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ENERGY ABSORPTION OF CAR CHASSIS RAILS UNDER IMPACT CONDITIONS

A. Otubushin

Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University.

To Susannah Otubushin
without whose early nurturing the seedlings would have been easily smothered by the weeds

and to Christopher Otubushin
for remaining a prince while surrounded by thieves

and to Jenny
for her love and understanding
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Abstract

Although the levels of safety offered to the occupants of cars has improved considerably in the recent past, car users still comprise more than 50% of all European road user fatalities. The predominant accident type experienced is a frontal impact with another car at speeds below 64 kmh (40 mph). This type of accident is reproduced by the new European offset deformable barrier frontal impact test requirement which supplements the fully distributed rigid barrier impact test that is still required in many countries around the world.

This thesis is concerned with the protection of car occupants, exposed to a variety of frontal impact conditions, via the structure ahead of the passenger compartment. The components that comprise that structure must be able to absorb energy by collapsing in a controlled and predictable manner throughout the range of impact speeds and regardless of the stiffness of the object struck. The front longitudinal chassis rails absorb much of the energy in a frontal impact and are therefore the focus of this thesis.

Analysis of the impact response of chassis rails is complicated by their non-uniform geometry and the fact that they are usually made of mild steel, a material whose mechanical properties depend on the rate of loading (strain-rate sensitivitiy). A generic chassis rail is therefore employed which takes the form of a uniform thin-walled tube subjected to axial impact, the likes of which have been studied both statically and dynamically in the literature.

In this research programme, non-linear finite element analysis is used to predict the response of mild steel tubular specimens to various impact regimes using both rigid targets and targets that also absorb energy. The numerical analysis method is first thoroughly validated by comparing predictions with previously available analytical estimates and experimental results. It is shown that energy absorption and collapse mode are well predicted if some form of imperfection is introduced in the numerical analysis which is otherwise too perfect.

Efficiency gains that could accrue from the careful choice of chassis rail geometry and the compromising effect of attached panels are shown. Strain-rate sensitivity and inertia effects are also de-coupled at various impact speeds to investigate changes in structural response when the mass and stiffness of the target is varied. The move to a deformable target could indeed alter the collapse mode and energy absorption distribution throughout the component. The inertia effect manifests itself as a change of mode shape but also a degree of axial plastic squashing was indicated. Practical design guidelines are given showing the impact speeds and mass ratios for which stable progressive collapse can be expected. Where this mode of collapse is not predicted, suggestions for encouraging it are given.
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Nomenclature

$A$ Material cross-sectional area of specimen, or area of contact segment
$A_o$ Cross-sectional area of specimen
$b$ Radius of toroidal shell element in the basic collapse element
$c$ Width of shorter side of rectangular tubular specimen
$C$ Circumferential length of a Basic Collapse Element
$d$ Width of longer side of a rectangular tubular specimen
$d_f$ Final displacement of drop-hammer
$D$ Coefficient in Cowper-Symonds relationship
$E$ Young's modulus or energy absorbed by a Basic Collapse Element
$E_t$ Strain hardening modulus
$E_c$ Energy absorbed in corner regions of specimen
$E_1$ Incident energy
$E_1, E_2, E_3$ Energy absorbed by various means
$E_a$ Maximum energy absorbed by specimen
$f_s$ Penalty stiffness factor in contact segment stiffness calculations
$G$ Shear modulus
$h$ Wall thickness of specimen
$h_d$ Drop height
$H$ Undeformed half-height of Basic Collapse Element
$K$ Spring stiffness, Kinetic Energy of drop-hammer prior to impact or Bulk modulus
$k$ Contact segment stiffness
$K_i$ Coefficient in Mahmood/Paluszny equation
$L$ Length of specimen
$l$ Characteristic length of contact surface on shell elements
$M$ Mass of drop-hammer
$M_o$ Fully plastic bending moment per unit width of sheet material $= \sigma_y h^2/4$
$m$ Mass of specimen
$p$ Coefficient in Cowper-Symonds constitutive equation
$p_m^s, p_m^d$ Static and dynamic mean crush load
$V_o$ Impact velocity
$V_m$ Mean velocity of striking mass during crush
$\beta$ Coefficient in Mahmood/Paluszny equation, or used to find buckling wavelength
$\delta_f, \delta$ Final crush distance
$\delta_1, \delta_2$ Effective crushing distance for modes I and II Basic Collapse Element
$e_{av}$ Average strain
$\dot{e}$ Strain-rate
\( \dot{\varepsilon}_{av} \) Average strain-rate
\( \dot{\varepsilon}_{\text{max}} \) Maximum strain-rate
\( \lambda \) Folding wavelength of a plate
\( \rho \) Mass density
\( \sigma_0 \) Average post-yield stress or flow stress, compensating for strain hardening
\( \tau \) \( E'/E \), material parameter used to calculate \( \lambda \)
\( \sigma^{s}_y, \sigma^{d}_y \) Quasi-static and dynamic yield stresses
\( \sigma^{s}_u, \sigma^{d}_u \) Quasi-static and dynamic ultimate tensile stresses
Chapter 1

Introduction
Road transport has become increasingly safe in the past few decades. Despite a 474% increase in vehicle population since 1951, the UK Department of Transport (DETR) [1] stated recently that there had only been a 35% increase in accidents resulting in personal injury. In Europe as a whole the trend in road traffic accident fatalities has in fact been generally downwards since the 1970's as shown in Figure 1.1 from the European Transport Safety Council (ETSC) [2]. However the DETR estimate that some 40,000 people were still killed on European roads in 1997, more than 21,000 of whom were car occupants. Several field studies [3, 4, 5, 6] have shown that car occupants comprise over 50% of all road user fatalities and the predominant accident type is a frontal impact with another car. Rather surprisingly, it was also shown by Pletschen et al. [7] that 90% of all injuries were sustained at energy equivalent speeds¹ less than 55 kmh (34 mph). Authorities seeking to reduce the road death toll would therefore do so most profitably by improving the level of protection offered in frontal impacts.

On the 1st October 1998 ECE Regulation 96 [8], a new European frontal impact test requirement came into force for manufacturers wishing to certify their cars for sale in the EU. In this test, cars travelling at 56 kmh (35 mph) strike a deformable barrier with a 40% overlap of the car's width towards the driver's side (Figure 1.2). The stiffness of the barrier simulates the front of a typical European car and two instrumented crash test dummies are seated in the front seats. In order to be certified for sale the maximum specified limits for the rearward and upward displacement of the steering wheel and trauma measurements from the dummies must not be exceeded.

Before October 1998 the frontal impact crashworthiness standard relating to cars manufactured for the European market had been ECE regulation 12 (ECE12) [9]. This standard specified steering wheel rearward and upward displacement limits for passenger cars travelling at 48 kmh (30 mph) when impacted against a rigid perpendicular barrier. The test originated from an early version of the Federal Motor Vehicle Safety Standard (FMVSS) 208 requirement in the USA [10] which has since been modified to include instrumented crash test dummies and a choice of approach angle varying between ±30° to the barrier normal direction.

The move to an offset deformable barrier impact requirement in Europe came as a result of data from accident reconstruction surveys during the 1980's. In some of these surveys researchers [11, 12] showed that the fully distributed frontal impact test specified in ECE12 was unrepresentative of real accident scenarios. Serious and fatal frontal impacts were predominantly car-to-car and an offset of 33-39% of the car's width was common. The

¹ Change in speed undergone by car during impact, estimated from residual crush.
researchers also found that there was generally an oblique nature to these impacts with the impact loads being concentrated to one side. Therefore, the introduction of ECE96 represents a step towards the real accident loadcase in the majority of frontal impacts, although FMVSS208 and ECE12 are still a good test of basic bodyshell strength. FMVSS208 and its derivatives are still current in the USA and in many other markets so car manufacturers need to ensure that their vehicles meet both ECE96 and FMVSS208.

Market forces have also played a part in further complicating the work of the crashworthiness engineer. The US Department of Transportation's National Highway Traffic and Safety Administration (NHTSA) conduct rigid barrier impacts on a selection of new cars and publish the results [13]. The impact speed for these tests is at 56 kmh (35 mph) leading to the current situation whereby manufacturers who want to avoid bad publicity make sure that their vehicles will pass this test at a higher speed. Usually they will set their in-house standard higher still at approximately 64 kmh (40 mph). The European New Car Assessment Programme (Euro-NCAP)² conduct the offset deformable barrier frontal impact test at the increased speed of 64 kmh, again with considerable embarrassment for any manufacturer who has not designed for this loadcase. There are also numerous insurance institution and motoring magazine frontal impact tests world-wide, all of which are derivatives of FMVSS208 and ECE96. The vehicle structure therefore must be able to collapse in a controlled and predictable manner throughout the range of impact speeds and stiffnesses of the object struck.

The uninitiated crashworthiness engineer might be tempted to design for the rigid barrier impact and assume that the car would automatically pass the deformable "softer" impact since, in the latter case, the impact energy is shared between the car and the barrier. Hobbs [14] recently showed that this would be an erroneous assumption because cars that successfully passed ECE12 with little or no passenger compartment intrusion sometimes suffered loss of passenger compartment integrity in a lower speed impact with another car. In the rigid barrier case the surface of the target ensures that the crash loads are channelled along the main structural members. The primary load paths for this type of impact are the two front longitudinal chassis rails, highlighted in Figure 1.3, and the passenger compartment bulkhead panel via direct loading on the engine.

In an offset impact only one of the chassis rails is generally involved and, in the majority of cases, the chassis rails of striking cars are not aligned causing them to strike relatively weak parts of the other car. Since these weak parts on both vehicles do not absorb much energy, structures further back, such as the passenger compartment must deform to bring the cars to

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² Co-sponsored by the EC, the Federation Internationale de l'Automobile (FIA), Alliance Internationale de Tourisme (AIT) and various consumer groups, Euro-NCAP regularly publish impact test results for the best-selling cars.
rest. Passenger compartment deformation is an undesirable outcome because it reduces the occupant survival space and leads to an increased risk of serious or fatal injury as documented by, for example, Rattenbury et al. [15] and Ward et al. [4].

In addition to the difference in the degree of offset between ECE12 and the ECE96 approximation to real impacts, there is also a difference in the deceleration levels. As both the car and the barrier absorb energy by crushing in ECE96 and the striking car travels a greater distance during the impact, this results in a lower deceleration than if it had hit a rigid barrier at the same closing speed. This will have some effect on the mode of deformation of the car structure due to the fact that it is usually made of steel, a material known to be sensitive to the rate of deformation (strain-rate sensitivity). One of the objectives of this study is to quantify that effect and to observe whether it is manifest at a component level. Understanding is therefore required as to whether the whole-car behaviour is reflected at the component level when comparing impacts with rigid and deformable targets.

**Scope and outline of this thesis**

This thesis is concerned with the protection of car occupants during frontal impacts. The greatest scope for occupant protection is achieved when the structure ahead of the passenger compartment absorbs the maximum amount of impact energy whilst transmitting the minimum possible force to the restrained occupants under a variety of impact conditions. In order to understand the counter-intuitive impact response of that complex structure it is necessary to acquire the crash characteristics of the components that constitute the system and control the impact performance. The thesis therefore concentrates in the chassis rail which is the primary energy absorbing structural component forward of the passenger compartment.

Due to engine bay packaging reasons, chassis rails often have a complicated geometry which makes them difficult to appraise using simple analytical methods. The thesis therefore concentrates on the behaviour of uniform section mild steel tubes as generic chassis rails subjected to axial impacts.

The study of the response of uniform prismatic tubes to dynamic impact is a well established method of identifying key factors that affect the complicated behaviour of non-uniform structures. However, theoretically exact appraisals of uniform section tubes undergoing large deformations under impact loading do not exist. The instability of the section and imperfection-sensitivity of sectional and material properties have resulted in a state-of-the-art where the dynamic response of structures are estimated from quasi-static approximations and empirical equations. Even in these cases the approximations are only valid for a small range of
impact speeds. In Chapter 2 experimental and analytical work that has been done in this area is reviewed. Important concepts such as mean load carrying capacity, energy absorption efficiency and approximate methods of modelling structural and material behaviour with large displacements are also introduced.

Scope for improved energy management in frontal impacts.
The majority of published work in this area does not include impacts with targets that themselves absorb energy. This thesis therefore seeks to extend knowledge in this area by utilising the capabilities of the non-linear finite element (FE) technique. FE analysis originally arose out of a desire to simplify the stress analysis of aircraft but only recently has the technique developed to a stage where it can be applied to crashworthiness problems with large deformations. The method is attractive because it works from first principles and offers the potential to capture transient, highly non-linear effects that are commonly found in crashworthiness problems.

Validation of method
In Chapter 3 the kinematical method of modelling the large scale collapse of prismatic tubes is described along with the theoretical foundations of the numerical analysis method used. The following chapter covers the thorough validation of the numerical analysis method comparing predictions with previously available analytical estimates and experimental results. It is shown that energy absorption and collapse mode are well predicted if care is taken to introduce some form of imperfection in the numerical analysis which is otherwise too perfect.

Evaluation of methods to improve energy absorption efficiency
The analysis technique is then used to show, in terms of efficiency, the advantages that could accrue from the careful choice of chassis rail cross-section and material thickness (Chapter 5). Another method of improving efficiency is to cut material out of areas that appear to be ineffective in a baseline analysis and guidelines are given as to the limits of this method. Flat plates of varying width were also added to the specimen to represent the inner wing attached to a chassis rail of a car and again the effects on efficiency are recorded.

A first attempt was made at analysing an impact with a deformable target in order to explore the effects on the efficiency of the specimen when the mass and stiffness of the target is varied. It was shown that the move to a deformable target could indeed alter the distribution of energy absorption throughout the component.
De-coupling of strain-rate and inertia effects

In Chapter 6 the inertia effects and strain-rate sensitivity of the material are de-coupled in the numerical analysis at various impact speeds. The inertia effect manifests itself as a change of mode shape as expected but also a variation in wall thickness is described indicating a degree of axial plastic squashing. Design curves are given for regions wherein a desirable controlled collapse can be expected for various mass ratios and impact speeds. Where an undesirable mode shape is to be expected a method of forcing the desirable mode is also given and evaluated. This method consists of impeding the progress of stress waves in the structure and appeared to be effective over a range of mass ratios and impact speeds. Future work would involve the recreating of these results under experimental conditions as described in the conclusions given in Chapter 7.
Figure 1.1. Trends in European road traffic fatalities and vehicle population since 1970. Source ETSC [2].

Figure 1.2. Offset deformable barrier impact test ECE96.
Figure 1.3  General location of front longitudinal chassis rails.
Chapter 2

Analysis tools for impact energy absorption through structural collapse
2.1 Introduction

In order to understand the response of a complex structure such as a motor car to the impulsive loads generated in a crash, it is necessary to acquire knowledge of the crash characteristics of structural components that make up the whole vehicle. The majority of car structures on today’s roads are fabricated from sheet steel; an exterior skin covers an assembly of thin-walled tubes of various cross-sections and stamped panels of diverse shapes and contours. The prediction of the response of these components to prescribed impacts is a task to be carried out prior to the construction of expensive prototype vehicles. In the past this predictive work was done exclusively by destructive component crush and bending tests at slow loading rates (quasi-static testing) and scaling the response by a factor to estimate the response at a fast loading rate (dynamic testing). Sub-assembly tests were subsequently carried out and finally full vehicle dynamic tests were performed using prototypes. It was not possible to apply classical theory to the predictive task because of the complex shapes and non-uniform sections employed on vehicles, the theory was used solely to guide the choice of sections and shapes.

Over the last two decades however, car manufacturers have been applying computer-aided methods of predicting component behaviour in an attempt to reduce the number of tests required before the construction of a prototype and thereby the development cost. Commercial computer programs such as CRASHCAD [16] have been used to aid the sectioning of structural members and other packages such as PAM CRASH, described by Haug et al.[17] and DYNA3D [18] to predict the dynamic response of components, sub-assemblies and sometimes full vehicles. These computer programs have much in common with classical analysis methods, a sound understanding of their strengths and limitations is required as is the use of good basic input data to realistically represent the material behaviour. For example, the well understood classical law due to Robert Hooke (1635-1702) relates the force experienced by a body to the deformation it undergoes via a linear constant. As long as that body exhibits what is termed linearly elastic behaviour, its response to a given force can be reliably and accurately predicted using this law. Hooke’s law is therefore extremely useful in predicting the elastic response of a wide range of structures made of various materials but it would not be valid to apply the law to the behaviour of say, wet clay which exhibits inelastic behaviour. Neither would the law apply to metals over their whole range of ductile deformation.

The above example serves to introduce the subject of approximation which is central to this thesis. Approximation is the process by which the human mind can be brought to understand the complexities of the physical world using relatively simple theories such as the above that appear to hold true for a certain set of circumstances. The bounds of application of the laws are then determined by experience and experiment through which new laws or modifications to the existing ones can be developed to satisfy a wider range of observed behaviour. This iterative
process is readily seen in the scientific and engineering literature where sometimes it is a new invention or technique that permits the proper observation of behaviour that had been previously misunderstood. Occasionally, however, the next iteration of theoretical development comes from the efforts of an individual, who asked questions that others had considered unimportant. The literature does not give an all-embracing theory that exactly predicts the behaviour of structures under all loading conditions but with intuition and experience, engineers have been able to choose which theories to apply for the problem in hand and most importantly to decide when the errors due to their approximations were significant and when they could be ignored. In this chapter the important approximations and concepts under-pinning this thesis are introduced. After briefly discussing the basic aims in frontal impact protection the historical development of impact energy management is presented. Classical theories and previous analytical approaches relevant to the present work are next discussed and compared with experimental studies. The chapter concludes by discussing the application of computers and finite element techniques to structural analysis.

2.2 Structural energy absorbing components - materials and geometry.

The aim when designing an energy-absorber is to maximise the work done during an impact. The following simple equation helps to illustrate the basic physics of the energy absorption process and the difficulties facing the crashworthiness engineer:

\[ E = P\delta. \] (2.1)

Here \( E \) is the energy absorbed by the structure when a force \( P \) forces it to, for example, crush through a distance \( \delta \). In order to maximise \( E \), either \( P \), \( \delta \) or both must be maximised. In a real structure the engineer must keep the forces transmitted through the structure below certain limits in order to protect the cargo being transported so an increase in \( P \) cannot generally be allowed. The ideal energy-absorber under these circumstances would therefore be very long and require a moderate force level to activate it. This luxury is seldom permitted for reasons of manoeuvrability, cost and styling among other factors so the crashworthiness engineer is left with a given crush distance and prescribed force limits within which to absorb the specified amount of energy. In order to achieve this the material invariably undergoes permanent (plastic) deformation as discussed in thorough reviews by Ezra and Fay [19] and Johnson and Reid [20, 21]. In these reviews thin-walled tubes with various cross-sectional shapes were shown to have received a great deal of theoretical and experimental attention due to their efficient energy absorption characteristics under axial loading. In these structures energy is dissipated through the formation of plastic hinges along which the material folds.
One of the earliest recorded applications of plasticity theory was by Baker [22] who designed the indoor "Morrison" air-raid shelter of World War II. This was a portal frame box structure which would absorb the energy of a collapsing house by the rotation of plastic hinges and still leave room for the occupants of the shelter located in the basement or ground floor. An earlier proposal which, as far as the author is aware, was not developed, was due to Fay [23] who proposed that expendable end sections be added to railway coaches and filled with the passengers' luggage. Space within the coaches would thereby be released for more profitable seating arrangements and the luggage in the end sections would serve the additional purpose of absorbing energy in the event of a crash.

However, more systematic transport-related studies into impact energy absorption by structural collapse began in earnest in the late fifties with the work reported by Pugsley and Macaulay [24], and Macaulay and Redwood [25] again in the locomotive industry. The collapse mode of circular and square tubes were found to be related to the thickness-to-radius or thickness-to-width ratio of the section and the speed of the impact. Under quasi-static loading, post-buckling collapse could begin anywhere in the specimen and often took the form of discrete folds separated by curved panel sections in square tubes. Under dynamic loading, collapse began at the struck end and the folds tended to be compact with no curved panel sections between them. Precise specification of desirable $h/c$ ratios were not given at the time but their specimen were of the order $h/c = 0.01$. Macaulay and Redwood proposed that the deformation pattern, distributed or concentrated, was determined by the longitudinal stress wave velocity in the material and the frequency of lateral vibrations.

A more recent study by Lowe et. al. [26] on idealised small scale motor coaches also revealed that different deformation patterns were obtained for quasi-static and dynamic loading conditions and this is discussed in detail in section 2.8. The mode of collapse is another important theme in this thesis because it not only affects how much energy a structure will absorb for a given crush but also the stability of the structure while crushing. In the compact crushing mode more material undergoes plastic deformation than in the non-compact mode for example. In order to quantify these observations the idea of structural energy absorbing efficiency was developed in the early eighties.

### 2.3 Energy absorbing efficiency

When assessing the usefulness of an energy absorber, it is helpful to be able to compare similar quantities for the various systems available. The efficiency of an energy absorber can be defined as the specific energy which is the energy absorbed per unit mass, or the volumetric
efficiency which is the volumetric ratio of the energy absorbing portion of the device to the total volume. For some devices this is equivalent to the stroke efficiency which is the usable stroke (shortening) of the device divided by the initial length. Ezra and Fay [19] and Coppa [27] define the ideal energy absorber as one which maintains the maximum allowable force throughout the stroke apart from elastic loading and unloading effects. Stroke efficiencies were given for several energy absorbers in [19] and axial crushing of tubes was found to be a suitable method of energy absorption for many practical applications, stroke efficiencies of 70% being possible.

Thornton et al. [28] supplement the above by suggesting that the product of the specific energy and the stroke efficiency is the appropriate method of defining the weight effectiveness of an energy absorber since this will highlight cases where an inefficient mode of collapse replaces the desired mode. They suggested that the more distributed deformation mode of folds separated by panel sections, as found in the early railway studies, could trigger global buckling around a local cross-section. Such global structural instability during collapse was shown to be an undesirable event since energy-absorbing efficiency was greatly reduced. That report was part of a series of conferences [29, 30, 31] dedicated to structural impact problems. Mahmood and Paluszny [32] also gave design criteria for tube geometries indicating that non-compact deformation characteristics were to be expected when crushing tubes with $h/c$ ratios less than 0.016.

2.4 Fundamental theories and their limitations

When a square tube is subjected to quasi-static axial loading the load required for a given deflection rises sharply to a peak after which, depending on the material properties and the tube geometry, further deflection may occur for a falling load. The peak is associated with the initiation of buckling failure of the flat plates forming the tube and a jump from an unstable compression to a more uniform buckling pattern under moderate load. Alternatively, after the peak, the tube may be unable to sustain further loading and bends around a cross-section in an inefficient and unstable buckling pattern. The precise outcome is dependent on the tube geometry and the material properties. The theories of elasticity, plasticity and elastic stability describe the deformation of solids under prescribed loading conditions and are based on experimental observations. Books by Johnson and Mellor [33] and Timoshenko and Gere [34] (Chapter 9) are chosen as particularly good sources of information on plasticity and elastic stability. Gerard [35] drew together previous work in a good introduction to structural stability theory and on page 24 he highlighted the importance of Shanley's work in settling the argument regarding the modelling of post-yield material behaviour in inelastic buckling of columns.
A fourth book that must be consulted as a starting point for studies in this area was written by Johnson [36] who concentrated on the response of materials to impacts. Of particular relevance to the present work was the introduction to stress wave propagation in uniform bars given in Chapter 5. All books are detailed and provide extensive references to other work.

The fundamental theories, as presented have a common shortcoming; they are generally valid only for small strains and curvatures of the order of the thickness of the material. In a concise presentation of theories and experimental work pertinent to structural impact, Jones [37] stated "Theoretical investigations into the influence of finite displacements or geometry changes on the static and dynamic behaviour of...structures have been hampered by the lack of simple theorems. Thus the response of various structural members has been explored on an ad-hoc basis." The elastic and inelastic buckling theories had reached the limits of their valid application and macroscopic models were required to predict the behaviour of structures exhibiting finite large displacements.

The method generally applied in the early eighties was one where the quasi-static load-deflection characteristics of the structure are assessed prior to examination of dynamic behaviour. A postulated collapse mechanism which matched experimentally observed collapse was used in making an energy (or work-rate) balance between the energy absorbed through plastic deformation and the external work done by an applied system of forces. An attempt was then usually made to factor the quasi-static response to fit experimentally observed dynamic behaviour. This process is greatly complicated by inertia and strain-rate effects as shall be discussed presently. It is first necessary to discuss some commonly used representations of the load-deflection characteristics of materials.

### 2.5 Approximate material models used in theories

The first task in a theoretical analysis of a mechanics problem such as that of a square tube is a method of representing the material behaviour. The stress-strain curve for mild steel, for example, is linear for loads up to the proportional limit so it would be straight-forward to represent this elastic regime mathematically by a linear relationship of the form of Hooke's law. Beyond the proportional limit, mild steel displays increasingly non-linear behaviour with discontinuities at the upper and lower yield stresses and various approximations must be made to render this complicated behaviour mathematically tractable. The most common of these approximations were given by Malvern [38] and Johnson [36] amongst others.
Four approximations of importance in this project are:-

a) rigid-plastic - where the material is assumed to be perfectly rigid (undergoes no elastic strain) until the yield stress is reached when it becomes perfectly plastic and undergoes infinite strain without further increase in load (no strain-hardening).

b) rigid, linear strain-hardening - this material is assumed to be rigid until the yield stress is reached after which it has a proportional relationship between stress and strain given by its strain-hardening modulus (post yield or plastic modulus).

c) elastic-plastic - this material deforms linearly up to the yield stress at which further deformation takes place at constant stress (no strain-hardening).

d) elastic, linear strain-hardening - the deformation of this material up till yield is linear followed by a linear strain-hardening of the material.

The rigid-plastic approximation is used most frequently in the theoretical appraisal of dynamic structural response. Studies which explore the accuracy of various approximations and simplifications were reviewed by Johnson [36], Symonds [39] and Jones [40]. More specifically to the crashworthiness problem, Yu [41] thoroughly re-examined the validity of the rigid-plastic approximation when analysing structures. Using three different analytical models, the case of the impulsively loaded cantilevered beam was used to show differences between the results obtained with dynamic elastic-plastic and rigid-plastic material approximations.

The long-standing energy ratio $R$ (input energy divided by the maximum energy that can be stored in the structure) was revealed not to be a unique index for judging whether elastic effects could be ignored. The mass ratio $\gamma$ (colliding mass/structural mass), significantly affected the deformation mechanism and energy dissipation pattern. The input energy ratio, $K_0/M_0$ ($K_0$ is the initial kinetic energy and $M_0$ the fully plastic moment) also played a significant role in the governing of dynamic elastic-plastic response of cantilevers. A table was given to illustrate how energy absorption in a cantilever was divided between two principal modes (rotating plastic hinge at root and travelling plastic hinge depending on the relative values of $\gamma$ and $R$. This is reproduced in Table 2.1.
Table 2.1 Modes of energy absorption related to impact conditions. Yu [41]

<table>
<thead>
<tr>
<th>Energy Ratio</th>
<th>Large $\gamma$</th>
<th>Small $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very large $R$</td>
<td>Similar to rigid-plastic approximation, travelling plastic hinge moves along the whole cantilever regardless of $\gamma$</td>
<td></td>
</tr>
<tr>
<td>$R (R = 100)$</td>
<td></td>
<td>Incomplete travelling hinge, root dissipates little energy</td>
</tr>
<tr>
<td>Large $R$</td>
<td>Most energy dissipated in root rotation.</td>
<td>Incomplete travelling hinge, root dissipates little energy</td>
</tr>
<tr>
<td>$(10 &lt; R &lt; 100)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Small $R$</td>
<td>Modal response, root dissipates almost all the energy.</td>
<td>Hinge travelling is delayed.</td>
</tr>
<tr>
<td>$(2 &lt; R &lt; 5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Very small $R$</td>
<td>Almost entirely elastic response with a little plasticity at root.</td>
<td>Very localised plastic deformation in region near tip.</td>
</tr>
<tr>
<td>$(.5 &lt; R &lt; 1)$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Yu concluded by stating that, for crashworthiness studies, the rigid-plastic idealisation was a good first order approximation only. For relatively large values of $R$ (say $R > 5$) less than 20% error could be expected. Analysts were warned that certain deformation modes may be missed by rigid-plastic analyses. The example given was of a transversely pulse-loaded beam that first deflected in the expected direction, then sprang back to oscillate around a mean deflection in the counter-intuitive direction. This type of anomalous response tended to occur for relatively small values of $R$.

The following section summarises the development of detailed analytical study into the energy-absorbing capability of prismatic tubes. Estimates for mean crush load, a useful measure of load bearing capacity of a tube, based on tube geometry, observed deformation pattern and material properties were developed by the various researchers and serve as a useful background to the present study.

### 2.6 The Basic Collapse Element

Wierzbicki and Abramowicz [42, 43] developed a kinematical method of analysing the crushing behaviour of thin-walled structures. Extensional and inextensional deformation paths for a flat sheet were shown by Hayduk and Wierzbicki [44] and are reproduced here in Figure 2.1. Combinations of these folding modes formed the Basic Collapse Elements (BCE) shown in Figure 2.2 which were used to predict the collapse behaviour of cruciform specimens. In addition to energy-dissipation through bending, extensional deformation was also permitted in the analysis and indeed these extensional contributions were found to account for a minimum of 30% of the total energy dissipated.
Wierzbicki and Abramowicz [43] observed that the symmetric (concertina) collapse mode for thin-walled square tubes consisted of four mode I BCEs. The energy absorbed by each of these was given and is summarised in Chapter 3. These energies were summed for the whole structure and the dimensionless mean crushing load under quasi-static conditions was then predicted as

\[ \frac{P_m}{M_o} = 38.27 \left( \sqrt[3]{\frac{c}{h}} \right) \]  \hspace{1cm} (2.2)

where

\[ M_o = \sigma_o \frac{h^2}{4} \]

is the fully plastic moment per unit width of sheet material. Abramowicz and Jones [45] used the kinematical method to analytically model the deformation observed in drop tests on square tube specimens. The above procedure was applied to symmetric collapse and two other experimentally observed collapse modes (asymmetric mixed A and B) and a postulated extensional collapse mode that was not observed at the time. All tube collapse modes were analysed using various combinations of mode I and II BCE and are shown in Figures 2.3 - 2.4. The authors then improved the definition of effective crushing distance for an axially crushed box column, \( \delta \), which was originally given by Abramowicz [46] as 70% of the initial length. It was observed that the dimensions of a flat sheet from which a corner developed was \( H\sqrt{2} \times 2\sqrt{2}H \) and not \( 2\sqrt{2}H \times 2\sqrt{2}H \) as given in [46]. Hence for mode I BCE's the effective crushing distance for symmetric deformation was

\[ \delta_1 = 0.73 \times 2H. \]  \hspace{1cm} (2.3)

For mode II BCE's \( \delta_2 \) was given by

\[ \delta_2 = 0.77 \times 2H. \]  \hspace{1cm} (2.4)

For symmetric axial crushing of square tube specimen in laboratory drop tests, the authors found fair agreement with average experimental values of 75% overall shortening of the specimen. It was also noted that asymmetric crushing tended to lead to global instability of the specimen. This was because of the different crushing distances of the mode I and II BCE's present in the asymmetric mixed collapse modes A and B.
2.7 Quasi-static crushing of rectangular and square tubes.

There have been several estimates for the mean crush load that can be expected for an axially crushed tube. Where possible they are expressed in dimensionless terms. Wierzbicki and Abramowicz [43] (for symmetric collapse mode):

\[
P_m^*/M_o = 38.27 \left( \frac{c+d}{2h} \right)^3
\]

Abramowicz and Jones [45], using an improved estimate of \( \delta_1 \), for symmetric collapse of square tubes:

\[
P_m^*/M_o = 52.22 \left( \frac{c}{h} \right)^3
\]

Wierzbicki [47]:

\[
P_m^*/M_o = 57.14 \left( \frac{c+d}{2h} \right)^3
\]

This was reached using the same kinematical approach but using different experimental observations of effective crush distance. The effective crush distance was stated as being 2/3 of the original length.

The most suitable value of flow stress to use in the equations above has been much discussed and is a matter for further research and judgement. For work-hardening materials, the yield stress may not be sufficiently accurate and some average value in the work-hardening range may be more suitable. Wierzbicki and Abramowicz [48] indicated that the difference between initial yield stress and ultimate strength could exceed 30% and gave an iterative method for obtaining \( \sigma_0 \) as a function of strain level in different parts of the BCE thus

\[
\frac{\sigma_0}{\sigma_\gamma} = 1.12 \left( \frac{h}{C} \right)^{0.044}
\]

where \( C \) was the circumferential length of the BCE.

Meng et al. [49] suggested:

\[
\sigma_0 = \sigma_\gamma + 5.66E_i \left( \frac{h}{c} \right) - 15.6E_i \left( \frac{h}{c} \right)^2
\]

2-9
where $E_t$ was the strain-hardening modulus. This was used to estimate the dimensionless mean load,

$$\frac{P_s}{\sqrt[3]{M_s}} = 32\sqrt{3}. \quad (2.10)$$

Thornton, Mahmood and Magee [28] used the tensile strength in their relationship for square tubes:

$$P_s = 16.98\sigma_s h^{1.8} c^{0.2}. \quad (2.11)$$

Mahmood and Paluszny [32]:

$$P_s = 2 \left[ \frac{K_i E}{\beta(1 - v^2)} \right]^{0.43} (1 + C/d) h^{1.86} d^{0.14} \sigma_s^{0.57} \quad (2.12)$$

Where values of $K_i$ were given in graphical form, plotted against aspect ratio $C/d$ and values of $\beta$ were given with respect to $h/d$. It was also stated that specimen with $h/c < 0.016$ would crush in a non-compact manner.

2.8 Dynamic plastic behaviour of structures

Returning to the study by Lowe et al. [26] on idealised small scale motor coaches where different deformation patterns were obtained for quasi-static and dynamic loading conditions, it was also noted that standard static methods of analysis with dynamic magnification factors were not always adequate to describe the response of the specimen to dynamic loads. Deformation mode changes such as these were believed to be a form of inertia effect and greatly complicated the theoretical analysis of energy absorption and crush characteristics. However the study of the uniform tube was seen as a useful method of gaining some insight into the behaviour of a wide range of vehicles and structural components and was adopted for this study.

2.8.1 Strain rate effects

Lindholm [50] defined ranges of strain-rate for which inertia forces, may be neglected. His chart is reproduced in Figure 2.5 and showed that for the range $10^{-1} < \varepsilon < 10^{1.5}$, mechanical resonance of the specimen was to be expected and inertia effects could not be neglected in any analysis. Strain-rates in typical car impacts are often considered to lie in this range and the
effects are highly non-linear and difficult to separate in any given situation. The post-yield flow of some materials is also sensitive to the rate of straining. This is known as material strain-rate-sensitivity. Sometimes referred to as viscoplasticity, this property was described by Campbell [51] in 1972. Viscoplasticity is independent of the structural geometry and is usually a strengthening effect; as the strain-rate increases, so does the resistance to yield. Various approximations can be made to the yield behaviour of materials at various strain-rates and the most common of these were discussed by Bodner [52] who summarised several fundamental experimental results [53, 54, 55, 56] where the tests were designed in such a way as to minimise inertia effects. Figure 2.6 shows Manjoine's [55] plot of stress-strain characteristics of mild steel at various strain-rates. The yield stress of mild steel may be double at strain-rates of the order present in car collisions. The ultimate tensile strength for mild steel, however, is not as sensitive to strain-rate as the yield strength as shown in Figure 2.7 which is a reproduction of Marsh and Campbell's [56] stress, strain, strain-rate surface for mild steel.

Symonds [57] also summarised the work of many previous experimental and analytical workers. With co-worker Cowper, Symonds developed a mathematical model for the behaviour of strain-rate sensitive materials such as mild steel [58]. The Cowper-Symonds constitutive equation may be written as:

\[ \dot{e} = D \left( \frac{\sigma}{\sigma_s} - 1 \right)^p \]  

(2.13)

or in its more common form

\[ \frac{\sigma_d}{\sigma_y} = 1 + (\dot{e}/D)^{\gamma_p} \]  

(2.14)

where \( D \) and \( p \) are empirical coefficients to best fit experimental stress-strain curves at different strain-rates. \( \sigma_s \) is the quasi-static yield stress and \( \sigma_d \) is the dynamic yield stress pertaining to the current value of strain-rate. For small strains in the neighbourhood of the yield stress, the values of \( D = 40.4 \text{ s}^{-1} \) and \( p = 5 \) for the empirical coefficients have generally been considered to work well. The value of \( \dot{e} \) to employ when trying to analytically model a crush tube is still an area requiring work and will be addressed in this thesis.

Perrone [59] noticed that the dynamic flow stress did not decrease significantly from the initial value until 90% of the incident energy had been absorbed. He therefore proposed that the initial strain-rate could be used in a constitutive equation to calculate a time-independent dynamic flow stress with reasonable accuracy. This is known as Perrone's rule and has been accepted as working well for modest deformations where there are no geometry changes.
An attempt to extend the rule for large deformations was made by Perrone and Bhadra [60] but complicated numerical analysis was required to solve a simple problem of a mass suspended by two horizontal massless wires made of rigid, plastic strain-rate-sensitive material and subjected to an impulsive velocity. It was also noticed that the dynamic flow stresses in the wires remained virtually constant with respect to the kinetic energy of the mass throughout the response except for relative short initial and final periods. It would therefore appear reasonable to use Perrone's rule but the dynamic flow stress corresponding to the initial strain-rate may not always be appropriate. A check should also be made for any possible inertia effects.

2.8.2 Inertia effects

As previously mentioned, inertia effects can alter the mode of deformation of a structure between quasi-static and dynamic loadcases. The effects are difficult to quantify but their magnitude depends on the impact velocity and the mass ratio of the colliding structures. Impulsively loaded cantilever studies by Parkes [61] and Bodner and Symonds [62] are good examples of the effect of inertia on a structure. Two phases of energy absorption were identified for this problem. In the first, a plastic hinge formed at the tip and travelled towards the root. In the second phase the remaining energy was dissipated by the rotation of a fixed plastic hinge at the root, the remainder of the beam behaving as a rigid body. In the analytical models presented in those reports, the two phases absorbed proportions of the incident energy according to the ratio of the tip mass to the beam mass. Strain-rate effects were also present in those studies however, and analysis was complicated by this. Inertia effects were recently more clearly defined by workers concerned with the scaling of impact problems between model and prototype [63, 64, 65, 66, 67].

2.8.3 Impact Scalability and the Concept of Micro-inertia.

Booth, Collier and Miles [65] found that the impact characteristics of thin, plated steel structures did not scale linearly from model to prototype. In an appendix to that paper, Calladine attributed the departure from linear scalability partially to the quasi-static characteristics of the structures. In a later paper, Calladine and English [66] developed non-dimensional displacement-impact velocity relationships for two types of structure based on idealised quasi-static force-deflection curves for each. The displacement-impact velocity relationships clearly showed that structures whose quasi-static load/deflection curve falls sharply after an initial "peak" (type 2) were much more velocity-sensitive than those with a "flat-topped" (type 1) curve. Figure 2.8 shows the two types of structure and their idealised quasi-static load-deflection curves.
Calladine and English [66] then carried out some drop-hammer tests which showed that inertia effects scaled differently from strain-rate effects. There was some scatter in the results but type 1 structures followed the theory quite well. With type 2 structures, approximately 25% of the impact energy was unaccounted for by the theory and it was suggested that another mechanism was responsible for the loss of this significant amount of energy. This phenomenon was further studied by Tam and Calladine [67].

2.8.4 The work of Tam, Calladine Zhang and Yu

In a recent study concentrating on type 2 structures, Tam and Calladine [67] introduced more variables and material types. Dimensional analysis was used in an attempt to discover why type 2 structures appeared to be more velocity sensitive than type 1 [66]. An important dimensionless energy ratio was developed:

\[ R = \frac{K}{SE} \] (2.15)

Where \( K \) was the kinetic energy of the striking mass prior to impact and \( SE \) was the static energy required to produce the same amount of deformation as obtained under dynamic conditions. This ratio thus included every effect, strain-rate sensitivity, inertia and degree of pre-failing. \( R \) was then tested against the three significant dimensionless groups that had been identified:

\[ \frac{GV^2}{\sigma_y L^3, \rho L^3, \theta_o} \] (2.16)

where \( G \) was the striking mass, \( V_0 \) the striking velocity and \( \theta_o \) represented the initial crookedness. Zhang and Yu [68] separately postulated that the impact event was split into two phases. Phase 1 was an instantaneous inelastic collision between the striking mass and the top of the specimen. Energy in phase 1 was dissipated by uniaxial "squashing" of the top of the specimen. The remaining energy \( T_2 \) was dissipated in Phase 2 of the impact by rotation of plastic hinges in the accepted manner. Thus if the mode of collapse was the same for static and dynamic conditions and the yield stress was insensitive to strain-rate \( (T_2 = SE) \), the amount of energy lost in Phase 1 could be calculated from:

\[ \frac{K}{T_2} = 1 + \frac{m}{120 \sigma_y G} \] (2.17)
If the mass ratio was defined as $\mu = \frac{G}{m}$ and the mode shape and strain-rate conditions applied, then

$$\frac{K}{SE} = 1 + \frac{1}{12\mu \theta^2}$$

(2.18)

Tam and Calladine [67] refined the theory of Zhang and Yu and found that some laboratory specimens exhibited a 1% overall shortening. It was concluded that regardless of the exact method by which the energy in phase I was "lost", the amount of energy was determined by momentum considerations only. It was indicated that up to 25% of the incident energy in their impacts was dissipated in phase 1 which was quite a substantial amount.

Further development of the analytical model has recently been described by Karagiozova and Jones [69, 70] and Su, et al. [71, 72] who separately included elasticity and strain-hardening into the models. Karagiozova and Jones showed that the material strain-hardening parameter influenced the critical impact velocity, determined by the maximum kinetic energy at the transition between stable and unstable behaviour after impact. Comparing a wholly elastic model to an elastic-plastic model with strain-hardening in ref. [69], it was observed that, for relatively small masses, the response of the elastic-plastic model was dominated by compression which absorbed a substantial part of the initial kinetic energy. With less energy available for subsequent overall bending therefore, higher critical velocities were achieved by the elastic-plastic model than for the elastic model. Conversely, large impact masses caused mainly an overall elastic-plastic bending and, due to the lower stiffness of the elastic-plastic model, instability occurred at lower impact velocities than for the fully elastic case.

In ref. [70], Karagiozova and Jones found reasonable agreement between their prediction and the results of Tam and Calladine. Further insight was gained into the elastic loading and unloading effects, a theme that was echoed by Su et al. [71] who stated that the inclusion of elasticity was necessary to determine the magnitude of the peak load. It was interesting to note that this model neglected strain-hardening and strain-rate effects but still showed a significant difference between quasi-static and dynamic structural behaviour. This difference was attributed to the inertia and led to the suggestion that these structures be renamed "inertia-sensitive" rather than "velocity-sensitive". In [72] the authors included strain-rate sensitivity in their elastic-plastic model and found even larger increases in peak force due to the greater amount of stored energy. Again reasonable agreement was found between the predictions of the rate-dependant model and the experiments of Tam and Calladine, especially for mild steel specimens.
These developments are important in improving understanding of the parts played by elasticity, inertia, strain-hardening and strain-rate sensitivity in the crash response of structures. It is clear that an analytical model that is capable of including all these aspects of material behaviour would be a powerful tool that could be used to further classify structures in terms of predicted behaviour. Given that uniform square tubes can be thought of as type II structures evidence of these phase 1 and phase 2 effects should be discernible to some degree in this study.

2.8.5 Wave effects
Stress waves become increasingly important as impact speed increases. The longitudinal stress wave speed in a material was given by Johnson amongst others [36]:

$$\sqrt{\frac{d\sigma}{de}}$$

where \(d\sigma/de\) is the slope of the stress-strain curve. Elastic stress waves will thus move at a speed of

$$\sqrt{\frac{E}{\rho}}$$

and they will be followed by a much slower plastic wave, if the impact force is sufficiently high, the speed of which is dependent on the post-yield slope of the stress-strain curve of the material. The elastic wave may rebound, be refracted or pass through other waves. In hypervelocity impacts elastic and plastic waves may join and form a shock wave. Until the 1990s it was generally assumed that, if the loading time was long in comparison with the propagation time of elastic waves in the structure, then wave effects could be ignored. This assumption should be re-visited in the light of recent research and the analyses described in Chapter 6 of this thesis.

2.8.6 Dynamic crushing of square and rectangular tubes
The literature presents many predictive estimates for the dynamic mean crush load the majority of which were based on experimental observations of tests at moderate speeds. These speeds ensured that the mode shape obtained was similar to the quasi-static mode (i.e. no inertia effects were assumed to occur). The predictive estimates were therefore predominantly obtained by simply factoring the observed quasi-static response. The tests were mostly of a drop-hammer nature where a tube was struck by a moving mass. None of the scaling factors
considered non-compact collapse of the specimen and the only one that permitted extensional contributions to energy absorption was that of Abramowicz and Jones [45].

Abramowicz and Jones proposed:

$$\dot{\varepsilon} = \frac{h V_m}{2 b \delta} \quad (2.21)$$

as a method of estimating strain-rate. $V_m$ was the mean velocity of the striking mass during crushing, $b$ the radius of the toroidal shell element described in Chapter 3.0 and $\delta$ the effective crushing distance. For symmetric collapse this reduced to

$$\dot{\varepsilon} = 0.33 \frac{V_o}{c} \quad (2.22)$$

where $V_o$ was the impact velocity. The use of the ultimate tensile strength in the Cowper-Symonds relation and new values for the empirical constants $D$ and $p$ were also suggested thus:

$$\frac{\sigma_u^d}{\sigma_u^s} = 1 + \left( \frac{\dot{\varepsilon}}{6844} \right)^{3.91} \quad (2.23)$$

The data of Campbell and Cooper [73] were chosen because a plot of UTS against strain-rate was given and the strains that were predicted by the analysis would put their specimens in the region of ultimate stress. Figure 2.9 shows their curve fitting exercise. Abramowicz and Jones [45] also gave a predictive equation for the dynamic dimensionless mean crush load for symmetric buckling of square tubes:

$$\frac{P^d}{M_o} = 52.22 \left(1 + \left(0.33 \frac{V_o}{c D}\right)^{3.3}\right) \left(\frac{c}{\delta}\right)^{3.5} \quad (2.24)$$

A selection of other estimates are given below in approximate chronological order:

Pugsley [24]:'

$$\frac{P^d}{P_m} = 1 + \frac{V_o}{50} \quad V_o \text{ in fts}^{-1} \quad (2.25)$$
Ohkubo, Akamatsu and Shirasawa [74]:

\[
\frac{P^{d}}{P^{m}} = 1 + 0.0668V_{o} \quad V_{o} \text{ in ms}^{-1} \quad (2.26)
\]

This analysis only permitted bending contributions to energy absorption.

Wimmer [75]:

\[
\frac{P^{d}}{P^{m}} = 1 + 0.07V_{o}^{0.82} \quad V_{o} \text{ in ms}^{-1} \quad (2.27)
\]

for a square tube with \( c = 50 \text{ mm} \) and \( h = 1.5 \text{ mm} \).

Wierzbicki and Akerström [76]:

\[
\frac{P^{d}}{P^{m}} = 1 + 0.11V_{o}^{0.7} \quad V_{o} \text{ in ms}^{-1} \quad (2.28)
\]

derived using \( p = 1, D = 300 \) in the Cowper-Symonds equation.

Wierzbicki, Molnar and Matolcsy [77]:

\[
\frac{P^{d}}{P^{m}} = 1 + 0.1V_{o}^{0.714} \quad V_{o} \text{ in ms}^{-1} \quad (2.29)
\]

The kinematical approach used by Abramowicz and Jones is considered to be the most thorough procedure and is detailed in Chapter 3. Recent work reported by Abramowicz and Jones [78] concerned a series of impact tests on circular and square section tubes of comparable length to automotive chassis rails. The object was to investigate the transition from an initial Euler-type buckling displayed by some of their specimens to a stable axial collapse and the dependence of this transition on column length. The specimens analysed in Chapter 6 of this thesis will be compared to some of those in that study.

### 2.9 The Finite Element Method

The method of analysis used in this research programme was the FE method which arose out of a desire to break down the stress analysis of aircraft into manageable components. Turner et al. [79] based their method of analysis on the principle of virtual work and equilibrium of
forces but Oden [80, 81] suggested a mathematical approach which has since been applied and further developed for several engineering fields as detailed by Zienkiewicz and Taylor [82] who have championed the method with many collaborators since the early sixties.

A continuum is represented as an assembly of elements connected together at discreet points on their boundaries called nodes; one-dimensional bar elements (for axial actions), beam elements (for bending actions), and frame elements (for axial, bending and torsional actions) were used in the early years. Two-and three-dimensional elements such as plates, shells and solids were later developments. The distribution of variables such as displacement or stress within each element was related to the value at the nodes by an interpolation function called a shape function. Displacement of a node in one element affected other elements sharing that node and so on through the structure.

In order to determine the distribution of displacements or stresses in a structure, a system of simultaneous equations would be assembled and solved for the whole structure to yield the nodal values of the desired variable for given external forces and restraints. The choice of nodal variable for evaluation depended on the application; so in stress analysis for example, the stress or the displacement may be chosen as a variable. The analysis would then be said to have been carried out using the "force" or "displacement" method respectively. Both methods were equally popular in the early years (see, for example, refs. [83, 84]) but the displacement method has since become the most widely applied and well documented.

Use of FE analysis was limited to linear analysis until the seventies by which time the large volume of published research in the area of non-linear constitutive equations undertaken in the previous two decades was widely understood. Until then finite element analysis was only applicable to structures composed of elastic materials and even with this restriction, the use of computers was necessary for the analysis of structures with many degrees of freedom. Computers were found to be particularly useful tools in FE analysis due to the large numbers of computations required when real structures with infinite degrees of freedom are approximated by an equivalent system (discretised) with a large but finite number of degrees of freedom. Books by Majid [85] and Crisfield [86] are early and more recent examples of texts dealing with non-linear structural analysis which contain computer program design guides for non-linear structural analysis. Inelastic buckling problems were thereby brought into the set of problems addressable by FE techniques but the large displacements with possible contacts that occur in crashworthiness problems were still only feasibly analysed using lumped parameter models such as that reported by McHenry [87] in 1965 and Kamal in 1970 [88]. These models were quite simple to implement and continue to be used for initial design exercises (see for example Skuse and Grew [89] ) but could not be used for accurate predictions of component behaviour based on geometry and constitution.
2.9.1 Implicit versus explicit time integration

In FE analysis of structures undergoing large deformations a set of non-linear partial differential equations of motion are solved given initial and boundary conditions. The equations are cast in the space-time domain and can either be coupled to or be independent of the material stress-strain relationship. The structure is first discretised in space using a weak variational formulation of the problem such as the principle of virtual work and assuming a continuous displacement field throughout the structure. The resulting second order partial differential equations are then discretised with respect to time using a method such as that due to Newmark [90]. The analysis is termed "implicit" if the solution integrals are chosen so as to couple the equations. For each time-step a stiffness matrix is assembled for the structure and the required stresses or displacements computed. Implicit analyses are unconditionally stable but require a large computational effort in the assembly procedure for each time-step. The equation solver is also less likely to converge on a particular solution in cases of high non-linearity using this method.

"Explicit" solvers de-couple the equations and do not require the assembly of system matrices at each time-step. However, the procedure is only stable if the time-step is less than a critical value proportional to the shortest time it takes a sound wave to travel between two adjacent nodes. Comparisons between the two integration schemes were given by Haug [17], Hallquist and Benson [91] and Winter and Pifco [92]. In computational terms therefore, explicit solvers require a large number of time-steps each of which are low in demand whereas implicit solvers require fewer time-steps each of which are high in demand. The time-step for implicit solvers is approximately 2-3 orders of magnitude greater than those for explicit ones. It may seem surprising then that, with the improving efficiency and capability of computers, explicit solvers have become the preferred choice for commercial crashworthiness FE codes. A possible explanation could be in the nature of crashworthiness work where wave-effects could be missed if the time-step is too large. Even with variable time-step integrators such as those described by Belytschko and Schoenbeule [93], FE codes with implicit time integration have not been as widely applied as those with explicit integration.

The equations of motion to be solved for explicit FE analysis can be written as

\[ m\ddot{x} + f^{(in)} - f^{(ex)} = 0 \]  

(2.30)

where \( m \) is the mass matrix of the structure, \( \ddot{x} \) is the nodal acceleration vector, \( f^{(in)} \) is the nodal force vector and \( f^{(ex)} \) is the external force vector.
The central difference method can be used to integrate the above set of equations with respect to time thus:

\[ \dot{x}^{(n)} = m^{-1} \left( f^{(ex)}(n) - f^{(int)}(n) \right) \]  
(2.31)

\[ x^{(n + \frac{1}{2})} = x^{(n - \frac{1}{2})} + \dot{x}^{(n)} \Delta t^{(n)} \]  
(2.32)

\[ x^{(n+1)} = x^{(n)} + \dot{x}^{(n + \frac{1}{2})} \Delta t^{(n + \frac{1}{2})} \]  
(2.33)

where \( \Delta t^{(n)} \) is the integration time-step at step \( n \) and

\[ \Delta t^{(n + \frac{1}{2})} = \frac{1}{2} \left( \Delta t^{(n)} + \Delta t^{(n+1)} \right). \]  
(2.34)

The geometry is updated by adding the displacement increments to the initial geometry

\[ d^{(n+1)} = d^0 + x^{(n+1)}. \]  
(2.35)

The Cauchy stress is updated for each element and the solution proceeds to the next time step until the termination condition is reached.

### 2.9.2 The development of DYNA3D

The development of an explicit finite element computer code that was capable of predicting the non-linear, large scale deformation of solids with self-contact was carried out at the Lawrence Livermore National Laboratories by Hallquist [94] in the mid- to late seventies. Continuous developments and additions to the program made during the eighties permitted the incorporation of discrete, lumped parameter elements such as springs and dampers, and improved contact search algorithms and shell formulations [95]. This has led to the increased flexibility of the program since it can be used from the initial stages of design through to the full prototype test. There are many other FE codes, some of which are based on the same Lawrence Livermore source programme. Their basic mode of operation is similar and differences that occur usually arise from the particular installation and data processing. Figure 2.10 shows a simplified flow chart of the operation of the code given by Holmes et al. [96]
2.9.3 Application of FE techniques to structural analysis

Wierzbicki and Abramowicz [97] outlined the development of a superfolding element based on the BCE. The advantage of this element was demonstrated by comparing the number of standard finite elements required to properly model a fold that could be replaced with a single superfolding element. It was indicated that coarse FE meshes led to a model that was too stiff leading to "membrane locking" of the elements and unreliable calculations. It was also suggested that poor shape representation during folding could lead to inaccurate modelling of self-contact and thus incorrect plastic folding wavelengths. A maximum element size of $8h$ was recommended for standard FE meshes which was comparable to the radius of curvature of a fold as given by Wierzbicki and Abramowicz [43]. A rather pessimistic conclusion was reached regarding standard FE mesh densities and is re-stated here: "It is likely that satisfactory accuracy in describing the fold lines could only be achieved by bringing down the mesh size to the (wall) thickness dimension."

The material model used for the superfolding element was a rigid-plastic one justified by their previous findings [98] that, for mild steel, the average flow stress in the plastically deforming zones falls between 90% and 95% of the ultimate stress. It was shown that the mean crushing force of the superfolding element was dependant on the initial angle of the sheet metal from which the BCE was formed, the highest value occurring when the angle was $2\pi/3$. Thus, it was indicated, hexagonal columns would have the highest energy absorption capabilities of all prismatic multi-corner columns of equivalent material cross-sectional area. This was perhaps an over-simplification since a less favourable initial corner angle could be offset by a greater number of corners as the section approached a circle. In spite of this the use of hexagonal sections seemed to represent a worthy avenue of investigation. If car manufacturers were prepared to move away from the traditional rectangular sections on chassis rails, there would seem to be scope for more efficient crash energy management. However, this would only apply to uniform sections that collapsed in the axisymmetric mode. Car chassis rails are seldom uniform in section along their entire length and additionally contain curved regions so as not to foul drive shafts. Simple analytical methods of appraising such sections under dynamic loading conditions have not been reported in the literature but some workers have begun to address the non-uniform section (an area of study to which superfolding elements are not suitable) using FE techniques.

Mahmood, Paluszny and Tang [99] attempted to speed up the computer analysis of non-uniform thin-walled sections and noted that stresses in the structure as a whole rarely reached the yield value. The force-deflection characteristics of the structure was therefore controlled by local buckling and subsequent collapse of the section. This process was divided into four regimes, linear, post-buckling, crippling and deep collapse. In the post-buckling regime, only
the part of the section that contributed to load-carrying was used to calculate section properties. In the deep collapse regime, section properties were calculated by considering an appropriate collapse mechanism (such as the BCE). The analysis procedure consisted of two stages. In the first a FE elastic analysis was carried out on the structure to find the locations of initial yield. The structure was then replaced by finite beam elements ensuring that a node was located at each yield location. Stiffness matrices were then developed for the beam structure for each of the four regimes, incremental loads applied and the response of the structure predicted. Maximum force and energy dissipation predictions compared well with commissioned bending and axial crush tests but poor deep collapse modelling was displayed due to inappropriate representation of the collapse mode. The two-stage process, though cumbersome had the appeal of simplifying a complex structure to a more manageable equivalent structure. However, analysis was limited to quasi-static crush conditions only. If a structure's response differed markedly between quasi-static and dynamic loading conditions the equivalent structure approach would not be appropriate.

Kitagawa, Hagiwara and Tsuda [100] also used a two-stage method to improve the crash performance of a FE model of a non-uniform chassis rail subject to a rigid barrier impact. A buckling analysis was first carried out with incremental loads and the determinant of the stiffness matrix was checked at each step. At the buckling point the mode shape was noted and sites for beneficial imperfections in the form of "beads" (swages) were chosen. The three types of bead used are shown in Figure 2.11. Individually, the edge, concave and convex beads were reported to reduce the initial peak load by 16% 14% and 14% respectively. By suitable sympathetic arrangement, the chassis rail (with attached 500 kg mass subjected to a 35 mph impact) deformed predominantly axially up to 15 ms whereas the baseline model started deforming in a predominantly bending mode at 7.5 ms. Figure 2.12 shows the improvement in buckling mode. Unfortunately no Figures were given for the overall improvement in energy absorption.

### 2.9.4 Whole car modelling

Whole car dynamic, non-linear FE analyses are now commonplace and widely reported. They are extremely resource-intensive requiring months of model preparation and over a day of central processor unit (CPU) time on expensive supercomputers. Developments in this area are therefore mostly on a co-operative basis between car manufacturers and consulting engineers or government departments. The following studies were chosen from the large quantity of published work in recent years partly because DYNA3D was involved in the analysis.

Miles et al. [101] reported on a detailed and revealing study using OASYS DYNA3D to investigate the crash response of a full vehicle to a variety of proposed frontal impact barrier
test scenarios. The models contained approximately 30,000 elements and required approximately 30 CPU hours to run a 100 millisecond analysis on a CRAY-YMP computer. The fidelity of the B-post acceleration histories were checked against laboratory tests and were satisfactory.

The front longitudinals were found to be the principal energy-absorbing members but when the energy was summed for the pair, they only absorbed 25-35% of the initial impact energy. This remained a consistent proportion for both offset and fully overlapping impacts against rigid and deformable targets. For fully overlapping rigid zero-degree frontal impacts, over half of the impact energy was absorbed by the front structure (excluding longitudinals), subframe, firewall and dash. This indicated that the effectiveness of the longitudinals was being hampered by "bridging", i.e. the engine bay "locks up" and forms a load path to the firewall. For a fully overlapping zero-degree impact into a deformable barrier, 45% of the input energy was absorbed by the front structure, subframe, firewall and dash, so a similar bridging effect also occurred for that test scenario.

Although the study was still at an early stage (different types of vehicle were yet to be modelled and the foam material used to simulate the deformable barrier was still under development), the value of FE analysis in non-linear dynamic situations was clearly demonstrated. Full vehicle simulations allowed the analyst to determine which structures controlled crash performance. It was also possible that a front longitudinal that functions well in isolation is compromised in its energy-absorbing function by other engine bay structures. This warning was given by Kaiser [102] who found that components such as sills, struts and frames frequently behaved differently in full car simulations than they did in isolation.

In a separate study considering only the front portion of a car, Sheh et al. [103] compared an FE model and a lumped parameter model with experimental results. The report gave details of some pitfalls to be avoided in detailed modelling and meshing. Different meshing and mesh refinements gave rise to different mode shapes though force-deflection characteristics were usually similar. Film analysis of crush sequence was used in conjunction with velocity and force histories to compare simulation and experiment. Good general agreement was found but the lumped parameter model was prone to resonance.
2.10 Summary of the chapter.

The study of uniform prismatic tubes subjected to axial impact has received much attention in the literature and the analysis of the impact performance of mild steel specimens has been complicated by two effects:

- the yield stress of mild steel may be twice as high at strain-rates of the order present in car impacts; chassis rails therefore do not routinely deform in the desired manner

- the inertia of some structures cause them to collapse in a different mode under dynamic loading than they do under quasi-static conditions.

These two effects are difficult to separate in any given structure but the study of uniform section square tubes is a useful first step towards understanding the complicated behaviour of non-uniform chassis rails. There are no straightforward general methods of mathematically describing inertia effects but they are known to scale differently between models and prototypes. A method of analysing full-sized structures that has the potential to also include these effects is the explicit non-linear FE analysis technique employed here.

Front longitudinals are jointly responsible for absorbing approximately 30% of frontal impact energy despite the fact that their function may be hampered by other engine bay structures. Improvements on this figure would therefore greatly contribute to occupant protection. Some of these improvements may be obtained by choice of section and the literature indicates that hexagonal section tubes are potentially more efficient than square ones.

Buckling analyses prior to dynamic impacts have been shown to help identify buckling points and aid the designer in locating beneficial imperfections that inhibit or delay Euler-type buckling. If changes in buckling mode from rigid to deformable impact can be quantified, it may be possible to modify the geometry of the chassis rail so that it will collapse efficiently for the variety of impact types found in real accidents.

Explicit non-linear finite element analysis has developed to a stage where exhaustive and expensive development tests in laboratories are unnecessary. However, accurate modelling of the mechanical properties of materials and their variation over time for large deformations is required to achieve useful results.
The literature presented here is by no means exhaustive. The author has tried to discuss the work that has been most influential in guiding the research given the original aims. The author is unaware of any analytical or experimental attempt to quantify the behaviour of uniform prismatic tubes under impact with non-rigid targets. This is not surprising since the mathematics for impacts with rigid targets is already sufficiently complex. However it is important to understand the effects on the energy absorption efficiency of a component when a transition is made from a rigid target to a deformable one. The suspicion is that the observed crash behaviour of cars in the field might stem from the change in behaviour of the components. This thesis therefore sets out to quantify the difference and make recommendations for future structural design. Since a numerical method of analysis is used, it seemed logical to present to the reader the theoretical basis for the method and that is done in the following chapter. Chapter 3 also contains other theoretical details that are used in the remainder of the thesis.
Figure 2.1. Extensional and inextensional deformation paths for flat sheets. Hayduk and Wierzbicki [44].

Figure 2.2. Basic Collapse Elements.
Figure 2.3  Uniform symmetric collapse mode. Abramowicz and Jones [45].

Figure 2.4  Asymmetric collapse mode. Abramowicz and Jones [45].
Figure 2.5  Lindholm [50]. Domain of strain-rate and inertia effects.

Figure 2.6  Manjoine [55]. Stress strain characteristics of mild steel at various strain-rates.
Figure 2.7. Interaction between stress, strain and strain-rate for mild steel. Marsh and Campbell [56].

Figure 2.8. Type I and Type II structures with their idealised quasi-static load deflection curves. From Calladine and English [66].
Figure 2.9. Curve fitting exercise of Abramowicz and Jones [45].

Figure 2.10. Simple flow chart of DYNA3D solver. Holmes et al. [96].
Figure 2.11. Three types of bead used to initiate crush by Kitagawa et al. [100].

Figure 2.12. Improvement in collapse mode found by Kitagawa et al. [100].
Chapter 3

*Mathematical and numerical representation of non-linear structural response with large displacements*
3.1 The kinematical method

This method of analysing the deep collapse of uniform prismatic sections arose principally out of the work of Wierzbicki, Abramowicz and Jones. A quarter segment of a mild steel tube with one fold of the symmetric type is shown in Figure 3.1a and an idealisation of this quadrant with a BCE is shown in Figure 3.1b. This idealisation was used by Wierzbicki and Abramowicz [43, 48] to theoretically appraise the collapse behaviour of the whole tube. A BCE can therefore be considered to be formed of two flanges with width c/2 and height 2H joined at a vertical edge (Figure 3.2a). In the following, a rigid-plastic approximation to the stress-strain curve of the material is assumed.

As the BCE is crushed downwards, moving plastic hinges must form to allow the joint of the flanges to travel to their new positions (Figure 3.2b). 2Ψ₀ defines the initial external angle between the two flanges forming the BCE and the remaining angles α, β and γ are related through trigonometry thus

\[
\tan \gamma = \frac{\tan \psi_0}{\sin \alpha}, \quad \tan \beta = \frac{\tan \gamma}{\sin \psi_0}. \tag{3.1.1}
\]

These angles can be used to describe the current geometry of the BCE based on the initial geometry and related to the vertical crush distance δ. Now the theory of perfect plasticity requires that displacements should be described by a smooth and continuous function across hinge lines that are in motion, so the straight lines in Figure 3.2b, which represent discontinuities in the displacement field, should be replaced by single curvature surfaces. A double curvature surface should also be introduced around the new corner point B such that the idealised BCE begins to resemble the observed fold lobe (Figure 3.2c) and consists of:

- **I** Four plane trapezoidal sections moving as rigid bodies.
- **II** Two sections of cylindrical surfaces at which continuous bending takes place without any extension.
- **III** Two sections of conical surfaces in which material is bent and straightened as it passes from one flange to the other.
- **IV** A section of a toroidal surface which produces extension in a circumferential direction and continuously changes principal curvature in the other direction.
There are therefore three regions within a BCE in which energy is dissipated. If the sum of
these internal energies is equated to the external energy required to crush the BCE an
equation of the following form may be written:

\[ 2HP_m^t = E_1 + E_2 + E_3 \]  

(3.1.2)

where \( E_1 \) is the energy dissipated in the toroidal region, \( E_2 \) is the energy dissipated in the
horizontal hinge lines and \( E_3 \) is the energy dissipated in the vertical hinge lines. Wierzbicki
and Abramowicz [43] estimated the energy absorbed by a BCE under quasi-static conditions
using the following version of Equation (3.1.2).

\[ E = M_0 \left( 16HI_1 b/h + 2\pi c + 4I_3 H^2 b \right) \]  

(3.1.3)

where the integral

\[
I_1(\psi_0) = \frac{\pi}{(\pi - 2\psi_0)\tan\psi_0} \int_0^{\pi/2} \cos \alpha \left\{ \sin \psi_0 \sin \left( \frac{\pi - 2\psi_0}{\pi} \right) \beta + \cos \psi_0 \left[ 1 - \cos \left( \frac{\pi - 2\psi_0}{\pi} \right) \beta \right] \right\} d\alpha
\]

(3.1.4)

and

\[
I_3(\psi_0) = \frac{1}{\tan \psi_0} \int_0^{\pi/2} \frac{\cos \alpha}{\sin \gamma} d\alpha.
\]

(3.5)

Wierzbicki and Abramowicz gave \( I_1 = 0.58 \) and \( I_3 = 1.11 \) for rectangular section tubes
\((\psi_0 = \pi/4)\). Alternative values for \( I_1 \) and \( I_3 \) were given as 0.555 and 1.148 respectively by
Abramowicz and Jones [45]. The discrepancy led the author to evaluate the integrals afresh.
The evaluation required numerical methods using a computation package for different values
of \( \psi_0 \). The results are shown in Table 3.1.

<table>
<thead>
<tr>
<th>Section</th>
<th>( \psi_0 )</th>
<th>( I_1 )</th>
<th>( I_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>( \pi/3 )</td>
<td>0.330</td>
<td>0.608</td>
</tr>
<tr>
<td>Square</td>
<td>( \pi/4 )</td>
<td>0.578</td>
<td>1.148</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>( \pi/6 )</td>
<td>1.038</td>
<td>2.390</td>
</tr>
<tr>
<td>Octagonal</td>
<td>( \pi/8 )</td>
<td>1.505</td>
<td>3.962</td>
</tr>
</tbody>
</table>

Table 3.1 Evaluation of integrals for various sections
Both integrals tended quite quickly toward 0 for large external angles although, strictly speaking, they were not valid for $\psi_0 > \pi/4$ since the postulated collapse mode no longer held. Table 3.1 resulted in a new combination of values for $I_1$ and $I_3$ but for the remainder of this thesis the values used by Abramowicz and Jones were adopted since $I_3$ was related to the more sensitive part of Equation (3.1.3). These values were used in an energy balance to estimate the mean force during the crush of a BCE. The values of $H$ and $b$ that corresponded to the minimum process force were given by Equations (3.1.6) and (3.1.7).

$$\frac{H}{h} = 0.99\left(\frac{c}{h}\right)^{2/3} \quad (3.1.6)$$

$$\frac{b}{h} = 0.72\left(\frac{c}{h}\right)^{1/3} \quad (3.1.7)$$

Under quasi-static crush conditions the energy absorbed by a BCE for a given wall thickness and side length can be readily estimated by substituting values of $H$ and $b$ from Equations (3.1.6) and (3.1.7) into Equation (3.1.3). Under dynamic conditions the strain-rate sensitive nature of mild steel is accounted for by use of the Cowper-Symonds [58] constitutive Equation which relates the yield stress under quasi-static conditions to the yield stress under dynamic conditions as shown in Equation (3.1.8).

$$\frac{\sigma_y^d}{\sigma_y} = 1 + \left(\frac{\dot{\varepsilon}}{D}\right)^p \quad (3.1.8)$$

with $D = 40.4 \text{ s}^{-1}$ and $p = 5$ for mild steel obtained from experimental specimens having relatively small strains in the neighbourhood of the yield value. The material undergoing plastic deformation in the horizontal and vertical hinge lines of a BCE however, undergo relatively large strains. Wierzbicki, again using a rigid-plastic analysis, gave

$$\varepsilon_{av} = \frac{h}{4b} \quad (3.1.9)$$

as an expression for the average strain in the vertical hinge lines during rolling deformations of the type under consideration. Equation (3.1.9) predicted strains between 11.2% and 13% for square tubes in the experimental study reported by Abramowicz and Jones [45] which led to the suggested use of $\sigma_y^d$ in the calculation of $M_o$ when applying Equation (3.1.3) under dynamic crush conditions. Furthermore, it was shown that Equation (3.1.8) re-written as

$$\frac{\sigma_y^d}{\sigma_u} = 1 + \left\{\frac{\dot{\varepsilon}}{6844}\right\}^{7.91} \quad (3.1.10)$$
gave a good fit to the ultimate stresses found experimentally by Campbell and Cooper [73].

Equation (3.1.11) shows an expression, also developed by Abramowicz and Jones [45], which was used to approximate the strain-rate in the specimen for substitution into Equation (3.1.10).

\[ \dot{\varepsilon} = 0.33 \frac{V}{c} \]  

(3.1.11)

It must be stated that the authors [45] were cautious of the degree to which this estimate of strain-rate could be relied upon but it seemed to be a reasonable estimate that was easy to use.

### 3.1.1 Application of the kinematical method

The characteristics of a uniform square tube such as specimen I21 [45] where \( c = 37.07 \) mm and \( h = 1.152 \) mm, can be substituted into Equations (3.1.6) and (3.1.7) to obtain values for \( H \) and \( b \) which, when substituted into Equation (3.1.3), result in Equation (3.1.12).

\[ E = M_o(699.269) \]  

(3.1.12)

A proof stress of 277.5 MPa can be used to obtain a value for \( M_o \) and the resulting value of \( E \) is shown in Equation (3.1.13).

\[ E = (277.5 \times 0.332)(699.269) = 64.38 \text{ J} \]  

(3.1.13)

Therefore, under quasi-static conditions and with uniform symmetric collapse, a BCE can be expected to absorb 64.38 J when squashed into an idealised fold similar to that observed experimentally.

Using Equation (3.1.11) and an impact velocity of 10.29 ms\(^{-1}\), the average strain-rate during dynamic collapse of the specimen can be estimated as 91.6 s\(^{-1}\). Upon substitution of \( \dot{\varepsilon} \) and \( \sigma_s^d = 330.5 \) MPa into Equation (3.1.10), a value of 440.16 MPa is obtained for \( \sigma_s^d \). Equations (3.1.3, 3.1.6 and 3.1.7) now give 102.186 J as the energy required to crush a BCE under dynamic conditions. This analytical estimate will be compared to the estimates for energy absorption obtained using the numerical analysis techniques described in chapter 4. However it is first necessary to provide a brief description of the theoretical foundation to the FE analysis technique employed.
3.2 Theoretical foundation of the numerical analysis method

Figure 3.3 shows a body B in a fixed rectangular Cartesian co-ordinate system which moves from B₀ to b. A particle in B, which was initially at \( X_\alpha \) (\( \alpha = 1, 2, 3 \)), moves to \( x_i (i = 1, 2, 3) \) in the same co-ordinate system and the time-dependant deformation of the particle is required. In the following development indicial notation is used, and the summation convention for Cartesian co-ordinates apply. Therefore, whenever the same letter subscript occurs twice in a term, that subscript is to be given all possible values and the results summed. A comma appearing before a subscript denotes a partial derivative (\( \partial F/\partial x_i = F_{,i} \)) and vectors and tensors are denoted with bold-faced type.

A Lagrangian formulation is used so the deformation can be expressed in terms of the convected co-ordinates \( X_\alpha \) and time \( t \).

\[
x_i = x_i(X_\alpha, t)
\]

(3.2.1)

At time \( t = 0 \) we have the following initial conditions

\[
x_i (X_\alpha, 0) = X_\alpha
\]

(3.2.2)

\[
x_i (X_\alpha, 0) = V_i(X_\alpha)
\]

(3.2.3)

where \( V_i \) defines the initial velocities.

The program solves the momentum equation

\[
\sigma_{ij,j} + \rho f_i = \rho \ddot{x}_i.
\]

(3.2.4)

The boundary conditions are

\[
\sigma_{ij} n_j = t_i(t)
\]

(3.2.5)

for traction forces on boundary \( \partial b_1 \),

\[
x_i (X_\alpha, t) = D_i(t)
\]

(3.2.6)
for displacement on boundary $\partial b_2$, and the contact discontinuity

$$ (\sigma_{ij}^+ - \sigma_{ij}^-)n_i = 0 $$

along an interior boundary $\partial b_3$ when $x_i^+ = x_i^-$. 

$\sigma_{ij}$ is the Cauchy Stress, $\rho$ is the present density, $f_i$ is the body force density, and $n_i$ is a unit outward normal to a boundary element of $\partial b$. Mass is conserved by ensuring that

$$ \rho V = \rho_0 $$

where $V$ is termed the relative volume which is the determinant of the deformation gradient matrix, $F_{ij}$

$$ F_{ij} = \frac{\partial x_i}{\partial x_j} $$

and $\rho_0$ is the initial density.

The energy equation

$$ \dot{E} = Vs_{ij}\dot{e}_{ij} - p\dot{V} $$

is integrated in time and used for equation of state evaluations and a global energy balance where $s_{ij}$ are the deviatoric stresses

$$ s_{ij} = \sigma_{ij} + p\delta_{ij} $$

and $p$ represents the pressure

$$ p = \frac{1}{3}\sigma_{ij}\delta_{ij} = -\frac{1}{3}\sigma_{kk}. $$

$\delta_{ij}$ is the Kronecker delta ($\delta_{ij} = 1$ if $i = j$; otherwise $\delta_{ij} = 0$) and $\dot{e}_{ij}$ is the strain-rate tensor.
A weak form of the equilibrium equations can be written as

\[
\int_{\Omega} (\rho \ddot{x}_i - \sigma_{ij,j} - \rho f \delta x_i) \, dv + \int_{\partial b_1} (\sigma_{ij} n_j - t_i) \delta x_i \, ds + \int_{\partial b_2} (\sigma_{ij}^+ - \sigma_{ij}^-) n_j \delta x_i \, ds = 0 \quad (3.2.13)
\]

where \( \delta x_i \) satisfies all boundary conditions on \( \partial b_2 \) and the integrations are over the current geometry. Application of the divergence theorem gives

\[
\int_{\Omega} (\sigma_{ij} \delta x_i)_{,j} \, dv = \int_{\partial b_1} \sigma_{ij} n_j \delta x_i \, ds + \int_{\partial b_2} (\sigma_{ij}^+ - \sigma_{ij}^-) n_j \delta x_i \, ds \quad (3.2.14)
\]

and noting that

\[
(\sigma_{ij} \delta x_i)_{,j} - \sigma_{ij,j} \delta x_i = \sigma_{ij} \delta x_{i,j}
\]

leads to

\[
\delta \pi = \int_{\Omega} \rho \ddot{x}_i \delta x_i \, dv + \int_{\Omega} \sigma_{ij} \delta x_{i,j} \, dv - \int_{\Omega} \rho f_i \delta x_i \, dv - \int_{\partial b_1} t_i \delta x_i \, ds = 0 \quad (3.2.15)
\]

which is a statement of the principle of virtual work.

The reference configuration is discretised into a mesh of finite elements, interconnected at nodes and the location of individual particles are recorded for the duration of the analysis as described in Equation (3.2.16).

\[
x_i(X_\alpha,t) = x_i(X_\alpha(\xi,\eta,\zeta),t) = \sum_{j=1}^{k} \phi_j(\xi,\eta,\zeta) x_j^f(t) \quad (3.2.16)
\]

Here, \( \phi_j \) are shape functions of the parametric co-ordinates \( (\xi,\eta,\zeta) \), \( k \) is the number of nodes defining an element and \( x_j^f \) is the nodal co-ordinate of the \( j \) th node in the \( i \) th direction. Summing over the \( n \) elements, \( \delta \pi \) may be approximated with

\[
\delta \pi = \sum_{m=1}^{n} \delta \pi_m = 0 \quad (3.2.17)
\]
leading to

\[
\sum_{m=1}^{n} \left\{ \int_{V_m} \rho \dot{x}_i \Phi_i^m dv + \int_{V_m} \sigma_{ij}^m \Phi_i^m dv - \int_{\partial V_1} \rho \dot{x}_i \Phi_i^m ds \right\} = 0 \quad (3.2.18)
\]

where

\[
\Phi_i^m = (\phi_1^m, \phi_2^m, \ldots, \phi_k^m)^T
\]

(3.2.19)

In matrix notation Equation (3.2.18) becomes

\[
\sum_{m=1}^{n} \left\{ \int_{V_m} \rho N^T N \dot{a}_v + \int_{V_m} B^T \sigma dv - \int_{V_m} \rho N^T b dv - \int_{\partial V_1} N^T t ds \right\}^m = 0
\]

(3.2.20)

where \( N \) is an interpolation matrix of shape functions, \( \sigma \) is the stress vector

\[
\sigma^T = (\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx})
\]

(3.2.21)

\( B \) is the strain-displacement matrix, \( a \) is the nodal acceleration vector

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{bmatrix} = \begin{bmatrix}
a_{x1} \\
ad_{y1} \\
\vdots \\
ad_{yk} \\
ad_{zk}
\end{bmatrix} = Na
\]

(3.2.22)

\( b \) is the body force vector,

\[
b = \begin{bmatrix}
f_x \\
f_y \\
f_z
\end{bmatrix}
\]

(3.2.23)

and \( t \) are the applied traction loads.

\[
t = \begin{bmatrix}
t_x \\
t_y \\
t_z
\end{bmatrix}
\]

(3.2.24)
3.2.1 The Belytschko-Lin-Tsay shell element formulation
The Belytschko-Lin-Tsay [104] shell element derives its computational efficiency from the use of a combined co-rotational and velocity strain formulation. A co-ordinate system is embedded within the element which avoids the complexities of non-linear mechanics and the choice of velocity strain (i.e. rate of deformation) facilitates the constitutive evaluation since the conjugate stress is the simple Cauchy stress. The following closely follows the notation of Belytschko, Lin and Tsay.

The mid-surface of the quadrilateral shell element is defined by the location of the nodes and forms the reference surface. An embedded co-ordinate system which deforms with the element is defined in terms of the nodal-co-ordinates as shown in Figure 3.4.

Construction of the co-rotational co-ordinate system begins by calculating a unit vector, normal to the main diagonal of the element.

\[
\hat{e}_3 = \frac{s_3}{\|s_3\|} \quad (3.3.1a)
\]

\[
\|s_3\| = \sqrt{s_{31}^2 + s_{32}^2 + s_{33}^2} \quad (3.3.1b)
\]

\[
s_3 = r_{31} \times r_{42} \quad (3.3.1c)
\]

The local x axis, \( \hat{x} \) is required to be approximately along the element edge between node 1 and 2. This definition is convenient for interpreting element stresses which are defined in the local \( \hat{x} - \hat{y} \) system. First a vector \( s_1 \) is defined which is nearly parallel to the vector \( r_{21} \).

\[
s_1 = r_{21} - (r_{21} \cdot \hat{e}_3)\hat{e}_3 \quad (3.3.2a)
\]

\[
\hat{e}_1 = \frac{s_1}{\|s_1\|} \quad (3.3.2b)
\]

The remaining unit vector is obtained from the vector cross product

\[
\hat{e}_2 = \hat{e}_3 \times \hat{e}_1 \quad (3.3.3)
\]

The unit vectors \( \hat{e}_1 \) and \( \hat{e}_2 \) are tangent to the mid-plane of the shell when all four nodes are co-planar. \( \hat{e}_3 \) is then in the fibre direction. On deformation of the shell, an angle may
develop between the actual fibre direction and the unit normal $\hat{e}_3$. Equation (3.3.4) shows an expression for the magnitude of this angle.

$$|\hat{e}_3 \cdot f - 1| < \delta$$  \hspace{1cm} (3.3.4)

Where $f$ is the unit vector in the fibre direction and the order of the Kronecker delta, $\delta$, is determined by the degree of strain. Belytschko et al. suggested that, for most engineering applications, values of $\delta$ of the order of $10^{-2}$ are acceptable. They further indicated that the difference between the rotation of the co-rotational co-ordinates $\hat{e}_i$ and the material rotation should be small if (3.3.4) is satisfied.

A transformation matrix can be defined using the global components of the element unit vectors $e_{ix}, e_{iy}, e_{iz}$ as in Equation (3.3.5).

$$[N] = \begin{bmatrix} e_{ix} & e_{iz} & e_{iz} \\ e_{iy} & e_{iy} & e_{iz} \\ e_{iz} & e_{iz} & e_{iz} \end{bmatrix}$$  \hspace{1cm} (3.3.5)

This matrix transforms vectors with global co-ordinates $A = (A_x, A_y, A_z)$ and element co-ordinates $\hat{A} = (\hat{A}_x, \hat{A}_y, \hat{A}_z)$ as follows:

$$\{A\} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} e_{ix} & e_{iz} & e_{iz} \\ e_{iy} & e_{iy} & e_{iz} \\ e_{iz} & e_{iz} & e_{iz} \end{bmatrix} \begin{bmatrix} \hat{A}_x \\ \hat{A}_y \\ \hat{A}_z \end{bmatrix} = [N] \{\hat{A}\} = [g]^T \{\hat{A}\}$$  \hspace{1cm} (3.3.6)

The transformation from local to global co-ordinates is achieved using the matrix transpose thus

$$\{\hat{A}\} = [N]^T \{A\}$$  \hspace{1cm} (3.3.7)

The above small rotation condition described by (3.3.4) only refers to out-of-plane deformations and thus on element strain. It does not restrict the magnitude of rigid body rotations. The velocity strain-displacement relations are also restricted to small strains. The displacement of any point in the shell is partitioned into a mid-surface displacement (nodal translations) and a displacement associated with rotations of the element fibres (nodal rotations). The Belytschko-Lin-Tsay shell element uses the Mindlin [105] theory of plates and shells to partition the velocity of any point in the shell as:
\[ v = v^m - \hat{z} e_3 \times \theta \]  

(3.3.8)

when \( v^m \) is the velocity of the mid-surface, \( \theta \) is the angular velocity vector and \( \hat{z} \) is the distance along the fibre direction. (thickness) of the shell element.

The corresponding co-rotational components of the velocity strain are given by

\[ \hat{d}_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \]  

(3.3.9)

Substitution of Equation (3.3.8) into the above gives the following velocity strain relations:

\[ \hat{d}_x = \frac{\partial v_x}{\partial x} + \hat{z} \frac{\partial \theta_y}{\partial x} \]  

(3.3.9a)

\[ \hat{d}_y = \frac{\partial v_y}{\partial y} - \hat{z} \frac{\partial \theta_x}{\partial y} \]  

(3.3.9b)

\[ 2\hat{d}_{xy} = \frac{\partial v_x}{\partial y} + \hat{z} \left( \frac{\partial \theta_y}{\partial y} - \frac{\partial \theta_x}{\partial x} \right) \]  

(3.3.9c)

\[ 2\hat{d}_{yx} = \frac{\partial v_y}{\partial x} - \hat{\theta}_x \]  

(3.3.9d)

\[ 2\hat{d}_{xz} = \frac{\partial v_x}{\partial x} + \hat{\theta}_y \]  

(3.3.9e)

These relations must be evaluated at the quadrature points within the shell. Standard bi-linear nodal interpolation is used to define the mid-surface velocity, angular velocity, and the element's co-ordinates (isoparametric representation). These interpolation relations are given by:

\[ v^m = N_1(\xi, \eta) v_1 \]  

(3.3.10a)

\[ \theta^m = N_1(\xi, \eta) \theta_1 \]  

(3.3.10b)

\[ x^m = N_1(\xi, \eta) x_1 \]  

(3.3.10c)
where the subscript I is summed over all the element's nodes and the nodal velocities are obtained by differentiating the nodal co-ordinates with respect to time i.e. \(v_i = \dot{x}_i\).

The bi-linear shape functions are:

\[N_1 = \frac{1}{4}(1 - \xi)(1 - \eta)\]  
\[N_2 = \frac{1}{4}(1 + \xi)(1 - \eta)\]  
\[N_3 = \frac{1}{4}(1 + \xi)(1 + \eta)\]  
\[N_4 = \frac{1}{4}(1 - \xi)(1 + \eta)\]

The velocity strains at the centre of the element \((\xi = 0, \eta = 0)\) are obtained by substitution of the above relations into the previously defined velocity-strain displacement relations (Equations 3.3.9a-e) which, after manipulation, yield

\[\dot{d}_x = B_{11}\dot{v}_{xI} + \dot{\xi}B_{11}\theta_{yI}\]  
\[\dot{d}_y = B_{21}\dot{v}_{yI} - \dot{\xi}B_{21}\theta_{xI}\]  
\[2\dot{d}_{xy} = B_{21}\dot{v}_{xI} + B_{11}\dot{v}_{yI} + \dot{\xi}(B_{21}\theta_{yI} - B_{11}\theta_{xI})\]  
\[2\dot{d}_{xz} = B_{11}\dot{v}_{xI} + N_1\theta_{yI}\]  
\[2\dot{d}_{yz} = B_{21}\dot{v}_{xI} - N_1\theta_{xI}\]

where

\[B_{11} = \frac{\partial N_1}{\partial \xi}\]  
\[B_{21} = \frac{\partial N_1}{\partial \eta}\]
The shape function derivatives $B_{al}$ are also evaluated at the centre of the element, i.e. at $\xi = 0$ and $\eta = 0$.

After constitutive evaluations using the above velocity-strains the resulting stresses are integrated through the thickness of the shell to obtain local resultant forces and moments.

The integration formula for the resultants are:

$$\hat{f}_{\alpha \beta}^R = \int \hat{\sigma}_{\alpha \beta} d\hat{\zeta}$$

$$\hat{m}_{\alpha \beta}^R = -\int \hat{\zeta} \hat{\sigma}_{\alpha \beta} d\hat{\zeta}$$

where the superscript $R$ indicates a resultant force or moment and the Greek subscripts emphasise the limited range of the indices for plane stress plasticity. The above element central force and moment resultants are related to the local nodal forces and moments by invoking the principle of virtual work and performing a one-point quadrature. The relations obtained thus are:

$$\hat{f}_x = A\left( B_{11}\hat{f}_{xx}^R + B_{21}\hat{f}_{xy}^R \right)$$

$$\hat{f}_y = A\left( B_{21}\hat{f}_{yy}^R + B_{11}\hat{f}_{xy}^R \right)$$

$$\hat{f}_z = A\kappa \left( B_{11}\hat{f}_{zz}^R + B_{21}\hat{f}_{yz}^R \right)$$

$$\hat{m}_{x \alpha} = A\left( B_{21}\hat{m}_{\alpha yy}^R + B_{11}\hat{m}_{\alpha xy}^R - \frac{\kappa}{4} f_{yx}^R \right)$$

$$\hat{m}_{y \alpha} = -A\left( B_{11}\hat{m}_{\alpha xx}^R + B_{21}\hat{m}_{\alpha xy}^R - \frac{\kappa}{4} f_{xy}^R \right)$$

$$\hat{m}_{z \alpha} = 0$$

where $A$ is the area of the element and $\kappa$ is the shear factor from the Mindlin [105] theory. In the Belytschko-Lin-Tsay formulation, $\kappa$ is used as a penalty parameter to ensure the Kirchoff normality condition as the shell becomes thin.
The above local nodal forces and moments are then transformed to the global co-ordinate system using the transformation relations given in Equation (3.3.7). The nodal forces and moments in the global co-ordinate system are then summed over all the nodes and the global equations of motion are solved for the next increment in nodal accelerations.

### 3.2.2 Hourglass control

A one-point quadrature is used in the plane of the element for all initial evaluations prior to the application of shape functions to obtain nodal conditions. Zero-energy hourglass deformation modes (Figure 3.5) that accompany one-point quadrature of this type are suppressed by the addition of hourglass viscosity stresses to the physical stresses at the local element level. These hourglass stresses are separate from the stresses derived from the constitutive evaluations and are imposed purely to maintain numerical stability.

The hourglass shape vector \( \tau_i \) is defined as

\[
\tau_i = h_i - (h_i \hat{x}_{al})B_{al} \tag{3.3.16}
\]

where

\[
h = \begin{bmatrix}
+1 \\
-1 \\
+1 \\
-1
\end{bmatrix} \tag{3.3.17}
\]

is the basis vector that generates the deformation mode that is neglected by one-point quadrature. In Equation (3.3.16) and the remainder of the section, Greek subscripts have a range of 2 e.g. \( \hat{x}_{al} = (\hat{x}_{11}, \hat{x}_{21}) = (\hat{x}_i, \hat{y}_i) \).

The hourglass shape vector then operates on the generalised displacements in a manner similar to Equations (3.3.12a - e) to produce the generalised hourglass strain-rates

\[
\dot{q}_i^B = \tau_i \hat{\theta}_{al} \tag{3.3.18}
\]

\[
\dot{q}_i^B = \tau_i \hat{v}_{al} \tag{3.3.19}
\]

\[
\dot{q}_i^M = \tau_i \hat{v}_{al} \tag{3.3.20}
\]
where the superscripts $B$ and $M$ denote bending and membrane modes respectively. The corresponding hourglass stress-rates are then given by

$$
\dot{Q}_\alpha^B = \frac{r_o E t^3 A}{192} B_{pl} B_{pl} \dot{q}_\alpha^B \\
\dot{Q}_3^B = \frac{r_w k G t^3 A}{12} B_{pl} B_{pl} \dot{q}_3^B \\
\dot{Q}_\alpha^M = \frac{r_m E t A}{8} B_{pl} B_{pl} \dot{q}_\alpha^M
$$

where $t$ is the shell thickness and the parameters $r_o$, $r_w$ and $r_m$ are generally assigned values between 0.01 and 0.05. The hourglass stresses are updated from the stress-rates thus

$$Q^{n+1} = Q^n + \Delta t \dot{Q}$$

and the hourglass resultant forces are

$$\hat{m}_{al}^H = \tau_I Q_\alpha^B$$
$$\hat{f}_{3i}^H = \tau_I Q_3^B$$
$$\hat{f}_{al}^H = \tau_I Q_\alpha^M$$

where the superscript $H$ emphasises that these are internal force contributions from the hourglass deformations.

These hourglass forces are added directly to the previously determined local internal forces due to deformations (Equations 3.3.15a - f).
3.2.3 Overview of stress update

The motion of the nodes relative to the initial conditions can be calculated and the strain-rate, angular velocity of the centre of gravity and Jaumann stress-rate for each element obtained as follows:-

\[ \dot{\varepsilon}_{ij} = \frac{1}{2} \left( \frac{\partial \nu_i}{\partial x_j} + \frac{\partial \nu_j}{\partial x_i} \right) \]  \hspace{1cm} (3.4.1a)

\[ \omega_{ij} = \frac{1}{2} \left( \frac{\partial \nu_i}{\partial x_j} - \frac{\partial \nu_j}{\partial x_i} \right) \]  \hspace{1cm} (3.4.1b)

\[ \sigma_{ij}^V = C_{ijkl} \dot{e}_{kl} \]  \hspace{1cm} (3.4.1c)

\( C_{ijkl} \) is the stress-dependent constitutive matrix.

3.2.4 Time step calculation

The time step must divide the period of the highest mode of oscillation of the model by at least \( \pi \). The highest mode is assumed to be a single element mode such as oscillation of a single brick, shell, beam or spring element. Equation (3.5.1) is used to calculate the maximum time step for a numerically stable analysis.

\[ \Delta t = 0.9 \frac{1}{c} \]  \hspace{1cm} (3.5.1)

Here, \( c \) is the speed of sound in the material and \( l \) is the characteristic length of the smallest element in the structure. The default characteristic length for shell elements is obtained by dividing the element area by the longest diagonal.

3.3 Summary of the chapter

The kinematical method has already been shown to predict experimental results well for moderate impact speeds. The results obtained using the numerical technique, the theory to which has been outlined here, can now be compared to both the predictions made using the kinematical method and the experimental results. With the FE technique however, care must be exercised in the choice of element size. If the elements are too large, not only will they be unable to accurately represent the shape of the structure during deformation, but transient wave effects may also be missed due to a large time step. In addition, large elements are prone to hourglass deformation modes which are inaccurate and out-of-plane bending, which
must be strictly limited when using the co-rotational shell formulation. In order to properly
analyse out-of-plane bending of a structure, several elements are required for a fold. This
will become clear in chapter 4 where the process involved in carrying out the numerical
analysis is first described. The predictions obtained using this method are then compared
with previous experimental and analytical results.
Figure 3.1. Basic Collapse Element (BCE) (a) quadrant of a mild steel tube (b) idealisation.

Figure 3.2. Dimensions and kinematics of a BCE (a) undeformed (b) deformed c) deformed with smooth and continuous displacement fields (after Wierzbicki and Abramowicz [43, 48]).
Figure 3.3 Notation used in governing equations of DYNA3D.

Figure 3.4. Construction of co-rotational element co-ordinates.
Figure 3.5. Hourglass deformation mode.
Chapter 4

Validation of the numerical analysis method
4.1 Introduction

In this chapter the creation of a typical analysis model is first described. Readers who are already familiar with non-linear finite element modelling may skip sections 4.2 - 4.2.5 which describe the pre-processing process. Experienced users of the pre-processor PATRAN who are about to embark on analysis using DYNA3D will find Appendix I useful since it contains some essential items. Two validation studies are then reported comparing results obtained from drop-hammer experiments with those predicted by the FE analysis. In the first study the experiments were conducted by Reid, Reddy and Gray [106] at the University of Manchester Institute for Science and Technology (UMIST) and in the second study the experiments were taken from the series reported by Abramowicz and Jones [45] at the University of Liverpool Impact Research Centre (IRC). Research papers written by the author concerning this work are given in the references [107, 108].

4.2 Creating a finite element model for crushing analysis

It is usual to create a FE model using a pre-processor which is a computer program that enables the user to create nodes, shell elements, thick shell elements and solid elements that are to be used in the analysis. A pre-processor cannot not generally be used to perform an analysis but helps the user to discretise their model, check that node and element orientations and connectivities are properly organised and to write out the large number of computer instructions required to perform an analysis. The pre-processor used for this study was PATRAN but the following five stages are involved in the creation of a FE model for non-linear analysis with large displacements regardless of pre-processor:

1. Creation of the model geometry
2. Spatial discretization (meshing) of the model geometry.
3. Creation of sliding interfaces (contact surfaces).
4. Specification of prescribed velocities or displacements (constraints) and restraints.
5. Specifying material and physical properties.

Stage 3 is necessary only when parts of the structure being modelled come into contact with other parts or other objects. Stages 4 and 5 are often just as easily carried out by editing the analysis input file written by the pre-processor, but are described here for completeness. Stages 1-3 must be carried out using the pre-processor due to the volume of data involved in meshing and contact surface creation.
4.2.1 Creation of the model geometry

The geometry model is an accurate 3-dimensional drawing of the structure to be analysed. Figure 4.1 shows a drawing of a Ford Sapphire chassis rail and the geometry model created from measurements taken while the vehicle was on a ramp. The geometry model was made up of "grid" instructions for geometry points which were entered in 3-D global Cartesian coordinates, "line" instructions which best-fit the grid points and "patch" instructions which defined surfaces bound by the lines. The "hyperpatch" instruction could also have been issued at this stage to define a solid region bound by patches. Grids, lines, patches and hyperpatches can be referred to as phase 1 entities and used solely for reference purposes when creating the phase 2 mesh. The phase 1 model was therefore an accurate former or guide around which the finite elements were to be placed. It must therefore be created first before meshing can take place.

4.2.2 Spatial discretization (meshing) of the model

During this stage, nodes and elements were specified for the model and overlaid on the geometry model described above. The number of elements per unit area is termed the mesh density and could be varied by the user. Figure 4.2 shows two identical patches meshed with different mesh densities. The first has a hundred elements of 10 x 10 units each, the second 16 elements of 25 x 25 units each. The first mesh will be able to follow contours more accurately but the second will be a great deal faster in terms of computing time. One of the skills of the analyst is to quickly find the mesh density that will yield results of acceptable accuracy without incurring unreasonable computing costs. This is often an iterative process, beginning with a coarse mesh and refining it until there is no longer a significant difference in the results. The following element types are usually used to describe structures: - beams, thin shell (3 and 4 nodes), thick shell (8 nodes), and solid, wedge, brick or tetrahedron (8 nodes) elements. Discrete elements such as point masses, translational and rotational linear and non-linear springs and dampers could also be used.

4.2.3 Creation of contact surfaces

Contact surfaces were required in order to prevent parts of a model passing through other parts during an analysis. For example, if an engine block was forced into the passenger compartment bulkhead, it would simply pass through it if the interaction between these two components was not anticipated and the correct contact conditions prescribed for their interaction. If there was insufficient displacement of the engine to force contact with the firewall, no contact surface specification would be required. The contact search algorithm [95] was therefore one of the features that had to be developed for large-scale deformation FE analysis codes.
All contacts must be predicted before analysis to avoid repeating the analysis with additional contact surfaces. Non-linear FE codes usually have the capacity to simulate six types of contact:

- **Sliding**, with or without separation and friction - allowed sliding between components which could also come apart. Friction between the components could also be specified.

- **Tied** - allowed two surfaces to be glued together. The surface defined as the "slave" was effectively pinned to the "master" surface, the nodes moved together and forces were transferred across the surface. Slave nodes on the edges of shell elements were also permitted to be glued to the master surface.

- **Single surface** - this was used where a structure folded and the folds contacted each other. It was used in every analysis carried out in this study.

- **Discrete nodes impacting a surface** - this type of contact definition was used when the contacting elements lay in different planes such as when an edge of a sheet metal component impinges on a flat plate structure.

- **Geometric contact** - contacts between arbitrary surfaces and geometric rigid entities such as flat planes, spheres, cylinders and ellipsoids could be specified but were not used for this study.

- **Stonewall** - this was used to represent a rigid barrier or anvil through which specified nodes or elements (slaves to the stonewall) were not permitted to pass.

An important general note on contact surfaces concerns the connectivity order of the elements forming the surface. A master/slave pair would only function if their outward normals were pointing towards each other. For single surface contacts, it was necessary to ensure that the outward normals for elements in each geometry region were pointing in a uniform direction as shown in Figure 4.3. Contact surfaces should also be kept to a minimum size. This speeds up the contact search operation which occurs at frequent intervals. When analysing a long crush tube, it would be better to have two zones of contact surfaces rather than one large zone. If uniform crumpling is anticipated, elements at one end of the tube are unlikely to come into contact with those at the other end so two contact surfaces could be employed. In order to ensure that no contacts are missed, it would be advisable to have a region of the model where the contact surfaces overlap as shown in Figure 4.4.
4.2.4 Specification of restraints and prescribed accelerations, velocities or displacements

It is common practice to use the pre-processor to specify restraints to any nodal degree of freedom for the duration of the analysis. Initial velocities could also be specified for the whole model or some part of it and prescribed motions (displacements, velocities or accelerations) assigned to specified nodes. For example, when analysing a stamping operation, the punch could be assigned a prescribed velocity history regardless of its interaction with the blank.

4.2.5 Specifying material and physical properties.

The specification of material properties, physical properties and constitutive laws to be obeyed is usually best carried out in a format suited to the particular solver being used to perform the analysis. It has been the author's experience that pre-processors can only be relied upon to write simple properties such as the density and the Young's Modulus correctly for an analysis. More complicated material laws and constitutive equations are not catered for in general pre-processors.

Having created the analysis model, specified material and physical properties, boundary and initial conditions, the solver should then be set to work for a short period of time, say 5 ms in order to shake down the model. After a few such shakedown analyses all typographical errors during model construction are usually found. Subsequent analyses can be carried out and the results checked for consistency.

4.3 Analysis of the UMIST drop-hammer experiment

The validation exercises reported in this section were carried out to develop the analysis techniques using the tool. Simple experiments were required that could be directly related to the crushing of a chassis rail in an car crash. Experience could therefore be gained regarding the representation of the experimental situation and how good a prediction could be expected from the FE analysis. The first laboratory experiment analysed was chosen from the work of Reid, Reddy and Gray [106] who were investigating the effectiveness of foam-filling thin-walled tubes as a means of stabilising the crushing process and thereby improving the tube's efficiency. Using a drop-hammer apparatus, mild-steel sheet metal rectangular section tubes were attached to the hammer and dropped vertically onto a rigid anvil. The tubes were 300 mm in length, had cross-sectional dimensions of 100 mm x 50 mm, a thickness of 0.83 mm and were stress relieved after fabrication. A load cell under the anvil measured the transient force response during tube crush and an accelerometer attached to the drop-hammer measured the deceleration of the drop-hammer as the tube collapsed.
Analysing the response of a foam-filled tube was considered to be too problematic for an early exercise so the rectangular tube without foam-filling and with an impact velocity of 9.9 ms$^{-1}$ shown in Table 1[106] was chosen to be modelled. The authors of that paper generously supplied the surviving load cell and accelerometer traces and data points were manually digitised from these traces which were marked TRY 7 (Figure 4.5) [109]. The mass was 81.36 kg and the drop height was 5 m. For the purposes of this study, the experimental results were considered to be the benchmark and the FE analysis code was being tested for its ability to predict how a structure would behave in a real impact. However, in a real situation, there would likely be a degree of scatter in any given set of test results because no two specimen are identical. It was therefore assumed that the overall shape of the experimental force and deceleration histories received were correct and would be reasonably repeatable as a characteristic response. An acceptable analysis would display a similar overall shape but would not necessarily have identical values to one particular experiment.

4.3.1 Material models and mesh densities

The analyses were all performed at Loughborough university of Technology using a Hewlett Packard 750 series computer. The orientation of the model was such that the $z$ axis lay parallel to the longitudinal axes of the specimen and the drop hammer travelled in the $+ve$ $z$ direction (Figure 4.6). The wide side of the cross section of the specimen was aligned with the $y$ axes. The dimensions of the drop-hammer were not known so they were chosen to be 90 mm x 140 mm x 50 mm in the $x$, $y$ and $z$ directions respectively. An eight-node solid element was used with rigid material specification to model the drop-hammer. This element was constrained to translate in the $z$ direction only, given a mass of 81.36 kg and an initial velocity of 9.9 ms$^{-1}$. Three mesh densities were tried for the specimen all of which employed four-node quadrilateral shell elements. The coarse mesh comprised 900 (10 mm x 10 mm) elements. The medium mesh employed two element sizes, 1350 (6.7 mm x 6.7 mm) elements on the wide sides and 630 (7.1 mm x 6.7 mm) elements on the narrow sides. The fine mesh tube model comprised 3600 (5 mm x 5 mm) elements. The choice of mesh densities was partially dictated by the specimen cross-section and the desire to have elements that were as square in shape as possible.

The Belytschko-Tsay shell formulation [104] was used with three through-thickness integration points. The whole specimen was covered by a type 4 (single surface) contact surface to enable self contact to be properly modelled. The default values of penalty stiffness calculation, initial penetration check and surface friction characteristics were used for the contact surfaces. All nodes were given an initial velocity of 9.9 ms$^{-1}$ in the $+z$ direction and the nodes closest to the drop-hammer (distal end) were attached to the rigid body that represented the drop-hammer.
The anvil was simulated by a stonewall which was fixed in space and had a stick condition applied which meant that nodes that came into contact with the stonewall were not permitted to slide along its surface but could rebound. The stonewall was normal to the longitudinal axis of the specimen and initially co-planar with the nodes at the struck (proximal) end of the specimen. The analysis would therefore begin as the specimen contacted the stonewall. All nodes in the analysis model were designated as slaves to the stonewall and, as such, would generate a reactive force if contact was made.

The basic material model used for the three mesh densities mentioned above was an isotropic elastic, linear strain-hardening model (DYNA3D material 3) with a Young's Modulus of 200 GPa, strain-hardening modulus of 1.4 GPa and a Poisson's ratio of 0.3. The given static yield stress of 280 MPa was used and some of the analyses additionally used the Cowper-Symonds [58] constitutive equation for calculating the instantaneous yield stress based on the strain-rate. Where this was the case, the medium mesh analysis with modified material specification was used and the results labelled “Cowper-Symonds”. The empirical constants $p = 3.91, D = 6844$ were used in the Cowper-Symonds analyses as discussed in Chapter 3. Finally, a uniform gravitational acceleration was applied to the models so that the analytical situation represented the laboratory situation as closely as possible.

4.4 Analysis method and results

The oscilloscope traces from the laboratory experiment were enlarged and carefully digitised. The upper trace shown in Figure 4.5 is the force history and the lower trace is the deceleration history. Calibrations were given as 10 kN per division and 20 g per division for the force and deceleration histories respectively and the time calibration was 10 ms per division. It was possible to take force and deceleration values at 1 ms intervals from the middle of the relatively broad lines that formed the traces. At this stage it became clear that precise numerical comparisons with the laboratory traces would be difficult. It was felt, however, that the traces provided a characteristic response of the specimen to the impact and this could be used as part of the development of the analysis technique. Data points from the FE analysis were thus written out more frequently (at 0.25 ms intervals) to ensure that all transients were captured.

In order to facilitate comparisons with FE analysis results, the digitised oscilloscope traces were manipulated using T/HIS [95] to obtain a force history in Newtons and a deceleration history in mms$^{-2}$ but no filters were applied to the traces. The deceleration history was then integrated, the initial velocity of 9.9E3 mms$^{-1}$ added to all ordinate values and integrated again to obtain the experimental drop-hammer displacement history. The displacement history comparisons for the initial set of four analyses are shown in Figure 4.7 and the force history
comparisons are shown in Figure 4.8. A FORTRAN program was written to calculate the mean dynamic force from the force history curve files written out using T/HIS (Appendix II). After each analysis a check was also performed to make sure that the energy added to the system to maintain contact surfaces and resist hourglassing during analysis did not exceed 5% of the total initial energy. The values of $P_m^d$ for the above four analyses are compared to the experimental values in Table 4.1 which also shows maximum displacement of the drop-hammer and the central processor unit (CPU) time required for each of the analyses. The deformed shape plots are shown in Figures 4.9-4.12 which should be read column-wise beginning at the top left. Each plot state shows the elapsed time in seconds at the lower right.

<table>
<thead>
<tr>
<th></th>
<th>$P_m^d$ (kN)</th>
<th>$\delta_{\text{max}}$ (mm)</th>
<th>CPU (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental</td>
<td>22.04</td>
<td>226</td>
<td>-- --</td>
</tr>
<tr>
<td>Coarse mesh</td>
<td>18.56</td>
<td>236</td>
<td>4203</td>
</tr>
<tr>
<td>Medium mesh</td>
<td>19.99</td>
<td>239</td>
<td>13290</td>
</tr>
<tr>
<td>Fine mesh</td>
<td>20.60</td>
<td>254</td>
<td>23720</td>
</tr>
<tr>
<td>Cowper-Symonds</td>
<td>17.91</td>
<td>221</td>
<td>14290</td>
</tr>
</tbody>
</table>

If the $P_m^d$ predictions given in Table 4.1 had been considered in isolation, the fine mesh analysis would have been the most accurate with an error of -6.5%. However, consideration of the force history comparisons in Figure 4.8 showed that the specimen in the fine mesh analysis bottomed out earlier than the others causing a rapid increase in force. This increase had a substantial influence on the $P_m^d$ prediction. In such circumstances the mean dynamic force is not a reliable measure of characteristic response for this type of structure subjected to an impact. All the force histories had an initial peak value that was more than 1.5 times that exhibited by the experimental traces and which also occurred earlier as will be discussed in section 4.5. Apart from this feature the force histories were judged to represent the experimental response characteristic reasonably well with distinct peaks for the folds and a relatively broad plateau at the tail end of the response. Maximum displacements were within 12.5% of the experimental value, the Cowper-Symonds model being least in error and the displacement histories from the first four analyses shown in Figure 4.7 followed the experimental characteristic faithfully. The computing power penalty for increasingly finer meshes is clear from Table 4.1. Doubling the linear dimensions of the elements reduced the
processing time by 82% for this specimen. No changes were made to the contact surface specification for the different analyses though in practice, the time penalty could have been mitigated by having two overlapping contact surfaces for the fine mesh. The Cowper-Symonds analysis took 14,290 CPU seconds (3hrs 58 mins) to perform which was 7.5% longer than the medium mesh analysis on which it was based.

4.4.1 Deformation mode
The thickness to width ratio of the specimen was given by:

\[
\frac{h}{c} = \frac{0.83}{\frac{1}{2}(50 + 100)} = 0.011
\]  

(4.1)

From the work of Mahmood and Paluszny [32] a non-compact folding pattern could be expected at the moderate speed of this impact. Such a deformation mode was displayed by the fine mesh and to some extent the Cowper Symonds analysis models. It is quite possible that this type of deformation would have led to global instability had constraints not been placed on the non-axial motion of the drop-hammer. The experimental quasi-static crush tests on specimens of this cross-section reported by Reid et al. [106] resulted in global buckling around a local cross-section although none of the dynamic experiments exhibited this Euler-mode of failure.

There were no experimental photographic results available at this stage of the project and no surviving specimen with which mode shape comparisons could have been made. The deformed shape plots for the first four analyses (Figures 4.9 - 4.12) were therefore compared to each other with the knowledge that the mode shape should not be the uniform symmetric type but none of the specimen exhibited the expected irregular mode of collapse. The complete folds in the coarse and medium mesh analyses started at the proximal end and progressed towards the distal end in a compact manner which may have wrongly led to the selection of this specimen geometry as an efficient one for energy absorption. In the fine mesh analysis though, discrete folds formed before the sections between them were crushed. On a more reassuring note, the half fold that consistently appeared at the distal end of the specimen was also reported in the experimental studies [106] where it was additionally indicated that folding sometimes started in the mid-region of the specimen. The Cowper-Symonds analysis model came the closest to displaying an irregular collapse mode.

4.4.2 Energy absorbed
The energy absorbed by the specimen was next considered. It was necessary to know whether the FE code could accurately predict the energy absorbed during the impact and the opportunity was also used to check for possible errors introduced during the digitisation process. Three
methods of determining the energy absorbed were identified. $E_1$ was calculated by integrating the force/displacement curve, $E_2$ was the internal energy of the specimen as reported by the FE code and $E_3$ was the product of $P_m^d$ and $\delta_{max}$ (Table 4.1). Values for $E_1$, $E_2$, and $E_3$ for the first four analyses are given in Table 4.2.

Table 4.2 Comparison of various methods of determining the energy absorbed

<table>
<thead>
<tr>
<th>Method</th>
<th>$E_1$ (kJ)</th>
<th>$E_2$ (kJ)</th>
<th>$E_3$ $P_m^d\delta_{max}$ (kJ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental [106]</td>
<td>4.72</td>
<td>--</td>
<td>4.98</td>
</tr>
<tr>
<td>Coarse mesh</td>
<td>4.16</td>
<td>4.28</td>
<td>4.38</td>
</tr>
<tr>
<td>Medium mesh</td>
<td>4.21</td>
<td>4.24</td>
<td>4.78</td>
</tr>
<tr>
<td>Fine mesh</td>
<td>4.21</td>
<td>4.34</td>
<td>5.23</td>
</tr>
<tr>
<td>Cowper-Symonds</td>
<td>4.04</td>
<td>4.23</td>
<td>3.96</td>
</tr>
</tbody>
</table>

The theoretical energy available for absorption in the impact can be expressed as:

$$E = (M + m)g(h_d + \delta_{max})$$ (4.2)

For an experimental maximum displacement of 226 mm and a specimen mass of 0.589 kg therefore, the energy for absorption could be calculated as 4.2 kJ. Most of this energy would be absorbed by the specimen as strain energy and some of the energy would be released on rebound but for the purposes of these comparisons the specimen was assumed to absorb all the incident energy. A comparison of the theoretical maximum energy of 4.2 kJ from Equation (4.2) and the experimental value of $E_1$ in Table 4.2 showed the latter to be 12.4% greater. The experimental value of $E_1$ had been obtained from digitised traces and it was possible that the digitisation process introduced a proportion of the error. It was also possible that the experimental impact speed was not exactly 9.9 ms$^{-1}$. Reid et al. [106] indicated that the pre-impact velocity was within 5% of the free-fall value so an exercise was carried out to quantify the effect of such a reduction. After the first integration of the digitised experimental acceleration trace, a boundary condition of 9.405E3 mms$^{-1}$ was added prior to the second integration to obtain experimental displacement history. When this displacement history was combined with the experimental force history and integrated the resulting experimental value
for $E_1$ was 4.26 kJ. Therefore, a 5% reduction in impact speed reduced the experimental $E_1$ error with respect to Equation (4.2) by 50%. The experimental $E_3$ calculation would also be exposed to the error sources above.

It will be noted that $\delta_{\text{max}}$ in Equation (4.2) was also obtained from the digitised traces but the equation is quite insensitive to errors in $\delta_{\text{max}}$ due to the relative orders of magnitude of $h$ and $\delta_{\text{max}}$. Small errors in $\delta_{\text{max}}$ would have little effect on the bracketed term on the far right in Equation (4.2) and a smaller effect on the resulting value of $E$. For example, a 20% increase in $\delta_{\text{max}}$ to 271 mm would result in less than a 1% increase in the value of $E$ in Equation (4.2).

The FE analytical predictions of $E_1$, $E_2$ and $E_3$ values were next compared directly to Equation (4.2). Although all analytical estimates of $E_1$ had an error no greater than 3.3% with respect to that equation, the models previously considered to be the most fidelic, namely the coarse mesh and the Cowper-Symonds models, were now less accurate than the medium and fine mesh models. The converse is true for the FE calculations of $E_2$ which show the coarse mesh and Cowper-Symonds analyses to be least in error when compared to the theoretical maximum amount of energy available to be absorbed (Equation (4.2)). The closest value was given by the Cowper-Symonds analysis with an error of +0.7%. Analytical values of $E_3$ were much more variable with the largest error being +24.5% for the fine mesh model. $E_1$ and $E_2$ would therefore seem to be the most reliable values to use when comparing the energy absorbed by the specimen to the theoretical maximum energy available for absorption. The coarse mesh and Cowper-Symonds analyses both underestimated this value but all the analyses were in good agreement.

4.5 Enhancement of the analysis model

The analysis results so far presented two areas of concern. These were the initial peak force calculation which seemed to be a gross over-estimation and the mode shape displayed by the collapsing FE specimen which, with the exception of the fine mesh model, tended to be compact. It was necessary to chose one of the four analyses as a baseline from which improvements could be sought. The fine mesh analysis was ruled out due to the time that would be required for each analysis and the coarse mesh model, though yielding good results, was eliminated because the elements were considered to be too large to properly model the expected non-compact collapse. The Cowper-Symonds model had actually increased the high value of initial peak force obtained in the medium mesh analysis and it was difficult to chose which of the two force histories was closer to the experimental characteristic overall. The Cowper-Symonds analysis was finally adopted as a baseline from which further improvements would be sought based on the predictions of energy absorption.
In an attempt to improve the accuracy of the calculation at the proximal end and therefore affect the initial peak force value, the mesh was refined over the proximal 50 mm of the specimen. Uniform and non-uniform mesh refinement schemes were used (Figure 4.13). In the uniform scheme, the longitudinal dimensions of the elements were reduced to 5 mm but their lateral dimensions were maintained at 6.7 mm and 7.1 mm on the wide and narrow sides respectively. The total number of elements increased by 132 in this scheme. The non-uniform mesh refinement scheme reduced the element size from 6.7 mm x 6.7 mm to 5 mm x 5 mm in stages employing one triangular element per side. Elements in this scheme were not necessarily rectangular and there was an increase of 268 in the total number of elements. Displacement and force histories for these two analyses are shown in Figure 4.14 and the deformed shapes are shown in Figures 4.15 and 4.16. The proximal mesh refinement schemes did not result in a lower value of initial peak force and uniform proximal mesh refinement decreased the level of overall fidelity in the force history. The half-fold at the distal end was not as pronounced in the mesh refinement analyses (Figures 4.15 and 4.16) which would seem to indicate that mesh refinement influenced the location of initial buckling.

Another attempt at lowering the initial peak force was made by introducing a geometric imperfection in the mesh. In a real structure there would be many imperfections that result in the structure buckling at a load lower than its theoretical limit. In the FE analyses discussed above all the nodes were perfectly aligned forming a perfect structure. Moving one node crudely out-of-plane by 1 mm would, it was hoped, be sufficient to initiate buckling at a lower value of force. The node chosen was on one of the narrow sides and close enough to the proximal end to be part of the first fold. Figure 4.17 shows the location of the imperfection and Figures 4.18 and 4.19 show the displacement and force histories and the deformed shape plots. The force history caused some alarm. A previous analysis run had been submitted with data points erroneously requested at 0.5 ms intervals. The results from that analysis had shown a reduction in peak force when compared with the perfect tube, yet when data was requested more frequently, the reduction had vanished. This effect can be seen in the force history comparison in Figure 4.18 where the experimental and analytical force histories first cross at approximately 1 ms, the experimental trace rising to, and the analytical trace falling from the initial peak. If the FE data had been requested at the same frequency as the digitised points from the experimental trace the initial slopes of the two curves would be identical up to 1 ms after which the analytical curve would fall while the experimental curve would keep rising to the peak. Comparison of analytical and experimental curves are therefore strongly influenced by the frequency with which the data were written from the FE code. The reader can verify from the turning points on the experimental force history curve (Figures 4.8, 4.14 and 4.18) that the initial peak force on all the FE analyses reported so far would be reduced to a value below the experimental if the data were requested at 1 ms intervals instead of the 0.25 ms...
used. However the author was curious as to the frequency of output at which no difference in
the FE curve would be discernible and this led to a closer inspection of the first 3 ms of the
impact. Data from analyses were requested at more frequent intervals until there was no
appreciable difference in the histories. 0.125 ms was thus determined to be the optimum
interval.

Additionally, the longitudinal location of the imperfection was varied in order to observe the
relationship, if any, between longitudinal location and initial peak force. In proximal to distal
order, nodes 365, 357, 349, 317, 181, 21 and 13 (Figure 4.20) were moved 1 mm out of
plane in successive analyses. The early force response for each case is shown in Figures 4.21
and 4.22 where a plateau can be seen before the initial peak. This initial plateau was believed
to be due to the discretisation of time employed by the analysis technique and could be expected
to tend towards a rounded peak on reduction of the time step. Notwithstanding this, it was
clear that imperfections introduced close to the boundaries, such as the stonewall or the drop-
hammer, resulted in a smaller effect on the post-plateau peak. In all cases the effect on this
peak was, at best, moderate and the study was not pursued further. Another reason for
curtailing investigations into the discrepancy between experimental and predicted peak force is
shown in Equation (4.3).

$$P_{sq} = \sigma_y A$$  \hspace{1cm} (4.3)

Here $P_{sq}$ is the static squash load for the specimen which can be readily calculated as 69.72
kN; almost double the value recorded on the experimental trace. Under dynamic conditions the
squash load could reasonably be expected to be a third higher (88.04 kN) leading to the
following explanations:-

a) the frequency of data acquisition was insufficiently high to capture the early response (0 - 1
ms) in the laboratory experiment, or

b) a feature of the impact or specimen weakened the specimen to such an extent that dynamic
collapse began at a significantly lower force level than the static squash load.

Clearly a combination of the above two explanations is also possible and examination of the
high speed film of the experiment was to yield evidence in support of explanation b) above.
The cine film record of the experiment was examined so that the mode of collapse could be
more accurately compared. It appeared that a side or corner of the specimen arrived at the anvil
ahead of the rest of the proximal section initiating the deformation in an irregular pattern. Such
an impact would result in the initiation of collapse at a force level lower than the dynamic
squash load due to the stress concentration in a smaller area. For example, if one of the wide
sides of the specimen arrived earlier at the anvil, presenting an area of 83 mm$^2$, the dynamic
squat load would be in the region of 30.21 kN ignoring the stiffening effects of the corner regions. A series of analyses were therefore carried out to determine whether rotation of the specimen with respect to the stonewall would affect the previous results. Analyses were performed using the Cowper-Symonds model at 0.25° intervals of rotation about the x and y axes. Combinations of angles about the axes were also tried but none of the rotations exceeded 1°. Figure 4.23 shows the sense of rotation and Figure 4.24 shows the displacement and force history comparison to the experimental results for 1° of rotation about the x and y axes which was typical of the other rotation analyses. Deformed shapes of the experimental specimen and the analysis specimen are compared in Figures 4.25 and 4.26. The peak force was always lower than that of the baseline Cowper-Symonds analysis but the overall fidelity of the force history was not greatly improved. However, the irregular folding patterns in the rotated analyses (Figure 4.25) were much closer to the type observed on the high speed film of the test than for the baseline simulation. The angled fold lines generated in the rotated FE specimen were also observed in the experimental specimen which lends credence to the observation above that the experimental specimen arrived off-axis at the anvil, a feature which had not been accounted for in the analyses to date.

The UMIST validation study was brought to a halt at this point several important lessons having been learned. The most important of these was that the FE structure and method of analysis, with all the approximations inherent in its construction, still tended to behave in a perfect manner which was unrealistic. It was therefore necessary to introduce some degree of imperfection in order to correctly predict the collapse mode of a real structure. In this case the experimental specimen arrived at the anvil off-axis so it was necessary to re-create this feature in the analysis before fully realistic results could be obtained. The medium mesh analysis with strain-rate enhancement as described gave good predictions of energy absorption, maximum crush and force history so the material model and mesh density would be considered for subsequent work.

4.6 Analysis of IRC drop-hammer experiments

The second validation study used a square specimen in a different impact configuration. Whereas the UMIST specimen was attached to the drop-hammer, the specimen described in the following was at rest on the anvil when struck by the drop-hammer. Analysis of this situation required a slightly different modelling scheme and the determination of a suitable interface stiffness between the drop-hammer and the specimen. The collapse mode of the specimen was uniform and symmetric so a three-way comparison could be made between experimental, FE and previously available analytical results using the BCE theory described in Chapter 3.
The experiments were chosen from the series of eighty-four drop-hammer experiments reported by Abramowicz and Jones [45] where various masses were dropped onto square section mild steel specimens that were resting on a rigid anvil. The impact velocity was measured using a timing gate and the velocity history during crush was measured in some cases from high speed film records and in others by a laser Doppler system.

4.6.1 FE Analysis Scheme

The FE analysis scheme for the IRC experiments is shown in Figure 4.27. A drop-hammer composed of four thin shell elements with a rigid material model was given an initial velocity in the downward (+z) direction. The specimen rested on a stonewall with a "stick" condition applied. Thin shell elements were used for the drop-hammer instead of a solid element because they enabled excessive penetration by the specimen to be observed more readily during model development.

The work performed on modelling the interface between the drop-hammer and the specimen for this set of analyses is described in detail in Appendix III and briefly summarised here. Due to the large difference in size between the elements representing the drop-hammer and those representing the specimen the contact surface stiffness calculations would result in a non-symmetric stiffness and incorrect force calculation. The guidelines in the FE code user manual suggested that for cases where the rigid element is much larger than the non-rigid element to be contacted, the Young's Modulus of the large rigid body should be scaled down by a factor given by the ratio of characteristic lengths for the two sets of elements. However the author found it necessary to scale the density of elements of the larger body rather than the Young's Modulus. For this reason it was necessary to first compare analyses of perfect structures of different mesh densities and no strain-rate enhancement to check for similar trends in $P_m^d$ as those found in the UMIST validation study. After this was done strain-rate enhancement and imperfections were added as will be described presently.

Mesh densities in the UMIST validation study described above were chosen somewhat arbitrarily but the medium mesh density was found to give reasonable results without requiring unreasonable computational effort. It was therefore decided to carry these mesh densities forward in a systematic fashion and check if similar findings would be obtained. For example $P_m^d$ was found to increase monotonically for a specimen on refinement of the mesh. A check was made for this trend in the current set of analyses.
The method used to obtain comparable mesh densities was a simple one: the value, for each mesh density, of the following fraction was maintained as far as possible:

\[
\frac{\text{surface area of element}}{\text{surface area of specimen}}
\]

This fraction will be referred to as the area fraction and the reason that exact fractions could not be maintained was because of the desire to have a whole number of equally sized elements in the mesh and thus minimise the risk of spurious results. The ideal area fractions calculated from the UMIST FE specimens were 11.10E-4, 5.05E-4 and 2.78E-4 for coarse, medium and fine meshes respectively. The surface area of the IRC specimen was multiplied by these values to determine the ideal surface area of the elements, then the element dimensions were chosen to give the closest value to the ideal surface area using a uniform mesh. An additional constraint was the aspect ratio of each element because it was considered desirable to keep the elements as square in shape as possible.

An isotropic elastic, linear strain-hardening material model was used (DYNA3D material 3) in keeping with the UMIST validation study. A Young's Modulus of 200 GPa, hardening modulus of 1.4 GPa and a Poisson's ratio of 0.3 were specified. The static yield stress specified was 277.5 MPa and 264.5 MPa for the 1.5'' and 2'' square specimen respectively. These were the average 0.2% proof stress values given in the experimental data and were derived from coupon tests of material cut from undeformed specimens. No strain-rate enhancement was used initially in the analysis so that direct comparisons could be made with the previous validation results with respect to mesh refinement. Excerpts from the experimental results [45] are shown in Tables 4.3 and 4.4.

**Table 4.3 Experimental results - Excerpt from Table 1 [45] for 1.5'' square specimen.**

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>L (mm)</th>
<th>M (kg)</th>
<th>V₀ (ms⁻¹)</th>
<th>K (kJ)</th>
<th>Δ⟩f (mm)</th>
<th>Pm (kN)</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>177.9</td>
<td>73.6</td>
<td>9.806</td>
<td>3.54</td>
<td>141.6</td>
<td>25.0</td>
<td>S</td>
</tr>
<tr>
<td>121</td>
<td>244.1</td>
<td>73.6</td>
<td>10.288</td>
<td>3.90</td>
<td>149.5</td>
<td>26.1</td>
<td>S</td>
</tr>
<tr>
<td>124</td>
<td>244.0</td>
<td>73.6</td>
<td>10.291</td>
<td>3.90</td>
<td>161.0</td>
<td>24.2</td>
<td>S</td>
</tr>
<tr>
<td>133</td>
<td>289.1</td>
<td>73.6</td>
<td>8.476</td>
<td>2.64</td>
<td>109.7</td>
<td>24.1</td>
<td>S</td>
</tr>
<tr>
<td>134</td>
<td>289.0</td>
<td>73.6</td>
<td>9.675</td>
<td>3.44</td>
<td>140.0</td>
<td>24.6</td>
<td>S</td>
</tr>
</tbody>
</table>

* S=Symmetrical collapse mode.
Table 4.4 Experimental results - Excerpt from Table 2 [45]
for 2" square specimen.

+0.06
\[ c = 49.31 \text{ mm } h = 1.63 \pm 0.02 \text{mm} \]
-0.14

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>( L ) (mm)</th>
<th>( M ) (kg)</th>
<th>( V_o ) (ms(^{-1}))</th>
<th>( K ) (kJ)</th>
<th>( \delta_f ) (mm)</th>
<th>( P_m^d ) (kN)</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>29</td>
<td>267.2</td>
<td>73.6</td>
<td>10.262</td>
<td>3.88</td>
<td>81.9</td>
<td>47.3</td>
<td>S</td>
</tr>
<tr>
<td>30</td>
<td>267.1</td>
<td>73.6</td>
<td>9.697</td>
<td>3.46</td>
<td>73.3</td>
<td>47.2</td>
<td>S</td>
</tr>
</tbody>
</table>

Table 4.5 shows the area fractions used for the coarse, medium and fine mesh analyses for each of the specimen. For each of the analyses, Table 4.6 shows the energy absorbed by the specimen, maximum crush and the mean dynamic force. A comment is also made regarding the degree to which the specimen was crushed during the impact. Force histories for the analyses are shown in Figures 4.28-4.31. Figures 4.32 and 4.33 show the deformed shape plots for runs ab17 and ab23 respectively. These were typical of the remaining deformed shape plots which are not included for brevity.

Table 4.5 Actual area fractions for the FE analysis specimens

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Specimen No.</th>
<th>( L ) (mm)</th>
<th>Mesh density</th>
<th>Area Fraction</th>
<th>Element dimension (mm)</th>
<th>No. of Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>ab27</td>
<td>19</td>
<td>177.9</td>
<td>C</td>
<td>10.83E-4</td>
<td>5.30 x 5.39</td>
<td>924</td>
</tr>
<tr>
<td>ab28</td>
<td>19</td>
<td>177.9</td>
<td>M</td>
<td>5.10E-4</td>
<td>3.71 x 3.63</td>
<td>1960</td>
</tr>
<tr>
<td>ab29</td>
<td>19</td>
<td>177.9</td>
<td>F</td>
<td>2.91E-4</td>
<td>2.65 x 2.74</td>
<td>3640</td>
</tr>
<tr>
<td>ab17</td>
<td>121/24</td>
<td>244.05</td>
<td>C</td>
<td>10.67E-4</td>
<td>6.17 x 6.26</td>
<td>936</td>
</tr>
<tr>
<td>ab19</td>
<td>121/24</td>
<td>244.05</td>
<td>M</td>
<td>5.89E-4</td>
<td>4.63 x 4.6</td>
<td>1696</td>
</tr>
<tr>
<td>ab20</td>
<td>121/24</td>
<td>244.05</td>
<td>F</td>
<td>2.95E-4</td>
<td>3.37 x 3.17</td>
<td>3389</td>
</tr>
<tr>
<td>ab21</td>
<td>133/34</td>
<td>289.05</td>
<td>C</td>
<td>9.69E-4</td>
<td>6.18 x 6.72</td>
<td>1030</td>
</tr>
<tr>
<td>ab22</td>
<td>133/34</td>
<td>289.05</td>
<td>M</td>
<td>4.96E-4</td>
<td>4.63 x 4.59</td>
<td>2016</td>
</tr>
<tr>
<td>ab23</td>
<td>133/34</td>
<td>289.05</td>
<td>F</td>
<td>2.98E-4</td>
<td>3.71 x 3.44</td>
<td>3360</td>
</tr>
<tr>
<td>ab24</td>
<td>29/30</td>
<td>267.05</td>
<td>C</td>
<td>11.90E-4</td>
<td>8.22 x 7.63</td>
<td>840</td>
</tr>
<tr>
<td>ab25</td>
<td>29/30</td>
<td>267.05</td>
<td>M</td>
<td>5.49E-4</td>
<td>5.48 x 5.24</td>
<td>1836</td>
</tr>
<tr>
<td>ab26</td>
<td>29/30</td>
<td>267.05</td>
<td>F</td>
<td>3.02E-4</td>
<td>4.11 x 3.87</td>
<td>3312</td>
</tr>
</tbody>
</table>

Mesh density: C=coarse, M=medium F=fine
Table 4.6 Preliminary FE analysis results for IRC specimen

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Specimen No.</th>
<th>$L$ (mm)</th>
<th>$V_0$ (m/s)</th>
<th>Mesh density</th>
<th>$E_a$ (kJ)</th>
<th>$\delta_f$ (mm)</th>
<th>$P_m^d$ (kN)</th>
<th>Collapse</th>
</tr>
</thead>
<tbody>
<tr>
<td>ab27</td>
<td>I9</td>
<td>177.9</td>
<td>9.806</td>
<td>C</td>
<td>3.610</td>
<td>144.2</td>
<td>27.219</td>
<td>C</td>
</tr>
<tr>
<td>ab28</td>
<td>I9</td>
<td>177.9</td>
<td></td>
<td>M</td>
<td>3.720</td>
<td>147.4</td>
<td>28.461</td>
<td>C</td>
</tr>
<tr>
<td>ab29</td>
<td>I9</td>
<td>177.9</td>
<td></td>
<td>F</td>
<td>3.678</td>
<td>149.3</td>
<td>29.348</td>
<td>C</td>
</tr>
<tr>
<td>ab17</td>
<td>I21/24</td>
<td>244.05</td>
<td>10.290</td>
<td>C</td>
<td>3.843</td>
<td>181.1</td>
<td>20.372</td>
<td>C</td>
</tr>
<tr>
<td>ab19</td>
<td>I21/24</td>
<td>244.05</td>
<td></td>
<td>M</td>
<td>3.947</td>
<td>188.4</td>
<td>22.074</td>
<td>C</td>
</tr>
<tr>
<td>ab20</td>
<td>I21/24</td>
<td>244.05</td>
<td></td>
<td>F</td>
<td>3.967</td>
<td>189.2</td>
<td>22.882</td>
<td>C</td>
</tr>
<tr>
<td>ab21</td>
<td>I33/34</td>
<td>289.05</td>
<td>9.076</td>
<td>C</td>
<td>3.050</td>
<td>139.8</td>
<td>19.964</td>
<td>I</td>
</tr>
<tr>
<td>ab22</td>
<td>I33/34</td>
<td>289.05</td>
<td></td>
<td>M</td>
<td>3.052</td>
<td>140.1</td>
<td>19.091</td>
<td>I</td>
</tr>
<tr>
<td>ab23</td>
<td>I33/34</td>
<td>289.05</td>
<td></td>
<td>F</td>
<td>3.048</td>
<td>157.8</td>
<td>17.307</td>
<td>I</td>
</tr>
<tr>
<td>ab24</td>
<td>29/30</td>
<td>267.05</td>
<td>9.980</td>
<td>C</td>
<td>3.594</td>
<td>73.9</td>
<td>46.373</td>
<td>I</td>
</tr>
<tr>
<td>ab25</td>
<td>29/30</td>
<td>267.05</td>
<td></td>
<td>M</td>
<td>3.650</td>
<td>82.5</td>
<td>36.998</td>
<td>I</td>
</tr>
<tr>
<td>ab26</td>
<td>29/30</td>
<td>267.05</td>
<td></td>
<td>F</td>
<td>3.688</td>
<td>82.7</td>
<td>38.132</td>
<td>I</td>
</tr>
</tbody>
</table>

Mesh density: C=coarse, M=medium F=fine
Collapse: C=complete, I=incomplete

Table 4.5 shows that it was possible to obtain reasonably close area fractions to the ideal whilst satisfying the aspect ratio constraint. Apart from analyses ab21 and ab19 which were 12.7% and 16.6% coarser respectively, the FE meshes were within 10% of the ideal mesh densities. Comparison of mesh refinement trends in the current set of analyses with those of the previous validation study was therefore carried out. The trend in analytical values of $P_m^d$ for coarse, medium and fine meshes fell into two categories. For specimen 19, 21 and 24 (analyses ab27-ab29 and ab17-ab20) $P_m^d$ increased as the mesh was refined. For the other specimens the opposite trend was evident. In analyses ab27-ab29 and ab17-ab20 there was no part of the specimen that remained undeformed. Complete collapse was followed in some cases by further compression of the folds after the formation of the last fold. This type of complete collapse will be referred to as bottoming out. For the other impacts analysed the specimen absorbed the impact energy without bottoming out. These two cases led to the difference in force history characteristics between Figures 4.28 and 4.29 on one hand and Figures 4.30 and 4.31 on the other. After the initial peak force, the response of the specimens shown in Figure 4.28 settled into a regular pattern until approximately 20 ms when the force began to build up due to the specimen bottoming out and becoming increasingly difficult to compress. Finer meshes seemed to model this process in more detail. The build-up in the force would strongly
influence the $P_m^d$ value as discussed in reference to the UMIST specimen and the characteristics shown in Figures 4.28 and 4.29 display a similar tail end plateau to that found for the UMIST specimen.

The force histories shown in Figures 4.30 and 4.31 do not display the build-up in force towards the end of the analysis and could reasonably be characterised by a mean value. It was therefore clear that the degree to which a specimen was crushed could affect the confidence with which $P_m^d$ could be used to characterise the force response.

The experimental values of $P_m^d$ (Tables 4.3 and 4.4) were obtained by dividing the kinetic energy of the drop-hammer prior to impact by $\delta_f$, the final crush distance. For uniform symmetric collapse this method of determining the mean dynamic force should give reasonably reliable results where there is no bottoming out of the specimen. Table 4.6 shows that the medium mesh analytical predictions of $P_m^d$ were consistently in error by more than 20% for the analyses with incomplete collapse and more than 12% for the others. The table also shows that the analytical predictions of $\delta_f$ for the medium mesh analyses were prone to large errors when compared to the averaged experimental values but predictions of $E_a$ were within 5% of the initial kinetic energy, $K$.

If the experimental values of $\delta_f$ were obtained by measurement post-impact, the analytical errors for this quantity could be partially attributed to elastic recovery of the specimen. In the analysis the maximum value of specimen crush was recorded and no estimate was made for elastic recovery. Experimental values of $\delta_f$ for specimens I21 and I24 showed a 7% variation for nominally identical impact velocities so it would be reasonable to attribute this much of the maximum 21.6% error (ab19) to experimental scatter. Analyses ab27-ab29 were least in error (maximum error of 2.6%) probably because the undeformed specimen was the shortest of the series analysed and it bottomed out. The final crush value would thus be related more to the volume of material from which the specimen was made than to crush behaviour during collapse. It was considered inappropriate at this stage however to conduct a detailed error analysis because no imperfections had been included in the structure and the material model did not include strain-rate enhancement. The deformed shape plots highlighted the need for the former, and the inclusion of the latter will be discussed subsequently.

The deformed shape plots shown in Figures 4.32 and 4.33 show that the initial folding patterns of the specimens in the FE analyses were extensional (all four sides folded outwards). This high energy mode of collapse was not observed in any of the experimental laboratory specimens in ref. [45] and was considered to be due to the perfect nature of the specimen in the analyses. As the analyses progressed the mode shape switched to the expected symmetrical
one. During the development of the analysis models the symmetrical mode was obtained when
the model structure was made imperfect by the out-of-plane displacement of a node near the
proximal end and the UMIST validation studies also highlighted the need to avoid perfect
analysis structures. No imperfections were introduced in this set of analyses in order to
confirm that the new modelling scheme, where the drop-hammer struck a specimen at rest, and
that the new contact surface representation would give similar trends to the previous UMIST
modelling scheme upon mesh refinement. It was therefore necessary to keep all other
modelling details similar between the two cases. With the confirmation complete, the strain-
rate enhancement was next incorporated into the material model and two imperfections placed
near the proximal end of the specimen as described in the following section.

4.6.2 Preparation of the experimental drop-hammer velocity
history result received

The detailed validation of the FE code was greatly aided by the supply of the experimental
drop-hammer velocity history for specimen 121 [110] and the remainder of this chapter
concentrates on this particular experiment. The FE material model was modified to include
strain-rate enhancement using the Cowper-Symonds constitutive equation, the co-efficients
used being the same as for the UMIST validation study. Two values of strain-hardening
modulus were used, 1 GPa which is usually quoted for mild steel and 1.4 GPa which was
quoted for seamless cold finished steel in BS6323 [111]. The strain-hardening modulus was
the only parametric change between the two FE analyses presented in the remainder of this
chapter.

Geometric imperfections were introduced in the form of nodes displaced 0.5 mm out of plane
in sympathy with the expected mode of collapse. The axial location of the nodes was
determined using the preliminary analysis deformed shapes to find the first plastic wave. The
global co-ordinates of two nodes on opposite sides of the specimen were altered before the
main analysis. Nodes were chosen to be as close as possible to the peak of the first plastic
wave and the middle of the specimen sides (Figure 4.34). The same initial geometry was
employed for both analyses. Random surface imperfections in the laboratory specimens were
measured to be less than 0.1 mm [112] but initial nodal displacements less than 0.25 mm in the
FE specimen resulted in an extensional collapse mode where all four sides folded outwards.
The specimen was therefore adopting a higher energy collapse mode than that described in
Chapter 3 because the two imperfections did not have a sufficient amplitude to overcome the
effects of perfectly aligned nodes in the rest of the specimen. The final amplitude chosen was
0.5 mm to ensure that perfect structure behaviour was avoided. Any errors, say in mean force
prediction, introduced by this form of pre-triggering were assumed to be limited to the first
layer of folds and would be masked by the remainder of the specimen response.
The velocity history for specimen I21 recorded the velocity of the drop-hammer from the point of contact with the specimen until it came to rest. The hard copy was scanned using a laser scanner and stored as a graphical image file which was displayed on a graphics computer screen and digitised by logging the screen cursor position in pixels while tracing the axes and the curve. Co-ordinates of the data points were then transformed from screen pixels to time and velocity. The digitised experimental record was subsequently stored in a readily accessible format for post-processor comparison with the analytical results. Figure 4.35 shows the velocity history as received and in digitised form.

4.6.3 Results

The velocity history (Figure 4.35b) was integrated to obtain crush history and the initial kinetic energy was divided by the maximum crush to obtain \( P_m^d \). Table 4.7 shows that, as expected, the results obtained from the digitised trace received were within 1.5% of the published results for that specimen (Table 4.3).

<table>
<thead>
<tr>
<th>Specimen No.</th>
<th>( V_o ) (ms(^{-1}))</th>
<th>( K ) (kJ)</th>
<th>( \delta_f ) (mm)</th>
<th>( P_m^d ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I21</td>
<td>10.29</td>
<td>3.90</td>
<td>151</td>
<td>25.83</td>
</tr>
</tbody>
</table>

Table 4.8 shows \( \delta_f \) and \( P_m^d \) for the two FE analyses. The Figures in brackets are errors with respect to the experimental values in Table 4.7. The analysis ab32 required 8.7 hours of processor time and ab34 required 8.1 hours of CPU time.

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Specimen No.</th>
<th>( K ) (kJ)</th>
<th>( \delta_f ) (mm)</th>
<th>( P_m^d ) (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ab32 ( E_T = 1 ) Gpa</td>
<td>I21</td>
<td>3.9</td>
<td>176 (+16.6%)</td>
<td>22.16 (-14.2%)</td>
</tr>
<tr>
<td>ab34 ( E_T = 1.4 ) Gpa</td>
<td>I21</td>
<td>3.9</td>
<td>165 (+9.3%)</td>
<td>23.64 (-8.5%)</td>
</tr>
</tbody>
</table>

The degree of correlation between experimental and analytical drop-hammer velocity history for the two analyses is shown in Figure 4.36 and Figure 4.37 shows the FE specimen force histories. Crush and energy histories for the analyses are shown in Figures 4.38 and 4.39 respectively. It will be noted that the FE specimens seemed to absorb more than 3.9 kJ of energy (Figure 4.39). The extra energy, which was always less than 3%, was used to maintain contact interfaces and was excluded from all error analyses discussed. Deformed
shape plots are given in Figures 4.40 and 4.41. The FE specimens folded with ten layers of folds (40 BCE) when $E_t$ was 1 GPa and nine layers of folds (36 BCE) when $E_t$ was 1.4 GPa.

In order to identify the shell elements that collectively formed a fold layer of four BCE all nodal displacements were factored by 0.3. An example of a deformed shape plot with factored displacements is shown in Figure (4.42a) where the nine layers of folds for analysis ab34 can be seen at 33 ms which was the time of maximum crush. Figure (4.42b) shows the strain energy density plot overlaid on the factored deformed shape for this analysis. For each layer of fold the strain energy density for the shell elements were written out. A FORTRAN program was written to sum the energy absorbed by the elements in each fold and this quantity was quartered to estimate the energy absorbed by a single BCE. Using a similar technique, the effective plastic strain for each shell element in a fold layer was used to estimate an average strain for a BCE. The effective plastic strain calculated by the analysis is given in Equation (4.4).

$$\varepsilon_{eff}^P = \sqrt{\frac{1}{2} \left( \frac{1}{2} \right) \left( \frac{\dot{\varepsilon}^P_{ij} \dot{\varepsilon}^P_{kl}}{\dot{\varepsilon}^P_{kl}} \right)^{1/2}$$

(4.4)

Here, $t$ is the current time and $\dot{\varepsilon}^P_{ij}$ is the plastic component of the total rate of deformation tensor for the FE structure.

The values obtained by the methods above are shown in Tables 4.9 and 4.10 where BCE No. 1 is in the top proximal fold. Two averages are given in the bottom rows of the tables. The first value is the average over all the BCE in the specimen and the second, in brackets, excludes the top and bottom BCE to minimise any effects that may be due to interface modelling. The tables also show the maximum and the average strain-rate for each BCE which was determined by writing the average BCE strains at one-millisecond intervals up to 34 ms (Figure 4.43). $\dot{\varepsilon}_{max}$ was the maximum slope of the strain history and $\dot{\varepsilon}$ was obtained by determining the start and end points of collapse and calculating the gradient between these points on the BCE strain histories. For example, the maximum slope of the strain history for BCE 1 in analysis ab32 was 133 s$^{-1}$ (Figure 4.43a) and the average strain-rate was calculated between 0 and 4 ms after which time the BCE was assumed to have collapsed. The strain at 4 ms was 0.18 giving an average strain-rate of 45 s$^{-1}$.
Table 4.9 FE analysis ab32 - $E_t=1$ GPa

<table>
<thead>
<tr>
<th>BCE No.</th>
<th>No. of Rows</th>
<th>Energy Absorbed (J)</th>
<th>Max. Strain (%)</th>
<th>$\dot{\varepsilon}_{\text{max}}$ (s$^{-1}$)</th>
<th>$\dot{\varepsilon}$ (s$^{-1}$)</th>
</tr>
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<tr>
<td>1</td>
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<td>127.10</td>
<td>18.5</td>
<td>133</td>
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<td>2</td>
<td>6</td>
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<td>11.8</td>
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<td>20</td>
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<td>3</td>
<td>6</td>
<td>105.59</td>
<td>11.9</td>
<td>40</td>
<td>19</td>
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<td>4</td>
<td>6</td>
<td>97.11</td>
<td>11.4</td>
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<td>21</td>
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<td>6</td>
<td>6</td>
<td>93.41</td>
<td>11.1</td>
<td>24</td>
<td>17</td>
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<td>7</td>
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<td>96.37</td>
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<td>15</td>
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<td>10.8</td>
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<td>8</td>
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<tr>
<td><strong>Average</strong></td>
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<td><strong>99.58</strong></td>
<td><strong>12.1</strong></td>
<td><strong>36</strong></td>
<td><strong>18</strong></td>
</tr>
</tbody>
</table>

Table 4.10 FE analysis ab34 - $E_t=1.4$ GPa

<table>
<thead>
<tr>
<th>BCE No.</th>
<th>No. of Rows</th>
<th>Energy Absorbed (J)</th>
<th>Max. Strain (%)</th>
<th>$\dot{\varepsilon}_{\text{max}}$ (s$^{-1}$)</th>
<th>$\dot{\varepsilon}$ (s$^{-1}$)</th>
</tr>
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<tbody>
<tr>
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<td>126.88</td>
<td>17.1</td>
<td>120</td>
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<td>2</td>
<td>6</td>
<td>114.82</td>
<td>11.9</td>
<td>34</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>105.45</td>
<td>11.0</td>
<td>29</td>
<td>20</td>
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<tr>
<td>4</td>
<td>7</td>
<td>116.73</td>
<td>10.4</td>
<td>27</td>
<td>19</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>96.43</td>
<td>10.0</td>
<td>28</td>
<td>15</td>
</tr>
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<td>6</td>
<td>6</td>
<td>105.47</td>
<td>10.5</td>
<td>31</td>
<td>14</td>
</tr>
<tr>
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<td>89.77</td>
<td>9.8</td>
<td>29</td>
<td>11</td>
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<tr>
<td>8</td>
<td>7</td>
<td>120.96</td>
<td>11.3</td>
<td>26</td>
<td>11</td>
</tr>
<tr>
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<td>10</td>
<td>127.18</td>
<td>9.1</td>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>6</strong></td>
<td><strong>111.52</strong></td>
<td><strong>11.2</strong></td>
<td><strong>38</strong></td>
<td><strong>18</strong></td>
</tr>
</tbody>
</table>

(97.14) (11.4) (27) (16)
4.6.4 Discussion of IRC validation study results

The procedure used to convert the laboratory traces to data points worked satisfactorily. It was assumed that any distortions that occurred in the photocopying and scanning would be mitigated by the digitising process which maintained the position of the curve relative to the axes. Comparison of the timing and magnitude of the main turning points in Figure 4.35 justified that assumption.

When validating a FE analysis technique it should not be the aim of the analyst to obtain a perfect match between the FE result and the laboratory result. Laboratory experiments with seemingly identical specimen rarely produce identical results and it was interesting to note that the two similar specimen 121 and 124 in Table 4.3 showed a 7.4% and 7.6% scatter around the mean value of $\delta_f$ and $P_m^d$ respectively. It would therefore be reasonable to accept errors of this magnitude in the predictions given by the FE analyses. For the FE specimen with $E_t = 1.4$ Gpa (ab34) the $\delta_f$ and $P_m^d$ predictions were within acceptable limits (9.3%) as shown in Table 4.8 but the specimen in ab32 required a 14.2% lower mean force to crush it than the experimental specimen crushing 16.6% more in order to absorb the incident energy. In addition to this poorer performance, ab32 incurred a 6.5% processing time penalty with respect to ab34. The longer analysis time was due to the larger degree of collapse of the specimen which required smaller time-steps to ensure a stable analysis.

The overall shape of a characteristic response such as the drop-hammer velocity history or force history should be similar between experiments carried out using like specimen. One of the objectives in this validation study was to obtain a similar velocity history between experimental and analytical cases and this was judged to have been achieved (Figure 4.36). Specimen 121 did not bottom out, the latter stages of the experimental velocity history showed a slight decrease in slope (Figure 4.35a) and this characteristic was reflected more in ab34 than in ab32 which tended to indicate a slight bottoming out towards the end of the analysis. This bottoming out characteristic can be more clearly seen by comparing the latter stages of the force histories of the specimens in Figure 4.37 where, for ab32, the force rose to a peak of 33 kN after the formation of the last fold. The velocity history comparisons were, on the whole, sufficiently encouraging to attempt a detailed comparison between FE results for energy absorbed and the BCE theory summarised in Chapter 3.

The theoretical axial length of fold (wavelength) for specimen 121 can be determined from Equation (3.6) in Chapter 3 as $2H = 23.08$ mm. Given an undeformed specimen length of 244.1 mm, complete collapse would result in 10.6 layers of folds. There would therefore be insufficient material to form 11 layers of folds with the above wavelength but 10 layers could
reasonably be expected. In addition, the axial dimension of the quadrilateral shell elements in
the FE analyses was 4.14 mm giving 5.6 elements to a fold. Each fold should therefore be no
more than 6 rows of elements long. In the FE analyses the specimen in run ab32 collapsed
with 10 layers of folds (Figure 4.40) and the specimen in run ab34 collapsed with 9 layers of
folds (Figure 4.41). Since the only parameter change between the two analyses was the
increased strain-hardening modulus for run ab34, the change in mode shape was attributed to
the post-yield material characteristics of the FE specimens. The number of rows of elements
that formed a BCE in run ab34 varied between 5 and 10 (Table 4.10) whereas all but the
proximal BCE were made up of 6 rows of elements in run ab32 (Table 4.9). The analysis with
the lower strain-hardening modulus therefore, seemed to result in a collapse mode that was
closer to the theoretical rigid-plastic analysis in terms of the uniformity and the dimensions of
the BCE.

In Chapter 3 Equation (3.1.3) was used to predict that a BCE in a specimen of the \( h/c \)
being considered here would absorb approximately 102.186 J of energy for an impact velocity
of 10.29 ms\(^{-1} \) (\( K = 3.897 \) kJ). Both specimens would therefore need to have developed 38
BCE of wavelength 23.08 mm if they were to fully absorb the impact energy. For uniform
symmetric collapse of a square specimen each layer of folds consist of 4 BCE so the specimen
would come closest to the theory by forming 36 or 40 BCE (9 or 10 layers of folds). The
specimen in ab32 formed 10 layers of folds and absorbed an average of 99.58 J per BCE, an
error of -2.6% with respect to the previous estimate. It is good practice to treat FE results at
the detailed level of elements with caution in the boundary regions so the most proximal and
distal BCE are excluded from further analyses. Considering BCE 2-9 therefore (Table 4.9), an
average of 97.14 J was absorbed by each BCE in ab32, under-predicting Equation (3.1.3) by
4.9%. The maximum individual error, excluding the BCE at the boundaries, for ab32 was
-10.3% and the minimum error was -1%.

The specimen in analysis ab34 formed 9 folds (Figure 4.41). An average of 107.09 J was
absorbed per BCE, over-predicting Equation (3.1.3) by 4.8% although, as mentioned
previously, there was considerable variation in the wavelengths predicted in this analysis.
Considering individual BCE, the maximum error in predicting energy-absorption was +18.4%
and the minimum was +3.2%.

The average strain in Table 4.9 reported for the BCE was 11.4%. and in Table 4.10 it was
10.7%. These figures compare favourably to the predicted value of 10.9% strain given by
Equation (3.1.9) in Chapter 3, using a value of \( b = 2.638 \) mm from Equation (3.1.7). The
analysis with the lower post-yield modulus (ab32) over-predicted average strain by 4.6% and
ab34 under-predicted average strain by 1.8%. The maximum error with regard to Equation
(3.1.9) was +9.2% and -10.1% for runs ab32 and ab34 respectively, the latter analysis giving
a wider range of error (-10.1% to +9.2%). Errors in average strain in ab32 ranged from (+1.8% to +9.2%). It was interesting to note that, with the exception of the proximal and distal BCEs, most of the BCEs reached similar final strains and energies. The final strains can be seen more graphically in Figure 4.43.

The precise method used to estimate strain-rate from the strain histories significantly affected the individual BCE results. For example, the maximum slope of the strain history for BCE 2 in both analyses was 31 s\(^{-1}\) and 34 s\(^{-1}\) as shown in column 5 of Tables 4.9 and 4.10. Estimating the strain-rate using that method led to a +55% and +41.6% difference with respect to the figures shown in column 6 which were calculated over the whole BCE collapse time as previously described. Differences of more than 100% can readily be seen for some of the other BCE. The overall specimen averages show that \(\dot{e}_{\text{max}}\) was almost twice as great as \(\dot{e}\) for both analyses but, at 27 s\(^{-1}\) and 29 s\(^{-1}\) for ab32 and ab34 respectively, the predictions were not comparable to the 91.6 s\(^{-1}\) obtained using Equation (3.1.11). It can be observed that, apart from the proximal and distal BCEs, the remainder appeared to collapse at a similar strain-rate of the order of 30 s\(^{-1}\). Using this as a characteristic strain-rate in the kinematical method led to an estimate of 95.858 J of energy absorbed by a BCE instead of the 102.186 J calculated using Equation (3.1.11) to estimate the strain-rate. If the average strain-rates were indeed of the order of 30 s\(^{-1}\), the error between previously available analytical predictions and the current prediction would be +1.3% and +11.17% for \(E_t = 1\) GPa and 1.4 GPa respectively.

As previously mentioned in Chapter 3, the authors of Equation (3.1.11) advised caution in the application of that estimate and it may be that a non-linear relationship exists between strain-rate in a given uniform section and impact velocity. It was an encouraging outcome of the detailed validation study that the average strain-rates predicted from the BCE fell within the range of 3 to 64 s\(^{-1}\) recently reported by McGregor et al. [113] for an aluminium crush tube subjected to a 10 kJ impact at approximately 11 ms\(^{-1}\).

**4.7 Concluding remarks**

The purpose of the validation studies was to develop an analysis method that would yield results that were close to previous estimates and experimental observations. The UMIST validation study had the modest objective of predicting an experimental force history and mean force and crush values for a rectangular mild steel specimen in a drop-hammer test. It was discovered however, that for that particular response, where the specimen bottomed out, the mean force may be correctly predicted by an analysis displaying an uncharacteristic force history. The frequency with which data were written out from the FE analysis or acquired from the laboratory equipment was also found to be especially important in the first 2
milliseconds where high amplitude fluctuations in force history occur. For example, elastic stress waves would traverse the length of the specimen several times during the first millisecond (equation 2.20) although it is likely to be the first plastic wave, travelling more slowly, that is associated with the initial peak force. For the UMIST validation it may be concluded that the laboratory equipment did not capture data at a sufficiently high frequency to record the initial peak value of force. The peak force recorded on the oscilloscope trace was approximately 39 kN whereas the calculated static squash load for the section was approximately 70 kN and the dynamic squash load would be higher still. This aside, the remainder of the experimental trace could be relied upon as a true characterisation of the specimen's response because the transients were much lower in both frequency and amplitude as could be seen from the experimental traces.

All the mesh densities in the UMIST validation study resulted in force histories that indicated a bottoming out of the specimen after two or three peaks, the tail end plateau increasing in magnitude with mesh refinement. The coarse and medium mesh were judged to reflect the experimental force characteristic well, as was the Cowper-Symonds analysis but the deformation mode of these FE specimen tended to be compact. The fine mesh analysis displayed a non-compact mode of collapse but was computationally intensive with a force history that was not judged to be as representative as the others.

Evidence of an off-axis arrival of the specimen at the anvil was discerned in the high speed film of the test (slides 2 and 3 figure 4.26). On rotating the FE specimen through a small angle relative to the stonewall the correct irregular collapse mode was displayed by the Cowper-Symonds analysis, the initial peak force was reduced to a justifiable value and the overall fidelity of the force history was not adversely affected. This was an important result because it clearly illustrated the danger of accepting all aspects of FE predictions without confirmation by experiment. The imperfections in the real life situation must be accounted for in the FE analysis and the two methods of achieving this that arose out of this study was to rotate the model or perhaps to move nodes out of plane prior to analysis.

Of the three methods of determining the energy absorbed by the FE specimen, two were shown to be reliable. These were the integral of the force-displacement curve and the internal energy for the specimen calculated directly by the analysis. Both methods predicted energy absorption values that were within 4% of the expected value.

The IRC validation study looked in more detail at the internal energy distribution in the specimen with specific reference to available estimates. It was reassuring to observe that the predictions of energy absorbed and BCE dimensions were closer to the rigid-plastic analysis estimates for the FE specimen with a lower strain-hardening modulus. The model with a
higher strain-hardening modulus however gave predictions of drop-hammer velocity history that were closer to the experimental case so this material model should be used when analysing real structures. Geometric imperfections were employed in this study to prevent the specimen behaving like a perfect structure. In this case two nodes were moved out of plane to obtain the experimentally observed collapse mode shape.

The validated FE code could now be used to explore methods of improving the performance of chassis rails and the following chapters detail the work done to this end.
Figure 4.1. Geometry model of a Ford Sierra Sapphire chassis rail.

Figure 4.2. Two mesh densities on an identical patch.
Figure 4.3  Outward normals for shell elements where self-contact is likely.

Figure 4.4  Example of overlapping contact surfaces.
Figure 4.5  Enlarged copy of the experimental force and deceleration traces for UMIST validation study

Figure 4.6  FE analysis scheme for UMIST validation study
Figure 4.7. Displacement history comparisons for the UMIST analyses.
Figure 4.8. Force history comparisons for the UMIST analyses.
Figure 4.9. Deformed shape plots - UMIST coarse mesh analysis.
Figure 4.10. Deformed shape plots - UMIST medium mesh analysis.
Figure 4.11. Deformed shape plots - UMIST fine mesh analysis.
Figure 4.12. Deformed shape plots - UMIST medium mesh Cowper-Symonds analysis.
Figure 4.13. (a) Uniform and (b) non-uniform proximal mesh refinement schemes.
Figure 4.14. Displacement and force history comparisons for uniform and non-uniform proximal mesh refinement analyses.
Figure 4.15. Deformed shape plots for uniform proximal mesh refinement analysis.
Figure 4.16. Deformed shape plots for non-uniform proximal mesh refinement analysis.
Figure 4.17. Location of geometric imperfection.

Figure 4.18. Displacement and force history comparisons for analysis with geometric imperfection.
Figure 4.19. Deformed shape plots for analysis with geometric imperfection.
Figure 4.20. Location of nodes used for early response geometric imperfection analyses.
Figure 4.21. Early force response for geometric imperfections at nodes (a) 365, (b) 357, (c) 349 and (d) 317.
Figure 4.22. Early force response for geometric imperfections at nodes (a) 181, (b) 21 and (c) 13.
Figure 4.23. Sense of rotation (exaggerated for clarity).
Figure 4.24. Displacement and force history comparisons for 1° rotation about x and y axes.
Figure 4.25. Deformed shape plots for 1° rotation about x and y axes.
Figure 4.26. Experimental deformed shapes. Stills taken from high speed film
Figure 4.27. FE analysis scheme for IRC experiments.
Figure 4.28. Force histories for preliminary analyses ab27-ab29.

Figure 4.29. Force histories for preliminary analyses ab17-ab20.
Figure 4.30. Force histories for preliminary analyses ab21-ab23.

Figure 4.31. Force histories for preliminary analyses ab24-ab26.
Figure 4.32. Deformed shape plots for preliminary analysis ab17.
Figure 4.33. Deformed shape plots for preliminary analysis ab23.
Figure 4.34  Cross-section of FE specimen showing geometric imperfections.
Figure 4.35. Drop-hammer velocity history, (a) as received and (b) digitised.
Figure 4.36. Experimental and analytical drop-hammer velocity histories, (a) ab32, $E_t=1$GPa and (b) ab34, $E_t=1.4$GPa.
Figure 4.37. Force histories for the FE analyses, (a) ab32, $E_t=1\text{GPa}$ and (b) ab34, $E_t=1.4\text{GPa}$.
Figure 4.38. Experimental and analytical specimen crush histories, (a) ab32, $E_t=1$GPa and (b) ab34, $E_t=1.4$GPa.
Figure 4.39. Energy histories for the FE analyses, (a) ab32, $E_t=1$ GPa and (b) ab34, $E_t=1.4$ GPa.
Figure 4.40. Deformed shape plot for FE analysis ab32, $E_t=1$ GPa.
Figure 4.41. Deformed shape plot for FE analysis ab34, $E_t=1.4$GPa.
Figure 4.42  Deformed shape of specimen in ab34 at maximum crush
(a) x, y, and z displacements factored by 0.3 and (b) overlay of strain energy density plot.
Figure 4.43. Strain histories for BCE
(a) ab32, $E_t=1$ GPa and (b) ab34, $E_t=1.4$ GPa.
Chapter 5

Influence of barrier deformation and specimen geometry on the structural response of a crush tube
5.1 Introduction

Experimental studies of axial impacts between thin-walled prismatic specimens and energy-absorbing targets have not been widely reported in the literature as far as the author is aware. Although energy losses in the anvil, drop hammer and isolating felt pad of an impact rig have been analysed and measured by Birch et al. [114], no indication was given of the effects that these losses might have on specimen response. Most workers assume that all the initial kinetic energy is absorbed by the specimen so application of the analytical method developed thus far in an exploratory manner was considered a potential means to gain new knowledge for the design of chassis rails.

That work is first reported in this chapter where a specimen, the response of which was known for a rigid impact, was subjected to a non-rigid impact. For this class of problem the target, hereafter referred to as a deformable barrier, was permitted to absorb significant quantities of energy on impact. Clearly there can be many types of non-rigid impact with an infinite number of combinations of mass and stiffness so a choice was needed in order to determine the combination that would be most representative of a real impact situation. The method of this choice is described in sections 5.2 and 5.3. The remaining sections of the chapter describe other studies that were carried out to quantify the effects of the cross-sectional choice for the specimen, flanges that may be attached or cut-outs that may be made. Short descriptions of the studies are given at the beginning of each section and each are discussed after the presentation of the results.

5.2 Deformable Barrier Impact Simulation

Figure 5.1 shows the FE analysis scheme used to represent an impact with a deformable barrier which was simulated by an eight-node solid element supported by a spring element. The specimen was 300 mm long, 100 mm x 50 mm in section, 0.83 mm thick and was in all respects apart from the mesh density the same as that used in the UMIST validation study described in section 4.3 above. A very coarse mesh density as shown in Figure 5.1 permitted rapid execution of analyses purely for the development of the deformable barrier. The value of spring stiffness, $K$, to use for the deformable barrier was addressed using a simple parametric study where the energy absorbed by the specimen was measured for various values of $K$. Initially, a linear elastic spring was used and values in the range $10 \text{GNm}^{-1} \geq K \geq 0.5 \text{kNmm}^{-1}$ were tried. The results for this study are shown in Table 5.1.
Table 5.1 Parametric study to determine deformable barrier spring stiffness

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>$K$ (Nmm$^{-1}$)</th>
<th>$E_a$ (kJ)</th>
<th>$\delta_f$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>de04</td>
<td>1E10 (1)*</td>
<td>3.16 (1)</td>
<td>165 (1)</td>
</tr>
<tr>
<td>de05</td>
<td>1E9 (0.1)</td>
<td>3.14 (0.994)</td>
<td>170 (1.03)</td>
</tr>
<tr>
<td>de06</td>
<td>1E8 (0.01)</td>
<td>3.15 (0.997)</td>
<td>166 (1.01)</td>
</tr>
<tr>
<td>de07</td>
<td>1E5 (1E-5)</td>
<td>3.13 (0.991)</td>
<td>168 (1.02)</td>
</tr>
<tr>
<td>de08</td>
<td>1E3 (1E-7)</td>
<td>3.12 (0.987)</td>
<td>178 (1.08)</td>
</tr>
<tr>
<td>de08a</td>
<td>0.5E3 (5E-8)</td>
<td>2.53 (0.801)</td>
<td>139 (0.84)</td>
</tr>
</tbody>
</table>

* Numbers in brackets are column ratios with respect to run de04.

The analyses were run for 45 ms each and the values of $E_a$ were those reported directly by the code. Values of $\delta_f$ were obtained by subtracting the barrier displacement history from the drop-hammer displacement history. The numbers shown in brackets are the results for the simulation runs normalised with respect to the stiffest case (analysis de04).

The table indicates that a reduction in barrier spring stiffness of seven orders of magnitude (de04-de08) resulted in energy absorption and maximum crush within 2% and 8% of the stiffest case respectively. It was interesting to note that specimens that were crushed against a weaker barrier than in run de04 showed greater amounts of crush although the coarseness of the mesh prevented confidence in that result. The results presented in Table 5.1 were used however to determine that a spring stiffness of the order of 1000 Nmm$^{-1}$ would absorb significant quantities of energy and could therefore influence the response of the specimen.

Further support for the choice of this range of stiffness was found in the internal energy, crush and force histories for the specimen in the various analyses which are shown in Figures 5.2-5.4. In Figure 5.2 the internal energy histories of the specimens in runs de04-de07 were almost indistinguishable. For runs de08 and de08a however, the deformable barrier began to absorb significant amounts of energy at approximately 5 ms. By the end of the analysis the total energy absorbed by the specimen in de08 was very similar to that obtained in the preceding runs although a different energy-absorption history was evident. Figures 5.3 and 5.4 revealed that the introduction of a greater degree of barrier compliance caused a delay in the specimen response but it was the energy absorption history (Figure 5.2), viewed in conjunction with deformed shape animations that brought the parametric study to a satisfactory conclusion.
Deformed shape plots from runs de04 and de08 are shown in Figures 5.5 and 5.6. These plots do not convey as much information as the animated graphics so a supplementary description is given below:

Run de04 (Figure 5.5) was the analysis that employed the stiffest barrier spring which did not deform appreciably as the specimen was crushed. The gross distortion of elements and contact surface penetration was consistent in all analytical specimens in this parametric study and this was an expected result. The restoration of energy to the specimen by the barrier spring during crush however, was not expected although with hindsight it should have been since a linear spring was employed. Close inspection of the last three slides in Figure 5.6 should reveal that the spring was expanding during the later stages of the analysis. Indeed the animations showed that there was a sequence when the barrier spring was expanding while the drop-hammer was still travelling downwards. On contact with the barrier, the specimen in run de08 began to crush before the inertia of the barrier (which had the same mass as the drop-hammer for these runs) was overcome. There was therefore little barrier movement in the early stages of specimen crush. Once the barrier began to crush, specimen collapse, and therefore energy absorption rate, slowed down. At approximately 22 ms the force in the barrier spring became greater than the force required to crush the specimen, so the barrier direction of motion was reversed although the drop-hammer was still travelling in its original direction of motion. The specimen was therefore being crushed between the upwards-moving barrier and the downwards-moving drop-hammer which carried on its downward motion until approximately 41 ms. After this time the drop-hammer, barrier and specimen were all travelling upwards.

The animation evidence coupled with the energy histories confirmed that significant quantities of energy were being restored to the specimen by the barrier. The elastic spring specified was clearly the incorrect choice to model the type of impact that would be experienced by a car in collision with another car. An irreversible spring characteristic was required in order to dissipate the energy absorbed by the barrier during impact.

In addition to the effects of spring stiffness, the results revealed that the mass of the barrier influenced the specimen response. The energy-absorption histories (Figure 5.2) show that for the first two or three milliseconds all the specimen responses were indistinguishable regardless of barrier spring stiffness. The force histories (Figure 5.4) show the same phenomenon. It could be reasonably postulated that, the important factor in this early part of the analysis was the barrier inertia and not the barrier spring stiffness. The force required to overcome the barrier inertia in the first few milliseconds in a deformable barrier case (analysis de8a) resulted in a specimen response that was almost identical to that obtained for a rigid barrier case (analysis de04).
An additional analysis was carried out using a non-linear barrier spring stiffness characteristic. This spring characteristic (DYNA3D type 6 general non-linear spring) not only permitted the specification of a yield load but allowed a separate unloading characteristic to be attributed to the spring. It was therefore possible to make the deformable barrier behave in a more realistic manner. The loading and unloading curves that were used for the non-linear spring are shown in Figure 5.7. The choice of yield load was based on previous experience obtained from the force histories above. A yield value of 50 kN was chosen which was less than the initial peak load. The post-yield loading stiffness was chosen to be 0.5E3 Nmm\(^{-1}\) which was half of the pre-yield value. This was done in order to simulate the work-hardening effect observed when metallic structures deform.

The unloading characteristic of the non-linear spring was rather more complicated than the loading characteristic since its displacement axis was not static as shown in Figure 5.7. The unloading curve could be translated along the displacement axis as the element was loaded and, on removal of the load, became activated. The curve fell very steeply to a value of 0.2 Nmm\(^{-1}\) on removal of loading. This shallow slope cannot readily be observed in Figure 5.7 so the five points on each graph are given in Table 5.2 beginning with the most negative value.

<table>
<thead>
<tr>
<th>Table 5.2 Points on the loading and unloading curve of the non-linear spring</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Loading Curve</strong></td>
</tr>
<tr>
<td>Displacement (mm)</td>
</tr>
<tr>
<td>-100</td>
</tr>
<tr>
<td>-50</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>50</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

The analysis using this spring characteristic (de09) was successfully carried out and the animation of the deformed shapes showed that the barrier did not return significant quantities of energy to the specimen. Deformed shape plots are shown in Figure 5.8 but the internal energy history comparison shown in Figure 5.9 makes the effect of the modification in spring characteristic clearer. The internal energy history for the specimen in analyses de08 and de09 are indistinguishable until the spring in the former run began to release the energy stored. This energy was used in further crushing of the specimen. The restoration phase did not occur in analysis de09 in which the specimen absorbed a maximum of 2.71 kJ and crushed by 148 mm.

With these results the simple study to determine a benchmark spring characteristic was brought to a close and a fine mesh model was imported to replace the very coarse mesh specimen.

5-4
5.3 Effect of the mass of the deformable barrier on energy absorption

The deformable barrier assembly with a non-linear dissipative spring characteristic described in the previous section was used in this study, the aim of which was to quantify the effect of barrier mass on specimen energy-absorption efficiency. The only variable was thus the mass of the barrier. Any energy not absorbed by the specimen was dissipated in the barrier spring.

The coarse mesh specimen was replaced by one with 1980 thin shell elements, essentially the same as the medium mesh density Cowper-Symonds model that had been used in the validation work (section 4.3). For this set of analyses the two columns of elements either side of the a corner of the specimen were designated "corner" elements, the remainder being termed "side" elements. This was done in order to evaluate any changes that may have occurred in energy absorption patterns when the target was changed from a rigid one to one that absorbed energy. The reader may recall that the experimental specimen collapsed in an irregular mode that required a degree of imperfection in the FE analysis to re-create. Since the objective of this study was to explore the effects of a change in loadcase no imperfections were introduced into the analyses.

Five cases were analysed. In the first case the barrier was fully clamped to represent an infinite mass. Barrier masses of 10 kg, 5 kg, 2 kg and 1 kg were then also tried. A value between 10 kg and 100 kg was not tried because of experience gained while developing the spring stiffness characteristic (section 5.2). It was understood that a barrier mass that was similar to the drophammer mass influenced the response of the specimen and could swamp any effect that may result from the switch from a rigid to a deformable barrier.

For each analysis the specific energy absorbed by the corner elements, the side elements and the specimen as a whole was determined and is shown in Table 5.3 which also shows the maximum crush, stroke efficiency (energy absorbed divided by the crush distance) and the proportion of incident energy absorbed in the corner regions termed \( E_c \). Results normalised with respect to the rigid barrier case are given in parentheses. For example, in analysis de14 the value of specific energy absorbed for the whole of the tube was 5.1 kJkg\(^{-1}\) and this was 0.72 of the value obtained for an impact with a rigid barrier (analysis de12).
Table 5.3 Efficiency comparisons for various barrier masses

Total specimen mass = 0.591 kg  Drop-hammer Mass = 81.5 kg  Mass of specimen corners = 0.217 kg
Mass of specimen sides = 0.374 kg  Initial Velocity = 9.9 ms\(^{-1}\)  Acceleration due to gravity = 9.81 ms\(^{-2}\)

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Mass Ratio</th>
<th>Specific Energy Absorbed (kJ kg(^{-1}))</th>
<th>(\delta_{\text{max}}) (mm)</th>
<th>Stroke eff. (E_{c}) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>Corners</td>
<td>Sides</td>
<td></td>
</tr>
<tr>
<td>de12</td>
<td>(\infty)</td>
<td>7.1</td>
<td>12.2</td>
<td>4.1</td>
</tr>
<tr>
<td></td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
<td>(1.00)</td>
</tr>
<tr>
<td>de14</td>
<td>16.92</td>
<td>5.1</td>
<td>8.8</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>(.72)</td>
<td>(.72)</td>
<td>(.71)</td>
<td>(.80)</td>
</tr>
<tr>
<td>de15</td>
<td>8.46</td>
<td>5.4</td>
<td>10.1</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>(.76)</td>
<td>(.83)</td>
<td>(.66)</td>
<td>(.85)</td>
</tr>
<tr>
<td>de16</td>
<td>3.38</td>
<td>6.2</td>
<td>11.1</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td>(.87)</td>
<td>(.91)</td>
<td>(.81)</td>
<td>(1.01)</td>
</tr>
<tr>
<td>de13</td>
<td>1.69</td>
<td>5.9</td>
<td>11.5</td>
<td>2.7</td>
</tr>
<tr>
<td></td>
<td>(.83)</td>
<td>(.94)</td>
<td>(.66)</td>
<td>(.94)</td>
</tr>
</tbody>
</table>

The raw data in columns 4 and 5 of Table 5.3 show that, regardless of mass ratio, the corner regions of the specimen absorb an average of 3.5 times more energy per unit mass than the side regions. Decreasing the barrier mass tended to increase the specific energy-absorption of the corners from 0.72 of the rigid impact case to 0.94. For the side regions however, the energy absorption rose from 0.71 of the rigid impact case to 0.81 then fell to 0.66 for the smallest mass ratio. When the total specific energy-absorption of the specimen is considered (column 3), the relative contributions of the corner and side regions to the overall trend in efficiency for decreasing mass ratio can be seen. The specific energy absorbed rose from 0.72 to 0.87 of the rigid impact case before falling back to 0.83 for the lowest mass ratio. It had been expected that the trend for the whole specimen would follow that for the corner regions more closely but the greater number of side region elements counteracted the high efficiency of the corner regions.

The stroke efficiencies did not vary by more than 4.3% as the mass ratio was reduced from 16.92. The average value was 0.88 of the rigid barrier case. The specimens were all crushed by different amounts so the energy absorbed was clearly closely related to the amount of maximum crush. This may be a somewhat artificial result since there would be an increased tendency towards Euler bending under experimental conditions for specimens exhibiting a non-compact or irregular collapse pattern. In any case the stroke efficiency would be expected to be
more sensitive to changes in specimen geometry than to barrier mass but this was not investigated at this point.

The trend in $E_c$ (column 8) was generally increasing for decreasing mass ratio although analysis de16 seemed to buck the trend. Indeed that analysis was found to be worthy of more attention since it seemed to blur otherwise clear trends that were developing in Table 5.3. An explanation for this was found on inspection of the deformed shapes.

The deformed shape plots for the specimens in Table 5.3 are shown in Figures 5.10-5.14. The two elements visible by the barrier were not structural, but were fixed in space purely to aid the observation of barrier deformation. In Figures 5.10-5.12 the deformation of the specimen began at the proximal (struck) end and progressed backwards in a non-compact mode. In analysis de16 however (Figure 5.13), collapse began in the mid-region of the specimen and was of a compact nature. This mode of collapse would result in an increase in specific energy-absorption since more folds would form and the material would be required to fold through greater curvatures (and hence greater strain) to form the compact folds. In analysis de13 (Figure 5.14) the first complete fold also began in the mid region of the specimen but due to a proximal fold that developed, subsequent collapse was in a mixture of non-compact and compact modes. Surprisingly for this analysis the corner elements exhibited a higher specific energy than for de12 which exhibited a greater degree of uniformity and collapse. This may be partly attributable to the mis-formed proximal fold visible between 20 and 28 ms in Figure 5.13.

Analyses de16 and de13 highlighted the need for care in the interpretation of results because a change in mode shape could unexpectedly occur. In this case the change was almost certainly a result of secondary impact with the lightweight barrier. On first contact, the barrier was so light in analyses de12 and de13 that it was accelerated away momentarily from the specimen without any significant buckling of the specimen. On the second contact buckling began in the middle and proximal regions. There were no imperfections in the specimens so initial buckling was due purely to the interaction of stress concentrations (stress waves). Figures 5.15 and 5.16 show the stress patterns at the onset of folding for a rigid barrier (analysis de12) and a 2 kg deformable barrier (analysis de16) respectively. For the latter analysis the specimen initially displayed a state of uniform stress until the barrier was accelerated away thereby unloading the specimen. The resulting pattern of stress concentrations on subsequent reloading meant that initial large amplitude collapse was confined to the mid-region of the specimen and continued in this pattern. These observations were made by re-running the analyses for 4 milliseconds but requesting deformed shapes and von Mises stress results for the elements at 0.25 ms intervals.
The results of this parametric study clearly indicated that, even without a change in collapse mode, a decrease in mass ratio resulted in a re-distribution of energy absorption in the structure. The re-distribution led to a greater proportion of energy being absorbed in the corner regions although the precise mechanism by which this occurred was not clear. This topic is covered in more detail in Chapter 6.

5.4 The effect of cross-section on energy absorption efficiency

A simple study is described to investigate the potential gains in energy absorption efficiency when the section is chosen carefully. It has been shown by von Karman et al. [115] amongst others that energy absorption in prismatic thin-walled tubes undergoing axial impact takes place predominantly in relatively thin strips near the corners of the section. It could therefore be expected that merely increasing the number of corners in a section would increase its efficiency until the section approached a circle. In order to adopt an efficient collapse mode shape however, the section must contain an even number of corners so that opposite sides can fold inwards while the others fold outwards. A further constraint is in the cost of additional forming required relative to the baseline square tube, a final decision over section choice depending on a cost-to-benefit ratio. Only the potential benefits are described here because the concern is pure research into methods by which efficiency can be increased.

Triangular, square and hexagonal sections were analysed using specimens with \( h = 1.25 \text{ mm} \) and \( L = 300 \text{ mm} \). Side width was 66.67 mm, 50 mm and 33.33 mm respectively and the specimens were at rest on a rigid surface when struck by a mass of 150 kg travelling at 10 \( \text{m s}^{-1} \). The results are presented in Table 5.4 below in which the useful crush is defined as the crush before the specimen began to bottom-out. The hexagonal specimen was not totally consumed by the impact so the useful crush was conservatively specified as the crush until the formation of the last fold. The mean dynamic force was calculated up to the point of useful crush and used to calculate the structural effectiveness \( \eta \) which is a ratio of the mean dynamic force and the dynamic squash load (Equation 5.1). The characteristic flow stress in this case was taken as 304 MPa.

\[
\eta = \frac{P_m^d}{A_0 \sigma_0} \tag{5.1}
\]

Geometric imperfections were introduced into the specimens prior to analysis as previously described although a uniform symmetric collapse mode was not expected for the triangular section.
Table 5.4 Results of cross-section analyses

<table>
<thead>
<tr>
<th>Section</th>
<th>Useful crush (mm)</th>
<th>$P_m$ (kN)</th>
<th>Specific energy (kJ/kg)</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>212</td>
<td>24.106</td>
<td>8.846</td>
<td>.317</td>
</tr>
<tr>
<td>Square</td>
<td>210</td>
<td>28.946</td>
<td>10.537</td>
<td>.381</td>
</tr>
<tr>
<td>Hexagonal</td>
<td>176</td>
<td>39.789</td>
<td>12.280</td>
<td>.524</td>
</tr>
</tbody>
</table>

The force and energy histories for the three analyses are shown in Figure 5.17 and the deformed shape plots are shown in Figures 5.18 - 5.20.

Additional analyses were carried out using the hexagonal section to identify the optimum wall thickness. Thicknesses of 0.75 mm, 1.0 mm, and 1.5 mm were used and Figure 5.21 shows the variation of specific energy, percentage crush, and structural effectiveness with non-dimensional thickness which in this case is expressed as the solidity ratio $\mathcal{O}_0 = A/A_0$. All but the thickest of these hexagonal sections (Figure 5.22) collapsed in a uniform symmetric mode. The collapse of the thickest specimen began in an extensional mode for two or three layers after which a global hinge formed, the specimen tilted to one side as collapse began at the distal end.

5.4.1 Discussion

The two efficiency descriptors chosen in Table 5.4 show the hexagonal section to be the most efficient of the three, absorbing 16.5% more energy per unit mass than the square tube. The triangular section was not expected to be highly efficient and crushed the most before bottoming out. The relatively low force level at which it crushed though, led to a 16.0% lower specific energy. The structural effectiveness of the hexagonal section reflects the high force level at which it deformed, a desirable characteristic in an energy absorber. It may be argued however that a section that deformed under a high load would simply transmit these loads to the vehicle occupants. The correct way to interpret the data however is that, given a mean dynamic force level for occupant protection, the most efficient way to absorb the incident energy using one of the three sections above is with a hexagonal section because other sections would have to be made of a heavier gauge or to a greater length to absorb the same amount of energy in a controlled manner.

The asymmetric nature of collapse adopted by the triangular section can be seen in Figure 5.18 where the two adjacent sides that are visible tended to fold the same way but when it was not possible a kink occurred in the corner. This mode of collapse is unstable and likely to lead to global bending. The square and hexagonal sections however were able to adopt the uniform symmetric mode of collapse as shown in Figures 5.19 and 5.20, the latter figure graphically
demonstrating the efficiency of the section which absorbed the incident energy without being totally consumed. The results clearly indicate the increased efficiency of a hexagonal section over a square one for an energy absorber, manufacturers must now weigh this benefit and the potential weight-saving against any extra fabrication costs.

Once the sectional choice has been made the thickness should be considered and Figure 5.21 shows that the optimum thickness for this section would be approximately 1.25 mm where the specific energy peaks and the structural effectiveness is highest for stable collapse. The figure shows a rapid fall-off in specific energy due to the change in mode shape that occurred for the thickest section. This specimen had a thickness-to-width ratio $h/c = 0.045$ which placed it in the approximate range $0.035 \leq h/c \leq 0.099$ given by Abramowicz and Jones [116] in which the asymmetric mixed mode controlled the collapse of square tubes under moderate impact speeds\(^3\). It would therefore be necessary to verify by dynamic test that the mode shape predicted by the FE code was also obtained in the laboratory.

### 5.5 The effect of a flitch panel on the specimen response to impact

The work presented so far has considered a uniform prismatic tube as an idealisation of part of a chassis rail. The tube has been considered in isolation whereas in reality the chassis rail is attached to a flitch panel (inner wing). The literature presents many studies relating to the crush performance of hat and double hat sections (for recent examples see Paik et al. [117] or White et al. [118, 119]), however the flange widths considered are generally only a fraction of the side width. A study was therefore conducted to extend the range of flange widths and quantify the effects of an idealised flitch panel on the energy absorption efficiency of a uniform tube. Parameters of the flitch panel that were postulated to be influential were the thickness and the width but it was also necessary to decide whether the drop-hammer should bear directly on the tube and flitch panel as an assembly or simply to load the tube. The prior case would perhaps be closer to a realistic impact case but the latter case would ensure that any effects were purely due to the interaction of the flitch panel and the tube rather than the flitch panel and the drop-hammer. If the drop-hammer were to load both the tube and the flitch directly, an increased squash load could be expected from the relation

$$P_{sq} = \sigma_y A.$$  \hspace{1cm} (5.2)

---

\(^3\) It should be noted that the symmetric and asymmetric mixed modes were reported to absorb similar amounts of energy in that study. Therefore the drop in specific energy observed here may be attributed to decreased deflection with no corresponding increase in structural effectiveness.
The post-yield loading characteristic for both cases would depend on which part of the structure was controlling the response and therefore was of greater interest in this study.

The study was simple in concept. The only parameter to be varied was flitch panel width. This reflected the assumption that the flitch and chassis rail were made from the same gauge material. The two impact conditions were:

A  Both flitch and tube were directly loaded by the drop-hammer
B  Only the tube was loaded by the drop-hammer

The tube used was the 50 mm square tube described in the cross-sectional area study in section 5.4. The performance of the baseline tube without a flitch panel was therefore known. Five widths of 0.5c, c, 1.5c, 2c and 4c were analysed. As in the previous studies using this square section tube initial imperfections were required. The positions of the imperfections on the tube were maintained but for the flitch, the axial location of the imperfection was adjusted according to the predicted inelastic buckling wavelength.

Bleich[120], in Table 26 gave values for the coefficient β used in calculating the inelastic folding wavelength λ of a plate loaded along two opposite edges with various support conditions on the other two edges according to the formula:

\[ \lambda = \beta c \sqrt{\frac{\tau}{E}} \]

where \( \tau = \frac{E}{E'} \).

It was interesting to note that there was no thickness term in the above relation. With one of the unloaded edges simply supported and the other free, it was indicated that λ would always be equal to the length of the plate, so no values were given for β but for one unloaded edge fixed and the other free, \( \beta = 1.680 \). Clearly neither of these cases perfectly describes the automotive flitch panel but the estimate served the purpose of locating initial imperfections close to the expected peak of a buckling wave.

A comparison was also made between the prediction of Bleich described above and that of Wierzbicki et al.[43] as discussed in Chapter 3 who gave

\[ \lambda = 1.98h \left( \frac{c}{h} \right)^{\frac{3}{5}}. \]

Table 5.5 lists theoretical estimates of buckling wavelength for the five flitch panel widths analysed in this study. They were obtained using the values of \( E = 200 \text{ GPa} \), \( E_t = 1.4 \text{ GPa} \) and a wall thickness of 1.25 mm. The elements were 5.66 mm long in the axial direction and the
closest node to the most proximal predicted half-wavelength in the middle of the flitch panel side was moved 0.5 mm out of plane. For a flitch panel width of 0.5c this was the 3rd node down although the estimate according to Bleich would have suggested the 2nd node. A trial analysis showed that when the 3rd node was displaced a more uniform initial collapse was promoted than when the 2nd node was displaced. For flitch panel widths of c, 1.5c and 2c the location of the 3rd, 4th and 5th nodes respectively were suitable for imperfections using both estimates for wavelength. For 4c the 7th node down was moved out-of-plane which was closer to the Wierzbicki estimate than the Bleich.

<table>
<thead>
<tr>
<th>Flitch panel width (c)</th>
<th>λ Bleich[120]</th>
<th>λ Wierzbicki et al.[43]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>12.15</td>
<td>17.88</td>
</tr>
<tr>
<td>1.0</td>
<td>24.30</td>
<td>28.24</td>
</tr>
<tr>
<td>1.5</td>
<td>36.45</td>
<td>36.91</td>
</tr>
<tr>
<td>2.0*</td>
<td>48.59</td>
<td>44.63</td>
</tr>
<tr>
<td>4.0*</td>
<td>97.19</td>
<td>72.94</td>
</tr>
</tbody>
</table>

* N.B. Non-compact folding expected at these widths

5.5.1 Results
Figures 5.23 and 5.24 show the results for conditions A and B respectively. For condition A, it will be remembered, the drop-hammer bore directly on the tube and flitch whereas for condition B the drop-hammer bore directly on the tube only. This was achieved by repeating the five analyses for condition A with a de-activated contact surface between the drop-hammer and the flitch. The mean force was calculated up to 150 mm and 180 mm crush and the dynamic squash load used in the calculation of structural effectiveness was obtained using a characteristic flow stress as discussed in previous chapters. Figures 5.25 - 5.34 show the deformed shape plots of the ten analyses with the drop-hammer removed for clarity. For condition B analyses (Figures 5.30 - 5.34) it will be noticed that the flitch was not constrained in the plane of the drop-hammer.

5.5.2 Discussion of flitch panel results
For condition A the specific energy of the tube and flitch assembly was greater than the baseline tube with no flitch panel for all but the largest flitch width (Figure 5.23a). It would therefore seem that the addition of a flitch panel was beneficial to energy absorption efficiency so long as its width was less than twice the tube width. Considering only the cases where there was a flitch panel present, it was clear that wider flitch panels led to lower specific energies for the total assembly. A beneficial effect can also be seen in the increasing specific energy of the tube for flitch widths up to 2c. The specific energy of the flitch itself however,
decreased monotonically with increasing width and it is this characteristic that is believed to be responsible for the downward trend shown by the whole assembly. The overall effect of attaching a flitch panel was to increase the specific energy of the tube by up to 26.5%.

Figure 5.23b shows a falling trend in structural effectiveness with increasing flitch width although the smallest flitch panel assembly increased \( \eta \) by 10.0% and 6.5% above the baseline for 150 mm and 180 mm crush respectively. The uniform decrease in \( \eta \) at larger crush distances would seem to indicate that the flitch panel induced folding at lower force levels (reduced mean force) as crushing progressed. This could result from irregular or non-compact folding.

For condition B, where the drop-hammer did not interact with the flitch panel, similar trends in specific energy and structural effectiveness \( \eta \) were evident (Figure 5.24). For the tube and flitch assembly a steadily decreasing trend can be observed for increasing flitch panel widths. The specific energy of the tube however, rises with increasing flitch width to a maximum of 9.814 kJ/kg for a flitch panel width of 2c before falling slightly to 9.804 kJ/kg at 4c. The overall effect of attaching a flitch panel was to increase the specific energy of the tube by up to 28.1% for a flitch width of 2c.

### 5.5.3 Discussion of deformed shape plots

The deformed shape plots for condition A (Figures 5.25 - 5.29) are first discussed followed by those for condition B (Figures 5.30 - 5.34). The baseline tube collapsed in a uniform symmetric manner which was known to be efficient. For condition A and a flitch panel width of 0.5c (Figure 5.25) the tube and flitch began collapse in a symmetric mode. There did not seem to be a conflict in wavelengths during the collapse until approximately 30 ms when the flitch folded outwards into a similar fold in the tube. The overall collapse remained stable however.

For a flitch width of 1.5c (Figure 5.26), the first fold was in the symmetrical mode although imperfectly formed and the subsequent four layers also included the flitch in the folds (up to 15 ms). However a distal fold formed which disrupted the progressive folding from the proximal end and subsequent folding was in the non-compact mode.

For a panel width of 1.5c (Figure 5.27), the first fold on the tube was a mixture of extensional and symmetric modes with the flitch panel in sympathy deforming towards the tube. By the fifth millisecond of folding a second layer of folds had formed in a symmetric mode and subsequent folds followed this pattern. In some instances the folding wavelength of the flitch seemed to be in conflict with that of the tube, for example at 15 ms, but this did not disrupt the folding pattern. The figure shows the distal end of the flitch to be freely sliding across the face
of the stonewall up to approximately 20 ms when a distal fold was about to form and this free
motion was believed to be the reason for the lack of conflict between flitch and tube. The distal
fold that did form was of the correct mode but of slightly longer wavelength than the previous
folds.

For a panel width of 2c (Figure 5.28), the tube began folding in an extensional mode. The
flitch panel folded towards the tube in the vicinity of the tube but switched folding direction
approximately five elements away. It seemed that the influence of the tube on the flitch was
only strong within 5 elements (0.5c) for this assembly. The subsequent four folds up to
approximately 15 ms were in the symmetrical mode with the flitch following the small
wavelength folds in the vicinity of the tube but adopting a longer wavelength further away.
The distal end of the flitch did not show the same motion across the face of the stonewall as in
the previous analysis but the proximal end folded away from the drop-hammer and as such was
not constrained (this analysis would therefore be similar to condition B since for most of the
impact, the drop-hammer bore only on the tube). A distal buckle began to form at
approximately 10 ms and collapse switched to this location at approximately 25 ms. A bending
moment was evident in this distal collapse and this was believed to be due partly to the
increased height of the extra folds in the flitch and the increased stiffness in the joint between
tube and flitch.

For a panel width of 4c (Figure 5.29), the initial tube collapse mode was again extensional with
a switch to symmetrical occurring in the second and third layers of folds. For this assembly
however the flitch panel could be seen to be controlling the response. It adopted a large
wavelength fold between 5 ms and 10 ms which formed a kink mid-length of the tube
preventing further uniform collapse. Contact between the drop-hammer and a large proportion
of the flitch panel ceased between 5 ms and 10 ms.

Attention was next focused on the deformed shape plots for condition B shown in Figures 5.30
- 5.34. For a flitch panel width of 0.5c (Figure 5.30), the tube collapsed symmetrically up to 6
layers of folds when the distal end of the tube began to deform. The distal end of the flitch
moved towards the tube and the assembly began to bend as the distal end of the tube collapsed
unevenly due to the increased stiffness of the tube/flitch joint. The bending in the latter stages
of the collapse was not witnessed in the same specimen under condition A.

For a panel width of c (Figure 5.31), symmetric collapse was observed throughout the
response of the structure. The distal end of the flitch began to move away from the tube at
approximately 20 ms when six layers of folds had been formed. It appeared that motion of the
distal end of the flitch in this direction did not interfere with the tube collapse whereas motion
towards the tube as in the previous case was in conflict with the collapse of the tube.
For a panel width of 1.5c (Figure 5.32), the extensional collapse mode was evident for the first three layers of folds but the flitch panel only followed the folds when the pattern switched to symmetrical between 10 ms and 15 ms. Between 15 ms and 20 ms though, the distal end of the flitch began to deform and a long wavelength fold formed in the flitch which interfered with the folding pattern of the tube. The last two layers of folds in the tube were of a longer wavelength than the previous folds because of the action of the flitch. This was similar to the response for condition A but in that case the flitch behaviour did not strongly influence the tube collapse.

For a panel width of 2c (Figure 5.33), the first fold was again extensional but subsequent folding was symmetrical with the flitch following the tube collapse pattern until the distal end became fixed between 10 ms and 15 ms. Collapse then switched to the distal end where the relatively stiff joint between the tube and the flitch caused a bending moment in the structure.

For a panel width of 4c, (Figure 5.34), the first two layers of folds were symmetrical unlike in previous cases. The flitch panel close to the joint followed the folding pattern of the tube but the remainder of the flitch adopted a large concave shell shape. On formation of the fifth layer of folds at approximately 15 ms the distal end of the flitch moved towards the tube and began to deform inducing extensional collapse in the distal end of the tube. this made the distal end of the tube stiffer so collapse transferred back to the advancing layers of folds. After the formation of the sixth fold, the remaining curved panel section began to collapse.

Conditions A and B did not lead to greatly differing results. The flitch panel was not in contact with the drop-hammer throughout the impact under condition A and only the region nearer the tube was effectively constrained by the drop-hammer. This was akin to condition B where the motion of the tube served to constrain the flitch panel in the neighbourhood of the joint. While either the proximal or distal end of the flitch panel was free to move the tube could be said to control the overall response of the structure but once the flitch panel had become fixed at the distal end due to the formation of a fold, there was an increased tendency to disrupt the folding pattern of the tube. This was particularly evident for flitch panels over 0.5c wide.

It can be concluded that the presence of a flitch panel made the tube absorb more energy but the flitch panel size should be limited to 1.5c otherwise the specific energy absorption of the assembly may be compromised. Methods for overcoming the effects of the flitch panel were not investigated in this project and are would be a useful subject for further work.
5.6 Investigation into the effects of cut-outs on energy absorption efficiency

For the final study reported in this chapter the non-linear FE code was used to determine the optimum location of cut-outs in a specimen. Cut-outs are often used in transport structures to save weight without compromising function. During weight-reduction exercises, a 2% weight-saving on a structure such as a chassis rail would be viewed as a great success amongst automotive engineers. As with previous work reported in this thesis, the cost of achieving the weight reduction was not considered, the desire being solely to quantify the potential gains setting aside financial concerns.

In specimens comprising uniform multi-corner sections subjected to axial impact most of the impact energy is dissipated in the corner regions so removal of material from these areas is likely to lead to a loss in efficiency. The only remaining areas are the mid-side regions of the specimen, the so-called ineffective strip of von Karman [115]. Once the width has been determined the length of the cut-out can then be chosen to increase specific energy without affecting the progressive symmetric collapse mode. Other studies have investigated the shape and location of cut-outs used for triggering collapse in the proximal region but none of which the author is aware were concerned with the contribution of cut-outs to energy absorption efficiency during large scale collapse. In the following, a specimen with a known collapse behaviour was the baseline from which improvements in specific energy were sought. The width of the ineffective strip was first determined after which analyses were carried out to determine the length and best position for the cut-outs.

5.6.1 Determination of cut-out dimensions

The square tube used in the comparison of cross-sections (section 5.4) was again adopted as the baseline for this study. Each side had nine columns of 5.56 mm x 5.66 mm elements with a wall thickness of \( h = 1.25 \) mm and the specimen was struck by a 150 kg mass travelling at 10 ms\(^{-1}\). At maximum crush the energy absorbed in each of the columns of elements was written out and is shown in Table 5.6 for all four sides.
Table 5.6 Energy absorbed by columns of elements

<table>
<thead>
<tr>
<th>Side</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>414</td>
<td>261</td>
<td>131</td>
<td>73</td>
<td>66</td>
<td>73</td>
<td>131</td>
<td>266</td>
<td>420</td>
</tr>
<tr>
<td>2</td>
<td>451</td>
<td>278</td>
<td>155</td>
<td>89</td>
<td>71</td>
<td>91</td>
<td>157</td>
<td>289</td>
<td>455</td>
</tr>
<tr>
<td>3</td>
<td>413</td>
<td>255</td>
<td>128</td>
<td>73</td>
<td>69</td>
<td>77</td>
<td>137</td>
<td>253</td>
<td>416</td>
</tr>
<tr>
<td>4</td>
<td>456</td>
<td>287</td>
<td>154</td>
<td>89</td>
<td>72</td>
<td>88</td>
<td>152</td>
<td>278</td>
<td>459</td>
</tr>
<tr>
<td>Ave.</td>
<td>433.5</td>
<td>270.3</td>
<td>142.0</td>
<td>81.0</td>
<td>69.5</td>
<td>82.3</td>
<td>144.3</td>
<td>271.5</td>
<td>437.5</td>
</tr>
<tr>
<td>Norm.</td>
<td>6.24</td>
<td>3.89</td>
<td>2.04</td>
<td>1.17</td>
<td>1.00</td>
<td>1.18</td>
<td>2.07</td>
<td>3.91</td>
<td>6.29</td>
</tr>
</tbody>
</table>

Sides 1 and 3 were opposite each other as were sides 2 and 4. Figure 5.35 shows the average energy distribution across the side of the specimen normalised with respect to the lowest value. Position 0 on the abscissa represents the middle column of elements, positions ±1 the neighbouring columns, and so forth with positions ±4 representing the columns of elements at the corners. A clear pattern of energy absorption across the width of the specimen can be seen which can be used to make a choice for cut-out width.

In choosing the axial length of a cut-out the author first determined the length of cut-out at which the collapse mode was disturbed. This was a length of 7 elements. The cut-outs were therefore to be kept to a maximum length of 6 elements and were placed on sides 1 and 3 so as not to interfere with the nodal imperfections on sides 2 and 4.

For the FE code to be of value in the determination of the axial location of a cut-out, the analyst should be able to detect differences in results between arbitrarily placed cut-outs and those placed in an informed manner. The baseline specimen with no cut-outs was therefore analysed by observing the strain energy density at the time of maximum crush (Figure 5.36) with the x, y and z displacements factored by 0.3. The units of the various energy density levels are mJ/mm³. It was observed that the pattern of energy absorption on opposite sides was very similar so cut-outs were placed in the same axial location on opposite each other. The cut-out schemes shown in Figure 5.37 were analysed and compared, the same volume of material being removed in all four schemes. The cut-out width was maintained at three elements in the central third of the sides as discussed presently. In scheme 1 single cut-outs 6 elements long were placed on sides 1 and 3 symmetrically about the middle of the specimen. In scheme 2 the single cut-outs were moved to areas in the proximal region that had been identified as absorbing relatively small amounts of energy. In scheme 3 half-sized cut-outs were placed symmetrically about the mid-point of the specimen and for scheme 4 the two sets of cut-outs were placed in relatively low energy absorbing areas of the proximal region where, in the baseline analysis, the material was considered to be moving as a rigid body.
Table 5.7 shows the energy absorbed by the material to be removed in each of the schemes as calculated from the baseline analysis. The calculations were made for different amounts of crush to observe any changes that might have occurred.

Table 5.7 Energy absorbed in proposed cut-out regions of the baseline specimen*

<table>
<thead>
<tr>
<th>Scheme</th>
<th>150 mm</th>
<th>190 mm</th>
<th>200 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>E (J)</td>
<td>22.2</td>
<td>22.1</td>
<td>22.2</td>
</tr>
<tr>
<td></td>
<td>22.5</td>
<td>22.5</td>
<td>22.5</td>
</tr>
<tr>
<td></td>
<td>34.0</td>
<td>35.6</td>
<td>35.9</td>
</tr>
<tr>
<td></td>
<td>21.7</td>
<td>21.6</td>
<td>21.7</td>
</tr>
</tbody>
</table>

Predicted rank 2 3 4 1

*Drawings are not to scale

The predicted ranking refers to the expected gains in specific energy absorption for the different cut-out schemes. A ranking of 1 was predicted for scheme 4 because the material removed had absorbed the least energy in the baseline analysis. It could therefore be expected that scheme 4 would have the highest specific energy of all the cut-out schemes.

5.6.2 Results

Specific energy results were calculated for 150 mm of crush which was half of the specimen length, 190 mm of crush and 200 mm of crush at which point the specimens generally began to bottom out. Table 5.8 shows the results for the four cut-out schemes compared to the baseline and Figures 5.38-5.41 show the strain energy density plots at maximum crush for the schemes with displacements factored by 0.3. In these Figures the first column shows sides 1 and 3 and the second shows sides 2 and 4 of the specimen. In addition, energy density values of 100 mJ/mm³ and above are all given the same contour colour in order to focus attention on the low energy absorbing areas.
### Table 5.8 Analysis results for cut-out schemes*

<table>
<thead>
<tr>
<th>Scheme</th>
<th>Baseline</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>150mm</td>
<td>7.66</td>
<td>7.84</td>
<td>8.04</td>
<td>8.02</td>
<td>7.93</td>
</tr>
<tr>
<td>E/m (kJ/kg)</td>
<td>190mm</td>
<td>9.54</td>
<td>9.58</td>
<td>10.02</td>
<td>9.91</td>
</tr>
<tr>
<td></td>
<td>200mm</td>
<td>9.95</td>
<td>10.18</td>
<td>10.49</td>
<td>10.36</td>
</tr>
<tr>
<td>Predicted rank</td>
<td>-----</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>Actual rank</td>
<td>-----</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

*Drawings are not to scale

**5.6.3 Discussion of cut-out results**

It was a straightforward matter to choose the width of the cut-outs. Figure 5.35 showed that material in the central third of the specimen was absorbing a good deal less energy than was being absorbed elsewhere. Removal of material from this portion of the specimen was therefore considered the most effective way to improve specific energy absorption.

All of the schemes exploring the best way to arrange a given total axial length of material removed improved specific energy absorption over the baseline, scheme 2 resulting in the largest improvement at approximately 5%. The actual rankings (Table 5.8) were insensitive to the degree of crush, being the same at all three crush distances. Table 5.8 also showed that the predicted rankings were not borne out by the analyses. Although the spread of specific energies for the four schemes was small at 2% of the highest value and smaller still between the schemes ranked 1 and 2 (0.2%), it was clear that the FE code could not be used to optimise the axial location of cut-outs using the method described here. Material in low-energy absorbing areas seemed to serve a restraining function for the surrounding material undergoing large deformation. Removal of the restraining material would therefore reduce the amount of deformation taking place in the surrounding areas. This would explain why the schemes with predicted ranks of 1 and 2 had actual ranks of 3 and 4. Further support for this hypothesis can be seen in Table 5.7 where, for schemes 1 and 4, evidence of slight elastic unloading can be seen in the material to be removed because the strain energy at 190 mm was less than at 150 mm and 200 mm of crush. In the other schemes the strain energy was either constant or increasing as crush progressed.
5.7 Concluding remarks

The deformable barrier parametric study was successful in identifying the order of magnitude of spring stiffness (1000 Nmm⁻¹) to use for the barrier spring. The inertia of the barrier was found to dominate the initial response of the specimen and the spring compliance had the effect of delaying the response characteristics when compared to a rigid barrier impact. Impacts with lighter barriers seemed to induce a change in mode shape that could affect the efficiency of the specimen. The results of the study clearly indicated that, even without a change in collapse mode, a decrease in mass ratio resulted in a re-distribution of energy absorption in the structure and this topic was revisited in more detail in the work presented in Chapter 6.

The comparison of cross sectional efficiencies showed that the up to 16.5% more energy per unit mass could be absorbed by moving from a square to a hexagonal section. An optimum thickness for the section analysed was found to be approximately 1.25 mm for stable collapse. A change in mode shape occurred for the thickest hexagonal section with a thickness-to-width ratio h/c = 0.045. It would therefore be necessary to verify by dynamic test that the mode shape predicted by the FE code was also obtained in the laboratory. An encouraging observation in this respect however was that this specimen lay in the approximate range 0.035 ≤ h/c ≤ 0.099 given by Abramowicz and Jones [116] wherein the asymmetric mixed mode controlled the collapse of square tubes under moderate impact speeds.

In addition to the higher efficiency of the hexagonal tube, other advantages can be shown relating to the overall stability of this section. The Euler formula for a fully clamped perfect column is given in Equation (5.4).

\[
\sigma_c^E = \frac{4\pi^2 EI}{AL^2} \tag{5.4}
\]

\( I \), for a square section can be approximated by

\[
I = \frac{2}{3} c^3 h \tag{5.5}
\]

and for a regular polygon with \( n \) sides by

\[
\frac{nc^3h}{8} \left[ \frac{1}{3} + \frac{1}{\tan^2 \theta} \right] \left[ 1 - \frac{3h \tan \theta}{c} + 4 \left( \frac{h \tan \theta}{c} \right)^2 - 2 \left( \frac{h \tan \theta}{c} \right)^3 \right] \tag{5.6}
\]

where

\[
2\theta = \frac{360}{n}
\]

and \( n \) is the number of sides. \( I \) was evaluated for the square and hexagonal tubes as 104166.67 mm⁴ and 107999.6 mm⁴ respectively which showed that the hexagonal section would resist 3.7% greater bending stresses than the square section. It will be observed that
Equation (5.6) suggests further gains for polygons of increasingly higher order but these were not analysed.

The addition of a flitch panel made the specimen absorb more energy but the flitch panel size should be limited to 1.5c otherwise the specific energy absorption of the assembly may be compromised. As the added panel increased in size there was evidence of a conflict in folding wavelength between the flitch and the specimen which tended to disrupt the uniform folding pattern of the specimen. Methods for overcoming the effects of the flitch panel were not investigated in this project and would be a useful subject for further work.

Cutting material out of the specimen was shown to increase the specific energy absorption by as much as 5% without affecting collapse mode as long as the width of the cut-out did not exceed 0.3c and the axial dimension did not exceed 0.6c approximately. Attempts to further optimise the axial location of cut-outs were unsuccessful. This was because material in low-energy absorbing areas seemed to serve a restraining function for the surrounding material undergoing large strains. Removal of the restraining material would therefore reduce the amount of deformation taking place in the surrounding areas.
Figure 5.1. FE analysis scheme for an axial impact with a deformable target.

Figure 5.2. Internal energy histories resulting from various barrier spring stiffnesses.
Figure 5.3. Crush histories resulting from various barrier spring stiffnesses.

Figure 5.4. Force histories resulting from various barrier spring stiffnesses.
Figure 5.5. Deformed shape plots for analysis de04.
Figure 5.6. Deformed shape plots for analysis de08.
Figure 5.7. Loading and unloading curves for the non-linear barrier spring.
Figure 5.8. Deformed shape plots for the analysis using a non-linear barrier spring.
Figure 5.9. Internal energy history comparison for linear and non-linear spring.
Figure 5.10. Deformed shape plots for a fixed barrier.
Figure 5.11. Deformed shape plots for a barrier mass of 10kg.
Figure 5.12. Deformed shape plots for a barrier mass of 5kg.
Figure 5.13. Deformed shape plots for a barrier mass of 2kg.
Figure 5.14. Deformed shape plots for a barrier mass of 1 kg.
Figure 5.15. Stress distributions during the first 4ms of collapse for analysis de12 - rigid barrier.
Figure 5.16. Stress distributions during the first 4ms of collapse for analysis de16 - 2kg deformable barrier.
Figure 5.17. Effect of the cross section of the specimen (a) Force history comparison, (b) Energy history comparison.
Figure 5.18. Deformed shape plots for specimen with a triangular cross section.
Figure 5.19. Deformed shape plots for specimen with a square cross section.
Figure 5.20. Deformed shape plots for specimen with a hexagonal cross section.
Figure 5.21. Variation of specific energy, percentage crush and structural effectiveness with non-dimensional thickness.
Figure 5.22. Deformed shape plots for the thickest hexagonal specimen.
Figure 5.23. Condition A - results for various flitch panel widths (a) Specific energy, (b) Structural effectiveness.
Figure 5.24. Condition B - results for various flitch panel widths (a) Specific energy, (b) Structural effectiveness.
Figure 5.25. Condition A - deformed shape plots for flitch panel width 0.5c.
Figure 5.26. Condition A - deformed shape plots for flitch panel width c.
Figure 5.27. Condition A - deformed shape plots for flitch panel width 1.5c.
Figure 5.28. Condition A - deformed shape plots for flitch panel width 2c.
Figure 5.29. Condition A - deformed shape plots for flitch panel width 4c.
Figure 5.30. Condition B - deformed shape plots for flitch panel width 0.5c.
Figure 5.31. Condition B - deformed shape plots for flitch panel width $c$. 
Figure 5.32. Condition B - deformed shape plots for flitch panel width 1.5c.
Figure 5.33. Condition B - deformed shape plots for flitch panel width 2c.
Figure 5.34. Condition B - deformed shape plots for flitch panel width 4c.
Figure 5.35. Normalised energy distribution across the sides of the specimen.
Figure 5.36. Baseline specimen – strain energy density plot at maximum crush with displacements factored by 0.3 to facilitate placement of cut-outs.

Figure 5.37. Sketch of the four cut-out schemes (not to scale).
Figure 5.38. Scheme 1 – strain energy density plot at maximum crush with displacements factored by 0.3.
Figure 5.39. Scheme 2 – strain energy density plot at maximum crush with displacements factored by 0.3.
Chapter 6

De-coupling of strain-rate and inertia effects in impacts with rigid and deformable targets
6.1 Introduction

In this chapter the knowledge gained from the work described in previous chapters is applied to chassis rails of realistic dimensions. A full-length uniform prismatic tube is first used to investigate the effects of de-coupling strain-rate and inertia effects under impact conditions with a rigid and deformable barrier. This is of course difficult under laboratory conditions so the application of the technique in this way represents a method of gaining new insight into the behaviour of structures. The influence of mass ratio was investigated by varying the barrier mass between 2 kg and 50 kg. A dimensionless combination of mass ratio and velocity was developed from the work of Tam and Calladine [67] which included material and geometric parameters as shown in Equation 6.1.

\[ \Gamma = \gamma V_o \left( \frac{48m}{\sigma_y AL} \right)^{1/2} \]  

(6.1)

The mass, length and material cross-section are given by \( m \), \( L \) and \( A \) respectively. The yield stress was used in this example but a characteristic flow stress could also be used. This parameter was used to organise the results into desirable and undesirable collapse mode zones and in the next set of analyses a method of forcing a desirable collapse mode was evaluated. The method was inspired by two items from the literature. The first was Macaulay and Redwood's [25] proposal that the deformation pattern observed in a specimen was perhaps linked to the stress wave velocity for the material. The second was the experimental work of Reid et al. [121] who found that discrete plates interspersed between rings altered the deformation mode of the assembly depending on the mass of the plates.

In addition to these two works the importance of stress wave effects on crush initiation and collapse mode was more recently demonstrated by Karagiozova and Jones [69] who used an elastic-plastic analytical model without strain-rate enhancement to predict the response of short aluminium cylinders to impact. They reported that the initiation of buckling was governed by stress wave effects while the inertia properties of the shell strongly influenced the post-buckling behaviour and final shape. In general the entire length of the shell was involved in the deformation process with regular shapes occurring in relatively thick shells. Increasing the impact velocity or decreasing the wall thickness resulted in a localisation of buckling.

In the works above, the impacts were essentially with rigid sleds or input and output Kolsky bars at fixed mass ratios. The application of the FE technique developed here permitted the study of the influence of a deformable element that could undergo large displacements during the impact. The repeatability of the technique also aided the study of the influence of mass ratio
on specimen response. The hypothesis behind the present method was that by impeding the progress of stress waves through the structure and causing partial reflection of the waves in the proximal region of the specimen, initial collapse in the proximal region could be guaranteed. The impeding of the waves and partial reflection was achieved by varying the section thickness along the length of the specimen thereby introducing a type of structural boundary.

\[ c = \sqrt{\frac{E}{\rho}} \]  

(6.2)

The first elastic wave would traverse the length of the baseline specimen in less than 0.2 ms and be reflected by the mass at the distal end. It is the interaction of the reflected waves and advancing ones that are believed to be responsible for crush initiation. By partially reflecting a wave before it has traversed the whole specimen, energy is removed from it leading to a reduction in the amount available to interact with other waves towards the distal end. The reflected part of the wave then interacts with advancing waves from the proximal end increasing the chances of crush initiation in this region. Increasing the mass of the middle portion of the tube also effectively increases the inertial forces resisting Euler bending. This inertial force is proportional to the impact speed however and will be of decreasing stability benefit as the crush progresses. With a thinner proximal section of the specimen, a lower squash load would also be expected in that region leading to an increased likelihood of proximal initial collapse.

6.2 Description of specimen and impact barrier

The specimen under consideration was based loosely on the chassis rail of a medium sized European family saloon car called the Ford Sierra Sapphire. It was 1m long, had a square section with \( c = 65 \text{ mm} \) and the thickness of 1.5 mm ensured that, for moderate impact speeds a uniform symmetric mode of collapse could be expected. Indeed, recent experimental work reported by Abramowicz and Jones [78] which considered specimen lengths of this order confirms that local buckling could reliably be expected for this specimen (their Equations (2) and (12) plotted as in their Figure 16). A mass of 500 kg was attached to the distal end of the specimen to represent half the car mass and the specimen was meshed using 4000 Belytschko-Tsay quadrilateral shell elements with the same material model as previously described. For some analyses however, the strain-rate enhancement was not included in the material model so the response was governed by initial geometry, plasticity and inertia effects only. The other variables for this study were impact speed and barrier stiffness.

6-2
The impact speed was varied between 20 mph and 40 mph (32.2-64.4 kmh) and the barrier was either fixed in space or permitted to deform as controlled by a non-linear spring. The barrier mass was initially 2 kg. The characteristics of the non-linear barrier spring were chosen to give a representative target. The guideline for this characteristic was the deformable barrier in the European offset frontal impact test specification due to take force in October 1998. That barrier has a nominal crush strength of 46 psi (317.16 kNm^{-2}). Therefore, a 1.3 kN force would be generated for a contacting surface of 65 mm x 65 mm. This is comparable to the crush modulus of 1.2 kNmm^{-1} given by Macmillan [122] (p29-30) for a medium car. Macmillan further gave an impact resistance factor (the force level required to cause plastic deformation) of 65 kN for a medium car. These two characteristics were used to specify the dissipative barrier spring whose turning points are given in Table 6.1.

<table>
<thead>
<tr>
<th>Loading Displacement (mm)</th>
<th>Force (kN)</th>
<th>Unloading Displacement (mm)</th>
<th>Force (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-500</td>
<td>-332.5</td>
<td>-55</td>
<td>-332.5</td>
</tr>
<tr>
<td>-54.17</td>
<td>-65</td>
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<td>0</td>
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<td>0</td>
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<tr>
<td>54.17</td>
<td>65</td>
<td>54.17</td>
<td>10</td>
</tr>
<tr>
<td>500</td>
<td>332.5</td>
<td>55</td>
<td>332.5</td>
</tr>
</tbody>
</table>

Two nodes were also moved 0.5 mm out of plane on opposite sides in the middle of the first expected proximal half wave as predicted by Wierzbicki et al. using the method described in Chapter 4 of this thesis.

The specimen was modified to test the hypothesis above using two schemes. In scheme 1 the wall thickness in the proximal 300 mm of the specimen was reduced to 1.0 mm, the next 300 mm was unchanged at 1.5 mm and the remainder of the specimen had a thickness of 2.0 mm. Therefore the total mass of the specimen was unchanged for this scheme. In scheme 2 the proximal 300 mm was again reduced to 1.0 mm but the remainder of the specimen was unchanged from the baseline thickness. In the second scheme therefore there was a 19% decrease in mass over the baseline and scheme 1.
6.3 Results

The typical free crush space on a real vehicle is less than 300 mm [123, 124]. Over this distance the crashworthiness engineer can reasonably expect to be given a free hand in the choice of geometry. In order to carry out a meaningful analysis of the results that can be related to a real vehicle therefore, specimen responses for 200 mm of crush were used. Results at 300 mm and 500 mm of crush were also analysed out of interest.

The energy absorbed for various amounts of crush is given in Tables 6.2 and 6.3 below. The crush mode given in the last column refers to the location of initial and progressive collapse. In analyses labelled ‘p’ collapse began in the proximal region, those labelled ‘d’ collapsed from the distal region and those labelled ‘m’ displayed initial collapse in the mid-region.

<table>
<thead>
<tr>
<th>Table 6.2 Results with strain-rate enhancement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier type</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>Rigid</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Deformable</td>
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<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

† with global bending

<table>
<thead>
<tr>
<th>Table 6.3 Results without strain-rate enhancement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Barrier type</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>Rigid</td>
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</tr>
<tr>
<td>Deformable</td>
</tr>
<tr>
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<td></td>
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</tbody>
</table>
For the lower impact speeds where the specimen did not crush up to the required distance within the 70 ms analysis time, no results could be given for the absorbed energy. During analysis lc09 the collapse mode switched to the Euler bending mode. Examples of the three different collapse modes observed are shown in Figures 6.1-6.3. Figures 6.4 and 6.5 are wall thickness plots for analyses lc04 and lc03 at 200 mm of crush with displacements factored.

Further analyses were carried out to more clearly determine the influence of mass ratio and impact velocity on the collapse mode of the specimen. The data in Tables 6.2 and 6.3 were augmented by choosing two impact speeds and varying the barrier mass between 5 kg and 50 kg. All results are shown in Figure 6.6 where the mass ratio $\gamma$ is defined as barrier mass/specimen mass. In Table 6.4 the augmented data set for the lower part of Table 6.2 shows the relationship between $\Gamma$ and location of initial collapse. This relationship is shown more graphically in Figure 6.7.

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>Mass (kg)</th>
<th>$\gamma$</th>
<th>$V_o$ (ms$^{-1}$)</th>
<th>$\Gamma$</th>
<th>$E_{200}$ (kJ)</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc03</td>
<td>2</td>
<td>0.65</td>
<td>17.88</td>
<td>0.43</td>
<td>9.44</td>
<td>m</td>
</tr>
<tr>
<td>lc09</td>
<td>2</td>
<td>0.65</td>
<td>13.41</td>
<td>0.32</td>
<td>9.31</td>
<td>d</td>
</tr>
<tr>
<td>lc10</td>
<td>2</td>
<td>0.65</td>
<td>11.18</td>
<td>0.27</td>
<td>9.00</td>
<td>d</td>
</tr>
<tr>
<td>lc11</td>
<td>2</td>
<td>0.65</td>
<td>8.94</td>
<td>0.22</td>
<td>8.38</td>
<td>d</td>
</tr>
<tr>
<td>lc20</td>
<td>50</td>
<td>16.13</td>
<td>17.88</td>
<td>10.67</td>
<td>10.01</td>
<td>p</td>
</tr>
<tr>
<td>lc21</td>
<td>10</td>
<td>3.23</td>
<td>17.88</td>
<td>2.14</td>
<td>10.21</td>
<td>p</td>
</tr>
<tr>
<td>lc22</td>
<td>5</td>
<td>1.61</td>
<td>17.88</td>
<td>1.07</td>
<td>10.07</td>
<td>p</td>
</tr>
<tr>
<td>lc23</td>
<td>50</td>
<td>16.13</td>
<td>11.18</td>
<td>6.67</td>
<td>9.25</td>
<td>p</td>
</tr>
<tr>
<td>lc24</td>
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<td>3.23</td>
<td>11.18</td>
<td>1.33</td>
<td>10.28</td>
<td>d</td>
</tr>
<tr>
<td>lc25</td>
<td>5</td>
<td>1.61</td>
<td>11.18</td>
<td>0.67</td>
<td>9.94</td>
<td>d</td>
</tr>
</tbody>
</table>

The full programme of analyses to test the method of controlling the location of initial collapse is shown in Table 6.5. The points of interest were at low impact speeds and low barrier masses (low mass ratios) so these were executed. The other checked analyses were carried out to guard against any anomalous responses that may be triggered by the modified specimen.
Table 6.5 Programme of analyses to investigate the forcing of proximal collapse through geometry

<table>
<thead>
<tr>
<th>Analysis No.</th>
<th>$V_0$ (ms$^{-1}$)</th>
<th>Barrier mass (kg)</th>
<th>Executed</th>
</tr>
</thead>
<tbody>
<tr>
<td>S lc32</td>
<td>17.88</td>
<td>2</td>
<td>✓</td>
</tr>
<tr>
<td>C lc33</td>
<td>17.88</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>H lc34</td>
<td>17.88</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>E lc35</td>
<td>17.88</td>
<td>50</td>
<td>✓</td>
</tr>
<tr>
<td>M lc36</td>
<td>11.18</td>
<td>2</td>
<td>✓</td>
</tr>
<tr>
<td>E lc37</td>
<td>11.18</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>lc38</td>
<td>11.18</td>
<td>10</td>
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</tr>
<tr>
<td>1 lc39</td>
<td>11.18</td>
<td>50</td>
<td>✓</td>
</tr>
<tr>
<td>S lc40</td>
<td>17.88</td>
<td>2</td>
<td>✓</td>
</tr>
<tr>
<td>C lc41</td>
<td>17.88</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>H lc42</td>
<td>17.88</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>E lc43</td>
<td>17.88</td>
<td>50</td>
<td>✓</td>
</tr>
<tr>
<td>M lc44</td>
<td>11.18</td>
<td>2</td>
<td>✓</td>
</tr>
<tr>
<td>E lc45</td>
<td>11.18</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>lc46</td>
<td>11.18</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2 lc47</td>
<td>11.18</td>
<td>50</td>
<td>✓</td>
</tr>
</tbody>
</table>

The two schemes were also run at the lowest speed and the lowest mass ratio condition to observe what was considered to be the worst case. Table 6.6 shows the mean dynamic force and location of initial collapse for the two schemes compared to the baseline. The energy-crush and force-crush characteristics are compared in Figures 6.8 - 6.12. Deformed shape plots are shown in Figures 6.13 - 6.22.
Table 6.6 Comparison of results from fractional set of analyses in Table 6.4 with baseline

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Scheme</th>
<th>$V_0$ (ms$^{-1}$)</th>
<th>$\gamma$</th>
<th>$P_x$ (kN)</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>lc03</td>
<td>Baseline</td>
<td>17.88</td>
<td>0.65</td>
<td>46.603</td>
<td>m</td>
</tr>
<tr>
<td>lc32</td>
<td>1</td>
<td>17.88</td>
<td>0.65</td>
<td>52.293</td>
<td>p</td>
</tr>
<tr>
<td>lc40</td>
<td>2</td>
<td>17.88</td>
<td>0.65</td>
<td>39.820</td>
<td>p</td>
</tr>
<tr>
<td>lc20</td>
<td>Baseline</td>
<td>17.88</td>
<td>16.13</td>
<td>47.437</td>
<td>p</td>
</tr>
<tr>
<td>lc35</td>
<td>1</td>
<td>17.88</td>
<td>16.13</td>
<td>52.618</td>
<td>p</td>
</tr>
<tr>
<td>lc43</td>
<td>2</td>
<td>17.88</td>
<td>16.13</td>
<td>40.435</td>
<td>p</td>
</tr>
<tr>
<td>lc10</td>
<td>Baseline</td>
<td>11.18</td>
<td>0.65</td>
<td>46.563</td>
<td>d</td>
</tr>
<tr>
<td>lc36</td>
<td>1</td>
<td>11.18</td>
<td>0.65</td>
<td>45.332</td>
<td>p</td>
</tr>
<tr>
<td>lc44</td>
<td>2</td>
<td>11.18</td>
<td>0.65</td>
<td>36.215</td>
<td>p</td>
</tr>
<tr>
<td>lc23</td>
<td>Baseline</td>
<td>11.18</td>
<td>16.13</td>
<td>44.241</td>
<td>p</td>
</tr>
<tr>
<td>lc39</td>
<td>1</td>
<td>11.18</td>
<td>16.13</td>
<td>42.781</td>
<td>p</td>
</tr>
<tr>
<td>lc47</td>
<td>2</td>
<td>11.18</td>
<td>16.13</td>
<td>35.638</td>
<td>p</td>
</tr>
<tr>
<td>lc11</td>
<td>Baseline</td>
<td>8.94</td>
<td>0.65</td>
<td>44.737</td>
<td>d</td>
</tr>
<tr>
<td>lc48</td>
<td>1</td>
<td>8.94</td>
<td>0.65</td>
<td>32.784</td>
<td>p</td>
</tr>
<tr>
<td>lc49</td>
<td>2</td>
<td>8.94</td>
<td>0.65</td>
<td>32.784</td>
<td>p</td>
</tr>
</tbody>
</table>

**6.4 Discussion**

Energy absorption

In the move from rigid to deformable barriers, energy dissipation is no longer confined to the specimen. The barrier absorbs energy and deforms according to the force generated by the impact given its own inertia and force-deflection characteristics. For this specific impact then, where the barrier was designed to be car-like in force-deflection characteristics, a quantity can be defined that relates the chassis rail performance under rigid impact conditions to that under deformable impact conditions. The performance criteria in this case is energy absorbed by the specimen for various amounts of crush.
In Table 6.2 the ratio between corresponding cells in the rigid and deformable parts of the table (Ed/Er) can be seen to reach a maximum of approximately 0.95 at 11.18 ms⁻¹ before dropping back to approximately 0.90 as the impact velocity increased. There was therefore up to a 10% difference in energy absorbed for a given amount of crush simply due to the move to a deformable barrier. The explanation was first thought to be due to the different strain-rate regimes of the two impacts. For the rigid barrier case, high initial strain-rates would lead to collapse at high forces. For a given crush distance therefore, more energy would be absorbed than in the deformable barrier case where the rate of straining was lower. However, the strain-rate soon falls after the initial folds and for the majority of the collapse process is at a level where the enhancement of the yield stress is somewhat lower. In Table 6.3 the strain-rate enhancement was switched off for the analyses so any effects that were observed from the transition from rigid to deformable barrier could be attributed to inertia and plasticity effects only. Comparing corresponding cells between rigid and deformable cases in this table led to a similar trend as observed in Table 6.2. Here though, the ratio peaks at 1.08 before falling back to 0.90. So at 11.18 ms⁻¹ the deformable barrier impact resulted in 8% more energy for 200 mm of crush of the chassis rail than was obtained with the rigid barrier impact albeit with an artificial material.

It therefore appeared that the chassis rail absorbed up to 10% less energy per unit crush when striking a deformable target than it did when striking a rigid target. The uniform symmetric collapse pattern was observed in both cases and the number of folds were also consistent so this aspect of the collapse mode did not explain the difference. Viscoplastic effects did not explain the difference either since the same behaviour was observed in the absence of strain-rate enhancement. The only remaining possible cause for the difference was considered to be uniaxial plastic compression which would occur to a greater degree for impacts with a rigid target than for impacts with a deformable one. Evidence for this compression was found in the thickness plots such as those shown in Figures 6.4 and 6.5. For the impact with a rigid barrier where the initial collapse was in the proximal region, increases in thickness of up to 22% could be observed in the corner regions where collapse had occurred (Figure 6.4). In addition, over approximately 13 rows of elements in the distal region, a 2.5-4% thickening could be observed. For an impact with a deformable barrier (Figure 6.5) this distal thickening only extended through approximately 6 rows of elements and was not as pronounced as in the previous case.

Thickening of the specimen, as an explanation for the analytical results, is entirely consistent with conclusions drawn by other workers[67, 69, 70, 71, 72, 125]. Grzebieta and Murray [125] found that local bearing failure on some of their specimens confounded attempts to equate theoretical and experimental observations. Their specimens were mild steel sheets with an initial imperfection in the form of a kink in the middle and were struck on their edges.
Plastic deformation was therefore expected to be limited to one large plastic hinge at the initial imperfection. However, evidence of plastic deformation was found at the distal end of the majority of their specimens and their rigid-plastic analysis under-predicted energy absorption by approximately 20% for thinner specimens and those with large initial imperfections. For the thicker and less imperfect specimens the under-prediction was greater.

Tam and Calladine [67] also found physical evidence of axial squashing in their specimens which had varying degrees of initial imperfection, were struck at a range of speeds by different masses. Their specimens were measured at 20 locations over the working length post-impact and showed a 0.5% mean increase in thickness which represented approximately 1% axial straining. More recent analytical work as discussed in section 2.8.4 has also improved understanding of this collapse mechanism and supports the findings in this study.

**Strain-rate enhancement and energy absorption**

The effect of strain-rate enhancement can be measured for this impact by comparing corresponding cells in Tables 6.2 and 6.3. For the analyses in Table 6.3 the strain-rate enhancement function was disabled leading to deformation at lower forces. In the rigid barrier impacts the energy absorbed for a given crush distance was between 17% and 20% lower when the strain-rate enhancement was disabled. For the deformable barrier impacts the shortfall was more variable and ranged between 6% and 21%. Considering the average effects for a given crush distance it would be expected that, as the crush progressed, the effect of strain-rate enhancement would become less noticeable as the drop-hammer slowed down. For rigid impacts the effect was quite steady being 18.6%, 18.4% and 18.3% reduction in energy absorbed for 200 mm, 300 mm and 500 mm of crush respectively. For impacts with the deformable barrier however the effect of strain-rate enhancement actually appeared to increase with crush distance. There was a 13.8%, 16.6% and 17.9% reduction in energy absorbed for 200 mm, 300 mm and 500 mm of crush. For impacts against a deformable barrier therefore the strain-rate effects were lower at the smaller crush values and approached the same level as for the rigid barrier impact at greater crush values. For the rigid barrier impacts the strain-rate effect on energy absorption was constant regardless of crush distance. A possible explanation for this counter-intuitive result would be the fact that, for the deformable impact case, the barrier displacement reduces the initial strain-rate of the specimen when compared to an impact with a rigid barrier. As the barrier bottoms out, the crushing of the specimen then speeds up with respect to the initial rate.

**Control of the location of initial collapse**

In Figure 6.6 a clear effect can be seen where, for lower mass ratios, the tendency was for non-proximal crush initiation. This was particularly so for the lower impact velocities. This phenomenon cannot be attributable to strain-rate effects alone since it was also observed for the
analyses where strain-rate enhancement of the yield stress was not applied (Figure 6.6b). Indeed strain-rate enhancement seemed to raise the $\gamma$ and $V_o$ threshold below which non-proximal collapse could be expected. Figure 6.6a can be used as a basic design guideline when selecting sections since it gives impact speed and mass ratio ranges for which the section will collapse from the proximal end rearward. This is the desirable, more stable mode of collapse and the figure can be readily reproduced for other sections.

An alternative method of using the data is given in Figure 6.7 where $\Gamma$ is plotted for various impact speeds. Again there is a clear region where proximal initial collapse can be expected and a simple first attempt can be made at defining the lower boundary by Equation 6.2 on which the lowest proximal point and the furthest proximal point to the left lie.

$$\Gamma = -0.8V_o + 19.25 \quad (6.2)$$

This equation is a simple guideline for the range of problems analysed here but the results would seem to confirm the importance of stress wave propagation even for deformable barrier impacts. This is contrary to the older assumption that, if the loading time is long in comparison to the propagation time of elastic waves in the structure, then wave effects may be ignored.

The two schemes designed to force proximal initial collapse for cases where it would not have been expected by inspection of Figure 6.7 were successful in this respect. The thinner proximal section lowered the initial peak force and also forced proximal initial collapse under a variety of impact conditions. Future work is needed in this area to support the analytical predictions with experimental results. This work should involve the precise definition and categorisation of deformable barriers in terms of mass and stiffness. The analysis will also require modification to ensure that all the elasticities and imperfections of the experimental facility are captured.

For scheme 2 the energy absorbed by the specimen was always less than the baseline for a given stroke. For scheme 1 this was also the case for the lower speed, lower mass ratio impacts (see for example Figures 6.8 and 6.12). At higher speeds and mass ratios when the thickest part of the specimen was involved in the crush, a cross-over in energy-absorption can be seen for scheme 1 specimens (Figures 6.10a and 6.11a). The loss in energy absorption at the proximal end could therefore be re-couped at the distal end if the impact was sufficiently violent and a stable collapse mode could be maintained.

In Table 6.6 the comparison of mean forces between the baseline and the two schemes showed that when the specimen was wholly consumed during impact the mean force was highest for scheme 1. This was due simply to the collapse of the thickest section at the distal end.
occurring at a sufficiently high force to over-compensate for the lower collapse force at the proximal end of the specimen. At 11.18 ms\(^{-1}\), when the specimens were not completely crushed, scheme 1 showed a mean force that was approximately 3% less than the baseline and the collapse force for scheme 2 was approximately 20% less than for the baseline. This difference was due to the involvement of part of the thickest section of the specimen in scheme 1 during the latter stages of the impact. In Figure 6.7a for example, a second rise in the force response can be seen as the specimen began to crush at the thickest section. It is this rise that accounted for the difference in mean force observed between the two schemes.

The exercise was successful in ensuring proximal initial collapse in all cases, demonstrating that the technique works (Figures 6.13 - 6.22). At very low speeds and mass ratios however a general buckling can be seen in the proximal region of the specimen before deep collapse (Figures 6.21 and 6.22). This buckling mode is similar to the dynamic plastic buckling mode described by Jones [37] for rigid impacts although it is usually observed for thicker specimens or at higher impact speeds. After this initial buckling, collapse began at the proximal end with an additional lobe at the thickness change boundary too. However, in one case (Figure 6.21) the section became distorted to the point where a transition to global bending could easily have been anticipated. There is therefore a suggestion of a lower limit to the effectiveness of this technique as applied. The length of the portion of the specimen with a lower wall thickness would have to be reduced to further confine initial collapse to the proximal end. However this would diminish the potential for weight-saving. Further work is required in this area.
Figure 6.1. Rigid barrier impact with initial collapse in the proximal region.
Figure 6.2. Deformable barrier impact with initial collapse in the mid-region.
Figure 6.3  Deformable barrier impact with initial collapse in the distal region.
Figure 6.4. Rigid barrier impact, thickness plot with displacements factored - analysis.
Figure 6.5  Deformable barrier impact, thickness plot with displacements factored - analysis lc03.
Figure 6.6  Interaction diagram of the location of initial collapse as a function of mass ratio and impact velocity, (a) with strain-rate enhancement, (b) without strain-rate enhancement.
Figure 6.7. Plot of $\Gamma$ as a function of impact velocity showing location of initial collapse.
Figure 6.8. Force- and Energy-crush results for $V_o=11.18$ ms$^{-1}$ and $\gamma=0.65$, (a) scheme 1, (b) scheme 2.
Figure 6.9. Force- and Energy-crush results for $V_o=11.18 \text{ ms}^{-1}$ and $\gamma=16.13,$ (a) scheme 1, (b) scheme 2.
Figure 6.10. Force- and Energy-crush results for \( V_0 = 17.88 \text{ ms}^{-1} \) and \( \gamma = 0.65 \), (a) scheme 1, (b) scheme 2.
Figure 6.11. Force- and Energy-crush results for $V_o=17.88$ ms$^{-1}$ and $\gamma=16.13$,
(a) scheme 1, (b) scheme 2.
Figure 6.12. Force- and Energy-crush results for $V_0=8.94$ ms$^{-1}$ and $\gamma=0.65$, (a) scheme 1, (b) scheme 2.
Figure 6.13. Deformed shape plots, $V_o = 17.88$ m s$^{-1}$ and $\gamma = 0.65$, scheme 1.
Figure 6.14. Deformed shape plots, $V_o = 17.88$ ms$^{-1}$ and $\gamma = 0.65$, scheme 2.
Figure 6.15. Deformed shape plots, $V_o = 17.88$ ms$^{-1}$ and $\gamma = 16.13$, scheme 1.
Figure 6.16. Deformed shape plots, $V_o = 17.88 \text{ ms}^{-1}$ and $\gamma = 16.13$, scheme 2.
Figure 6.17. Deformed shape plots, $V_o = 11.18$ m$^{-1}$ and $\gamma = 0.65$, scheme 1.
Figure 6.18. Deformed shape plots, $V_o = 11.18 \text{ ms}^{-1}$ and $\gamma = 0.65$, scheme 2.
Figure 6.19. Deformed shape plots, $V_o = 11.18 \text{ ms}^{-1}$ and $\gamma = 16.13$, scheme 1.
Figure 6.20. Deformed shape plots, $V_o = 11.18$ ms$^{-1}$ and $\gamma = 16.13$, scheme 2.
Figure 6.21. Deformed shape plots, $V_o = 8.94$ ms$^{-1}$ and $\gamma = 0.65$, scheme 1.
Figure 6.22. Deformed shape plots, $V_o = 8.94 \text{ ms}^{-1}$ and $\gamma = 0.65$, scheme 2.
Chapter 7

Conclusions and future work
This programme of research has used a simplified chassis rail in the form of a uniform prismatic tube to improve understanding of realistic impact events. The full and controlled crush of chassis rails during car collisions is often hampered by the necessary compromises of engine bay packaging. It is therefore doubly important to ensure that every millimetre of free crush space is utilised efficiently.

The numerical form of analysis used was first thoroughly validated using several laboratory experiments at different speeds. The frequency with which data were written out from the FE analysis or acquired from the laboratory equipment was found to be important, especially in the first 2 milliseconds where high amplitude fluctuations in force history occur. For the UMIST validation it may be concluded that

- the laboratory equipment did not capture data at a sufficiently high frequency to record the initial peak value of force accurately

- the specimen arrived off-axis at the platen.

The peak force recorded on the oscilloscope trace was approximately 39 kN whereas the calculated static squash load for the section was approximately 70 kN. The two conditions above could account for the difference and both represent some form of procedural imperfection of which the analyst must be aware when developing structures. Procedural and physical imperfections in the real-life situation must be accounted for in the FE analysis and the two methods of achieving this that were successfully applied in this study was to rotate the model or to move nodes out of plane prior to analysis.

A method of determining the effects of imperfections could be to conduct repeat laboratory tests and it was observed that two of the experiments with seemingly identical specimens showed approximately 7.5% scatter around the mean value of $\delta_f$ and $P_m^d$. It would therefore be reasonable to accept errors of this magnitude in the predictions given by the analyses. The analytical predictions of final crush and mean dynamic force were within 9.3% of the test result in the IRC validation study.

In addition to mean force and final crush results the overall shape of a characteristic response was also well predicted. One of the objectives in the validation studies was to obtain a similar velocity history between experimental and analytical cases and this was judged to have been achieved.
Another objective was to carry out a detailed comparison between the analytical predictions and the established BCE theory detailed in Chapter 3. It was observed that the post-yield behaviour of the material model in the FE analysis affected the collapse mode of the structure. When comparing the dimensions of a BCE, the analysis with the lower strain-hardening modulus seemed to result in a collapse mode that was closer to the previously available rigid-plastic analysis. However, the material with the higher strain-hardening modulus gave predictions of drop-hammer velocity history that were closer to the experimental case. The FE analysis technique used here therefore represented an improvement over the previously available approximations based on a rigid-plastic material model.

The IRC validation study also looked in more detail at the internal energy distribution in the specimen with specific reference to available estimates. Once again it was observed that the predictions of energy absorbed were closer to the rigid-plastic analysis estimates for the FE specimen with a lower strain-hardening modulus. This analysis under-predicted the established theory by 4.9% whereas the analysis with a higher strain-hardening modulus over-predicted the established theory by 4.8% with a good deal more scatter around the average. The average strain in a BCE was also well predicted comparing favourably to the previously available estimate of 10.9% at maximum crush.

**Estimation of strain-rate**

Two methods of estimating strain-rate from the analysis were developed, one used the maximum slope of the strain histories of individual BCE and the other used a mean slope. The technique using the maximum slope yielded a strain-rate of the order of 30 s\(^{-1}\) approximately. This was not comparable to the previously available estimate of 96 s\(^{-1}\) but fell within the range of 3 to 64 s\(^{-1}\) recently reported for an aluminium crush tube subjected to a 10 kJ impact at approximately 11 ms\(^{-1}\). It was observed that, apart from the proximal and distal BCEs, the remainder appeared to collapse at a similar strain-rate of the order of 30 s\(^{-1}\). Using this as a characteristic strain-rate in the previously available kinematical analysis method led to an estimate of 95.858 J of energy absorbed by a BCE instead of the 102.186 J calculated using Equation (3.1.11) to estimate the strain-rate. If the average strain-rates in the experiment were indeed of the order of 30 s\(^{-1}\), the error between previously available analytical predictions and the current prediction would be +1.3% and +11.17% for \(E_t = 1\) GPa and 1.4 GPa respectively. However there may not be much to be gained by attempting to improve the agreement between the low strain-hardening FE analysis and the rigid-plastic kinematical analysis due to the neglect of inertia effects in the latter.
Improvements in energy management

Once validated the analysis technique was used to explore methods of improving the performance of chassis rails. The comparison of cross sectional efficiencies showed that the up to 16.5% more energy per unit mass could be absorbed by moving from a square to a hexagonal section. An optimum thickness for the particular section analysed was found to be approximately 1.25 mm for stable uniform symmetric collapse. A change in mode shape occurred for the thickest hexagonal section analysed which had a thickness-to-width ratio \( h/c = 0.045 \). This placed it in the approximate range \( 0.035 \leq h/c \leq 0.099 \) given by Abramowicz and Jones [116] wherein the asymmetric mixed mode controlled the collapse of square tubes under moderate impact speeds. Laboratory tests would confirm whether this also held for hexagonal columns. Another advantage of using a hexagonal section for energy management was shown to be the 4% increase in second moment of area over a square section of identical wall thickness and material cross-sectional area. It would thus be able to withstand greater bending moments generated during axial impacts before yield. Moments generated by an impact that was slightly off-axis would also be better resisted by a hexagonal section.

The addition of a flitch panel made the specimen absorb more energy but the flitch panel size should be limited to 1.5c otherwise the specific energy absorption of the assembly may be compromised. As the added panel increased in size there was evidence of a conflict in folding wavelength between the flitch and the specimen which tended to disrupt the uniform folding pattern of the specimen. Methods for overcoming the effects of the larger flitch panels were not investigated in this research programme and would be a useful subject for further work. It is possible that cut-outs could be made in the flitch panel parallel to the axis of the chassis rail at a suitable distance away. The chassis rail could therefore dominate the response of flitch panel material close by and material further away could adopt a different collapse mode.

Cutting material out of the chassis rail itself was shown to increase the specific energy absorption by as much as 5% without affecting collapse mode as long as the width of the cut-out did not exceed 0.3c and the axial dimension did not exceed 0.6c approximately. Essentially an ineffective strip width and length was identified for a specimen undergoing dynamic axial loading which now needs careful experimental corroboration. Attempts to further optimise the axial location of cut-outs were unsuccessful. This was because material in low-energy absorbing areas seemed to serve a restraining function for the surrounding material undergoing large strains. Removal of the restraining material would therefore reduce the amount of deformation taking place in these surrounding areas.
Quantification of an inertia effect

In the move from a rigid to a deformable barrier the stroke efficiency of a specimen was found to be up to 10% lower. The explanation was first thought to be due to the different strain-rate regimes of the two impacts but was finally considered to be uniaxial plastic compression which would occur to a greater degree for impacts with a rigid target than for impacts with a deformable one and some evidence for this compression was found in the thickness changes observed in the analyses. For the impact with a rigid barrier where the initial collapse was in the proximal region, increases in thickness of up to 22% could be observed in the corner regions where collapse had occurred. In addition, over approximately 13 rows of elements in the distal region, a 2.5-4% thickening could be observed. For an impact with a deformable barrier this distal thickening only extended through approximately 6 rows of elements and was not as pronounced as in the previous case. This kind of thickening is very difficult to measure experimentally although some workers have remarked that their specimens have shown evidence of its presence. In this case then, the inertia effect manifested itself as a possible change in collapse mode accompanied by a reduction in stroke efficiency of up to 10%.

Quantification of a strain-rate effect

In the analysis of the rigid barrier impacts in Chapter 6 the energy absorbed for a given crush distance was between 17% and 20% lower when the strain-rate enhancement was disabled. For the deformable barrier impacts the shortfall was more variable and ranged between 6% and 21%. Considering the average effects for a given crush distance it would be expected that, as the crush progressed, the effect of strain-rate enhancement would become less noticeable. However, for the rigid barrier impacts, the strain-rate effect on energy absorption was constant regardless of crush distance. For impacts against a deformable barrier the strain-rate effects were lower at the smaller crush values and approached the same level as for the rigid barrier impact at greater crush values. These results represent an isolation of the strain-rate effects from the inertia effects which, as far as the author is aware, has not been attempted before and is difficult to achieve under laboratory conditions in this range of impact speeds.
Control of the location of initial collapse

Encouraging results were also achieved when an attempt was made to investigate inertia effects by varying the mass ratio and impact speed. For lower mass ratios and lower impact speeds, the tendency was for non-proximal crush initiation. In reality it is a combination of inertia, strain-rate, strain-hardening and stress wave effects that influence the location of initial collapse. This very complex interrelationship is also further complicated by the degree of imperfection in a real structure. However, a basic design guideline was presented which gave impact speed and mass ratio ranges for which initial collapse in the proximal region was expected for a particular structure. A non-dimensional parameter, $\Gamma$, which combined mass ratio and impact speed with material and physical properties was developed. $\Gamma$ was plotted for various impact speeds and there was a clear region where proximal initial collapse could be expected. An initial recommendation was made to ensure that material and sectional choice resulted in a point in $\Gamma-V_o$ space above the following line approximately:

$$\Gamma = -0.8V_o + 19.25.$$ 

For specimens lying below this line two schemes were designed to force proximal initial collapse. They both worked successfully under a variety of impact conditions but both absorbed less energy than the baseline up to 200 mm of crush.

The results in this part of the study would seem to indicate that stress wave propagation should not be neglected when analysing automotive components for crashworthiness purposes including deformable barriers. This conclusion is supported by recent findings by several workers and is contrary to the older assumption that, if the loading time is long in comparison to the propagation time of elastic waves in the structure, then wave effects may be ignored. Future work is needed in this area to support the analytical predictions with experimental results. This work should involve the investigation of a change of section for the proximal region to redress the loss in energy absorption when forcing initial proximal collapse. The precise definition and categorisation of deformable barriers in terms of mass and stiffness is also required. The analyses will require modification to ensure that all the elasticities and imperfections of the experimental facility are captured.
References


124. T. Zeguer, Personal communication, Jaguar Cars Ltd., 1996.

I.1 Essential pre-processing items for users of PATRAN for DYNA3D analyses

The following notes are for the benefit of advanced users of PATRAN who wish to create FE models for analysis in DYNA3D:

**Nodes** - It is important that all nodes are written in global Cartesian co-ordinates.

**Material Number** - The Oasys DYNA3D translator, N/CODE only uses the material-id (MID) to set up physical and material properties. Therefore the same MID must not be used for different thicknesses of the same material. The material thickness should be the second data value on the element property data card.

**Contact surfaces** - The N/code translator accepts stonewalls and three types of contact surface.

- DYNA3D contact surface type 3 - sliding with contact and separation
- DYNA3D contact surface type 4 - single surface contact
- DYNA3D contact surface type 5 - slave nodes impacting on a master surface.

On translation, the user is permitted to change these types into other types.

For types 3 and 4, quadrilateral or triangular shell elements must be created to 'cover' the areas of the model requiring contact surfaces. This can be done by copying the phase 2 model using the NAME command and translating the region with a displacement of 0/0/0 (i.e. NAME,TUBE_CON,TR,0/0/0,TUBE). Patches, grids and lines (Phase 1 entities) created by this copying must be deleted and then the new named component re-named again to, say, TUBE_CON1 or overwritten. Modifications to the property-ID (PID) and configuration numbers of the shell elements can be carried out by reference to their patch numbers (ELEMENT command). The geometry must then be equivalenced so that N/CODE recognises that the new named component is in fact a contact surface.

The property-id (PID) identifies the master/slave pair. The surfaces must therefore have the same PID. The configuration number of the QUAD or TRI elements indicates whether they are a master or slave surface. QUAD/4/11 for four-node slave quadrilateral elements QUAD/4/10 for four-node master quadrilateral elements. Single surface contact elements (type 4) can be defined as slave or master surface.
For slave nodes impacting a master surface (type 5), the master surface is defined as above and the slave nodes are placed in a named component called CONTACT_n where n is the same as the PID of the master surface. The nodes in the group must not be connected into an element. The NODE command can be used to modify the PID of the nodes in the group before the phase 1 items are deleted.

**Stonewalls** - These have slave nodes defined which are not permitted to penetrate the wall. These nodes should be placed in a named component "STONE_n" where n is the PID of the elements forming the stonewall.

**I.2 Specifying material and physical properties.**

PATRAN permitted the use of one set of material property specifications for several parts of a model; i.e. different thicknesses of a component could have different physical data but the same material property data. Oasys DYNA3D however used just one number, the material number, to describe both material and physical data. The N/CODE translator read all material and physical properties described in the pre-processor and, if necessary, created dummy DYNA3D material numbers to ensure that all parts of a model were translated.
Appendix II
II.1 Computer programs written during the course of the work

Programe to calculate mean dynamic force from T/HIS curve file.

```fortran
PROGRAM MEAN
  OPEN (UNIT=12, FILE='re3lforce.cur', STATUS='OLD')
  I=0
  XSUM=0
  YSUM=0
  100 CONTINUE
  READ(12,*,ERR=101)X,Y
  * The next line increments the number of points read for non-zero Y only
  * This avoids inclusion of the origin in the calculations and also the
  * end points.
    IF (Y.NE.0.0) I=I+1
    YSUM=YSUM+Y
  GOTO 100
  101 YMEAN=YSUM/I
  CLOSE(12)
  PRINT*, 'ARITHMETIC MEAN', YMEAN
END
```

The above module was used as a building block for evaluating the average strain in a BCE at a given time thus

```fortran
PROGRAM MEAN1
  OPEN (UNIT=12, FILE='ab34bce2.e01', STATUS='OLD')
  I=0
  XSUM=0
  YSUM=0
  100 CONTINUE
  READ(12,*,ERR=101)X,Y
  * The next line increments the number of points read for non-zero Y only
  * This avoids inclusion of the origin in the calculations and also the
  * end points.
    IF (Y.NE.0.0) I=I+1
    YSUM=YSUM+Y
  GOTO 100
  101 YMEAN=YSUM/I
  CLOSE(12)
  PRINT*, 'ARITHMETIC MEAN STRAIN ab34bce2.e01', YMEAN
  * OPEN (UNIT=12, FILE='ab34bce2.e02', STATUS='OLD')
    I=0
    XSUM=0
    SUM=0
  200 CONTINUE
  READ(12,*,ERR=201)X,Y
  * The next line increments the number of points read for non-zero Y only
  * This avoids inclusion of the origin in the calculations and also the
  * end points.
    IF (Y.NE.0.0) I=I+1
    YSUM=YSUM+Y
  GOTO 200
  201 YMEAN=YSUM/I
  CLOSE(12)
  PRINT*, 'ARITHMETIC MEAN STRAIN ab34bce2.e02', YMEAN
etc...
```

II-1
Program written to sum the energy absorbed by the elements in a BCE

```
PROGRAM SUM
OPEN (UNIT=12, FILE='ab17bce4.nrg', STATUS='OLD')
I=0
XSUM=0
YSUM=0
100 CONTINUE
   READ(12, *, ERR=101) X, Y
* The product of the energy density (Y) and the volume of the element * is to be evaluated for each line read in. the sum of these products * is then to be evaluated. The volume of the element is 44.495mm³.
   IF (Y.NE.0.0) I=I+1
   YSUM=YSUM+Y*44.495
   GOTO 100
101 YMEAN=YSUM/I
CLOSE(12)
PRINT*, 'ARITHMETIC MEAN ENERGY DENSITY', YMEAN
PRINT*, 'SUM OF ENERGY FOR BCE4', YSUM
END
```

In a similar manner to the program MEAN, the module for finding the BCE energy was added to others in order to evaluate the various BCE in the structure.
Appendix III
III.1 Calculation of interface stiffness between specimen and barrier

The young's Modulus $E$ and Poisson ratio, $\nu$ specified for a rigid body at input are used to calculate the interface stiffness of contact surfaces attached to that body in the numerical analysis. $E$ and $\nu$ should be similar for non-rigid bodies that come into contact with the rigid body unless the elements of the rigid body are much larger than those of the non-rigid body.

In the model under consideration the barrier element was covered by a single contact surface segment the area of which was approximately 50 times larger than the deformable elements that came into contact with it. In such a case the authors of the FE code suggest that $E$ for the rigid body be scaled down so that the product of $E$ and the element length is similar for the rigid and deformable elements. This is referred to as symmetric stiffness.

The following is an example of contact stiffness calculation using the coarse mesh model employed to develop the deformable barrier:

$$ k = \frac{f_s \times A^2 \times K}{\text{Volume}} $$  \hspace{1cm} (III-1)

for segments on solid elements and

$$ k = \frac{f_s \times A \times K}{\text{Minimum diagonal}} $$  \hspace{1cm} (III-2)

for segments on shell elements. The penalty factor $f_s$ is 0.1 by default and users are advised not to adjust it in order to maintain numerical stability.

Now $K = \frac{2G(1-\nu)}{3(1-2\nu)}$ and $G = \frac{E}{2(1-\nu)}$

$$ \Rightarrow \quad K = \frac{E}{3(1-2\nu)} $$ \hspace{1cm} (III-3)

For the shell elements, $E$ was set at $2.0E5$ Nmm$^{-2}$ and $\nu = 0.3$

$$ \therefore \quad K = \frac{2E5}{2 \times 0.4} = 1.67E5 \text{Nmm}^{-2} $$ \hspace{1cm} (III-4)
For the barrier, $E$ was scaled by

$$\frac{\sqrt{25^2 + 33.3^2}}{\sqrt{90^2 + 140^2}} = \frac{41.46}{166.43} = 0.25$$  \hspace{1cm} (III-5)$$

and substitution of the scaled value of $E$ into equation (III-3) gave $K = 4.17 \times 10^4$ Nmm$^{-2}$. For the contact segments on the barrier therefore, the stiffness is given by equation (III-6).

$$k = 0.1 \times (140 \times 90) \times 4.17 \times 10^4 \div (140 \times 90 \times 9.9) = 5.31 \times 10^6$$  \hspace{1cm} (III-6)$$

Equation (III-7) below gives the stiffness for contact segments on the shell elements that come into contact with the barrier

$$k = \frac{0.1 \times (25 \times 33.3) \times 1.67 \times 10^5}{\sqrt{25^2 + 33.3^2}} = 3.34 \times 10^5$$  \hspace{1cm} (III-7)$$

When the analysis was carried out with the values of $E$ used to derive $K$ above, the interface forces were found to be too great causing the specimen and drop hammer to bounce off the barrier before significant deformation could take place. Numerical instabilities also caused the analysis to fail shortly after the start. A process of trial and error was therefore used to find the value of $E$ for the barrier that would give a realistic response.

Figure III-1 shows a typical set of deformed shapes for a value of $E$ that was too low. The specimen passes through the interface until a sufficiently large contact force is generated to repel the nodes. A value of $E = 20$ Nmm$^{-2}$ for the material representing the barrier was found to give a reasonably realistic response given the coarseness of the mesh. That value was therefore used in the exercise to determine spring stiffness.

### III.2 Further work on Interface stiffness specification

In order to identify penetration of the drop hammer more accurately, thin shell elements were used to model the drop-hammer for the IRC work. This had the added bonus of simplifying the interface calculations since the same equation held for segments on the master and the slave surfaces. The barrier was 100 mm square and the specimen was 37.07 mm square. It was recognised that equation (III-2) could be simplified to equation (III-8)

$$k = f_s \times l \times K$$  \hspace{1cm} (III-8)$$
and advantage was taken of the simplicity of the geometry by choosing the characteristic lengths to be the side lengths. For symmetric stiffness therefore equation (III-9) was maintained since the penalty stiffness factors were not to be altered.

\[
\frac{K_{\text{mass}}}{K_{\text{specimen}}} = \frac{37.07}{100}
\]

Equation (III-3) led therefore to the scaling of \( E \) for the drop-hammer by a factor of 0.0371.

First attempts at scaling the drop hammer \( E \) led to deformation sequences such as shown in Figure III-2 for run ab12. Collapse began at the distal end in an extensional mode and progressed towards the proximal end. None of the observed experimental specimen displayed this type of performance.

It was only when the value of Drop-hammer density was scaled that the initial collapse switched to the proximal end as observed in the experiments.
Figure III-1    Excessive penetration resulting from a low value of $E$ for the barrier.
Figure III-2  Deformed shape plots for analysis ab12