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OPTIMAL CONTROL INVESTIGATION

OF A HVDC TRANSMISSION SYSTEM

BY

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A Doctoral Thesis
Submitted in Partial Fulfilment of the Requirements for
the Award of the Degree of Doctor of Philosophy
of the Loughborough University of Technology
May, 1981

Department of Electronic & Electrical Engineering

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"The only way
to predict the future
is to have the power
to shape the future"

Erich Hoffer

"καὶ τὸ μὴ κακῶς φρονεῖν,
θεοῦ μέγατον δῶρον"
'
'Αγαμέμνονος 927/8
Αἰσχύλος

"and being optimist,
is God's best gift"

Agamemnon 927/8
Aischilos
TO MY PARENTS

AND MY WIFE MARY
ACKNOWLEDGEMENTS

I wish to express my gratitude to my supervisor, Professor I.R. Smith, at Loughborough University of Technology for recommending me for the studentship, for his concern in the progress of my work, for his guidance and useful recommendations, and for reading and commenting upon the thesis manuscript.

I would like to thank the Scholarship Foundation of Greece and the Public Power Corporation for the financial support to carry out this research project.

I also wish to express my sincere thanks to Dr. Stephen Williams for his helpful recommendations, his continuous help, for his deep interest in my work, his critical discussion and suggestions and for commenting upon the thesis manuscript.

Thanks are also due to the Head and the staff of the Electronic and Electrical Engineering Department at Loughborough University of Technology for the facilities provided throughout this work.

Acknowledgement is made to the staff of the Computer Centre of the University and especially to Dr. B. Negus. Thanks are due to the operators for their cooperation and help.

Thanks are also due to Miss J.M. Briers for typing the manuscript.
SYNOPSIS

This thesis describes an investigation into the design of an optimal control system for an A.C.-D.C. Power Transmission System.

The steady-state operation of the plant is considered first, and the relationships between some important parameters in the system are established. The dynamic performance of the system is then considered, and an optimal controller for the system is designed using Pontryagin's Principle.

Several computer programs were written to analyse and confirm the mathematical models developed in the thesis and good agreement was obtained between the computed values and the results obtained from the literature, over a range of operating and transient conditions.
LIST OF PRINCIPAL SYMBOLS

1. VOLTAGES

\[ V_0 = \text{rms value of the voltage of the generator supplying converter assembly; line-to-neutral value.} \]
\[ E_r = \text{rms value of the voltage, line-to-neutral, on the A.C. side} \]
\[ E_l = \text{rms value of the voltage, line-to-line, on the A.C. side} \]
\[ E_m, V_m = \text{maximum value of the voltage} \]
\[ E_d, E_{do} = \text{average value of direct voltage of rectifier} \]
\[ E_x = \text{reduction of average value of D.C. Voltage caused by trigger commutation} \]
\[ E_a = \text{reduction of average value of D.C. Voltage caused by trigger angle} \]
\[ V_t = \text{rms value of the fundamental wave of the voltage at the point of connection of the transformer} \]
\[ E_d', E_{do}', V_o', V', E_t', E_x' \] are the expressions of the above values on the inverter side
\[ e = \text{instantaneous value of voltage} \]

2. CURRENTS

\[ i = \text{instantaneous value of current} \]
\[ I_d = \text{average value of D.C. current in load circuit} \]
\[ I_1 = \text{rms value of the fundamental wave of the line current at the point of connection to the converter} \]
\[ I = \text{rms value of the fundamental wave of the line current in the A.C. network} \]

3. POWER QUANTITIES

\[ P_w = \text{effective power of the fundamental on the A.C. side} \]
\[ P_r = \text{reactive power of the fundamental on the A.C. side} \]
\( P_t \) = apparent power of the fundamental on the A.C. side
\( P_{\text{DC}} \) = power in the D.C. line
\( P_{\text{DIS}} \) = power dissipation on the D.C. line
\( P_{\text{WO}} \) = the apparent power

4. OTHER QUANTITIES

\( \alpha \) = firing delay angle (or trigger angle) of the rectifier
\( \varphi \) = commutating angle on the rectifier
\( \gamma \) = angle of commutation on the inverter side
\( \delta \) = angle between the end of commutation in an inverter and the voltage zero
\( \beta \) = advance angle of valve firing of an inverter
\( \phi \) = power factor angle
\( q \) = number of pulses (identical intervals of operation) per cycle
\( f \) = frequency of supply system
\( \omega \) = angular frequency (= 2\( \pi \)f)
\( L_t \) = commutating inductance of one phase of a transformer
\( X_t \) = \((\omega L_t)\): leakage reactance per phase of the transformers for the rectifier
\( X'_{t} \) = leakage reactance per phase of the transformers for the inverter
\( X_R \) = reactance per phase of the network, on the rectifier side
\( X_C \) = reactance per phase of the capacitors
\( R_d \) = ohmic resistance on the D.C. line
\( L \) = inductance on the D.C. line
5. ABBREVIATIONS

Avg = average
A = Amperes
V = Volts
Ω = Ohms
H = Henrys
s, sec = seconds
Ap = Appendix
eq(s) = equation(s)
emf = electromotive force
Fig. = Figure

6. CONTROL ELEMENTS

I₀ = the current order
Iₜ = the D.C. current given by a transducer, Iₜ = Iₜ
k = the static gain
g, T₁ = time constants of derivation and integration of the regulator
T₂ = the time constant of the filter
E = the output voltage regulator
V_F = the A.C. voltage from the filter for the comparator
s = the Laplace operator

7. OPTIMAL CONTROL

A = matrix coefficient of state variables of system
B = " " of controls
x = state variable vector
u = driving function vector
H₀ = the optimal Pontryagin state function
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CHAPTER 1

INTRODUCTION
The construction of powerful generating stations far removed from the centres of consumption involves the transmission of large quantities of electrical energy over long distances. While both A.C. and D.C. systems have been used in recent years, and D.C. dates from the earliest days of electrical transmission, it has become of renewed importance within the last 20 years or so.

Economic arguments of HVDC versus A.C. for short distances - on the basis of equal circuit loadings - have given rise to considerable controversy amongst engineers for many years.\(^{(4)}\) However, for sufficiently long distances and high powers, an HVDC system is undoubtedly more economic than the corresponding A.C. system.\(^{(22)}\)

Naturally, economic as well as engineering aspects of the problem, such as stability and reliability, are of crucial importance in the choice of an HVDC or an A.C. transmission system.

In comparing the economics of the two possibilities, the high initial cost of the converter stations has to be weighed against the savings in the cost of the transmission systems. One incentive to the use of HVDC is the interconnection of Power Systems of different frequencies and the establishment of power lines, or the unification of separate energy systems, as in the cases of the Swedish mainland Gotland\(^{(29)}\) scheme and the English Channel crossing.\(^{(54)}\)

In addition, a D.C. system is inherently more flexible than an A.C. system. Since the transmitted power is capable of being controlled in any desired manner, a D.C. system is less subject to influence from disturbances from the A.C. system.\(^{(28)}\)

At the present time HVDC has become firmly established as a reliable and economic means of transmitting large blocks of power, usually over long distances or links involving submarine cables.\(^{(26)}\)

The increased interest in HVDC transmission has fostered the development of techniques for the study of D.C. systems in parallel with A.C.
systems. Further investigation of A.C.-D.C. systems has established that D.C. transmission may also be used to stabilize long A.C. transmission systems.\(^{(28),(30)}\)

Since the dynamic performance of a HVDC transmission system depends considerably on the controllers, many of the operational properties of the complete transmission system are determined by its control scheme.\(^{(42)}\) The design of a suitable control system is therefore of crucial importance for a HVDC system, remembering that stability and protection are basic technical requirements for a HVDC transmission system.

In order to keep the direct current within prescribed limits, and to protect the system from over-voltages (caused by disturbances in the A.C. system, external sources or even during normal control operations) two current regulators are needed one at each end of the system.\(^{(39)}\) These also ensure that the HVDC link is operated in a stable mode, and provide for the reversal of power flow.

In recent years, considerable progress has been made in the theoretical study of HVDC systems. Classical methods such as Bode diagrams and Nyquist plots have been used to design the converter controllers\(^{(41),(42),(52)}\), but these techniques are not convenient for designing controllers to provide optimum system performance.

The concept of control system optimization comprises the selection of a criterion of optimization known as a Performance Index (abbreviated to PI) and this leads to a design which yields a control system within the limits imposed by physical constraints. A control system is considered optimal if the values of parameters are chosen so that the selected PI is either a minimum or a maximum. The use of optimal controllers in electrical power systems is not new. Elgerd\(^{(58)}\) and Yu\(^{(47)}\) have used optimal control techniques in the design of frequency and voltage controllers. State regulators and tracking techniques have also been used to design controller feedback, so as to improve the performance of HVDC systems.\(^{(23)}\)
In order to perform these studies, some form of model is required to represent the system. The HVDC system is being approximated by a mathematical model, which is solved on a digital computer. This approach offers flexibility, accuracy and high speed of calculation and could additionally be used for the design of a suitable controller. (51)

The overall aim of this project is to investigate the dynamic performance of a HVDC system. In particular, it is intended to investigate the transient response of the inverter of a HVDC system, subject to different forms of optimal control.

The criteria selected for optimization are a minimum settling time and a minimization of the error signal, since a fast and smooth response is of crucial importance in the performance of a HVDC power transmission system.

Thereafter the analysis of the system is considered in two parts; the steady-state performance, and the transient response following a system disturbance.

The objective of the steady-state investigation was mainly to develop a mathematical representation of a HVDC system on a digital computer which would give accurately the characteristics of the converter at any operating point for different network parameters. These characteristics are needed by the appropriate controller at any point of operation of the system.

In the investigation of the transient response of the inverter, the first step was an analysis of the dynamic performance of the control system on the rectifier side, as described in references (1) and (5). The study is then extended to the design of an optimal system based on a quadratic performance index, in order to minimize the error signal. This gives rise to state feedback control.

This feedback was added to the control system in order to enhance the dynamic performance of the plant, and the system was analysed by applying the classical criteria such as stability and speed of response.

Thereafter, since in practice any control is constrained in magnitude -
a concession to the practical fact that no system has unbounded control available - a time-optimal current control system was designed using the Pontryagin Minimum Principle.
CHAPTER 2

THE STEADY-STATE HVDC TRANSMISSION SYSTEM
In this chapter the equations describing the steady-state behaviour of a converter are produced and a numerical solution of the resulting non-linear algebraic equations describing the plant is performed using a digital computer.

2.1 GENERAL CONSIDERATIONS OF A.C. AND D.C. SYSTEMS

2.1.1 Advantages of HVDC Systems

1. D.C. transmission becomes advantageous compared with an A.C. system, where it is needed to transmit power using cables rather than overhead lines, since in practice long A.C. cable systems have limitations due to problems associated with charging current. The use of a D.C. system thus enables power to be transmitted under special local conditions, (e.g. across large areas of water, islands, channels), or where environmental conditions become important, e.g. dense industrial centres such as London in the United Kingdom. \(^{(3),(22)}\)

2. The possibility of using D.C. substations and lines operating at a higher voltage than is possible with A.C. at the present state of high voltage engineering. \(^{(65)}\)

3. On the basis of the same current and insulation, the power transmitted can be increased by about 40% with the line losses reduced by about 30% \(^{(3)}\).

4. The absence of any intermediate stabilising stations or apparatus for any length of line.

5. Skin effect is completely absent and a better utilisation of the conductor is obtained.
6. A.C. systems operating at different frequencies may be interconnected by a HVDC link, i.e., a "back-to-back" connection.

2.1.2 Limitations

1. Difficulties in the design of switchgear are a major limitation of D.C. systems.\(^{(3)}\)

2. The steady-state reactive power requirements of the inverter may be of the order of 50% of the real power.\(^{(40)}\)

2.1.3 Comparison of Relative Merits of A.C.-D.C. Systems

A D.C. system has lower line costs than a corresponding A.C. system, but needs two converter terminal stations, which are from 2- to 3 times more expensive than the corresponding A.C. transformer stations, and far more complicated from an engineering viewpoint.

Various calculations have been performed which establish the minimum distance for which D.C. transmission is the most economic. For overhead lines the critical distance is about 300 miles for a 400 kV (line voltage) 750 MW system. For submarine cables it varies from 30 to 40 km for a 200 kV, 100 to 200 MW scheme.\(^{(3)}\) Thus the longer a transmission system is the more favourable is the case for HVDC.

Further studies show that the transient stability of an A.C. system, in the event of A.C. faults, can be improved, by temporarily increasing the power of a joint A.C.-D.C. system.\(^{(30)}\)
2.2 ANALYSIS OF THE 3-PHASE BRIDGE RECTIFIER

The wide variety of operating conditions and component characteristics on the one hand and the desire to keep the model small and simple on the other, demands that some simplifying assumptions are made in the analysis of any rectifier circuit. Thus, instead of a real 3-phase rectifier system, a theoretical model is investigated; one which operates under ideal conditions and consists only of components with ideal characteristics. It is convenient therefore to make the following assumptions\(^{3},^{(64)}\):

1. All resistors, capacitors and inductances are ideal, with linear characteristics.
2. The load on the D.C. circuit has a sufficiently large inductance for the D.C. current ripple to be negligible.
3. The rectifying elements are ideal diodes with zero resistance in the forward direction, zero conductivity in the reverse direction and no inductance or capacitance.
4. The A.C. voltages are undistorted and balanced.
5. All phases operate identically and are equally controlled.
6. The magnetisation current and the resistance of the transformers is neglected.

The 3-phase bridge rectifier is shown in figures 2.11 and 2.12; the circuit is derived by combining two three pulse (i.e. three identical intervals of operation during each cycle) commutating groups, these being illustrated in figures 2.13 and 2.14.

2.2.1 Operating Characteristics of the Rectifier

2.2.1.1 D.C. Output Voltage and Current (see Appendix I)

Assuming a star connected secondary to the rectifier supply transformer, the average D.C. voltage at no load of an uncontrolled (diode) bridge is
\[ E_{do} = \frac{3 \sqrt{3} E_r}{\pi} \quad (2.2.1) \]

Delaying the current transfer from the outgoing to the incoming phase by an angle \( \alpha \) (i.e. by using controlled devices), reduces the average value of the D.C., so that

\[ E_d = E_{do} \cos \alpha \quad (2.2.2) \]

It is assumed in the above expressions that the current transfer from one phase to another takes place instantaneously as soon as the anode voltage of the incoming phase becomes more positive than that of the outgoing phase. Normally this transfer takes a finite time, because of inductance in the line and other system components and this period of commutation is frequently referred to as an angle of overlap.\(^3\)

For a given overlap (or commutation) angle \( u \), the new value of \( E_d \) is

\[ E_d = \frac{E_{do}}{2} \{ \cos \alpha + \cos (\alpha + u) \} \quad (2.2.3) \]

so that the reduction in \( E_d \) caused by commutation is

\[ E_x = \frac{E_{do}}{2} \{ \cos \alpha - \cos (\alpha + u) \} \quad (2.2.4) \]

The D.C. current is, (see appendix 1, section 1.3)

\[ I_d = \frac{\sqrt{3} E_r}{\sqrt{2} \omega L} \{ \cos \alpha - \cos (\alpha + u) \} \quad (2.2.5) \]

and combining equations (2.2.4) and (2.2.5), the voltage drop caused by commutation is

\[ E_x = \frac{3}{\pi} \omega L I_d = \frac{3}{\pi} X_t I_d \quad (2.2.6) \]

Combining relations (2.2.3), (2.2.4) and (2.2.6), the average D.C. voltage is

\[ E_d = E_{do} \cos \alpha - \frac{3}{\pi} X_t I_d \quad (2.2.7) \]

In the steady-state:

\[ E_d = R_d I_d \quad (2.2.8) \]

and substituting (2.2.8) into (2.2.7)

\[ E_d = \frac{E_{do}}{1 + \frac{3X_t}{\pi R_d}} \cos \alpha \quad (2.2.9) \]
A computer program was written to calculate the normalised relationship \( \frac{E_d}{E_{do}} \) for different values of \( X_t \) and \( R_d \), and for phase control variations from zero to \( 90^\circ \). The results obtained are shown in figure 2.15 and are in accordance with existing literature. (2)

2.2.1.2 The Power Relationships of a Converter

The alternating current input to a rectifier can be interpreted as a combination of a fundamental and various harmonics; the latter usually being filtered to decrease the waveform distortion. (6)

The fundamental component of the current \( I_1 \), gives the effective power

\[
P_{AC} = 3 E_r I_1 \cos \phi
\]

which under any operating conditions is in balance with the power delivered to the D.C. circuit. Thus,

\[
P_{AC} = P_{DC}
\]

where

\[
P_{DC} = E_d I_d
\]

and combining equations (2.2.2), (2.2.10), (2.2.11), (2.2.12) gives:

\[
3 E_r I_1 \cos \phi = E_{do} I_d \cos \alpha
\]

Substituting (2.2.1) into (2.2.13) and assuming \( \mu \) is zero gives:

\[
I_1 = \frac{\sqrt{6}}{\pi} I_d
\]

For a commutation angle \( \mu \), the power factor is (7) (see appendix, 1)

\[
\cos \phi = \frac{1}{2} \{ \cos \alpha + \cos (\alpha + \mu) \}
\]

Another useful expression for the power factor of the system, as shown in Appendix 1, is

\[
\cos \phi = \cos \alpha - \frac{\pi}{6} \frac{X_t I_1}{V_t} = \cos \alpha - \epsilon
\]

Relation (2.2.15) is true under the assumption that the rectifier is not able to absorb or store any energy.

In practice, the total power factor is slightly lower than the value of \( \cos \phi \); but only \( \cos \phi \) has practical importance; the total power factor being generally only of theoretical interest. (31)
2.2.2 The Commutation Process

During the time the current is transferring from one phase to another, two phases and their corresponding valves, or SCRs, conduct simultaneously. The current is decaying in the outgoing phase (while it is rising in the incoming one) with a time constant determined by the two short-circuited phases.

Further analysis of the commutation phenomenon shows that:

1. The voltage drop due to commutation is independent of the delay angle \( \alpha \) and is proportional to the current, as indicated by equation (2.2.6).

2. Keeping all other characteristics of the network constant (i.e. \( E, I_d, \omega, L \)) but increasing the delay angle, results in a decrease in commutation time as indicated by equation (2.2.5). The physical interpretation of this is that increasing the trigger angle means delaying the point of firing so that the commutating voltage therefore increases; in consequence the current is forced to transfer in a shorter time, which means a decrease of the overlap angle. \(^{(3)} \)

3. If the D.C. current increases, while all the other parameters (i.e. \( E, \alpha, L, \omega \)) are constant, then \( u \) changes correspondingly, since the characteristics remain constant. The mathematical basis for the above comments can easily be deduced from relation (2.2.5).

4. A sudden fall in the output voltage for a constant output current, tends to reduce the commutating voltage so that the angle \( u \) increases, as is confirmed by equations (2.2.4) to (2.2.6). The reason for this is that decreasing the output voltage, for the same current, means that more time is required to transfer the current from the preceeding phase to the succeeding one. Phase control thus not only alters the amplitude of the D.C. voltage but also the angle of overlap.

Thus, the commutation angle \( u \) is related to the delay angle \( \alpha \); and the complete rectifier and inverter characteristics can be derived from relation (2.2.5):

For some values of \( I_d \), with a trigger angle \( \alpha = 0 \), there is a
corresponding angle \( u_0 \):

\[
I_d = \frac{\sqrt{3} E_r}{\sqrt{2} \omega L} (1 - \cos u_0)
\]  

(2.2.5a)

For the same current \( I_d \), a set of curves of trigger angle \( \alpha \) against the angle \( u \) (on the inverter side normally referred to as the commutation angle \( \gamma \)) can be plotted for fixed values of \( u_0 \), as shown in figure 2.21. These characteristics are obtained by combining equations (2.2.5) and (2.2.5a) so that:

\[
\alpha = A \cos \left\{ \cos \alpha - \frac{2}{\pi} \frac{R_d}{X_t + 1} \right\} - \alpha
\]  

(2.2.17)

The change-over from rectification to inversion occurs at the point where the mean output D.C. voltage is zero, that is where the angles \( \alpha \) and \( u \) are such that

\[
\alpha + u/2 = \pi/2
\]  

(see Appendix 3)  

(2.2.18)

On the inverter side there is a limitation on the angle \( \alpha \) beyond which successful commutation is not possible, usually called the "commutation limit line"(3). This prevents the commutation being delayed beyond point S in figure 2.16, because after this the voltage of phase A becomes negative with respect to that of phase B. Thus, for example if \( \gamma = 18^0 \), the angle \( \alpha \) cannot exceed \( 162^0 \), as shown in figure 2.21 (see Appendix 4, section 4.2)

Taking into account the deionisation angle \( \delta_0 \) of the valves the delay and commutation angles must satisfy the relation(3):

\[
\alpha + \gamma + \delta_0 = \pi
\]  

(2.2.19)

or

\[
\alpha = (\pi - \delta_0) - \gamma
\]  

(2.2.20)

In the case of a rectifier only, the commutation angle \( u \) is defined from the relation: (see Appendix 3, section 3.2)

\[
\cos(\alpha + u) = \left\{ 1 - \frac{6X_t}{\pi R_d} \cdot \frac{1}{1 + \frac{3}{\pi R_d}} \right\} \cos \alpha
\]  

(2.2.21)

which is in agreement with equation (14) of reference (17).
2.3 **The Inverter**

Let us first consider a 3-phase bridge working as a rectifier and neglect commutation. As the firing angle increases $E_d$ decreases, and beyond $\alpha=60^\circ$ it is seen from figure 2.17a that there is some negative output voltage. At $\alpha=90^\circ$ the positive and negative volt-second areas become equal and the average D.C. voltage is zero (figure 2.17b).

Any further delay of the firing angle gives a negative average voltage and at $\alpha=180^\circ$ we obtain from relation (2.2.2):

$$E_d = E_{do} \cos 180^\circ = -E_d$$  \hspace{1cm} (2.3.1)

Now, if an external D.C. voltage is applied at the output terminals of the rectifier and grid control is applied to force the valve to conduct, current will flow from anode to cathode in opposition to the induced e.m.f. in the transformer secondaries of the inverter, as shown in figure 2.18. This means that power is being supplied to the A.C. system, and the rectifier has therefore become an inverter.\(^{(3)}\)

The firing point of the valves is controlled by the voltage of their grids. Thus valve 1 is prevented from firing up to point C by the negative bias on its grid, although its anode is positive with respect to its cathode (loop 1-A-B-S in figure 2.18).

After firing, the current in valve 5 starts to fall while it starts to rise in valve 1 at point C (figure 2.16), and commutation ends after an angle $\gamma$, at point D. After a further angle $\delta_0$ deionisation is complete in valve 5 so that its grid regains control and the valve is blocked to prevent the flow of reverse current after point S.

From the consideration outlined above, it is clear that a minimum angle $\beta_0 = \gamma + \delta_0$ is necessary for inverter operation, and hence that the inverter should be supplied with some reactive power. The deionisation angle $\delta_0$ is usually called the "extinction angle" and it is of crucial importance in the successful operation of an inverter.\(^{(2)}\)
The inverter equations can be obtained directly from those in equations (2.2.3) to (2.2.7) for the rectifier, by substituting \( \alpha = \pi - \beta \)

\[
\begin{align*}
E'_{do} &= \frac{E'_1}{2} (\cos \delta + \cos \beta) = E'_{do} \cos \delta - \frac{3}{\pi} X'_d I_d \\
E'_{x} &= \frac{E'_1}{2} (\cos \delta - \cos \beta) = \frac{3}{\pi} X'_d I_d \\
I_d &= \frac{\sqrt{3} E'_1}{\sqrt{2} X'_t} (\cos \delta - \cos \beta) \\
\cos \phi' &= \frac{1}{2} (\cos \delta + \cos \beta)
\end{align*}
\] (2.3.2) \hspace{1cm} (2.3.3) \hspace{1cm} (2.3.4) \hspace{1cm} (2.3.5)

As stated in section 2.2.2 the process of commutation is a short-circuit between two phases. It is clear that as soon as commutation between valves 1 and 3 begins, phases A and C are short-circuited and consequently the line voltage \( E_1 \) (see figure 2.18) goes to zero.

During commutation in a 3-phase system, the other two voltages automatically adjust themselves so that their sum is equal to zero. This results in a phase displacement of \( \pm 30^\circ \) together with an amplitude reduction of \( \sqrt{3}/2 \) as shown in figures 2.19 and 2.20.
2.4 operation of a HVDC Link

The operation of a system of two converters as a transmission link, implies selection of the converter characteristics to meet the requirements of regulation and protection. Converters which satisfy these requirements are called "compounded".

2.4.1 Required Regulation

1. The valves should be operated strictly within their current ratings, since there is a risk of damage if the current is increased beyond this for even a short time. Thus the current is usually kept within prescribed limits.

2. For satisfactory operation of the inverter, a minimum extinction angle $\delta_0$ is required, as mentioned in section 2.3, otherwise commutation failure will occur.

3. A change in the power transmitted in accordance with the requirements of the receiving system is also necessary if, for example, the receiving system is large compared with the D.C. link, then this will normally be used to feed constant power into the A.C. system.

2.4.2 Rectifier Compounding

The characteristics and the equivalent circuit of a compounded rectifier are shown in figures 2.24 and 2.25 respectively. The top characteristic is the rectifier output voltage when $\alpha=0^\circ$ and is usually called the "Natural-Voltage Characteristic" (abbreviated to NV). A vertical line is called a "Constant Current Characteristic" (abbreviated to CC) and gives the output voltage of the rectifier for a set current, as the delay angle varies from zero to $90^\circ$. The NV characteristic is described by:

$$E_d = E_{do} - \frac{3X_t}{\pi} I_d$$  \hspace{1cm} (2.4.1)

The slope of this characteristic being

$$\frac{E_d - E_{do}}{I_d} = \frac{3X_t}{\pi}$$  \hspace{1cm} (2.4.2)
The voltage at the end of the transmission line is

$$V_d = E_{do} \cos \alpha - \left(\frac{3X_t'}{\pi} + R_d\right)I_d$$  \hspace{1cm} (2.4.3)$$

The D.C. current depends on the voltages of the rectifier and inverter at any instant, and

$$I_d = \frac{E_d - E_d'}{R_d}$$  \hspace{1cm} (2.4.4)$$

Combining equations (2.2.7), (2.3.7) and (2.4.4) gives:

$$I_d = \frac{3\sqrt{6}}{\pi} \frac{E_r \cos \alpha - E_r' \cos \delta}{R_d + \frac{3}{\pi} (X_t - X_t')}$$  \hspace{1cm} (2.4.5)$$

From equation (2.4.3) it can be seen that even if the inverter back voltage collapses entirely - due for instance to a fault on the inverter side - the current will not increase to a value greater than $I_{dm}'$ and it will be sustained by a rectifier voltage of reduced value $V_{df}$; thus for $V_d = 0$:

$$V_{df} = E_{do} \cos \alpha = \left(\frac{3X_t'}{\pi} + R_d\right)I_{dm}$$  \hspace{1cm} (2.4.6)$$

### 2.4.3 Inverter Compounding

Because of the deionisation time the operation of the inverter is associated with the consumption of a certain minimum amount of reactive power. The A.C. current and voltages are liable to undergo considerable changes and hence some extra angle greater than the deionisation angle $\delta_0$ has to be provided, to take account of these changes, in order to ensure successful commutation (3) (see figure (2.16)). Safety obtained in this way is obviously at the expense of reactive power.

The inverter equivalent circuit is shown in figure 2.26, and by combining equations (2.3.2) and (2.3.3) the back voltage of the inverter can be defined as:

$$V'_d = E_{do} \cos \beta + \frac{3}{\pi} X'_t I_d = E_{do}' \cos \delta - \frac{3}{\pi} X'_t I_d$$  \hspace{1cm} (2.4.7)$$

The above equations are represented graphically in figure 2.27.

In addition, when the inverter A.C. voltage drops $E_d'$ also drops and the D.C. line capacitance discharges through the inverter; in this case
the inverter current will rise momentarily even if the rectifier has been
given a constant current characteristic.

These considerations demand an angle of advance $\beta > \gamma + \delta_0$ to meet
reasonable safety requirements and yet every attempt must be made to
economise the amount of reactive power. As a compromise, the inverter
may be compounded in such a way that suitable variations in the firing
angle are provided in accordance with the instantaneous values of the
voltage and current.\(^{(3)}\) This involves the use of a computer to calculate
exactly and at every instant, the angle which is required. Such an
arrangement has been used in the Gotland Scheme.\(^{(29)},(55)\)

There is no doubt that a low value of $\beta$ is more desirable than a
large one, since this not only requires less kVA\(_r\) for the inverter but
also means operation with a lower value of $I_d$, thus minimising the losses
in the D.C. transmission system.\(^{(35)}\)

In the uncompounded inverter there is no provision for changing $\beta$;
this angle will therefore have to be sufficiently large to avoid any
commutation failure. However, this results in unnecessary reduction of the output voltage, as illustrated by eq(24), and consequently in poor utilisation
of the installed converter.

The inverter operation with constant angle $\delta$ (equation 2.4.7), will
usually be referred to as "Constant Extinction Angle Control" (abbreviated
to CEA Control)\(^{(24)},(38)\) and the inverter is said to be compounded.

As has been shown, compounding of an inverter not only provides
greater safety but also saves reactive power.

2.4.4 Current Regulation of a HVDC Transmission Link

It can be seen from the discussion above that power transmission should
be carried out with adequate safety, as well as in a way which minimises
the amount of reactive power.

Under steady-state conditions the current $i_d$ is always defined by
equation (2.4.5); in other words, the operating point of the system is the
intersection of the characteristics of the rectifier and the inverter.

However, the most desirable mode of operation of a HVDC system, to ensure that the necessary commutation margin is maintained is for the rectifier to be working with CC characteristics, (i.e. to be provided with a constant current regulator), while the inverter is under CEA control.

The combined transmission characteristics are as shown in figure 2.28. The operating point A is defined such that the delay angle \( \alpha \) in the rectifier is small so that sufficient active power is transmitted.

For working in this manner, the rectifier voltage characteristic must be higher than those of the inverter, so that

\[
E_{do} - (\frac{3X}{\pi t} + R_d)I_d > E_{do} \cos \delta_0 - \frac{3X}{\pi t}I_d \quad (2.4.8)
\]

This is the normal working condition, since operating an inverter on CC regulation (the rectifier then works with zero delay angle on NV characteristics) implies a large angle of advance \( \alpha \) and hence a large reactive power consumption.\(^{(3)}\)

The protection of the rectifier and inverter against faults in the A.C. system is another problem.\(^{(44)}\)

In order to keep the current within prescribed limits, two current regulators are needed; one on the rectifier side and the other on the inverter side. The current regulator of the rectifier ensures that the current does not rise above the set value, but it does not ensure that it will not fall too low.\(^{(2)}\) On the other hand, a constant current regulator on the inverter side ensures that the current does not fall below the lower limit and hence avoids a "running down" of the system.

It may also be seen that, in all cases, the current setting on the inverter has to be lower than that of the rectifier, and the inverter regulator takes over only if there is a fall in the rectifier voltage. This so-called "margin" has to be sufficient to give a large enough difference in the peak voltages, as defined in equation (2.4.).
avoid both regulators operating simultaneously, since this operation may lead to instability\(^{(3)}\).

Care must also be taken in the design of the controllers. A very sensitive controller (with a steep characteristic) has the tendency to cause oscillations in the system - usually called the "hunting phenomenon" - and these disturb the operating condition during sudden D.C. voltage changes. This phenomenon has been demonstrated by Busemann.\(^{(3)}\)

### 2.4.5 Reactive Power Requirements and Compensation

A rectifier bridge necessarily absorbs reactive power, because the bridge current is allowed to flow at only certain intervals\(^{(6)}\) during each supply cycle. However, when operated with a lagging current, a rectifier consumes little reactive power compared with an inverter, and it can be run at nearly unity power factor. The inverter on the other hand, because of the inherent deionisation angle, draws a considerable reactive power at a leading power factor.\(^{(3)}\)

In practice, the reactive power required increases in proportion to the active power\(^{(40)}\), and it may be about 0.5 to 0.7 kVAR per kW of active power supplied to the receiving A.C. system.\(^{(3)}\)

The reactive power of converters is usually compensated by shunt or series capacitors in the supply circuit as shown in figure 2.29 because of their lower cost and energy losses, rather than synchronous condensers which are found mainly at inverter substations. The presence of these capacitors results in a certain decrease in the duration of commutation.\(^{(22)}\)

The possibility remains of compensating for reactive power by means of artificial commutation, but various investigations have shown that the expense involved in achieving it, exceeds that of providing a reactive power source on the A.C. side.\(^{(3)}\)
The distortion factor is defined as

\[ \mu = \frac{\text{rms amplitude of fundamental current}}{\text{rms amplitude of real current}} \]

For any of the four arrangements of delta and Wye connections, \( \mu = 0.955 \).\(^{(45)}\)

For more powerful rectifier installations, a 12-pulse performance is desirable. This is accomplished by combining two 3-phase bridge units, which can be connected together in series or in parallel through an interphase transformer.\(^{(22)}\) This combination reduces the ripple of the D.C. voltage and improves the shape of the A.C. currents, making them more sinusoidal. A 12-pulse system has a distortion factor in the primary lines of 0.99.\(^{(45)}\)
2.5 **BRIDGE CONNECTIONS**

A rectifier connection presents a special kind of load to an A.C. supply system. The valves are switching elements and the current wave-shape is discontinuous, with a flat top in the conducting regions, due to the large inductance on the D.C. side. This arises because the supply currents balance the D.C. load, and the manner in which the D.C. current is reflected through the rectifier connection to the A.C. side changes every time the current is commutated from one phase to another. \(^{(64)}\)

Grid control and commutation are the factors which determine the phase shift between the fundamental components of voltage and current as well as the form of the distortion of the current and voltage curves in the A.C. system. Thus, a choice of an appropriate connection is the first and fundamental step in the design of a large converter.

Midpoint connections are the building blocks of more complex rectifier systems. For power conversion the 3-phase bridge connection - which consists of two commutating groups in series (see fig.2.11)-is widely used for reasons of economy. \(^{(64)}\)

For a bridge converter circuit, the transformer may be connected in any of four ways, involving delta, star and mixed connections. \(^{(3)}\)

Connections with a Wye secondary have a D.C. component of \(1/3 I_d\) in the secondary current which cannot be balanced by the primary current; but this can sometimes be tolerated since it does not saturate the transformer core significantly. \(^{(64)}\)

For connections with a Wye primary, there is another phenomenon in addition to the D.C. unbalance which distorts the secondary voltages and this is termed excitation unbalance. This occurs because of the magnetic characteristic of the transformer and the requirement (due to the Wye connection) that the currents must sum to zero. In order to avoid this voltage distortion on the secondary, the primary windings are preferably connected in delta, but with the ratio of the conductor areas increased by \(\sqrt{3}\). \(^{(64)}\)
2.6 IMPROVEMENT IN WAVEFORM

The conversion of A.C. current to D.C. through the converter is associated with distorted currents and voltages, as explained in section 2.5, and the harmonics introduced imply a need for filtering.

An effective way of reducing the harmonics in the A.C. circuits is usually to increase the number of phases of the converter. However, it is not economical to increase the number beyond twelve, and other methods must then be adopted to bring about a further reduction of the harmonics.\(^{(3)}\)

The inductance of the system and the power factor correcting capacitors act as an L-C filter and resonance between them can occur. Care must thus be taken in the determination of the capacitor values so that the resonance occurs at a frequency much lower than that of the lowest converter harmonic.\(^{(3)}\)

One possible method of preventing undesirable resonance is the provision of a reactor in series with each capacitor, so that series resonance is obtained with the lowest generated harmonic,\(^{(3)},(54)\) (e.g. 5th harmonic for 6-phase operation). For the elimination of still higher harmonics, additional tuned circuits must be provided.

2.6.1 The Inverter Output Waveform

On the D.C. side of a converter, the output voltage waveshape has a form which depends upon the number of phases, the firing angle and the angle of commutation.\(^{(3)}\) The output voltage can be considered as a D.C., voltage on which is superimposed a number of harmonics, which can be determined using Fourier analysis.

The selection of a method for improving the inverter output voltage waveform depends on several factors, which must be carefully weighed up in order to determine the most advantageous approach for a given inverter system. The most important of them are the inverter voltage and current rating, the range of the load, the operating frequency range and the acceptable total harmonic content.\(^{(56)}\)
The terms "series or parallel capacitor - commutated inverter" or briefly, "series or parallel capacitor inverter" are used to indicate an inverter that is commutated by a capacitor in series or in parallel with the load, as shown in Figure 2.29.

Using a "series-capacitor inverter", the load voltage may approach a sinusoid. Since the voltage across the capacitor depends upon the magnitude of the current, the capacitor voltage at low currents is insufficient to keep the power factor angle constant and consequently, a reduced percentage supply of reactive power ensues\(^3\); which is accompanied by an increase in the reactive power requirements of the inverter. The series-capacitor inverter can produce a very nearly sinusoidal output voltage waveform, without requiring external filtering only when supplying a relatively constant load over a limited frequency range.\(^{66}\)

The parallel-capacitor inverter is one of the best known inverters, and can use valves which do not have the ability to interrupt forward current, that is the transfer of current from one phase to another is obtained by the action of the commutating capacitor. With a large capacitance the parallel-capacitor inverter can provide sufficient filtering to produce a nearly sinusoidal output voltage, but the inverter has a relatively large oscillating kVA with respect to its power output.\(^{66}\)
2.7 THE STEADY-STATE COMPUTER MODEL

The objective is to design a model for the steady-state performance of the system, which will give the characteristics of the converter at any chosen operating point, for different combinations of circuit parameters.

A range of computer solutions have been produced, which have started from the simplest rectifier model and gradually developed more complicated models, taking into account the presence of the inverter.

The software of the computer models is based on the simultaneous solution of the non-linear equations relating to the rectifier and the link, as defined in Sections 2.2 and 2.4.

The results of the computer solutions have been compared with those values derived from the classical equations, to establish the accuracy of the computer models.

2.7.1 The Simplified Rectifier Model

The first computer program was written to calculate the values of $E_d$ for a simplified rectifier, as shown in fig. 2.12, when the delay angle changes from zero up to $90^\circ$. The results are in accordance with published material\cite{17} and are shown in fig. 2.15.

Another computer program was written for the simultaneous solution of the linearised equations of the system, assuming $\Delta V_o=0$. These equations can be written, (as shown in Appendix 2, section 2.1)

\begin{align*}
\Delta d &= \frac{R_e}{E_{do}} \sqrt{1+\left(\frac{\Delta i}{R_e}\right)^2} \quad \Delta i = E_1 \cdot \Delta i \quad (2.7.1) \\
\Delta d &= -\tan \alpha \cdot \Delta \alpha - E_2 \cdot \Delta i \quad (2.7.2) \\
\Delta \phi &= \Delta i - \tan \alpha \cdot \Delta \alpha \quad (2.7.3)
\end{align*}

Combining equations (2.7.1) and (2.7.2) gives

\begin{align*}
\Delta i &= -\tan \alpha \frac{E_1 + E_2}{E_1 + E_2} \Delta \alpha \quad (2.7.4)
\end{align*}

where

\begin{align*}
E_2 &= \frac{X_L \cdot I_d}{\sqrt{6} \cdot V_t \cdot \cos \alpha}
\end{align*}
Example - Results

As an example of the use of the program a system with a smoothing choke
of 100 H and a frequency of 50 Hz, was simulated and it was found that
the D.C. current ripple is practically smoothed, since there is a fluctuation
not greater than 1% for a range of firing angle $0^\circ$ up to $60^\circ$ for a trigger
angle deviation $\Delta \alpha = s^0 \text{ and } 10^0$.

This deviation increases considerably as the operating point moves
towards larger delay angles ($70^\circ - 80^\circ$). This was expected, since the slope
of the tangent at the operating point increases as the delay angle increases,
as indicated by eq. (2.7.4). This remark applies to the other two variables $\Delta p$ and
$\Delta d$. Some results are shown in Table 2.1, and figs. 2.30 and 2.31.

2.7.2 The Rectifier and Inverter Characteristics Model

This computer program was written to determine the rectifier and inverter
characteristics of the trigger angle against commutation angle, for fixed
values of $u_o$. The flow chart and the listing are shown respectively in
figures A4.21 and A4.22 of Appendix 4. A family of curves for different
values of transformer leakage inductance, is shown in fig. 2.21 and these
curves are in accordance with references (2) and (8). The right hand end
of these curves outline, approximately, the "limitation line" for $\delta_o = 0^0$,
as mentioned in Section 2.2.2. (see Appendix 3, section 3.3).

The changeover from rectification to inversion for a given D.C. current,
occurs at the points where the mean output voltage of the rectifier is zero, as
explained in Section 22.2. A computer program has been written to give the
changeover points for different values of inductance $X_t$.

When the rectifier is considered alone, a similar method to that used
in the previous program, gives the curves $u=f(\alpha)$, as explained in Appendix
3, and these are shown in fig. 2.22.
**TABLE 2.1**

Values of Variables and Parameters of Model in Section 2.7.1

\[ V_0 = 250V \quad R_d = 10\Omega \]

<table>
<thead>
<tr>
<th>Angle</th>
<th>( \Delta a = -5^\circ )</th>
<th>( X_t = 1.4\Omega )</th>
<th>( \Delta a = 10^\circ )</th>
<th>( X_t = 10\pi/9\Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta i )</td>
<td>( \Delta d )</td>
<td>( \Delta p )</td>
<td>( \Delta i )</td>
</tr>
<tr>
<td>5°</td>
<td>0.000</td>
<td>0.008</td>
<td>0.008</td>
<td>0.000</td>
</tr>
<tr>
<td>20°</td>
<td>0.000</td>
<td>0.032</td>
<td>0.032</td>
<td>0.000</td>
</tr>
<tr>
<td>45°</td>
<td>0.000</td>
<td>0.087</td>
<td>0.088</td>
<td>0.000</td>
</tr>
<tr>
<td>60°</td>
<td>0.001</td>
<td>0.151</td>
<td>0.152</td>
<td>0.001</td>
</tr>
<tr>
<td>70°</td>
<td>0.002</td>
<td>0.241</td>
<td>0.241</td>
<td>0.002</td>
</tr>
<tr>
<td>80°</td>
<td>0.006</td>
<td>0.494</td>
<td>0.501</td>
<td>0.008</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>( \Delta a = +10^\circ )</th>
<th>( X_t = 1.4\Omega )</th>
<th>( \Delta a = 10^\circ )</th>
<th>( X_t = 10\pi/9\Omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \Delta i )</td>
<td>( \Delta d )</td>
<td>( \Delta p )</td>
<td>( \Delta i )</td>
</tr>
<tr>
<td>5°</td>
<td>-0.000</td>
<td>-0.015</td>
<td>-0.015</td>
<td>-0.000</td>
</tr>
<tr>
<td>20°</td>
<td>-0.000</td>
<td>-0.064</td>
<td>-0.064</td>
<td>-0.000</td>
</tr>
<tr>
<td>30°</td>
<td>-0.000</td>
<td>-0.101</td>
<td>-0.101</td>
<td>-0.000</td>
</tr>
<tr>
<td>45°</td>
<td>-0.000</td>
<td>-0.174</td>
<td>-0.175</td>
<td>-0.001</td>
</tr>
<tr>
<td>60°</td>
<td>-0.001</td>
<td>-0.302</td>
<td>-0.304</td>
<td>-0.002</td>
</tr>
<tr>
<td>70°</td>
<td>-0.003</td>
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<td>-0.004</td>
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<tr>
<td>80°</td>
<td>-0.013</td>
<td>-0.988</td>
<td>-1.003</td>
<td>-0.015</td>
</tr>
</tbody>
</table>
2.7.3 The Complete Rectifier Model

This model is the first step towards building up a complete representation of the converter system and includes the capacitance and inductance of the network as shown in fig. 3.12.

A confirmation of the correctness and accuracy of the model can be determined by comparing the computer results with the approximate equations of the system, taking into consideration only the fundamental line current. Combining equations (2.2.2), (2.2.8) and (2.2.9) with (2.2.14), the line current is given by:

\[ I_1 = \frac{18V_t \cos \alpha}{\pi (\pi R_d^2 + 3X_t)} \]  

(2.7.5)

Thereafter, using equations (2.2.16), (2.2.7) and (2.2.8) and combining these with equation (2.2.11) gives

\[ I_1 (\cos \alpha - \frac{X_t I_1}{V_t}) = \frac{18V_t^2 R (\cos \alpha)^2}{(\pi R_d^2 + 3X_t)^2} \]  

(2.7.6)

In the particular case when the network parameters give

\[ \frac{X_t}{V_t} \approx \cos \alpha \]  

the above relation simplifies to

\[ I_1 \approx \frac{18V_t R_d}{(\pi R_d^2 + 3X_t)^2} \cos \alpha \]  

(2.7.7)

Solution of the system of equations (2.7.7) and (A2.8) in Appendix 2, gives \( I_1 \) and \( V_t \) for different values of network parameters (i.e. \( X_t \) and \( \alpha \)).

The linearized equation (2.7.3), using equation (A1.17), is given by

(see Appendix 2 equation A2.13)

\[ \Delta p = \Delta v_t + (1 - \frac{2\epsilon}{\cos \alpha}) \Delta i - \tan \alpha \Delta \alpha \]  

(2.7.8)

and assuming \((1 - \frac{2\epsilon}{\cos \alpha}) \approx 1\) the above equation (2.7.8) reduces to:

\[ \Delta p = \Delta v_t + \Delta i - \tan \alpha \Delta \alpha \]  

(2.7.9)

For delay angles up to 30° - the range over which HVDC systems are usually operated - equation (2.7.9) provides a very good approximation since it gives an error less than 2%, compared with the value given by equation (2.7.8); i.e. for \( \alpha = 30^\circ \), \( V_o = 330V \) and \( X_t = 4\Omega \), then \((1 - \frac{2\epsilon}{\cos \alpha})\) is equal to 0.001.
Results

A computer program which solved the equation (2.7.7), showed that there was a deviation from equation (2.2.14) of about 9% up to 12% over a range of delay angles from zero to $85^0$, for different network parameters as shown in Table 2.3 and figure 2.23.

2.7.4 The Simplified Converter Model

This converter model is designed to take into account the back voltage of the inverter, and it is an extension of the model described in Section 2.7.3.

This new model is intended to improve the accuracy of the results, taking into account some terms which were omitted or taken as approximations in Section 2.7.3; for instance using equation (2.7.6) instead of equation (2.7.7) for better accuracy.

The program is based on the simultaneous solution of equations (2.2.7), (2.3.1), (2.2.10) to (2.2.12), (2.4.4) and (A2.8). This results in the solution of a second-order system of equations (2.7.8) to (2.7.10) shown below using a polynomial solving package (see Appendix 2). The system is described by the relations:

\[
V_o^2 = V_o^2 \left(1 - \frac{X_R}{X_C}\right)^2 + 2V_o X_R I_1 \left(1 - \frac{X_R}{X_C}\right) \sin \phi + \frac{X_R^2}{R I_1} \tag{2.7.10}
\]

\[
V t_{I1} \cos \phi = \frac{18R_d}{(\pi R_d + 3(X_t - X_t'))^2} \left(V_t \cos \alpha - V_{t'} \cos \gamma\right)^2 \tag{2.7.11}
\]

\[
\cos \phi = \frac{\sqrt{6}}{\pi} \frac{I_d}{I_1} \left(\cos \alpha - \frac{X_t I_d}{\sqrt{6} V_t}\right) \tag{2.7.12}
\]

where $\sin \phi$ is taken as an approximation of the Taylor expansion (see Appendix 4, section 4.3).

Results

1. The accuracy of this model may be considered as satisfactory and there is good agreement between the values of $I_{IC}$ which the computer model
### TABLE 2.2

Values of Variables and Parameters of Model in Section 2.7.3

<table>
<thead>
<tr>
<th>Angle</th>
<th>$V_t$ (V)</th>
<th>$\epsilon / I_{1C} \times 10^3$ (1)</th>
<th>$I_{1C}$ (A) (3)</th>
<th>$I_1$ (A) (4)</th>
<th>$\Delta i$ (%) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>200.3</td>
<td>2.58</td>
<td>30.4</td>
<td>33.3</td>
<td>8.7</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>186.5</td>
<td>2.77</td>
<td>27.9</td>
<td>30.6</td>
<td>8.8</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>178.0</td>
<td>2.90</td>
<td>25.4</td>
<td>27.8</td>
<td>8.6</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>175.9</td>
<td>2.93</td>
<td>18.9</td>
<td>20.7</td>
<td>8.7</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>187.6</td>
<td>2.75</td>
<td>14.3</td>
<td>15.6</td>
<td>8.3</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>202.0</td>
<td>2.56</td>
<td>10.5</td>
<td>11.5</td>
<td>8.7</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>223.2</td>
<td>2.22</td>
<td>5.9</td>
<td>6.4</td>
<td>7.8</td>
</tr>
<tr>
<td>$85^\circ$</td>
<td>236.7</td>
<td>2.18</td>
<td>3.1</td>
<td>3.4</td>
<td>8.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Angle</th>
<th>$V_t$ (V)</th>
<th>$\epsilon / I_{1C} \times 10^3$ (1)</th>
<th>$I_{1C}$ (A) (3)</th>
<th>$I_1$ (A) (4)</th>
<th>$\Delta i$ (%) (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>205.3</td>
<td>3.57</td>
<td>29.1</td>
<td>33.0</td>
<td>11.8</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>191.3</td>
<td>3.83</td>
<td>26.7</td>
<td>30.3</td>
<td>11.9</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>182.5</td>
<td>4.01</td>
<td>24.3</td>
<td>27.6</td>
<td>12.0</td>
</tr>
<tr>
<td>$45^\circ$</td>
<td>179.7</td>
<td>4.07</td>
<td>18.0</td>
<td>20.4</td>
<td>11.8</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>190.9</td>
<td>3.84</td>
<td>13.5</td>
<td>15.3</td>
<td>11.8</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>204.8</td>
<td>3.57</td>
<td>9.9</td>
<td>11.2</td>
<td>11.6</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>224.9</td>
<td>3.26</td>
<td>5.5</td>
<td>6.3</td>
<td>12.7</td>
</tr>
<tr>
<td>$85^\circ$</td>
<td>237.7</td>
<td>3.08</td>
<td>2.9</td>
<td>3.3</td>
<td>12.1</td>
</tr>
</tbody>
</table>

1. $\epsilon = \frac{\pi}{6} \frac{I_1}{X_t V_t}$
2. $\Delta i = 100 \frac{I_1 - I_{1C}}{I_{1C}}$
3. $I_{1C}$: defined from eq (2.7.7)
4. $I_1$: " " or (2.2.14)
gives and those obtained from the classical equation (2.2.14), for a range of
delay angles between 0° to 70°, and this showed that this model is better
than the previous one of section 2.7.3.

2. For \( X_t = 0 \), the powers \( P_{W} \) and \( P_{D1} \) (as defined in Appendix 4) are equal,
as expected.
The various expressions for power factor angle, as defined by equations
(2.2.16) and (A4.2) in Appendix 4, gave the same numerical values and
this is a further verification of the accuracy of the model.

2.7.5 The Mathematical Model of the Converter

This mathematical model is intended to give an accurate solution of the
non-linear equations of a converter, using a digital computer. It provides
the steady-state characteristics of a converter system when a step-change
is made in the back voltage of the inverter. The difference between the
previous model and that described here is that the expressions which describe
the system are used without simplification, resulting in a more accurate
solution.

The plant is described by the system of equations (2.2.1), (2.2.7),
(2.2.10) to (2.2.12), (2.3.2), (2.4.4) and (2.7.10). The solution firstly
evaluates the line current and voltage \( (V_t \text{ and } I_1) \) as defined by equations
(2.7.10) and (2.7.11) where \( \sin \phi \) and \( \cos \phi \) are forced to be consistent
using an iteration technique (as described in Appendix 9).

Example - Results

The accuracy of this mathematical model may be considered satisfactory,
since it provides results in good agreement with results obtained on a
laboratory model (56), as shown below, and also with results previously
published (1), (20), (49).

1. Once measured network parameters of the experimental model (56) have
been substituted into the program, the results show that the mathematical
model gives a satisfactory accuracy, as demonstrated by the good agreement evident in Table 2.3.

2. The relationship which exists between the D.C. current \( I_d \) and the fundamental A.C. current \( I_1 \), in equation (2.2.14) give a very good approximation when compared with the real values of the currents, according to reference (49). The error in the current \( I_1 \) is greatest when \( \alpha=0^\circ \) but even then is less than 1.5%. The steady-state computer solution which calculates the currents \( I_1 \) and \( I_d \) directly gives a deviation at only about 1% for \( \alpha=3^\circ \).

3. Further analysis of the mathematical model demonstrates that approximating the power factor by the expression \( \cos 2\psi \cos \alpha \), is acceptable for most situations, when the system is working with a firing delay angle not larger than \( \alpha=30^\circ \). Thus for the steady-state values given in reference (1) - chosen to represent approximately a scaled-down real system, as shown in Table 3.7 - the deviation between angles \( \alpha \) and \( \phi \) is very small as indicated in Table 2.4.

The transmitted power \( P_{TR} \) increases considerably compared with the dissipated power \( P_{DIS} \) as the difference between the rectifier D.C. voltage and the back voltage of the inverter decreases, i.e. as this difference decreases from 10% to 5% the ratio \( P_{TR}/P_{DIS} \) increases from 16 to 64. The Celilo Sylman HVDC project in Los Angeles (33) is working with a ratio of 27 as shown in Table 3.7.
TABLE 2.3

Values of Variables and Parameters of a D.C. Simulator and the Mathematical Model in Section 2.7.5

<table>
<thead>
<tr>
<th>Hardware Model</th>
<th>Mathematical Model</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. DATA</strong> (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A.C. Voltage $V_0(V)$</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Rectifier leakage inductance $X_{t}(\Omega)$</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>Inverter leakage inductance $X^*_{t}(\Omega)$</td>
<td>0.18</td>
<td>0.18</td>
</tr>
<tr>
<td>Line Resistance $R_d(\Omega)$</td>
<td>13.6</td>
<td>13.67</td>
</tr>
<tr>
<td>Line Inductance $L(H)$</td>
<td>0.23</td>
<td>0.23</td>
</tr>
<tr>
<td>Delay angle, $\alpha(°)$</td>
<td>0°</td>
<td>0.1°</td>
</tr>
<tr>
<td>Extinction angle $\gamma(°)$</td>
<td>20°</td>
<td>20°</td>
</tr>
<tr>
<td><strong>2. RESULTS</strong> (1)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct Current, $I_d(A)$</td>
<td>1.0</td>
<td>1.034</td>
</tr>
<tr>
<td>Voltage at the point, $V_t(V)$ of connection of the transformer</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Commutation angle, $\mu(°)$</td>
<td>3.24</td>
<td>2.97</td>
</tr>
<tr>
<td>Power factor $\cos \phi$</td>
<td>0.9992</td>
<td>0.9993</td>
</tr>
<tr>
<td>Angle of power factor, $\delta(°)$</td>
<td>2.29°</td>
<td>2.17°</td>
</tr>
<tr>
<td>Inverter back voltage, $V_d(V)$ (2)</td>
<td>93</td>
<td>94.13</td>
</tr>
</tbody>
</table>
### TABLE 2.4

**Deviation Between Firing Delay Angle and Power Factor Angle (section 2.7.5)**

<table>
<thead>
<tr>
<th>Firing delay angle $\alpha$</th>
<th>$15^\circ$</th>
<th>$23^\circ$</th>
<th>$25^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power factor angle $\phi$</td>
<td>16.81</td>
<td>23.89</td>
<td>25.95</td>
</tr>
<tr>
<td>Power factor $\cos \phi$</td>
<td>0.9572</td>
<td>0.9143</td>
<td>0.8991</td>
</tr>
<tr>
<td>$\cos \alpha$</td>
<td>0.9659</td>
<td>0.9205</td>
<td>0.9063</td>
</tr>
<tr>
<td>Deviation ($\frac{\cos \phi - \cos \alpha}{\cos \phi}$)</td>
<td>0.0091</td>
<td>0.0068</td>
<td>0.0080</td>
</tr>
</tbody>
</table>
CHAPTER 3

THE DYNAMIC PERFORMANCE AND SIMULATION

OF THE CONVERTER SYSTEM
3.1 OPERATION AND CONTROL OF THE SYSTEM

In this chapter the dynamic performance of the converter system is analyzed. A first step is a preliminary investigation into the transient response of the control system on the rectifier side. Thereafter the work mainly concentrates on the mathematical representation of the inverter control systems using a digital computer.

3.1.1 The Aim of the Control System

Protection and stability are basic technical requirements in the design of a control system for a converter. The overcurrent protection required for valves and the current regulation, both necessitate a current controller on the rectifier side. Additionally, in order to avoid any collapse of the system and to keep the direct current within prescribed limits two current regulators are needed, (38), (49), (57) one on the rectifier side and the other on the inverter side.

The problem of how the transmission stability is affected by the control systems at each end of a D.C. link, has been analysed by Reider (3), (11) and the following overall conclusions are stated:

A. The operation of a D.C. transmission line with only a current regulator on the rectifier is stable for all values of the delay angle.

B. The operation of the system with regulators on a compounded inverter only (no regulator on the rectifier), is unstable.

C. The operation of the system with both regulators is stable.

The control system of an HVDC system is a very important part of the plant. In fact, many important operational properties of the whole transmission system are determined by the control system. (42)
Since the direct current in the link results from the difference between two voltages - the rectified d.c. voltage and the d.c. voltage provided by the invertor - the control equipment adjusts the firing angles in relation firstly to the current in the link and secondly to the voltage at the primary terminals of the converter transformer.\(^{(5)}\)

3.1.2 **The A.C.-D.C. System Representation**

A converter with a control system on the rectifier side, is shown in Fig. 3.16. A transducer is used to feed the comparator of the amplifier.

The system is normally operated with constant current regulation and the necessary regulators are included in the analysis described in this chapter. The elements enclosed within the solid lines of Fig. 3.16 make up the power circuit, and those within the dotted lines represent the control circuitry.

3.1.3 **The Mathematical Model of the System**

Although the operation of a HVDC system is basically non-linear, in practice it can often be considered as linear within a limited operating range. This approximation reduces the complexity of the system and, most importantly, the principle of superposition is still applicable. It is this principle that allows complicated solutions to be built up from simple ones.

Although linear theory is a powerful method for analysing HVDC systems - which are by their nature, complex - it is important to ensure that under all situations considered, it is a valid analysis technique.

In this context, the work described by Clade and Lacoste\(^{(5)}\), is of particular significance. These authors investigated the stability of a d.c. link with a control system which enabled the link to respond within 30msec. They concluded that although firing pulses can only be given to the valves at discrete intervals, these intervals are short enough so that, even for six-pulse operation their converter control could be considered
as a continuous operation. If this is so, one may utilise the classical well-known methods of analysis to investigate HVDC control systems.

Further justification of the use of linear theory is provided by Freris, who has considered a number of models for a typical HVDC system. These were a sample and hold model, an impulse model and a continuous system, and each was compared with corresponding experimental waveforms. In the case of the continuous model, Freris showed that the error between the model and experimental results, was in the order of only 4% for a system which responds to transients within about 20msec.

Other authors, for instance Sachdev, Fleming and Claud, have considered linear models, and have applied optimal control theory to the design of feedback systems, to improve system performance. Their conclusions on the validity of linear theory and the use of optimal control were based upon a system responding in about 25msec.

The accuracy of the digital computer studies reported by Brewer, Luini and Young provides further support for the use of linear theory. These authors considered the behaviour of a HVDC system in clearing a d.c. line fault. They established that estimated fault clearance times of about 15msec were obtained in practice with a well-designed control system.

All these studies and their experimental results, lead us to the belief that a continuous model will give results of acceptable accuracy, even for fast control systems.

Assuming small perturbations about a fixed operating point, the equations of operation can therefore be linearised in terms of the perturbed quantities, these being expressed in terms of the per unit steady-state values. The dynamic behaviour of the system can therefore be described by a set of linear differential equations.
The overall transfer function of the link can be derived from the equations relating the power and control circuits and involves two experimentally verifiable quantities, the control current $I_0$ and the d.c. current $I_d^{(5)}$.

3.1.3.1 The Control Elements

The elements in the control assembly are: $(5), (43)$

A. A Regulator (a comparator and an amplifier)
B. A Filter for the calculator
C. A Calculator for the control of trigger angle.
A. The Regulator

The purpose of this is to keep the d.c. current in the link always equal to the set value current $I_0$. By comparing the two currents the comparator provides an error signal of

$$y = I_0 - I_d t,$$  \hspace{1cm} (3.1.1)

the amplifier then modifies this to give an output voltage, according to the relationship

$$f(s) = K \frac{1 + gs}{1 + T_1 s},$$  \hspace{1cm} (3.1.2)

B. The Calculator

The calculator receives an output voltage $E$ from the regulator, and an A.C. voltage $V_f$ from the filter and provides the appropriate control signal to the controlled devices. In the steady-state regime, the angle $\alpha$ is given by

$$\frac{E - V_f}{V_f} = \cos \alpha,$$  \hspace{1cm} (3.1.3)

C. The Filter

The filter provides from the A.C. voltage of the network $V_t$, a filtered voltage $V_f$ from which harmonics have been removed and it is assumed therefore that the residual harmonic content is negligible.

The transfer function of the filter is given by:

$$G(s) = \frac{V_f}{V_t} = \frac{1}{1 + T_2 s},$$  \hspace{1cm} (3.1.4)

3.1.3.2 The Linearized Equations

A. Power Circuit Equations

From the phasor diagram of the A.C. network, shown in fig. 3.11 it can be shown that (see Appendix 2):

$$V_o^2 = V_t^2 \left(1 + \frac{X_R}{X_C}\right)^2 + 2V_t X_C R_1 (1 - \frac{X_R}{X_C}) \sin \theta + \frac{X_R^2}{X_C R_1},$$  \hspace{1cm} (3.1.5)
Equations (2.2.7), (2.4.4) and (3.1.5) can all be linearised in terms of perturbed quantities, about an operating point:

\[
\Delta V_t = \frac{\Delta V_t}{V_t}; \quad \Delta V_F = \frac{\Delta V_F}{V_F}; \quad \Delta d = \frac{\Delta E_d}{E_{do}}
\]

\[
\Delta i = \frac{\Delta I_d}{I_d}; \quad \Delta i_o = \frac{\Delta I_o}{I_o}; \quad \Delta e = \frac{\Delta E}{E}; \quad \Delta V_o = \frac{\Delta V_o}{V_o}
\]  

(3.1.6)

From the above equations it can be deduced that:

\[
\Delta d = \Delta V_t - \tan \alpha \Delta \alpha - \frac{X_{t} I_d}{\sqrt{E} \cos \alpha} \Delta i
\]  

(3.1.7)

\[
\Delta V_o = A \Delta V_t + B \Delta i + C \Delta \alpha
\]  

(3.1.8)

\[
\Delta d = r(1+Ts) \Delta i
\]  

(3.1.9)

where the quantities A, B, C, are defined in appendix 2.

B. The Control Circuit Equations

The control elements are set out in Section 3.1.3. From eqs. (3.1.2) to (3.1.4), the following relations in per unit values are obtained, (see Ap. 2):

\[
\Delta e = K \frac{I_o}{E} \frac{1+gs}{1+T_1 s} (\Delta i_o - \Delta i)
\]  

(3.1.10)

\[
\Delta \alpha = \frac{1+\cos \alpha}{\sin \alpha} (\Delta V_F - \Delta e)
\]  

(3.1.11)

\[
\Delta V_F = \frac{1}{1+T_2 s} \Delta V_t
\]  

(3.1.12)

3.1.3.3 The State-Space Form of the System

For the analysis of a complicated system it is essential to reduce the complexity of the mathematical expressions. The state-space approach to system analysis is well-suited from this viewpoint. The state-space form of system representation is used to obtain the required transfer function. These in turn can be used to perform studies on any of the plant controllers. (77), (79), (80)

Computationally, the state-space technique is particularly suited to digital computer implementation. (21)
Using $\Delta i, \Delta v_f$ and $\Delta e$ as the state variables and $\Delta v_0, \Delta i_0$ as the forcing functions and assuming that $\frac{d}{dt} \Delta i_0 = 0$ (since $i_0$ is assumed to be a step function), the equations of Section 3.1.3.2 can be put in the state-space form:

$$\dot{x} = Ax + Bu$$

where

$$\begin{bmatrix}
\dot{\Delta i} \\
\dot{\Delta v_f} \\
\dot{\Delta e}
\end{bmatrix} =
\begin{bmatrix}
\Delta i \\
\Delta v_f \\
\Delta e
\end{bmatrix},
\begin{bmatrix}
\dot{x}
\end{bmatrix} =
\begin{bmatrix}
A_0 \\
A_i \\
A_o
\end{bmatrix},
\begin{bmatrix}
u
\end{bmatrix} =
\begin{bmatrix}
\Delta v_0 \\
\Delta i_0
\end{bmatrix},
$$

$A$: is a 3x3 matrix and $B$: is a 3x2 matrix.

For the present analysis the following steady-state values are used:

- $V_T = V_F = V_0 = 330V$, $R_d = 7$ ohm, $L = 2.68H$
- $X_t = X'_t = 4.0$ ohm, $X_R = 8$ ohm, $X_C = 700$ ohm
- $I_d = I_0 = 1.25A$, $a = 25^\circ$, $T_2 = 0.025s$, $g = 0.02$
- $T_1 = 0.01s$ and $K = 800$.

These values come from reference (1) and are used to test the correctness of the computer program - described in Appendix 5 - which gives the state-space form of the system.

This numerical example enables matrices $A$ and $B$ to be calculated:

$$A = \begin{bmatrix}
-4.69 & -465.24 & 465.24 \\
-0.39 & -44.29 & 4.29 \\
-142.97 & 1468.18 & -1568.18
\end{bmatrix},
B = \begin{bmatrix}
215.03 & 0.0 \\
40.55 & 0.0 \\
-678.56 & 157.79
\end{bmatrix}$$

The values of the elements are similar to those in the literature\(^{(1)}\), i.e. the corresponding matrices are:

$$A = \begin{bmatrix}
-4.8 & -464.7 & -464.7 \\
-0.41 & -44.3 & 4.36 \\
-142.4 & 1463.7 & -1568.7
\end{bmatrix},
B = \begin{bmatrix}
218.4 & 0.0 \\
41.24 & 0.0 \\
-687.9 & 157.48
\end{bmatrix}$$

This demonstrates that the program is working well and can be used for further investigation of the converter system.
3.2 THE BUILD-UP OF THE RECTIFIER CONTROL SYSTEM

The step by step build-up of the control system on the rectifier side, is firstly to confirm that the expression for the overall transfer function of the system provide results in agreement with published results, and secondly to act as the first step in a preliminary investigation of the dynamic response of the system.

The objective of this section is to analyse a rectifier control system of the type described in reference (1). The work is later extended to include the design of a control system for the inverter side of a HVDC system. The analysis is made to assist in the understanding of the make up of a control system and to consider how the individual parts affect the dynamic behaviour of the HVDC system.

The design starts from the simplified current regulator, described in section 3.2.1 up to section 3.2.4, where a complete current regulator is analyzed; and digital computer programs have been written for them.

The static error and the settling time are used as criteria in judging the response of each system, which has been analyzed for different values of parameters, to show how they affect the dynamic behaviour of the control system.

This investigation showed that computer programs can be developed to give satisfactory accuracy; in which case they may be extended to investigate the control systems used on the inverter side.

The parameter values used in the investigation are taken from the literature (1), (5), and represent typical real systems. Thus the model gives values of $I_d = 1.25\text{amps}$ and $V_d = 706\text{V}$, whereas the corresponding values for real systems are $I_d = 1.288\text{A}$ and $V_d = 766\text{kV}$, as shown in Table 3.7.

The equations for the power and control circuits, developed in section 3.1.3, may be expressed in condensed form, so that the transfer function of
the link can be produced more simply. Thus

\[ \Delta d = \Delta A \Delta \alpha + B \Delta i + \Delta V_t \]  

(3.2.1)

corresponding to equation (3.1.7).

Once the supply voltage is assumed constant, i.e. \( \Delta V_0 = 0 \), relation 3.1.8 can be rewritten in condensed form as

\[ \Delta V_t = P \Delta i + Q \Delta \alpha \]  

(3.2.2)

Similarly, equations (3.1.9) to (3.1.12) can be expressed as

\[ \Delta d = C \Delta i \]  

(3.2.3)

\[ \Delta e = H(\Delta i_o - \Delta i) \]  

(3.2.4)

\[ \Delta \alpha = F(\Delta V_f - \Delta e) \]  

(3.2.5)

\[ \Delta V_f = S \Delta V_t \]  

(3.2.6)

The two quantities of the link that are amenable to experimental verification, \( \Delta i_o \) and \( \Delta i \) are related through a transfer function, and this can be derived from the signal flow diagram of the link, using Mason's formula.(21)

3.2.1 The Simplified Model

The first design of a current regulator is shown in Fig. 3.12. The control circuit consists of a regulator and a simple multiplier which satisfies the relation

\[ \Delta e = K_x \Delta \alpha \]  

(3.2.7)

where \( K_x < 0 \).

Using equations (3.2.1) to (3.2.4) and (3.2.7), represented by the graph in Fig. 3.13, we may produce the closed-loop transfer function relating the d.c. current and the reference current, \( I_o \), of the regulator (see Appendix 6, section 6.1):

\[ G(s) = \frac{\Delta i}{\Delta i_o} = \frac{H K_x (A+Q)}{H K_x (A+Q) - (P+B-C)} \]  

(3.2.8)

A verification of the above relation is that it can be deduced from the corresponding transfer function in the literature(5), by putting \( S=0 \) and \( F=-K_x \).
A computer program was written to produce the transfer function of the system for different values of the static coefficients of the multiplier and the amplifier.

**Example - Results**

1. **The system with a rectifier only**

   Analysis of the rectifier a.c.-d.c. system, using a digital computer, showed that a static coefficient of the multiplier (eq. 3.2.7) can be defined, such that a good margin of static error can be achieved, as well as an improvement in the settling time.

   Thus for $K_x = -1$

   \[ G(s) = \frac{2892(50+s)}{s^2 + 3001s + 145562} \]

   giving a static coefficient $e_{ss} = 0.993$, and the inverse Laplace transform is:

   \[ f(t) = 0.993 - 0.0635e^{-47t} - 0.980e^{-314t} \]

   Since the response of the system varies for different loads, as shown in Table 3.1, an adjustment of the above coefficients is needed in order for each system considered, to maintain the same performance.

   Further analysis showed that the static error of the system is not affected considerably as the gain $k$ changes, for small values of the resistance $R_d$. For example when $R_d = 7\Omega$, which is the value for the plant being investigated, the steady-state error is given by $e_{ss} = 0.99$ (see Figures 3.21 to 3.24).

   Similar comments can be made for variation in the coefficient $K_x$ (see equation 3.2.7), as shown in Figure 3.25.
### TABLE 3.1

Values of the steady-state error of the simplified model (Section 3.2.1), with and without an inverter

(INVERTER OMITTED)

<table>
<thead>
<tr>
<th>$R_d$ ($\Omega$)</th>
<th>$I_o = I_d$ (A)</th>
<th>$e_{ss}$</th>
<th>$K_x = -1$</th>
<th>$K_x = -0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K=800$</td>
<td>$K=1500$</td>
<td>$K=800$</td>
</tr>
<tr>
<td>564.2</td>
<td>1.25</td>
<td>0.573</td>
<td>0.716</td>
<td>0.736</td>
</tr>
<tr>
<td>300.0</td>
<td>2.34</td>
<td>0.722</td>
<td>0.829</td>
<td>0.838</td>
</tr>
<tr>
<td>70.0</td>
<td>9.6</td>
<td>0.927</td>
<td>0.960</td>
<td>0.962</td>
</tr>
<tr>
<td>20.0</td>
<td>29.8</td>
<td>0.979</td>
<td>0.988</td>
<td>0.988</td>
</tr>
<tr>
<td>7.0</td>
<td>65.7</td>
<td>0.988</td>
<td>0.993</td>
<td>0.993</td>
</tr>
<tr>
<td>5.0</td>
<td>80.5</td>
<td>0.989</td>
<td>0.994</td>
<td>0.994</td>
</tr>
<tr>
<td>1.0</td>
<td>147.4</td>
<td>0.992</td>
<td>0.999</td>
<td>0.996</td>
</tr>
</tbody>
</table>

### TABLE 3.2

(INVERTER INCLUDED)

<table>
<thead>
<tr>
<th>$R_d$ ($\Omega$)</th>
<th>$I_o = I_d$ (A)</th>
<th>$e_{ss}$</th>
<th>$I_o = I_d = 1.25A$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$K_x = -3$</td>
<td>$K_x = -1$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$K=800$</td>
<td>$K=800$</td>
</tr>
<tr>
<td>100</td>
<td>0.088</td>
<td>0.700</td>
<td>0.707</td>
</tr>
<tr>
<td>70</td>
<td>0.126</td>
<td>0.765</td>
<td>0.908</td>
</tr>
<tr>
<td>20</td>
<td>0.439</td>
<td>0.907</td>
<td>0.968</td>
</tr>
<tr>
<td>7</td>
<td>1.255</td>
<td>0.956</td>
<td>0.985</td>
</tr>
<tr>
<td>5</td>
<td>1.758</td>
<td>0.960</td>
<td>0.987</td>
</tr>
<tr>
<td>1</td>
<td>8.788</td>
<td>0.976</td>
<td>0.992</td>
</tr>
</tbody>
</table>
2. The system with a rectifier and an inverter

In this case the inverter voltage is chosen as $E_d' = 701.78V$ in order to make $I_d = I_o$. The transfer function for $K_x = -1$ is then given by:

$$G(s) = \frac{\Delta i}{\Delta i_o} = \frac{573. (50+s)}{s^2 + 678.4 + 29153}, \quad \text{and}$$

$$e_{ss} = 0.985,$$

A digital computer study of this system showed that the steady-state deviation from a set current value is affected by the value of $R_d$ as shown in Table 3.2 and Fig. 3.26. Therefore the greater the resistance $R_d$, the larger the steady-state deviation (i.e. the lower the accuracy of the system).

In any case, this deviation becomes smaller and smaller as the value of the resistance $R_d$ becomes smaller than $10\Omega$, the value chosen for the system is $R_d = 7\Omega$ and it is clear that the steady-state error is nearly 0.99. This relationship is clearly seen in Figures 3.26 and 3.27.

It seems that the steady-state value used for $R_d = 7\Omega$ is a good choice since it gives a satisfactory dynamic behaviour of the control system, while keeping the steady-state characteristics within the working areas of real systems.

3.2.2 The Inverse Transfer Function of the Simplified System

The capabilities of the controller designed in section 3.2.1 were tested using a sudden step increase of load. For an analysis in this case, the inverse transfer function is needed (see Ap.7) and this can be obtained from the signal flow diagram of Fig. 3.14, i.e.
Example - Results

For the system considered in section 3.2.1, the computer solution showed that for a unit step input, the response of the system improves as $K$ increases, i.e. with,

1. $K = \frac{1}{K^x}$, where $K^x = -1$ (see eq. 3.2.7), the final value of the system for a unit step input is $Y(t=\infty) = 1.021$.

The system transfer function is:

$$G'(s) = \frac{\Delta i}{\Delta_i} = \frac{HK_x(A+Q)-(P+B-C)}{HK_x(A+Q)}$$

Taking the inverse Laplace transform

$$y(t) = 1.02 + 0.076e^{-50t}$$

showing an overshoot of 1.096.

2. $K = -10$, the overshoot is 1.009 and $e_{ss} = 1.002$.

From the above results it is clear that with a suitable gain it is possible to achieve any required steady-state error, Table 3.3b. As $K$ increases the roots of the characteristic equation move to the left hence improving the settling time of the system. These are shown in Table 3.3.

3.2.3 The System with an Improved Controller

The next step is the design of a real controller which includes the fluctuation of the a.c. voltage. In this case equation (3.2.5) is used instead of equation (3.2.7) and the transfer functions of the system are given by (see appendix 6):
TABLE 3.3
Position of the roots of the characteristic equation for model in Section 3.2.2

<table>
<thead>
<tr>
<th>Kx</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>-40</td>
<td>-13</td>
<td>-107</td>
</tr>
<tr>
<td>-20</td>
<td>-16.4</td>
<td>-118.4</td>
</tr>
<tr>
<td>-10</td>
<td>-25.2</td>
<td>-138.4</td>
</tr>
<tr>
<td>-5</td>
<td>-33.8</td>
<td>-187.2</td>
</tr>
<tr>
<td>-3</td>
<td>-40</td>
<td>-257.3</td>
</tr>
<tr>
<td>-1</td>
<td>-47</td>
<td>-633</td>
</tr>
</tbody>
</table>

TABLE 3.3 (S)
Values of the steady-state error of the simplified model

(Section 3.2.2)

<table>
<thead>
<tr>
<th>Kx</th>
<th>e_{ss}</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.20</td>
<td>1.107</td>
</tr>
<tr>
<td>-0.33</td>
<td>1.064</td>
</tr>
<tr>
<td>-0.50</td>
<td>1.042</td>
</tr>
<tr>
<td>-1</td>
<td>1.021</td>
</tr>
<tr>
<td>-2</td>
<td>1.010</td>
</tr>
<tr>
<td>-10</td>
<td>1.002</td>
</tr>
</tbody>
</table>
\[ G(s) = \frac{HF(A+Q)}{HF(A+Q) + (P+B-C) - QF(B-C) + A FP} \]  
(3.2.12)

Equation (3.2.12) can be deduced immediately from the signal flow diagram of Fig. 3.15 by putting \( S=1 \) in equation (3.2.13), since the filter is omitted, (see section 3.2.4).

For the parameters defined in section 3.1.3.3, the system was investigated using a computer program.

**Example - Results**

By keeping all other parameters constant and varying only the resistance \( R_d \), the following results were obtained:

1. For the case of a rectifier only the analysis showed that at low currents the steady-state error \( e_{ss} \) is outside the required margin (see Table 3.4) i.e. for \( I_d = 1.25A \)

\[ G(s) = \frac{1326 \ (50+s)}{s^2 + 1636s + 87294} \]  
and \( e_{ss} = 0.76 \)

But for relatively large values of d.c. current, \( e_{ss} \) is satisfactory, although the system becomes sluggish (i.e. a longer settling time) since the roots are moving towards the imaginary axis as shown in Figure 3.28, i.e. for \( R_d = 7\Omega, \ I_d \) becomes 65.7A and,

\[ G(s) = \frac{820.5 \ (50+s)}{s^2 + 923s + 41301} \]  
and \( e_{ss} = 0.994 \)
### TABLE 3.4
Values of the steady-state error and position of the roots of the characteristic equation for the model (INVERTER OMITTED) for $K=800$

<table>
<thead>
<tr>
<th>$R_d (\Omega)$</th>
<th>$I_d=I_o (A)$</th>
<th>$e_{ss}$</th>
<th>Roots Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>560</td>
<td>1.25</td>
<td>0.76</td>
<td>Root A</td>
</tr>
<tr>
<td>300</td>
<td>2.34</td>
<td>0.85</td>
<td>-55</td>
</tr>
<tr>
<td>70</td>
<td>9.62</td>
<td>0.96</td>
<td>-52</td>
</tr>
<tr>
<td>20</td>
<td>29.8</td>
<td>0.98</td>
<td>-52</td>
</tr>
<tr>
<td>7</td>
<td>65.7</td>
<td>0.994</td>
<td>-49</td>
</tr>
<tr>
<td>3</td>
<td>104.2</td>
<td>0.998</td>
<td>-48</td>
</tr>
</tbody>
</table>

### TABLE 3.5
(INVERTER INCLUDED) for $K=800$

<table>
<thead>
<tr>
<th>$R_d (\Omega)$</th>
<th>$I_d=I_o (A)$</th>
<th>$e_{ss}$</th>
<th>Roots Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.126</td>
<td>0.968</td>
<td>-49.6</td>
</tr>
<tr>
<td>20</td>
<td>0.439</td>
<td>0.993</td>
<td>-49.0</td>
</tr>
<tr>
<td>7</td>
<td>1.255</td>
<td>0.998</td>
<td>-48.3</td>
</tr>
<tr>
<td>5</td>
<td>1.758</td>
<td>1.00</td>
<td>-47.9</td>
</tr>
<tr>
<td>3</td>
<td>2.929</td>
<td>1.002</td>
<td>-47.6</td>
</tr>
<tr>
<td>1</td>
<td>8.788</td>
<td>1.003</td>
<td>-47.4</td>
</tr>
</tbody>
</table>
2. When the inverter is included, the following results are obtained (see Table 3.5).

1. The steady-state error is satisfactory for a d.c. current range from 0.12A up to 1.76A, and approaches unity as \( I_d \) increases, i.e., for \( I_d = 1.25A \)

\[
e_{ss} = \lim_{s \to 0} \frac{1326 (50+s)}{s^2 + 1426s + 66349} = 0.998
\]

2. It seems that the steady-state error is practically independent of the load, as is evident from Table 3.5, and Figure 3.32, also it is obvious that the accuracy of the system is much better at low current levels, than it is when the inverter has been omitted (Tables 3.4 and 3.5).

3. It is clear that the gain \( K \) affects considerably the dynamic behaviour of the system as shown in Figures 3.29, 3.30 and 3.31, 3.32.

4. For a value of the gain of \( K = 800 \) which is a reasonable value for the control system since the roots of the system move only slowly towards the imaginary axis as load increases, as shown by Figure 3.33, so that the time response of the system may therefore be considered as invariant.

5. Further analysis of the system showed that for a sufficiently small static coefficient of the amplifier (\( K = 80 \) instead of 800), there is a slight deviation of the steady-state error near the operating point (\( I_d = 1.25A \)), but this deteriorates significantly as the current decreases while the settling time is improving (see Table 3.6 and Figure 3.32 and 3.34).
### TABLE 3.6
Values of the steady-state error and position of the roots (poles) of the model in section 3.2.3 (INVERTER INCLUDED) for \( K=80 \)

<table>
<thead>
<tr>
<th>( R_d (\Omega) )</th>
<th>( I_d=I_o (A) )</th>
<th>( e_{ss} )</th>
<th>Roots Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>0.126</td>
<td>0.744</td>
<td>-42.7</td>
</tr>
<tr>
<td>20</td>
<td>0.439</td>
<td>0.927</td>
<td>-36.1</td>
</tr>
<tr>
<td>7</td>
<td>1.255</td>
<td>0.992</td>
<td>-33.8</td>
</tr>
<tr>
<td>5</td>
<td>1.758</td>
<td>1.020</td>
<td>-33.0</td>
</tr>
<tr>
<td>3</td>
<td>2.929</td>
<td>1.013</td>
<td>-32.2</td>
</tr>
<tr>
<td>1</td>
<td>8.788</td>
<td>1.026</td>
<td>-30.3</td>
</tr>
</tbody>
</table>
3.2.4 The System with a Complete Current Controller

In this sophisticated controller, a filter for the a.c. voltage harmonics is included (Fig. 3.16).

3.2.4.1 The Transfer Function of the System

The overall transfer function of the system may be determined from the signal flow diagram of Fig. 3.15 by applying Mason's formula, and with all the equations of section 3.2 taken into account (see Appendix 8):

\[ G(s) = \frac{\Delta i}{\Delta i_0} = \frac{HF(A+Q)}{F(A+Q)(H+SP)+(P+B-C)(1-FQS)} \]  

(3.2.13)

3.2.4.2 Simulation of the System

The transient response of the control system has been simulated using a digital computer, and various subroutines have been used to plot the results. A computer program, described in Appendix 11, has been written to determine the transient response - for both step and impulse inputs - of 3rd and 2nd-order systems, with real poles and zeros, since this is the form of the transfer function of the converter control system.

A first application of the above digital computer program was the determination of the transient response of the rectifier control system. After the formulation of the transfer function of the control system, a verification of the correctness of the designed mathematical model was needed.
Example - Results

For the parameters defined in section 3.1.3.3, a computer program gave the closed-loop transfer function (see Appendix 8):

\[ G(s) = \frac{1468 (50+s) (40+s)}{s^3+1617s^2+137059s+2938880} \]  

(3.2.13a)

The simulation of the above 3rd-order optimal system on the rectifier side(18) showed that there is good agreement between the computed values and those presented in the literature.(5) Thus for an impulse input the system comes to rest in 0.188s as shown in Figure 3.20.

The poles of the system described by the transfer function of eq.(3.2.13a) are the same as those of the state-space representation of the system in section 3.1.3.3. This provides further verification of the accuracy of the solution.

Applying the final-value theorem, the output for a unit step input is \( Y(t \to \infty) = 1 \).

In addition the system can be shown to be stable at the operating point, by application of Routh's stability criterion.(21)
3.3 THE INVERTER CONTROL SYSTEM

In order to design a current control system on the inverter side, it is necessary to determine the equations of both the power and the control circuits. Additionally, the state-space form of the transfer function of the system should be determined, as mentioned in section 3.1.3.3. The main parameters used in the computer model were taken to be the average of those found in typical real systems, as indicated in Table 3.7, while the control parameters were derived from references (1), and (5).

3.3.1 Power and Control Circuits Relations

Applying the theory of small perturbations allows the dynamic performance of the system to be described by relations similar to those for the rectifier side, and shown in section 3.1. From the phasor diagram for the A.C. network on the inverter side, given in figures 3.17 and 3.18, it is possible to deduce that

\[ V_o^2 = (1 + \frac{X_R^2}{X_L^2 + R_L^2}) V_T^2 + \frac{X_R^2}{X_L^2 + R_L^2} I_1^2 + 2V_T I_1 \left[ \frac{X_R L}{X_L + R_L} \cos \theta - (1 + \frac{X_R L}{X_L + R_L}) \sin \theta \right] \]  

(3.3.1)

Linearizing about the point of operation gives:

\[ \Delta V_o = A_1 \Delta V_t + B_1 \Delta i + C_1 \Delta \phi \]  

(3.3.2)

where \( A_1, B_1 \) and \( C_1 \) are defined in Appendix 10.

Similarly, it is possible to obtain, as on the rectifier side, the following control circuit equations:

\[ \Delta d = \frac{R_i I_d}{V_d} (1 + s \frac{L}{R_i}) \Delta i \]  

(3.3.3)

\[ \Delta d = - \tan \gamma \Delta \alpha + \Delta v_t - \frac{X+I_d}{\sqrt{V_t}} \sin \gamma \Delta i \]  

(3.3.4)

\[ \Delta \gamma = \frac{1 + \cos \gamma}{\sin \gamma} (\Delta v_t - \alpha e) \]  

(3.3.5)
<table>
<thead>
<tr>
<th>PROJECT</th>
<th>DIRECT CURRENT $I_d$ (A)</th>
<th>DC VOLTAGE $V_d$ (KV)</th>
<th>POWER ($P_{TR}$) TRANSMITTED (MW)</th>
<th>POWER ($P_{DIS}$) DISSIPATED (MW)</th>
<th>TOTAL POWER (MW)</th>
<th>( \left[ \frac{V_{dm}}{V_d} \right]_{1d} ) ( \times I_d ) (A)</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Nelson-River Canada 1972-77</td>
<td>1800</td>
<td>900</td>
<td>1620</td>
<td>-</td>
<td>-</td>
<td>1.41</td>
<td>(1) The direct current value for a scale-down model according to the D.C. voltage, $V_{dm}=706V$, in the item 9.</td>
</tr>
<tr>
<td>2. Stalingrad-Donbass USSR 1958</td>
<td>900</td>
<td>800</td>
<td>720</td>
<td>-</td>
<td>-</td>
<td>0.79</td>
<td></td>
</tr>
<tr>
<td>3. Kingsnorth UK 1973</td>
<td>1200</td>
<td>532</td>
<td>640</td>
<td>-</td>
<td>-</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>4. Celilo-Sylmar Los Angeles USA 1967</td>
<td>1800</td>
<td>800</td>
<td>1350</td>
<td>50</td>
<td>1400</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>5. Pacific-Intertie USA 1970</td>
<td>1800</td>
<td>800</td>
<td>1440</td>
<td>-</td>
<td>-</td>
<td>1.59</td>
<td></td>
</tr>
<tr>
<td>6. Skagerrak Denmark 1976-77</td>
<td>1000</td>
<td>500</td>
<td>500</td>
<td>-</td>
<td>-</td>
<td>1.41</td>
<td></td>
</tr>
<tr>
<td>7. CU-Project USA 1978</td>
<td>1250</td>
<td>800</td>
<td>1000</td>
<td>-</td>
<td>-</td>
<td>1.10</td>
<td></td>
</tr>
<tr>
<td>8. Inga-Shaba Zaire 1976</td>
<td>560</td>
<td>1000</td>
<td>560</td>
<td>-</td>
<td>-</td>
<td>0.40</td>
<td></td>
</tr>
<tr>
<td>Average Values</td>
<td>1288</td>
<td>766</td>
<td>935</td>
<td>-</td>
<td>-</td>
<td>1.23 (2)</td>
<td>(2) Average value of the items 1 to 8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.35 (3)</td>
<td>(3) Average value of the items 1 to 7. The value of item 8 is excluded since the deviation is large from the mean value of the group.</td>
</tr>
<tr>
<td>9. Mathematical Computer Model</td>
<td>1.25A</td>
<td>706V</td>
<td>872.6MW</td>
<td>10.9W</td>
<td>883.5W</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
3.3.2 State-Space Form of the System Equations

Using $\Delta i_t$, $\Delta v_t$, and $\Delta e$ as the state variables and $\Delta v_o$, $\Delta i_o$ as the forcing functions, equations (3.3.2) to (3.3.7) can be put into a state-space form as explained in section 3.1.3.3; using the parameters defined in that section it is found that

$$A = \begin{bmatrix}
-5.36 & -425.83 & 425.83 \\
0.02 & -36.85 & -3.15 \\
-140.88 & 1343.82 & -1443.82
\end{bmatrix},
B = \begin{bmatrix}
97.47 \\
18.38 \\
-307.60 & 157.79
\end{bmatrix}$$

by using the computer program described in Appendix 5.

The poles of the state-space form and those derived from the transfer function of the system were exactly the same, and this provides verification of the accuracy of the solution.

For the simplified dynamic simulation of a HVDC system, with a stiff A.C. supply system, filters may be neglected. In this case $T_2=0$, when matrices $A$ and $B$ reduce to:

$$A = \begin{bmatrix}
-5.58 & 462.19 \\
-140.15 & -1558.56
\end{bmatrix},
B = \begin{bmatrix}
-114.93 & 0 \\
362.70 & 157.78
\end{bmatrix}$$

3.3.3 Transfer Function of the HVDC Link

From the equations of section 3.3.1 it is possible to define the closed-loop transfer function of the system relating the direct current $I_d$ to the control current $I_o$. Applying the same technique as in section 3.2.4, the overall transfer function of the link can be evaluated using the computer program described in Appendix 8, as

$$G(s) = \frac{\Delta i}{\Delta i_o} = \frac{1343.8(50+s)(40+s)}{s^5+1486.0s^4+125370.3s+2708221.4}$$

$$= \frac{1343.8(50+s)(40+s)}{(s+40.68)(s+1597.71)(s+47.63)}$$

(3.3.8)
This is the transfer function of the converter system as represented in Figure 3.17.

By changing the load \((R_L, X_L)\) on the receiving end, the two poles of the 3rd order system move symmetrically, as shown in Figure 3.35, while the value of the third pole, which is the largest, changes only slightly; this implies no effect on the sensitivity of the control system.

With the filter omitted, the system reduces to one of 2nd order, and the transfer function to:

\[
G(s) = \frac{1458.6 (50+s)}{s^2 + 1564.1s + 73486.7} \tag{3.3.9}
\]

In any case, the 3rd-order system of eq.(3.3.8) may be considered as very close to a 2nd-order system, since a closely-located pole-zero pair will effectively cancel.

Similarly by changing the load, the two poles of the above 2nd-order system remain practically invariant - as shown in Figure 3.36 - which means the load does not affect the sensitivity of the control system.

Further analysis of the system showed that lowering the natural time constant of the d.c. line, (i.e. by decreasing the line inductance), causes the system to oscillate as the numerator factor, \(g\), of the transfer function of the amplifier decreases. Thus, for \(\frac{L_d}{R_d} = 0.54H = 0.064s\) and \(g=0.0067\) the system becomes unstable, and complex poles exist at \(p_{1,2} = -262.4 \pm j66.58\). By altering the time constant of the amplifier, \(T_1\), the poles of the system become very large and far removed from typical real systems e.g. for \(T_1=0.0033s\), and holding the other parameters constant, the poles of the system are \(p_1=-47.52\) and \(p_2=-734.8\).

On the other hand, for large values of \(T_1\) the system becomes sluggish, as expected e.g. for \(T_1=0.02s\), \(p_1=-50.0\) and \(p_2=-734.8\).
3.3.4 Simulation of the System

The transient response of the control system has been simulated using the same computer programs described in section 3.2.4.

The system on the receiving end has been tested for various transient conditions (i.e. for impulse disturbances and step changes of current) and as expected it has been shown that the response of this uncompensated system is similar to that on the rectifier side, as shown in Figures 3.20 and 3.37.

In addition another program has been written for the case when the transfer function has complex roots (see appendix 11, section 11.2). The graphical subroutines of the Loughborough University of Technology Computer Centre have been used to demonstrate the oscillation of a system for both step and impulse inputs as shown in Figures 3.38 and 3.39.
CHAPTER 4

DESIGN OF A CLOSED-LOOP OPTIMAL CONTROL SYSTEM
This chapter describes the design of an optimal controller for the inverter side of a HVDC system, based on a quadratic performance index. The aim is to minimize the error signal and the control effort (i.e. the energy required to achieve the control action).

4.1 INTRODUCTION

The control system of a HVDC transmission link is a very important part of the plant. In fact, most of the operational properties of the whole system are determined by the control system. However, it is not always an easy task to design a fast and stable control system. The control process is rather complicated, since all parameters of the main and control circuits in the two terminals, as well as those of the D.C. link, must be considered.

In the classical approach to systems design, frequency domain techniques such as Nyquist plots and Bode diagrams are used to design control systems with an acceptable performance. In addition, a modern approach demands that the performance is not only acceptable but also optimal.

Problems of optimal control have received a great deal of attention during the past twenty years, owing to the increasing demand for systems of high performance and the ready availability of the digital computer. In fact, when applying modern control theory to the design of an optimal control system, large and fast digital computers are needed in order to determine the solution.

In this chapter, the selected criterion of optimization is the minimization of the error signal owing to an A.C. disturbance, since a smoothed response is desirable in the performance of HVDC power transmission systems. A current controller has been designed according to the regulation requirements of a HVDC system, as explained in Section 2.4.1.

The State Regulator Technique has been applied which gives a state control system. This method has the advantage of giving a control which is
a function of the system states, so that the feedback can be used to control the HVDC system. The disadvantage of this approach is the difficulty of the numerical solution.
4.2 SYSTEM EQUATIONS

The system may be described by a set of linear differential equations, as defined in Section 3.3.1. Assuming a stiff A.C. system, the operation of the filters may be neglected, when the system reduces to the second-order form indicated in Sections 3.3.2 and 3.3.3.

The system will contain derivatives of the driving function, since there are zeros in the transfer function. In view of this, a change of base is applied in order to eliminate these derivatives, and the matrix $[B]$ is altered as shown in Appendix 14.

The evaluation of the new matrix $[B]$ is included in a digital computer program which calculates the control-law for the optimal system (see Appendix 13).
4.3 DESIGN OF THE CONTROL SYSTEM

The state-space equations presented in Section 3.3.3 describe the system mathematically and have the form

\[ \dot{x} = Ax + Bu \]  

(4.1)

In order to consider an optimal performance, it is necessary to specify methods for determining the quality of performance of a system. In a modern approach this is most often achieved by means of an integral performance index.\(^{17}\)

A minimization of the system errors, regardless of the control energy required, is undesirable since all real systems are subject to physical constraints such as saturation.\(^{21}\)

Since we are interested in minimizing the error signal and the energy required, the quadratic form is a suitable performance index, i.e.,

\[ \text{PI} = \int_0^t (x^TQx + u^TPu) \, dt \]  

(4.2)

The design of the optimal control law for the system given by eq(4.1), subject to the above PI has the practical significance that the resulting system is a compromise between the requirements of minimizing the integral error and minimizing the control energy.

A characteristic of this performance index with its quadratic terms, is that it weights large errors more heavily than small ones. This implies that a system designed by this criterion tends to rapidly decrease large initial errors, leading to a fast response.

Since \(u\) is the control vector and \(x\) is the deviation of the state \(x(t)=0\) in relation 4.2, the purpose of minimizing a PI is to minimize the control effort and error response of the system after a disturbance.\(^{47}\)

In the state regulator, the state variables should be fed back not just to the output as in the case of unity feedback classical control theory, but to the system states which are used to generate a control feedback action such that the chosen Performance Index is minimized.
It is known that for an optimal solution, the Pontryagin $H$ function has to satisfy the relation

$$\frac{\partial H}{\partial u} = 0$$ (4.3)

where $H$ is defined as

$$H(x,u,\mathbf{v},t) = \mathbf{v}^T(Ax+Bu)+x^TQx+u^TPu$$ (4.4)

and $\mathbf{v}$ is a quadratic form variable, i.e.

$$\mathbf{v} = \mathbf{v}(x,t) = x^T\mathbf{R}(t)x$$ (4.5)

where

$$\mathbf{v}^T = \left(\frac{\partial \mathbf{v}}{\partial x_1}, \frac{\partial \mathbf{v}}{\partial x_2}, \ldots, \frac{\partial \mathbf{v}}{\partial x_n}\right)$$ (4.6)

Combining eqs. (4.3) and (4.4), the optimal control law is given by

$$u = -\frac{1}{2}P^{-1}\mathbf{v}$$ (4.7)

where $P$ must be a positive definite real symmetric matrix and $Q$ at least a semi-definite real symmetric matrix; the control vector $u$ is unconstrained.

For a closed-loop control law the Hamilton-Jacobi equation should be satisfied, i.e.

$$H^0(x,\mathbf{v},t) + \frac{\partial \mathbf{v}}{\partial t} = 0$$ (4.8)

where $H^0$ is derived from eq(4.4) by substituting into it the control $u$ obtained from eq (4.7).

Taking into account eq.(4.5) and (4.6), equation (4.8) can be written:

$$\dot{\mathbf{R}}(t) + Q - R(t)BP^{-1}B^TR(t) + R(t)A + A^TR(t) = 0$$ (4.9)

which is the matrix Riccati equation.

When the upper limit of the integral in relation (4.2) increases without bound, $T=\infty$. The $R$ matrix becomes constant (usually defined as $R_0$), and equation (4.9) reduces to

$$Q - R_0BP^{-1}B^TR_0 + R_0A + A^TR_0 = 0$$ (4.10)

where $R_0$ should be a positive definite matrix.

The elements of the matrix $R_0$ are defined from the solution of the non-linear matrix algebraic equation (4.10). A digital computer program
calculates these elements by solving a system of algebraic equations, as described in Appendix 13).

The optimal control is then given by

$$u_0(x) = -p^{-1}B^T R o x$$

where

$$u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \Delta v_0 \\ \Delta i_0 \end{bmatrix} \quad \text{and} \quad x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \Delta i \\ \Delta o \end{bmatrix}$$

This optimal control feedback was added to the current controller to enhance the system dynamic performance, minimizing the error signal, as shown in fig. (4.1) and explained in Appendix 12.

In many cases some of the feedback gains of the control vector may be considerably smaller than others, and may therefore be neglected. These approximations provide the so-called sub-optimal (i.e. nearly optimal) controller design. (23)

Thereafter, the system has been analysed using a digital computer, for both step and impulse disturbances in current, as shown in Appendix 11.
4.4 RESULTS

The parameters of the system under investigation are given in section 3.1.3.3. The matrices Q and P are chosen as

\[
P = \begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}, \quad Q = \begin{bmatrix}
1 & 0 \\
0 & \lambda
\end{bmatrix}
\]

where \( \lambda = 1 \).

These matrices determine the relative weighting of the error and the expenditure of the power of the control signal.

4.4.1 Evaluation of Matrix \( R_0 \)

The evaluation of the matrix \( R_0 \) as defined in Section 4.3, eq(4.10), is given by a digital computer program as explained in Appendix 13.

Sylvester's criterion, \(^{21}\) i.e., for a positive definite matrix, has been taken into account in the evaluation of the constant feedback matrix \( R_0 \), and a unique solution is obtained.

The computer solution gives:

\[
R_0 = \begin{bmatrix}
0.01034 & 0 \\
0 & 0.00033
\end{bmatrix}
\]

(4.12)

4.4.2 The Transfer Function of the Optimal System

The state feedback, as defined by equation (4.11), was added to the plant and the parameters of an optimal control system were determined as given in Appendix 12.

The eigenvalues of the new matrix

\[
[A_c] = [A - BR_0] = \begin{bmatrix}
-5.55 & -452.06 \\
140.17 & -1556.92
\end{bmatrix}
\]

(4.13)

as defined in Appendix 12, gives the poles of the optimal system.

The transfer function of the optimal system is then:

\[
G(s) = \frac{\Delta i}{\Delta i_0} \frac{1459(50+s)}{s^2+1562s+73400}
\]

(4.14)
4.4.3 The Time Response of the System

Thereafter the sub-optimal system can be treated for step and impulse disturbances in current using the digital computer program described in Appendix 12.

The results show that the response of the system may be considered as satisfactory compared with those of the optimal system on the rectifier side. Thus the system moves to the desired state in less than 0.15 sec, as indicated in Table 4.3.1. Additionally, no oscillations occur as shown in figure (4.2).
### TABLE 4.3.1

**TIME RESPONSE OF THE CONTROL SYSTEM**

<table>
<thead>
<tr>
<th></th>
<th>Settling Time in Seconds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Step input</td>
</tr>
<tr>
<td>ΔI=0.99I₀</td>
<td>ΔI=0.01I₀</td>
</tr>
<tr>
<td>Optimal Control System on the Rectifier side</td>
<td>0.038</td>
</tr>
<tr>
<td>Uncompensated system on the Inverter side</td>
<td>0.052</td>
</tr>
<tr>
<td>Optimal control system on the Inverter side</td>
<td>0.041</td>
</tr>
</tbody>
</table>

**NOTE:** Since the network on the inverter side is slightly different from that on the rectifier side, but the controller is of the same type, it is expected that the results will be near those given by the relevant optimal control system on the rectifier side.
CHAPTER 5

THE BOUNDED TIME-OPTIMAL CONTROLLER FOR A HVDC TRANSMISSION SYSTEM
This chapter describes the design of an open-loop time-optimal current controller, constrained in magnitude, for the inverter side of a HVDC converter.

Minimum settling time is used as a criterion of optimization.

5.1 INTRODUCTION

In practice the control signal of a HVDC transmission system is constrained in magnitude - a concession to the practical fact that no system has unbounded control available. The control is always limited by constraints such as saturation and limited power source.\(^{(17 46)}\)

The minimization of the transition time - from an initial to a final state - usually called the minimum time (or brachistochrone) problem, is of considerable importance for a power transmission system since a reduction in this time improves the response of the system. Therefore, the minimum settling time criterion is used as the basis of the optimization and the modern approach is formulated exclusively in the time domain.

Optimal control is applied to initial conditions rather than to an input. This is quite different from the classical methods of design which assume all initial conditions to be zero, enabling the transfer function concept to be used.\(^{(17)}\)

Pontryagin\(^{(18)}\) has shown that regardless of any constraint (such as saturation, limited power, etc.) that exists on the control \(u\), the optimal control must still be chosen to minimize the Hamiltonian\(^{(17)}\) of the system.

Applying modern control theory, an attempt has been made to design an open-loop optimal current controller for the inverter side of a HVDC converter system.

The controller will be constrained in magnitude and the design will employ the state-function of Pontryagin.
5.2 ANALYSIS OF THE CONTROL SYSTEM

The linearized model presented in this chapter is intended for use in the design of a converter time-optimal control system.

Linear control theory has been used for some time in the design and development of converter control systems. Feedback control also has been applied, since the converter control may be considered as a continuous operation and the time domain response of the system can be obtained by mathematical modelling techniques.

Digital computer programs have been developed for the calculation of the transfer function of the control loop and to demonstrate that a digital representation of d.c. transmission lines and their control is possible.

In fact, there are disturbances on HVDC systems, i.e. persistent faults - such as a sustained single-phase short-circuit in the a.c. system - and transient faults, such as a lightening impulse, which can produce an over-voltage factor as high as 2.2 per unit on the unfaulted pole.

The problem of changing the state of a dynamic plant from one value to another in the shortest time, when the control effort is limited, has been treated extensively in the literature.

Very fast control systems have been designed, whose time response have been in the order of 0.015 to 0.020s.
These results mean that during unbalanced conditions a current control loop has been optimized in such a way that for a pole-to-ground fault the current returns to the desired state in about 12ms to 15ms. In addition, very fast control systems are used to help circuit breakers eliminate a fault within 12ms, 20), (70), (71)

The use of combined voltage and current feedback for converter control has been investigated. Results of the linearized analysis showed that the use of voltage feedback appears to offer greater damping of system oscillations and considerably faster large signal response than the use of a conventional current regulator. (69)

Recently, open and closed-loop controllers have been used for multi-terminal HVDC transmission systems. (74)
5.3 **SYSTEM EQUATIONS**

The equations which describe the transient response of the converter have been defined in section 3.3.1 and modified in section 4.2. They consist of a set of linear differential equations of the form:

$$\dot{x} = Ax + Bu$$  \hspace{1cm} (5.1)

Assuming the control \(u(t)\) is constrained, the components \(u_i(t)\) of the control vector \(u(t)\) are dependent and constrained by the relation:

$$\sqrt{u_1^2 + u_2^2 + \ldots + u_n^2} = \left[ \sum_{i=1}^{n} u_i^2(t) \right]^{\frac{1}{2}} \leq m, \text{ for all } t$$  \hspace{1cm} (5.2)

where \(m\) is an integer bounded by the voltage and current.

For the plant as defined in section 3.3.1:

$$\begin{align*}
\dot{x} &= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \Delta i \\ \Delta e \end{bmatrix} \\
u &= \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \Delta v_o \\ \Delta i_o \end{bmatrix}
\end{align*}$$  \hspace{1cm} (5.3)

5.4 **DESIGN OF THE TIME-OPTIMAL CONTROL SYSTEM**

The first step in the design of an optimal system is to consider whether the system is controllable.

The concepts of controllability and observability were introduced by Kalman(17) and play a central role in modern control theory. If a plant is completely controllable it is possible to drive it to a desired (final) equilibrium state in a finite time. (21)

Even if the real control system is controllable the linearized model of the system may not be controllable and a test for controllability is therefore necessary in any attempt to design an optimal system.

The Performance Index of the time optimal problem which is required to be minimized is an integral:

$$\text{PI} = \int_{0}^{T} 1 \, dt$$  \hspace{1cm} (5.4)

The Hamiltonian for this problem is:

$$H(x,u,\lambda,t) = 1 + \lambda^T (Ax + Bu)$$  \hspace{1cm} (5.5)

By the minimum principle it is necessary that
which gives the relation
\[ \lambda^T Bu^0 \leq \lambda^T Bu \] (5.7)

Equation (5.7) means that whatever the value of \( \lambda(t) \), the time optimal control must minimize the scalar product
\[ \langle u^0, B \lambda \rangle \] (5.6)

This leads to the following time-optimal control \( u^0(t) \) (see Appendix 15):
\[ u^0(t) = -m \frac{B^T \lambda}{|B^T \lambda|} \] (5.8)

It can be shown(17) that a generalized boundary condition can be established:
\[ \left. \left( \frac{\partial S(x,t)}{\partial x} - \lambda \right) \right|_{T=t_f}^T + \left. \left( H^0(x,\lambda,t) + \frac{\partial S(x,t)}{\partial t} \right) \right|_{T=t_f}^T = 0 \] (5.9)

Since the final time is unspecified and \( x(t) \) is specified, it follows that \( dt \neq 0 \) and \( dx = 0 \), which leads to the requirement,
\[ \left. H^0(x,\lambda,t) \right|_{T=t_f} = 0 \] (5.10)

According to the Pontryagin procedure(59) an optimal system is described by the 2n equations (where, in this case \( n = 2 \))
\[ \frac{\partial H^0}{\partial \lambda} = \dot{x} = Ax + Bu \] (5.11)
\[ \frac{\partial H}{\partial x} = -\lambda = A^T \lambda \] (5.12)

and the given initial \( x(0) \) and terminal \( x(T_0) \) boundary conditions.

The differential equations (5.11) are the same as the plant equations (5.1), as expected.

Subject to the requirement of equation (5.10) a time-optimal bounded controller can be defined from the system of equations (5.11) and (5.12). This is given by relation (5.8).

Although the formulation of the problem for the solution has been completed, the actual computation difficulties of obtaining it is far more complicated. In fact the techniques for solving the optimal linear problem require, in general, the use of a digital computer for all but trivial problems.
The solution may be obtained only by numerical methods and then only if elaborate computational procedures are used. Usually these methods are highly sophisticated procedures that tax large computers. This is an unfortunate feature of the modern optimal-control approach, and it is the price that must be paid for demanding the best performance from a system.

A first step in the solution—after the mentioned test for controllability, shown in a computer program in appendix 17 (Figure A17.21 and A17.22)—is to diagonalise the matrix \( A \), since this facilitates the solution of the differential equations (5.11) and (5.12) which describe the optimal system.

For the evaluation of the new matrices of the equivalent system the following transformation has been applied:

\[
\dot{y} = P^{-1}APy + P^{-1}Bu
\]

where
\[
y = P^{-1}x
\]

In this case the new matrix becomes diagonal, i.e.

\[
P^{-1}AP = \text{diag}(S_1, S_2)
\]

where \( S_1 \) and \( S_2 \) are the poles of the system.

The solution of the costates (5.12) is then given by (see appendix 16):

\[
\lambda_1(t) = \lambda_1(0)e^{-S_1t}
\]

\[
\lambda_2(t) = \lambda_2(0)e^{-S_2t}
\]

where \( \lambda_1(0) \) and \( \lambda_2(0) \) are the unknown initial costates corresponding to the initial state \( x(0) \).

If equations (5.16), (5.17) and (5.8) are substituted into the plant equations (5.11), since these differential equations are linear, the general solution is given by integration as (see appendix 16):

\[
x_1(t) = e^{S_1t} [x_1(0) + \int_0^T e^{-S_1t} F_1(t) \, dt]
\]

\[
x_2(t) = e^{S_2t} [x_2(0) + \int_0^T e^{-S_2t} F_2(t) \, dt]
\]

where

\[
F_1(t) = \frac{B_{11}\lambda_1(t) + B_{12}\lambda_2(t)}{\sqrt{\lambda_1(t)^2 + \lambda_2(t)^2}}
\]
and
\[ F_2(t) = \frac{B_{21} \lambda_1(t) + B_{22} \lambda_2(t)}{\sqrt{\lambda_1(t)^2 + \lambda_2(t)^2}} \]  

(5.21)

Let \( T_0 \) be the minimum (optimal) time required to force the initial state \( x(0) \) to the final state \( x(T_0) \), then equations (5.18) and (5.19) must be satisfied simultaneously.

The analytical expression for the optimal open-loop controller as defined in equation (5.8) is given by (see appendix 15):

\[ u_1^0(t) = -m \frac{B_{11} \lambda_1(t) + B_{21} \lambda_2(t)}{||B^T \lambda||} \]  

(5.22)

\[ u_2^0(t) = -m \frac{B_{12} \lambda_1(t) + B_{22} \lambda_2(t)}{||B^T \lambda||} \]  

(5.23)

Difficulties arise in the analytic solution of equations (5.18) and (5.19) since the initial costates \( \lambda_1(0) \) and \( \lambda_2(0) \) are unknown. Additionally the integrals themselves may be somewhat complicated to evaluate. Thus the analytical solution of the two-point boundary-value problem is often extremely difficult.\(^{(18)}\)

A first step for the numerical solution is to define the admissible area in which the initial costates should lie. Since the Hamiltonian should be zero at the final time, \( H(T_0) = 0 \) and a relationship between \( \lambda_1(0) \) and \( \lambda_2(0) \) can be obtained. This relationship, equation (5.10), has been analyzed in appendix 15.

Thereafter, the simultaneous solution of equations (5.18) and (5.19) depends on the evaluation of the integrals that appear in them. For the evaluation of these integrals a trial-and-error approach has been used where the initial costates and the final time (optimal) vary.

A digital computer program has been written, as described in appendix 17, (figures A17.21 and A17.22) which determines the optimal (minimum) time in which the plant returns to rest.
5.5 RESULTS

Before turning to the solution of the control problem, another computer program will be described which was written to solve a similar problem for the simple system shown in reference (13).

The evaluation of simple integrals, for example a numerical approximation to π=3.1459 allows subroutines to be tested and checks that the computer programs work correctly.

The time optimal solution can be obtained by an analytical evaluation of the integrals that describe the system, i.e. equations (10.21) & (10.22) of reference (18). If then the equations are solved simultaneously, an optimal solution will result.

Thereafter comes the state-space form of the system, described in section 3.3.1, using the similarity transformation of section 5.4, i.e. equations (5.13) and (5.14):

\[
\begin{align*}
\dot{Y} &= \begin{bmatrix} -48.49 & 0 \\ 0 & -1515.66 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} -4.04 & 49.92 \\ 378.93 & 169.85 \end{bmatrix} \begin{bmatrix} \Delta v_0 \\ \Delta i_0 \end{bmatrix} \\
Y &= \begin{bmatrix} 1.0337 & 0.3164 \\ 0.0999 & 1.0764 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}
\end{align*}
\]

(5.24)

(5.25)

It is clear that in the above state-space form of the system, matrix A becomes diagonal and has as elements the system poles.

Therefore the differential equations of the costates, eqs.(5.16) and (5.17) can be written as

\[
\begin{align*}
\dot{\lambda}_1(t) &= \begin{bmatrix} -48.49 & 0 \\ 0 & -1515.66 \end{bmatrix} \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix} \\
\dot{\lambda}_2(t) &= \begin{bmatrix} -48.49 & 0 \\ 0 & -1515.66 \end{bmatrix} \begin{bmatrix} \lambda_1(t) \\ \lambda_2(t) \end{bmatrix}
\end{align*}
\]

(5.26)

and the solution of eqn.(5.26) is given by (see appendix 16):

\[
\begin{align*}
\lambda_1(t) &= \lambda_2(0)e^{48.49t} \\
\lambda_2(t) &= \lambda_2(0)e^{1515.61t}
\end{align*}
\]

(5.27)

(5.28)

The above expressions of the costate variables are linear of one unknown, and this helps in the solution of the differential equation describing the plant, eqs.(5.18) and (5.19), and hence the optimum controller.

The sensitivity of the control system has been tested by assuming that a disturbance in the a.c. system results in a sudden increase of the normalized d.c. current to the unit, i.e.,

\[
\begin{align*}
x_1(0) &= 1 \\
x_2(0) &= 0
\end{align*}
\]

(5.29)
this gives the initial conditions for the system, as

\[ \chi^{(0)} = \begin{bmatrix} y_1^{(0)} \\ y_2^{(0)} \end{bmatrix} = \begin{bmatrix} 1.0337 & 0.3164 \\ 0.0999 & 1.0764 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.034 \\ 0.100 \end{bmatrix} \]  

(5.30)

and as \( m=1 \), for the normalized perturbed quantities a numerical solution may be undertaken for the system given by equations (5.25), (5.27), (5.28) and (5.10).

A digital computer program for the solution of the above system, as shown in appendix 17, has been used to determine the optimal (minimum) time in which the system will return to rest. This program evaluates the initial costates as well, and these are given as

\[ \lambda_1^{(0)} = 0.0122, \quad \lambda_2^{(0)} = 0.600 \times 10^{-11} \]  

(5.31)

Substituting the above values into relations (5.22) and (5.23), describing the optimal open-loop controller, gives:

\[ u_0^1(t) = -0.0495 e^{48.49t} + 2.91 \times 10^{-9} e^{1515.61t} \]  

(5.32)

\[ u_0^2(t) = -0.612 e^{48.49t} + 1.027 \times 10^{-9} e^{1515.61t} \]  

(5.33)

where \( ||B^T \lambda|| \) is given by relation (A17.5).

The above calculations have also been checked by hand.

The details of the controller are given in Table T.5 and it is clear that the defined values will never exceed the constraint implied by equation (5.2).

A graph of the magnitude \( u=f(t) \) of the optimal controller against time is shown in Figure 5.2.

The minimum and therefore optimal time is determined by the digital computer program described previously, and \( t=0.015s \) which is a fast response, compared with the time of interruption of normal circuit-breakers approaching one cycle or \( 0.02s \).\(^{(12)}\),\(^{(60)}\)

The above results are not far from those produced by other authors working on similar models - as given in section 5.2 and defined in previous publications.
Studies on large a.c./d.c. systems using a digital computer have shown that the response of the system in clearing a d.c. line fault requires a settling time of approximately 0.015 s.\(^{(23),(27),(43)}\)

If the time delay in the feedback part of the control system is much shorter than the time between the firing of successive valves, then the linearized approximation is acceptable.\(^{(81)}\) When the time delay of the transducer is neglected, then only the transfer function of the amplifier is normally taken into account. For this model the time constant is \(T_1 = 0.010\) sec (a fast amplifier).

In any case, the current changes from the preceding valve to the succeeding one in a 3-phase bridge, every \(\frac{2\pi}{6}\) of the cycle, i.e. every 3.3 ms, and this allows a fast control system to affect the plant strongly.

The discontinuity has not been taken into account by authors who have applied optimal control theory, and this implies a continuous system.

The time constant of the d.c. line affects the behaviour of the control system, but the circuit of the d.c. line cannot be separated from the whole plant (i.e. the power circuitry and control system) since it is a constituent part of the plant. Thus, for example, though the time constant of a d.c. line of the system, represented in reference (18) is \(T = \frac{L}{R} = \frac{2.68H}{75} = 0.38\) s, the system returns to rest in 0.18 s, a time less than one half the natural time constant of the isolated d.c. line.

Besides, it is possible to achieve even a faster time response than 0.015 s but there are other engineering constraints.\(^{(82)}\)

A time-optimal bounded control system has been designed. The system returns to rest when a fault occurs in the line in less than 0.02 s; this is achieved with no oscillations since there are no complex roots. This time is considerably better than the uncompensated system which has a settling time of 0.107 s as shown in Figures (5.1),(5.2) and Table T.5.
### TABLE T.5

The Bounded Time-Optimal Controller

<table>
<thead>
<tr>
<th>Time (in msec)</th>
<th>$u_1^0(t)$ (normalized voltage controller)</th>
<th>$u_2^0(t)$ (normalized current controller)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0807</td>
<td>-0.9967</td>
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<td>5</td>
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<td>-0.9967</td>
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<tr>
<td>7</td>
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</tr>
<tr>
<td>15</td>
<td>0.4090</td>
<td>-0.9125</td>
</tr>
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</table>

**NOTE:** At any time $\sqrt{u_1^0(t)^2 + u_2^0(t)^2} = 1$ (equation 5.2).
CHAPTER 6

CONCLUSIONS
The objective of this research project was to analyze a typical HVDC transmission system, to study aspects of its performance and to design an optimal control system on the inverter side to improve the dynamic performance. To this end, the differential equations describing the converter plant were solved using a digital computer, and several computer programs were written to analyze and to confirm the mathematical relationship established for the HVDC system. Both steady-state and transient response characteristics were predicted, and the results obtained showed generally good agreement, both with results obtained experimentally and also with results shown in the literature.

The analysis of the system was divided into two parts, the first of these being concerned with the steady-state performance of the converter and the second with its dynamic performance.

A steady-state mathematical model for the converter system (i.e. the rectifier, the transmission line and the inverter) was established, and when solved on a digital computer this can accurately determine the characteristics of a converter at all operating points. Good agreement was obtained between the computed results and those obtained during a previous experimental project in the Department, generally within 3%; from an engineering aspect this may be considered to be of sufficient accuracy.

The investigation was then extended to the design of an optimal current controller for an inverter. This was required to minimize the control effort and to smooth the response, and to this end a state regulator technique, based on the Reduced Matrix-Riccati Equation, was applied. Thus a state feedback regulator was designed which drives the system, without oscillations, to the desired state in about 0.14s (the non-optimal time being 0.15s). This improvement between optimal and non-optimal system seems to be slight, as the system which was initially chosen for study already contained elements which were near optimal, and it was therefore not surprising that the improvement in performance should be slight. If this original system had not already been close to optimal, the improvement would naturally have been much greater.
Additionally, this time is close to results obtained from the literature for an optimal control system applied to the rectifier side.

The minimization of the transition time is of considerable importance for a power transmission system. Therefore, the minimum settling time criterion is used as the basis of the optimization and the modern approach is formulated exclusively in the time domain.

In practice, it is assumed that any real control is constrained in magnitude, and a bounded time-optimal control system was therefore designed, based on the Pontryagin Minimum Principle. A numerical solution using a digital computer, was established to derive the parameters of time-optimal control, which drives the system from the initial to the final state in a minimum time of about 0.015s. This time is much less than 0.14s obtained using the feedback technique, but the reason is that in the first case is concerned with a minimization of the error signal, while in the latter case we are only interested in minimizing the settling time.

Similar very fast control systems have been designed with time responses less than 20ms by applying modern control theory. In addition, this time-optimal control system is in line with the minimum time of interruption of present a.c. circuit-breakers, which approach one cycle, i.e. 20ms.

In any case to obtain a better performance from a system, using modern optimal control methods involves complicated mathematical techniques, or in other words there is a trade-off between computation efforts and improvement in system performance.

The work described in this thesis could be extended to include feedback compensation, based on the Matrix-Riccati Equation, where the final time is a fixed, rather than a variable parameter, and in this case a numerical solution on a digital computer is needed. However, this would
take a considerable amount of time and effort. In the case when the final time is assumed to be the minimum, as defined in the time-optimal control problem, then a control system may be found which satisfies the requirements of minimum settling time and smooth response; however this solution may give values of parameters, which are so large, since the control is unbounded, that they have no practical meaning from an engineering aspect. A comparison between this method and the Reduced Matrix-Riccati Equation which has already been applied, would act as a confirmation of the parameters of the state feedback control system.

In addition, under the assumption that an electrical system - usually considered as a black box - has more than one input and output, (i.e. current and voltage), further studies of an optimal nature based on multivariable theory may be applied to a HVDC system. In this case, if a solution can be found, a sophisticated and sensitive control system could be derived, where non-interaction between outputs can be achieved. This implies that every reference input affects only one output, in other words makes the output variables independent of each other.
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APPENDICES
APPENDIX 1

THE RECTIFIER PARAMETERS

1.1 D.C. VOLTAGE

The average value of d.c voltage can be derived from the time integral of a half pulse, \( q \), since the waveform is symmetrical. Then from fig. 2.13:

\[
\frac{E_d}{q} = \frac{E_m}{q} \int_{0}^{\pi/q} \cos \omega t \, dt = \frac{q}{\pi} E_m \sin \frac{\pi}{q} \quad (A1.1)
\]

For a 3-phase bridge connection \( q=6 \) and

\[
E_m = \sqrt{3} E_L = \sqrt{2} \sqrt{3} E_r \quad (A1.1a)
\]

Substituting the above relations into eq. (A1.1) gives:

\[
E_d = \frac{3\sqrt{6}}{\pi} E_r \quad (A1.2)
\]

1.2 ALTERNATIVE APPROACH

From fig. 2.13 by integration we have:

\[
E_d = \frac{1}{\pi/6} \int_{\pi/6}^{\pi/3} \left( E_1 \sin \omega t - E_2 \sin (\omega t - \frac{2\pi}{3}) \right) dt = \frac{6}{\pi} E_m \left[ \cos \omega t + \cos (\omega t - \frac{2\pi}{3}) \right]_{\pi/6}^{\pi/3} = \frac{6}{\pi} E_m \frac{\sqrt{3}}{2} = \frac{3\sqrt{6}}{\pi} E_r
\]

With phase control the time integral of the voltage \( E_a \) is given by:

\[
E_a = \frac{1}{2\pi/q} \int_{0}^{\alpha} E_m \sin \omega t \, dt = \frac{\alpha}{2\pi} E_m (1-\cos \alpha) = \frac{3\sqrt{6}}{\pi} E_r (1-\cos \alpha)
\]

then

\[
E_d = E_d - E_a = E_d \cos \alpha \quad (A1.3)
\]

Taking into account the reactive voltage drop, \( E_x \), the average voltage \( E_d \) can be obtained from the time integral of the voltage (see fig. 213) as follows:
\[ \delta V_x = \frac{1}{2} \int_a^{a+u} E_m \sin \omega t \, d\omega t = \frac{E_m}{2} (\cos \alpha - \cos (a+u)) \]  

(A1.4)

Then

\[ E_x = \frac{1}{2\pi/6} \delta V_x = \frac{E_{do}}{2} (\cos \alpha - \cos (a+u)) \]

(A1.5)

but

\[ E_d = E_{do} - E_a - E_x \]

(A1.6)

Substituting eqs. (A1.3) and (A1.5) into (A1.6) gives:

\[ E_d = \frac{E_{do}}{2} (\cos \alpha + \cos (a+u)) \]

(A1.7)

The average value of a sinusoidal wave is defined by:

\[ E_{av} = \frac{1}{\pi} \int_0^\pi e \, d\phi, \text{ where } \phi = \omega t \text{ and } e = E_m \sin \phi \]

1.3 D.C. CURRENT

Neglecting the resistance of the windings and the arc drop, which is very small compared to the leakage inductance \( L \) of each phase of the winding, the short circuit current is given by:

\[ 2L \frac{di}{dt} = E_m \sin \omega t \]

(A1.8)

Integrating gives:

\[ i = \frac{E_m}{2L} \int \sin \omega t \, d\omega t + C = \frac{E_m}{2\omega L} (-\cos \omega t) + C \]

(A1.9)

for \( \omega t = \alpha, \; i = 0 \) then

\[ C = \frac{E_m}{2\omega L} \cos \alpha \]

(A1.10)

Combining eqs. (A1.9) (A1.10) and (A1.1a) gives:

\[ i = \frac{\sqrt{3}}{\sqrt{2}\omega L} E_r (\cos \alpha - \cos \omega t) \]

(A1.11)

For \( \omega t = a+u, i = I_d \) and the above relation can be written:

\[ I_d = \frac{\sqrt{3}E_r}{\sqrt{2}\omega L} (\cos \alpha - \cos (a+u)) \]

(A1.12)

which is in accordance with reference (2).

1.4 POWER FACTOR ON THE RECTIFIER SIDE

Assume

\[ P_{AC} = \frac{2}{3} E_r I_1 \cos \phi \]

(A1.13)

and since the A.C. and D.C. powers must balance, i.e.

\[ P_{AC} = P_{DC} = E_d I_d \]

(A1.14)
then it is easy to determine the power factor on the rectifier side.

Combining relations (A1.2), (A1.5), (A1.13), (A1.14) and (2.2.14), gives:

$$\cos \phi \cong \frac{1}{2} (\cos a + \cos(a+u))$$  \hspace{1cm} (A1.15)

Again, combining eqs. (2.2.6), (2.2.14), (A1.2) and (A1.5) we obtain

$$\cos a - \cos(a+u) = \frac{\pi X_t I_1}{3 E_r}$$  \hspace{1cm} (A1.16)

Substituting eq. (A1.16) into (A1.15) gives:

$$\cos \phi = \cos a - \frac{\pi X_t I_1}{6 V_t} = \cos a - \varepsilon,$$  \hspace{1cm} (A1.17)

where \( \varepsilon = \frac{\pi X_t I_1}{6 V_t} \),

which is the same as given in the reference (5).
APPENDIX 2

THE LINEARISED EQUATIONS FOR THE CONTROL AND POWER CIRCUITS OF A CONVERTER

2.1 POWER CIRCUIT EQUATIONS

From the phasor diagram of the A.C. network, fig. 3.11 it follows that:

\[ \begin{align*}
V_o^2 &= V_t^2 + (X_R I)_t^2 - 2V_t X_R I \cos \beta \\
\text{but } \cos \beta &= \cos \left( \frac{\pi}{2} + \phi_1 \right) = -\sin \phi_1
\end{align*} \tag{A2.1} \]

Combining eqs. (A2.1) and (A2.2) gives:

\[ V_o^2 = V_t^2 + (X_R I)_t^2 + 2V_t X_R I \sin \phi_1 \tag{A2.3} \]

In order to express relation (A2.3) as a function of the fundamental line current, \( I_1 \), and the phase angle between it and the voltage, we consider the triangle ABG

\[ I \sin \phi_1 = I_1 \sin \phi - \frac{V_t}{X_C} \tag{A2.4} \]

\[ I \cos \phi_1 = I_1 \cos \phi \tag{A2.5} \]

Combining eqs. (A2.4) and (A2.5) it follows that

\[ \tan \phi_1 = \tan \phi - \frac{V_t}{I_1 X_C \cos \phi} \tag{A2.6} \]

Substituting eqs. (A2.4), (A2.5) and (A2.6) into relation (A2.3) and from trigonometry

\[ \sin^2 \phi = \frac{\tan^2 \phi}{1 + \tan^2 \phi} \tag{A2.7} \]

It therefore follows that:

\[ V_o^2 = V_t^2 \left( 1 - \frac{X_R}{X_C} \right)^2 + 2V_t X_R I_1 \left( 1 - \frac{X_R}{X_C} \right) \sin \phi + \frac{X_R^2 I_1^2}{X_C^2} \tag{A2.8} \]

Assuming small variations about the operating point, it is possible to linearise the above equations; taking the first derivative from the Taylor expansion:
\[ V(X_0 + \delta X) = V(X_0) + \frac{\partial V}{\partial X} \left| _{X=X_0} \right. \delta X + \frac{1}{2} \left. \frac{\partial^2 V}{\partial X^2} \right| _{X=X_0} \delta X^2 + \ldots \]

\[ \Delta V = V(X_0 + \delta X) - V(X_0) = \left. \frac{\partial V}{\partial X} \right| _{X=X_0} \delta X \]

then,

\[ 2V_0 \Delta V_o = 2V_t (1- \frac{X_R}{X_C})^2 \Delta V_t + 2X_R (1- \frac{X_R}{X_C}) (I_1 \sin \phi \Delta I_1 + \Delta V_t + V_t \sin \phi \Delta I_1 + V_t I_1 \cos \phi \Delta \phi) \Delta I_1 + 2X_R^2 \Delta I_1 \]

In terms of the perturbed quantities at the operating point (see eqs. 3.1.6) and assuming \( \Delta \approx \alpha \):

\[ \Delta V_o = A \Delta V_t + B \Delta I + C \Delta \alpha \quad (A2.10) \]

where

\[ A = \frac{1}{V_0} \left[ V_t^2 (1- \frac{X_R}{X_C})^2 + V_t X_R I_1 (1- \frac{X_R}{X_C}) \sin \alpha \right] \]

\[ B = \frac{1}{V_0} \left[ X_R^2 I_1^2 + V_t X_R I_1 (1- \frac{X_R}{X_C}) \sin \alpha \right] \]

\[ C = \frac{1}{V_0} \left[ -X_t X_R I_1 (1- \frac{X_R}{X_C}) \cos \alpha \right] \]

Combining eqs. (2.4.4) and (2.3.2) gives,

\[ E_d - E_d' = R I_d \text{ or } E_d - E_d' = (R- \frac{3}{\pi} t) I_d = R e I_d \quad (A2.11) \]

For small variations of \( E_d \), relation (A2.11) gives

\[ \Delta (E_d - E_d') = \Delta E_d = R e \Delta I_d + \frac{1}{I_d} \frac{d}{dt} \Delta I_d \]

taking Laplace transformation and normalising according to the relationship (3.1.6) gives:

\[ \Delta d = r(1+R) e \Delta I_d \quad (A2.12) \]

where \( r = \frac{R e I_d}{D_o} \), \( D_o = \frac{3 \sqrt{6}}{\pi} V_t \cos \alpha \) and \( s \) is the Laplace operator.

Again, linearisation of eq. (2.2.7) gives;

\[ \Delta E_d = \frac{3 \sqrt{6}}{\pi} (V_t (-\sin\alpha) \Delta \alpha + \cos \alpha \Delta V_t) - \frac{3}{X_t} \Delta I_d \]

\[ \Delta d = \frac{\Delta E d}{D_o} = -\tan \alpha \Delta \alpha + \Delta \phi \frac{X_t I_d}{\sqrt{6} V_t \cos \alpha} \quad (A2.13) \]
for the simplified model in Section (2.7.1.) $X_R = 0$

and relation (A2.13) gives:

$$
\Delta d = -\tan \alpha \Delta i - \frac{X_t I_d}{\sqrt{6} V_t \cos \alpha} \Delta i
$$  \hfill (A2.13a)

In the steady-state, for a frequency $\omega$, eq. (A2.12) gives:

$$
\Delta d = \frac{R_e I_d}{D_o} \sqrt{1 + \left(\frac{\omega L}{R_e}\right)^2} \Delta i = E_1 \Delta i
$$  \hfill (A2.14)

Also linearisation of power eq. (2.2.10) assuming $\cos \phi = \cos \alpha$, gives:

$$
\Delta P = 3(\Delta V_t I_1 \cos \alpha + V_t J_1 \Delta \cos \alpha - V_t I_1 \sin \alpha \Delta \alpha),
$$

normalising at the operating point, where $P_o = 3V_t I_1 \cos \alpha$ gives

$$
\Delta P = \frac{\Delta P}{P_o} = \Delta V_t + \Delta i - \tan \alpha \Delta \alpha
$$  \hfill (A2.15)

Taking into account eq. (A1.17) for small variations and differentiating it, $\Delta (\cos \phi) = \Delta (\cos \alpha) - \Delta (\epsilon)$

where $\epsilon = \frac{\pi}{6} \frac{X_t}{V_t}$ and

$$
\Delta (\epsilon) = \frac{\pi}{6} \frac{X_t}{V_t} \left(-I_1 \frac{\Delta V_t}{V_t^2} + \frac{\Delta I_1}{V_t}ight)
$$  \hfill (A2.17)

combining eq. (A2.16) and (A2.17), gives

$$
\Delta \phi = \frac{1}{\sin \phi} (\sin \alpha \Delta \alpha + \epsilon \Delta i - \epsilon \Delta V_t)
$$  \hfill (A2.18)

which agrees with reference (5).

According to the transformation (A2.16), eq. (A2.15) changes to

$$
\Delta p = \Delta V_t + (1 - \frac{2\epsilon}{\cos \alpha}) \Delta i - \tan \alpha \Delta \alpha
$$  \hfill (A2.19)

2.2 CONTROL CIRCUIT EQUATIONS

Combining eqs. (3.1.1) and (3.1.2) gives

$$
E = K \frac{1 + gs}{l + I_1 s} (I_o - I_1 dt)
$$

and linearising about the operating point, gives:

$$
\Delta E = K \frac{1 + gs}{l + I_1 s} (\Delta I_o - \Delta I)
$$  \hfill (A2.20)
Again from eq. (3.1.3) \[ \frac{\Delta (E - V_F)}{V_F} = \Delta (\cos \alpha) \]

\[ \frac{\Delta E - \Delta V_F}{V_F} + (E - V_F) \frac{\Delta V_F}{(-1)V_F^2} = -\sin \alpha \Delta \alpha \] (A2.21)

Normalising the above variables and combining eqs. (3.1.3) and (A2.21) gives:

\[ \Delta \alpha = \frac{1 + \cos \alpha}{\sin \alpha} (\Delta \varphi - \Delta \psi) \] (A2.22)

Similarly, eq. (3.1.4) gives immediately

\[ \Delta \varphi = \frac{1}{1 + T_2^s} \Delta v_t \] (A2.23)
APPENDIX 3

RECTIFIER AND INVERTER CHARACTERISTICS

3.1 THE CHANGE-OVER OF TRANSMISSION

The change-over from rectification to inversion occurs when \( E_d = 0 \), hence from eq. (2.2.3):
\[
\cos \alpha + \cos (\alpha + u) = 0
\]
or,
\[
\cos \alpha = -\cos (\alpha + u) = \cos (\pi - (\alpha + u))
\]
Relation (A3.1) is satisfied if:
\[
\alpha = 2k\pi + (\pi - (\alpha + u)) \quad \text{for} \quad k = 0, \pm 1, \pm 2, \ldots
\]
for \( k = 0 \), \( 2\alpha + u = \pi \) or \( \alpha + \frac{u}{2} = \frac{\pi}{2} \) (A3.2)

3.2 THE CHARACTERISTIC OF A RECTIFIER CONVERTER

In the case of a rectifier only, the relation \( u = f(\alpha) \) can be deduced by combining eqs. (2.2.4), (2.2.6) and (2.2.8) and (2.2.9).
This gives:
\[
\cos \alpha - \cos (\alpha + u) = \frac{6X_t}{\pi R_d} \cdot \frac{\cos \alpha}{1 + \frac{3X_t}{\pi R_d}}
\]
or
\[
\cos (\alpha + u) = \cos \alpha \left(1 - \frac{6X_t}{\pi R_d + 3X_t}\right)
\]
\[
= \frac{\pi R_d - 3X_t}{\pi R_d + 3X_t} \cos \alpha
\]
with the ratio \( \frac{X_t}{R_d} \) defined as \( K_x \), the above equation is the same as eq. (14) of reference (2).

3.3 THE RECTIFIER AND INVERTER CHARACTERISTICS

For the same current and a trigger angle \( \alpha = 0 \), relation (A3.3) gives
\[
\cos u = \frac{\pi R_d - 3X_t}{\pi R_d + 3X_t}
\]
(A3.4)
where \( u_0 \) is the commutation angle for \( \alpha = 0^\circ \).
For a constant d.c current $I_d$, from eq. (2.2.5) it can be deduced immediately that

$$1 - \cos u_0 = \cos \alpha - \cos (\alpha + u)$$

(A3.5)

Combining eq. (A3.4) with eq. (A3.5) gives:

$$u = a \cos \left( \cos \alpha - \frac{\frac{2}{\pi R_d}}{1 + \frac{d}{3 \chi_t}} \right) - \alpha$$

(A3.6)
4.1 THE RECTIFIER MODEL IN SECTION 2.7.3

This digital computer program gives the solution of the system of equations described in section 2.7.3. The flow chart for the computer is given in figure A4.11.

4.2 THE RECTIFIER AND INVERTER CHARACTERISTICS

This computer program computes the rectifier and inverter curves of trigger angle against commutation angle as explained in section 2.7.2. The flow diagram and the listing of the program are given in figures A4.21 and A4.22.

4.3 THE SIMPLIFIED CONVERTER MODEL (SECTION 2.7.4)

The solution of the system of equations derived in section 2.7.4 results in the determination of the unknowns \( V_t \) and \( I_1 \) using eqs. (2.7.7), (2.7.8) and (A1.17) or in the case of a rectifier only the following relation:

\[
\cos \phi = \frac{1}{3X_t} \cos \alpha \frac{1+\pi R_d}{1}
\]

This expression can be deduced by combining eqs. (A3.3) and (2.2.15).

A verification of the accuracy of relation (2.7.11) for the case \( V'_t=0 \) and \( X'_t=0 \), can be attained by comparing the computed results with those of the model in section 2.7.3.

Another expression of the power factor can be derived by combining eqs. (2.2.3), (2.2.4), (2.2.6) and (2.2.10) to (2.2.12) of section 2.2, i.e.,
\[ 3V_t I_1 \cos \phi = \frac{F_{do}}{2} \left( \cos \alpha + \cos (\alpha + \mu) \right) I_d \quad \text{or} \]

\[ \cos \phi = \frac{\sqrt{6}}{2\pi} \frac{I_d}{I_1} \left( \cos \alpha + \cos (\alpha + \mu) \right) = \frac{\sqrt{6}}{\pi} \frac{I_d}{I_1} \left( \cos \alpha - \frac{X_t I_d}{\sqrt{6}V_t} \right) \]  

(A4.2)

The apparent fundamental power is defined as \[ P_{wo} = 3V_t I_1 \]  

(A4.3)

The a.c. power for the case where \( u = 0 \) is defined as \[ P_{do} = E_{do} I_d' \]  

(A4.4)

The quantity \( \sin \phi \) was approximated by a Taylor series expansion combining eq. (A1.17) and using the binomial relationship

\[ (1+x)^n \approx 1+nx \]

\[ \sin \phi = \sqrt{1-\cos^2 \phi} = \sin \alpha \left( 1 - \frac{\pi}{6} \frac{X_t I_d}{V_t} \cos \alpha \right) \]  

(A4.5)

The flow chart in fig. A4.31 represents the software of the computer program.
APPENDIX 5

The State-Space Representation of the System

The system of eqs. (3.1.7) to (3.1.12) may be written in a condensed form, as follows:

\[ F_1(\Delta e, \Delta e, \Delta i, \Delta i_o, \Delta i_f, \Delta i_t) = 0 \quad \text{(eq. 3.1.10)} \]
\[ F_2(\Delta v_f, \Delta a, \Delta e) = 0 \quad \text{(eq. 3.1.11)} \]
\[ F_3(\Delta v_f, \Delta v_f, \Delta v_t) = 0 \quad \text{(eq. 3.1.12)} \]
\[ F_4(\Delta v_o, \Delta v_t, \Delta i, \Delta a) = 0 \quad \text{(eq. 3.1.8)} \]
\[ F_5(\Delta i, \Delta i, \Delta v_t, \Delta a) = 0 \quad \text{(combination of eqs. (3.1.7) and (3.1.9))} \]

Separating forcing functions and state variables as mentioned in section (3.1.3), and eliminating the undesirable variables, enables the system to be expressed in the form of eq. (3.1.13).

A computer program solves the system of the above variables and prints them in a state-space form, as shown in the flow chart and listing of the program, figures A5.11 and A5.12.
APPENDIX 6

TRANSFER FUNCTION OF A SIMPLIFIED CONVERTER SYSTEM

6.1 THE TRANSFER FUNCTION OF THE SYSTEM IN SECTION 3.2.1

From the signal flow diagram in fig. 3.13, using Mason's formula (21), the transfer function of the closed-loop system can be deduced, i.e.,

(i) Forward paths gain: $P_1 = \frac{AHK}{C}, P_2 = \frac{HQK}{C}$

(ii) Individual loops: $L_1 = -\frac{HKQ}{C}, L_2 = \frac{HKA}{C}, L_3 = P/C, L_4 = B/C$

(iii) Determinant of the graph:

$D = 1 - (L_1 + L_2 + L_3 + L_4) = 1 + \frac{HK}{C} (A+B) - \frac{1}{C} (P+B)$

(iv) The cofactors: $D_1 = 1, D_2 = 1$

Therefore, the closed-loop transfer function is given by:

$$G(s) = \frac{A_i}{A_i_0} = \frac{1}{D} \left(P_1 D_1 + P_2 D_2\right) = \frac{HK}{C} \frac{(A+Q)}{(A+Q) - (P+B-C)}$$

(A6.1)

6.2 THE TRANSFER FUNCTION OF THE SYSTEM IN SECTION 3.2.3

Using the same procedure as mentioned above, the following can be deduced from the signal flow diagram, figure 3.15:

(a) $P_1 = -\frac{HAF}{C}, P_2 = -\frac{HQF}{C}$

(b) $L_1 = \frac{HOF}{C}, L_2 = \frac{HAF}{C}, L_3 = P/C, L_4 = B/C, L_5 = QF, L_6 = \frac{AFP}{C}$

(c) $D = 1 - \sum_{i=1}^{6} L_i + \sum_{q=1}^{5} \sum_{m=1}^{4} L_m L_n = 1 - \frac{HF(A+Q)}{C} - \frac{P+B}{C} + \frac{QF(B-C)}{C} - \frac{AFP}{C}$

(d) $D_1 = 1, D_2 = 1$

(e) $G(s) = \frac{HF(A+Q)}{AF(A+Q) + (P+B-C) - QF(B-C) + AFP}$

(A6.2)
APPENDIX 7

THE INVERSE TRANSFER FUNCTION OF THE SIMPLIFIED SYSTEM
(SECTION 3.2.2)

The inverse transfer function of the system is defined as the transfer function which results when the output is considered as the input and vice versa.

In order to use Mason's gain formula in the inverse direction, the relations (3.2.1) and (3.2.4) in section 3.4.2 have to be re-arranged:

\[
\Delta a = L \Delta d + M \Delta i + N \Delta v t
\]  \hspace{1cm} (A7.1)

where

\[
L = \frac{1}{A}, \quad M = -\frac{B}{A}, \quad N = \frac{1}{A}
\]

\[
\Delta i_o = \frac{1}{H} \Delta e + \Delta i
\]  \hspace{1cm} (A7.2)

From the signal flow diagram of fig. 3.4 using the same symbols as in Appendix 6, gives:

(i) \( P_1 = \frac{CLK}{H} \times, P_2 = \frac{PNK}{H} \times, P_3 = \frac{MK}{H} \times, P_4 = 1 \)

(ii) \( L = QN \)

(iii) \( D = 1 - L = 1 - QN \)

(iv) \( D_1 = D_2 = D_3 = 1, D_4 = D \)

(v) The overall transfer function of the system is:

\[
G'(s) = \sum_{i=1}^{4} \frac{1}{D_i} (P_i D_i)
\]  \hspace{1cm} (A7.3)

or

\[
G'(s) = \frac{K (CL + PN + M) + H(1-QN)}{H(1-QN)}
\]  \hspace{1cm} (A7.4)

Substituting the values of \( L, M, N \) from (A7.1) we obtain:

\[
G'(s) = \frac{HK (A+Q) - (P+B-C)}{HK (A+Q)}
\]  \hspace{1cm} (A7.5)
APPENDIX 8

TRANSFER FUNCTION OF THE SYSTEM

WITH A COMPLETE CURRENT CONTROLLER

From the equations in section 3.4 and the corresponding signal flow diagram of fig. 3.15, the overall transfer function of the system can be deduced by applying Mason's formula (21):

(a) Forward path gains: \( P_1 = -AHF \), \( P_2 = -HFQ \)

(b) Individual loops: \( L_1 = \frac{HFQ}{C} \), \( L_2 = FQS \), \( L_3 = \frac{P}{C} \), \( L_4 = \frac{APSE}{C} \), \( L_5 = \frac{AHF}{C} \), \( L_6 = \frac{B}{C} \)

(c) Product of the non-touching loops: \( L_2 L_6 = BFQS \)

(d) Determinant of the graph: \( D = 1 - \sum_{i=1}^{6} L_i + L_2 L_6 \)

(e) Cofactors: \( D_1 = 1 \), \( D_2 = 1 \)

(f) Therefore, the closed-loop transfer function is

\[
G(s) = \frac{1}{D} \sum_{i=1}^{2} \frac{P_i D_{i1}}{F(A+Q)(H+SP)+(P+B-C)(1-FQS)}
\]

(A8.1)

This transfer function agrees with reference (5).

Thereafter, a computer program was developed to solve the system of eqs. (3.2.1) to (3.2.6) in section 3.2, assuming that there is no fluctuation of the supply voltage (i.e. \( \Delta V_o = 0 \)). This system establishes a relationship between the quantities \( \Delta i \) and \( \Delta i_0 \).

The flow chart and the listing are shown in figures A8.2 and A8.3.
This appendix describes the digital computer program which solves accurately the steady-state mathematical model of a HVDC system. The system of equations to be solved are those of appendix 4.3, but the quantities $\sin \phi$ and $\cos \phi$ described by eqs. A1.17 and A4.5 are forced to be consistent, using an iteration technique based upon the trigonometric relation

$$\cos^2 \phi + \sin^2 \phi = 1 \tag{A9.1}$$

The accuracy of the solution can be chosen by the user, and depends on how closely equation (A9.1) is required to be satisfied.

The flow chart and the listing of the program are given in figures A9.2 and A9.3 respectively.
APPENDIX 10

POWER CIRCUIT RELATIONS ON THE INVERTER SIDE

The mathematical expressions describing the power circuit on the inverter side of a converter can be established as follows:

From equation (2.4.4) taking into account equations (2.2.7) and (2.3.1), it can be deduced immediately that,

\[ E_{d0} V_d = (R_d + \frac{3}{\pi} X_L) I_d = R_I I_d \quad (A10.1) \]

Taking small variations about the operating point, gives in per unit values:

\[ \Delta d = \frac{R_I I_d}{V_{d0}} (1 + \frac{L}{R_I}) \Delta i \quad (A10.2) \]

From the phasor diagram, figure 3.19 of the fundamental frequency of the A.C. network, it is possible to deduce that

\[ V_o^2 = V_t^2 + (X_R I_g)^2 = 2 V_t (X_R I_g) \cos \beta \quad (A10.3) \]

from the diagram

\[ I_g \cos \theta = I_L \cos \phi - I_1 \cos \phi_L \] and

\[ \cos \phi_L = \frac{R_L}{\sqrt{X_L^2 + R_L^2}}, \quad I_L = \frac{V_t}{\sqrt{R_L^2 + X_L^2}} \]

therefore

\[ I_g \cos \theta = \frac{V_t}{X_L^2 + R_L^2} \frac{R_L + I_1 \cos \phi}{X_L^2 + R_L^2} \]

\[ V_o^2 = (1 + \frac{X_R^2}{X_L^2 + R_L^2}) V_t^2 + X_R I_1^2 + \]

\[ + 2V_t X_R I_1 \left[ \frac{X_R R_L}{X_L^2 + R_L^2} \cos \phi - \frac{X_R X_L}{X_L^2 + R_L^2} \sin \phi \right] \quad (A10.4) \]

Linearizing eqn. (A10.4) about a point of operation gives:

\[ \Delta V_o = A_1 \Delta V_t + B_1 \Delta I + C_1 \Delta \phi \quad (10.5) \]

where
\[ A_1 = \frac{1}{V_0^2} \left( \frac{X_R^2}{Z_L^2} + \frac{X_R X_L}{Z_L^2} \right) V_t^2 + \left[ \frac{X_{RL}^2}{Z_L^2} \cos \phi - \left( \frac{X_{RL}^2}{Z_L^2} + X_R \right) \sin \phi \right] V_t I_1 \]

\[ B_1 = \frac{1}{V_0^2} \left[ \frac{X_{RL}^2}{Z_L^2} + \left[ \frac{X_R^2}{2 Z_L^2} \cos \phi - \left( \frac{X_{RL}^2}{Z_L^2} + X_R \right) \sin \phi \right] V_t I_1 \right] \]

and

\[ C_1 = \frac{1}{V_0^2} \left[ \frac{X_{RL}^2}{Z_L^2} \cos \phi + \left( \frac{X_{RL}^2}{Z_L^2} + X_R \right) \sin \phi \right] V_t I_1 \]

where

\[ Z_L = \sqrt{X_L^2 + R_L^2} \]

In practice \( \cos \phi \approx \cos \gamma \) and since the A.C. network is considered stiff, \( \Delta V_o=0 \), and eq. (A10.5) can be written in condensed form as

\[ \Delta v_t = \Delta \phi \Delta i + Q \Delta \gamma \]  \hfill (A10.6)

For the case when \( R=0 \) equation (A10.4) reduces to:

\[ V_o^2 = V_t^2 \frac{X_R}{X_L} \frac{1}{1+ \frac{X_R}{X_L}} + (X_R I_1)^2 - 2V_t X_R I_1 \frac{X_R}{X_L} \frac{1}{1+ \frac{X_R}{X_L}} \sin \phi \]  \hfill (A10.7)

Equation (A10.7) is similar to that describing the power circuit on the rectifier side, as given in reference (18).
APPENDIX 11

DIGITAL COMPUTER PROGRAMS FOR THE TIME-RESPONSE
OF A CONTROL SYSTEM

This appendix describes the method of solution employed by a computer program to determine the time response of a control system for step and impulse disturbances.

The software of the program is based on the Heaviside's theorem and on the partial fraction expansion method for finding the Inverse Laplace Transform $f(t)$.

11.1 SYSTEM WITH REAL POLES

Both methods have been used in order to find the time response - for step and impulse inputs - of a system in which the numerator is not of higher order than the denominator.

The numerator coefficients of the expanded polynomial, according to Heaviside's theorem are given by:

$$k_i = \frac{(s+s_i)F(s)}{s=-s_i}, \quad i=1,2,...,n \quad (A11.1)$$

where

$$F(s) = \frac{A(s)}{B(s)} = \frac{(s+a_1)(s+a_2)...(s+a_m)}{(s+s_1)(s+s_2)...(s+s_n)} = \frac{k_1}{s+s_1} + ... + \frac{k_i}{s+s_i} + ... + \frac{k_n}{s+s_n} \quad (A11.2)$$

In the case of the partial fraction expansion method the definition of the coefficients $k_i$ requires the solution of an algebraic system with $n$ linear equations, where $n$ is the order of the transfer function. Thus in the case of an impulse input to a $3^{rd}$ order system, the evaluation of the coefficients is given by applying Cramer's method to the following linear system:
The flow chart and the listing of the relevant digital computer program, are shown in figures A11.4 and A11.5.

11.2 SYSTEM WITH COMPLEX POLES

A digital computer program has been written which evaluates the inverse Laplace transform, when the denominator $F(s)$ involves complex conjugate poles. The time response of the system is plotted using graphical output subroutines, for step and impulse inputs.

The flow chart and the listing of the program are shown in figures A11.6 and A11.7.
APPENDIX 12

STATE FEEDBACK COMPENSATION

This appendix describes the method of shifting the poles of a control system by applying the state feedback compensation technique.

Given the system
\[ \dot{x} = Ax + Bu \]  
\[ y =Cx \]  
(A12.1)  
(A12.2)

The transfer function is defined by:
\[ G(s) = C(sI-A)^{-1}B \]  
(A12.3)

The poles of this system, eq. (A12.1), are the eigenvalues of the matrix A.

When state feedback compensation is applied (see fig. 4.1)
\[ u = v - kx \]  
(A12.4)

and the system (A12.1) gives
\[ \dot{x} = Ax + B(v-kx) \]  
(A12.5)

The matrix \((A-Bk)\) gives the new poles of the compensated system.

The new transfer function is then:
\[ G_c(s) = C \frac{\text{adj}(sI-A_C)}{\text{determinant}(sI-A_C)} \]  
(A12.6)

where \(A_C = A-Bk\) and I is the unit matrix.

Application

After the definition of the matrix \(R_o\) given in equation (4.12),
the new matrix \(A_C = [A-B.R_o]\) is defined:
\[
A_C = \begin{bmatrix}
-5.59 & 462.19 \\
-140.16 & -1558.56
\end{bmatrix}
- \begin{bmatrix}
0.042 & -0.13 \\
-0.015 & 1.64
\end{bmatrix}
= \begin{bmatrix}
-5.55 & -462.06 \\
140.17 & -1556.92
\end{bmatrix}
\]  
(A12.7)
APPENDIX 13

COMPUTER PROGRAM FOR THE SOLUTION OF THE
REDUCED MATRIX RICCATI EQUATION (CHAPTER 4)

In this appendix a digital computer program is presented for the solution of the algebraic equations describing the control law which makes the system optimal. In addition, the program calculates the constraints of the state optimal control system.

The program is described in the form of a flowchart and a listing as shown in figures A13.1 and A13.3.

The Numerical Algorithms Group (NAG) library subroutines used are mentioned in Appendix 18.
Consider the control system:
\[ \dot{x} = Ax + Bu + ku \]  
(A14.1)

Assuming
\[ z = Px + Qu \]  
(A14.2)

then
\[ \dot{x} = P^{-1}[z - Qu] \]  
(A14.3)

and
\[ \dot{z} = P\dot{x} + Qu \]  
(A14.4)

Combining eqs. (A14.1) and (A14.4) gives:
\[ \dot{z} = P[Ax + Bu] + [P.k + Q]u \]  
(A14.5)

In order to eliminate the derivatives \( \dot{u} \), in eq. (A14.5), put:
\[ [P.k + Q] = 0 \]  
(A14.6)

Combining eqs. (A14.3), (A14.5) and (A14.6) gives
\[ \dot{z} = [PAP^{-1}]z + [PB - PAP^{-1}Q]u \]  
(A14.7)

When \( P=I \) equation (A14.6) gives:
\[ Q = -k \]  
(A14.8)

and relation (A14.7) becomes
\[ \dot{z} = Az + [B + Ak]u \]  
(A14.9)

where derivatives of the driving function have been eliminated.
APPENDIX 15

PONTRYAGIN'S STATE FUNCTION

1. DEFINITION OF THE OPTIMAL CONTROLLER

From the Hamiltonian eq. (5.6) and relation (5.5) it follows that

\[ \lambda^T Bu^o \leq \lambda^T Bu \]  \hspace{1cm} (A15.1)

or as a scalar product

\[ <\lambda^T B, u^o> \leq <\lambda^T B, u> \]

i.e.

\[ <u^o, B^T \lambda> \leq <u, B^T \lambda> \] \hspace{1cm} (A15.2)

For an optimal solution it is required to minimize the left hand side of relation (A15.2):

\[ \min <u^o, B^T \lambda> \] \hspace{1cm} (A15.3)

Clearly to minimize the scalar product (A15.3) the vector \( u^o \) must be of opposite sign to the vector \( B^T \lambda \), and the control \( u^o \) must be as large as possible,

\[ ||u|| \leq m \]

Then

\[ u^o = -m \cdot \text{sign}(B^T \lambda) \] \hspace{1cm} (A15.4)

But

\[ \text{sign}(B^T \lambda) = \frac{B^T \lambda}{||B^T \lambda||} \]

where

\[ ||B^T \lambda|| = \left\{ \left[ b_{11} \lambda_1(t) + b_{21} \lambda_2(t) \right]^2 + \left[ b_{12} \lambda_1(t) + b_{22} \lambda_2(t) \right]^2 \right\}^{1/2} \] \hspace{1cm} (A15.5)

therefore

\[ u^o(t) = -m \frac{B^T \lambda}{||B^T \lambda||} \] \hspace{1cm} (A15.6)

2. EVALUATION OF THE INITIAL COSTATES

The choice of a pair of initial costates is not arbitrary.

Equation (5.10) establishes a relationship between the two costates \( \lambda_1(0) \) and \( \lambda_2(0) \):
Considering eq. (A15.7) as a quadratic of one unknown, either $\lambda_1(0)$ or $\lambda_2(0)$, and analysing it for real roots (its characteristic should be greater or equal to zero, i.e. $Q=b^2-4ac\geq0$) a locus is defined within which each of the initial costates should lie.

After manipulation eq. (A15.7) has the form:

\[
(F_1 e^{-S_1 T_0})p_1^2 + 2(F_3 e^{-S_1 T_0} p_2) + [(F_2 e^{-S_1 T_0} p_2 - 1)^2 = 0
\]

(A15.8)

where

\[
F_1 = B_{11}^2 + B_{12}^2, \quad p_1 = \lambda_1(0), \quad S_1, S_2 = \text{the poles of the system}
\]

\[
F_2 = B_{21}^2 + B_{22}^2, \quad p_2 = \lambda_2(0),
\]

\[
F_3 = B_{11}B_{21} + B_{12}B_{22}
\]

The relationship between $p_1$ and $p_2$ is given from eq. (A15.8), as

\[
p_1 = e^{-S_1 T_0} \left[ -F_3 p_2 \pm \sqrt{F_1^2 p_2 - 2F_1 F_2 e^{-2S_1 T_0} p_2 - 1} \right]
\]

(A15.9)

similarly,

\[
p_2 = e^{-S_2 T_0} \left[ -F_3 p_1 \pm \sqrt{F_1^2 p_1 - 2F_1 F_2 e^{-2S_2 T_0} p_1 - 1} \right]
\]

(A15.10)

where

\[
F = B_{11}B_{22} - B_{12}B_{21}
\]

Since the quantity inside the square root should be greater than or equal to zero, the initial costates $\lambda_1(0)$ and $\lambda_2(0)$ from eq. (A15.8) are constrained by

\[
-\frac{\sqrt{F_1}}{B} e^{-S_1 T_0} \leq \lambda_1(0) \leq \frac{\sqrt{F_2}}{B} e^{-S_1 T_0}
\]

(A15.11)

\[
-\frac{\sqrt{F_2}}{B} e^{-S_2 T_0} \leq \lambda_2(0) \leq \frac{\sqrt{F_1}}{B} e^{-S_2 T_0}
\]

(A15.12)

The vector equation (A15.6) can be expanded:

\[
u_1^o(t) = \frac{B_{11} \lambda_1(t) + B_{21} \lambda_2(t)}{||B\lambda||}
\]

(A15.13)

\[
u_2^o(t) = \frac{B_{12} \lambda_1(t) + B_{22} \lambda_2(t)}{||B\lambda||}
\]

(A15.14)

where $B_{ij}$, the elements of the matrix $B$, as given in equation (5.24).
THE SOLUTION OF THE DIFFERENTIAL EQUATIONS DESCRIBING THE PLANT (CHAPTER 5)

Substituting equations (5.8), (5.16) and (5.17) into eq. (5.1), gives the differential equations which describe the system. These equations are linear and are of the form:

\[ \dot{x} + kx = f(x) \]  \hspace{1cm} (A16.1)

The solution of eq. (A16.1) can be found since the general solution of the related homogeneous equation is easy to define. Thereafter the particular solution is of the form

\[ y = v_1(x) \cdot y_1(x) \]  \hspace{1cm} (A16.2)

where \( y_1(x) \) is the solution of the related homogeneous equation. \( 63 \)

Then assuming

\[ x(t) = v_1(t) \cdot x_1(t) \]  \hspace{1cm} (A16.3)

and differentiating gives,

\[ \dot{x} = v_1 \dot{x}_1 + v_1 \dot{x}_1 \]  \hspace{1cm} (A16.4)

Substituting eq. (A16.3) into eq. (A16.1), gives

\[ v_1 \ddot{x}_1 + v_1 \dot{x}_1 + kv_1 x_1 = f(t) \]

or

\[ v(\ddot{x}_1 + kx_1) + v_1 \dot{x}_1 = f(t) \]

Since \( x_1 \) and \( \dot{x}_1 \) satisfy the homogeneous relationship

\[ \ddot{x}_1 + kx_1 = 0 \]  \hspace{1cm} (A16.5)

leading to

\[ \dot{v}_1 = \frac{f(t)}{x_1(t)} \]  \hspace{1cm} (A16.6)

Therefore

\[ v_1 = \int_{x_1(t)} ^{x(t)} \frac{f(t)}{x_1(t)} dt = \int_{x_1(0)} ^{x(t)} \frac{f(t)}{x_1(0)e^{kt}} \]

and the general solution is given by:

\[ x(t) = x_1(0)e^{kt} + x_1(t)v_1(t) \]

or

\[ x(t) = x_1(0)e^{kt} + e^{kt} \int_{0} ^{t} e^{-kt} f(t) dt \]  \hspace{1cm} (A16.7)
In this appendix three digital computer programs are presented for the solution of the time-optimal problem of a HVDC transmission system, as described in Chapter 5.

The first program, named KALMANTEST, derives the controllability of the system. Figures A17.11 and A17.12 give respectively the flowchart and listing of the program.

The second program, named BOUNDED, calculates the optimal (minimum) time the system takes to travel to rest. The flowchart and the listing are shown in figures A17.21 and A17.22.

The third program, named HAMILTON, computes the optimal control functions, \( u^o(t) \), in the time interval \([0,T_0]\), checking in the meantime the accuracy of the solution, as shown in figures A17.31 and A17.33.
APPENDIX 18

SUBROUTINES OF THE COMPUTER LIBRARY USED IN SEVERAL PROGRAMS

The following subroutines contained in the computer library of Loughborough University of Technology, have been used in the computer programs of this thesis:

The graph plotter subroutines

VTPOP, VTP4A, VTP4B and VTPCL.

In addition the following Numerical Algorithms Group (NAG) library routines have been used - these routines are available on all British universities and many industrial computers.

C02AEF The roots of a 4th order polynomial
F02AFF The eigenvalues of a real matrix
C06ABF Calculates a finite Fourier transform for complex data values
F03AAF The determinant of a matrix
C02ACF The roots of a 2nd order polynomial
F01AAF Matrix inversion
F02AGF The eigenvectors and eigenvalues of a real unsymmetric matrix
F01CKF Matrix multiplication
F04AEF Accurate solution of real linear equations
D01ABF Calculate an integral.
FIGURES
Figures 2.11 & 2.12: The 3-Phase Bridge Connection

Figure 2.13: A Bridge Rectifier

Figure 2.14: Current of Phase A
   i) Zero Phase Control and \( u = 0^\circ \)
   ii) Phase Control only
   iii) Phase Control and \( u \neq 0^\circ \)

Figure 2.14b: Voltage of Phase B
Figure 2.15 $\frac{E_d}{E_0} = f(\alpha)$ for various values of $K_x = \frac{X_c}{R_d}$

1: $K_x = 0$
2: $K_x = \frac{\pi}{30}$
3: $K_x = \frac{\pi}{9}$
Figure 2.16: Operation of an Inverter

Figure 2.17: Inverter Output Voltage

Figure 2.18: A Bridge Inverter
Figure 2.19: Commutation in a Single Bridge Converter.

Figure 2.20: Voltage Across Any Particular Valve in a Single Bridge Inverter.
Figure 2.21: Complete Rectifier and Inverter Characteristics
\[ V_0 \cos \alpha \]

\[ V_0 \cos \alpha = \frac{3Xt}{\pi} I_d \]

\[ V_0 \cos \alpha = (\frac{2Xt}{\pi} + R_d) I_d \]

Figure 2.24 Rectifier-Equivalent Circuit.

\[ E_d = V_d - \left( \frac{2Xt + R_d}{\pi} \right) I_d \]

\[ E_d = V_0 \cos \alpha - \left( \frac{2Xt}{\pi} + R_d \right) I_d \]

Figure 2.25 Rectifier Constant Current Computation.

\[ d.c. \text{ voltage from rectifier} \]

\[ V_0 \cos \beta + I_d \frac{3\omega L}{\pi} \]

Angle $\beta$ constant

\[ d.c. \text{ voltage from rectifier} \]

\[ V_0 \cos \delta - I_d \frac{3\omega L}{\pi} \]

Angle $\delta$ constant
Figure 2.27: Characteristics of an Inverter

Figure 2.28: Combined Rectifier and Inverter Characteristics with Current Regulation from Both Sides

Figure 2.29: Single-Bridge Converter System with Capacitor Connected in Parallel and in Series.
1. $\Delta i = f(\alpha)$ for $X_L = 1.4$ Ohm and $\Delta \alpha = -5^\circ$
2. $\Delta i = f(\alpha)$ for $X_L = 1.4$ Ohm and $\Delta \alpha = -10^\circ$

Figure 2.30 Bridge Rectifier variables (see table 2.1)
1. \( \Delta p = f(\alpha) \) for \( X_t = 1.4 \) Ohm and \( \Delta \alpha = -5^\circ \)

2. \( \Delta p = f(\alpha) \) for \( X_t = 1.4 \) Ohm and \( \Delta \alpha = -10^\circ \)

Figure 2.31 Bridge Rectifier variables (see table 2.1)
Figure 3.11: Diagram of Voltages and Currents in the A.C. Circuit.

Figure 3.12: A.C./D.C. Representation of the Simplified Model (section 3.2.1).

Figure 3.13: Signal Flow Diagram of the Simplified Model (section 3.1).
Figure 3.15: Signal Flow Diagram of the Complete System

Figure 3.14: Signal Flow Diagram of Equations in Section 3.2.2

Figure 3.16: A.C./D.C. System Representation
Figure 3.17: Simplified equivalent diagram of the power circuit in a DC link.

Figure 3.18: AC-DC Transmission System Representation.

Figure 3.19: Diagram of Voltages and Currents in the A.C. System.
Figure 3.21 Values of the steady-state error (Inverter omitted, Table 3.1).
Figure 3.22  Values of the steady-state error (Table 3.1)
Figure 3.23 Values of the steady-state error (Table 3.1)
Figure 3.24 Values of the steady-state error (Table 3.1)
Figure 3.26. Values of the steady-state error (Table 3.2)
Figure 3.27 Values of steady-state error
Figure 3.28 Position of the Roots (Table 3.4)
Figure 3.29 Values of the steady-state error
Figure 3.31 Position of the Root B (section 3.2.3)
Figure 3.32 Values of the steady-state error (section 3.2.3)
Figure 3.33. Position of the Roots (Table 3.5)
Figure 3.36 Position of the poles (section 3.3.3)
Figure 3.37: Time Response of the Inverter
Control System for an Impulse Input (uncompensated)
Figure 4.31: Time for Curve for Sample Network of a...
Figure 6.11: Transient Test of a Step Input of a System with Complex Poles
Figure 4.1: The State Feedback Form of the Control System
Figure 4.2: Time Response of the Optimal Control System of the Inverter for an Impulse input.
1: The Uncompensated System
2: The Optimal System

Figure 5.1: Time Response of the System for an Impulse Input

U₁(t) (normalized)

1: The current Control
2: The Voltage Control

Figure 5.2: The Time-Optimal Bounded Control

U₂(t) (normalized)
FIGURE A4.11

Flow Diagram of the Computer Program
for Model in Section 2.7.3

START

READ parameters of
the system:
\[ X_R, X_C, R_d, L, V_0, \alpha, \omega \]

CALCULATE
\[ E_{do}, \text{eq. (2.2.1)}, E_d, \text{eq. (2.2.2)} \]
\[ I_d, \text{eq. (2.2.5)}, I_1, \text{eq. (2.2.14)} \]

SOLUTION of the system:
\[ I_{1C} = f_{1}(V_t, \alpha), \text{eq. (2.7.5)} \]
\[ f(V_o, V_t, I_{1C}, \alpha) = 0, \text{eq. (A2.8)} \]
\[ P_{Wo} = 3I_{1C}.V_t, \text{eq. (A4.3)} \]
\[ P_{do} = E_{do}.I_d, \text{eq. (A4.4)} \]
\[ P_W = P_{Wo}.\cos \alpha \]
\[ P_R = P_{Wo}.\sin \alpha \]

WRITE: \[ I_1, I_{1C}, P_{Wo}, P_{do}, P_W, P_R \]

FINISH
Flow Chart of the Computer Program for Rectifier and Inverter Characteristics (section 2.7.2)

START

READ parameters of the system: $X_t, R_d, V_o, \alpha$ and $u$

$i = 1$

CALCULATE

$E_{do} = \frac{3\sqrt{3}V_o}{\pi}, eq.(2.2.1)$

$XX = \frac{\pi R_d + 3X_t}{\pi R_d + 3X_t}$

$XX_i = \cos(\alpha_i - XX) \text{ eq.}(2.2.17)$

$\text{EPS} = 1.000001$

If $\text{ABS}(XX) \geq \text{EPS}$

$\text{YES}$

$W_i = \text{Acos}(XX_i) - \alpha_i$

$V_i = \frac{180}{\pi} W_i$

Angle I = $\alpha_i + V_i$

Angle II = $\alpha_i + \frac{V_i}{Z}$

$V_{di}$: eq.(2.2.3)

$\text{NO}$

A

B

$i = i + 1$
WRITE \( E_{d_i}, \text{Angle I, Angle II} \)

IF \( \text{ABS}(X_{K_i,j}) \geq \text{EPS} \)

\( V_{i,j} = f(\alpha_j) \)  

eq. (2.2.17)

IF \( \text{ABS}(X_{K_i,n}) \geq \text{EPS} \)

\( V_{i,n} = f(\alpha_n) \)

NO

IF \( i = M \)

\( i = i + 1 \)

CALL

PLOT SUBROUTINES

VTPOP, VTP4A, VTP4B, VTPCL

YES

FINISH
Figure A4.22: The Listing of the Rectifier and Inverter Characteristics Computer Program

MASTER COMMUTATION

THIS PROGRAM PLOTS THE COMPLETE RECTIFIER AND INVERTER CHARACTERISTIC:

```
DIMENSION CK(40), ED(40), A(40), YD(40), W(40), CC(40), XT(10), RD(10), X1(10), XX(10), U(40), W1(30), U2(30), W5(30), U4(30), W6(30), U1(30), U3(30), U4(30), U5(30), U6(30), R(1), R1(1)
DATA PI, R1/1.0, HIU/
READ(1,21) VO, HL
WRITE(2,22) VO, HL
READ(1,23) (CK(I), I=1,30)
WRITE(2,23) (CK(I), I=1,30)
READ(1,24) (XT(I), I=1,6)
WRITE(2,35)
WRITE(2,24) (XT(I), I=1,6)
READ(1,29) (RD(I), I=1,6)
WRITE(2,36)
WRITE(2,29) (RD(I), I=1,6)
P=4.0, *ATAN(1.0)
EDO=3.0*SQR(T(6.0)*VU/P)
DO 5 IL=1,6
X1(IL)=3.0*XT(IL)/(P*RD(IL))
XX(IL)=6.0*XT(IL)/(P*RD(IL)+3.0*XT(IL))
WRITE(2,27) XT(IL), RD(IL)
WRITE(2,41) EDO, X1(IL), XX(IL)
WRITE(2,25)
DO 9 I=1,30
A(I)=CK(I)*ATAN(1.0)/45.
ED(I)=EDO*COS(A(I))*(1.0+X1(I))
YD(I)=ED(I)/EDO
XK=COS(A(I))-XX(IL)
EPS=1.000001
IF(ABS(XK).GT.EPS) GO TO 9
U(I)=ACOS(XK)-A(I)
U1(I)=U(I)*180.0/P
ANGLE=CK(I)+U(I)/2.
EDRR=EDO*(COS(A(I))+COS(A(I)+W(I)))/2.
WRITE(2,26) CK(I), U(I), YD(I), ED(I), EDRR, ANGLE
9 CONTINUE
5 CONTINUE
C PLOT SET OF CURVES OF ANGLE A AGAINST COMMUTATION ANGLE U.
DO 12 I=1,30
XK1=COS(A(I))-XX(1)
IF(ABS(XK1).GT.EPS) GO TO 52
W1(I)=ACOS(XK1)-A(I)
U1(I)=U1(I)*180.0/P
ANGLE=CK(I)+U1(I)/2.
EDRR=EDO*(COS(A(I))+COS(A(I)+W(I)))/2.
WRITE(2,26) CK(I), U1(I), YD(I), ED(I), EDRR, ANGLE
52 XK2=COS(A(I))-XX(2)
IF(ABS(XK2).GT.EPS) GO TO 53
W2(I)=ACOS(XK2)-A(I)
U2(I)=U2(I)*180.0/P
L2=I
53 XK3=COS(A(I))-XX(3)
IF(ABS(XK3).GT.EPS) GO TO 54
W3(I)=ACOS(XK3)-A(I)
U3(I)=U3(I)*180.0/P
L3=I
54 XK4=COS(A(I))-XX(4)
```

IF (ABS(XX(5)) .GT. EPS) GOTO 55
U5(1) = ACOS(XX(5)) - A(I)
U5(1) = U5(1) * 180. / P
L5 = 1
55 XX(5) = COS(A(I)) - XX(5)
IF (ABS(XX(5)) .GT. EPS) GOTO 56
U5(1) = ACOS(XX(5)) - A(I)
U5(1) = U5(1) * 180. / P
L5 = 1
56 XX(6) = COS(A(I)) - XX(5)
IF (ABS(XX(6)) .GT. EPS) GOTO 12
U6(1) = ACOS(XX(6)) - A(I)
U6(1) = U6(1) * 180. / P
L6 = 1
12 CONTINUE
CALL UTOP
CALL UTP4A(CK, U1, L1, 1)
CALL UTP4B(CK, U2, L2, 1)
CALL UTP4B(CK, U3, L3, 1)
CALL UTP4B(CK, U4, L4, 1)
CALL UTP4B(CK, U5, L5, 1)
CALL UTP4B(CK, U6, L6, 1)
CALL UTP4B(CK, U1, L1, 3)
CALL UTP4B(CK, U2, L2, 3)
CALL UTP4B(CK, U3, L3, 3)
CALL UTP4B(CK, U4, L4, 3)
CALL UTP4B(CK, U5, L5, 3)
CALL UTP4B(CK, U6, L6, 3)
CALL UTPCL
21 FORMAT(2F7.1)
22 FORMAT(FX, 3H000=F7.1, 5X, 3H00=F7.2, //)
23 FORMAT(1OF7.1)
24 FORMAT(1GF7.2)
25 FORMAT(1X, 6H A, 16H U , 7H YD , 7H FD , 7H X, 8H EDRR, 3X, 8H ANGLE , //)
27 FORMAT(1X, 25X, 3HXT=, F7.3, 5X, 3HDR=, F7.3, //)
28 FORMAT(1GF7.2)
35 FORMAT(1X, 3X, 3HDR=)
36 FORMAT(1X, 3X, 3HDR=)
41 FORMAT(1X, 6X, 2HEDO=, F7.3, 5X, 3HHX=, F7.3, 5X, 3HHX=, F7.5, //)
STOP
END
FIGURE A4.31

Flow Chart of the Computer Program for the
Steady-State Converter Model (Section 2.7.4)

START

DATA: Given parameter of the system

CALCULATE coefficients of the system of eqs:
(2.2.1), (2.2.7), (2.2.10) to (2.2.12)
(2.2.16), (2.3.1) to (2.3.7) (2.44) and (2.7.10)

COMPUTE coefficients of the equivalent 2nd order system of equations (2.7.10) and (2.7.11)

CALL subroutine POLYONYMO to define the coefficients of the resulting 4th order equation (unknown $V_t$)

CALL subroutine COZAEF of the computer package to define the roots of the above 4th order polynomial

PRINT roots

CHOOSE a REAL and POSITIVE ROOT $V_t$
COMPUTE other unknowns:
\[ I_1, E_d, I_d', P_{AC}', P_{DC} \]
\[ \cos \phi: \text{eqs. (A4.1) and (A4.2)} \]

COMPUTE the unknowns:
\[ \Delta d, \Delta i, \Delta p \text{ and } \Delta v_t \]
system of eqs. (2.7.1) to (2.7.3) and (2.7.8)

PRINT results

CHECK 1. The correctness of the calculations, i.e.,
quantity \( \text{CHECK} = A_0 \Delta v_t + B \Delta i + C \Delta \alpha \)
should be zero (eq. A2.10).

2. The accuracy of the solution:
2.1 comparisons: \( I_d \), eqs (2.4.4) and (2.4.5)
\[ I_1 = f(V_t) \text{ eq. (2.7.11)} \]
\[ I_1 = f(I_d) \text{ eq. (2.2.14)} \]
2.2 deviation between left and right hand side terms of equations (2.2.11) and (A2.8).

FINISH
Flow Chart of the State-Space Form Computer Program

START

DATA: Given parameters of the plant

PRINT DATA

COMPUTE the coefficients of eqs. (3.1.7) to (3.1.12)

COMPUTE the elements of matrices A and B of the system

PRINT the matrices A and B in a state-space form

COMPUTE the elements of the new matrix B after derivatives of the driving function have been eliminated

PRINT the new matrix B in a state-space form

CALL SUBROUTINE F02AFF to find the eigenvalues of matrix A
PRINT eigenvalues to check the correctness of the solution of comparing the defined poles of the Transfer Function of the system.

FINISH
Figure A5.12: The Listing of the State-Space Form Computer Program.

This program gives the initial state-space matrices $A$ and $B$ of the rectifier control system.

The form of the system is $\dot{x} = [A]x + [B]u$.

A change of basis in order to eliminate the undesirable derivatives of the driving functions, gives the new state-space form which is of the form: $\dot{x} = [A]x + [B]u$.

MASTER STATE-SPACE

```plaintext
DIMENSION AA(3,3), RR(3), RL(3), INT(9)
WRITE(2,79)
READ(1,21) XT, CID, CL, VO, VT, XC, XR
READ(1,23) XT1, CID, K, K, AK
WRITE(2,27)
WRITE(2,21) XT, CID, CL, VO, VT, XC, XR
WRITE(2,23) XT1, CID, K, K, AK
WRITE(2,25) T1, T2, BG
P=4.*ATAN(1.)
CIL=SIGN(6.)*CID/P
Z1=1. -XR/XC
Z12=Z1**2
Z2=VT*CIL*XR
Z3=(XR*CIL)***2
Z4=1. /(VO**2)
Z5=VT**2
A0=ALK/P/100.
A=Z4*(Z5*Z12+Z2*Z1*SIN(AG))
B=Z4*(Z3+Z2*Z1*SIN(AG))
C=Z4*Z2*Z1*COS(AG)
E=VT*(1. +COS(AG))
C1=K*C10/E
VD0=3.*SIGN(6.)*VT*COS(AG)/P
RE=R-(3./P)*XT1
RS=RE*C10/VD0
T=CL/RE
C2=(1. +COS(AG))/SIN(AG)
C3=T/K5
C4=XT*C10/(SIGN(6.)*VT*COS(AG))
C5=C2*TAN(AG)/C3
C6=(C4+K5)/C3
C7=C5*K5
C8=1. /C7
C9=C/C7
WRITE(2,40)
WRITE(2,41) CIL, A, B, C
WRITE(2,44)
WRITE(2,45) E, C1, VD0, RE, RS, T
WRITE(2,40)
A11=(C0+C8)
A12=(C5+C9*C2)
A13=0.
B11=1. /C7
B12=0.
A21=-h/(A*T)
```
A22 = \frac{(1 + C \cdot C2 / A)}{T2}
A23 = C \cdot C2 / (A \cdot T2)
B21 = 1 / (A \cdot T2)
B22 = 0.
A31 = C1 \cdot \left(1 + B1 \cdot A11 / T1\right)
A32 = C1 \cdot B1 \cdot A12 / T1
A33 = -C1 \cdot B1 \cdot A13 \cdot C1 / T1
B31 = B1 \cdot B11 \cdot C1 / T1
B32 = C1 / T1
WRITE(2, 32)
WRITE(2, 49)
WRITE(2, 33) A11, A12, A13, B11, B12
WRITE(2, 33) A21, A22, A23, B21, B22
WRITE(2, 33) A31, A32, A33, B31, B32
WRITE(2, 50)
CUR = C1 * B1 / T1
B13 = A15 * CUR
B23 = A25 * CUR
B33 = A35 * CUR
B12 = B14 + B13
B22 = B24 + B23
B32 = B34 + B33
WRITE(2, 32)
WRITE(2, 49)
WRITE(2, 33) A11, A12, A13, B11, B12
WRITE(2, 33) A21, A22, A23, B21, B22
WRITE(2, 33) A31, A32, A33, B31, B32
AA(1, 1) = A11
AA(1, 2) = A12
AA(1, 3) = A13
AA(2, 1) = A21
AA(2, 2) = A22
AA(2, 3) = A23
AA(3, 1) = A31
AA(3, 2) = A32
AA(3, 3) = A33
WRITE(2, 37)
WRITE(2, 39) ((AA(I, J), J = 1, 3), I = 1, 3)
CALL F02AFF(AA, 3, 3, INT, 0)
WRITE(2, 69)
WRITE(2, 89) (R1(J), R1(J), J = 1, 3)
21 FORMAT(F15.1, 2F5.2, 15, F5.1)
23 FORMAT(F15.1, 2F5.2, 15, F5.1)
25 FORMAT(F17.3, 2F7.3)
27 FORMAT(1H-, 10X, 5H XT, 5H C10, 5H L, 5H VO, 5H V1, 5H XC, 5H 1 X, /)
29 FORMAT(1H0, 10X, 5H AT1, 5H C10, 5H R, 5H K, 5H AK, /)
31 FORMAT(1H0, 10X, 7H T1, 7H 12, 7H BG, /)
32 FORMAT(1H0)
33 FORMAT(10X, 3F10.2, 20X, 2F10.2, /)
37 FORMAT(46X, 'THE MATRIX AA', /)
39 FORMAT(3X, 3F9.2, /)
40 FORMAT(1H0, 10X, 7H CIL, 8H A, 8H B, 8H C, /)
41 FORMAT(1H0, 10X, 7H CIL, 8H A, 8H B, 8H C, /)
42 FORMAT(10X, 5H L, 7H C1, 7H VDO, 7H RE, 8H RS, 1H T, /)
45 FORMAT(10X, F5.1, F7.4, F7.1, F6.3, F8.5, F7.4, /)
46 FORMAT(10X, 6H C2, 9H C3, 9H C4, 7H C5, 7H C6, /)
47 FORMAT(10X, F6.4, 2F9.6, F7.2, F7.2, /)
49 FORMAT(49X, 'THE MATRIX A1', 34X, 'THE MATRIX B', /)
50 FORMAT(1HU,20X,'THE NEW SPACE FORM OF THE SYSTEM',20X,'AFTER ELIMINATING THE DERIVATIVES OF THE DRIVING FUNCTIONS',/)
50 FORMAT(20X,'THE RULES OF THE SYSTEM',/)
70 FORMAT(0X,0X,)
80 FORMAT(3(0X,6HRR(I)=.FO.3,5X,6HR(I)=.FO.3,/))
STOP
FINISH
FIGURE A8.2

Flow Chart of the Transfer Function Computer Program for the System in Section 3.4.

START

DATA: Given parameters of the system

PRINT DATA

CALCULATE coefficients of equations (3.2.1) to (3.2.6)

DO 3
COMPUTE the coefficients of the polynomials of the numerator and denominator of the Transfer Function G(S) for different loads

PRINT G(S) as a Laplace transformation \( G(S) = \frac{A(S)}{B(S)} \)

CALL subroutine C02AEF to define the poles of the system, ie the roots of the denominator

PRINT ROOTS
CALCULATE the elements of the matrices A and B to define a state-space form of the system.

CALL subroutine F02AFF to define the eigenvalues of matrix A.

PRINT eigenvalues to check the correctness of the solution, by comparing the poles of the $G(S)$ and the eigenvalues of matrix A.

FINISH.
THE TRANSFER FUNCTION OF THE CONTROL SYSTEM, ON THE INVERTER SIDE.

A. INCLUDING A FILTER ON THE A.C. SIDE, GIVES A THIRD ORDER SYSTEM.

B. In case the filter is omitted the system reduces to a second order.

Then giving 12.40 or 22.60 in the data, results in a third or a second order system correspondingly.

Using the state-space form expansion of the $G(s)$, define the matrices $A$ and $B$.

The eigenvalues of the matrix $A$ should be the poles of the system.
FC=(1+COS(AG))/SIN(AG)
C01=C*K*5.10/L
VD0=3.*SQR(T(6.)*VT*COS(AG))/P
P1=R0+C*Y*T
R5=R1+C10/VDP
Y=C/L/F1
C5=T1+F1
C4=XT*C10/(SQR(6.)*VT*COS(AG))
WRITE(2,40)
WRITE(4,41) CIL,A1,B1,C1
WRITE(2,44)
WRITE(2,45) E,C01,VDP,11,RS,T
WRITE(6,42)
WRITE(2,43) C3,C4
P1=-B1/A1
G=-C1/A1
C11=-TA:(AG)+1
C13=U1*C11*FC
C14=PP-C4-13
WRITE(2,51) C11,C13,C14
D11=C5*FC*Q
D12=C14*FC*C
D13=C11*FC*PP
D14=D13-D12+C14
D15=D17-C3+C14*T2
D16=-C3*T2
D6=5*IK=4.4
T1=T1*(IK-1.)/3.
D6=7*JK=3.3
BG=(GJK+(JK-1.))/2.
WRITE(2,91) T1,T2,BG,IK,JK
C10=D16+T1
E11=D16+D15+T1+C13*T2*BG
E12=D15+D14+T1+C13*T2+C13*BG
E13=D14+C13
BGR=1.12:G
IF(T2,3,4,0) GO TO 31
E1=E11/10
F2=E12/10
F3=E13/10
C1K=C10/E10
WRITE(2,71) F10,E1,E2,E3
WRITE IF FINAL FORM OF G(S)
WRITE(2,81)
WRITE(2,77)
WRITE(2,51) CK,BG,T2
WRITE(2,52)
WRITE(2,76) E1,E2,E3
C1K=C*K*1.14*T2
T2=1.1/T2
WRITE(2,77)
WRITE(2,101) CK,BG,T2
WRITE(2,51)
WRITE(2,76) E1,E2,E3
C11=C11*1.14*T2
WRITE(2,101) SSR
A2(1)=1.
AC(1)=2
AC(2)=1
AC(3)=2
AC(4)=2
191

CALL CUPAFF (AC, L, X1, Y1, DD, 0)
WRITE (4, 61)
WRITE (4, 69)
WRITE (4, 89) (X1(J), Y1(J), J=1, 3)
WRITE (4, 50)
A1 (1, 1) = 0.
A1 (1, 2) = 1.
A1 (1, 3) = 0.
A1 (2, 1) = 0.
A1 (2, 2) = 0.
A1 (2, 3) = 1.
A1 (3, 1) = -E3
A1 (3, 2) = -E2
A1 (3, 3) = -E1

A0 = 0.
B1 = 1. * CKX
B2 = CKX * (12) * 8GR
B3 = CKX * (12) * 8GR
B (1) = E1 + E2 + D0
B (2) = E2 + E1 * E (1) + E2 * B0
B (3) = E3 - E1 * E (2) + E2 * A (1) - E3 * B0
WRITE (4, 37)
WRITE (4, 39) ((A5, R1, J, J=1, 3), B (1), JJ=1, 3)
CALL F3AFLF (AC, 3, 3, RR, RIM, INT, 0)
WRITE (4, 69)
WRITE (4, 50) (R (1), RIM (J), J=1, 3)
GO TO 81

81 CKX = C15 / E11
THE G'S OF THE REDUCED SECOND ORDER SYSTEM
E1 = E12 / E11
E2 = E13 / E11
WRITE (4, 68) 1111, E1, E2
WRITE (4, 61)
WRITE (4, 67)
WRITE (4, 58)
WRITE (4, 60) 1111, E1, E2
CKX = CKX * 8G
WRITE (4, 61)
WRITE (4, 65) CKX, BGR
WRITE (4, 56)
WRITE (4, 50)
WRITE (4, 60) 1111, E2
SBR = CKX * BGR / E2
WRITE (4, 16) SBR
AS (1) = 1.
AS (2) = E1
AS (3) = E2
BD = 0 < A (X Y)
L = 3
CALL CUPAFF (AC, L, X1, Y1, DD, 0)
WRITE (4, 61)
WRITE (4, 89) (X1(J), Y1(J), J=1, 2)
WRITE (4, 50)
A2 (1, 1) = 0.
A2 (1, 2) = 1.
A2 (2, 1) = -E2
A2 (2, 2) = -E1

L = 3
1. S + (0.1, 0.1, 0.1)

74 FORMAT (17X, T109M108K, 8X, F5.1, 8H + S)* (F5.1, 8H + S)

77 FORMAT (4X, "THE TRANSFER FUNCTION OF THE SYSTEM", I7)

78 FORMAT (8X, "THE TRANSFER FUNCTION OF THE REDUCED SECOND ORDER SYST")

161 FORMAT (5X, "THE STEADY STATE ERROR FOR A STEP INPUT=", F9.5, I7)

166 FORMAT (5X, "THE POLES OF THE OPTIMAL SYSTEM", I7)

STOP

END
FIGURE A9.2

Flow Chart of the Computer Program of the
Steady-State Mathematical Model
(Section 2.7.5)

START

DATA: Given parameters of the system

PRINT DATA

CALCULATE coefficients of the eqs:
(2.2.1), (2.2.7), (2.2.10) to
(2.2.12)
(2.2.16), (2.3.1) to (2.3.7),
(2.4.4) and (2.7.10)

COMPUTE coefficients of the equivalent 2nd order system
of eqs. (2.7.10) and (2.7.11)

CALL subroutine POLYNOM to define the coefficients of the resulting 4th order
equation of one unknown $V_t$

CALL subroutine VO2AEF of the computer package to find the roots of the above polynomial

A
PRINT ROOTS

CHOOSE a real and positive value of \( V_t \)

APPLY an ITERATION technique to make \( \cos \phi \) and \( \sin \phi \) consistent

COMPUTE the rest of the characteristics of the system

PRINT RESULTS

CHECK the accuracy of the solution
1. Comparisons:
   \[ I_d, \text{(eq.2.4.4) and (2.4.5)} \]
   \[ I_1=f(V_t), \text{eq.}(2.7.11) \text{ and } I_1=f(I_d), \text{eq.(2.2.14)} \]
2. Deviation between left and right hand side terms of eqs.\((2.2.11)\) and \((A2.8)\)

END
THIS PROGRAM GIVES THE STEADY-STATE CHARACTERISTICS OF AN AC-DC TRANSMISSION SYSTEM AT ANY OPERATING POINT, BY SOLVING THE MATHEMATICAL MODEL WHICH DESCRIBE THE PLANT.

DEFINITE THE COEFFICIENTS OF THE POLYNOMIAL

\[ p = 4, \quad A_T(A_1(1)) \]
\[ D_U = 1, 2, 2 \]
\[ D_S = 1, 5, 2 \]
\[ \text{READ} = 3, \quad * (X_T(I_1) - X_T(I_1))/P \]
\[ \text{READ} = 3, \quad * X_T(I_1)/P \]
\[ \text{READ} = 1, \quad * (F_H * H_L / R_1) * 2 \]
\[ \text{WRITE} = 29, \quad X_H(I_1), X_C(I_1), X_L, X_T(I_1), X_T(I_1), V_T(I_1), V_0 \]
\[ D_U = 1, 10, 4 \]
\[ \text{WRITE} = 222, \quad C_K(I), C_1K(I) \]
\[ \text{MARAKI} = 10 \]
\[ K_2 = 1 \]
\[ A(1) = C_K(I) * A_T(A_1(1))/45 \]
\[ A_1(1) = C_1K(I) * A_T(A_1(1))/45 \]
\[ D_A(I) = C_C(I) * D / 180 \]
\[ A_1(1) = X_H(I_1)/X_C(I_1) \]
\[ A_2 = A_1(1) \]
\[ A_3 = 2 * X_H(I_1) / A_1(1) * \sin(A(I)) \]
\[ A_3 = A_3 \]
\[ A_4 = X_1(R) \cdot 2 \]
\[ A_5 = -V_0 \cdot 2 \]
\[ C_6 = \cos(A_1) / (\sin(A_1) \cdot 2) \]
\[ C_1 = P \cdot X_1(R) / 6 \]
\[ F_1 = C_1 \cdot C_1 \]
\[ X_K = 1.0 \]
\[ A_0 = A_5 \cdot F \]
\[ A_0 = A_4 \cdot A_0 \]
\[ A_K = 18 / (P \cdot ((P \cdot R_1 \cdot 3 \cdot X_1(R) - X_1(R))) \cdot 2) \]
\[ NIF = 0.0 \]

109 KDF = 10, *DF
WRITE(2, 971) X, K, C0
WRITE(2, 972) FT
C3 = \cos(A_1) \]
C2 = A_K \cdot C2
C4 = A_3 \cdot (3 \cdot X_1(R) + X_1(R)) \cdot (C_3 \cdot 2)
C5 = A_3 \cdot C5
D1 = A_3 \cdot C1 + A_0 \cdot C3
D2 = A_3 \cdot C2 + A_2 \cdot C3
D3 = A_3 \cdot C4
D4 = A_3 \cdot C5 + C_3 \cdot A_5
E1 = A_2 \cdot C1 - A_0 \cdot C2
E2 = A_0 \cdot C3 + A_3 \cdot C1
E3 = A_0 \cdot C4
F4 = A_5 \cdot C1 - A_0 \cdot C5
D1 = 0.0, Y1 \cdot D1
D2 = 0.0, Y1 \cdot D2
D3 = 0.0, Y1 \cdot D3
D4 = 0.0, Y1 \cdot D4
E1 = 0.0, Y1 \cdot E1
E2 = 0.0, Y1 \cdot E2
F3 = 0.0, Y1 \cdot F3
F4 = 0.0, Y1 \cdot F4
F4 = 3 \cdot LF
F6 = E1 / E2
E7 = E4 / E2
CALL POLYNY(:)(D1, D2, D3, D4, E5, E6, E7)

FIND THE ROOTS OF THE POLYNOMIAL

A1(1) = A1
A3(2) = A3
A4(3) = A4
A5(4) = A5
D2 = 0.0, R \cdot A \cdot (x)
L = 1
CALL C0201, F(x), X, Y, ID, 0
Y1 = Y(1) \cdot 160,
Y2 = Y(2) \cdot 160,
Y3 = Y(3) \cdot 160,
Y4 = Y(4) \cdot 160.
IF (((X(1). GT. 0.). AND. (KY(1). EQ. 0.)). AND. (X(1). GE. V1T(IR))) GO TO 81
IF (((X(2). GT. 0.). AND. (KY(1). EQ. 0.)). AND. (X(2). GE. V1T(IR))) GO TO 82
IF (((X(3). GT. 0.). AND. (KY(1). EQ. 0.)). AND. (X(3). GE. V1T(IR))) GO TO 83
IF (((X(4). GT. 0.). AND. (KY(1). EQ. 0.)). AND. (X(4). GE. V1T(IR))) GO TO 84
GO TO 195
IF (((X(1). GT. 0. ). AND. (KY(2). EQ. 0.)). AND. (X(1). LT. V1T(IR))) GO
IF (((X(2). GT. 0. ). AND. (KY(2). EQ. 0.)). AND. (X(2). LT. V1T(IR))) GO
IF (((X(3). GT. 0. ). AND. (KY(3). EQ. 0.)). AND. (X(3). LT. V1T(IR))) GO
IF (((X(4). GT. 0. ). AND. (KY(4). EQ. 0.)). AND. (X(4). LT. V1T(IR))) GO
GO TO 194
31 VT=X(1)
GO TO 301
33 VT=X(2)
GO TO 301
35 VT=X(3)
GO TO 301
37 VT=X(4)
301 CONTINUE
C1(I) = E5*E6*VT*I7/VT
C1D(I) = 1.*SQR(T(6.)*VT*COS(A(1)) - V1T(IR)*COS(A1(1)))/(R1*P)
C1D=SQRT(6.)*C1D(I)/P
E0(I) = 3.*SQR(T(6.)*VT*COS(A(I)))/P
E1(I) = ED0(I) - 3.*XT(IR)*C1D(I)/P
V0(I) = (ED1(I) / ED0(I)) * COS(A(I))
ED1(I) = E0(I) - 3.*XT(IR)*COS(A(1(I)))/P
C1D = (ED1(I) - 11(I))/RD
Z2 = P*(XT(IR)*C1(I))/VT
pC = COS(A(I)) = 72
pC = COS(A(I))
D1C0SF = 5.*DC/ATAN(1.)
E2 = 5.*410(I) + XL/ED0(I)
V2 = 1. / (V0***2)
V1 = XR(I) * C1(I) * VT * A11
B2 = B1 * SIN(DC0)
B3 = 1.TDCUSF
B4 = (VT**2) = A2
B5 = (XR(I) * C1(I))**2
F3 = XT(I) * C1D(I) / (SQR(T(6.)*VT*COS(A(I)))*JUMP=1.0.D00*(A3-A33)
WRITE(4,371) JUMP
IF (JUMP) 314,313,314
313 CONTINUE
EVALUATING THE REST OF THE CHARACTERISTICS
A3= 12.*(F4+F7)
A4= V1*(A5+A7)
C1= V2*6.3
F3= A3*E2+E3)+B
DC1//= (CC+AA*TAN(A(I)))/E5*UA(IK)
S1T(I) = (CC+AA*TAN(A(I)))/E5-C+DA(IK)/AA
SA(I) = L*DC1(I)
SPV(I) = V1T(I) + DC1(I) - TAN(A(I)) * DA(IK)
C1D = A1+...+C1(I) + D1(I) + C1C(I)
WRITE(6,79)
WRITE(4,77) A11, A2, A3, A4, A5, A6, A7
WRITE(4,317) E5, E6, E7
WRITE(4,41) G1, G2, G3, G4, G5
WRITE(4,29)
WRITE(4,30) RE,EE,HL,E3,VT(1),CT1,CID,YD(1),ED(1),PD(1),FD(1),FR(1),DVT(I)
P0O(I)=ED(I)*CID(I)/COS(A(I))
PWO(I)=3.*CI(I)*VT
CD(I)=E1(I)*CID(I)
ZR=(1./R(I)**2)

CHECK THE CORRECTNESS OF THE SOLUTION

RRCOF=COS(A(I))
RRCOF=ACOS(RRCOF)
RRACOF=4.*RRCOF/ATAN(1.)
PH(I)=PWW(I)*DCOSF
PWR(I)=PWW(I)*SIN(DCO)
WRITE(4,40) VT(I)
WRITE(4,31)
WRITE(4,33) CK(I),CC(I),V0,ED(I),CI(I),CID(I),VT,DV(I),DPU(I),DC
1(I),CHECK,PW(I),PWR(I),PWO(I),PWW(I),PD(I)
PGIS=R*CIC(I)**2)
TRANS=FDC(I)+PGIS
WRITE(4,401) PGIS,PW(I),PWR(I),TRANS
CONTINUE
ZR=ED(I)*CID(I)/3.
ZL=VT*C1(I)*DCOSF
ZL2=ZL-ZR
ZV=VU**2
ZV=1./ZL-ZR
ZI1=1./ZR(1)/XC(I)
ZI2=ZI1**2

AN ITERATION TECHNIQUE MAKES COSINE AND SINE CONSISTENT

2Y2=2A2*(VT**2)
2Y3=(2X2(I)**2)
2Z2=(2X2(I)**2)
ACOSF=VU1*(COS(A(I))-RZ2)
EPS=1.000001
IF(ACOSF,GE,EPS) GO TO 19
WRITE(4,112) ZR2,ACOSF,Z2,DCOSF,DCOSF
IF(KDIS.EQ.E) GO TO 299
GO TO 13
2PD,DISIE=SIN(A(I))*XK+FIT*CI(I)/VT
GO TO 17
13 PS=DIFF
CPS=DIFF
17 WRITE(4,170) PSINF
IF(ABS(CPSINF).GE,EPS) GO TO 141
CPS=ASIN(PSINF)
RCOSF=UCOSF**2
PSINF=PSINF**2
PSINF=5.*PSINF/ATAN(1.)
UCOSF=DCOSF/PSINF
WRITE(4,15) UCOSF,DCOSF,UNIT
WRITE(4,151) CK(I),DACOSF,PSINF
ACOSF=5.*XK/ATAN(1.)
PSINF=5.*PSINF/ATAN(1.)
UNIT=UNIT
WRITE(4,117) UNIT,CK(I),DACOSF,ACOSF,RRACOF
IF(KDIS.EQ.E) GO TO 317
317 CONTINUE

DEFINE THE ACCURACY OF THE SOLUTION

FINA=A33(U=1.000)
KFINA=10000*FINA
WRITE(2,303) KFINA
IF(MARAKI.EQ.24) GO TO 0
IF(KFINA.GT.0) GO TO 201
GO TO 577
19 WRITE(2,44)
RCOSF=0.0
GO TO 95
141 WRITE(2,145)
KAZO=0
CKO=1.0+IT*C I(1)/VT-0.9/SIN(A(I))
XX=0.0+CKO
A3=A33*XX
GO TO 109
201 WRITE(2,136)
KAZO=0
WRITE(2,131) DSINF, DASINF, DACOSF
KSI=190.0+DASINF
KCOS=190.0+DACOSF
KFA=KSKI-KCOS
IF(KIFA.EQ.2307,71)
71 DIF=(C51+C51)/2.
CKO=1.0+IT*C I(1)/VT-SIN(DIF)/SIN(A(I))
XX=0.0+CKO
A3=A33*XX
ACODIF=45.0+DIF/ATAN(1.)
WRITE(2,53) ACODIF
GO TO 95
357 WRITE(2,306)
WRITE(2,136) R2COS, R2SINF, R2A
WRITE(2,131) DSINF, DASINF, DACOSF
DZ=1.0+D1(DIF)
A3=1.0+D1(DIF)
MARAKI=24
WRITE(2,24) MARAKI
KAZO=1
WRITE(2,67) KAZO
GO TO 515
132 FORMATION, ZEZERO1 = .F10, 2, 9X, ZEZERO2 = .F10, 2, />
135 FORMATION, 'SIDE AND COSINE NOT CONSISTENT, ITERATION CONTINUES' />
139 FORMATION, 'SIDE OUTSIDE THE RANGE OF ±1 -1' />
155 FORMATION, 'THE TRANSMISSION IS REVERSED. D.C. CURRENT NOW FLOWS 
FROM THE INVERTER TO THE RECTIFIER.' />
222 FORMATION, 'X, SYSTEMS OPERATING POINT AT DELAY ANGLE F5, 1, 5X7 
*AND AT EXTINCTION ANGLE F5, 1, />
395 FORMATION, 'X, CONFINA = .15, />
399 FORMATION, 'SIDE AND COSINE ARE CONSISTENT: ITERATION FINISHED' />
310 FORMATION, 'X, ZF= .F9, 3, 5X, 3HE6 = .F9, 3, 5X, 3HE7 = .F9, 3, />
321 FORMATION, 'X, XJUMP = .15, />
401 FORMATION, 'POWER DISSIPATION = .F9, 1, 1/9X, 'POWER EFFECTIVE ='.F9, 1, 
1/9X, 'POWER REACTIVE ='.F9, 1, 1/9X, 'POWER TRANSMITTED ='.F9, 1, />
STOP
END
SUBROUTINE POLYNOMIAL(D1,D2,D3,D4,E5,E6,E7)
COMMON G1,G2,G3,G4,G5
G1=0.2*D1*(E6*X2)
G2=0.2*D1*(E5*X1+E6*D3)
G3=0.1*(E5*X2+E6*X1+E7)*D4
G4=0.1*E5*X1
G5=0.1*(E7*X2)
RETURN
END
FIGURE A11.4

Flow Chart of the Time Response Computer Program
for a Control System with Real Poles

START

DATA: Static coefficient, time constants and poles of the system

PRINT DATA

CALL SUBROUTINE MATRIX FORM to find the coefficients of the G(S) broken up into simple components

COMPUTE mentioned coefficients by applying the residue corollary

PRINT coefficients of the expanded polynomial to check the correctness of the solution by comparing the results of the applied methods

COMPUTE time-response curve of the system for a unit-impulse using the inverse Laplace Transformation

PRINT defined points of the curve
CALL SUBROUTINES VTPOP, VTP4A and VTP4B to plot the response curve

COMPUTE time-response curve of the system for a step-input

PRINT defined points of the curve

CALL SUBROUTINES VTPOP, VTP4A, VTP4B and VTPCL to plot the response curve

END
SUBROUTINE MATRIX FORM

DATA (COMMON)

COMPUTE the elements of the matrices related to Cramer's method, to define the unknown coefficients of the given algebraic equations

PRINT matrices in a state-space form

CALL subroutine F03AAF to calculate the determinants of the above matrices

CALCULATE the coefficients of the expanded polynomial and PRINT them

RETURN

FINISH
THE TIME RESPONSE OF A THIRD ORDER SYSTEM WITH TWO ZEROS,
FOR IMPULSE AND STEP INPUTS.

MASTER TIME RESPONSE
DIMENSION PD(3,3), AS1(3,3), RS(3,3), CS(3,3), TIM(300), YUNIT(300), R1(1), H(1), WURSP(3), PD5(3,3), VSSTEP(300)
COMMON AS1, RS, CS, AA, DB, CC
DATA RA1/AA, nE_$/HCURPENT l
REA(1,2 )St? 1, S 02, SD3, CKX, RG, 12
WRITE(2,79)
WRITE(2,70)
WRITE(2,71) SD1, SD2, SD3, CKX, RG, 12
PD(1,1):=1,
P(1,2) :=1,
P(1,3):=1,
P(2,1):=SD2+SD3
P(2,2):=SD1+SD3
P(2,3):=SD1+SD2
P(3,1):=SD2+SD3
P(3,2):=SD1+SD3
P(3,3):=SD1+SD2
WRITE(2,79)
WRITE(2,131)
WRITE(2,161)
WRITE(2,102)(PD(1,J), J=1,3), I=1,5
CM13:=1,
CM2:=9+T2
CM3:=6+T3
CALL MATRIXFORM(PD, CM11, CM21, CM31)
CHECK THE CORRECTNESS OF THE VALUES OF THE COEFFICIENTS.
ARCH: (T1-SD1)*(RG-SD1)/(SD2-SD3)*(SD3-SD2)
BCR: (T1-SD2)*(RG-SD2)/(SD1-SD3)*(SD3-SD1)
CCR: (T1-SD3)*(RG-SD3)/(SD1-SD2)*(SD2-SD3)
WRITE(2,79)
WRITE(2,113)
WRITE(2,114) ARCH, BCR, CCR
THE PROGRAM PLOTS THE TIME RESPONSE OF THE SYSTEM
FOR THE TIME INTERVAL 0 UP TO 0.2 SECONDS.
A. THE TIME RESPONSE FOR AN IMPULSE INPUT, D10=1.
CKA=CKX*AA
CKB=CKX*BB
CKC=CKX*CC
WRITE(2,117) CKA, CKB, CKC
WRITE(2,115)
DO 1=1,120
TIM(J)=0.5*UNIT(J)
UNIT(J)=CKA*EXP(-SD1*TIM(J))+CKB*EXP(-SD2*TIM(J))+CKC*EXP(-SD3*TIM(J))
WRITE(2,117) TIM(J), UNIT(J), J
J=J+1
1 CONTINUE
9 CONTINUE
3 K=47
CALL UPOP
CALL ULP4A(YUNIT), TIM(157), 0.0, 23.0, 0.0, R1, R1, 1)
CALL UTP4H(TIN(4), YUNIT(4), K, T)

CONTINUE

C       THE TIME RESPONSE FOR A STEP INPUT, DT0=1/S.

P = 1.0 + 1/(SM1*SD2*SD3)
CH1 = 0
CM2 = 1.0 - 1/(SM1*SD2*SD3)
CM3 = (BG + T2) - 1/(SM1*SD2*SD1*SD3*SD2*SD3)
AT = -AA/SD1
BT = -AA/SD2
CT = -CC/SD3
DT = 0

CALL MATRIXFORM(PD, CM1, CM2, CM3)

C       CHECK THE CORRECTNESS OF THE VALUES OF THE COEFFICIENTS.

WRITE(2, 79)
WRITE(2, 133)
WRITE(2, 118)
WRITE(2, 114) AT, BT, CT
WRITE(2, 114) DT
CKA = CKX*AA
CKB = CKX*BB
CKC = CKX*CC
WRITE(2, 117) CKA, CKB, CKC
CKM = CKX*D
WRITE(2, 119) CKA, CKB, CKC, CKM
WRITE(2, 115)
DO 10 J= 1, 201
TIM(J) = 0.001*(J-1)
YUNIT(J) = CKA*EXP(-SD1*TIN(J)) + CKB*EXP(-SD2*TIN(J)) + CKC*EXP(-SD3*TIN(J)) + 1
YSTEP(J) = YUNIT(J) + CKD
WRITE(2, 116) TIM(J), YSTEP(J), J
IF (J.EQ.201) GO TO 31
10 CONTINUE

31 CONTINUE

CALL UTP4K(TIM(201), YSTEP(1), J1)
CALL UTP4K(TIM, YSTEP, J, T1)

181 CONTINUE

14 FORMAT(6X, 'RT= ', F9.4, /)
20 FORMAT(6X, 'S01', 'S02', 'S03', 'S04', 'S5X', 'CKX', 'BG', '7X', 'T2', /)
21 FORMAT(3F10.3, 5F10.2)
70 FORMAT(9H)
101 FORMAT(6X, 'PRINT PD', /)
102 FORMAT(6X, 'PRINT C', /)
114 FORMAT(6X, 'R= ', F9.4, 5X, 'C= ', F9.4, /)
115 FORMAT(6X, 'T1', 5X, 'Y-OUTPUT', 5X, 'STEP-', /)
116 FORMAT(6X, 'C= ', F12.6, 5X, 'CK= ', F12.6, 5X, 'CKC= ', F12.6, /)
117 FORMAT(6X, 'THE COEFFICIENTS ARE USING THE RESIDUE COROLLARY', /)
119 FORMAT(6X, 'CKD= ', F12.6, 5X, 'CKE= ', F12.6, 7X, 'KD= ', F11.6, /)
131 FORMAT(6X, 'THE TIME RESPONSE OF THE SYSTEM', '/9X, 'FOR AN impulse
1 INPUT, DT0=1/S, /)
133 FORMAT(6X, 'THE TIME RESPONSE OF THE SYSTEM', '/9X, 'FOR A step
1 INPUT, DT0=1/S', /)
STOP
END
SUBROUTINE MATRIXFORM(PD,CM1,CM2,CM3)
DIMENSION PD(3,3),BS2(3,3),CS3(3,3),TIM(300),AS1(3,3),PD(3,3),WOR

COMMON AS1,BS2,CS3,AA,BB,CC

DO 19 I=1,3
DO 20 J=1,3
PD(I,J)=PD(I,J)
20 CONTINUE
19 CONTINUE
WRITE(2,103)
WRITE(2,102)((PD(I,J),I=1,3),J=1,3)
CALL F03AAF(PD,3,3,PDFT,WORSP,0)
WRITE(2,104) PDFT
AS1(1,1)=CM1
AS1(2,1)=CM2
AS1(3,1)=CM3
DO 3 1=1,3
DO 4 J=1,3
AS1(I,J)=PD(I,J)
3 CONTINUE
4 CONTINUE
WRITE(2,105)
WRITE(2,102)((AS1(I,J),I=1,3),J=1,3)
IFAIL=1
CALL F03AAF(AS1,3,3,AS1DFT,WORSP,IFAIL)
IF(IFAIL,1,0) GO TO 30
GO TO 39
30 AS1DFT=0,0
WRITE(2,49)
39 CONTINUE
WRITE(2,107) AS1DFT
AA=AS1DFT/PDFT
WRITE(2,108) AA
BS2(1,2)=CM1
BS2(2,2)=CM2
BS2(3,2)=CM3
DO 5 1=1,3
DO 6 J=1,3
BS2(I,J)=PD(I,J)
5 CONTINUE
6 CONTINUE
WRITE(2,109)
WRITE(2,102)((BS2(I,J),I=1,3),J=1,3)
IFAIL=1
CALL F03AAF(BS2,3,3,BS2DFT,WORSP,IFAIL)
IF(IFAIL,1,0) GO TO 31
GO TO 46
31 BS2DFT=0,0
WRITE(2,49)
46 CONTINUE
WRITE(2,110) BS2DFT
BL=BS2DFT/PDFT
WRITE(2,111) BL
CS3(1,3)=CM1
CS3(2,3)=CM2
CS3(3,3)=CM3
DO 7 1=1,3
DO 8 J=1,3
CS3(I,J)=PD(I,J)
7 CONTINUE
8 CONTINUE
7 CONTINUE
  WRITE(*,111)
  WRITE(*,112)((CS3(I,J),J=1,3),I=1,3)
  IFAIL=1
  CALL FOSAAF(CS3,3,3,CS3DET,WORSP,FAIL)
  IF(FAIL.EQ.1) GO TO 32
  GO TO 41
32 CS3DET=0.0
  WRITE(*,113)
41 CONTINUE
  WRITE(*,114) CS3DET
  CC=CS3DET/DET
  WRITE(*,115) CC
  WRITE(*,116) AA, BB, CC
40 FORMAT(9X,'THE DETERMINANT IS SINGULAR',/
102 FORMAT(9X,3F12.2,/
103 FORMAT(34X,'MATRlX FDS',/
104 FORMAT(20X,'DETERMINANT OF DENOMINATOR=',F4.5,/
105 FORMAT(24X,'MATRlX AS1',/
106 FORMAT(20X,'COEFFICIENT C=',F9.4,/
107 FORMAT(20X,'DETERMINANT FOR COEFFICIENT A=',E14.5,/
108 FORMAT(20X,'COEFFICIENT A=',F9.4,/
109 FORMAT(24X,'MATTRX B52',/
110 FORMAT(20X,'DETERMINANT FOR COEFFICIENT B=',F14.5,/
111 FORMAT(20X,'COEFFICIENT B=',F9.4,/
112 FORMAT(24X,'MATTRX CS3',/
113 FORMAT(20X,'DETERMINANT FOR COEFFICIENT C=',E14.5,/
114 FORMAT(9X,'AA=',F9.4,5X,'BB=',F9.4,5X,'CC=',F9.4,/
RETURN
FINISH
FIGURE A11.6

Flow Chart of the Time-Response Computer Program for a Control System with Complex Poles

START

DATA: Static coefficients, time constants and poles of the system

PRINT DATA

COMPUTE time-response curve for a unit-impulse using the Inverse Laplace Transformation

PRINT points of the curve

CALL SUBROUTINES VTPOP, VTP4A and VTP4B to plot the curve

COMPUTE time-response curve for a step-input

CALL SUBROUTINE CO2ACF to find the coefficients of the expanded polynomial with conjugate poles

PRINT points of the curve
CALL SUBROUTINES VTP4A, VTP4B and VTPCL to plot the response curve

FINISH
Figure A II.7: The Listing of the Time-Response Computer Program for a Control System with Complex Poles.
WRITE(2,79)
WRITE(2,60)(X(I1),Y(I1),I1=1,2)
SG1=COMPLEX(X(1),Y(1))
SG2=COMPLEX(X(2),Y(2))
COL=-(SG1+1)/(-SG1)
RR=REAL(COL)
NJ=INT(COL)
A1=-NJ/Y(1)
A2=RR+NJ*X(1)
AK=CK*A1
CS=U1/E2
CSK=CK*CS
BE=(A2-A1*A1)*W
B=CK*BB
WRITE(2,79)
WRITE(2,115)
DO 9 1=1,N
TIM(1)=0.02*(1-1)
TS=TIM(1)
TS=-EXP(-TT)*COS(TS)*BK*EXP(-TT)*SIN(TS)
VSTEP(1)=VUNIT(1)+CSK
WRITE(2,110) TIM(1),VSTEP(1),1
YY(1)=1.0
9 CONTINUE
CALL UTP4A(TIM(1),TIM(K),-2.0,0.2,20.,10.,R,1,R1,1)
CALL UTP4B(TIM,YSTEP,K,1)
CALL UTPCL
21 FORMAT(F5.1,3X,F7.2,F10.2)
22 FORMAT(9X,C=1.,F7.1,10X,:E1=1.,F7.2,3X,:E2=1.,F7.2,9X,:P=1.,F7.2,1/

74 FORMAT(1H0)
80 FORMAT(5X,NK(1)=,F9.3,3X,SHY(1)=,F8.4,1/

112 FORMAT(5X,A=1.,F7.2,3X,:AK=1.,F7.2,7X,:BB=1.,F7.2,5X,

1 RADIUS: SPEED=1.,F7.2,1 RAD/SEC,1/

115 FORMAT(5X,TIME,5X,7X,:OUTPUT,

116 FORMAT(7X,13.5,9)
STOP
END
FIGURE A13.1

Flow Chart of the Computer Program for the Solution of the Reduced Matrix Riccati Equation

START

DATA: Read parameters and matrices of the system

PRINT DATA

Matrices [A], [B] and [C] to be stored

CALL subroutine F02AGF to find eigenvalues and eigenvectors of matrix [A]

FORM the matrix [P] for the diagonalisation of matrix [A]

CALL subroutine F01AAF to invert matrix [P]

CALL subroutine F01CKF to compute the diagonal matrix [P^{-1}AP] and CHECK the correctness of the solution by comparing the eigenvalues of matrices [P^{-1}AP] and [A]
CALL subroutine F01CKF to compute the matrix \([P^{-1}B]\)

CALL subroutine F02AGF to define the eigenvalues and eigenvectors of another state-space form related matrix \(A_S\) of the same system

CHECK the correctness of the above state-space form by comparing the diagonal matrices \([P^{-1}A_S P]\) and \([P^{-1}AP]\)

CALL subroutine RICATIEQ\(T\) to define the constant matrix \([R_0]\) and the state optimal control system

END
This subroutine calculates the elements of matrix $[R_0]$ and the matrix of the state optimal control system.
IF matrix \([R_0]\) satisfy the requirements for a symmetric positive and definite matrix

YES

PRINT matrix \([R_0]\)

CALL subroutine FOICKF to define the optimal controller

PRINT the matrix of the controller

PRINT the optimal control system

RETURN

FINISH

NO

WRITE a message

STOP
Figure A 13.3: The Listing of the Computer Program for the Solution of the Matrix Riccati Equation (chapter 4).

**This Program Define The Control-Law Which Makes The System Optimal For Given Quadratic Performance Index**

**MASTER RICCATI SOLUTION**

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A(3,3), AR(3), AI(3), VR(3,3), VI(3,3), INT(3), B(3,3)</td>
<td></td>
</tr>
<tr>
<td>A(3,3), BB(3,3), CC(2,2), D(2,2), BUC(2,2), BUD(2,2), PB(2,2)</td>
<td></td>
</tr>
<tr>
<td>PB(2,2), P(2,2), P1(2,2), P2(2,2), P12(2,2), Z(1), IZ(1), AINT(3)</td>
<td></td>
</tr>
<tr>
<td>CAR(2), CAL(2), CVR(2,2), CVI(2,2), PBS(2,2)</td>
<td></td>
</tr>
<tr>
<td>CC(2,2), DD(2,2), PP(2,2), PP2(2,2), PPC(2,2), P12D(2,2)</td>
<td></td>
</tr>
<tr>
<td>BAGC(2,2), BADA(2,2), BCP1(2,2), BPI(4,2), BUCS(2,2), CP21(2,2)</td>
<td></td>
</tr>
<tr>
<td>PA(3,3), PPA(3,3), PS(3,3), PPS(3,3), PUA(3,1), PUS(3,1)</td>
<td></td>
</tr>
<tr>
<td>PS1(3,1), PBA(3,1), PBS(3,1), PBS(3,1), PPA(3,3), PPS1(3,3), PA(3,3)</td>
<td></td>
</tr>
</tbody>
</table>

**COMMON** S1, S2, CM, PI, X2, X3

READ(1, 19) S1, S2, CM
READ(1, 39) X2, X3
WRITE(2, 79)
K=1
N=2
M=3
GO TO 97
READ(1, 24)(( A(I, J), J=1, 3), I=1, 3)
WRITE(2, 22)
READ(2, 34)(( A(I, J), J=1, 3), I=1, 3)
WRITE(2, 23)
READ(1, 111)(( B(I, J), J=1, 3), I=1, 3)
WRITE(2, 121)
READ(2, 111)(( BUA(J, 1), J=1, 3)
WRITE(2, 122)
READ(2, 111)(( BUS(J, 1), J=1, 3)
WRITE(2, 123)
READ(2, 111)(( BUSS(J, 1), J=1, 3)
WRITE(2, 111)(( BUSS(J, 1), J=1, 3)
CONTINUE
READ(1, 27)(( C(I, J), J=1, 2), I=1, 2)
DO 91 I=1, 2
DO 92 J=1, 2
CC(I, J)=C(I, J)
DO(1, J)=D(I, J)
91 CONTINUE
92 CONTINUE
WRITE(2, 30)
WRITE(2, 37)(( C(I, J), J=1, 2), I=1, 2)
WRITE(2, 50)
WRITE(2, 37)(( CC(I, J), J=1, N), I=1, N)
WRITE(2, 32)
WRITE(2, 37)(( D(I, J), J=1, 2), I=1, 2)
WRITE(2, 37)(( DD(I, J), J=1, N), I=1, N)
READ(1, 27)(( BUC(I, J), J=1, 2), I=1, 2)
WRITE(2, 75)
WRITE(2, 37)(( BUC(I, J), J=1, 2), I=1, 2)
READ(1, 27)(( BUD(I, J), J=1, 2), I=1, 2)
WRITE(*,76)
WRITE(*,77)(( BUD(I,J),J=1,2),I=1,2)
READ(*,77)(( BUD(I,J),J=1,2),I=1,2)
WRITE(*,77)
WRITE(*,78)(( BUD(I,J),J=1,2),I=1,2)
GO TO 90
WRITE(*,124)
DO 11 I=1,M
DO 12 J=1,M
AA(I,J)=A(I,J)
BB(I,J)=A(I,J)
12 CONTINUE
11 CONTINUE
WRITE(*,22)
WRITE(*,24)(( A(I,J),J=1,M),I=1,M)
WRITE(*,25)
WRITE(*,24)((AA(I,J),J=1,M),I=1,M)
WRITE(*,23)
WRITE(*,24)(( B(I,J),J=1,M),I=1,M)
WRITE(*,26)
WRITE(*,24)((BB(I,J),J=1,M),I=1,M)
K=1
CALL F02AGF(A,3,M,AR, AI, VR,3,VI,3,INT,K)
IF (K.EQ.1) GO TO 17
WRITE(*,79)
WRITE(*,43)
WRITE(*,21)(AR(I),AI(I),I=1,M)
WRITE(*,79)
WRITE(*,24)
DO 3 I=1,M
WRITE(*,21)(VR(J,I),VI(J,I),J=1,M)
WRITE(*,79)
3 CONTINUE
DO 13 I=1,M
DO 14 J=1,M
PA(I,J)=VR(I,J)
PPA(I,J)=VR(I,J)
14 CONTINUE
13 CONTINUE
WRITE(*,113)
WRITE(*,49)((PA(I,J),J=1,M),I=1,M)
WRITE(*,112)
WRITE(*,49)((PPA(I,J),J=1,M),I=1,M)
C
C FIND THE EINGENVALUES OF THE SYSTEM
C
K=1
CALL F02AGF(B,3,M,AR, AI, VR,3,VI,3,INT,K)
IF (K.EQ.1) GO TO 17
WRITE(*,79)
WRITE(*,41)
WRITE(*,21)(AR(I),AI(I),I=1,M)
WRITE(*,79)
WRITE(*,42)
DO 5 I=1,M
WRITE(*,21)(VR(J,I),VI(J,I),J=1,M)
WRITE(*,79)
5 CONTINUE
C
C ELIMINATION OF THE UNDESIRABLE DERIVATIVES FROM THE
DRIVING FUNCTIONS.

DO 15 I=1,M
DO 16 J=1,M
PS(I, J) = VR(I, J)
PPS(I, J) = VR(I, J)
16 CONTINUE
15 CONTINUE
WRITE(2, 116)
WRITE(2, 117) (PS(I, J), J=1, M), I=1, M
WRITE(2, 118)

TEST THAT THE ABOVE TRANSFORMATION WAS CORRECT, BY TAKING
INTO ACCOUNT THE INVARIANCE OF THE EIGENVALUES.

WRITE(2, 119)
WRITE(2, 120) ((PS(I, J), J=1, M), I=1, M)
CALL F01PAF(PA, 3, 3, PAI, 3, AINT, 0)
CALL F01PAF(P5, 3, 3, PSI, 3, AINT, 0)
DO 103 I=1, M
DO 104 J=1, M
PPAI(I, J) = PAI(I, J)
PSI(I, J) = PSI(I, J)
P2SI(I, J) = PSI(I, J)
104 CONTINUE
103 CONTINUE
WRITE(2, 121)
WRITE(2, 122)((PAI(I, J), J=1, M), I=1, M)
CALL F01CKF(PIA, PPA, 3, 3, 1, 1)
CALL F01CKF(DIAG, PIA, PPA, 3, 3, 1, 1)
WRITE(2, 123)
WRITE(2, 124)((PSI(I, J), J=1, M), I=1, M)
CALL F01CKF(P5, PSI, 3, 3, 1, 1)
CALL F01CKF(DIAG, P5, PSI, 3, 3, 1, 1)
WRITE(2, 125)
WRITE(2, 126)( (PAI(I, J), J=1, M), I=1, M)
CALL F01CKF(PB, PAI, BUA, 3, 1, 1, 1, 1)
WRITE(2, 127)
WRITE(2, 128)( (PSI(I, J), J=1, M), I=1, M)
CALL F01CKF(P5, PSI, BSS, 3, 1, 1, 1, 1)
WRITE(2, 129)
WRITE(2, 130)( (PSJ(I, J), J=1, M), I=1, M)
CALL F01CKF(PPS, PSS, 3, 1, 1, 1, 1)
WRITE(2, 131)( (PSI(I, J), J=1, M)
CALL F01CKF(PSS, P2SI, BSS, 3, 1, 1, 1)
WRITE(2, 132)
CONTINUE
WRITE(2, 79)
WRITE(2, 80)
WRITE(2, 81)((C(I, J), I=1, N), J=1, N)
K=1
CALL F02AGF(C, 2, N, CAR, CAI, CVR, 2, CVI, 2, INT, K)
17 K, EQ, 1) GO TO 17
WRITE(2, 79)
WRITE(2, 51)
WRITE(2, 52)( (CVR(I, J), CVI(J, I), J=1, N)
UfITE: (2, 79)

CONTINUE

DO 8 I = 1, N
    DO 1 J = 1, N
        P(I, J) = CVR(I, J)
        PP(I, J) = CVR(I, J)
    1 CONTINUE

UNITE (1, 6)

WRITE(ý, 37)((P(I, J), J = 1, N), I = 1, N)
WRITF(2, 70)
WPI TE(1, 37) ((PP(I, J), J = 1, N), I = 1, N)
CALL F01A AF(P, 2, 2, P1, 2, AINT, 0)
WRITE(2, 35)
WRITE(2, 39) (PI1(I, J), J = 1, N), I = 1, N)
DO 71 J = 1, N
    CPI(I, J) = PI1(I, J)
    CP21(I, J) = PI1(I, J)
72 CONTINUE

WRITE(2, 65)
WRITE(2, 39) (CP1(I, J), J = 1, N), I = 1, N)
CALL F01CKF(P1, CPI, 1, P2, 2, 2, 2, 2, 2, 2, 2, 1, 0)
WRITE(2, 75)
WRITE(2, 37) (BUC(I, J), J = 1, 2), I = 1, 2)
WRITE(2, 79)
CALL FO1CKF(P21, CPI, BUC, 2, 2, 2, 2, 2, 2, 1, 0)
WRITE(2, 31)
WRITE(2, 39) (PB1(I, J), J = 1, 2), I = 1, 2)
WRITE(2, 77)
WRITE(2, 37) (BUCS(I, J), J = 1, 2), I = 1, 2)
WRITE(2, 79)
CALL FO1CKF(P2CS, CP21, BUCS, 2, 2, 2, 2, 2, 1, 0)
WRITE(2, 47)
WRITE(2, 39) (PUC5(I, J), J = 1, 2), I = 1, 2)
WRITE(2, 124)
WRITE(2, 79)
WRITE(2, 32)
WRITE(2, 37) (P(I, J), J = 1, N), I = 1, N)
CALL FO2AGF(D, 2, N, CAR, CAL, CVR, 2, CVI, 2, AINT, K)
IF (K, .LE. 1) GO TO 17
WRITE(2, 79)
WRITE(2, 54)
WRITE(2, 37) (CAR(I), CAL(I), I = 1, N)
WRITE(2, 79)
WRITE(2, 55)
DO 1 I = 1, N
    WRITE(2, 37) (CVR(I, J), CVI(J, I), J = 1, N)
1 CONTINUE

CONTINUE

DO 2 I = 1, N
    DO 18 J = 1, N
        P2(I, J) = CVR(I, J)
        PP2(I, J) = CVR(I, J)
18 CONTINUE
CONTINUE
WRITE(2,46)
WRITE(2,37)((P2(I,J),J=1,N),I=1,N)
WRITE(2,40)
WRITE(2,37)((P2(I,J),J=1,N),I=1,N)
CALL Fortran(p2,2,Z,P12,2,A14T,0)
WRITE(2,45)
WRITE(2,39)((P12(I,J),J=1,N),I=1,N)
DO 82 J=1,N
DO 83 I=1,N
DPI(I,J)=P12(I,J)
83 CONTINUE
82 CONTINUE
WRITE(2,66)
WRITE(2,39)((DPI(I,J),J=1,N),I=1,N)
CALL Fortran(p12,P12,DD,2,2,2,2,2,12,1,0)
CALL Fortran((DAGD,P12,PP2,2,Z,1Z,1,0)
WRITE(2,95)
WRITE(2,66)((DAGD(I,J),J=1,N),I=1,N)
CALL Fortran(p2,DP1,UD,2,2,2,2,1Z,1,0)
WRITE(2,79)
WRITE(2,40)
WRITE(2,39)(( p2(I,J),J=1,2),I=1,2)
GO TO 9
17 WRITE(2,20)
9 CONTINUE
CALL RICCATIEQUAT(PB1)
X2=0.0
X3=0.0
CALL RICCATIEQUAT(PB2)
X2=0.0
X3=0.0
CALL RICCATIEQUAT(PB2)
19 FORMAT(3F10.2)
20 FORMAT(1HO,31X,'NO SOLUTION',//)
21 FORMAT(1HO,20X,'THE MATRIX A',//)
22 FORMAT(1HO,20X,'THE MATRIX B',//)
23 FORMAT(7X,3F11.2)
24 FORMAT(1HO,20X,'THE MATRIX AA',//)
25 FORMAT(1HO,20X,'THE MATRIX AB',//)
26 FORMAT(1HO,20X,'THE MATRIX BB',//)
27 FORMAT(10X,2E12.5)
28 FORMAT(2F10.2)
30 FORMAT(1HO,17X,'THE MATRIX C',//)
31 FORMAT(9X,'THE MATRIX PB1=PI*BUC',//)
32 FORMAT(17X,'THE MATRIX P',//)
33 FORMAT(14X,'12.2',//)
34 FORMAT(7X,3F12.8)
35 FORMAT(18X,'THE MATRIX PI',//)
36 FORMAT(18X,'THE MATRIX P',//)
37 FORMAT(10X,2E10.2)
38 FORMAT(10X,2E12.4)
40 FORMAT(2X,'THE EIGENVALUES OF MATRIX B',//)
41 FORMAT(1HO,18X,'THE EIGENVALUES OF MATRIX B',//)
42 FORMAT(1HO,18X,'THE EIGENVECTORS OF MATRIX B',//)
SOLVE THE NON LINEAR ALGEBRAIC EQUATIONS WHICH EVALUATE
THE ELEMENTS OF THE CONSTANT MATRIX P01 OF THE
REDUCED MATRIX RICCATI EQUATION.

CALL RICCATIEQUAT(PB1)
X2=0.0
X3=0.0
CALL RICCATIEQUAT(PB2)
X2=0.0
X3=0.0
CALL RICCATIEQUAT(PB2)
STOP
THIS SUBROUTINE CALCULATES THE ELEMENTS OF THE ABOVE MATRIX RO IN THE MASTER PROGRAM.

SUBROUTINE RICCATIEQUAT(B)

C DIMENSION T0(3,1), DF(3,3), RO(2,2), CR(3,3), CBB(3,1), B(2,2)
C DIMENSION ET(2,2), GBT(2,2), .Z(1), L1(1), DX(3,1), WSP(5)
C COMMON S1, S2, CM, P1, X2, X3
C
WRITE(2,79)
X1=-1./3.81
X1=1.0E+14
WRITE(2,26) X1, X2, X3
WRITE(2,25)
WRITE(2,24) ((PI(I,J), J=1,2), I=1,2)
WRITE(2,32)
WRITE(2,24) ((B (I,J), J=1,2), I=1,2)
DO 11 I=1,2
DO 12 J=1,2
BT(I,J) = B(I,J)
12 CONTINUE
11 CONTINUE
WRITE (2,20)
WRITE (2,33) ((BT(I,I), J=1,2), ll=1,2)
CALL F01CKF(BBT, BBT, 2, 2, 0)
WRITE (2,28)
WRITE (2,33) ((BBT(I,I), J=1,2), ll=1,2)
G11 = BBT(1,1)
G22 = BBT(2,2)
GX = BBT(2,1)
GXX = BBT(1,2)
WRITE (2,81) GX, GXX
X11 = X1**2
X22 = X2**2
X33 = X3**2
X12 = X1*X2
X13 = X1*X3
X23 = X2*X3
IK = 0
9 IK=IK+1
IF(((IK.EQ.100).OR.(IK.EQ.1000)).OR.(IK.EQ.2000)) GO TO 111
IF(((IK.EQ.3000).OR.(IK.EQ.4000)).OR.(IK.EQ.5000)) GO TO 111
IF(((IK.EQ.6000).OR.(IK.EQ.7000)).OR.(IK.EQ.9000)) GO TO 111
GO TO 101
111 CONTINUE
WRITE (2,41) F1, F2, F3, E
WRITE (2,27) (DX(I1,1), ll=1,3)
WRITE (2,27) DX1, DX2, DX3
WRITE (2,29) IK, X1, X2, X3
101 CONTINUE
F1=G11*X1+D2*X2/C+G3*X3+51*X1**2-2.*S1*X1-1.
F2=G3*X2+G1*X1+G2*X3+(S1+S2)*X2+F3
F3=G11*X2+G22*X3+33*E.+G3*X2**2-2.*S2*X3-CM
F0(1,1) = F1
F0(2,1) = F2
F0(3,1) = F3
C
BUILD UP THE MATRIX [DF].
C
\[
\begin{align*}
DF_{11} &= 2 \cdot g_{11} x_1 + 2 \cdot g_{22} x_2 - 2 \cdot s_1 \\
DF_{12} &= 2 \cdot g_{22} x_2 + 2 \cdot g_{11} x_1 \\
DF_{13} &= 0, \\
DF_{21} &= g_{11} x_1 + 2 g_{11} x_2 + g_{22} x_3 - (s_1 + s_2) \\
DF_{22} &= g_{22} x_2 + g_{11} x_1 + g_{22} x_3 - (s_1 + s_2) \\
DF_{23} &= g_{22} x_3 - 2 \cdot g_{22} x_2 - 2 \cdot s_2 \\
DF_{31} &= 0, \\
DF_{32} &= 2 \cdot g_{11} x_2 + 2 \cdot g_{22} x_3 \\
DF_{33} &= 2 \cdot g_{22} x_3 + 2 \cdot g_{11} x_2 - 2 \cdot s_2 \\
DF(1,1) &= DF_{11} \\
DF(1,2) &= DF_{12} \\
DF(1,3) &= DF_{13} \\
DF(2,1) &= DF_{21} \\
DF(2,2) &= DF_{22} \\
DF(2,3) &= DF_{23} \\
DF(3,1) &= DF_{31} \\
DF(3,2) &= DF_{32} \\
DF(3,3) &= DF_{33} \\
E &= 1.0 \cdot E^{-7} \\
E &= 1.0 \cdot E^{-10} \\
AF_1 &= \text{ABS}(F_1) \\
AF_2 &= \text{ABS}(F_2) \\
AF_3 &= \text{ABS}(F_3) \\
\text{IF}((AF_1 \cdot \text{LE} \cdot E) \cdot \text{AND} \cdot (AF_2 \cdot \text{LE} \cdot E) \cdot \text{AND} \cdot (AF_3 \cdot \text{LE} \cdot E)) \text{ GO TO 10} \\
C \text{ CALL SUBROUTINE OF NAG TO SOLVE THE EQUATION } \{DF\} \cdot \{DX\} = -\{FO\}.
\end{align*}
\]

C \text{ CALL FOU4AEF(DF;3,FO,3,3,1,DX,3,WSF,CR,3,CRB,3,IFAIL) }

C \text{ IFAIL}=9

C \text{ GO TO 9}

C \text{ WRITE}(2,41)(F_1,12,F_3,E)

C \text{ WRITE}(2,27)(\{DX\}(1,1),II=1,3)

C \text{ WRITE}(2,27)(DX_1,DX_2,DX_3)

C \text{ WRITE}(2,29)1K,X_1,X_2,X_3

C \text{ XMAX}=(S_1**2+G_{11})/(2.0*G_{22})*S_1

C \text{ XMIN}=-1.0/(-2.0*S_1)

C \text{ WRITE}(2,30)XMAX,XMIN

C \text{ WRITE}(2,70)

C \text{ CHECK IF THE EVALUATED MATRIX MEETS THE REQUIREMENT }

C \text{ FOR A REAL SYMMETRIC AND POSITIVE DEFINITE MATRIX.}

C \text{ DET}=-X_1*X_3-X_2**2

C \text{ WRITE}(2,51)\text{DET}

C \text{ IF(\text{DET} .LE. 0.) GO TO 55}

C \text{ WRITE}(2,108)

C \text{ GO TO 91}
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
C DEFINE THE OPTIMAL CONTROLLER OF THE SYSTEM, WHICH
C MINIMIZES THE GIVEN PERFORMANCE INDEX.
CALL FO1CKF(U0,BT,RO,2,2,2,12,1,0)
WRITE(2,97)
WRITE(2,71)(( UO(I,J),J=1,7),I=1,2)
DO 1 J=1,2
UO(I,J)=UO(I,J)
2 CONTINUE
FIGURE A17.11

Flow Chart of the Computer Program to State Controllability and Observability of a System

START

DATA: Read parameters and matrices $A, B$ and $C$

PRINT DATA

CALL subroutine FO1CKF to form the vectors $AB$ and $A^2B$

FORM the matrix $[B | AB | A^2B]$

CALL subroutine FO3AAF to calculate the associated determinant

PRINT matrix $[B | AB | A^2B]$

WRITE a suitable message for system's controllability

TRANSPOSE matrix $A$

CALL subroutine FO1CKF to form the vectors $A^T C, (A^T)^2 C$
FORM the matrix $[C^T, A^T, C^T (A^T)^2 C^T]$

PRINT matrix $[C^T, A^T, C^T (A^T)^2 C^T]$

CALL subroutine F03AAF to calculate the associated determinant

WRITE suitable message for system's observability

FINISH
MASTER KALMAN TEST  

This program tests the controllability and the observability of a third order system.

Dimension A(3,2), B(3,2), C(3,1), I(1,1), A2P(2,3), WORS(3), CT1(3,3), A(3,2), AT(3,3), C(3,3), CT(3,3), TAC(3,3), TA2(3,1), OBS1(3,3)

WRITE (2, 79)
N=3
M=3
READ (1, 6) ((A(I, J), J=1, N), I=1, N)  
WRITE (2, 20)
WRITE (2, 21) ((A(I, J), J=1, N), I=1, N)
IP=2
IK=1
READ (1, 10) ((B(I, J), J=1, IP), I=1, M)
WRITE (2, 22)
WRITE (2, 23) ((B(I, J), J=1, IP), I=1, M)
IT=1
READ (1, 0) ((C(I, J), J=1, N), I=1, IT)
WRITE (2, 24)
WRITE (2, 25) ((C(I, J), J=1, N), I=1, IT)
CALL F01CKF (AB, A, B, 3, 1, 3, 1, 0)
WRITE (2, 79)
WRITE (2, 30)
WRITE (2, 31) ((AB(I, J), J=1, IP), I=1, N)
CALL F01CKF (A2B, A, AB, 3, 2, 3, 2, 12, 1, 0)
WRITE (2, 24)
WRITE (2, 25) ((A2P(I, J), J=1, IP), I=1, N)
DO 1 1=1, N
1 1=1, N
DO 3 J=1, IP+1
IF (J .GT. IP) GO TO 31
CT1(I, J)=B(I, J)
GO TO 3
31 CONTINUE
   DO 4 J=1, N
4 CONTINUE
   CT1(I, J)=AB(I, IK)
   3 CONTINUE
   DO 4 J=1, N
4 CONTINUE
   AT(I, J)=A(J, I)
   4 CONTINUE
   DO 5 J=1, IT
5 CONTINUE
   DO 6 I=1, N
6 CONTINUE
   CT(I, J)=C(J, I)
   6 CONTINUE
   WRITE (2, 35)
   WRITE (2, 21) ((CT1(I, J), J=1, IP+1), I=1, N)
   CALL F03AAF (CT1, 3, 3, DETER, WORS, 0)
   WRITE (2, 32) DETER
   WRITE (2, 34)
   WRITE (2, 79)
   DETER=0
   WRITE (2, 29) DETER
   WRITE (2, 36)
   IF (.NOT. DETER) GO TO 40
   GO TO 12
40 WRITE (2, 35)
   IF (IK .EQ. IP) GO TO 12
   IK=IK+1
   GO TO 13
12 CONTINUE
   WRITE (2, 41)
WRITE (2, 21) (AT(I, J), J=1, N), I=1, N
WRITE (2, 42)
WRITE (2, 27) (CT(I, J), J=1, IT), I=1, N
CALL FUSTCKF(TAC, AT, CT, 3, 1, 3, 2, 12, 1, 0)
WRITE (2, 44)
WRITE (2, 27) (TAC(I, J), J=1, IT), I=1, N
CALL FUSTCKF(TA2C, AT, TAC, 3, 1, 3, 2, 12, 1, 0)
WRITE (2, 45)
WRITE (2, 27) (TA2C(I, J), J=1, IT), I=1, N
IL=IT+1
DO 7 J=1, N
DO 8 I=1, IT
IF(J, GT, IT) GO TO 51
OBS1(I, J) = CT(I, J)
GO TO 54
51 IF(J, GT, IL) GO TO 52
OBS1(I, J) = TAC(I, 1)
GO TO 54
52 OBS1(I, J) = TA2C(I, 1)
GO TO 54
7 CONTINUE
8 CONTINUE
WRITE (2, 28)
WRITE (2, 21) (OBS1(I, J), J=1, N), I=1, N
CALL FUSTCAAF(OBS1, 3, 3, DETOB, WORSF, 0)
WRITE (2, 79)
 WRITE (2, 39) DETOB
IF(DETOB . EQ. 0.0) GO TO 49
WRITE (2, 55)
GO TO 14
49 WRITE (2, 57)
14 CONTINUE
9 FORMAT (3F9.2)
10 FORMAT (2F9.2)
20 FORMAT (24X, 'MATRIX A1', /)
21 FORMAT (3(9X, 3F12.2, /))
22 FORMAT (24X, 'MATRIX B1', /)
23 FORMAT (3(14X, 2F12.2, /))
24 FORMAT (18X, 'THE MATRIX A*B=A*(A*B)', /)
25 FORMAT (3(14X, 2E12.4, /))
27 FORMAT (24X, 3F9.2, /)
28 FORMAT (24X, 'THE MATRIX OBS1', /)
29 FORMAT (OX, 'DETER', 'F6.2', /)
30 FORMAT (18X, 'THE MATRIX AR=A*B', /)
32 FORMAT (19X, 'DETERMINANT OF CT1=', E14.5, /)
33 FORMAT (24X, 'THE MATRIX CT1', /)
34 FORMAT (OX, 'THE SYSTEM IS COMPLETELY CONTROULABLE, ACCORDING TO
1 KALMAN THEOREM1', /)
35 FORMAT (OX, 'THE PROCEDURE CONTINUES', /)
36 FORMAT (OX, 'THE SYSTEM IS NOT COMPLETELY CONTROULABLE', /)
39 FORMAT (44X, 'DETERMINANT OF OBS1=', E14.5, /)
41 FORMAT (44X, 'THE TRANSPPOSE MATRIX AT', /)
42 FORMAT (44X, 'THE TRANSPPOSE MATRIX CT1', /)
44 FORMAT (44X, 'THE MATRIX TAC=AT*LT', /)
45 FORMAT (44X, 'THE MATRIX TA2C=AT*(AT*CT)', /)
47 FORMAT (24X, 'MATIC C1', /)
55 FORMAT (OX, 'THE SYSTEM IS COMPLETELY OBSERVABLE ACCORDING TO
1 KALMAN THEOREM1', /)
57 FORMAT (OX, 'THE SYSTEM IS NOT COMPLETELY OBSERVABLE', /)
79 FORMAT (OX, '!', /)
STOP
FIGURE A17.21

Flow Chart of the Computer Program Calculating the Optimal Time (Chapter 5)

START

DATA: Given poles of the system and matrix A and B

PRINT DATA

CALCULATE coefficients of the integral quantities, i.e. relations (5.20) and (5.21)

CALL subroutine D01ABF to compute the integrals of eqs. (5.18) and (5.19) for different values of the upper limits, $T_0$, and the initial costates $j_1(0)$ and $j_2(0)$

COMPUTE the costates $j_1(t)$ and $j_2(t)$

PRINT the costates $j_1$ and $j_2$, the optimal time $T_0$, and the values of the integrals

END
SUBROUTINE FUN(T)

DATA (COMMON)

COMPUTE coefficients of the integral quantities, i.e. eq(5.18) and (5.19)

RETURN

FINISH
Figure A 17.22: The Listing of the Computer Program which Calculates the Optimal Time (chapter 5).

This Program Gives a Numerical Solution of the Time-Optimal Controller, Constrained in Magnitude, According to the State-Function of Pontryagin. The Optimal (Minimum) Time Has Been Evaluated as Well as the Unknowns Initial Costates for the Analytical Expression of This Open-Loop Controller.

Master Bounded

**Dimension Uc(2, 2), Ra(2, 2), P(2, 2), r(2, 2), z(1), Iz(1)**

**Real Inc**

Common to, S1, S2, INC, P1, PZ, OX

Common R11, R12, R21, R22

**Comm** F1, F2

External Fun1, Fun2

**Read (1, 2) S1, S2, INC**

**Write (2, 22) S1, S2, INC**

Real (1, 24) ((P(1, J), J=1, 2), I=1, 2)

**Write (2, 36)**

Write (2, 24) ((P(1, J), J=1, 2), I=1, 2)

**Read (1, 24) (B(1, J), J=1, 2), I=1, 2**

**Write (2, 37)**

Write (2, 25) ((BA(I, J), J=1, 2), I=1, 2)

**Call for KF(B, P1, RA, 2, 2, 2, 1Z, 10)**

**Write (2, 20)**

Write (2, 25) ((B(I, J), J=1, 2), I=1, 2)

B11=R(1, 1)

B12=R(1, 2)

B21=R(2, 1)

B22=R(2, 2)

**Write (2, 333) B11, B12, R21, R22**

Evaluation of the two integrals of the plant equations, in order to define the upper integration limit which satisfy simultaneously the given initial conditions of the system.

F11=6(1)+B11+B12+R12

F22=F21*B21+B22+B22

**Write (2, 331) F11**

**Write (2, 331) F22**

F1=F11+F22+R11+F22

**Write (2, 331) F1**

N=1.0

ACC=1.0+4

**Write**

Me=1.0

Bc=1.0+1.0

A=0.0

**Write**

I=N+0, 0.<N<.1-1.)

**Write (2, 79)**

**Write (2, 51)**

Bc+J=1.1

**Write**

DO J=1, L

**Write**

DO J=1, L/4

**Write**

P1=-2*DB+1N0+EXP(S1*1O)

**Write**

P0=-S1*D
P1 = P1 + P1
G = 9A2 + P12 + P22 * EXP(-2 * S1 * T0)
IF (G < 1.0, 5 GO TO 14
QX = SORT(G)
101 CONTINUE
P2 = EXP((S2 - S1) * T0) * (-PI1 + GX) / FS2
WRITE(2, ?1) PI, PI, INC, I, F1, F2
FAIL = 1
CALL ERROR1, TO, FUN2, ACC, NMAX, K, ANS, IFAIL
WRITE(2, ?1) ANS, K, J, TO
IF (FAIL) GE 50, 47
IF (ANS, LT, 0.007) GO TO 12
GO TO 7
12 WRITE(2, ?9)
WRITE(2, ?1)
ANS = 0
TO = 0.02
NMAX = N
ACC = 1.0 + 4
WRITE(2, ?1)
PRINT 0000
F = 0
DO 20 J = 1, K
T = TO - (J - 1) / FLOAT(N)
P1 = (-F12 / DP1 + INC) * EXP(ST * T0)
P1 = F1 * S0
P1 = P1 + F1
Q = (-H(1) * P12 + P12 + EXP(-2 * S1 * T0)
IF (ANS, LT, 0.0) GO TO 114
QX = SORT(0)
11 CONTINUE
P2 = EXP((S2 - S1) * T0) * (-PI1 + GX) / FS2
WRITE(2, ?1) PI, PI, INC, I, F1, F2
FAIL = 1
CALL ERROR1, TO, FUN2, ACC, NMAX, K, ANS, IFAIL
WRITE(2, ?1) ANS, K, J, TO
IF (FAIL) GE 50, 47
IF (ANS, LT, 0.007) GO TO 99
GO TO 9
9 WRITE(2, ?2)
GO TO 9
14 WRITE(2, ?2)
GO TO 4
9 CONTINUE
99 WRITE(2, ?2)
END (2, 01)
END (2, 01)
END (2, 01)
13 END (2, 01)
11 END (2, 01)
10 END (2, 01)
9 END (2, 01)
8 END (2, 01)
7 END (2, 01)
6 END (2, 01)
5 END (2, 01)
4 END (2, 01)
3 END (2, 01)
2 END (2, 01)
1 END (2, 01)
*F7.4, //
24 FORMAT(2F10.4)
25 FORMAT(Z7X, ZF11.2)
31 FORMAT(9X, 'SEARCHING FOR THE FINAL TIME WHERE THE', //, 9X,
  'VALUE OF THE INTEGRAL BECOMES EQUAL TO X1(0)', //)
32 FORMAT(1HO, 9X, 'INTEGRAL DID NOT REACH THE REQUIRED ACCURACY', //)
33 FORMAT(9X, 'SEARCHING FOR THE FINAL TIME WHERE THE', //, 9X,
  'VALUE OF THE INTEGRAL BECOMES EQUAL TO X2(0)', //)
36 FORMAT(1HO, 9X, 'THE INVERSE MATRIX PI', //)
37 FORMAT(1HO, 9X, 'THE MATRIX BA', //)
41 FORMAT(9X, 'THE SEARCHING TECHNIQUE FOR THIS INTEGRAL HAS
  FINISHED', //)
71 FORMAT( 9X, 'P1=', F9.7, 3X, 'P2=', E12.6, 3X, 'IC=', F7.5, 5X, 'STEP=',
  '13.3X, 'F1=', F12.9, 3X, 'F2=', E12.9, //)
72 FORMAT(1HO, 9X, 'DATA ARE NOT CORRECT: P2 NOT REAL', //, 9X, 'O=', E16.9,
  '/*')
79 FORMAT(1HO)
531 FORMAT(9X, 'O1=', F9.5)
533 FORMAT(1HO, 9X, 'THE NEW MATRIX B', //, 7X, 2F11.2, //, 7X, 2F11.2, //)
STOP
END
EVALUATION OF THE TWO INTEGRALS OF THE PLANI EQUATIONS, IN ORDER TO DEFINE THE UPPER INTEGRATION LIMIT WHICH SATISFY SIMULTANEOUSLY THE GIVEN INITIAL CONDITIONS OF THE SYSTEM.

FUNCTION FUN1(T)
REAL INC
COMMON TO,S1,S2,INC,P1,P2,QX
COMMON B11,B12,B21,B22
COMMON F1,F2
EX1=EXP(-S1*T)
EX2=EXP(-S2*T)
FF1=B11*P1*EX1+B21*P2*EX2
FF2=B21*P1*EX1+B22*P2*EX2
RTL=SORT(FF1**2+FF2**2)
F1=(B11*FF1+B12*FF2)/RTL
F2=0
FUN1=FX1*F1
GO TO 33
33 CONTINUE
RETURN
END
FUNCTION FUN2(T)
REAL INC
COMMON T0,S1,S2,INC,P1,P2,QX
COMMON B11,B12,B21,B22
COMMON F1,F2
EX1=EXP(-S1*T)
EX2=EXP(-S2*T)
FF1=K11*P1*EX1+B21*P2*EX2
FF2=K21*P1*EX1+B22*P2*EX2
RTL=SQRT(FF1**2+FF2**2)
F1=0.0
F2=(B21*FF1+B22*FF2)/RTL
FUNC=EX2*F2
GO TO 33
CONTINUE
RETURN
END
FIGURE A17.31

Flow Chart of the Computer Program which Calculates
the Optimal Control Function (Chapter 5)

START

DATA: Given poles of the system and matrices A and B

PRINT DATA

CALCULATE initial costates $j_1(0)$ and $j_2(0)$, eqs. (5.16) and (5.17) and PRINT them

COMPUTE: the optimal control $u^0$, i.e. eqs. (5.22) and (5.23) in the time interval $[0, T_0]$, and Hamiltonian, eq. (5.10)

PRINT $u^0(t), H^0(T_0)$

CHECK: The accuracy of the solution:
1. Satisfaction of eq. (5.2)
2. Deviation of the evaluated Hamiltonian, relation 5.5, at any step in the time interval $0$ to $T_0$

PRINT RESULTS

FINISH
**Figure A17.33: The Listing of the Computer Program to Calculate the Optimal Control Function (Chapter 5)**

```plaintext
MASTER HAMILTON
DIMENSION B2(2,2), EA(2,2), P1(2,2), E(2,2), IZ(1)
REAL INC
READ (1,21) S1, S2, INC
WRITE (2,22) S1, S2, INC
READ (1,24) (P1(I,J), I=1,2), J=1,2
WRITE (2,36)
WRITE (2,24) (P1(I,J), I=1,2), J=1,2
READ (1,24) (EA(I,J), I=1,2), J=1,2
WRITE (2,37)
WRITE (2,25) (EA(I,J), I=1,2), J=1,2
CALL F01CF(B, P1, EA, E, IZ, I0)
WRITE (2,20)
WRITE (2,25) (B(I,J), J=1,2), I=1,2
B11=B(1,1)
B12=B(1,2)
B21=B(2,1)
B22=B(2,2)
WRITE (2,333) B11, B12, B21, B22

FS1=B11*B11+B12*B12
FS2=B21*B21+B22*B22
F22=SORT(FS2)
FS3=B11*B11+B12*B12
DB=B11*B22-B21*B12
WRITE (2,35) DB
DB2=DB**2
P10=F22/DB
P20=(-FS3/(F22*DB))
P20C=(-FS3/(F22*DB))*EXP(S2*T0)
WRITE (2,41) P10, P20, P20C
DO 17 JJ=1,5
INC=INC+0.050*(JJ-1.)
WRITE (2,99) INC
END
5 X0=14.15
TC=0.001*K0
WRITE (2,44) 10
P11=(-F22/DB+INC)*EXP(S1*T0)
P12=P11*2
Pp1=FS3/P11
Qe=-E/p1+P12+FS2*EXP(-2.*S1*T0)
IF(1,1,T,0.) GO TO 14

10 CONTINUE
GX=SQRT(T0)
P22=EXP((S2*S1*T0)+(-PP1-QX)/FS2)
WRITE (2,49) P11, P22
DO 11 I=1, K0
T<=-0.01*KT
P1=U11*EXP(-S1*T5)
P2=U12*EXP(-S2*T5)
U11=U11+P1+U21+P2
U21=P12+P12+P2
U12=U12+P2
U22=U22+P2
END
```

WRITE(2,133) U12,U22,HAM
WRITE(2,131) U11,U21,ROOT
WRITE(2,161) P11,P1,P22,P2
GO TO 11
14 WRITE(2,72) 0
OX=U,
GO TO 10 101
11 CONTINUE
U10=B11*P10+H21*P20
U20=B12*P10+H22*P20
U1X=U10**2
U2X=U20**2
ROTA=SORT(U1X+U2X)
HAM1=1.-ROTA
WRITE(2,77) FO01,HAM,ROTA,HAMIT
UNU=-U10/ROTA
U1U=-U20/ROTA
US1=U11**2
US2=U22**2
UII=1.(US1+US2)
WRITE(2,31) U1U,U1U,U20,UNIT
CONTINUE
1K.FUNC=U,OX50*(JJ-1.)
17 CONTINUE
26 FORMAT(1HO,YX,'THE MATRIX B'1/) 0
21 FORMAT(2F10.4,FY.5)
22 FORMAT(1HO,YX,'ROOT S1=1.,F9.2,5X,'ROOT S2=1.,F9.2,10X,'INCREMENT=', F7.4,1/)
24 FORMAT(2F10.4)
25 FORMAT(1HO,YX,'THE INVERSE MATRIX P1'1/) 0
27 FORMAT(1H0,YX,'THE MATRIX BA'1/)
31 FORMAT(9X,'U10=',F9.5,5X,'U120=',F9.5,5X,'UNIT=',F9.5,1/)
34 FORMAT(9X,'P10=',F12.5,5X,'P20=',F12.5,5X,'P20C=',E12.5,1/)
36 FORMAT(9X,'OPTIMAL TIME=',F9.5,5X,3X,1/)
72 FORMAT(9X,'L=',F12.6,1/)
77 FORMAT(9X,'ROOT=',F9.5,5X,'HAM=',F9.5,5X,3X,'ROOT=',F9.5,5X,3X)
131 FORMAT(9X,'U11=',F9.5,5X,'U21=',F9.5,5X,'P10=',F9.5,5X,1/)
99 FORMAT(9X,'INCR=',F9.5,5X,3X,1/)
132 FORMAT(9X,'U1=',F9.5,5X,'U2=',F9.5,5X,'P1=',F9.5,5X,'TIME=',F7.4,1/)
133 FORMAT(9X,'U1=',F9.5,5X,'U2=',F9.5,5X,'P2=',F9.5,5X,1/)
* HAM ITO
161 FORMAT(9X,'P1=',F12.6,5X,'P1=',F12.6,5X,1/)
169 FORMAT(9X,'P1=',F12.6,5X,'P2=',F12.6,5X,1/)
331 FORMAT(9X,'THE NEW MATRIX BS=',7X,2F11.2,1/)
333 FORMAT(9X,1/) STOP
END