**ICT and economic growth: a dynamic non-parametric approach**

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**Metadata Record:** [https://dspace.lboro.ac.uk/2134/7248](https://dspace.lboro.ac.uk/2134/7248)

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ICT and Economic Growth:
A Dynamic Non-parametric Approach

By

Bin Wang

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy

Of

Loughborough University

November 2010
ABSTRACT

One of important issues of the policymakers is to improve output and/or productivity growth associated with information and communication technology (ICT) adoption, where total factor productivity (TFP) growth related with ICT in the 1990s appeared in the US but not in the UK (Jorgenson and Stiroh, 2000; Oliner and Sichel, 2000). The general agreement is that ICT can raise output and/or productivity growth via an increase in productivity growth in the ICT-producing sectors due to rapid technological progress, through capital deepening driven by high levels of investment in ICT equipments, and via increases in efficiency in ICT-using sectors that successfully adopt this new technology by ICT spillover effects (David, 1990). Due to the small size of ICT-producing industries and relatively low level of ICT investments in the UK (Colecchia and Schreyer, 2001; Daveri, 2002; Visselaar and Albers, 2002), the utilization of ICT spillover effects was crucial to improving output and/or productivity growth for the UK. However, in most of the previous studies, while many concluded ICT spillover effects existed in the US, they had mixed results as to whether ICT spillover effects existed in the UK (Schreyer, 2000; Basu et al., 2003; Inklaar et al., 2005; Jorgenson et al., 2005).

The objective of this thesis is to contribute to the existing literature by investigating the existence of ICT spillover effects in the US and the UK and exploring the reasons for the different effects between them. This thesis argues that the mixed findings in the previous studies are due to the ignorance of the General-purpose technology (GPT) theory and weakness in methodology. Thus, the first step is to build a new framework of measuring ICT spillover effects to solve the problems from the existing studies.
The main ignorance of the GPT theory is the lack of guidance for the proxy of co-invention related to ICT investments and for the length of lag. The new framework no longer has this ignorance because it uses efficiency as a proxy of co-invention and captures the length of lag by years with negative return on ICT capital. The methodology employed in the previous studies was inappropriate mainly because of the small sample size taken in the ICT study, the two-stage approach used to explore the effect of the environmental variables on efficiency and the linear and concavity assumptions on the frontiers without taking account of ICT as a GPT. The new framework uses Bayesian technique, one-stage approach and non-parametric frontiers to avoid these three drawbacks. In addition, the new framework introduces the persistent level of inefficiency, using a first-order autoregressive (i.e. AR(1)) structure of inefficiency itself, as one of factors that influence ICT spillover effects.

In order to model the new framework which takes into account the non-parametric frontiers for capturing negative return of ICT capital, an AR(1) structure of inefficiency, the small sample size and factors that influence ICT spillover effects, this thesis has developed two non-parametric dynamic stochastic frontier analysis (SFA) models with an AR(1) structure and performed the analysis via Bayesian inference. The first model was a semi-parametric dynamic stochastic frontier with a time-variant non-parametric frontier at the basic level along with a time-invariant linear function for the technical inefficiency at the higher-level. The second model relaxed the time-invariant linear functional form for technical inefficiency at the higher level.

The results of the new framework showed strong ICT spillover effects in the US with a lag of about 6-8 years during 1982-83 to 1988-89, while relatively weaker ICT spillover effects in the UK. This can be evidenced by the fact that the UK has been in the process
of organizational adjustment up to 2000 due to a longer lag. Thus, in the 1990s, there was a lack of TFP growth in the UK. Related to the different ICT spillover effects between the US and the UK, the results from the new framework suggested that the various persistent levels of inefficiency between the two countries was important, apart from the different levels of ICT investment between them mentioned in the previous studies (Inklaar, O’Mahony and Timmer, 2003).

JEL Classifications: C51, E13, O30, O33

Keywords: ICT, Spillovers, GPT, Efficiency, Non-parametric, Bayesian, SFA
ACKNOWLEDGMENTS

First, I would like to show my deepest gratitude to my supervisors, Dr. Hailin Liao and Dr. Baibing Li, who are respectable, responsible, and resourceful scholars that have provided me with valuable guidance during each stage of the writing for this thesis. Without their enlightening instruction, impressive kindness and patience, I could not have completed my thesis. Their keen and vigorous academic observation enlightens me not only in this thesis, but also in my future study.

I shall extend my thanks to Professor David Llewellyn, Professor Tom Weyman-Jones and Dr. Paul Turner for all their kindness and support. I would also like to thank all my teachers who have helped me develop the fundamentals and the important sense of academic competence.

Finally, my sincere appreciation also goes to my family, who has always stood behind me.
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<tr>
<td>DEA</td>
<td>Data Envelope Analysis</td>
</tr>
<tr>
<td>GPT</td>
<td>General Purpose Technology</td>
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<td>ICT</td>
<td>Information and Communication Technology</td>
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<tr>
<td>MCMC</td>
<td>Markov Chain Monte Carlo</td>
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<tr>
<td>NCS</td>
<td>Natural Cubic Spline</td>
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<tr>
<td>SBOC</td>
<td>Skill-biased Organizational Change</td>
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<tr>
<td>SBTC</td>
<td>Skill-biased Technical Change</td>
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<tr>
<td>SF</td>
<td>Stochastic Frontier</td>
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<tr>
<td>SFA</td>
<td>Stochastic Frontier Analysis</td>
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<tr>
<td>TE</td>
<td>Technical Efficiency</td>
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<td>TEC</td>
<td>Technical Efficiency Change</td>
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<td>TFP</td>
<td>Total Factor Productivity</td>
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Chapter 1 Introduction

Recently, there has been much research on Information and communication technology (ICT) spillover effects in the 1990s in the US and the UK (Basu et al., 2003, 2007; Schreyer, 2000; Inklaar et al., 2005; Jorgenson et al., 2005). ICT spillover effects mean the contributions of ICT capital to TFP growth and thus output growth through the increase in the efficiency in ICT-using sectors that successfully adopted ICT (Basu et al., 2003, 2007). Since the UK do not have a large ICT-producing sector and a high level of ICT investment relative to the US (Colecchia and Schreyer, 2001; Daveri, 2002; Vijselaar and Albers, 2002), a deeper understanding of the following two questions about ICT spillover effects is very important for the UK’s policymakers who want to increase productivity and output via ICT. First is whether there are ICT spillover effects in the UK, and the second is why the ICT spillover effects in the UK are relatively smaller than that in the US?

The objective of this thesis is to answer these two questions. The challenge of the first question is that the current empirical studies on ICT spillover effects are weak both in their economic theoretical underpinning and their methodology. Theoretically, since the neoclassical growth accounting approach does not allow a direct linkage between ICT capital and total factor productivity (TFP) growth in ICT-using industries (Stiroh, 2002), the GPT theory has been developed to analyse ICT spillover effects (Brynjolfsson and Hitt, 2002; Howitt, 1998; Basu et al., 2003, 2007). This theory suggests that ICT capital contributes to TFP growth in ICT-using industries through cooperation between some unobserved complementary co-invention, but this contribution results in a lag time
because the co-invention needs time to develop. However, the GPT theory itself gives little guidance on the length of the lag period and the measurements of this co-invention. Previous empirical studies usually assumed a lag to be 5-15 years and used the organizational capital indicated by the lagged ICT capital growth as a proxy for the co-invention (Brynjolfsson and Hitt, 2002; Howitt, 1998; Basu et al., 2003, 2007).

In addition, from the perspective of econometric methodology, there are three main problems in the existing studies, i.e. the small sample size taken in the ICT study, the two-stage approach used to explore the effect of the environmental variable on efficiency, and the linear and concavity assumptions on the frontiers without taking account of ICT as a GPT. Small ICT data sets have always hindered development of robust, accurate estimates of ICT spillover effects (Koop et al., 1999). A two-stage approach is usually used to measure the effect of influencing factors of ICT spillovers, which may lead to a biased estimate due to the econometric problems (Koop et al., 2000). In addition, a pre-specified production functional form is also problematic (Lin and Shen, 2002; Lin, 2009). Different empirical studies using different assumptions for the stochastic frontier (SF) have obtained conflicting results on the same data when efficiency as a component of TFP is used to explore the linkage between ICT capital and an organizational structure adjustment as indicated by the improvement of efficiency (Shao and Lin, 2001, 2002; Lin, 2009).

To fill the theoretical and methodological gaps in the current literature, this thesis combines the GPT theory and the concept of efficiency to develop a framework to measure ICT spillover effects and their influencing factors. The characteristics of this framework include: (i) that efficiency is treated as a measurement of co-invention,
according to the argument of the GPT theory that co-invention reflects unobserved accumulations of intangible organizational capital or an organizational structure adjustment like increasing the skilled labour, or both; (ii) that a non-parametric dynamic SF is used to capture the length of the lags by years with negative return of ICT capital, according to the GPT hypothesis that the association of co-invention with ICT capital might lead to initially negative returns on the ICT capital before it could positively contribute to the output; (iii) that Bayesian inference is drawn for the two-level models with a non-parametric dynamic SF as the basic level and a parametric/non-parametric dynamic efficiency function as the higher level that gives accurate estimates on small samples; (iv) that a one-stage approach is employed to analyse the effect of influential factors of ICT spillovers; and (v) that the persistent level of inefficiency is considered as an endogenous factor that influences ICT spillovers.

Based on this framework, a country will have ICT spillover effects if ICT-using industries’ efficiencies increase during (or shortly after) the lags. The results for the UK and the US, based on the non-parametric SF functions at the basic-level, showed a strong ICT spillover effects in the US with a lag of 6-8 years during 1982/83-1988/89. The UK seemed to have ICT spillover effects, but it had been in the process of organizational adjustment up to 2000 due to a long lag with 12-13 years during 1988/89-2000. The long lag contributed to little TFP growth in the UK in the 1990s.

After investigating the pattern of ICT spillover effects in the UK and the US, the focus of this thesis turns to the second question and discusses what leads to the different ICT spillover effects between the UK and the US. The framework in this thesis uses

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1 If considering a possible lagged appearance of efficiency changes, a country may have ICT spillover effects if ICT-using industries’ efficiencies increase after the lags.
efficiency as a proxy of co-invention and takes into account of both the endogenous and exogenous factors related to efficiency. The endogenous factor is the persistent level of inefficiency itself measured by an AR(1) structure of inefficiency in the efficiency function at the higher level of the framework. The exogenous factors include two traditional factors associated with organizational adjustment, which are software investment and skilled labour. Based on the parametric/non-parametric efficiency function at the higher level of the framework, the results showed that the main reason for different ICT spillover effects between the US and the UK is a stronger persistent technical inefficiency in the UK than that in the US. Meanwhile, the strong endogenous persistent inefficiency in the UK may further limit the influence of exogenous factors on inefficiency, such as the supply of skilled labour and software investment. Thus, the results from the new framework presented that the various persistent levels of inefficiency between the two countries was important, apart from the different levels of ICT investment between them mentioned in the previous studies (Inklaar, O’Mahony and Timmer, 2003).

This thesis contributes to the existing literature in three aspects. First, a new dynamic non-parametric SF approach has been developed and the corresponding Bayesian inference has been investigated using Markov Chain Monte Carlo. Consequently a one-stage approach for statistical inference to measure ICT spillover effect and its influential factors can be used.

The second aspect concerns the literature of ICT spillover effects, where this thesis built a new framework to measure the length of lag associated with ICT spillover effects by combining the GPT theory and the concept of efficiency. The contributions on this aspect
can be highlighted as follows: (i) finding negative return of ICT capital initially; (ii) measuring the length of the lag by years with negative return of ICT capital; (iii) using efficiency as a proxy of co-invention and getting evidence that efficiency increases during the lag period; (iv) introducing the persistent level of inefficiency as one of influential factors of ICT spillover effects; and (v) exploring the effects of both the persistent level of inefficiency as an endogenous factor and skills and software as the exogenous factors of ICT spillovers in a one-stage approach, and finding the endogenous factor is dominant.

The third aspect of the contributions is related to the literature in explaining different pattern of ICT spillover effects between the US and the UK, where (i) weak ICT spillover effects in the UK is suggested, relative to the US; (ii) a reasonable explanation has been explored for the question why little TFP growth in the UK in 1990s was found. Finally, the persistent level of inefficiency has been considered as a reason of different ICT spillover effect between the US and the UK apart from the level of ICT investment mentioned by the previous studies.

The remainder of this thesis is organized as follows. Chapter 2 reviews the existing literature on ICT spillover effects using neoclassical growth theory, the GPT theory and the frontier framework associated with the efficiency. Chapter 3 reports the data used in this thesis. Chapter 4 presents two two-level non-parametric SF dynamic models and investigates their Bayesian inference. One has a parametric time-invariant efficiency function at the higher level while the other has a non-parametric time-variant efficiency function; both have a non-parametric dynamic SF at the basic level. Based on these two-level models, Chapter 5 develops a new framework to measure ICT spillover effects and
their influences through cooperation between the GPT theory and efficiency. This framework is then employed to investigate ICT spillover effect for the US and the UK and the reason for the different ICT spillover effects in the US and the UK in Chapter 6. Chapter 7 presents the conclusions.
Chapter 2 Literature reviews

In the field of the engineering and information science, the ICT innovation is making significant progress. In economics, empirical studies using the neoclassical growth model show that ICT significantly impacted the measured TFP growth of ICT-producing sectors in the US in the 1990s (Schreyer, 2000; Jorgenson, Ho and Stiroh, 2005; Triplett and Bosworth, 2004; Inklaar, O’Mahony and Timmer, 2003). However, the neoclassical growth model does not show whether ICT contributed to measured TFP growth of ICT-using sectors by ICT spillover effect (e.g. Stiroh, K. J., 2002; Jorgenson, et al., 2005; Triplett and Bosworth, 2004; Vijselaar and Albers, 2002; O’Mahony and Vecchi, 2005; Jalava and Pohjola 2007). Recent research on the ICT spillover effect has followed two directions. One is to use the GPT theory to investigate the unobserved accumulation of intangible organizational capital associated with ICT usage in ICT-using sectors (e.g. Basu et al., 2003, 2007). The other is to use the productive technical efficiency as an economic measure of ICT benefits in organizational performance for those ICT-using sectors based on the frontier approach (Shao and Lin, 2001, 2002; Lin, 2009).

Based on this background, this chapter firstly reviewed the analysis of ICT spillover effects based on the neoclassical growth models, the GPT theory and the frontier framework. The empirical studies of ICT spillover effects in the US and the UK were then highlighted.
2.1 ICT spillover effects based on the neoclassical growth models

Conceptually, ICT can raise output and/or productivity growth via several routes: (1) an increase in productivity growth in the ICT-producing sectors themselves due to rapid technological progress, and/or an increase in the size of the fast-growth ICT-producing sectors in the economy; (2) capital deepening driven by high levels of investment in ICT equipment and (3) increases in efficiency in ICT-using sectors that successfully adopt this new technology through ICT spillover effects. Most empirical studies of the impact of ICT on output and/or productivity growth have been based on the neoclassical growth models, even though they cannot clearly explain ICT spillover effects.

2.1.1 Neoclassical growth models

2.1.1.1 Neoclassical growth model

The neoclassical growth model (Solow, 1956) assumes that there is a (value added) production function that relates output to labour, capital and technology.

This model claims that:

- Both endogenous input changes and exogenous technological progress indicated by TFP growth contribute to output growth.
- The economy will converge to a steady state.
- Exogenous technological progress is the only factor driving economic growth.

(1) Both input changes and TFP growth contribute to output growth
The Solow model based on an aggregate production function is

\[ Y = AF(K, L) \]  \hspace{1cm} (2.1)

where \( K \) is the capital stock, \( L \) is the labour and \( A \) reflects a Hicks-neutral technical change. \( F(.) \) presents the relationship between input and output, which is a constant return to scale. In practice, two commonly used forms of \( F(.) \) are the Cobb-Douglas and the Translog forms. Eq. (2.1) indicates that output growth can be from input accumulation (i.e. increases in \( K \) and \( L \)) and technology progress \( A \). \( A \) is usually measured by the residual, with the economic meaning that TFP has a permanent effect on output growth. In contrast, input accumulation has a temporary effect on output growth due to the inputs’ diminishing return. The contribution of the inputs is measured by the coefficients of these inputs.

In empirical studies, the labour productivity rather than output is usually employed to analyse the economic performance. Therefore, Eq. (2.1) can be rewritten as

\[ \frac{Y}{L} = AF(K/L, L) \text{ or } y = Af(k) \]  \hspace{1cm} (2.2)

where \( y = \frac{Y}{L} \) is the output per worker and \( k = K/L \) is capital stock per worker. Eq. (2.2) presents the labour productivity as a function of capital per worker and technology.

(2) The economy converges to a steady state

Based on Eq. (2.1), in a closed economy, if there is a constant savings rate \( s \), a constant population growth rate \( n \), and a constant depreciation rate \( \delta \) then the net rate of increasing capital per worker is

\[ \dot{k} = sf(k) - (n + \delta)k \]  \hspace{1cm} (2.3)

\(^2\) If knowledge enters the production function with capital, that is \( Y=F(AK,L) \), the technology progress is capital-augmenting. If knowledge enters in the form \( Y=F(K,AL) \), the technology progress is labour-augmenting.

\(^3\) If we consider a production function with labour augmentation, i.e. \( Y=F(K,AL) \), the break-even investment is \((n+\delta+g)k\), in which \( g \) is the constant growth rate of the technology.
where \( \dot{k} \) is the rate of change of the capital stock per worker, which is determined by two factors. On the supply side, \( sf(k) \) is the actual investment per worker, which relates to the output per worker and the proportion of the output used to invest \((s)\). On the demand side, \((n+\delta)k\) is the break-even investment, which depends on the depreciation of existing capital and the capital required by the increased quantity of labour. The break-even investment is the minimum investment in an economy needed to prevent \( k \) from falling.

Eq. (2.3) shows that there is a steady state for an economy on the condition \( k=k^* \), where \( k^* \) is the capital stock per worker in the equilibrium economy. Note here \( \dot{k}=0 \) when \( k=k^* \).

For example, if \( k \) is initially less than \( k^* \), the actual investment \( sf(k) \) will exceed the break-even investment \((n+\delta)k\). Thus, \( \dot{k} \) is positive and \( k \) increases. In contrast, \( \dot{k} \) is negative when \( k \) exceeds \( k^* \). Thus, \( k \) converges to \( k^* \) regardless of the position where \( k \) starts, and therefore, the economy ultimately reaches equilibrium at \( \dot{k}=0 \).

(3) Exogenous technological progress is the only driver of the economic growth rate

A production function with the labour augmentation \( Y = F(K, AL) \) is based on the following assumptions. If the constant growth rate of labour is \( n \), the constant growth rate of exogenous variable knowledge is \( g \), i.e. \( \frac{\dot{L}}{L} = n, \frac{\dot{A}}{A} = g \), then the growth rate\(^4\) of \( K \) at \( k = k^* \) is \( n + g \). The growth rate of \( Y \) at \( k = k^* \) is also \( n + g \) because \( Y = F(K, AL) \) is the constant scale return\(^5\). The growth rate of output per worker at \( k = k^* \)

\[
\frac{\dot{Y}}{Y} = n + g. 
\]

\(^4\) The growth rate of \( K \) (here \( K = ALk \)) at \( k = k^* \) is

\[
\frac{\dot{K}}{K} = \frac{(ALk) + A\dot{L}k + A\dot{k}L}{ALk} = \frac{\dot{A}L + L\dot{k}}{A} = n + g + 0 = n + g. 
\]

\(^5\) At the point where \( k = k^* \), the growth rate of \( K \) is \( n + g \) and the growth rate of \( AL \) is also \( n + g \). Thus,

\[
\frac{\dot{Y}}{Y} = n + g. 
\]
the output per worker grows at the rate of the exogenous technology growth, \( g \). In other words, the sustained economic growth has a constant rate \( g \), which is determined by the exogenous technology progress. In the Solow model, the effect of this exogenous technology progress is usually explained as the growth of total factor productivity (TFP).

### 2.1.1.2 Neoclassical Growth accounting

In practice, the neoclassical growth accounting is usually used to measure the effect of inputs and TFP on output growth. In this approach, the growth rate of output can be written as the sum of the growth rate of each inputs weighted according to their production elasticities and the growth rate of TFP. Based on Eq. (2.1), two approaches can be used in the neoclassical growth accounting. One is the index number approach and the other is econometric method.

**Index method**

With some conditions including (i) constant returns to scale, (ii) neutral technological progress, (iii) producers are price takers in both output and input markets, (iv) firms maximize profits and (v) factor markets and product markets are perfectly competitive, the elasticity of output with respect to capital is equal to the share of the capital cost in the total output, the elasticity of output with respect to labour is equal to the share of the labour cost in the total output. Both parts are directly observable. Moreover, constant returns to scale imply that the elasticities of the input factors add up to one. In this approach, the residual is the total factor productivity (i.e. TFP) growth, which includes everything that cannot be explained by input changes.

**Econometric method**
This approach estimates the elasticities of inputs by assuming a production function form. Then natural log TFP is simply the estimated sum of the constant and the residual. For example, for a natural log-form Cobb-Douglas production function

\[ \Delta y = \Delta c + a\Delta k + b\Delta l \]  

(2.4)

where \( \Delta k \) is the growth of the log-form capital stock, \( \Delta l \) is the growth of the log-form labour. In this case, \( a \) and \( b \) are the elasticity of capital and labour respectively and \( \Delta c \) is TFP growth. However, TFP growth is a biased estimate in this approach due to the existence of the noise. To solve this problem, Fuentes et al. (2006) assumed that TFP growth was exponential in time trend, with the stochastic function of the growth of the per worker product then given by:

\[ \Delta y_i = \gamma + \alpha\Delta k_i + \varepsilon_i \]  

(2.5)

where \( \gamma \) is the average TFP growth rate, \( \varepsilon_i \) is error term and \( \Delta k_i \) is the growth of the capital per worker.

Growth accounting is a common approach to analyse ICT spillover effects in the previous studies. The relevant empirical works of ICT spillover effects on this approach were reviewed as follows.

**2.1.2 ICT spillover effects based on the neoclassical growth model**

According to the third conclusion that exogenous technological progress is the only driver of the economic growth rate (i.e. the third conclusion of the neoclassical growth model mentioned earlier), the neoclassical growth model predicts no direct relationship between ICT and TFP growth for ICT-using industries. Thus, this model can only capture the effects of ICT on output from the first two channels (i.e. an increase in TFP growth in
the ICT-producing sectors and capital deepening driven by high levels of investment in ICT equipment), but cannot measure the ICT spillover effects. Other conceptual framework is needed to interpret the effect of ICT on TFP growth for ICT-using industries. Some such frameworks like the GPT will be reviewed in the next section.

Before considering such frameworks, it is necessary to describe the failure of the neoclassical growth accounting when this approach is used to study ICT spillover effects. For the purpose of analyzing the impact of ICT on output, Eq. (2.4) can be rewritten as:

\[ \Delta y = \varepsilon_{\text{ICT}} \Delta k_{\text{ICT}} + \varepsilon_{\text{Non-ICT}} \Delta k_{\text{Non-ICT}} + \varepsilon_l \Delta l + \Delta TFP \]  

(2.6)

where \( \Delta y \), \( \Delta k_{\text{ICT}} \), \( \Delta k_{\text{Non-ICT}} \) and \( \Delta l \) are the growth of the natural log-form real value added, ICT-related capital, other forms of capital and labour, respectively, and \( \Delta TFP \) is the true TFP growth. \( \varepsilon \) represents the output elasticity of each input, which equals to the observed factor share of each input for the relevant assumptions mentioned in the previous section. For ICT capital, this means

\[ \varepsilon_{\text{ICT}} = \frac{P_{k,\text{ICT}} K_{\text{ICT}}}{p Y} = \alpha_{\text{ICT}} \]  

(2.7)

where \( \varepsilon_{\text{ICT}} \) is the output elasticity of the ICT capital, \( \alpha_{\text{ICT}} \) is the ICT capital’s factor share, \( P_{k,\text{ICT}} \) is the rental price of ICT capital and \( p \) is the output price.

The neoclassical growth accounting function given by Eq. (2.6) has no direct relationship between TFP growth and the growth of ICT capital because TFP growth is the output growth that is not explained by input growth (Stiroh, K. J., 2002). Thus, output growth of ICT-using sectors associated with ICT investment is attributed to ICT capital deepening rather than TFP growth, which is supported by previous studies. For example, Baily and Gordon (1988) argued that there is no TFP growth from the use of ICT. Thus, the neoclassical growth accounting cannot be directly used to measure ICT spillover effects.
Although no direct relationship between TFP growth and ICT investment for ICT-using industries is predicted by the neoclassical growth accounting, the empirical evidence from the neoclassical growth accounting shows that TFP accelerated in a small set of intensive ICT-using industries in the US in the 1990s. For example, Jorgenson et al. (2005) showed acceleration of TFP in retail trade and a deceleration in wholesale trade in the US in 1990s, Triplett and Bosworth (2004) found acceleration in both. Basu et al. (2003) concluded the TFP accelerated in finance and wholesale trade in the US in the 1990s. Timmer et al. (2005) showed a larger set of such industries than Basu et al. (2003).

This evidence seems to imply a relationship between TFP growth and ICT capital for ICT-using industries, which violates the assumption of the neoclassical growth accounting. Strioh (2002) argued that this failure could reflect production spillovers. In this case, ICT spillover effects lead to the elasticity of ICT capital exceeding ICT capital’s measured input share, i.e. $\varepsilon_{ICT} > \alpha_{ICT}$. If $\varepsilon_{ICT} = \alpha_{ICT} + w$, where $w$ is a wedge between the unobserved elasticity and the observed factor share, the measured TFP growth is given by

$$dTFP_{\text{measured}} = dTFP_{\text{true}} + wd\kappa_{ICT}$$

Eq. (2.8) suggests that the conventionally measured TFP growth will be positively correlated with ICT capital if the elasticity exceeds the factor share for ICT capital. Thus, failures of the neoclassical growth accounting might imply a potential link between ICT capital deepening and measured TFP growth.

This explanation stresses a relationship between current ICT capital and current TFP growth if we relax some assumptions of the traditional neoclassical growth accounting. Although failures of the conventional growth-accounting studies can explain ICT
spillover effects, Strioh (2002) did not provide a reasonable theoretical basis for this explanation. Strioh presented that the linkage between ICT capital and TFP growth may be attributed to embodied technical progress, investment-led organizational change, learning-by-doing, technology-induced capital accumulation and positive feedback effects.

Stiroh’s work is a representative study of ICT spillover effects in the early stage. Other similar studies include Caselli (1999), Greenwood and Yorukoglu (1997), Hobijn and Jovanovic (2001). The main problem of the studies in this stage is that the neoclassical growth accounting studies generally lack a conceptual framework to interpret movements in TFP. Although some studies try to look for the evidence of a “new economy” in which ICT has indirect effects on measured TFP in ICT-using industries, in the absence of clear theoretical guidance, it is not clear that many would know if they had, in fact, found such effect (see Basu et al., 2003). Recently, economists have attempted to use the GPT theory as a conceptual framework to explain ICT spillover (e.g. Basu et al., 2003, 2007). This framework stresses that the current TFP growth is positively correlated to the lagged ICT investment, which is different from the explanation that the current TFP is related to current ICT capital as presented by Eq. (2.8).

2.2 ICT spillover effects based on the GPT model

Recently, economists have attempted to use the GPT theory to explore ICT “spillover” effect (David, 1990; Brynjolfsson & Hitt, 2000; Hall & Trajtenberg, 2004). They argued that ICT could be identified as a general-purpose technology that spurs further innovation over time in a wider range of industries and ultimately boost growth in their TFP. Since the GPT theory predicts that ICT increases productivity of ICT-using industries with a
The notion of General Purpose Technologies (GPTs), first introduced by Bresnahan and Trajtenberg in a conference contribution in 1991, was later published as Bresnahan and Trajtenberg (1995). The notion of the GPT in Bresnahan and Trajtenberg (1995) is as follows.

At any point in time, there are a handful of ‘generic’ or ‘general purpose’ technologies (i.e. GPT’s) characterized by their pervasiveness (i.e. they can be used as inputs in a wide range of downstream sectors), and by their technological dynamism.

About the pervasiveness of GPT, they pointed out that the whole technology system (or technological tree) is structured as two levels in any given period. The top level is the GPT, which provides some generic function that is vital to the functioning of a large segment of existing or potential products and production systems. The bottom level includes a large number of product classes or sectors that make use of those GPT. The sharing of the GPT among an increasing number of application sectors represents a
horizontal spillover, which may reduce the cost of these application sectors (e.g. ICT’s network effect).

Secondly, the characteristic of GPT’s technological dynamism means inherent potential for technical improvements. In other words, the efficiency of the GPT has been improved over time by continuous innovation efforts, which is presented as reductions in the price of the produce or as the qualitative improvements in the components embodied in the GPT. This leads to the lower costs of the application sectors that used the GPT’s as input. Meanwhile, the productivity of R&D in the application sectors increases as a consequence of innovation in the GPT, which reflects the GPT related to innovation complementarities. This characteristic is a vertical spillover of the GPT.

Clearly, ICT can be treated as a GPT since it satisfies these two GPT characteristics (David, 1990; Brynjolfsson & Hitt, 2000; Hall & Trajtenberg, 2004). Firstly, ICT goods as an input are widely used by nearly all industries in the whole economy, which represents the ICT horizontal spillover. This spillover allows TFP growth of ICT-using industries through the channels of network effects. The network effect means that an individual ICT user will obtain productivity gain when more and more other ICT users appear (Strioh, 2002; Vuijlsteke et al., 2007). For example, if more suppliers and customers of an ICT-using industry use the Internet, this industry will improve its productivity due reduced communication cost and increased efficiency.

Secondly, ICT-using industries will make more complementary innovations associated with ICT for the purpose of maximizing the benefit of ICT investments, which shows the ICT vertical spillover (Vuijlsteke et al., 2007). For example, ICT-using firms do not just use computers for the purpose of calculating or word-processing, they also facilitate
reorganization of production systems, thereby creating ‘complementary innovation’ in the form of organizational knowledge. Since creating such complementary innovation needs a long period, ICT-using industries should experience a lag in obtaining the final potential benefits of ICT. This lag time is usually explained as an ICT spillover process.

Accordingly, treating ICT as a GPT gives two expectations. One is that ICT contributes to TFP growth in ICT-using sectors due to ICT horizontal spillover. The other is that the effect of ICT on TFP growth in ICT-using sectors should face a long lag due to ICT vertical spillover process.

2.2.2 ICT spillover effects based on GPT theory

Empirical studies using the GPT theory use two approaches to measure ICT spillover effects. Some try to measure ICT spillover effects within a neoclassical growth framework with an augmented neoclassical growth model (e.g. Basu et al., 2003 and Basu and Fernald, 2007), while others give up the neoclassical growth framework and employ an endogenous growth model (e.g. Vuijlstteke et al., 2007). The empirical results of these two approaches suggest the existence of ICT spillover effects in the US. However, the length of the lag is still a freely set parameter in these empirical studies.

*Augmented neoclassical growth model:*

Basu et al. (2003) and Basu and Fernald (2007) tried to interpret ICT’s general-purpose nature in the spirit of the neoclassical growth framework since they assumed that the GPT arrives exogenously. That is, they argued that ICT as a GPT stems from the technological progress of ICT production and, therefore, this GPT is exogenous for those ICT-using industries. In addition, Basu, et al. (2001) found little role of productivity acceleration for deviations from constant returns and perfect competition, which gives confidence to
using the neoclassical growth framework.

Basu et al. (2003, 2007) used an augmented neoclassical model to measure ICT spillover effects by combining the GPT theory and the traditional neoclassical growth model. The first challenge was to seek a suitable proxy for the “unobserved complementary innovation” predicted by the GPT theory in the world of the neoclassical growth framework. In the augmented neoclassical growth model developed by Basu and Fernald (2007), “unobserved complementary innovation” is interpreted as a form of capital, because many microeconomic, firm-level, and anecdotal studies suggest that ICT-users respond in a neoclassical way as firms respond to faster, more powerful computers and software by reorganizing and accumulating intangible organizational capital. Thus, measured TFP, which omits this intangible organizational investment as output and the service flow from organizational capital as an input, is affected.

In their augmented neoclassical growth model, the value added by ICT-using industries is given by

\[
Q_{it} = Y_{it} + A_{it} = F(Z_t, G(K_{it}^{IT}, C_{it}), K_{it}^{NT}, L_{it}), i = 1, ..., N, t = 1, ..., T \tag{2.9}
\]

where \( Y_{it} \) is measured output and \( A_{it} \) is the intangible organizational investment flow which reflects the time and resource costs of training and creating new business structures, \( Z_t \) is a technology term (i.e. ICT technological progress from ICT-producing sectors) that each industry takes as exogenous; \( K_{it}^{IT} \), \( K_{it}^{NT} \) and \( L_{it} \) are ICT capital stock, non-ICT capital stock and labour; \( C_{it} \) is individually complementary capital accumulated by ICT-using industries which represents business and organizational models or ICT training (i.e. the “unobserved complementary innovation” predicted by the GPT theory).

Since both \( A \) and \( K_{it}^{NT} \) investment goods cost the same to produce, the economic
difference between the two types of capital is that they interact in different ways with ICT capital. In addition, the difference in terms of $Y$ and $A$ is that $Y$ is observable by national accountants but $A$ is not

Since complementary organizational capital, $C_u$, is unobservable, the next step of this augmented model is to seek an observable proxy for $C_u$. In Basu and Fernald (2007) model, observed growth in ICT capital was treated as a reasonable proxy of the growth in complementary capital, which is given by:

$$
\Delta c_i = \Delta k_{i,IT} + \sigma \Delta \ln(P_{k,IT}/P_{k,C}),
$$

(2.10)

where $\Delta c_i$ is the growth in complementary capital and $\Delta k_{i,IT}$ is the growth of observed ICT capital. $P_{k,IT}/P_{k,C}$ is the relative rental rate of ICT capital to complementary capital (see Basu et al., 2007, Eq. (8)). Eq. (2.10) links observed ICT capital growth and the growth of “unobserved complementary innovation” predicted by the GPT theory.

After taking account of the effect of $C_u$ and $A_u$ (Note that $\Delta a_i$ can be obtained through $\Delta c_i$), the correct measured TFP is reported as (Basu et al., 2007, Eq. (10)):

$$
\Delta TFP = F(\tilde{k}_i, \tilde{k}_{i-1}, \Delta z,\ldots)
$$

(2.11)

where $\tilde{k} = s_{K,ICT}\Delta \ln k_{K,ICT}$ for computers, software and communications equipment is a proxy of ICT use. (The interpretations of $s_{K,ICT}$ and $\Delta \ln k_{K,ICT}$ are described later); $\Delta z$ is the growth of the exogenous technology. In this model, purposeful innovations are lumped into $C$ with the assumption that all purposeful innovations are closely linked to ICT and all other ‘exogenous’ increases in technology, including the component of

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6 Basu, et al. (2001) found a noticeable role for traditional adjustment costs associated with ICT investment, which encouraged to use the observed growth in ICT capital as a proxy for $C_u$. 
organizational change that spills over as an externality from the sector of origin (e.g. ICT’s network effect), are captured by \( Z \).

This augmented model based on the GPT theory described by Eq. (2.11) shows that TFP growth (i.e. \( \Delta TFP \)) is correlated to ICT use (i.e. \( \tilde{k} \)) for ICT-using industries due to the existence of complementary capital and related investment flows (i.e. \( C_n \) and \( A_n \)). In addition, this model states that TFP growth depends on both current and lagged \( \tilde{k} \) (i.e. \( \tilde{k}_t, \tilde{k}_{t-1} \)), which differs from the conclusion of Stiroh’s work in the neoclassical growth model as mentioned in the previous section. Stiroh argued that current TFP growth depends on current ICT capital for ICT-using sectors. Finally, this augmented model shows that the bias of the measured TFP is small for ICT-using industries with small shares of ICT capital or slow ICT capital growth rates. In Eq. (2.11), the correct “proxy” for ICT-use (i.e. \( \tilde{k} = s_{K, ICT} \Delta \ln k_{K, ICT} \)) involves the interaction of ICT-intensity (\( s_{K, ICT} \) is the share of computers and software in the gross output) and the growth rate \( \Delta \ln k_{K, ICT} \).

Intuitively, if ICT capital grows quickly but its share is small or the share is large (implying complementary capital is likely important) but the growth of ICT capital is small, then there is probably not much complementary capital to cause wrong measurements.

This understanding of the augmented model based on the GPT theory was then used to understand its empirical results in Basu et al. (2007). Empirical results based on industry-level data in the US for this augmented model suggest the existence of ICT spillover effects in the US. The U.S. industry results show that the acceleration after the mid-1990s was broadly based - located primarily in ICT-using industries rather than ICT.
producing industries. Furthermore, industry TFP acceleration in the 2000s is positively correlated with industry ICT capital growth in the 1990s and negatively correlated with increases in ICT usage in the 2000s. These results provide evidence for the conclusion that measured TFP rises in ICT-using sectors (reflecting either unobserved accumulation of intangible organizational capital, spillover or both), but with a long lag. In addition, this evidence also suggests the conclusion of ICT as a GPT because the two expectations of ICT as a GPT mentioned in section 2.2.1 are satisfied.

Although this augmented model gives evidence to the theory of ICT as a GPT in the US, it is unclear how long the lags are between ICT investment and complementary investments. Thus, the lag length is a free parameter in this model and theory gives little guidance. The lagged $\tilde{k}$ may be last year’s ICT capital accumulation or last decade’s.

*Endogenous growth model:*

Solow (1956, 1957) developed the neoclassical growth model and drew the conclusion that long run economic growth just depends on exogenous technological progress rather than anything else like saving rates. A natural extension was to move this exogenous technological progress into the growth model, i.e. technological progress is determined by variables in the model. Intuitively, technology innovations should come from knowledge accumulation. For the question how knowledge accumulation impacts technical progress, Arrow (1962) concluded that economic growth comes from “learning by doing”\(^7\), which indicated that the knowledge accumulation was from the experience of producing the new capital goods rather than the result of long research effort. However, Romer (1986, 1990) argued that technology is improved by a firm’s investment in

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\(^7\) Arrow (1962) assumed the stock of knowledge ($A$) is a function of total capital stock ($K$). For example $A=K^c$ and, thus, the production function becomes $Y=K^c(AL)^{1-c}=K^c(KL)^{1-c}=K^{c+1}L$. When $c>0$, the economy shows ever-increasing growth rather than converging to a balanced growth path.
research and development (R&D) for a competitive purpose. In his model, R&D investment is in the production function. Although a firm’s investment in knowledge still faces diminishing returns in his model, the aggregate social level return to knowledge can increase. This reflects the knowledge spillover effect. Recent research on endogenous growth theory has highlighted several factors including human capital, innovation, imperfect competition and creative destruction (Lucas, 1998; Romer, 1990; Grossman and Helpman, 1991; Aghion and Howitt, 1992).

International Futures (IFs) is one endogenous growth model to measure ICT spillover effects. As mentioned by Vuijlsteke et.al (2007), IFs covers 164 countries for each of which there are seven models: population, economic, agricultural, energy, socio-political sub-model, international political sub-model, environmental, implicit technology model. For each country the economic model includes six sectors (agriculture, energy, primary materials, manufactures, services, and ICT) and two household types (high-skilled and low-skilled). Each country is modelled through a dynamic general equilibrium seeking-model: meaning that the system is not in equilibrium at each point in time, it rather converges toward equilibrium over time. In this model, ICT can exert its impact on the economy through different channels rather than being modelled as a simple input in a standard production function. IFs model uses a Cobb-Douglas production function with disembodied technology as TFP, capital and labour. Meanwhile, TFP of this model is acted as a stock, which can change endogenously in the models. Vuijlsteke et.al (2007) used this model to suggest that a more rapid rate of adoption and diffusion of ICT in the intensive ICT-using sector lead to a different but rising trend-cycle profile of the TFP growth rate.
Comparing to the idea of Basu et al. (2007) that ICT capital needs to cooperate with the complementary organizational co-invention with a type capital for contribution of TFP growth for ICT-using sectors, Vuijlsteke et.al (2007) argued that ICT can influence TFP growth of ICT-using sectors by more channels due to the existence of seven elements in their model. Thus, the mismatches between these elements may cause the different potential level of adopting ICT in the model of Vuijlsteke et.al (2007), but different level in both ICT investment and the complementary organizational capital leads to various ICT spillover effect in Basu et al. (2003, 2007).

2.3 ICT spillover effects and the efficiency

Beside the GPT theory, efficiency is also linked to the relationship between ICT and productivity of ICT-using industries under the frontier framework. Many management information system (MIS) studies have followed this direction (Shao and Lin, 2001, 2002; Lin, 2009). These studies highlight the value of ICT in a business production process, which reflects the contribution of ICT to organizational performance indicated by efficiency. Thus, the efficiency instead of TFP becomes a performance measure of ICT value in these studies. However, these studies show that the question of whether the ICT capital is positively correlated to productive efficiency of an industry/country depends on the specifications of the stochastic production model.\textsuperscript{8}

Compared with the GPT theory, efficiency studies emphasize that ICT capital immediately contributes to the organizational performance of ICT-using industries as indicated by efficiency, while the GPT studies stress that ICT improves TFP of ICT-using industries.

\textsuperscript{8} The stochastic production frontier is widely believed to be theoretically and empirically better for measuring production efficiency than deterministic approaches like DEA (Schmidt, 1985; Shao and Lin, 2002; Lin 2009).
industries with a lag due to the accumulation of the unobserved complementary organizational capital indicated by lagged ICT capital growth. In other words, unlike in the GPT theory, the frontier framework does not predict any lag between ICT investment and efficiency change.

2.3.1 Efficiency

2.3.1.1 Introduction of efficiency

Efficiency is associated with the production frontier, which is introduced here. Production theory focuses on the transformation process in which an industry/firm utilizes different resources as inputs such as capital and labour and produces tangible goods or intangible services as outputs. The non-frontier production such as the production function in the traditional neoclassical growth model assumes that all industries/firms are fully effective and, thus, the production ignores efficiency term, which is clearly unreasonable. In contrast, the production frontier specifies the ideal output level with maximum output realizable from a given combination of inputs. Conceptually, a production frontier characterizes the minimum input bundles required to produce various outputs, or the maximum output producible with various input bundles, and a given technology (Kumbhakar and Lovell 2000).

The definition of efficiency is then given based on this frontier. The difference between the ideal and actual output levels is deemed the productivity inefficiency (Lin 2009). The producer who operates on the production frontier is technically efficient, while the producer that is below the production frontier is technically inefficient (Kumbhakar and Lovell 2000).
The simplified figure shown in Fig 2.1 illustrates the concepts of a typical production frontier $f$ with inputs $k$ and $l$ and output $y$. According to the production frontier, over time, the inefficient firm can catch up to the frontier by becoming less inefficient (efficiency change). However, the frontier itself also could shift up over time due to technological progress (technical change). In Fig 2.1, the curves in period 1 ($t=1$) and in period 2 ($t=2$) are the frontiers for period 1 and period 2. The distance between them is $TP$ due to the technology progress from period 1 to period 2. Suppose the inefficient firm stay at point A at period 1. The inefficiency is the distance between point A and the frontier of period 1, which is label as ‘$TE_A$’ in Fig 2.1. During period 2, this firm improves its efficiency to point B and the distance between point B and the frontier of period 2 is the inefficiency in period 2 labelled as ‘$TE_B$’ in Fig 2.1. The change of efficiency between the two periods (i.e. $TEC$) is the vertical distance between ‘$TE_A$’ and ‘$TE_B$’.
2.3.1.2 Efficiency and ICT spillover effects

Two factors related to ICT spillover effects are ICT investment and TFP growth of ICT-using industries. Efficiency is linked to these two factors; thus, it can possibly be used to investigate ICT spillover effects in a country or an industry. First, efficiency is a component of TFP and, thus, it is one important factor in deciding a firm’s TFP.

Technical efficiency and TFP are two related but different concepts in production theory. Shao and Lin (2001: 449) described these two concepts as follows.

‘With the production technology as given, technical efficiency pertains to getting the most out of a set of input resources. Productivity, on the other hand, refers to the effective usage of overall resources, without making any assumption for the production technology. Therefore, an essential relationship exists between these two constructs: productivity growth is the net effect of the change in technical efficiency and the shift in the production frontier.’

This could be also presented as:

\[
\text{Productivity growth} = \text{technical efficiency change} \times \text{technological change} \quad (2.12)
\]

Secondly, technical efficiency is only related to the potential utilization of input resources. A firm with full production efficiency means that it produces the ideal output for a given input bundle and technology. In other words, this firm can obtain all the potential contribution of the input due to a better organizational structure (Shao and Lin 2002). For example, for an ICT-using firm, an organizational structure based on decentralized decision-making may be a better organizational structure to obtain all the potential contribution of ICT capital (Caroli and Van Reenen 2001).

Thus, efficiency as a part of TFP can be used to reflect the potential utilization of input resource. According to the definition of ICT spillover effects that ICT capital can contribute to TFP growth through the increase in the efficiency in ICT-using sectors that
successfully adopted ICT, most previous studies try to attribute efficiency change of ICT-using industries to the adopt of ICT for the purpose of seeking evidence of ICT spillover effect (Shao and Lin, 2001, 2002; Lin, 2009). Thus, these existing studies predicted a positive contribution of ICT capital to the efficiency change for ICT-using industries and attempted to find the relevant evidence. However, there are mixed results for this prediction. Some studies have attributed this to unsuitable measurements of efficiency. For example, Lin (2009) argued that using a linear frontier with both/either the Cobb-Douglas and/or the Translog specification(s) alone might lead to misleading conclusions in terms of the contributions of ICT investment to productive efficiency. 

2.3.2 Measurement of the efficiency

2.3.2.1 Production frontiers

The two main economic approaches used to estimate the production frontier are deterministic production frontiers and stochastic production frontiers. The former does not consider random errors and treats the difference between ideal and actual output levels as the technical inefficiency. Hence, statistical noise is absorbed into the inefficiency, which leads to measured inefficiencies biased from what their actual value. The stochastic production frontier takes into account both the technical inefficiency and random errors and, therefore, avoids the problem of the deterministic production frontier. Thus, Schmidt (1986), Shao and Lin (2001), (2002) and Lin (2009) claimed that the stochastic production frontier is a better approach for measuring technical efficiency than

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9 Traditional SF models assuming various functional forms may yield mixed findings for the relationship between ICT investment and the efficiency of ICT users. Lin (2009) carried out the estimations and comparisons among four SF models with various model specifications, i.e. the generalized Cobb-Douglas (CD), Box-Cox (BC), Box-Tidwell (BT) and Translog (TL), details of which can be found in Appendix A in Lin (2009). The results show that estimates based on the BC and BT frontiers indicate that the presence of IT capital as an additional input to the production process does not result in a higher efficiency, while the use of IT capital along with ordinary capital and labour does lead to efficiency gains in models with CD and TL specifications.
the deterministic production frontier.

2.3.2.2 Stochastic production frontiers

The stochastic frontier (SF) model first developed by Aigner et al. (1977) and Meeusen and van den Broeck (1977) accommodates both a symmetric distributed error and a one-sided inefficiency random component. The statistical noise reflects the random operating environment faced by producers, while the one-sided random error captures different types of inefficiencies like organizational failure. The noise is generally a normal distribution with zero mean, while the inefficiency can have various distributions. Thus, the distribution form of the inefficiency and the functional form need to be assumed in advance before estimating the stochastic frontier.

Many forms of distributions have been employed for the inefficiency error in previous studies. For example, Aigner et al. (1977) suggested half-normal distributions and exponential distributions, Stevenson (1980) used a truncated normal distribution and Greene (1990) proposed a Gamma distribution. However, the form of the distribution seems to not have a big influence on the inefficiency estimates. For example, Greene (1990) found similar efficiency rankings for firms using the half-normal, truncated normal, exponential and Gamma distributions.

In addition, many function forms for the frontier have been used in existing studies. For example, linear, log linear (Cobb-Douglas) and constant elasticity of substitution (CES) production frontiers were employed in the early stage. However, they have been criticized for some inherent restrictions on the production function. For example, although the Cobb-Douglas production function satisfies the basic characteristic of quasi-
concavity and monotonicity, it imposes a fixed return to scale and unitary elasticity of substitution on the production structure. Then, based on the quadratic Box-Cox model developed by Appelbaum (1979) and Berndt and Khaled (1979), the generalized Leontief, normalized quadratic and squared-root quadratic production frontiers were used in the previous studies. However these all maintain a quasi-homotheticity of the underlying technology. The translog functional form has been widely applied recently since it does not include such restriction. The translog function in complete form can be reported as:

\[ \ln y_{it} = \alpha_0 + \sum_j \alpha_j \ln x_{jit} + \frac{1}{2} \sum_j \sum_l \beta_{jl} \ln x_{jit} \ln x_{jit} + v_{it} - u_{it} \]

where \( y_{it} \) is the observed output of firm \( i \) at time \( t \), and \( x \) variables are inputs. \( j \) and \( l \) are the indexes of the inputs for firms. \( u_{it} \) and \( v_{it} \) are efficiency and noise. For a case with three inputs, non-ICT capital (NK), ICT capital (IK) and labour (L), the above translog function can be written as:

\[ \ln y_{it} = \alpha_0 + \alpha_L \ln L_{it} + \alpha_{NK} \ln NK_{it} + \alpha_{IK} \ln IK_{it} \\
+ \frac{1}{2} \beta_{LL} (\ln L_{it})^2 + \frac{1}{2} \beta_{NNK} (\ln NK_{it})^2 + \frac{1}{2} \beta_{IKK} (\ln IK_{it})^2 \\
+ \beta_{LNN} (\ln L_{it})(\ln NK_{it}) + \beta_{LIK} (\ln L_{it})(\ln IK_{it}) + \beta_{NKK} (\ln NK_{it})(\ln NK_{it}) \\
+ v_{it} - u_{it} \]

See also Christensen, Jorgenson, and Lau (1971), Griliches and Ringstad (1971) and Koop et al. (1999).

Besides research on the distribution form of the inefficiency and the functional form, the original SF model was extended to obtain a producer-specific inefficiency (Jondrow et al. 1982) in a cross-sectional model. Furthermore, the SF model was also applied to panel data to obtain consistent estimates of the technical inefficiency and to relax distributional assumptions on the inefficiency component (Koop, 2003). Then, the panel extension of SF models generates an important issue of how to model the (unobservable)
heterogeneity in the inefficiency and the (observable) heterogeneity in the frontier parameters in a panel setting.

In the first stage, modelling the (unobservable) heterogeneity in the inefficiency focuses on how to handle the possibility of a time variation in the inefficiency in a model using panel data. Three models are commonly employed to resolve this issue. The first uses a time-invariant technical inefficiency with panel data, i.e. $u_{it} = u_i$ for all $t = 1,\ldots,T$, where $u_i$ is the inefficiency. The second assumes an independent and identically distributed inefficiency, namely $u_{it} = u$ for all $t = 1,\ldots,T$. The third assumes that the technical inefficiency varies deterministically over time, i.e. $u_{it} = f(t) \times u_i$ where $f(t)$ is some deterministic function of time. For example, Battese and Coelli (1992) used $f(t) = \exp(\eta(t - T))$ where $\eta$ is an unknown parameter with its sign determining whether the function is non-increasing or non-decreasing. Pitt and Lee (1981), Battese and Coelli (1988), Schmidt and Sickles (1984) and Cornwell et al. (1990) attempted other functions. Recently, heteroscedastic inefficiency terms have been highlighted (e.g. Hadri, 1999 and Hadri et al., 2003). Tsionas (2006) proposed a dynamic inefficiency model based on the foundations of existing SF models, in which past values of inefficiency determine the current values of inefficiency. This issue will be discussed in Chapter 4.

Further extension of the panel SF models concerns the frontier parameters– observable heterogeneity issue. Most previous studies stay in the familiar constant parameter linear world where $f(x_{it}, b_{it}) = x_{it} b$, where $b$ is a parameter. One direction of this extension is to assume that parameters are time-invariant but allow for cross-sectional heterogeneity through random coefficients (e.g. $f(x_{it}, b_{it}) = \mu_i b_j$) (Tsionas, 2002), while another is to assume that the parameters are constant cross-sectional but vary over time due to
technical progress (e.g. \( f(x_{it}, b_i) = x_{it} b_i \)) (Koop et al. 2000; Han, et al., 2005). Recently, some have extended the panel SF models to a non-parametric frontier model (Hajargasht, 2004), details of which are given in Chapter 4.

With these SF models, the next question is how to make estimates with these models. When the estimating efficiency and/or productivity is chosen from an array of methodologies, several sources of variation should be considered: factor price heterogeneity, which in the model can be interpreted as optimisation errors or high frequency productivity shocks, noise, and differences in production technologies. There is a wide range of approaches, including OLS, ML, GMM, and Bayesian. OLS and ML are the more traditional approaches, the GMM-SYS estimator provides the robust productivity level and growth estimates of the parametric models with many measurement errors or some technological heterogeneity. But accurate estimates from these approaches depend on the sample size, particularly in the panel data case. The Bayesian approach is the best for small samples. The main challenge of ICT studies is the limited data. Thus, the Bayesian inference on the SF model is used in this study of ICT spillover effects\(^{10}\).

2.3.3 ICT spillover effects on the efficiency

ICT spillover effects mean that ICT investments positively correlate to TFP growth of ICT-using industries due to organizational adjustments. When the efficiency change is

\(^{10}\) In general, if the number of observations is less than 30, it is regarded as a small sample (Kreft 1996, Bock and Sergeant 2002). In my case showed later, the panel data for both the US and UK studies include 24 industries observed over a time period of 21 years. Hence, if a linear CD function is used for each year, in total there are eight parameters to be estimated (the parameters at the basic level include the constant, and the coefficients of non-ICT capital, ICT capital and labour. Likewise, the parameters at the higher level include the constant, the coefficients of the persistent level of inefficiency, skilled labour and software). It is clear that the number of observations is small in comparison with the number of parameters to be estimated. This is also the case for the non-parametric approach investigated in this thesis where the entire functions (the returns of inputs) are to be estimated. My case used three non-parametric functions and thus estimate returns of input directly rather than parameters. There are only 24 observations (i.e. 24 industries data) for each input in each year (e.g. ICT capital in each year) in my case.
used as a proxy of organizational change, the SF models should present a positive relationship between ICT investment and efficiency change according to the hypothesis of ICT spillover. However, previous empirical studies using traditional SF models have shown mixed results (e.g. Lin, 2009). The divergent conclusions can be mainly attributed to the different models employed by these studies due to the inherent restrictions of the SF models including the assumed frontier form and the inefficiency error distribution.

The two models most commonly used in ICT spillover studies are: (i) the two-stage approach that obtains the efficiency in the first stage and relates it to the ICT investment in the second stage (Shao and Lin 2001, 2002, Lee and Guo, 2004), and (ii) the approach that compares the efficiency with IT spending as a production factor and the efficiency without IT spending in the SF (Lin and Shao 2006, Lin 2009). The first approach is based on the (unobservable) heterogeneity in the inefficiency and the second approach is based on the traditional linear SF models with the assumed frontier form. The results of the first approach have been shown a positive relationship between ICT capital and efficiency, while the results of the second approach has shown that this conclusion depends on the SF specification.

The persistent level of inefficiency related to the heterogeneity inefficiency has also been studied. This may contribute to the divergent conclusion because both two models ignore this factor.

Two-stage approach for the heterogeneity in the inefficiency term

The traditional SF models include the implicit assumption that the inefficiency error term
is homoskedastic\textsuperscript{11}. That is, the variance of the inefficiency term is constant across observations. However, this assumption may be unreasonable. For example, some believe that the sources of inefficiency change with the firm size, therefore, the inefficiency error varies with the firm size rather than being constant across firms with different sizes. In the case of the heterogeneity inefficiency term, Caudill et al. (1995) argued that heteroscedasticity in a one-sided inefficiency error resulted in biased parameter estimates for the SF. He found that the heteroscedasticity of inefficiency leads to overestimation of the intercept and underestimation of the slope for the production frontier.

The exogenous effects on heteroscedastic inefficiency have been examined using functions that link the exogenous variables and factors related to the heteroscedastic inefficiency error like the mean or variance. For example, Reifschneider and Stevenson (1991) did an empirical study of firm-specific inefficiencies in the electric utility industry, which incorporated heteroscedasticity into the composite error by allowing the mean of the one-side error to change. In addition, Caudill et al. (1995) incorporated size-related heteroscedasticity into frontier models by building a function relating the variance of the inefficiency error and variables related to firm size. He found that rankings of firms by their inefficiency measures are markedly affected by the correction for heteroscedasticity with the possible reason that larger firms have more “under their control”.

The exogenous effect on the heteroscedastic inefficiency provides a possible measure of the effect of ICT capital as an exogenous variable on efficiency. The two-stage approach based on the traditional linear frontier models is a common approach to measure this effect. The existing literature suggests that ICT investment is positively related to

\textsuperscript{11} Heteroskedastic noise may exist. However, the heteroskedastic noise does not generate serious problems in the panel data case. Caudill et al. (1995) argued that OLS estimation yields a mean regression that is not affected by symmetric dispersions around the estimate.
efficiency increases. For example, Shao and Lin (2001) in their first stage used two stochastic production frontiers with the Cobb-Douglas function and the translog function to measure technical efficiency scores for firms and then in their second stage related the efficiency scores to the corresponding IT investments of the firms. Significant statistical evidence suggests that IT exerts a significant favourable impact on technical efficiency, which in turn contributes to productivity in an organization.

However, there are two serious econometric problems associated with this two-stage formulation as pointed out by Coelli et al. (1998) and Kumbhakar and Lovell (2000). First, the assumption of non-correlation between environmental/exogenous variables and production frontier input variables must stand. If they are correlated, the estimated inefficiency in the first stage is biased, through the calculation involving those biased parameter estimates representing the production frontier due to the omission of the relevant environmental/exogenous variables in the first stage estimation. The second problem lies in the contradiction in the treatment of inefficiencies. Inefficiencies are modelled as a function of exogenous/environmental variables in the second stage which is conflicting to the presumed assumption of an independently identically distribution in the first stage. Based on the Monte Carlo experiment, Wang and Schmidt (2002) stated that this bias could be very severe. Furthermore, Koop et al. (1999) argued that this two-stage approach is unsatisfactory, as it would lead to substantial (generated) regressor problems due to the use of estimated means as input data in the second stage, and ignorance of large uncertainty in the point estimates.

*Traditional SF model with the assumptions of frontier form*

Later studies, instead of using a two-stage approach, measured the effect of ICT on efficiency by comparing the efficiency with and without IT spending as a production
factor. Other frontier models beyond the Cobb-Douglas function and the Translog function have also been tested on industry-level and country-level data.

These extensions have lead to mixed results for the positive relationship between ICT investment and efficiency for ICT users. For example, Lin and Shao (2006) estimated the IT business value in terms of the impact of IT on technical efficiency, based on the constant elasticity of substitution (CES) stochastic production frontier model, at three level in firms, in an industry and in an industrial sector. They found that the relationship between technical efficiency and IT investment is not robust with respect to the specifications of the CES production frontier, which contradicts the conclusions reached by Shao and Lin (2001, 2002) and Brynjolfsson and Hitt (1996, 2000). Lin (2009) tested four other linear SF models for the same purpose, the generalized Cobb-Douglas (CD), Box-Cox (BC), Box-Tidwell (BT) and Translog (TL), whose specifications can be found in appendix A in Lin (2009). Lin compared the estimate of the CD, BC, BT and TL models to show that the estimates based on the BC and BT frontiers indicate that the presence of IT capital as an additional input to the production process does not result in a higher efficiency, while the CD and TL frontiers use of IT capital along with ordinary capital and ordinary labour leads to efficiency gains. Thus, Lin concluded that the CD or TL models alone might lead to misleading conclusions concerning the contributions of IT investment to productivity efficiency. However, the CD and TL functions have been widely used in applied research (e.g. Dewan and Kraemer, 2000, Lee and Guo 2005.) with excellent results.

Persistent level of inefficiency due to heterogeneity in the inefficiency

In addition to the exogenous effects on the heteroscedastic inefficiency used in the two-
stage approach, other studies have looked at the endogenous effects on the heteroscedastic inefficiency. Unlike the exogenous effects, the endogenous effects emphasise the effect of inefficiency itself. For example, Tsionas (2006) use an AR(1) structure of inefficiency to capture the effect of the past inefficiency on the current inefficiency. In this case, the heteroscedastic inefficiency may be due to different historical developments of inefficiency for different firms. The economic explanation of this endogenous effect is that firms need to pay adjustment costs when they improve their efficiency. A high adjustment cost may then lead to a persistent level of inefficiency for a firm. Different persistent level for inefficiency across firms becomes another possible reason of heteroscedastic inefficiency, in addition to the exogenous environmental variables.

The endogenous effect is important when investigating the relationship between ICT investment and efficiency change. Based on the consensus that the ICT spillover effect is related to the adjustment cost, the endogenous effect due to the adjustment cost should be considered when modelling. For example, Brynjolfsson, et al. (2002) argued that there was a $9 complementary cost for every $1 of ICT investment and they explained this complementary cost as the organizational adjustment cost. However, there are few empirical studies on this issue. The main reason may be that the estimates are too complex. Although Tsionas (2006) built a dynamic SF model with included endogenous factors, this model has not been employed to analysis ICT spillover effects.

2.4 ICT spillover effects in the US and the UK

This section reviews the empirical studies of ICT spillover effects in the US and the UK using the three analysis frameworks including the neoclassical growth model, the GPT
theory and frontier framework with efficiency. The effect of ICT spillover in the UK is uncertain in all three frameworks, while both the neoclassical growth model and the GPT theory confirm ICT spillover effects in the US.

2.4.1 Evidence based on the neoclassical growth models

Empirical studies using the neoclassical growth accounting have shown that TFP acceleration appeared in a small set of ICT-using industries with heavy ICT investment in the US in 1990s, in particular in the service sector of the economy, such as finance and distributive trade sectors (Basu et al., 2003; Triplett and Bosworth, 2004; Jorgenson et al. 2005; Inklaar et al., 2005), although the neoclassical growth accounting cannot determine whether this TFP growth is due to ICT capital growth. In contrast, studies in the UK have yielded mixed findings. Schreyer (2000) reported little evidence for a link between ICT capital and TFP growth of ICT-using industries for the G7 countries. Inklaar et al. (2005) found that TFP accelerated in a small set of service industries such as retail and finance in the US, but not in the UK, France, Germany or the Netherlands in the 1990s. Jorgenson, et al. (2005), Triplett and Bosworth (2004), Vijselaar and Albers (2002), O’Mahony and Vecchi (2005) and Jalava and Pohjola (2007) found similar conclusions. However, Van Ark et al. (2008) concluded that TFP accelerated growth in the retail trade sector of the UK and in wholesale trade in the Netherlands after 1995. These observations imply that growth accounting studies do not accurately capture the presence of ICT spillover effects in the UK.

2.4.2 Evidence based on the GPT models

The GPT theory argues that ICT capital contributes to TFP growth with a lag due to
organizational adjustments. By using lagged ICT capital growth as a proxy for unobservable complementary organizational co-invention, Basu et al. (2007) obtained evidence of ICT spillover effects in the US in the 1990s, with the current TFP growth positively correlated with lagged ICT investment, but negatively correlated with current ICT investment. However, Basu et al. (2003) found that TFP growth in the UK did not appear correlated with lagged ICT capital (stock growth, which suggests that lagged ICT capital growth is a poor proxy for unobserved UK complementary capital accumulation or that no ICT spillover effect exists in the UK, unlike the US. They argued that the UK has ICT spillover effects when using ICT investment growth as a proxy for unmeasured investment in complementary capital. However, they found that ICT could not explain the observed TFP slowdown because the net effect of ICT on TFP was positive in the 1990s. Indeed, these findings do not provide strong evidence for ICT spillover effects in the UK. Other studies have reported similar conclusions for the UK (Daveri, 2002, van de Wiel et al. 2005, Hagemann 2008). Thus, further studies should be done for this issue.

2.4.3 Evidence based on the efficiency

Lin (2009) tested the contribution of ICT investment to efficiency for the UK and the US. His results reported in TABLE 2.1 provide mixed findings. For example, the result of the Cobb-Douglas SF model showed that the IT investments lead to an increase in production efficiency for both the UK and the US over the period of 1993-1999. The level of IT investment effect on efficiency in the US was moderate (8.51%), less than Germany’s (15.76%) but larger than the UK’s (5.37%). However, evidence from the Box-Cox model

---

12 Basu et al. (2003) used ICT investment growth rather than ICT capital stock growth to act as a proxy of the complementary capital in the UK. They argued that the proxy of lagged ICT capital stock ($k_{i,t-1}$) used in the US was suitable for the case of ICT spillover effects with very long lags, but the proxy of lagged ICT investment ($I_{i,t-1}$) should work well for the ICT spillover effects with a short lag (e.g. the reorganization was contemporaneous with the ICT investment).
indicated that only the US has a positive effect of ICT investment on efficiency, which is consistent with the neoclassical growth model’s common conclusion that ICT spillover effects appear in the US but not in the UK (Susiluoto 2003, Inklaar, et al., 2007). Thus, we should seek a more robust analysis framework of ICT spillover effects.

<table>
<thead>
<tr>
<th>Country</th>
<th>CD %</th>
<th>BC %</th>
<th>BT %</th>
<th>TL %</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>I 8.51</td>
<td>I 3.73</td>
<td>R -10.05</td>
<td>R -1.12</td>
</tr>
<tr>
<td>FR</td>
<td>I 0.08</td>
<td>R -9.54</td>
<td>R -21.51</td>
<td>R -0.44</td>
</tr>
<tr>
<td>GM</td>
<td>I 15.76</td>
<td>R -43.90</td>
<td>R -35.91</td>
<td>R -1.44</td>
</tr>
<tr>
<td>UK</td>
<td>I 5.37</td>
<td>R -17.54</td>
<td>R -18.14</td>
<td>I 1.85</td>
</tr>
</tbody>
</table>

Note: (i) CD, BC, BT and TL represent the generalized Cobb-Douglas, Box-Cox, Box-Tidwell and translog SF models. (ii) ‘R’ means ‘reduce’ and ‘I’ is ‘increase’.

2.5 Concluding remarks

The literature review shows that the study of ICT spillover effects is still in an early stage. On the theoretical side, although the GPT theory provides a theoretical explanation for the ICT spillover, the GPT theory gives little guidance for the measurement of the lag time and the proxy for the intangible complementary reorganization. Chapter 5 will build a new framework of measuring ICT spillover effects to bridge these two gaps by combining the GPT theory and efficiency.

Concerning methodologies, as another concept related to the ICT spillover effects, efficiency seems not to be positively related to ICT investment due to the methodology problems that include the assumed frontier form, the biased estimate of the two-stage approach due to exogenous heteroscedastic inefficiency terms, ignoring of endogenous heteroscedastic inefficiency terms and limited data. Chapter 4 will present a dynamic semi-parametric SF model and a dynamic nonparametric SF model to deal with the first three problems with Bayesian inferences of these models employed to obtain an accurate estimate from small data sets.
Empirical studies have shown the US seems to have ICT spillover while the UK is not clear. The reason of the mixed findings for the UK is weakness of the three frameworks used by the previous studies. For example, the existing framework on the GPT theory cannot obtain a compatible result for the US and the UK due to little guidance of the length of lag and the proxy of co-invention, and the existing frontier framework is similar due to the assumption of frontier forms and the weakness of two-stage approach. Chapter 6 will investigate ICT spillover effects of the US and the UK and examine the reason for the different ICT spillover effects between the UK and the US through the more robust framework built in Chapter 5.
Chapter 3 Data descriptions

This thesis focuses on ICT spillover effects in the US and the UK using industry-level data for ICT-using industries. Data is taken from the EU KLEMS Project Database (http://www.euklems.net), which provides the panel industry-level information including value added, real gross fixed capital stock of ICT and non-ICT assets, total hours worked by persons engaged, high skilled labour share and software share.

The panel data includes annual time-series data for 24 ICT-using industries from the UK and the US during 1980-2000. The reason for using just the data during 1980-2000 is that computers played a limited role in ICT-usage before 1980. Jorgenson and Stiroh (1995) found ICT equipment of the US provided just 0.56% of total flow of real services from non-residential producers of durable equipment stock in 1979, but this rose to 13.8% by 1990 (Cohen 2006). Since the objective of this study is ICT spillover effects in ICT-using industries, ignoring data from before 1980 should have no effect on the conclusions. The 24 industries and their SIC classification numbers are listed in TABLE 3.1.

The raw data series are value added, real gross fixed capital stocks of ICT and non-ICT assets, total hours worked by persons engaged, highly skilled labour share (share in total hours) and software share (share in ICT assets). The variables used in this thesis are output, ICT capital stock, non-ICT capital stock, labour, highly skilled labour share and software share, which are measured as follows.
### 3.1 Output

The raw series of the value added is measured in the local currency unit at the current prices for the UK. For the purpose of comparison between the UK and the US, these raw values needed to be changed to prices for the same base year in dollars for both countries. First, because the raw data for the real gross fixed capital stock of ICT, non-ICT and software assets use 1995 prices, the value added data for both countries was converted into constant prices based on the year 1995 using the industry GDP deflators from the GGDC database (www.GGDC.net). Then, the value added data for the UK at the constant 1995 prices in the local currency unit are transferred to value added data for the

<table>
<thead>
<tr>
<th>Industries</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>MARKET SERVICES, EXCLUDING POST AND TELECOMMUNICATIONS</td>
<td>MSERV</td>
</tr>
<tr>
<td>Sale and repair of motor vehicles and motorcycles; retail sale of fuel</td>
<td>50</td>
</tr>
<tr>
<td>Wholesale trade and commission trade</td>
<td>51</td>
</tr>
<tr>
<td>Retail trade, except of motor vehicles and motorcycles</td>
<td>52</td>
</tr>
<tr>
<td>Transport and storage</td>
<td>60t63</td>
</tr>
<tr>
<td>Financial intermediation</td>
<td>J</td>
</tr>
<tr>
<td>Renting of machinery and equipment and other business activities</td>
<td>71t74</td>
</tr>
<tr>
<td>Hotels and restaurants</td>
<td>H</td>
</tr>
<tr>
<td>Other community, social and personal services</td>
<td>O</td>
</tr>
<tr>
<td>GOODS PRODUCING, EXCLUDING ELECTRICAL MACHINERY</td>
<td>GOODS</td>
</tr>
<tr>
<td>Food products, beverages and tobacco</td>
<td>15t16</td>
</tr>
<tr>
<td>Textiles, textile products, leather and footwear</td>
<td>17t19</td>
</tr>
<tr>
<td>Manufacturing; recycling</td>
<td>36t37</td>
</tr>
<tr>
<td>Wood and products of wood and cork</td>
<td>20</td>
</tr>
<tr>
<td>Pulp, paper, paper products, printing and publishing</td>
<td>21t22</td>
</tr>
<tr>
<td>Coke, refined petroleum products and nuclear fuel</td>
<td>23</td>
</tr>
<tr>
<td>Chemicals and chemical products</td>
<td>24</td>
</tr>
<tr>
<td>Rubber and plastics products</td>
<td>25</td>
</tr>
<tr>
<td>Other non-metallic mineral products</td>
<td>26</td>
</tr>
<tr>
<td>Basic metals and fabricated metal products</td>
<td>27t28</td>
</tr>
<tr>
<td>Machinery</td>
<td>29</td>
</tr>
<tr>
<td>Transport equipment</td>
<td>34t35</td>
</tr>
<tr>
<td>Mining and quarrying</td>
<td>C</td>
</tr>
<tr>
<td>Electricity, gas and water supplies</td>
<td>E</td>
</tr>
<tr>
<td>Construction</td>
<td>F</td>
</tr>
<tr>
<td>Agriculture, hunting, forestry and fishing</td>
<td>AtB</td>
</tr>
</tbody>
</table>
UK at the 1995 prices in US dollars using the purchasing power parity (PPP) exchange rates from the GGDC database (www.GGDC.net) or the ESDS database (www.esds.ac.uk). The natural log of the value added at the 1995 price in US dollars for both countries is then used as the output variable in this thesis.

### 3.2 Labour

The EU KLEMS Project Database provides various labour measurement data. The simplest measurement of labour variable is a head count of employees. However, this reflects neither changes in the average hours worked per employee nor changes in the labour quality. A first refinement to this measure is to use number of hours actually worked which bears a closer relationship to the amount of productive services provided by workers than simple head counts. Thus, the natural log of the raw series of the total hours worked by persons engaged is used as the labour measurement in this thesis.

### 3.3 ICT capital stock and non-ICT capital stock

The EU KLEMS Project Database provides raw data for the real gross fixed capital stock for ICT and non-ICT assets for the US and the UK. However, they are measured in local currency units at constant 1995 prices for both countries. For the cross-country comparison, the local currency units in these two data series in the UK are converted to US dollars based on the purchasing power parity (PPP) exchange rates from the GGDC database (www.GGDC.net) or the ESDS database (www.esds.ac.uk). The natural log of the real gross fixed capital stock for the ICT and non-ICT assets in 1995 US dollars for both countries is then used as the ICT capital stock and non-ICT capital stock variables in this thesis.
3.4 Skilled labour share and software share

The EU KLEMS Project Database provides raw data for the hours worked by highly skilled persons (as share of total hours). Thus, the natural log of the hours worked by highly skilled persons (as share of total hours) is used as the highly skilled labour share variable in this thesis. However, the definitions of higher education differ between the US and the UK. TABLE 3.2 given a short overview of the definitions used for highly skilled in the US and the UK. The software share is simply the proportion of the software capital stock relative to the ICT capital stock for the UK and the US.

<table>
<thead>
<tr>
<th>Country</th>
<th>Definition of highly skilled labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>College graduate and above</td>
</tr>
<tr>
<td>UK</td>
<td>University degree</td>
</tr>
</tbody>
</table>
Appendix

A summary of descriptive statistics for the two samples in this thesis

Table 3.3 displays a summary of descriptive statistics for the two samples used in Chapter 6 of this thesis. These two samples consist of a balanced panel of annually time-series data for 24 ICT-using industries during 1980-2000 for the US and UK respectively.

<table>
<thead>
<tr>
<th>Variables</th>
<th>US All 24 industries</th>
<th>UK All 24 industries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output (millions of $)</td>
<td>160,442</td>
<td>26,995</td>
</tr>
<tr>
<td></td>
<td>133,522</td>
<td>18,860</td>
</tr>
<tr>
<td></td>
<td>22,014</td>
<td>2,990</td>
</tr>
<tr>
<td></td>
<td>573,976</td>
<td>85,266</td>
</tr>
<tr>
<td>Labour(millions hours)</td>
<td>5,838</td>
<td>1,157</td>
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<tr>
<td></td>
<td>5,147</td>
<td>928</td>
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<tr>
<td></td>
<td>347</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>21,866</td>
<td>4,037</td>
</tr>
<tr>
<td>Non-ICT capital (millions of $)</td>
<td>229,740</td>
<td>40,244</td>
</tr>
<tr>
<td></td>
<td>204,606</td>
<td>34,418</td>
</tr>
<tr>
<td></td>
<td>30,775</td>
<td>4,478</td>
</tr>
<tr>
<td></td>
<td>814,969</td>
<td>120,565</td>
</tr>
<tr>
<td>ICT capital (millions of $)</td>
<td>20,849</td>
<td>2,733</td>
</tr>
<tr>
<td></td>
<td>40,047</td>
<td>4,158</td>
</tr>
<tr>
<td></td>
<td>553</td>
<td>60</td>
</tr>
<tr>
<td></td>
<td>183,041</td>
<td>17,360</td>
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<tr>
<td>Skilled labour(%)</td>
<td>19</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>22</td>
</tr>
<tr>
<td>Software share(%)</td>
<td>37</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>17</td>
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<td>4</td>
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<tr>
<td>Sample counts</td>
<td>504</td>
<td>504</td>
</tr>
</tbody>
</table>

General method to measure the aggregate capital stock

In general, the aggregate capital stock can be written as:

$$K_t = \sum_{s=0}^{n} (1-d_s)I_{t-s}$$

where \(d_s\) is the decay factor for an investment \(s\) years old and \(I_{t-s}\) is the real gross investment of vintage \(s\). On the condition that the rate of decay is constant over time (i.e. geometric rate of decay, \(d\)), the aggregate capital stock is \(K_t = I_t + (1-d)K_{t-1}\). Oulton and Srinivasan (2003) argued that the geometric rate of decay, \(d\), is equal to the depreciation rate, \(\delta_t\). Based on the definition that depreciation measures the difference between the price of a new and a one-year old asset at time \(t\), \(\delta_t\) is given as:

$$\delta_t = \frac{p_{t,0} - p_{t,j+1}}{p_{t,j}}$$
where $p_{t,j}$ is the price of an asset of age $j$ at time $t$ and the numerator $p_{t,j} - p_{t,j+1}$ is the capital gains/losses ($f_{t,j}$) that is the change in the price of a new asset between periods $t-l$ and $t$. If the depreciation rate of the asset is constant, $\delta = (p_j - p_{j+1}) / p_j$.

Considering depreciation and capital gains/losses, the rental price ($\rho_{t,j}$) for the capital services of a capital input of age $j$ at time $t$ is:

$$\rho_{t,j} = r_j \times p_{t-1,j} + \delta \times p_{t,j} - f_{t,j}$$

where $r_j$ is the nominal rate of return during period $t$. The rental price is the price a company is willing to pay to rent the capital goods instead of buying them. A profit-maximizing firm will hire capital goods at the point when the rental price is equal to the marginal revenue of the product of the capital good (Draca et al., 2006).

**Depreciation Rate of ICT Assets**

ICT assets experience dramatic price and quality changes over short times. Thus, nominal ICT asset flows must be carefully converted into real flows for ICT assets since ICT assets have special depreciation rates than non-ICT assets. The depreciation rate is the rate of value decay of different vintages of an investment. In the US, Bureau of Economic Analysis (BEA) and Bureau of Labour Statistics (BLS) developed quality-adjusted prices for computer equipment (Grimm et al., 2002). Since the 1990s, the deflators used by BEA for computers have been derived from the producer price index (PPI) and the import price index. BLS uses hedonic techniques to adjust the quality for ICT assets (Holdway 2001).

---

13 In BEA, before the 1999 revision, the estimated depreciation rates for computers were cohort and specific according to the study by Oliner (1993), but after 1999, a new revision of the National Income and Product Accounts (NIPA) based on Lane (1999) gave a new depreciation rate of 0.3119 for PCs only. Since 2003, the depreciation rate has been adjusted again from 0.3119 to 0.34 based on Doms et al. (2004).
EU statistics for ICT assets lag behind the US since EU members use different country specific industry level data sets for ICT investment flows. Van Ark et al., (2002) combined official statistics on ICT flows at the industry level for EU members using the US methodology (e.g. depreciation patterns and hedonic prices). First, they use the price index harmonization method (Schreyer 2002) to make country specific data deflators for EU members. Then they used a US deflator to adjust for the inflation in each country to obtain the investment flow. Finally, they calculated the capital stocks by PIM using the US depreciation rates from Jorgenson and Stiroh (2000). This database is used in this study as relative data for the UK.

**Perpetual Inventory Method (PIM)**

Both the BEA and BLS use real investment figures to develop capital stocks using the Perpetual Inventory Method (PIM), for assets and industry. The standard perpetual inventory method (PIM) is

\[
K_{i,t+1} = K_{i,t} + I_{i,t+1} - \delta * K_{i,t}
\]

where \(K_{i,t}\) is the capital stock of sector \(i\) at period \(t\), \(I_{i,t}\) is the capital formation/investment and \(\delta\) is the depreciation rate. If only investment data is available, following Pham et al. (2002) and Young (1995), the raw capital stock series are initialised by assuming that the investment growth rate in the first five years of the national accounts investment series is representative of the growth of investment prior to the beginning of the series. That is,

\[
K_{i,0} = \sum_{t=0}^{\infty} I_{i,t-1} (1 - \delta)^t = \sum_{t=0}^{\infty} I_{i,0} (1 + g_i)^{t-1} (1 - \delta)^t = I_{i,0} / (g_i + \delta)
\]

\(^{14}\) BLS uses a hyperbolic age-efficiency function (see http://www.bls.gov/web/mprcaptl.htm).
where $I_{t,0}$ is the first year of investment data, $g_t$ is the average growth in the first five years of the investment series and $\delta$ is the depreciation rate. Past studies have shown that given positive rates of depreciation and a sufficiently long investment series, the PIM is insensitive to the level of capital used to initialise the series\textsuperscript{15}.

\textsuperscript{15} This assumption is valid if there is a sufficient burn-in period during which the capital stock is calculated but not used in the estimating equation.
Chapter 4 Methodology: Bayesian inference for the stochastic frontier models

As mentioned in the literature review in Chapter 2, the traditional stochastic frontier analysis is based on a specific parametric SF form and suffers from bias due to endogenous and exogenous heteroscedastic inefficiency terms. Apart from that, it is difficult to draw statistical inference using traditional stochastic frontier analyses when samples are small. Thus, this chapter develops new econometric methodologies for stochastic frontier analyses. Two two-level stochastic frontier models are developed and their Bayesian inference will be investigated.

First, a semi-parametric dynamic stochastic frontier model will be considered. It has a time-variant non-parametric frontier at the basic level and a time-invariant linear equation for the technical inefficiency at the higher level so that both frontiers and technical inefficiencies can be modelled simultaneously in one stage. Then in the second model, the time-invariant linear function at the higher level for the technical inefficiency will be further extended to a time-variant non-parametric form. These two models can be seen as a generalization from the existing specific models, that is, they can be applied to more general scenarios. In particular, each of the two models collapses to a one level non-parametric time-variant frontier model investigated in Hajargasht (2004) if the higher level is ignored, and boils down to the two-level model investigated in Tsionas (2006) if a linear time-invariant structure is used at the basic level.
Both models can address these problems in the following ways: (a) the non-parametric frontier at the basic level relaxes the assumption of the frontier form; (b) the higher level takes into account the endogenous heteroscedastic inefficiency terms through an AR(1) structure of inefficiency; and (c) the two-level one-stage model avoids the econometric bias of the two-stage approach due to exogenous heteroscedastic inefficiency terms.

Overall, statistical inference for stochastic frontiers may be classified into two different categories: the frequentists’ approach using the maximum likelihood method and the method of moments, and the Bayesian approach. In recent years, considerable attention has been paid to the Bayesian approach. The Bayesian approach incorporates prior knowledge into the analysis. Consequently, statistical analysis can be carried out accurately on the basis of the posterior distributions of the parameters of interest, so that it does not rely on asymptotic theory and the assumption of large samples. This is particularly useful when the data sample is small. This chapter highlights the Bayesian inference for the stochastic frontier analysis.

The remainder of this chapter is organized as follows: we first summarize the existing stochastic models using Bayesian approaches and then develops the two new models.

4.1 Summary of existing approaches

This section briefly summarizes some important stochastic frontier models involving Bayesian analysis, including model specifications, derivations of posterior distributions and computational issues.
4.1.1 Bayesian analysis and MCMC

First, a brief introduction to Bayesian inference and MCMC is given. This forms the basis for the rest of the discussions on Bayesian analysis for stochastic frontier analysis in this chapter.

4.1.1.1 Introduction to the Bayesian approach


The key point in the Bayesian approach is the concept of prior distributions and posterior distributions. For a vector of unobservable parameters $\theta = (\theta_1, ..., \theta_k)'$ that seeks to explain data, $y$, $\theta$ is treated as random with a specified prior distribution, $p(\theta)$, in the Bayesian approach, whereas $\theta$ is an unknown constant vector in the classical method.

Then, a model for the data is assumed: $P(y | \theta)$ for the given $\theta$. This difference leads to the classical method focusing on the information in the data (sample) while the Bayesian approach considers the information contained in both the data and the prior distribution, where the prior distribution contains non-data information available about $\theta$. The posterior distribution $p(\theta | y)$ is derived via Bayes’ law as follows:

$$p(\theta | y) \propto p(\theta)P(y | \theta)$$

where ‘$\propto$’ stands for proportionality. Bayes’ law shows how the subjective probability distribution of $\theta$ is modified by the observation of $y$.

Bayes’ law shows that the posterior distribution combines information from the prior distribution and the current data, thus it summarizes all we know about $\theta$ after (i.e.
posterior to) seeing the data. Bayesian inference for a model is based on the posterior distribution only (Koop, 2003).

4.1.1.2 Gibbs-sampler

In Bayesian analysis, statistical inference for a $k$-dimensional vector of the parameters of interest $\theta = (\theta_1, \ldots, \theta_k)'$ is drawn using the posterior distribution $p(\theta | y)$. The marginal posterior density of $\theta_i$ is in principle defined by integrating the joint posterior density of $\theta$ with respect to all elements of $\theta$ other than $\theta_i$. However, this integration generally cannot be computed analytically.

Computational issues for Bayesian analysis are usually addressed via a simulation tool referred to as a Markov chain Monte Carlo (MCMC), including the importance sampling technique, Gibbs-sampler, Metropolis-Hastings algorithm, and data augmentation technique. For instance, van Den Broeck, et al. (1994) use importance sampling to reveal features of the posterior distribution of $p(\theta | y)$. Koop et al. (2000) use the Gibbs sampler to draw information from the posterior distribution.

Gibbs sampler can be used to carry out posterior simulations if we know the full conditional posterior distributions, which define a posterior for each block conditional on all the other blocks. Specifically, if $\theta$ is partitioned into several blocks as $\theta = (\theta_1', \ldots, \theta_B')'$ where $\theta_{(j)}$ is a scalar or vector $(j = 1, 2, \ldots, B)$, the Gibbs sampler involves the following steps:
Step 1. Randomly choose a starting value, $\theta^{(0)}$. Under regularity conditions (Koop, 2003), initial draws do not influence that the Gibbs sample converges to a sequence of draws from $p(\theta | y)$ when enough replications are taken and the first $s_0$ burn-in replications are dropped. The remaining $s_1$ draws are retained for statistical inference such as calculating $E\{g(\theta) | y\}$.

Step 2. Take a random draw, $\theta^{(s)}_{(1)}$, from $p(\theta^{(1)} | y, \theta^{(s-1)}_{(2)}, \theta^{(s-1)}_{(3)}, ..., \theta^{(s-1)}_{(B)})$ and then take a random draw, $\theta^{(s)}_{(2)}$, from $p(\theta^{(2)} | y, \theta^{(s)}_{(1)}, \theta^{(s-1)}_{(3)}, ..., \theta^{(s-1)}_{(B)})$. Continue this process until $\theta^{(s)}_{(B)}$ is taken from $p(\theta^{(B)} | y, \theta^{(s)}_{(1)}, \theta^{(s)}_{(2)}, ..., \theta^{(s)}_{(B-1)})$.

Step 3. For the obtained set of draws, $\theta^{(s)}$, drop the first $s_0$ to eliminate the effect of the initialization on $\theta^{(0)}$. By the weak law of large numbers, if $\tilde{g}_n = \frac{1}{s_1 + s_0} \sum_{s=s_0+1}^{s_1+s_0} g(\theta^{(s)})$, then $\tilde{g}_n$ converges to $E\{g(\theta) | y\}$ as $s_1$ goes to infinity.

Thus, the key step in the Gibbs sampler is to derive the full conditional posterior distributions.

4.1.1.3 Metropolis-Hastings algorithm

In the Gibbs sampler it is assumed that we can take random draws from the full conditional posterior distributions. When random draws from a conditional posterior distribution are difficult to take; the Metropolis-Hastings algorithm can be used instead to simulate the posterior.
The Metropolis-Hastings algorithm uses a candidate generating density for the draws rather than drawing directly from the posterior distribution. Note that not all candidate draws are accepted, but they are accepted based on the Metropolis-Hastings algorithm. The Metropolis-Hastings algorithm involves the following steps:

Step 1. Choose a starting value, \( \theta^{(0)} \).

Step 2. Take a candidate draw, \( \theta^* \) from the candidate generating density, \( q(\theta^{(s-1)}, \theta) \).

Step 3. Calculate an acceptance probability, \( \alpha(\theta^{(s-1)}, \theta^*) \), where the acceptance probability is given by 
\[
\alpha(\theta^{(s-1)}, \theta^*) = \text{min}\left[ \frac{p(\theta = \theta^* | y)q(\theta^*; \theta = \theta^{(s-1)})}{p(\theta = \theta^* | y)q(\theta^*; \theta = \theta^{(s-1)})}, 1 \right], \quad p(\theta = \theta^* | y)
\]
is the posterior density evaluated at the point \( \theta = \theta^* \), and \( q(\theta^*; \theta = \theta^{(s-1)}) \) is the density of random variable \( \theta \) evaluated at the point \( \theta = \theta^{(s-1)} \).

Step 4. Set \( \theta^{(s)} = \theta^* \) with probability \( \alpha(\theta^{(s-1)}, \theta^*) \) and set \( \theta^{(s)} = \theta^{(s-1)} \) with probability \( 1 - \alpha(\theta^{(s-1)}, \theta^*) \).

Step 5. Repeat steps 2-4 \( s_0 + s_1 \) times.

Step 6. Drop the first \( s_0 \) to eliminate the effect of the initialization on \( \theta^{(0)} \). Then take the average of the \( s_1 \) draws \( g(\theta^{(s_0 + 1)}), \ldots, g(\theta^{(s_0 + s_1)}) \). The weak law of large numbers
can be invoked to show that, if \( \hat{g}_{n} = \frac{1}{s_{1}} \sum_{s_{1} = n_{1}}^{\infty} g(\theta^{(s_{1})}) \), then \( \hat{g}_{n} \) converges to \( E\{g(\theta) \mid y\} \) as \( s_{1} \) goes to infinity.

A combination of the Gibbs sampler and the Metropolis-Hastings algorithm can be used in practice. This is useful when some blocks of the parameters are easy to draw using the Gibbs sampler but others are not so that the Metropolis-Hastings algorithm must be used.

### 4.1.1.4 Differences between the Bayesian and Maximum-likelihood approaches

Both the Bayesian approach and classical econometric methods like MLE can be used to estimate frontiers and inefficiencies. There are many advantages to using Bayesian analysis compared with classical econometric methods, such as its flexibility in dealing with missing data. In particular, for the purpose of this thesis, the major advantages of the Bayesian approach include: (a) it takes into account uncertainties in the parameters of interest by assigning probability distributions; (b) it enables exact small sample results because the Bayesian approach does not depend on the assumption of large samples with a normal approximation; rather, it uses the posterior simulation to obtain a posterior estimate and (c) in stochastic frontier analysis, it derives a full posterior distribution of any firm-specific efficiency or function of efficiencies. We can thus compute the exact standard deviations and make inference about whether the efficiency of one firm is different from that of other firms. Based on these advantages, the Bayesian approach was used in this thesis.
4.1.2 Simple one-level linear model

To facilitate the exposition, a simple time-invariant linear stochastic frontier model is introduced as a starting point (e.g. Shao et al. 2001; Koop et al. 1999). In recent years this simple stochastic frontier model has been extended to various forms, such as the non-linear frontier models in Hajargasht (2004) and Kumbhakar et al., (2007) and the two two-level models in Koop et al., (2000) and Tsionas (2006). The model developed by Koop et al. (2000) used binary dummies to explain variation in the technical inefficiency. The model developed by Tsionas (2006) is more general combining the simple stochastic frontier model with a linear time-invariant structure for the technical inefficiency.

Consider the following production frontier for panel data:

\[ y_{it} = x_{it}' \beta + v_{it} - u_{it}, \quad i = 1,.., N; t = 1,.., T. \]  

(4.1)

Equation (4.1) can also be rewritten in matrix form as

\[ \mathbf{y} = \mathbf{x} \beta + \mathbf{v} - \mathbf{u}, \]  

(4.2)

where \( \mathbf{y} = (y_{1}',..., y_{T}')' \) is an \( NT \) vector for the log output, \( \mathbf{x} = (x_{1}',..., x_{T}')' \) is an \( NT \times k \) matrix with \( x_{it}' = (x_{it1}',..., x_{itk}')' \) and \( x_{it} = (x_{it1},..., x_{itk})' \) is a \( k \) vector of regressors, \( \beta = (\beta_1, ..., \beta_k)' \) is a \( k \) vector of parameters, \( \mathbf{v} = (v_{1}',..., v_{T}') \) is an \( NT \) vector of two-sided noise and \( \mathbf{u} = (u_{1}',..., u_{T}') \) is an \( NT \) vector of non-negative technical inefficiency errors. \( y_{i} = (y_{i1}',..., y_{iN}')' \) and \( v_{i} \) and \( u_{i} \) are defined similarly.

It is commonly assumed that the technical inefficiency errors and the noise follow an exponential distribution\(^{16}\) with the parameter \( \lambda^{-1} \) and a normal distribution with mean

\(^{16}\) Koop et al. (1999) pointed out the exponential distribution was a fairly flexible one-sided distribution with the ability to capture a wide variety of inefficiency behaviours.
zero and variance $\sigma^2$ (i.e. $v_\mu \sim N(0,\sigma^2)$), respectively. Further, $v_\mu$ and $u_\mu$ are assumed to be mutually independent as well as independent of $x_\mu$. Hence, the joint distribution of the technical inefficiency errors is given by

$$\prod_{i=1}^{T} \prod_{t=1}^{N} f_G(u_{it} \mid 1, \lambda^{-1}).$$

In this simple model, $\beta$ is a constant vector and, thus, the frontier is unchanged over time, which reflects the time-invariant feature of this linear model, and it is a common frontier for all $N$ industries. One of the major drawbacks of this simple model is that it does not fully use the information in the panel data\(^{17}\).

To perform Bayesian inference with this linear time-invariant model, the prior is usually specified as

$$p(\sigma^-2)p(\lambda^{-1})p(\beta) \quad (4.3)$$

where $p(\beta) \propto 1$. $p(\sigma^-2)$ and $p(\lambda^{-1})$ are gamma distributions $f_G(\cdot \mid a, b)$ with the hyper-parameters $a$ and $b$. For example, Koop et al. (1999) assume $p(\sigma^-2) = f_G(\sigma^-2 \mid 0, 10^{-6})$ to make the prior on $\sigma^-2$ very close to the ‘usual’ non-informative prior. In addition, in their model, $p(\lambda^{-1}) = f_G(\lambda^{-1} \mid 1, -\ln(\tau^*))$ with $\tau^* = 0.75$, which is the median of the inefficiency distribution.

Applying Bayes’s rule to combine the prior (4.3) with the model (4.1) and the distribution of the technical inefficiency errors, the posterior distribution is proportional to

\(^{17}\) The assumption of independence for the frontiers over time in a time invariant model does not fully exploit the panel data since panel data contains repeated observations on each producer. As shown later, data used in this thesis include observations from 24 industries over 21 years. It is clear that the frontier may shift over time and thus a simple time-invariant linear stochastic frontier model cannot fully exploit the panel data.
\[
f_N^k(y \mid x\beta - u, \sigma^2 I_{NT})p(\sigma^{-2})p(\lambda^{-1}) \prod_{i=1}^{T} \prod_{j=1}^{N} f_G(u_{ij} \mid 1, \lambda^{-1}), \tag{4.4}
\]

where \(f_N^k(. \mid a, b)\) is the density function of the \(k\)-variate normal distribution with mean \(a\) and covariance matrix \(b\).

The conditional posterior distributions for the Gibbs sampler can be obtained from the posterior distribution listed in Eq. (4.4) (Koop et al., 1999):

\[
p(\beta \mid y, x, u, \sigma^{-2}, \lambda^{-1}) = f_N^k(\beta \mid (x'x)^{-1}x'(y + u), \sigma^2(x'x)^{-1}),
\]

\[
p(\sigma^{-2} \mid y, x, \beta, u, \lambda^{-1}) = f_G(\sigma^{-2} \mid \frac{n_o + TN}{2}, \frac{1}{2}[a_o + (y - x\beta + u)'(y - x\beta + u)]),
\]

\[
p(u \mid y, x, \beta, \sigma^{-2}, \lambda^{-1}) = f_{TN}^{NT}(u \mid x\beta - y - \sigma^{-2} / \lambda, \sigma^2 I_{TN}) \prod_{i=1}^{T} \prod_{j=1}^{N} I(u_{ij} \geq 0), \tag{4.5}
\]

\[
p(\lambda^{-1} \mid y, x, \beta, u, \sigma^{-2}) = f_G(\lambda^{-1} \mid 1 + NT, -\ln(\tau^*) + \sum_{i=1}^{T} \sum_{j=1}^{N} u_{ij}).
\]

The vectors \(\beta\) and \(u\) can be estimated using the Gibbs sampling technique on the data drawn from the posterior distribution.

Note, however, this is a one-level model that does not consider which factors affect the technical inefficiency. Furthermore, even at this basic level for frontiers, this model imposes an assumption of time-invariant linear frontiers. As will be shown in the next subsection, Hajargasht (2004) and Kumbhakar et al. (2007) addressed the linear assumption issue for frontiers with time-invariant non-parametric stochastic frontier models, although they were still one-level models.
4.1.3 One-level non-parametric frontier model

Hajargasht (2004) used the spline technique to specify the time-invariant linear stochastic frontier model in equation (4.1) in non-parametric form. He used the following frontier model with a univariate input \( x_{it} \):

\[
y_{it} = f(x_{it}) - u_i + v_{it}, \quad i = 1, \ldots, N; t = 1, \ldots, T,
\]

(4.6)

where \( y_{it}, x_{it}, u_i, \) and \( v_{it} \) are defined as in the previous section. \( f(.) \) is a time-invariant frontier with an unknown functional form.

Hajargasht (2004) assumed that the noise had a normal distribution, \( v_{it} \sim N(0, \sigma^2) \) and that the technical inefficiency errors had an exponential distribution with the parameter \( \lambda^{-1} \). Furthermore, \( v_{it} \) and \( u_i \) were assumed to be mutually independent as well as independent of \( x_{it} \).

The major difference between this model and the linear time-invariant model in Eq. (4.1) lies in its non-parametric frontier. Clearly, Eq. (4.6) will collapse to Eq. (4.1) when \( f(.) \) is a linear function. Technically, this feature of the non-parametric frontier makes the Bayesian inference much more complicated because an appropriate prior for the non-parametric \( f(.) \) has to be specified.

Hajargasht (2004) uses cubic splines to fit the model in Eq. (4.6), thus, a brief introduction to the natural cubic spline method is provided.
4.1.3.1 An introduction to the spline technique

Economists often need to estimate a production function/frontier based on a set of observations in the output-input space. A common approach is to assume one particular parametric production functional form (e.g. Cobb-Douglas and Translog) and to estimate the parameters of inputs. These traditional production functions impose some certain structure on the production functional form and on the noise term, which may be unreasonable, and thus the commonly used approaches may be not suitable for some specific cases.

First, in the traditional approach it is assumed that the production function yields the maximum output. However, Lovell et al (1997) argued that efficiency plays a role in the production function and only the frontier with fully efficiency presents the maximum output. Tsionas (2006) argued that the high adjust cost of reorganization may lead to an inefficiency firm for a long time, which has addressed the issue why we need to consider the role of inefficiency when we estimate the production function.

Secondly, these traditional production functions assume that all marginal productivities are positive and thus a production function is monotonic. Many of them further assume the production function is concave. However, these assumptions could be wrong sometimes. Chambers (1988) gave some obvious exceptions by the example that the production of potatoes on a parcel of ground that is sufficiently small so that the addition of more units of labour leads to such overcrowding that the works produce less. We also note that the GPT story has argued that the input associated with GPT may lead to a negative return of output initially due to the accumulation of co-invention.
In this thesis, the estimated production function associated with ICT is related to the GPT story and efficiency. This leads to use a non-parametric approach to estimate the frontier rather than the commonly used parametric production functions with the monotonicity and concavity properties. Fig 4.1 illustrates the estimates obtained using a parametric frontier and a non-parametric frontier. We consider some observations in an output-input space (y-x space) in Fig 4.1. The traditional parametric frontier (e.g. the logarithm Cobb-Douglas function form) with the assumption of monotonicity can be estimated by OLS, which is represented by the thin straight line in Fig 4.1. It is clear that it cannot reflect the negative elasticity of the input. In contrast, the non-parametric frontier represented by the smooth curve in the figure can capture the negative elasticity of the input.

There are two commonly used splines techniques that can be employed to nonparametrically estimate the frontier. One is the B-splines and the other is the smoothing splines. The B-splines approach uses the basis function to estimate the non-
parametric function where the knots have been pre-selected. The knot selection procedure is usually complicated and may lead to overestimation (Smith and Kohn 1996). On the other hand, the smoothing splines approach does not have to pre-select the knots. It takes into account of both the goodness-of-fit to the data and the roughness of the fitted curve, which is important for the measurement of negative return of inputs, such as ICT capital in this study.

The idea of the smoothing splines is illustrated as follows. If we simply focus on the goodness-of-fit to the data, the best estimate of the frontier is to connect every two successive observations by a straight line, which is displayed by the dash line in the above Fig 4.1. Obviously, the dash line cannot reflect the true relationship between the input and the output because the random noise is not taken into account. When both two criterions (i.e. the goodness-of-fit to the data and the roughness of the fitted curve) are taken into consideration via a weight parameter $\lambda$, the smoothing splines technique leads to the curved real line in the figure, showing a negative relationship between the input and the output.

Smoothing splines technique is not without limitations. It uses all the observations as knots and thus leads to a higher computationally cost. In addition, the generalization to the multivariate function estimation (except for the additive and partially linear model) is not straightforward when the number of observations is large (Hajargasht, 2004).

The smoothing splines technique is particularly appropriate for this study. It does not pre-impose a structure on the functional form of the production functions as the traditional models do, thus is able to capture the true relationship of the initial negative
returns of ICT capital when ICT is viewed as a GPT. In addition, its limitations are not a serious issue since a small sample and the additive model are used in this study. As a specific smoothing spline technique, the natural cubic spline technique is briefly introduced as follows.

Assume $n$ distinct knots $x_i$ on the interval $[a, b]$ such that $a < x_1 < \ldots < x_n < b$. Use a polynomial $S_i(x)$ with degree $k$ for each knot to seek a spline function $S(x)$ with degree $n$. That is

$$S(x) = \begin{cases} S_1(x) & x \in [x_1, x_2] \\ S_2(x) & x \in [x_2, x_3] \\ \vdots \\ S_n(x) & x \in [x_{n-1}, x_n] \end{cases}$$

where each $S_i(x)$ is a polynomial of degree $k$.

There are different types of spline interpolation, including linear spline interpolation ($k = 1$), quadratic spline interpolation ($k = 2$) and cubic spline interpolation ($k = 3$). Linear spline interpolation connects the successive data points by straight lines. Quadratic spline interpolation uses a derivative approximation with the curves passing through all the data points, which is susceptible to severe oscillations when the signal quickly changes since this method uses just two points to calculate the next iteration’s curve. Thus, cubic spline interpolation is common selected in practice.

$S_i(x)$ on the interval $[a, b]$ is a cubic spline if two conditions are fulfilled. First, $S_i(x)$ is a cubic polynomial on each of the intervals $[x_1, x_2]$, $[x_2, x_3]$, $\ldots$, $[x_{n-1}, x_n]$. Secondly, the polynomial pieces fit together at the points $x_i$ in such a way that $S_i(x)$ itself and its first
and second derivatives are continuous at each \( x_i \) and, thus, on the interval \([a,b]\). In other words,

\[
S'_{i-1}(x_i) = S_i(x_i), i = 1, \ldots, n - 1,
\]

\[
S''_{i-1}(x_i) = S''_i(x_i), i = 1, \ldots, n - 1,
\]

and \( S''_{i-1}(x_i) = S''_i(x_i), i = 1, \ldots, n - 1 \).

Since a third degree polynomial (i.e. \( k = 3 \)) needs four conditions to choose the curve, the \( n \) cubic polynomials \( S_i(x) \) are determined by \( 4n \) conditions. The interpolating property \( (S(x_i) = y_i, i = 1, \ldots, n) \) gives \( n \) conditions. The condition on the interior data points gives \( n - 2 \) data points each, summing to \( 4n - 2 \) conditions. Then, only two other conditions are required. The natural cubic spline (NCS) uses \( S''(x_i) = S''(x_n) = 0 \), which gives two other conditions for a total of \( 4n \) conditions. Thus, each \( S_i(x) \) can be determined to obtain the spline function \( S(x) \).

An NCS can be specified by its value and second derivative at each of the knots \( x_i \).

Define \( \alpha_i = S(x_i) \) and \( \beta_i = S''(x_i) \) for \( i = 1, \ldots, n \), and let \( \mathbf{a} = (\alpha_1, \ldots, \alpha_n)' \) and \( \mathbf{b} = (\beta_1, \ldots, \beta_{n-1})' \). Note here for NCS, \( \beta_1 = \beta_n = 0 \). The necessary and sufficient condition for these two vectors to represent a natural cubic spline on the given knot sequence depends on two band matrices \( \mathbf{Q} \) and \( \mathbf{R} \). The matrix \( \mathbf{Q} \) is an \( n \times (n - 2) \) matrix with elements \( q_{ij} \) (\( i = 1, \ldots, n \) and \( j = 2, \ldots, n - 1 \)), where

\[
q_{i-1,j} = \frac{1}{x_j - x_{j-1}}, q_{jj} = -\frac{1}{x_j - x_{j-1}} - \frac{1}{x_{j+1} - x_j}, q_{j+1,j} = \frac{1}{x_{j+1} - x_j},
\]

and \( q_{ij} = 0 \) for \( |i - j| \geq 2 \). \( \mathbf{R} \) is an \( (n - 2) \times (n - 2) \) matrix with elements \( r_{ij} \) (\( i = 2, \ldots, n - 1 \) and \( j = 2, \ldots, n - 1 \)), where
\[ r_{i,i} = \frac{1}{3} (x_i - x_{i-1}) + \frac{1}{3} (x_{i+1} - x_i) \text{ for } i = 2, \ldots, n-1, \]
\[ r_{i,i+1} = \frac{1}{6} (x_{i+1} - x_i) \text{ for } i = 2, \ldots, n-2, \]
and \[ r_{ij} = 0 \text{ for } |i - j| \geq 2. \]

According to Green and Silverman (1994, Theorem 2.1, p.13), the vectors \( \alpha \) and \( \beta \) specify a natural cubic spline \( S(x) \) if and only if the condition \( Q^T \alpha = R \beta \) is satisfied.

Based on the condition of \( Q^T \alpha = R \beta \), the roughness of the NCS is given by
\[
\int_a^b S''(x) = \beta^T R \beta = \alpha^T K \alpha
\]
where \( K = QR^{-1} Q^T \) for the condition that \( R \) is strictly positive-definite in the sense that \( |r_{ii}| > \sum_{j \neq i} |r_{ij}| \) and \( R \) is strictly diagonally dominant.

To obtain a smooth curve from observations by spline interpolation, we need to take into account both the goodness-of-fit to the data and the roughness. Thus the following penalized sum of the square is minimized:
\[
J(S) = \sum_{i=1}^{n} (y_i - S(x_i))^2 + \lambda \int [S''(x_i)]^2 dx
\]
The criterion function \( J(S) \) ensures the estimator of curve \( S(x) \) is determined both by its goodness-of-fit to the data as given by the mean square error \( \sum_{i=1}^{n} (y_i - S(x_i))^2 \) and its roughness \( \lambda \int [S''(x_i)]^2 dx \). The smoothing parameter \( \lambda \) is the “rate of exchange” between residual and roughness (i.e. the weight of the two components) with a larger \( \lambda \) giving a smoother curve. Using matrix algebra, \( S(x) \) for the minimum \( J(S) \) is given by
\[ S(x) = F(\lambda)y \]

where \( F(\lambda) = (I + \lambda K)^{-1} \) and \( y = (y_1, \ldots, y_n)' \).

### 4.1.3.2 Bayesian inference

Following Hastie and Tibshirani (1990), Hajargasht (2004) used the following prior for the non-parametric frontier for equation (4.6):

\[
 f \sim N(0, K^{-1} \tau^2), \tag{4.7}
\]

where \( K \) is an \( NT \times NT \) matrix associated with the cubic splines as defined in the previous subsection. \( \tau^2 \) is a smoothing parameter.

In addition, Hajargasht (2004) also specified the following prior distributions:

\[
 p(\sigma^{-2}) = f_G(a_0, b_0),
\]

\[
 p(\tau^{-2}) = f_G(a_1, b_1), \tag{4.8}
\]

\[
 p(\lambda^{-1}) = f_G(a, b),
\]

where \( a_0, b_0, a_1, b_1, a \) and \( b \) are the hyper-parameters of the gamma distributions. Hajargasht (2004) argued that Fernandez et al. (1997) noticed it was not easy to define the posterior distribution if a non-informative prior for \( \sigma^{-2} \) and \( \lambda^{-1} \) were used. Thus, he used gamma priors instead.

Bayes’s rule is used to combine the model in Eq. (4.6) with these priors to give the posterior distribution (Hajargasht, 2004):
\[
p(f, \sigma^{-2}, \tau^{-2}, \mathbf{u}, \lambda^{-1} | \mathbf{y}, \mathbf{x}) \propto \sigma^{-2(2N_T + a_n - 1)} \exp\left\{-\frac{2b_0 + \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} + u_t - f(x_{it}))^2}{2\sigma^2}\right\} \times \tau^{-2(N-2)+a_n - 1} \exp\left\{-\frac{2b_1 + \int \mathbf{K} f}{2\tau^2}\right\} \times \lambda^{(N+\sigma-1)} \exp\left\{-\lambda^{-1}(b + \sum_{i=1}^{N} u_i)\right\}
\]

(4.9)

Based on the above posterior distribution, the conditional distributions for the Gibbs sampler are given by (Hajargasht, 2004):

\[
p(f | \mathbf{y}, \mathbf{u}, \sigma^{-2}, \tau^{-2}, \lambda^{-1}) = f_N^T \{\mathbf{S}[\mathbf{y} + \mathbf{u}], \sigma^2\} \quad \text{where} \quad \mathbf{S} = \{I + (\sigma/\tau)\mathbf{K}\}^{-1}.
\]

\[
p(\sigma^{-2} | \mathbf{y}, \mathbf{u}, \tau^{-2}, f, \lambda^{-1}) = f_G \{a_0 + \frac{N^2T}{2}, b_0 + \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} + u_t - f(x_{it}))^2}{2}\},
\]

\[
p(\tau^{-2} | f, \mathbf{u}, \sigma^{-2}, \lambda^{-1}) = f_G \{a_1 + \frac{(N-2)T}{2}, b_1 + \frac{\int \mathbf{K} f}{2}\},
\]

(4.10)

\[
p(u_i | \mathbf{y}, f, \sigma^{-2}, \tau^{-2}, \lambda^{-1}) = f_N \{\sum_{t=1}^{T} (f(x_{it}) - y_{it} - \lambda^{-1} \sigma^2)^T T \lambda^{-1} \sigma^2, T \sigma^2\}, u_i \geq 0
\]

\[
p(\lambda^{-1} | f, \sigma^{-2}, \tau^{-2}, \mathbf{u}) = f_G (n + a, b + \sum_{i=1}^{N} u_i).
\]

Hajargasht (2004) further extended the model in Eq. (4.6) to a finite additive model with multiple inputs: \(y_{it} = \sum_{j=1}^{k} f_j(x_{ij}) - u_t + v_{it},\)

\(i = 1,..,N; j = ICT, non - ICT, labour; t = 1,..,T.\)

The Bayesian inference is drawn in the same way as outlined above.

Kumbhakar et al., (2007) also constructed a non-parametric model using the local kernel technique. The approach for statistical inference, however, was based on the maximum
conditional local likelihood rather than Bayesian analysis. We thus do not discuss this approach in detail.

All these models are time-invariant with only one level. Thus, they fail to explain: (a) the variability in the changes of frontiers and (b) the variability in the technical inefficiency. The next section discusses two-level models developed in the pioneering research of Koop et al., (2000) and Tsionas (2006).

4.1.4 Two-level linear models

This section summarizes the two-level stochastic frontier models developed by Koop et al. (2000) and Tsionas (2006). A two-level model allows us to simultaneously measure the frontier at the basic level and examine the effect of the environmental variables on the technical inefficiency at the higher level. Tsionas (2006) also included autocorrelation of the technical inefficiency using an AR(1) structure at the higher level.

Koop et al. (2000) develop a complicated two-level model based on the simple linear model in Eq. (4.1), by introducing binary dummies to explain the variation in the technical inefficiency. The basic level is used to capture the variability in the frontier. The higher level assumes that some exogenous environmental variables affect the technical inefficiency via the mean of the exponential distribution for the efficiency term:

\[ \lambda_w = \prod_{j=1}^{2} \phi_j^{-w_0}, \]
where \( w_{ij} \) are the environmental variables. Due to the technical difficulties, however, the environmental variables were restricted to be binary dummies only (i.e. 0 or 1) in Koop et al. (2000).

Recently, Tsionas (2006) extended this simple two-level model to a more general structural model. The basic level for the frontier is kept the same as equation (4.1) but at the higher level the log of the technical inefficiency \( u_{it} \) is modelled by regression which includes some environmental variables and an AR(1) structure:

\[
\log u_{it} = z_i' \gamma + \rho \log u_{i,t-1} + \epsilon_{it}, \quad \text{for } t = 2, \ldots, T, \tag{4.11}
\]

\[
\log u_{i1} = z_i' \gamma_1 + \epsilon_{i1}, \quad \text{for } t = 1,
\]

where \( \epsilon_{it} \sim N(0, \omega_{it}^2) \) for \( t = 2, \ldots, T \), and \( \epsilon_{i1} \sim N(0, \omega_{i1}^2) \) for \( t = 1 \). \( z_{it} \) is an \( m \)-vector of environmental variables for \( i = 1, \ldots, N \). \( \gamma \) and \( \gamma_1 \) are \( m \)-vectors of parameters and \( \rho \) reflects the time correlation of the technical inefficiency. As before, \( v_{it} \) and \( u_{it} \) are assumed to be mutually independent as well as independent of \( x_{it} \) and \( z_{it} \).

At the higher level of this model, the systematic part \( z_i' \gamma + \rho \log u_{i,t-1} \) (\( t=2, \ldots, T \)) reflects the expected log-inefficiency source and \( \epsilon_{it} \) captures the ‘unexpected part’. The technical inefficiency is related to both the environmental variables and the lagged technical inefficiency in the ‘expected part’.

Tsionas’ pioneering work (2006) has three important features. First, the model considers the effect of environmental variables on the technical inefficiency by investigating the relationship between the technical inefficiency and the environmental variables. In addition, unlike in the model developed in Koop et al. (2000), there is no restriction in
Tsionas’ model on the types of the environmental variables. They can be binary as well as continuous which allows analysis of variables affecting the technical inefficiency. The second feature is that the model includes autocorrelation of the technical inefficiency via an AR(1) structure. This not only reduces the bias when estimating the technical inefficiency, but also gives the rate of change in the technical inefficiency over time. The third feature is that, from a statistical perspective, Tsionas’ research takes into account technical inefficiencies and the impact of environment variables on technical inefficiencies in a one-stage model, which significantly reduces the biases in the estimation.

The model developed by Tsionas (2006) also has some limitations with a linear frontier structure and the equations at both levels are assumed to be time-invariant. These assumptions will be relaxed in the rest of this chapter.

4.1.5 Discussion

The review of the existing Bayesian approaches for stochastic frontier analysis shows that they have three major shortcomings. First, most models impose a linear structure on the frontiers. Secondly, the frontiers are assumed to be time-invariant. Finally, due to the complexity, only a few models consider a two-level structure to explain how both the frontiers and the technical inefficiency simultaneously change over time. Various researchers have noted these problems and begun to address these issues, including the non-parametric approaches by Hajargasht (2004) and Kumbhakar et al. (2007), and the two-level models by Koop et al. (2000) and Tsionas (2006).
The rest of the chapter develops the two two-level models as (a) a semi-parametric dynamic stochastic frontier model with a time-variant non-parametric frontier at the basic level and a time-invariant linear function for the technical inefficiency at the higher level and (b) a non-parametric dynamic stochastic frontier model having a time-variant non-parametric form at both the higher level and the basic level.

4.2 Two-level dynamic semi-parametric SFA model

In the past two decades, considerable attention has been paid to the problem of how to obtain robust and accurate measurements of frontiers and the technical (in)efficiency in efficiency research.

For measuring frontiers, the traditional SFA approach usually stipulates a linear functional form for the frontier in advance. In addition, this frontier is often assumed to be time-invariant like the one in Eq. (4.1). The former assumption may lead to mixed findings for the frontier estimates while the latter ignores changes in the frontiers over time. To illustrate these issues, consider the curved frontier represented in Fig. 4.2. When a linear structure for the frontier is imposed, three different types of estimates of the frontier may be obtained depending on the measurement noise and/or estimation techniques and thus completely different conclusions. For instance, the estimated frontier 1 has a positive slope, whereas the slope of the estimated frontier 3 is negative. The estimated frontier 2 may not be statistically significant. Clearly none of these estimates reflect the real frontier and thus mixed findings obtained for the same frontier.
FIGURE 4.2 Illustration of several estimated linear frontiers

For time-invariant frontiers, when a frontier is assumed to be unchanging over time, there is no role for technology progress (TP). For instance, Hajargasht (2004) estimated the non-parametric frontier using the spline technique while Kumbhakar et al. (2007) used the kernel technique. They both assumed a time-invariant frontier, which is unrealistic in practice. To accommodate TP, it is necessary to consider a time-variant frontier at the basic level. For instance, Koop et al. (1999) used a linear trend frontier, which assumed a frontier having a time-variant form given by \[ y = x'\beta + v - u \] with \( \beta_t = \beta' + \beta'' t \). The model discussed in the following shall relax the assumption of linear time-invariant frontiers at the basic level.

The measurement of technical inefficiency in panel data is a more complicated problem. For one-level models (i.e. ignoring the effect of environmental variables on inefficiency), there are four commonly used specifications for the technical inefficiency in the literature. The first model is to assume that the measurements of the technical inefficiency are independent and identically distributed. The second model assumes a time-invariant firm-specific technical inefficiency for the panel data, namely \( u_{it} = u_i \) for all \( t = 1, \ldots, T \). An extension of the second model stipulates a firm-specific technical inefficiency which
varies deterministically over time, i.e. $u_\mu = \Phi(t)u_\tau$, where $\Phi(t)$ is a given deterministic function of time. Finally, some researchers have also considered firm-specific time-variant technical inefficiencies with the condition $u_\mu \geq 0$.

The first two models are not suitable choices for panel data. The third approach imposes a constant ratio of $u_\mu / u_\mu$, which may be very restrictive for many applications. The last model is flexible and, thus, will be used in this study. Due to the limitation of a one-level model, however, it is difficult to explore why and how the technical inefficiency changes over time\textsuperscript{18}.

The effect of environmental variables can be investigated with a two-level structure. For instance, Koop et al. (2000) used binary dummies to explain variation in the technical inefficiency. Tsionas (2006) employed a linear time-invariant function of some continuous environmental variables to explain the variability in the technical inefficiency.

As in measuring frontiers, imposing a linear time-invariant functional form of the environmental variables at the higher level for the technical inefficiency may produce a biased estimate. Thus, a non-parametric time-variant function should also be used to examine the effect of environmental variables at the higher level. In addition, following Tsionas (2006), the autocorrelation of the technical inefficiency will be used to examine the effect of lagged technical inefficiencies on the current value in this thesis.

\textsuperscript{18} Although a one-level model can use two-stage approach to measure the effect of environmental variables on technical inefficiency, this approach is criticized due to its econometric problem (Koop et al.1999).
This section presents a two-level semi-parametric dynamic stochastic frontier model as a stepping-stone to the final model. This model has a time-variant non-parametric frontier at the basic level and a time-invariant linear function at the higher level to explain the variability in the technical inefficiency. The linear equation at the higher level for the technical inefficiency will be further extended to a time-variant non-parametric form in the next section.

### 4.2.1 Model

Consider the following production frontier for the panel data:

$$y_{it} = \sum_{j=1}^{k} f_{jt}(x_{jt}) - u_{it} + v_{it}, \quad \text{for } i = 1, \ldots, N; t = 1, \ldots, T,$$

(4.12)

where $y_{it}$ is the log output of industry $i$ in time period $t$, $x_{jt}$ is the $j$th log input of industry $i$ in time $t$, $v_{it}$ is a two-sided measurement error, $u_{it}$ is a non-negative technical inefficiency error, and $f_{jt}(.)$ is a time-variant unknown function of a specific input.

As before, assume that $v_{it}$ and $u_{it}$ are mutually independent as well as independent of $x_{jt}$.

The measurement error is assumed to follow a normal distribution, $v_{it} \sim N(0, \sigma^2)$, for $i = 1, \ldots, N; t = 1, \ldots, T$.

The basic level of the model in Eq. (4.12) assumes a non-linear frontier via an additive model. The additive model structure is not too restrictive because the relationship
between an output and a set of inputs in the productivity analysis is usually assumed to be a multiplicative model. An additive model then follows by a log-transformation.

Following Tsionas (2006), a linear time-invariant model is assumed for the technical inefficiency:

\[
\log u_t^i = z_t' \gamma + \rho \log u_{t-1}^i + \varepsilon_t^i, \text{ for } t = 2, \ldots, T, \tag{4.13}
\]

\[
\log u_1^i = z_1' \gamma_1 + \varepsilon_1^i, \text{ for } t = 1,
\]

where \( \varepsilon_t^i \sim N(0, \omega_t^2) \) for \( t = 2, \ldots, T \) and \( \varepsilon_1^i \sim N(0, \omega_1^2) \) for \( t = 1 \). \( z_t^i \) is an \( m \)-vector of environmental variables for \( i = 1, \ldots, N \). \( \gamma \) and \( \gamma_1 \) are \( m \)-vectors of the parameters and \( \rho \) reflects how the technical inefficiency changes over time. \( z_t' \gamma + \rho \log u_{t-1} \) captures the expected log-inefficiency source and \( \varepsilon_t^i \) captures the ‘unexpected part’.

The basic level of this two-level model uses a non-parametric time-variant frontier; thus, this model should give an estimated frontier closer to reality. The non-parametric structure relaxes the linear assumption on the functional form of the frontiers in the traditional SFA. The time-variant frontiers ensure that we can examine the technology progress. The higher level can estimate the effects of the environmental variables and examine how the level of technical inefficiency changes over time. However, use of a linear time-invariant structure at the higher lever may result in bias.

After specifying the model, its Bayesian inference is investigated. Following Tsionas (2006), first consider the joint distribution \( p(y, u | X, Z, \theta) \), where \( \theta \) is the parameter vector consisting of all unknown parameters in the model, \( y = (y_1', \ldots, y_T')' \) with \( y_t = (y_{1t}, \ldots, y_{Nt})' \) and similarly for \( X \) and \( Z \), as given by
\[
p(y, u | X, Z, \theta) = (2\pi\sigma^2)^{\frac{NT}{2}} \exp\left\{ -\frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - \sum_{j=1}^{p} f_{jt}(x_{it}) + u_{it})^2}{2\sigma^2} \right\} \\
\times (2\pi\omega^2)^{\frac{NT}{2}} \exp\left\{ -\frac{\sum_{i=2}^{N} \sum_{t=1}^{T} (\log u_{it} - z'_{it} \gamma - \rho \log u_{i,t-1})^2}{2\omega^2} - \log u_{it} \right\} \\
\times (2\pi\alpha^2)^{\frac{N}{2}} \exp\left\{ -\frac{\sum_{i=1}^{N} (\log u_{it} - z'_{it} \gamma)_{t-1}^2}{2\alpha^2} - \log u_{it} \right\}
\] (4.14)

The first line is due to the normality assumption on \( y_{it} | (x_{it}, u_{it}) \). The second line is due to log-normality of \( u_{it} | (z_{it}, u_{i,t-1}), \) and the third line is due to the log-normality of \( u_{i,t} | z_{it} \).

The likelihood function can be obtained from the joint distribution

\[
p(y | X, Z, \theta) = \int p(y, u | X, Z) du,
\]

where the latent variables are defined in \( \mathbb{R}_+^{NT} \). Tsionas (2006) pointed out that this integral couldn’t be computed analytically. Thus, Eq. (4.14) is treated as an augmented likelihood function and the technical inefficiencies \( u_{it} \) are treated as latent variables.

### 4.2.2 Bayesian inference

The basic level of this model assumes a non-parametric time-variant frontier with the cubic spline technique used to estimate the frontier. Following Hastie and Tibshirani (1990) and Hajargasht (2004), consider the prior for the non-parametric function \( f_{jt}(.) \) of each input in each period:

\[
f_{jt} \sim N(0, K^{-1}_{jt} \tau^2_j), \text{ for } j = 1, \ldots, p, t = 1, \ldots, T
\] (4.15)
where $\mathbf{K}_j$ is an $N \times N$ matrix defined in Section 4.1.3. $\tau_j^2$ is a smoothing parameter, which is assumed to be constant over time. The prior (4.15) can be rewritten as

$$p(f_{\mu_j}) = \left(2\pi \tau_j^2\right)^{-\frac{NT}{2}} |\mathbf{K}_j|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2\tau_j^2} \frac{\mathbf{f}'_{\mu_j} \mathbf{K}_j \mathbf{f}_{\mu_j}}{2}\right\}$$

(4.16)

where $|\mathbf{K}|$ is the determinant of matrix $\mathbf{K}$.

In addition, following Hajargasht (2004) and Tsionas (2006), the prior distributions of the scale parameters are specified by the gamma distributions:

$$p(\sigma^{-2}) = f_G(\sigma^{-2} | a_0, b_0) \propto (\sigma^{-2})^{a_0-1} \exp\left(-\frac{b_0}{\sigma^2}\right),$$

$$p(\tau_j^{-1}) = f_G(\tau_j^{-1} | a_j, b_j),$$

(4.17)

$$p(\omega^{-2}) = f_G(\omega^{-2} | n_\omega, q_\omega),$$

$$p(\omega_1^{-2}) = f_G(\omega_1^{-2} | n_1, q_1),$$

where $a_0$, $b_0$, $a_j$, $b_j$, $n_\omega$, $q_\omega$, $n_1$ and $q_1$ are prior hyper-parameters to be chosen.

Non-informative priors are specified for the location parameters.

$$p(\gamma) \propto 1,$$

$$p(\gamma_1) \propto 1,$$

(4.18)

$$p(\rho) \propto 1.$$

This model has location parameters such as $\gamma$, $\gamma_1$ and $\rho$, and scale parameters including $\sigma$, $\omega_1$, $\omega$ and $\tau_j$. The location parameters are assumed to be independent of the scale parameters \textit{a priori}.
Let $\theta$ denote the vector of parameters consisting of both the location parameters and the scale parameters. Our research interest is to estimate the parameter vector $\theta$ and the latent vector $u$. An applying Bayes’ rule to combine the augmented likelihood (4.14) with the priors of $\theta$ gives the posterior distribution

$$p(\theta, u | y, X, Z) \propto p(y, u | X, Z) \times p(\theta)$$

(4.19)

With Eq. (4.15) to Eq. (4.18), the joint posterior (4.19) can be rewritten as

$$p(f_j, \tau_j, \gamma, \omega, \rho, u | y, X, Z) \propto p(y, u | X, Z)p(f_j | \tau_j)p(\gamma)p(\omega)p(\rho)p(\tau_j)$$

$$= \sigma^{-\frac{NT}{2}} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \sum_{j=1}^{T} \left[ \sum_{i=1}^{T} f_{ij}(x_{ij}) + u_{ij} \right]^2 + 2b_0 \right\} \times \omega^{-\frac{N}{2}} \exp\left\{ -\frac{1}{2\omega^2} \sum_{i=1}^{N} \left[ \gamma_i \right] \right\} \times \rho^{-\frac{N}{2}} \exp\left\{ -\frac{1}{2\rho^2} \sum_{i=1}^{N} \left[ \log y_{ij} - \log \gamma_i \right]^2 \right\}$$

$$= \sigma^{-\frac{NT}{2}} \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \sum_{j=1}^{T} \left[ f_{ij}(x_{ij}) + u_{ij} \right]^2 \right\} \times \exp\left\{ -\frac{1}{2\sigma^2} \sum_{i=1}^{N} \left[ \sum_{j=1}^{T} f_{ij}(x_{ij}) + u_{ij} \right]^2 + 2b_0 \right\} \times \exp\left\{ -\frac{1}{2\omega^2} \sum_{i=1}^{N} \left[ \gamma_i \right] \right\} \times \exp\left\{ -\frac{1}{2\rho^2} \sum_{i=1}^{N} \left[ \log y_{ij} - \log \gamma_i \right]^2 \right\}$$

4.2.3 MCMC

This section describes how to draw a sample from the posterior distribution (4.19) using the MCMC. Since the posterior distribution (4.19) is not a standard density function, a mixture of the Gibbs sampler and the Metropolis-Hastings algorithm is used to draw samples from the posterior (Tsionas 2006).

4.2.3.1 Gibbs sampling

The conditional distributions for the non-parametric functions at the basic level are:

$$p(f_j | y, u, \sigma, \tau_j, \gamma, \omega, \rho) \sim N\{S_j[y_{ij} - \sum_{k \neq j} f_{kj}(x_{ik}) + u_{ij}], \sigma^2 S_j\}, \quad (4.20)$$
where $S_\beta = \{I + (\sigma^2 / \tau_j^2)K_j\}^{-1}$. Hence, according to the Gibbs sampler outlined in Section 4.1.1.2, $f_\beta$ can be drawn from Eq. (4.20) for any given values of the technical inefficiency $u_i$, with the other parameters.

The parameters $\gamma, \gamma_1$ and $\rho$ are found as in Tsionas (2006) with $\xi = \begin{bmatrix} \gamma \\ \rho \end{bmatrix}$ and $H = (H_1, ..., H_T)'$ as a $N(T-1) \times m$ matrix with $H_\eta' = (H_{\eta_1}, ..., H_{\eta_N})$ and $H_\eta = [z_{\eta_1}', \log u_{\eta_1}']$. Then the conditional distributions are

$$p(\xi | \gamma, \rho, \omega) \sim N((H'H)^{-1}H' \log u, \omega^2 (H'H)^{-1})$$

$$p(\gamma_1 | u_i, z_i, \omega_i) \sim N((z_i'z_i)^{-1}z_i' \log u_i, \omega_i^2 (z_i'z_i)^{-1}) \quad (4.21)$$

Then $\xi$ and $\gamma_1$ can be drawn from (4.21).

The conditional distributions for the scale parameters are:

$$p(\sigma^{-2} | y, u, f) = f_G \{a_0 + \frac{NT}{2}, b_0 + \frac{\sum_{i=1}^{T} \sum_{i=1}^{N} (y_{ji} - \sum_{j=1}^{g} f_{ji} (x_{ji}) + u_{ji})^2}{2}\}$$

$$p(\tau_j^{-2} | y, f) = f_G \{a_1 + \frac{NT}{2}, b_1 + \sum_{i=1}^{T} f_{ji}' K_{ji} f_{ji}\}$$

$$p(\omega^{-2} | H, u, \xi) = f_G \{n_\omega + \frac{N(T-1)}{2}, q_\omega + \frac{(\log u - \xi)^2}{2}\}$$

$$p(\omega_1^{-2} | u_i, z_i, \gamma_1) = f_G \{n_1 + \frac{N(T-1)}{2}, q_1 + \frac{(\log u_i - z_i'\gamma_1)^2}{2}\} \quad (4.22)$$

Samples of the scale parameters can be drawn from the conditional posterior distributions in Eq. (4.22).
4.2.3.2 Sampling using the Metropolis-Hastings algorithm

Drawing of the technical inefficiency data directly from the conditional posterior distributions via the Gibbs sampler can be difficult. Hence, the Metropolis-Hastings algorithm will be used in this subsection.

For \( i = 1, \ldots, N \), Eq. (4.19) can be rearranged to give:

\[
p(u_i | z_i, \ldots, z_N) \propto \exp \left\{ \frac{(u_i - R_i)^2}{2\sigma^2} - \frac{(\log u_i - z_i')^2}{2\sigma^2} - \frac{(\log u_i - z_i')^2}{2\omega^2} - \log(y_i) \right\}, \text{for } t = 1,
\]

\[
p(u_i | z_i, \ldots, z_N) \propto \exp \left\{ -\frac{(u_i - R_i)^2}{2\sigma^2} - \frac{(\log u_i - z_i')^2}{2\sigma^2} - \frac{(\log u_i - z_i')^2}{2\omega^2} - \log(u_i) \right\}, \text{for } 1 < t < T,
\]

\[
p(u_{it} | z_i, \ldots, z_N) \propto \exp \left\{ -\frac{(u_{it} - R_{it})^2}{2\sigma^2} - \frac{(\log u_{it} - z_{i}'')^2}{2\sigma^2} - \frac{(\log u_{it} - z_{i}'')^2}{2\omega^2} - \log(u_{it}) \right\}, \text{for } t = T.
\]

where \( R_i = [y_i - \sum_{j=1}^{k} f_{ij}(x_{ij})] \).

Following Tsionas (2006), rewrite these conditional distributions with lognormal terms and “complete the square” as:

\[
p(u_i | z_i, \ldots, z_N) \propto \exp \left\{ -\frac{(u_i - R_i)^2}{2\sigma^2} - \frac{(\log u_i - M^*)^2}{2V^*} - \log(u_i) \right\}, u_i \geq 0, \quad (4.23)
\]

where

\[
M^* = M V_2 + M V_1, \quad V^* = \frac{V_1 V_2}{V_1 + V_2}, \quad \text{for } t < T
\]

\[
M^* = z_{i'}' + \rho \log u_{i-1}, \quad V^* = \omega^2, \quad \text{for } t = T.
\]

The constants are defined as:
\[ M_1 = z_1' \gamma, M_2 = \frac{\log u_2 - z_2' \gamma}{\rho}, V_1 = \frac{\omega_1}{\rho}, V_2 = \frac{\omega_2}{\rho_2} \quad \text{for } t = 1, \]

\[ M_1 = z_t' \gamma + \rho \log u_{t-1}, M_2 = \frac{\log u_{t+1} - z_{t+1}' \gamma}{\rho}, V_1 = \frac{\omega_1}{\rho}, V_2 = \frac{\omega_2}{\rho_2} \quad \text{for } 1 < t < T. \]

Thus, we obtain

\[ p(u_t | .) \propto f_N(u_t | R_u, \sigma^2) f_{LN}(M^*, V^*), \quad u_t \geq 0, \quad (4.24) \]

where \( f_N(u_t | R_u, \sigma^2) \) is the normal distribution and \( f_{LN}(M^*, V^*) \) is the distribution of lognormal random variables with location \( M^* \) and scale \( V^* \). The inefficiency \( u_t \) in this model is related to the data and the other parameters, \( u_{t-1} \) and \( u_{t+1} \).

The Metropolis-Hastings algorithm can then be used to draw \( u_t \). Specifically, let \( u_t^{(0)} \) denote the current draw and \( u_t^{(\text{log})} \) be a draw from a lognormal distribution with parameter \( M^* \) and \( V^* \). Tsionas assumed that \( u_t^{(\text{log})} \) is accepted with the probability

\[
P(u_t^{(0)}, u_t^{(\text{log})}) = \min\{1, \frac{f_N(u_t^{(\text{log})} | R_u, \sigma^2)}{f_N(u_t^{(0)} | R_u, \sigma^2)}\} \\
= \min\{1, \exp[-\frac{(u_t^{(\text{log})} - R_u)^2 - (u_t^{(0)} - R_u)^2}{2\sigma^2}]\}
\]

while \( u_t^{(0)} \) is accepted with probability \( 1 - P(u_t^{(0)}, u_t^{(\text{log})}) \).

However, the above acceptance probability resulted in unreasonable estimates on the dataset of this thesis. Thus, in this thesis the standard Metropolis-Hastings algorithm was used to correct the probability of \( P(u_t^{(0)}, u_t^{(\text{log})}) \).
The simulation method used in this thesis is described as follows. First, a proposal distribution is required for the Metropolis-Hastings algorithm. Each proposal \( u^{(\text{log})}_i \) is generated as \( u^{(\text{log})}_i = \exp\{\tilde{u} / a\} \), where \( \tilde{u} \) is a standard normal variate with zero mean and unit variance, and \( a \) is a tuning parameter which is determined during the simulation to ensure the acceptance rate is not low. Hence, the proposal distribution is a lognormal distribution. Let \( q(u^{(\text{log})}_i | u^{(0)}_i) \) denote the density of the proposal distribution.

Next we note that the acceptance probability of the standard Metropolis-Hastings algorithm is

\[
\tilde{P}(u^{(0)}_i, u^{(\text{log})}_i) = \frac{q(u^{(0)}_i | u^{(\text{log})}_i) p(u^{(0)}_i, u^{(\text{log})}_i)}{q(u^{(\text{log})}_i | u^{(0)}_i) p(u^{(\text{log})}_i, u^{(0)}_i)}.
\]

Hence the following acceptance probability is used in the simulation:

\[
\tilde{P}(u^{(0)}_i, u^{(\text{log})}_i) = \min \left\{ 1, \frac{q(u^{(0)}_i | u^{(\text{log})}_i) f_N(u^{(\text{log})}_i | R_i, \sigma^2) f_N(u^{(\text{log})}_i | M^*, V^{**})}{q(u^{(\text{log})}_i | u^{(0)}_i) f_N(u^{(0)}_i | R_i, \sigma^2) f_N(u^{(0)}_i | M^*, V^{**})} \right\}.
\]

Note that according to the standard Metropolis-Hastings algorithm, 
\( q(u^{(0)}_i | u^{(\text{log})}_i) / q(u^{(\text{log})}_i | u^{(0)}_i) \) is equal to 1 if the proposal density is symmetric. However, in the present method, the proposal distribution \( q(u^{(\text{log})}_i | u^{(0)}_i) \) is not symmetric. Thus, \( P(u^{(0)}_i, u^{(\text{log})}_i) \) has to be further modified to include the ratio of the two proposal distributions, i.e. \( q(u^{(0)}_i | u^{(\text{log})}_i) / q(u^{(\text{log})}_i | u^{(0)}_i) \). Thus,

\[
P(u^{(0)}_i, u^{(\text{log})}_i) = \min \left\{ 1, \frac{f_N(u^{(\text{log})}_i | R_i, \sigma^2) f_N(u^{(\text{log})}_i | M^*, V^{**}) f_N(u^{(0)}_i | 0, \sigma^2)}{f_N(u^{(0)}_i | R_i, \sigma^2) f_N(u^{(0)}_i | M^*, V^{**}) f_N(u^{(\text{log})}_i | 0, \sigma^2)} \right\}.
\]

Consequently, in each iteration of the MCMC, draws \( u_i \) are first obtained using the Metropolis-Hastings algorithm conditional on the rest of the parameters. Then the rest of
the parameters are drawn using the Gibbs sampler for the given $u_{it}$. This process continues until convergence.

### 4.2.4 Summary

A two-level semi-parametric stochastic frontier model was developed and its Bayesian inference was investigated. This model is flexible in terms of capturing the variability in the frontiers. The model includes many existing models as special cases. In particular, the model collapses to a one-level non-parametric time-variant frontier model if the higher level is eliminated and reduces to the model in Tsionas (2006) if a linear time-invariant structure is used at the basic level.

A linear time-invariant structure is kept at the higher level of this model, which may produce a biased estimate of the technical inefficiency. To address this issue, the next section extends the semi-parametric stochastic frontier model to a non-parametric model with a non-parametric time-variant structure at the higher level.

### 4.3 Two-level dynamic non-parametric SFA model

This section extends the semi-parametric model in section 4.2 to a non-parametric time-variant model.
4.3.1 Model

The higher level of the semi-parametric model in Eq. (4.13) assumes a linear, time-invariant equation for the log of the technical inefficiency, which may lead to misleading estimates. Thus, in this section, the linear, time-invariant structure in equation (4.13) is replaced by a non-parametric time-variant structure. This produces a complete non-parametric time-variant structure at both the basic level and the higher level in a dynamic non-parametric stochastic frontier model. This model then gives more exact estimates of the frontier, the technical inefficiency and other parameters of interest.

The time-variant, nonparametric frontier in the basic level given by Eq. (4.12) is not changed. A non-parametric time-variant structure is then used for the technical inefficiency at the higher level as given by:

\[ y_{it}^u = \sum_{j=1}^{q} f_{jt} (x_{ijt}) - u_{it} + v_{it}, \quad \text{for } i = 1, \ldots, N; \ t = 1, \ldots, T, \tag{4.25} \]

\[ \log u_{it} = \sum_{j=1}^{q} g_{jt} (z_{ijt}) + \rho \log u_{it-1} + \epsilon_{it}, \quad \text{for } t = 2, \ldots, T, \]

\[ \log u_{i1} = \sum_{j=1}^{q} h_{ji} (z_{ij1}) + \epsilon_{i1}, \quad \text{for } t = 1, \]

where \( g_{jt} (.) \) and \( h_{ji} (.) \) are time-specific, non-parametric functions. The other variables are defined similarly to those in Eq. (4.12) and Eq. (4.13).

The model in Eq. (4.25) is more general than the models discussed in the previous sections. It will collapse to the semi-parametric model if a linear time-invariant structure is used at the higher level. The advantage of this model is that it uses a non-parametric
time-variant structure at both the basic level and the higher level to give more robust estimates of the frontier and technical inefficiency.

The Bayesian inference will be investigated using an augmented likelihood function. Compared with equation (4.14), the joint distribution \( p(y, u | X, Z, \theta) \) is

\[
p(y, u | X, Z, \theta) = \frac{1}{(2\pi\sigma^2)^{\frac{NT}{2}}} \exp \left\{ -\frac{\sum_{i=1}^{N} \sum_{t=1}^{T} \left( v_{it} - \sum_{j=1}^{q} f_{j}(x_{ij}) + u_{it} \right)^2}{2\sigma^2} \right\}
\times \frac{1}{(2\pi\omega^2)^{\frac{N(T-1)}{2}}} \exp \left\{ -\frac{\sum_{t=2}^{T} \sum_{i=1}^{N} \left( \log u_{it} - \sum_{j=1}^{q} g_{j}(z_{ij}) - \rho \log u_{i(t-1)} \right)^2}{2\omega^2} \right\}
\times \frac{1}{(2\pi\alpha^2)^{\frac{N}{2}}} \exp \left\{ -\frac{\sum_{i=1}^{N} \left( \log u_{it} - \sum_{j=1}^{q} h_{j}(z_{ij}) \right)^2}{2\alpha^2} \right\}
\]

(4.26)

where \( \theta \) is the parameter vector consisting of all unknown parameters in the model. The first line is the same as in Eq. (4.14) since these two models use the same form of the non-parametric time-variant structure at the basic level. The non-parametric time-variant functional form at the higher level is given by the log-normality of \( u_{it} | (z_{it}, u_{i(t-1)} \) in the second line and the log-normality of \( u_{it} | z_{it} \) in the third line.

### 4.3.2 Bayesian Inference

The prior distributions for the Bayesian inference are then given by the prior of \( f_{j}(\cdot) \) specified as the same as in Eq. (4.15), but with \( g_{j}(\cdot) \) and \( h_{j}(\cdot) \) specified according to the spline technique for the unknown form of the technical inefficiency function at the higher level. Following Hastie and Tibshirani (1990), consider the following priors:
\[ p(f_{\mu}) = \left(2\pi \tau_{\mu}^2 \right)^{-\frac{NT}{2}} \left\{ \mathbf{K}_{\mu} \right\}^{-\frac{1}{2}} \exp\left\{ -\frac{\mathbf{f}_{\mu}^{\mathrm{T}} \mathbf{K}_{\mu} \mathbf{f}_{\mu}}{2\tau_{\mu}^2} \right\}, \]

\[ p(g_{\mu}) = \left(2\pi \eta_{\mu}^2 \right)^{-\frac{NT}{2}} \left\{ \mathbf{W}_{\mu} \right\}^{-\frac{1}{2}} \exp\left\{ -\frac{\mathbf{g}_{\mu}^{\mathrm{T}} \mathbf{W}_{\mu} \mathbf{g}_{\mu}}{2\eta_{\mu}^2} \right\} \tag{4.27} \]

\[ p(h_{j}) = \left(2\pi \delta_{j}^2 \right)^{-\frac{N}{2}} \left\{ \mathbf{P}_{j} \right\}^{-\frac{1}{2}} \exp\left\{ -\frac{\mathbf{h}_{j}^{\mathrm{T}} \mathbf{P}_{j} \mathbf{h}_{j}}{2\delta_{j}^2} \right\}, \]

where \( \mathbf{W}_{\mu} \) and \( \mathbf{P}_{j} \) are \( N \times N \) matrices reflecting the prior knowledge for the roughness of \( g_{\mu}(\cdot) \) and \( h_{j}(\cdot) \). \( \eta_{j}^2 \) and \( \delta_{j}^2 \) are two smoothing parameters assumed to be constant over time.

This model includes the location parameters, \( f_{\mu}(\cdot), g_{\mu}(\cdot), h_{j}(\cdot) \) and \( \rho \), and the scale parameters, \( \sigma, \omega_{i}, \omega, \eta_{j}, \delta_{j} \) and \( \tau_{j} \). Let \( \mathbf{\theta} \) denote the vector of both the location parameters and the scale parameters and assume that the location parameters are independent of the scale parameters \( \textit{a priori} \). All the scale parameters are specified to have a gamma prior:

\[ p(\sigma^{-2}) = f_{\mathcal{G}}(\sigma^{-2} \mid a_{0}, b_{0}) \propto (\sigma^{-2})^{a_{0}-1} \exp(-\frac{b_{0}}{\sigma^{2}}), \]

\[ p(\tau_{j}^{-1}) = f_{\mathcal{G}}(\tau_{j}^{-1} \mid a_{j}, b_{j}), \tag{4.28} \]

\[ p(\eta_{j}^{-1}) = f_{\mathcal{G}}(\eta_{j}^{-1} \mid c_{j}, d_{j}), \]

\[ p(\delta_{j}^{-1}) = f_{\mathcal{G}}(\delta_{j}^{-1} \mid e_{j}, l_{j}), \]

\[ p(\omega^{-2}) = f_{\mathcal{G}}(\omega^{-2} \mid n_{o}, q_{o}), \]

\[ p(\omega_{i}^{-2}) = f_{\mathcal{G}}(\omega_{i}^{-2} \mid n_{i}, q_{i}), \]

where \( a_{0}, b_{0}, a_{j}, b_{j}, c_{j}, d_{j}, e_{j}, l_{j}, n_{o}, q_{o}, n_{i} \) and \( q_{i} \) are prior hyper-parameters to be chosen.
The prior of $\rho$ is specified as before, i.e. $p(\rho) \propto 1$.

Bayes’s rule is used to combine the augmented likelihood in Eq. (4.26) with the priors of $\theta$ to get the posterior distribution

$$p(f, g, h, \sigma, \tau, \omega, \delta, \omega, \alpha, \omega, \rho, u|y, X, Z) \propto p(y, u|X, Z) p(f|\tau)p(g|\eta)p(h|\omega)p(\sigma)p(\tau)p(\omega)p(\delta)p(\alpha)p(\omega)p(\rho)p(\rho)$$

(4.29)

$$= \sigma^{-2\left(NT+\alpha_0-1\right)} \exp\left\{-\frac{\sum_{i=1}^{T} \sum_{i=1}^{N} (y_{it} - \sum_{j=1}^{p} f_{jt}(x_{it}) + u_{it})^2 + 2b_0}{2\sigma^2}\right\}$$

$$\times \tau^{-2\left(NT+\alpha_0-1\right)} \left\{K_{\mu}\right\}^\frac{1}{2} \exp\left\{-\frac{f_{\mu}' K_{\mu} f_{\mu} + 2b}{2\tau_{\mu}}\right\}$$

$$\times \omega^{-2\left(NT+\alpha_0-1\right)} \left\{\omega \right\}^\frac{1}{2} \exp\left\{-\frac{2q_\omega + \sum_{i=1}^{T} \sum_{i=1}^{N} (\log u_{it} - \sum_{j=1}^{q} g_{j}(z_{it}) - \rho \log u_{it-1})^2}{2\omega^2}\right\}$$

$$\times \eta_j^{-2\left(NT+\alpha_0-1\right)} \left\{\eta_j \right\}^\frac{1}{2} \exp\left\{-\frac{g_{\mu}' \omega \eta \eta_{\mu} g_{\mu} + 2d}{2\eta_j^2}\right\}$$

$$\times \omega_h^{-2\left(NT+\alpha_0-1\right)} \exp\left\{-\frac{2q_h + \sum_{i=1}^{T} \sum_{i=1}^{N} (\log u_{it} - \sum_{j=1}^{q} h_{ji}(z_{it}))^2}{2\omega_h^2}\right\}$$

$$\times \delta_{\mu}^{-2\left(NT+\alpha_0-1\right)} \left\{\delta_{\mu} \right\}^\frac{1}{2} \exp\left\{-\frac{h_{\mu}' \omega \delta \delta_{\mu} h_{\mu} + 2d}{2\delta_j^2}\right\}$$

Note here that the differences between Eq. (4.29) and Eq. (4.19) of the semi-parametric model are in the second and third lines of the right hand side. The second line includes the non-parametric function $g_{\mu}(.)$, thus, $\eta_j$ is required. Similarly, the third line includes the non-parametric function $h_{ji}(.)$ with the parameter $\delta_{ji}$.
4.3.3 MCMC

Samples are drawn from the joint posterior in Eq. (4.29) based on conditional posterior distributions. At the basic level, the conditional distributions for the non-parametric functions of the inputs are the same as in Eq. (4.20)

\[
p(f_j | y, u, \sigma, \tau_j) \sim N\{S_{\mu_j}[y_u - \sum_{k \neq j} f_{\mu_j}(x_{uk}) + u_{uk}], \sigma^2 S_{\mu_j}\},
\]

(4.30)

where \( S_{\mu} = (\mathbf{I} + (\sigma^2 / \tau_j^2)K_{\mu})^{-1} \).

At the higher level, the conditional distributions of the non-parametric functions of the environmental variables \( z \) and \( z_1 \) are given by:

\[
p(g_{\mu_j} | \log u, z, \omega, \rho, \eta)
\sim N\{Q_{\mu_j}[\log u - \rho \log u_{u-1} - \sum_{k \neq j} g_{\mu_j}(z_{uk})], \omega^2 Q_{\mu_j}\},
\]

(4.31)

\[
p(h_{j1} | \log u_{u1}, z_1, \omega_1, \rho, \delta) \sim N\{O_{j1}[\log u_{u1} - \sum_{k \neq j} h_{j1}(z_{uk})], \omega^2 O_{j1}\},
\]

(4.32)

where \( Q_{\mu} = (\mathbf{I} + (\omega^2 / \eta^2)W_{\mu})^{-1} \) and \( O_{j1} = (\mathbf{I} + (\omega_1^2 / \delta^2)P_{j1})^{-1} \).

In addition, the conditional distribution of the parameter \( \rho \) is:

\[
p(\rho | \log u, g_{\mu_j}, \omega) \sim N\{\bar{\rho}, \omega^2 (\log u_{u-1} \log u_{u-1})^{-1}\},
\]

where \( \bar{\rho} = (\log u_{u-1} \log u_{u-1})^{-1} \log u_{u-1} - \sum_{j=1}^{n} g_{\mu_j}(z_{uj}) \} \).

The conditional distributions for the scale parameters can be obtained as:

\[
p(\sigma^2 | y, u, f) = f_G \{a_0 + NT, b_0 + \sum_{i=1}^{T} \sum_{j=1}^{N} (y_{j} - \sum_{j=1}^{P} f_{\mu_j}(x_{ji}) + u_{ji})^2 \}
\]

\[
+ \sum_{i=1}^{T} \sum_{j=1}^{N} (y_{j} - \sum_{j=1}^{P} f_{\mu_j}(x_{ji}) + u_{ji})^2 \}
\]

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The use of non-parametric functions with environmental variables \( z \) and \( z_i \) results in some difference between Eq. (4.33) and Eq. (4.22) of the semi-parametric dynamic model. First non-parametric model uses the two new conditional distributions \( \eta_j^2 \) and \( \delta_j^2 \). Secondly, the parameters \( \omega^{-2} \) and \( \omega_i^{-2} \) of the higher level are updated. However, the conditional distributions of the parameters \( \tau_j^2 \) and \( \sigma^2 \) at the basic level are unchanged between these two equations. Therefore, for any given values of the technical inefficiency \( u_\mu, f_\mu, g_\mu, h_j, \rho \), and all the scale parameters can be drawn using the Gibbs sampler outlined in Section 4.1.1.2.

Next we consider how to draw the technical inefficiency, \( u_\mu \), using the Metropolis-Hastings algorithm. The conditional posterior distributions for the technical inefficiency are given by:

\[
p(\tau_j^2 \mid y, f) = f_G \{ a_j + \frac{NT}{2} b_j + \frac{\sum_{j=1}^{T} f'_{\mu j} K_{j} f_{\mu j}}{2} \}
\]

\[
p(\delta_j^2 \mid \log u_{ji}, h) = f_G \{ e_j + \frac{N}{2} l_j + \frac{\sum_{j=1}^{T} h'_{j} p_{ji} h_{ji}}{2} \}
\]

\[
p(\omega^{-2} \mid \log u_\mu, g, z) = f_G \{ n_\omega + \frac{N(T-1)}{2} q_\omega + \frac{\sum_{j=1}^{T} \sum_{i=1}^{N} (\log u_\mu - \sum_{j=1}^{P} g_{\mu j}(z_{ji}) - \rho \log u_{i,j-1})^2}{2} \}
\]

\[
p(\omega_i^{-2} \mid \log u_{i}, h, z_i) = f_G \{ n_i + \frac{N}{2} q_i + \frac{\sum_{i=1}^{N} (\log h_i - \sum_{j=1}^{P} h_{j i}(z_{ji}))^2}{2} \} \quad (4.33)
\]
\begin{align*}
p(u_i | .) \propto \exp \left\{ -\frac{(u_{i1} - R_{i1})^2}{2\sigma^2} - \frac{(\log u_{i2} - \sum_{j=1}^q g_{j1}(z_{ji}) - \rho \log u_{i1})^2}{2\omega^2} \right\}, \text{ for } t = 1, \\
&\quad - \frac{(\log u_{i1} - \sum_{j=1}^q h_{j1}(z_{ji}))^2}{2\omega_i^2} - \log u_{i1} \} \\
p(u_u | .) \propto \exp \left\{ -\frac{(u_{u1} - R_{u1})^2}{2\sigma^2} - \frac{(\log u_{u2} - \sum_{j=1}^q g_{j1}(z_{ji}) - \rho \log u_{u1})^2}{2\omega^2} \right\}, \text{ for } 1 < t < T, \\
&\quad - \frac{(\log u_{u1} - \sum_{j=1}^q g_{j1}(z_{ji+1}) - \rho \log u_{u1})^2}{2\omega_i^2} - \log u_{u1} \} \\
p(u_{it} | .) \propto \exp \left\{ -\frac{(u_{it} - R_{it})^2}{2\sigma^2} - \frac{(\log u_{it} - \sum_{j=1}^q g_{jt}(z_{jt}) - \rho \log u_{it-1})^2}{2\omega^2} \right\}, \text{ for } t = T, \\
&\quad - \log u_{it(T)} \}
\end{align*}

where \( R_i = -(y_i - \sum_{j=1}^p f_{jt}(x_{ji})) \).

Similar to the semi-parametric model, rewrite these conditional distributions as:

\begin{equation}
p(u_u | .) \propto \exp \left\{ -\frac{(u_u - R_u)^2}{2\sigma^2} - \frac{(\log u_u - M^*)^2}{2\omega^*} - \log u_u, \quad u_u \geq 0, \quad (4.34)\right\}
\end{equation}

where \( R_u = -(y_u - \sum_{j=1}^p f_{jt}(x_{j1})) \) and \( M^* = \frac{M_1 V_2 + M_2 V_1}{V_1 + V_2}, \quad V^* = \frac{V_1 V_2}{V_1 + V_2}, \text{ for } t < T \).

\[ M^* = \sum_{j=1}^q g_{jt}(z_{jt}) + \rho \log u_{u-1}, \quad V^* = \omega^2, \text{ for } t = T, \]

where the constants are defined as:

\begin{align*}
&\text{For } t = 1, \quad M_1 = \sum_{j=1}^q h_{j1}(z_{j1}), \quad M_2 = \frac{\sum_{j=1}^q g_{j1}(z_{j1})}{p}, \quad V_1 = \omega_1^2, \quad V_2 = \frac{\omega_2}{\rho_2}.
\end{align*}

For \( 1 < t < T \),
\[ M_1 = \sum_{j=1}^{q} g_j(z_{it}) + \rho \log u_{it-1}, \quad M_2 = \frac{\log u_{it+1} - \sum_{j=1}^{a} g_j(z_{it+1})}{\rho}, \quad V_1 = \sigma_1^2, \quad V_2 = \frac{\sigma_2^2}{\rho_2} \]

Based on Eq. (4.34), we have
\[ p(u_{it} | .) \propto f_N(u_{it} \mid R_{it}, \sigma^2) f_{LN}(M^*, V^*), u_{it} \geq 0, \]
where \( f_N(u_{it} \mid R_{it}, \sigma^2) \) is a normal distribution and \( f_{LN}(M^*, V^*) \) is the distribution of a lognormal random variable with location parameter \( M^* \) and scale parameter \( V^* \). As in the semi-parametric model, the inefficiency \( u_{it} \) in this model is related to the data and the other parameters, \( u_{it-1} \) and \( u_{it+1} \). Hence, the Metropolis-Hastings algorithm is used to simulate \( u_{it} \).

4.4 Concluding remarks

This chapter starts with a traditional time-invariant and linear SFA model to develop a dynamic semi-parametric SFA model and a dynamic non-parametric SFA model. These two models both have two-level structures. Consequently, the frontier can be measured at the basic level and the effect of both environmental variables and lagged technical inefficiency on the current technical inefficiency can be measured at the higher level in a one-stage model. These two models relax many assumptions made in the traditional approaches. Thus, they provide more robust and accurate measurements of the frontier and technical inefficiency, which is one of the main purposes of this thesis. Chapter 5 will build a new framework of measuring ICT spillover effects based on these two models and Chapter 6 will compare these two models with other traditional models based on an empirical study of ICT spillover effects.
Chapter 5 A new framework to measure ICT spillover effects

5.1 Introduction

As mentioned in the literature review chapter, the lack of theoretical guidance and the drawbacks of methodology were two main limitations for measuring ICT spillover effects in the previous studies. In Chapter 4, a semi-parametric SFA model and a non-parametric SFA model were developed to overcome the drawbacks of the methodology. In this chapter, a new framework (hereafter called “new framework”) was developed to measure ICT spillover effects and its influential factors by combining the GPT theory with the efficiency concept in order to fill the gaps of theoretical guidance (i.e. measurement of lag and co-invention). The new framework is based on the models developed in Chapter 4. Therefore, the framework has two levels to match both levels of models in Chapter 4 (i.e. SF as a basic level and efficiency function as a higher level).

The basic level of the new framework focused on measuring ICT spillover effects. The efficiency behaved as a proxy of co-invention, indicated by organizational improvement according to the GPT theory. The non-parametric SF models developed in Chapter 4 were used to capture the length of the lag according to the GPT hypothesis which states that an organizational adjustment of ICT-using industries can cause an initial negative return of ICT capital before turning a profit (i.e. ICT capital as an abnormal input).
At the basic level, a country has ICT spillover effects when the efficiency increases during lag\(^19\), according to the GPT hypothesis that ICT-usage requires the cooperation with an organizational adjustment process before ICT can contribute positively to the industrial output. Furthermore, the length of lag could indicate the strength of ICT spillover effects in a country. The shorter the lag, the stronger the ICT spillover effect becomes because short lag is an indication that a country can obtain the potential output growth earlier due to faster organizational adjustment in response to its accumulation of ICT capital.

At the higher level, the factors that influence ICT spillover effects were investigated. According to the GPT theory, the influential factors of ICT spillover effects were related to co-invention. Since efficiency was a proxy of co-invention on the basic-level, the higher level was comprised of both endogenous and exogenous factors that influence efficiency. The endogenous factor was the persistent level of inefficiency. Among several of the exogenous factors mentioned in previous studies, this study considered two exogenous factors: software shares and supply of skilled labour.

After building the new framework, this framework will be used to analyse ICT spillover effects in the US and the UK in the next Chapter. The remainder of this chapter was organized as follows. Section 2 constructs the new framework of measuring ICT spillover effects. Section 3 lists the relevant model specifications, which include the semi-parametric/non-parametric two-level models developed in Chapter 4 and three traditional linear one-level models to compare the results of this framework on semi/non-parametric two-level models with the results of traditional models. Section 4 gives concluding remarks.

\(^{19}\) If considering a possible lagged appearance of efficiency changes, a country may have ICT spillover effects if ICT-using industries’ efficiencies increase after the lags.
5.2 A new framework of measuring ICT spillover and its influential factors

In this section, by combining the GPT theory and the efficiency concept, a new two-level framework, used to measure ICT spillover effects and its influential factors, was developed for three purposes: (i) testing ICT spillover effects, (ii) measuring the strength of ICT spillover effects, and (iii) investigating the factors that influence ICT spillover effects. The basic level of the new framework focused on the first two purposes. The higher level focused on the third purpose.

5.2.1 Measuring ICT spillover effects at the basic-level

The GPT theory could explain ICT spillover effects. However, two problems are still existing: lack of an appropriate proxy of co-invention and the absence of guidance for the identification of the lag period.

5.2.1.1 Efficiency as a proxy of co-invention

This study follows Basu et al. (2003) to model ICT spillovers within the GPT framework, but with different proxy of co-invention. Efficiency concept is used to measure accumulation of co-invention in this study, while Basu et al. (2003) employed the lagged ICT capital as a proxy of co-invention.

As mentioned in section 2.2.1, GPT are characterized by pervasiveness, technological dynamism, and innovation complementarities with other forms of advancement
(Bresnahan and Trajtenberg, 1995). One of evidence of ICT as a GPT is huge ICT investment in the downstream industries (Stiroh 2002, Vuijlsteke et al., 2007). However, these ICT investments by these downstream industries may lead to an initially negative growth (Basu et al. 2003). The GPT theory attributes this initially negative output growth to accumulation of co-invention and argued co-invention is the necessary redesigns of physical capital, re-skilling of human capital, and etc. (Carlaw and Lipsey 2006). Since no clear definition of co-invention, most existing empirical studies agreed co-invention is related to reorganization of these downstream industries due to ICT investment. For example, Basu et al. (2003) wrote “investments in ICT may in fact be associated with lower TFP as resources are diverted to reorganization and learning”. Thus, this study defines co-invention as organization adjustment. Based on this definition, the accumulation of co-invention is a process of reorganization.

Some empirical studies (Baus et al. 2003, 2007) used the organizational capital as a proxy of co-invention in their studies. But the results show organizational capital seems not to be an appropriate proxy for co-invention because organizational capital is unobservable and no good approach can be used to measure organizational capital. Pervious studies usually employed observable lagged ICT capital instead of organizational capital in their studies. But lagged ICT capital works well only for US data, not for the UK case (Basu et al. 2003).

Differ from the work of Basu et al. (2003, 2007), this study use efficiency as a proxy of co-invention within the GPT framework for the following reasons. Firstly, according to the frontier theory, the efficiency is a good measurement of organizational adjustment. Most previous literature measured the contribution of ICT to organization at firm level,
industry-level, and country-level using efficiency as a proxy of organization (Lin and Shao 2001, Shao and Lin 2006 and Lin 2009). Second is that efficiency can be directly estimated, although it is a latent variable. Fig 5.1 illustrates the definition of co-invention and its two proxies (i.e. the proxy of efficiency and the proxy of organizational capital).

FIGURE 5.1 The definition of co-invention and its two proxies

Using efficiency as a proxy of co-invention, this study combines the GPT theory and the frontier theory to explain the initially negative growth due to ICT investment in Fig 5.2, where the abscissa is the ICT capital given Non-ICT capital and labour and the ordinate is the output.

FIGURE 5.2 Initially negative growth due to ICT investment in the frontier framework

In Fig 5.2, the curves of period 1 \((t=1)\), of period 2 \((t=2)\) and of period 3 \((t=3)\) are the frontiers for period 1, period 2 and period 3, which are labelled as \(f1\), \(f2\) and \(f3\).
distance between $f1$ and $f2$ is TP1 due to the technology progress from period 1 to period 2, which is negative (i.e. frontier falls) since resources are diverted to reorganization. Then the frontier rises to $f3$ at period 3 and thus TP2 is positive and larger in absolute term than TP1 since the new technology matured at period 3. Suppose an inefficient firm stays at point A in period 1. The inefficiency is the distance between point A and the frontier of period 1 (i.e. $f1$), which is labelled as ‘TE1’ in Fig 5.2. During period 2, this firm improves its efficiency and moves itself to point B with inefficiency ‘TE2’ - the distance between point B and $f2$. In period 3, the efficiency of this firm rises further, reflecting the reposition from point B to point C. Technical efficiency change (TEC) can be used to rank the magnitude of change in efficiency over time, which can be defined as $TE_t - TE_{t-1}$. In a sum, while the frontier initially falls from $f1$ to $f2$ due to the accumulation of co-invention (i.e. organizational adjustment) and finally jumps to the peak level in period 3 when the ICT technology matures, efficiency increases from period 1 to period 3 reflecting a continual organizational adjustment process.

Even though efficiency as a proxy of co-invention appears to be better than organizational capital, they both face the same problem when used to measure ICT spillover effects: the length of the lag could not be determined. When efficiency was treated as a proxy of co-invention, the efficiency captured by SF on the basic-level should be increased during the lag, assuming ICT spillover effects existed in a country, according to the hypothesis of the GPT theory that the use of ICT required the cooperation with a reorganization process before ICT could contribute positively to the industrial output. When organizational capital is indicated by lagged ICT capital as a proxy of co-invention, lagged ICT capital should relate positively to current TFP growth, assuming a country has ICT spillover effects in accordance with the GPT theory. Thus,
both cases require information on the length of the lag for a country when examining the possibility of ICT spillover effects in a country.

5.2.1.2 Using non-parametric SF to measure the length of lags

Concerning the estimation of lags, since there was no direct guidance of lags in the GPT theory, the previous empirical studies considered a range of 5-15 years as the duration of the lag period but did not specify how long it was precisely (Howitt 1998; Brynjolfsson and Hitt 2002; Basu et al. 2003, 2007). The lag selected in this manner was clearly inaccurate. Although no direct guidance for lag selection was provided by the GPT theory, an indirect guidance could be employed. According to the GPT hypothesis proposed by Carlaw and Lipsey (2006), the impact on growth of a new technology, such as ICT, is not immediately positive but could initially reduce growth and/or productivity due to the necessary redesigns of physical capital, re-skilling of human capital, and etc. That is, the length of a period with negative return on growth due to introduction of the GPT may indicate the length of a lag that stems from an organizational adjustment process. For ICT spillover effects, the negative impact on growth could be measured as the initial number of years with negative return on ICT capital, which is the lag in this study (hereafter called ‘lag’). The growth and productivity slowdowns are reversed when the technology matures and the introduction of the GPT eventually restores its growth with long-term growth benefits. Following this GPT hypothesis, the length of lags could be captured by the length of years with negative return of ICT capital to output.

The previous empirical studies with traditional parametric production functions did not use this approach because this approach implied that ICT capital was a capital that could generate temporary negative return to output (hereafter called ‘abnormal capital’). In the
case of ICT capital as an abnormal capital, the traditional parametric production functions do not work due to its assumptions of monotonicity and concavity for the underlying production functions. Thus, to deal with this problem, a non-parametric SFA is needed to relax these assumptions. By using a non-parametric SFA, the ICT capital can be treated as an abnormal input with the condition that negative return only appears in ICT capital but not in any other inputs. Consequently the years with negative return of ICT capital before the year with positive return of ICT capital can be treated as the lag. In other words, the number of years with a negative return of ICT capital indicates the length of lag for a country with ICT spillover effects.

5.2.1.3 Measuring ICT spillover effects at the basic-level

The basic level of the new framework measured ICT spillover effects for a country using two steps. The first step was to examine the existence of lags and measure the length of lags. The hypothesis of the GPT theory argued that ICT capital needed to cooperate with organizational change indicated by efficiency change before ICT could contribute positively to output growth. This means that the return of ICT capital to output was related to both observed ICT investment and the unobserved organizational change. Furthermore, this also indicated that the return of ICT capital may be abnormal that generated negative return to output the before ICT could contribute positively to output growth due to organization adjustment. Based on this argument, the lag could be captured by the number of years with negative return of ICT capital. Furthermore, the length of such years was an indication of the length of lags. Accordingly, the Hypothesis 1 can be written as:
Hypothesis 1: ICT capital is an abnormal capital that leads to the initially negative return of ICT capital to output for a country with ICT spillover effects.

The second step was to test whether efficiency increased during lags. If considering a possible lagged appearance of efficiency change, it is reasonable to test whether efficiency increased during and after lags. As mentioned above, the ICT capital may face an initial negative return due to an organizational adjustment process. Thus, organizational adjustment indicated by efficiency increase should be observed during and/or after the period of initially negative return of ICT capital. Thus, Hypothesis 2 was given as follows:

Hypothesis 2: Efficiency increases during and/or immediately after the lag for a country with ICT spillover effects.

Finally, the difference between the basic-level of the new framework and the previous empirical work based on the GPT theory was emphasized. The former assumed ICT capital as an abnormal input and measured ICT spillover effects on a non-parametric SFA by checking whether efficiency increased during and after lags with certain length. The latter argued ICT capital as a normal input and explored ICT spillover effects on a parametric production function to test whether ICT capital growth with undefined lags was positively correlated to the current TFP growth. The advantages of the former include: (i) from the theoretical perspective, it considered two characteristics of ICT spillovers, i.e. ICT as an abnormal input and organizational adjustment due to ICT investment. Thus, it can capture the strength of ICT spillover effects using the length of lags; (ii) from the perspective of methodology, the semi/non-parametric approach was more flexible in terms of less assumptions being made (e.g. monotonicity and concavity)
than a traditional parametric production function and allowed the capturing of the return of ICT capital when ICT capital was as an abnormal input.

5.2.2 Measuring influential factors of ICT spillover effects at the higher-level

After measuring ICT spillover effects at the basic level, the factors that influence ICT spillover effects were examined at the higher level. In the case of efficiency as a proxy of co-invention, the factors that influenced ICT spillover effects should be related to efficiency change. A direct factor was the persistent level of inefficiency, referred to as endogenous factor. For exogenous variables on the efficiency, this study considered two factors: software share and supply of skilled labours.

5.2.2.1 Endogenous factor

According to the rule of “survival of the fittest”, firms cannot survive in the long term unless they are technically efficient. However, if technical inefficiency was due to factors that were under the influence of firms but the factors could not be adjusted without cost, then the improvement of efficiency necessarily depends on the cost of adjustment (Tsionas 2006). A persistent technical inefficiency may appear if such costs are high.

For the case of ICT usage, many previous studies suggested that ICT investment required a large amount of complementary costs to finish a reorganization process in order to utilize the potential benefits of the ICT investment and finally obtain growth. For example, Yang and Brynhjolfsson (2001) reported that the cost of an Enterprise Resource Planning (ERP) project incurred within the first year was five times the cost of the hardware and software licenses. Brynjolfsson, Hitt, and Yang (2002) had argued that
there were $9 of complementary cost for every $1 of ICT investment and explained the complementary cost was an organizational adjustment cost. Empirical studies on the GPT theory argued that these costs were a type of intangible organizational capital (Basu et al. 2003, 2007). Thus, the persistent level of inefficiency should be considered when investigating the factors that influence (in)efficiency related to ICT spillover effects.

However, it was difficult to measure this persistent level of inefficiency on stochastic frontier analysis due to the heteroscedastic inefficiency terms. Tsionas (2006) proposed a dynamic inefficiency model based on the existing SFA models, in which the current value of inefficiency depends on past values of inefficiency. This study will follow this approach to measure the persistent level of inefficiency in the US and UK.

Specifically, consider the current inefficiency which is related to its one-year lagged value via an AR(1) structure. The coefficient of the lagged inefficiency was between 0 and 1, which represents the time correlation at lag one. Clearly a coefficient close to 1 indicated that the inefficiency of a country was persistent due to high adjustment costs, and thus the country has a limited room for the effect of exogenous environmental variables. On the other hand a coefficient close to 0 indicated that the efficiency of the country can more be increased by the exogenous environmental variables.

Now considering the persistent level of inefficiency related to adjustment cost as a influential factor of ICT spillover effects, a country with strong ICT spillover effects should have a lower persistent level of inefficiency since this country is able to adjust organization with low cost to digest ICT investment. Hence, the following hypothesis is proposed:
Hypothesis 3: A low persistent level of inefficiency appears for a country with strong ICT spillovers.

5.2.2.2 Exogenous factors

A number of studies investigated the exogenous factors that influenced efficiency, including software investment, intangible education investment, skilled labours, R&D level, marketing of new products, etc. (Bernstein, J. I. and Mohnen, P. 1994; Cincera, M. and Potterie, B. 2001; Shao and Lin 2001; Doi, J. and Mino, K. 2005; Pohjola, M. 2001; Oulton, N. 2002; Garbacz, C. 2007). Among these factors, skilled labours and software investment were considered in this thesis.

Skilled labour

Skilled labour was chosen to test whether ICT, skilled labour and organizational adjustment complement each other in the industry-level. Skill-biased Technical Change (SBTC) theory suggests technological change increases the demand for skilled labour with respect to unskilled labour. Existing empirical studies suggest a correlation between ICT and the skilled labour at the worker level, firm level and industry level (Krueger 1993; Doms et al. 1997; Dunne et al. 1997). Besides these direct influences, some researchers have argued that technical progress also has an indirect effect on the skilled labour (Bresnahan et al., 2002). Skill-biased Organizational Change (SBOC) theory relates the organization and the skilled labour to point out that the adoption of new organizational systems (e.g. ICT) based on decentralized decision-making calls for more skilled labour (Milgrom and Roberts 1990; Caroli and Van Reenen 2001).

Later, Bresnahan (1999) argued that the SBTC and SBOC theories are two sides of the same coin. Bresnahan introduces the concept of organizational complementarities
between ICT and skilled labour. In this perspective, technological change, and particularly the adoption of ICT increases the demand for skilled labour which supports the idea that the adoption of ICT is more effective in organizations with more skilled labour and with decentralised workplace organizations (Machin and van Reenen, 1998; Autor, et al., 1998; Acemoglu, 1998; Bresnahan, et al., 2002; Card and Lemieux 2001; etc.). Thus, a larger supply of skilled labour should positively contribute to organizational adjustment (indicated by efficiency change) which stem from ICT spillover effects. Most previous studies mentioned above agreed with this conclusion, but were generally based on firm-level data with organizational data collected by a survey among firms.

Based on SBTC and SBOC, it is expected a significantly positive contribution of skilled labour to efficiency increased in and after the lag for a country that has strong ICT spillover effects with low persistent level of inefficiency. Hence the following hypothesis is proposed:

*Hypothesis 4: Skilled labour positively affects the efficiency during lag for a country with strong ICT spillover effects.*

**Software share**

The software share was close to organizational change. On the one hand, the ‘organizational computing’ was very important for the reduction of both the transaction cost and the management cost for the firms of ICT-using sectors, which was closely linked with the software itself. For example, enterprise resource planning (ERP), material requirement planning (MRP), database management systems (DBMS), and customer resource management (CRM) were widely used by all sectors of the entire economy (Steinmueller 1995). Thus, a high software share should increase efficiency due to the
reduction of the relevant cost of firms. On the other hand, software share can also increase the demand for skills labour and thus increase efficiency as well (Roach, 1991; Berndt et al., 1992; Stiroh, 1998). Accordingly this relationship the following hypothesis is proposed:

*Hypothesis 5: Software share positively affects the efficiency during the lag for a country with strong ICT spillover effects.*

Despite the hypothesis, existing literature revealed mixed findings. For example, Shao and Lin (2006) and Lin (2009) argued that IT spending did not necessarily improve production efficiency in a country, which was contrary to the conclusions reached by Hitt and Brynjolfsson (1996) and Lin and Shao (2000). As mentioned by Lin (2009), one possible reason for this was that a parametric frontier such as a Cobb-Douglas or Translog frontier alone might have caused misleading conclusions as far as the contributions of IT investment to productivity or productive efficiency are concerned. Therefore, a non-parametric time-variant frontier was employed in this study.

5.2.2.3 Measuring the influential factors of efficiency on one-stage approach

A one-stage approach was employed in this framework to avoid potential bias in the estimates obtained using the two-stage approach in many previous studies. At the higher-level, in the efficiency function where inefficiency was a dependent variable together with lagged inefficiency and two exogenous factors as independent variables, a negative coefficient of exogenous factors indicated a positive relationship between exogenous factors and an increased efficiency. A relatively small coefficient value of lagged inefficiency suggests a low persistent level of inefficiency.
5.2.3 Summary

This framework focuses on the lag period and efficiency change at the basic-level, and investigates the persistent level of inefficiency and the effects of software share and skilled labour on inefficiency at the higher-level. Based on these, the ICT spillover effects and the factors that affected the ICT spillover effects can be investigated by addressing the following three issues.

First, does a country have ICT spillover effects? A positive conclusion can be drawn if the two conditions are satisfied: (i) a lag exists and (ii) efficiency increases during or immediately after this lag. The first condition is linked to the GPT theory argument that ICT capital is an abnormal input that may decrease growth as resources are diverted to reorganization and learning (Basu et al. 2003). The second condition is based on the hypothesis of the GPT theory that ICT investments lead to reorganization in ICT-using industries (Basu et al. 2007). Both conditions can be tested via hypothesis 1 and 2 at the basic-level.

Secondly, how strong are a country’s ICT spillover effects? If a country has ICT spillover effects, then the length of the lag can be estimated. Based on the GPT theory, a country that obtains a positive return of ICT capital and output growth earlier than others indicates that this country has strong ICT spillover effects. Thus, the length of the lag is a measurement of the strength of ICT spillover effects, where the length of the lag is measured by the length of years with negative return of ICT capital at the basic-level.

Finally, what is the underlying factors driving ICT spillover effect? Since the persistent level of inefficiency dominates the organization situation, it consequently plays a major role of determining the strength of capability to adjust the organization, which is tested
through hypothesis 3 at the higher level. The persistent level of inefficiency was captured by the coefficient of the lagged inefficiency in the efficiency function of the higher level where the current inefficiency was a dependent variable and inefficiency that lagged a year was an independent variable. In addition, the effects of exogenous factors that influenced efficiency are to be investigated via testing hypotheses 4 and 5 at the higher level.

### 5.3 Model specifications

The two-level framework mentioned above was based on two models developed in Chapter 4. For the purpose of comparison, three other traditional models were also considered. The most commonly used approach to measuring the effects of ICT on productivity/output was the time-invariant growth accounting, i.e. Model 1 below. This approach does not take into account for the efficiency. The time-invariant parametric stochastic frontier model, Model 2, was also considered in which the efficiency could be estimated. Model 3 assumed a time-variant parametric stochastic frontier. Note that all these three models were one-level models, and thus they were unable to investigate the effects of environmental variables on efficiency.

In the literature the two-stage approach was usually used to solve this problem by adding an efficiency function used to measure the effect of environmental variables (e.g. Lin and Shen, 2002). From the statistical perspective, however, this may lead to a biased estimate and/or a lower statistical efficiency in inference (Koop et al. 1999). In contrast, a one-stage approach, Model 4, will be considered. It is the semi-parametric model developed in Chapter 4, including a non-parametric time-variant stochastic frontier at the basic-level and a parametric time-invariant efficiency function at the higher-level. Further, Model 5
extends the higher level of Model 4 to a non-parametric time-variant efficiency function. Detailed model specifications are depicted as follows.

5.3.1 One-level Models

Model 1: Time-invariant linear model

The traditional parametric model can be used to analyse the impact of ICT on output under the neoclassical growth accounting. Under some certain assumptions, a log-form Cobb-Douglas production function could be written as

\[ y = c + ak_{\text{non-ICT}} + bk_{\text{ICT}} + dl + v \]  

(5.1)

where \( k_{\text{non-ICT}} \) is the log of the non-ICT capital, \( k_{\text{ICT}} \) is the log of the ICT capitals, \( l \) is the log of labour, \( c \) is constant and \( v \) is residual. The output elasticity with respect to non-ICT capital, ICT capital, and labour \((a, b, d)\) can be estimated by Eq. (5.1) based on the observed \( y, k_{\text{non-ICT}}, k_{\text{ICT}}, l \).

For the industry-level panel data \((i = 1, ..., N; t = 1, ..., T)\), Eq. (5.1) could be rewritten as a time-invariant linear model for:

\[ y_{it} = \beta_0 + \beta_{nk} n_{it} + \beta_{ik} i_{it} + \beta_l l_{it} + v_{it}, \]  

(5.2)

where \( y_{it} \) is the log of the value added, \( n_{it} \) is the non-ICT capital stock, \( i_{it} \) is the ICT capital stock, and \( l_{it} \) is the log of labour for period \( t \) in industry \( i \) for an individual country. Eq. (5.2) could also be written as:

\[ y = \beta_0 + \beta_{nk} n + \beta_{ik} i + \beta_l l + v, \]  

(5.3)

where \( y = (y'_1, y'_T) \), \( n, i, l \) are \( NT \) vectors, \( v = (v'_1, v'_T) \) is an \( NT \) vector with two-sided noise, and \( \beta_0, \beta_{nk}, \beta_{ik} \) and \( \beta_l \) are time-invariant parameters. This model
assumed that the production function had the Cobb-Douglas form that allowed direct estimates of production function parameters. Although this traditional model has been widely used in previous studies, it has a limitation: it assumes that all industries in a country operate at full efficiency, which was clearly unreasonable in a cross-country comparison study. Thus a time-invariant linear stochastic frontier model was also used as described next.

Model 2: Time-invariant linear stochastic frontier model

When the industries in a country have different efficiencies, an efficiency term should be included into the production function. Eq. (5.2) and Eq. (5.3) could be rewritten as the following inefficiency function:

\[ y_{it} = \beta_0 + \beta_{nk} n_{it} + \beta_{ik} k_{it} + \beta_l l_{it} + v_{it} - u_{it}, \]  

or

\[ y = \beta_0 + \beta_{nk} n + \beta_{ik} k + \beta_l l + v - u, \]  

where \( u = (u_1, \ldots, u_T) \) is an \( NT \) vector of non-negative technical inefficiency errors. The other variables were defined similarly as in Eq. (5.2). \( v_{it} \) and \( u_{it} \) were assumed to be mutually independent as well as independent of \( nk_{it}, ik_{it} \) and \( l_{it} \). The Model in Eq. (5.4) collapses to Model in Eq. (5.2) without the inefficiency term. The drawback of Model 2 was that it assumed a time-invariant linear production function, which could not be used to measure technological progress (TP). Hence, a time-variant linear stochastic frontier model is considered next.

Model 3: Time-variant linear SF model

The time-variant form of Eq. (5.4) is:
\[ y_{it} = \beta_{0,t} + \beta_{nk,i} n_{ik} + \beta_{ik,i} k_{it} + \beta_{l,i} I_{it} + v_{it} - u_{it}, \]  

(5.6)

where \( \beta_{0,t}, \beta_{nk,i}, \beta_{ik,i} \) and \( \beta_{l,i} \) are time-variant parameters. This model will reduce to the time-invariant linear model in Eq. (5.4) if the time-variant parameters become time-invariant. It is important to note that this model allows for frontier shifts over time and thus, \( \beta_{0,t} - \beta_{0,t-1} \) represents technological progress (TP).

Two-stage Approach

Models 1, 2, and 3 are one-level models that cannot measure the impact of exogenous environmental variables on the efficiency or TFP (Note that Model 1 does not include an inefficiency term). Previous studies have used a two-stage approach to solve this problem. In the two-stage approach, the efficiency was estimated from model 2 or model 3 with the environmental variables omitted in the first-stage. Then, the efficiencies obtained from the first stage regressed on the environmental variables. However, Koop (1999) argued that the two-stage approach generated biased estimates because it used posterior means as data in the second stage and thus this ignored the uncertainties in the point estimates. An example of the two-stage approach is:

\[ y_{it} = \beta_{0,t} + \beta_{nk,i} n_{ik} + \beta_{ik,i} k_{it} + \beta_{l,i} I_{it} + v_{it} - u_{it}, \]  

(5.7)

\[ u_{it} = \gamma_{\alpha} + \gamma_{hl} h_{lt} + \phi_{0} + \phi_{ss} s_{lt} + \epsilon_{it}, \]

where \( u = (u_1, \ldots, u_T) \) is an \( NT \) vector of non-negative technical inefficiency errors, \( h_{lt} \) is the share of skilled labour, \( \gamma_{hl} \) represents the coefficient of variable \( h_{lt} \), \( s_{lt} \) is the share of software, and \( \phi_{ss} \) is the coefficient of variable \( s_{lt} \). The other variables were already defined for Eq. (5.6).
5.3.2 Two-level Models

The one-level models could not be used to explore the effects of the environmental variables on TFP or efficiency without the accompaniment of the two-stage approach. The two general stochastic frontier models developed in Chapter 4 were two-level models that investigated the efficiency and the effect of exogenous environmental variables on the efficiency in one-stage. The first model was a semi-parametric dynamic stochastic frontier with a time-variant non-parametric frontier at the basic level and a time-invariant linear function for technical inefficiency at the higher level. The second model relaxed the linear functional form for technical inefficiency at the higher level.

Model 4: Semi-parametric dynamic stochastic frontier model

\[
\begin{align*}
\gamma_{t} &= f_{nk,t}(nk_{t}) + f_{lk,t}(ik_{t}) + f_{l,t}(l_{t}) - u_{t} + \nu_{t}, \\
\log u_{t} &= \gamma_{hl,t}h_{l_{t}} + \phi_{ss}s_{s_{t}} + \rho \log u_{t-1} + \epsilon_{t}, \text{ for } t = 2, \ldots, T, \\
\log u_{1} &= \gamma_{hl,1}h_{l_{1}} + \phi_{ss,1}s_{s_{1}} + \epsilon_{1}, \text{ for } t = 1,
\end{align*}
\]

where \( u = (u_1', \ldots, u_T') \) is an \( NT \) vector of non-negative technical inefficiency errors. The other variables were already defined in Eq. (5.6). Note that Eq. (5.8) does not assume that \( u_{1} \) and \( \{u_{t}, t > 1\} \) originate from the same process, but allows for the possibility that the first observation is different. In Eq. (5.8), \( h_{l_{t}} \) is the share of skilled labour, \( \gamma_{hl,1} \) is the coefficient of \( h_{l_{1}} \) during the first year (1980), and \( \gamma_{hl} \) is the coefficient of \( h_{l_{t}} \) during the other years. Similarly, \( s_{s_{t}} \) is the share of software, \( \phi_{ss,1} \) is the coefficient of \( s_{s_{1}} \) during the first year (1980), and \( \phi_{ss} \) is the coefficient of \( s_{s_{t}} \) during the other years.

Model 5: Non-parametric dynamic stochastic frontier model
\[ y_{it} = f_{nk,t}(nk_{it}) + f_{ik,t}(ik_{it}) + f_{l,t}(l_{it}) - u_{it} + v_{it}, \]
\[ \log u_{it} = g_{hl,t}(h_{it}) + g_{ss,t}(ss_{it}) + \rho \log u_{i,t-1} + \varepsilon_{it}, \quad \text{for } t = 2, \ldots, T, \quad (5.9) \]
\[ \log u_{it} = h_{hl,t}(h_{it}) + h_{ss,t}(ss_{it}) + \varepsilon_{it}, \quad \text{for } t = 1, \]

where \( h_{hl,t}() \) and \( h_{ss,t}() \) are the non-parametric time-variant functions that represent the effect of skilled labour and software for the first year (1980), and \( g_{hl,t}() \) and \( g_{ss,t}() \) are the non-parametric time-variant functions for the subsequent years. The other variables were already defined in Eq. (5.8).

Based on the two-level models, the new framework could measure the lag and efficiency change at the basic level, and the persistent level of inefficiency and the effects of software and skilled labour on inefficiency at the higher level as follows: (i) the lag time was measured by the number of years with negative return of ICT capital. The negative return of ICT capital was captured by the return of ICT capital \((f_{ik,t})\) on the non-parametric frontier at the basic-level in either model 4 or model 5. For example, based on the data of 24 ICT-using industries in the US during 1980-2000, if there is a negative trend of ICT capital return \((f_{ik,t})\) over successive years in the \(\log Y_{it} - \log K_{ICT}\) space, then the length of the lag captured by the number of these successive years; (ii) the efficiency change during the lag could be examined by estimating the inefficiency term \((u_{it})\) at the basic-level in either model 4 or model 5. A decrease of \(u_{it}\) during the lag indicated ICT spillover effects for a country; (iii) the persistent level of inefficiency could be captured by \(\rho\) at the higher level in Model 4 or Model 5. A relatively small value of \(\rho\) indicates a weak persistent level of inefficiency; (iv) the effects of skilled labour and software share on inefficiency could be explored by the coefficients of the exogenous supply of skilled labour \((\gamma_{hl,t} \) and \(\gamma_{hl}\)) and software \((\phi_{ss,t} \) and \(\phi_{ss}\)) at the higher level in...
model 4. The non-parametric efficiency function in model 5 revealed the effect of the exogenous supply of skilled labour and software share by \((h_{hl,t}, g_{hl,t})\) and \((h_{ss,t}, g_{ss,t})\). For strong ICT spillover effects indicated by shorter lag, it is expected a value of \(\rho\) that would be far less than one, and statistically significant and negative values for \((\gamma_{hl,t} \text{ and } \gamma_{hl})\) and \((\phi_{ss,t} \text{ and } \phi_{ss})\) in Model 4. In Model 5, a negative trend of \(g_{hl,t}\) and \(g_{ss,t}\) during successive years in the \(\text{log}\ u_t - \text{log} \text{ (skilled labour share)}\) (i.e. \(\text{log} u_t - hl_a\)) space and the \(\text{log}\ u_t - \text{log} \text{(software share)}\) (i.e. \(\text{log} u_t - ss_a\)) space are expected.

5.4 Concluding remarks

Based on the GPT theory, this chapter built a new framework to measure ICT spillover effects by choosing efficiency as a proxy of co-invention. The framework could capture the length of the lag of ICT spillover effects as a proxy of strength of ICT spillover effects by using a dynamic non-parametric SFA model. Furthermore, the framework could also measure the effects of factors that influenced ICT spillover effects by considering the persistent level of inefficiency as an endogenous factor and exogenous factors as environmental variables of efficiency. This framework will be used to measure ICT spillover effects in the US and the UK respectively and to explore the reasons of different ICT spillover effects between them in the next chapter.
Chapter 6 ICT spillover in the US and UK

6.1 Introduction

Based on the GPT theory, Chapter 5 built a new framework of measuring ICT spillover effects to fill the gaps of theoretical guidance. This chapter used this framework to investigate ICT spillover effects and explored those factors that might contribute different ICT spillovers in the US and UK.

ICT spillover effects in the US were first examined by testing Hypothesis 1&2 proposed in Chapter 5. Strong ICT spillover effects indicated by a short lag in the US were observed, which was consistent with the conclusion in the previous studies (Basu et al. 2007). Furthermore, the new framework gave the measurement of the lag with 6-8 years in the US.

Then, the ICT spillover effects in the UK were investigated in section 6.3. The result of a weak ICT spillover indicated by a long lag in the UK was in line with the conclusion of Basu et al. (2003). Furthermore, the longer lag of 12-13 years in the UK means the UK had been in the process of organizational adjustment up to 2000 and thus contributed to the slowdown in TFP growth of the UK, which could not be explained by Basu et al. (2003) due to an inappropriate proxy of co-invention employed in their work. Comparison of ICT spillovers in the UK in relation to the US was presented in section 6.4.
Finally, the reasons for the different ICT spillover effects between the US and the UK were explored in section 6.5. A stronger persistent level of technical inefficiency in the UK than that in the US suggested that the persistent level of inefficiency was a crucial factor for the divergent ICT spillover effects between the UK and the US, apart from the distinguished ICT investment level as commonly mentioned in previous studies (Schreyer, 2000, Daveri, 2002 and Basu et al. 2003). Having a strong persistent level of inefficiency in the UK further limited the influence of exogenous factors on inefficiency, such as the supply of skilled labour and the software investment.

6.2 ICT spillover effects in the US at the basic-level

This section focuses on the measurement of ICT spillover effects in the US by testing hypotheses 1 and 2 proposed in Chapter 5 at the basic-level of this framework, whereas the ICT spillover effects for the UK will be investigated in the next section.

6.2.1 Measurement of the lag time

*Hypothesis 1: ICT capital is an abnormal capital that can lead to the initially negative return of ICT capital to output for a country with ICT spillover effects.*

This hypothesis can be tested using the US data over the period of 1980-2000. Clearly a model with traditional parametric production function was incapable to address this issue due to its implicit assumptions of monotonicity and concavity for the underlying production function, for example in the form of Cobb-Douglas or translog. After comparing with Models 1-3 with parametric production functions, this sub-section used
Models 4 and 5 with a non-parametric stochastic frontier to identify the abnormal ICT capital and measure the lag time for the US.

Most previous studies treated ICT capital as a normal input and used a linear or log-linear production function to measure its positive return to output. For example, Dewan and Kraemer (2000) estimated a Cobb-Douglas function with GDP as the output and IT capital, non-IT capital and labour hours as the inputs using pooled annual data from 36 countries during 1985-1993. Their result showed that output elasticities of non-IT capital, IT capital, and labour for the 22 developed countries were 0.16, 0.057 and 0.823, respectively. Based on the US data over the period of 1980-2000, Model 1 with the time-invariant linear production function produced quantitatively similar results. As shown in TABLE 6.1, output elasticities of non-ICT capital, ICT capital, and labour were significantly positive with the values 0.18, 0.16, and 0.62, respectively.

| TABLE 6.1 Output elasticities of inputs estimated by Model 1 for US |
|-----------------|-----------------|-----------------|
| Non-ICT capital | ICT capital     | Labour          |
| 0.18* (0.02)    | 0.14* (0.01)    | 0.62* (0.02)    |

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates p<0.05.

However, not all models with a linear production function can ensure a significantly positive elasticity of ICT capital. In other words, the estimate of positive elasticity of ICT capital depends on models with various specifications. For example, Pohjola (2002) reported that ICT capital was not statistically significant at the cross-sectional estimate of economic growth in 42 countries during 1985-1999. When the time-invariant production function in Model 1 changed to the time-variant linear production frontier in Model 3 and applied to the present US data, the obtained results were similar to that of Pohjola (2002). As shown in TABLE 6.2, output elasticities of ICT capital were insignificant over some years, e.g. 1982-1988 with one anomaly in 1983.
<table>
<thead>
<tr>
<th>Year</th>
<th>Non-ICT</th>
<th>ICT</th>
<th>Labour</th>
<th>Year</th>
<th>Non-ICT</th>
<th>ICT</th>
<th>Labour</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
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<td>0.10*</td>
<td>0.84*</td>
<td>1990</td>
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<td>0.12*</td>
<td>0.67*</td>
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<td>(0.04)</td>
<td>(0.08)</td>
<td></td>
<td>(0.09)</td>
<td>(0.06)</td>
<td>(0.07)</td>
</tr>
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<td>1991</td>
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<td>0.14*</td>
<td>0.65*</td>
</tr>
<tr>
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<td>(0.04)</td>
<td>(0.08)</td>
<td></td>
<td>(0.09)</td>
<td>(0.06)</td>
<td>(0.08)</td>
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<td>0.81*</td>
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<td>0.14*</td>
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<td></td>
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<td>(0.06)</td>
<td>(0.07)</td>
</tr>
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<td>0.08</td>
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<td>0.20*</td>
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<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>1985</td>
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<td>0.69*</td>
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<td>0.23*</td>
<td>0.54*</td>
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<td>(0.09)</td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>1986</td>
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<td>(0.07)</td>
<td>(0.07)</td>
</tr>
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<td>(0.07)</td>
<td>(0.08)</td>
</tr>
<tr>
<td>1988</td>
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<td>0.10</td>
<td>0.61*</td>
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</tr>
<tr>
<td>1989</td>
<td>0.18</td>
<td>0.11*</td>
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<td>1999</td>
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<td>0.27*</td>
<td>0.44*</td>
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<tr>
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<td>(0.07)</td>
<td>(0.07)</td>
</tr>
<tr>
<td>1990</td>
<td>0.19*</td>
<td>0.12*</td>
<td>0.67*</td>
<td>2000</td>
<td>0.23*</td>
<td>0.24*</td>
<td>0.47*</td>
</tr>
<tr>
<td></td>
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<td>(0.06)</td>
<td>(0.07)</td>
<td></td>
<td>(0.10)</td>
<td>(0.07)</td>
<td>(0.08)</td>
</tr>
</tbody>
</table>

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates \( p < 0.05 \).

The insignificant results on some output elasticities of ICT capital in the US might attribute to the fact that the possible negative return of ICT capital cannot be captured in the linear relationship specified in Model 3 due to the implicit monotonicity and concavity restrictions. This can be supported by some previous studies that argued that traditional parametric especially linear production function is an inappropriate function form to explore the input-output relationship. For example, Costa and Markellos (1997) using a neural network approach found the so-called ‘congested area’ with a negative slope between inputs and outputs in the London underground from 1970 to 1994 in terms of fleet size and workers (inputs) and the millions of train km per year covered by the fleet (outputs). Some researchers in their educational production function analysis have also found that traditional restrictive specifications failed to capture potential non-linear
effects of school resources (Baker, 2001). Thus, a non-parametric production function without explicit assumption of monotonicity and concavity is more appropriate to capture the negative return of these abnormal inputs.

Indeed, the results of Model 4 with the non-parametric time-variant production function presented a negative relationship between ICT capital and the return of ICT capital to output in some years for the US. For example, the negative return of ICT capital in 1987 in the US was shown in Fig 6.1, where the abscissa is the log ICT capital stock for 24 ICT-using industries in 1987 (i.e. $k_{i,1987}$ where $i=1,...,24$ in Eq. (5.8)) and the ordinate is the return on the log ICT capital of the 24 ICT-using industries in 1987 (i.e. $f_{ik,1987}(ik_{i,1987})$ where $i=1,...,24$ in Eq. (5.8)).

![FIGURE 6.1 Non-linear return of ICT capital in 1987 in the US estimated by Model 4](image)

Note the curve displayed in Fig 6.1 is the normalised return of log ICT capital rather than a true return of log ICT capital. Thus, Fig 6.1 does not give a quantitative relationship
between log ICT capital and the return of log ICT capital. Rather, it shows the qualitative linkage between log ICT capital and its return. In other words, Fig 6.1 only addresses the issue whether the return of ICT capital rises or falls when ICT capital increases. The detail is given in Appendix B.

Fig 6.2 presented the return of log ICT capital in relation to log ICT capital in the US for each year over the period of 1981-2000. Similar to Fig 6.1, in each sub-plot, the abscissa was the annual log ICT capital stock for 24 ICT-using industries and the ordinate was the annual return of the log ICT capital of these 24 ICT-using industries.

Fig 6.2 showed that a negative return of ICT capital existed in the US during 1982-1989 and then this negative return of ICT capital turned into a positive return in 1990 and the
US had kept the positive return up to 2000. In other words, there was initially negative return of ICT capital in the US.

For comparison purpose, annual returns of non-ICT capital and labour over the period 1981-2000 were also presented in Fig 6.3 and Fig 6.4. It can be seen from Fig 6.2, Fig 6.3, and Fig 6.4 that the return of normal input, such as non-ICT capital and labour, was positive during all periods, while the return of ICT capital was negative in some periods. Therefore the empirical evidence is in favour of the claim that the ICT capital is an abnormal capital, hence supports Hypothesis 1 that the ICT capital is an abnormal capital and the initially negative return of ICT capital to output existed in the US.

The results from Model 5 gave the same conclusion as Fig 6.5 showed initially negative returns to ICT capital during 1983-1988.
Apart from the conclusion of ICT capital as an abnormal capital, further remarks can be made as follows. First, the analysis provided empirical evidence to the claim that ICT as a GPT may generate initial periods with negative contribution to output due to the relevant organization adjustment. The existence of the years with negative return of ICT capital provides a measurement of lag period as the length of years with negative return to output. It can also been seen that, to come to the conclusion that the ICT capital is an abnormal capital and to capture the lag period, a non-parametric production function/frontier which relaxes the assumptions of monotonicity and concavity is crucial in the analysis.

**Discussion**

Two issues related to the negative return of ICT capital should be noted. The first was whether the negative return of ICT capital in Fig 6.2 was purely a reflection of heterogeneity in ICT capital across industries. O'Mahony et al. (2005) argued that traditional industry panel data analysis could fail to discover a positive contribution mainly due to heterogeneity in ICT across industries, particularly in the time dimension. Therefore, negative elasticity of ICT capital estimated in their studies was mainly due to ICT capital heterogeneity across industries.

In O'Mahony et al. (2005), the average ICT to total capital ratio (ICT/TK) was used to measure ICT capital heterogeneity across industries. Fig 6.6 showed the average ICT to total capital ratio (ICT/TK) by industry for the United States over the period 1980-2000. It was evident that the magnitude of heterogeneity across industries was not weakened over time, but strengthened. Following their logic, if the result of negative return of ICT capital stemmed from the bias of heterogeneity, then a longer lag in the US during 1980-

Indeed, as shown in Fig 6.2, there was a clear trend that negative return of ICT capital was gradually transforming to a positive one, which indicated that the non-linear negative return of ICT capital was not determined by ICT capital heterogeneity across industries.

The second issue was whether the application of this non-linear model leads to the negative return of ICT capital in the US. If this is true, then the return of all three inputs should be affected. The return of non-ICT capital and labour listed in Fig 6.3 and Fig 6.4 presented traditional positively returns of the inputs in these non-linear models. In other words, the returns of traditional inputs remained positive in a non-linear model, which provides the support to the argument that negative return of ICT capital were not due to the usage of a non-parametric model.
6.2.2 Estimates of efficiency

Hypothesis 2: Efficiency increases during and/or immediately after the lag for a country with ICT spillover effects.

This hypothesis can be tested using the US data over the period of 1980-2000. In subsection 6.2.1, the results of Model 4 and Model 5 showed an 8-years time lag during 1982-1989 and a 6-years time lag during 1983-1988 in the US, respectively, in which the ICT capital presented a negative return. According to the GPT theory, this lag reflected a reorganization process if the US had ICT spillover effects during this lag. A reorganization process was experienced during and/or after the lag, which can be evidence by the increase in efficiency. Thus, it was possible to observe an improvement of efficiency in and/or after the lag period, if the US had ICT spillover effects.

TABLE 6.3 showed the efficiency results for the US data during 1980-2000 estimated by Model 4. The sign of return on ICT capital in 1980-2000 was also listed in TABLE 6.3 for the purpose of indicating the lag. From the results of Model 4, the lag period was from 1982 to 1989. TABLE 6.3 showed that the efficiency in the US increased from 0.62 in 1982 to 0.66 in 1989 and reached the peak 0.68 in 1986. Furthermore, immediately after the lag period, the efficiency was substantially increased: the average efficiency rose to 0.70 in 1990s compared to the average efficiency level of 0.65 in the lag period in 1980s. Hence the empirical evidence supported Hypothesis 2.
### TABLE 6.3 Efficiencies in the US estimated by Model 4

<table>
<thead>
<tr>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>+</td>
<td>0.67* (0.20)</td>
<td>1990</td>
<td>+</td>
<td>0.70* (0.15)</td>
</tr>
<tr>
<td>1981</td>
<td>+</td>
<td>0.64* (0.22)</td>
<td>1991</td>
<td>+</td>
<td>0.70* (0.15)</td>
</tr>
<tr>
<td>1982</td>
<td>-</td>
<td>0.62* (0.20)</td>
<td>1992</td>
<td>+</td>
<td>0.70* (0.14)</td>
</tr>
<tr>
<td>1983</td>
<td>-</td>
<td>0.61* (0.18)</td>
<td>1993</td>
<td>+</td>
<td>0.68* (0.14)</td>
</tr>
<tr>
<td>1984</td>
<td>-</td>
<td>0.62* (0.17)</td>
<td>1994</td>
<td>+</td>
<td>0.68* (0.14)</td>
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<tr>
<td>1985</td>
<td>-</td>
<td>0.62* (0.16)</td>
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<td>0.76* (0.14)</td>
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<tr>
<td>1986</td>
<td>-</td>
<td>0.68* (0.15)</td>
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<td>0.74* (0.13)</td>
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<tr>
<td>1987</td>
<td>-</td>
<td>0.65* (0.14)</td>
<td>1997</td>
<td>+</td>
<td>0.68* (0.13)</td>
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<td>1988</td>
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<td>1998</td>
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<tr>
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<td>1999</td>
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<td>0.71* (0.13)</td>
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<tr>
<td>1990</td>
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<td>2000</td>
<td>+</td>
<td>0.69* (0.14)</td>
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<tr>
<td>Average</td>
<td>In 1980s</td>
<td>0.64</td>
<td>Average</td>
<td>In 1990s</td>
<td>0.70</td>
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</table>

*Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates p<0.05.*

Model 5 used a different efficiency function than Model 4 at the higher level but Model 5 generated a similar outcome as Model 4, as shown in TABLE 6.4. The result of Model 5 suggested a 6-year lag during 1983-1988. TABLE 6.4 also showed that the efficiency of the US increased from 0.65 in 1983 to 0.70 in 1986, although efficiency was 0.64 in 1988. The average efficiency in 1980s with the lag time was 0.65, which was less than the average efficiency (0.69) in 1990s. This result also supported Hypothesis 2.
TABLE 6.4 Efficiencies in the US estimated by Model 5

<table>
<thead>
<tr>
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<th>Return of ICT capital</th>
<th>Efficiency</th>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
</tr>
</thead>
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<td>1990</td>
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<td>0.71*</td>
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<tr>
<td></td>
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<td>1992</td>
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<tr>
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<td>-</td>
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<td>1998</td>
<td>+</td>
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</tr>
<tr>
<td>1989</td>
<td>+</td>
<td>0.66*</td>
<td>1999</td>
<td>+</td>
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<tr>
<td>1990</td>
<td>+</td>
<td>0.71*</td>
<td>2000</td>
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<td></td>
<td>(0.16)</td>
<td></td>
<td></td>
<td>(0.14)</td>
</tr>
</tbody>
</table>

Average In 1980s 0.65 In 1990s 0.69

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates p<0.05.

For the purpose of comparison between the non-parametric models and the parametric models, the efficiency change using the traditional parametric SF models were also investigated. Model 2 with the time-invariant stochastic frontier (SF) and Model 3 with time-variant SF were employed to estimate the efficiency on the same data of the US used by Models 4 and 5. Again the results confirmed that the parametric models were not applicable to investigate ICT spillover effects.

The efficiency estimated by Models 2 and 3, as shown in TABLE 6.5, presented a stable efficiency during 1980-2000, which suggested no significant efficiency change during 1980-2000. It can be seen from TABLE 6.5 that the efficiency was within the ranges of 0.96-0.97 and 0.81-0.83, respectively, during the all periods of Model 2 and Model 3. That is, the model with a parametric frontier did not capture the efficiency change.
associated with ICT spillover effects. This further justifies the use of non-parametric models in the study of ICT spillover effects.

**TABLE 6.5 Efficiencies in the US estimated by Models 2 and 3**

<table>
<thead>
<tr>
<th>Year</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Year</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1980</td>
<td>0.96* (0.03)</td>
<td>0.81* (0.12)</td>
<td>1990</td>
<td>0.96* (0.01)</td>
<td>0.83* (0.07)</td>
</tr>
<tr>
<td>1981</td>
<td>0.96* (0.03)</td>
<td>0.80* (0.12)</td>
<td>1991</td>
<td>0.96* (0.01)</td>
<td>0.83* (0.06)</td>
</tr>
<tr>
<td>1982</td>
<td>0.96* (0.03)</td>
<td>0.80* (0.12)</td>
<td>1992</td>
<td>0.97* (0.01)</td>
<td>0.83* (0.06)</td>
</tr>
<tr>
<td>1983</td>
<td>0.96* (0.02)</td>
<td>0.81* (0.11)</td>
<td>1993</td>
<td>0.97* (0.01)</td>
<td>0.84* (0.06)</td>
</tr>
<tr>
<td>1984</td>
<td>0.96* (0.02)</td>
<td>0.81* (0.10)</td>
<td>1994</td>
<td>0.97* (0.01)</td>
<td>0.84* (0.06)</td>
</tr>
<tr>
<td>1985</td>
<td>0.96* (0.02)</td>
<td>0.82* (0.10)</td>
<td>1995</td>
<td>0.97* (0.01)</td>
<td>0.84* (0.06)</td>
</tr>
<tr>
<td>1986</td>
<td>0.96* (0.02)</td>
<td>0.82* (0.09)</td>
<td>1996</td>
<td>0.97* (0.01)</td>
<td>0.84* (0.06)</td>
</tr>
<tr>
<td>1987</td>
<td>0.96* (0.01)</td>
<td>0.82* (0.08)</td>
<td>1997</td>
<td>0.97* (0.01)</td>
<td>0.83* (0.07)</td>
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<tr>
<td>1988</td>
<td>0.97* (0.01)</td>
<td>0.82* (0.08)</td>
<td>1998</td>
<td>0.97* (0.01)</td>
<td>0.83* (0.07)</td>
</tr>
<tr>
<td>1989</td>
<td>0.97* (0.01)</td>
<td>0.83* (0.07)</td>
<td>1999</td>
<td>0.96* (0.01)</td>
<td>0.83* (0.07)</td>
</tr>
<tr>
<td>1990</td>
<td>0.96* (0.01)</td>
<td>0.83* (0.07)</td>
<td>2000</td>
<td>0.97* (0.01)</td>
<td>0.83* (0.07)</td>
</tr>
</tbody>
</table>

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates p<0.05.

In addition, it can be seen that the estimated efficiencies from Models 2 and 3 were substantially larger than those estimated by Models 4 and 5. One possible explanation for this was that Models 4 and 5 used a non-parametric frontier and thus could measure the efficiency more accurately. For example, Fig. 6.7 showed how the measurement of the technical efficiency based on the non-parametric frontier can be either larger or smaller than the technical efficiency estimated by a model with a linear frontier. In Fig 6.7, line AB represents a linear parametric frontier and curve CD represents a non-parametric frontier. A firm increased the efficiency from $u_E$ at point E to $u_F$ at point F with the parametric frontier AB. This firm could also present a decrease in efficiency from $U_E$ at
point E to $U_F$ at point F with the non-parametric frontier CD. Thus, the efficiency estimated by Models 4 and 5 with non-parametric frontier can be lower than the efficiency estimated by Models 2 and 3 with a linear frontier.

![Illustration of larger efficiency estimated by Models 2 and 3](image)

**FIGURE 6.7 Illustration of larger efficiency estimated by Models 2 and 3**

### 6.2.3 Summary

Having tested Hypotheses 1&2 proposed in Chapter 5, this section has examined ICT spillover effects in the US with the new framework. As mentioned in section 5.2.3, the two conditions on the ICT spillover effects for a country are the existence of a lag period and the increase of efficiency in and after this lag. For the first condition, the test of Hypothesis 1 in section 6.2.1 showed that ICT capital as an abnormal capital led to the initially negative output of ICT capital in the US and thus the existence of lag in the US. For the second condition, based on the test of Hypothesis 2 in section 6.2.2, it was concluded that the increased efficiency during and after lag with negative return of ICT capital could be obtained in the US. With the two conditions being satisfied in the US case, it was argued that the US had ICT spillover effects in 1980-2000, based on the new framework developed in section 5.2. This conclusion is consistent with most other literature (e.g. Bresnahan and Trajtenberg 1995, Basu et al. 2003, 2007).
Further, apart from the existence of ICT spillover effects in the US, the new framework also contributed to the previous literature by providing an approach to measuring the length of the lag period in the US. As mentioned in the Chapter of literature review, the previous empirical work on the GPT theory could not clearly explain ICT spillover effects since they could not measure the length of lags. A possible reason could be the neglect of ICT capital as an abnormal input and thus there was mismatch between their assumption and the reality. According to the GPT theory, the true return of ICT capital may be negative during lag where ICT is treated as a GPT. However, the previous empirical studies typically assume a parametric production function that cannot capture the negative return of ICT capital (due to the parametric production function’s characteristic of monotonicity and concavity), as demonstrated earlier using Models 1, 2, and 3.

6.3 ICT spillover effects in the UK at the basic-level

With regard to the existence of ICT spillovers in the UK, there were mixed findings in previous literature. Some argued that there were no ICT spillover effects at all in the UK, which was a reason why the growth in the UK lagged behind the US combing the fact that relatively small size of ICT-producing sectors and relative low ICT investment (Stiroh 2002a; O’Mahony and Vecchi 2005). Others suggested that ICT did appear as a GPT with different pattern of ICT spillover effects in both the UK and US (Base et al. 2003).

The mixed findings may be due to the biased measurement of an incorrect model. The new framework developed in Chapter 5 can be used to fill in the theoretical gap and the
methodology gap. Section 6.2 showed that this framework worked well for the US case. Thus, this section used the new framework to investigate ICT spillover effects in the UK in order to have a clear picture of ICT spillover effects in the UK.

6.3.1 Measurement of the lag time

The models 4 and 5 were applied to analyse the UK data during the period of 1980 to 2000. The results of Models 4 and 5 showed that the ICT capital was also an abnormal input for the case of the UK. For example, there was a negative return of ICT capital in the UK in 1982 in Model 5. This was shown in Fig 6.8, where the abscissa was the log ICT capital stock (i.e. $i_{k_{1,24}}$ where $i=1,..,24$ in Eq. (5.9)) and the ordinate was the return on the log ICT capital (i.e. $f_{i_{k_{1,24}}}$ where $i=1,..,24$ in Eq. (5.9)) for the 24 industries using ICT in the year mentioned above.

Since the ICT capital had acted as an abnormal input for the UK, the lag of the UK can be captured by the years with negative return of ICT capital. This is summarized in TABLE 6.6, where “+” indicates a positive return of ICT capital and “-” reflects a negative return.
of ICT capital. The results of Model 4 shown in TABLE 6.6 indicated a lag period in the UK during 1989-2000 with one anomaly in 1984. Similarly, the result of Model 5 shown in TABLE 6.6 suggested that the UK’s the lag period was during 1988-2000, with a couple of anomalies in 1982, 1984-85, and 1993. The anomalies in the early 1980s could reflect the fact that during these initial years the ICT spillover just started to take effect but was still quite marginal.

<table>
<thead>
<tr>
<th>Year</th>
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<th>Year</th>
<th>Return of ICT capital</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Model 5</td>
<td></td>
</tr>
<tr>
<td>1980</td>
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<td>1995</td>
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<tr>
<td>1986</td>
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<td>+</td>
<td>1996</td>
</tr>
<tr>
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<td>+</td>
<td>+</td>
<td>1997</td>
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<tr>
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<td>-</td>
<td>1998</td>
</tr>
<tr>
<td>1989</td>
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<td>-</td>
<td>1999</td>
</tr>
<tr>
<td>1990</td>
<td>-</td>
<td>-</td>
<td>2000</td>
</tr>
</tbody>
</table>

The lag in the UK seems to be much longer compared to the US and to be held to year 2000, i.e. a successive 12-year lag during 1989-2000 in Model 4 and a successive 13-year lag in 1988-2000 in Model 5. This supported the Hypothesis 1 for the UK.

### 6.3.2 Estimate of efficiency

Another condition that should be satisfied by a country with ICT spillover effects was the increased efficiency during and after lag. TABLE 6.7 presents the estimates of efficiencies during 1980-2000 for the UK estimated by Model 4. The results of Model 4 showed that the UK had increased efficiency during the lag period. For instance the UK improved its efficiency from 0.55 in 1989 to 0.59 in 2000.
TABLE 6.7 Efficiencies in the UK estimated by Model 4

<table>
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<tr>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
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<tr>
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<td>0.41*</td>
<td>1991</td>
<td>-</td>
<td>0.55*</td>
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<td>1993</td>
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<tr>
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<td>1998</td>
<td>-</td>
<td>0.54*</td>
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</table>

Note: (i) “+” indicates a positive return of ICT capital and “-” reflects a negative return of ICT capital. (ii) Posterior means and posterior standard deviations in parentheses. (iii) * indicates \( p<0.05 \).

The results from Model 5 were displayed in TABLE 6.8, which are similar to the results from Model 4. It can be seen from TABLE 6.8 that the efficiency increased in the lag period. For instance it was 0.63 in 1988 and then increased to 0.68 in 2000. Since the UK had been in the lag period in year 2000, it could not examine whether efficiency increased after lag due to the data of 1980-2000 used in this study.

Both the results of Models 4 and 5 supported Hypothesis 2 in Chapter 5 for the UK. That is, efficiency increased during the lag for the UK. It can also been seen from tables 6.7 and 6.8 that the efficiency estimates differed when different models were used. Since at the higher level of modelling, Model 4 is more restrictive with a linear structure whereas
a non-parametric structure is specified in Model 5, it seems that the efficiency estimates in TABLE 6.8 are more reliable.

<table>
<thead>
<tr>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
<th>Year</th>
<th>Return of ICT capital</th>
<th>Efficiency</th>
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</thead>
<tbody>
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<td>1990</td>
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<td>0.62*</td>
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<td>1997</td>
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</table>

Note: (i) “+” indicates a positive return of ICT capital and “-” reflects a negative return of ICT capital. (ii) Posterior means and posterior standard deviations in parentheses. (iii)* indicates p<0.05.

### 6.3.3 Summary

As shown in Tables 6.6, 6.7 and 6.8, the UK had a lag and efficiency increase in the lag period. In other words, the UK had ICT spillover effects during 1980-2000 because it satisfied the two conditions of ICT spillover effects. These results provided evidence for the argument that ICT spillover effects appeared in the UK.

The previous literature that claimed there were no ICT spillover effects in the UK was usually based on the traditional growth accounting method. As mentioned in Chapter 2,
the growth accounting studies showed that during the 1990s, there was TFP growth in some industries using ICT in the US, but not in the EU including the UK (Schreyer 2000, Jorgenson, Ho and Stiroh 2005, Triplett and Bosworth 2004, Inklaar, O’Mahony and Timmer 2003, Vecchi and Albers 2002, O’Mahony and Vecchi 2005). However, those who believed in the GPT theory argued that the measured TFP in the above results of the traditional growth accounting method was biased due to the neglect of ICT investment’s co-invention. According to the GPT theory, the measured TFP should fall during the initial period of ICT investment as resources are diverted for reorganization and learning (Basu et al. 2003, 2007). Thus, co-invention, such as the unobserved complementary organizational capital, should take a place into the production function. Their conclusion based on this idea suggested that the measured TFP was positively related to those organizational capitals, but was negatively related to ICT capital for the industries using ICT in the US, which indicated ICT spillover effects in the US. Although the empirical studies on the GPT theory also concluded the existence of ICT spillover effects in the UK, they found that the proxy of co-invention adopted by the US could not be used in the UK case. For example, Basu et al. (2007) used lagged ICT capital growth as a proxy of co-invention (i.e. complementary organizational capital) in the US case. However, Basu et al. (2003) found that unlike the US case, TFP growth did not correlate with lagged ICT capital in the UK. Thus, for the UK, they had to change the proxy of co-invention from ICT capital stock to ICT investment. In other words, the conclusion in the previous empirical studies on the GPT theory was also not robust for the issue of the existence of ICT spillover effects in the UK due to some weakness of the measurement tool.

Accordingly, when Basu et al.(2003) followed the GPT theory to take account of the co-invention ignored by the neoclassical growth accounting in order to reduce the biased of
the measurement of TFP in the neoclassical growth accounting, they still ignored the role of efficiency and the possibility of ICT capital as an abnormal capital. This resulted in a mixed finding for ICT spillover effects in the UK and led to their work generating a biased TFP measurement as the traditional growth accounting does. The new framework reduced the bias due to these two aspects and thus gave a compatible result for both the US and the UK. As a result, the new framework’s conclusion of ICT spillover effects in the UK is more creditable.

In sum, this result of the existence of ICT spillover effects in the UK based on the new framework contributed to the existing literature from two aspects. First, it provides evidence to the argument of ICT spillover effects in the UK for which there has been a debate in the existing studies (e.g. Basu et al. 2003, O’Mahony and Vecchi 2005). Secondly, it suggested the developed framework was a robust tool for a country-level study of ICT spillover effects since it worked well for the US and the UK.

6.4 Comparison of ICT spillover effects between the UK and US

This section has investigated the difference in ICT spillover effects between the US and the UK. As mentioned in Chapter 5, the new framework can be used to examine the strength of ICT spillover effects by measuring the length of lag. The results of new framework in sections 6.2 and 6.3 showed that the UK has weaker ICT spillover effects compared with the US since the 12-13 years lag of the UK is much longer than 6-8 year lag of the US.
From the perspective of empirical studies based on the GPT theory, the result provides precise lag measurements. The existing GPT studies usually assume an approximately equal length of lag period for the UK and the US as a method to estimate the lag length was methodological unavailable. For example, Basu et al. (2003) suggested that ICT appeared as a GPT in the UK, resulting in different ICT spillover effects between the US and the UK with much shorter lag passing by the UK than in the US, when assuming that an equal lag was required by both economies. In contrast, our new framework relaxes the assumption of an equal lag in different countries and a shorter lag in the US is observed from the results with a more precise lag measurements, which gives stronger evidence to draw the conclusion that there are stronger ICT spillover effects in the US according to the hypothesis of the GPT that lag reflects initially negative return of ICT capital before ICT capital can positively contribute to the output.

Further, apart from the length of lag, the new framework is also able to identify the period of lag (or the distribution of the lag from the statistical view). The result of the new framework showed a big difference in the period of lag between both countries. The lag of the US appeared in 1982/83-1988/89, while the UK’s lag was from 1988/89 up to 2000. That is, the lag of the UK started at the end of US’s lag and remained up to 2000. This gave a clear indication of the different performances of TFP growth between the UK and the US. Based on the GPT explanation, the process of organizational adjustment due to ICT investments had not finished yet in the UK before 2000, and thus it could not have obtained a TFP growth in the 1990s. The reason behind this was as follows. The productivity growth is the net effect of the change in technical efficiency and the shift in the production frontier (i.e. TFP = TP+TEC) (Shao and Lin, 2001). The initially negative
return of ICT capital in the period from 1988/89 up to 2000 in the UK contributed to fall in TP when the efficiency increased in this period as shown by TABLE 6.7 and 6.8. Thus the UK’s lag up to 2000 contributed little to the UK’s TFP growth in 1990s by its fall of TP. In contrast, the lag of the US was over within 1980s and the US obtained positive TP in 1990s. Thus, both TP and TEC in the US in 1990s contributed to TFP growth in the US in 1990s. This explanation about the different TFP performances between the UK and the US in 1990s was completely different from the explanation on the neoclassical growth accounting and the argument of the existing empirical GPT studies. This explanation argued that there were ICT spillover effects in both the US and the UK, but the US obtained an acceleration of TFP in the 1990s due to the end of its lag in 1988, while the UK did not due to a persistent lag up to year 2000. However, the explanation of the neoclassical growth accounting argued that the reason of the existence of TFP growth only in the US was ICT spillover effects existed in the US rather than the UK (Jorgenson, et al. 2005, Triplett and Bosworth 2004). The ignorance of the possibility of ICT as a GPT in the traditional growth accounting led to the biased measurement of TFP and thus an inappropriate interpretation. It should be noted that the existing empirical GPT studies (e.g Basu et al. 2003) that treated ICT as a GPT also suggests that ICT cannot explain the observed TFP slowdown in the UK. The possible reason is that they used inappropriate proxy of co-invention and ignored the possible initially negative return of ICT capital.

6.5 Why ICT spillover effects in the UK differ from that in the US

This section focuses on exploring the reasons why ICT spillover effects in the UK were weaker than that in the US. The strength of ICT spillover effects for industries using ICT
could be attributed to two factors: the level of ICT investment and the capability of organizational adjustment to digest these ICT investments. The previous GPT studies with lagged ICT capital as a proxy of co-invention argued that lagged ICT capital negatively related to TFP growth and current ICT capital positively correlated to TFP growth. Thus, they concluded that the reason of the difference between the US and the UK was the low level of lagged ICT capital and high level of current ICT capital in the UK (e.g. Basu et al. 2003).

Efficiency change, for the first time, was introduced in our new framework as a proxy of co-invention. Thus, in new framework, it was noted a correlation between the different capability of organizational adjustment indicated by efficiency change from the US to the UK and the factors that could influence efficiency change. At the higher level of the new framework, the persistent level of inefficiency was considered as an endogenous factor, with software investment and skilled labour together being two exogenous factors.

This section investigated effects of these factors in the US and in the UK by testing the Hypotheses 3, 4 and 5 formulated in Chapter 5, and then explored the reasons why ICT spillover effects in the UK were weaker than the US.

6.5.1 Influential factors of ICT spillover in the US at the Higher-level

The new framework used efficiency as a proxy of co-invention and included the endogenous and exogenous factors that influenced efficiency, which in turn affected ICT spillover effects. For the case of the US, this sub-section tested the persistent level of
inefficiency as an endogenous factor and software investment and skilled labour as two exogenous factors via hypotheses 3, 4, and 5.

6.5.1.1 Effect of the persistent level of inefficiency in the US

Hypothesis 3: A low persistent level of inefficiency appeared for a country with strong ICT spillover effects.

The previous empirical GPT literature argued that only the US obtained TFP growth in 1990s. The results at the basic-level also suggested that the US had a short 6-8 year lag and the efficiency increased. This suggests that the US have strong ICT spillover effects, which indicated a strong capability of increase efficiency by successfully adoption ICT in ICT-using sectors of the US according to the definition of ICT spillover effects. Thus a weak persistent level of inefficiency is expected for the US.

In Models 4 and 5, the persistent level of inefficiency as the endogenous effect of inefficiency was captured by $\rho$ in the AR(1) structure for the inefficiency in Eqs. (5.8)-(5.9) at the higher-level. Applying Models 4 and 5 to analyse the US data, the obtained estimate of $\rho$ was 0.59 and 0.61 respectively, as shown in TABLE 6.9. The relatively small value of $\rho$ supported the Hypothesis 3.

| TABLE 6.9 Persistent level of inefficiency in the US estimated by Models 4 and 5 |
|---------------------------------|-----------------|-----------------|
| $\rho$                         | Model 4         | Model 5         |
|                                | 0.5919*         | 0.6172*         |
|                                | (0.0379)        | (0.0236)        |

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii)* indicates $p<0.05$. 
6.5.1.2 Effect of skilled labour on inefficiency in the US

Hypothesis 4: Skilled labour positively affects the efficiency during lag for a country with strong ICT spillover effects.

For the strong capability to increase efficiency by successful adoption of ICT in ICT-using sectors of the US, apart from the persistent level of inefficiency as one of endogenous factors, the exogenous factors that affected the reduction of inefficiency were also investigated. In this thesis skilled labour was considered as one of such exogenous factors. The previous firm-level studies for the US suggest a larger supply of skilled labour should positively contribute to organizational adjustment (indicated by efficiency change) that stem from ICT spillover effects (Machin and van Reenen, 1998,Autor, et al., 1998, Acemoglu, 1998, Bresnahan, et al., 2002, Card and Lemieux 2001, etc.). Thus, skilled labour is expected to positively affect the efficiency during the lag for the US.

To test Hypothesis 4, both Models 4 and 5 were applied. The evolution of inefficiency was captured at the higher level of Models 4 and 5. Model 4 assumes a time-invariant linear structure for the efficiency function at the higher level, whereas Model 5 has a time-variant non-parametric efficiency function that is more flexible than Model 4. Hence, Model 4 can measure the total effect of the influential factors in a whole period of 1980 to 2000, but is not able to measure the effect in an inter-period (e.g. the lag during 1983-1988). Thus, Model 5 was used to test hypothesis 4.

In Model 5, the effect of log of skilled labours on inefficiency in the US in 1980 and during 1981-2000 could be measured by $h_{hl,1}()$ and $g_{hl,2}()$ respectively. The results of
$h_{it}()$ and $g_{it}()$ are shown in Fig 6.9 and Fig 6.10, respectively. In Fig 6.9, the abscissa was the log skilled labour share of 24 ICT-using industries in 1980 (i.e. $h_{iit}$, where $i=1,\ldots,24$ in the third equation in Eq. (5.9)) and the ordinate was the effect of the log of skilled labour on the inefficiency. Similar as the other figures, the range of the horizontal axis in Fig 6.9 is the same as the range of the corresponding sample data, i.e. the logarithm of skilled worker share ($h_{iit}$).

The result of $h_{iit}()$, as shown in Fig 6.9, showed that skilled labour negatively affect inefficiency in 1980 in the US. In other words, skilled labour contributed positively to efficiency growth in 1980 in the US.

In Fig 6.10, the effect of log of skilled labour on inefficiency over the period 1981-2000 was plotted with 20 sub-plots and titled ‘ghl(year)’. In each sub-plot, the abscissa was the log skilled labour share for 24 ICT-using industries in each year (i.e. $h_{it}$ where $i=1,\ldots,24, t=1,\ldots,21$ in Eq. (5.9)) and the ordinate was the effect of log of skilled labour share on the inefficiency. The fact that skilled labour was negatively related to inefficiency during 1983-1988 with one anomaly in 1986 supports Hypothesis 4, which
was consistent with the argument of some previous literature on firm-level data (e.g. Bresnahan et al. 2002, Gretton et al. 2002).

**FIGURE 6.10 Effect of log of skilled labour in 1981-2000 in the US estimated by Model 5**

### 6.5.1.3 Effect of software on inefficiency in the US

**Hypothesis 5:** Software share positively affects the efficiency during the lag for a country with strong ICT spillover effects.

Potentially software share could be another exogenous factor that influenced inefficiency. The previous studies stressed that software improved the efficiency (Becchetti et al. 2003, Roach, 1991, Berndt et al. 1992 and Stiroh 1998). Thus, the software investment should be expected positively affect efficiency for the US with strong ICT spillover effects.
In Model 5, the effect of log of software share on inefficiency in 1980 and from 1981-2000 could be measured by $h_{ss,I}(\cdot)$ and $g_{ss,\mu}(\cdot)$. The results of $h_{ss,I}(\cdot)$ and $g_{ss,\mu}(\cdot)$ were shown in Fig 6.11 and Fig 6.12, respectively. In Fig 6.11, the abscissa was the log software share for 24 ICT-using industries in 1980 (i.e. $ss_{It,i}$ where $i=1,..,24$ in the third equation in Eq. (5.9)) and the ordinate was the effect of the log software share of the 24 ICT-using industries in 1980 (i.e. $h_{ss,I}(ss_{It,i})$ where $i=1,..,24$ in Eq. (5.9)). Similar as the other figures, the range of the horizontal axis in Fig 6.11 is the same as the range of the corresponding sample data, i.e. the logarithm of software share. Fig 6.11 showed a negative contribution of software to inefficiency in 1980 in the US, which indicated that the software had increased the efficiency in 1980.

![FIGURE 6.11 Effect of software share in 1980 in the US estimated by Model 5](image)

In Fig 6.12, the effect of log software share on inefficiency over the period 1981-2000 was plotted with 20 sub-plots and titled ‘gss(year)’. In each sub-plot, the abscissa was the annual log software share for 24 ICT-using industries (i.e. $ss_{\mu}$ where $i=1,..,24,t=1,..,21$)
in Eq. (5.9)) and the ordinate was the annual effect of the log software share on inefficiency for these 24 ICT-using industries (i.e. $g_{ss_i}(ss_u)$ where $i=1,...,24, t=1,...,21$ in Eq. (5.9)). Fig 6.12 showed that the software was negatively related to inefficiency during 1983-1988, except in 1986. In other words, the software was negatively related to inefficiency during the lag, which supported Hypothesis 5.

![FIGURE 6.12 Effect of software share during 1981-2000 in the US estimated by Model 5](image)

As mentioned in the Chapter of literature review, there were conflicting results in the previous empirical studies about the effect of ICT capital on efficiency. The results of Model 5 that software investment positively correlated with the efficiency seem to suggest that part of ICT capital that was associated with software investment indeed contributed the increase in efficiency, thus partially supporting the existing studies that argued ICT capital positively related to efficiency (e.g. Shao and Lin 2001, 2002 and Hitt and Brynjolfsson 1996).
6.5.1.4 Effects of the skilled labour and software share estimated by different models

Note that the two two-level models, Models 4 and 5, showed consistent results on the effects of the exogenous environment variables. For the effect of skilled labour, the results obtained from Model 4 (see Eq. (5.8)) showed that $\gamma_{hl,1}$ was insignificant (-0.0876), which suggested that skilled labour had no correlation with the inefficiency in 1980. However, Model 4 showed that $\gamma_{hl}$ was statistically significant (-0.2742), which indicated that the supply of skilled labour increased the efficiency in the US during 1981-2000.

Likewise, for the effect of software share, Model 4 (see Eq. (5.8)) showed that $\phi_{ss,1}$ was insignificant (-1.3), which suggested that ICT capital had no correlation with the inefficiency in 1980. However, Model 4 also showed that $\phi_{ss}$ was statistically significant (-0.4317), which indicated that the software increased the efficiency in the US during 1981-2000.

As mentioned in the section on model specifications of Chapter 5, one-level models such as Models 1-3 cannot be used to measure the impact of environmental variables on the efficiency. Although the two-stage approach could be used as a step stone to explore this issue, it may lead to a biased estimate on the impact of environmental variable on efficiency.

For instance, consider Model 3 with a time-variant Cobb-Douglas stochastic frontier and a two-stage approach. When applying this approach to analyse the US data and estimate the effect of skilled labour on inefficiency, the results showed the coefficient of skilled labour in the US was –0.002 with a standard deviation of 0.0005. Although this result
indicated that the skilled labour negatively correlated with inefficiency (or positively correlated with efficiency) in the US during 1980-2000, the value of –0.002 was too small to have economic significance. Similarly, the coefficient of software in the US was –0.0023, which was also too small to be used to explain the effect of software. Thus, the two-stage approach used to replace one-stage model was not suitable to analyse ICT spillover effects.

### 6.5.1.5 Summary

This thesis has considered the persistent level of inefficiency as an endogenous factor, software share and skilled labour as two exogenous factors to address the issue of ICT spillovers determinants. Based on the tests of Hypotheses 3, 4, and 5, the results of the new framework showed that both the endogenous and exogenous factors influenced ICT spillover effects in the US during 1980-2000. The lower persistent level of inefficiency together with the strong effects of supply of skilled labour and high level of software investment influenced ICT spillover in the US internally and externally, which were consistent with the results of firm-level empirical works on SBTC/SBOC (e.g. Bresnahan et al. 2002, Gretton et al.2002) and of firm-level empirical studies on ICT investment and efficiency (Becchitti et al. 2003, Roach, 1991, Berndt et al. 1992 and Stiroh 1998, Shao et al. 2001, 2002, 2003) respectively.

### 6.5.2 Comparison of influential factors in the UK and US

In Models 4 and 5, the endogenous persistent effect of inefficiency is captured through \( \rho \) in the AR(1) structure for inefficiency in Equation (5.8) and (5.9) at the higher-level.
These two models were also applied to analyse the UK data to investigate how inefficiency evolved over time. The results are displayed in TABLE 6.10. For comparison purpose, the persistent effect of inefficiency $\rho$ in the US estimated in section 6.4.1 was also given in TABLE 6.10.

As shown in TABLE 6.10, the result on $\rho$ for the UK’s persistent effect of inefficiency was significant. Clearly, in comparison with the persistent effect of inefficiency in the US, the UK has a much higher level of $\rho$ with 0.92 and 0.95, respectively, obtained from Models 4 and 5. The higher persistent level of inefficiency in the UK indicated the lower ability of increasing efficiency by adoption ICT in ICT-using sectors of the UK, which contributed to the weak of ICT spillover effects in the UK.

<table>
<thead>
<tr>
<th>Country</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>0.59*</td>
<td>0.62*</td>
</tr>
<tr>
<td></td>
<td>(0.04)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>UK</td>
<td>0.92*</td>
<td>0.95*</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
</tbody>
</table>

Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates $p<0.05$.

For the two exogenous environmental variables, skilled labour and software share, Model 5 gave unclear results on whether these variables contributed to the efficiency growth in the lag in the UK. Among the UK’s 13 year of lag, there were 9 years in which skilled labour contributed to efficiency and 6 years in which the software share contributed to efficiency. In other words, Model 5 did not suggest a robust result whether these exogenous variables contributed to efficiency growth in the lag in the UK.

Model 4 was also used to test whether these variables contributed to efficiency growth during the entire time period of 1980-2000. Based on Eq. (5.8), Model 4 uses $\gamma_{hl,t}$ and $\gamma_{hl}$ to capture the contribution of skilled labour to the efficiencies in 1980 and over the
period 1981-2000, respectively. Also, Model 4 uses $\phi_{ss,t}$ and $\phi_{st}$ to measure the effect of software share on the efficiency in 1980 and over the period 1981-2000, respectively. The results of Model 4, as shown in TABLE 6.11, indicated that the two exogenous variables had no statistically significant contribution towards the efficiency change in the UK with weak ICT spillover effects. In contrast, both the supply of skilled labour and software investment contributed towards efficiency in the US in 1981-2000.

**TABLE 6.11 Effects of exogenous variables in the US and UK estimated by Model 4**

<table>
<thead>
<tr>
<th>Country</th>
<th>US</th>
<th></th>
<th>UK</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_{hl,t}$</td>
<td>-0.0876</td>
<td>(0.7920)</td>
<td>-0.3009</td>
<td>(0.6697)</td>
</tr>
<tr>
<td>$\gamma_{hl}$</td>
<td>-0.2742*</td>
<td>(0.1279)</td>
<td>0.0086</td>
<td>(0.0433)</td>
</tr>
<tr>
<td>$\phi_{ss,t}$</td>
<td>-1.3046</td>
<td>(0.5712)</td>
<td>0.8036</td>
<td>(1.5406)</td>
</tr>
<tr>
<td>$\phi_{st}$</td>
<td>-0.4317*</td>
<td>(0.1251)</td>
<td>0.0509</td>
<td>(0.0746)</td>
</tr>
</tbody>
</table>

*Note: (i) Posterior means and posterior standard deviations in parentheses. (ii) * indicates $p<0.05$.*

This result was in line with the previous studies on the effect of skilled labour between the US and the UK. For example, most empirical existing studies have shown that only the US supported the complementarities between SBTC and SBOC, while data for the UK did not suggest that SBTC and SBOC work together (Giuri et al., 2006, Bresnahan et al., 2002, Gretton et al., 2002, Berthek and Kaiser 2004). This result also indicated that the significant effects of both the supply skilled labour and software investment in the US might be part of the reasons why there were different ICT spillover effects in the UK and the US.

In addition, since the two exogenous variables, skilled labour and software share, did not contribute to the increase of efficiency in the UK, it is not surprising that the persistent level of inefficiency was dominant in the evolution equation Eq. (5.8) in the UK.
To sum up, the stronger persistent level of inefficiency in the UK relative to the US was the main reason why there were different ICT spillover effects between the UK and the US. In addition, the strong endogenous persistent inefficiency in the UK further limited the influence of exogenous factors on inefficiency, such as the supply of skilled labour and software. This finding differed from the previous literature’ that argued the divergence of ICT investment was the main reason (Inklaar, O’Mahony and Timmer, 2003).

6.6 Concluding remarks

Based on the new framework developed in Chapter 5, this chapter has been argued that there were ICT spillover effects in the US, which is consistent with the previous studies (e.g. Basu et al, 2007). Further, the results of new framework gave an accurate measurement on the lag period for the US as 6-8 year.

Compared with the US, the UK showed a weaker ICT spillovers as indicated by a longer lag, which gave further evidence to the existing GPT literature that argued the UK had ICT spillover effects (e.g. Basu et al. 2003). Further, unlike Basu et al. (2003), this thesis also explained why little TFP growth in the UK in 1990s. The new framework used two dynamic non-parametric models to capture the length of lag by the years with negative return of ICT capital and thus showed that UK obtained little TFP growth in the 1990s due to a persistent lag up to year 2000.
To address the issue why there were different ICT spillover effects between the UK and the US, it was argued that stronger persistent level of inefficiency in the UK relative to the US was crucial. In the US, exogenous factors such as the supply of skilled labour and software played a significant role to increase efficiency and thus reduced the persistent level of inefficiency, whereas this did not happen in the UK. This finding differed from the previous studies that argued the divergence of ICT investment was the main reason (Inklaar, O’Mahony and Timmer, 2003).

Finally, the compatible results in the US and the UK obtained based on the new framework also showed the new framework was a more general tool to measure ICT spillover effects, which was more appropriate for a comparison study of ICT spillover effects among different country than the existing models (e.g. Basu et al. 2003).
Appendix A

The implementation of the developed algorithms and empirical results

In this appendix A, the implementation of the algorithms developed in Chapter 4 will be investigated and some detailed empirical results will be discussed. We first outline the commonly used criterion for the convergence of MCMC. Then we turn to consider the details of the implementation of the developed algorithms and the empirical analysis based on Model 4 and Model 5.

A1. Convergence rule for MCMC

As mentioned in Chapter 4, there are two different approaches in econometrics: the frequentists’ and Bayesian approaches. For the frequentists’ approach, it is well known that the maximum likelihood method seeks a vector of fixed parameters of interest that maximises the likelihood. A solution is obtained during each iteration of the algorithm, and the convergence is attained if the sequence of the solutions obtained in the iterations approaches to a fix point in the parameter space that maximises the likelihood when the number of the iteration increases. Unlike the case of the maximum likelihood method, however, the convergence for MCMC used in the Bayesian approach is related to the concept concerning a distribution of the parameter vector rather than a point, i.e. it is the convergence of the entire posterior distribution that is of interest. Once the convergence is attained, the draw (realization) of the parameter vector is simulated in each iteration from the distribution.
To monitor the convergence for MCMC, the most commonly used method in statistics and econometrics is by checking the R-value during the process of iterations (see, e.g. Gelman et al., 2003) that is defined below.

Consider a scalar parameter of interest (denoted as \(a\)). Suppose \(m\) parallel sequences of draws, each of length \(n\), are simulated which are denoted as \(a_{ij}\) (\(i = 1, \ldots, n; j = 1, \ldots, m\)). Let \(B\) and \(W\) denote the between-sequence variance and within-sequence variance respectively:

\[
B = \frac{n}{m-1} \sum_{j=1}^{m} (\bar{a}_{,j} - \bar{a}_{,..})^2, \text{ where } \quad \bar{a}_{,j} = \frac{1}{n} \sum_{i=1}^{n} a_{ij}, \quad \bar{a}_{,..} = \frac{1}{m} \sum_{j=1}^{m} \bar{a}_{,j}
\]

\[
W = \frac{1}{m} \sum_{j=1}^{m} s_{j}^2, \text{ where } \quad s_{j}^2 = \frac{1}{n-1} \sum_{i=1}^{n} (a_{ij} - \bar{a}_{,j})^2
\]

Then the marginal posterior variance of the estimator is computed by a weighted average of \(W\) and \(B\):

\[
\text{vár}^* (a \mid y) = \frac{n-1}{n} W + \frac{1}{n} B
\]

Gelman et al. (2003) suggest using the R-value defined to be \(\hat{R} = \sqrt{\frac{\text{vár}^* (a \mid y)}{W}}\) to monitor the convergence. In theory it declines to 1 as \(n\) approaches to infinity. In practice, it is recommended that R-values below 1.1 are acceptable.

Once \(\hat{R}\) is near 1 for all scalar estimators, the draws obtained before convergence are discarded (the corresponding period before the convergence is termed burn-in period), and the draws obtained in the remaining \(n^*\) iteration are retained. These \(mn^*\) simulations of all the sequences are gathered together and treated as a sample from the target posterior distribution.
A2. The implementation of the developed algorithms

In Chapter 4 two algorithms were developed for the proposed semi-parametric and non-parametric models, i.e. Model 4 and Model 5. Currently there is no software available for the Bayesian nonparametric analysis required in this study. Code was therefore written using the MATLAB programming language to perform the empirical analysis for the US and UK data, as outlined in this chapter.

The empirical results were obtained using the algorithm developed in Chapter 4 with 12,000 draws. Following Hajargasht (2004), the first 2000 iterations was used as burn-in passes. In other words, for different starting values, there were 12,000 passes, among which the first 2,000 were discarded to eliminate possible start-up effects, and the draws obtained from the remaining 10,000 iterations were retained for the subsequent analysis.

Although we set the number of iterations a priori, ultimately the convergence rule is the benchmark to determine whether the estimates are robust. This will be checked using the R-value as discussed above.

The next section will report Bayesian analysis of Model 4 and Model 5 in detail. First, the priors for the relevant parameters are specified, and then the change in each parameter at the final iteration is displayed and finally the convergence of the parameters and nonlinear functions will be investigated.
A2.1 Model 4

A2.1.1 The priors of the hyper-parameters

Eq. (4.17) gives the prior distributions of the scale parameters, i.e. the variance of the measurement error at the basic level ($\sigma^{-2}$), the smoothing parameter of various independent variables $j$ at the basic level ($\tau_j^{-1}$), the variance of the measurement error for the first period at the higher level ($\omega_1^{-2}$) and the variance of the measurement error for the remaining periods at the higher level ($\omega^{-2}$). In Chapter 4, these prior distributions are specified as the gamma distributions, each having two hyper-parameters, which are denoted by $a_0, b_0, a_j, b_j, n_1, q_1, n_{\omega}$ and $q_{\omega}$ respectively.

As no prior information is available in the analysis, these priors were set as non-informative prior distributions. More specifically, the Bayesian analysis carried out in this chapter set $a_0=0, b_0=1, n_1=0, q_1=1, n_{\omega}=0$ and $q_{\omega}=1$. Since Model 4 has three independent variables (i.e. Non-ICT capital, ICT capital and labour), $a_j=0, b_j=1$ for $j=1,2,3$ are used. Similar to Hajargasht (2004), the obtained empirical results were not sensitive to small changes in the priors. Starting values for the prior of inefficiency and the nonlinear functions to be estimated such as the return of Non-ICT capital were obtained using a simple OLS estimator.

A2.1.2 The change in each parameter at the final iteration

Following Hajargasht (2004), 12,000 draws were obtained and the first 2,000 were discarded as the burn-in period. The final analysis was based on the samples simulated in
the remaining \( n^* = 10,000 \) iterations. Following Gelman et al. (2003), the obtained posterior means were used as point estimates of the parameters.

To save space, here focus on the parameters rather than the non-parametric functions used in Model 4, and report below the change in each parameter at the final two iterations (otherwise there will be too much information to display here. For example, there are 24 industries over 21 years. If the nonlinear function of the return of Non-ICT capital at the basic level were reported, then there would be \( 21 \times 24 = 154 \) values).

At the higher level of Model 4, there are five parameters, i.e. \( \gamma_{hl,1}, \gamma_{hl}, \phi_{ss,1}, \phi_{ss}, \) and \( \rho \), where \( \gamma_{hl,1} \) is the coefficient of skilled labour share \( (hl_g) \) during the first year (1980), and \( \gamma_{hl} \) is the coefficient of \( hl_g \) during the other years. \( \phi_{ss,1} \) is the coefficient of is the software share \( (ss_g) \) during the first year (1980), and \( \phi_{ss} \) is the coefficient of \( ss_g \) during the other years. \( \rho \) is the persistent level of inefficiency to reflect how the technical inefficiency changes over time. The changes in these five parameters at the final two iterations are displayed in TABLE 6.12.

**TABLE 6.12 Changes in the draws for parameters in Model 4 at the final two iterations**

<table>
<thead>
<tr>
<th>US</th>
<th>( \gamma_{hl,1} )</th>
<th>( \phi_{ss,1} )</th>
<th>( \gamma_{hl} )</th>
<th>( \phi_{ss} )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>iteration 11,999</td>
<td>-0.74</td>
<td>-1.36</td>
<td>-0.24</td>
<td>-0.44</td>
<td>0.62</td>
</tr>
<tr>
<td>iteration 12,000</td>
<td>-0.80</td>
<td>-0.91</td>
<td>-0.31</td>
<td>-0.20</td>
<td>0.64</td>
</tr>
<tr>
<td>Change at the final iteration</td>
<td>-0.06</td>
<td>0.46</td>
<td>-0.07</td>
<td>0.24</td>
<td>0.03</td>
</tr>
<tr>
<td>Posterior means</td>
<td>-0.09</td>
<td>-1.30</td>
<td>-0.27*</td>
<td>-0.43*</td>
<td>0.59*</td>
</tr>
<tr>
<td>Posterior standard deviations</td>
<td>0.79</td>
<td>0.57</td>
<td>0.13</td>
<td>0.13</td>
<td>0.04</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>UK</th>
<th>( \gamma_{hl,1} )</th>
<th>( \phi_{ss,1} )</th>
<th>( \gamma_{hl} )</th>
<th>( \phi_{ss} )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>iteration 11,999</td>
<td>-0.90</td>
<td>0.17</td>
<td>-0.08</td>
<td>0.10</td>
<td>0.92</td>
</tr>
<tr>
<td>iteration 12,000</td>
<td>-0.88</td>
<td>0.71</td>
<td>-0.07</td>
<td>0.08</td>
<td>0.91</td>
</tr>
<tr>
<td>Change at the final iteration</td>
<td>0.02</td>
<td>0.54</td>
<td>0.01</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>Posterior means</td>
<td>-0.30</td>
<td>0.80</td>
<td>0.009</td>
<td>0.051</td>
<td>0.92*</td>
</tr>
<tr>
<td>Posterior standard deviations</td>
<td>0.67</td>
<td>1.54</td>
<td>0.043</td>
<td>0.075</td>
<td>0.02</td>
</tr>
</tbody>
</table>

* indicates \( p < 0.05 \)
From the above table, it can be seen that at the iterations 11,999 and 12,000, the draws for \( \rho \) were 0.62 and 0.64 respectively for the US data. According to the theory of Bayesian statistics, they were two different realizations of \( \rho \) drawn from its posterior distribution. The estimated mean and standard deviation of the posterior were 0.59 and 0.04 respectively. Because of the variability of the posterior distribution, these two draws were not necessarily close to each other. Note that the concept of convergence for MCMC applies to the convergence of the entire distribution.

As mentioned earlier, the estimates of these five parameters (e.g. \( \rho \)) are taken as the means of the posterior distributions (see Gelman et al., 2003). The posterior estimates of these five parameters are also displayed in TABLE 6.10 and TABLE 6.11.

\textit{A2.1.3 Convergence}

Next, the issue of convergence will be addressed. To investigate the convergence of the carried out MCMC a plot of R-value versus the corresponding number of the iterations for some of the parameters and non-parametric functions is given.

First, for Model 4, the R-value for \( \rho \) and the return of ICT capital in 1980 are displayed in Fig 6.13 for the US data (upper left and upper right respectively), and for the UK data (lower left and lower right respectively). The results for the other parameters and nonlinear functions are similar, and thus are not shown here.
The R-value for $\rho$ in the US               The R-value for $f_2(1980)$ in the US

The R-value for $\rho$ in the UK                The R-value $f_2(1980)$ in the UK

FIGURE 6.13 The R-value versus the number of iterations for some parameters and nonlinear functions in Model 4

As displayed in Fig 6.13, the R-values increase (or decrease) rapidly. After about the first 2000 iterations, the R-values became close to 1. This indicates the convergence of the posterior distributions (Gelman et al., 2003).

A2.2   Model 5

Next, we turn to Model 5. Again we first specify the priors.

A2.2.1   The priors of the hyper-parameters
Similar to the case of Model 4, there are a number of the prior distributions with hyperparameters as given by Eq. (4.28), i.e. $a_0, b_0, a_j, b_j, c_j, d_j, e_j, l_j, n_i, q_1, n_a$ and $q_a$.

Again these prior distributions are as non-informative since no prior knowledge about them during the analysis. So I set $a_0=0, b_0=1, a_j=0, b_j=1, c_j=0, d_j=1, e_j=0, l_j=1, n_1=1, q_i=0, n_a=1$ and $q_a=0$. The results were not sensitive to small changes in the priors.

\section*{A2.2.2 The change in each parameter at the final iteration}

The parameter $\rho$ is the only parameter in Model 5. The draws for $\rho$ at the final two iterations for the US and the UK data are displayed in TABLE 6.13.

\begin{table}[ht]
\centering
\begin{tabular}{|l|c|c|}
\hline
 & \textbf{US} & \textbf{UK} \\
\hline
iteration 11,999 & 0.63 & 0.96 \\
iteration 12,000 & 0.61 & 0.95 \\
\hline
Change at the final iteration & 0.02 & 0.01 \\
Posterior means & 0.62* & 0.95* \\
Posterior standard deviations & 0.02 & 0.01 \\
\hline
\end{tabular}
\caption{Changes in the draw for $\rho$ at the final two iterations in Model 5}
\end{table}

It can be seen that at the iterations 11,999 and 12,000, the draws for $\rho$ were 0.63 and 0.61 respectively for the US data. They are two different realizations of $\rho$ from its posterior distribution. The estimated mean and standard deviation of the posterior were 0.62 and 0.02 respectively. As mentioned earlier, the estimate of the parameter $\rho$ was taken as the mean of the posterior distribution.

\section*{A2.2.3 Convergence}
For the empirical analysis for Model 5, I just report here the R-values for the estimate of $\rho$ and the return of ICT capital in 1980 in the US and the UK data for illustration purposes in Fig 6.14. The R-values for the others in both countries were similar.

![Graphs showing R-values for $\rho$ and $f_2(1980)$ in the US and UK](image)

**FIGURE 6.14** The R-value versus the number of iteration for some parameters and nonlinear function in Model 5

It can be seen in Fig 6.14, the R-values increase (or decrease) rapidly and after about the first 2000 iterations, the R-values became close to 1. This suggests the convergence of the posterior distributions (Gelman et al., 2003).
Appendix B
Details on plotting of the estimated non-parametric functions

In this appendix B, the details on the plotting of the estimated non-parametric functions are discussed. Throughout Appendix B, we focus on Fig 6.1 only but the discussion also applies to the other graphs displayed in this chapter.

B1. The curve displayed in Fig 6.1 is the normalised return of log ICT capital.

On one hand, the scale of the y-axis in Fig 6.1 does not reflect a true return of log ICT capital. This is because in the above additive model (i.e. Model 4), each unknown function $f()$ is identifiable only up to an additive constant. This is a property that holds for all additive models (see, e.g. Hastie and Tibshirani, 1990).

For instance, the output of the non-parametric frontier function in Model 4 includes the returns of three inputs (i.e. Non-ICT capital, ICT capital and labour). Therefore if a constant value, $C$, is added to $f_{nk,t}(nk_t)$ and at the same time a constant value of $C$ is taken away from $f_{nk,t}(nk_t)$, then the output remains unchanged. For example, consider the case in 1987. If $(y+u)_{1987} = 10$, where $f_{nk,1987} = 2$, $f_{ik,1987} = 5$ and $f_{l,1987} = 3$, is the real situation (where $f_{nk,t}$, $f_{ik,t}$, $f_{l,t}$ are estimates of the return of non-ICT capital, ICT-capital and labour in each period), the situation of $f_{nk,1987} = 15$, $f_{ik,1987} = -30$ and $f_{l,1987} = 25$ also satisfies the requirement of this model (i.e., $(y+u)_{1987} = f_{nk,t} + f_{ik,t} + f_{l,t} = 10$).

To address this issue, $f_{nk,t}$, $f_{ik,t}$ and $f_{l,t}$ have to be normalised so that it always passed a fixed point (roughly it is set as in the middle of the range of the input in this thesis).
Consequently Fig 6.1 does not give a quantitative relationship between log ICT capital and the return of log ICT capital. Rather, it shows the qualitative linkage between log ICT capital and its return. In other words, Fig 6.1 only addresses the issue whether the return of ICT capital rises or falls when ICT capital increases.

On the other hand, the curve displayed in Fig 6.1 is neither the elasticity of ICT capital nor the rate of return of ICT capital as well. This is because elasticity is defined to be the ratio of the percentage change in one variable to the percentage change in another variable. So the output elasticity is the percentage change of output divided by the percentage change of an input. Obviously, no calculation of percentage change of output and input was carried out here. Moreover, the nonparametric function of log ICT capital cannot be used to calculate elasticity straightforwardly due to the fact that no actual coefficient is involved in the log ICT capital.

**B2. How to set the range of the horizontal axis in Fig 6.1?**

The range of the horizontal axis in Fig 6.1 is set to be the same as the range of the corresponding sample data, i.e. between the minimum and maximum of the logarithm of the ICT capital. Technically, since this study uses a spines technique, it assumes $n$ distinct knots $x_i$ on the interval $[a, b]$ such that $a < x_1 < \ldots < x_n < b$. Because the bound of splines equal to the range of the sample data here, $a =$ minimum value of sample data (e.g. logarithm of ICT capital) and $b =$ the maximum value of sample. For instance, the range of logarithm of ICT capital ($\log_{10}$) in the US in 1987 is from 5.0013 to 10.367, which is the range of the horizontal axis in Fig 6.1.

**B3. How to use MATLAB to plot Fig 6.1?**
ICT capital and its normalised return of 24 ICT-using industries in a year (e.g. 1987) are highlighted. Based on Eq. (5.8), Bayesian inference gives the estimated $f_{i,k,t}(ik_{u})$ in the year over the range of the ICT capital $ik_{u}$. Fig 6.1 is used to show this relationship graphically. Technically, the following steps were taken when using MATLAB to plot Fig 6.1:

(a) To reorder $ik_{u}$ and $f_{i,k,t}(ik_{u})$ of 24 industries in 1987 by the ascending order of $ik_{u}$.

(b) The estimated $f_{i,k,t}(ik_{u})$ is normalised due to the problem of identifiability as explained in the first issues mentioned above.

(c) The estimated curve is plotted on the graph using the formulas in Green & Silverman (1994:22) where the scale of the y-axis reflects the values after the normalization.

**B4. The return of ICT capital in each year has a threshold point.**

It can be seen from Fig 6.1 that there exists a threshold point. The return of ICT capital falls as the amount of the ICT capital increases when the latter is less than this threshold, whereas it rises when the ICT capital is larger than this threshold. In other words, there is a negative relationship between ICT capital and its return under the condition that the amount of ICT capital is less than the threshold. Clearly the assumption of monotonicity and concavity associated with the commonly used production functions are not satisfied here, and thus it provides evidence to the story of ICT as a GPT.
Chapter 7 Concluding remarks and future work

This thesis focuses on an important issue: whether the UK has ICT spillover effects, as US does, through developing a new framework of measuring ICT spillovers based on a dynamic non-parametric approach. The debate on this issue is important for the UK policymakers because ICT spillover effects lead to output/productive growth in the ICT-using sectors. Since the statistical data showed accelerated output/productivity of ICT-using sectors only within the US, the UK’s policymakers could learn from the US’ experience by focusing on the difference and causes of ICT spillover effects between the US and the UK. However, the current literature has not reached a consensus as to how ICT spillover effects work on the output/productivity. There were mixed findings related to relevant questions, such as whether the UK had ICT spillover effects and the reasons for the difference in ICT spillover effects between the US and the UK.

As a main theoretical basis of studying ICT spillover effects in the existing studies, the GPT theory did not give guidance on which proxy to be used as co-invention associated with ICT investment and on determining the lag period. This thesis chooses the efficiency as a proxy of co-invention and used a dynamic non-parametric frontier to capture the lag after accounting for the possibility of ICT as an abnormal input with negative return. In addition, this thesis applied the Bayesian inference on two dynamic non-parametric SFA models to obtain an accurate estimate for small samples. Based on these work, a new framework was built to measure ICT spillover effects and to further investigate the
effects of the persistent level of inefficiency and the environmental variables, such as skilled labour and software investment on efficiency.

On the basis of the developed framework and the analyses for the UK and the US, this thesis has the following findings:

1. ICT capital was a capital with initially negative return. This result provides evidence to the GPT hypothesis proposed by Carlaw and Lipsey (2006) about the new technological (like ICT) impact on growth that may not be immediately positive and could initially reduce growth and/or productivity due to necessary redesigns of physical capital, re-skilling of human capital, and etc.

2. There were ICT spillover effects for the UK in the 1990s. The evidence was the increase of the UK’s efficiency during the lag with negative return of ICT capital. This finding supports the previous studies that argued for ICT spillover effects in the UK (e.g. Basu et al., 2003).

3. The measured lag of the US was 6-8 years during 1982/83-1988/89, while the measured lag of the UK was 12-13 years during 1988/89-2000. The measured lag of the US and the UK seems to be reasonable since it indicates stronger ICT spillover effects in the US than that in the UK and this conclusion is in line the previous studies (e.g. Basu et al., 2003). This finding provides a way to capture the lag predicted by the GPT theory.

4. The reason TFP did not exist in the UK in 1990s was because it was still in the process of ICT spillover effects with negative return of ICT capital. In contrast, the traditional growth accounting studies argued the reason was because no ICT spillover effect in the UK at all (Jorgenson, et al. 2005, Triplett and Bosworth 2004). This
suggests that the GPT theory is more appropriate than the growth accounting for analysing ICT spillover effects. The former took into account of the possibility that the ICT contribution to productivity with a lag, in which ICT capital initially negative return to output. But the latter simply accounted for whether TFP growth existed together with the current ICT investment as a benchmark to determine whether a country has ICT spillover effects.

5. Of the endogenous factors such as the persistent level of inefficiency and the exogenous factors such as skilled labour and software investment, the persistent level of inefficiency was the more dominating factor that determined the level of ICT spillover effects for a country. Based on this finding, the different ICT spillover effects between the US and the UK was mainly due to the different persistent level of inefficiency, apart from the divergent level of ICT investment between them, as argued by previous empirical studies (e.g. Schreyer, 2000; Daveri, 2002). In this regard, the UK’s policymakers could enhance their capability of organizational adjustment through lowering the persistent level of inefficiency.

6. The methodology plays a crucial role in the analysis of ICT spillover effects. First, the non-parametric production function/frontier developed in this thesis captured the negative return of ICT capital by relaxing the assumption of monotonicity and concavity, and thus obtained evidence about ICT capital as an abnormal input with an initial negative return. Secondly, the Bayesian approach provided the possibility to investigate the effects of the persistent level of inefficiency (see Tsionas 2006) and obtain the evidence of the persistent level of inefficiency as the main reason for the divergent ICT spillover effects between the UK and the US for small samples.
This thesis has contributed to the existing literature in three aspects. The first is on methodology. This thesis has developed two non-parametric dynamic SF models with an AR(1) structure, investigated their Bayesian inference using Markov Chain Monte Carlo and used a one-stage approach to measure ICT spillover effect and to explore the influential factors.

The second aspect concerns the literature of ICT spillover effects. This thesis has developed a new framework to measure the length of lag period associated with ICT spillover effects by combining the GPT theory and the concept of efficiency. The contribution on this aspect can be highlighted as follows: (i) finding negative return of ICT capital initially; (ii) measuring the length of lag by years with negative return of ICT capital; (iii) using efficiency as a proxy of co-invention and getting evidence that efficiency increases during the lag period; (iv) introducing the persistent level of inefficiency as one of influential factors of ICT spillover effects and (v) exploring the effects of both the persistent level of inefficiency as an endogenous factor and skills and software as the exogenous factors of ICT spillover effects in a one-stage approach and finding the endogenous factor is more dominant.

The third aspect is related to the literature in explaining different pattern of ICT spillover effects between the US and the UK. First, this thesis suggests weak ICT spillover effects in the UK relative to the US. Secondly it has provided some reasonable explanations for the question why there was little TFP growth in the UK in 1990s from the perspective of ICT spillovers. Finally this thesis has suggested that the persistent level of inefficiency as the main reason of different ICT spillover effect between the US and the UK, apart from
the level of ICT investment mentioned by the previous studies (Inklaar, O’Mahony and Timmer, 2003).

However, further study could be carried out in future. On the empirical study, since the strength of ICT spillover effects was determined by the ICT investment level and the strength of capability of organizational adjustment, a clearer quantitative relationship between them could be further explored via a function linking the ICT investment level to the strength of capability of organizational adjustment as the independent variables and the strength of ICT spillover effects as the dependent variable. Secondly, the non-parametric dynamic model showed mixed results for the effect of exogenous environmental variables, such as skilled labour and software share, on inefficiency in the lag in the UK, but not in the US. Further study could be taken to investigate these problems. On the methodological side, there were at least two further issues that could be addressed. The first is to extend the AR(1) structure of inefficiency in the current non-parametric dynamic SFA model to a structure with the longer lag inefficiency on the current inefficiency. The second is to relax the assumption of the distribution of inefficiency. In the interest of policymakers, this study did not consider other factors that directly influence the persistent of inefficiency, such as the market structure, the regulation environment, etc. Clearly these could be included into the analysis in future.

Although these improvements could be made, the current study contributed the current literature substantially in relation to these three aspects.
References


