System identification for crash victim simulation

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System Identification for Crash Victim Simulation

by

Roisin Hopkins

A Doctoral Thesis

Submitted in Partial Fulfilment of the Requirements for the Award of Doctor of Philosophy of the Loughborough University of Technology

October 1995

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Dedication.

I would like to dedicate this thesis to my parents for their love, guidance and support, thank you.
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Abstract

The work presented in this thesis concerns the identification of vehicle occupant models. Mathematical models of the vehicle occupant are used in the preliminary design and development phase of vehicle design. In the design phase, the model is used to guide the decision on restraint system feasibility. In the development phase the model is used to suggest solutions to problems associated with the dummy trajectory or restraint system performance.

Current methods used to determine such models involve independent component testing. The conditions under which the components are tested are often not typical of a crash test, hence iterations of the computer model are needed to successively improve model and test correlation.

In order to address these problems which cause inaccurate specification of the mathematical models, an alternative method of data set assembly for crash victim models is suggested. This alternative method is based on the techniques of system identification which allow unknown system parameters to be determined from experimental input/output data.

Initially the viability of using system identification techniques to develop a valid mathematical model of the vehicle occupant and restraint system was investigated. This initial study used input and output measurements from computer simulations of the occupant in frontal impact, as source data for the identification. Effects of simulated disturbances (noise corrupted output signals) and the effects of simplified model structure on the identification are also investigated. Several methods for analysing the likely errors in the identified parameters are defined and discussed in this simulation study.

Results relating to the identification of seat contact and seat belt characteristics from physical tests are also presented and these are interpreted in light of the simulation results.
Acknowledgements

The author owes immense gratitude to Dr Tim Gordon of Loughborough University, for his continuous guidance, invaluable suggestions and recommendations throughout the course of this research and to Professor Geoff Callow for his continual interest, encouragement and support.

Further thanks are due to Paul Wellicome and Dr Viv Stephens for their invaluable support with the experimental work carried out during the research.

The work was jointly funded by the Motor Industry Research Association and the DTI. The support of both is acknowledged with thanks.
List of Symbols

\( m_b \)  
Mass of the dummy thorax

\( m_f \)  
Mass of the dummy femur

\( I_b \)  
Inertia of the dummy thorax

\( I_f \)  
Inertia of the dummy femur

\( \ddot{x}_b \)  
Horizontal acceleration of the thorax

\( \ddot{y}_b \)  
Vertical acceleration of the thorax

\( \ddot{x}_f \)  
Horizontal acceleration of the femur

\( \ddot{y}_f \)  
Vertical acceleration of the femur

\( \dot{\theta}_b \)  
Angular acceleration of the thorax

\( \dot{\theta}_f \)  
Angular acceleration of the femur

\( \theta_b \)  
Angle of the thorax

\( \theta_f \)  
Angle of the femur

\( \theta_{seat} \)  
Angle of the seat

\( g \)  
Acceleration due to gravity

\( l_b \)  
Distance from the hip joint to the thorax centre of gravity

\( l_f \)  
Distance from the hip joint to the femur centre of gravity

\( \alpha_b \)  
Angle between the vertical and \( l_b \), when \( \theta_b = 0.0 \)

\( \alpha_f \)  
Angle between the horizontal and \( l_f \), when \( \theta_f = 0.0 \)

\( x_2, y_2 \)  
Horizontal and vertical distance of pelvis contact point from femur centre of gravity

\( x_3, y_3 \)  
Horizontal and vertical distance of knee contact point from femur centre of gravity

\( M_\theta \)  
Friction torque at the hip joint

\( R_2 \)  
Reaction force at the pelvis/seat contact

\( R_3 \)  
Reaction force at the knee/seat contact

\( F_2 \)  
Friction force at the pelvis/seat contact

\( F_3 \)  
Friction force at the knee/seat contact

\( F_{e2} \)  
Elastic force at the pelvis/seat contact

\( F_{e3} \)  
Elastic force at the femur/seat contact

\( F_{e4} \)  
Elastic force at the thorax/seat back contact

\( F_{d2} \)  
Damping force at the pelvis/seat contact

\( F_{d3} \)  
Damping force at the femur/seat contact

\( F_{s2} \)  
Friction force at the pelvis/seat contact

\( F_{s3} \)  
Friction force at the femur/seat contact

\( F_b \)  
Elastic Belt Force

\( F_{bx} \)  
Horizontal component of belt force

\( F_{by} \)  
Vertical component of belt force

\( C_{d2} \)  
Damping co-efficient at the pelvis/seat contact

\( C_{d3} \)  
Damping co-efficient at the femur/seat contact

\( C_{d_p} \)  
Damping co-efficient of the pelvis/seat contact

\( C_{d_k} \)  
Damping co-efficient of the knee/seat contact

\( \mu_p \)  
Friction co-efficient of the pelvis/seat contact

\( \mu_k \)  
Friction co-efficient of the knee/seat contact
\( \mu_2 \) Friction co-efficient at the pelvis/seat contact
\( \mu_3 \) Friction co-efficient at the femur/seat contact
\( K_b \) Stiffness of the belt
\( C_f \) Friction torque co-efficient
\( \Delta \) Penetration
\( V_n \) Normal velocity at contact point
\( \dot{\alpha} \) Relative angular velocity
\( \delta l_o \) Relative elongation
\( C_{f_1} - C_{i_0} \) Force term co-efficients
\( E \) Cost Function
\( y \) Response
\( a \) Parameters
\( \phi \) Basis functions
\( A \) Regressor matrix
\( U \) Left singular vectors
\( S \) Matrix of singular values
\( V \) Right singular vectors
\( \sigma \) Standard deviation
\( y' \) Response in transformed co-ordinates
\( a' \) Parameters in transformed co-ordinates
\( a_{\phi} \) Pre-assigned parameter value
\( W \) Weighting parameter
\( \hat{a} \) Estimated parameter values
\( \mu \) Bias
\( C \) Covariance matrix
\( \chi \) Parameter Condition Number

**Units**

Unless otherwise stated all quantities have SI units.
# Chapter 1

An Overview of System Identification and Crash Victim Simulation.

1.1 Introduction

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1.3 Research Objectives

1.4 Outline of Thesis

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1. An Overview of System Identification and Crash Victim Simulation

1.1 Introduction

Crash Victim Simulation (CVS) programs have become a valuable tool in the research and development of vehicle occupant safety. Over thirty years have elapsed since McHenry proposed one of the first mathematical simulation models to describe the dynamic response of a vehicle occupant in a collision event in 1963. Since then, many more sophisticated models have been developed for simulation of occupant kinematics. A computer model gives an insight into the principal influences on the occupant kinematics, and allows modifications to be investigated before tests are conducted.

Mathematical models can be divided into two classes: lumped parameter models and distributed parameter models. The difference between these classes of model is that the variables of interest for the lumped parameter systems are functions of time only, whilst the distributed parameter model accounts for both time and spatial variations in the physical process. The dynamic behaviour of lumped parameter systems is very often modelled by ordinary differential equations for continuous systems, or difference equations for discrete systems. Distributed parameter models are generally modelled by partial differential equations, integro-differential equations or delay-differential equations. Models can either be stochastic, where the relation between variables is given in terms of statistical distributions, or deterministic where the probability of events does not appear in the equations. One may also distinguish between linear and non-linear, time-invariant and time-varying models.

Vehicle occupant models may comprise of a series of connected rigid bodies, in which case the dynamics would be modelled by ordinary differential equations. If we allow flexibility in the model, for example a flexible beam element to model the spine, then the model would be described using partial differential equations. In most cases vehicle occupant models are non-linear.

In practical situations, mathematical models may be difficult to formulate from purely physical arguments because of some unknown phenomena. Creation of the input data sets for CVS models requires detailed knowledge of the various system parameters. Current practice in determining such parameters involves independent component testing. This standard approach can give variable results for the parameter values, also the conditions under which the components are tested may not be typical of a crash test. Hence iterations of the computer model are typically needed to successively improve model and test correlation. In such situations, we may resort to system identification techniques where input and output measurements from the system under test are used to determine an appropriate mathematical model.
Parameter identification may be defined as the determination, from experimental input/output data, of a set of unknown parameters in a mathematical model of a physical process, such that the model outputs over a desired range of conditions are acceptably close to the process outputs under analogous inputs and initial conditions. Very often engineering experience can provide valuable information about the most suitable form for the mathematical model whose parameters are to be identified from observed input/output data. In practice experimental data may be corrupted with noise, and this may result in severe degradation of the parameter estimates. In such situations special identification methods are used to eliminate, as far as possible, the influences of disturbances in order to yield adequate estimates. While these techniques are well established for the identification of linear systems, results for non-linear systems are generally less satisfactory; in such situations error analysis techniques may be implemented to help identify which of the parameter estimates are poorly conditioned (ie those that are sensitive to error), and which are well conditioned.

The expected advantage of using the formal techniques of system identification for crash victim simulation, is that a model can be fitted to the data generated from the physical crash test, speeding up the modelling process and making it potentially more accurate.

The following section of this chapter gives a brief overview of the work done over the last few years on the subject of system identification and identifies some of the crash victim simulation programs developed over the past twenty years. The system identification techniques required for crash victim simulation are highlighted and we also highlight potential problems and original features of application of system identification in crash victim simulation.

1.2 Literature Survey

1.2.1 System Identification

The parameter identification problem has received much attention over the past twenty years. During this time many techniques, ranging from frequency response analysis to various sophisticated time domain identification approaches, have been proposed. When formulating the identification problem, an error criterion is introduced to give a measure of how well a model fits the experimental data, minimisation of this criterion with respect to the parameters then yields model parameter estimates. This formulation leads to an optimisation problem, the methods used to solve it are classified with respect to the type of model and the type of signals being used, for example continuous or discrete, deterministic or stochastic, linear or non-linear. They are also classed with respect to the specific algorithm used to minimize the chosen criterion.
An Overview of System Identification and Crash Victim Simulation

(i) Comparisons of Different Identification Methods

Several attempts have been made to compare different identification and parameter estimation methods, Iserman et al (1974), Saridis (1974), Ljung and Glover (1981), and Ljung (1985).

Isermann et al compare the performance, computation time and overall reliability of six recursive identification and parameter estimation methods, using three simulated processes. These methods are least squares, generalised least squares, instrumental variables, stochastic approximation, correlation analysis and Fourier analysis. They conclude that all on-line methods tested using the same \textit{a priori} knowledge, result in comparable performance. The main differences between the identification methods are therefore seen in the kind of input signals, in the computational expense, overall reliability and in the assumption of specific \textit{a priori} known factors.

Saridis also discusses six popular parameter identification algorithms developed mainly for linear discrete systems, namely the cross-correlation technique, first-order and second-order stochastic approximation methods, maximum likelihood, maximum \textit{a posteriori} probability filter and extended Kalman filter. Their computational properties are compared and their convergence is tested on two fourth-order discrete time systems. An overall evaluation of the methods is also presented. The conclusion drawn was that the very poor results were demonstrated by the maximum \textit{a posteriori} filter.

The basic techniques of time domain and frequency domain identification including the maximum entropy methods are outlined by Ljung and Glover (1981). Connections and distinctions between the methods are explored. They conclude that the techniques are complementary rather than competing.

Presented by Ljung (1985), are the close conceptual relationships between basic approaches to the estimation of the transfer function of linear systems where the methods of frequency and spectral analysis are shown to be related to the time domain methods of predictor error type. The asymptotic properties of the estimates obtained by the respective methods are described and discussed and asymptotic expressions for the estimate of mean square error are shown to be similar for both types of method.

(ii) Surveys

An explicit description of all known methods of system identification is beyond the scope of this brief review. We shall therefore restrict ourselves to discuss only the major techniques used in parameter identification algorithms. Explicit descriptions can be found in Norton (1986), Ljung (1987), Eykhoff (1974) and Soderstrom and Stoica (1989).
Further surveys have been carried out on identification and parameter estimation techniques. Nieman et al (1971), present a survey of literature in the area of process identification and parameter estimation techniques applicable to lumped parameter, deterministic, dynamical systems, before 1971. They review statistical estimation techniques, direct and indirect methods based on optimal control theory, functional expansion, impulse response and frequency response. They also outline the general characteristics of the different methods and gives a guide to their application in specific problems.

Young (1981) reviews the progress of research on parameter estimation for continuous time models of dynamic systems over the period 1958-1980. Major developments are considered particularly in the discipline of research in the control and systems field. The paper surveys the choice of cost function and the classification of estimation methods.

Astrom and Eykhoff (1971), present a survey on the general properties and the classification of identification problems. For linear-in-parameters models the survey explains solutions obtained by repeated and generalised least squares, maximum likelihood, instrumental variable and the Tally principle. Non-linear, on-line and real time identification is also reported. The authors conclude that there are good techniques available for the identification of linear, time-invariant systems. For non-linear systems the techniques that are currently used convert the identification problem to an approximation problem by postulating a structure.

A brief introduction and overview of some of the better known algorithms for the identification of non-linear systems is given by Billings (1985). These algorithms form three distinct approaches to identification of non-linear systems, they are based on the functional series expansions, block orientated systems, and non-linear differential or difference equation models. It is stated that the identification of non-linear systems is a difficult problem, much work remains to be done to unify the methods, devise new algorithms and apply the results to practical systems.

Sinha and Qijie (1985) consider three direct methods for the identification of a linear, continuous time system from the samples of input and output observations. The methods are block pulse functions, trapezoidal pulse functions and cubic spline functions. Each method is tested in the absence and presence of measurement noise. Sinha and Qijie conclude that all methods were reasonably good for the noise-free case, but not so good for the noise corrupted case. A more recent survey in the subject of system identification was reported by Unbehauen (1985) where an introduction to identification methods using parameter estimation was given. The author discusses four steps into which the identification procedure can be divided:

- planning and analysing the experimental conditions
- selection of an approximate model structure
- estimation of the parameter models
- model validation
(iii) Deterministic Models

Strejc (1981) discusses methods used for identification of linear and non-linear systems, stochastic and deterministic approaches are distinguished and specific features concerning parameters, structures and state estimation are discussed with respect to the possible advantages and difficulties for identification. The significance of the uncertainty in structure, parameters and noise, and the possible application of a priori knowledge of the analysed system are taken into consideration in this paper. The most important class of deterministic parameter identification is the least squares technique because of its simplicity and ease of practical implementation.

In Strejc (1980), analytical and numerical approaches to the application of least squares for the estimation of system parameters are described. The model of the system dynamics is assumed in the form of a regression model and the following least squares methods are implemented: instrumental variable method, generalised least squares and extended least squares. Solutions are discussed for the cases of white noise and correlated noise corrupting the useful output signal of the system. It was shown that the most important factors influencing the success of identification schemes are the excitation of the system to be identified, the choice of output variables that can be measured, the precision of measurements, the signal to noise ratio, the method of identification, the numerical procedure of calculation, the redundancy of parameters in the regression model and the sampling period. Other techniques used for the same purpose include gradient search methods which are recursive schemes for finding the minimum error criterion (Eykhoff (1974)) and methods based on the property of orthogonal polynomials.

An operational matrix for the integration of the shifted Legendre polynomial functions was developed and applied to parameter identification of time invariant linear systems by Chang and Wang (1982). By employing operational matrix of shifted Legendre polynomials to approach the identification problem, algorithms for computation proved to be effective and straightforward. The results obtained were more accurate than those obtain by Laguerre functions.

Soderstrom and Stoica (1985), give an overview of the asymptotic properties of and the general implementation problems associated with instrumental variable techniques. They conclude that the principle leads to a simple estimation algorithm of low complexity and that the instrumental variable parameter estimates are consistent under reasonably weak assumptions. The authors also reviewed optimal instrumental variable methods, the accuracy of these methods were shown to be comparable with those of the prediction error methods, though they require more complex algorithm for implementation than a simple instrumental variable scheme.

An integral equation approach to the problem of parameter identification in continuous linear SISO (Single Input Single Output), MIMO (Multi Input Multi Output) and linear-in-parameters non-linear systems, is presented by Whitfield and
An Overview of System Identification and Crash Victim Simulation

Messali (1987). The method can be used on-line or off-line, the effect of deterministic disturbances at the system input and output are considered. A feature of the technique is that care needs to be taken in the choice of test input signal, with certain standard inputs such as impulses, steps and ramps being excluded from consideration in the presence of non-zero initial conditions.

(iv) Stochastic Models

Stochastic methods comprise statistical and probabilistic approaches. The most important class of parameter estimation methods for stochastic models are based on the following principles: least squares parameter estimation where the effect of identification errors is analysed, maximum likelihood and prediction error methods, Bayesian approaches, correlation methods and non-linear filtering techniques.

Astrom (1980), discusses the maximum likelihood and prediction error methods. The application to estimation of parameters in dynamical systems is treated in detail using the problem of estimating parameters in a continuous system using discrete time measurements. Various ways of minimising the likelihood function are discussed together with alternative methods for computing the likelihood function, its gradient and its Hessian. Computational aspects are discussed and theoretical results covered.

Puthenpura and Sinha (1986), present a method for the robust estimation of system parameters based on the censoring of data and employing the maximum likelihood estimation. They address the problem of occurrence of large measurement errors (outliers) where common methods such as generalised least squares and ordinary maximum likelihood methods have failed to converge. Simulation results show that this modified maximum likelihood method works well where others have failed.

The theory and application of correlation methods are discussed by Godfrey (1980), the use of cross-correlation to determine weighting functions for linear systems is emphasised. The corresponding frequency domain expressions are derived. Correlation methods have been applied widely in engineering, the author discusses problems associated with them. Godfrey concludes that cross-correlation is a useful approach for determining weighting functions of noisy linear systems provided that precautions are taken to recognise and include the effects of non-linearities and to provide appropriate inputs which continually excite the system over its frequency range of interest.

The use of neural networks in system identification has become of interest more recently. Sjoberg et al (1994) present a tutorial paper to explain how artificial neural networks can be used to solve problems in system identification. Neural networks are non-linear black-box structures, to be used with conventional parameter estimation methods. They have good general approximation capabilities for reasonable non-
linear systems. When estimating the parameters in these structures there is also a good adaptability to concentrate on those parameters that have the most importance for a particular data set.

(v) Identifiability

The problem of identifiability has also received much interest and is important in the identification procedure. Identifiability determines whether the parameters of a model can be identified, uniquely or with several solutions, from a specified input-output experiment.

Godfrey and Distefano (1985) consider the identifiability initially from perfect data, looking at five methods of deterministic identifiability analysis for linear, time invariant models, the methods are: Laplace transform, Taylor series, Markov parameter matrix, modal matrix and exhaustive modelling approaches. Only one of the methods, Taylor series, is then applicable to non-linear models or time-varying systems. The paper also deals with identifiability in the presence of real noisy data, often known as a posteriori identifiability, and this is essentially the problem of parameter estimation accuracy.

(vi) Application of System Identification in Industry

System identification is a powerful tool in engineering and much work has been done in the application of identification and parameter estimation techniques to engineering problems. Natke (1988) brings together a course of papers to describe state of the art in specific application areas, such as estimation of eigenquantities, noise source detection, fault detection by investigation of dynamic properties such as machine sound characteristics, and the identification of the dynamic behaviour of flow induced systems (e.g., aeroelastic problems). The book contains demonstrations of several methods.

Hollandsworth and Busby (1989) apply the general inverse technique to the prediction of single concentrated impact forces from a single measurement. The authors state that application of this technique to impact force identification can achieve good results although the problem is not necessarily unique in that there may be more than one solution which fits the noisy data.

Mattsson (1985) uses least squares parameter estimation techniques in the frequency domain for the identification of wind turbine dynamics. Good agreement between the estimated and physical models was obtained with simple physical models.

A Gauss Newton algorithm for the estimation of physical parameters in non-linear loudspeaker models is applied by Knudsen (1994), to minimise the squared error.
between input/output measurements. The paper describes how the loudspeaker parameters can be estimated directly from the measured diaphragm displacement and the output of a simulated model.

Hahn and Nierbergall (1994) use linear least squares estimation algorithms for the simultaneous experimental identification of the ten inertia parameters of a rigid body. Non-linear model equations based on the equations of motion of the rigid body are used for the simultaneous identification of the inertia parameters of the rigid body in space. The model is non-linear in state variables but linear in the parameters to be identified. Good comparisons of measured and estimated results were obtained.

Application of identification techniques is popular in the aeronautical industry, the naval industry, the rail industry and particularly in the automotive industry. Bourassa and Marcos (1991) propose a parameter identification technique for a human pilot driver. Parameters are identified based on use of the minimum cost function based on human pilot and simulated pilot state vectors. The technique was found to be useful in identifying cases of understeer, neutral and oversteer behaviours.

Maximum likelihood estimation methods are used by Jategaonker and Plaetrehke (1985) to identify parameters in non-linear flight mechanics. Two methods of optimisation were tested: quasi-linearisation and minimum search. The authors state that the minimum search methods were found to require long computation times and in addition did not provide directly the information about the accuracy of the estimates, however the quasi-linearisation method led to a practical implementation of the estimation algorithm.

Time and frequency domain identification methods are applied to the identification of the fuel flow to shaft speed dynamics of a gas turbine by Evans et al (1994). Poor quality time domain models were obtained, a result of using a measured input signal for estimation in order to exclude actuator dynamics, violating the underlying assumption of discrete time estimation, that the input signal is piecewise constant. High quality models were obtained using frequency domain techniques.

Trankle (1985) describes on-line and off-line algorithms for processing navigation-quality inertial data to identify non-linear hydrodynamic models of ships. The on-line algorithm implements an extended Kalman filter for the estimation of ship velocity and acceleration. The off-line algorithms use maximum likelihood methods to tune parameters of the on-line filter and to estimate ship non-linear hydrodynamic coefficients.

Michelberger et al (1985) discuss the identification of multivariable models for vehicle structures excited by stochastic road profiles. The identified models are used to determine the dynamic stress and stress statistics for arbitrary road profile excitation and speed levels within the examined range. Use of recursive on-line parameter estimation techniques provide good results when applied to the estimation of the centre of gravity height of a vehicle - see Germann and Isermann (1994).
An Overview of System Identification and Crash Victim Simulation

A time domain direct identification method is used for the identification of system physical parameters for vehicle systems with non-linear components, by Lin and Kortum (1991). The method is based on least square type of error costs which are constructed from the system dynamic equations. The estimated parameters are then tuned to minimise the cost functions. Applied to a four degree of freedom model, with differing non-linear suspension components, such as cubic springs, non-linear hydraulic and frictional dampers, the author states that the method identifies all the unknown system parameters with sufficient accuracy, low memory and computation time.

Butsuen et al (1986) apply the direct system identification method to engine mount systems. The technique allows accurate and quick determination of two groups of properties that exercise dominant effects on low frequency vibration of a vehicle body. The first group is the rigid body properties of an engine, the second is the properties of each engine mount. The authors suggest that use of this method can contribute to the upgrading of the accuracy and reliability of the prediction analysis for the entire vehicle characteristics.

(vii) Application of System Identification Techniques in Vehicle Safety

Radwan and Hollowell (1990) and Mentzer et al (1992), apply system identification techniques to vehicle crash models. Radwan and Hollowell develop a formulation for determining the non-linear stiffness and damping characteristics of structures subjected to crash loading environments. The system identification is accomplished using adaptive time domain, constrained minimisation techniques. Motivation for this work is to develop lumped mass models of automobiles from acceleration and barrier load cell data collected during frontal barrier crash testing. The technique proves that there is enough information in a crash test to identify a useful multi-load path model with minimal constraints on the behaviour of the load paths. However the identified load paths may exhibit certain non-physical behaviour which affects the match between the model and the actual crash data. An extension of this work is presented by Mentzer et al, where the SISAME methodology for extracting one dimensional lumped parameter vehicle crash models from non-oblique crash test data is defined. Model extraction is based on constrained least squares optimisation of an over-determined system of target equations for the model parameters.

Gabler, Hollowell and Hitcock (1994) present a methodology for global optimisation of vehicle impact designs taking into account maximisation of passenger safety and minimisation of aggressiveness to other vehicles. Results relating to a design optimisation of a passenger car are discussed, illustrating potential design modifications which simultaneously improve passenger protection while reducing aggressivity.
Goualou, Vittecoq and Faidy (1993) discuss a data fitting methodology for frontal crash victim simulation. They present a methodology for fitting all interactions between a dummy and a car during frontal impact using standard crash experimental measurements with rigid multibody models of the dummy. An iterative approach is used to fit the most important parameters.

1.2.2 Crash Victim Simulation

Human response to vehicle accidents has been studied in laboratory experiments which are aimed at representing real-life accidents. In such experiments it is rarely possible to obtain human response to trauma by using living human subjects. Hence substitutes, such as animals and human cadavers, have been used to obtain human responses to impacts and to determine trauma tolerance levels. Experiments in which human volunteers have been exposed to tolerable levels of trauma have also contributed to the understanding of the impact biomechanics of the human body (Nahum and Melvin, 1993).

The knowledge gained of occupant dynamics in car crashes has been used to design biofidelic mechanical occupant substitutes; anthropomorphic test devices or crash test dummies. An additional difficulty in achieving biofidelic dummy responses is that there are requirements that the dummy should be durable, repeatable and reproducible. Currently the best available dummy for frontal impact testing is the Hybrid III dummy, available in three sizes, 5th percentile female and the 50th and 95th percentile male. The gross mass distribution of the Hybrid III is human like, as well as its joint locations and chest deflection properties. Also available are the EuroSid, BioSid and USSid dummies, specifically designed for side impact response.

The risk of injury in a specific accident is assessed by means of Injury Criteria. The most commonly used injury criterion in frontal impact tests is the Head Injury Criterion (HIC) (SAE, 1984). This criterion is based on the resultant linear acceleration of the centre of gravity of the head, HIC values exceeding 1000-1500 indicate risk of injury to the brain. It has been observed that head injury is unlikely to occur if the head does not contact another object, in order to differentiate between impacts where the head does and does not contact, the algorithms for the HIC have been set so that the time span between $t_1$ and $t_2$ is limited to 36 ms. Head displacement itself also constitutes an important parameter, since this is strongly related to the risk of the head striking the vehicle interior.

$$HIC = \frac{1}{(t_2-t_1)^2} \int_{t_1}^{t_2} a \, dt$$

(1.2.1)

where, $a$ represents the resultant linear acceleration at the centre of gravity of the head, $t_1$ and $t_2$ should be chosen so that the integral is maximised.
Another part of the body often injured in car accidents is the chest. The risk of thoracic injury in frontal impact is today assessed by means of the maximum resultant linear acceleration of the centre of gravity of the chest over more than three continuous milliseconds. The most commonly used threshold value for this criterion is 60g. More sophisticated chest injury criteria, such as the viscous criterion (V*C), have been developed for high velocity chest trauma, such as those occurring in side impacts.

In order to investigate injuries to the occupants in car accidents, the vehicle interior and the accident itself must also be modelled. A frontal impact is normally represented by accelerating a vehicle to a pre-determined road speed before impact with a flat concrete test barrier. Other impact scenarios use standardised vehicle-form barriers or production vehicles to investigate the effect upon the test vehicle. Another means of modelling impacts in a particular direction is the crash sled; deceleration and acceleration sleds are available and they are generally used for restraint system development. The crash sled has fewer degrees of freedom than a vehicle and so repeatability in sled tests is higher than in full scale crash tests. Another advantage of the sled test is that it is non-destructive.

In addition to the physical models described, several mathematical models aimed at describing the car crash scenario have also been developed. These models are based mainly on multibody systems techniques and finite element methods. Since the mathematical models are deterministic and therefore repeatable, they play a major role in the vehicle development program. The repeatability enables the user to resolve effects of small changes to a system and to examine large parameter spaces in a short time. The main disadvantage of mathematical models is that all the physical properties have to be described in detail, numerically.

Computer programs which automatically calculate rigid body dynamic equations have been available since the early 1970's. In the field of vehicle safety, these programs are known as crash victim simulation (CVS) programs. The first CVS program developed was the MVMA2D, released in 1972 and based on work by McHenry and Naab at the Cornell Aeronautical Laboratories (Robbins et al). As suggested in its title, the program was capable of simulating two dimensional crash scenarios. The first three dimensional CVS program developed was the CAL3D model, developed at CALSPAN by Fleck and Butler. The analytical formulation for the CAL3D model was a combination of Newtonian and Lagrangian. MADYMO is a more recent the CVS modeller, developed by TNO Road Vehicles Research Institute, the first version was released in 1975. Existing in both 2D and 3D versions, the analytical formulation was originally Lagrangian; later versions introduced a new multibody module which used a more advanced mathematical formulation. Recent versions of MADYMO include a finite element module, which enables the modelling of airbag fabric and seat belt webbing. Several other purpose written CVS programs have been developed (Schmid 1984, Hoffmann et al. 1989, Lestrelin et al. 1984); comparisons and overviews of these models have been carried out by Prasad (1984, 1985).
We should also consider the automated mathematical modelling of general multibody systems and in particular the research and development of symbolic multibody codes (Sharp 1994). The first approaches to the automation of the multibody process were numerical, the most widely used multibody codes being ADAMS and DADS, which have become the standards for modelling in the automotive industry. Unlike the above numerical multibody codes, symbolic codes provide the user with the system equations rather than solutions to the equations. The priorities for symbolic codes are that the equations prepared may be linked with standard integration methods and that the solution process will be as fast as possible. There are several symbolic codes that are now commercially available: AUTOSIM (Sayers, 1990), AUTOLEV (Schiehlen, 1990) and SD/FAST (Rosenthal et al, 1983), use Kanes equations for the multibody formulation whereas MESA VERDE and NEWEUL (Schiehlen, 1990) use the Newton-Euler formalism. All these symbolic codes are intended for use by reasonably expert dynamicists and are aimed at producing ready-to-compile simulation programs. AUTOSIM also employs the artificial intelligence language COMMON LISP, utilizing the symbol manipulation capabilities of that language to further enhance its capabilities.

To summarize, numerical codes prepare equations in number form only, solve them and post-process the results to give graphs and animations; typically they include their own numerical solvers, plotters and animators. Symbolic codes simply yield equations which need to be linked with a numerical processing package in order to solve the equations. Symbolic equations are more difficult to obtain than numerical ones, however once they have been obtained for a particular system, they need not be regenerated for similar applications; they are available for use in the same way as manually derived equations. Hence with regard to the mathematical modelling and simulation of vehicle occupant models either numerical codes, such as MADYMO, or symbolic codes, such as AUTOSIM, may be preferred, according to the skill of the user and the nature of the application.

1.3 Research Objectives

The research undertaken in this thesis was motivated by a specific engineering need. A comprehensive data set is required to create a mathematical model of a vehicle occupant and its restraint system. It will include geometrical data, inertial data, contact interaction characteristics and component data, such as seat belt characteristics or airbag data. To simulate an impact, crash-pulse time-histories are also required. The geometrical and inertial data are easily obtained by direct measurement, and the crash-pulse time-histories are obtained via full-scale crash tests on the prototype vehicle; however good contact interaction data is less easy to determine, and in particular velocity dependent terms are difficult to determine. Current methods in obtaining such data rely on quasi-static component tests, which, although in common use, can give variable results for parameter values and the conditions under which the components are tested are often not typical of a crash test. Hence the mathematical
An Overview of System Identification and Crash Victim Simulation

model described by such data may need lengthy iterations to improve model and test correlation. Furthermore the model so created is not always valid for all sets of initial conditions; model and test results may only correlate for one particular set of initial conditions. It would be of significant value to vehicle safety development to automate further the construction of valid mathematical models of the occupant and its restraint system, by formally identifying the contact interaction data. Application of system identification techniques should enable us to identify these characteristics using input and output measurements from the system under test.

System identification analysis could also be used to define ideal configurations, in order to be confident of extracting accurate parameters of interest from the test output data. Although this approach may lead to a valid mathematical model it would add significant cost to the safety development process. There is therefore a practical constraint on the system identification to use data collected from legislative or routine tests carried out during typical development programmes.

The main objective of this research project is therefore to investigate the feasibility of determining mathematical models of occupants and restraint systems using input and output measurements from routine impact tests, carried out during typical vehicle development programmes. The purpose of the mathematical model so identified would be for use in further development of the vehicle restraint system. The identified model should correlate well with acceleration, velocity and displacement time histories obtained from the impact test program, to ensure accurate calculation of injury criteria and to define whether or not the occupant contacts the vehicle interior.

In this work the vehicle occupant and its restraint system are modelled as a lumped parameter system and hence via sets of ordinary differential equations. Such models are deterministic and non-linear. The differential equations turn out to be linear in external forces, but these forces generally involve non-linear characteristics such as contact friction and, as will be seen, leads to parameter identification that is non-linear in parameters.

The research presents several challenges: Little successful work has been carried out on identification of system parameters from non-linear models which are also non-linear-in-parameters. To add to the complexity of this problem we are aiming to restrict experimental conditions to a limited number of standard test situations. Adding to this the likely occurrence of noise on the test signals may lead to the well documented problems of ill-conditioning in the parameter estimates.

The aim is thus to make best use of the available experimental data to assist with the modelling, but to recognise that additional a priori knowledge may need to be composed. The general aim is then to define, justify and validate such procedures.
1.4 Outline Of Thesis

The work is arranged in eight chapters. Chapter 1 has given an overview of the work done over the past few years in the field of system identification and a brief introduction to crash victim simulation. The objectives of the research have been defined and the system identification techniques required for crash victim simulation were highlighted. Potential problems and original features of the application of system identification in crash victim simulation were also noted.

Mathematical models for vehicle occupants are featured in Chapter 2. An overview of an industry standard CVS program is given and introductory models for system identification are defined. Use of symbolic multibody code for development of vehicle occupant models is also introduced, with a view to using such code for the purposes of system identification.

Chapter 3 defines the techniques required for system identification of vehicle occupant models. Computer simulations are used to provide the source data to assess the viability of extracting system parameters from impact test data. Mathematical models of a modified Hybrid II dummy in frontal impact scenarios are considered.

Chapter 4 investigates the identification of system parameters from disturbed observations where the kinematic data is corrupted with 'experimental noise'. The minimisation procedure and identified models are assessed. This chapter also demonstrates a way in which the identified parameter values and models can be improved.

Chapter 5 addresses the problems associated with simplified model structure, that is, when the model assumed for the identification is a simplification of the model from which the kinematic data is produced. As in Chapter 4 the minimisation procedure and identified models are assessed.

A more general analysis of the likely errors in parameter estimates is discussed in Chapter 6. Several standard methods of analysis are reviewed and the Parameter Condition Number, a measure of the worst case error in each parameter estimate, is defined. Each of the error analysis techniques is applied to parameters identified using both frontal and rear impact data.

Identification of the contact characteristics from the frontal impact of the Hybrid II manikin is featured in Chapter 7. General background to impact simulation and data analysis procedures are given. Results of the identification of contact and belt characteristics are interpreted in light of the observations made in Chapters 4 and 5.

Chapter 8 summarises the work presented in previous sections. The achievements of the research are defined and options for taking the work forward are discussed.
# Chapter 2

## Mathematical Modelling

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Mathematical Modelling

2.1 Crash Victim Simulation (CVS)

This section outlines the nature of CVS programmes and also defines current practice in the creation of CVS models with particular emphasis on MADYMO, which is a current standard in the industry. In Section 2.2 background information on MADYMO is given. Section 2.3 defines a simple mathematical occupant model which is used for the initial investigation for application of system identification for crash victim simulation. This simple introductory model is defined in MADYMO and an explicit model is also created to allow confirmation of the MADYMO model formulation. In Section 2.4 more complex occupant models are considered and the use of symbolic code to create such models for system identification is introduced.

Although CVS models do not provide direct simulation of human impact response, they still provide reliable information on occupant impact dynamics, since the dummies are deemed to possess certain human-like behaviour.

The benefits of using CVS programmes are as follows:

- They provide fundamental data on impact phenomena
- They alleviate the tremendous work involved in conducting impact tests.
- They provide an appropriate means of evaluating the occupant protection system of a vehicle by predicting occupant loads, acceleration levels and injury indices.
- They provide an economical method for conducting parametric and optimisation studies to guide the re-design of the restraint system.

In order to generate the computer model, the user must supply the following data:

- Geometry of the vehicle interior eg. initial seating position, anchor locations of the seat belt.
- Mechanical properties of the occupant ie. dimensions, masses, moments of inertia, joint characteristics.
- Characteristics of the contact interaction between the segments of the body and the vehicle interior.

CVS programmes are used in the preliminary design and development phases of the prototype vehicle. In the preliminary phase, the model is set up using the vehicle interior measurements and some typical contact interaction data. Dummy properties are provided from standard databases, supplied for use in the CVS programmes. The basic model is sufficient to guide the decision on restraint system feasibility.
Mathematical Modelling

In the development phase, a programme of static and dynamic tests are performed on components of the restraint system in order to provide more accurate contact interaction data missing from the basic model data set. For example, the information required to create a data set for a belted occupant in a frontal impact situation may compromise of the following:

- topography of the driver's side vehicle interior
- knee bolster contact interaction data
- seat cushion contact interaction data
- windscreen contact interaction data
- steering wheel contact interaction data
- crash pulse time history
- toeboard and dashboard intrusion time histories
- seat belt data
- airbag data

The geometrical data are easily obtained by direct measurement. The contact interactions are provided by sub-system tests. The seat belt and airbag data are measured or obtained from the manufacturers. The crash pulse and intrusion time histories are obtained via full scale crash tests on the prototype vehicle.

The resulting model is then validated and refined by means of impact sled tests. Model and test outputs are correlated, where the degree of correlation is judged from the peak values, the injury indices, durations and shapes associated with impact response acceleration and load time histories as well as the trajectory of the occupant. The model is then typically refined in an ad-hoc way, adjusting physical parameters within reasonable bands to give some kind of 'best fit'. Because of the large number of free parameters involved this process can be lengthy, and the results heavily depend on the experience of the modeller.

The refined model is then used to suggest solutions to problems associated with the dummy trajectory or restraint system performance. In particular, comprehensive legislation must be complied with, for example the Federal Motor Vehicle Safety Standard (FMVSS) 208, concerning occupant protection. Once solutions to problems have been realised theoretically, they are then physically built into the prototype vehicle and tested, first on a HYGE or similar sled and then with full scale crash testing to ensure compliance with the appropriate safety standard.
Mathematical Modelling

The processes of sub-system testing used to generate a CVS data set can be relatively costly in terms of time and money, and this approach, though in common use, can give variable results for parameter values. Also, the conditions under which the components are tested may not be typical of a crash test, further increasing the need to rely on the computer model.

2.2 Example CVS Program - MADYMO

MADYMO is an industry standard CVS programme for rigid body modelling. MADYMO stands for MAthematical DYnamic MOdels and is supplied by TNO-Road-Vehicles Research Institute. It is supplied in both 2D and 3D versions, (TNO 1994).

MADYMO allows analysis of the dynamic response of systems undergoing large displacements. This is achieved by considering such structures to be a number of rigid bodies connected by joints, or (in the 3D version) an assemblage of finite elements. The multibody module generates the contribution of the inertia of the bodies to the equations of motion; applied loads are generated by specific force models such as springs, dampers, airbags, seat belts and contacts.

The MADYMO multibody formalism for generating the equations of motion is suited for multiple systems of rigid bodies with a tree structure. A system of bodies has a tree structure when it does not have a closed chain. Figure 2.2.1(a) shows a tree structure and Figure 2.2.1 (b) shows a closed chain.

![Multibody Structures](image)

Figure 2.2.1 Multibody Structures: (a) Tree Structure  (b) Closed Chain

For the 2D models, connections between bodies are always rotational (pin) joints. Eight kinematic joints are available for the 3D models.
Mathematical Modelling

The motion of a system of joint-connected rigid bodies is defined by the action of applied loads. MADYMO has the following set of standard force models:

- The acceleration field model calculates the forces at the centres of gravity of bodies due to a homogeneous acceleration field. This model is particularly useful for the simulation of the acceleration forces on a vehicle occupant during impact. Planes/lines and ellipsoids/ellipses can be attached to the rigid bodies to represent the occupant's shape. These planes and ellipsoids are also used to model contact with other bodies or with the surroundings. The contact surfaces are of major importance in the description of the interaction between the occupant and the vehicle interior. The elastic contact forces, including hysteresis, are a function of penetration of the contact surfaces. In addition to elastic contact forces, damping and friction characteristics can be specified.

- Three types of massless spring-damper elements are available: the Kelvin element is a uniaxial element which simulates a spring in parallel with a damper; the Maxwell element is a uniaxial element which simulates a spring in series with a damper; the point restraint can be considered as a combination of 3 spring/damper elements each parallel to one of the axes of an orthogonal co-ordinate system.

- Simple belt systems can be modelled with reasonable success using Kelvin elements. However, more complex belt systems require the use of a specifically written belt model which considers initial belt slack or pre-tension, as well as rupture of belt segments.

- In MADYMO 2D an empirical airbag model is available. The geometry of the airbag is represented by an ellipsoid, elliptical cylinder or an arbitrary shape in the plane of the simulation.

A multibody algorithm yields the second time derivatives of each of the degrees of freedom of the model in explicit form. The number of computer operations is directly proportional to the number of rigid bodies in the case where all joints in the model have the same number of degrees of freedom. At the start of the integration the initial state of the systems of bodies has to be specified. Time integration of second time derivatives of the degrees of freedom gives the degrees of freedom and their first time derivatives at a new point in time. For the time integration, two explicit numerical integration methods are available, namely a fourth order Runge Kutta method with fixed time step, and a fifth order Runge-Kutta Merson method which uses a variable time step that is controlled by a local truncation error.
Mathematical Modelling

MADYMO uses NEWTON-EULER equations of motion of a rigid body. The equations of motion of a rigid body \( i \) referred to its centre of gravity are:

\[ M_i \ddot{\mathbf{r}}_i = \mathbf{F}_i \]  (2.2.1)

\[ J_i \ddot{\omega}_i + \omega_i \times J_i \omega_i = \mathbf{T}_i \]  (2.2.2)

\( M_i \) is the mass, \( J_i \) is the inertia tensor with respect to the centre of gravity, \( \omega_i \) is the angular velocity vector, \( \mathbf{F}_i \) is the resultant torque vector relative to the centre of gravity. The unknown joint forces and torques can be eliminated using the Principal of Virtual Work. Equations (2.2.1) and (2.2.2) are multiplied by a variation of the position vector, \( \delta \mathbf{r}_i \), and a variation of the orientation, \( \delta q_i \), and the resulting equations are summed for all bodies of the system.

\[ \sum \delta \mathbf{r}_i (M_i \ddot{\mathbf{r}}_i - \mathbf{F}_i) + \delta q_i (J_i \ddot{\omega}_i + \omega_i \times J_i \omega_i - \mathbf{T}_i) = 0 \]  (2.2.3)

In case the variations \( \delta \mathbf{r}_i \) and \( \delta q_i \) of connected bodies are such that the constraints caused by the joint are not violated, the constraint forces and torques in joints will cancel.

2.3 Introductory Model for System Identification

MADYMO is proprietary software, not suited to the development of the system identification procedures. One could abandon it entirely, in favour of more flexible symbolic codes, except for the fact that, as a standard in the industry, any application of the system identification would necessarily come back to using MADYMO. To start with, a highly simplified system was required, so that explicit equations of motion could be formed 'by hand'. The reasons for this were:

- To validate the MADYMO model. Given that the results of this research project will eventually be measured by comparing MADYMO output with experimental data, some independent confirmation of the correctness of that code is required.
Mathematical Modelling

- Explicit models of simple two dimensional systems would assist in an initial assessment of the feasibility of system identification of the occupant restraint system.

- Once the 'correctness' of MADYMO has been established - especially concerning the algorithms used for contact forces - more complex MADYMO models can be used in turn to validate independent symbolic (AUTOSIM) CVS models.

2.3.1 Two-Dimensional Model

It was decided to use the TNO 10 manikin as a base for the initial mathematical models, since it has fewer degrees of freedom than the more complex Hybrid II and Hybrid III dummies.

The TNO 10 adult manikin shown in Figure 2.3.1 is constructed from a series of steel and aluminium plates, blocks and bars. Each section of the dummy is covered with skinned polyurethane foam forms to give the dummy size, mass and moment of inertia characteristics representative of a human. The TNO 10 manikin is primarily used for restraint system legislative tests. The TNO 10 has the mass of the arms combined in the dummy thorax and the lower legs are combined into one limb.

A jointed 2-body model, based on the femurs and thorax of the TNO 10 adult manikin was chosen for the initial study, this model is shown in Figure 2.3.2. Simple two-dimensional models were chosen for the following reasons:

- A mathematical model of a two-part dummy in frontal impact can be developed with relative ease. To write original code for a full dummy would require much more substantial resources.

- Identification of the jointed two body model in a frontal impact scenario creates sufficient complexities to allow a deep understanding of any problems to be gained before moving on to the even more complex model of a whole dummy.
Figure 2.3.1 TNO 10 Adult Manikin
2.3.2 2-Body Model Formulation

For the purposes of modelling, the TNO 10 was represented as a pair of two-dimensional systems, that is two rigid bodies representing the femurs and thorax of the dummy. The 2-body model was represented by four ellipses to give it a representative geometry; one ellipse to shape the thorax, three ellipses to shape the femur, the pelvis and the knee as the three were considered to be lumped together as one rigid body known hereafter as the femur. The vehicle seat, on which the manikin is seated, is represented by 2 planes; seat base and the seat back.
Using the Newton-Euler equations of motion for a rigid body, the following differential equations were derived:

\[ m_b \ddot{x}_b + m_f \ddot{x}_f = -(F_2 + F_3) \cos \theta_{seal} - (R_2 + R_3) \sin \theta_{seal} \quad (2.3.1) \]

\[ m_b \ddot{y}_b + m_f \ddot{y}_f = -(m_b + m_f)g -(F_2 + F_3) \sin \theta_{seal} + (R_2 + R_3) \cos \theta_{seal} \quad (2.3.2) \]

\[ I_{\theta_b} = m_b \ddot{x}_b l_b \cos (\theta_b + \alpha_b) + (m_b \ddot{y}_b + m_f g) l_b \sin (\theta_b + \alpha_b) + M_f \quad (2.3.3) \]

\[ I_{\phi_f} = (-m_f \ddot{y}_f - m_f g - (F_2 + F_3) \sin \theta_{seal}) l_f \cos (\theta_f + \alpha_f) + ((R_2 + R_3) \cos \theta_{seal}) l_f \cos (\theta_f + \alpha_f) + ((R_2 + R_3) \sin \theta_{seal}) l_f \sin (\theta_f + \alpha_f) \]

\[ = -(R_2 \ddot{x}_2 + F_2 \ddot{y}_2) \cos \theta_{seal} - (R_3 \ddot{y}_3 + F_3 \ddot{x}_3) \cos \theta_{seal} - (R_2 \ddot{y}_2 - F_2 \ddot{x}_2) \sin \theta_{seal} - (R_3 \ddot{y}_3 - F_3 \ddot{x}_3) \sin \theta_{seal} - M_f \quad (2.3.4) \]

Here the contact force is split into the following three components:

\[ R = \text{Elastic Force} + \text{Damping Force} \]

\[ F = \text{Friction Force} \]

The following section describes the plane-ellipse contact model adopted for this model.

**Plane-Ellipse Contact Model**

Since one of the objectives of the user-defined model is to verify the MADYMO formulation, it was necessary to adopt the same contact force algorithms in the user-defined model as are used in MADYMO 2D. Figure 2.3.3 shows the penetration of an ellipse contact surface with a plane contact surface as described in MADYMO 2D. The contact force between the two surfaces is modelled as consisting of three components which act at the contact point P. Both contact surfaces are assumed not to deform as a result of their interaction and so retain their size and shape even though they penetrate one another. Hence, centres of mass and moments of inertia of elements remain fixed at all times.
The three contact force components are defined as:

- **An Elastic Force,** $F_e$. This force is defined as a function of the penetration, $\Delta$, of the ellipse with the plane. It acts in a normal line through the contact point so as to oppose the penetration. Calculation of the contact point is given in Appendix A.

\[ F_e = F_e(\Delta) \hat{k}, \quad \Delta > 0.0 \]  

(2.3.5)

- **A Damping Force,** $F_d$. This is defined (during penetration) as a function of the normal relative velocity $\Delta V_{\text{norm}}$ between the plane and ellipse at the contact point. It again acts in a normal line through the contact point and opposes the normal velocity vector.

\[ F_d = -\text{sign}(\Delta V_{\text{norm}})F_d(\Delta V_{\text{norm}}) \hat{k} \]  

(2.3.6)

- **A Friction Force,** $F_f$. This is defined as a function of the normal contact force (given by the elastic and damping forces) at the contact point. It acts in a tangential line, ie perpendicular to the normal force and opposes the direction of the tangential relative velocity vector $\Delta V_{\text{tang}}$.

\[ F_f = -\text{sign}(\Delta V_{\text{tang}})F_f(|F_e + F_d|) \hat{j} \]  

(2.3.7)
2.3.3 Model Correlations

Once the formulated the model was coded in FORTRAN and NAG library routine F04AJF (NAG, 1991) was used to carry out the integration, using a Runge-Kutta variable time step method. The model was then tested by comparing its output with that of a MADYMO model, using identical inputs to both. The contact interaction data was invented and not representative of any particular situation. The dummy mass, inertia, joint and geometrical data was obtained via the manufacturer, direct measurements and sub-system tests.

The seated dummy was subjected (in simulation) to an acceleration pulse, taken from an actual impact test. This relatively weak pulse had a peak acceleration of 27.5 ms\(^{-2}\) and a peak velocity of 7 ms\(^{-1}\). Three sets of initial conditions were used, as shown in Table 2.3.1.

<table>
<thead>
<tr>
<th>TEST</th>
<th>(\theta_b (\degree))</th>
<th>(\theta_f (\degree))</th>
<th>(\theta_{seal} (\degree))</th>
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<tr>
<td>A</td>
<td>15</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B</td>
<td>25</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C</td>
<td>15</td>
<td>-5</td>
<td>-5</td>
</tr>
</tbody>
</table>

Table 2.3.1 Model Simulation Initial Conditions

Figures 2.3.4 - 2.3.5 show comparisons of the MADYMO output with the FORTRAN model output, for test A. Slight discrepancies in the two model outputs are seen. These were due to different numerical precisions being defined in the two models.
Figure 2.3.4 Acceleration Time Histories
Figure 2.3.5 Displacement Time Histories
2.4 Multibody Occupant Models

Although we are able to build simple mathematical models of the occupant and its restraint system, when the models become more complex (for example, more than two rigid bodies or three dimensional models), the model building process becomes very time consuming. It was therefore considered necessary to use a symbolic multibody software package, for this purpose.

The symbolic code, AUTOSIM (Sayers, 1990), is ideal for this purpose; the source code generated by AUTOSIM is completely accessible, can be easily modified and allows hand-written subroutines and auxiliary variables to be included, as required for the calculation of the contact forces.

2.4.1 The Modified Hybrid II Manikin

It is worthwhile showing a validation of model output from a MADYMO model with that of the same model built in AUTOSIM.

Since we were looking for a tool that could model more complex systems, it was decided to model the frontal impact of a Hybrid II dummy, shown in Figure 2.4.1. However we restricted the motion to two dimensions, so as to not bring too many complications into the system identification. It was therefore decided to remove the arms from the dummy and to join the lower legs together, giving essentially a model composed of four rigid bodies. The mathematical model is shown in Figure 2.4.2. The MADYMO data set and AUTOSIM model build commands, used to create the dynamic models are detailed in Appendix B.

The MADYMO and AUTOSIM models were run with identical inputs and sample outputs are shown in Figure 2.4.3. Slight discrepancies in the models are inevitable due to the different integrating routines used and round-off errors. However the correlations shown are clearly very good.
Figure 2.4.1 Hybrid II Manikin
Figure 2.4.2 Modified Hybrid II Model
Figure 2.4.3 Angular Acceleration Time Histories
2.5 Discussion

Although the industry standard CVS programme is well developed and is constantly being improved, for the purposes of system identification it cannot be immediately used. The equations of motion can be defined and coded for simple models, but to develop code for more complex rigid body models would be time consuming and is unnecessary - a suitable generic modeller already exists in AUTOSIM. With a good understanding of system dynamics we are able to build simple or complex models in relatively short periods of time.

The underlying symbolic and mathematical tools are now sufficient to allow us to approach the System Identification problem.
## Chapter 3

**System Identification - Theory and Example CVS Applications.**

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3.1 Introduction

System identification can be defined as the process of constructing a mathematical model of a dynamical system from observations and prior knowledge. The application of system identification in CVS modelling is to estimate the unknown parameters of an occupant restraint system by use of measured geometrical, inertial and kinematic data taken from an impact test upon that restraint system. Using the equations of motion of the occupant model, and the displacements, velocities and accelerations of the dummy mass centres for each time point, we are able to develop an over-determined set of non-linear differential equations. Optimisation techniques may then be used to extract the optimal set of unknown parameters, that is those parameters that best fit the test data.

The unknown parameters to be identified from our occupant models are the vehicle seat contact characteristics, that is the stiffness, damping and friction terms, and the lap belt characteristics. The process for defining such parameters by ad-hoc simulation and parameter variation can be relatively costly in terms of time and use of facilities, and the approach used, although in common use, can give extremely variable results for parameter values. Also the conditions under which the components are tested may not be typical of a crash test, hence iterations of the computer model are typically needed to improve model and test correlations. For rate dependent terms, that is damping and friction coefficients, the modeller often estimates values for the CVS data set based upon previous experience.

The potential advantage of combining the two processes of occupant modelling and impact testing by the use of system identification techniques is that a model can be fitted to the data generated from the physical crash test.

To investigate the viability of extracting restraint system parameters from an occupant model it is useful to adopt an ‘ideal case’ scenario. In such a scenario the mathematical test model is used to obtain the kinematic data required for the optimisation; contact data is invented and used in the model, the model is then subjected to an acceleration pulse (to simulate frontal impact) in order to obtain the acceleration, velocity and displacement time histories of the dummy mass centres. Optimisation techniques are then used to extract, from the occupant-kinematic data of the model, the contact parameters that must have been used to create it. A fitted model is then created using the extracted contact parameters plus masses, inertias and geometry as used in the original model. A forward run of the fitted model will give occupant kinematic data to compare with data from the test model.

When the structure of the fitted model corresponds exactly to the test model (ie. there are no errors due to simplified model structure, eg dummy flexibility) near perfect correlations should be expected in the absence of extraneous error signals.
This chapter looks at the least squares methods for extracting restraint system parameter. While techniques are well established for system identification of linear systems (see Section 2), the results for non-linear systems are less satisfactory: the system identification of vehicle occupant models is a fully non-linear problem. Both linear and non-linear least squares methods are investigated here.

Section 3.2 of this chapter introduces the linear least squares problem involved in this work and describes two methods of solution to the problem: (i) solution using the Normal Equations and (ii) solution using the singular value decomposition technique. Section 3.3 moves on to define the non-linear least squares problem and discusses a solution to the problem using the Levenberg-Marquardt method.

Having defined solutions to the least squares optimisation problems, Sections 3.4 and 3.5 apply these solutions to simulation data. Section 3.4 applies linear least squares solutions to identify the optimal restraint system parameters for the simple TNO10 model described in Section 2.3. Section 3.5 implements the Levenberg-Marquardt method for solving non-linear least squares problems to identify the restraint system parameters from the more complex Modified Hybrid II model which was described in Chapter 2.4. Identification of the Hybrid II model brings with it further complexity as multibody software code is used to generate the model and the expressions for the equations of motion generated by this code are extremely complicated. We introduce a straightforward method of handling these equations for any identification problem.

3.2 Linear Least Squares

With knowledge of centre of mass accelerations, velocities, and displacements, at sampled time points from the test model, we seek to determine the unknown contact forces. It was assumed that these forces can be expressed in terms of basis functions $\phi$, for example the elastic force can be expressed as:

$$ F_e = \sum_{k=1}^{n} a_k \phi_k(\Delta) $$

(3.2.1)

Here $a_k$ is a set of unknown coefficients to be determined, that is the parameters to be estimated in this case the contact stiffness, and $\phi_k(\Delta)$ are a set of basis functions dependent, in this case, on the penetration depth of the occupant with the vehicle seat. The damping and friction force functions can be described similarly to span the whole range of contact interactions.

Let $y_i (i=1, 2, \ldots, m)$ denote acceleration time history data. The problem becomes one of solving an overdetermined linear system of equations of the form:
which becomes \( y = Aa \) when written in matrix form.

Mathematically these overdetermined equations are equivalent to a least squares minimisation problem, where the following error criterion is to be optimised:

\[
E = |Aa - y|^2
\]  

We define as optimal those parameters that minimise \( E \), giving the least squares solution of equation (3.2.2). For the unconstrained problem the minimum occurs uniquely, where the derivative of \( E \) with respect to all \( n \) parameters \( a_k \) vanishes. The computational procedures outlined here (Sections 3.2 and 3.3) closely follow those given by Press et al (1989).

### 3.2.1 Solution by use of the Normal Equations

There are several ways of minimising the criterion (3.2.3). If we take the derivative of equation (3.2.3) with respect to the parameters \( a_k \), we obtain the following set of \( m \) equations for the \( n \) unknown parameters.

\[
0 = \sum_{i=1}^{m} \left[ y_i - \sum_{j=1}^{n} a_j \phi_j(x_i) \right] \phi_k(x_i) \quad k = 1, \ldots, n
\]  

Interchanging the order of summations we can write equation (3.2.4) as the matrix equation

\[
\sum_{j=1}^{n} \alpha_k a_j = \beta_k
\]  

Here

\[
\alpha_k = \sum_{i=1}^{m} \phi_j(x_i) \phi_k(x_i) \quad \text{or} \quad [\alpha] = A^T A
\]  

is an \( n \times n \) matrix, and
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\[ \beta_k = \sum_{i=1}^{m} y_i \phi_k(x_i) \quad \text{or} \quad [\beta] = A^T b \quad (3.2.7) \]

a vector of length \( m \).

Equations (3.2.4) and (3.2.5) are called the normal equations of the least squares problem. They can be solved for the vector of parameters \( a \) by standard methods, such as LU decomposition and back substitution or Gauss-Jordan elimination. In some applications, the normal equations are perfectly adequate for linear least squares problems, however in many cases the normal equations are very close to being singular. A zero pivot may be encountered during the solution of the linear equations, in which case one would obtain no solution at all; or a very small pivot may occur, in which case the solutions are likely to be highly sensitive to errors. These problems are most severe whenever \( m > n \), in which case more subtle methods of linear algebra are required.

3.2.2 Solution by Singular Value Decomposition

A factorisation method for solving linear least squares problems, which indicates exactly where the cause of any ill-conditioning lies is Singular Value Decomposition (SVD) (Maia, 1988). The technique is based on decomposing the regressor matrix \( A \), that is the matrix of basis functions, into the form

\[ A = U S V^T \]

where \( U \) and \( V \) are orthogonal matrices, i.e.,

\[ U^T U = U U^T = V V^T = V V^T = I \]

and

\[ U^T = U^{-1} \quad V^T = V^{-1} \]

\( S \) is a real matrix with elements \( S_{ij} = s_i \) for \( i = j \) and \( S_{ij} = 0 \) for \( i \neq j \). The values \( s_i \) are called the singular values of matrix \( A \). Without loss of generality we shall assume them to be arranged in decreasing order \( s_1 > s_2 > \ldots > s_n \).
Applying SVD to equation (3.2.2), in matrix form, we obtain

\[ USV^T a = y \]  \hspace{1cm} (3.2.11)

or

\[ SV^T a = U^T y \]  \hspace{1cm} (3.2.12)

which may be written as

\[ Sa' = y' \]  \hspace{1cm} (3.2.13)

with

\[ a' = V^T a \]  \hspace{1cm} (3.2.14)

\[ y' = U^T y \]  \hspace{1cm} (3.2.15)

Equation (3.2.13) represents a set of \( m \) uncoupled equations with \( n \) unknowns. From equation (3.2.13), we have

\[ s_j a'_j = y'_j \quad \text{for } j \leq n \text{ and } s_j \neq 0 \]

\[ 0 = y'_j \quad \text{for } j > n \]  \hspace{1cm} (3.2.16)

Parts 1 and 2 of equation (3.2.16) will only be consistent if \( y'_j = 0 \) for \( s_j = 0 \) or \( j > n \). The range of \( A \) implies that \( y'_j \) has to be zero for \( s_j = 0 \) if \( j \leq n \) or for \( j > n \). If this does not happen, then \( y \) does not belong to the range of \( A \) and equation (3.2.14) has no exact solutions. In this case, \( a'_j \) cannot be determined from part 2 of (3.2.16), although approximate solutions can be obtained by setting \( a'_j \) to zero whenever \( s_j = 0 \). This corresponds to the shortest solution in a least squares sense. Having calculated \( a' \), the vector \( a \) can be recovered from (3.2.14), i.e.

\[ a = Va' \]  \hspace{1cm} (3.2.17)
3.3 Non-linear Least Squares

As vehicle occupant models become more complex, as with the modified Hybrid II model, we no longer have a model which depends linearly on the unknown parameters $a_i$ and linear least squares techniques no longer apply. The problem becomes one of solving a set of over-determined equations, non-linear in its unknown parameters.

Solution of the non-linear least squares problem may be approached in a similar way as the linear least squares problem. An error criterion $E$ is defined and the optimal parameter values are determined by minimisation of this function. In this case however the function has non-linear dependencies and an iterative approach to the optimisation is applied. Given trial values for the unknown parameters, a procedure which improves the trial solution is developed. This procedure is repeated until the cost function, $E$, stops decreasing.

Suppose the model to be fitted is

$$y = y(x; a) \quad (3.3.1)$$

and the error criterion $E$ is

$$E(a) = \sum_{i=1}^{N} [y_i - y(x_i; a)]^2 \quad (3.3.2)$$

As the minimum is approached, the error criterion is expected to approximate the following quadratic form:

$$E(a) = y - d^T a + \frac{1}{2} a^T D a \quad (3.3.3)$$

where $d$ is a vector length $n$ and $D$ is an $n \times n$ matrix. If the approximation is a good one, it is possible to jump from the current trial parameters $a_{\text{cur}}$ to the minimizing ones $a_{\text{min}}$ in a single step, in the following way

$$a_{\text{min}} = a_{\text{cur}} + D^{-1}[-\nabla E(a_{\text{cur}})] \quad (3.3.4)$$

On the other hand if the approximation is poor, it makes more sense to take a step down the gradient, as in the steepest descent method

$$a_{\text{next}} = a_{\text{cur}} - \text{constant} \times \nabla E(a_{\text{cur}}) \quad (3.3.5)$$
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where the constant is small enough not to exhaust the downhill direction.

Application of equations (3.3.4) and (3.3.5), requires the ability to compute the gradient of the cost function $E$ for any set of parameters $a$. To use equation (3.3.4) also requires the matrix $D$, which is the second derivative matrix (Hessian matrix) of the cost function, at any $a$.

The gradient of $E$ with respect to the parameters $a$, is given as

$$
\frac{\partial E}{\partial a_k} = -2 \sum_{i=1}^{N} [y_i - y(x_i;a)] \frac{\partial y(x_i;a)}{\partial a_k} \quad k=1,2,...,M
$$

(3.3.6)

Taking an additional partial derivative gives

$$
\frac{\partial^2 E}{\partial a_k \partial a_l} = 2 \sum_{i=1}^{N} \left[ \frac{\partial y(x_i;a)}{\partial a_k} \frac{\partial y(x_i;a)}{\partial a_l} - [y_i - y(x_i;a)] \frac{\partial^2 y(x_i;a)}{\partial a_l \partial a_k} \right]
$$

(3.3.7)

Removing the factors of 2 by defining

$$
\beta_k = -\frac{1}{2} \frac{\partial E}{\partial a_k} \\
\alpha_{kl} = \frac{1}{2} \frac{\partial^2 E}{\partial a_k \partial a_l}
$$

(3.3.8)

gives $[\alpha] = \frac{1}{2} D$ in equation (3.3.4), and therefore that equation can be rewritten as the set of linear equations

$$
\sum_{i=1}^{M} \alpha_{kl} \delta a_l = \beta_k
$$

(3.3.9)

This set is to be solved for the increments $\delta a_l$ that, added to the current approximation, give the next approximation. We shall call this the 'Inverse Hessian' solution.

The steepest descent formula, given in equation (3.3.5) now takes the following form

$$
\delta a_i = constant \times \beta_i
$$

(3.3.10)
The components $a_{ij}$ of the Hessian matrix depend both on the first derivatives and on the second derivatives of the basis functions with respect to their parameters. Second derivatives occur because the gradient already has a dependence on $\delta y/\delta a_i$, so the next derivative by definition must contain terms involving $\delta^2 y/\delta a_i \delta a_k$. The second derivative term can be dismissed when it is zero (as in the linear case), or small enough to be negligible when compared to the term involving the first derivative.

There are various methods that can be used to solve the non-linear least squares problem, such as quasi-Newton methods, downhill simplex, simulated annealing and Levenberg-Marquardt. The following section describes the method chosen for this work.

### 3.3.1 Solution Using the Levenberg-Marquardt Method

The Levenberg-Marquardt Method (L-M) is frequently used for solving non-linear least squares problems and is reported to work well in practice (Press et al., 1989); for this reason it has become a standard method for solving non-linear least squares problems.

The method automatically combines the Inverse Hessian and Steepest Descent methods to solve the non-linear least squares problem. Far from the minimum the steepest descent formula is used, moving towards the Inverse Hessian method as the minimum is approached.

The method is based on the following:

- Consider the constant term in equation (3.3.10); we cannot obtain any information on what value it should take or even what magnitude it should have from the gradient, as this only gives us information of the slope, not how far the slope extends. We can however obtain some information on the order of magnitude of the problem from the components for the Hessian matrix. $E$ is non-dimensional (from its definition, equation (3.3.5)) and $\beta_k$ has dimensions of $1/a_k$. Parameter $a_k$ may well be dimensional, in fact each component of $a_k$ may have a different dimension and hence similarly $\beta_k$. The constant of proportionality between $\beta_k$ and $\delta a_k$ must therefore have the dimensions of $a_k^2$ and looking at the matrix $\alpha$ (equation 3.3.8) the only obvious components which have the dimensions of $a_k^2$ are $1/\alpha_{kk}$. This may therefore set the scale of the constant in equation (3.3.10). As it stands this scale may be too big, and we therefore divide the constant by a non-dimensional scale factor $\lambda$, with the option of setting $\lambda \gg 1$ so enabling a small step size if required. The constant in equation (3.3.10) is therefore given as:

$$constant = \frac{1}{\lambda \alpha_{kk}}$$

(3.3.11)
and equation (3.3.10) then becomes

\[ \lambda a_{ij} \delta a_i = \beta_i \]  

(3.3.12)

Equations (3.3.12) and (3.3.9) can be combined by defining a new matrix \( a' \) as

\[ a'_{ij} = a_{ik} (1 + \lambda) \]

\[ a'_{ij} = a_{jk} \quad (j \neq k) \]

(3.3.13)

thus equations (3.3.12) and (3.3.9) can be written as

\[ \sum_{i=1}^{m} a'_{ij} \delta a_i = \beta_k \]

(3.3.14)

When \( \lambda \) is very large, equation (3.3.14) is forced into being diagonally dominant and takes the form of equation (3.3.12), but as \( \lambda \) approaches zero equation (3.3.14) switches to the form of equation (3.3.9). The idea is that far from the minimum the steepest descent formula is used with large steps towards the solution, as the minimum is approached \( \lambda \) tends to zero and the inverse Hessian method is used with small steps approaching the solution. Hence after each iteration, if there is no reduction in the error, \( E \), then an increase in \( \lambda \) is required, \( \lambda_{\text{new}} = \lambda_{\text{old}} \times 10 \). If there is a reduction in \( E \), then a reduction in \( \lambda \) is required, \( \lambda_{\text{new}} = \lambda_{\text{old}} \times 1/10 \). However, this only works if the error function is "well-behaved", especially if there are no multiple local minima. If multiple local minima are present then this can actually cause some technical problems - see Section 4.2.

### 3.4 Identification via Linear Least Squares

#### 3.4.1 2-Body Model in Frontal Impact

The aim is to generate a fitted model of the system directly from test model output data using the above criteria. Referring to the contact forces shown in Figure 2.3.2, we may write

\[ R_2 = F_{d2} + F_{s2} \]

\[ R_3 = F_{d3} + F_{s3} \]

\[ R_2 + R_3 = F_s + F_d \]

(3.4.1)

where
Using these definitions and mathematical manipulations on equations (2.3.1)-(2.3.4), the equations of motion of the 2-body model can be expressed as:

\[
F_e + F_d = (m_b \ddot{y}_b + m_f \ddot{y}_f + (m_b + m_f)g) \cos \theta_{seat} - (m_b \dot{\theta}_b + m_f \dot{\theta}_f) \sin \theta_{seat}
\]
\[
F_f = -(m_b \ddot{y}_b + m_f \ddot{y}_f + (m_b + m_f)g) \sin \theta_{seat} - (m_b \ddot{\theta}_b + m_f \ddot{\theta}_f) \cos \theta_{seat}
\]
\[
M_f = I_b \ddot{\theta}_b - m_b \ddot{y}_b \dot{\theta}_b \cos (\theta_b + \alpha_b) - (m_b \ddot{y}_b + m_f \ddot{y}_f) l_b \sin (\theta_b + \alpha_b)
\]

Equations (3.4.4) define the motion of the system in terms of the motions of the centres of mass of the femur and body. With knowledge of mass centre accelerations, velocities and displacements, at sampled time points, we seek to determine the unknown functions, \( F_e(\Delta) \), \( F_d(V_b) \), \( F_f(R) \), \( M_f(\alpha) \).

It is assumed that these functions are expressed linearly in terms of basis functions \( \phi \).

\[
F_e = \sum_{k=1}^{n_1} a_{ek} \phi_{ek}(\Delta)
\]

(3.4.5)

\[
F_d = \sum_{k=1}^{n_2} a_{ak} \phi_{ak}(V_b)
\]

(3.4.6)

\[
F_f = \sum_{k=1}^{n_3} a_{fk} \phi_{fk}(R)
\]

(3.4.7)

\[
M_f = \sum_{k=1}^{n_4} a_{mk} \phi_{mk}(\alpha)
\]

(3.4.8)

Here \( a_{ij} \) are the unknown coefficients to be determined.
For a series of sampled time points, \( t_i, i=1, \ldots, j \), equations (3.4.4) can be expressed as a set of simultaneous linear equations, in the unknown coefficients \( \alpha \):

\[
\sum_{k=1}^{n_1} a_{sk}\phi_{sk}(\Delta(t_i)) + \sum_{k=1}^{n_2} a_{dk}\phi_{dk}(V_n(t_i)) = b_1(t_i) \quad (3.4.9)
\]

\[
\sum_{k=1}^{n_2} a_k\phi_k(R(t_i)) = b_2(t_i) \quad (3.4.10)
\]

\[
\sum_{k=1}^{n_3} a_{mk}\phi_{mk}(\ddot{\alpha}(t_i)) = b_3(t_i) \quad (3.4.11)
\]

where \( b_1, b_2 \) and \( b_3 \) each represent the expression on the right hand sides of the three equations (3.4.4), respectively.

The problem looks as though it is linear in parameters since the right hand side of each equation is known; however \( R(t_i) \) in equation (3.4.10) is unknown unless both \( F_e(t) \) and \( F_d(t) \) have been identified. Hence in order to be able to use linear least squares methods to solve the equations a two stage optimisation is required, firstly to identify \( F_e \) and \( F_d \) and secondly, with \( F_e \) and \( F_d \) known, to identify \( F_r \). This two stage approach generally gives a sub-optimal solution to the stated minimisation problem but for the case of consistent equations (zero noise) this is of no consequence.

### 3.4.2 Choice of Basis Functions

The basis functions, \( \phi \), were defined for the identification as follows: note that subscript \( P \) denotes contact at the pelvis and subscript \( K \) denotes contact at the knee.

For the elastic force representation, \( \phi_{ek}(\Delta_P) \) and \( \phi_{ek}(\Delta_K) \), were chosen as piecewise linear functions, given by
where $x_k$ is a set of break points. Values of the dependent variables $F_{e2}$ and $F_{e3}$, at the break points are given by

$$F_{e2}(i) = \sum_{k=1}^{n_1} a_{e2k} \Phi_{e2}(\Delta_p(i)) \quad i=1,...,n_1 \tag{3.4.13}$$

$$F_{e3}(i) = \sum_{k=1}^{n_2} a_{e3k} \Phi_{e3}(\Delta_k(i)) \quad i=1,...,n_2$$

The break points were chosen to be at (0, 0.5, 1.0, 1.5, 2.0, ... )mm, and $n_p$ and $n_k$ chosen sufficiently large so as to represent the maximum penetration depth. These break points, chosen for the identification, differ to the break points used for the test model. This makes an exact solution impossible, and introduces a low level of 'noise' into the identification problem.

The functions representing the damping force, friction force and joint friction torque, $\phi_{dP}(V_{nP})$, $\phi_{dK}(V_{nK})$, $\phi_{fP}(R_P)$, $\phi_{fK}(R_K)$ and $\phi_{mk}(\dot{\alpha})$, were chosen to be simple linear functions, that is:

$$\phi_{dP} = V_{nP}$$
$$\phi_{dK} = V_{nK}$$
$$\phi_{fP} = R_P$$
$$\phi_{fK} = R_K$$
$$\phi_{mk} = \dot{\alpha} \tag{3.4.14}$$

3.4.3 Correlation of Identified Model and Test Model

The output data from each of the frontal impact simulations, A, B and C, (see Table 2.3.1) were used independently, for the identification of the system parameters. The linear least squares problem formulated above was solved via SVD, and Table 3.4.1 shows the resulting parameter estimates for each of the test models, together with the prescribed damping, friction and friction torque values, as used in the test models.
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Figure 3.4.1 shows comparisons of the prescribed elastic force/displacement characteristic with those derived from the identified stiffness parameters. It can be deduced from these graphs that despite the choice of break points the identified characteristics closely resemble the prescribed characteristics, except where the data is sparse or non-existent - for example in test A, the pelvis has a penetration depth exceeding 2mm for the majority of the simulation.

Estimates of the system parameters were used to create the fitted models of each test. Figures 3.4.2 - 3.4.4 show the comparisons between fitted model and test model trajectories, for Tests A, B and C; it is clear that the fitted models give trajectories which closely fit the test model trajectories. The occasional loss of accuracy in the contact force data has not seriously upset the predicted kinematics.

<table>
<thead>
<tr>
<th>Penetration (m)</th>
<th>Pelvis Stiffness (N)</th>
<th>Knee Stiffness (N)</th>
<th>Cdₚ/Cdₛ (Nm/s)</th>
<th>μₛ/μₓ</th>
<th>Joint Torque (N)</th>
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<td>0.0</td>
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<td>Identified Values (Test A)</td>
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</tr>
<tr>
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<td></td>
<td></td>
</tr>
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<td>Identified Values (Test B)</td>
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<td>Identified Values (Test C)</td>
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<td>788.25</td>
<td>671.63</td>
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Table 3.4.1 Prescribed and Identified Parameter Values
Figure 3.4.1 Comparisons of Prescribed and Estimated Contact Stiffness Functions
Figure 3.4.2 Correlation of Fitted Model and Test Model Results for Test A.
Figure 3.4.3 Correlation of Fitted Model and Test Model Results for Test B.
Figure 3.4.4 Correlation of Fitted Model and Test Model Results for Test C.
3.5 Identification via Non-linear Least Squares

3.5.1 Modified Hybrid II Model in Frontal Impact

As for the earlier TNO10 2-body model, the aim is to generate a fitted model of the system directly from test model output data. For the (modified) Hybrid II model described in Chapter 2, the aim is again to identify the contact interaction data and also the lap belt characteristics. Values for joint torques, masses and moments of inertia were taken from a validated dummy database (TNO, 1992(b)). Appendix B contains the model build commands for both the MADYMO and AUTOSIM Hybrid II models.

Unlike the explicit analytical formulation of the 2-body model, the Hybrid II model is described in the multibody software code AUTOSIM and the equations of motion are generated in AUTOSIM format. The equations of motion are defined in such a way that makes manipulation of the equations less simple than for the earlier model. For the Hybrid II model we can still write each equation of motion in the following form, which is explicitly linear in the contact force variables:

\[
\dot{Y} = C_1 F_{x_2} + C_2 F_{x_2} + C_3 F_{d_3} + C_4 F_{a_3} + C_5 F_{t_4} + C_6 F_n + C_7 F_{\beta} + C_8 F_{ex} + C_9 F_{by} + C_{10}
\]

(3.5.1)

The coefficients \(C_i\) can be found as explicit functions of system displacements, velocities, inertias etc. via symbolic differentiation. However, the expressions are extremely complicated, being optimised (in AUTOSIM) for numerical computation speed. Hence it is more straightforward to calculate \(C_i(t_j)\) numerically by 'interrogating' the numerical subroutine for \(\dot{Y}\). Thus at any time instant \(t_j\), formally set all forces to zero, to evaluate \(\dot{Y} = C_{10}\). Next set \(F_{x_2} = 1\) with all other forces still set to zero and evaluate \(\dot{Y} = C_1 + C_{10}\) which therefore yields the value \(C_1\). Continuing in this fashion with the other force variables allows all the remaining coefficients \(C_i\) to be determined.

We then seek to determine the unknown functions, \(F_{x_2}, F_\phi, F_f\) and \(F_b\). Once again, piecewise linear basis functions are used and the formulation is the same as in Section 3.4.2. The belt force is represented as a function of the relative elongation, \(\delta l_\phi\), and is defined, in terms of basis functions, as:

\[
F_b = \sum_{k=1}^{n_b} a_{bk} \phi_{bk}(\delta l_\phi)
\]

(3.5.2)
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For a series of sampled time points $t_i$, each equation of motion may be expressed as a set of simultaneous non-linear equations in the unknown coefficients $a$. Since the system to solve is non-linear in the unknown parameters (due to the frictional force terms), non-linear least squares techniques are implemented to solve for the set of unknown parameters $a$.

For the Hybrid II model there are 6 equations of motion to represent vertical, horizontal and rotational motion of the femur and rotation of the head, thorax and lower leg. With several break points for each stiffness characteristic there is a total of 14 parameters to identify. The data spans a time period of 0.2 seconds and is sampled every millisecond, giving 200 time points for each equation of motion.

3.5.2 Identification of Contact and Belt Characteristics

The Levenberg-Marquardt method, for solving these non-linear equations in a least squares sense, was adopted. The structure of the program used for finding the optimal parameter values is given in Appendix C.

The mass centre displacement, velocity and acceleration time history data and rotational motion of each rigid body, generated from the simulation of the modified Hybrid II model in frontal impact, was used as the 'test data', for the identification of the optimal parameter values. Since the model used for the identification is the same as the test model, in this case even the break points are identical, we would expect the identified parameters to be very good estimates of the prescribed values. However we must be aware that in the case of non-linear least squares the optimiser may potentially find local minima, and hence the identification represents an early test of the numerical procedure.

To ensure that the optimiser is converging, one should check that $\beta=0$ (equation (3.3.8)). If $\beta=0$, then a minimum has been reached, whether it be a local minimum or a global minimum. If $\beta \neq 0$, the numerical routine has failed to converge.

Another point to be aware of, is the case when a part of a piecewise linear function is not tested, that is there is no data available in that interval, which will lead to a zero element on the diagonal of the Jacobian matrix and therefore matrix inversion in equation (3.3.14) is impossible. This problem can be addressed in two ways. We can decide not to identify the parameter causing the problem and hence remove it from the identification routine, or we can employ a mathematical technique to remove singularity. The first approach is simpler and is adopted temporarily here.
3.5.3 Correlation of Identified Model and Test Model

The identification routine was run several times, using the test model output data, with different initial guesses for the unknown parameters. The optimiser converged to the known solution each time, therefore confirming the convergence to a global minimum. Table 3.5.1 rather trivially shows the resulting extracted parameter values along with the true values as used in the test model. In view of the exact fit, simulated comparisons are clearly unnecessary here.

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>( \Delta )</th>
<th>( F_2 ) (N)</th>
<th>( F_3 ) (N)</th>
<th>( F_4 ) (N)</th>
<th>( C_{d2}/C_{d3} ) (Nm/s)</th>
<th>( \mu_2/\mu_3 )</th>
<th>( K_b ) (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prescribed Values</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td>0.4</td>
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</tr>
<tr>
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<td>500.0</td>
<td>500.0</td>
<td>500.0</td>
<td>100.0</td>
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</tr>
<tr>
<td>0.050</td>
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<td>800.0</td>
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<td>2900.0</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Identified Values</td>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>100.0</td>
<td>0.4</td>
<td>70000</td>
</tr>
<tr>
<td>0.025</td>
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<td>100.0</td>
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<tr>
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<tr>
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<td>2900.0</td>
<td>-</td>
<td>100.0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5.1 Prescribed and Identified Parameter Values

3.5.4 Weighting Parameter Solution to the Singular Matrix Problem

As seen in Table 3.5.1, the last parameter of the pelvis/seat contact stiffness and the last two parameters of the thorax/seat back contact stiffness, were not identified. As mentioned, the reason for this was lack of data in those regions of the stiffness characteristic, hence these parameters were omitted from the identification routine. If these parameters are included, the optimisation fails through singular matrix inversion. The approach proposed in this section avoids this failure of the optimisation routine, and is therefore more appropriate for general application.

The numerical problem is to minimise the error criterion in equation (3.3.2). If however the contribution to the sum from one of the parameters, \( a \), is zero, this will cause a zero element on the diagonal of the Jacobian matrix. To avoid this situation consider minimising the following function:
where $a_i$ represents a pre-assigned value for the parameter and $W$ represents a positive weighting parameter. We must assign a value to this weighting parameter, but even the smallest value will avoid a singular matrix problem provided that it is within the floating point accuracy of the computer being used. To verify the use of this weighting parameter, all four parts of the pelvis/seat contact interaction were now included in the model. Including the weighting parameter in the optimisation and hence defining the function given in equation (3.5.4), the pelvis/seat contact stiffness was accurately identified.

3.6 Discussion of Identification Results

In an ideal case scenario the non-linear identification procedure accurately extracts the relevant system parameters for the modified Hybrid II model. These optimal values are achieved from various initial value guesses for the parameters and we are therefore confident of numerical convergence in this ideal case.

The use of the weighting parameter has proved beneficial in that we may pre-assign values to parameters, which we could otherwise not identify, without having any significant effect on the identified system parameters. The weighting matrix at the same time solves the problem of occurrence of a singular matrix.

This technique may be useful when identifying from experimental data. For example we may see a contact in reality but the identification model does not (generally because of simplifications in the model structure, for example differences in geometry between the mathematical model and the physical model). If we assumed there was a contact in the identification model and did not apply the weighting matrix, we would find the singular matrix problem occurring and could not identify the system parameters. If we assumed no contact, then the errors in the model would increase and the identified parameters would most likely be poorly conditioned. In applying the weighting parameter we can assume there is a contact and pre-assign sensible values for those contact parameters, hence obtaining optimal parameter values.

It is no surprise that we are able to extract accurate values for the system parameters, when the identification model is identical to the mathematical model, and the identification input data has come directly from the mathematical model. We would not expect the presence of errors, except the possibility of round-off errors.
The occurrence of local minima was not observed in the ideal case identification, although the range of initial value guesses for the parameter values was relatively small. The problem of local minima is encountered and discussed in the next chapter, when parameters are identified from simulated 'noisy' data.

Although results obtained using linear least squares methods proved to be accurate, the method is ad-hoc and not very general. For these reasons linear least squares methods are not included any further in this work.
### 4.1 Introduction

When using experimental data, rather than simulation data, for input into the identification process, the identified parameter values will be subject to error. Sources of error are many, including electrical noise, extraneous mechanical vibrations and other unmodelled dynamics.

Typical sources are as follows:

- Mounting of accelerometers on the dummy - errors in location and alignment of the transducer and flexibility in the mounting
- Accelerometer cross sensitivity
- Vibrations due to the impact sled pulse and brakes
- Post processing of the test data - for example round off errors
- Flexibility of the dummy which is not accounted for in the mathematical model

Standard test instrumentation is normally calibrated and is accurate to specified tolerances, and for standard experiments, the associated noise levels are negligible. However, for the purposes of identification, tighter tolerances on the signal noise and a more comprehensive data set may be required to ensure adequate estimation of the parameter values.

It would be impossible to quantify or model all the noise sources realistically. The objective of this chapter is to use simulation studies to gain some understanding of how errors in the experimental data might effect the identified parameter values. We concentrate on the identification of the contact parameters from the modified Hybrid II model described in Chapter 3. All relevant parameters are normalised for the identification procedure so that the reference value for each parameter estimate is 1.0. Definitions of the parameters, their physical value and their normalised value are given in Table 4.1.1.

The effect that different levels of noise have on the identification process is to be investigated. Two amplitudes of noise corruption are applied to the kinematic data obtained from the frontal impact simulation of the modified Hybrid II manikin. The noise corruption is obtained as follows:

\[ n_i(t_i) = 0.01 \ p \ r(t_i) \]
\[ n_h(t_i) = 0.025 \ p \ r(t_i) \]  \hspace{1cm} (4.1.1)

where \( i = 1, 2, \ldots, n \), for \( n \) time points. Here \( r(t_i) \) represents a vector of random numbers, generated using a pseudo-random number generator which is normally distributed with a mean of zero and standard deviation of 1.0. \( n_i \) and \( n_h \) refer to 'low' and 'high' noise cases respectively and \( p \) represents the peak value from the relevant time history.
### Identification in the Presence of Simulated Disturbances

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Physical Value</th>
<th>Normalised Value</th>
</tr>
</thead>
<tbody>
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Table 4.1.1 Parameter Assignment for Modified Hybrid II Identification

The exact kinematic data $y_e(t)$ is therefore corrupted in the following way:

$$y_k(t) = y_e(t) + n_l(t) \quad (lownoise) \quad (4.1.2)$$
Identification in the Presence of Simulated Disturbances

\[ y_{hc}(t) = y_0(t) + n_h(t) \quad \text{(highnoise)} \]  

(4.1.3)

Each of the velocity and acceleration time histories were subjected to this noise corruption before they were input into the identification procedure. Examples of the resulting noise corrupted signals are shown in Figure 4.1.1 (Note the clipped acceleration value on the measured data).

![Figure 4.1.1 Head Acceleration Data (a) measured (b) simulated - low noise (c) simulated - high noise](image)

The displacement data was not corrupted in the same way as high frequency variations are not usually present in experimentally measured data. Instead, quantisation was used to incorporate a small amount of error into the displacement data, rounding down to the nearest mm (for linear displacements) or $10^3$ rad (for angles):

\[ y_c(t) = \frac{\text{INT}(y_0(t) \times 1000))}{1000} \]  

(4.1.4)

Section 4.2 assesses the results of the non-linear least squares parameter identification with these two levels of noise corruption present. Identified parameter values are used to create fitted models, and the performance of these models is addressed in Section 4.3. Section 4.4 discusses the implication of these results. Section 4.5 then looks at an extended use of the weighting parameter defined in Chapter 3.
Identification in the Presence of Simulated Disturbances

4.2 Non-linear Least Squares Results

With the presence of errors in the identification input data, occurrence of local minima was anticipated. To increase the probability of finding the global minimum it was necessary to repeat the optimisation several times, changing the initial trial values for the unknown parameters each time. Fifty non-linear least squares optimisations were run and initial normalised parameter values were set using a random number generator, normally distributed with a mean of zero and a standard deviation of 1.0.

To eliminate the problems of singular matrix inversion, equation (3.5.4) was implemented in the non-linear least squares procedure, with a weighting parameter of $W=10^4$ and pre-assigned parameter values of $a_0=1.0$. Again, in order to be sure of convergence, the gradient of the cost function, $\beta$, of each optimisation was checked against zero on completion.

Consider the 'low noise' case. Figure 4.2.1 shows the values of the identified normalised parameter values for each of the 50 optimisations. The parameter values have been ranked in order of increasing final cost so that we can easily identify how many different 'minima' were found and how many times the optimiser converged to each of these 'minima'. Plot 16 of Figure 4.2.1 shows the cost function values $E$, which correspond to the identified parameters. Four different cost function values and corresponding parameter sets are observed.

The optimiser actually failed to converge, ($\beta \neq 0$), for 11 out of the 50 trial values. The choice of initial trial values for the unknown parameters thus seems to have a large effect on the speed of convergence and the converged solution. This non-convergence is a problem with the Levenberg-Marquardt method. However if one is prepared to run a reasonably large number of optimisations with varying initial trial values, the non-convergence effect is not a major problem. For the purposes of this work several optimisations were required in any case to reduce problems of local minima; hence the additional problems with non-convergence were not considered too significant.

It was observed that $\beta$ was zero in two of the four sets of 'solutions' and hence the optimiser had failed to converge to a minimum in two cases. Converged solutions were found with $E=439122$ and $E=445100$, that is for optimisation numbers 1-37 and 38-39 respectively. It is therefore reasonable to suppose that optimisation numbers 1-37 converge to a global minimum, with local minima being found by optimisation numbers 38 and 39. As can be seen in Figure 4.2.1 the parameter values associated with these two solutions are in some cases significantly different; hence the following section assesses the identified characteristics and corresponding fitted model simulation results.
The results for 'high noise' corrupted data demonstrate similar behaviour as the low noise optimisation. Figure 4.2.2 shows the identified parameter values and corresponding cost function values. The parameters are again ranked in order of increasing cost function. First observations indicate that four minima are found, however when the gradient, $\beta$, was examined the optimiser had converged in two out of the four cases. Optimisation numbers 10-13 ($E=1544070$) and 14-49 ($E=1689556$) resulted in converged solutions. In the high noise case the majority of optimisations converge to the same solution ($E=1689556$) which, unlike the low noise case, is not the smallest cost function value. The following section will identify criteria to decide which is the 'best' parameter set to use. The non-converged parameter values were not considered, in spite of the fact that lower costs and a relatively stable parameter set was obtained in optimisation numbers 1-9.
Figure 4.2.1 Low Noise - Normalised Parameter and Cost Function Values vs Ranked Optimisation Number
Figure 4.2.2 High Noise - Normalised Parameter and Cost Function Values vs Ranked Optimisation Number.
4.3 Performance of Fitted Models

For both levels of noise corruption, the optimiser converged to two different solutions. The corresponding normalised parameter sets were converted back into the characteristics required for the modified Hybrid II model - see Tables 4.3.1 and 4.3.2. The reference characteristics are those used in the original model of the modified Hybrid II to provide source data for the identification.

It can be seen that in both cases the identified parameter values lead to some unphysical characteristics. Negative stiffnesses for contact between the seat base and the femur and pelvis, are identified in all cases. A negative stiffness would give the effect of the dummy being 'sucked' into the seat. In some cases negative damping and friction values are also identified, hence putting energy into the system rather than taking it out. Identified values for the lap belt stiffness compare well with the reference value in every case, the worst estimate being 83% of the true value.

In spite of this, the four sets of identified characteristics shown in Tables 4.3.1 and 4.3.2 were used to recreate four modified Hybrid II models. Output from simulations of these models were compared with output from the corresponding reference model. Figures 4.3.1 and 4.3.2 show examples of these correlations (model L1 is created using the identified parameter set with the lowest cost function value, model L2 is created using the second identified parameter set).

Despite the unphysical properties displayed by the identified parameters, the fitted model outputs correlate reasonably well with the original model results. In the low noise case, model L1 gives better correlation with the original model than model L2. In the high noise case, however, output from model H1 does not correlate as well as the output from model H2. The majority of optimisations, however, did converge to the solution providing model H2 and this may in some way explain its better performance.

Inspection of Tables 4.3.1 and 4.3.2 and Figures 4.3.1 and 4.3.2 highlights that higher noise levels give less satisfactory parameter values and fitted model performance. This is fairly obvious; however as mentioned in the Introduction, the noise levels used to corrupt the simulation data were not particularly high when compared to those expected with experimental data. It is therefore anticipated that parameters identified from experimental data may also have unphysical values, and the fitted models created may give kinematics which deviate significantly from the experimental results. The identification method therefore requires further development in order to improve correlation and, more critically, to eliminate the problem of unphysical parameter values.
Identification in the Presence of Simulated Disturbances

<table>
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<tr>
<th>Parameter Values</th>
<th>(\Delta) (m)</th>
<th>(F_{Fe2}) (N)</th>
<th>(F_{Fe3}) (N)</th>
<th>(F_{Fe4}) (N)</th>
<th>(Cd_2/Cd_3) (Nm/s)</th>
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Table 4.3.1 Low Noise Corruption - Identified Parameters

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Table 4.3.2 High Noise Corruption - Identified Parameters
Figure 4.3.1 Low Noise - Fitted Model and Reference Model Time Histories
Figure 4.3.2 High Noise - Fitted Model and Reference Model Time Histories
Identification in the Presence of Simulated Disturbances

4.4 Discussion

The above results have highlighted several issues:

- **Occurrence of local minima**
  In both cases of high and low noise the non-linear least squares optimisation found more than one converged minimum value, this was not a problem when the optimisation input data was largely free from corruption.

- **Unphysical identified parameter values**
  Negative stiffness, damping and friction characteristics were identified on several occasions. These non-physical parameters were experienced by Radwan and Hollowell (1990), however in their case the non-physical identified load paths did seriously affect the match between the model and the actual crash data.

- **Good fitted models despite poor parameter values**
  Despite the unphysical characteristics identified by the optimisation procedure, simulations of the fitted models created from these unphysical characteristics can give acceptable output correlations with the original model output data. However unphysical parameters are bound to cause problems for different initial conditions or dummy restraint system optimisation, we would not expect to obtain such accurate fitted models.

- **Severe loss of accuracy at moderately high noise levels**
  Fitted models identified from the low noise corrupted data gave better correlation with the original model results than the fitted models identified from the high noise corrupted data.

The main question now is how to reduce the possibility of identifying unphysical parameter values. One possibility is to apply constraints on the parameter values and use a constrained non-linear least squares technique to identify the optimal parameter values. Applying constraints would place limits on the optimisation procedure and may increase the computing time required to find the optimal solution. Another possible approach is to extend the implementation of equation (3.5.4). Instead of using a very small value for the weighting parameter $W$, we can look for a value of $W$ which, when implemented, provides physical parameter values and accurate fitted models. However great care is needed to avoid simply imposing some pre-assigned parameter set in preference to using the measured data.
4.5 Extended Implementation of the Weighting Parameter

By including the weighting parameter we minimise the following function:

$$E(a) = \sum_{i=1}^{N} [y_i - y(x_i; a)]^2 + \sum_{i=1}^{N_p} W_i \left( \frac{a_{ip} - a_i}{a_{ip}} \right)^2$$  \hspace{1cm} (4.4.1)

this may be written

$$E = E_{dat} + W E_{par}$$  \hspace{1cm} (4.4.2)

$E_{dat}$ represents error due to the input data and $E_{par}$ represents "error" due to the parameter value deviating from the pre-assigned value.

It is useful to analyse the effects of increasing $W$ on $E_{dat}$ and $E_{par}$. Note that there is no assumption that the pre-assigned values are actually correct. Assume that $a_0$ minimises $E$ at $W_0$, $a_1$ minimises $E$ at $W_1$ etc., then the following inequalities apply:

$$E_{dat}(a_0) + W_0 E_{par}(a_0) \leq E_{dat}(a_1) + W_0 E_{par}(a_1)$$  \hspace{1cm} (4.4.3a)

$$E_{dat}(a_1) + W_1 E_{par}(a_1) \leq E_{dat}(a_0) + W_1 E_{par}(a_0)$$  \hspace{1cm} (4.4.3b)

Hence

$$E_{dat}(a_0) - E_{dat}(a_1) \leq W_0 (E_{par}(a_1) - E_{par}(a_0))$$  \hspace{1cm} (4.4.4a)

$$E_{dat}(a_0) - E_{dat}(a_1) \geq W_1 (E_{par}(a_1) - E_{par}(a_0))$$  \hspace{1cm} (4.4.4b)

So

$$W_0 (E_{par}(a_1) - E_{par}(a_0)) \geq W_1 (E_{par}(a_1) - E_{par}(a_0))$$  \hspace{1cm} (4.4.5)

hence

$$(W_1 - W_0)(E_{par}(a_1) - E_{par}(a_0)) \leq 0$$  \hspace{1cm} (4.4.6)
Identification in the Presence of Simulated Disturbances

Assuming now that \( W_t \geq W_o \), this implies

\[
E_{\text{par}}(a_1) - E_{\text{par}}(a_0) \leq 0
\]  

(4.4.7)

We conclude that \( E_{\text{par}} \) is a monotonically non-increasing function of \( W \), provided the parameter sets are globally optimal.

Similarly for \( E_{\text{dat}} \), consider equation (4.4.4):

\[
\frac{E_{\text{dat}}(a_o) - E_{\text{dat}}(a_1)}{W_0} \leq \frac{E_{\text{dat}}(a_o) - E_{\text{dat}}(a_1)}{W_1}
\]  

(4.4.8)

Giving

\[
\left( \frac{1}{W_0} - \frac{1}{W_1} \right) (E_{\text{dat}}(a_o) - E_{\text{dat}}(a_1)) \leq 0
\]  

(4.4.9)

So if again we assume \( W_o \leq W_t \)

then

\[
\frac{1}{W_0} \geq \frac{1}{W_1}
\]  

(4.4.10)

and

\[
E_{\text{dat}}(a_1) \geq E_{\text{dat}}(a_0)
\]  

(4.4.11)

and \( E_{\text{dat}} \) must therefore be monotonically non-decreasing with respect to \( W \).

Hence if we consider a range of weighting parameter values, as \( W \) increases \( E_{\text{par}} \) should tend towards zero and \( E_{\text{dat}} \) should increase. Any deviation from monotonicity implies sub-optimal fit i.e. the non-linear least squares procedure has not converged to a global minimum.

We now turn to the problem of choosing a suitable value for \( W \). If it is too small \( E_{\text{par}} \) will be large and unphysical parameters will result (as in Section 4.3). If \( W \) is too large, the test data will be ignored, in preference of the somewhat arbitrary pre-
assigned parameters. We therefore look for a ‘transition zone’ in $W$, where the value given by $E_{\text{par}}$ has decreased by a relatively large amount and the $E_{\text{dat}}$ term is still relatively small. In order to find such a transition zone, the optimisation must be run over a range of weighting values.

We can now implement equation (4.4.1) and identify possible transition weightings for the identification of the Hybrid II model for both cases of low and high levels of signal disturbance.

$W$ was set to vary from $10^{-10}$ to $10^{10}$, increasing by powers of ten and the pre-assigned parameter values were set to 1.0. As before 50 runs of the non-linear least squares optimisation were executed for each value of $W$. Checks for convergence and local minima were made in each optimisation. The parameter set with the lowest converged cost from the 50 optimisations was noted for each value of weighting parameter.

Consider the low noise case; Figure 4.5.1 shows how the terms $E_{\text{par}}$ and $E_{\text{dat}}$ vary with the weighting parameter. Figure 4.5.2 shows the corresponding identified parameters. As expected very small values of $W$ have little or no effect on the identified parameters or the cost function. As $W$ increases a large relative change in the parameters is observed, $E_{\text{dat}}$ begins to increase and $E_{\text{par}}$ begins to decrease. As $W$ becomes very large, the parameters are pushed to the pre-assigned values, $E_{\text{dat}}$ increases and $E_{\text{par}}$ tends to zero.

We are looking for a value of $W$, which gives physically realistic parameter values, a small relative change in $E_{\text{dat}}$ and a large relative change in $E_{\text{par}}$. From Figures 4.5.1 and 4.5.2, a transition parameter of $W=10^6$, seems reasonable. Table 4.5.1 shows the parameter values identified with this transition weighting, model LT1, along with the reference values.

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Table 4.5.1 Parameters for Reference model and model LT1
Identification in the Presence of Simulated Disturbances

Compared with earlier results (Table 4.3.1), the identified characteristics no longer display unphysical behaviour and in some cases they compare very well with the original model characteristics.

The transition parameter values were used to create a fitted model of the modified Hybrid II. Example simulation results from model LT1, model L1 and the reference model are shown in Figure 4.5.3. As expected, model LT1 gives much better correlation with the reference model than model L1.

It could be argued that because the pre-assigned parameter values are set to the reference parameter values, it is no surprise that, as the weighting parameter increases and the unknown parameters are pushed towards their reference values, the transition weighting gives improved identified characteristics and hence improved fitted model results. To avoid this criticism the non-linear least squares optimisation was repeated with the pre-assigned normalised parameter values set to $a_\varphi=2.0$.

Figures 4.5.4 and 4.5.5 show the variation of error with the weighting parameter and the corresponding identified parameters when pre-assigned parameter values of 2.0 are used. Again a transition weighting of $W=10^4$, was chosen. Table 4.5.2 shows the parameters identified with the transition weighting, model LT2, and the reference model values. Improvement in the identified parameters is again observed. These transition parameters were again used to create a fitted model of the Modified Hybrid II and Figure 4.5.6 shows example correlations of the model LT2, model L1 and the reference model simulation results. The transition model simulation results compare very well with the reference model results for all displacement, acceleration and velocity time histories.

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Table 4.5.2 Parameters Identified in the Low Noise Case - Pre-assigned Values 2.0, Transition Weighting
Identification in the Presence of Simulated Disturbances

A similar story is true with the case of high noise corruption. A transition weighting of $W=10^4$ is again chosen for both pre-assigned values of 1.0 and 2.0. Table 4.5.3 shows the characteristics identified with this transition weighting for pre-assigned values of 1.0 (model HT1) and 2.0 (model HT2). The transition model simulation results correlate well with the reference model results, in much the same way as with the low noise corruption.

<table>
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<th>$\Delta$ (m)</th>
<th>$F_{e1}$ (N)</th>
<th>$F_{e2}$ (N)</th>
<th>$F_{e3}$ (N)</th>
<th>$F_{e4}$ (N)</th>
<th>$C_{d2}/C_{d3}$ (Nm/s)</th>
<th>$\mu_2/\mu_3$</th>
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Table 4.5.3 Parameters Identified in the High Noise Case - Transition Weighting.

Having obtained the optimal parameter sets for each of the weighting parameters it is interesting to consider how critical the actual transition value, $W_T$, is. We expect that if the transition weighting values are too high then the 'simulation error' would increase. We therefore want to check that the transition values identified from observations of $E_{dat}$ and $E_{par}$, conform to transition values based on simulation. Ten fitted models were created using the identified characteristics from optimisations with high noise corrupted data and pre-assigned parameters of $a_p=2.0$, for weighting parameters of $W=1$ to $W=10^4$, model HW1-HW11. Simulation results from these fitted models and from the reference model, were used to calculate the 'simulation error' between fitted model results and reference model results at each of the weighting values. The dynamic simulation error was calculated in the following way:

$$e = \sum (y_{orig} - y_{fitted})^2$$ (4.5.1)

Where $y$ represents the relevant time history signal, for example femur vertical displacement.
Figure 4.5.7 shows how the simulation error varies with increasing weighting parameter for models HW1-HW11. It is observed that for low values of $W$ there is a relatively large fitting error, as $W$ increases the fitting error decreases to a minimum, at $W=10^5$, as $W$ increases further the fitting error begins to increase again. Choice of transition weighting based on $E_{dat}$ and $E_{par}$ observations led to a value of $W=10^4$, but we could have quite easily have chosen $W=10^5$ which would have conformed with the results shown in Figure 4.5.7. It is feasible in practice to use this method to determine $W_T$ as the simulations are relatively cheap in cpu time.
Figure 4.5.1 Variation of $E_{\text{dat}}$ and $E_{\text{par}}$ with increasing Weighting Parameter, $W$. Low Noise Case. $a_0=1.0$
Figure 4.5.2 Variation of Identified Parameters with increasing Weighting Parameter, $W$. Low Noise Case. $a_p = 1.0$
Figure 4.5.3 Correlation of Transition, Fitted and Reference Model Results Low Noise Case.
Figure 4.5.4 Variation of $E_{dat}$ and $E_{par}$ with increasing Weighting Parameter, $W$. Low Noise Case. $a_{fp}=2.0$
Figure 4.5.5 Variation of Identified Parameters with increasing Weighting Parameter, $W$. Low Noise Case. $a_p=2.0$
Figure 4.5.6 Correlation of Transition, Fitted and Reference Model Results
Figure 4.5.7 Fitting Error of models HW1 - HW11 for Increasing W
4.6 Chapter Summary and Conclusions

Corrupting the kinematic data with random noise has a significant effect on the identified parameter values. Direct use of non-linear least squares fitting results in the identification of unphysical parameters in all cases of noise corruption. The fitted models created using these poor parameter estimates do however give generally good correlation with the original model output data.

The use of a transition weighting parameter, to improve the values of the estimated parameters, has proved successful. The use of constrained least squares techniques was considered, to force the parameters to 'realistic values'. Constraining the parameters would however slow down the optimisation procedures. The use of the transition weighting has the added advantage of providing a check that physically sensible parameter values can be achieved within the transition zone.

It was simple to define the pre-assigned parameter values for the theoretical situation, even incorrect ones, since we know what the 'true' parameter values are; this is less simple when we move to experimental data, when the parameters are unknown. However the realistic aim is to make best use of the available information to assist with the modelling but to recognise that additional a priori knowledge may need to be composed. In this case engineering judgement is required to define 'sensible values' for the pre-assigned parameter values. It has been seen in the above that accurate pre-assigned values are not necessary to find a sensible transition weighting and provide accurate fitted model results. Figure 4.5.7 also demonstrates that if $W$ is too large the simulation error may be large.
# Chapter 5

Complex Model Identification

<table>
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<th>Section</th>
<th>Title</th>
<th>Page</th>
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<tbody>
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<td>85</td>
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<td>5.2</td>
<td>Baseline Results using Transition Weightings</td>
<td>89</td>
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<tr>
<td>5.3</td>
<td>Approximated Contact Interactions</td>
<td>95</td>
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<td>Sensitivity to Choice of Pre-assigned Parameter Values</td>
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Complex Model Identification

5.1 Introduction

Crash test dummies are physical models which are built in such a manner that, under impact conditions, they represent human behaviour. However, it is not possible for a crash dummy to fully mimic human behaviour. Reasons for this are many, including inability to model muscle extension and flexion, differences in joint articulation, inability to model reflex actions. Hence the behaviour of a crash dummy is only an approximation of human response. Similarly, mathematical models of the crash dummy do not exactly mimic the physical model, for example the dummy flexibility is not usually included in the mathematical model and complex interactions between the dummy and its environment are often simplified in the mathematical formulation.

In Chapters 3 and 4 the structure of the model assumed for the identification process exactly matched the structure of the model from which the identification source data was obtained. The aim of this chapter is to investigate the effect that simplified model structure has on the identification process.

To achieve this a model which is ‘more complex’ than the modified Hybrid II model, was required to provide the source data for the identification. The identification process would assume, as in Chapters 3 and 4, the less complex modified Hybrid II model structure. This enables one to assess the effect of a simplified model structure on the success of the identification procedure.

The ‘Complex Model’ is shown in Figure 5.1.1. It has more degrees of freedom and more contact interactions than the modified Hybrid II model, with the spine, pelvis and neck defined as individual rigid bodies, giving seven degrees of freedom for 2 dimensional motion. Figure 5.1.2 shows the contact interactions with the vehicle seat.

In order to highlight the differences between the modified Hybrid II model, 4-body model, and the Complex Model Figure 5.1.3 shows correlation of simulation results for the two models when subjected to identical inputs. The simplified model structure gives noticeable differences when compared to the Complex Model. Using Complex Model output data for will therefore introduce significant errors in the identification.

As in Chapter 4 all the parameters we wish to identify are conveniently normalised to unity - see Table 5.1.1. Note that these occur as exact ‘physical’ parameters in the complex test model; in the simplified identification model no ‘exact’ values exist.
Figure 5.1.1 Complex Model

Figure 5.1.2 Contact Interactions of the Complex Model
Figure 5.1.3 Complex Model and 4-Body Model Displacement Time Histories
# Complex Model Identification

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<th>Normalised Value</th>
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<td>Seat Belt Stiffness</td>
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Table 5.1.1 Parameter Assignment for the Complex Model
5.2 Baseline Results using Transition Weightings

In this section we implement equation (4.4.1) in the hope of finding a transition weighting and pre-assigned parameter values, which will then lead to the identification of realistic parameter values and accurate fitted models.

As executed in Chapter 4, the weighting parameter $W$ was set to vary from $10^{-10}$ to $10^{10}$. Pre-assigned values of 1.0 and 2.0 were chosen. Once again to ensure convergence and to reduce problems with local minima, 50 runs of the non-linear least squares optimisation, were executed for each of the 20 weighting parameter values. Again the optimal sets of identified parameters from the 50 optimisations were noted for each value of the weighting parameter.

From Figures 5.2.1 to 5.2.4, a transition weighting of $W=10^6$ was chosen for both pre-assigned values ($a_p=1.0$ and $a_p=2.0$). Table 5.2.1 shows the transition model parameters for pre-assigned values of 1.0 and 2.0 (model T1 and model T2 respectively). Table 5.2.1 shows that there is no indication of unphysical parameter values, the seat stiffnesses seem to have quite high values, but generally the identified parameters seem to be sensible. This indicates that the optimisation method with the transition weightings still works in the case of a simplified model structure.

The characteristics shown in Table 5.2.1 were used to create transition fitted models to correlate with the Complex Model. Figure 5.2.5 shows angular acceleration and displacement correlations of the Complex Model with model T1 and model T2. Both transition models correlate reasonably well with the Complex model, confirming the chosen transition value of the weighting parameter.

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<th>$\mu_2/\mu_3$</th>
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Table 5.2.1 Transition Weighting - Identified Characteristics
Figure 5.2.1 Variation of Error with Weighting Parameter, W. For the case $a_0 = 1.0$
Figure 5.2.2 Variation of Identified Parameters with Weighting Parameter, $W$. For the case $\alpha_p = 1.0$
Figure 5.2.3 Variation of Error with Weighting Parameter, $W$. For the case $\alpha_p = 2.0$
Figure 5.2.4 Variation of Identified Parameters with Weighting Parameter, $W$. For the case $a_p=2.0$
Figure 5.2.5 Transition Fitted Model Correlations
5.3 Approximated Contact Interactions

In the case of experimental data, acceleration, velocity and displacement time history data of the dummy mass centres, are likely to be available for the identification procedure. Accurate information on contact penetration depths is less likely to be available and may be calculated using mass centre displacement and dummy geometry. It was considered important to investigate how this calculation of the contact interaction data effects the non-linear least squares results. If the effects cause excessive deterioration of the identified model, it might be concluded that localised instrumentation will be required for direct measurement of contact deflection. Optimisations were therefore run with this approximated penetration data (calculated from mass centre positions and rigid body geometry of the simplified model). Figure 5.3.1 shows correlation of the accurate and approximated penetration data, for the three contact interactions: seat/pelvis, seat/femur and seat back/thorax. The contact interactions between the thorax and the seat back are virtually identical, however this is not the case for the femur/seat and pelvis/seat contact interactions, by the end of the simulation the calculated penetrations have deviated significantly from the accurate penetrations.

![Figure 5.3.1 Correlation of Contact Penetration Depths](image-url)
Complex Model Identification

Figures 5.3.2 and 5.3.3 show results relating to the variation of error terms and the identified parameters with respect to the weighting parameter, for pre-assigned values of 2.0, when using this approximated contact interaction data for input into the identification procedure. Observations from Figures 5.3.2 and 5.3.3 lead to the choice of $10^4$ for the transition weighting value.

Table 5.3.1 shows the transition model parameters identified with pre-assigned values of 1.0 and 2.0, model TA1 and model TA2 respectively. It is interesting to note that although the friction co-efficient for model TA1 has a negative value, generally the characteristics are very similar to those identified with the accurate penetration data (see Table 5.2.1). Characteristics shown in Table 5.3.1 were used to create transition fitted models to compare with the Complex model. Figure 5.3.4 shows the angular acceleration and displacement correlations of the Complex model with these transition models, models TA1 and TA2. As with the case of accurate penetration data, models TA1 and TA2 show reasonable correlation with the Complex model, although in some cases they are less accurate than models T1 and T2.

### Table 5.3.1 Transition Weighting - Identified Characteristics

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<td>0.0</td>
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<td>-</td>
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</table>

Despite the significant differences between the accurate and calculated penetrations, there seems to be relatively good correlation of the resulting fitted models when using the calculated penetration data. This leads to the conclusion there is no overwhelming need for localised measurement of contact penetration at least when sensor noise is negligible.
Figure 5.3.2 Variation of Error with Weighting Parameter. Case of Calculated Data
Figure 5.3.3 Variation of Identified Parameters with Weighting Parameter, W. Case of Calculated Data
Figure 5.3.4  Transition Fitted Model Correlations
5.4 Integrated Least Squares

The fitted models identified in the previous section in some cases show poor correlation with the Complex Model results. This is the first time that such correlations are observed when the transition weighting parameter is included in the identification. It was decided that a different error criterion should be implemented to try to improve the fitted model performance. The error criterion chosen was an integrated least squares criterion:

\[
E(a) = \sum_{i=1}^{N} \left( y_i - \int y(x_i; a) \right)^2
\]  

Equation (5.4.1) was applied to the case of calculated data, as in the previous section. Optimisations were run with a low weighting and also with increasing weighting to find the transition value, \( W_T \), with pre-assigned values of 1.0. For the case of low weighting, two minima were found and the resulting identified characteristics are shown in Table 5.4.1. A transition weighting of \( W_T = 10^6 \) was chosen, the parameters identified with this weighting are also shown in Table 5.4.1. The parameters identified with the transition weighting are a vast improvement on the parameters identified with a low weighting, however the belt stiffness characteristic is still negative.

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>( \Delta ) (m)</th>
<th>( F_{e2} ) (N)</th>
<th>( F_{e3} ) (N)</th>
<th>( F_{e4} ) (N)</th>
<th>( C_{d2}/C_{d3} ) (Nm/s)</th>
<th>( \mu_2/\mu_3 )</th>
<th>( K_b ) (N)</th>
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<td>995.0</td>
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</tr>
</tbody>
</table>

Table 5.4.1 Parameters Identified Using Integrated Least Squares
The characteristics shown in Table 5.4.1 were used to create fitted models to compare with the Complex Model: Figure 5.4.1 shows resulting model correlations. Although the transition characteristics appear to be more realistic the transition fitted model gives worse correlation with the Complex model than the low weighting fitted models. When compared with the correlation seen in Figure 5.3.4 the models identified with the integrated least squares criterion give worse results.

Figure 5.4.1 Fitted Model Correlations, Integrated Least Squares.
Complex Model Identification

5.5 Sensitivity to Choice of Pre-assigned Parameter Values

As observed in the identification from noise corrupted data, implementation of the weighting parameter improves both the identified parameter values and the fitted model correlations. However one must rely on using engineering judgement to (a) set the pre-assigned parameter values and (b) to choose the 'correct' transition weighting. Ideally there would be a transition weighting which gives similar identified parameter values for a range of pre-assigned parameter values.

In the case of identification of the Complex Model, for the accurate penetration data, four pre-assigned values were used to observe how the identified parameters vary with different pre-assigned values. Figure 5.5.1 shows the correlations of the variation of identified parameters with the weighting parameter for different pre-assigned values. There is a weighting value at which all pre-assigned values give approximately the same set of identified parameter values. This weighting parameter, $W=10^6$, can therefore be used as the transition weighting parameter. This value coincides well with the earlier choice, made on the basis of the variation of error terms with increasing weighting value. This result strengthens the hypothesis that transition weightings can be chosen objectively in practice.
Figure 5.5.1a Correlation of Identified Parameters using Various $q_y$. 
5.6 Discussion

Chapter 5 has demonstrated that, when using the transition weighting value, the identification procedure can automatically extract acceptable fitted models despite errors due to simplification in model structure.

The case of approximated contact interaction data gave less accurate results than the case with accurate interaction data, with the optimisation still identifying negative values for the contact friction. This highlights the need for an accurate and comprehensive set of data from physical crash simulation.

In Chapters 4 and 5 we have looked at the effect of corrupted data and unmodelled effects on the identification procedure and resulting fitted models. We have been able to assess the effect of these simulated errors by comparing the reference parameters and models with the identified parameters and fitted models. In reality this method of assessment is not possible as we do not know what the reference values are. More importantly, the sources of error assumed have been quite specific, and a more general method of assessing the likely errors in parameters is required. The following section looks at several methods of error analysis and relates the results with those found in Chapter 4.
Chapter 6

Analysis of Errors in Identified Parameters

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6.1 Introduction

The analysis of the effect of errors in identification has so far depended on some very specific assumptions about errors in the test data. A more general analysis of the likely errors in the identified parameters is therefore desirable. This chapter looks at several methods for analysing the accuracy of the estimated parameters.

- Sensitivity analysis, introduced in Section 6.2, is used as a simple way of analysing the accuracy of the estimated parameters.

- Statistical methods outline how the covariance matrix is used to provide a measure of error on each of the parameter values. Section 6.3 defines the covariance matrix.

- Worst case analysis, that is the worst possible errors that can occur in the parameters under given error bands, is introduced in Section 6.4. Two measures of worst case error are discussed, the standard System Condition Number (SCN) and the Parameter Condition Number (PCN).

In Section 6.5 these methods are applied to parameters identified from frontal and rear impact simulation data. Both frontal and rear impacts are considered for two reasons:

- In frontal impact the contact interaction between the dummy and the seat back is of little significance compared with the same interaction in a rear impact. Hence the seat back contact parameters identified from the frontal impact simulation data would normally be less reliable than those extracted from rear impact data. The various error analysis techniques should highlight these measures of reliability in the identified parameter values.

- We can combine simulation data from the frontal and rear impact simulations to assess whether information from multiple tests improves the conditioning of the identified parameter values.

Thus each error analysis method is applied under three conditions: (i) frontal impact data (ii) rear impact data (iii) combined (frontal and rear impact) data.

The effect of corrupting poorly conditioned parameter estimates on the simulation results is considered in Section 6.5.2 and in Section 6.5.3 the transition weighting parameter is implemented to demonstrate its effect on the parameter condition number of each of the estimated parameters. Results are discussed and interpreted in view of results from Chapter 5, in Section 6.6.
6.2 Sensitivity Analysis

The simplest way to analyse how 'accurate' the estimated parameters $\hat{a}$, is to evaluate the increase in the cost function $E$ resulting from applying an artificial bias $\mu$ in any one parameter. This analysis involves no assumption regarding noise but simply tells us how important each parameter is to the optimisation. We would expect that a parameter, whose bias away from $\hat{a}$ has little effect on $E$ would be poorly estimated, compared to another parameter having a large effect on $E$.

Following on from Chapter 3.2, in the absence of bias

$$\hat{a} = (A^TA)^{-1}A^Ty$$  \hspace{1cm} (6.2.1)

If one parameter is artificially perturbed then

$$a_i = \hat{a}_i + \mu$$  \hspace{1cm} (6.2.2)

More generally

$$an = \hat{a}n + \mu$$  \hspace{1cm} (6.2.3)

where $n$ represents a unit vector.

Here it is assumed that the remaining part of the parameter vector is re-optimised to gauge the true effect of the imposed bias, the resulting sub-optimal parameter vector will be denoted $\tilde{a} = \tilde{a}(n, \mu)$.

We wish to minimise

$$E = |Aa - y|^2$$  \hspace{1cm} (6.2.4)

subject to equation (6.2.3). Equivalently we can introduce a Lagrange multiplier $\lambda$ and minimise

$$E_\lambda = |Aa - y|^2 - 2\lambda(a - \hat{a})n$$  \hspace{1cm} (6.2.5)

Differentiating, for $a = \hat{a}$ we have

$$\frac{\partial E_\lambda}{\partial a} = 2A^T(Aa - y) - 2\lambda n = 0$$  \hspace{1cm} (6.2.6)
Analysis of Errors in Identified Parameters

The inverse matrix (where it exists) $C = (A^T A)^{-1}$ is closely related to the probable uncertainties of the estimated parameter and is known as the covariance matrix - see Section 6.3.

So if we write

$$C = (A^T A)^{-1} \quad \text{and} \quad C_{nn} = n^T C n$$  \hspace{1cm} (6.2.7)

where $C_{nn}$ is a scalar. Substituting this into equation (6.2.6)

$$\bar{a} = C(A^T y + \lambda n)$$  \hspace{1cm} (6.2.8)

so

$$\bar{a} = \bar{a} + \lambda C n$$  \hspace{1cm} (6.2.9)

Substituting into equation (6.2.3) gives an expression for the Lagrange multiplier

$$\lambda n^T C n = \mu$$  \hspace{1cm} (6.2.10)

$$\lambda = C_{nn}^{-1} \mu$$  \hspace{1cm} (6.2.11)

Thus finally

$$\bar{a} = \bar{a} + \mu C_{nn}^{-1} C n$$  \hspace{1cm} (6.2.12)
Analysis of Errors in Identified Parameters

The corresponding increase in $E_*$ is given by

$$\Delta_n = |A(\bar{a} - a)|^2 = |\mu C_n^{-1} A C n|^2 = \mu^2 C_n^{-2} n^T C A^T A C n$$

(6.2.13)

Substituting in equation (6.2.7) this simplifies to

$$\Delta_n = \mu^2 C_n^{-1}$$

(6.2.14)

The tolerance in individual parameters can then be assessed by assuming a fixed uncertainty, $\Delta$, in $E$ and setting $n = e_i$ ($i = 1, 2, \ldots, n$):

$$\delta a_i = \sqrt{\Delta} \sqrt{C_{ii}}$$

(6.2.15)

Equation (6.2.14) can also be used to assess the effect of 'deleting' a parameter to determine its significance. Consider deletion of parameter $a_i$. Set $a_i = 0.0$ and the other parameters are then re-optimised. The cost increase due to that parameter deletion is given by

$$\Delta_i = \frac{\delta_i^2}{C_{ii}}$$

(6.2.16)

This provides a means of ranking the parameters in order of 'significance to the fit'. Note that $\Delta_i$ is sometimes called the 'regression sum of squares' for parameter $a_i$.

6.3 Statistical Analysis of Errors

The results given in equations (6.2.15) and (6.2.16) involve the covariance matrix $C$, which is more traditionally associated with the covariance of parameter estimation errors arising from least squares optimisation, in the case where measurement errors are assumed to be statistically independent and normally distributed [(Press et al, 1989), (Strang, 1980), (Norton J.P, 1986)].
Analysis of Errors in Identified Parameters

A brief derivation of this fact is now given. Consider the statistical model

$$\eta = A \xi \quad \text{and} \quad y = \eta + \epsilon$$

(6.3.1)

where $A$ is the matrix of basis functions, or regressors, and

- $\xi$ = exact model parameters
- $\eta$ = exact response for model
- $\epsilon$ = corruption in measurements
- $y$ = corrupted response
- $\epsilon_i \sim N(0, \sigma^2)$, independent, normally distributed with constant standard deviation $\sigma$

The analysis is simplified by using transformed co-ordinates, obtained from the SVD of $A$ (Section 3.2.2)

$$\epsilon = Ue' \quad \eta = U\eta' \quad \xi = V\xi'$$

(6.3.2)

Note that $\epsilon'$ has the same properties as $\epsilon$, that is $\epsilon' \sim N(0, \sigma^2)$.

Here $U$ and $V$ are left and right hand singular vectors from the singular value decomposition of $A$. Using the singular value decomposition of $A$ in equation (6.3.1) gives the following;

$$\eta' = S\xi'$$

(6.3.3)

Taking the first $n$ elements of the vector, gives

$$\eta_1' = D\xi'$$

(6.3.4)

where $D$ is the diagonal matrix of singular values. Note that in the new co-ordinates, equation (6.2.7) becomes

$$C' = D^{-2} \quad \text{and} \quad C'_{nn} = C_{nn} = n^T D^{-2} n'$$

(6.3.5)
Analysis of Errors in Identified Parameters

Here $C_m$ is a scalar, which is unchanged in the co-ordinate transformation.

The estimated model parameters $\hat{a}'$ are then given by

$$
\hat{a}' = D^{-1}y_i'
= D^{-1}(\eta_i' + \varepsilon_i')
= \xi' + D^{-1}\varepsilon_i'
$$

Therefore the expected values of the model parameters are

$$
E(\hat{a}') = \xi'
$$

Transforming back to original co-ordinates

$$
E(\hat{a}) = \xi
$$

i.e. the estimate is unbiased. The covariance of errors is given as

$$
COV(\hat{a}' - \xi) = COV(D^{-1}\varepsilon_i')
= D^{-1}E(\varepsilon_i'\varepsilon_i'^T)D^{-1}
= D^{-1}(\sigma^2 I)D^{-1}
= \sigma^2 D^{-2}
$$

Transforming back to the original co-ordinate system, equation (6.3.9) becomes

$$
COV(\hat{a} - \xi) = \sigma^2 V D^{-2} V^T
= \sigma^2 C
$$

The diagonal elements of $C$ are the variances (squared uncertainties) of the fitted parameters. The off-diagonal elements $C_{jk}$ are the covariances between parameters $a_j$ and $a_k$. The covariance matrix, $C$, can also be evaluated via singular value decomposition. It follows that the standard deviations of an individual parameter is proportional to the square root of the corresponding diagonal element in $C$, in complete agreement with equation (6.2.15).
Analysis of Errors in Identified Parameters

6.4 Worst Case Analysis of Errors

6.4.1 The System Condition Number

The system condition number (SCN) is another standard measure of accuracy of parameter estimates, (Strang, 1980), based on a worst case assumption about the corrupting measurements. The SCN for $A$, is defined numerically as

$$\text{cond}(A) = \frac{s_1}{s_n}$$ (6.4.1)

where $s_n$ and $s_1$ are the smallest and largest singular values respectively, from the singular value decomposition of $A$.

The interpretation of the SCN is as follows. About an exact solution, the response is defined as

$$y = \eta + \epsilon$$ (6.4.2)

and the estimated parameters are given by

$$\hat{a} = \xi + \delta$$ (6.4.3)

but in this case $\eta$ and $\epsilon$ are varied for worst case conditions. Working in transformed co-ordinates,

$$\delta' = D^{-1} \epsilon_1$$ (6.4.4)

hence

$$|\delta| \leq s_n^{-1} |\epsilon_1| \leq s_1^{-1} |\epsilon|$$ (6.4.5)

also,

$$\xi' = D^{-1} \eta_1$$ (6.4.6)

$$|\xi| \leq s_1^{-1} |\eta_1| = s_1^{-1} |\eta|$$ (6.4.7)
Analysis of Errors in Identified Parameters

The relationship between the relative error in parameters and the relative error in response is given by

\[
\frac{\| \delta \|}{\| \xi \|} \leq \frac{s_1}{s_n} \frac{\| e \|}{\| \eta \|} \tag{6.4.8}
\]

The error in the parameters caused by error in the response is therefore bound by the SCN, equation (6.4.1).

The worst case errors occur when the corrupting errors are parallel to the nth right singular vector, \( e \| U_n \) and the uncorrupted response is parallel to the first right singular vector, \( \eta \| U_1 \).

The advantages of using the system condition number for analysis of errors are:

- it is independent of the response, hence no knowledge of the 'true' solution is required
- it is independent of any modelling of actual errors; rather it provides overall limits on how susceptible the parameter estimates are to corruption.

The disadvantages of the system condition number are:

- it gives very limited information - one scalar value for a whole set of parameters
- the system condition number is dependent on the units chosen for the parameters and the response.

6.4.2 The Parameter Condition Number

In an attempt to overcome the above disadvantages, the parameter condition number (PCN) is introduced here to define the worst case magnification of error possible in each parameter estimate, in the least squares solution.

Seek the worst case change in the parameters \( \delta \) due to corruption \( e \).

Let \( \Delta \) be the parameter error, as in equation (6.4.3), and let \( n \) be a unit vector in the parameter space as before. The parameter error associated with direction \( n \) is then

\[
\delta a = n^T \delta \\
= n^T \delta'
\]  

(6.4.9)
Analysis of Errors in Identified Parameters

then for equation (6.4.4)

\[ \delta a = n^T D^{-1} e' \]  \hspace{1cm} (6.4.10)

For worst case errors, assume \(|e| = |e'|| \) is specified, but the direction of this error vector is known. \(|\delta a|\) is maximised if \(e'\) is nonzero in only its first \(n\) components, so that \(|e'| = |e|\), and,

\[ e \parallel D^{-1} n', \text{ say } e = k D^{-1} n' \]  \hspace{1cm} (6.4.11)

Then

\[ |e'|^2 = k^2 n^T D^{-2} n' \]
\[ = k^2 C_{nn} \]

hence

\[ k = \frac{|e|}{\sqrt{C_{nn}}} \]  \hspace{1cm} (6.4.13)

Substituting for \(e\) and \(k\), in equation (6.4.12) and noting equation (6.3.5) gives

\[ \delta a = |e| \sqrt{C_{nn}} \]  \hspace{1cm} (6.4.14)

The relationship between the relative error in each parameter estimate, \(\delta a_i\), and the relative error in response is then given by setting \(n = e_i\), and dividing by the exact parameter values. For general errors \(e\) we then have

\[ \frac{|\delta a_i|}{|\xi_i|} \leq \frac{\sqrt{C_{ii}}}{|\xi_i|} |e_i| \leq \frac{|e|}{|\eta|} \]  \hspace{1cm} (6.4.15)
Analysis of Errors in Identified Parameters

The bound here is given by the parameter condition number, $\chi$,

$$\chi_i = \frac{\sqrt{C_{ii}}}{|\xi_i|} \eta_i$$  \hfill (6.4.16)

The PCN shares the same advantages as the SCN with the following additional advantages:

- The parameter condition number is independent of the units of the parameters and response data
- The parameter condition number gives a worst case error for each individual parameter, not just on the parameter set.

The PCN defined here does however rely on knowledge of an exact solution $\xi_i$ and is therefore mainly useful for simulation studies. It is noted that the three methods giving information about individual parameters all give results that are proportional to $\sqrt{C_{ii}}$, and are expected to give similar results for parameter accuracy. Because the assumptions concerning the errors are quite different in these three cases, the results are likely to apply under all but very specialised error conditions.

6.5 Analysis of Errors in Parameters Identified from Frontal and Rear Impact Simulation

This section applies the above error analysis techniques to parameters identified from frontal and rear impact of the Modified Hybrid II manikin. Three situations are considered (1) use of frontal impact data only, (2) use of rear impact data only, (3) use of combined frontal and rear impact data, for input into the identification routine. The parameters estimated from each of the three identification procedures are firstly ranked in order of significance to the fit.

The 'ideal case' situation was considered for the identification, that is there was no noise corruption imposed on the test data. Descriptions of the unknown parameters are given in Table 6.5.1. Results obtained using the four measures of error are displayed and discussed in Section 6.5.1. Parameters which are identified as being poorly conditioned in Section 6.5.1 are corrupted and used to create 'corrupted fitted models' in Section 6.5.2. The resulting 'corrupted models' are run and the simulation results are analysed and compared with the original data.

In Section 6.5.3 the transition weighting parameter is implemented and it's effect on the conditioning of each parameter is assessed.
### Analysis of Errors in Identified Parameters

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<th>Parameter</th>
<th>Definition</th>
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Table 6.5.1 Definition of Parameter Values
6.5.1 Application of the Measures of Error

Considering each of the measures of error described in the previous sections give essentially the same information, with exception to the SCN, it is only necessary to apply one of the measures of error to the parameters identified from frontal, rear and combined impact test data. The parameters were ranked in order of significance by calculating the increase in cost due to deletion of each parameter: the largest value represents a poorly conditioned parameter and takes the ranking value of 18.

Figure 6.5.1 shows how the measures of error give different ranking values for frontal, rear and combined impact. As expected some of the parameters which are well conditioned when using frontal impact data, are poorly conditioned when using just rear impact data. An example of this is parameter 12 which is the last part of the seat back/thorax contact stiffness curve. This parameter has very poor conditioning when only frontal impact data is used for the identification, with ranking values of 16/17/18, but is very well conditioned when only rear impact data is used where ranking values of 2 and 4 are given. Using Figure 6.5.2 we can also assess the effect of combining both frontal and rear impact data for the identification. The ranking values for combined data generally lie between the ranking values for frontal and rear impact, but they do occasionally have higher ranking values.

As the system condition number is a single value we cannot rank each of the parameters when using this as a measure of error. Table 6.5.1 shows the SCN for identification from each of frontal, rear and combined impact data. The SCN indicates that it is best to use combined impact data in order to obtain the best conditioned system. SCN values for frontal and rear impact data are very similar.

<table>
<thead>
<tr>
<th>Impact Data</th>
<th>System Condition Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frontal</td>
<td>2138</td>
</tr>
<tr>
<td>Rear</td>
<td>2179</td>
</tr>
<tr>
<td>Combined</td>
<td>1598</td>
</tr>
</tbody>
</table>

Table 6.5.1 System Condition Numbers
Figure 6.5.1 Ranking Values for Frontal, Rear and Combined Impact Data
6.5.2 Corruption of Ill Conditioned Parameters

In the previous section we saw how each of the measures of error can be used to identify which of the identified parameters are poorly conditioned and which are well conditioned. One of the main reasons for a parameter being poorly conditioned is that there is not enough information in the data to identify that parameter accurately. In this case one could argue that the parameter has no significance in that particular simulation and could take any value.

In this section we assess the effect on the identified model, of corrupting the parameters which were shown, in the previous section, to be poorly conditioned. The case of identification using frontal impact data is considered and Table 6.5.2 shows the parameters that are corrupted along with their true and corrupted values.

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>$\Delta$ (m)</th>
<th>$F_{e_2}$ (N)</th>
<th>$F_{e_3}$ (N)</th>
<th>$F_{e_4}$ (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Model</td>
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<td>0.0</td>
<td>0.0</td>
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</tr>
<tr>
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</tr>
<tr>
<td></td>
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<td>800.0</td>
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</tr>
<tr>
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<td>0.075</td>
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<td>1300.0</td>
<td>1300.0</td>
</tr>
<tr>
<td></td>
<td>0.100</td>
<td>2900.0</td>
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<tr>
<td>Corrupted Model</td>
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<td>0.100</td>
<td>4900.0</td>
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<td>4900.0</td>
</tr>
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</table>

Table 6.5.2 Corrupted Model Characteristics

The corrupted set of parameter values were used to create a corrupted model for frontal impact simulation. The resulting acceleration, velocity and displacement time histories were compared with those obtain from the original frontal impact simulation. Figure 6.5.3 compares the original and corrupted model trajectories. Considering the large amount of corruption on the parameter values the corrupted model trajectories closely match those given by the original model. Thus demonstrating that poorly conditioned parameters have little significance for that particular impact scenario. If however the crash pulse or initial conditions were changed for some reason we could not necessarily believe that the same parameters were poorly conditioned.
Figure 6.5.3 Correlation of Original and Corrupted model Trajectories
6.5.3 Implementation of the Transition Weighting Parameter

In Chapters 4 and 5 the weighting parameter was implemented to reduce problems with non-physical values for the identified parameters. In this section we investigate the effect that the transition weighting parameter has on the conditioning of the identified parameters.

The identification procedure was run using frontal impact data and a transition weighting parameter of \( W = 10^4 \). The parameter condition number was calculated for the resulting identified parameters and was compared with the PCN obtained without the transition weighting. Figure 6.5.4 shows this comparison, it can be seen that the PCN is reduced in most cases when the transition weighting parameter is implemented. There are three parameters which have a slightly higher PCN, these are parameters 4, 11 and 12. In this case these three parameters are not used in the frontal impact and so they are never going to be accurately identified.

![Figure 6.5.4 Comparison of PCN for parameters identified with and without the transition weighting parameter](image)

Figure 6.5.4 Comparison of PCN for parameters identified with and without the transition weighting parameter
6.6 Discussion and Summary

Use of error analysis techniques enables us to look at the conditioning of the identified parameter values and determine which parameters are believable and which are subject to error. This chapter has identified several methods for estimating how well conditioned the identified parameters are. Sections 6.2 - 6.4 demonstrated that the different measures of error tell us basically the same thing, with the exception of the SCN which can only give us a measure of well conditioned the whole system is rather than the conditioning of each individual parameter.

Parameters identified from the frontal, rear and combined impact of the modified Hybrid II were ranked in order of significance. The parameters that were least significant to the fit were those parameters which were not relevant to the simulation, for example parts of the contact force/penetration characteristics. In all three cases the parameter with the most significance to the fit was the seat belt stiffness, which is not unexpected in a frontal impact but a little surprising in a rear impact scenario.

The system condition number indicates that the most information would be gained by using combined data (frontal and rear impact) for the identification process. The parameter condition number defines the worst case magnification of errors possible in the least squares solution, for each individual parameter. In a similar fashion to ranking the parameters, we can easily identify which parameters are well conditioned and which are ill conditioned. The covariance matrix, though based on a very different set of assumptions, gives a similar measure of error.

The relatively high parameter condition numbers seen in Figure 6.5.4 indicate that parameters identified from physical crash data could be subject to large amounts of error, particularly if the transition weighting parameter is not implemented during the identification.

One must be careful when implementing the four methods of error analysis included in this chapter as the methods are not general to all identification problems, they apply to exact model and theoretical noise assumptions. Figure 6.6.1 shows a correlation of the rankings for each of the parameters identified from the frontal impact along with the parameters identified from the low noise case (LNT1) which are ranked according to their percentage error. We cannot compare the ranking values directly as there are different numbers of parameters in the two models; the frontal impact scenario has seat back friction and damping whereas the low noise model does not. In order to enable some comparison we assume that LNT1 has the same number of parameters as in the case of frontal impact and assign the highest ranking values to those parameters which we are not able to identify (parameters 11, 12 and 15), that is those parameters that are least important to the fit. It can be seen from Figure 6.6.1 that the trends are generally the same with the belt stiffness parameter has a low ranking value in both cases: the percentage error was very low and the theoretical analysis shows that this parameter is well conditioned and most
Analysis of Errors in Identified Parameters

significant to the fitting error.

As an additional application the error analysis techniques described in this chapter are particularly useful for simulation studies. They would be a great advantage to help define a matrix of impact tests which would give maximum information for use in identification.

Figure 6.6.1 Comparison of Ranking Values for Frontal Impact and Model LNT1 Identification.
Chapter 7

Experimental Study

7.1 Introduction

7.2 Impact Simulation
   7.2.1 The HyGe Acceleration Rig
   7.2.2 Instrumentation
   7.2.3 Data Acquisition

7.3 Frontal Impact Simulation of the Hybrid II Manikin
   7.3.1 Test Set-Up
   7.3.2 Data Acquisition and Analysis

7.4 Identification of Seat Contact and Seat Belt Characteristics
   7.4.1 Identification using Film Derived Displacements
   7.4.2 Identification using Accelerometer Derived Displacements

7.5 Discussion
7.1 Introduction

As outlined in Chapter 3, the application of system identification in CVS data set assembly is to estimate the unknown parameters of an occupant restraint system by use of geometrical, inertial and kinematic data, taken from an impact test upon that restraint system. The unknown parameters to be identified for our CVS data set are the vehicle seat characteristics, that is stiffness, damping and friction terms, and the lap belt characteristics. Chapters 3, 4 and 5 assessed the viability of the extraction of such parameters from simulated test data, and established the techniques required for identification in CVS data set assembly. This chapter looks at using data obtained from real impact tests on the dummies and restraint systems modelled in Chapter 2.

Section 7.2 outlines how a vehicle impact is reconstructed in the laboratory, the various methods available and which are the most reliable. Some explanation of the HYGE sled is given, since this was the method of impact simulation used for this work, where impact tests were carried out on the modified Hybrid II dummy. The test set-up is described in Section 7.3, which also describes the instrumentation used in the test programme. The raw data obtained from the impact test requires a certain amount of post-processing to make it suitable for input in the identification routine and this post-processing is also described in Section 7.3. Identification of the seat contact and seat belt parameters is described in Section 7.4.

The parameters identified from the experimental data were then used to create a fitted mathematical model and this is assessed in Section 7.5. The results from the fitted model are compared with the experimental data to assess the accuracy of the identified parameters and the fitted model results are interpreted in light of the simulations of Chapters 4 and 5.

7.2 Impact Simulation

There are two types of crash sled, the deceleration rig and the acceleration rig. The deceleration rig (often known as bungee sled or gravity sled), stops the sled at a known velocity by use of crush tubes. The acceleration rigs take the sled from rest and accelerate it in the reverse direction to a known velocity. The shape of the acceleration pulse can be arranged to be dynamically similar to that seen in the full scale crash test. The accuracy and repeatability of the pulse obtained from the acceleration rig makes it a better tool than the deceleration rig. The acceleration rig was used for our experimental work.
7.2.1 The HYGE Acceleration Rig

The acceleration rig used for this research is situated at the Motor Industry Research Association (MIRA) and is known as the HYGE: Hydraulically-actuated Gas Energised System. Figure 7.2.1 shows a diagram of the HYGE gun.

The HYGE gun divides into two halves, separated by an orifice plate. In the first half there is a ram with a thrust plate, which carries a metering pin protruding through the orifice plate in order to vary the cross sectional area. The front volume of the gun is charged with oxygen-free nitrogen. This controls the duration of the pulse, and decelerates the internal mechanism of the gun once the pulse is complete. The volume of gas delivered is varied by changing the position of a floating piston, which is moved by hydraulic fluid.

The rear volume, which can also be varied by means of a hydraulically actuated floating piston, is charged with compressed air. This provides the energy for the pulse.

By varying the stored gas volumes and pressures, and by altering the shape of the metering pin, the amplitude, duration and shape of the acceleration can be accurately determined.

The shape of the metering pin controls the shape of the acceleration pulse. Metering pins can be designed to reproduce crash pulses seen in full scale crash tests. An envelope, within which the HYGE pulse must lie, is defined from crashed vehicle tests. The envelope is usually defined using the crash pulse observed at the base of the left and right B post and the rear of the transmission tunnel of the body shell.

To define the shape of the pin required to reproduce the crash pulse, the gas equations through the gun are defined. An iterative process is used to define the shape of the pin; this process involves solving the supersonic flow equations for known geometrical data, accelerations, pressures and volumes at discrete time steps.

In normal use the gun would typically generate peak acceleration of $200\text{ms}^{-2}$ over a period of 30ms. A velocity of $10\text{ms}^{-1}$ would generally be achieved after 200ms. For the purposes of this work only this initial 200ms period is considered to be of interest.
Figure 7.2.1 HYGE Gun
Each impact test can be filmed using high speed photographic cameras. The cameras have a variety of lenses and can run at a variety of speeds between 250 and 3000 frames per second. The cameras can be placed onboard, offboard or overhead, and an offboard video camera is used to give instant playback at 200 frames per second.

A variety of transducers can be used to measure various parameters during impact tests. Table 7.2.1 shows a list of data that could be obtained from a typical HYGE test. All such measurements are made in the time domain: accelerations are measured directly using piezo-resistive strain gauge-type accelerometers, forces are measured using strain gauge-type load cells and displacements are generally measured directly by means of film and/or video image analysis. Other kinematic quantities such as linear velocity and angular velocity are derived from accelerometer and film measurements. The transducers used in this work are described in Section 7.3.

<table>
<thead>
<tr>
<th>Object</th>
<th>Measurement</th>
<th>Transducer</th>
<th>Film/Video</th>
</tr>
</thead>
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<td></td>
</tr>
<tr>
<td></td>
<td>Linear Velocity</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angular Acceleration</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Angular Velocity</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Linear Displacement</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Angular Displacement</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Neck</td>
<td>Load</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Chest</td>
<td>Linear Acceleration</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shoulder Trajectory</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>Sternum Displacement</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Pelvis</td>
<td>Linear Acceleration</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Pelvis Trajectory</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>Femurs</td>
<td>Load</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td>Knees</td>
<td>Degree of Facia Penetration</td>
<td></td>
<td>x</td>
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<td>Feet</td>
<td>Contact Events</td>
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<td>Seat Belt System</td>
<td>Retractor Reel out</td>
<td>x</td>
<td>x</td>
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<tr>
<td></td>
<td>Belt Loads</td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2.1 Measurements from a typical sled test.
7.2.3 Data Acquisition

Transducer signals have to be stored in a form suitable for signal analysis. The data acquisition system used in this experimental work was a Kayser-Threde system. The data acquisition system has 54 channels onboard and samples at 10 KHz for 2.5 seconds. This system consists of a data encoder, a MICROVAX 2 computer and a SUN SPARC based analysis system. The encoder capture data during the impact test; it takes the transducer signal, amplifies and digitises the signal, stores it in memory and then transfers the signal to the MICROVAX. The MICROVAX acts as a data server and also as a man machine interface. The system carries out analysis to produce output of accelerations, displacements, velocities and injury criteria, and it is also used to present graphical output of the test data.

7.3 Frontal Impact Simulation of the Modified Hybrid II Manikin

A frontal impact test was carried out using the modified Hybrid II manikin. The arms of the dummy were removed and the lower legs were bolted together, to reduce the number of degrees of freedom in the impact. The dummy was restrained in an aircraft seat by the use of a lap belt, essentially reducing the frontal impact of the modified dummy to two dimensional motion. Figure 7.3.1 shows the acceleration pulse used for the test. In this section we define the test set up and instrumentation used for the test, and also outline the post-processing required to obtain the identification input data.

7.3.1 Test Set Up

Figure 7.3.2 shows a photograph of the test set up. The instrumentation used in this test consisted of the following:

- Tri-axial linear accelerometers
- Tri-axial angular accelerometers - this was the first time that such instrumentation was used in the MIRA HYGE laboratory. To obtain confidence with regard to their accuracy some preliminary tests were carried out.
- High speed film, digitisation markers

The linear accelerometers were placed in the head, chest, femur and lower leg of the dummy, and also on the HYGE sled. Angular accelerometers were placed at the femur and thorax mass centres. Markers were placed on the dummy head, chest, femur and lower leg, and on the seat. Three high speed cameras were used; two offboard cameras (one on either side of the sled), and one onboard camera with a close up view of the femur/seat contact interaction. Detailed descriptions of the transducers can be found in Appendix D.
Figure 7.3.1 Acceleration Pulse
7.3.2 Data Acquisition and Analysis

Accurate kinematics of the femur, thorax, head and lower leg of the Hybrid II dummy were required. We now consider the experimental data obtained from the impact test and outline the post-processing involved in transforming the experimental data into the required format for system identification.
Experimental Study

For input into the identification routine, accelerations, velocities and displacements of the following degrees of freedom were required:

- Horizontal motion of the femur mass centre
- Vertical motion of the femur mass centre
- Absolute angular motion of the femur
- Angular motion of the thorax with respect to the femur
- Angular motion of the head with respect to the thorax
- Angular motion of the leg with respect to the femur

The kinematic data obtained from the impact test were as follows:

(a) Acceleration Measurements

The accelerations obtained from the transducers were given in the accelerometer local co-ordinate system, these needed transforming into a global (lab) co-ordinate system for the identification routine. Corrections for gravity terms were also required: Appendix E gives the calculation for correction for gravity and the method used to convert from local to global co-ordinates. The following data was obtained from accelerometers:

- Horizontal acceleration of the impact sled
- Horizontal and vertical acceleration of the femur mass centre
- Horizontal and vertical acceleration of the thorax mass centre
- Horizontal and vertical acceleration of the head mass centre
- Horizontal and vertical acceleration of the leg mass centre
- Angular acceleration of the femur
- Angular acceleration of the thorax

(b) Displacement Data

The following were obtained from high speed film:

- Horizontal and vertical displacement of the seat
- Horizontal and vertical displacement of the hip
- Horizontal and vertical displacement of the femur
- Horizontal and vertical displacement of the knee
- Horizontal and vertical displacement of the thorax
- Horizontal and vertical displacement of the head
- Horizontal and vertical displacement of the leg
Using the above linear displacements, the following angles were calculated, assuming a rigid-body geometry of the manikin:

- Absolute angle of the femur
- Absolute angle of the thorax
- Absolute angle of the head
- Absolute angle of the leg

(c) Derived Data

From the above it is seen that angular accelerations for the lower leg and the head were not available, and all velocity data needed to be derived. Appendix F shows the method used to calculate the head and lower leg angular accelerations. All velocity terms were obtained by integrating the acceleration signals.

7.3.3 Sources of Error

Potential sources of error were discussed in general terms in Chapter 4. Due to the necessity of using integrated and double integrated acceleration data in order to complete the input data set required for the identification, the frequency response of the accelerometer is of particular relevance.

When subject to an input sinusoidal motion of known amplitude and frequency, an output electrical voltage sinusoid is obtained of measurable amplitude, frequency and phase shift relative to the input. Using complex amplitudes the frequency response may be defined as the ratio of output to input. In practice, the peak output, for a given frequency, may be measured for several peak inputs within a designated operational limit, and the best-fit line (in a least squares sense) to the data gives the coefficient of sensitivity. This exercise may be repeated over a range of frequencies, giving a sensitivity coefficient at each frequency. These coefficients may be averaged over the frequency range to give a calibration factor of the transducer in question. This calibration factor is used in the data processing software to convert the accelerometer output to units of acceleration. Frequency dependent deviations, in sensitivity, from the mean are not accounted for and frequency dependent phase shifts are also neglected. The size of error so induced depends on the frequency content of the acceleration being measured. Lower frequencies contribute more to an integrated signal than the higher frequencies.
From consideration of the potential sources of error, we have to accept that for this type of accelerometer the measured acceleration is subject to corruption by noise, with a noise to signal ratio of approximately 1-5%. It could therefore be dangerous to rely on double integrated acceleration for displacement if optimal accuracy in displacement measurement is required. To illustrate this problem Figure 7.4.1 shows correlations of double integrated acceleration signals with the displacements taken directly from high speed film. The 2 signals correlate relatively well for horizontal femur displacement, thorax angle and lower leg angle. The vertical femur displacement, femur angle and head angle do not correlate so well.

### 7.4 Identification of Seat Contact and Seat Belt Characteristics

This section now applies the techniques developed in Chapters 3 - 5 to identify the contact interaction parameters directly from impact test data, taken from the frontal impact of the Modified Hybrid II dummy.

To consider the effects of using accelerometer derived displacement data, the following two optimisation studies were considered:

- Optimisation using film derived displacement, accelerometer derived velocities and accelerations.
- Optimisation using accelerometer derived displacements, velocities and accelerations.

Figures 7.4.2 and 7.4.3 show the accelerometer derived velocity and acceleration time history data, used for input into the identification routine.

#### 7.4.1 Identification using Film Derived Displacements

The characteristics we wished to identify from the frontal impact data of the manikin were the same as those defined in Chapter 4 (see Table 4.1.1). Although the 'real' values of the parameters were not known in this case, the parameters were normalised using the same reference values as given in Chapter 4, so that each identified parameter would be of a similar order of magnitude.

In this section we implement equation (4.4.1) in the hope of finding a transition weighting and pre-assigned parameter values, which will lead to the identification of realistic parameter values and accurate fitted models.
Figure 7.4.1 Correlation of Film Derived and Accelerometer Derived Displacements
Experimental Study

As in Chapters 4 and 5, optimisations with a range of weighting parameters were executed. $W$ was set to vary from $10^4$ to $10^7$ and pre-assigned values of 1.0 were set for each parameter. Again, to ensure convergence and reduce problems of local minima, 50 optimisations were carried out for each of the weighting parameter values.

Figure 7.4.4 shows how the error terms vary with $W$, from this we choose a weighting parameter of $W=10^3$. Table 7.4.1 shows the transition model parameters. The seat contact stiffnesses for the femur and pelvis are very high, whilst the seat belt stiffness appears to be very low. Unphysical parameter values are also apparent, this is not entirely surprising as some negative friction parameters were observed when using the calculated penetration data in Chapter 5.

The characteristics shown in Table 7.4.1 were used to create a transition fitted model so that we could compare its output with the impact test data. However when the simulation of this model was executed it became unstable, hence we cannot correlate model and test output. For this reason this model was not used any further in this study.

7.4.2 Identification using Accelerometer Derived Displacement Data

The above procedure was adopted to identify the parameter values using input data which consisted of accelerometer derived accelerations, velocities and displacements, as shown in Figures 7.4.1 - 7.4.3.

Figure 7.4.5 shows how the error terms vary with weighting parameter. From Figure 7.4.5 a transition weighting of $W=10^5$ is chosen. Table 7.4.2 shows the transition model parameters. As seen in the previous section some unphysical parameter values are again identified. However the seat belt and seat base contact stiffnesses are much more realistic than those seen in the previous section, these parameters were used to create the transition fitted model.

Figures 7.4.6 - 7.4.8 show the acceleration, velocity and displacement time histories from the transition fitted model compared with the impact test data. Considering the identification input data was taken from the accelerometer one would expect the fitted model displacements to follow the accelerometer derived displacements, however, as can be seen in Figure 7.4.6, this is not always the case: the fitted model horizontal displacement, thorax angle and lower leg angle all show good correlation with both film derived and accelerometer derived displacements, the fitted model vertical displacement and the head angle correlate well with the film derived data and not the accelerometer derived data. The fitted model femur angle shows poor correlation with both accelerometer and film derived data. The fitted model velocity and acceleration time histories show lesser degrees of correlation than the fitted model displacements.
Figure 7.4.2 Velocity Time Histories
Figure 7.4.3 Accelerometer Time Histories
**Experimental Study**

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>$\Delta$ (m)</th>
<th>$F_{e2}$ (N)</th>
<th>$F_{e3}$ (N)</th>
<th>$F_{e4}$ (N)</th>
<th>$C_{d2}/C_{d3}$ (Nm/s)</th>
<th>$\mu_2/\mu_3$</th>
<th>$K_b$ (N)</th>
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**Table 7.4.1 Film Derived Displacements - Identified Characteristics**

<table>
<thead>
<tr>
<th>Parameter Values</th>
<th>$\Delta$ (m)</th>
<th>$F_{e2}$ (N)</th>
<th>$F_{e3}$ (N)</th>
<th>$F_{e4}$ (N)</th>
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<th>$\mu_2/\mu_3$</th>
<th>$K_b$ (N)</th>
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<tbody>
<tr>
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**Table 7.4.2 Accelerometer Derived Displacements - Identified Characteristics**
Figure 7.4.4 Film Derived Displacement - Variation of Error with W
Figure 7.4.5 Accelerometer Derived Displacement - Variation of Error with W
Figure 7.4.6 Correlation of Fitted Model and Test Model Displacements
Figure 7.4.7 Correlation of Fitted Model and Test Model Velocities
Figure 7.4.8 Correlation of Fitted Model and Test Model Accelerations
7.5 Discussion

In this chapter we have applied the techniques developed in Chapters 3-6 to the identification of seat contact and seat belt characteristics from impact test data.

In Section 7.3.3, the use of integrated and double integrated acceleration data was discussed and considered to be a potential source of error if used as input data for the identification. However, the results seen in Section 7.4 show that, for this particular case, using the accelerometer derived data (acceleration, velocity, and displacement) for the identification gives slightly better results than seen when using the film derived displacement data. In fact, when using the film derived data, the resulting fitted model becomes unstable.

The transition fitted model results obtained when using the accelerometer derived data show generally good correlation with the experimental data. The displacement correlations are particularly good, with one exception; the angle of the femur. One reason for this poor correlation could be that there was no contact specified in the model for the interaction between the manikin's feet and the floor. This lack of contact in the model would have an effect on the motion of the lower leg and hence the femur.

It is also seen that where the film derived displacements and accelerometer derived displacements differ, the fitted model results fit the film derived results even though the fitted model was identified using solely accelerometer derived data.

Velocity and acceleration time histories do not correlate as well as the displacement time histories. In particular, the initial accelerations on the fitted model are not zero; the model is not in equilibrium at $t=0.0s$. A way to eliminate this problem would be to apply a high weighting to the first time point in each equation of motion. This would force the initial conditions of the fitted model to match the initial conditions of the test. It is also noted from Figure 7.4.3 that the femur angular acceleration exceeds the range of the transducer for a period of approximately 10ms - this will also have a slight effect on the identification results.

Comparing the results obtained in this chapter with those seen in Chapters 4 and 5; the degree of displacement correlation is very similar to the degree of correlation seen in Chapter 5 when using the calculated penetration data. The Complex model results are good to use as a comparison since the model used in this chapter has a simplified structure of the manikin. The acceleration and velocity results do however need improving if they are to be used for calculating injury criteria.
Chapter 8

Discussion and Conclusions
Discussion and Conclusions

8 Discussion and Conclusions

In this chapter the results and knowledge gained during this study are discussed and summarised. Ideas for further work which will build on the foundations laid here are also identified.

This study has been based on the identification of relatively simple two-dimensional occupant models. However, the identification procedure has been developed as a general tool which can be applied to the identification of general multibody models. The symbolic code used to define our models was AUTOSIM (Sayers, 1991); again the identification procedure is not specifically dependent on this code - alternative symbolic codes could be used instead.

Three dimensional dynamics have not been considered in this work; however with the identification procedure established as a general tool, in principle there are no restrictions in the move from 2D models to 3D models. Extra instrumentation would of course be required to and the detailed requirements for this would need careful consideration.

One of the objectives of this research was to investigate the viability of identifying contact interaction parameters when only limited test data is available. Identification from limited experimental data, noisy observations and simplified model structure data were assessed in Chapters 3-6. In each case this led to identification of an ill-conditioned problem. Using standard nonlinear least squares optimisation techniques unphysical parameter values were then identified. One method of solving the problem of unphysical parameter values is to apply constraints to the unknown parameters during optimisation. However this has serious limitations if limits are set a priori. If these are set at the limits of likely physical ranges, the ill-conditioning will lead to a relatively large number of parameters being identified on these limits. If the limits are set narrowly, the problem becomes over-constrained and the experimental data becomes redundant. Compared to the weighting parameter approach, where only one scalar value is adjusted, there is also a large number of free parameters, and hence arbitraries, in any constraint procedure.

In Chapter 4 the simulated data was corrupted with two levels of added 'noise' and in Chapter 5 errors were introduced into the physical model by assuming a simplified model structure for the identification. Although this gave us some information on ill-conditioning and parameter accuracy, the specific assumptions made about the corrupting noise were in no way considered realistic. Therefore, in Chapter 6 a more general analysis of errors in the identified parameters was given. Several measures of error were defined and it was seen that in each case the predicted errors in individual parameters were proportional to $\sqrt{C_{ii}}$, where $C$ is the covariance matrix.
Discussion and Conclusions

The assumptions concerning the errors in each of these measures are quite different and hence the results are likely to apply under all but very specialised error conditions. The methods are not restricted to linear least squares applications; they can be applied in non-linear least squares in the idealised case where very small errors are assumed.

Throughout this work a sum of squared equation errors criterion is implemented. In most cases this proved satisfactory for the identification, however there is always the possibility of improving the fitted model results, for this reason an integrated least squares criterion was implemented in Chapter 5. However in this case no improvement in the fitted model correlation was seen.

The approach taken in this work was to base the optimisation on the physical model structure. Another approach would be to base the optimisation explicitly on the dummy kinematics, that is to minimise the error between the model kinematics and the test kinematics. This Black Box approach would be achieved using an iterative procedure, changing the parameter values on each iteration to minimise the error between the model and test kinematics.

This approach may be useful to correlate an individual test, it has little use however if the model is required for parametric studies. If you optimise solely on the kinematic data then the method has little to do with the physical model hence the critical contact interactions are not considered and the model would be of little use in a simulation study.

This study demonstrates a methodology for the identification of vehicle occupant models which, in its early stages of development, has shown encouraging results under both theoretical and experimental conditions. The methodology is not limited to use in crash victim simulation, it can be applied to the identification of general multi-body models.

The transition weighting parameter, first implemented in Chapter 4, has proved critical to the success of the identification procedure. Without this transition weighting, identification from limited test data would become an ill-conditioned problem: leading to the identification of unphysical parameter values and potentially poor fitted models.

The short experimental study carried out in Chapter 7 highlights the need for further work. Generally good correlation is seen on the dummy displacement time histories but less impressive results are observed with the velocity and acceleration time histories. The identified characteristics also still show some unphysical parameter values and hence we suspect that under different conditions the fitted mathematical model would not perform so well. These results could potentially be improved by including a contact interaction between the floor and the dummies feet, and by increasing the number of degrees of freedom of the identification model.
Discussion and Conclusions

As indicated in the previous section the foundations have been laid for the identification of vehicle occupant models and the methodology works well when applied to theoretical models. Further work must concentrate on the experimental study.

One area for further development would be to develop a procedure for obtaining the identification input data from experimental output data, that is a standard method for post processing of the test data. One should also consider developing a technique for optimising the instrumentation required for the identification, so that the maximum information is obtained with the least instrumentation possible.

To be useful in the field of vehicle safety the identification of three dimensional models must be considered. This would enable airbag interactions and three point belts must be included in the models.
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Appendix A

Calculation of the Contact Point for Plane Ellipse Contact Interaction.

Figure A shows an ellipse of semi-axes $a$ and $b$, and a principal axis co-ordinate system $(x_e, y_e)$. The ellipse is rotated through an angle of $\theta_e$ with respect to a global co-ordinate system $(x, y)$. The point of contact between the ellipse and the plane (rotated through an angle $\theta_p$) is given in the ellipse co-ordinate system as $(x_o, y_o)$ and as $(x_{op}, y_{op})$ in the global co-ordinate system.
Appendix A

Calculation of the Contact Point

The equation of an ellipse is given as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Differentiating this equation with respect to x gives

$$\frac{dy}{dx} = -\frac{x a^2}{y b^2}$$

In the ellipse co-ordinate system, the following holds at the contact point \((x_o, y_o)\)

$$\frac{dy}{dx} = \tan(\theta_p - \theta_e)$$

and

$$\frac{dy}{dx} = \frac{x_o a^2}{y_o b^2}$$

hence

$$\frac{x_o^2 a^4}{y_o^2 b^4} = \tan^2(\theta_p - \theta_e)$$

Substituting for x from Eqn(A1) gives

$$y_o = \pm \left[ \frac{b^4}{a^2 \tan^2(\theta_p - \theta_e) + b^2} \right]$$

Then

$$x_o = -\frac{b^2}{a^2} y_o \tan(\theta_p - \theta_e)$$

Transforming into global coordinates gives

$$x_{op} = x_o \cos \theta_e - y_o \sin \theta_e$$
$$y_{op} = x_o \sin \theta_e + y_o \cos \theta_e$$
Appendix B

MADYMO and AUTOSIM Model Build Commands for Modified Hybrid II Model
**Modified Hybrid II - MADYMO Model**
**2-Dimensional**
**6 Degrees of Freedom**

---

**Modified Hybrid II**

**Version 1**

| 0.000 0.1 |
| 0.0 0.001 |
| 0.0 0.5 0.010 0.1 |

---

**NULL SYSTEM**

**SEAT PLANES**

| 0 0 0.45 0.1 0 0 SEAT BASE |
| -0.09549 0.5416 0.0 0.2 0 0 SEAT BACK |

---

**FUNCTIONS**

*Dummy to Seat Base Stiffness*

5

| 0.000 0.0 |
| 0.01778 1779.2 |
| 0.02794 4670.4 |
| 0.03429 8451.2 |
| 0.04084 15564.0 |

---

**SEAT BACK STIFFNESSES**

5

| 0.00 0.0 |
| 0.03 1779.2 |
| 0.05 4670.4 |
| 0.07 8450.2 |
| 0.08 15560.0 |

---

**MOTION**

**POSITION**

| 0 0 |
| 0 0 0.0 |
| 0.005 0.0 |
| 0.01 0.001 |
| 0.02 0.00492 |
| 0.03 0.012382 |
| 0.04 0.024682 |
| 0.05 0.043661 |
| 0.06 0.069946 |
| 0.07 0.104526 |
| 0.08 0.180554 |
| 0.09 0.200737 |
| 0.10 0.262243 |
| 0.11 0.326549 |
| 0.12 0.400670 |
| 0.13 0.483429 |
| 0.14 0.572065 |
| 0.15 0.665687 |
| 0.16 0.763407 |
| 0.17 0.864142 |
| 0.18 0.967066 |
| 0.19 1.070892 |
| 0.20 1.174062 |
| 0.30 1.174062 |
| 0.40 1.174062 |

---

**CONFIGURATION**

3 2 1

---

**GEOMETRY**

| 0.000 0.000 0.3209 0.0829 |
| 0.1139 0.0829 -0.05576 0.2879 |
| -0.019 0.594 0.006 0.152 |
| 0.5189 0.0829 0.016 -0.272 |

---

**INERTIA**

| 19.36 0.2774 |
| 32.11 1.2775 |
| 5.30 0.03 |
| 0.84 0.2542 |

---

**JOINTS**

| 1 0 0 0 12.00 78.0 |
| 2 0 0 0 10.00 26.0 |
| 3 0 0 0 1.0 0.0 |

---

**FUNCTIONS**

| -1.71 1080 -0.71 80.0 0.0 0.0 1.57 0.0 2.57 -1000 0 |
| -1.73 1000 -0.7200 0.0 0.0 2.570 -1000 |
| -2.26 -1000 -1.26 0.3 12.0 4.12 1000 |
| -1 -1000 0.0 0 1 20 2 70 2.3 200 3.3 1200 |
| 0 000 0 1.000 317 |
| 0 000 0 1.000 154 |

---

**ELLIPSES**

| 2 0.115 0.115 -0.039 0.06927 2.0 0.0 0.0 LOWER TORSO |
| 2 0.110 0.110 -0.06 0.210 2.0 0.0 0.0 SPINE |
| 2 0.120 0.175 -0.06 0.439 2.0 0.0 0.0 UPPER TORSO |
| 3 0.040 0.065 0.000 0.042 2.0 0.0 0.0 NECK |
| 3 0.090 0.115 0.0225 0.149 2.0 0.0 0.0 HEAD |
| 1 0.2775 0.080 0.3354 0.0829 2.0 0.0 0.0 UPPER LEG |
| 4 0.060 0.270 0.000 -0.180 2.0 0.0 0.0 LOWER LEG |

---

**INITIAL CONDITIONS**

| 0.03 0.05 |
| -1 0.1745 |

---

**ORIENTATIONS**

| 1 0 0 1 |

---

**FORCE MODELS**

| 1 0 0 1 |

---

**ACCELERATION FIELDS**

| 1 0 0 1 |

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FUNCTIONS

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END OUTPUT CONTROL PARAMETERS

END INPUT DATA

END CONTACT INTERACTIONS

KELVIN ELEMENTS

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END FORCE MODELS

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| 1 | 2 | -0.05576 | 0.2879 | -1 0 THORAXCO |
| 1 | 3 | 0.026 | 0.152 | -1 0 HEADCO |
| 1 | 4 | 0.016 | -0.272 | -1 0 LUMGCO |
| 1 | 2 | 0.0 | 0.0 | -1 0 peljnt |

LINACC

| 1 1 | 0.3209 | 0.0829 | 0 0 0 FEMURCO |

ANGACC

| 1 | 1 | 0 FEM |
| 2 | 0 FVOR |
| 3 | 0 HEAD |
| 4 | 0 LEG |

ANGVEL

| 1 | 1 |
| 2 | 2 |
| 3 | 3 |
| 4 | 4 |

ANGLES

| 1 2 1 1 |
| 1 3 1 2 |
| 1 4 1 1 |

FORCES

| 1 1 1 |
| 2 1 1 |
| 3 1 1 |
| 4 1 1 |
(setsym velyuleg "dot([ny],vel(upperlegcm)))")
(setsym velxtorax "dot([mx],vel(thoraxcm)))")
(setsym velytorax "dot([ny],vel(thoraxcm)))")

(setsym th2 "rq(upperleg)")
(setsym thrileth2 "rq(thorax)")

(add-subroutine difeqn contactp [a2 b2 a3 b4 thesat thsback
 @nuleg @yuleg @thtorax @xtorax @xpeljnt @ypelj
 nt
 xop2 yop2 xop3 yop3 xop4 yop4])

(add-subroutine difeqn REACFORC (K2 K3 K4 CD2 CD3 CD4
 thesat thsback
 @nuleg @yuleg @xtorax @xtorax
 #setidis #setvel
 @velxtorax @velxtorax @velxtorax
 xop2 yop2 xop3 yop3 xop4 yop4
dxop2 dyop2 dxop3 dyop3 dyop4 dyop4
fe2 fd2 fe3 fd3 fe4 fd4)

(add-subroutine difeqn FRICFORC (MUZ M14 MU4
 dyop1 thesat thsback
 @nuleg @yuleg @xtorax @xtorax
 #setidis #setvel
 @xpeljnt @ypelj
 dxop2 dyop2 dxop3 dyop3 dyop4 dyop4
fe2 fd2 fe3 fd3 fe4 fd4
tyfe2 fyfe2 tyfe3 fyfe3 tyfe4 fyfe4))

(add-subroutine difeqn BELTPORCE (xanchor1 yanchor1 xanchor2 yanchor2
 #nuleg #yuleg kb #setdis ypl lo that tab flax f
 bey))

(add-subroutine difeqn ELASTORQ (ETP E1K ETH))

(add-subroutine difeqn DAMPTORQ (CTDP CDTK CTHM DTP DTK DTH))

(add-subroutine difeqn FRICFORC (CFTP CFTK FTP FTK FTH))

(add-line-force r2 :name "react-force-pelvis/seatsbase" :direction "cos(thesat)\(n\y\)sin(thesat)\(n\x\)"
 :magnitude "fe2+fd2" :point1 p6)

(add-line-force r3 :name "react-force-uleg/seatsbase" :direction "cos(thesat)\(n\y\)sin(thesat)\(n\x\)"
 :magnitude "fe3+fd3" :point1 p7)

(add-line-force r4 :name "react-force-thorax/seatsback" :direction "cos(thesat)\(n\x\)+sin(thesat)\(n\y\)"
 :magnitude "fe4+fd4" :point1 p8)

(add-line-force f2 :name "fric-force-pelvis/seatsbase" :direction "cos(thesat)\(n\x\)+sin(thesat)\(n\y\)"
 :magnitude "ff2" :point1 p6)

(add-line-force f3 :name "fric-force-uleg/seatsbase" :direction "cos(thesat)\(n\x\)+sin(thesat)\(n\y\)"
 :magnitude "ff3" :point1 p7)

(add-speed-constraint "tu(seat) - #setvel")
(add-position-constraint "tq(seat) - #setdis")

(setsym xtorax "dot([nx],pos(thoraxcm)))")
(setsym ytorax "dot([ny],pos(thoraxcm)))")
(setsym xuleg "dot([nx],pos(upperlegcm)))")
(setsym yuleg "dot([ny],pos(upperlegcm)))")
(setsym xpeljnt "dot([nx],pos(thoraxcm)))")
(setsym ypeljnt "dot([ny],pos(thoraxcm)))")

(setsym velxuleg "dot([nx],vel(upperlegcm)))")
(add-line-force f4 :name *fric-force-back* :direction *(cos(thback))*(ny)-(sin(thback))*(nx))
  :magnitude *ff4* :point1 p8)

(add-line-force fbex :name *elastic-belt-forceax* :direction *[nx])
  :magnitude *fbex* :point1 p4 :point2 p3)

(add-line-force fbey :name *elastic-belt-forceay* :direction *[ny])
  :magnitude *fbye* :point1 p4 :point2 p3)

(add-moment ETORQP :name elastic-torque-pelvis :direction [nx]
  :magnitude ETP :body1 thorax :body2 upperleg)

(add-moment ETORQH :name elastic-torque-head :direction [nx]
  :magnitude ETH :body1 head :body2 thorax)

(add-moment ETORKK :name elastic-torque-knee :direction [nx]
  :magnitude ETK :body1 lowerleg :body2 upperleg)

(add-moment DTORQP :name damping-torque-pelvis :direction [nx]
  :magnitude DTP :body1 thorax :body2 upperleg)

(add-moment DTORQH :name damping-torque-head :direction [nx]
  :magnitude DTH :body1 head :body2 thorax)

(add-moment DTORKK :name damping-torque-knee :direction [nx]
  :magnitude DTK :body1 lowerleg :body2 upperleg)

(add-moment FTORQP :name friction-torque-pelvis :direction [nx]
  :magnitude FTP :body1 thorax :body2 upperleg)

(add-moment FTORQH :name friction-torque-head :direction [nx]
  :magnitude FTN :body1 head :body2 thorax)

(add-moment FTORKK :name friction-torque-knee :direction [nx]
  :magnitude FTK :body1 lowerleg :body2 upperleg)

(finish)

(write-to-file write-sim "hinsim.f"
Appendix C

Program Structure for the Identification of the Hybrid II Model

INPUT DATA
Read in the kinematic data

INITIAL VALUE ESTIMATE
For non-linear optimisation
a set of initial values must
be set for the unknown parameters

DIFFERENTIAL EQUATIONS
This routine calculates the
force coefficients, C, and is
therefore called several times,
setting each force to zero in
turn. The penetrations and
velocity terms required for the
basis functions, are also calculated
within this subroutine.

CALCULATE BASIS FUNCTION VALUES
This subroutine calculates the values
of the basis functions at each timestep
and for all equations of motion.

OPTIMISER
The kinematic data, basis function values
and the initial value estimates are sent to
this subroutine. Levenberg/Marquardts method
for solution of non-linear least squares is applied
within this routine. The routine returns optimal
parameter values and corresponding cost function.
Piezoresistive Accelerometer

Model 2262A
- Rugged, Fluid Damped
- DC Response
- 25 to 2000 g Full Scale
- 500 mV Full Scale Output
- Hermetically Sealed

DESCRIPTION
The ENDEVCO® Model 2262A accelerometers are rugged, fluid damped transducers of the piezoresistive type. ENDEVCO'S PIEZITE® Type P-11 semiconductor strain gage elements are used in a bridge configuration, providing a low impedance output with 10 Vdc excitation. The output is high enough to drive most tape recorders and low frequency galvanometers directly, without amplification.

A unique system of overrange stops (2262A-25, -100, and -200) limits the movement of the seismic element allowing the units to withstand shock up to 2000 g's without calibration shift. Viscous damping extends their useful frequency range and reduces the effect of spurious, high frequency vibrations.

Typical applications for these accelerometers include transportation environmental testing, transient accelerations on structural members and combined environments of steady state acceleration plus transient inputs.

ENDEVCO Model 106/109 Dual Channel System or Model 68207 BCSTM Computer Controlled System are recommended as signal conditioner and power supply.

SPECIFICATIONS

PERFORMANCE CHARACTERISTICS: All values are typical at +75°F (+24°C) and 10 Vdc excitation unless otherwise stated. Calibration data, traceable to the National Institute of Standards and Technology (NIST), is supplied.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Model 2262A-25</th>
<th>-100</th>
<th>-200</th>
<th>-1000</th>
<th>-2000</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RANGE</strong></td>
<td>g pk</td>
<td>± 25</td>
<td>± 100</td>
<td>± 200</td>
<td>± 1000</td>
</tr>
<tr>
<td><strong>SENSITIVITY (at 100 Hz)</strong></td>
<td>mV/g Typ</td>
<td>20</td>
<td>5</td>
<td>2.5</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(Min)</td>
<td>(16)</td>
<td>(4)</td>
<td>(2)</td>
<td>(0.38)</td>
</tr>
<tr>
<td><strong>FREQUENCY RESPONSE</strong></td>
<td>Hz</td>
<td>0 to 650</td>
<td>0 to 1300</td>
<td>0 to 1800</td>
<td>0 to 1500</td>
</tr>
<tr>
<td>(±5% max, ref. 100 Hz)</td>
<td>% Max</td>
<td>+ 5</td>
<td>+ 5</td>
<td>+ 5</td>
<td>+ 5</td>
</tr>
<tr>
<td>Maximum Deviation (0 Hz to 2000 Hz)</td>
<td>% (Min)</td>
<td>(- 12)</td>
<td>(- 7)</td>
<td>(- 8)</td>
<td>(- 7)</td>
</tr>
<tr>
<td><strong>MOUNTED RESONANCE FREQUENCY</strong></td>
<td>Hz Typ</td>
<td>2500</td>
<td>5000</td>
<td>7000</td>
<td>8000</td>
</tr>
<tr>
<td>(Min)</td>
<td>(2000)</td>
<td>(4000)</td>
<td>(5600)</td>
<td>(6400)</td>
<td>(8000)</td>
</tr>
<tr>
<td><strong>DAMPING RATIO</strong> [2]</td>
<td></td>
<td>0.707</td>
<td>0.707</td>
<td>0.707</td>
<td>0.707</td>
</tr>
<tr>
<td><strong>NON-LINEARITY AND HYSERESIS</strong></td>
<td>(% of reading, to full range)</td>
<td>% Max</td>
<td>± 1</td>
<td>± 2</td>
<td>± 2</td>
</tr>
</tbody>
</table>

Endevco Corporation
30700 Rancho Viejo Road
San Juan Capistrano, CA 92675
(714) 493-8181 FAX (714) 661-7231
Piezoresistive Accelerometer

SPECIFICATIONS—continued

PERFORMANCE CHARACTERISTICS—continued

<table>
<thead>
<tr>
<th>Units</th>
<th>Model</th>
<th>-100</th>
<th>-200</th>
<th>-1000</th>
<th>-2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANSVERSE SENSITIVITY</td>
<td>% Max</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>THERMAL ZERO SHIFT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>From 0°F to 200°F (-18°C to +93°C)</td>
<td>mV Max</td>
<td>± 20</td>
<td>± 20</td>
<td>± 20</td>
<td>± 20</td>
</tr>
<tr>
<td>THERMAL SENSITIVITY SHIFT</td>
<td>% Typ</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>WARM-UP TIME</td>
<td>Minutes Max</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

ELECTRICAL

| | | | | | |
| INSULATION RESISTANCE | 100 megohms minimum at 100 Vdc; all leads to case |

PHYSICAL

| CASE, MATERIAL | Stainless steel (416 CRES) |
| ELECTRICAL, CONNECTIONS | ENDEVCO Model 3022B-30 (supplied) |
| IDENTIFICATION | Manufacturer's logo, model number and serial number |
| MOUNTING/TORQUE | Hole for 10-32 UNF x 1/8 inch mounting stud/18 lbf-in (2 Nm) |
| WEIGHT | 28 grams (cable weighs 18 grams/meter) |

ENVIRONMENTAL

| ACCELERATION LIMITS (in any direction) | g | 250 | 1000 | 1000 | 1000 | 2000 |
| Base Strain Sensitivity (at 250 microstrain) | | 0.005 | 0.005 | 0.005 | 0.05 | 0.05 |
| TEMPERATURE | | | | | | |
| Operating | 0°F to 200°F (-18°C to +93°C) |
| Storage | -20°F to +220°F (-29°C to +104°C) |
| HUMIDITY | Unaffected. Unit is hermetically sealed. |

ALTITUDE

| Unaffected |

CALIBRATION DATA SUPPLIED

| SENSITIVITY (at 100 Hz and 10 g pk) | mV/g |
| FREQUENCY RESPONSE | 20 to 2000 Hz for 2262A-25, to 4000 Hz for -100, to 6000 Hz for -200, to 5000 Hz for -1000 and -2000; % deviation reference 100 Hz |
| ZERO MEASURAND OUTPUT | mV |
| MAXIMUM TRANSVERSE SENSITIVITY | % of sensitivity |
| MOUNTED RESONANCE FREQUENCY | Hz |
| INPUT AND OUTPUT RESISTANCE | Ohms |

ACCESSORIES

| 2981-3 | Mounting Stud (10-32 UNF-2A) |
| 3022B-30 | Cable Assembly |

OPTIONAL ACCESSORIES

| 2950 | Triaxial Mounting Block |
| 2981-4 | Mounting Stud (M5-0.8) |
| 3022B-XX | Cable Assembly (XX identifies cable length in inches) |

NOTES

1. The sensitivity increase at the mounted resonant frequency is less than 10%, reference 100 Hz.
2. Damping ratio is 2.2/0.2, typical, at 0°F/200°F (-18°C/+93°C).
3. Zero Measurand Output (ZMO) is the transducer output with 0 acceleration applied.

4. Rated excitation is 10.0 Vdc. The strain gage elements have a positive temperature coefficient of resistance of approximately 0.5% per °F. Power supply current capability (regulation) should be carefully considered when operating at low temperature extremes, especially when exciting more than one transducer from a single power supply.
5. Other excitation voltages may be used to 15.0 Vdc. Specify at time of order to obtain a more accurate calibration.
6. Measured at approximately 1 Vdc. Bridge resistance increases with applied voltage due to heat dissipation in the strain gage elements.

NOTE: Tighter specifications available on special order.
Piezoresistive Accelerometer

Model 7267A
- Triaxial Accelerometer
- DC Response
- 1500 g Full Scale
- Replaceable Sensors
- Undamped

DESCRIPTION
The ENDEVCO® Model 7267A is a replaceable-element triaxial accelerometer designed to measure acceleration in three mutually-perpendicular axes. Although designed for installation in anthropomorphic test dummies used in automotive crash studies, it has application wherever triaxial accelerometers are used for steady state or long duration pulse measurements. The Model 7267A uses ENDEVCO's PIEZITE® piezoresistive elements in half-bridge configuration and meets SAEJ211 specifications for anthropomorphic dummy instrumentation.

The three sensors are mutually perpendicular and are positioned so that theoretical lines drawn through the centers of the seismic masses intersect at a single point.

Each sensor is replaceable. It is held in place by a single screw for easy installation or removal by the user. Solder pins are provided for electrical connection of an easily replaced nine-conductor cable. Both side and top cable entry holes are provided. Accessories include a 10 ft. (3.05 m) cable and a mounting base. Sensors, housing and cable clamp are available as replacement components.

ENDEVCO® Model 106/109 Dual Channel System or Model 68207 BCATM Computer Controlled System are recommended as signal conditioner and power supply.

SPECIFICATIONS

<table>
<thead>
<tr>
<th>Performance Characteristics</th>
<th>Model 7267A</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ranging (g pk)</td>
<td>±1500</td>
<td></td>
</tr>
<tr>
<td>Sensitivity (at 100 Hz)</td>
<td>0.15</td>
<td>mV/g Typ</td>
</tr>
<tr>
<td>(Min)</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>Frequency Response [1]</td>
<td>0 to 2000 Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>(± 5% max., ref. 100 Hz)</td>
<td>0 to 1200 Hz</td>
<td>Hz</td>
</tr>
<tr>
<td>Mounted Resonance Frequency</td>
<td>14 000</td>
<td>Hz Typ</td>
</tr>
<tr>
<td>(Min)</td>
<td>(10 000)</td>
<td></td>
</tr>
<tr>
<td>Damping Ratio</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>Non-linearity and Hysteresis</td>
<td>±2</td>
<td>% Max</td>
</tr>
</tbody>
</table>

Endevco Corporation
30700 Rancho Viejo Road
San Juan Capistrano, CA 92675
(714) 493-8181 FAX (714) 661-7231
### SPECIFICATIONS—continued
#### PERFORMANCE CHARACTERISTICS—continued

<table>
<thead>
<tr>
<th>Units</th>
<th>Model 7267A</th>
</tr>
</thead>
<tbody>
<tr>
<td>TRANSVERSE SENSITIVITY [3]</td>
<td>% Max 3</td>
</tr>
<tr>
<td>ZERO MEASURAND OUTPUT</td>
<td>mV Max ± 25</td>
</tr>
<tr>
<td>THERMAL ZERO SHIFT [4]</td>
<td>mV Max ± 15</td>
</tr>
<tr>
<td>From -10°F to +150°F (-23°C to +66°C)</td>
<td></td>
</tr>
<tr>
<td>THERMAL SENSITIVITY SHIFT</td>
<td>% Typ ± 3</td>
</tr>
<tr>
<td>At -10°F and +150°F (-23°C and +66°C)</td>
<td></td>
</tr>
<tr>
<td>WARM-UP TIME</td>
<td>Minutes Max 2</td>
</tr>
</tbody>
</table>

#### ELECTRICAL
- INPUT RESISTANCE [5] [7] 1000 ohms
- INSULATION RESISTANCE 100 megohms minimum at 100 Vdc pin to case

#### PHYSICAL
- CASE, MATERIAL Stainless steel
- ELECTRICAL, CONNECTIONS [8] Integral cable, nine conductor No. 32 AWG, Teflon® insulated leads, braided shield, silicone rubber jacket
- IDENTIFICATION Manufacturer's logo, model number and serial number
- MOUNTING/TORQUE Holes for two 4-40 mounting screws/6 lbf-in (0.7 Nm)
- WEIGHT 50 grams

#### ENVIRONMENTAL
- ACCELERATION LIMITS (in any direction)
  - Static 4000 g
  - Sinusoidal Vibration 1000 g pk below 2000 Hz
  - Shock (half-sine pulse) [1] 4000 g, 500 μsec or longer
- TEMPERATURE
  - Operating -10°F to +150°F (-23°C to +66°C)
  - Storage -100°F to +300°F (-73°C to +149°C)
- HUMIDITY Unaffected. Individual sensors are hermetically sealed.
- ALTITUDE Unaffected

#### CALIBRATION DATA SUPPLIED (X, Y and Z axes)
- SENSITIVITY (at 100 Hz and 10 g pk) mV/g
  - 20 Hz to 2000 Hz, % deviation reference 100 Hz
- ZERO MEASURAND OUTPUT mV
- MAXIMUM TRANSVERSE SENSITIVITY % of sensitivity
- INPUT RESISTANCE Ohms

#### ACCESSORIES
- 23699 Cable, 10 Ft. (3.0 mm). Cable is factory-installed through top entry. Side entry on special order.
- 23700 Cable Clamp
- 23898 Mounting Base

#### OPTIONAL ACCESSORIES
- 23937 Housing
- 24236 Sensor (Includes Installation Hardware Kit 24356)
- 2874M1 Triax Calibration Fixture

#### NOTES
1. In shock measurements, minimum pulse duration for half sine or triangular pulses should exceed 0.25 milliseconds to avoid excessive high frequency ringing.
2. Mounting is in the Z axis. It is normal for accelerometers with multi-axes to have reduced frequency response performance in the axes perpendicular to the mounting.
3. Transverse sensitivity is factory adjusted to be less than 3% before shipment. Replacement sensors must be measured and adjusted to ensure comparable performance.
4. Thermal 2-25 Zero Shift millivolts specified are at 10°F to 150°F (-23°C to +66°C), reference 75°F (24°C).
5. Rated excitation is 10.0 Vdc. The strain gage elements have a positive temp coefficient of resistance of approximately 0.5% per °F. Power supply current regulation capability should be carefully considered when operating at low temperature extremes.

6. Other excitation voltages may be used to 15.0 Vdc. Specify time of order to obtain a more accurate calibration.
7. Half-bridge input resistance measured across the excitation leads. It does not include external bridge completion resistance. Measured at approximately 1 Vdc. Bridge resistance increases with applied voltage due to heat dissipation in the strain gage elements.
8. Three pin solder terminations on each of three recessed surfaces. Cable entry holes for either side or top cable entry.

NOTE: Tighter specifications available on special order.
LINEAR AND ANGULAR SERVO ACCELEROMETERS

- DC Input — DC Output
- Small Size — Light Weight
- Electrical Self-test Feature
- High Reliability
- TSO Qualified Units
- MIL Qualified Units

Schaevitz servo accelerometers are DC-operated closed-loop force balance transducers for the measurement of acceleration. They are more stable and accurate than open-loop accelerometers by several orders of magnitude. Undesirable characteristics inherent in open-loop accelerometers such as sensitivity to supply voltage, non-linearity in the acceleration-to-position pickoff, and high thermal coefficients of scale factor and zero shift are negligible in closed-loop force balance accelerometers.

Typical applications of Schaevitz servo accelerometers are in high reliability guidance systems for missiles, torpedoes, or related military devices; stable platforms for spacecraft and shipboard requirements; monitoring and controlling deceleration in mass transit systems; and in roadbed analysis and fault detection equipment for high speed railways. Low range instruments can be used to level gun platforms, monitor the angle-of-elevation of naval guns or field artillery, and check slope and grade in earthmoving or paving operations.

Many Schaevitz servo accelerometers have been MIL-qualified for use in a broad variety of weapons systems. Schaevitz also offers servo accelerometers that have been TSO-qualified by the FAA for flight avionics. For nearly twenty years, high reliability for such critical applications has been a significant feature of Schaevitz inertial products. Depending on the application, the MTBF for Schaevitz servo accelerometers ranges between 25,000 and 100,000 hours.

In addition to the instruments offered in this catalog, Schaevitz Engineering designs and produces servo accelerometers and accelerometer systems (i.e., dual/triaxial/redundant), specifically for a customer’s particular application. These custom-designed units can be manufactured and tested in conformity with current military standards, utilizing the quality assurance procedures described in MIL-Q-9858A or MIL-Q-45208A.

Schaevitz servo accelerometers are DC-operated closed-loop force balance transducers for the measurement of acceleration. They are more stable and accurate than open-loop accelerometers by several orders of magnitude. Undesirable characteristics inherent in open-loop accelerometers such as sensitivity to supply voltage, non-linearity in the acceleration-to-position pickoff, and high thermal coefficients of scale factor and zero shift are negligible in closed-loop force balance accelerometers.

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PHYSICAL CHARACTERISTICS

SM Series

LSXH

SB Series

BENDIX CONNECTOR
TYPE PT1H:10-6P
MATING CONNECTOR PT06A:10-6SR
WHEN SEATED ADDS 1.76 TO HEIGHT

Schaevitz Engineering
U.S. Route 130 & Union Avenue
Pennsauken, N.J. 08110
Tel: (609) 662-6000
Fax: (609) 662-6281
Telex: (710) 892-0714

Schaevitz EM Ltd
543 Ipwich Road
Slough Berkshire, SL1 4EG England
Tel: (0753) 37622
Fax: (0753) 823563
Telex: 847818 G.

Schaevitz Sensing Systems
21640 N. 14th Avenue
Phoenix, AZ 85027-2839
Tel: (602) 545-3243
Fax: (602) 582-3520
Telex: (502) 32SENSYS

Circumstances may necessitate changes in specifications and/or prices without prior notice.
SB SERIES

SB series accelerometers are intended for general use in the measurement of acceleration, guidance control systems, vehicle ride analysis and a variety of other applications. A durable inertial sensor and miniature electronic module are integrated within common housing, leaving space for several optional electronic circuits including telemetry output, biased output and low impedance output. These accelerometers incorporate a built-in test capability and are available with pin or connector terminations.

Specifications at 20°C

<table>
<thead>
<tr>
<th>LSB Linear</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5</td>
<td>70</td>
<td>10</td>
<td>± 200</td>
<td>0.2</td>
<td>70</td>
</tr>
<tr>
<td>± 1.0</td>
<td>100</td>
<td>5</td>
<td>± 500</td>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>± 2.0</td>
<td>140</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 5.0</td>
<td>125</td>
<td>5</td>
<td>± 100</td>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>± 10.0</td>
<td>140</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 20.0</td>
<td>160</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 50.0</td>
<td>200</td>
<td>7</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
</tbody>
</table>

*Consult factory for other ranges*

<table>
<thead>
<tr>
<th>ASB Angular</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range*</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td>Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5</td>
<td>70</td>
<td>10</td>
<td>± 200</td>
<td>0.2</td>
<td>70</td>
</tr>
<tr>
<td>± 1.0</td>
<td>100</td>
<td>5</td>
<td>± 500</td>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>± 2.0</td>
<td>140</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 5.0</td>
<td>125</td>
<td>5</td>
<td>± 100</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

SM SERIES

SM series accelerometers have a volume of approximately one cubic inch and are intended for use where space is limited. Typical applications include tightly packed aerospace control devices such as inertial platforms and re-entry vehicles. The self-contained miniature electronic module utilizes a proprietary hybrid and is housed together with a rugged inertial sensor providing maximum performance and reliability in a minimum envelope. SM series accelerometers incorporate a built-in test capability.

Specifications at 20°C

<table>
<thead>
<tr>
<th>LSB Linear</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5</td>
<td>70</td>
<td>10</td>
<td>± 200</td>
<td>0.2</td>
<td>70</td>
</tr>
<tr>
<td>± 1.0</td>
<td>100</td>
<td>5</td>
<td>± 500</td>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>± 2.0</td>
<td>140</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 5.0</td>
<td>125</td>
<td>5</td>
<td>± 100</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

*Consult factory for other ranges*

<table>
<thead>
<tr>
<th>ASB Angular</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range*</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td>Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 0.5</td>
<td>70</td>
<td>10</td>
<td>± 200</td>
<td>0.2</td>
<td>70</td>
</tr>
<tr>
<td>± 1.0</td>
<td>100</td>
<td>5</td>
<td>± 500</td>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>± 2.0</td>
<td>140</td>
<td>2.5</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
<tr>
<td>± 5.0</td>
<td>125</td>
<td>5</td>
<td>± 100</td>
<td>1.0</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LSBH Linear</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td>Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 2.0</td>
<td>30</td>
<td>&lt;100</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
</tbody>
</table>

LXSH

The LXSH linear accelerometer is specifically designed for flight control applications. It features Schaeffel fluid filled for optimal flexure suspension system in a compact rectangular unit. The LXSH is exceptionally rugged, capable of withstanding extreme shock (1500 g) and vibration (35 g rms) environments. It incorporates a built-in test capability.

Specifications at 20°C

<table>
<thead>
<tr>
<th>LSBH Linear</th>
<th>Nominal Natural Output</th>
<th>Frequency Impedance</th>
<th>Range</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>Hz</td>
<td></td>
<td>Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>± 2.0</td>
<td>30</td>
<td>&lt;100</td>
<td>± 1000</td>
<td>2.0</td>
<td>120</td>
</tr>
</tbody>
</table>

LXSH Linear

<table>
<thead>
<tr>
<th>Nominal Natural Output</th>
<th>Range</th>
<th>Hz</th>
<th>kOhm</th>
</tr>
</thead>
<tbody>
<tr>
<td>± 2.0</td>
<td>30</td>
<td>&lt;100</td>
<td></td>
</tr>
</tbody>
</table>

LSXH

The specifications are based on best fit straight line generated by method of least squares. Note 2. Full range is defined as 'minus full scale to plus full scale'. Note 3. Cross-axis sensitivity, independent of alignment errors. Note 4. Multiple options may require a dimensionally non-standard housing.

Note 1. Linearity is based on best fit straight line generated by method of least squares.

Note 2. Full range is defined as 'minus full scale to plus full scale'.

Note 3. Static cross-axis sensitivity, independent of alignment errors.

Note 4. Multiple options may require a dimensionally non-standard housing.
Appendix E
Correction for Gravity Effects and of Transformation from Local to Global Co-ordinates

Before transforming the accelerometer data into global co-ordinates, initial correction for gravitational effects is required.

Note that subscript $r$ represents accelerometer reading in local co-ordinates and that subscript $t$ represents actual acceleration in local co-ordinates, subscript $tg$ represents actual acceleration in global co-ordinates and $g = -9.81\text{ms}^{-2}$.

$$\begin{align*}
\ddot{x}_r &= \ddot{x}_t + g \sin \theta \\
\ddot{y}_r &= \ddot{y}_t + g \cos \theta
\end{align*}$$

At $t=0$ the accelerometer readings are zeroed, giving

$$\begin{align*}
\ddot{x}_r &= 0 \\
\ddot{y}_r &= 0
\end{align*}$$

and

$$\begin{align*}
\ddot{x}_t &= \ddot{x}_r - g \sin \theta \\
\ddot{y}_t &= \ddot{y}_r - g \cos \theta - g
\end{align*}$$

Transforming into global co-ordinate gives

$$\begin{align*}
\ddot{x}_{tg} &= \ddot{x}_t \cos \theta - \ddot{y}_t \sin \theta \\
\ddot{y}_{tg} &= \ddot{x}_t \sin \theta + \ddot{y}_t \cos \theta
\end{align*}$$
Appendix F

Calculation of Head and Lower Leg Angular Acceleration

Calculation of Head Angular Acceleration

Linear Acceleration of the neck:

\[ X_n = x_b - \sin \theta_b \]
\[ \dot{X}_n = \dot{x}_b \]
\[ \ddot{X}_n = x_b - \cos \theta_b \dot{\theta}_b \]
\[ \dddot{X}_n = x_b + \sin \theta_b \dot{\theta}_b^2 - \cos \theta_b \ddot{\theta}_b \]
\[ Y_n = y_b + \cos \theta_b \]
\[ \dot{Y}_n = \dot{y}_b \]
\[ \ddot{Y}_n = y_b - \sin \theta_b \dot{\theta}_b \]
\[ \dddot{Y}_n = y_b - \cos \theta_b \dot{\theta}_b^2 - \sin \theta_b \ddot{\theta}_b \]

Linear Acceleration of the head:

\[ X_h = x_n - \sin \theta_h \]
\[ \dot{X}_h = \dot{x}_n \]
\[ \ddot{X}_h = x_n - \cos \theta_h \dot{\theta}_h \]
\[ \dddot{X}_h = x_n + \sin \theta_h \dot{\theta}_h^2 - \cos \theta_h \ddot{\theta}_h \]
\[ Y_h = y_n + \cos \theta_h \]
\[ \dot{Y}_h = \dot{y}_n \]
\[ \ddot{Y}_h = y_n - \sin \theta_h \dot{\theta}_h \]
\[ \dddot{Y}_h = y_n - \cos \theta_h \dot{\theta}_h^2 - \sin \theta_h \ddot{\theta}_h \]
Appendix F

Re-arranging the above gives:

\[
\ddot{\theta}_h = \frac{\ddot{X}_h + b\sin\theta_h \dot{\theta}_h^2 - \ddot{X}_h}{b}
\]

\[
\ddot{\theta}_h = \frac{\ddot{Y}_h - b\cos\theta_h \dot{\theta}_h^2 - \ddot{Y}_h}{b}
\]

Multiplying the top equation by \(\cos\theta_h\) and the bottom equation by \(\sin\theta_h\) and adding gives the angular acceleration of the head:

\[
\ddot{\theta}_h = \frac{\ddot{X}_h \cos\theta_h - \ddot{X}_h \cos\theta_h + \ddot{Y}_h \sin\theta_h - \ddot{Y}_h \sin\theta_h}{b}
\]

Calculation of Lower Leg Angular Acceleration

Linear Acceleration of the knee:

\[
X_k = X_f + a\cos\theta_f
\]

\[
\dot{X}_k = \dot{X}_f - a\sin\theta_f
\]

\[
\ddot{X}_k = \ddot{X}_f - a\cos\theta_f \dot{\theta}_f^2 - a\sin\theta_f \ddot{\theta}_f
\]

\[
Y_k = Y_f + a\sin\theta_f
\]

\[
\dot{Y}_k = \dot{Y}_f + a\cos\theta_f \dot{\theta}_f
\]

\[
\ddot{Y}_k = \ddot{Y}_f - a\sin\theta_f \dot{\theta}_f^2 + a\cos\theta_f \ddot{\theta}_f
\]

Linear Acceleration of the Lower Leg:

\[
X_l = X_k + b\sin\theta_l
\]

\[
\dot{X}_l = \dot{X}_k + b\cos\theta_l \dot{\theta}_l
\]

\[
\ddot{X}_l = \ddot{X}_k - b\sin\theta_l \dot{\theta}_l^2 + b\cos\theta_l \ddot{\theta}_l
\]

\[
Y_l = Y_k - b\cos\theta_l
\]

\[
\dot{Y}_l = \dot{Y}_k + b\sin\theta_l \dot{\theta}_l
\]

\[
\ddot{Y}_l = \ddot{Y}_k + b\cos\theta_l \dot{\theta}_l^2 + b\sin\theta_l \ddot{\theta}_l
\]

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Appendix F

Re-arranging the above gives:

\[
\ddot{\theta}_I = \frac{\ddot{X}_I + b\sin\theta_i\dot{\theta}_I^2 - \ddot{X}_k}{b}
\]

\[
\ddot{\theta}_I = \frac{\ddot{Y}_I - b\cos\theta_i\dot{\theta}_I^2 - \ddot{Y}_k}{b}
\]

Multiplying the top equation by \(\cos\theta_i\) and the bottom equation by \(\sin\theta_i\) and adding gives the angular acceleration of the lower leg:

\[
\ddot{\theta}_I = \frac{\dot{X}_I\cos\theta_i - \dot{X}_k\cos\theta_i + \dot{Y}_I\sin\theta_i - \dot{Y}_k\sin\theta_i}{b}
\]