Reconfigurable and closely coupled frequency selective surfaces

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RECONFIGURABLE AND CLOSELY COUPLED FREQUENCY SELECTIVE SURFACES

by

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A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy by Loughborough University

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To my Mother and my late Father
ABSTRACT

The performance of a planar Frequency Selective Surface (FSS) cannot be changed or adapted once the manufacturing process has been completed. In practice, however, it would be advantageous to be able to do so, in order to increase flexibility of performance in multiband systems for example. This thesis examines a novel electromagnetic technique that has been developed, whereby the frequency and/or the angular response of FSS's can be tuned in situ over a wide range of frequencies and/or steering angles. The technique employed is passive and relies upon the displacement of closely separated (and therefore closely coupled) arrays with respect to each other. A global loading of the array results so that the reconfigurable FSS (RFSS) will produce a broadband and/or multibeam response without altering the individual array design.

The experience and understanding gained during this work was subsequently used to produce FSS responses of extreme angular stability. In this case a static, double layer structure has been used to make use of the high coupling between the layers i.e. two FSS's printed on a single dielectric substrate to form a close coupled FSS (CCFSS). It was found that the coupling between the two layers was highly dependent on the relative displacement between arrays. This displacement is introduced statically during the manufacture of the FSS. The cases described use two identical layers. A further development of this concept makes use of complementary conducting and aperture elements giving rise to a complementary FSS (CFSS). The CFSS is also manufactured on a common dielectric and produces ultra stable resonant frequencies for both TE and TM oblique incidences.

Theoretical verification of the measured results has been achieved, and the measured and predicted results agreed very closely. Modal analysis, using a novel coupled integral equation technique, has been used to predict the response of the RFSS and CFSS. The correlation between the predicted and measured transmission response of the RFSS was very good and it was discovered that operational stability of the bandwidths and band spacing ratios were significantly improved over conventional static FSS.
PUBLICATIONS FROM THE RESEARCH


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CHAPTER 1

Introduction to Reconfigurable Frequency Selective Surfaces

1.1 Introduction to Frequency Selective Surfaces

Frequency selective surfaces (FSS) are, in basic form, passive microwave filters. They consist of arrays of thin metallic [1,2] or aperture [3] elements contained within periodically distributed unit cells. The elements can have many forms depending on the particular application and performance required. Examples of elements used for FSS include rings [4], square loops [5], tripoles [6], Jerusalem cross [7] and cross dipoles [8]. The nature of the array elements, be they conductors supported by a dielectric substrate [2], or apertures within a metallic sheet, largely determines the resonant properties of the FSS (also referred to as dichroic surfaces). In the former case the array is capacitive and at resonance the FSS is highly reflective, whereas the latter case produces an inductive effect.
Fig. 1.1. Transmission response of an FSS array of conducting elements

Fig. 1.2. Transmission response of an FSS array of aperture elements
and the FSS is very highly transparent at the resonant frequency. Example transmission responses of the two types of FSS are shown in Figs. 1.1 and 1.2. The difference between the stopband and passband characteristics can be clearly seen. In each case $f_r$ represents the resonant frequency and is determined primarily by the dimensions of the element used. Some examples of FSS elements are shown in Fig. 1.3.

Fig. 1.3. Examples of FSS elements

The primary applications for FSS are in antenna systems, be they fixed or mobile, for example [8,9]. The ability of an FSS to allow multiband transmission from a single, fixed system was widely used in satellite applications [10,11], and curved FSS have been considered for use in radome technology, for example, [12,13]. In recent years the general
subject area of FSS has expanded to such an extent that two books have recently been published on the subject [14,15].

The work presented in this thesis is concerned with the combination of two FSS in close proximity and the transmission responses of such structures. Previously, multilayer FSS have been studied in order to produce a multiband response for specific applications at fixed frequencies. There are certain limitations associated with this approach insofar as the frequencies of operation cannot be changed for a particular FSS, and the necessary stability of the response with the angle of incidence of plane wave illumination is difficult to achieve. This thesis addresses these problems in two ways. Firstly, a reconfigurable FSS (RFSS) is introduced whereby the frequency response of a two layer FSS structure is tuned over a certain frequency range, as shown in Fig. 1.4.

![Fig. 1.4 Tuning range of RFSS with conducting elements](image-url)
The RFSS has an upper and lower frequency limit, $f_u$ and $f_l$ respectively, which will be largely defined by the element and lattice dimensions, and the separation between the layers. Theoretically, any frequency between these limits should be obtainable as the resonant frequency of the RFSS. This is achieved by introducing a variable, lateral relative displacement between the two layers, thereby altering the coupling between them [16]. The coupling developed between the layers using this dynamic system can be maximised for any displacement by permanently aligning the layers during the manufacturing process. This structure has been called a close coupled FSS (CCFSS). Although this then becomes a static system, the FSS exhibits a very stable frequency response [17,18]. The RFSS and CCFSS can be implemented with either conducting or aperture elements in each layer, depending on whether one wishes to have a stopband or passband at resonance. A development on this theme is a variation to the CCFSS, in which a layer of conductors and a layer of apertures are placed in close proximity to one another giving a complementary FSS (CFSS).

1.2 Organisation of the Thesis

This thesis is comprised of five chapters with the main thrust of the work described in Chapters 2, 3, and 4.

Chapter 1 acts as an introduction to the work carried out in the body of the thesis. Initially some brief background comments are made regarding the overall subject of FSS, although for a fuller description one could, for example refer to [14]. The area of multilayer FSS is introduced thereby giving an outline of the motivation behind the thesis, and the aims are set out. Also included in Chapter 1 is a description of the concept of the RFSS, which gives an outline of the principles involved.

Chapter 2 considers multilayer FSS using conducting elements as both RFSS and CCFSS. The underlying theory of the modal analysis used to predict the FSS behaviour is detailed in order to provide a sound understanding of the theory of the specific cases considered in the thesis. A mathematical model is developed which allows for the prediction of the transmission properties of both RFSS and CCFSS combining two distinct arrays, each
backed by a supporting dielectric substrate. The model also includes the capability to include a relative lateral displacement between the arrays, by which the coupling between them can be varied, and hence the transmission response is varied also. Comparisons between measured and predicted results are presented for representative dipole array designs and the performance of the arrays in terms of tuning ranges and stability with respect to the angle of incidence of the incident plane wave radiation is discussed.

Chapter 3 describes the use of two layers of aperture elements as multilayer FSS. A model is developed whereby two layers of apertures can be analysed in both RFSS and CCFSS configurations in terms of their transmission response. Results are presented for various dipole arrays and comparisons between measured and predicted values are drawn.

Chapter 4 covers the Complementary FSS (CFSS) part of the thesis, where the combination of a layer of apertures and a layer of conductors in close proximity is analysed in the CCFSS configuration. Comparisons between measured and predicted results are presented for both dipole and ring arrays and discussed in terms of their angular stability at highly oblique angles of incidence.

Finally, Chapter 5 draws conclusions from the work presented in the thesis and analyses the results as a whole. Considerations are made as to future developments and improvements that could be implemented.

An appendix is included at the end that contains some information about piezoelectric materials that were considered as a means to achieving a RFSS.

### 1.2 Concept of Reconfigurable Frequency Selective Surfaces

The initial period of research investigated the potential materials and mechanisms that could be used to implement a passive RFSS [19]. Active RFSS which involve, for example, the use of diodes placed between the array elements [20] are only practical for microwave devices that require small surface areas. The reasons for this are due to cost and reliability, both of which stem from the engineering complexity involved in having so many small, active devices grouped so closely together.
The RFSS concept involves the passive reconfiguring of the plane wave transmission response of a two layer FSS. A novel technique is employed whereby the frequency response of an FSS can be adjusted over a wide range by altering the relative displacement between two closely coupled FSS arrays. This is shown in Fig. 1.5.

Dipoles are depicted, but any element could be used in the RFSS. The arrays are separated by a distance $S$, and are displaced relative to one another by a distance $DSY$, along the $y$-axis. When $DSY$ is zero, the effect on an incident plane wave is essentially that of a single layer FSS. As $DSY$ is introduced and then increased, the coupling between the arrays is altered and changes the overall transmission response of the structure. As a result it is possible to tune the resonant frequency over a given range, where the frequency shift is proportional to the relative displacement. In order that significant tuning can be achieved it
is important to consider the separation distance, $S$, between the arrays. Theoretical and experimental data has shown that this needs to be of the order of 0.03 wavelengths or less.

A passive RFSS has many applications, typically where the need arises for variable transmission characteristics over large surface areas which becomes impractical and costly with active RFSS designs. These include tuneable absorbers for use in anechoic chambers and other applications where tuneable filtering characteristics are desired. A passive RFSS could be used in a beam scanning application where variable geometry elements on one surface are moved relative to fixed geometry elements on a second surface so that the direction of the beam is variable, so behaving in the same manner as a phased array. An RFSS could be used in a radome application where it could be synchronised to the PRF (pulse repetition frequency) of the radar system.
REFERENCES


CHAPTER 2

Two Layer Frequency Selective Surfaces With Conducting Elements

2.1 Introduction

This chapter examines the properties of two layer FSS with conducting elements on each layer. Conducting elements have been widely studied as FSS elements both in single and double layer situations [1-4] although the work in this chapter limits itself to linear dipole elements only. A modal analysis method is used to predict the behaviour of the two layer structure and whilst the theory has been developed elsewhere [5], the salient features are included here as they provide the essential background to the work in chapters 3 and 4. The theory developed is used for both the Close Coupled FSS (CCFSS) and Reconfigurable FSS (RFSS).

A model has been developed such that predictions can be made of the transmission properties of two layers of conducting dipoles in one of two configurations. In the first case
each array is printed onto a separate dielectric substrate layer. One array is fixed while the second is free to move relative to it. The coupling achieved between the elements on the different arrays is influenced by the relative displacement of the arrays. By manipulating the displacement, the stopband of the FSS can be dynamically tuned over a wide frequency range (around 50%) and effectively deliver reconfigurable FSS (RFSS) performance. The second scenario involves the arrays being printed on either side of a single dielectric sheet. In this way the coupling between layers is maximised, leading to a close coupled FSS (CCFSS), although obviously there is no facility to tune the response in this static situation. However, it has been found that the sensitivity of the resonant frequency to the angle of incidence can be markedly reduced for angles of incidence up to 45°.

Section 2.2 gives details of the RFSS design with attention paid to the techniques used to provide the relative shift between the layers. The process of etching the arrays will also be addressed in this section, as it is important to understand the limitations in the manufacturing phase, and design considerations will be discussed. Section 2.3 details the theory involved in the modelling of the RFSS. The Floquet modal representation of the fields is introduced as the basis for all the theory in chapters 3 and 4 and two, coupled, electric field integral equations (EFIEs) are formed in terms of the unknown currents on the elements. These are solved using the method of moments [5-6] to obtain the current coefficients from which the transmission response can be predicted. Section 2.4 presents a range of measurement results to firstly verify the model and secondly to explore the properties of both the RFSS and CCFSS in terms of tuning capabilities and stability. Finally in this chapter, section 2.5 draws conclusions from the work presented.

2.2 RFSS Concept and Design

An RFSS was initially proposed in a UK patent application in 1990 [7]. The underlying principle is the dynamic tuning of the frequency response of a multi-layer FSS array such that the stopband or passband can be adjusted over a range of frequencies. Whilst there has been much work published on the theory of multi-layer FSS [3,4,8-10], previously this had been in a static situation and largely involved the study of the effect of the separation between arrays and surrounding dielectric support. Also this work uses a novel, coupled integral equation method to solve for the unknown element currents and hence determine
the transmission properties of the structures. The RFSS, on the other hand, benefits from being adaptable and tuneable over a given range of frequencies determined primarily by the array geometries. The system is mechanically simple because the electromagnetic performance of the RFSS relies only on the interaction between passive dipole elements, whilst still allowing both tuning the resonant frequency and switching on and off of the stopband/passband response. Other schemes have been proposed whereby the switching on and off of the stopband of an FSS is achieved by the integration of active diodes into the unit cells [11]. It has been proposed that the array is printed on a ferrite substrate which is then biased to effect some tuning [12], and even liquids have been proposed as exchangeable substrates [13]. There has also been attention paid to a reconfigurable reflector antenna [14] whereby the beam pattern can be adjusted by altering the shape of a mesh reflector. The scheme described in this thesis has advantages over those mentioned in that there is only one moving part, being the array that is displaced relative to the fixed one. The displacement is achieved by means of a stepping linear actuator and is described in section 2.2.2.

Figs. 2.1 and 2.2 show schematically the principle of the RFSS and CCFSS. Fig. 2.1 shows the arrangement of the two layers of conducting dipole elements for the RFSS. Each array is printed on a thin dielectric substrate, which acts to support the individual elements. The arrays are separated by an air-gap, $S$, which is exaggerated in this instance for clarity, although in practice the arrays are in close proximity to enhance the coupling between them. Whilst the arrays need not be identical, they are in this case both in terms of the element dimensions and, more significantly, the lattice parameters, where complications can arise if non identical lattices are used [15]. The relative displacement between the arrays is shown to be along the length of the elements. The displacement produces a perturbation of the high fields that exist between the arrays, resulting in a global loading such that an RFSS is achieved without the need to change the arrays themselves. Fig 2.2 shows a schematic of a CCFSS consisting of an array of linear dipoles printed each side of a dielectric substrate. This arrangement allows for maximum coupling between the arrays for any desired displacement, which is set during the manufacturing process. The manufacturing process and experimental set up are described in the following sections.
Dielectric Layers

Conducting Dipole Elements

Relative Displacement

$S$

Fig. 2.1. RFSS Using Conducting Element Arrays
Fig. 2.2. CCFSS Using Conducting Element Arrays
2.2.1 Array Manufacture

The fundamental building blocks of this thesis are the arrays themselves. The manufacturing of the arrays is an issue which should not be ignored, as it is important to realise the limitations of the process in terms of element definition and alignments between two layers (for CCFSS).

The FSS arrays considered throughout this thesis are manufactured in the same way using a chemical etching technique. The initial material is a laminate of copper, adhesive and dielectric, which can be either polyester or Kapton (which has a lower loss tangent). The copper is 1/4oz. which has a thickness of 9μm and the dielectric comes in a variety of thicknesses from 21μm to 100μm for the single copper layer and 70μm for the double sided copper laminate.

For a single layer array the laminate must be mounted on a rigid board using a polimide tape backed by a silicon thermosetting adhesive. It must be fixed copper side up and sealed on all four sides to prevent water and chemicals leaking under. The surface of the copper is cleaned using a scrub cleaner; a process that removes any oxide deposits and creates slight abrasion to promote improved adhesion for the dry film photo resist. When dry, the laminate is placed in an oven pre-heated to 80°C prior to laminating with the photo resist. Once at 80°C the copper is laminated with a negative dry film resist using a hot roll laminator. A photographic artwork of the desired array design is then placed on top of the photo resist and exposed to ultra violet light. The artwork is removed and the photo resist can be developed in a spray developer using a 1% solution of sodium carbonate. The final process involves the etching away of the unwanted copper in a spray etching machine using a solution of ammonium persulphate, and the removal of the remaining dry film resist in a dry film stripping solution.

For the manufacture of a double layer FSS the same process is followed except that two photographic artworks are produced and pre-aligned with the appropriate displacements. Great care is needed during this alignment procedure to ensure the double layer array will perform as expected.
2.2.2 Experimental set up

The plane wave measurements presented in this thesis have been performed in an indoor anechoic chamber. Standard gain pyramidal horns have been used for transmitting and receiving over three bands: J-band (12-18GHz), K-band (18-26GHz) and Q-band (26-40GHz). In all cases the FSS is in the far field of the horns used such that $r > \frac{2D^2}{\lambda}$, where $r$ is the distance between the horn and the FSS, $D$ is the largest dimension of the horn aperture and $\lambda$ is the wavelength of the incident field. The measurement equipment consists of a HP 8758A scalar network analyser, HP 8350B sweep oscillator and HP 83596A RF plug-in (2.4-40GHz). The equipment is partially computer controlled via an HPIB link so that the measured data can be electronically stored.

An experimental prototype has been built that uses a computer controlled micropositioning test-bed for producing an integrated RFSS structure. Fig 2.3 shows a representation of the experimental set-up used to control the displacements of the RFSS. It depicts the two arrays being supported by their own individual support frames. These frames are necessary to achieve maximum flatness of each array and to ensure that the air-gap between them is kept as constant as possible over the entire area. Each array measures 280mm x 280mm and is produced by the process detailed in section 2.2.1. It is important to note that because of the thickness of the support frames (5mm) the front and rear arrays are mounted, respectively, on the backward facing and forward facing planes of their support frames. In practice the two supports would, themselves, be fixed to an overall support frame, which is removed here for clarity. The rear array is fixed and cannot move, and is also attached to the stepping linear actuator, which provides the necessary movement between the two arrays.

The actuator, from RS, consists of four 12Vdc windings and a permanent magnet rotor construction and is designed for unipolar operation. It is driven, via computer control, by a RS four phase unipolar stepper motor drive board. The step increment is 25µm but this can be improved, if necessary, to 12.5µm by taking advantage of the half step facility. In practice a 25µm step size was found to be sufficient. With the actuator fixed, its rotational output is transferred to the lead screw. However, with the lead screw also fixed to the front
array, the ultimate movement is a linear displacement of the front array, in 25μm steps. The computer control allows the movement to be defined as a particular distance or in single steps, both forwards and backwards.

Fig. 2.3. Schematic representation of RFSS micropositioning jig.
Fig. 2.4 depicts a unit cell of the two layer array and shows the displacements in the perpendicular $x$ and $y$ directions. The underlying principle behind the RFSS is the ability to
displace one layer with relation to another. When the separation distance, $S$, is included a three dimensional unit cell can be produced as in Fig. 2.4. The relative displacements in $x$ and $y$ are $DSX$ and $DSY$ respectively, and the introduction of these parameters from the case when there is no displacement will alter the performance of the array as a whole. Altering the separation distance will also alter the response of the RFSS and for practical considerations it must be kept as uniform as possible and this is most critical near the centre of the structure. The separation region is air in the cases studied here, and the front and back layers identified in Fig. 2.4 are printed on a dielectric substrate, although these are omitted from Fig. 2.4. The separation region is crucial in the RFSS, as it is where the evanescent mode coupling takes place, and directly contributed to the level of reconfigurability. It is important, therefore, that the model used to predict the responses is inherently able to account for these modes. The method used here is a coupled integral equation (CIE) method and is well suited to the RFSS, as opposed, for example, to the scattering matrix approach which can break down for small separation distances [16].

2.3 Theory of RFSS

The theoretical aspects of the RFSS covered in this section are of fundamental importance to the theory covered in chapters 3 and 4. The FSS's studied are treated as being infinite arrays, with each element being located in a periodic unit cell. The periodicity enables the unknown fields and currents to be expanded in terms of a set of Floquet modes. Currents are excited on the elements (or fields within them) when the arrays are excited by an incident plane wave. An integral equation can be formed, by the application of appropriate boundary conditions, in terms of the unknown currents or fields. In the two layer structures studied in this thesis the currents and fields on each array are linked together by the formation of two coupled integral equations (CIEs), that contain terms for the currents or fields in each array. The method of moments (MOM) is then used to reduce the CIEs to a system of linear equations in matrix form, in which the currents or fields are represented by a set of basis functions. These equations are then solved for the unknown coefficients.
2.3.1 Infinite array configuration

A planar FSS can be represented as an infinite array [5], which is represented in Fig. 2.5.

![Diagram of FSS array](image)

Fig. 2.5. Geometry of FSS array of arbitrary periodicity

The individual unit cells are arranged periodically in the x-y plane, where the vectors $D_1$ and $D_2$ are given by

$$D_1 = D_1 (\cos \alpha, \hat{x} + \sin \alpha, \hat{y})$$  \hspace{1cm} (2.1)

and
\[ D_2 = (D_2 \cos \alpha_2 \hat{x} + D_2 \sin \alpha_2 \hat{y}) \] (2.2)

where

\[ \alpha_2 = \alpha_1 + \alpha \] (2.3)

and

\[ D_1 = |D_1| \text{ and } D_2 = |D_2| \] (2.4)

The unit cell area is defined by,

\[ A = D_1 \times D_2 \] (2.5)

The incident plane wave, represented by \( E' \) in Fig. 2.5, can be considered as a linear combination of transverse electric (TE) and transverse magnetic (TM) components. Pure TE incidence has no electric field component in the direction of propagation (z) whereas pure TM incidence has no magnetic field component in the direction of propagation.

### 2.3.2 Floquet Modes

The tangential fields on the array can be expressed in terms of a set of Floquet modes [17], which are given by,

\[ \Theta_{pq}(r, z) = \Psi_{pq}(r)e^{-jr\cdot r'} \] (2.6)

\[ p, q = 0, \pm 1, \pm 2, \ldots \]

Where

\[ \Psi_{pq} = e^{jk_{pq}r} \] (2.7)
\[ r = x\hat{x} + y\hat{y} \]  

(2.8)

The propagation constants \( k_{pq} \) and \( y_{pq} \) are functions of the lattice geometry. \( p \) and \( q \) are the Floquet indices and the polar incidence angles, \( \theta \) and \( \phi \) are shown in Fig 2.6.

\[ k_{pq} = k_t + p k_1 + q k_2 \]
\[ = k_x \hat{x} + k_y \hat{y} \]  

(2.9)

where

\[ k_t = k_0 \sin \theta \cos \phi \hat{x} + k_0 \sin \theta \sin \phi \hat{y} \]
\[ = k_{0x} \hat{x} + k_{0y} \hat{y} \]  

(2.10)

Fig. 2.6. Polar incidence angles
\begin{align*}
k_0 &= \left(\frac{2\pi}{\lambda}\right) \quad (2.11) \\
k_1 &= -\frac{2\pi}{A} \hat{z} \times D_2 \quad (2.12) \\
k_2 &= \frac{2\pi}{A} \hat{z} \times D_1 \quad (2.13) \\
k_x &= k_{0x} + pk_{1x} + qk_{2x} \quad (2.14) \\
k_y &= k_{0y} + pk_{1y} + qk_{2y} \quad (2.15)
\end{align*}

The z-directed propagation constant, \( \gamma_{pq} \), is given by:

\[
\gamma_{pq} = \sqrt{(k^2 - (k_x^2 + k_y^2))} \quad (2.16)
\]

where,

\[
k = k_0 \sqrt{\varepsilon_r} \quad (2.17)
\]

\( \varepsilon_r \) is the relative permittivity of the dielectric.

For propagating waves, \( k^2 \geq (k_x^2 + k_y^2) \), and \( \gamma_{pq} \) is real and positive (or zero).

For evanescent waves, \( k^2 < (k_x^2 + k_y^2) \), and \( \gamma_{pq} \) is negative and imaginary.

Waves that propagate and have \( p,q \neq 0 \) are referred to as grating responses and are generally considered to be undesirable, due to their impact on the main beam of the array.
The Floquet mode orthogonality is used to establish the relationship between the Floquet mode amplitude and the induced fields or currents. The orthogonality of the Floquet modes is expressed as follows:

\[ \int_A \Psi_{pq}(r) \Psi_{pq'}^*(r) \, dr = \Lambda \delta_{pp'} \delta_{qq'} \]  \hspace{1cm} (2.18)

where,

\[ \delta_{ab} = \begin{cases} 
1 & \text{if } \alpha = \beta \\
0 & \text{otherwise}
\end{cases} \]

The tangential fields on the array can be expressed as a summation of both TE and TM vector Floquet modes, which are detailed below. A new subscript is introduced, \( m \), where \( m=1 \) for TM modes and \( m=2 \) for TE modes. The TM vector mode is given by:

\[ \Phi_{1pq}(r, z) = \Psi_{pq}(r, z) K_{1pq} \]  \hspace{1cm} (2.19)

where,

\[ K_{1pq} = \frac{k_{pq}}{|k_{pq}|} \]  \hspace{1cm} (2.20)

and the TE vector mode is given by:

\[ \Phi_{2pq}(r, z) = \Psi_{pq}(r, z) K_{2pq} \]  \hspace{1cm} (2.21)

where,

\[ K_{2pq} = z \times K_{1pq} \]  \hspace{1cm} (2.22)
The TM and TE modal admittances are described respectively by:

\[ \eta_{1pq} = \frac{k}{\gamma_{pq}} \eta \]  \hspace{1cm} (2.23)

and

\[ \eta_{2pq} = \frac{\gamma_{pq}}{k} \eta \]  \hspace{1cm} (2.24)

where

\[ \eta = \sqrt{\frac{\varepsilon}{\mu}} \]  \hspace{1cm} (2.25)

and \( \varepsilon \) and \( \mu \) are the permittivity and permeability respectively of the relevant medium.

2.3.3 Formulation of the integral equations

This section covers the formulation of two Coupled Electric Field Integral Equations (EFIEs) which can then be solved in order to determine the current coefficients on each of two layers of conductors. Each layer of conductors is printed on a dielectric substrate and these are accounted for in the following analysis. The Coupled Integral Equations (CIEs) are developed assuming the same number of Floquet modes for the unit cells on each array, such that the same Floquet indices \( p \) and \( q \) denote the same order Floquet mode in each unit cell on the two arrays. The two-layer array structure is illustrated in Fig. 2.7 together with the notation for the reflected and transmitted fields for the different regions. It is assumed that the lattices in the two arrays are identical, which is the case in all the results covered in section 2.4. Although the EFIEs are developed, the analysis has been covered fully elsewhere [5] and so some intermediate steps are omitted. The formulation of the CIEs is achieved by the application of the appropriate boundary conditions at \( z=0 \) and \( z=z_2 \). The equations are coupled by the presence of a term in each relating to the currents on the array at which the other was formed.
Fig. 2.7. Cross sectional view of RFSS with conducting elements
The tangential fields are first expanded in terms of TE and TM Floquet modes as described in section 2.3.2, as follows:

For $z \leq 0$

$$E^1(r, z) = E^{inc} + \sum_{mpq} R_{mpq}^{-} e^{j\beta_{mpq} z} \Psi_{pq}(r) K_{mpq}$$  \hspace{1cm} (2.26)

$$H^1(r, z) = H^{inc} - \sum_{mpq} \eta_{mpq}^{-} R_{mpq}^{-} e^{j\beta_{mpq} z} \Psi_{pq}(r) \hat{z} \times K_{mpq}$$  \hspace{1cm} (2.27)

For $0 \leq z \leq z_1$

$$E^1(r, z) = \sum_{mpq} \left( T_{mpq}^1 e^{-j\beta_{mpq} z} + R_{mpq}^1 e^{j\beta_{mpq} z} \right) \Psi_{pq}(r) K_{mpq}$$  \hspace{1cm} (2.28)

$$H^1(r, z) = \sum_{mpq} \eta_{mpq}^1 \left( T_{mpq}^1 e^{-j\beta_{mpq} z} - R_{mpq}^1 e^{j\beta_{mpq} z} \right) \Psi_{pq}(r) \hat{z} \times K_{mpq}$$  \hspace{1cm} (2.29)

For $z_1 \leq z \leq z_2$

$$E^2(r, z) = \sum_{mpq} \left( T_{mpq}^2 e^{-j\beta_{mpq} z} + R_{mpq}^2 e^{j\beta_{mpq} z} \right) \Psi_{pq}(r) K_{mpq}$$  \hspace{1cm} (2.30)

$$H^2(r, z) = \sum_{mpq} \eta_{mpq}^2 \left( T_{mpq}^2 e^{-j\beta_{mpq} z} - R_{mpq}^2 e^{j\beta_{mpq} z} \right) \Psi_{pq}(r) \hat{z} \times K_{mpq}$$  \hspace{1cm} (2.31)

For $z_2 \leq z \leq z_3$

$$E^3(r, z) = \sum_{mpq} \left( T_{mpq}^3 e^{-j\beta_{mpq} z} + R_{mpq}^3 e^{j\beta_{mpq} z} \right) \Psi_{pq}(r) K_{mpq}$$  \hspace{1cm} (2.32)
\[ H^3(r, z) = \sum_{\text{mpq}} \eta_{\text{mpq}}^2 \left( T^{\pm}_{\text{mpq}} e^{-j\alpha z} - R^\pm_{\text{mpq}} e^{j\alpha z} \right) \Psi_{pq}(r) \hat{z} \times \kappa_{\text{mpq}} \]  

(2.33)

For \( z \geq z_3 \)

\[ E^+(r, z) = \sum_{\text{mpq}} T^{+}_{\text{mpq}} e^{-j\alpha z} \Psi_{pq}(r) \kappa_{\text{mpq}} \]  

(2.34)

\[ H^+(r, z) = \sum_{\text{mpq}} \eta_{\text{mpq}}^2 T^{+}_{\text{mpq}} e^{-j\alpha z} \Psi_{pq}(r) \hat{z} \times \kappa_{\text{mpq}} \]  

(2.35)

where the incident field, \( E^{\text{inc}} \), is given by:

\[ E^{\text{inc}} = \sum_{m=1}^{2} \sum_{\text{m00}}^{\text{inc}} e^{-j\alpha z} \Psi_{00} \kappa_{m00} \]  

(2.36)

The integral equations can now be formed by the application of the boundary condition that the electric field is zero on the conducting parts of the unit cell.

At \( z=0 \),

\[ E'(r, 0) = 0, \quad r \in A' \]  

(2.37)

\[ E^{\text{inc}} + \sum_{\text{mpq}} R^-_{\text{mpq}} \Psi_{pq}(r) \kappa_{\text{mpq}} = 0 \]

and at \( z=z_2 \)

\[ E^2(r, z_2) = 0 \]
\[
\sum_{mpq} \left( T_{mpq}^2 e^{-jr_{pq}^{i}z_2} + R_{mpq}^2 e^{jr_{pq}^{i}z_2} \right) \psi_{pq}(r) \kappa_{mpq}
\] (2.38)

With some algebraic manipulation the EFIEs can now be formed.

At \( z=0 \) the EFIE is given by:

\[
\frac{1}{A} \sum_{mpq} \omega_{mpq}^1 \left( \omega_{mpq}^2 \Omega_{mpq} \tilde{J} + \tilde{r} \right) \psi_{pq}(r) \kappa_{mpq} = \sum_{m00} \left( 1 + R_{m00}^i \right) T^{inc}_{m00} \psi_{00} \kappa_{m00}
\] (2.39)

and at \( z=z_2 \) the EFIE is:

\[
\frac{1}{A} \sum_{mpq} \omega_{mpq}^3 \left( \omega_{mpq}^2 \Omega_{mpq} \tilde{J} + \omega_{mpq}^2 \Omega_{mpq} \tilde{r} \right) \psi_{pq} \kappa_{mpq}
\]

\[
= \sum_{m00} \omega_{m00}^2 \omega_{m00}^3 \left( 1 + R_{m00}^i \right) T^{inc}_{m00} \psi_{00} \kappa_{m00}
\] (2.40)

where,

\[
\omega_{mpq}^1 = \frac{1 + R_{mpq}^i}{2\eta_{mpq}^0}
\] (2.41)

\[
\omega_{mpq}^2 = \frac{e^{-jr_{pq}^{i}z_2} + \tau_{mpq}^1 e^{jr_{pq}^{i}z_2}}{1 + \tau_{mpq}^1}
\] (2.42)

\[
\omega_{mpq}^3 = \frac{e^{-jr_{pq}^{i}z_2} + \tau_{mpq}^2 e^{jr_{pq}^{i}z_2}}{e^{-jr_{pq}^{i}z_2} + \tau_{mpq}^2 e^{jr_{pq}^{i}z_2}}
\] (2.43)
\[ R_{mpq}^{s} = \left( \frac{\eta_{mpq}^{0} - \eta_{mpq}^{1} \tau_{mpq}}{\eta_{mpq}^{0} + \eta_{mpq}^{1} \tau_{mpq}} \right) \]  
(2.44)

\[ \tau_{mpq} = \frac{1 - \tau_{mpq}^{1}}{1 + \tau_{mpq}^{1}} \]  
(2.45)

\[ \tau_{mpq}^{1} = e^{-2j\rho_{mpq}z_{1}} \left( \frac{\eta_{mpq}^{1} - \varphi_{mpq}^{1} \eta_{mpq}^{0}}{\eta_{mpq}^{1} + \varphi_{mpq}^{1} \eta_{mpq}^{0}} \right) \]  
(2.46)

\[ \varphi_{mpq}^{1} = \frac{e^{-j\rho_{mpq}z_{1}} - \tau_{mpq}^{2} e^{j\rho_{mpq}z_{1}}}{e^{-j\rho_{mpq}z_{1}} + \tau_{mpq}^{2} e^{j\rho_{mpq}z_{1}}} \]  
(2.47)

\[ \tau_{mpq}^{2} = e^{-2j\rho_{mpq}z_{2}} \left( \frac{\eta_{mpq}^{0} - \varphi_{mpq}^{2} \eta_{mpq}^{2}}{\eta_{mpq}^{0} + \varphi_{mpq}^{2} \eta_{mpq}^{2}} \right) \]  
(2.48)

\[ \varphi_{mpq}^{2} = \frac{e^{-2j\rho_{mpq}z_{2}} - \tau_{mpq}^{3} e^{2j\rho_{mpq}z_{2}}}{e^{-2j\rho_{mpq}z_{2}} + \tau_{mpq}^{3} e^{2j\rho_{mpq}z_{2}}} \]  
(2.49)

\[ \tau_{mpq}^{3} = e^{-2j\rho_{mpq}z_{3}} \left( \frac{\eta_{mpq}^{2} - \eta_{mpq}^{0}}{\eta_{mpq}^{2} + \eta_{mpq}^{0}} \right) \]  
(2.50)

and

\[ \Omega_{mpq} = \left[ \omega_{mpq}^{3} \omega_{mpq}^{3} \left( \omega_{mpq}^{1} \omega_{mpq}^{2} + \frac{e^{j\rho_{mpq}z_{3}} - e^{-j\rho_{mpq}z_{3}}}{2\eta_{mpq}} \right) \right] \]  
(2.51)

\( \vec{I} \) and \( \vec{J} \) are the Floquet transforms of the electric currents, \( I \) and \( J \), excited on the elements at \( z=0 \) and \( z=z_{2} \) respectively. They are defined by the inner products:
\[ I_{mpq} = \langle J(r) \cdot \kappa_{mpq}, \Psi_{pq}(r) \rangle_A = I_{pq} \cdot \kappa_{mpq} \] (2.52)

and

\[ J_{mpq} = \langle J(r) \cdot \kappa_{mpq}, \Psi_{pq}(r) \rangle_A = J_{pq} \cdot \kappa_{mpq} \] (2.53)

The EFIEs, Eqns. 2.39 and 2.40 form a coupled pair and it is interesting to note the interchange of the coefficient \( \omega_{mpq}^1 \omega_{mpq}^3 \omega_{mpq}^3 \) between \( \vec{J} \) at \( z=0 \) and \( \vec{I} \) at \( z=z_2 \).

The transmission coefficient is given by:

\[ T^+ = e^{i \kappa m z_3} \Omega_{mpq}^4 \Omega_{mpq}^3 \left[ \omega_{mpq}^1 (1 + R_{mpq}^s) T_{m00}^{inc} - \omega_{mpq}^2 \Omega_{mpq}^1 \frac{\overline{I}}{A} - \Omega_{mpq}^2 \frac{\overline{J}}{A} \right] \] (2.54)

where,

\[ \Omega_{mpq}^4 = \frac{e^{-j \kappa m z_3} + \zeta_{mpq}^2 e^{j \kappa m z_3}}{e^{-j \kappa m z_3} + \zeta_{mpq}^2 e^{j \kappa m z_3}} \] (2.55)

### 2.3.4 The Method of Moments

The Method of Moments (MOM) [5,6] is frequently used to solve integral equations like those of Eqns. 2.39 and 2.40 which can be coupled together in an equation of the form:

\[ \frac{1}{A} \sum_{m00} \left[ \begin{array}{c} \Xi_{11} \Xi_{12} \\ \Xi_{21} \Xi_{22} \end{array} \right] \left[ \begin{array}{c} \overline{I} \\ \overline{J} \end{array} \right] \Psi_{pq} \kappa_{mpq} = \sum_{m00} \left[ \begin{array}{c} \Xi_{10} \\ \Xi_{20} \end{array} \right] T^{inc}_{m00} \Psi_{00} \kappa_{m00} \] (2.56)
where, \( \Xi_{12} \) and \( \Xi_{21} \) represent the coupling between the arrays. The MOM enables Eqn. 2.56 to be reduced to a linear system of simultaneous equations by expanding the unknown currents as an infinite series of basis functions.

\[
I(x) = \sum_{n=1}^{\infty} c_n^1 h_n^1(x) \quad L_n \in A' \quad \text{at } z=0 \tag{2.57}
\]

and

\[
J(x) = \sum_{n=1}^{\infty} c_n^2 h_n^2(x) \quad L_n \in A' \quad \text{at } z=z_2 \tag{2.58}
\]

It is assumed here that the same elements are used on each layer. In order that the system of equations can be solved the series of bases cannot, in reality, be infinite, so a finite series is chosen to approximate the induced currents, say \( N \). The system can now be solved, being of finite order.

By substituting Eqns. 2.57 and 2.58 into Eqn. 2.56 and taking the inner product with testing functions \( h_i^1 \) and \( h_i^2 \) leads to the matrix equation:

\[
\begin{bmatrix}
[X_{11}] & [X_{12}]
\end{bmatrix}
\begin{bmatrix}
[C_1]
\end{bmatrix}
= \begin{bmatrix}
[S_1]
\end{bmatrix}
\begin{bmatrix}
[X_{21}] & [X_{22}]
\end{bmatrix}
\begin{bmatrix}
[C_2]
\end{bmatrix}
= \begin{bmatrix}
[S_2]
\end{bmatrix} \tag{2.59}
\]

The square matrices \( X_{11} \) and \( X_{22} \) are similar to those that would be produced in the case of individual layers and \( X_{12} \) and \( X_{21} \) represent the coupling between the layers. In this case:

\[
S_{11} = \sum_{m=0}^{\infty} (1 + R_{m0}^s) T_{m0}^{inc} \tilde{h}_1^1(x_{r0}) \tag{2.60}
\]
Generally, the testing functions need not be the same as the basis functions, but throughout this thesis the testing functions and basis functions are the same. This is a special case of the MoM known as the Ritz-Galerkin procedure. Eqn. 2.59 is solved for the unknown coefficients $C_j$ and $C_2$, is achieved by matrix inversion using an elimination technique, Crout's factorisation with partial pivoting.

It is apparent that the size of the matrices in Eqn. 2.59, and therefore the complexity of the solution depends on $N$ and the number of Floquet modes chosen. It is worth mentioning, at this point, something about the convergence [18] of the solution, because the series of basis
functions cannot be infinite, and we cannot have an infinite number of Floquet modes the
solution cannot be exact. $N$ has been chosen as the finite number of basis functions and so
a finite limit is also set to the order of the Floquet indices $(p,q)$, say $(P,Q)$. When
attempting to establish the solution for a particular element geometry care needs to be
taken to ensure that sufficient Floquet harmonics are included to cover at least the main
lobe of the spectra of the basis functions. As a rule of thumb, to check for convergence, the
number of Floquet modes should be increased while keeping $N$ constant. A figure of merit
has been introduced [5], such that $\Delta = PQ/N$, where $PQ$ is the total number of Floquet
modes. If $N$ needs to be increased then $\Delta$ should be investigated further, but generally, the
best way to ensure the best recipe in the predictions is through experimentation.

2.4 Results From Representative Arrays

In this section results are presented both for the RFSS and CCFSS, for angles of incidence
up to TE: 45°. The arrays consist of linear dipole elements, with the same dimensions on
each layer, and the two layer structure is depicted in Fig. 2.8. The two arrays are shown in
plan form, one on top of the other. The two arrays have the same lattice parameters $DX$
and $DY$, and the shaded elements represent the dipoles on layer 1 and the dotted elements
those on layer 2. In the cases studied in this section $L1=L2$, as the elements are identical on
the two layers as well. When $DSY=0$ the elements on the respective arrays are displaced
only in the $z$ direction and the array behaves similarly to a single layer array of dipoles of
the same dimensions. There would be a small effect due to the separation between the
layers, most observable in the case of the RFSS, but primarily, the main effect of the
incident electromagnetic plane wave would be that of a single layer. As a point of
reference, the transmission responses for a single layer array are shown in Figs. 2.9a-c. The
array dimensions are $DX=DY=5$mm, $L=3.5$mm and $W=0.3$mm. The array is printed on a
polyester substrate with $S=70\mu$m and $\varepsilon_r =3$. A crucial point to note is the shift in the
resonant frequency from 37GHz for normal incidence to 33GHz at TE: 45° incidence (a
shift of about 10%). This is characteristic of the impact of the grating lobe region on the
resonance stability and will be discussed in more detail in section 2.4.2.
Fig. 2.8. Plan view of two layer FSS array showing the relative shift, $DSY$
Fig. 2.9a. Transmission response for single layer dipole array: normal incidence

Fig. 2.9b. Transmission response of single layer dipole array: TE: 30° incidence.
2.4.1 RFSS with Conducting Elements

This section gives some results of the performance of the RFSS in which the frequency response is dynamically tuned over a wide range. This type of operation would be very desirable in situations where multi-band filters are employed, as it would enable just one such filter to be used. Conventionally, an FSS would be designed for a specific filtering response (such as a subreflector) which would be determined primarily by the dimensions of the array elements with respect to wavelength of operation. This is a distinct disadvantage when the surface will often be permanently fixed and aligned to the antenna assembly.

Using the micropositioning jig shown in Fig. 2.3, two FSS arrays are placed in close proximity to one another, separated by the distance, $S$. Fig. 2.8 shows the plan view of the geometry of the RFSS, and indicates that the displacement is only along the y axis. However, depending on the array geometries, the displacement could be along any arbitrary vector. When the arrays are directly aligned i.e. $DSY=0$, the performance is
essentially that of a single layer array. The rear layer would be predominantly masked by
the presence of the front one. Increasing the displacement, \( DSY \), introduces a lateral shift
between the arrays so that the gaps between the elements on the rear layer are increasingly
covered up by the first. Looking at the RFSS from the incidence side, an effective increase
in the length of the elements is observed and this is accompanied by an alteration of the
electromagnetic coupling between the arrays. The change in coupling alters the overall
performance of the RFSS and produces significant performance enhancements on the
single layer case.

Figs. 10a-c display the effect of increasing \( DSY \) on the transmission response for normal,
TE:30° and TE:45° incidences. The two arrays are identical with each having
\( DY=DX=5\text{mm}, L=3.5\text{mm} \) and \( W=0.3\text{mm} \). They are printed on a polyester substrate of

![Diagram of transmission response](image)

**Fig. 2.10a.** Transmission response of RFSS: normal incidence
Fig. 2.10b. Transmission response of RFSS: TE: $30^\circ$ incidence

Fig. 2.10c. Transmission response of RFSS: TE: $45^\circ$ incidence
thickness 70µm and \( \varepsilon_r=3 \). The incident electric field is parallel to the y axis. In the predicted results the separation distance between the arrays is set to 50µm. The basis functions employed to model the induced currents on the dipoles are entire domain sinusoids and cosinusoids. It is important that the bases chosen are zero ended to simulate the practical situation existing on the dipole. Five basis functions were employed and they are:

\[
\cos\left(\frac{n\pi}{L}\right), \sin\left(\frac{2n\pi}{L}\right), \cos\left(\frac{3n\pi}{L}\right), \sin\left(\frac{4n\pi}{L}\right), \text{ and } \cos\left(\frac{5n\pi}{L}\right),
\]

With these bases it was found that 15 Floquet modes was sufficient to achieve convergence (see section 2.3.4). The relative shift between the arrays is achieved by moving the Floquet modes of the front array by a distance \( DSY \) in the y direction such that for the front array Eqn. 2.10 becomes:

\[
k_r = k_{0x} \hat{x} + k_{0y} (DSY) \hat{y}
\]

(2.66)

Figs. 2.10a-c show the dramatic effect of increasing \( DSY \) from zero to \( DY/2 \) (2.5mm). Fig. 2.10a shows for normal incidence that there is an available tuning range from around 36GHz to 17.5GHz which represents a percentage shift of 51%. The original resonance at 36GHz can be shifted to 31.5GHz with a \( DSY \) value of 1mm, but the dipoles on the two layers have not yet started to overlap so the increase in coupling is not maximised. However, a maximum frequency shift can be achieved by making \( DSY \) maximum (\( DY/2 \)) at 2.5mm. If \( DSY \) is increased further, then the dipoles begin to return to the state when \( DSY=0 \), and the resonance moves back up in frequency until the dipoles once more cover one another and the RFSS behaves in similar fashion to a single layer array. The results for TE:30° and TE:45° incidences perform in a similar manner although with slightly smaller frequency shifts. For instance, for TE:45° incidence the overall shift from \( DSY=0 \) to \( DSY=2.5 \) mm is about 45%, which is still a significant range. Also, there is a marked increase in bandwidth for all incidences as \( DSY \) is increased from zero to 2.5mm. If the normal and TE:45° incidence cases are compared it can be seen that for \( DSY=0 \) the resonant frequency shifts from 36GHz to 32GHz as the angle of incidence is increased, which amounts to about 11%. However, when \( DSY=2.5 \) mm there is a shift from 17.5GHz to 18.5GHz which represents only about a 5% shift. This increase in angular stability is a key feature of the RFSS and comes about because the resonance when \( DSY=2.5 \) mm has
been moved far away from the grating lobe region. These undesirable, non-zero order, propagating modes have the effect of destabilising the resonance and are largely dependent on the lattice periodicity, $DX$. At normal incidence they are far enough from the resonance not to have an effect but as the angle of incidence increases, $DX$ effectively becomes smaller as $\cos\theta$, so that the grating lobes start to impinge on the stability of the resonance. Fig. 2.10c clearly illustrates the grating lobes at about 35GHz and their effect on the resonance, when $DSY=0$, is large. When $DSY=2.5\text{mm}$, however, the resonance has been forced far enough away from the grating region to render its effect small.

A figure of merit that has been introduced previously [19] in conjunction with arrays of conventional and convoluted elements is the periodicity to resonant wavelength ratio, $\lambda/D$. When $DSY=0$, $\lambda/D=1.6$, which is fairly typical for an ordinary dipole array. When $DSY=2.5\text{mm}$, $\lambda/D$ increases to about 3.4, and with $D=5\text{mm}$ this suggests that the high coupling between the arrays has produced a resonance whose wavelength is larger than the array lattice. This, of course, is not physically possible, but what it does show is that the RFSS is capable of producing responses that simulate those of electrically large elements, when the elements themselves are electrically small. It has achieved a resonance at a frequency where the dipole elements are more than twice as large electrically as they are physically. This could be very useful in applications where a low frequency filter is required but physical space is limited.

### 2.4.2 Close Coupled FSS With Conducting Elements

The results for the RFSS in the previous section showed the advantages of forcing the resonant frequency of the RFSS structure to be far away from the grating lobe region. Because this is achieved by introducing a shift between the elements on the different layers, thereby increasing the electromagnetic coupling between the arrays, a new structure has been investigated in which the coupling between arrays is maximised. In this section results are presented whereby extremely angularly stable responses can be achieved, again using simple dipole elements. The technique could be applied to any element, but the simplicity and linearity of the dipole makes it an ideal element to investigate. The double
layer CCFSS structure, described in section 2., is used to implement the design and allows one to take advantage of the maximum coupling between arrays as the critical part of the design. Results are presented for two array designs which, according to the nomenclature in Fig. 2.8 have the following dimensions: Array 1 has $DX=DY=5\text{mm}$, $L1=L2=4\text{mm}$ and $W1=W2=0.2\text{mm}$; Array 2 is the same as array 1 but with $DSY=1.7\text{mm}$. Array 3 has $DX=DY=4\text{mm}$, $L1=L2=2.8\text{mm}$ and $W1=W2=0.2\text{mm}$. Array 4 is the same as array 3 but with $DSY=1.5\text{mm}$. Each double sided array is printed onto a dielectric (polyester) substrate of thickness 70\text{\textmu}m and $\varepsilon_r=3$.

The comparisons of measured and predicted results can be seen in Figs. 2.11a-c (arrays 1 and 2) and Figs. 2.12a-c (arrays 3 and 4) for normal, TE: $30^\circ$ and TE: $45^\circ$ incidences. The first thing to note is the excellent agreement between the measured and predicted responses for all cases. Taking Fig. 11 firstly, it can be seen that when $DSY=0$ the resonance is

![Image of transmission response graph]

Fig. 2.11a. Transmission response of CCFSS arrays 1 and 2: normal incidence
Fig. 2.11b. Transmission response of CCFSS arrays 1 and 2: TE:30° incidence

Fig. 2.11c. Transmission response of CCFSS arrays 1 and 2: TE:45° incidence
at 32.5 GHz which, as in the case of the RFSS is very similar to a single layer array of the same element and lattice geometries. Increasing the angle of incidence to 30° and then to 45° moves the resonant frequency down to 31 GHz and 30 GHz respectively. This represents shifts of 4.6% and 7.7%. The grating lobes are evident near 35 GHz and it would be largely due to the presence of these higher order propagating modes, close to the resonance, that the resonance is unstable. The situation is completely different when a displacement of $DSY=1.7 \text{ mm}$ is introduced, while all the other parameters remain unchanged. The resonance has been shifted down in frequency to 16 GHz and remains very stable as the angle of incidence is increased to 45°. This verifies the earlier comment of the effects of the grating lobes on resonance stability. The further the resonance is away from the grating lobe region, the more stable the response is to the effects of changes in the angle of incidence. If $DSY$ were increased to 2.5 mm ($DY/2$), the resonance moves further down in frequency to around 10 GHz. This, however, was outside the prescribed measurement range, so $DSY$ was set at 1.7 mm in this case.

The angular stability is better than the RFSS although the displacement is in the same direction. This is because the coupling between arrays is maximised due to the proximity of the arrays. Electrically there is a linear dipole element whose length produces approximately a $\lambda/2$ resonance, whereas the physical length is of the order of $\lambda/5$ for the case shown in Fig. 2.11. The results in Fig. 2.12 show the comparison between measured and predicted transmission responses for arrays 3 and 4. Because the elements are shorter, the resonance at normal incidence is at a higher frequency, around 36.5 GHz. This shifts to about 35.5 GHz and 34.5 GHz when the incidence is increased to TE: 30° and TE: 45° respectively. This represents, in percentage terms, shifts of 2.7% and 5.5%. This is appreciably less than that observed for array 1 (unshifted) in the previous case (4.6% and 7.7%). This can be explained by the grating lobes being further from the resonance in array 3. Figs. 2.11a-c clearly show the grating lobes coming close to the (unshifted) resonance as the angle of incidence is increased, until they are prominent around 35 GHz in Fig. 11c. There are no such signs of the grating lobes in Fig. 2.12 however. This is because the periodicity is smaller in Fig. 2.12 (4 mm) than Fig. 2.11 (5 mm). Because the grating lobes are frequency dependent, this decrease in periodicity has the effect of moving them up in frequency. This puts them further from the resonance, so their destabilising effect is reduced.
Fig. 2.12a. Transmission response of CCFSS arrays 3 and 4: normal incidence

Fig. 2.12b. Transmission response of CCFSS arrays 3 and 4: TE: 30° incidence
When $DY=1.5\text{mm}$, the resonance at normal incidence moves down in frequency to around 21.5GHz. Once again, the angular stability is greatly improved due to this displacement, and the resonant frequency remains stable around 21GHz when the angle of incidence is increased to 45°.

In order to quantify the effects of the element length and periodicity, a theoretical study has been carried out to determine the effect that the element length to lattice periodicity, $L/D$, has on the $\lambda/D$ ratio, where $\lambda$ is the resonant wavelength at normal incidence. In general, the larger the $\lambda/D$ ratio, the more stable an arrays response will be. Remembering that the grating lobes are frequency dependent, and effectively appear at a wavelength corresponding to $D$, it can be seen that if $\lambda/D >> 1$, this will put the resonance at a much lower frequency than the grating lobes, where the resonance stability will not be hugely
affected by the grating lobes. In the study, the effect that $L/D$ has on $D/\lambda$ and the instability factor is assessed. The instability factor is introduced here and defined as $\{1-(f_F - f_i)/f_i\}$, where $f_F$ and $f_i$ are the resonant frequencies at normal and extreme oblique angles of incidence respectively. Table 2.1 shows the results for oblique incidence up to TE:45°.

With no displacement between the arrays ($DSY=0$) there is an expected improvement in the instability factor from 25.9% to 8.1% as $L/D$ increases from 0.65 to 0.95. The instability factor is significantly improved when a displacement, $DSY=D/2$ is introduced. In fact, the instability factor is below 1% for all $L/D$ ratios considered here, and is $\leq 0.3$ for $L/D$ of 0.85 and above. The introduction of the relative displacement is the key element in the stability of the CCFSS, due to the strong coupling between the dipoles on each layer. The fact that $\lambda/D$ ratios of 9.55 can be achieved indicates the potential of this structure to obtain responses that a far removed from the negative influence of the grating lobe region.

<table>
<thead>
<tr>
<th>L/D</th>
<th>$\lambda/D$</th>
<th>Instability Factor (%)</th>
<th>$\lambda/D$</th>
<th>Instability Factor (%)</th>
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<tr>
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<td>5.46</td>
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<td>1.57</td>
<td>21.9</td>
<td>6.71</td>
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<td>7.46</td>
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<td>8.02</td>
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<td>1.86</td>
<td>12.4</td>
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<td>1.99</td>
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<td>&lt;0.3</td>
</tr>
<tr>
<td>0.95</td>
<td>2.13</td>
<td>8.1</td>
<td>9.55</td>
<td>&lt;0.3</td>
</tr>
</tbody>
</table>

Table 2.1. Effect of displacement on resonance and angular stability
This chapter has examined a two layer FSS of conducting elements. Their performance has been assessed both in RFSS and CCFSS configurations. A model has been developed which caters for both situations, and which relies upon the formulation of two, coupled, electric field integral equations in terms of the unknown currents on the conductors. These were then solved using the Method of Moments, with the currents represented by a set of entire domain sinusoids and cosinusoids. The model also has the ability to simulate a lateral displacement between the arrays, whereby the spectrum of the Floquet modes used to expand the fields is displaced, thereby enabling a relative shift between arrays to be achieved. The objectives were to investigate the effects of a relative shift between layers to achieve a dynamically tuned response (RFSS) and much improved angular stability (CCFSS).

Using simple dipole elements in square lattice arrays, these objectives have largely been met. The RFSS has demonstrated the ability to dynamically tune the frequency response of a two layer array by over 50%. This has been achieved by introducing a relative displacement between the arrays, in the direction of the dipoles’ long dimension, using a specially designed micropositioning jig. This ability to reconfigure filtering responses over such a wide range would be advantageous in situations where multiband performance is required and space constraints are a serious consideration. The RFSS also demonstrated enhanced angular stability up to TE:45° incidence because the resonance has been moved out of the range of influence of the destabilising grating lobes. The CCFSS has demonstrated the effects of having maximum coupling between the arrays. This was achieved by printing the arrays on each side of a single dielectric substrate. With the displacement included during manufacture it has been demonstrated that the angular stability up to TE:45° incidence can be improved from 7.7% to less than 1%. A parallel parametric study has indicated that figures of less than 0.3% are attainable.
REFERENCES


CHAPTER 3

Two Layer Frequency Selective Surfaces with Aperture Elements

3.1 Introduction

This chapter presents the work undertaken with respect to a study into the behaviour of arrays of aperture elements as two layer FSS. An aperture FSS consists of a metal sheet perforated periodically with specific shapes, which act as the resonant elements. In the absence of any dielectric supporting layers, an aperture array is the Babinet compliment of an array of conductors with identical element and unit cell dimensions [1]. Whereas the conducting element arrays studied in Chapter 2 produced a stopband at resonance the aperture arrays investigated in this chapter produce a passband. In some filtering applications it is deemed beneficial to have a passband rather than a stopband. For example, airborne radome technology, requires both selection and rejection of the frequencies to be transmitted or rejected. Aperture arrays have been previously studied both with and without dielectric support [2-4] but generally in a static, single layer environment. This chapter considers them as RFSS and CCFSS, as a complementary study
to that detailed in Chapter 2. Comparisons will be drawn between the RFSS [5] and close coupled scenarios [6] for the conductors with specific attention paid to the CCFSS

Section 3.2 will introduce the concept of the aperture RFSS and discuss the fundamental differences between it and the conducting case in Chapter 2. The structure of the aperture arrays will be detailed and manufacturing issues will be discussed. Section 3.3 covers the theoretical aspects and electromagnetic analysis of the aperture RFSS, whereby two magnetic field integral equations (MFIEs) [7] are formed in order to solve for the unknown fields within the aperture regions using the Method of Moments [8]. Section 3.4 looks at some results that have been produced and compares theory with measured data. Results are presented for both RFSS and close couple structures although most attention will be paid to the close coupled. Assessments are made of the stability of the passbands as well as the theoretical and measured bandwidths. The results shown are for linear slot elements, which are modelled using entire domain basis functions comprising of waveguide modes [9]. Waveguide modes were employed to allow for the possibility of incorporating two-dimensional patch elements.

3.2 Aperture RFSS Concept

The results discussed in Chapter 2 demonstrated the effect of introducing a relative lateral shift between two arrays of conducting elements. The primary result was to greatly improve the angular stability of the plane wave transmission response when compared to the single layer case. This was especially true in the case of the CCFSS, whilst the RFSS demonstrated the capability to tune the resonance over a range greater than 50%. Also, the shift between layers led to an increase in bandwidth between the unshifted and shifted cases. It is desirable in some FSS applications, such as radomes and satellite systems, to have a passband at resonance rather than the stopband produced by the conducting arrays. To facilitate this arrays of aperture elements can be used, and by combining two separate arrays additional flexibility can be introduced into their operation. The flexibility allows for manipulation of the passband and also enables narrower passbands and more stable responses to be obtained than are available from a single layer. The key feature once again
Fig. 3.1 RFSS Aperture Arrays
Copper Arrays

Dielectric Support

Fig. 3.2 Close Coupled Aperture arrays
is the close proximity of the arrays which leads to a high level of coupling between the fields generated within the aperture regions by the incident plane wave radiation. When corresponding elements between the two layers are directly aligned, coupling between them is weak and the overall structure behaves essentially like a single layer array. When a relative displacement between the arrays is introduced, however, the dislocation of the overlapping aperture fields causes them to interact very strongly. This effect is directly analogous to the conducting element case where the currents on the conductors interact, so invoking the effect of having an electrically large element when in fact the elements are physically small. This will be covered in more detail in section 3.4.

Fig. 3.1 shows a schematic view of the aperture RFSS, where the thickness of the dielectric support layers has been exaggerated. The two arrays are placed in close proximity to one another with the elements on each aligned. Although in this case the dielectric is not needed for supporting the metallic elements, it is included due to ease of manufacture, and in the case of the front array, to prevent shorting between them. The two arrays need not be identical, either in terms of the elements used or the lattice geometries, but having different lattice parameters introduces complications in the modelling of such structures [11]. Whilst one array is fixed the second is free to move, linearly, in one dimension, thereby achieving a relative displacement between the arrays. The mechanism is the same computer controlled stepping linear actuator described in Section 2.2.2. In principle this should enable the passband to be both dynamically scanned over a prescribed frequency range and/or switched on and off at specific frequencies. A drawback to this technique is a problem associated with keeping the air-gap between the arrays constant. The sheets are relatively rigid, their mechanical properties being largely determined by the copper itself. The thickness of the copper used (1/4oz.) is approximately 9μm, and great care must be taken when handling the sheets to avoid any permanent deformation.

Fig 3.2 is a representation of a close coupled aperture FSS. The structure is made from a single sheet of two sided copper clad dielectric. As such, there is no dynamic reconfigurability, but maximum coupling between arrays can be achieved in this case thereby demonstrating the optimum RFSS performance. The relative displacement can be in any direction but the limit here is in two perpendicular directions, which adhere to the square nature of the lattices involved. The separating dielectric acts not only to prevent shorting but also maintains a uniform array separation, improving on the performance of
the dynamically tuned RFSS for any given displacement. The structures are very robust and lend themselves to being shaped in any desired way. The displacement between arrays is determined prior to manufacture by the careful positioning of two photographic masks on the copper foil, which is then etched with the desired two sided pattern, as described in Section 2.2.1.

3.3 Theory of Two Layer Aperture FSS

As in the case of the conducting RFSS the structures investigated here are highly coupled and therefore the coupled integral equation method is again used in conjunction with a vector Floquet mode analysis technique, [7], to predict the transmission responses. Two magnetic field integral equations are derived which are then coupled to form a single matrix equation to solve for the unknown fields within the apertures. This procedure is different to that described in Section 2.3 in that the boundary conditions are not the same as those used to solve for the unknown currents on the conducting elements. Fig 3.3 shows a cross sectional view of the two layer aperture RFSS structure used in the model. The arrays are located at \( z=0 \) and \( z=z_2 \) and the transmitted and reflected fields are shown in the three dielectric regions and free space. The arrows in Fig. 3.3 indicate the direction of travel of the fields if they are propagating. The notation follows that of Chapter 2, with the superscript denoting the dielectric region concerned or free space.

3.3.1 Tangential Field Expansions

Following the same notation used in Section 2.3, the tangential electromagnetic fields in each region are expressed as TE and TM Floquet modes as follows:

For \( z \leq 0 \)

\[
E^- (r, z) = E^{inc} + \sum_{m,p,q} R_{mpq} e^{jnpz} \Psi_p (r) \kappa_{mpq} \tag{3.1}
\]
\[
H^-(r, z) = H^{inc} - \sum_{mpq} \eta_{mpq}^0 P_{mpq}^0 e^{i\nu_{pq}z} \Psi_{pq}(r) \hat{z} \times K_{mpq}
\] (3.2)

Fig. 3.3. Cross section of aperture RFSS
For $0 \leq z \leq z_1$

$$E^1(r, z) = \sum_{mpq} \left[ T^1_{mpq} e^{-jr^2 z} + R^1_{mpq} e^{jr^2 z} \right] \psi_{pq} \kappa_{mpq}$$

(3.3)

$$H^1(r, z) = \sum_{mpq} \eta_{mpq} \left[ T^1_{mpq} e^{-jr^2 z} - R^1_{mpq} e^{jr^2 z} \right] \psi_{pq} \hat{z} \times \kappa_{mpq}$$

(3.4)

For $z_1 \leq z \leq z_2$

$$E^2(r, z) = \sum_{mpq} \left[ T^2_{mpq} e^{-jr^2 z} + R^2_{mpq} e^{jr^2 z} \right] \psi_{pq} \kappa_{mpq}$$

(3.5)

$$H^2(r, z) = \sum_{mpq} \eta_{mpq} \left[ T^2_{mpq} e^{-jr^2 z} - R^2_{mpq} e^{jr^2 z} \right] \psi_{pq} \hat{z} \times \kappa_{mpq}$$

(3.6)

For $z_2 \leq z \leq z_3$

$$E^3(r, z) = \sum_{mpq} \left[ T^3_{mpq} e^{-jr^2 z} + R^3_{mpq} e^{jr^2 z} \right] \psi_{pq} \kappa_{mpq}$$

(3.7)

$$H^3(r, z) = \sum_{mpq} \eta_{mpq} \left[ T^3_{mpq} e^{-jr^2 z} - R^3_{mpq} e^{jr^2 z} \right] \psi_{pq} \hat{z} \times \kappa_{mpq}$$

(3.8)

For $z \geq z_3$

$$E^4(r, z) = \sum_{mpq} T^+_{mpq} e^{-jr^2 z} \psi_{pq} \kappa_{mpq}$$

(3.9)

$$H^4(r, z) = \sum_{mpq} \eta^0_{mpq} T^+_{mpq} e^{-jr^2 z} \psi_{pq} \hat{z} \times \kappa_{mpq}$$

(3.10)
3.3.2 The Fields Within the Dielectric Regions

Two coupled magnetic field integral equations (MFIE's) are formed in terms of $\tilde{E}^1$ and $\tilde{E}^2$ which are the Floquet transforms of the fields within the apertures at $z=0$ and $z=z_2$ respectively. In order to correctly form the MFIE's the transmitted and reflected field amplitudes $T_{mpq}^1$, $T_{mpq}^2$, $R_{mpq}^1$, and $R_{mpq}^2$ must first be evaluated in terms of the modal propagation constants and admittances. This is achieved by expressing the tangential electric and magnetic fields at the various boundaries between the dielectric layers.

At $z=0$, we find

$$R_{mpq}^1 + T_{mpq}^1 = \frac{\tilde{E}_{mpq}^1}{A}$$

(3.11)

at $z=z_L$, $\tilde{E}_1^1(r,z) = \tilde{E}_2^2(r,z)$

$$R_{mpq}^1 e^{j\beta_{pq}z_1} + T_{mpq}^1 e^{-j\beta_{pq}z_1} = R_{mpq}^2 e^{j\beta_{pq}z_1} + T_{mpq}^2 e^{-j\beta_{pq}z_1}$$

(3.12)

and

$$\eta_{mpq}^{-1} \left(T_{mpq}^1 e^{-j\beta_{pq}z_1} - R_{mpq}^1 e^{j\beta_{pq}z_1} \right) = \eta_{mpq}^{-1} \left(T_{mpq}^2 e^{-j\beta_{pq}z_1} - R_{mpq}^2 e^{j\beta_{pq}z_1} \right)$$

(3.13)

at $z=z_2$

$$R_{mpq}^2 e^{j\beta_{pq}z_2} + T_{mpq}^2 e^{-j\beta_{pq}z_2} = \frac{\tilde{E}_{mpq}^2}{A}$$

(3.14)

These equations can be manipulated to yield the required expressions for the transmitted and reflected fields as follows:
Eqn. (3.12) multiplied by $\eta_{mpq}^2$ and added to eqn. (3.13) gives

$$R_{mpq}^1 e^{j\eta_{mpq}^2} (\eta_{mpq}^2 - \eta_{mpq}^1) + T_{mpq}^1 e^{-j\eta_{mpq}^2} (\eta_{mpq}^2 + \eta_{mpq}^1) = 2\eta_{mpq}^2 T_{mpq}^2 e^{-j\eta_{mpq}^2}$$

(3.15)

multiplying eqn. (3.12) by $\eta_{mpq}^1$ and adding eqn (3.13) gives

$$R_{mpq}^1 e^{j\eta_{mpq}^1} (\eta_{mpq}^1 - \eta_{mpq}^2) + T_{mpq}^1 e^{-j\eta_{mpq}^1} (\eta_{mpq}^1 + \eta_{mpq}^2) = 2\eta_{mpq}^1 T_{mpq}^1 e^{-j\eta_{mpq}^1}$$

(3.16)

multiplying eqn. (3.12) by $\eta_{mpq}^2$ and subtracting eqn. (3.13) gives

$$R_{mpq}^1 e^{j\eta_{mpq}^2} (\eta_{mpq}^2 + \eta_{mpq}^3) + T_{mpq}^1 e^{-j\eta_{mpq}^2} (\eta_{mpq}^2 - \eta_{mpq}^1) = 2\eta_{mpq}^2 R_{mpq}^2 e^{j\eta_{mpq}^2}$$

(3.17)

and multiplying eqn. (3.12) by $\eta_{mpq}^1$ and subtracting eqn (3.13) gives

$$R_{mpq}^2 e^{j\eta_{mpq}^1} (\eta_{mpq}^1 + \eta_{mpq}^2) + T_{mpq}^2 e^{-j\eta_{mpq}^1} (\eta_{mpq}^1 - \eta_{mpq}^2) = 2\eta_{mpq}^1 R_{mpq}^1 e^{j\eta_{mpq}^1}$$

(3.18)

By simple manipulation these four equations can be made to give expressions for the required field amplitudes.

From eqn (3.15)

$$\frac{R_{mpq}^1}{T_{mpq}^2} = R_{mpq}^1 e^{j\eta_{mpq}^2} (\eta_{mpq}^2 - \eta_{mpq}^1) + T_{mpq}^1 e^{-j\eta_{mpq}^2} (\eta_{mpq}^2 + \eta_{mpq}^1)$$

$$2\eta_{mpq}^2 e^{-j\eta_{mpq}^2}$$

(3.19)

From eqn. (3.17)

$$R_{mpq}^2 = R_{mpq}^1 e^{j\eta_{mpq}^1} (\eta_{mpq}^1 + \eta_{mpq}^2) + T_{mpq}^1 e^{-j\eta_{mpq}^1} (\eta_{mpq}^1 - \eta_{mpq}^1)$$

$$2\eta_{mpq}^2 e^{j\eta_{mpq}^1}$$

(3.20)
\[ T_{mpq}^1 = \frac{R_{mpq}^2 e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^1 - \eta_{mpq}^2) + T_{mpq}^2 e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2)}{2\eta_{mpq}^1 e^{j\alpha \lambda_{mz1}}} \]  

(3.21)

From eqn. (3.18)

\[ R_{mpq}^1 = \frac{R_{mpq}^2 e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2) + T_{mpq}^2 e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^1 - \eta_{mpq}^2)}{2\eta_{mpq}^1 e^{j\alpha \lambda_{mz1}}} \]  

(3.22)

Substituting eqns. (3.19) and (3.20) into eqn. (3.14) gives

\[
\begin{align*}
&\left\{ R_{mpq}^1 e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2) + T_{mpq}^1 e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^1 - \eta_{mpq}^2) \right\} \\
&+ e^{-j\alpha \lambda_{mz1}} \left\{ R_{mpq}^1 e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^2 - \eta_{mpq}^1) + T_{mpq}^1 e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2) \right\} = \frac{E_{mpq}^2}{A}
\end{align*}

(3.23)

From which is obtained

\[
\begin{align*}
&\left\{ R_{mpq}^1 e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2) e^{j\alpha \lambda_{mz2}} + \frac{e^{j\alpha \lambda_{mz1}} (\eta_{mpq}^2 - \eta_{mpq}^1) e^{-j\alpha \lambda_{mz2}}}{2\eta_{mpq}^2 e^{j\alpha \lambda_{mz1}}} \right\} + \\
&T_{mpq}^1 \left\{ e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^2 - \eta_{mpq}^1) e^{j\alpha \lambda_{mz2}} + \frac{e^{-j\alpha \lambda_{mz1}} (\eta_{mpq}^1 + \eta_{mpq}^2) e^{-j\alpha \lambda_{mz2}}}{2\eta_{mpq}^2 e^{-j\alpha \lambda_{mz1}}} \right\} = \frac{E_{mpq}^2}{A}
\end{align*}

(3.24)

This can be simplified for manipulation by renaming the coefficients of \( T_{mpq}^1 \) and \( R_{mpq}^1 \) such that

\[ R_{mpq}^1 R_{mpq}^D + T_{mpq}^1 R_{mpq}^y = \frac{E_{mpq}^2}{A} \]  

(3.25)
Substituting eqns. (3.21) and (3.22) into (3.11) yields

\[
\frac{R_{mpq}^2 e^{j\gamma_{mpq}^2} (\eta_{mpq}^1 + \eta_{mpq}^2) + T_{mpq}^2 e^{-j\gamma_{mpq}^2} (\eta_{mpq}^1 - \eta_{mpq}^2)}{2\eta_{mpq} e^{j\gamma_{mpq}^2}} + \frac{R_{mpq}^2 e^{j\gamma_{mpq}^2} (\eta_{mpq}^1 - \eta_{mpq}^2) + T_{mpq}^2 e^{-j\gamma_{mpq}^2} (\eta_{mpq}^1 + \eta_{mpq}^2)}{2\eta_{mpq} e^{j\gamma_{mpq}^2}} = \frac{\bar{E}_{mpq}^1}{A}
\]

Rearranging gives

\[
R_{mpq}^2 \left\{ \frac{e^{j\gamma_{mpq}^2} (\eta_{mpq}^1 + \eta_{mpq}^2)}{2\eta_{mpq} e^{j\gamma_{mpq}^2}} + \frac{e^{-j\gamma_{mpq}^2} (\eta_{mpq}^1 - \eta_{mpq}^2)}{2\eta_{mpq} e^{-j\gamma_{mpq}^2}} \right\} + T_{mpq}^2 \left\{ \frac{e^{-j\gamma_{mpq}^2} (\eta_{mpq}^1 - \eta_{mpq}^2)}{2\eta_{mpq} e^{j\gamma_{mpq}^2}} + \frac{e^{j\gamma_{mpq}^2} (\eta_{mpq}^1 + \eta_{mpq}^2)}{2\eta_{mpq} e^{-j\gamma_{mpq}^2}} \right\} = \frac{\bar{E}_{mpq}^1}{A}
\]

which can be simplified to

\[
R_{mpq}^2 R_{mpq}^L + T_{mpq}^2 V_{mpq}^R = \frac{\bar{E}_{mpq}^1}{A}
\]

Expressions can now be formed for \( T_{mpq}^1 \), \( T_{mpq}^2 \), \( R_{mpq}^1 \) and \( R_{mpq}^2 \) in terms of the modal admittances and propagation constants.

Substituting \( R_{mpq}^1 \) from eqn. (3.11) in eqn (3.25)
\[ R_{mpq}^D \left( \frac{\tilde{E}_{mpq}^1}{A} - T_{mpq}^1 \right) + T_{mpq}^1 R_{mpq}^y = \frac{\tilde{E}_{mpq}^2}{A} \] (3.29)

which gives

\[ T_{mpq}^1 = \frac{1}{A} \left( \frac{\tilde{E}_{mpq}^2 - R_{mpq}^D \tilde{E}_{mpq}^1}{(R_{mpq}^y - R_{mpq}^D)} \right) \] (3.30)

Similarly, substituting \( T_{mpq}^1 \) from eqn (3.11) into eqn (3.25)

\[ R_{mpq}^1 \left( R_{mpq}^D - R_{mpq}^y \right) = \frac{\tilde{E}_{mpq}^2}{A} - R_{mpq}^y \frac{\tilde{E}_{mpq}^1}{A} \] (3.31)

\[ R_{mpq}^1 = \frac{1}{A} \left( \frac{\tilde{E}_{mpq}^2 - R_{mpq}^y \tilde{E}_{mpq}^1}{(R_{mpq}^D - R_{mpq}^y)} \right) \] (3.32)

Now, from eqn (3.14)

\[ T_{mpq}^2 = \frac{\tilde{E}_{mpq}^2}{A} e^{j\gamma z^2} - R_{mpq}^2 e^{2j\gamma z^2} \] (3.33)

which, when substituted in eqn. (3.28) gives

\[ R_{mpq}^2 R_{mpq}^L + \left( \frac{\tilde{E}_{mpq}^2}{A} e^{j\gamma z^2} - R_{mpq}^2 e^{2j\gamma z^2} \right) = \frac{\tilde{E}_{mpq}^1}{A} \] (3.34)

yielding

\[ R_{mpq}^2 = \frac{1}{A} \left( \frac{\tilde{E}_{mpq}^1 - E_{mpq}^2 R_{mpq}^L e^{j\gamma z^2}}{R_{mpq}^L - R_{mpq}^2 e^{2j\gamma z^2}} \right) \] (3.35)
Similarly, from eqn. (3.14)

\[
R_{mpq}^2 = \frac{\vec{E}_{mpq}^2}{A} e^{-j\gamma\text{m} z_2} - T_{mpq}^2 e^{-2j\gamma\text{m} z_2}
\]  

(3.36)

which, when substituted into eqn. (3.28) gives

\[
R_{mpq}^L \left( \frac{\vec{E}_{mpq}^2}{A} e^{-j\gamma\text{m} z_2} - T_{mpq}^2 e^{-2j\gamma\text{m} z_2} \right) + T_{mpq}^2 R_{mpq}^V = \frac{\vec{E}_{mpq}^1}{A}
\]

(3.37)

yielding

\[
T_{mpq}^2 = \frac{1}{A} \left( \frac{\vec{E}_{mpq}^1 - \vec{E}_{mpq}^2 R_{mpq}^L e^{-j\gamma\text{m} z_2}}{R_{mpq}^V - R_{mpq}^L e^{-2j\gamma\text{m} z_2}} \right)
\]

(3.38)

Expressions also need to be obtained for \( T_{mpq}^3 \) and \( R_{mpq}^3 \) to form the MFIE at \( z = z_2 \).

At \( z = z_3 \)

\[
R_{mpq}^3 e^{jr\text{m} z_3} + T_{mpq}^3 e^{-jr\text{m} z_3} = T_{mpq}^+ e^{-jr\text{m} z_3}
\]

(3.39)

and

\[
\eta_{mpq}^3 \left( T_{mpq}^3 e^{-jr\text{m} z_3} - R_{mpq}^3 e^{jr\text{m} z_3} \right) = \eta_{mpq}^0 T_{mpq}^+ e^{-jr\text{m} z_3}
\]

(3.40)

Multiplying eqn. (3.38) by \( \eta_{mpq}^0 \) and subtracting eqn. (3.39) leads to

\[
R_{mpq}^3 e^{jr\text{m} z_3} \left( \eta_{mpq}^0 + \eta_{mpq}^3 \right) + T_{mpq}^3 e^{-jr\text{m} z_3} = 0
\]

(3.41)
3.3.3 Formulation of the Integral Equations

It is now possible to form the two MFIE's at \( z=0 \) and \( z=z^2 \) by the application of the appropriate boundary condition. The boundary condition applied is that at \( z=0 \) and \( z=z^2 \) the tangential electric and magnetic fields are continuous across the aperture regions. The continuity of the magnetic field at \( z=0 \) is used and the following equation is applied:

\[
H^- (r,0) = H^* (r,0)
\]  

(3.43)

When this condition is applied using eqns. (3.2) and (3.4), and \( R_{mpq}^- \) is substituted in terms of \( T_{mpq} \), \( R_{mpq}^l \) and \( T_{m00}^{inc} \) the following expression is derived for the MFIE at \( z=0 \)

\[
2 \sum_{m00} \frac{\eta_{m00}^{0} T_{m00}^{inc} \Psi_{00}(r)}{\sqrt{\kappa_{m00}}} \hat{z} \times \kappa_{m00} \\
= \sum_{mpq} \left( T_{mpq}^l \left( \eta_{mpq}^{l} + \eta_{mpq}^{0} \right) + R_{mpq}^l \left( \eta_{mpq}^{0} - \eta_{mpq}^{l} \right) \right) \Psi_{pq}(r) \hat{z} \times \kappa_{mpq}
\]  

(3.44)

Using eqns. (3.30) and (3.32) one arrives at
\[ 2 \sum_{m00} \eta_{m00}^0 T_{m00}^{inc} \Psi_{00}(r) \hat{z} \times \kappa_{mpq} \]

\[ = \sum_{mpq} \frac{\tilde{E}_{mpq}^2}{A} \left( \left( \frac{\eta_{mpq}^1 + \eta_{mpq}^0}{(R_{mpq}^Y - R_{mpq}^D)} \right) + \left( \frac{\eta_{mpq}^0 - \eta_{mpq}^1}{(R_{mpq}^D - R_{mpq}^Y)} \right) \right) \Psi_{mpq}(r) \hat{z} \times \kappa_{mpq} \]

\[ - \sum_{mpq} \frac{\tilde{E}_{mpq}^3}{A} \left( \frac{R_{mpq}^D (\eta_{mpq}^1 + \eta_{mpq}^0)}{(R_{mpq}^Y - R_{mpq}^D)} + \frac{R_{mpq}^Y (\eta_{mpq}^0 - \eta_{mpq}^1)}{(R_{mpq}^D - R_{mpq}^Y)} \right) \Psi_{mpq}(r) \hat{z} \times \kappa_{mpq} \]

(3.45)

where,

\[ R_{mpq}^D = \frac{e^{i\gamma \zeta_1} (\eta_{mpq}^1 + \eta_{mpq}^0) e^{i\beta \zeta_2}}{2\eta_{mpq}^2 e^{i\gamma \zeta_1}} + \frac{e^{i\gamma \zeta_1} (\eta_{mpq}^1 - \eta_{mpq}^0) e^{-i\beta \zeta_2}}{2\eta_{mpq}^2 e^{-i\gamma \zeta_1}} \]

(3.46)

and

\[ R_{mpq}^Y = \frac{e^{-i\gamma \zeta_1} (\eta_{mpq}^2 - \eta_{mpq}^1) e^{i\beta \zeta_2}}{2\eta_{mpq}^2 e^{-i\gamma \zeta_1}} + \frac{e^{-i\gamma \zeta_1} (\eta_{mpq}^1 + \eta_{mpq}^2) e^{-i\beta \zeta_2}}{2\eta_{mpq}^2 e^{-i\gamma \zeta_1}} \]

(3.47)

Eqn. (3.45) is the MFIE at \( z=0 \)

To form the MFIE at \( z=z_2 \), the boundary condition applied is

\[ H^2(r, z_2) = H^3(r, z_2) \]

(3.48)

from which, by equating eqns. (3.6) and (3.8), one obtains
The expression on the right hand side of eqn. (3.48) can be reduced to one containing known quantities and $\vec{E}_{mpq}^2$ by using eqn. (3.41) and the continuity of the tangential electric field at $z=z_2$. This results in:

$$\sum_{mpq} \eta_{mpq}^3 \left( T_{mpq} e^{-j\lambda_2 z_2} - R_{mpq} e^{j\lambda_2 z_2} \right) \Psi_{pq}(r) \hat{z} \times K_{mpq}$$

$$= \sum_{mpq} \eta_{mpq}^3 \left( T_{mpq} e^{-j\lambda_2 z_2} - R_{mpq} e^{j\lambda_2 z_2} \right) \Psi_{pq}(r) \hat{z} \times K_{mpq}$$

(3.49)

The MFIE at $z=z_2$, therefore becomes

$$\sum_{mpq} \eta_{mpq}^2 \left( T_{mpq} e^{-j\lambda_2 z_2} - R_{mpq} e^{j\lambda_2 z_2} \right) \Psi_{pq}(r) \hat{z} \times K_{mpq}$$

$$= \sum_{mpq} \eta_{mpq}^3 \frac{\vec{E}_{mpq}^2 R_{mpq}^H}{AR_{mpq}^G} \Psi_{pq}(r) \hat{z} \times K_{mpq}$$

(3.54)
Substituting eqns. (3.35) and (3.38) leads to

\[
\sum_{mpq} \eta_{mpq}^3 \frac{E^2_{mpq} R^H_{mpq}}{AR^G_{mpq}} \Psi_{pq}(\mathbf{r}) \hat{\mathbf{z}} \times \hat{\mathbf{K}}_{mpq} \\
= \sum_{mpq} \frac{E^2_{mpq}}{A} \left[ \frac{\eta_{mpq}^2 e^{-jr^2_{mz_2}}}{R^V_{mpq} - R^L_{mpq} e^{-2jr^2_{mz_2}}} - \frac{\eta_{mpq}^2 e^{jr^2_{mz_2}}}{R^L_{mpq} - R^V_{mpq} e^{2jr^2_{mz_2}}} \right] \Psi_{pq}(\mathbf{r}) \hat{\mathbf{z}} \times \hat{\mathbf{K}}_{mpq} \\
- \sum_{mpq} \frac{E^1_{mpq}}{A} \left[ \frac{R^L_{mpq} \eta_{mpq}^2 e^{-2jr^2_{mz_2}}}{R^V_{mpq} - R^L_{mpq} e^{-2jr^2_{mz_2}}} - \frac{R^V_{mpq} \eta_{mpq}^2 e^{2jr^2_{mz_2}}}{R^L_{mpq} - R^V_{mpq} e^{2jr^2_{mz_2}}} \right] \Psi_{pq}(\mathbf{r}) \hat{\mathbf{z}} \times \hat{\mathbf{K}}_{mpq}
\]

(3.55)

Rearranging and collecting terms, produces the MFIE at z=z_2

\[
\sum_{mpq} \frac{E^1_{mpq}}{A} \left[ \frac{\eta_{mpq}^2 e^{-jr^2_{mz_2}}}{R^V_{mpq} - R^L_{mpq} e^{-2jr^2_{mz_2}}} - \frac{\eta_{mpq}^2 e^{jr^2_{mz_2}}}{R^L_{mpq} - R^V_{mpq} e^{2jr^2_{mz_2}}} \right] \Psi_{pq}(\mathbf{r}) \hat{\mathbf{z}} \times \hat{\mathbf{K}}_{mpq} \\
- \sum_{mpq} \frac{E^2_{mpq}}{A} \left[ \frac{\eta_{mpq}^2 R^L_{mpq} e^{-2jr^2_{mz_2}}}{R^V_{mpq} - R^L_{mpq} e^{-2jr^2_{mz_2}}} - \frac{\eta_{mpq}^2 R^V_{mpq} e^{2jr^2_{mz_2}}}{R^L_{mpq} - R^V_{mpq} e^{2jr^2_{mz_2}}} + \frac{\eta_{mpq}^3 R^H_{mpq}}{R^G_{mpq}} \right] \Psi_{pq}(\mathbf{r}) \hat{\mathbf{z}} \times \hat{\mathbf{K}}_{mpq} = 0
\]

(3.56)

where

\[
R^L_{mpq} = \frac{e^{jr^2_{mz_1} \left( \eta_{mpq}^1 + \eta_{mpq}^2 \right)}}{2\eta_{mpq}^1 e^{jr^2_{mz_1}}} + \frac{e^{jr^2_{mz_1} \left( \eta_{mpq}^1 - \eta_{mpq}^2 \right)}}{2\eta_{mpq}^1 e^{-jr^2_{mz_1}}}
\]

(3.57)

and

\[
R^V_{mpq} = \frac{e^{-jr^2_{mz_1} \left( \eta_{mpq}^1 - \eta_{mpq}^2 \right)}}{2\eta_{mpq}^1 e^{jr^2_{mz_1}}} + \frac{e^{-jr^2_{mz_1} \left( \eta_{mpq}^1 + \eta_{mpq}^2 \right)}}{2\eta_{mpq}^1 e^{-jr^2_{mz_1}}}
\]

(3.58)
The transmission coefficient is given by:

\[
T^+ = \frac{e^{i\sigma_1}}{A} \left( R_{mpq}^F e^{i\sigma_2} + e^{-i\sigma_2} \right) \left( \bar{E}_m \chi_{mpq}^A - \bar{E}_m \chi_{mpq}^B \right) \quad (3.59)
\]

where,

\[
\chi_{mpq}^A = \frac{R_{mpq}^V e^{i\sigma_2}}{R_{mpq}^G \left( R_{mpq}^L - R_{mpq}^V e^{2i\sigma_2} \right) + R_{mpq}^G \left( R_{mpq}^L - R_{mpq}^V e^{-2i\sigma_2} \right)} \quad (3.60)
\]

and

\[
\chi_{mpq}^B = \frac{R_{mpq}^V e^{2i\sigma_2}}{R_{mpq}^G \left( R_{mpq}^L - R_{mpq}^V e^{2i\sigma_2} \right) + R_{mpq}^G \left( R_{mpq}^L - R_{mpq}^V e^{-2i\sigma_2} \right)} \quad (3.61)
\]

The two MFIE’s can now be coupled into a single matrix equation, which is then reduced to a system of linear equations by the Method of Moments. The procedure is the same as that outlined in Section 2.3.4.

### 3.3.4 Waveguide Modes as Basis Functions for Rectangular Apertures

The linear dipole conductors in Chapter 2 were represented by entire domain sinusoidal and cosinusoidal basis functions but for the apertures waveguide modes have been employed. These allow for the modelling of two-dimensional patch elements, which is why they were chosen, although only linear dipoles have been considered for the results. This section outlines the waveguide modes used to represent the aperture dipoles and the equivalence between the sinusoids is noted. Fig. 3.4 shows the conventions adopted in the use of the waveguide modes.
The tangential electric fields within the aperture regions are represented as a summation over both TE and TM waveguide modes, which are described as follows:

**TM modes:**

\[
E_{y_{TM}}^{TM} = -\left(\frac{2}{a}\right)\left(\frac{m}{c}\right)\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right) \tag{3.59}
\]

and

\[
E_{x_{TM}}^{TM} = -\left(\frac{2}{b}\right)\left(\frac{n}{c}\right)\sin\left(\frac{m\pi x'}{a}\right)\cos\left(\frac{n\pi y'}{b}\right) \tag{3.60}
\]

**TE modes:**

\[
E_{x_{TE}}^{TE} = \sqrt{\frac{\epsilon_m \epsilon_n}{b}}\left(\frac{n}{c}\right)\cos\left(\frac{m\pi x'}{a}\right)\sin\left(\frac{n\pi y'}{b}\right) \tag{3.61}
\]

and
\[ E_{\text{TE}}^{m,n} = -\frac{\sqrt{e_m e_n}}{a} \left( \frac{m}{c} \right) \sin \left( \frac{m\pi x'}{a} \right) \cos \left( \frac{n\pi y'}{b} \right) \] (3.62)

where \( x' = x - \frac{a}{2}, y' = y - \frac{b}{2} \), \( m \) and \( n \) are the waveguide mode indices, \( e_m \) and \( e_n \) are the Neumann factors such that:

\[
e_{m,n} = 1: m, n = 0
\]
\[
e_{m,n} = 2: m, n \neq 0
\] (3.63)

and

\[
c = \sqrt{\frac{m^2 b}{a} + \frac{n^2 a}{b}}
\] (3.64)

These waveguide modes have equivalent sinusoids which enables the same modes to be modelled as for the conducting dipoles i.e. the zero ended sinusoids and cosinusoids. The waveguide modes and their sinusoid equivalents are set out in table 3.1.

<table>
<thead>
<tr>
<th>Waveguide mode</th>
<th>Corresponding Sinusoid</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE_{10}</td>
<td>\cos(\pi x/a)</td>
</tr>
<tr>
<td>TE_{20}</td>
<td>\sin(2\pi x/a)</td>
</tr>
<tr>
<td>TE_{30}</td>
<td>-\cos(3\pi x/a)</td>
</tr>
<tr>
<td>TE_{40}</td>
<td>-\sin(4\pi x/a)</td>
</tr>
<tr>
<td>TE_{50}</td>
<td>\cos(5\pi x/a)</td>
</tr>
<tr>
<td>TE_{60}</td>
<td>Sin(6\pi x/a)</td>
</tr>
</tbody>
</table>

Table 3.1. Waveguide modes and equivalent entire domain sinusoids
3.4 Representative Results

In this section results are presented for linear dipole elements in both the RFSS and close coupled configurations. It will be seen that, in general, the performance is not as good as that achieved using conducting elements insofar as losses are introduced with the relative displacement of one layer to another. The same terminology applies as in Chapter 2 with reference to element and lattice dimensions but because the elements are rotated 90° the displacements $DSY$ and $DSX$ now apply to the width and length of the elements respectively. This is depicted in Fig. 3.5.

![Fig. 3.5. Plan view of two-layer aperture FSS. Dotted elements represent layer 2](image)

$DSX$ and $DSY$ indicate the possibility of a two dimensional shift, and this is investigated in the close coupled results. The RFSS results rely only on $DSX$ (along the element length). The incident electric field is polarised along the $y$ direction as is required to induce the electric field in the apertures to be along the small dimension.
3.4.1 Results for Reconfigurable Aperture FSS

The experimental set up used to test the performance of the RFSS apertures is the same used for the conductors and was detailed in Section 2.4 so will not be revisited here. The arrays considered were identical in each layer being linear dipoles of length 3.5mm, width 0.3mm arranged on a square lattice of 5mm periodicity. Each was etched from a 9µm copper sheet supported on a polyester dielectric substrate of thickness 50µm, relative permittivity 3 and tanδ of -0.002. They were placed in the positioning jig and every effort was made to ensure a uniform separation (air gap) between the arrays. In these results DSY is zero at all times and the effect of only DSX is considered. In the model 17 Floquet modes for each layer were found to be sufficient to produce convergence of the results. The following modes were used for the fields within the aperture regions: TE_{10}, TE_{20}, TE_{30}, TE_{40} and TE_{50}. These represent the entire domain basis functions as shown in table 3.1. Fig. 3.6 shows comparisons between measured and predicted results for normal and TE:45° incidence with DSX =0. It can be seen from Fig. 3.6a that at normal incidence the agreement is very good in the higher frequency regions especially around the passband at about 37GHz and the null at about 39GHz. The agreement is not quite so good, however, at lower frequencies below about 30GHz where the measurements exhibit undulations. This is not surprising when the multilayer nature is considered. The fact that the arrays are apertures etched from a copper sheet will allow the possibility of reflections developing within the RFSS and possibly outside as well if the sheets are not extremely flat. This emphasises the importance of ensuring all surfaces are completely flat and the air-gap is uniform.

Fig. 3.6b shows the effect of changing the incidence of the electric field to TE:45°. The instability of the passband is shown and there is a loss at the inband resonance of about 2.5dB at 35GHz. This is because the grating lobes have been brought into play by the increased angle of incidence, thereby destabilising the resonance. The agreement between measured and theoretical data is still very good above about 30GHz with the model accurately following the trends of the measurement. Below 30GHz, however, the effects of the reflections within the structure have been amplified by the increased angle of incidence.
Figs 3.7a and b show the same two scenarios, but with a displacement, $DSX$, of 1.5mm introduced between the two layers. Fig. 3.7a shows the transmission response at normal incidence and exhibits a distinct narrowing of the passband at around 35.5GHz. Unfortunately there is now a loss of about 7dB on the measured result which cannot all be accounted for by the loss tangent as the model only predicts about 3dB loss. The excess 4dB is likely to be due to dissipation within the dielectrics and air-gap due to the very strong coupling fields between the arrays. Increasing the incidence to TE:45° in this instance completely destabilises the resonance and introduces a loss of more than 10dB when compared to the unshifted, normal incidence case in Fig. 3.6a. The undulations are again prominent but have now moved higher in frequency and start at about 32GHz.

![Graph showing transmission coefficient vs frequency]

**Fig.3.6a. RFSS Apertures: $DSX=0$, Normal Incidence**
Fig. 3.6b. RFSS Apertures: $DSX=0$, TE: $45^\circ$ incidence

Fig. 3.7a. RFSS Apertures: $DSX=1.5\text{mm}$, Normal incidence
3.4.2 Results for Close Coupled Aperture FSS

Arrays of the same dimensions as in the previous section were used to study the close-coupled aperture FSS. This was to enable a detailed study of the effects of the relative displacement to be undertaken. Four different close couple FSS are considered here, the differences between them being in $DSX$ and $DSY$ and the substrate. Table 3.2 gives details of them. Arrays A, B and C are etched from 1/4oz copper on a 75μm polyester substrate whilst array D is etched on a considerably lower loss Kapton substrate.

Array A is aligned so that it resembles, optically, a single layer. Figs. 3.8a-c display the measured and predicted transmission responses for this array for normal, TE:45° and TM:45° incidences. At normal incidence the array resonates at around 36GHz and it's behaviour is similar to that expected of a single layer with the same element and lattice geometries. There is an inband loss of 0.5dB at 36GHz and the percentage bandwidth is
11.4%. An interesting feature is the pronounced step at around 24GHz, which is not observed in the single layer case. The predictions give an excellent match to the measured results across the frequency range of interest. As the angle of incidence is increased there is

<table>
<thead>
<tr>
<th>Close Coupled Array</th>
<th>DSX(mm)</th>
<th>DSY(mm)</th>
<th>Substrate thickness(µm)</th>
<th>Loss tangent</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td>0</td>
<td>75</td>
<td>-0.004</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>2.5</td>
<td>75</td>
<td>-0.004</td>
</tr>
<tr>
<td>C</td>
<td>2.5</td>
<td>0</td>
<td>75</td>
<td>-0.004</td>
</tr>
<tr>
<td>D</td>
<td>2.5</td>
<td>0</td>
<td>108</td>
<td>-0.002</td>
</tr>
</tbody>
</table>

Table 3.2. Description of displacement parameters for four close coupled aperture arrays

Fig. 3.8a. Close Coupled Array A: normal incidence
Fig. 3.8b. Close Coupled Array A: TE: 45° incidence

Fig. 3.8c. Close Coupled Array A: TM: 45° incidence
a marked disruption to the shape of the curve. This is true for both TE:45° and TM:45° incidences. For TE:45° the array is very much more lossy than at normal incidence, and the fact that the model has not captured all the loss seems to indicate that it is largely due to dissipated losses within the dielectric. However, at TM:45°, there is no increase in the loss but there is more disruption to the shape of the curve as the passband has moved from 36GHz to 32.5GHz. This instability is due to the encroachment of the grating lobe, which can be seen clearly near 35GHz. In both cases there is good agreement between measured and predicted data, although this is especially true for TM:45°, but at lower frequencies below about 25GHz, the measured data exhibits similar undulations to those observed in the RFSS case. Nonetheless the theory follows the measured results very well.

Figs. 3.9a-c show the effect of introducing a shift of $DSY=2.5\text{mm}$ ($DY/2$). Compared to Fig. 3.8 the effect is extreme, moving the passband to 18GHz, which represents a shift of about 50%. There is also a reduction in bandwidth from 11.4% to 3.1%. There are, however, large losses in the passband of about 12dB. This loss is probably due to two factors. The strong fields within the separation region will amplify any losses due to the dielectric within the passband which has a high Q. A further contribution, though smaller, may come from the contribution of adhesive layers used in the manufacturing process to bond the copper to the dielectric. In order to match the measured with predicted data it is necessary to model a loss tangent of 0.02, which is five times greater than the quoted value. A loss tangent of 0.004 reduces the loss to about 1.8dB and Fig. 3.9a clearly demonstrates the effect of a zero loss substrate where there is zero loss at resonance and a bandwidth of less than 1%. Figs. 3.9b and 3.9c show that the passband stability has increased due to the introduction of $DSX$. It is interesting to note that compared to the normal incidence case TM:45° incidence exhibits less loss at around 10dB whilst TE:45° incidence exhibits more at around 15dB. In most cases the agreement between measurement and theory is very good with the only exception being for the TM:45° case where there is a pronounced peak in the measurements at about 34GHz.
Fig. 3.9a. Close Coupled Array B, normal incidence

Fig. 3.9b. Close Coupled Array B, TM:45° incidence
The final set of results looks at arrays C and D. They have the same geometries with $DSY=0$ and $DSX=2.5\text{mm}$ but they are etched on different substrates, with array D being on the low loss Kapton dielectric. Figs. 3.10a and 3.10b show the comparison between arrays C and D at normal incidence. The first thing to note is that there are two passbands with one at about 38GHz and the other at about 19GHz, whereas Fig. 3.8a shows the unshifted array to resonate at 36GHz. The main resonance is that at 19GHz which will be seen to be more stable than that at 38GHz due to the proximity of the grating lobe region. The low frequency side of the higher resonance could just be seen between 35 and 40 GHz in Figs. 3.9a and b. In terms of the resonant frequency and response shapes the predicted and measured results match up very closely. The arrays resonate at the same frequencies but there is a difference of around 3.5dB in their respective losses, due to the Kapton used for array D having much lower loss than the polyester used for array C. The effect that a very
Fig. 3.10a. Close Coupled Array C, normal incidence

Fig. 3.10b. Close Coupled Array D, normal incidence
lossless substrate would have is also shown by the inclusion of the predicted response with a zero loss tangent. The same bandwidth savings have been made as that with array B i.e. from 11.4% to 3.1%.

When the incidence is changed to TE:45° (Figs. 3.11a and b) and TM:45° (Figs. 3.12a and b) there is a marked change and deterioration in array performance except for Array D TM:45°. For TE:45° the two passbands generally tend to move together and coalesce and there is no resonance really to speak of. The upper resonance, which is affected by the grating lobes reduces to virtually nothing. For array C the lower resonance shifts up in frequency by about 14% and the loss is increased by about 5dB whilst array D experiences the same frequency shift but only about 2.5dB of increased loss. For TM:45° incidence the lower resonance of array C shifts down in frequency by about 8% but again there is an increased loss of about 2.5dB. Array D, on the other hand, demonstrates a reasonably stable response at this incidence. There is a downward frequency shift of about 4% but no significant increase in the passband loss. Primarily, this is due to the fact that the Kapton is a lower loss substrate than the polyester but also that the shift in DSY is less susceptible to oblique TM incidence than TE.

Overall the results for the four close coupled aperture arrays indicate that narrow passbands and increased stability are achievable with this method, although the direction of the relative displacement between arrays is significant. This is highlighted by the difference in performance between arrays B and D. Whilst the passband performance of array D is generally better, it appears that the most desirable displacement would be in DSY rather than DSX. The benefits are that there are no higher frequency effects near the main resonance. Figs 3.9 a-c clearly show that there is very little interference from these sources except that for TM:45° incidence which has been commented upon. This is probably due to the displacement DSY being in the same direction as the incident (and aperture) electric fields. It was noted in Chapter 2 that in order to produce the maximum effects it was necessary to displace one array in the same direction as the incident field and that seems to be the same in this aperture case.
Fig. 3.11a. Close Coupled Array C. TE:45° incidence

Fig. 3.11b. Close Coupled Array D, TE:45° incidence
Fig. 3.12a. Close Coupled Array C, TM: 45° incidence

Fig. 3.12b. Close Coupled Array D, TM: 45° incidence
3.5 Conclusions

In this chapter a two layered aperture FSS has been examined. Two configurations were discussed; a reconfigurable FSS making use of two separate single layer arrays and a close-coupled FSS that uses two arrays on a single dielectric sheet. The objective was to take advantage of the strong coupling between the tangential electric fields within the aperture regions of the arrays in a similar fashion to the currents on the conducting elements studied in Chapter 2. A model has been developed to cater for both scenarios, which relied on the formulation of two, coupled, magnetic field integral equations in terms of the unknown fields within the apertures. These are then solved using the Method of Moments with the unknown fields represented by two-dimensional waveguide modes.

Results have been presented for both the reconfigurable and close-coupled FSS's. The results for the reconfigurable case did not produce the same magnitude of effect as was observed in Chapter 2 with the conductors. Certain problems have been identified associated with this situation. Primarily these are the necessity to keep the predominantly copper arrays very flat and also to keep the air-gap between them constant across the whole array. Further, there was a lot of inband loss observed in almost all cases and this cannot be accounted for by the substrate losses themselves, although the impact of a very low loss substrate has been discussed. A detailed examination of the fields within the dielectric separation region would be needed to accurately assess the origin of these losses, as it is likely to be due to the strong fields set up with the introduction of $DSX$ and $DSY$. The close-coupled results were very promising, and have been published [6]. The advantage over the reconfigurable FSS being that the relative displacements are incorporated during manufacture and maximum coupling is encouraged by the integrated nature of the structure. Results have shown that it is possible to achieve percentage bandwidth reductions from 11.4%, in the undisplaced case, to 3.1%, and although this is accompanied by substantial inband losses it has been shown theoretically that with a lossless dielectric the bandwidth could be further reduced to about 1.25%. The angular stability of the structures has been calculated at about 4% at TM:45° for array D and the significance of the displacement being in the same direction as the incident electric field has been commented upon.
REFERENCES


CHAPTER 4

Complementary Frequency Selective Surfaces

4.1 Introduction

In this chapter aspects of a complementary frequency selective surface (CFSS) are examined. As its name suggests the CFSS is, effectively, a hybrid of the close coupled FSS's covered in Chapters 2 and 3 whereby a layer of aperture elements and a layer of conducting elements are etched either side of a supporting dielectric substrate. Little work has been published on the use of complementary elements, although similar structures have been investigated [1]. The elements need not necessarily be the same either side of the substrate but in the cases studied here they are. The CFSS takes advantage of the interaction between the different elements to produce very strong fields in the separation region and a resonant frequency that is much lower than that of a single layer array (aperture or conductor). As a result, the resonance is far from the grating region and excellent angular stability can be achieved for both TE and TM incidences.

Section 4.2 will introduce, and give an overview of, the CFSS concept and will highlight the fundamental differences from the RFSS and CCFSS discussed in Chapters 2 and 3. The structure of the CFSS is, in fact, similar to the close coupled FSS in that two arrays are
supported on the same substrate. For the CFSS, however, there is one layer of conductors complemented by a layer of apertures. It will be appreciated that for polarised elements, such as the linear dipole or tripole, the conductors and apertures must be rotated by 90° with respect to each other in order that they will both be resonant. This does not apply to symmetrical elements like rings or square loops. When the electromagnetic boundary conditions are applied at each interface, this gives rise again to two integral equations. In this case a MFIE is formed for the fields within the apertures and an EFIE for the currents on the conductors. These are solved in the normal fashion using the Method of Moments [2] for the unknown fields and currents. The electromagnetic analysis is detailed in section 4.3.

Representative results are presented in section 4.4. Results obtained from the model are compared with those measured for different CFSS’s of linear dipoles and single rings. Comparisons are made for both TE and TM incidences of 30° and 45°, as well as for normal incidence. The angular stability is compared to that achieved with the close-coupled FSS in the previous chapters and found to be much superior. The CFSS, as described in this section is the subject of a UK patent application [3].

4.2 CFSS Concept

It has been seen in the previous chapters that by placing two identical arrays in close proximity very stable responses can be achieved. The hybrid resonance appears far from the influence of the grating lobe region, which is related to the array periodicity. In effect, the resonance is due to that of an element which is electrically much larger than the elements employed in the design. The CFSS again makes use of this phenomenon but the interaction between the complementary elements results in ultra stable responses for both TE and TM incidences due to the contributions from both the elements in the unit cell.

The basic CFSS structure is the same as that of the close coupled FSS. Array patterns are etched from the copper glued to either side of a polyester substrate, which acts to support both layers. A schematic representation of the CFSS is shown in Fig. 4.1. In Fig. 4.1 the
aperture and conducting elements are shown, not to scale, either side of the supporting dielectric. In the case of each set of elements, they would be about 9μm thick, much thinner than the dielectric, but are depicted as being thick here for clarity.

Fig. 4.1. Representation of Complementary FSS Structure
Fig. 4.1. shows the principle that the CFSS is made up of two separate, complementary arrays. In isolation, when illuminated by an incident plane wave both arrays become resonant due to the currents and fields induced; and if the two sets of array dimensions are identical, as in the cases studied here, the resonant frequencies are the same. When they are in very close proximity the transmission responses produced by the two layers interact with one another across the frequency range. At the resonant frequency, when the conducting array is almost totally reflective and the aperture array almost totally transmissive, the total response is mainly reflective. This is true over most of the frequency range. However, at the frequencies where the two transmission responses intersect, they are of equal amplitude but out of phase. At these points the arrays interact very strongly and high field magnitudes are generated in the dielectric region which lead to the elements, and hence the CFSS becoming resonant. Because there are two intersections two resonances are produced (f_i and f_u) this is shown in Fig. 4.2.

![Fig. 4.2. Interaction of two arrays to produce CFSS response](image-url)
This shows the overlapping of the transmission responses of the different single layer arrays which independently would each have a resonant frequency at \( f_i \), provided the elements were identical in geometry. In the CFSS the arrays are very closely linked and the overall effect is to produce two separate and narrow passband responses at roughly the frequencies where the two transmission curves intersect, \( f_i \) and \( f_u \). The behaviour of these passbands is the key of the CFSS because they behave very differently. The lower band, centred around \( f_i \), is termed the main resonance for our purposes, as this is the one that exhibits the outstanding angular stability. This is due to the fact that the resonant state of the CFSS creates this band far away from the grating lobe region at the higher frequencies. The upper, or secondary, band at \( f_u \) is much closer to the grating lobes and they therefore have a considerable influence on it at oblique angles of incidence in terms of \( f_u \) shifting significantly when the angle of incidence is varied.

### 4.3 CFSS Theory

The procedure for obtaining the integral equations for the CFSS is similar to that in Chapters 2 and 3 in that the standard electromagnetic boundary conditions are applied to the structure at the boundaries between the different regions. A section of the CFSS is shown in Fig. 4.3 and this illustrates the conventions adopted for the various transmitted and reflected fields in the analysis. Included in the analysis is a supporting dielectric layer and the parameters relating to this are denoted by the superscript '1'. The parameters of the region outside the dielectric region, which is considered here to be air, have the superscript '0'.

Two integral equations are formed in terms of \( \vec{J} \) and \( \vec{E} \) which are the Floquet transforms of, respectively, the currents on the conductors at \( z=0 \), and the fields within the apertures at \( z=d \). The derived equations are solved for the unknown fields and currents by means of the method of moments [2].
The relationships of the fields in the different regions are as follows:

For \( z \leq 0 \):

\[
\mathbf{E}^- (r, z) = \mathbf{E}^{\text{inc}} - \sum_{mpq} R_{mpq} e^{j\frac{m}{l}\pi z} \psi_{pq}(r) \kappa_{mpq} \tag{4.1}
\]

\[
\mathbf{H}^- (r, z) = \mathbf{H}^{\text{inc}} - \sum_{mpq} \eta_{mpq} R_{mpq} e^{j\frac{m}{l}\pi z} \psi_{pq}(r) \hat{z} \times \kappa_{mpq} \tag{4.2}
\]

For \( 0 \leq z \leq d \):

\[
\mathbf{E}^I (r, z) = \sum_{mpq} \left( T_{mpq} e^{-j\frac{m}{l}z} + R_{mpq} e^{j\frac{m}{l}z} \right) \psi_{pq}(r) \kappa_{mpq} \tag{4.3}
\]

\[
\mathbf{H}^I (r, z) = \sum_{mpq} \eta_{mpq} \left( T_{mpq} e^{-j\frac{m}{l}z} - R_{mpq} e^{j\frac{m}{l}z} \right) \psi_{pq}(r) \hat{z} \times \kappa_{mpq} \tag{4.4}
\]

For \( z \geq d \):

\[
\mathbf{E}^+ (r, z) = \sum_{mpq} T_{mpq} e^{-j\frac{m}{l}z} \psi_{pq}(r) \kappa_{mpq} \tag{4.5}
\]

\[
\mathbf{H}^+ (r, z) = \sum_{mpq} \eta_{mpq} T_{mpq} e^{-j\frac{m}{l}z} \psi_{pq}(r) \hat{z} \times \kappa_{mpq} \tag{4.6}
\]
where,

\[
E^\text{inc} = \sum_{m=1}^{2} T^\text{inc}_{m00} e^{-j\beta_0 z} \Psi_{00} K_{m00} \tag{4.7}
\]

Fig. 4.3. Cross section of CFSS showing dielectric sandwiched between two arrays
4.3.1 The Fields Within the Dielectric

In order to derive the two coupled integral equations, expressions for the electric and magnetic, transmitted and reflected, fields within the dielectric region \((T_{mpq} \text{ and } R_{mpq})\) must first be obtained. These are found by the application of appropriate boundary conditions and the matching of the fields at \(z=0\) and \(z=d\). The boundary conditions are the same as those used in chapters two and three, but in this case they are used in conjunction. They are:

1. At \(z=0\), the tangential electric field is continuous and the tangential magnetic field is discontinuous. Further, the total electric field is zero on the conductors.

2. At \(z=d\), both the tangential electric and magnetic fields are continuous across the aperture regions.

These conditions can now be used to initially establish expressions for the fields within the dielectric region.

At \(z=0\),

\[
E^{inc}(r,0) = E^1(r,0) \tag{4.8}
\]

\[
T^{inc}_{m00} + R^-_{mpq} = T_{mpq} + R_{mpq} \tag{4.9}
\]

and

\[
H^-(r,0) = H^1(r,0) \tag{4.10}
\]
\[ \eta_{m00}^0 T_{m00}^{\text{inc}} - \eta_{mpq}^0 R_{mpq} = \eta_{mpq}^1 T_{mpq} - \eta_{mpq}^1 R_{mpq} + \frac{\tilde{J}_{mpq}}{A} \]  \hspace{1cm} (4.11) \\

Multiplying Eqn. (4.9) by \( \eta_{mpq}^0 \) and then adding to Eqn. (4.11) gives

\[ 2\eta_{m00}^0 T_{m00}^{\text{inc}} = T_{mpq} (\eta_{mpq}^1 + \eta_{mpq}^0) - R_{mpq} (\eta_{mpq}^1 - \eta_{mpq}^0) + \frac{\tilde{J}_{mpq}}{A} \]  \hspace{1cm} (4.12) \\

Now, at \( z=d \)

\[ E^1(r,d) = E^+(r,d) \]  \hspace{1cm} (4.13) \\

\[ T_{mpq} e^{-j \omega t_d^d} + R_{mpq} e^{j \omega t_d^d} = T_{mpq}^+ e^{-j \omega t_d^d} = \frac{\tilde{E}_{mpq}}{A} \]  \hspace{1cm} (4.14) \\

From Eqn. (4.14)

\[ T_{mpq} = \frac{\tilde{E}_{mpq}}{A} e^{j \omega t_d^d} - R_{mpq} e^{2j \omega t_d^d} \]  \hspace{1cm} (4.15) \\

Substituting Eqn. (4.15) in Eqn. (4.12) yields

\[ 2\eta_{m00}^0 T_{m00}^{\text{inc}} = \left[ \frac{\tilde{E}_{mpq}}{A} e^{j \omega t_d^d} - R_{mpq} e^{2j \omega t_d^d} \right] (\eta_{mpq}^1 + \eta_{mpq}^0) - R_{mpq} (\eta_{mpq}^1 - \eta_{mpq}^0) + \frac{\tilde{J}_{mpq}}{A} \]  \hspace{1cm} (4.16)
\[
R_{mpq} = \frac{\tilde{E}_{mpq}}{A \Omega_{mpq}} e^{j \omega t} (\eta_{mpq} + \eta_{mpq}^0) + \frac{j_{mpq}}{A \Omega_{mpq}} - \frac{2\eta_{m00}^0 T_{m00}^{inc}}{\Omega_{mpq}}
\] (4.18)

where,

\[
\Omega_{mpq} = e^{2j \omega t} (\eta_{mpq} + \eta_{mpq}^0) + (\eta_{mpq} - \eta_{mpq}^0) = (e^{2j \omega t} + \rho_{mpq}) (\eta_{mpq} + \eta_{mpq}^0)
\] (4.19)

and,

\[
\rho_{mpq} = \frac{\eta_{mpq}^1 - \eta_{mpq}^0}{\eta_{mpq} + \eta_{mpq}^0}
\] (4.20)

Having established an expression for \( R_{mpq} \) in terms of \( \tilde{J} \) and \( \tilde{E} \), an expression for \( T_{mpq} \) can now be developed. Dividing Eqn. (4.12) by \( (\eta_{mpq} + \eta_{mpq}^0) \) and rearranging,

\[
T_{mpq} = R_{mpq} \rho_{mpq} - \frac{j_{mpq}}{A (\eta_{mpq}^1 + \eta_{mpq}^0) + 2\eta_{m00}^0 T_{m00}^{inc}} + \frac{j_{mpq}}{\eta_{mpq}^1 + \eta_{mpq}^0}
\] (4.21)

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By substituting for $R_{mpq}$ from Eqn. (4.18) one gets,

$$T_{mpq} = \left[ \frac{\tilde{E}_{mpq}}{A\Omega_{mpq}} e^{i\rho_{mpq}} (\eta_{mpq}^1 + \eta_{mpq}^0) + \frac{\tilde{J}_{mpq}}{A\Omega_{mpq}} - \frac{2\eta_{m00}^0 T_{m00}^{inc}}{\Omega_{mpq}} \right] \rho_{mpq}$$

(4.22)

Collecting terms gives,

$$T_{mpq} = \frac{\tilde{E}_{mpq}}{A\Omega_{mpq}} e^{i\rho_{mpq}} (\eta_{mpq}^1 - \eta_{mpq}^0) + \frac{\tilde{J}_{mpq}}{A} \left[ \frac{\rho_{mpq}}{\Omega_{mpq}} - \frac{1}{\eta_{mpq}^1 + \eta_{mpq}^0} \right] + 2\eta_{m00}^0 T_{m00}^{inc} \left[ \frac{1}{\eta_{mpq}^1 + \eta_{mpq}^0} - \frac{\rho_{mpq}}{\Omega_{mpq}} \right]$$

(4.23)

which leads to the following expression for $T_{mpq}$

$$T_{mpq} = \frac{\tilde{E}_{mpq}}{A\Omega_{mpq}} e^{i\rho_{mpq}} (\eta_{mpq}^1 - \eta_{mpq}^0) - \frac{\tilde{J}_{mpq}}{A\Omega_{mpq}} e^{2i\rho_{mpq}} + 2\eta_{m00}^0 T_{m00}^{inc} e^{2i\rho_{mpq}}$$

(4.24)
4.3.2 Formulation of the Integral Equations

Having derived the expressions for $R_{mpq}$ and $T_{mpq}$ the boundary conditions can now be applied at $z=0$ and $z=d$ to obtain the integral equations related to each layer. Firstly, at $z=0$, we force the total electric field on the conducting regions of the unit cell to be zero, in order to derive the Electric Field Integral Equation (EFIE).

At $z=0$,

$$\vec{E}(r,0) = 0 \quad (4.25)$$

$$\sum_{mpq} (T_{mpq} + R_{mpq}) \Psi_{pq}(r) K_{mpq} = 0 \quad (4.26)$$

From Eqns. (4.18) and (4.24),

$$T_{mpq} + R_{mpq} = \frac{2\eta_{mpq}^i \vec{E}_{mpq} e^{jr_{mpq}^d}}{A\Omega_{mpq}} + \frac{\bar{T}_{mpq}}{A\Omega_{mpq}} (1 - e^{2jr_{mpq}^d}) \frac{2\eta_{m00}^{inc} T_{m00}^{inc}}{\Omega_{mpq}} \left(e^{2jr_{mpq}^d} - 1\right) \quad (4.27)$$

So that the (EFIE) at $z=0$ is

$$\sum_{m=1}^{2\eta_{m00}} \frac{2\eta_{m00}^0 T_{m00}^{inc}}{\Omega_{m00}} (1 - e^{jr_{mpq}^d}) \Psi_{m0}(r) K_{m00}$$

$$= \sum_{mpq} \left[ \frac{2\eta_{mpq}^i \vec{E}_{mpq} e^{jr_{mpq}^d}}{A\Omega_{mpq}} + \frac{\bar{T}_{mpq}}{A\Omega_{mpq}} (1 - e^{2jr_{mpq}^d}) \right] \Psi_{pq}(r) K_{mpq} \quad (4.28)$$
The Magnetic Field Integral Equation (MFIE) at $z=d$ is obtained by applying the condition that the magnetic field is continuous across the apertures, which leads to

$$
\eta_{mpq}^1 T_{mpq} e^{j \gamma_{mpq} d} - \eta_{mpq}^0 R_{mpq} e^{j \gamma_{mpq} d} - \eta_{mpq}^0 \frac{\vec{E}_{mpq}}{A} = 0
$$

(4.29)

By substituting for $R_{mpq}$ and $T_{mpq}$ and collecting terms we get,

$$
\frac{\vec{E}_{mpq}}{A} \left[ \rho_{mpq} \left( \eta_{mpq}^1 - \eta_{mpq}^0 \right) - e^{2 j \gamma_{mpq} d} \left( \eta_{mpq}^1 + \eta_{mpq}^0 \right) \right]
$$

(4.30)

$$
- \frac{2 j \eta_{mpq} \eta_{mpq}^1 e^{j \gamma_{mpq} d}}{A \Omega_{mpq}} + \frac{4 \eta_{mpq}^1 \eta_{mpq}^0 T_{m00} \Omega_{mpq}}{\Omega_{mpq}} e^{j \gamma_{mpq} d} = 0
$$

And the MFIE at $z=0$ is,

$$
\sum_{m=1}^{2} \frac{2 \eta_{m00}^1 \eta_{m00}^0 T_{m00} \Omega_{m00}}{\Omega_{m00}} e^{j \gamma_{m00} d} \Psi_{00}(r) z \times \kappa_{m00}
$$

(4.31)

$$
= \sum_{mpq} \left[ \frac{2 \eta_{mpq}^1 \eta_{mpq}^0 e^{j \gamma_{mpq} d}}{A \Omega_{mpq}} - \rho_{mpq} \overline{E}_{mpq} \left( \eta_{mpq}^1 - \eta_{mpq}^0 \right) - e^{2 j \gamma_{mpq} d} \left( \eta_{mpq}^1 + \eta_{mpq}^0 \right) \right] \Psi_{pq}(r) z \times \kappa_{mpq}
$$

The MFIE and the EFIE can now be coupled into a single matrix equation, which follows the same procedure as that described in Chapter 2. In this case it is the coefficient of $\overline{E}$ in
the EFIE and $\tilde{J}$ in the MFIE which describe the coupling between the conductors and apertures on the different layers.

4.4 Measurements of Representative Arrays

The key aspect of the CFSS is the coupling of the evanescent fields within the dielectric region which is achieved by having the elements closely spaced so that the fields due to each being resonant are forced to interact. Two elements will be examined in the CFSS configuration, being linear dipoles and single rings. Dipoles have been widely discussed in chapters two and three so will not be looked at in detail again here but comparisons will be drawn between theory and measurement in Section 4.4.1. The ring element has not been encountered previously and will be discussed in section 4.4.2 where comparative results will also be presented.

4.4.1 CFSS with Dipole Elements

Of all the potential elements for use in the CFSS the simplest is the linear dipole that has already been employed for the close coupled and RFSS in both conductor and aperture cases. Here, a conducting dipole is coupled with its free space Babinet complement the aperture dipole. Babinet's principle states, relating to optics "when the field behind a screen with an opening is added to the field of a complementary structure, the sum is equal to the field when there is no screen". Although the presence of the dielectric in this case means that the separate elements are not direct Babinet compliments the principle is preserved [4]. Babinet's principle in optics also takes no account of the polarisation of the radiation but such an omission cannot be made in this case since the orientation of the electric field is of fundamental importance.

Fig. 4.4 shows the array geometry and element orientation with respect to the incident electric field in this case. The apertures are rotated 90° with respect to the conductors so
that both elements will be polarised with the electric field as depicted and therefore resonant. The material used in the construction of the arrays consists of sheets approximately 280mm x 300mm which are made up of a layer of dielectric (polyester) 25μm thick sandwiched between two layers of 1/4oz copper (9μm). The dielectric has a relative permittivity of 3. The CFSS’s are etched using the techniques described in Section 2.2.1.

The unit cells of each array are identical being on a square lattice with periodicities $D_x$ and $D_y$ equal to 4mm. The element dimensions are also identical having $W_1=W_2=0.3\text{mm}$ and $L_1=L_2=3.5\text{mm}$. In the computer model three basis functions were used for each layer. For the conductors $\cos(\pi y/L_1), \sin(2\pi y/L_1)$ and $\cos(3\pi y/L_1)$ were used and for the apertures the
waveguide modes described in section 3.4.1 were used to reproduce the sinusoidal modes \( \cos(\frac{\pi}{L2}), \sin(2\frac{\pi}{L2}) \) and \( \cos(3\frac{\pi}{L2}) \). In each layer 13 Floquet modes were found to be sufficient per unit cell to cover the spectra of these bases. The maximum coupling between the dipoles is achieved when their centres are coincident because the maxima of the dominant (cosinusoidal) basis functions is at this point and it is desirable for optimum performance to have current and field maxima as proximate as possible. Due to this, there is no need for a relative shift of the spectra of the Fourier transforms of the basis functions on different layers to facilitate coupling so \( DSX \) and \( DSY \) are both zero in the case of the dipoles, although this is not the case for the rings.

Figs. 4.5a-4.5c show a comparison of measured and theoretical data for the dipole CFSS described above at normal incidence and oblique TE:30° and TE:45° incidences.
Fig. 4.5b. Dipole CFSS; TE: 30° Incidence

Fig. 4.5c. Dipole CFSS; TE: 45° Incidence
It can be seen that the measured results have been closely predicted for all incidences. The passband resonance is stable at around 13.2GHz with the null between 23 and 24GHz. The resonant frequency displays remarkable angular stability to TE: 45° and is, in fact, stable to within 1% which is superior to the close coupled FSS (conductor or aperture). There is, however, in all cases a loss at resonance of around 3dB. The loss tangent of the dielectric is about -0.003 but using this figure in the simulations results in a loss of only a few tenths of a dB. The actual value needed to match the measurements is -0.03, which is clearly not the case in practice. The observed loss is probably due to a combination of two things: the high fields in the dielectric may indeed cause higher than normal dissipative losses in the material, but there is likely to be a contribution from mismatch and reflection losses at the CFSS itself which was observed in the case of the close coupled aperture arrays in chapter 3. It is a characteristic of the closely coupled structures that have been studied to exhibit a loss in excess of that which can be accounted for in the simulations by the inclusion of the nominal loss tangent of the dielectric.

Figs. 4.6a and b show the comparisons for TM: 30° and TM: 45° incidences of measured and predicted results. It is interesting that in the case of the CFSS, extreme angular stability is demonstrated for oblique TM incidence as well as TE incidence, which was not observed for other closely, coupled structures. It is also interesting to note that the resonant loss for TM: 45° incidence is only about 2dB which is the lowest of all the dipole CFSS's and contrasts with the TE: 45° case, where the loss is about 4dB, which is the worst case. The fact that the loss changes with incidence supports the supposition that it is primarily due to mismatching at the CFSS which would be inclined to vary with changing angles of incidence. Dissipative loss in the dielectric is likely to be constant for all incidences, even if higher than suggested by the given loss tangent, because the fields within the dielectric appear to be fairly constant for all incidences as witnessed by the maintaining of the angular stability to within 1%, relative to normal incidence. It is this ultra angular stability that is the trump card of the CFSS as it exceeds anything achievable by using two arrays of identical elements.

It was noted that the numbers of basis functions and Floquet modes chosen were adequate to model the dipole CFSS. As an exercise the model was run using just one current mode in each layer \((\cos \pi y/L1)\) and \(\cos(\pi x/L2)\), which are the dominant modes, to
Fig. 4.6a. Dipole CFSS; TM: 30° incidence

Fig. 4.6b. Dipole CFSS; TM: 45° Incidence
see what the effect would be. It was found that the model correctly predicted the passband resonant frequency but those phenomena at higher frequencies were not accurately identified when compared to the measurements. Specifically, the null, which is at about 23GHz when using three current modes in each layer was broadened and centred at about 29GHz, and that the positive slope of the upper (secondary) resonance was constantly about 10dB below its correct level – a consequence of the higher frequency effects being predicted at a much higher frequency than they physically occur. This verified that the higher order basis functions are necessary to accurately represent the currents and field generated on and within the conductors and apertures.

4.4.2 CFSS with Single Ring Elements

The attention so far in the thesis has been on the use of linear dipole elements and this has been due largely to the fact that they lend themselves to the close coupled and RFSS work studied earlier when displacement in one direction was desirable and needed to be easily controlled. However, with the CFSS, there is slightly more scope in terms of the elements that may be used due primarily to the fact that interaction between electric currents and magnetic fields is the physical property to be harnessed, and non-linear elements provide more freedom to achieve this.

In this section a CFSS from single ring elements is investigated. Rings have been extensively studied as elements for FSS, for example [5,6]. They exhibit good cross polar performance and allow close packing of the elements. It is this second property that is taken advantage of in the CFSS with ring elements. Because the rings are not polarised in the same way as dipoles they cannot be arranged to be co-centric on the different arrays, as this does not generate the required level of coupling between layers. Instead, a shift of one layer with respect to the other is introduced at the manufacturing stage, so that there is an overlapping of the apertures and conductors and hence coupling between the fields and currents. Fig. 4.7 shows the arrangement used for the CFSS. The identical elements are arranged on identical square lattices on the different layers. Coupling is maximised by introducing the shifts in two directions – DSX and DSY. The overlapping effect this produces can be clearly seen.
The dimensions used in this case are $W=0.15\text{mm}$; $R_{in}=1.1\text{mm}$; $D_x=D_y=3\text{mm}$. The relative shifts are of half a lattice period in each of the two perpendicular directions such that $D_{sx}=D_{sy}=1.5\text{mm}$. The dielectric was $75\mu\text{m}$ thick with $\varepsilon_r=3$, as previously.

In modelling the rings the linear sinusoidal basis functions are clearly no good so a particular basis function is used that takes account of the ring’s shape. They are of the form:
\[ h_n(\alpha) = N \cos m\alpha \]

and

\[ h_n(\alpha) = N \sin n\alpha \]

(4.32)

where \( N \) is a constant function and \( \alpha \) represents the angular position of a point on the circumference of the ring element from 0 to \( 2\pi \). \( m \) and \( n \) are integers representing the order of the basis function. In this particular case \( m=1,2,3 \) and \( n=1,2,3 \) in both arrays and it was found that 13 Floquet modes were again sufficient to cover the spectra in each layer.

Figs. 4.8a – 4.8c show a comparison of measured and predicted results for the array described, at normal incidence, TE:45° and TM:45°. The first thing that is evident about these results is that the ring CFSS is not as ultra stable as the dipole. The primary passband
Fig. 4.8b. Ring CFSS: TE: 45° Incidence

Fig. 4.8c. Ring CFSS: TM: 45° Incidence
is centred at about 13.8GHz for normal incidence, but at TM:45° incidence this moves to 14.2GHz. This represents a percentage shift in frequency of just under 2%, and whilst not as good as the dipole CFSS this is far superior to a conventional planar FSS. Further, the better than 2% stability holds for both TE and TM cases which was not the case for the close coupled surfaces. It is also interesting to note the slight change in bandwidth as the incidence conditions change. From normal incidence there is a narrowing of the bandwidth with oblique TE incidence but a widening for oblique TM incidences, although the change in each case is very small.

Once again the CFSS exhibits loss at resonance but it is more consistent than was the case with the dipoles at around 2-2.5dB. This will be explained again by a combination of dissipative loss within the dielectric and reflection losses at the CFSS face due to mismatching between the structure and free space. The effect of the higher order grating lobes is more apparent for the rings because the secondary resonance is much closer to their frequency of influence. At normal incidence, the upper passband is out of the measurement range but it moves to 33GHz for 45° oblique incidence for both TE and TM polarisation. This clearly demonstrates the destabilising effect of the grating lobes and highlights the advantage of having a resonance as far away from this region as possible. Overall the agreement between theory and measurement is very good but an artificially high value of loss tangent is needed to correctly model the loss observed.

4.5 Conclusions

The transmission and angular stability properties of a new, complementary, FSS structure have been assessed in this Chapter. A mathematical analysis was performed and implemented using a modal analysis technique. It was found that the CFSS delivered very powerful performance in terms of angular stability with all cases of oblique plane wave incidence, up to 45°, resulting in a frequency shift at resonance of less than 2% when compared to the case at normal incidence. This is due to the primary passband being situated suitably far enough from the grating lobe region so as the destabilising effect does not interfere, even at oblique angles of incidence. The CFSS creates electrically large elements from physically small ones to such an extent that a \( \lambda/2 \) resonator in free space at
the CFSS resonant frequency would be over three times longer than the periodicity employed in the CFSS. The dipole CFSS, for example, resonated at around 13GHz using elements that are 3.5mm in length. In theory these elements (in the absence of dielectric) would normally resonate at around 42GHz. The grating lobes would strongly influence the resonance at this frequency but the physical resonance at 13GHz is virtually unaffected. This makes the CFSS a very attractive proposition for use in applications where filtering is required over a narrow bandwidth and the signals to be filtered are not from the same direction. This would be true, for instance, in an aircraft radome application where steady angles of incidence could not be realistically maintained. It was observed that due to the highly coupled evanescent fields within the separating dielectric region, the CFSS was inherently lossy and that this loss was a product of dissipative loss and mismatch loss. This could certainly be overcome to a certain extent by the use of a matching layer in front of the CFSS which would eliminate the reflection losses, but this would still leave the problem of the dissipative losses which could only be reduced by the use of a very low loss dielectric substrate.
REFERENCES


CHAPTER 5

Conclusions

This thesis has described a study into the theory and practical demonstration of a new class of FSS. Specifically two layer FSS have been examined in three different configurations: two layers of conducting elements in close proximity, two layers of aperture elements in close proximity and a layer each of conducting and aperture elements. For each of the first two cases they have been studied in both dynamic, reconfigurable configurations (RFSS) and static close-coupled configuration (CCFSS). The third case was examined only in the static situation and due to the complementary nature of the arrays is known as a complementary FSS (CFSS).

The initial thrust behind the work was to achieve a RFSS response insofar as the stopband of a RFSS of conducting element arrays was aimed to be tuneable over a certain frequency range. The method chosen to achieve this was to keep one array fixed while the second was moved, in a controlled manner, in relation to it so that the coupling between them could be altered and hence the transmission response would be tuneable in terms of the resonant frequency. Initial studies examined the possibility of using piezoelectric materials to facilitate the relative displacement of the arrays.
Polyvinylidene Flouride (PVDF) is a piezoelectric polymer that was investigated. It is produced in thin sheets and so lends itself to the sort of radome applications that the RFSS was considered for. In the end, however, it proved unsuccessful. This was largely due to its flimsy nature and the associated problems of keeping both arrays flat. This is an important consideration because the separation between arrays is a critical aspect of the RFSS, and above all else must be maintained as uniform as possible over the area of overlap (generally about 250x250mm$^2$). A further issue with the PVDF was one of safety: applied voltages of more than 3.3kV were needed to produce a linear movement of less than 100µm. Considering that a main intention of the RFSS was to be able to scan the whole available frequency range, i.e. to have displacements of one period (about 5mm), the PVDF was unable to deliver the performance window required.

Instead, a mechanically operated, micropositioning test-bed was developed and made use of a stepping linear actuator to provide the displacements. Using this device a RFSS was realised using two layers of conducting dipole arrays. Nominally, a displacement increment of 25µm was available, leading to 200 available steps over a unit cell of 5mm periodicity. In practice, however, only a few specific positions were necessary to prove the principle of the RFSS. With two arrays of linear dipoles, on square lattices, it was found that the RFSS could be dynamically tuned from a resonant frequency of 36GHz with no displacement, to 17.5GHz with a displacement of half a period. This represents a greater than 50% tuning range, and would be advantageous in multiband switching applications.

When the transmission response of the RFSS was observed for TE:45° incidence it was noted that the angular stability, relative to the angle of incidence, had significantly improved with the half period shift. This seemed to indicate, that with the resonance now much lower than the unshifted case, the coupling between arrays had changed the electrical properties of the elements. To investigate this it was decided to make an RFSS type structure with maximum coupling between the arrays (CCFSS). Two dipole arrays were etched on either side of a single substrate in both the unshifted and shifted configurations. It was found that in this case, with coupling maximised, the angular stability improved hugely at oblique angles of incidence, when the arrays were
displaced. With no displacement between arrays, when the incidence conditions changed from normal to TE:45° there is a shift in the resonant frequency of 7.7%. However, when the same change in incidence change was made to the CCFSS with a relative shift between the arrays, (less than half a period), it was discovered that the resonant frequency shifted by less than 1%. A further theoretical study found that with half period displacements the resonant frequency could be kept to within less than 0.3% of its original value, for incidence angles up to TE:45°.

The measured results were accurately predicted using a model developed in parallel with the experimental work. The model involves using a modal analysis technique to predict the transmission responses of the structures. The fields close to the arrays are expanded as a set of Floquet modes, and when the appropriate electromagnetic boundary conditions are applied, two, coupled integral equations are derived. These are be solved by the method of moments, using entire domain sinusoids and cosinusoids as basis functions representing the unknown currents on the conductors.

The next phase was to try to implement the RFSS and CCFSS using arrays of aperture elements. This was an entirely different problem, both practically and theoretically. Practically, the two aperture arrays have a passband transmission response, and are etched from copper sheets. This makes them prone to permanent or semi-permanent deformation and also makes the attainability of the necessary uniform separation difficult to achieve. Consequently, the results for the aperture RFSS did not produce the same performance as those for the conducting case, although nearly 10dB of switching in the passband was achieved, when the relative displacements were introduced.

The results for the CCFSS with aperture elements were very promising and demonstrated that narrow bandwidth arrays could be produced with a reduction in bandwidth from 11.4% in the case of the unshifted CCFSS to 3.1% when the arrays are displaced. A drawback to this is that fairly substantial inband losses are introduced with the displacements, It was demonstrated that with a very low loss substrate these losses could be reduced to nearly zero, but in practice this would be difficult to achieve. However, by using Kapton as the substrate instead of the more lossy polyester, the
inband loss was reduced to 8dB from about 12dB. It was considered that the high fields generated within the dielectric may be contributing to the losses observed although it is not obvious how these could be alleviated as the coupling between layers is the keystone of the RFSS and CCFSS performance. Further studies should involve the use of elements other than the linear dipole to assess their suitability in these structures.

The final part of the thesis dealt with the CFSS, in which an array of conductors and an array of apertures was organised in the CCFSS configuration. The CFSS delivered very powerful performance in terms of angular stability, similar to the conducting CCFSS, but crucially for both oblique TE and TM incidences. This characteristic could prove a real advantage where the application demands not just one polarisation. The shift in resonant frequency from normal to both TE and TM:45° incidences was less than 2%. The origins of this improved stability can be found in the fact that the main passband is too far from the grating lobe region for them to significantly influence it. However, the CFSS also suffered from inband losses, although to a much lesser degree than the aperture CCFSS, and would probably benefit from the inclusion of a lower loss substrate.

This thesis has shown that the coupling between two, closely separated, FSS arrays, is critical in determining the performance of such structures. Two arrays cannot be thrown together and hope to produce useful performance if the coupling between them is not carefully assessed. It has been shown that much improved stability of the resonant frequency of a two layer FSS can be achieved by harnessing the coupling between them, and that this coupling can be maximised by etching both arrays on opposite sides of the same dielectric substrate. It has also been shown that it is possible to tune the resonant frequency of an RFSS structure by displacing one relative to the other, so adjusting the coupling between arrays. Essentially these techniques produce electrically large elements from physically small ones and as a consequence push the resonance out of the influence of the grating lobe region, which otherwise would have a detrimental effect of the transmission performance.
APPENDIX

The emphasis of the initial RFSS research was centred on the use of piezoelectric materials. During initial investigations the most suitable was found to be Polyvinylidene Flouride (PVDF). PVDF is a piezoelectric polymer available in relatively large sheets, which because of the potential applications makes it attractive for use in the RFSS, although its rather flimsy nature renders it physically not ideal. A second piezoelectric material considered was Lead Zirconate Titanate (PZT). This, however, is a piezoelectric ceramic and as such is very brittle, so it is inappropriate in applications where large areas are considered. It was, though, considered for use as a spacer between array layers where the separation between the layers could be varied, so producing the required reconfigurability.

Investigation of PVDF

Piezoelectric materials are mechanically deformed when an electric field is applied to them. Conversely, when they have a mechanical force applied, so as to produce a compressive or tensile strain, they generate an electric charge. They are described by either the mechanical-electro stress constant, $g$, or the electro-mechanical strain constant, $d$, which is the property considered for the PVDF. The various modes in which the PVDF can operate are described by Fig. A.1 and Table A.1. The piezoelectric strain constant is defined as:

$$d_{ab} = \frac{\text{strain}}{\text{applied field}} = \frac{\varepsilon}{E} \quad (A.1)$$

where $a$ and $b$ are the strain indices described in Table A.1.
The PVDF can operate in four different modes: $d_{31}$ (length extension), $d_{33}$ (thickness extension), $d_{14}$ (face shear) and $d_{15}$ (thickness shear) although the mode considered here was $d_{31}$. In order to produce the relative displacement between arrays the $d_{31}$ mode was considered for the PVDF, utilising the mechanical strain, $\varepsilon$, which is defined by:
\[ \varepsilon = \frac{\text{length extension}}{\text{original length}} = \frac{\delta l}{l} \quad (A.2) \]

Combining Eqns. A.1 and A.2 leads to

\[ \delta l = d_{31} \times E \times l \quad (A.3) \]

It can be seen that to maximise the extension, the applied field, strain constant and original length must all be as large as possible. \( d_{31} \) for PVDF is 23pm/V, which suggests that a large applied field would be necessary to generate any meaningful extension. The electrodes on the PVDF are provided by a nickel on copper film, which is sputtered on at the time of manufacture, and has a thickness of around 250-500Å. This is very thin and makes arcing a real hazard with high voltages. In fact, for a PVDF sheet of thickness 28μm and length 120mm, to achieve an extension of 80μm requires a voltage of 840V. Similarly, for PVDF which is 110μm thick, a voltage of 3.3kV is required for the same extension.

When the PVDF sheet is energised, the further any point on the sheet is away from the energising voltage source, the further that point will extend. This is because the extension is a cumulative effect along the length of the sheet. In order to achieve a uniform relative displacement, two sheets were energised in such a way that they extended in opposite directions. The principle here was that identical relative displacement of the sheets could be achieved over the whole area of overlap of the sheets. However, when the sheets were energised they did not remain flat; they tended to ripple rather than slide smoothly over one another which realistically meant that their functionality as an RFSS was limited.

The modelling performed during the investigation of the PVDF sheets did, however, suggest that the principle of the relative lateral displacement of one array with another provided the desired reconfigurability. This led to the mechanically driven micropositioning test bed described in Chapter 2, which was successful in producing the desired displacements.