauges to measure the displacements in the incident and transmitter bars was first reported by Hausér et al (1961) and has now become normal practice in most systems. Watson (1970) has attached strain gauges directly to the specimen under test.

A number of optical methods have been used to measure sample strain throughout the test. Sharpe and Hoge (1972) used interferometry to measure the strain in a sample whose cylindrical surface was marked with a set of closely spaced grooves. Griffiths et al (1979) have used an optical method in which shutters are connected to the bars
adjacent to the sample. Sample displacement is then determined by the amount of light incident on a photodetector. Albertini and Montagnani (1977) and Gorham (1979) have used high speed photography to observe specimen displacements during an SHPB test.

All the work reviewed above refers to uniaxial compressive testing which is the most popular type performed using the SHPB. However, SHPB systems have been developed to produce other modes of deformation. These include shear (Campbell and Dowling, 1970), torsion (Duffy, 1974), pure uniaxial strain in which radial displacement is constrained (Bhushan and Jahsman, 1978), biaxial (Stiebler et al, 1989) and tension (Lindholm and Yeakley, 1968; also see Chapter 6).

4.2.2 SHPB Theory

In this section expressions for specimen stress and strain are derived by consideration of the motion of the bar faces at the specimen. The problem is best appreciated with reference to Figure 4.2 which indicates the strain pulses and specimen face displacements caused by the arrival of the incident strain pulse, $\varepsilon_I$.

Assuming the pressure bars remain elastic, then the force, $F$, stress, $\sigma$ and strain, $\varepsilon$ in the bars are related simply by:

$$ F = \sigma A = E_b A \varepsilon $$

(4.1)

where $E_b$ is the elastic modulus of each bar and $A$ is the bar's cross-sectional area.

Elastic wave propagation theory shows that:

$$ \sigma = \rho C_0 \dot{u} $$
\[ \dot{u} = \frac{\sigma}{\rho C_o} = \frac{\varepsilon E_b}{\sqrt{E_b / \rho}} = \varepsilon C_o \quad (4.2) \]

where \( u \) is the particle displacement and \( \dot{u} \) is particle velocity. \( C_o \) is the longitudinal wave velocity in the bar.

Let \( \varepsilon_I, \varepsilon_R \) and \( \varepsilon_T \) represent the strain amplitudes of the incident, reflected and transmitted pulses respectively and subscripts 1 and 2 refer to the end faces of the incident and transmitter bar adjacent to the specimen as depicted in Figure 4.2.

Then, forces on bar faces

\[ F_1 = E_b A (\varepsilon_I + \varepsilon_R) \]
\[ F_2 = E_b A \varepsilon_T \]

Velocity of bar faces

\[ \dot{u}_1 = C_o (\varepsilon_I - \varepsilon_R) \]
\[ \dot{u}_2 = C_o \varepsilon_T \]

Displacements of the bar faces:

\[ u_1 = C_o \int_0^t (\varepsilon_I - \varepsilon_R) \, dt' \]
\[ u_2 = C_o \int_0^t \varepsilon_T \, dt' \]

where \( t \) is time.

Hence for the sample:
\[ \sigma_s = \frac{F_1 + F_2}{2A_s} = \frac{E_B A_s}{2A_s} (\varepsilon_I + \varepsilon_R + \varepsilon_T) \] (4.3)

\[ \varepsilon_s = \frac{u_1 - u_2}{k_s} = \frac{C_0}{k_s} \int_0^t (\varepsilon_I - \varepsilon_R - \varepsilon_T) \, dt' \] (4.4)

where \( l_s \) is the length of the sample and \( A_s \) is its cross-sectional area.

Assuming that the forces at the specimen ends are in equilibrium (i.e. the time for wave propagation through the specimen is negligible) gives

\[ F_1 = F_2 \]

or

\[ \varepsilon_T = \varepsilon_I + \varepsilon_R \] (4.5)

Thus

\[ \sigma_s = \left( \frac{A_s}{A_s} \right) E_B \varepsilon_T \] (4.6)

\[ \varepsilon_s = \frac{-2C_0}{l_s} \int_0^t \varepsilon_R \, dt' \] (4.7)

\[ \varepsilon_s = \frac{-2C_0}{l_s} \varepsilon_R \] (4.8)

Using these equations, corresponding points of specimen stress and strain can be matched to give actual stress versus strain curves. In the practical situation, the strain pulses \( \varepsilon_T \) and \( \varepsilon_R \) are sampled at small time intervals, \( \Delta t \) so that equation (4.7) is used in the form:

\[ \varepsilon_s = \frac{-2C_0}{l_s} \sum_0^t \varepsilon_R \, \Delta t \] (4.9)
4.3 DESCRIPTION OF THE LOUGHBOROUGH SHPB SYSTEM

4.3.1 Historical Development

For over fifteen years the SHPB technique has been used in the Physics Department at Loughborough University to study the dynamic mechanics of a variety of materials. In the early days the incident bar was loaded by impact from a standard 0.22" calibre bullet and sample strains were recorded by an optical shutter method (Griffiths and Martin, 1974). Years later, a compact gas gun was developed to fire a short steel rod projectile at the free end of the incident bar (Parry and Griffiths, 1979). This gas gun, driven solely by atmospheric pressure was a great improvement on the 0.22" bullet system since it produced uniform stress pulses which could be consistently repeated.

Ellwood (1983) used a gas gun of almost identical design except that it was longer 2.4m (cf 1.32m) and had a 20% larger bore at 2.5" which meant that a larger projectile could be used to achieve incident stress waves of larger amplitude.

Ellwood, Griffiths and Parry (1982) modified the apparatus to a 3 bar system in which a 'dummy' specimen was placed in the split between the first two bars while the actual specimen under test was placed between the second and third bars. The result of this modification was to shape the incident loading pulse in such a way as to produce a constant strain-rate during each test.

Walker (1987) devised an optical fibre based system for measuring the impact velocity of the projectile which replaced the mechanical triggering system used by Ellwood. Walker also improved and expanded the software for the Commodore Pet computer, first introduced by Ellwood, to analyse the SHPB data (Parry and Walker, 1988).
B) SHPB bar arrangement

**FIG4.3** Overall arrangement of the Split Hopkinson Pressure Bar System

A) The gas gun
Thus in the passage of time many advancements have been made in the Loughborough SHPB materials testing facility. The current project has not gone without contributing to these improvements, most notably in the design of a new infra-red projectile velocity measuring device (Dixon, 1986). The following sections describe the SHPB arrangement used to test type 224 steel.

4.3.2 Overall View of the SHPB System

Figure 4.3 shows an overall view of the current basic SHPB apparatus. It can be seen that there are two principal components; the gas gun and a 4.5m length of pressure bars.

The gas gun accelerates the projectile until it impacts with the free end of the incident bar thereby creating a stress pulse in the bar of duration

$$T = \frac{2l_p}{C_o}$$  \hspace{1cm} (4.10)

where $l_p$ is the projectile length and $C_o$ is the velocity of longitudinal waves in the bar. The amplitude of the incident wave, $\varepsilon_I$, is expressed by

$$\varepsilon_I = \frac{V}{2C_o}$$  \hspace{1cm} (4.11)

where $V$ is the projectile velocity upon impact.

The incident pulse propagates along the incident bar and a reflected and transmitted pulse are created at the specimen. All three waves are detected by pairs of strain gauges mounted at equidistant points 40 cm either side of the sample. The momentum bar at the end of the line flies away from the rest of the bars after the transmitted pulse has been reflected back as a tensile wave. It is trapped by a plasticine filled box thereby dissipating the energy released by the system without causing damage to the apparatus or injury to personnel.
FIG 4.4 Photograph of the projectile
The following sections describe the components of the Loughborough SHPB facility in more detail.

4.3.3 The Projectile

Figure 4.4 is a photograph of a 25 cm projectile. The essential part of it is a length of 431 stainless steel rod of 1/2" diameter, equal to that of the bars. The rod is held centrally in a cotton reel-shaped body made from PTFE. 'O' rings and brass locating rings allow the rod to slip through the PTFE body upon impact with the Hopkinson bar. The PTFE collars of the projectile allow it to slide easily along the polished bore of the gas gun and also provide two convenient edges from which velocity measurements can be made (see Section 4.4). The projectile was usually cooled in a refrigerator so that it could move freely along the interior of the gas gun without sticking. If left on the laboratory bench for any length of time, the projectile was apt to swell or contract according to the ambient temperature. This would lead to inconsistency of projectile velocity and on warm Summer days the projectile would become jammed in the gun if not cooled before loading.

4.3.4 The Gas Gun

The gas gun depicted on the left hand side of Figure 4.3 is based on the original smaller version designed by Parry and Griffiths (1979). Since it operates by the use of atmospheric pressure only, it is far safer and more convenient to operate than most other gas guns used in materials testing which usually require very high pressures.

The gas gun itself is basically a 2.4m long 321 stainless steel tube having a polished bore of diameter 2.5". The tube is joined to a rotary-type vacuum pump at both its ends via a set of four diaphragm valves. The gun operates by loading the projectile at the end furthest from the Hopkinson bar, evacuating the tube to a pressure below 1 mb
and then suddenly exposing the projectile to atmospheric pressure by the release of a lever.

The lever moves a flat end plate which covers, and thus seals, an aperture plate situated at the end of the gun. The aperture plate has various sized holes (2, 4, 6, 8, 12 mm and two of 20 mm diameter) drilled through it and each of these may be sealed with small easily removable nylon plugs. When the gun is fired (the lever released), atmospheric pressure enters the tube through these holes.

Hence by varying the number and size of the holes unplugged the force on the projectile from atmospheric pressure may be varied and this in turn determines the impact velocity of the projectile on the incident bar. For safety's sake the gun will not fire without the aperture plate in place.

At the impact end of the gun three brass fibre optic mounts are situated (A, B and C) 10 cm apart. These brass mounts form vacuum-proof seals with the wall of the gas gun cylinder and allow three pairs of optical fibres to protrude into the wall of the gun without disturbing the motion of the projectile or the internal vacuum. The height of the optical fibres above the PTFE collars on the projectile can be easily adjusted and is currently set for optimum reflection of infra-red from the collars as the projectile passes. This reflective optical system permitted the measurement of the impact velocity of the projectile (described in Section 4.4) and held the advantage over a transmissive optical system in requiring only three holes in the wall of the gas gun as opposed to six.

A rubber bung with its central core removed is located at the end of the gun. It protects the PTFE guide of the projectile from being smashed against the steel end plate as it slides along the projectile rod after impact.
4.3.5 The Bars
All the bars in the SHPB system are of 1/2" diameter. The incident bar actually comprises two separate 1m long bars. The first of these, into which the projectile collided, is made of 431 martensitic stainless steel (yield strength = 700 MPa). The second bar, which is in intimate contact with the first at one end and with the specimen at the other end, is made of a high strength maraging steel (yield strength = 1300 MPa). The reason for this two bar, incident bar structure was two-fold. Firstly, it provided a convenient split in which to place pulse shaping samples as discussed by Ellwood et al (1982). Secondly, the initial 431 bar tended to attenuate the Pochhammer-Chree oscillations (high frequency stress components discussed in Section 4.7.2) superimposed on top of the loading stress waves which are generated upon impact of the projectile. This meant that the stress wave incident upon the specimen was clearer and more uniform than if the 431 bar had been omitted.

The transmitter bar was again a 1m length of maraging steel rod, while the momentum bar was 1.5m of 431 steel. Before each experimental test on a sample, the alignment of the bars was checked by shining a diffuse light from a small hand-held torch behind each bar/bar and bar/specimen interface. In this way any chinks of light observed revealed a misalignment which would then be corrected. It was important to ensure that all bar end faces were flush and parallel so that damage to the bars was prevented and unwanted reflections from the bar interfaces were avoided. Strong elastic cords were used to pull together any bar/bar or bar/specimen interfaces (except for the momentum bar) to avoid small gaps occurring.

4.3.6 The Mechanical Supports
The gas gun and the Hopkinson bar rest on two separate mild steel supports so that unwanted vibrations from the gas gun as it is being
fired are not transmitted to the strain gauges mounted on the bar network.

The first Hopkinson bar is coupled to the gas gun via a pair of vacuum tight 'O-rings' which permit a certain amount of movement of the bar through them immediately after impact. These O-rings prevent any vibrations generated in the initial stages of the projectile launch from being transmitted to the bars themselves.

The bars themselves rest on 'V'-shaped nylon clamps which are mounted on commercial optical bench stands. These may be placed in any adjustable position along a 4m length of optical bench. The stands have a screw driven movement in two perpendicular directions allowing variation in height and transverse distance from the centre of the optical bench. This arrangement enabled precise alignment of the bars and also permitted bars of different lengths and configurations to be used.

It was important not to overtighten the V-shaped nylon blocks which clamped the bars as this would have caused spurious reflections to occur during the passage of the loading stress wave. For this reason it was ensured that for every test the bars were able to slide freely through the nylon blocks after the initial projectile impact.

4.3.7 The Strain Gauges

The strain gauges used in this experiment were type FLA-6-17 (Tokyo Sokki Kenkyujo Co Ltd) designed to give a linear change in resistance with strain up to 2%. The strain gauges had gauge lengths of 6 mm and were affixed to the bars in diametrically opposite pairs so that the effects of bending waves would cancel and also to double the size of signals from them. Each pair was mounted at a distance of 40 cm either side of the sample on bar surfaces slightly abraded with 400 grit
silicon carbide paper and cleaned with methyl-pentone. A cyanoacrylate superglue adhesive (type CN-2) was used to fix them to the bars, as recommended by the supplier of the strain gauges (Techni Measure Ltd).

The gauges have a nominal resistance, $R_s$ of 120Ω. The relationship between change in resistance, $dR_s$ and strain is given by:

$$
\epsilon = \frac{1}{F} \frac{dR_s}{R_s}
$$

(4.12)

where $F$ is the gauge factor. For the batch of gauges used here $F$ equalled 2.12.

4.3.8 Specimens and Specimen Lubrication

Two sizes of solid cylindrical specimens were used: 10 mm (± 0.1 mm) diameter x 5 mm (± 0.05 mm) length and 8 mm (± 0.1 mm) diameter and 4 mm (± 0.05 mm) length. Details of measurements of surface flatness and parallelism of the samples were presented in Section 3.2. The specimens were machined on a lathe in the Physics Department's mechanical workshop. A coolant was used to ensure that high temperatures which may have disturbed the internal structure of the steel were not incurred. After cutting, the specimens were finely ground to produce smooth flat parallel faces. They were not heat treated in any way before being mechanically tested in compression. Details of the original locations of the specimens within the type 224 steel billets supplied by the UKAEA (Winfrith) are described at the beginning of Chapter 8. The same specimen types were used for compression testing at lower strain rates (Chapter 3).

The importance of lubrication in SHPB tests has been clearly demonstrated [Ellwood (1983) and Follansbee et al (1984)]. To corroborate this point, the first twenty samples to be tested using the SHPB were not lubricated, thereafter ALL samples were lubricated.
FIG. 4.5 A Schematic Diagram of the old Projectile Velocity Measuring System.
with silicone grease for tests at room temperature and $+150^\circ C$. At $-40^\circ C$ and $-110^\circ C$ PTFE spray was used while at $+300^\circ C$ molybdenum disulphide ('Molyslip') was used. The effects of lubrication are discussed at greater length in the results in Chapter 8.

4.4 RECORDING INSTRUMENTATION AND DATA ANALYSIS

4.4.1 System for Measuring Impact Velocity of Projectile

At the start of the current investigation a new infra-red system was designed to measure the velocity of the projectile just before impact with the Hopkinson bar (Dixon, 1986). Schematically it is depicted in Figure 4.5. The projectile velocity is determined from the time taken for it to travel two consecutive distances (i.e. AB and BC), each of 10 cm, immediately before impact.

Three pairs of optical fibre probes are inserted through holes in the wall of the gas gun at points A, B and C. In each pair, one of the fibres transmits infra-red radiation from a GaAlAs LED (400 µW at 820 35 nm) while the other fibre acts as a receiver of any radiation reflected from the collars of the PTFE projectile guide as it passes the probe.

The ends of each transmitter-receiver fibre pair are cut at an angle $\gamma$ as shown in Figure 4.6. This causes refraction of the emitted beam from the transmitting fibre and also refraction of the reflected beam into the receiving fibre as indicated by the ray lines. By this method it was found that more radiation could be recaptured after reflection from the PTFE guide than if the ends had been left flat (i.e. $\gamma = 0^\circ$). A series of simple experiments using fibres cut at various angles, showed that the greatest return of radiation through the receiving fibre was achieved if $\gamma$ was equal to $40^\circ$. 
FIG 4.6 Showing the Refractive Effect of cutting the Optical Fibres at a non-perpendicular angle to the fibre-axis.
It was also found that the distance \( d \) of the fibre ends from the PTFE reflecting surface is critical. The maximum return signal is detected when \( d \) is such that the centre of the emerging beam is reflected back into the centre of the receiving fibre as illustrated in Figure 4.6. Hence in the current fibre optic system all three fibre pairs have \( \gamma = 40^\circ \) and \( d \) has been set for optimum reflection (i.e. \( d = 1.2 \text{ mm} \)).

The electronics of the projectile velocity measuring system are shown in Figure 4.7A (the transmitter circuit) and 4.7B (the receiver circuit). The transmitter circuit consists quite simply of a 2% tolerance metal oxide resistor of low temperature coefficient in series with an infra red 'sweet spot' emitter diode (RS 303-309). This LED was specially selected from a range of possible alternatives since it contained a small glass bead lens which focused the emitted infra red into the connecting optical fibre with greatly improved efficiency. The series resistor was chosen to be 60\,\Omega so that the current through the LED was \( (55 \pm 2) \text{ mA} \) which was close to the specified optimum operating conditions for the diode.

Figure 4.7B shows the receiver circuit. When infra red radiation is reflected from one of the PTFE projectile collars, it is passed via the receiving optical fibre to an RS 303-292 'sweet spot' photodiode. For optimum sensitivity this device also embodied a small glass bead lens to focus infra red from the optical fibre onto the photosensitive area.

The first stage of the receiver circuit behaves as a transimpedance amplifier since the CA3140 op-amp acts as a current to voltage converter. When infra red radiation falls on the photodiode a current, \( i \), is generated. Virtually all this current flows through the 100 \,\text{k}\Omega feedback resistor thereby producing a voltage of \( 10^5 \times i \)
FIG4.7A The transmitter circuit

FIG4.7B The New Receiver Circuit
volts at the output of the first stage. The signal is then amplified by a factor of ten by the 2nd stage so that it reaches a level of approximately +2V for maximum reflection of infra red from the PTFE guide.

CA3140 FET op-amps were chosen for the first and second stages for their high input impedance and low noise characteristics. The two 47 pF capacitors have been included to minimise the amplification of unwanted diode noise which may cause erroneous triggering in stage 3.

In the third stage of the circuit a 311 comparator compares the amplified input signal arriving at its inverting input, A, with the +0.56V level on its non-inverting input, B. As long as the voltage on the inverting input is lower than this threshold level, the output, C of the comparator remains at a fixed +5V. However as soon as the input goes higher than the threshold, when light is being reflected back from the PTFE guide, the comparator output drops to zero.

The 74LS00 inverter may be switched into the circuit so that the pulses generated by the passage of the projectile are all positive going (0 → +5V) as indicated in Figure 4.7B. The sum of the response time and rise time of the whole circuit is no more than 100 μS. In other words if reflected infra-red radiation falls on the photodiode in stage 1 the output level of the circuit (inverter switched in) will reach +5V in less than 100 μS later. Since the projectile velocity never exceeds 40 ms⁻¹ this means that the maximum error in the velocity measurement of this system is about 4% and is smaller than this for the lower projectile velocities. The percentage error in the velocity measurement may be calculated roughly as 0.1 x the actual recorded velocity (in ms⁻¹).
**FIG 4.8 Output pulses from amplifier of Fig 4.7B (point C)**

**Vertical scale**: 5V/div.

**Horizontal scale**: 2ms/div.

**Projectile velocity** \(\approx 35 \text{ms}^{-1}\)}
The complete receiver comprises three identical and separate amplifier channels each one being the circuit of Figure 4.7. The outputs from the three channels are fed into two Racal-Dana 9901 timer/counters which record the two times for the projectile to traverse the fibre optic probes A B and B C. In this way an indication is obtained of whether the projectile is accelerating or decelerating before impact with the Hopkinson bar.

Figure 4.8 shows typical output pulses produced by channels A and B of the receiver circuit for a projectile velocity of 35 ms\(^{-1}\) with the inverter on the output stage switched out. Note that the start of the second pulse in the upper trace is coincident with the start of the first pulse on the lower trace since the leading edges of the front and back collars of the projectile are 10 cm apart which is the same distance as between the fibre optic probes.

A calibration was made of projectile impact velocity as measured by the infra red system described above for different gun apertures. The resulting curve is shown in Figure 4.9. Average projectile velocities were calculated from the means of the two times recorded in each 'shot' as it was found that there was little or no acceleration of the projectile just before impact.

Along the horizontal axis of the graph in Figure 4.9 is plotted the sum of the radii squared of the unplugged holes (i.e. the square of the effective aperture radius, \(ER^2\)) when the gun is fired. This quantity equals the total aperture area divided by \(\pi\). The error bars drawn in Figure 4.9 represent the extreme variations of velocity for each aperture combination. A best fit curve has been drawn through the points.
The calibration curve proved useful in all subsequent work using the SHPB since it allowed the author to choose appropriate aperture combinations to achieve a desired strain rate in the specimen. The maximum projectile velocity recorded was 39 ms$^{-1}$ at an effective radius of 10 mm. No attempt was made to exceed this speed as this may have resulted in an impact stress of more than 8 Kbar which would cause damage to the 431 stainless steel pressure bar.

The variability in velocity for a given aperture combination (indicated by the error bars in Figure 4.9) occurs because of the difficulty in reproducing the projectile firing conditions. For two 'shots' to produce the same projectile velocity the initial gun pressure and projectile temperature must be identical in each case. There is more variation and less consistency in projectile velocity when more than two aperture holes are used.

4.4.2 Data Acquisition
The SHPB data acquisition system is outlined in schematic form in Figure 4.10. The strain gauge pairs used to detect the incident and reflected stress curves and the transmitted stress wave are denoted by SG1 and SG2 respectively. Each pair of total resistance 240 $\Omega$, were wired in series and formed part of a simple strain gauge circuit as shown in Figure 4.11. The circuit is a simple potential divider which includes a 2.2K$\Omega$ ballast resistor and a 90V d.c. stabilised power supply (Farnell type E350) as well as the gauge pair. The battery polarity is configured in such a way that a compressive strain pulse reduces $R_s$ and thereby produces a positive going output voltage, $V_s$.

It can be shown (e.g. Ellwood, 1983) that at the low strain levels in the bars (<0.3%) the change in voltage, $dV_s$ across the strain gauges is linearly proportional to the strain, $\epsilon$, in the gauges according to the equation
FIG 4.10 Block diagram of Data Acquisition System

FIG 4.11 Strain gauge potential divider circuit

- Power supply $E = 90V$
- Ballast resistor $R_b = 2.2k\Omega$
- Strain gauge resistance $R_g = 120\Omega$
- Resistance of strain gauge pair $R_s = 240\Omega$
\[
\varepsilon = \left( \frac{n + 1}{n \times F^2} \right)^2 dV_s
\]  

(4.13)

where \( n \) is the ratio of ballast resistance to total gauge resistance \( (= R_b/R_s) \), and \( F \) is the gauge factor as defined by equation (4.10) and \( E \) is the power supply voltage.

The voltages across SG1 and SG2 are fed to linear high bandwidth (> 20 MHz) amplifiers contained in the plug-in units of a Tektronix CRO (Tektronix type 556 dual beam oscilloscope). In compressive testing gains of 10 were usually used, but for the tensile tests where the signal for the transmitted stress pulse from SG1 was small gains of X25 or even X100 were employed.

The amplified signals are fed to the two input channels of a Datalab DL912 8-bit amplitude resolution transient recorder which digitises and stores each strain gauge signal in separate 4 kbyte digital memories. Hence the total electronic gain of the system is determined by the combination of the gain of the oscilloscope amplifier plus that of the transient recorder.

The full scale input voltage on both input channels of the transient recorder could be selected between 0.1V and 10V. Hence the voltage resolution of the transient recorder was determined as the selected full scale input voltage divided by 255. Usually before a shot, the full scale voltage setting was chosen to be as low as possible without the likelihood of saturation occurring. This allowed the maximum amount of memory to be used and therefore meant that the voltage resolution was also at a maximum. However a certain amount of experience was required before optimum full scale voltages could be selected reliably.
Throughout this thesis a number of typical traces are presented. On some of the earlier ones (such as Figure 4.16 later) a small amount of 'stepping' on the traces may be observed which illustrates the digitising action of the transient recorder.

The temporal resolution, $\Delta t$ of the system is dependent on strain gauge length, $L_g$ and the longitudinal wave velocity, $C_o$ in the pressure bars. This is given by:

$$\Delta t = \frac{L_g}{C_o}$$  (4.14)

Since $L_g = 6 \text{ mm}$ and $C_o = 5 \text{ mm/\mu s}$, $\Delta t$ is approximately $1 \mu s$ and hence the optimum sampling rate is $1 \text{ MHz}$. At this sampling rate 4 ms of data are recorded in each channel of the transient recorder.

A polaroid photograph is taken from the Tektronix oscilloscope screen of the incident, reflected and transmitted waveforms captured by the transient recorder for every experimental shot. This meant that if the stored data was lost or became corrupted an analysis could still be made from the photograph. The photographs also proved useful in establishing the starting points of the strain pulses (see Section 4.4.4).

4.4.3 Data Transfer

The software currently used for data transfer from the transient recorder to a Commodore 3032 microcomputer (commonly known as 'PET') and subsequent data analysis was developed by Walker (1987). The transfer of 400 data bytes from each digital memory of the transient recorder occurs via an IEEE interface. From the 'PET' the data is transferred for permanent storage on a floppy disk.
4.4.4 Data Analysis

The data analysis program starts by identifying each of the strain pulses $\varepsilon_I$, $\varepsilon_R$ and $\varepsilon_T$ and their starting points. Next the baseline or zero strain level is determined for each channel. The baseline for channel 1 was evaluated by averaging the 10 data points which occur 130 $\mu$s after the start of the reflected pulse. In a similar fashion the baseline for channel 2 was calculated as the mean of the 10 points which occur 130 $\mu$s after the start of the transmitted pulse. If necessary at this stage two modifications may be made to the standard analysis:

a) In the case of imperfect alignment between the pressure bars and the test specimen (i.e. when one or both faces of the specimen are only in partial contact with the face of the pressure bar) a delay will occur in the arrival of the strain pulse at SG2 compared with the arrival of the reflected pulse at SG1. The initial part of the reflected pulse will be of high amplitude due to the closing up of the air gap during compression and this in turn will result in an over-estimation of the initial specimen strain. Figure 4.12(a) and (b) schematically compares the type of reflected pulses produced by a perfectly aligned and imperfectly aligned specimen. The program allows the operator to modify the start of the reflected pulse so that the portion of the pulse which arrived too soon is removed and the pulse becomes modified to the new shape illustrated by the dashed line in Figure 4.12(b). Once the new starting point has been chosen the computer effectively draws a line parallel to the original initial slope of the pulse but beginning at this new start point. Hence the premature part of the original pulse which may be ascribable to a misalignment effect is discarded. This is a very valuable feature of the analysis program since it means that data which may have been discarded due to an artificially large reflected peak can be corrected and sensibly analysed.
(a) Reflected pulse from perfectly aligned specimen
(b) Reflected pulse from imperfectly aligned specimen

FIG 4.12

(a) Reflected pulse sloping, baseline correction

FIG 4.13
Reflected pulse sloping baseline correction
b) Often in SHPB tests the baseline from SG1 does not return to zero after the passage of the incident strain pulse so that the reflected pulse may commence or terminate at a strain level above or below the 'zero' strain level of the baseline. In such a case, the program permits a correction which introduces a sloping baseline so that the values of strain at each end of the reflected pulse are set to zero. Figure 4.13 illustrates an exaggerated correction where the dotted line is the previously estimated baseline and the dashed line is the modified sloping baseline.

In handling the analysis software the operator is able to override any incorrect computer decisions due to non-ideal pulse shapes. At each microsecond interval, values of engineering strain, strain-rate and stress are calculated according to equations (4.6) to (4.9) which are then converted to true stress, strain and strain-rate by equations (3.6) and (3.7). Numerous manual analyses from polaroid results of the original waveforms have established the validity of the computer method.

4.4.5 Graph Plotting
The last stage of the computer routine is the plotting of the calculated true strain, strain-rate and stress in graphical form. This is done on a high resolution digital XY plotter (JJ Lloyds PD4) coupled to the 'PET'. The main Hopkinson bar program plus the graph plotting program are presented in Appendix I.

4.5 ELEVATED TEMPERATURE TESTING (150 AND 300°C)

Chiddister and Malvern (1963) were the first to report elevated temperature tests in the SHPB method. They used a cylindrical 12" ceramic furnace placed round the Hopkinson bar to heat aluminium
specimens up to temperatures of 550°C. They found that the heat generated in the pressure bars altered the shape of the stress waves and thus had to apply a correction for this.

Since then numerous tests have been conducted at elevated temperatures using the SHPB method, most workers favouring some form of electrical coil furnace to heat the specimen although quartz lamps (Watson and Ripperger, 1969 and induction heating (Rosenberg et al, 1986) have also been used. The main concern in such tests, is the effect of the increased temperature of the bars on the velocity and shape of the strain pulses. Frantz et al (1984) have devised a system which overcomes this problem by separately heating the specimen up to the desired temperature and then bringing the pressure bars into contact with it for less than 150 ms before the actual test.

In the current SHPB system the Nichrome wire coil heater described in Section 3.11.1 was used to perform tests at specimen temperatures of 150°C and 300°C. As in the low strain-rate testing a K-type thermocouple mounted on the bars at a point within 2 mm of the specimen was used to monitor the temperature. The relatively short length of the heater and the fast heating time (300°C in 4 minutes) meant that the temperature of the bars outside the ceramic tube did not rise appreciably. Hence no thermal effect of the bars on the shape of the incident, reflected and transmitted waveforms was observed. This was consistent with the findings of Ellwood et al (1984) who used the same heating method for investigating 321 stainless steel up to a temperature of 600°C.

4.6 LOW TEMPERATURE TESTING (-110°C AND -40°C)

Low temperature work was carried out using a liquid nitrogen based system similar to that described in Section 3.10.1 used in the low
FIG 4.14 Photograph of cooling box for SHPB compression tests.
strain-rate tests. A rectangular shaped copper box (15 mm long x 6 mm high x 9 mm wide) was made which enclosed the Hopkinson bar plus sample (see photograph in Figure 4.14). Cooling was achieved by filling the hollow walls of the box with liquid nitrogen which had the effect of cooling the air around the bars and specimen. The whole of the box was well lagged with polystyrene during a test. Since heat conduction through the bars was small temperatures as low as -110°C could be obtained with ease. The lowest temperature recorded using this system was -140°C but this required several liquid nitrogen refills of the cooling chamber.

4.7 FACTORS WHICH AFFECT THE ACCURACY OF THE SHPB METHOD

Since the early development (Kolsky, 1948) of the SHPB in its current form the validity of the method has been questioned by many investigators. Nowadays it is accepted that the SHPB can produce results which represent the true dynamic mechanical behaviour of the material under test provided that due consideration has been paid to the effects of friction, inertia, wave dispersion and stress equilibrium in the sample. Recently, Follansbee (1987b) has reviewed the importance of these effects in SHPB work. This section deals with such effects and considers their significance with regard to the Loughborough SHPB system.

The issue of stress equilibrium within the specimen is a necessary requirement of equation (4.3) and is discussed in the next Chapter in which a computer program to test the validity of this assumption for a variety of materials is described.

4.7.1 Friction and Inertia Effects
Friction can be a problem in all compression tests, regardless of strain-rate, if proper consideration is not given to specimen
dimensions and lubrication. Its effect is to restrict the radial movement of a specimen and cause an overestimate of its uniaxial stiffness and strength. Axial and radial inertia too may cause errors in dynamic tests by opposing the equilibration of stresses.

In his original work, Kolsky (1949) attempted to minimise the effects of axial inertia by using thin specimens of length to diameter ratio \(l_s/d_s\) of only 0.1 or less. However, Davies and Hunter (1963) showed that in order to minimise frictional effects the sample aspect ratio should be:

\[
\frac{l_s}{d_s} = \frac{\nu_s \sqrt{3}}{2}
\]

(4.15)

where \(\nu_s\) is the Poisson's ratio for the sample material. These two investigators also derived a condition for which inertial effects can be considered negligible, i.e.

\[
\frac{d\sigma}{dt} > \frac{\pi^2 \rho_s l_s^2}{T^2}
\]

(4.16)

where \(T\) is the rise-time of the loading stress wave and \(\rho_s\) is the density of the specimen material. Davies and Hunter recommended that the test data should be rejected if this inequality is violated.

The choice of optimum \(l_s/d_s\) ratio for an SHPB sample, then, is a compromise between what is best in terms of minimising frictional effects and what is best for the minimisation of inertial effects. One dimensional analyses of the problem have been carried out by Chiu and Neubert (1967) and Jahsman (1971) but the most comprehensive computer analysis in two dimensions was carried out by Bertholf and Karnes (1975). These investigators concluded that the optimum \(l_s/d_s\) ratio is 0.5 which agrees reasonably with that quoted by Davies and Hunter (equation 4.15).
Bertholf and Karnes also proposed that, based on inertia, the experimental results from an SHPB test are valid provided that:

\[ d_s < 5 \times 10^3 \text{ cm/s} \]  

(4.17)

and that the loading pulse has a rise time, \( T \), such that

\[ T/d_s > 16 \mu \text{s cm}^{-1} \]  

(4.18)

In the Loughborough SHPB system used here \( d_s = 0.8 \) or 1 cm and \( T \) is of the order of 15 \( \mu \)s with the 431 and maraging steel bars. Thus equations (4.17) and (4.18) confirm the validity of the Loughborough method up to strain-rates of 5000 s\(^{-1}\). It has been found that equation (4.18) is consistent with equation (4.16) for most metals. Hence a ramped loading wave is desirable in all SHPB experiments.

4.7.2 Wave Dispersion and Pochhammer-Chree Oscillations

Figure 4.15 shows four different SHPB compression shots on type 224 steel specimens recorded in the Loughborough system for four different projectile velocities. In each photograph the upper trace depicts the incident and reflected waves as recorded by SG1 (in Figure 4.10) while in every lower trace can be seen the transmitted wave recorded by SG2. As expected from the theory the reflected and transmitted waves are coincident in time.

The important feature to notice in Figures 4.15B and C is that high frequency oscillations are present in the pulses. The amplitude and frequency of these oscillations are dependent on the nature of the initial projectile impact, the bar material and any misalignments along the length of the bars. Misalignments or small air gaps at the bar/bar or bar/specimen interfaces tend to accentuate the oscillation.
FIG4.15 4 Typical SHPB Shots at 4 Different Projectile Impact Velocities

All Horizontal Scales: - 40μs/div

A) Impact velocity = 17ms⁻¹; Vertical scales: Upper Trace 0.06%/div
Lower Trace 0.12%/div

B) Impact velocity = 26ms⁻¹; Vertical scale for both traces: 0.12%/division
C) Impact velocity = 31 ms\(^{-1}\); Vertical scale for both traces: 0.12% per division

D) Impact velocity = 38 ms\(^{-1}\); Vertical scale for both traces: 0.12% per division
The Pochhammer-Chree phenomenon is a direct consequence of the Fourier components of a pulse suffering from elastic wave dispersion, i.e. from the fact that the velocity of a wave (Fourier component) in a bar depends on its wavelength relative to the bar diameter. Thus if a near perfect trapezoidal incident stress pulse is created upon impact of the projectile on the bar then by the time the pulse reaches SG1, its higher frequency components will be lagging behind the leading edge.

The mathematical description for this type of elastic wave behaviour was first derived by Pochhammer (1876) and Chree (1889) and their solutions were applied to elastic pressure bars by Davies (1948). The appearance of Pochhammer-Chree oscillations in SHPB data is a nuisance since they mask the true behaviour of the material under test. This is especially true for materials which show a definite discontinuous yield point such as 224-steal in which case the upper and lower yield points may be easily over and under estimated respectively.

Pochhammer-Chree oscillations can be 'smoothed' out by electronic filtering or by the application of Fourier correction techniques (Follansbee and Frantz, 1983), but it should always be remembered that the recorded incident pulse represents the actual stress profile of the pulse which loads the specimen. Thus the presence of Pochhammer-Chree oscillations in the incident pulse may give rise to a sequence of unloading and reloading fronts within the specimen so that localised regions may experience stress and strain histories which differ substantially from the average behaviour over the whole specimen.

For this reason, the philosophy of Loughborough is to produce incident pulses which are as clean and oscillation free as possible. It has been observed (Dixon, 1988) that high strength maraging steel is particularly good transmitter of the Pochhammer-Chree oscillations,
FIG4.16 An oscillation free trace produced for carbon 224 samples (Impact velocity: 35ms⁻¹. Sample size: 10 x 5 mm. The actual SHPB pulses for this record are shown in Figure 3).

Projectile Impact velocity = 35ms⁻¹
Vertical Scale = -1% strain/division
Horizontal Scale = 40μs/division
resulting from rapid projectile impact. To minimise these oscillations, the first bar to be struck by the projectile in the Loughborough system is 1m of 431 stainless steel. This is followed by 1m of maraging steel bar which acts as the usual incident bar, and on which the strain gauges SG1 are affixed. The 431 bar has a lower yield strength than the maraging steel and has the effect of damping the oscillations generated by projectile impact before reaching the specimen.

As mentioned previously, at each bar/bar and bar/specimen interface restoring forces are applied by pieces of elastic material attached to the bars and tied around the nearest convenient optical support. This improves the overall alignment of the system and removes the possibility of air gaps being present at the interfaces due to gas-gun generated vibrations. With such a simple elastic band arrangement it is possible to record traces which are almost devoid of any oscillations such as the one shown in Figure 4.16. For this particular test the upper and lower yield points in the computer plot of true stress versus true strain are determined unequivocally.

4.8 SUMMARY

This Chapter has dealt with the historical development and the theory behind the split Hopkinson pressure bar method which has now become a well established technique for testing materials at high strain-rates. In particular a detailed account of the Loughborough SHPB system has been given. Investigations carried out during this project have proved that the system at Loughborough is a valid one from which meaningful results may be obtained. Factors which affect the accuracy of the Loughborough system have been discussed.

Results of SHPB tests on type 224 steel carried out during this project using the Loughborough system are presented in Chapter 9.
5.1 INTRODUCTION

When a stress wave is incident on a sample in the SHPB system, reflections will occur at both interfaces, bar/sample and then sample/bar. Reflections from the second interface (sample/bar) will return to the first interface (bar/sample) and will again be partially transmitted and reflected upon reaching it. The process continues so that multiple reflections will occur within the Hopkinson Bar sample and a succession of reflected waves become 'trapped' inside the sample propagating back and forth between the two interfaces.

Theoretically, a reflected wave 'trapped' in this manner will undergo an infinite number of reflections between the interfaces; however at each reflection the intensity of stress will decrease since a fraction of the wave is always transmitted. After only a few reflections the wave will have decayed to a negligible amplitude.

The effect of these multiple reflections within the sample is to produce a dispersion of the incident wave. Thus if the incident pulse has a sharp rise time before reaching a constant maximum stress, the transmitted pulse rise time will be less sharp. The increase in rise time apparent in the transmitted pulse will depend on the severity of the mismatch between bar and sample and can lead to imprecision in the analysis in the yield region.

Furthermore, in Chapter 4 it was shown that the theory upon which the SHPB analysis is based assumes that
where $\varepsilon$ is the strain produced in the pressure bars and the subscripts, T, I and R refer to the transmitted, incident and reflected pulses respectively. The condition defined by equation (4.5) above is only true if the stresses and therefore the forces are equal in equilibrium on either side of the sample. This equilibrium condition will not arise immediately a stress wave is incident on a SHPB specimen, but will occur after several reflections have taken place inside the sample (Hauser, 1966).

Lindholm (1964) investigated the equilibrium condition experimentally for a variety of specimen materials and lengths by adding $\varepsilon_R$ and $\varepsilon_T$ in a differential amplifier and comparing the sum with $\varepsilon_I$. He observed that the differences recorded were within the experimental accuracy and thus could be safely neglected.

Follansbee (1986), concerned about the doubtful validity of using equation (4.5), elected instead to use equations (4.3) and (4.4) which involved making a precalibration of the timing of the reflected pulse with no transmitter bar, and the timing of the transmitted pulse with sample present, both relative to the leading edge of the incident pulse. In this way the timing between $\varepsilon_I$, $\varepsilon_R$ and $\varepsilon_T$ are more accurately established during the actual test but the analysis becomes lengthier.

In this Chapter, a computer program is described which can be used to investigate the significance of multiple reflections in establishing the rise and fall times of the transmitted and reflected waves respectively. This would prove useful in designing dummy samples for pulse shaping in the three bar, SHPB system introduced by Ellwood et al (1982).
Furthermore, the program has been used to calculate the time taken for stress equilibrium between both faces of the specimen to be achieved. This has allowed an assessment of the validity of the Loughborough system for the current experiment on 224 steel as well as for other materials.

5.2 ONE DIMENSIONAL THEORY OF STRESS WAVES IN AN SHPB SAMPLE

5.2.1 Initial Assumptions

In this section the theory behind the generation of multiple reflections within a SHPB sample is developed starting with a general consideration of a wave incident at an interface between two bars of different material and diameter. The equations so deduced are then applied to the SHPB sample having two interfaces.

The theory assumes that:

1. the stress distribution is uniform radially through the bars and sample;
2. the cross-sectional area of bars and the sample remain constant throughout the passage of the stress waves;
3. the bar and sample materials remain homogeneous and elastic at all times.

Before commencing it is worth bearing in mind the following figures. The incident stress pulses in the Loughborough SHPB system typically have rise times of between 5 and 10 μs and total duration 100 μs. The traverse time of a wave in a 5 mm long steel sample is approximately 1 μs.
FIG5.1 Reflection and transmission of a stress wave at an interface, AB.

FIG5.2 Transmission and reflection of stress at the interfaces NN and 00 of a SHPB sample.
5.2.2 Reflection and Transmission of Stress at a Discontinuity

Consider the situation of an elastic wave of stress \( \sigma_I \), incident on a discontinuity as depicted in Figure 5.1 where

\[
\begin{align*}
A_1 &= \text{cross-sectional area of } S_1, \\
A_2 &= \text{cross-sectional area of } S_2 \\
\rho_1 &= \text{density of } S_1, \\
\rho_2 &= \text{density of } S_2 \\
C_1 &= \text{velocity of sound in } S_1, \\
C_2 &= \text{velocity of sound in } S_2
\end{align*}
\]

- \( \sigma_I \) = the stress wave incident on AB
- \( \sigma_R \) = reflected stress wave which may be tensile or compressive
- \( \sigma_T \) = transmitted stress wave

\( V_{I,R,T} \) are the particle velocities in the materials \( S_1 \) and \( S_2 \) due to the incident, reflected and transmitted stress waves respectively.

Part of the wave will be transmitted with stress, \( \sigma_T \) and part of it reflected with stress, \( \sigma_R \). \( \sigma_T \) and \( \sigma_R \) may be calculated as fractions of \( \sigma_I \) if the following conditions are satisfied at AB:

i) the forces on plane AB acting from \( S_1 \) and \( S_2 \) are at all times equal, and

ii) the particle velocity in plane AB for \( S_1 \) and \( S_2 \) are equal.

According to (i) we have, assuming \( \sigma_I, \sigma_R, \sigma_T \) are compressive:

\[
A_1 (\sigma_I + \sigma_R) = A_2 \sigma_T \tag{5.1}
\]

and, by (ii)

\[
V_I - V_R = V_T \tag{5.2}
\]

Now, in general, stress (\( \sigma \)) is related to density (\( \rho \)), sound speed (\( C \)) and particle speed (\( V \)) by
\[ \sigma = \rho CV \]

then from (5.2):

\[ \frac{\sigma_I}{\rho_1 C_1} - \frac{\sigma_R}{\rho_1 C_1} = \frac{\sigma_T}{\rho_2 C_2} \quad (5.2a) \]

From (5.1) and (5.2a):

\[ \sigma_T = \frac{2 \rho_1 \rho_2 C_2}{\rho_2 C_2 + \rho_1 C_1} \quad \sigma_I \quad (5.3) \]

\[ \sigma_R = \frac{\rho_2 C_2 - \rho_1 C_1}{\rho_2 C_2 + \rho_1 C_1} \quad \sigma_I \quad (5.4) \]

\( \rho C \) is often referred to as the mechanical impedance, \( Z \).

5.2.3 Multiple Reflections in an SHPB Specimen

The above theory can be readily applied to the situation of a sample sandwiched between the incident and transmitter bars in the SHPB system (Figure 5.2) where:

\[ \rho_B = \text{density of bars} \quad \quad C_B = \text{sound velocity in bars} \]
\[ \rho_S = \text{density of sample} \quad \quad C_S = \text{sound velocity in sample} \]
\[ d_B = \text{diameter of bar} \quad \quad L = \text{length of sample} \]
\[ d_S = \text{diameter of sample} \]

Hence the cross-sectional area of the bars and samples are given by:

\[ A_B = \pi d_B^2 / 4 \quad \text{and} \quad A_S = \pi d_S^2 / d \quad \text{respectively} \quad (5.5) \]
FIG 5.3 Reflection and transmission of stress from face NN of an SHPB Sample at time, $T = 0$.

FIG 5.4 Reflection and transmission of stress from faces 00 and NN of an SHPB sample at time, $T = L/C_s$

FIG 5.5 Reflection and transmission of stress in a SHPB sample at time, $T = 2L/C_2$
The mechanical impedances of the bars and sample are given by:

\[ Z_B = \rho_B C_B \quad \text{and} \quad Z_S = \rho_S C_S \text{ respectively} \quad (5.6) \]

Now consider an elastic stress wave incident on the first bar-sample interface NN. It is convenient to allow the first reflection from this interface to occur at time, \( T = 0 \) (Figure 5.3), where

\[ \sigma_I|_0 = \text{incident stress} \quad \sigma_{T1} = \text{stress transmitted into sample} \]

\[ \sigma_R|_0 = \text{reflected stress} \]

If the incident stress wave has a finite duration, then the stress, \( \sigma_I \), may be time dependent.

Using equations (5.3), (5.4) and (5.6) above we obtain:

\[ \sigma_R|_0 = \left( \frac{A_S Z_S - A_B Z_B}{A_S Z_S + A_B Z_B} \right) \sigma_I|_0 \quad (5.7) \]

(the subscript '0' refers to \( T = 0 \))

and the stress transmitted to the sample:

\[ \sigma_{T1} = \left( \frac{2 A_B Z_S}{A_S Z_S + A_B Z_B} \right) \sigma_I|_0 \quad (5.8) \]

It is important to note at this stage that if \( \sigma_I|_0 \) is compressive stress (+ve) then according to (5.8), \( \sigma_{T1} \) will also be compressive (+ve). However, from (5.7) it can be seen that \( \sigma_R|_0 \) may be compressive (+ve) or tensile (-ve) depending on the mechanical impedance, \( Z_S \), of the sample and its cross-sectional area \( A_S \). For most tests performed on a SHPB system \( A_B Z_B > A_S Z_S \) so that the reflected waves tend to be tensile (-ve).
After time $T = \frac{L}{C_S}$ the transmitted stress wave from NN will have reached the second bar-sample interface 00 where further reflection and transmission will occur (Figure 5.4). Typically $L/C_S = 1$ µs when the sample is a 5 mm long steel cylinder. In Figure 5.4, $\sigma_I |_1 \sigma_R |_1$ and $\sigma_T |_1$ represent the stress intensities of the incident, reflected and transmitted stress waves after one traverse time ($1 \times L/C_S$); hence the subscript '1'. $\sigma_{R1}$ is the reflected stress from 00.

Applying equations (5.3) - (5.6) again yields:

$$
\sigma_R |_1 = \left( \frac{A_SZ_S - A_BZ_B}{A_SZ_S + A_BZ_B} \right) \sigma_I |_1
$$

which is the same equation as (5.7) but $L/C_S$ later in time.

Also

$$
\sigma_T |_1 = \left( \frac{2A_SZ_B}{A_SZ_S + A_BZ_B} \right) \sigma_{T1}
$$

i.e.

$$
\sigma_T |_1 = \left( \frac{2A_SZ_B}{A_SZ_S + A_BZ_B} \right) \times \left( \frac{2A_BZ_S}{A_SZ_S + A_BZ_B} \right) \sigma_I |_0
$$

The component, $\sigma_{R1}$, reflected from 00 at time $T = L/C_S$ is given by:

$$
\sigma_{R1} = \left( \frac{A_BZ_B - A_SZ_S}{A_SZ_S + A_BZ_B} \right) \times \left( \frac{2A_BZ_S}{A_SZ_S + A_BZ_B} \right) \sigma_I |_0
$$

It is worth noting here that according to equation (5.10) $\sigma_T |_1$ will have the same sign as $\sigma_I |_0$ (+ve for a compressive wave). On the other hand, $\sigma_{R1}$ may be tensile or compressive depending on whether $A_SZ_S > A_BZ_B$; comparing equations (5.9) and (5.11) it is easy to see that $\sigma_{R1}$ will be of opposite sign to that of $\sigma_R |_1$. 
After time $T = 2L/C_S$ this reflected wave will reach the interface NN where again reflection and transmission will occur (Figure 5.5).

The transmitted component of $\sigma_{R1}$ incident on NN, $\sigma_{T2}$ has the opposite sign to that of $\sigma_{R1}$, so that $\sigma_{R1} < \sigma_{R1}$. In general for further reflections within the sample, it can be shown that at time $T = nL/C_S$ where $n$ is the number of reflections made by the wave.

$$\sigma_{R1} < \sigma_{R1}$$

This means that the reflected stress wave tends to zero with time. Accordingly it is found that

$$\sigma_{Tn} = \sigma_{In}$$
as $n$ becomes larger.

After $T = 2L/C_S$, reflection and transmission will occur at both interfaces NN and OO for the whole duration of the incident stress pulse and for some time afterwards. Theoretically an infinite number of stress reflections will be created, although their intensities will become negligible after several traverses of the sample.

The reflection/transmission analysis of Figures 5.3 to 5.5 may be continued ad infinitum. The equations are simplified if one considers a normalised 'step' function incident stress wave (of value unity) of the form shown in Figure 5.6.

For convenience, if the incident wave is allowed to start at $T = 0$ then $\sigma_{In} = 0$ for $n < 0$ and $\sigma_{In} = 1$ for $n > 0$. (Negative $n$ and $T$ are not considered here but refer to time BEFORE the incident pulse). Using this incident stress wave as a basic starting point, it is
FIG 5.6 Normalised (value unity) 'step' function incident stress wave.

FIG 5.8 Ramp incident stress wave having a risetime of T2.
possible to generate an infinite series of terms for the reflected and
transmitted waves from the sample.

Table 5.1 shows the first eight terms in such a series from which a
general pattern may be deduced.

The transmission coefficients are:

\[ Y = \frac{2ABZS}{A_SZ_S + A_BZ_B} \]  
(5.12)

\[ X = \frac{2ASZ_B}{A_SZ_S + A_BZ_B} \]  
(5.13)

and the reflection coefficient is

\[ P = \frac{A_SZ_S - A_BZ_B}{A_SZ_S + A_BZ_B} \]  
(5.14)

Table 5.1 demonstrates some important features. The transmitted wave
starts \( L/C_S \) after the reflected wave since it takes this amount of
time for the transmitted part of the incident stress wave to reach the
second interface \( 00 \) from \( NN \).

Also it should be noticed that for the step impact, both the reflected
and transmitted wave intensities change abruptly at \( 2L/C_S \) intervals,
rather than changing smoothly throughout time. It is true that for a
more realistic incident stress wave, having a finite gradual rise in
stress intensity to some maximum value, the resulting reflected, \( \sigma_R \)
and transmitted, \( \sigma_T \) stress waves would have a more continuous change
in intensity with time. However, there would still be marked changes
in the slopes of \( \sigma_R \) and \( \sigma_T \) versus time at regular \( 2L/C_S \) intervals.
This 'jumpy' nature of the build-up of \( \sigma_T \) and decline of \( \sigma_R \) is made
more apparent in the graphs of the results Section 5.4.
TABLE 5.1

This shows the first eight stress terms of the reflected and transmitted waves in the bars generated by a unit step, function stress wave incident on a Hopkinson bar sample.

<table>
<thead>
<tr>
<th>Time</th>
<th>Reflected Wave</th>
<th>Transmitted Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = 0$</td>
<td>$\sigma_R</td>
<td>_0 = P$</td>
</tr>
<tr>
<td>$T = \frac{L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_1 = P$</td>
</tr>
<tr>
<td>$T = \frac{2L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_2 = P(1 - XY)$</td>
</tr>
<tr>
<td>$T = \frac{3L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_3 = P(1 - XY)$</td>
</tr>
<tr>
<td>$T = \frac{4L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_4 = P[1 - XY(1 + P^2)]$</td>
</tr>
<tr>
<td>$T = \frac{5L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_5 = P[1 - XY(1 + P^2)]$</td>
</tr>
<tr>
<td>$T = \frac{6L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_6 = P[1 - XY(1 + P^2 + P^4)]$</td>
</tr>
<tr>
<td>$T = \frac{7L}{C_S}$</td>
<td>$\sigma_R</td>
<td>_7 = P[1 - XY(1 + P^2 + P^4)]$</td>
</tr>
</tbody>
</table>

From Table 5.1 it may be deduced that the general expressions for $\sigma_R|_n$ and $\sigma_T|_n$ are:

for $n$ odd:  \[ \sigma_T|_n = \sigma_T|_{n-2} + XYP^{(n-1)} \]  \[ (5.15) \]

for $n$ even:  \[ \sigma_T|_n = \sigma_T|_{n-1} \]  \[ (5.16) \]

for all $n$:  \[ \sigma_R|_n = P[1 - \sigma_T|_{n-1}] \]  \[ (5.17) \]

Equations (5.15) - (5.17) above have been incorporated in an initial program which calculates and plots the rise of the transmitted and fall of the reflected wave, for different mechanical impedances $Z_S$, $Z_B$ and cross-sectional areas $A_S$, $A_B$. A second program was then written.
Do you want to plot out y-axis?

Yes → "YAXIS" No → Do you want to plot out x-axis?

Yes → "XAXIS" No → Do you wish to consider a step input?

Yes → "STEPY" No → "CONS" "CONS" → "PAW34" → END

"CONS" is a data file.

FIG5.7 Flow diagram of the program. Rectangular boxes represent sub-programs.
based on the first but modified so that the incident pulse could start with a finite linear rise time (> 2 μs), followed by a constant unit stress rather than an instantaneous rise from zero to unit stress. The fundamental equations used in this second program are similar to those derived above but have been adjusted to accommodate the added complication that $\sigma_t$ changes linearly with time during the first part of the stress wave. The next section describes the workings of both programs.

5.3 THE PROGRAM

The program was written and run on a BBC Model B microcomputer upgraded via a second processor to have more versatility and more available RAM. The computer was linked to a 'linear graphics' plotter which was accompanied by its own control software which had to be incorporated within the main plotting programs.

Figure 5.7 is a flow diagram of the main program, which is subdivided into five smaller programs (rectangular boxes) each one having its own particular task.

'CONS' is a data file which is created to store bar and sample parameters, $A_B$, $A_S$, $Z_B$, $Z_S$, $L$ and the transmission reflection coefficients $X$, $Y$ and $P$. 'PARAS' is a program which allows the program operator to enter the crucial bar and sample parameters after which they are transferred to the data file 'CONS'.

'YAXIS', as implied by the name, plots out the y-axis and similarly 'XAXIS' draws the x-axis. 'STEPY' calculates the reflected and transmitted waves for a step input and 'PAN34' does the same for a ramp input.
Initially the software was written as one continuous program, but it was found that the BBC would often run out of memory whilst running the program. Thus it was necessary to break it down into five smaller programs which loaded each other in succession. The program listings are given in the appendices. The five are discussed in turn below.

5.3.1 Program 1: 'PARAS'

This is the first program loaded in the chain of five. It allows the program operator to input the important physical parameters for the Hopkinson bar and sample, which appear in the first column of Table 5.2. The table gives figures for ρ and CS for the materials considered in this Chapter.

### TABLE 5.2: PARAMETERS ENTERED FROM KEYBOARD

<table>
<thead>
<tr>
<th></th>
<th>Maraging Steel</th>
<th>431 Steel</th>
<th>Aluminium</th>
<th>Copper</th>
<th>224 Carbon Steel</th>
<th>High-Density Poly-ethylene</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of bars, dB (mm)</td>
<td>12.7</td>
<td>12.7</td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Diameter of sample, dS (mm)</td>
<td></td>
<td></td>
<td>10</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Density of bars, ρB (g cm⁻³)</td>
<td>8.05</td>
<td>7.80</td>
<td>2.70</td>
<td>8.93</td>
<td>7.50</td>
<td>0.95</td>
</tr>
<tr>
<td>Density of sample, ρS (g cm⁻³)</td>
<td></td>
<td></td>
<td>2.70</td>
<td>8.93</td>
<td>7.50</td>
<td>0.95</td>
</tr>
<tr>
<td>Velocity of sound in bars, CB (m s⁻¹)</td>
<td>4818</td>
<td>5240</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Velocity of sound in sample, CS (m s⁻¹)</td>
<td></td>
<td></td>
<td>6374</td>
<td>4759</td>
<td>5164</td>
<td>1600</td>
</tr>
<tr>
<td>Length of sample, L (mm)</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Throughout all the program runs, L was fixed at 5 mm since this is the length of nearly all Loughborough SHPB samples.
From these the program computes the values of $t$ (sample traverse time $= \frac{L}{C_S}$), $Z_B$, $Z_S$, $\lambda$, $Y$, $X$, $P$ and together with $L$, places them in a data file 'CONS' which is then used subsequently by the other programs. Thus the multiple reflection scenario may be investigated for a whole variety of materials.

5.3.2 Program 2: 'YAXIS'
As implied by its name, this program plots out the y-axis for each stress intensity versus time plot. On nearly all the plots produced by the program so far the y-axis is simply a numbered linear scale covering the range $0+1$.

5.3.3 Program 3: 'XAXIS'
This program, when run, plots out a linear x-axis which represents time. Marks are made along this axis at intervals of $t$ (sample traverse time) and the total length of the axis is printed as a multiple number of $t$ just above the far right-hand side of the axis. The length of the x-axis plotted in terms of time may be chosen by the program operator or, if not, is automatically selected by the computer as the time taken for $\sigma_R$ or $\sigma_T$ to reach a steady value.

5.3.4 Program 4: 'STEPY'
This program actually calculates the stress $\sigma_R$ and $\sigma_T$ of the reflected and transmitted stress waves for a 'step' input stress wave, i.e. $c_{I\mid n}^1 = 1$ for $n > 0$ using equations 5.15 to 5.17 developed in Section 5.2. The stress values are calculated at each interval of $t$ ($= \frac{L}{C_S}$) and the results may be printed out in the form of a table or plotted as stress versus time graphs.

The program has been written in such a way that the operator may choose to plot out the following as a function of time:
a) transmitted wave only  
b) reflected wave only  
c) both  
d) the ratio $\frac{\sigma_I}{\sigma_R + \sigma_T}$

Of course, any number of curves may be plotted on the same axes.

The calculations within the program are iterative and are stopped once $\sigma_T > 0.999$ to avoid needlessly wasting computer time.

5.3.5 Program 5: 'PAW34'

The title of this, the most significant of the five programs, may seem a little odd; it is derived from 'Pulse draW' and the program itself is the thirty-fourth revised version.

'PAW34' is similar in structure to 'STEPY'; again $\sigma_T$, $\sigma_R$ and $\sigma_T/(\sigma_I + \sigma_R)$ may be plotted against time. The main difference between the two programs is that with 'PAW34' the operator is able to enter a rise time ($T_2$) for the incident stress wave. Thus, instead of being a step input the incident stress wave has the more realistic form shown in Figure 5.8.

The program accepts any value of $T_2 > 2$ μs. Typically, in the Loughborough SHPB system pulses incident on the sample have rise times of about 5 to 10 μs. As expected, the changes in reflected and transmitted waves are more continuous with a ramp input than with a step input. The program divides the waves into points at $(T_2/500)$ intervals in time. Thus the time resolution $T_4$ can be defined by:

$$T_4 = \frac{T_2}{500}$$

(5.18)

The number of points $N$ occurring in one traverse time of a wave in the SHPB sample is given by:
\[ N = \frac{T_0}{T_4} \quad (5.19) \]

where \( T_0 \) is the computer notation for the traverse time \( t (= L/C_S) \).

Thus, as the program calculates values of the points of the reflected and transmitted waves, a change in slope of the stress is seen to occur at every \( 2N \)th point.

The equations which describe the change in stress intensity of the reflected and transmitted waves for a ramp input pulse are modified forms of (5.15) to (5.17).

Let

\[ q = \text{INT} \left( \frac{x}{N} \right) \quad (5.20) \]

where \( q \) is the largest integer smaller than \( x/N \).

Then for \( q \) even:

\[ \sigma_T \big|_{x} = \sigma_T \big|_{x-2N} + x\eta \theta \quad (5.21) \]

for \( q \) odd:

\[ \sigma_T \big|_{x} = \sigma_T \big|_{x-2N} + x\eta \theta (q-1) \quad (5.22) \]

and for all \( x \) and \( q \):

\[ \sigma_R \big|_{x} = P \left[ \sigma_I \big|_{x} - \sigma_T \big|_{x-N} \right] \quad (5.23) \]

The subscript \( x \) refers to each point of the wave which is separated in time from each neighbouring point by \( T_4 \). All the equations above are used in 'PAW34' to calculate the stresses for the reflected and transmitted waves. Since the analysis is performed point by point, there are \( N \) times the number of calculations involved in 'PAW34' than in 'STEPY'. For some plots of \( \sigma_T \) and \( \sigma_R \) with an incident wave having a 2 \( \mu \)s rise time values of \( \sigma_T \big|_{x} \) and \( \sigma_R \big|_{x} \) have been calculated up to \( x = 15,000 \) which takes about 10 minutes!
Values of \( \sigma_T \big|_x \) and \( \sigma_R \big|_x \) are plotted as soon as they have been calculated so that it is not necessary for the computer to store vast quantities of data.

The program continues to compute values of \( \sigma_T \) and \( \sigma_R \) after the initial rise time \( T_2 \) of the incident wave, \( \sigma_I \), and ceases calculations once \( \sigma_T > 0.999 \). The input wave, \( \sigma_I \), has a initial linear rise from zero to one in time \( T_2 \) after which it maintains a constant value of unity as depicted in Figure 5.8. Hence \( \sigma_I \) can be expressed mathematically as:

\[
\begin{align*}
\text{for } 0 < x < 500 & \quad \sigma_I \big|_x = \frac{x}{500} \\
\text{for } x > 500 & \quad \sigma_I \big|_x = 1
\end{align*}
\]

and these equations are used in conjunction with equations (5.20) to (5.22) in the 'PAW34' program.

### 5.4 GRAPHICAL RESULTS

The program may be used to produce any number of stress versus time curves for different \( A_S, Z_S \) values of the sample. It is the initial rise time of the transmitted and fall time of the reflected wave which is of interest, i.e. in most cases the first 10 to 30 \( \mu s \). Below is a summary of the graphical results possible.

1. For a step input:
   a) Transmitted and reflected rises with \( Z_B, Z_S \) constant, \( A_B/A_S \) variable;
   b) Transmitted and reflected rises with \( A_B/A_S \) constant, \( Z_S \) variable;
FIG 5.10 Transmitted wave profiles produced when a 'step' stress wave is incident on an SHPB sample.
c) Transmitted and reflected rises with both $A_B/A_S$ and $Z_S/Z_B$
variable;

d) A plot of $\sigma_T/(\sigma_T+\sigma_R)$ against time for the conditions outlined
in a, b and c. The significance of this ratio will be
realised in the next section.

2. For a ramp input:

e) Plots a, b, c and d for incident pulses having a variety of
rise times.

5.5 DISCUSSION OF MAIN FEATURES OF THE COMPUTED GRAPHS

The trend for all the computed results is for the transmitted stress
wave to tend to '1' and for the reflected wave to vanish to zero. The
amount of time this takes clearly depends on the cross-sectional area
and the mechanical impedance of the sample and bars.

5.5.1 Results for Step Input Wave

In all the graphs plotted for a step wave input the x-axis is labelled
in units of $t$ (i.e. the traverse time through the sample) which means
that all the graphs apply to a sample having any fixed length $L$.

Figure 5.9 shows reflected and transmitted waves in the bars generated
by a 'step' input stress wave incident on a SHPB sample, whose
diameter is one third the diameter of the bars themselves ($A_B/A_S = 9$
and $Z_S = Z_B$). For the conditions given which represent a considerable
mismatch between the bars and the sample, it takes about 12 traverses
for the transmitted wave to reach 95% of its maximum value.

Figure 5.10 shows the build up in the transmitted stress for a range
of $A_B/A_S$ ratios ($Z_S = Z_B$). Figure 5.11 shows the reflected wave over
the same range. Clearly, the smaller the mismatch between bar and
The reflected step wave for various $A_B/A_S$ ratios; $Z_B = Z_S$

**FIG 5.11** Reflected wave profiles produced when a 'step' stress wave is incident on an SHPB sample.
sample areas, the shorter the rise and fall times of the transmitted and reflected waves respectively. At the limit when $A_B/A_S = 1$, there is no reflected wave and all of the incident step wave is transmitted through the sample.

Figure 5.12 displays the ratio of
\[
\frac{\sigma_T}{\sigma_I + \sigma_R}
\]
versus time for three values of $A_B/A_S$. This ratio eventually tends to '1' as $\sigma_T + \sigma_I = 1$. Referring again to Figure 5.2, $\sigma_T + \sigma_R$ is the total stress on the first bar face NN while $\sigma_I$ is the stress on the second bar face OO, so that the ratio
\[
\frac{\sigma_T}{\sigma_I + \sigma_R}
\]
at any time is an indication of the stress difference between the two faces. Hence the plot of Figure 5.12 shows the time for stress equilibrium to occur within the sample after the 'step' stress wave is incident. Logically, stress equilibrium occurs fastest when the difference between $A_B$ and $A_S$ is small.

Figure 5.13 is another plot of
\[
\frac{\sigma_T}{\sigma_I + \sigma_R}
\]
against time for various sample materials. $A_B/A_S$ in this figure has a value of 1.6 since this is the ratio of bar to sample area normally used in the Loughborough system. $t$ on the x-axis equals 1.05 ms which represents the traverse time for copper in a 5 mm thick sample. Since $C_S$ is different for each one, so too is the traverse time for stress waves in the sample, and this determines the real time for stress equilibrium to be reached. The combination of low wave speed (1600 m/s) and relatively low density (0.95 g/cm$^3$) of HDPE means that a considerable amount of time elapses before the stress on each face
$\sigma_T / (\sigma_I + \sigma_R)$ plots for various materials

$A_B / A_S = 1.6$

$t =$ traverse time for Cu (1.05\$\mu$s)

Al = Aluminium ; Cu = Copper

HDPE = High Density Polyethylene

224 = 224 carbon steel

**FIG 5.13** Stress equilibrium curves for a variety of materials. (In each case the incident stress wave is the unit step function of Fig. 6).
FIG5.14 Transmitted stress waves for a variety of risetimes for the incident wave $A_B/A_S = 9; Z_B = Z_S$. 

$\sigma_T$ for a variety of risetimes

$A_B/A_S = 9; Z_B = Z_S$
$\sigma_T$ plots for a variety of risetimes

$A_B/A_S = 4; Z_B = Z_S$

FIG5.15 Transmitted stress waves for a variety of risetimes for the incident wave

$A_B/A_S = 9; Z_B = Z_S; \quad A_B/A_S = 4; Z_B = Z_S$
FIG 5.16: Transmitted stress waves for a variety of risetimes for the incident wave $Z_B = Z_S$. $A_B/A_S = 1.6$ which is the ratio most commonly used in the Loughborough SHPB system.
of an SHPB sample of that material balance. In the case of aluminium stress equilibrium is attained within 1% after 5 or 6 \( \mu s \) or about 5 or 6 traverses of stress waves across the sample, whereas for the copper and 224 carbon steel, equilibrium

\[
\frac{\sigma_T}{\sigma_I + \sigma_R} = 0.99
\]

is achieved after about 3 \( \mu s \).

5.5.2 Results for Incident Waves Having Various Rise Times

Figures 5.14 to 5.16 are comparisons of the rise in the transmitted stress for a variety of rise times of the incident stress waves for three different \( A_B/A_S \) ratios, \( A_B/A_S = 1.6 \) being the ratio in the Loughborough system when 10 x 5 mm samples are used.

Figure 5.17 gives the shape of transmitted and reflected stresses produced for an incident wave having a rise time of 10 \( \mu s \), \( A_B/A_S = 9 \) and \( Z_B = Z_S \). The reflected stress intensity curve shows a nice build up to a maximum when \( \sigma_I \) reaches its maximum after 10 \( \mu s \) and then a gradual exponential type decay to zero thereafter.

Figure 5.18 shows the curves of

\[
\frac{\sigma_T}{\sigma_I + \sigma_R}
\]

versus time for three different rise times of the incident wave (\( A_B/A_S = 9 \) and \( Z_B = Z_S \)). All three curves are identical for the first two microseconds and the curves for incident rise times of 5 \( \mu s \) and 10 \( \mu s \) separate only after 5 \( \mu s \). All three curves reach a maximum value for

\[
\frac{\sigma_T}{\sigma_I + \sigma_R}
\]

which is slightly greater than '1' at \( t \) after their corresponding rise times, i.e. at 3 \( \mu s \), 6 \( \mu s \) and 1 \( \mu s \).
\[ \sigma_i, \sigma_R, \sigma_T \text{ for } A_B/A_S = 9; Z_B = Z_S \text{ and incident risetime } = 10 \mu s \]

**Figure 5.17** Time profiles of incident stress wave (\(\sigma_i\)), reflected stress wave (\(\sigma_R\)) and transmitted stress wave (\(\sigma_T\)).

Normalized Stress

Time (in units of traverse time, t)
\[ \sigma_1 / (\sigma_1 + \sigma_R) \] for 3 different risetimes 

\[ A_B/A_S = 9, \quad Z_B = Z_S \]

Time (in units of traversal time, \( t \))

FIG. 18 Stress equilibrium curves, \( \sigma_1 / (\sigma_1 + \sigma_R) \) for 3 different risetimes of the incident stress wave, \( \sigma_1 \)
\( \sigma_T/(\sigma_I + \sigma_R) \) for various \( A_R/A_S \) ratios

10\( \mu \)s input risetime; \( Z_B = Z_S \)

**FIG5.19** Stress equilibrium curves, \( \sigma_T/(\sigma_I + \sigma_R) \) for 4 different \( A_R/A_S \) ratios. 1.6 is the ratio most often employed in the Loughborough SHPB system.
Looking at Figure 5.18, it would seem that stress equilibrium occurs more rapidly for the shorter rise time incident waves. However, if one considers Figure 5.12 again for a step input, one soon realises that this can only be true up to a point, since for the instantaneous step rise in $\sigma_I$, the ratio

$$\frac{\sigma_T}{\sigma_I + \sigma_R}$$

overshoots the equilibrium value of 1 by a considerable amount. 'Overshooting' begins to have a detrimental effect when the rise time of the incident wave is less than the traverse time (i.e. $< 1 \mu s$).

Figure 5.19 displays

$$\frac{\sigma_T}{\sigma_I + \sigma_R}$$

versus time for a range of $A_B/A_S$ values ($Z_B = Z_S$ and rise time = 10 $\mu s$). The ratio tends to be closer to '1' during the rise time of the incident pulse for the larger $A_B/A_S$ values. Although at first this seems surprising, it can be explained by the fact that the greater the mismatch in cross-sectional area between the bar and sample the larger the reflected stress, $\sigma_R$. This is seen in Figure 5.11. Since $\sigma_R$ is tensile (-ve) the total compressive stress $\sigma_I + \sigma_R$ on the first face is made smaller and hence the ratio

$$\frac{\sigma_T}{\sigma_I + \sigma_R}$$

is larger for increasing area mismatch. When $A_B = A_S$ (100% transmission/no reflection) the curve is determined by the delay time $t$ between $\sigma_T$ and $\sigma_I$, i.e. the transmitted wave lags the incident wave by one traverse time, about 1 $\mu s$. Figure 5.19, then, reveals an unsuspected advantage of using samples of small cross-sectional area in that they should reach stress equilibrium faster.
\[ \sigma_t / (\sigma_I + \sigma_R) \] plots for various materials

\[ A_B / A_S = 1.6; \text{ incident risetime: 10us} \]

- \( t \) = traverse time for Cu (1.05\( \mu s \))
- Al = Aluminium; Cu = Copper
- HDPE = High Density Polyethylene
- 224 = 224 carbon steel

**FIG 5.20** Stress equilibrium curves, \( \sigma_t / (\sigma_I + \sigma_R) \) for 4 different materials
Figure 5.20 is the $\sigma_T/(\sigma_I + \sigma_R)$ ratio plotted for four materials. For the metals aluminium, copper and 224 carbon steel, total (100%) stress equilibrium is achieved after about 12 µs, which is reasonable when compared to the 10 µs rise time of the incident pulse. Even after only 3 µs after the arrival of the incident wave about 80% stress equilibrium is achieved in 224 steel while at the same point in time stress equilibrium in aluminium is better than 90%.

Figure 5.20 should be compared with Figure 5.13 which illustrates the time taken for equilibrium to build up in samples of the same group of materials when a 'step' wave is incident. The main difference between the two graphs is the marked decrease in 'overshoot' (i.e. the ratio $\sigma_T/(\sigma_I + \sigma_R)$ being greater than one) when a ramp input pulse is incident. In Figure 5.20 none of the curves for the metals rises above 1 unlike the corresponding curves in Figure 5.13.

The reduction in 'overshoot' is especially demonstrated by the two curves for an HDPE specimen. On the basis of the step input analysis of Figure 5.13 one might conclude that the Loughborough SHPB method is not valid for a specimen of such material since the stress on the transmitter face of the specimen rises to nearly double that on the incident face. Furthermore it takes a considerable amount of time before stress equilibrium is approached. However from the HDPE curve in Figure 5.20 which has been calculated for an incident wave having a 10 µs rise time and thus more closely resembles a real incident wave in the SHPB system, it is seen that about 95% stress equilibrium is reached after only 6 µs. Admittedly, beyond this point the stress equilibrium ratio $\sigma_T/(\sigma_I + \sigma_R)$ does oscillate slightly about unity but the disparity from true stress equilibrium remains less than 20%.
FIG 5.21 A comparison of the profiles of the transmitted stress waves for two different bar materials (maraging and 431 steel) used in the Loughborough SHPB system. The curves were computed assuming a ramp input stress wave of risetime 10 μs incident on a 5mm long sample of carbon 224 steel. \( \frac{A_B}{A_S} = 1.6 \)
The HDPE curve in Figure 5.20 indicates that the Loughborough SHPB method may be valid for such a material. This is an enlightening observation since it means that the method can work for materials whose mechanical impedances, $Z_s$, do not match that of the bars, $Z_b$, since the large reflected component of the incident wave which is produced in such a case helps to establish an overall stress equilibrium across the sample more rapidly.

Figure 5.21 is the curve for 224 steel taken from Figure 5.20 but plotted for the two types of Hopkinson bar steels (maraging and 431) used in the Loughborough system. Clearly, there is little difference between them and so it may be safely assumed that the type of steel bars used will have a negligible effect on the incident, reflected and transmitted wave shapes.

5.6 SUMMARY

A computer program has been developed which predicts the profiles of the transmitted and reflected waves created when a flat-topped stress wave of known rise time is incident on a 5 mm thick SHPB sample. The program indicates the time taken for stress equilibrium to occur in a sample, i.e. when the stress on the first face of the sample equals that on the second for a given set of bar and sample parameters. For example, in the case of a 224 carbon steel sample sandwiched between two maraging steel Hopkinson bars we should expect complete equilibrium to occur within three or four microseconds after the initial rise time of the incident pulse (Figures 5.13 and 5.20). Even after only 2 μs (or 2 traverse times) the equilibrium is about 90% when a ramp input wave is incident. From these figures the Loughborough SHPB method would seem to be valid.
Consideration of the time it takes to achieve equilibrium is highly pertinent to SHPB testing, since the theory assumes it is achieved instantaneously. Thus the results of the program may be able to explain any observed differences in the rate of build up of the transmitted and reflected stress pulses.

In the standard SHPB theory the transmitted stress, \( \sigma_T \), is regarded as being proportional to the actual specimen stress. Clearly this cannot be true before stress equilibrium has been achieved. Perhaps a more realistic measure of the specimen stress in the first few microseconds after the arrival of the incident wave at the specimen is the mean of the stress on the left hand \( (\sigma_L + \sigma_R) \) and right hand \( \sigma_R \) faces of the specimen. With this point in mind it is interesting to note that in the current investigation the elastic modulus of the 224 steel is determined by the computer analysis was always less than the actual modulus of 200 GPa. It is very likely that this miscalculation of the initial modulus is due to the non-equilibrium conditions which prevail during the first few microseconds.

The program has been based entirely on a 10 x 5 mm specimen size since only this size of specimen had been used in the Loughborough system at the time the program was developed. Since then the use of smaller 8 x 4 mm samples has proved beneficial in producing higher strain rates and greater final strains in SHPB tests. The findings of this chapter suggest that these 8 x 4 mm specimens should achieve equilibrium more quickly than the 10 x 5 mm specimens when a stress wave of finite rise time is incident because of the smaller cross-sectional area and the shorter traverse time for waves through the smaller specimens.

The program described above has only dealt with the case of elastic waves. At some point of course the specimen will become plastic with the result that the stress wave velocity in the specimen, \( C_s \) and hence
its mechanical impedance, $Z_s$ will be reduced. In the case of most metals, the lowest $C_s$ will fall will be to the bulk wave speed roughly a third of the elastic wave speed and hence $Z_s$, too, will fall to about 33% of its elastic value at most. The net effect of this onset of plasticity will most likely be a prolongation of the time for stress equilibrium to occur due to the increased traverse time of stress waves through the specimen. However for strong metals such as 224 steel a high degree of equilibrium should already exist by the time the incident stress is large enough to produce plastic deformation.

For non-metals such as HDPE the picture of how stress equilibrium is affected when plastic flow commences is less clear to see without modifying the program to accommodate changes in $C_s$ and $Z_s$. Any shortening of the time to equilibrium due to a lower $Z_s$ (and thus greater mismatch between bars and sample) will be offset by the increased traverse time to to a lower $C_s$.

Many of the aspects considered in this chapter are highly pertinent to the current Loughborough SHPB method since choosing the point in time at which to start comparing the reflected ($\sigma_R$) and transmitted ($\sigma_T$) is critical. As has been implied by the program, the largest experimental errors in the calculation of true stress and strain occur in the first few microseconds of the test.

In setting the incident and transmitter strain gauges to be equidistant from the specimen (Figure 4.3) takes no account of the delay in the arrival of the stress wave at the transmitter strain gauges due to the traverse time in the sample. In the case of a type 224 carbon steel sample this time is only ~1 µs and so is unlikely to produce a large error. However in non-metallic materials this traverse time cannot be neglected and will have a significant bearing
on the interpretation of the incident and transmitted stress waves as will any non-equilibrium effects. The program described above could prove helpful in deciding the appropriate point at which the transmitted stress may be compared with the reflected stress for a valid computation of true stress versus strain and true strain or stress versus time.

Three of the main conclusions from this work may be summarised as:

a) Stress equilibrium in a sample occurs more rapidly when a ramped input wave is incident than when a step input wave is.

b) Equilibrium conditions are achieved more quickly the shorter the rise time of the incident wave as long as it is not shorter than a single traverse time.

c) Stress equilibrium can be attained in a shorter time when smaller specimens are used.

Only the first of these (a) was suspected before the writing of the program described here.

In the future this program may be used to design a suitable pulse shape for incident stress pulses arriving at the sample in order to control the strain rate. At the moment the typical rise times of incident pulses in the Loughborough SHPB system are between 5 and 10 µs, since the impact between the projectile and the first bar is never instantaneous but lasts for a finite time and also because of wave dispersion effects as the incident wave travels along the bar. Pulses may be given still longer rise times (leading to lower strain rates within the sample) by placing a dummy metal sample in an extra split in the Hopkinson bar before the actual sample under test. The program could be useful in predicting the likely profiles of pulses shaped in this way (Ellwood, 1983).
CHAPTER 6
QUASISTATIC TENSILE TESTS

6.1 INTRODUCTION

In order to compare tensile and compressive properties, dynamic and quasistatic tensile testing was carried out on 24 tensile test pieces fabricated in the workshop of the University's Manufacturing Engineering Department. They were made according to the specifications detailed in Figure 6.1, a design recommended specifically for dynamic testing by JCR, Ispra (Italy) and AWRE (Foulness).

Preliminary tensile SHPB tests (to be described in Section 6.2) on some of these specimens revealed that the strains measured by strain gauges mounted on the sample tended to be greater than those calculated by the usual SHPB analysis programme. This suggested that the sample gauge length might be smaller than the assumed value of 8.3 mm based on the work of Ellwood (1983) and Walker (1987), and so a detailed investigation into the "effective gauge length", \( l_g \) was undertaken at quasistatic strain rates (\( \varepsilon = 10^{-3}s^{-1} \) and 0.04s\(^{-1}\)) using the Instron machine described in Section 3.3 previously.

6.2 GAUGE LENGTH, \( l_g \)

Figure 6.2 is a clearer representation of the tensile specimens used in the experiments. It can be seen that the central parallel region of 3 mm diameter is only 5 mm long adjoined on either side by 4 mm long tapered sections which give a gradual transition in diameter from 3 mm to the 5 mm diameter of the threaded ends. This gradual change in cross-section prevents any localised concentrations of stress from building up during testing.
FIG 6.1. DYNAMIC TENSILE TEST SPECIMEN

FIG 6.2 Tensile Specimen
(Dimensions in mm. Magnification x 4)
The length of specimen between the grips of the Instron machine, and between the bars in the dynamic SHPB tests, is 16 mm. Obviously during a test the total change in length of the sample $\Delta x$, from the original 16 mm, will be the sum of the change in length $\Delta l_m$ of the middle 5 mm, plus the smaller change in length of the tapered ends $\Delta l_e$. The main point of interest is the strain, $\varepsilon_m$, in the middle section.

The specimen gauge length, $l_g$ is defined by equation 6.1 below:

$$\varepsilon_m = \frac{\Delta x}{l_g}$$

i.e.

$$l_g = \frac{\Delta x}{\varepsilon_m}$$  \hspace{1cm} (6.1)

or

$$l_g = \frac{\Delta l_e}{\varepsilon_m} + \frac{\Delta l_m}{\varepsilon_m}$$  \hspace{1cm} (6.2)

Ideally $l_g$ should remain constant with increasing strain in the sample and this may be assumed to be true if the contribution from $\Delta l_e$ is small. For this reason, British Standard No 18 'Tensile Testing of Metals' (British Standards Institute, 1987) recommends that the gauge length of a specimen should be made to satisfy

$$l_g = 5.65 \sqrt{S_o}$$  \hspace{1cm} (6.3)

where $S_o$ is the cross-sectional area of the mid-section.

It also recommends that the parallel region of the sample should be longer than $l_g$ so that 'end effects' are negligible.
Such design criteria are most suitable for specimens tested at quasistatic strain rates since they allow valid comparison of results from test pieces of different sizes. However in dynamic tests, the long gauge length inferred by equation 6.3 would not be suitable where stress wave dispersion effects can play a significant role. According to this British Standards specification (equation 6.3) a circular cross-section sample of diameter 3 mm would require a gauge length of 15 mm while for a 5 mm diameter, \( l_g \) would have to be 25 mm.

All previous work in the Physics Department, Loughborough University on tensile samples of copper (Walker, 1987) and stainless steel (304, 316, 321 and 325) (Ellwood, 1983) has indicated a gauge length of 8.3 mm for a specimen of dimensions depicted in Figures 6.1 and 6.2, and this value has been used in the SHPB analysis to date. Both workers found no significant change in \( l_g \) with increasing strain but few of their measurements proceeded beyond a strain of 10%. Ellwood, however, did observe that the "overall trend is for effective gauge length to decrease with increasing strain".

For the case of 224 carbon manganese steel (the present material under investigation), which is stronger than the copper and the stainless steel tested in earlier work, and which shows a definite yield point unlike the latter two, one might expect this trend to be enhanced. A probable explanation for this trend is that up to the yield point the elastic contribution from the tapered ends makes a substantial contribution to the overall length increase of the sample; this is in addition to the elastic elongation produced in the central 5 mm parallel region. However, immediately after yielding, a progressively greater proportion of the elongation is due to the central region since the plastic modulus here is so much lower than the elastic modulus which still applies to most of the end section. Thereafter, as the strain increases, the plastic modulus (proportional to \( \frac{\sigma}{\varepsilon} \))
decreases thereby further enhancing the effect. As a result, the effective gauge length, \( l_g \), decreases with increasing strain even though the length of plastically deformed material in the centre section increases as the material work hardens and the stress increases.

Experimental verification and quantification of the dynamic gauge length \( l_g \) is given in the following sections.

6.3 QUASISTATIC TENSILE TESTING ARRANGEMENT

The quasistatic tensile tests required a modification of the Instron 1026 test machine used for some of the compression tests. The general arrangement is shown in Figure 6.3 from which it can be seen that the specimens were screwed at each end into 1/2" diameter cylindrical adaptors or 'grips' of 431 stainless steel. These were attached to the machine by two fixing pins, the top one being coupled to a universal joint which enabled correct alignment of the two ends of the specimen thereby ensuring uniform uniaxial stress throughout the test.

Two extension rates were used, 0.5 mm per minute and 20 mm per minute, and all tests were carried out at room temperature.

6.4 MACHINE COMPLIANCE

Unlike the compression tests, in tensile testing, it is not possible to take into account machine compliance by comparing Instron records from runs carried out with and without a sample. To assess the compliance of the machine plus grips it was necessary to introduce a dummy sample of known elastic properties. Type 431 stainless steel was chosen as the high strength material for this purpose.
FIG 6.3 Static tensile testing arrangement.

FIG 6.4 431 dummy samples used to check machine compliance
The elastic modulus of 431 steel has been measured as 211 GPa by Ellwood (1983) from the observation of resonances of longitudinal waves in a 1m length bar of 431 and by measurement of the density and pulse propagation velocity in 431 steel. The author's own investigations, using the same methods, confirmed this figure of 211 GPa and so it was used in the calculation of elastic displacement of the dummy specimens.

Two 431 specimen types (a) and (b) were used; these are shown in Figure 6.4. Type (a) which consisted of a uniform cross-section of 5 mm diameter was tested entirely elastically. Type (b) was of dogbone design (only two of which were made) and was tested to fracture. Originally, type (b) was manufactured merely to provide practice samples for the actual 224 carbon steel tests, but in the end yielded a considerable amount of useful information about the testing procedure.

Figure 6.5 shows the load versus displacement curve for a 431 dogbone sample plus machine tested at 0.5 mm per minute. The record is drawn up to only 500 kg (producing a stress of approximately 700 MPa in the mid-section) since this represents the linearly proportional limit for this steel. Also plotted on the same graph is a straight line of gradient, 13.8 kg/µm, passing through the origin. This represents the elastic elongation produced in the 16 mm long sample alone, calculated from the sum of the elastic strain in the 8 mm mid-section assumed to be uniform in cross-section and that in the 8 mm total length of end sections, assuming an elastic modulus of 211 GPa.

By subtracting this line from the load versus displacement curve for the machine and grips plus sample, it was possible to deduce the compliance for the machine and grips only as is also shown on the graph. The second sample was used to repeat the measurements at the
FIG 6.5 Instron tensile compliances using 431 dogbone sample

FIG 6.6 Instron tensile compliances using uniform 5mm sample
maximum Instron displacement rate; 20 mm per minute. The dashed line in Figure 6.5 shows the appropriate compliance curve at this rate. The sample used was of the dogbone type (Figure 6.4b) but in this case was fully screwed into the grips so that only the central 8 mm length of 3 mm diameter section was tested. The elastic component for this specimen geometry was subtracted as before.

The same process was repeated for the uniform 5 mm diameter 'dummy' samples (type (a), Figure 6.4) at both rates: 0.5 mm and 20 mm per minute. The resultant curves A and B for the compliance of the machine plus grips were found to agree exactly with those produced from the dogbone tests and are drawn in Figure 6.6. Again the machine was observed to be softer at the higher speed of 20 mm per minute. Since the continuous diameter samples could withstand much higher forces (>800 kg) without plastically deforming, they produced compliance curves to higher loads which could be appropriately applied to the dogbone curves beyond the elastic limit to obtain true stress versus strain graphs for the 431 steel. These are discussed in Section 6.1.6.

6.5 METHODS OF MEASURING DYNAMIC STRAIN AND GAUGE LENGTH, \( l_g \) IN 224 STEEL TENSILE TEST PIECES

Had there been no variation in gauge length at different levels of strain, the strain in the central region of the sample could have been obtained directly from the Instron record by simple division by the previously assumed 8.3 mm gauge length after subtraction of the machine compliance (equation 6.1). However, since the gauge length was apparently not constant for the present tensile specimens an independent method of measuring strain in the sample was employed in 8 out of 10 tensile tests performed at quasistatic rates. Four methods of strain determination were used and are described below.
Out-of-balance Wheatstone bridge used for tensile strain measurements. The measuring arm AB consists of a pair of 120 Ω strain gauges connected electrically in series as shown; the three other arms were 240 Ω wire-wound resistors (± 1 Ω). V is the out-of-balance output which was fed to a chart recorder during the tensile tests.

FIG 6.7

FIG 6.8 Calibration of Wheatstone bridge by two methods
a) Post-Yield Strain Gauge Technique
This was probably the most accurate and reliable method of the four, for establishing the actual dynamic strain in the central region of the sample. A set of post-yield etched strain gauges (Sokki Kenkyujo YFLA-Z) able to measure strains of up to 15% were used. The active length of these gauges was only 2 mm which made them ideal for fixing on the parallel section of certain samples. The gauges were mounted diametrically opposite each other to eliminate the possibility of bending strains being recorded.

A simple out-of-balance Wheatstone bridge circuit was constructed to monitor the change in resistance of the gauge pair (see Figure 6.7). The stabilised supply voltage of 18V produced a current of 38 mA through the strain gauges, which permitted a high sensitivity without any danger of overheating. A pre-calibration of the circuit was made by inserting a 0-10 K resistance box (resolution: ±1Ω) in series with a strain gauge pair on arm AB of the bridge. A 'zero' correction was made for the contact resistance of the box itself. The calibration curve is drawn in Figure 6.8. Calibration of the bridge by the introduction of a parallel resistance, $R_c$ to the arm AB was less accurate.

Up to about a strain of 5% the bridge output is virtually linear giving 95 mV per 1% strain.

During the tensile tests in which the gauges and bridge were used, the out of balance output was fed to a JJ Instruments CR652S chart recorder. By synchronising the Instron chart record with the JJ recorder at the start of a test it was possible to compare specimen displacement with specimen strain, and hence deduce the gauge length according to equation 6.1. The technique was employed at both machine speeds i.e. 0.5 mm per minute and 20 mm per minute.
b) 'Static' Scratch Mark Method

Two specimens of type shown in Figure 6.2 were mounted so that the total length between their fixed ends was 17 mm as in the SHPB tests. They were tested at 0.5 mm per minute, two parallel scratch marks having been made 4 mm apart in the mid-section of each. The tests were terminated after two and four minutes respectively (i.e. 8.8% and 31.8% engineering strain respectively, as determined by the change in distance between the scratch marks) and the permanent extension of the distance between the scratch marks was measured on a travelling microscope accurate to ±0.01 mm. The strain measured in this way was then compared with the total sample elongation recorded by the Instron machine to give a value for $i_g$.

In the case of the two minute duration test the overall change in length of the specimen between the grips was 0.49 mm whereas in the four minute test the length change was 1.75 mm as measured by a metal rule (accuracy ±0.25 mm). These distances agreed with the total sample displacement calculated from the crosshead speed multiplied by the test time recorded by the Instron minus the Instron compliance.

c) 'Dynamic' Scratch Mark Method

On another sample a second pair of parallel marks were made in a similar manner to (b). This time, however, the distance between the marks was measured 'dynamically' throughout the whole test until fracture. A travelling telescope (accuracy ±0.01 mm) mounted at a distance of 1m from the tensile test piece was used to measure this distance. It was necessary to stop the test periodically for about 15s whilst maintaining the load to allow each measurement to be taken. No creep was observed to occur during such stationary intervals. Thirteen measurements were made in this way throughout the time to fracture, which was roughly 7 minutes at a rate of 0.5 mm per minute. The increased amount of data acquired by this method compared to only the
simple measurement in method (b) was offset by a reduced accuracy in the length measurements themselves at small strains since the distances involved were smaller.

d) Constant Volume Method
In two tests, again at 0.5 mm per minute, the diameter of the tensile piece was measured by digital callipers (accuracy ± 0.01 mm) at 30s intervals. As in the last method (c), this meant necessarily stopping the Instron machine for brief periods of no longer than 15s. The method for determining strain was founded on the assumption that for plastic deformation of the test piece the central region should maintain a constant volume i.e. Poisson's ratio should equal exactly 0.5. Thus by measuring the radial strain, assuming the axial strain was exactly double, and comparing this last figure with the Instron displacement record, it was possible to estimate \( \varepsilon_g \). This method was the least accurate of the four because at low strains the change in diameter was only just measurable and also since radial strain was half the longitudinal strain the error in measurement was double.

6.6 PRELIMINARY RESULTS FROM 431 'DOGBOLE' TEST SAMPLES

As mentioned earlier in Section 6.1.4 two 'dogbone' shaped 431 stainless steel test pieces (Figure 6.4b) were fabricated initially to check the correct operation of the Instron machine in tension (Figure 6.3). On one of these samples a pair of post-yield strain gauges was mounted. This also enabled a preliminary assessment of the combined performance of the Wheatstone bridge and chart recorder system to be made. The first test was carried out at a rate of 0.5 mm per minute. Figure 6.9 shows the engineering strain versus time curve (I) as recorded by the system.
At 4.15% strain the bonding between the gauges and the sample failed and further strain values were no longer sensible. Up to 48s the strain increased linearly with time, and the measured strain values, when compared with their corresponding stress values from the synchronised Instron record, were found to be consistent with a modulus of 211 GPA.

Curve II is a corresponding strain curve deduced from the total sample displacement recorded by the Instron machine. In calculating this curve several assumptions had to be made. Firstly, since it was presumed that uniform force acted throughout the specimen, then since the ratio of the cross-sectional areas of the end sections to that of the middle section was 25/9, the stress and thus strain in the mid-section was 25/9 times that at the specimen ends, for as long as the whole specimen remained elastic. This means that providing the force acting on the test piece is not large enough for any plastic deformation to occur in the mid-section, one should expect the two ends of the specimen to accommodate about 26% of the total sample strain, the remainder being in the 8 mm mid-section.

On this basis then, it was assumed that for all times less than 48s only 74% of the total sample displacement, $\Delta x$ (mm), from the Instron record (having subtracted machine compliance) occurred in the central region of the original 8 mm length (ignoring the slight taper which was 1.5 mm on each side of the specimen). Hence elastic strain $\varepsilon_m$ in the central region was calculated from

$$\varepsilon_m = \frac{0.74\Delta x}{8}$$  \hspace{1cm} (6.4)

Beyond 48s, when the force was sufficient to deform the central region, plastically the elastic strain in the ends was considered negligible compared to the plastic strain in the middle, $\varepsilon_m$. The rest of the strain/time curve was then completed by putting
\[ \varepsilon_m = \Delta x / 8 \]  
\[ (6.5) \]

Curve II agreed perfectly with the strain gauge curve I in the elastic region revealing the validity of the assumptions made for this region. However, after yielding had commenced it can be seen that curves I and II begin to diverge as time, and therefore strain, increases. This implies that the gauge length of 8 mm used in equation 6.5 was too large for accurate determination of strain in the plastic regime.

A more realistic figure for the apparent gauge length was estimated from a measurement of the plastic strain rate, \( \dot{\varepsilon} \), in the specimen at the point of strain gauge failure as recorded by the strain gauge.

Now the time dependent form of equation 6.1 is:

\[ \dot{\varepsilon} g = \frac{dx}{dt} \]  
\[ (6.6) \]

where \( \frac{dx}{dt} \) is the rate of elongation of the whole specimen. It is convenient here to take \( \frac{dx}{dt} \) as being the crosshead displacement speed i.e. 0.5 mm per minute, ignoring the small amount of displacement due to the machine plus grips. The contribution from the machine compliance to the total movement of machine plus grips and sample should be small once the sample has deformed plastically.

Measurement of the gradient of the strain-time curve at the point of gauge failure gives \( \dot{\varepsilon} p = 1.5 \times 10^{-3} s^{-1} \), hence from equation 6.6

\[ \dot{\varepsilon} g = 5.4 \text{ mm} \]

This is a surprisingly low value for the gauge length which was expected to be nearer 8 mm. The reasons for this small gauge length
FIG6.10 Plots of stress versus strain for 431 dogbone tests
are not entirely clear. In the calculations above it was assumed that there was an abrupt change in diameter of the specimens from 5 mm to 3 mm whereas in actual fact the change was more gradual. On either side of the central parallel region there was a 1.5 mm tapered section producing this gradual change in diameter. Since no account of tapering was made it is to be expected that the actual gauge length of the specimen is less than 8 mm.

Another possible reason for the small gauge length could be that the onset of necking may have occurred in the central region thereby producing a localised yield. This too would tend to shorten the gauge length of the specimen as a whole.

Curve III represents strain versus time calculated from the Instron record based on \( l_g = 5.4 \) mm. Clearly curve III agrees well with the actual strain gauge curve I at plastic strains above 1% but is severely in error for the elastic regime. The strain gauge should give the most reliable representation of the actual engineering strain within the central region of the test piece. Thus the discrepancy between curve I with curves II and III provided the first indication that the gauge length \( l_g \) was a function of strain even for the simple specimen geometry of Figure 6.4b.

Figure 6.10 shows the true stress versus true strain curve (A) for the test described above. Engineering stress, \( \sigma_0 \), was calculated directly from the Instron record while engineering strain \( \varepsilon_0 \) was the strain measured by the strain gauges up to 4% (solid curve) after which it was calculated from the Instron record assuming \( l_g = 5.4 \) mm (dotted portion of curve). True stress \( \sigma_T \) and true strain \( \varepsilon_T \) were obtained from the standard equations:

\[
\varepsilon_T = \ln (1 + \varepsilon_0)
\]  

(6.7)
Also shown in Figure 6.10 is the true stress-strain curve (B) for the second 431 dogbone specimen tested at a speed of 20 mm per minute (\(\dot{\varepsilon}_p = 0.03 s^{-1}\)). Since no strain gauges were affixed to this specimen during the test a change in effective gauge length could not be observed directly. Logically, a gauge length of 5.4 mm was assumed for the parallel central section of the test piece on the basis of the results described above for the first 431 test piece.

While curve B is initially above A, as should be expected for a higher strain rate test, it falls below A beyond a strain of 6%. The onset of necking is responsible for the levelling out of curve B beyond this point and once again a small gauge length is implied.

Curve A agrees well with similar records recorded by Ellwood (1983) on 431 steel at approximately the same strain rate \(1.5 \times 10^{-3} s^{-1}\). The change in \(\ell_g\) with strain as shown by the two results described above was then investigated for the 224 steel tensile samples.

6.7 224 STEEL RESULTS AND GAUGE LENGTH ANALYSIS

All the 224 steel specimens had the same geometry as delineated in Figures 6.1 and 6.2. Four of these specimens had pairs of post-yield strain gauges mounted on them so that strain could be observed directly in the central parallel region. The experience gained from the 431 tests led to an improvement in the method of adhesion of the strain gauges to the surfaces of the specimens so that in three out of four specimen tests, strains of around 10% were recorded before the bonding failed. The cement used for the purpose was Tokyo Sokki Kenkyujo Co Ltd CN adhesive.
FIG. 6.1 Comparison of strain versus time by 2 methods at 20 mm min⁻¹.
Fig. 6.12 Tensile sample strain versus time (0.5 mm min⁻¹)

X = Instron record assuming \( I_0 = 8.3 \text{ mm} \), B
O = Actual strain gauge record, A
The specimen surfaces were first prepared by abrasion using a fine grade sandpaper and then were cleaned by the application of methyl-pentanone.

Figure 6.11 shows two strain versus time curves for a tensile test carried out at the higher Instron speed of 20 mm per minute. A is the actual strain in the sample as measured by a pair of affixed strain gauges while B represents strain calculated from the Instron record assuming a gauge length of 8.3 mm. For times <0.6s the whole sample is elastic and A and B agree perfectly. For times >0.6s the specimen deforms plastically. Curve A now lies above curve B, there being a gradual divergence of the two. This once more suggests that the gauge length, \( l_g \), for the specimen decreases with strain.

A similar pattern is seen in Figure 6.12 which shows a direct strain versus time (curve A) measurement from a pair of strain gauges and B, the corresponding Instron curve assuming \( l_g = 8.3 \) mm, this time at a testing speed of 0.5 mm per minute. A and B agree quite well up to a strain of about 7% after which B falls away from A with increasing strain, and again a decrease in \( l_g \) is implied.

Figure 6.13 is the actual true stress versus true strain graph for the same test ('shot 260') for which the strain-time measurement of Figure 6.12 was made. The curve up to a strain of 11% is founded on the engineering strain measured directly from the strain gauges and stress from the Instron record. The errors in stress and strain values determined in this way should be small and therefore this initial portion of the curve is believed to be a reliable representation of the material properties.

At 11% strain the gauge bonding failed. The rest of the curve in Figure 6.13, beyond this point, is based on strain calculated as the
FIG. 6.3 Analysis of shot260 on RA2 0.5mm min⁻¹
FIG6.14 Analysis of shot260 on RA2 for 4 different gauge lengths:

a) $l_g = 5.7\text{mm}$   b) $l_g = 6.5\text{mm}$   c) $l_g = 7.6\text{mm}$   d) $l_g = 8.3\text{mm}$
total sample displacement (from Instron records) divided by $l_g = 6.5$ mm. This value was derived from the plastic strain rate just before gauge failure (Figure 6.12) together with the crosshead displacement velocity $\frac{dx}{dt} = 0.5$ mm per minute, in accordance with equation 6.6. The compliance of the machine has been neglected as it makes only a small contribution in the plastic region. The second half of the stress-strain curve in Figure 6.13 forms a harmonious continuation of the initial curve composed from actual strain gauge measurement. However, there is a mismatch between the two when the derived strain is extrapolated below 10%.

Figure 6.14 demonstrates the effect of gauge length, $l_g$ on the shape of the true stress versus true strain graph for the 'shot 260' at 0.5 mm per minute described above. In this diagram the respective curves have been drawn up entirely from data taken from the Instron record, for four different gauge lengths.

The graph shows that at low strains below 15%, the level of stress at a given strain is higher if a larger value of $l_g$ is used, while above a strain of 18% the opposite of this is true. At a certain point in between, close to 16.5% strain, the stress-strain coordinate appears to be independent of the gauge length chosen. The pattern of behaviour is easily understood by consideration of the true strain and true stress relations, equations 6.7 and 6.8 respectively.

Below 15% strain the true strain equation 6.7 has a more dominant effect over the shape of the curve than equation 6.8. Thus the larger $l_g$ the smaller the strain and hence the larger the shift of the whole curve to the left of the stress-strain graph. Above 18% strain the true stress equation 6.8 dominates so that although a smaller gauge length means a larger true strain the tendency for the curve to be shifted to the left is more than compensated by the increase in true stress level.
FIG 6.15 Gauge length, $l_g$ versus strain, $\varepsilon_g$

- $l_g = 7.8 \text{ mm (for } \varepsilon_g < 10\%)$
- $l_g = 8.2 - \varepsilon_g / 10 \text{ (for } \varepsilon_g > 10\%)$

Symbols:
- $\Delta$ = shot 259
- $\Delta$ = shot 260
- $\triangle$ = shot 263
- $+ = $ static scratch method
- $\square$ = shot 274
- $\times$ = shot 275
- $\times = $ dynamic scratch marks

Strain gauge measurements

Gauge strain, $\varepsilon_g$, Gauge strain(%)
Figure 6.14 shows the importance of establishing a precise and realistic figure for gauge length for a tensile test piece since this can significantly influence the resulting stress versus strain relationship. Curves (c) and (d) for gauge lengths of 7.6 mm and 8.3 mm respectively, most closely follow the actual strain gauge curve in Figure 6.13 up to 11%. However, above 18% strain both (c) and (d) show a marked decrease in true stress with increasing true strain which hardly seems a genuine trend in the light of the compression results (Chapter 8). Hence, Figure 6.14 shows that whereas a gauge length of between 7.6 mm and 8.3 mm is reasonable below about 10% strain, a much lower or steadily decreasing gauge length should apply at strains beyond this point.

Figure 6.15 is probably the most important graph in the sequence as it illustrates the variation in gauge length \( \ell_g \) at various strain levels as measured by the four techniques described in Section 6.1.5. There is considerable scatter in the data, especially at low strains where displacement and strain are hardest to determine accurately particularly for Instron measurements.

Below an engineering strain of 10% the points are randomly distributed about a mean gauge length, \( \bar{\ell}_g \) of 7.8 mm (standard deviation = \( \pm 1.03 \)). Beyond this level of strain there is an unequivocal fall in \( \ell_g \) with increasing strain, providing clear evidence that \( \ell_g \) actually decreases during a tensile test. A least squares linear regression has been carried out for the 11 points above 10% strain and this gave a best fit straight line of equation

\[
\ell_g = 8.2 - 0.1 \epsilon_m \tag{6.9}
\]

where \( \epsilon_m \) is the percentage engineering strain at the centre of the specimen.
and \( l_g \) is the gauge length in mm.

Standard Error in the slope (0.1) = ±0.025  (95% confidence)
Standard Error in the intercept (8.2) = ±0.52

Due to the large scatter in the data the error in the gradient is considerably large at ±25%. Extrapolating the line back to zero strain gives an intercept at \((8.2 \pm 0.5)\) mm which is consistent with the average \( l_g = 7.8 \) mm for the points below 10%, and also compares favourably with the previously believed constant gauge length of 8.3 mm (Ellwood, 1983 and Walker, 1987).

With this knowledge, then, it was possible to analyse the eight quasistatic tensile test records in a logically sound manner. Below engineering strains of 10%, true stress and true strain were calculated by the use of equations 6.1, 6.7 and 6.8, assuming the mean value for \( l_g \) of 7.8 mm.

Above 10% engineering strain an interesting feature arose in the calculations caused by the employment of equation 6.9. According to equation 6.1 the percentage engineering strain, \( \varepsilon_m \) in the mid-section of the sample is given by:

\[
\varepsilon_m(\%) = \frac{100 \Delta x}{l_g} \quad (6.10)
\]

\( \Delta x \) being the total sample displacement from the Instron record. However \( l_g \) itself is a function of \( \varepsilon_m \) (equation 6.9) so that

\[
\varepsilon_m = \frac{100 \Delta x}{8.2 - 0.1 \varepsilon_m}
\]

or

\[
-\varepsilon_m^2 + 82 \varepsilon_m - 1000 \Delta x = 0 \quad (6.11)
\]
Strain gauge record

Instron record assuming $l_g = 7.5$ mm.

New analysis using $l_g = 8.2 - 0.1\varepsilon$

Mean curve at 0.5 mm min. from fig. 6.17.
Hence equation 6.11 was solved to find $\epsilon_m$ from Instron records where the strain was above 10%.

By using,

$$\epsilon_m = \frac{100 \Delta x}{7.8}$$

for $\epsilon_m < 10\%$ and equation 6.11 for $\epsilon_m > 10\%$, Figure 6.16 shows the appreciable effect this new analysis has on the shape of the stress-strain curve when compared to the old constant $l_g$ analysis which formerly used equation 6.10 and $l_g = 8.3$ mm. As indicated on the diagram, a point which would have been drawn at (16.6%, 617 MPa) is shifted to (26.2%, 680 MPa) under the new analysis.

Figure 6.17 shows the new analysis of six tensile tests carried out at a speed of 0.5 mm per minute producing a mean strain rate of $10^{-3}$s$^{-1}$. A mean curve has been drawn through the set which shows a high degree of consistency. It represents the most definitive true stress versus true strain relationship obtained throughout the tensile work for a particular set of conditions.

Figure 6.18 shows the mean results from two tests at 20 mm per minute giving a strain rate of 0.04 s$^{-1}$ (i.e. the final true sample strain divided by the total test time). For the purposes of comparison the mean curve from Figure 6.17 has been added. Up to about 15% strain both curves agree with compression curves (Chapter 8) obtained at similar rates.

The results displayed in Figures 6.17 and 6.18 are discussed in more detail with respect to the compression test and higher strain rate tensile test results in Section 9.3.
6.8 DUCTILITY AND STRENGTH OF 224 STEEL

A useful measure of ductility for a metal, which is often quoted in tensile tests, is the 'reduction of area', q (see for example Dieter, 1985) which is given by:

\[ q = \frac{A_0 - A_f}{A_0} \]  

(6.12)

Measured values of q for a selection of the tensile tests at three different testing rates are presented in Table 6.1. It appears that q is insensitive to strain rate over the reasonably narrow range covered by the tests here. Also shown are figures for the ultimate tensile strength (UTS) (i.e. engineering stress at maximum load) and the corresponding true stress at maximum load, \( \sigma_u \). The values for UTS comply well with the specification of British Standard No 1501, British Standards Institution, 1980, which states an acceptable range of 430 MPa to 550 MPa. The trend is for UTS to increase at the higher strain rates which fits in well with the thermal activation theory discussed in Chapter 2.

<table>
<thead>
<tr>
<th>Test No</th>
<th>Test Machine Speed</th>
<th>Plastic Strain Rate</th>
<th>Ultimate Tensile Strength UTS</th>
<th>( \sigma_u ) (MPa)</th>
<th>q(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>260</td>
<td>0.5</td>
<td>( 10^{-3} )</td>
<td>509</td>
<td>621</td>
<td>74</td>
</tr>
<tr>
<td>274</td>
<td>0.5</td>
<td>( 10^{-3} )</td>
<td>523</td>
<td>621</td>
<td>71</td>
</tr>
<tr>
<td>275</td>
<td>0.5</td>
<td>( 10^{-3} )</td>
<td>516</td>
<td>662</td>
<td>73</td>
</tr>
<tr>
<td>MEAN</td>
<td></td>
<td></td>
<td>515</td>
<td>634</td>
<td>73</td>
</tr>
<tr>
<td>262</td>
<td>20</td>
<td>0.03</td>
<td>545</td>
<td>642</td>
<td>76</td>
</tr>
<tr>
<td>263</td>
<td>20</td>
<td>0.03</td>
<td>547</td>
<td>647</td>
<td>73</td>
</tr>
<tr>
<td>MEAN</td>
<td></td>
<td></td>
<td>546</td>
<td>645</td>
<td>75</td>
</tr>
<tr>
<td>264</td>
<td>50</td>
<td>0.08</td>
<td>552</td>
<td>*</td>
<td>75</td>
</tr>
</tbody>
</table>

* For this record the machine speed was too great for a reliable load-displacement record to be obtained. Hence there is no measured strain and thus no true stress.

TABLE 6.1: SHOWING DUCTILITY AND STRENGTH OF 224 C-Mn STEEL IN TENSION
FIG 6.19 Comparison of mean tensile curve for 224 steel with previous work on 321 steel (Ellwood, 1983) & Cu (Walker, 1987) at $10^3$ s$^{-1}$
(mean grain sizes in brackets)
6.9 COMPARISON OF PRESENT TENSILE RESULTS WITH PREVIOUS WORK

The most striking difference between the results obtained here and tensile results from 321 stainless steel and copper recorded by previous workers at Loughborough at 20°C (Ellwood, 1983 and Walker, 1987, respectively) is that the observed change in gauge length with increasing strain as described above. The reason for this is the difference in relative strength between the three materials. Ellwood's work on 321 stainless steel did not proceed beyond a strain of 10% and so he was unlikely to see a change in gauge length during the test.

Figure 6.19 shows tensile true stress versus true strain curves for all three materials at a rate of approximately $10^{-3}s^{-1}$. Copper is clearly weaker than the two steels which have similar yield stresses, $\sigma_y$ (i.e. $\sigma_y321 = 258$ MPa and $\sigma_y224 = 270$ MPa (LYP)). The graph also demonstrates that the rate of work hardening $\frac{\partial\sigma}{\partial\varepsilon}$ is largest for the 224 steel studied here. 224 steel is the only material of the three which showed an upper and lower yield point, an indication of inhomogeneous deformation associated with the propagation of Luders bands.
CHAPTER 7
DYNAMIC TENSILE TESTS

7.1 A REVIEW OF THE DYNAMIC TENSILE TEST

Tensile tests are very conveniently performed at low strain rates (quasi-static) of the order of $10^{-2}\text{ s}^{-1}$ or less using conventional screw driven Instron type machines (Davis et al., 1982). Above this rate, up to about $100 \text{ s}^{-1}$, servo-hydraulic type machines can be employed based on similar principles to the slower Instron machines. At these low and intermediate strain rates the tensile test holds the advantage over the equivalent compression test in that, provided the system is correctly aligned, the specimen is unaffected by friction. Also, in tension a wider variety of sample geometries can be used as long as the gauge length is known or remains constant throughout the test (Dieter, 1985).

At strain rates above $100 \text{ s}^{-1}$ tensile tests become increasingly more difficult to perform with precision. For a start, it is difficult to achieve perfect axiality of loading of the specimen without introducing bending moments. Moreover effects due to wave propagation and inertia become increasingly more significant at higher rates. This is why most dynamic testing of materials has favoured a compressive method using smaller samples.

In carrying out tensile tests above $10^3 \text{ s}^{-1}$ the effects of inertia are a matter of some considerable concern and should not be ignored in the analysis. Regazzoni and Montellellet (1985) have reviewed the important factors which affect the results from uniaxial tensile tests at high strain rates between $10^3$ and $3.5 \times 10^3 \text{ s}^{-1}$. In 1986 Regazzoni and his co-workers made a theoretical study of the dynamic tensile test at
high rates concentrating particularly on inertial effects. They identified three important features caused by inertia:

i) it induces a radial component to the stress that may not be negligible under certain conditions;
ii) it causes non-uniformity of deformation resulting in heterogeneous distribution of stress, strain and strain rate;
iii) it affects the elongation stability (i.e. the onset and growing of the neck).

Of these, probably (ii) is of the most consequence in influencing the resultant stress-strain analysis. The effect may be simply illustrated for the case of the Loughborough SHPB by considering a tensile test at the highest strain rate \( \approx 1500 \text{ s}^{-1} \) on a 224 steel specimen of gauge length 8.3 mm. Assuming a longitudinal wave speed of 5 mm/\( \mu \text{s} \) the elastic wave will take 1.7 \( \mu \text{s} \) in travelling from the loading end of the gauge length to the stationary end of the gauge, in which time the specimen will have undergone an elongation of 0.25% within the gauge length itself. This is, however, small compared to the total 10% permanent strain expected at the end of such a test, and is also less than the yield strain. Furthermore the elastic strain rate will be substantially less than 1500 s\(^{-1}\), so the figure 0.25% can be regarded as a maximum.

Since the pioneering work of Harding and his colleagues at Oxford in 1960, the dynamic tensile test has become increasingly popular as a method of determining material properties at high strain rates. Before that time little of the dynamic tensile experimentation produced details of true stress versus true strain. The paper by Harding et al (1960) reviews much of the earlier work in this field.
<table>
<thead>
<tr>
<th>AUTHOR(S)</th>
<th>YEAR</th>
<th>TEST METHOD</th>
<th>TEST DURATION OR MAX STRAIN</th>
<th>STRAIN RATE (s⁻¹)</th>
<th>TEMPERATURE (K)</th>
<th>MATERIAL</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Brookes &amp; Reddaway</td>
<td>1957</td>
<td>Piston driven by compressed gas</td>
<td>To fracture .126 mS</td>
<td>2400</td>
<td>Room temp.</td>
<td>Steel</td>
<td>The system measures energy absorbed to cause fracture. No stress v strain.</td>
</tr>
<tr>
<td>2 Harding, Wood &amp; Campbell</td>
<td>1960</td>
<td>Modified SHPB</td>
<td>80 to 160 μS</td>
<td>1000 (for Fe)</td>
<td>Room temp.</td>
<td>Fe, Al, Mo</td>
<td>Pioneering modification of SHPB for tensile testing</td>
</tr>
<tr>
<td>3 Lindholm &amp; Yeakley</td>
<td>1968</td>
<td>Modified SHPB</td>
<td>20% strain</td>
<td>780 to 960</td>
<td>Room temp.</td>
<td>Aluminium</td>
<td>'Top hat' shaped specimen</td>
</tr>
<tr>
<td>4 Harding</td>
<td>1971</td>
<td>Modified SHPB</td>
<td>50 μS</td>
<td>1000</td>
<td>Room temp.</td>
<td>Copper</td>
<td>Tensile pulse induced by 50 μS magnetic pulse</td>
</tr>
<tr>
<td>5 Albertini &amp; Montagnani</td>
<td>1977</td>
<td>Modified SHPB</td>
<td>2.5 mS</td>
<td>100 to 1000</td>
<td>Room temp.</td>
<td>Austenitic Stainless Steels</td>
<td>Loading bar is pre-stressed almost to elastic limit then is suddenly relaxed</td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>TEST METHOD</td>
<td>TEST DURATION OR MAX STRAIN</td>
<td>STRAIN_RATE (s(^{-1}))</td>
<td>TEMPERATURE (K)</td>
<td>MATERIAL</td>
<td>COMMENTS</td>
</tr>
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<td>------------------------------</td>
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<td>---------------------------------</td>
<td>--------------------------------------------------------------------------</td>
</tr>
<tr>
<td>6  <strong>Ito</strong> &amp; Oba</td>
<td>1979</td>
<td>Explosively driven piston</td>
<td>To fracture &gt; 6 mS</td>
<td>100 to 1000</td>
<td>Room temp. 670, 820</td>
<td>316 Stainless Steel</td>
<td>Strain measured via 'on-off' series of copper brushes</td>
</tr>
<tr>
<td>7  <strong>Kawata, Hashimoto</strong></td>
<td>1979</td>
<td>Pendulum and rotating disc</td>
<td>30% strain</td>
<td>420 to 2600</td>
<td>Room temp. 0.45% C-steel 2014-T6 Al</td>
<td>Measure energy absorption upon fracture</td>
<td></td>
</tr>
<tr>
<td>8  <strong>Dormeau &amp; Stelly</strong></td>
<td>1979</td>
<td>Crossbow</td>
<td>500 S</td>
<td>300 to 1200</td>
<td>Room temp.</td>
<td>Copper</td>
<td>Strain in specimen measured by high speed streak photography</td>
</tr>
<tr>
<td>9  <strong>Nicholas</strong></td>
<td>1981</td>
<td>Modified SHPB</td>
<td>300 S</td>
<td>100 to 1000</td>
<td>Room temp.</td>
<td>Various alloys of Al, Ti, Steel</td>
<td>Very similar to Loughborough set-up</td>
</tr>
<tr>
<td>10 <strong>Ellwood, Griffiths &amp; Parry</strong></td>
<td>1982</td>
<td>Modified SHPB</td>
<td>100 S 15% strain</td>
<td>70 to 1510</td>
<td>Room temp.</td>
<td>Austenitic Stainless Steels</td>
<td>Loughborough system</td>
</tr>
<tr>
<td>11 <strong>Ellwood</strong></td>
<td>1983</td>
<td>Modified SHPB</td>
<td>100 S 15% strain</td>
<td>70 to 1510</td>
<td>Room temp.</td>
<td>Austenitic Stainless Steels</td>
<td></td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>TEST METHOD</td>
<td>TEST DURATION OR MAX STRAIN</td>
<td>STRAIN RATE (s⁻¹)</td>
<td>TEMPERATURE (K)</td>
<td>MATERIAL</td>
<td>COMMENTS</td>
</tr>
<tr>
<td>---------------------------------------</td>
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</tr>
<tr>
<td>Regazzoni &amp; Montheillet</td>
<td>1984</td>
<td>Crossbow</td>
<td>To fracture</td>
<td>3 x 10³</td>
<td>Room temp.</td>
<td>Copper and Tantalum</td>
<td>Same system as used by Dorneval and Stelly (1979)</td>
</tr>
<tr>
<td>Meyer</td>
<td>1984</td>
<td>Pendulum, flywheel and 'Shlag-Harmer'</td>
<td>30% strain</td>
<td>20 to 8x10³</td>
<td>120 to 300</td>
<td>Stainless Steel x45 CrSi 93</td>
<td>Tests thermal activation theory. Investigates effect of $\varepsilon$ and T on UTS and ductility</td>
</tr>
<tr>
<td>Sturges, Parsons &amp; Cole</td>
<td>1984</td>
<td>Flywheel</td>
<td>To fracture</td>
<td>1000 s⁻¹</td>
<td>Room temp.</td>
<td>Copper</td>
<td>Tests performed under hydrostatic pressure of up to 200 MPa</td>
</tr>
<tr>
<td>Albertini, Boone &amp; Montagnani</td>
<td>1985</td>
<td>Modified SHPB</td>
<td>To fracture</td>
<td>25 to 50</td>
<td>Room temp.</td>
<td>Various</td>
<td>Similar to Albertini and Montagnani (1977) but in this case both incident and transmitter bars are pre-stressed thereby doubling strain-rate in specimen</td>
</tr>
<tr>
<td>Kobayashi, Hashimoto, Lilah Wang, Toba</td>
<td>1985</td>
<td>Impacting hammer</td>
<td>30% strain</td>
<td>450 to 1500</td>
<td>Room temp.</td>
<td>Zircaloy</td>
<td>Hammer attached to rotating disc or fixed to pendulum</td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>TEST METHOD</td>
<td>TEST DURATION OR MAX STRAIN</td>
<td>STRAIN RATE (s⁻¹)</td>
<td>TEMPERATURE (K)</td>
<td>MATERIAL</td>
<td>COMMENTS</td>
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<td>----------------------------------------------------</td>
</tr>
<tr>
<td>Warnes, Karpp, 1985</td>
<td></td>
<td>Expanding ring</td>
<td>30% strain</td>
<td>5 x 10³ to 2.3 x 10⁸</td>
<td>Room temp.</td>
<td>Copper</td>
<td>Comparison with SHPB</td>
</tr>
<tr>
<td>Kussmaul, Zimmermann, Issler, 1985</td>
<td>Rotating disc Impact</td>
<td>To fracture</td>
<td>4 to 500</td>
<td>Room temp.</td>
<td>20 MnMoNi Steel</td>
<td>High energy machine 33 MJ produces only 1% fall in speed of disc after impact so that strain-rate is virtually constant</td>
<td></td>
</tr>
<tr>
<td>Nojima, Ogawa</td>
<td>1985</td>
<td>Modified SHPB</td>
<td>8% strain</td>
<td>200 to 1000</td>
<td>Room temp.</td>
<td>Al and 0.45% C steel</td>
<td>Tensile pulse is transmitted through an anvil into loading bar</td>
</tr>
<tr>
<td>Rajendran &amp; Bless</td>
<td>1986</td>
<td>Modified SHPB</td>
<td>To fracture &gt; 70%</td>
<td>800 to 9500</td>
<td>Room temp.</td>
<td>HY100 Steel 1020 Steel and Al</td>
<td>Dynamic study of 'necking' and application of Bridgeman correction</td>
</tr>
<tr>
<td>Walker</td>
<td>1987</td>
<td>Modified SHPB</td>
<td>10%</td>
<td>250 to 1300</td>
<td>Room temp. 473, 673</td>
<td>Copper</td>
<td>Loughborough system</td>
</tr>
<tr>
<td>Ahmad</td>
<td>1988</td>
<td>Expanding Cylinder</td>
<td>90% hoop strain</td>
<td>10⁴ to 9 x 10⁴</td>
<td>Room temp.</td>
<td>Various polymers</td>
<td>Exploding copper wire used to load cylindrical specimens</td>
</tr>
<tr>
<td>AUTHOR(S)</td>
<td>YEAR</td>
<td>TEST METHOD</td>
<td>TEMPERATURE (K)</td>
<td>STRAIN RATE (s⁻¹)</td>
<td>MAX STRAIN</td>
<td>MATERIAL</td>
<td>COMMENTS</td>
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</tr>
<tr>
<td>23 Gourdin</td>
<td>1989</td>
<td>Expanding Ring</td>
<td>Room temp.</td>
<td>10⁴</td>
<td>To ductility limit</td>
<td>Tantalum and Copper</td>
<td>Virtually constant strain rate</td>
</tr>
<tr>
<td>24 Kusznial, Dambler &amp; Kleck</td>
<td>1989</td>
<td>Rotating Disc Impact</td>
<td>Room temp.</td>
<td>10 to 1000</td>
<td>To fracture</td>
<td>Perlitic steel</td>
<td></td>
</tr>
<tr>
<td>25 Harding, Li, Saka &amp; Taylor</td>
<td>1989</td>
<td>Modified SHPB</td>
<td>Room temp.</td>
<td>290</td>
<td>1.58 fracture strain</td>
<td>Woven carbon/epoxy laminate</td>
<td></td>
</tr>
</tbody>
</table>
FIG 7.1 Hammer to impact block method of dynamic tensile testing
Nowadays, the dynamic tensile test has become more common. Table 7.1 reviews some of the main techniques which have been used in the last ten years and some of the pioneering work before that. It emerges from this table that there are three main types of dynamic test most commonly used and these may be classed, loosely, as modified SHPB, hammer to impact block (or the one bar method, Figure 7.1), and expanding ring or cylinder. Of these, it appears that some form of modified SHPB system is more often favoured.

Obviously by placing the methods reviewed in Table 7.1 into these three categories does not mean to say that the methods in each category are identical but merely that they share a common philosophy and are based on similar principles. They are summarised briefly below.

a) Expanding ring/cylinder test [Refs 17, 22 and 23]
Generally this method is used at strain rates of $10^4$ s$^{-1}$ or greater, above the strain rate range covered by experiments described in this thesis. Most expanding ring or cylinder methods use some form of explosive loading such as might be produced by the vaporisation of a copper wire through which a high current has been discharged. Recently Ahmad (1988) has reviewed the development of the expanding cylinder/ring test.

b) Hammer to impact block technique [Refs 1, 6, 7, 8, 12, 13, 14, 16, 18 and 24]
This is a fairly general method for dynamic tensile testing at strain rates between 10 and 1000 s$^{-1}$. A typical schematic arrangement is shown in Figure 7.1. The moving hammer may be an explosively driven piston, a pendulum or connected to a constant speed rotating disc. The block is usually supported in some form of guide so that after impact both hammer and block continue moving together thus conserving
linear momentum and thereby pulling the specimen apart. From the
dynamic strain, $\varepsilon_g(t)$ measured in the gauge attached to the output bar
it is possible to calculate the dynamic tensile stress $\sigma(t)$ and
strain, $\varepsilon(t)$ in the specimen from the following equations (Kobayashi
et al, 1985):

$$\sigma(t) = \left(\frac{A_0}{A}\right) E_b \varepsilon_g (t + \frac{\Delta}{C}) \quad (7.1)$$

$$\varepsilon(t) = \frac{1}{k} \int_0^t \left[ V(\tau) - c \varepsilon_g (\tau + \frac{\Delta}{C}) \right] d\tau \quad (7.2)$$

where $k$ and $\lambda$ are the gauge length and minimum cross-sectional area of
the specimen, $A_0$, $E_b$ and $c$ are the cross-sectional area, Young's
modulus and longitudinal elastic wave velocity of the output bar. $V$ is
the impact block velocity. $\varepsilon_g (\tau + \frac{\Delta}{C})$ is the time dependent strain
measured by the gauge. The main disadvantage of this method is that
very often the strain rate is variable throughout the test.

c) Modified SHPB for dynamic tension [Refs 2, 3, 4, 5, 9, 10, 11, 15,
19, 20, 21 and 25]

It is far easier to produce a compressive wave of short rise time in a
bar than a tensile one. Hence many modified SHPB systems convert
compressive waves, produced by impact in the normal way, into tensile
waves by the introduction of a special mechanical joint at some point
along the length of the loading bar. Harding et al (1960) were the
first to do this by arranging the tensile sample plus inertia bar
inside a hollow tube which acted as the loading bar. A tensile wave
was produced at a 'yoke' which connected the sample to the tube.

In 1971 Harding, using a different SHPB system, produced a tensile
wave in the incident bar by sudden application of a 50 $\mu$s electromagnetic pulse. Albertini and Montagnani, 1977 and 1985, similarly
achieved a tensile wave in the incident bar by first compressing it almost to the elastic limit and then suddenly allowing it to relax. Unfortunately in all three systems described above long rise times were observed in the tensile waves incident on the specimen.

Alternatively, the simplest modification to a SHPB arrangement to allow tensile testing is to replace the compression sample with a tensile test piece and a surrounding protective collar. The test is then performed by allowing the normal compression pulse to pass through the specimen plus collar and then reflect from the free end of the bar line as a tensile pulse. The incident bar then becomes the transmitter bar and vice versa. Such systems were developed independently by Ellwood et al (1982b) and Nicholas (1981). Near perfect reflection at the free end means that there is very little wave dispersion in producing the tensile wave. However the method does have the disadvantage that the amplitude of the loading wave is limited by the maximum compressive stress sustainable in the tensile test piece without it yielding.

This latter form of modified SHPB is currently used at Loughborough and is described below.

7.2 GENERAL DESCRIPTION OF THE SPLIT HOPKINSON PRESSURE BAR ARRANGEMENT FOR TENSILE TESTING

The current SHPB tensile testing arrangement at Loughborough has been developed and described in some detail by Ellwood (1983) and Walker (1987). The object of the experiments described in this thesis were to test the validity of the technique so developed, and to apply it to the 224 carbon steel samples.
A. Tensile version of the SHPB apparatus

B. Enlarged view of tensile specimen and collar

C. FIG7.2 Idealised strain pulses recorded by strain gauges SG1 and SG2
The modification to the compressive SHPB system described in Chapter 4 required for tensile testing was very simple. The incident bar comprising a one metre length of 431 bar plus one metre of maraging steel bar was replaced by a 2m maraging steel bar which had a screw-threaded hole at one end in which to grip the tensile specimens. The transmitter bar was replaced by the 1m length of maraging steel bar which formed the second half of the incident bar in the compressive arrangement. This too had a threaded hole for gripping the tensile test pieces. The momentum bar, no longer needed, was removed completely.

The only other addition required was the collar, a cylinder of bar material which surrounds the specimen without touching it and is sandwiched between the incident and transmitter bars in much the same way as a compression sample.

Schematically the tensile version of the SHPB is shown in Figure 7.2. A compressive wave, $\varepsilon_C$, is produced in the normal manner by impact of the projectile with bar 1. This compressive wave travels through bar 1 and through the collar plus sample into bar 2 where it is reflected back as a tensile wave, $\varepsilon_I$, from the free end as illustrated in Figure 7.2a. The tensile wave upon arriving back at the collar plus specimen acts solely on the specimen, the collar falling since it is not fixed to the bars (Figure 7.2b). At this point reflected $\varepsilon_R$ and transmitted, $\varepsilon_T$, waves are created according to the degree of strain produced in the specimen, and the equations used in Chapter 4 for compression may be applied here.

Figure 7.2c shows idealised signal-time traces for the recording strain gauges SG1 and SG2. It can be seen that modification from the compressive system merely converts the incident bar to transmitter bar and vice versa.
The function of the collar (Figure 7.2b) is to protect the sample from the initial incident compressive wave. The total compression in the specimen is limited to the strain produced in the collar and this imposes a limit on the size of the compressive pulse which can be incident at the collar without producing work hardening in the specimen. In turn this fact imposes a limit on the maximum specimen strain rate of approximately $10^3$ s$^{-1}$.

Two sizes of collar have been made. The first by Ellwood (1983) has an external diameter of 12.7 mm to match that of the bars as depicted in Figure 7.2b. The internal bore of this collar is 6 mm so that its total cross-sectional area is 78% of that of the pressure bars. Hence, in compression, the collar undergoes only slightly more strain than the bars.

The Walker (1987) collar has the same internal diameter of 6 mm but double the external diameter i.e. 25.4 mm. This latter collar was made to increase the maximum achievable tensile strain rate for which no predeformation in copper samples occurred. This was done by allowing a larger initial compressive pulse which was partially reflected at the collar. Both collars were 17 mm long made from 431 steel. A pair of 1 mm diameter grooves were cut in each to allow electrical connection of strain gauges mounted on specimens in certain tests. In all but the very last two tests described here the 'Ellwood collar' was used.

Figure 7.3 shows a record from an actual tensile test on 224 steel. The upper trace indicates the incident tensile wave, $\varepsilon_I$, followed by the compressive reflected wave, $\varepsilon_R$. The lower trace, synchronised with the upper, shows the transmitted wave, $\varepsilon_T$. Clearly the noise level on the transmitted signal is greater than that on the incident and reflected waves. This is a natural consequence of the fact that most of the incident tensile wave is reflected due to the large cross-sectional area mismatch between bar and specimen.
FIG 7.3 Typical SHPB tensile test (shot 225 T) on 224 carbon steel.

FIG 7.4 Lagrangian diagram of strain pulses in a SHPB tensile test.
A higher magnification of the transmitted signal is then required with an overall reduction in signal to noise ratio. The noise on the transmitted channel is not white but is predominantly of 100 kHz frequency which suggests that it may be interference from the PET computer or transient recorder circuitry or from the mass spectrometer in the neighbouring laboratory.

7.3 EXPERIMENTAL CONSIDERATIONS

Figure 7.4 is a Lagrangian diagram of the stress waves propagated in the SHPB tensile system. It can be seen that the actual tensile test occurs about 800 μs after the initial impact of the projectile with bar 1. The diagram also shows what happens to the portion of the initial compression wave reflected from the collar after 400 μs. It is not possible to make this small reflection zero even when using the Ellwood collar since the combined areas of the collar plus sample does not equal that of the bars. However by having the first bar 2m long means that the reflection does not return to the first pair of strain gauges at the same time as the transmitted tensile wave generated by the test.

Due to the small signal size of the transmitted tensile wave (Figure 7.3) it was even more important to minimise the effects of bending waves generated at impact and of spurious waves created at the bar supports. Hence these supports were made as loose and free as possible while special care was taken in mounting the strain gauges in each pair exactly diametrically opposite each other on the bars to cancel out the effects of bending waves.

Due to the relatively high noise level on the transmitted wave traces from SG1, various oscilloscope amplifier gains were tried until an optimum signal to noise level was achieved. For the most part the SG1 amplifier gain was set to x25 while that for SG2 was x10.
On the first few trial tests using 5 mm continuous diameter 431 specimens, it was found that false upper and lower yield point effects could be produced at the screw threads if they were not tightly gripped to the bars. For this reason, all the 224 dogbone specimen threads were packed with PTFE tape before testing.

It was essential to ensure that the collar was correctly aligned with the pressure bars on either side of it. The existence of an air gap between the collar and the bars would not only cause problems with large reflections being created upon arrival of the initial compressive pulse, but also it would result in the slight deformation of the specimen before the arrival of the tensile wave. Correct alignment was not always easy to achieve since the 2m bar 1 had a bend in it.

7.4 THEORETICAL LIMITATIONS ON STRAIN AND STRAIN-RATE IN THE TENSILE TEST

The experiments of Nicholas (1981) and Ellwood et al (1982) proved that the total compression of the specimen in response to the initial compressive pulse is limited to the strain produced in the collar. However, they did not go on to consider that the strain throughout the specimen is not uniform. Since the cross-sectional area of the tensile specimen varies from its ends to the central parallel region, the compressive stress will be greater in this region than at the ends. Consequently, there is effectively amplification of the stress and hence strain in the passage of the initial compressive pulse from the first pressure bar through the specimen plus collar. It is instructive to evaluate the amount of strain amplification and compare this with experimental observations.
Walker (1987) paid particular attention to this problem for the case of copper tensile specimens. Theoretically he determined that the ratio of compressive strain, $\varepsilon_m$, in the mid-section of a copper specimen to the strain in the first pressure bar, $\varepsilon_b$, should lie within the range 1.35 to 4.38. However, in two particular tests in which strain gauges were affixed to the specimens he measured this ratio ($\varepsilon_m/\varepsilon_b$) as 4.2 and 7.1 respectively.

The alarming disparity between this second empirical value of $\varepsilon_m/\varepsilon_b$ and the theoretically predicted range is a point of some concern. In Walker's theoretical analysis two rather vague parameters are introduced to describe the behaviour of the whole specimen. These are: (i) "effective area", $A'$, which may take some value between the cross-sectional area of the central parallel region (7.1 mm$^2$) to the cross-sectional area of the ends of the specimen (19.6 mm$^2$), and (ii) "effective modulus", $E'$, whose value is somewhere between the elastic modulus for copper (129 MPa) and zero (for plastic flow).

In the text below an alternative analysis is presented for the case of 224 carbon steel specimens and the Ellwood collar based on purely elastic conditions of sample and collar. Since we are concerned that the strain in the central parallel region, $\varepsilon_m$, does not surpass the elastic limit, the reasoning here is felt to be valid for determination of $\varepsilon_m/\varepsilon_b$ and the maximum strain rate, $\varepsilon_m/\Delta t$.

Consider Figure 7.5 which is a schematic picture of the collar and specimen arrangement in the SHPB system. Let force, stress, strain elastic modulus, cross-sectional area, length and change in length be denoted by $F$, $\sigma$, $\varepsilon$, $E$, $A$, $L$ and $\Delta L$ respectively and the subscripts b, c, e and m denote the bars, collar, ends and middle of specimen respectively.
**FIG 7.5** Diagram of 'Ellwood' collar and specimen arrangement in the SHPB system.

$F_b$, $F_c$, $F_e$ and $F_m$ represent the equilibrium forces on the faces of the bars, collar, ends and mid-section of the specimen respectively during the passage of a compressive wave. $l_c$ is the length of the collar which equals the length, $l_m$ of the central 3 mm diameter parallel region plus the sum of the two 'end' sections, $l_e$. In brackets are the original static values of $l_m$, $l_e$ and $l_c$.

$A_c = 98.4 \text{ mm}^2$, $A_h = 28.3 \text{ mm}^2$, $A_e = 19.6 \text{ mm}^2$, $A_m = 7.1 \text{ mm}^2$.

**FIG 7.6** Initial compressive, $\varepsilon_b$ in the first pressure bar (left-hand-side) and compressive pulse, $\varepsilon_m$ in the collar with no sample present (right-hand-side).

Ratio of pulse amplitudes $\varepsilon_c/\varepsilon_b = 1.29$
Now for equilibrium of forces at the interfaces between each of the pressure bars and the collar plus sample:

\[ F_b = F_c + F_e \quad (7.3a) \]

If the force through the specimen is homogeneous, then

\[ F_e = F_m \]

so that

\[ F_b = F_c + F_m \quad (7.3b) \]

hence

\[ \sigma_{b} A_b = \sigma_{c} A_c + \sigma_{m} A_m \quad (7.4) \]

and since the regions are all elastic

\[ \epsilon_{b} E_{b} A_b = \epsilon_{c} E_{c} A_c + \epsilon_{m} E_{m} A_m \quad (7.5) \]

Looking at equations (7.3) through to (7.5) it may be noted that for a relatively soft specimen, such as copper, where \( E_m < E_c \) most of the force is taken up by the collar and consequently the stress and strain in the collar is increased. For \( E_m = 0 \) i.e. no specimen, we have from equation (7.5)

\[ \frac{\epsilon_c}{\epsilon_b} = \frac{E_b A_b}{E_c A_c} \quad (7.6) \]

This defines the maximum strain in the collar, \( \epsilon_c/\max \) in terms of the bar strain.

With \( E_b = E_c \), \( A_b = 126.7 \text{ mm}^2 \) and \( A_c = 98.1/4 \text{ mm}^2 \) we obtain

\[ \frac{\epsilon_c}{\epsilon_b} \bigg|_{\max} = 1.29 \quad (7A) \]

For the case of 224 carbon steel testing, and indeed for many other steels, we have the situation in which the collar and sample are of
roughly equal modulus, i.e. $E_C = E_m = E_b$. Under the condition of equal moduli, equation (7.5) may be written,

$$126.7e_b = 98.4e_C + 7.1e_m \quad (7.7)$$

since $A_m = \pi(1.5)^2 = 7.1 \text{ mm}^2$.

Since the total collar compression must equal the combined change in length of the ends and middle of the specimen we have:

$$\Delta l_c = \Delta l_m + \Delta l_e \quad (7.8)$$

Recalling equation (6.1):

$$\epsilon_m = \frac{\Delta l}{l_g}$$

where $l_g$ is the gauge length, gives

$$\epsilon_m = \frac{\Delta l_m + \Delta l_e}{l_g} = \frac{l_m \epsilon_m + l_e \epsilon_e}{l_g}$$

i.e.

$$\epsilon_m = \frac{5\epsilon_m + 12\epsilon_e}{l_g}$$

For small strains $l_g = 8.3 \text{ mm}$ so that

$$\frac{\epsilon_e}{\epsilon_m} = 0.275 \quad (7.8)$$

Equation (7.8) may be rewritten as

$$l_c \epsilon_C = l_m \epsilon_m + l_e \epsilon_e$$

or

$$17e_C = 5e_m + 12e_e \quad (7.9)$$
Combining equations (7.7) and (7.9) with the result of (7B) gives

\[ \frac{\varepsilon_m}{\varepsilon_b} = 2.3 \quad (7C) \]

and

\[ \frac{\varepsilon_c}{\varepsilon_b} = 1.12 \quad (7D) \]

Thus the compressive strain in the middle of the tensile sample is about 2.3 times the compressive strain in the bars for the incident compressive pulse and we should not expect it to be greater than this.

The analysis can be extended in order to determine the maximum strain rate achievable with this SHPB system which does not pre-deform the specimen before the test itself.

From the compression results on the SHPB it was found that for strain rates above 1000 s\(^{-1}\) the yield strain was not less than 0.3%. Hence the corresponding maximum strain of the initial compressive pulse is from equation (7C):

\[ \varepsilon_b \big|_{\text{max}} = \frac{0.3}{2.3} \% \]

i.e.

\[ \varepsilon_b \big|_{\text{max}} = 0.13\% \quad (7E) \]

Now the compression strain amplitude, \( \varepsilon_b \), in the first pressure bar is related to the steel projectile velocity \( V \) by

\[ \varepsilon_b = \frac{V}{2C_o} \]

where \( C_o \) is the longitudinal wave speed in the bars (= 5 mm/\( \mu \)s). Thus we can write:

\[ V_{\text{MAX}} = 13 \text{ ms}^{-1} \quad (7F) \]

This defines the maximum velocity at which the projectile may impact with the first bar without compressively work hardening the tensile specimen.
When the compressive wave reaches the free end of the bar line it is reflected and becomes the incident tensile wave for the test. Assuming that this reflection is perfect, and that the collar causes no reduction in the initial incident pulse, the maximum amplitude of tensile pulse, $\varepsilon_I|_{\text{MAX}}$ will equal $\varepsilon_b|_{\text{MAX}}$:

i.e.  

$$\varepsilon_I|_{\text{MAX}} = 0.13\%$$

In a typical tensile test at a high strain rate (> 1000 s$^{-1}$) roughly 90% of the incident wave is reflected so that:

$$\varepsilon_R|_{\text{MAX}} = 0.9 \times 0.13\%$$

i.e.  

$$\varepsilon_R|_{\text{MAX}} \approx 0.12\% \quad (7G)$$

Now strain rate in the specimen is proportional to the reflected wave strain, $\varepsilon_R$, according to:

$$\dot{\varepsilon}_s = \frac{-2C_0}{\ell_g} \varepsilon_R$$

Substituting in result (7G), $\ell_g = 8.3$ mm and $C_0 = 5$ mm/μs yields:

$$\dot{\varepsilon}_s|_{\text{MAX}} \approx 1400 \text{ s}^{-1} \quad (7H)$$

This, then, is the maximum allowable strain rate in 224 steel specimens in the tensile SHPB system using the Ellwood collar (Figure 5.23). If the projectile has an impact velocity greater than 13 ms$^{-1}$, the specimen will yield compressively before the start of the tensile test.
7.5 EXPERIMENTAL VERIFICATION OF THE TENSILE SHPB METHOD

A number of experimental measurements were performed to test the theory proposed in the last section (7.4). Particular attention was paid to the unavoidable compressive loading of the specimen before the actual start of the tensile test.

The strain in the collar, and in numerous samples, was measured directly by pairs of 1 mm gauge length strain gauges (Sokki Kenkyujo Co Ltd FLE-1-11) attached to them. The gauges, placed 180° apart, were connected in series and accommodated into two extra ballast resistance type circuits identical to the one shown in Figure 4.11. A Philips PM3302 8 kbyte digital storage oscilloscope was used to record the strain gauge signals in most of the tests. Since this oscilloscope was sensitive enough to record signals down to less than 5 mV it was unnecessary to amplify the signals from the gauges (which were typically of the order of 25 mV).

Roughly half of the 'shots' failed due to either breaking of wires under impact or short circuiting of them with the steel bars or collar. As there was only 1 mm clearance between the specimen and the collar, it was technically quite difficult to fit the connecting wires to the strain gauges into this space without upsetting the orientation of the collar which consequently led to problems of correct alignment. Nevertheless, from the shots which were successful some enlightening information was obtained.

First of all, the response of the pair of strain gauges on the collar to the initial compressive pulse was checked with no tensile specimen present. The pair of gauges on the collar were connected to channel two of the normal SHPB circuit replacing SG2. Figure 7.6 shows the pulses recorded in this preliminary shot (TT2). On the right hand
A) Incident compressive pulse in bar

B) Incident compressive pulse in collar

FIG 7.7 Comparison of the incident compressive strain in the first pressure bar (A) with the compressive strain in the collar when a continuous 5 mm diameter tensile specimen is present. $\frac{\varepsilon_c}{\varepsilon_b} = 1.29$. 
side of the oscilloscope photograph is the stress wave in the collar, the larger of the two, while the one on the left hand side is the incident compressive wave in the bar. The ratio $\varepsilon_c/\varepsilon_b$ as measured from the photograph is 1.29 in perfect agreement with the theoretical result (7A). A second test (TT4) was performed under the same conditions and $\varepsilon_c/\varepsilon_b$ was measured as 1.31 agreeing very well with the first result, within experimental error.

The next stage was to test the system with a dummy tensile specimen plus collar. The 431 steel, continuous 5 mm diameter type specimens which had been used to determine the compliance of the Instron machine (Section 6.4) were used again here. Figure 7.7 shows the compressive pulses recorded for the first of these tests (TT3). Again the ratio $\varepsilon_c/\varepsilon_b$ was measured as 1.29 which suggested that the specimen offered little or no resistance to the compressive pulse, most of the loading stress being taken up by the collar and virtually none by the specimen. The strains in the specimen and the collar should be equal. Further tests using 431 continuous 5 mm diameter samples verified this; all of them produced an $\varepsilon_c/\varepsilon_b$ ratio close to 1.29.

In the 431 tests which had strain gauge pairs affixed to the centre of the specimen no significant compressive pulse was ever recorded. Shot TT5 whose oscilloscope record is presented in Figure 7.8 is a typical example. The explanation of this observation is simple. All tensile samples, even if compressed, will fail at their weakest point. For the normal 'dogbone' shaped specimen this should be at the centre, but for the 431 continuous diameter specimens the point of failure is where they are screwed into the bars. Since the incident compressive strain is so small the threads may not have been in full contact with the bars and therefore all the displacement may be accommodated at the threads and thus no compressive strain in the specimen.
FIG 7.8  Comparison of strain recorded in the collar and 431 specimen due to the passage of the initial incident compressive wave. No significant pulse is seen in the sample trace.

FIG 7.9  Oscilloscope record of tensile shot No 221T at a strain rate of 833 s⁻¹.
Table 7.2 presents the measured ratios of $\varepsilon_c/\varepsilon_b$ for the initial compressive pulse in the early shots described above. The mean of the first six $\varepsilon_c/\varepsilon_b$ values is 1.30 which compares remarkably well with the theoretically predicted value of 1.29 for no sample in (7A). At the base of the table is shown the measured ratio obtained from a shot on a 224 tensile test piece. The result of $\varepsilon_c/\varepsilon_b = 1.17$ is only slightly larger than the 1.12 estimated in (7D) which was the ratio expected for all the 431 test pieces in Table 7.2. This indicates a greater uniformity of compressive strain in the 224 steel sample than in the 431 solid specimens and no initial slackness in the threads of the 224 piece.

<table>
<thead>
<tr>
<th>Shot No</th>
<th>Sample</th>
<th>Incident Compressive Strain $\varepsilon_b$ (%)</th>
<th>$\varepsilon_c/\varepsilon_b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT2</td>
<td>None</td>
<td>0.102</td>
<td>1.29</td>
</tr>
<tr>
<td>TT4</td>
<td>None</td>
<td>0.088</td>
<td>1.31</td>
</tr>
<tr>
<td>TT3</td>
<td>431</td>
<td>0.112</td>
<td>1.29</td>
</tr>
<tr>
<td>TT5</td>
<td>431</td>
<td>0.082</td>
<td>1.33</td>
</tr>
<tr>
<td>TT6</td>
<td>431</td>
<td>0.108</td>
<td>1.28</td>
</tr>
<tr>
<td>TT7</td>
<td>431</td>
<td>0.110</td>
<td>1.31</td>
</tr>
<tr>
<td>MEAN</td>
<td></td>
<td></td>
<td>1.30</td>
</tr>
<tr>
<td>221T</td>
<td>224</td>
<td>0.074</td>
<td>1.17</td>
</tr>
</tbody>
</table>

TABLE 7.2: MEASUREMENTS OF THE RATIO $\varepsilon_c/\varepsilon_b$ FOR THE INITIAL COMpressive wave IN THE SHPB SYSTEM

As great consistency had been achieved in the measurement of the amplitude of the stress pulse in the collar relative to that in the first pressure bar, attention was next concentrated on the measurement of both compressive and tensile strain in the 224 steel specimens.
FIG 7.10 Strain gauge response to tensile wave in specimen and collar.

Strain gauge trace

FIG 7.11 True strain versus time plots for three SHPB tensile tests at a mean strain rate of 636 s⁻¹.
The first attempt to measure the compressive strain in a 224 test piece proved only moderately successful as poor electrical contact of the leads from the ballast circuitry to the strain gauge pair on the sample resulted in a rather noisy record. Furthermore a slight misalignment between the collar and the bars initially led to a small plastic deformation (0.047%) in the specimen before the tensile pulse arrived. Nevertheless it was possible to determine an estimate for $\varepsilon_m/\varepsilon_b$ of 1.59, well below the theoretical prediction of 2.3 (result (7C)).

The next successful shot was 221T whose pressure bar strain pulses are shown in Figure 7.9. A pair of gauges attached to the collar in this test recorded a ratio $\varepsilon_c/\varepsilon_b$ of 1.17 for the initial compressive pulse as quoted in Table 7.2. A second pair of gauges on the central parallel region of the sample monitored the tensile strain only. Figure 7.10 shows the strain gauge signal in response to the tensile wave. As expected, there is no tensile strain signal in the collar. The gauges affixed to the sample broke 52 $\mu$s into the test (i.e. half way) at a strain of 3.6%.

In Figure 7.11 the signal trace from the specimen gauges (Figure 7.10) has been superimposed on the graph of strain versus time for the shot 221T from the computer analysis. The computer calculations were based on a gauge length of 7.8 mm for the test piece, based on the quasistatic results of Section 6.7. Up to a time of 20 $\mu$s the strain gauge record agrees reasonably well with the computer analysis. Thereafter, however, the gauge signal attains levels of strain higher than at the corresponding lines in the computer plot, which suggests that the gauge length, $l_g$, has been overestimated.
From Figure 7.10 it is possible to determine the strain rate at the time when the strain gauges broke which is \( \dot{\varepsilon}_m = 833 \text{ s}^{-1} \). It is appropriate here to recall the SHPB equation (4.8)

\[
\dot{\varepsilon}_m = \frac{2C_O}{\dot{\varepsilon}_g} \varepsilon_R
\]

where \( \dot{\varepsilon}_g \) is the specimen gauge length and \( C_O \) is the longitudinal wave speed in the bars.

Rearranging:

\[
\dot{\varepsilon}_g = \frac{2C_O}{\dot{\varepsilon}_m} \varepsilon_R \tag{7.10}
\]

Taking \( C_O = 5 \times 10^3 \text{ ms}^{-1} \), \( \varepsilon_R = 0.57 \times 10^{-3} \) and \( \dot{\varepsilon}_m = 833 \text{ s}^{-1} \) as deduced above, gives the value of the gauge length as

\[
\dot{\varepsilon}_g = 6.8 \text{ mm} \tag{71}
\]

This result is appreciably lower than the figure of 7.8 mm expected from the results of the quasistatic tests in Section 6.7.

Following shot 221T it was decided to carry out further tests at slightly lower strain rates. For this it was necessary to replace the steel projectile with a duralumin one of the same length, 25 cm. Measurements concentrated on both the compressive and tensile strain in the central parallel region of the specimens, using both channels of the Philips RM3302 storage oscilloscope.

Figure 7.12 shows the specimen strains recorded in shot 223T. Figure 7.12A shows both the initial compressive pulse, and the tensile wave arriving 400 \( \mu \text{s} \) later; B and C are x10 expansions (on the time axis).
FIG 7.12 Compressive and tensile strains recorded in a tensile specimen during a SHPB test at 292 s⁻¹.
Considering first the compression pulse in Figure 7.12B it is clear that the pulse is not level topped which means that there must have been some reflection of the incident wave due to misalignment between the collar and bars to begin with. Also the strain level does not return immediately to the base line but flattens out at a level of 0.048% strain. It is likely that this represents a small amount of plastic deformation before the arrival of the tensile wave. There may have been a small amount of plastic deformation due to the passage of the compressive wave though this is not certain since the lower strain gauge signal in Figure 7.12A does return to the baseline before the arrival of the tensile wave. The highest compressive strain towards the end of the pulse is 0.12% which makes the ratio $\varepsilon_m/\varepsilon_b = 2.64$ in good agreement with the theoretically predicted value of 2.3 (result (7C)).

Moving on to the tensile strain (Figure 7.12C), it can be seen that the strain gauge pair remained perfectly intact throughout the 100 μs test and beyond. The final strain acquired at the end of the test was 3.7%. The strain rate, $\dot{\varepsilon}_m$ over the second half of the test has been measured from the photograph as 439 s$^{-1}$. Applying equation (7.10) again here confirms the gauge length calculated for shot 221T.

\[ l_g = 6.8 \text{ mm} \quad (7I) \]

The last test in this series was shot 225T, again performed at a relatively low strain rate generated by the duralumin projectile. The strain pulses recorded in the pressure bars from this particular test were presented previously in Figure 7.3. This represents one of the best tensile results obtained. Unfortunately both the incident tensile wave and reflected wave (upper trace) are not level topped but show a steady rise in strain amplitude, which is a sign that a
Strains recorded in specimen

Vertical scales:

- tensile (upper trace) = 1.18%/div
- compression (lower trace) = 0.03%/div

Compressive strain
- upper trace 1.18%/div
- lower trace 0.03%/div

Tensile strain
- Vertical scale 1.18%/div

FIG 7.13 Compressive and tensile strains recorded in a tensile specimen during a SHPB test at 308 s\(^{-1}\).
significant reflection occurred at a discontinuity between the bar and the collar earlier in the shot.

This is confirmed in the pictures of Figure 7.13 which show the compressive and tensile strains recorded in the specimen. Figure 7.13B shows the strain produced in the sample by the initial compressive pulse. The lower trace shows a compressive strain of about 0.22% before exceeding the storage range of the CRO. The upper trace indicates that a maximum strain of about 0.3% has been reached, although the accuracy of measurement is lower. The compressive pulse does not return to the baseline at the end of this loading but registers a strain of 0.066%. The high initial compressive strain and the residual strain indicate that the sample has been slightly deformed plastically by the compressive pulse.

In terms of change in length over the whole sample, this plastic strain corresponds to about 11 μm. In many practical instances it would be possible not to notice an air gap of this order between the collar and the bars which reduces the effectiveness of the collar.

The non-return of the specimen compressive strain to zero was a feature of all three shots for which the measurement was made, and is now believed to be due to the presence of a small air gap or slight misalignment between the collar and the bars rather than due to the actual compressive pulse amplitude being too large. The author believes that this reason may explain the large values of $\varepsilon_m/\varepsilon_b$ (i.e. 4.2 and 7.1) measured experimentally by Walker (1987) in copper samples. For shot 225T the ratio $\varepsilon_m/\varepsilon_b$ was measured as 5.4 as a result of the misalignment described above.

Figure 7.13 shows the tensile record in which the strain in the mid-section of the specimen reaches 4.07% by the end of the test. In the
FIG 7.14 True strain versus time plots for four SHPB tensile tests at a mean strain rate of 382 s\(^{-1}\).

FIG 7.15 Compressive and tensile waves in the pressure bars for shot 225 T.
second 50 μs of the tensile pulse, the strain rate, $\dot{\varepsilon}_m$ is 480 s$^{-1}$ and by applying equation (7.10) once more, gives:

$$tg = 6.6 \text{ mm} \quad (7J)$$

Again this is a consistently low value for the gauge length, in disagreement with $l_g$ determined at quasistatic strain rates. Figure 7.14 compares the tensile strain-time records from the strain gauges with those determined by the computer analysis based on a gauge length of 8.3 mm. This figure illustrates the considerable difference in strain and hence strain rate obtained by the two methods. The present quasistatic tests indicated a gauge length of 7.8 mm for strain < 10%. Some comment is needed as to why the value of $l_g$ should be lower in the higher strain rate SHPB tests than at quasistatic rates. One possible reason is that since the yield point is more pronounced at higher strain rates and occurs at a higher stress level, the plastic strain in the specimen is concentrated more in the central parallel region.

It is unlikely that the error in measurement of strain in the central parallel region using strain gauges is very large. In the last shot, 225T the final strain recorded by the gauges in Figure 7.13A (i.e. 3.95%) after the passage of multiple tensile and compressive pulses agreed well with the strain deduced from the change in distance between a couple of parallel scratch marks made in the middle of the specimen (i.e. 3.85%), thereby justifying the reliability in the strain gauge measuring technique.

It is instructive to note what happens to the specimen after the passage of the primary 100 μs tensile pulse through it (Figure 7.13A) and is best understood by consideration of the various compressive and tensile pulses travelling up and down the bars (Figure 7.15).
latter figure represents the same test as Figure 7.3 but has a recording time of ten times longer to include more stress pulses. Initially, in the test, the specimen strain reaches 4.07% (Figure 7.13A) and stays roughly at this level until the reflected wave, $\varepsilon_R$, returns back from the free end 400 $\mu$s later, as a second tensile pulse which increases the strain to 4.65%. Another 400 $\mu$s after that a third tensile wave arrives at the specimen at the same time as a compressive wave produced by the reflection of the primary transmitted tensile wave $\varepsilon_T$ from the impact end of the SHPB line. The compressive wave dominates since now the sample is unprotected by the collar and the strain falls back to 3.7%. Thereafter, compressive and tensile waves arrive at the specimen simultaneously thereby cancelling each other out. As the waves in the pressure bars decay exponentially, the specimen strain levels out to 3.95%. Ogawa (1985) has made use of these multiple stress waves in a similar SHPB arrangement to perform dynamic tension-compression tests on steel and aluminium. He found that the strain rate effect in the tensile loading (or reverse loading) was almost the same as that during the initial compressive loading. It was also found that the tensile loading was always associated with a reduction of yield stress.

7.6 CONCLUDING SUMMARY

This chapter has reviewed the more important methods of tensile testing materials at dynamic strain rates used at the present time. A description of how the Loughborough SHPB system can be modified for tensile testing has been given and a detailed assessment of the accuracy and limitations of this system has been made. It has been found that a gauge length, $l_g$, of 6.8 mm, as opposed to 7.8 mm, is more appropriate to the strain analysis of the 224 steel tensile specimens.
The actual true stress versus true strain results for the tensile tests described here are presented in Chapter 9 where they are compared with the compression results at room temperature and corresponding strain rates.
CHAPTER 8
MEASUREMENT OF ADIABATIC TEMPERATURE RISE IN COMPRESSION TESTS

8.1 INTRODUCTION

When material specimens are compressed beyond their yield point, virtually all the work of plastic deformation is converted into heat. If the deformation takes place slowly, \( \dot{\varepsilon} \leq 10^{-2} \text{ s}^{-1} \) say, then any heat generated within the specimen will be lost at the same rate to the surroundings by means of natural cooling and hence there will be no overall change in temperature of the sample itself. A compression test under these conditions may be considered ISOTHERMAL. However, if the deformation takes place at a high strain rate, as in a SHPB test (duration \( \approx 100 \mu s \), \( \dot{\varepsilon} > 100 \text{ s}^{-1} \) in the Loughborough system) then there will be insufficient time for the heat to escape and the specimen temperature will rise according to the level of strain. A compression test may be said to be ADIABATIC if the strain-rate, \( \dot{\varepsilon} \), is sufficiently high that no heat is lost during the test itself.

Few workers have considered the importance of adiabatic heating in compression testing at high strain rates but in some materials the temperature rise may be large enough to cause an appreciable reduction in flow stress. Follansbee (1986) has calculated that in a Hopkinson Bar test at \( \dot{\varepsilon} = 5000 \text{ s}^{-1} \) on a sample of Nitronic 40 stainless steel the decrease in flow stress, \( \Delta \sigma_y \), at a strain of 20% may be as much as 168 MPa (where \( \sigma_y = 1200 \text{ MPa} \)) due to the expected temperature rise of only +62°C.

Clearly then, it is important to remember the difference between isothermal and adiabatic testing when comparing \( \sigma \) versus \( \varepsilon \) curves at different strain rates since their shape in the plastic region may be
influenced by the thermal condition of the test piece. This idea is developed further in Chapter 9.

8.2 ADIABATIC STRESS AND ADIABATIC STRAIN-RATE, $\dot{\varepsilon}_A$

Adiabatic shear is said to occur at the point at which the loss of strength due to adiabatic heating exceeds the gain in strength due to strain hardening and strain-rate hardening. The effect is usually localised. Frost and Ashby (1982) have derived an expression for the critical strain, $\varepsilon_c$, above which adiabatic shear may occur:

$$\varepsilon_c = \frac{-n C_p^l}{\left| \frac{\partial \sigma_y}{\partial T} \right| \dot{\varepsilon}}$$  \hspace{1cm} (8.1)

where $n$ is the work hardening exponent, $C_p$, the volume specific heat at constant pressure, $\sigma_y$ is flow stress and $T$ is temperature.

For carbon steel: $C_p = 3.69 \times 10^3 \text{ J/m}^3 \text{ °C}$ (from 'Tables of Physical and Chemical Constants' by Kaye and Laby).

For carbon steel, $n = 0.27$ for $\dot{\varepsilon} = 3.8 \times 10^{-4} \text{ s}^{-1}$.

Culver (1973) gives a typical value of $\partial \sigma_y / \partial T$ for mild steel of 633 kN/m$^2$ °C. Substituting these values into equation (8.1) above gives

$$\varepsilon_c = 1.57$$

This is high, corresponding to an engineering compressive strain of about 80% and is hardly a realistic value of $\varepsilon_c$ as one would not expect $\partial \sigma_y / \partial T$ to remain constant at such high strain levels. However, the large value of $\varepsilon_c$ calculated strongly suggests that under most
compressive testing conditions the softening of 224 carbon steel and similar mild steels due to adiabatic heat generation will never equal the level of strain hardening. This is indeed the case for all tests described in this paper but at high strain-rates adiabatic thermal effects may account for a significant reduction in flow stress, \( \sigma_y \).

The condition for adiabacity (i.e. no heat loss from the test specimen to the surroundings) is met when the strain rate exceeds a critical value, \( \dot{\varepsilon}_A \). The value of \( \dot{\varepsilon}_A \) may be calculated by a number of methods but all of them involve solving the heat balance equation which relates the plastic work input with temperature rise in the sample and heat lost to the surroundings.

Figure 8.1 shows a cross-sectional view of the general compression testing arrangement which is for the ESH method. In ESH testing the heat sinks shown in the figure are the one inch steel rollers referred to in Section 3. For the SHPB method the heat sinks are the two ends of the half inch diameter Hopkinson bars themselves.

At first sight it may appear that the determination of the temperature profile in time with the simple geometry of Figure 8.1 is trivial, however a correct solution requires the solving of the expression:

\[
\frac{1}{V} \frac{\partial}{\partial t} T + \frac{\dot{q}_v}{K} = \frac{1}{D} \frac{\partial^2 T}{\partial t^2}
\]

where \( T \) = temperature of the sample, \( \dot{q}_v \) = rate of heat generated in the sample per unit volume, \( K \) = thermal conductivity of the sample, \( t \) = time and \( D \) = thermal diffusivity of the sample which is defined by:

\[
D = \frac{K}{\rho c_p}
\]
FIG 8.1 Diagram showing the general testing arrangement for cylindrical specimens.

- $T$ = temperature of specimen
- $T_s$ = temperature of surroundings

FIG 8.2 Expected temperature rise, $\Delta T_f$ versus final strain, $\varepsilon_f$ in a SHPB sample at $i = 2088s^{-1}$

$\Delta T_f = 2.1i\varepsilon_f - 4$
where \( \rho = \) density and \( C_p = \) mass specific heat capacity of material at constant pressure. Note the difference between \( C_p \) above and the volumetric heat capacity, \( C'_p \) of equation (8.1); the two are related simply by:

\[
C'_p = \rho C_p
\]  

(8.4)

Detailed solutions of equation (8.2) as applied to cylindrical systems may be found in the book by Carslaw and Jaeger (1959). Here it suffices to deal only with simple approximations.

The rate of heat loss, \( \dot{q} \) of a test specimen will be proportional to the difference, \( \Delta T \), between its temperature, \( T \), and the temperature of the surroundings, \( T_s \). Since conduction into the platens is the major heat loss, mathematically this may be written most simply as:

\[
\dot{q} = Ah\Delta T(t)
\]  

(8.5)

where \( A = \) cross-sectional area of the sample, \( h = \) overall heat transfer coefficient and \( \Delta T(t) \) may be written:

\[
\Delta T(t) = T(t) - T_s
\]  

(8.6)

Geiger and Poirrer (1973) show that for a cylindrical system similar to that of Figure 8.1, \( h \) is approximately given by:

\[
h = \frac{2K}{R}
\]  

(8.7)

where \( R = \) radius of the cylindrical sample.

The heat balance equation for a sample under compression can now be written:
where \( V \) = volume of sample. Substituting in for \( \dot{q} \) and integrating:

\[
Vc_p' \frac{dT}{dt} + \dot{q}dt = Vc_y(\varepsilon)d\varepsilon
\]

(8.8)

where \( V \) = volume of sample. Substituting in for \( \dot{q} \) and integrating:

\[
Vc_p' \left( T_F - T_s \right) + 2\pi R K \int_0^{t_F} \Delta T(t)dt = V \int_0^{\varepsilon_F} c_y(\varepsilon)d\varepsilon
\]

(8.9)

where subscript 'F' refers to final sample temperature, strain etc, and \( t_F \) is the duration of the test.

Frost and Ashby (1982) continue their analysis with \( \varepsilon_c \) to solve equation (8.9) and find the critical strain-rate \( \dot{\varepsilon}_A \) for adiabaticity is very approximately

\[
\dot{\varepsilon}_A = \frac{4\varepsilon_c K}{C_p R^2}
\]

(8.10)

Substituting in the values \( \varepsilon_c = 1.57, K = 48 \ W/\text{m}^2\text{C}, C_p' = 3.7 \ \text{MJ}/\text{m}^3\text{OC} \) and \( R = 5 \text{ mm} \) gives:

\[
\dot{\varepsilon}_A = 3.3 \text{ s}^{-1}
\]

which is close to the maximum rate achievable on the ESH machine!

It is instructive to note that for the adiabatic case the middle term in equation (8.9) disappears and one should expect the total temperature rise, \( \Delta T_F \) in the sample to be given by:

\[
\Delta T_F = T_F - T_s = \frac{1}{C_p} \int_0^{\varepsilon_F} c_y(\varepsilon)d\varepsilon
\]

(8.11)

The equation above (8.11) has been used to determine the graph of \( \Delta T_F(\text{OC}) \) versus \( \varepsilon(\%) \) at \( \varepsilon = 2088 \text{ s}^{-1} \) in Figure 8.2. The integral on the right hand side of the equation was evaluated by measuring the area.
under the $\sigma$ versus $\varepsilon$ curve (Figure 9.9) at $\dot{\varepsilon} = 2088 \text{ s}^{-1}$ for various strain levels extrapolating to $\varepsilon = 40\%$.

It can be seen that the curve of Figure 8.2 is virtually linear and this linearity may be represented quite accurately ($\pm 0.5^\circ\text{C}$) by

$$\Delta T_{FP}(^\circ\text{C}) = 2.1\varepsilon(\%) - 4$$

(8.12)

for $\varepsilon > 10\%$, $\dot{\varepsilon} = 2088 \text{ s}^{-1}$.

$\dot{\varepsilon} = 2088 \text{ s}^{-1}$ is just about the highest strain rate achievable with 224 carbon steel 10 x 5 mm samples tested on the Loughborough SHPB and thus should produce the largest temperature rise. It is important to note that the 10 x 5 mm samples are not usually strained beyond 16%, thus one should expect the temperature rise to be below $+30^\circ\text{C}$ for most shots. On the other hand, the 8 x 4 mm specimens which have been used can reach true strains of between 32-36% at their highest strain-rate, of 4500 $\text{ s}^{-1}$ and so may reach temperatures up to $+70^\circ\text{C}$ above ambient, in which case significant flow stress reduction, $\Delta\sigma_y$ may occur.

8.3 EXPERIMENTAL PROCEDURE FOR MEASURING THE TEMPERATURE RISE IN SAMPLES DURING COMPRESSION

During the elevated temperature testing at $150^\circ\text{C}$ on the ESH machine in which a K-type thermocouple was strapped firmly with several loops of wire to the surface of the sample it was observed that the surface temperature would rise by a few degrees during compression. This initial observation was followed up by drilling 0.77 mm bore holes through the sides of ten 10 x 5 mm samples, mounting 36 SWG (diameter 0.2 mm) K-type thermocouple wires inside them and hence recording the temperature rise, $\Delta T_{ESH}$ during compression at a variety of rates.
FIG8.3 Thermocouple output during an E.S.H. compression of a 10 x 5mm sample at 2mm/s. Temperature rise = +25°C. Test duration = 1 second; Permanent sample strain = 33.1%.
The thermocouple was held fast in each hole by the application of a thin smear of 'Araldite' to the two wires before insertion after which the orifice of the hole was sealed off with a larger mass of 'Araldite'. It was found that 0.75 mm was the smallest diameter of hole which could be drilled without breaking the drill itself, though this inevitably meant that the thermocouple only occupied about 15% of the volume of the hole the rest being mainly air. Later on it was found that this particular configuration was not ideal but nevertheless formed a useful starting point.

The leads of the thermocouple were connected directly to a JJ Instruments 6525 recorder which had a maximum voltage resolution of ±2 μV on its most sensitive scale and so was sufficiently accurate to monitor temperature changes from the thermocouple (output = 40 μV/°C). An advantage of the ESH machine was that since its output gave load versus displacement for each compression run it was a simple matter to calculate the expected temperature rise, ΔTESH, from the area under the curve (= energy input) divided by specific heat, C_p. In all these thermal tests ΔTESH = (71 ± 1)°C assuming adiabaticity, for a final strain of 36%.

8.4 RESULTS FROM ESH THERMAL TESTS

Figure 8.3 shows a typical chart recorder trace of thermocouple emf versus time at a displacement rate of 2 mm/s (≈ 0.4 s⁻¹). Since the total displacement was only 2 mm the duration of the test was a single second which is precisely the time for which the thermocouple output rises before decaying away because of conductive cooling of the sample through the steel rollers. The slight kink in the upward rise of thermocouple emf was a feature of most of these thermal records and is thought to be due to an effect caused by the hole in the sample closing up.
Vertical scale = 0.1mV/division
= 2.5°C/division
Horizontal scale = 0.5s/division

FIG8.4 Thermocouple output during an E.S.H. compression of a 10 x 5mm sample at 20mm/s. Test duration = 0.1s; Permanent sample strain = 32%.
Figure 8.4 shows the trace for which $\Delta T_{ESH}$ was greatest, i.e. $+43^\circ C$. This was recorded at a displacement rate of 20 mm/s ($\dot{\epsilon} = 3.4 \text{ s}^{-1}$) in which the test duration was only 0.1s. Some time afterwards it was discovered that the maximum writing speed of the chart recorder, 50 cm s$^{-1}$ was insufficiently fast to cope with such a rapid rate of rise, evidenced by the fact that the trace in Figure 8.4 takes 0.3s, instead of 0.1s, to reach maximum temperature. Hence it must have taken the chart recorder an extra 0.2s to catch up with the thermocouple emf in which time, of course, the sample would have cooled appreciably. It is possible to determine a rough estimate of the temperature the sample actually reached by fitting an exponential cooling curve to the trace for the first 0.2s after the maximum $\Delta T_{ESH}$ is reached and then extrapolating this back for the 0.2s before maximum $\Delta T_{ESH}$ is reached. When this approximate calculation is performed it is found that $\Delta T_{ESH} = +66^\circ C$ is a more representative temperature rise at this strain-rate.

The best method of checking this would have been to use a transient recorder in conjunction with a dc amplifier to amplify the thermocouple signal; however since no suitable amplifier was available for use with the ESH machine this was not possible.

The corrected rise of $+66^\circ C$ indicates that the process is almost adiabatic at this strain rate, as $+71^\circ C$ is predicted from the actual load displacement curve. Figure 8.5 is a graph of mean values of $\Delta T_{ESH}$ versus strain rate from the ten ESH 'thermal' tests and includes the correction. Extrapolation of the curve to intersect with $\Delta T_{ESH} = 71$ gives the critical adiabatic strain rate:

$$\dot{\epsilon}_A = 4 \text{ s}^{-1}$$

This value compares very well with the theoretical value deduced earlier i.e. $\dot{\epsilon}_A = 3.3 \text{ s}^{-1}$ from equation (8.10). The fact that $\dot{\epsilon}_A$ as measured from the graph is larger merely suggests that the actual heat
FIG 8.5 $\Delta T_{esh}$ versus $\varepsilon (s^{-1})$
transfer coefficient, $h$, for the sample is slightly higher than given by equation (8.7). Certainly the rapid cooling in Figure 8.4 shows that the conductive heat transfer from the sample to the steel rollers is very efficient.

8.5  THE MEASUREMENT OF TEMPERATURE RISE IN HOPKINSON BAR TESTS

8.5.1 Chart Recorder Results
The first three shots on the Hopkinson bar to measure the temperature rise in the sample, $\Delta T_{\text{SHPB}}$ were carried out under similar conditions to the ESH 'thermal' shots, the thermocouple being stuck in 0.75 mm holes with 'Araldite' and connected directly to the same chart recorder as used previously. All three were performed at maximum $\dot{\varepsilon} = 2088 \text{ s}^{-1}$. The temperature rises measured in this way were in very good agreement with those predicted by equation (8.12), see Table 8.1.

<table>
<thead>
<tr>
<th>Dimensions of Samples (mm)</th>
<th>Final True Strain (%) $\varepsilon_F$</th>
<th>Recorded Temperature Rise ($^\circ$C), $\Delta T_{\text{SHPB}}$</th>
<th>Expected Temperature Rise ($^\circ$C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) 10 x 5</td>
<td>12.3</td>
<td>+21</td>
<td>+21.8</td>
</tr>
<tr>
<td>ii) 10 x 5</td>
<td>13.0</td>
<td>+22</td>
<td>+23.3</td>
</tr>
<tr>
<td>iii) 8 x 4</td>
<td>31.7</td>
<td>+60</td>
<td>+62.6</td>
</tr>
</tbody>
</table>

* Expected $\Delta T$ has been calculated from $\varepsilon_F$ using equation (8.12).

TABLE 8.1: SHOWING TEMPERATURE RISES IN THREE HOPKINSON BAR SAMPLES RECORDED USING A JJ INSTRUMENTS CR6525 CHART RECORDER

Figure 8.6 shows the actual record for the 8 x 4 mm sample in Table 8.1. The traces for the 10 x 5 mm sample shots were very similar in shape except of course that the level of thermocouple emf was lower.
FIG 8.6 Thermocouple trace recorded during a SHPB shot on an 8 x 4mm sample, at 36ms\(^{-1}\) projectile velocity. Thermocouple eventually indicates a rise of +60°C after which there is slow air cooling of sample detached from the bar. Test duration = 100μs. Permanent sample strain = 27.2%.
All three records had the characteristic rapid rise to a peak followed by a dip then a more gentle rise to the maximum $\Delta T_{\text{SHPB}}$. The very slow rise time (approximately 9s in Figure 8.6) for the thermocouple emf trace to reach maximum $\Delta T_{\text{SHPB}}$ is surprising since on the time scale of the chart recorder (0.5s per cm) one should expect the signal to reach the maximum after $\approx$0.35 ms (the fastest chart response) as the test duration time is only 100 $\mu$s, after which no further work is done on the sample.

The presence of a peak in the record is believed to be due to an effect caused by the hole closing up as in the ESH records. It is important to note also that the rate of cooling after the maximum temperature has been reached is much slower here than in the ESH arrangement. This is because, a short time after the incident stress pulse has passed through the SHPB sample, the latter drops free of the bars and becomes suspended in mid-air while still attached to the thermocouple leads. Hence thermal contact with the bars, which act as good heat sinks, is lost and the convective cooling in air is much slower.

8.5.2 The Response Time of K-Type ('Chromel-Alumel') Thermocouples

The extremely long rise time of Figure 8.6 calls into doubt the response time of the thermocouple itself and poses the question of whether it is reasonably possible to measure temperature rises in SHPB samples inside 100 $\mu$s using such a device. In answering this problem careful consideration must be paid to several aspects of the thermocouple arrangement described in use so far. As well as the response time of the bare wire thermocouple, the effects of the 'Araldite' and air in the hole need to be assessed. These features are discussed broadly in the following pages.
Only a small amount of literature exists on the response time of thermocouples and its measurement. Palmer and Turner (1966) have investigated the response of thermocouple junctions to shock waves of pressures between 50 and 300 Kbar but always found that the thermocouple emf's were far greater than could be expected from the temperature rise alone. Wormser (1960) has measured the response time of various thermocouples by immersion in a water bath at constant temperature.

Theoretical determination of the response time of a thermocouple involves the solving of the heat balance equation (8.2) without the heat generation term:

\[ \nabla^2 T = \frac{1}{\rho} \frac{\partial T}{\partial t} \]  

(8.13)

Benedict (1977) shows that the solution for a temperature sensor (such as a thermocouple) may be written:

\[ T = C e^{-t/\tau} + \frac{1}{\tau} \int_0^t T_e e^{t/\tau} \, dt \]  

(8.14)

where \( T \) = temperature of thermocouple and \( T_e \) is the temperature of the environment, both at time \( t \), \( C \) is a constant of integration and \( \tau \) is a first order thermal time constant. \( \tau \) is often used to characterise the response time of a thermal sensor and is defined for all thermal sensors by

\[ \tau = \frac{\rho V C_p}{h A} \]  

(8.15)

where \( \rho \) is density, \( V \) is volume, \( C_p \) is specific heat of the sensor, \( A \) is the area of fluid film around the sensor and \( h \) is the heat transfer coefficient. Clearly \( h \) will depend on the thermal contact between the
A) A bare 36swg K-type thermocouple immersed in a water bath at 72°C, from room temperature.

Horizontal scale: 10ms/division
Vertical scale: 0.5mV/division = 12.5°C/division

Total temperature rise indicated = +46°C

B) Same thermocouple as above but embedded in araldite and immersed into a water bath at 80°C, from room temperature.

Horizontal scale: 400ms/division
Vertical scale: 1mV/division = 25°C/division

Total temperature rise indicated = +50°C

FIG 8.7 The time response of three thermocouples when immersed suddenly into a constant temperature water bath.
C: The response of 0.1mm K-type thermocouple wire when immersed suddenly into a water bath at 80°C. (from room temperature).
Horizontal scale: 2ms/division
Vertical scale: 1mV/division = 25°C/division
Indicated temperature rise = +59°C

FIG8.7 (cont): The Time Response of Three Thermocouples when Immersed Suddenly into a Constant Temperature Water Bath.
sensor and the medium whose temperature it is trying to measure. For the thermal measurements described in this paper \( h \) will depend on the air/'Aralditel' content in the holes.

For an "instantaneous" step change in temperature \( T_1 + T_e \) (such as when a thermocouple is plunged into a constant temperature bath) equation (8.14) reduces to:

\[
T_e - T = (T_e - T_1)e^{-t/\tau}
\]  

(8.16)

\( \tau \) is the time for the thermocouple temperature, \( T \) to reach 63\% of \( T_e \) and for all practical purposes \( T \) will equal \( T_e \) after about 5\( \tau \).

A simple experiment was performed to observe the difference in response between a bare wire 36 SWG K-type thermocouple, the same thermocouple surrounded by a layer of 'Aralditel' and a fine wire (diameter = 0.1 mm) K-type thermocouple when suddenly immersed in a constant temperature water bath. The results are displayed in Figure 8.7.

All three responses have exactly the shape one would expect from equation (8.16). Table 8.2 gives the values of \( \tau \) as measured from the three photographs.

<table>
<thead>
<tr>
<th>A) 36 SWG bare wire thermocouple (dia = 0.2 mm)</th>
<th>B) 36 SWG thermocouple coated in 'Aralditel'</th>
<th>C) Bare Fine Wire thermocouple (dia = 0.1 mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau ) (ms)</td>
<td>10</td>
<td>400</td>
</tr>
</tbody>
</table>

TABLE 8.2: VALUES OF \( \tau \) FOR THERMOCOUPLES PLUNGED IN CONSTANT TEMPERATURE WATER BATH
Table 8.2 illustrates the marked effect that 'Aralditel' has on slowing the response time of the 36 SWG thermocouple. In the light of these elementary results it was decided to desist from using Araldite to fix the thermocouples in sample holes and to make more use of the fine 'Chromel-Alumel' wire which had now become available. The figures in Table 8.2 do not represent the absolute response times of the thermocouples in question since their immersion into the constant temperature bath is far from being instantaneous. Ultimately the value of \( \tau \) by this method of experimental analysis depends on the 'immersion velocity' i.e. \( h \) is time dependent.

An idea of the response time of K-type thermocouples to a step temperature input \( (T_1 + T_e) \) may be gained by calculating theoretically the time it takes for the chromel and alumel leads to come into thermal equilibrium with the surroundings. Consider each wire as being an infinitely long cylinder of radius, \( r = a \). Appropriate boundary conditions are that at time \( t = 0 \) the temperature at the surface of the cylinder \( (r = a) \) is maintained at \( T = T_e \). The solution of equation (8.13) as applied to these conditions is given by Carslaw and Jaeger (p.199) as:

\[
T = T_e - \frac{2T_e}{\Lambda} \sum_{n=1}^{\infty} e^{-D\alpha_n^2 t} \frac{J_0(\alpha_n r/a)}{\alpha_n J_1(\alpha_n)}
\]

(8.17)

where \( J_0 \) and \( J_1 \) are Bessel functions with roots \( \alpha_n \) and \( D \) = thermal diffusivity of the cylinder.

Carslaw and Jaeger present the solution to the equation (8.17) above as a family of curves relating the ratio \( T/T_e \) with \( r/a \). Here we are concerned with the temperature, \( T_e \), at the centre of the wires, at \( r = 0 \). Table 8.3 gives the solution at this point.
FIG 8.8 Temperature $T_c$ at the centre of thermocouple wire versus time, $t$, when temperature, $T_e$, is instantaneously applied to the wire surface at $t = 0$. 
FIG8.9: Theoretical response of 36SWG K-type thermocouple in a 0.1mm coating of "araldite"
TABLE 8.3: SHOWING THE RELATIONSHIP BETWEEN THE RATIO $T_c/T_e$ AND THE DIMENSIONLESS PARAMETER WHICH IS PROPORTIONAL TO TIME, $t$ [From Carslaw and Jaeger, p.200]

<table>
<thead>
<tr>
<th>$T_c/T_e$</th>
<th>$\beta = \frac{Dt}{a^2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.98</td>
<td>0.80</td>
</tr>
<tr>
<td>0.95</td>
<td>0.60</td>
</tr>
<tr>
<td>0.84</td>
<td>0.40</td>
</tr>
<tr>
<td>0.72</td>
<td>0.30</td>
</tr>
<tr>
<td>0.50</td>
<td>0.20</td>
</tr>
<tr>
<td>0.34</td>
<td>0.15</td>
</tr>
<tr>
<td>0.155</td>
<td>0.10</td>
</tr>
<tr>
<td>0.08</td>
<td>0.08</td>
</tr>
<tr>
<td>0.03</td>
<td>0.06</td>
</tr>
</tbody>
</table>

In Table 8.3 is given by:

\[
\beta = \frac{Dt}{a^2} \quad (8.18a)
\]

Rearranging:

\[
t = \frac{a^2 \beta}{D} \quad (8.18b)
\]

Hence it can be seen that the response time of a thermocouple varies as the square of its radius and the reciprocal of its thermal diffusivity. From Table 8.3, curves of $T_c/T_e$ versus time, $t$, have been drawn up for the 36 SWG ($a = 9.5 \times 10^{-3}$ cm) and the fine ($a = 5.5 \times 10^{-3}$ cm) thermocouple wire in Figure 8.8. Figure 8.9 shows the approximate theoretical response of the 36 SWG thermocouple surrounded
<table>
<thead>
<tr>
<th>Material</th>
<th>Thermal Conductivity, K (W/m°C)</th>
<th>Specific Heat, Cp (J/kg°C)</th>
<th>Density, ρ (kg/m³)</th>
<th>Thermal Diffusivity D (10⁻⁶ m²/s⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+Ve &quot;Chromel&quot; (90%Ni+10%Cr)</td>
<td>19.3</td>
<td>449</td>
<td>8730</td>
<td>4.92</td>
</tr>
<tr>
<td>-Ve &quot;Alumel&quot; (95%Ni+5%Al+1%Si)</td>
<td>0.19</td>
<td>525</td>
<td>8600</td>
<td>0.12</td>
</tr>
<tr>
<td>Soft Solder (54%Sn+45%Pb)</td>
<td>48</td>
<td>480</td>
<td>7690</td>
<td>13</td>
</tr>
<tr>
<td>224 Carbon Steel</td>
<td>51</td>
<td>176</td>
<td>9000</td>
<td>32</td>
</tr>
</tbody>
</table>
uniformly by a 0.1 mm cylindrical layer of 'Araldite'. The physical data necessary to deduce D in the determination of all three curves is given in Table 8.4.

\[
\begin{array}{|c|c|c|}
\hline
 \text{A) 36 SWG wire} & \text{B) 36 SWG thermocouple} & \text{C) Fine Wire} \\
\text{thermocouple} & \text{coated in a 0.1 mm layer of Araldite} & \text{thermocouple} \\
& & (\theta = 0.1 \text{ mm}) \\
\hline
\text{t}_{98} \text{(ms)} & 1.28 & 65.6 & 0.428 \\
\hline
\end{array}
\]

**TABLE 8.5: THEORETICAL TIMES (t\textsubscript{98}) FOR CENTRE OF THERMOCOUPLES TO REACH 98\% OF THE SURFACE TEMPERATURE T\textsubscript{e} WHEN A STEP RISE IN TEMPERATURE IS APPLIED T\textsubscript{1} \rightarrow T\textsubscript{e}**

Table 8.5 gives the theoretical times for the thermocouples under consideration here to come into thermal equilibrium with their surroundings when a step rise in temperature is applied. The figures serve as rough estimates of the likely response time of the thermocouples in practical use when transient temperatures are present. The 'actual' response may be faster than the values suggest since a welded thermocouple junction will begin responding first to its surface temperature change and not at the internal centre of the junction. However to offset this one should remember that the heat transfer from the surroundings to the thermocouple wires will never be 100\% efficient. Hence it is felt that Table 8.5 serves as a useful guide to the typical response times likely to be met with in practice. We should not expect, therefore, that the thermocouples considered here be able to reproduce the temperature-time profile inside a 100 \mu s Hopkinson bar test, though they should record the maximum temperature reached.

It is gratifying to note that the times in Table 8.5 are roughly in proportion to the time constants, \( \tau \) of Table 8.2, i.e.
\[ \frac{5\tau}{t_{98}} = 30 \pm 8 \]

Since \( 5\tau \) is the time to reach 98%, this ratio should be 1 in theory.

8.5.3 Transient Recorder Results

In the light of the preceding work and the dubiety which existed in the chart recorder results (Table 8.1) it was decided to carry out further temperature rise, \( \Delta T_{SHPB} \), measurements using a similar but improved experimental arrangement incorporating a Datalab DL902 transient recorder. For these tests 0.75 mm bore holes were drilled right through 10 x 5 mm and 8 x 4 samples which were then filled by a mass of soft solder. The samples were heated on a large iron to a temperature of about 200°C at which point the solder melted and the bare wire thermocouples (absolutely no Araldite) were placed directly into the molten solder, removed from the heat and allowed to set. Both the 0.1 mm fine and 36 SWG wire were used in the tests.

In this fashion it was hoped to banish all air from the holes and ensure good thermal contact (high value of \( h \)) between the thermocouple and the surrounding steel. In fact solder is a particularly good conducting material to use for this purpose since it has the highest thermal diffusivity of all substances considered here (see Table 8.4).

During the compression shots themselves the thermocouple signals were amplified 25X through one channel of a 556 Textronix oscilloscope before being passed on to the transient recorder. Shots were fired at the highest projectile velocity, i.e. 37 ± 3 ms\(^{-1} \) so that the largest strain and hence largest possible temperature rise was observed. The resulting photographic records are shown in Figure 8.10.
FIG8.10 The response of a K-type thermocouple to a SHPB shot at $\dot{e} \sim 2100 \text{s}^{-1}$ (for further details refer to TABLE8).
E) Horizontal scale: 4ms/division  
Vertical scale: 1mV/division = 25°C/division

F) Horizontal scale: 4ms/division  
Vertical scale: 1mV/division

FIG8.10 (Cont.)
G) Horizontal scale: 4ms/division
Vertical scale: 1mV/division = 25°C/division

H) Horizontal scale: 4ms/division
Vertical scale: 1mV/division = 25°C/division

FIG8.10 (Cont.)
I) Horizontal scale: 0.2ms/division
Vertical scale: 1mV/division = 25°C/division

J) Horizontal scale: 0.2ms/division
Vertical scale: 1mV/division = 25°C/division

FIG8.10 (Cont.)
Horizontal scale: 0.4 ms/division
Vertical scale: 1 mV/division = 25°C/division
<table>
<thead>
<tr>
<th>SHOT</th>
<th>SAMPLE DIMENSIONS (mm)</th>
<th>THERMOCOUPLE TYPE</th>
<th>MEASURED THERMOCOUPLE TIME CONSTANT, ( \tau (\text{ms}) )</th>
<th>FINAL TRUE STRAIN ( \varepsilon_{\text{f}}(%) )</th>
<th>PREDICTED TEMPERATURE RISE ( \Delta T_{\text{F}}(\degree \text{C}) )</th>
<th>MEASURED TEMPERATURE RISE ( \Delta T_{\text{SHPB}}(\degree \text{C}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8 x 4</td>
<td>36 swg</td>
<td>&lt; 40</td>
<td>26.7</td>
<td>52</td>
<td>50 ± 4</td>
</tr>
<tr>
<td>B</td>
<td>8 x 4</td>
<td>36 swg</td>
<td>&lt; 4</td>
<td>32.1</td>
<td>63</td>
<td>62 ± 3</td>
</tr>
<tr>
<td>C</td>
<td>8 x 4</td>
<td>Fine Wire</td>
<td>8</td>
<td>23.1</td>
<td>45</td>
<td>40 ± 5</td>
</tr>
<tr>
<td>D</td>
<td>8 x 4</td>
<td>36 swg</td>
<td>immeasurable</td>
<td>31.7</td>
<td>63</td>
<td>57 ± 10</td>
</tr>
<tr>
<td>E</td>
<td>8 x 4</td>
<td>Fine Wire</td>
<td>3.2</td>
<td>24.9</td>
<td>48</td>
<td>45 ± 2</td>
</tr>
<tr>
<td>F</td>
<td>10 x 5</td>
<td>Fine Wire</td>
<td>5.2</td>
<td>18.1</td>
<td>34</td>
<td>33 ± 5</td>
</tr>
<tr>
<td>G</td>
<td>10 x 5</td>
<td>36 swg</td>
<td>immeasurable</td>
<td>13.3</td>
<td>24</td>
<td>25 ± 10</td>
</tr>
<tr>
<td>H</td>
<td>8 x 4</td>
<td>Fine Wire</td>
<td>&lt; 1</td>
<td>35.5</td>
<td>71</td>
<td>70 ± 5</td>
</tr>
<tr>
<td>I</td>
<td>10 x 5</td>
<td>Fine Wire</td>
<td>Noise rather than sensible</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>10 x 5</td>
<td>36 swg</td>
<td>immeasurable</td>
<td>14.7</td>
<td>27</td>
<td>30 ± 15</td>
</tr>
<tr>
<td>K</td>
<td>8 x 4</td>
<td>Fine Wire</td>
<td>0.32</td>
<td>35.5</td>
<td>69</td>
<td>69 ± 3</td>
</tr>
</tbody>
</table>

Table 3.5 The concise results of 11 SHPB shots performed to compare the experimentally measured temperature rise, \( \Delta T_{\text{SHPB}} \), with the expected rise \( \Delta T_{\text{F}} \), predicted by equation (5.12).
On the noise free records the temperature was measured at the maximum in the signal. On the records where noise was present the temperature was read off at the most 'stable' point on the trace. In photograph I it was not possible to measure any sensible temperature as this particular record was 'swamped' in noise.

(A) was the first thermal shot recorded in this way with the transient recorder timebase set to a slow value (≈ 4s total sweep) comparable with the chart recorder timebase used previously. Clearly on this time scale the rise is "instantaneous". After this first shot the following tests were carried out with progressively shorter timebases so that in the last shot (K) the total sweep was just 2 ms.

On a number of the traces, most notably (D), (G) and (J), a varying amount of noise appeared before the thermocouple reached a stable dc emf. This made it difficult to measure the time constant \( \tau \) at the point at which the thermocouple signal reached 63\% of its stable level, and in the cases named above was impossible. Strangely enough, most noise was always produced in the 36 SWG thermocouple. The noise was caused by the mechanical shock to the thermocouple, though the varying magnitude and nature of it is difficult to explain.

The important features of the photographic results are summarised in Table 8.6. The most impressive outcome is how well the measured temperature rises agree with those predicted from the final strains, \( \varepsilon_F \) (equation 8.12). The worst discrepancy occurs in shot (D) where the measured \( \Delta T_{SHPB} \) is 6\(^\circ\)C lower than \( \Delta T_F \). Most values of \( \Delta T_{SHPB} \) are lower than \( \Delta T_F \) as one would suppose that heat transfer from the steel to the thermocouple is not 100\% efficient and also that there may be some slight conductive cooling in the time it takes for the thermocouple to respond. The excellent agreement between \( \Delta T_{SHPB} \) and \( \Delta T_F \) means that equation (8.12) may be used to predict temperature rises in future tests with confidence.
Lastly, it is pleasing to see that values of $\tau$ in the fine wire thermocouple tests are of the same order as suggested by the empirical and theoretical considerations of Section 8.4.2.

8.6 SUMMARY

This chapter has demonstrated the significance of adiabatic temperature rise in test samples during compression at room temperature. Reliable temperature measurements inside specimens have been made, the results of which agree remarkably well with the theory indicating that nearly all the work of plastic deformation is converted into heat. The critical strain-rate, $\dot{\varepsilon}_A$ for adiabatic heat generation to occur in compression tests using the Loughborough facilities has been shown to be about 4 s$^{-1}$.

An interesting aside to the experiments described in this chapter has been the determination of the time constant, $\tau$, of a fine wire thermocouple - something which is practically difficult to achieve.

In the next chapter, the effect of adiabatic temperature rise during compression on the flow stress is investigated and is shown to cause a marked reduction in strength at low test temperatures.
CHAPTER 9

RESULTS

9.1 INTRODUCTION

This Chapter presents all the stress-strain results from 224 steel obtained throughout the course of the project covering a strain-rate range of $10^{-4}$ s$^{-1}$ to 5000 s$^{-1}$ and a temperature range 163K to 573K. Here the results are collated and analysed in some detail. Comparison with the results of other workers on steels of similar composition indicates that thermally activated mechanisms play an important part in the mechanical behaviour of 224 steel at high strain-rates. First, the source of the specimens used in these tests should be indicated.

In January 1987 two billets of carbon 224 steel were received from the UKAEA (Winfirth). Figure 9.1 shows photographs of both billets which were near semi-circular slices of steel and labelled 1Bl and 2Bl.

Each billet was divided up into clearly defined segments from which the specimens were extracted. A logical cylindrical coordinate system $(Z,R,\theta)$ was devised so that the exact original location in the billet of each sample was known. The system is illustrated in Figure 9.2.

Two types of sample were cut out from two orthogonal planes within each segment denoted by the coordinate $\theta$. Samples were termed 'Parallel Axis' (PA) for those cut in a direction parallel to the central axis of the billet and 'Radial Axis' (RA) for those samples cut in a radial direction as shown. Out of the 500 specimens extracted in total only 24 were tensile test pieces of the type shown in Figure 5.2 and all of these were removed from segment 9 (i.e. $\theta = 9$), 12 being cut in the radial direction, the other 12 being cut in the parallel direction.
FIGURE 9.1A Photograph of the two steel billets 1B1 and 2B1 received from U.K.A.E.A. (Winfrith).
Photograph of 2B1 showing how it was divided into segments for the extraction of small cylindrical samples.
FIG9:2: The PA(Z,R,θ) and RA(Z,R,θ) coordinate scheme in the 2B1 billet.
As can be seen from Figure 9.2 no samples were taken from within 15 mm of the outer circumference of the billet since the strength of steel in this region may have been susceptible to effects of inhomogeneous cooling and composition during manufacture. Initially, the plan was to test samples from both billets (1B1 and 2B1) and compare the results from each. However, an expanded testing program over a larger range of temperatures and strain-rates than originally anticipated meant that there was only sufficient time to test samples from 2B1.

The cylindrical compression specimens were manufactured in the Department of Physics' mechanical workshop. Two sizes were made, 8 mm diameter x 4 mm length and 10 mm diameter x 5 mm length. The smaller samples proved useful in producing a larger strain rate in the SHPB tests and a larger final strain in the Instron tests. Samples were machined on a lathe which used coolant to ensure that high temperatures which may have disturbed the internal structure of the steel were not incurred. After cutting the samples were finely ground to produce smooth flat parallel faces. They were not heat treated in any way before being mechanically tested in compression. An account of the quality of the surface finish of the samples was given in Section 3.2.

The tensile test pieces were prepared in the Department of Manufacturing Engineering and similarly were not heat treated before testing.

9.2 THE SPLIT HOPKINSON PRESSURE BAR (SHPB) RESULTS

9.2.1 Room Temperature Compression Results

In Chapter 4, Figure 4.15, photographs were shown of 4 typical sets of SHPB pulses recorded at 4 different projectile impact velocities. Figure 4.16 showed the oscillation free quality of pulses achievable
FIG 7: Set of SHPB results at a mean strain rate of 2088 s⁻¹, 293 K.

FIG 9.8: Typical plots of true strain versus time records. (corresponding stress-strain curves given in Fig 9.7)
in the Loughborough SHPB system when a type 431 martensite steel bar was used before the loading bar with elastic bands providing a restoring force between all the bar and specimen faces. The accompanying true stress-true strain computer plot in Figure 4.16 shows how there can be no uncertainty in the level of stress when such 'clean' pulses are recorded. More important is the unequivocal definition of upper and lower yield point.

Unfortunately, it was not possible to rid every SHPB test of Pochhammer-Chree oscillations completely. Figures 9.3 to 9.7 show a complete set of SHPB results at room temperature over the strain-rate range 83 s\(^{-1}\) to 2317 s\(^{-1}\) on 10 x 5 mm specimens. Figures 9.3 to 9.7 represent the definitive set of SHPB results at room temperature and were obtained only after preliminary investigations had been carried out on a batch of unlubricated 10 x 5 mm samples and a batch of 8 x 4 mm samples.

Clearly the stress-strain curves in Figures 9.3 to 9.7 have been grouped according to the strain-rate, \(\dot{\varepsilon}\), at which each curve was obtained. The strain-rates corresponding to the curves in each group are listed at the top of each graph. Strain-rate as used here represents the mean plastic strain-rate for each test and was calculated from the difference in strain at 20 \(\mu\)s and 80 \(\mu\)s divided by 60 \(\mu\)s. Figure 9.8 illustrates the calculation for the group of strain versus time curves which resulted from the tests in which the group of stress versus strain curves in Figure 9.7 were obtained.

Figure 9.8 is typical of all SHPB results grouped in this way at different temperatures and strain-rates. Throughout the 100 \(\mu\)s duration of each test the strain-rate is fairly constant apart from the initial part where the specimen is still elastic. The time for which the specimen is elastic at the beginning of the test depends
<table>
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<tr>
<th>Impact Velocity $(\pm 3)/\text{ms}^{-1}$</th>
<th>Elastic $\dot{\varepsilon}_e$ $(\text{s}^{-1})$</th>
<th>Plastic $\dot{\varepsilon}_p$ $(\text{s}^{-1})$</th>
<th>Final Strain ($%$)</th>
<th>Elastic $\dot{\varepsilon}_e$ $(\text{s}^{-1})$</th>
<th>Plastic $\dot{\varepsilon}_p$ $(\text{s}^{-1})$</th>
<th>Final Strain ($%$)</th>
</tr>
</thead>
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<td>215</td>
<td>1257</td>
<td>9.3 $\pm$ 2</td>
</tr>
<tr>
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<td>788</td>
<td>7.5 $\pm$ 2</td>
<td>250</td>
<td>2286</td>
<td>18 $\pm$ 2</td>
</tr>
<tr>
<td>31</td>
<td>303</td>
<td>1390</td>
<td>10 $\pm$ 2.5</td>
<td>267</td>
<td>3571</td>
<td>26 $\pm$ 4</td>
</tr>
<tr>
<td>37</td>
<td>298</td>
<td>2009</td>
<td>15 $\pm$ 5</td>
<td>271</td>
<td>4683</td>
<td>30 $\pm$ 5</td>
</tr>
<tr>
<td>(All $\pm$ 30)</td>
<td></td>
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<td></td>
<td>(All $\pm$ 30)</td>
</tr>
<tr>
<td>Quasistatic</td>
<td>$10^{-3}$</td>
<td>7.6</td>
<td>0.8</td>
<td>$10^{-3}$</td>
<td>19.8</td>
<td>1.3</td>
</tr>
</tbody>
</table>
largely on the yield stress which in turn depends on the test temperature and so may vary between 10 \( \mu \)s at around 573K and 20 \( \mu \)s at 163K. Hence the slope of the strain-time curve for the time between 20 \( \mu \)s and 80 \( \mu \)s should give a reasonable mean strain-rate for each test under all temperature and strain-rate conditions considered here.

It is possible to determine the mean elastic strain-rate, \( \dot{\varepsilon}_e \) corresponding to the portion of the test before yield. This was done in the preliminary trial tests on a batch of 10 x 5 mm samples (unlubricated) and a batch of lubricated 8 x 4 mm samples (Dixcn, 1988). For each test the upper yield stress was noted and divided by the Young's Modulus for the 224 steel (i.e. 200 GPa at room temperature) to give the yield strain, \( \varepsilon_y \). This figure was then divided by the time, \( \tau_y \) taken for the upper yield stress to be reached, thus giving elastic strain-rate, \( \dot{\varepsilon}_e \). This method of determining the strain at upper yield was used because of the difficulties inherent in the SHPB method for determining accurately the small strain associated with the elastic responses. Then

\[
\dot{\varepsilon}_e = \frac{\varepsilon_y}{\tau_y}
\]  

Values of \( \dot{\varepsilon}_e \) were averaged for each strain-rate group and are presented in Table 9.1 alongside the mean plastic strain-rates.

There are several important points to note from Table 9.1. There is a substantial difference between \( \dot{\varepsilon}_e \) and plastic strain-rate, \( \dot{\varepsilon}_p \) and the disparity increases at larger plastic strain-rates. Furthermore, the elastic strain-rate is only moderately sensitive to projectile impact velocity and between impact velocities of 31 and 37 ms\(^{-1}\) there is virtually no difference in \( \dot{\varepsilon}_e \) whereas the plastic strain-rate increases by 44\% over this range for the 10 x 5 mm batch. For this reason and because \( \dot{\varepsilon}_e \) depends on the upper yield stress which may be
distorted by Pochhammer-Chree oscillations, \( \dot{\varepsilon}_e \) can be considered a poor and unreliable measure of strain-rate. Thus in all further discussion the term 'strain-rate' may be assumed to be the mean plastic strain-rate as determined graphically (Figure 9.8) unless otherwise stated.

Table 9.1 displays the obvious advantage of using 8 x 4 mm specimens so that higher strain-rates and final strains can be achieved.

In Figures 9.3 to 9.7 it can be seen that each stress versus strain curve is labelled by an eight character code, for example in Figure 9.6 the codes for the two curves are 34SP513Z and 30SR423Y. The first two numbers in the code indicate the projectile impact velocity for the shot in ms\(^{-1}\). The third character is either 'S' or 'D' indicating whether a steel or duralumin projectile was used. In actual fact the steel projectile was used in all the SHPB compression tests while the duralumin projectile was only employed in the last few tensile tests.

The next letter in the code is either 'R' or 'P' denoting the type of specimen tested, either radial axis or parallel axis respectively. The three numbers which follow this represent the \((Z,R,\theta)\) coordinates of the original specimen location within the 2B1 billet according to the scheme depicted in Figure 9.2. The last character in the code is always a letter and was chosen to indicate the number of shots performed at a given set of conditions.

Thus returning to one of the codes quoted above, 30SR423Y indicates that this particular test was performed using a steel projectile whose impact velocity was 30 ms\(^{-1}\) (± 1 ms\(^{-1}\)) and the sample used was of radial axis type i.e. RA (4,2,3). Figures 9.3 to 9.7 show that stress versus strain results for the two different types of sample, radial axis and parallel axis, were in agreement with each other at a given
strain-rate. Hence the material properties appear to be isotropic and independent of sample orientation within the 2B1 billet. This was first concluded after the preliminary tests on a set of 10 x 5 mm and 8 x 4 mm samples (Dixon, 1988) and therefore in all the analyses on SHPB results the curves from 'radial-axis' and 'parallel-axis' have been grouped together according to a common strain-rate and not separately.

Figures 9.3 through to 9.7 also show that although Pochhammer-Chree oscillation is sometimes present the consistency of the curves for a given plastic strain-rate is good. The total variation in stress for a particular group of curves is usually less than 30 MPa which is approximately 5% of the maximum flow stress at room temperature. In each figure a mean curve has been fitted by judgement of eye after first shifting all the curves in each set over to the left so that their respective upper yield points all lay on the straight line of equation:

\[ \sigma = E \varepsilon \] (9.2)

where \( E \) is the Young's modulus of 224 steel (i.e. 200 GPa at room temperature).

The shifting process is not shown in Figures 9.3 to 9.7 but the resulting mean curves are. It was felt that this subjective method of determining mean curves was more accurate than any numerical averaging procedure which may have produced final stress versus strain curves distorted by Pochhammer-Chree oscillations present in the original records.

The mean curves drawn in Figures 9.3 to 9.7 are shown collectively in Figure 9.9 together with a mean quasistatic curve from the Instron tests. Figure 9.9 is the definitive set of compression results at room temperature.
FIG 9.9 Most recent set of true stress versus true strain results from 10 x 5 mm samples. Strain rates indicated (s⁻¹) 293K
9.2.2 Effects of Lubrication

In the preliminary trials, the first twenty one 10 x 5 mm specimens were not lubricated before being tested. The mean true stress versus true strain curves for this initial batch are shown in Figure 9.10 and were obtained in an identical manner as described for the mean curves in Figure 9.9. Comparing Figure 9.10 with Figure 9.9 one finds that the curves for the unlubricated specimens (Figure 9.10) are consistently above the corresponding curves at approximately the same strain-rates in Figure 9.9. The upper and lower yield points in Figure 9.9 are typically of the order of 80 MPa (≈ 13%) above their counterparts in Figure 9.10 while the flow stresses beyond yield are about 50 MPa higher (≈ 8%). These observations are consistent with the findings of Ellwood (1983) on 321 stainless steel specimens. He found that light silicone grease was the most effective lubricant at room temperature. Thus, in the experiments described here a light silicone spray was used to lubricate specimens in tests at room temperature and at +150°C (423K).

The highest test temperature, +300°C was above the upper limit of the specified working range of the silicone grease and so molybdenum disulphide ('Molyslip') was used instead.

The current investigation was the first to undertake SHPB tests at temperatures below 0°C in the Physics Department at Loughborough and consequently there was no experience to call upon regarding the most suitable lubricant to use at these temperatures. A study was made of the lubricating properties of graphite powder and PTFE spray at the lowest testing temperature -110°C (163K) in SHPB tests at a mean strain-rate of 1100 s⁻¹. The resulting true stress versus true strain curves are shown in Figure 9.11. Clearly the level of stress was lowest for samples lubricated with PTFE spray thus indicating that this material was a more efficient lubricant than graphite under
these conditions. Consequently PTFE spray was used in all subsequent tests at -110°C (163K) and -40°C (233K).

9.2.3 Effect of Specimen Size

As mentioned earlier, two specimen sizes were used during the course of experimentation presented here. These were cylindrical specimens of dimensions: 10 mm diameter x 5 mm length and 8 mm diameter x 4 mm length.

As part of the preliminary testing programme a batch of twenty lubricated 8 x 4 mm specimens were tested at room temperature on the SHPB and quasistatically. The resultant mean curves are presented in Figure 9.12. When curves in this graph are compared with corresponding curves at similar strain-rates in Figure 9.9 (determined for 10 x 5 mm specimens) they are found to agree well. This provided evidence that if 8 x 4 mm specimens were used in the Hopkinson bar apparatus instead of 10 x 5 mm specimens there should be no alteration in the resulting stress-strain curve. Since the use of 8 x 4 mm samples was advantageous in producing higher strain-rates and larger final strains (see Table 9.1) they were used wherever possible in SHPB tests. However, because they proved to be more difficult to manufacture in the Department of Physics workshop, 8 x 4 mm specimens were less readily available.

9.2.4 Comparison of SHPB Compression Results with SHPB Tensile Results at Room Temperature

Figures 9.13 to 9.15 show the SHPB tensile results which were obtained by the methods described in Chapter 7. Due to the particular dimensions of the tensile specimens, it was not possible to apply the coding system of Figure 9.2. Instead they were classified simply as radial (R) or parallel axis (P) and hence the shorter codes in Figures 9.13 to 9.15.
Obviously there is considerably more noise on these tensile stress versus strain curves than recorded in the compressive case for reasons explained in Chapter 7. The presence of this noise swamps out any upper and lower yield point features which should be present in the curves.

The computer plotted curves in Figures 9.13 to 9.15 were based on the original tensile specimen gauge length, $l_g = 8.3$ mm while the work of Chapter 7 showed that $l_g = 6.8$ mm was a more appropriate value for these tensile tests. Thus, it has been necessary to correct the true strain axes in each graph to make amends for this discrepancy.

Mean compression true stress-strain curves for corresponding strain-rates from Figure 9.9 have been superimposed upon the graphs of Figures 9.13 to 9.15. Generally the agreement between the compression and tension curves is reasonably good, especially so in Figure 9.14. The agreement is not so good in Figure 9.15, for the highest mean tensile strain-rate of 1200 s$^{-1}$. However, the noise level in this case is so dominant over the general shape of the stress-strain curves, it is difficult to draw any sensible conclusions.

From Figure 9.13 and Figure 9.14 it can be seen that although there is a significant amount of inaccuracy in the SHPB tensile results and the yield points are not well defined, the level of flow stress recorded is consistent with that of compression curves at similar strain-rates. This is as should be expected since there is no reason why, for this particular steel, the mechanical properties in tension should be different from those in compression at a given strain-rate.
FIG 9.16: Instron results at 150°C (423 K) and \( \dot{\varepsilon} = 1 \times 10^{-3} \, \text{s}^{-1} \) using 10 x 5 mm samples
9.3 INSTRON AND ESH RESULTS (4 x 10^{-4} s^{-1} to 4 s^{-1})

In Figure 3.8 a set of three true stress-strain curves from ESH compression tests at room temperature was presented. They showed the degree of consistency possible with this type of test. The Instron tests proved to be equally consistent as illustrated in Figure 9.16 which shows three results obtained directly from three Instron tests at 150°C (423K).

The typical variation in flow stress seen in the Instron results for a particular temperature and strain-rate was ±10 MPa. This equals the variation observed in the ESH results. On the other hand, in the dynamic SHPB tests the scatter in the results was considerably larger at ±25 MPa due to Pochhammer-Chree oscillations and the fact that mean strain-rates for the tests could not be repeated exactly.

Both 8 x 4 mm and 10 x 5 mm samples were used with the Instron machine and were found to give the same results at a given temperature and strain-rate. Since the maximum load capacity of the Instron machine was only 5000 kg, the 8 x 4 mm samples were used whenever possible to achieve a larger final strain.

Strain-rate in all these quasistatic tests on both the ESH and Instron machines was calculated by dividing the final specimen strain (having subtracted machine compliance) by the total test time. In a similar manner to the SHPB results the ESH and Instron results have been grouped together graphically according to strain-rate and temperature and hence a mean curve deduced. For every condition the mean curve is the average of at least three experimental stress-strain curves.
FIG 9.17 Room Temperature Stress versus Strain curves from E.S.H. tests. 10x5mm

Key to Strain-Rates

- 20 : \( \dot{\varepsilon} = 3.4 \text{s}^{-1} \)
- 2 : \( \dot{\varepsilon} = 0.4 \text{s}^{-1} \)
- 0.2 : \( \dot{\varepsilon} = 3.8 \times 10^{-2} \text{s}^{-1} \)
- 0.02 : \( \dot{\varepsilon} = 3.8 \times 10^{-3} \text{s}^{-1} \)
- 0.002 : \( \dot{\varepsilon} = 3.8 \times 10^{-4} \text{s}^{-1} \)
- 0 : \( \dot{\varepsilon} = 1.67 \times 10^{-3} \text{s}^{-1} \)
9.3.1 Results at Room Temperature (20°C, 293K)

Figure 9.17 shows the full set of mean ESH curves obtained at room temperature at every decade of strain-rate between $3.8 \times 10^{-4}$ s$^{-1}$ and 3.4 s$^{-1}$. The upper and lower yield stresses and the flow stress all increase with increasing strain-rate but the difference between the curves at strain-rates of $3.8 \times 10^{-3}$ s$^{-1}$, $3.8 \times 10^{-2}$ s$^{-1}$ and 0.4 s$^{-1}$ is small. In fact the curve at $3.8 \times 10^{-2}$ s$^{-1}$ overlaps the curve at 0.4 s$^{-1}$ above 14% true strain. One possible explanation for this is that frictional effects are more prominent in the lower rate tests at the higher strains since there is more time available for the expulsion of lubricant from between the sample and the platens.

The closeness of the curves at 0.4 s$^{-1}$, $3.8 \times 10^{-2}$ s$^{-1}$ and $3.8 \times 10^{-3}$ s$^{-1}$ shows that there is little strain-rate sensitivity of flow stress in this region. The yield stress, however, does increase with increasing strain-rate. It was decided at other temperatures to only conduct tests at 4 s$^{-1}$ and $3.8 \times 10^{-3}$ s$^{-1}$ and to only carry out additional tests if there proved to be a significant difference in the stress versus strain curves at these two rates.

The mean curve from several Instron tests at a rate of $10^{-3}$ s$^{-1}$ has been superimposed onto the graph of Figure 9.17 and is labelled 'Q'. It agrees within experimental precision with the nearest equivalent mean ESH curve ($\dot{e} = 3.8 \times 10^{-3}$ s$^{-1}$) which implies that the mechanical properties observed here are independent of the type of testing machine used.

9.3.2 Results at -40°C (233K)

Figure 9.18 shows mean true stress versus strain curves for the low strain rate tests performed at -40°C on the ESH machine. Instron tests were carried out first and it was found that as soon as each test began the temperature rose sharply by approximately 10°C in 30s
FIG 9.18: Mean stress versus strain curves for low strain rate tests at 
-40°C (233 K) using 10 x 5 mm samples
and thereafter more steadily so that by the end of the test the sample temperature could be as high as -20°C. This upward drift in temperature during the Instron tests meant that the results from them could not be relied upon and so they have been discarded from further analysis while the ESH have not, as explained below.

The reason for the sharp temperature rise was due to enhanced conduction of heat from the surroundings to the specimen through the crosshead and platen assembly as soon as a load was applied.

With this experience from the Instron tests a small preload was applied before the start of the ESH tests during the cooling down process. This small preload established good thermal contact between the platens and the ESH rams so that thermal equilibrium was attained before the start of each test and, more importantly, the temperature did not rise above -40°C during the test. It was the fact that thermal conduction through the rams and crosshead assemblies was so efficient in both machines which prevented the possibility of achieving temperatures lower than -40°C using the simple cooling system described in Section 3.10. In contrast, the small amount of heat conduction through the 1/2" diameter bars in the SHPB system allowed sample temperatures of -110°C and below to be attained with relative ease.

It is worth noting here that there is a marked convergence of the ESH curve at 4 s⁻¹ and that at 4 x 10⁻³ s⁻¹ in Figure 9.18 and there is the possibility that they might overlap at high strains. This may be attributable to frictional effects or 'adiabatic softening' to be explained in depth in Chapter 10.
FIG 9.18: Initial stress versus strain curves from ESH tests at 150°C (423 K) using 10 x 5 mm samples.

Key to Strain-Rates
- 0.002: \( \dot{\varepsilon} = 4 \times 10^{-2} \text{s}^{-1} \)
- 0.02: \( \dot{\varepsilon} = 3.8 \times 10^{-3} \text{s}^{-1} \)
- 0.2: \( \dot{\varepsilon} = 3.8 \times 10^{-4} \text{s}^{-1} \)
- 0.002: \( \dot{\varepsilon} = 3.8 \times 10^{-5} \text{s}^{-1} \)
9.3.3 Results at 150°C (423K)

Figure 9.19 shows the mean true stress versus strain curves for the initial ESH tests carried out at 150°C. Although the yield stresses for these curves increase with increasing strain-rate, above a strain of 3% there is a tendency for the flow stress to be larger the lower the strain-rate for the three strain-rates less than 4 s⁻¹. At higher strains beyond 24% the lower strain-rate curves (i.e. 3.8 x 10⁻² s⁻¹, 3.8 x 10⁻³ s⁻¹, 3.8 x 10⁻⁴ s⁻¹) overlap with the 4 s⁻¹ curve.

Previous workers (for example Manjoine, 1944 or Keh et al, 1968) have reported negative strain-rate sensitivities in iron and low carbon steel similar in nature to this. However for the initial results presented in Figure 9.19 it was not certain whether the trends were due to genuine material behaviour or due to errors in the testing method.

The furnace used for these initial ESH tests was a large ceramic cylinder which completely housed the specimen, platens and ram. The furnace had a large thermal inertia and so took a considerable time to reach 150°C. Unfortunately it was designed to operate automatically at temperatures above 250°C and so for use at 150°C it had to be controlled manually. For this reason the time taken to bring the specimen to the correct temperature before the test began varied between 15 and 30 minutes in which time the silicone grease lubricant may have dried up and become less effective. The peculiar order of the curves in Figure 9.19 could merely be a reflection of the variation in friction between specimen and platen faces due to variations in the heating up times before the start of the tests. Furthermore, the heating up period (15 to 30 minutes) is long enough to introduce ageing effects.
FIG 9.20: ESH results at +150°C (425 K) and $\dot{\varepsilon} = 4$ s$^{-1}$ using 10 x 5 mm samples
FIG 9.21: Comparison of results from initial ESH tests (solid curves from FIG 9.19) with repeat tests (dashed curves) at +150°C (423 K) using small furnace.
In view of these uncertainties tests were carried out at $10^{-3} \text{ s}^{-1}$ using the Instron machine (results shown in Figure 9.16) and repeat tests at $4 \text{ s}^{-1}$ using the ESH machine. In both machines the small hand-made furnace described in Section 3.11 was incorporated and this was able to attain a steady $150^\circ\text{C}$ sample temperature within two minutes thereby avoiding the problems described in the paragraph above.

The results for the repeat ESH tests at $4 \text{ s}^{-1}$ are displayed in Figure 9.20 which includes the mean curve (dashed line) from the Instron results of Figure 9.16. Evidently the results for the three ESH tests at $4 \text{ s}^{-1}$, $150^\circ\text{C}$ show a high degree of consistency. The mean Instron curve lies underneath them but its closeness indicates that the strain-rate sensitivity between $10^{-3}\text{s}$ and $4 \text{ s}^{-1}$ is low.

The mean true stress-true strain curves of Figures 9.19 and 9.20 are compared together in Figure 9.21 where it is found that the curves for the Instron and the repeat ESH tests (dashed curves) lie above the corresponding curves from the initial ESH tests (solid curves). It is believed that due to the improved heating arrangement used, the dashed curves represent a more realistic description of the 224 behaviour at $150^\circ\text{C}$. The small difference between them does suggest, however, that at strain-rates between $10^{-3} \text{ s}^{-1}$ and $4 \text{ s}^{-1}$ overlapping of flow stress curves or negative strain-rate sensitivity may be possible. Such real effects in iron and steel have been described by dynamic strain ageing (Keh et al, 1968) which is discussed further in Chapter 10.

Adiabatic softening may also be responsible in part for a reduction of the flow stress at the strain rate of $4 \text{ s}^{-1}$. 

9.3.4 Results at 300°C (573K)

Figure 9.22 shows the mean true stress versus strain curves from the Instron and ESH tests at 300°C. The three Instron curves from which the mean curve in Figure 9.22 was determined were the only ones in the whole testing series (Figures 9.16 to 9.22) to show a serrated yielding effect in the early stages. This phenomenon is known as the Portevin-Le Chatelier effect and is often observed in low carbon steels (see Section 2.5.4). In some of the ESH tests at elevated temperatures sudden drops in load (and therefore stress) were observed but it was not possible to tell whether these were genuine material characteristics or whether they merely emanated from background electronic noise from nearby machinery. The size (10 to 15 MPa) and low frequency nature of the serrations seen in the Instron tests meant that they could be unequivocally ascribed to actual material behaviour.

The fact that no such obvious serrations appeared in the ESH tests at similar strain-rates, together with the fact that the curves for the ESH tests in Figure 9.22 are between 35 and 60 MPa below the mean Instron curve, suggests that conditions in the Instron tests may have been different to those in the ESH tests. In both cases, the small hand-built furnace was used to bring the specimen up to 300°C in less than 5 minutes before the start of the test so that problems due to drying up of the silicone lubricant before the test were minimised.

It is possible that a poor location of the monitoring thermocouple in the Instron tests, for example in contact with the walls of the furnace, may have resulted in a higher temperature being recorded than that of the specimen itself. If such were the case then a misleadingly large flow stress would result. It is important to note, however, that the work of Keh et al (1968) on dynamic strain ageing showed that the presence of serrations in a stress strain record was always accompanied by a dramatically increased rate of work hardening.
The ESH curves of Figure 9.22 show a similar pattern of behaviour to that of Figure 9.19 at 150°C. Below 4% strain the flow stress is larger for the higher strain rate but above about 15% strain in Figure 9.22 the flow stress is larger the lower the strain-rate. Since the improved heating arrangement was used for these tests the explanation given in Section 9.3.3 for the overlapping of stress v strain curves or negative strain-rate sensitivity is not a valid argument here.

An alternative explanation comes from a consideration of the duration of the tests up to 26% strain which on average was 0.07s, 60s and 540s for $\dot{\varepsilon} = 4 \text{ s}^{-1}$, $4.3 \times 10^{-3} \text{ s}^{-1}$ and $4.8 \times 10^{-4} \text{ s}^{-1}$ respectively. Hence the lower the strain-rate for the test the more time there is available for the expulsion and drying up of lubricant between the specimen and platen faces. This would cause an increase in frictional forces and thus a higher stress would be required to continue the deformation. This view was supported by visual inspections of the samples after testing.

The above is not the only explanation. At elevated temperatures of 150°C and 300°C we are dealing with a thermal region in which the rate of diffusion of interstitial atoms (C and N) is comparable with the mobility of dislocations at strain-rates between $10^{-4}$ and 1 s$^{-1}$. Many authors (e.g. Manjoine, 1944) have shown that such conditions are able to produce negative strain-rate sensitivity trends similar to those depicted in Figures 9.16 and 9.22. It is important therefore not to dismiss the curves of Figures 9.16 and 9.22 as not being representations of the mechanical behaviour of 224 steel.

9.3.5 Comparison of Quasistatic Tension and Compression Results

Figure 9.23 shows the mean tensile curves for room temperature tests at strain rates of $10^{-3} \text{ s}^{-1}$ and 0.03 s$^{-1}$. For the purposes of comparison the nearest equivalent compression curves for rates of 0.038 s$^{-1}$ and $3.8 \times 10^{-3} \text{ s}^{-1}$ are also presented. Clearly the
FIG 9.23: Comparison of tensile (T) and compression (C) results (10 x 5 mm samples) at quasistatic strain rates and room temperature. Curves drawn are MEAN curves (experimental scatter = ±12 MPa).
agreement between the compression and tensile results is excellent below a strain of 10%. Up to 12% strain the compression and tensile curves agree within the experimental error for both.

Beyond 12% strain the compression curves diverge above their tensile counterparts. Among the possible reasons for this could be (i) the presence of friction in compression testing (there is none for tension testing), which tends to increase the level of stress required to continue the deformation at high strains; (ii) the location of the samples in the original billet. The tensile pieces were extracted from segment 9, a mean distance of 10 cm from most of the room temperature compression samples.

At 26% strain the difference between the stress in the compression curve for \( \dot{\varepsilon} = 0.038 \) s\(^{-1} \) and that in the tensile curve for \( \dot{\varepsilon} = 0.03 \) s\(^{-1} \) is 20 MPa while the stress difference between the compression and tensile curve at \( 3.8 \times 10^{-3} \) s\(^{-1} \) and \( 10^{-3} \) s\(^{-1} \) respectively is 35 MPa. This could be a further indication that frictional effects might be enhanced at the lower strain-rates in which there is more time for the expulsion of lubricant at the specimen/platen interface.

9.3.6 Barrelling
Up to 25% true strain in all compression tests 'barrelling' was negligible. Beyond this level of strain the onset of barrelling could be observed indicating the increasing importance of frictional effects. The author attempted to quantify the degree of barrelling in samples compressed beyond 25% true strain via the introduction of a 'barrelling quotient', \( B \), defined by

\[
B = \frac{d - \frac{1}{2}(d_u + d_l)}{d} = 1 - \frac{d_u + d_l}{2d}
\]  

\text{(9.3)}
FIG 9.24 Diagram of barrelling in a compressive sample
where d is the maximum diameter measured across the specimen at its widest point and \( d_u \) and \( d_l \) are the diameters at the upper and lower end faces of the specimen respectively (see Figure 9.24).

B was measured for a selection of specimens compressed in the ESH machine to a final true strain of \((40 \pm 3)\%\). Representative specimens were measured for every single test temperature and strain-rate. It was found that the value of B varied from 2% to 4% but there was no overall trend for B to increase at higher test temperatures or lower strain-rates.

Most Instron tests were terminated below true strains of 30% so it was the ESH tests which were most affected by barrelling. In examining the compression results, Figures 9.16 to 9.23, it should be remembered that the shape of the true stress-strain curves may be influenced by friction effects above 12% true strain.
CHAPTER 10
ANALYSIS OF RESULTS

10.1 FUNDAMENTAL ANALYSIS

10.1.1 Compilation of Results
In Figure 9.9 the full set of compression results at room temperature (293K) were presented. In a similar fashion, Figures 10.1, 10.2, 10.3 and 10.4 show sets of mean true stress versus strain curves for compression tests at -110°C, -40°C, 150°C and 300°C respectively. As in Figure 9.9 the curves have been shifted to the left where necessary so that the initial elastic region up to the upper yield point is a straight line (equation 9.2) whose slope is the Young's modulus, E, for the 224 steel at each given temperature. Table 10.1 shows that there was only a small variation in E throughout the temperature range considered here.

<table>
<thead>
<tr>
<th>Temperature/K</th>
<th>Young's Modulus/GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>163</td>
<td>209</td>
</tr>
<tr>
<td>233</td>
<td>204</td>
</tr>
<tr>
<td>293</td>
<td>200</td>
</tr>
<tr>
<td>423</td>
<td>191</td>
</tr>
<tr>
<td>573</td>
<td>180</td>
</tr>
</tbody>
</table>

TABLE 10.1: VALUES OF YOUNG'S MODULUS, E, FOR 224 STEEL AT VARIOUS TEMPERATURES. (Data supplied by Sheffield Forgemasters Ltd)

Figure 9.9 and Figures 10.1 to 10.4 form the basis of the analysis presented in the following sections. It is worth noting that the mean variation in flow stress about these mean curves was ±15 MPa for the
FIG 10.1: Mean SHPB curves at -110°C (163 K) 8 x 4 mm samples.
FIG 10.2: Set of mean stress versus strain curves at -40°C (233 K)
FIG 10.5A: Plots of true stress versus linear \( \dot{e} \) at various strain levels for the experimental data recorded at \(-110^\circ\)C (163 K) using 8 x 4 mm samples.
SHPB tests ($\dot{\varepsilon} > 80$ s$^{-1}$) and ±6 MPa for the Instron and ESH tests ($\dot{\varepsilon} \leq 4$ s$^{-1}$).

10.1.2 Stress Versus Strain-Rate

A useful manner in which to present the results from mechanical tests over a variety of strain rates is to plot stress at constant strain versus linear strain-rate. This has been done in Figure 10.5 for the SHPB results at $-110^\circ$C. The points plotted on this graph have been taken directly from the original computer plots of stress versus strain, not just from the mean curves, and hence the experimental scatter is clearly illustrated. The lines drawn on the graph have been fitted by eye.

When the range of strain-rates covered is large, stress at constant strain can be plotted against the logarithm of strain-rate. Figures 10.5, 10.6, 10.7, 10.8 and 10.9 show both graphs of stress versus linear $\dot{\varepsilon}$ (A) and log $\dot{\varepsilon}$ (B) from the SHPB results except where indicated. The log $\dot{\varepsilon}$ graphs have incorporated mean values of stress at given $\dot{\varepsilon}$ from the linear $\dot{\varepsilon}$ plots. Unfortunately at $-110^\circ$C there were no quasistatic results as explained in Section 3.10.

The curves delineated in these figures are most useful since they allow one to produce graphs of stress versus strain at any given strain-rate within the range presented. This is done most simply by drawing a vertical line at a chosen $\dot{\varepsilon}$ and recording the points at which the constant strain curves intersect this line. (Typically lower yield stress, $\sigma_{LYP}$ occurs at 0.5% strain while upper yield stress, $\sigma_{UYP}$ will occur at $(\sigma_{UYP}/E) \times 100$% strain where E is the Young's modulus at the given temperature (see Table 10.1).
FIG 10.6A: Plot of true stress versus linear $\dot{\varepsilon}$ at various strain levels for the experimental data recorded at $-40^\circ$C (233 K) using 8 x 4 mm samples.
Fig 10.6B: Plot of true stress versus log ε using data presented in Fig 10.6A.
FIG 10.7A Plot of true stress versus linear strain-rate for experimental data recorded at room temperature. All points below a strain-rate of 3000 s\(^{-1}\) were derived from 10 x 5 mm samples, all those above 3000 s\(^{-1}\) are from 8 x 4 mm samples.
FIG 10.7B Plot of true stress versus log strain-rate for data plotted in FIG 10.7A.
FIG 10.8A: Plot of true stress versus linear $\dot{\varepsilon}$ at various strain levels for the experimental data recorded at +150°C (423 K) using 8 x 4 mm samples.
FIG 10.8B: Plot of true stress versus log \( \varepsilon \) using data presented in FIG 10.8A. +150°C (423 K)
FIG 10.9A: Plot of true stress versus linear ε at various strain levels for the experimental data recorded at +300°C (573 K) using 8 × 4 mm samples.
FIG 10.9B: Plot of true stress versus log ε using data presented in FIG 10.9A. +300°C (573 K)
<table>
<thead>
<tr>
<th>Temperature</th>
<th>Strain-Rate ((s^{-1}))</th>
<th>UYP</th>
<th>LYP</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>25%</th>
<th>Mean over all strains</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110°C (163K)</td>
<td>2000</td>
<td>150</td>
<td>73</td>
<td>183</td>
<td>141</td>
<td></td>
<td>137</td>
<td></td>
</tr>
<tr>
<td>-40°C (233K)</td>
<td>4x10^{-3}</td>
<td>24</td>
<td>23</td>
<td>12</td>
<td>12</td>
<td>8</td>
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<td>185</td>
<td>185</td>
<td>205</td>
<td>191</td>
<td></td>
</tr>
<tr>
<td>20°C (293K)</td>
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<td>9</td>
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<tr>
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</tr>
<tr>
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<td>120</td>
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<td>0</td>
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<td>71</td>
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</tr>
<tr>
<td>150°C (423K)</td>
<td>4x10^{-3}</td>
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<td>8</td>
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<td>80</td>
<td>18</td>
<td>43</td>
<td>66</td>
<td></td>
</tr>
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<td>300°C (573K)</td>
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<td>-2</td>
<td>-3</td>
<td>-3</td>
<td>-8</td>
<td>-3</td>
</tr>
<tr>
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<td>7</td>
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<td>1</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>2000</td>
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<td>95</td>
<td>21</td>
<td>16</td>
<td>13</td>
<td>10</td>
<td>42</td>
</tr>
</tbody>
</table>

**TABLE 10.2 VALUES OF STRAIN-RATE SENSITIVITY, \(\lambda_e (\text{MPa/log } (s^{-1}))\) AT A VARIETY OF TEMPERATURES AND STRAIN-RATES**

**Note:** All figures in the main body of the table are correct to ±5 MPa unless otherwise stated.

LYP = lower yield point (0.5 to 1% strain)

UYP = upper yield point (0.1 to 0.4% strain)
At a given strain level, \( \varepsilon \), the strain-rate sensitivity, \( \lambda_\varepsilon \), may be defined by:

\[
\lambda_\varepsilon = \left. \frac{\partial \sigma}{\partial \log \varepsilon} \right|_{\varepsilon, T}
\]

(10.1)

This quantity serves as a useful indication as to what strain rates are important in the material mechanical behaviour. Table 10.2 shows a series of values of \( \lambda_\varepsilon \) estimated from the gradients of the curves in Figures 10.5 to 10.9.

Several important features emerge from Table 10.2. Generally \( \lambda_\varepsilon \) increases with increasing \( \dot{\varepsilon} \) especially at SHPB rates. Excluding the UYP and LYP data which may be sensitive to a number of factors then, there is little change in \( \lambda_\varepsilon \). This is obvious from the shapes of the \( \sigma \) versus \( \log \dot{\varepsilon} \) curves in Figures 10.6 to 10.9. \( \lambda_\varepsilon \) decreases as the test temperature increases. In fact at 300°C (573K) and \( \dot{\varepsilon} = 4.3 \times 10^{-3} \) s\(^{-1}\) small negative values of \( \lambda_\varepsilon \) have been recorded.

The figures in Table 10.2 also show that the upper and lower yield points are more strain-rate sensitive than the other strain levels.

The trends of Table 10.2 and Figures 10.5 to 10.9 are similar to those discovered by Campbell and Ferguson (1970) for mild steel (see Figure 2.13) and indicate that at the dynamic strain-rates above 4 s\(^{-1}\) the deformation of 224 steel is controlled by thermally activated mechanisms. This idea is developed in detail in Section 10.2.

10.1.3 Stress Versus Temperature

Zener and Hollomon (1944) were the first to demonstrate and quantify the reciprocal effects of temperature and strain-rate on the stress-strain curves for iron and low carbon steels. Generally, the stress at a given strain will increase with increasing strain-rate but will
FIG 10.10  Stress at various strain levels versus temperature at a mean strain-rate of $10^{-3} \text{s}^{-1}$.
FIG 10.10 Stress at various strain levels versus temperature at a mean strain-rate of $10^{-3}$ s$^{-1}$.
FIG 10.11 Stress at various strain levels versus temperature at a mean strain-rate of 4 s⁻¹
FIG 10.12 Stress at various strain levels versus temperature at a mean strain-rate of 2000 s$^{-1}$.
decrease as the temperature is increased. This trend is outlined by Figures 10.10, 10.11 and 10.12 which show the stress at various constant strains versus temperature at strain-rates of 10^{-3} \text{ s}^{-1}, 4 \text{ s}^{-1} and 2000 \text{ s}^{-1} respectively.

In a similar way to strain-rate sensitivity, thermal sensitivity, \( \beta_\varepsilon \) for a given strain and strain-rate, can be defined as

\[
\beta_\varepsilon = \frac{\partial \sigma}{\partial T} \bigg|_{\varepsilon, \dot{\varepsilon}}
\]  

(10.2)

where \( \sigma \) is the stress at constant strain, \( \varepsilon \) and \( T \) is the temperature. From Figures 10.10 to 10.12 a set of values of \( \beta_\varepsilon \) have been calculated and are presented in Table 10.3. The values of \( \beta_\varepsilon \) specified for a temperature of 150°C (423K) were calculated from the tangent of the slopes at 150°C. In many cases the tangent is constant over the range 293K to 573K.

In the SHPB results at strain-rates of 1000 s^{-1} (not shown) and 2000 s^{-1} it was possible to determine \( \beta_\varepsilon \) at -40°C (233K) since data were obtained from tests at -110°C (163K). Values of \( \beta_\varepsilon \) at -40°C were about five times the mean value at 150°C. This increased thermal sensitivity occurring in the low temperature results is apparent in Figure 10.12 which shows stress versus temperature curves at a strain-rate of 2000 s^{-1}. All the curves for different strain levels exhibit the same behaviour in which there is a dramatic change in slope at (or close to) room temperature. At 10^{-3} \text{ s}^{-1} and 4 \text{ s}^{-1} no experimental data were obtainable below -40°C and the curves in Figures 10.10 and 10.11 show a more or less constant slope (\( \beta_\varepsilon \)) throughout their temperature range.
<table>
<thead>
<tr>
<th>Strain-Rate (s⁻¹)</th>
<th>Temperature</th>
<th>Strain Level</th>
<th>UYP</th>
<th>LYP</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
<th>20%</th>
<th>25%</th>
<th>Mean over all strains</th>
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<td>10⁻³</td>
<td>150°C</td>
<td></td>
<td>-0.18</td>
<td>-0.18</td>
<td>-0.20</td>
<td>-0.30</td>
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<td>-0.36</td>
<td>-0.38</td>
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</tr>
<tr>
<td></td>
<td>150°C</td>
<td></td>
<td>-0.47</td>
<td>-0.47</td>
<td>-0.40</td>
<td>-0.48</td>
<td>-0.56</td>
<td>-0.56</td>
<td>-0.58</td>
<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>150°C</td>
<td></td>
<td>-0.60</td>
<td>-0.51</td>
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<td></td>
<td></td>
<td></td>
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<td>-0.50</td>
</tr>
<tr>
<td></td>
<td>-40°C</td>
<td></td>
<td>-2.76</td>
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<td></td>
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<td>-2.67</td>
</tr>
<tr>
<td></td>
<td>150°C</td>
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<td>-0.65</td>
<td>-0.44</td>
<td>-0.35</td>
<td>-0.44</td>
<td>-0.44</td>
<td></td>
<td></td>
<td>-0.46</td>
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<td></td>
<td>150°C</td>
<td></td>
<td>-0.77</td>
<td>-0.75</td>
<td>-0.57</td>
<td>-0.65</td>
<td>-0.67</td>
<td>-0.95</td>
<td>-0.94</td>
<td>-0.76</td>
</tr>
</tbody>
</table>

**TABLE 10.3: VALUES OF THERMAL SENSITIVITY, $\beta_e$ (MPa/K) AT A VARIETY OF TEMPERATURES AND STRAIN-RATES**

Values of $\beta_e$ specified at a temperature of 150°C represent the mean value of $\beta_e$ over the range 20°C to 300°C, while those specified at a temperature of -40°C represent the mean from -110°C to 20°C.
FIG 10.13  Upper yield stress versus temperature
FIG 10.14  Lower yield stress versus temperature

- $\circ = 2000 \text{s}^{-1}$
- $\times = 1000 \text{s}^{-1}$
- $\square = 4 \text{s}^{-1}$
- $\bullet = 4 \times 10^{-3} \text{s}^{-1}$

Mean of 3000 & 1000 s$^{-1}$
The curves in Figures 10.10 to 10.12 are nearly all approximately parallel and there seems to be only a small variation in $\beta_c$ at 150°C over all strains and strain-rates. The range of mean values of $\beta_c$ at 150°C for the various strain-rates considered is $-0.28$ to $-0.76$ MPa K$^{-1}$, the overall mean being $-0.50$ MPa K$^{-1}$. This value compares well with the $-0.633$ MPa K$^{-1}$ quoted by Culver (1973) for mild steel and which was used in equation 8.10 for the estimation of the critical strain-rate for adiabacity.

Figures 10.13 and 10.14 show the upper and lower yield stresses versus temperature for four different strain rates. Both figures show a common trend of behaviour. In both cases the curves for strain-rates of $4 \times 10^{-3}$ s$^{-1}$ and 4 s$^{-1}$ are more or less linear within the total test range; 233K to 573K. The higher strain-rates curves ($\dot{\varepsilon} = 1000$ s$^{-1}$ and 2000 s$^{-1}$) show a very distinct change in thermal sensitivity at room temperature as was seen in Figure 10.12 for curves of different strain levels.

10.1.4 Adiabatic Thermal Softening

The subject of Chapter 8 was the measurement of the adiabatic temperature in compression specimens tested at room temperature. There it was shown that the theoretical temperature rise as predicted by equation 8.11 agreed with the experimental measurements and could be as much as 60 to 70°C in the dynamic tests. It was also shown both empirically and theoretically (equation 8.10) that at a critical strain-rate, roughly 3 or 4 s$^{-1}$, the test could be considered adiabatic. The question which arises naturally from all this is, what effect does the rise in temperature in a sample have on the shape of the resulting true stress versus true strain curve?

Generally for most materials, and indeed all steels, the rise in temperature during deformation produces a softening of the specimen so
that the level of flow stress recorded is less than if the test had been entirely isothermal. The subject of adiabatic plastic deformation has been studied for over forty years and is well reviewed by Rogers (1979) and Stelly and Dormeval (1986). Holzer and Wright (1981) have pointed out that there are two types of adiabatic softening:

a) A general softening which occurs throughout the whole of the deforming region of the specimen which has a substantial effect on the flow stress. Loosely, this may be termed "bulk softening".

b) A more dramatic softening on a localised scale, described as "catastrophic softening", which may occur simultaneously with "bulk softening" and has an even more pronounced effect on the flow stress.

"Catastrophic softening" is usually present in tensile testing especially in the necked region of the sample just before fracture. In such tests on steels large temperature rises can occur even at low strain-rates as evidenced by the infrared thermometer pictures of Sachdev and Hunter (1982).

For the compression results on 224 steel presented in this Chapter, it is type (a) "bulk softening" which influences the shape of the resulting stress-strain curves.

In spite of the obvious importance of adiabatic softening surprisingly few workers have calculated the magnitude of the effect on actual stress versus strain results. Exceptions include Follansbee (1986) who calculated (using a form of equation 8.11) that the temperature rise in a specimen of Nitronic 40 stainless steel compressed by 20% in a SHPB test at 5000 s⁻¹ should be 62°C. He then stated that such a temperature rise could account for a reduction in flow stress of 168
MPa i.e. 14% of the observed flow stress at 20% true strain. Follansbee then suggested that this degree of thermal softening could explain an overlap of quasistatic stress-strain curves with dynamic curves at true strains between 20 and 40%.

Holzer and Wright (1981) have applied thermal softening corrections to experimental results from compression tests on AISI 1025 steel and have shown that the effect can account for negative values of strain-rate sensitivity, \( \lambda_e \) at strain-rates between 0.1 and 20 s\(^{-1}\). Muller (1972) corrected his dynamic test data from iron and nickel specimens and hence presented effective 'isothermal curves' and Oyane et al (1967) have done the same for S35C carbon steel (0.33% C, 0.75% Mn) at elevated temperatures between 650 and 850\(^{\circ}\)C.

The reason why adiabatic effects are often omitted from analyses of dynamic stress versus strain data is that to make an accurate assessment of the size of such effects requires a detailed knowledge of the material's mechanical properties over a wide temperature range. Of particular importance is the thermal sensitivity \( \partial \sigma / \partial T \) which must be known if adiabatic corrections are to be made. Some values of \( \partial \sigma / \partial T \) for metals can be found in the literature (e.g. Culver, 1973). However, for an accurate determination of the reduction in stress, \( \Delta \sigma \), due to thermal softening the thermal sensitivity must be established by actually carrying out mechanical tests at various temperatures.

The information presented in the last section, especially Figures 10.10 to 10.14 and Table 10.3, is sufficient for a calculation of the thermal softening effects in 224 steel. The first step is to work out the dependence of bulk temperature rise in the sample on the compressive strain for each test condition. Figure 8.2 showed the variation of \( T \) with true strain for SHPB tests at room temperature (293K) and a strain-rate of 2088 s\(^{-1}\). This relationship was
FIG 10.15 Variation of specific heat capacity, $C_p$, with temperature for EN 14 steel
determined from equation 8.11 in which the integral (energy per unit volume) was evaluated by measuring the area under the true stress versus strain curve (Figure 9.9) at various strain levels. It was necessary to extrapolate this curve parallel to the quasistatic curve in order to deduce likely temperature rises at strains beyond 20%.

In order to predict adiabatic temperature rises at temperatures other than room temperature the variation of specific heat capacity, $C_p$, with temperature was taken into account by use of the straight line relationship plotted in Figure 10.15. This graph actually shows the variation in specific heat with temperature for the carbon-manganese steel En 14 (Smithells, 1976). As the composition of En 14 (0.23% C, 1.51 Mn) is close to that of 224 steel the specific heats for the two should be virtually the same.

Figure 10.16 shows the relationship between bulk temperature rise in the specimen versus true strain for a variety of test temperatures and strain-rates. The family of curves shown have been calculated from the areas under the appropriate curves of stress versus strain presented in Chapter 9, for a mixture of 10 x 5 mm and 8 x 4 mm sample data labelled A and B respectively. The data for 10 x 5 mm samples SHPB tested at room temperature produced maximum true strains of the order of 20% thus the curve for these conditions in Figure 10.16 has been extrapolated beyond 20% strain by assuming that the dynamic stress-strain curve continues beyond 20% parallel to the quasistatic curve at room temperature.

Clearly the slope of the curves ($^\circ C$ temperature rise per percentage strain) is larger for the higher strain rates and lower temperatures. The reason for this is that at lower temperatures and higher strain-rates larger flow stresses are recorded so that the area under the stress-strain curves are generally greater implying that more energy is absorbed during the test therefore producing larger temperature rises. The fact that specific heat $C_p$ becomes smaller with decreasing
FIG 10.16 Adiabatic temperature rise, ΔT versus compressive strain

A = 10x5mm
B = 8x4mm
temperature also means that the temperature rise, $\Delta T$, during a test should be larger at lower temperatures according to equation 8.11. Note that the curves in Figure 10.16 are independent of sample size, whether 10 x 5 mm or 8 x 4 mm as these dimensions are implicit within the integral of equation 8.11.

The reduction in stress, $\Delta \sigma$ due to thermal softening is related to the temperature rise, $\Delta T$ during a test by the thermal sensitivity, $\beta_\varepsilon$ of the 224 steel at a given temperature and strain-rate. To deduce figures for $\Delta \sigma$ from the curves of $\Delta T$ versus $\varepsilon$ in Figure 10.16 appropriate values of $\beta_\varepsilon$ at 5% strain (i.e. $\beta_{5\%}$) were chosen from Table 10.3 which were then used as multiplying factors.

<table>
<thead>
<tr>
<th>Test Temperature $T$</th>
<th>Test Strain Rate, $\dot{\varepsilon}$ (s$^{-1}$)</th>
<th>Final Strain Reached $\varepsilon_F$ (%)</th>
<th>Calculated Temperature Rise, $\Delta T$ at $\varepsilon_F$ (°C)</th>
<th>Total Reduction of flow stress $\Delta \sigma$ (MPa)</th>
<th>$\Delta \sigma$ as a % at $\varepsilon_F$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110°C (163K)</td>
<td>3000</td>
<td>28</td>
<td>85</td>
<td>208</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>1000</td>
<td>11</td>
<td>29</td>
<td>57</td>
<td>6</td>
</tr>
<tr>
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<td>14</td>
</tr>
<tr>
<td></td>
<td>1800</td>
<td>15</td>
<td>32</td>
<td>73</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>60</td>
<td>50</td>
<td>6</td>
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<td>20°C (293K)</td>
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</tr>
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<td></td>
<td>4</td>
<td>34</td>
<td>68</td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>150°C (423K)</td>
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<td>5</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td>15</td>
<td>20</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>45</td>
<td>18</td>
<td>3</td>
</tr>
<tr>
<td>300°C (573K)</td>
<td>4407</td>
<td>50</td>
<td>69</td>
<td>40</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>30</td>
<td>33</td>
<td>13</td>
<td>2</td>
</tr>
</tbody>
</table>

**TABLE 10.4:** ESTIMATED TEMPERATURE RISES AND REDUCTION IN FLOW STRESSES DUE TO THERMAL SOFTENING FOR SHPB TESTS ON 224 STEEL
FIG 10.18  Isothermal stress versus strain curves at -40°C (233 K)
FIG 10.21  Isothermal stress versus strain curves at +300°C (573 K)
\[ \Delta \sigma = \beta_{5\%} \Delta T \] (10.3)

\(\beta_{5\%}\) was selected since it was felt that at 5% true strain the 224 steel mechanical properties were most reliably known. Figure 9.9 and Figures 10.1 to 10.4 show that 5% strain is large enough for the flow stress at this point to be not influenced by the lower yield point. Also at 5% true strain only a small amount of work hardening will have occurred and no appreciable temperature rise will have taken place. Equation 10.3 in conjunction with Figure 10.16 and Table 10.3 has been used to determine \(\Delta \sigma\) for all dynamic results at \(\dot{\varepsilon} \geq 4\) s\(^{-1}\).

Figures 10.17, 10.18, 10.19, 10.20 and 10.21 show the dynamic true stress-strain curves (dashed) corrected for the effects of adiabatic thermal softening at test temperatures of \(-110^\circ\text{C}, -40^\circ\text{C}, 20^\circ\text{C}, 150^\circ\text{C}\) and \(300^\circ\text{C}\) respectively. For comparison purposes the original true stress-strain curves have also been plotted. The dashed curves of Figures 10.17 to 10.21 may be referred to as ISOTHERMAL CURVES since they represent the stress versus strain curves which would have resulted had the test conditions been completely isothermal at these strain-rates. Table 10.4 shows estimated values of \(\Delta \sigma\) and \(\Delta T\) for the SHPB results presented in the first part of this Chapter (Figures 9.9 and 10.1 to 10.4).

Figure 10.17 shows isothermal curves drawn for the results at \(-110^\circ\text{C}\), the temperature at which adiabatic effects are most pronounced because of the high thermal sensitivity, \(\beta_{\varepsilon} = -2.59\). At 3000 s\(^{-1}\) and 28% true strain thermal softening accounts for a reduction in stress of 208 MPa which represents 20% of the actual observed flow stress at this strain.
Figure 10.18 shows isothermal stress-strain curves at -40°C. From Table 10.4 it can be seen that at a strain-rate of 4100 s\(^{-1}\), \(\Delta T = 91^\circ\text{C}\), thus taking the bulk temperature of the sample from -40°C to 51°C. This is precisely the temperature range in which the thermal sensitivity, \(\beta_c\), changes markedly from -2.59 to -0.57. To accommodate this change, values of \(\Delta \sigma\) for \(\dot{\varepsilon} = 4100\text{ s}^{-1}\), \(T = -40^\circ\text{C}\) were calculated using \(\beta_c = -2.59\) until the estimated temperature of the sample reached 20°C after which \(\beta_c = -0.57\) was applied.

Also plotted in Figure 10.18 is the true stress-strain curve at \(\dot{\varepsilon} = 4 \times 10^{-3}\text{ s}^{-1}\) and at a strain close to 30\% which overlaps with the curve for \(\dot{\varepsilon} = 4\text{ s}^{-1}\). However the corrected isothermal curve at 4 s\(^{-1}\) runs almost parallel to the isothermal \(4 \times 10^{-3}\text{ s}^{-1}\) curve thereby demonstrating that the overlapping of the original curves can be ascribed to thermal softening effects at 4 s\(^{-1}\).

A similar trend is apparent in Figure 10.19 in which the dynamic room temperature curves for \(\dot{\varepsilon} = 4\text{ s}^{-1}\) and 2088 s\(^{-1}\) are presented. The original uncorrected curves at these two rates tend to converge at a strain of 20\%. However, in the case of the corresponding isothermal curves, although there is still a slight convergence, the difference in stress level between them is greater than in the original curves.

Figures 10.20 and 10.21 show isothermal curves for the elevated temperature results at 150°C and 300°C. Clearly the magnitude of \(\Delta \sigma\) at a given strain is reduced at these higher temperatures but the effects of thermal softening are no less important. In Figure 10.21 overlapping occurs at about 34\% strain in the original true stress-strain curves for 4407 s\(^{-1}\) and \(4.3 \times 10^{-3}\text{ s}^{-1}\) which is isothermal anyway. The 4407 s\(^{-1}\) isothermal curve, however, does not overlap with the \(4.3 \times 10^{-3}\text{ s}^{-1}\) curve until strains above 50\% are approached by which stage friction will play a major role.
In summary, the work described above has shown the significant influence of adiabatic effects on the shape of resultant true stress-strain curves from dynamic tests ($\dot{\varepsilon} = 4 \text{ s}^{-1}$). The curves of Figures 9.9 and 10.1 to 10.4 represent the actual mechanical behaviour of 224 steel at the specified test temperatures and strain-rates. It should be remembered that inherent in the dynamic curves is the lowering of the flow stress due to thermal softening. The detailed mechanical data obtained over the temperature range -110°C to 300°C (Figures 10.10 to 10.14) has made it possible to extricate the reduction of flow stress due to this effect and hence draw up stress-strain curves which are effectively isothermal. This isothermal description of the material behaviour in one sense represents the true mechanical behaviour at constant temperature and allows useful comparisons to be made with lower strain-rate curves at the same temperature.

10.1.5 Work Hardening

The basic true stress versus strain graphs of Figures 9.9 and 10.1 to 10.4 all have similar shaped curves. Beyond the yield point, all the curves share the familiar characteristic in which increasing stress is required to produce further deformation and hence strain. The effect is commonly known as work or strain hardening. The discussion of the preceding sections has illustrated how in the post yield plastic region, thermal and mechanical effects are dependent on each other. At a given strain-rate the change in flow stress with strain may be expressed by the following differential equation:

$$\frac{d\sigma}{d\varepsilon} = \frac{\partial \sigma}{\partial \varepsilon} \bigg|_T + \frac{\partial \sigma}{\partial T} \frac{dT}{d\varepsilon} \quad (10.14)$$

Incorporating equation 10.2:

$$\frac{d\sigma}{d\varepsilon} = \frac{\partial \sigma}{\partial \varepsilon} \bigg|_T + \beta \frac{dT}{d\varepsilon} \quad (10.5)$$
where \( \frac{\partial \sigma}{\partial \varepsilon} \bigg|_T \) is the isothermal rate of work hardening and \( \frac{\partial T}{\partial \varepsilon} \) represents the gradients of the near linear plots of Figure 10.16. The term \( \beta \frac{\partial T}{\partial \varepsilon} \) on the right hand side of equation 10.5 is negative and describes the reduction in flow stress due to thermal softening. Logically this term is zero for an isothermal deformation test (i.e. one at low strain rate \( < 0.1 \text{ s}^{-1} \)).

It is interesting to note here that when \( \frac{\partial \sigma}{\partial \varepsilon} = 0 \) (e.g. at the onset of necking in tensile tests) \( \frac{\partial \sigma}{\partial \varepsilon} \) will equal \( -\beta \frac{\partial T}{\partial \varepsilon} \) which is the condition for catastrophic thermal softening (Rogers, 1979).

Generally for many metals and especially steels, the plastic work hardening region of the stress-strain curve may be described by:

\[
\sigma = K \varepsilon^n
\]

(10.6)

where \( n \) is the work hardening exponent and \( K \) is the strength coefficient (MPa), a term which is proportional to ultimate tensile strength (Kocks, 1982).
<table>
<thead>
<tr>
<th>Test Temperature</th>
<th>K (MPa)</th>
<th>Strain-Rate (s(^{-1}))</th>
<th>n</th>
<th>Test Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110°C (163K)</td>
<td>1140</td>
<td>1000</td>
<td>0.12</td>
<td>SHPB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3000</td>
<td>0.080</td>
<td></td>
</tr>
<tr>
<td>-40°C (233K)</td>
<td>1050</td>
<td>8 x 10(^{-4})</td>
<td>0.22</td>
<td>ESH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 x 10(^{-3})</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>500</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1800</td>
<td>0.14</td>
<td>SHPB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4100</td>
<td>0.094</td>
<td></td>
</tr>
<tr>
<td>20°C (293K)</td>
<td>1050</td>
<td>1.67 x 10(^{-3})</td>
<td>0.24</td>
<td>Instron</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.8 x 10(^{-3})</td>
<td>0.28</td>
<td>ESH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.24</td>
<td>(post-calibration)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>642</td>
<td>0.20</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>908</td>
<td>0.19</td>
<td>SHPB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2088</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>150°C (423K)</td>
<td>940</td>
<td>10(^{-3})</td>
<td>0.24</td>
<td>Instron</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4</td>
<td>0.23</td>
<td>ESH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>800</td>
<td>0.21</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2000</td>
<td>0.19</td>
<td>SHPB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4700</td>
<td>0.17</td>
<td></td>
</tr>
<tr>
<td>300°C (573K)</td>
<td>820</td>
<td>4.8 x 10(^{-4})</td>
<td>0.20</td>
<td>ESH</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.3 x 10(^{-3})</td>
<td>0.23</td>
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<td>4</td>
<td>0.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>872</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>2140</td>
<td>0.16</td>
<td>SHPB</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4407</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 10.5: WORK HARDENING EXONENTS, n AND STRENGTH COEFFICIENTS, K FOR 224 STEEL AT VARIOUS TEMPERATURES AND STRAIN-RATES**
FIG 10.23  Work hardening curves at -40°C (233 K)
Log Plot of all room temperature $\sigma$ versus $\varepsilon$ results from 10 x 5mm samples
N.B., In A two lines have been drawn through the points corresponding to $\dot{\varepsilon} = 3.4s^{-1}$ to indicate the change in gradient, n, which occurs at this rate.
\( \sigma = 4 \text{s}^{-1}, \quad + = 3.8 \times 10^{-3} \text{s}^{-1} \)

FIG 10.24B  Work hardening curves for room temperature ESH results after calibration of machine had been made.
FIG 10.25  Work hardening curves at +150°C (423 K)
FIG 10.26  Work hardening curves at +300°C (573 K)

A  ESH RESULTS

○ = 4s⁻¹ (● = isothermal)
+ = 4.3x10⁻³ s⁻¹
☐ = 4.8x10⁻⁴ s⁻¹

B  SHPB RESULTS

× = 872s⁻¹
□ = 2140s⁻¹
◇ = 4407s⁻¹ (●: isothermal)
### TABLE 10.6: WORK-HARDENING EXPONENTS, n AND STRENGTH COEFFICIENTS, K FOR DYNAMIC ISOTHERMAL CURVES

<table>
<thead>
<tr>
<th>Test Temperature</th>
<th>K (MPa)</th>
<th>Strain-Rate (s⁻¹)</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>-110°C (163K)</td>
<td>1300</td>
<td>1000</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>1520</td>
<td>3000</td>
<td>0.17</td>
</tr>
<tr>
<td>-40°C (233K)</td>
<td>1300</td>
<td>4</td>
<td>0.29</td>
</tr>
<tr>
<td></td>
<td>1800</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td>4100</td>
<td></td>
<td>0.16</td>
</tr>
<tr>
<td>Room Temperature</td>
<td>No appreciable change from values given in Table 10.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20°C (293K)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>150°C (423K)</td>
<td>1000</td>
<td>4</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>2000</td>
<td></td>
<td>0.21</td>
</tr>
<tr>
<td></td>
<td>4700</td>
<td></td>
<td>0.20</td>
</tr>
<tr>
<td>300°C (573K)</td>
<td>No appreciable change from Table 9.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figures 10.22 to 10.26 show logσ versus logε plots for all the compression results obtained here. From these graphs it is an easy matter to find values for K and n. n is the gradient of each line drawn while K is the stress when ε = 1. Table 10.5 shows figures for K and n found in this manner.

Two main trends emerge from the table. K decreases with increasing temperature as the steel becomes softer. n decreases as the strain rate increases. This happens because the yield stress always increases for higher strain rates while the ultimate strength of the material is far less sensitive to strain-rate. As a result the slope of the work hardening line which joins the two points decreases.
'Isothermal' points have also been included in the plots of Figures 10.22 to 10.26 taken from the isothermal data of Figures 10.17 to 10.21. Values of \( K \) and \( n \) for these isothermal points are presented in Table 10.6.

At room temperature and 300°C the figures for \( K \) and \( n \) are not appreciably different from those at the same temperature in Table 10.5. There is, however, a marked improvement in the overall linearity of these isothermal points at the higher strains. Furthermore, the values of \( n \) obtained from these isothermal points seem to be in closer agreement with values of \( n \) for the low strain rate data (< 0.1 s\(^{-1}\)) which is 'naturally' isothermal.

10.2 THERMAL ACTIVATION ANALYSIS

10.2.1 Introduction

In Section 2.10 the fundamentals of thermal activation theory as applied to the plastic deformation of carbon steel were introduced. In this section, the thermal activation theory is expanded further and is used to analyse the results of this project.

The relationship between the plastic strain rate, \( \dot{\varepsilon}_p \) and absolute temperature, \( T \) can be represented by the Arrhenius type equation:

\[
\dot{\varepsilon}_p = \dot{\varepsilon}_o \exp \left( \frac{-\Delta G (\sigma^*)}{kT} \right)
\]

(2.15)

\[
(10.7)
\]

where \( k \) is the Boltzmann constant; \( \Delta G \) is the Gibbs free energy of activation, which is a function of the local thermal stress, \( \sigma^* \); and \( \dot{\varepsilon}_o \) is the frequency factor (see for example Hull and Bacon, 1984).

Now,

\[
\dot{\varepsilon}_o = \phi b \Lambda \rho Y_o
\]

(10.8)
where $\phi$ is an orientation factor, $b$ is the Burgers vector, $A$ is the average area swept out by a dislocation after it overcomes a barrier, $\rho$ the dislocation density and $\gamma_0$ is the average frequency at which attempts are made to overcome the barrier. Since $\gamma_0$ depends on the dislocation density it necessarily depends on the thermal and mechanical history of the material.

The Gibbs free energy may be expressed as:

$$\Delta G = \Delta G_0 - V \sigma^*$$

where $V$ is the activation volume, $\Delta G_0$ is the total energy required for dislocations to overcome the barrier and the product $V \sigma^*$ is the mechanical work done by the applied load. If a rectangular barrier is assumed then $V$ is constant.

In Section 2.10 it was shown that the friction stress, $\sigma_i$, in the Hall-Petch relation (equation 2.12) could be split into an athermal component and thermal component. Since the effects of grain size have not been evaluated in this experiment it is convenient to simplify the analysis by condensing $\sigma_i$ and the $K \gamma d^{-1/2}$ term into a single athermal stress term, $\sigma_a$. Hence the applied stress, $\sigma$ may be written as:

$$\sigma = \sigma_a + \sigma^*$$

and equation 2.18 reduces to:

$$\sigma = \sigma_a + \Delta G_0/V + (kT/V) \ln \left( \dot{\varepsilon}_P / \dot{\varepsilon}_0 \right)$$

Equation 10.11 represents the most basic relationship between applied stress, $\sigma$, strain-rate, $\dot{\varepsilon}_P$ and temperature, $T$, for the thermally activated region. The deformation map of Figure 2.12 illustrated that
there are four distinct regions in which a particular type of deformation mechanism dominates. For the temperature and strain-rate ranges covered in this experiment it is the athermal (I) and thermal activation (II) regions which should be of most importance. However, at low temperatures and high strain-rates there may be some overlapping with regions III and IV respectively. The point to realise is that the boundaries between the regions are not abrupt or discontinuous but that there is a gradual change in predominant dislocation mechanism in going from one region to the next. This means that in the thermal activation region (II) for certain experimental conditions one or more other types of dislocation mechanism may have an influence on the observed macroscopic mechanical behaviour. It is important to be aware of this fact when attempting to formulate a thermal activation model which fits the experimental data.

The purpose of this section is to apply the most up to date thermal activation theories to the experimental data which was presented in the first half of this chapter and Chapter 9. In order to do this successfully it is necessary to find equivalent points on the stress-strain curves for the whole range of temperatures and strain-rates which can then be compared against each other.

The upper yield point is unsuitable since it is too sensitive to the method of testing. Many authors, including Campbell and Ferguson (1970) have chosen to use the lower yield point in thermal activation analysis. However, the lower yield point may not be considered entirely ideal since it can also be sensitive to the method of testing. In Chapter 3 it was demonstrated that at low and intermediate strain-rates the stiffness of the testing machine could have a significant effect on the extent to which the lower yield point is detected.
Another feature of the lower yield point in the experimental data collated in Sections 9.2 and 9.3 is that for the true stress-strain curves derived from the Instron and ESH machines at low strain-rates, the lower yield point appears typically at strains less than 0.5% whereas in the SHPB results the lower yield point occurs at strains between 0.5 and 3%. Due to this possible difference in true strain it may not be entirely valid to directly compare the lower yield points from the high and low strain-rate results with each other. Harding (1977) in his thermal activation analysis on four alloy steels avoided any complications due to lower yield points by considering only flow stresses at 2.5% strain. For the current experimental data a similar sort of analysis may be done by collating flow stresses at 5% strain. At this strain level the stress should not be influenced by the lower yield point even for the SHPB results and, furthermore, only a small amount of work hardening will have occurred. However the amount of work hardening, though small, will be greater for the low strain rate results (ESH and Instron) than for the high strain-rate results (SHPB) - a fact which may affect the comparison.

What is required is a level of stress, just beyond the yield of the material, which is structurally comparable over the range of strain rates. In metals which show no lower yield point, the flow stress at an appropriately small strain (say 0.5%) may be chosen. However, for the type 224 carbon steel under investigation here a suitable flow stress is not so easily determinable because of the existence of the lower yield point.

In order to make the analysis as thorough as possible both the lower yield stress and the stress at 5% strain have been extracted from the original true stress-strain curves of Sections 9.2 and 9.3 to test how well thermal activation theory applies to them.
FIG 10.27 Rectangular barrier opposing dislocation motion

FIG 10.32 A more realistic dislocation barrier shape (Gaussian)
10.2.2 The Basic Thermal Activation Model (1) (Rectangular Barrier)

Equation 10.11 is the most fundamental thermal activation relationship which is often applied to experimental data over a range of strain-rates and temperatures. It is based on the assumption that the obstacles to dislocation motion may be represented by a rectangular barrier (see Figure 10.27). In Figure 10.27, \( F \) is the force which opposes the motion of a dislocation when it encounters a barrier. If the obstacles are roughly equally spaced along a dislocation line at distances, \( l \), apart then the force on the line per obstacle due to the applied shear stress is \( \tau^{*}bl \).

To overcome this barrier completely, the line must move from a position \( x_1 \) to \( x_2 \). In order for this to happen the energy required is \( \Delta G_0 \) given by

\[
\Delta G_0 = \int_{x_1}^{x_2} F \, dx
\]

(10.12)

Part of this energy is provided in the form of mechanical work done by the applied load and is equal to \( \tau^{*}bl \, (x_2 - x_1) \). The remainder of the required energy is labelled 'thermal' and is equal to the Gibbs free energy \( \Delta G \) given by equation 10.9. Hence the activation volume, \( V \), for the process can be written as:

\[
V = bl \, (x_2 - x_1)
\]

(10.13)

Thus the significant feature of this simple theory upon which equation 10.11 is founded, is that the activation volume is assumed constant and does not vary with the applied stress.

10.2.3 Fitting the Simple Thermal Activation Model (1) to the Experimental Data

Taking the natural logarithm of the plastic strain rate \( \dot{\varepsilon}_p \) in equation 10.7 gives:
\[ \dot{\varepsilon} = 775\text{MPa}, \sigma = 650\text{MPa}, \varnothing = 475\text{MPa}, \]
\[ \times = 375\text{MPa}, \square = 425\text{MPa} \]

FIG 10.28A \( \dot{\varepsilon} \) versus 1000/T for lower yield stress data
FIG 10.28B  $\dot{\varepsilon}$ versus $1000/T$ for stresses at 5% strain

$\bullet = 850\,\text{MPa}, \, X = 750\,\text{MPa}, \, \circ = 650\,\text{MPa}, \, \otimes = 550\,\text{MPa}, \, \boxtimes = 500\,\text{MPa}$
\[ \sigma_{\text{LYP}} \text{ (MPa)} \]

- \( \square = 163K \), \( \otimes = 233K \), \( \bullet = 293K \),
- \( \circ = 423K \), \( \times = 573K \)

FIG 10.29A \( \sigma_{\text{LYP}} \) versus \( kT \ln(\dot{\varepsilon}_0/\dot{\varepsilon}_p) \)
FIG 10.29B  \( \sigma_{5\%} \) versus \( kT \ln(\dot{\varepsilon}_o/\dot{\varepsilon}_p) \)

\( \sigma(0) = 1300 \text{MPa} \)

- Square: 163K
- Cross: 233K
- Circle: 293K
- Circle with dot: 423K
- Cross with minus: 573K
Figure 10.28 A and B show plots of \( \log \dot{\varepsilon} \) versus \( \frac{1000}{T} \) for stresses at the lower yield point (LYP) and at 5\% strain respectively. The points plotted in Figure 10.28 were extracted from the graphs of stress versus \( \log \dot{\varepsilon} \) at 5 different temperatures (Figures 10.5 to 10.9).

According to the simple model (1) considered here these plots should comprise two straight lines; i.e. a line of negative slope representing the thermal activation region which converts at a critical point to a horizontal line representing the athermal region (see idealised diagram in Figure 2.14). The fact that the points plotted in Figure 10.29 form continuous curves reveals the inadequacy of the model in describing the variation of the activation volume, \( V \), with applied stress.

It can be seen that the points for a particular stress level lie approximately on a straight line and each line converges to \( \dot{\varepsilon}_0 \) at the intercept with the \( \log \dot{\varepsilon}_p \) axis. From Figure 10.28 A and B values of \( \dot{\varepsilon}_0 \) for the LYP and 5\% data are:

\[
\dot{\varepsilon}_0 \bigg|_{\text{LYP}} = (5.4 \pm 0.5) \times 10^5 \text{ s}^{-1}
\]

and

\[
\dot{\varepsilon}_0 \bigg|_{5\%} = (9 \pm 1) \times 10^5 \text{ s}^{-1}
\]

There is more scatter and slightly less consistency in the points taken at 5\% strain and hence the larger error in \( \dot{\varepsilon}_0 \). The fact that the lines in Figure 10.28 tend to converge at a single value of \( \dot{\varepsilon}_0 \) supports one of the fundamental ideas behind the theory that \( \dot{\varepsilon}_0 \) should be independent of the applied stress.
The points of LYP data form a hyperbolic shaped curve. The 5% data forms a less well defined trend as the points are more scattered. Nevertheless, it is possible to fit straight lines to the points for
\[ kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right) < 3 \times 10^{-20} \text{ J}. \] This has been done in both cases as shown by the lines labelled (1). Hence equation 10.11 describes the material behaviour at high strain-rates (SHPB results) and low temperatures.

At elevated temperatures and low strain-rates the applied stress required for plastic deformation should equal the athermal stress, \( \sigma_a \). Indeed the points plotted in Figure 10.29 tend to flatten out to a constant stress level at large values of \( kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right) \) (i.e. \( > 12 \times 10^{-20} \text{ J} \)). These stress levels may be approximated to \( \sigma_a \). Hence:

\[
\begin{align*}
\sigma_a \bigg|_{\text{LYP}} & = (230 \pm 10) \text{ MPa} \quad \text{and} \quad \sigma_a \bigg|_{5\%} = (410 \pm 20) \text{ MPa} \\
\end{align*}
\]

According to equation 10.11 the gradient of the straight lines (1) drawn in Figure 10.29 should equal the reciprocal of the activation volume. Hence by direct measurements from the graphs:

\[
V_{\text{LYP}} = (3.62 \pm 0.05) \times 10^{-29} \text{ m}^3 \quad \text{and} \quad V_{5\%} = (3.69 \pm 0.1) \times 10^{-29} \text{ m}^3
\]

The intercepts of the two lines (1) with the \( y \)-axis gives the flow stress at OK i.e. \( \sigma(0) \). Thus for the two sets of data we obtain:

\[
\begin{align*}
\sigma(0) \bigg|_{\text{LYP}} & = 1160 \text{ MPa} \quad \text{and} \quad \sigma(0) \bigg|_{5\%} = 1300 \text{ MPa} \\
\end{align*}
\]

At OK the Gibbs free energy is zero since there is no thermal energy available to overcome barriers. Thus by equation 10.9

\[
\Delta G_0 = V \sigma^\ddagger(0) \quad \text{(10.15)}
\]
\[ + = 4 \times 10^{-3} \text{s}^{-1}, \bigcirc = 4 \text{s}^{-1}, \times = 1000 \text{s}^{-1}, \]
\[ \square = 2000 \text{s}^{-1}, \square_m = \text{mean of 1000\&3000s}, \]
\[ \lozenge = 4470 \text{s}^{-1}, \lozenge = 4470 \text{s}^{-1} \text{extrapolated data}. \]

**THERMAL ACTIVATION**

**ATHERMAL**

**FIG 10.30A Lower yield stress versus temperature**
FIG 10.30B  Stress at 5% versus temperature
Using equation 10.10 to determine $\sigma^*(0)$ and the values of $V$, $\sigma(0)$ and $\sigma_a$ quoted above we obtain:

$$\Delta G_{\text{LYP}} = 3.37 \times 10^{-20} \text{J} \ (0.21 \text{ eV}) \quad \text{and} \quad \Delta G_{\text{5%}} = 3.28 \times 10^{-20} \text{J} \ (0.21 \text{ eV})$$

These energy levels at roughly 0.21 eV are of the correct order for the interaction of a dislocation with a short range barrier such as with another dislocation or with solute atoms (Cottrell, 1953).

Figure 10.30 A and B show $\sigma_{\text{LYP}}$ and $\sigma_{\text{5%}}$ plotted against temperature respectively at various strain rates. Extrapolation of the curves for high strain rates (SHPB) give intersections with the stress axes at values of $\sigma(0)$ consistent with those quoted above. The basic thermal activation theory (1) predicts that for high test temperatures and low strain-rates the curves in Figure 10.30 should flatten out to a horizontal level, $\sigma_a$. The fact that they do not means that the flow stress is not entirely temperature independent for the prescribed conditions. Thus further tests at lower strain rates and higher temperatures are required to see if true athermal behaviour is to be found and hence a more precise value for $\sigma_a$ established.

A similar temperature dependence of flow stress at low strain-rates was observed by Harding (1977) for both annealed and normalised mild steels and also by Monteiro et al (1970) for $\alpha$-titanium. The latter ascribed the effect to a mechanism of dynamic strain ageing (see Section 10.3). For the results of this investigation it is likely that the onset of the 'true' athermal region must occur close to 573K so that the values of $\sigma_a$ quoted above represent a reliable indication of the athermal flow stress.
If a rectangular barrier to dislocation motion is assumed, as depicted in Figure 10.27, then the Gibbs free energy may be expressed as (e.g. Hull and Bacon, 1984):

\[ \Delta G = \Delta G_0 \left( 1 - \frac{\tau^*(T)}{\tau^*(0)} \right) \]  

(10.16)

Since the thermal shear stress, \( \tau^* \) is related to the thermal stress, \( \sigma^* \) by

\[ \sigma^* = \bar{M} \tau^* \]  

(10.17)

where \( \bar{M} \) is the Taylor factor (\( ^{1/3} \), Harding, 1977) then equation 10.16 is equivalent to

\[ G = G_0 \left( 1 - \frac{\sigma^*(T)}{\sigma^*(0)} \right) \]  

(10.18)

Substituting this into equation 10.7 gives:

\[ \frac{\sigma^*(T)}{\sigma^*(0)} = \frac{kT}{\Delta G_0} \ln \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_0} \right) + 1 \]  

(10.19)

At a certain temperature, \( T = T_c \) the thermal energy of the material is not able to assist dislocations in overcoming long range barriers and the flow stress reduces to \( \sigma_a \). At this point \( \sigma^* (T_c) = 0 \) so that from equation 10.19 we obtain:

\[ T_c = \frac{\Delta G_0}{k \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right)} \]  

(10.20)

Table 10.7 presents values of \( T_c \) calculated for both the lower yield point and 5% data. It is interesting to note that values of \( T_c \) tend to converge at the low strain rates. The loci of \( T_c \) points are plotted...
in Figure 10.30 and represent the boundary between the athermal (I) and thermal activation (II) regions according to the Rosenfield and Hahn deformation map (Figure 2.12).

<table>
<thead>
<tr>
<th>( \dot{\varepsilon}_p ) (s(^{-1}))</th>
<th>LYP ( T_C ) (K)</th>
<th>5% ( T_C ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4470</td>
<td>502</td>
<td>448</td>
</tr>
<tr>
<td>2000</td>
<td>430</td>
<td>389</td>
</tr>
<tr>
<td>1000</td>
<td>382</td>
<td>349</td>
</tr>
<tr>
<td>4</td>
<td>204</td>
<td>193</td>
</tr>
<tr>
<td>( 4 \times 10^{-3} )</td>
<td>129</td>
<td>124</td>
</tr>
</tbody>
</table>

TABLE 10.7: VALUES OF CRITICAL TEMPERATURE, \( T_C \) FOR VARIOUS STRAIN RATES, \( \dot{\varepsilon}_p \)

In a similar fashion, at a given temperature \( T \), there will be a critical strain rate, \( \dot{\varepsilon}_c \) which marks the transition from athermal (I) to thermal activation (II) type behaviour. A simple rearrangement of equation 10.20 gives:

\[
\dot{\varepsilon}_c = \dot{\varepsilon}_o \exp \left( -\frac{\Delta G_o}{kT} \right)
\]  

(10.21)

Values of \( \dot{\varepsilon}_c \) are tabulated in Table 10.8.

<table>
<thead>
<tr>
<th>( T ) (K)</th>
<th>LYP ( \dot{\varepsilon}_c ) (s(^{-1}))</th>
<th>5% ( \dot{\varepsilon}_c ) (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>163</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>233</td>
<td>18</td>
<td>33</td>
</tr>
<tr>
<td>293</td>
<td>147</td>
<td>270</td>
</tr>
<tr>
<td>423</td>
<td>1830</td>
<td>3266</td>
</tr>
<tr>
<td>573</td>
<td>8100</td>
<td>14216</td>
</tr>
</tbody>
</table>

TABLE 10.8: VALUES OF CRITICAL STRAIN RATES, \( \dot{\varepsilon}_c \) AT VARIOUS TEMPERATURES
Equation 10.21 has been plotted as a straight line (long dashes) in Figure 10.28 and once again represents the boundary between the athermal (I) and thermal activation (II) regions.

In Section 10.2.9 the simple thermal activation model (1) incorporating the parameters found in this section is fitted to the $\sigma_{\text{LYP}}$ and $\sigma_{5\%}$ versus $\dot{\varepsilon}_p$ data from Figures 10.5 to 10.14 and is compared with the other models dealt with here.

10.2.4 Harding Semi-Empirical Analysis - Model (2)

Harding (1981) in his work on several alloy steels obtained a trend of points similar to that depicted in Figure 10.29 and found that the trend could be described adequately by a single curve of equation:

$$\sigma = \sigma_a + (\sigma_o - \sigma_a) \left( \frac{\varepsilon}{\varepsilon_o} \right)^D$$  \hspace{1cm} (10.22)

where $D$ is a constant. Campbell (1975) showed that an equation of the form 10.22 corresponded to an inverse dependence of activation volume, $V$, on the thermal component of the applied shear stress, $\tau^*$, such that

$$V \tau^* = D$$  \hspace{1cm} (10.23)

The inverse proportionality between $V$ and $\tau^*$ described by equation 10.23 is an attractive feature of the theory since many workers [e.g. Conrad (1961, 1963), Harding (1977) and Ellwood et al (1984)] have found that the activation volume $V$ increases markedly as the thermal component of stress $\sigma^*$ decreases at small values of $\sigma^*$. On the other hand when $\sigma^*$ is large (> 100 MPa), $V$ is virtually constant. This behaviour is reflected in the shape of the curve of experimental points plotted in Figure 10.29a.

Substituting equations 10.23 and 10.17 for $D$ and $\tau^*$ respectively into equation 10.22 gives
\[ \ln(\sigma - \sigma_a) \] versus \( kT \ln(\dot{\varepsilon}/\dot{\varepsilon}_p) \times 10^{-20} \text{J} \)

- \( \square = 163K \)
- \( \bigotimes = 233K \)
- \( \bullet = 293K \)
- \( \bigodot = 423K \)
- \( \bigtimes = 573K \)

FIG 10.31A \( \ln(\sigma_{LYP} - \sigma_a) \) versus \( kT \ln(\dot{\varepsilon}/\dot{\varepsilon}_p) \)
FIG 10.31B \( \ln(\sigma_{5k} - \sigma_a) \) versus \( kT \ln(\dot{\epsilon}_o/\dot{\epsilon}_p) \)
which is equivalent to equation 2.19 introduced in Chapter 2. The form of equation 10.24 implies a Gibbs' free energy of activation given by:

\[
\Delta G = E \{1 - \ln \left[\frac{(\sigma/\sigma_a)}{(\sigma/\sigma_a) - 1}\right]\}
\]  

where \(E\) is a constant.

Rearranging and taking the natural logarithm of both sides of equation 10.22 gives:

\[
\ln (\sigma - \sigma_a) = \ln (\sigma_0 - \sigma_a) - \frac{kT}{D} \ln \left(\frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_{\text{P}}}ight)
\]  

In Figure 10.31A and B \(\ln(\sigma - \sigma_a)\) has been plotted against \(kT \ln (\dot{\varepsilon}_0/\dot{\varepsilon}_{\text{P}})\) for the \(\sigma_{\text{LYP}}\) and \(\sigma_{\text{SF}}\) data respectively. The \(\sigma_{\text{SF}}\) data (Figure 10.31) shows a large amount of scatter and is not particularly linear hence the points bear little agreement with an equation of the form of 10.26. On the other hand the LYP data (Figure 10.31A) shows better consistency and a least squares regression has been carried out to find the best fit straight line to the points. This regression line has been rotated slightly about the median point so that the intercept with the \(y\)-axis occurs at a point which agrees with the flow stress at 0K found previously, i.e. \(\sigma(0) = 1160\) MPa. This also has the benefit of providing a better fit to points for low temperature and high strain rates. From the gradient of the line plotted in Figure 10.31A we obtain:

\[D = (2.33 \pm 0.3) \times 10^{-20} \text{J} \quad (0.15 \pm 0.2 \text{ eV})\]
From this value of $D$ an approximate hyperbolic curve of $V$ versus thermal stress, $\tau^*$ could be drawn up.

The regression analysis just described has been used to fit a curve of equation 10.24 to the LYP data plotted in Figure 10.29 and is labelled (2). Quite clearly this Harding semi-empirical model (2) provides a better fit to the experimental points over a wide range of strain-rate than does the elementary model (1) since the changing activation volume $V$ is implicit in equation 10.24. Model 2 is discussed further in Section 10.2.9.

10.2.5 The 'Kocks' Model - Model (3)

In the simplest thermal activation theory (1) a rectangular shaped variation of the force, $F$, opposing dislocation motion versus distance $x$ was assumed. This over-simplification is bound to produce shortcomings in the model to accurately describe the mechanical behaviour within the thermally activated region (II). Davidson and Lindholm (1974) have investigated the effect of barrier shape in thermal activation theory and have proved it to be a critical factor in matching up thermal activation mechanisms to the macroscopic behaviour of the material over a wide range of temperature and strain-rate.

Figure 10.32 shows a more realistic barrier profile for the variation of the resistive force, $F$, which opposes dislocation motion with distance, $x$. Kocks et al (1975) have shown that such a profile may be described by an equation for the Gibbs free energy of the form:

$$\Delta G = \Delta G_0 \left(1 - \left[\frac{I_x(T)}{I_x(0)}\right]^p\right)^q \quad (10.27)$$

where $p$ and $q$ must lie within the ranges:

$$0 \leq p \leq 1 \quad \text{and} \quad 1 \leq q \leq 2$$
Note that if \( p = q = 1 \) then we have equation 10.16 again and the barrier is rectangular. A barrier profile which most closely resembles that depicted in Figure 10.32 is obtained when \( p = 1/2 \) and \( q = 3/2 \), Ono (1968). For greater precision the thermal variation of \( \Delta G_0 \) should be considered and accounted for in equation 10.27.

10.2.6 The Mechanical Threshold Stress Model

This model which involves defining the structural state of a material has been developed over the last few years by the 'high strain-rate mechanics' group working at Los Alamos (e.g. Follansbee et al (1985), Regazzoni et al (1987) and Armstrong et al (1989)). Follansbee (1986) has shown that \( \sigma(0) \) is dependent on the strain-rate and thus it is not strictly valid to compare stress levels at constant strain for different strain-rates. The problem may be overcome by introducing a structure sensitive parameter, the mechanical threshold stress, \( \hat{\sigma} \) which is the stress at which the material in a given structural state would begin to first deform at OK.

In this model the Gibbs free energy, based on equation 10.27, is expressed as:

\[
\Delta G = g_0 \mu b^3 \left( 1 - \frac{\sigma^k}{\sigma} \right)^{1/2} \frac{3}{2} \tag{10.28}
\]

where \( g_0 \) is the normalised Helmholtz free energy, \( \mu \) is the temperature dependent shear modulus, \( b \) is the Burgers vector. The product \( g_0 \mu b^3 \) describes the temperature dependence of \( \Delta G_0 \).

To use this model (Follansbee, 1986) involves straining the material to a given strain level at each strain-rate and temperature investigated and thereafter quasistatically reloading the material at three low temperatures: 76K, 180K and 295K. Extrapolation of these three experimental points gives the value of the mechanical threshold stress.
stress, \( \hat{\sigma} \) for the original test conditions. \( \hat{\sigma} \), then, defines the structural state of the material.

Clearly, to use the approach outlined above would require a considerable amount of extra experimental work on top of the numerous tests over the wide range of temperatures and strain rates normally needed. In this project there was insufficient time to carry out the additional tests required to assess this particular model. Quasistatic tests at liquid nitrogen temperatures would have proved a major difficulty in themselves with the existing facilities at Loughborough.

10.2.7 Application of the Kock's Model (3) to the Current Data
In Sections 10.2.3 and 10.2.4 where models (1) and (2) were fitted to the experimental data it was seen that generally there was more scatter in the \( \sigma_{5\%} \) points than in the \( \sigma_{\text{LYP}} \) points. The reason for this may be due to differing amounts of work hardening in the \( \sigma_{5\%} \) data especially between the low strain-rate and high strain-rate results. In this section the \( \sigma_{5\%} \) data has not been considered in order to avoid the duplication of graphs. It is felt from the foregoing discussion that the lower yield stress, \( \sigma_{\text{LYP}} \) serves as a more reliable indicator of material behaviour and is therefore more suitable for the fitting of thermal activation theories.

In this section we deduce a model based on the discussion of the two preceding sections 10.2.5 and 10.2.6. Replacing shear stress, \( \tau^* \) with normal stress, \( \sigma^* \) in equation 10.27 gives:

\[
\Delta G = \Delta G_0 \left(1 - \left(\frac{\sigma^*}{\sigma^*_0}\right)^{1/2}\right)^{3/2}
\]

(10.29)

where

\[
\sigma^*_0 = \sigma^*(0) = \sigma(0) - \sigma_a
\]

(10.30)
Combining equation 10.29 with the Arrhenius rate equation 2.15 gives:

\[
\frac{1}{2} = \frac{(\sigma_o^*)^{1/2} - (\sigma_o^*)^{1/2}}{\Delta G_o^{2/3}} \left[ kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right) \right]^{2/3} (10.31)
\]

Hence a plot of \((\sigma - \sigma_a)^{1/2}\) versus \([kT \ln (\dot{\varepsilon}_o/\dot{\varepsilon}_p)]^{2/3}\) should produce a straight line. Such a plot has been made in Figure 10.33 for the LYP data using \(\sigma_a = 230\) MPa and \(\dot{\varepsilon}_o = 5.3 \times 10^5\) s\(^{-1}\) as established in Section 10.2.3.

At first sight it is clear that the points plotted in Figure 10.33 do not form a single straight line but rather constitute a curve with a marked knee in it close to its mid-point. Nevertheless, an approximate linear regression fit has been made for the experimental points, slightly adjusted so that the intercept with the y-axis occurs at \((\sigma_{LYP} - \sigma_a)^{1/2} = \sqrt{930}\) in accordance with a result from Section 10.2.3 (see intercept on Figure 10.30A). This line has been drawn in Figure 10.33 and is labelled (3A). Its equation is:

\[
(\sigma - \sigma_a)^{1/2} = 30.5 - 1.43 \times 10^{14} \left[ kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right) \right]^{2/3} (10.32)
\]

Comparison of 10.32 with 10.31 yields:

\[
\Delta G_o = 9.85 \times 10^{-20} J (0.62\ eV)
\]

which is of the correct order for the energy of activation of a dislocation with a short range barrier.

The line (3A) serves as a first approximate fit to the results depicted in Figure 10.33. Although it lies close to some of the low strain-rate points it does not fit some of the high strain-rate points
\[
\left( \sigma_{\text{yp}} - \sigma_a \right)^{1/2} \text{ versus } \left( kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right) \right)^{2/3}
\]

- □ = 163K
- ○ = 233K
- ● = 293K
- ○ = 423K
- × = 573K

FIG 10.33 \( \left( \sigma_{\text{yp}} - \sigma_a \right)^{1/2} \) versus \( (kT \ln \left( \frac{\dot{\varepsilon}_o}{\dot{\varepsilon}_p} \right))^{2/3} \)
so well. It is at high strain-rates where we should expect thermal activation models to agree more closely with the experimental data and so we require something better here.

The bend in the line of points plotted in Figure 10.33 suggests that it may be more appropriate to fit two straight lines of equation 10.31 to the data, one for high strain rates and one for low strain rates. Accordingly, best fit lines have been drawn on the graph and are labelled (3B) and (3C) respectively. They intersect at \( \{kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right) \}^{2/3} = 12.4 \times 10^{-14} \left( j^{2/3} \right) \). The equation of line (3B) is:

\[
(s-\sigma_a)^{1/2} = 34 - 1.97 \times 10^{14} \left( kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right) \right)^{2/3} \tag{10.33}
\]

giving \( \Delta G_\Omega = 7.17 \times 10^{-20} J \) (0.45 eV).

For line (3C):

\[
(s-\sigma_a)^{1/2} = 17 - 5.7 \times 10^{13} \left( kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right) \right)^{2/3} \tag{10.34}
\]

giving \( \Delta G_\Omega = 1.61 \times 10^{-19} J \) (1 eV).

Hence the combination of equation 10.33 for \( \{kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right) \}^{2/3} < 12.4 \times 10^{-14} \left( j^{2/3} \right) \) and equation 10.34 for \( \{kT \ln \left( \frac{\dot{\varepsilon}_0}{\dot{\varepsilon}_p} \right) \}^{2/3} > 12.4 \times 10^{-14} \left( j^{2/3} \right) \) provides a reasonable description of the experimental results presented here. One good feature of this semi-empirical model is that at high strain rates where short range obstacles play a significant role the 'barrier energy', \( \Delta G_\Omega = 0.45 \) eV, while at low strain rates and elevated temperatures where long range obstacles play the major part in the dislocation dynamics, \( \Delta G_\Omega \sim 1 \) eV. Thus the combination of lines (3B) and (3C) reflect the changing nature of the obstacles to dislocation motion.
If the point of intersection between lines (3B) and (3C) is considered to mark the transition point between the thermal activation region II and the 'athermal' region I, then it is possible to determine \( T_c \) and \( \dot{\varepsilon}_c \) as was done in Section 10.2.3 by noting that at the 'transition point' \( \{ kT \ln (\dot{\varepsilon}_0/\dot{\varepsilon}_p)\}^{2/3} = 12.4 \times 10^{-14} (\dot{\varepsilon}^{2/3}) \). Values of \( T_c \) and \( \dot{\varepsilon}_c \) calculated in this way are presented in Table 10.9.

<table>
<thead>
<tr>
<th>( \dot{\varepsilon}_p ) (s(^{-1}))</th>
<th>( T_c ) (K)</th>
<th>( T ) (K)</th>
<th>( \dot{\varepsilon}_c ) (s(^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4470</td>
<td>661</td>
<td>163</td>
<td>2 \times 10^{-3}</td>
</tr>
<tr>
<td>2000</td>
<td>566</td>
<td>233</td>
<td>0.7</td>
</tr>
<tr>
<td>1000</td>
<td>503</td>
<td>293</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>268</td>
<td>423</td>
<td>303</td>
</tr>
<tr>
<td>4 \times 10^{-3}</td>
<td>157</td>
<td>573</td>
<td>2149</td>
</tr>
</tbody>
</table>

**TABLE 10.9**: VALUES OF CRITICAL TEMPERATURE \( T_c \) AND CRITICAL STRAIN-RATE, \( \dot{\varepsilon}_c \) DEFINED BY THE INTERSECTION OF LINES (3B) AND (3C) AT \( \{ kT \ln (\dot{\varepsilon}_0/\dot{\varepsilon}_p)\}^{2/3} = 12.4 \times 10^{-14} (\dot{\varepsilon}^{2/3}) \)

In Section 10.2.9 model (3A) and the combination (3B)+(3C) are discussed further and are compared with models (1) and (2).

10.2.8 Empirical Linear Regression Fits (4)

Figure 10.34A and B shows two plots of \( \sigma_{LYP} \) and \( \sigma_{SS} \) respectively versus strain-rate \( \dot{\varepsilon} \) for all the SHPB tests carried out in this project. It should be expected that in this high strain-rate regime thermal activation should be the dominant controlling mechanism for dislocation motion. Hence the plots of experimental points in Figure 10.34 may be used to assess how well the models (1) to (3) discussed in the preceding sections fit the actual material behaviour.
FIG 10.34A  Lower yield stress versus strain rate
\[ \sigma = 163K, \quad \circ = 233K, \quad \bullet = 293K, \quad \circ = 423K, \quad \times = 573K. \]

**FIG 10.34B Stress at 5% versus strain rate**
The simplest way of dealing with the data in Figure 10.34 to determine an approximate relationship between strain-rate and stress is to find the best fit straight line through the points at each temperature. The straight lines drawn in Figure 10.34 (labelled (4)) have been calculated by the least squares method and their equations are given in Table 10.10.

A LYP Data (Error in c = ±20 MPa, error in m = ±0.01 MPa.s)

<table>
<thead>
<tr>
<th>Test Temperature</th>
<th>Regression Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>163K</td>
<td>( \sigma_{LYP} = 758 + 0.016 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>233K</td>
<td>( \sigma_{LYP} = 582 + 0.032 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>293K</td>
<td>( \sigma_{LYP} = 430 + 0.040 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>423K</td>
<td>( \sigma_{LYP} = 335 + 0.025 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>573K</td>
<td>( \sigma_{LYP} = 270 + 0.020 \dot{\varepsilon} )</td>
</tr>
</tbody>
</table>

B 5% Data (Error in c = ±30 NlPa, error in m = ±0.014 Wa.s)

<table>
<thead>
<tr>
<th>Test Temperature</th>
<th>Regression Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>163K</td>
<td>( \sigma_{5%} = 802 + 0.015 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>233K</td>
<td>( \sigma_{5%} = 607 + 0.05 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>273K</td>
<td>( \sigma_{5%} = 530 + 0.025 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>423K</td>
<td>( \sigma_{5%} = 475 + 0.024 \dot{\varepsilon} )</td>
</tr>
<tr>
<td>573K</td>
<td>( \sigma_{5%} = 410 + 0.019 \dot{\varepsilon} )</td>
</tr>
</tbody>
</table>

TABLE 10.10: LINEAR REGRESSION EQUATIONS FOR (A) LYP DATA AND (B) 5% DATA PLOTTED IN FIGURE 10.34 m = GRADIENT OF LINE AND c = INTERCEPT ON THE STRESS AXIS.

It can be seen that all the equations in Table 10.10 take the form:

\[
\sigma = \sigma_b + \eta \dot{\varepsilon} \tag{10.35}
\]

where \( \sigma_b \) and \( \eta \) are constants. Equation 10.35 is usually used to describe material behaviour at high strain rates when viscous drag is the controlling mechanism in the dislocation motion (see Campbell and
Ferguson, 1970) and usually only applies to region IV in the Rosenfield-Hahn deformation map (Figure 2.12). We should not expect, however, dislocation drag effects to become dominant until strain rates of $10^4 \, \text{s}^{-1}$ are approached. Such a strain-rate is outside the range of this experiment but since the transition from region II to IV is gradual there may be some slight dislocation drag effects present in the SHPB results of Figure 10.34.

Inspection of Figure 10.34 does show that the straight lines (4) drawn there do provide a reasonable fit to the data. These best fit straight lines which are purely empirical provide a standard by which to compare the thermal activation models (1), (2) and (3) described in the previous sections.

10.2.9 Comparison of the Models (1), (2), (3) and (4)

In this section the theoretical models (1) and (3A), the semi-empirical models (2) and (3B)+(3C) and the purely empirical linear fits (4), which were discussed in the preceding sections, are assessed and are compared with each other. Only the LYP data is considered here since the 5% strain data was not thought to be as reliable. In Figure 10.34A the SHPB results were presented in the linear plot of $\sigma_{\text{LYP}}$ versus $\dot{\varepsilon}_p$. As well as the experimental points the curves deduced from models (3A) and (3B) were also plotted along with the best fit straight lines (4). Figure 10.35 is a repeat plot of the experimental data in Figure 10.34A only this time the curves due to models (1) and (2) have been included. As all five models apply to the thermally activated regime only experimental data recorded at strain rates above 200 $\, \text{s}^{-1}$ will be analysed in this section.
<table>
<thead>
<tr>
<th>Temperature (K)</th>
<th>md $\times 10^3$</th>
<th>msd $\times 10^3$</th>
<th>md $\times 10^3$</th>
<th>msd $\times 10^3$</th>
<th>md $\times 10^3$</th>
<th>msd $\times 10^3$</th>
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</thead>
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<td>51.7 3.48</td>
<td>32.6 2.40</td>
<td>1.18 .411</td>
<td></td>
</tr>
<tr>
<td>233</td>
<td>28.7 4.60</td>
<td>3.58 1.35</td>
<td>4.67 1.21</td>
<td>18.5 1.90</td>
<td>2.5 .714</td>
<td></td>
</tr>
<tr>
<td>293</td>
<td>22.6 2.01</td>
<td>-62.2 4.31</td>
<td>-62.3 4.28</td>
<td>-23.7 .792</td>
<td>-1.44 .249</td>
<td></td>
</tr>
<tr>
<td>423</td>
<td>81.5 6.88</td>
<td>61.0 3.92</td>
<td>-77.0 6.23</td>
<td>-19.8 .817</td>
<td>1.73 .164</td>
<td></td>
</tr>
<tr>
<td>573</td>
<td>88.3 8.85</td>
<td>-47.6 2.62</td>
<td>-53.2 3.48</td>
<td>-3.23 .381</td>
<td>1.14 .252</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>44.6 4.78</td>
<td>41.6 2.89</td>
<td>49.8 3.74</td>
<td>19.6 1.26</td>
<td>1.60 .358</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 10.11: MEAN DEVIATION (md) AND MEAN SQUARE DEVIATION (msd) OF EXPERIMENTAL POINTS FROM MODELS (1), (2), (3A), (3B) AND THE LINEAR REGRESSION FITS (4)**

Assessing how well the models (1) to (4) fit the experimental results is perhaps best done by measuring the distance of the points from the curves plotted in Figures 10.34 and 10.35. Two convenient measures of this distance averaged for all points are the mean deviation (md) and the mean square deviation (msd) defined by the equations below:

$$\text{md} = \frac{1}{n} \sum \Delta x_i$$  \hspace{1cm} (10.36)

and

$$\text{msd} = \frac{1}{n} \sum (\Delta x_i)^2$$  \hspace{1cm} (10.37)

where $n$ = total number of points and $\Delta x_i$ is the vertical distance of the $i^{th}$ point from the curve of fit. If this point lies below the curve then $\Delta x_i$ is negative, if above then $\Delta x_i$ is positive.
FIG 10.36 Fitting of models 1, 2 & 3 to graph of $\sigma_{yf}$ versus In
FIG 10.37 Combination of equations 3B and 3C fitted to a plot of lower yield stress versus $\log \dot{\varepsilon}$. 3B applies down to $\dot{\varepsilon}_x$, while 3C applies below $\dot{\varepsilon}_x$. 

$\square = 163K$, $\otimes = 233K$, $\bullet = 293K$, $\bigcirc = 423K$, $\times = 573K$. 

I. THERMAL ACTIVATION

II. THERMAL ACTIVATION

I. A THERMAL
Of the two, msd is the better indicator since discrepancies above the curve do not cancel those below. Values of md and msd are displayed in Table 10.11 for each test temperature. By inspection of the row of mean md and msd figures it is immediately obvious that the best fit straight lines (4) are closest to the experimental points. (3B) appears to be the best of the thermal activation models.

Figure 10.36 is a plot of the $\sigma_{LYP}$ data plotted over the complete strain-rate range covered in this project. On top of the experimental points has been drawn the curves from models (1), (2) and (3A). Model (1) appears as a straight line on this log-plot and only applies to the high strain-rate points. There appear to be large discrepancies between the curves and the corresponding experimental points and in places it is difficult to tell which curves belong to which points. Curve (2) for the Harding semi-empirical model gives a fair representation of the experimental trend over the whole range of strain rates but is perhaps more accurate at low strain-rates than at high strain rates where it should be better.

Figure 10.37 is a repeat plot of the points in Figure 10.36 but here the lines drawn were deduced from the (3B)+(3C) combination. It is pleasing to see how well this last thermal activation model fits the experimental data. Also plotted on this graph is the locus of $\varepsilon_C$ for the five different temperatures determined from the values given in Table 10.9. This line marks the boundary between the thermal activation region II in which the dislocation motion is determined by obstacles of energy 0.45 eV and the athermal region I in which long range obstacles of energy 1 eV operate.

Clearly, no thermal activation model can be expected to satisfy the experimental data without disparity. The work of this section of the Chapter has revealed the importance of the barrier shape and how the
model can be improved by choosing a barrier profile similar to that outlined in Figure 10.32 [equation 10.29] than the 'rectangular' barrier, Figure 10.27 [equation 10.18]. Apart from barrier shape the thermal activation theory is also vulnerable in making the simplifying assumptions that obstacles are evenly distributed and that only one type of obstacle operates at a given instance. As well as it being highly likely that more than one type of obstacle opposes the movement of dislocations, it is also likely that more than one dislocation mechanism operates under a given set of conditions.

Nonetheless the good agreement of the curves in Figure 10.37 with the experimental points is a vindication of the (3B)+(3C) combination model and demonstrates that thermal activation plays the major part in the material behaviour at high strain-rates between 200 s\(^{-1}\) and 5000 s\(^{-1}\).

10.3 A NOTE ON SERRATIONS (THE PORTEVIN-LE CHATELIER EFFECT)

Figure 9.22 displayed the set of low and intermediate strain-rate results from the ESH and Instron tests at 300\(^\circ\)C. As discussed earlier these curves show some interesting trends: the yield stress is independent of strain-rate, the collection of curves show a negative strain-rate sensitivity (i.e. flow stress decreasing with increasing strain-rate), and the Instron curve was serrated at low strain. All these effects are signs of dynamic strain ageing and the multiple yield observed in the Instron curve is known as the Portevin-Le Chatelier effect or sometimes is referred to as "blue brittleness".

Cottrell (1953b) has noted how discernible serrations, or multiple yield points actually are dependent on the testing machine. In 'hard' machines where sudden plastic deformation causes stress relaxation, the serrations appear as repeated oscillations of the applied stress. On the other hand, in 'soft' machines where the stress cannot relax,
serrations are seen as alternate vertical and horizontal steps on the curve corresponding to periods in which the rate of plastic flow is either practically zero or very large. It is possible that in the ESH machine which is 'hard', the Portevin-Le Chatelier effects in the results obtained at elevated temperatures may have been overlooked or disregarded as electrical noise from surrounding equipment. On the two occasions where serrations were observed using the Instron machine ('soft') there was no interfering noise on the chart recorder trace so that the presence of a genuine Portevin-Le Chatelier effect was unequivocal.

Numerous studies have been made on the Portevin-Le Chatelier effect in iron and mild steel, for example, Sleeswyk (1958), Blakemore and Hall (1966) and Keh and Leslie (1963) using an electron microscope. A particularly comprehensive investigation has been carried out by Keh et al (1968) on a 0.035% carbon steel. They found that serrations would start to appear on the load-displacement curves at temperatures between 500°C and 1000°C and would disappear again at temperatures between 200°C and 300°C. It was proposed that the material behaviour between these two bounds could be described by an Arrhenius type equation:

\[ \dot{\varepsilon} = A \exp \left( -\frac{Q}{RT} \right) \]  

(10.38)

where A is a constant, R is the universal gas constant and Q is the activation energy for the process.

At the point where serrations just begin to appear the activation energy is \( Q_D \) - the activation energy required for the diffusion of solute C and N atoms. At the point where they start to disappear the activation energy is \( Q_D + \Delta E \) where \( \Delta E \) is the binding energy for a C or N atom with a dislocation core. Hence by observing the onset and disappearance of serrations measurements of \( Q_D \) and \( \Delta E \) can be made. To
do so though requires a large number of tests to be performed between room temperature and \( \sim 300^\circ C \). Unfortunately in this project insufficient experimental data was collected over this temperature range to allow an accurate determination of \( \Delta E \) and \( Q_d \). Nonetheless this is an interesting consideration for future work.

What the graph of Figure 9.22 does show however is that the Portevin-Le Chatelier effect definitely does influence the mechanical behaviour of type 224 carbon steel at elevated temperatures in common with iron, mild steel and related metals.

10.4 SUMMARY

This Chapter is arguably the most important of the whole thesis for in it we have taken the complete set of results from the project (Chapter 9) and analysed them in considerable detail. The competing effects of adiabatic thermal softening and work hardening in the plastic region have been investigated and quantified. It has been found that adiabatic softening accounts for a substantial reduction in the observe flow stress, particularly at low temperatures and high strain-rates.

The second half of the Chapter (10.2) has dealt with the subject of thermal activation. It has been shown that the material behaviour in the SHPB tests can be explained largely in terms of thermal activation mechanisms. Three such models have been applied to the experimental data starting with the simple rectangular barrier model (1) which is usually applied. It has been found that a modified curved barrier shape (3A+3B) provides the best description of the SHPB results presented here.
CHAPTER 11
METALLOGRAPHY

11.1 INTRODUCTION

The use of optical and electron microscopy plays a very important role in the study of materials mechanics. Such microscopic techniques allow the experimenter to gain a detailed knowledge of the initial microstructure of a material which can then be compared with changes in that structure after or even during the application of a known deformation. In this manner the relationship between mechanical properties and the underlying microscopic mechanisms may be determined. Over recent years the electron microscope has been an invaluable aid in the development, understanding and confirmation of dislocation behaviour in metals.

Many studies in materials mechanics, especially those in which new materials are concerned, place the main emphasis on understanding the microstructure and consequently the use of electron and optical microscopes feature more prominently. In the present investigation into the mechanical properties of 224 steel, the primary aim was to establish these properties accurately over a wide range of conditions and so only a small portion of the project time was dedicated to microscopic analysis. This approach was felt to be justifiable since the steel was not altered from the 'as received' condition in any of the tests performed. Had the effect of grain size on the mechanical properties been investigated, then greater use of optical microscopy would have been required.

Nevertheless, this chapter presents photographs taken from both optical and electron microscopes which give a pictorial insight into
FIG 11.1 Showing orientations in which two halves of each specimen were mounted in a resin block before examination under an optical microscope.
L is the view from the top of the sample
T is the transverse section through the sample.
Fig 11.2 Micrographs of sample RA(1,1,3)
10x5mm untested
Fig 11.3 Micrographs of sample RA(2,5,1) 10x5mm

after quasistatic testing at a strain-rate of $10^{-1}$s$^{-1}$
Fig 11.4 Micrographs of sample PA(2,3,1) 10x5mm

SHPB tested at 26ms$^{-1}$
Fig 11.5 Micrographs of sample RA(2,1,1) 10x5mm

SHPB tested at 37ms⁻¹
Fig 11.6 Micrographs of sample PA(5,2,2) 8x4mm
Untested
Fig 11.7 Micrographs of sample RA(6,3,2) 8x4mm
Tested Quasistatically

$\varepsilon \sim 10^{3.1}$
Fig 11.8 Micrographs of sample RA(2,1,2) 8x4mm
Tested on SHPB at 25m\textper秒^-1
Fig 11.9 Micrographs of sample PA(3,1,2) 8x4mm
Tested on SHPB at 36ms$^{-1}$
the microstructure of 224 carbon steel. They reveal a field in which a considerable amount of further work could have been carried out had more time been available.

11.2 OPTICAL MICROSCOPY

All the optical microscopy in this project was undertaken by a local metallurgy firm, Charnwood Consultants Limited. Four 8 x 4 mm and four 10 x 5 mm cylindrical samples were sent to them for analysis. In both groups of four the conditions of the samples were untested, quasistatically tested (\(\dot{\varepsilon} = 10^{-3} \text{ s}^{-1}\)), SHPB tested at 25 ms\(^{-1}\) impact speed and SHPB tested at 36 ms\(^{-1}\) impact speed. At Charnwood Consultants each sample was cut in half and the two halves were then mounted in an epoxy resin block, in two perpendicular orientations as illustrated in Figure 11.1.

After mounting the specimens were polished and etched. Micrographs of the etched sample surfaces at a magnification of x100 are shown in Figures 11.2 to 11.5 and Figures 11.6 to 11.9 for the 10 x 5 mm and 8 x 4 mm specimens respectively.

One thing which is apparent upon examination of the micrographs is that there is little variation in the grain structure throughout the whole series which embodies a vast range of testing conditions as well as two orthogonal orientations.

The sketch of Figure 11.10 illustrates the typical grain structure seen in the micrographs of Figures 11.2 to 11.9. The areas of white are pure ferrite grains which are usually surrounded along their boundaries by the darker material which is pearlite (ferrite plus 0.8% C in the form of Fe\(_3\)C) and cementite (Fe\(_3\)C). The 'greyer' grains indicated by diagonal shading in Figure 11.10 may be all pearlite else
FIG 11.9A Sketch of typical grain structure as seen in the micrograph of FIG 11.9.
the greyness could be due to etching effects. For type 224 steel with a carbon content of 0.18% the pearlite content should amount to 23%. The fact that on most of the micrographs the dark pearlite area is roughly equal to the white ferrite area is an indication of over-etching of the samples.

The ferrite grain size analysis made by Charnwood Consultants is given in Table 11.1.

<table>
<thead>
<tr>
<th>Specimen</th>
<th>Grain Size</th>
<th>ASTM No.</th>
<th>Specimen</th>
<th>Grain Size</th>
<th>ASTM No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>RA (1,1,3) L</td>
<td>4 equi-axed</td>
<td></td>
<td>RA (5,2,2) L</td>
<td>3 equi-axed</td>
<td></td>
</tr>
<tr>
<td>Untested T</td>
<td>5 banded</td>
<td></td>
<td>Untested T</td>
<td>4 banded</td>
<td></td>
</tr>
<tr>
<td>RA (2,5,1) L</td>
<td>5 banded</td>
<td></td>
<td>RA (6,3,2) L</td>
<td>3 equi-axed</td>
<td></td>
</tr>
<tr>
<td>Quasistatic T</td>
<td>4 banded</td>
<td></td>
<td>Quasistatic T</td>
<td>4 banded</td>
<td></td>
</tr>
<tr>
<td>PA (2,3,1) L SHPB 26 ms⁻¹ T</td>
<td>4 equi-axed</td>
<td></td>
<td>RA (2,1,2) L SHPB 25 ms⁻¹ T</td>
<td>4 equi-axed</td>
<td></td>
</tr>
<tr>
<td>RA (2,1,1) L SHPB 37 ms⁻¹ T</td>
<td>6 equi-axed</td>
<td></td>
<td>PA (3,1,2) L SHPB 36 ms⁻¹ T</td>
<td>4 equi-axed</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 11.1: GRAIN SIZE ANALYSIS CONDUCTED BY CHARNWOOD CONSULTANTS LIMITED ON 8 TYPE 224 STEEL SPECIMENS

The ASTM numbers, N for grain size, correspond to a certain number of grains per square inch, n, in accordance with the relationship:

\[ n = 2^{N-1} \]  

(11.1)

The ASTM numbers in Table 11.1 have been estimated for the most frequently occurring grain size in each sample. The range of grain sizes in any one sample is approximately ±1 ASTM. There is not much grain size variation in the complete set of micrographs, Figures 11.2 to 11.9, and it would be reasonable to describe the batch as a whole
as "ASTM 3 to 5". Table 11.2 presents figures for grain size diameter, d, for the range of ASTM numbers observed in this batch of specimens.

<table>
<thead>
<tr>
<th>ASTM</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grains/in²</td>
<td>4</td>
<td>8</td>
<td>16</td>
<td>32</td>
</tr>
<tr>
<td>Grain diameter (µm)</td>
<td>140</td>
<td>100</td>
<td>70</td>
<td>50</td>
</tr>
</tbody>
</table>

**Table 11.2:** SHOWING RELATIONSHIP BETWEEN ASTM NUMBER, NUMBER OF GRAINS PER SQUARE INCH AND GRAIN DIAMETER

On some of the micrographs of Figures 11.2 to 11.9 and especially on those for high rate deformation (Figures 11.5 and 11.9) thin needle like structures or laths about 5 to 10 µm wide are observable. It is possible that these may be growths of martensite or bainite formed during the normalisation process during the steel's manufacture. More likely, however, they represent the formation of "twins" during the deformation tests. "Twinning" is usually prevalent at high strain rates and thus it is not surprising to find evidence for the effect in the micrographs of Figures 11.5 and 11.9 for which the samples were compressed at the highest projectile velocity in the SHPB system. Apart from this indication of "twinning" there are no signs that the various rates of deformation of the samples have altered the microstructure of the steel from its original non heat treated untested state (Figures 11.2 and 11.6).

11.3 TRANSMISSION ELECTRON MICROSCOPY (TEM)

11.3.1 Sample Preparation

A microstructural examination was undertaken in the electron microscope unit of the University's Institute of Polymer Technology and Materials Engineering (IPTME). Four 10 x 5 mm specimens were
chosen for the purpose, each one representing one of the following conditions: untested, quasistatically tested ($\dot{\varepsilon} = 10^{-3} \text{ s}^{-1}$), SHPB tested at an impact speed of 25 ms$^{-1}$ ($\dot{\varepsilon} = 1000 \text{ s}^{-1}$) and SHPB tested at 37.5 ms$^{-1}$ ($\dot{\varepsilon} = 2000 \text{ s}^{-1}$). From these samples 0.1 mm thick discs of 3 mm diameter were cut out by electrical discharge machining ('spark erosion') along transverse planes through each cylindrical sample, parallel to its central axis. The discs were then electropolished at 50V with a circulating sulphuric acid-ethanol electrolyte in a twin jet electropolisher at room temperature. Polishing was stopped when a hole first appeared in the disc as detected by transparency to light. In this way foils of thickness about 1000 Å were formed around the holes which were then observed in a TEM operated at 100 kV.

11.3.2 Electron Micrographs

Using a transmission electron microscope it is often possible to measure dislocation densities, $\rho$ and the Burgers vector, $\mathbf{b}$ in metals. To do this necessitates taking pictures of the same area of foil at three different orientations. However when this was attempted with the type 224 steel foils it was found that the magnetic nature of the steel interacted with the electron beam so that clear pictures of the foil were possible at ONE orientation only. Hence it was not possible to determine $\mathbf{b}$ and $\rho$ here. The way round this problem which is common to all ferritic steels is to build small copper frames which replace most of the steel in the 3 mm diameter disc apart from the thin material at the centre. Since the amount of magnetic matter is substantially reduced by this construction, the interference with the electron beam is lowered so that clear pictures at 3 orientations may be found.

Unfortunately, the process of mounting foils on copper frames can be time consuming and insufficient time was available to do this during the project. This meant that a quantitative analysis of dislocation
Fig 11.10 Electron micrograph x208,000 magnification showing complex tangle of edge & screw dislocations in a mechanically untested specimen.
Fig 11.11 Electron Micrograph x271,000 magnification in an Instron tested specimen. The picture shows the interaction of screw dislocations (thin straight lines) with an obstacle (dark diagonal shadow).
Fig 11.12 Electron micrograph x271,000 magnification in a SHPB tested sample at 25ms⁻¹. The picture shows screw dislocations on the 112 plane, slip direction<111>. 
Fig 11.13 Electron Micrograph x300,000 magnification from a sample SHPB tested at impact speed of 37.5 ms\(^{-1}\). The picture shows dislocations on the 112 plane, slip direction <111>.
Fig 11.14 Electron Micrograph x 100,000 magnification on sample SHPB tested at 37.5ms. The picture shows twinning
densities for the four different deformation conditions of the specimens was not possible. Nevertheless the micrographs presented in this section do give a qualitative view of dislocations. The area shown in each micrograph is of the order of one square micron that is about 0.1% of a single grain area.

Figure 11.10 shows a complex tangle of screw and edge dislocations observed in the untested specimen. These are forest or primary dislocations. Such tangled masses of dislocations were seen over large areas of the specimens including those which had been plastically deformed. It was not possible to tell whether the density of such tangles increased with increasing deformation rate.

Figure 11.11 shows screw dislocations (thin straight lines) interacting with an obstacle (dark diagonal shadow) which may be a precipitate at a grain boundary.

Figures 11.12 and 11.13 show regular arrays of screw dislocations in the \{112\} plane, slip direction \{111\} in samples SHPB tested at 25 ms\(^{-1}\) and 37.5 ms\(^{-1}\) respectively. These pictures are slightly marred by the 'blotchiness' of the background which is due to rusting which occurred even though the foils were stored in a vacuum.

Figure 11.14 is a relatively lower magnification (x 100,000) micrograph of foils from the specimen SHPB tested at 37.5 ms\(^{-1}\). The picture shows a lath type structure which has been formed by twinning. This was confirmed by X-ray diffraction which showed that the twins had been formed on the \{112\} plane. Only on foils from this specimen which had been deformed at the highest rate (\(
abla \dot{\varepsilon} \approx 2000\) s\(^{-1}\)) was this type of lath structure observed.
11.4 CONCLUDING REMARKS

This Chapter has qualitatively touched upon a subject of great pertinence to the mechanical testing of materials. Under different circumstances, at least half of the project time might have been dedicated to the metallographic investigations described above. It would have been especially interesting to have observed the effect of heat treatment on the microstructure of 224 steel and hence its mechanical properties and also to have determined the amount of dislocation multiplication taking place in high-strain rate tests by accurately measuring dislocation densities. In future SHPB studies at Loughborough it is recommended that the electron microscope facilities in IPTME are exploited to the full as well as their optical microscopes which are capable of x1000 magnifications.
CHAPTER 12
CONCLUSIONS AND RECOMMENDATIONS FOR FURTHER WORK

The work described in this thesis has successfully achieved its principal objective which was to establish the mechanical properties of type 224 carbon manganese steel (BS 1503 - 430 grade) at strain-rates between $10^{-4}$ s$^{-1}$ and 4000 s$^{-1}$ and at temperatures between $-110^\circ$C and $+30^\circ$C. The results have shown that within these temperature and strain-rate ranges there are two discernible regions of mechanical behaviour: an athermal region I (low $\dot{\varepsilon}$, high $T$) in which yield stress and flow stress are relatively insensitive to either strain-rate and temperature, and region II (high $\dot{\varepsilon}$, low $T$) in which the mechanical deformation of the steel is controlled by thermally activated mechanisms. It has been shown that a reasonable fit to data in the thermal activation region is provided by a modified form of the Kock's model (equation 10.27) which incorporates a 'bell' shaped barrier to dislocation motion rather than a rectangular one, which is more often used.

On route to producing the comprehensive set of experimental data in Chapter 9, a number of important achievements have been made. Most notable has been the successful measurement of adiabatic temperature rises in SHPB samples, which can be as high as $+90^\circ$C in the low temperature tests ($-110^\circ$C and $-40^\circ$C). Measured temperature rises within the specimens agree with theoretical expectations assuming that virtually all the plastic work of deformation is converted into heat. From these measured and calculated temperature rises it has been possible to draw up effective 'isothermal' stress versus strain curves for the Hopkinson bar data. When these are compared with the original adiabatic curves it is seen that thermal softening has an appreciable effect on the flow stress.
The project has established the validity of using three materials testing systems (Instron, ESH and SHPB) at Loughborough to determine the mechanical properties of 224 steel and similar materials. Some important aspects of tensile testing have been evaluated - using the SHPB and Instron methods. It has been shown that for the tensile test pieces used here the gauge length decreases with increasing strain above 10% and it was necessary to quantify this change before the records from the tensile tests could be interpreted sensibly. In general it was found that both tensile and compression specimens used throughout the course of the work, although optimally designed for high strain-rate conditions, were not of the optimum design for quasistatic testing. In future work it may prove wise to include a second set of tensile (dimensions complying with BS 18) and compression (2 > 1/d > 1) test pieces which are better suited to low strain-rate testing. The advantages are that tensile test pieces which comply with BS 18 have a nearly constant gauge length which hardly varies throughout a test and compression samples specified above are less sensitive to frictional effects.

A mainly qualitative metallographic examination on tested and untested samples has been carried out. There is great scope for further work in this area which might concentrate on using the transmission electron microscope to measure dislocation densities at various strains and strain-rates. Such experimental studies could be compared with existing theories of dislocation dynamics and could lead to a further refinement of the thermal activation models discussed in Section 10.2. Measurements and structural analysis using the optical microscope are equally important especially when specimens are heat treated before testing. This would also be another worthwhile exercise.
Unfortunately in this experiment it was not possible to obtain quasistatic results at temperatures lower than -40°C due to the high thermal conductivity of the steel platens and machine jaws which held the specimens in place. In future it should be possible to design a cryogenic system capable of sustaining temperatures of -110°C and lower, perhaps, by incorporating low thermal conductivity ceramic platens instead of steel ones.

In future experiments on this type of steel or on other materials use can be made of the computer program described in Chapter 5. The program could be suitably employed in the design of dummy samples between the first maraging steel bar and the 431 bar to shape the incident SHPB pulse so that it has a longer initial rise time and hence produces a lower initial loading rate in the specimen. Furthermore, the program is useful in estimating the time for stress equilibrium to be achieved in SHPB samples of different materials and also could be applied to tensile test samples.

The use of the ESH machine has gone a considerable way in bridging the gap in strain-rate between quasistatic rates of the order $10^{-3}$ s$^{-1}$ and the lowest SHPB rate of approximately 100 s$^{-1}$. However there is still a gap between 100 s$^{-1}$ and the highest achievable rate from the ESH machine of about 4 s$^{-1}$. The strain-rate sensitivity seems to increase within this very range, thus it would be useful to have a testing machine capable of delivering strain-rates from about 1 s$^{-1}$ to 100 s$^{-1}$ and producing final strains in the sample of at least 10%. In the lowest SHPB strain-rate tests ($\sim 100$ s$^{-1}$) the permanent strain in the sample is seldom above 1.5%.

A suitable machine for the task, a drop weight tower, is currently under development in the Physics Department at Loughborough University. The early indications are that this drop weight apparatus
may produce permanent strains of 10 to 20% in the strain-rate region 50 to 300 s\(^{-1}\) for carbon steel samples, but the method has the disadvantage that the strain-rate during the test is not constant.

It would be interesting to conduct further studies into dynamic strain ageing and the Portevin-Le Chatelier effect (serrations) in type 224 steel at low strain-rates, by collecting data at narrower temperature intervals between \(-100^\circ\text{C}\) and \(+400^\circ\text{C}\).

Also of interest would be the measurement of adiabatic temperature rises in specimens of other materials by the thermocouple methods developed in Chapter 8.

It is hoped that the results and discussion presented in this thesis will make a worthwhile contribution to the safe transportation of radioactive materials.
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4015 FOR J=BT-15 TO BT-1
4020 TFP=KP(J)
4022 NEXT
4030 IF TFP \= TP:15
4035 IF TF-INT(TFP) >= 0.5 THEN TFP=INT(TFP)+1
4040 TF=INT(TFP)
4045 PRINT "BASELINE FOR CHANNEL "$C DATA IS "$TF
4046 PRINT "ALTER BASELINE(Y OR N)";AF
4047 IF AF \= "N" THEN 4350
4048 IF AF \= "Y" THEN 4346
4049 PRINT "NEW BASELINE VALUE";TF
4050 FOR J=BT-20 TO FT+20
4055 AF=AF(J) \= TP
4057 NEXT J
4060 PRINT "ANALYSING TRANSMITTED PULSE DATA"";
4071 KK=2 IF CF="T" THEN KK=1
4072 TF=ES*10D5\12(ES12)
4073 FOR J=BT TO FT
4077 J=J-BT
4080 AF=KK(J)
4085 SS(J)=\=CF#TF
4090 NEXT J
4095 PRINT "ANALYSIS COMPLETED"
4096 GOSUB 14000
4097 IF CF="T" THEN ZZ=2
4098 PRINT "CHANNEL "$C DATA INCIDENT & REFLECTED PULSES";
4100 RR=DS(J) \= "A4+4#
4105 RF=RF#\="SEG.READ";
4110 J=1\=ID
4115 GOSUB 10000
4120 PRINT "FINDING START OF INCID PULSE";
4125 CH="INCID" PP=1; B=20
4130 IF CF="T" THEN PP=1
4135 IF AF=1 AND CF="T" THEN PP=1
4140 IF AF=1 AND CF="C" THEN PP=1
4145 IF J=1 AND CF="C" THEN PP=1
4150 IF J=1 AND CF="T" THEN PP=1
4155 PRINT "FINDING START OF REFLE PULSE";
4160 GOSUB 14000
4165 B=\=BP; FR\=PR+FJ
4170 AS=40
4175 GOSUB 2400
4180 PRINT "FIND AVG OF 20 PTS AT END OF REFL PULSE"
4185 FOR J=PR TO FR+13
4190 YES=AF(J)\=ID
4195 NEXT J
4200 AS=20
4205 IF AS \= INT(AS)=0.5 THEN AS=INT(AS)+1
4210 YES=INT(AS)
4215 PRINT "BASELINE FOR CHANNEL "$C DATA IS "$AS
4220 PRINT "ALTER BASELINE(Y OR N)";AF
4225 IF AF \= "N" THEN 4470
4230 IF AF \= "Y" THEN 4457
4235 PRINT "NEW BASELINE VALUE";AS
4240 ENDD=BT
4245 PRINT "REFLECTED PULSE STARTS "ED MICROSECS AFTER TRANS PULSE";
4250 IF DE \= ED THEN 4556
4255 IF DE \= ED THEN 4556
4260 PRINT "THIS DELAY SHOULD BE "DE MICROSECS"
4265 IF DE \= ED THEN 4556
4270 PRINT "DO YOU WANT TO MODIFY START OF REFL PULSE(Y OR N)";AF
4275 IF AF \= "Y" THEN 4556
4280 IF AF \= "Y" THEN 4556
18000 INPUT "WHAT MATERIAL TESTED":MM
18010 INPUT "TEST TEMP.(E.G. 200)":TM
18020 INPUT "ANNEALING TEMP.(E.G. 300)":AN
18030 INPUT "APPROX MAX STRAIN:(E.G. 20)":MC
18040 INPUT "IDENTITY LETTER(E.G. A)":IL
18050 IF IL<"A" THEN 18080
18060 IF IL<"Z" THEN 18080
18070 IF IL<4 THEN 18080
18080 IF IL<2 THEN 18080
18090 IF IL<2 THEN 18080
18100 IF IL<2 THEN 18080
18110 IF IL<2 THEN 18080
18120 IF IL<2 THEN 18080
18130 IF IL<2 THEN 18080
18140 IF IL<2 THEN 18080
18150 IF IL<2 THEN 18080
18160 IF IL<2 THEN 18080
18170 IF IL<2 THEN 18080
18180 IF IL<2 THEN 18080
18190 IF IL<2 THEN 18080
18200 IF IL<2 THEN 18080
18210 IF IL<2 THEN 18080
18220 IF IL<2 THEN 18080
18230 IF IL<2 THEN 18080
18240 IF IL<2 THEN 18080
18250 IF IL<2 THEN 18080
18260 IF IL<2 THEN 18080
18270 IF IL<2 THEN 18080
18280 IF IL<2 THEN 18080
18290 IF IL<2 THEN 18080
18300 IF IL<2 THEN 18080
18310 IF IL<2 THEN 18080
18320 IF IL<2 THEN 18080
18330 IF IL<2 THEN 18080
18340 IF IL<2 THEN 18080
18350 IF IL<2 THEN 18080
18360 IF IL<2 THEN 18080
18370 IF IL<2 THEN 18080
18380 IF IL<2 THEN 18080
18390 IF IL<2 THEN 18080
18400 IF IL<2 THEN 18080
18410 GOTO 4910
20000 REM END OF PROGRAM
52299 DIM UK(1004)
52300 FOR G=1 TO 1000
52301 U(G)=130
52302 NEXT
52303 FOR G=400 TO 440 STEP 2
52304 R01(G)=R01(G-2)-1:R01(G-1)=R01(G)
52305 NEXT
52306 FOR G=440 TO 521 STEP 3
52307 R01(G)=R01(G-3)-1:R01(G-1)=R01(G)+R01(G-2)=R01(G)
52308 NEXT
52309 FOR G=522 TO 532
52310 R01(G)=R01(G-1)+1
52311 NEXT
52312 OPEN 8.8.8."C:TOT02050815821. SEQ.WRITE"
52313 J=1 K=320
52314 LOUSU 3600
52315 CLOSES
52316 PRINT"TRANSFERED DATA TO DISK"
52317 READY.

10 REM GRAPHICS - VERSION 09-10-84 15.40 20 REM MAX VALUES OF X7 SUB 4000
20 REM MAX VALUES OF Y7 SUB 4000 30 REM CALC AXES LIMITS SUB 5000 40 REM DRAW 1 LABEL AXES SUB 3000 50 REM CONV DATA X SUB 7000 60 REM CONV V DATA SUB 8000 70 REM PLOT GRAPH SUS SUB 9000 80 REM READ 50 DATA SUB 10000 95 IF Pm=1 THEN 100
100 IF Gm=1 THEN 200 110 DIM Z(140),X(150) 150 Cw=1 160 SI="TRUE STRESS"; S2="(MPA)"
161 E1= "TRUE STRAIN"; E2= "(R)"; E3="STRAIN GAUGE DATA"; E4="TRUE STRAIN RATE"
162 ES=":V/(SEC)" 163 T1="TIME" T2=":MICROSECS"
200 SYS 2*4096
210 PRINT "GRAPH PLOTTER ACTIVATED"
310 S=1/12.8
320 R=11.87
350 NO=22 Y0=20
390 PRINT "ORIGIN AT X = "NO "MM AND Y = "VO "MM"
400 INPUT "CHANGE THESE COORDS(X OR Y)";A1
424 IF A1="N" THEN 400
425 IF A1=NO THEN 222
426 INPUT "NEW ORIGIN COORD(X, Y)";NO,Y0
500 PRINT "DIRECTORY OF GRAPHS"
550 PRINT 1. STRESS V. STRAIN"
590 PRINT 2. STRESS V. TIME"
580 PRINT 3. STRAIN V. TIME"
560 PRINT 4. STRAIN RATE V. TIME"
570 PRINT 5. STRAIN GAUGE DATA V. TIME"
500 PRINT 6. BAR STRAIN V. TIME"
530 PRINT 7. INCIDENT PULSE V. TIME"
540 PRINT 8. REFLECTED PULSE V. TIME"
550 PRINT 9. TRANSMITTED PULSE V. TIME"
560 PRINT 10. WORK BAR MENU"
570 PRINT "SELECT CODE"
600 INPUT NC1
700 IF NC1=0 OR NC2=10 THEN 350
808 NLONGH:
899 ON NO GOSUB 2200.2300.2400.2500.2250.2250.3000.3100.3200.25000
1000 IF Y0=105 THEN 2000
2000 IF Y0=105 THEN GOSUB 10000
2010 INPUT "DO YOU WANT TO DRAW AXES(Y OR N)"; A1
2011 IF A1="Y" OR A1=11 THEN 2100
2110 GOTO 1010
2120 GOTO 1012
2130 INPUT "DO YOU WANT TO LABEL AXES(Y OR N)"; L1
2140 IF L1="Y" OR L1=11 THEN 2110
2150 GOTO 1013
2160 INPUT "DO YOU WANT TO PRINT TITLE(Y OR N)"; T1
2170 IF T1="Y" OR T1=11 THEN 2100
2180 GOTO 1016
2190 GOTO 1018
3T "XAXIS"
7 REM "XAXIS"
10 INPUT "HOW MANY TIME INTERVALS(t) DO YOU WISH TO CONSIDER ON THE X-AXIS", T
20 REM NEXT BIT WORKS OUT APROPRIATE X-SCALE AND PLOTS IT
30 #RUN H.FLTMATE
40 Pax=1400DIVT
50 S=1400-(1400MODT)
60 MOVE100,0
70 DRAW 100+6,0
80 FOR J=1 TO T
90 MOVE 100+(J*PAX),0
A0 DRAW 100+(J*PAX),10
10 NEXT J
20 VDUS
30 MOVE 1170,40
40 PRINT T"t"
50 #RUN H.DISABLE
60 PRINT"DO YOU WISH TO CONSIDER STEP INPUT?"
70 IF GETS="Y" THEN CHAIN "STEPY"
80 PRINT "NOW THIS BRINGS US ONTO THE CONTINUOUS RISE TIME PROBLEM"
90 CHAIN "PAWJ4"

5T "YAXIS"
15 LIST
20 REM "YAXIS"
30 REM DRAWS THE Y-AXIS
40 #RUN H.FLTMATE
50 MOVE 100,0
60 DRAW 100,1000
70 FOR I=1 TO 10
80 MOVE 100,100*I
90 DRAW 110,100*I
A0 VDUS
B0 MOVE -200,100*I+15
C0 PRINT 0.1*I
D0 NEXT I
E0 #RUN H.PENPARK
F0 #RUN H.DISABLE
G0 PRINT "DO YOU WANT PLOT OUT X-AXIS?"
H0 IF GETS="Y" THEN CHAIN "XAXIS"
I0 PRINT "DO YOU WISH TO CONSIDER STEP INPUT?"
J0 IF GETS="Y" THEN CHAIN "STEPY"
K0 PRINT "NOW THIS BRINGS US ONTO THE CONTINUOUS RISE TIME PROBLEM"
L0 CHAIN "PAWJ1"
150 PRINT "DO YOU WANT A TABLE OF RESULTS?" 
160 IF GET$="Y" THEN 170 
170 INPUT "HOW MANY T DIVISIONS ON X-AXIS?", T 
180 AX=1400DIVT 
190 S=1400-(1400MODT) 
200 CLS 
210 PRINT "DO YOU WISH TO CONSIDER" 
220 PRINT "a)transmitted pulse ONLY" 
230 PRINT "b)reflected pulse" 
240 PRINT "c)both" 
250 PRINT "d)deep rise (T/I+R)" 
260 PRINT "Enter A,B,C or D:"
270 IF GET$="A" THEN 310 
280 IF GET$="B" THEN 350 
290 IF GET$="C" THEN 400 
299 PRINT "DO YOU WANT TO END IT ALL?" 
300 IF GET$="Y" THEN 340 
310 PROCSUM 
320 PRINT "REFLECTED PULSE AS WELL?" 
330 PROCREF 
340 END 
350 DEF PROCINPUT 
360 REM INPUT STARTING PARAMETERS 
370 PRINT "DIAMETER OF BARS (mm)?" 
380 INPUT DI 
390 PRINT "DIAMETER OF SAMPLE (mm)?" 
400 INPUT D2 
410 PRINT "DENSITY OF BARS (g/cm3)?" 
420 INPUT R1 
430 PRINT "DENSITY OF SAMPLE (g/cm3)?" 
440 PRINT "SPEED OF SOUND IN BARS (ms-1)?" 
450 PRINT "SPEED OF SOUND IN SAMPLE (ms-1)?" 
460 INPUT C1 
470 PRINT "SPEED OF SOUND IN SAMPLE (ms-1)?" 
480 INPUT C2 
490 PRINT "SAMPLE LENGTH (mm)?" 
500 INPUT L 
510 ENDSUM 
520 DEF PROCSUMS 
530 REM CALCULATE TRAVERSE TIME OF PULSE IN SACOSPLE AND USE THIS AS A CONVENIENT INTERVAL TIME, TO 
540 T0=(L/C2)*1000 
550 PRINT "THE TRAVERSE TIME, t, FOR A PULSE TO PROPAGATE ACROSS THE SAMPLE IS " 'TO"US" 
560 PRINT "" 
570 PRINT "WE SHALL CONSIDER CHANGES IN THE PULSES AT INTERVALS T APART" 
580 REM WORK OUT R&T COEFFICIENTS 
590 A1=0.25*PI*(L1^2) 
600 A2=0.25*PI*(L2^2) 
610 Z1=RI1*C1 
620 ZZ=RI1*C2
10 REM "STEPY"
20 REM DIM ARRAYS FOR INCIDENT, P: REFLECTED, R & TRANSMITTED, T PULSES
30 MODE 7
40 *DISC
50 HINEM=15000
60 REM DIM ARRAYS FOR INCIDENT, P: REFLECTED, R & TRANSMITTED, T PULSES
70 DIM I(500)
80 DIM TR(500)
90 DIM REF(500)
100 PROCINPUT
110 PROCSUMS
120 PRINT "DO YOU WANT A TABLE OF RESULTS? "
130 IF GET$="Y" THEN 140 ELSE 150
140 PROCTABLE
150 IF GET$="N" THEN 170
160 CHAIN "'AXIS"
170 INPUT "HOW MANY T DIVISIONS ON X-AXIS? " T
180 *TAX=1400DIVT
190 S=1400-(1400MODT)
200 CLS
210 PRINT "DO YOU WISH TO CONSIDER"
220 PRINT "a)transmitted pulse ONLY"
230 PRINT "b)reflected pulse"
240 PRINT "c)both"
250 PRINT "d)equ rise (T/I+R)"
260 PRINT "Enter A,B,C or D:"
270 IF GET$="B" THEN 330
280 IF GET$="D" THEN PROCSEMGPH
290 PRINT "DO YOU WANT TO END IT ALL?"
300 IF GET$="Y" THEN 340
310 PROCGRAPH
320 PRINT "REFLECTED PULSE AS WELL? "
330 PROCREFGRAPH
340 END
350 DEF PROCINPUT
360 REM INPUT STARTING PARAMETERS
370 PRINT "DIAMETER OF BARS (mm)?"
380 INPUT D1
390 PRINT "DIAMETER OF SAMPLE (mm)?"
400 INPUT D2
410 PRINT "DENSITY OF BARS (g/cm³)?"
420 INPUT R1
430 PRINT "DENSITY OF SAMPLE (g/cm³)?"
440 INPUT R2
450 PRINT "SPEED OF SOUND IN BARS (m/s-1)? "
460 INPUT C1
470 PRINT "SPEED OF SOUND IN SAMPLE (m/s-1)? "
480 INPUT C2
490 PRINT "SAMPLE LENGTH (mm)? "
500 INPUT L
510 ENDFRGC
520 DEF PROCSUMS
530 REM CALCULATE TRAVERSE TIME OF PULSE IN SACOSMPE AND USE THIS AS A CONVENIENT INTERVAL TIME. TO
540 TO=(L/C2)*1000
550 PRINT "THE TRAVERSE TIME, T, FOR A PULSE TO PROPAGATE ACROSS THE SAMPLE IS " TO"US"
560 PRINT "" 
570 PRINT "WE SHALL CONSIDER CHANGES IN THE PULSES AT INTERVALS T APART"
580 REM WORK OUT R&T COEFFICIENTS
590 A1=0.25*F1*(D1²)
600 A2=0.25*F1*(D2²)
610 Z1=R1*C1
620 Z2=R2*C2
630 \( \text{W} = (A1*Z1)^*(A2*Z2) \)
640 \( \text{Y} = (2*A1*Z1)/\text{W} \)
650 \( \text{X} = (2*A2*Z1)/\text{W} \)
660 \( \text{F} = (A2*Z2)^*(A1*Z1)/\text{W} \)
670 REM ACTUAL FORMULATION OF THE SERIES REPRESENTING PULSE CHANGES
680 \( \text{REF}(0) = \text{F} \)
690 \( \text{REF}(1) = \text{F} \)
700 \( \text{TR}(1) = \text{Y} * \text{X} \)
710 FOR \( i = 2 \) TO 100
720 REM DISTINGUISH BETWEEN ODD & EVEN I
730 \( \text{E} = i/2 \)
740 IF \( \text{E} < \text{INT} (\text{E}) \) THEN 770
750 \( \text{TR}(i) = \text{TR}(i-1) \)
760 GOTO 880
770 REM CALCULATE TRANSMITTED PULSE ODD POINTS
780 \( \text{U} = i-1 \)
790 REM LETS ELIMINATE ALL FUTILE CALCULATIONS (i.e too small) OF TR(P-U)
800 IF \( i = 3 \) THEN 830
810 IF \( \text{F} > 10^{-4} \) THEN 860
820 IF \( \text{U} > 63 \) THEN 860
830 \( \text{F} = \text{TR}(1)^*(\text{P-U}) \)
840 \( \text{TR}(1) = \text{TR}(1-2) + \text{F} \)
850 GOTO 880
860 \( \text{TR}(i) = \text{TR}(i-2) \)
870 REM REFLECTED POINTS
880 \( \text{REF}(i) = \text{F}*(1-\text{TR}(1-1)) \)
890 IF \( \text{TR}(i) > 0.999 \) THEN 910
900 NEXT I
910 LET \( T = 1 \)
920 ENDPROC
930 DEF PROCetable
940 CLS
950 REM PRINT A TABLE OF RESULTS ON SCREEN AND PRINTER
960 #FX5.1
970 VDU2
980 #FX3.0
990 PRINT TAB(10,10);"HERE WE GO!"
1000 PRINT TAB(7);"TIME";TAB(17);"REFLECTED";TAB(36);"TRANSMITTED"
1010 FOR \( j = 0 \) TO 150
1020 PRINT J;"";TAB(17);"REF(J);TAB(36);TR(J)
1030 IF TR(J) > 0.999 THEN 1060
1040 NEXT J
1050 CLS
1060 PRINT "IS THIS THE END?"
1070 VDU3
1080 ENDPROC
1090 DEF PROCGRAPH
1100 REM LETS SEE IF WE CAN PLOT OUT THE DAMN GRAPH
1110 #RUN H.PLTMATE
1120 PLOT4,1000
1130 PLOTS,100+PAX,0
1140 PLOT5,100+PAX,TR(1)*100
1150 LET \( K = 1 \)
1160 REPEAT
1170 K = K+1
1180 IF \( \text{TR}(K) > \text{TR}(K-1) \) THEN PLOT5,K*PAX+100,TR(K-1)*1000
1190 PLOT5,K*PAX+100,TR(K)*1000
1200 IF \( \text{TR}(K) > 0.999 \) THEN PLOT 5,1500,1000:SDT01220
1210 UNTIL \( K = \text{PAX} \)
1220 #RUN H.PENPARK
1230 #RUN H.DISABLE
1240 ENDPROC
1250 DEF PROCFREGRAPH
1260 REM step plot for reflected pulse
1270 #RUN H.PLTMATE
1280 PLOT4,1000: PLOT5,1000-(REF(0)*1000)
1290 LET P=0
1300 REPEAT
1310 K=K+1
1320 IF REF(K)<REF(K-1) THEN PLOTS,K*PAX+100,-(REF(K-1)*1000)
1330 PLOTS,K*PAX+100,-(REF(K)*1000)
1340 UNTIL K*PAX>1400
1350 *RUN H.PENPARK
1360 *RUN H.DISABLE
1370 DEF PROCEQGRAPH
1380 *RUN H.PLTMAT
1390 MOVE100,0
1400 PLOTS,100+PAX,0
1410 LET K=0
1420 REPEAT
1430 K=K+1
1440 PLOTS,K*PAX+100,TR(K)*500/(1+REF(K))
1450 PLOTS,(K+1)*PAX+100,TR(K)*500/(1+REF(K))
1460 IF TR(K)>0.999 THEN PLOTS,1500,500
1470 IF TR(K)>0.999 THEN END
1480 UNTIL K*PAX>1400
1490 *RUN H.PENPARK
1500 *RUN H.DISABLE
1510 ENDP
530 PRINT "Enter A,B,C or D";
540 IF GET$="D" THEN PROCEDEGRAPH
550 IF GET$="B" THEN 390
560 PROCEDURE
570 PRINT "REFLECTED PULSE ASWELL?"
580 IF GET$="N"THEN 400
590 PROCEDURE
600 PRINT "DO YOU WISH TO CONSIDER"
610 PRINT "a transmitted pulse ONLY"
620 PRINT "b) reflected pulse"
630 PRINT "c) both"
640 PRINT "d) eqn rise (T/I*R)"
650 PRINT "Enter A,B,C or D";
660 A$=GET$;
670 IF GET$="A" AND GET$="B" AND GET$="C" AND GET$="D" THEN 450
680 CLS
690 PROCESTRISE
700 PROCEDURE
710 PRINT "DO YOU WANT TO REPEAT THE PLOT FOR A DIFFERENT RISETIME?"
720 IF GET$="Y" THEN 490
730 IF GET$="N" THEN 520
740 END
750 DEF PROCESTRISE
760 REM LETS CONSIDER A LINEAR RISETIME AT THE START OF THE INCIDENT PULSE
770 LOCAL H,J,K,G,F
780 INPUT "WHAT IS THE RISETIME OF THE PULSE?","T2"
790 LET T4=T2/500
800 REM T4 THEN IS THE TIME RESOLUTION
810 REM SET UP INCIDENT PULSE DATA
820 FOR K=1 TO 500
830 I(K)=K/500
840 NEXT K
850 REM TO TRAVERSE TIME, T SO THAT NO OF TIME DIVISIONS IN T IS:
860 N=INT(T0/T4)
870 REM FOR NEXT LOOP SETS TR TO ZERO FOR TIMES BETWEEN 0 AND IT
880 FOR H=0 TO N-1
890 TR(H)=0
900 IF A$="A" THEN 730
910 REF(H)=M0(H)
920 NEXT H
930 REM REPEAT LOOP GENERATES TR IN THE TIME BETWEEN T AND IT
940 H=N-1
950 REM
960 H=H+1
970 TR(H)=TR(H-1)+M0(H)
980 IF H=500 THEN 960
990 REM NESTED FOR NEXT LOOP CALCULATES TR IN THE REMAINING TIME, i.e., T0-T2-T1;
1000 FOR H=500 TO 500
1010 TR(H)=TR(H-1)+M0(H)
1020 IF H=H/0.001 THEN 920
1030 IF P$="G" THEN 920
1040 IF A$="A" THEN 920
1050 NEXT G
1060 REM NEXT H
1070 REF(H)=M0(H)-TR(H-1)
1080 PRINT "PRINT REF(H)"
1090 NEXT H
1100 REM IF HOPE IT WORKS!
1110 REM TABULATE THE ABOVE CALCULATIONS
1000 PRINT "DO YOU WANT A TABLE OF VALUES OF THE TRANSMITTED PULSE AT 1us INTERVALS?"
990 IF GET$="N" THEN 1100
1000 IF GET$="Y" THEN GOTO 980
1010 N=INT(1/T4)
1020 REM ABOVE CORRESPONDS TO NO. OF TIME INTERVALS IN 1us
1030 REM CONVERTS TR TO T() AT SUITABLE 1us INTERVALS TO USE IN PROCTABLE
1040 FOR J=0 TO 10
1050 T(J)=TR(J*10)
1060 R(J)=REF(J*10)
1070 NEXT J
1080 T(11)=1
1090 PROCTABLE
1100 ENDFRC
1110 DEF PROCRISEREGRA
1120 RUN H,PLTMATE
1130 REM T HAS BEEN DEFINED EARLIER AS THE TOTAL NO. OF TIME DIVISIONS ON THE X
1140 REM TOTAL TIME ON X-AXIS IS;T*10
1150 REM T4=TIME RESOLUTION
1160 REM HENCE FRACTION OF DISTANCE TO BE PLOTTED FOR EACH TIME RES/DIV IS;
1170 FR=(T4/(T(T0))*S
1180 PRINT "YES"
1190 MOVE100,0
1200 IF A$="B" THEN 1300
1210 IF A$="D" THEN 1360
1220 FOR K=0 TO 500
1230 REM NEXT BIT IS A ROUNDED FRIG SO THAT WE PLOT TO NEAREST POINT
1240 LET FRIG=FRIG(100
1250 IF FRIG-INT(FRIG)>0.5 THEN FRIG=1+INT(FRIG) ELSE FRIG=INT(FRIG)
1260 PLOTS,FRIG,TR(K)*1000
1270 NEXT K
1280 IF A$="A" THEN 1420
1290 MOVE100,0

1300 FOR K=0 TO 500
1310 LET FRIG=FRIG+100
1320 IF FRIG-INT(FRIG)>0.5 THEN FRIG=1+INT(FRIG) ELSE FRIG=INT(FRIG)
1330 PLOTS,FRIG,TR(K)*1000
1340 NEXT K
1350 IF A$="D" THEN 1420
1360 MOVE100,0
1370 FOR K=1 TO 500
1380 LET FRIG=FRIG+100
1390 IF FRIG-INT(FRIG)>0.5 THEN FRIG=1+INT(FRIG) ELSE FRIG=INT(FRIG)
1400 REM above calculates the eqn curve
1410 PLOTS,FRIG,(TR(K)*1000)/(1(K)*REF(K))
1420 NEXT K
1430 RUN H,PENPARK
1440 RUN H,DISABLE
1450 ENDFRC
1460 DEF PROCREISTRIP
1470 RUN H,PLTMATE
1480 REM moves pen to position left after PROCREISTRIP
1490 MOVE FRIG,TR(500)*1000
1500 LET FRIGIT=FRIG
1510 L=0
1520 REM plots first N points after initial rise;thereafter I=I
1530 REPEAT
1540 L=L+1
1550 I(L)=XY!!(500-N*L)+(P-2)*TR(500-(2*N)+L)
1560 IF A$="A" THEN REF(L)=P(1-TR(500-N)L)
1570 PRINT TR(500-(2*N)+L)+"",I(L)
1580 TR(L)=1(L)
1590 IF A$="B" THEN 1550
1600 IF A$="D" THEN 1550
1610 LET FRIG=FRIGIT+(FF*L)
1620 PRINT FRIG
2290 NEXT K
2300 IF A="A" THEN 2460
2310 IF A="D" THEN 2390
2320 MOVE FRIGIT,-(REF(1)*1000)
2330 FOR K=1 TO 2*N
2340 LET FRIG=FRIGIT+FRIT
2350 IF FRIG-INT(FRIG)>0.5 THEN FRIG=1+INT(FRIG) ELSE FRIG=FRIG
2360 PLOTS,FRIG,-(REF(K)*1000)
2370 NEXT
2380 IF A<>"D" THEN 3328
2390 MOVE FRIGIT,(TR(1)*1000)/(1+REF(I))
2400 REM t/r plot again
2410 FOR K=1 TO 2*N
2420 LET FRIG=FRIGIT+FRIT
2430 IF FRIG-INT(FRIG)>0.5 THEN FRIG=1+INT(FRIG) ELSE FRIG=FRIG
2440 PLOTS,FRIG,(TR(K)*1000)/(1+REF(K))
2450 NEXT
2460 UNTIL FRIG=1499
2470 *RUN H.FENPARK
2480 *RUN H.DISABLE
2490 ENDPROC