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SMALL FIRM EFFECTS IN THE UK STOCK MARKET

by

Patricia Lorraine Chelley-Steeley

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of

Doctor of Philosophy in Economics of the Loughborough University of Technology

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I WOULD LIKE TO THANK, MY HUSBAND JIM, FOR ALL HIS SUPPORT DURING MY Phd.
To Jim
ABSTRACT

This thesis will be concerned with investigating the empirical characteristics of stock returns, for UK firms which are distinguished by market value. The primary aim of this work is to identify whether there are differences between the behaviour of large and small firm returns.

A substantial amount of attention has recently focused upon how firm size influences the behaviour of stock returns in US markets, but, the role that firm size might have in determining the behaviour of stock returns in UK markets has received very little attention. The aim of this thesis is to redress this imbalance.

The first part of this study will be concerned with showing that the returns of small firms are more predictable than the returns of large firms. The second part of this study will show that the relationship between risk and return depends on firm size. The third and final part of this thesis will show that not only are the mean returns of large and small firms different but that there are also important differences in the conditional variances of large and small firms. In all three parts of this thesis, important differences between the behaviour of large and small firm returns are documented for the first time.
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CHAPTER ONE
The Aims and Objectives of this Study

1.1 Background

The belief that financial markets are efficient is widely held among financial economists. For example, Jensen (1986) says of market efficiency that

"there is no better documented proposition in any of the social sciences." (p95).

As we shall see in Chapter 2, one definition of market efficiency\(^1\) is that "share prices must reflect all available information" Fama (1970) (p387). There is good reason to expect stock markets to be efficient in this way. Stock markets consist of a large number of investors which are in competition with, millions of other self interested and well informed investors. Therefore, if prices do not reflect publicly available information there will be enough investors who will attempt to profit from this inefficiency, and therefore, drive prices to a level where they really do reflect the available information. However, the existence of persistent regularities such as the "small firm effect" have led a number of researchers to suggest that stock markets are not as efficient as we once thought.

Following the pioneering research, which was undertaken by Banz (1981), evidence has now accumulated to indicate that the returns of small firms, in a number of major stock markets, including that of the UK,\(^2\) are apparently substantially more predictable than the returns of large firms.

Subsequently empirical work, has uncovered a second regularity in the behaviour of small firm returns, which appears to be equally puzzling. Not only are small firm returns predictable, but a large proportion of the predictability appears (in the US) to be associated with one month of the year, January. See for example, the work of Roll (1983) and Keim (1983) which documented that over half of the US small firm premium was due to the performance of small firms during the first few trading days of January. Such evidence is puzzling because regularities such as these, which are now documented on over half a century of data, appear to suggest that markets are inefficient.

In an attempt to help our understanding of why small firms behave differently to large firms, subsequent empirical work has searched for additional empirical differences, between the behaviour of small and large firms. A large number of papers, which will be reviewed in

---

1. As Chapter 2 will show there are also many other definitions of an efficient market.
2. See, for example, Levis (1985), Fong (1992), or Strong and Xu (1994).
Chapter 3, have considered the role that firm size might have in determining the extent to which stock returns are predictable. Generally, these papers have found that the returns of small firms are substantially more predictable than the returns of large firms.

An example of this work is provided by Lo and MacKinlay (1988, 1990a), who found that portfolios of small firms were substantially more autocorrelated, and therefore more predictable, than their large firm counterparts. Furthermore, Lo and MacKinlay (1990a) also discovered that the returns of small firm portfolios were cross serially correlated with the returns of large firms but the reverse was not true. This implies that the returns of large firms predict the future returns of small firms but the returns of small firms do not predict the returns of large firms.

Most of the studies which have looked at differences in the behaviour of small and large firms have directed their attention to comparisons of mean returns. Relatively little work has been concerned with exploring differences in the behaviour of the second moments of large and small firms. An exception is Conrad, Gultekin and Kaul (1991), who found for the US market that, the conditional variances and returns of small firms could be predicted from volatility shocks (unexpected returns) experienced by large firms, but the reverse was not true, shocks to small firms did not predict future volatility or return in the large firms.

Clearly, a substantial amount of research effort has been directed at discovering differences in the empirical behaviour of small firms, although most of these studies have been directed at companies listed on the US stock markets. In contrast relatively few studies have considered the behaviour of small and large firms listed on the UK stock market. The aim of this thesis is therefore to provide a comprehensive study of "small firm effects" in the UK stock exchange and extend the empirical models which have been used in this field. Primarily, this study will show that the first and second moments of small firm returns listed on the UK stock market are more predictable than those of the large firms.

1.2 Objectives of This Study

This thesis will begin by defining an efficient market. This is an important starting point because a re-current question which has to be addressed in later chapters is whether the returns of companies investigated in this study are consistent with market efficiency. Although, defining an efficient market appears to be straightforward, this is not the case because there has been a substantial degree of controversy surrounding exactly what constitutes an efficient market and what are the testable implications of such a market. Traditionally as we shall see in Chapter 2, it has been recognised that one testable implication of the efficient markets hypothesis is that
stock returns should vary in a random fashion. Although, as we shall see in that chapter the existence of microstructure frictions implies that return predictability can exist even if markets are efficient.

Chapter 3 reviews the empirical literature that has sought to test whether stock market returns are predictable or not. This is necessary to provide assistance in establishing the contribution made by the empirical work presented in later chapters of this thesis. Since this study will primarily focus upon differences between small and large firms, Chapter 3 where possible, will highlight the major empirical differences which have been observed.

Chapter 4 contains the first part of empirical work. This chapter confirms the existence of a small firm premium in the UK over the period investigated in this study. This in itself is important because it indicates that the small firm effect, previously observed in the UK, is not an artifact of the sample periods previously investigated (although evidence presented in this chapter suggests that the size premium has diminished in magnitude in recent years). Autocorrelation tests, which are performed on portfolios of large and small firms, confirm that portfolio returns of small firms are more autocorrelated, and therefore more predictable, than the portfolio returns of large firms. In contrast, the individual security returns of small firms, like those of large firms, appear to be uncorrelated. This indicates that there are some important differences between the returns of individual securities and portfolios of different capitalisations. This chapter also uses co-integration tests to identify predictability in individual security prices, and this is the first example of these tests having been applied to a study of the small firm effect.

As was mentioned in Section 1.1 return seasonality has been shown to have an important influence over the magnitude of the size premium. Chapter 5 will investigate the extent to which monthly return regularities can explain the UK size premium. Although this chapter finds substantial evidence of return seasonality, this does not appear to have a strong influence over the size premium. The focus of this chapter will be on extending the work of Tinic and West (1984,1986), which found that high January returns in the US appeared to be associated with the pricing of risk. Essentially, Tinic and West (1984) found that systematic risk was not priced during any month of the year other than January. Surprisingly, the role that firm size might have on determining the magnitude of the risk premium, particularly the systematic risk premium, during the different months of the year, and the relationship this might have with the size premium, has not previously been investigated.

Few US studies, and no UK studies have been concerned with identifying differences in the second moments of the returns of large and small firms. Chapters 7 and 8 will respond to this by providing a substantial amount of information on the characteristics of second moments.
An important objective of Chapters 7 and 8 is to show that the return predictability such as that identified in Chapters 4 and 5 does not provide evidence of market inefficiency, if such predictability is also consistent with time variation in expected returns.

Primarily, Chapter 7 will be concerned with testing a conditional version of the Capital Asset Pricing Model to establish whether the return autocorrelation for large and small firms can be explained by a time varying systematic risk premium. Although, a number of papers have tested whether the systematic risk premium is time varying, no study has considered the role that firm size has on the conditional co-variances/variances of portfolio returns.

Chapter 8 will be concerned with identifying the impact that volatility shocks (unexpected returns) might have on the future volatility and return performance of portfolios. This chapter investigates whether there are volatility spillover shocks in the UK stock market, which requires testing whether a volatility shock to one portfolio can predict future volatility in another portfolio. The aim of these investigations is to establish whether there are differences in the transmission of shocks across portfolios of different capitalisations. For example, do shocks to large firms predict future volatility for small firms and vice versa. This chapter will also investigate whether there is any evidence of a leverage effect for portfolios that have been organised on the basis of firm size. Although, Nelson (1991) for the US and Poon and Taylor (1992) for the UK find some evidence of a leverage effect on market indexes no paper has previously investigated whether a leverage effect exists for portfolios formed on the basis of firm size. Finally, Chapter 9 provides a summary and some conclusions to the study.

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1.3 As chapter 8 will show the leverage effect suggests that volatility is related to changes in the gearing ratio of companies.
CHAPTER TWO
The Efficient Markets Model

2.1 Introduction

This chapter will be concerned with reviewing the development of the efficient markets model. This is an important starting point for this study, because one issue which this thesis attempts to address is whether the behaviour of the UK stock market is consistent with market efficiency. Considering whether markets are efficient is a worthwhile area of research, because stock prices, like prices in any other market, are signals for the allocation of resources. For resources to move to companies which can make the best use of investment funds, prices must be a fair reflection of the value of a company and therefore its future earnings potential. Since this study will be concerned with the empirical behaviour of stock prices it is important to identify the testable implications of an efficient market, in order to establish whether the UK market which will be studied in this thesis is efficient or not.

Essentially, there are three phases to focus upon when considering the development of the efficient markets hypothesis. Firstly, there was what LeRoy (1989) called the "prehistory" which was a period when significant developments in the empirical work were being made, but a theoretical framework to explain the empirical findings was lacking. LeRoy (1989), suggests that this prehistory ended with the Fama (1970) paper, which formalised the concept of an efficient market and comprehensively surveyed the empirical literature. Although, Fama (1970) was fundamentally important in generating interest in efficient markets, it was arguably the paper by Samuelson (1965) that acts as the boundary between the prehistory and modern efficient markets because of the shift this paper caused in turning the emphasis of testable predictions away from the random walk model to the martingale model.

The second phase in the development of the efficient markets hypothesis, stretches from Samuelson’s (1965) paper to approximately the end of the 1970’s. During this period a large number of empirical papers appeared which attempted to test empirically whether markets were efficient. Most of these empirical studies, many of which are surveyed in Fama (1970), strongly supported market efficiency. In this survey paper, which is entitled "Efficient Capital Markets" Fama concludes

"In short, the evidence in support of the efficient markets model is extensive, and (somewhat uniquely in economics) contradictory evidence is sparse" (1970, p.416)
The third and most recent phase stretches from the end of the 1970's to the present day. In this most recent phase, new tests have been developed that have indicated that stock prices are much more predictable than was first realised, see for example Shiller (1981a), DeBondt and Thaler (1985) and Lo and MacKinlay (1988). A comprehensive survey of this work follows in Chapter 3. So overwhelming has been the contribution of these new tests in suggesting that stock markets are inefficient, that even Fama (1991) in "Efficient Capital Markets II" (which was a follow up to his survey paper to Efficient Capital Markets) had to admit that perhaps markets were not quite as efficient as they had first appeared. This phase has also been important for directing attention towards the importance of microstructure factors and inadequate risk measurement, which can introduce empirical predictability into stock prices even when markets are informationally efficient.

The remainder of this chapter is set out as follows. In Section 2.2 the random walk model, which was initially believed to be the model underpinning the efficient markets model is discussed. The focus of this section will be on discussing the historical development of the random walk model of stock prices. In Section 2.3 the martingale model, which was introduced by Samuelson (1965) and Mandelbrot (1966) will be discussed. The first attempt at formally linking the martingale model to the behaviour of stock prices was undertaken by Fama (1970). The nature of the model discussed by Fama was controversial and has subsequently caused a lengthy debate regarding its appropriateness. This controversy is discussed in Section 2.4. Alternative definitions of market efficiency, which unlike the martingale model, do not assume perfect markets are discussed in Section 2.5. In Section 2.6, the efficient markets model will be reconciled to stock valuation models, which explain movements in the intrinsic value of stocks. Section 2.7 will be concerned with discussing the role of asset pricing models in tests of market efficiency. Section 2.8 discusses the role of microstructure frictions which can introduce predictability into the behaviour of share prices even in an informationally efficient market.

2.2 The Random Walk Model

2.2.1 Background

It is customary to suggest that a stock market is efficient if share prices reflect all available information. As such we would expect realised share prices to fluctuate in a random fashion, reflecting the random arrival of new information. This is a description of an efficient market that lacks rigour and does not tell us what the testable characteristics of an efficient market are. In order to test the efficient markets hypothesis we need more than this vague statement.
Prior to the development of a rigorous theoretical framework for the efficient markets model, that is in what LeRoy (1989) calls the "prehistory" of efficient markets, research on the behaviour of speculative prices had little formal content. Primarily, research had been directed at documenting the empirical characteristics of speculative prices, usually in the expectation of identifying predictable patterns. At this time, the behaviour of speculative prices became to be associated with the random walk model, because of the seemingly random behaviour of prices which had been documented in empirical studies. An example of this work is provided by Kendall (1953) who found that successive changes in the value of stock indexes appeared to be random.

A random walk is a very restrictive example of a stochastic process. Essentially, the random walk model assumes that the probability distribution of a stochastic process such as \( \{x_t\} \) is independent and identically distributed such that the distribution must be the same for all time, \( t \). Such a distribution is illustrated below

\[
f(x_{t+1} | \Phi_t) = f(x_{t+1})
\]

where \( f(x_{t+1} | \Phi_t) \) is the conditional probability distribution of \( x_{t+1} \) conditional on \( \Phi_t \), which is the information set available in time \( t \), and \( f(x_{t+1}) \) is the unconditional distribution of \( x_{t+1} \).

In the simplest example of a random walk process, which is a zero mean random walk, each successive change in \( x_t \) is assumed to be drawn independently from a probability distribution with a zero mean. Thus \( x_t \) is determined by

\[
x_t = x_{t-1} + e_t
\]

\[
E(e_t) = 0 \quad E(e_t^2) = \sigma^2 \quad E(e_t e_s) = 0 \quad t \neq s
\]

The random walk model has the following important properties. If we knew the past history \( x_1, x_2, \ldots, x_t \) and we wanted to obtain a forecast \( \hat{x}_{t+1} \) the forecast would be given by

\[
\hat{x}_{t+1} = E(x_{t+1} | x_t, \ldots x_1)
\]

which is the expected value of \( x_{t+1} \) conditional on the previous values of \( x_t \) which were \( (x_t, \ldots, x_1) \).

Since, \( x_{t+1} = x_t + e_{t+1} \) is independent of \( (x_t, \ldots, x_1) \) the forecast one period ahead becomes

\[
\hat{x}_{t+1} = x_t + E(e_{t+1}) = x_t
\]

so that all the information required to make a forecast of the future value of \( x_t \) is contained in its most recent observation. Similarly, the forecast \( n \) periods ahead is also \( x_t \),

\[
\hat{x}_{t+n} = x_t + E(e_{t+n}) = x_t \quad n = 1, 2, 3, 4, \ldots
\]

1. A stochastic process is a variable whose values are random at successive points in time.
which can be interpreted as meaning that the optimal predictor of $x_{t+n}$ can be obtained from $x_t$.

If a stochastic process is a random walk, successive changes in $x_t$ must be uncorrelated since

$$ (x_{t+1} - x_t) = e_{t+1} \quad (2.6) $$
$$ (x_t - x_{t-1}) = e_t \quad (2.7) $$
$$ Cov[(x_{t+1} - x_t), (x_t - x_{t-1})] = 0 \quad (2.8) $$

This is not only true for the covariance between successive changes in $x_t$ but is also true for the covariance between $x_t$ and $x_{t+n}$.

$$ (x_{t+n} - x_t) = e_{t+n} \quad n = 1, 2, 3, \ldots \quad (2.9) $$
$$ (x_t - x_{t-n}) = e_t \quad n = 1, 2, 3, \ldots \quad (2.10) $$
$$ Cov[(x_{t+n} - x_t), (x_t - x_{t-n})] = 0 \quad n = 1, 2, 3, \ldots \quad (2.11) $$

The random walk model therefore appeared to be a good description of how stock prices should behave in an efficient market because it described well the apparently random behaviour of stock prices.

The assumption of independence inherent in the random walk model requires that each $x_t$ is drawn from a probability distribution which repeats itself identically over time. This requires that dependence not only between the first moments, but also between higher moments of $x_t$ such as the conditional variance, must also be ruled out. This is a key weakness of the random walk model if this model is to be used to explain the behaviour of stock prices because a substantial amount of evidence has accumulated to indicate that higher moments are not independent.

An example of such work was provided by Fama (1965), Mandelbrot (1966) and Brealey (1970) who demonstrated that the unconditional distribution of short-horizon returns was characterised by excess kurtosis (that is it is fat tailed). Such a distribution will be caused by the tendency for large returns to follow large returns and small returns to follow small returns, which suggests there is a changing conditional variance. A consequence of this is that returns are characterised by an excessive number of returns clustered around the expected return and a large number of returns which are extreme returns, which give rise to the fat tails. In this case a random walk would not be a good description of short-horizon returns in an efficient market because a distribution with excess kurtosis indicates that the conditional variance rather than being constant is time varying. The nature of the excess kurtosis in the stock returns is examined in detail in Chapter 7 and 8.
2.2.2 The Development of the Random Walk Model of Speculative Prices

The first statement and test of the random walk model was provided by Bachelier (1900). Bachelier asserted that the current value of a speculative price was an unbiased estimator of the price for any future date, the increment in the price was independent of previous values and successive prices were Gaussian² with zero mean and a variance proportional to \( t \). This was the first description of a random walk applied to speculative prices, although, Bachelier never actually used that terminology. The genius of this work went largely unappreciated and unacknowledged until subsequent empirical work, which is reviewed later in this chapter, showed that prices actually did behave randomly and therefore in a way consistent with Bachelier’s descriptions.

One of the developments which was important in vindicating Bachelier’s work was the empirical study undertaken by Kendall (1953). This work aimed to discover whether patterns existed in speculative prices such as commodity or stock index prices which mirrored alleged trade cycle patterns in economic data. The finding that there appeared to be virtually no correlation between successive prices led Kendall to conclude that the “data behaved almost like a wandering series”. At this stage a convincing rationale for the discovery could not be provided. Although similar findings were reported by Roberts (1959) and Osborne (1959).

Roberts (1959) found that weekly changes in the Dow Jones index resembled a time series generated from a sequence of random numbers. The implication of the work undertaken by Roberts was that price changes were independent of their past history. Osbourne (1959) found that stock price movements were very similar to the random Brownian motion of physical particles, since the logarithm of their prices appeared to be independent of each other.

This transformation to the use of log prices has been justified on both empirical and theoretical grounds. Granger and Morgenstern (1970) provide a full account. A summary of their main arguments follows. Theoretically, the distribution of prices is bounded from below by zero but is unbounded from above. The logarithm transformation results in a distribution which is symmetrically unbounded and hence may be more symmetric about its mean. Empirically, transformed data have more symmetric and more nearly normal histograms. They appear to have more time invariant first and second sample moments and appear to be much closer to being independent observations from a random process.

It was confirmation in this early empirical work that share prices behaved randomly that caused speculative prices, in efficient markets, to be linked to the random walk model. Although, as the work of Samuelson (1965) and Mandelbrot (1966) showed, this association was premature.

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² The Gaussian distribution is a normal distribution. The normal distribution is symmetric and bell shaped. The distribution can be fully described by the mean and the variance.
2.3 Martingales

Samuelson (1965) and Mandelbrot (1966) were the first to link market efficiency to the martingale model. So simple was the exposition provided by Samuelson that he wrote of the martingale, it seemed

"so general that I must confess to having oscillated over the years in my own mind between regarding it as trivially obvious (and almost trivially vacuous) and regarding it as remarkably sweeping" (1965, p.45).

Despite this initial uncertainty regarding the value of Samuelson’s work, the contributions made by Samuelson and Mandelbrot in shifting attention away from the random walk model, towards the martingale model are now viewed as one of the most important developments in Finance.

2.3.1 Defining a Martingale

A stochastic process \( \{x_t\} \) is a martingale with respect to a sequence of information sets \( \phi_t \) if \( x_t \) has the property

\[
E(x_{t+1} | \phi_t) = x_t
\]

and a stochastic process \( y_t \) is a fair game if it has the property

\[
E(y_{t+1} | \phi_t) = 0
\]

where, \( E(y_{t+1} | \phi_t) = (x_{t+1} - x_t) \).

Here if \( x_t \) is a martingale, the best forecast of \( x_{t+1} \) that could be made based on currently available information \( \phi_t \) would be \( x_t \). This is true for any possible value of the information set \( \phi_t \). Similarly, if \( y_t \) is a fair game the forecast of \( y_{t+1} \) would be zero for any value of \( \phi_t \). Thus \( x_t \) is a martingale if and only if \( (x_{t+1} - x_t) \) is a fair game. At first glance these properties look surprisingly similar to a random walk. Indeed, this is the case because a random walk is a more restrictive example of a martingale. The martingale model imposes the restriction that successive changes in the value of a stochastic process \( x_t \) are uncorrelated, but in the martingale model the distribution of \( x_t \) is not assumed to be identical and independent. The martingale model does not therefore impose any restrictions on the higher moments of the distribution so that dependence in the conditional variance of \( x_t \) can not be ruled out.

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3. This exposition was made in terms of commodity futures prices, it was not until later that the martingale model was applied to stock prices.
2.3.2 The Sub-Martingale Model

A model related to the pure martingale is the sub-martingale model. The sub-martingale assumes that the stochastic variable $x_{t+1}$ has an expected value greater or equal to $x_t$. In this case $x_t$ is a sub-martingale if $y_t$ has an expected value greater than zero. Therefore, the sub-martingale model assumes that $x_t$ grows larger each period.

$$E(x_{t+1} | \phi_t) \geq x_t$$  \hspace{1cm} (2.14)

$$E(y_{t+1} | \phi_t) \geq 0$$  \hspace{1cm} (2.15)

where, $y_{t+1} = (x_{t+1} - x_t)$.

Since the expected returns of risky assets such as stock investments must be positive, stock prices must follow a sub-martingale, to ensure that stock prices grow over time to provide positive expected returns. From equation (2.14) we can see that if expected stock prices must rise then security prices must be a sub-martingale, and can be written as follows

$$\frac{E(p_{t+1} | \phi_t) - p_t}{p_t} \geq 0$$  \hspace{1cm} (2.16)

where $p_t$ = the price of a security in time $t$ (assuming expected returns are not negative). By assuming that expected returns are constant, returns themselves should be a martingale.

Turning now to abnormal returns, we can define an abnormal return as the difference between a realised return and an expected return as follows.

$$R_t - E(R_t) = e_t$$  \hspace{1cm} (2.17)

$$R_t = E(R_t) + e_t$$  \hspace{1cm} (2.18)

$$E(e_t) = 0 \quad E(e_t e_s) = 0 \quad t \neq s$$

where, $E(R_t)$ is the expected return for an asset, $R_t$ is the realised return in time $t$, so that $e_t$ is the abnormal return.

Expected returns $E(R_t)$ will be set by an appropriate equilibrium asset pricing model to ensure that investors are provided with compensation for the time value of money and the amount of risk associated with the investment. Since $E(e_t) = 0$, abnormal returns must be a fair game. Chapter 3 will review empirical evidence which has tested whether $E(e_t) = 0$. 

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2.3.3 The Fama and LeRoy Controversy

It was Fama (1970) who formerly linked the martingale model to the behaviour of stock prices in an efficient market. The framework used by Fama is notable because of both its subsequent popularity and the degree of controversy it attracted. Fama’s interpretation of an efficient market assumes that market equilibrium can be expressed in terms of expected returns and that all currently available information is used by the market in forming equilibrium expected returns and thus current equilibrium prices.

This enables Fama to represent the conditions of market equilibrium in terms of conditional expected returns as follows.

\[ E(p_{t+1} | \phi_t) = [1 + E(r_{t+1} | \phi_t)]p_t \]  

(2.19)

Fama then states that equivalent representations of equilibrium expected prices and returns can be made in terms of a fair game as follows

\[ y_{t+1} = p_{t+1} - E(p_{t+1} | \phi_t) \]  

(2.20)

\[ E(y_{t+1} | \phi_t) = 0 \]  

(2.21)

where \( y_{t+1} \) is a fair game with respect to the information set \( \phi_t \) because the expected difference between the future share price, \( p_{t+1} \), and the future conditional expected equilibrium price, \( E(p_{t+1}) \), given the information set \( \phi_t \), is zero.

Similarly, returns can be written in terms of an equilibrium condition as follows,

\[ z_{t+1} = r_{t+1} - E(r_{t+1} | \phi_t) \]  

(2.22)

\[ E(z_{t+1} | \phi_t) = 0 \]  

(2.23)

where \( z_{t+1} \) is a fair game with respect to the information set \( \phi_t \) because the expected difference between the future return, \( r_{t+1} \), and the future equilibrium expected return, \( E(r_{t+1}) \), given the information set \( \phi_t \), is zero, in which case abnormal returns would be a fair game.

In which case,

\[ E(y_{t+1} | \phi_t) = E(z_{t+1} | \phi_t) = 0 \]  

(2.24)

Following Roberts (1967) Fama defines a weak form efficient market as one where share prices reflect all past information, a semi-strong efficient market as one where share prices reflect all currently available information and a strong form efficient market as one where share prices reflect all information.
According to Fama the representation of returns in this way rules out the possibility of implementing trading rules which are based on available information (however defined) to obtain abnormal returns. In this context a martingale means that on average, across a large number of samples, the actual return on an asset equals its expected return. But, as LeRoy (1976,1989) subsequently argued, Fama is rejecting the requirement that rates of return themselves be a martingale. Because, if they were then the covariance between returns must be zero as we saw earlier. In Fama’s representation, however, it is only the deviation of the price from its conditional expectation that is a fair game, so that under this definition returns themselves may be serially correlated.

A focus of LeRoy’s (1976,1989) criticisms is on what appears to be the tautological nature of Fama’s interpretation of the martingale. Any stochastic process such as \( \{r_t\}, \{p_t\}, \{y_t\} \) and \( \{z_t\} \) that is related by equation (2.20) and (2.22) and the return definition

\[
\begin{align*}
    r_{t+1} &= \frac{p_{t+1} - p_t}{p_t} \\
    \text{(2.25)}
\end{align*}
\]

implies that equation (2.19) and (2.24) must also follow. Appendix 1 provides a proof of the tautology.

LeRoy (1976) argues that with this definition of an efficient market it is possible for any return distribution to be consistent with market efficiency, because if the expected return is not correct then the equality proposed by Fama could hold even if prices are informationally inefficient. In which case the equations do not imply any restrictions on the data so that any market capital market may be efficient, and can not provide any testable implications concerning market efficiency.

It is often suggested, see for example Ball (1989), that LeRoy missed the spirit of Fama’s original definition, which almost certainly intended the unconditional expected return to be its correct value. Even though Fama (1976a) refuted that his definitions were a tautology LeRoy’s criticisms demonstrated that the definition of market efficiency needed to be made more precise. In an attempt to rectify the ambiguity, Fama (1976a,1976b) introduced an alternative definition of market efficiency which will be described below.

### 2.4 Alternative Definitions of Market Efficiency

In a later attempt to clarify the definition of an efficient market Fama (1976a,1976b) defined an informationally efficient market as one where the market uses all information that is available to determine security prices at time \( t - 1 \). Let us call this condition 1.

\[
\phi_{t-1}^\omega = \phi_{t-1} \quad \text{(2.26)}
\]
where $\phi_{t-1}$ is the information that the market uses to set security prices in time $t-1$, and $\phi_t$ is all the information that was available in time $t-1$.

Condition 1 indicates that the market uses all available information. Market efficiency then implies a second condition, let us call this condition 2 which can be expressed as

$$f_n(p_{t+1} | \phi_t^n) = f(p_{t+1} | \phi_t)$$

(2.27)

where $f(p_{t+1} | \phi_t)$ is the actual probability distribution of stock prices, and $f_n(p_{t+1} | \phi_t^n)$ is the market probability distribution of prices.

Condition 2 states that the market understands the implications of the available information for the joint probability distribution of returns. Together these two conditions state that the market is efficient if it behaves as if the actual probability distribution used in setting current prices at time $t$ is the true marginal distribution implied by all the information available to the market at time $t$.

A major conceptual problem with Fama's revised definition of an efficient market is that it makes use of the term "the market". According to Fama, it is the market that assesses the joint probability distribution of future prices, and it is the market that sets current prices, but it is not clear what the market represents. A related problem with Fama's second definition is that it refers to the true probability distribution of security prices implied by the information available. Theoretically this is unsatisfactory. In a world where individuals have heterogeneous beliefs and differential information, the relationship between the beliefs of individuals and the beliefs of the market are not well defined; nor are the concepts of the true probability distribution of security prices or the "information available".

Partly in an attempt to overcome these difficulties, Beaver (1981) has advanced an alternative definition of market efficiency which is concerned with making market efficiency dependent on the precise amount of information available. Beaver suggests that a securities market is efficient with respect to a signal, if and only if the configuration of security prices is the same as it would be in an otherwise identical economy (that is, with an identical configuration of preferences and endowments), except that every individual receives the defined signal as well as that individual's own information. Beaver also provided a more comprehensive definition of efficiency, defined with respect to the system that produces the observed signal and thus with respect to the set of all feasible signals. He termed this "information system efficiency" as distinct from "signal efficiency". The advantage of Beaver's definition is that the information set available can be partitioned to reflect almost any conceivable information set and assumes that investors do not have the same information set. In the Beaver model, market efficiency is defined
for markets which may or may not be perfect 4. Unfortunately, this approach does not provide us with empirically testable predictions because market prices reflect the beliefs of a large number of market participants, so it is impossible to model these heterogeneous beliefs. A detailed account of Beavers contribution, on which the analysis provided by this section draws, can be found in Strong and Walker (1989).

Jensen (1978) was also concerned with defining market efficiency when markets were less than perfect because Jensen considered how efficient markets might behave in the presence of transaction costs. The definition of an efficient market provided by Jensen is much more practical because it recognises the importance of transaction costs. Jensen suggests

"A market is efficient with respect to an information set \( \phi \), if it is impossible to make economic profits by trading on the basis of the information set \( \phi \). By economic profits, we mean risk adjusted returns net of all costs" (1978, p96)

This definition has the advantage of being a workable, economically meaningful interpretation of market efficiency, which does not need to assume that markets are perfect. It also reflects an important conceptual development. If markets are predictable then investors would only want to know about the source of the predictability to use that information to trade and make above normal profits. Sources of predictability which can not be exploited to provide abnormally high profits can not violate market efficiency. Ball (1989) has argued that this concept is vague because it does not suggest what level of predictability is acceptable or how transactions costs should be defined, or what level of transaction costs should be incorporated when testing market efficiency. These issues are not a serious impediment, it is important to be aware of a potential source of predictability even if its influence can not be measured precisely. Empirically, it is quite straightforward to establish an upper bound for transaction costs. If a predictable variation in stock returns exists above the level of transaction costs then markets would be considered as being inefficient.

2.5 Stock Valuation Models.

This section will be concerned with reconciling the behaviour of stock prices in an efficient market to fundamental valuation models which show that a risky asset’s equilibrium price can be interpreted as the present value of the risk adjusted future cash flows that investors expect.

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4. Much of the theoretical foundations of an efficient market describe a perfect market. In a perfect market individual agents are assumed to have homogeneous beliefs, and use information rationally. Meanwhile, individual agents are price takers and there are assumed to be no market frictions such as transaction costs or microstructure considerations.
The fundamental value of a stock is determined by the cash flows that accrue to investors over the life span of a company. Cash flows accrue in the form of dividends, which implies that the intrinsic value of a stock should be linked to the discounted value of future dividends as follows.

\[ p_0^* = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+r)^j}, \quad t = 1, 2, 3, \ldots, \infty \]  

(2.28)

where,

- \( p_0^* \) = the current intrinsic value of the share,
- \( \sum_{j=1}^{\infty} \frac{D_j}{(1+r)^j} \) = the discounted value of dividends over the whole lifespan of the company, and
- \( r \) = the discount rate which allows future cash flows to be compared with current cash flows.

Equation (2.28) says that if investors knew the future value of discounted dividends the equilibrium price would be \( p_0^* \). The equilibrium price could not be below \( p_0^* \) because at this price investors would want to buy the stock, and benefit from the cost of the investment being less than the discounted value of future cash flows arising from the investment. Buying action would therefore tend to increase the price. Neither can the equilibrium price of the stock lie above \( p_0^* \) because no rational investor will pay more for an investment than they can earn from the investment. In which case, the equilibrium price must be where the share price is equal to the discounted value of future dividends.

If the distributions from which future dividends are drawn are not identical and investors do not know these distributions with certainty, then as dividends are observed, new information is incorporated into investors' estimates of the expected future dividends to be paid. Future dividends are therefore not known but are random variables in time \( t \), and their ex-ante expected values are not observable. The prevailing share price, in an efficient market is deemed to be the best guess of the future fundamental streams, given the information set available as shown in equation (2.29) but at any point in time, the stock price may deviate from its fundamental value owing to a forecast error \( u_t \), as shown in equation (2.30).

\[ p_0 = \left[ \sum_{j=1}^{\infty} E_t \frac{D_j}{(1+r)^j} \mid \phi_t \right] \]  

(2.29)

\[ \frac{\sum_{j=1}^{\infty} D_j}{\sum_{j=1}^{\infty} (1+r)^j} - E \left[ \frac{\sum_{j=1}^{\infty} D_j}{\sum_{j=1}^{\infty} (1+r)^j} \mid \phi_t \right] = p_0^* - p_0 = u_t \]  

(2.30)

5. If the share price was above \( p_0^* \) investors would want to sell the share because the price is above the future discounted value of the cash flows. In this case the selling pressure would tend to reduce the price.
where \( p_0 = \) the current value of the stock, conditional on the information available, and \( u_t = \) the difference between the actual value of future discounted dividends and the expected value of future discounted dividends.

In this setting, the current market price is a forecast of the expected discounted value of future dividend streams, and as such, the current price may deviate from the underlying fundamental value owing to a forecast error. However, the assumption of market efficiency requires that on average, actual market prices equal fundamental prices and when actual market prices deviate from fundamental values, available information can not be used to predict the difference between the actual market price and the fundamental price. If the difference between the fundamental price and the actual price can be predicted then it becomes possible to identify future price movements so the market in this case is not efficient.

2.6 Equilibrium Asset Pricing Models and the Joint Hypothesis Problem

A large number of the empirical tests which have attempted to test market efficiency have been concerned with testing whether abnormal returns (the difference between realised and expected returns) are predictable. To establish whether abnormal returns can be forecast, it is necessary to be able to identify the equilibrium expected return. A consequence of this is that many tests of market efficiency have been joint tests, of both an equilibrium asset pricing model, and market efficiency. The problem with a joint hypothesis such as this is that it is impossible to refute market efficiency unless there is no uncertainty concerning the appropriateness of the equilibrium model being used. But as this section will show such certainty does not exist.

Initially, there was little controversy surrounding the adoption of the Capital Asset Pricing Model (CAPM) as the appropriate model from which to generate expected returns. This model was developed into a theory of equilibrium asset pricing by Sharpe (1964), Treynor (1965), Lintner (1965) and Mossin (1966) from ideas put forward by Markowitz (1952,1959). These works resulted in the development of the relationship between return and risk, summarized in what has been called the security market line of the Capital Asset Pricing Model. The security market line, says that the equilibrium expected return of an asset is a linear function of its systematic risk which can be captured by \( \beta_i \).

\[
E(R_i) = r_f + \beta_i [E(R_m) - r_f]
\]

where,
\( E(R_i) = \) the expected return of security \( i \),
\( E(R_m) = \) the expected return of the market,
\( r_f \) = the risk free return which compensates for the time value of money, and

\[ \beta_i = \frac{\sigma(R_i, R_m)}{\sigma^2(R_m)}. \]

Equations (2.31) and (2.32) says that the required ex-ante return of investors on any asset is equal to the return, \( r_f \) on a risk free asset plus a risk premium, \( \beta_i[E(R_m) - r_f] \). The Capital Asset Pricing Model (CAPM) states that the risk premium is proportional to the systematic risk of the asset, where \( \beta \) captures the amount of systematic risk and \( E(R_m) - r_f \) is the market risk premium. The implications of CAPM are therefore as follows: securities with larger betas, that is, more systematic risk, should have higher expected returns, returns should be linearly related to beta and nonsystematic risk should not be priced by the market.

CAPM can be written in terms of a fair game as follows

\[ y_{i,t} = R_{i,t} - E(R_{i,t} | \beta_{i,t}) \]  \hspace{1cm} (2.33)

\[ E(R_{i,t} | \beta_{i,t}) = r_{f,t} + [E(R_{m,t} | \beta_{m,t}) - R_{f,t}] \beta_{i,t} \]  \hspace{1cm} (2.34)

where \( \beta_{i,t} \) is the estimated amount of systematic risk.

From this presentation the nature of the joint hypothesis problem becomes obvious. If it is assumed that the CAPM is the appropriate equilibrium model then \( E(y_{i,t}) \) must be a fair game, if markets are efficient. However, if tests of \( E(y_{i,t}) \) reveal that this is not the case then either markets are inefficient (so that a predictable difference between the actual and expected return exists) or CAPM is not the true model of equilibrium returns (in this case predictability of \( E(y_{i,t}) \) is due to the wrong model being used).

A large number of papers have tested the properties of CAPM empirically. A discussion of the most important findings follows. The early tests of CAPM were generally supportive of the model, Sharpe and Cooper (1972) found that an almost perfect monotonic relationship existed between the average beta of a security and its return in the following year.

Tests by Lintner (1965) and Douglas (1968) utilised cross-section regression analysis to identify the relationship between beta and return. The results of these papers appeared to refute CAPM because residual risk and total risk appeared to be positively priced by the market.
Although since the work of Miller and Scholes (1972) we have known that the results provided by Lintner and Douglas are probably due to misspecification of the tests.

More sophisticated investigations, which used both time series and cross-section tests such as those undertaken by Black, Jensen and Scholes (1972) and Fama and MacBeth (1973) found confirmation of a linear relationship between systematic risk and return but the absence of a relationship between un-systematic risk and return. However, in the case of the tests presented by Fama and MacBeth during a number of their sub-periods there appeared to be no relationship between systematic risk and return.

More recently, the validity of CAPM has been questioned. Tinic and West (1984,1986) repeated the Fama and MacBeth study, but tested the relationship between risk and return on a month by month basis, in order to establish whether systematic risk was priced in some months of the year but not in others. They found that the positive trade off between systematic risk and return was exclusively due to the relationship between systematic risk and return in January. Systematic risk did not appear to be priced in any other month other than January. Additionally, in a later paper Tinic and West (1986) demonstrated that the relationship between systematic risk and return was sensitive to the specification being tested. When un-systematic risk was added to the model, the positive relationship between systematic risk and return in January was eliminated and replaced by a positive relationship between un-systematic risk and return.

In the late 1970’s a number of papers appeared which indicated that variables other than beta could predict cross sectional returns. Basu (1977) showed that price earnings ratios could predict future excess returns, while Banz (1981) found that firm size could predict future excess returns. Leverage, has also been found to be an important predictor of excess returns. Bhandari (1988) found expected stock returns to be positively related to the ratio of debt to equity after controlling for beta and firm size. Fama and French (1992) also challenged CAPM because they found that the book to market equity ratio has strong explanatory power; after controlling for beta, higher book to market ratios are associated with higher expected returns. But this study is particularly important, because, not only was the book to market ratio found to be important,

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6. The work of Miller and Scholes identified three possible sources of bias in estimates of the relationship between beta and return. Firstly they suggested that the correlation between the asset and the market should be estimated using the market risk premium form as follows.

\[ R_{i,t} = r_{f,t} + \beta_i (R_{m,t} - r_{f,t}) \]

This ensures that if \( r_{f,t} \) fluctuates over time, and is correlated with \( R_{m,t} \), a missing variable bias will not be incorporated into test results. Secondly, Miller and Scholes also advise testing for non-linearity in the relationship between beta and return. Finally they also draw attention to the possibility that heteroscedasticity might introduce biased estimates.

7. This paper is particularly important because it identified a methodology which eliminated the "errors in the variables" problem. As Black et al (1972) noted when portfolios are formed on the basis of beta, cross-sectional errors across equations will be correlated. This is due to selection bias, stocks with high observed betas would be likely to have a positive measurement error while stocks with low betas will be more likely to have negative measurement errors. For firms or portfolios with high betas this would introduce a positive bias into estimates of beta. The methodology suggested by Black, Jensen and Scholes (1972) to eliminate the problem required introducing an instrumental variable so that in one period portfolios should be ranked on the basis of portfolio beta (the instrument). Then, in a subsequent period the betas should be re-estimated and using these betas the relationship between risk and return should be tested.
Fama and French found that in addition to the book to market ratio, beta had no explanatory power at all. UK studies appear to support these results because Miles and Timmermann (1993) and Strong and Xu (1994) both find that the book to market ratio explains some of the cross-sectional behaviour of expected returns. The apparent empirical failure of CAPM has questioned whether this really is the appropriate model with which to capture equilibrium returns, which makes the joint hypothesis issue particularly important when undertaking tests of market efficiency.

Partly as a response to dissatisfaction with the ability of CAPM to predict expected returns alternative models of equilibrium returns have been developed. One such alternative is the Arbitrage Pricing Theory (APT) which was developed by Ross (1976). The APT is based on the concept that assets which have the same risk characteristics sell for the same price, and derives asset prices from arbitrage arguments. The APT requires that each asset is linearly related to a set of factors, each factor represents a form of systematic risk which might influence returns. The relationship between the factor, and the asset return depends on the factor loading $b_{ij}$ which captures the sensitivity of the return to changes in the systematic factor.

$$ R_i = \alpha_i + b_{i,1}I_1 + b_{i,2}I_2 + \ldots + b_{i,j}I_j + e_i $$

(2.35)

$$ E(R) = \lambda_0 + \lambda_1 b_{1,1} + \lambda_2 b_{1,2} + \ldots + \lambda_j b_{ij} $$

(2.36)

where,

- $R_i$ = the realised return for security $i$,
- $E(R_i)$ = the expected return to security $i$,
- $\alpha$ = the return of stock $i$ if all the factors/index have a value of zero,
- $I_j$ = the value of the $j^{\text{th}}$ index that influences the return of stock $i$,
- $b_{i,j}$ = the sensitivity of stock $i$'s return to the $j^{\text{th}}$ index,
- $e_i$ = a random error with a zero mean.
- $\lambda_i$ = the increase in the expected return which occurs as a result of a one unit increase in $b_{ij}$, that is, $\lambda_i$ represents returns for bearing risk, and
- $\lambda_0$ = $r_f$ or $R_e$ if the zero beta version is being tested.

8. Merton (1973) also describes a multifactor model. Rubinstein (1976) and Breeden (1979) develop models in which a proxy for the volatility of the (unobserved) value of the portfolio of wealth of individuals is used. Breeden’s insight is that with efficient capital markets, intertemporal optimising decisions by individuals would make consumption expenditure closely linked to the present value of the total wealth of individuals. Here total wealth includes both human capital and the expected productivity of future investments. As Breeden shows there is a strong relationship between changes in consumption and unexpected changes in the value of portfolios of assets.

9. Because of the impossibility of observing a truly risk free asset Black (1972) suggested using a zero asset beta rather than a risk free rate in tests of CAPM. A zero beta asset is an asset which is not correlated with the market return.
There has been considerable success in testing the APT empirically although, because the APT does not specify how many factors should be included, or what they are there have been criticisms of how tests are actually performed. Roll and Ross (1984) use factor analysis to test the APT, factor analysis is used to extract common factors in returns, and then tests are performed which identify whether expected returns are explained by the cross-sectional factor loadings of security returns on the factors. Roll and Ross (1984) conclude that three factors appear to be significant in explaining expected returns.

The primary concern of tests of a multifactor model is that the factor analysis approach which has been used to test APT only indicates how many common factors there are and not what the common factors are.

Chen, Roll and Ross (1986) use an alternative approach to factor analysis, they identify economic variables which are correlated with stock returns and then test whether the factor loading of returns are related to the crosssection of expected returns. This approach therefore constrains the variables which could be included in the multifactor model.

Chan, Chen and Ross (1986) examine a range of "business conditions" that might be related to returns because they are related to shocks to expected future cash flows or discount rates. They find that the most important variables are the growth rate of industrial production and the difference between the returns on long term, low grade bonds and long term Government bonds (which is a proxy for changes in the risk premium). They find that when β is added to the multifactor model it has no additional explanatory power. A study by Poon and Taylor (1991) finds no evidence that these macroeconomic variables have an influence over the pricing of UK stocks.

In a paper by Chan, Chen and Hsieh (1985) it is found that in addition to the business condition factors which were identified, beta had no explanatory role. This paper also found that the small firm premium could be explained by the APT model tested. This of course suggested that the small firm anomaly, which indicates that there is a predictive difference between the actual return of small firms and the expected returns of small firms, may be a consequence of using CAPM as the equilibrium model rather than APT. Although, in a later paper Lehmann and Modest (1988) found that a fifteen factor model the multifactor model left an unexplained size effect, that is expected returns were too high relative to the model for small stocks, and to low for large stocks.

10. Factor analysis provides estimates of the factor loadings $b_{ij}$'s and the factors $f_i$'s. In the factor analysis approach $f_i$ is an index consisting of a (different) weighted average of the securities on which factor analysis is performed. A specific factor analysis is performed for a given number of factors, the process is then repeated as additional factors are added until the covariance between residual returns is minimised. Since factor analysis provides estimates of the factor loading $b_{ij}$, these are then used in a second pass regression so that $\lambda_i$ can be estimated.
Since the work of Roll (1977) forceful rejections of CAPM have been difficult to make. Roll demonstrated that empirical tests of CAPM are unable to tell whether the market portfolio is inefficient or whether the wrong proxy for the market is being used because the market portfolio is empirically unobservable. To overcome this problem, Shanken (1986) suggests refining statistical techniques which are used when choosing a market portfolio: he terms this "Living with the Roll Critique". His work develops an empirical framework in which prior beliefs about the correlation between a proxy and the true market portfolio can be explicitly incorporated. The usual notation of a proxy is expanded to accommodate a vector of variables, which, together account for much of the variation in the market portfolio's return. Shanken finds that if the statistical evidence of the proxy's inefficiency is sufficiently strong, then the inefficiency of the true market may be correctly inferred and CAPM rejected.

Stambaugh (1982) argues that the relationship between risk and return does not appear to be sensitive to the proxy for the market portfolio which is used, because the relationship between risk and return does not appear to be any different even when four different market portfolios are utilised in tests of CAPM. Although, in a recent paper Jagannathan and Wang (1994) find that a value weighted index is a poor proxy for the market portfolio, following Mayers (1973) they argue that a measure of human capital should be included in the market index. When the Fama and French (1992) study which was reviewed earlier is replicated by Jagannathan and Wang, but where the betas are allowed to vary across the business cycle and are derived from a market index which includes human capital, beta is found to explain as much as 50% of the cross section of expected returns.

This section has shown that tests of market efficiency and equilibrium models appear to be inseparable in many tests of market efficiency. Furthermore, given the empirical evidence which has been discussed in this section, it is not obvious that previous tests of market efficiency that appear to reject market efficiency, but use CAPM as the equilibrium model, can not provide us with a definitive answer on this issue.

2.7 Microstructure Factors

In an important paper Cohen, Maier, Schwartz and Whitcomb (1983) (see also Cohen et al 1986), demonstrated that microstructure frictions have a strong influence on the behaviour of stock returns. Three aspects of the microstructure literature are believed to be fundamentally important for this study and will consequently be discussed in detail. The first discussion will be concerned with the bid-ask spread, and the role that this has in introducing spurious negative autocorrelation into security returns. This will be followed by a discussion of how non-synchronous trading can introduce positive autocorrelation into portfolio returns, and cause the β estimates to be biased. The discussion of microstructure considerations will conclude with
a discussion of why beta estimates are dependant on the return interval. These three issues must be discussed because they demonstrate that predictability can be introduced into stock returns even if prices are set in accordance with market efficiency. That is why, in this study, predictability in itself will not be interpreted as providing evidence of market inefficiency.

2.7.1 Bid-Ask Spread

The bid-ask spread is the difference between the price at which investors can undertake buy and sell trades. This spread represents the cost to market makers, the wholesalers of securities, of providing predictable immediacy to participants in the market. When transaction prices are used in empirical studies the transaction price records the trade price whether, it is a purchase trade and therefore on the bid side of the margin, or a sell trade which is on the ask side of the spread. As Niederhoffer and Osbourne (1966) originally discussed, the use of transaction prices when a bid-ask spread exists means that some negative serial dependence in observed price changes should be anticipated. In a later paper, Roll (1984) has shown formally how the bid-ask spread can introduce spurious negative autocorrelation into successive changes in security prices. A summary of Roll’s explanation, for why the bid-ask spread introduces negative autocorrelation into returns that are calculated from transactions prices follows.

If at time \( t - 1 \) the market maker undertook a buy trade, the price of the stock would have been at the bid price. This means that (assuming no new information arrives) the next trade will be either at the bid price or the ask price with equal probability\(^{11}\). A similar, but opposite pattern will exist if the price at \( t - 1 \) happened to have been a sell trade, so that the price was an ask price. In the next period the price could be either at the bid or the ask price with equal probability.

This means that the joint probability of successive price changes in trades initiated other than by new information, depends on whether the last transaction was at the bid or at the ask. If the previous trade took place at the bid(ask) price, the next price change can not be negative(positive) because there is no new information. Similarly, there is no probability that two successive price increases(declines) can take place.

This means that the covariance between successive price changes, as Appendix 2 demonstrates, can not be zero. The correlation between two successive price changes, when there is no new information must be

\[
\text{Cov}(\Delta p_t, \Delta p_{t+1}) = 1/8(-S^2 - S^2) = -S^2/4 \quad (2.37)
\]

\(^{11}\) It is assumed that in the absence of no new information then there is no incentive for the price to be at the bid or the ask. The probability of the price being at either the bid or the ask is therefore a random event.
where $\Delta p_t$ is the change in the transaction price of a security between time $t$ and time $t+1$, and $S$ is the bid-ask spread.

Thus the covariance between successive price changes is minus the square of one half the bid-ask spread. Similarly, as Appendix 2 demonstrates, the variance of $\Delta p$ is $S^2/2$. This predicts an autocorrelation coefficient of $-1/2$, assuming no new information arrives. However, observed autocorrelations might be smaller than this because the estimated covariance is divided by the sample variance. The variance, is likely to be dominated by the effect of new information, whereas the covariance between successive changes will not be influenced by new information if markets are efficient.

Roll (1984) tested empirically whether there really was an inverse relationship between the magnitude of the bid-ask spread and the first order autocorrelation coefficient. One of the most important findings presented by Roll was that the bid-ask spread and therefore the expected negative autocorrelation was larger for small firms than for large firms. In a later paper Kaul and Nimalendran (1990) also highlighted the importance of the bid-ask spread in empirical tests. Kaul and Nimalendran found that the bid-ask error in transaction prices could account for the observed short-run price reversals which appeared to take place for NASDAQ firms. The bid-ask spread therefore appears to be responsible for the observed short-run overreaction of security prices. As Chapter 3 will show, work by Conrad and Kaul (1990) finds that the bid-ask spread component is able to explain as much as 5% of the variation in individual security returns. Surprisingly, unlike Roll (1984) Conrad and Kaul do not find that firm size influences the magnitude of the variation in realised returns which can be explained by variations in the bid-ask spread.

2.7.2 Thin Trading

Frequently, when empirical work is undertaken, the assumption is made that prices are sampled synchronously when in fact they are sampled non-synchronously. For example, prices quoted in the daily financial press are usually closing prices, that is prices at which the last transaction in each of these securities occurred on the previous business day. The closing prices of distinct securities need not be recorded simultaneously, but traditionally few studies have taken this into account, see for example, Fama and MacBeth (1973). Failing to take into account the non-synchronous nature of prices can alter substantially the characteristics of asset returns, as this section will illustrate.

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12. NASDAQ firms are companies traded through the National Association of Securities Dealers Automated Quotation system. This is a dealer market (over the counter market) which accounts for about 25% of the market value of shares on the New York Stock Exchange.
The first to recognise the importance of non-synchronous price adjustments was Fisher (1966), who found that indexes which gave greater weight to small firms were more positively autocorrelated. More detailed models of nontrading have been developed by Scholes and Williams (1977) Cohen et al (1986) and Dimson (1979). Whereas, these early studies were concerned with the effects of thin trading on empirical applications of the Capital Asset Pricing Model and the Arbitrage Pricing Theory, although, more recent attention, for example, Lo and MacKinlay (1990b) and Boudoukh, Richardson and Whitelaw (1994) has focused on the effect of thin trading as a cause of spurious autocorrelation.

Non-synchronous trading in stock markets exists when some stocks trade more frequently than others. Suppose that the two stocks i and j are independent but i trades less frequently than j. If some systematic news arrives in time t, it is more likely that stock j will reflect the news before stock i because it trades more frequently. Eventually, stock i will reflect the new information, but only when it trades. Because the price of stock i responds with a lag to the information, the prices of stock i and j will be cross serially correlated. As a result, a portfolio containing both i and j will be serially correlated.

Lo and MacKinlay (1990b) introduce a model of non-synchronous trading which illustrates well why non-synchronous trading introduces positive autocorrelation into portfolio returns. A brief description of their model follows.

In each period t, there is some chance that security i does not trade with probability \( p_i \). If it does not trade, its observed return is zero. Although, the true or virtual return will reflect any systematic shocks. In the next period \( t+1 \), there is again some chance that security i does not trade, also with probability \( p_i \). If security i does trade in period \( t+1 \) and did not trade in time t, then the realised return is the sum of the two previous virtual returns for all previous consecutive periods in which stock i has not traded. In fact, the observed return in any period is the sum of all virtual returns for consecutive periods in which the stock failed to trade.

When a portfolio contains both stocks i and j, the virtual returns of i and j will be contemporaneously correlated but serially uncorrelated. Observed returns, however, will be autocorrelated because the virtual return of i and j is observed at different intervals. As long as i and j have positive betas so that they react in the same way as the market to a systematic shock, the portfolio return will be positively autocorrelated. This means that identifying autocorrelation in portfolio returns does not imply that the portfolio returns are predictable in the sense that such autocorrelation implies market inefficiency.
2.7.2.1 Risk Measurement and Thin Trading

Dimson (1979) and Scholes and Williams (1977) were the first to recognise that non-synchronous trading will bias estimates of systematic risk. Estimates of $\beta$ which were discussed in the following section are usually obtained from the market model (which provides an estimate of the correlation between the return of a security and the market return). The market model can be written as

$$R_{i,t} = \alpha_i + \beta_i R_{m,t} + e_i$$

(2.38)

where,

$R_{i,t}$ = the return to security $i$ in time $t$,

$R_{m,t}$ = the market return in time $t$, and

$\beta_i$ = the estimate of systematic risk for stock $i$ or portfolio $i$ which indicates the extent to which stock $i$ covaries with the market return, this can also be calculated as $\frac{\sigma(R_i, R_m)}{\sigma^2(R_m)}$.

This equation measures the extent to which a variation in the market is also reflected in stock $i$. If a stock which is thinly traded, has a trade in time $t$, but not in time $t-1$ then the effects of a systematic shock in time $t$ will not be observed in the security return until it trades. This means that the relationship between the market and an individual security will be understated in cases where stock $i$ is thinly traded. In which case the $\beta$ estimates will be biased downwards.

To overcome this problem Dimson (1979) proposed an aggregate coefficient model which allows us to make unbiased estimates of betas in the presence of thin trading. This aggregate coefficients model can be written as

$$R_{i,t} + \alpha_i + \beta_i R_{m,t} + \sum_{\tau=1}^{T} \beta_{i,\tau} R_{m,t-\tau} \quad \tau = 1, 2, 3..T$$

(2.39)

where,

$R_{m,t-\tau}$ = the past market return in time $t-\tau$

$\beta_0$ = the covariability of the return of stock $i$ in time $t$ with the market return in time $t$

$\beta_k$ = the covariability of the return of stock $i$ in time $t$ with the market return in time $t - \tau$.

If a stock fails to trade in time $t$, the effect on stock $i$ of a change in the market index will only be observed when stock $i$ trades. If stock $i$ trades in time $t+1$, the effect of a systematic shock in time $t$ will be captured by the relationship between the market return in time $t-1$ and the security return in time $t$. Dimson therefore, suggests using an aggregate coefficients model where lagged market returns are allowed to enter the market model. The coefficients associated
with the lagged market return will capture the effect of systematic changes which are not observed in the security stock return because a stock is thinly traded. For thinly traded stocks, betas should be estimated as the sum of the contemporaneous beta that is $\beta_0$ and the $\beta_i's$ from equation (2.39).

Non-synchronous trading introduces a second problem when estimating betas. Frequently, traded stocks will also have betas which are biased. In this case the bias is introduced because the market index contains infrequently traded stocks. This means that the relationship between the frequently traded security return and the market index (which contains some infrequently traded securities) is overstated. An adjustment for this can be made by including leading market returns in aggregate coefficient market model.

2.7.3 The Intervalling Effect

The "intervalling effect" recounts the empirical observation that risk measurements are sensitive to the return interval being used. Although, changes in the measurement of beta as the return interval increases have been documented extensively, see for example Scholes and Williams (1977) Dimson (1979) and Cohen, Hawawini, Maier, Schwartz and Whitcomb (1983). Handa, Kothari and Wasley (1989) focused on beta changes stemming purely from varying the return interval. They showed that as the return interval varies from one day to one year beta estimates vary in a predictable way.

The main arguments put forward by Handa et al were as follows. As an earlier section showed, a security beta can be calculated as $\frac{\sigma(R_i, R_m)}{\sigma^2(R_m)}$. Betas are sensitive to the return interval because $\sigma(R_i, R_m)$ and $\sigma^2(R_m)$ do not change proportionately as the return interval is varied. As Appendix 3 demonstrates, this means that for securities with a beta less than one, as the return interval increases the estimated betas get smaller, but for securities with betas greater than 1 the estimated betas get larger. Smith (1978) provided empirical confirmation of this. Ten portfolios were created on the basis of estimated beta and as the return interval increased the difference between the beta of the first and the last portfolio is found to widen considerably.

Handa Kothari and Wasley (1989) also found empirical support for a relationship between the magnitude of a security beta and the return interval. Handa et al investigated twenty portfolios which had been stratified on the basis of market value. They found that beta estimates were closely associated with the return interval. To illustrate, they found that the portfolio with the smallest beta, when returns were sampled annually was 0.56 but this rose to 0.90 when returns were sampled daily. In contrast, the portfolio with the highest beta, when returns were sampled annually was found to be 1.66 but when returns were sampled daily this fell to only 0.99.
As this section has demonstrated, microstructure considerations are crucially important when trying to understand how stock prices behave. As we shall see in the next chapter, when the empirical work, that has investigated the behaviour of stock prices is discussed, the microstructure behaviour of stock prices has an important role to play in explaining the variation of stock prices.

2.8 Summary and Conclusions

This chapter has introduced the concept of market efficiency which will be a central theme in this study. Consequently, it is important to establish what the testable implications of stock market efficiency are, in order to test the hypothesis. This requires identifying a model with testable implications. Initially, as we saw at the start of the chapter the testable implications of the efficient markets hypothesis was initially associated with the random walk model. This model was readily accepted at the time because there was growing evidence that stock returns were uncorrelated, suggesting that stock return distributions were independent.

In later work this conclusion was challenged, because although, mean returns appeared to be uncorrelated, this did not appear to apply to higher moments. Samuelson (1965) and Mandelbrot (1966) linked the efficient markets model to the martingale model, which is a less restrictive model. The martingale model requires that means are uncorrelated but no assumptions are imposed concerning higher moments. Essentially, it is the martingale property of returns that empirical research now tests when investigating whether capital markets behave efficiently.

The application of the martingale to the behaviour of capital market efficiency was made first by Fama (1970) who demonstrated that abnormal returns in an efficient market should have an expected value of zero. Although, this application has been controversial as we shall see in the next chapter a vast amount of empirical work has been concerned with testing whether abnormal returns really do follow a martingale.

In recent years a number of practical problems have been identified which makes it very difficult to conclude that violations of the martingale suggest market inefficiency. Fama suggested that the difference between realised and equilibrium expected returns should be a fair game. Testing capital market efficiency in this way implies that equilibrium returns can be identified. This chapter has shown that a great deal of uncertainty exists concerning which equilibrium model should be used. Because, of the empirical failure of CAPM it is no longer clear that this is the appropriate equilibrium model to use in tests of market efficiency. In which case tests of market efficiency are joint tests of both an equilibrium model and market efficiency.
As we saw towards the end of this chapter, even if the equilibrium model is identified then we would only expect abnormal returns (or returns according to LeRoy) to be a martingale if market imperfections and microstructure frictions are ruled out. This makes testing stock market efficiency very difficult indeed. As we shall see later in this study perhaps the best procedure under these conditions is to test for stock market efficiency and then if there is predictability present, test to see how much of the predictability can be accounted for by market imperfections or microstructure frictions.

As we shall see from the next chapter which surveys the empirical work which has been undertaken on market efficiency there is a substantial amount of evidence to suggest that stock prices are predictable, but as we shall see little of the evidence suggests that markets are inefficient.
Appendix 1: Proof of the LeRoy Tautology

Assume equation (2.20) of the text,

\[ y_{t+1} = p_{t+1} - E(p_{t+1} | \phi_t) \]  \hspace{1cm} (A1.1)

Taking expectations of both sides provides,

\[ E(y_{t+1}) = E(p_{t+1}) - E(p_{t+1} | \phi_t) = 0 \]  \hspace{1cm} (A1.2)

Hence the tautology.

Now looking at returns, taking equation (2.22) of the text,

\[ z_{t+1} = r_{t+1} - E(r_{t+1} | \phi_t) \]  \hspace{1cm} (A1.3)

and taking expectations of both sides provides

\[ E(z_{t+1}) = E(r_{t+1}) - E(r_{t+1} | \phi_t) = 0 \]  \hspace{1cm} (A1.4)

Taking equation (2.19) of the text

\[ E(p_{t+1} | \phi_t) = [1 + E(r_{t+1} | \phi_t)p_t] \]  \hspace{1cm} (A1.5)

By definition, rates of return are given by

\[ r_{t+1} = \frac{(p_{t+1})}{p_t} - 1 \]  \hspace{1cm} (A1.6)

Applying a conditional expectations operator to equation (A1.6) gives

\[ E(r_{t+1} | \phi_t) = E\left[ \frac{p_{t+1}}{p_t} - 1 | \phi_t \right] = \frac{E(p_{t+1} | \phi_t)}{p_t} - 1 \]  \hspace{1cm} (A1.7)

Multiplying through by \( p_t \) gives,

\[ E(r_{t+1} | \phi_t)p_t = E(p_{t+1} | \phi_t) - p_t \]  \hspace{1cm} (A1.8)

Rearranging gives

\[ E(p_{t+1} | \phi_t) = p_t[E(r_{t+1} | \phi_t) + 1] \]  \hspace{1cm} (A1.9)

which provides another tautology.
Appendix 2: Bid-Ask Spread as a Cause of Negative Autocorrelation

1) The Covariance Between Successive Changes in Prices

If a stock is trading at the bid price in time $t-1$, in the absence of any new information, assuming markets are efficient, market prices can not fall. This means that $\Delta p_t$ can only be zero (if the price stays at the bid) or be positive (if the price moves from the bid to the ask). If $p_{t-1}$ is at the bid price, $\Delta p_t$ must have been either positive or zero with equal probabilities.

The joint probabilities of $\Delta p_t$ and $\Delta p_{t+1}$ being in any direction can be found by taking the probability of each occurrence as follows $\sum_i (\Delta p_t, \Delta p_{t+1})p_i$, where $p_i$ is the probability of a price change in any direction. The probability of a successive price change being in a particular direction is $0.5 \times 0.5 = 0.25$ because there is a 50% chance that if the price in $t-1$ is at the bid, then in the next period there is a 50% chance of it being at the bid again (a zero price change) or a 50% chance that the price will rise from the bid to the ask which would be a price rise. The sequence of successive price changes with probabilities is indicated in Table A.1

<table>
<thead>
<tr>
<th>$\Delta p_t$</th>
<th>$\Delta p_{t+1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>+S</td>
<td>0.25</td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>+S</td>
<td>0</td>
</tr>
</tbody>
</table>

If the price is at the bid in $t-1$

In Table A.1 above, +S indicates a price move to the ask price, while -S indicates a price move to the bid price. As we can see, there is a 25% chance that $\Delta p_t$ is zero and $\Delta p_{t+1}$ is also zero (this implies that prices stay at the bid price). There is a 25% chance that if $\Delta p_t$ is zero $\Delta p_{t+1}$ will be positive (this implies that the price stays at the bid price in time $t$ but moves to the ask price in time $t+1$). Meanwhile, there is a 25% chance that if at time $t-1$ the price is at the bid then $\Delta p_t$ is positive and $\Delta p_{t+1}$ is negative (the price moves to the ask price in time $t$ and moves to the bid in time $t+1$). Finally, there is also a 25% chance that if $\Delta p_t$ is positive then $\Delta p_{t+1}$ is zero (the price moves to the ask in time $t$ but stays at the ask in time $t+1$). Because the price is at the bid price in time $t-1$ it is impossible for $\Delta p_t$ to be negative, since the price can
not fall below the bid price, in the absence of new information. This rules out successive price falls. In a similar way it is impossible to observe successive price rises in the absence of new information.

**If the price at \( t - 1 \) is at the ask**

Similarly, if a stock is trading at the ask price in time \( t - 1 \) then \( \Delta p_t \) must be negative or zero with equal probabilities. The joint probability of \( \Delta p_t \) and \( \Delta p_{t+1} \) being in any direction must again be \((0.5) \times (0.5) = 0.25\). The possible combination of sequences is summarised in Table A.2. There are 25% probabilities that a) the price will stay at the ask in time \( t \), but move to the bid in time \( t + 1 \) b) the price will stay at the ask in time \( t \) and stay at the ask in time \( t + 1 \) c) the price will move to the bid in time \( t \) and stay at the bid in time \( t + 1 \) d) the price will move to the bid in time \( t \) and move to the ask in time \( t + 1 \).

Table A.2

<table>
<thead>
<tr>
<th>( \Delta p_t )</th>
<th>(-S)</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-S)</td>
<td>0</td>
<td>0.25</td>
</tr>
<tr>
<td>0</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>(+S)</td>
<td>0.25</td>
<td>0</td>
</tr>
</tbody>
</table>

Since prices may be at the bid or the ask price with equal probabilities the probability of two successive changes taking place in a particular combination must be \((0.25) \times (0.25) = 0.125\). Thus, Tables A.1 and A.2 combine to give Table A.3.

Table A.3

<table>
<thead>
<tr>
<th>( \Delta p_t )</th>
<th>(-S)</th>
<th>0</th>
<th>(+S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-S)</td>
<td>0</td>
<td>0.125</td>
<td>0.125</td>
</tr>
<tr>
<td>0</td>
<td>0.125</td>
<td>0.25</td>
<td>0.125</td>
</tr>
<tr>
<td>(+S)</td>
<td>0.125</td>
<td>0.25</td>
<td>0</td>
</tr>
</tbody>
</table>

The means of \( \Delta p_t \) and \( \Delta p_{t+1} \) must be zero in the absence of new information. This implies the middle rows and columns of Table A.3 can be ignored and the covariance between \( \Delta p_t \) and \( \Delta p_{t+1} \) becomes,
\[ Cov(\Delta p_t, \Delta p_{t+1}) = (\Delta p_{b,t} - E(\Delta p_t)) (\Delta p_{a,t+1} - E(\Delta p_{t+1}))0.125 \]
\[ + (\Delta p_{a,t} - E(\Delta p_t)) (\Delta p_{b,t+1} - E(\Delta p_{t+1}))0.125 \]

where \(\Delta p_{b,t}\) is a price change from the ask to the bid, and \(\Delta p_{a,t}\) is a price change from the bid to the ask. This simplifies to

\[ [(-S^2(0.125)) + (-S^3(0.125))] = \frac{2(-S^2)}{0.125} = \frac{-S^2}{4} \]  

2) The Variance of Price Changes

The possible range of price movements in time \(t\) are summarised below in Table A.4.

<table>
<thead>
<tr>
<th>Price (t-1)</th>
<th>Bid</th>
<th>Ask</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bid</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Ask</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>

From Table A.4 we can see that there is a 50% of there being no price change at all, and a 25% chance that the price change will be positive, and a 25% chance that the price change will be negative. Thus, the variance of \(\Delta p_t\) can be written as

\[ Cov(\Delta p_t, \Delta p_t) = [\Delta p_{a,t} - E(\Delta p_t)]^20.25 + [\Delta p_{b,t} - E(\Delta p_t)]^20.25 \]
\[ = [S^2](0.25) + [S^2](0.25) = \frac{S^2}{2} \]

3) The Autocorrelation of Successive Price Changes

The first order autocorrelation statistic of the price changes is

\[ \frac{Cov(\Delta p_t, \Delta p_{t+1})}{\text{var}(\Delta p)} = \frac{-S^2/4}{S^2/2} = \frac{-1}{2} \]
Appendix 3: The Intervalling Effect

Define returns as price relatives, and let $R_i(n)$ be the $n$-period buy and hold return from time 0 to time $n$ for security $i$, defined as

$$R_i(n) = \frac{P_{i,n}}{P_{i,n-2}} \cdots \frac{P_{i,k}}{P_{i,k-1}} \cdots \frac{P_{i,1}}{P_{i,0}}$$

where the series of one period returns, $R_i(1)$, are given by

$$R_i(1) = \frac{P_{i,k}}{P_{i,k-1}} \quad \forall k$$

Define the expected buy and hold return over the period from time 0 to time $n$ for security $i$, $E(R_i(n))$, as $\mu_i(n)$. Then, assuming constant expected (one period) returns, it follows that

$$\mu_i(n) = \mu_i^n$$

where $\mu_i$ is the expected one period return, $E(R_i(1))$.

Now consider the covariance between the $n$-period returns on security $i$ and those on security $j$, $\sigma_{ij}(n)$, which is defined as

$$\sigma_{ij}(n) = E[(R_i(n) - E(R_i(n)))(R_j(n) - E(R_j(n)))]$$

This can be simplified as follows

$$\sigma_{i,j}(n) = E[R_i(n)R_j(n)] - E[R_i(n)]E[R_j(n)]$$

$$= E[R_i(n)R_j(n)] - \mu_i \mu_j$$

Similarly, the covariance between the one period returns of security $i$ and security $j$, $\sigma_{ij}$, is defined as follows

$$\sigma_{i,j} = E[R_i(1)R_j(1)] - E[R_i(1)]E[R_j(1)]$$

$$= E[R_i(1)R_j(1)] - \mu_i \mu_j$$

Now define $\eta_{ij} = E[R_i(1)R_j(1)]$, and $\eta_{ij}(n) = E[R_i(n)R_j(n)]$. If the expected one period returns on each security are constant, then $\eta_{ij}$ will also be constant. It then follows that

$$\eta_{ij}(n) = \eta_{ij}^n$$

$$E[R_i(n)R_j(n)] = E[R_i(1)R_j(1)]^n$$

This means that equation (A3.5) can be simplified further to

$$\sigma_{i,j}(n) = E[R_i(1)R_j(1)]^n - \mu_i \mu_j^n$$

and by substitution of equation (A3.6) gives
\[ \sigma_{i,j}(n) = [\sigma_{ij} - \mu_i \mu_j]^n - [\mu_i \mu_j]^n \]  
(A3.9)

Similarly, by assuming that the expected returns on the market portfolio are constant over time, the covariance between the \( n \)-period returns of security \( i \) and those of the market, \( \sigma_{i,m}(n) \), can be shown to be

\[ \sigma_{i,m}(n) = [\sigma_{i,m} + \mu_i \mu_m]^n - [\mu_i \mu_m]^n \]  
(A3.10)

where \( \sigma_{i,m} \) is the covariance between the one period returns of security \( i \) and the market portfolio.

It follows that the beta of security \( i \) using \( n \) period returns is

\[ \beta_i(n) = \frac{(\sigma_{i,m} + \mu_i \mu_m)^n - (\mu_i \mu_m)^n}{(\sigma_m^2 + \mu_m^2)^n - (\mu_m^2)^n} \]  
(A3.11)

while the beta of security \( i \) using one period returns is \( \beta_i = \sigma_{im}/\sigma_m^2 \).

Dividing the \( n \)-period return beta by the one period return beta gives the following ratio

\[ \frac{B_i(n)}{\beta_i} = \frac{(\sigma_{i,m} + \mu_i \mu_m)^n - (\mu_i \mu_m)^n}{(\sigma_m^2 + \mu_m^2)^n - (\mu_m^2)^n}/\sigma_{im}^2 \]  
(A3.12)

Important inferences can be made by looking at the ratio as a function of \( \mu_i \) and \( n \). Firstly, for a given return interval as \( \mu_i \) increases then so does the beta ratio. Secondly, as the return interval, \( n \), increases, for a given \( \mu_i \), the beta ratio increases for one-period betas greater than one. For one-period betas less than one the ratio declines as \( n \) increases. As the return interval increases the spread between high and low betas must increase.

The conclusion of this analysis is important. For securities which have betas less than one, then as the return interval, which is used to calculate betas, decreases then so does the estimated beta. Meanwhile, when betas are greater than one the estimated security beta increases with the return interval.
CHAPTER THREE
The Predictability of Stock Market Returns

3.1 Introduction

This chapter reviews the empirical literature which has investigated stock market predictability. The first section considers the evidence which has explored whether short-horizon returns are serially correlated or not. This will initially study the work on market wide indexes and individual securities. The discussion will then progress to consider the behaviour of portfolios which have been formed in a number of different ways. A great deal of attention has currently been directed at the behaviour of portfolios because portfolio returns appear to be autocorrelated and therefore much more predictable than is the case for individual securities. Autocorrelation tests have played an important role in determining whether markets are efficient or not because they directly test the martingale property of returns. However, as Section 2.7 of Chapter 2 indicated, predictability in itself does not imply market inefficiency, predictability can exist even if markets are efficient.

The second section of this chapter will review the evidence which has suggested that stock markets overreact to new information. There are three strands to this literature, which have all developed independently of each other. Firstly, there is the mean-reversion literature, which began with a paper by Summers (1986). This literature has demonstrated that over long horizons stock returns are negatively correlated. Secondly, a similar theme is considered by DeBondt and Thaler (1985,1987) in the "winner loser hypothesis". DeBondt and Thaler have shown that companies that experience abnormally bad returns in one period provide excessively high returns in future years. Meanwhile, companies that are extreme winners in one period, because they have abnormally high returns, become extreme losers in subsequent years. Thirdly, variance bounds tests, which were pioneered by Shiller (1981a) have demonstrated that share prices appear to be much more volatile than the streams of fundamental information, such as dividends, which share prices are meant to represent. This appears to suggest that contrary to what we would expect in an efficient market, share prices tend to react to things other than changing expectations about future dividends.

The final section of this chapter will examine the empirical evidence which suggests that stock markets are characterised by persistent regularities. The most important of these "anomalies" are the calendar and size anomalies which will be discussed in detail because they

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1. It is usual to characterise long-horizon returns as returns which are over one year or more.
are particularly important for this study. The calendar regularities indicate that stock returns are predictable during certain calendar periods, while the size effect demonstrates that returns to small firms are higher than returns to large firms, even after controlling for risk.

A review of this empirical literature is important for a number of reasons. Firstly, a full review of the empirical literature is necessary to identify the issues which are important to consider when evaluating whether a market is efficient or not. Secondly, studying previous work in this way will allow us to gauge the extent to which the empirical work, which is undertaken in this study, adds to our understanding of how stock markets behave. Finally, discussing the extensive empirical literature will allow us to determine whether perhaps previous work which rejected market efficiency was a little premature in doing so.

3.2 Short Horizon Returns

Short-horizon returns exist when the differencing interval between the current share price and the previous share price is considerably less than one year. For example, daily, weekly or monthly returns would be considered as being short-horizon returns. Due to the very different empirical characteristics of short and long-horizon returns they will be discussed separately. This section will focus upon the empirical properties of short-horizon returns.

3.2.1 Autocorrelation Tests

If prices reflect all available information, as they would in an efficient stock market, then future stock returns should not be predictable from past stock returns\(^2\). One way of testing whether stock returns are predictable is by undertaking an autocorrelation test which allows us to test the relationship between current and past returns.

Assume that \(\hat{\rho}_t\) is the sample autocorrelation statistic which captures the average relationship between returns in time \(t\) and time \(t - \tau\). This statistic can be defined as

\[
\hat{\rho}_\tau = \frac{\sum_{t=1}^{n-\tau} (r_t - \bar{r})(r_{t-\tau} - \bar{r})}{\sum_{t=1}^{n} (r_t - \bar{r})^2} \quad \tau = 1, 2, ...
\]

where, \(r_t\) is the return in time \(t\), and \(\bar{r}\) is the mean return over the sample period, that is,

\[
\bar{r} = \frac{1}{n} \sum_{t=1}^{n} r_t.
\]

\(^2\)Assuming that expected returns are constant.
A finding of positive autocorrelation between successive price changes would indicate that positive (negative) returns tend to follow positive (negative) returns. In contrast, negative autocorrelation would indicate that negative (positive) returns tend to follow positive (negative) returns. If the autocorrelation is significant and sufficiently strong, it would suggest that investors may be able to use these autocorrelation patterns to devise trading rules which would provide abnormally high returns.

One of the earliest documentations of an autocorrelation test, being applied to the stock market, is provided by Kendall (1953). In this study Kendall sought to identify whether long term trends in the behaviour of stock market prices could be distinguished from day to day price fluctuations. Kendall investigated the correlation coefficient between successive weekly changes in the stock indexes of a number of industrial sectors. These computations produced correlation coefficients which were all close to zero, although, the serial correlation coefficient of the Financial Times All Share index\(^3\), which was positive, appeared to be considerably larger than the correlation coefficient for the industrial sectors that were tested.

The first study to consider the autocorrelation structure of both individual securities and market wide indexes was conducted by Fama (1965). Fama investigated the daily serial correlation properties of the thirty stocks in the Dow Jones Industrial Average \(^4\) which were listed over the period 1957-1962. The results of these tests indicated that only a small number of the serial correlation coefficients were statistically significant. Furthermore, the magnitude of those coefficients which were found to be statistically significant were so small that little more than 1\% of the variation in stock returns could be explained by the autocorrelation pattern. This meant that after transaction costs, it would not be possible to use the information provided by the autocorrelation patterns to outperform the market. This led Fama to conclude that the US stock market appeared to be efficient.

A similar study was undertaken by Dryden (1970a,1970b) who tested the autocorrelation properties of UK daily stock returns. The results reported by Dryden for the UK market were strikingly similar to those reported by Fama. The chief findings were as follows. Autocorrelation tended to be more likely to be significantly positive for one period lags, particularly for indexes. Autocorrelation coefficients greater than one lag tended to have a tendency to being negative, although again the coefficients appeared to be very small, suggesting that any predictability which existed could not be used in a trading rule to provide excess profits after transaction costs.

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3. This index consists of over 500 companies which are listed on the London stock exchange.
4. The Dow Jones index is a composite of the thirty largest companies listed on the New York Stock Exchange.
Working (1960) was the first person to draw attention to the importance of non-synchronous sampling of stock price information, in an attempt to explain why stock market indexes had relatively more autocorrelation than commodity indexes. Working showed that spurious autocorrelation can be introduced into stock market indexes when average prices are calculated from prices taken at different points in time. Working had argued that this was the cause of the small amount of positive serial correlation which had been discovered previously by Kendall. A similar issue is discussed by Brealey (1970) who tested for serial correlation in the FT 30 share index. Brealey constructed his own index comprising of 29 of the shares in the FT 30 share index. Prices were taken at 2pm exactly each day over the sample period February to November 1968 which avoided the spurious autocorrelation which was discussed by Working. The magnitude of the autocorrelation coefficient reported by Brealey was approximately 50% smaller than the one reported by Kendall, and therefore Brealey appeared to provide support for Working’s argument that some of the autocorrelation identified by Kendall, in indexes, was spurious.

Unfortunately, Working did not discuss the impact that persistent thin trading (where companies fail to trade for at least one full trading day) would have had on the autocorrelation patterns of market indexes. Intra-day non-synchronous trading is likely to cause much less autocorrelation than persistent thin trading. In which case, it is difficult to ascertain whether the low degree of autocorrelation reported by Brealey is due to synchronous sampling times or is a consequence of Brealey using very large firms in the index, which would trade every day, and have lower autocorrelations irrespective of whether prices were sampled at the same time during the day or not.

Unfortunately, none of these early studies included small firms in their sample. Consequently, these studies have a selection bias in favour of finding no autocorrelation. The firms investigated are all high profile companies and very large. This means they are likely to get more coverage in the financial press and will have more intense analyst following. In which case prices are much more likely to reflect fundamentals and therefore be uncorrelated.

3.2.2 Filter Tests

Although, the autocorrelation tests which were discussed in the previous section appeared to indicate that returns could not be forecast from previous returns, the only way to test whether an autocorrelation pattern is economically meaningful is by testing a trading rule based on the autocorrelation pattern. Alexander (1961) provided the first major evidence on the use of trading

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5. The FT 30 index is a composite of the 30 largest shares listed on the London stock exchange.
rules. The development of filter trading rules by Alexander was an advancement over the autocorrelation tests described in the previous section because filter rules are able to capture non-linearities which might exist between successive price changes.

An Alexander filter rule is based on price signals where buy (sell) signals are generated by some upward (downward) price movements from recent troughs (peaks). This requires that if the price of a security rose by at least x%, buy and hold the security, until its price moves down at least x% from a subsequent high, at which time the rule requires that the trader sells short. The short position should be maintained until the price rises at least x% above a subsequent low, at which time one covers the short position and buys. Moves of less than x% in either direction are ignored. In the absence of serial correlation a trading rule should not provide abnormal profits. In a later paper, Fama and Blume (1966) compared the profitability of various filters to a buy and hold policy 6 for the Dow-Jones Industrial Average stocks. The filter rules of both Alexander (1961) and Fama and Blume (1966) demonstrated that it was impossible to make profits from a filter rule. For very small filters such as 0.5%, 1.0% and 1.5% it becomes possible to devise trading rules which on average can out perform buy and hold strategies but the number of trades required for such small filter sizes are so large that very high transactions costs are incurred, which erode the profits.

Dryden (1970a,1970b) tested a filter rule on the UK market and found that a filter rule of 0.1% was capable of providing average profits of 32.9% per year, while a buy and hold policy provides average profits of only 2.6% a year. Transaction costs were specifically discussed by Dryden, assuming transactions costs of only 2% (which is very conservative since Stoll and Whaley (1983) find that average transaction costs in the US lie in the range 2-6%, while Thomas (1989, p77) also reports transaction costs as being higher than 2% in the UK) the after transaction cost return to the filter rule was found to be on average -17.1% a year.

Cunningham (1973) and Girms and Damant (1975) also provided evidence of trading rules which generated profits. However Cunningham failed to compare the profits to a buy and hold strategy, which is necessary if the filter profits are to be compared with normal profits. Furthermore, a weakness of both papers is that neither considered whether transactions costs would have eliminated the filter profits.

Recent evidence by Chelley and Steeley (1995) has shown that profits from filter rules are overstated, particularly when filters are small, if some of the stocks which are used in the sample are thinly traded. An overestimate of profits occurs in ex-post tests of a trading rule because the decision to buy (sell) a stock assumes that another party is willing to undertake the reverse trade. This will not always be possible if at least some of the stocks in the sample are

6. The return to a buy and hold strategy is calculated to capture normal returns, which for risky assets like stock investments should be positive. Only if a filter rule provides a higher return than the buy and hold return is the filter rule providing abnormally high profits.
thinly traded. In a sample of UK large and small firms they report that the profits from a trading rule based on the autocorrelation structure of portfolio returns will overstate profits for small firms by as much as 20%. Without further evidence on the trading characteristics of the stocks used in these earlier studies it is impossible to gauge the size of the potential pre-transaction cost profit with any reliability.

3.2.3 Microstructure Frictions and Portfolio Autocorrelation

The two previous sections have demonstrated that there is little evidence to suggest that either individual security returns or market wide returns are predictable from their own previous values. In these early studies, no attention was directed at the serial correlation patterns of portfolios. This section will be concerned with the recent discovery that portfolio returns are much more autocorrelated and therefore more predictable than market wide indexes or individual securities.

Since the seminal work of Fisher (1966) and the empirical applications of the Capital Asset Pricing Model by Scholes and Williams (1977) and Dimson (1979) it has been known that indexes which give greater weight to small firms are characterised by greater serial correlation. Furthermore, a number of recent studies have documented a similar pattern among portfolios of stocks, where the component companies have been sorted on the basis of their capitalisation value. For example, Perry (1985) and Lo and MacKinlay (1990a) for the US market and Levis (1985) for the UK market, have found that portfolios which contain stocks with low capitalisation values have an autocorrelation coefficient substantially greater then the autocorrelation coefficient of portfolios which contain stocks with high capitalisation values. For example, Lo and MacKinlay (1990a) found that the first order autocorrelation statistic for a portfolio containing the largest and smallest firms in their sample was 4% and 33% respectively, indicative of pronounced differences between the return autocorrelations of large and small firms.

A cause of serial correlation in indexes and portfolios that are heavily weighted by the stocks of small capitalisation firms was identified by Fisher (1966). He observed that small firms tended to trade less frequently than large firms. As a result, their prices would not adjust immediately to new information. So, for example, the return today of stock A may be reflecting an adjustment to information that was fully reflected in the stock of B yesterday. Thus, the return of stock A today will be correlated with the return of B yesterday, and so stocks A and B are cross serially correlated. Consider a portfolio which contains both stocks A and B. An equally-weighted portfolio return is the average of the return of the two stocks. Because stocks A and B are cross serially correlated, the return of an equally-weighted portfolio today (which depends on the return of A) will be correlated with the return of the portfolio yesterday (which
depends on the return of B). Thus, cross serial correlation among stocks can generate autocorrelation in a portfolio. So, in portfolios or indexes, which are heavily weighted by stocks with non-synchronous price adjustments, we would anticipate positive return autocorrelation.

Despite the intuitive appeal of this argument there has been extensive academic debate recently concerning the cause, or causes, of portfolio serial correlation. Atchison, Butler and Simonds (1987) appeared to find evidence to suggest that non-synchronous trading could not account for the high autocorrelation in portfolio returns. They derive an estimate of the implied theoretical portfolio autocorrelation for portfolios of different sizes from the Scholes and Williams model of non-synchronous trading. Their results indicate that the implied autocorrelations are much lower than observed levels. They conclude therefore that high index or portfolio autocorrelation is not due to thin trading. Lo and MacKinlay (1990b) support this argument, the authors constructed a model of non-synchronous trading (this model was discussed in Section 2.7.2.1 of Chapter 2) which was tested empirically with almost 30 years of portfolio return data for the USA. They concluded that non-synchronous trading could account for only a small fraction of the observed levels of portfolio serial correlation. However, a recent paper by Boudoukh, Richardson and Whitelaw (1994) has shown that by relaxing the assumption in the Lo and MacKinlay model that the average probability of thin trading is equal across all securities within a portfolio, an assumption which is unlikely unless it is imposed when portfolios are formed, then the amount of autocorrelation introduced by thin trading becomes close to observed levels.

Because thin trading was initially dismissed as a plausible explanation of portfolio return autocorrelation, a number of other authors have focused on the importance of bid-ask bounce (the importance of this was explained in Section 2.7.1 of Chapter 2) and time variation of expected returns as a possible cause of portfolio serial correlation. Outstanding contributions in this field have been made by Conrad and Kaul (1988) and Conrad, Kaul and Nimalendran (1991).

Conrad and Kaul (1988) find evidence of substantial time variation in the returns of ten portfolios which have been formed on the basis of firm size. They show that weekly expected returns are well represented by a first order autoregressive process as follows,

\[ E_{t-1}(R_t) = \phi E_{t-2}(R_{t-1}) + u_{t-1} \]  \hspace{1cm} (3.2)

where, \( E_{t-1}(R_t) \) is the expectation at time \( t-1 \) of the next periods weekly return, and \( \phi \) is the autoregressive coefficient and \( u_{t-1} \) is a random error. If expected returns follow an AR(1) process as follows then if \( \phi \) is large enough autocorrelation in realised returns will be implied. It is now possible to write realised returns in terms of the expected return and the error as follows,

\[ R_t = E_{t-1}(R_t) + e_t \]  \hspace{1cm} (3.3)
where, $R_t$ is the realised return in time $t$, and $e_t$ is a random error.

Using a Kalman filter, Conrad and Kaul (1988) test whether weekly expected returns for the ten portfolios are constant or time varying. This requires equation (3.3) being estimated as,

$$R_t = \alpha_t Con + e_t$$

$$\alpha_t = \phi \alpha_{t-1} + u_t$$

where, $Con$ has a value of 1 for all $t$, $\alpha_t$ is the expected return at time $t$ which can be time varying. Both $e_t$ and $u_t$ are white noise errors. The coefficient $\phi$ captures the autoregressive component in expected returns. In this test $\alpha_t$ captures the expected return, if $\alpha_t$ is a constant then expected returns are constant, but if $\alpha_t$ is an AR(1) process then expected returns are time varying. The $R^2$ of equation (3.4) indicates how much of the variation in returns can be explained by the variation in $\alpha_t$ over time and, therefore the proportion of the variation in realised returns which is caused by the time variation of expected returns.

The results reported by Conrad and Kaul indicate that weekly expected returns are time varying, and can explain a substantial proportion of the variation in weekly realised portfolio returns. Meanwhile, the magnitude of the estimated time variation appeared to be monotonically associated with firm size. As we move from the portfolio containing the smallest firms to the portfolio containing the largest firms the magnitude of the estimated time variation declines. To illustrate, the $R^2$ of equation (3.4) for the portfolio which contains the smallest firms is 26% but for the portfolio containing the largest firms in the sample the $R^2$ of equation (3.4) is only 1%, suggesting that very little of the variation in portfolio realised returns can be explained by time varying expected returns of the large firms.

The failure by Conrad and Kaul (1988) to find any evidence of time variation in the returns of large firms appears to be a weakness of the study. Firstly, if the expected returns of the large firms are not time varying then they will not be well represented by an AR(1) model. Their results for the large firms may therefore be due to misspecification. Furthermore, their results conflict with Bollerslev, Engle and Wooldridge (1988) and Harvey (1989) who certainly do find evidence of time variation in the expected returns of large firms. These studies will be discussed in detail in Chapter 7 and 8. Finally, the approach used by Conrad and Kaul assumes market efficiency so it is not possible to rule out that at least some of the predictability found for small firms is caused by market inefficiency.
Research on the autocorrelation patterns of individual security and portfolio returns had discovered a striking pattern, although, portfolio returns appeared to be autocorrelated this did not appear to be reflected in the autocorrelations of individual securities, which seemed puzzling until Lo and MacKinlay (1988) developed a model to reconcile these findings.

The Lo and MacKinlay model assumes that stock returns are made up of three components, a random component, a bid-ask spread and a component which captures time variation. The three components are summarised in the following equation,

\[ R_t = E_t + B_t + U_t \]

where \( R_t \) is the realised security return, \( E_t \) is the positively autocorrelated time varying expected return, \( B_t \) is the negatively autocorrelated bid-ask bounce, and \( U_t \) is a random component.

As shown by the work of Roll (1984), which was discussed in Chapter 2, the existence of a bid-ask spread can introduce negative autocorrelation into security returns when a stock price moves from the bid (ask) to the ask (bid). In a portfolio, some securities will be moving from the bid to the ask, while others will be moving from the ask to the bid, so the effects of bid-ask bounce are diversified away. This means that although, individual securities may contain negatively correlated components because of bid-ask bounce, portfolio returns will not be negatively correlated.

Meanwhile, time variation of expected returns will introduce positive autocorrelation into security returns. Since all securities are characterised to some extent by positive autocorrelation, introduced because expected returns are time varying, there is no diversification effect and so portfolios will be autocorrelated. When the effects of bid-ask bounce and time variation are combined, at a security level the negative autocorrelation of the bid-ask bounce is offset by the positive autocorrelation caused by time variation. Consequently, individual securities will display little autocorrelation. In the case of portfolios, which will not be characterised by a bid-ask effect, and therefore have no offsetting negative autocorrelation, portfolio returns will be positively autocorrelated.

Using a similar test procedure to the one developed by Conrad and Kaul (1988) Conrad, Kaul and Nimalendran (1991) assess the extent to which variations in the realised returns of individual securities can be explained by the components identified by Lo and MacKinlay. Conrad et al find that time varying expected returns and bid-ask errors are found to explain substantial proportions of the variation in weekly realised security returns. Again, the magnitude of the variation in weekly returns which can be explained by time variation is associated with firm size. For the smallest firms in the sample time variation can explain 19% of the variation in weekly realised returns but only 5.8% of the variation in weekly realised returns for the largest firms.
When comparing that study to Conrad and Kaul (1988) no attempt is made to explain why time variation appears to be less important in that study for small firms but more important for large firms, although, this might be a consequence of how securities are grouped. In that paper Conrad et al study only three size groupings not ten so the results may reflect the degree of aggregation.

Meanwhile, for both large and small firms, bid-ask bounce appears to explain approximately 5% of the variation in weekly returns. Again, this seems somewhat strange because the bid-ask spread tends to be related to firm size, see for example Stoll and Whaley (1983), so perhaps we might expect the bid-ask component to explain rather more of the variation in the returns of small firms. Finally, Conrad et al (1991) agree with Lo and MacKinlay (1988) that at a security level the bid-ask spread will offset any time variation. However, it is not clear why the security returns of small firms are not moderately positively autocorrelated because clearly the time variation component explains much more of the variation in small firm security returns than the bid-ask component. Furthermore, there is certainly no evidence to suggest that the returns of individual securities are positively autocorrelated, actually Lo and MacKinlay (1988) find that the security returns of small firms appear to be negatively not positively autocorrelated.

Despite their flaws, the papers by Conrad and Kaul (1988) and Conrad, Kaul and Nimalendran (1991a) have been fundamentally important because they have been instrumental in turning attention towards microstructure considerations and time variation in expected returns as a potential cause of observed portfolio autocorrelation. Moreover, the work of Conrad and Kaul has demonstrated that if stock returns are to be modelled correctly then these factors must be taken into account where necessary.

3.3 Market Volatility

Until relatively recently, it was argued that although standard statistical tests, such as autocorrelation tests which were discussed in the previous section were generally unable to reject the null hypothesis of market efficiency this could not be taken as evidence in favour of the hypothesis. This principle applies to all scientific theories. Experiments can falsify a theory by contradicting one of its implications, but the verification of one of its predictions cannot be taken to prove or establish the theory.

In response to this, new and more powerful ways of testing stock market predictability were introduced by Shiller (1981): variance bounds tests; DeBondt and Thaler (1985): tests of long-run overreaction; Poterba and Summers (1988), Fama and French (1988); tests of mean-reversion. Although, these tests were developed independently of each other they can be
loosely grouped together because they are all concerned with testing whether stock prices over-react to new information. A useful summary of the overreaction literature is provided by Forbes (1994).

3.3.1 Variance Bounds Tests

The first of these new type of tests, which sought to test stock market predictability were the so called variance bounds tests. The variance bounds test is based on intrinsic value models of stock returns such as the dividend valuation model, which was discussed in Section 2.5 of Chapter 2. As we saw this model links the intrinsic value of a stock to the future value of expected discounted dividends. Variance bounds tests fall into two groups, the original tests known as first generation tests Shiller (1979,1981), Shiller and Grossman (1981), LeRoy and Porter (1981) and the modified second generation tests, such as West (1985) and Mankiw, Romer and Shapiro (1985) which were developed primarily as a response to criticisms of the early models which were made by Flavin (1983) Kleidon (1986) and Merton and Marsh (1986,1987).

The dividend valuation model implies that the fundamental value of a stock is related to the future value of discounted dividends as follows.

\[ p_t^* = \sum_{j=1}^{\infty} \frac{D_{t+j}}{(1+r)^j} \]  

(3.6)

where \( p_t^* \) is the fundamental or intrinsic value of a stock at time \( t \), \( r \) is the required discount rate which makes future cash flows comparable to cash flows which accrue in time \( t \), and \( D_t \) are the dividends which accrue in time \( t \).

Future dividends, are random variables so their expected values are not observable. Although, as dividends are observed, new information is incorporated into investors’ expectations about future dividends. The current market price \( p_t \) is therefore a forecast of the expected discounted value of the future dividend stream as follows.

\[ p_t = E_t(p_t^* | \phi_t) = \sum_{j=1}^{\infty} E_t(D_{t+j} | \phi_t) \frac{1}{(1+r)^j} \]  

(3.7)

where \( \phi_t \) is the information set currently available. The current market price may deviate from this intrinsic price by an error. Since in an efficient market stock prices must be an optimal forecast of future fundamentals, \( p_t \) should be the best guess of \( p_t^* \) given the information set available \( \phi_t \).
Unless investors have perfect foresight, the markets guess of this intrinsic stream will differ from the actual out-turn. This difference will represent the forecast error $u_t$ such that $u_t = p_t^* - p_t$. In efficient markets the market share price must be an optimal forecast of $p_t^*$ which requires that $u_t$ must be uncorrelated with the forecast from equation (3.7).\footnote{Correlation of $u_t$ with $p_t$ implies that information in $u_t$ can be used to predict future values of $p_t$ in which case $p_t$ can not be the best forecast of $p_t^*$.}

Since stock market investors do not have perfect foresight the actual dividend series can be expressed as the forecast plus the forecast error as follows,

$$p_t^* = p_t + u_t \tag{3.8}$$

Similarly, taking the variance of each of the terms means that equation (3.8) can be written as follows,

$$\text{var}(p_t^*) = \text{var}(p_t) + \text{var}(u_t) \tag{3.9}$$

This says that the variance of $p_t^*$ will be equal to the sum of the variance of the forecast plus the variance of the forecast error $u_t$. Since a variance can not be negative the variance of the series $p_t^*$ can not be greater than the variance of the forecast $p_t$,

$$\text{var}(p_t^*) \leq \text{var}(p_t) \tag{3.10}$$

If these variance bounds equalities are not met it implies that actual stock prices are more volatile than the underlying fundamental series which they are supposed to be linked to. That is stock prices over-react to new information.

### 3.3.1.1 Empirical Evidence

A casual observation of the behaviour of major stock markets over quite short horizons reveals that stock prices appear to be very volatile, for example during October 1987 stock prices fell by approximately 20% in one day. Such pronounced periods of volatility lead Shiller (1981) to comment on the volatility of the Standard and Poor index\footnote{The Standard and Poor index is an equally-weighted index which is made up of approximately 500 companies.} as follows.


Shiller (1981a) and LeRoy and Porter (1981) performed the variance bounds tests which were outlined in the previous section on both the Dow Jones and the Standard and Poor index. Shiller, found that the Standard and Poor was 6 times more volatile than $p_t^*$ while the Dow Jones...
index appeared to be 13 times more volatile than the series $p_t^*$ (the construction of the series $p_t^*$ is described in Appendix 1). Similar tests have been performed on the UK market by Bulkley and Tonks (1991) which have indicated that UK stock prices are also excessively volatile.

Initially, violations of the variance bound inequalities were interpreted as striking evidence of market inefficiency. Although now, as we shall see, these tests are viewed at best as being very controversial, and it is not clear that they tell us anything about market efficiency.

A recurrent criticism of the first tests was that the dividend and price series used in tests were both assumed to be stationary, in which case the variance estimates of both $p_t$ and $p_t^*$ would have been consistent. Using Monte Carlo simulation, Kleidon (1986) found that even if prices reflect future discounted dividends, but if the price and dividend series are not stationary then the variance bounds relationship can be violated.

Flavin (1983) is also concerned with the statistical methodology used in the variance bounds tests. In large samples the estimated variances approach the actual variances, but if a small sample is used to calculate the mean of the time series, and the series is autocorrelated then the estimated variance of the series will be downwards biased. Flavin argues that the typical 100 observations used in the volatility tests was insufficient to provide unbiased results, which implies the tests may be biased in favour of finding excess volatility.

Conceptual problems with the nature of the variance bounds tests have been discussed by Kleidon (1986) and Merton and Marsh (1986,1987). Both papers demonstrate that the violation of the variance bounds may not imply excess volatility. Merton and Marsh, (1986,1987), for example, summarise this by arguing that the variance bounds tests are really tests of different dividend process specifications, particularly stationary versus non-stationary alternatives under the assumption of a constant discount rate. It has been known since the early work on dividends by Lintner (1956) that managers set dividends in relation to permanent rather than realised earnings. This means that if dividends are smoothed in this way the dividend series will be autocorrelated. This implies that even small changes in the current dividend can cause considerable changes in the total value of dividends payable, and therefore large changes in the current share price. However, the perfect foresight price $p_t^*$ is calculated from the ex-post dividend series, in which case there is no uncertainty about whether dividend changes were long or short lived.

9. The price series $p_t$ is autocorrelated because it is the weighted sum of future dividends which is itself a series which is autocorrelated. $p_t^*$ is autocorrelated because it is a weighted average of both the weighted sum of dividends and the a weighted sum of future prices. $p_t^*$ is therefore more autocorrelated than $p_t$ and will correspondingly be more downwards biased.
To illustrate this criticism, Shiller argues that a decline in dividends at the time of the 1929 crash were short lived and could therefore not account for the large fall in share prices which took place at this time. This assumes that investors knew that dividends declines would not persist. Since investors in 1929 did not have this foresight violent changes in prices experienced at that time can not be assumed to be excessively volatile.

Tonks and Bulkley (1989) model the persistence of changes in the dividend series by respecifying the way in which \( p_t^{*} \) is calculated. Allowing each observation of \( p_t^{*} \) to reflect even small dividend changes. Although, the results of Bulkley and Tonks provide confirmation of excessive volatility the extent of the breach is far less pronounced than the one reported by Shiller. Although, it should also be noted that Bulkley and Tonks tested a different market so their results may reflect the fact that the UK market is less volatile anyway.

### 3.3.1.2 Second Generation Tests

Mankiw, Romer and Shapiro (1985) developed an alternative test which is both unbiased in small samples and overcomes the problem of non-stationarity. The Mankiw et al test assumes that a naive forecast of the fundamental stream \( p_t^{*} \) exists and is naive because it only utilises past dividend information. Let us call this \( p_t^{0} \). The difference between the perfect foresight price, \( p_t^{*} \) and the naive forecast \( p_t^{0} \) must be equal to

\[
p_t^{*} - p_t^{0} = (p_t^{*} - p_t) + (p_t - p_t^{0})
\]

Squaring both sides then taking expectations provides

\[
E_t(p_t^{*} - p_t^{0})^2 = E_t(p_t^{*} - p_t)^2 + E_t(p_t - p_t^{0})^2
\]

which implies the following inequalities, centred around the naive forecast rather than the mean.

\[
E_t(p_t^{*} - p_t^{0})^2 > E_t(p_t^{*} - p_t)^2
\]

\[
E_t(p_t^{*} - p_t^{0})^2 > E_t(p_t - p_t^{0})^2
\]

The first inequality requires that the forecast error of the naive forecast must be larger than the error of actual market prices. The second inequality says that the variation in the fundamental price around a naive forecast must be greater than the variation in actual prices around a naive forecast. Neither of these conditions depend on the dividend process being stationary. A later paper by Mankiw, Romer and Shapiro (1989) defines the distributional properties of this test and finds that the variance bounds which are set out in equation (3.13) and (3.14) are only marginally violated.

The variance bounds tests are a joint hypothesis of market efficiency and the hypothesis that fundamental stock prices are determined by the constant discount dividend valuation model. Time variation in discount rates may cause the variance bounds conditions to be rejected even
if markets are efficient. This follows because changes in the discount rate will cause the
discounted value of expected dividends to fluctuate even when the raw dividend series itself is
constant. This means that if discount rates are assumed to be constant when they are really time
varying then prices will fluctuate in response to changes in the value of discounted dividends
but this will not be reflected in the series \( p_t \). Mankiw et al (1989) test a time varying discount
rate model and find that allowing for time varying discount rates increases the volatility of the
ex-post rational price series substantially.

Given the wealth of evidence which now exists to suggest that discount rates are time
varying, which will be discussed in Chapter 7 and 8 it is not clear how results from the variance
bounds tests should be interpreted. Certainly, it can no longer be claimed that the violation of
the variance bounds relationship provides evidence of market inefficiency. Violation of the
variance bounds relationship can only suggest that stock prices are more volatile than would be
suggested by an efficient market where discount rates are assumed to be constant, which as
Chapter 7 and 8 will indicate, is an unrealistic assumption.

3.3.2 The Overreaction Hypothesis

Although the variance bounds tests which were discussed in the previous section indicated
that perhaps, stock prices overreacted to new information, these tests did reveal whether it was
possible to make money from the overreaction. It was not until the work of DeBondt and Thaler
(1985,1987) that it was demonstrated that a trading rule could successfully exploit the
overreaction of share prices. Their empirical work has subsequently spawned a branch of the
market efficiency literature concerned with testing the so called "overreaction hypothesis"\textsuperscript{10}.

Essentially, the overreaction hypothesis demonstrates that securities which are extreme
losers in one period, that is those which are characterised by exceptionally bad performance,
become extreme winners in subsequent periods and vice versa. The overreaction literature, has
shown that a contrarian strategy, where winners are sold short, to finance the purchase of losers
typically provides substantial abnormal profits. DeBondt and Thaler (1989) argued that the
power of overreaction tests is stronger than most other weak form tests of market efficiency.
This stems from the fact that overreaction tests concentrate on examining periods when some
abnormal behaviour is most likely to take place, calm periods, where little news is released, are
generally ignored.

An overreaction effect is usually tested for in the following way. Initially the performance
of securities is measured over a period, usually no more than a couple of years. On the basis of
this performance stocks are then ranked. Securities are then grouped according to whether they

\textsuperscript{10}There is also evidence of a short-run overreaction effect, see for example Lehmann (1990).
are winners or losers, from these groups portfolios are created which consist of the extreme winners and losers.\footnote{Studies concentrate on extreme performances in the formation period because these securities will be the ones investors are most likely to over-react to.} In subsequent periods, the portfolio returns of the winners and losers are then compared. DeBondt and Thaler find that over a five year period their loser companies outperform their winners by almost 32% suggesting that it is possible to make money from the overreaction.

Transaction costs can not reduce the high performance of the loser portfolio because they will be very small for a contrarian strategy such as the one outlined by DeBondt and Thaler. Their results indicate that the difference between the performance of the loser and the winner portfolios peaks after five years. This suggests that profits from the contrarian strategy are maximised if the loser portfolio is purchased and held for about five years, a strategy which would incur only very small transaction costs. DeBondt and Thaler argue that differences in risk can not account for the relative performance of the two portfolios since the average beta of the winner is slightly higher than the average beta for the loser portfolio. However, their results are based on the testing of a joint hypothesis because the Capital Asset Pricing Model is assumed to be the equilibrium model of returns.

The evidence presented by DeBondt and Thaler has not gone unchallenged. One criticism concerns the role of the January effect\footnote{The January effect describes the calendar regularity which finds that stock prices appear to be persistently higher in January than in other months of the year. The January effect will be discussed in detail in section 4 of this chapter.} in explaining the overreaction results. DeBondt and Thaler found that up to 84\% of the discrepancy between winners and losers comes from the behaviour of returns in January, furthermore, return reversals continue to be important up to five Januaries after the portfolio formation data. Because, of the strong performance of losers in January it is clearly important to rule out that the overreaction effect is no more than an alternative way of capturing the January effect. Despite this close association, it is not obvious that a strong overreaction effect in January is merely further evidence of the January effect or the small firm anomaly. January is the month in which the most corporate earnings information is announced because firms tend to have tax years which coincide with the calendar year. If investors are going to revise expectations about the performance of a company then these revisions are most likely to take place in January because there is an abundance of new information during this month. An overreaction in January is therefore very likely for reasons which may or may not be associated with tax loss trading.

Other critics such as Zarowin (1989,1990,1991) believe that the overreaction effect is no more than another manifestation of the small firm effect. Since loser firms tend to be smaller than the winners, if small firms outperform large firms then losers will outperform winners even if there is no overreaction taking place. Although, DeBondt and Thaler (1989) accept that losers
are smaller than winners they dispute that there is not also an independent loser effect. When they form portfolios which are ranked on the basis of size and compare the return performance of these with the portfolios which were created on the basis of performance they find that the portfolio containing the smallest firms never outperforms the portfolio containing the loser firms, which indicates that even when loser companies are not small, losers outperform winners.

This conclusion is contradicted by Zarowin (1989) who matches firms which have similar capitalisations but disparate earnings profiles, and in a second exercise matches firms which have similar earnings profiles irrespective of capitalisation value. To support stock market overreaction Zarowin argues that we would expect significant return differences between the portfolios matched by size but insignificant return differences between the portfolio matched by earnings. Verification of the importance of firm size is found to exist because when poor earners are matched with good arrears of similar capitalisation values there is little evidence of differential performance.

Zarowin (1990) continues to provide evidence of the link between firm size and overreaction. In this third paper portfolios are again matched by market capitalisation, but this time two sub-periods are examined. The first is a sub-period when winners are larger than losers. In the second sub-period winners are smaller than losers. It is found that when losers are smaller than winners, in the portfolio formation period, they subsequently outperform the winner portfolio. However, if losers are bigger than winners in the portfolio formation period then they do not outperform winners in subsequent months. This is a serious criticism of the overreaction papers because it implies that overreaction, rather than being a new discovery is capturing the same empirical patterns which had been discovered previously in tests of the small firm effect such as those of Banz (1981) and of those which investigate tax loss trading, for example, Roll (1981) and Ritter (1988) who found that losers over the previous year became winners in January.

Vermaelen and Vestringe (1986) test for overreaction on the Belgian stock market, and although overreaction is supported, they open up a new line of inquiry by suggesting that the superior performance of the loser portfolio can be accounted for by changes in the riskyness of the loser portfolio which takes place between the pre and post ranking period. The original work undertaken by DeBondt and Thaler (1985) assumed that the beta of a security was the same in the period prior and subsequent to the ranking procedure, despite extensive volatility in the capitalisation value, gearing and security price which would be experienced between these two periods. For example, DeBondt and Thaler find that extreme losers fall in value by 45%, while winners rise in value by 365%. Such large changes in the capitalisation value of companies is likely to have a pronounced influence over the size of security betas in the post ranking period. However, DeBondt and Thaler (1989) are unconvinced by the importance of time varying risk.
They find that in the post ranking period the betas of losers are only 0.22 higher than the beta of winners which is not enough to explain the difference in the returns of winner and loser portfolios.

DeBondt and Thaler (1987) attempt to cast further doubt on the importance of time variation because the betas of the loser portfolio appear to be smaller than the betas of winners when markets are falling but larger than the betas of winners when markets are rising. This seems to suggest that time variation can not explain divergences in the performance of winner and loser portfolios because loser portfolios should be more risky in down markets if time variation is to explain their better performance in subsequent periods.

Chan (1988) also investigates the role of time varying betas and provides further support for the winner loser effect being an artifact of beta non-stationarity. Chan finds that at the beginning of the ranking period winners have larger betas than losers, but losers become riskier than winners after portfolio formation. The difference between the beta of losers and winners in this later period was found to be 0.453 and therefore substantial, but still not large enough to explain fully the overreaction effect.

Ball and Kothari (1989) also provide support for the notion that inappropriate calculations of systematic risk fail to capture changes in equilibrium required returns. They show that when betas are estimated using annual returns the betas of the extreme losers are higher than the betas of the winners by 0.76, with a market risk premium of 14-15% this implies that only 3% of the overreaction effect is left unexplained. However, as we know from discussions of the intervaling effect, undertaken in Chapter 2, the return interval which is used to calculate betas influences the magnitude of beta itself. But, we have no guidance as to whether a short or a long interval is preferred, therefore, we do not know whether it is DeBondt and Thaler or Ball and Kothari that use the appropriate return interval.

Conrad and Kaul (1993) suggest that papers such as those of DeBondt and Thaler which have tested for an overreaction effect, over a long period of time have possibly provided biased estimates regarding the extent to which markets overreact. They argue that short horizon returns are upwardly biased (see for example, Blume and Stambaugh (1983) or Roll (1984)). Conrad and Kaul (1993) show that measurement errors in observed prices due to bid-ask errors and non-synchronous trading, introduce spurious returns in long horizon investment strategies when long term returns are calculated as the cumulation of a series of short-term returns. If this is done, the cumulated long-term return also accumulates the measurement error, which becomes substantial over long horizons. Conrad and Kaul demonstrate that over long horizons the performance of previous losers is overstated which gives rise to the overreaction effect.

13. This bias is a problem in any study which cumulates abnormal returns and therefore applies to event study work also.
Recently, Stroyny and DeBondt (1994) have considered the role that heteroscedastic disturbances might have on the estimation of betas. They argue that fan shaped heteroscedasticity biases estimates of beta when betas and the sample mean are simultaneously determined. This happens because if a disturbance is large and positive (negative), it results in too strong an upward (downward) influence on both the estimated slope of the regression (the beta) and the sample mean. As a result positive errors in the mean occur along with positive (negative) errors in the slope (beta). Furthermore, Stroyny and DeBondt argue that some of the beta changes which are observed between pre and post ranking periods are caused by rank period estimates being biased, using simulation techniques they find that Chan (1988) overstates differences in the risk premium between the pre and post ranking period by up to 40%.

3.3.3 Mean Reversion

The concluding discussion of market volatility will be concerned with reviewing the mean-reversion literature, which was introduced by Summers (1986). The contribution of the mean-reversion literature is two fold. Firstly, it documents that long-horizon stock returns are negatively autocorrelated. For example, Fama and French (1988) undertake long-horizon autocorrelation tests and find that up to 45% of the variation in five year returns can be explained by the previous five year return. However, the mean-reversion literature is also important because it formally explains why transitory components can introduce negative autocorrelation into long-horizon returns, and why this negative autocorrelation can explain such a large proportion of the variation in long-horizon returns, even though, in short-horizon returns predictable components go largely undetected.

Mean-reversion requires that stock prices are driven by both random and stationary components. The stationary component introduces negative autocorrelation, but because it is slowly decaying, traditional short-horizon autocorrelation tests are dominated by the effect of the random component. In which case the predictability introduced by the mean reverting component goes undetected. In long-horizon tests, the effect of the stationary component, because it is slowly decaying, begins to swamp the random component and can therefore be detected.

Summers (1986) demonstrates that typical tests of market predictability, such as autocorrelation tests have low power in detecting departures from randomness, when stock prices contain both a transitory and a random component. Summers is able to show that even when observed prices deviate from fundamentals by as much as 30%, tests of weak form market

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14. A stock price can be above or below its fundamental value. If a stock price is mean reverting then it is not at its fundamental value but is reverting towards its fundamental value.

15. As a rule of thumb long-horizon returns refer to return horizons of approximately one year or more.
efficiency can still fail to reject the null that stock prices are random 50% of the time. This was obviously an important finding because it drew attention to the possibility that previous tests, such as those which were discussed in Section 3.2, had failed to find evidence of market predictability not because markets were unpredictable but because the tests previously used had very lower power.

Fama and French (1988) provide a comprehensive proof of why it is that traditional autocorrelation tests have such a low power. They consider a model where the logarithm of stock prices $p_t$ contains both a random walk $q_t$ and a stationary component $z_t$ as follows,

$$p_t = q_t + z_t$$  \hspace{1cm} (3.15)

$$q_t = q_{t-1} + u + n_t$$  \hspace{1cm} (3.16)

where $u$ is a drift term, and $n_t$ is a white noise process.

The continuously compounded return from $t$ to $T$ is

$$r_{t+T} = p_{t+T} - p_t$$  \hspace{1cm} (3.17)

which can be written as the sum of the change in the random component, and the change in the stationary component as follows,

$$r_{t+T} = (q_{t+T} - q_t) + (z_{t+T} - z_t)$$  \hspace{1cm} (3.18)

The $T$ period autocorrelation coefficient for changes in $z_t$ can be written as follows,

$$\rho_T = \frac{\sigma(z_{t+T} - z_t, z_t - z_{t-r})}{\sigma^2(z_{t+T} - z_t)}$$  \hspace{1cm} (3.19)

As Appendix 3 shows, stationarity of $z_t$ implies that the numerator of equation (3.19) will approach $-\sigma^2(z)$ as $T$ increases. Meanwhile, as Appendix 4 demonstrates, stationarity of $z_t$ implies that as $T$ increases the denominator of equation (3.19) approaches $2\sigma^2(z)$.

For large $T$ this implies that the slope of the regression of $z_{t+T} - z_t$ on $z_t - z_{t-r}$ approaches $-\sigma^2(z)/2\sigma^2(z)$ that is -0.5 for large $T$. Within this framework it is now possible to see why it is difficult to capture autocorrelation of returns in conventional tests, when returns contain a slowly decaying stationary component.

If $z_t$ is an AR(1) process then, as Appendix 5 shows, the $T$ period ahead forecast of $z_t$ is

$$E_t(z_{t+T}) = \phi^T z_t$$  \hspace{1cm} (3.20)

If $z_t$ is an AR(1) process the expected change from $t$ to $T$ as Appendix 5 also demonstrates, is equal to
\[ E_t(z_{t+T} - z_t) = \phi^{T-1} z_t. \]  
(3.21)

This means that the slope of the regression between \( E_t(z_{t+T} - z_t, z_t - z_{t-T}) \) is equal to minus the ratio of the variance of the expected change in \( z_t \) to the variance of the actual change, as shown in equation (3.22), a derivation of which is provided in Appendix 6.

\[ \beta_z = -\frac{\sigma^2[E_t(z_{t+T} - z_t)]}{\sigma^2(z)} \]  
(3.22)

where \( \beta_z \) is the slope of a regression between \( E_t(z_{t+T} - z_t, z_t - z_{t-T}) \).

\(-\sigma^2[E_t(z_{t+T} - z_t)]\) is the variance of the expected change in \( z_t \), and \( \sigma^2(z) \) is the variance of the actual change in \( z_t \).

This means that when \( \phi \) is close to 1.0, the expected change in an AR(1) process slowly approaches \(-z_t\) as \( T \) increases, and the process is said to be stationary but slowly decaying. For short horizons therefore, the slope of \( \beta_z \) is close to 0.0 but slowly approaches -0.5 as the return interval increases. Since it is not possible to observe \( z_t \), it is necessary to infer its properties from the behaviour of returns. If \( \beta_r \) is the slope of the regression of the return \( r_{t+T} \) on \( r_t \), if changes in the random and stationary components are uncorrelated then the slope of the regression of \( r_{t+T} \) on \( r_t \) as Appendix 7 demonstrates is

\[ \beta_r = -\frac{\sigma^2[E_t(z_{t+T} - z_t)]}{\sigma^2(r_t)} \]  
(3.23)

\( \beta_r \) therefore measures the proportion of the variance of \( T \) period returns explained by the mean-reversion of a slowly decaying stationary component \( z_t \). If stock prices do not have a stationary component, the slopes \( \beta_r \) are 0.0 for all \( T \). If stock prices do not have a random component but do have the slowly decaying stationary component \( z_t \), \( \beta_r = \rho_T \) and the slopes approach -0.5 for large values of \( T \).

If stock prices contain both a random and a stationary component, then mean-reversion of the stationary component tends to push the slopes \( \beta_r \) to -0.5 for long-horizon returns, while the variance of the white noise component pushes the slope of \( \beta_r \) towards zero. Since the variance of \( z_{t+T} - z_t \) approaches \( 2\sigma^2(z) \) as the return horizon increases and the white noise variance grows with \( T \), the white noise component eventually dominates when \( T \) is very large. Thus, if stock prices have both random and slowly decaying stationary components, the slopes of the regression of \( r_{t+T} \) on \( r_t \) might be U shaped starting about 0.0 for short horizons becoming more negative as \( T \) increases then moving back towards 0.0 as the white noise variance begins to dominate over very long horizons.
Empirical support for long-horizon mean-reversion was provided by Fama and French (1988) who estimate long-horizon return regressions for return horizons of 1 through to 10 years. These regressions are estimated for both the value-weighted and equal-weighted CRSP indexes 16, for portfolios of stocks formed according to capitalisation value and industry classification as well as for individual stocks. Fama and French find evidence of a U shaped pattern in the $\beta_T$ estimates consistent with the Summers (1986) model of mean-reversion. Their estimates suggest a mean reverting component in stock prices which can account for up to 30% of the variation in three year returns, and up to 45% of the time variation in five year returns.

As the small sample properties of $\beta_T$ are not known, Fama and French use Monte Carlo simulation to determine the statistical significance of their results. Their simulation results suggest that for the three and five year returns estimates of $\beta_T$ are more than two standard deviations from zero and therefore statistically significant.

Interestingly, Fama and French also discover that the magnitude and persistence of the mean reverting component is associated with firm size. Fama and French report that for the small firms they investigate the effect of the stationary component takes five years to maximise the effect it has on the predictability of realised returns. But, for the largest firms the stationary component reaches its peak after only three years. Meanwhile, in the case of small firms the mean reverting component is able to explain a greater proportion of the variation in realised returns than is the case for the large firms. Although Fama and French document this empirical finding, they do not suggest what might be causing the difference in the behaviour of the large and small firms.

From a comparison of the patterns for individual stocks and portfolio returns, Fama and French conclude that the mean reverting component is likely to be due to a common factor experienced by all stocks rather than being a firm specific component of returns, which they argue is consistent with a time varying expected return that is autocorrelated. This argument is consistent with predictable components in long horizon returns because a change in expected returns today must cause a change in the rate at which future dividends must be discounted. In which case, current dividend changes, must cause an offsetting price movement 17. If expected returns are time varying, and autocorrelated, then a change in expected returns today will predict an offsetting change in prices in the future (when required returns change). Share prices will therefore contain predictable components which will be understated in a constant discount rate model.

16. The CRSP (Centre for Research in Security Prices) is compiled by Chicago Business School, the index consists of all stocks listed on the New York Stock Exchange (NYSE) and the American Stock Exchange (AMEX).

17. If required returns rise then the share price must fall and vice versa.
3.3.3.1 Variance Ratios

An alternative test which is capable of detecting autocorrelation patterns in either long or short-horizon returns is the variance ratio test, which was derived by Cochrane (1988) and applied to financial data by Poterba and Summers (1988) and Lo and MacKinlay (1988).

The simplest variance ratio test is a test of the null hypothesis that prices follow a random walk. If stock prices follow a random walk with drift then,

$$\ln(p_t + D_t) = \ln(p_{t-1} + u + e_t)$$

$$e_t \sim IID(0, \sigma^2)$$

where $p_t$ is the share price in time $t$, $D_t$ are the dividends paid in time $t$, $u$ is the drift, and $e_t$ is a white noise error.

In this case returns, $r_t$, are

$$r_t = \ln(p_t + D_t) - \ln(p_{t-1})$$

Because innovations in a random walk are independent, the variance of a random walk process grows linearly with the differencing interval. This implies that if log prices are a random walk then the variance of $[\ln(p_t) - \ln(p_{t-1})]$ grows linearly with $T$ and $\sigma^2[\ln(p_t) - \ln(p_{t-1})] = T\sigma^2$. However, if log prices are not a random walk but are instead a stationary process then as shown in the previous section, the variance of $[\ln(p_t) - \ln(p_{t-1})]$ approaches a constant, which is twice the variance of the series $\sigma^2[\ln(p_t) - \ln(p_{t-1})] = 2\sigma^2$.

These properties are utilised by the variance ratio statistic, $VR(T)$, which can be written as follows,

$$VR(T) = \frac{\sigma^2(R_T^T)/T}{\sigma^2(R_{12}^T)/12}$$

where $\sigma^2(R_T^T)/T$ is the average variance of the $T$ period return, and $\sigma^2(R_{12}^T)/12$ is the average variance of twelve month returns.

If stock prices are random then the variance of returns will be proportional to the return horizon for all $T$. In this case equation (3.26) will have a variance ratio equal to 1 so that

$$\sigma^2(R_T^T)/T = \sigma^2(R_{12}^T)/12$$

But, if any of the price changes over $T$ periods are due to the influence of a negatively autocorrelated transitory component, then as Section 3.3.2 demonstrated autocorrelations at
some lags must be negatively autocorrelated. In this case if any of the $T$ period changes are due to the influence of a transitory component then $\sigma^2(R_t^T)$ will grow at a rate less than $\sigma^2(R_t^{12})$ in which case

$$\sigma^2(R_t^T)/T < \sigma^2(R_t^{12})/12$$

(3.28)

Consequently, the variance ratio statistic will be less than one if there are negatively autocorrelated transitory components but greater than one when there are positively autocorrelated transitory components.

Empirical work on mean-reversion which utilises the variance ratio statistic has been undertaken by Poterba and Summers (1988) who estimate variance ratio statistics for the US and 18 other countries. Their results suggest that stock prices display substantial mean reverting tendencies. In the US, stock prices exhibit positive autocorrelation at return intervals of less than a year, and negative autocorrelation over longer horizons. Consistent with the results of Fama and French (1988) Poterba and Summers finds that for long-horizon returns, a mean reverting component in stock prices appears to explain a sizeable proportion of the variation in returns. The evidence for indexes tested in a range of other countries including the UK also broadly supports mean-reversion.

Because the variance ratio test appears to have much stronger power in detecting autocorrelation Lo and MacKinlay (1988) reconsider the predictability of short-horizon returns using the variance ratio statistic. The contribution made by Lo and MacKinlay is that they derive the following approximations which allow them to use overlapping data and take advantage of the technical improvements which are associated with large samples.

Assume that $p_0, p_1, \ldots, p_n$ is a series of stock prices, the first difference (when $p$ is sampled weekly, monthly or annually etc) has an unknown mean $\mu$ and variance $\sigma^2$ but these can be estimated with maximum likelihood as follows

$$\hat{\mu} = \frac{1}{2n} \sum_{k=1}^{2n} \left( p_k - p_{k-1} \right)$$

(3.29)

$$\hat{\sigma}_u^2 = \frac{1}{2n} \sum_{k=1}^{2n} (p_k - p_{k-1} - \hat{\mu})^2$$

(3.30)

$$\hat{\sigma}^2 = \frac{1}{2n} \sum_{k=1}^{n} (p_k - 2\hat{\mu} - 2\hat{\sigma}^2)^2$$

(3.31)

where $\hat{\mu}$ is the maximum likelihood estimator of the unknown $\mu$, $\hat{\sigma}_u^2$ is the maximum likelihood estimator of $\sigma^2$, and $\hat{\sigma}$ is also an estimator of $\sigma^2$, but where stock prices are sampled biweekly bi-monthly or bi-annually.
The general formulation for the variance of the $q_{th}$ differenced stock price series is given by

$$\hat{\sigma}^2_b(q) = \frac{1}{nq} \sum_{k=q}^{nq} (p_k - p_{k-q} - q \hat{u})^2$$

\hspace{1cm} (3.32)

where $\hat{\sigma}^2_b$ now becomes a function of $q$ which is the differencing period. Under the null hypothesis of a Gaussian random walk $\hat{\sigma}^2_b$ and $\hat{\sigma}^2_b(q)$ will be close in value, so a test statistic for a random walk is given by some measure of the difference between the two.

Such a measure, $[\hat{\sigma}^2_b(q)/\hat{\sigma}^2_b] - 1$, is constructed by Lo and MacKinlay (1988,1989) as a variation on the variance ratio test proposed by Cochrane. They then go on to define a statistic which is able to utilise overlapping returns. A new estimator of $\sigma^2$ is described as

$$\hat{\sigma}^2_v(q) = \frac{1}{nq} \sum_{k=q}^{nq} (p_k - p_{k-q} - q \hat{u})^2$$

\hspace{1cm} (3.33)

in which case the variance ratio statistic becomes

$$VR(q) = \frac{\hat{\sigma}^2_v(q)}{\hat{\sigma}^2_b} - 1$$

\hspace{1cm} (3.34)

In the Lo and MacKinlay test, if the variance ratio statistic is negative then negative autocorrelation is implied, if the variance ratio statistic is positive then positive autocorrelation is implied.

Autocorrelation tests based on the variance ratio test are conducted by Lo and MacKinlay on both individual security returns and portfolio returns which utilise overlapping observations. For individual stocks they find that weekly returns are negatively autocorrelated, but insignificantly so. In contrast, for portfolios strong positive autocorrelation is discovered. For example, when portfolios are formed on the basis of market capitalisation it is found that for the portfolio containing the smallest firms positive autocorrelation in the order of 42% was documented but for the largest firms the positive autocorrelation was found to be 14% which provides results comparable to studies which have used simple autocorrelation tests.

Surprisingly, Lo and MacKinlay dismiss the possibility that thin trading can account for the high positive autocorrelation in the short-horizon returns, which is found for the portfolios. Instead, they suggest that portfolio returns of well diversified portfolios contain transitory components and therefore suggest that perhaps the autocorrelation reflects market inefficiency.
A substantial amount of UK evidence also exists to suggest that UK stock prices are mean reverting. Fraser and Power (1993), examine persistence in the excess returns, of ten investment trust shares, over the period 1970-1989. The key findings are that the excess returns appear to have a predictable component because the variance ratios for return intervals of between 2 to 60 months have dissimilar patterns. For return horizons of less than 24 months there is a tendency towards the variance ratios being greater than one, indicating positive autocorrelation in returns. Meanwhile, for return intervals greater than this the variance ratios were all less than one indicative of negative autocorrelation.

In an alternative paper which tests for predictability in UK stock returns MacDonald and Power (1993) investigate the autocorrelation properties of ten UK size based portfolios using the variance ratio approach and find that short-horizon returns are significantly autocorrelated, although like Fama and French (1988), the strength of the positive autocorrelation appeared to be related to firms size. However, the study by MacDonald and Power can be criticised because each portfolio only contains ten securities and does not therefore reflect a fully diversified portfolio.

Although, the variance ratio test, because of its stronger power, has been useful in identifying autocorrelation, it is difficult to assess how important short-horizon variance ratio tests are as tests of market efficiency. As we saw in Chapter 2, there are a number of reasons why we would expect short-horizon, portfolio returns to be positively autocorrelated. But, the variance ratio test does not allow us to identify whether there is predictability greater than that introduced by such considerations, the variance ratio test can not therefore be used to reject stock market efficiency only to confirm its existence.

The most important criticisms of the mean-reversion literature has been directed at questioning the empirical assumptions which have underpinned the tests. Kim, Nelson and Startz (1991), for example, question the true significance of the mean-reversion studies. They re-calculate the mean-reversion test statistics on sub-samples provided by Poterba and Summers and Fama and French and find that the statistical evidence provided by Fama and French is generated by data from the 1930s. Using an alternative procedure to Monte Carlo simulation called randomisation stratification they find that the standard error estimates used in mean-reversion studies are understated, consequently, mean-reversion can only be supported at a 60% significance level.

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18. Steeley (1995) has demonstrated that as more securities are added to a portfolio the predictability of the portfolio increases because of serial diversification.

19. Random stratification involves taking the population of a group and splitting it into sub-groups from which to estimate parameters. The advantage of random stratification is that estimates may be both more efficient and less biased.
Nelson and Kim (1990) discuss two important sources of bias which may have introduced spurious statistical significance into mean-reversion tests. Long-horizon estimates require the use of overlapping observations. The use of overlapping returns, introduces positive autocorrelation, and autocorrelated data may have produced an upwardly biased estimate of $R^2$. Autocorrelated errors may have also biased $t$ statistics, and although the studies corrected for autocorrelation the properties of the $t$ statistics are not well known.

Kim and Nelson also argue that the coefficient associated with a lagged dependant variable, is biased towards zero in small samples. In mean-reversion tests the bias has the potential to create the illusion that prices are mean reverting when they are really unpredictable. Because the bias is stronger in small samples, long-horizon tests will be the most biased and will appear to be more mean reverting than short-horizon tests. Kim and Nelson find that for return horizons of ten years over 20% of the apparent variation in stock returns seems to be caused by bias, which would mean that over half of the apparent predictability in ten year returns can be explained by the bias.

In conclusion, this section has studied a number of the most recent studies which have suggested that stock markets over-react to new information. The results of these tests have been interpreted by many as finally providing clear cut evidence of stock market inefficiency. Although, as this section has argued, rational explanations and weaknesses in the test procedures can also explain these findings.

3.4 Anomalies

Market anomalies can be described as unexplainable return regularities, which make stock returns predictable. There are two anomalies which will be discussed in depth in this section because they are particularly important for the empirical work which is to be presented in later chapters of this study. The first part of this section will be concerned with the so called calendar regularities which indicate that stock returns over different calendar periods are predictable. The second part of this section will be concerned with the so called small firm effect which recounts the fact that even after adjusting for differences in systematic risk, small firms appear to provide a premium over large firms. A third anomaly will also be discussed ie the earnings anomaly. This anomaly is not directly relevant for this thesis but it may be of interest to provide some background information concerning this topic because the earnings anomaly has been closely linked to the size effect.

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20 Other anomalies which have been identified but will not be discussed here are the price earnings anomaly and the dividend yield anomaly. The price earnings ratio anomaly suggests that even after adjusting for systematic risk companies with high price earnings ratios earn risk adjusted premiums. The dividend yield anomaly indicates that companies with high dividend yields also earn risk adjusted premiums. Since this study will focus on the calendar and size effects only these anomalies will be discussed.
Although, much of the work which documents the anomalies has been conducted on US data, we are becoming increasingly aware of similar patterns in other major markets such as the UK, which appears to rule out data snooping as a potential cause of the regularities.

3.4.1 The January Effect

The January effect describes the empirical observation that stock returns appear to be higher in January than in other months of the year. The January effect was first observed by Watchel (1942) and was investigated more fully by Rozeoff and Kinney (1976) who found that returns to an equally-weighted portfolio tended to be higher in the month of January than in other months. Another dimension to the January effect indicates that a substantial proportion of the January effect was caused by the performance of small firms, see for example, Ikenberry and Lakonishok (1989) who found that over the period 1926-1986 small firms outperformed large firms in the month of January by about 5.5%. Furthermore, out of the 71 years which were investigated in this study small firms have only underperformed large firms in 6 of the Januaries. An excellent summary of the issues involved with the January effect are provided by Haugen and Lakonishok (1990). Although, we have known about the January effect for some time, there still exists an exiting lack of consensus regarding its causation as we shall see in the following sections.

3.4.1.1 Tax Loss Trading

An explanation for the January effect is provided by Keim and Roll (1982) who link the superior performance of firms in January to tax loss trading. Tax loss selling exists when investors sell stocks before the start of the new tax year, if they have declined in value over the preceding year. This ensures that capital losses are realised so that capital gains tax is minimised. As loser stocks are offloaded onto the market additional price depreciations may take place, induced by excessive selling pressure. At the beginning of the new tax year as portfolios are re-balanced, buying pressure pushes up prices creating a January price rise which is most obvious during the first few trading days of January. Ritter (1985) has provided empirical support for tax loss trading in the US because he finds that buy/sell ratios tend to be higher in January but lower in December. Reinganum (1983) also supports the tax loss trading hypothesis because an empirical link between firm size, the January premium and tax loss trading is discovered. Reinganum reports that firms which depreciate in value by the same amount provide diminishingly higher returns in January as firm size increases, indicating that small firms are more influenced by tax loss trading than large firms.

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21. Data snooping refers to using the same data set to empirically measure different predictable patterns. If the observed predictability is an artifact of the data set used, and can not be replicated on alternative data sets then data snooping may have taken place.
Ritter (1988) explained why tax loss trading influences small firms more than large firms. Individual investors have stronger tax loss trading incentives because institutional investors are governed by different tax codes. Proportionately, individual investors also own more stocks in small companies. The securities of small firms tend to be more volatile, a consequence of this is that small stocks are more likely to be tax loss candidates, since they are more likely to have declined in value over the previous year. Reinganum (1982) confirms this empirically because he finds that 60% of firms which had a depreciating share price over the previous year were to be found in the quartile of firms with the lowest capitalisation value. In contrast, less than 10% of the firms in the largest quartile experienced significant price declines.

What appears puzzling in the tax loss literature is why shareholders wait until December to sell off their badly performing shares and why non tax paying investors do not pick up the bargain shares before the start of January. These are important questions which have not yet been answered by those who promote tax loss trading as a rational explanation for the turn of the year effect.

If the January regularities are caused only by tax loss selling, countries with tax years which do not start in January should experience a regularity at the start of their tax year, but not in January. Meanwhile, countries that do not tax capital gains should not experience a new tax year rally whatsoever. In which case, the UK, with an April/March tax year should experience an April seasonality. Australia with a July/June tax year should experience a July seasonal but, in Japan, where capital gains are not taxed, no seasonal should be observed at the start of its tax year. An examination of average monthly returns in countries with different tax years provides little support for the January effect having been only caused by tax loss trading. Kato and Shallheim (1985) provide evidence of high January and June returns in Japan despite the lack of incentive to undertake tax loss trading. In Australia as Brown et al (1983) discover it is January returns which appear to be abnormally high not July returns. Finally, seasonalties in the UK market can not be explained completely by resorting to the tax loss trading explanation. The UK market is characterised by a January return which is substantially higher than the April regularity, see for example Levis (1985). The pattern of monthly returns in these countries does not appear to be consistent with the January effect, unless we assume that foreign investors are exerting sufficient pressure on domestic markets to introduce a January effect.

3.4.1.2 Return Seasonality and the Pricing of Risk

Although, the tax loss trading explanation is the one that has received most of the attention, an attempt has been made by Tinic and West (1984) to link the January effect to the pricing of systematic risk. Tinic and West repeat the CAPM tests originally undertaken by Fama and MacBeth (1973), but organise the tests into two groups. The first group tests CAPM only
in the month of January, the second group tests CAPM using observations for the other eleven months of the year. Tinic and West find that the positive relationship between systematic risk and return which is found by Fama and MacBeth is caused by the dominating influence of January. For the eleven months of the year other than January no relationship between systematic risk and return was found to exist. This appears to indicate that the January premium in the US can be linked to the provision of a systematic risk premium.

Similarly, when Corhay, Hawawini and Michel (1987) undertake similar tests for the UK, no relationship between systematic risk and return was found to exist for any month other than April. It appears that in the UK systematic risk is only ever priced during the month of April. The work of Tinic and West and Corhay et al shifts the monthly regularities from being a return anomaly to be an equally puzzling risk anomaly. Chapter 6 will attempt to shed some light on this puzzle.

3.4.1.3 Day of the Week Effects

Not only are expected returns different across the months of the year, they also appear to be different across the days of the week. Cross (1973) was the first to discover that the US stock market appeared to be characterised by day of the week effects since the Standard and Poor appeared to rise on Fridays but not on Mondays. In a later paper French (1980) found that average returns on a Monday were negative and statistically different to average returns during other days of the week. Meanwhile, returns on a Wednesday and a Friday were found to be significantly higher than returns on a Tuesday or Thursday. Similar results were also reported by Keim and Stambaugh (1984). Data snooping does not appear to be the cause of the day of the week effects since Lakonishok and Smidt (1985) confirm the finding for alternative data sets. It should also be noted that Board and Sutcliffe (1988) find evidence of a weekend effect for the UK in the FT all share index over the period 1962 to 1986. This paper is unusual in that it actually tests a trading rule based on the weekend effect. Board and Sutcliffe (1988) find that by selling short one million pounds worth of shares on a Friday to buy back on a Monday provides an average profit before transaction costs of three thousand pounds. Choy and O'Hanlon (1989) also find evidence of day of the week effects in the UK.

Rogalski (1984) argues that the so called weekend effect appears to be linked to both the size and the January effect. Rogalski reports that in January, average Monday returns are found to be positive, unlike Monday returns in other months. A comparison of average day of the week returns for ten capitalisation based portfolios over the period 1963-1982 indicated that a negative Monday effect failed to exist except for the smallest firms in the sample.
Although, it is clear that returns are not equal across the different days of the week, Harris (1986) disputes that this is caused by returns being lower on a Monday. Instead Harris argues that the fall in Monday prices may be caused by the behaviour of prices during the weekend, and therefore prior to the open of the market on a Monday. Studies which attempt to document the weekend effect invariably calculate returns as close to close returns which means it is impossible to gauge whether the weekend effect takes place before the market opens or after it opens. To clarify when the weekend effect takes place Harris calculates day of the week returns for ten equal-weighted capitalisation based portfolios for securities quoted on the New York Stock Exchange. Two sets of portfolio returns are compared, returns which are calculated from open to close prices and those which are calculated from close to open prices. The comparisons of the two sets of returns revealed that the Monday negative return was much stronger for large firms when returns were calculated from close to open prices but was larger for small firms when returns were calculated from open to close prices. This indicated that for large firms the Monday negative accrued before Monday trades began but for small firms the Monday negative accrued after the start of Monday trading.

### 3.4.2 The Small Firm Effect

The previous section demonstrated that returns appear to be predictable in certain calendar periods. An equally puzzling anomaly is the small firm effect, which recounts the fact that after accounting for systematic risk small firms appear to significantly outperform large firms. Work on the small firm effect was pioneered by Banz (1979) who found that over the period 1936-1975 a portfolio of the smallest securities in the sample outperformed the Standard and Poor index by an average 8.4% a year, even after adjustments for differences in systematic risk had been made. Even more striking evidence of a size effect was provided by Reinganum (1981,1982) who found that the difference between the return to a portfolio of small and large companies was as high as 30%.

An interesting facet of the small firm effect is that the small firm premium varies significantly across the various months of the year. Work undertaken by Keim (1983) And Roll (1983) indicates that approximately half of the small firm premium accrues during the first five trading days of January. While, Reinganum (1982) reports that the difference between a portfolio of large and small firms on the first trading day of January is as high as 3%.

The small firm effect has also been reported for a number of other major exchanges around the world, including the London stock exchange. Levis (1985) finds that a portfolio of the smallest firms in his sample provides a risk adjusted small firm premium of about 6%. In contrast to the US, the UK small firm premium does not appear to be linked to either the turn of the year, or the start of the new tax year. Typically, large firms perform better in both of these
months. This difference between the behaviour of small firms in the UK and the US is important because it might suggest that the underlying causes of the small firm effect in the UK are not the same as in the US.

We do not have to resort to assuming that the market is inefficient to be able to explain the small firm effect. Stoll and Whaley (1983) have suggested that the higher transaction costs associated with trading small firms can account for the difference in the performance of large and small firms. Stoll and Whaley report that the bid-ask spread of small stocks in their sample is 2.93% but only 0.69% for large value stocks. Meanwhile, average broker commissions were reported to be 3.84% for the smallest firms but only 2.02% for the high valued stocks.

A number of explanations for the small firm effect consider the possibility that either the Capital Asset Pricing model is not the appropriate asset pricing model to use or alternatively measurements of systematic risk which are used to estimate a portfolio expected returns are biased. A discussion concerning the recent lack of empirical support for the Capital Asset Pricing Model was undertaken in Chapter 2 and as we saw Chan (1985) et al find that the small firm premium can be explained within the context of an APT framework.

It has been argued that the small firm effect arises because the betas of small firms tend to understate the true risk of small firms. In which case, expected returns of small firms are understated and excess returns overstated. In some studies the impact of the thin trading which exists is frequently understated which will introduce a bias in favour of finding a small firm effect. Levis (1985) for example, uses a three lag aggregate coefficient model to estimate the beta for a portfolio of small firms on the London stock market. Levis then appears to be surprised that the portfolio of small stocks is less risky than the portfolio of large stocks. In the UK market, it is not unusual for the smallest 10% of stocks to fail to trade for 5 or 6 months, considering the level of disaggregation that Levis considers (deciles) it seems likely therefore that three lags with monthly data is an insufficient adjustment for thin trading.

Blume and Stambaugh (1983) draw attention to the importance of measuring returns correctly so that any potential bias in the calculation of returns is minimised. Blume and Stambaugh demonstrate that when an equally-weighted portfolio return is calculated there is an inherent source of bias caused by the bid-ask spread. For example, if a portfolio is constructed one year ago, so that the portfolio contains a large number of stocks. After the construction date, whenever the prices of securities within a portfolio change, so that at least some security returns are non zero, the portfolio ceases to be an equally-weighted portfolio unless the portfolio weights are re-aligned. This is the case because as prices change the weights attached to each individual security must also change. The return from the equally-weighted portfolio is therefore biased because an equally-weighted portfolio would require that stocks are traded each period to re-align the weights, consequently a bid-ask spread cost would be incurred by investors.
Since the bid-ask spread tends to be substantially higher for small stocks, (see the work of Stoll and Whaley which was discussed earlier) then the returns of small stocks will be inflated relative to the returns of large firms. Blume and Stambaugh find evidence that about 50% of the small firm premium is due to this bias. In which case the small firm premium is substantially overstated. Fong (1992) tests the small firm premium to buy and hold portfolios (which in this case are portfolios which are forced to maintain equal weights) in the UK but finds that the small firm premium is about 6% and therefore very similar to the one reported by Levis (1985).

More recently, the importance of the return intervalling effect has been recognised in a number of studies. Handa, Kothari and Wasley (1989) for example, argue that the size effect is sensitive to the length of the return interval used to estimate betas, irrespective of the trading frequency of stocks and, the small firm effect becomes statistically insignificant when risk is measured using annual returns.

3.4.3 Earnings Announcements

Using the event study methodology22, Ball and Brown (1968) discovered a puzzling anomaly associated with earnings announcements. Ball and Brown discovered that companies appear to suffer from "post announcement drift", i.e. after an earnings announcement has been made, stock returns appear to move in the same direction as the earnings announcement. This means that companies which announce good earnings figures appear to experience abnormally high returns, after the earnings announcement has been made, contrary to what we would expect if a market is semi-strong efficient. Meanwhile, companies which announce bad earnings figures appear to experience abnormally bad returns for several months after the earnings announcement. A review of the earnings announcement literature by Ball (1978) considers over twenty papers which had investigated and found evidence of the post announcement drift phenomenon.

Watts (1978) studies a sample of U.S. firms during the period 1957-1968 and found that there was an average abnormal return of 3.2% following an earnings announcement. Watts argued that a trading rule on the basis of this apparent inefficiency would be unprofitable, assuming transactions costs of 4%, which was believed by Watts to be appropriate at the time.

Rendleman, Jones and Latane (1982) showed that the magnitude of the post announcement drift appeared to depend on the magnitude of the abnormal earnings announcement. That is, firms which have the largest unexpected earnings have the strongest post announcement drift. Rendleman et al constructed ten portfolios of stocks for the twenty days prior and the ninety days following an earnings announcement. The first portfolio contained

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22 An event study investigates the return performance of a group of companies around the announcement of some information which has fundamental relevance. If a market is efficient then share prices should react to the new information on the day the announcement is made but on the days after the announcement the returns of the companies should not show signs of abnormal behaviour.
the firms which provided the highest unexpected earnings, down to portfolio 5, which contained firms with the lowest positive earnings announcement. Portfolio 6 contained firms with the lowest negative unexpected earnings, while portfolio 10 contained firms with the largest negative unexpected earnings figures. Rendleman et al. found that the magnitude of the post announcement drift was monotonically associated with the size of the unexpected earnings figures. In this study the mean abnormal return of the extreme portfolios, i.e. portfolio 1 and 10, in the month following the earnings announcement was found to be 6%. This suggests that in order to maximise trading rule profits based on earnings announcements, investors should restrict the use of their trading rule to only firms which announce extreme earnings figures.

Foster, Olsen and Shevlin (1984) are among many that have subsequently replicated this phenomenon and tested a trading rule. Foster et al. estimated that over the 60 trading days subsequent to an earnings announcement, a long position in stocks with unexpected earnings in the highest decile, combined with a short position in stocks in the lowest decile, yielded an annualised abnormal return of 25% before transaction costs.

There are two competing explanations for the existence of post announcement drift. One class of explanations suggests that at least some portion of the abnormal performance is due to some delay in the way prices react to the new information. This would occur if traders failed to assimilate available information or if transactions costs introduced a wedge between the intrinsic price and the observed price following the release of new information.

The transaction costs theory does not appear to be supported, alternative studies have shown an immediate price reaction to other types of news, see for example Fama, Fisher, Jensen, and Roll (1969) who investigated the reaction of share prices to stock splits. An alternative explanation is that estimates of market risk, which have been used are biased, so that abnormal returns are understated for firms which announce abnormally high earnings figures, but understated for firms which announce abnormally bad earnings figures. This possibility was first discussed by Ball (1978) who argued that a number of studies had measured market risk inadequately.

Bernard and Thomas (1990) suggest that there is good reason to suggest that misspecification of CAPM might be causing the post announcement drift since there are a vast number of studies in existence which suggest that betas are time varying and sensitive to the arrival of new information (a detailed review of this literature is undertaken in Chapter 7). For example, Ball, Kothari and Watts (1988) suggested that betas shift upward for firms with high unexpected earnings and shift down for companies with low unexpected earnings. Assuming stationary betas in the post announcement period, under these conditions would give rise to a post announcement drift.
Bernard and Thomas (1990) attempt to discriminate between the two possible explanations. Using betas which are allowed to be time varying they find that risk mis-measurement can not explain fully the magnitude of the post announcement drift. Although there is extensive documentation regarding the post announcement drift its causations are not fully understood and therefore it remains an anomaly.

3.5 Conclusions

This chapter has reviewed the most important empirical research which has addressed the question of whether stock market prices are predictable or not. Up until the end of the 1970's empirical research appeared to indicate that stock prices were unpredictable. The focus of the empirical work up until this time had been concerned with testing whether future stock prices could be predicted from their past behaviour. This was done chiefly by undertaking autocorrelation tests and testing trading rules. Generally the results provided from such tests indicated that stock prices were not predictable and therefore consistent with market efficiency.

In the period from the end of the 1970's onwards it became clear that with the development of new, stronger ways of testing for stock price predictability, markets appeared to be far less efficient that they had first appeared. One factor which led to this discovery was a move away from testing the behaviour of short-horizon returns to long-horizon returns. The empirical work which has now been amassed suggests that long-horizon returns are much more predictable than short-horizon returns. A theoretical framework for this discovery was provided by the mean-reversion literature, which considered how stock returns would behave if prices contained transitory components. This literature predicted that long-horizon returns would be substantially (negatively) autocorrelated. Although, this was deemed by many to provide evidence of market inefficiency a competing explanation for this result is that expected returns are time varying. Furthermore recent work has shown that short horizon returns are characterised by time variation which appears to be able to explain a substantial amount of the variation in short horizon returns.

Another important discovery which has put the efficient markets hypothesis under close scrutiny has been the discovery of the so called anomalies such as the size effect which has suggested that stock returns are more predictable for small firms, while the calendar anomalies have suggested that stock returns are characterised by regularities which allow investors to earn higher returns during certain periods within a calendar year.

The findings of the tests, which have been surveyed in this chapter, have placed the efficient markets hypothesis close scrutiny. Although these tests are also important because they have caused research to focus on alternative, rational explanations for return predictability such as microstructure frictions and time variation in expected returns. The possibility that expected returns are time varying and autocorrelated so that realised returns are autocorrelated is
The findings of the tests, which have been surveyed in this chapter, have placed the efficient markets hypothesis close scrutiny. Although these tests are also important because they have caused research to focus on alternative, rational explanations for return predictability such as microstructure frictions and time variation in expected returns. The possibility that expected returns are time varying and autocorrelated so that realised returns are autocorrelated is investigated in Chapter 7 and Chapter 8 of this study. A recurrent comment about the empirical work, which has been discussed in this chapter, is that many tests, which claim to be tests of market efficiency are really only tests of price or return predictability which does not necessarily in itself imply market inefficiency.

This chapter is also important because it documents the extent to which the behaviour of small firm returns differs from that of large firms. Section 3.4 demonstrated that the returns of small firms tend to outperform the returns of large firms but clearly this is not the only difference between small and large firms that has been identified. Lo and MacKinlay (1988, 1990a) has shown that both the serial cross-correlation and the autocorrelation patterns of short horizon portfolio returns indicate that the returns of small firms are more predictable than the returns of large firms. Conrad and Kaul (1988) and Conrad et al (1991) provide evidence to suggest that small firms contain a more predictable component because their expected returns, at both portfolio and security level, are characterised by more time variation than the returns of large firms. Furthermore, as we saw from the work on long-horizons, a small firm effect also appears to exist here because much more of the variation in small firm long horizon returns can be explained in comparison with large firms. Although, they do not develop this issue, the pattern of results provided in this paper indicates that the expected returns of the small and the large firms appear to behave differently. The following chapters of this thesis will be concerned with extending the search for differences in the behaviour of small firm and large firm returns. In Chapter 4 and Chapter 5 this thesis will be considering the extent to which the small firm returns are more predictable than the returns of large firms.
Appendix 1: The Construction of the Series \( p_i^* \) in Shiller (1981a)

The series \( p_i^* \) which is the ex-post rational share price is constructed as follows.

A terminal value, that is a value for \( p_i^* \) is calculated for the last period of the sample being tested. In the Shiller tests this is calculated as the average value of the actual share price over the full sample period.

A discount rate is then chosen. This discount rate should reflect the required return to investing in the stock market. Shiller calculates this as the average dividend divided by the average price.

Using these values and actual dividend outcomes \( p_i^* \) are estimated recursively working backwards from the terminal value.

Appendix 2: Shocks to a Random Walk have a Permanent Effect

When stock prices are driven by purely random factors a shock will have a permanent effect on the price. The current stock price \( p_i \) is just the last periods price plus the current periods error. Assume that time starts in time 0, if \( p_i \) is a random walk then

\[
p_i = p_0 + e_i,
\]

the price one period ahead is

\[
p_{i+1} = p_0 + e_{i+1} + e_i
\]

the price in \( T \) periods ahead is therefore

\[
p_{i+T} = p_0 + e_i + e_{i+1} + \ldots + e_{i+T}
\]

If stock prices are driven by purely random factors then the current periods price is equivalent to

\[
p_i = p_0 + \sum_{j=0}^{t-1} e_{i-j}
\]

In this case a shock will have a permanent effect on the share price because as \( e_i \) varies then so does \( p_i \). If the stock price contained a temporary component then a shock would not have a permanent effect on the price.
Appendix 3: Proof that the Numerator of Equation (3.19) is $-\sigma^2(z)$

Starting with the numerator of equation (3.19) in the text,

$$\text{cov}(z_{t+T} - z_t, z_t - z_{t-T})$$

and using the properties of covariances, which permit the covariance to be written as a combination of its component parts, equation (A.31) can be split up into the following four terms.

\begin{align*}
\text{cov}(z_{t+T}, z_t) & \quad \text{(A.32)} \\
-\text{cov}(z_{t+T}, z_{t-T}) & \quad \text{(A.33)} \\
\text{cov}(z_t, z_{t-T}) & \quad \text{(A.34)} \\
-\text{cov}(z_t, z_t) & \quad \text{(A.35)}
\end{align*}

Because the span between $+T$ and $-T$ is twice the span between $+T$ and $t$, so the distance between $+T$ and $-T$ is $2T$, equation (A.33) therefore becomes $-\text{cov}(z_t, z_{t+2T})$. The terms $\text{cov}(z_{t+T}, z_t)$ in equation (A.32) and (A.34) and $\text{cov}(z_t, z_{t-T})$ are equivalent, so together they represent $2\text{cov}(z_t, z_{t+T})$, and $-\text{cov}(z_t, z_t)$ in equation (A.35) is equivalent to $-\sigma^2(z)$. Taking all four terms together it is possible to write the numerator covariance of equation (3.19) of the text as,

$$-\sigma^2(z) + 2\text{cov}(z_t, z_{t+T}) - \text{cov}(z_t, z_{t+2T})$$

(A.36)

Assuming that $z_t$ is covariance stationary then collecting terms provides the numerator of equation (3.19) $-\sigma^2(z)$
Appendix 4: Proof that the Denominator of Equation (3.19) is $2\sigma^2(z)$

Again taking the denominator of equation (3.19) in the text

$$\sigma^2(z_{t+T} - z_t) \quad (A.41)$$

and using the fact that a variance of a process can be written in terms of a covariance with itself, gives

$$\text{cov}[(z_t - z_{t+T})(z_t - z_{t+T})] \quad (A.42)$$

Because a covariance can be written in terms of its component parts, this equation can be decomposed by the properties of covariances into four constituent covariances, as follows.

$$\text{cov}(z_{t+T}, z_{t+T}) \quad (A.43)$$
$$-\text{cov}(z_{t+T}, z_t) \quad (A.44)$$
$$\text{cov}(z_t, z_t) \quad (A.45)$$
$$-\text{cov}(z_t, z_{t+T}) \quad (A.46)$$

Taking equations (A.43) to (A.46) collectively, the denominator of equation (3.19) in the text can be written as

$$-\text{cov}(z_{t+T}, z_{t+T}) - \text{cov}(z_{t+T}, z_t) + \text{cov}(z_t, z_t) - \text{cov}(z_t, z_{t+T}) \quad (A.47)$$

and

$$-\text{cov}(z_{t+T}, z_t) + \text{cov}(z_{t+T}, z_t) = 0 \quad (A.48)$$

the denominator of equation (3.19) is therefore

$$\text{cov}(z_{t+T}, z_{t+T}) + \text{cov}(z_t, z_t) = 2\sigma(z)$$
Appendix 5: Proof of Equation (3.21)

If \( z_t \) follows an AR(1) process, then the one period ahead forecast of \( z_t \) is

\[
z_{t+1} = \phi z_t + e_t \tag{A.51}
\]

\[
E_t(z_{t+1}) = \phi z_t \tag{A.52}
\]

The \( T \) period ahead forecast is

\[
z_{t+T} = \phi^T z_t + e_t \tag{A.53}
\]

\[
E_t(z_{t+T}) = \phi^T z_t \tag{A.54}
\]

\[
E_t(z_{t+T} - z_t) = \phi^{T-1} z_t \tag{A.55}
\]

This is equation (3.21) of the text

Appendix 6: Proof of Equation (3.23)

The expected difference between \( (z_{t+T}) \) and \( z_t \) when \( z_t \) follows an AR(1) process is as follows

\[
E_t(z_{t+T} - z_t) = \phi^T (z_t) - z_t = (\phi^T - 1) z_t \tag{A.61}
\]

The numerator of the autocorrelation coefficient of equation (3.19) when \( z_t \) follows an AR(1) process is

\[
cov(z_{t+T} - z_t, z_t - z_{t-T}) \tag{A.62}
\]

which is equivalent to

\[
Cov(\phi^T z_t - z_t, z_t - \phi^T z_t) \tag{A.63}
\]
Just as in the previous example, this can also be broken up into its component parts as follows

\[
\begin{align*}
\text{cov}(\phi^T z_t, z_t) &= -\phi^T \sigma^2(z) \\
\text{cov}(-z_t, z_t) &= -\sigma^2(z) \\
\text{cov}(-z \phi^T_t, z_{t-r}) &= -\phi^T \sigma^2(z) \\
\text{cov}(\phi^T z_t, \phi^T_{t+1}) &= (-\phi^T)^2 \sigma^2(z)
\end{align*}
\]

Collecting terms gives

\[
(1 + 2\phi^T \sigma^2(z)
\]

or,

\[
(1 - 2\phi^T + \phi^2 \sigma^2(z)
\]

Let \( \phi^T = x \), then

\[
\begin{align*}
-(1 - 2x + x^2)\text{var}(z) \\
-(1 - x)(1 - x)\text{var}(z) \\
-(1 - x)^2\text{var}(z) \\
-(1 - \phi^2)^2\text{var}(z)
\end{align*}
\]

This provides us with the numerator of equation (3.22) of the text which is the numerator of the return autocorrelation when \( z_t \) is an AR(1) process.

The denominator is just the variance of \( z_t \). This means that the coefficient \( \beta(z) \) of a regression of \( E(z_{t+T} - z_t) \) on \( z_t - z_{t-r} \) will be

\[
\beta_z = \frac{-\sigma^2 E(z_{t+T} - z_t)}{\sigma^2(z)}
\]

this is equation (3.22) of the text.
Appendix 7: Proof of Equation (3.23)

The slope of the regression of \( r_{t+T} \) on \( r_t \) is

\[
\beta_T = \frac{\text{cov}(r_{t+T}, r_t)}{\sigma^2(r_t)} \tag{A.71}
\]

This can be written as

\[
\frac{\sigma^2(z_{t+T} - z_t)(z_t - z_{t-T})}{\sigma^2(z_{t+T} - z_t)} + \frac{\text{cov}(q_{t+T} - q_t)(q_t - q_{t-T})}{\sigma^2(q_{t+T} - q_t)} \tag{A.72}
\]

Because \( \text{cov}(q_{t+T} - q_t)(q_t - q_{t-T}) = 0 \) then

\[
\beta_T = \frac{\rho_T \sigma^2(z_{t+T} - z_t)}{\sigma^2(z_{t+T} - z_t) + \sigma^2(q_{t+T} - q_t)} \tag{A.73}
\]

This provides us with equation (3.23) of the text

\[
B_T = \frac{-\sigma^2 E_z(z_{t+T} - z_t)}{\sigma^2(r_t)} \tag{A.74}
\]
CHAPTER FOUR
The Predictability of UK Portfolio and Security Returns.

4.1 Introduction

This thesis will be concerned with investigating the empirical characteristics of a selection of UK securities and portfolios, primarily, with the aim of identifying differences between the behaviour of large and small firms. The beginning of this chapter describes the source and quality of the data used in this study, to assess its strengths and weaknesses. Section 4.2 examines in detail some important characteristics of the short-horizon portfolio and security returns which are used in this study. Particular attention in this section will be focused upon the return autocorrelation patterns. This section finds that the returns of portfolios appear to be autocorrelated, unlike the returns of individual securities. Furthermore, the portfolio autocorrelation patterns also appear to be associated with firm size. The autocorrelation statistics when a portfolio contained small firms were found to be considerably higher than when a portfolio contained large firms. In contrast, the returns of individual companies, irrespective of size, did not appear to be autocorrelated. The final section of this chapter, Section 4.3, will investigate whether individual stock prices are predictable using co-integration tests. Co-integration offers an alternative way to the autocorrelation test of identifying predictability. This test is concerned with identifying long run relationships between stock prices and has the advantage of being able to capture both serial and cross serial correlation between security prices in a single test.

4.1.1 The Data

The data used in this study have been obtained from two sources: the London Share Price Database (LSPD) and Datastream. An objective of this study is to investigate the behaviour of returns at different sampling intervals and for different sized companies. This requires more than one data source. A set of monthly interval data was obtained from LSPD while a set of weekly interval data was obtained from Datastream.

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1. A paper which was based on this chapter was published in Applied Financial Economics in December 1994.
2. Serial correlation tests applied to stock returns investigates whether it is possible to predict the future value of a stock return to security \( i \) or portfolio \( i \) from its own previous values. In contrast, cross serial correlation tests applied to stock returns are concerned with whether it is possible to predict the future return of security or portfolio \( i \) from past values of the return to security or portfolio \( j \).
3. Although, it would have been possible to use Datastream to obtain both weekly and monthly prices which are used in this study, at the start of this thesis Datastream was not available to me.
4.1.2 LSPD Data

The LSPD database provides monthly share price and return information for all stocks listed on the London Stock Exchange, a survey of the data held on LSPD is provided by Board, Pope and Skerratt (1991). The LSPD tapes provide information on currently listed companies, and on de-listed companies (if they were previously included in the LSPD database). This means that it is possible to use the price or return information of a company even if a company no longer exists. This indicates a particular advantage for using the LSPD tapes, because samples can be created which are free of survivorship bias 4. From the information available on LSPD a sample of return and price information, for randomly selected stocks which were listed on LSPD was collected, to provide a sample period ranging from January 1975 to December 1991.

The prices provided by LSPD are monthly transaction prices. These are the prices at which the last trade took place for a given month, irrespective of whether the trade was a buy or a sell trade. If a stock fails to trade on the last day of the month then the most recent transaction price before the month end is provided. LSPD returns are calculated as:

\[ r_{i,t} = \ln \left( \frac{p_{i,t} + d_{i,t}}{p_{i,t-1}} \right) \]  

where,

- \( r_{i,t} \) = the return of security \( i \) in time \( t \),
- \( p_t \) = the last recorded price in month \( t \) for security \( i \),
- \( p_{t-1} \) = the last recorded price in the previous month for security \( i \),
- \( d_t \) = any dividends which might have been paid between \( t-1 \) and \( t \).

Since, LSPD records transaction prices, if a stock trades at the bid(ask) price but next month trades at the ask(bid) price then as we saw in Section 2.7.1 of Chapter 2 as long as some securities move from the bid(ask) to the ask(bid), then in the absence of new information, negative autocorrelation will be introduced into the security returns. Although, for a monthly return series the bid-ask spread will be small relative to the return and therefore the magnitude of the spurious autocorrelation which might be introduced would be relatively small. Furthermore, since most of the empirical work in this thesis will consider portfolios the impact of the bid-ask spread bias is likely to be small.

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4. Survivorship bias is caused when only companies that have survived over a sample period are used in empirical tests. Survivorship bias exists when such a sample behaves differently to a sample which also contains companies that have entered or exited during the sample period.
Information for two groups of individual securities were obtained from LSPD. No restrictions are imposed on the first cohort of companies, other than return and price information must be available for at least one calendar year, during the sample period. The second group contains companies which were continuously listed between 1975 and 1991. For these companies there may be some survivorship bias, because the sample does not include companies that have gone bankrupt or have been taken over during the sample period. Using market capitalisation data (number of shares times share price) each of the companies were then ranked on the basis of market capitalisation, every January over the sample period. This allows the firms to be segregated on the basis of market value.

Table 4.1 provides a return analysis of all companies listed on the London Share Price Database over the period January 1976 to December 1991 and for two sub-periods, January 1976 to December 1984 and from January 1985 to December 1991.

As we can see from Table 4.1 the returns of the LSPD companies provided on average, an annual return of about 13% over the period 1976-1991. However, during the sub-periods 1976-1984 and 1985-1991 the returns were strikingly different. The mean return during 1976-1984 was approximately 16% but during 1985-1991 the mean monthly return to LSPD companies was only about 10%. The lower return in the second sub-sample reflects the influence of the stock market crash during October 1987 and poor stock market performance during the economic recession of 1988-1991. The lowest return recorded for the LSPD group of companies was -0.4405 which was recorded for October 1987 and reflects the impact that the stock market crash of 1987 had on the London stock market.

The volatility of the LSPD companies was slightly higher during the second sub-period, this is probably due to the influence of the stock market crash which increased volatility and therefore the dispersion of stock market returns for 1987.

4.1.3 Datastream

Some of the empirical work which will be undertaken in this study will require the use of high frequency data (weekly). Since high frequency data was not available from LSPD an alternative data source was used. Datastream can provide price information for all currently existing companies listed on the London stock market. This means that companies which start up during the sample period can be utilised but companies which exit during the sample period, for whatever reason, can not be included. It is clear therefore that the chief shortcoming of the Datastream database is that there is an inherent source of survivorship bias.

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5. It is possible to buy the de-listed codes from Datastream, but this facility is very expensive and was not used.
In this study the price information provided by Datastream are the weekly (Wednesday to Wednesday) closing mid-point prices for individual securities. The decision to select Wednesday to Wednesday prices reflected the need to ensure that the data set was not influenced by any day of the week effects, such as those identified by French (1980) for the US and Board and Sutcliffe (1988) for the UK, who found that share prices tend to be abnormally high on a Friday but abnormally low on a Monday. These closing values are calculated by the Stock Exchange reporting department. The Stock Exchange mid-price for a stock is the average of the bid and ask prices obtained from two market-makers selected daily on a "most suitable basis". The market-makers are unaware of whether their prices are being used in a particular day.

Weekly security returns were calculated from the security prices as the change in the logarithm of successive Wednesday closing prices as follows

$$r_{i,t} = \ln(p_{i,t}) - \ln(p_{i,t-1})$$

(4.2)

To provide comparability with the LSPD data, the securities selected from Datastream as far as possible, were consistent with those chosen from LSPD.

Clearly, there is a difference between the way returns were calculated for the LSPD sample and the Datastream sample. The LSPD sample includes dividends, while the Datastream sample excludes dividends. This would be a problem if this study was to undertake tests of the small firm return premium using the Datastream sample. If dividends are omitted from Datastream returns, mean returns of the Datastream sample would be biased downwards because a positive component of returns is being excluded. If the large and small companies in the sample had different dividend returns then it would be impossible to make return comparisons across size based portfolios.

In this study, Datastream returns are never used in chapters which are concerned with investigating first moments. It is unlikely that omitting dividends from the return series will have any impact on higher moments such as volatility. We know from the work of Lintner (1965) that companies tend to smooth dividends. Dividends are smoothed because managers dislike reducing dividends since this sends pessimistic signals to the market about future performance. Investors are therefore only likely to receive a dividend increase if managers feel confident that the dividend can be maintained. The smoothing of dividends in this way ensures that dividends do not contribute to return volatility. Consequently, using a log price difference return when investigating volatility should provide unbiased estimates about the behaviour of volatility. Confirmation of this is provided by Poon and Taylor (1992) who investigate the conditional variance of the returns to the FT all share index. The similarity in the results provided by Poon

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6. No outliers appeared in the LSPD data set, although, three outliers were deleted from the Datastream sample. These three companies were deleted because they appeared to have erroneous prices or returns.
and Taylor for two samples, the FT index which includes dividends and the FT index which does not include dividends demonstrates that volatility is not influenced by the exclusion of dividends.

The Datastream companies are survivorship biased. In 1991, the end of the sample period there were approximately 2,500 stocks listed on Datastream. Since 1976 approximately 2,600 stocks have been de-listed from Datastream, which represents a de-listing rate of about 5% per year. The history of the dead companies, available from datastream suggests that the average beta of the dead companies was about 1.58\textsuperscript{7}. The two extreme betas were provided by SEKONG which was the de-listed company with the lowest beta at 0.025. The company with the highest beta was DUALVES which had a beta of 2.88. The securities which were included in the survivorship biased sample were selected in 1992. Since 1992 a number of these companies have now been de-listed. For portfolios 1, 2 and 3, 4 and 3 and 5 securities have been de-listed respectively. For portfolio 4 however 8 securities have been de-listed. This suggests that for small firms there is a reduced likelihood that they will survive.

4.1.4 Characteristics of Security and Portfolio Returns

This section will consider the characteristics of weekly and monthly returns for portfolios and individual securities which have been grouped together on the basis of firm size. Considering first the monthly returns, four equally-weighted portfolios were formed from the individual securities as follows. Each year the returns of randomly selected securities were obtained from LSPD. Every January throughout the sample period each security was ranked on the basis of market capitalisation (share price times number of shares issued). The fifty smallest stocks each year were then sorted into a portfolio, the next smallest into another portfolio and so on until the final portfolio contained the largest fifty stocks. The same procedure was adopted for the weekly return portfolios \textsuperscript{8}. In all four portfolios were created, portfolio P\textsubscript{1} contained the fifty smallest firms, P\textsubscript{2} contained the fifty small-intermediate companies, P\textsubscript{3} contained the fifty large-intermediate companies and P\textsubscript{4} contained the fifty largest firms.

The return to each of the equally-weighted portfolios was calculated as an equally-weighted average of the weekly returns of the component stocks as follows.

\[
R_{p,t} = \frac{1}{50} \sum_{i=1}^{50} r_{i,t}, \quad i = 1, 2, ..., 50 \quad p = 1, 2, 3, 4
\]

where,

---

\textsuperscript{7}The average beta is the average beta of all stocks contained in the dead files.

\textsuperscript{8}In this case weekly share price information was collected from Datastream, and the security return was calculated as the log price difference as explained in the previous section.
where,
\( R_{p,t} \) = the return of portfolio \( p \) in time \( t \), and
\( r_{i,t} \) = the return of one of the \( i = 1, 2, 3,...50 \) component stocks in the portfolio.

4.2 Summary Statistics

To provide information about the characteristics of the various data sets used in this study a selection of summary statistics, for both portfolio and individual security returns are provided in Table 4.2 and Table 4.3 respectively.

4.2.1 Mean Returns

The mean monthly return for the four size-based portfolios ranges from 1.37 percent to 1.04 percent. If these returns are annualised, large firms provide an annual average return over the sample period of approximately 12.48 percent but the portfolio containing the smallest firms provides an average annual return of about 15 percent. This suggests that the difference between the large and small firm returns provides an average small firm premium of about 2.5 percent per year. Although, it should also be noted that the small firm portfolio is not the portfolio which provides the highest return in this sample, this is provided by \( P_3 \) the portfolio which contains the large-intermediate companies. This conflicts with the findings reported by Banz (1981) and Levis (1985) who found that as firm size diminishes portfolio returns increase.

A similar pattern is observed for the weekly returns of the four size-based portfolios. The portfolio containing the smallest firms provides a return of about 15.89 percent a year, whereas the portfolio containing the largest stocks provides a return of about 9.41 percent a year. The annualised small firm premium is therefore about 6.5 percent. Most of the difference between the performance of the two portfolios appears to be due to the lower performance of the portfolio containing the large stocks.

In every year of this data set prior to 1987 the small firms outperformed the larger firms. In contrast from 1988 onwards, the small firms have always underperformed the large firms irrespective of whether returns are sampled at weekly or monthly intervals.

The small firm premium reported here is smaller than has been reported for an earlier sample period in the UK, see for example Levis (1985), who also used monthly interval data and, reports that over the sample period 1958-1982 the smallest firms in the sample outperform the largest firms in the sample by an average annual 6.5 percent. The reason for the fall in the small firm premium appears to be due to the relatively weak performance of small firms during the last five years of the sample period studied here. In particular, during this period interest rates were very high. This had a prolonged adverse effect on the UK economy, especially on
small firms, which tend to be more heavily reliant on debt as a source of finance, see for example, The Wilson Committee Report (1980), Mitchell (1980) or Bannock (1981). At the same time that stock returns were low, because of the adverse effect that interest rates had on the profitability of small firms, small investors who proportionately invest more in small firms, were attracted away from stock investment to interest bearing investment because at this time interest bearing accounts provided relatively high returns for little risk. This was particularly true because of the introduction of new interest bearing accounts such as TESSA's (Tax Exempt Saving Accounts). These accounts not only provided high interest rates but also provided a tax break for small investors. It is not surprising therefore that the returns of small firms performed so badly during the period 1988-1991.

4.2.2 Standard Deviations

Comparisons of the return standard deviations for the portfolios, when returns are sampled at both weekly and monthly intervals can be made by looking at Table 4.2. The results presented in this table imply that the return standard deviation for the portfolio containing the smallest stocks appears to be lower than the return standard deviation of large firms for both weekly and monthly returns. This would suggest that the portfolio containing the small firms is less risky than the portfolio which contains the large firms. To illustrate, the standard deviation of monthly returns for the portfolio containing the smallest firms is 0.00449 but the standard deviation for the portfolio containing the largest firms is 0.00652. A similar pattern exists when returns are sampled weekly. The standard deviation of $P_1$, the portfolio containing the smallest firms, is 0.0149 while the standard deviation of $P_4$, which is the portfolio containing the largest firms, is 0.0238. Initially, this appears surprising because it is large firms not small firms which have the greatest scope for internal diversification. However, it is possible to reconcile the portfolio standard deviations to the relative thin trading characteristics of large and small firms.

As Markowitz (1957, 1959) demonstrated if a portfolio is well diversified, the variance of the portfolio will be determined by the average of the covariances between securities rather, than by the average of the security variances. When a stock is thinly traded there will be an abundance of zeros in the return time series, particularly, when the returns are observed at frequent intervals. The covariance between two random variables will be zero if one of the variables is a constant. The presence of constant segments within a time series of a random variable will reduce the measured covariance with another random variable. Thus, the average covariance between the returns of thinly traded stocks is likely to be understated relative to that

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9. External diversification occurs when different securities are combined in a portfolio. The effect of this will be to reduce the amount of systematic risk in the portfolio. If a company diversifies by operating in different markets, or by acquiring different outlets for its business, internal diversification takes place. When internal diversification takes place, diversifiable risk is reduced because poor performance in one market is offset by superior performance in an alternative market. This will reduce the overall return variability of a company's stock.
of frequently traded stocks. Since small firms are characterised by more thin trading than large firms, the average covariance of small stocks will be more downwards biased than the average covariance of large firms. A portfolio of small firms, which have identical virtual return distributions to a portfolio of large firms, except that the small firm stocks trade less frequently will have a lower standard deviation than the portfolio of large firms. The bias which is introduced into the covariances because of thin trading explains why the small firm portfolio in this sample appears to have a lower return standard deviation. This issue is also discussed in Lo and MacKinlay (1990a, 1990b) but rarely is it that in the US the bias is strong enough to indicate that the variances or standard deviations of small firm portfolios are lower than for large firms, see for example, Lo and MacKinlay (1990a). Although this is not surprising since US markets are not characterised by such acute thin trading as UK markets.

Confirmation that thin trading has biased the portfolio standard deviations of the small firms is provided by comparing the average return standard deviations of the individual securities within each portfolio. These are presented in Table 4.3. For both weekly and monthly return intervals, the average standard deviations of the smallest firms in the sample are considerably larger than the average return standard deviations for the largest firms. When returns are sampled monthly the average return standard deviations for the smallest firms is 0.0305, but the average standard deviation for the largest firms is 0.0189. When returns are sampled at weekly intervals the average standard deviations are 0.0739 and 0.0468 respectively. This indicates, that counter to what the portfolio standard deviations suggest the small firms are indeed more risky than the large firms. Meanwhile, it should also be noted that the average standard deviation of the individual securities is substantially larger than the portfolio standard deviation. This is to be expected as each of the portfolios is fully diversified and should therefore be less risky because most of the idiosyncratic which is risk present in the individual stocks would have been diversified away at a portfolio level.

### 4.2.3 Autocorrelations

The estimated autocorrelations for the portfolios are reported in Table 4.2. These autocorrelations mirror the findings of previous studies such as those undertaken by Lo and MacKinlay (1988) which have shown that short-horizon portfolio returns are highly autocorrelated. Although the first order autocorrelations are large for all of the portfolios, an obvious pattern associated with firm size appears to exist. The first order portfolio autocorrelation coefficient for the smallest firms in the sample, when returns are sampled weekly is about 40 percent but only 14.8 percent for the portfolio containing the largest firms. This pattern is also reflected in the portfolios where returns are sampled monthly. This time the first order autocorrelations are 48 percent and 17 percent respectively. Since thin trading introduces
autocorrelation into portfolio returns the high autocorrelations associated with the small firm portfolios is not surprising. This issue was discussed in detail in Section 2.7.2 of Chapter 2 and Section 3.2.3 of Chapter 3.

An examination of autocorrelations for lags greater than one period suggests that the autocorrelation decays fairly rapidly, especially when returns are sampled monthly. For monthly returns, the second order autocorrelation coefficient is insignificant for all but the portfolio containing the smallest firms. For \( P_1 \) which is the portfolio containing the smallest firms, the autocorrelation takes substantially longer to decay than is the case for the other portfolios, for example, the sixth order autocorrelation coefficient of the small firm portfolio when returns are sampled weekly is still significant.

The average of the autocorrelations of the component securities in each of the four capitalisation-based portfolios are presented in Table 4.3. These are calculated as

\[
\bar{\rho}_t = \frac{1}{50} \sum_{i=1}^{50} \rho_{i,t}
\]

(4.4)

where, \( \rho_{i,t} \) are the \( \tau \)th order autocorrelations for component security \( i \).

Examining the average of the first order security autocorrelations, reveals that for all four groups, at both weekly and monthly return intervals the autocorrelations of individual securities are very small. For monthly returns the average of the security first order autocorrelations for the small firms is only 0.008 and -0.040 for the largest group of firms. For weekly returns the average of the security first order autocorrelations are 0.055 and 0.039 respectively. For every size grouping the average of the security autocorrelations appear to be substantially smaller than is the case for the portfolios. It is possible to conclude that, although, portfolio returns appear to be autocorrelated individual security returns are not. This is an important empirical characteristic to identify because it indicates that portfolio returns are much more predictable than security returns. This does not imply that the UK stock market is inefficient, what the results suggest is that there are important differences between securities and portfolios. If this predictability was due to market inefficiency then at least some of it should be able to be identified in the security returns. This suggests that the predictability in the portfolio returns is more likely to have been caused by microstructure frictions and time variation of expected returns.

"Volatility clustering" describes what Fama (1965) and Mandelbrot (1966) identified as the tendency for small returns to follow small returns and large returns to follow large returns. Both Fama and Mandelbrot found that the unconditional distribution of returns had a leptokurtotic appearance. Such a distribution can be identified by the fat tailed and peaked appearance of the distribution. The fat tails capture the tendency for large returns to follow large returns, while the peaked appearance captures the tendency for small returns to be followed by
small returns. Autocorrelation in the squared and absolute returns may be indicative of "volatility clustering" which would imply a changing conditional variance. Meanwhile, as Granger and Anderson (1978) demonstrate autocorrelation of squared returns may suggest that the conditional variance of returns is autocorrelated.

For monthly portfolio returns, the autocorrelation in the squared and absolute returns are all very close to the autocorrelations in the pure return series. For the weekly portfolio returns, the autocorrelations of all the portfolios are substantially greater than the autocorrelations of the return series. For example, for the portfolio containing the smallest firms the first order autocorrelation coefficient of weekly squared returns is 0.604. For the portfolio containing the largest firms the first order autocorrelation coefficient for squared returns is 0.428. In the weekly return series strong support exists for a changing conditional variance and or variance persistence. This issue will be investigated in detail in Chapter 7 and Chapter 8.

Interestingly, in the case of the individual securities there appears to be no evidence of autocorrelation in either the squared or the absolute values of monthly returns. The average of the autocorrelations for the squared and absolute security returns provide autocorrelations which are all close to zero. In contrast, for weekly security returns, the average first order security autocorrelation is strikingly larger for both the squared and absolute returns than is the case for the pure return series. First order autocorrelation in the absolute weekly returns for each of the four portfolios lies between the ranges of 15.2 percent and 18.5 percent. For weekly individual returns there may be enough evidence to suggest that the conditional variance may be time varying. The possibility that second moments are time varying is important, because, it suggests that expected risk premiums rather than being a constant may be time varying which provides additional support for the possibility that the autocorrelation in the portfolio return series is at least partially reflecting a time varying expected return, which is autocorrelated.

4.3 A Predictability Test of Share Prices using Co-integration Analysis

In the previous section it was demonstrated that some of the variation in the return of a portfolio can be predicted from its most recent values, although, this did not appear to be the case for individual securities. This section will examine the predictability of individual stock prices by applying co-integration tests. Co-integration tests provide an alternative way, to the autocorrelation test of capturing predictability in prices. The chief advantage of undertaking co-integration tests is that this approach allows two sources of predictability, that is serial and cross serial correlation, to be captured in a single test. This will be demonstrated in Section 4.2.
4.3.1 Co-integration and Stationarity

Many examples of a time series may be non-stationary, displaying the absence of a constant mean or variance, but some examples of such a time series move together. If there is a linear relationship between a set of non-stationary variables but they are related to each other, so that in the long run they do not drift too far apart, they are said to be co-integrated. In this case the non-stationarity is due to a common factor. A detailed explanation of the co-integration methodology and its applications can be found in Engle and Granger (1991), Banerjee, Dolado, Galbraith and Hendry (1993) and Charemza and Deadman (1993).

A key consideration in the discussion of co-integration is stationarity. A stationary stochastic process is one where the conditional distribution of the random variable is invariant with respect to displacement in time. Weak stationarity, which is less restrictive, implies that the mean, variance and covariance of the process are invariant to displacement in time as follows. The mean of this process must be constant, so that

\[ E(x_t) = E(x_{t+m}) \quad t = 1, 2, \ldots \quad m = 1, 2, 3; \quad (4.5) \]

the variance of the series,

\[ \sigma^2_x = E(x_t - \mu_x)^2 \quad (4.6) \]

must be constant, so that

\[ E(x_t - \mu_x)^2 = E(x_{t+m} - \mu_x)^2; \quad (4.7) \]

the covariance of the series

\[ Cov(x_t, x_{t+k}) = E(x_t - \mu_x)(x_{t+k} - \mu_x) \quad k = 1, 2, 3, \ldots \quad (4.8) \]

must also be invariant with respect to displacement in time, so that

\[ Cov(x_t, x_{t+k}) = Cov(x_{t+m}, x_{t+m+k}). \quad (4.9) \]

If a time series has a joint probability distribution and conditional probability distribution which are both invariant to time, the time series is said to be a strictly stationary series. The opposite of a stationary process is one which is said to be non-stationary. In which case the series will have a time-varying mean, variance and covariance so that it becomes impossible to refer to the mean, variance or covariance of the time series without knowing a particular time period for reference.

The most straightforward example of a non-stationary variable is a random walk without drift which is represented by equation (4.10), and was discussed earlier in Section 2.2 of Chapter 2.
\[ x_t = x_{t-1} + e_t \]  

\[ E(e_t) = 0 \quad \text{and} \quad E(e_t e_s) = 0 \quad t \neq s \quad E(e_t^2) = \sigma^2 \]

which can be expressed as

\[ x_t = x_0 + \sum_{j=0}^{t-1} e_{t-j} \quad t = 1, 2, \ldots, T \]  

so that if \( x_0 = 0 \) then \( x_t \) is the sum of its own past innovations. The variance of \( x_t \) is \( t\sigma^2 \) and therefore becomes infinitely large as \( t \to \infty \). Under these conditions \( x_t \) has little meaning as a time series. Such a series would have a different mean, variance and covariance at different points in time.

A time series is said to be \( I(d) \) if it is integrated of the order \( d \), so that the time series is non-stationary in its level but becomes stationary after differencing \( d \) times. Thus, a series \( \{x_t\} \) is \( I(d) \) if \( x_t \) is non-stationary but \( \Delta^d x_t \) is stationary. Usually, if we take a linear combination of two non-stationary series such as \( \{x_t\} \) and \( \{y_t\} \); each integrated to a different order, then the resulting series will be integrated at the highest of the two orders of integration. This can be demonstrated below. Assume \( x_t - I(d), y_t - I(e) \) where \( e > d \). If \( z_t \) is a linear combination of the two series \( x_t \) and \( y_t \) such that

\[ z_t = \beta x_t + \alpha y_t \]

where, \( z_t \) is a linear combination of \( x_t \) and \( y_t \), while \( \alpha \) and \( \beta \) are scaling constants.

If \( z_t \) is differenced \( d \) times the resulting series would be

\[ \Delta^d z_t = \beta \Delta^d x_t + \alpha \Delta^d y_t \]  

\( \beta \Delta^d x_t \) is stationary, since \( x_t - I(d) \) but the second term on the right hand side \( \alpha \Delta^d y_t \) is not since to make it stationary it must be differenced \( e \) times, and \( e > d \). It therefore requires further differencing. Since \( \Delta^d z_t \) is the sum of a stationary series \( \beta \Delta^d x_t \) and a non-stationary series \( \alpha \Delta^d y_t \), then \( \Delta^d z_t \) will also be non-stationary.

In general any linear combination of stochastic time series variables has an order of integration equal to the highest order of the component series. The exception to this rule are co-integrated variables. We would normally expect a linear combination of two \( I(1) \) variables such as \( x_t \) and \( y_t \) to be \( I(1) \) also.

\[ z_t = \beta x_t - \alpha y_t \]  

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If, however, the linear combination is stationary such that \( z_t \) in equation 4.14 is I(0), and therefore stationary, \( x_t \) and \( y_t \) are said to be co-integrated. If this is the case, \( \alpha \) is known as the constant of co-integration, and is unique in the two variable case when \( \beta \) is normalised to equal 1. This means equation (4.14) can be written as follows.

\[
z_t = x_t - \alpha y_t
\]  

(4.15)

In the case of more than two variables, \( \alpha \) becomes the co-integrating vector.

The concept of co-integration captures the long run properties of a stochastic time series. More specifically, co-integration indicates that in the long run \( x_t \) and \( y_t \) can not drift too far apart since \( z_t \), the difference between \( x_t \) and \( y_t \), is stationary with zero mean and therefore due to a common factor. Co-integration therefore implies a long run, or equilibrium relationship, such that \( x \) will return to the value \( \alpha y \) in the long run and \( y \) will return to the value \( \alpha^{-1}x \) in the long run. A lack of co-integration suggests that variables have no long run link and in principle they can wander arbitrarily far away from each other.

### 4.3.2 Co-integration and Error Correction Mechanisms

An error correction model exists when the value of a series such as \( \{x_t\} \) adjusts to equilibrium at a rate proportional to the degree of disequilibrium in the previous period. An important relationship between co-integrated variables and error correction processes was introduced in Granger (1983) and more formally discussed in Engle and Granger (1987). The so called Granger Representation Theorem demonstrates that if two variables are co-integrated then the following error correction process must also exist.

\[
\phi(L)(1-L)X_t = -\alpha'X_{t-1} + \theta(L)\epsilon_t
\]  

(4.16)

where, \( \theta(L) \) and \( \phi(L) \) are finite polynomials. The \( L \) is the lag operator, \( X \) is an \( N \times 1 \) vector, and \( \alpha \) is the co-integrating vector such that \( \alpha'X_{t-1} = 0 \).

More specifically, for the two-variable case, where \( x_t \) and \( y_t \) are co-integrated of the order (1,1) the following error correction model must exist.

\[
\Delta x_t = a_x + \rho_x z_{t-1} + \sum_{\tau=1}^{T} \gamma_{\tau} \Delta y_{t-\tau} + \sum_{\tau=1}^{T} \delta_{\tau} \Delta x_{t-\tau} + \epsilon_{x,t} \tau = 1, 2, 3, \ldots T
\]

(4.17)

\[
\Delta y_t = b_y + \rho_y z_{t-1} + \sum_{\tau=1}^{T} \theta_{\tau} \Delta y_{t-\tau} + \sum_{\tau=1}^{T} \iota_{\tau} \Delta x_{t-\tau} + \epsilon_{y,t} \tau = 1, 2, 3, \ldots T
\]

(4.18)

where, \( z_{t-1} = x_{t-1} - \alpha y_{t-1} \). \( \Delta x_t \) and \( \Delta y_t \) are the lagged first differences of \( x_t \) and \( y_t \). The term \( \rho_x + \rho_y \neq 0 \) which requires that their sum is non zero. This implies that \( z_{t-1} \) must have an influence over at least one of the variables \( x_t \) or \( y_t \).
The error correction model specified above indicates that the current change in at least one of the variables $x_t$ or $y_t$ can be predicted from $x_{t-1} - \alpha y_{t-1}$, which means that changes in either $x_t$ or $y_t$ can be predicted from past values of at least one of the series. This implies that if two variables are co-integrated then they must either be cross-serially correlated or serially correlated. For speculative assets such as stock prices the error correction representation has an important application. If $x_t$ and $y_t$ are two share prices, then the existence of an error correction model of this form suggests that at least one of the share prices is predictable from the extent to which the two prices were away from their long-run relationship in the previous period. In which case co-integration can be used as a test of stock market predictability because co-integration between share prices implies that the current value of a share price can be predicted from past values of at least one other price. In the case of stock prices, which are likely to be characterised by predictability for reasons unassociated with market inefficiency, this test is not a test of market inefficiency but is a test of price predictability.

4.3.3 Testing For Co-integration

In a bi-variate co-integration test the first step is to establish that the two variables being tested are both I(1) variables. This can be done using the Dickey-Fuller (1979) test. For a stochastic time series such as $\{x_t\}$ Dickey-Fuller tests are concerned with discovering whether the value of $\alpha$ in equation (4.19)

$$x_t = \alpha x_{t-1} + e_t$$ (4.19)

is equal to one (which implies $x_t$ is non-stationary) or less than one (in which case $x_t$ is stationary). If $\alpha$ is assumed to equal 1 equation 4.19 becomes a random walk. Since the variance of a random walk tends to infinity as $t$ increases, the assumptions underlying the standard t-tests for least squares will not be valid. In this case equation (4.19) must be transformed as follows,

$$x_t - x_{t-1} = (\alpha - 1)x_{t-1} + e_t$$ (4.20)

Such a transformation is necessary if the variance of $x$ is assumed to be constant. This equation can be re-paramaterised to provide

$$\Delta x_t = \beta x_{t-1} + e_t$$ (4.21)

where $\beta = \alpha - 1$, the test is now valid using least squares but the test is now a test of whether $\beta$ is zero. If $\beta$ is negative then it implies that $\alpha$ is less than one and then $x$ is stationary. The critical values for this test, which are non standard, are provided by Fuller (1976). A modification of this test, which adapts the test for autocorrelated variables is provided by Dickey-Fuller (1981). The modification requires introducing the lagged dependant variable into equation (4.21) to eliminate autocorrelation in $x_t$ so that the residuals in equation (4.21) are white noise. The
appropriate order of this test, is determined by the number of lags of $x_i$ which must be introduced to eliminate the autocorrelation which is present. These tests, have as the null hypothesis that $x_i$ is non-stationary. Alternative unit root tests which can be used when $x_i$ is autocorrelated are provided by Phillips and Perron (1988). The sensible approach, is to first of all test the data in its level. If the data appears to be non-stationary, the data should then be differenced and tested for stationarity once again. This will identify whether after differencing the series becomes stationary which would provide confirmation that the undifferenced series is I(1).

One way of testing for co-integration is to use the two step regression technique proposed by Engle and Granger (1987). The first step requires the estimation of the following regression for two variables which are both I(1).

$$x_t = \alpha y_t + z_t$$  \hspace{1cm} (4.22)

where, $x_t$ and $y_t$ are both I(1) variables, so that after differencing once they become stationary. The term $z_t$ is the residual from the co-integrating equation. This can be interpreted as the extent to which $x_t$ and $y_t$ are away from their long-run equilibrium relationship.

The second step is then to test the residuals $z_t$ from the co-integration regression in order to establish whether they are stationary. If these residuals are stationary then the series is said to be co-integrated. A serious problem with this procedure is that a finding of co-integration or no co-integration may depend on the order in which the variables $x_t$ and $y_t$ are regressed.

In this chapter all the co-integration tests which have been performed have been bi-variate Johansen (1988) co-integration tests. A brief description of this procedure follows.

Consider an $N \times 1$ vector of I(1) variables so that the elements in $P$ include the share prices of $N$ companies. The Johansen approach to co-integration begins by expressing the data generation process of $P$ as an autoregressive process. In the tests which are performed here seven lags have been included, although, the results do not appear to be sensitive to the number of lags chosen in the autoregressive representation.

$$P_t = \pi_1 P_{t-1} + \pi_2 P_{t-2} + \ldots + \pi_k P_{t-k} + e_t \hspace{1cm} \tau = 1, 2, 3...7$$  \hspace{1cm} (4.23)

where, $\pi_k$, are coefficient matrices, and $e_t$ is an $N \times 1$ vector which has zero mean and constant variance.

The long-run equilibrium corresponding to 4.23 is $\Pi P = 0$, where the long run co-efficient matrix $\Pi$ is defined as

$$\Pi = I - \pi_1 - \pi_2 - \pi_3 - \ldots - \pi_k$$  \hspace{1cm} (4.24)

where,
\[ \Pi = \text{an } N \times 1 \text{ matrix whose rank}^{10} \text{ determines the number of distinct co-integrating vectors} \]

between the variables in \( P \), \( I = \text{the identity matrix.} \)

If the matrix \( \Pi \) has rank \( r < N \), then it is possible to define two \( N \times r \) matrices \( \alpha \), and \( \beta \), such that

\[ \Pi = \alpha \beta' \] (4.25)

The rows of \( \beta \) form the \( r \) distinct co-integrating vectors and \( \alpha \) is called the "loading" or "adjustment" variable whose value reflects the speed of adjustment to the long run equilibrium. Although, the values of \( \alpha \) and \( \beta \) cannot be estimated directly, since they are over-parameterised, \( r \) can be identified in the following way. The residuals from two regressions are collected. These are the residuals from regressing both \( P_t - k \) and \( \Delta P \), on lagged differences. The squares of the residuals of the canonical correlation coefficients between the two sets of residuals are computed.\(^{11} \) Then, Johansen shows that the likelihood ratio LR statistic or trace test for the hypothesis that there are at most \( r \) co-integrating vectors is

\[ LR = -T \sum_{i=r+1}^{N} \ln(1 - \hat{\lambda}_i^2) \] (4.26)

where, \( \hat{\lambda}_{r+1}, \ldots, \hat{\lambda}_N \) are the smallest \( N - r \) squared canonical correlation coefficients. Johansen shows that this statistic will have a non-standard distribution under the null hypothesis and provides approximate critical values for the statistic which have been generated from Monte Carlo simulation.

4.3.4 Previous Work

Copeland (1991) tested the extent to which a sample of foreign exchange spot prices were predictable by using the co-integration methodology. No co-integrating relationships were found to exist between any of the spot markets investigated. The paper therefore concluded that the foreign exchange spot prices which were investigated appeared to be unpredictable and therefore efficient. In a later paper, Copeland (1993) uses the same methodology to test the efficiency of the forward market. These tests were concerned with whether the forward market rate was an unbiased predictor of future spot rates, as we would expect if the forward prices were set efficiently. This time the forward prices tested were found to be predictable and therefore seemingly inefficient. Co-integration tests have also been applied to metal prices on the London Metal Exchange. MacDonald and Taylor (1988), for example, use the co-integration approach.

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10. The rank of a matrix is the number of linearly independent rows or columns.

11. Canonical correlations may be defined in the following manner. Given two sets of data defined by the matrices \( Y = [Y_1, Y_2, \ldots, Y_j] \) and \( X = [X_1, X_2, \ldots, X_k] \) where \( i \geq k \), the object of the procedure is to find those linear combinations of \( Y \) and \( X \) which show the highest degree of correlation.
to confirm that metal prices on the London Metal Exchange are unpredictable. Because most
of these studies claim that the prices investigated are unpredictable the authors conclude that
the markets investigated must be efficient.

Although, co-integration tests have been used extensively in the foreign exchange market
they have been used less extensively as a way of testing the predictability of share prices.
Essentially, there are two ways of testing for predictability in stock market prices using
cointegration. Either, the predictability of the stock prices can be tested directly by ascertaining
whether one stock price can be predicted from another. MacDonald and Power (1993) have used
cointegration to test for stock price predictability in this way for a selection of security prices.
In this paper they argue that some of the prices which are tested must be inefficient because in
multiple co-integration tests some evidence of cointegration is found to exist. Clearly, in view
of the microstructure work which was discussed in Section 2.7 of Chapter 2 this is not the case.
The paper of MacDonald and Power has unrealistic aims, they suggest

"unit root and co-integration techniques are to test the concept of static efficiency for
individual share prices"\(^{12}\) MacDonald and Power (1993,p251)

The co-integration test can not indicate whether markets are efficient or inefficient, they
can only indicate whether there is a serial or cross-serial relationship between share prices. Such
a relationship need not exist because of market inefficiency but can be introduced by
non-synchronous trading. This possibility is not even discussed by MacDonald and Power and
seems a much more likely explanation for their results. No other study has investigated whether
stock markets are predictable using co-integration in this way.

Alternatively, co-integration techniques can be used to test the validity of some model
of stock price behaviour, such as the present value model. Tests of the present value model such
as those conducted by Campbell and Shiller (1987,1989) were concerned with finding a
stationary difference between stock prices and dividends (or some other measure of fundamental
cash flows such as earnings). Since stock prices, in an efficient market, represent an optimal
forecast of future expected dividends, stock prices and dividends should move together in the
long run. The two series should therefore be co-integrated. If they are not co-integrated, it
suggests that there is an unstable differential between the two series, that is they are not moving
together so prices could not be an optimal forecast of future expected dividends. Campbell and
Shiller (1987) conclude that stock prices do not appear to be consistent with the dividend
valuation model, prices appear to react to factors other than changing expectations about
dividends, stock prices therefore appear to be inefficient.

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\(^{12}\)Static market efficiency is described as share prices reflecting all available information but assuming that expected returns are constant.
4.3.5 Stock Price Predictability, Co-integration and Firm Size.

The aim of this section is to test for predictability in a sample of UK individual stock prices using the Johansen (1988) co-integration approach. The tests which are conducted in this section are concerned with whether stock prices can be predicted from past share price values. The primary objective when undertaking the tests is to identify whether the prices of small or large firms are more predictable using the co-integration methodology.

The monthly share price data which is used in this section has been collected from LSPD. Price data for eighteen individual companies, which were selected randomly from the LSPD database, for the sample period January 1975 to December 1989 were utilised in this study. On the basis of the market capitalisation of the companies in January 1975, the stocks were ranked on the basis of firm size. The eighteen firms used in this study are listed in Table 4.4 along with their capitalisation values at the start of the sample period. For ease of reference the companies have been coded from L1 which signifies the largest company in the sample to L18 which signifies the smallest.

All of the co-integration tests which have been conducted in this chapter have been bi-variate co-integration tests. A finding of co-integration would indicate that in the long run stock prices are predictable, meanwhile, a finding of substantially more co-integration associated with small firm stocks would imply that small firm stocks are more predictable than those of large firms.

4.3.6 Results

The security prices were transformed into the logarithm of the price and then using the Augmented Dickey-Fuller (1981) approach both the logarithm of the share prices and the first difference of the logarithm of the share prices were tested for stationarity.

The results of the Augmented Dickey-Fuller tests are reported in Table 4.5. In every case it is clear that we can not reject the null hypothesis $H_0$ that the share prices are I(1) against the alternative $H_A$ that the share prices are I(0). Since we can not reject that the share prices are I(1) we can now test whether the log prices, once differenced are I(1) against the alternative that they are I(0). If the null hypothesis is rejected then it would indicate that the differenced share prices were I(0) but prices in their levels were I(1).

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13. The aim of these tests is not to provide evidence of market efficiency or inefficiency, as discussed previously co-integration tests can not provide this evidence. The tests here are used as a diagnostic to indicate that some predictability in the individual share prices does exist. Even when only 18 share prices are tested, 153 co-integration tests are performed. Testing a larger number of share prices might therefore obscure any pattern which might emerge.

14. This was done because the log price difference in the error correction will be a return.
The results of this second set of Augmented Dickey-Fuller tests are provided in Table 4.6. The results indicate that in every case we can not accept the null hypothesis \( H_0 \) that the differenced log prices are \( I(1) \) and therefore reject the null hypothesis \( H_0 \) that the share prices once differenced are \( I(1) \). Since the Augmented Dickey-Fuller tests indicate that all of the share prices are \( I(1) \) it is possible to go ahead and test for co-integration between the share prices.

Since there are 18 share prices in this sample, bi-variate tests require that each of these 18 prices are tested against each of the other 17 share prices. This means that in all 153 co-integration tests are performed. The results from these tests are summarised in Tables 4.7(a), 4.7(b) and 4.8. Out of the 153 tests performed it was found that 41 of the tests provided a Johansen trace statistic which was significant at a 95 percent confidence level, Meanwhile, at a 90 percent confidence level 55 examples of co-integration were found to exist. The very large number of examples of co-integration which appear to exist suggest that many of the stock returns are predictable although this can not be taken as evidence of market inefficiency.

An aspect of interest in this chapter is whether there is a link between the predictability of the share prices and firm size. If Table 4.7(a) and Table 4.7(b) are studied it is possible to see that there are more co-integrating vectors identified for the small companies than for the large companies. Looking at Table 4.7(a), for the nine largest firms there are only 24 cases where a co-integrating vector has been identified, out of a maximum of 117 possible cases.

For the smallest six firms, out of eighty-seven possible examples of co-integration 35 co-integrating vectors were identified. In 40 percent of all cases at a 95 percent confidence level share prices appear to be predictable, rising to 44 cases and over 50 percent, at a 90 percent confidence level as we can see from Table 4.7(b). This appears to suggest that the stock prices of individual small firms are more predictable than is the case for the large firms.

Since proportionately small investors are more heavily represented in small firms the results presented here may suggest that small investors are less well informed. If small firm investors are less well informed than investors in large firms (this is possible because relatively little information is published about the performance of small firms) then the return behaviour of large firms provides information about the future return performance of small firms. If the behaviour of large firms provides information about the performance of small firms then current small firm returns will be related to the past returns of large firms, which could lead to a co-integrating relationship between small and large firms.

This pattern is further substantiated when the larger firms are investigated more closely. Most of the examples of co-integration which are identified for large firms arise because the large firm prices are co-integrated with small firm prices. Out of a possible 66 examples of co-integration between two companies that are both in the top twelve largest companies only
six were found to exist which represents only 9 percent. In contrast, of the seventy-two possible examples of co-integration between the smallest six companies and the largest companies twelve 26 cases (36%) are identified at the 90 percent confidence level.

4.3.7 Identifying the Direction of Causality

The presence of co-integration has been identified for a number of the stock prices used in this sample, although, the direction of causality has not been established. The pattern in the co-integration appears to indicate that the prices of the small firms are more predictable than is the case for the large firms, although it is worthwhile to carry out further tests to obtain additional information.

Causality can be identified by undertaking Granger (1969) causality tests which take the form of an augmented unrestricted vector autoregression represented by equation (4.27) and (4.28)

\[
\Delta p_{j,t} = \alpha_j + \beta_1 p_{j,t-1} + \beta_2 \Delta p_{k,t-1} + \sum_{i=1}^{2} \theta_i \Delta p_{j,t-\tau} + \sum_{i=1}^{2} \gamma_i \Delta p_{k,t-\tau} + e_{j,t} \quad \tau = 1, 2 \tag{4.27}
\]

\[
\Delta p_{k,t} = \alpha_k + \beta_3 p_{j,t-1} + \beta_4 \Delta p_{k,t-1} + \sum_{i=1}^{2} \phi_i \Delta p_{j,t-\tau} + \sum_{i=1}^{2} \delta_i \Delta p_{k,t-\tau} + e_{k,t} \quad \tau = 1, 2 \tag{4.28}
\]

where,

- \(\Delta p_{j,t}\) = the change in the logarithm of the share price of stock \(j\) in time \(t\). Because stock prices are tested in their log form \(\Delta p_{j,t}\) is the return of stock \(j\),
- \(\Delta p_{k,t}\) = the change in the share price of stock \(k\) in time \(t\). Stock \(k\) is a stock which is co-integrated with stock \(j\),
- \(\beta_1\) and \(\theta_i\) capture the effect that the past share price and returns of stock \(j\) have on the current return of stock \(j\),
- \(\beta_2\) and \(\gamma_i\) capture the effects that the past share price and the past returns of stock \(k\) have on the current return of stock \(j\),
- \(\beta_3\) and \(\phi_i\) capture the effect that the past share price and returns of stock \(j\) have on the current return of stock \(k\), and
- \(\beta_4\) and \(\delta_i\) capture the effect that the past share price and the past returns of stock \(k\) have on the current return to stock \(k\).

In this model only one lag in the price level and two lags in the price difference series were chosen because higher order lags were never found to be significant in preliminary testing which is not reported here.
The Granger test requires testing the restriction in equation (4.27) that \( \beta_2 + \sum_{i=1}^{2} \gamma_i = 0 \). This establishes whether collectively \( p_{k,t-1} \) and \( \sum_{i=1}^{2} \Delta p_{k,t-\tau} \) has a causal influence over \( \Delta p_{j,t} \). In equation (4.28) the restriction \( \beta_3 + \sum_{i=1}^{2} \phi_i = 0 \) is tested to establish whether collectively \( p_{j,t-1} \) or \( \sum_{i=1}^{2} \Delta p_{j,t-\tau} \) has a causal influence over \( \Delta p_{k,t} \).

For each pair of co-integrated price series Granger causality tests were performed in the manner described above. The results of both an F test and a Lagrange Multiplier (LM) variable addition test for each of the examples of co-integration are reported in Table 4.9. Looking at the results presented in Table 4.9 we can see that quite a weak pattern emerges. Typically, it seems that when there is evidence of co-integration, the prices of large firms appear to be able to predict the prices of small firms. In 19 of the causality tests the results indicate that the prices of small firms can be predicted from the past behaviour of large firm prices. Although, it should also be noted that the results of 9 of the causality tests indicate that the prices of small firms appear to predict the prices of large firms, and in 13 cases the predictability is so weak it can not be identified in the causality test at all. Broadly speaking the results from the causality tests appear to confirm the pattern which was suggested by the co-integration results that is, the prices of small firms are more predictable than the prices of large firms. In a large number of cases causality does not appear to have been identified in either direction. This appears to suggest that co-integration is a more powerful test of the existence of causality than procedures which use Ordinary Least Squares. The tests which have been performed in this section indicate that the individual security prices of small firms appear to be more predictable that the security prices of large firms.

A caveat to the results presented here should be noted. The aim of the co-integration tests which have been performed in this chapter has been to establish whether a stationary difference between two non-stationary sets of share prices exists. Over time, this means that if for example, the stocks \( L_1 \) and \( L_2 \) were co-integrated and furthermore that stocks \( L_1 \) and \( L_3 \) were co-integrated, then \( L_1 \) and \( L_3 \), should also be co-integrated. In the results presented here this is not the case. A triangular relationship between the co-integration results fails to hold in many cases as Table 4.7(a) and Table 4.7(b) suggests. The probable explanation for the lack of a triangular relationship may be due to the existence of Type I and Type II errors. The existence of Type I and Type II errors suggests that co-integration is being accepted sometimes when it should be rejected and
in other cases co-integration is being rejected when it should be accepted. If co-integration is being accepted when it should be rejected and vice versa a triangular relationship will fail to hold.

4.4 Conclusion

In this chapter a preliminary discussion of the empirical characteristics of short-horizon UK security and portfolio returns was undertaken. This chapter tested the autocorrelation properties of four capitalisation stratified portfolios and for four groups of individual securities which had been organised on the basis of firm size. It was found that the portfolios of small firms are much more autocorrelated than portfolios of large firms. Not only were the first lag autocorrelations found to be larger for small firms but the autocorrelations also tended to be significant for longer lags than was the case for the large firms. This suggests that the returns of portfolios which contain small firms are more predictable than portfolios which contain large firms.

When the cross-sectional averages of autocorrelations for the individual securities were estimated it became obvious that the portfolio autocorrelation patterns were not shared by individual securities, even for the small firms. This suggests that inefficiency is probably not the cause of the return predictability at a security level because we would expect to observe some of this predictability at a portfolio level also. This suggests that the predictability may be due to some other cause such as non-synchronous trading or time variation in expected returns, or both.

The squared and absolute autocorrelations for the portfolios, and for individual returns when they were sampled at weekly intervals suggested that the conditional variance was changing. This provides some support for the possibility that expected returns are time varying which will be investigated more fully in Chapter 7 and Chapter 8.

In the second set of predictability tests, which were undertaken in this chapter, a set of individual security prices were tested using the co-integration approach. The co-integration test provides an alternative way of testing for predictability. The co-integration tests which are performed here concentrate on identifying whether there is an equilibrium relationship between two share prices. In which case, if the equilibrium relationship is disturbed temporarily at least one of the share prices must react in a predictable way to restore the equilibrium relationship.

The results of the co-integration tests suggested that the securities which were tested were in some cases predictable. Furthermore, the predictability appeared to be associated with firm size. The conclusion of this chapter is that the share prices of small firms appear to be more predictable than the share prices of large firms. Although, the nature of this predictability does not appear to be detectable in traditional autocorrelation tests. The results from these
co-integration tests are not assumed to provide evidence of market inefficiency, as was stated in the previous chapter predictability does not necessarily assume that markets are inefficient, a higher burden of proof is required than is provided by these tests.

Further research should be directed at investigating the causation of portfolio and individual security return autocorrelation. Although, Conrad and Kaul (1988) (1990) have attempted to estimate the causes of predictable variations in expected returns, caused by time variation of expected returns and other factors in US portfolio and security returns, a detailed analysis of UK portfolio returns or UK individual security returns has never been undertaken. Further research should also be directed at ascertaining the role that firm size has to play in determining the magnitude of autocorrelation introduced by microstructure considerations and reconciling any relationships to theoretical microstructure models.

One study has attempted to gauge the magnitude of negative autocorrelation introduced by bid-ask bounce. Pagano and Roell (1990) investigated stocks which were listed on both the London stock exchange and the Paris Bourse. In order to estimate the magnitude of the bid-ask spread on the London stock exchange Pagano and Roell (1990) estimated the first order serial correlation coefficient for dually listed stocks. They found evidence of strong negative autocorrelation, negatively related to firm size.

As this chapter has shown co-integration is not always a very useful test to be employed if markets are being tested to ascertain whether they are efficient or not. Despite this the use of co-integration tests has been widespread. Further research needs to investigate whether the information provided by co-integration analysis can be used in a trading rule. Only if the information provided by co-integration can be used in a trading rule which provides profits in excess of buy and hold returns can we be sure that the existence of co-integration has provided evidence of market inefficiency.
Table 4.1

where, Mean is the mean monthly return for all stocks listed on the LSPD database, \( \sigma \) refers to the standard deviation of the LSPD returns. Min and Max refer to the minimum and the maximum return experienced during a given sample period. Skewness is the skewness statistic.

<table>
<thead>
<tr>
<th>Sample</th>
<th>Mean</th>
<th>( \sigma )</th>
<th>Min</th>
<th>Max</th>
<th>Skewness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1976-1991</td>
<td>0.111</td>
<td>0.06</td>
<td>-0.441</td>
<td>0.151</td>
<td>-2.61</td>
</tr>
<tr>
<td>1976-1984</td>
<td>0.013</td>
<td>0.05</td>
<td>-0.217</td>
<td>0.140</td>
<td>-0.95</td>
</tr>
<tr>
<td>1985-1991</td>
<td>0.008</td>
<td>0.07</td>
<td>-0.441</td>
<td>0.113</td>
<td>-3.50</td>
</tr>
</tbody>
</table>
Table 4.2

This table contains summary statistics for monthly realised returns, squared returns, and absolute returns for four equally weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 to December 1991. %R is the mean percentage return and \( \sigma(R) \) is the return standard deviation, but because this is a small value to prevent confusion this has been raised by three decimal places. \( \hat{\rho}_c \) are the estimated autocorrelation coefficients at lag \( \tau \). Under the null hypothesis that these coefficients are zero their standard errors are \( \frac{1}{\sqrt{n}} = 0.035 \) for weekly returns and 0.0722 for monthly returns.

### Monthly Returns

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>%R</th>
<th>( \sigma(R) \times 10^3 )</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
<th>( \hat{\rho}_3 )</th>
<th>( \hat{\rho}_4 )</th>
<th>( \hat{\rho}_5 )</th>
<th>( \hat{\rho}_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>1.25</td>
<td>4.493</td>
<td>0.476</td>
<td>0.141</td>
<td>0.011</td>
<td>0.031</td>
<td>0.072</td>
<td>0.106</td>
</tr>
<tr>
<td>R₂</td>
<td>1.09</td>
<td>3.992</td>
<td>0.361</td>
<td>0.112</td>
<td>0.019</td>
<td>-0.020</td>
<td>-0.072</td>
<td>0.029</td>
</tr>
<tr>
<td>R₃</td>
<td>1.37</td>
<td>5.420</td>
<td>0.228</td>
<td>-0.005</td>
<td>-0.091</td>
<td>-0.067</td>
<td>-0.040</td>
<td>-0.023</td>
</tr>
<tr>
<td>R₄</td>
<td>1.04</td>
<td>6.516</td>
<td>0.169</td>
<td>-1.00</td>
<td>-0.154</td>
<td>-0.071</td>
<td>0.027</td>
<td>-0.056</td>
</tr>
<tr>
<td>( (R₁)^2 )</td>
<td>0.22</td>
<td>0.517</td>
<td>0.419</td>
<td>0.010</td>
<td>0.236</td>
<td>0.387</td>
<td>0.185</td>
<td>-0.007</td>
</tr>
<tr>
<td>( (R₂)^2 )</td>
<td>0.17</td>
<td>0.347</td>
<td>0.253</td>
<td>-0.052</td>
<td>0.051</td>
<td>0.253</td>
<td>0.069</td>
<td>-0.039</td>
</tr>
<tr>
<td>( (R₃)^2 )</td>
<td>0.31</td>
<td>0.650</td>
<td>0.314</td>
<td>-0.005</td>
<td>0.107</td>
<td>0.226</td>
<td>0.135</td>
<td>-0.023</td>
</tr>
<tr>
<td>( (R₄)^2 )</td>
<td>0.43</td>
<td>1.06</td>
<td>0.172</td>
<td>0.049</td>
<td>0.002</td>
<td>-0.007</td>
<td>-0.009</td>
<td>-0.037</td>
</tr>
<tr>
<td>| R₁ |</td>
<td>3.41</td>
<td>3.17</td>
<td>0.345</td>
<td>0.027</td>
<td>0.137</td>
<td>0.217</td>
<td>0.217</td>
<td>-0.019</td>
</tr>
<tr>
<td>| R₂ |</td>
<td>3.13</td>
<td>2.29</td>
<td>0.265</td>
<td>-0.019</td>
<td>-0.009</td>
<td>0.152</td>
<td>0.047</td>
<td>-0.053</td>
</tr>
<tr>
<td>| R₃ |</td>
<td>4.10</td>
<td>3.38</td>
<td>0.218</td>
<td>0.073</td>
<td>0.049</td>
<td>0.163</td>
<td>0.120</td>
<td>-0.024</td>
</tr>
<tr>
<td>| R₄ |</td>
<td>4.83</td>
<td>4.49</td>
<td>0.194</td>
<td>0.047</td>
<td>0.030</td>
<td>0.027</td>
<td>0.041</td>
<td>-0.035</td>
</tr>
</tbody>
</table>

### Weekly Returns

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>%R</th>
<th>( \sigma(R) \times 10^3 )</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
<th>( \hat{\rho}_3 )</th>
<th>( \hat{\rho}_4 )</th>
<th>( \hat{\rho}_5 )</th>
<th>( \hat{\rho}_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>R₁</td>
<td>0.3046</td>
<td>14.886</td>
<td>0.401</td>
<td>0.258</td>
<td>0.145</td>
<td>0.118</td>
<td>0.134</td>
<td>0.095</td>
</tr>
<tr>
<td>R₂</td>
<td>0.3130</td>
<td>16.865</td>
<td>0.386</td>
<td>0.216</td>
<td>0.136</td>
<td>-0.127</td>
<td>-0.058</td>
<td>0.056</td>
</tr>
<tr>
<td>R₃</td>
<td>0.3331</td>
<td>22.047</td>
<td>0.298</td>
<td>-0.164</td>
<td>-0.052</td>
<td>-0.095</td>
<td>-0.016</td>
<td>-0.036</td>
</tr>
<tr>
<td>R₄</td>
<td>0.1809</td>
<td>23.765</td>
<td>0.148</td>
<td>-0.109</td>
<td>-0.021</td>
<td>-0.059</td>
<td>-0.039</td>
<td>-0.004</td>
</tr>
<tr>
<td>( (R₁)^2 )</td>
<td>0.23</td>
<td>0.752</td>
<td>0.604</td>
<td>0.343</td>
<td>0.134</td>
<td>0.062</td>
<td>0.052</td>
<td>0.048</td>
</tr>
<tr>
<td>( (R₂)^2 )</td>
<td>0.29</td>
<td>0.889</td>
<td>0.465</td>
<td>0.233</td>
<td>0.112</td>
<td>0.056</td>
<td>0.074</td>
<td>0.031</td>
</tr>
<tr>
<td>( (R₃)^2 )</td>
<td>0.49</td>
<td>1.363</td>
<td>0.379</td>
<td>0.150</td>
<td>0.052</td>
<td>0.058</td>
<td>0.007</td>
<td>0.004</td>
</tr>
<tr>
<td>( (R₄)^2 )</td>
<td>0.56</td>
<td>1.693</td>
<td>0.428</td>
<td>0.077</td>
<td>0.014</td>
<td>0.049</td>
<td>0.020</td>
<td>0.046</td>
</tr>
<tr>
<td>| R₁ |</td>
<td>10.55</td>
<td>10.924</td>
<td>0.344</td>
<td>0.282</td>
<td>0.146</td>
<td>0.090</td>
<td>0.069</td>
<td>0.077</td>
</tr>
<tr>
<td>| R₂ |</td>
<td>11.97</td>
<td>12.319</td>
<td>0.379</td>
<td>0.261</td>
<td>0.163</td>
<td>0.115</td>
<td>0.092</td>
<td>0.053</td>
</tr>
<tr>
<td>| R₃ |</td>
<td>15.83</td>
<td>15.651</td>
<td>0.305</td>
<td>0.203</td>
<td>0.067</td>
<td>0.106</td>
<td>0.030</td>
<td>0.040</td>
</tr>
<tr>
<td>| R₄ |</td>
<td>17.35</td>
<td>16.329</td>
<td>0.194</td>
<td>0.124</td>
<td>0.035</td>
<td>0.124</td>
<td>0.057</td>
<td>0.098</td>
</tr>
</tbody>
</table>
This table contains cross-sectional average summary statistics for weekly realised returns, squared returns, and absolute returns for four groups of securities, where each group of individual securities contains the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest stocks on the UK stock exchange. These groups were formed by ranking stocks according to market value of equity outstanding at the beginning of January of each year, January 1976 to December 1991. \( \sigma(\bar{R}) \) are the cross-sectional averages of the security return standard deviations. The \( \bar{\rho}_t \) are the average autocorrelations at lag \( t \) for the component securities within a portfolio. The standard errors for the weekly autocorrelations are approximately 0.035 for the monthly autocorrelations, the standard errors are approximately 0.0722.

### Monthly Returns

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(\bar{R}) \times 10^3 )</th>
<th>( \bar{\rho}_1 )</th>
<th>( \bar{\rho}_2 )</th>
<th>( \bar{\rho}_3 )</th>
<th>( \bar{\rho}_4 )</th>
<th>( \bar{\rho}_5 )</th>
<th>( \bar{\rho}_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>30.51</td>
<td>0.008</td>
<td>0.002</td>
<td>0.019</td>
<td>0.009</td>
<td>-0.001</td>
<td>-0.003</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>25.79</td>
<td>0.037</td>
<td>0.011</td>
<td>0.001</td>
<td>0.007</td>
<td>0.002</td>
<td>0.021</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>21.38</td>
<td>0.015</td>
<td>-0.041</td>
<td>-0.025</td>
<td>-0.019</td>
<td>-0.013</td>
<td>-0.006</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>18.88</td>
<td>-0.040</td>
<td>0.025</td>
<td>0.049</td>
<td>-0.008</td>
<td>-0.028</td>
<td>0.029</td>
</tr>
<tr>
<td>( (R_1)^2 )</td>
<td>0.008</td>
<td>-0.008</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.005</td>
<td>0.010</td>
<td></td>
</tr>
<tr>
<td>( (R_2)^2 )</td>
<td>0.095</td>
<td>0.073</td>
<td>0.060</td>
<td>0.067</td>
<td>0.069</td>
<td>0.053</td>
<td></td>
</tr>
<tr>
<td>( (R_3)^2 )</td>
<td>-0.042</td>
<td>-0.046</td>
<td>0.090</td>
<td>-0.055</td>
<td>-0.075</td>
<td>-0.061</td>
<td></td>
</tr>
<tr>
<td>( (R_4)^2 )</td>
<td>0.037</td>
<td>0.043</td>
<td>-0.003</td>
<td>-0.026</td>
<td>-0.002</td>
<td>0.024</td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>R_1</td>
<td>)</td>
<td>0.002</td>
<td>-0.009</td>
<td>0.005</td>
<td>0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td>(</td>
<td>R_2</td>
<td>)</td>
<td>0.121</td>
<td>0.041</td>
<td>0.041</td>
<td>0.047</td>
<td>0.051</td>
</tr>
<tr>
<td>(</td>
<td>R_3</td>
<td>)</td>
<td>0.098</td>
<td>0.055</td>
<td>0.043</td>
<td>0.051</td>
<td>0.034</td>
</tr>
<tr>
<td>(</td>
<td>R_4</td>
<td>)</td>
<td>0.049</td>
<td>0.030</td>
<td>-0.012</td>
<td>-0.041</td>
<td>-0.014</td>
</tr>
</tbody>
</table>

### Weekly Returns

<table>
<thead>
<tr>
<th></th>
<th>( \sigma(\bar{R}) \times 10^3 )</th>
<th>( \bar{\rho}_1 )</th>
<th>( \bar{\rho}_2 )</th>
<th>( \bar{\rho}_3 )</th>
<th>( \bar{\rho}_4 )</th>
<th>( \bar{\rho}_5 )</th>
<th>( \bar{\rho}_6 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_1 )</td>
<td>73.91</td>
<td>0.055</td>
<td>0.032</td>
<td>0.006</td>
<td>0.029</td>
<td>0.003</td>
<td>0.015</td>
</tr>
<tr>
<td>( R_2 )</td>
<td>69.90</td>
<td>0.066</td>
<td>0.024</td>
<td>0.008</td>
<td>0.001</td>
<td>0.011</td>
<td>-0.003</td>
</tr>
<tr>
<td>( R_3 )</td>
<td>50.05</td>
<td>0.106</td>
<td>0.045</td>
<td>0.001</td>
<td>0.010</td>
<td>-0.008</td>
<td>-0.014</td>
</tr>
<tr>
<td>( R_4 )</td>
<td>46.81</td>
<td>0.039</td>
<td>0.031</td>
<td>-0.034</td>
<td>-0.018</td>
<td>-0.034</td>
<td>0.004</td>
</tr>
<tr>
<td>( (R_1)^2 )</td>
<td>0.051</td>
<td>0.037</td>
<td>0.029</td>
<td>0.030</td>
<td>0.109</td>
<td>0.055</td>
<td></td>
</tr>
<tr>
<td>( (R_2)^2 )</td>
<td>0.085</td>
<td>0.046</td>
<td>0.054</td>
<td>0.035</td>
<td>0.037</td>
<td>0.019</td>
<td></td>
</tr>
<tr>
<td>( (R_3)^2 )</td>
<td>0.122</td>
<td>0.076</td>
<td>0.045</td>
<td>0.037</td>
<td>0.031</td>
<td>0.032</td>
<td></td>
</tr>
<tr>
<td>( (R_4)^2 )</td>
<td>0.185</td>
<td>0.105</td>
<td>0.067</td>
<td>0.065</td>
<td>0.049</td>
<td>0.059</td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>R_1</td>
<td>)</td>
<td>0.179</td>
<td>0.111</td>
<td>0.090</td>
<td>0.078</td>
<td>0.072</td>
</tr>
<tr>
<td>(</td>
<td>R_2</td>
<td>)</td>
<td>0.168</td>
<td>0.115</td>
<td>0.098</td>
<td>0.081</td>
<td>0.080</td>
</tr>
<tr>
<td>(</td>
<td>R_3</td>
<td>)</td>
<td>0.185</td>
<td>0.125</td>
<td>0.086</td>
<td>0.076</td>
<td>0.066</td>
</tr>
<tr>
<td>(</td>
<td>R_4</td>
<td>)</td>
<td>0.152</td>
<td>0.113</td>
<td>0.084</td>
<td>0.092</td>
<td>0.065</td>
</tr>
</tbody>
</table>

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Table 4.4
Companies Used in The Co-integration Study

The names of the companies which were used in the co-integration tests are provided below. Company refers to company name. Code refers to the code no given to each company name, L1 is the largest company at the start of the sample period while L18 is the smallest. Market capitalisation refers to the market capitalisation of companies in pounds as recorded for January 1976.

<table>
<thead>
<tr>
<th>Company</th>
<th>Market Capitalisation</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hawker Siddeley Group</td>
<td>62,831,700</td>
<td>L1</td>
</tr>
<tr>
<td>Rugby Group PLC</td>
<td>31,390,835</td>
<td>L2</td>
</tr>
<tr>
<td>Blue Circle Industries</td>
<td>20,597,880</td>
<td>L3</td>
</tr>
<tr>
<td>Pilgrom House Group</td>
<td>9,382,680</td>
<td>L4</td>
</tr>
<tr>
<td>Kalon Group</td>
<td>6,826,100</td>
<td>L5</td>
</tr>
<tr>
<td>Silentnight Holdings</td>
<td>6,375,795</td>
<td>L6</td>
</tr>
<tr>
<td>Manders (Holdings)</td>
<td>3,465,486</td>
<td>L7</td>
</tr>
<tr>
<td>Hunting Assoc Pl. Ind</td>
<td>2,543,620</td>
<td>L8</td>
</tr>
<tr>
<td>Wholesale Fittings</td>
<td>1,308,720</td>
<td>L9</td>
</tr>
<tr>
<td>Reylon Group PLC</td>
<td>1,107,111</td>
<td>L10</td>
</tr>
<tr>
<td>Clarke (T) and Co LTD</td>
<td>863,635</td>
<td>L11</td>
</tr>
<tr>
<td>Fortnum and Mason</td>
<td>825,000</td>
<td>L12</td>
</tr>
<tr>
<td>Toothill (R and W) LTD</td>
<td>688,512</td>
<td>L13</td>
</tr>
<tr>
<td>Dewhurst PLC</td>
<td>295,000</td>
<td>L14</td>
</tr>
<tr>
<td>Arlen PLC</td>
<td>260,000</td>
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Table 4.5
The Results of the Stationarity Tests on The Share Prices

This table provides the results of the Augmented Dickey Fuller unit root tests which were conducted to establish the order of integration for each of the share prices. Tests are conducted using the logarithm of the share price. ADF(1) is the Augmented Dickey Fuller test with one lag, ADF(3) is the Augmented Dickey Fuller test with three lags. The critical value at a 95% confidence level is -3.4373 for trended variables. An ADF(1) and an ADF(3) test was used to ensure that all of the autocorrelation in the return series was purged from the residuals.

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Table 4.6
The Results of the Stationarity Tests for The Differenced Share Prices

This table provides the results of the Augmented Dickey Fuller unit root tests which were conducted to establish the order of integration for each of the differenced share prices. Tests are conducted using the logarithm of the share price. ADF(1) is the Augmented Dickey Fuller test with one lag, ADF(3) is the augmented Dickey Fuller test with three lags. The critical value at a 95% confidence level is -3.4373 for trended variables. An ADF(1) and an ADF(3) test was used to ensure that all of the autocorrelation in the return series was purged from the residuals.

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Table 4.7(a)
Summary of Co-integration Results at a 95% Confidence Level

The following table provides a summary of the co-integration tests at a 95% confidence level. Y indicates the existence of a co-integrating vector, n indicates the absence of a co-integrating vector.

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Table 4.7(b)
Summary of Co-integration Results at a 90% Confidence Level

The following table provides a summary of the co-integration results at a 90% level. y indicates the existence of a co-integrating vector, n indicates the absence of a co-integrating vector.

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Table 4.8
Results of the Johansen Trace Test

This table provides the results of the Johansen trace test for a bi-variate regression between the share price of the firm in the stub of the table to those above the statistic. Critical values are 15.410 at a 95% confidence level and 13.325 at a 90% confidence level. The Johansen tests were undertaken with 7 lags specified in the vector autoregression. It was also assumed that the stock prices tested here are trended variables.

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117
Table 4.9

Results of the Granger Causality Tests

The results in the following table relate to a variable addition test for the following unrestricted vector autoregression model.

\[ \Delta p_{j,t} = \alpha_j + \beta_j p_{j,t-1} + \beta_j p_{k,t-1} + \sum_{i=1}^{2} \theta_i \Delta p_{j,t-i} + \sum_{i=1}^{2} \gamma_i \Delta p_{k,t-i} + e_{j,t}, \quad \tau = 1, 2 \]

\[ \Delta p_{k,t} = \alpha_k + \beta_k p_{j,t-1} + \beta_k p_{k,t-1} + \sum_{i=1}^{2} \phi_i \Delta p_{j,t-i} + \sum_{i=1}^{2} \delta_i \Delta p_{k,t-i} + e_{k,t}, \quad \tau = 1, 2 \]

where \( p_j \) refers to the logarithm of the share price of stock \( j \). \( p_k \) refers to the logarithm of the share price of stock \( k \). \( F \) refers to the results from an \( F \) variable addition test. \( LM \) refers to the results from a Lagrange Multiplier, \( LM \) variable addition test. The \( F \) and \( LM \) refer to the results of the variable addition test where \( p_{k,t-1}, \Delta p_{k,t-1}, \) and \( \Delta p_{k,t-2} \) are tested to see if they have a causal influence over \( \Delta p_{j,t} \). Meanwhile, \( F_2 \) and \( LM_2 \) refer to the results of the variable addition test where \( p_{j,t-1}, \Delta p_{j,t-1}, \) and \( \Delta p_{j,t-2} \) are tested to identify whether they have a causal influence over \( \Delta p_{k,t} \). The superscript \( l \) adjacent to the \( LM \) probability indicates that large firms predict the price of a smaller company, while the superscript \( s \) indicates that a small firm price predicts a large firm price.

| \( I_j = L1 \) | \( I_k = L13 \) | \( I_j = L1 \) | \( I_k = L15 \) | \( I_j = L1 \) | \( I_k = L17 \) | \( I_j = L2 \) | \( I_k = L3 \) | \( I_j = L2 \) | \( I_k = L10 \) | \( I_j = L2 \) | \( I_k = L12 \) | \( I_j = L2 \) | \( I_k = L13 \) | \( I_j = L2 \) | \( I_k = L15 \) | \( I_j = L3 \) | \( I_k = L7 \) | \( I_j = L3 \) | \( I_k = L15 \) | \( I_j = L4 \) | \( I_k = L13 \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( \beta_j + \sum_{i=1}^{2} \gamma_i = 0 \) | \( \beta_j + \sum_{i=1}^{2} \phi_i = 0 \) | \( F_1 \) | \( LM_1 \) | \( F_2 \) | \( LM_2 \) | \( F_1 \) | \( LM_1 \) | \( F_2 \) | \( LM_2 \) | \( F_1 \) | \( LM_1 \) | \( F_2 \) | \( LM_2 \) | \( F_1 \) | \( LM_1 \) | \( F_2 \) | \( LM_2 \) | \( F_1 \) | \( LM_1 \) | \( F_2 \) | \( LM_2 \) |
| 5.73 | 11.01 | 1.095 | 2.221 | 2.459 | 7.362 | 0.291 | 0.599 | (0.06) | (0.06) | (0.75) | (0.75) | 0.068 | 0.140 | 3.01 | 5.99 | 2.603 | 7.78 | 5.483 | 15.615 | (0.05) | (0.05) | (0.00) | (0.00) | 3.080 | 9.19 | 0.291 | 0.599 | (0.03) | (0.03) | (0.00) | (0.00) | 3.45 | 6.898 | 0.16 | 0.301 | (0.03) | (0.03) | (0.85) | (0.85) | 1.05 | 0.51 | 3.72 | 7.34 | (0.59) | (0.60) | (0.02) | (0.02) | 1.05 | 0.51 | 21.743 | 49.082 | (0.59) | (0.60) | (0.00) | (0.00) | 0.349 | 0.692 | 3.11 | 6.15 | (0.70) | (0.71) | (0.04) | (0.04) |
Table 4.8 (cont.)

\[ \beta_2 + \sum_{i=1}^{2} \chi_i = 0 \quad \beta_1 + \sum_{i=1}^{2} \phi_i = 0 \]

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CHAPTER FIVE
Mean Reversion in Short Horizon Returns

5.1 Introduction

This chapter will be concerned with examining whether forecasts of future monthly portfolio returns, which are made from a weighted sum of past weekly return observations can predict future monthly returns better than past monthly returns. This chapter is therefore concerned with whether there are rapidly decaying predictable components in UK short-horizon portfolio returns so that short-horizon returns are mean-reverting.

Section 3.3.3 of Chapter 3 indicated that as much as 45 percent of the variation in long-horizon returns appears to be predictable, because, share prices contain transitory components that are very persistent. In contrast, the conclusion of studies that seek to gauge predictability in short-horizon returns (such as daily, weekly, or monthly frequencies) indicate that predictable components of short-horizon returns can only explain a small fraction of the variation in realised portfolio returns. For example, short-horizon autocorrelation tests, such as those undertaken by Lo and MacKinlay (1988, 1990a), which were performed on capitalisation based portfolio returns, showed that at most only about 6 percent of the future return to a portfolio could be predicted from its previous value.

However, Conrad and Kaul (1988) demonstrate that like long-horizon returns, weekly portfolio returns are also characterised by substantial predictability, which had been understated in other studies. More importantly, the predictable component in weekly expected returns seemed to dissipate rapidly (within a few weeks) and therefore appeared to be quite different from, and perhaps unrelated, to the slow movements in long-horizon expected returns which had been documented by Fama and French and Porterba and Summers.

A uniting characteristic of the work undertaken by Conrad and Kaul, on short-horizon returns and the work of Fama and French, on long-horizon returns was the discovery that there was a relationship between the size of firms within a portfolio and the extent to which portfolio returns were predictable. Both sets of studies found that as firms within a portfolio became progressively smaller, the portfolio return became increasingly more predictable. To illustrate this, the tests performed on short-horizon returns by Conrad and Kaul found that for the portfolio containing the largest firms only 0.7 percent of the variation in portfolio returns was predictable.

In Chapter 3 long horizon mean-reversion was discussed in detail. As this chapter pointed out share prices are mean reverting if they are away from their fundamental value but reverting towards their fundamental value. A stock return which is moving towards its fundamental value as an AR(1) process will be positively autocorrelated over short horizons even though over long horizons returns will be negatively autocorrelated.
However, as the firms within a portfolio got progressively smaller, the proportion of the variation in a portfolio's return which could be explained increased. For the portfolio containing the smallest firms as much as 26.5 percent of the portfolio realised weekly return was found to be predictable. Similarly, Fama and French (1988) found that for a portfolio of small firms, 45 percent of the variation of five year portfolio returns was predictable but, for the largest firms in the sample, only 7 percent of five year returns was predictable.

To demonstrate that short-horizon returns were highly predictable Conrad and Kaul (1989) developed a simple model of monthly expected returns that relied on the rapidly decaying structure of weekly expected returns. They proposed, that because of the rapidly decaying nature of the autocorrelations in short-horizon returns, the most recent weekly returns, will have more information about the subsequent month’s return than the most recent monthly return.

If short-horizon returns contain predictable components that decay rapidly, then the most recent weekly return will provide more information about the subsequent return performance of a security than is provided by less recent weekly return observations. Monthly return autocorrelation statistics can be used to forecast monthly returns from past monthly returns, in such a test equal weights are given to the previous four weekly realised returns of a month. In this case, the forecast based on the autocorrelation pattern will understate the rapid decay in the predictable component and will therefore be a poor forecast of the future monthly return.

Conrad and Kaul (1989) successfully test a model which utilises the information in the most recent weekly returns of a month to predict future monthly returns to capitalisation based portfolios, which comprise of stocks listed on the New York Stock Exchange. They find that the first order autocorrelation statistic of monthly returns could explain less than 4 percent of the variation in monthly returns. Meanwhile, a forecast of monthly portfolio returns based on a weighted sum of the past four week’s realised returns which attaches greater weight to the most recent return observations, is able to explain substantially more of the variation in realised portfolio excess returns. This indicated that short-horizon portfolio returns contained predictable components, which were mean reverting.

Conrad and Kaul found that for the smallest firms in a portfolio the autocorrelation pattern of monthly returns could only explain approximately 4 percent of the variation in realised monthly returns. In contrast, their model which forecasted the future month’s return from a weighted sum of past weekly returns, where greater weight was given to the most recent weekly returns was able to explain up to 25 percent of the variation in realised portfolio returns for the portfolio containing the smallest firms. The proportion of the variation in the portfolio return explained by past weekly returns was found to decline with firm size. In the case of the large
firms, none of the monthly variation in the portfolio return could be explained by the behaviour of the expected return series. This suggests that as firm size gets smaller the magnitude of the predictable component in the portfolio return gets progressively larger.

The remainder of this chapter is set out as follows. In Section 5.2 the time series model which forecasts future returns from prior weekly returns will be described. Since this model forecasts monthly returns from prior weekly returns the procedure for identifying the weights to be attached to previous weekly returns is also described. In Section 5.3 the data used in this study is examined. In Section 5.4 the results from the estimation of the ARMA(1,1) model of returns will be explored to establish how to calculate the weights which are to be imposed on past weekly returns. This section also provides results which indicate that the model of expected returns tested here predicts future monthly returns better that the autocorrelation pattern of monthly returns. Section 5.5 modifies the tests to take account of the turn of the year effect. Section 5.6 will provide a summary and some conclusions to the chapter.

5.2 The Model

5.2.1 The ARMA(1,1) Model of Realised Returns

Section 2.3.2 of Chapter 2 showed that realised returns can be written in terms of the expected return and a random error as follows

\[ R_t^w = E_{t-1}(R_t^w) + e_t^w \]  
\[ e_t^w \sim iid \ N(0, \sigma_e^2) \]  

Section 3.2.3 of Chapter 3 discussed the work of Conrad and Kaul (1988) which showed that weekly expected returns are well represented by the following first order autoregressive process.

\[ E_{t-1}(R_t^w) = \phi E_{t-2}(R_t^w) + u_{t-1}^w \]  
\[ u \sim iid \ N(0, \sigma_u^2) \]  

where, \( R_t^w \) is the realised return of a security for week \( t \) and \( E_{t-1}(R_t^w) \) is the expected return for a security at time \( t - 1 \) for a return due in time \( t \), where \( 0 < \phi < 1 \).

As Appendix 1 demonstrates, if expected returns follow an AR(1) process then realised returns can be written in the following way.

\[ R_t^w = \phi R_{t-1}^w + e_t^w + \phi e_{t-1}^w + u_{t-1}^w \]
which, because of the properties of $e_i$ and $u_i$ above, can be written as a familiar ARMA(1,1) process.

$$R_i^w = \phi R_{i-1}^w + a_i^w - \theta a_{i-1}^w$$ (5.4)

### 5.2.2 Characterising Monthly Expected Returns

Let us define the one month realised return $R_t^m$ as the sum of the four weekly realised returns which occur during a given month.

$$R_t^m = R_t^w + R_{t-1}^w + R_{t-2}^w + R_{t-3}^w = \sum_{\tau=0}^{3} R_{t-\tau}^w \quad \tau = 0, 1, 2, 3$$ (5.5)

where, $R_t^w$ is the weekly return which is realised during the last week of the month, $R_{t-3}^w$ is the weekly realised return for the first week of the month, and $R_{t-\tau}^w$ is the return for week $t - \tau$.

Since weekly expected returns follow an AR(1) process, and weekly realised returns follow an ARMA(1,1) process, the conditional expected monthly return which exists at the end of the previous month for the following month’s return, that is, $E_{t-4}(R_t^m)$ can be written in terms of the four expected weekly returns which make up the following month’s expected return as shown in Equation (5.6), which is derived in Appendix 2.

$$E_{t-4}(R_t^m) = E_{t-4}(\sum_{\tau=0}^{3} R_{t-\tau}^w) = (1 + \phi + \phi^2 + \phi^3)E_{t-4}(R_{t-3}^w) \quad \tau = 0, 1, 2, 3$$ (5.6)

If expected returns follow an AR(1) process and realised returns follow an ARMA(1,1) process then the expected weekly return for the following week can be written as an infinite order autoregressive process as follows.

$$E_{t-1}(R_t^w) = \psi_1 R_{t-1}^w + \psi_2 R_{t-2}^w + \psi_3 R_{t-3}^w + \ldots$$ (5.7)

where, $\psi_i$ are the weights which are attached to each of the past realised returns, these are derived in Appendix 3. This means that the infinite AR representation for monthly expected returns can be written in terms of the four weekly expected returns of a given month, as shown in the following equation, which is derived in Appendix 4.

$$E_{t-4}(R_t^m) = \pi_1 R_{t-4}^w + \pi_2 R_{t-5}^w + \ldots$$ (5.8)

$$\pi_i = \theta^{i-1}(\phi - \theta)(1 + \phi + \phi^2 + \phi^3) \quad i = 1, 2, 3, \ldots \infty$$ (5.9)

---

2 Although monthly returns usually refers to the return which accrues over a calendar month in this study monthly return will be synonymous with a four weekly return.
where, \( \pi_i \) are the weights to be attached to the past realised weekly returns, such that \( \pi_i \) is the weight given to the most recent weekly return when predicting the following month’s return.

Equation (5.8) demonstrates that as long as short-horizon expected returns contain predictable components, which decay rapidly, then a better prediction of monthly returns can be obtained from past weekly returns, than can be obtained from past monthly returns. The intuition is that the quality of the time-series forecasts are a function of the weights placed on past observations, and monthly time-series models put the same weights on all weeks within a particular month. However, if there is rapid mean-reversion in weekly expected returns, the weights placed on past weeks should decay exponentially (and rapidly) because the most recent weeks of a particular month contains a disproportionately large amount of information about the subsequent month’s return. This means that when you are trying to forecast future monthly returns the degree to which future monthly returns are predictable will be understated, unless greater weight is given to the most recent weekly return observations.

If expected monthly returns are to be forecast from past weekly returns the forecasting model to be used will contain the four expected weekly returns which coincide with a given month as follows.

\[
E_{t-4}(R'_m) = w_1R'_{t-4} + w_2R'_{t-5} + w_3R'_{t-6} + w_4R'_{t-7},
\]

(5.10)

In this example, the expected future monthly return at the end of the previous month \( E_{t-4}(R'_m) \) is forecast from the previous four weekly return series’ which are \( R'_{t-4}, R'_{t-5}, R'_{t-6}, R'_{t-7} \). Since the monthly conditional expected return is a weighted average of the four past weekly returns the sum of the weights \( w_1, w_2, w_3, w_4 \) which are attached to the previous four weekly returns must sum to unity so that only the past four weekly returns are used in the forecast. The weights attached to each of the previous weekly realised returns so that the weights sum to unity will be as follows

\[
\sum_{i=1}^{4} w_i = \sum_{i=1}^{4} \left( \Phi - \Phi + \Phi^2 + \Phi^3 \right) = 1
\]

(5.11)

so that \( w_i = \Phi^{i-1}(1 - \Phi) \), a derivation of this is provided in Appendix 5.

5.3 Data and Summary Statistics

5.3.1 Data

Over the sample period January 1976 to December 1991, weekly Wednesday share prices of randomly selected companies were obtained from Datastream. Every January the companies
which had been selected were ranked on the basis of market capitalisation (share price times number of shares issued) and then sorted into four portfolios, stratified on the basis of capitalisation value. Each portfolio contained fifty stocks. From the weekly security prices, four equally weighted portfolio returns were calculated in the manner described in Section 4.1.4 of Chapter 4. Portfolio \( P_1 \) contains the smallest firms, \( P_2 \) contains the small-intermediate firms, \( P_3 \) contains the large-intermediate firms and \( P_4 \) contains the largest firms in the sample. The portfolio excess return was then calculated as the portfolio weekly return less the weekly Treasury Bill Rate. It should be noted that in this study returns will be synonymous with excess returns. In this chapter excess returns have been tested to ascertain whether there is predictability in the realised risk premium as well as the pure return series.

A set of monthly (that is four weekly) returns was then calculated for each of the individual securities. These monthly security returns were calculated as the log price difference over a four week period as follows.

\[
R_t^m = \ln(P_t) - \ln(P_{t-4})
\]

From the security returns an equally weighted portfolio return was calculated for four capitalisation based portfolios in the same manner as described for the weekly returns. Portfolio \( P_1^m \) contains the portfolio containing the smallest firms, \( P_2^m \) contains the small-intermediate firms, \( P_3^m \) contains the large-intermediate firms and \( P_4^m \) contains the large firms.

### 5.3.2 Autocorrelations

Table 5.1 presents the autocorrelation statistics from for the weekly and monthly excess returns obtained for the period January 1976 to December 1991. As expected, the autocorrelations for excess returns mirror those reported in Table 4.1 of Chapter 4 which were estimated for the pure return series. For weekly returns the autocorrelations are large and significant and the higher order autocorrelations, although, significant decay at longer lags. The weekly autocorrelation structure is characterised by a consistent pattern as we move from the smallest portfolio \( P_1 \) to the largest \( P_4 \). The magnitude and persistence of the autocorrelations decline monotonically. In the case of the weekly autocorrelations the previous periods’ return for the portfolio containing the smallest firms’ explains about 16 percent \((0.41)^2\) of the variation in portfolio returns. For the portfolio containing the largest firms the first order autocorrelation can only explain about 2 percent \((0.15)^2\) of the variation in portfolio returns.

A similar pattern is displayed by the monthly return series since the autocorrelations are also large and significant. In all cases the first order autocorrelation is significant. The monthly autocorrelations decline much more quickly, the second order autocorrelation is insignificant for all but the portfolio containing the smallest firms.
5.4 Parameter Estimates of the Model

This section will provide the results from the estimation of the ARMA(1,1) model of expected returns. From the estimates of $\theta$ and $\phi$, weights are calculated and attached to the previous weekly returns. Using these weights, forecasts of the following month’s return are made. This section shows that the monthly return autocorrelation test understates the extent to which monthly returns can be predicted from the previous four weekly returns.

5.4.1 Weekly Estimates of the ARMA(1,1) Model of Weekly Realised Returns.

In this section an ARMA(1,1) model of realised weekly returns is estimated using the non-linear procedure of Box and Jenkins (1970) for each of the four portfolios.

\[ R_{p,t}^w = \phi_p R_{p,t-1}^w + \theta_p a_{p,t-1} + \alpha_{p,t} \]  
\[ p = 1, 2, 3, 4 \]  

(5.13)

where, $R_{p,t}^w$ are realised returns in time $t$, $\phi_p$ is the autoregressive coefficient which captures the relationship between the previous weekly return and the current weekly return of portfolio $p$, $\theta_p$ is the moving average coefficient for portfolio $p$ which captures the relationship between the previous periods error and the following periods weekly return. The term $\alpha_{p,t}$ is a white noise error.

Diagnostic tests which are performed on the residuals obtained from the estimation of the ARMA(1,1) model are presented in Table 5.2. A stationary autoregressive process for expected returns appears to be well specified. The residuals from the ARMA(1,1) model, for three of the portfolios, behave like white noise (the fourth is close to being white noise). For all four portfolios there is an absence of autocorrelation in the residuals obtained from the estimation of the ARMA(1,1) model, for example the first order autocorrelations are all less than 1 percent. Further support for the model is provided by the Ljung-Box (1978) Q statistic, which tests the hypothesis that all the autocorrelations up to lag 6 are jointly zero. For all three of the portfolios this is insignificant and is only marginally significant for the fourth. The ARMA(1,1) model for realised excess returns therefore appears to be well specified.

Estimates of $\phi$ are significantly different from zero and unity for all portfolios. The magnitude of the $\phi$ is monotonically associated with the size of firms in the portfolio. As we move from the portfolio containing the smallest firms, that is $P_1$, to the portfolio containing the largest firms $P_4$ the size of $\phi$ diminishes. For the portfolio containing the smallest firms the estimated value of $\phi$ is 0.668 but for the portfolio containing the largest firms $\phi$ is only 0.488. This pattern is entirely consistent with the autocorrelation pattern of the portfolio returns, since we would expect the size of $\phi$ to reflect the magnitude of the first order autocorrelation. The
estimated values of $\phi$ presented here are consistent with those presented by Conrad and Kaul (1988) who estimated an ARMA(1,1) model for portfolios which contained stocks listed on the New York Stock Exchange.

So far it has been established that there are predictable components in short-horizon returns which dissipate quickly, this has implications for the relative importance of different weeks within a particular month when predicting the subsequent month’s return. The pattern discussed here suggests that the most recent week’s realised return contains more information about the subsequent month’s return than is provided by more distant returns. This means that when forecasting the future month’s return from the previous weekly return more weight should be given to the most recent weekly return as this contains more information about next month’s returns.

5.4.2 Calculating the Weights

Using equation (5.10) and estimates of $\theta$ which are provided in Table 5.2, it is possible to construct the weights to be attached to each of the four weekly returns of the previous month. The procedure for calculating the weights was outlined in detail in Appendix 5, this procedure is followed exactly in this section. As Table 5.3 indicates, to predict monthly portfolio returns a disproportionately large weight should be given to the fourth, and therefore the most recent week of the previous month when attempting to forecast future monthly returns.

For example, in the case of the smallest portfolio the weights given to the return of the fourth, third, second and first week of the previous month are as follows 0.68 0.23 0.07 and 0.02. This means that the monthly expected return series for the small firm portfolio can be calculated as

$$E_{t-4}(R^m_{p,t}) = 0.68(R^w_{p,t-4}) + 0.23(R^w_{p,t-5}) + 0.07(R^w_{p,t-6}) + 0.02(R^w_{p,t-7})$$

where, $E_{t-4}(R^m_{p,t})$ is the monthly conditional expected return of portfolio $p$ which is expected at the end of the previous month, that is, week $t-4$ of the previous month, and $e_{p,t}$ is a random error.

For all four portfolios the weights attached to the previous weekly returns decline rapidly, actually the rate of decline in all four portfolios is very similar. For example, for the large firms the weights are 0.66 0.23 0.08 and 0.03 and are therefore very similar to the weights of $P_1$ which is the portfolio containing the smallest firms. This reflects the changing values of $\theta$ across the four portfolios because as $\theta$ gets smaller, a greater weight is attached to the most recent weekly return of the previous month.
5.4.3 Forecasts of Monthly Returns

To establish whether the expected monthly return obtained from the weighted sum of expected weekly returns predicts future monthly returns better than the previous month’s return, the regression which is described by equation (5.15) is estimated. This is a regression of the following month’s realised return on a constant and the expected monthly return which is anticipated in week \( t - 4 \). Where the monthly expected return is calculated as a weighted sum of the previous four weekly realised returns using the weighting scheme outlined in the previous section.

\[
R_{p,t}^m = \alpha_p + \beta_p E_{t-4}(R_{p,t}^m) + n_{p,t}
\]  

where, \( R_{p,t}^m \) is the realised monthly return for portfolio \( p \), \( E_{t-4}(R_{p,t}^m) \) is the expected monthly return for portfolio \( p \) which is expected at the end of the previous month, while \( n_{p,t} \) is a white noise error.

The \( R^2 \) of this regression will indicate the proportion of the variation in realised returns explained by the model of expected returns. Of course, if the \( R^2 \) indicates that more than about 10 percent of the variation in portfolio returns can be explained, confirmation will be provided that the model of expected returns tested here has more predictive power than the autocorrelation patterns which could explain no more than about 10 percent of the variation in a portfolio’s return.

For all portfolios the slope coefficients are close to one and the constant is close to zero. This indicates that expected returns are an unbiased predictor of future realised returns. Although in some cases the t-statistics associated with the constant are significant, when the constant is significant the magnitude of the coefficient is far too small to have any predictive power. The presence of heteroscedasticity in the residuals appears to be ruled out. In all cases the ARCH(6) statistic, which can detect up to sixth order heteroscedasticity indicates that the null hypothesis of no heteroscedasticity in the residuals can not be rejected in all four cases.

The \( R^2 \) of this equation indicates that a much higher proportion of the variation in realised monthly returns can be explained from the model of expected returns than can be explained by using past realised monthly returns. The \( R^2 \) of \( P_1, P_2 \) and \( P_3 \) are all approximately 15 percent. This means that the model of expected monthly returns which attaches greater weight to the most recent weekly expected returns can explain substantially more of the variation in portfolio returns than can the previous realised monthly return. This indicates that there is mean-reversion in the portfolio returns for all but the portfolios containing the largest firms.
In the case of the large firms the $R^2$ is only 0.008 percent which suggests that the model tested here does not predict any of the variation in realised monthly returns. This suggests that for the large firms short-horizon returns do not contain predictable components and are therefore not mean-reverting over short-horizons.

In this study, the $R^2$ is very similar for all but the portfolio containing the largest firms. As such, the results presented here for the UK contrast with the findings of Conrad and Kaul who found that for US stocks the $R^2$ became progressively larger as the size of firms within a portfolio declined.

5.5 The January Effect

In an attempt to improve the explanatory power of expected returns a second model of expected returns was tested, which takes account of the calendar anomalies which are a characteristic of the UK stock market. This model not only takes advantage of the time series properties of the ARMA(1,1) model but also exploits any seasonal patterns which exist, but are not already accounted for. The two most important seasonalities for the UK stock market occur in January and April. For example, Levis (1985) reports that returns for stocks listed on the London stock exchange are higher in both of these months. Seasonality work which will be presented in Section 6.3 of Chapter 6 confirms this pattern over the sample period used in this study. If the time series coefficients obtained from the estimation of the ARMA(1,1) model fail to capture the presence of these seasonalities then expected returns will understate realised returns during these months.

To improve the forecasts of realised returns a number of models with seasonal dummies were estimated. Initially, in order to establish which seasonals might provide an additional source of predictability an ARMA(1,1) model with dummies for each of the weeks in January (to capture the turn of the year effect) and dummies for each of the first two weeks in April(to capture the effect of tax loss trading) was estimated. Once the model was estimated it became clear that the only dummies which provided additional explanatory power were the three dummies which represented the first three weeks of January. Consequently, the ARMA(1,1) model was re-estimated, this time with the inclusion of only the dummies which represented each of the first three weeks of January.

$$R_{p,t} = \delta_{1,p} + \delta_{2,p} Jan_1 + \delta_{3,p} Jan_2 + \delta_{4,p} Jan_3 + \phi R_{p,t-1} - \theta a_{p,t-1} + a_{p,t}$$

(5.16)

where,

$R_{p,t}$ is the return to portfolio $p$ in time $t$. 

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Jan1 is a dummy variable which takes on the value of 1 if it is the first week of January but a zero otherwise.

Jan2 is a dummy variable which takes on the value of 1 if it is the second week of January but a zero otherwise.

Jan3 is a dummy variable which takes on the value of 1 if it is the third week of January but a zero otherwise.

δ1,p is the constant coefficient.

δ2,p, δ3,p, δ4,p is the coefficient attached to the January dummies, which indicates how much above or below the constant returns are in January.

The coefficients obtained from this model are presented in Table 5.5. In the case of the portfolio of large firms the addition of the January dummies makes a considerable difference to the estimated coefficients. The coefficient of the autoregressive component declines from 0.488 to 0.064. Meanwhile, for the large firms, the moving average coefficient also declines from 0.344 to 0.254. In the case of the large firms, the January dummies are economically meaningful. For example, the dummies for January reveal that average returns for the portfolio of large firms is approximately 0.75 percent higher if the month is a January, although, most of this accrues during the first week of January. The inclusion of the January dummies is likely to improve the forecasting ability of the model for the large firm portfolio. This is not the case for the other portfolios. For all of the other portfolios the magnitude of the coefficients associated with the January dummies are very small. For the other three portfolios the addition of the January dummy is therefore likely to add very little to the forecasting ability of the ARMA model.

Expected returns for the large firm portfolio were calculated as follows.

\[ E_{p,t-4}(R_{p,t}^w) = \delta_{1,p} + w_1(R_{p,t-4}^w) + w_2(R_{p,t-5}^w) + w_3(R_{p,t-6}^w) + w_4(R_{p,t-7}^w) \quad p = 4 \]  

\[ E_{p,t-4}(R_{p,t}^w) = 0.002 + 0.75R_{p,t-4}^w + 0.19R_{p,t-5}^w + 0.05R_{p,t-6}^w + 0.01R_{p,t-7}^w \quad p = 4 \]  

Additionally, if the future month’s return falls during January expected returns will be inflated by approximately 0.0064, 0.0004 and 0.0008 to reflect that the future month’s return coincides with the first second or third week of January. A similar procedure was followed to estimate the expected returns for each of the three other portfolios.
The results of undertaking a regression of realised returns on expected returns derived from this model are presented in Table 5.6. It is clear that for the portfolio of large firms the inclusion of the January dummies substantially improves the forecast of future monthly returns. Expected returns can now explain almost 8 percent of the variation in the returns of $P_4$. This confirms results which will be presented in Section 6.3 of Chapter 6 which suggests that the January effect in the UK is more pronounced for the returns of large firms. For the other three portfolios no additional explanatory power is obtained by including the January dummies. This indicates that the ARMA(1,1) model, for these three portfolios, captures very well the seasonal behaviour of the portfolio returns. In some cases the inclusion of a constant January premium reduces the performance of expected returns slightly. In all cases except one, the results from an ARCH(6) test for heteroscedasticity revealed that the null hypothesis of no ARCH effects could not be rejected.

5.6 Summary and Conclusions

This study has shown that short-horizon returns, in this case monthly returns are more predictable than is suggested by the autocorrelation pattern of realised monthly returns. Exploiting the autocorrelation patterns of realised monthly returns allows no more than about 10 percent of the variation in portfolio returns to be predicted. Autocorrelation tests assume that each weekly return within the previous month contains the same amount of information. This will not be the case if short-horizon returns contain predictable components which dissipate very rapidly so that they display a mean-reverting pattern. In this case, the most recent week may say a lot more about the future monthly return than is suggested by less recent weeks. In which case, when predicting future monthly returns more weight should be given to the most recent weeks of the previous month, because, the most recent weekly returns provide the most information about the subsequent months performance.

In this study an ARMA(1,1) model of weekly portfolio returns was estimated, in order to derive weights which can be attached to previous weekly returns. These weights indicate that more importance should be placed on the most recent weekly return, when trying to predict future monthly returns. It is found that the model tested in this chapter, which gives greater weight to the most recent return information, can predict up to 15 percent of the variation in the future month’s portfolio return. Which is a substantial improvement over the proportion which can be explained by autocorrelation tests.

This chapter finds that there is only a weak relationship between the size of firms within a portfolio and the extent to which portfolio returns are mean-reverting. For all but the portfolio containing the largest firms the model of expected returns tested here performs very well. This suggests that for all but the largest firms in the sample there is evidence of mean-reversion in
short-horizon returns. For the largest firms in the sample this was not the case because almost none of the variation in portfolio returns could be explained by the model of expected returns tested here. This suggests that for large firms there is no evidence of mean-reversion in short-horizon returns.

The results presented here are consistent with at least two competing hypotheses. Firstly, it is possible, but unlikely, that these predictable components reflect market inefficiency. As this study has suggested previously predictability in stock returns does not suggest that markets are inefficient unless time variation in expected returns and the presence of microstructure frictions can be ruled out. An alternative hypothesis is that the returns in this sample are time varying and autocorrelated, which introduces the predictable component into realised returns. This amounts to saying that if expected returns rise today the effects of this rise will influence future returns but will decay rapidly so that the predictability has a short life-span. The results for the large firms are not inconsistent with this hypothesis, although, the nature of the time variation would have to be such that there is very little variation over a period of about a month in the expected returns of large firms. In which case, the information provided by the most recent weekly return will not be any better than the information provided during other weeks of the month. The results of an investigation into the time varying properties of the systematic risk premium which is undertaken in Chapter 7 suggest that differences in the characteristics of the time varying systematic risk premium are consistent with this hypothesis.

The aims of further research should be directed at determining whether, out of sample, the autocorrelation patterns and the associated weights provide any real predicative power. This could be achieved by testing this model during the sample period 1992-1996. This would suggest whether such models really do allow us to predict ex-ante the behaviour of portfolio stock returns.

Since this chapter has suggested some interesting differences between the predictability of UK and US stock returns a detailed comparative study which investigates the UK and US stock markets might be able to suggest reasons for why in the UK there is much weaker evidence of short horizon mean reversion.
Appendix 1, Proof Of Equation (5.3) in the text.

Taking equation (5.1) of the text

\[ R_t^w = E_{t-1}(R_t^w) + e_t^w \]  \hspace{1cm} (A1.1)

and lagging this equation by one period

\[ R_{t-1}^w = E_{t-2}(R_{t-1}^w) + e_{t-1}^w \]  \hspace{1cm} (A1.2)

adding \( \phi \) to both sides

\[ \phi R_{t-1}^w = \phi E_{t-2}(R_{t-1}^w) + \phi e_{t-1}^w \]  \hspace{1cm} (A1.3)

Since expected returns follow an AR(1) process

\[ E_{t-1}(R_t^w) = \phi E_{t-2}(R_{t-1}^w) + u_{t-1}^w \]  \hspace{1cm} (A1.4)

rearranging equation (A1.4) gives

\[ \phi E_{t-2}(R_{t-1}^w) = E_{t-1}(R_t^w) - u_{t-1}^w \]  \hspace{1cm} (A1.5)

Taking equation (A1.3) and substituting in \( E_{t-1}(R_t^w) - u_{t-1}^w \) for \( \phi E_{t-2}(R_{t-1}^w) \) provides.

\[ \phi(R_{t-1}^w) = E_{t-1}(R_t^w) - u_{t-1}^w + \phi e_{t-1}^w \]  \hspace{1cm} (A1.6)

Then subtracting equation (A1.1) from (A1.6) gives

\[ R_t^w - \phi R_{t-1}^w = E_{t-1}(R_t^w) - E_{t-1}(R_t^w) + e_t^w + \phi e_{t-1}^w - u_{t-1}^w \]  \hspace{1cm} (A1.7)

which simplifies to

\[ R_t^w - \phi R_{t-1}^w = e_t^w + \phi e_{t-1}^w - u_{t-1}^w \]  \hspace{1cm} (A1.8)

or

\[ R_t^w = \phi R_{t-1}^w + e_t^w + \phi e_{t-1}^w - u_{t-1}^w \]  \hspace{1cm} (A1.9)

Equation (A1.9) is equation (5.3) of the text.

Appendix 2, Proof of Equation (5.6) in the text.

The following month’s return which captures returns from time \( t - 3 \) to time \( t \) will include \( R_t^w, R_{t-1}^w, R_{t-2}^w, R_{t-3}^w \) but at time \( t - 4 \) these are expected returns. If expected returns follow an AR(1) process then

\[ E_{t-4}(R_{t-3}^w) = \phi E_{t-5}(R_{t-4}^w) \]  \hspace{1cm} (A2.1)

\[ E_{t-4}(R_{t-2}^w) = \phi^2 E_{t-5}(R_{t-3}^w) = \phi E_{t-4}(R_{t-3}^w) \]  \hspace{1cm} (A2.2)

\[ E_{t-4}(R_{t-1}^w) = \phi^3 E_{t-5}(R_{t-4}^w) = \phi^2 E_{t-4}(R_{t-3}^w) \]  \hspace{1cm} (A2.3)

\[ E_{t-4}(R_t^w) = \phi^4 E_{t-5}(R_{t-4}^w) = \phi^3 E_{t-4}(R_{t-3}^w) \]  \hspace{1cm} (A2.4)

This means that the monthly return \( E_{t-4}(R_t^w) \) which is the sum of the four expected weekly returns can be written as the sum of equations (A2.1) to (A2.4), that is

\[ E_{t-4}(R_t^w) = E_{t-4}(R_{t-3}^w) + E_{t-4}\phi(R_{t-3}^w) + E_{t-4}\phi^2(R_{t-3}^w) + E_{t-4}\phi^3(R_{t-3}^w) \]  \hspace{1cm} (A2.5)

Collecting terms gives us Equation (5.6) in the text, that is,

\[ E_{t-4}(R_t^w) = E_{t-4}\left(\sum_{\tau=0}^{3} R_{t-\tau}^w\right) = (1 + \phi + \phi^2 + \phi^3)E_{t-4}(R_{t-3}^w) \]  \hspace{1cm} (A2.6)

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Appendix 3, Proof of Equation (5.7) in the Text

Taking an ARMA(1,1) model such as that of equation (5.4) in the text

\[ x_t = \phi x_{t-1} + a_t - \theta a_{t-1} \]  
(A3.1)

where \( x_t \) is a variable which follows an ARMA(1,1) process, and \( a_t \) is a white noise error process.

Writing this equation in terms of the lag operator \( L \) gives

\[ (1 - \theta L)x_t = (1 - \phi L)a_t \]  
(A3.2)

Divide by \( 1 - \theta L \) gives

\[ \frac{1 - \theta L}{1 - \phi L} x_t = a_t \]  
(A3.3)

which can also be written as

\[ (1 - \theta L)^{-1}(1 - \phi L)x_t = a_t \]  
(A3.4)

Expanding the term in the second bracket provides

\[ (1 - \phi L)(1 + \phi L + \phi^2 L^2 + \ldots)x_t = a_t \]  
(A3.5)

Rearranging gives

\[ x_t + (\phi - \theta)x_{t-1} + \theta(\phi - \theta)x_{t-2} + \ldots + a_t \]  
(A3.6)

\[ x_t = \psi_1 x_{t-1} + \psi_2 x_{t-2} + \ldots + a_t \]  
(A3.7)

The ARMA(1,1) model thus leads to an autoregressive representation having an infinite number of weights. The \( \psi \) weights converge for \( |\theta| < 1 \).
Appendix 4, Proof of Equations (5-8) and (5.9) in the Text

As Appendix 2 demonstrated we can express the following month’s return in terms of the four component weekly returns so the sum of the four weekly expected returns provides the monthly expected return

\[ E_{t-4}(R_t^m) = E_{t-4} \left( \sum_{\tau=0}^{\tau=3} R^w_{t-\tau} \right) \quad \tau = 0, 1, 2, 3 \]  

(A4.1)

If expected returns follow an AR(1) process then the following week’s expected return is \( \phi \) times the previous week’s return while the \( n \) period ahead forecast is \( \phi^n \) times the previous weekly return so that

\[ E_{t-4}(R_t^w) = \phi E_{t-4}(R_{t-4}^w) + \phi^2 E_{t-4}(R_{t-5}^w) + \phi^3 E_{t-4}(R_{t-6}^w) \]  

(A4.2)

Since the following month’s return can be expressed in this way each of the four expected weekly returns can be expressed as an infinite order moving average process as follows.

\[ E_{t-4}(R_t^w) = \psi_1(R_{t-4}^w) + \psi_2(R_{t-5}^w) + \ldots \]  

(A4.3)

\[ E_{t-4}(R_t^{w-1}) = \psi_1 \phi(R_{t-4}^{w-1}) + \psi_2 \phi^2(R_{t-5}^{w-1}) + \ldots \]  

(A4.4)

\[ E_{t-4}(R_t^{w-2}) = \psi_1 \phi^2(R_{t-4}^{w-2}) + \psi_2 \phi^3(R_{t-5}^{w-2}) + \ldots \]  

(A4.5)

\[ E_{t-4}(R_t^{w-3}) = \psi_1 \phi^3(R_{t-4}^{w-3}) + \psi_2 \phi^4(R_{t-5}^{w-3}) + \ldots \]  

(A4.6)

Taking equations (A.4.3) to (A4.6) and collecting terms it is possible to \( \pi_i \) as the weight attached to each of the weekly returns when trying to forecast future monthly returns as follows.

\[ \pi_i = \theta^{-1}(\phi - \theta)(1 + \phi + \phi^2 + \phi^3) \]  

(A4.7)

which is equation (5.9) in the text.

Appendix 5. Proof Of How The Weights Are Calculated

We know that the weight \( \pi_i \) for a particular past weekly return is

\[ \pi_i = \theta^{-1}(\phi - \theta)(1 + \phi + \phi^2 + \phi^3) \quad i = 1, 2, 3, 4 \]  

(A5.1)

Because information in only the previous four weekly returns is to be used to predict monthly returns the optimal weight to be attached to the four weekly realised returns must sum to unity. This requires calculating the weights proportional to the sum of the weights.

\[ w_i = \frac{\pi_i}{\sum \pi_i} \]  

(A5.2)

Substituting equation (A5.1) and simplifying gives

\[ w_i = \frac{\theta^{-1}}{(1 + \phi + \phi^2 + \ldots)} \]  

(A5.3)

which can be written as

\[ w_i = \theta^{-1}(1 - \theta) \]  

(A5.4)
Table 5.1: Autocorrelations of Excess Portfolio Returns

This table contains the autocorrelations for the excess portfolio returns. Excess portfolio returns are calculated as the portfolio return less the risk free return (the Treasury Bill Rate). Excess returns are calculated for four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares listed on the London stock exchange in the sample. The portfolios were formed from the rankings of market value of equity outstanding at the beginning of January each year for the sample period January 1976-December 1991. $R_1$ is the return of the small firm portfolio and $R_4$ is the return of the portfolio which contains the largest firms, $\hat{\rho}_t$ are the estimated autocorrelation coefficients at lag $\tau$. Under the hypothesis that these coefficients are zero, their standard errors are approximately 0.035 for the weekly autocorrelations and approximately 0.0722 for the monthly autocorrelations, $Q$ are the Ljung-Box statistics testing the hypothesis that all autocorrelations up to lag 6 are jointly zero their p values are shown in parentheses.

### Weekly autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
<th>$\hat{\rho}_3$</th>
<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.407</td>
<td>0.266</td>
<td>0.545</td>
<td>0.277</td>
<td>0.143</td>
<td>0.104</td>
<td>242.54 (0.00)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.391</td>
<td>0.225</td>
<td>0.143</td>
<td>0.345</td>
<td>0.066</td>
<td>0.0633</td>
<td>198.95 (0.00)</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.301</td>
<td>0.166</td>
<td>0.055</td>
<td>0.098</td>
<td>-0.013</td>
<td>0.039</td>
<td>109.06 (0.00)</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.148</td>
<td>0.109</td>
<td>-0.021</td>
<td>0.060</td>
<td>-0.038</td>
<td>0.004</td>
<td>33.03 (0.00)</td>
</tr>
</tbody>
</table>

### Monthly autocorrelations

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
<th>$\hat{\rho}_3$</th>
<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>0.321</td>
<td>0.183</td>
<td>0.012</td>
<td>0.008</td>
<td>-0.01</td>
<td>-0.049</td>
<td>29.29 (0.00)</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.288</td>
<td>0.054</td>
<td>0.039</td>
<td>-0.016</td>
<td>-0.024</td>
<td>0.011</td>
<td>18.53 (0.00)</td>
</tr>
<tr>
<td>$R_3$</td>
<td>0.200</td>
<td>-0.026</td>
<td>-0.008</td>
<td>-0.038</td>
<td>-0.014</td>
<td>0.019</td>
<td>9.03 (0.15)</td>
</tr>
<tr>
<td>$R_4$</td>
<td>0.148</td>
<td>-0.155</td>
<td>-0.074</td>
<td>-0.041</td>
<td>0.010</td>
<td>-0.049</td>
<td>11.68 (0.08)</td>
</tr>
</tbody>
</table>
This table contains the autocorrelations of the residuals which are obtained the estimation of the following ARMA(1,1) model of realised excess returns for four equally-weighted portfolios. Where $P_1$ is the portfolio containing the smallest firms while $P_4$ is the portfolio which contains the largest companies in the sample.

$$R_{p,t} = \phi R_{p,t-1} - \theta a_{p,t-1} + a_{p,t}$$

Under the hypothesis that these coefficients are zero, their standard errors are approximately 0.035. The $Q$ are the Ljung-Box statistics which test the hypothesis that all autocorrelations up to lag 6 are jointly zero, their p values are shown in parentheses. $\phi_1$ and $\theta_1$ are the estimated AR and MA coefficients respectively which are obtained from the estimation of the ARMA(1,1) model, t-statistics for these coefficients are reported in parenthesis.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
<th>$\hat{\rho}_3$</th>
<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
<th>$Q$</th>
<th>$\hat{\phi}$</th>
<th>$\hat{\theta}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.003</td>
<td>0.001</td>
<td>-0.047</td>
<td>-0.010</td>
<td>0.068</td>
<td>0.046</td>
<td>7.7</td>
<td>0.668</td>
<td>-0.318</td>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.25)</td>
<td>(2.91)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.005</td>
<td>-0.027</td>
<td>-0.017</td>
<td>0.056</td>
<td>-0.012</td>
<td>0.029</td>
<td>4.367</td>
<td>0.613</td>
<td>-0.264</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.63)</td>
<td>(6.06)</td>
<td>(-2.58)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>-0.001</td>
<td>0.011</td>
<td>-0.055</td>
<td>0.079</td>
<td>-0.059</td>
<td>0.054</td>
<td>13.34</td>
<td>0.539</td>
<td>-0.264</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.04)</td>
<td>(9.29)</td>
<td>(-3.58)</td>
</tr>
<tr>
<td>$P_4$</td>
<td>-0.009</td>
<td>0.045</td>
<td>-0.069</td>
<td>0.059</td>
<td>-0.046</td>
<td>0.020</td>
<td>10.86</td>
<td>0.488</td>
<td>-0.344</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(11.53)</td>
<td>(-4.31)</td>
</tr>
</tbody>
</table>
Table 5.3: The Weights Attached to Past Weekly Realised Returns.

This table provides estimates of the weights to be attached to each of the previous four weekly portfolio realised returns which are to be used to forecast the future monthly portfolio return. As explained in Section 2 of the text and in Appendix 5 these are calculated as $\theta (1 - \theta)$.

$$E_{\tau - 4}(R_t^*) = w_1(R_{\tau - 4}^*) + w_2(R_{\tau - 3}^*) + w_3(R_{\tau - 2}^*) + w_4(R_{\tau - 1}^*)$$

<table>
<thead>
<tr>
<th>$\Phi - \theta$</th>
<th>$w_1$</th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 - \theta$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P_1$</td>
<td>0.22</td>
<td>0.68</td>
<td>0.23</td>
<td>0.07</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.74</td>
<td>0.74</td>
<td>0.19</td>
<td>0.05</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.38</td>
<td>0.74</td>
<td>0.19</td>
<td>0.05</td>
</tr>
<tr>
<td>$P_4$</td>
<td>0.22</td>
<td>0.66</td>
<td>0.23</td>
<td>0.08</td>
</tr>
</tbody>
</table>
Table 5.4:
Estimates from Regressing Monthly Realised Returns on Expected Monthly Returns

The following table provides estimates obtained from the regression of realised monthly returns \( R^m_p \) on expected monthly returns \( E_{t-\delta}(R^m_p) \) over the sample period January 1976 to December 1991. \( \alpha \) is the coefficient for the constant, \( \beta \) is the coefficient which captures the effect that a 1% increase in expected returns will have on realised returns. T-statistics are provided in parenthesis. An ARCH(6) test on the residuals captures the effect of sixth order heteroscedasticity in the residuals from this model, \( p \) values for this statistic are shown in parentheses. In all cases the null that there are no ARCH effects can not be rejected.

\[
R^m_{p,t} = \alpha_p + \beta_p E_{t-\delta}(R^m_{p,t}) + \epsilon_{p,t}
\]

<table>
<thead>
<tr>
<th></th>
<th>( \hat{\alpha} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{R^2} )</th>
<th>ARCH(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.005</td>
<td>1.19</td>
<td>0.148</td>
<td>11.72</td>
</tr>
<tr>
<td></td>
<td>(1.97)</td>
<td>(6.05)</td>
<td>(0.68)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.006</td>
<td>1.27</td>
<td>0.153</td>
<td>6.71</td>
</tr>
<tr>
<td></td>
<td>(2.1)</td>
<td>(6.19)</td>
<td>(0.34)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.007</td>
<td>1.17</td>
<td>0.150</td>
<td>4.94</td>
</tr>
<tr>
<td></td>
<td>(2.09)</td>
<td>(6.02)</td>
<td>(0.55)</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.009</td>
<td>0.98</td>
<td>0.008</td>
<td>5.34</td>
</tr>
<tr>
<td></td>
<td>(2.50)</td>
<td>(1.63)</td>
<td>(0.50)</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.5: Estimates from the ARMA(1,1) model with January Dummies.

This table reports the coefficients obtained from the estimation of the ARMA(1,1) model which has dummies to capture the effect of the month of January on a portfolio return.

\[ R_{p,t}^* = \delta_{1,p} + \delta_{2,p}Jan_1 + \delta_{3,p}Jan_2 + \delta_{4,p}Jan_3 + \phi R_{p,t-1} + \theta a_{p,t-1} + a_{p,t} \]

\( \delta_{p,j} \) is a constant, \( Jan_1, Jan_2, Jan_3 \) are dummies which take on a value of one if the weekly return occurs in the first, second or third week of January respectively but a zero otherwise. \( \delta_{p,2}, \delta_{p,3}, \delta_{p,4} \) are the coefficients for the dummies and indicate how much above the constant returns are in either of these weeks in January. Because these coefficients are very small in magnitude, in the tables they have been raised by five decimal places. \( w_1, w_2, w_3, w_4 \) are the new set of weights to be attached to the previous four weekly returns respectively. These are calculated as \( \theta(1 - \theta) \) as explained in Section 2.

<table>
<thead>
<tr>
<th></th>
<th>( \delta_{1,p} \times 10^4 )</th>
<th>( \delta_{2,p} \times 10^4 )</th>
<th>( \delta_{3,p} \times 10^4 )</th>
<th>( \delta_{4,p} \times 10^4 )</th>
<th>( \phi )</th>
<th>( \theta )</th>
<th>( w_1 )</th>
<th>( w_2 )</th>
<th>( w_3 )</th>
<th>( w_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_1 )</td>
<td>2.10</td>
<td>2.16</td>
<td>5.31</td>
<td>3.04</td>
<td>0.428</td>
<td>-0.193</td>
<td>0.81</td>
<td>0.16</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(5.28)</td>
<td>(16.36)</td>
<td>(-4.54)</td>
<td>(-15.07)</td>
<td>(0.048)</td>
<td>(2.18)</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>( P_2 )</td>
<td>1.79</td>
<td>6.57</td>
<td>5.34</td>
<td>9.27</td>
<td>0.338</td>
<td>-0.156</td>
<td>0.84</td>
<td>0.13</td>
<td>0.03</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>(3.97)</td>
<td>(10.17)</td>
<td>(-3.90)</td>
<td>(-21.24)</td>
<td>(2.23)</td>
<td>(1.48)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_3 )</td>
<td>2.46</td>
<td>8.24</td>
<td>8.84</td>
<td>5.73</td>
<td>0.235</td>
<td>-0.272</td>
<td>0.72</td>
<td>0.21</td>
<td>0.06</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>(3.81)</td>
<td>(6.59)</td>
<td>(-5.37)</td>
<td>(-20.67)</td>
<td>(1.67)</td>
<td>(2.54)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>( P_4 )</td>
<td>199.5</td>
<td>643.8</td>
<td>41.4</td>
<td>78.1</td>
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<td>(-20.41)</td>
<td>(1.99)</td>
<td>(2.03)</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
Table 5.6: 
Estimates from Regressing Returns on Expected Monthly Returns with Dummies

The following table provides estimates obtained from the regression of realised monthly returns ($R_i$) on expected monthly $E_{i-4}(R_i)$ returns over the sample period January 1976 to December 1991. Expected returns this time are a weighted sum of the weekly returns from each of the previous four weeks, if the monthly return interval falls in a January then the expected weekly return will be inflated by the return predicted by the January dummy. $\alpha$ is the coefficient from the constant. $\beta$ is the coefficient from the expected return, t values are indicated in parenthesis. An ARCH(6) test on the residuals captures the effect of sixth order heteroscedasticity in the residuals from this model, p values for this statistic are shown in parentheses. In all cases (except one) the null that there are no-ARCH effects can not be rejected.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$\bar{R}^2$</th>
<th>ARCH (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>0.005</td>
<td>1.07</td>
<td>0.148</td>
<td>14.41</td>
</tr>
<tr>
<td></td>
<td>(2.07)</td>
<td>(6.05)</td>
<td></td>
<td>(0.03)</td>
</tr>
<tr>
<td>$P_2$</td>
<td>0.006</td>
<td>1.17</td>
<td>0.155</td>
<td>(8.61)</td>
</tr>
<tr>
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<td>(6.21)</td>
<td></td>
<td>(0.20)</td>
</tr>
<tr>
<td>$P_3$</td>
<td>0.007</td>
<td>1.17</td>
<td>0.121</td>
<td>5.15</td>
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<tr>
<td></td>
<td>(2.05)</td>
<td>(5.96)</td>
<td></td>
<td>(0.52)</td>
</tr>
<tr>
<td>$P_4$</td>
<td>-0.000</td>
<td>0.79</td>
<td>0.090</td>
<td>2.52</td>
</tr>
<tr>
<td></td>
<td>(-0.58)</td>
<td>(4.6)</td>
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<td>(0.86)</td>
</tr>
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</table>
CHAPTER SIX
Risk, Seasonality and the Asymmetric Behaviour of Stock Returns

6.1 Introduction

This chapter will be concerned with investigating the seasonal properties of returns, for portfolios which have been formed on the basis of market capitalisation. This chapter will provide evidence to suggest that for both large and small firms stock returns are abnormally high during the months of January and April. The focus of this chapter will be on testing whether abnormally high returns during these months can be explained by the pricing of systematic risk. This chapter finds that risk is not priced every month of the year but instead investors receive a systematic risk premium in only three months of the year, January, April and July. Furthermore, the magnitude of the systematic risk premium appears to be closely associated with firm size, when systematic risk is priced large firms always provide a higher systematic risk premium than small firms. It is also found that the risk premiums associated with total and residual risk are significantly different from zero for a number of months, including January. This provides little evidence in support of the Capital Asset Pricing Model, at least on a month by month basis.

The remainder of this chapter is set out as follows. Section 6.2, provides some background information concerning the return behaviour of small firms. Section 6.3, examines the data set which is used in this study and discusses the month by month performance of the different portfolios. Section 6.4, examines the Fama and MacBeth (1973) methodology which is used in this study to test the relationship between risk and return. Section 6.6, provides the results which are obtained when the relationship between systematic risk and return is tested for each month of the year. Section 6.5, investigates the relationship between non-systematic risk and return. Section 6.7, provides a summary and offers some conclusions to the paper.

6.2 Background

6.2.1 The Small Firm Premium

The existence of a small firm premium has now been established on considerably more than half a century of US and UK data. Empirical support for a size premium was first discovered by Banz (1981) who found that the average return from holding small firms long but executing a short position on large firms provided a return of about 19.8 percent a year. The small firm

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1. The work presented in this chapter is the basis of two published papers appearing in Journal of Business Finance and Accounting and European Journal of Finance respectively.
premium has now been identified empirically for a large number of countries, for the UK; by Levis (1985,1989) and Fong (1992), Canada; Berges McConnell and Schlarbaum (1985), Australia; Brown, Kleidon and Marsh (1983) and Japan Kato and Shallheim (1985).

6.2.2 Return Seasonality and The Small Firm Effect

A thorough analysis of the size premium has been clouded somewhat by the issue of return seasonality, which has been closely linked to the size issue. For example, Keim (1983) and Roll (1983) find that in the US, half of the small firm premium can be explained by the behaviour of returns during the last trading day of December and the first five trading days of January. Because of the surge in prices, experienced at the start of the new tax year, the US small firm effect has inevitably been linked to tax-loss trading, see for example, Roll (1983), Reinganum (1983) and Ritter (1988). Interestingly, the small firm premium in the UK is dominated neither by the behaviour of January returns or the new tax year rally. Indeed, counter to the US experience large firms rather than small firms appear to perform slightly better in both of these months. For example, over the period 1958-1982 Levis (1985) reports that the difference between the return of the smallest and the largest firms in the sample is only significant during the month of May, Levis finds that small firms outperform large firms on average by about 2.4 percent during this month and therefore termed this the May effect. There is little evidence of this persisting for later samples, see for example Limmack (1995). It is possible therefore that the May effect previously observed for small firms was really an April effect, but, because thin trading exists for many small firms, the April return rise was not observed until May. 

6.2.3 Explaining Return Seasonality

An important contribution to our understanding of the causes of the January effect in the US market was made by Tinic and West (1984,1986) and Corhay, Hawawini, and Michel (1987,1989). Tinic and West (1984) investigated whether monthly return seasonalities could be linked to the Capital Asset Pricing Model (CAPM). This was done by testing whether monthly return regularities could be accounted for by month to month variations in the pricing of systematic risk. When Tinic and West (1984) re-estimated the Fama and MacBeth (1973) CAPM regressions which were discussed in Section 2.6 of Chapter 2, they found that the positive trade-off between systematic risk and return was exclusively due to what happened in the month of January. When the eleven months, other than January were tested separately, no positive

2. In recent years and particularly since big bang volume of trading on the London Stock Exchange has increased substantially, see for example Thomas (1989).
relationship between systematic risk and return was found to emerge in any period. Initially, these findings were interpreted as an indication that investors obtained a systematic risk premium in only one month of the year, January.

In a later paper Tinic and West (1986) extended their work to examine the relationship between monthly returns and other forms of risk such as unsystematic (residual) and non-linear systematic risk. The results reported in this second paper suggest that the positive relationship between January returns and systematic risk, found in the earlier paper was not robust to the inclusion of alternative forms of risk. When they tested the relationship between monthly returns and risk, for a four parameter model which included alternative measures of risk, a negative, but insignificant relationship between systematic risk and return was found for January. They report instead that residual risk is positively priced during January.

Corhay, Hawawini and Michel (1987,1989) reported similar results for the UK. Their 1989 paper tested the relationship between monthly returns and the systematic risk of securities traded on the London Stock Exchange. They found that the systematic risk premium was only significantly different from zero and positive during the month of April. Like Tinic and West, Corhay, Hawawini and Michel (1987) also investigated the relationship between monthly returns and other forms of risk. Their results also appeared to be sensitive to the precise specification being tested. Interestingly, like Tinic and West (1986) they also found when January was excluded from the tests a negative relationship between systematic risk and return appeared to exist for the other eleven months of the year. Although the ability of the Capital Asset Pricing Model to predict expected returns has been questioned there is nothing to explain why a negative relationship between systematic risk and return should exist.

Although, these papers made a very important contribution to our understanding of both the causation of monthly return seasonalties and, the month by month relationship between risk and return in the US and the UK, it is important to extend the work of Tinic and West and Corhay et Al and investigate the effect that firm size might have on the monthly relationship between risk and return.

Surprisingly, the contribution that firm size might have on monthly risk premiums has not previously been investigated. The aim of this chapter is to investigate the monthly relationship between risk and return for firms of different capitalisations. The Chapter will seek to establish whether firm size contributes to the magnitude of monthly risk premiums and whether monthly return seasonalties can be linked to the pricing of risk for both large and small firms.
6.3 Data and Summary Statistics

Monthly return data was obtained from the London Share Price Data Base covering the period 1976 to 1991. Monthly returns were used to minimise the effect that microstructure biases, such as those described by Roll (1983) Dimson (1979) and Cohen, Maier and Schwartz (1986), might have on the results. These were discussed at length in Section 2.7 of Chapter 2.

Companies selected are those which existed in 1976 and survived until the end of the sample period. In all return information for 150 stocks was utilised. In 1976 all 150 companies were ranked on the basis of their January market value. From these 150 stocks three portfolios were constructed, The first portfolio \( P_1 \) contained the 50 smallest firms, the second portfolio \( P_2 \) contained the 50 medium sized companies, and portfolio three \( P_3 \) contained the 50 smallest companies. The average market value of companies contained in the largest portfolio was approximately £85m but the average value of companies in the smallest portfolio was only £1m. In this Chapter only three portfolios have been tested in order that differences between the behaviour of large and small firms can be isolated in the empirical tests which are performed.

It should be noted that this sample uses a survivorship biased sample of companies. As Chapter Four noted, a survivorship biased sample of companies may not behave in the same way as a sample of companies which contain survivors and non-survivors. The most obvious problem associated with the use of survivor companies is the possibility that the returns of surviving companies may be biased upwards. This problem would arise if non-survivor companies experienced lower returns, perhaps because they were experiencing financial distress. In this case bankruptcy would be the probable cause of their de-listing. However, companies might also disappear if they merged with another company or were the subject of a takeover, in this case we might expect the non-surviving companies to perform better, not worse, than the surviving companies at the time of the takeover.

It is possible that small firms are more influenced by the survivorship problem. Small firms are more likely to be taken over because financing a takeover or merger when a company is small is easier than when the company is large, extensive resources are necessary to fund a takeover/merger of a large firm. It is also true that small firms are more likely to experience financial distress, see for example the work of Castanias (1984) which shows that as firm size decreases companies are more likely to experience failure. The corporate finance literature, for example see Harris and Raviv (1992) suggests that this is due to small firms having to rely more heavily on debt finance as a source of finance.

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3. Using monthly observations will reduce the effect of the bid-ask bias, it is also important to use monthly data so that the effect of thin trading on estimates of security \( \beta \)'s is minimised.
4. This is the average market value of companies over the period 1976 to 1991.
Although this chapter does undertake a comparison of the returns of large and small firms the difference between the large and the small firm returns, over the same sample is robust even when a non survivorship sample is used. This appears to suggest that the survivorship problem does not have a large impact on the returns of UK companies, at least over the period 1976-1991.

The chief aim of this chapter is to investigate the relationship between risk and return. This study is therefore more concerned about the impact that the survivorship properties of the data has on the betas, other risk estimates and their relationship to returns. A sample of companies which are survivorship biased may have lower(higher) betas than a similar set of non survivorship biased companies, furthermore, the relationship between risk and return for survivorship biased companies may be different. Unfortunately, in a study such as the one undertaken in this chapter it is impossible to use a sample of companies which are entirely survivorship free. It is necessary for a company to exist for at least five years to allow enough time to calculate betas and then test the relationship between risk and return. However, as Chelley-Steeley (1996) demonstrates the results presented in this chapter are substantially unchanged, even if the only requirement placed on companies is that they must exist for at least five years. This suggests that the results presented in this chapter are not an artifact of using a surviving sample of companies.

Table 6.1 reports the mean monthly return for the three portfolios. The mean monthly return over the full sample period was found to be 1.54 percent for small firms, 1.1 percent for the medium sized firms and 1.13 percent for the large firms. This can be converted into a small firm annualised premium relative to the large firms of about 3.0 percent. This small firm premium is considerably lower than the premium reported in the US or the premium reported for an earlier sample period by Levis (1985). The reason for this is that small firms have substantially underperformed large firms during the period 1988-1990, which has been a period of very unusual economic activity.

The mean monthly return, for both large and small firms associated with each month of the year individually are also reported in Table 6.1. There is considerable evidence of variation in the mean monthly returns both between the largest and the smallest portfolio and across the different months of the year. January, which provides a return of about 5 percent in all cases, is the month which provides the highest mean return. Returns are also found to be relatively high for all three size-based portfolios in February, March and April.

During April the mean monthly return for the largest firms is approximately 3 percent but only 1.7 percent for the smallest firms which indicates that large firms outperform small firms by a statistically significant average of 1.3 percent during this month. This is in conflict with results reported by Keim (1983) and Roll (1983) for the US who both document a strong small firm premium during the start of the new tax year. The conventional tax-loss trading
small firm premium during the start of the new tax year. The conventional tax-loss trading framework, which suggests that small firms earn a premium because small investors are more heavily engaged in tax-loss rebalancing, Ritter (1988), does not therefore appear to be consistent with the UK evidence.

Generally, the pattern of monthly returns reported for the UK in this paper are similar to those which are presented by Levis (1985) for an earlier observation in the UK. A distinguishing characteristic of the results presented here, however, is what appears to be a strengthening of the January effect, particularly for large firms, this has also been noted by Limmack (1995). During the 1980’s there has been an increased internationalisation of stock markets, encouraged by the removal of exchange controls, see for example, Taylor and Tonks (1987) and Chelley, Pentecost and Steeley (1994). This has encouraged an increased co-movement between national stock markets because of the additional pressure foreign investors now put on domestic markets. The strengthening of the January effect in the UK is therefore consistent with an increased comovement between the UK and US stock market.

6.4 The Fama and MacBeth Methodology

The procedure used to test the relationship between systematic risk and return is based on the Fama and MacBeth (1973) methodology which requires the estimation of equation (6.1) below. Each year using the preceding two years of data, each of the three size-based portfolios are divided into a further five portfolios, stratified this time on the basis of security beta, in all this provides fifteen portfolios, five for each size grouping. In the next two year period security betas are re-estimated, from these the mean sub-portfolio beta is then calculated, one for each of the fifteen portfolios. This is done by taking the arithmetic average of the betas for the individual securities making up a portfolio. The relationship between the preceding years portfolio beta and the following years’ realised monthly return is then tested. This entire procedure is updated each year to provide a time series with which to test the risk-return relationship as follows,

\[
R_{p,t} = \gamma_{1,t} + \gamma_{2,t} \beta_{p,t-1} + \bar{u}_{p,t} \quad p = 1,2...15
\]

where, \( R_{p,t} \) is the realised return on the \( p \)th sub-portfolio of stocks within one of the three size groupings in month \( t \), and \( \beta_{p,t-1} \) is the average of the estimated \( \beta \)'s associated with the stocks contained within each sub-portfolio. The individual component betas are calculated from the market model, which was described in Section 2.7.2.1 of Chapter 2, using the preceding two years of data. The \( \gamma_{1,t} \) and \( \gamma_{2,t} \) are coefficients, \( \tilde{\gamma}_{1,t} \) is an estimate of the constant return (return

5. It should be noted that the data which is used to test this model has twelve years of time series observations and fifteen cross-sectional observations each year.
unassociated with systematic risk) for portfolio \( p \) in time \( t \), while \( \tilde{y}_{2,t} \) is an estimate of the systematic risk premium for portfolio \( p \) in time \( t \). \( \tilde{u}_{p,t} \) is the error or unexpected return in time \( t \).

Since this study aims to discover whether both calendar and size effects exist in the risk-return relationship, the following extended version of the Capital Asset Pricing Model was tested. This model, which is represented by equation (6.2), will allow us to identify specific months of the year in which systematic risk is priced, and establish whether the risk premium for large, medium and small firms is equivalent.

\[
R_{p,t} = \bar{\gamma}_{t} + \tilde{y}_{2,t} \beta_{p,t-1} + \sum_{j=2}^{3} \bar{\lambda}_{j,t} \delta_{j,t} + \sum_{j=2}^{3} (\delta_{j,t} \tilde{\beta}_{p,t-1}) + \tilde{u}_{p,t} \quad j = 2, 3 \quad p = 1, 2, \ldots 15 \tag{6.2}
\]

where, \( \bar{\gamma}_{t}, \tilde{\gamma}_{t}, \bar{\lambda}_{j,t}, \delta_{j,t} \) are all coefficients. The \( \tilde{\beta}_{p,t-1} \) are the average beta of stocks within portfolio \( p \).

To control for the effects of thin trading\(^6\), the Dimson (1979) estimator for \( \beta \) has been used throughout\(^7\). This was discussed fully in Section 2.7.2.1 of Chapter 2. The \( \delta_{j,t} \) are dummy variables representing firm size. The first, \( \delta_{2} \), takes on a value of 1 if the \( p^{th} \) portfolio consists of medium sized firms, but a value of zero otherwise. The second, \( \delta_{3} \), takes on a value of 1 if the \( p^{th} \) portfolio consists of small firms but a zero otherwise. The terms \( \bar{\lambda}_{j,t} \) are coefficients which indicate how much above or below the \( \bar{\gamma}_{t} \) of the large firms this coefficient is for the medium and small firms respectively. The \( (\delta_{j,t} \tilde{\beta}_{p,t}) \) are a set of slope dummies. These represent the beta of the \( p^{th} \) portfolio times each of the two size dummies in turn. The coefficient \( \bar{\lambda}_{j} \) will capture differences in the systematic risk premium of the medium or small firms relative to the systematic premium of the large firms. This coefficient will therefore tell us whether the systematic risk premium for the medium and small firms differs to that of the large firms.

Using the Fama and MacBeth approach, this model is then tested, for all three portfolios, for each individual month of the year. This means that the time series of return and beta observations are segregated on the basis of calendar month. For a given year the beta of a portfolio is regressed against the monthly return for a given month. Although the betas during the course of the year are the same, the returns over the different months are vary. The results of the cross-sectional tests then allow us to gauge if on average the beta of a portfolio is related to the portfolio returns of a given month and whether this relationship is stronger for some months in

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6. Thin trading associated with the smallest companies in the cohort is substantial. A detailed analysis of the data reveals that on average 28.6 percent of the returns in the portfolio containing the smallest stocks are stale.

7. A Dimson model with five lags for the smallest portfolio has been used, this ensures that thin trading durations of up to five months will be captured by beta estimates. To illustrate the importance of adjusting for thin trading in this way, the estimated contemporaneous portfolio beta for the smallest firm was estimated and compared to the beta estimate obtained from a five lag aggregate coefficient model. The contemporaneous portfolio beta was found to be 0.531, meanwhile the aggregate coefficient beta was found to be 1.07. Since thin trading is less severe in medium firms a model with only three lags was utilised for this group of firms and no lags for the largest firms.
comparison to others. This will allow individual months of the year, when systematic risk is or is not priced to be identified. Furthermore, the model will also allow us to establish whether firm size influences the magnitude of the risk premium in any of the months.

6.5 The Results

The results from the estimation of this model, which are provided in Table 6.2 suggest that both the month of the year and firm size have an important influence over the systematic risk premium. In only January and April does systematic risk appear to be priced. For the other ten months of the year no persistent relationship between systematic risk and return appears to exist.

Focusing on the coefficients associated with the two slope dummies \((\delta_j, \beta_p)\), these coefficients indicate that the systematic risk premium is not equal for large and small firms. Both slope dummies are found to be significantly negative in both January and April. This suggests that large firms earn a higher systematic risk premium than either the medium or small firms. Therefore, not only is the systematic risk premium dependant on the month of the year but it is also dependant on firm size. For a given amount of risk, large firms will on average earn a higher systematic risk premium, than investments in small or medium sized companies. Actually, the results indicate that both the medium and the small firms obtain a small negative systematic risk premium during these two months. Strong and Xu (1994) also find some evidence to indicate that betas are negatively priced cross-sectionally in the UK, although, they do not investigate whether this finding is related to firm size.

The results for July suggest that during this month yet again small and medium firms earn a lower systematic risk premium than the large companies. Since \(\gamma_2\) is only significant at a 10 percent level, but \(\delta_j, \beta_p\) is significant in both cases at a 5 percent level, it is possible that the systematic risk of large firms is not priced in this month, but that the systematic risk of small and medium firms is priced.

In Table 6.2 the results of an ARCH(6) test are presented. This tests for up to sixth order heteroscedasticity in the residuals and except for the month of May and August the null-hypothesis of no heteroscedasticity can not be rejected. The results of a Ljung-Box (1978) Q test for up to sixth order autocorrelation are also reported in Table 6.2. In a number of cases the Q statistic is found to be significant. This is not surprising, since the CAPM model being tested here appears to be misspecified, at least for the months other than January, April and July.

The \(R^2\) for all months is quite low which indicates that this model can not explain much of the variation in the returns of a given month. As we can see from Table 6.2 when all months are estimated together the \(R^2\) indicates that no more than about 2 percent of the variation in
realised portfolio returns can be explained by the model tested here. When the months are estimated separately, the $R^2$ in a number of months is much higher. The month for which the model explains the greatest proportion of the variation in monthly returns is August, during this month the model explains about 8 percent of the variation in realised portfolio returns, in April 5.5 percent can be explained and in January approximately 4 percent.

The findings reported in this section appear to shed a little more light on monthly calendar and size effects which were reported in the previous section. April and January are the two months which provide the highest returns for the UK stock market. These are also the only two months for which there is a relationship between systematic risk and return. The return profile of stocks during this month may be related to the systematic risk premium. Since the systematic risk premium is higher for large firms it is not surprising that large firms outperform small firms in these months. Although, it is puzzling why the returns of small firms do not appear to be more closely related to systematic risk.

6.5.1 Seasonality in the Systematic Risk Premium

Having found that systematic risk provides a persistent risk premium in only two months, it is also important to determine whether the magnitude of the risk premium varies across the months of the year by examining a second model which is represented by Equation (6.3). The aim of the model is to discover whether the magnitude of the coefficients $\gamma_1$ and $\gamma_2$ are significantly different across the months of the year. Each year the coefficient $\gamma_1$ and $\gamma_2$ for each individual month, was estimated annually over the sample period\(^8\). These coefficients were then tested for the existence of monthly seasonalities using the following model.

$$
\tilde{\gamma}_{j,i} = \tilde{\alpha}_i + \sum_{i=1}^{12} \tilde{\phi}_{j,i} D_{i,i} + \sum_{i=1}^{12} \tilde{\theta}_{j,i} (D_{i,i} \delta_{2,i}) + \sum_{i=1}^{12} \tilde{\theta}_{j,i} (D_{i,i} \delta_{3,i}) + \tilde{u}_{j,i}, \quad j = 1, 2
$$

where, $D_1 - D_{12}$ are a set of dummy variables one each for the months of the year January to December, but excluding July\(^9\). The $\tilde{\alpha}_{j,i}$ are coefficients which are estimates of the difference between the monthly means of $\tilde{\gamma}_j$ from that of July. The $(D_{i,i} \delta_{2,i})$ are a set of dummies, each one represents the month dummy times the medium firm size dummy, while $(D_{i,i} \delta_{3,i})$ represents each of the month dummies in turn times the small firm size dummy. The coefficients associated with these dummies, $\tilde{\phi}_{j,i}$ and $\tilde{\theta}_{j,i}$, will capture differences in $\tilde{\gamma}_{j,i}$ for month $i$ which are related to firm size. This will allow us to determine whether the systematic risk premium $\tilde{\gamma}_2$ which is associated with medium and small firms is different to that of large firms in any month.

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8. This required taking estimates $\gamma_i$ and $\gamma_i$ from equation (6.1) so that it was possible to test for size effects in specific months.

9. This test was performed using a selection of control months. The results seem to be robust whatever month is used as the control month.
Taking the coefficient associated with the intercept $\tilde{\gamma}_1$ first, there is little evidence to suggest that $\tilde{\gamma}_1$ displays a seasonal pattern associated with the months of the year. The only month dummy found to be significant was that of June, since the sign is a negative one this indicates that the June intercept of the large firms is below that of July. The results also suggest that the medium firms obtain a premium unrelated to risk in January which is significantly higher that the July constant of the large firms.

When the test is performed on $\tilde{\gamma}_2$ a number of interesting results are uncovered. Even after controlling for firm size a positive January seasonality in $\tilde{\gamma}_2$ is found to exist. The systematic risk premium in January for the large firms is significantly higher than the risk premium for July. The magnitude of the systematic risk premium for large firms also appears to be higher during June than for July, suggesting that the risk premium in this month is also on average higher than for July. Although, as the previous section indicated no persistent relationship existed between risk and return for this month. For April, the month dummy is insignificant, this indicates that the magnitude of the risk premium for large firms in April is not higher than in July. Although there is a persistent relationship between risk and return for large firms during April the size of the premium is modest. In contrast, the months of October and December provide systematic risk premiums for the large firm portfolio which are lower than during the month of July. These results appear to indicate that not only is systematic risk priced during January but the premium paid for systematic risk is also higher during this month.

6.6 Non-Systematic Risk and Return Seasonality

Since this chapter is concerned with identifying the causes of return seasonality, an extended version of equation (6.3) is also tested. This model investigates whether alternative sources of risk such as residual or total risk are priced by the market, and if they are, can they account for any return seasonality. These tests are also important to undertake because, as Section 6.2.3 of this chapter outlined, the work undertaken by Tinic and West (1986) found that the positive pricing of systematic risk, in certain months of the year, does not appear to be robust when extended versions of the Capital Asset Pricing Model are tested.

The full model tested in this section is represented by equation (6.4) below. This model is tested each month separately, providing twelve different estimates of each of the risk premiums, one for each month of the year.

$$R_{p,t} = \tilde{\gamma}_t + \tilde{\gamma}_2 t \hat{\beta}_{p,t-1} + \tilde{\gamma}_3 t \text{var}_{p,t-1} + \tilde{\gamma}_4 t \hat{r}_{p,t-1} + \tilde{\gamma}_5 t \left( \sum_{j=2}^{3} \hat{\phi}_j (\text{var}_{p,t-1}, \delta_{j,t}) + \sum_{j=2}^{3} \phi_j (\hat{r}_{p,t-1}, \delta_{j,t}) + \sum_{j=2}^{3} \phi_j (\hat{r}_{p,t-1}, \delta_{j,t}) + \mu_{p,t} \right)$$

(6.4)
where, \((\hat{\beta}_{p,t-1}, \delta_{j,i})\) is the estimated systematic risk premium of the \(p^{th}\) portfolio in time \(t-1\) times each of the two size dummies in turn. In this model \(\tilde{\lambda}_2\) is an estimate of the difference in the systematic risk premium of medium firms from that of large firms, while, \(\tilde{\lambda}_3\) will provide an estimate of the difference in the systematic risk premium of the smallest firms from that of large firms. The term \(r\hat{e}s\) is the estimate of unsystematic risk for the \(p^{th}\) portfolio and is measured as the mean, standard deviation of the security residuals extracted from the aggregate coefficient market model, which was used to estimate security betas. The term \(v\text{ar}\) is the estimate of total risk for the \(p^{th}\) portfolio and is measured as the average return variance of securities within portfolio \(p\). Security variances were calculated using the previous two years of data. The coefficients \(\tilde{\gamma}_{4,t}\) and \(\tilde{\gamma}_{3,t}\) can be interpreted as the residual and total risk premium for large firms in time \(t\) respectively. The term \((r\hat{e}s_{p,t-1}, \delta_{j,i})\) represents the estimated residual risk of portfolio \(p\) in time \(t-1\) times the medium firm size dummy and the small firm size dummy in turn. This allows \(\tilde{\theta}_{2,t}\) to capture the difference in the unsystematic risk premium of medium firms from that of the large firms in time \(t\). The coefficient \(\tilde{\theta}_{3,t}\) captures the difference in the unsystematic risk premium of the small firms from that of the large firms in time \(t\). \((v\text{ar}_{p,t-1}, \delta_{j,i})\) represents the estimate of variance risk for portfolio \(p\) times the size dummy for medium and small firms in turn so that \(\tilde{\phi}_{2,t}\) can be interpreted as the difference in the estimated total risk premium of medium firms from that of the large firms while \(\tilde{\phi}_{3,t}\) can be interpreted as the difference between the estimated total risk premium of small firms from that of the large firms in time \(t\).

The results from the estimation of equation (6.4), for each individual month of the year are reported in Table 6.4. The ARCH(6) test statistic for up to sixth order heteroscedasticity appear to rule out the presence of heteroscedasticity except during the month of August, but again the Q statistic reveals that there is significant autocorrelation in the residuals for some months, suggesting that the model is still not fully specified.

Looking at the results associated with systematic risk first it can be observed that systematic risk is only ever priced during the months of January, April and July. In no other months of the year is there a positive relationship between systematic risk and return. The results presented here indicate that unlike previous studies the positive pricing of the systematic risk premium is robust to the inclusion of other risk variables such as variance or residual risk. One possible reason for this difference is that aggregate coefficient betas have been used in this study, Tinic and West (1984,1986) and Corhay et Al (1987,1989) only use contemporaneous betas, which might have biased their estimates because biased downwards estimates of beta will bias downwards the estimated risk premium.
The coefficients $\tilde{\alpha}_2$ and $\tilde{\alpha}_3$ are both significantly negative in the months of January, April and July which suggests that in these months the systematic risk premium of the large firms is greater than the premium for either small or medium firms. In January, the systematic risk premium for large firms is approximately 25.6 percent but only 15.8 percent for the medium firms and 20.5 percent for the smallest companies. In April, the systematic risk premium for the large firms is 16.4 percent, but only 6.7 percent for the medium firms and 11.8 percent for the smallest firms. In July, the systematic risk premium for the large firms is 16.4 percent, but only 6.1 percent and 13 percent for the medium and small firms respectively. As we can see, the findings which were reported in Section 5 are robust to the inclusion of other risk variables. These results show that the monthly systematic risk premium of large capitalisation firms is higher than for firms with medium to low capitalisation values. This finding is therefore consistent with the higher raw returns of large firms in the months of January and April.

Residual risk is found to be negatively priced for large firms during the months of January, February, June and July but positively priced during the month of October. For medium sized firms, the premium to unsystematic risk is only significantly different from the premium associated with the large firms in April. Small firms earn a less negative residual risk premium than large firms during the month of February and June, a slightly positive premium in July and October. Although, the premium is positive in October it is lower than the premium associated with large firms.

The results also indicate that during January and September for large firms variance risk is positively priced. Small firms earn a January variance risk premium which is lower than the premium to large firms. Meanwhile, during the month of April the medium firms obtain a variance risk premium which is lower than the premium of large firms.

The size dummies reveal that, on a month by month basis, firm size is frequently priced. During January and April, even after controlling for differences in the size of the risk premiums of small and medium firms, firm size still appears to be an important factor determining returns in these two months. In both January and April the smallest firms earn a size premium of approximately 4.7 percent and 3.6 percent respectively.

January is the month which stands out as being different from other months of the year. In this month all three forms of risk investigated provide a positive risk premium, the only month in which this happens. The positive systematic and variance risk premium during this month appears to be able to explain some of the January return regularity observed in the UK.

To isolate the impact January has on the risk-return relationship the same model was tested across all months of the year, and then tested again with the exclusion of January. The results from these tests are reported in Table 6.4. Looking at the results for all the months together first. It is found that there is a systematic risk premium for large firms which is positive and
significantly different from zero. Small firms are found to earn a systematic risk premium which is significantly below that of the large firms. The systematic risk premium for large firms is found to be 6.7 percent a month but only 4.7 percent for medium firms and 5.1 percent for the smallest firms.

If we look at the results for the sample which excludes January then the systematic risk premium of large firms is only significant at a 10 percent level. This time the systematic risk premium is only found to be significant for the smallest firms. This demonstrates the dominating effect that the month of January has on the relationship between systematic risk and return. When all months are tested variance risk is found to be positively priced for large firms which becomes insignificant when January is excluded from the tests. The size premium to the smallest firms in the sample is found to be approximately 0.14 percent when January is included but fails to be significant at a 5 percent level when January is excluded from the sample.

The $\overline{R^2}$ from these equations are also reported in Table 6.4. Again, over the full sample the $\overline{R^2}$ of the model is very low. In these tests the highest explanatory power for the model occurs in July, during this month the $\overline{R^2}$ is over 20 percent, there is also a considerable increase in explanatory power during January and April, in both cases the $\overline{R^2}$ is approximately 10 percent.

6.7 Summary and Conclusions

This chapter has investigated the monthly return behaviour of capitalisation based portfolios. It has been shown that there is a great deal of variation in the mean monthly returns both between the largest firms and the smallest firms and across the different months of the year. Although, it is clear that the month which provides the highest return to each of the portfolios is January.

This chapter has explored one possible explanation for this high January return. The Capital Asset Pricing Model has been tested on a month by month basis to identify whether the high January return is related to the systematic risk premium. The novelty of the work undertaken in this chapter is that this study has focused on whether differences in the size of the risk premiums for firms of different capitalisations can account for differences in the return behaviour of large and small firms.

Consistent with the findings of Tinic and West (1984,1986) for the US and Corhay (1987,1989) for the UK, this chapter has demonstrated that systematic risk is not priced every month of the year. Actually systematic risk is only ever priced consistently in two months of the year, January and April, which are also the months which provide the highest raw returns. High returns in January and April may therefore be related to the behaviour of the systematic risk premium.
This asymmetric relationship between risk and return across the different months of the year appears to be supported for both large and small firms. Firm size is found to make an important contribution to the magnitude of the systematic risk premium. When systematic risk is priced, the largest firms always receive a higher risk premium than smaller firms. For small firms only a very weak relationship between systematic risk and return appears to exist.

An analysis of the behaviour of the relationship between risk and return during all months of the year and all months excluding January reveals that the positive systematic risk premium found when all months are tested together is the result of the dominating effect that January has on the results. When all months are tested with the exclusion of January then no significant relationship between systematic risk and return is found to emerge, except for the smallest firms in the sample.

This chapter also finds that in several months of the year variance and residual risk is priced by the markets. This provides evidence against the validity of the Capital Asset Pricing Model at least on a month by month basis.

In light of the results presented in this chapter a number of avenues for future research present themselves. Firstly, perhaps now that the size and seasonality are well documented research should focus on the possible causation of size and calendar anomalies in the UK. Perhaps such research should focus on institutional portfolio behaviour during the various months of the year. Clearly, in the UK a substantial proportion of stock turnover is instigated by institutional portfolio holders. To understand return seasonality and/or size anomalies we have to understand the trading behaviour of the institutional investors.

Clearly, the way in which beta is estimated is important if betas are to provide unbiased estimates of market risk. As Chapter 7 will demonstrate the assumption of constant betas is unrealistic. Currently there exist an array of ways in which beta estimates can be obtained. Research needs to be employed to provide a definitive way of estimating betas and suggesting which method should be employed to measure accurately the cross-section of expected returns.

Finally, Fama and French (1992) have demonstrated that in the presence of the book to market ratio there is no relationship between beta and return. It would be interesting to discover whether the relationship between book to market and return is influenced at all by the calendar month. Since the existence of return seasonality is usually identified within a capital asset pricing model framework it would also be interesting to see whether the relationship between risk and return in alternative models such as the arbitrage pricing theory framework leads to seasonal relationships between risk and return.
Table 6.1

\( P_1 \) is the mean return for month \( i \) accruing to the portfolio of small firms. \( P_2 \) and \( P_3 \) is the mean portfolio return for month \( i \) accruing to the portfolio of medium and large firms respectively. \( P_3 - P_1 \) is the mean return in month \( i \) accruing to largest portfolio less the smallest portfolio. FTA is the monthly return to the FT All-Share index, the representative market portfolio used in this study. * indicates significance at a 10% level, ** indicates significance at a 5% level.

<table>
<thead>
<tr>
<th>Month</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_3 - P_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Months</td>
<td>0.0133*</td>
<td>0.0113**</td>
<td>0.0154**</td>
<td>-0.0025</td>
</tr>
<tr>
<td></td>
<td>(1.97)</td>
<td>(2.8)</td>
<td>(4.36)</td>
<td>(1.83)</td>
</tr>
<tr>
<td>January</td>
<td>0.058**</td>
<td>0.051**</td>
<td>0.051**</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>(4.64)</td>
<td>(4.20)</td>
<td>(3.60)</td>
<td>(0.84)</td>
</tr>
<tr>
<td>February</td>
<td>0.014**</td>
<td>0.033**</td>
<td>0.036**</td>
<td>-0.022</td>
</tr>
<tr>
<td></td>
<td>(3.9)</td>
<td>(2.5)</td>
<td>(2.07)</td>
<td>(-0.84)</td>
</tr>
<tr>
<td>March</td>
<td>0.034**</td>
<td>0.036**</td>
<td>0.024**</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>(3.0)</td>
<td>(3.03)</td>
<td>(2.2)</td>
<td>(0.35)</td>
</tr>
<tr>
<td>April</td>
<td>0.030**</td>
<td>0.010**</td>
<td>0.017**</td>
<td>0.013**</td>
</tr>
<tr>
<td></td>
<td>(3.5)</td>
<td>(2.26)</td>
<td>(3.5)</td>
<td>(2.5)</td>
</tr>
<tr>
<td>May</td>
<td>0.020</td>
<td>0.009</td>
<td>0.004</td>
<td>0.016</td>
</tr>
<tr>
<td></td>
<td>(1.35)</td>
<td>(0.85)</td>
<td>(0.01)</td>
<td>(1.35)</td>
</tr>
<tr>
<td>June</td>
<td>0.010</td>
<td>0.003</td>
<td>0.013</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(1.52)</td>
<td>(0.28)</td>
<td>(0.87)</td>
<td>(-0.09)</td>
</tr>
<tr>
<td>July</td>
<td>0.005</td>
<td>0.004</td>
<td>0.018</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>(0.98)</td>
<td>(0.42)</td>
<td>(0.9)</td>
<td>(-0.66)</td>
</tr>
<tr>
<td>August</td>
<td>0.019</td>
<td>0.006</td>
<td>0.004</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(1.11)</td>
<td>(0.44)</td>
<td>(1.26)</td>
<td>(0.87)</td>
</tr>
<tr>
<td>September</td>
<td>-0.005</td>
<td>-0.002</td>
<td>0.009</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td>(-0.98)</td>
<td>(-0.12)</td>
<td>(0.90)</td>
<td>(-1.28)</td>
</tr>
<tr>
<td>October</td>
<td>-0.030</td>
<td>-0.017</td>
<td>-0.002</td>
<td>-0.028**</td>
</tr>
<tr>
<td></td>
<td>(-0.11)</td>
<td>(-0.02)</td>
<td>(-1.1)</td>
<td>(-2.08)</td>
</tr>
<tr>
<td>November</td>
<td>-0.005</td>
<td>-0.009</td>
<td>0.000</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(-0.59)</td>
<td>(-0.56)</td>
<td>(-0.89)</td>
<td>(-1.5)</td>
</tr>
<tr>
<td>December</td>
<td>0.010</td>
<td>0.011</td>
<td>0.011**</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(1.36)</td>
<td>(1.17)</td>
<td>(2.25)</td>
<td>(-1.31)</td>
</tr>
</tbody>
</table>
Table 6.2

The Monthly Relationship Between Systematic Risk and Return

This table provides the results from the estimation of the following model:

\[ R_{p,t} = \tilde{\gamma}_1, t + \tilde{\gamma}_2, t \beta_{p,t-1} + \sum_{j=2}^{3} \tilde{\gamma}_j, t \delta_j, t + \sum_{j=2}^{3} \tilde{\lambda}_j, t (\delta_j, t \beta_{p,t-1}) + \tilde{\mu}_t \]

Where \( \tilde{\gamma}_1 \) and \( \tilde{\gamma}_2 \) are the average coefficients associated with the intercept and systematic risk, respectively. \( \tilde{\lambda}_j \) is the average coefficient associated with the two size dummies \( \delta_2 \) and \( \delta_3 \). \( \tilde{\lambda}_j \) and \( \tilde{\lambda}_3 \) are the average coefficients associated with the two slope dummies. * indicates significance at 10%, ** indicates significance at 5%. ARCH(6) is the statistic from anARCH test which tests for up to sixth order heteroscedasticity in the residuals from this model. Q is the Ljung-Box test for up to sixth order autocorrelation, the p values from these tests are given in parentheses.

<table>
<thead>
<tr>
<th>Month</th>
<th>( \tilde{\gamma}_1 )</th>
<th>( \tilde{\gamma}_2 )</th>
<th>( \tilde{\gamma}_3 )</th>
<th>( \tilde{\lambda}_2 )</th>
<th>( \tilde{\lambda}_3 )</th>
<th>Q</th>
<th>ARCH(6)</th>
<th>( \bar{R}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Months</td>
<td>-0.050**</td>
<td>0.068**</td>
<td>-0.022**</td>
<td>0.021**</td>
<td>0.0148**</td>
<td>7.7</td>
<td>3.56</td>
<td>0.019</td>
</tr>
<tr>
<td></td>
<td>(-2.45)**</td>
<td>(2.78)**</td>
<td>(-5.43)**</td>
<td>(-3.56)**</td>
<td>(3.45)**</td>
<td>(2.08)**</td>
<td>(0.24)</td>
<td>(0.80)</td>
</tr>
<tr>
<td>January</td>
<td>-0.1482**</td>
<td>0.2021**</td>
<td>0.1999**</td>
<td>0.1936**</td>
<td>-0.2078**</td>
<td>-0.2061**</td>
<td>8.6</td>
<td>3.31</td>
</tr>
<tr>
<td></td>
<td>(-2.36)</td>
<td>(3.10)</td>
<td>(3.11)</td>
<td>(3.03)</td>
<td>(-3.14)</td>
<td>(-3.13)</td>
<td>(0.19)</td>
<td>(0.76)</td>
</tr>
<tr>
<td>February</td>
<td>-0.0660</td>
<td>0.0931</td>
<td>0.1090</td>
<td>0.1090</td>
<td>-0.0971</td>
<td>-0.1060</td>
<td>31.3</td>
<td>6.98</td>
</tr>
<tr>
<td></td>
<td>(-0.94)</td>
<td>(1.27)</td>
<td>(1.51)</td>
<td>(1.52)</td>
<td>(-1.32)</td>
<td>(-1.44)</td>
<td>(0.00)</td>
<td>(0.32)</td>
</tr>
<tr>
<td>March</td>
<td>0.0537</td>
<td>-0.0281</td>
<td>-0.0374</td>
<td>-0.0269</td>
<td>0.0364</td>
<td>0.0185</td>
<td>35.13</td>
<td>5.82</td>
</tr>
<tr>
<td></td>
<td>(0.79)</td>
<td>(-0.40)</td>
<td>(-0.54)</td>
<td>(-0.39)</td>
<td>(0.51)</td>
<td>(0.26)</td>
<td>(0.00)</td>
<td>(0.44)</td>
</tr>
<tr>
<td>April</td>
<td>-0.0978**</td>
<td>0.1246**</td>
<td>0.1314**</td>
<td>0.1168**</td>
<td>-0.1409**</td>
<td>-0.1357**</td>
<td>10.51</td>
<td>9.29</td>
</tr>
<tr>
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<td>(2.35)</td>
<td>(2.51)</td>
<td>(2.24)</td>
<td>(-2.61)</td>
<td>(-2.53)</td>
<td>(0.10)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>May</td>
<td>0.0832</td>
<td>-0.0871</td>
<td>-0.0694</td>
<td>-0.05712</td>
<td>0.0789</td>
<td>0.0669</td>
<td>26.46</td>
<td>16.52</td>
</tr>
<tr>
<td></td>
<td>(1.31)</td>
<td>(-1.32)</td>
<td>(-1.07)</td>
<td>(-0.88)</td>
<td>(1.18)</td>
<td>(1.00)</td>
<td>(0.00)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>June</td>
<td>-0.0412</td>
<td>0.0626</td>
<td>0.0712</td>
<td>0.0569</td>
<td>-0.0730</td>
<td>-0.0736</td>
<td>15.44</td>
<td>2.8</td>
</tr>
<tr>
<td></td>
<td>(-0.59)</td>
<td>(0.87)</td>
<td>(1.00)</td>
<td>(0.81)</td>
<td>(-1.00)</td>
<td>(-1.01)</td>
<td>(0.02)</td>
<td>(0.83)</td>
</tr>
<tr>
<td>July</td>
<td>-0.1157</td>
<td>0.1310*</td>
<td>0.1459**</td>
<td>0.1497**</td>
<td>-0.1373*</td>
<td>-0.1536**</td>
<td>18.46</td>
<td>1.44</td>
</tr>
<tr>
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<td>(1.78)</td>
<td>(2.01)</td>
<td>(2.07)</td>
<td>(-1.84)</td>
<td>(-2.07)</td>
<td>(0.005)</td>
<td>(0.96)</td>
</tr>
<tr>
<td>August</td>
<td>-0.0915</td>
<td>0.1072</td>
<td>0.1060</td>
<td>0.1235*</td>
<td>-0.1269*</td>
<td>-0.1383*</td>
<td>63.64</td>
<td>21.34</td>
</tr>
<tr>
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<td>(-1.35)</td>
<td>(1.53)</td>
<td>(1.53)</td>
<td>(1.78)</td>
<td>(-1.78)</td>
<td>(-1.94)</td>
<td>(0.00)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>September</td>
<td>-0.0491</td>
<td>0.0310</td>
<td>0.0254</td>
<td>0.0654</td>
<td>-0.0197</td>
<td>-0.0523</td>
<td>32.71</td>
<td>11.90</td>
</tr>
<tr>
<td></td>
<td>(-0.62)</td>
<td>(0.38)</td>
<td>(0.31)</td>
<td>(0.81)</td>
<td>(-0.24)</td>
<td>(-0.63)</td>
<td>(0.00)</td>
<td>(0.06)</td>
</tr>
<tr>
<td>October</td>
<td>-0.0519</td>
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<td>0.0433</td>
<td>0.0571</td>
<td>-0.0503</td>
<td>-0.0649</td>
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</tr>
<tr>
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<td>(-0.45)</td>
<td>(0.41)</td>
<td>(0.37)</td>
<td>(0.49)</td>
<td>(-0.41)</td>
<td>(-0.54)</td>
<td>(0.002)</td>
<td>(0.29)</td>
</tr>
<tr>
<td>November</td>
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<td>0.0304</td>
<td>0.0260</td>
<td>-0.0046</td>
<td>-0.0440</td>
<td>-0.0153</td>
<td>6.15</td>
<td>6.77</td>
</tr>
<tr>
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<td>(0.34)</td>
<td>(0.30)</td>
<td>(-0.05)</td>
<td>(-0.48)</td>
<td>(-0.17)</td>
<td>(0.41)</td>
<td>(0.34)</td>
</tr>
<tr>
<td>December</td>
<td>0.1158</td>
<td>-0.1033</td>
<td>-0.1387</td>
<td>-0.0865</td>
<td>0.1179</td>
<td>0.0769</td>
<td>5.54</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>(1.03)</td>
<td>(-0.89)</td>
<td>(-1.21)</td>
<td>(-0.77)</td>
<td>(0.99)</td>
<td>(0.65)</td>
<td>(0.48)</td>
<td>(0.99)</td>
</tr>
</tbody>
</table>
Table 6.3

Seasonality and the Magnitude of the Risk Premium

This table provides the results from the estimation of the following model.

\[
\tilde{\gamma}_{j,t} = \tilde{\alpha} + \sum_{i=2}^{12} \tilde{\alpha}_{i,j} D_{i,t} + \sum_{i=2}^{12} \tilde{\delta}_{i,j} (D_{i,t} \delta_{i,j}) + \sum_{i=2}^{12} \tilde{\theta}_{i,j} (D_{i,t} \delta_{i,j}) + \tilde{\nu}_{j,t}
\]

Where, \(\tilde{\gamma}\) and \(\tilde{\nu}\) are the coefficients estimated from equation (1) for each month, annually over the sample period. \(\tilde{\alpha}\) is the constant and represents the mean return accruing to portfolios in July which is the control month. \(\tilde{\alpha}\) are the average coefficients associated with the eleven month dummies. \(\tilde{\delta}\) are the average coefficients associated with the eleven month dummies times the dummy for medium firms. \(\tilde{\theta}\) are the coefficients for eleven month dummies times the small firm dummy. ** indicates significance at a 5% level while * indicates significance at a 10% level.

<table>
<thead>
<tr>
<th>Month</th>
<th>(\tilde{\alpha}_{i,j})</th>
<th>(\tilde{\delta}_{i,j})</th>
<th>(\tilde{\theta}_{i,j})</th>
<th>(\tilde{\alpha}_{z,i})</th>
<th>(\tilde{\delta}_{z,i})</th>
<th>(\tilde{\theta}_{z,i})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.0031</td>
<td>0.0113</td>
<td></td>
<td>0.0780**</td>
<td>-0.0895**</td>
<td>-0.0591</td>
</tr>
<tr>
<td></td>
<td>(-0.62)</td>
<td>(-0.39)</td>
<td></td>
<td>(2.10)</td>
<td>(-2.38)</td>
<td>(-0.74)</td>
</tr>
<tr>
<td>January</td>
<td>-0.0449</td>
<td>0.0939**</td>
<td>0.0494</td>
<td>0.0780**</td>
<td>-0.0895**</td>
<td>-0.0591</td>
</tr>
<tr>
<td></td>
<td>(-0.50)</td>
<td>(4.0)</td>
<td>(1.20)</td>
<td>(2.10)</td>
<td>(-2.38)</td>
<td>(-0.74)</td>
</tr>
<tr>
<td>February</td>
<td>-0.0022</td>
<td>0.0528</td>
<td>0.0394</td>
<td>0.0066</td>
<td>-0.0285</td>
<td>-0.0252</td>
</tr>
<tr>
<td></td>
<td>(-0.70)</td>
<td>(1.20)</td>
<td>(1.60)</td>
<td>(1.50)</td>
<td>(-0.47)</td>
<td>(-0.46)</td>
</tr>
<tr>
<td>March</td>
<td>-0.0259</td>
<td>0.0365</td>
<td>0.0118</td>
<td>0.0406</td>
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<td>(0.14)</td>
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<td>-0.0947</td>
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<td>-0.0195</td>
<td>0.0260</td>
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<td>(-0.48)</td>
<td>(1.73)</td>
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<td>(0.61)</td>
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<td>-0.0053</td>
<td>0.0247</td>
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<td>0.0213</td>
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<td>(1.10)</td>
<td>(-0.63)</td>
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<td>-0.0869</td>
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<td>(1.32)</td>
<td>(-0.96)</td>
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<td>-0.0579</td>
<td>-0.0436</td>
<td>0.0247</td>
<td>0.0334</td>
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<tr>
<td></td>
<td>(-0.52)</td>
<td>(-0.14)</td>
<td>(-0.90)</td>
<td>(-1.23)</td>
<td>(0.99)</td>
<td>(0.37)</td>
</tr>
<tr>
<td>December</td>
<td>0.0676</td>
<td>-0.0669</td>
<td>-0.1373</td>
<td>-0.0718**</td>
<td>-0.0684**</td>
<td>0.1071**</td>
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<tr>
<td></td>
<td>(0.30)</td>
<td>(-0.40)</td>
<td>(-0.20)</td>
<td>(2.43)</td>
<td>(2.28)</td>
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Table 6.4
The Monthly Relationship Between Risk and Return

The average coefficients from the estimation of the following model over the sample period 1980-1991:

\[ R_{p,t} = \gamma_0 + \gamma_1 \delta_{p,t-1} + \gamma_2 \nu_{p,t-1} + \frac{1}{\delta_0} \left( \sum_{j=2}^{3} \delta_j (\beta_{p,t-1,j} - \bar{\beta}_p) + \frac{1}{\delta_0} \left( \sum_{j=2}^{3} \delta_j (\nu_{p,t-1,j} - \bar{\nu}_p) + \frac{1}{\delta_0} \left( \sum_{j=2}^{3} \delta_j (\bar{\delta}_p) + u_{p,t} \right) \right) \right) \]

Where \( R_{p,t} \) is the return to the \( p \) portfolio, \( \beta \) is the estimate of systematic risk for the \( p \) portfolio, \( \nu \) is the estimate of unsystematic risk for the \( p \) portfolio, \( \nu \) is the estimate of variance risk for the \( p \) portfolio. \( \delta_0 \) and \( \delta_2 \) are dummies which take on a value of one if the firm is a medium or small firm respectively, but a zero otherwise. \( \bar{\beta}_p \) and \( \bar{\nu}_p \) are the systematic, variance and unsystematic risk premiums of the large firms respectively. \( \bar{\delta}_p \) captures the difference between the constant of the medium and small firms respectively from that of the large firms. \( \gamma_0 \) captures the difference between the constant of the medium and small firms respectively from the constant of the large firms. * indicates significance at a 10% level while ** indicates significance at a 5% level. ARCH(6) is the statistic from an ARCH test which tests for up to sixth order heteroscedasticity in the residuals from this model. Q is the Ljung-Box test for up to sixth order autocorrelation, the p values from these tests are given in parentheses. All refers to all months, and XJan refers to all months excluding January.

<table>
<thead>
<tr>
<th>Month</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
<th>( \bar{\beta}_p )</th>
<th>( \bar{\nu}_p )</th>
<th>( \bar{\delta}_p )</th>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>( \gamma_2 )</th>
<th>( \gamma_3 )</th>
<th>( \bar{\beta}_p )</th>
<th>( \bar{\nu}_p )</th>
<th>( \bar{\delta}_p )</th>
<th>Q (ARCH(6))</th>
<th>ARCH(6)</th>
<th>Notes</th>
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<td>Jan</td>
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<td>0.256</td>
<td>2.880</td>
<td>-0.130</td>
<td>-0.098</td>
<td>-0.051</td>
<td>-1.528</td>
<td>-0.501</td>
<td>1.64</td>
<td>0.098</td>
<td>0.098</td>
<td>0.047</td>
<td>7.52</td>
<td>2.42</td>
<td>0.101</td>
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<td>Feb</td>
<td>-0.088</td>
<td>0.120</td>
<td>-0.158</td>
<td>-0.139</td>
<td>-0.052</td>
<td>-0.025</td>
<td>-0.191</td>
<td>-0.361</td>
<td>0.352</td>
<td>0.125</td>
<td>0.068</td>
<td>0.026</td>
<td>26.23</td>
<td>6.42</td>
<td>0.056</td>
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</tr>
<tr>
<td>Mar</td>
<td>0.079</td>
<td>-0.033</td>
<td>-1.276</td>
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<td>0.019</td>
<td>0.005</td>
<td>0.145</td>
<td>0.250</td>
<td>-0.844</td>
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<td>-0.001</td>
<td>-0.014</td>
<td>35.28</td>
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<td>0.744</td>
<td>0.019</td>
<td>-0.031</td>
<td>-0.042</td>
<td>-3.364</td>
<td>0.077</td>
<td>3.967</td>
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<td>0.067</td>
<td>0.054</td>
<td>25.60</td>
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<td>-1.526</td>
<td>-0.049</td>
<td>0.046</td>
<td>0.016</td>
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<td>-0.253</td>
<td>0.067</td>
<td>-0.023</td>
<td>-1.434</td>
<td>25.60</td>
<td>7.64</td>
<td>0.000</td>
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<tr>
<td>Jun</td>
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<td>0.827</td>
<td>-0.276</td>
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<td>Jul</td>
<td>-0.143</td>
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<td>-0.034</td>
<td>-1.89</td>
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<td>-0.065</td>
<td>-0.034</td>
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<td>4.194</td>
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<td>0.030</td>
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<td>10.75</td>
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<td>0.065</td>
<td>0.551</td>
<td>0.439</td>
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<td>3.46</td>
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<td>-0.085</td>
<td>0.034</td>
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<td>6.42</td>
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<td>-1.023</td>
<td>0.008</td>
<td>0.524</td>
<td>0.102</td>
<td>-0.012</td>
<td>-0.002</td>
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<td>7.50</td>
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<td>0.088</td>
<td>0.022</td>
<td>-2.203</td>
<td>-0.038</td>
<td>4.062</td>
<td>-0.039</td>
<td>-0.241</td>
<td>-0.016</td>
<td>4.96</td>
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CHAPTER SEVEN

Firm Size and Time Variation in Expected Return

7.1 Introduction

Asset pricing models put few, if any restrictions on the behaviour of expected returns over time. However, in the implementation of most tests of market efficiency, and, or, a particular equilibrium model, expected returns are assumed to remain constant over some period of time. In this case, risk is measured using the historical distribution of returns which is assumed to be constant. This implies that the risk premium associated with a particular investment is invariant to the passage of time, irrespective of the behaviour of the assets return or the market return.

The Capital Asset Pricing model which was originally proposed by Sharpe (1964), Lintner (1965), Mossin (1965), and Treynor (1965) following the suggestions of mean variance optimisation proposed by Markowitz (1952, 1959), has provided a simple but compelling theory of asset market pricing for almost 30 years. In its classical form, the theory predicts that the expected return to an asset, above the risk free rate, is proportional to the amount of non-diversifiable risk, which is measured by the covariance of the assets' return with a portfolio composed of all available assets in the market.

Recently, a catalogue of empirical evidence has been collected to suggest that expected returns are time varying, and are able to explain a large proportion of the variation in realised returns. As Section 3.2.3 of Chapter 3 indicated, Conrad and Kaul (1988) have shown, that as much as 26 percent of the variation in realised portfolio returns may be caused by time variation in expected returns. This is an important discovery because if expected returns are time varying and, autocorrelated, returns should be modelled accordingly.

As Section 3.4 of Chapter 2 showed, a number of tests have now found that variables, other than the covariance, appear to be able to predict risk premiums. In particular, firm size, seasonalities, gearing ratios and price-earnings ratios all appear to be able to explain expected returns better than the Capital Asset Pricing Model. One possible reason for the empirical failure of the Capital Asset Pricing Model is that perhaps the classical version is empirically misspecified, because, risk rather than being a constant, is instead time varying. In that case, these type of tests, many of which were discussed in Chapter 2 and 3, can only show us that a constant discount rate model is rejected, they can not tell us that CAPM is the wrong asset pricing model. Neither can such tests tell us that markets are inefficient, this can only be stated if a time varying model of expected returns is also rejected.

1. The Capital Asset Pricing Model is a one period model and as such can not be theoretically misspecified by having a constant risk premium. However, in a multi-period application empirically risk may not be constant.
This chapter will be concerned with testing whether the systematic risk premium for a group of UK portfolios, which have been organised on the basis of firm size, is constant or time varying. The focus in this chapter will therefore be on identifying any distinguishing characteristics in the conditional risk premium of large and small firms, an issue which has received very little attention. Exceptions have been provided by Harvey (1989) and Boudurtha and Mark (1988) who find evidence of time varying conditional covariances for both large and small firms. However, both papers consider the US market, while neither Harvey (1989) or Boudurtha and Mark (1988) directly focus on the differences between the large and small firm portfolios. Despite this Harvey reports evidence of a "clientele effect" because the market price of risk, is found to be higher for large firms than for small firms, and also reports that the conditional version of the CAPM tested tends to overstate expected returns for large firms but not for small firms.

The remainder of this chapter is set out as follows. Section 7.2 will consider the static and the conditional version of the Capital Asset Pricing Model. Section 7.3, describes the ARCH family of models which are used to test the conditional CAPM. Section 7.4, describes the version of the conditional CAPM tested in this chapter and provides a detailed discussion of how the procedure is modified to take account of the characteristics of the data set used in this study. Section 7.5, provides results which indicate that the risk premium of large and small firms is time-varying but that there are also important differences between the behaviour of the large and small firms. Section 7.6, investigates the behaviour of the conditional risk premium and once again finds some important differences between large and small firms. Section 7.7, undertakes some sub-period analysis. Section 7.8, is concerned with how the January effect influences the risk premium and provides further support for a systematic risk premium which is higher during the month of January. Section 7.9 provides a summary and conclusions to the paper.

7.2 The Sharpe-Lintner Capital Asset Pricing Model

According to the Sharpe-Lintner Capital Asset Pricing Model, the expected return to any asset is a function of the risk free rate of return and a risk premium which compensates investors for undertaking risky investments. The distinguishing characteristic of the Capital Asset Pricing Model is that the market only compensates investors for non-diversifiable, or market risk. Other forms of risk are not priced. Furthermore, the Capital Asset pricing Model suggests that there is a linear relationship between market risk and expected return. This means that the expected

---

2. The risk free rate of return reflects the fact that even when investments are risk free investors will require a positive rate of return. For a risk free investment, the positive required return compensates investors for the opportunity cost of the funds which are used for investment. When money is tied up in an investment, investors are unable to use the funds on goods or services, which provide positive utility. To encourage investors not to consume funds, but invest a positive required return is provided.
return for a portfolio of securities would be fully described by the risk free return and, a risk premium proportional to the amount of systematic risk. The implications of the Capital Asset Pricing Model, for an asset such as a portfolio of securities can be summarised by equation (7.1) below.

\[ E(R_p) = r_f + \beta_p (E(R_m) - r_f) \]  

(7.1)

where, \( E(R_p) \) is the expected return of portfolio \( p \), \( r_f \) is the risk free rate of return. The \( \beta_p \) is the amount of systematic risk for portfolio \( p \) which can be estimated as \( \sigma(R_p, R_m) / \sigma^2(R_m) \) which is the covariance of the portfolio return with the market return divided by the variance of the market return.

We can rearrange equation (7.1) as follows,

\[ E(R_p) = r_f + \lambda (\sigma(R_p, R_m)) \]  

(7.2)

where \( \lambda \) is \( (E(R_m) - r_f) / \sigma^2(R_m) \) and is the market price of risk which captures how many units of excess market return investors require in compensation for one unit of market risk. \( \sigma(R_p, R_m) \) is the amount of market risk associated with portfolio \( p \). Then \( \lambda \) times \( \sigma(R_p, R_m) \) is the total risk premium for portfolio \( p \).

The expected return of the market will also be a function of the amount of risk and the price of risk. The amount of market risk is given by

\[ \sigma^2(\theta_m) \]

and the price of market risk is given by \( \lambda \). This means that in a similar way the market risk premium is a function of the price of risk and the amount of risk.

A fundamental assumption of the Capital Asset Pricing Model described here is that the return distributions are constant over time. In which case the variances and covariances and therefore the \( \beta \)'s are fixed. This is an assumption which appears to be increasingly unrealistic. For example, from the seminal work of Modigliani and Miller (1958) we know that as the gearing ratio of a company increases, investors require a proportional increase in the return to equity. The empirical specification of CAPM contradicts this statement, because, in the static version of CAPM the future gearing ratio of a company does not influence the required return of investors because the amount of risk is a constant.

7.2.1 The Conditional Capital Asset Pricing Model

In an attempt to explain why the Capital Asset Pricing Model does not appear to explain expected returns very well empirically, attention has focused on the possibility that the risk premium, rather than being constant, is instead time varying. In which case, \( \beta_p \) would depend upon when \( \beta_p \) was sampled. In this case, the beta of a security or portfolio last year might be
very different to the beta of a security or portfolio today. The transformation from the classical Capital Asset Pricing Model described in the previous section to the Conditional Capital Asset Pricing Model, in which the systematic risk premium is allowed to be time varying is straightforward. What is required is that the expectations of agents, regarding the means, variances and covariances of returns, rather than being fixed become conditional expectations, and are therefore random variables.

Since it seems reasonable that investors would seek to utilise as much information as possible, investment decisions which are made in time \( t \) would utilise all information up until time \( t-1 \). In which case the conditional Capital Asset Pricing Model makes expected returns, variances and covariances conditional on the information available at \( t-1 \) as follows.

\[
E(R_{p,t} \mid \phi_{t-1}) = r_{f,t-1} + \beta_{p,t}(E(R_{m,t} \mid \phi_{t-1}) - r_{f,t-1})
\]  

\[
\beta_{p,t} = \frac{\sigma(R_{p,t}, R_{m,t} \mid \phi_{t-1})}{\sigma^2(R_{m,t} \mid \phi_{t-1})}
\]

where, \( \phi_{t-1} \) is the information set available to agents when forming expectations regarding the mean, variance and covariance of an investment in time \( t \). \( E(R_{p,t}) \) is the expected return to portfolio \( p \) in time \( t \), and \( \beta_{p,t} \) is the systematic risk of a portfolio in time \( t \).

Drawing on the analysis presented in the previous section, we can now write the expected return to a portfolio in terms of the conditional moments as follows.

\[
E(R_{p,t} \mid \phi_{t-1}) = r_{f,t-1} + \lambda(\sigma(R_{p,t}, R_{m,t} \mid \phi_{t-1}))
\]

In a similar way the market expected return can be written as

\[
E(R_{m,t} \mid \phi_{t-1}) = r_{f,t-1} + \lambda(\sigma^2(R_{m,t} \mid \phi_{t-1}))
\]

As Section 2.3.2 of Chapter 2 showed, realised portfolio returns can be expressed in terms of the expected portfolio return and an error which captures the difference between the realised and the expected return as follows.

\[
R_{p,t} = E(R_{p,t} \mid \phi_{t-1}) + u_{p,t} = r_{f,t-1} + \lambda(\sigma(R_{p,t}, R_{m,t} \mid \phi_{t-1})) + u_{p,t}
\]

\[
E(u_{p,t}) = 0 \quad E(u_{p,t}^2) = \sigma^2(R_{p,t} \mid \phi_{t-1})
\]

In a similar way the market realised return can be written in terms of the expected market return and the error as follows

\[
R_{m,t} = E(R_{m,t} \mid \phi_{t-1}) + u_{m,t} = r_{f,t-1} + \lambda(\sigma^2(R_{m,t} \mid \phi_{t-1})) + u_{m,t}
\]

\[
E(u_{m,t}) = 0 \quad E(u_{m,t}^2) = \sigma^2(R_{m,t} \mid \phi_{t-1})
\]

where, \( u_{p,t} \) is the error which reflects the difference between the realised return and the expected return of portfolio \( p \) such that,
\[ u_{p,t} = R_{p,t} - E(R_{p,t} | \phi_{t-1}) \]  
(7.9)

and \( u_{m,t} \) is the error which reflects the difference between the realised market return and the expected market return,

\[ u_{m,t} = R_{m,t} - E(R_{m,t} | \phi_{t-1}) \]  
(7.10)

### 7.3 Modelling Time Varying Moments using ARCH Models

#### 7.3.1 Introduction to ARCH Models

While it has been recognised for some time that the uncertainty of stock prices, as measured by the variances and covariances, are changing through time. The work of Fama (1965) and Mandelbrot (1966) which was discussed in earlier chapters indicated that there was a tendency for large returns to follow large returns and small returns to follow small returns. This would give rise to a changing conditional variance. However, it was not until recently that financial research began to model time variation in the second or higher moments. This section will be concerned with describing Autoregressive Conditional Heteroscedasticity (ARCH) related models which allow us to model the conditional moments of a time series. Three of the most prominent theories in asset pricing, the Capital Asset Pricing Model which was discussed earlier, the Arbitrage Pricing Theory (APT) Ross (1976) and the Consumption based CAPM model of Breeden (1979) have all found empirical implementations using ARCH models. Indeed, later in this chapter the Conditional Capital Asset Pricing Model, which was discussed earlier, will be tested using a multivariate ARCH related model for a selection of UK portfolios.

#### 7.3.2 The ARCH Model

The ARCH model was first introduced by Engle (1982). This was a major breakthrough for the modelling of financial time series because ARCH models provide a framework with which uncertainty, in the variance and covariance of a series can be modelled. In the ARCH process, the variance of a time series, an example of which might be a time series of stock returns, rather than being constant is allowed to be time varying. Thus, the variance of returns in time \( t \) becomes conditional upon the past squared unexpected return of the time series. An example of an ARCH(1) \(^3\) model applied to stock returns is provided below.

---

3. The model described here is called an ARCH(1) model because the conditional variance is a function of the one period past errors squared. An ARCH model can include any number of lagged squared errors. An ARCH(q) process is an ARCH model which contains q lags in the squared errors.
In the ARCH model, the conditional variance in time $t$ becomes a function of the squared unexpected return in time $t - 1$, which is a proxy for the previous periods volatility. A return in time $t$ which is very different from the expected return, makes the squared error very large. The conditional variance in the next period must therefore become larger. In this case, because the distribution in time $t + 1$ flattens (because it has a higher variance) so it becomes more likely that the next periods return will also be a long way from the expected value. In contrast, a return in time $t$, which is very close to the expected return, causes the squared error to be very small, in which case the conditional variance is small (so the distribution narrows) making it more likely that in the next period another return will be observed which is also very close to the expected return. An ARCH process therefore allows us to capture volatility clustering, which causes the tendency for large returns to follow large returns and small returns to follow small returns, a characteristic of short-horizon returns which was identified by Fama (1965) and Mandelbrot (1966).

An extension of this model which was proposed by Engle, Lilien and Robbins (1987) was the ARCH in mean or ARCH-M model which allows the conditional mean to be a function of the conditional variance. This has an important application for finance because it allows us to investigate whether the past volatility of returns, as captured by the past squared errors, predicts the future value of $R_{p,t}$. This allows us to establish directly whether past volatility predicts future returns.

---

4. Assuming that the coefficient on the past errors squared is positive.
An example of an ARCH (1)-M model is provided below.

\[ R_{p,t} = \alpha_{p,1} + \alpha_{p,2}x_{p,t} + \beta_p h_{p,t} + u_{p,t} \]  
\[ h_{p,t} = \gamma_{p,1} + \gamma_{p,2}u^2_{p,t-1} \]  
\[ u_{p,t} \mid \phi_{t-1} \sim N(0, h_{p,t}) \]  

where, \( \beta_p \) captures the extent to which the one period ahead portfolio return is related to the volatility shock, and other terms are as previously defined.

A further extension of the ARCH model was proposed by Bollerslev (1986). Bollerslev modelled the conditional variance as a function, of not only the past errors squared, but also as a function of the past conditional variance. Such a model is known as a Generalised Autoregressive Heteroscedastic Model or GARCH model an example a GARCH(1,1) model is provided below.

\[ R_{p,t} = \alpha_{p,1} + \alpha_{p,2}x_{p,t} + u_{p,t} \]  
\[ h_{p,t} = \gamma_{p,1} + \gamma_{p,2}u^2_{p,t-1} + \gamma_p h_{p,t-1} \]  
\[ u_{p,t} \mid \phi_{t-1} \sim N(0, h_{p,t}) \]  

where, stationarity of the variance process requires that \( \gamma_2 + \gamma_3 < 1 \), \( h_{p,t} \) in this model is conditional on the past values of \( u^2_{p,t-1} \) and \( h_{p,t-1} \).

The GARCH model has become very important in financial time series work because it allows researchers to model a conditional variance process which is persistent. If, for example, the conditional variance followed an ARCH process and was autocorrelated (that is the conditional variance process was persistent), it would indicate that a shock to the time series would have an influence on the conditional variance for some time. Such persistence would be captured by the lagged conditional variance which is analogous to the moving average term in an ARMA model.

In a later paper Bollerslev, Engle and Wooldridge (1988) developed the conditional GARCH-M model. In such a model the conditional variance is determined by a GARCH process, but in this case the conditional variance is allowed to influence the conditional mean of the time series.

5. A GARCH(1,1) model has the one period lagged conditional variance and the one period lagged squared errors in the conditional variance. In the GARCH model any number of lags can be included in the conditional variance. A GARCH(p,q) model has p lagged conditional variance terms and q lagged squared error terms.
The applications of ARCH related models in finance have been numerous, a comprehensive survey of ARCH models and their application in finance is provided by Bollerslev, Chou and Kroner (1992).

One of the first studies to investigate the relationship between expected returns and stock market volatility was undertaken by French, Schwert and Stambaugh (1987) who examined the relationship between returns to the value weighted NYSE index and the past volatility of market returns. Using a GARCH(1,1)-M model to capture volatility French et al found that there was a positive relationship between expected volatility and the risk premium to the market index.

Bollerslev, Engle and Wooldridge (1988) investigated whether the systematic risk premium, derived from the Capital Asset Pricing Model was time varying, which would give rise to time varying expected returns. Essentially, Bollerslev et al test the conditional Capital Asset Pricing Model which was described in the previous section where the covariances between portfolio returns and the market return, are conditional on the past volatility of returns. Using a GARCH(1,1)-M model to capture volatility Bollerslev et al concluded that the estimated systematic risk premium was indeed time varying and closely related to the previous periods unexpected return. Over the sample period that is investigated it was found that the estimated risk premium was quite variable, so that the quarterly percentage risk premium for stocks ranged from a low of -2% to a high of about 1.5%.

Results which have been found to be consistent with the findings of Bollerslev et al have been provided for a number of different asset markets and for a number of different national stock markets. Using the same GARCH-M framework developed by Bollerslev et al, De Santis and Sorbone (1990) find evidence of a time varying systematic risk premium for the Italian stock market. Engle and Rodriguez (1986) finds that the world systematic risk premium, obtained from an international Capital Asset Pricing Model is time varying, and related to the past volatility of the world market portfolio. While studies, such as those conducted by Attansio and Wadhwani (1989), have found evidence of time varying covariances in the APT model.

Using the same framework which was developed by Bollerslev et al (1988) a study by Hall, Miles and Taylor (1989) investigated whether the systematic risk premium of UK portfolios, formed on the basis of industrial classification, were characterised by time variation. The results provided by Hall et al for the UK were comparable to the results provided by Bollerslev et al for the US. For the four portfolios investigated, β's were found to be time varying and again quite volatile. For example, over the sample period used by Hall et al which was 1976-1986, the beta of the portfolio of companies which were from the chemical industry varied from about 0.77 in 1981 to about 1.06 in 1977. A similar degree of dispersion was also recorded for three of the portfolios in the sample, but not for the financial sector which appeared to have a much less volatile conditional risk premium than the other three portfolios.
The research which has been reviewed in this section is important because it demonstrates that time varying risk must be modelled when testing certain hypotheses about stock market behaviour, otherwise, it is possible to make erroneous conclusions. For example, DeBondt and Thaler (1985) and Lo and MacKinlay (1988) argue that stock markets appear to be inefficient because stock returns appear to be predictable. However, there is now growing evidence to suggest that time variation in expected returns can account for a substantial proportion of the predictability which is identified in these studies, see for example, Chan (1988) and Ball and Kothari (1989) which were discussed in Section 3.3.2 of Chapter 3. Furthermore, anomalies such as the book to market ratio and the small firm effect, which were discussed in Section 2.6 of Chapter 3, appear to be the result of an empirical misspecification of CAPM rather than indicating that abnormal returns are predictable. Support for this possibility is provided by Jagannathan and Wang (1994) who re-estimate the Fama and French (1992) regressions (which found that in addition to the book to market ratio systematic risk did not appear to explain any of the variation in expected returns). Instead of using constant betas, Jagannathan and Wang estimate these regressions using betas which are time varying. Whereas, Fama and French found no relationship between systematic risk and return Jagannathan and Wang found that conditional beta has strong explanatory power, even after controlling for the book to market ratio. Furthermore, Attanasio and Wadhwani (1989) find that the predictability of stock returns from lagged dividend yields, see also, Shiller (1984), Levis (1989), Fama and French (1992) or Fama and French (1995) can be explained by a time varying measure of risk.

In this chapter, the Conditional Capital Asset Pricing Model, which allows the covariances to be conditional on past volatility so that estimates of the systematic risk are allowed to be time varying, is tested for four portfolios which have been formed on the basis of firm size. The aim of these tests is to establish whether portfolio risk premiums, for portfolios which have been formed on the basis of firm size are time varying. This study is specifically looking for differences in the behaviour of the risk premiums of large and small firms, which distinguishes the work undertaken in this study from earlier research.

### 7.4 The Estimation of The Conditional Capital Asset Pricing Model

#### 7.4.1 Data and Summary Statistics

The weekly, Wednesday to Wednesday returns for four equally weighted portfolios over the sample period January 1976-December 1991 are utilised in this study. This is the same weekly portfolio return data which was described in Section 4.1.4 of Chapter 4. The four portfolios have been formed on the basis of market capitalisation as described in Chapter 4. $P_1$ denotes the portfolio which contains the smallest companies, while $P_4$ denotes the portfolio...
which contains the largest firms in the sample. Summary statistics for all four portfolios were provided in Table 4.1 of Chapter 4. In this study, the FT All Share index is used as the representative market portfolio and the one week Treasury Bill rate is used as a proxy for the risk free rate of return.\(^6\)

### 7.4.2 Modelling the Conditional Moments

In this section, a GARCH(1,1)-M version of the Conditional Capital Asset Pricing Model, which is to be estimated in Section 5, is described. To specify a conditional Capital Asset pricing Model, in an economy with many assets, and allow for time varying conditional moments, the GARCH-M model described in the previous section has to be extended to a multivariate framework. In the conditional CAPM described here, as in the static CAPM, it is assumed that the risk premium is proportional to the covariance of each assets return with the return on the market. It is also assumed that all investors share the same expectations about the first two moments of the conditional distribution of returns. The distinguishing characteristic of the model tested here is that expectations are conditional on the squared lagged one period unexpected returns (which captures volatility) and the degree of persistence in the conditional variance.\(^7\)

In Chapter 4, it was noted that the squared returns of all four portfolios used in this study were substantially autocorrelated, which indicated that when specifying the conditional variance this should be taken into account. This persistence could be captured by modelling the conditional variance as an ARCH model with a large number of autoregressive components. But, this would require the estimation of a large number of parameters. Instead, the GARCH specification of the conditional variance was used which allows persistence to be captured in a single parameter.

The conditional version of the CAPM which allows for a time varying conditional covariance obtained from a GARCH(1,1)-M model can be written as follows,

\[
R_t = \alpha + \lambda (H_t) e + U_t \tag{7.17}
\]

\[
vech(H_t) = A_0 + A_1 vech(U_{t-1}, U_{t-1}') + B_1 vech(H_{t-1}) \tag{7.18}
\]

\[
U_t | \phi_{t-1} \sim N(0, H_t)
\]

where, \(R_t\) is a vector of portfolio and market excess returns so that \(R_t = (R_{p,t}, R_{m,t})'\) where the excess returns have been calculated as the realised return less the risk free return. The \(\alpha\) is a \(2 \times 1\) coefficient vector of constants which captures the weekly expected return which is not associated with market risk. The \(H_t\) is the conditional variance/covariance matrix of these returns.

---

6. This data is obtained from Datastream.

7. The GARCH (1,1) is an ARMA(1,1) model in the variance. The past squared errors represent the autoregressive component, while the lagged conditional variance captures the moving average component, see for example, Hamilton (1995).
given the information available in time $t-1$. $H_t$ is estimated using the past squared errors from equation (7.19) and (7.20) and its own lagged values. The $vech$ is the column stacking operator of the lower triangular portion of a symmetric matrix. $U_t = (u_{p,t}^t u_{m,t}^t)'$ which are the unexpected return of the portfolio and the market respectively. $A_1$ is a $2 \times 2$ symmetric coefficient matrix, these estimated coefficients provide an indication of the extent to which the conditional variance is related to past squared errors from the conditional mean equation. The $A_0$ is a $2 \times 1$ coefficient vector. This vector of estimated constants gives an estimate of the long run volatility because the coefficients indicate the amount of volatility present even if the squared errors and lagged conditional variance had no influence upon the conditional variance. $B_1$ is a $2 \times 2$ symmetric coefficient matrix. The co-efficient estimates in this matrix indicate the impact that the past conditional variance has upon its one period ahead future value.

The coefficient estimate $\lambda$ is a point estimate of the market price of risk. In this study the market price of risk is assumed to be a constant throughout time, except if the month of the year is a January. Investigative empirical work not reported in this thesis suggests that this is a realistic assumption. A model which allowed $\lambda$ to vary in relation to volatility was found underperform the presented model ($\lambda$ was never found to be significant even at a 20% level, while in this case the variance process became explosive). A model where the market price of risk was allowed to increase if past returns were negative was also found to add nothing to the model. Finally, sub-period analysis which allowed the current model to be estimated over three non overlapping periods suggested that the market price of risk was not time varying.

In the model which is estimated in this chapter, diagonality is assumed. This means that the elements in $H_t$ depend only on the past squared residuals and an autoregressive component, while the covariance between the market return and the portfolio return depends only upon the past cross products of residuals and an autoregressive component as follows

$$H_t = \begin{pmatrix}
(h_{11,t}) & (h_{12,t}) \\
(h_{21,t}) & (h_{22,t})
\end{pmatrix} = \begin{pmatrix}
\sigma^2(R_{p,t} \mid \phi_{t-1}) & \sigma(R_{p,t}, R_{m,t} \mid \phi_{t-1}) \\
\sigma(R_{p,t}, R_{m,t} \mid \phi_{t-1}) & \sigma^2(R_{m,t} \mid \phi_{t-1})
\end{pmatrix}
$$

$$= \begin{pmatrix}
A_{0,11} + A_{1,11}(u_{p,t}^2) + B_{1,11}h_{11,t-1} & A_{0,12} + A_{1,12}(u_{p,t-1}^2 u_{m,t-1}^2) + B_{1,12}h_{12,t-1} \\
A_{0,12} + A_{1,12}(u_{p,t-1}^2 u_{m,t-1}^2) + B_{1,12}h_{12,t-1} & A_{0,22} + A_{1,22}(u_{m,t}^2) + B_{1,22}h_{22,t-1}
\end{pmatrix}
$$

It is also assumed that $A_1$ and $B_1$ are estimated scalar coefficients rather than vectors. This restriction is imposed because it implies that across the portfolios investors attach the same relative importance to past events when forming expectations about volatility of prices. This amounts to investors using similar forecasting rules for similar problems because it is assumed that a shock to one portfolio has the same impact on the conditional variance of that portfolio as a shock to another portfolio has on the conditional variance of its portfolio.
amounts to investors using similar forecasting rules for similar problems because it is assumed that a shock to one portfolio has the same impact on the conditional variance of that portfolio as a shock to another portfolio has on the conditional variance of its portfolio.

The bi-variate specification outlined in this section, requires the simultaneous estimation of the following system of five equations. It should be noted that since $\lambda$ is the price of market risk its estimated value has been restricted in equation (7.20) and (7.21) to be equal. Other variables and coefficients are as previously described in the previous page.

\[
R_{p,t} = \alpha_p + \lambda(h_{12,t}) + u_{p,t} \tag{7.19}
\]
\[
R_{m,t} = \alpha_m + \lambda(h_{22,t}) + u_{m,t} \tag{7.20}
\]
\[
h_{11} = A_{011} + A_{111}(u_{p,t-1}^2) + B_{111}(h_{11,t-1}) \tag{7.21}
\]
\[
h_{22} = A_{022} + A_{122}(u_{m,t-1}^2) + B_{122}(h_{22,t-1}) \tag{7.22}
\]
\[
h_{12} = A_{012} + A_{112}(u_{p,t-1}u_{m,t-1}) + B_{112}(h_{12,t-1}) \tag{7.23}
\]

### 7.4.3 Estimates of the Risk Premium and Thin Trading

Section 7.2 of this chapter showed that the systematic risk premium for a portfolio of assets can be estimated as follows

\[
\beta_p = \frac{\sigma(R_{p,t}, R_{m,t})}{\sigma^2(R_m)}(R_{m,t} - r_{f,t}) \tag{7.24}
\]

which is equivalent to

\[
\frac{(R_{m,t} - r_{f,t})}{\sigma^2(R_m)} \sigma(R_{p,t}, R_{m,t}) \tag{7.25}
\]

As Section 2.7.2.1 of Chapter 2 indicated, Dimson (1979) has demonstrated that estimates of the systematic risk premium in the presence of thin trading will be biased downwards, because the covariance between the portfolio return and the market return will be understated. This happens because if a stock fails to trade in time $t$ the effect of a systematic shock in time $t$ will not be observed until the stock trades. Consequently, the extent to which the contemporaneous observed returns covary with the market will be understated.

Dimson (1979), suggested an adjustment, which provides unbiased estimates of the risk premium when prices are set non-synchronously. Empirically, this adjustment requires that the estimated beta is measured as the sum of the contemporaneous and lagged serial covariances which provides an estimate of the risk premium in the presence of thin trading as follows

\[
\frac{\sigma(R_{p,t}, R_{m,t})}{\sigma^2(R_m)}(R_{m,t} - r_{f,t}) + \sum_{\tau=1}^{T} \frac{\sigma(R_{p,t}, R_{m,t-\tau})}{\sigma^2(R_m)}(R_{m,t-\tau} - r_{f,t}) \tag{7.26}
\]

$\tau = 1, 2, 3, \ldots T$
This aggregate coefficient beta provides an aggregate coefficient estimate of the risk premium as follows,

\[
\frac{(R_{m,t} - r_{f,t})}{\sigma^2(R_m)} \sigma(R_{p,t}, R_{m,t}) + \ldots + \sum_{\tau=1}^{T} \frac{(R_{m,t} - r_{f,t})}{\sigma^2(R_m)} \sigma(R_{p,t}, R_{m,t-\tau})
\]  

(7.27)

In this study a similar adjustment is proposed to correct estimates of the risk premium for the effect of thin trading. This is because, in the GARCH-M model estimated here, the conditional covariance will also be biased downwards, if stocks within a portfolio are characterised by thin trading. This bias would provide an estimate of the market price of risk which was upwards biased but an estimate of the conditional covariance which was downwards biased. Consequently, failing to adjust for thin trading would cause the estimated risk premium of small firms to be downwards biased. In order to correct for this understatement a conditional version of the aggregate coefficient model is proposed which allows the contemporaneous and the lagged covariances to enter the conditional mean equation as follows.

\[
R_{p,t} = a_p + \lambda_0(\sigma(R_{p,t}, R_{m,t} | \phi_{t-1})) + \sum_{\tau=1}^{T} \lambda_{\tau}(\sigma(R_{p,t}, R_{m,t-\tau} | \phi_{t-1})) + \epsilon_{p,t} \quad \tau = 1, 2, 3, \ldots
\]  

(7.28)

where, \( \lambda_0 + \sum_{\tau=1}^{T} \lambda_{\tau} \) are collectively estimates of \( \lambda \) which is the market price of risk. The term \( \sigma(R_{p,t}, R_{m,t} | \phi_{t-1}) \) is the contemporaneous conditional covariance between portfolio \( p \) and the market and it is \( \sigma(R_{p,t}, R_{m,t-\tau} | \phi_{t-1}) \) which captures the covariance between the return of portfolio \( p \) and the lagged return of the market, so that the terms \( \sum_{\tau=1}^{T} \sigma(R_{p,t}, R_{m,t-\tau} | \phi_{t-1}) \) capture the understatement in the conditional covariance caused by thin trading. It should be noted that estimates of the risk premium will only be unbiased if sufficient lags are incorporated into the model. It should be noted that in this model \( \sigma(R_{p,t}, R_{m,t-\tau} | \phi_{t-1}) \) is an estimate of the conditional covariance between the market and portfolio \( p \).

In this study, all but the portfolio containing the largest firms in the sample are characterised by thin trading. For \( P_3 \), which is the portfolio containing the large-intermediate firms, one lag has been included in the conditional mean equation, for \( P_2 \) which is the portfolio containing the small-intermediate firms four lags have been included and finally \( P_4 \) the portfolio which contains the smallest firms six lags have been incorporated. Choosing the correct number of lags is difficult. As more lags are used the estimate of the covariance becomes less biased, however, as more lags are used the estimate of the covariance becomes increasingly inefficient, because of the effect that random variations have. The precise number of lags chosen here
reflected the decay in the autocorrelation coefficients which would be a good guide for the number of thin trading periods which would have a statistically significant impact on the covariances.

7.5 Results From the Estimation of The Conditional CAPM Model

For each portfolio in turn a bi-variate GARCH-M model which is similar to the one described in the previous section was estimated using Maximum Likelihood. There were two modifications in the specification of the model which is estimated here, the first reflects the adjustments which were made to account for thin trading, these were discussed in the previous section, secondly, a dummy variable was included for the month of October 1987 in order to control for the effect of the crash. The dummy variable takes on a value of one if it is October 1987 but a zero otherwise. This extended model is as follows.

\[
R_i = a + \lambda(H_i) + bC + U_i
\]

\[
vech(H) = A_0 + A_1v \text{ech}(U_{i-1}U_{i-1}^\prime) + B_1v \text{ech}(H_{i-1})
\]

\[
U_i | \phi_i \sim N(0, H_i)
\]

where \(b\) is a 2 x 1 coefficient vector which captures the effect of the crash on the conditional returns of the market and portfolio \(p\) respectively, the estimated coefficient values provided by this vector indicate the impact that the stock market crash had on the market portfolio and portfolio \(p\) respectively. \(C\) is a vector of dummy variables as explained above. The \(\lambda(H_i)\) is a point estimate which now reflects the adjustment for thin trading.

The coefficients which are obtained from the estimation of this model are provided in Table 7.1. Looking at the conditional mean equations first, in only one case is the constant significant. This provides some support for the conditional CAPM model being tested here because it indicates that the Treasury Bill rate, the proxy for the risk free rate, fully captures the risk free component of realised portfolio returns.

Not surprisingly, in all cases the crash dummy variable is very significant. This indicates that expected returns during this period were substantially lower than expected. The magnitude of the crash coefficient appears to be associated with the size of firms within a portfolio. As we move from the portfolio containing the smallest firms to the portfolio containing the largest firms the magnitude of the crash coefficient gets progressively larger. This does not indicate that the crash had less of an impact on small firms but reflects that the crash had less of a once and for all impact on the returns of small firms. Many of the small firms influenced by the crash
did not trade until some time after the crash, consequently, for such a portfolio the crash
coefficient would be lower. An alternative specification, which tested the model using a crash
dummy in the conditional variance equation, found that this was inappropriate because the
dummy was insignificant for all four portfolios.

The value of $\lambda$ for all four portfolios is close to 12. This means that for each unit of market
risk that is accepted by investors 12 units of market return are required in compensation. This
estimate is rather high compared to values provided by French et al (1987) and Miles et al
(1989) who report coefficients close to 5. It should be noted that these studies looked at the
market and value weighted portfolios respectively. However, Harvey (1989) reports values of
$\lambda$ similar to those reported here. Since the study undertaken by Harvey examined capitalisation
based portfolios it appears that the high value of $\lambda$ found here appears to be a consequence of
the portfolio formation method. The same tests were performed on monthly data which provided
estimates of $\lambda$ in the region of 7. The value of $\lambda$ obtained from these tests provided an annual
risk premium which was similar in magnitude to the risk premium which was obtained from the
weekly estimates. Estimates of the model without a crash dummy were also made. The value
of $\lambda$ was found to be very sensitive to the inclusion of the crash dummy because when this was
excluded the estimate of $\lambda$ was found to lie in the range 3.5 to 5. Although the estimates of $\lambda$
presented here appear to be high, it should be noted that the model without the crash dummy
performed very badly, because the residuals had, in some cases, as much autocorrelation as the
pure return series which suggests that the conditional mean in this case was badly specified.

Turning now to the conditional variance equations, it is possible to see that for all four
portfolios there is little evidence of any long run volatility because the magnitude of the constant,
although, statistically significant is very small.

In all cases the coefficients from the volatility shock (squared unexpected return) and
the lagged conditional variance are statistically very significant. This provides evidence that the
conditional variances and covariances examined here are time varying. Furthermore, because
the coefficient on the lagged conditional variance/covariance is significant, support is provided
for a conditional variance/covariance that is autocorrelated.

For all four portfolios the coefficients from the squared lagged one period unexpected
returns are close 0.1, and range from 0.1302 to 0.0869. Meanwhile, the magnitude of the
coefficients appear to be related to firm size. As we move from the portfolio containing the

---

8. The returns of the small firm portfolios suggest that at the time of the crash small firm portfolio returns fell by less than the returns of
large firms. But, whereas the returns of large firms recovered in the week after the crash, the returns of small firms tended to continue to be
significantly negative for several weeks after the crash.
small firms to the portfolio containing the large firms the magnitude of $A_i$ gets smaller. This indicates that an unexpected return experienced by small firms increases the conditional covariance by more than does a shock to large firms.

The $B_i$ coefficients are much more dispersed ranging from 0.5071 to 0.7613. Again, the magnitude of the coefficient is associated with firm size. As we move from the portfolio containing the smaller firms to the portfolio containing the larger firms the magnitude of the coefficient gets progressively larger. This means that shocks to large firms appear to influence the conditional variance for longer than is the case for small firms. Surprisingly, this results in a shock to large firms lasting longer than a shock to small firms. This result is very interesting, because it is small firms and not large firms that are perceived as reacting more inertly to shocks. Further support for this is found in Chapter 7 of this study, which also investigates volatility but within an alternative framework.

The sum of the $A_i$ and the $B_i$ coefficients are all close to but less than one. This means that a volatility shock is very persistent, having an almost permanent impact on the conditional variance. An indication of the degree of persistence in a shock is provided by the half life of a shock, which is the amount of time it takes for half of the shock to dissipate. This can be calculated as

$$\frac{\ln 0.5}{A_i + B_i} \quad (7.31)$$

Estimates of the half life of a shock for each of the four portfolios are also presented in Table 7.1. For the portfolio containing the smallest firms the half life is 11 trading days and is therefore substantially less than the half life of the other portfolios. For example, the portfolio containing the largest firms has a half life of only 25 trading days.

Table 7.2, provides the results of diagnostic tests which were performed on the residuals of these four models. Autocorrelation tests from one to six lags indicates that the autocorrelation which was present in the return series is substantially reduced, although, not entirely eliminated. The high autocorrelation appears to indicate that the conditional mean is not fully specified, at least for the portfolios which do not contain the largest firms in the sample. Although, an attempt has been made in this study to correct for the influence of thin trading, it seems likely that this adjustment has not been complete. Despite this, further lags in the conditional mean equation did not appear to improve the performance of the model.

The kurtosis statistics for the weekly portfolio returns and for the residuals after the fitting of the GARCH model are presented in Table 7.2. As we can see for all four portfolios
the excess kurtosis has been eliminated indicating that the conditional variances of the residuals are no longer characterised by the volatility clustering which gives rise to a time varying conditional variance.

The $R^2$ from the conditional version of the CAPM model tested here indicates that a large proportion of the variation in portfolio returns can be explained. In all cases the model can explain more than 10 percent of the variation in portfolio returns, although, as we move from the portfolio containing the largest firms to the portfolio containing the smallest firms the value of $R^2$ increases. For the smallest firms the conditional version of the CAPM tested here is able to explain as much as 30 percent of the variation in portfolio returns which reflects a substantial improvement in explanatory power in comparison to static versions of CAPM such as the one estimated in the previous chapter.

Interestingly, these figures are comparable to those provided by Conrad and Kaul (1988) for the US (although, it should be noted that they model expected returns as a purely parsimonious time series process rather than using an asset pricing model). This result is important because it suggests that for all capitalisation based portfolios time variation in expected returns explains a substantial proportion of the variation in realised returns. Consequently, such time variation should be modelled when undertaking tests of market efficiency or asset pricing models. Like Conrad and Kaul, this study finds that for small firms time variation explains a larger proportion of the variation in realised returns than is the case for large firms. This is the first documentation of such a pattern for the UK stock market.

7.6 Estimates of the Time Varying Risk Premium

From the estimates presented in Table 7.1 and the conditional covariances obtained from the GARCH-M model, the portfolio risk premium can be calculated as follows.

$$\lambda(h_{12,t})$$

where, $h_{12,t}$ is the conditional covariance between the return of portfolio $P$ and the market return in time $t$. In this study the risk premium of the large firm portfolio has been calculated in this way. Because, the other portfolios have been adjusted for the effects of thin trading the risk premiums for these portfolios have been calculated as follows

$$\lambda_0(h_{12,t}) + \lambda_1(u_{p,t-1}, u_{m,t-2}) + \ldots + \lambda_4(u_{p,t-1}, u_{m,t-4}) \quad \tau = 1, 2, 3, 4...$$

(7.33)

For portfolio $P_1$ there is one lag term, for portfolio $P_2$ there are four lagged terms and for portfolio $P_4$ there are six lagged terms included in the model.
The estimated mean weekly risk premium for the four portfolios are presented in Table 7.3. The expected risk premiums vary across the portfolios. The mean weekly risk premium for the large firm portfolio is approximately 0.0054 while the mean weekly risk premium of the portfolio containing the smallest firms is only 0.0021. This difference may confirm the suggestion made earlier that the effects of thin trading on the covariances has not been fully captured. Although, it should also be noted that in the previous chapter it was found that systematic risk provided large firms with a higher risk premium than was the case for small firms. The difference in the magnitude of the risk premium reported here for large and small firms may reflect this finding, that is, for an equivalent amount of systematic risk large firms have a higher systematic risk premium than small firms which appears to be supported by Harvey (1989) who also finds that the conditional systematic risk premium of larger firms is higher than for small firms.

The expected risk premium for the four portfolios is graphed in Fig 1 to 4. As we can see the risk premium in all cases is quite variable, although, there appears to be much greater week by week variability in the expected risk premium of the smaller firm portfolios. This reflects the magnitude of the ARCH coefficients which indicated that a volatility shock influenced the conditional variance of the small firm portfolio to a greater extent than was the case for large firms. In this case the expected risk premium is more sensitive to volatility shocks than are the large firms in the sample.

By looking at these graphs we can see that specific periods can be identified in which the risk premium is unusually high and these periods appear to be consistent across the portfolios. The highest risk premium which is recorded is for 1977. During 1977 the expected risk premium for each of the portfolios is as much as three times the mean risk premium over the full sample period. Although, for the smallest firms in the sample this period provides a much more modest rise in the conditional risk premium. The risk premium is also exceptionally high for all portfolios during 1980, 1982, 1988 and 1991, while for the smallest firms the period during 1982 provides the highest risk premium.

Although, this study considers a different sample period to the one considered by Hall et al (1989), because of some overlap between samples some comparisons can be made. Hall et al finds that the beta of the chemical, financial and the electrical sector peaks in 1977 while for the mechanical sector the beta peaks at the end of 1981. This is consistent with the results reported in this study because all of these periods provide unusually high conditional risk premiums for the four portfolios which are investigated in this study.

The diagrams illustrate well the finding that a change in the conditional covariance of large firms has a more persistent impact on the risk premium than is the case for small firms. A rise in the risk premium lasts longer for large firms than is the case for small firms where a rise
in the risk premium disappears very quickly. This is illustrated by the fatter appearance of the peaks in Fig 1 and 2 relative to Fig 3 and Fig 4 which shows that a rise in the conditional risk premium of large firms tends to last longer than for small firms.

7.6.1 The Behaviour of Excess Returns

The excess return is the difference between the realised return and the expected return predicted by the conditional version of the CAPM tested here. The average excess return over the sample period is negative, which indicates that the model tested here tends to overestimate the risk premium. For example, the mean weekly excess return for the smallest firms in the sample is -0.000202 but for the largest firms in the sample it is -0.001480. This implies an annual overstatement in the risk premium of approximately 1.5 percent for small firms and 7.5 percent for large firms.

As we can see from these figures, the risk premium of the larger firms in the sample is overstated by more than for small firms. This finding is consistent with Harvey (1989) who also reports that the conditional version of the CAPM overstates the risk premium, but, particularly for large firms. Figures 5 to 8 present a graphical account of the behaviour of excess returns over the sample period. These graphs illustrate the tendency for the larger firms to overstate the magnitude of the risk premium because there are rather more instances of a negative excess return for the larger firms than is the case for the smaller firms.

7.6.2 Autocorrelation of the Risk Premium

Table 7.3 provides estimates of the autocorrelation coefficients for each of the portfolio conditional expected risk premiums. It can be seen clearly that expected returns for all portfolios are very autocorrelated, although, the magnitude of the autocorrelation appears to be closely associated with firm size. For the portfolio containing the largest firms the autocorrelation in the expected risk premium is exceptionally high. For example, the first order autocorrelation coefficient for the large firm portfolio is approximately 86 percent (and therefore very close to a random walk model) which indicates that a change in the conditional expected risk premium has almost a permanent effect on the conditional expected risk premium. In contrast, the first order autocorrelation coefficient for the portfolio containing the smallest firms is only 46 percent, indicating that a change in the conditional risk premium of small firms does not have such a long impact on the conditional risk premium as is the case for large firms. Given the strong autocorrelation in the expected risk premiums found here it is not surprising that realised portfolio returns appear to be so strongly autocorrelated. The results for the large firms suggests that there is not much variation in the conditional risk premium of large firms on a week by week basis, that is, the expected return this week is not very different from the expected return next week.
This is therefore consistent with the results presented in Section 5.4.3 of Chapter 5 which indicated an absence of short-horizon mean reversion for large firms. It appears that the reason for this is that the most recent weekly return of large firms provides very little additional information about the future performance of the portfolio return because the effect of a change in the expected return has an almost permanent effect on future returns. In which case this week's expected return therefore provides almost as much information as next weeks.

In the case of the large firms at least 10 percent of the autocorrelation appears to be a consequence of time variation in the systematic risk premium. For the smaller firms it is more difficult to identify the component of autocorrelation caused by time variation because a substantial amount of the portfolio autocorrelation will have been caused by thin trading.

It should be noted that the expected risk premium appears very much more autocorrelated than realised returns. This is expected because realised returns will also be influenced by random variations in the share price which will reduce the degree of autocorrelation in the time series of realised portfolio returns. Although, the error process between realised and expected returns of large and small is very different. The error process for the large firms is clearly much more random, but this probably reflects the different thin trading probabilities of the portfolios.

7.7 Sub-Period Analysis of the Systematic Risk Premium

Since the estimation of a GARCH model requires a very large sample this model has not been re-estimated over different sub-samples. However, to provide a gauge as to whether there have been shifts in the magnitude of the conditional risk premium over the full sample the mean weekly conditional risk premium has been calculated for four separate sub-periods. The sub-periods range from the first week in 1976 to the last week of 1980, the first week of 1981 to the last of 1983, the first week of 1984 to the last week of 1987 and from the first week of 1988 to the last week of 1991. The results from this analysis are provided in Table 7.4. Generally it seems that as we move from the beginning of the sample period towards the end of the sample period the magnitude of the risk premium tends to decline, but only slightly. For example, for the portfolio of large firms, the mean weekly risk premium in the first sub-period is approximately 0.00606 but in the last sub-period it is only approximately 0.00507. This pattern is also reflected in the portfolios of smaller companies, for example, for the portfolio containing the smallest firms in the sample the estimated systematic risk premium is on average approximately 0.00224 in the first sub-period but only 0.0019 in the second sub-period. Although, to some extent the difference in the magnitude of the conditional risk premium between the first and subsequent periods reflects the higher conditional risk premium present in 1977.
7.8 Seasonality and the Risk Premium

One issue which has received little attention is whether the conditional expected risk premium varies on a seasonal basis, or whether the January effect can be explained within the context of a conditional asset pricing model.

In this section the relationship between the conditional expected return and seasonality will be investigated. This section shows that conditional expected returns are higher during January which provides evidence to suggest that the January anomaly is not due to market inefficiency but instead reflects changes in required returns.

7.8.1 The Month by Month Expected Risk Premium

By looking at the graphical representations of the conditional risk premium which are presented in Figures 1 to 4, it is clear that at the start of the year the conditional risk premium appears to increase, this can be identified by the jump in the conditional risk premium which appears to occur at the start of the year. This pattern appears to exist for all four of the portfolios.

Table 7.5 provides estimates of the mean weekly conditional risk premium for each month of the year and for the other months of the year excluding January. By looking at this table it is possible to observe that for all four portfolios the risk premium is highest during the month of January. For the small firm portfolio the mean weekly risk premium during the month of January is approximately 0.00282 but the mean risk premium for the other eleven months of the year is only 0.00201. Similarly, for the portfolio containing the largest firms in the sample the mean weekly risk premium during January is approximately 0.0077 but the mean weekly risk premium for the other months of the year is only 0.00520.

In no other month of the year does the conditional systematic risk premium appear to be comparable to the January systematic risk premium. For example, although realised returns during the month of April appear to be higher than any month other than January this is not the case for the conditional expected risk premium. During April the conditional risk premium appears to be indistinguishable from the risk premium during other months. This appears to indicate that in the UK the January and April regularity may be caused by different factors. The January risk premium appears to be related to the conditional systematic risk premium but the April seasonality does not.

A comparison of the two means indicates that there is a significant difference between the risk premium in January and the risk premium for the other months of the year. Consistent with the results which were reported in Section 6.5 and 6.6 of Chapter 6 there appears to be an extraordinary relationship between systematic risk and return during the month of January.
7.8.2 January and The Market Price of Risk

One possible explanation for the higher January risk premium is that the price of risk during January may be higher than during other months of the year. This would follow if investors perceived January as being more inherently risky than other months of the year.

In order to test this possibility the GARCH-M version of the CAPM model which was tested in Section 7.5 of this chapter is re-estimated with a slope dummy variable to capture the price of risk during January. The dummy variable takes on a value of 1 if the portfolio or market return takes place during a January but a zero otherwise. This new version of the conditional mean equation is represented below.

\[
R_t = a + \lambda(H_t)e + d(H_{Jan,t})e + bC + U_t \quad (7.34)
\]

\[
vech(H_t) = A_0 + A_1vech(U_{t-1}U_{t-1}') + B_1vech(H_{t-1}) \quad (7.35)
\]

where, \(d\) is a scalar which indicates how many additional units of market risk are required by investors during January\(^9\). Other variables are as previously defined.

The results from the estimation of this model for all four portfolios are presented in Table 7.6 which demonstrates that the price of risk in all cases is significantly higher in January than for other months of the year. These results imply that investors do perceive investments in January as being more risky than during other months of the year. For the large firms the market price of risk is approximately twice as high relative to the other months of the year, but, for the smallest firms the market price of risk is only slightly larger than during the other months of the year. This pattern confirms the work reported in Section 6.5 and 6.6 of Chapter 6 which found that during January the systematic risk premium of large firms was higher than the systematic risk premium of either smaller firms or the systematic risk premium during other months of the year.

7.9 Summary and Conclusions

This chapter has tested whether the systematic risk premium of four portfolios formed on the basis of firm size are time varying. Using a GARCH-M model framework to estimate the conditional covariances this study finds strong support for the conditional version of CAPM for both large and small firms. Although, there appear to be some interesting differences in the behaviour of the conditional moments of small and large firms.

\[^9\] This has also been adjusted for the effects of thin trading in the same way as \(\lambda\).
It is found that for all four size-based portfolios which are investigated the systematic risk premium is sensitive to changes in the conditional covariance between the market and the portfolio return. This means that for both large and small firms the possibility that the risk premium is constant is rejected. Furthermore, it is found that in this study the $R^2$ of the model tested is able to explain a large proportion of the variation in realised returns. Although there appears to be an inverse relationship between the magnitude of the variation in realised returns which can be explained and the size of firms within a portfolio.

When the relationship between the return and volatility shock is investigated it is found that a number of differences exist between the behaviour of large and small firms. It appears that a shock to the conditional covariance of small firms causes more of a change in the conditional risk premium than is the case for large firms. However, it is found that a shock to large firms is much more persistent than is the case for small firms. This means that a shock of equal size which influences a portfolio of large and small firms will have a longer lasting effect on the portfolio of large firms.

In this study the relationship between January returns and the magnitude of the conditional systematic risk premium is investigated. It is found that the conditional risk premium of all four portfolios investigated in this study are considerably higher during the month of January. This provides further support for linking high January returns to an extraordinary relationship between risk and return rather than assuming the January anomaly is evidence of market inefficiency. Again, an interesting relationship between firm size and the behaviour of the risk premium during the month of January appears to exist. It is found that for large firms, during the month of January the estimated market price of risk is considerably higher than for other months of the year, but for small firms, although, the market price of risk appears to be higher it is only moderately so. This appears to indicate that investors perceive January as a more risky month than the rest of the year. However, there is also evidence of a "clientele effect" because investors in large firms view January as being more risky than is the case for small firms.

An interesting finding in this chapter is that the conditional risk premium of the four portfolios is heavily autocorrelated. Consequently, it is not surprising that realised portfolio returns have some autocorrelation. The autocorrelations in the conditional risk premium display a pattern which appears to be associated with firm size. As we move from the portfolio containing the smallest firms to the portfolio containing the largest firms the magnitude of the autocorrelation increases. For the large firm portfolio the conditional risk premium has an autocorrelation coefficient of 86 percent. This means that for large firms the predictable component in realised returns which is caused by time variation in expected returns does not decay very rapidly. This is an important finding because it helps to explain the lack of mean-reversion in short-horizon returns of large firms which was documented in Section 5.4 of Chapter 5. The reason there is no mean-reversion for large firms is that this weeks expected
anomalies we still do not understand why market premiums appear to be higher during January for countries which do not have tax years that end in December. Rather than documenting further the seasonal behaviour of risk premiums more work should be directed at investigating the institutional considerations which might cause January to be more risky than any other month.

Future research could also be gainfully employed at investigating the impact that of thin trading on the conditional risk premium. Work presented in this chapter suggests that models of the expected risk premium should be modified in the presence of thin trading. Further research could be done in this area to establish more fully through the use of simulation models the impact that thin trading might have on estimates of the conditional variance/covariance relationships and the market price of risk.
### Table 7.1
Estimates from the GARCH-M Conditional CAPM

The following table provides the results from the estimation of the following bi-variate GARCH(1,1)-M model.

\[
R_t = a + \lambda R_{t-1} + bC + U_t \\
vech(H_t) = A_0 + A_1 vech(U_{t-1}) + B_1 vech(H_{t-1}) \\
U_t | \phi_{t-1}, H_{t-1} = 0
\]

\(a\) is a vector of constants, which should all be insignificantly different from zero if the conditional version of the CAPM here is fully specified. The coefficient \(\lambda\) is a point estimate of the price of risk, and reflects how many units of market return investors require for additional units of market risk. The \(H_t\) is the conditional variance matrix, \(b\) is the coefficient vector for the crash which is denoted by the vector \(C\). \(A_0\) are constant coefficients which captures the degree of long run volatility in either the covariances or variances, \(A_1\) is the scalar coefficient associated with the past errors squared. This indicates how a shock influences the covariance or the variance, the scalar \(B_1\) is the coefficient on the one period lagged conditional variance and indicates whether there is a relationship between the current conditional variance/covariance and its previous value. The coefficients which are obtained from the estimation of the conditional mean equation for the portfolios are sub-scripted by \(p\). \(P_1\) to \(P_4\) are the portfolios containing the firms ranked on the basis of size so that portfolio \(P_1\) contains the smallest firms in the sample down to \(P_4\) which contains the largest firms in the sample. The t-statistics are provided in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>(P_1)</th>
<th>(P_2)</th>
<th>(P_3)</th>
<th>(P_4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a_p)</td>
<td>-0.0009</td>
<td>-0.0012</td>
<td>-0.0069</td>
<td>-0.0051</td>
</tr>
<tr>
<td></td>
<td>(-0.91)</td>
<td>(-0.99)</td>
<td>(-0.01)</td>
<td>(-0.13)</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>12.7999</td>
<td>11.22</td>
<td>12.22</td>
<td>12.49</td>
</tr>
<tr>
<td></td>
<td>(2.16)</td>
<td>(1.88)</td>
<td>(1.97)</td>
<td>(2.11)</td>
</tr>
<tr>
<td>(b_p)</td>
<td>-0.078</td>
<td>-0.066</td>
<td>-0.1085</td>
<td>-0.1291</td>
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<tr>
<td></td>
<td>(-7.91)</td>
<td>(-14.55)</td>
<td>(-3.33)</td>
<td>(-2.76)</td>
</tr>
<tr>
<td>(A_{011})</td>
<td>0.00006</td>
<td>0.00004</td>
<td>0.00006</td>
<td>0.00008</td>
</tr>
<tr>
<td></td>
<td>(9.30)</td>
<td>(4.88)</td>
<td>(5.77)</td>
<td>(4.07)</td>
</tr>
<tr>
<td>(A_{012})</td>
<td>0.00005</td>
<td>0.00004</td>
<td>0.00005</td>
<td>0.00008</td>
</tr>
<tr>
<td></td>
<td>(6.95)</td>
<td>(4.74)</td>
<td>(6.03)</td>
<td>(4.10)</td>
</tr>
<tr>
<td>(A_{022})</td>
<td>0.00017</td>
<td>0.00009</td>
<td>0.00007</td>
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</tr>
<tr>
<td></td>
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<td>(4.65)</td>
<td>(5.42)</td>
<td>(4.15)</td>
</tr>
<tr>
<td>(A_1)</td>
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<td>0.1005</td>
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<td>0.077</td>
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<tr>
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<td>(4.88)</td>
<td>(5.28)</td>
<td>(6.07)</td>
<td>(4.07)</td>
</tr>
<tr>
<td>(B_1)</td>
<td>0.5071</td>
<td>0.6963</td>
<td>0.7613</td>
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<td></td>
<td>(10.58)</td>
<td>(13.12)</td>
<td>(24.49)</td>
<td>(14.45)</td>
</tr>
<tr>
<td>Half Life in days</td>
<td>11</td>
<td>21</td>
<td>29</td>
<td>25.5</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.1287</td>
<td>0.1369</td>
<td>0.2610</td>
<td>0.3094</td>
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Table 7.2
Diagnostics for the Residuals Obtained from the Conditional CAPM

This table provides the results of diagnostic tests which were performed on the residuals from the estimation of the model presented in the previous table. Kurt Before refers to the kurtosis in the return series while Kurt After refers to the kurtosis in the residuals after fitting the GARCH-M model. The estimated autocorrelations in the residuals from lag one to six are captured by $\hat{\rho}_1$ to $\hat{\rho}_6$. Standard errors for these are approximately 0.035. The $Q$ is the sixth order Ljung-Box statistic which identifies autocorrelation from lags one to six. In all cases the probability values for the $Q$ test were zero.

<table>
<thead>
<tr>
<th></th>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
<th>$\hat{\rho}_3$</th>
<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
<th>$Q$</th>
<th>Kurt Before</th>
<th>Kurt After</th>
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<td>-0.1117</td>
<td>0.1173</td>
<td>0.0654</td>
<td>0.0556</td>
<td>0.0981</td>
<td>0.0584</td>
<td>38.94</td>
<td>10.63</td>
<td>2.19</td>
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<tr>
<td>$P_2$</td>
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<td>0.0833</td>
<td>0.0523</td>
<td>0.0950</td>
<td>-0.0199</td>
<td>0.0394</td>
<td>20.98</td>
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<tr>
<td>$P_3$</td>
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<td>0.0698</td>
<td>0.1353</td>
<td>-0.0042</td>
<td>0.0596</td>
<td>88.93</td>
<td>6.38</td>
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<tr>
<td>$P_4$</td>
<td>0.0669</td>
<td>0.1054</td>
<td>-0.0144</td>
<td>0.0757</td>
<td>-0.0327</td>
<td>0.0268</td>
<td>19.55</td>
<td>7.35</td>
<td>1.377</td>
</tr>
</tbody>
</table>
Table 7.3  
Information About the Conditional Systematic Risk Premium

This table provides information on the conditional risk premium. The conditional risk premium for portfolio 4, which contains the largest firms in the sample is calculated as $\lambda(h_{12,t})$. For the other portfolios which have adjustments to account for thin trading, the conditional expected risk premium has been calculated as

$$
\lambda_0(h_{12,t}) + \lambda_1(u_{p,t-1}, u_{m,t-2}) + \ldots + \lambda_6(u_{p,t-1}, u_{m,t-1})
$$

$\text{Prem}$ is the mean weekly conditional risk premium over the sample period. $\text{Prem}_H$ and $\text{Prem}_L$ are the highest and lowest recorded weekly conditional risk premiums respectively. The $\hat{\rho}_1$ to $\hat{\rho}_6$ are the autocorrelation statistics for the conditional expected risk premium from lags one to six, standard errors for these are approximately 0.035. $Q$ is the Ljung-Box statistic for the presence of autocorrelation from lags one to six in all cases the probability values are zero. $P_1$ to $P_4$ are the portfolios containing the firms ranked on the basis of size so that portfolio $P_1$ contains the smallest firms in the sample down to $P_4$ which contains the largest firms in the sample.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\text{Prem}$</th>
<th>$\text{Prem}_L$</th>
<th>$\text{Prem}_H$</th>
<th>$\hat{\rho}_1$</th>
<th>$\hat{\rho}_2$</th>
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<th>$\hat{\rho}_4$</th>
<th>$\hat{\rho}_5$</th>
<th>$\hat{\rho}_6$</th>
<th>$Q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
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<td>-0.0002</td>
<td>0.0137</td>
<td>0.4839</td>
<td>0.3297</td>
<td>0.2383</td>
<td>0.2108</td>
<td>0.1381</td>
<td>0.1358</td>
<td>403</td>
</tr>
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<td>$P_2$</td>
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<td>0.0115</td>
<td>0.3875</td>
<td>0.2553</td>
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<td>0.2472</td>
<td>0.1271</td>
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</tr>
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<td>0.5791</td>
<td>0.5033</td>
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Table 7.4

Sub-Period Analysis of the Estimated Systematic Risk Premium.

The following table provides estimates of the mean conditional risk premium for each of the four portfolios, estimated for four different sub-periods. Each sub-period begins during the first week of the year and ends at the last week of the year. The conditional risk premium for the four portfolios was estimated in the same way as reported in Table 7.3. The t-statistics are reported in parentheses. The $P_1$ to $P_4$ are the portfolios containing the firms ranked on the basis of size so that portfolio $P_1$ contains the smallest firms in the sample down to $P_4$ which contains the largest firms in the sample.

<table>
<thead>
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</thead>
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<td>(0.0012)</td>
<td>(0.0018)</td>
<td>(0.0008)</td>
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<td>(0.0014)</td>
<td>(0.0008)</td>
</tr>
<tr>
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<td>(0.0016)</td>
<td>(0.0017)</td>
<td>(0.0013)</td>
</tr>
<tr>
<td>$P_4$</td>
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<td>0.00518</td>
<td>0.00507</td>
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<td>(0.0051)</td>
</tr>
</tbody>
</table>
Table 7.5
The Month by Month Conditional Systematic Risk Premium

The following table provides estimates of the systematic risk premium on a month by month basis for all four portfolios. Feb-December provides the mean weekly return for each week in the sample except those that fall during a January. Standard errors are shown in parentheses. The \( P_1 \) to \( P_4 \) are the portfolios containing the firms ranked on the basis of size so that portfolio \( P_1 \) contains the smallest firms in the sample down to \( P_4 \) which contains the largest firms in the sample.

<table>
<thead>
<tr>
<th>All Portfolios</th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>0.0054362</td>
<td>0.002818</td>
<td>0.003067</td>
<td>0.008151</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00147)</td>
<td>(0.00499)</td>
<td>(0.00217)</td>
</tr>
<tr>
<td>February</td>
<td>0.0039437</td>
<td>0.001954</td>
<td>0.002391</td>
<td>0.005142</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00050)</td>
<td>(0.00149)</td>
<td>(0.00135)</td>
</tr>
<tr>
<td>March</td>
<td>0.0037227</td>
<td>0.001958</td>
<td>0.002255</td>
<td>0.005103</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00033)</td>
<td>(0.00136)</td>
<td>(0.00080)</td>
</tr>
<tr>
<td>April</td>
<td>0.0035977</td>
<td>0.001872</td>
<td>0.002239</td>
<td>0.004841</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00041)</td>
<td>(0.00139)</td>
<td>(0.00096)</td>
</tr>
<tr>
<td>May</td>
<td>0.0037647</td>
<td>0.002093</td>
<td>0.002435</td>
<td>0.004952</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00070)</td>
<td>(0.00242)</td>
<td>(0.00151)</td>
</tr>
<tr>
<td>June</td>
<td>0.0032020</td>
<td>0.001872</td>
<td>0.001803</td>
<td>0.004229</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00018)</td>
<td>(0.00071)</td>
<td>(0.00061)</td>
</tr>
<tr>
<td>July</td>
<td>0.0033087</td>
<td>0.001953</td>
<td>0.002170</td>
<td>0.004090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00043)</td>
<td>(0.00117)</td>
<td>(0.00144)</td>
</tr>
<tr>
<td>August</td>
<td>0.003031</td>
<td>0.001856</td>
<td>0.001684</td>
<td>0.003913</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00016)</td>
<td>(0.00056)</td>
<td>(0.00042)</td>
</tr>
<tr>
<td>September</td>
<td>0.0038807</td>
<td>0.002741</td>
<td>0.002395</td>
<td>0.005067</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00024)</td>
<td>(0.00092)</td>
<td>(0.00196)</td>
</tr>
<tr>
<td>October</td>
<td>0.0034615</td>
<td>0.001951</td>
<td>0.002113</td>
<td>0.004648</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00039)</td>
<td>(0.00101)</td>
<td>(0.00082)</td>
</tr>
<tr>
<td>November</td>
<td>0.0031447</td>
<td>0.001847</td>
<td>0.001802</td>
<td>0.004160</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00029)</td>
<td>(0.00088)</td>
<td>(0.00034)</td>
</tr>
<tr>
<td>December</td>
<td>0.0034225</td>
<td>0.002020</td>
<td>0.001944</td>
<td>0.004707</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00046)</td>
<td>(0.00182)</td>
<td>(0.00102)</td>
</tr>
<tr>
<td>Feb-December</td>
<td>0.003487</td>
<td>0.002010</td>
<td>0.002111</td>
<td>0.004629</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.00101)</td>
<td>(0.00124)</td>
<td>(0.00211)</td>
</tr>
</tbody>
</table>
Table 7.6  
Estimates from the Multivariate GARCH-M Model with a January in Mean Dummy

The following table provides the results from the estimation of the following bi-variate GARCH(1,1)-M model.

\[ R_t = a + \lambda H_t e + \lambda H_{jm}, e + U_t \]

\[ \text{vech}(H) = A_0 + A_1 \text{vech}(U_{t-1}, U_{t-1}) + B_1 \text{vech}(H_{t-1}) \]

\[ U_t | \theta_{t-1}, \theta_{H} \]

\( a \) is a vector of constants, which should be insignificantly different from zero if the conditional version of the CAPM here is fully specified. The coefficient \( \lambda \) is a point estimate of the price of risk, and reflects how many units of market return investors require for each additional unit of market risk. \( d \) provides an estimate of how many additional units of market risk are required by investors during the month of January. \( H_t \) is the conditional variance matrix. \( A_0 \) are constant coefficients which capture the degree of long run volatility in either the covariances or variances, \( A_1 \) is the coefficient from the past errors squared, \( B_1 \) is the coefficient on the one period lagged conditional variance and indicates whether there is a relationship between the current conditional variance/covariance and its previous value. \( P_1 \) to \( P_4 \) are the portfolios containing the firms ranked on the basis of size so that portfolio \( P_1 \) contains the smallest firms in the sample down to \( P_4 \) which contains the largest firms in the sample. The t-statistics are shown in parentheses.

<table>
<thead>
<tr>
<th></th>
<th>( P_1 )</th>
<th>( P_2 )</th>
<th>( P_3 )</th>
<th>( P_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( a )</td>
<td>-0.0014</td>
<td>-0.0016</td>
<td>-0.0017</td>
<td>-0.0055</td>
</tr>
<tr>
<td></td>
<td>(-1.43)</td>
<td>(-1.26)</td>
<td>(-0.83)</td>
<td>(-2.26)</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>21.26</td>
<td>10.87</td>
<td>11.08</td>
<td>12.64</td>
</tr>
<tr>
<td></td>
<td>(1.99)</td>
<td>(1.87)</td>
<td>(2.09)</td>
<td>(2.18)</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.81</td>
<td>13.30</td>
<td>11.25</td>
<td>11.98</td>
</tr>
<tr>
<td></td>
<td>(2.23)</td>
<td>(2.01)</td>
<td>(1.96)</td>
<td>(1.93)</td>
</tr>
<tr>
<td>( b_p )</td>
<td>-0.078</td>
<td>-0.066</td>
<td>-0.123</td>
<td>-0.129</td>
</tr>
<tr>
<td></td>
<td>(-7.77)</td>
<td>(-12.65)</td>
<td>(-23.75)</td>
<td>(-0.96)</td>
</tr>
<tr>
<td>( A_{011} )</td>
<td>0.00006</td>
<td>0.00004</td>
<td>0.00006</td>
<td>0.00007</td>
</tr>
<tr>
<td></td>
<td>(9.14)</td>
<td>(4.85)</td>
<td>(5.72)</td>
<td>(40.07)</td>
</tr>
<tr>
<td>( A_{012} )</td>
<td>0.00005</td>
<td>0.00004</td>
<td>0.00005</td>
<td>0.00007</td>
</tr>
<tr>
<td></td>
<td>(6.77)</td>
<td>(4.72)</td>
<td>(5.93)</td>
<td>(77.10)</td>
</tr>
<tr>
<td>( A_{022} )</td>
<td>0.00017</td>
<td>0.00009</td>
<td>0.00007</td>
<td>0.00007</td>
</tr>
<tr>
<td></td>
<td>(7.96)</td>
<td>(4.63)</td>
<td>(5.34)</td>
<td>(39.79)</td>
</tr>
<tr>
<td>( A_1 )</td>
<td>0.1283</td>
<td>0.0986</td>
<td>0.0948</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>(4.90)</td>
<td>(5.16)</td>
<td>(6.12)</td>
<td>(5.44)</td>
</tr>
<tr>
<td>( B_1 )</td>
<td>0.4969</td>
<td>0.6914</td>
<td>0.7502</td>
<td>0.7620</td>
</tr>
<tr>
<td></td>
<td>(9.14)</td>
<td>(10.58)</td>
<td>(23.05)</td>
<td>(44.27)</td>
</tr>
<tr>
<td>( \overline{R^2} )</td>
<td>0.1383</td>
<td>0.1259</td>
<td>0.2672</td>
<td>0.2713</td>
</tr>
</tbody>
</table>
Fig 7.1
The Weekly Conditional Systematic Risk Premium for Portfolio $P_4$
Fig 7.2
The Weekly Conditional Systematic Risk Premium for Portfolio $P_3$

![Graph showing the weekly conditional systematic risk premium for portfolio $P_3$.]
Fig 7.3
The Weekly Conditional Systematic Risk Premium for Portfolio $P_2$
Fig 7.4
The Weekly Conditional Systematic Risk Premium for Portfolio $P_1$
Fig 7.5
The Weekly Conditional Excess returns for Portfolio $P_4$
Fig 7.6
The Weekly Conditional Excess returns for Portfolio $P_3$
Fig 7.7
The Weekly Conditional Excess returns for Portfolio $P_2$
Fig 7.8
The Weekly Conditional Excess returns for Portfolio $P_1$. 

![Weekly Conditional Excess Return for Portfolio $P_1$](image)
CHAPTER EIGHT
Volatility, Leverage and Firm Size: The UK Evidence

8.1 Introduction

Recent evidence for the US stock market, such as that provided by Lo and MacKinlay (1990a) and Conrad, Gultekin and Kaul (1991), has uncovered a number of asymmetries in the dynamic behaviour of size-based portfolio returns. It had long been known that portfolios or indexes that give greater weight to small capitalisation firms were characterised by greater serial correlation than portfolios or indexes that gave little weight to small firms, see for example, the seminal paper by Fisher (1966), and the studies of the empirical application of the Capital Asset Pricing Model by Scholes and Williams (1977) and Dimson (1979) which were discussed more fully in section 3.2.3 of Chapter 3.

Lo and MacKinlay (1990a) pointed out that this was not the only difference between the serial correlations of size-based portfolio returns. For example, they systematically documented the cross serial correlations between the portfolio returns. They found that while the past returns of a portfolio of large firms were strongly correlated with the current return of a portfolio of small firms, the reverse was not the case. To illustrate, Lo and MacKinlay reported that the cross serial coefficient between the returns of a portfolio containing the smallest firms and a portfolio containing the largest firms in their sample was 27.6 percent if the returns of the large firms lead the returns of the small firms by one period. In contrast, it was found that the cross serial correlation coefficient was only 2 percent when large firms lead small firms by one period. Results consistent with these findings are also reported by Conrad et al (1991). This means that the returns of large firms help to predict the returns of small firms but the reverse is not true, the returns of small firm do not help to predict the returns of large firms.

The discovery of asymmetries in the empirical characteristics of large and small firms was enhanced by the work of Conrad et al (1991) who found that unexpected returns to large firm portfolios were important determinants of the future volatility and returns of small firms, but that the reverse was not the case. Using a GARCH (1,1) model to capture volatility, Conrad et al found that the unexpected returns of portfolios containing large firms predicted the future volatility for a portfolio of smaller firms. In contrast, the unexpected returns of small firms did not predict the future volatility of a portfolio of large firms. The aim of this chapter is to examine whether there are similar asymmetries in the dynamics of the returns of size-based portfolios for UK stocks, and to extend the search for size-related regularities to the leverage effect.

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1 This work has been undertaken jointly with James Michael Steeley.
The leverage effect, which was introduced by Black (1976) and Christie (1982), refers to
the tendency for returns and volatility to be negatively correlated. That is, negative returns are
more likely to be associated with greater volatility than positive returns. There is good reason
to expect a leverage effect in stock markets. Negative stock returns are caused by a fall in the
share price of a company, which necessarily induces a fall in the capitalisation value, therefore
causing an increase in the gearing ratio making the equity more risky. Consequently, investors
will require higher returns, which can only be achieved by a fall in the share price, which
introduces additional volatility.2

In this study a number of aspects to the leverage effect will be tested. Firstly, this chapter
will seek to determine whether the leverage effect is influenced by firm size. Essentially, this
means identifying whether an unexpectedly bad return causes more future volatility than an
unexpectedly good return and, furthermore, looking at whether the magnitude of the predicted
future volatility depends upon firm size. Secondly, this chapter tests whether the sign of a shock
to one portfolio is important for the volatility and returns of another portfolio, and whether this
relationship is itself size-based.

The remainder of this chapter is set out as follows. Section 8.2 describes the characteristics
of the data set used in this study. In Section 8.3, a specification of the generalised autoregressive
conditional heteroscedasticity (GARCH) family of statistical processes is used to identify the
influence that portfolio return shocks have on the future volatility of a portfolio. Section 8.4
investigates whether volatility shocks are transmitted across portfolios so that volatility
spillovers exist across portfolios. Section 8.5 reports the results of an investigation into the
relationship between the leverage effect and firm size, using an asymmetric conditional variance
model. Section 8.6 provides a summary and conclusions to the chapter.

8.2 Data and Summary Statistics

The weekly returns for four size-based portfolios of UK stocks spanning the period from
January 1976 to December 1991 are tested in this chapter. These portfolio returns were the same
as the ones used in Chapter 4 and Chapter 7. A detailed account of how the four portfolios were
formed is provided in Section 4.1.4 of Chapter 4.

As Chapter 4 demonstrated and, specifically by looking at Table 4.1 the autocorrelations
for the portfolio weekly returns, squared returns, and absolute returns for the four
equally-weighted portfolios indicated that there were striking differences among the
autocorrelations of the portfolio returns. Furthermore, these differences appeared to be to be

2 The impact that higher gearing ratios have on the risk and therefore the required return of investors is well documented. Modigliani and
Miller (1958), for example, discuss the fact that as the gearing ratio of a company increases the probability of the company experiencing
bankruptcy increases. The occurrence of bankruptcy is bad for shareholders because when a company is liquidated the shareholders are the
last to receive their investment and are only paid after creditors and debt holders.
related to the capitalisation ranking of the portfolios. The portfolios of relatively small firms tended to have return autocorrelation coefficients which were greater than those for portfolios of relatively larger firms. Although, for all portfolios the magnitudes of the autocorrelation coefficients decayed, at higher orders. More than half of the reported coefficients were statistically significant. All of the portfolios had significant first and second order autocorrelation, and all of the reported coefficients for the small firm portfolio were significant. The small firm portfolio had greater autocorrelation than the large firm portfolio in both squared and absolute returns. Autocorrelation coefficients were generally smaller as the lag length increases.

In contrast to the autocorrelation observed for the underlying returns series, the coefficients for squared and absolute returns were generally much larger at lags 1 and 2. For example, first order autocorrelation coefficients for squared returns were 60 percent and 43 percent for the small firm portfolio and large firm portfolio, respectively. Strong autocorrelation in squared and absolute returns is a symptom of changing unconditional or conditional variances. In particular, these results suggest that the volatility of small firm portfolio returns is more clustered than the volatility of large firm portfolio returns. That is, large absolute returns are more likely to be followed by large absolute returns than by small absolute returns.

Table 8.1 provides summary information on the debt/equity ratios of component securities, for each of the portfolios used in this chapter. As we can see from table 8.1 all of the portfolios have a reasonably similar mean debt/equity ratio over the period 1976-1991. For portfolio 1 to 3 the mean debt/equity ratio is in the region of 0.3. The variance of the debt/equity ratio is much higher for the portfolios containing the smaller firms. For example, for the portfolio containing the smallest companies the variance of the debt/equity ratio is about 0.127 but for the larger firms the variance is only 0.026. This means that the debt/equity ratio for large firms is reasonably homogeneous but for small firms, some have high debt/equity ratios while others have more modest ratios.

Table 8.1 also provides information on the mean debt/equity ratio for the years 1976, 1985 and 1991. What is noticeable from these figures is the downward trend in debt/equity ratios of smaller firms during 1991 but the rise in the ratios for large firms.

In Table 8.3(a) of this chapter, the estimated cross serial correlations between the returns of each of the portfolios are reported. These cross serial correlations indicate a similar pattern to that documented by Lo and MacKinlay for the US stock market. It should be noted that the leading diagonal terms in the sections of Table 8.3(a) correspond to the autocorrelations reported in Table 4.1 of Chapter 4. The terms above the diagonal tend to be larger than the corresponding terms below the diagonal. This means that there is a stronger correlation between the past returns of portfolios which contain relatively large firms and the current portfolio returns of relatively
small firms than there exists in the opposite direction. For example, the previous week’s return of the large firm portfolio and the current return for the small firm portfolio provide a correlation coefficient of 0.383. In contrast, the previous week’s return of the small firm portfolio and the current week’s return of the large firm portfolio provides a correlation coefficient of only 0.096. As with the autocorrelations, the cross serial correlations decay as the lag length increases. For example, the cross serial correlation coefficients at three lags, when the portfolio returns of large firms lead the portfolio returns of small firms is 0.154 but is only -0.063 when small firms lead the large firms.

8.3 Asymmetries in the Conditional Variance of Portfolio Returns

The strong autocorrelation in the portfolio returns indicates that a good way of modelling the returns for the portfolios would be to use a stationary first-order autoregressive moving-average model, ARMA(1,1), as follows

\[ R_{i,t} = \mu_i + \phi R_{i,t-1} + \epsilon_{i,t} - \theta \epsilon_{i,t-1} \quad i = 1, 2, 3, 4 \]  

where, \( R_{i,t} \) are the returns of one of the four portfolios, \( \epsilon_{i,t} \) is the one period lagged error from portfolio \( i \), and \( \mu_i \) is a constant.

The coefficients which are obtained from the estimation of this model are given in Table 8.2. The autoregressive coefficient, \( \phi \), captures the first order autocorrelation in the returns series. The coefficient is largest and most significant for the small firm portfolio returns, where the return autocorrelation is strongest. The moving-average coefficient, \( \theta \), provides a parsimonious representation of the decay in the autocorrelation function of the returns and, as such, is less dissimilar among the different portfolio returns series. These characteristics have already been noted in Chapter 5. In the spirit of Box and Jenkins (1976), the autocorrelation structure of the residuals was analysed as a specification test. These residual autocorrelations are also reported in Table 8.2 and, with the slight exception of \( P_3 \), there is little evidence of significant autocorrelation. So, the ARMA(1,1) process provides a well-specified form for the conditional mean process, and will be used as the basis for the examination of the conditional variance.

Table 8.2, reports strong evidence of serial correlation among the squared residuals of the ARMA(1,1) model of returns. The pattern of autocorrelation in the squared residuals is consistent

---

3. Alternative specifications were investigated, but did not appear to perform very well. For example, a number of MA and AR models were investigated but these tended to provide quite high autocorrelations in the residuals.

4. Alternative conditional variance specifications such as the GARCH(1,2) were also tested but such models did not perform very well since higher order lags in the AR and MA component of the GARCH specification tended to be insignificant.
with an ARMA(1,1) specification of the conditional variance process. Hence, volatility is modelled as a GARCH(1,1) process. As Section 7.3 showed such a model allows us to capture the clustering and persistent nature of the conditional variance.

$$h_t = m + b \varepsilon_{t-1}^2 + ch_{t-1}$$

(8.2)

As Section 7.3 of the previous chapter showed, in this model, the coefficient $b$ measures the tendency of the conditional variance to cluster, while the coefficient $c$ measures the degree of persistence in the conditional variance process. A tendency for volatility clustering will also give rise to leptokurtosis in the return series, which is present in this data set. The kurtosis figures are reported for each of the four portfolios in Table 8.2. As we can see, the kurtosis statistic for the residuals, for all four portfolios, is a long way from 3 which would indicate normality. For example, for the small firm portfolio the kurtosis is 8.234 but for the large firm portfolio the kurtosis is 5.941.

In this study realised returns have been estimated as a purely parsimonious time series process rather than being modelled with reference to an asset pricing model such as CAPM. The reason for this is that it is important that the spillover results presented in this chapter are not due to mis-specification of the asset pricing model which is used. This would be a strong possibility in view of the evidence presented in Chapters 6 and 7 which indicated that CAPM did not explain realised returns very well for the portfolios examined in this study. Utilising the ARMA(1,1) model instead, implies that expected returns follow an AR(1) process but no equilibrium conditions are imposed. This means that as long as the ARMA(1,1) model appears to be well specified the behaviour of expected returns will be well captured by the model.

The specification of the conditional mean equation which is estimated in this study is modified slightly from the model presented in equation (8.1). This modification is necessary to take into account the specific characteristics of the data set used in this study which otherwise might lead to a mis-specification in the conditional variance equation. Since the sample period overlaps with the equity market crash of October 1987, a dummy variable is included to cover this event. The dummy variable takes on a value of one if it is October of 1987 and a zero otherwise. Furthermore, as Chapters 6 and 7 demonstrated the UK stock market is characterised by a significant turn-of-the-year effect, a dummy variable is also included for this. This dummy variable takes on a value of one if the portfolio return occurs in a January but a zero otherwise. In later specifications, the volatility shocks of one portfolio will be included in the conditional variance of another portfolio. To ensure, that these relationships are not obscured by the cross serial correlations in the returns, which were identified in Table 8.3(a), additional terms in the form of lagged returns of each of the other portfolio returns are included in the conditional mean.

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5. Dummies to account for the start of the tax year in April were found to be insignificant and were not included in the final version which was estimated.
equation. Since the cross serial correlation between the portfolios is a symptom of thin trading (see Section 2.7.2 which discussed this issue in detail), including the lagged one period returns of each of the other portfolios will ensure that this specification will purge from the data any remaining influence that thin trading might have which has not already been eliminated by the ARMA(1,1) specification. The full model tested is as follows:

\[ R_{i,t} = \mu_i + \beta_i h_{i,t} + \phi_i R_{i,t-1} - \theta_i \varepsilon_{i,t-1} + \sum_{j=1}^{4} \gamma_{i,j} R_{j,t-1} + \delta_{i,1} D_1 + \delta_{i,2} D_2 + \varepsilon_{i,t}, \quad i, j = 1, 2, 3, 4 \]  

\[ h_{i,t} = m_i + b_i \varepsilon_{i,t-1}^2 + c_i h_{i,t-1} \]  

where \( \sum_{j=1}^{4} \gamma_{i,j} R_{j,t-1} \) refers to the one period lagged return of each of the three other portfolios and the coefficients associated with these lagged one period returns. \( D_1 \) is a dummy variable which takes on a value of 1 if it is a week in October of 1987. \( D_2 \) is a dummy which takes on a value of 1 if it is a week in January but a zero otherwise. \( \delta_1 \) and \( \delta_2 \) are coefficients which indicate how much above or below the constant returns are at this time.

The results which have been obtained from the estimation of equation (8.3) and (8.4) are provided in Table 8.4. As a specification test of the model presented in equations (8.3) and (8.4), the Ljung-Box (1978) sixth order statistic has been calculated for the normalised residuals of each of the four portfolio expected return series. In each case, there was no significant evidence of serial correlation which indicates that the extended ARMA(1,1) model of equation (8.3) is fully specified.

The dummy variable for the crash was found to be statistically significant for each portfolio. While, the turn-of-the-year dummy variable was significant for all portfolios except the one containing the largest firms.

Focusing now on the estimated conditional variance coefficients, it can be seen that they are all strongly significant. These results indicate a clear pattern, the past squared errors have more influence over the conditional variance of the small firm portfolio than they do over the conditional variance of the large firm portfolio. The coefficient \( b_i \) takes the value 0.243 for the small firm portfolio, but has a value of only 0.121 for the large firm portfolio. These coefficients also reflect the relative levels of kurtosis in the return series of the portfolios. The small firm
portfolio returns are more leptokurtotic because they have the greater tendency toward clustering, this is reflected in the kurtosis statistic because the clustering introduces an excessive number of extreme returns which cause the fat tails in a leptokurtotic distribution.

The estimated values of the coefficient $c_i$ are 0.356 and 0.730 respectively, for the small and large firm portfolios. The combination of these two features suggests that although shocks to the volatility of large firm portfolios have less impact than shocks to the volatility of small firm portfolios, they are much more persistent. This is an interesting finding because we usually think of the small firms as being characterised by more persistence, whether it is due to more inertia in the reaction of investors, or whether it is caused by microstructure frictions. These results confirm those reported in Section 7.5 of Chapter 7 which showed that a systematic shock had an influence on the conditional variance of large firms for longer than was the case for small firms.

For small firms, but not for large firms, volatility implies stock return predictability. Because, both the GARCH in mean term and the components in the conditional volatility equation are all statistically significant it is possible to infer that volatility predicts stock returns for all but the largest portfolios. The estimated $\beta_i$ coefficients on the GARCH-M terms also display an interesting pattern. The coefficients have a greater magnitude and statistical significance in portfolios of smaller firms. This implies, that the smaller are the firms in a portfolio, the more valuable is information concerning the current value of the volatility. This feature is entirely consistent with the previous results concerning the relative persistence of volatility. Information on current volatility is most important for portfolios of small firms exactly because it has a big impact and then decays quickly.

The results which are provided in this section indicate that if investors are concerned about risk, then during periods of high volatility they should hold portfolios of large firm stocks. This follows because during periods of high market volatility, it is the returns of small firms which will become the most volatile. Although, investors who hold portfolios of large firms do not obtain return compensation for this because volatility does not predict returns.

It is possible to obtain a clearer picture of the persistence in the volatility processes by calculating the half-life of a shock to the process, that is, the time that it takes for half of the shock to have dissipated. This can be calculated as follows

\[
\text{Half-life}_i = \frac{\ln(0.5)}{\ln(c_i + b_i)}
\]

(8.5)

In the case of the small firm portfolio, the half-life is about 7 trading days in length, whereas for the large firm portfolio, the half-life is about 20 trading days. These estimates are almost identical to those reported in Section 7.5 of Chapter 7 which looked at the half life of a systematic

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shock. This section suggests that volatility persistence is stronger for large firms than for small firms. We normally expect news to take longer to be reflected in the share prices of small firms not large firms. However, the results presented here may reflect the trading patterns of institutional shareholders better represented by the behaviour of large firms. When a large firm stock provides an unexpectedly high or low return this might cause large institutional investors to rebalance their portfolios. The result of the portfolio rebalancing, if it has price implications is that future volatility during the rebalancing period is increased. Small investors more active in small firm stocks are less likely to rebalance in response to immediate price pressure because of higher transaction costs see for example, Pagano and Roell (1990) who show that round trip transaction costs for small transactions are about 4% but only 1% for large transactions.

8.4 Volatility Spillovers

Although the residuals from the ARMA(1,1) models did not possess significant autocorrelation, however, there is some evidence that the residuals from the ARMA(1,1) models, for different portfolio returns are cross-autocorrelated, see Table 8.3(b). This suggests that return shocks, that is, unexpected returns experienced by one portfolio, may be important in determining the volatility and expected returns of another portfolio. Again there appears to be an asymmetry whereby the terms above the leading diagonal are larger than the terms below the diagonal, which suggests that shocks to large firm portfolios may influence small firm portfolios, but not the reverse. This idea of examining volatility spillovers was pioneered by Hamao et al (1990), who considered the influence that stock market volatility in one country might have on the behaviour of the volatility and returns of the stock market in another country. Some evidence is provided by Hamao for spillovers of volatility from New York to Tokyo and London to Tokyo, but not from Tokyo to either New York or London.

This idea of a volatility shock being transmitted across portfolios was introduced into the framework of portfolio returns by Conrad et al (1991), who examined whether volatility shocks were transmitted across capitalisation based portfolios in the US markets. Using a GARCH-M framework to capture volatility, Conrad et al found that a shock to large firms could predict future volatility in the portfolio returns of small firms but not vice versa, that is, volatility surprises to small firms did not predict future volatility or returns for large firm portfolios. This finding indicated an interesting asymmetry in the behaviour of the conditional variance of large and small firms.

In Table 8.5, maximum likelihood estimates of the model represented by equation (8.6) below are estimated. This equation is a modification of the conditional variance equation represented by equation (8.4) in the previous section. In equation (8.6) the conditional variance also includes volatility spillover terms. This means that the past volatility of one portfolio,
(measured by the past errors squared) is allowed to influence the conditional variance equation of another portfolio. This allows us to test whether a volatility shock to one portfolio can predict volatility in another portfolio. Specifically, the aim of these tests is to identify whether there is a spillover relationship associated with firm size.

\[ h_{it} = m_i + b_i \varepsilon_{it-1}^2 + c_i h_{i,t-1} + k_{i,j} \varepsilon_{j,t-1}^2 \quad j \neq i \]  

where, \( \varepsilon_{i,t-1}^2 \) is the squared unexpected return of portfolio \( j \), \( k_{i,j} \) is the coefficient which measures the impact of past return shocks to portfolio \( j \) on the conditional volatility of portfolio \( i \), and other terms are as previously defined.

Looking at the results from the estimation of this model, which are reported in Table 8.5, it can be seen that the estimated values of \( k_{i,j} \) tend to be more statistically significant above the leading diagonal, which is in cases where the volatility spillover is from a portfolio of relatively large firms to a portfolio of relatively small firms. The past volatility of the large firm portfolio is important in determining the future volatility of the small firm portfolio, but the reverse is not so. To illustrate, Table 8.5 shows that the coefficient on \( k_{i,j} \), when the spillover is coming from the small firm portfolio into the portfolios which contain the larger firms, in only one case is this coefficient statistically significant. This indicates that a shock from small firms does not predict the future volatility of larger firms. An exception, is the case of \( P_2 \) which contains the small-intermediate firms\(^6\). In contrast, when a shock is observed for the largest firms in the sample, this shock always predicts future volatility for the smaller firms, \( k_4 \) is statistically significant for portfolio \( P_1 \) and \( P_2 \), the portfolios which contain the smaller companies in the sample. While, this general pattern is supported throughout.

The estimated \( \beta \) coefficients on the GARCH-M terms maintain the same picture which was obtained without the inclusion of the spillover terms. That is, the volatility of relatively small firm portfolios is important in determining the conditional mean return of small firm portfolios, but the volatility of the large firm portfolio does not have a statistically significant influence on the mean return of the large firm portfolio. This feature, in combination with the spillover results, means that the volatility shocks to the large firm portfolio are important in determining both the volatility and the mean return of the small firm portfolio. However, the reverse is not the case; shocks to small firms do not appear to spillover into large firms.

The estimated values of \( b_i \) and \( c_i \) display the same clustering and persistence features as documented in Table 8.4. Small firm portfolio returns are more clustered, but shocks are less persistent, than for large firm portfolio returns. We can use the \( c_i \) coefficients in combination

\(^6\) In this case a shock from the smallest firms in the sample does appear to predict future volatility since the coefficient is 0.062 and statistically significant.
with the $k_{i,j}$ to determine the half-life of a spill-over volatility shock, in the same manner as described in equation (8.5). Since the volatilities of larger firm portfolios display more persistence, we would expect that the half life of spillover shocks from small firm portfolios to large firm portfolios would be longer than those in the opposite direction. The calculated half-lives confirm this picture with the half-lifes being between 7 and 20 trading days for spill-over shocks from small firm portfolios to large firms portfolios, and between 3 and 7 days for spill-over shocks from large firm portfolios to small firm portfolios.

The results presented in this section suggest that the return performance of large firms provides some new information about the return performance of small firms. This is consistent with findings presented elsewhere in this thesis which suggests that because small investors lack information about the their investments small investors may use the return performance of large firms to guess what is to happen next to the return performance of small firms.

### 8.5 Conditional Volatility and the Leverage Effect

This section tests for a leverage effect in UK capitalisation based portfolios. As the introduction stated, the leverage effect predicts that an unexpected return, when it is unexpectedly bad, causes more future volatility than an unexpected return when it is unexpectedly good. This phenomenon has been called the leverage effect since it has been explained in terms of the financial leverage of companies, see for example, Black (1976) and Christie (1982) who propose the following arguments. Increases in financial leverage increase both the required return and the risk of equity. The increase in the required rate of return leads to a decrease in the stock price, which gives the negative relationship between stock price movements and volatility.

The linear GARCH model is unable to capture the leverage effect, because, in this model the conditional variance is only linked to past conditional variances and squared innovations, hence the sign of return plays no role in determining volatility. This limitation of the standard ARCH or GARCH model is one of the primary motivations for the exponential or EGARCH model. The differences between the standard GARCH and the EGARCH model are that in the EGARCH model the effect of volatility is captured by two variables and is in logarithmic form. The first volatility term is the unexpected return relative to the contemporaneous conditional standard deviation. The second term is the absolute unexpected return relative to the conditional standard deviation. In this model a negative coefficient associated with the actual volatility implies that positive shocks generate less volatility than negative shocks. Nelson (1991) uses this framework to find evidence of a Leverage effect in the US market while Engle and Ng (1990) confirm the leverage effect using an EGARCH model for the Japanese stock market.

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7. The required return of a security can only go up if the stock price of the security goes down, ceteris paribus.
The existence of a leverage effect in the UK market has been investigated by Poon and Taylor (1992). Poon and Taylor confirm the leverage effect in the UK market, because they find that an unexpected return to the FT All share index predicts more future volatility if it unexpectedly bad than when it is unexpectedly good (although statistical significance is not always supported). Previous studies, have concentrated on whether there is evidence of a leverage effect in stock market indexes. But the issue of whether there is a leverage effect for capitalisation based portfolios, or whether the magnitude of the leverage effect is associated with firm size has not previously been addressed. This section will be concerned with whether there is a leverage effect, for portfolios of large and small firms. Also, of interest in this study, is whether the magnitude of the leverage effect is dependant upon firm size.

In order to test for a leverage effect it is necessary to isolate the impact that positive and negative unexpected returns have on the conditional variance. Isolating the impact of the sign of the shock from the pure volatility can be achieved by the following re-specification of the conditional variance model which is similar to a model by Taylor (1986).

\[
h_{i,t} = m_i + b_i | \varepsilon_{i,t-1} | + c_i h_{i,t-1} + d_i \varepsilon_{i,t-1}
\]  

(8.7)

where, \( \varepsilon_{i,t} \mid \phi_{i-1} \sim N(0, h_{i,i}) \), \( R_{i,t} \) is the return on portfolio \( i \) in week \( t \), and \( \phi_{i-1} \) is the set of all available information at time \( t - 1 \).

In this specification, the past squared volatility shock in the conditional variance is replaced by the absolute value of the shock. The absolute value of a shock will capture the effect that an unexpected return will have on the conditional variance. In addition to this, the actual value of the shock is also included in equation (8.7) which allows us to isolate the additional impact that the sign of the shock will have on the conditional variance.

If the coefficients \( b_i \) are positive and \( d_i \) is negative, then the impact of a positive shock will be dampened, while the impact of a negative shock will be enhanced. This implies the existence of a leverage effect. However, if the coefficients \( b_i \) and \( d_i \) in equation (8.7) are both positive, then there would be the reverse of a leverage effect because the coefficients would suggest that a positive shock would create more volatility that a negative shock. This specification has been suggested previously by Taylor (1986, p.78). Such a model can capture the autocorrelation structure of the absolute residuals shown in Table 8.2. But, the mathematical properties of this model are less well known than for the symmetric GARCH model. However, because this model does not explicitly model the autocorrelation in squared residuals, it will not capture the leptokurtosis in the returns distribution.  

Table 8.6, contains maximum likelihood estimates of the coefficients from the estimation of the model which is represented by equations (8.3) and (8.7). The specification of the conditional mean includes dummy variables for the turn of the year and the stock market crash, the lagged returns of other portfolios, and the GARCH-M term, in the underlying ARMA(1,1) structure. As expected, the properties of the normalised residuals from this model suggest that the specification is relatively poor. Nevertheless, the magnitudes, signs and significance of the estimated coefficients that feature in both the symmetric and asymmetric ARMA-GARCH-M specifications shown in Tables 8.5 and 8.6 respectively, are remarkably similar. Furthermore, the estimated coefficients in the asymmetric specification are generally statistically significant and contain interesting results regarding the leverage effect.

The estimated values of the coefficient \( c_i \) are 0.286 and 0.637, respectively, for the small and large firm portfolios. These figures confirm the feature that shocks to large firm portfolios are more persistent than shocks to small firm portfolios. The values of all of the estimated \( b_i \) coefficients (on the absolute shock) are positive, whereas the values of all of the estimated \( d_i \) coefficients (on the actual shock) are negative. This means that a positive shock will have a smaller impact on the volatility, than a negative shock. This establishes the existence of a strong leverage effect in the UK stock market. The difference between the \( b_i \) and \( d_i \) is greatest for the large firm portfolio, which means that the leverage effect will be more pronounced for large firms than for small firms. Collectively, the coefficients indicate that the leverage effect diminishes with firm size.

### 8.5.1 Spillover Volatility and the Leverage Effect

In this section a spillover version of the asymmetric GARCH model is tested. The aim of the spillover leverage tests is to allow us to gauge whether a shock to another portfolio predicts more future volatility in another portfolio if that shock is a negative one. The spillover leverage effect has not previously been tested on any data set. Specifically, this study is concerned with testing whether there is a leverage spillover effect which is associated with firm size.

Spill-over volatility shocks can be decomposed into the magnitude and the sign of the shock, by including in equation (8.7) both the absolute and actual value of the volatility shock of other portfolios. Such a transformation is provided by equation (8.8) below

\[
h_{i,t} = m_i + b_i \left| e_{i,t-1} \right| + c_i h_{i,t-1} + d_i e_{i,t-1} + k_{i,j} |e_{j,t-1}| + l_{i,j} e_{j,t-1} \quad j \neq i
\]

where, \( |e_{j,t-1}| \) is the one period lagged absolute unexpected return from portfolio \( j \), and \( e_{j,t-1} \) is the one period lagged unexpected return from portfolio \( j \).
Table 8.7 reports the maximum likelihood estimates of the asymmetric volatility spill-over coefficients. Consistent with the spill-over model for the symmetric GARCH model, the past shocks to the volatility of large firms tend to influence the current volatility of relatively small firms, but there is little statistically significant effect in the reverse direction. Furthermore, in the cases where there is a significant spill-over shock, there is also a distinct leverage effect. That is, not only is the past volatility shock to large firms important for the volatility of small firms, but impact of the shock will be much greater if it is negative.

To illustrate this more clearly we can look more closely at Table 8.7. If we look at the $k_i$ coefficients then we can see that typically below the diagonal the coefficients are insignificant. This means that typically a volatility shock experienced by small firms does not predict future volatility in large firms. Above the diagonal however, the magnitude and sign of the coefficients indicate that there is a relationship between the volatility of large firms and the future volatility of small firms. Looking now at the $l_i$ coefficient, a pattern emerges here which also appears to be associated with firm size. Typically, the coefficients below the diagonal are positive, but since the corresponding $k_i$ coefficients were insignificant no evidence of a leverage spillover effect is provided. This means that there is not a spillover leverage effect coming from small firms and spilling over into large firms. In contrast, if we focus on the $l_i$ coefficients above the diagonal then we can see that typically the coefficients are statistically significant and negative, indicating that a spillover volatility effect is being dampened if it is a positive shock.

One possible explanation for the results presented in this chapter is that the large companies are displaying signs of short-termism pressures. In the UK a large proportion of shares are traded by pension funds and other large institutional investors. These investors invest for short term gain, hoping to make a capital gain on stocks within the portfolio. On average pension funds tend to turn over their portfolio every 6 mths. The consequence of this is that large portfolio holders will only be willing to hold a stock if it is expected to perform well over the following months. If a stock performs badly, this may encourage portfolio holders to sell their holdings of a badly performing stock further depressing the price. Such behaviour will cause a leverage effect of the nature described in this chapter. A stock that provides an unexpectedly bad return may well experience further volatility in the future as portfolio managers reduce their holdings of such a stock, depressing the share price further.

It is not surprising that the leverage effect is more pronounced for the large firms if we consider the composition of large institutional portfolios. Institutional investors tend to trade only large value stocks and ignore small firm stocks. Proportionately, small investors tend to hold more small firm stocks. Small investors behave very differently to large institutional portfolio holders. Small investors tend to hold stocks for long term gains rather than for short term capital appreciation. This means that if a stock a small investors holds falls in price it is
unlikely to trigger a desire to rebalance the portfolio. Large transaction costs payable on small trades would suggest that even a substantial fall in the price of a stock would not encourage individual investors to sell. For small stocks there would be less of a tendency for a price fall to be followed by further price declines. As a result, negative volatility in one period would generate less volatility in future periods because small investors react more inertly to the price fall than do large investors.

8.6 Summary and Conclusions

In this chapter two conditional variance models have been tested to examine the influence that firm size might have on the relationship between volatility and return. The tests which have been performed in this chapter have sought to investigate a number of issues. Such as whether returns are time varying and related to past return volatility, whether volatility shocks spillover from one portfolio to another and whether there is any evidence of a leverage effect in the UK stock market for capitalisation based portfolios.

In the initial specification tested, which is an ARMA(1,1)-GARCH(1,1)-M model, it is found that the return conditional volatilities for portfolios of relatively large firms are more persistent than those of the returns on portfolios of relatively small firms. However, a shock to the returns of a small firm portfolio has a much larger impact on the volatility of a portfolio than is the case for large firms. This confirms the same pattern of results which was discovered when the conditional CAPM model was tested in the previous chapter. Moreover, shocks to large firms appear to spillover to small firms because volatility shocks in large firms appear to predict future volatility in small firms, although, the reverse is not the case. Volatility shocks in small firms do not appear to spillover into large firms, a shock to small firms does not typically predict volatility in large firms.

In the asymmetric specification of the conditional volatility, for the returns on UK stock portfolios, the model picks up a distinct leverage effect. This means that a negative unexpected return has a much greater impact on volatility than a positive unexpected return. Furthermore, this leverage effect is more pronounced for relatively large firm portfolios. The spill-over of volatility shocks is examined within this specification. Again, there are significant spill-overs from large firm portfolios to small portfolios. In these cases, it is appears that there is a distinct cross-leverage effect. A negative volatility shock will have a much greater impact than a positive volatility shock, when the shock spills over from relatively large to relatively small firm portfolios.

This chapter has therefore identified a number of important differences in the conditional variances of large and small firms. Further research can be directed at investigating why the return volatility of small firm stocks appears to react to past return volatility experienced by
large firms. It is possible that return surprises in large firms is news to investors in small firms. Perhaps the small firm investors are less well informed than investors in large firms so that volatility in large firms provides news about the behaviour of small firms. This needs to be ascertained empirically.

Further research also needs to discover whether the institutional environment in which shares are held encourages the short-termism motives which were described earlier. It is important if this argument has any validity for it to be established empirically whether small investors do have different trading patterns to large firm investors and the impact this has on the price performance of equity portfolios.
Table 8.1
Sample Portfolio Debt/Equity Ratios 1976-1991

where, $\bar{u}$ refers to the mean Debt/Equity ratios either for the full sample or for one of the three years 1976, 1985 or 1991. The var refers to the variance of the Debt/Equity ratio over the full sample period or for one of the three years 1976, 1985 or 1991.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>$\bar{u}$</th>
<th>var</th>
<th>$\bar{u}76$</th>
<th>var76</th>
<th>$\bar{u}85$</th>
<th>var85</th>
<th>$\bar{u}91$</th>
<th>var91</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.387</td>
<td>0.027</td>
<td>0.475</td>
<td>0.053</td>
<td>0.375</td>
<td>0.051</td>
<td>0.205</td>
<td>0.41</td>
</tr>
<tr>
<td>2</td>
<td>0.314</td>
<td>0.061</td>
<td>0.279</td>
<td>0.029</td>
<td>0.315</td>
<td>0.035</td>
<td>0.544</td>
<td>0.143</td>
</tr>
<tr>
<td>3</td>
<td>0.315</td>
<td>0.041</td>
<td>0.243</td>
<td>0.041</td>
<td>0.303</td>
<td>0.052</td>
<td>0.145</td>
<td>0.080</td>
</tr>
<tr>
<td>4</td>
<td>0.237</td>
<td>0.127</td>
<td>0.341</td>
<td>0.102</td>
<td>0.369</td>
<td>0.085</td>
<td>0.172</td>
<td>0.163</td>
</tr>
</tbody>
</table>
Table 8.6
Asymmetric Conditional Variance Models for Portfolio Returns

This table contains the estimated coefficients from the model

\[ R_{t,i} = \mu + \phi R_{t-1,i} + \beta h_{t-1,i} + \sum_{j=1}^{4} \gamma_{ij} R_{t-j,i} + \delta_{ij} D_{t,i} + \delta_{ij} D_{2,i} + \epsilon_{t,i} - \theta_{ij} \epsilon_{t-1,i} \quad i, j = 1, 2, 3, 4 \]

where \( \epsilon_{t,i} \sim N(0, h_{t,i}) \)

\[ h_{t,i} = m_i + b_i |\epsilon_{t,i-1}| + c_i h_{t,i-1} + d_i \epsilon_{t,i-1} \]

where, \( R_{t,i} \) are the weekly returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. All the estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t-statistics. The skewness and kurtosis figures are for the normalised residuals of the model. The Q are the Ljung-Box statistics testing the hypothesis that all autocorrelations in the normalised residuals up to lag 6 are jointly zero. Their p-value is given in brackets, below each coefficient. \( D_1 = 1 \) For each week during September and October of 1987, and \( D_1 = 0 \) for all other weeks. \( D_2 = 1 \) For the first two weeks in January and the last two weeks in December, and \( D_3 = 0 \) for all other weeks.

| Portfolio | Log likelihood | \( \hat{\mu} \times 10^3 \) | \( \hat{\phi} \) | \( \hat{\beta} \) | \( \hat{\gamma}_1 \) | \( \hat{\gamma}_2 \) | \( \hat{\gamma}_3 \) | \( \hat{\delta}_1 \times 10^3 \) | \( \hat{\delta}_2 \times 10^3 \) | \( \hat{n} \times 10^3 \) | \( \hat{\sigma} \times 10^3 \) | \( \hat{d} \times 10^3 \) | Skew. | Kurt. | Q |
|-----------|----------------|-----------------|----------------|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-------|
| 1         | 3252.693       | -1.281          | 0.790          | 0.670           | 10.269         | 0.081          | -0.009          | 0.048           | -0.040          | 0.243           | 0.482           | 0.675           | 0.286          | -0.196 | -1.156 | 9.461 | 18.341 |
|           | (-2.693)       | (17.936)        | (13.252)       | (3.102)         | (-3.181)       | (0.261)        | (1.583)         | (-0.556)        | (3.587)         | (5.913)         | (7.321)         | (3.939)         | (-3.299)       | [0.005] |
| 2         | 3150.812       | -0.814          | 0.584          | 0.565           | 7.711          | 0.089          | 0.007           | 0.141           | -1.267          | 2.548           | 1.962           | 0.605           | 0.594          | 0.027  | 0.096  | 3.295 | 8.681  |
|           | (-1.357)       | (8.672)         | (8.628)        | (2.461)         | (2.281)        | (0.169)        | (4.093)         | (-5.717)        | (2.659)         | (2.805)         | (6.875)         | (9.587)         | (0.453)       | [0.192] |
| 3         | 2891.316       | -1.989          | 0.505          | 0.486           | 7.939          | 0.071          | -0.119          | 0.243           | -1.419          | 0.538           | 0.026           | 0.972           | 0.570          | -0.204 | -0.485 | 2.719 | 13.742 |
|           | (-2.347)       | (6.569)         | (6.349)        | (3.469)         | (1.29)         | (-2.216)       | (4.893)         | (-4.903)        | (4.024)         | (1.715)         | (6.015)         | (8.802)         | (-2.561)      | [0.033] |
| 4         | 2762.706       | -1.157          | 0.793          | 0.634           | 3.611          | -0.0790        | -0.183          | 0.025           | -0.986          | 0.411           | 0.063           | 0.733           | 0.637          | -0.439 | -1.435 | 11.131 | 21.259 |
|           | (-1.614)       | (7.824)         | (6.258)        | (1.895)         | (-1.456)       | (-3.28)        | (0.377)         | (-3.733)        | (2.544)         | (2.985)         | (4.777)         | (9.911)         | (-5.235)      | [0.0020] |
Table 8.2
Summary Statistics of the Residuals from an ARMA(1,1) Model of Portfolio Returns

This table contains summary statistics for the residuals, squared residuals, and absolute residuals from the model in which portfolio returns follow a stationary ARMA(1,1) process. The model is

\[ R_{it} = \mu_i + \phi R_{i,t-1} + \epsilon_i - \theta \epsilon_{i,t-1} \quad i = 1, 2, 3, 4 \]

and \( R_{it} \) are the returns for four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. The variable \( \bar{x} \) are the sample means for variable \( x \). The \( \hat{\rho}_i \) are the estimated autocorrelation coefficients at lag \( i \). Under the hypothesis that these coefficients are zero, their standard errors are approximately 0.035. The \( Q \) are the Ljung-Box statistics testing the hypothesis that all autocorrelations up to lag 6 are jointly zero. Their p-value is given in brackets. The figures in parentheses below the sample means and the estimated coefficients of the ARMA(1,1) model for the portfolio returns are t-statistics.

<table>
<thead>
<tr>
<th>Variable (x)</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
<th>( \hat{\rho}_3 )</th>
<th>( \hat{\rho}_4 )</th>
<th>( \hat{\rho}_5 )</th>
<th>( \hat{\rho}_6 )</th>
<th>( Q \times 10^3 )</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>( \hat{\phi} )</th>
<th>( \hat{\theta} )</th>
<th>( \mathfrak{F} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_i )</td>
<td>-0.002</td>
<td>-0.013</td>
<td>-0.066</td>
<td>-0.022</td>
<td>0.062</td>
<td>0.040</td>
<td>8.745</td>
<td>1.341</td>
<td>-0.757</td>
<td>8.234</td>
<td>0.696</td>
<td>0.339</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>-0.002</td>
<td>-0.043</td>
<td>-0.031</td>
<td>0.046</td>
<td>-0.019</td>
<td>0.024</td>
<td>5.040</td>
<td>1.633</td>
<td>-0.468</td>
<td>5.781</td>
<td>0.648</td>
<td>0.295</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>-0.008</td>
<td>0.001</td>
<td>-0.062</td>
<td>0.076</td>
<td>-0.063</td>
<td>0.052</td>
<td>13.772</td>
<td>1.826</td>
<td>-0.524</td>
<td>4.671</td>
<td>0.565</td>
<td>0.282</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>-0.013</td>
<td>0.042</td>
<td>-0.072</td>
<td>0.058</td>
<td>-0.047</td>
<td>0.019</td>
<td>10.995</td>
<td>1.334</td>
<td>-0.828</td>
<td>5.941</td>
<td>0.507</td>
<td>0.359</td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>0.396</td>
<td>0.151</td>
<td>0.158</td>
<td>0.201</td>
<td>0.039</td>
<td>0.039</td>
<td>207.566</td>
<td>0.184</td>
<td>0.000</td>
<td>2.861</td>
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<td></td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>0.343</td>
<td>0.083</td>
<td>0.177</td>
<td>0.164</td>
<td>0.104</td>
<td>0.039</td>
<td>163.083</td>
<td>9.244</td>
<td>0.000</td>
<td>0.582</td>
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</tr>
<tr>
<td>( \epsilon_i )</td>
<td>0.339</td>
<td>0.068</td>
<td>0.093</td>
<td>0.127</td>
<td>0.030</td>
<td>0.017</td>
<td>121.365</td>
<td>0.442</td>
<td>0.000</td>
<td>11.407</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \epsilon_i )</td>
<td>0.368</td>
<td>0.056</td>
<td>0.042</td>
<td>0.070</td>
<td>0.034</td>
<td>0.059</td>
<td>125.161</td>
<td>0.551</td>
<td>0.000</td>
<td>10.407</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lvert \epsilon_i \rvert )</td>
<td>0.288</td>
<td>0.207</td>
<td>0.138</td>
<td>0.121</td>
<td>0.086</td>
<td>0.065</td>
<td>143.005</td>
<td>9.594</td>
<td>0.000</td>
<td>28.809</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lvert \epsilon_i \rvert )</td>
<td>0.260</td>
<td>0.152</td>
<td>0.138</td>
<td>0.165</td>
<td>0.137</td>
<td>0.043</td>
<td>131.984</td>
<td>11.055</td>
<td>0.000</td>
<td>28.718</td>
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<td></td>
</tr>
<tr>
<td>( \lvert \epsilon_i \rvert )</td>
<td>0.201</td>
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<td>0.090</td>
<td>0.144</td>
<td>0.067</td>
<td>0.053</td>
<td>86.598</td>
<td>15.058</td>
<td>0.000</td>
<td>29.587</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \lvert \epsilon_i \rvert )</td>
<td>0.167</td>
<td>0.109</td>
<td>0.069</td>
<td>0.142</td>
<td>0.077</td>
<td>0.105</td>
<td>68.522</td>
<td>17.239</td>
<td>0.000</td>
<td>31.206</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

216
This table contains the weekly estimated cross-serial correlations, for lags $\tau = 0, 1, 2, 3$, among the returns of the four equally-weighted size-ranked portfolios of shares on the UK stock exchange, January 1976 - December 1991. Under the hypothesis that these cross-serial correlation coefficients are zero, their standard errors are approximately 0.035.

### Table 8.3a

<table>
<thead>
<tr>
<th>Variable</th>
<th>$R_{1,t}$</th>
<th>$R_{2,t}$</th>
<th>$R_{3,t}$</th>
<th>$R_{4,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{i,t}$</td>
<td>1.000</td>
<td>0.743</td>
<td>0.694</td>
<td>0.599</td>
</tr>
<tr>
<td>$R_{i,t-1}$</td>
<td>0.743</td>
<td>1.000</td>
<td>0.798</td>
<td>0.702</td>
</tr>
<tr>
<td>$R_{i,t-2}$</td>
<td>0.694</td>
<td>0.798</td>
<td>1.000</td>
<td>0.879</td>
</tr>
<tr>
<td>$R_{i,t-3}$</td>
<td>0.599</td>
<td>0.702</td>
<td>0.879</td>
<td>1.000</td>
</tr>
</tbody>
</table>

This table contains the weekly estimated cross-serial correlations, for lags $\tau = 0, 1, 2, 3$, among the residuals from the model in which portfolio returns follow a stationary ARMA(1,1) process. The model is

$$R_{i,t} = H + \phi R_{i,t-1} + \theta \epsilon_{i,t-1} + \epsilon_{i,t},$$

and $R_{i,t}$ are the returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year. January 1976 - December 1991. Under the hypothesis that these cross-serial correlation coefficients are zero, their standard errors are approximately 0.035.

### Table 8.3b

<table>
<thead>
<tr>
<th>Variable</th>
<th>$\epsilon_{1,t}$</th>
<th>$\epsilon_{2,t}$</th>
<th>$\epsilon_{3,t}$</th>
<th>$\epsilon_{4,t}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_{i,t}$</td>
<td>1.000</td>
<td>0.658</td>
<td>0.774</td>
<td>0.884</td>
</tr>
<tr>
<td>$\epsilon_{i,t-1}$</td>
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<td>1.000</td>
<td>0.774</td>
<td>1.000</td>
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<tr>
<td>$\epsilon_{i,t-2}$</td>
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<td>0.774</td>
<td>1.000</td>
<td>0.884</td>
</tr>
<tr>
<td>$\epsilon_{i,t-3}$</td>
<td>0.884</td>
<td>0.884</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>
Table 8.4
ARMA(1,1)-GARCH(1,1)-M Models for Portfolio Returns

This table contains the estimated coefficients from the model

\[ R_{it} = \mu_i + \delta h_{it-1} + \beta h_{it} + \sum_{j=1}^{4} \gamma_j R_{t-j} + \delta h_{t-j} D_{t-j} + \epsilon_{t-j} \]  

where \( h_{it} = m_t + \beta \epsilon_{t-1}^2 + \epsilon_{it-1} \)

where, \( R_{it} \) are the weekly returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK Stock Exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. All the estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t-statistics. The skewness and kurtosis figures are for the normalised residuals of the model. The \( Q \) are the Ljung-Box statistics testing the hypothesis that all autocorrelations in the normalised residuals up to lag 6 are jointly zero. Their p-value is given in brackets, below each coefficient. \( D_1 = 1 \) for each week during September and October of 1987, and \( D_1 = 0 \) for all other weeks. \( D_2 = 1 \) for the first two weeks in January and the last two weeks in December, and \( D_2 = 0 \) for all other weeks.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Log Likelihood</th>
<th>( \hat{\mu} \times 10^5 )</th>
<th>( \hat{\phi} )</th>
<th>( \hat{\theta} )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\gamma}_1 )</th>
<th>( \hat{\gamma}_2 )</th>
<th>( \hat{\gamma}_3 )</th>
<th>( \hat{\gamma}_4 )</th>
<th>( \hat{\delta}_1 \times 10^3 )</th>
<th>( \hat{\delta}_2 \times 10^3 )</th>
<th>( \hat{\mu} \times 10^4 )</th>
<th>( \hat{\epsilon} )</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3271.424</td>
<td>-0.615</td>
<td>0.725</td>
<td>0.650</td>
<td>6.513</td>
<td>0.071</td>
<td>0.011</td>
<td>0.073</td>
<td>-0.683</td>
<td>0.199</td>
<td>0.614</td>
<td>0.243</td>
<td>0.356</td>
<td>-0.793</td>
<td>6.260</td>
<td>15.647</td>
</tr>
<tr>
<td></td>
<td>(1.712)</td>
<td>(13.764)</td>
<td>(10.912)</td>
<td>(2.777)</td>
<td>(2.155)</td>
<td>(0.318)</td>
<td>(2.551)</td>
<td>(-4.338)</td>
<td>(2.548)</td>
<td>(6.342)</td>
<td>(5.060)</td>
<td>(4.202)</td>
<td>[0.016]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3153.688</td>
<td>-0.614</td>
<td>0.564</td>
<td>0.547</td>
<td>6.244</td>
<td>0.110</td>
<td>-0.019</td>
<td>0.167</td>
<td>-1.143</td>
<td>0.264</td>
<td>0.517</td>
<td>0.249</td>
<td>0.512</td>
<td>-0.209</td>
<td>3.370</td>
<td>6.520</td>
</tr>
<tr>
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<td>(1.208)</td>
<td>(7.498)</td>
<td>(6.999)</td>
<td>(2.436)</td>
<td>(2.761)</td>
<td>(-0.449)</td>
<td>(4.674)</td>
<td>(-5.538)</td>
<td>(2.644)</td>
<td>(4.160)</td>
<td>(5.265)</td>
<td>(5.855)</td>
<td>[0.368]</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2886.138</td>
<td>-1.175</td>
<td>0.536</td>
<td>0.566</td>
<td>5.303</td>
<td>0.050</td>
<td>-0.091</td>
<td>0.271</td>
<td>0.000</td>
<td>0.515</td>
<td>0.772</td>
<td>0.215</td>
<td>0.597</td>
<td>-0.448</td>
<td>2.533</td>
<td>13.078</td>
</tr>
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<td></td>
<td>(1.599)</td>
<td>(6.646)</td>
<td>(7.241)</td>
<td>(2.530)</td>
<td>(0.874)</td>
<td>(1.666)</td>
<td>(5.427)</td>
<td>(-4.574)</td>
<td>(3.658)</td>
<td>(4.644)</td>
<td>(4.399)</td>
<td>(8.272)</td>
<td>[0.042]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2776.849</td>
<td>-0.922</td>
<td>0.851</td>
<td>0.694</td>
<td>3.199</td>
<td>0.041</td>
<td>-0.013</td>
<td>-0.038</td>
<td>-1.494</td>
<td>0.239</td>
<td>0.726</td>
<td>0.121</td>
<td>0.730</td>
<td>-0.548</td>
<td>2.668</td>
<td>10.954</td>
</tr>
<tr>
<td></td>
<td>(1.042)</td>
<td>(7.467)</td>
<td>(6.399)</td>
<td>(1.674)</td>
<td>(0.067)</td>
<td>(-2.209)</td>
<td>(-0.053)</td>
<td>(-3.692)</td>
<td>(1.305)</td>
<td>(3.748)</td>
<td>(4.021)</td>
<td>(12.373)</td>
<td>[0.090]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 8.5
ARMA(1,1)-GARCH(1,1)-M Models for Portfolio Returns with Pairwise Volatility Spillovers

This table contains the estimates of the pairwise volatility spillover effects using the model

\[ R_{i,t} = \mu_t + \phi R_{i,t-1} + \beta_t h_{i,t} + \sum_{j=1}^{4} \gamma_{i,j} R_{j,t-1} + \delta_{i,1} D_{1,t} + \delta_{i,2} D_{2,t} + \varepsilon_{i,t} - \theta_t \varepsilon_{i,t-1} \]

where \( R_{i,t} \) are the weekly returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. The coefficients \( \delta_{i,j} \) measure the impact of past volatility surprises to portfolio \( j \), \( j \neq i \), on the conditional variance of portfolio \( i \). The table also reports estimates of \( \beta_{i,j} \), \( b_{i,j} \), and \( c_{i,j} \), which are the key volatility coefficients in the model, when there are volatility spillovers from portfolio \( j \) to portfolio \( i \). All the estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t-statistics. \( D_1 = 1 \) for each week during September and October of 1987, and \( D_1 = 0 \) for all other weeks. \( D_2 = 1 \) for the first two weeks in January and the last two weeks in December, and \( D_2 = 0 \) for all other weeks.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>( \hat{\beta}_1 )</th>
<th>( \hat{\beta}_2 )</th>
<th>( \hat{\beta}_3 )</th>
<th>( \hat{\beta}_4 )</th>
<th>( \hat{\beta}_5 )</th>
<th>( \hat{\beta}_6 )</th>
<th>( \hat{\beta}_7 )</th>
<th>( \hat{\beta}_8 )</th>
<th>( \hat{\epsilon}_1 )</th>
<th>( \hat{\epsilon}_2 )</th>
<th>( \hat{\epsilon}_3 )</th>
<th>( \hat{\epsilon}_4 )</th>
<th>( \hat{k}_1 )</th>
<th>( \hat{k}_2 )</th>
<th>( \hat{k}_3 )</th>
<th>( \hat{k}_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.390</td>
<td>6.069</td>
<td>6.301</td>
<td>0.221</td>
<td>0.205</td>
<td>0.219</td>
<td>0.332</td>
<td>0.293</td>
<td>0.261</td>
<td>0.031</td>
<td>0.043</td>
<td>0.034</td>
<td>1.299</td>
<td>2.609</td>
<td>2.550</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.651)</td>
<td>(2.855)</td>
<td>(2.992)</td>
<td>(3.914)</td>
<td>(3.766)</td>
<td>(3.718)</td>
<td>(3.704)</td>
<td>(3.302)</td>
<td>(2.796)</td>
<td>(1.979)</td>
<td>(2.840)</td>
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<tr>
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<td>6.072</td>
<td>6.286</td>
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<td>0.310</td>
<td>0.169</td>
<td>0.539</td>
<td>0.575</td>
<td>0.538</td>
<td>0.062</td>
<td>0.069</td>
<td>0.041</td>
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</tr>
<tr>
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<td>5.259</td>
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<td>0.214</td>
<td>0.213</td>
<td>0.163</td>
<td>0.592</td>
<td>0.590</td>
<td>0.011</td>
<td>0.013</td>
<td>0.048</td>
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</tr>
<tr>
<td>4</td>
<td>3.126</td>
<td>3.322</td>
<td>3.358</td>
<td>0.114</td>
<td>0.119</td>
<td>0.023</td>
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<td>0.699</td>
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<td>0.020</td>
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<td>0.152</td>
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<td>(3.551)</td>
<td>(0.789)</td>
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<td>(10.478)</td>
<td>(10.108)</td>
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<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Table 8.6
Asymmetric Conditional Variance Models for Portfolio Returns

This table contains the estimated coefficients from the model

\[ R_{t,i} = \mu_i + \phi_i R_{t-1} + \beta_i h_{t,i} + \sum_{j=1}^{5} \gamma_{t,j} R_{t-j} + \delta_{t,1} D_{1,t} + \delta_{t,2} D_{2,t} + \epsilon_{t,i} - \Theta_i \epsilon_{t,i-1} \quad i,j = 1, 2, 3, 4 \]

where \( \epsilon_{t,i} | \Omega_{t-1} \sim N(0, h_{t,i}) \)

\[ h_{t,i} = m_i + b_i \mid \epsilon_{t,i-1} \mid + c \mid h_{t,i-1} + d_i \epsilon_{t,i-1} \]

where, \( R_{t,i} \) are the weekly returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. All the estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t-statistics. The skewness and kurtosis figures are for the normalised residuals of the model. The \( Q \) are the Ljung-Box statistics testing the hypothesis that all autocorrelations in the normalised residuals up to lag 6 are jointly zero. Their p-value is given in brackets, below each coefficient. \( D_1 = 1 \) for each week during September and October of 1987, and \( D_2 = 0 \) for all other weeks. \( D_2 = 1 \) for the first two weeks in January and the last two weeks in December, and \( D_2 = 0 \) for all other weeks.

<table>
<thead>
<tr>
<th>Log</th>
<th>Portfolio likelihood</th>
<th>( \hat{\mu} \times 10^3 )</th>
<th>( \hat{\phi} )</th>
<th>( \hat{\theta} )</th>
<th>( \hat{\gamma}_1 )</th>
<th>( \hat{\gamma}_2 )</th>
<th>( \hat{\lambda} )</th>
<th>( \hat{\delta}_1 \times 10^3 )</th>
<th>( \hat{\delta}_2 \times 10^3 )</th>
<th>( \hat{\Sigma} \times 10^6 )</th>
<th>( \hat{\beta} \times 10^3 )</th>
<th>( \epsilon )</th>
<th>( \delta \times 10^3 )</th>
<th>Skew.</th>
<th>Kurt.</th>
<th>( Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3252.693</td>
<td>-1.281</td>
<td>0.790</td>
<td>0.670</td>
<td>10.269</td>
<td>0.081</td>
<td>-0.009</td>
<td>0.048</td>
<td>0.040</td>
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<td>-1.156</td>
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</tr>
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<td>(2.693)</td>
<td>(17.396)</td>
<td>(13.252)</td>
<td>(3.102)</td>
<td>(3.181)</td>
<td>(0.261)</td>
<td>(1.583)</td>
<td>(0.556)</td>
<td>(3.587)</td>
<td>(5.913)</td>
<td>(7.321)</td>
<td>(3.939)</td>
<td>(3.299)</td>
<td>(0.005)</td>
<td>(0.192)</td>
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</tr>
<tr>
<td>2</td>
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<td>0.584</td>
<td>0.565</td>
<td>7.711</td>
<td>0.089</td>
<td>0.007</td>
<td>0.141</td>
<td>-1.267</td>
<td>2.548</td>
<td>1.962</td>
<td>0.605</td>
<td>0.594</td>
<td>0.027</td>
<td>-0.096</td>
<td>3.295</td>
</tr>
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<td>(1.357)</td>
<td>(6.272)</td>
<td>(6.268)</td>
<td>(2.461)</td>
<td>(2.281)</td>
<td>(0.169)</td>
<td>(4.093)</td>
<td>(5.717)</td>
<td>(2.659)</td>
<td>(2.805)</td>
<td>(6.875)</td>
<td>(9.587)</td>
<td>(0.453)</td>
<td>(0.192)</td>
<td>(0.033)</td>
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</tr>
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<td>-0.986</td>
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<td>11.131</td>
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<td>(-3.733)</td>
<td>(2.544)</td>
<td>(2.985)</td>
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<td>(9.911)</td>
<td>(-5.235)</td>
<td>(0.0020)</td>
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Table 8.7
Asymmetric Conditional Variance Models for Portfolio Returns with Pairwise Volatility Spillovers

This table contains the estimates of the pairwise volatility spillover effects using the model

\[
R_{it} = \mu_i + \phi_i R_{it-1} + \beta_i h_{i,t} + \sum_{j=1}^{4} \gamma_{ij} R_{jt-1} + \delta_{ij} D_{it} + \epsilon_{it} - \theta_{ij} e_{jt-1}, \quad i, j = 1, 2, 3, 4 \quad \text{where } e_{it} | \Omega_{it-1} \sim N(0, h_{it})
\]

\[
h_{it} = m_i + b_i | \epsilon_{it-1} | + c_i h_{it-1} + d_i e_{it-1} + k_{ij} | \epsilon_{jt-1} | + l_{ij} e_{jt-1}, \quad j \neq i
\]

where, \(R_{it}\) are the weekly returns on four equally-weighted portfolios of the 50 smallest, 50 small-intermediate, 50 large-intermediate, and the 50 largest shares on the UK stock exchange, formed by rankings of market value of equity outstanding at the beginning of January of each year, January 1976 - December 1991. The coefficients \(k_{ij}\) and \(l_{ij}\) measure the impact of the magnitude and direction, respectively, of past volatility surprises to portfolio \(j, j \neq i\), on the conditional variance of portfolio \(i\). All the estimated parameters are denoted by a caret, and the numbers in parentheses below the estimated coefficients are t-statistics. \(D_1 = 1\) For each week during September and October of 1987, and \(D_2 = 0\) for all other weeks. \(D_2 = 1\) For the first two weeks in January and the last two weeks in December, and \(D_2 = 0\) for all other weeks.

<table>
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<th>Portfolio</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
<th>(\beta_3)</th>
<th>(\beta_4)</th>
<th>(k_1 \times 10^1)</th>
<th>(k_2 \times 10^1)</th>
<th>(k_3 \times 10^1)</th>
<th>(k_4 \times 10^1)</th>
<th>(l_1 \times 10^1)</th>
<th>(l_2 \times 10^1)</th>
<th>(l_3 \times 10^1)</th>
<th>(l_4 \times 10^1)</th>
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<td>6.613</td>
<td>6.795</td>
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<td>(2.709)</td>
<td>(1.056)</td>
<td>(3.754)</td>
<td>(3.311)</td>
<td>(0.367)</td>
<td>(-2.021)</td>
<td>(-2.465)</td>
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<tr>
<td>2</td>
<td>9.362</td>
<td>8.273</td>
<td>7.141</td>
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<td>0.400</td>
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<td>-0.194</td>
<td>-0.132</td>
<td>-0.149</td>
<td>-0.211</td>
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<td></td>
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<td>(2.716)</td>
<td>(2.859)</td>
<td>(2.248)</td>
<td>(5.087)</td>
<td>(3.089)</td>
<td>(-2.877)</td>
<td>(-2.771)</td>
<td>(-2.439)</td>
<td>(-2.507)</td>
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<td>0.244</td>
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<td>-0.231</td>
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<td>0.722</td>
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<td></td>
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<td>(2.523)</td>
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<td>(3.779)</td>
<td>(4.006)</td>
<td>(5.203)</td>
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CHAPTER NINE
Summary and Conclusions

1 Summary and Implications

The chief objective of this study was to investigate the role that firm size might have in determining the empirical characteristics of UK stock returns. Whereas previous studies, which have considered both the US and the UK stock market, have focused primarily upon differences in the mean returns of large and small firms, this study has paid particular attention to noting differences in the higher moments of firms' returns. The main empirical findings are presented in Chapters 4 to 8, these chapters document a number of previously undiscovered differences between large and small firm returns.

This thesis began by introducing the concept of an efficient market. Under the restrictive assumptions that markets are perfect, and in the absence of market frictions, it was demonstrated that returns should be uncorrelated. This definition now seems rather simplistic, because, we know that market frictions such as non-synchronous trading and bid-ask bounce can introduce autocorrelation into stock returns. Furthermore, as Chapter 3 suggested, autocorrelated time varying expected returns, can also introduce predictability in stock returns. This was a particularly important issue to review because chapters 7 and 8 were concerned with the characteristics of time varying expected returns for UK small and large firms.

A survey of the existing literature, that has investigated whether stock market prices are predictable, was discussed in Chapter 3. Most of these studies indicated that the returns of small firms were much more predictable than the returns of large firms. Although, most of these studies investigated the US stock market, Chapters 4 and 5 confirm that the stock returns of small firms listed on the UK market are also more predictable than the returns of large firms.

Chapter 4 began by considering the autocorrelation patterns of portfolios and individual securities, which had been grouped according to firm size. These autocorrelations revealed that the portfolio returns of small firms were indeed much more predictable than was the case for the large firms. In contrast, the returns of individual companies, irrespective of firm size did not appear to be predictable from their return autocorrelations. This reflected the findings of previous studies for the US, which were discussed in Chapter 3. Chapter 4 also contained some co-integration analysis of the prices of a selection of UK individual securities. The results suggest that the individual security prices of small firms are more
predictable than for large firms, although, the nature of this predictability does not appear to be detectable in autocorrelation tests. This was the first time that these tests have been applied to tests of the small firm effect.

The autocorrelation patterns which were identified for the portfolios in Chapter 5 indicated that the returns of small firms contained predictable components which decayed rapidly. This implies that when forecasting future monthly returns additional weight should be given to the most recent weekly return of a month. This means that in autocorrelation tests, which give equal weights to each of the four weekly returns of the previous month, the extent to which future monthly returns are predictable will be understated. Empirical work undertaken in Chapter 5 reveals that as much as 15% of the variation in monthly portfolio stock returns can be explained by the previous periods monthly portfolio return for all but the portfolio containing the largest firms in the sample.

Since previous work in the US has associated the small firm premium with monthly return regularities, Chapter 6 considered whether there were persistent regularities in the portfolio returns of UK firms. While, regularities appeared to exist during the months of January and April, in no month did small firms persistently outperform large firms. Although, during April, the returns of large firms appear to persistently outperform the returns of small firms.

The January effect, in the UK stock market is particularly puzzling, unlike the January effect in the US it can not be linked to tax loss trading. This study has shown that the January effect in the UK appears to be influenced by the pricing of systematic risk. When the relationship between risk and return is tested during each month of the year separately it is found that for both large and small firms systematic risk appears to be priced in only a few months of the year, that is January, April and July. Furthermore, it is found that when systematic risk is priced, large firms always provide a higher risk premium than small firms. This partly explains why large firms outperform small firms during both January and April. The results presented in this chapter are important for two reasons. Firstly, the results indicate that the January and April anomaly are linked to an extraordinary relationship between risk and return during these months. Secondly, there are important month by month differences in the magnitude of the systematic premiums of large and small firms, a feature which had not previously been documented.

The existence of an unusual relationship between systematic risk and return was also supported in Chapter 7. In this chapter it was found that the conditional systematic risk premium for large and small firms appeared to be higher during the month of January.
Furthermore, it was found that during the month of January the market price of risk was considerably higher than during other months. This suggests that investors perceive January as inherently more risky than other months, consequently investors require higher returns during this month. Although, interestingly for large firms, the market price of risk appeared to be substantially higher than for small firms during this month and therefore explains why the risk premium was higher during January for large firms. This pattern had not previously been documented.

Chapter 7 and 8 are both concerned with investigating the conditional variances and covariances of large and small firms. Chapter 7 found support for the conditional version of the CAPM for both large and small firms. It was reported that for both large and small firms expected returns are conditional on the past behaviour of the market. Essentially, during periods of high market volatility investors require higher risk premiums to compensate them for the additional risk they face. This rejects the notion that the CAPM can be explained within a framework in which beta and therefore risk remains constant. If expected returns are time varying, these characteristics should be modelled in tests of asset pricing models and in tests of market efficiency.

Both chapter 7 and 8 note some interesting differences in the second moments of small and large firms. The effect of an unexpected return, that is, a volatility shock, appears to have more of an impact on the conditional variance of small firms than is the case for large firms. However, the effect of a volatility shock on large firms tends to last for much longer than is the case for small firms. This is the first time this pattern has been documented for UK portfolios.

Chapter 7 found that the conditional expected risk premium of portfolios was autocorrelated, which implies that realised returns would also be autocorrelated. Indeed, for large firms nearly all the autocorrelation in realised returns appears to be explained by a conditional version of CAPM, although, for smaller firms which are also characterised by thin trading it is impossible to disentangle how much of the autocorrelation is due to time variation and how much is due to thin trading.

The autocorrelations of the conditional expected risk premium reveal an interesting pattern. Typically, there is a positive relationship between the extent to which the conditional risk premium is autocorrelated and the size of firms within a portfolio. This is a particularly important finding because it explains why Chapter 5 found that the short-horizon returns of large firms were not mean reverting and why previous studies by Conrad and Kaul (1988) and Conrad, Kaul and Nimalendran (1991a) found that realised returns of large firms were
not well explained by a time varying model of expected returns. The expected returns of large firms are very highly autocorrelated, unlike the expected returns of small firms. This means that a change in the expected return of large firms takes a long time to decay, in which case the most recent weekly returns of large firms do not provide much additional information about the future weekly returns of large firms because the effect of a change in the expected return of large firms is so slowly decaying.

Chapter 3 demonstrated that the portfolio returns of small firms listed in US markets were cross serially correlated with the returns of large firms, but the reverse was not true. This pattern is confirmed for the portfolios investigated in this study. However, the primary aim of Chapter 8 was to explore whether volatility shocks are transmitted across portfolios. Essentially, this required investigating whether a volatility shock to one portfolio could be transmitted to another portfolio and vice versa and, whether there was a relationship associated with firm size. This chapter found that there are asymmetric volatility shocks operating in the UK stock market. The asymmetry exists because shocks to large firms predict future volatility in small firms but the reverse is not true. Shocks to small firms do not allow us to predict volatility in large firms.

Having established the existence of spillovers in the UK market Chapter 8 also contained an investigation into whether there is a leverage effect operating in the UK market. Although, previous studies have examined the leverage effect on market wide indexes, no study has explored whether the leverage effect is associated with firm size. This chapter finds that for all size-based portfolios there is strong evidence of a leverage effect. However, yet another asymmetry is identified in this chapter. An unexpectedly bad volatility shock predicts more future volatility for large firms than is the case for small firms. The conclusion of this work is that a stronger leverage effect exists for large firms than for small firms.

Chapter 8 concludes by investigating whether there are leverage spillover shocks in the UK market. This involves testing whether the magnitude of a spillover shock which is transmitted to another portfolio depends upon the sign of the shock. It is found that the sign of a volatility shock does influence the strength of the spillover. When a shock is experienced by large firms, the magnitude of the shock that is transmitted to smaller firms increases if the shock is a negative shock.

In conclusion this thesis set out to explore whether the empirical characteristics of UK large and small firm returns differed. This was an important area of research to investigate because unlike the US market, very little attention has been focused on the size effect in the UK. This study found that a large number of differences exist between large and small firms,
many of which have not previously been identified on any data set. This is important information to participants in both the primary and secondary markets in the UK, where huge resources are allocated into and between risky assets.
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