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Turbulent Diffusion in Channels of Complex Geometry

by

Dominic Kearney

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy of Loughborough University

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I would like to thank my supervisors Professor Koji Shiono and Dr. Cecil Scott for their support and guidance at all stages in the completion of this thesis, and all the staff in the Department of Civil and Building Engineering. I would also like to thank my family and friends for their continuing love and support. Finally, I would like to thank Slater for being the best punk band in the East Midlands.
ABSTRACT

This thesis examines turbulent diffusion processes in rectangular and compound open channels, with particular attention to the effect of secondary flow and the relationship between eddy viscosity and eddy diffusivity. Three dimensional velocities and concentration were measured using 3 component Laser Doppler Velocimetry (LDV) combined with Laser Induced Fluorescence (LIF) from three laboratory flumes: one rectangular simple channel and a deep and a shallow compound channel. To facilitate the analysis four finite difference numerical models were used: a linear k-ε model, a non-linear k-ε model, and two Algebraic Stress Models. Turbulent intensity, Reynolds’ stress, and production of turbulent kinetic energy were examined, and the most significant contributors identified. It was found that secondary flow was relatively insignificant in the production of turbulence, compared with bed and wall generated turbulence. In the rectangular channel dye tracer study, it was found that the near to the channel side-wall, an isotropic turbulent Schmidt number of 1.0 prevailed. In the centre of the channel, an anisotropic turbulent Schmidt number arrangement of $\sigma_z=2.3$ in the vertical direction and $\sigma_y=0.5$ in the lateral direction existed. The non-linear k-ε model gave the best predictions overall in the rectangular channel. In the deep compound channel dye tracing study, Noat and Rodi’s ASM with a lateral turbulent Schmidt number of $\sigma_y=0.44$ and a vertical turbulent Schmidt of $\sigma_z=1.66$ gave the best results for dye injected in the main channel and the flood plain. However, the turbulent Schmidt number was 1.0 in both directions for dye injected at the main channel/flood plain interface. In the shallow compound channel, a double concentration peak was observed, but only when the dye is injected close to the main channel/floodplain interface with strong secondary flow.
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NOTATION

\( \overline{cv} \) lateral flux
\( \overline{cw} \) vertical flux
\( u' \) streamwise turbulent intensity (fluctuating velocity)
\( v' \) lateral turbulent intensity (fluctuating velocity)
\( w' \) vertical turbulent intensity (fluctuating velocity)
\( \ell \) mixing length
\( \rho v^2 \) lateral normal Reynolds stress
\( \rho w^2 \) vertical normal Reynolds stress
\( \rho uv \) lateral Reynolds shear stress
\( \rho uw \) vertical Reynolds shear stress
\( \rho vw \) Reynolds shear stress
\( \delta \) kronecker delta
\( C'_\mu \) constant in Kolmogorov-Prandtl expression for eddy viscosity
\( C_\mu \) constant
\( \varepsilon \) rate of dissipation of turbulent kinetic energy
\( \rho \) fluid density
\( \Omega \) vorticity
\( \tau \) shear stress
\( \lambda \) non-dimensional eddy viscosity
\( \kappa \) von Karman’s constant (=0.4)
\( \sigma_k \) turbulent Schmidt number
\( \nu_t \) eddy viscosity
\( a, b \) constants in the pressure strain model
\( B \) width of channel
\( C \) mean concentration
\( C_D \) constant
\( C_V \) constant
\( D \) eddy diffusivity
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>wall proximity function</td>
</tr>
<tr>
<td>$f_2$</td>
<td>water surface proximity function</td>
</tr>
<tr>
<td>$F_u$</td>
<td>flatness of lateral concentration distribution</td>
</tr>
<tr>
<td>$g$</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>$H$</td>
<td>water depth in main channel</td>
</tr>
<tr>
<td>$h$</td>
<td>water depth over flood plain</td>
</tr>
<tr>
<td>$k$</td>
<td>turbulent kinetic energy</td>
</tr>
<tr>
<td>$L_x$</td>
<td>streamwise distance from dye injection point</td>
</tr>
<tr>
<td>$L_y$</td>
<td>lateral distance from dye injection point</td>
</tr>
<tr>
<td>$L_z$</td>
<td>vertical distance from dye injection point</td>
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<tr>
<td>$P$</td>
<td>mean pressure</td>
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<tr>
<td>$Q$</td>
<td>channel flow</td>
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<tr>
<td>$r$</td>
<td>radial distance from dye injection nozzle</td>
</tr>
<tr>
<td>$r_c$</td>
<td>half width radius of mean concentration profile</td>
</tr>
<tr>
<td>$s$</td>
<td>longitudinal bed slope</td>
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<tr>
<td>$S_u$</td>
<td>skewness of lateral concentration distribution</td>
</tr>
<tr>
<td>$U$</td>
<td>mean streamwise velocity</td>
</tr>
<tr>
<td>$U_*$</td>
<td>friction velocity</td>
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<tr>
<td>$V$</td>
<td>mean lateral velocity</td>
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<tr>
<td>$W$</td>
<td>mean vertical velocity</td>
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<tr>
<td>$x$</td>
<td>streamwise co-ordinate</td>
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<tr>
<td>$y$</td>
<td>lateral co-ordinate</td>
</tr>
<tr>
<td>$z$</td>
<td>vertical co-ordinate</td>
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<tr>
<td>$z_o$</td>
<td>roughness height</td>
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1 INTRODUCTION

In the past few years there has been an increase in the public's awareness of pollution problems and in the health risks associated with various forms of water pollution. These forms of water pollution are mainly sewage discharged into the sea, cooling water discharges from coal fired and nuclear power stations, fertiliser and desalination plants, industrial and radioactive waste from chemical plants and nuclear power stations and farming waste.

This increase in public awareness of environmental issues and different types of water pollution has been taken up by the civil engineering industry and has led to an increase in work relating to the feasibility and design studies of coastal, estuarine, river and reservoir water quality. As a result the civil engineer has had to diversify his expertise to include an overall understanding of complex hydrodynamic, meteorological, chemical, biological and ecological processes.

One of the results of this has been an increasing interest amongst engineers in predicting solute processes for controlling pollution levels in rivers. Most natural rivers have a flood plain that extends laterally away from the main channel at a gentle gradient or a series of terraces. River channels such as this are typically called compound channels. When flooding occurs, the fast flow in the main channel is retarded by the slower moving flow on the flood plain, thereby causing a large lateral exchange of momentum. The large shear layers generated by the difference in velocity affects the turbulent structure and vortices develop in the longitudinal axis. This is known as secondary flow, and is driven by the inhomogenity and anisotropy of turbulence between the main channel and the flood plain. Figure 1.1 illustrates simply the relationship between the secondary flow and the main streamwise channel flow. To understand fully the flow mechanisms in compound channels, three-dimensional flow measurements and computations are needed.
Experimental techniques such as three-component laser Doppler velocimetry (LDV) and laser induced fluorescence (LIF) have recently been developed and can provide researchers with the most accurate measurements to date. In tandem with advances in measurement techniques, advances in turbulence modelling allow secondary flow and solute transport to be predicted ever more accurately.

Experimental work carried out by Shiono and Knight (1991), Tominaga and Nezu (1991) and Naot et al. (1993), for example, have found that secondary currents are significant in the influence of momentum transfer, so that they could not only convect solute but also affect the momentum and solute exchange coefficients. Wood and Liang (1989) measured tracer concentration in a series of experiments and also developed a two dimensional semi-analytical model. Djordjevic (1993) developed a two dimensional numerical model to predict the unsteady solute transport rates in a compound channel and verified his model experimentally. A three dimensional eigenfunction solution for solute dispersion in wide rectangular channels was presented by Nokes and Wood (1988) and extended by Nokes and Hughes (1994) to include compound channel flows. Prinos (1992) developed a three dimensional model in conjunction with the linear k-ε model to study solute transport rates in compound channel flow. These methods do not take into account the effect of secondary flow on pollutant transport as these models do not produce secondary flow.

A numerical model should produce secondary flow in order for its effect on solute transport to be modelled. Models that produce secondary flow in compound channels, for example, Naot and Rodi (1982) have initially developed the algebraic Reynolds stress model for a rectangular open channel which was modified for compound channel flow by Naot et al. (1993). A non-linear k-ε model (Speziale, 1987) has been modified for open channel flow by Shiono and Lin (1992) and Pezzinga (1994). The full Reynolds-stress transport model has been developed by Cokljkat and Younis (1995) to simulate flows in open compound channels and a large eddy simulation has also been applied to compound channels (Thomas & Williams, 1995). Lin and Shiono (1995) used linear and non-linear k-ε models to predict the solute transport rates measured by Wood and Laing (1995).
They speculate on that it may be inaccurate to assume one Schmidt number for all points in the channel and for both vertical and lateral diffusion. This hypothesis was taken a step further by Simoes and Wang (1997) who used an anisotropic arrangement of Schmidt number in a compound channel and obtained preferable results.

Most recently, experimental studies carried out at Loughborough University by Feng (1996) have provided new data to help describe the turbulent flow structure and its effect on diffusion of solute. Using combined LDV and LIF, 3-D velocity, turbulent intensities, Reynolds shear stresses, concentration and flux have been measured.

This main objectives of the present study are to investigate and understand the effects of secondary flow on solute transport processes in rectangular and compound channels using numerical models and experimental data. Four numerical models have been selected for this purpose: linear and non-linear k-ε models, originally developed by Speziale (1987), modified for compound channels by Shiono and Lin (1992), and two algebraic stress models developed by Launder and Ying (1973) and Naot and Rodi (1982). Chapter 2 of this thesis is a literature review of previous studies in similar and related subjects. These include turbulence modelling, turbulent diffusion and dispersion and secondary flow. Chapter 3 includes the details of the governing equations of flow, i.e. the conservation mass and momentum, and the solute transport model derived from Fick’s first law of diffusion. Details of the numerical models and the experimental method are also included in Chapter 3 together with some preliminary benchmark testing of the numerical models. Chapter 4 provides the results, analysis and discussion for the hydrodynamics of the rectangular channel study. The analysis of the rectangular channel solute transport is detailed in Chapter 5. Statistical analysis is applied to the solute transport results, and the influence of turbulent Schmidt number is examined. The results, analysis and discussion for the compound channel hydrodynamics are presented in Chapter 6. Experimentally measured Reynolds stresses, $-\rho uv$, $-\rho vw$ and $-\rho vw$ are compared with the numerical predictions. The third component, $-\rho vw$, was not measured in the experiments by Tominaga and Nezu (1991) as $-\rho uv$ and $-\rho uw$ were.
measured in the experiments by Tominaga and Nezu (1991) as \( \bar{\rho}uv \) and \( \bar{\rho}uw \) were, and so this thesis provides a more complete picture of the factors influencing turbulence. Experimental and numerical three-dimensional turbulence intensities are also compared. Chapter 7 discusses the solute transport results of two compound channels: one deep and one shallow. Again the influence of turbulent Schmidt number is focused upon and the models are calibrated using mixing length relationships. The conclusions of the thesis are outlined in Chapter 8, together with some recommendations for further study.

**Figure 1.1 Compound channel flow**
2. LITERATURE REVIEW

2.1 TURBULENCE MODELLING

Turbulence models practically solve the Navier-Stokes equations for the mean properties of the flow. Reynolds (1883) introduced the turbulence fluctuations into the Navier-Stokes equations. The unknowns introduced as a result of the average turbulence properties are known as the Reynolds stresses. There is no direct way for obtaining the Reynolds stresses; their consequences must be modelled using flow parameters that are either known or knowable. A turbulence model is therefore a set of differential equations or algebraic formulae, which allow determination of the Reynolds stresses and hence close the time-averaged equations of fluid motion.

2.1.1 Eddy Viscosity Concept

The eddy viscosity concept (Boussinesq, 1877) was conceived by assuming an analogy between the molecular motion, leading to Stokes’ viscosity law in laminar flow, and the turbulent motion. For general flow situations, this concept may be expressed as

\[-\overline{u_iu_j} = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_i^j \]  

(2.1)

\(\nu_t\) is the turbulent or eddy viscosity which, unlike the molecular viscosity, is not a property of the fluid but one of the flow and is dependent on the state of turbulence. The eddy viscosity concept assumes that the Reynolds stresses, which arise from the turbulent nature of the flow, are linear functions of the appropriate velocity gradients. In a simple unidirectional flow this may be expressed as
\[ \tau = \rho \nu_t \frac{\partial U}{\partial y} \]  

(2.2)

where \( \tau \) is the shear stress, \( \rho \) is the density, \( \nu_t \) is the eddy viscosity, \( U \) is the velocity and \( y \) is the lateral distance.

If the general 3-D equations are integrated through the water column and the flow assumed to be steady then an equation that describes the lateral distribution of flow across a straight channel is derived (not shown here).

At this point a model must be assumed for the lateral eddy viscosity. Wark and Samuels (1992) found the following model acceptable in many situations.

\[ \nu_t = \lambda U \cdot H \]  

(2.3)

\( U \) is the local shear velocity, \( H \) is the water depth, and \( \lambda \) is a constant, but the problem here is establishing a value for \( \lambda \), the non-dimensional eddy viscosity. This model was found to give reasonable results for a wide range of data from small-scale laboratory channels to full scale gauging data. The best values for the non-dimensional eddy viscosity were in the range of 0.08 and 0.24 and further work is required to identify general values, which are appropriate in different conditions. Wark and Samuels (1992) also identify the need for more basic research to quantify bed friction in natural channels.

### 2.1.2 Eddy Diffusivity Concept

In direct analogy to the turbulent momentum transport, the turbulent mass transport is often assumed to be related to the gradient of the transported quality (Rodi, 1993),
\[-u_i c = D \frac{\partial C}{\partial x_i}\]  \hspace{1cm} (2.4)

where \(D\) is the turbulent diffusivity of the mass transport, \(\bar{u}_i\) is the mass flux term, and \(C\) is the mean concentration of mass. Similar to the eddy viscosity, \(D\) is not a fluid property but is dependent on the state of turbulence in the flow. It is assumed that the eddy diffusivity, \(D\), is closely related to the eddy viscosity, \(\nu_t\), thus:

\[D = \frac{\nu_t}{\sigma_k}\]  \hspace{1cm} (2.5)

where \(\sigma_k\) is the turbulent Schmidt number. Research has been carried out to establish the value of the ratio \(\sigma_k\), and it has been found to vary between 0.5 and 1.0 (Lin and Shiono, 1995). However, Simões and Wang (1997) found that better results were achieved from a numerical model using anisotropic Schmidt ratios.

Gibson and Rodi (1981) applied a Reynolds-stress closure model of turbulence applied to the calculation of a highly curved mixing layer. A calculation based on the Reynolds-stress equations reproduces the main features of a turbulent shear layer subjected to strong stabilising curvature. The Reynolds stress model was previously developed for plane flow. The effect of strong curvature are predicted without modification to the basic closure hypothesis or changes in the model constants whose values are established by reference to experimental data from simple shear layers.

2.2 DISPERSION AND DIFFUSION

Holley (1969) defined diffusion and dispersion as convective transport mechanisms. He proposed that the term "diffusion" be used for transport that is associated primarily with the time average of velocity fluctuations and that "dispersion" be used for transport associated with primarily with the spatial average velocity variations.
Some of the earliest research on turbulence and turbulent diffusion was carried out by Hagen (1839), Boussinesq (1877), and Reynolds (1883). Taylor (1921) developed a solution for turbulent diffusion by continuous movements. Prandtl (1925), von Karman (1930) and others carried out other similar research around this time. Mixing mass in shear flows was investigated by Taylor (1953) who reported that when a substance is introduced into a fluid flowing slowly through a small bore tube, it spreads out under combined action of molecular diffusion and variation of velocity over the cross section. He showed analytically that the distribution of concentration produced in this way is centred on a point, which moves with the mean speed of flow, and is symmetrical about it in spite of the asymmetry of the flow. This dispersion can be described by means of a “virtual” coefficient of diffusion, also known as the longitudinal dispersion coefficient.

Taylor’s paper described the importance of the shear on the mass transport in the flow direction as it contributes to the longitudinal mixing much more than turbulent transport. Following this, Taylor (1954) investigated the dispersion of matter in turbulent flow through a pipe and calculated the virtual diffusion coefficient for that case. Extending the analysis of Taylor (1954), diffusion of a marked fluid in the turbulent flow in an open channel was studied by Elder (1958). The longitudinal and lateral dispersion coefficient was presented as a function of the depth and the friction velocity of the channel. The lateral diffusion coefficient was reported to be 3 times larger than the value obtained by assuming isotropy. Fischer (1966) investigated longitudinal dispersion in natural streams. He remarked that the lateral velocity had an effect on the longitudinal dispersion coefficient by use of Taylor’s modeling.

Taylor (1954) assumed that the coefficient of longitudinal dispersion is equal to the coefficient of lateral diffusion and showed that under certain circumstances dispersion in turbulent flow can be described by the one-dimensional diffusion equation. The equation only applies when complete vertical and transverse mixing has occurred. Taylor (1954) also assumed that after an initial period of time the rate of advective transport, either upstream or downstream from a plane moving at velocity U, depends only on tracer concentration gradient and the longitudinal dispersion coefficient. E. Before this
unspecified time period, Taylor’s assumption is not valid. Orlob (1959) applied Taylor’s theory of continuous movements and the Einstein equation of diffusion to eddy diffusion of particles in a two dimensional field of homogenous turbulence produced by a broad shallow channel of extreme bottom roughness.

2.2.1 Lateral Diffusion

Lateral diffusion is approximately 5 times smaller than longitudinal dispersion (Elder, 1959). The lateral diffusion can be described by a diffusion coefficient since a Gaussian distribution of concentration implies the existence of a diffusion coefficient. Elder (1959) revealed by experiment that the non-dimensionalised lateral diffusion coefficient $D_y/U\cdot h$ to be 0.23, where $U\cdot$ is the friction velocity and $h$ is the channel depth, and showed that particles disperse some 70% quicker than for the isotropic case. This result showed that assumption that longitudinal dispersion and lateral diffusion are isotropic (Taylor, 1954) when calculating the lateral diffusion is in error by a factor of three. The isotropic assumption was expected to produce too low a value of $D_y$ since lateral turbulent intensity always exceeds the vertical intensity (Klebanoff, 1954). Okoye (1970) carried out a large number of experiments varying the aspect ratio and friction factor and concluded that $D_y$, non-dimensionalised by shear velocity and flow depth, tended to decrease as the aspect ratio increased and $D_y/U\cdot h$ varied between 0.09 and 0.235. Prych (1970) found $D_y/U\cdot h$ to range between 0.136 and 0.162. Miller & Richardson (1974) described turbulent diffusion as depending on the motion of individual fluid particles and stated that the diffusion equation describes a Lagrangian process for vertical and lateral longitudinal mixing. However, longitudinal dispersion in the direction of the flow is a bulk one-dimensional equation to describe the process by which a flowing stream spreads and dilutes a pollutant. Dispersion describes the process averaged over the entire cross section whereas turbulent diffusion considers the mixing process at each point in the flow field. Miller and Richardson (1974) concluded that the lateral turbulent diffusion coefficient to be $0.23R\cdot U\cdot$, $R$ being the hydraulic radius, which agrees with other
experiments. e.g. Sayre and Chang, who found $D_y = 0.17U * h$, which, when $h$ is converted to $R$, becomes $D_y = 0.23RU$. Also Fischer found lateral diffusivity to be equal to $0.24RU$. It was also found by Miller and Richardson (1974) that neither the secondary flow nor the large range in resistance to the flow affected $D_y$. An increase in velocity, slope and turbulence intensity caused $D_y$ to increase and with constant velocity. An increase in the resistance to flow, slope and turbulent intensity also increased $D_y$. Longitudinal diffusion $D_y$ was small compared with longitudinal dispersion. Lau and Krishnappen (1977) reviewed 9 studies and included results from their own and found no consistent trend in the variation of aspect ratio with $D_y/U.h$ except in the results of their own study. Lau and Krishnappen then non-dimensionalised $D_y$ with width and not depth and concluded that $D_y/U.B$ was the better non-dimensionalisation, where $B$ is the channel width, and the principle mixing mechanism was secondary flow and not turbulence. The results of Webel and Schatzmann’s (1984) study differed from those of Lau and Krishnappen. Over 80 runs of varying Froude number, Reynolds number, friction factor and aspect ratio were carried out with smooth, transitional and rough conditions. It was reported that $\varepsilon_y$ is independent of aspect ratio except in small channels ($B/h \leq 5$), agreeing with Kaleris and Schatzmann (1981) but contradicting the secondary flow attribution of Lau and Krishnappen. This reasoning agrees with Rouse (1975) and Naot and Rodi (1981) who found that turbulence driven secondary flow declines rapidly with a distance of a few water depths away from the channel walls. McNulty and Wood (1984) developed an algorithm to solve the diffusion equation for any velocity and diffusivity distribution enabling a comparison to be made with previous approximate methods and to assess their capabilities. This has shown that assumptions of uniform flow are conservative in terms of dispersion. Nokes and Wood (1988) concluded from their experiments that non-dimensionalised $D_y$ is independent of all flow parameters except friction factor and that lateral turbulence is due to bed and wall generated turbulence only. Details of mixing in rivers can also be seen in Rutherford (1994). A summary of the experimentally measured values of non-dimensionalised $D_y$ from the literature is shown in Table 2.1.
2.2.2 Laser Induced Fluorescence

Rosensweig et al. (1961) first used the technique based on the light scattering properties of solid particles and Shaughnessy and Morton (1977) measured the particle concentration in a turbulent jet using this technique. Kotsovinos (1977) combined one-dimensional laser Doppler velocimetry (LDV) with fast response thermisters to measure simultaneously the fluctuations of a scalar and the velocity. Simultaneous temperature and velocity measurements by means of two hot wires were carried out by Chevray and Tutu (1978), obtaining the cross-correlation between temperature and velocity in a turbulent jet. Owen (1976) was the first to implement simultaneous laser velocimetry and laser induced fluorescence (LIF) measurements, in order to investigate the mixing of coaxial jets. Pananicolaou and List (1988) studied turbulent buoyant jets by means of a combined LDV and LIF system, providing a measurement of the turbulent mass flux profile in the jet. An experimental determination of the turbulent mass flux and the turbulent diffusivity has been performed in a turbulent submerged jet by Lemoine et al. (1996), by combining LIF of rhodamine B and LDV in an easy to implement experimental arrangement. There is a large amount of literature concerning investigations of turbulent mass or temperature fluxes in jets and shear flows, but very little concentrating on the turbulent diffusion processes in isotropic turbulence fields. Gad-el-Hak and Morton (1979) attempted to determine the concentration-velocity cross-correlation by means of a combined LDV and light-scattering technique, but the experimental results were too varied to be significant. Simeons and Ayrault (1994) studied the jet of a passive contaminant submerged in an isotropic homogeneous turbulent channel by using two combined imaging techniques: particle image velocimetry and planar LIF. Considerable work has been done by Nakamura et al. (1987) on mass diffusion using grid-generated turbulence.

2.3 RECTANGULAR CHANNEL

McQuivey et al. (1969) carried out turbulence measurements in open channel flow using hot film sensing elements. Lauder and Ying (1972) measured secondary flow in a duct
with rough surfaces. It was concluded that secondary flow has a larger proportion of axial flow than it does in smooth ducts. However, normalised by $U^*$ it has the same magnitude as for smooth ducts. Melling and Whitelaw (1976) measured secondary flow using LDA in a rectangular duct. Steffler et al. (1985) measured mean velocity and some turbulence properties, including turbulence intensities $u'$ and $w'$. Reynolds shear stress $-\overline{uw}$ and turbulent eddy viscosity, in smooth rectangular channels using an LDA. Tominaga et al. (1989) investigated 3-D turbulent structure in rectangular and trapezoidal open channels measuring velocities using a hot-film. Three components of turbulent intensity were measured along with Reynolds stress terms $-\overline{uv}$ and $-\overline{uw}$. They report differences in the secondary flow pattern between rectangular and trapezoidal channels. Kirkgöz (1989) measured velocity using a laser-Doppler anemometer in rough and smooth rectangular channels, finding a value 0.41 for the von Karman constant. Shiono & Lin (1992) predicted secondary flow, Reynolds stresses and turbulent intensities in a simple straight rectangular channel using algebraic stress models (ASM) of Naot and Rodi (1982) and of Launder and Ying (1973) and the non-linear k-ε model (NLKE) of Speziale (1987). The ASM and NLKE models. The ASM and NLKE model predicted flow structure and turbulent parameters well except near the free surface region. This was due to the vertical component of the turbulent intensity not being suppressed sufficiently to increase both the longitudinal and lateral components of the intensity at the free surface. Turbulence characteristics in rough uniform open channel flow were reported by Kironoto and Graf (1994) in experimental studies measuring velocities and turbulence intensities and Reynolds stress profiles using hot film probes.

Jobson and Sayre (1970) investigated vertical exchange of momentum transfer, fluid mass and suspended sand particles. Vertical concentration profiles of dye injected at the surface were measured and the turbulent Schmidt number was found to be 1.0 for the vertical exchange coefficient in a rough boundary open channel. Lemoine et al. (1997) conducted an experimental investigation in a grid-generated turbulent flow using a combination of LDV and LIF in a rectangular channel.
2.4 COMPOUND CHANNEL

2.4.1 Introduction

Zhekaengakov (1950) first carried out experimental studies in the interaction of main channel and flood plain flows and found that a reduction of flow velocity and discharge in main channel with respect to in bank flow occurred, especially at low depths of over bank flow.

Flow in a compound channel has a three dimensional structure. The main hydraulic processes are bed-generated turbulence, shear generated turbulence at the main channel-flood plain junction, and helical secondary currents acting perpendicular to the main flow direction (Samuel & James, 1992). Some of these processes are illustrated in Figure 2.1 (Shiono & Knight, 1991), showing over bank flow in a compound trapezoidal channel. Natural rivers add the complexity of the geometry and boundary roughness influences these processes further.

The horizontal momentum transfer between the faster flowing main channel and the slower flowing flood plain is an important aspect of the over bank flow situation in compound channels. This transfer phenomenon is especially apparent in the region of the main channel-flood plain junction. The strong transverse gradient of longitudinal velocity in this region causes a series of vortices rotating on a vertical axis to be present along the line of the junction. Sellin (1964) was the first to show that this existed and used a visualisation technique. A considerable amount of research has been done in this area in the last 30 years in order to confirm the significance of the transfer of longitudinal momentum between the main channel and the flood plain when over bank flow occurs in a compound channel. Fukuoka and Fujita (1989) demonstrated from visualisation studies that large vortices or eddies convect the water with the highest momentum from the main channel to the flood plain, as illustrated in Figure 2.2. This has been confirmed using flow visualisation by Imatoto and Ishigaki (1983, 1990) and Pasche and Rouvé (1985).
2.4.2 Longitudinal Velocity

Among the research conducted on longitudinal velocity in compound channels, Rajaratnam and Ahmadi (1979, 1981), Shiono and Knight (1989) and Tominaga and Nezu (1991) showed that the vertical profiles of mean longitudinal velocity for the middle of the main channel and outer regions of flood plain coincided well with the standard logarithmic law, except in the vicinity of the free surface. In this region flow is essentially two-dimensional. At the intersection of the main channel and the flood plain they found that the deviation of the velocity profile from the logarithmic law increased and the flow becomes three-dimensional. The main longitudinal velocity is affected by the large exchange of momentum between the main channel and the flood plain in the region of the channel step. All researchers concluded this.

2.4.3 Secondary Flow

Thomson (1878) had discovered the importance of secondary flow in open channel flow in the last century. His argument was that these currents caused the maximum velocity in open channel flows to lie below the water surface, as had been observed on many occasions. The secondary motion near the free surface transports fluid with relatively low longitudinal momentum towards the centre of the central region of the channel, and thus reducing the velocity at the surface. A similar process occurs in non-circular ducts where the fluid with relatively high momentum is carried by the secondary flow towards the corners, causing a bulge in the velocity contours towards the corners (Nikuradse, 1926). “Secondary flow of the second kind” (Prandtl, 1952) was indicated to occur at corners of rectangular channels as a result of anisotropic turbulence. The corner induced secondary motion also causes the bed shear stress to rise towards corners before dropping sharply extremely close to the corner, as measured by Leutheusser (1963) in a square closed channel. Brundrett & Baines (1964), Gressner (1964), Gressner and Jones (1965) and Launder and Ying (1972) quantified these flows with improved accuracy using Hoagland’s (1960) hot wire technique. Melling and Whitelaw (1976) used a laser-Doppler anemometer which allowed the measurements of the three components of the mean velocity and the corresponding Reynolds stresses without the flow interference that
can occur with Hoadland's hot-wire technique. The following sections review the experimental and numerical investigations into secondary flow.

Humphrey et al. (1981) investigated secondary flow in a square duct with a strong curvature and it was found that the main effect of the bed as to induce strong cross-stream motions that develop into two counter rotating vortices in the longitudinal direction. They concluded that the driving force for this secondary motion was the centrifugal force radial pressure gradient imbalance, which acts upon the slowly moving fluid along the side walls of the bend and displaces it along the side walls from the outer to the inner curvature walls. Secondary flow due to curvature has also been investigated by Booij & Tukker (1994) using a 3-D laser Doppler velocimeter in a curved flume. The curvature of the flow in riverbeds gives rise to flow components perpendicular to the main flow (i.e. secondary flow). This has important consequences for the form of alluvial beds since sediment is directed towards the inside riverbank and as a result the outside bank is eroded and undermined. Brundrett and Baines (1963) wrote that secondary flows in non-circular ducts are accompanied by a longitudinal component of vorticity. Gessner and Jones (1965) looked at secondary flow in turbulent flow in rectangular channels and concluded that secondary flow velocities, relative to the centreline mean flow velocity, decrease with an increase in Reynolds number. Mechanisms, which initiate secondary flow in developing turbulent flow along a corner, were examined on the basis of energy and vorticity considerations. Gessner (1973) investigated secondary flow in turbulent flow along a corner. Tracy (1965) investigated turbulent flow in a three-dimensional channel. It was concluded that secondary motions are present in long channels of non-circular shape and that one of the boundaries may be a free surface. The turbulence intensities $v'$ and $w'$ are of primary importance to the establishment of secondary flow. The magnitude of the velocity fluctuation is inhibited by the boundary. Gessner's (1970) results showed that transverse flow is initiated and directed towards the corner as a direct result of turbulent shear stress gradients normal to the bisector. Anisotropy of the turbulent normal stresses does not play a major role in the generation of secondary flow. Rajaratnam and Ahmadi (1981) performed experiments in channels with flood plains.
Knight and Abril (1996) refined the calibration for the depth averaged model originally developed by Shiono and Knight (1988).

Secondary flow in a straight open channel as investigated by Imamoto et al. (1983) using 3-D laser Doppler anemometer (LDA) measurements, flow visualisation and algebraic stress modelling. Three kinds of secondary flow were investigated: the first being flow due to velocity difference in compound channels with non-uniform bed roughness, the second due to flow boundaries, and the third due to bed generated turbulence, the latter being detected only by the visualisation technique. An investigation into secondary currents using fibre optic laser Doppler anemometer (FLDA) to obtain accurate measurements in fully developed open compound channel flow was carried out by Tominaga & Nezu (1991). They found that strong inclined secondary currents are generated from the main channel/flood plain junction edge toward the free surface and a pair of longitudinal vortices are recognised on both sides of the inclined upflow. Also, when the ratio of flood plain depth to main channel depth is large (0.75), the main channel vortex is not clearly recognised, while the flood plain vortex expands in the spanwise direction. The study also stated that the interaction between the main channel and flood plain flow is important because the contribution of secondary currents is very large near the junction. Wood and Laing’s (1989) investigations into compound channel flow showed that the point at which effluent is released has an effect on its dilution. It was reported that releasing effluent in the main channel as opposed to the flood plain gives a much more rapid initial dilution. More minor effects were reported such as that releasing effluent in the main channel near the junction leads to the maximum concentrations moving towards the step and that releasing effluent in the flood plain near the junction leads to maximum concentration moving away from the step.

Laser Doppler Anemometer techniques have also been used by Nezu and Rodi (1986). A powerful two-component LDA system was used to accurately measure the longitudinal and vertical velocity components in two dimensional fully developed open channel flow.
over smooth beds. These experiments were carried out to verify the log law often applied to open channels. It was found that the log law can only strictly be applied to the near wall region. Knight and Shiono (1990) took turbulence measurements in a compound channel, including primary velocity, turbulent intensities, kinetic energy and Reynolds stresses in the region of strong lateral shear at the main channel/flood plain interface. The upwelling at the interface caused considerable temporal variations in velocities $U$, $V$ and shear stress $\tau_{xy}$. Turbulent open channel flows in compound channels have been experimentally investigated by Shiono & Knight (1991) using an LDA system and analysed by an analytical model. The analytical model was found to give the lateral distributions of depth-mean velocity together with the effects of bed generated turbulence, lateral shear turbulence and secondary flows.

An investigation into secondary currents (Tominaga & Nezu, 1991) used fibre optic laser Doppler anemometer (FLDA) to obtain accurate measurements in open compound channel flow. It was found that strong inclined secondary currents are generated from the main channel/flood plain junction edge toward the free surface and a pair of longitudinal vortices are recognised on both sides of the strong inclined upflow. Also, when the ratio of flood plain depth to main channel depth is large (0.75), the main channel vortex is not clearly recognised, while the flood plain vortex expands in the spanwise direction. The study also stated that the interaction between the main channel and flood plain flow is important because the contribution of secondary currents is very large near the junction. The results of Tominaga and Nezu's (1991) experiments are shown in Figures 2.3 to 2.5.

In an attempt to clarify the flow structure in the main channel/flood plain interface region, Imamoto and Ishigaki (1990), performed laboratory experiments using a flow visualisation technique. The results obtained show that the secondary flow plays an important role in the characterising the flow structure. The lateral distribution of mean velocity near the boundary is closely related to the turbulent shear stress, and indicates the existence of secondary flow. There is strong secondary flow and weaker vortices up
welling from the vicinity of the flood plain's edge to the water surface, intermittently observed and producing secondary flow cells along the flow.

2.4.4 Numerical modelling of secondary flow

The momentum exchange between the main channel and flood plain is produced locally by turbulence and can be model by a standard k-ε model. For secondary flow, and isotropic eddy viscosity model is not sufficient. A full Reynolds-stress model would be adequate, although a much more computationally simpler method would be to use an Algebraic Stress Model. The Launder and Ying (1973) ASM is good for computing secondary flows in rectangular ducts, although it does give the wrong magnitude of the Reynolds stresses themselves. Another problem is that it does not take the wall effects into account. Near walls, the normal Reynolds stresses are redistributed so that the fluctuations parallel to the walls are enhanced at the expense of the fluctuations perpendicular to the wall. This is not import in duct flow but in channel flow, where there is a free surface it is important that this effect is taken into consideration. The free surface acts in basically the same effect on the normal stresses, as do solid walls.

The first successful attempt at solving the free surface problems of ASM was presented by Naot and Rodi (1982). Naot and Rodi (1982) produced algebraic expressions for the Reynolds stresses $\rho u^2$, $\rho v^2$ and $-\rho uv$ in the momentum equations for the secondary velocities by simplifying modelled transport equations for the Reynolds stresses. The standard eddy viscosity relation used in the standard k-ε model was employed for the shear stresses, $-\rho uw$ and $-\rho uv$ in the longitudinal momentum equation. The model correctly simulated the eddy viscosity distribution and predicted the separation between $\rho u^2$ and $\rho v^2$, which drives the secondary motion. The main features of their model (ASM+) are two empirical wall proximity functions, one of which is specifically designated to account for the free surface.

A two-dimensional model (Keller & Rodi. 1988) was developed to calculate flow characteristics in channels of compound cross-section. Two areas of improvement were
identified: the determination of the bed shear stress, and the correct modelling of turbulence production on submerged vertical or strongly inclined walls. Keller and Rodi (1988) stated that the coupling of the bed shear stress to a vertically depth averaged velocity distribution is clearly inadequate where the transverse bed slope is significant. Modelling of a submerged vertical wall by a very steep transverse bed gradient results in the wall shear stress being linked with an assumed vertical velocity distribution instead of a lateral velocity distribution. Also identified was the lack of direct measurements of the transverse dimensionless eddy diffusivity $D_y$. Uncertainty remains as the situation for natural compound channels and there exists a need for accurate experimental data from such channels to enable more extensive testing of the model.

Cokljat and Younis (1993) used a second order closure of turbulence to predict the behaviour of turbulent flows in open channels of both simple and compound cross sections. Using data from Tominaga et al. (1989) for the simple cross section, Tominaga and Ezaki (1988) for the symmetric compound channel and Tominaga and Nezu (1991) for the asymmetric compound channel, it was found that the model predicted fairly satisfactorily the principal features of the flow in both main channel and flood plain. Naot et al (1993) also calculated secondary flow in compound channels, using an algebraic stress model, comparing the computed results with experimental results by Tominaga and Nezu (1991). Shiono and Lin (1992) applied algebraic stress and non-linear k-$\epsilon$ models to an asymmetric compound channel to predict turbulence parameters. Generally the features of the flow structure agreed well with the experimental data of Tominaga and Nezu (1991) although the Reynolds stresses and turbulent intensities near the boundary were overpredicted.

Lin and Shiono (1995) used a linear and non-linear k-$\epsilon$ model to predict turbulent transport in compound channels using experimental data from Wood and Liang (1989). They found that the non-linear k-$\epsilon$ model predicts very well the twin vortices at the main channel/flood plain junction. The bed shear stresses predicted by both models agreed well with measured results. The non-dimensional eddy viscosity predicted by the non-linear
model was 10% smaller than that predicted by the linear model near the M/F region. The depth averaged λ was about 20% higher than that of a 2-D flow at the centre of the main channel and similarly at the centre of the flood plain.

Cokljat and Younis (1995) used a full Reynolds stress transport model of turbulence based on one developed by Launder (1975) for computing flow in asymmetric compound channels with rectangular main channel. Thomas and Williams (1995) used a large eddy simulation (LES) of turbulent flow in a compound open channel. Comparing results with the experimental data of Tominaga and Nezu (1991) it was found that the agreement was good, and that the mean velocity and secondary circulation fields and the bed and lateral stress distributions could be presented in detail. Sofialidis and Prinos (1998) used non-linear low-Reynolds k-e models, since often the flood plain flow is not fully turbulent and, for low relative depths the law of the wall and the wall function approach may not be valid, especially in the interaction region. Comparing the experimental results of Tominaga and Nezu (1991) with two versions of a model developed by Craft et al. (1993) and optimised by Suga (1995). It was found that the strength of secondary currents was not predicted well, but turbulent shears stresses – \( \rho \bar{uw} \) and – \( \rho \bar{uv} \) were predicted well.

### 2.4.5 Turbulence Intensities

Investigation of turbulence characteristics in compound channels have been carried out by Townsend (1968), Prinos et al. (1985), Tominaga and Nezu (1991), Shiono and Knight (1989), and Knight and Shiono (1990). Townsend (1968) concluded that for small flood plain depths, both longitudinal and transverse turbulent intensities at the main channel/flood plain interface were considerably higher under compound channel flow than the corresponding values in separate isolated channels. Prinos et al. (1985) examined the structure of turbulence in a compound channel under wide and narrow conditions. It was concluded that the longitudinal and vertical turbulence intensities were significantly higher than in the channel step region than in the centre of the main channel. In addition the longitudinal and vertical turbulence intensities in the step region increase with an increase of relative boundary roughness parameter or a decrease of relative flow
Tominaga and Nezu (1991) observed that all three components of turbulent intensity increase in the vicinity of the main channel flood plain interface. Shiono and Knight (1989) found that as the relative flow depth decreases both longitudinal and transverse turbulent intensities increase considerably whereas the vertical turbulent intensity does not.

### 2.4.6 Turbulent Diffusion in Compound Channels

Kay (1987) used an analytical solution for the diffusion of dye injected into a compound open channel. Arnold et al (1989) experimentally measured turbulent Schmidt number and found that it varied between 0.5 and 0.9 across a compound channel. Djordjevic (1993) also carried out experimental tracer investigations in a compound laboratory channel. Dynamic measurements of tracer concentration were done by photoelectric colorimeter developed by Simic et al. (1987) and agreed well with a depth averaged mathematical model. Prinos (1993) conducted experiments and numerical modelling of solute transport in compound open channel and duct flows. Lin and Shiono (1995) developed adapted the linear and non linear k-ε turbulence models of Speziale (1987) for compound channels, comparing the flow calculations with the experimental results of Tominaga and Nezu (1991) and the solute transport calculations with the experimental data of Wood and Laing (1989) using a turbulence Schmidt number of 0.5. Lin and Shiono commented that a local value depth averaged eddy may not be correctly predicted or that the assumption of constant Schmidt number for the whole area is erroneous. The tracer concentration was better predicted by the non-linear k-ε model. Spence et al (1997) used Kay’s two-dimensional analytical solution to produce predictions of tracer concentrations in compound channels, and found that turbulent mixing for in-bank flows was correctly predicted for overbank flows, transverse mixing is significantly underestimated. Simoes and Wang (1997) developed a numerical model to simulate turbulent flows and the transport of dissolved materials. Using two arrangements of turbulent Schmidt number, isotropic and anisotropic the results were compared with the simulations of Lin and Shiono (1995) and found that the best results were achieved with
the anisotropic Schmidt number arrangement of 0.5 for horizontal mixing and 1.0 for vertical mixing.
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† Webel and Schatzmann altered the width in their channel for each run obtaining the same values of $\varepsilon_d/U^*d$ in each case. (a) $d/w = 0.0495, 0.0680$; (b) $d/w = 0.0330, 0.0452$. 0.01235; (c) $d/w = 0.0220, 0.0302, 0.0826$.

Table 2.1 The lateral diffusivity coefficient calculated in previous research.
Figure 2.1 Hydraulic parameters associated with overbank flow in a two stage channel (Shiono & Knight, 1991).

Figure 2.2 Large scale eddy structure associated with compound channel flow (Lin and Shiono, 1992)
Figure 2.3 Secondary current vectors as measured by Tominaga and Nezu (1991).

Figure 2.4 Isolines of Reynolds stresses $-uv$ and $-uw$ (Tominaga and Nezu, 1991).
Figure 2.5 Isolines of turbulence intensities $u'$, $v'$ and $w'$ (Tominaga and Nezu, 1991).
3. EXPERIMENTAL AND MODEL DETAILS

3.1 MEAN FLOW EQUATIONS

The governing equations of flow, i.e. the conservation of mass and momentum equations have to be solved in three dimensions in order to predict the velocity field and turbulent parameters in a compound channel for steady uniform turbulent flow. The continuity equation is shown in Equation (3.1).

\[
\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0
\]  

(3.1)

and the momentum equations, ignoring the rate of change of Reynolds stresses in the \(x\)-direction, \(- \frac{\partial u'^2}{\partial x}\), \(- \frac{\partial u v}{\partial x}\) and \(- \frac{\partial u w}{\partial x}\) since the flow is uniform, are Equations (3.2) to (3.4).

\[
U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} + W \frac{\partial U}{\partial z} = \left( -g \frac{dH}{dx} + g \sin \theta \right) - \frac{\partial}{\partial y} \frac{v u}{\partial x} - \frac{\partial}{\partial z} \frac{w u}{\partial x}
\]  

(3.2)

\[
U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\partial^2 v}{\partial y} - \frac{\partial w}{\partial z}
\]  

(3.3)

\[
U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} - \frac{\partial^2 v}{\partial z} - \frac{\partial w}{\partial y}
\]  

(3.4)

where \(x\) is the streamwise co-ordinate, \(y\) is the transverse co-ordinate, \(z\) is the vertical co-ordinate, \(U\) is the mean streamwise velocity, \(V\) is the mean lateral velocity, \(W\) is the mean vertical velocity, \(g\) is acceleration due to gravity, \(\rho\) is fluid density, \(H\) is local channel depth, and \(- \frac{\rho u w}{\partial x}, - \frac{\rho u v}{\partial x}\), and \(- \frac{\rho v w}{\partial x}\) are Reynolds stresses, and \(v\) and \(w\) are the fluctuations against the mean values, \(V\) and \(W\), respectively.
3.2 EDDY VISCOSITY

The eddy viscosity concept (Bousinesq, 1877) is based on an analogy between molecular motion, leading to Stokes' viscosity law in laminar flow, and the turbulent motion. For general flow situations, this concept may be expressed in the tensor form (Rodi, 1993)

\[-u_iu_i = \nu_t \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) - \frac{2}{3} k \delta_{ij} \]  

(3.5) or (2.1)

\( \nu_t \) is the turbulent or eddy viscosity which, unlike molecular viscosity, is not a property of the fluid but one of the flow and is dependent on the state of turbulence. The eddy viscosity concept assumes that the Reynolds stresses which arise from the turbulent nature of the flow are linear functions of the appropriate velocity gradients.

The equation for streamwise vorticity, \( \Omega \), is derived by cross differentiation of equation (3.3) and (3.4) and is given as follows:

\[
V \frac{\partial \Omega}{\partial y} + W \frac{\partial \Omega}{\partial z} = \frac{\partial}{\partial \zeta} \left( v' z - w' z \right) + \left( \frac{\partial^2}{\partial \zeta^2} - \frac{\partial^2}{\partial \zeta \partial z} \right) v' w' + \nu V^2 \Omega \]    

(3.6)

[A]    [B]    [C]    [D]

where \( \Omega = \frac{\partial W}{\partial y} - \frac{\partial V}{\partial z} \)

Equation (3.6) governs the secondary motion exactly since if does not contain primary velocity, \( U \). In this equation, term [A] indicates the advection of vorticity. Einstein and Li (1958) first concluded that the secondary motion would not exist if term [A] were zero. Term [B] promotes the generation of secondary currents and term [C] suppresses it. The viscous term [D] is negligible except for regions near the wall. It has been verified both experimentally (Nezu and Nakagawa, 1984) and numerically (Demuren and Rodi, 1984) that the generation term [B] and the Reynolds stress term [C] are the dominant terms. These terms have opposite signs and are much higher than the advection term [A].
The linear k-ε Turbulence Model (LKE Model)

The standard or linear k-ε turbulence model as classified as a two equation model since it uses two transport equations to characterise the turbulence. It makes use of the eddy viscosity concept and the Kolmogorov-Prandlt expression. The Kolmogorov-Prandtl expression states that (Rodi, 1993 and Versteeg & Malasekera, 1995)

\[ \nu_t = C'_\mu \sqrt{k\ell} \]  
\[ (3.7) \]

where \( C'_\mu \) is a constant, \( k \) is the turbulent kinetic energy, and \( \ell \) is the mixing length. From dimensional considerations, the turbulent energy dissipation is given by

\[ \varepsilon \propto \frac{u^3}{L} \]  
\[ (3.8) \]

where \( u \) and \( L \) are local reference velocity and corresponding length scales; or

\[ \varepsilon = C_D \frac{k^{3/2}}{L} \]  
\[ (3.9) \]

where \( u = \sqrt{k} \) and \( C_D \) is an empirical constant. Substituting (3.8) into (3.9) to eliminate \( L \) gives

\[ \nu_t = C'_\mu \frac{k}{\varepsilon} \]  
\[ (3.10) \]

where \( C'_\mu \) is a further empirical constant.

The linear k-ε model has two model equations, one for \( k \) and one for \( \varepsilon \). The variables \( k \) and \( \varepsilon \) are used to determine eddy viscosity, \( \nu_t \), in equation (3.8). At any point in the flow this same \( \nu_t \) is used in all flow directions, i.e. for all Reynolds stress
components. This usage is equivalent to the assumption of a local isotropy in the turbulence. At high Reynolds numbers where local isotropy prevails, the rate of dissipation is equal to the (molecular) kinetic viscosity multiplied by the fluctuating vorticity (Tenekes and Lumley, 1972), i.e.

\[ \varepsilon = \nu \sqrt{\left( \frac{\partial U_i}{\partial x_j} \right) \left( \frac{\partial U_j}{\partial x_i} \right)} \]  

(3.11)

An exact transport equation can be derived from the Navier-Stokes equations of the fluctuating vorticity, and thus for the dissipation \( \varepsilon \). This equation contains complex correlations whose behaviour is little known and for which fairly drastic model assumptions must be introduced in order to make the equation tractable. The outcome of the modelling is the \( \varepsilon \)-equation, i.e. equation (3.11), ((Rodi, 1993)

\[
U \frac{\partial k}{\partial x} + W \frac{\partial k}{\partial z} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial x} \left( \frac{v_t \frac{\partial k}{\partial x}}{\sigma_k \frac{\partial x}{\partial x}} \right) + \frac{\partial}{\partial y} \left( \frac{v_t \frac{\partial k}{\partial y}}{\sigma_k \frac{\partial y}{\partial y}} \right) + \frac{\partial}{\partial z} \left( \frac{v_t \frac{\partial k}{\partial z}}{\sigma_k \frac{\partial z}{\partial z}} \right) + \frac{\partial}{\partial x} U \frac{\varepsilon}{k} + \frac{\partial}{\partial y} U \frac{\varepsilon}{k} + \frac{\partial}{\partial z} \frac{\varepsilon}{k} \pi - \frac{\varepsilon}{k} \]  

(3.12)

\[
U \frac{\partial \varepsilon}{\partial x} + W \frac{\partial \varepsilon}{\partial z} + V \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial x} \left( \frac{v_t \frac{\partial \varepsilon}{\partial x}}{\sigma_k \frac{\partial x}{\partial x}} \right) + \frac{\partial}{\partial y} \left( \frac{v_t \frac{\partial \varepsilon}{\partial y}}{\sigma_k \frac{\partial y}{\partial y}} \right) + \frac{\partial}{\partial z} \left( \frac{v_t \frac{\partial \varepsilon}{\partial z}}{\sigma_k \frac{\partial z}{\partial z}} \right) + \frac{\partial}{\partial x} U \frac{\varepsilon}{k} + \frac{\partial}{\partial y} U \frac{\varepsilon}{k} + \frac{\partial}{\partial z} \frac{\varepsilon}{k} \pi - \frac{\varepsilon}{k} \]  

(3.13)

where \( \pi = \) production of energy by the mean velocity gradients given as

\[
\pi = \nu \left( \frac{\partial U_i}{\partial x_i} \right)^2 + \nu \left( \frac{\partial U_j}{\partial y_j} \right)^2 
\]  

(3.14)

the coefficients in the equation (3.13) are given by Naot and Rodi (1982)

\[
C_{et} = 1.44; \quad C_{et} = 1.44; \quad \sigma_k = 1.225; \quad \sigma_\varepsilon = 1.225
\]

the eddy viscosity is defined as:

\[
\nu_t = \frac{C_{et} k^2}{\varepsilon}
\]  

(3.14)
3.3 SECONDARY FLOW MODELS

3.3.1 Background

The linear k-ε models of turbulence have been shown to be incapable of accurately predicting normal Reynolds stresses and hence making the description of secondary flows by these types of models impossible (Speziale, 1987).

There are a number of other numerical methods for generating secondary flow in an open channel. Three models are used to specify the normal Reynolds stresses and the cross Reynolds stress in the present thesis.

3.3.2 Launder and Ying ASM (LY Model)

Launder and Ying (1973) derived algebraic expressions for the Reynolds stresses in the momentum equation for secondary motion by simplifying the transport equations of the Reynolds stress. The following expression for the normal and cross Reynolds stresses were obtained:

\[ \overline{v^2} = -C' \nu \frac{k}{\varepsilon} \left( \frac{\partial U}{\partial y} \right)^2 + C'_k k \]  

(3.15)

\[ \overline{w^2} = -C' \nu \frac{k}{\varepsilon} \left( \frac{\partial U}{\partial x} \right)^2 + C'_k k \]  

(3.16)

\[ \overline{vw} = \overline{wv} = -C' \nu \frac{k}{\varepsilon} \left( \frac{\partial U}{\partial y} \right) \left( \frac{\partial U}{\partial x} \right) \]  

(3.17)

The values for the empirical constants are \( C'_k = 0.522 \) and \( C' = 0.037 \). The \( C' \) value controls the secondary flow magnitude.
3.3.3 Naot and Rodi ASM (NR Model)

Launder, Reece and Rodi (1975) proposed making empirical constants in the pressure strain model functions of the distance from a surface. Shir (1973) extended this to include a unit vector normal to the surface, and another function of distance from the surface. The suggestion made by Launder et al. (1975) is not very suitable near a free surface where mean velocity gradients are not important, and only the first term in pressure strain is significant. Shir’s (1973) correction does produce the difference between normal and parallel velocity gradients, but is not easy to apply wherever the surface is not plane since it does not allow more than one unit vector to pass through a point in the flow domain. Noat and Rodi (1982) combined Shir’s method for the free surface and Launder et al.’s (1975) method for walls. Launder et al. (1975) assumed a linear function of length scale to distance from a surface, which gives an unrealistically strong influence of remote surfaces. Noat and Rodi (1982) introduced quadratic relationships, which reduce the distance over which the surface has influence. Another difference between Launder and Ying’s ASM and Naot and Rodi’s ASM is that the latter has a higher value for the coefficient $2\beta/C_1$, resulting in higher secondary flow. The equations for the Reynolds stresses governing secondary flow used by Naot and Rodi (1982) are:

$$
\overline{v^2} = \frac{k}{C_1 + 2C_3f_2} \left[ \frac{2}{3} (\frac{1}{2} \alpha - \beta + C_1 - 1) + \frac{\beta}{\varepsilon} \left( \frac{\partial u}{\partial y} - \frac{\partial w}{\partial z} \right) \right] - 2v_1 \frac{\partial v}{\partial y} \quad (3.18)
$$

$$
\overline{w^2} = \frac{k}{C_1} \left[ \frac{2}{3} (\frac{1}{2} \alpha - \beta + C_1 - 1) + C_3f_2 \frac{v_w}{\rho k} \frac{\beta}{\varepsilon} \left( \frac{\partial u}{\partial z} - \frac{\partial w}{\partial y} \right) \right] - 2v_1 \frac{\partial w}{\partial z} \quad (3.19)
$$

$$
\overline{vw} = \overline{wv} = \frac{k}{C_1 + \frac{3}{2} C_3 f_2} \frac{v_w}{\varepsilon} \left( \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} \right) - v_1 \left[ \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right] \quad (3.20)
$$

The empirical values are: $\alpha = 0.7636 - 0.06f_1$, $\beta = 0.1091 + 0.06f_1$, $C_1 = 1.50 - 0.5f_1$, $C_3 = 0.1$. The wall proximity function $f_1$, has been taken as the absolute value of the gradient of the length scale, $l$ suggested by Ilegbusi (1985) and $f_2$ was determined by the Naot and Rodi formula.
3.3.4 Speziale Non-Linear k-ε Model (NLKE Model)

Speziale (1987) demonstrated a special case of the complex non-linear eddy viscosity model obtained by Yoshizawa (1984), namely the non-linear k-ε model. The approach consisted of deriving asymptotic expansions for the Reynolds stresses which maintain terms that are quadratic in velocity gradients. Like the ASM, this model can account for the secondary flow in fully developed non-circular duct flow. The normal stresses and cross Reynolds stresses are as follows:

\[-w_w = v_i \frac{\partial U}{\partial \zeta}\]  
\[-v_v = v_i \frac{\partial U}{\partial y}\]  
\[-w^2 = -k^{\frac{1}{2}}l \frac{\partial W}{\partial \zeta} - C_D l^2 \left[ \frac{1}{12} \left( \frac{\partial U}{\partial \zeta} \right)^2 - \frac{1}{6} \left( \frac{\partial U}{\partial y} \right)^2 \right] - C_E l^2 \frac{1}{3} \left[ \left( \frac{\partial U}{\partial \zeta} \right)^2 + \left( \frac{\partial U}{\partial y} \right)^2 \right]\]  
\[-v^2 = -k^{\frac{1}{2}}l \frac{\partial V}{\partial y} - C_D l^2 \left[ \frac{1}{12} \left( \frac{\partial U}{\partial y} \right)^2 - \frac{1}{6} \left( \frac{\partial U}{\partial \zeta} \right)^2 \right] - C_E l^2 \frac{1}{3} \left[ \left( \frac{\partial U}{\partial \zeta} \right)^2 + \left( \frac{\partial U}{\partial y} \right)^2 \right]\]  
\[\bar{w}v = \bar{v}w = -\frac{1}{2} k^{\frac{1}{2}} \left( \frac{\partial V}{\partial \zeta} + \frac{\partial W}{\partial y} \right) - \frac{1}{4} C_D l^2 \left( \frac{\partial U}{\partial \zeta} \right) \left( \frac{\partial U}{\partial y} \right)\]

where \(k\) = turbulent kinetic energy, \(\varepsilon\) = dissipation rate of turbulent energy, \(v_i\) = eddy viscosity, \(l = 2C_{\mu} k^{\frac{1}{2}} \varepsilon^{-1}\), \(C_{\mu} = 0.09\), \(C_D, C_E\) = coefficients determined from experimental data and both found to be 1.68 using the duct flow data by Speziale (1987). The turbulent energy \(k\) and dissipation rate \(\varepsilon\) are computed using equations (3.26) and (3.27).

\[U \frac{\partial k}{\partial \zeta} + W \frac{\partial k}{\partial y} + V \frac{\partial k}{\partial y} = \frac{\partial}{\partial \zeta} \left( \frac{v_i \frac{\partial k}{\partial \zeta}}{\sigma_k \frac{\partial k}{\partial \zeta}} \right) + \frac{\partial}{\partial y} \left( \frac{v_i \frac{\partial k}{\partial y}}{\sigma_k \frac{\partial k}{\partial y}} \right) + \pi - \varepsilon\]

(3.26)
where $\pi = \text{production of energy by the mean velocity gradients given as:}$

$$\pi = n_{10} \left( \frac{\partial U}{\partial x} \right)^2 + n_{10} \left( \frac{\partial U}{\partial y} \right)^2$$

The coefficients in the equations (3.27) are given by Naot and Rodi (1982):-$

$$C_{\varepsilon_1} = 1.44; \quad C_{\varepsilon_2} = 1.92; \quad \sigma_k = 2.225; \quad \sigma_\varepsilon = 1.225$$

The eddy viscosity $n_{10}$ is defined as:-

$$n_{10} = \frac{C_{\mu} k^2}{\varepsilon}$$

### 3.4 SOLUTE TRANSPORT MODEL

Assuming the Fick’s law to be valid and that the scale of random turbulent motion is very much greater than that of the molecular motion, the steady, three-dimensional solute transport equation of a passive contaminant can be written as:

$$U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} + W \frac{\partial C}{\partial z} = \frac{\partial}{\partial y} \left( D_y \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left( D_z \frac{\partial C}{\partial z} \right)$$

except that the turbulent diffusion term in the streamwise direction is ignored in the hydrodynamic model. The boundary condition to be imposed for solute transport is that no material is lost through the boundaries of the flow and is written as:-

$$\frac{\partial C}{\partial n_i} = 0$$
where $n_i$ is the $i^{th}$ component of the outward boundary unit normal to the boundary.

### 3.4.1 Boundary and initial conditions

There are three types of boundary condition in open channel flow.

- Plane of symmetry: the velocity component normal to the symmetric plane is zero, while the gradient normal to the plane is taken as zero for all other quantities.

- Solid wall: the wall function is adopted such that a resultant streamwise velocity, $U$, is expressed by a local friction velocity at a first grid point to the wall of the computational domain:

- Free surface: a plane of symmetry where the conditions are applied for all variables except for the rate of turbulent energy dissipation. The expression for the dissipation rate given by Noat and Rodi (1982) at a first grid point below the surface was adopted here.

\[
\varepsilon = \frac{C_{\mu}^{3/4} \kappa^{3/2}}{\kappa} \left( \frac{1}{Y'} + \frac{1}{0.07H} \right)
\]  

(3.32)

### 3.5 EXPERIMENTAL PROCEDURE

The experimental measurements were carried out at the Department of Civil and Building Engineering, Loughborough University by Feng and Shiono (1995, 1998). Three dimensional velocities, intensities and Reynolds stresses were measured using a Laser Doppler Anemometer (LDA) with the tracer concentrations measured using a Laser Induced Fluorescence (LIF). The tracer used was a fluorescence dye, injected at a constant rate from a reservoir and via a 2mm diameter nozzle.
3.5.1 Rectangular Channel

The rectangular channel had dimensions of width $B = 1.2$ metres and depth of flow $H = 0.085$ metres. The discharge $Q = 40.4$ l/s, and the bed slope $s = 1/2000$. The tracer was continuously injected at a single point. Two different injection locations were tried, hereon referred to as Case R1 and Case R2. The position of injection are illustrated in Figure 3.2 and the co-ordinates of the injection points are given in Table 3.1. The experimental measurements for velocity were taken over a 10 mm grid between the channel wall and 180 mm laterally towards the centre. The concentration measurements for each case are also shown in Table 3.1.

3.5.2 Compound Channel

Tests were carried out with two different water depths. The first with a depth of 100 mm, and the second shallow case with a depth of 75 mm. The compound channel geometry and dimensions are shown in Figure 3.3. The two discharges were $Q = 46.1$ l/s for the deep channel, and $Q = 22.6$ l/s for the shallow channel, both with a bed slope $s = 1/2000$. The positions of the tracer injection and measurement points are tabulated in Table 3.2. The tracer was injected in three positions in the deep channel, hereafter referred to as Case C1, Case C2, and Case C3. In the shallow channel, only one injection point was used, referred to as S1 in Table 3.2. The measurements were taken 1 metre downstream of the injection nozzle and the dye injection flow rate was 54 ml/minute with an initial concentration 2500 ppb.

3.6 MODELLING METHODOLOGY

The models detailed in Section 3.3 use the SIMPLE algorithm (Versteeg & Malalasekera, 1995) as the solution method, shown in Figure 3.4. The computer code used to execute these models was written in FORTRAN by Lin & Shiono (1992). The author adapted the code for the present study by calibrating the bed roughness. and experimenting with the eddy viscosity model.
3.6.1 Preliminary Benchmark Test

3.6.1.1 Rectangular Channel

Initially the mesh size of the numerical scheme was set at 2 mm, to coincide with the 2mm diameter injection nozzle used in the laboratory experiments. However this grid size required considerable computation time, and therefore a 5 mm grid size was chosen. This grid size enabled much quicker computation and meant that the nodes could coincide with the points of measurement in the experiments. The secondary flow field measured in the laboratory is shown in Figure 3.5. The secondary flows for the LY model, shown in Figure 3.6, shows a large vortex centred at y=100 mm and z=65 mm and a smaller one at y=40 and z=25 mm. The Naot and Rodi model secondary flow predictions, shown in Figure 3.7, show a laterally elongated vortex at the water surface, and a smaller vortex at the channel bed, similar to that predicted by Launder and Ying’s model. Figure illustrates the secondary flow as predicted by the NLKE model. The magnitudes of the secondary velocities are lower than predicted by the ASM models, with a vortex at the water surface in the region of the channel wall. The magnitudes of the computationally predicted secondary flows agree well with the measured data (Feng, 1997).

The vertical streamwise velocity profile at y = 45 mm and y=125 mm are shown examples in Figure 3.9. The linear k-ε model predicted velocity is much higher than the measure values. The model assumes a smooth channel when calculating the friction velocity, $U_*$, with

$$\frac{U}{U_*} = \frac{1}{\kappa} \ln \left( \frac{zU_*}{u} \right)$$  \hspace{1cm} (3.33)

where $U_*$ is friction velocity, $U$ is streamwise velocity, $\kappa$ is von Karman's constant, z is distance from bed and $u$ is kinematic viscosity. However, taking the hypothesis that the channel is not smooth, then an alternative velocity profile has to be used, such as:
\[ U = 2.5U * \ln \left( \frac{30z}{z_0} \right) \] (3.34)

where \( z_0 \) is the bed roughness height. A roughness height of 0.00069 m was chosen by fitting a theoretical rough bed velocity profile to the measure profile at \( y=170 \) mm. The model predictions using the rough bed velocity profile are shown in Figure 3.10 where marked improvement in agreement can be seen when compared with the smooth bed velocity shown in Figure 3.5. The agreement is better further from the wall than it is close to the wall. At \( y=45 \) mm, the measured data shows a negative curve near the surface and this is due to the action of secondary flow. The linear \( k-\varepsilon \) (LKE) model does not calculate this and so the velocity profile straightens near to the surface. The non-linear \( k-\varepsilon \) model (NLKE) results are better than the linear model results because the secondary flow effect is modelled although the negative curve exhibited by the experimental data at \( y=45 \) mm has not been predicted very accurately by this model. Better results were obtained from the Noat and Rodi (NR) model, where secondary flow is stronger than that predicted by the NLKE model and has been shown in the present study to represent secondary flow structure well.

3.6.1.2 Compound Channel

A grid mesh size of 5 mm was chosen for the computation domain which allowed the grid nodes to coincide with the points where the flow properties were measured experimentally. The computational and measured results for secondary flow are shown in Figure 3.11 to Figure 3.14. It can be seen in all the compound channel secondary flow figures that the characteristic twin vortices at the main channel/flood plain interface are present. Better secondary flow is predicted by the NLKE model (Figure 3.14) and the NR model (Figure 3.13) than by the LY model (Figure 3.12).
### Table 3.1 Co-ordinates of the dye injection points in the rectangular channel.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\ell_x$ (m)</th>
<th>$\ell_y$ (mm)</th>
<th>$\ell_z$ (mm)</th>
<th>Measurement points (mm)</th>
<th>Injection concentration (ppb)</th>
<th>Injection flow rate (l/hour)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R1</td>
<td>1.0</td>
<td>170</td>
<td>83</td>
<td>$y = 125, 140, 155, 170$</td>
<td>$z = 60, 70, 80$</td>
<td>2500</td>
</tr>
<tr>
<td>R2</td>
<td>1.0</td>
<td>85</td>
<td>83</td>
<td>$y = 45, 65, 85, 105, 125$</td>
<td>$z = 60, 70, 80$</td>
<td>2500</td>
</tr>
</tbody>
</table>

### Table 3.2 Co-ordinates of the dye injection points in the compound channel.

<table>
<thead>
<tr>
<th>Case</th>
<th>$\ell_x$ (m)</th>
<th>$\ell_y$ (mm)</th>
<th>$\ell_z$ (mm)</th>
<th>Measurement Points (mm)</th>
<th>Injection concentration (ppb)</th>
<th>Injection flow rate (ml/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>1.0</td>
<td>50</td>
<td>108</td>
<td>$y = 15, 30, 50, 75, 100$</td>
<td>$z = 70, 90, 105$</td>
<td>2500</td>
</tr>
<tr>
<td>C2</td>
<td>1.0</td>
<td>100</td>
<td>108</td>
<td>$y = 50, 75, 100, 125, 150$</td>
<td>$z = 70, 90, 105$</td>
<td>2500</td>
</tr>
<tr>
<td>C3</td>
<td>1.0</td>
<td>150</td>
<td>108</td>
<td>$y = 100, 125, 150, 170$</td>
<td>$z = 70, 90, 105$</td>
<td>2500</td>
</tr>
<tr>
<td>S1</td>
<td>1.0</td>
<td>100</td>
<td>73</td>
<td>$y = 100$</td>
<td></td>
<td>2500</td>
</tr>
</tbody>
</table>
Figure 3.1 Co-ordinate system of channels

Figure 3.2 Rectangular channel with injection points.

Figure 3.3 Compound channel with injection points.
START

Initial guess $p^*, u^*, v^*, \phi^*$

STEP 1: Solve discretised momentum equations
\[
\begin{align*}
&\alpha_{i,j} u^*_{i,j} = \sum_{m,n} (p^*_{i,j} - p^*_{i,m}) d_{i,j} + b_{i,j} \\
&\alpha_{i,j} v^*_{i,j} = \sum_{m,n} (p^*_{i,j} - p^*_{i,n}) d_{i,j} + b_{i,j}
\end{align*}
\]

$u^*, v^*$

STEP 2: Solve pressure correction equation
\[
\begin{align*}
&\alpha_{i,j} p'_{i,j} = \alpha_{i,j} p^*_{i,j} + \alpha_{i,j} p^*_{i,j} + \alpha_{i,j} p^*_{i,j} + \alpha_{i,j} p^*_{i,j} + b_{i,j}
\end{align*}
\]

$p'$

STEP 3: Correct pressure and velocities
\[
\begin{align*}
&p_{i,j} = p^*_{i,j} + p'^{'}_{i,j} \\
&u_{i,j} = u^*_{i,j} + \alpha_{i,j} (p^*_{i,j} - p^*_{i,j}) \\
&v_{i,j} = v^*_{i,j} + \alpha_{i,j} (p^*_{i,j} - p^*_{i,j})
\end{align*}
\]

$p, u, v, \phi^*$

STEP 4: Solve all other discretised transport equations
\[
\begin{align*}
&\alpha_{i,j} \phi_{i,j} = \alpha_{i,j} \phi_{i,j} + \alpha_{i,j} \phi_{i,j} + \alpha_{i,j} \phi_{i,j} + \alpha_{i,j} \phi_{i,j} + \alpha_{i,j} \phi_{i,j} + b_{i,j}
\end{align*}
\]

$\phi$

Convergence?

Yes

STOP

No

Figure 3.4 SIMPLE Algorithm
Figure 3.5 Measured secondary flow for rectangular channel

Figure 3.6 LY model secondary flow for rectangular channel
Figure 3.7 NR model secondary flow for rectangular channel

Figure 3.9 NLKE secondary flow for rectangular channel
Figure 3.9 Vertical streamwise velocity with smooth bed at $y=45$ mm and $y=125$ mm

Figure 3.10 Vertical streamwise velocity with rough bed at $y=45$ mm and $y=125$ mm
Figure 3.11 Measured secondary flow for compound channel

Figure 3.12 LY model secondary flow for compound channel
Figure 3.13 NR model secondary flow for compound channel

Figure 3.14 NLKE secondary flow for compound channel
4. RECTANGULAR CHANNEL FLOW

4.1 INTRODUCTION

In this chapter the properties of turbulent flow measured in the rectangular channel are described and compared with the numerical model predictions. The degree of accuracy of the numerical models is commented on, especially the properties most affecting the magnitude of turbulence, and in the regions where turbulent diffusion is being examined.

4.2 TURBULENCE INTENSITIES

4.2.1 Streamwise Turbulence Intensity

The measured streamwise turbulence intensity, normalised by friction velocity $U^*$, is shown in Figure 4.1. The highest turbulence intensities lie at the channel bed and at the sidewall, with the maximum recorded dimensionless value being 1.8 at the channel bed. The turbulence intensity at the channel bed tends to increase as $y/B$ increases. The isolines bulge towards the corner, where the channel bed meets the channel wall, and high turbulence intensities were measured in this region. The values of $u'$ are lowest in the region of the water surface and away from the channel wall.

Figure 4.2 illustrates the LY model’s prediction of streamwise turbulence intensities. As with the measured results the predicted turbulence intensities are highest on the channel bed and at the sidewall. In these regions the LY model shows some resemblance to the experimental data and the LY model predicted isolines also exhibit the bulge towards the corner, albeit not as pronounced as with the measured data. However the LY model has generally over-predicted the magnitude of the intensities.
The numerical prediction of the NR model is shown in Figure 4.3. A high turbulence intensity of 2.0 is predicted on the channel bed, and the sidewall predicted magnitude is 1.6. The bulge of the isolines towards the corner is not well pronounced with this model’s predictions and the isolines are more widely spaced in comparison with the LY model and the measured data. The NR model has not predicted the large area of mainly constant intensity measured close to the water surface.

The NLKE model predictions are presented in Figure 4.4. The predicted intensities on the channel bed agree reasonably well with experimental data, with a maximum magnitude of 1.9. At the channel wall the magnitude of intensity also agrees well with experimental data. However, like the other numerical predictions, the intensity isolines are much more widely space and do not bulge towards the corner as with the measured intensity.

4.2.2 Lateral Turbulence Intensity

The measured lateral turbulent intensity $v'/U*$ is shown in Figure 4.5. The intensity is highest on the channel bed, steadily becoming lower as $z/H$ increases, until the water surface is reached and the value is lowest. Slightly higher values are found close to the channel wall. The lateral intensity predicted by the LY model is shown in Figure 4.6. The $v'$ intensity in the channel is higher than that measured, and is also higher at the channel wall. Like the measured results, the turbulent intensity reduces as the water surface is approached, and as the distance from the wall increases. Figure 4.7 shows the NR model prediction for $v'/U*$. High intensity has been predicted again on the bed and at the sidewall. The intensity decreases more slowly than the LY model prediction as the water surface is neared. A region of high intensity is predicted in the corner of the water surface and the wall, which is similar to the intensity measured in that region. The $v'$ prediction of the NLKE model is presented in Figure 4.8. High intensity at the bed is predicted again, although the wall intensity is lower from the model. The model most accurately predicting the measured results is the NR model in both magnitude and the shape and spacing of the intensity of the isolines. The NR model has a more refined function for the distance of wall influence, and the
The vertical turbulent intensity, \( \frac{w'}{U^*} \), is illustrated in Figures 4.9 to 4.12. First to be shown is the measured intensity in Figure 4.9. The highest intensity occurs at the channel wall, as opposed to the bed where the high lateral intensity occurred. This suggests that the intensity is reduced by the solid boundaries running perpendicular to the vertical plane. Figure 4.10 shows the LY models vertical intensity prediction. High values are predicted along the channel bed as well as near the wall, which is unlike the measured results. An intensity of 1.4 was predicted at the bed near \( y/B=0.14 \). The lowest intensity is predicted close to the water surface, similar to the measured turbulent intensity. Figure 4.11 shows that the NR model predicts the highest turbulent intensity at the channel wall, agreeing with the measured. Elsewhere in the channel the intensity reaches high values in the mid-depth region, becoming less in the vicinity of the channel bed or the water surface. The NLKE model turbulent intensity prediction is illustrated in Figure 4.12. This model predicts a low intensity of 0.69 at the water surface, and a high intensity at the channel bed and wall. Again it is the case that the NR model best predicts the turbulent intensity. This is again due to the NR model's improvement on intensities close to surfaces.

4.3 REYNOLDS STRESSES

4.3.1 Lateral Reynolds Stress

The measured turbulent Reynolds stresses \( \rho \overline{uv} \), normalised by \( U^* \), are shown in Figure 4.13. The maximum Reynolds stress occurs adjacent the channel wall at \( y=0.6H \). As \( y/B \) increases the values of \( \rho \overline{uv} \) decrease. This decrease is gradual in the mid depth region, but very rapid in the bed/wall corner and the wall/surface.
corner. A negative value of 0.04 occurs at the bed where \(y=0.03B\), and more negative values exist in the regions shown furthest away from the wall.

The LY model predictions of \(-\overline{\rho_{uv}}\) are shown in Figure 4.14. The maximum Reynolds stress of 0.89 occurs close to the sidewall the stresses becoming progressively lower with increasing \(y/B\) similarly to the measured data. However the LY model computed Reynolds stresses are generally higher than was measured in the laboratory experiments and does not predict any negative values of \(-\overline{\rho_{uv}}\). The stresses at the channel bed/side wall junction are lower than at the sidewall at mid depth, and likewise where the wall meets the water surface, although \(-\overline{\rho_{uv}}\) at the water surface is higher than at the channel bed.

Figure 4.15 shows the NR model Reynolds stress predictions. A pattern similar to that predicted by the LY model can be seen, with stresses becoming progressively smaller away from the channel wall. Again the highest stress 0.8 occurs by the channel wall at approximately \(z/H=0.5\). This agrees very well with the experimental data where \(-\overline{\rho_{uv}} = 0.52U^2\) at \(y/B = 0.01\) and \(z/H = 0.6\). A value of \(-3.7E-9\) is predicted in the region \(z/H < 0.8\) and \(0.11 < y/B < 0.14\) and this corresponds with the location of the measured negative stresses. However the measured negative values are four times the magnitude of those predicted by the NR model.

The NLKE model predictions are illustrated in Figure 4.16. Again the highest Reynolds stresses are close to the channel wall, and steadily decrease with \(y/B\). The \(-\overline{\rho_{uv}}\) isolines appear close together in the region of the corners, and more spread out elsewhere in the channel. The magnitude of the NLKE model predictions is higher than that of the measured data and is more akin to the LY model predictions. However, the NLKE model predicts an area of negative \(-\overline{\rho_{uv}}\) similar to the NR model, although the magnitude of the Reynolds stresses in this area are lower than those that were measured.
All numerical models predict a similar pattern of isolines for $-\bar{\rho}uv$. The $-\bar{\rho}uv$ isolines trend to be packed closely together in the vicinity of the channel wall and especially so in the corners where the wall meets the bed and the water surface. Also the highest values of $-\bar{\rho}uv$ occur near the wall and these steadily reduce as $y/B$ increases. The measured results include a region of negative $-\bar{\rho}uv$ where $y/B > 0.10$ and $z/H < 0.7$ and this is approximately predicted by the NR and NLKE models, but not the LY model. The closest model prediction in magnitude is the NR model, with the LY and NLKE models both over-predicting the turbulent Reynolds stress $-\bar{\rho}uv$. Therefore the computer model prediction that agrees best with the measured data is that of the NR model since the magnitudes are very similar and in addition predicts the area of negative $-\bar{\rho}uv$.

4.3.2 Vertical Reynolds Stress

The experimental and numerical results for the Reynolds stress $-\bar{u'w'} U^*$ are shown in Figures 4.17 to 4.20. The experimentally measured values are shown in Figure 4.17. At the water surface and where $0.02 < y/B < 0.06$ there is an area of negative $-\bar{\rho}uw$ of value $-0.11$ and surrounding this there is a much lower negative value of $-0.02$. The majority of other values measured are positive. These values start at a magnitude of $0.07$ and increase as $y/B$ increases and $z/H$ decreases. The highest value of $-\bar{\rho}uw$, $0.61$, was measured at the channel bed, at the approximate lateral co-ordinate of $y/B = 12.5$.

Figure 4.18 shows the numerical predictions computed by the LY model. A region of negative Reynolds stress of $-0.13$ is predicted close to the water surface, similar to the values measured. The difference between the LY prediction the measured results at this point is that the LY predicts this negative region to be slightly further from the water surface and closer to the channel wall. Elsewhere in the channel the LY model's predicted $-\bar{\rho}uw$ isolines follow a pattern similar the isolines of the measured $-\bar{\rho}uw$ increase as $y/B$ increases and $z/H$ decreases. At the corner point where the channel
bed meets the channel wall, the $\rho_{\text{uw}}$ isolines are tightly packed increase from 0.04 to 0.89 in the space of $y/B$ increasing from zero to 0.01. This rapid change in $\rho_{\text{uw}}$ at this point agrees with the experimental data, although the numerically predicted change is more dramatic.

The NR model prediction for $\rho_{\text{uw}}$ is shown in Figure 4.19 and follows a similar trend to that of the measured and LY data. Like the measured and LY model results, the NR model results show a region of negative $\rho_{\text{uw}}$ close to the water surface and the channel wall. However the area of this region is larger than both the LY model and measured results, and the magnitudes of $\rho_{\text{uw}}$ are also greater. The positive isolines radiate from the bed/wall corner, as they did with the measured and LY model results, and also like the latter two data sets, are closely grouped together in this region. A high $\partial(-\rho_{\text{uw}})/\partial z$ can be observed at the channel bed.

The numerical prediction for the NLKE model is shown in Figure 4.20 and the familiar $\rho_{\text{uw}}$ isoline pattern can be observed. The negative $\rho_{\text{uw}}$ region has higher stress than had been measured and this region does not extend to the water surface as shown for the measured data in Figure 4.17. The highest positive values of occur at the channel bed where there is a maximum value of 1.4.

It is evident that all the numerical models predict similar patterns of the distribution of $\rho_{\text{uw}}$ to the measured data. The LY model predicts the area of negative stress at the water surface better than the other two numerical models. All three models generally over-predict the magnitude of $\rho_{\text{uw}}$ quite significantly.

### 4.3.3 Secondary Flow Reynolds Stress

The turbulent Reynolds stresses $\rho_{\text{vw}}$, normalised by $U^2$, are shown in Figures 4.21 to 4.24. Figure 4.21 shows the experimentally measured values of $\rho_{\text{vw}}$. In the top left hand corner of Figure 4.21, at the point where the water surface meets the
channel sidewall, there is a small region of negative $-\rho \bar{v}w$ values. The largest positive value of $-\bar{v}w$, 0.08, occurs where approximately $y/B = 0.11$ and $z/H = 0.5$.

The values of $-\rho \bar{v}w$ predicted by the LY model are shown in Figure 4.22. An area of negative $-\rho \bar{v}w$ is evident close to the water surface and the channel wall with a maximum magnitude of $-0.010$. This is the only area of negative $-\rho \bar{v}w$ in the part of the channel shown and corresponds to the clockwise rotation of secondary flow in the region $0.05 < y/B < 0.12$. The peak positive $-\rho \bar{v}w$ occurs in the channel bed/wall region, and has a value of $0.017U^2$. It corresponds to the anti-clockwise vortex of secondary flow in the channel wall/bed region. Where $y/B > 0.06$, $-\rho \bar{v}w$ does not vary as much as it does in the vicinity of the wall.

Figure 4.23 shows $-\rho \bar{v}w$ as predicted by the NR model. This model's predictions show similarities between it and those of the LY model. In particular, there is an area of negative $-\rho \bar{v}w$ in the water surface/channel wall vicinity, however the magnitudes predicted here are much higher than those predicted by the LY model. The maximum value of $-\rho \bar{v}w$ in this region is 0.06, six times greater than the magnitude in the same region predicted by the LY model. Also the shape of the concentric isolines differs from the previous model's predictions in that they are more vertically elongated and closer together. Near the channel bed and the channel wall the NR model has predicted some positive values $-\rho \bar{v}w$, similar to the LY model. There is a high $-\rho \bar{v}w$ value of 0.04 occurring at the bottom corner of the channel and this corresponds to the anti-clockwise secondary flow vortex predicted by the same model.

Finally, model predictions shown for $-\rho \bar{v}w$ are by the NLKE model in Figure 4.24. Again the familiar region of negative $-\rho \bar{v}w$ is visible in the water surface/channel wall region. The magnitude of stresses in this region peak at $-0.07$, corresponding to the clockwise secondary circulation. A major difference between this model prediction and the two algebraic stress models is that the bulge of isolines, protruding
at an angle 45° to the bed from the bottom corner of the channel, stretches almost to the water surface. On examination of the secondary flow (Figure 3.10), it is observed that there is a strong current directed towards the corner from the water surface at an approximate angle of 45° to the bed. The maximum $-\rho\bar{v}w$ has a magnitude of 0.13 in the bottom corner of the channel. A region of negative Reynolds stress occurs in the top corner near the water surface.

4.4 TURBULENT KINETIC ENERGY PRODUCTION

The turbulent kinetic energy production due to the Reynolds shear stresses, normalised by $U^3/H$, is shown in Figures 4.25 to 4.36.

The measured turbulent kinetic energy production due to $u\bar{v}$ is shown in Figure 4.25. The highest turbulent kinetic energy production occurs close to the channel wall with a maximum value of 4.4. Turbulent kinetic energy production beyond 0.1B is significantly lower.

The LY model prediction of turbulent kinetic energy production is illustrated in Figure 4.26. The values of turbulent kinetic energy production calculated from the results of this model are considerably higher than the measured values. The highest value of 36 occurs very close to the wall. Measurements this close to the wall were not possible due to physical restrictions. Bearing this in mind, the lower value of 4.0 occurring around 0.02B is similar to measured values at that same point.

Figure 4.27 shows the turbulent kinetic energy production predicted by Naot and Rodi’s (1982) ASM. The distribution of turbulent kinetic energy production differs from that observed in the measured results and in the LY model prediction.

The prediction of the NLKE model has a similar formation to the NR model prediction, as can be seen in Figure 4.28. This model predicts a maximum turbulent
kinetic energy production of 31, higher than the NR model, but lower than the LY model.

The distributions of turbulent kinetic energy production due to the Reynolds stress $\overline{\rho u w}$ are shown in Figure 4.29 to 4.32. This turbulent kinetic energy production is governed by the Reynolds stress $\overline{\rho u w}$ and the rate of change of streamwise velocity with respect to $z$. The measured turbulent kinetic energy production, shown in Figure 4.29, is largest at the channel bed, with the highest value being 6.2 at the channel bed.

The LY model prediction, shown in Figure 4.30, is larger than the measured values by a factor of 10 with the values peaking at 62. However, if one regards the position of the measured data compared to that of the model data, it can be seen that the prediction and measurement agree. Measurements very close to the bed were not possible because of size of the LDV probe causing similar physical restrictions as encountered for measurements near the wall.

The NR model turbulent kinetic energy production results are shown in Figure 4.31 and again they are much greater than the measured values, over-predicting the amount of turbulence produced.

Figure 4.32 shows the turbulent kinetic energy production predicted by the NLKE model. Very high values are apparent at the channel bed; similar in magnitude to the NR model predictions and considerably over-predicted compared to the measured values.

The turbulent kinetic energy production of the Reynolds stress $\overline{\rho v w}$ is shown in Figures 4.33 to 4.35. The measured values of turbulent kinetic energy production are shown in Figure 4.33. The values close to the channel bed are in the region of $-0.015$ and are generally much lower than the turbulent kinetic energy productions of the other Reynolds stresses $\overline{\rho u v}$ and $\overline{\rho u w}$. Near the water surface and the channel wall there is a region of high production with a magnitude of -0.027. For most of the
channel section pictured the turbulent kinetic energy production remains mainly constant.

Figure 4.34 shows the LY model predictions of turbulent kinetic energy production. Like the measured results, there is a region of high negative production (-0.024) in the water surface/channel wall region. There are also high negative values in the region of the channel bed/wall corner, which was not calculated from the measured results, again due to restrictions of LDV probe movement. The isolines in that area radiate outwards from the corner.

In Figure 4.35 and Figure 4.36 this pattern is also apparent indicating that the NR and NLKE models also predict a large amount of turbulence being produced from the bottom corner of the channel.

4.5 CONCLUSIONS

In conclusion, all models reasonably predict the longitudinal turbulence intensities well, the LY model giving the best prediction. For lateral turbulence intensity, the models all predict well at the channel wall and water surface, but all tend to over-predict at the channel bed. The NR model gives the best prediction for lateral turbulent intensity. For vertical intensity all models predict well at the sidewall and the water surface, but like the lateral intensity, all over-predict at the channel bed. The vertical fluctuations are more inhibited at the bed in the NR model and NLKE model predictions. The influence of the wall is not as far reading in the NR model predictions due to its quadratic function of distance from the wall. The NR model damps vertical fluctuations more than the LY model, which is evidence of the former’s modelled free surface effect.

The measured Reynolds stress $\overline{\rho uv}$ is greatest near the channel sidewall and is of the magnitude of 0.60 in that region. This Reynolds stress is generally slightly smaller than the Reynolds stress $\overline{\rho uw}$, which reaches magnitudes of approximately 0.61
near the channel bed. The Reynolds stress $-\rho\bar{vw}$ has its highest magnitude in the corners of the channel where the water surface and the bed meet the channel sidewall. This Reynolds stress is much lower in magnitude than the Reynolds stresses $-\rho\bar{uv}$ and $-\rho\bar{uw}$, and this leads to the conclusion that $-\bar{vw}$ is the smallest contributor to the turbulence flow resistance.

The numerical models show, confirmed by the measured data, that the most turbulence is generated by the channel bed, which is approximately double that generated by the channel sidewall. The secondary flow does not contribute significantly to the production of turbulence, which has been demonstrated by the $-\bar{vw}$ turbulent kinetic energy production, being of the order of 1000 times smaller than the bed or wall generated turbulent kinetic energy.
Figure 4.1 Measured turbulent intensity $u'/U$.

Figure 4.2 LY model turbulent intensity $u'/U$.

Figure 4.3 NR model turbulent intensity $u'/U$.

Figure 4.4 NLKE turbulent intensity $u'/U$. 
Figure 4.5 Measured turbulent intensity $v'/U$.

Figure 4.6 LY model turbulent intensity $v'/U$.

Figure 4.7 NR model turbulent intensity $v'/U$.

Figure 4.8 NLKE turbulent intensity $v'/U$. 
Figure 4.9 Measured turbulent intensity $w'/U$.

Figure 4.10 LY model turbulent intensity $w'/U$.

Figure 4.11 NR model turbulent intensity $w'/U$.

Figure 4.12 NLKE turbulent intensity $w'/U$. 
Figure 4.13 Measured Reynolds stress
\(-\bar{uv}/U^2\)

Figure 4.14 LY model Reynolds stress
\(-\bar{uv}/U^2\)

Figure 4.15 NR model Reynolds stress
\(-\bar{uv}/U^2\)

Figure 4.16 NLKE Reynolds stress
\(-\bar{uv}/U^2\)
Figure 4.17 Measured Reynolds stress
\[-uw/U *2\]

Figure 4.18 LY model Reynolds stress
\[-uw/U *2\]

Figure 4.19 NR model Reynolds stress
\[-uw/U *2\]

Figure 4.20 NLKE Reynolds stress
\[-uw/U *2\]
Figure 4.21 Measured Reynolds stress 
$-\overline{vw}/U^2$

Figure 4.22 LY model Reynolds stress 
$-\overline{vw}/U^2$

Figure 4.23 NR model Reynolds stress 
$-\overline{vw}/U^2$

Figure 4.24 NLKE Reynolds stress 
$-\overline{vw}/U^2$
Figure 4.25 Measured turbulent kinetic energy production $-\frac{u \nu \partial U}{\partial y} \left/ \frac{U \ast^3}{H} \right.$

Figure 4.26 LY model turbulent kinetic energy production $-\frac{u \nu \partial U}{\partial y} \left/ \frac{U \ast^3}{H} \right.$

Figure 4.27 NR model turbulent kinetic energy production $-\frac{u \nu \partial U}{\partial y} \left/ \frac{U \ast^3}{H} \right.$

Figure 4.28 NLKE turbulent kinetic energy production $-\frac{u \nu \partial U}{\partial y} \left/ \frac{U \ast^3}{H} \right.$
Figure 4.29 Measured turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / H$

Figure 4.30 LY model turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / H$

Figure 4.31 NR model turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / H$

Figure 4.32 NLKE turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / H$
Figure 4.33 Measured turbulent kinetic energy production $-\bar{v}_w\left(\frac{\partial N}{\partial z} + \frac{\partial \mathcal{W}}{\partial y}\right)/U*^3 \over H$

Figure 4.34 LY model turbulent kinetic energy production $-\bar{v}_w\left(\frac{\partial N}{\partial z} + \frac{\partial \mathcal{W}}{\partial y}\right)/U*^3 \over H$

Figure 4.35 NR model turbulent kinetic energy production $-\bar{v}_w\left(\frac{\partial N}{\partial z} + \frac{\partial \mathcal{W}}{\partial y}\right)/U*^3 \over H$

Figure 4.36 NLKE turbulent kinetic energy production $-\bar{v}_w\left(\frac{\partial N}{\partial z} + \frac{\partial \mathcal{W}}{\partial y}\right)/U*^3 \over H$
5. TURBULENCE DIFFUSION IN A RECTANGULAR CHANNEL

5.1 INTRODUCTION
In the previous chapter, the characteristics of the flow were examined. In this chapter the turbulent diffusion of dye injected into the same channel as discussed in Chapter 4 is studied. This chapter includes firstly, a statistical analysis of the measured and computed data from the present study, followed by a comparison with the results from a previous study by Lemoine et al. (1997). Secondly, a calculation of diffusivity establishing mixing lengths using the measured data, and subsequently, a quantitative study of the turbulent Schmidt number.

5.2 STATISTICAL ANALYSIS

5.2.1 Theory
The horizontal distribution of concentration can be analysed through comparison with theoretical distributions. The deviation of the concentration distribution from the normal distribution can be assessed using two quantities: skewness and flatness. Skewness, $S_u$, is defined thus:

$$S_u = \frac{\bar{u}^3}{u'^3} = \int \bar{u}^3 p(\bar{u}) d\bar{u}$$  (5.1)

and flatness, $F_u$,

$$F_u = \frac{\bar{u}^4}{u'^4} = \int \bar{u}^4 p(\bar{u}) d\bar{u}$$  (5.2)
\[ \bar{u} = \frac{u}{u'} \]

and

\[ p(\bar{u}) = G(\bar{u}) \left\{ 1 + \frac{1}{6} Q_{30} (\bar{u}^3 - 3\bar{u}) + \frac{1}{24} Q_{40} (\bar{u}^4 - 6\bar{u}^2 + 3) \right\} \quad (5.3) \]

where

\[ G(\bar{u}) = \frac{1}{\sqrt{2\pi}} \exp \left\{ -\frac{\bar{u}^2}{2} \right\}, \quad Q_{30} = S_u, \quad \text{and} \quad Q_{40} = F_u - 3 \quad (5.4) \]

Skewness indicates the deviation laterally from the symmetry of a normal distribution. A normal distribution yields a skewness zero. Flatness indicates the vertical deviation of the peak of a distribution from that of a normal distribution. A flatness of 3.0 is given by a normal distribution.

5.2.2 Analysis of Data from Present Study

For injection Case R1 the values of skewness of the experimental data and computational data are shown in Figure 5.1. The skewness values measured from the experimental concentration profile are considerably higher than their numerical model counterparts. The skewness for the LKE model reduces as the distance from the bed increases, as it does for the NR and NLKE models. The skewness for the LY model increases with z. Examination of the horizontal velocity components at this lateral position (y=170 mm), as shown in Figure 5.54, Figure 5.55 and Figure 5.56, it is revealed that the experimentally measured velocity has high negative values at z=80 mm at around the region of y=170 mm and this causes the large skew in the concentration profile. In comparison to the measured data, the velocities predicted by the computational models are much lower in this region, hence the skewness of the lateral concentration profiles are much lower.

The flatness, a measure of how much the concentration has spread laterally across the channel is, for Case R1, shown in Figure 5.2. The flatness values of the experimental concentration distribution are considerably higher than the corresponding values for the numerical models. Since the flatness is a measure of
how well the dye is mixed laterally in a particular part of the channel, it can be said that the numerical models are over-predicting the turbulent mixing.

Skewness for injection Case R2 is shown in Figure 5.3. Immediately noticeable is the increasing skewness as the distance from the bed increases from 60 to 80 mm. At $z=80$ mm the LY model produces a skewness of 0.7926 considerably higher than all the other skewness values at that level. The non-linear $k-\varepsilon$ model is close to a normal distribution for $z=60$ mm and $z=70$ mm, and all models have skewed towards the negative. The experimental data however gives positive results for skewness at $z=60$ mm and 70 mm, implying that the transport of dye by the secondary flows is in the opposite direction than that is predicted by the models when assuming turbulent intensities on both sides are similar.

The flatness for Case R2 is shown in Figure 5.4. A value of 3.0 for flatness implies a normal distribution, and here the values for all models and the measured data congregate around that mark. All flatness values tend to be greater than 3 except for the NR model at $z=80$ mm, implying that most the computational data tends to agree with the measured data. The data most closely resembling the normal distribution is that of the NLKE model and the furthest from this is the LY model.

5.2.3 Comparison with Study by Lemoine et al. (1997)

Previous studies of the concentration field downstream of a jet include Shaughnessy and Morten (1977) and Lemoine et al. (1996). The spread of a dye flume due to grid generated turbulence flow has been investigated by Gad-el-Hak & Morton (1979), Nakamura et al. (1987), Simeons and Ayrault (1994) and Lemoine et al. (1997). It is the most recent paper (Lemoine et al.,1997) in grid generated turbulence diffusion that is being compared with the present results.

In the case of isotropic turbulence the theory predicts that the mean concentration field downstream of the grid is Gaussian (Nakamura et al. 1987, Hinze, 1975). Lemoine et al (1997) performed experiments in a 60 mm square channel generating
turbulence using a square grid in a 10 mm mesh. Rhodamine B dye was injected from a nozzle 50 mm downstream of the grid. The results were obtained from measurements 20 mm downstream of the nozzle. Measuring first the concentration along the nozzle axis, Lemoine et al found that the concentration $C(r, x)$ followed a hyperbolic law as shown in Equation 5.5,

$$\frac{1}{C_e(x)} = 1.27 \frac{x}{M} - 0.94,$$  \hspace{1cm} (5.5)

where $x$ is the distance downstream of the injection point, and $M$ is the grid mesh size. This result agreed with the findings of Nakamura et al. (1987).

Table 5.3 below indicates the skewness of experimental and computational data. Immediately apparent is the fact that the skewness value for Lemoine’s data is equal to zero, implying no skewness and the concentration is normally distributed, and evenly spread either side of the injection point. Lemoine assumed that the concentration curve obtained from the study followed the Gaussian law:

$$\frac{C}{C_e(x)} = e^{-\frac{2}{r^2} (r/r_e)^2}$$  \hspace{1cm} (5.6)

where $r$ is the radial distance from the nozzle axis and $r_e$ half-width radius of the mean concentration profile.

The results of the Lemoine et al. (1997) study are compared with the present study in Figure 5.5. The concentrations have been normalised by their maximum values, but their relative positions have been preserved. In the experimental data from the present work, the skew was calculated as 0.2376 at 80 mm from the channel bed. This indicates that there is a shift of the concentration curve to the right of the centre. All the computational models indicate the same shift but to varying degrees. The highest skew is given by the NR model with a value of -0.26516.
A flatness of 3 indicates a normal distribution but a flatness less than 3 indicates a narrower less dispersed distribution, and conversely a flatness greater than 3 implies a flatter, more widely spread distribution. Table 5.4 shows the flatness of experimental data and numerical data. Lemoine’s results give a flatness of 2.96, which is very close to a normal distribution (flatness = 3) with regards to the spreading of dye. All the model results and the present experimental data have flatnesses greater than 3 for their respective horizontal mean concentration distributions. This implies that the concentration is less widely distributed than normal and consequently less widely distributed than the dye concentration measured by Lemoine et al. (1997). The closest data from the present study to that of Lemoine’s data is the experimental data, with a flatness of 3.18.

The influence of secondary flow on the position of the peak concentration can clearly be seen by the difference in skewness between the Gaussian distribution of Lemoine’s data, to the experimental data of the present study and the numerical models’ predictions. The measured secondary flow from the present study, illustrated in Figure 5.6, is responsible for the shift in peak observed in the experimental measured concentration curve. The secondary flows predicted by the numerical models are shown in Figure 5.7, Figure 5.8 and Figure 5.9. The strong secondary flow produced by the LY model and NR model, as shown in Figure 5.7 and Figure 5.8, moved the point of peak concentration the further from the central point. Similarly, the lack of secondary flow in grid generated turbulence yielded a near Gaussian distribution.

The skewness indicates the shift of the concentration distribution peak and maybe an indication of the magnitude of the transverse secondary velocity. With this in mind it can be assumed that a skewness value of zero, with a flatness of 3, then the concentration distribution is Gaussian, implying that there is little or no secondary flow acting on the dye plume.
5.3 MIXING LENGTH INVESTIGATION

The horizontal and vertical turbulent diffusivities, $D_y$ and $D_z$, can be represented by the equations

$$D_y = -\frac{cv}{\partial C/\partial y} \quad \text{and} \quad D_z = -\frac{cw}{\partial C/\partial z} \quad (5.1)$$

The turbulent diffusivities were calculated using the measured fluxes and concentrations. The turbulent kinetic energy was calculated using the NLKE model results for turbulent intensity, thus

$$k = \frac{1}{2} \sqrt{u'^2 + v'^2 + w'^2} \quad (5.2)$$

It was decided to use the numerically predicted turbulent kinetic energy rather than the measured kinetic energy because the numerical results were much less scattered. The diffusivity and kinetic energy are linked in the relationship

$$D = \ell \sqrt{k} \quad (5.3)$$

where $\ell$ is the turbulent mixing length. The mixing length model, $D = \ell \sqrt{k}$, can be used in the solute transport model to calculate the eddy viscosity, since

$$\nu_t = \frac{D}{\sigma_k} \quad (5.4)$$

where $\nu_t$ is eddy viscosity and $\sigma_k$ is the turbulent Schmidt number.

The values of diffusivity calculated from the measured data were plotted against the numerical turbulent kinetic energy, and the gradient of which providing the mixing length. This is illustrated in Figures 5.10 to 5.13. These mixing lengths were used in the numerical models in the eddy diffusivity equation 5.3. Various mixing lengths.
ranging from 2 mm to 20 mm, were experimented with, and optimum values giving the best concentration fits were obtained, as shown in Tables 5.5 and 5.6. The standard deviation between the measured and numerical concentration was used to find the best fit. Two examples are shown in Figures 5.14 and 5.15.

The next step was to relate the optimum mixing length to the eddy viscosity. The relation of the eddy viscosity concept to the kinetic turbulent energy is given by the Kolmogorov-Prandtl expression,

$$v_t \propto \ell \sqrt{k} \quad (5.5)$$

The relationship between rate of turbulent kinetic energy dissipation and length scale with velocity scale can be given by

$$\varepsilon \propto \frac{u^3}{\ell} \quad (5.6)$$

through dimensional analysis where \( u \) and \( \ell \) are local reference velocity and length scales. In terms of the turbulent kinetic energy \( k \), it can be written in the form of

$$\varepsilon = C_D \frac{k^{3/2}}{\ell} \quad (5.7)$$

where \( u \propto \sqrt{k} \) with an empirical constant. Eliminating the length scale using the above relationships gives

$$v_t = C_C C_D \frac{k^2}{\varepsilon} = C' \frac{k^2}{\varepsilon} \quad (5.8)$$

Taking the results of the non-linear k-\( \varepsilon \) model mixing and plotting mixing length, \( \ell \) versus calculated \( k^{3/2} \varepsilon^{-1} \), the constant \( C_D \) was provided by the gradient (Rodi, 1986). This is shown in Figure 5.16. Since the eddy diffusivity, instead of the eddy
viscosity, can be written as $D = C_v k \sqrt{\ell}$, the constant $C_v$ can be obtained by plotting eddy viscosity versus $k \sqrt{\ell}$, as shown in Figure 5.17. Obtaining the two constants, $C_v$ and $C_D$, the constant $C_\mu$ can be calculated thus:

$$C_\mu = C_v C_D$$
$$= 4.11528 \times 0.00949$$
$$= 0.039$$

This indirectly gives the turbulent Schmidt number equal to 2.3 since

$$\nu_t = C_v C_D \frac{k^2}{\varepsilon} = C_\mu \frac{k^2}{\varepsilon} \text{ and } C_\mu = 0.09 \text{ used in the numerical model.}$$

### 5.4 TURBULENT SCHMIDT NUMBER

#### 5.4.1 Introduction

Analysis detailed in Section 5.3 has indicated a turbulent Schmidt number $\sigma_z = 2.3$ in the vertical direction. Traditionally, modellers have used an isotropic turbulent Schmidt number of 1.0. In the light of this result in the present study, an investigation of the turbulent Schmidt number was carried out. Firstly, the traditional isotropic arrangement of 1.0 was tested, referred to as Run RA. Following this, the value $\sigma_z = 2.3$ in the vertical direction was tested, in conjunction with lateral turbulent Schmidt numbers, $\sigma_y$, of 1.0, 1.5 and 0.5, referred to as Run RB, Run RC and Run RD respectively.

#### 5.4.2 Run RA: $\sigma_z=1.0$, $\sigma_y=1.0$

##### 5.4.2.1 Case R1

The Case R1 concentration and flux at $z=80$ mm are shown in Figure 5.20. The computational predictions are similar in terms of magnitude, and agree with the experimental data in that sense, although mixing seems to be under predicted in the
region $0.06B$ to $0.12B$. The peak of the experimental concentration has shifted laterally by $+0.02B$. This has not been predicted by the numerical models. The numerically predicted fluxes, $\overline{cv}$, are equal to zero at the point of maximum concentration, but the numerical magnitudes are two thirds of the experimentally measured flux. This may imply that the models are predicting less mixing.

The vertical mixing in Case R1 is shown in Figure 5.21 and Figure 5.22. At the injection point, $y=0.142B$ (170 mm), the mixing is over-predicted by the numerical models, as can be seen by comparing the profiles in Figure 5.21 with Figure 5.19. The flux is greatly over-predicted. In Figure 5.22, which shows concentration at $y=0.117B$, the mixing is again over-predicted, the largest over-prediction being given by the linear k-ε model. The flux is over-predicted by all models, except the linear k-ε model, which yields are fairly good prediction where $z<0.75H$.

5.4.2.2 Case R2

The horizontal concentration profile at $z=0.94H$ (80 mm) is shown in Figure 5.23. All mixing is under predicted by the numerical models in this isotropic turbulent Schmidt number case. The spread of the experimental concentration is much wider, and peak much lower, than the predictions. The NR model predicts the largest peak shift. The flux is predicted reasonably well in the region $0.05B<y<0.09B$. The flux is under predicted on the right hand side of the injection point. On the left hand side the magnitude is predicted well but the experimental data shows a shift in the peak flux away from the injection point by $0.01B$. Figure shows the experimental and computational concentration and flux at $z=0.75B$. The concentration is predicted very well by the models. The gradient $\partial C/\partial z$ of the experimental data is matched by the numerical data, and the peak concentrations also roughly coincide. The flux is also well predicted in terms of both magnitude and position. The NR models predictions of both flux and concentration show a slight shift to the right of the injection point.

The vertical concentration and flux profiles for Case R2 are shown in Figure 5.25. Figure 5.26 and Figure 5.27. The results at $y=0.104B$ (125 mm) are shown in Figure
5.25. The numerical models all over-predict the mixing, the most over-prediction being made by the two k-ε models. The flux is under predicted, mostly by the k-ε models. This under prediction is a result of \( \frac{\partial C}{\partial z} \) being under predicted. At the injection point, \( y=0.07B \) (85 mm), the concentration prediction is quite good, fairly closely matching the mixing where \( z<0.75B \), as illustrated in Figure 5.26. The experimental data is most closely matched by the non-linear k-ε model. The lateral flux is also predicted well, the peak measured flux at 0.75H being predicted well by the NLKE and LY models. At \( y=45 \) mm, shown in Figure 5.27, the experimental concentration has been measured incorrectly, as a comparison with the concentration measured horizontally in this case will confirm. The experimental flux is very scattered. The numerically predicted concentration peaks at approximately 0.5 ppb. The flux predicted by the models shows a profile similar to the horizontal flux, with positive and negative peaks, illustrating that the concentration gradient changes direction at one point.

5.4.3 Run RB \( \sigma_{kz}=2.3 \) \( \sigma_{ky}=1.0 \)

5.4.3.1 Case R1

The turbulent Schmidt number arrangement consists of vertical \( \sigma_{kz}=2.3 \) and \( \sigma_{ky}=1.0 \). Figure 5.28 shows that, at 0.94H, the magnitude of concentration is over-predicted as a result of the horizontal mixing being under predicted. This is apparent due to the width of the concentration curve. The predicted peak concentrations are all similar, but the position of the LY model peak is shifted approximately 0.01B to the right. The flux predictions at this point, also shown in Figure 5.28, agree fairly well with the experimental data in terms of magnitude, but the predicted positions of the flux curves do not match the data. At 0.82H the mixing is again under predicted, and the concentration magnitudes are too high. The flux is also over-predicted, and the predicted position of maximum flux is out of place by 0.02H. At 0.71H, the concentration magnitudes are lower than the measured data, and the flux values are over-predicted, as shown in Figure 5.29.
The vertical concentration distributions at \( y = 0.142B \) (170 mm) are shown in Figure 5.30. The gradient, \( \partial C / \partial z \), is predicted well by all the numerical models, implying that the numerical models turbulent mixing in the vertical direction is similar to the actual mixing in the laboratory channel. The vertical flux, \( \bar{c}w \), is well predicted where \( z < 0.7H \), but elsewhere the models over-predict the flux by 100%. At \( y = 0.117B \) the concentration is predicted well where \( z < 0.7H \), as shown in Figure 5.31, but is under predicted elsewhere. The closest prediction is produced by the linear \( k-\varepsilon \) model, and the worst prediction is yielded by the LY model. The flux is predicted well where \( z < 0.8H \), and again the best prediction is produced by the LKE model.

5.4.3.2 Case R2

The measured concentration and flux are shown in Figure 5.32. It can be seen that the mixing is greatly under predicted by all the models with this anisotropic turbulent Schmidt number arrangement. This under prediction is signified by the predicted gradients \( \partial C / \partial y \) being much higher than the same gradient for the measured concentration. The flux is over-predicted on both sides of the injection point. The measured flux is more skewed to the right than the models predict. The maximum flux magnitude is predicted by the NLKE model, and the highest shift on the right hand side of the injection point is predicted by the NR model. On the left hand side the predicted peak magnitudes occur at approximately 0.07B, whereas the measured flux peak occurs at around 0.06B. At \( z = 60 \) mm (0.7211), shown in Figure 5.33, the concentration predictions do not agree with the experimental data as well as they did with the isotropic turbulent Schmidt number, the mixing being over-predicted. The flux on the left hand side of the injection point is predicted well, but it is under predicted on the right hand side.

Figure 5.35 shows the vertical distributions for concentration, \( C \), and flux, \( \bar{c}w \). Where \( z < 0.8H \) the agreement is good, but the mixing is still over-predicted where \( z > 0.8H \), especially by the NLKE and LKE models. The flux prediction by the two algebraic stress models is also good where \( z < 0.8H \). The vertical concentration
distributions at the injection point, $y=0.07B$ (85 mm), are shown in Figure 5.36. The mixing is under predicted, as can be seen by comparing the profile of the concentration curves with the idealised diagram in Figure 5.34. The peak concentration predicted by the NR model is closest to the peak experimental concentration. The vertical flux, $cw$, also shown in Figure 5.36, is over-predicted by the models, with the LY model producing the largest over-prediction.

### 5.4.4 Run RC $\sigma_{kz}=2.3 \sigma_{ky}=1.5$

#### 5.4.4.1 Case RI

At $z=80$ mm, Figure 5.37 shows that the peak concentration is over-predicted by the numerical models, implying an under prediction of the turbulent mixing. The other indication of this is that the gradient $\partial C/\partial y$ for the experiment data is much lower than that for the numerical predictions. The flux at $z=80$ mm is over-predicted on both sides of the injection point. The concentration is slightly over-predicted at $z=60$ mm, as Figure 5.38 shows.

Figure 5.39 illustrates the concentration and flux at $y=0.142B$ (170 mm). The models all predicted values of concentration higher than the experimental data, although the gradient $\partial C/\partial z$ of the experimental concentration seems to match that of the numerical data, especially where $0.85<z/H<0.95$. The prediction is better than the prediction with the horizontal turbulent Schmidt number, $\sigma_{kz}=1.0$. The predicted flux $cw$, also shown in Figure 5.39, is much greater than the measured flux. The NR model produced the closest agreement with the experimental data, and the other three models predicted relatively high values for flux.

At $y=0.117B$ (140 mm) the numerical predictions are satisfactory where $z<0.75H$, but over-predict mixing where $z>0.75H$, as illustrated in Figure 5.40. The best prediction is yielded by the NLKE model followed closely by the NR model. The flux also shows a similar pattern, agreeing more with the experimental data below
5.4.4.2 Case R2

The horizontal concentration and flux at $z=0.71H$ (60 mm) can be seen in Figure 5.38. The NLKE and NR model predictions are shown to be reasonable since the spreading and the peak magnitude are approximately of the order of the experimental data. The NR model predicts the largest shift to the left as a result of secondary flow action. The LY and LKE models under predict the magnitude of concentration. The flux predictions are good on the left hand side of the injection point, the best prediction being provided by the NLKE model. On the right hand side of the injection point the numerical models under predict the flux. The NR model predicts the greatest peak shift to the right.

The vertical concentration and flux distributions for $y=0.0875H$ (105 mm) are shown in Figure 5.42. There is very good agreement from the LKE model prediction for concentration. The other models under predict the mixing, and hence give higher values of concentration. The flux values predicted by algebraic stress models are much higher than the experimental values, but the two k-ε models give reasonable flux predictions. All models give reasonable flux predictions. The concentration and flux at $y=0.071B$ (85 mm), the injection point for Case R2, are shown in Figure 5.43. The values of concentration are reasonably well predicted by the NR model where $z>0.75H$, but the shape of the curves indicates that the mixing is generally being under predicted by the models. The flux is fairly well predicted by the two algebraic stress models, but over-predicted by the k-ε models, especially where $z>0.75H$.

5.4.5 Run RD, $\sigma_{kz}=2.3, \sigma_{ky}=0.5$

5.4.5.1 Case R1

Figure 5.44 shows the lateral concentration distributions at $z=0.94H$ (80 mm). All the models predicted a concentration distribution shifted considerably to the right of
the experimental data. The LY model predicts the largest shift to the right. The mixing seems to be predicted well, since the gradient $\partial C/\partial y$ of the experimental data matches that of the computational predictions well. The computational peak magnitudes are slightly higher than the experimental peak. The shift to the right is also noticeable in the lateral flux profiles, again the LY model predicts the largest shift to the right. The magnitude of flux is slightly under predicted by the numerical models.

The vertical concentration and flux distributions are shown in Figure 5.45 and Figure 5.46. The predicted concentration agrees very well with the experimental data at the injection point, $y=0.142B$ (170 mm). The gradient $\partial C/\partial z$ predicted by all models matches that of the experimental data, and the concentration magnitudes also coincide. Figure 5.46 shows that, at $y=0.117B$ (140 mm), the NLKE and NR models agree very well with the experimental concentration data. The LKE and LY models agree with the experimental data where $z<0.75H$, but at depths greater than this the agreement, especially the LY model, is poor.

5.4.5.2 Case R2

At $z=0.94H$ (80 mm), as illustrated in Figure 5.47 the numerical models have all under predicted mixing, the computed peak concentrations being approximately twice those of the experimental data. The flux has been over-predicted by all the models, more so on the left hand side of the injection point that on the right. However, Figure 5.48 shows that, at $z=0.82H$ (70 mm), the computational predictions are good, especially by the NLKE model, which matches the experimental data well in terms of peak magnitude, concentration gradient, $\partial C/\partial y$, and position of the concentration curve. The flux, however, is over-predicted by the all models.

Figure 5.49 shows that the vertical mixing in this case is under predicted by the models with this turbulent Schmidt number arrangement. The peak experimentally measured concentration is much lower than any of the predicted peak concentrations, and the gradient $\partial C/\partial z$ of the experimental data is lower than that of the numerical
data. The numerical flux, also shown in Figure 5.49, does not agree with the experimental flux, although the shapes of the curves are similar. It appears that there could be experimental error, since the measured flux does not reach zero when the measured concentration does.

5.5 DISCUSSION

5.5.1 Lateral Mixing

In Case R1, Run RA the peak concentrations are predicted well by the models, but there is a substantial shift in peak position, and this is due to the lateral velocity. Figure 5.56 shows that the experimental lateral velocity at the injection point is much greater than the numerical prediction. Figure 5.61 shows how the magnitude of lateral velocity, \( V \), directly relates to the shift in peak concentration, \( y \), in the relationship \( y = \frac{Vx}{U} \), where \( x \) is the longitudinal distance from injection point to measurement point, and \( U \) is the longitudinal velocity.

In Run RB, the numerical peak concentration magnitudes are over-predicted. The difference between Run RB and Run RA is that the vertical turbulent Schmidt number was increased from 1.0 to 2.3. The consequence of this is that the predicted peak concentrations have increased hence showing that the vertical Schmidt number of 2.3 does not give as good a prediction as \( \sigma_k=1.0 \). This increase has apparently increased the peak concentration, but has not affected the horizontal mixing, since the lateral turbulent Schmidt number has remained the same. The increase in peak concentration was a result of a reduction in vertical mixing which left a higher concentration of dye in the region of the injection nozzle, instead of spreading it vertically. However, the numerical models have over estimated the mass flow rate.

In Run RC, the turbulent Schmidt number arrangement was \( \sigma_k=2.3 \) and \( \sigma_v=1.5 \). The Case R1 predicted horizontal concentration peaks are much higher than the experimental data, and higher than the predictions in Run RB and Run RA. The
increase in peak was expected due to the reduction in lateral eddy diffusivity that the increase the lateral turbulent Schmidt number induces. In Run RD, where the lateral turbulent Schmidt number of 0.5 was used, the peak concentration is much lower, due to the increased lateral mixing. At \(z=0.94H\) and \(0.82H\) (80 mm and 70 mm respectively), the agreement between numerical and experimental data for Run RD is better of than Run RB and Run RC.

In Case R2, Run RA, the predicted peak concentrations were much higher than the experimental data at \(z=0.94H\) (80 mm), but agreed well at \(z=0.71H\) (60 mm). When the vertical turbulent Schmidt number, \(\sigma_z\), was increased from 1.0 to 2.3, the peak of the predicted horizontal concentration distributions increased in magnitude. This was not an effect of reduced lateral diffusivity, but a result of reduced vertical diffusivity increasing the amount of dye present in the injection region. At \(z=0.71H\), the predicted peak concentrations have reduced, the opposite of the occurrence at \(z=0.94H\). This is because the vertical profile of concentration has changed towards the under prediction of mixing profile, as demonstrated in Figure , producing high concentrations near the surface and the injection point, which rapidly reduce with \(z\), resulting in lower concentrations at \(z=0.71H\) (60 mm). In Run RC, the lateral turbulent Schmidt number was increased from 1.0 to 1.5. This resulted in a large increase in peak concentrations at \(z=0.94H\), due to the cumulative effect of a reduction in both lateral and vertical diffusivity. At \(z=0.71H\) the effect of increasing lateral turbulent Schmidt number has improved the predictions, when comparing Run RC to the predictions in Run RB, since the predicted peak concentrations have increased due to the reduced lateral mixing.

Reducing the lateral turbulent Schmidt number to 0.5 increases lateral diffusivity. The results in Run RD demonstrate this, with the peak concentration at \(0.94H\) being reduced compared to all previous runs. However, for the predictions to agree with the experimental data, it appears that a much lower lateral turbulent Schmidt number is needed in this region of the channel. However, at \(z=0.82H\) (70 mm), the agreement between numerical and experimental data is very good, implying that the turbulent Schmidt number arrangement used in Run RD is correct for this particular point. However, this is not quite confirmed by the lateral flux results, which are
over-predicted by the numerical models. This could be that, although the proportion of eddy diffusivity to eddy viscosity is producing good concentration predictions, the eddy viscosity is being over-predicted by the numerical models, resulting in the over-prediction of lateral flux seen.

The present study demonstrates that the peak concentration can be controlled with variation of the lateral turbulent Schmidt number, however the mass flow rate seems to be over estimated by the numerical models. It has been shown that when the vertical turbulent Schmidt number was increased to 2.3, there was an increase in concentration near the injection location, but this increase was less in regions away from the injection point.

5.5.2 Vertical Mixing

The vertical mixing in injection Case R1 is over-predicted by the numerical models at both lateral points shown, $y=0.142B$ (170 mm) and $y=0.117B$ (140 mm). Comparisons of the experimental with the numerically modelled eddy diffusivity, in Figure 5.57, show that, at $y=0.142B$, the numerical models predicted a higher diffusivity than was obtained from the experimental data, causing the over-prediction of the dye mixing.

The vertical mixing in Run RB is better predicted than it was in Run RA. The increase of vertical turbulent Schmidt number from 1.0 to 2.3 has meant that the vertical diffusivity has decreased, and consequently the numerically predicted eddy diffusivity is closer to the experimental eddy diffusivity, as can be seen by comparing Figure 5.57 and Figure 5.58. In Run RC the lateral turbulent Schmidt number was increased from 1.0 to 1.5. The effect of this was to increase the peak concentration close to the injection point, but reduce the peak concentration away from the injection point. This effect was due to the reduction in lateral mixing causing more of the dye to stay in the region of the injection point, rather than being spread horizontally. In Run RD, with the lateral turbulent Schmidt number equal to 0.5, the models give the best prediction of mixing, suggesting that the arrangement
of $\sigma_z=2.3$ and $\sigma_y=0.5$ is the best arrangement tested in the present study for this region of the channel.

In Case R2, the increase in vertical turbulent Schmidt number has not improved the agreement between models and experimental data for the vertical mixing. Whereas in Case R1, the increment of vertical turbulent Schmidt number from 1.0 to 2.3 improved the agreement of numerical and experimental data. In Case R2 this is not so. The numerical results agreed well with the measured data in Run RA, but with Run RB, the mixing is under predicted. Inspection of the eddy diffusivity, shown in Figure 5.59 and Figure 5.60 reveals that the numerical eddy diffusivity for the isotropic turbulent Schmidt number arrangement is more agreeable with the experimental eddy diffusivity when compared with the anisotropic turbulent Schmidt number arrangement used in Run RB. When the lateral turbulent Schmidt number was increased to 1.5, as in Run RC, the agreement of the numerical data with the experimental did not improve at $y=0.071B$ (85 mm). In Run RD, the lateral turbulent Schmidt number was 0.5. At $y=125$ mm (0.104B), the agreement between the experimental data and the LY model and NR model predictions improves, although the NLKE and LKE models still do not predict well. The increase in lateral mixing has spread the dye further from the injection point, making more dye available at $y=125$ mm for vertical mixing.

This implies that, at 170 mm from the channel side wall, a turbulent Schmidt number of 2.3 is good, but at 85 mm, a turbulent Schmidt number of 1.0 prevails.

5.5.3 Demonstration of Secondary Flow Effect on Solute Transport

Figure 5.62 shows the concentration contours predicted by the LKE model in Case R1. It can be seen that the contours spread out evenly laterally and downwards from the injection point. This spreading is purely due to turbulent diffusion, since this model does not predict secondary flow. Figure 5.63 shows the measured concentration curves for Case R1. It is noticeable that the peak concentration has
shifted laterally towards the wall. This is an effect of the secondary flow (see Figure 5.6).

Figure 5.64 shows the LKE model prediction of concentration in Case R2. As with this model's prediction in Case R1, the contours spread out evenly from the injection point. The measured concentration contours for Case R2 are shown in Figure 5.65. Here the effect of the secondary flow is illustrated well. The contours have been distorted laterally towards the wall and vertically downwards. Comparison of Figure 5.65 with measured secondary flow plot, shown in Figure 5.6 reveals how the secondary flow vectors are acting on the dye.

5.6 CONCLUSIONS

It has been demonstrated that the concentration of dye at a point in a channel is affected by four mechanisms: the lateral eddy diffusion, vertical eddy diffusion, secondary flow and the value of turbulent Schmidt number. The effects of vertical and lateral diffusion are not independent of one another, but have a mutual effect since the dye available to be mixed in one direction is affected by the amount of mixing in the other direction. The secondary flow affects the position of the peak concentration since the action of secondary flow transports the dye in space in the channel without causing mixing itself. The turbulent Schmidt number has been found to vary throughout the channel, and this value affects the amount of mixing obtained from a given eddy viscosity. At 170 mm from the channel side wall, a turbulent Schmidt number of 2.3 is good, but at 85 mm, a turbulent Schmidt number of 1.0 prevails.
Table 5.1 Skewness of concentration curve Case R1

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Table 5.2 Flatness of concentration curve Case R1

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Table 5.3 Skewness of concentration curve Case R2

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Table 5.4 Flatness of concentration curve Case R2

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Table 5.5 Optimum mixing lengths for computational results in Case R1

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Table 5.6 Optimum mixing lengths for computational results in Case R2

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Figure 5.1 Skewness of Concentration Curve Case R1
Figure 5.2 Flatness of Concentration Curve Case R1

Figure 5.3 Skewness of Concentration Curve Case R2
Figure 5.4 Flatness of Concentration Curve Case R2

Simple Channel, z=80
L_x=1.0 m L_y=85 mm L_z=82 mm

Figure 5.5 Case R2, normalised concentration curves of present and previous study (Lemoine et al, 1997)
Figure 5.6 Experimental Secondary Flow

Figure 5.7 Launder-Ying ASM Secondary Flow

Figure 5.8 Noat-Rodi ASM Secondary Flow

Figure 5.9 Non-Linear k-\( \varepsilon \) Model Secondary Flow
Rectangular channel, 
$L_x = 1.0$ mm, $L_y = 155$ mm, $L_z = 82$ mm

**Figure 5.10** Diffusivity vs. Kinetic Energy, Case R1, $y = 155$ mm

Rectangular channel, 
$L_x = 1.0$ mm, $L_y = 170$ mm, $L_z = 82$ mm

**Figure 5.11** Diffusivity vs. Kinetic Energy, Case R1, $y = 170$ mm
Figure 5.12 Diffusivity vs. Turbulent Kinetic Energy, Case R2, y=85 to 105 mm

Figure 5.13 Diffusivity vs. Turbulent Kinetic Energy, Case R2, y=125 mm
Figure 5.14 Performance of models for Case R1 with varying mixing length at y=170 mm

Figure 5.15 Performance of models for Case R2 with varying mixing length at y=85 mm

Figure 5.16 Mixing length versus $k^{3/2}/\varepsilon$
**Figure 5.17** Eddy viscosity versus $\ell \sqrt{k}$

**Figure 5.18** Empirical function

$$C_\mu = f\left(\frac{P}{\varepsilon}\right)$$

**Figure 5.19** Over and under prediction of concentration
Figure 5.20 Run RA, Case R1, concentration (ppb) and flux (ppb.m/s), z=80 mm
Rectangular channel, y=170 mm, $\sigma_{x} = 1.0$, $\sigma_{y} = 1.0$
$L_x = 1.0$ mm, $L_y = 170$ mm, $L_z = 82$ mm

Figure 5.21 Run RA Case R1 Concentration (ppb) and flux (ppb.m/s), $y=170$ mm

Rectangular channel, y=140 mm, $\sigma_{x} = 1.0$, $\sigma_{y} = 1.0$
$L_x = 1.0$ mm, $L_y = 170$ mm, $L_z = 82$ mm

Figure 5.22 Run RA Case R1 Concentration (ppb) and flux (ppb.m/s), $y=140$ mm
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Figure 5.24 Run RA Case R2 concentration (ppb) and flux (ppb.m/s), z=60 mm
Rectangular channel, \( y=125 \text{ mm} \), \( \sigma_z=1.0, \sigma_y=1.0 \)

\( L_x=1.0 \text{ mm}, \ L_y=85 \text{ mm}, \ L_z=82 \text{ mm} \)

Figure 5.25 Run RA Case R2 Concentration (ppb) and flux (ppb.m/s), \( y=125 \text{ mm} \)

Rectangular channel, \( y=85 \text{ mm} \), \( \sigma_z=1.0, \sigma_y=1.0 \)

\( L_x=1.0 \text{ mm}, \ L_y=85 \text{ mm}, \ L_z=82 \text{ mm} \)

Figure 5.26 Run RA Case R2 Concentration (ppb) and flux (ppb.m/s), \( y=85 \text{ mm} \)
Figure 5.27 Run RA Case R2 Concentration (ppb) and flux (ppb.m/s), y=45 mm
Figure 5.28 Run RB, Case R1, concentration (ppb) and flux (ppb.m/s), z=80 mm
Rectangular channel, $z=60$, $\sigma_z=2.3$, $\sigma_{yz}=1.0$
$L_x=1.0\ m$ $L_y=170\ mm$ $L_z=82\ mm$

Figure 5.29 Run RB, Case R1, concentration (ppb) and flux (ppb.m/s), $z=60\ mm$
Rectangular channel, \( y = 155 \text{ mm}, \sigma_{x_y} = 2.3, \sigma_{y_y} = 1.0 \)
\( L_x = 1.0 \text{ mm}, L_y = 170 \text{ mm}, L_z = 82 \text{ mm} \)

Figure 5.30 Run RB Case R1 Concentration (ppb) and flux (ppb.m/s), \( y = 155 \text{ mm} \)

Rectangular channel, \( y = 140 \text{ mm}, \sigma_{x_y} = 2.3, \sigma_{y_y} = 1.0 \)
\( L_x = 1.0 \text{ mm}, L_y = 170 \text{ mm}, L_z = 82 \text{ mm} \)

Figure 5.31 Run RB Case R1 Concentration (ppb) and flux (ppb.m/s), \( y = 150 \text{ mm} \)
Figure 5.32 Run RB Case R2 concentration (ppb) and flux (ppb.m/s), z=80 mm
Rectangular channel, $z=60$, $\sigma_{k_z}=2.3$, $\sigma_{k_y}=1.0$

$L_x=1.0$ m $L_y=85$ mm $L_z=82$ mm

Figure 5.33 Run RB Case R2 concentration (ppb) and flux (ppb m/s), $y=60$ mm
Rectangular channel, \( y=125 \text{ mm}, \sigma_k = 2.3, \sigma_{y} = 1.0 \)
\( L_x = 1.0 \text{ mm}, L_y = 85 \text{ mm}, L_z = 82 \text{ mm} \)

Figure 5.34 Run RB Case R2 Concentration (ppb) and flux (ppb.m/s), \( y=125 \text{ mm} \)

Rectangular channel, \( y=105 \text{ mm}, \sigma_k = 2.3, \sigma_{y} = 1.0 \)
\( L_x = 1.0 \text{ mm}, L_y = 85 \text{ mm}, L_z = 82 \text{ mm} \)

Figure 5.35 Run RB Case R2 Concentration (ppb) and flux (ppb.m/s), \( y=105 \text{ mm} \)
Rectangular channel, y=85 mm, $\sigma_{xz}=2.3$, $\sigma_{xy}=1.0$
$L_x=1.0 \text{ mm}, L_y=85 \text{ mm}, L_z=82 \text{ mm}$

Figure 5.36 Run RB Case R2 Concentration (ppb) and flux (ppb.m/s), y=85 mm
Figure 5.38 Run RC, Case R1, concentration (ppb) and flux (ppb.m/s), z=60 mm
Rectangular channel, \( y=170 \) mm, \( \sigma_{x}=2.3, \sigma_{y}=1.5 \)
\( L_{x}=1.0 \) mm, \( L_{y}=170 \) mm, \( L_{z}=82 \) mm

Figure 5.39 Run RC Case R1 Concentration (ppb) and flux (ppb.m/s), \( y=170 \) mm

Rectangular channel, \( y=140 \) mm, \( \sigma_{x}=2.3, \sigma_{y}=0.5 \)
\( L_{x}=1.0 \) mm, \( L_{y}=170 \) mm, \( L_{z}=82 \) mm

Figure 5.40 Run RC Case R1 Concentration (ppb) and flux (ppb.m/s), \( y=140 \) mm
Rectangular channel, $z=60$, $\sigma_{xz}=2.3$, $\sigma_{yz}=1.5$

$L_x=1.0$ m, $L_y=85$ mm, $L_z=82$ mm

Figure 5.41 Run RC Case C2 Concentration (ppb) and flux (ppb.m/s), $z=60$ mm
Rectangular channel, \( y=170 \) mm, \( \sigma_x=2.3, \sigma_y=0.5 \)
\( L_x=1.0 \) mm, \( L_y=170 \) mm, \( L_z=82 \) mm

Figure 5.45 Run RD Case R1 Concentration (ppb) and flux (ppb.m/s), \( y=170 \) mm

Rectangular channel, \( y=140 \) mm, \( \sigma_x=2.3, \sigma_y=0.5 \)
\( L_x=1.0 \) mm, \( L_y=170 \) mm, \( L_z=82 \) mm

Figure 5.46 Run RD Case R1 Concentration (ppb) and flux (ppb.m/s), \( y=140 \) mm
Rectangular channel, $z=80$, $\sigma_{k_z}=2.3$, $\sigma_{k_y}=0.5$
$L_x=1.0$ m $L_y=85$ mm $L_z=82$ mm

![Graph showing concentration and flux](image)

Figure 5.47 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s)
Rectangular channel, $z=70$, $\sigma_{k_z}=2.3$, $\sigma_{k_y}=0.5$

$L_x=1.0 \text{ m}$ $L_y=85 \text{ mm}$ $L_z=82 \text{ mm}$

Figure 5.48 Run RD Case R2 Concentration (ppb) and flux, $z=70 \text{ mm}$
Rectangular channel, $y=125$ mm, $\sigma_k=2.3$, $\sigma_y=0.5$
$L_x=1.0$ mm, $L_y=85$ mm, $L_z=82$ mm

Figure 5.49 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s), $y=125$ mm

Rectangular channel, $y=105$ mm, $\sigma_k=2.3$, $\sigma_y=0.5$
$L_x=1.0$ mm, $L_y=85$ mm, $L_z=82$ mm

Figure 5.50 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s), $y=105$ mm
Rectangular channel, $y=85$ mm, $\sigma_x=2.3$, $\sigma_y=0.5$
$L_x=1.0$ mm, $L_y=85$ mm, $L_z=82$ mm

Figure 5.51 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s), $y=85$ mm

Rectangular channel, $y=65$ mm, $\sigma_x=2.3$, $\sigma_y=0.5$
$L_x=1.0$ mm, $L_y=85$ mm, $L_z=82$ mm

Figure 5.52 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s), $y=65$ mm
Rectangular channel, $y=45$ mm, $\sigma_{z}=2.3$, $\sigma_{y}=0.5$
$L_{x}=1.0$ mm, $L_{y}=85$ mm, $L_{z}=82$ mm

Figure 5.53 Run RD Case R2 Concentration (ppb) and flux (ppb.m/s), $y=45$ mm

Figure 5.54 Lateral velocity at $z=60$ mm
Figure 5.55 Lateral velocity at z=70 mm

Figure 5.56 Lateral velocity at z=80 mm
Figure 5.57 Run RA, experimental and computed eddy diffusivity at $y=170$ mm

Figure 5.58 Run RB, experimental and computed eddy diffusivity, $y=170$ mm

Figure 5.59 Run RA, experimental and computed eddy diffusivity, $y=85$ mm

Figure 5.60 Case RB, experimental and computed eddy diffusivity, $y=85$ mm
Figure 5.61 Effect of secondary flow on position of peak concentration

\[ V = 0.006687 \text{ m/s} \]
\[ y = \text{lateral distance} \]

\[ U = 0.432 \text{ m/s} \]
\[ x = 1.0 \text{ m} \]

\[ \frac{y}{1.0} = \frac{0.006687}{0.432} \]
\[ y = 0.015 \text{ m} \]
Figure 5.62 Linear k-ε model concentration contours for injection at 0.14B.

Figure 5.63 Measured concentration contours for injection at 0.14B.
Figure 5.64 Linear k-ε model concentration contours for injection at 0.07B.

Figure 5.65 Measured concentration contours for injection at 0.07B.
6. COMPOUND CHANNEL FLOW

6.1 INTRODUCTION
This chapter describes the flow structure and turbulent properties in compound channel flow, and compares the measured with the computational results to validate the flow regime predicted by the numerical models. It is not intended to make criticisms of the numerical methods employed.

6.2 VELOCITY
Streamwise velocities are an important indicator of secondary currents and lateral velocity gradients (Nokes and Wood, 1988). Figure 6.1 shows the measured isovels for streamwise velocity, normalised by friction velocity, \( U^* = \sqrt{gRS_0} \) where \( g \) is acceleration due to gravity, \( R \) is the hydraulic radius, and \( S_0 \) is the bed slope. The bulging of the isovels towards the main channel away from the edge of the flood plain is characteristic of flows where secondary currents are present. The velocity maxima occur at mid depth in the main channel and on the flood plain. It is noticed that the lowest velocity was recorded near the water surface at the main channel side wall. This is obviously incorrect data. The streamwise velocity predicted by the linear k-ε (LKE) model is shown in Figure 6.2. This model does not produce the secondary flow, and accordingly its influence is not reflected in the streamwise velocity contours. There is no bulge in the isovels at the channel junction and the maximum velocity occurs at the surface. This maximum velocity of 0.31 m/s is also higher than that measured in the laboratory. The velocities predicted by the Launder and Ying (LY) algebraic stress model are plotted in Figure 6.3. This model produces secondary flow and their characteristics are evident in the streamwise velocity distribution. The isoline of the maximum velocity of 20 bulges to approximately 0.8\( H \) near the channel step, but then drops to 0.4\( H \) in centre of the main channel. This is a fairly accurate prediction of the experimental data. Figure 6.4 shows the
Naot and Rodi (NR) model predictions for streamwise velocity. Again the characteristics of the compound channel flow are predicted well, and are in good agreement with the experimental data. The velocity predictions of the NLKE model, shown in Figure 6.5 agree less well with experimental data than the two algebraic stress models. The velocity in the main channel is under predicted, and the bulge at the channel step is less pronounced.

6.3 TURBULENCE INTENSITY

Figure 6.6 shows the measured streamwise turbulence intensities, normalised by the friction velocity, \( U^* \). It is noticeable that the peak values occur in the region of the flood plain/main channel interface, the highest value being 2.0. Other high values occur close to the solid boundaries. At the water surface the turbulence intensity is at its lowest, with a value of 1.0 being recorded. The streamwise turbulence intensity predicted by the LY model is plotted in Figure 6.7. The prediction of this model is very similar to that of the experimental data, with maximum intensity of 2.4 being found close to the flood plain bed. There is also a region of high intensity near the main channel wall similar to that measured in the laboratory. The region of low intensity between mid-depth and near the water surface around the main channel is also predicted well by this model. In Figure 6.8 the predictions of Naot and Rodi's ASM are shown. A similar scene is illustrated here, with high intensity on the flood plain bed, and lower intensities in the middle of the main channel and near the water surface. One noticeable difference is that the region of high intensity at the main channel wall is small and further from the water surface than predicted by the Launder Ying model. Finally, the prediction from the non-linear \( k-\varepsilon \) model is shown in Figure 6.9. The intensities on the flood plain bed are similar to those predicted by the LY model and therefore greater than was measured in the laboratory flume, with a peak of 2.4. The intensities near the main channel wall are relatively high, reaching a peak of 2.7. At the opposite end of the scale, the intensities near the free surface are lower than those measured or those predicted by the LY model, and similar to the predictions of the NR model. The best overall prediction for \( u'/U^* \) was given by the LY model.
The measured lateral turbulence intensity, normalised by $U^*$, is shown in Figure 6.10. The peak intensities occur on the flood plain and main channel beds. The intensity in the centre of the main channel is fairly constant, at 0.88 for a large portion of the flow. Figure 6.11 shows the lateral turbulence intensity predicted by the LY model. The values of $v'/U_*$ on the flood plain bed are larger than those measured, peaking at 1.8. The intensity reduces more rapidly, indicated by the closeness of the contour lines, at the walls of the channel than at the bed. This is because the lateral intensity is inhibited by the proximity of the vertical side wall. At the water surface the values are lowest, 0.72 being predicted, close to the measured value. The values of $v'/U^*$ predicted by the NR model, shown in Figure 6.12, are generally slightly lower than both the measured values and the LY model predictions. Close to the water surface 0.70 is predicted compared to 0.72 and 0.82 from the LY model and the experimental data respectively. The values on the flood plain are similar to those predicted by the LY model, and hence they are higher than the values measured in that region. At the side walls of the channel the predicted intensities are very low, (0.44). At the centre of the main channel the intensity is reasonably constant. Figure 6.13 shows the predicted lateral turbulence intensities of the non-linear $k$-$\varepsilon$ model (NLKE). A value of 1.7 on the flood plain bed is very close to the measured value in that area, as is the 1.1 intensity at the flood plain side wall. At the water surface the model slightly over-predicts the intensity with 0.9, but agrees well near the main channel side wall with $v'/U^* = 1.4$. This model is the most agreeable with the measured data for lateral turbulence intensity.

In Figure 6.14, the measured vertical turbulence intensity contours, $w'/U^*$, are plotted. A noticeable difference between these values and the previous lateral intensities is the relative high vertical intensity close to the channel side walls. The intensity is lower near the surface (0.8) and this low intensity protrudes deeper in the centres of the main channel and the flood plain as indicated by the contours. The vertical intensities predicted by the LY model are shown in Figure 6.15. It agrees reasonably well with the measured data, predicting low intensity at the surface (0.72) and high intensity on the flood plain bed (1.7). The LY model predicted intensity
isoline equal to 1.0 penetrates deeper into the main channel than the corresponding measured isoline. Figure 6.16 shows the NR model predictions for vertical turbulence intensities. The magnitudes of vertical turbulence intensity predicted by this model are lower than the measured results and the predictions of the LY model. At the water surface and on the flood plain bed the turbulence intensity drops to 0.68. However, along the main channel side wall the intensity peaks at 1.9, higher than predicted by the LY model and more agreeable with the measured data. The non-linear k-ε model predictions are shown in Figure 6.17. The turbulence intensity at the channel step is relatively high, peaking at 1.4. This is in agreement with the measured data at the channel step. The highest intensities are at the main channel side-wall, also in accordance with the experimental values. The vertical turbulence intensity drops to 0.84 at the water surface over the flood plain, slightly over-predicting the measured value.

6.4 REYNOLDS SHEAR STRESS

The isolines of the Reynolds stress terms $-\overline{uv}$ and $-\overline{uw}$ normalised by the average friction velocity $U^*$ are shown in Figure 6.18 to Figure 6.21. The Reynolds shear stresses are generally related to gradients for the streamwise mean velocity. The measured Reynolds shear stress, $-\overline{\rho uv}$, is shown in Figure 6.18. There is a high negative value near the junction of the main channel and flood plain. The values of $-\overline{uv}/U$. stay positive along the bed of the flood plain, but are negative where $y/H > 0.6$ over the flood plain. Along the main channel side all the values are positive, and remain so while $y/B < 0.2$. They then, however, become negative as far as the flood plain at $y = 0.5B$. Tominaga and Nezu (1990) stated that the sign of the Reynolds stress $-\overline{\rho uv}$ corresponds to the sign of the velocity gradient $\partial U/\partial y$ since

$$-\overline{uv} = D\left(\frac{\partial U}{\partial y} + \frac{\partial V}{\partial x}\right)$$

and $\partial V/\partial x$ is assumed to be small. However, the sign of $-\overline{uv}/U$. is also related to the direction of the secondary flow circulation. The negative region between $y/B=0.2$ and the flood plain corresponds to the anti-clockwise secondary flow adjacent to the wall in the main channel. This is similar to
the findings of Tominaga and Nezu (1990). The positive region over the flood plain where $0.5 < y/B < 0.65$ corresponds to the clockwise secondary flow in this region as shown in Figure 3.10. In Figure 6.19, the values of $-\overline{uv}/U_*$ predicted by the LY model are shown. There is a positive region along the main channel side wall, extending as far as $y/B=0.2$, corresponding to the clockwise secondary flow vortex produced by the LY model, demonstrated in Figure 3.11. In the main channel and where $y/B>0.25$ the values of $-\overline{uv}/U_*$ are negative, the secondary flow vortex is anti-clockwise. The isoline equalling 0.3 that proceeds from the corner of the channel step is inclined at a similar angle to that of the inclined up flow in Figure 3.11. Above the inclined up flow, the value of $-\overline{uv}/U_*$ turns positive as the secondary flow vortex in this region is in the opposite direction to the latter region. Where $y>0.6B$ the secondary flow vortex is anti-clockwise again, demonstrated by the negative values of $-\overline{uv}/U_*$ in that region. Figure 6.20 shows the values of $-\overline{uv}/U_*$ produced by the NR model. Again the correlation between the sign of the $-\overline{uv}/U_*$ values and the direction of the secondary flow vortices, shown in Figure 3.10, are apparent. The values of $-\overline{uv}/U_*$ near the main channel side wall are higher than those predicted by the LY model, but the longitudinal vortex in this area is less pronounced in the NR model. At the main channel step the negative values of $-\overline{uv}/U_*$ are present, but again higher than predicted by the LY model. The gradient of the $-0.38$ isoline is in accordance with that of the inclined up flow shown in Figure 3.12. The values around the channel step agree very well with the experimental data. On the flood plain the NR model $-\overline{uv}/U_*$ values are much greater than the LY values but closer to the experimental than the LY model predictions, with the NR model value of 0.9 being close to the experimental value of 0.71. The $-\overline{uv}/U_*$ values computed by the NLKE model are shown in Figure 6.21. Near the main channel side wall the predicted magnitude is 2.0, the same as the LY model prediction, but greater than the NR model. All the models over-predict the Reynolds stress at this point, the peak measured Reynolds shear stress at the side wall being 1.1. Inspection of the secondary flow in this region (Figures 3.10 to 3.13)
reveals that the magnitude of the measured secondary flow is not as high as predicted computationally.

The negative value of the measured Reynolds stress $-\bar{\rho}u'w'$, shown in Figure 6.22, appears in an area underneath the outline of the inclined upflow close to the channel step and also in the water surface region where $y/B < 0.2$. The negative value near the edge of the flood plain is very significant in compound open channel flow (Tominaga and Nezu, 1990). The surface region condition has been noted in studies in rectangular channel flow, such as Nezu and Rodi (1985) and Nezu et al. (1989). These negative regions correspond well to the regions where $\partial U/\partial z$ is negative. Figure 6.23 shows the LY model $-\bar{\omega}w'/U^*$ prediction. The Reynolds shear stresses are large on the main channel and flood plain beds, peaking at 2.2. There is a positive Reynolds shear stress at the main channel/flood plain interface, where a negative value was measured. Since $-\bar{\omega}w' = D\left(\frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\right)$ and $\partial W/\partial x$ is assumed to be small, the sign of $-\bar{\omega}w'/U^*$ is dependant on the sign of $\partial U/\partial z$. It can be inferred that the LY model predicts a positive value of $\partial U/\partial z$ in this region, and this is confirmed from inspection of the streamwise velocity shown in Figure 6.2. Negative values of $-\bar{\omega}w'/U^*$ are present in the region of the free surface where $y/B < 0.2$ and $y/B > 0.7$. The values of $-\bar{\omega}w'/U^*$ on the flood plain are all positive and the isolines are close together, peaking at 2.2 at $0.8B$ and at the junction but dropping to 1.3 at the flood plain bed. The NR model (Figure 6.24) predicts a negative value of $-\bar{\omega}w'/U^*$ just below the inclined upflow (see Figure 3.12), and also negative values at two water surface regions: $y/B < 2.5$ and $y/B > 0.6$. As with the LY model, the isolines on the flood plain bed region are very closely packed, and peak at 1.9. The NLKE model predictions of $-\bar{\omega}w'/U^*$ are shown in Figure 6.25. The negative values of $-\bar{\omega}w'/U^*$ occur at two water surface regions: $y/B < 1.5$ and $y/B > 0.85$. There is no negative region near the inclined up flow region. However, the flood plain bed values are positive and peaking at 2.2.
The sign of Reynolds stress $-\overline{\rho vw}$ is not only governed by the direction of the secondary flow but also the distortion of the cell, since it is governed by both streamwise velocity gradients, $\partial \overline{V}/\partial y$ and $\partial \overline{W}/\partial z$ in the relation

$$-\overline{vw} = \nu \left( \frac{\partial \overline{V}}{\partial z} + \frac{\partial \overline{W}}{\partial y} \right).$$

For example, a clockwise rotating secondary flow cell, as shown in Figure 6.26, produces a positive value for $-\overline{\rho vw}$, and the anti-clockwise rotating cell, shown in Figure 6.27, produces negative values of $-\overline{\rho vw}$. However, the vertically elongated clockwise rotating cell shown in Figure 6.28 produces negative $-\overline{\rho vw}$, and the anti-clockwise rotating, vertically elongated cell, as shown in Figure 6.29, yields positive $-\overline{\rho vw}$. The measured and computed Reynolds shear stresses, $-\overline{\rho vw}$, normalised by the squared friction velocity $U^2$ are shown in from Figure 6.30 to Figure 6.33, the measured values being illustrated in Figure 6.30. Positive values of 0.08 prevail over the flood plain and negative $-\overline{vw}/U^2$ values of $-0.08$ are prominent in the main channel. A high value of 0.4 was recorded in the free surface main channel side wall corner. The LY model predicted values of $-\overline{vw}/U^2$ are shown in Figure 6.31. The model values are much lower than those measured in the laboratory. There is a positive value in the region of the twin secondary flow vortices at the channel step. In this region the gradients $\partial \overline{V}/\partial y$ and $\partial \overline{W}/\partial z$ are in opposing directions since the vortices are in opposing circulatory directions, and this can be confirmed by examination of the secondary flow vectors shown in Figures 3.10 to 3.13. On the flood plain, the value of $-\overline{vw}/U^2$ is 0.07 at the bottom, which corresponds to a clockwise circulation of secondary flow. In the free surface floodplain side wall region, $-\overline{vw}/U^2$ peaks at -0.02 corresponding to an anti-clockwise circulation. Near the surface and adjacent to the main channel side wall, a maximum of 0.025 is predicted for $-\overline{vw}/U^2$ corresponding to the anti-clockwise circulation. The $-\overline{vw}/U^2$ values computed by the NR model are shown in Figure 6.32. Negative values of $-\overline{vw}/U^2$ equalling 0 and -0.03 are computed in the region of the main secondary flow vortices by this model, slightly higher than the
measured values in the same vicinity. The positive $-\overline{vw}/U^2$ values in the main channel/free surface corner region are lower in magnitude than were recorded experimentally. On the flood plain, values of 0.6 are present near to the bed, and -0.3 in the side wall/free surface corner. In Figure 6.33 the NLKE model Reynolds shear stress $-\overline{vw}/U^2$ prediction is shown. There are areas of positive $-\overline{vw}/U^2$ near the main channel twin vortex (see Figure 3.13). An area of negative values exists in the region of the flood plain twin vortex. Positive values are present again approximately where $y/B > 0.7$, but a negative value exists at the flood plain side wall/surface corner. The magnitudes of the positive and negative values near the twin vortices are much lower than the other computer models in all regions of the compound channel.

6.5 TURBULENT KINETIC ENERGY PRODUCTION

The turbulent kinetic energy production was calculated from the Reynolds shear stresses, $-\overline{wv}$, $-\overline{uw}$ and $-\overline{vw}$. The turbulent kinetic energy production due to $-\overline{wv}$ was calculated $-\overline{wv}\frac{\partial U}{\partial y}$, assuming $\frac{\partial V}{\partial x} = 0$ because the flow is uniform. Turbulence kinetic energy production due to the $-\overline{uw}$ term was calculated from $-\overline{uw}\frac{\partial U}{\partial z}$, assuming $\frac{\partial W}{\partial x} = 0$. The relationship giving the turbulence kinetic energy production due to the $-\overline{vw}$ term is written as $-\overline{vw}\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right)$. The results were all normalised by $\frac{U^3}{H}$.

There are areas of very high turbulent kinetic energy production due to the $-\overline{wv}$ term close to the channel side walls, peaking at 120 in the main channel and 46 on the flood plain, as shown in Figure 6.34. High magnitudes of turbulent kinetic energy production were measured close to the main channel/flood plain interface wall (120). The LY model predicts the same high peak areas, as can be seen in Figure 6.35, but
with lower turbulent kinetic energy production of 83 at the main channel walls, and 64 at the flood plain wall. The turbulent kinetic energy production due to the $-\overline{uv}$ term predicted by the NR model, shown in Figure 6.36, is also very high in the side wall regions, and predicts the same value as the measured results at the main channel side wall and low values in the mid channel regions. Similar magnitudes, shown in Figure 6.37, have been predicted by the NLKE model. The high areas of turbulent kinetic energy production occur in the regions where secondary flow is relatively low, but Reynolds shear stress, $-\rho\overline{uv}$, is high.

The turbulent kinetic energy production due to the shear stress $-\overline{uw}$, $-\overline{uw}\frac{\partial U}{\partial z}/\overline{U}^3$, is shown in Figure 6.38 to Figure 6.41. High magnitudes were measured on the flood plain bed, near the main channel bed, the flood plain side wall and in the region of the main twin vortices, as shown in Figure 6.38. It can be seen that the highest values of turbulent kinetic energy production are on the flood plain close to the channel wall and close to the main channel/flood plain interface. Figure 6.39 shows that the LY model predicts very high values of turbulent kinetic energy production on the main channel and flood plain beds, although these values are lower than the measured. Turbulent kinetic energy production predicted by the NR model is also high on the main channel bed, but lower on the flood plain bed, as Figure 6.40 shows. The turbulent kinetic energy production predicted by the NLKE model is shown in Figure 6.41. High turbulent kinetic energy production is predicted on both the main channel and flood plain beds. The values on the flood plain are higher than predicted by the NR model, but lower than the LY model prediction and the measured data. The main channel bed values are lower than predicted by the all models and the measured data.

The turbulent kinetic energy production due to $-\overline{vw}$ is very much lower than those due to $-\overline{uv}$ or $-\overline{uw}$. The highest magnitudes occur on the flood plain bed and near the corners. The highest measured value of $-\overline{vw}$ induced turbulent kinetic energy production, -1.9, occurs at the corner of the main channel bed (6.0) and above the flood plain bed (-6.0), as shown in Figure 6.42. High values also occur in the bottom
right hand corner of the main channel. The LY model predicts values that are lower than the measured results, but the highest values are in the same regions as the highest measured values. Specifically, these are the corners where the walls meet the beds, and just above the main channel/flood plain interface as Figure 6.43 illustrates. The predictions from the NR model, shown in Figure 6.44, produce values of -1.9 in the water surface/side wall corner region of the main channel. At the bottom right hand corner of the main channel, a relatively high value of -1.3 is predicted. On the flood plain side the turbulent kinetic energy production was -0.7 on the bed, and 1.1 near the main channel/flood plain interface. Finally, the NLKE model predictions are shown in Figure 6.45. Once again, high values are apparent in the corners and at the main channel/flood plain interface, where secondary flow is strong.

6.6 CONCLUSIONS

The measured results and the LY, NR and NLKE model predictions all show in the influence of secondary flow in their streamwise velocity predictions. The magnitude of streamwise velocity is predicted well by the numerical models.

The streamwise turbulent intensity is high at the walls and the beds, and especially high at the bed/wall corners. All models slightly over-predict the experimental results, but their predictions are satisfactory. The best prediction is produced by the NR model. The lateral turbulent intensity is lower close to the side walls than it is at the channel bed. The models tend to over-predict in the main channel, but predict well on the flood plain and near the surface. The vertical turbulent intensity tends to be lower at the beds than at the side walls. The models all predict the vertical turbulence intensity well at the channel walls, the main channel bed, the flood plain and the water surface. The NR model tends to under predict the magnitudes, but is good at the main channel side wall.
The Reynolds shear stresses, $-\bar{\rho uv}$, are highest at the vertical surfaces, and the Reynolds shear stresses, $-\bar{\rho uw}$, are highest at the horizontal. These two stresses are much higher than the relatively small Reynolds stress, $-\bar{\rho vw}$. The values of $-\bar{\rho vw}$ are relatively high in areas of strong secondary flow.

The highest values of turbulent kinetic energy production are in the same regions as the highest values of Reynolds shear stress. Turbulence is therefore predominantly generated by the walls, although bed generated turbulence is also a significant source of turbulence production. Secondary flow generated turbulence is negligibly small, as expected from the values of $-\bar{\nu w/U^2}$ recorded.
Figure 6.1 Measured streamwise velocity $U/U$.

Figure 6.2 LKE model Streamwise isovels $U/U$.

Figure 6.3 LY ASM Streamwise isovels $U/U$.

Figure 6.4 NR ASM Streamwise isovels $U/U$.

Figure 6.5 NLKE model Streamwise isovels $U/U$. 
Figure 6.6 Measured streamwise turbulence intensity $u'/U$.

Figure 6.7 LY ASM streamwise turbulence intensity $u'/U$.

Figure 6.8 NR ASM streamwise turbulence intensity $u'/U$.

Figure 6.9 NLKE model streamwise turbulence intensity $u'/U$. 
Figure 6.10 Measured lateral turbulence intensity $v'/U$.

Figure 6.11 LY ASM lateral turbulence intensity $v'/U$.

Figure 6.12 NR ASM lateral turbulence intensity $v'/U$.

Figure 6.13 NLKE model lateral turbulence intensity $v'/U$. 
Figure 6.14 Measured vertical turbulence intensity $w'/U_*$.

Figure 6.15 LY ASM vertical turbulence intensity $w'/U_*$.

Figure 6.16 NR ASM vertical turbulence intensity $w'/U_*$.

Figure 6.17 NLKE model vertical turbulence intensity $w'/U_*$.
Figure 6.18 Measured shear stress $-\overline{uv}/U^2$

Figure 6.19 LY ASM shear stress $-\overline{uv}/U^2$

Figure 6.20 NR ASM shear stress $-\overline{uv}/U^2$

Figure 6.21 NLKE model Reynolds shear stress $-\overline{uv}/U^2$
Figure 6.22 Measured Reynolds shear stress $-\frac{\overline{uw}}{U} \ast^2$

Figure 6.23 LY ASM Reynolds shear stress $-\frac{\overline{uw}}{U} \ast^2$

Figure 6.24 NR ASM Reynolds shear stress $-\frac{\overline{uw}}{U} \ast^2$

Figure 6.25 NLKE model Reynolds shear stress $-\frac{\overline{uw}}{U} \ast^2$
Figure 6.26 Clockwise secondary flow cell producing positive $-v_w$

Figure 6.27 Anti-clockwise secondary flow cell producing negative $-v_w$

Figure 6.28 Clockwise secondary flow cell producing negative $-v_w$

Figure 6.29 Anti-clockwise secondary flow cell producing positive $-v_w$
Figure 6.30 Measured Reynolds shear stresses $-\bar{vw}/U^2$

Figure 6.31 LY ASM Reynolds shear stress $-\bar{vw}/U^2$

Figure 6.32 NR ASM Reynolds shear stress $-\bar{vw}/U^2$

Figure 6.33 NLKE model Reynolds shear stress $-\bar{vw}/U^2$
Figure 6.34 Measured Turbulent kinetic energy production $-u v \frac{\partial U}{\partial y} / U^3$

Figure 6.35 LY model Turbulent kinetic energy production $-u v \frac{\partial U}{\partial y} / U^3$

Figure 6.36 NR model turbulent kinetic energy production $-u v \frac{\partial U}{\partial y} / U^3$

Figure 6.37 NLKE turbulent kinetic energy production $-u v \frac{\partial U}{\partial y} / U^3$
Figure 6.38 Measure turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / U^3$.

Figure 6.39 Turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / U^3$.

Figure 6.40 NR model Turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / U^3$.

Figure 6.41 NLKE Turbulent kinetic energy production $-u_w \frac{\partial U}{\partial z} / U^3$. 
Figure 6.42 Measured turbulent kinetic energy production \(-\nu_w \left( \frac{\partial N}{\partial x} + \frac{\partial W}{\partial y} \right) \frac{U^*}{H}\)

Figure 6.43 LY model turbulent kinetic energy production \(-\nu_w \left( \frac{\partial N}{\partial x} + \frac{\partial W}{\partial y} \right) \frac{U^*}{H}\)

Figure 6.44 NR model turbulent kinetic energy production \(-\nu_w \left( \frac{\partial N}{\partial x} + \frac{\partial W}{\partial y} \right) \frac{U^*}{H}\)

Figure 6.45 NLKE turbulent kinetic energy production \(-\nu_w \left( \frac{\partial N}{\partial x} + \frac{\partial W}{\partial y} \right) \frac{U^*}{H}\)
7. TURBULENCE DIFFUSION IN COMPOUND CHANNEL

7.1 INTRODUCTION

In this chapter, turbulence diffusion in a compound channel for two relative depths is discussed. For the deep compound channel, the experimental data for concentration and flux are compared with the computational predictions. The anisotropy of eddy diffusivity is examined using results of a calibration exercise to calculate turbulent Schmidt numbers, $\sigma_k = \frac{V}{D}$. The first case to be considered is isotropic diffusivity with a turbulent Schmidt number equal to one. Following this two anisotropic arrangements of Schmidt number are discussed and the best numerical predictions are identified.

In the shallow compound channel case (see Table 3.2 and Figure 3.3), the effect of secondary flow on the turbulent diffusion is examined in detail, with particular reference to the injection position.

7.2 ISOTOPIC DIFFUSIVITY

7.2.1 Lateral Mixing

7.2.1.1 Case C1, Run CA $\sigma_{xy} = 1.0$, $\sigma_{xz} = 1.0$

The dye concentration distributions for Case C1 are shown in Figure 7.1. The experimental data shows a curve that peaks at approximately 8 ppb. All the numerical models, except for the linear k-\(\epsilon\) model (LKE) predict higher peaks. but the spread of dye is less than in the experimental data. The linear k-\(\epsilon\) model produces more or less a normal distribution, i.e. skewness = 0 and flatness = 3, as expected as the result of no secondary flow. However, the distribution is a little skewed. This is caused by the variation of the eddy diffusivity in the lateral direction. The models predict lower
concentration in the regions \( y < 0.1B \) and \( y > 0.4B \) implying that the lateral mixing is being under predicted by the models. The statistical analysis, illustrated in Figure 7.2 and Figure 7.3, shows that the skewness of the experimental data is much higher than that of the numerical predictions. This implies that the experimental data concentration curve is more off centre than the models' predictions, and this is an effect of secondary flow. The flatness, which is a measure of the spreading of the dye, is shown in Figure 7.3. The closer the flatness value is to 3, the closer the distribution is to a normal distribution. The LY and NLKE models' predictions have an almost perfect normal distribution in terms of the flatness, but not for skewness. The experimental data and the NR model each have a flatness of about 3.5, and the LKE model has a flatness of approximately 2.7. The experimental data curve is flatter than the numerically predicted curves, which illustrates that the models are under predicting the spreading of dye.

The lateral Reynolds flux, shown in Figure 7.1 is predicted well by the non-linear \( k-\varepsilon \) model (NLKE) where \( y < 0.2B \). However, in the region \( 0.3B < y < 0.4B \) the NLKE model, LKE model and LY model all under predict the flux.

7.2.1.2 Case C2, Run CA, \( \sigma_{y} = 1.0, \sigma_{z} = 1.0 \)

The distributions of the concentration and lateral Reynolds flux for Case C2 are shown in Figure 7.4. The experimental concentration peaks at approximately 12 ppb, and the distribution is skewed to the right. The LY model most closely matches the experimental data in peak magnitude and peak position. The NR model predicts the highest peak, and is the furthest displaced from the injection point. The displacement of the concentration peak has been quantified as the skewness, plotted in Figure 7.5. The experimental data skewness is a little over -1.0 and the LY model skewness is the closest model to that. The LKE results have a very small skewness and the NR and NLKE model skewnesses are in between. The NR model predicted the greatest lateral displacement of peak concentration, and the highest peak concentration. The LKE model has predicted the lowest peak concentration, followed by the NLKE model. The gradient \( \partial C / \partial y \) of the concentration is an indication of the turbulent mixing. The gradient is steeper in the right hand side of the injection point, implying that there is
less dye mixing than to the left of the injection point. The gradient of dye is predicted well by the LY and NR models on both sides of the injection point, but the LKE and NLKE models do not predict the concentration gradient well. The flatness of the curves, representing the overall spreading of the dye, are all of a similar magnitude, as shown in Figure 7.6. The closest predicted flatness to the experimental data is provided by the NLKE model.

The lateral Reynolds flux is also shown in Figure 7.4. The experimental flux is higher than the predicted fluxes, in both the positive and negative peaks. The flux is linked to the concentration gradient and the eddy viscosity by the following relationship,

$$\overline{\nabla v} = \frac{\nu_t}{\sigma_{ky}} \frac{\partial C}{\partial y},$$

where $\nu_t$ is the eddy viscosity and $\sigma_{ky}$ is the turbulent Schmidt number. The turbulent Schmidt number $\sigma_{ky} = \frac{\nu_t}{D_y}$, where $D_y$ is the eddy diffusivity for the lateral component. On the right hand side of the injection point, the measured flux peaks at 0.04 ppb.m/s, but peaks at just less than -0.03 on the left hand side of the injection point. The steeper concentration gradient is linked to the higher flux observed on the right hand side of the injection point. The point of maximum flux coincides with the point of steepest concentration gradient and zero flux with the peak concentration. The NR model predicts a flux whose peaks coincide with the experimental flux peaks, and on the left had side of the injection point the flux magnitudes are predicted well. On the right hand side of the injection point, all the models under predict the flux.

### 7.2.1.3 Case C3, Run CA, $\sigma_{ky}=1.0, \sigma_{kz}=1.0$

In this case, as can be seen in Figure 7.7, the measured concentration peaks at 12 ppb. at lateral distance $y=0.75B$, which is in line with the injection point. The NR model predicts a higher peak concentration of over 14 ppb, and the LKE model predicts the lowest peak concentration of 9 ppb. The LY model best predicts the concentration peak magnitude. The concentration gradients $\frac{\partial C}{\partial y}$ are predicted well by all the models except the NR model. The steeper concentration gradient is left of the
injection point, and the lower gradient on the right side of injection point and both sides are accurately predicted.

The skewness results are shown in Figure 7.8. The experimental data skewness is 0.29, the LY model and NLKE model relatively close to that value. The highest skewness is predicted by the NR model, and the LKE prediction has a very low skewness, close to zero. The flatness values are shown in Figure 7.9. The experimental data flatness is a little over 3.0. The other models are all around the value of 3.0.

The measured lateral Reynolds flux, shown in Figure 7.7, peaks at 0.04 ppb.m/s on the right of the injection point. The numerical models all under predict this peak, but in the region y>85B, the models over-predict the flux. In the region 0.5B<y<0.7B the linear and non-linear k-ε models predict the magnitude of flux reasonably well. In the same region, the NR model under predicts the flux, and at y=0.7B, over-predicts. The NR model’s flux curve is shifted much further to the right, compared with the experimental data and the other models’ predictions.

7.2.2 Vertical Mixing

7.2.2.1 Case C1, Run CA, σ_kz=1.0, σ_kε=1.0

The experimental results and numerical predictions at y=0.075B are shown in Figure 7.10. The experimental data shows a maximum concentration of approximately 4.0 ppb. This concentration is not predicted by any of the models. The models all over-predict the mixing, and therefore produce lower peak concentrations, and lower concentration gradients ∂C/∂z. The flux seems to be predicted well by the numerical models.

At y=0.15B, the numerical models again over-predict the mixing, displaying the typical over-predicting profile, as shown in Figure 7.11. The experimental Reynolds flux data is very scattered, and this does not make it easy to say whether the numerical
data is good or not. It seems that the numerical predictions produce values within the range of the experimental values, and therefore it could be assumed that the predictions are satisfactory.

At $y=0.5B$, shown in Figure 7.12, the main channel/flood plain (M/F) interface, the experimental data is well scattered, but it is clear that the models over-predict the mixing. However, the flux predictions are good.

7.2.2.2 Case C2, Run CA, $\sigma_y=1.0, \sigma_z=1.0$

Figure 7.13 shows the concentration and flux at $y=0.25B$. The experimental concentration peaks at around 3.0 ppb, but the numerical predicted concentrations are much lower implying that the models assume more mixing is taking place. At $y=0.50B$, shown in Figure 7.14, the numerical models predict lower concentration than the experimental data, but the concentration gradients of models and experiment data are similar, implying that vertical mixing has been predicted well. The flux predictions and the experimental flux also agree well in support of this.

Figure 7.15 illustrates the concentration and flux at $y=0.75B$. The experimental data predicts a maximum concentration of 4.5 ppb. All the predictions over-predict the mixing, giving relatively low concentrations, but the NR model gives a slightly better prediction.

7.2.2.3 Case C3, Run CA, $\sigma_y=1.0, \sigma_z=1.0$

The concentration and flux at $y=0.5B$ for Case C3 are shown in Figure 7.16. The experimental concentration curve appears to have two peaks, one at $z=0.6H$ (0.35 ppb) and one at $0.925H$ (5.0 ppb). The numerical models do not predict the double peak, although the NR model predicts a peak at $z=0.6H$, which coincides with one of the experimental data’s peak. The linear $k$-$\varepsilon$ model predicts the peak concentration at $0.9H$ and predicts the highest magnitude, the non-linear $k$-$\varepsilon$ model at $0.825H$, slightly lower in magnitude than that of the LKE model, and the LY model at $0.775H$, lower in magnitude still.
The measured flux is very scattered, with positive and negative values reflecting the changing gradient direction of the concentration curve. The magnitude of the numerical models' flux seems reasonable.

### 7.2.3 Demonstration of Secondary Flow Effect on Solute Transport

Figure 7.20 shows concentration contours of the LKE model prediction for Case C1. The contours show that the dye spreads out evenly away from the injection point since this model does not predict secondary flow. The contours for concentration measured between $y=0.075B$ and $y=0.5B$ for Case C1 are shown in Figure 7.21. It is illustrated that the dye is transported downwards and towards the side wall by the secondary flow.

In Case C2, the LKE model predicts the dye spreading from the injection point as shown in Figure 7.22. For the same injection point, Figure 7.23 shows the measured concentration contours where $0.25B<y<0.75B$. It is apparent that the dye has been transported onto the flood plain by the secondary flow (See Figure 3.11). The concentration on the flood plain is higher than in the main channel. This is because the eddy viscosity on the flood plain is smaller, and so dye is mixed less (see Figure 7.32).

Figure 7.24 shows the concentration contours as predicted by the LKE model. As with the LKE model prediction in Case C1, the contours indicate an even spreading of dye in all directions from the surface. Figure 7.25 shows that, with the measured concentration data between $y=0.5B$ and $y=B$, the dye is transported downwards from the surface and towards the wall by the secondary flow.

### 7.2.4 Discussion

The numerical predictions of concentration for Case C1, shown in Figure 7.1, are all higher than the experimental data between $y=0.1B$ and $y=0.4B$, but lower elsewhere in the channel. This effect is a cause of the spreading of the dye laterally in the channel. The more the dye spreads, then the lower the concentration gradient. The numerical models are predicting less lateral spreading than has been measured
experimentally. The flux predictions, also shown in Figure 7.1, agree well with the experimental data. The relationship between the concentration gradient and the lateral flux is given in the equation,

$$- \frac{\bar{c}_v}{\bar{c}_v} = - \frac{v_t}{\sigma_y} \frac{\partial C}{\partial y},$$

The under prediction of the mixing is due to either the predicted eddy viscosity being too low, or the turbulent Schmidt number being too high. The non dimensional depth averaged eddy viscosity is shown in Figure 7.18. The linear k-ε model predicts the highest eddy viscosity across the channel width, and the LY model the lowest. There is a sharp decrease in eddy viscosity at the M/F interface. The turbulent Schmidt number used in this run was equal to 1.0 and isotropic. Lin and Shiono (1995) calculated a Schmidt number of 0.72, assuming a non-dimensional depth averaged eddy diffusivity of 0.134, as calculated by Nokes and Wood (1988). Assuming the non-dimensional depth averaged eddy viscosity of 0.134, the Schmidt number should be range between 0.30 and 0.44. These values are arrived at through the relationship between the Schmidt number and eddy diffusivity and eddy viscosity, $\sigma_k = \frac{v_t}{D}$, where $v_t$ is the average eddy viscosity, and $D$ is the average eddy diffusivity, and using the highest and lowest computational predictions for eddy viscosity, as shown in Table 7.1. These values are lower than have been found in previous studies, such as Arnold et al. (1985) who experimentally measured Schmidt numbers between 0.4 and 1.0. The most recent study (Simoes and Wang, 1997) used a Schmidt number of 0.5.

In Cases C2 and C3, the prediction of mixing is generally good, with concentration gradients being predicted well, as already mentioned and illustrated in Figure 7.4 and Figure 7.7. In Figure 7.7 however, there is under prediction in the lateral flux, $\bar{c}_v$, in the region $0.7<y/B<0.85$, and over-prediction in the region $y/B>0.85$. Given that the concentration gradient $\partial C/\partial y$ is predicted well, the disagreement of $\bar{c}_v$ has to be a result of the eddy viscosity or Schmidt number. The eddy viscosity is much smaller on
the flood plain side than it is in the main channel, and hence the steeper concentration gradient on the flood plain side of the injection point.

The effect of secondary flow is more noticeable in Cases C2 and C3 than was in Case C1. The position of the peak concentration predicted by the NR model is significantly shifted laterally away from the injection point. The strong secondary flow vortices in this region can transport dye to other points in the channel, but do not directly affect the mixing. The concentration prediction for the NR model seems to be the least well mixed of the four models, and yet the eddy viscosity of that model is not the lowest. This is an example of how the secondary flow can indirectly affect the mixing by transporting the dye to another region of the channel where the eddy viscosity may be different. In this case, the dye is being carried onto the flood plain, where the eddy viscosity is significantly lower, causing the steepening of the concentration gradient and the increment of the peak concentration.

The vertical concentration profiles give a good indication of whether the mixing is being over-predicted. As shown in Chapter 5, the concentration profiles shown in Figure 7.19 illustrate how over and under prediction of mixing can be identified. From this we can infer that, in Case C1, at $y=0.075B$ and $y=0.15B$, the mixing is over-predicted by all the numerical models. At $y=0.25B$, the flux is markedly over-predicted possibly indicating that the Schmidt number controlling vertical mixing is too low.

In Case C2, $y=0.25B$, the mixing is again over-predicted. In the same case, at $y=0.5B$ the numerically predicted concentrations are lower than the measured concentrations, but the concentration gradients $\partial C/\partial z$ and the vertical fluxes are all very similar. This implies that mixing has been well predicted. In this instance, the experimental data is unreliable, as can be confirmed if the measured concentrations in Figure 7.14 are cross referenced with those in Figure 7.4. This is an example of how numerical models can be used as a good checking facility of experimental data.
7.2.5 Conclusions

The skewness can be used as a measure of the effect of the secondary flow. The flatness a measure of the horizontal mixing. The secondary flow produces more skewness than any due to the turbulence diffusion. It appears that in the vertical concentration measurements, the models consistently over-predict the mixing, whereas in the horizontal concentration measurements, the models tending to under-predict mixing. This suggests that the turbulent diffusion is anisotropic. This agrees with a study by Simoes and Wang (1997) that found the best results were obtained with an anisotropic eddy diffusivity that used a turbulent Schmidt number of 0.5 for the horizontal mixing, and 1.0 for the vertical mixing coefficient. The LY model most consistently gave the best results.

7.3 ANISOTROPIC EDDY DIFFUSIVITY

As in Chapter 5, a mixing length model was used to calibrate the vertical eddy diffusivity. The optimum mixing lengths are shown in Table 7.2. These mixing lengths were plotted against $\frac{k^{3/2}}{\ell}$ to give a value for $C_D$. In addition, computed eddy viscosity was plotted against $\ell\sqrt{k}$ to give a value for $C_V$. These two constants were able to provide values for the mixing length.

The relationships showing $\ell = C_D \frac{k^{3/2}}{\varepsilon}$ and $\nu_i = C_V \ell\sqrt{k}$ are shown in Figure 7.26 to Figure 7.29. These relationships yielded the following value for the vertical turbulent Schmidt number, $\sigma_z=2.5$ at the M/F intersection, and $\sigma_{zk}=1.66$ elsewhere in the channel. Figures 7.30 to 7.32 will be referred to later in this chapter.

First to be examined is the Schmidt number arrangement of $\sigma_z=1.66$, $\sigma_{zk}=0.44$, referred to as Run CB. Concentration and Reynolds flux is discussed for the three injection cases C1, C2 and C3.
7.3.1 Case C1, Run CB, $\sigma_z=1.66, \sigma_y=0.44$

The vertical experimental and numerical data at $y=0.075B$ and $y=0.15B$ are shown in Figure 7.33 and Figure 7.34 respectively. At $y=0.075B$, the magnitude and gradient of the computed concentrations seem to fit in the somewhat scattered experimental data, and is an improvement on the prediction with the isotropic Run CA. The flux predictions agree well with the experimental data where $z<0.8H$. In the region between $z=0.8H$ and the water surface the experimental flux seems shifted to the left, and is possibly experimental error, since the curve of the experimental is similar to that of the numerical predictions. The magnitudes and gradient of numerically predicted concentration agree well with the experimental data at $y=0.15B$. The NR model over estimates the concentration, and under predicts the mixing, but generally there is an improvement on the isotropic Schmidt number arrangement. The flux predictions all agree with the experimental data, although the experimental data is rather scattered.

7.3.2 Case C2, Run CB, $\sigma_z=1.66, \sigma_y=0.44$

The vertical measurements, shown in Figure 7.35 and Figure 7.36, show that the agreement between the experimental data has not improved on the isotropic Schmidt arrangement. At $y=0.5B$, the experimental data does not coincide with numerical predictions, but it can be seen that the gradient does not match as well as it did with the vertical Schmidt number equal to 1.0.

The results of lateral diffusion and flux are shown in Figure 3.37. The highest peak concentration is predicted by the NR model, and is just slightly lower than the measured peak concentration. The other models all predict peak concentrations lower than the experimental data. The gradient of the concentration curve is predicted well by the NR model on the right of the injection point, but the measured concentration gradient is steeper than the predictions on the left side of the injection point. The peak values for flux are under predicted by all the models. The point of peak flux is predicted well by the LY model and magnitude is also quite well predicted. On the left
of the injection point the two k-ε models do not predict the point of peak flux satisfactorily.

7.3.3 Case C2, Run CC, σ_z=2.5, σ_y=0.44

The lateral results yielded by vertical Schmidt number of 2.5, found from the calibration exercise, at this injection location, coupled with the lateral Schmidt number of 0.44 in an anisotropic arrangement can be seen in Figure 7.38. The NR model predicts a peak concentration higher than the measured concentration, and higher than the Run CB prediction. The LY model predicts concentration very well. The peak magnitude agrees with the experimental data, and the gradient \( \partial C/\partial y \) is predicted very well between 0.6\( B \) and 0.8\( B \). The NR model also predicts the concentration gradient well, although the peak magnitude is higher than the experimental data, and the lateral shift is more. The NR model is the better prediction on the left hand side of the injection point, but the LY model is the better prediction on the right. The NLKE model predicts a peak concentration that is lower than the experimental peak, and the spread of dye is considerably more. This is echoed to a greater extent by the KE model.

The magnitude of lateral flux has been predicted well by the LY model, as is the rate of change of flux, and the point of zero flux. The NR model also predicts the magnitude and gradient \( \partial \bar{C}/\partial y \) well, but the position of the NR model curve is shifted to the right. The two k-ε models do not predict flux as well, their peak fluxes being both under predicted in magnitude, and out of position.

The vertical distributions of concentration and flux are shown in Figure 7.39 and Figure 7.40. At \( y=0.5B \), the predictions of vertical concentration and flux are not as good as Run CB, or Run CA because the mixing has been under predicted. At \( y=0.375B \) the predicted concentration gradients agree fairly well with the measured data close to the water surface, but elsewhere the match is not very good.
7.3.4 Case C3, Run CB, $\sigma_z=1.66$, $\sigma_y=0.44$

The lateral concentrations and flux for Case C3, Run CB are shown in Figure 7.41. The LY model again predicts the gradient well, but slightly under predicts the peak concentration. The dye spreading is also predicted reasonably well by the two $k$-$\varepsilon$ models. The NR model does not predict the mixing especially well in this case, despite the peak magnitude being similar to that of the experimental data.

The lateral flux magnitude is predicted well by the NR model in the region $y>0.70B$, but its peak is shifted towards the right by $0.1B$. The LY model does not predict the magnitude of the flux as well, but is not as far shifted towards the flood plain side wall. The NLKE model predicts the magnitude and peak position of the flux well at $y=0.6B$, also the point of zero flux, but does not predict the experimental data well where $y>0.7B$. The LKE model under predicts the lateral flux, and is the least satisfactory prediction.

The vertical distributions of concentration and flux at $y=0.625B$ are shown in Figure 7.42. The concentration is predicted very well by all models except the NR model above the line $z=0.7H$. Below this line none of the models predict the concentration well. The measured concentration gradient changes direction below $z=0.7H$ and the concentration increases, and this is not predicted by the numerical models. This is shown by the vertical flux $\bar{c}w$ which becomes positive where $z<0.7H$. A similar effect is present in the experimental data at $y=0.75B$, as shown in Figure 7.43. The agreement of the model data with the experimental data is good where $z>0.8H$, but below this line, the measured concentration begins to increase again, whereas the numerically predicted concentrations continue to decline. Again the measured flux reflects this effect by the positive region below $z=0.8H$.

7.3.5 Discussion

In Case C1 the anisotropic Schmidt number arrangement clearly gives much better computational agreement than the isotropic case. This is in accordance with the
findings of Simoes and Wang (1997). The value 1.66 for the vertical Schmidt number, calibrated from the experimental data, improved the agreement of the models with the experimental data, as was expected.

The value of the Schmidt number for injection Case C2 was calculated to be higher (2.5) than found elsewhere in the channel (1.66). However, the value of 2.5 for Case C2 did not consistently give better results than the value 1.66. In fact, the value of 1.66 did not yield better agreement than the original Schmidt number of 1.0 in Case C2. From inspection of the horizontal concentration distributions it was noted that the vertical Schmidt number has an effect on the lateral concentration distribution. This is because, the mixing of dye vertically affects the concentration of dye present at the lateral position of injection, therefore governing the amount of dye available to be mixed laterally. Hence, when the vertical Schmidt number was high (2.5), the vertical mixing was lower and therefore at the injection point a higher concentration of dye was present, hence to increase peak concentration observed in the lateral distribution of Case C2, Run CC.

The Schmidt number 1.66 works well for vertical mixing in Case C3, although the numerical models do not predict the concentration accurately at all values of z. The measured concentration increases close to the flood plain bed. This is an effect of the secondary flow transporting dye down towards the bed and increasing concentration there. The numerical models do not predict this, despite the predicted secondary flow being strong, and in some instances stronger, than the measured secondary flow. A possible reason for this is that the transport of dye is very sensitive to the position of the injection point relative to the secondary flow vortices. If the predicted secondary flow is not precisely duplicating the experiments, then the dye may not be transported in the numerical model. Evidence of this is demonstrated in Section 7.4 of this chapter.

At the M/F interface, the eddy viscosity is lower, and the value of vertical Schmidt number, being much larger than that of the lateral one can be explained by comparing the vertical and lateral eddy diffusivities. The values of the vertical eddy diffusivity range being around a fifth to a third of the lateral eddy diffusivity.
The non dimensional value of eddy diffusivity used to calculate the lateral turbulent Schmidt number in this investigation was 0.134U*H (Nokes and Wood, 1988). This value provided the Schmidt number of 0.44, implying that the diffusivity is approximately twice the eddy viscosity. Figure 7.30 shows the vertical non-dimensional eddy viscosity, included for completion. Figure 7.31 shows that the non-dimensional eddy diffusivity is approximately 0.05, much lower than found by Nokes and Wood. However, calculations carried out on the experimental data indicate that the experimental eddy viscosity is higher than the computational prediction, as shown in Figure 7.32. The value of 0.05 for the eddy diffusivity yields the turbulent Schmidt number of approximately 1.0. This, however, has been seen to give less accurate results when compared to the anisotropic case where $\sigma_{k_y}=0.44$.

7.3.6 Conclusions

In Case C1 the anisotropic Schmidt number arrangement used in Run CB worked better than the isotropic arrangement, Run CA. In Case C2, the anisotropic arrangement did not work as well as in isotropic arrangement. It was noted that the effects of vertical and horizontal mixing are not mutually exclusive, and one has an effect on the amount of dye being spread by the other. In Case C3 the anisotropic arrangement in Run CB yielded the best results. It was noted that the transport of dye by secondary flow is very sensitive to the position of injection.

7.4 SHALLOW COMPOUND CHANNEL

7.4.1 Introduction

The section describes and discusses the turbulent diffusion of dye injected in a shallow compound channel. The geometry of the channel is the same as for the deeper compound channel, but the water depth was reduced to 75 mm. In the laboratory experiments only one injection case was investigated. This was at the main
channel/flood plain interface, \( y=0.5B \). In the numerical model simulation, the Schmidt number \( \sigma_k = \frac{v_k}{D} = 1.0 \) was used.

### 7.4.2 Results

The measured and computed concentration distributions for the shallow water compound channel are shown in Figure 7.48. The experimental result shows that two points of peak concentration occur when the dye is injected at the main channel/flood plain (M/F) interface. This phenomenon is possibly caused by the secondary flow. Inspection of the measured secondary flow vectors in Figure 7.54 show that at the injection point of 0.5B, there are two vortices flowing in opposite directions. This could cause the dye to be transported laterally in both directions, thus resulting in two concentration peaks. None of the numerical models predict these two peaks, instead showing the one peak concentration as found with the deeper compound channel. A possibility of this is the magnitude of secondary flow is too small to transport the dye in both directions. A second possibility is that the dye transport is very sensitive to the injection position and must be at the point in between the two secondary flow vortices at the M/F interface. The effect of the change of the injection point on the concentration can be shown in Figure 7.49 to Figure 7.52. It is clearly seen from Figure 7.51 that at \( y=0.475B \), the NR model predicts two peak concentrations, similar to the experimental data. From all the figures it can clearly be seen that the value of position of peak concentration varies greatly with the position of the injection point. When dye is injected at \( L_y=0.45B \), i.e. on the main channel side of the M/F interface, the peak concentration is relatively low and the dye relatively wide spread. However, when dye is injected a small distance further toward the M/F interface, i.e. \( y/B=0.475 \), the peak concentration is much higher, has moved considerably further right than the injection point, and dye spreading is less. When the dye is injected at the M/F interface, and positions beyond on the flood plain, the peak concentration becomes higher and the spreading less. This demonstration shows the importance that the secondary flow is correctly predicted in terms of an injection location.

The concentration distributions for the shallow channel when dye is injected at \( y=0.475B \) mm are shown in Figure 7.53. The experimentally measured concentration...
profile has two peak concentrations occurring at $y=0.5B$ with a magnitude of 4.0 ppb and $y=0.675B$ with a magnitude of 5.0 ppb, the dye having spread over 70% of the channel width. The linear k-$\varepsilon$ model predicts a concentration distribution with one peak, at $y=0.5B$ and a magnitude of 7.5 ppb. The peak concentration is higher than predicted because the spreading of the dye across the channel is less. This implies lateral mixing is underpredicted by the model. The non-linear k-$\varepsilon$ model also predicts just one peak, the concentration at the peak being 7.6 ppb. The lateral mixing is again underpredicted since the concentration distribution curve is less flat than the measured distribution. The LY model also predicts just one peak, but slightly more lateral spreading than the two k-$\varepsilon$ models, hence the lower peak concentration, but still underpredicts the lateral spreading when compared to the experimental data. The NR model predicts two concentration peaks, similar to the experimental data, although the predicted peaks are higher and the spreading of dye is less.

### 7.4.3 Discussion

In the laboratory experiments, dye was injected at $y=0.5B$, at the M/F interface, and has been transported by the secondary flow in both lateral directions resulting in the two peaks shown. The only model to have shown this is the NR model injecting at $y=0.475B$, and so it could be concluded that this is because the secondary flow predicted is similar to the measured secondary flow. From the secondary flow plots (see Figure 7.54 to Figure 7.57) it is apparent that the NR model predicts a high secondary flow, compared to the other models, and is most similar to the measured secondary flow. Therefore this model predicts the solute transported in two directions, when the injection point is between the twin secondary flow vortices at the M/F interface. However, the transportation and spreading of the dye is sensitive to the position of the injection point. This variation in peak concentration is due to the variation in the direction and magnitude of secondary flow, which in turn affects the way the dye is transported across the channel. The dye mixing at a point in the channel is proportional to the eddy viscosity at that point. Therefore, the variation in mixing observed is a combination of the lateral variation of eddy viscosity, and the transportation of dye across the channel between areas of high and low eddy viscosity.
The variation of eddy viscosity across the channel is shown in Figure 7.58. It can be seen that the eddy viscosity in the main channel is approximately twice the magnitude it is on the flood plain. This explains why, when the dye is transported laterally onto the flood plain by the secondary flow, the mixing of the dye decreases causing the peak concentration to rise.

Previous experiments (Wood and Liang, 1989) concluded that releasing an effluent in the main channel as opposed to the flood plain produces a higher degree of mixing. This has been confirmed in the present experiments and by the model predictions, the cause being identified as the higher eddy viscosity in the main channel as compared to the flood plain. When dye is released on the flood plain, the peak concentration is higher and is transported away from the M/F interface by the action of secondary flow.

7.4.4 Conclusions

It can be concluded that the secondary flow and the position of injection play a major role in the transport of solute across the channel. A double peak is observed only when the dye is injected close to the main channel/floodplain interface and the secondary flow is strong. Where the secondary flow is weaker or when the dye is injected in a position away from the step, only a single peak is observed. The position of the peak concentration is therefore governed by the magnitude of the secondary flow, but the magnitude of the peak is governed by the eddy viscosity. The eddy viscosity near the water surface on the flood plain is approximately half that in the main channel. This explains why, when the dye is transported by the secondary flow laterally onto the flood plain, the mixing of the dye decreases, resulting in an increase in peak concentration. It has been shown that the lateral diffusivity, $D_L$, needs to be large compared with the vertical diffusivity, $D_z$. For shallower water compound channel flow, the turbulence, shear and large horizontal eddies are dominant.
In further studies the prediction of eddy diffusivity needs to be improved, or the turbulent Schmidt number needs to be investigated. It appears that a smaller turbulent Schmidt number is needed, which would imply large values of eddy diffusivity.

Table 7.1 Range of turbulent Schmidt number for Compound Channel

<table>
<thead>
<tr>
<th>Computational Prediction</th>
<th>$\nu_t$</th>
<th>$D$</th>
<th>$\sigma_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LY Model (lowest)</td>
<td>0.048</td>
<td>0.134</td>
<td>0.30</td>
</tr>
<tr>
<td>LKE Model (highest)</td>
<td>0.059</td>
<td>0.134</td>
<td>0.44</td>
</tr>
</tbody>
</table>

Table 7.2 Optimum mixing lengths

<table>
<thead>
<tr>
<th>$y$ (mm)</th>
<th>Mixing Length (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Case 2</td>
</tr>
<tr>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>75</td>
<td>36</td>
</tr>
<tr>
<td>100</td>
<td>46</td>
</tr>
<tr>
<td>125</td>
<td>24</td>
</tr>
<tr>
<td>150</td>
<td>--</td>
</tr>
</tbody>
</table>
compound channel, $z=105$, $\sigma_{kz}=1.0$, $\sigma_{ky}=1.0$

$L_x=1.0$ m $L_y=50$ mm $L_z=108$ mm

Figure 7.1 Run CA, Case C1, Concentration and Reynolds Flux $\overline{c v}$
Skewness for Run CA Case C1

Figure 7.2 Skewness of Concentration Distribution, Run CA, Case C1.

Flatness for Run CA Case C1

Figure 7.3 Flatness of Concentration Distribution, Run CA, Case C1
Figure 7.4 Run CA, Case C2, Concentration and lateral Flux
Figure 7.5 Skewness of Concentration Distribution, Run CA, Case C2

Figure 7.6 Flatness of Concentration Distribution, Run CA, Case C2
PAGE NUMBERING AS ORIGINAL
compound channel, \( y = 15 \text{ mm} \), \( \sigma_x = 1.0 \), \( \sigma_y = 1.0 \)
\[ L_x = 1.0, L_y = 50 \text{ mm}, L_z = 108 \text{ mm} \]

Figure 7.10 Run CA, Case C1, Concentration (ppb) and vertical flux \( \bar{cw} \)
(ppbm/s), \( y = 0.075B \)

compound channel, \( y = 30 \text{ mm} \), \( \sigma_x = 1.0 \), \( \sigma_y = 1.0 \)
\[ L_x = 1.0, L_y = 50 \text{ mm}, L_z = 108 \text{ mm} \]

Figure 7.11 Run CA, Case C1, Concentration (ppb) and vertical flux \( \bar{cw} \)
(ppbm/s), \( y = 0.15B \)
Figure 7.8 Skewness of Concentration Distribution, Run CA, Case C3

Figure 7.9 Flatness of concentration distribution, Run CA, Case C3
Figure 7.7 Run CA, Case C3, Concentration and lateral flux

compound channel, $z=105$, $\sigma_{xz}=1.0$, $\sigma_{wy}=1.0$

$L_x=1.0 \, \text{m} \quad L_y=150 \, \text{mm} \quad L_z=108 \, \text{mm}$

- measured data
- linear $k$-$\varepsilon$ model
- non-linear $k$-$\varepsilon$ model
- launder-ying asm
- naot-rodi asm
Figure 7.12 Run CA, Case C1, Concentration (ppb) and vertical flux $cw$ (ppbm/s), $y=0.5B$.

Figure 7.13 Run CA, Case C2, Concentration (ppb) and vertical flux $cw$ (ppbm/s), $y=0.5B$. 

compound channel, $y=100 \text{ mm}$, $\sigma_x=1.0$, $\sigma_y=1.0$
$L_x=1.0, L_y=50 \text{ mm}, L_z=108 \text{ mm}$

compound channel, $y=50 \text{ mm}$, $\sigma_x=1.0$, $\sigma_y=1.0$
$L_x=1.0, L_y=100 \text{ mm}, L_z=108 \text{ mm}$
Figure 7.14 Run CA, Case C2, Concentration (ppb) and vertical flux $\bar{c}_w$ (ppbm/s), $y=0.75B$

Figure 7.15 Run CA, Case C3, Concentration (ppb) and vertical flux $\bar{c}_w$ (ppbm/s), $y=0.25B$
Figure 7.16 Run CA, Case C3, Concentration (ppb) and vertical flux $cw$ (ppb.m/s), $y=0.5B$.

Figure 7.17 Run CA, Case C3, Concentration (ppb) and vertical flux $cw$ (ppb.m/s), $y=0.75B$. 
Figure 7.18 Non-dimensional depth averaged eddy viscosity

Figure 7.19 Typical concentration profiles
Figure 7.20 Case C1 concentration contours for LKE model

Figure 7.21 Case C1 measured concentration contours
Figure 7.22 Case C2 LKE model concentration contours

Figure 7.23 Case C2 measured concentration contours
Figure 7.24 Case C3 LKE concentration contours

Figure 7.25 Case C3, measured concentration contours
Figure 7.26 Case C2, $\ell$ versus $k^{3/2}/\varepsilon$

Figure 7.27 Case 2, eddy viscosity versus $\ell \sqrt{k}$
Figure 7.28 Case C3, mixing length, $\ell$ versus $k^{(3/2)}/\varepsilon$.

Figure 7.29 Case C3, $\nu_t$ versus $\ell \sqrt{k}$. 
Figure 7.30 Experimental non-dimensional eddy diffusivity
Figure 7.31 Non dimensional eddy diffusivity at z=105 mm

Figure 7.32 Non dimensionalised eddy viscosity at z=105 mm
Figure 7.33 Run CB, Case C1, Concentration (ppb) and flux (ppb.m/s) y=0.075B.

Figure 7.34 Run CB, Case C1, Concentration (ppb) and flux (ppb.m/s) y=0.15B
Figure 7.35 Run CB, Case C2 Concentration (ppb), Flux (ppb.m/s)

Figure 7.36 Run CB, Case C2, Concentration (ppb), Flux (ppb.m/s) y=0.5B
compound channel, $z=105$, $\sigma_z=1.66$, $\sigma_y=0.44$
$L_x=1.0$ m $L_y=100$ mm $L_z=108$ mm

Figure 7.37 Run CB, Case C2, Concentration (ppb), Flux (ppb.m/s) $y=0.5B$
compound channel, $z=105$, $\sigma_{xz}=2.5$, $\sigma_{xy}=0.44$

$L_x=1.0 \text{ m} \ L_y=100 \text{ mm} \ L_z=108 \text{ mm}$

---

Figure 7.38 Run CC, Case C2, Concentration (ppb) and lateral flux (ppb.m/s)
Figure 7.39 Run CC, Case C2, Concentration (ppb), Flux (ppb.m/s) y=0.375B

Figure 7.40 Run CC, Case C2, Concentration (ppb), Flux (ppb.m/s) y=0.5B
compound channel, $z=105$, $\sigma_{xz}=1.66$, $\sigma_{zy}=0.44$

$L_x=1.0 \text{ m} \ L_y=150 \text{ mm} \ L_z=108 \text{ mm}$

Figure 7.41 Case C3, Run CB, Concentration (ppb) and flux (ppb.m/s)
Figure 7.42 Run CB, Case C3, Concentration (ppb), Flux (ppb.m/s) $y=0.625B$

Figure 7.43 Run CB, Case C3, Concentration (ppb), Flux (ppb.m/s) $y=0.75B$
Figure 7.44 Experimental data secondary flow

Figure 7.45 LY model secondary flow
Figure 7.46 NR model secondary flow

Figure 7.47 NLKE model secondary flow
Figure 7.48 Non-dimensional concentration, injection point = 0.5B.
Figure 7.49 Effect of injection position on linear k-ε model prediction

Figure 7.50 Effect of injection position on LY Model prediction
Figure 7.51 Effect of injection position on NR Model prediction

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Figure 7.54 Experimentally measured secondary flow
Figure 7.55 Launder-Ying ASM secondary flow

Figure 7.56 Naot-Rodi ASM secondary flow

Figure 7.57 Non-linear k-ε model secondary flow
shallow compound channel.
$L_x = 1.0$ mm, $L_y = 100$ mm, $L_z = 73$ mm

Figure 7.58 Non-dimensional eddy viscosity at $z=75$ mm
8 CONCLUSIONS

8.1 INTRODUCTION

This chapter summarises the main findings for the present study and makes some recommendations for further work. The conclusions drawn from the rectangular study are presented first, followed by the compound channel study conclusions. Finally some recommendations for further study are put forward.

8.2 RECTANGULAR CHANNEL STUDY

8.2.1 Model Prediction of Hydrodynamics

All models have demonstrated that they predict the flow in a rectangular channel very well. Most turbulence is generated by the channel bed, which is about twice that generated by the channel side wall.

The secondary flow did not contribute significantly to the production of turbulence, which has been demonstrated by the $-\bar{vw}$ turbulent kinetic energy production, being of the order of 1000 times smaller than the bed or wall generated turbulent kinetic energy.

8.2.2 Turbulent Transport Coefficient Anisotropy

The turbulent Schmidt number has been found to vary throughout the channel, and this value affects the amount of mixing obtained from a given eddy viscosity. For the rectangular channel, the anisotropic turbulent Schmidt number arrangement, $\sigma_{kz}=2.3$ and $\sigma_{ky}=0.5$ gave the best results for mid-channel injection. The isotropic arrangement, $\sigma_{kz}=\sigma_{ky}=1.0$, was found to be best for near wall injection. This suggests that away from the side wall, there is less vertical mixing and more horizontal mixing. Because the dye was injected near the surface, the
damping effect of the water surface reduced the vertical mixing, hence the anisotropic arrangement of turbulent Schmidt number for mid channel injection. However, near the side wall, the wall generated turbulence increase the vertical mixing, and hence the isotropic turbulent Schmidt number was better.

8.2.3 Model Prediction of Concentration Field

It has been demonstrated that the concentration of dye at a point in a channel is affected by three mechanisms: the lateral eddy diffusion, vertical eddy diffusion, and secondary flow. The effects of vertical and lateral diffusion are not independent of one another, but have a mutual effect since the dye available to be mixing in one direction is affected by the amount of mixing in the other direction. The secondary flow affects the position of the peak concentration since secondary flow transports the dye in space in the channel without causing mixing itself.

The non-linear k-ε model most consistently gave the best results in the prediction of the concentration field. Hence, when modelling rectangular channels, this study recommends the use of a non-linear k-ε model, using an anisotropic turbulent Schmidt number arrangement of $\sigma_{kz}=2.3$ and $\sigma_{ky}=0.5$ mid channel, and an isotropic turbulent Schmidt number of 1.0 near the wall.

8.3 COMPOUND CHANNEL STUDY

8.3.1 Model Prediction of Hydrodynamics

The flow and turbulence properties were predicted well by all models. In the deep compound channel, the bed and wall generated turbulence were approximately equal. The NR model was the best prediction overall.

8.3.2 Turbulent Transport Coefficient Anisotropy

In order to investigate mixing, the skewness was used as a measure of the effect of secondary flow and the flatness as a measure of horizontal mixing. In the main channel, experimentally measured secondary flow was stronger than the
numerical predictions in the main channel, indicated by the experimental skewness of 0.5 compared to nearly 0.3 for the NR model and approximately 0.1 for the LKE, NLKE and LY models. Hence, the NR model most accurately predicted the effect of secondary flow. Horizontal mixing was reasonably well predicted by all models, but the flatness values indicated that the NR model prediction was the best.

At the main channel flood plain interface, the experimental skewness was 0.9, higher than that of all the model data, implying higher secondary flow. The predicted data flatness values were all higher than the measured data flatness, implying that the models under predicted mixing.

On the flood plain, the skewness value of -1 for the experimental data was similar to most of the numerical model flatness values. The NR model results yielded a higher skewness, due to its prediction of higher secondary flow. The NR model gave the best results in Case C1, and the LY model for Cases C2 and C3.

It appeared that in the vertical concentration measurements, the models consistently over predict the mixing, whereas in the horizontal concentration measurements, the models tending to under predict mixing. This suggests that the turbulent diffusion is anisotropic. This agrees with the results obtained by Simoes and Wang (1997) using an anisotropic eddy diffusivity with a turbulent Schmidt number of 0.5 for the horizontal mixing, and 1.0 for the vertical mixing coefficient.

8.3.3 Model Prediction of Concentration Field

From the results of optimization of the turbulent Schmidt number, it was found that the NR model produced the best results with the following arrangements of turbulent Schmidt number.
When dye was injected at the centre of the main channel, a lateral turbulent Schmidt number of $\sigma_{ky}=0.44$ and a vertical turbulent Schmidt number of $\sigma_{kz}=1.66$ gave the best results.

For dye injection at the main channel/flood plain interface, vertical and lateral Schmidt number of 1.0 was best.

When dye was injected on the flood plain, the anisotropic Schmidt number arrangement of $\sigma_{ky}=0.44$ and $\sigma_{kz}=1.66$ yielded the best results.

### 8.3.4 Shallow Compound Channel

From the shallow compound channel study, the following were concluded.

The secondary flow and the position of injection play a major role in the transport of solute across the channel. A double peak is observed only when the dye is injected close to the main channel/flood plain interface and the secondary flow is strong.

Where secondary flow is weaker or when the dye injection is not close to the main channel/flood plain interface, only a single peak is observed. The position of the peak concentration is therefore governed by the secondary flow.

The predicted concentration peaks were higher when the injection position used in the models was on the main channel side, compared to with the simulations of dye injected on the flood plain side. The eddy viscosity near the water surface on the flood plain is approximately half that in the main channel. This explains why, when the dye is transported by the secondary flow laterally onto the flood plain, the mixing of the dye decreases, resulting in an increase in peak concentration.
8.4 RECOMMENDATIONS FOR FURTHER STUDY

8.4.1 Rectangular Channel

The lateral turbulent Schmidt number has been demonstrated to be lower than the vertical turbulent Schmidt number for dye injected in the mid channel region. Further study could apply these Schmidt numbers to other experimental data to examine whether these values are universal, or limited to certain channels.

8.4.2 Compound Channel

As with the rectangular channel, the limitations of the Schmidt numbers presented in this study should be tested by studying further sets of experimental data.

It was noted in the shallow compound channel study that the numerical models under predicted the horizontal mixing, implying that the lateral turbulent Schmidt number needs reducing. Due to lack of experimental data in the shallow compound channel, a comprehensive turbulent Schmidt number optimization was not possible, as it was with the rectangular and deeper compound channels.
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