Solar thermoelectric system for small scale power generation

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Solar Thermoelectric System for Small Scale Power Generation

by

Siddig Adam Omer

Doctoral Thesis

Submitted in partial fulfillment of the requirement for the award of
Doctor of Philosophy
of Loughborough University

May, 1997

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Abstract

This thesis is concerned with the design and evaluation of a small scale solar-thermoelectric power generation system. The system is intended for electricity generation and thermal energy supply to small scale applications in developing countries of the sunny equatorial regions. Detailed design methodologies and evaluations of both the thermoelectric device and the solar energy collector, which are parts of the combined system, are presented.

In addition to experimental evaluations, three theoretical models are presented which allow the design and evaluation of both the thermoelectric module and the solar energy collector. One of the models (a unified thermoelectric device model) concerns the geometrical optimization and performance prediction of a thermoelectric module in power generation mode. The model is unified in the sense that it accounts for the effect of all the parameters that contribute to the performance of the thermoelectric module, a number of which are ignored by the available design models. The unified model is used for a comparative evaluation of five thermoelectric modules. One of these is commercially available and the others are assumed to have optimum geometry but with different design parameters (thermal and electrical contact layer properties). The model has been validated using data from an experimental investigation undertaken to evaluate the commercial thermoelectric module in power generation mode. Results showed that though the commercially available thermoelectric cooling devices can be used for electricity generation, it is appropriate to have modules optimized specifically for power generation, and to improve the contact layers of thermoelement accordingly. Attempts have also been made to produce and evaluate thermoelectric materials using a simple melt-quenching technique which produces materials with properties similar to those of the more expensive crystalline materials.
The remaining two models presented here have been developed for the design and evaluation of solar concentrating system designed specifically for providing a suitably high temperature gradient across the thermoelectric device.

A detailed design methodology has been given for a two stage solar concentrator comprising a primary parabolic trough concentrator (PTC) and a secondary compound parabolic concentrator (CPC), a combination which makes use of the compactness of the PTC and the maximum concentration efficiency of the CPC. Different techniques for performance evaluation have been used which cover the range of operational conditions that the collector system may experience from fully evacuated, in order to suppress heat losses, to a collector at atmospheric pressure. The former is evaluated using a specifically developed analytical model which can predict solar energy collector performance under particular operational conditions. The model has been validated using experimental data. The thermodynamically more complex unevacuated operation, is modeled numerically (i.e. using computational fluid dynamics). With this approach the temperature distribution and flow field inside the solar energy receiver can be directly calculated.

An experimental prototype has been constructed and tested, and the results have been used to validate the analytical model. Results from the evaluation show that the system perform well at tilt angles up to about 30 degrees, which is well suited to the latitudes of regions for which the system is intended. The design is also found to be tolerable to the incoming sun rays to an extend that permits tracking adjustment to be made once every 15 to 20 days.

An economic analysis has been completed which shows that, a solar thermoelectric generation system can be cost effective source of energy, when compared to the existing alternatives.
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CHAPTER ONE

1 Introduction

1.1 General

In addition to the common methods of solar energy conversion by photovoltaic cells and the conventional solar thermal processes, solar energy can also be converted to electricity by using thermoelectric devices. Thermoelectric technology which goes back 175 years, to the discovery of Seebeck effect, though it has not gained widespread use for terrestrial power generation mainly due to its low conversion efficiency (heat to electricity of typically 1% to 5%) [Freedman, 1966; Fuschill et al, 1965] has advantage of operation at elevated temperatures, which makes it attractive for applications with concentrated solar energy. By making use of concentrated solar energy, the power per unit of material, and hence the power per unit cost, can be increased significantly. It also offers a distinct advantage when used in a co-generation system, where a solar thermoelectric generation system simultaneously provides electrical power and useful heat; extracted from the thermoelectric device. This combination is expected to improve the overall energy conversion efficiency of the solar thermoelectric system and increase its cost effectiveness.

In view of the above, and to explore the potential of thermoelectric devices for solar energy conversion, a combination of solar energy concentration and thermoelectric conversion is investigated as an alternative renewable energy power generation technology for use in the remote sunny areas, where electricity supply is not readily available. In most of such areas, particularly in the developing countries, a self-sustaining electricity supply for lighting is a predominant demand. Light from non-electric sources is of poor quality, expensive and is a potential fire hazard. Electricity is also needed for radio/cassette players and television operation. In some cases, electricity may be used to drive refrigerators. With this combination, the required refrigeration cooling load can be provided by operating an absorption refrigeration system [Duffie and Beckman, 1974] using the waste heat from the thermoelectric device. Fig. (1.1) illustrates the solar energy utilization with this combination.
1.2 Thermoelectric Generators, Background

The fast growth which took place in the development of semiconductor materials during the last three to four decades has offered thermoelectric technology a distinctive application potential. Since its discovery until about the early 1950s, thermoelectricity was mainly applied to the measurement of temperature and energy fluxes by thermopiles. The most significant studies on thermoelectricity took place afterwards, following results of Ioffe [Ioffe, 1957] and his co-workers in the field of thermoelectric materials. Following this basic research on compound semiconductors for transistors and photo-conducting devices there has been a resurgence of interest in the application of thermoelectricity [Dissmukes, 1965].
A thermoelectric device is in essence a simple heat engine which converts heat energy directly into electricity or the reverse. Its operation is based on the three well known interrelated phenomena of Seebeck, Peltier and Thomson effects [Ioffe, 1957].

In its simplest form a thermoelectric generator is formed of a $p$- and $n$-type pellets connected electrically in series and thermally in parallel, as shown in Fig. (1.2). The hot junction temperature is maintained by a heat flux from a heat source while the cold junction temperature is maintained by a heat sink. The generator operates as an electron-gas Rankine cycle, where the energy level of the electrons is raised at the hot junction and lowered at the cold junction.

![Fig. 1.2 Schematic diagram of a thermoelectric element](image)

Due to the temperature difference, charge carriers with energy level near the Fermi-energy level at the hot end will be easily excited to a high energy level and can lower
their energies by diffusing to the cold end. Similarly those at the cold end can diffuse toward the hot end at a rate dependent on their energies. Thus, a net current of charge carriers will result. This flow will cause the charge carriers to pile up at one end of the element, usually the cold end, and thereby produces an electromotive force (emf).

Thermoelectric devices have a broad range of applications including power generation using the Seebeck effect, and cooling making use of the reverse phenomena known as the Peltier effect. As Peltier coolers, they have attracted more general commercial interest and have found a potential use in the electronics industry (for electro_optics and small volume space cooling) [Marlow and Burke, 1995]. This has led to an expanding market for thermoelectric devices, and now a wide variety of Peltier cooling modules are available off the shelf.

As generators, thermoelectric devices received significant development for modestly sized power generation during early 1960s, when specific requirements for autonomous sources of electrical power arose from the exploration of space and the expansion of remote unattended applications. Several units operated by fossil and isotope fuels were developed in Russia and the United States [Freedman, 1966]. Units operated by gas-burners (propane or methane) have been produced commercially, which provided a silent maintenance-free dc-power source for a wide range of applications including military and some civilian applications such as cathodic protection of pipelines, remote automatic data gathering and applications that require precise voltage (DC-DC converters and output power conditioning) [Hall, 1995]. Thermoelectricity has also showed a great promise for near and long term space missions. For example, "Pioneer-10" and "Pioneer-11" were powered by lead-telluride thermoelectric generators. Similarly, most of "SNAP" satellites were provided with thermoelectric generators based on telluride technology [Henry et al., 1990]. The current space mission power generating systems are based on the Radioisotope Thermoelectric Generators (RTG). The "Voyagers-I" and "Voyage-II" space crafts are using silicon-germanium based thermoelectric converters. Similarly, the "Galileo" spacecraft is using configuration of RTGs which provides a total of 570 watts of electricity [Henry et al., 1990]. Currently over 20 different models of thermoelectric
ranging from about 10 to 550 watts [McNauhton, 1995], most of which are operated by gaseous fuels such as propane (C₃H₈) or natural gas (CH₄), and few by liquid fuel (diesel).

However, application of solar energy thermoelectric generation was initially curtailed by concerns about its low conversion efficiency and high material cost. As a result very little research effort has been devoted to the use of thermoelectric devices for solar energy in general and terrestrial solar energy conversion in particular. The limited attempts which have been made were to explore the potential of solar energy thermoelectric conversion in exploratory space missions using flat plate solar thermoelectric generators. A system based on bismuth-telluride alloys was developed by NASA for near-earth orbit. An electrical power output of 32 watts per square meter and heat to electrical conversion efficiency as high as 4% at temperature difference of about 170 degrees has been reported [Fuschillo et al., 1966]. Most of the recent research activities on applications of thermoelectric power generation have been directed towards utilization of industrial waste heat [Min and Rowe, 1992; Matsuura et al., 1991].

A number of thermoelectric materials are available commercially [Goldsmid, 1965]. One of the earliest thermoelectric materials to be studied, was the compound lead-telluride which has found use in power generation, and is useful in the temperature range 500 - 800 K [Freedman, 1966]. Compounds based on bismuth-telluride are useful over the temperature range from room temperature to about 400 K [Goldsmid, 1965]. For temperature range above 800 K, the best materials are the germanium-silicon alloys [Dissmukes, 1965]. Both lead-telluride and germanium-silicon thermoelements are widely used for power generation, particularly auxiliary power supply in space satellites [Bennett, 1995], while bismuth-telluride thermoelements are mainly used for cooling purposes and low scale power generation.

Most of the available commercial thermoelectric devices are based on alloys made of bismuth, telluride, antimony and selenium optimized for operation in the temperature range 180 - 400 K. They are primarily developed for Peltier cooling, however, they are
also proposed for power generation [Burke, 1983]. A module which produces about one watt of electrical power when operating at a temperature difference of around 80 K, costs about $20, or $10 when ordered in large quantities [Min and Rowe, 1992].

1.3 Solar Energy Collectors in Perspective

There are a number of solar energy systems that can be used for the collection and concentration of solar energy. For low temperature applications, below 70 °C, solar ponds and flat plate collectors are generally used. Flat plate collectors are well developed and are widely used throughout the world, mainly for hot water provision. They have relatively high efficiency at a low operating temperature, however, their performance drops dramatically at high temperature due to the large absorbing plate surface that is exposed to the cold surrounding environment [Russell, 1967].

For temperatures above 70 °C, as is generally required to generate electricity, a system for concentrating solar radiation is essential. Three common concentrating systems are the parabolic-troughs, parabolic-dishes and central towers. Central towers use sun-tracking mirrors (heliostats) to concentrate solar radiation onto a receiver located on top of a high tower. They achieve temperatures ranging from 500 to 1500 °C [Alpert, 1993]. Studies suggest that a near-term 100 MWe central receiver plant could generate electricity for as low as 8 to 16 cents per KWh. As a result of further improvements in design and capacity, the levelised (annual) electricity cost for 200 MWe power plants may well reach less than 5 cents per KWh by 2010 [Pascal et al., 1992]. They are considered promising utility-scale solar thermal electric systems, in the capacity range above 10 megawatts.

Parabolic-dish systems use a tracking dish reflector to concentrate sunlight onto a receiver/engine mounted at the focal point of the dish. They concentrate the solar radiation on to a single absorbing point like, and achieve the highest concentration ratio and the highest annual solar energy collection rate, due to two axis tracking to keep the aperture normal to the sun rays [Groswani, 1987]. A system based on parabolic-dish concentrators, usually consists of a collector and receiver unit that can either
function independently or as part of a large system of dishes. In the latter approach, thermal energy from each dish is transported to a centralized power conversion system. The former uses a small heat engine (for example, a Stirling engine) coupled to a generator to produce electricity at each dish. Although, these modular dish-Stirling systems have the highest solar-thermal electric conversion efficiency, maintenance costs remain highly uncertain. In addition, the requirement for high accuracy two axis tracking adjustment is one of the major concerns of these concentrators when considered for small scale applications. A projected levelised energy cost for a remote power system of 5 to 10 kW is estimated to be between 15 to 33 cents per KWh [Pascal et al., 1992].

Parabolic-trough systems are the most fully developed of the solar-thermal technologies, and major installations for both process heat and electric power production exist. They operate at relatively low temperatures, in the range between 100 and 400 °C. Parabolic-troughs concentrators focus solar radiation along central long axis receiver, often a pipe enclosed in evacuated quartz envelope. Heat is collected by fluid flowing through the pipes to the thermal processing point, where steam is raised to be used for electric power generation. Research work on this system has matured considerably, and the technology has received wide scale application. Parabolic trough systems are available for use with costs that are comparable with non-solar alternatives for large-scale electrical power generation [Albert, 1993]. Due to the requirement for only single axis tracking, parabolic trough concentrators are also suitable for use in small scale power plants.

Photovoltaic solar cells on the other hand offer a potentially attractive means for the direct conversion of sunlight into electricity with high reliability and low maintenance as compared with solar-thermal systems. The major disadvantage is the cost of the electricity generated compared to other alternative technologies, which can only be reduced by using concentrated solar energy. However reduction in the conversion efficiency accompanying the temperature rise due to concentration of sunlight is one of the limiting factors. Although development of the technology is underway, cells using
concentrated solar energy have not reached the stage where they can be usefully employed commercially [Strebkov et al., 1993; Mills, 1979].

1.4 The Proposed System

The electrical energy requirement of rural households for lighting and the supply of television and radio/cassette players is in the vicinity of 300 Wh/day. This energy is sufficient to supply 2 to 3 sets of 20-watts fluorescent lamps for three to four hours a day, a 12-volt television set for about three hours, and radio/cassette set for several hours a day. For night time or off sunshine periods, a 12-volt, 50-Ah car battery is usually sufficient to provide the night load [Bierman, et al., 1992], by storing the extra electricity.

The above range of power requirement can be met by using the commercial thermoelectric devices operating at temperature in the vicinity of 400 K. This temperature range can be obtained by concentrating solar energy to about 20X using line focusing parabolic trough concentrators (PTC), which can satisfactorily concentrate solar radiation by rotating the concentrator about a single axis to produce alignment of the reflector's vertex with the sun rays. When set on an east-west plane, can be manually adjusted in order to face the sun.

However to operate a parabolic trough concentrator efficiently, high tracking accuracy is essential [Rabl, 1976a]. This has been an issue of research activity for several years since Tabor [1958] first explored the concept of a stationary solar collector, and reached the disappointing conclusion that, the maximum possible concentration ratio that can be obtained by a stationary solar concentrator is only about 3.

The development of what is described as an ideal cylindrical light concentrator using a compound parabolic arrangement, which approaches the maximum theoretical concentration efficiency has offered the possibility of solar energy concentrators free of tracking concerns, particularly at low concentration ratios [Winston, 1975]. The compound parabolic concentrator (CPC) is characterized by its half acceptance angle which determines the maximum attainable concentration ratio. It is a result of
improvement on the cone concentrator concept through application of the edge-ray principle [Winston, 1974]. According to this principle all the rays entering through the aperture of the collector at an extreme angle known as the half acceptance angle emerge through the exit aperture, either directly or after some reflections, as shown in Fig. (1.3). The concentrator is formed of two parabolas whose axes are inclined at angle $\theta$ with respect to the optical axis of the concentrator. Concentration ratio as high as 10 have been claimed without diurnal tracking [Winston, 1974]. When set on north-south plane, only seasonal tracking adjustments may be required for concentration ratios up to about 3.

Fig. (1.3) Concentrating principle of a compound parabolic concentrator
However, the compound parabolic concentrator has the major drawback that the overall depth of the collector is determined by the extreme rays about the symmetry axis of the concentrator. The depth of the concentrator must be sufficient to allow both extreme rays to pass from the inlet aperture to the exit aperture. Therefore the ratio of the concentrator depth to the reflector aperture becomes excessive and impractical at high concentration ratios. This is the main economic disadvantage of the CPC, since too much reflector material is needed for a given concentration factor, compared to the simple parabolic concentrator. For example, a concentration factor of 10 requires reflector to aperture area ratio of about 11 (for one square meter of aperture, 11 square meters of reflector material is required) [Rabl, 1976b].

In general, a solar energy collector based on a compound parabolic concentrator, although it approaches the maximum theoretical concentration ratio, is impractical except for very low concentration ratios. At the same time, as mentioned earlier, a solar energy collector based on parabolic trough concentrator requires high level of tracking accuracy, which is difficult and expensive.

A promising approach is to design the PTC for a low concentration factor and use it in conjunction with a wider secondary to collect and further concentrate the radiation onto smaller absorber plate, making in effect a multi-stage concentrator. This allows a prescribed maximum tracking error without a significant reduction in performance. Hence a two stage design made of a primary PTC and a secondary CPC, which combines the compactness of the PTC and the high concentration efficiency of the CPC is likely to be an effective combination. Several articles are available in the literature which describe designs based on this concept [Rabl 1976a; Mills, 1980; 1995; Collares et al., 1991; Brunotte et al., 1996]

A two stage concentrator using a second stage and a primary Fresnel mirror field for central receiver has been described by Rabl [1976a]. More recently, Brunotte et al., [1996] have described a two stage arrangement giving concentration ratios up to 300, using a north-south polar axis primary PTC with a row of filled dielectric non-imaging 3-D concentrators, designed for photovoltaic conversion. Although the design achieves a high concentration ratio, its tolerance to the incoming rays is very limited, and
therefore the major problem of frequent tracking requirement remains. A combination of a primary PTC and an intermediate asymmetrical CPC secondary, providing concentration ratios in the range 9 to 12, was described by Mills [1980; 1995]. The design is suggested for both photovoltaic conversion using optical prisms and for thermal processes using tubular receivers.

Collares, et al. [1991] has described a two stage concentrator, using a second stage composed of several multi-segments fabricated side by side with each segment facing a different portion of the primary concentrator. However, the awkward complex design and fabrication complexity of the multi-segmented compound parabolic concentrators may limit the applicability of this option, except for very high concentration ratios.

For the purpose of the system we are concerned with here, and with consideration to the design simplicity and cost effectiveness, it is suggested that a symmetrical CPC secondary is adequate to meet the solar energy concentration requirements (i.e. about 20X). The design developed here is based on a wider receiver than usual for the primary concentrator, allowing interception of the misaligned incoming rays within the angular region (±5) defined as the tolerance of the concentrator to the incoming rays.

The geometrical description of the proposed system in a cross section is shown in Fig. (6.1). In addition to the advantage of improved concentration efficiency and limited tracking requirements, this configuration is sought to help reducing the heat losses from the absorber plate by trapping the hot air in the space confined by the CPC, the absorber plate and the aperture window of the glass tube when the receiver system is not adequately evacuated to suppress the convective currents. The solar energy concentrator and the thermoelectric generator are part of a small scale combined heat and power unit intended for use in rural areas of developing countries.

1.5 General Theme of the Thesis

This thesis comprehensively details design optimization procedure, evaluation and economic justification of thermoelectric elements and of the solar energy concentrating
system proposed to meet the requirement of a combined heat and power unit for rural households in the developing countries of the sunny regions.

Following the historical and the technical background of the thermoelectric devices, given above, the basic principles of thermoelectric devices and the performance equations in power generation mode are outlined in Chapter 2. In this chapter, also a brief discussion on the thermoelectric materials, parameters affecting materials selection and the common preparation methods of thermoelectric elements are given. Results of laboratory work to produce and evaluate some thermoelectric elements are also presented in Chapter 2.

Chapter 3 presents a unified theoretical thermoelectric model developed for the purpose of geometrical optimization of thermoelectric elements, and prediction of the device performance in power generation mode. The model has an advantage of taking into account the effect of all the parameters which contribute to the heat transfer process associated with the thermoelectric device. The model is also used to predict and evaluate performance of a commercial thermoelectric device in comparison with other devices of optimum thermoelements and with a varying electrical and thermal contact layer properties.

Chapter 4 presents an experimental investigation of a commercially available thermoelectric device in power generation mode. Results of the experiment are used to validate the unified theoretical thermoelectric model.

Chapters 5, 6, 7, 8 and 9 are mainly devoted to the design and evaluation of the solar energy collecting system. Chapter 5 presents a design methodology and procedure for the two stage solar energy concentrating system. Descriptions of the system and the appropriate evaluation methods are given in Chapter 6. Different evaluation techniques are considered which simulate the range of conditions that would be normally expected to be experienced during the operational life of the system. Since the proposed design seems well suited for operation under vacuum as well as under residual pressure, two possible operational conditions for the receiver system are considered: one evacuated in order to suppress the convective heat loss, and one not. The former is modeled using a
specifically developed analytical method, presented in Chapter 8. This model allowed prediction of the temperature distribution and the performance of the system under vacuum.

The unevacuated condition, which is thermodynamically more complex, is modeled using numerical techniques involving computational fluid dynamics, and is presented in Chapter 7. The numerical analysis has been performed using the propriety computational fluid dynamic code (FLUENT), which uses a control volume/finite difference based solution procedure. This model allowed prediction of the flow field, and the temperature distribution in the receiver of the solar energy collector.

Chapter 9 describes the construction and experimental evaluation of a small model of the proposed two stage solar energy concentrator. Results of this experiment are used to validate the analytical model.

An economic analysis of the proposed system is given in Chapter 10.
CHAPTER TWO

2 Thermoelectric Generators

This Chapter describes the basic principle of thermoelectric devices in power generation mode, and presents the basic performance equations and factors which affect the performance of these devices. A short review of thermoelectric materials, their properties and preparation methods, is also given. Finally, results from laboratory work to fabricate and evaluate thermoelectric materials are presented.

2.1 Basic Principles of Thermoelectric Devices

The direct conversion of the heat energy into the electrical energy or the reverse in a solid conductor is controlled by three interrelated phenomena; the Seebeck, Peltier and Thomson effects. These have been widely covered in the literature [Freedman, 1966; Pollock, 1985; Cadoff & Miller 1960] so here only a brief description is given.

- Seebeck coefficient

The Seebeck effect concerns the generation of the electromotive force (emf) in a circuit composed of two different conductors whose junctions are maintained at different temperatures. It is quantified as the open circuit voltage per unit temperature difference between the two junctions; namely the hot and the cold ends of the conductors. It can be explained by considering an element comprised of two conductors connected in a simple loop, with one end at a temperature \( T_h \), and the other at a temperature \( T_c \), where \( T_h > T_c \) as shown in Fig. (2.1).

Due to the temperature difference, charge carriers with energy level near the Fermi-energy level at the hot end will be easily excited to a high energy level and can lower their energies by diffusing to the cold end. Similarly those at the cold end can diffuse toward the hot end at a rate dependent on their energies. Thus, a net current of charge carriers will result. This flow will cause the charge carriers to pile up at one end of the element, usually the cold end, and thereby produces an electromotive force.
BASIC PRINCIPLES OF THERMOELECTRIC DEVICES

The \textit{emf} induced across the loop is the function of the junction's temperature and is given [Cadoff \& Miller, 1960; Pollock, 1985] by

\[ V_{12}(T_h, T_c) = S_{12}(T) \Delta T \]  \hspace{1cm} (2.1)

where, \( S_{12}(T) = S_1(T) - S_2(T) \), \( S_1(T) \) and \( S_2(T) \) are the absolute Seebeck coefficient of conductors (1) and (2) respectively.

\[ \text{Conductor-1} \]

\[ T_c \]

\[ \text{Conductor-2} \]

\[ T_h \]

\[ \text{Fig. (2.1) \hspace{1cm} A simple thermoelectric circuit} \]

Equation (2.1) indicates that, the \textit{emf} of a thermoelectric element can be determined whenever the thermoelectric coefficients \( S_1 \) and \( S_2 \) are known, over the temperature range under consideration, and for a given conductors, it depends solely upon the temperature of the two junctions, and is independent of the shapes of the elements.

- Peltier effect

The Peltier effect was identified by Jean C. A. Peltier 1834 [Pollock, 1985], and refers to the heat generation at a junction between two dissimilar conductors, when an electric current passes through the junction. When electrons flow from material-1 to material-2, as shown in Fig. (2.2), they affect the amount of heat energy and the difference is either absorbed or released, depending on the direction of the current flow at the junction. This heat exchange is known as the Peltier heat [Freedman, 1966].
The Peltier heat at a junction of the two dissimilar conductors $I$ and $2$ is given by the following equation [Freedman, 1966]

$$Q_{12}(T) = I \Pi_{12}(T) \quad (2.2)$$

where $\Pi_{12}$ is the Peltier coefficient and is defined as the rate of heat generated per second per unit current flow through the loop, and $I$ is the electrical current passing the junction. $\Pi_{12}$ is taken to be positive when the cooling occurs at the junction ($I-2$) and when the current flow is from conductor-$I$ to conductor-$2$ as shown in Fig. (2.2).

- Thomson effect

The Thomson effect involves the generation of heat in a single current carrying conductor along which a temperature gradient is maintained. It takes its name from William Thomson (1854) [Cadoff & Miller, 1960], who postulated the existence of a reversible generation of heat when a temperature gradient is applied to a current carrying conductor. The Thomson effect can be explained by considering a single conductor with temperatures $T_c$ and $T_h$, as shown in Fig. (2.3), where, $(T_h > T_c)$.

As the current flows through the conductor, electrons flowing past the point with temperature $T_h$ will absorb energy and increase their potential energy. Electrons flowing in the same direction as the thermal gradient (towards decreasing
temperature) at a point with temperature $T_i$, will give up their energy. As a result, temperatures of the two points will change as will their heat contents. These changes in the heat content of the conductor are the Thomson heats.

![Thomson heat in a current carrying conductor](image)

**Fig. (2.3)** Thomson heat in a current carrying conductor

Thomson heat per unit length of the conductor is given by the relation [Freedman, 1966]

$$\frac{d Q_T}{d x} = \tau I \frac{d T}{d x}$$

(2.3)

where, $\tau$ is the Thomson heat coefficient, and is taken as positive, if the heat is absorbed when the current and the temperature gradient are in the same direction.

By using the first and the second laws of thermodynamics, Lord Kelvin was able to define the three thermoelectric effects upon each other as follow [Cadoff & Miller, 1960]

The Peltier coefficient and the Thomson coefficient are related to the Seebeck coefficient as follows

$$\Pi_{l_2}(T) = T S_{l_2}(T)$$

(2.4)

$$\tau = \frac{T d S_{l_2}}{d T}$$

(2.5)
Therefore all the thermoelectric heat terms can be determined by knowing only the temperature range of the element's junctions and the corresponding values of the Seebeck coefficient.
2.2 Formulation of Thermoelectric Generator's Performance

The performance of a thermoelectric generator is dependent upon several factors, including the thermoelectric properties of the material and their useful temperature range, the hot and the cold junctions temperatures and the structural arrangement of the generator. The performance can be evaluated in terms of the thermal-electrical conversion efficiency, which may be defined as the ratio of the useful electrical energy output to the heat energy absorbed by the device from the heat source.

The input energy to the thermoelectric device is evaluated by balancing the various energy transfer processes that are taking place within the device. The traditional method of evaluation of this heat term, assumes that all the heat passes from the hot junction to the cold junction only through thermoelement legs, and therefore neglects any heat leakage to the cold junction through the space around element legs. If the space between the two ceramic plates is evacuated, there will only be radiation heat exchange between the hot and the cold plates, in addition to the conductive heat through side sealant. However if it is not, heat flow by conduction through the gas filled space between the two ceramic plates should also be considered. Normally, the gap is too small for convective heat transfer to be established.

In the model presented in Chapter 3 and which is used to optimize thermoelement length and predict its performance in power generation mode, considerations are given to these heat terms. The rest of this section describes the basic formulation of thermoelectric performance using the traditional method and with consideration to the contribution of the Thomson heat term.

Consider a thermoelectric circuit, consisting of two dissimilar elements joined by metallic strips as shown in Fig. (2.4). An external load (R) is connected to complete the circuit at the cold ends of the thermoelement. The Temperature $T_h$ is maintained by a heat source, while the temperature $T_c$ is maintained by a heat sink at $T_c$.

The following parameters can also be defined
FORMULATION OF THERMOELECTRIC PERFORMANCE

$L_1$ and $L_2$ are the lengths of the element legs (1) and (2) respectively, (cm)

$A_1$ and $A_2$ are the cross section areas of the element legs, assuming uniform cross-sections, (cm$^2$)

$k_1$ and $k_2$ are the thermal conductivities of the element legs, (W/cm.°K)

$\rho_1$ and $\rho_2$ are the electrical resistivities of legs materials, (Ohm-cm)

$S$, $\tau$ and $\Pi$ are the Seebeck, Thomson and Peltier heat coefficients respectively of the element legs.

$I$ is the electric current flowing through the circuit of the element, (Amperes)

$V_{oc}$ is the open circuit voltage induced across the loop, (Volts)

![Fig. (2.4)](image) A schematic drawing of a thermoelectric circuit

Once the temperature gradient is maintained across the two junctions of the element, conduction heat $Q_c$ flows through the element's legs by direct conduction. Also,
Seebeck *emf* will be induced across the loop, and consequently, Peltier heat $Q_p$ will be generated at each of the two junctions. The element absorbs heat at a rate equal to $(S_h T_h I)$ at the hot junction at temperature $T_h$ and rejects heat at a rate equal to $(S_e T_e I)$ at the cold junction at temperature $T_e$. At the same time, Thomson heat $Q_t$ will be generated in one of the element legs and absorbed in the other depending on the direction of the current flow. Also heat will be generated along the element legs due to the Joule heating $Q_j$ in each leg.

Based on the assumption that, heat transported from the hot junction to the cold junction through medium other than the element legs is neglected (traditional method), heat entering the hot junction, is the sum of the heat input from the hot source $Q_{in}$ and portions of the Joule and the Thomson heats presumably transported to the hot junction, while heat leaving the hot junction is equal to the sum of the conduction heat transported to the cold junction, and the Peltier heat absorbed by the charge carriers. In most of the available derivations, the contribution of the Thomson heat is also neglected, on assumption that it is relatively small, particularly when the device is operated at relatively low temperature difference and for materials which do not show significant variation in the Seebeck effect with temperature. However, in the case of power generation at high temperatures, the effect of the Thomson heat is appreciable and should be taken into consideration.

Therefore the energy balance gives

$$Q_{in} = Q_{c(h)} + Q_{p(h)} - f_t Q_t - f_j Q_j$$

(2.6)

where, $Q_c = -KVT$, is the conduction heat through elements legs at the hot junction, $Q_p = S_h I T_h$, is Peltier heat at the hot junction, $Q_t = I^T \Delta T$, is the Thomson heat, and $Q_j = I^2 \rho L / A$, is the Joule heating, $f_j$ and $f_t$ are the fractions of the Joule heat and Thomson heat respectively fed back to the hot junction.

Similarly, the heat rejected at the cold junction ($Q_r$) can be estimated as follow
FORMULATION OF THERMOELECTRIC PERFORMANCE

\[ Q_r = Q_{c(c)} + Q_{P(o)} + f_i Q_i + f_j Q_j \]  \hspace{1cm} (2.7)

where, \( Q_{c(c)} = -KVT \) is the conduction heat through elements legs at the cold junction and \( Q_{P(o)} = S_c IT_c \) is Peltier heat.

The rate of Peltier heat absorbed and evolved at the hot and the cold junctions respectively can be readily evaluated from the values of \( T_h \) and \( T_e \) and the corresponding values of \( S_h \) and \( S_e \) at the hot and the cold junctions respectively. However, the other terms of the heat rates involve the temperature gradient at the hot and the cold ends, and thus, require solution of a differential equation for the temperature distribution in a current carrying conductor in the presence of a temperature gradient. This issue has been repeatedly treated [Talaat, 1960; Burshtein, 1958; Domenicali, 1954; Talaat, 1962], and will be summarized here for the sake of completeness.

By using the energy conservation principle in any cross section of the rod, solution of the differential equations were found by Burshtein [1958]. The effective thermoelectric material properties of the different heat terms in equations (2.8) and equation (2.9) are estimated over the operating temperature range. A possible approach which avoids cumbersome solutions for differential equations of the temperature distribution along the thermoelectric legs in order to estimate the effective values of the temperature dependent parameters, is simply to substitute the average values of these parameters over the temperature range. This procedure has been found to give results within a reasonable accuracy [Egli, 1960].

Accordingly, expressions for the input heat rate \( Q_{in} \) and also the rate of heat rejection \( Q_r \) are given as follows [Talaat, 1960]

\[ Q_{in} = S_h T_h I + K(T_h - T_e) - I \overline{T} \Delta S f_i - I^2 r f_j \]  \hspace{1cm} (2.8)

and
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\[
Q_r = IS_{12}T_e + K(T_h - T_c) + I \overline{T} \Delta S(1 - f) - I^2 r(1 - f) \tag{2.9}
\]

where, \( \overline{T} = 0.5(T_h + T_c) \), \( \Delta S = S_{12(h)} - S_{12(c)} \), \( r \) is the total electrical resistance of the thermoelement.

By evaluating the Thomson heat interactions for the legs, it has been shown [Burshtein, 1958; Cohen, 1963] that, half the Thomson heat appears at both the hot and the cold junctions (i.e. \( f_i = 0.5 \)). Similarly, it has been shown [Cohen, 1963] that, the temperature distribution along the thermoelectric legs is parabolic, indicating that, the conduction heat is not directly transferred from hot junction to the cold junction. Instead, the Joule heat generated in the element is transferred to both ends of the element legs (i.e. \( f_j = 0.5 \)).

The current \( (I) \) is given by

\[
I = \frac{V_{12}}{R_{TT}} = \frac{S_{12}(T_h - T_c)}{(R + r)} = \frac{S_{12}(T_h - T_c)}{r(M+1)} \tag{2.10}
\]

where, \( S_{12} \) is the effective Seebeck effect of the thermoelectric couple at the applied temperature range, \( R_{TT} = (R + r) \), is the total resistance of the circuit and \( M = R/r \), is the ratio of the load resistance to the total internal resistance of the thermoelement.

Substituting for \( I \) in each heat term in equation (2.8), the heat \( Q_{in} \) added to the hot junction from the heat source is given by

\[
Q_{in} = K\Delta T + \frac{S_{12}^2 T_h(T_h - T_c)}{r(M+1)} - \frac{1}{2} \frac{S_{12}(T_h - T_c)}{r(M+1)} \overline{T}(\Delta S_{12}) - \frac{1}{2} \frac{S_{12}^2 (T_h - T_c)^2}{r(M+1)^2} \tag{2.11}
\]

The useful electrical power produced by the device is given by
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\[ P_e = \int R = \frac{S_{12}^2 (T_h-T_c)^2}{r^2 (M+1)^2} R = S_{12}^2 (T_h-T_c)^2 \frac{M}{r(M+1)^2} \]  \hspace{1cm} (2.12)

The efficiency of the thermoelectric generator, is given by

\[ \eta = \frac{P_e}{Q_{in}} = \frac{(T_h-T_c)}{T_h} \left( 1 - \frac{T_h}{S_{12}} \frac{\Delta S_{12}}{T_h} + \frac{K_r (M+1)}{2T_h} \frac{(T_h-T_c)}{2T_h} \right) \frac{1}{(M+1)} \]  \hspace{1cm} (2.13)

Equation (2.13) is obtained by substituting for \( P_e \) and \( Q_{in} \) and dividing both the numerator and the denominator by \( S_{12}^2 T_h (T_h-T_c)(r(M+1))^{-1} \) and rearranging. The difference between equation (2.13) and those available in the literature (Ioffe, 1957) is the presence of the Thomson heat term here.

It can be seen that the efficiency is determined by two terms, a thermodynamic Carnot efficiency of a heat engine; defined by the ratio of the temperature difference to the higher temperature of the device, which can be denoted by \( \eta_c \) and a term which involves, in addition to the temperatures terms, parameters determined by the thermoelement properties, and can be denoted by \( \eta_{TE} \).

\( \eta_c \) can be improved by increasing the temperature difference between the two junctions of the device, while \( \eta_{TE} \) can only be maximized by selection of the thermoelectric material with the best properties in the operating temperature range.

One of the terms involved in \( \eta_{TE} \), is \( (K_r) \), which should be minimized in order to maximize \( \eta_{TE} \). Thus

\[ K_r = (k_1 \frac{A_1}{L_1} + k_2 \frac{A_2}{L_2})^* (\rho_1 \frac{L_1}{A_1} + \rho_2 \frac{L_2}{A_2}) \]
FORMULATION OF THERMOELECTRIC PERFORMANCE

For arbitrary values of $\rho$ and $k$, a parameter $(x = \frac{l_1 A_2}{l_2 A_1})$ can be defined to describe the geometry of thermoelement's legs. Therefore, the value of $K_r$ can be minimized by setting $\frac{d}{dx}(K_r) = 0$. Thus

$$\frac{d}{dx}\left(\frac{k_1 \rho_1 + k_2 \rho_2}{x} + x k_1 \rho_1 + k_2 \rho_2\right) = 0$$

This will give the optimum value of the geometrical ratio $(x)$ as

$$x = \frac{k_1 \rho_2}{\sqrt{k_2 \rho_1}}$$ (2.14)

The value of $(K_r)$ for the optimum ratio of the lengths to areas becomes

$$K_{r_{\text{min.}}} = \frac{(\sqrt{\rho_1 k_1} + \sqrt{\rho_2 k_2})^2}{2S_{12}}$$ (2.15)

The reciprocal of the term $K_r S_{12}^2$ is known as the figure-of-merit of the thermoelectric generator and for the minimum value of $K_r$, it is defined as the optimum Figure-of-Merit $(Z)$, and is given by

$$Z = (S_{12})^2 \left(\sqrt{\rho_1 k_1} + \sqrt{\rho_2 k_2}\right)^{-2}$$ (2.16)

It can be seen that $Z$ is independent of the geometry of the element's legs, as long as the lengths and areas of the elements are kept proportional according to equation (2.14) for arbitrary values of $\rho$ and $k$.

Another variable that can be adjusted to improve the value of $\eta_{\text{re}}$, is the ratio of load resistance to the internal resistance of the generator $(M = R/r)$. Its optimum value can be obtained by differentiating $\eta_{\text{re}}$ with respect to $M$ and equating to zero. Thus, by putting
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\[
\frac{d\eta_{TE}}{dM} = \frac{d}{dM} \left( M \left( M+1 \right) \left( 1 - \frac{T}{T_s} \Delta S_{a_{_n}} + \frac{1}{Z} \left( \frac{M+1}{T_s} - \frac{(T_s-T)}{2T_s} \right) \right) \right) = 0
\]

will give an optimum value of \( M \) as follows

\[
M = \sqrt{1 + ZT - ZT \frac{dS_{12}}{S_{12}}}
\]

(2.17)

where, \( M \) is the optimum value of the load resistance to the internal resistance of the thermoelectric generator.

Similarly, equation (2.17) differs from those available in the literature due to the existence of the contribution of the Thomson heat term, which is necessary in order to avoid overestimation of the thermoelectric performance, particularly for power generation which requires high temperature difference between the hot and the cold junctions.

By substituting for \( \bar{T} \frac{dS_{12}}{S_{12}} \) in equation (2.13) and rearranging, the maximum value of the conversion efficiency can now be written as

\[
\eta = MZ \Delta T \left( M(M+1)^2 - \left( \frac{Z}{2} (M+1) - 1 \right) \Delta T \right)^{-1}
\]

(2.18)

Sometimes, the device may be operated at the maximum power output rather than the maximum efficiency. In such situations, we have

\[
P_e = IV = I \left( S_{12} \Delta T - I r \right)
\]

(2.19)

The optimum current for maximum output power can be obtained by setting

\[
\frac{d(p_e)}{dI} = 0 = S_{12} \Delta T - 2I r
\]

which implies that,
FORMULATION OF THERMOELECTRIC PERFORMANCE

\[ I = \frac{S_1 \Delta T}{2r} \]  \hspace{1cm} (2.20)

Comparing equation (2.20) with equation (2.10) will give the optimum ratio of the load resistance to the internal resistance of the thermoelectric generator for the ideal current at the maximum power output, i.e. \( M = R/r = 1 \).

Substituting the value of \( I \) at this optimum load ratio in equation (2.19), gives the maximum output power as

\[ P_{e\text{max.}} = \frac{S_1^2 R \Delta T^2}{4r^2} = \frac{S_1^2}{4r} \Delta T^2 \]  \hspace{1cm} (2.21)

Equations from 2.6 to 2.21 derived in this section, though they consider the contribution of the Thomson heat, are based on an ideal system which assumes that, the heat flows only through thermoelement legs, and neglects all the stray heat leakage through both the space between the two ceramic plates and through the sealant material. The maximum values of the conversion efficiency and the power output given by equations (2.18) and (2.21) respectively are based on the optimum ratio of the element legs dimensions \((UA)\) which optimize the value of \( Kr \), but do not give any indications about the optimum values of these dimensions. The model presented in Chapter 3 introduces a procedure which allows optimization of the thermoelement length, based on the maximum power output with consideration to the stray heat and Thomson heat as well.
2.3 Parameters Affecting Thermoelectric Device Performance

Investigation of the equation (2.18) shows that, aside of the junctions temperatures, the parameters which determine the performance of a thermoelectric generator is the figure-of-merit and the load ratio. There are some other auxiliary properties that do not directly affect the performance, but determine the design limitation of the device, such as the chemical stability and melting point of the material. These properties limit the useful operating temperature range of the device.

Parameters that determine the figure-of-merit, are the Seebeck coefficient, the thermal conductivity and the electrical conductivity of the material. A large figure-of-merit requires a high Seebeck coefficient, large electrical conductivity and low thermal conductivity. The optimization methods applicable to these parameters are fully explained in the literature [Pollock, 1985; Cadoff & Miller, 1960] and here will be briefly for the sake of completeness.

At a given temperature difference, the magnitude of the Seebeck coefficient directly determines the voltage induced across the element under consideration. It is apparent that for a large output voltage, a promising material should have a large Seebeck coefficient. Seebeck coefficient is an electron transport property of the material and therefore is dependent on the electron energy level (Fermi-level), and the concentration of charge carriers in the material [Pollock, 1985; Klein, 1960]. The Seebeck coefficient varies from extremely small values for metals which have a very high charge carriers concentration to a very large values for insulators which have a very low charge carriers concentration. Semiconductors have intermediate values, and can be altered by addition of impurities or adjustment of the stoichometry of the substance [Ioffe, 1957; Cadoff & Miller, 1960].

The electrical conductivity of a semiconductor is a function of the charge carriers concentration and their mobility [Klein, 1960]. High electrical conductivity requires large concentration of charge carriers and higher mobility.
PARAMETERS AFFECTING THERMOELECTRIC DEVICE PERFORMANCE

The thermal conductivity \((k)\) of a material is formed of two components, lattice and an electronic. The lattice component is due mainly to the vibration of atoms in a crystal when brought into intense agitation by applying heat. It is given by the specific heat per unit volume of the substance and the mean free path of the phonon, and is independent of the carrier concentration. The electronic component is related to the electrical conductivity and hence on the concentration and mobility of the charge carriers.

From the above, it is clear that, thermoelectric properties; \(S, \sigma\) and \(k\) which determine the figure-of-merit of thermoelectric devices, are all dependent on the charge carriers concentration. The Seebeck coefficient increases with decreasing \(n\), which is in a good agreement with the quantity \(k^{-1}\), since a large figure-of-merit requires larger values of both \(S\) and \(k^{-1}\) and hence lower carriers concentration. In contrast, \(\sigma\) directly increases with the carriers concentration. For thermoelectric devices it is the product \((S^2 k^{-1} \sigma)\) that is important and not the individual values of \(S, \sigma\) and \(k\). For insulators the electrical conductivity is very low, while \(S\) is large, consequently the value of \((S^2 k^{-1} \sigma)\) is also small. While for metals \(\sigma\) is large and \(S\) is very small and therefore the product \((S^2 k^{-1} \sigma)\) is also small. Semiconductors are found to have a value of \((S^2 k^{-1} \sigma)\) approaching its maximum, approximately between \(n\) equals \(3 \times 10^{18}\) and \(3 \times 10^{19}\) per \(cm^3\) [Klein, 1960], and which can be altered by addition of impurities or adjustment of the stoichiometry of the substance [Ioffe, 1957].

On the other hand the load ratio is a design parameter which need to be optimized for the maximum performance. It is given by the internal resistance of the thermoelement, which include, the resistance of the thermoelement leg and the contact layers. These in turn are dependent on the energy transport properties of the thermoelectric material, its geometry and the quality of the electrical contact points.

Auxiliary properties can be as important as the thermoelectric properties themselves. They determine the useful operating temperature range of the thermoelectric material. The most crucial auxiliary property is the melting point. Although some of the materials may have high melting points, when heavily doped, alloys often result with
PARAMETERS AFFECTING THERMOELECTRIC DEVICE PERFORMANCE

low melting points. When selecting thermoelectric materials, the melting point should be safely higher than the operating temperature.

Another important auxiliary property is thermal diffusion. When a thermoelectric material is placed in a thermal gradient, inhomogeneity maybe caused in the material's constituents due to diffusion. Increase in the concentration of impurities due to diffusion towards the hotter portion of the specimen may not only alter the thermoelectric properties, but also may affect the other auxiliary properties such as the melting point.

Oxidization is another problem that affects the function of thermoelectric material. Most thermoelectric materials when exposed to oxygen of air, form an oxidized material layer on the outer surface, or alternatively oxygen may diffuse into the interior of the material destroying its thermoelectric behavior. This process may occur during preparation stages if the specimen is exposed to air. This oxidized layer may increase the contact resistance if not removed. Polishing may be adequate for removal of this layer; alternatively, an adequate heat treatment may be necessary [Klein, 1960]. Oxidization may also take place when the material is operated at high temperature in presence of air or other oxidizing gasses. Therefore, it may be appropriate to operate thermoelectric devices in an evacuated or an unoxidizing gas environment.

Most of the thermoelectric materials are brittle, and may not be able to withstand the compression and thermal shocks. The coefficient of thermal expansion of the thermoelectric material must match well that of the electrical contacts, otherwise fracture of the joint between them may result under severe temperature change. To prevent fracture, the thermoelectric legs must be allowed to expand freely under changing operational temperatures.
2.4 Thermoelectric Materials in Prospective

Most of the significant work on thermoelectricity was completed during 1950s and 1960s following results of Ioffe and his co-workers [Ioffe, 1957]. During this period most of the presently known thermoelectric material’s classification, development and theory was been established [Airapetians et al., 1958; Gordiakova et al., 1958; Smith, 1962; Yim et al., 1966; Horst and William, 1980; Scherrer et al., 1990; Carle et al., 1991; Rudolf et al., 1991; Amin et al., 1992].

In principle, combinations of elements from groups II-V, IV-VI and V - VI of the periodic table are regarded as suitable for thermoelectric purposes [Cadoff & Miller, 1960]. In particular, alloys made from compounds of Bismuth-Telluride, Bismuth-Selenide, Bismuth-Antimony, Lead-Telluride, Lead-Selenide, Lead-Sulphide, Silver-Antimony-Telluride, Germanium-Telluride, Mercury-Telluride, Germanium-Silicon and Indium-Antimony were evaluated as potential thermoelectric materials.

Recent development in thermoelectric material added further groups of materials to the above list, such as Borides of phosphate (BP) and transition metal Silicides (FeSi₂, CoS₃) using chemical vapor deposition [Matsubara et al., 1995]. Although most of the above compounds are unlikely to meet the requirements of the thermoelectric and auxiliary properties for the broad range of the practical applications, some of them showed good results for a limited applications range [Goldsmid,1964]. For instance, Bismuth-Telluride and its related alloys showed an advantage for low temperature application. Lead-Telluride and its alloys are recommended for power generation at intermediate temperature ranges, while Silicon-Germanium alloys are well suited for higher temperature ranges [Fuschill and Gibson, 1967; Dismukes et al., 1965; Abeles and Chen, 1964]. Brief description of these materials is given below.

- Germanium-Silicon

Germanium, which has an energy gap of 0.67 eV, was one of the first pure semiconductors to be the subject of a detailed study. Its high Seebeck coefficient drew
attention to the possibility of exploiting it in thermoelectric conversion. But its high lattice thermal conductivity (63 W/m°C at room temperature) [Hogarth, 1965] results in a low Figure-of-Merit. However by alloying the Germanium with Silicon, its thermal conductivity was found to be reduced considerably [Steele and Rossi, 1958], due to a decrease in the phonon mean free path. Also it was found [Borshchevsky and Fleurial 1991] that, doping with solvents including combinations of elements from groups III-VI and III-V of the periodical table such as Ga, In and P considerably improves the Figure-of-Merit. Efficiency as high as 10% with a hot junction temperature of 1040 °C has been reported [Cody, et al., 1990].

Most of the efforts made regarding the use of Germanium-Silicon alloys is for space applications using radioisotope fuels. Heavily doped silicon-germanium alloys have been the exclusive choice for NASA’s radioisotope thermoelectric generator (RTG) needs [Vining and Fleurial 1991; Cody et al., 1990].

• Lead-Telluride

Lead-Telluride ($Pb_2Te_3$), is one of the thermoelectric materials to be used extensively in thermoelectric generators. It has an energy level of about 0.3 $eV$, enabling it to remain extrinsic even at higher temperatures. Both Lead and Tellurium are soluble in $Pb_2Te_3$ to a few hundredths of one percentage. Therefore the material can be prepared in either p-type or n-type form by adding either of the two elements, and the carrier concentration of either type can be varied by suitable doping. Excess Lead results in an n-type material, while excess Tellurium produces p-type properties.

The lattice thermal conductivity of Lead-Telluride can be reduced by alloying with Tin-Telluride [Dismukes, 1965]. An alloy which consists of 75% of $Pb_2Te_3$ and 25% $SnTe$ has been found [Goldsmid, 1965] to be the best n-type material for use in the temperature range between 250 to 550°C. A Figure-of-Merit of 1.5E-3 per degree Kelvin has been reported [Putley, 1965].
- Bismuth-Telluride

Bismuth-Telluride ($\text{Bi}_2\text{Te}_3$) is one of the compounds that has received much attention as a material for thermoelectric cooling. The crystals of $\text{Bi}_2\text{Te}_3$ are made of layers of like atoms [Lougher, 1960], where the Tellurium and Bismuth layers are connected by strong covalent ionic bonds, while the neighboring $\text{Te}$ layers are held together only by weak Van der Waals-type forces [Goldsmid, 1965]. Therefore, the $\text{Bi}_2\text{Te}_3$ crystals can be easily cleaved along planes perpendicular to the trigonal axis, giving it an advantage of easy fabrication. Bismuth-Telluride can be prepared as a p-type or a n-type material by introducing suitable impurities to the solid solution of Bismuth-Telluride. As a general role, it was found [Gordiakova et al., 1958] that, the addition of halides of elements in the left subgroups of the first group (AgCl, AgBr, .. CuBr,...) as well as Tellurium and Selenium to the solid solution of $\text{Bi}_2\text{Te}_3$-$\text{Bi}_2\text{Se}_3$, produces donor effect “n-type”, while the addition of elements from groups 2, 3, 4 and 5 of the Mendeleeev’s periodic table as well as Bi and Pb creates an acceptor effect “p-type” material.

Based on measurements of a number of alloys, it was concluded [Goldsmid, 1964; Yim et al., 1966] that, an alloy with a composition $\text{Bi}_{0.5}\text{Sb}_{1.5}\text{Te}_3$ forms the best p-type material with a Figure-of-Merit of about $3.3 \times 10^{-3}$ per degree Kelvin at 20 °C, while the best n-type material has the composition $\text{Bi}_2\text{Te}_{2.7}\text{Se}_{0.3}$ with Figure-of-Merit $3 \times 10^{-3}$ per degree Kelvin.

Most important to $\text{Bi}_2\text{Te}_3$ thermoelements is the anisotropic behavior of the thermal and the electrical properties. It has been reported [Lougher, 1960; Sherrer et al., 1990] that the figure-of-merit is maximum in a direction parallel to the cleavage plane where it has a value twice that perpendicular to the cleavage plane. It is therefore very important that all the grains be oriented parallel to the cleavage planes to obtain the maximum figure-of-merit. This highlights the importance of the preparation method.
In general the figure-of-merit is very much dependent on the method of preparation which determines the type and size of the grains grown and composition achieved.

2.5 Preparation Methods of Thermoelectric Materials

Several techniques are available to produce thermoelectric alloys, including directional freezing by employing the Bridgeman method; crystal pulling by the Czochralsky method; Zone melting; traveling solution or traveling heater method; pulverized and intermixing elements sintering method; fine-grained production employing powder metallurgy techniques and thin films [Borshchevsky, 1995].

Directional freezing is one of the simplest techniques that have been used in aligning thermoelectric semiconductors. A tube containing the molten compound is maintained in a horizontal furnace which has a temperature gradient from one end to the other. As the overall power input to the furnace is reduced, freezing starts at one end and the interface gradually moves towards the other end. This technique was modified by Bridgeman, in which a vertical double furnace is used. This method was successfully employed to produce lead-telluride and bismuth-telluride thermoelectric elements [Borshchevsky, 1995], however, factors such as the rate of movement and the temperature gradient are crucial in determining the quality of the material produced [Horst and William, 1980].

Similarly, crystal pulling employing the Czochralsky method, is also widely used to produce single crystals of semiconductors. In this method the material to be grown is melted in a crucible, and a crystallographically oriented seed is then dipped into the melt and slowly pulled up by rotating the seed or the crucible or both in a counter-direction as appropriate. The technique has been employed successfully for the growth of crystals of Lead-Telluride and Bismuth-Telluride based thermoelectric materials [Borshchevsky, 1995]. However, in all the techniques which involves crystal growth it is not possible to control the stoichiometric compositions, particularly in the case of bismuth-telluride because its maximum melting point is not at stoichiometric composition [Tokiaia et al., 1990].
Zone melting and as well zone leveling and traveling solution or traveling heater method are all similar techniques which tend to remove and redistribute impurities where dopant concentration is expected to be relatively high and permit a uniform doping by implementing an annealing condition at the solidus line [Borshchevsky, 1995]. The success of these methods lies in the precise knowledge of the phase diagram and the rate of the traveling used [Fleurial, 1991; Scherrer et al., 1990 and Carle et al., 1991].

On the other hand, fine-grained production produces finely grained polycrystalline samples by employing either cold pressing or hot pressing or sintering. The main disadvantage of this method is that, the specimens obtained consist of small un-oriented crystals, and due to the anisotropic properties of some material such as Bismuth-Telluride, the material may exhibit poor electrical conductivity. However it has been shown [Amin et al., 1992] that, this is compensated for by the decrease in the thermal conductivity as a result of phonon scattering at grain boundaries, for grain sizes that are comparable to the mean free path of phonons. It has been possible to produce material with properties comparable to those of single crystals produced by the Bridgeman method [Amin et al., 1992; Rudolf et al., 1991]. This method, followed by annealing at a temperature just under the melting point will further improve the quality of the thermoelectric properties by homogenizing the material.

Pulverized and intermixed elements sintering (PIES) method, on the other hand is a process which has been proposed [Tokiaia et al., 1990; Ohta et al., 1990] to overcome some of the difficulties associated with the earlier methods. The PIES method uses a planetary ball mill which generates strong centrifugal force, pulverizes and mixes the elements resulting in solid solution formation.

Although, a number of papers on the preparation of thermoelectric films have been published in the literature [Matsubara et al., 1995], fabrication of thermoelectric semiconductors using this approach is still new and requires further investigation. In general, the technique is of a considerable interest from the industrial point of view.
Preparation of thermoelectric elements using thin films techniques will save much of the time and waste material associated with the current preparation methods.
2.6 Laboratory Preparation of Thermoelectric Elements

There are several techniques that can be employed to produce thermoelectric elements, including directional freezing from the melt, crystal pulling, fined-grained powder technology, zone melting, transfer solvent or heater methods, the pulverised and intermixing element sintering technique and thin films techniques. Methods which involve crystal growth from the melt can produce single crystals of good electrical properties, however parameters such as the Seebeck coefficient are difficult to control, mainly due to the segregation of the impurities. Thermal conductivity of materials elaborated using crystal growth techniques tends to be very high, resulting in a poor figure-of-merit. Most of the other techniques suffer from similar problems. Interesting but not reproducible results have been obtained with most of these techniques [Scherrer, 1990].

In general, it is strongly emphasised that, regardless of the technique employed, any successful process requires a precise knowledge of composition-temperature phase diagrams [Horst, 1980]. This helps selection of an appropriate preparation method, apparatus and processing procedure.

Recent publications [Borschchevsky, 1995] indicated that Bi-Te-Sb-Se melt-quenching followed by proper annealing could be effective for thermoelectric manufacturing. The melt quenching process is a simple technique and allows fast production of thermoelements. Due to this disadvantage, this technique was selected for the laboratory preparation of \((\text{Bi}, \text{Sb})_2(\text{Te}, \text{Se})_3\) based thermoelements.

Samples were prepared in the Department of the Chemistry, while the measurement of material properties was undertaken in the Department of the Physics, where appropriate instrumentation and facilities were available. Whilst, some success has been achieved in producing several samples, and measurement of the electrical conductivity was carried on one of the samples, the work was curtailed because of concerns about the toxicity of some elements associated with the compounds used.
Preparation of samples

The starting materials which included powder forms of Bismuth, Tellurium, Antimony and Selenium were obtained from Mining and Chemical Products (MCP) Ltd, UK. The purity level and the price list are shown in Table (2.1). The exact amounts of the compositions were prepared for each sample in a controlled environment, using a high precision electronic balance.

<table>
<thead>
<tr>
<th>Material</th>
<th>Purity (%)</th>
<th>Price (£/kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bismuth (Bi)</td>
<td>99.99%</td>
<td>210</td>
</tr>
<tr>
<td>Antimony (Sb)</td>
<td>99.5%</td>
<td>210</td>
</tr>
<tr>
<td>Tellurium (Te)</td>
<td>99.8%</td>
<td>210</td>
</tr>
<tr>
<td>Selenium (Se)</td>
<td>99.5%</td>
<td>210</td>
</tr>
</tbody>
</table>

The p-type material has the chemical formula of \((Bi_2Te_3)_{0.25}(Sb_2Te_3)_{0.75}\) or \(Bi_{10}Te_{60}Sb_{30}\) with 2.3 wt% extra Te as an impurity. The n-type material has a chemical formula of \((Bi_2Te_3)_{0.9}(Sb_2Te_3)_{0.05}(Sb_2Se_3)_{0.05}\) with addition of 0.08 wt% \(SbI_2\) as an impurity material. Table (2.2) shows some of the properties and quantities required for a sample of 7.5 grams.

<table>
<thead>
<tr>
<th>Material</th>
<th>Bismuth</th>
<th>Tellurium</th>
<th>Antimony</th>
<th>Selenium</th>
<th>Iodine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atomic weight</td>
<td>208.98</td>
<td>127.6</td>
<td>121.75</td>
<td>78.96</td>
<td>126.9</td>
</tr>
<tr>
<td>Melting point (°C)</td>
<td>271.442</td>
<td>449.5</td>
<td>630.775</td>
<td>220</td>
<td>113.7</td>
</tr>
<tr>
<td>Density (gm/cc)</td>
<td>9.8</td>
<td>6.24</td>
<td>6.6</td>
<td>4.8</td>
<td>4.69</td>
</tr>
<tr>
<td>P-type wt (gm)</td>
<td>1.17</td>
<td>4.459</td>
<td>2.045</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>N-type wt (gm)</td>
<td>3.636</td>
<td>3.515</td>
<td>0.25</td>
<td>0.114</td>
<td>0.045</td>
</tr>
</tbody>
</table>
Samples were introduced into prepared clean quartz ampoules of 0.50 cm bore and about 7 cm long, shown in Fig. (2.5). After sufficient mixing by rocking the ampoule, the samples are connected to a vacuum system. A vacuum of about 8.0E-05 torr was maintained for all P-type samples. However for the N-type samples, some difficulties was encountered due to continuous evaporation of Iodine vapour during evacuation. To overcome these, a procedure was employed in which the Iodine is frozen at the bottom of the ampoule using Nitrogen liquid. By this procedure a vacuum of about 1.0E-04 torr was achieved. The ampoules were sealed circumferentially using propane gas torch. The samples were melted using an electrical heater furnace, kept at about 755\degree C for six hours and then water quenched.

![Fig. (2.5) Alloying ampoule preparation of thermoelectric materials, (A) Before sealing, (B) After sealing](image-url)
Measurement of the thermoelectric properties

Thermoelectric materials can be evaluated by measurement of the properties which determine the figure-of-merit, namely, the thermoelectric power $\alpha$, the electric resistivity $\rho$, and the thermal conductivity $k$ as a function of temperature. However, due to the practical difficulties encountered, only the measurement of the electrical resistivity of a p-type material was undertaken.

The sample was cut to specified dimensions (1.5mm×1.5mm×20mm) and prepared for the measurement of the electrical resistance. To eliminate the effect of contact resistance, a four-probes technique was employed. The probes were set about 4 mm apart from each other as shown in Fig. (2.6). The outer probes were connected to a power source which allowed a small current to pass through the sample and the standard resistor in forward and reverse directions, while the inner probes were used to measure the voltage drop across a piece of the sample. The probes leads were spring loaded on the sample to achieve proper contact.

The system was provided with a heater and a cooling device, allowing adjustment of the sample temperature to a predetermined value, using a temperature controller. The heating, cooling and the measurement were accomplished using a computer aided and controlled procedure. This instrument and the associated facilities were provided by the Department of the Physics. The procedure involved measurement of the voltage across the sample and across the standard resistor in forward and backward directions by reversing the direction of the current.

The basic defining equation for the experimental determination of the resistivity ($\rho$) of the sample is

$$\rho = \frac{\Delta V A}{I dx}$$

where, $\Delta V$ is the average voltage drop across the sample, $I$ is the current through the sample, $A$ is the cross section area and $dx$ is the length of the sample between the
voltage probes. Substituting for the current by the potential drop across the standard resistor ($R_s$), the expression for sample resistivity becomes

$$\rho = R_s \frac{\Delta V}{V_s} \frac{A}{L}$$

where, $L = dx$ is the length of the specimen, and $V_s = I R_s$ is the voltage across the standard resistor.

**Fig. (2.6)** Measurement of the electrical resistivity of a thermoelement

- **Results and discussion**

Measurement was undertaken for a temperature range from the ambient temperature to 65 K. Fig. (2.7) shows the variation of the electrical resistivity with the temperature for p-type material of the chemical formula of $(\text{Bi}_2\text{Te}_3)_{0.25}(\text{Sb}_2\text{Te}_3)_{0.75}$. A representative sample of results are shown in table (2.3).
The electrical resistivity of the sample was found to vary approximately as a function of $T^{4.6}$ for the temperature range $200<T<300$ °K, and as a function of $T^{4.1}$ for $70<T<200$ °K. These results are in good agreement with those published by [Yim et al., 1966].
which showed that, \( \rho \) for a p-type material prepared using vertical Bridgeman technique varied as a function of \( T^{4.5} \) for the temperature range 200 - 300 °K and approached \( T^{4.0} \) with decreasing temperature. A figure-of-merit in excess of \( 3 \times 10^{-3}/\text{degree} \) at room temperature has been reported for p-type alloys made from the same compositions.

However, these results do not imply that, the material would have a similar figure-of-merit, mainly because of the lack of knowledge of the other parameters; namely the thermal conductivity and the Seebeck coefficient. Low electrical resistivity indicates for high carriers concentration and high mobility. Important in this connection is the theoretical consideration [Ioffe, 1957], regarding the formation of solid solution alloying, which could improve the figure-of-merit of semiconductors, by decreasing the lattice thermal conductivity, without affecting the thermoelectric power and the electrical conductivity.

In general, the results indicated the possibility of good material production using melt-quenching technique, which is in practice simple, and economical.
CHAPTER THREE

3 Theoretical Modeling of a Thermoelectric Generator

3.1 Introduction

Semiconductors materials are relatively expensive, and therefore the design aim is to improve the power output per unit material. This can be achieved by optimizing the device geometrically for maximum performance for a given thermoelectric material.

Usually the design optimization is guided by matching the load resistance to achieve either the maximum conversion efficiency or the maximum power output, both of which are dependent upon the thermoelectric geometry for a given material. The optimum geometry necessary to give maximum power differs from that for maximum conversion efficiency. For thermoelectric generation, the geometry has to be optimized in a way to minimize the cost of the electricity generated. The cost of the thermoelectric module is related to the power output, while the conversion efficiency determines the running cost.

In the case of solar energy, where the input energy is essentially free, the conversion efficiency is not an overriding consideration, and the thermoelement geometry should be optimized for maximum electrical power output. This procedure has been found [Min and Rowe, 1992] to result in a design with a significant increase in the electrical power output accompanied by a minor reduction in the conversion efficiency, compared to a device optimized for maximum efficiency.

The traditional optimization method, described in chapter 2, allows only the determination of the optimum ratio of lengths to areas of the elements leg, for the minimum value of the term (Kr), however, it does not give the exact element length. It also does not consider the effect of factors such as the contact layers resistances and as well the contribution of the heat leakage to the cold junction through the space around element legs on the performance of the device.
In a real module, some heat leakage takes place between the heat source and the heat sink. For example, heat transfers from the hot plate to the cold plate through the space between the two plates around thermoelement legs (stray heat leakage) by radiation and as well by the conduction or convection of the gas particles in the space (if it is not evacuated). This heat leakage is regarded as heat losses and has to be added to the heat absorbed by the device at the hot plate, and therefore, the input energy to the device, given by equation (2.6) has to be corrected to account for this heat leakage.

The contact layers (ceramic plates and the electrical contact strips) result in thermal and electrical contact resistances at the two ends of the thermoelectric element. The thermal resistance and as well the stray heat leakage reduces the effective temperature difference across the elements, and consequently the thermoelectric power, while the contact electrical resistance adds to the internal electrical resistance of the elements legs, thus affects the maximum power point of the device.

Recently, Min and Rowe [1992] have used a procedure which considers the effect of the contact layers resistances for the optimization of thermoelement leg, though, the effect of the Thomson heat and as well the stray heat leakage are not taken into consideration. The above procedure is reasonable for devices, which are intended for low grade industrial waste heat recovery. The operating temperature range for such applications is low enough, and therefore the effect of the Thomson heat on the performance may not be significant, depending on the material used. However, at high operating temperature range, the effect of the Thomson heat term becomes significant and must be taken into consideration when predicting the performance of a thermoelectric device. Similarly, the stray heat is determined by the thermoelement length, particularly when the space between the ceramic plates is not evacuated, and therefore, should be considered in the optimization of the element leg. The radiation heat term itself is not directly given by the element length, however, they are interrelated to each other by junctions temperatures, which are dependent on both. Neglecting the above heat terms would tend to overestimate the performance of the thermoelement. This is realized from the efficiency curves versus element length,
which showed that, the conversion efficiency continues to increase with element length.

In this study, a unified theoretical model is developed which allows geometrical optimization of the thermoelectric elements and the prediction of their performance, with all the parameters discussed above taken into account. This model simulates the real conditions of a thermoelectric module, and therefore is regarded as a unified and real model.

For the sake of comparison, the results of element length optimization are used in a comparison evaluation of several thermoelectric modules. One of the modules is a commercial thermoelectric device known as (Dura TEC Thermoelectric Cooler DT 1089) manufactured by Marlow Industrial Inc.(USA), for cooling purpose, and is also suggested for power generation. The commercial device is evaluated using the current geometrical and contact layer properties, as a base case, and then with optimum element length. The other modules are assumed to have similar geometrical structure, but with various contact layers properties and optimum thermoelement lengths. All the modules are based on bismuth-telluride thermoelectric alloys doped to give the best p-type and n-type materials [Freedman, 1966].

The unified model is validated using data from an experimental investigation of the commercial module under the same operating conditions, given in Chapter 4.
3.2 A Unified Thermoelectric Model

The thermoelectric element, shown in Fig. (3.1), is used to establish a set of formulae for the unified model. The module is formed of \( N \) thermoelectric couples, connected electrically in series and thermally in parallel, and sandwiched between two thermal conducting and electrically insulating ceramic plates.

The temperature dependency of the thermoelectric material properties is considered by using the average values over the operating temperature range, i.e. the hot and the cold junctions temperatures. These have been found give results within a reasonable accuracy [Egli, 1960]. The p- and the n-type thermoelectric pellets are assumed to be of uniform cross section and equal in length.

Assuming that the Seebeck coefficients of the p-type and n-type materials at the hot junction are defined by \( S_p(h) \) and \( S_n(h) \) respectively, and at the cold junction by \( S_p(c) \) and \( S_n(c) \) respectively, the corresponding average values over the temperature range can be estimated as follows:

\[
\overline{S}_p = \frac{1}{2} (S_p(h) + S_p(c)) \quad (3.1a)
\]
\[
\overline{S}_n = \frac{1}{2} (S_n(h) + S_n(c)) \quad (3.1b)
\]
\[
\overline{S} = |\overline{S}_p| + |\overline{S}_n| \quad (3.1c)
\]
\[
\Delta S = S_{pn}(h) - S_{pn}(c) \quad (3.1d)
\]

Similarly, it can be assumed that, for the p-type and the n-type materials, the electrical resistivities at the hot junction are \( \rho_p(h) \) and \( \rho_n(h) \) respectively, and at the cold junction are \( \rho_p(c) \) and \( \rho_n(c) \) respectively. The thermal conductivities of the p- and the n-type elements at the hot junction are \( k_p(h) \) and \( k_n(h) \) respectively, and at the cold junction are \( k_p(c) \) and \( k_n(c) \) respectively. The average values can be obtained as follows:

\[
\overline{\rho}_p = \frac{1}{2} (\rho_p(h) + \rho_p(c)), \quad \overline{\rho}_n = \frac{1}{2} (\rho_n(h) + \rho_n(c)), \quad (3.2a, b)
\]
\[
\bar{k}_p = \frac{1}{2} (k_p(h) + k_p(c)), \quad \bar{k}_n = \frac{1}{2} (k_n(h) + k_n(c)) \tag{3.3a, b}
\]

\[\text{Fig. (3.1) Schematic diagram of thermoelectric module: (A) thermoelements connected electrically in series and thermally in parallel, (B) Details of junction's temperatures and contact points (c) Equivalent thermal network}\]
By introducing the average value $\overline{S}$ in equations (2.20) and (2.21), the electrical current ($I$) through the circuit, the terminal voltage ($V_{mc}$) per a couple and the power output ($P_{mc}$) per unit couple at the maximum power point as a function of thermoelement geometry are estimated as follows

$$I_m = \frac{AS \Delta T}{2pL}$$  \hspace{1cm} (3.4a)

$$V_{mc} = \frac{1}{2} \overline{S} \Delta T$$  \hspace{1cm} (3.5a)

$$P_{mc} = \frac{AS^2}{2pL} \left(\frac{\Delta T}{2}\right)^2$$  \hspace{1cm} (3.6a)

where, $A$ and $L$ are the cross-sectional area and the length of the thermoelectric element, $\rho = (\rho_p + \rho_n)/2$ is the bulk electrical resistivity of the thermoelectric material and $\Delta T$ is the temperature difference across the thermoelements.

Equations (3.4a) and (3.6a) indicate that the current and the power output approach infinity as the length of the thermoelements goes to zero. However, in an actual device this limit can not be realized, because of the thermal and electrical resistance of the contact layers [Min and Rowe, 1992]. The contact layers (ceramic plates and the electrical contact strips) result in thermal and electrical contact resistances at the two ends of the thermoelectric element. So, as mentioned earlier, the thermal resistance causes the effective temperature difference across the elements to be $\Delta T_e$ rather than $\Delta T$, as shown in Fig. (3.1B), while the contact electrical resistance adds to the electrical resistance of the elements legs.

Taking into account the contact electrical resistance of the junctions layers, the total internal resistance of the thermoelement can be expressed as follows [Ioffe, 1957]

$$r = 2r_c + 2\rho \frac{L}{A} = \frac{2\rho}{A} \left(\frac{r_c A}{\rho} + L\right) = \frac{2\rho}{A} \left(\frac{\rho_c}{\rho} + L\right)$$
A UNIFIED THERMOELECTRIC MODEL

\[ \frac{2p}{A} \left( \rho_{oc} + L \right) \]  \quad (3.7)

where, \( \rho_{oc} = \frac{\rho_c}{p} \).

The quantity \( (p_c = r_c A) \) characterizes the quality of the junction. \( r_c \propto A^{-1} \), is the contact resistance at each junction (hot or cold).

Now let us introduce the effect of the heat leakage from the hot plate to the cold plate through the space between the two plates around thermoelement legs (stray heat leakage) by radiation and as well by the conduction of the gas particles in the space.

Considering the composite section of the thermoelectric element shown in Fig. (3.1b), heat flows from the heat source through the hot ceramic plate, then through thermoelectric arms, the gas between the plates and the sealant, and finally through the cold ceramic plate to the heat sink. It can be assumed that, the thermoelectric legs and the gas filled space between the plates are at a mean temperature \( (T_m = 0.5(T_h + T_c)) \) for which their thermal properties can be evaluated. The analysis is done by considering \( T_h \) and \( T_c \) to be the temperatures of the outer surface of the hot and the cold ceramic plates respectively, and \( T_{hi} \) and \( T_{cj} \) the corresponding inner surfaces temperatures at the metallic bridges respectively.

To take account of stray heat leakage and the contact layers thermal resistance, a procedure is employed which estimates the temperature distribution across the thermoelectric device, allowing the effective temperature difference \( (\Delta T_e) \) across the hot and the cold junctions to be evaluated.

For simplicity, one-dimensional heat flow is assumed, since the ceramic plates are usually thin enough, and therefore heat flow along the plates due to local temperature gradient can be neglected compared to heat flow rate across the plates. Heat losses from the side edges are also neglected, since the heat flow is assumed to be one directional. Considering an open circuit condition, a simplified equivalent thermal network for the thermoelectric element is shown in Fig. (3.1C).
The heat flux \( Q_{FL} \) from the hot source to the heat sink through the thermoelectric device is given by

\[
Q_{FL} = \left( T_h - T_c \right) \sum R_{th}
\]

where,

\[
\sum R_{th} \text{ is the total thermal resistance to the heat flow through the thermoelectric device, given by}
\]

\[
\sum R_{th} = R_{hc} + \left( \frac{1}{R_{TE}} + \frac{1}{R_g} + \frac{1}{R_s} \right)^{-1} + R_{cc}
\]

where, \( R_{TE} \), \( R_{hc} \), \( R_{cc} \), \( R_g \) and \( R_s \) are the thermal resistances to the heat flow through thermoelectric legs, hot ceramic plate, cold ceramic plate, gas filled space and the sealant respectively, and are given as follows

\[
R_{TE} = \left( \frac{L}{A k} \right)_{TE}, \quad R_{hc} = R_{cc} = R_c = \left( \frac{L_c}{A_c k_c} \right), \quad R_s = \left( \frac{L}{A_s k_s} \right)
\]
A UNIFIED THERMOELECTRIC MODEL

\[ R_g = \left( \frac{A_g k_g}{L} + \frac{\varepsilon \alpha A_g}{2 - \varepsilon} \left( T_{lj}^2 + T_{cj}^2 \right) \left( T_{lj} + T_{cj} \right) \right)^{-1} \]

where,

\( L \) is the length of the thermoelectric arm, \( L_e \) is the thickness of the ceramic plate, \( k \) and \( A \) in each term are the thermal conductivity and the cross-section area of the heat passage respectively for each component \( k_{TE} = k_p + k_n \), \( \sigma = 5.67 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4} \) is the Stefan-Boltzmann constant, and \( \varepsilon \) is the emissivity of the ceramic plate surface. For simplicity, the hot and the cold ceramic plates surfaces are assumed to have the same emissivity, and are approximated as infinitely long parallel plates.

Under steady state and open circuit conditions, the heat flow rate through the hot and through the cold ceramic plates is the same and equals to the heat flux through the device. Thus

\[ Q_{hc} = \frac{\Delta T_{hc}}{R_{hc}} = Q_{cc} = \frac{\Delta T_{cc}}{R_{cc}} = Q_{FL} = \frac{\Delta T}{\sum R_{th}} \quad (3.10) \]

where, \( \Delta T_{hc} = (T_h - T_{lj}) \), \( \Delta T_{cc} = (T_{cj} - T_c) \) and \( \Delta T = (T_h - T_c) \) are the temperature differences across the hot ceramic plate, across the cold ceramic plate and across the thermoelectric device respectively.

From equation (3.10), temperatures at the hot and the cold contact points can be estimated as follows

\[ T_{lj} = T_h - \frac{R_c \Delta T}{\sum R_{th}} \quad (3.11) \]

\[ T_{cj} = T_c + \frac{R_c \Delta T}{\sum R_{th}} \quad (3.12) \]

The effective temperature difference \( \Delta T_e \) is given by
\[ \Delta T_e = (T_{hj} - T_{cj}) = \left( 1 - 2 \frac{R_c}{\sum R_{th}} \right) \Delta T \quad (3.13a) \]

where, the ratio of thermal resistances \( R_c / \sum R_{th} \) is obtained by rearranging equation (3.9) as follows

\[ \frac{R_c}{\sum R_{th}} = \left( 2 + \frac{L}{L_c} \frac{k_{oo}A_{oo}}{U} \right)^{-1} \quad (3.14) \]

where,

\[ U = 1 + k_{oo}A_{oo} + k_{og}A_{og} + \frac{\varepsilon A_{oo}}{2 - \varepsilon} \frac{L}{k} \left( T_{hj}^2 + T_{cj}^2 \right) \left( T_{hj} + T_{cj} \right) \quad (3.15a) \]

\[ k_{oc} = \frac{k_c}{k}, \quad k_{oo} = \frac{k_s}{k}, \quad k_{og} = \frac{k_g}{k}, \quad A_{oo} = \frac{A_s}{A}, \quad \text{and} \quad A_{og} = \frac{A_g}{A}, \]

Equation (3.15a) can be solved iteratively together with equations (3.11), (3.12) and (3.14) for \( U \), by using initial guess values for \( T_{hj} \) and \( T_{cj} \) until the solution converged. Alternatively, equation (3.15) can be solved for \( U \) by replacing values of \( T_{hj} \) and \( T_{cj} \) from equations (3.11) and (3.12) respectively. This latter solution results in an equation which also contains the ratio \( R_c / \sum R_{th} \), and which is again a function of \( U \), and is solved for \( U \) as follow (Appendix 1):

\[ U = \frac{3}{\sqrt[3]{z}} + \frac{1}{9} \frac{b^2 - 3ac}{a^2 \sqrt[3]{z}} - \frac{1}{3} \frac{b}{a} \quad (3.15b) \]

where,

\[ a = C_3^2, \quad b = 2C_3 - C_1C_3^2 - C_2, \quad c = 1 - 2 C_1C_3, \]

\[ z = \frac{1}{18a^2} \sqrt{12a^2 - 3(2bc)^2 + 54abcd + 81(a^2d^2 - 12db^3)} - \frac{b}{54a^3} (2b^3 - 9abc - 27a^2d) \]

and \( d = C_1 \),
A UNIFIED THERMOELECTRIC MODEL

where,

\[
C_1 = 1 + k_\alpha A_\alpha + k_{og} A_{og} + \frac{2\epsilon\sigma A_\alpha}{2-\epsilon} \frac{L}{k} (T_n^2 + T_e^2) T, \\
C_2 = \frac{4\epsilon\sigma A_{og}}{2-\epsilon} \frac{1}{Lk} \left( \frac{L_c}{A_{oc} k_{oc}} \right)^2 (\Delta T)^2 T \\
C_3 = 2 \frac{1}{L A_{oc} k_{oc}},
\]

Substituting for \( U \) in equation (3.14) and for the ratio \( (R_\delta/\Sigma R_{th}) \) in equation (3.13a) and rearranging, the effective temperature difference across the thermoelectric element (\( \Delta T_e \)) is given by

\[
\Delta T_e = \left( 1 + 2 \frac{L_c}{L} \frac{U}{k_{oc} A_{oc}} \right)^{-1} \Delta T
\]  
(3.13b)

Now, the current \( I_m \), the voltage \( V_m \) per a module and the power output \( P_m \) in watts per a module delivered to the matched load, given by equations (3.4a), (3.5a) and (3.6a) respectively can be modified to account for the effect of the contact layers resistances and the stray heat leakage by using the effective temperature difference \( (\Delta T_e) \) as follow:

\[
I_m = \frac{A S}{4\rho} \left( \rho_{oc} + L \right)^{-1} \Delta T_e
\]  
(3.4b)

\[
V_m = \frac{N S}{2} \Delta T_e
\]  
(3.5b)

\[
P_m = \frac{N \rho S}{8\rho} \left( \rho_{oc} + L \right)^{-1} (\Delta T_e)^2
\]  
(3.6b)

where, \( N \) is the number of couples of thermoelements in a module.
Under the matched load conditions and at given hot and cold junctions temperature, the current, the terminal voltage and the maximum power output are given by equations (3.4b), (3.5b) and (3.6b) respectively. For the calculation of the conversion efficiency, the input energy \( (Q_i) \) absorbed by the thermoelectric element at the hot junction can be determined by modifying the heat terms of equation (2.6) to account for the contact resistance and the stray heat leakage, as follow:

The conduction heat \( (Q_c) \) through thermoelement legs is given by

\[
Q_c = \frac{NAk}{L} \Delta T_e 
\]  

The Peltier heat \( (Q_p) \) dissipated at the hot junction is given by

\[
Q_p = N IS_h T_{ij} 
\]

\[
= \frac{NA\bar{S}}{4\rho} (\rho_o + L)^{-1} \left( T_h - \frac{L_e}{L} A_{oc} k_{oc} \Delta T_e \right) (\Delta T_e) 
\]  

Portion of the Thomson heat delivered to the hot junction is given by

\[
Q_t = 0.5NI\Delta \bar{S}_{pm} \bar{T} 
\]

\[
= \frac{N\bar{S}}{8\rho} (\rho_o + L)^{-1} \left( \bar{T} \Delta \bar{S} \right) \Delta T_e 
\]  

Portion of the Joule heat delivered to the hot junction is given by

\[
Q_j = 0.5NI^2r 
\]

\[
= \frac{N\bar{S}^2}{16\rho} (\rho_o + L)^{-1} (\Delta T_e)^2 
\]  

Heat leakage through the gap (space) around the thermoelectric legs shown in Fig (3.1B) from the hot junction to the cold junction \( (Q_g) \) is given by
Heat leakage through the sealant from the hot junction to the cold junction \( Q_s \) is given by

\[
Q_s = N A k \frac{k_{oc} A_{oc}}{L} \Delta T_e \quad (3.21)
\]

The energy input to the thermoelectric device can be obtained by substituting for \( Q_c \), \( Q_p \), \( Q_o \), \( Q_j \), \( Q_g \) and \( Q_s \) in equation (2.6). Using equations (3.6b) and the modified form of equation (2.6), the thermoelectric heat to electrical power conversion efficiency as a function of gross temperature difference \( \Delta T \) of the device, is given by

\[
\eta = \frac{P_M}{Q_{in}} = \frac{\Delta T}{\frac{1}{F} \left( \frac{2}{T_h} \frac{S_h}{S} \left( \frac{\Delta T}{T} \right) - \frac{\Delta S}{S} \frac{T}{T_h} + \frac{8k_D}{S^2 T_h} \frac{BD}{F} - \frac{\Delta T}{2FT_h} + E D L \frac{D_T}{S^2 T_h} \left( \frac{T_h^2 + T_c^2 + \frac{2}{V^2} \Delta T^2} \right) \right)^{-1}}
\]

where,

\[
F = \left( 1 + 2 \frac{L_c}{L} \frac{U}{k_{oc} A_{oc}} \right), \quad B = 1 + k_{os} A_{os} + k_{og} A_{og}, \quad V = F \frac{L}{L_c} \frac{A_{oc} k_{oc}}{U},
\]

\[
D = \frac{\rho_o + L}{L}, \quad E = \frac{16 \epsilon A_{po}}{2 - \epsilon}
\]

Equation (3.22) is valid when the device is operated at maximum power output and with the contact resistance at the junctions and stray heat leakage taken into account.

The set of equations from 3.7 to 3.22 form the basis of the unified model, which allows determination of optimum thermoelectric element length for the maximum...
power output and the prediction of the device performance with the effect of all the parameters involved accounted for.

In practice, a range of standard modules having specified characteristics are available from manufacturers, and due to economic reasons, one may have to select the one which best suits the design requirement. In order to meet the power requirement using standard modules, it may become necessary to interconnect a number of modules in series and in parallel to meet the voltage and the load current specifications.

The relationship between the number of modules connected in series and in parallel with the load voltage, the load current and the power delivered to the load, are given by the following expressions

\[ I_L = N_p I_M \] (3.4c)

\[ V_L = N_s V_M \] (3.5c)

\[ P_L = N_s N_p P_M \] (3.7c)

where, \( V_L \), \( I_L \) and \( P_L \) are the load voltage, current and the power requirement respectively. \( V_M \), \( I_M \), and \( P_M \) are the voltage at maximum power from the module, the current and the power respectively per a module. \( N_s \) and \( N_p \) are the number of modules connected in series and in parallel respectively.

For an example, using a module which has 127 couples and produces 1 watt at 1 volt at a temperature difference of 75 °C, the arrangement to provide 110 watts at 18 volts will involve 7 strings of 18 modules in series.
3.3 Result of the unified Model

3.3.1 Optimum Thermoelement length

Economic use of the thermoelectric material suggests that, both A and L should be as small as possible, provided that their required ratio to minimize the parameter $K_T$ is maintained. The reduction of $L$ is expedient as long as the junction resistance does not become comparable with the resistance of the arms.

In order to avoid an appreciable temperature drop across contact points and the outer surface of the ceramic plates, the following analogous condition should be satisfied: $L \gg L_c/k_{oc}$, that the heat flow through the contact layer should be high enough compared to that across the thermoelement.

The optimum length of the thermoelectric pellets can be obtained by plotting the power delivered to the matched load against the length ($L$). The cross-section area of the thermoelectric element pellets and the number of couples in a module needed to meet the requirement of a given power can be estimated using equations (3.4b) and (3.5b) respectively.

The calculations are based upon the temperature dependent properties of bismuth telluride materials, and the contact layer properties available in the literature [Freedman, 1966; Min & Rowe 1992]. In order to outline the significance of the effect of the contact layer resistance on the optimum length, a range of contact thermal and electrical resistances are also investigated.

The p-type material is made of 25%Bi$_2$Te$_3$ - 75%Sb$_2$Te$_3$ with 2.3% excess tellurium and the n-type material is made of 75% Bi$_2$Te$_3$ - 25%Bi$_2$Se$_3$ doped with CuBr. The properties of these alloys are shown in Fig. (3.2).

The common ceramic plates materials used in construction of thermoelectric modules are aluminum oxide (alumina) or beryllium oxide, due to their good thermal conductivity and electrical isolation properties [Marlow and Burke, 1995]. They are
RESULTS OF THE UNIFIED MODEL

categorized as aluminum oxide ceramics (AC), beryllium oxide ceramics (BC) or a combination of aluminum and beryllium oxide ceramics (AB).

The metallic strips are made of copper conductors affixed to the ceramics by a variety of methods. The most common is to print and fire a circuit pattern using molymanganese or copper. The copper conductors are then soldered to this circuit pattern [Marlow and Burke, 1995]. The elements are soldered to the electrical conductors using bismuth tin solder (58%Bi-42%Sn; melting point = 138 °C) or tin lead solder (63%Sn-37%Pb; melting point = 183 °C). The junction quality is determined by the quality of the contact surface and soldering.

The range of operational temperature, ceramic plates and contact properties considered for geometry optimization are as follows:

The hot junction temperature \( T_h \) is kept at 127 °C, while the cold junction temperature \( T_c \) varied from 27 to 67 °C. The thickness of the ceramic plate \( L \) is 0.07 cm, and the thermal conductivity \( k_c \) varied from 0.065 to 0.26 W cm\(^{-1}\) deg\(^{-1}\), equivalent to \( k_c \) between 2.5 and 20 approximately.

The contact electrical resistivity \( \rho_c \) varied from 1.0E-04 to 1.0E-05 Ω-cm, equivalent to resistivity ratio \( \rho_{oc} \) approximately between 0.1 and 0.01.

Other surface area and conductivity ratios are considered to be equal to those of the commercial thermoelectric modules \( k_{oc} = 0.0385, A_{oc} = 2.0, A_{oc} = 0.5, k_{og} = 0.011, A_{og} = 0.5 \).
RESULTS OF THE UNIFIED MODEL

Fig. (3.2)  Thermoelectric properties of bismuth telluride thermoelement
(A) P-type material (B) N-type material
The computation of the unified model is performed using a solution algorithm written for Turbo Pascal, presented in Appendix 2. Results of the unified model computation for hot junction temperature of 127 °C, with $k_\infty = 5$ and $p_\infty = 0.1$, for a varying cold junction temperature are presented in Fig. (3.3). As can be seen, the overall temperature difference across the device essentially determines the performance, however it does not affect the optimum value of the thermoelement length for given contact properties.

Fig. (3.4) illustrates the variation of the power output per unit area with the length of the thermoelement for varying thermal and electrical contact layer resistances at a temperature difference of 70 °C.

Fig (3.5) shows the variation of the maximum power output and the corresponding optimum thermoelement length with the contact layer resistances. The importance of using good contact layers is clearly shown.

It can be observed that, the optimum thermoelement length which gives the maximum power varies substantially with both the electrical and the thermal contact resistances. This evidently indicates the sensitivity of the contact resistances to the length dependence of the power output. High contact conductivities (both electrical and thermal) allow the use of a very short thermoelement, while achieving high power per unit area.

Poor contact layers result in a device that requires long thermoelement and yields low power per unit area. This is because, low thermal conductivity results in a high temperature gradient at the contact points, and a consequent drop in the effective temperature difference between the two junctions. On the other hand, contact layers with high electrical resistivity mean that, the internal resistance of the device will be dominated by the contact resistance, and a consequent drop in the power output.

Figures (3.6), (3.7) and (3.8) show the variation of the conversion efficiency, the current per unit area and the voltage per unit couple respectively with the length of the thermoelement for varying contact resistances at hot junction temperature of 127
RESULTS OF THE UNIFIED MODEL

°C and cold junction temperature of 57 °C. Sample results of the optimum element length and the corresponding device performance for the cases investigated are presented in table (3.1). For the sake of comparison, similar samples results at the maximum efficiency are presented in table (3.2).

It can also be observed that, the maximum efficiency is obtained at element length longer than that which gives the maximum power. So by using element length to give maximum power, will resulted in a few percentage drop in the conversion efficiency and vice versa. For example, at a temperature difference of 70 °C the optimum length for maximum power is 0.075 cm, and the optimum length which gives maximum efficiency is 0.13 cm (about 1.73 times longer). Using optimum length for maximum power (0.075 cm) will result in a 21% increase in the power output, accompanied by 8% drop in the efficiency compared to optimum length for maximum efficiency conditions. For improved contact layers, the above figures will even be higher. Therefore, the power output per unit material and hence the cost effectiveness can be improved substantially.

For the sake of a comparison, Fig. (3.9) and Fig. (3.10) compares results of the unified model with results of a model which does not take the Thomson heat and the stray heat leakage into consideration [Min & Rowe, 1992], for, \( k = 2.5 \), \( \rho = 0.1 \), and hot and cold junctions temperatures of 127 °C and 27 °C respectively. It can clearly be seen that neglecting these two heat terms results in a longer thermoelement length and overestimation of device performance. More important is the efficiency curve of the existing models, which shows a continuous increase of the efficiency with thermoelement length. This is because, the stray heat is determined by the effective temperature difference which is dependent on the thermoelement length, and both determine the rate of heat flux through the device. Neglecting the stray heat leakage will affect the heat flux. The big difference between the unified model and the previously published models at high element length is due to the fact that, thermal resistance to conduction heat flow increases with element length, so with a model that neglects the stray heat, particularly radiation heat exchange, high temperature difference is predicted and thus high power output. While in a real device, radiation
heat exchange dominates as the temperature difference increases, and thus lower power output. This fact is the reason that the unified model presented here, is more realistic.

Table (3.3) compares the results at optimum thermoelement length for the unified model with and without stray heat leakage and Thomson heat being considered. As can be seen, neglecting these heat terms results in an overestimation of the optimum element length, power output and the conversion efficiency by about 18%, 33% and 57% respectively, compared to the predictions of the unified model.

Assuming that all thermoelements in a module are connected in series, and that all modules are connected in series, the number of couples and the cross-section area required to meet a given load requirement (110 Watts at 18 volts) as a function of thermoelement length for a varying contact resistance are shown in Fig. (3.11) and Fig. (3.12) respectively. The minimum cross section area and the number of couples obtained by using this model can be recalculated and adjusted according to the connection (series and/or parallel) as discussed earlier. Most important is that, the minimum number of couples and the cross section area can be further reduced by using improved contact layers, which in turn will result in an improved cost effectiveness of the thermoelectric device.

Overall, it can be concluded that, the performance of thermoelectric generators can be improved substantially by using better contact layers and optimum element length based on the maximum power output. Results also indicated the importance of the parameters such as the stray heat leakage through the device in the optimizations of the thermoelement.
RESULTS OF THE UNIFIED MODEL

Fig. (3.3) Results of the unified model solution versus thermoelement length for varying junction temperature difference for $T_h = 127 \, ^\circ C$, $k_\infty = 5$ and $\rho_\infty = 0.1$; (A) Power per unit area, (B) Conversion efficiency, (C) Voltage per unit couple, (D) Current per unit area.
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Fig. (3.4) Power output as a function of the thermoelement length for varying contact layer resistance: (A) $k_{oc} = 2.5$, (B) $k_{oc} = 5$, (C) $k_{oc} = 10$ and (D) $k_{oc} = 20$. 

---

$w_U = 0.5$

$PLO = 0.5$

$Thermoelement length (cm)$

$Power per unit area (W/cm^2)$
Fig. (3.5) Effect of the electrical and the thermal contact layers resistances on the maximum power and the optimum thermoelement length
(A) Thermoelement length, (B) Power output.
Fig. (3.6) Conversion efficiency versus the thermoelement length for varying contact layer resistance, $\Delta T = 70\,^\circ\text{C}$: (A) $k_{oc} = 2.5$, (b) $k_{oc} = 5$, (C) $k_{oc} = 10$ and (D) $k_{oc} = 20$. 
Fig. (3.7) Current per unit area versus the thermoelement length for varying contact layer resistance $\Delta T = 70 \, ^\circ\text{C}$: (A) $k_c = 2.5$, (b) $k_c = 5$, (C) $k_c = 10$ and (D) $k_c = 20$. 

- $r_{oc} = 0.1$
- $r_{oc} = 0.05$
- $r_{oc} = 0.025$
- $r_{oc} = 0.01$
RESULTS OF THE UNIFIED MODEL

Fig. (3.8) Variation of the voltage with thermoelement length for varying contact layers thermal conductivity.

Fig. (3.9) Comparison between power out of the unified model and a model which neglects the contribution of Thomson heat and the stray heat leakage Ref. [Min & Rowe, 1992] for \( k_{\text{oc}} = 2.5, \rho_{\text{oc}} = 0.1, T_h = 127^\circ \text{C} \) and \( T_c = 27^\circ \text{C} \).
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**Fig. (3.10)** Comparison between the efficiency of the unified model with and without the contribution of Thomson heat and the stray heat leakage for $k_{oc} = 2.5$, $\rho_{oc} = 0.1$, $T_h = 127^\circ C$ and $T_c = 27^\circ C$.

**Fig. (3.11)** Number of couples required to meet 110 W at 18 voltage load requirement versus thermoelement length for varying contact layers thermal conductivity.
Fig. (3.12) Variation of the cross section area of an element required for 100 W at 18 volts load with thermoelement length for varying contact layers resistance: (A) $k_{cc} = 2.5$, (B) $k_{cc} = 5$, (C) $k_{cc} = 10$ and (D) $k_{cc} = 20$. 

- $r_{oc} = 0.1$
- $r_{oc} = 0.05$
- $r_{oc} = 0.025$
- $r_{oc} = 0.01$
Table (3.1) Optimum thermoelement length and the corresponding maximum power and conversion efficiency at $\Delta T = 70 \, ^\circ C$, for varying contact layers properties.

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.075</td>
<td>0.055</td>
<td>0.04</td>
<td>0.035</td>
</tr>
<tr>
<td>0.05</td>
<td>0.06</td>
<td>0.04</td>
<td>0.03</td>
<td>0.025</td>
</tr>
<tr>
<td>0.025</td>
<td>0.04</td>
<td>0.03</td>
<td>0.02</td>
<td>0.015</td>
</tr>
<tr>
<td>0.010</td>
<td>0.03</td>
<td>0.02</td>
<td>0.01</td>
<td>0.0095</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.17</td>
<td>0.26</td>
<td>0.37</td>
<td>0.46</td>
</tr>
<tr>
<td>0.05</td>
<td>0.23</td>
<td>0.4</td>
<td>0.58</td>
<td>0.78</td>
</tr>
<tr>
<td>0.025</td>
<td>0.31</td>
<td>0.56</td>
<td>0.88</td>
<td>1.22</td>
</tr>
<tr>
<td>0.010</td>
<td>0.4</td>
<td>0.78</td>
<td>1.31</td>
<td>2.08</td>
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</table>

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.58</td>
<td>0.64</td>
<td>0.69</td>
<td>0.72</td>
</tr>
<tr>
<td>0.05</td>
<td>0.77</td>
<td>0.87</td>
<td>0.92</td>
<td>0.98</td>
</tr>
<tr>
<td>0.025</td>
<td>0.87</td>
<td>1.05</td>
<td>1.10</td>
<td>1.20</td>
</tr>
<tr>
<td>0.010</td>
<td>1.01</td>
<td>1.21</td>
<td>1.30</td>
<td>1.35</td>
</tr>
</tbody>
</table>
Table (3.2) Sample results of the maximum conversion efficiency and the corresponding element length and power and at $\Delta T = 70 \, ^\circ C$.

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.63</td>
<td>0.77</td>
<td>0.87</td>
<td>0.92</td>
</tr>
<tr>
<td>0.05</td>
<td>0.82</td>
<td>1.01</td>
<td>1.15</td>
<td>1.24</td>
</tr>
<tr>
<td>0.025</td>
<td>0.98</td>
<td>1.23</td>
<td>1.43</td>
<td>1.56</td>
</tr>
<tr>
<td>0.010</td>
<td>1.15</td>
<td>1.48</td>
<td>1.74</td>
<td>1.93</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
<th>5</th>
<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.13</td>
<td>0.11</td>
<td>0.10</td>
<td>0.095</td>
</tr>
<tr>
<td>0.05</td>
<td>0.10</td>
<td>0.09</td>
<td>0.08</td>
<td>0.07</td>
</tr>
<tr>
<td>0.025</td>
<td>0.08</td>
<td>0.06</td>
<td>0.055</td>
<td>0.05</td>
</tr>
<tr>
<td>0.010</td>
<td>0.06</td>
<td>0.05</td>
<td>0.045</td>
<td>0.04</td>
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</table>

<table>
<thead>
<tr>
<th>$\rho_{oc}$</th>
<th>$K_{oc} = 2.5$</th>
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<th>10</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>0.14</td>
<td>0.23</td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td>0.05</td>
<td>0.20</td>
<td>0.33</td>
<td>0.45</td>
<td>0.56</td>
</tr>
<tr>
<td>0.025</td>
<td>0.27</td>
<td>0.48</td>
<td>0.70</td>
<td>0.86</td>
</tr>
<tr>
<td>0.010</td>
<td>0.35</td>
<td>0.64</td>
<td>1.00</td>
<td>1.24</td>
</tr>
</tbody>
</table>

Table (3.3) Comparison between the results of the unified model and a model which neglects the contribution of Thomson heat and the stray heat leakage [Min & Rowe, 1992] for $k_{oc} = 2.5$, $\rho_{oc} = 0.1$, $T_b = 127 \, ^\circ C$ and $T_c = 27 \, ^\circ C$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unified model</th>
<th>[Min &amp; Rowe, 1992]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optimum thermoelement length (cm)</td>
<td>0.075</td>
<td>0.0915</td>
</tr>
<tr>
<td>Maximum power (W/cm²)</td>
<td>0.334</td>
<td>0.499</td>
</tr>
<tr>
<td>Efficiency at maxim power point (%)</td>
<td>0.913</td>
<td>2.163</td>
</tr>
</tbody>
</table>
3.3.2 Comparative Performance of Different Modules

The comparative results presented here are obtained from the unified model for five different thermoelectric modules. The first module is the commercial module with its current geometrical and contact layer properties. The second module is similar to the first one, but with optimum thermoelement length. The other three modules are assumed to have optimum lengths and with varying contact layer properties (improved). These results give a general comparative idea of the performance of thermoelectric devices under various design conditions, including the contact layer resistance and the thermoelement length. The results are validated using data from an experimental investigation to the commercial thermoelectric device.

The commercial device considered for comparative evaluation, is principally developed for Peltier cooling as mentioned earlier, however, it has been also suggested for power generation [Burke, 1983]. Materials used to prepare this device has been claimed to have the highest thermoelectric figure-of-merit. The device is a single stage thermoelectric module comprising of a matrix of 127 thermoelectric couples (p- and n-type) sandwiched between two aluminum oxide plates and connected in series using eutectic BiSn solder which melts approximately at 138 °C. Therefore, the maximum operation temperature of the device is limited to this temperature range. The space between the two ceramic plates was filled with dry nitrogen gas and sealed using sealant (RTV) to improve its reliability in condensing environments. The details of the geometrical description and the technical specifications of the device are given in Table (3.4) below.

Table (3.4) Specification of DuraTEC Thermoelectric Cooler DT1089

<table>
<thead>
<tr>
<th>Property</th>
<th>AC Ceramic</th>
<th>RTV Sealant</th>
<th>TE Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm)</td>
<td>40&lt;sub&gt;MD&lt;/sub&gt;</td>
<td>-</td>
<td>1.4&lt;sub&gt;MD&lt;/sub&gt;</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>40&lt;sub&gt;MD&lt;/sub&gt;</td>
<td>1.5&lt;sub&gt;MD&lt;/sub&gt;</td>
<td>1.2&lt;sub&gt;MD&lt;/sub&gt;</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>0.7&lt;sub&gt;MD&lt;/sub&gt;</td>
<td>1.5&lt;sub&gt;MD&lt;/sub&gt;</td>
<td>1.2&lt;sub&gt;MD&lt;/sub&gt;</td>
</tr>
<tr>
<td>k (Wcm&lt;sup&gt;-1&lt;/sup&gt;deg&lt;sup&gt;-1&lt;/sup&gt;)</td>
<td>0.065&lt;sup&gt;MR&lt;/sup&gt;</td>
<td>0.001&lt;sup&gt;MI&lt;/sup&gt;</td>
<td>0.013&lt;sup&gt;FR&lt;/sup&gt;</td>
</tr>
</tbody>
</table>

<sub>MD = Measured, MR = [Min, Gao, R 1992], FR = [Freedman, 1966.]</sub>
The thermoelectric properties used for the calculation are based on bismuth-telluride thermoelectric alloys doped to give the best p-type and n-type materials [Freedman, 1966]. The typical values of the temperature dependent thermoelectric properties of these materials are given in Fig. (3.2). The temperature variation of thermoelectric properties are taken into account by using average values over the operating temperature range.

The geometrical and the other technical parameters of the five modules which are considered for comparative evaluation are presented in table (3.5).

<table>
<thead>
<tr>
<th>MODULE</th>
<th>Length(mm)</th>
<th>$k_\text{oC}$</th>
<th>$\rho_\text{oC}$</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-1</td>
<td>1.4</td>
<td>2.5</td>
<td>0.1</td>
<td>Commercial module</td>
</tr>
<tr>
<td>Case-2</td>
<td>0.75</td>
<td>2.5</td>
<td>0.1</td>
<td>Case-1, $L$(optimum)</td>
</tr>
<tr>
<td>Case-3</td>
<td>0.4</td>
<td>10</td>
<td>0.1</td>
<td>$k_\text{c}$ improved, $L$(optimum)</td>
</tr>
<tr>
<td>Case-4</td>
<td>0.3</td>
<td>2.5</td>
<td>0.01</td>
<td>$\rho_\text{c}$ improved, $L$(optimum)</td>
</tr>
<tr>
<td>Case-5</td>
<td>0.1</td>
<td>10</td>
<td>0.01</td>
<td>both $k_\text{c}$ and $\rho_\text{c}$ improved, $L$(optimum)</td>
</tr>
</tbody>
</table>

The performance of the five modules are predicted over a wide temperature range using the unified model described in section 3.2. The power delivered to the matched load, the terminal voltage, the current and the conversion efficiency as function of junctions temperatures for the five modules are given in figures (3.13), (3.14), (3.15) and (3.16) respectively. A representative sample of the results for the five modules for hot junction temperature 127 °C and cold junction temperature 27 °C are presented in table (3.6).

<table>
<thead>
<tr>
<th>Module</th>
<th>Power (W)</th>
<th>Efficiency (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case-1</td>
<td>0.22</td>
<td>0.63</td>
</tr>
<tr>
<td>Case-2</td>
<td>0.26</td>
<td>0.58</td>
</tr>
<tr>
<td>Case-3</td>
<td>0.57</td>
<td>0.69</td>
</tr>
<tr>
<td>Case-4</td>
<td>0.61</td>
<td>1.01</td>
</tr>
<tr>
<td>Case-5</td>
<td>2.01</td>
<td>1.30</td>
</tr>
</tbody>
</table>
Fig. (3.13)  Power output as a function of junctions temperatures for the different modules
Fig. (3.14) Voltage as a function of junctions temperatures for the different modules
Fig. (3.15) Current as a function of junctions temperatures for the different modules
Fig. (3.16) Efficiency as a function of junctions temperatures for the different modules
Results in table (3.6) show that, using optimum thermoelement length (about 46% shorter than the current Peltier element length) with present contact layers properties, increases the output power by about 18% at 70 °C temperature difference. This improvement in the power output is accompanied by a decline in the efficiency; about 8%. However, the power output from a thermoelectric module can be improved substantially by using good contact layers and optimizing the element length, as shown in table (3.6), and in Fig. (3.17).

The conversion efficiency, however can only be improved by reducing the electrical resistance of the contact layers, as shown in Fig. (3.18).

Fig. (3.19) compares the variation of the gross temperature difference (ΔT) and the effective temperature difference (ΔTe) for the commercial module (case-1) and a module with improved thermal contact layers and optimum thermoelement length (case-3), with the hot junction temperature at cold junction temperature 27 °C. The difference between the two effective temperature differences is clearly shown.

Fig. (3.17) Variation of the power delivered to the matched load with the hot junction temperature for the five modules at cold junction temperature 27 °C.
Fig. (3.18)  Variation of the conversion efficiency with hot junction temperature for the five modules at cold junction temperature 27 °C.

Fig. (3.19)  Variation of the effective and the gross temperature differences across the thermoelement with the hot junction temperature, for cold junction temperature 27 °C.
Fig. (3.20) compares the relative magnitudes of the heat transfer terms associated with the thermoelectric generator under the matched load as functions of junctions temperatures for the commercial module. The results are presented in two groups due to the large difference in the range of the two groups. It can be seen that, though, the Thomson heat and the Joule heating represent only a small fractions of the heat supplied to the device at the hot junction, compared to other heat terms, Thomson heat contribution is very close to that of Joule heating. Since Joule heating is one of the major parameters of thermoelectric design optimization, this emphasizes that, Thomson heat is not of less concerns. However, the stray heat leakage is the most dominant heat term associated with the thermoelectric device as shown in Fig. (3.20A). This heat term dominates the heat transfer process particularly at high temperature difference.
Fig. (3.20) Percentage of the different heat transfer terms associated with a thermoelectric module for the commercial module as function of hot junction temperatures, for cold junction temperature 27 °C. [A] Conduction through element leg, Peltier heat and heat flux through the gap. [B] Joule heat, Thomson heat and conduction through the sealant.
3.4 Conclusion

A Unified theoretical model is developed, which allows geometrical optimization of the thermoelement leg and prediction of the optimum device performance in power generation mode. The model is regarded as unified and more realistic because it considers the effect of all the parameters that contribute to the heat transfer process associated with thermoelectric device in power generation mode, some of which have been ignored in previous models. The optimization of the element length performed here is based on maximized power output from the device.

The unified model is used to optimize the thermoelement leg under various contact layer properties, and to predict the theoretical performance of a commercial Peltier cooling device in power generation mode. This is compared with four modules based on optimum thermoelement length and with varying (improved) contact layer resistances, in power generation mode. The comparison indicated that, the performance of the commercial Peltier device is inferior to those based on optimum element length, particularly those with improved contact layers. This could be attributed to the fact that the commercial Peltier device has been optimized for maximum efficiency when operated as a cooler. This emphasizes the importance of optimizing thermoelectric generators on the basis of maximizing power, rather than efficiency, particularly for designs proposed for solar energy applications where the cost of the energy input is free.

The results also indicated the importance of using devices of optimum length and the improvement of the contact layers properties. This will improve the power per unit material and consequently the cost effectiveness of these devices. Result also showed that the unified model simulates the actual energy transfer mechanisms associated with thermoelectric device in a manner that gives improved agreement with experimental results.
CHAPTER FOUR

4 Experimental Evaluation of Thermoelectric Device

4.1 Introduction

In this chapter a thermoelectric cooling device manufactured by Marlow Industries Inc. in USA is evaluated experimentally in power generation mode. Although, the device was primarily developed for Peltier cooling, it has been considered suitable also for power generation. The prime goal of the experiment is to demonstrate the performance of the current Peltier devices to produce electrical power when operated in power generation mode. It also allowed validation of the performance of the commercial Peltier device predicted using the unified thermoelectric model presented in Chapter 3.

Five modules of the type (DuraTEC thermoelectric cooler DT1089) were purchased from the Marlow Industries Incorporation for test purposes. Description and the technical specifications of the module are given in Chapter 3.

4.2 Experiment Set-up & Measurements

The thermoelectric device DT1089 was assembled in a test rig as shown schematically in Fig. (4.1). The device was attached to a liquid heat exchanger (heat sink) of a rectangular shape, made of aluminum plate. Prior to the mounting, the device was prepared by cleaning the outer surface of the cold ceramic plate using acetone to allow good thermal contact to the heat sink surface. A thin layer of thermal grease is then applied gently on the contact surfaces. The outer surface of the hot ceramic plate was painted black to improve the absorptivity of the incident radiation.

The heat source was maintained by concentrating light from a 175 watts infrared heat lamp using a conical aluminum concentrator as shown in the Fig. (4.1C). The conical concentrator which has a concentration factor of approximately 6X was designed to
provide a uniform radiation on the surface of the device. It has been possible with this concentration factor to obtain a radiant power of about 20895 watts per square meter on the surface of the ceramic plate (about 33 watts). This level of radiation can be obtained using solar concentration ratio in the vicinity of 20.

**Fig. (4.1A)** Schematic description of the experimental set-up

**Fig. (4.1B)** Details of the liquid heat exchanger and the mounting surface
A variable load resistance was connected across the terminals of the device to allow measurement of the current-voltage characteristics of the device by changing the load resistance.

The cold junction temperature was maintained by circulating cold water through the liquid heat exchanger. A flow indicator (rotameter) was used for the measurement of the water flow rate which was controlled by hand operated valves (V1 & V2). The inlet and outlet temperatures of the cooling water were measured using two thermocouples inserted into the inlet and outlet of the liquid heat exchanger.

The hot junction temperature of the device was measured by a (T-type) thermocouple attached to the outer surface of the ceramic plate. The measurements are verified using an infrared sensing thermometer. The cold junction temperature was also measured by a (T-type) thermocouple mounted between the outer surface of the cold ceramic plate and the liquid heat exchanger via a V-groove made into the heat exchanger skin.

In order to reduce the heat leakage to the surrounding environment, the whole assembly was insulated on all the sides except the hot ceramic surface which was exposed to the concentrated light.
The test was carried out for the hot junction and the cold junction temperatures varied between 77 °C to 127 °C and 27 °C to 62 °C respectively. The terminal voltage and the current were measured as a function of junctions temperatures. The current-voltage characteristics of the device were obtained by varying the load resistant (R) from open circuit to short circuit. The maximum electrical power output was obtained from the measurements of the current and the voltage, while the thermal heat rejected from the cold junction was estimated from measurements of the flow rates and the temperature difference between the outlet and the inlet fluid temperatures. The input energy is the concentrated light, and is measured on the plane of the hot ceramic front surface.
4.3 Results and Discussions

By analogy to the current-voltage characteristics curves of photovoltaic cells, which indicate the quality of the cell to produce a useful electrical power, the current-voltage relationship was also used to describe the quality of a thermoelectric element.

In Fig. (4.2) are shown the current-voltage characteristics of the device with varying hot and cold junctions temperatures. As can be seen, the current-voltage characteristics of the thermoelectric device in power generation mode show a linear relationship from the short circuit current to the open circuit voltage, basically, due to the series nature of the internal resistance associated with the device.

Figures (4.3) and (4.4) show the electrical power output versus the terminal voltage for varying hot and cold junctions temperatures. As can be seen, the power curves are parabolic, with maximum power at almost half the open circuit voltage. This matches very well with equation (2.21).

Fig. (4.5) shows a detailed current-voltage and power curve for hot and cold junctions temperatures of 127 °C and 27 °C respectively. At 100 °C temperature difference, the open circuit voltage and the short circuit current are 2 volts and 0.6 Ampere respectively. Unlike photovoltaic modules, because of the linear voltage-current relationship, the maximum power of the thermoelectric module is obtained at half the open circuit voltage, i.e. corresponding to fill factor of 0.5 only, regardless of the operation conditions. This emphasis the role which the internal resistance plays on the performance of the thermoelement, and supports the idea of maximizing the power output as a function of thermoelectric material, in effect to improve the energy cost per unit thermoelectric material.

Fig. (4.6) shows the variation of the maximum power delivered to the matched load, the voltage, the current and the efficiency at the maximum power point with the hot junction temperature for a varying cold junction temperature. While the voltage at the maximum power continues to increase with the temperature difference, the current at
the maximum power increases rapidly, reaches a maximum value between 70 - 90 degree Celsius and then declines with increasing junction temperature difference. As a result the power delivered to the matched load and consequently the conversion efficiency follow the same pattern. The maximum test temperature was limited by the temperature of the solder, i.e. 138 degree Celsius. The maximum power output obtained at 100 °C temperature difference is about 0.3 watts with solar to electricity conversion efficiency of about 0.9%.

It is concluded that, the performance of the current commercial Peltier cooling device is limited by a temperature difference of about 75 degree Celsius, though, the voltage continues to increase above this temperature difference, mainly because, of its temperature dependency. The maximum limiting temperature difference is known as the maximum no-load temperature difference, when the device is operated as a cooler. One of the factors that determines this maximum temperature difference is the thermoelectric material itself, and its electron transport properties. The material may change it status and become intrinsic after a certain temperature. Other factor which is not directly related to the thermoelectric material itself, but limit the maximum operating temperature, is the solder material, which in this case is limited to 138 °C.

Tables (4.1) and (4.2) preset sample results of the power delivered to the matching load and the corresponding conversion efficiency of the device in power generation mode for varying junction temperatures.
### Experimental Results of a Thermoelectric Device

#### Table (4.1) Power delivered to the matched load in watts

<table>
<thead>
<tr>
<th>$T_c$ $^\circ$C</th>
<th>Hot junction temperature $^\circ$C</th>
<th>77</th>
<th>87</th>
<th>97</th>
<th>107</th>
<th>117</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>0.158</td>
<td>0.190</td>
<td>0.238</td>
<td>0.274</td>
<td>0.288</td>
<td>0.301</td>
<td></td>
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<tr>
<td>32</td>
<td>0.140</td>
<td>0.179</td>
<td>0.225</td>
<td>0.258</td>
<td>0.280</td>
<td>0.290</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.116</td>
<td>0.171</td>
<td>0.218</td>
<td>0.250</td>
<td>0.270</td>
<td>0.281</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>0.104</td>
<td>0.163</td>
<td>0.209</td>
<td>0.241</td>
<td>0.261</td>
<td>0.268</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>0.089</td>
<td>0.139</td>
<td>0.198</td>
<td>0.232</td>
<td>0.252</td>
<td>0.259</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>0.080</td>
<td>0.129</td>
<td>0.180</td>
<td>0.216</td>
<td>0.245</td>
<td>0.249</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.071</td>
<td>0.121</td>
<td>0.158</td>
<td>0.203</td>
<td>0.235</td>
<td>0.239</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>0.064</td>
<td>0.111</td>
<td>0.146</td>
<td>0.192</td>
<td>0.218</td>
<td>0.227</td>
<td></td>
</tr>
</tbody>
</table>

#### Table (4.2) The conversion efficiency at the maximum power point

<table>
<thead>
<tr>
<th>$T_c$ $^\circ$C</th>
<th>Hot junction temperature $^\circ$C</th>
<th>77</th>
<th>87</th>
<th>97</th>
<th>107</th>
<th>117</th>
<th>127</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>0.48%</td>
<td>0.57%</td>
<td>0.72%</td>
<td>0.83%</td>
<td>0.87%</td>
<td>0.91%</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>0.42%</td>
<td>0.54%</td>
<td>0.68%</td>
<td>0.78%</td>
<td>0.85%</td>
<td>0.88%</td>
<td></td>
</tr>
<tr>
<td>37</td>
<td>0.35%</td>
<td>0.52%</td>
<td>0.66%</td>
<td>0.75%</td>
<td>0.81%</td>
<td>0.85%</td>
<td></td>
</tr>
<tr>
<td>42</td>
<td>0.31%</td>
<td>0.49%</td>
<td>0.63%</td>
<td>0.73%</td>
<td>0.79%</td>
<td>0.81%</td>
<td></td>
</tr>
<tr>
<td>47</td>
<td>0.27%</td>
<td>0.42%</td>
<td>0.60%</td>
<td>0.70%</td>
<td>0.76%</td>
<td>0.78%</td>
<td></td>
</tr>
<tr>
<td>52</td>
<td>0.24%</td>
<td>0.39%</td>
<td>0.54%</td>
<td>0.65%</td>
<td>0.74%</td>
<td>0.75%</td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.21%</td>
<td>0.37%</td>
<td>0.48%</td>
<td>0.61%</td>
<td>0.71%</td>
<td>0.72%</td>
<td></td>
</tr>
<tr>
<td>62</td>
<td>0.19%</td>
<td>0.33%</td>
<td>0.44%</td>
<td>0.58%</td>
<td>0.66%</td>
<td>0.69%</td>
<td></td>
</tr>
</tbody>
</table>
EXPERIMENTAL RESULTS OF A THERMOELECTRIC DEVICE

Fig. (4.2) Current-voltage characteristics of the thermoelectric device in Power generation mode for varying hot and cold junctions temperatures

* $T_c = 27$  \( \triangle \) $T_c = 37$  \( \circ \) $T_c = 47$  \( + \) $T_c = 57 \, ^\circ$C
Fig. (4.3) Electrical power output versus the terminal voltage for varying hot and cold junction temperature
EXPERIMENTAL RESULTS OF A THERMOELECTRIC DEVICE

![Graph of Power versus Voltage for different hot junction temperatures](image1)

**Tc = 27 °C**

**Fig. (4.4)** Electrical power output versus the terminal voltage for varying hot junction temperature and cold junction temperature 27 °C

![Graph of Current-Voltage characteristics and Power output](image2)

**Fig. (4.5)** Current-Voltage characteristics and the power output for hot and cold junctions temperatures of 127 °C and 27 °C respectively
EXPERIMENTAL RESULTS OF A THERMOELECTRIC DEVICE

Fig. (4.6) Performance of the device at maximum power point
(A) Power (B) Terminal voltage (C) Current (D) Efficiency

* Tc = 27  △ Tc = 37  ○ Tc = 47  + Tc = 57 °C
4.4 Validation of the Unified Theoretical Model

Chapter 3 gives results for the theoretical performance of the commercial Peltier cooling device in power generation mode in comparison with typical other modules based on optimum thermoelement length and improved contact layer resistance. The comparison indicated that, the performance of the commercial Peltier device is inferior to those based on optimum element length, particularly those with improved contact layers. This could be attributed to the fact that, the commercial Peltier device has been optimized for maximum efficiency when operated as a cooler.

In this section a validation of the unified model used for prediction of the theoretical results is given. Data has been collected as part of an experimental investigation of the commercial Peltier device in power generation mode. In this regard Fig. (4.7) and Fig. (4.8) compare the predicted and the measured values of the power delivered to the matched load and the corresponding conversion efficiencies respectively as function of the hot junction temperature, for a cold junction temperature of 27°C.

![Graph](image)

**Fig. (4.7)** Power delivered to the matched load versus hot junction temperature at cold junction temperature 27°C.
VALIDATION OF THE UNIFIED MODEL

It can be seen that, whilst the predicted power output (labeled Model) increases continuously with junction temperature up to the maximum temperature considered in this evaluation, the measured power reaches a maximum value at a hot junction temperature just over 100 °C. However, at low temperatures, there is a good agreement between the predicted and the measured power output.

However, there is a reasonable agreement between the predicted and the measured values of the conversion efficiency in general. The discrepancy between the values varied between 1.2% to 17% depending on the junctions temperatures, as can be seen in table (4.3).

As can be seen, for both the power output and the conversion efficiency, the discrepancy is high at higher temperatures. This can be attributed to several factors. The power is proportional to the product of the square of temperature difference and the square of the Seebeck coefficient. So, as the temperature difference increases, the term \((\Delta T^2)\) dominates over the value of the predicted power output. The decrease in

Fig. (4.8) Conversion efficiency at the maximum power versus hot junction temperature at cold junction temperature 27 °C
the Seebeck coefficient used in the model is too small to compensate for this increase. The difference between the predicted and the measured performance of the device at higher temperatures may also be due to material properties, since the actual material properties of the commercial device are not known due to company restriction.

It is likely that, the material properties change at higher temperature, and possibly the material becomes intrinsic, resulting in a drop of of the power output, mainly because the thermal conductivity will increase [Cadoff & Miller, 1960]. Another important factor, can be related to the fact that, the commercial module is optimized for cooling purposes and hence for maximum temperature difference of about 75°C, when operated as a cooler. This means that, the device performance drops beyond this temperature difference, as can be seen in Fig (4.7).

Also, at higher temperatures, both the thermal and the electrical contact layers resistivity are expected to change, and this will certainly affect the performance of the device as shown in table 3.6, for the cases 3, 4 and 5. This is considered in the unified theoretical model, with the curves labeled (Modified), where an increase in the electrical contact layers resistivity with the temperature at a rate proportional to 0.005T is assumed (T is the temperature in °C). As can be seen, this increment in the contact layer resistivity is sufficient to overcome the discrepancy, particularly in the predicted values of the conversion efficiency, though the predicted power output at high temperatures is still higher than the measured one, due to the reason mentioned above.

Another source of error, can be related to the thermocouples sensors, particularly the one used for the measurement of the hot junction temperature. The sensor was exposed directly to concentrated radiation, and therefore would have resulted in an overestimated temperature. Some attempts were made to shield it but without much success because of the limited space.
VALIDATION OF THE UNIFIED MODEL

Table (4.3) Comparison between the measured and predicted performance of the thermoelectric device in power generation mode.

<table>
<thead>
<tr>
<th>T(hot) °C</th>
<th>Model</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Unified Model</td>
<td>Unified Model(modified)</td>
</tr>
<tr>
<td></td>
<td>Power(W)</td>
<td>η(%)</td>
</tr>
<tr>
<td>77</td>
<td>0.12</td>
<td>0.58</td>
</tr>
<tr>
<td>87</td>
<td>0.18</td>
<td>0.67</td>
</tr>
<tr>
<td>97</td>
<td>0.24</td>
<td>0.76</td>
</tr>
<tr>
<td>107</td>
<td>0.31</td>
<td>0.84</td>
</tr>
<tr>
<td>117</td>
<td>0.38</td>
<td>0.91</td>
</tr>
<tr>
<td>127</td>
<td>0.47</td>
<td>0.98</td>
</tr>
</tbody>
</table>

4.5 Conclusion

An experimental evaluation of a Peltier cooling device manufactured by Marlow Industries Inc. (USA) in power generation mode is given. Results showed that, the current Peltier cooling devices can be used to produce electrical power, though they are not optimized for maximum power output. Some of the short comings of these devices are that, the performance of the device is limited to a maximum temperature difference of about 75 °C, which is known as no-load maximum temperature difference when the device is used as a cooler. Also, the device can only be operated safely below 138 °C, determined by the solder material used in the device.

Data obtained from the experiment was used to validate the unified thermoelectric model developed in Chapter 3. Results of the validation are reasonably acceptable, given the assumptions made for the model and the conditions at which the experiment was performed.

It is concluded that the model can be improved by considering the temperature dependency of the contact layers properties.
5 Design of the Solar Energy Collecting System

This chapter details a design methodology for a two stage solar concentrator, sufficient to drive a thermoelectric device. The concentrator comprises of a primary one axis parabolic concentrator, and a second stage symmetrical compound parabolic concentrator, mounted at the focus of the primary. The key requirement of the concentrator design is to be tolerant of tracking misalignment, so as to minimize the tracking requirements. A proposed configuration is also sought which reduces the overall heat loss coefficient of the system.

5.1 Theoretical Background

The relationship between the size of the sun disk and its distance from the earth indicates that, the angle subtended by the solar disk is about 32 minutes to an absorber on the surface of the earth [Pol Duwez, 963]. Thus the theoretical image of the sun disk created by a simple concentrating optical system will have a finite size, which is dependent on the size of the solar disk and the optical characteristics of the concentrating system.

The minimum diameter ($D_s$) of the sun image formed at the focus of a parabolic reflecting surface with a focal length ($f$), is given by [Pol Duwez, 1963] as

\[ D_s = 2f \tan(\delta) \]  

(5.1)

where, $\delta$ is half the angle subtended by solar disk ($\tan\delta \approx 0.00464$).

Equation (5.1) indicates that, the size of the sun image that can be produced by a simple optical system depends on the focal length of the reflecting optics. In the case of line focusing concentrator, $D_s$ is the minimum possible width of the sun image, that can be directly obtained by a simple optical system. For an optical system with a focal
length equal to one meter, the diameter of the sun image at the focal point will be about 9 mm.

When aberration are absent and the reflecting surface is perfect, all the radiant power of the sun intercepted by the reflector will be directed towards the image at the focal point. The concentration ratio can therefore be determined from the relation

$$C_r = \frac{\text{flux in image}}{\text{flux intercepted}} = \frac{\text{intercepted area}(A)}{\text{image area}(A')}$$

(5.2)

For a high concentration factor, maximum amount of radiant power must be collected and focused into the sun image. From equation (5.1) and (5.2), high concentration also implies that the focal length of the mirror should be as small as possible compared to the aperture of the mirror. In the case of parabolic concentrators, the aperture of the parabola is determined by the focal length and the angle (\(\phi\)) between the axis of the parabola and the line joining the focal point to the edge of the parabola (rim point). Therefore it may become appropriate to relate the concentration ratio to the rim angle (\(\phi\)). Initial question concern how the heat flux within the sun image varies with the rim angle, and how much of the reflected radiant power is concentrated into the sun image as a function of the rim angle.

As Fig. (5.1) demonstrates, for a parabolic concentrator, a cone of light falling on an element of the parabola at point Q is reflected as a cone having an angle equal to that subtended by the sun disk. If \(\varphi\) is the angle PFQ, the angle of incidence of the axis of the cone with the normal to the element of parabola at point Q is \(\varphi/2\). The intersection of the reflected cone and the focal plane is an ellipse, and the major axis of the ellipse increases with the angle \(\varphi\) and also with the distance FQ. This means that, the outer portion of the parabola reflects longer and wider ellipses around the sun image in the focal plane, and as a consequence, the amount of radiation focused into the sun image by an element of the parabola at point Q decreases as the angle \(\varphi\) increases. In fact, only parts of the parabola located near the vertex will reflect the sun rays into the sun image with diameter \(D_v\), given by equation 5.1.
The above indicate that, a parabola with large rim angle produces a larger light image at the focal plane, which varies in intensity from the focal point decreasing outwards along the focal plane. To increase the concentration efficiency, a wider receiver is
required to intercept more of the reflected light, which in turns means a lower concentration ratio.

The total radiant power concentrated at the focus can be obtained by integrating the contributions of all the reflecting elements of the parabola over its surface. Consider an element of reflector around point Q, at a distance $r$ from the focal point and at an azimuth angle $\varphi$. The radiation contributed by this element is the ratio between the area of the sun image, and the area of the ellipse resulting from the intersection of the reflected cone and the focal plane. Considering that, the reflector is a cylindrical parabolic trough of unit length, the total radiant power concentrated at the focus can be obtained by integrating the contributions of all the reflecting elements of the parabola over its surface. The concentration factor $C$, and the concentration efficiency $\eta_{(2D)}$ defined as the ratio of the total radiation received within the sun image to the total radiation received at the plane of the reflector, can readily be obtained, following the general procedure of the 3-Dimensional arrangement [Pol Duwez, 1963] as follows

$$C_{(2D)} \equiv 215 \rho \sin \varphi \quad (5.3)$$

$$\eta_{(2D)} = (1 + \cos \varphi)/2 \quad (5.4)$$

For a given reflectance, equations (5.3) and (5.4) indicate that the concentration factor and the concentration efficiency of a cylindrical parabolic trough concentrator are dependent on the rim angle of the concentrator.

For a perfect reflector, $C_r$ has the maximum value of about 215 at $\varphi = \pi/2$, corresponding to $\eta_{(2D)}$ equal to 50%. As can be seen, although, cylindrical parabolic concentrators (PTC) have a maximum theoretical concentration ratio of about 215, only 50% of the incident radiant power will be intercepted by the image at the focal point.
DESIGN OF THE SOLAR COLLECTOR, THEORETICAL BACKGROUND

The concentration ratio and concentration efficiency given by equations (5.3) and (5.4) respectively, assume that, the sun, the vertex and the focal plane of the concentrator are all in a good alignment. A small misalignment beyond the 16 minutes angle, will cause the focused radiation to fall off the focal point and a consequent drop in the concentration ratio and the concentration efficiency. In addition, because the curvature of the reflector is never perfect, the rays coming from the sun are not unidirectional and due to diffraction to the radiation, the actual concentration ratio will be reduced by several factors. These considerations emphasize the need of an accurate tracking mechanism for this types of solar energy concentrator.

On the other hand, the compound parabolic concentrator (CPC), which can attain the maximum theoretical concentration efficiency (Winston, 1975), and which can operate free of tracking concerns, particularly at low concentration ratio, has a major drawback that the overall depth of the collector is determined by the extreme rays about the symmetry axis of the concentrator. Therefore the ratio of the concentrator's depth to the reflector aperture becomes excessive and impractical at high concentration ratios. Since excessive reflector material is needed for a relatively small concentration factor, compared to the simple parabolic concentrator, the CPC arrangement tends to be uneconomic except for very low concentration ratios.

A promising approach to improve the concentration efficiency of a PTC without tracking requirement, is to design the PTC for a low concentration factor and use it in conjunction with a wider secondary receiver to collect and further concentrate the radiation onto a smaller absorber plate, making in effect a multi-stage concentrator. This allows a prescribed maximum tracking error without a significant reduction in performance. Hence, a two stage design made of a primary PTC and a secondary CPC, which combines the compactness of the PTC and the high concentration efficiency of the CPC is likely to be an effective combination.

Several articles are available in the literature which describe designs based on this concept (Rabl 1976a; Rabl, 1976b; Mills, 1980 & 1995; Collares_Pereira et al., 1991; Brunotte et al., 1996).
A two stage concentrator using a second stage CPC and a primary Fresnel mirror field for central receiver has been described by (Rabl A. 1976a). More recently, Brunotte et al., (1996) have described a two stage arrangement giving concentration ratio up to 300, using a north-south polar axis primary PTC with a row of filled dielectric non-imaging 3-D concentrators, designed for photovoltaic conversion. Although the design achieves a high concentration ratio, its tolerance to the incoming rays is very limited. A combination of a primary parabolic trough and intermediate asymmetrical CPC secondary, providing concentration ratios in the range 9 to 12, was described by (Mills, 1980 & 1995). The design is suggested for both photovoltaic conversion using optical prisms and thermal processes.

As already mentioned, an overall solar concentration factor of 20 is suited to commercially available thermoelectric applications. This can be achieved by using a two stage concentrator, combining a primary one axis PTC with a second stage made of a symmetrical CPC, mounted at the focus of the primary [Omer and Infield 1997]. The design developed here is based on a wider receiver than usual for the primary concentrator, allowing interception of the misaligned incoming rays within the angular region (±δ) defined as the tolerance of the concentrator to the incoming rays.
5.2 Design Formulation of a Two-Stage Concentrator

The design proposed for the solar thermoelectric generator is comprised of a two-stage concentrator, with a primary parabolic trough concentrator (PTC) and a second stage made of a compound parabolic concentrator (CPC) mounted at the focus of the primary, as shown in Fig. (5.2). The approach used here is that, the focusing is assumed to be on a wider receiver rather than the sun image, allowing interception of some of the misaligned rays within the angular region (±δ) defined as tolerance of the concentrator to the incoming rays.

First, a relation between the geometrical concentration ratio, the half the angle subtended by the arc of the parabola (rim angle) and the tolerance angle δ is formulated for the PTC. The contribution of the second stage concentrator is then dealt with. Finally, optimal rim and tolerance angles are identified that provide the required concentration ratio, with due consideration to the other practical constraints.

The system is configured as shown in Fig. (5.2), where AA' is the aperture plane of the primary concentrator. The entrance aperture BB' of the CPC, shown in the figure, acts as a receiver to the PTC. Efficient concentration requires that, BB' should be sufficiently wide to intercept all the rays reflected from the primary within the angular region specified as ±δ from the normal. B and B' are located at the intersections of the extreme rays reflected from A and A', to C and C' respectively.

Shading resulting from the existence of the secondary CPC on the focal line of the primary PTC, can be neglected at this stage, as BB' will be reasonably small compared to the overall aperture width for the applications considered here. Therefore, the geometrical concentration ratio of the PTC is given simply by

\[ C_{r(\text{PTC})} = \frac{AA'}{BB'} = \frac{a}{b} \]  

(5.5)
where \( a \) is half the aperture width of the PTC \( AA' \) and \( b \) is half the aperture width of the receiver \( BB' \).

**Fig. (5.2)** Schematic diagram of a two stage concentrator with a primary PTC and a single symmetrical CPC secondary.
DESIGN FORMULATION OF A TWO STAGE SOLAR COLLECTOR

The equation for the surface of the parabola in Cartesian co-ordinates is [Pettofrezzo, 1970]:

\[ x^2 = 4fy \]  \hspace{1cm} (5.6)

where the y-axis is the axis of the PTC, and \( f \) its focal length. The radius \( r \) from a point on the rim of the parabola to the focal point, is given by the surface equation in polar co-ordinates [Pettofrezzo, 1970] as :

\[ r = 2f(1 + \cos \varphi)^{-1} \]  \hspace{1cm} (5.7)

where \( \varphi \) is the rim angle. The aperture width of the PTC is the projection of AA' onto the plane normal to the incident rays; (i.e. normal to the axis of the parabola) and is given by

\[ a = r \sin \varphi = 2f \sin \varphi(1 + \cos \varphi)^{-1} \]  \hspace{1cm} (5.8)

From Fig. (5.3), simple trigonometry implies that,

\[ b = a - 2f \cos \varphi(1 + \cos \varphi)^{-1} \tan(\varphi - \delta) \]

Substitution for \( a \) from equation (5.8) gives

\[ b = 2f(1 + \cos \varphi)^{-1}(\sin \varphi - \cos \varphi \tan(\varphi - \delta)) \]  \hspace{1cm} (5.9)

Thus the concentration ratio \( C_{(\text{RTC})} \) is given by

\[
C_{(\text{RTC})} = \frac{a}{b} = \frac{2f \sin \varphi}{1 + \cos \varphi} \left( \frac{2f}{1 + \cos \varphi} \left( \sin \varphi - \cos \varphi \tan(\varphi - \delta) \right) \right)^{-1}
\]

\[ = \left( 1 - \frac{\tan(\varphi - \delta)}{\tan \varphi} \right)^{-1} \]  \hspace{1cm} (5.10)
Fig. (5.3)  Schematic diagram of a parabolic trough concentrator with a planar receiver.
Equation (5.10) shows that, for a given rim angle, the geometrical concentration factor of the primary concentrator depends on $\delta$.

Now, consider a compound parabolic concentrator CPC with entrance and exit apertures $BB'$ and $CC'$ respectively, separated so that, an extreme ray $BC'$ or $B'C$ makes the maximum collecting angle $\theta$ with the axis of the concentrator, as shown in Fig. (5.4). The focus of the right-hand side parabola is at the base of the left-hand side and vice-versa. The axis of each parabola is inclined to the vertical optical axis by an angle $\theta$. The Profile of the CPC between $B'$ and $C'$ is a parabola with axis parallel to $BC'$ and with focus at $C$. By using polar co-ordinates $(r, \alpha)$ for the parabola, the concentration ratio $C_{(CPC)}$ and the depth $D_{(CPC)}$ of the CPC are given by (Welford & Winston, 1978) as

$$C_{(CPC)} = \sin \theta^{-1}$$  \hspace{1cm} (5.11)

$$D_{(CPC)} = (b + c)\cot \theta$$  \hspace{1cm} (5.12)

where $\theta$ is the half acceptance angle.

Equation (5.11), shows that, $C_{(CPC)}$ depends solely on the acceptance angle. To maximise $C_{(CPC)}$, the acceptance angle $\theta$ must be as small as possible. However, the CPC does not help much at high concentration ratio, because it becomes too deep. In addition, its acceptance angle must match to the incident angle of the source radiation (radiation reflected by the primary PTC). So, selection of these two factors is key to determining the optimum design of the two stage concentrator.

Referring to the Fig. (5.4), a second stage CPC with entrance aperture $2b$ and exit aperture $2c$, has a concentration ratio given by equation (5.11), with the half acceptance angle ($\theta = (\varphi - \delta)$). Thus

$$C_{(CPC)} = (\sin(\varphi - \delta))^{-1}$$  \hspace{1cm} (5.13)
The total concentration ratio of the combined concentrator \((Cr_{(\text{TOT})})\) is the product of the concentration ratios of the primary and the secondary, thus

\[
Cr_{(\text{TOT})} = Cr_{(\text{PRC})} \times Cr_{(\text{CPC})} = \left(\sin(\phi - \delta) \left(1 - \frac{\tan(\phi - \delta)}{\tan \phi}\right)\right)^{-1}
\]  

(5.14)
DESIGN FORMULATION OF A TWO STAGE SOLAR COLLECTOR

The concentration ratio is a function of the rim angle and the permissible degree of tolerance to misaligned incoming rays. For a given concentration ratio $\varphi$ has to be optimised for maximum $\delta$.

For a given parabolic reflector, $\delta$ is dependent on the width of the receiver. High concentration factors are achieved by using smaller receivers, and consequently smaller tolerance angles which require frequent tracking adjustment. However, the CPC is problematic at high concentration ratios because it becomes too deep. In addition, its acceptance angle must match the incident angle of the source radiation (radiation reflected by the primary PTC). Careful selection of these factors is essential to design of an effective concentrator.
5.3 Design Optimization

The aim of the design is to improve the performance of the solar energy collector, by appropriate selection of design parameters taking into account the design and the operational simplicity. One of these is the concentration ratio needed to give the specified operating conditions. As shown in equation (5.14), the concentration ratio is a function of the rim angle and the permissible degree of tolerance to misaligned incoming sun rays. This in turn determines the precision of tracking adjustment, which has to be provided for efficient concentration.

The variation of the concentration ratio with the rim angle, for various combinations of PTC and CPC, is plotted in Fig. (5.5). These curves are obtained by solving equation (5.10) for \( \delta \), for each combination, and substituting its value in equation (5.14). Higher concentration ratios can be obtained by using a small rim angle, but at the expense of other factors. One of these is the depth of the CPC, which needs to be limited to be accommodated inside an appropriate glass tube. Another is the mechanical stability of the concentrating system. A small rim angle implies a large ratio of focal length to aperture width for the PTC. This means that the receiver will be far away from the centre of mass of the reflector. Such a system can be mechanically unstable, making rotation and tracking adjustment more difficult.

To improve the mechanical stability of the concentrating system, it is often appropriate to use a primary PTC with a large rim angle. For a given aperture, a large rim angle implies that the receiver is closer to the centre of mass of the collector. As a general rule, the optimum rim angle of parabolic trough concentrators for flat receivers, is in the range 40° to 60° [Rabl, 1982]. CPCs are suitable for acceptance half angles in the range up to about 40° [Collares-Pereira, et al., 1991]. This suggests that the matching angle should be around 40°. Large rim angles can be achieved by using a second stage composed of several multi-segments fabricated side by side [Collares-Pereira et al., 1991], with each segment facing a different portion of the primary concentrator. A key short coming of the multistage design is that some of the
reflected rays will impact on the gaps between the multiple secondaries and will not reach the receiver. In addition the awkward complex design and fabrication complexity of the multi-segmented compound parabolic concentrators may limit the applicability of this option, except for very high concentration ratios.

![Graph](image)

**Fig. (5.5)** Total concentration ratio of the combined concentrating system versus the rim angle for various combinations of PTC and CPC

The variation of the resulting tolerance with the rim angle, for various combinations of PTC and CPC, is plotted in Fig. (5.6). The locus of $\delta$ for different combinations with a concentration ratio of 20 is also shown. The maximum allowable tolerance that can be obtained by a single PTC concentrator with this concentration ratio is 1.47 degrees. Introduction of a second stage will boost the tolerance angle, for instance, to 1.96, 2.43, 2.54, 2.80 and 3.12 degrees for combinations with a PTC of concentration factor 15, 12, 10, 8 and 6 respectively. In general, a larger tolerance can be obtained by combining a PTC with small concentration ratio and a CPC of a relatively large concentration ratio, when other factors such as depth of the CPC, the matched CPC...
acceptance and PTC rim angles, and the size of the receiver system are taken into consideration.

![Locus of δ for Cr(PTC) of 20X](image)

**Fig. (5.6) Tolerance of incident rays as a function of PTC rim angle**

The variation of the CPC depth with the rim angle for varying PTC and CPC combinations for a total concentration ratio of 20 is illustrated in Fig. (5.7). The depth of the CPC follows a pattern very similar to the combined concentration ratio. The depth of the CPC is reasonably small at large rim angles, and increases with decreasing rim angle to become excessive at low rim angles. The design requires a compromise to be made by choosing a combination that best meets the requirements outlined above.
To determine which combination should be selected to give a total concentration ratio of 20, the following steps are followed. From Fig. (5.6), the rim angle at which each combination gives the required concentration ratio is determined. The corresponding degree of tolerance for each combination is determined using the locus of $\delta$ from Fig. (5.6). The final selection is based on the equivalent depth of the CPC that allows accommodation of the receiver system inside an appropriate glass tube.

An important aspect of the overall design is heat loss from the absorber. This can be reduced substantially by enclosing the receiver system inside a glass envelope. This has the additional benefit of protecting the second stage reflector surface from the environment. With evacuated tubes, bare silver surface reflectors can be used. The size of the glass envelope is of concern, bearing in mind that a minimum air gap of about 6 mm may be necessary to avoid heat losses in cases when the glass tube is not adequately evacuated to suppress convective heat loss [Omer 1994; Ratzel et al., 1979]. Therefore the CPC depth should be such as to allow accommodation inside the glass tube with the minimum air gap.
The availability of the glass tube to accommodate the receiver system, is an important practical consideration. Commercial light wall tubing glass that would suit solar energy applications is available in sizes between 65 - 100 mm in diameter. For a total concentration of 20X, a combination of a PTC with a concentration of 12X and a CPC with a concentration of 1.67X is proposed, allowing a tolerance of about 2.45 at a rim angle of 40°. Fig. (5.8) shows the variation of the tolerance angle and the corresponding depth of the CPC across a range of primary concentrators for a total concentration of 20X. Assuming an average daily change in the latitude of the sun at the noon of about 0.2 degrees per day, typical of equatorial regions, the tracking adjustment is needed only every two weeks at most. If one can afford glass tube of larger diameter, a design with higher tolerance can be selected, though the corresponding rim angle will be smaller and thus less stable concentrator.
Fig. (5.8) Variation of the tolerance angle and the depth of the CPC with the concentration ratio of the PTC, showing design parameters.
5.4 CONCLUSION

A design procedure has been described for a two stage solar concentrator proposed for thermoelectric power generation. The concentrator comprises a primary one axis parabolic trough concentrator and a second stage symmetrical compound parabolic concentrator, mounted at the focus of the primary. The key requirement of the concentrator design is to be tolerant of tracking misalignment, so as to minimize the tracking requirements taking into account the design and the operational simplicity required by the target end users. A design methodology is followed which allows interception of rays within an angular region (±δ). This results in a wider receiver for the parabolic trough concentrator than would usually be used for a similar concentration ratio. The design has the advantage of providing efficient concentration of solar radiation without the need for frequent tracking adjustment. Overall, the system is well suited to small scale applications in parts of the world with a high direct beam component of solar radiation.

The above methodology can be applied to other concentration factors. The approach is very versatile, and allows design of any two stage concentrator based on a primary PTC and a symmetrical secondary CPC using flat absorbers. Even for a circular receiver the same approach applies, with consideration to equations which relate the concentration ratio to the rim and tolerance angles. To achieve a very high concentration ratio, it may be preferable to use a multistage secondaries, regardless of its complexity.
CHAPTER SIX

6 Description and Evaluation Methods of Solar Energy Collector

This chapter describes the basic geometry of the solar thermoelectric system and briefly the surface radiation properties of key components of the concentrating system. The chapter also presents the overall framework on which the evaluation of the system is based, and the appropriate evaluation methods which suit the various operating conditions of the system.

6.1 Geometrical Description of the System

The two-stage concentrator designed in chapter (5) is considered suitable for solar thermoelectric system based on bismuth-telluride and lead-telluride thermoelements. These thermoelements operate in low to medium temperature range that can be met by concentrating solar energy to a factor of about 20. As described in chapter 5, a two stage concentrator composed of a primary parabolic trough concentrator PTC and a secondary compound parabolic concentrator CPC of concentration factors 12 and 1.64 respectively is found to be a suitable configuration to meet the above concentration requirements. This combination is optimized for a PTC with a 40° rim angle and a CPC with a 37.5° half acceptance angle.

To improve the performance of a solar energy system by combating the heat losses, the absorber system including the thermoelectric device and the secondary concentrator has to be enclosed inside a glass tube. This gives the potential benefit of using highly selective surfaces for both the secondary concentrator and the absorber plate. With an evacuated glass tube, a bare silver surface of solar reflectivity 0.91-0.96 [Collare-Pereira et al., 1991], can be used for the CPC reflector, instead of anodized aluminum sheet metal or thin films of reflectivity 0.80 - 0.85, typical of external reflector design [O’Gallagher et al., 1982]. In general, the optical characteristics of the concentrating system need to be carefully chosen in order to ensure good optical performance.
The basic design configuration of the system is illustrated in Fig. (6.1). The secondary concentrator is fitted in a concentric glass tube. The thermoelectric device is placed at the focus of the CPC with the absorber plate (hot junction) and a liquid cooling tube fitted to the cold junction. The cooling tube which is made from copper tube, protrudes from both ends of the glass tube, and is to be held in place using glass-metal bonding.

The summary of the geometrical parameters of the combined concentrating system is given in Table (6.1).
Fig. (6.1a) Solar energy thermoelectric power generation system
Fig. (6.1b) Geometrical description of the receiver system

Table (6.1) Geometrical design parameters of the solar concentrating system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTC aperture width, cm</td>
<td>39.4 cm</td>
</tr>
<tr>
<td>Rim angle of the PTC, degree</td>
<td>40°</td>
</tr>
<tr>
<td>PTC focal length, cm</td>
<td>27.0 cm</td>
</tr>
<tr>
<td>PTC radius at rim points, cm</td>
<td>30.7 cm</td>
</tr>
<tr>
<td>primary concentration ratio</td>
<td>12</td>
</tr>
<tr>
<td>Secondary concentration ratio</td>
<td>1.64</td>
</tr>
<tr>
<td>CPC aperture window, cm</td>
<td>3.28 cm</td>
</tr>
<tr>
<td>CPC half acceptance angle, degree</td>
<td>37.55°</td>
</tr>
<tr>
<td>Absorber plate width, cm</td>
<td>2.0 cm</td>
</tr>
<tr>
<td>Diameter of the glass tube, cm</td>
<td>7.0 cm</td>
</tr>
<tr>
<td>Depth of the CPC, cm</td>
<td>3.54 cm</td>
</tr>
<tr>
<td>Diameter of the cooling tube, cm</td>
<td>2.2 cm</td>
</tr>
<tr>
<td>thermoelement Depth, cm</td>
<td>0.5 cm</td>
</tr>
<tr>
<td>Total concentration ratio</td>
<td>19.7</td>
</tr>
</tbody>
</table>
The overall length of the collector is determined by the length of the glass tube that could be vacuum processed. Longer collector will offer an advantage of reducing the effect of the early morning and late afternoon radiation losses, reflected off the absorber plate for a east-west oriented collector. The number of collector modules for a given installation is determined by the electrical power requirement. For example, a 50 watts system, employing thermoelectric device at 2.5% efficiency, requires about 2.5 m² solar collector aperture for an average insolation of 800 Wm⁻². About 4 strips of modules will be required for a 1.6m long collector.

The receiver system will be mounted along the focal line of the primary concentrator, using a structural support allowing one axis movement of the assembly to track the sun. In the case of east-west axis mounted collector, an adjustment at least every ten days is required. Horizontal arrangement will require use of a pump to circulate the cooling fluid. This can be avoided by tilting the concentrating system in order to provide thermosiphon head across the collector. East-west axis orientation will in general cause the ends of the collector not to be illuminated during the early morning and late afternoon.

Use of a north-south axis oriented collector, may help avoiding the above problems of a pumping requirement and non-effective usage of the collector’s ends, if diurnal tracking can be afforded.
6.2 Optical Characteristics of the Solar Concentrator

The optical properties of a solar concentrator are the major parameters that determine its performance. These parameters include, the response of the surfaces to the electromagnetic waves, surface fabrication, receiver placement and tracking misalignment, etc. The latter factors are crucial in determining the concentration efficiency and are regarded as optical errors of the concentrator. In general, parameters that affect the optical efficiency of a solar concentrator can be categorized in two main groups.

1. Material properties and manufacturing capability to produce spectrally optimum reflecting and absorbing surfaces that would convert the incident solar radiation into a useful energy.

2. Variables associated with the design optimization such as the design concentration ratio, the rim angle and the resulting intercept factor. For a given concentration ratio, the rim angle must be chosen to maximize the intercept factor. The intercept factor is the parameter that embodies the effects of all the optical errors, and therefore becomes an important factor in the design of the solar concentrator. In principle, the intercept factor can be improved by using a wider receiver, as described in Chapter 5.

In this section, we are concerned with the first group of variables, which includes the optical properties of the glass envelope, the surface of the absorber plate and the reflecting surfaces of the primary and the secondary concentrators.

- Reflecting surfaces

Several reflecting surfaces have been evaluated for use in solar energy concentration [Duffie & Beckman, 1974]. Metallized plastic films, such as aluminized and silvered polymer films, have been used experimentally. Silvered polymer films with a specular reflectance greater than 90% has an expected lifetime of about 5 years [Daniel, 1993]. Other surfaces such as the anodized aluminum sheets have also been
used in experimental units [Granqvist, 1991]. For the surface of the PTC, an aluminized polymeric films (aluminized acrylic film) which has been evaluated for durability in real weather conditions [Duffie & Beckman, 1974], can be used.

The reflecting wall of the CPC can be made of thin layer vacuum deposited silver with high reflectivity, since there is no concern about deterioration by the weather.

- **Absorber plate**

It is desirable to obtain an absorbing surface with a combination of high absorptance for the solar radiation and low emittance for long wave radiation, by employing selective surface approach. The common means to enhance the requirement of selective surfaces (coating) is by the monochromatic reflectance of the surfaces, since there is only a little overlap in wavelength between incoming solar radiation and the thermal emitted long wave-length radiation. About 98% of the incoming solar radiation is at wavelength below 3 μm, while only about 2% of the thermal emitted long wave-length radiation for a black surface at about 500K is below 3 μm [Granqvist, 1991].

Among the coating surfaces that may be considered as practically viable, are electroplated black chrome, nickel pigmented anodized aluminum and black nickel, and black copper prepared by chemical conversion [Granqvist, 1991]. Black chrome is a widely used selective surface for solar energy absorption. It is a complex composite of metallic chromium and dielectric Cr₂O₃. In the low temperatures range, commercial electroplated black chromium coatings produced by Energie Solaire (ES) in Switzerland (coating backed by stainless steel) and by Mt. Solar Inc. in the USA (coating baked by nickel-covered copper) which reported [Granqvist, 1991] to have solar absorptance of 0.94 and 0.97 and thermal emittance of 0.2 and 0.09 respectively at 100 °C, are suitable coatings.

For temperatures above 300 °C, a good selective coating is the sputtered Molybdenum (Mo) based composite which showed an excellent high temperature
stability [Schmidt et al., 1965], and has solar absorptivity of about 0.97 and thermal emissivity of about 0.17 at 350°C. Other commercial manufactured coatings include metallic Ni particles embedded in anodic Al₂O₃, black nickel made by electroplating, graded stainless steel carbon coatings developed for high performance tubular solar collectors, copper oxide coatings and metallic-filled colored stainless steel surfaces [Granqvist, 1991].

- **Glass tube**

Glass tube enclosures play a major role in determining the performance of solar energy collectors. By protecting the surfaces from the environment, glass tubes allow use of highly selective surfaces that possess a high ratio of solar absorptivity to infrared emissivity. They also offer the advantage of combating heat losses by convection by facilitating a certain level of vacuum. Radiation transmission heat losses can also be reduced by applying special thin coatings on the glass surface. Borosilicate glass which has excellent radiation and thermal properties is the most suitable glass ware for tubular receivers.
6.3 Framework of the Collector Performance

As shown earlier in Chapter 3 and Chapter 4, the performance of a thermoelectric device is critically dependent on its hot and cold junction temperatures, which are primarily determined by the thermal performance of the solar energy concentrating system. This in turn is dependent on collector concentration (optical) efficiency and the collector's thermal resistance to heat flow to the surrounding environment.

Under ideal conditions of surface and curvature quality, the concentration efficiency of a solar energy concentrator can be estimated straightforward. The actual concentration efficiency depends on the manufacturing capability to produce perfect reflecting surfaces with parabolic curvature, and perfect tracking.

Regarding the thermal performance, the key evaluation point concerns the effect of the secondary CPC and its role in providing a thermal insulation to both the thermal radiative and the convective heat losses from the absorber plate. In addition to its role in improving the concentration efficiency, the CPC of the proposed design also contributes to trapping both the convective currents and the infrared radiation within the region confined by the absorber plate, the CPC reflector and the glass aperture window, resulting in a reduced overall heat loss coefficient.

The performance of a solar energy collector in general can be described by an energy balance equation that relates the incident solar energy to the useful energy and heat losses. To describe the thermal performance of the concentrating system, let us assume a cross section of the receiver system as shown in Fig. (6.2). Where, the heat transfer terms are defined as follow

\[ Q_{r(SO)} \] = longwave radiation from the absorber plate to the glass tube directly,

\[ Q_{r(SR)} \] = longwave radiation from the absorber plate to CPC reflector,

\[ Q_{r(RG)} \] = longwave radiation from CPC reflector to the glass tube,

\[ Q_{r(CG)} \] = longwave radiation from the cooling tube to the glass tube,
$Q_r(GA) = $ longwave radiation from the glass tube to the surrounding environment,

$Q_c(RG) = $ convective heat flow from CPC reflector to the glass tube,

$Q_c(GA) = $ convective heat flow from the absorber plate to CPC reflector,

and $Q_u = $ useful heat extracted from the cold junction.

The input energy to the receiver system is the beam radiation reflected and concentrated by the PTC, intercepted by the aperture window of the CPC and further concentrated onto the absorber plate. As will be shown later on in Chapter 8, this portion of the beam radiation will undergo multiple reflection between the absorber plate and the aperture window of the glass tube due to the CPC, until absorbed or until it re-emerge through the aperture window. As a result, the net optical properties of the surfaces involved should be corrected to account for the contribution of this multiple reflection.

Under the assumptions outlined later on in Chapter 8, the solar radiation per unit absorber area absorbed by the absorber plate, is given by

$$I_s = \eta \cdot F_b \cdot G(C_R)$$  \hspace{1cm} (6.1)

where,

$C_R$ is the geometrical concentration ratio of the concentrating system,

$F_b$ is the beam radiation factor,

$G$ is the total solar radiation measured in the plane of the PTC aperture,

$K$ is a factor that corrects for the effect of the shading due to existence of the receiver on the focal line of the PTC.
Fig. (6.2)  Energy transfer associated with the receiver system.
The term $\eta_0$ represents the optical efficiency of the concentrator, and involves all the effects of the optical behavior of the concentrating system. In its general form, the optical efficiency of a solar concentrator can be expressed as

$$\eta_0 = \rho_p (\tau \alpha)_\text{eff} \gamma \cos (z)$$

(6.2)

where,

$\rho_p$, is the reflectance of the PTC reflector surface,

$(\tau \alpha)_\text{eff}$, is the effective transmittance-absorptance product of the receiver system,

$z$, is defined as the angle between the unit vector $\hat{e}_n$ and $\hat{e}_s$, where, $\hat{e}_n$ is directed from a point on the PTC's vertex axis through the collector's focal line and is perpendicular to both the focal line and the vertex axis, while $\hat{e}_s$ is directed from a point on the axis of the parabola through the focal line of the collector and points towards the sun. If sun rays, the PTC and the CPC are all in good alignment, the value of $(z)$ is zero.

$\gamma$, is the intercept factor defined as the fraction of the rays incident upon the aperture of PTC that reach the receiver. It is the property of the concentrator and its orientation relative to the incident solar radiation and of the absorber and its positioning relative to the reflector (receiver placement and tracking errors).

Other factors that may contribute to the value of the intercept factor, include the irregularity in the reflector surface which may cause dispersion and enlargement of the solar image. In principle, the value of $(\gamma)$ can be improved by using a wider receiver, an approach which has led to the design of the two stage concentrator.

The term $(\tau \alpha)_\text{eff}$ in equation (6.2) is the effective transmittance-absorptance product, and is given by

$$(\tau \alpha)_\text{eff} = \rho_r \alpha_s \tau_w F_1$$

(6.3)
where, $\alpha_s$, $\tau_w$ and $\rho_r$ are the absorptivity of the absorber plate, the transmittivity of the glass aperture window, and the reflectivity of the CPC reflector respectively, for the solar range.

$F_i$ is a correction factor accounting for the multiple reflection contribution to the absorbed solar radiation. According to the derivation given in Chapter 8, $F_i$ is given by

$$F_i = [1 - (1 - \alpha_s)\rho_w\rho_r^2]^{-1}. \quad (6.4)$$

Solar radiation absorbed in the receiver system will raise the temperature of the various components of the receiver system, giving rise to an longwave radiation, conduction and convection heat exchange between these components on one side, and to the surrounding environment on the other side, as shown in Fig. (6.2).

By analogy to the equation that defines the performance of a flat plate solar collector, the performance of a receiver system in steady state conditions, can be described as

$$Q_u = I_s(A_s) - A_s U_r (T_s - T_a) \quad (6.5)$$

where, $Q_u$ is the useful energy extracted from the solar concentrating system, $I_s$ is incident solar radiation reduced by the optical losses of the concentrating system given by equation (6.1), $U_r$ is the overall heat loss coefficient of the receiver system, and $T_s$ and $T_a$ are the receiver and the ambient air temperatures respectively.

Under normal operation conditions, the absorbed solar radiation will be divided into useful electrical energy delivered to the load, thermal energy extracted by the circulating fluid and thermal losses to the surrounding environment. The thermal resistance to these heat transfer terms can be represented by an electrical network, as shown in Fig. (6.3), where, $R_{SG}$ is thermal resistance to heat flow from the absorber plate to the glass tube directly, $R_{TSO}$ is total thermal resistance to heat flow from the absorber plate to the glass tube directly and via the CPC reflector, $R_{SR}$ is thermal resistance to heat flow from the absorber plate to CPC reflector, $R_{RG}$ is thermal...
resistance to heat flow from CPC reflector to the glass tube, $R_{CG}$ is thermal resistance to heat flow from the cooling tube to the glass tube, $R_{GA}$ is thermal resistance to heat flow from the glass tube to the ambient air and $Q_{\text{loss}}$ is heat loss to the outside environment.

If the glass tube is evacuated, the above terms will represent the thermal resistance to the infrared radiation exchange only. However, a general case of an un-evacuated glass tube will be considered at this stage, and will be simplified later for any other specific conditions as appropriate.

As can be seen, at the absorber surface with temperature $T_S$, solar radiation $I_S$ is absorbed and distributed to thermal losses at temperature $T_A$, either directly through the glass tube at temperature $T_G$ or by the help of the CPC reflector at temperature $T_R$, and the cooling tube at temperature $T_C$. The rest of the absorbed energy is extracted as a useful heat from the cold junction.

The useful energy consists of two parts, the electrical energy delivered to the load, and the thermal energy extracted from the cold junction. Therefore, it can also be expressed as

$$Q_u = IV + mC_p(T_o - T_{in})$$ \hfill (6.6)\\

The first term on the right hand side is the electrical energy produced by the thermoelectric device. For simplicity, it is advisable to analyze the thermal performance of the concentrating system under open loop conditions and therefore, the term $IV$ can be neglected. This term is in any case small due to the low efficiency of thermoelectric devices and will not affect the thermal analysis anyway.
Fig. (6.3) Electrical network analog for heat flow in the receiver system
(A) Representative network, (B) Simplified network.
Useful thermal energy is extracted in the form of heat by the fluid flowing inside the cooling tube. The cooling tube itself is in essence, a heat exchanger which satisfies

\[
Q_u = m C_p (T_o - T_{in}) = U_{cf} A_{cf} (T_c - T_f) \tag{6.7}
\]

where, \( m C_p \) is the thermal capacity of the cooling fluid, \( T_{in}, T_o \) and \( T_f \) are the inlet, the outlet and the mean temperatures of the cooling fluid, and \( A_{cf} \) is the surface area of the heat passage from the tube to the cooling fluid. Details of thermoelectric device and cooling tube contact is shown in Fig. (6.4).

![Schematic diagram of thermoelectric and cooling tube attachment](Image)
The heat transfer coefficient $U_{cr}$ accounts for both the conduction across the cooling tube wall and convective heat transfer to the fluid inside the flow passage. $U_{cr}$ can be formulated as follows

$$U_{cr} = \left( \frac{b_c}{k_c} + \frac{1}{h_{ct}} \right)^{-1}$$  \hspace{1cm} (6.8)

where, $b_c$ and $k_c$ are the thickness and the thermal conductivity of the cooling tube wall respectively, and $h_{ct}$ is the convective heat transfer coefficient of the fluid inside the flow passage, given by

$$h_{ct} = \frac{k_f}{D_{cf}} \frac{N_u}{D_{cf}}$$  \hspace{1cm} (6.9)

where, $k_f$ is the thermal conductivity of the cooling fluid, $D_{cf}$ is the diameter of the cooling tube, and $N_u$ is the Nusselt number.

Since the diameter of the cooling tube is very small compared to its length, the flow is expected to be laminar, allowing the use of the empirical relation for the Nusselt number ($N_u = 4.364$) for constant heat flow [Kays, 1993].

The value of the overall heat loss coefficient $U_L$ in equation (6.5) depends on the magnitude of the convective, radiative and conductive heat losses. The conductive heat losses may not play a major role, since there is no a significant contact between the heated components and the glass tube except at the glass-metal bonding at the ends of the collector. Conduction through the air gap can be eliminated by allowing sufficient air gap between the heated elements and the glass tube, or by maintaining very low pressure. An air gap of about 6 - 8 mm. is sufficient to suppress the heat flow by conduction [Omer, 1994].

Convective heat losses are proportional to the square root of the pressure, and so can be greatly reduced by maintaining some level of vacuum inside the glass tube. A
pressure level of 25 to 50 mm Hg will be enough to suppress the convective heat losses [Duffie & Beckman, 1991].

In this design, however, the proposed geometrical configuration is intended to help in reducing the convective heat losses from the absorber plate, by trapping the hot air between the space confined by the absorber plate and the CPC reflector. It is one purpose of this study to determine to what extent the second stage CPC will contribute in improving the thermal performance of the system in this way.

The radiative heat losses depend on the radiative properties of each surface involved and the operating temperature. The radiative heat losses can be reduced by using selective surfaces with low emittance for the long wave radiation as discussed earlier. The proposed design has an advantage of reduced view factor of the hot absorber surface to the surrounding environment due to the presence of the CPC reflector. In addition, the absorber surface is facing the PTC, which is at a relatively higher temperature compared to the temperature of the sky in the case of the ordinary upward-facing solar collectors. These conditions can be expected to reduce the radiative heat losses from the receiver system.

The heat flow rate from the absorber plate to the cold junction can be estimated by

\[ Q_{sc} = A_s U_{sc} (T_s - T_C) \tag{6.10} \]

where, \( U_{sc} \) is the overall heat transfer coefficient from the absorber plate to the cold junction, given by

\[ U_{sc} = (A_s R_{sc})^{-1} \]

The derivation of total thermal resistance to heat flow from hot junction to the cold junction through thermoelectric element \( R_{sc} \) is described in Chapter 3, and only its final form will be given here.
FRAMEWORK OF COLLECTOR PERFORMANCE EVALUATION

\[
R_{SC} = R_{hc} + \left( \frac{1}{R_{TEL}} + \frac{1}{R_{esp}} + \frac{1}{R_{rsp}} + \frac{1}{R_{seal}} \right)^{-1} + R_{cc}
\]  

where, \( R_{hc}, R_{TEL}, R_{SEAL} \) and \( R_{cc} \) are the thermal resistances to heat flow through the hot ceramic plate, the thermoelectric legs, the sealant, and the cold ceramic plate respectively, while \( R_{esp} \) and \( R_{rsp} \) are the thermal resistances to convective/conductive heat flow and infrared radiation respectively between the hot and the cold junctions of the thermoelectric device through the space between the two plates around thermoelectric legs.

The individual thermal resistances are given as

\[
R_{TEL} = \left( \frac{L}{Ak} \right)_{TEL}, \quad R_{hc} = \left( \frac{L}{Ak} \right)_{hc}, \quad R_{cc} = \left( \frac{L}{Ak} \right)_{cc}, \quad R_{seal} = \left( \frac{L}{Ak} \right)_{seal}
\]

\[
R_{esp} = \left( \frac{L}{Ak} \right)_{esp} \quad \text{and} \quad R_{rsp} = \left( \sigma \left( \frac{1}{\varepsilon_h} + \frac{1}{\varepsilon_c} \right)^{-1} \left( T_s^2 + T_c^2 \right)(T_s + T_c) \right)^{-1}
\]

where, \( L, A \) and \( k \) in each represent the length, area of heat flow passage and the thermal conductivity of each element respectively. \( T_s, T_c, \varepsilon_h \) and \( \varepsilon_c \) are the temperature and the emissivity of the hot and the cold plates surfaces respectively.

On the other hand, the rate of heat loss from the receiver system can be estimated as

\[
Q_{loss} = A_s U_L (T_s - T_A)
\]  

(6.12)

From the simplified thermal network representation shown in Fig. (6.4), \( U_L \) can be estimated as follows

\[
U_L = U_{GA} \left( 1 + \frac{U_{GA} U_{SC}}{U_{CG} U_{SC} + U_{CG} U_{SGT} + U_{SC} U_{SGT}} \right)^{-1}
\]  

(6.13)
where, \( U_{(SG)T} \) is the total heat transfer coefficient from the absorber surface to the glass tube, both directly and by the help of the CPC reflector as illustrated by the electrical network analog in Fig. (6.4). \( U_{(SG)T} \) can be estimated from the relation

\[
U_{(SG)T} = A_S^{-1} \left( \frac{1}{R_{SG}} + \frac{1}{R_{SR}} + R_{RG} \right)
\]

where,

\[
R_{SG} = A_S (h_{r(SG)} + h_{C(SG)})^{-1}, \quad R_{SR} = A_S (h_{r(SR)} + h_{C(SR)})^{-1},
\]

\[
R_{RG} = A_R (h_{r(RG)} + h_{C(RG)})^{-1}, \quad U_{CG} = A_C (R_{CG})^{-1} = (h_{r(CG)} + h_{C(CG)}),
\]

\[
U_{GA} = A_G (R_{GA})^{-1} = (h_{r(GA)} + h_{e(GA)})
\]

The heat transfer coefficients with subscript \( (r(**)) \) refer to radiation heat transfer coefficients, and those with subscripts \( (c(**)) \) refer to convective/conductive heat transfer coefficients.

\[
h_{r(SG)} = \sigma \frac{(T_S^2 + T_G^2)(T_S + T_G)}{1 - \varepsilon_S + \frac{1}{F_{SG}} + \frac{1}{A_S (1 - \varepsilon_G)}}, \quad h_{r(SR)} = \sigma \frac{(T_S^2 + T_R^2)(T_S + T_R)}{1 - \varepsilon_S + \frac{1}{F_{SR}} + \frac{1}{A_S (1 - \varepsilon_R)}},
\]

\[
h_{r(RG)} = \sigma \frac{(T_R^2 + T_G^2)(T_R + T_G)}{1 - \varepsilon_R + \frac{1}{F_{RG}} + \frac{1}{A_R (1 - \varepsilon_G)}}, \quad h_{r(CG)} = \sigma \frac{(T_C^2 + T_G^2)(T_C + T_G)}{1 - \varepsilon_C + \frac{1}{F_{CG}} + \frac{1}{A_G (1 - \varepsilon_G)}},
\]

\[
h_{r(GA)} = \varepsilon_G \sigma \frac{(T_G^4 - T_m^4)}{(T_G - T_m)},
\]

where, \( F_{SG}, F_{SR}, F_{RG} \) and \( F_{CG} \), are the view factors from the absorber plate to the glass aperture window, from the absorber plate to the CPC reflector, from the CPC reflector to the glass tube and from the cooling tube to the glass tube respectively,
\( \varepsilon_s, \varepsilon_g, \varepsilon_r \) and \( \varepsilon_c \) are the emmisivities of the absorber plate, the glass tube, the CPC reflector and the cooling tube respectively.

From equations (6.1) and (6.5), The efficiency of the concentrating system can be estimated by

\[
\eta = \phi_s \varepsilon_s - \frac{U_L}{K G(C_R)} (T_s - T_A) \tag{6.14}
\]

Another common way of rewriting \( Q_u \) is to express it in terms of the fluid inlet temperature \( T_{in} \) by introducing the heat removal factor \( (F_R) \) [Duffie & Beckman, 1974], as follow

\[
Q_u = A_s F_R [I_s - U_L (T_{in} - T_A)] \tag{6.15}
\]

where, \( F_R \) is the heat removal factor, and is defined as the proportion of the absorbed radiant energy transferred to fluid. It relates the actual useful energy gain of a solar collector to the useful gain if the entire collector surface were at the fluid inlet temperature. From equation (6.7) and (6.14), \( F_R \) is given as

\[
F_R = \frac{m C_p (T_o - T_{in})}{A_s [I_s - U_L (T_{in} - T_A)]} \tag{6.16}
\]

Therefore, the efficiency of the solar energy collector can also be rewritten as function of inlet temperature as

\[
\eta = \phi_s \varepsilon_s F_R - \frac{U_L F_R}{K G(C_R)} (T_{in} - T_A) \tag{6.17}
\]

where, \( \phi_s \) is the optical efficiency of the concentrating system. This is a convenient representation when analyzing solar energy systems, since the inlet fluid temperature is usually known. equation (6.17) is known as the Hottel-Whillier-Woertz-Bliss (HWWB) formula [Hottel et al., 1958].
Equations (6.14) and (6.17), are extremely useful and apply to essentially all solar energy collecting systems. The terms $\eta R$ and $U_t F_r$ are two important parameters that describe how the collector performs. $\eta R$ is an indication of how solar energy is absorbed and transferred to useful energy, while $U_t F_r$ is an indication of how the absorbed energy is lost.

The values of $\eta$, $U_t$, and $F_r$ can be determined theoretically or experimentally from a plot of collector performance against the terms $(T_a - T_h)/KGC$, and $(T_s - T_h)/KGC_r$.

It should be noted that, from the experimental data, the solar energy collecting efficiency can be evaluated by using the simple relation

$$\eta = \frac{m C_p (T_a - T_in)}{G A_PTC}$$

(6.18)

6.4 Evaluation Methods

Two complementary approaches have been considered for the thermal evaluation of the concentrating system. One of them is an experimental investigation to determine the thermal performance of the solar energy collecting system. This includes the useful heat energy that can be extracted from the cold junction of the thermoelectric element and the temperature distribution inside the receiver system, which will enable the determination of the collector performance parameters. This evaluation is given in Chapter 9.

The other approach is based on the theoretical predictions. Since the proposed design seems well suited to operation under vacuum as well as under residual pressure, two complementary theoretical methods are employed. When the receiver system is adequately evacuated, heat losses may occur only by the thermal radiation. Under such conditions an analytical solution is adequate to estimate the temperature distribution of the receiver system by solving the energy balance equations of the various components of the receiver system, under given operation conditions. From
the knowledge of the temperature distribution, the performance of the solar concentrating system can then be predicted. This evaluation is given in Chapter 8. However, in view of the complexity of the situation when the glass tube is not adequately evacuated to suppress the convective heat losses, a numerical solution to predict the flow field and the temperature distribution was considered essential and is given in Chapter 7.
CHAPTER SEVEN

7 Numerical Modeling of the Solar Collector

7.1 Introduction

As outlined earlier in Chapter 5, the purpose of the proposed configuration for the solar energy concentrating system, beside improvement of the concentrating efficiency, is to minimize the heat losses, particularly the convective heat losses from the absorber plate. Complete suppression of these heat losses can be achieved by evacuating the glass tube, a case which has been evaluated in Chapter 8. However, in view of the complexity of the situation when the glass tube is not fully evacuated, a numerical solution was considered essential. The numerical solution is confined to the heat transfer analysis inside the glass tube (receiver system), i.e. it excludes the incoming solar radiation.

The main purpose of the numerical solution is to evaluate the role of the second stage concentrator in reducing the convection currents inside the receiver system, by studying the flow field and the temperature distribution at different tilt angles. This can be achieved by solving the Navier-Stokes equations subject to continuity of mass, momentum and energy. A numerical solutions has been found using the proprietary computational fluid dynamic FLUENT code, which uses a control volume/finite difference based procedure.

FLUENT is a computer program developed by Fluent Inc., for modeling fluid flow, heat transfer, and chemical reaction. It incorporates up-to-date modeling techniques for simulating numerous types of fluid flow and heat transfer problems. These models are accessible through an interactive user interface for problem definition, computation and graphical post-processing, and is well suited to the type of problems we are concerned with here. FLUENT is a two part program consisting of a preprocessor; either PreBFC or GeoMesh, and a main module; FLUENT. PreBFC is used for geometry definition and grid generation, while FLUENT is used for description of the physical models, fluid/material properties, boundary conditions and calculation.
The numerical technique considered in this study, involves the subdivisions of the domain of interest into a finite number of control volumes or cells. The partial differential equations representing the fluid flow equations are discretized and the resulting sets of algebraic relations are simultaneous solved together for air flow, temperature distribution and heat fluxes in the receiver system.

7.2 Geometrical Description

The receiver system shown in Fig. (7.1) is used for the numerical analysis. It is assumed as an infinitely long cavity comprising the cooling tube (CT), the thermoelectric device (TED) and the CPC reflector assembled in one unit and enclosed inside a glass tube. By assuming the length to be infinitely long compared to the cross section, the effect of the receiver’s ends on the heat transfer process is eliminated, and consequently a two dimensional problem is considered. The rest of the dimensions are given in Chapter 6.

Fig. (7.1) Descriptive geometry of the receiver system of the solar energy concentrating system.
7.3 The Mathematical Model

The heat transfer process associated with the receiver system involves convection, conduction and radiation. These three heat transfer processes can be numerically described by the conservation laws of continuity of mass, momentum and energy in Cartesian coordinates. Equations of these laws are well established in the literature, so here will be briefed to clarify the approximations that has been introduced in order to specify the mathematical models which will be solved by FLUENT.

The set of Navier-Stokes equations of continuity of mass, momentum and energy conservation [Kays, 1993] were reduced by appropriate assumptions and boundary layer approximations as follows:

Since convection currents in the glass tube are normally expected to be of low velocity, a laminar flow is assumed. Therefore, thin thermal boundary layers at hot and cold surfaces can be assumed. These boundary layers are typically very small [Incropera, 1981], allowing a boundary layer approximation, that the velocity components in the direction along a surface are much larger than those normal to it, and that velocity gradients normal to a surface are larger than those along it. These approximations eliminate the terms containing the normal and the shear stresses from momentum equations. Similarly, since the velocities are typically low, energy dissipation due to pressure and viscous forces are neglected. Body forces are acting only in the vertical direction.

An assumption is also made that, the receiver system is in a thermal equilibrium. This will permit assumption of a steady state condition to be made. Consequently, for a two-dimensional flow in Cartesian coordinates \((x, y)\), the general forms of the conservation equations; continuity of mass, \(x\)-momentum, \(y\)-momentum and energy respectively are approximated as follows:

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \tag{7.1}
\]

\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} \right) \tag{7.2}
\]
\[ \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x} \left( \mu \frac{\partial u}{\partial x} \right) + \frac{\partial}{\partial y} \left( \mu \frac{\partial v}{\partial y} \right) - g(p - p_{\text{ref}}) \]  
(7.3)

\[ \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + S_h \]  
(7.4)

where, \( u \) and \( v \) are the mass velocity in the \( x \) and \( y \)-directions respectively, \( \rho \) and \( \mu \) are the mass density and dynamic viscosity respectively of the fluid, \( \rho_{\text{ref}} \) is the fluid density at the reference point, \( g \) is the gravitational acceleration, \( p \) is the static pressure, \( h \) is the enthalpy, \( k \) is the thermal conductivity of the fluid or surface, \( T \) is the absolute temperature and the source term \( S_h \) in equation (7.4), represents the radiation input.

The surface to surface radiation exchange is predicted using the Discrete Transfer Radiation Model (DTRM) available in FLUENT code, which treats the radiation energy transfer as follows:

In the absence of a participating medium, the radiation intensity approaching a point on a wall surface is integrated to yield the incident radiation heat flux \( (q_{\text{rad}}^-) \) as:

\[ q_{\text{rad}}^- = \int_{\Omega} I^- d\Omega \]  
(7.5)

where, \( \Omega \) is the hemispherical solid angle and \( I^- \) is the intensity of the rays. The emissive power \( (q_{\text{em}}) \) of the surface is estimated as:

\[ q_{\text{em}} = e_w \sigma T_w^4 \]  
(7.6)

where \( e_w \) and \( T_w \) are the wall emissivity and surface temperature respectively at the point under consideration. The net radiation heat flux from the surface \( (q_{\text{rad}}^+) \) is then computed as a sum of the reflected portion of the \( q_{\text{rad}}^- \) and the emissive power of the surface as:
This radiation heat flux is then incorporated in the energy equation for the prediction of the wall surface temperature. It also provided the surface boundary conditions for the radiation intensity \( I \) of a ray emanating from that point as \( I = \frac{q_{\text{rad}}}{\pi} \).

Other essential equations to be solved together with the above four conservation equations are the equations of state and the thermodynamic enthalpy relation. In an ideal gases, the equation of the state relates the density to the pressure and temperature as:

\[
\frac{P}{R} = \rho T
\]  

(7.8)

The enthalpy is defined as function of the specific heat and the temperature as follows

\[
h = \int_{T_{\text{ref}}}^{T} C_p \, dT
\]  

(7.9)

where \( C_p \) is the specific heat at constant pressure and \( T_{\text{ref}} \) is the reference temperature.

Other properties of the fluid (air) i.e. viscosity, specific heat and thermal conductivity are given as a function of the temperature.

### 7.4 Control Volume Grids

As mention earlier, the numerical solution is based on solution of simple algebraic expressions obtained from discretization of the conservation equations described above, by integrating them over specified control volume node \( P \) as shown in Fig. (7.2). Points \( E \) and \( W \) (denoting east and west) are the x-direction neighbors, while \( N \) and \( S \) (denoting north and south) are the y-direction neighbor points. The lower case letters \( e \), \( w \), \( n \) and \( s \) denote the eastern, the western, the northern and the southern faces of the control volume at point \( P \). The size of these control volumes determines the accuracy of the solution, though there is no general rule that determines the best grid size. However, usually errors will decrease as grids lengths are reduced.
The preprocessor PreBFC package was used to define the geometry and the structural grid for the model. This is done by dividing the domain into control volume grids, using mapping grid coordinate generation techniques. In this way the geometry of the domain is defined in terms of curves and arcs. Grid points are then determined on the geometric boundaries of the computational model, during a process called mapping. Each region is mapped to obtain the required grid distribution by interpolation. First, a coarse grid was generated to identify solution patterns, and in particular, regions of high variation. These regions were re-modeled using a higher grid density. Fig. (7.3) shows the final (70×70) grid system used for the numerical analysis.
Figure 7.2 Grid system used for the Numerical Solution
Grid (70 x 70)

Fig (7.3) Grid System developed for the numerical Analysis
7.5 Boundary Conditions

The equations of the control volume nodes with boundary faces at wall boundaries, contain the boundary conditions and therefore require specification of these boundary conditions, in order to be introduced into the numerical solution scheme.

The receiver system is divided into five heat transfer zones or walls, each with specified boundary conditions. W1 is the glass tube and is specified as an external heat transfer and radiation wall. W2 is the CPC reflector and is assumed as a conduction wall. W3 is the thermoelectric element and is specified as a conducting wall. W4 is the absorber plate, and is specified as a constant temperature wall. W5 is the cooling water tube and is specified as a constant temperature wall. The air inside the glass tube is regarded as a continuum, with temperature dependent properties.

The velocity boundary conditions for the walls are specified as follows:

<table>
<thead>
<tr>
<th>Zone</th>
<th>U-Velocity</th>
<th>V-Velocity</th>
<th>k(W/m/K)</th>
<th>Emmisivity</th>
<th>temperature, K</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td>0.80</td>
<td>Ext. Rad/H-T</td>
</tr>
<tr>
<td>W2</td>
<td>0.00</td>
<td>0.00</td>
<td>205</td>
<td>0.74</td>
<td>Cond. Wall</td>
</tr>
<tr>
<td>W3</td>
<td>0.00</td>
<td>0.00</td>
<td>1.4</td>
<td>0.60</td>
<td>Cond. Wall</td>
</tr>
<tr>
<td>W4</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
<td>0.60</td>
<td>355</td>
</tr>
<tr>
<td>W5</td>
<td>0.00</td>
<td>0.00</td>
<td>-</td>
<td>0.90</td>
<td>400</td>
</tr>
<tr>
<td>W6</td>
<td>0.00</td>
<td>0.00</td>
<td>f(T)</td>
<td>0.10</td>
<td>Cond. Wall</td>
</tr>
</tbody>
</table>

The external radiation boundary temperature and emmisisvity are assumed to be 281K and 1.0 respectively, while the external heat transfer coefficient boundary is 295K. Rest of the parameters and boundary conditions are listed in Appendix 3.
The external heat transfer coefficient to the ambient air \( (U) \), comprised of tube wall conductance and external convection heat transfer coefficient is estimated as follows [Fluent User Manual]:

\[
\frac{1}{U} = \frac{1}{h_{ga}} + \left( \frac{t}{k} \right)_a
\]  

(7.10)

where, \( t \) and \( k \) are the thickness and the thermal conductivity of the glass tube respectively, and \( h_{ga} \) is the external convection heat transfer coefficient and is given as follows

\[
h_{ga} = \frac{k}{D} Nu
\]  

(7.11)

where \( D \) is diameter of the glass tube and \( Nu \) is the Nusselt number, which for free convection of air at atmospheric pressure is given by [Churchill et al., 1975] as

\[
Nu = \left( 0.6 + 0.32 Ra^{1/4} \right)^2
\]  

(7.12)

where, \( Ra \) is the Rayleigh number given by

\[
Ra = \frac{g D^3}{\gamma^2 P_r \Delta T \bar{T}}
\]  

(7.13)

where, \( \Delta T = (T_g - T_\infty) \), \( \bar{T} \) is the mean temperature, \( \gamma \) is the kinematic viscosity of the air, \( P_r \) is the Prandlt number and \( g \) is the gravitational acceleration.

### 7.6 Fluent Solution

The FLUENT code is used for solution of the flow rate, temperature, and the heat fluxes. First, the pre-processor PreBFC was used to define the geometry and a structured grid for the model as outlined earlier. The information is then transferred from the pre-processor to the main module FLUENT via a grid file. Following this transfer, FLUENT was used to define physical models, fluid/material properties, and the boundary conditions that describe
the problem. This information is added to the grid information and stored in a case file that is a record of all the inputs defining the problem. All the calculations are performed and post-processed in FLUENT, which stores the results of the calculation in a data file. Fig. (7.4) describes the FLUENT program structure.

![FLUENT program structure](image)

**Fig. (7.4) FLUENT program structure**

The overall solution was produced using a twin loops iteration procedure: an inner loop undertaken by the FLUENT until solution convergence, and an outer loop for the external heat flow interaction which iterates the external heat transfer coefficient as shown in Fig. (7.5). The overall external heat transfer coefficient is first pre-estimated using a guessed value for the glass tube temperature and subsequently re-adjusted following FLUENT calculations until convergence is achieved.

Different techniques were adopted to accelerate the solution procedure. These included relaxation factors, and sweep direction alteration to feed the boundary information into the calculation more effectively. A summary of these options and solution parameters used, for the case of a vertically aligned receiver, is given in Appendix 3.
Fig. (7.5) Procedure for the numerical solution

Definition of boundary conditions & Initial properties

Guess $T_G$, & estimate $U$

Re-adjust $U$

Begin loop

FLUENT SOLUTION

Update properties

Repeat

Check for conversion

Evaluate $T_G$, & $U$

Exit loop

End
7.7 Results And Conclusion

Since the purpose of the numerical solution is to study the flow field and temperature distribution of the receiver system, the solution was performed only for hot and cold junctions temperatures of 127 °C and 82°C. The other boundaries are specified as explained earlier. Tilt angles are varied from zero to 40 degrees since this range covers the latitudes of most of places where solar energy has a potential. For the sake of completeness, the pressure level inside the glass tube was also varied, and only results of the internal heat transfer coefficients of the glass tube are presented.

Solutions convergence for most of the tilt angles are achieved after about 360 iterations, for the exact external heat transfer coefficient. Residuals of the different variable at the convergence are listed in Appendix 3.

The results of the numerical solution can be evaluated from the plots of the stream functions (contours of constant velocity), flow velocity vectors and the isotherms (contours of constant temperature) shown in figures (7.6), (7.7) and (7.8) respectively.

Generally, it can be observed that, the flow field and the isotherms are determined by the tilt angle of the collector. Four main thermal regions can be observed. These are the region confined by the CPC reflector, aperture window and the absorber plate, the region between the cooling tube and the glass tube at the top, and finally the two side regions between the glass tube and the CPC reflector. The former is the region where heat may be transferred from the absorber plate to the glass tube directly or by the help of the CPC. At the vertical position, this region is dominated by a large central stable thermal plume of low multi-cellular flow and low temperature gradients. Both the thermal gradients and the flow in this region start to increase and distort with the tilt angle. The multi-cellular pattern decreases with the tilt angle forming nearly complete circulation at about 40 degrees tilt. As can be seen from the velocity vectors, the hot air gets its way out from this region through the right hand gap between the CPC reflector and the glass tube.
The latter two regions form a unicellular flow with high upward and downward flow velocity for a vertically aligned receiver. These two regions are dominated by high temperature gradients near the CPC reflector and the glass tube, which are determined by thermal properties of the CPC material and external heat transfer coefficient. As can be seen the temperature of the CPC reflector is very uniform from top to bottom, forming a uniform boundary layers regardless of the tilt angle. This generally accelerates the heat flux from the whole CPC surface to the air around it and to the glass tube, by the help of the convex shape of the CPC. The center of these regions have somewhat stagnant flow which is not strongly dependent on the tilt angle. At the vertical position or at small tilt angles, heat flow to this regions is expected to be mainly by conduction through the CPC reflector. As tilt angle increases, additional heat will flow through the right hand gap. The magnitude of this latter portion is dependent on the width of the gap, which is usually very small because of the concerns about the glass sizes. Generally this portion of heat flow is small even at high angles, as can be seen in Fig. (7.6E), at tilt angle of 40 degrees, most of the flow is still circulating inside the first region.

The upper region, above the cooling tube, also exhibits very low flow velocity and high temperature gradients, indicating that the heat transfer is dominated by conduction. This is because the gap between the cooling tube and the glass is too small for convection currents to become established. However, the rate at which the heat flow through this region is determined by the temperature of the cooling tube which is dependent on the cooling flow rate. In real system, the temperature of the upper part of the cooling tube will be quite low due to the temperature gradient across the tube, therefore, the temperature gradient in this region will be lower than it looks, where the cooling tube is assumed to have a uniform temperature.

Fig. (7.9) shows the variation of the average heat transfer coefficient to the glass tube with the tilt angle, based on the average temperature of air, at atmospheric pressure and 50 mbar. Further drops in the pressure level does not show any change in the local heat transfer coefficients, indicating the absence of convective currents below 50 mbar.
Generally there is a substantial drop in the heat transfer coefficients at 50 mbar compared to those at atmospheric pressure, however, they are little affected by the tilt angle. This essentially show that, the heat loss through the gap between the lower edges of the CPC and the glass tube is little affected by the tilt angle. This suggests that most of the heat transfer from region confined by the CPC, glass tube and the absorber plate is by conduction through the CPC wall, which may necessitates use of a material with low thermal conductivity for the CPC. Alternatively the convection heat transfer from the CPC to the glass can be suppressed by filling the space between them by a mesh of insulating material.
FIGURE 7.6A  STREAM FUNCTIONS OF THE FLOW FIELD
WITH THE RECEIVER AT VERTICAL POSITION (M2/SEC)
Max = 4.19E-04   Min = -4.379E-04

Jan 07 1997
Fluent 4.32
Fluent Inc.
FIGURE 7.6D: STREAM FUNCTION OF THE FLOW FIELD WITH THE RECEIVER AT 30 DEG. TILT (M2/SEC)

Max = 3.44E-04 Min = -4.732E-04
FIGURE 7.6E  STREAM FUNCTIONS OF THE FLOW FIELD WITH THE RECEIVER AT 40 DEG. TILT(M2/SEC)
Max = 3.16E-04  Min = -4.69E-04
FIGURE 7.7A VELOCITY VECTORS OF THE FLOW
WITH THE RECEIVER AT VERTICAL POSITION (M/SEC)
Max = 8.324E-02  Min = 1.213E-04

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FIGURE 7.7B  VELOCITY VECTORS OF THE FLOW
WITH THE RECEIVER AT 12 DEG. TILT(M/SEC)
Max = 8.737E-02  Min = 1.371E-04

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FIGURE 7.7D VELOCITY VECTORS OF THE FLOW WITH THE RECEIVER AT 30 DEG. TILT(M/SEC)
Max = 8.835E-02  Min = 1.237E-04
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FIGURE 7.7E VELOCITY VECTORS OF THE FLOW WITH THE RECEIVER AT 40 DEG. TILT(M/SEC)
Max = 8.770E-02  Min = 4.231E-05

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FIGURE 7.8A  CONTOURS OF THE ISOTHERMS WITH THE RECEIVER AT VERTICAL POSITION (DEG. KELVIN)
Max = 4.000E+02  Min = 3.106E+02
FIGURE 7.8B  CONTOURS OF THE ISOTHERMS WITH THE RECEIVER AT 12 DEG. TILT (DEG. KELVIN)
Max = 4.000E+02  Min = 3.149E+02
FIGURE 7.8C CONTOURS OF THE ISOTHERMS WITH THE RECEIVER AT 20 DEG. TILT (DEG. KELVIN)
Max = 4.000E+02 Min = 3.112E+02

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FIGURE 7.8D CONTOURS OF THE ISOHERMS WITH THE RECEIVER AT 30 DEG. TILT(DEG. KELVIN)
Max = 4.000E+02  Min = 3.110E+02

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Variation of the internal heat transfer coefficient of the glass tube with the tilt angle.
8 Analytical Modeling of the Receiver System

8.1 Introduction

This chapter presents an analytical model specifically developed for the prediction of the thermal performance of the solar energy concentrator. Analytical modeling is appropriate for a situation where the receiver system is evacuated to a level capable of suppressing the convective heat transfer inside the glass tube. Under these conditions, the dominant heat transfer mechanism associated with the receiver system is radiative. Conductive heat transfer may also take place between the different components which are in direct contact. The model allows prediction of the thermal performance of the solar energy concentrating system from the temperature distribution, obtained by solution of the energy balance equations of the various components of the receiver system.

To formulate the energy balance equations, the receiver system is divided into four main parts, namely, the absorber plate S which has the same temperature as the hot junction of thermoelectric device, the CPC reflector R, the glass tube which includes the aperture window and the rest of the glass envelope, and finally the cold plate which includes the cooling tube and the cold junction of the thermoelectric element. The receiver system is assumed to be in thermal equilibrium, and all the components of the receiver are assumed to be thin and have no heat capacities. For simplicity, each of the above parts is assumed to have a uniform temperature, though the temperatures of the cooling and the glass tubes are expected to vary from bottom to top. The temperature of the cooling tube will be higher at bottom where it receives heat from the thermoelectric device and lower at the top where it loses heat to the glass tube. Also, due to the variation of the external local heat transfer coefficients of the glass tube as a result of natural convection, its temperature is expected to be higher at the
bottom and cooler at the top. However the assumption of uniformity of temperature is necessary for derivation of the energy balance equations.

The analysis is done under assumption that, both the PTC and the CPC reflectors have perfect parabolic reflecting surfaces and are free from fabrication errors, allowing the achievement of the geometrical concentration ratio in each case. The sun rays, the PTC and the CPC are all assumed to be in a good alignment, so that sun’s rays are focused and reach the CPC at an angle $|\beta| \leq (\theta_{an} - \phi - \delta)$. Since there will not be much variation in the temperature distribution of each component, it is reasonable to assume averaged optical properties over the solar spectrum, and for the longwave radiation as well.

In order to facilitate the evaluation of the multiple reflection processes, the solar radiation is assumed to be absorbed diffusely by all the surfaces involved, i.e. absorptivity does not depend on the direction of the incident rays.

The energy transfer paths associated with the receiver system are shown in Fig (8.1), and can be summaries in the following:

1. Solar radiation transfer between the aperture window $W$ of the glass tube, the inner surface of CPC reflector $R$ and the absorbed plate $S$.

2. Thermal radiation exchange between the absorber plate $S$, the inner surface of the CPC reflector $R$ and the aperture window of the glass tube $W$, between the outer surface of the CPC reflector $R$, the glass envelope, the upper surface of the cooling tube, between the hot and the cold junctions of the thermoelectric device through the space around the thermoelement legs, and between the glass tube and the surrounding environment. It is assumed that there is no direct solar radiation reaching the rest of the glass envelop, except the radiation reflected by the PTC and intercepted by the aperture window $W$. However, in practice the receiver will be exposed to radiation from all sides, if not shaded from the top.
3. Conductive heat transfer from the absorber plate S to the CPC reflector R through the contact points, and to the cooling tube through thermoelement legs.

4. Useful heat extracted by the circulating cooling fluid.

5. Heat transferred to the surrounding environment by thermal convection.

The heat transfer analysis is made under open circuit conditions. Therefore, the energy transfer by thermoelectric mechanisms (Peltier, Thomson and Joulian heat flow) are considered negligible. The effect of the thermoelectric device on the thermal performance of the system will be insignificant in any case, because of its low conversion efficiency. The energy balance equation of each surface is derived separately, and then the four equations are solved together for the temperature distribution.
Fig. (8.1) A cross section of the receiver system along with the path of the sun rays, showing the heat transfer paths
8.2 Analysis of the Energy Terms

a. Solar radiation

Since the CPC is capable of capturing all rays incident within its half acceptance angle, all the rays that hit the CPC at an angle $|\beta| \leq \theta_{acc}$ will be accepted and reach the absorber plate $S$ directly or via the CPC reflector. Radiation outside the acceptance angle $\theta_{acc}$ may be rejected, and will not contribute to the energy transfer in the receiver system. The beam rays reflected by the PTC reach the aperture window of the CPC with a varying incident angle. A certain fraction of this radiation will reach the absorber plate directly and the rest via the CPC reflector. The estimation of each of these fractions can only be obtained by employing the Ray-Tracing method. However, because on the worst conditions, radiation incident at an acceptance angle $\theta_{acc}$ will be brought to the focus at the edge of the absorber plate after one reflection from the CPC reflector, a simple approximation can be made by assuming a uniform distribution of the reflected radiation on the aperture window of the CPC, over all the incident angles $|\beta| \leq \theta_{acc}$ and that all the radiation may reach the absorber plate after one reflection from the CPC reflector. This allows assumption to be made that, the radiation throughout the passage from the aperture window of the glass tube $W$ to absorber plate $S$ and vice versa reaches each of them after reflection from the CPC reflector, provided that the incident angles of the rays do not exceed the acceptance half angle of the CPC, no matter how many repeated reflections may have been occurred. Radiation leaving the receiver system through the aperture window will be lost.

Now, the solar radiation input to the receiver system $Q_{br}$, is the portion reflected by the PTC and incident on the aperture window of the CPC within its acceptance angle $(\phi - \delta)$. The calculation is made per unit length of the collector. Therefore the solar radiation incident on the aperture window $W$ can be estimated by

$$Q_{br} = \gamma \varepsilon F_b S C \rho_r G$$

(8.1)
where,

$C_r$ is the concentration factor, $K = (1 - D_s/A)$, is the correction factor for the shading of the receiver on the PTC, $\gamma$ is the intercept factor of the radiation reflected by the PTC on the aperture window of the CPC, $F_\text{b}$ is the beam radiation factor, $G$ is the solar radiation measured on the plane of the PTC aperture, $A$ is the width of the PTC, $D_g$ is the diameter of the glass envelope, $S$ is the width of the absorber plate, $\rho_p$ is the reflectance of the PTC surface.

$q_{\text{ap}}$ transmits through the aperture window of the CPC, hits the reflector surface of CPC reflector $R$, reflects and hits the surface of the absorber plate. A portion of this radiation will be reflected back to $W$ by the help of $R$. This later portion will continue travelling forwards and backwards between the aperture window of the glass tube $W$ and the absorber surface $S$ until absorbed or emerging through $W$. This approximation makes it possible to add the contributions from the multiple reflections. After each round trip, the radiation is attenuated by a factor $(P_W P_R^2 P_S)$, provided that the incident radiation on $W$ and $S$ is not exceeding the half acceptance angle of the CPC, no matter how many round trips between them may have occurred. $\rho_w$, $\rho_r$, and $\rho_s$ are the reflectivities of $W$, $R$, and $S$ respectively. Therefore, a factor ($F_1$) which need to be introduced to account for the multiple reflection process between the aperture window and the absorber plate is given by

$$F_1 = 1 + \rho_w \rho_r^2 \rho_s + (\rho_w \rho_r^2 \rho_s)^2 + (\rho_w \rho_r^2 \rho_s)^3 + \ldots + (\rho_w \rho_r^2 \rho_s)^n + \ldots = (1 - \rho_w \rho_r^2 \rho_s)^{-1}$$ (8.2)

Therefore, the net solar radiation absorbed by the absorber surface $S$, can be estimated by

$$q_{s(\text{b})} = \alpha_{s(B)} \tau_{W} \rho_r F_1 q_{\text{BP}}$$

$$= C_{s(\text{b})} q_{\text{BP}}$$ (8.3)
where, \( C_{SB} = \alpha_S \tau_w \rho_R F_1 \), is the fraction of the incident radiation absorbed by the absorber plate, taking in consideration the contribution of the multiple reflection process.

The solar radiation incident on \( W \), is the sum of the radiation incident on the outer surface of \( W \) while entering the receiver system, and the radiation from the absorber surface incident on the inner surface of \( W \) after reflection from CPC. Solar radiation leaving the \( W \) is the sum of the radiation reflected and the radiation transmitted through \( W \), both inwards and outwards. Assuming that, the optical properties of the inner and the outer surfaces of \( W \) are equal, the net solar radiation absorbed by \( W \), can be estimated by

\[
Q_{W(B)} = \alpha_w(1 + \tau_w \rho_s \rho_R^2 F_1) Q_{BP}
\]

\[
= C_{W(B)} Q_{BP}
\]

\( (8.4) \)

where, \( C_{W(B)} = \alpha_w(1 + \tau_w \rho_s \rho_R^2 F_1) \).

Similarly, since the solar radiation during its journey from the aperture window to the absorber surface and vice versa hits the CPC reflector surface, the solar radiation absorbed by the CPC reflector \( Q_{R(B)} \), is the product of \( Q_{BP} \) and a factor \( C_{R(B)} \) given by

\[
C_{R(B)} = \tau_w(I - \rho_R)\{1 + \rho_s \rho_R + \rho_s^2 \rho_R^2 \rho_w + \rho_s^3 \rho_R^3 \rho_w + \rho_R^4 \rho_s^4 \rho_w^2
\]

\[
+ \rho_R^5 \rho_s^5 \rho_w^2 + \ldots + \rho_R^{2n} \rho_s^{2n} \rho_w^n + \rho_R^{2n+1} \rho_s^{n+1} \rho_w^n + \ldots\}
\]

\[
= \tau_w(I - \rho_R)\{1 + \rho_s \rho_R \}^{n}\{1 + \rho_R^2 \rho_s^2 \rho_w + (\rho_R^2 \rho_s^2 \rho_w)^2 + \ldots + (\rho_R^2 \rho_s^2 \rho_w)^n + \ldots\}
\]

\[
= \tau_w(I - \rho_R)(1 + \rho_R \rho_s)[1 + \rho_R^2 \rho_s^2 \rho_w + (\rho_R^2 \rho_s^2 \rho_w)^2 + \ldots + (\rho_R^2 \rho_s^2 \rho_w)^n + \ldots]
\]

\[
= \tau_w(I - \rho_R)(1 + \rho_R \rho_s)(1 - \rho_R^2 \rho_s \rho_w)\}
\]

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Therefore, $Q_{R(B)}$ can be estimated by

$$Q_{R(B)} = C_{R(B)} Q_{BP}$$  \hspace{1cm} (8.6)

b. **Thermal radiation transfer**

As a result of the absorption of the solar radiation, the temperature of surfaces involved in the heat transfer process will rise, resulting in thermal radiation exchange between the various surfaces of the receiver system. The thermal radiation exchange will take place between $S$, $W$ and the inner surface of $R$ on one side, and between the rest of the glass envelope $E$, the outer surface of the reflector $R'$ and the upper part of the cooling tube $C$ on the other side. Infrared radiation exchange also takes place between the inner surfaces of the cold and hot plates of the thermoelectric element. While the outer surface of the glass tube will exchange thermal radiation with the surrounding environment at temperature $T_m$.

In analogy to the multiple reflection process that applied to the solar radiation, it can also be applied to the longwave radiation as it travels between $S$ and $W$ and vice-versa, with the optical properties replaced by the corresponding longwave radiation range values. The factor $F_l$ for the longwave radiation will be denoted by $F_l$, while $\rho_R$, $\rho_s$ and $\rho_w$ will be replaced by $\rho_{RT}$, $\rho_{ST}$ and $\rho_{WT}$ respectively.

For simplicity, it can be assumed that, the portion of thermal radiation emitted by any one of the CPC reflector couples and incident on the other couple, will reach neither the aperture window nor the absorber plate. It will continue travelling between the surfaces of the two couples until be absorbed by the CPC itself. Because both sides of CPC reflectors are symmetric, will have the same temperature, and therefore the net heat exchange between them will be zero any way.
1. **Emissive power of the absorber surface**

As mentioned earlier, due to absorption of the solar radiation, the absorber plate surface will attain a certain temperature resulting in an emissive power \( E_s \) which can be estimated by the formula [Incropera, 1981]

\[
E_s = A_s \varepsilon_s \sigma T_s^4
\]  

(8.7)

where, \( \varepsilon_s \) is the emissivity of the surface, \( \sigma \) is the Stefan-Boltzmann constant and \( T_s \) is the absolute temperature of the absorber plate surface.

\( E_s \) will transfer to the aperture window \( W \), either directly or via the CPC reflector \( R \). A portion proportional to the view factor \( F_{sw} \) of \( S \) on \( W \) will reach \( W \) directly, part of which (proportional to \( \rho_{wt} \)) will be reflected back to reach \( S \) by the help of \( R \). This last portion will be reflected back again by \( S \) and will continue travelling forward and backward until absorbed or emerging from the receiver through \( W \). Therefore, the net amount of this portion of the emissive power, which leaves the absorber surface \( S \) is estimated as a product of \( E_s \) and a factor \( F_{S1} \) given by

\[
F_{S1} = F_{sw}[1 + \rho_{st} \rho_{rt} \rho_{wt} + \rho_{st}^2 \rho_{rt}^3 \rho_{wt}^2 + \rho_{st}^3 \rho_{rt}^4 \rho_{wt}^3 + ... + \rho_{st}^n \rho_{rt}^{n+1} \rho_{wt}^n + ...]
\]  

\[
= F_{sw}[1 + \rho_{st} \rho_{rt} \rho_{wt}(1 + \rho_{st}^2 \rho_{rt}^2 \rho_{wt}^2 + \rho_{st}^3 \rho_{rt}^4 \rho_{wt}^3 + ... + \rho_{st}^n \rho_{rt}^{2n} \rho_{wt}^n + ...)]
\]  

\[
= F_{sw}[1 + \rho_{st} \rho_{rt} \rho_{wt}(1 - \rho_{st} \rho_{rt}^2 \rho_{wt}^2)^{-1}]
\]  

\[
= F_{sw}[1 + \rho_{st} \rho_{rt} \rho_{wt} F_T]
\]  

(8.8)

where, \( F_T \) is a factor that accounts for the contribution of the multiple reflection of the longwave radiation from the absorber surface to the aperture window of the glass tube via the CPC reflector as explained earlier, and is given by

\[
F_T = (1 - \rho_{st} \rho_{rt}^2 \rho_{wt}^2)^{-1}
\]  

(8.9)
Similarly, the other part of the radiation which is proportional to the view factor \((F_{SR})\) of \(S\) on \(R\) will reach \(W\) by the help of the CPC reflector \(R\), and will undergo the same process until absorbed or emerging from the receiver through the aperture window.

The total amount of this portion of emissive power which leaves the absorber surface \((S)\) is the product of \(E_s\) and a factor \(F_{SR2}\) given by

\[
F_{SR2} = F_{SR}[1 + \rho_{st}^2 \rho_{wt}^2 + \rho_{st}^4 \rho_{wt}^4 + \rho_{st}^6 \rho_{wt}^6 + \ldots + \rho_{st}^{2n} \rho_{wt}^{2n} + \ldots]
\]

\[
= F_{SR} (1 - \rho_{st}^2 \rho_{wt}^2)^{-1}
\]

\[
= F_{SR} F_T \quad (8.10)
\]

The net amount of the emissive power that is emitted by the absorber surface and leaves it, can therefore be approximated as

\[
E_{TS} = E_s(F_{SR1} + F_{SR2})
\]

\[
= \varepsilon_s A_s \sigma T_s^4 [F_{sw}(1 + \rho_{st} \rho_{wt} F_T) + F_{SR} F_T]
\]

\[
= \varepsilon_s \sigma A_s F_{TS} T_s^4 \quad (8.11)
\]

where, \(F_{TS} = F_{sw}(1 + \rho_{st} \rho_{wt} F_T) + F_{SR} F_T\); is the total contribution from the multiple reflection process.

2. **Emissive power of the inner surface of the aperture window**

By the above analogy, the emissive power leaving the inner surface of the aperture window can be estimated as

\[
E_{TW} = \varepsilon_w \sigma A_w F_{TW} T_w^4 \quad (8.12)
\]

where, \(\varepsilon_w\), \(T_w\) and \(A_w\) are the emissivity, absolute temperature and the surface area respectively of the aperture window of the glass tube \(W\), and the factor \(F_{TW} = F_{ws} (1 +\)
ρ_{st}ρ_{ri}ρ_{wt}F_T + F_{wr}F_T accounts for the contribution from the multiple reflection process.

3. Emissive power of the rest of the glass tube

The inner surface of the glass envelope excluding the aperture window, exchanges thermal radiation with the outer surface of the reflector R' and the cooling tube. It is assumed that the average temperature of the glass envelope is the same as that of the aperture window (T_w). And by assuming that the outer surface of the cooling tube faces the glass envelope only, and similarly does the outer surface of the reflector R', due to their geometries, the view factors of the cooling tube to the glass envelope (F_{CE}) and of the reflector R to the glass envelope (F_{RE}) will both be equal unity.

Therefore, the emissive power of the inner surface of the glass envelope which leaves it can be estimated as,

\[E_{TE} = \varepsilon_e \sigma A_e T_w^4 \left(1 + \rho_{ET}(\rho_{CT}F_{EC} + \rho_{ROT}F_{ER}) + \rho_{ET}^2(\rho_{CT}F_{EC} + \rho_{ROT}F_{ER})^2 + \cdots + \rho_{ET}^n(\rho_{CT}F_{EC} + \rho_{ROT}F_{ER})^n + \cdots\right)\]

or,

\[E_{TE} = \varepsilon_e \sigma A_e F_{TE} T_w^4 \left(1 - \rho_{ET}(\rho_{CT}F_{EC} + \rho_{ROT}F_{ER})^{-1}\right)\]

\[= \varepsilon_w \sigma A_e F_{TE} T_w^4 \quad (8.13)\]

where, \(F_{TE} = \left(1 - \rho_{ET}(\rho_{CT}F_{EC} + \rho_{ROT}F_{ER})^{-1}\right)\), accounts for the contribution from the multiple reflections. For simplicity, it is assumed that the emissivity of the rest of the glass envelope is the same as that of the aperture window of the glass tube. \(A_e\) is the surface area of the rest of the glass envelope.

The outer surface of the glass tube (glass envelope plus the aperture window) also exchanges thermal radiation (\(E_{oa}\)) with the outside environment. For simplicity it can
be assumed that, the emissivity of the outer surface of the glass tube is the same as that of the inner surface ($\varepsilon_0$), thus,

$$E_{GA} = \varepsilon_0 \sigma A_G (T_w^4 - T_e^4)$$

where, $A_G$ is the surface area of the glass tube.

4. **Emissive power of the CPC reflector**

The inner surface of the CPC reflector emits thermal radiation $E_R$, which can be estimated by

$$E_R = \varepsilon_R \sigma A_R T_r^4$$

where, $A_R$, $\varepsilon_R$ and $T_r$ are the surface area of the CPC reflector, emissivity of the CPC reflecting surface and the temperature respectively.

Portions proportional to $F_{RW}$ and $F_{RS}$ of emissive power $E_R$ transfer to the aperture window of the glass tube and to the absorber surface respectively. According to the assumptions made earlier, the remainder from each two surfaces of the CPC reflector hits the other and continues to travel forwards and backwards between the two surfaces of the reflector $R$, until absorbed by the CPC. The former two portions also continue to travel between the aperture window and the absorber plate by the help of CPC reflector $R$ until absorbed or emerging through the aperture window as described earlier.

The total emissive power of the inner surface of the CPC reflector and which leaves it, is estimated by

$$E_{TR} = \varepsilon_R \sigma A_R T_r^4 \left[1 + F_{RS} \left\{ \rho_{RT} \rho_{ST} (1 + \rho_{RT} \rho_{ST} \rho_{WT} + (\rho_{RT} \rho_{ST} \rho_{WT})^2 + \ldots) + \right. \right.$$

$$\left. \rho_{RT}^2 \rho_{ST} \rho_{WT} (1 + \rho_{RT} \rho_{ST} \rho_{WT} + (\rho_{RT} \rho_{ST} \rho_{WT})^2 + \ldots) \right\} \right]$$

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\[
F_{\text{RW}} \{ \rho_{\text{RT}} \rho_{\text{WT}} (1 + \rho_{\text{RT}} \rho_{\text{ST}} \rho_{\text{WT}} + (\rho_{\text{RT}} \rho_{\text{ST}} \rho_{\text{WT}})^2 + \cdots) + \\
\rho_{\text{RT}}^2 \rho_{\text{ST}} \rho_{\text{WT}} (1 + \rho_{\text{RT}} \rho_{\text{ST}} \rho_{\text{WT}} + (\rho_{\text{RT}} \rho_{\text{ST}} \rho_{\text{WT}})^2 + \cdots) \}
\]

or

\[
E_{\text{TRI}} = \varepsilon_\sigma A_R T_R^4 [1 + F_F (\rho_{\text{RT}} \rho_{\text{ST}} F_{\text{RS}} (1 + \rho_{\text{RT}} \rho_{\text{WT}}) + \rho_{\text{RT}} \rho_{\text{WT}} F_{\text{RW}} (1 + \rho_{\text{RT}} \rho_{\text{ST}}))] 
\]

Where, \( F_{\text{TR}} = 1 + F_F (\rho_{\text{RT}} \rho_{\text{ST}} F_{\text{RS}} (1 + \rho_{\text{RT}} \rho_{\text{WT}}) + \rho_{\text{RT}} \rho_{\text{WT}} F_{\text{RW}} (1 + \rho_{\text{RT}} \rho_{\text{ST}})) \), accounts for the contribution from the multiple reflection process.

5. **Emissive power of the outer surface of the CPC reflector**

The outer surface of the CPC reflector \( R \), exchanges thermal radiation with the glass envelope. For simplicity and because \( R \) is thin, it is assumed that, the temperature of outer surface of the reflector equals that of the inner surface. The outer surface of the CPC reflector will emit radiation \( (E_{\text{RO}}) \), which amounts to

\[
E_{\text{RO}} = \varepsilon_\sigma A_R T_R^4 
\]

where, \( \varepsilon_\sigma \) is the emissivity of the outer surface of the CPC reflector.

\( E_{\text{RO}} \) will continue travelling between the outer surface of the CPC reflector and the glass envelope until absorbed or transmitted through the glass to the outside surroundings. The total emissive power leaving the outer surface of the CPC reflector can be estimated as

\[
E_{\text{TRO}} = \varepsilon_\sigma A_R T_R^4 (1 + F_{\text{ER}} \rho_{\text{ET}} \rho_{\text{ROT}} + (F_{\text{ER}} \rho_{\text{ET}} \rho_{\text{ROT}})^2 + \cdots + (F_{\text{ER}} \rho_{\text{ET}} \rho_{\text{ROT}})^n + \cdots) 
\]

\[
= \varepsilon_\sigma A_R T_R^4 F_{\text{TRO}} 
\]
where, $F_{\text{TR}_0} = (1 - F_{\text{ER}} \rho_{\text{ET}} \rho_{\text{ER}}) \frac{1}{1}$, is the contribution from multiple reflections, and $F_{\text{ER}}$ is the view factor of the glass envelope to the CPC reflector.

6. **The emissive power of the cooling tube**

The upper surface of the cooling tube exchanges thermal radiation with the glass tube, depending on the temperature of each and the view factors between them. It is assumed that, the cooling tube has a uniform temperature across its cross section, and is equal to the cold junction temperature of the thermoelectric element.

By analogy to the procedure used to drive the emissive power of the outer surface of the CPC reflector, the total emissive power of the cooling tube is estimated by

$$E_{\text{TC}} = \varepsilon_{\text{C}} \sigma A_{\text{C}} F_{\text{TC}} T_{\text{C}}^4$$

(8.18)

where, $F_{\text{TC}} = (1 - F_{\text{EC}} \rho_{\text{ET}} \rho_{\text{EC}}) \frac{1}{1}$, and $F_{\text{EC}}$ is the view factors of C on E.

7. **Thermal radiation exchange between the hot and the cold junctions**

The thermal radiation exchange between the hot and the cold junctions of the thermoelectric device, occurs through the space between the two junction around the thermoelement legs. Since the distance between the two junctions is very short, the effect of the view factors between either the hot and the cold junction surfaces and the thermoelectric legs can be neglected. The emissive power ($E_{\text{hj}}$) which leaves the inner surface of the hot junction amounts to

$$E_{\text{hj}} = \varepsilon_{\text{hj}} \sigma A_{\text{hj}} F_{\text{hj}} T_{\text{h}}^4$$

(8.19)

Similarly, the emissive power ($E_{\text{cj}}$) which leaves the cold junction surface amounts to

$$E_{\text{cj}} = \varepsilon_{\text{cj}} \sigma A_{\text{cj}} F_{\text{cj}} T_{\text{c}}^4$$

(8.20)

where, $F_{\text{hj}} = F_{\text{cj}} = (1 - \rho_{\text{cj}} \rho_{\text{hj}})$.
Using the above emissive power terms, and following the same approach used for derivation of solar radiation absorbed by each part of the receiver system, the net amounts of the emissive power leaving each of the four surfaces are estimated as follow

1. The net emissive power leaving the absorber plate is given by

\[ E_{SN} = C_{S1}T_s^4 - C_{W1}T_w^4 - C_{R1}T_R^4 - C_{C1}T_C^4 \]  \hspace{1cm} (8.21)

2. The net emissive power leaving the glass tube is given by

\[ E_{WN} = C_{W2}T_w^4 - C_{S2}T_s^4 - C_{R2}T_R^4 - C_{C2}T_C^4 - C_{A2}T_A^4 \]  \hspace{1cm} (8.22)

3. The net emissive power leaving the CPC reflector is given by

\[ E_{RN} = C_{R3}T_R^4 - C_{W3}T_w^4 - C_{S3}T_s^4 \]  \hspace{1cm} (8.23)

4. The net emissive power leaving the cooling tube is given by

\[ E_{CN} = C_{C4}T_C^4 - C_{S4}T_s^4 - C_{W4}T_w^4 \]  \hspace{1cm} (8.24)

where the coefficients \( C_{ij} \) (i = S, W, R and C, and j = 1, 2, 3 and 4) are defined as follow

\[ C_{S1} = \sigma (\varepsilon_s F_{TS} + \varepsilon_h A_{ST} F_{th}) \]

\[ C_{w1} = \sigma \alpha_{ST} \varepsilon_w F_{TW} F_{sw} A_s \]

\[ C_{R1} = \sigma \alpha_{ST} \varepsilon_r F_{TR} F_{sr} A_s \]

\[ C_{w2} = \sigma (A_w \varepsilon_w F_{TW} + A_g \varepsilon_g + A_e \varepsilon_e F_{TE}) \]

\[ C_{R2} = \sigma \frac{A_R}{A_s} (\alpha_{TR} F_{TR} F_{RW} + \alpha_{ET} \varepsilon_{ET} F_{R0}) \]

\[ C_{C2} = \sigma \alpha_{CT} \varepsilon_c F_{TC} \]

\[ C_{A2} = \sigma \varepsilon_g A_g \]

\[ C_{S3} = \sigma \alpha_{ST} \varepsilon_s F_{SR} F_{TS} A_s \]
c. Heat transferred by convection and/or conduction mechanisms

The glass tube is assumed to be evacuated to a vacuum pressure level that will be capable of suppressing any convective heat transfer inside the tube. The heat transfer by convection is from the outer surface of the glass tube to the ambient air and from the cooling tube to the cooling fluid only. Other heat transfer is by conduction, from the hot junction (absorber plate) to the CPC reflector and to the cold junction through the contact points and the thermoelectric element respectively. The heat flow to the cold junction will be through the thermoelectric legs and through the space between the two junctions. The derivation is given in Chapter 6 by equation (6.11).

1. Convective heat from the glass tube to the ambient air

The glass tube is considered to be a long horizontal circular tube, and is assumed to have a uniform temperature distribution circumferentially.

Heat transfer by convection from the glass tube to the air is estimated by

\[ Q_{GA} = A_G h_{GA} (T_W - T_A) \]  

(8.25)

where \( h_{GA} \) is the heat transfer coefficient and is given by

\[ h_{GA} = k_A N_u D^{-1} \]  

(8.26)
$T_A$ is the temperature of the ambient air and $(k_a)$ is the thermal conductivity of the air.

On assumption that, the air speed around the glass tube is zero (still air), the following expression for Nusselt number is recommended by [Churchi & Chu, 1975]

$$Nud = (0.6 + 0.32R_s^{\frac{1}{2}})^2$$

(8.27)

where $R_s$ is the Raleigh number, and is given by

$$R_s = g\beta\Delta T D^3\nu^{-2}P_r$$

(8.28)

$P_r$ is the Prandtl number and for the $P_r = 0.71$,

$\nu$, is the kinematics viscosity of the air.

g, is the gravitational acceleration,

$\beta = \frac{1}{T}$, is the coefficient of the thermal expansion,

$\Delta T$, is the temperature difference between the glass tube and the ambient air, and

$D$, is diameter of the glass tube.

2. Other heat transfer

Heat transfer from the absorber plate to the CPC reflector through the contact points is estimated as

$$Q_{k(SR)} = U_{conSR} A_{conSR} (T_S - T_R)$$

(8.29)

where,

$U_{conSR} = (K_{conSR}/L_{conSR})$, is the heat transfer coefficient between S and R. $A_{conSR}$ is the cross section area of the contact points.
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Other energy terms include the electrical energy output from the thermoelectric element and the heat extracted from the cooling tube. The former is zero since the analysis is considered under open circuit conditions. The later is the useful heat output from the collector, and is given by equation 6.8. The cooling tube is a heat exchanger, and by neglecting thermal resistance of the tube skin, the useful heat can be estimated as

\[ Q_u = m C_p \left( T_o - T_i \right) = h_{cf} A_{cf} \left( T_c - T_f \right) \]  (8.30)

where, \( m C_p \) is thermal capacitance of the fluid, \( T_o \) and \( T_i \) are the outlet and the inlet temperatures of the fluid, \( T_f = 0.5(T_o + T_i) \) is the fluid mean temperature, and \( h_{cf} \) is the heat transfer coefficient from the tube to the fluid. \( A_{cf} \) is the surface area of the copper tube.

The outlet temperature of the water as a function of \( T_c \) and \( T_i \), can be found by replacing the value of \( T_f \) in equation (8.30), and solving for \( T_o \). Thus

\[ T_o = \left( m C_p - C_F \right) \left( m C_p + C_F \right)^{-1} T_i + 2C_F \left( m C_p + C_F \right)^{-1} T_c \]  (8.31)

where, \( C_F = 0.5 h_{CF} A_{CF} \)

Substituting for \( T_o \) in equation 8.30, the heat extracted by the cooling water is given by

\[ Q_u = A_{CF} U_{CF} \left( T_c - T_i \right) \]  (8.32)

where,

\[ U_{CF} = \frac{m C_p h_{CF}}{m C_p + \frac{1}{2} h_{CF} A_{CF}} \]  (8.33)

The value of \( U_{CF} \) is also given by equation (6.8), however equation (8.33) gives its value as a function of the mass flow rate. The value of \( h_{CF} \) is given by equation (6.9).
8.3 Energy Balance Equations

The energy terms analysed in section (8.2), give the basis to develop the energy balance equations for the four components of the receiver system. Considering the energy input and the energy output for each component, the energy balance equations are as follows:

- **Energy balance of the absorber plate**

Energy input to the absorber plate $S$ is the net solar radiation absorbed by the absorber surface; given by equation (8.3). The energy output from the absorber plate is the sum of the net emissive power leaving the absorber plate, given by equation (8.21), the energy leaving the absorber plate to the CPC reflector by conduction, given by equation (8.29) and to the cooling tube through the thermoelectric device, given by equation 6.10. Thus

$$Q_{S(B)} - E_{SN} - Q_{K(SR)} - Q_{K(SC)} = 0$$

Substituting values of these heat terms and rearranging, the energy balance equation for the absorber plate will be

$$C_{s1}T_s^4 + C_{w1}T_w^4 + C_{r1}T_R^4 + C_{c1}T_C^4 + C_{s01}T_s$$

$$+ C_{w01}T_w + C_{r01}T_R + C_{c01}T_C = C_1$$

(8.34)

where,

$$C_{s01} = -(U_{SCA_s} + U_{SCSR}A_{SCSR}), \quad C_{w01} = 0, \quad C_{r01} = (U_{CSR}A_{CSR}), \quad C_{c01} = U_{SCA_s}, \quad \text{and}$$

$$C_1 = - C_{S(B)} Q_{BP}.$$
Energy balance of the glass tube

Energy input to the glass tube is the net solar radiation absorbed by the aperture window of the glass tube, given by the equation (8.4). The energy output from the glass tube is the sum of the net emissive power leaving the glass tube, given by the equation (8.22), and the energy leaving the glass tube to the ambient air by convection, given by the equation (8.25). Thus

\[ Q_{w(B)} - E_{WN} - Q_{OA} = 0 \]

Substituting for these heat terms and rearranging gives the energy balance equation of the glass tube,

\[
C_{s2}T_s^4 + C_{w2}T_w^4 + C_{k2}T_k^4 + C_{c2}T_c^4 + C_{so2}T_s + C_{wo2}T_w + C_{k02}T_k + C_{c02}T_c = C_2
\]

(8.35)

where,

\[
C_{so2} = 0, \quad C_{wo2} = -A_Gh_{GA}, \quad C_{k02} = 0, \quad C_{c02} = 0, \quad \text{and}
\]

\[
C_2 = -((C_{w(B)}Q_{BP} + A_Gh_{GA}T_\infty) + \sigma e_G A_G T_\infty^4).
\]

Energy balance of the CPC reflector

Energy input to the CPC reflector is the sum of the net solar radiation absorbed by the CPC reflector, given by the equation (8.5), and the heat transferred to CPC from the absorber plate by conduction through the contact points, given by equation (8.29). The energy output from CPC is the sum of the net emissive power leaving CPC, given by the equation (8.23) and the heat leaving CPC to the cooling tube through the contact points by conduction. Thus

\[ Q_{r(B)} - E_{RN} + Q_{k(SR)} = 0 \]

Therefore by substituting these heat terms and rearranging, the energy balance equation of the CPC reflector becomes,
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\[ C_{s3}T_s^4 + C_{w3}T_w^4 + C_{r3}T_R^4 + C_{c3}T_c^4 + C_{so3}T_s + \]

\[ C_{wo3}T_w + C_{ro3}T_R + C_{co3}T_c = C_3 \] (8.36)

where, \( C_{s3}, C_{w3} \) and \( C_{r3} \) are defined earlier, \( C_{c3} = 0, C_{so3} = U_{conSR}A_{conSR}, C_{wo3} = 0 \),

\( C_{ro3} = -(U_{conSR}A_{conSR}), C_{co3} = 0 \), and \( C_3 = -C_{R(B)}Q_{in} \).

• Energy balance of the cold junction

Energy input to the cold junction is the sum of heat transferred to the cold junction from the absorber plate, given by the equation (6.10) and from the CPC reflector through the contact points. The energy output from the cold junction is the sum of the heat extracted by the circulated cooling liquid, given by the equation (8.32), and the net emissive power leaving the cooling tube, given by equation (8.24). Thus

\[ Q_{K(SC)} + Q_{K(RC)} - Q_U - E_{CN} = 0 \]

Substituting for these heat terms and rearranging, the energy balance equation of the cold junction becomes,

\[ C_{s4}T_s^4 + C_{w4}T_w^4 + C_{r4}T_R^4 + C_{c4}T_c^4 + C_{so4}T_s + \]

\[ C_{wo4}T_w + C_{ro4}T_R + C_{co4}T_c = C_4 \] (8.37)

where,

\( C_{s4} = 0, C_{so4} = U_{SCA_S}, C_{wo4} = 0, C_{ro4} = 0, C_{co4} = -(U_{SCA_S} + U_{CFACF}), \) and

\( C_4 = -U_{CFACF}T_{in} \).

• General form of the energy balance equation

The four non-linear energy balance equations of the absorber plate, the glass tube, the CPC reflector and the cold junction can be expressed in the following generalised form

\[ C_{sj}T_s^4 + C_{wj}T_w^4 + C_{sj}T_R^4 + C_{cj}T_c^4 + C_{soj}T_s + \]
where \( j = 1, 2, 3 \) and 4 refers to the energy balance equation of a given surface (1 for absorber plate surface, 2 for glass tube, 3 for CPC reflector and 4 for cold junction, and the cooling tube).

\( C_{sj}, C_{wj}, C_{cj}, \) and \( C_j \) are defined with respect to the surface areas involved and account for the longwave radiation properties and contribution of the multiple reflections process, while \( C_{soj}, C_{woj}, C_{roj}, \) and \( C_{coj} \) account for convective/conductive heat transfer coefficients. The above four non-linear equations, which are valid only under the steady state and open circuit voltage conditions are solved simultaneously, together with the equation (8.31) to determine the temperatures \( T_S, T_w, T_R, T_C \) and \( T_o \) for a given inlet temperature \( T_i \), ambient temperature and solar radiation.

### 8.4 Geometrical View Factors

The geometrical view factors between the absorber plate surface \( S \), the inner surface of the CPC reflector and the aperture window of the glass tube \( W \) are obtained by using view factor algebra and the Hottel’s crossed-string method [Modest, 1993], as described in Appendix 4. The rest of view factors are obtained using the reciprocity and summation relationships [Incropera, 1981]

First, the following are defined:

\( S = \) width of the absorber plate; exit aperture of the CPC.

\( W = S (\sin \theta_{acc})^{-1} \), is the width of the entrance aperture of the CPC.

\( R = \) overall length of the CPC reflector along the two portions of its parabolic curves.

The length \( R \) of the CPC reflector is estimated by describing the CPC in a two dimensional co-ordinate and is given by the equation (see Appendix 4):
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\[ R = S(1 + \sin \theta) \left[ \frac{\cos \theta}{\sin^2 \theta} + \ln \left( \frac{(1 + \sin \theta)(1 + \cos \theta)}{\sin \theta\left(\cos \theta + \sqrt{2(1 + \sin \theta)}\right)} \right) - \frac{\sqrt{2} \cos \theta}{(1 + \sin \theta)^{3/2}} \right] \]  (8.39)

where \( \theta \) is the acceptance half angle of the CPC.

The depth \( D \) of the CPC is given by equation (5.12) in Chapter 5 as

\[ D_{\text{CPC}} = \frac{1}{2}(W + S)\cot \theta \]  (8.40)

\[ = \frac{S(1 + \sin \theta)}{2 \sin^2 \theta} \cos \theta \]  (8.41)

The view factor \( F_{sw} \) of the absorber plate to the aperture window of the glass tube, is estimated by using the Hottel’s Crossed-String method described in Appendix 4, and is found to be

\[ F_{sw} = \frac{1}{2 \sin \theta} \left( \frac{1}{\sin \theta} + 1 - \sqrt{(1 + \sin \theta)^2 \left(\frac{\cos \theta}{\sin \theta}\right)^2 + (1 - \sin \theta)^2} \right) \]  (8.42)

Similarly, the view factor \( F_{wr} \) of the aperture window to the CPC is given by considering long wedge-shaped groove (Appendix 4) as follow

\[ F_{wr} = 1 + x + \sqrt{\sin \theta + \frac{1}{4} x^2} \]  (8.43)

where, \( x = \sqrt{(1 + \sin \theta)^2 \left(\frac{\cos \theta}{\sin \theta}\right)^2 + (1 - \sin \theta)^2} \)

The view factor \( F_{sr} \) of the absorber plate to the CPC reflector is

\[ F_{sr} = 1 - F_{sw} \]  (8.44)

The view factor \( F_{rs} \) of the aperture window to the absorber plate surface is
ANALYTICAL MODELING

\[ F_{RS} = \frac{A_s}{A_w} F_{SW} \]  \hspace{1cm} (8.45)

the view factor \( F_{RW} \) of the CPC reflector to the absorber plate surface is

\[ F_{RS} = \frac{A_s}{A_r} F_{SR} \]  \hspace{1cm} (8.46)

The view factor \( (F_{RW}) \) of the CPC reflector to the aperture window is given by

\[ F_{RW} = \frac{A_w}{A_r} F_{WR} \]  \hspace{1cm} (8.47)

For the rest of the receiver system it can be assume that the upper surface of the cooling tube views the glass envelope only, as does the outer surface of CPC reflector. This implies that the view factors of the cooling tube \( F_{CE} \) and the outer surface of the CPC reflector \( F_{RE} \) to the glass envelope are as follows

\[ F_{CE} = 1 \]  \hspace{1cm} (8.48)

\[ F_{RE} = 1 \]  \hspace{1cm} (8.49)

The view factors of the glass envelope to the upper surface of the cooling tube \( F_{EC} \), and to the outer surface of CPC reflector \( F_{ER} \) are estimated by employing the reciprocity rules and are found to be

\[ F_{EC} = \frac{A_c}{A_e} \]  \hspace{1cm} (8.50)

\[ F_{ER} = \frac{A_r}{A_e} \]  \hspace{1cm} (8.51)

8.5 Results of the Analytical Model

The four non-linear energy balance equations (8.34), (8.35), (8.36) and (8.37) were solved together with equation (8.31) for the temperatures \( T_s, T_w, T_r, T_c \) and \( T_e \) for a given inlet fluid temperature...
and solar radiation. The solution is accomplished using the proprietary (Maple) solver. The solution
algorism written for Maple is presented in Appendix 5.

The overall external heat transfer coefficient from the glass tube to the ambient air is pre-estimated
by assuming a value for the mean temperature of the glass tube. This value is then iteratively re-
adjusted using the results from the interior solution until an overall solution is obtained, when the
difference between the assumed value and the computed one is insignificant. Simultaneously, the
same procedure is employed for the film coefficient of the cooling tube by pre-assuming a value for
the fluid mean temperature.

The geometrical dimensions and the thermophysical properties of receiver's components used in the
solution, are same as those of the experimental solar thermoelectric model described in chapter (7).
Some of these parameters are listed in table (8.1) below.

Table (8.1) Thermal and solar properties of receiver system components

<table>
<thead>
<tr>
<th>Site Specification</th>
<th>CPC reflector</th>
<th>Thermoelectric ceramic plates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar radiation (W/m²): 1180</td>
<td>Material: Anodised aluminium</td>
<td>Material: Aluminium oxide</td>
</tr>
<tr>
<td>Beam radiation factor: 0.85</td>
<td>reflectivity(solar): 0.86</td>
<td>Conductivity(W m⁻¹°C⁻¹): 0.065</td>
</tr>
<tr>
<td>Parabolic trough concentrator</td>
<td>emmisivity: 0.74</td>
<td>emmisivity: 0.69</td>
</tr>
<tr>
<td>Material: Polymer reflecting film</td>
<td>Thermolectric ceramic plates</td>
<td>Absorber plate</td>
</tr>
<tr>
<td>Surface reflectivity(solar): 0.85</td>
<td>Material: Aluminium oxide</td>
<td>absorptivity: 0.88</td>
</tr>
<tr>
<td>Glass tube</td>
<td>Conductivity(W m⁻¹°C⁻¹): 0.065</td>
<td>Cooling tube</td>
</tr>
<tr>
<td>Material: Pyrex glass</td>
<td>emmisivity: 0.69</td>
<td>Material: copper</td>
</tr>
<tr>
<td>Size(O/D) (cm): 7.06</td>
<td>Absorptivity: 0.88</td>
<td>emmisivity: 0.5</td>
</tr>
<tr>
<td>Thickness (cm): 0.2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmittance(solar): 0.85</td>
<td></td>
<td></td>
</tr>
<tr>
<td>emmisivity: 0.85</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The results of the solution for a mass flow rate 60 ml per minute are presented below.
Figures (8.2) and (8.3) show the variation of the temperature distribution and the
temperature difference across the thermoelectric device with inlet cooling water temperature respectively.

Fig. (8.2) Temperature distribution as a function of the inlet cooling water temperature.

Fig. (8.3) Temperature difference across the thermoelectric device as a function of the inlet cooling water temperature.
The instantaneous performance of the solar energy concentrators as function of the terms \((T_s - T_a)/G\) and \((T_i - T_R)/G\) are shown in Fig. (8.4) and (8.5) respectively. The key parameters describing the collector performance \((\eta_o, U_L\) and \(F_R\)) are obtained using least square fitting in the form of equations (6.14) and (6.19). The optical efficiency \((\eta_o)\) of the concentrating system, the heat removal factor \((F_R)\) and the overall heat loss coefficient \((U_L)\) are found to be 0.51, 0.78 and 2.01 Wm\(^{-2}\)K\(^{-1}\) respectively. The thermal conversion efficiency which varied between 40% to 34% for inlet water temperature between 65 °C and 20 °C respectively, is favourably comparable to results available in the literature [Hsieh, 1981; O’Gallagher, 1982], for conventional concentrating solar energy collectors using CPC, which give efficiency in the range 30 - 45% for absorber plate temperature between 100 and 150 °C. The instantaneous performance curve generally is quite flat indicating that the efficiency is insensitive to the parameter \((T_i - T_a)/G\).

Tables (8.2) and (8.3) present sample of the solution results

![Graph](image)

**Fig. (8.4)** Instantaneous performance of the solar concentrating system as a function of the term \((T_s - T_a)/G\).
Fig. (8.5) Instantaneous performance of the solar concentrating system as a function of the term \((T_i - T_s)/G\).

Table (8.2) Temperature distribution of the receiver system

<table>
<thead>
<tr>
<th>Temperature , °C</th>
<th>Inlet water</th>
<th>Outlet water</th>
<th>Hot junction</th>
<th>Cold junction</th>
<th>CPC</th>
<th>Glass tube</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>31</td>
<td>85</td>
<td>49</td>
<td>65</td>
<td>27.0</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>36</td>
<td>89</td>
<td>53</td>
<td>66</td>
<td>28.0</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>41</td>
<td>93</td>
<td>57</td>
<td>67</td>
<td>28.5</td>
<td></td>
</tr>
<tr>
<td>35</td>
<td>46</td>
<td>97</td>
<td>62</td>
<td>69</td>
<td>29.0</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>51</td>
<td>101</td>
<td>66</td>
<td>70</td>
<td>30.0</td>
<td></td>
</tr>
<tr>
<td>45</td>
<td>55</td>
<td>105</td>
<td>70</td>
<td>72</td>
<td>30.5</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>60</td>
<td>109</td>
<td>75</td>
<td>73</td>
<td>31.0</td>
<td></td>
</tr>
<tr>
<td>55</td>
<td>65</td>
<td>113</td>
<td>79</td>
<td>75</td>
<td>31.5</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>70</td>
<td>117</td>
<td>83</td>
<td>76</td>
<td>32.0</td>
<td></td>
</tr>
<tr>
<td>65</td>
<td>75</td>
<td>121</td>
<td>88</td>
<td>77</td>
<td>32.5</td>
<td></td>
</tr>
</tbody>
</table>
8.6 Conclusion

The analytical model developed here is successfully used to predict the thermal performance of the receiver system under vacuum condition. The solution is obtained by solving the energy balance equations of the various components of the receiver system.

The solar to thermal energy conversion efficiency varied between 34% to 40% for the inlet water temperature between 65 and 20 °C respectively. The key parameters describing the collector performance $\eta_o$, $U_L$ and $F_R$ are found to be 0.51, 0.78 and 2.01 Wm⁻²K⁻¹ respectively. These results are favourably comparable with those available in the literature, for the conventional solar energy concentrators using compound parabolic concentrators. This in effect show that the combined solar thermoelectric power generation system, can provide thermal energy at the same efficiency as the conventional solar collectors, whilst also provides additional electrical power to the user. As a result the overall energy conversion efficiency and the cost effectiveness of the combined system will be improved.
CHAPTER NINE

9 Experimental Evaluation of the Solar Concentrator

9.1 Introduction

The experimental work is aimed at the provision of thermal performance data which will enable assessment of the thermal characteristics of the solar energy concentrating system and validation of the theoretical models presented in Chapters 8. The main task which the experimental investigation tackles is the role of the second stage concentrator (CPC) in improving the performance of the solar energy concentrating system. The test considers the effect of three main parameters, namely the tracking misalignment, the collector inclination and the vacuum level inside the glass tube. The experiment was performed indoors using a solar simulator.

9.2 Construction of the Concentrator

The experimental solar energy concentrator was constructed using a two stage concentrator comprising a primary parabolic trough concentrator (PTC) of concentration ratio 12 and a secondary compound parabolic concentrator (CPC) of a concentration ratio 1.64. The PTC was formed by forcing an aluminum sheet onto a pre-shaped structural frame, fabricated to fit an approximate curvature of a parabola of a focal length 33 cm and a rim angle 40°, as shown in Fig. (9.1A). The reflecting surface was provided by lining an aluminized polymer film (mirror) on the aluminum sheet.

The CPC was made of an anodized aluminum sheet, cut to fit approximately segments of a parabola as shown in Fig. (9.1B). The CPC was attached to the cooling tube using small pieces of metallic strips. The cooling tube was made of (22 mm. bore) copper tube, flattened on one side to allow mounting of the thermoelectric device with minimum thermal resistance.
Several Peltier thermoelectric cooling devices were cut into small (2 cm. by 4 cm.) pieces and mounted on the flat surface of the cooling tube, using an acrylic adhesive heat sink bonder. These devices are not intended to function as power generator, but to provide correct thermal properties between the absorber plate and the cooling tube. It worth mentioning that, since the efficiency of these devices is very low, the heat flow will not be affected much. The absorber surface was provided using black paint on the outer surface of the hot ceramic plate.

The glass tube was prepared from Pyrex glass using quick fit facilities. The tube is made of two pieces of glass tube, each with a collar/flange at one end. This allowed the assembly of the receiver system as shown in Fig. (9.2). The flange joint was held in place using a metallic spring clamp and a vacuum sealing ring in between. The two ends of the glass tube were drawn down to fit the cooling tube, and sealed to it using a sleeve gasket and hollow screw caps made of heat resistance phenolic plastic.
9.3 Experimental Setup and Measurement

The receiver system was mounted along the focal line of the PTC using a structural support allowing one axis movement of the assembly to track the light source, as shown in Fig. (9.3). The rig was mounted on a wheeled support, with an elevated water tank located about two feet above the receiver level to permit natural circulation of the cooling water in a case of low flow rates. A small submersible dc
motor driven pump located at the bottom of the tank was used to circulate the cooling water when a high flow rate was required.

The flow rate of the cooling water was measured using two liquid flow indicators (rotameters), connected in parallel, and selected by a hand operated valve allowing switching between the low flow (zero to 20 ml per minute) to the high flow (60 - 550 ml per minute) indicator. Both rotameters were calibrated using a measuring cylinder and a stop watch to ±2% accuracy in the temperature range 22 - 60°C. Details of the experimental setup are shown in photos (9.1) to (9.4).

Fig. (9.3) Schematic description of the experimental Rig, (A) Solar energy collecting system (B) Solar simulator.
The solar radiation was simulated using two 1000W Compact Source Iodide (CSI) lamps mounted on an adjustable support allowing vertical and inclined positioning as shown in Fig. (9.3B).

A LabMaster power sensor head (LM3) obtained from the Coherent Company was used for the measurement of the radiation. The technical specifications of the LM3 are given in Appendix 6.

The temperatures were measured using k-type thermocouples (accuracy $\pm 3^\circ C$ in the temperature range zero to 400 $^\circ C$), attached at the locations indicated in Fig. (9.4). Surface temperatures were measured using self adhesive k-type thermocouples with a small metallic disc (7 mm diameter). The inlet and the outlet cooling water temperatures were measured using long stainless steel probe k-type thermocouples (300 mm long and 3 mm diameter), and were controlled using a submersible electrical heater located inside the water tank.
**Photo (9.1)** Details of the cooling tube and the CPC assembly
Photo (9.2) Details of the glass tube
Photo (9.3) Assembly of the experimental setup
Photo (9.4)  View of the experimental setup and the solar simulator
Fig. (9.4) Locations of temperature sensors in the receiver system.

A PC driven data acquisition system using a (PC-LabCard 818HG) supported by a signal conditioning and channel multiplexer daughter board (PCL-789D) was used for data recording. A program was developed which allowed data monitoring, collection and calculation of the performance of the solar concentrating system. The chart in Fig. (9.5) describes the procedure of the program algorithm used. The algorithm of the program is presented in detail in Appendix 7.
Fig. (9.5) Flow chart of the program algorithm used for data measurement, monitoring and processing
9.4 Results of the Experiment and Discussion

The performance of the concentrating system was determined at steady state conditions under various operating conditions, by displaying the data on the computer monitor. The variable parameters included the inlet water temperatures, mass flow rates, tracking misalignment angles, collector inclination angles and pressure levels inside the glass tube.

The concentration ratios of the PTC and the CPC were determined by measuring the intensity of the light at the aperture and the focus of each concentrator separately. The measured concentration factor of the PTC was found to be about 7, or about 58% of the theoretical value (12). This low value can be attributed to several factors, one of them is the curvature of the reflecting surface, which only approximates to a parabola. Another factor can be related to the light spectrum of the solar simulator. Although the CSI lamp acceptably coincides with the solar spectrum for the important band 0.4 - 0.7 μm [Schmid & Kursi, 1983], it emits too much energy near the IR region (λ > 1.0 μm). This combined with the incident angle of the incoming rays (rays from the lamp are not parallel), are sufficient to reduce the actual concentration efficiency of the PTC to the above value.

On the other hand the concentration ratio of the CPC was found to be about 1.4, i.e. about 85% of the theoretical value (1.64). Hence, the total concentration ratio of the combined concentrator is 9.8 or about 50% of the total geometrical concentration ratio designed for (19.7).

The above values are found using a single CSI lamp located about three meters away from the aperture plane of each concentrator's reflector, which allowed approximately uniform radiation distribution on the aperture plane of each. In the actual experimental setup, the distance between the light source and the aperture plane of the primary concentrator was adjusted to 1.6 meters. Consequently, the resultant radiation distribution on the aperture plane of the concentrating system is as shown in Fig. (9.6).
RESULTS OF EXPERIMENT AND DISCUSSION

The dimensions of the x- and the y-axis shown in Fig. (9.6) are in arbitrary units chosen to divide the aperture area into equal sections. The gross dimensions of the PTC are 50 cm length by 40 cm width, however the effective length (the middle part), where the radiation distribution is relatively uniform is 30 cm in length. With this arrangement an average radiation intensity of about 1180 Wm\(^{-2}\) was obtained on the effective aperture plane of the PTC. About 78% of the illuminated area is within \(\pm 25\%\) of the average radiation, while only about 22% meets the standard uniformity requirement recommended by ASHRAE [ASHRAE 93-77] i.e. \(\pm 10\%\) of the average radiation. This is likely to affect the overall concentration efficiency of the system, however, this was the only possible arrangement.

The distribution of the concentrated light along a line perpendicular to the focal line of the PTC is shown in Fig. (9.7). Almost all the reflected radiation (area under the curve) is intercepted by the glass tube (7 cm. diameter). About 75% of this radiation would be intercepted by a CPC of an aperture width 4 cm, while only about 51% of the reflected radiation would be intercepted by a single receiver (2 cm diameter) if the second stage CPC was not used (typical case of a single PTC concentrator). However, with the current secondary CPC (aperture width of 3.28 cm), the intercept factor is about 67%.
RESULTS OF EXPERIMENT AND DISCUSSION

Fig. (9.7) Distribution of the radiation along a line perpendicular to the focal line of the PTC

In situations when the solar energy concentrator does not track the radiation source perfectly, the intercept factor will fall dramatically, depending on the degree of misalignment. Fig. (9.8) and Fig. (9.9) show the effect of the tracking misalignment on the concentrated radiation at the focus of the primary concentrator and the temperature distribution inside the receiver system respectively.

Fig. (9.8) Effect of the tracking misalignment on the concentrated radiation at the focus of the PTC.
RESULTS OF EXPERIMENT AND DISCUSSION

It can be observed that, without a secondary concentrator, a very small misalignment will result in a rapid reduction of radiation intensity at the focal line. At a misalignment of 2 degrees, the radiation level dropped to about one quarter. However with the two stage concentrator, the concentrating system will tolerate higher misalignment angles. As can be seen in Fig. (9.9), the temperatures of the different components of the receiver system are not affected much by the misalignment for angles up to about 4 degrees, except the temperature of the lower part of the glass tube, which is directly exposed to concentrated light. As mentioned earlier, the glass tube absorbs most of the long wave radiation emitted by the CSI lamp, and because it is directly exposed to the concentrated light, it tends to become too hot when the receiver system is perfectly aligned with the PTC. However, as the misalignment increases, it intercepts less concentrated light and a consequent drop in its temperature.

![Graph showing temperature distribution](image)

**Fig. (9.9)** Effect of the tracking misalignment on the temperature distribution inside the receiver system at atmospheric pressure

Fig. (9.10) shows the variation of the temperature difference across the thermoelectric device with collector misalignment angle at atmospheric pressure and mass flow rate of 60 ml/minute. The graph essentially indicates the sensitivity of the temperature difference across the thermoelectric device to the tracking misalignment, particularly at large angles.
RESULTS OF EXPERIMENT AND DISCUSSION

Mass flow rate = 60 ml/min.
Inlet water temperature = 23 °C
at atmospheric pressure

Fig. (9.10) Temperature difference across the thermoelectric device versus collector misalignment angle.

On the other hand, the thermal conversion efficiency of the concentrating system is less sensitive to the tracking misalignment, for angles less than 4°, as shown in Fig. (9.11).

Fig. (9.11) Thermal conversion efficiency of the concentrating system versus collector misalignment angle.
In general, the design can tolerate misalignments up to about 4° without a significant reduction in the thermal conversion efficiency of the concentrating system.

Another important parameter which has been investigated is the tilt angle of concentrating system. Fig. (9.12) shows the temperature distribution of the receiver system as a function of the collector tilt angle at atmospheric pressure. As can be seen, the temperatures are almost constant for collector tilt angle less than 20°.

The variation of the absorber plate temperature with the tilt angle for varying mass flow rates is shown in Fig. (9.13). Again, it can be observed that, regardless of the flow rate, the absorber plate temperature remains almost constant for collector tilt angles less than about 20°. Similarly, the temperature difference across the thermoelectric device is also almost constant for collector tilt angle less than 20°, as shown in Fig. (9.14). Regardless of the cooling water flow rate, the maximum temperature difference across the thermoelectric device was found to be about 35 degrees. This may necessitate use of multistage thermoelectric elements in order to increase the overall temperature difference by increasing the thermal resistance of the device.

Fig. (9.12) Temperature distribution inside the receiver system as a function of the collector tilt angle at atmospheric pressure.
RESULTS OF EXPERIMENT AND DISCUSSION

**Fig. (9.13)** Variation of the absorber plate temperature with the collector tilt angle at atmospheric pressure and inlet water temperature 23 °C.

**Fig. (9.14)** Temperature difference across the thermoelectric device versus collector tilt angle at atmospheric pressure.

---

$m = 14 \text{ ml/min.}$
$m = 18 \text{ ml/min.}$
$m = 60 \text{ ml/min.}$
$m = 175 \text{ ml/min.}$
$m = 450 \text{ ml/min.}$
RESULTS OF EXPERIMENT AND DISCUSSION

The thermal conversion efficiency of the concentrating system, however, is not affected much by the tilt angle for angles up to about 30°, as shown in Fig. (9.15). However for tilt angle greater than 30°, the drop in the efficiency is appreciable.

![Graph showing efficiency vs. tilt angle](image)

**Fig. (9.15)** Efficiency of the concentrating system versus tilt at atmospheric pressure and inlet temperature of 23°C.

The variation of the temperature distribution of the receiver system with the pressure level inside the glass tube is shown in Fig. (9.16). Although, the effect of the tilt angle is considerable, the temperature distribution is not affected much by the pressure level, except at pressure levels below 200 mbar, where a slight improvement can be observed. It is worth mentioning that, the lowest pressure obtained in the experiment was 50 mbar.

The temperature of the air in the region confined by the absorber plate, CPC and the aperture window of the glass tube is overestimated, because the thermocouples were directly exposed to concentrated radiation, and do not necessarily indicates the temperature profile of the air in that region. Some attempts were made to shield these thermocouples but without success, mainly due to the limitation of the space.
Fig. (9.16)  Variation of temperature distribution with pressure level, (A) Tilt angle = zero degree, (B) Tilt angle = 30°.

Continue Fig. (9.16).
RESULTS OF EXPERIMENT AND DISCUSSION

The variation of the temperature difference across the thermoelectric device with the pressure level inside the glass tube is shown in Fig. (9.17). Similarly, it can be observed that the temperature difference across the thermoelectric device is little affected by the pressure level for pressures greater than 200 mbar. However, the temperature difference is very much determined by the tilt angle.

Fig. (9.18) shows the variation of the thermal conversion efficiency with the pressure inside the glass tube, with the receiver system at the vertical position and at 30° tilt angle. Although, the effect of the tilt angle is obvious, the effect of the pressure level is insignificant except at very low pressure.

Figures (9.19), (9.20) and (9.21) present the instantaneous performance of the concentrating system for varying tilt angle and pressure level. The efficiency curves are generally flat, and have not shown much sensitivity to both the pressure level and the term $\frac{(T_i - T_a)}{G}$). However, the effect of the collector tilt is appreciable.

The key performance parameters of the solar energy concentrating system ($\eta_o$, $U_L$ and $F_R$) were obtained from the set of graphs in Fig. (9.19) and Fig. (9.21), using least square fitting to the data assuming forms of equations (6.14) and (6.19). Variation of $U_L$ and $F_R$ with the pressure for varying tilt angles are shown in Fig. (9.22) and Fig. (9.23) respectively.

The average optical efficiency ($\eta_o$) is found to be about 50.4%. The overall heat loss coefficient ($U_L$) varied slightly with both the pressure level and the tilt angle, however, its dependency on the pressure level was not significant, particularly at low tilt angles. The overall heat loss coefficient varied from about 4.2 Wm$^{-2}$ K$^{-1}$ at 50 mbar to about 4.7 Wm$^{-2}$ K$^{-1}$ at atmospheric pressure, with the receiver at vertical position, and from about 6.2 Wm$^{-2}$ K$^{-1}$ at 50 mbar to about 7.5 Wm$^{-2}$ K$^{-1}$ at atmospheric pressure, with the receiver tilted 30° from the vertical position. Reduction of the pressure from atmospheric to 50 mbar is accompanied by a 11% reduction in $U_L$ at zero tilt angle, while the corresponding reduction at 30° tilt angle is

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RESULTS OF EXPERIMENT AND DISCUSSION

about 17%. Tilting the collector by $30^\circ$ from the vertical position increases $U_L$ by about 60% at atmospheric pressure and by about 48% at 50 mbar.

Fig. (9.17) Temperature difference across the thermoelectric device versus pressure level.

Fig. (9.18) Thermal conversion efficiency versus the pressure level for varying tilt angles.
RESULTS OF EXPERIMENT AND DISCUSSION

(A) Tilt angle = zero

(B) Tilt angle = 30°

Fig. (9.19) Performance of the solar energy concentrating system at different pressure levels, (A) Tilt angle = 0 degree, (B) Tilt angle = 30°.

Continue Fig. (9.19).
Fig. (9.20) Performance of the solar energy concentrating system at different tilt angles, (A) Atmospheric pressure, (B) Pressure = 50 mbar.

Continue Fig. (9.20).
Fig. (9.21) Performance of the solar energy concentrating system as a function of the term \((T_s - T_a)/G\) at different tilt angles, (A) Tilt angle = 0°, (B) Tilt angle = 30°.

Continue Fig. (9.21).
RESULTS OF EXPERIMENT AND DISCUSSION

Fig. (9.22) Heat loss coefficient as a function of pressure.

Fig. (9.23) Heat removal factor as a function of pressure.
9.5 Validation of the Analytical Model

The numerical model presented in Chapter 7 is used mainly to determine the variation of the flow pattern inside the receiver system with collector tilt angle. Although the model gives the local heat fluxes and heat transfer coefficients at walls of the various components of the receiver system in addition to the flow pattern and temperature distribution, it is itself not suitable for the direct prediction of system performance. For this reason comparison is made instead between the analytical results and experimental.

The most useful conclusion from results of the CFD is that a pressure level of about 50 mbar is sufficient to suppress convective heat losses. Results obtained at pressure levels of 50 mbar and 15 mbar are found to be similar. This permits a useful comparison of the analytical results given in Chapter 8, which relate to vacuum conditions, with the experimental results taken at 50 mbar.

The relationship between the absorber plate temperature and the inlet water temperature, as measured and calculated, is given in Fig. (9.24). Fig. (9.25) compares the predicted and the measured temperature difference across the thermoelectric device.

![Graph](image)

**Fig. (9.24)** Variation of the absorber plate temperature with the inlet water temperature
VALIDATION OF THE ANALYTICAL MODEL

Fig. (9.25) Variation of the temperature difference across thermoelectric device with the inlet water temperature

Fig. (9.26) compares the predicted and the measured performance of the solar energy concentrating system. At lower operating temperatures the measured and the predicted results agrees very well. The difference between the predicted and the measured results at higher temperature difference can be attributed to several factors. As outlined earlier, the CSI lamp emits too much energy in the IR region ($\lambda>1.0$ $\mu$m), and because the Pyrex glass is opaque to the IR radiation, a significant amount of the incoming radiation is absorbed by the aperture window of the glass tube. This can be seen from the temperature of the lower part of the glass tube compared to the sides and the upper part of the glass envelope, shown in figures (9.9), (9.12) and (9.16). In the theoretical models the temperature of the glass envelope was assumed to be uniform which, according to FLUENT predictions will allow a uniform downward temperature gradient to be established throughout the air mass inside the tube as shown in Fig. (7.8). The measurement showed that the temperature gradient over the glass envelope itself is upward due to absorption of the radiation by the lower part of the glass tube. At high cooling rates, the temperature of the absorber plate was found to be lower than those of the CPC, the lower part of the glass tube and even the air in
the region confined by the absorber plate, the CPC and the aperture window of the glass tube, as shown by Fig. (9.16).

Another factor that could explain the difference between the theoretical and the measured results, can be related to errors in the reflector surface, mounting and tracking of the concentrating system. As shown in Fig. (9.8), misalignment of about 1 degree is sufficient to reduce the radiation level at the focal line of the PTC concentrator by about 30%. With a two stage concentrator, this effect will be reduced to some extent, but it would still be sufficient to account for the difference found.

**Fig. (9.26)** Comparison between the theoretical and the experimental performance of the solar energy concentrating system.

Table (9.1) compares theoretical and experimental results for the key parameters under vacuum and 50 mbar respectively for a vertically aligned solar collector. It is worth mentioning that the maximum vacuum obtained in the experiment was 50 mbar.
VALIDATION OF THE ANALYTICAL MODEL

Table (9.1) Solar energy collector performance key parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical</th>
<th>Experimental</th>
<th>discrepancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_R \eta_o$</td>
<td>0.40</td>
<td>0.41</td>
<td>2.4%</td>
</tr>
<tr>
<td>$F_R U_L$ (Wm^{-2}K^{-1})</td>
<td>1.61</td>
<td>3.4</td>
<td>52%</td>
</tr>
</tbody>
</table>

The 2.4% discrepancy in the $F_R \eta_o$ value can be attributed to the tracking error and the low transmittance of glass in the IR range as explained earlier. However, the large discrepancy in $F_R U_L$ values in addition to the above factors, can be related to several factors. One of these factors is that, the distribution of the radiation on the plane of the PTC is not uniform, as explained earlier, Fig (9.6). So the distribution of the concentrated radiation on the receiver itself is not uniform. It was outlined earlier that only 22% of the illuminated area of the PTC aperture is within 10% of the average uniformity recommended by the ASHRAE Standards [ASHRAE Standards 93-77]. This will result in some points with radiation intensity higher than others. Depending on the relative positions of the thermocouples with respect to these high radiation spots, these at higher spots will read higher temperatures. This can be justified by the high temperature readings at the bottom of the glass tube compared to other parts, Fig. (9.9) and Fig. (9.16). Due to these high spots points, the temperature distribution of the glass tube will vary substantially, and as a result the radiation properties of the glass tube particularly the emissivity will vary accordingly, resulting in higher radiative heat loss from these high spot points. Most of the radiant energy absorbed at these high spots point, particularly at the lower part of the glass tube will easily be lost by either radiation or convection to the surrounding environment, due to the high temperature difference, resulting in an increased heat loss coefficient. It worth mentioning that, in the analytical solution, the temperature of the glass tube and as well all the other components are assumed to have a uniform temperature.

Other factor which may relate to the high discrepancy between the predicted and the measured results at high operating temperatures, is the pressure level inside the glass tube. The analytical solution is assumed to be in a complete vacuum, while the measurements at made 50 mbar. However, it was clear that, some components of the
receiver system, such as the black paint and the adhesive material which was used to glue the thermoelectric devices to the cooling tube, are producing some gases when heated. This was realized as the pressure level was changing during the experiment and the vacuum pump is to be operated from time to time maintain the required pressure level during the experiment. Therefore, some convective heat transfer must be expected under such conditions.

Considering all these factors which may have contributed to the discrepancy in the values of the predicted and the measured heat transfer coefficient, the comparison between the other results as shown in Fig. (9.25) and Fig. (9.26) is reasonably acceptable, particularly at lower operating temperatures, where the effect of the above factors is not dominant.

9.6 Conclusion

The results showed that the design is capable of providing efficient concentration of solar radiation without the need for frequent tracking adjustment. Misalignments up to about 4° can be tolerated without a significant reduction in the thermal conversion efficiency of the concentrating system. This permits operation for about 20 days without the need for tracking adjustment. Results also showed that the system can be operated under vacuum or under residual pressure without significant heat losses, particularly for collector tilt angles less than 30 degrees.

In general the discrepancy between the measured and the predicted values is reasonable, considering the difference in the pressure level and the assumptions regarding the radiation source for the experiment and the theoretical model.
10 Economic Evaluation of the Solar Thermoelectric Generator

10.1 Methods and Considerations

Although solar power plants are capital intensive, they are relatively cheap to operate because fuel and maintenance costs are almost non-existent. In the case of solar thermal electrical power plants, capital costs are usually dominated by the solar energy collector, which effectively represents the plant's lifetime fuel supply. The next most expensive item is the power-conversion system followed by the energy storage and control systems. For the system under evaluation, the thermoelectric device represents the energy conversion system and its cost is the most concerning issue. It is only by utilising concentrated solar energy, that the cost of the thermoelectric component for a given power output may be reduced.

As indicated earlier, the system is aimed at the provision of a minimal electricity supply for lighting, television and radio/cassette players for rural households, which may amount to about 0.3 to 0.4 kWh electrical power per day. This power demand can be supplied by a 50 watts system operating for 6 to 8 hours per day. With a thermoelectric device having an overall solar to electricity conversion efficiency of about 2.5%, about 2.5 square meters of solar energy collector aperture will be required. A 12-volt 50-Ah car battery will be sufficient to cover demand during non daylight hours. The waste heat which will amount to about 3.6 to 4.8 kWh per day from the cold junction of the thermoelectric device may be useful for the direct supply of heating loads.

The economic evaluation is made on an annual energy cost basis. This allows comparison to be made with various alternative systems, including solar photovoltaic, solar thermal power plants and conventional fossil fuelled system. Other economic methods, such as the payback period and the internal rate of return
are not considered at this stage because they are probably not appropriate to the scale of the system in question.

Assuming that \( N_m \) is the number of thermoelectric modules per unit aperture area of the solar collector required for a power generation system, \( P_m \) is the power output per module in watts, \( C_m \) is the cost per module, and \( C_S \) is the cost of the solar energy concentrator per unit aperture area, the total capital cost of the system (\( C_C \)) is estimated by

\[
C_C = AC_S + C_B + AN_m C_m
\tag{10.1}
\]

where, \( C_B \) is the cost of the battery and the control unit, and \( A \) is the total aperture area of the solar collector in square meters.

If the annual running cost of the system is \( C_r \), the levelled cost of unit power output (\( C_p \)) is given by

\[
C_p = \frac{1}{365(N_m P_m + \eta G) \Delta t} \left( \left( C_s + \frac{C_B}{A} + C_m N_m \right) CR_i^n + \frac{C_r}{A} \right)
\tag{10.2}
\]

where, \( \eta \) is the solar to thermal energy conversion efficiency of the solar system, \( G \) is the solar radiation in watts per square meter, on the plane of the solar collector, \( \Delta t \) is the number of operating hours per day (sun-shine hours), \( CR_i^n \) is the capital recovery factor, \( n \) is the number of years, and \( i \) is the real rate of interest.

10.2 Cost Estimation

1. Cost estimation of thermoelectric device

Current costs for thermoelectric devices are high due to lack of mass production. The prediction of the future cost of thermoelectric devices is subject to many uncertainties due to the many factors involved. These include the availability of raw materials and the processes by which those raw materials are prepared, the expected
technological improvements of the production techniques of these devices and the consequent performance improvement, the size and the rate of growth of the world market for thermoelectric related products, and the extent to which other competing technologies will affect the related market.

It appears that the raw materials for thermoelectric elements are expensive, particularly those of high purity. Current prices of thermoelectric materials vary substantially depending on the purity and the quantity. For example, prices of bismuth and tellurium vary from $210 per kg for 99.99% purity, for quantities over 500 grams [Mining & Chemical Products Ltd] to about $900 per kg for 99.999% purity, for 50 grams amounts [Koch Chemicals Ltd]. Bismuth, the major element of these devices is abundant [Burton, 1990], whilst tellurium another important element, is a relatively rare element which constitutes only about $10^{-7}$% of the earth’s crust. The total world production of tellurium has been estimated about 300 tonnes per annum. About 55% of the world production is used as an alloying element in the production of low carbon steels, while the electronic industry consumes about 10% of the world supply for fabrication of solar modules, infrared detectors and thermoelectric devices [Burton, 1990]. This rate is likely to increase in view of electronic industry growth. Antimony is also a relatively rare elements constituting one part per million of the earth’s crust. It is principally used as an alloying ingredient for imparting strength to lead [William, 1966]. Selenium is also important element in this context, and constitutes about $10^{-5}$% [William, 1966] of the earth crust and is widely distributed in the earth crust’s.

Other materials such as germanium, silicon and lead which are abundant, also significantly contribute to the future price of thermoelectric materials, particularly for power generation. Various other materials such as disilicide, magnesium and manganese compounds which have been investigated as low cost alternative thermoelectric materials, have a low figure-of-merit, and are unlikely to find application in the near future.
Currently, the use of thermoelectric materials is for Peltier cooling for picnic coolers, and for the fibre optic industry which are high volume markets. The latter is a growth market, although there are expectations that this will mature quickly [Burton, 1990]. The high number of military, aerospace and high reliability applications does not seem to increase the thermoelectric market sufficiently to affect the current material prices. Unless there is breakthrough in technological development, it is unlikely that neither the current supply-demand pattern, nor the prices, will change significantly in the near future.

The interconnection technology and the method of assembly are significant factors in the overall production cost of a module. The production methods for thermoelectric modules have not changed much in the last 20 years [Burton, 1990]. Module production methods that might be expected to cut the production cost, probably will employ sintering, thin films or melt-quenching techniques. Production methods of thermoelectric devices using thin film techniques still need further investigation before they can be practically considered. Production processes using sintering and melt-quenching, followed by adequate annealing, produce material with thermoelectric quality very similar to those of crystalline materials. Sintered thermoelectric material is more amenable to mechanical handling and it appears that if a good sintered process for manufacturing could be devised, then an automated production line could be designed. There have been some attempts at automated production for thermoelectric modules [Burton, 1990], but few of these have come to fruition.

Another factor that may determine the future cost of thermoelectric devices is the expected improvement in the performance of the module. Commercial modules are based on thermoelectric Peltier devices which are made using bismuth-telluride alloys. Currently, several manufacturers are producing these modules at various sizes according to the load requirements. A Peltier module which produces about 1 We, when operated at a temperature difference of 80 °C, costs about $20 per module or about $10 for large quantity [Matsuura et al., 1991].
As concluded in chapter 4, when the module is optimised geometrically for power generation, its output will be improved satisfactorily, and with improved contact layers, or by using highly doped material, the output can be boosted by several times. Improvement of production methods, and performance will result in cheaper modules at high power output. The effect of these factors is considered in the economic calculations by varying both the cost and the power output of a module within the range expected, as shown in chapter 3.

The life time of these devices is another crucial factor. It is not yet certain for how long such a thermoelectric device will last under the expected operational conditions. There is insufficient information regarding the life time of thermoelectric devices, except for the use of space applications. It was reported [Bennet, 1995] that a 52 We PbTe thermoelectric generator, developed for SNAP-3B spacecraft, has operated over the 15 years since it was launched. Similarly, a 85 We PbTe thermoelectric generator operated for over 18 years in the SNAP-19 spacecraft. Several other systems also have operated satisfactorily over extended periods of time, including the Radioisotope Thermoelectric Generator (RTG) developed for Galileo spacecraft, which continues to perform according to the predictions [Bennet, 1995], and all the indications are that it will provide enough power to enable the spacecraft to complete its mission.

Operational conditions such as the humidity vary sufficiently between space and terrestrial applications. Another factor is the operating temperature, which must be kept safely below the melting point to avoid deterioration of the material. It may also be necessary to maintain an inert environment or vacuum to avoid long-term oxidisation. If these factors are attended to, there is no reason why a thermoelectric generator on the surface of the earth should not operate for similar periods to those experienced in space. In view to the above, a lifetime between 15 to 20 years can be considered reasonable.
ECONOMIC EVALUATION

2. Cost estimation of the solar energy concentrator

The main components of the solar energy concentrator are the primary concentrator, the secondary concentrator and the glass envelope. The technical specifications of these components are given in chapter (6). Cost estimation of the materials for these components can be undertaken directly, since most of them are available commercially. The fabrication cost is also an important issue, and its estimation is subject to uncertainties, since there are no standard components available off-the-shelf.

Although, the surface finishing of the reflectors is very important for an efficient concentrator. Generally both the primary and the secondary concentrators can easily be constructed in any workshop. The construction cost is estimated at 30% of the material cost. The lifetime of the reflecting film is of concern. There is currently, significant research underway, on the development of durable reflective films. The performance goal of the Solar Energy Research Institute, USA, for silvered polymer films is a five-year life with a specular reflectance greater than 90%. The cost of the film varies between $2 to $17 per square meter depending on the supplier and the quantity [Alpert, 1993]. However, the glass tube requires a special consideration, and can be prepared only in workshops with glass blowing facilities, which are generally limited, and there is consequently a high preparation cost, unless mass production is considered. Details of the cost breakdown for the solar energy concentrator per square meter area of solar collector aperture are shown in table (10.1).

In addition to the above, other costs include the cost of the 12-volt, 50 AH battery and a battery control unit (BCU), which may need to be replaced every four to five years [Bierman, et al., 1995]. The costs of battery and the BCU are estimated at $100/kWh capacity and $0.5/Wp respectively [Omer & Taha, 1993]. Additional costs include that of replacing the reflecting film for the PTC, assumed at $17 per square meter every five years [Alpert, 1993].
### Table (10.1) Cost estimation of the solar concentrator per unit collector aperture

<table>
<thead>
<tr>
<th>Component</th>
<th>Description</th>
<th>Cost ($/m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTC</td>
<td>Aluminium sheet</td>
<td>30.00</td>
</tr>
<tr>
<td></td>
<td>Reflecting mirror (polymerised aluminium)</td>
<td>10.00</td>
</tr>
<tr>
<td></td>
<td>Support structure (aluminium angle)</td>
<td>38.50</td>
</tr>
<tr>
<td></td>
<td>Construction cost (@30%)</td>
<td>23.55</td>
</tr>
<tr>
<td>CPC</td>
<td>Anodised aluminium sheet (0.175 m²)</td>
<td>01.50</td>
</tr>
<tr>
<td></td>
<td>Construction cost (@30%)</td>
<td>00.45</td>
</tr>
<tr>
<td>Glass envelope</td>
<td>Pyrex glass (7.5 mm bore) @20.65/m length</td>
<td>82.50</td>
</tr>
<tr>
<td></td>
<td>Preparation (@100%)</td>
<td>82.50</td>
</tr>
<tr>
<td>Cooling tube</td>
<td>Copper tube (2.20 cm bore) @$1.36/m length</td>
<td>03.40</td>
</tr>
<tr>
<td></td>
<td>Preparation cost (@30%)</td>
<td>01.02</td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td>273.40</td>
</tr>
<tr>
<td>Others</td>
<td>@ 5% of the total cost</td>
<td>013.67</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>287.10</td>
</tr>
</tbody>
</table>

Other maintenance costs are small in general, and include the yearly replacement of the fluorescent tube lamps, and acid for the battery, estimated at 5% of the battery cost [Omer & Taha, 1993]. No other parasitic costs are considered, since the system is assumed to operate under thermosiphon pressure, and the tracking adjustment will be done manually by the user.

### 10.3 Results and Discussion

The analysis is performed for the solar thermoelectric power generation system over a period of 20 years and with a 6% real rate of return. Two cases are considered: one where the waste heat is not utilised, and the electricity from the thermoelectric device represents the only useful energy output, and a second case of a co-generation system where in addition to the electricity, the waste heat is also used.

The calculated cost of the electricity generated using the concentrated solar thermoelectric system is shown in Fig. (10.1). The cost is dependent on the prices of thermoelectric module and its output. For commercial modules, which are claimed to produce about 1 watt at temperature difference of 75 °C, the results at the current
prices compare very favourably with the prices of about $1.0 per kWh [Vicharangan, 1993] for electricity generated by photovoltaic systems for rural villages. Even Peltier modules of a predicted output power of 0.54 watts, the cost is still comparable with that of photovoltaic systems for rural villages. However, as shown in table (10.2), modules with an output of 0.25 watts which was measured in the experiment undertaken in this study, the cost of $1.77 per kWh is too high compared to the alternatives. If modules are optimised geometrically for power generation, or modules with improved contact layers are used at reduced price, the cost of the electricity generated by concentrated solar thermoelectric generator is comparable with photovoltaic systems in general.

The capital cost of the solar thermoelectric generation system at the current prices of thermoelectric modules is about $1283, which about 60% is the cost of solar energy concentrator. If mass production of solar concentrator is considered, this may reduce the prices substantially. Under assumption that mass production of the solar concentrators will reduce its cost to half, the capital cost of the system will be reduced by about 28%. Fig. (10.2) compares the cost of the electricity at the current prices with the cost under mass production of the solar concentrator. Also shown in table (10.3) is the capital cost and the corresponding levelised energy cost under mass production. As can be seen, only at the reduced thermoelectric module prices, may the electricity cost from the solar thermoelectric generation compare with conventional utility based solar thermal electrical power plants, which produce electricity at 7.5 to 27 cents per kWh [Pascal et al., 1992]. Overall, the cost of electricity from solar thermoelectric generators is generally higher than the 6 cents per kWh predicted by [Matsuura, 1991], for a thermoelectric power generation using industrial waste heat.

However, when the system is used a co-generation system where both the electricity from the thermoelectric device and the waste heat are utilised, the picture may look quite differently. Fig. (10.3) presents the cost of the combined energy produced by the system. As can be seen, the range of the combined energy cost is quite different.
than that for electricity supply alone. The cost of the energy at the current thermoelectric module prices is in the range $0.044 to $0.134 per kWh, depending on module output. The cost in general compares very well with that of conventional fuel systems.

Results of the combined energy cost for varying module cost and module output power are shown in Table (10.4). Effect of the mass production of solar concentrator is not considered for this case since the cost compares very well with most of the alternatives anyway.

The above results are for thermoelectric modules using bismuth telluride based materials, which operate at relatively low temperatures. If higher temperatures are considered at high solar energy concentration, that would be of an advantage, since materials such as lead-telluride can be used. This allows further reduction in the number of the thermoelectric modules required per unit solar collector area, and hence less cost. However, this depends on the several factors which have been outlined earlier regarding the prices of modules.

![Fig. (10.1) Cost of electricity produced by the solar thermoelectric power generation system for unit production prices](image-url)
Table (10.2) Cost of electricity from solar energy thermoelectric generator for unit solar concentrator production prices ($/kWh)

<table>
<thead>
<tr>
<th>Module cost ($/module)</th>
<th>Capital cost (US$)</th>
<th>No. of modules per m²</th>
<th>Power output W/module</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>1.77</td>
<td>2783</td>
<td>80.0</td>
</tr>
<tr>
<td>10</td>
<td>1.47</td>
<td>1783</td>
<td>40.0</td>
</tr>
<tr>
<td>7.5</td>
<td>1.17</td>
<td>1449</td>
<td>26.7</td>
</tr>
<tr>
<td>5</td>
<td>0.87</td>
<td>1283</td>
<td>20.0</td>
</tr>
<tr>
<td>2.5</td>
<td>0.87</td>
<td>1183</td>
<td>16.0</td>
</tr>
<tr>
<td>1.77</td>
<td>0.80</td>
<td>1116</td>
<td>13.3</td>
</tr>
<tr>
<td>1.17</td>
<td>0.87</td>
<td>1068</td>
<td>11.4</td>
</tr>
<tr>
<td>1.05</td>
<td>0.75</td>
<td>1033</td>
<td>10.0</td>
</tr>
<tr>
<td>0.97</td>
<td>0.72</td>
<td>1005</td>
<td>8.9</td>
</tr>
<tr>
<td>0.91</td>
<td>0.74</td>
<td>983</td>
<td>8.0</td>
</tr>
<tr>
<td>0.87</td>
<td>0.72</td>
<td>949</td>
<td>6.7</td>
</tr>
<tr>
<td>0.84</td>
<td>0.71</td>
<td>908</td>
<td>5.0</td>
</tr>
<tr>
<td>0.81</td>
<td>0.66</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.77</td>
<td>0.65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.72</td>
<td>0.63</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Based on mass production (@$2.5)
Based on mass production (@$10)
Based on unit production (@$10)

Fig. (10.2) Comparison between the cost of the electricity for unit and mass production of the solar concentrators
**ECONOMIC EVALUATION**

**Table (10.3) Cost of the electricity with mass production of the solar concentrators**

<table>
<thead>
<tr>
<th>Module output power (watts)</th>
<th>Mass production of solar concentrator</th>
<th>Capital cost (US$)</th>
<th>Energy cost ($/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cm = $10</td>
<td>Cm = $2.5</td>
<td>Cm = $10</td>
</tr>
<tr>
<td>0.25</td>
<td>2424</td>
<td>924</td>
<td>1.55</td>
</tr>
<tr>
<td>0.50</td>
<td>1424</td>
<td>674</td>
<td>0.96</td>
</tr>
<tr>
<td>0.75</td>
<td>1091</td>
<td>591</td>
<td>0.76</td>
</tr>
<tr>
<td>1.00</td>
<td>924</td>
<td>549</td>
<td>0.66</td>
</tr>
<tr>
<td>1.25</td>
<td>824</td>
<td>524</td>
<td>0.60</td>
</tr>
<tr>
<td>1.50</td>
<td>757</td>
<td>507</td>
<td>0.56</td>
</tr>
<tr>
<td>1.75</td>
<td>710</td>
<td>495</td>
<td>0.53</td>
</tr>
<tr>
<td>2.00</td>
<td>674</td>
<td>486</td>
<td>0.51</td>
</tr>
<tr>
<td>2.25</td>
<td>646</td>
<td>479</td>
<td>0.49</td>
</tr>
<tr>
<td>2.50</td>
<td>624</td>
<td>474</td>
<td>0.48</td>
</tr>
<tr>
<td>3.00</td>
<td>591</td>
<td>466</td>
<td>0.46</td>
</tr>
<tr>
<td>4.00</td>
<td>549</td>
<td>455</td>
<td>0.43</td>
</tr>
</tbody>
</table>

**Fig. (10.3)** Cost of the combined energy production by the solar thermoelectric power generation system
Table (10.4) Cost of the combined energy from solar thermoelectric generation system ($/kWh)

<table>
<thead>
<tr>
<th>Module cost ($/module)</th>
<th>Power W/module</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>0.230</td>
<td>0.134</td>
</tr>
<tr>
<td>0.134</td>
<td>0.086</td>
</tr>
<tr>
<td>0.102</td>
<td>0.070</td>
</tr>
<tr>
<td>0.086</td>
<td>0.062</td>
</tr>
<tr>
<td>0.076</td>
<td>0.057</td>
</tr>
<tr>
<td>0.070</td>
<td>0.054</td>
</tr>
<tr>
<td>0.065</td>
<td>0.051</td>
</tr>
<tr>
<td>0.062</td>
<td>0.050</td>
</tr>
<tr>
<td>0.059</td>
<td>0.048</td>
</tr>
<tr>
<td>0.057</td>
<td>0.047</td>
</tr>
<tr>
<td>0.054</td>
<td>0.046</td>
</tr>
<tr>
<td>0.050</td>
<td>0.044</td>
</tr>
</tbody>
</table>

10.3 Conclusion

It is concluded that, with bismuth-telluride thermoelements at the current prices, a solar thermoelectric generation can be cost effective as a source of electricity when compared to photovoltaic systems. With some improvement of the performance by optimizing the geometry for power generation and/or using better contact layers, the cost of electricity from solar thermoelectric generation is even comparable with utility based solar thermal electric power plants. However, if use is made of the heat as well, the combined system is competitive against those based on conventional fuel.
CHAPTER ELEVEN

11 Overall Conclusion

A detailed design description and evaluation has been given for a two stage solar concentrator using a thermoelectric device for power generation. The design has an advantage of providing efficient concentration of solar radiation without the need for frequent tracking adjustment, whilst also minimizing the convective heat losses in circumstances where the glass tube is not adequately evacuated, particularly for collector tilt angle less than 30 degrees. Thus, the system can be operated under vacuum or under residual pressure without significant heat losses. The system can simultaneously provide electrical and thermal energy, hence the overall solar to useful energy conversion efficiency and cost effectiveness can be improved.

A unified theoretical thermoelectric model has been developed, which allows geometrical optimization of the thermoelectric elements legs and prediction of the performance of the optimum device in power generation mode. As an advantage over the available methods, the unified model takes into account the effect of the parameters which contribute to the heat transfer process associated with the thermoelectric device. Thus the model is regarded as a definite improvement on those previously available.

Taking into account the experimental conditions at which both the thermoelectric module and the solar energy collector were tested, results of design and evaluation models presented here showed a reasonable agreement with the experimental data, indicating that these models can be used with confidence for the purpose intended.

Although, commercially available Peltier devices can be used to produce electricity, the predicted and the measured results of the thermoelectric modules evaluated in this study indicated the need to have modules optimized specifically for power
 generation, and outlined the importance of using improved contact layers at thermoelement junctions.

The plots of the instantaneous performance of the solar concentrator showed that, the slope of the efficiency curve is quite insensitive to the operating conditions tested for the collector (pressure level and inlet fluid temperature), indicating that the efficiency remains high at high operating temperatures. Results also showed that the system perform well at tilt angles up to about 30 degrees, which is consistent with applications at the latitudes of the regions intended for which the system has been developed.

An economic analysis showed that with bismuth-telluride thermoelements at the current prices, a solar thermoelectric generation can be cost effective as a source of electricity when compared to photovoltaic systems. With module geometry optimized for power generation and/or by using better contact layers, the cost of electricity from solar thermoelectric generation could be comparable even with those of the utility based solar thermal electrical power plants. However, if use is made of the heat as well, the combined system would be competitive with existing utilities' supplies. Overall the design appears to be well suited for small scale electricity and thermal energy generation in the sunny climates regions.

The tools developed as part of this research, in particular the two stage concentrator design methodology, the unified model and the associated evaluation models are of general applicability. They should find widespread application in the design and assessment of solar thermoelectric power generation.
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Appendix 1

Estimation of the U value for the effective temperature difference across the thermoelectric element.

Referring to Fig. (3.1), $R_{TE}$, $R_{hc}$, $R_{cc}$, $R_g$ and $R_s$ are the thermal resistances to the heat flow through thermoelectric legs, hot ceramic plate, cold ceramic plate, gas filled space and the sealant respectively, and are given as follows

$$R_{TE} = L/(k_{TE}A_{TE}), \quad R_{hc} = R_{cc} = R_c = L_c/(k_cA_c), \quad R_s = L/(k_sA_s),$$

$$R_g = R_s = \left( \frac{A_s k_s}{L} + \frac{cA_s}{2} \left( T_{hj}^2 + T_{cj}^2 \right) \left( T_{hj} + T_{cj} \right) \right)^{-1}$$

where,

$L_c$ is the length of the thermoelectric arm, $L_c$ is the thickness of the ceramic plate, $k$ and $A$ in each term are the thermal conductivity and the cross-sectional area of the heat passage respectively for each component ($k_{TE} = k_p + k_n$), $\sigma = 5.67 \times 10^{-8}$ Wm$^{-2}$K$^{-4}$ is the Stefan-Boltzmann constant, and $\epsilon$ is the emissivity of the ceramic plate surface.

The thermal conductivity and area ratios are defined as

$$k_{oc} = k_c / k_{TE}, \quad k_{os} = k_d / k_{TE}, \quad k_{og} = k_d / k_{TE}, \quad A_{os} = A_d / A_{TE}, \quad \text{and} \quad A_{og} = A_d / A_{TE},$$

The heat flux ($Q_{FL}$) from the hot source to the heat sink through the thermoelectric device is given by

$$Q_{FL} = \frac{(T_h - T_c)}{\sum R_{th}} \quad (1)$$

where,

$\sum R_{th}$, is the total thermal resistance to the heat flow through the thermoelectric device, given by

$$\sum R_{th} = R_{hc} + \left( \frac{1}{R_{TE}} + \frac{1}{R_g} + \frac{1}{R_s} \right)^{(-1)} + R_{cc} \quad (2)$$
Under steady state and open circuit conditions, the heat flow rate through the hot and through the cold ceramic plates is the same and equals to the heat flux through the device. Thus

\[ Q_{hc} = \frac{\Delta T_{hc}}{R_{hc}} = Q_{cc} = \frac{\Delta T_{cc}}{R_{cc}} = Q_{FL} = \frac{\Delta T}{\sum R_{th}} \]  

(3)

where, \( \Delta T_{hc} = (T_h - T_{bj}) \), \( \Delta T_{cc} = (T_{cj} - T_c) \) and \( \Delta T = (T_h - T_c) \) are the temperature differences across the hot ceramic plate, across the cold ceramic plate and across the thermoelectric device respectively.

From equation (3), temperatures at the hot and the cold contact points can be estimated as follows

\[ T_{bj} = T_h - \frac{R_c}{\sum R_{th}} \Delta T \]  

(4)

\[ T_{cj} = T_c + \frac{R_c}{\sum R_{th}} \Delta T \]  

(5)

From equation 4 and 5, the effective temperature difference \( \Delta T_e \) is given by

\[ \Delta T_e = (T_{bj} - T_{cj}) = \left(1 - 2 \frac{R_c}{\sum R_{th}} \right) \Delta T \]  

(6)

The ratio of thermal resistances \( (R_c / \sum R_{th}) \) can obtained by rearranging equation (2) as follows

\[ \frac{R_c}{\sum R_{th}} = \left(2 + \frac{L}{L_c} \frac{k_{oc} A_{oc}}{U} \right)^{-1} \]  

(7)

where,

\[ U = 1 + k_{oa} A_{oa} + k_{og} A_{og} + \frac{\epsilon \sigma A_{og}}{2 - \epsilon} \frac{L}{k} (T_{bj}^2 + T_{cj}^2)(T_{bj} + T_{cj}) \]  

(8)

\( U \) can be obtained by substituting for values of \( T_{bj} \) and \( T_{cj} \) from equations (4) and (5) respectively as follow:
\[ T_{hj}^2 + T_{cj}^2 = T_h^2 + T_c^2 + 2 \left( \frac{R_c}{\Sigma R_c} \Delta T \right)^2 \]  

(9)

and

\[ (T_{hj} + T_{cj}) = 2 \bar{T} \]

equation (8) can be rewritten as

\[ U = 1 + k_a A_{oa} + k_{og} A_{og} + \frac{2 \varepsilon \alpha A_{og} L}{2 - \varepsilon} \left( \frac{T_h^2 + T_c^2}{T_h^2 + T_c^2} \right)^2 \bar{T} \]

Substituting for the ratio \((R \sqrt{2} \Sigma R_{th})\), and solving for \(U\) will result in the following

\[ U = C_1 + C_2 \left( C_3 + U^1 \right)^2 \]  

(10)

where,

\[ C_1 = 1 + k_a A_{oa} + k_{og} A_{og} + \frac{2 \varepsilon \alpha A_{og} L}{2 - \varepsilon} \left( \frac{T_h^2 + T_c^2}{T_h^2 + T_c^2} \right)^2 \bar{T} \]

\[ C_2 = \frac{4 \varepsilon \alpha A_{og} 1}{Lk} \left( \frac{L_c}{A_{oc} k_{oc}} \right)^2 (\Delta T)^2 \bar{T} \]  

and

\[ C_3 = 2 \frac{1}{L} \frac{L_c}{A_{oc} k_{oc}} \]

Equation (10) can be rearranged as follows

\[ aU^3 + bu^2 + cu - d = 0 \]  

(11)

where, \(a = C_3^2\), \(b = 2C_3 - C_1C_3 - C_2\), \(c = 1 - 2C_1C_3\), and \(d = C_1\),

Solution of equation (10) gives \(U\) as follows

\[ U = \frac{3}{3z} + \frac{1}{9} \frac{b^2 - 3ac}{a^2 3z} - \frac{1}{3a} \]

(12)

where

\[ z = \frac{1}{18a^2} \sqrt{\left( 12ac^3 - 3(bc)^3 + 54abcd + 81(ad)^2 - 12db^3 \right) - \frac{b}{54a^3} \left( 2b^3 - 9abc - 27a^2 d \right)} \]
Appendix 2

PROGRAM NOVELI-1;
USES crt, dos;
CONST
main_disp : ARRAY [0..14] OF STRING =
'----------------------------------------------------------------------------------------
* Unified Thermoelectric Model Routine * SA. Omer - Feb 1996
----------------------------------------------------------------------------------------
PROGRAM NOVELI-1;
USES crt, dos;
CONST
message2 : STRING =('* Saving Data Arrays *');
message3 : STRING =('Data Saved In File *');
messclr : STRING =('');
messclr1 : STRING =('');
MinTcold = 0; MaxTcold = 5;
Lc = 0.07; {thickness of the ceramic plate, cm } 
Aoc = 2.00; {plate to thermoelement area ratio }
Aos = 0.5; { Ratio of sealant's area to thermoelement }
Aog = 0.5; { Ratio of gas's area to thermoelement }
kog = 0.011; kos = 0.0385; {ratio of gas and sealant conductivities to kpn}
Voltage = 18; PowerT = 110;
{------------------ geometrical parameters --------------------------}
VAR
ij,z,ch,k, temp : INTEGER ;
koc, roc : REAL ; {conductivity ratios}
Tj : ARRAY[0..5] OF REAL;
Thot, TA, dT, Current : REAL;
dTe : ARRAY[0..60] OF REAL;
Sph, Spc, Snh, Snc,Spn, Th, Tc : ARRAY[0..60] OF REAL; {Seebeck Coe}
kp, kn : ARRAY[0..60] OF REAL; {Conductivit}
roph, ronh, ropc, ronc, ropn : ARRAY[0..60] OF REAL; {resistivity}
kpn, kph, kpc, knh, knc ARRAY[0..60] OF REAL; {average values }
dSpn : ARRAY[0..60] OF REAL; {delta sebeek}
kc, rc, ro : REAL; {contact layer properties}
C1, C2, C3, a, b, c, d, E1, E2, E3 : REAL;
U, AMP, VOLT, POWER, EFFICIENCY : ARRAY[0..60] OF REAL;
L : ARRAY[0..61] OF REAL;
AREA, NUMBER : ARRAY[0..60] OF REAL;
Qc, Qp, Qt, Qj, Qg, QS, Qin, Qtp, Qgp : ARRAY[0..60] OF REAL;
filename : TEXT ;
ascname : STRING[10];
APPENDIX 2

Ke_PR, Answer, Answer1, Answer2 : Char;
Year, Month, Day, Dayofweek : word;

{--------------------------------------------------------------------------------------}
PROCEDURE Time_Date; (set the date)
Begin;
ClrScr;
GetDate(Year, Month, Day, Dayofweek);
End; {End procedure Get_Date }

{----------------- Initialise all data ------------------}
PROCEDURE Initialise;
BEGIN
{----- Draw screen display ----------------------------------}
ClrScr;
GoToXY(33, I); Write(Dayofweek, '-', day, ',', month, ',', Year);
FOR z := 0 TO 14 DO
BEGIN
GoToXY(7, 2+z); Write(main_disp[z]); {Write display line }
END;
FOR temp := 0 TO MaxTcold DO
Begin
Tj[temp] := 0;
end;
FOR i := 0 TO 60 DO
Begin
Sph[i] := 0; Snh[i] := 0; kph[i] := 0.013; knh[i] := 0.013;
orph[i] := 0; ronh[i] := 0; Th[i] := 0; Tc[i] := 0;
Spec[i] := 0; Snc[i] := 0; kpc[i] := 0.013; knnc[i] := 0.013;
orph[i] := 0; ronh[i] := 0; ronp[i] := 0; ronnc[i] := 0;
Spn[i] := 0; dSpn[i] := 0; knp[i] := 0.013; ronpn[i] := 0;
Amp[i] := 0; Vlt[i] := 0; Power[i] := 0;
Efficiency[i] := 0; Area[i] := 0; NUMBER[i] := 0;
Qc[i] := 0; Qp[i] := 0; Qt[i] := 0; Qj[i] := 0; Qg[i] := 0;
Qs[i] := 0; Qin[i] := 0;
End;
Thot := 0;
Lop := 0; Lop := 0; Vop := 0; Pop := 0; Effop := 0; Aop := 0; Nop := 0;
{------------------- Setup file for data arrays ----------------------}
IF ParamStr(l) = " THEN
ascname := 'NOVEL1.dat'
ELSE
ascname := ParamStr(l);
Assign(filename, ascnname);
Rewrite(filename);
{Open and reset ascii data file}
Write(filename, ' Date: ', Day, ',', Month, ',', Year);
Writeln(filename, ' RESULTS OF THE GEOMETRICAL OPTIMIZATION OF
 THERMOELEMENT LENGTH');
END; {END PROCEDURE Initialise }

{----------------------------------------}
PROCEDURE Update_Display; (Update display)
BEGIN
GoToXY(25, 10); Writeln(' PLEASE WAIT!!');
GoToXY(33, 6); Write(Aoc: 1: 4); GoToXY(53, 7); Write(koc: 1: 4);
GoToXY(33, 7); Write(Aos: 1: 4); GoToXY(53, 8); Write(kog: 1: 4);
GoToXY(33, 8); Write(Lc: 2: 3); GoToXY(53, 6); Write(roc: 1: 4);

2-2
PROCEDURE CAI, _Tj;
BEGIN
  Th[i] := Thot - dT/(2 + L[i]/Lc*koc*Aoc/U[i]);
  Tc[i] := Tj[temp] + dT/(2 + L[i]/Lc*koc*Aoc/U[i]);
END; {end procedure CAI-Tj }

PROCEDURE CAL_U;
BEGIN
  CI := 1 + kog*Aog + kos*Aos + 2*emm*Aog/(2 - emm)*segma*L[i]/kpn[i]*(Thot*Thot + Tj[temp]*Tj[temp])*TA;
  C2 := 4*emm*segma*Aog/(2 - emm)*sqr(Lc/koc/Aoc*dT)/kpn[i]/L[i]*TA;
  C3 := 2*Lc/L[i]/koc/Aoc;
  a := C3*C3; b := 2*C3 - CI*sqr(C3) - C2; c := 1 - 2*CI*C3; d := CI;
  E1 := 1/18/sqr(a)*sqr(12*a*e*c*c - 3*sqr(c*b) + 54*a*b*c*d + 81*sqr(a*d) - 12*d*b*b*b); 
  E2 := 1/54/(a*a*a)*(-9*a*b*c - 27*a*a*d + 2*b*b*b);
  E3 := E1 - E2;
  U[i] := EXP(1/3*ln(E3)) + 1/9*(b*b - 3*a*c)/a/a/EXP(1/3*ln(E3)) - 1/3*b/a;
END; {End Procedure CAL_U }

PROCEDURE Read_Temp;
Begin
  GoToXY(30,12); Writeln('Enter T(hot)');
  GoToXY(48,12); Readln(Thot);
  GoToXY(30,12); Write(messclr);
  GoToXY(43,5); Write(Thot: 4: 1);
END; { End Procedure Read_Temp }

PROCEDURE Change_KocOnly;
Begin
  GoToXY(18,12); Write(messclr); GoToXY(20,13); Write('koc = ', koc: 2: 2);
  GoToXY(20,13); Write('Enter new koc 1); 
  Readln(koc);
  GoToXY(18,12); Write(messclr); GoToXY(20,13); Write(messclr);
  End; { End procedure Change_Koc }

PROCEDURE Change_Koc;
Begin
  GoToXY(18,12); Write(messclr); GoToXY(20,13); Write(messclr);
  GoToXY(20,12); Write('koc = ', koc: 2: 2);
  GoToXY(20,13); Write('Enter new koc 1); 
  Readln(koc);
  GoToXY(18,12); Write(messclr); GoToXY(20,13); Write(messclr);
  End; { End procedure Change_Koc }

PROCEDURE Change_Roc;
Begin
  GoToXY(18,12); Write(messclr); GoToXY(20,13); Write(messclr);
  GoToXY(20,12); Write('roc = ', roc: 2: 4);
  GoToXY(20,13); Write('Enter new roc ');
Readin(roc);
GoToXY(18,12); Write(messclr); GoToXY(18,13); Write(messclr);
End; {End Procedure Change_Roc }

(--------------------------Change_Ratios-------------------------------------)
PROCEDURE Change_Ratios;
Begin
GoToXY(18,12); Write('Do you want to contact layer resistance ratios ?');
GoToXY(31,13); Write(' Y/N ? ');
Readln(Ke-Pr);
IF (Ke_Pr = 'Y') OR (Ke_Pr = 'y') THEN
Begin
GoToXY(18,12); Write(messclrl); GoToXY(20,13); Write(messclr);
GoToXY(20,12); Write('Which Ratio, r (OR) k ? ');
Read(Answer);
IF (Answer = 'Y') OR (Answer = 'R') THEN
Begin
  Change_Roc;
  GoToXY(18,12); Write('Do you want to change (koc) as well? ');
  GoToXY(31,13); Write(' Y/N ? ');
  Readln(Answer1);
  GoToXY(18,12); Write(messclrl); GoToXY(20,13); Write(messclr);
  IF (Answer1 = 'Y') OR (Answer1 = 'y') THEN
  Begin
    Change_Koc;
    GoToXY(18,12); Write(messclrl); GoToXY(20,13); Write(messclr);
  end
  ELSE
    Change_KocOnly;
    GoToXY(18,12); Write('Do you want to change (roc) as well? ');
    GoToXY(31,13); Write(' Y/N ? ');
    Readln(Answer2);
    GoToXY(18,12); Write(messclrl); GoToXY(24,13); Write(messclr);
    IF (Answer2 = 'Y') OR (Answer2 = 'y') THEN
    Begin
      Change_Roc;
      GoToXY(18,12); Write(messclrl); GoToXY(20,13); Write(messclr);
    end
    ELSE
      END
    ELSE
      GoToXY(18,12); Write(messclrl); GoToXY(20,13); Write(messclr);
END; {End Change_Ratio Procedure }

(-------------------------- calculate thermal conductivity -------------------)
PROCEDURE CAL_kpn;
BEGIN
  kph[i] := (3.541978E-15)*EXP(6*Ln(Th[i])) - (7.974502E-12)*EXP(5*Ln(Th[i]))
    + (7.409253E-9)*EXP(4*Ln(Th[i])) - (3.638502E-6)*EXP(3*Ln(Th[i]))
    + (9.9651E-4)*Sqr(Th[i]) - 0.1443649*Th[i] + 8.65532;
  knh[i] := -(3.974821E-16)*EXP(6*Ln(Th[i])) + (1.002398E-12)*EXP(5*Ln(Th[i]))
    - (1.041759E-9)*EXP(4*Ln(Th[i])) + (5.695937E-7)*EXP(3*Ln(Th[i]))
    - (1.72512E-4)*Sqr(Th[i]) + (2.742616E-2)*Th[i] - 1.776681;
  kpc[i] := (3.541978E-15)*EXP(6*Ln(Tc[i])) - (7.974502E-12)*EXP(5*Ln(Tc[i]))
    + (7.409253E-9)*EXP(4*Ln(Tc[i])) - (3.638502E-6)*EXP(3*Ln(Tc[i]))
    + (9.9651E-4)*Sqr(Tc[i]) - 0.1443649*Tc[i] + 8.65532;
END; {End Procedure CAL_kpn }

2-4
+ (9.9651E-4)*Sqr(Tc[i]) - 0.1443649*Tc[i] + 8.65532;

knc[i] := -(3.974821E-16)*EXP(6*Ln(Tc[i])) + (1.002398E-12)*EXP(5*Ln(Tc[i])) 
- (1.041759E-9)*EXP(4*Ln(Tc[i])) + (5.695937E-7)*EXP(3*Ln(Tc[i])) 
- (1.72512E-4)*Sqr(Tc[i]) + (2.742616E-2)*Tc[i] - 1.776681;

kpn[i] := ((kph[i] + kpc[i])/2 + (knh[i] + knc[i])/2);
IF kpn[i] = 0 THEN kpn[i] := 0.013;

END; {End procedure CAL_kpn }
PROCEDURE CAL_Performance;
BEGIN
    dTe[i] := dT/(1 + 2*Lc/L[i]/koc/Aoc*U[i]);
    AMP[i] := Spn[i]*0.001/4/ropn[i]/(roc + L[i])*dTe[i];
    VOLT[i] := Spn[i]*0.001/2*dTe[i];
    POWER[i] := AMP[i]*VOLT[i];
    AREA[i] := PowerT/Voltage/AMP[i];
    NUMBER[i] := Voltage/VOLT[i];
    Qc[i] := kpn[i]/L[i]*dTe[i];
    Qp[i] := Sph[i]*Spn[i]*Sqrt(0.001)/4/ropn[i]/(roc + L[i])*dTe[i]*Th[i];
    Q[i] := Spn[i]*0.001/8/ropn[i]/(roc + L[i])*dTe[i]*TA*DSpn[i]*0.001;
    Q[i] := SQR(Spn[i]*0.001)/16/ropn[i]/(roc + L[i])*Sqr(dTe[i]);
    Qg[i] := qpn[i]*(kog*Aog/L[i] + 2*emm*segma*Aog/kpn[i]/(2 - emm)*TA*(SQR(Thot) +
    SQRT(T[temp])) + 2*SQR(dTe[i]*Lc/L[i]/koc/Aoc*U[i]))*dTe[i];
    Qs[i] := kpn[i]*kos*Aos/L[i]*dTe[i];
    EFFICIENCY[i] := POWER[i]/Qc[i] - Q[i] - Qg[i] + Qs[i] + Q[i]*100;
    Qtp[i] := Q[i]/Qc[i] + Qp[i] - Q[i] - Qg[i] + Q[i] + Qs[i]*100;
    Qgp[i] := Qg[i] + Q[i] + Q[i] - Q[i] - Q[i] + Qg[i] + Qs[i]*100;
END;  {END PROCEDURE CAL_Performance }

PROCEDURE Save_Data;  {Save data arrays }
BEGIN
    GoToXY(30,20); Write(message1);
    GoToXY(30,21); Write(message1);
    GoToXY(30,12); Write(message2);
    Writeln(filename);
    Write(filename, 'Thot = ', Thot - 273): 4: 1, %
    Writeln(filename, 'T(cold) = ', Tj[temp] - 273): 4: 1, %
    Write(filename, 'koc = ', koc: 2: 2, ', 1);
    Writeln(filename, 'roc = ', roc: 1: 4, ');
    Writeln(filename, 'length', ', ', ' DTe', ', ', 'Outpuf, l, ', 'voltage', ', ', 'Current', ', ');
    Write(filename, 'eff. (%)', ', ', 'Area', ', ', Numbers, ', ');
    Write(filename, 'Q(Thomson)', ', ', 'Q(gap) I, ');
    FOR i := 0 TO 60 DO
        Write(filename, L[i]: 5: 5, ', '); Write(filename, Th[i] - Tc[i]: 3: 2, ', '); Write(filename, POWER[i]: 2: 3, ', '); Write(filename, VOLT[i]: 2: 4, ', '); Write(filename, AMP[i]: 4: 2, ', '); Write(filename, EFFICIENCY[i]: 2: 4, ', '); Write(filename, AREA[i]: 2: 3, ', '); Write(filename, NUMBER[i]: 6: 0, ', '); Write(filename, Qtp[i]: 2: 3, ', '); Write(filename, Qgp[i]: 2: 3, ', ');
    END;
    GoToXY(30,13); Write(message3);
    Delay(500); GoToXY(30,12); Write(message2); GoToXY(30,13); Write(message2);
END;  {procedure Save_Data}

{-------------------------------------------}
BEGIN
    Time_Date;
    Initialise;
    koc := 5.0;
roc := 0.1;
Read_Temp;
Change_Ratios;
Update_Display;
FOR temp := MinTcold TO MaxTcold DO
  Begin
    Tj[temp] := 10*temp + 300;
    GoToXY(35,11); Writeff('T(cold) = ', Tj[temp]:4:0);
    TA := 0.5*(Thot + Tj[temp]);  \{Average temeperature \}
    dT := (Thot - Tj[temp]);  \{Temperation difference \}
  FOR i := 0 TO 60 DO
    Begin
      Th[i] := Thot;
      Tc[i] := Tj[temp];
      IF i = 0 THEN L[i] := 0.0001;
      REPEAT
        Begin
          If L[i] = 0 THEN L[i] := 0.0001;
          CAL_kpn;
          CAL_U;
          CAL_Tj;
        End;
        UNTIL (Abs(kpnn[i] - kpno[i])/kpnn[i] < 0.001);
    End;
    Update_Properties;
    CAL_Performance;
    IF L[i] < 0.01 THEN L[i+1] := L[i]*1.2;
    IF L[i] >= 0.01 THEN L[i+1] := L[i] + 0.01;
    End;
  END. \{main program\}
{---------------------------------------------------------------------------------}
APPENDIX 2

{--- Thermoelectric Unified model routine ---}
{--- geometrical parameters -------------------}

**CONST**

- MinTcold = 0; MaxTcold = 4; Thmin = 0; Thmax = 10;
- Lc = 0.07; \( \text{thickness of the ceramic plate, cm} \)
- Aoc = 2.00; \( \text{plate to thermoelement area ratio} \)
- Aos = 0.5; \( \text{Ratio of sealant's area to thermoelement} \)
- Aog = 0.5; \( \text{Ratio of gas's area to thermoelement} \)
- kog = 0.077; kos = 0.0385; \( \text{ratio of gas and sealant conductivities to kpn} \)
- Voltage = 18; PowerT = 110; Nte = 127; Ate = 0.0121;

{--- Radiation properties -------------------}
- segma = 5.67E-8; \( \text{Boltzmann constant, (W/cm}^2\text{/k}^4\) \}
- emm = 0.10; \( \text{emissivity of the ceramic plate} \)

**VAR**
- Lte : REAL;

{--- Procedure Time_Date ---}

**PROCEDURE** Time_Date; \{set the date\}

**Begin**;

**ClrScr**;

FOR temp := 0 TO MaxTcold DO

**Begin**

Tj[temp] := 0;

**end**;

FOR i := 0 TO 10 DO

**Begin**

Sph[i] := 0; Snh[i] := 0; kph[i] := 0.013; knh[i] := 0.013;

roph[i] := 0; ronh[i] := 0; Th[i] := 0; Tc[i] := 0;

Spc[i] := 0; Snc[i] := 0; kpc[i] := 0.013; knc[i] := 0.013;

roph[i] := 0; ronh[i] := 0; ropc[i] := 0; ronc[i] := 0;

Snp[i] := 0; dSnp[i] := 0; kpn[i] := 0.013; ropn[i] := 0;

AMP[i] := 0; VOLT[i] := 0; POWER[i] := 0;

EFFICIENCY[i] := 0; Thj[i] := 0; Tcj[i] := 0;

Qc[i] := 0; Qp[i] := 0; Qt[i] := 0; Qj[i] := 0; Qg[i] := 0;

Qs[i] := 0; Qin[i] := 0;

**End**;

Lte := 0;

Lop := 0; Iop := 0; Vop := 0; Effop := 0; Aop := 0; Nop := 0;

**END PROCEDURE** Initialise

**PROCEDURE** CA1ý_Tj;
BEGIN
Th[i] := Th[i] - dT/(2 + Lte/Lc*koc*Aoc/U[i]);
Tc[i] := Tj[temp] + dT/(2 + Lte/Lc*koc*Aoc/U[i]);
END; {end procedure CAL-Tj }

{---------------------- Read Temperatures ----------------------}
PROCEDURE Read_OLength;
Begin
  GoToXY(25,12); Writeln('Enter Element length');
  GoToXY(48,12); Readln(Lte);
  GoToXY(25,12); Write(messclr);
  GoToXY(43,5); Write(Lte: 1: 4);
END; ( End Procedure Read_Ternp)

{--------------------- CAL-PERFORMANCE ---------------------}
PROCEDURE CAL_Performance;
BEGIN
  dTe[i] := dT/(1 + 2*Lc/Lte/koc/Aoc*U[i]);
  IF (Th[i] > 373) THEN
    roc := rocc*(l + 0.005*(Tb[i]- 373));
  ELSE
    roc := rocc;
  End;
  AMP[i] := Ate*Spn[i]*0.001/4/ropn[i]/(roc + Lte)*dTe[i];
  VOLT[i] := Nte*Spn[i]*0.001/2*dTe[i];
  POWER[i] := AMP[i]*VOLT[i];
  Qc[i] := Nte*Ate*Spn[i]*Spn[i]*sqr(0.001)/4/ropn[i]/(roc + Lte)*dTe[i]* (Th[i] - Lc/Lte/koc*Aoc*U[i])*dTe[i]);
  Qt[i] := Nte*Ate*Spn[i]*Spn[i]*sqr(0.001)/4/ropn[i]/(roc + Lte)*dTe[i]*TA*dSpn[i]*0.001;
  Qp[i] := Nte*Ate*Spn[i]*Spn[i]*sqr(0.001)/4/ropn[i]/(roc + Lte)*dTe[i]*f(Tb[i] + 2*emm*segma*Aog/kpn[i]1/(2 - emm))TA*(SQR(Th[i]) + SQR(Tj[temp]) + 2*SQR(dTe[i]*Lc/Lte/koc*Aoc*U[i])*dTe[i]);
  Qs[i] := Nte*Ate*kpn[i]*kos*Aos/Lte*dTe[i];
  EFFICIENCY[i] := POWER[i]/(Qc[i] + Qp[i] - Qt[i] - Qg[i] + Qs[i])*100;
  Qtp[i] := Q[i]/(Qc[i] + Qp[i] - Q[i] - Q[1] + Qg[i] + Qs[i])*100;
  Qgp[i] := Qg[i]/(Qc[i] + Qp[i] - Q[i] + Qg[i] + Qs[i])*100;
END; (END PROCEDURE CAL_Performance )

{----------------------------------------- Save Data }
PROCEDURE Save_Data;
BEGIN
  GoToXY(30,20); Write(messclr);
  GoToXY(30,21); Write(messclr);
  GoToXY(30,12); Write(message2);
  Writeln(filename);
  Writeln(filename, 'Element length = ', Lte: 1: 4, ', ');
  Writeln(filename, 'Cold junction temp. = ', (Tj[temp] - 273): 2: 2, ', ');
  Writeln(filename, 'Conductivity ratio = ', koc: 2: 2, ', ');
  Writeln(filename, 'resistivity ratio = ', rocc: 2: 4, ', ');
  Writeln(filename, 'Amps.', 'eff. (%)', ', ');
  Writeln(filename, 'Power', ', ');
  Writeln(filename, 'Volt', ', ');
  Writeln(filename, 'Q(Thomson)', 'Q(gap)', '; ');
END; (END PROCEDURE Save_Data )

APPENDIX 2
Writeln(filename,'--------------------------------------------------------------------------'); FOR i := 0 TO 10 DO BEGIN Write(filename, (Th[i] - 273):3:1,' '); Write(filename, (Th[i] - Tcj[i]):3:1,' '); Write(filename, POWER[i]:2:2,' '); Write(filename, VOLT[i]:2:2,' '); Write(filename, AMP[i]:2:2,' '); Write(filename, EFFICIENCY[i]:2:2,' '); Write(filename, Qtp[i]:2:3,' '); Writeln(filename, Qgp[i]:2:3,' '); END; GoToXY(30,13); Write(message3); Delay(500); GoToXY(30,12); Write(messclr); GoToXY(30,13); Write(messclr); END; {procedure Save_Data} SUBMAIN PROGRAM {BEGIN Time_Date; Initialise; koc := 2.5; rocc := 0.1; Read_OLength; Change_Ratios; Update_Display; FOR temp := MinTcold TO MaxTcold DO Begin Tj[temp] := 10*temp + 300; GoToXY(35,11); Write("r(cold) =', Tj[temp]:4:0); FOR i := Thmin TO Thmax DO Begin Th[i] := 350 + 10*i; Tb[i] := Tb[i]; Thj[i] := Tb[i]; Tqj[i] := Tj[temp]; TA := 0.5*(Th[i] + Tj[temp]); {Average temperature } dT := (Th[i] - Tj[temp]); {Temperature difference } CAL_kpn; CAL_U; CAL_Tj; Update_Properties; CAL_Performance; Save_Data; {Save data array } End; END. {main program} {--------------------------------------------------------------------------}
Appendix 3

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I OUTPUT PRODUCED BY VERSION 4.32  HEAT TRANSFER INSIDE THE RECEIVER SYSTEM (TILT = ZERO) I

*******************************************************************************************************

** FLUENT (V4.32) Fluid Flow Modeling **

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* (800) 445-4454 *

*******************************************************************************************************

- UNITS SYSTEM -

- GEOMETRY -

BOUNDARY FITTED COORDINATES

NI =  71     NJ =  71     NK =  1

- MULTI-GRID PARAMETERS -

PRESSURE IS SOLVED BY MULTI-GROUP METHOD.
TERMINATION CRITERION: .1
RESIDUAL REDUCTION RATE: .6999999
MAX. LEVEL OF BLOCK CORRECTIONS IN I-DIR.: 0
MAX. LEVEL OF BLOCK CORRECTIONS IN J-DIR.: 0
MAX. LEVEL OF BLOCK CORRECTIONS IN K-DIR.: 0

ENTHALPY IS SOLVED BY MULTI-GROUP METHOD.
TERMINATION CRITERION: .1
RESIDUAL REDUCTION RATE: .6999999
MAX. LEVEL OF BLOCK CORRECTIONS IN I-DIR.: 0
MAX. LEVEL OF BLOCK CORRECTIONS IN J-DIR.: 0
MAX. LEVEL OF BLOCK CORRECTIONS IN K-DIR.: 0

MAXIMUM NO. OF FINE GRID ITERATIONS: 30
MAXIMUM NO. OF ITERATIONS PER LEVEL: 500
COARSE GRID SPACING IN I-DIRECTION: 2
COARSE GRID SPACING IN J-DIRECTION: 2
COARSE GRID SPACING IN K-DIRECTION: 2
MONITOR MG SOLVER: NO
MAX.-MG-LEVEL: 10

- VELOCITY BOUNDARY CONDITIONS -

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<th>U-VEL.</th>
<th>V-VEL.</th>
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- TEMPERATURE BOUNDARY CONDITIONS -

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<tr>
<td>W3</td>
<td>COND. WALL</td>
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<tr>
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<tr>
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- SPECIAL TEMPERATURE BOUNDARIES -

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EXT. RAD

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CONDUCTING WALL ZONE PROPERTIES:

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<td>1.4000E+00</td>
<td>N/A</td>
<td>N/A</td>
<td>.0000E+00</td>
</tr>
<tr>
<td>W6</td>
<td>T**0 -2.4500E-03</td>
<td>N/A</td>
<td>N/A</td>
<td>.0000E+00</td>
</tr>
<tr>
<td>T**1</td>
<td>1.13000E-04</td>
<td>N/A</td>
<td>N/A</td>
<td>1.13000E-04</td>
</tr>
<tr>
<td>T**2</td>
<td>-6.2870E-08</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX 3

T**3  1.8910E-11

- BODY FORCE -

IMPROVED TREATMENT OF BODY FORCE IN DISCRETE EQNS.- YES
INCLUDE BODY FORCE TERMS IN VELOCITY INTERPOLATION - YES

GRAVITATIONAL ACCELERATIONS:
X = .000E+00,  Y =-9.810E+00,  Z = .000E+00

REFERENCE DENSITY LOCATION:
I = 2,  J = 2,  K = 1

- ZONAL EMISSIVITIES (RADIATION MODEL) -

ZONE  EMISSIVITY
------  ---------
W1  8.0000E-01
W2  7.4000E-01
W3  6.0000E-01
W4  6.0000E-01
W5  9.0000E-01
W6  1.0000E-01

- DISCRETE TRANSFER RADIATION MODEL CONSTANTS -

ABSORPTION COEFFICIENT = 1.000E-07
NUMBER OF RADIATING SURFACES = 526
NUMBER OF RAYS IN THETA = 2
NUMBER OF RAYS IN PHI = 2

- DENSITY IS COMPUTED FROM THE IDEAL GAS LAW
- THE OPERATING PRESSURE = 1.0133E+05
MOLECULAR WEIGHT = 2.8970E+01,  MOLECULAR WEIGHT = 2.8970E+01
VISCOITY = 1.355E-06 + 6.738E-08*T**1 - 3.808E-11*T**2 + 1.183E-14*T**3
CP = 1.051E+03 - 3.645E-01*T**1 + 8.388E-04*T**2 - 3.848E-07*T**3
ENTHALPY REFERENCE TEMPERATURE = 3.0000E+02
- THERMAL CONDUCTIVITY DEFINITION
  K = -2.450E-03 + 1.130E-04*T**1 - 6.287E-08*T**2 + 1.891E-11*T**3

- SOLUTION CONTROL PARAMETERS -

SOLVER SWEEP DIRECTION = I-DIRECTION
ALTERNATE SWEEP DIRECTION = YES
SOLUTION METHOD = SIMPLE
ALLOW PATCHING OF BOUNDARY VALUES = NO
CONVERGENCE/DIVERGENCE CHECK ON = YES
MINIMUM RESIDUAL SUM = 1.000E-03
MINIMUM ENTHALPY RESIDUAL = 1.000E-06
NORMALIZE RESIDUALS = YES
CONTINUITY CHECK = YES

- INTERPOLATION SCHEME ON CELL FACES -

FOR DENSITY = UPWIND
FOR PRESSURE = MOMENTUM WEIGHTED
FOR VELOCITY - LINEAR

TEMPERATURE CHANGE LIMITER - 1.00E+00
MONITOR SOLVER - YES
COMPRESSIBLE FLOW - NO
SUPERSONIC INFLOW - NO
SUPERSONIC OUTFLOW - NO
FIX VARIABLE OPTION ENABLED - NO
SET PRESSURE REFERENCE LOCATION - NO
VISCOUS DISSIPATION - NO

DIFFERENCING SCHEME - POWER LAW
REFERENCE PRESSURE LOCATION: I = 2, J = 2

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>SOLVED</th>
<th>BLOCK CORRECT NO. SWEEPS</th>
<th>UNDERRELAX 1</th>
<th>UNDERRELAX 2</th>
<th>RESIDUAL AT 359 ITERATIONS</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRESSURE</td>
<td>YES</td>
<td>NO</td>
<td>5</td>
<td>3.0000E-01</td>
<td>1.8765E-04</td>
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<tr>
<td>U-VELOCITY</td>
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<td>NO</td>
<td>1</td>
<td>2.0000E-01</td>
<td>2.8729E-04</td>
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<tr>
<td>V-VELOCITY</td>
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<td>NO</td>
<td>1</td>
<td>2.0000E-01</td>
<td>5.1309E-04</td>
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<tr>
<td>ENTHALPY</td>
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<td>NO</td>
<td>1</td>
<td>1.0000E+00</td>
<td>2.6897E-07</td>
</tr>
<tr>
<td>PROPERTIES</td>
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<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>VISCOSITY</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>2.0000E-01</td>
<td>N/A</td>
</tr>
<tr>
<td>TEMPERATURE</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>1.0000E+00</td>
<td>N/A</td>
</tr>
<tr>
<td>RADIATION</td>
<td>YES</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

RADIATION SOLVED EVERY 10 ITERATIONS

MAXIMUM NUMBER OF DTRM ITERATIONS = 100
DTRM ITERATION TOLERANCE = 1.00E+03
FLOW FIELD AFTER 359 ITERATIONS--

HEAT TRANSFER COEFFICIENTS (FLUID AVERAGE TEMP. = 3.424E+02 K)

<table>
<thead>
<tr>
<th>ZONE</th>
<th>HEAT FLUX  = HT COEFF * (T(SURFACE) - T(AVE))</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1:</td>
<td>-1.262E+02 = 4.820E+00 * -2.618E+01</td>
</tr>
<tr>
<td>W2:</td>
<td>1.326E+02 = 7.031E+00 * 1.886E+01</td>
</tr>
<tr>
<td>W3:</td>
<td>8.657E+01 = 2.429E+00 * 3.564E+01</td>
</tr>
<tr>
<td>W4:</td>
<td>9.722E+01 = 7.719E+00 * 1.259E+01</td>
</tr>
<tr>
<td>W5:</td>
<td>3.313E+02 = 5.752E+00 * 5.759E+01</td>
</tr>
<tr>
<td>W6:</td>
<td>7.899E+01 = 2.547E+00 * 3.102E+01</td>
</tr>
</tbody>
</table>

TOTAL HEAT RATE = 2.427E+00, W
TOTAL HEAT TRANSFER AREA = 4.390E-01, M2
AVERAGE HEAT FLUX = 5.528E+00, W/M2
AVERAGE SURFACE TEMPERATURE = 3.401E+02, K
OVERALL HEAT TRANSFER COEFFICIENT = -2.356E+00, W/M2-K
Appendix 4


This method allows the determination of view factors between any two arbitrary surfaces in a long rectangular enclosure (regular or non-regular) with constant cross section and is called the Crossed-Strings method [Modest, 1993].

Diagonals $d_1$ and $d_2$ are drawn as shown in the Fig. (1), in which the surfaces $S$, $R$ and $W$ are assumed to be extended infinitely far along the plane of the figure. For simplicity, the surfaces of the reflector $(R)$ are estimated by straight lines $CB$ and $C'B'$. Applying the Crossed-String method for the view factors, the followings are obtained:

![Diagram](image)

Fig. (1) Geometrical dimensions of the CPC

The view factor of $S$ onto $W$ is given by
\[ F_{sw} = \frac{(d_1 + d_2) - (R_1 + R_2)}{2S} \]

where, \( d_1 = d_2 \) and \( R_1 = R_2 \), therefore

\[ F_{sw} = \frac{d_1 - R_1}{S} \]  \hspace{1cm} (1)

From the figure, simple trigonometry, implies that

\[ d_1 = \frac{S}{2} \left( \frac{1 + \sin \theta}{\sin \theta^2} \right) \]  \hspace{1cm} (2)

\[ R_1 = \frac{S}{2 \sin \theta} \sqrt{(1 + \sin \theta)^2 \left( \frac{\cos \theta}{\sin \theta} \right)^2 + (1 - \sin \theta)^2} \]  \hspace{1cm} (3)

Substituting for \( d_1 \) and \( R_1 \) from 2 and 3 results in

\[ F_{sw} = \frac{1}{2 \sin \theta} \left( \frac{1}{\sin \theta} + 1 - \sqrt{(1 + \sin \theta)^2 \left( \frac{\cos \theta}{\sin \theta} \right)^2 + (1 - \sin \theta)^2} \right) \]  \hspace{1cm} (4)

Similarly, the view factor \( F_{WR} \) of the aperture window to the CPC reflector can be obtained by considering two dimensional wedge-shaped long groove geometry formed of sides \( W, R_1 \) and \( d_2 \) with an open groove angle \( \alpha \), as shown in Fig. (1). Therefore, employing the special method of view factor algebra [Modest, 1993], the view factor \( (F_{WR}) \) of the aperture window to the CPC reflector is estimated as

\[ (F_{WR})_{couple} = \frac{1}{2} \left( 1 + \frac{\alpha}{W} - \sqrt{1 - 2 \frac{\alpha}{W} \cos \gamma + \left( \frac{\alpha}{W} \right)^2} \right) \]  \hspace{1cm} (5)

where \( \cos \gamma = \frac{\left( \frac{W - S}{2} \right)}{R_1} \)

therefore, the view factor of the aperture window to the whole CPC reflector is given by

\[ W_r = \left( 1 + \frac{\alpha}{W} - \sqrt{1 - \frac{W - S}{W} + \left( \frac{\alpha}{W} \right)^2} \right) \]

From the concentration ratio of the CPC, \( \frac{W}{S} = \frac{1}{\sin \theta} \).

Therefore, substituting for \( R_1 \) and \( W \), equation 8.5 yields into
\[ F_{wr} = 1 + x + \sqrt{\sin \theta + \frac{1}{4} x^2} \]  
(6)

where, \[ x = \sqrt{(1 + \sin \theta)^2 \left(\frac{\cos \theta}{\sin \theta}\right)^2 + (1 - \sin \theta)^2} \]

2. Estimation of the CPC reflecting surface length

Consider a compound parabolic concentrator as shown in Fig. (2). The parabola is given by equation

\[ y = \frac{1}{2 f} x^2 \]  
(7)

At the rim point \((x_s, y_s)\), the rim angle is \((\pi/2 + \theta)\), and hence, the equation of the right branch parabola, in polar coordinates is given by

\[ f = \frac{S}{2} (1 + \sin \theta) \]  
(8)

Substituting for \(f\) equation (7), yields

\[ y = \frac{x^2}{2S(1 + \cos \theta)} \]  
(9)

and the end points are determined by

\[ x_s = S \cos \theta \]  
(10)

\[ y_s = C(1 - \sin \theta) \]  
(11)

\[ x_w = (S + W) \cos \theta \]  
(12)

\[ y_w = \frac{S}{2} (1 - \sin \theta)(1 + 1/\sin \theta)^2 \]  
(13)

If the arc length along the curve is estimated by a parameter \(A\), then the reflector area for unit trough length is

\[ A_r = 2 \int_{A_s}^{A} dA \]  
(14)

where, \[ dA = \sqrt{1 + (y')^2} \, dx \]  
(15)

and
\[ y' = \frac{x}{S(1 + \sin \theta)} \] (16)

Integration of \( A_R \) overall the length of the right branch CPC reflector yields the following [Rabel, 1976]

\[
A_R = A_s (1 + \sin \theta) \left[ \frac{\cos \theta}{\sin^2 \theta} + \ln \left( \frac{(1 + \sin \theta)(1 + \cos \theta)}{\sin \theta \left( \cos \theta + \sqrt{2(1 + \sin \theta)} \right)} \right] - \sqrt{2} \frac{\cos \theta}{(1 + \sin \theta)^{3/2}} \right) \] (17)

\[
\text{Axis of the parabola}
\]

\[
\text{Fig. (2) Coordinate system used for describing CPC}
\]
Appendix 5

#### Optical Properties of the Solar Concentrating system####

# writeto(output);
# Solar radiation
Radiation := 1180;
# Beam radiation Factor (Fb)
bF := 0.85;
## Intercept factor (gama)
gama := 1;
## Initial guess mean and fluid temperatures ###
Tfi := 65 + 273;
To := 75 + 273;
Tm := 104 / 273;  # Tm = (Ts + Tc)/2
Tf := (Tfi + To)/2;
ml := 0.06;  ###liter/minute
## Ambient air temperature
Ta := 22 + 273;
## Initially gussed glass temperature ###
Tga := 32.5 + 273;
Tsky := 22 + 273;
Tgam := (Ta + Tga)/2;
##### Parabolic Trough Concentrator
rPTC := 0.85;
##### Glass Tube and aperture window
Digits := 2:
tWb := 0.850;  aWb := 0.10;
tW := 0.19;    aW := 0.80;
#### Absorber Plate
Digits := 2:
aSb := 0.88;
eS := 0.960;
aHj := 0.69;
### CPC reflector(R)
Digits := 2:
rRb := 0.86;
eR := 0.74;
eRO := 0.74;
### Cold Juntion(C)
aCj := 0.69;
aC := 0.5;
##### Calulated properties
rWb := 1 - (tW + aW);
rW := 1 - (tW + aW);  eW := aW;
tE := tW;           aE := aW;  rE := (1 - tE - aE);  eE := aE;
tG := tW;           aG := aE;  rG := (1 - tG - aG);  eG := aG;
rSb := 1 - aSb;
rS := 1 - eS;        aS := eS;
rHj := 1 - aHj;      eHj := aHj;
arB := 1 - rRb;
ar := eR;           rR := 1 - aR;
aRO := eRO;        rRO := 1 - aRO;
rCj := 1 - aCj;  eCj := aCj;
rC := 1 - aC;      eC := aC;
##### Geometrical Description

5-1
## PTC rim angle(phy)

\[
PI := \text{evalf}(\pi); \quad e := \text{evalf}(E); \quad Phy := 40 \times PI/180;
\]

## solar inclination(z)

\[
Z := 0;
\]

## total concentration ratio

\[
Cr := 19.7;
\]

## Length of the element(dL)

\[
dL := 0.3;
\]

## Width of the absorber plate(S)

\[
S := 0.02;
\]

## Half acceptance angle (theta)

\[
theta := 0.654; \quad ss := \sin(\theta); \quad cs := \cos(\theta);
\]

## Diameter of the glass tube

\[
Dg := 0.0706;
\]

## Effective aperture width of the PTC(A)

\[
A := Cr*S*Dg;
\]

## Circumference of the rest of glass envelop

\[
CirE := Dg*(\pi - theta);
\]

## the Compound parabolic Concentrator

\[
W0 := S/ss; \quad W1 := Dg*theta; \quad DCPC := S/2*(1+W0/S)/\tan(theta);
\]

## length of the CPC reflector (R)

\[
ob := S*(1+ss); \quad ib1 := cs*(ss)^2; \\
ib2 := (1+ss)*(1+cs)/ss/(cs+sqrt(2*(1+ss))); \\
ib3 := sqrt(2)*cs/(1+ss)^2;
\]

## CirCf := PI*dL - sin(S/dL)*dL;

## Properties of the ambient air

\[
tca := 10^(-3)*(0.113*Ta - 6.287*10^(-5)*Ta^2 + 1.891*10^(-8)*Ta^3 - 2.45); \\
dva := 10^(-7)*(13.554 + 0.6738*Ta - 3.808*10^(-4)*Ta^2 + 1.183*10^(-7)*Ta^3); \\
dena := 348.59/Ta;
\]

## Calculation of the heat transfer coefficient from glass tube to ambient air

\[
Raa := 9.81*Dg^3*Pma*(Tga - Ta)/Tgam/(kva^2); \\
Nua := (0.6 + 0.32*Raa^1/6)^2; \\
hga := Nua*tca/Dg;
\]

## Properties of the water

\[
tcf := 5.847*10^(-3)*Tf - 0.733188*10^(-5)*Tf^2 - 0.4806; \\
dvdf := 10^(-6)*(35.6602*10^(-3) - 272.757*Tf + 0.70777*Tf^2 - 0.618833*10^(-3)*Tf^3); \\
denf := 766.17 + 1.80396*Tf - 3.4589*10^(-3)*Tf^2; \\
Cpf := 10^(-9)*(3.6158 - 9.0277*10^(-3)*Tf + 14.177*10^(-6)*Tf^2);
\]
\[ Pmf := \frac{Cpf \cdot dvf}{tcf}; \]
\[ kvf := \frac{dvf}{denf}; \]
\[ Digits := 4; \]
\[ Nuf := 3.66; \]
\[ Digits := 5; \]
\[ hcf := \frac{tcf \cdot Nuf}{df}, \]
\[ Ucf := \frac{mf \cdot Cpf \cdot hcf}{mf \cdot Cpf + hcf \cdot Acf/2}; \]

## Thermal conductivity of thermoelectric element
\[ tcTE := 1.4; \]

## Thermal conductivity of the seal
\[ tcSEAL := 0.1; \]

## Thermal conductivity of the ceramic plate
\[ tcHC := 7; \]
\[ tcCC := tcHC; \]

## Thermal conductivity of the contact material
\[ tccon := 0.33; \]

## Thermal conductivity of the dry nitrogen
\[ tcN := 3.72254 \times 10^(-14) \cdot Tm^6 + 1.082841 \times 10^{-10} \cdot Tm^7 - 1.296404 \times 10^{-7} \cdot Tm^12 + 1.238202 \times 10^{-4} \cdot Tm - 0.002145446; \]

## calculate the U values
\[ RTE := \frac{tcTE \cdot Ateff}{Lte}, \]
\[ RHC := \frac{tcHC \cdot S \cdot dL/LHC}{RI}, \]
\[ RCC := \frac{tcCC \cdot S \cdot dL/LCC}{RI}, \]
\[ Rseal := \frac{tcSEAL \cdot 2 \cdot bseal \cdot dL/Lseal}{RI}, \]
\[ Rspace := \frac{tcN \cdot Aspace/Lspace}{RI}, \]
\[ USC := \frac{(S \cdot dL \cdot RSC)}{RI}, \]
\[ USR := \frac{tccon}{LcSR}, \]

## Calculate the view factors of the CPC
\[ XSQ := (((1 + ss) \cdot 2 \cdot (cs/sS)^2 + (1 - ss)^2)^1/2); \]
\[ ###FSW := 0.5 \cdot ss/(1 + ss + 1 - XSQ); \]
\[ FSW := (Dig1 - R1)/S; \]
\[ FSR := 1 - FSW; \]
\[ FWR := 1 + XSQ + (ss + 0.25 \cdot XSQ^2)^1/2); \]
\[ FWR := 1 + R1/W0-sqrt(1-2*R1/W0*(W0-S)/2+(R1/W0)^2); \]
\[ FR := \frac{S/R1/2*FSR}{FW}; \]
\[ FWR := \frac{W0/R1/2*FSR}{FW}; \]
\[ FEC := \frac{CirCf/CirE}{FER}; \]
\[ FER := \frac{R/CirE}{FER}; \]
\[ FE := \frac{1/(1 - rE*rRO*FER)}{FE}; \]
\[ FC := 1/(1 - rE*Fcj*FEC); \]
\[ FCj := 1/(1 - rCj*FHj); \]
\[ FHj := 1/(1 - rCj*FHj); \]

### calculations of the area's
\[ \sigma := 5.67 \times 10^{-8}; \]
\[ CSb := aSb*tWb\cdot rRb\cdot FB; \]
\[ CWb := aWb*(1 + tWb\cdot rSb\cdot rRb\cdot FB); \]
\[ CRb := tWb*(1 - rRb)*(1 + rRb\cdot rSb)*FB; \]

### Solar energy from the PTC and which is intercepted by CPC
\[ Qpb := \gamma*bF\cdot ArP\cdot Radiation\cdot cos(Z); \]

### Calculate the coefficients of the energy balance equations
\[ CS1 := -1*(S\cdot ArS\cdot FS + (ArS-Atef)\cdot eHj\cdot FHj); \]
\[ CW1 := ArS\cdot aS\cdot eW\cdot FW\cdot FSW; \]
\[ CR1 := ArS\cdot eR\cdot FR\cdot FSR; \]
\[ CC1 := aS\cdot eCj\cdot Fcj\cdot (ArS-Atef); \]
APPENDIX 5

CS01 := -1/segma*(USR*AconSR + USC*ArS);
CW01 := 0;
CR01 := 1/segma*USR*AconSR;
CC01 := 1/segma*USC*ArS;
C1 := -1/segma*CSb*Qpb;

# Coefficients of the glass tube energy balance equation
CS2 := aW*eS*ArS*FS*FSW;
CW2 := -1*(ArW*eW*FW + ArG*eG + ArE*eE*FE);
CR2 := ArR*(aW*eR*FR*FRW + aE*eRO*FRO);
CC2 := aE*ArC*eC*FC;
CS02 := 0;
CW02 := -1/segma*ArG*hga;
CR02 := 0;
CC02 := 0;
C2 := -(1/segma*(CWb*Qpb + ArG*hga*Ta) + ArG*eG*Tsky^4);

# Coefficients of the CPC reflector energy balance equation
CS3 := aR*eS*ArS*FS*FSR;
CW3 := (ArW*aR*eW*FW*FWR + ArE*aRO*eE*FE*FER);
CR3 := -1 *ArR*(eR*FR + eRO*FRO);
CC3 := 0;
CS03 := 1/segma*USR*AconSR;
CW03 := 0;
CR03 := -1/segma*USR*AconSR;
CC03 := 0;
C3 := -1/segma*CRb*Qpb;

# Coefficients of the cold junction energy balance equation
CS4 := aq *Aspace*eHj*FHj;
CW4 := aC*ArE*eE*FE*FEC;
CR4 := 0;
CC4 := -(ArC*eC*FC + Aspace*eCj*FCj);
CS04 := 1/segma*USC*ArS;
CW04 := 0;
CR04 := 0;
CC04 := -1/segma*(USC*ArS + Ucf*Acf);
C4 := -1/segma*Ucf*Acf*Tfi;

EQU1 := CS1*TS^4 + CW1*TW^4 + CR1*TR^4 + CC1*TC^4 + CS01*TS + CW01*TW +
CR01*TR + CC01*TC = C1;
EQU2 := CS2*TS^4 + CW2*TW^4 + CR2*TR^4 + CC2*TC^4 + CS02*TS + CW02*TW +
CR02*TR + CC02*TC = C2;
EQU3 := CS3*TS^4 + CW3*TW^4 + CR3*TR^4 + CC3*TC^4 + CS03*TS + CW03*TW +
CR03*TR + CC03*TC = C3;
EQU4 := CS4*TS^4 + CW4*TW^4 + CR4*TR^4 + CC4*TC^4 + CS04*TS + CW04*TW +
CR04*TR + CC04*TC = C4;

equationset := {EQU1, EQU2, EQU3, EQU4};
variablesset := {TS, TW, TR, TC};

# solve the energy balance equations
solutionset := fsolve(equationset, variablesset);
assign(solutionset);

# solve the energy equation for TO
Ci05 := (mf*Cpf - hcf*Acf/2)/(mf*Cpf + hcf*Acf/2);
Cco5 := hcf*Acf/(mf*Cpf + hcf*Acf/2);
EQU5 := TO - Ci05*Tfi - Cco5*TC = 0;
TO := fsolve(EQU5, TO);

# calculate the efficiency
efficiency := mf*Cpf*(TO - Tfi)/ArP/Radiation;

# Useful heat extracted at the cold junction
Quseful := mf*Cpf*(TO - Tfi);
# write the results
appendto(result);
print('================================================================');
print(' Results of the analytical solution ');
print('================================================================');
print('Solar insolation, W/sq_m Radiation');
print('Mass flow rate, mlitre/min = ', mf*1000*60/denf);
print('Fluid Inlet temperature,(K) = ', Tfi - 273);
print('Ambient Temperature,(K) = ', Ta - 273);
print('heat transfer coefficient (glass tube/air) = ', hga);
print('-------------------------------------------------------------------------------');
print(round(TS - 273), ', round(TW - 273), ', round(TR - 273), ', round(TC - 273), ' , round(TO - 273));
print('-------------------------------------------------------------------------------');
print('Overall efficiency of the concentrator = ', (100*efficiency));
print('Useful energy extracted, watts = ', Quseful); quit;
# save the output file to result and quit maple
Appendix 6

Technical specification of the LabMaster power sensor (LM3).

<table>
<thead>
<tr>
<th>Specrail response</th>
<th>0.3 - 10.16 μm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>±2%</td>
</tr>
<tr>
<td>Aperture size</td>
<td>19 mm</td>
</tr>
<tr>
<td>Maximum power</td>
<td>3 watts</td>
</tr>
<tr>
<td>Maximum power density</td>
<td>200 W/cm²</td>
</tr>
<tr>
<td>Sensor type</td>
<td>thermopile</td>
</tr>
</tbody>
</table>
Appendix 7

{--------------------------}
[ SCTESTNN - ThermoCouple Reading Software ]
{--------------------------}
PROGRAM SCTESTNN;
USES crt, dos;
CONST
main_disp : ARRAY [0..14] OF STRING =
('ThermoCouple Reading Program
  TC #0 = xxx.XX0C
  TC #1 = xxx.XX0C
  TC #2 = xxx.XX0C
  TC #3 = xxx.XX0C
  TC #4 = xxx.XX0C
  TC #5 = xxx.XX0C
  TC #6 = xxx.XX0C
  TC #7 = xxx.XX0C
  CJC = XXX.XX0C
  Centre for Renewable Energy Systems Technology

{--------------------------}
message1 : STRING = (* Reading ThermoCouples Signals * );
message2 : STRING = (* Saving Data Arrays * );
message3 : STRING = (* Data Saved In File * );
message4 : STRING = (* Data Appended In File * );
message5 : STRING = (? Press Return key to Save These Data ??);
message6 : STRING = (??? OR <e> to End The Test. ???);
messclr : STRING = (' '.
comment1 : STRING = ('Temperature Distribution inside the Receiver System');
base = $300; (I/O port base address )
gain =1000; [ PCLD 789D GAIN SETTING ]
{--- geometrical parametres of the collector ---}
S = 0.02; [ Width of the absorber plate ]
A = 0.46; [ width of the PTC ]
dL = 0.3; [ length of the collector ]
Cr = 19.7; [ total concentration ratio ]
Dg = 0.07; [ diameter of the glass tube ]
df = 0.022; [ diameter of the cooling tube ]
{--------------------------}
{ --- Temperature conversion table for K-type thermocouples ---}
table : ARRAY[0..9] OF REAL = (-0.05115307103, 24850.28007, -382662.2822, 99661056.73, -10820623570.0, 603928552400.0, -19108999620000.0, 347823473000000.0, -3399102821000000.0, 1.382851398E+16);
tablec : ARRAY[0..21] OF REAL = (-0.0000008167743593, 0.00003963927586, 1.640341085E-08);
{--------------------------}
VAR
i,j,ch,status, stage, stage_final, No_Enter : INTEGER;
datal,datab, datalc, datahtc : BYTE;
data, datatc : WORD;
cjctemp, tctemp, volt, TV : REAL;
TS, TR, TC, TW, TIN, TOUT, Tf, TAIR, TAM : REAL;
Radiation, Quse, Efficiency : real;
Distance, mf, Cpf : real;
Year, Month, Day, Dayofweek : word;
temp : double;
temperature, TM : ARRAY[0..15] OF REAL;
filename : TEXT;
ascname : STRING[10];
Ke-PR : Char;

PROCEDURE Time_Date;  \{ set the date \}
Begin;
ClrScr;
GetDate(Year, Month, Day, Dayofweek);
End; \{ End procedure Get_Date \}

PROCEDURE Initialise; \{ initialise machine & variables \}
BEGIN
---- Draw screen display ------------------------------------------
ClrScr;
GoToXY(33,1); Write(Dayofweek, '-', day,, month, P, Year);
FOR i := 0 TO 14 DO
BEGIN
GoToXY(7,2+i); Write(main_disp[i]); \{ Write display line \}
END;
---- Initialise all data buffers -------------------------------------
FOR ch := 0 TO 15 DO temperature[ch] := 0; cjctemp := 0; stage := 0;
END; \{ procedure Initialise \}

PROCEDURE Read_Data; \{ Read ThermoCouples \}
BEGIN
----- read CJC data -----------------------------
GoToXY(20,17); Write(message1);
port[base+2] := 7;
port[base+1] := 8; \{ set A/D range code \}
port[base+2] := 7;
Delay(1);
port[base+8] := 0;
port[base] := 0; \{ trigger A/D converter \}
REPEAT
status := port[base+8]; \{ wait for data \}
UNTIL ((status AND $ 10) <> $ 10);
Delay(1);
datal := port[base]; \{ read A/D low byte & channel \}
datah := port[base+1]; \{ read A/D high bytes \}
data := datah*16 + datal DIV 16; \{ read CJC data into indata \}
volt := 20.0 * data / 4096 +(-10); \{ convert data to voltage \}
cjctemp := volt * 1000 / 24.4; \{ convert data to temperature \}

FOR ch := 0 TO 15 DO
BEGIN
port[base+3] := ch; \{ select next thermocouple channel \}
Delay(3); \{ for for circuit to stabilize \}
port[base+2] := 0;
port[base+1] := 8; \{ set A/D range code \}
port[base+2] := 0;
Delay(1);

--- read thermocouple data ---------------

--- Read CJC data ----------------------

--- Initialise all data buffers ------------

--- Draw screen display ----------------------

--- Get Date ----------------------------

--- Initialise machine & variables ----------

port[base+8] := 0;
port[base] := 0;  
[trigger A/D converter]
REPEAT
  status := port[base+8];  
  [wait for data]
UNTIL (status AND $10) <> $10);
Delay(1);
  dataltc := port[base];  
  [read A/D low byte & channel]
datathc := port[base+1];  
  [read A/D high bytes]
datatec := dataltc*16 + dataltc DIV 16;  
  [read TC data into ipdata]
if datatec > 4090 then datatec := 0;  
  [avoid numerical overflow]
if datatec < -4090 then datatec := 0;  
  [avoid numerical overflow]
volttc := (20*datatec / 4096+(-10)) / gain;  
  [convert data to voltage]
  T := 0;  
  V := 1;
FOR i := 0 TO 2 DO
  BEGIN
    1 := V * tablec[i] + T;
    V := V * cjctemp;
  END;
tctemp := T + volttc;
  T := 0;  
  V := 1;
FOR i := 0 TO 9 DO
  BEGIN
    1 := V * tablet[i] + T;
    V := V * tctemp;
  END;
temperature[ch] := T + 0.005;  
  [save converted temperature data]
END;
Delay(200);
GoToXY(20,17); Write(messclr);
END;  
[procedure Read_Data]
PROCEDURE Update_Display;  
  [Update display]
BEGIN
  GoToXY(42,5) -
  Write(cjctemp: ý: 2);  
  [Display CJC temperature]
  FOR ch := 0 TO 15 DO
    BEGIN
      GoToXY(20 * (ch DIV 8) + 33, (ch MOD 8) + 6);  
      Write(temperature[ch]: 6: 2);  
      [Display next TC temperature]
    END;
  END;  
  [procedure Update_Display]
PROCEDURE CALCULATE_performance;  
  [Calculation procedure]
BEGIN
  GoToXY(20,20); Write(messclr); GoToXY(20,21); Write(messcir);
  For ch := 0 to 15 do
    Begin
      TM[ch] := temperature[ch] + 273;
    End;
  TIN := TM[0];  
  TOUT := TM[1];  
  TS := TM[2];  
  TC := (TM[3] + TM[4])/2;
  delay(100);
  GoToXY(10,20); Write('?? Enter the Ambient temperature in celsius ??');
  Readln(TAIR);
  GoToXY(10,21); Write('?? Enter the tilt angle/misalignment. ??');
  Readln(angle);
  GoToXY(10,22); Write('?? Enter the water flow rate liter/min ??');
Readln(mfl);
GoToXY(10,23); Write('?? Enter the radiation level ??');
Readln(Radiation);
Tf := (TOUT + TIN)/2; \{average temperature of the cooling water\}
Cpf := IOE3*(5.6158 - 9.0277*IOE-3*Tf + 14.177*IOE-6*Tf*Tf); \{specific heat of water\}
denf := 766.17 + 1.80396*Tf - 3.4589*IOE-3*Tf*Tf; \{density of the cooling water\}
mf := mfl*denf/1000/60; \{mass flow rate in kg per second\}
Quse := mf*Cpf*(TOUT - TIN); \{useful heat extracted\}
Efficiency := Quse/Radiation/S/A/dL; \{calculate the thermal conversion efficiency\}
GoToXY(10,20); Write(messclr); GoToXY(10,21); Write(messclr); GoToXY(10,22);
Write(messclr);
delay(100);
END; \{PROCEDURE CALCULAnNG\}
PROCEDURE Save_Data; \{Save data arrays\}
BEGIN
GoToXY(20,20); Write(messclr); GoToXY(20,21); Write(messclr);
\{---- Setup file for data arrays \}
IF ParamStr(1) = '' THEN
ascname := 'SCTESTN.dat'
ELSE
ascname := ParamStr(1);
Assign(filename,ascname);
Rewrite(filename); \{Open and reset ascii data file\}
Writeln(filename, ' Date: ', Day, ',', Month, ',', Year);
Writeln(filename, 'Experimental Setting number [', stage, ']; \{Save header\}
Writeln(filename, 'Channel No., \', Temperature(K));
FOR ch := 0 TO 15 DO
BEGIN
Writeln(filename, '[', ch: 2, '] =', temperature[ch]: 6: 2, ', '); \{Save thermocouple data\}
WriteLn(filename); \{Write CR-LF to end data\}
END;
Writeln;
Writeln(filename, 'Usefull heat extracted = ', Quse : 6: 5, ' Watts');
Writeln(filename, 'Thermal efficiency = ', Efficiency : 6: 5);
Writeln(filename, 'Water Flow Rate = ', mf : 6: 5, ' Litre/min');
Writeln(filename, 'Air Temperature TAIR = ', TAIR : 6: 2 );
Writeln(filename, 'Specific heat of water = ', Cpf : 6: 2 );
Close(filename); \{Close ascii data file\}
Stage := stage + 1;
delay(200);
GoToXY(20,20); Write(messclr);
GoToXY(20,20); Write(message3, ascname);
delay(200); GoToXY(20,20); Write(messclr);
END; \{procedure Save_Data\}
PROCEDURE Append_data;
BEGIN
GoToXY(20,20); Write(message4);
GoToXY(20,20); Write(messclr);
GoToXY(20,20); Write(message4);
Append(filename);
writeln(filename);
writeln(filename, '"Experiment Setting: [', stage, ']"'); (Save header)
writeln(filename, '-------------------------------------- 0);
Writeln(filename, 'Channel No. ', ' ', Temperature(K));
Writeln(filename, '-------------------------------------- 1);
FOR ch: = 0 TO 15 DO
BEGIN
Writeln(filename, ', ch: 2, ', ' %temperature [ch]: 6: 2, '); (Save thermocouple data)
END; Writeln;
Writeln(filename, 'Usefull heat extracted Quse: 6: 5, ' Watts);
Writeln(filename, 'Thermal efficiency Efficiency: 6: 5);
Writeln(filename, 'Mass Flow Rate mfl: 6: 5, ' Litre/min);
Writeln(filename, 'Air Temperature TAIR : 6: 2
Writeln(filename, 'Specific heat of water Cpf : 6: 2);
Close(filename); (Close ascii data file)
Stage := stage + 1;
delay(200);
GoToXY(20,20); Write(messclr);
GoToXY(20,21); Write(message5, ascname);
delay(200); GoToXY(20,21); Write(messclr);
END; {procedure Append_Data}
BEGIN
stage := 0;
Time_Date; { GET TIME & DATE }
Initialise; { Initialise machine & program }
REPEAT {Repeat reading data and updating screen}
Begin
Repeat
Begin
Read_Data; {Read ThermoCouples}
Update_Display; {Display ThermoCouple data}
GoToXY(20,20); Write(message6);
GoToXY(20,21); Write(message7);
end;
Until keyPressed;
End;
Ke_Pr := Readkey;
IF ((Ke_Pr <> 'E') and (Ke_Pr <> 'e')) then begin
CALCULATE_Performance;
if Stage = 0 then
Save_Data {Save data array}
else
Append_Data;
end
ELSE
stage := stage_final;
UNTIL stage = stage_final;
GoToXY(20,20); Write(messclr);
GoToXY(20,21); Write(messclr);
END. {main program}

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