The mechanics of twisting somersaults

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THE MECHANICS OF TWISTING SOMERSAULTS

BY

MAURICE RAYMOND YEADON

A Doctoral Thesis
Submitted in partial fulfilment of the requirements
for the award of Doctor of Philosophy
of the Loughborough University of Technology
December, 1984.

Supervisors: Dr. J. Atha,
Professor F.D. Hales

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ABSTRACT

THE MECHANICS OF TWISTING SOMERSAULTS

by Maurice Raymond Yeadon

Loughborough University of Technology, December 1984.

Twisting movements are categorised into three mechanical types, named as DIRECT, COUNTER-ROTATION and TILT TWIST. Twisting techniques are studied using mathematical models.

A mathematical inertia model is constructed to enable the determination of segmental inertia parameters from anthropometric measurements. A film analysis program is developed so that the angles, which specify the orientation and configuration of the body, may be derived from digitised film data. A computer simulation model, comprising 11 segments and 17 degrees of freedom, is constructed to represent the human body in free fall. The combined use of the three computer programs results in maximum errors of 3% for somersault and 9% for twist in ten filmed movements.

The mechanics of twisting techniques are explained using simple mathematical models. An analysis of rigid body motions shows that there are two distinct modes of motion, named as the ROD MODE and the DISC MODE. It is shown that it is possible to change from one mode to the other by varying the angle of pike and this permits the twist to be increased or stopped or even reversed.

The capacities of twisting techniques are determined using simulations. For twists from a piked position, delaying the extension from the pike can increase the twist rate although this does depend upon the particular technique used and the initial direction of somersault.

The contributions of twisting techniques used in the filmed movements are determined using simulations based upon modifications of the film data. It is found that counter-rotation techniques made small contributions and that aerial techniques, which increased the angle of tilt, were the major contributors, even in movements where the twist was apparent at take off.

Using the simulation model it is shown that the build up of twist in the unstable double layout somersault may be controlled by means of small asymmetrical arm movements during flight.
STATEMENT OF RESPONSIBILITY

I hereby certify that I am responsible for the work submitted in this thesis, that the original work is my own except as specified in acknowledgements, and that neither the thesis nor the original work contained therein has been submitted to this or any other institution for a higher degree.

Maurice Raymond Yeadon
Loughborough University of Technology
December 1984.
DEDICATION

To Lili and Cyrus
I wish to express my thanks to:

My supervisors Dr. Atha and Professor Hales for their advice and continued encouragement,

Gill Statham, John Cryer and Carl Furrer for performing the trampoline movements,

The departments of: Human Sciences
Physical Education and Sport Science
Chemical Engineering

the Computer Centre
and Audio Visual Services

for providing advice, technical assistance and facilities,

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CHAPTER 1

INTRODUCTION

In July 1969 I attended a trampolining course given by G.S. Aaron at Loughborough Summer School. It was here that I saw a piked rudi-out fliffus for the first time. In this movement, which consists of 2 somersaults and $\frac{1}{2}$ twists, a piked forward somersault is followed by a straight somersault with $\frac{1}{2}$ twists. I found the movement exciting and also remarkable since the origin of the twist was far from obvious. Whilst some mechanically sound explanations of the twist were put forward, they were incompatible with the manner in which the movement was performed. I therefore set out to find a satisfactory mechanical explanation of the twist.

Having obtained an explanation, more than fifteen years later, I now find the performance of such movements to be even more remarkable.

At the First International Week for Acrobatic Sports, held in August 1981 at Dinard in France, a film was shown of Richard Tison (World Trampoline Champion, 1974, 1976). The film showed Tison performing a double backward somersault in the straight position, with a full twist in the second somersault. It appeared that a symmetrical body configuration was maintained throughout the movement. None of the international coaches present could explain how the twist was produced.

Without such an understanding a coach has no foundation upon which to base his advice to a performer. So long as a competitor has no difficulty in learning new movements, a coach may restrict his attention to improving form and presentation. Once a performer is experiencing difficulty, and even World Champions have problems, technical advice is needed. The correct advice can often produce good results within seconds, whereas poor advice will aggravate the problem and may place the performer in a dangerous situation.
Thus the need for an understanding of the mechanics of twisting somersaults is clear and the insights into twisting techniques may be applied, not only to trampolining, but also to diving, gymnastics and acrobatic skiing.

The variety and complexity of such human movement comprises an interesting but difficult field of investigation.

Careful observation by the experienced coach can establish the techniques which appear to be employed in a particular movement. The coach may then identify those techniques which are associated with increased skill and may cultivate modifications to improve performance. Through this process of observation, analysis and experiment the coach progressively becomes experienced.

The same process may be employed by the sports scientist who uses cameras to record movements, analyses the film data thus obtained using statistical techniques and forms hypotheses to be tested by experiment. Unfortunately, it is not possible to have complete control in such experiments since a performer, who attempts to alter some aspect of a movement, may introduce additional changes. It is this effect that dilutes the power of experiment and prevents the resolution of much of the controversy amongst coaches about twisting technique.

Cinematographical investigations have been used to determine the techniques responsible for the production of twist in particular movements. As will be discussed in Chapter 2, this aim cannot be achieved without an accompanying quantitative mechanical analysis and so the conclusions of such investigations should be regarded as informed opinions rather than established results.

Experimental studies have shown that the production of twist whilst airborne is indeed possible but the techniques responsible are largely a matter of conjecture.

Of all the mathematical models used in theoretical studies, only those of Kosa and Kamimura (1972) and Van Gheluwe (1981a,b) were validated using data taken from actual performances. Thus the only
results on twisting techniques that have been conclusively established are:

(1) One cycle of hula movement of the hips is capable of producing a half twist.

(2) Asymmetrical arm movements are capable of producing a full twist in both piked and straight backward somersaults.

Whilst much of the literature comprises opinions rather than properly supported conclusions, the views expressed do raise a number of questions which may be listed as follows:

Twist may be started prior to take off by turning the arms, chest or the whole body in the direction of the twist (Aaron, 1977, p.10; Smith, 1980,1981; Moorse, 1966). However Valliere (1976) stated that the common belief that the arms initiate the twist is incorrect.

**Question (1)**

What are the relative capabilities of the arms, chest and whole body for initiating twist prior to take off?

Whilst some movements certainly do appear to start the twist before contact is lost (Winter, 1966; Mood, 1968), it may be that aerial twisting techniques make substantial contributions.

**Question (2)**

In movements where the twist is apparent at take off, how much of the subsequent twist is a result of the momentum built up during the contact phase?

Rackham (1960) stated that piking during a twisting somersault will slow but cannot stop the twist, whereas Nazarov (1978) showed that there is some theoretical basis for the contention that piking can remove the twist.
Question (3)

Is it possible to stop the twist by piking during a twisting somersault?

Eaves (1960b, 1960c) and Biersterfeldt (1974a) noted that, when twist and somersault are present at take off, the twist axis becomes tilted out of the vertical somersault plane when the body is inverted. This effect will produce undesirable tilt during the entry phase of twisting somersault dives.

Question (4)

Is it possible to remove the tilt in a twisting 1½ somersault dive either by piking or by some other method?

If neither twist nor somersault is present at take off, it is still possible to twist whilst airborne as the experiments of McDonald (1961) have shown. Kosa and Kamimura (1972) demonstrated that a hula cycle of the hips can produce a half twist. However, the ability of the arms to produce aerial twist has not been established. Whilst Bunn (1972, p.220), LaDue and Norman (1967, p.87) and Batterman (1974, p.75) advocated counter-rotation of the arms; Lanoue (1940), Eaves (1969, p.81) and Valliere (1976) held the view that the arms by themselves are incapable of producing rotation.

Question (5)

How much twist can be produced by counter-rotation of the arms?

Frolich (1979) showed that it is theoretically possible to convert a plain somersault into a twisting somersault by tilting the twist axis out of the vertical somersault plane. Van Gheluwe (1981b) demonstrated that a full twist may be produced during a backward somersault using asymmetrical arm movements, whilst Pike (1980) concluded that it should be possible to introduce a full twist into a plain dive by means of such arm movements.
Question (6)

Are asymmetrical arm movements capable of producing a full twisting dive with a final tilt angle of zero?

Another means of producing twist during a somersault was proposed by Rackham (1970) and Batterman (1974, p. 77) who stated that turning the shoulders and chest, when the body is piked, will result in twist once the body extends.

Question (7)

Can twist be produced during a somersault by torsion of the chest relative to the hips prior to the extension from a piked position?

Aaron (1977, pp. 28-30) stated that a 'pike-extension hip rotation' movement will produce twist in a somersault. This hip rotation could be interpreted as a hula movement of the hips.

Question (8)

Can a sustained twist be produced in a somersault by using a hula movement of the hips as extension is made from a piked position?

Whilst Barrow (1959c) and Aaron (1977, p. 29) held the view that the production of a sustained twist slows the somersault, Bunn (1972, p. 220) thought that the somersault rate increased and Rackham (1970) argued that the somersault rate would remain unchanged.

Question (9)

What effect does the production of a sustained aerial twist have on the somersault rate?

Straight somersaults about the lateral axis are theoretically unstable (Nigg, 1974) and this will lead to a build up of twist.

Question (10)

How much somersault rotation must occur before the build up of
twist becomes noticeable in the unstable layout somersault?

Hinrichs (1978) hypothesised that performers make small corrective movements during flight to prevent the build up of twist.

Question (11)

How may the build up of twist be controlled during straight somersaults by making in-flight corrections?

The aims of this study are to answer these questions and to provide quantifiable mechanical descriptions of twisting techniques. These aims are summarised in the following statement of purpose.

STATEMENT OF PURPOSE

(a) To present quantifiable mechanical explanations of the operation of twisting techniques.
(b) To establish quantitatively the capacities of different techniques for producing and controlling twist.
(c) To establish the contributions of the various techniques to the twist produced in actual performances.

In order to realise these objectives a computer simulation model of the human body in free-fall will be developed. The model will permit the simulation of modified filmed performances as well as the creation of hypothetical movements. By this means it will be possible to observe the effect of a single configurational change.

The mechanics of twisting techniques will be explained using mathematical models which permit analytical solutions.

The capacities of different techniques for producing and controlling twist will be established using a range of simulations.

The contributions of the various techniques to the twist produced in filmed movements will be determined by the effects that modifications of the filmed configurations have on simulated performance.
CHAPTER ORGANISATION

Chapter 2 first reviews the literature on the description of twisting techniques. Three main categories of twist are identified as DIRECT TWIST, COUNTER-ROTATION TWIST and TILT TWIST. The results of cinematographic, experimental and theoretical studies are critically examined to see which conclusions can be justified. A summary of investigative techniques provides the background to the following three chapters.

Chapter 3 develops the inertia model which is used to determine segmental inertia parameters from anthropometric measurements.

Chapter 4 details the film analysis procedures for determining the orientation angles.

Chapter 5 develops the simulation model and describes the ways in which it may be used.

Chapter 6 presents the validation of the combined use of the computer programs developed in the previous three chapters.

Chapter 7 investigates twisting techniques using the simulation model.

Chapter 8 provides a summary of the results obtained.
TWISTING TECHNIQUES AND THEIR MECHANICAL DESCRIPTIONS

In this section twisting techniques are classified into three distinct categories: viz. DIRECT TWIST, COUNTER-ROTATION TWIST and TILT TWIST. Whilst there is a measure of general agreement regarding these categories the terminology is by no means universal and so other terms in use are given at the beginning of the subsection on each category. Since many of the explanations of how twist is produced are unclear or questionable, the selection of quotations has been restricted to those which present a clear point of view and the description of each category of twist has been structured with the intention of enlightening rather than merely presenting a number of conflicting views.

DIRECT TWIST: Twist produced whilst in contact with the apparatus

A direct twist is started whilst the feet are in contact with the springing medium (diving board, trampoline, floor, gymnastic beam) or whilst the hands are in contact with the gymnastic apparatus (rings, high bar, parallel bars, asymmetric bars, vaulting horse). Other terms for direct twist include:

- transfer of momentum twist (Rackham, 1960)
- torque twist (Frolich, 1979)
- angular momentum twist (Frolich, 1980)
- twist from the board (Eaves, 1969)
- inertial twist (Kosa and Kamimura, 1972)

The Amateur Swimming Association Manual of Diving (1963) described twisting by momentum transfer (p.25):

'Twist by transfer of momentum takes the form of a continuous rotation of the whole body about the long axis lasting throughout
the flight... It is achieved by starting the upper trunk twisting during the take-off, i.e. whilst the feet are in contact with the board, usually by turning the shoulders about the long axis as the feet give their final path.

When the feet leave the board the momentum stored in the upper body will be transferred to the whole, and it will rotate in the original direction of rotation of the shoulders. The body will continue to rotate (twist) about its long axis throughout the flight without any further effort on the diver's part. He can control the rate at which he twists, i.e. his angular velocity about the long axis, by altering his moment of inertia about that axis, but he cannot stop himself twisting'.

This last statement appears to conflict with the subsequent description of a twisting dive (pp.31-32):

'... a diver performing a back 1½ somersault with 2½ twists will frequently take part of the twist from the board. Directly he has left the board he will start to straighten his body and bring his hands in close to the most favourable position for producing rapid twists. When he has nearly completed the required number of twists he will go into the half lever or open pike position with arms extended sideways. This will reduce his rate of twisting to approximately a seventh of what it was but this remaining twist will appear to the onlooker as a side somersaulting motion, not as a twist!'.

The last sentence is self-contradictory since upon piking either the twist will continue but at a slower rate or the twisting motion will be replaced by side somersault. If the latter is the case then it may be concluded that the diver has indeed stopped himself twisting. The opposite view was held by Rackham (1960), who stated that a diver cannot stop himself twisting by stretching his arms out sideways and piking his body, and by Kosa, Kamimura and Hayashi (1973) who stated that stopping or reversing the twist is impossible.

Eaves (1960b,1960c) stated that taking twist from the board leads to cartwheeling or 'going over sideways' at the end of a dive:
'... it can be shown that the amount of going over sideways... in a forward dive straight with half twist is... roughly 8°'. When discussing somersaults with twist taken from the floor, Biesterfeldt (1974a) noted the same effect and stated that the twist axis does not remain in a vertical plane since it describes a cone. He presented a diagram which showed that the tilt of the longitudinal axis away from the vertical plane reaches a maximum after half a somersault and disappears after one somersault.

In gymnastics and trampolining most twisting movements are either single or double somersaults for which the tilt will disappear at the completion of the movement. In diving however, most movements involve an odd number of half somersaults for which the tilt will tend to be a maximum at entry into the water. Eaves (1969, p. 87) stated that the combination of twist from the board and somersault will invariably lead to going over sideways so that the only cure is to eliminate the practice of taking twist from the board. On the other hand Barrow (1959a) noted that piking into the entry of the 1½ twisting back 1½ somersault dive helps to avoid any tendency for the legs to drop sideways on finishing the dive.

The initiation of the twist during take off is achieved by turning the upper body (Batterman, 1974, p. 75) or the head, arms and shoulders (Aaron, 1977, p. 10) in the direction of twist. Smith (1980, 1981) stated that the legs are also turned in the direction of twist during the contact phase. Regarding the contribution of the arm movements Valliere (1976) commented: 'Contrary to what certain people still believe, it is not the transfer of momentum from the arm to the rest of the body that initiates the movement'.

It is accepted that this type of twist occurs in diving (Orner, 1960; Rackham, 1960; Frolich, 1980), trampolining (Hay, 1962; LaDue and Norman, 1967; Aaron, 1977) and gymnastics (George, 1980; Smith, 1982) and there is general agreement in the descriptions of how the twist is produced although there are differences of opinion about the contributions made by the movements of the various body segments. Whilst it is agreed that the twist may be accelerated by bringing the arms close to the body and by extending the body, there is dis-
agreement as to whether piking will stop the twist or will merely slow the twist. Movements such as the forward 2½ somersaults with 2 twists in the first somersault certainly appear to stop twisting when the body becomes piked. Whether the twist is stopped or only slowed during the piked phase, it might be expected that twist would again occur when the body extends prior to the entry into the water. The presence of tilt prior to entry also presents the diver with a problem and it remains an open question whether piking can reduce or eliminate the tilt.

COUNTER-ROTATION TWIST: Twist produced by continued muscular action whilst airborne

A counter-rotation twist is produced by the relative movements of body segments while the body is free from support and when the relative movements cease so does the twist. Other terms for counter-rotation twist include:

- action-reaction twist (LaDue and Norman, 1967)
- contrary motion twist (Eaves, 1969)
- non-inertial twist (Kosa and Kamimura, 1972)
- torque-free twist with zero angular momentum (Frolich, 1979)

A clear description of how a cat may land on its feet when dropped from an inverted position was given by Lanoue (1940):

'In Figure 1, A shows the cat dropping freely with front and back legs straight. B shows the front legs down to the chest and rear legs extended. Of course, when the front legs are twisted to the right the hind legs must go to the left, but although the centres of mass move the same linear distance, that of the front legs moves through a greater angle, as the center of mass of the hind legs is way out near the knees and that of the front legs is close to the shoulders. Then in C the operation is reversed and the hind legs twist further around than the front ones. By repeating these movements, the number of times depending on his agility and flexibility, the cat eventually arrives at an upright position'.

For astronaut reorientation Kulwicki, Schlei and Vergamini (1962)
Figure 1. Twisting of a cat (Lanoue, 1940)

Figure 2. Twisting of a rabbit (Dyson, 1973)
described a similar technique in which the moments of inertia of the upper and lower body are varied by adducting and abducting the arms and legs. Although the movement was referred to as the 'Cat Reflex', Scher and Kane (1969) observed that the technique does not appear to be used by cats. Even if the technique could produce a half twist, the repeated abduction and adduction of the legs would render it unsuitable as a gymnastic movement for aesthetic reasons.

A different technique which enables an animal to twist in the air was described by Dyson (1973, p.106):

Initially the animal pikes so that there is an angle between the longitudinal axes of the upper and lower body. Successive twists are considered about the two longitudinal axes. When twisting the trunk through 180°, the hind parts are displaced in the opposite direction but through a much smaller angle because of their greater moment of inertia about the trunk axis. When the hind quarters twist about their axis, the reaction of the trunk is small due to the large moment of inertia of the upper body about this new axis. At the completion of one such sequence a half twist has been produced (Figure 2).

Rackham (1960) gave a very similar account of how a diver can twist by performing successive rotations about two axes and noted that these rotations can be made simultaneously (Figure 3).

When rotations about the longitudinal axes of the upper and lower body are made simultaneously, the body moves from a forwards pike, through a side pike, into a back arch, through a side pike on the other side and returns to a forward pike position. Since the hips move with a circular motion similar to Hawaiian hula dances, Van Gheluwe and Duquet (1977) have used the term 'hula twist'. A mechanical description of how twist results from the hula movement was given by Van Gheluwe (1981a) who explained that in order for the angular momentum to be conserved the whole system must rotate in the opposite direction.

Three distinct techniques have been described and they may be referred to as:
Figure 3. Hula action of a diver (Rackham, 1960)

Figure 4. Somersault with half twist (Dyson, 1973)
(a) varying moments of inertia twist
(b) two-axes twist
(c) hula twist

Accounts of twisting techniques often incorporate combinations of (a), (b) and (c). The explanation of Batterman (1974, p.75), of how a cat can twist by varying the moments of inertia of the front and rear legs and by circumducting the rear legs in a direction opposite to the desired twist, uses (a) and (c). Dyson (1973, p.108) used (a) and (b) when explaining how a forward somersault with half twist can be performed by twisting the trunk with arms adducted and the body piked and then twisting the legs with arms abducted and the body straight (Figure 4).

The two-axes technique used by the rabbit (Figure 2) is quite distinct from the hula technique since the rabbit employs nearly 180 degrees of spinal torsion (Figure 2c). For humans however, the amounts of spinal torsion attainable are less so that a number of successive rotations about the two axes will have to be made and the technique will become very similar to the hula twist in which the rotations about the two axes are made simultaneously.

The two-axes explanation (Rackham, 1960) of how twist is produced may be regarded as an alternative to Van Gheluwe's simple explanation of the hula twist. Unfortunately the frames of reference of the upper and lower body are not inertial frames and the motions are not referred to the mass centre of the system. The explanations of Lanoue and Dyson are also deficient in this respect, although their accounts do provide a clear description of the relative segmental movements and the resulting motion.

Hopper (1973, p.167) and Frolich (1980) overcame the deficiency of the two-axes explanation of hula twist by considering two equal cylinders which twist about their own axes whilst the whole system counter-rotates about the line joining the two mass centres so that the total momentum is conserved. This three-axes explanation is equivalent to that of Van Gheluwe (1981a) and leads directly to a valid equation of motion (Frolich, 1979) whereas the two-axes explanation does not.
Counter-rotation of the arms has also been proposed as a means of producing a mid-air twist. Bunn (1972, p. 220) wrote 'the movement of the arms in one direction about the longitudinal axis will cause rotation of the body in the opposite direction'. A similar description was given by LaDue and Norman (1967, p. 87) who noted that the twist stops when the arms reach the end of their swing, but also stated that dropping the arms to the sides shortens the radius of gyration and adds speed to the twist. It should be observed however that the angular momentum associated with the arm movements decreases as the arms move close to the body and this will tend to reduce the resulting twist rate.

Batterman (1974, p. 75) stated that a half twist can be produced in a dive by counter-rotation of the arms, whereas Lanoue (1940), Eaves (1969, p. 81) and Valliere (1976) held the view that the arms by themselves are incapable of producing rotation.

It is accepted that relative motions of the upper and lower body segments produce twist in diving (Eaves, 1969; Rackham, 1960; Batterman, 1974), trampolining (Aaron, 1977) and gymnastics (Bajin, 1972; Biesterfeldt, 1974b) although just how much twist can be produced is an open question. On the other hand, there is disagreement as to whether counter-rotation of the arms is capable of producing even a half twist.

**TILT TWIST**: Twist resulting from tilting the twist axis out of the somersault plane

Tilt twist is employed whenever a plain somersaulting motion is changed into a twisting somersault. There has been much controversy as to whether arm movements are necessary and whether such a twist is even possible as well as disagreements about the mechanical explanations put forward. Other terms for tilt twist include:

- twist by somersault transfer (Rackham, 1970)
- twist by trading momentum (Hay, 1978)
- torque-free twist with angular momentum (Frolich, 1979)
Commenting upon the descriptions of twisting dives given by Barrow (1959a, 1959b), Orner (1959) stated that when twist is not taken from the diving board it can only be produced by action-reaction and '... any postulated technique, in which the diver leaves the board without angular momentum about the desired axis of rotation and then has him 'Spinning', is faulty'.

Barrow (1959c) replied to Orner and showed how the total angular momentum may be resolved about tilted body axes:

'In Figure 5 the line OX represents the angular momentum vector of the somersault before any twist is commenced. It can be seen at once that this is equivalent to two component vectors OA and OB. If OB is the angular momentum vector of the twist and OA is the angular momentum vector of the somersault during the twist, the Total Angular Momentum of the system \( \text{OX} \) is exactly equivalent to the vector sum of these two. Consequently the system may be in either state without violating the principle of Conservation of Angular Momentum. In practice this means that the axis of somersault rotation is tilted slightly and the rate of somersault rotation is decreased slightly during the twist'.

Eaves (1960a) observed: 'Mr. Barrow offers no mechanism whereby the body is tilted... and the only feasible mechanism is a series of rapid rotations of the arms across the front of the body in the opposite direction to the tilt. It can be shown that this cannot be done quickly enough to be effective...'. He concluded that even if the body were tilted this would not give rise to a twist and the body would simply describe a double cone about the axis OX (Figures 5, 6).

However, since a rigid body will spin steadily only about a principal axis (Synge and Griffith, 1959, p.378), it is not possible for a rigid body to describe a double cone without twisting and the conclusion of Eaves is without foundation.

Batterman (1974) and Frolich (1979) described how the body may be tilted by raising one arm sideways and lowering the other (Figure 7) and, like Barrow (1959c), they showed that when the body is tilted out of the somersault plane there will be a component of momentum
Figure 5. Resolution of angular momentum (Barrow, 1959b)

Figure 6. Double cone arising from tilt (Eaves, 1960a)

Figure 7. Tilt produced by asymmetric arm movement (Batterman, 1974)
along the twist axis.

A different explanation was proposed by Yefimova (1963), Horne (1978) and George (1980) who stated that having one arm raised and the other lowered causes one side of the body to somersault faster so that a twist is produced. Travis (1968) observed that after a quarter twist the same argument implies that the process must start to reverse itself so that a sustained twist will not be produced.

LaDue and Norman (1967, p.92) and Hay (1962) stated that, if an arm is swung across the body when piked, the momentum will be transferred to the whole body as it extends and a twist will occur. Jennett (1954, 1967) proposed a similar transfer of momentum by twisting the trunk and then extending from the pike. Whilst such body movements may produce tilt and result in twist when somersault is present, the explanations in terms of transfer of momentum must be faulty since they imply that a sustained twist can be obtained from a piked jump.

The ability of the arms to convert a plain somersault into a twisting somersault has been questioned by Eaves (1960a), Rackham (1958) and Aaron (1977, p.30). Rackham wrote: 'If the body is in the air in a horizontal position with both arms stretched above the head, the moving of one arm out to a sideways position will not cause the body to roll over on its side and create a half-twist movement as some divers believe'. Subsequently, after experiments in which divers produced such a twist, Rackham (1970) reversed his position and agreed with Travis (1968) that asymmetrical arm movements can product twist in a somersault.

Frolich (1980) observed that in addition to arm movements any relative movement of body segments that produces tilt will result in twist. Rackham (1970) stated that turning the shoulders in one direction when the body is piked will cause the legs to swing in the opposite direction (Figure 8) so that when the body extends it will be tilted. Batterman (1974, p.77) also proposed a twisting of the upper body when in the piked position (Figure 9). Aaron (1977, pp.28-30) described a similar 'pike-extension, hip rotation' technique, in which the trunk is turned in the direction of the
Figure 8. Torsion of trunk to produce tilt (Rackham, 1970)

Figure 9. Torsion of trunk to produce tilt (Batterman, 1974)
twist and the hips are rotated as the body extends to bring the legs into line with the trunk. The hip rotation could be interpreted as a hula movement of the hips.

When the required number of twists are near completion, appropriate arm movements will remove the tilt and stop the twist. If an asymmetrical arm movement is used to produce the tilt in a dive with an even number of half twists, then a reversal of this arm movement will remove the tilt (Hay, 1978, p.157; Batterman, 1974, p.113). This is probably the reason why divers twist with one arm above the head. For movements with an odd number of half twists the arm action required for the removal of the tilt is identical to that which initially produces the tilt (Travis, 1968). Thus for a 1½-out fliffus on trampoline it is a disadvantage to leave the leading arm above the head and this may be the reason why trampolinists use symmetrical arm positions during twists. When the tilt has been produced by relative movements of the upper and lower body, it may again be removed by asymmetrical arm movements (Rackham, 1970) but presumably the same technique that produced the tilt is also capable of removing it.

Whilst Barrow (1959c) and Aaron (1977, p.29) held the view that the introduction of twist slows the somersault, Bunn (1972, p.220) wrote: '... when twisting movements are employed in a dive the turning speed is increased'. Rackham (1970) reasoned: 'As the body is now tilted over, its moment of inertia about the lateral somersaulting axis is slightly less so that the rate of spin is increased so balancing out the loss of somersaulting angular momentum into twist momentum'. In other words whilst the momentum available for somersault is reduced the moment of inertia is also reduced, so it is not clear whether the somersault rate will increase or decrease.

There are a number of movements in diving, trampolining and gymnastics in which the performer completes a somersault before starting to twist (e.g. Figure 18) and it is accepted that aerial twist is used in such movements, although there is disagreement about the technique employed.
INVESTIGATIONS OF TWISTING MOVEMENTS

Cinematographic, experimental and theoretical studies of twisting movements have been made but each approach has its weaknesses.

An observational study can record what happens but needs to be used in conjunction with a mechanical analysis before conclusions concerning twisting technique can be drawn.

An experimental study needs to be controlled since otherwise it reduces to an observational study. However the constraints imposed by a controlled experiment may affect the performance of movements to such an extent that the results are not applicable to a free movement.

A theoretical study must be shown to be a valid representation of the system that it models before conclusions can be drawn concerning actual performance.

Thus a combination of techniques is required before conclusions may be properly drawn. Unfortunately the conclusions of a number of investigators tend to be opinions rather than the end products of a line of reasoning based upon marshalled evidence.

CINEMATOGRAPHIC STUDIES

Observation of a twisting somersault can establish the orientation of the body throughout the movement together with the internal configuration of the system. This information may suggest that a particular mechanical type of twist was used but without a quantitative mechanical analysis the conclusions that can be drawn are limited. If a diver has completed some of his twist and has a twisting velocity at the instant of takeoff, it seems reasonable to conclude that the direct twist technique is responsible for the production of the twist. Such a conclusion is premature. If a somersault without twist is analysed the amounts of twist and twisting velocity at takeoff will not be exactly zero and so it is a matter of the mechanical significance of these values. Additionally
any actual performance will exhibit configurational changes which may be interpreted as showing that a particular technique contributes to the twist. Without some way of determining quantitatively the contributions of each technique to the twist, any conclusion reached on the origin of the twist is likely to reflect the investigator's personal opinions. As McDonald (1961) wrote: '... how easy it is to have a preconceived theory and interpret what you see so that it fits'.

DIRECT TWIST

Winter (1966) filmed four twisting dives with one 16mm cine-camera, which was positioned to the side of the diving board, in order to determine whether the divers had initiated the twist before takeoff. For the forward 1¼ somersaults with 2 twists he concluded that the twist was started after the diver had left the board. For the remaining dives: the reverse 1¼ somersaults with 1½ twists, the backward 1¾ somersaults with 1½ twists and the forward dive with half twist, he concluded that the twist was started whilst the diver was in contact with the board. In these three dives the divers were in layout position at take-off and twist occurred early in each dive. In the forward 1¼ somersaults with 2 twists the diver left the board in a piked position. If twist was taken from the board then the piked position would have ensured that the initial twist rate was small. Moreover Winter (p.21) stated: 'While his body is in a piked position, he has begun his twisting rotation about the long axis of the body'. Thus the evidence is consistent with the alternative hypothesis that the twist was taken from the board.

Three 16mm cine-cameras, situated to the side of, in front of and above the diving board, were used by Mood (1968) to film six dives performed by each of three divers. The twist angle was taken to be the apparent angle between the shoulder line and the front edge of the diving board, as given by the front or top camera, depending on the amount of somersault. Such a procedure will give only rough approximations and may account for Mood's observation that some twists reversed in direction. He concluded that, in all but one of
the dives, the twisting rotation of the trunk was started whilst the diver was still in contact with the springboard.

Bangerter and Leigh (1968) used two 16mm cine-cameras to film three springboard divers executing the forward 1 ½ somersaults with 2 twists and the backward 1 ½ somersaults with 1½ twists and three trampolinists performing the forward somersault with 1½ twists and the backward somersault with 2 twists. After viewing the films they concluded that in all cases the twist was taken from the springing medium whereas most of the coaches and performers who completed a questionnaire held the opposite view.

The studies of full and double twisting backward somersaults from the floor by Wiley (1964), Seidel (1976) and Al-Haroun (1980) concluded that the twist began whilst the gymnasts were still in contact with the floor.

Figure 10 shows a double twisting somersault dismount from the high bar. Biesterfeldt (1974b) held the opinion that there was little tilt and concluded that the twist was a result of counter-rotation rather than direct twist. Figure 11 shows that one cycle of hula movement has occurred so that, if Biesterfeldt's interpretation is correct, a single hula cycle is capable of producing a double twist. On the other hand, it may be that the twist is taken from the bar and that the observed hula cycle is incidental to the movement. Gluck (1979) presented sequences showing that the tilt increased with the number of twists (Figure 12) and this is consistent with the hypothesis that direct twist is used (Hopper, 1973, p.164).

**TILT TWIST**

Van Gheluwe and Duquet (1977) filmed two gymnasts performing full twisting backward somersaults from the floor and minitrampoline. The purpose of the study was to determine to what extent the two-axes and hula techniques contributed to the twist action. Torsion of the shoulders relative to the pelvis was observed as the twist
Figure 10. Double twisting dismount from high bar (Biesterfeldt, 1974b)

Figure 11. Hula movement during dismount (Biesterfeldt, 1974b)
Figure 12. Dismounts from high bar (Gluck, 1979)
started and two cycles of hula movement occurred during the twist. It was concluded that neither theory is contradicted by the evidence. On the other hand, Figures 13 and 14 show that both performers employed asymmetrical arm movement and obtained tilt, so that the hypothesis that tilt twist is the underlying mechanism is also supported by the available evidence.

Overhead and side views of a gymnast performing full twisting backward somersaults were obtained by Borms, Duquet and Hebbelinck (1973). It was concluded that no twist was taken from the floor and that 'the arms started the twisting action, using the gyroscopical effect'. Although a number of graphs of the paths of joint centres were given, none appears to provide support for the conclusions.

A single camera was used by McCormick (1954) to film the forward 1½ somersaults with full twist, the camera being positioned on the right, on the left and in front of the diver for successive performances. He concluded that the twist starts in the air from the open pike position with the body remaining extended throughout the twist. The faster twists were performed by Bob Clotworthy who lowered his left arm in the sagittal plane prior to an asymmetrical arm movement in the frontal plane (Figures 15 and 16). It is to be expected that such a sequence of arm movements will produce more tilt than that obtained by simply raising one arm and lowering the other, so that the twist rate will be correspondingly greater.

In the illustrations of Frolich (1980), based upon film of the 1979 U.S. Outdoor Diving Championships, asymmetrical arm movement is evident in the triple twisting 1½ somersaults (Figure 17) whilst the arm movement is more restricted in the double twisting 2½ somersaults (Figure 18).

Al-Haroun (1980) used two phase-locked Locam 16mm cameras to film six gymnasts performing full twisting backward somersault dismounts from the rings. Of the six gymnasts, three initiated the twist before releasing first one ring and then the other, whilst the remaining three released the two rings simultaneously and initiated the twist after release. Of the latter three, one gymnast exhibited
Figures 13, 14. Back somersaults with full twists
(Van Gheluwe and Duquet, 1977)
Figure 15. Arm action of Bob Clotworthy

Figure 16. Full twisting forward 1½ by Clotworthy (McCormick, 1954)
Figure 17. Triple twisting forward 1½ somersaults (Frolich, 1980)

Figure 18. Double twisting forward 2½ somersaults (Frolich, 1980)
both asymmetrical arm movement and spinal flexion, another asymmetrical arm movement with little spinal flexion and the third showed spinal flexion with little asymmetrical arm movement (Figure 19).

Hinrichs (1978) filmed a tucked barani-out fliffus on trampoline using two cameras and calculated the three-dimensional coordinates of segment endpoints from their digitised coordinates. The directions of the principal axes and the values of the principal moments of inertia were then determined at times throughout the movement. It was found that, at the time the twist was initiated, the maximum and intermediate whole body principal inertias converged to a common value and small deviations from body symmetry produced a marked shift in the orientation of the maximum principal axis away from the angular momentum vector. It should be noted, however, that such movements of the principal axes will occur whenever the body moves from a tucked to an extended position, so that the observed shift in the directions of the principal axes at the time the twist starts may not be related to the twist. A series of stick figures depicting the movement show the presence of tilt during the twist and this suggests that a tilt twist technique was used.

THE STABILITY OF NON-TWISTING SOMERSAULTS

Hinrichs (1978) determined the principal axes and principal moments of inertia during a tucked forward somersault, a piked forward somersault and a layout backward somersault. In the tucked and piked somersaults the principal axis corresponding to maximum moment of inertia remained close to the angular momentum vector, whilst for the layout somersault the intermediate principal axis remained close to the angular momentum vector (Figures 20, 21, 22). Since rotations of a rigid body about the intermediate principal axis are unstable, the apparent stability of the layout somersault suggested that the performer must have made corrective movements during flight.

Whilst few, if any, of the results of the above cinematographic studies can be regarded as conclusive, it does appear that the direct and tilt twist methods are frequently used whilst the contribution
Figure 19. Full twisting dismount from rings (Al-Haroun, 1980)

Figures 20, 21, 22. Paths of lateral principal axes during tucked, piked and layout somersaults (Hinrichs, 1978)
of counter-rotation twist seems less certain. If the capabilities of each twisting technique could be established then it may be possible to interpret the information obtained from film with some degree of confidence.

EXPERIMENTAL STUDIES

In an experimental study it is necessary to have a measure of control over the performance. Such control may take the form of an instruction to the subject to perform a movement in a certain way or may involve physical restraints which restrict freedom of movement. It should be noted that, as the constraints on the subject increase, the performance is less likely to be representative of a free movement.

Whilst an experimental study may show that a certain movement is possible, caution should be used when interpreting the results of an experiment in which subjects have little success, especially when the subjects are of moderate gymnastic ability.

DIRECT TWIST

Moorse (1951) studied full twisting backward somersaults on trampoline. The performer's freedom of movement was restricted using plaster casts, splints and a head brace. As more parts of the body were immobilised the performer twisted further whilst in the bed, used a more transverse arm sweep and brought the arms closer to the body. It was found that the full twisting backward somersault could be executed, even when the body was almost entirely immobilised, provided the subject initiated the twist from the bed.

COUNTER-ROTATION TWIST

In the study of Lanoue (1936) several experienced divers dropped from various heights and attempted to twist. They had very little success in twisting unless they were aided by an initial impulse.
Subsequently, however, Lanoue (1940) wrote: 'Basically all twist dives are executed by making an angle greater or less than 180 degrees by bending the body out of a straight line position into either a pike or an arch ... The arms by themselves are incapable of producing rotation, as may be proved by immobilising the body with splints to eliminate flexion or extension at the hips and then trying to do twists'.

Beauchamp (1968) attempted to refute this last statement of Lanoue. A subject performed straight jumps on a trampoline and a command was given while the subject was airborne. Upon hearing the command the subject circled one arm through 720 degrees about a vertical axis and this resulted in 70 degrees of twist in the opposite direction whereby it was concluded that the arms alone are capable of producing twist. Since the smallest amount of twist recognised in diving is 180 degrees, the conflicting conclusions of Lanoue and Beauchamp may be just different interpretations of similar experimental results.

In experiments on trampoline Stepantsov, Yeremin and Alekperov (1966) found that 70 degrees of twist could be produced using one arm whilst 130 degrees of twist could be produced using both arms.

In the study of Hay (1957) subjects jumped from a box with their feet tied together and looked over a rug to observe a hand signal indicating the required direction of twist. The maximum twist achieved was 90 degrees.

When cats were dropped in an inverted position by Frederick (1971) from a height of four feet, they easily completed a half twist in midair. The experiment was repeated with human subjects by dropping them from the one-metre diving board but most of the subjects did not twist at all and none achieved a half twist.

In the inverted cat dropping experiments of McDonald (1960) a cat in which the whole inner ear mechanism was missing performed a half twist to land on its feet but made no attempt to turn over when it was blindfolded and dropped. Other blindfolded cats were successful in righting themselves but one cat was accidentally
dropped in an upright position and promptly turned over and landed on its back.

Subsequently McDonald (1961) carried out experiments with the Olympic diver Brian Phelps who jumped from the one-metre spring-board and whilst airborne was instructed to twist to the left or to the right. The maximum twist obtained was $\frac{1}{4}$ revolutions and it took about 0.3 seconds for each half twist. Figure 23 shows that one cycle of hula movement occurred during a half twist.

In another experiment the cat drop was mimicked. The subject hung by his hands and feet from the three-metre diving board then on command he dropped off, turned over and entered the water. Figure 24 shows a hula cycle in which the body moved from flexing forwards to flexing left, then backwards, then right and finally forwards again. Untrained subjects who tried the twisting experiments were unsuccessful.

Other experiments have shown that a cycle of hula movement is capable of producing a half twist whilst the subject is airborne (Hay, 1978) or hanging from a ring (Eaves, 1960b; Biesterfeldt, 1974a) or standing on a turntable (Homma, Kosa and Haruyama, 1966) as shown in Figure 25 although Harper (1966) found it impossible to twist on a turntable.

Rackham (1970) described an experiment which introduced twist into an inward dive straight: 'Divers were asked to execute an inward dive straight and place one arm in the customary manner above the head on entry but the other arm by the side. No mention of twist entered the instructions at any time. The results varied but with most divers this arm action at entry produced a rapid half-twist. The look of amazement on their faces as they surfaced was obvious proof that they were taken completely by surprise'.

In order to ensure that direct twist was not used, Bartee (1977) required that divers took off from a swivel board placed on the seven
Figure 23. Hula cycle during half twist jump (McDonald, 1961)

Figure 24. Hula cycle during cat twist (McDonald, 1961)
Figure 25. Half twist on turntable (Honma, Kosa & Haruyama, 1966)

Figure 26. Layout back somersault with twist (Bartee, 1977)
metre diving platform. 'If the twisting action was attempted before
the diver was free of support, the board would spin and his attempt
would be aborted and another attempt would be made'. All three
divers were successful in producing twists during the forward dive
straight, forward somersault piked and backward somersault straight
without disturbing the swivel board. The maximum twist produced
was $\frac{1}{4}$ revolutions.

Bartee obtained the following results:

(a) A skilled diver can twist once free of support
without outside force
(b) Greater transverse speeds are more effective for
greater twisting speeds
(c) Greater twisting speeds occur when prior transverse
rotation is established from a piked position rather
than a layout position

and concluded that skilled divers can transfer rotation from the
transverse axis to the longitudinal axis (Bartee and Dowell, 1982).

Although it was stated that the arms are necessary for the
initiation of effective twisting, the asymmetrical arm action used
by each diver when twisting from a layout backward somersault
(Figure 26) had the opposite direction to that indicated by theory
and so some other technique must have been responsible for producing
the required tilt.

Thayer (1980) studied the use of two arm actions used in
full twisting forward somersaults from the one-metre springboard.
The asymmetrical arm action was described as high or low according
to whether the lower arm lay across the chest or abdomen. One of
the two divers studied obtained a faster twist using the high arm
action whilst the other showed no difference in twisting speed when
using the two techniques.
In 1958 the U.S.A. launched its first satellite. This satellite, named Explorer I, was an elongated, inertially axi-symmetric body designed to spin about its symmetry axis which was the principal axis of least moment of inertia. Although a rigid body should spin stably about such an axis, the initial spinning motion of Explorer I was transformed into a coning motion with a half angle of about 60 degrees after one orbit (Likins, 1973, p.484). The reason for this lay in the flexibility of the satellite antennae and the resulting energy dissipation. Since no physical body is truly rigid, it might be expected that rotations about the principal axis of minimum moment of inertia are unstable in general.

Whitsett (1963, p.24) described an experiment conducted under weightless conditions which were achieved for periods of up to 30 seconds in a jet transport plane flying parabolic trajectories. In order to demonstrate the instability of a non-rigid body rotating about the axis of minimum moment of inertia, a subject was spun about the longitudinal axis by means of a rope wound around the waist. In the straight position twist rates of up to two revolutions per second were achieved and appeared stable for the impact-free periods of between five and eight seconds. Thus it seems that such rotations are in fact stable over a short time interval.

It may be concluded from these experimental studies that twist can be produced solely by the direct method (Moorse, 1951), the counter-rotation method (McDonald, 1961) and the tilt method (Bartee, 1977). There is some evidence that the arms do not contribute greatly to counter-rotation twists (Beauchamp, 1968) and the comments concerning arm action in the review of Bartee's study indicate that it is possible to produce tilt twist without using the arms.

THEORETICAL STUDIES

Some caution should be exercised when interpreting results obtained from a mathematical model since the level of accuracy is
often unknown. Where similar results are obtained using different approaches there can be a measure of confidence in the results, although the simplifying assumptions made in a mathematical model can only be shown to be justified by validating the model using data taken from actual performances.

**TORQUE-FREE MOTION OF A RIGID BODY**

During phases of twisting somersaults the body often appears to maintain a fixed configuration and so a rigid body should be a reasonable model for these phases. When the body is straight with the arms held close, it may be approximated by a cylinder with principal moments of inertia $A, A, C$ where $A > C$.

Travis (1968) used Euler's equations of motion to obtain:

$$p = \Omega \frac{(A/C - 1)}\sin \theta$$

where $p$ is the twist rate, $\Omega$ the somersault rate and $\theta$ the angle of tilt.

Frolich (1979) used a vector diagram to obtain:

$$\omega_t = \omega_s \frac{(A/C)\sin \theta}{\omega_s}$$

where $\omega_t$ is the twist rate, $\omega_s$ the somersault rate and $\theta$ the angle of tilt.

Eaves (1969, p.76) also stated an equation equivalent to:

$$\omega_3 = \omega_1 \frac{(A/C)\tan \theta}{\omega_1}$$

where $\omega_3$ is the twist angular velocity, $\omega_1$ the somersault angular velocity and $\theta$ the angle of tilt.

Whilst these three equations do differ they all yield similar values for the number of twists per somersault when $A/C$ is large and $\theta$ is small.

On the other hand Eaves (1971) used a Poinsot construction to obtain a relation equivalent to:
where $p$ is the twist rate, $\Omega$ the somersault rate and $\theta$ the angle of tilt.

This equation gives much lower values for the number of twists per somersault than those obtained using the previous three equations and is suspect since the moments of inertia $A$ and $C$ do not appear.

For a general rigid body with unequal principal moments of inertia Synge and Griffith (1959, pp.377-380) showed that there are two distinct modes of motion. In the first mode the twist steadily increases, whilst in the second the twist angle oscillates.

These two modes are evident in the study of Grantham (1961) which simulated the effects of shifting masses within a rotating space station. A steady somersaulting motion about a principal axis was disturbed by a change in the inertia tensor and the subsequent motion was either a twisting somersault or a somersault with wobble.

Nazarov (1978) used the construction of Poinsot (described in Synge and Griffith, 1959, pp.375-377) to describe the two modes of motion and noted that, when a sportsman moves from a layout position to a deep pike, a twisting somersault will become a somersault without twist. Thus there is some theoretical basis for the contention that piking can stop the twist.

**COUNTER-ROTATION TWIST**

Three segment orientation models have shown that one cycle of arm movements can produce 29 degrees of twist (Riddle and Kane, 1968, p.63) or 33 degrees of twist (Kane and Scher, 1970). In the two segment models of Byers, Pocklington and Eaves (1951); Stepantsov, Yeremin and Alekperov (1966); and Beauchamp (1968, p.35) the angular momentum is calculated about the shoulder centre. Since the shoulder centre moves relative to the mass centre of the system, the total angular momentum about the shoulder centre is not constant so that
these analyses cannot be regarded as valid.

Kulwicki et al. (1962) modelled the Cat Reflex where the torso is twisted relative to the legs with arms adducted and legs abducted and then untwisted with arms abducted and legs adducted. It was found that an initial torsion angle of 90 degrees lead to 54 degrees of twist.

The hula movement of a cat was simulated by Kane and Scher (1969) using a model which comprised two equal cylinders and permitted varying amounts of spinal flexion so that forward bending could be greater than backward bending. For an average flexion angle of 70 degrees one hula cycle produced a half twist.

A similar model was used by Kosa (1968) to calculate the twist resulting from one hula cycle of the human body. He found that 40 degrees of flexion gave rise to a half twist whilst full flexion gave a theoretical limit of one twist, although it should be noted that it is anatomically impossible to perform a hula cycle with the body fully flexed.

Subsequently Kosa and Kamimura (1972) derived an equation of motion for the hula motion of two unequal cylinders. They implicitly assumed that the line joining the two mass centres is a principal axis of the system. Since this is not true in general their equation is exact for a smaller class of two segment systems than they supposed. Using the equation the total momentum of a trampoline performance was evaluated at seven different times and found to be approximately zero throughout. Each hula cycle gave rise to half a twist.

Eaves (1969, p.107) calculated the momenta of the upper and lower body, during leg circling, about the longitudinal axis of the upper body. Since this axis is not at rest relative to the mass centre of the system, the total momentum about the axis is not constant and so the analysis is invalid.

Stepantsov et alii (1966) considered successive rotations about the longitudinal axes of the trunk and the legs. They argued that since the moment of inertia of the legs is about 8 times that of the
trunk about the longitudinal axis of the trunk, the trunk would rotate through an angle 8 times greater than that of the legs. Again such an analysis is incorrect since it implicitly assumes that the axis under consideration is fixed in space. Thus the popular relative moments of inertia explanation of the cat twist fails to translate into a valid quantifiable description of the motion.

Gluck (1979, 1982) reported: 'Research done in both Russia and the U.S. related to the space programs indicate that one complete hula-hoop action is required to make each full twist if the body is inclined or bent in the range of 20-30 degrees off the vertical'. Of the above studies only the invalid analysis of Stepantsov supports this statement.

In summary it may be concluded that a cycle of hula movement is capable of producing a half twist and a cycle of arm movement can produce 30 degrees of twist.

**TILT TWIST**

Jutsum and Barrow (1960) showed that if a contortion of the body into an asymmetrical rigid posture changed the directions of the principal axes then a plain somersault would be converted into a sustained twisting somersault.

Travis (1968) used his equation of motion for an axially symmetric rigid body to show that if the ratio of the principal moments of inertia is assumed to be 14:1 then 18 degrees of tilt are required to produce 4 twists per somersault. If it can be shown that asymmetrical arm movements are capable of producing such a tilt angle, it may be concluded that the technique is viable.

The reorientation model of Passerello and Huston (1971) showed that the abduction of one arm through 90 degrees followed by 135 degrees of flexion at the elbow produced 4 degrees of tilt. Since the moments of inertia about the lateral and frontal axes are similar, it is reasonable to expect that the amount of somersault produced by a half circle of both arms will be comparable to the
amount of tilt produced when one arm is abducted through 180 degrees and the other is adducted through 180°. For a half circle of both arms with a straight body Passerello and Huston (1971) obtained 12 degrees of somersault whereas Kane and Scher (1970) obtained 6 degrees of somersault using a more restricted arm movement. Since more rotation will be produced when the body is tucked these results indicate that asymmetrical arm movements are capable of producing tilt angles of the order of magnitude needed for a rapid twist.

Scher and Kane (1969) used a two segment model to show that it is theoretically possible to convert a pure twist into a pure somersault by repeatedly abducting and adducting one arm through 135 degrees. Since it required eight such cycles of arm movement to produce the necessary 90 degree change in the tilt angle it may be concluded that the abduction of one arm through 135 degrees is capable of producing $5\frac{1}{2}$ degrees of tilt during a somersault.

In order to show that a full twist could be introduced into a plain dive by means of asymmetrical arm movements, Pike (1980) used eight simulations of a five segment model. Although none of the simulations achieved a full twist, one produced 0.94 twists with 8 degrees of tilt after 0.46 somersaults and it was concluded that a suitable combination of initial conditions and arm movements would lead to a full twist with a final tilt angle of zero.

Liu (1984) explained how twist will result from raising an arm during a somersault by considering the Coriolis force on the arm. He defined a 'Coriolis Index' which was proportional to the moment of the Coriolis force about the longitudinal axis and used this index to identify the direction of twist for movements of each arm in each quadrant of twist. However as the arm is raised parallel to the longitudinal axis, the remainder of the body moves in the opposite direction relative to the mass centre of the system and so there will be an equal and opposite Coriolis force acting at the mass centre of the remainder of the body. It is the combination of these two forces that gives the Coriolis couple. In addition Liu assumed that his single Coriolis force acted at the shoulder rather than at the mass centre of the arm so that his Coriolis index did not reflect
the greater Coriolis couple resulting from a wide arm action.

The index was evaluated throughout an initial phase of a full twisting \(\frac{1}{2}\) somersault dive and it was concluded that the arm movement could be improved to produce a faster twist.

The six segment model of Van Gheluwe (1981b) was validated using film data of three performances of somersaults with full twist to define the relative segmental movements and comparing the simulated values of somersault, tilt and twist with the corresponding film values. Two modifications of each performance were obtained by restricting movement at the hips and by restricting the arm movements. The resulting simulations showed that the removal of hula movement by fixing the hips reduced the amounts of twist only slightly whereas fixing the arms resulted in little twist. It may be concluded that the twist in the three performances was produced primarily by the asymmetrical arm action during flight. Whilst this result suggests that asymmetrical arm action is more efficient than hula movement in producing twist it may be that the two performers did not make full use of the hula technique and other performers may use hip movement in preference to arm movement. As a consequence Van Gheluwe's conclusion that arm movement is more efficient than hip movement should be viewed with reserve.

**INSTABILITY OF THE LAYOUT SOMERSAULT**

Nigg (1974) investigated the stability of a rigid body rotating about a principal axis. He determined the direction of the angular momentum vector in the body by considering the intersection of the energy ellipsoid with the momentum spheroid. Rotations about the principal axes corresponding to maximum and minimum inertias were found to be stable so that if the instantaneous axis was initially close to such a principal axis it would remain close throughout the motion. Rotation about the median principal axis was found to be unstable. Nigg concluded that the layout somersault was unstable and suggested that the arms be extended laterally to minimise the influence of the instability. Whilst it is true that extending the
arms will reduce the twist rate, the problem of instability remains and this method can at best only delay the inevitable build up of twist.

Hinrichs (1978) used Euler's equations of motion for a rigid body to establish the stability of rotations about the maximum and minimum principal axes and the instability of rotations about the median principal axis. The analysis followed closely that of Marion (1965, pp. 407-410). Hinrichs hypothesised that in the layout somersault the performer makes small corrective movements to prevent the build up of twist.

It might be argued that, although a back somersault in the straight position is theoretically unstable, in practice the body is not straight at the beginning and end of the movement so that the instability has insufficient time to produce noticeable twist. When double and triple somersaults in the straight position are considered this argument loses much of its force. The issue might be resolved, however, if it could be shown that a small asymmetry of body position leads to a quarter twist after a certain number of somersaults.

**SUMMARY OF RESEARCH FINDINGS**

Cinematographical investigations have been used to determine the techniques responsible for the production of twist in particular movements in diving, trampolining and gymnastics. Without an accompanying quantitative mechanical analysis such investigations are incapable of fulfilling this aim. As a consequence the conclusions reached owe much to the investigators' personal opinions and whilst such conclusions may indeed be true they have not been demonstrated to be so. Although none of the studies can be regarded as conclusive it does appear that both direct and tilt twist techniques are in use.

An experimental study can show what is possible but cannot demonstrate that something is impossible. Thus the lack of success experienced by Lanoue (1936), Hay (1957) and Frederick (1971) in producing counter-rotation twist merely demonstrates that the chosen subjects were unsuccessful. The success of the experiments of
McDonald (1961) with the diver Brian Phelps showed that it was possible to produce a full twist in a jump from the one-metre springboard. It is for this reason that the experiments of Beauchamp (1968) and Stepantsov et al. (1966), in which arm movements produced little twist, should not be regarded as conclusive. Bartee (1977) demonstrated that it is possible to produce a full twist in forward and backward somersaults without taking twist from the board.

Before a mathematical model can be used with confidence the model must be shown to be a valid representation of the system under consideration. Of the many mathematical models used in theoretical studies of twisting and reorientation only two were validated using data taken from actual performances. Because of this only two results may be stated with confidence.

(1) One cycle of hula movement is capable of producing a half twist (Kosa and Kamimura, 1972)

(2) Asymmetrical arm movement is capable of producing a full twist in a backward somersault (Van Gheluwe, 1981b).

TECHNIQUES OF INVESTIGATION

This section provides a background to the development of the simulation model which occurs in Chapters 3, 4 and 5. Various techniques in the areas of inertia parameters, film analysis, curve fitting and simulation models are described and compared.

DETERMINATION OF SEGMENTAL INERTIA PARAMETERS

Any quantitative mechanical analysis of human movement requires values for the inertia parameters of the body segments. A general three-dimensional simulation model will need a complete set of segmental inertia parameters. For each segment the mass, location of mass centre, inertia tensor and location of the joint centres must be specified.
The various techniques which have been used for obtaining segmental inertia values are grouped under three headings.

1. Experimental methods for segments in situ
2. Cadaver studies and linear regression analyses
3. Mathematical inertia models.

EXPERIMENTAL METHODS FOR SEGMENTS IN SITU

The only method of segmental mass determination which could be described as direct is the gamma radiation technique described by Brooks and Jacobs (1975). Gamma radiation from a cobalt-60 source was passed through legs of lamb in quarter inch square sections. The radiation readings, which are indicative of the electron density, together with a scaling factor based on the ratio of electrons to protons and neutrons were used to calculate the mass of each section. The estimation of total mass by the radiation technique differed from the mass values obtained by weighing by less than 1%.

Segment volumes have been determined by water displacement (Dempster, 1955; Contini, Drillis and Bluestein, 1963). Jones and Pearson (1969) stated their error as 50ml which is not as good as the 0.5% given by Clauser, McConville and Young (1969) for volumes of cadaver segments. For segments in situ, however, there is the problem of choosing the two planes which define the segment. Katch, Michael and Amuchie (1973) weighed the displaced water and calculated the volume using the density of water at the recorded temperature. The standard error for a single measurement was stated as 1.5ml but this is in conflict with the value 30ml given by Katch and Weltman (1975) when referring to the same study. Cleaveland (1955) used the Principle of Archimedes to obtain the weight of displaced water as the change in the weight of a subject when the segment is immersed in water.

Accuracy in the calculations of mass from volume measurements is dependent upon the assumed value for segment density. The
variation in the density of a particular segment in the study of cadaver segments by Chandler, Clauser, McConville, Reynolds and Young (1975) is around 2%. When the cadaver studies of Dempster (1955) and Clauser et al. (1969) are included, this figure rises to about 5%.

Katch et al. (1973) obtained leg weight by suspending the limb from its mass centre using a spring balance. Errors can result from muscle torques at the hip in addition to incorrect positioning of the suspension system.

The method of reaction change (Hay, 1973) can determine either the mass or the mass centre location of a segment providing the other is known. The subject is placed on a board supported by a fixed base at one end and a weighing scale at the other. Scale readings are taken for two different orientations of the segment and the principle of moments is employed. Hay noted that several studies have extended the method using larger boards and two or more sets of scales. Such techniques determine two planes passing through the mass centre, given a knowledge of the segmental mass. For a body comprising n segments the specification of (n-1) masses or (n-1) mass centre locations is sufficient for the determination of the remaining (n+1) parameters. Ji-Chun (1983) showed that an arbitrary choice of mass values led to a set of mass centre locations which produced the correct whole body mass centre for any orientation of the segments.

Dainis (1980) defined the 'structure parameters' of an n-link system in terms of the segmental masses and mass centre positions and showed that the whole body mass centre can be determined from the n structure parameters. He demonstrated that if the coordinates of the system mass centre can be expressed as polynomial functions of time with unknown coefficients, then a least squares technique may be used to find the structure parameters and the coefficients from the observed motion of the segmental endpoints. Once the structure parameters have been found then a knowledge of either the segmental masses or mass centre locations permits the calculation of the other set.
The method was applied using a 3-link representation of a vault and a 6-link representation of a tucked somersault. Errors in the calculated acceleration values of the system mass centre were less than 0.1 ms$^{-2}$ whilst independent calculations based upon the segmental parameters of Dempster (1955) lead to errors of up to 1.2 ms$^{-2}$. Segmental mass centre locations were calculated using Dempster's segmental mass ratios and the determined structure parameters. The mass centre locations of the larger segments were comparable with positions given by Dempster whilst those of the feet and forearms lay outside their segments. The accuracy of the acceleration values would seem to indicate that the errors in mass centre location result from the use of incorrect segmental masses rather than from inadequate film data.

Hay (1973) observed: 'Bernstein... concluded that... the centre of mass of a body segment can be considered coincident with the centre of volume'. Clauser et al. (1969), on the other hand, have shown that if the mid-volume were to be used to approximate the location of the centre of mass of body segments, the estimated centre of mass would be proximal to its true location.

This apparent discrepancy is easily explained. Contini (1972) defined centre of volume in terms of volume moments. The description of Bernstein's method in Drillis, Contini and Bluestein (1964) indicates that Bernstein used this definition of centre of volume. Bernstein's conclusion that volume centres and mass centres of cadaver segments are practically coincident implies that the density along a segment may assumed to be constant for the purpose of calculating mass centres. On the other hand Clauser et al. (1969, p.9) take centre of volume to mean mid-volume level which will be coincident with the mid-mass level for a segment with constant density. Since the mid-mass level will coincide with the mass centre of a segment only when the mass centres of the two parts are equidistant from the mid-mass level, it is only to be expected that the mid-mass level will be nearer the proximal end of a body segment than the mass centre is.

Drillis et al. (1964) described the compound pendulum method of
Nubar (1960) by which the effective point of suspension, the mass centre and the moment of inertia of a segment may be determined. If damping moments are neglected the three unknowns may be calculated from three time periods of oscillation for which different masses are attached to the segment. The inclusion of a viscous moment in the analysis necessitates the measurement of the amplitude decrement in successive oscillations.

Contini et al. (1963) described how the compound pendulum method may also be used with plaster casts of the segments. This technique assumes that the density of a body segment may be taken to be constant.

A torsion pendulum technique was used by Tichonov (1976) to obtain moments of inertia of the arms and legs about their lateral axes. Two periods of oscillation were determined. In the first the limb was located along the axis of the pendulum. Subsequently the limb was raised to a horizontal position while the remainder of the body did not change position. The moment of inertia of the limb was taken to be the difference in the moments of inertia of the two systems. Since this calculation implicitly assumes the moment of inertia about the longitudinal axis is negligible, Tichonov's stated error of 2% is optimistic.

Drillis et al. (1964) showed that, providing the segmental axis on which the mass centre lies can be clearly positioned, the torsion table may be used to find both the mass and mass centre of a segment.

The quick release method was used by Bouisset and Pertuzon (1968) to determine the moment of inertia of the forearm-hand segment about the elbow joint. Torque and angular acceleration were calculated from measurements of force and linear acceleration and the moment of inertia was evaluated as the quotient. Hatze (1975) noted that the method neglected damping components and changes in the muscle torque.

A passive oscillation method was developed by Hatze (1975) to determine the mass centre, moment of inertia and damping coefficient of a segment. The limb was suspended from a spring arrangement and performed oscillations about a horizontal position. Using the time
period and amplitude decrement the moment of inertia and damping coefficient can be calculated whilst the expression given for mass centre location assumes that segmental mass is known. Hatze suggested that the method could be extended in the form of a torsion pendulum so that moments of inertia of segments about their longitudinal axes could be obtained.

Vaughan and Andrews (1981) used a least squares technique, similar in principle to that of Dainis (1980), to minimise the difference between calculated and measured distal extremity kinetics. The segmental parameters obtained in this way were found to be different from values derived using the regression equations of cadaver studies.

Of all the experimental studies described, none provides a method for finding the inertia parameters of central segments such as the pelvis and only the suggestion of Hatze provides the possibility of determining segmental inertias about longitudinal axes.

CADAVER STUDIES AND LINEAR REGRESSION ANALYSES

Barter (1957) used the data of three dissection studies, involving 12 cadavers, to express segmental weight as a linear function of total body weight. He believed that the equations would provide better estimates of segment weights than estimates obtained using mean ratio values. Clauser et al. (1969) obtained similar equations for certain segments of 13 cadavers whilst Chandler et al. (1975) derived linear regression equations for the segmental weights of 6 cadavers. A comparison of the equations for upper arm weight reveals that the slopes range from 0.016 to 0.040 whereas the average ratios of segmental weight to body weight range from 0.026 to 0.029. This indicates that such regression equations are likely to be less accurate than mean ratios when used outside the small sample from which the equations were derived.
Clauser et al. (1969) also used multiple linear regression equations to express segment weight as a linear combination of incommensurate measures such as body weight, segment length and circumference. Such a procedure is even more likely to produce poor estimates when used outside the original sample. A better procedure may be to consider the dimensions of the quantities involved which suggest that segment weight is proportional to the product of segment length and the square of the circumference.

Contini (1972) reported the distance of the mass centre of a segment from the proximal joint as a ratio of segment length for each of five dissection studies. Hay (1973) presented the ratios of Clauser et al. (1969) while Chandler et al. (1975) provided yet another set. The ratios lie between 0.35 and 0.50 for the various body segments with standard deviations of around 0.03 which is equivalent to 0.07 of the distance from mass centre to proximal joint.

Chandler et al. (1975) noted that previous determinations of segmental moments of inertia had assumed that the axes used were principal axes. A compound pendulum technique was used to determine the moments of inertia of each segment about six axes so that principal moments and the directions of principal axes could be found. In general the longitudinal principal axis of a segment was found to lie within 10 degrees of the line joining joint centres whilst the other two principal axes, which corresponded to the larger principal moments of inertia, exhibited considerable variation in direction. For limb segments this may be explained by the fact that the two larger principal moments were close in value. The principal moments of inertia of the six torsos are given in Table 1.

Since the moments of inertia about three mutually orthogonal axes $x, y, z$ may be defined as:

$$I_{xx} = \sum m(y^2 + z^2), \quad I_{yy} = \sum m(x^2 + z^2), \quad I_{zz} = \sum m(x^2 + y^2)$$

where $m$ is an elemental mass situated at the point $(x,y,z)$ it is clear that:

$$I_{xx} \leq I_{yy} + I_{zz}$$
Table 1. Principal moments of inertia of six torsos (Chandler, Clauser & McConville, 1975)

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>Ixx</th>
<th>Iyy</th>
<th>Izz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.44</td>
<td>0.93</td>
<td>0.26</td>
</tr>
<tr>
<td>2</td>
<td>2.04</td>
<td>1.43</td>
<td>0.50</td>
</tr>
<tr>
<td>3</td>
<td>2.31</td>
<td>1.81</td>
<td>0.62</td>
</tr>
<tr>
<td>4</td>
<td>1.36</td>
<td>0.90</td>
<td>0.23</td>
</tr>
<tr>
<td>5</td>
<td>1.25</td>
<td>0.66</td>
<td>0.30</td>
</tr>
<tr>
<td>6</td>
<td>1.31</td>
<td>0.79</td>
<td>0.35</td>
</tr>
</tbody>
</table>
with equality occurring only when the mass distribution lies in the plane \( x=0 \).

Table 1 shows that only torso number 3 satisfies this inequality. It may be concluded that the errors in the inertia values are substantial.

In the study of Santschi, DuBois and Omoto (1963) two thirds of the whole body inertia triads of the 66 subjects also fail the inequality. This indicates that there is a systematic error in the compound pendulum technique.

Santschi et al. (1963) produced multiple linear regression equations of whole body moments of inertia against stature and weight, ignoring the fact that the dimension of moment of inertia is mass times length squared.

This fact was taken into account by Widule (1976) who scaled the transverse segmental moments of inertia obtained by Dempster (1955) using the factor \( wh^2 \) for body weight \( w \) and stature \( h \). The average values obtained could then be used to provide personalised segmental moments of inertia using the height and weight of a subject. This method was extended by Dapena (1979) who assumed that the square of the radius was proportional to \( w/h \) so that the scaling factor \( w^2/h \) could be used for longitudinal moments of inertia.

Whilst admittedly crude, the procedure of Widule and Dapena may be better than poorly constructed regression equations in the same way that average percentage values of body weight are superior to Barter's equations for estimating segmental weights. However the fact that body dimensions are poorly correlated (Daniels, 1952) means that the attempt to obtain personalised segmental parameters from the height and weight of a subject is unlikely to be successful.

**MATHEMATICAL INERTIA MODELS**

The human body may be modelled as a system of rigid segments of simple geometric form linked at joint centres. The dimensions of
the geometric forms may be conveniently determined from anthropometric measurements of an individual so that, providing segmental density distribution can be modelled, the segmental inertia parameters may be evaluated.

However segments of the body are not linked at single points and the divisions between segments are not well-defined. Segments change their shape and mass distribution when muscles contract and the instantaneous centres of rotation move relative to the linked segments. As Whitsett (1963) noted: 'man is a non-symmetrical, fluid-filled sack of variable shape containing a large air bubble'. Thus there are inherent limitations in any such analytical representation.

Skerlj (1954) calculated segmental volumes from perimeter and length measurements using the formula \( v = kp^2l \) with \( k = 0.075 \). Since the volume of a circular cylinder of length \( l \) and perimeter \( p \) is given by \( v = kp^2l \) with \( k = 1/4\pi = 0.0796 \) it is clear that Skerlj had introduced a correction factor to allow for the fact that a circle encloses a greater area than other curves of equal perimeter. Clauser et al. (1969) incorrectly stated \( k \) as 0.75 which they interpreted as an approximation to \( \pi/4 \).

Contini (1972) determined correction factors for each segment 'by applying the equation to five subjects for whom the volume of the various body segments was known'.

Katch, Weltman and Gold (1974) modelled each segment by a number of truncated circular cones. Whilst the calculated volumes of limbs were in close agreement with the values obtained by water displacement the volumes of a mid-torso section were overestimated by 30%. This is only to be expected since a circle is a poor representation of a torso cross-section.

Whitsett (1963) used truncated cones, elliptical cylinders, ellipsoids and rectangular blocks to model body segments, each segment being represented by a single geometric solid (Figure 27). An average man was defined by the mean of the anthropometric measurements given by Hertzberg, Daniels and Churchill (1954).
Figure 27. The inertia model of Whitsett (1963)

Figure 28. The inertia model of Hanavan (1964)
Daniels (1952) showed that if a subject was regarded as average for a particular anthropometric measurement when his measurement lay between the 35th and 65th percentiles then not one of the 4063 subjects in the study of Hertzberg et al. was average in all of 10 selected linear measurements. Thus the idea that an average man is representative of a population with respect to a number of characteristics is unfounded.

Whitsett used the regression equations of Barter (1957) to obtain segmental weights and defined the locations of mass centres using the data of Dempster (1955). Some of the dimensions of the segmental solids were defined using the average man data whilst others were calculated using segmental density values. Segmental and whole body moments of inertia were calculated about three orthogonal axes.

Hanavan (1964) produced a 15 segment model of the human body similar in many respects to Whitsett's 14 segment model (Figure 28). Whilst Barter's equations were again used to obtain segment weights from body weight, the dimensions of the solids were all obtained from anthropometric measurements. This enabled density values to be calculated and compared with those of Dempster (1955). The calculated densities varied widely from 0.72 to 2.14 presumably as a result of poor volume estimation since the standard errors of segment weights as given by Barter's equations are not of this order of magnitude. Whilst average mass centre locations were within 15% of Dempster's average values, individual locations differed from Dempster's averages by up to 30% of the distance from the proximal joint. These large variations in segmental densities and mass centre locations are much greater than the variation found by Dempster (1955) and although Hanavan considered the agreement to be good the evidence clearly favours the procedures of Whitsett.

Hanavan calculated whole body principal moments of inertia and compared them with the data of Santschi et al. (1963). The quartile error values were less than 25%. Whole body moments of inertia of Hanavan's median man and Whitsett's average man in the standing position show a difference of 5%.
The Hanavan model was modified by Tiebner and Lindemuth (1965) using revised regression equations for segmental weights and a hollow torso segment. The errors were reduced slightly.

Miller and Morrison (1975) used the original Hanavan model and a modified version based on the regression equations of Clauser et al. (1969) to calculate the segmental weights, densities and moments of inertia of 30 male athletes. Their expectation that Clauser's regression equations would provide better estimates of segmental weights could not be demonstrated since the only criterion available was segmental density which the Hanavan model estimates badly due to the poor modelling of segmental volumes. Failure to recognise that Hanavan included the neck with the head segment increased the error in head density. Despite the fact that Barter's equations estimated the sum of segmental weights with greater accuracy it was recommended that Clauser's equations be used since they took segment lengths and circumferences into consideration. However Clauser's statement about his multi-step regression equations being 'more accurate than simple ratios' may not be true outside Clauser's cadaver population.

The cadaver study of Chandler et al. (1975) compared the segmental mass and inertia estimates of Tiebner and Lindemuth's modified Hanavan model with experimental values. The average segmental mass deviations were less than 25% and the moments of inertia average deviations were less than 35% for all segments except the hands and feet. Whilst the agreement is not good it should be remembered that Chandler's experimental results contain large errors and in any case Hanavan constructed his model in order to evaluate whole body moments of inertia rather than segmental parameters.

Kosa and Kamimura (1972) used 18 solids to model the body. The dimensions of the solids representing the head, hands and feet were calculated from the segmental volumes obtained by water displacement. This procedure overcomes the difficulties encountered when using a simple geometric solid to model a body segment of irregular shape. No difficulties were reported regarding the determination of head volume.
Weinbach (1938) used front and side view photographs to determine segment diameters at 19 levels. Assuming that segmental cross-sections were elliptical in shape, he calculated the area at each level and used graphical techniques to obtain whole body mass, mass centre location and moment of inertia. Dempster (1955) used the method with 50 equally spaced levels and found that many of these levels could be eliminated without significant loss of information. He listed 21 levels based on body landmarks from which a realistic estimate of area distribution could be made.

Weinbach's photogrammetric method was used by Jensen (1976) to section the body into elliptical discs 2cm thick (Figure 29). He calculated the segmental inertia parameters of a child by this procedure and by Hanavan's method and concluded that there were considerable differences. Jensen (1978) found that the errors in the estimation of the body masses of three children were less than 2%.

Hatze (1979) observed that the moment of inertia of an elliptical disc about an equatorial axis was \( I = m\left(3b^2+h^2\right)/12 \) where \( m \) is the mass, \( b \) the length of a semi-axis and \( h \) the thickness. He noted that Jensen had neglected to include the \( h^2 \) term and stated that this introduced an error of 33% when \( h = b \). Although Jensen's implicit assumption that the discs are infinitely thin is unnecessary, the resulting 25% error in moment of inertia is for a single disc rather than a segment. If 12 such discs are used to represent a segment, then the application of the theorem of parallel axes with Jensen's formula gives the segmental moment of inertia as \( (3mb^2+143mh^2) \) rather than \( (3mb^2+144mh^2) \). Thus when \( h = b \) the error is less than 1% so that Hatze's criticism is misleading.

The ellipse has been used to model cross-sections of the body because it is mathematically convenient to do so. Figure 30 shows a cross-section of the thorax obtained by Cornelis, Van Gheluwe, Nyssen and Van Den Berghe (1978). It would appear that the rounded-off rectangle of Sady, Freedson, Katch and Reynolds (1978) in Figure 31 bears more resemblance to the torso cross-section than an ellipse of equal width and depth (Figure 32).
Figure 29. The elliptical zone model of Jensen (1978)
Figure 30. Cross-section of thorax (Cornelis et al., 1978)

Figure 31. Rounded-off rectangle representing thorax cross-section

Figure 32. Elliptical representation of thorax cross-section
Sady et al. (1978) defined solids bounded by two such rounded-off rectangles of different dimensions. Such a procedure should produce more accurate results than the use of cylindrical solids since the cross-section of each solid is permitted to vary. However Sady used a formula based on the volume of a frustum of a pyramid and this gives the correct volume of the solid only when the bounding curves have the same ratio of width to depth.

Herron, Cuzzi and Hugg (1976) obtained detailed models of the cadaver segments of Chandler et al. (1975) using three stereocameras (Figure 33). The segmental inertia parameters were calculated using many sided polygons to approximate each cross-section and assuming that segments were of uniform density.

McConville and Clauser (1976) compared the segmental parameters calculated by Herron et al, with the experimental values obtained by Chandler et al. They were unable to explain why a constant density value of 1.0 produced better estimates of segmental masses than values obtained using the segmental densities of Chandler et al. Since the use of the cadaver average densities systematically overestimated the segment weights, it is clear that the stereometric technique systematically overestimated the segment volumes. On the other hand torso volumes were systematically underestimated suggesting that the volumes of body parts may have changed in the time interval separating stereophotography and the experimental determination of volume.

Herron et al. (1976) also took stereophotographs of the cadavers before segmentation took place. The bodies were analytically sectioned into 5 segments and segmental volumes and principal moments of inertia were calculated. A comparison with experimental values (pp.191-193) shows that, when the analytical planes of dissection were close to the physical dissections, the errors in volume were less than 4% and the errors in moments of inertia were less than 10% when a constant density value of 1.0 was used.

Hatze (1979, 1980) developed a very detailed model of mass distribution of the human body. Inertial parameters of the 17
Figure 33. Stereometric model of a body segment (Herron et al., 1976)

Figure 34. The inertia model of Hatze (1979)
segments were calculated from 242 anthropometric measurements made directly on the subject. Segmental volumes and moments of inertia were within 5% of experimentally determined values.

Limb segments were sectioned into 10 elliptical discs; the thorax comprised elliptical plates into which parabolic plates of low density were inserted; the head was modelled as an elliptic octoparaboloid and the shoulders were included as individual segments (Figure 34). The model took into account variations in density along a segment, the sex of the subject and the subcutaneous fat content of segments.

When describing the shoulder segments Hatze (1979, p. 40) made the curious statement that moment of inertia values about the lateral axis were not needed since no rotation takes place about this axis. These values are in fact necessary for the mechanical analysis of any movement which includes transverse rotations of the trunk.

**SUMMARY OF THE DETERMINATION OF INERTIA PARAMETERS**

Segmental inertia parameter values can be determined experimentally for living subjects by a number of methods. Such techniques are time consuming and do not appear to be capable of determining the full set of inertia parameters for all segments. Complete sets of segmental inertia parameters have been determined experimentally for dissected cadavers. These values can be personalised to some extent using regression equations or scaling techniques but it is doubtful whether such procedures will give accurate values outside the particular cadaver sample used. A mathematical model of the human body can be used to obtain the full set of segmental inertia parameters from anthropometric measurements of an individual. The time required for the measurements is much less than that involved in experimental methods and, providing the model is sufficiently detailed, the values obtained are in close agreement with experimentally determined values. Of the methods available only mathematical modelling provides a convenient means of accurately determining a complete set of segmental inertia parameters for a given individual.
TECHNIQUES OF CINEMATOGRAPHIC ANALYSIS

A quantitative mechanical analysis of human movement requires the time histories of the orientation angles which specify body configuration. In addition the equations of motion will involve the rates of change of these angles. For planar motion, angles may be determined by use of a single cine-camera or the polarised light device of Mitchelson (1973). For general three-dimensional body configurations, Noss (1967) stated that the angle between two lines may be evaluated as the average of the apparent angles given by three cameras with orthogonal optical axes. Spray (1973) and Putnam (1979) gave examples showing that the formula of Noss is incorrect. Walton (1981) commented 'the expressions for which Noss was searching do exist, but they are more complex than a simple average'. If this means that the angle between two lines may be calculated from the magnitudes of the three projections then even Walton's statement is incorrect:

For simplicity consider the case in which the three cameras are situated at infinity so that the apparent angles are the same as the orthogonal projections on the three coordinate planes. The four points O(0,0,0), A(1,1,½), B(1,1,1) and C(1,1,2) define angles BOA and BOC (Figure 35). These two angles have equal projections on each of the coordinate planes but are not equal to each other.

The orientation angles may be calculated once the spatial coordinates of the joint centres are known. This section reviews the techniques available for obtaining spatial coordinates from two or more synchronised views, for obtaining synchronous data sets and for fitting curves to data in order to obtain rates of change.

DETERMINATION OF SPATIAL COORDINATES

For planar motion a single camera may be used in conjunction with a reference frame placed in the plane of motion so that a scale factor or multiplier may be obtained to convert distances in the projected image into actual distances. Noble and Kelley (1969) obtained three-dimensional coordinates using three cameras aimed
Figure 35. The unequal angles BOA and BOC have equal projections.

Figure 36. The geometry of a simple two-camera system.
along the axes of an orthogonal reference frame. By using the multiplier technique for each camera they obtained two estimates for each coordinate of a point and used the average as their estimate. This technique was also used by Walton (1970a) but does not make any correction for perspective errors.

Miller (1970) used two cameras A, B aimed along two of the axes of an orthogonal reference frame with origin O (Figure 36). Using the multiplier technique she obtained the coordinates of the projections P1, P2 of an object point on the coordinate planes. The camera distances AO, BO were measured so that it was merely a matter of geometry to find the intersection of the rays AP1 and BP2. Miller obtained the equations of the four planes AP1S1, AP1T1, BP2S2, BP2T2 which in general define a tetrahedron but do not have a common point due to errors in locating A, B, P1 and P2. By omitting the equation of plane BP2T2 the coordinates of point Q1 were obtained using three equations in three unknowns.

Martin and Pongratz (1974a, 1974b) obtained the point Q2 by omitting the plane AP1T1 and took the midpoint of Q1Q2 to be their estimate of the object point.

Spray (1973) compared this method with the Susanka technique which is detailed in Walton (1981) and uses the midpoint of the common perpendicular R1R2 of the rays AP1 and BP2. It was found that the midpoints of Q1Q2 and R1R2 had equal z-coordinates by applying the two methods to the same data. Whilst this is not true in general it will be approximately true for large camera distances.

Walton (1981) suggested that the point which had the minimum sum of squared distances from Miller's four planes should be used as the best estimate of the object point. However his least squares formulation (pp.258-260) implicitly weights the distances and gives a different point.

It should be observed that the midpoint of the common perpendicular is itself a least squares solution in the sense that the sum of squared distances from the rays AP1 and BP2 is minimised. In addition Miller's choice of pairs of planes defining these rays is
entirely arbitrary whereas a choice of pairs of orthogonal planes would have a least squares solution coincident with the midpoint of the common perpendicular.

A system using three cameras with axes at 120° was described by Miller and Petak (1973). They provided equations similar to Miller (1970) for calculating spatial locations using partial information from two cameras. The purpose of using three cameras was to overcome the problem of the obscured landmark which occurs when a point is not in camera view.

The Susanka technique was extended by Bergemann (1974) to include arbitrary camera placement, requiring only that the optical axes intersect. This restriction was subsequently removed by Penrose, Wood and Blanksby (1976) who reported that a theodolite was essential for obtaining accurate locations of the cameras and reference points.

Van Gheluwe (1974) back-calculated camera positions from the image coordinates of a set of reference points of known location. However the method required that the camera axes intersect and the focal lengths of the lenses be measured.

Subsequently Van Gheluwe (1975) derived expressions for correcting image coordinates to allow for errors arising from camera misalignments. The expressions are not of general use since they are restricted to the correction along a single axis arising from a misalignment on that axis.

Van Gheluwe (1978) produced a generalised method whereby both camera location and orientation were calculated from the image coordinates of known reference points. The two equations relating object coordinates and image coordinates involved 9 coefficients which could be determined for each camera using 5 non-coplanar reference points. Once the coefficients had been determined the image data from each camera defined two planes which theoretically contained the object point.

The power of such a method arises from the use of least squares techniques to calculate the undetermined coefficients and object co-
ordinates. This permits accuracy to be increased by using more than 5 reference points and more than 2 cameras.

Shapiro (1978) described a similar method involving 11 undetermined coefficients which required the use of at least 6 reference points. He stated the equations connecting the image coordinates \((u,v)\) and the object coordinates \((x,y,z)\) in the form:

\[
\begin{align*}
u + c_1x + c_2y + c_3z + c_4 + c_9ux + c_{10}vy + c_{11}uz &= 0 \\
v + c_5x + c_6y + c_7z + c_8 + c_9vx + c_{10}vy + c_{11}vz &= 0
\end{align*}
\]

Van Gheluwe's equations are the same but with \(c_4 = c_8 = 0\). This condition is equivalent to the requirement that \(u = v = 0\) when \(x = y = z = 0\). In other words the origin of the image coordinate system is coincident with the image of the origin of the reference frame. This requirement is easily fulfilled either by zero adjustments on the digitising apparatus or by a simple translation of origin.

Miller, Shapiro and McLaughlin (1980) and Walton (1981) gave extensions of the method which took into account lens distortion, digitiser non-linearities and film deformations. Walton (p. 142) reported: 'the errors associated with film deformations are far less significant than those associated with improper identification of body landmarks'.

These generalised methods which permit an arbitrary arrangement of cameras without field measurements are particularly useful in small laboratories and at sporting events where constraints may be placed on camera location. However the accuracy of such techniques appears to be no better than others which require field measurements. Penrose et al. (1976) obtained an accuracy of 5mm in a field of 3 metres which was similar to the accuracy of earlier techniques. Shapiro (1978) using the method of undetermined coefficients also obtained average deviations of 5mm in a field of 3 metres for points within the reference region. However the average error in estimating the length of a metre rule was an excessive 26mm.

The reason for this is that the back-calculation of camera location is not accurate so that points some distance away from the
reference set will be poorly estimated. Both Miller et al. (1980) and Walton (1981) observed that the reference points should be distributed throughout the region of interest.

Since the manual digitising of film is time consuming and liable to human error other systems have been developed. The SELSPOT system uses light emitting diodes as body markers in conjunction with light sensitive electronic detectors whilst the CODA sweeps the field of view with a fan of light and uses retro-reflecting prisms as markers. Such systems may be able to give more accurate estimates of the locations of body markers but these markers are attached to the subject's skin which moves relative to the joint centres. The shoulder and hip centres are about 5 to 10 centimetres beneath the skin and present particular problems. On the other hand the manual digitisation of film provides the opportunity to use body markers as guidelines only and to make a subjective judgement regarding the location of the joint centre.

Once the three-dimensional locations of the joint centres have been determined, orientation angles may be calculated. The direct use of inverse trigonometric functions, as employed by Gervais and Marino (1983), is inappropriate however, since the orientation angles are obtained as discontinuous functions of time.

SYNCHRONISATION OF CINE-CAMERAS

Data sets obtained from two or more views may be combined to produce accurate estimates of spatial coordinates only when they correspond to the same instant of time. In the SELSPOT system the active markers are pulsed sequentially at 10 kHz so that the data for a particular marker are automatically time-matched, although the spatial coordinates of the various markers are obtained at different instants of the interval containing a full sequence. The CODA obtains data from three views occurring within a 3 millisecond interval, although the three planes defining a particular landmark are obtained at different times within the interval. Cine-cameras with electronically synchronised shutters produce time-matched views
A single camera used in conjunction with a mirror can produce synchronised pairs of images on each frame of film although a large mirror would be required for a recording of human movement.

Failing such physical synchronisation it is necessary to obtain time-matched data sets indirectly.

Miller (1970) used a pulse generator to put timing marks along the sides of the film in each camera. Frames were then chosen to be as close as possible to corresponding timing marks at 30 ms intervals. This rather crude technique gave data sets at times which differed from each other by less than the interval between frames. For Miller's frame rates of around 100 Hz this produced errors of up to 10 milliseconds.

A better technique is to record the time at which each frame occurs and then use mathematical interpolation to estimate data values at the required time intervals. Walton (1981) suggested using interpolation on the data obtained from the cameras with lower frame rates so that one set of original data values could be preserved. Dapena (1979) and Van Gheluwe (1979) used cubic and quintic splines to fit all data sets and then obtained interpolated values at convenient time intervals.

The time at which each frame of film is exposed may be obtained by putting a clock or timing device in the field of view of all cameras. The rotating cone of Blievernicht (1967) and the electronic unit of Walton (1970b) can give time values to within 5 ms and 1 ms respectively. If the interval between frames is around 10 ms the coarseness of the time estimates contribute to the interpolation error. Dapena (1979) overcame this by using only initial and final time values and assuming that frame rates were constant during each filmed sequence. A better procedure would be to make use of all the frame time values and fit a straight line to them assuming again that the intervals between frames remained constant. For spring driven cameras the frame rates may not remain constant so that a cubic or quintic spline, which uses an estimate of the accuracy of the time values, would be better than a linear fit.
The differential equations of motion of a human mechanical system involve the orientation angles and their rates of change. Numerical integration techniques with variable step length require the evaluation of the angles and their derivatives at arbitrary times so that finite difference methods (Lees, 1980) are inappropriate. This and the requirement that angle and derivative values should be consistent favour the use of analytic or piecewise analytic functions to approximate the data sets specifying the angles. Such curve fitting techniques permit smoothing and interpolation in addition to the evaluation of derivatives so that they can be applied to the original displacement data sets to reduce the effects of random errors and to obtain interpolated values for synchronised data sets.

Pezzack, Norman and Winter (1977) found that least squares polynomial fits produced large errors in the second derivative whilst digital filtering followed by a finite difference technique gave close agreement with accelerometer values.

Cappozzo, Leo and Pedotti (1975) determined the order of the best finite Fourier series by examining unbiased estimates of the mean square error. These estimates were in fact biased since the expression representing the number of degrees of freedom was incorrect. The computer program listed in Cappozzo and Gazzani (1983) does not suffer from this error.

Miller, Shapiro and McLaughlin (1980) applied Cappozzo's Fourier fit to data with unequal endpoint values and obtained second derivative values which were so poor that the authors chose not to state the error estimate.

This difficulty was overcome by Hatze (1981a) by subtracting a linear function from the data values so that the endpoint values were zero. The new values were approximated by a Fourier sine series and good agreement obtained with the second derivative data of Pezzack et al. (1977). Such a representation using sine functions produces zero values for the second derivatives at the endpoints so that in general the agreement will be poor at the endpoints. The
problem may be overcome by subtracting higher order polynomials from the data so that derivatives at the endpoints are equal (Lanczos, 1966).

Cubic splines are defined as piecewise cubic polynomials which satisfy certain continuity conditions at the knots defining the endpoints of the intervals. Wold (1974) gave guidelines for choosing the knot positions of least squares cubic spline approximations. The drawback of manual selection of knot positions was removed by Powell (1967) who developed an automatic knot selection algorithm. However Parker (1970) noted that there are dangers in automatic knot placement when analysing experimental data.

For a set of data points \((x_i, y_i)\) and estimates \(\delta y_i\) of the errors in the ordinates \(y_i\) Reinsch (1967,1971) obtained the cubic spline \(g(x)\) for which:

\[
\sum_{i=1}^{n} \frac{(g(x_i) - y_i)^2}{\delta y_i^2} \leq n
\]

and \(\int_{x_1}^{x_n} (g''(x))^2 \, dx\) is a minimum.

The problem of knot selection had been avoided by taking a knot at each data point and the method was automatic so long as \(\{\delta y_i\}\) could be provided.

The endpoint condition \(g''(x)=0\) of the Reinsch natural spline leads to problems when obtaining acceleration values from displacement data. Philips and Roberts (1983) stated that the addition of single data points at the beginning and end of the interval removed the problem whilst Zernicke, Caldwell and Roberts (1976) suggested three points and McLaughlin, Dillman and Lardner (1977) recommended at least twenty such points at each end of the interval.

Wold (1974) recommended the use of Reinsch's spline for the smoothing and differentiation of 'well behaved' data with almost equidistant \(x\)-values if there are at least 10 points between any two inflection or extremum points. He suggested that \(\{\delta y_i\}\) could be estimated by dividing the data into different groups and using spline
simulations to estimate the errors in a group by fitting the data set comprising the other groups.

Wood and Jennings (1979a, 1979b) used an automatic routine which examined the residuals for trends and rescaled the $\delta y_i$. They compared the Reinsch cubic spline with the quintic equivalent, which has the endpoint condition $g'''(x)=0$ rather than $g''(x)=0$, and concluded that quintic splines are superior to cubic splines for fitting biomechanical data.

When data is obtained by digitising film, accuracy estimates can be obtained merely by repeated digitisation. Whilst this procedure might be time consuming it is likely to be safer than the automatic routine used with the general cubic and quintic spline routines of Jennings.

Even when the purpose of spline fitting is to obtain interpolated values, the approximating spline with $\delta y_i$ obtained as above is preferable to the interpolating spline which corresponds to $\delta y_i=0$. Besides the oscillatory problems associated with spline interpolation (Brodlie, 1980), choosing $\delta y_i=0$ does not lead to the best estimates of the interpolated values.

If splines are to be fitted to the data sets of the orientation angles then some method of estimating the $\delta y_i$ must be devised. If repeated digitisation has already been used to provide accuracy estimates of the displacement data then more than one estimate of each angle value can also be obtained to estimate the error of each angle.

**SIMULATION MODELS OF HUMAN MOVEMENT**

Simulation models of man in free-fall conditions have been developed to evaluate techniques for the reorientation of astronauts in space. Riddle and Kane (1968) used a 3 segment model to determine the effects of arm movements. The arm movements relative to the body were used as input data and the computer simulation program output the orientation of the body in terms of three Eulerian angles.
McCrank and Seger (1964) used a more general 9 segment model which permitted the angular momentum to be non-zero but gave angular velocities of the trunk as output values. Passerello and Huston (1971) developed a 10 segment model in which the internal orientation angles were defined piecewise using endpoint values as input and described the motion of the system using three output angles to give the orientation of the trunk. Such a system has the advantage of being easy to use and the results are readily interpreted.

In order to be a valid representation of human movement the model must have sufficient detail but as complexity increases the ease of use decreases. The model of Hatze (1981b) required that the activity of 46 muscle groups be specified as input parameters. Since it is not possible to obtain such information directly, Hatze initially guessed the muscle activity during a long-jump takeoff and then made adjustments until the simulated trajectory agreed with that of the filmed performance. In addition Hatze stated that 'models which disregard the intricate internal dynamics of the muscle are, a priori, doomed to failure'. Whilst this may be true in situations which require maximum muscular effort it is certainly not true in general. The results obtained for astronaut reorientation are independent of the muscle forces employed to produce the required paths of limb movements although the physiological limitations of the human body will place lower bounds on the duration of such movements. Thus whilst it might be expected that a model of the contact phase of a sports movement should include the mechanics of muscles, a model of the flight phase may prove adequate without the inclusion of muscle mechanics.

If information obtained from film is to be used as input data then it may not be possible to obtain complete information about internal orientation angles. Dapena (1979, p.135) noted that it was difficult to determine the rotations of limb segments about their longitudinal axes. He assumed that the angular momentum associated with such rotations could be taken to be zero and modelled the limbs as thin rods with moments of inertia about the longitudinal axes equal to zero. There was no necessity for such a procedure since the angular velocities about the longitudinal axes could have
been set to zero to give zero momentum. In addition the thin rod approximation leads to large errors in the moment of inertia about the longitudinal axis of a combined leg segment.

Before any credence may be given to the results obtained from a computer simulation model it is necessary to establish the accuracy of the representation. This validation of a model should be accomplished by comparing the output parameters with values obtained from actual performances in the areas in which the model is to be used (Panjabi, 1979).

None of the reorientation models were validated in this way. Pike (1980) obtained a partial check on her computer program by comparing the output for some simple mechanical systems with results obtained analytically or from other computer programs. Miller (1970) compared the somersault values output by her 4 segment model of non-twisting dives with data obtained from film. The agreement was good for dives in which the body held a fixed position but poor for the piked dives during which the hip angles changed considerably.

Dapena (1979) used his model to simulate the high jump but included some trampoline movements in the validation studies. For a double somersault with 180 degrees of twist the model was in error by 140 degrees.

Van Gheluwe (1979, 1981a, 1981b) was more successful in simulating back somersaults with full twist (Figure 37). He used manually fitted trigonometric splines to approximate the values of the internal angles. It would be better to remove this subjective element by using an automatic curve fitting technique so that the validations would not be dependent upon choices made by the researcher.

Once the model has been validated, simulations of hypothetical movements may be obtained by modifying the data obtained from film (Dapena, 1979; Van Gheluwe, 1981b) or by arbitrarily specifying the internal movements. If the latter procedure is adopted then some constraints must be placed upon the speed and duration of the internal movements to ensure that they are not impossible. Hatze (1983) was more specific and stated that 'the muscular properties of
Figure 37. Comparison of simulation values (solid lines) of (a) somersault, (b) twist and (c) tilt with estimates obtained from film (discrete points) for a piked backward somersault with full twist (Van Gheluwe; 1981a, b)
individual athletes vary so greatly that any model which does not take full account of these individual differences is a priori doomed to failure'. Any results which are performer-specific to the degree indicated by Hatze can only be applied to the one particular individual. Of greater import are general results which are not dependent on the particular characteristics of the performer but have wider application. It is in this wider field that this study will seek answers.
CHAPTER 3

DEVELOPMENT OF THE INERTIA MODEL

INTRODUCTION

This chapter details the development of the computer program ISEG which calculates the segmental inertia parameters from a series of anthropometric measurements. ISEG provides segmental values which are used by the program FILM to calculate orientation angles from film data and by the program SIM which simulates an aerial movement.

The accuracy with which ISEG estimates total body mass is given in Table 6 at the end of this chapter. The accuracy of the combined use of the three programs ISEG, FILM and SIM is evaluated in Chapter 6.

SEGMENTATION OF THE HUMAN BODY MODEL

The inertia model of the human body comprises 11 rigid segments (Figure 38). The notation used for the segments is:

- C: chest-head
- T: thorax
- P: pelvis
- A1: left upper-arm
- A2: left forearm-hand
- B1: right upper-arm
- B2: right forearm-hand
- J1: left thigh
- J2: left calf-foot
- K1: right thigh
- K2: right calf-foot
Figure 38. The 11 segments of the inertia model
ANTHROPOMETRIC MEASUREMENTS DEFINING THE SUBSEGMENTS

Measurements are taken at the levels L∅ through L8 which define the planes separating subsegments S1-S8 (Figure 39). The levels LI (I=0,8) are measured from L∅ so that L∅=0.0. At level LI perimeter is denoted by PI, width by WI and depth by DI. The set of measurements used comprises:

levels: L∅ L1 L2 L3 L4 L5 L6 L7 L8
perimeters: P∅ P1 P2 P3 P5 P6 P7
widths: W∅ W1 W2 W3 W4
depths: D4

At level L4 the width W4 is defined as the distance between shoulder joint centres and since the perimeter P4 cannot be measured the depth D4 is used. At level L5, which separates the shoulder subsegment S5 from the neck-jaw subsegment S6, the perimeter P5 is measured around the lowest part of the neck. For subsegment S5 the upper width W5 is defined to be equal to W4 and the depth D5 is obtained from the neck perimeter P5.

The pelvic segment P comprises subsegments S1 and S2, the thorax T comprises S3 and the chest-head segment C comprises S4 through S8.

The subsegments S11-S15 are separated by planes at the levels L1∅-L14 (Figure 40). The levels LI (I=10,14) are measured from L1∅ so that L1∅=0.0. The perimeter P1∅ is measured as high up the arm as possible since the perimeter cannot be measured at the shoulder centre level L1∅. The remaining perimeters PI (I=11,14) are measured at the corresponding levels LI (I=11,14). The subsegment S15 representing the hand requires the volume V15 to be measured and the length H15 is obtained as twice the distance from the wrist to knuckle III. The set of measurements used comprises:

levels: L1∅ L11 L12 L13 L14
perimeters: P1∅ P11 P12 P13 P14
volume: V15 length: H15

The upper-arm segment comprises subsegments S11, S12 and the forearm-hand segment comprises S13 through S15.
Figure 39. Segmentation levels LI of the subsegments SI which comprise the torso-head segments.
Figure 40. Segmentation levels LI of the subsegments SI which comprise the limb segments
The subsegments S21-S26 are separated by planes at the levels L20-L25 (Figure 40). The levels LI (I=20,25) are measured from L20 so that L20=0.0. The set of measurements used comprises:

- levels: L20 L21 L22 L23 L24 L25
- perimeters: P21 P22 P23 P24 P25
- volume: V26
- length: H26

The perimeter P20 at the hip centre level L20 cannot be measured and is obtained instead from an assumed diameter of \( W/2 \). The perimeter P25 is measured at the level which produces the smallest value rather than at the ankle centre level L25. The subsegment S26 representing the foot requires the volume V26 to be measured and the length H26 is taken as the distance from the heel to toe H.

The thigh segment comprises subsegments S21-S23 and the calf-foot segment comprises subsegments S24-S26.

**MEASUREMENT PROCEDURES**

The levels of the joint centres are marked on the skin surface. The elbow joint centre is taken to be at the intersection of the mid-lines of the upper arm and forearm (Figure 41). This procedure is chosen since it is consistent with the simplifying assumption of the simulation model that joint centres lie on the longitudinal axes of the limb segments. The same method is used to determine the joint centres of the knee (Figure 42), wrist and ankle. The shoulder centre S is taken to be the point on the mid-line of the upper arm for which SX=SZ (Figure 41). The shoulder method is used to determine the joint centre of the hip in the flexed position and the joint centre level L is marked on the floor (Figure 42). After lowering the leg the level L is marked on the skin surface. This procedure is necessary since the skin at the level of the hip joint moves relative to the joint as the hip is extended.

The remaining levels LI are marked on the skin and then all levels are transferred on to a roll of paper on which the subject lies. The outline of the subject is drawn on the paper and the
Figure 41. Location of joint centres of elbow and shoulder

Figure 42. Location of joint centres of knee and hip
depth D4 at the shoulder level is measured using anthropometric measuring callipers with the subject lying supine. The mid-lines of segments are then drawn on the paper using the body outline as a guide and the levels L1 are measured along the mid-lines. The remaining widths and perimeters are measured using callipers and measuring tape with the subject standing.

Hand and foot volumes are determined by placing the segment in a container and adding water until the surface is level with the joint centre. The water level is marked on the container, the segment removed and measured volumes of water are added until the water surface reaches the previous level.

**GEOMETRICAL SOLIDS USED TO MODEL THE SUBSEGMENTS**

The measurements that may be made at a particular segment level are perimeter p, width w and depth d. If the cross-section is modelled by an ellipse or some other convenient shape, two of the three measurements are sufficient to define the contour. Of the three measurements the depth d is probably the most difficult to measure accurately so that calculations based upon perimeter p and width w may be expected to be more accurate than calculations based upon measured values of d.

The perimeter of an ellipse is not a simple algebraic function of width and depth and so it is difficult to calculate d from values of p and w. This difficulty may be overcome by using an approximate relation connecting p, w and d providing that the accuracy obtained is acceptable. In general the errors in such expressions increase with the eccentricity of the ellipse and so comparisons will be made for an ellipse with a high value of w/d. Figure 30 shows a thorax cross-section with the ratio w/d equal to 1.8 so a choice of w/d equal to 2.0 is convenient and high but not unreasonable.

**CALCULATION OF THE DEPTH OF AN ELLipse FROM THE PERIMETER AND WIDTH**

The perimeter p of an ellipse of width 2a, depth 2b and
eccentricity \( e \) where \( b^2 = a^2(1-e^2) \) is given by \( p = 4aE(e) \) where \( E(e) \) is the complete elliptic integral of the second kind with modulus \( e \) (Bowman, 1953, p. 26). When \( b = 0.5a \), \( e^2 = 0.75 \) and \( E \) may be obtained from tables as \( E = 1.2111 \) (Jahnke, Emde and Losch, 1960, p. 58) giving \( p = 4.8444a \).

Taking \( p = 4.8444a \) a number of expressions will be used to calculate \( b \) which should equal \( 0.500a \).

The simplest approximation to the perimeter of an ellipse is given by \( p = \pi(a+b) \). Taking \( p = 4.8444a \) and \( \pi = 3.1416 \) a simple calculation gives \( b = 0.542a \).

Hatze (1979, p. 42) used the approximation \( p^2 = 2\pi^2(a^2+b^2) \) to calculate \( b \) for cross-sections of limb segments. When \( p = 4.8444a \) the expression gives \( b = 0.435a \).

Bowman (1953, p. 24) stated the complete elliptic integral 
\[ E = \pi F \left( \frac{1}{2}, \frac{1}{2}, 1, e^2 \right) \] where the eccentricity \( e \) is the modulus and \( F \) is the hypergeometric function which may be evaluated using the infinite hypergeometric series (Whittaker and Watson, 1962, p. 281). Using the first three terms of the series results in the expression 
\[ p = 2\pi a \left( 1 - e^2/4 - 3e^4/64 \right) \] which does not agree with Bowman, since he stated the third term incorrectly. This expression is quadratic in \( e^2 \) so that when \( p = 4.8444a \) use of the quadratic formula gives \( e^2 = 0.7969 \) and substitution in \( b^2 = a^2(1-e^2) \) results in \( b = 0.451a \).

Weinbach (1938) gave the expression \( p = \pi(a+b)k \) where \( k = 1 + m^2/4 + m^4/64 + \ldots \) and \( m = (a-b)/(a+b) \). Using the first two terms of the series for \( k \) produces the equation 
\[ 4(a+b)p = \pi \left( 4(a+b)^2 + (a-b)^2 \right) \] which is quadratic in \( b \). When \( p = 4.8444a \) use of the quadratic formula gives \( b = 0.500a \) to three decimal places. Note that in the extreme case of \( b = 0 \) the equation gives the perimeter as \( p = 1.25\pi a = 3.927a \) which is in error by less than 2% of the correct value \( p = 4a \).

Since the area \( A \) of an ellipse is calculated from \( A = \pi ab \) the first three approximations are unacceptable since they produce errors of around 10%. The Weinbach approximation is good but although ellipses were used to represent torso cross-sections in the early development of this model, the comparison of Figures 30,
31 and 32 lead to the adoption of the rounded-off rectangle as discussed in the literature review.

A possibility which was not explored but does appear promising is to use all three measurements (perimeter p, width w and depth d) to define the contour of a cross-section. One way of doing this would be to calculate the perimeter of an ellipse or rounded-off rectangle from the measured values of w and d and then adjust the values of w and d by multiplying by the measured perimeter and dividing by the calculated perimeter.

MODELLING THE TORSO SUB SEGMENTS

The rounded-off rectangle of width 2(r+t) and depth 2r shown in Figure 43 will be referred to as a stadium since it is the shape of an athletic stadium. Each of the subsegments SI (I=1,5) will be modelled by a solid bounded by two stadia (Figure 44).

An advantage of the stadium solid is that cylinders, cones and truncated cones, which are used to model the limb segments, are included as special cases so that the same computer subroutine may be used to calculate the inertia parameters of these solids.

MODELLING THE SUB SEGMENTS OF THE HEAD

Subsegments S6 and S7 are modelled as truncated right circular cones and the cranium S8 is modelled as a semi-ellipsoid of revolution (Figure 39).

MODELLING THE SUB SEGMENTS OF THE LIMBS

The limb segments SI (I=11,14; I=21,25) are modelled as truncated right circular cones (Figure 40).
Figure 43. Stadium shape with parameters $r$ and $t$

Figure 44. Solid bounded by two stadia
MODELLING THE HANDS AND FEET

Since the hands and feet are irregular in shape measurements of volume are used so that the calculated masses are independent of the choice of solid. Subsegments S15 and S26 are modelled as circular cones since the resulting mass centre locations are in close agreement with Dempster (1955).

Dempster (1955, p.192) stated that the distance of the mass centre of the hand from the wrist axis was 0.506 of the distance from the wrist to knuckle III. Since the mass centre of a cone with height $H_{15}$ lies $0.25*H_{15}$ from the base, $H_{15}$ is defined to be equal to twice the distance from the wrist to knuckle III. This choice of $H_{15}$ also approximates the distance from the wrist to the tip of finger III.

Dempster stated that the distance of the mass centre of the foot from the ankle axis was 0.249 of the distance from the heel to toe II. Defining the height $H_{26}$ to be equal to the distance from the heel to toe II results in a mass centre location which is in close agreement with Dempster.

During twisting somersaults the ankles are plantar-flexed and as a result the mass centre of a foot will lie close to the longitudinal axis of the lower leg. Thus it is reasonable to assume that the axes of S24, S25 and S26, which comprise the calf-foot segment, are collinear (Figure 40).

SEGMENTAL DENSITY VALUES

Density values of cadaver segments were obtained by Dempster (1955), Clauser et al. (1969) and Chandler et al. (1975) but only Dempster found individual values for shoulders, thorax and pelvis. The values used in the inertia model are taken from Dempster (pp.195, 196) and are listed in Table 2 together with the corresponding subsegments. It should be noted that the value for the density of Dempster's thorax segment is used with subsegments S3 and S4 whereas segment T, which has been termed the thorax segment for convenience, comprises only S3.
Table 2. Segmental density values (Dempster, 1955)

<table>
<thead>
<tr>
<th>Segment</th>
<th>Subsegments</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>head-neck</td>
<td>S6, S7, S8</td>
<td>1.11</td>
</tr>
<tr>
<td>shoulders</td>
<td>S5</td>
<td>1.04</td>
</tr>
<tr>
<td>thorax</td>
<td>S3, S4</td>
<td>0.92</td>
</tr>
<tr>
<td>abdomen-pelvis</td>
<td>S1, S2</td>
<td>1.01</td>
</tr>
<tr>
<td>upper arm</td>
<td>S11, S12</td>
<td>1.07</td>
</tr>
<tr>
<td>forearm</td>
<td>S13, S14</td>
<td>1.13</td>
</tr>
<tr>
<td>hand</td>
<td>S15</td>
<td>1.16</td>
</tr>
<tr>
<td>thigh</td>
<td>S21, S22, S23</td>
<td>1.05</td>
</tr>
<tr>
<td>lower leg</td>
<td>S24, S25</td>
<td>1.09</td>
</tr>
<tr>
<td>foot</td>
<td>S26</td>
<td>1.10</td>
</tr>
</tbody>
</table>
DERIVATION OF FORMULAE FOR THE INERTIA PARAMETERS OF THE SOLIDS

In this section the mass, location of mass centre and the principal moments of inertia of a stadium solid and a semi-ellipsoid are obtained in terms of the dimensions and uniform density of each solid.

STADIUM SOLID

A stadium is defined as a rectangle of width $2t$ and depth $2r$ with an adjoining semi-circle of radius $r$ at each end of its width (Figure 43).

The perimeter $p=4t+2\pi r$ and width $w=2t+2r$ so that $r$ and $t$ may be calculated using the equations:

$$r = \frac{(p-2w)}{(2\pi-4)} \quad \text{and} \quad t = \frac{(\pi w-p)}{(2\pi-4)}$$

A stadium solid bounded by parallel stadia (Figure 44) is defined as follows: Let the lower stadium have parameters $r_0$ and $t_0$, the upper stadium parameters $r_1$ and $t_1$ and let $h$ be the distance separating the stadia.

Consider a point $P$ which lies on the boundary of the first quadrant of a stadium with parameters $r$ and $t$ (Figure 45). If $P$ has coordinates $(x, y)$ then:

for $0 \leq x \leq t$, $x = \lambda t$ ($0 \leq \lambda \leq 1$) and $y=r$;

for $x > t$, $x = t+r\cos\theta$ and $y=r\sin\theta$ ($0 \leq \theta \leq \pi/2$).

A correspondence may now be defined between the points on the boundaries of the first quadrants of the upper and lower stadia. Points on the straight sections correspond when they have equal $\lambda$ values and points on the curved sections correspond when they have equal $\theta$ values. In a similar way a correspondence may be defined for the remaining quadrants.

Suppose that $P_0$ and $P_1$ are corresponding points of the lower and upper stadia. If $p_0$ and $p_1$ are position vectors of $P_0$ and $P_1$ relative to some origin then the point $P$ with position vector
Figure 45. A quadrant of a stadium

Figure 46. A section of a stadium solid
p=(1-z)p_0+z p_1 lies on the line segment P_0 P_1 when 0≤z≤1 and divides it in the ratio z:(1-z). If z is held constant and P_0 and P_1 move around the lower and upper stadia, the point P moves around the boundary of an intermediate section of the stadium solid. This procedure defines the stadium solid and it will now be shown that each intermediate section is itself a stadium and is parallel to the bounding stadia.

Let the coordinate system (x, y, z) have origin at the centre of the lower stadium with the z-axis lying along the line joining the centres of the bounding stadia (Figure 46).

The points P_0(λ t_0, r_0, 0) and P_1(λ t_1, r_1, h) on the straight sections correspond to P((1-z) t_0+z t_1, (1-z) r_0+z r_1, z h) which is equivalent to P(λ t(z), r(z), z h) where t(z)=(1-z) t_0+z t_1 and r(z)=(1-z) r_0+z r_1.

Similarly the points P_0(t_0+r_0 \cos θ, r_0 \sin θ, 0) and P_1(t_1+r_1 \cos θ, r_1 \sin θ, h) correspond to P(t(z)+r(z) \cos θ, r(z) \sin θ, z h) where t(z) and r(z) are as above.

Thus as P_0 and P_1 move around the bounding stadia the point P moves around a parallel stadium which has parameters t(z) and r(z) and is at a distance z h from the lower bounding stadium.

It should be noted that although the above treatment does not assume that the z-axis is normal to the bounding stadia only right stadium solids will be considered in the following derivations.

**SECOND MOMENTS OF AREA OF A STADIUM**

The calculation of the second moments of area of a stadium will require the evaluation of a number of integrals. For convenience these definite integrals will be evaluated prior to the calculation of second moments.

\[
\int_0^{\frac{1}{2}\pi} \sin^2 \theta d\theta = \int_0^{\frac{1}{2}\pi} \frac{1}{2} (1-\cos 2\theta) d\theta = \frac{1}{2} [\theta - \sin 2\theta]_0^{\frac{1}{2}\pi} = \frac{1}{4} \pi
\]

\[
\int_0^{\frac{1}{2}\pi} \sin^4 \theta d\theta = \int_0^{\frac{1}{4}\pi} \frac{1}{4} (1-\cos 2\theta)^2 d\theta = \frac{1}{4} \int_0^{\frac{1}{4}\pi} (1-2\cos 2\theta + \cos^2 2\theta) d\theta
\]
\[
\int_0^{\frac{3\pi}{2}} [1-2\cos 2\theta + \frac{1}{4}(1+\cos 4\theta)]d\theta = \frac{1}{4}\left(\frac{3\pi}{2}\right) = \frac{3\pi}{16}
\]

\[
\int_0^{\frac{1}{4}} \sin^2 \theta \cos \theta d\theta = \int_0^{\frac{1}{4}} \sin^2 \theta . d(\sin \theta) = \left[\frac{1}{3} \sin^3 \theta\right]_0^{\frac{1}{4}} = \frac{1}{3}
\]

\[
\int_0^{\frac{1}{4}} \cos^2 \theta \sin \theta d\theta = \int_0^{\frac{1}{4}} (1-\sin^2 \theta) \sin \theta d\theta = \int_0^{\frac{1}{4}} \sin^2 \theta d\theta - \int_0^{\frac{1}{4}} \sin^4 \theta d\theta
\]

\[
= \frac{\pi}{4} - \frac{3\pi}{16} = \frac{\pi}{16}
\]

A point lies on the boundary of the first quadrant of a stadium (Figure 43) providing either \(0 \leq x \leq t\) and \(y = r\)

or \(x = t + r\cos \theta\) and \(y = r\sin \theta\) \((0 \leq \theta \leq \frac{\pi}{2})\)

Thus the area of the stadium is \(A\) where:

\[
\frac{1}{4}A = \int_0^{t} ydx + \int_0^{\frac{\pi}{2}} y \frac{dx}{d\theta} d\theta \text{ where } \frac{dx}{d\theta} = \frac{dx}{d\theta}
\]

\[
= \int_0^{t} rdx + r^2 \int_0^{\frac{\pi}{2}} \sin^2 \theta d\theta
\]

\[
= rt + \frac{1}{4} r^2 = \frac{\pi}{16} \text{ so that } A = 4rt + \pi r^2 \text{ as expected.}
\]

Let \(J_x, J_y\) and \(J_z\) be the second moments of area of the stadium in Figure 43 about the \(x, y, z\) axes where the \(z\)-axis is orthogonal to both the \(x\) and \(y\) axes.

\[
J_x = 4 \int_0^{t} \frac{1}{3} y^2 . ydx = \frac{4}{3} \int_0^{t} y^2 dx \text{ the integration occurring over a single quadrant.}
\]

Now \[
\int_0^{t} y^2 dx = \int_0^{t} y^2 dx + \int_0^{\frac{\pi}{2}} y^2 x d\theta
\]

\[
= \int_0^{t} r^3 dx + r^4 \int_0^{\frac{\pi}{2}} \sin^4 \theta d\theta
\]

\[
= r^3 t + 3\pi r^4 /16
\]

Thus \[
J_x = 4r^3 t/3 + \pi r^4 /4.
\]
\[ J_y = 4 \int x^2 \, y \, dx \]

and
\[
\int x^2 \, y \, dx = \int_0^t x^2 \, y \, dx + \int_{\pi}^0 x^2 \, y \, r \, d\theta
\]
\[
= \int_0^t x^2 \, r \, dx + \int_0^{\pi} (t + r \cos \theta)^2 \cdot r^2 \sin^2 \theta \, d\theta
\]
\[
= r \left[ \frac{x^3}{3} \right]_0^t + r^2 t^2 \int_0^{\pi} \sin^2 \theta \, d\theta + 2r^3 t \int_0^{\pi} \sin^2 \theta \cos \theta \, d\theta
\]
\[
+ r^4 \int_0^{\pi} \cos^2 \theta \sin^2 \theta \, d\theta
\]
\[
= r \left[ \frac{t^3}{3} \right] + r^2 t^2 \left[ \frac{\pi}{4} \right] + 2r^3 t \left[ \frac{1}{3} \right] + r^4 \left[ \frac{\pi}{16} \right]
\]

so that \[ J_y = 4rt^3/3 + \pi r^2 t^2 + 8r^3 t/3 + \pi r^4/4 \]

For a lamina the theorem of perpendicular axes states that \[ J_z = J_x + J_y \] so that \[ J_z = 4rt^3/3 + \pi r^2 t^2 + 4r^3 t + \pi r^4/2 \].

**THE MASS AND LOCATION OF MASS CENTRE OF A STADIUM SOLID**

It has been shown that the intermediate stadium of Figure 44 is at a distance \( z_h \) from the lower bounding stadium and has parameters \( r \) and \( t \) where \( r = r_0 + z(r_1 - r_0) \) and \( t = t_0 + z(t_1 - t_0) \).

Thus \( r \) and \( t \) may be expressed as \( r = r_0 (1 + az) \) and \( t = t_0 (1 + bz) \) where \( a = (r_1 - r_0)/r_0 \) and \( b = (t_1 - t_0)/t_0 \).

If the intermediate stadium has density \( D \), area \( A(z) \) and thickness \( h \, dz \) the mass will be \( DA(z)h \, dz \) so that the mass of a stadium solid of uniform density is given by \( M = \int_0^1 DA(z)h \, dz \).

The first moment of mass about the lower plane is \( \int_0^1 hzDA(z)h \, dz \) and the distance \( \bar{z} \) of the mass centre from the lower plane is given by:
\[
\bar{z} = \int_0^1 Dh^2zA(z)dz/M.
\]

Define the functions \( F_1, F_2, F_3 \) by the equations:
\[
F_1(a, b) = 1 + (a + b)/2 + ab/3
\]
\[
F_2(a, b) = \frac{1}{2} + (a + b)/3 + ab/4
\]
\[
F_3(a, b) = \frac{1}{3} + (a + b)/4 + ab/5
\]
Area $A(z) = 4rt + \pi r^2 = 4r_0(1+az)t_0(1+bz)+\pi r_0^2(1+az)^2$

so that

$$
\int_0^1 Adz = 4r_0 t_0(1+(a+b)/2+ab/3)+\pi r_0^2(1+2a/2+a^2/3) = 4r_0 t_0 F_1(a, b) + \pi r_0^2 F_1(a, a)
$$

and

$$
M = dh \int_0^1 Adz = dhr_0[4t_0 F_1(a, b)+\pi r_0^2 F_1(a, a)]
$$

Now $zA = 4r_0 t_0(z+(a+b)z^2+abz^3)+\pi r_0^2(z+2az^2+a^2z^3)$

so that

$$
\int_0^1 zAdz = 4r_0 t_0(\frac{1}{2}z + (a+b)/3+ab/4) + \pi r_0^2(\frac{1}{2} + 2a/3+a^2/4)
$$

and

$$
\bar{z} = dh \int_0^1 zAdz/M = dh [4r_0 t_0 F_2(a, b)+\pi r_0^2 F_2(a, a)]/M
$$

Note also

$$z^2A = 4r_0 t_0(z^2+(a+b)z^3+abz^4)+\pi r_0^2(z^2+2az^3+a^2z^4)
$$

so that

$$
\int_0^1 z^2Adz = 4r_0 t_0(\frac{1}{3}z + (a+b)/4 + ab/5) + \pi r_0^2(\frac{1}{3} + 2a/4+a^2/5)
$$

and

$$
\bar{z}^2 = d\int_0^1 z^2Adz/M = d[4r_0 t_0 F_3(a, b)+\pi r_0^2 F_3(a, a)]/M
$$

This result will be used in conjunction with the theorem of parallel axes in the next section to calculate moments of inertia.

**MOMENTS OF INERTIA OF A STADIUM SOLID**

The moment of inertia of a stadium solid about the z-axis (Figure 44) is $I_z = \int_0^1 Dzhdz$ where the second moment of area of a stadium has been obtained as $J_z = 4rt^2/3+\pi r^2/2 + 4t^2/2$. Thus

$$
I_z = Dh[(4/3)\int_0^1 rt^3dz + t\int_0^1 r^2tdz + 4\int_0^1 r^3dz + \frac{1}{2} \pi \int_0^1 r^4dz]
$$

Define $F_4(a, b) = 1+(a+3b)/2+(3ab+3b^2)/3+(3ab^2+b^3)/4+ab^3/5$

and $F_5(a, b) = 1+(2a+2b)/2+(a^2+4ab+b^2)/3+2ab(a+b)/4+a^2b^2/5$
Now \[ rt^3 = r_0 t_0^3 (1+az)(1+bz)^3 \]
\[ = r_0 t_0^3 (1+3bz+3b^2 z^2 + b^3 z^3) \]
\[ = r_0 t_0^3 (1+(a+3b)z+(3ab+3b^2)z^2+(3ab^2 + b^3)z^3 + ab^3 z^4) \]
so that \[ \int_0^1 t^3 dt = r_0 t_0^3 \cdot F_4(a, b) \]

Similarly \[ \int_0^1 r^3 dr = r_0 t_0^3 \cdot F_4(b, a) \]

Whilst \[ r^2 t^2 = r_0 t_0^2 (1+az)^2 (1+bz)^2 \]
\[ = r_0^2 t_0^2 (1+2az+a^2 z^2)(1+2bz+b^2 z^2) \]
\[ = r_0^2 t_0^2 (1+(2a+2b)z+(a^2 + 4ab+b^2)z^2+2ab(a+b)z^3+a^2 b^2 z^4) \]
so that \[ \int_0^1 r^2 t^2 dt = r_0^2 t_0^2 \cdot F_5(a, b) \]

and \[ r^4 = r_0^4 (1+az)^4 \]
\[ = r_0^4 (1+4az+6a^2 z^2+4a^3 z^3 + a^4 z^4) \]
so that \[ \int_0^1 r^4 dr = r_0^4 \cdot F_4(a, a) \] and:

\[ I_z = Dh \left[ 4r_0 t_0^3 F_4(a, b)/3 + r_0^2 t_0^2 F_5(a, b) + 4r_0^3 t_0 F_4(b, a) + r_0^4 F_4(a, a)/2 \right] \]

The moment of inertia of a stadium section about the y-axis (Figure 44) is given by the theorem of parallel axes as \( J_y = J_y' + (h\bar{z})^2 \cdot A \) where \( J_y \) is the second moment of area, \( D \) the density, \( h\bar{z} \) the thickness, \( h\bar{z} \) the distance from the y axis and \( A \) the area.

If \( I_y^0 \) denotes the moment of inertia of the stadium solid about the y-axis (which lies in the lower face) then:

\[ I_y^0 = \int_0^1 J_y Dhd\bar{z} + \int_0^1 (h\bar{z})^2 ADhd\bar{z} \]
\[ = Dh \int_0^1 J_y dz + Dh \int_0^1 z^3 Adz \] where:
\[ \int_{0}^{1} \frac{1}{y} \, dz = \int_{0}^{1} \left( \frac{4r^3}{3} + \frac{\pi r^2 t^2}{2} + \frac{8r^3}{3} + \frac{\pi r^4}{4} \right) \, dz \]
\[ = 4r_0 t_0^3 F_4(a, b) / 3 + \pi r_0^2 F_5(a, b) + 8r_0^3 t_0 F_4(b, a) / 3 + \pi r_0^4 F_4(a, a) / 4 \]

using the results obtained during the calculation of \( I_z \) and

\[ \int_{0}^{1} z^2 \, dz = 4r_0 t_0 F_3(a, b) + \pi r_0^2 F_3(a, a) \]

having been derived in the section on the calculation of \( z \).

If \( I_y \) denotes the moment of inertia about a parallel axis through the mass centre then the theorem of parallel axes gives

\[ I_y = I_0 - Mz^2. \]

Similarly the moment of inertia \( I_x^o \) about the x-axis is given by:

\[ I_x^o = \int_{0}^{1} J_x \, dh \, dz + \int_{0}^{1} (hz)^2 A \, dh \, dz \]
\[ = Dh \int_{0}^{1} J_x \, dz + Dh^3 \int_{0}^{1} z^2 \, Adz \]

where

\[ \int_{0}^{1} J_x \, dz = \int_{0}^{1} \left( \frac{4r^3}{3} + \frac{\pi r^4}{4} \right) \, dz \]
\[ = 4r_0 t_0^3 F_4(a, b) / 3 + \pi r_0^4 F_4(a, a) / 4 \]

and the moment of inertia \( I_x \) about a parallel axis through the mass centre is given by

\[ I_x = I_x^o - Mz^2. \]

SUBROUTINE SS(Z, V, M, XI, YI, ZI, R0, T0, R1, T1, H, D) in the computer program ISEG uses the above formulae to calculate mass centre distance \( Z \), volume \( V \), mass \( M \) and moments of inertia \( XI, YI, ZI \) about axes through the mass centre from the dimensions \( R0, T0, R1, T1, H \) and density \( D \).

A stadium solid becomes a truncated cone when \( T_0 = T_1 = 0 \) but since the parameter \( b = (T_1 - T_0) / T_0 \) SUBROUTINE SS cannot be used with \( T_0 = 0 \). This difficulty is overcome by SUBROUTINE CC which sets \( T_0 = T_1 = 0.001 \) cm prior to calling SS so that a very thin trapezium is added to the truncated cone. CC calculates the radii \( R0, R1 \) from the perimeter values \( P0, Pl \).
A truncated cone becomes a cone when either $r_0 = 0$ or $r_1 = 0$ but since the parameter $a = (r_1 - r_0)/r_0$, SUBROUTINE CC cannot be used with $P_0 = 0.0$. SUBROUTINE FF calculates the perimeter $P_0$ from the volume $V$ and height $H$ using $P_0 = 2\pi R$ where $V = \pi R^2 H/3$ determines $R$. FF then sets $P_1 = 0.0$ prior to calling CC to obtain the cone parameters.

**SEMI-ELLIPSOID OF REVOLUTION**

The semi-ellipsoid of height $h$ and base radius $r$ shown in Figure 47 is symmetrical about the $z$-axis. The cross-section at a distance $zh$ from the base is a circle of radius $x$. The $x$-$z$ plane meets the semi-ellipsoid in a semi-ellipse so that $x$ is given by the equation $x^2/r^2 + (zh)^2/h^2 = 1$, i.e. $x^2 = r^2(1-z^2)$ where $0 \leq z \leq 1$. If the circular cross-section of radius $x$ has density $D$ and thickness $h_0 x$ the mass will be $m = D\pi x^2 h_0 dz$ so that the mass of the semi-ellipsoid will be $M$ where:

$$M = \int_0^h D\pi x^2 h_0 dz.$$

Thus

$$M = \int_0^h D\pi x^2 h_0 dz = D\pi h^2 \int_0^1 (1-z^2) dz = D\pi h^2 [z - z^3/3]_0^1 = 2D\pi h^2/3$$

The first moment of mass about the base is $\int zhD\pi x^2 h_0 dz$ and so the distance $\bar{z}$ of the mass centre from the base is given by:

$$\bar{z} = \frac{\int_0^1 D\pi h^2 x^2 z dz}{M}$$

$$= \frac{\int_0^1 D\pi h^2 x^2 z dz}{D\pi h^2 \int_0^1 (1-z^2) dz} = D\pi h^2 [z^2/2 - z^4/4]_0^1 = D\pi h^2/4$$

Thus

$$\bar{z} = (D\pi h^2/4)/(2D\pi h^2/3) = 3h/8.$$

The moment of inertia of the circular disc about the $z$-axis is $I_{zx} = D\pi x^4 h_0 dz$ so that the moment of inertia of the semi-ellipsoid about the $z$-axis is given by:

$$I_z = \frac{1}{2}D\pi h x^4 dz$$

$$I_z = \frac{1}{2}D\pi h r^4 \int_0^1 (1-z^2)^2 dz = \frac{1}{2}D\pi h r^4 \int_0^1 (1-2z^2 + z^4) dz$$

$$= \frac{1}{2}D\pi h r^4 \left[ 1 - \frac{2}{3} + \frac{1}{3} \right] = 4D\pi h r^4/15 = 2Mr^2/5.$$
Figure 47. A section of a semi-ellipsoid
Using the theorem of parallel axes the moment of inertia of the circular disc about the x-axis is \( \frac{1}{4}m\pi^2 + mzh^2 \) so that the moment of inertia of the semi-ellipsoid about the x-axis is \( I_x^0 \) where:

\[
I_x^0 = \int_0^1 D\pihx^2 \left( \frac{1}{4}x^2 + z^2h^2 \right) dz = \frac{1}{2}\pi z^2 + D\pi h^2 \int_0^1 (1-z^2)z^2 dz
\]

\[
= Mr^2/5 + D\pi r^2 h^2 \left[ \frac{1}{3} - \frac{1}{5} \right] = M(r^2 + h^2)/5
\]

The moment of inertia \( I_x \) about a parallel axis through the mass centre is obtained using the theorem of parallel axes as \( I_x = I_x^0 - Mz^2 \)
giving \( I_x = M(r^2 + h^2)/5 - 9Mh^2/64 \).

By symmetry the moment of inertia \( I_y \) is obtained as \( I_y = I_x \).

**THE COMPUTER PROGRAM ISEG**

The interactive program ISEG requests the subjects' name PERF and reads the anthropometric data from file PER whose name comprises the first three characters of the subjects' name. ISEG outputs segmental link lengths to file PERF for use with the program FILM which calculates orientation angles from film data. ISEG also outputs segmental inertia parameter values to file PERF for use with the simulation program SIM. A listing of the program ISEG is presented in Appendix A.

Since it is not the intention of this study to obtain results about twisting somersaults which may be dependent upon differences between the left and right limbs, the input data file PER uses averages of the left and right limbs. The input data set has been described earlier and is listed in Table 3 with distances in centimeters and volumes in millilitres.

File PERF contains the segmental link lengths \( L_P, L_2P, LT, LJ, LJ_1 \) in metres. These link lengths are defined in the next chapter.

File PERF contains the mass, principal moments of inertia, mass centre location and link length of each segment in SI units. The
sequence of data is given in Table 5.

ANTHROPOMETRIC DATA OF THREE SUBJECTS

The anthropometric data files GIL, JOH and CAR of the three subjects GILL, JOHN AND CARL who performed the filmed movements are presented in Table 3.

INERTIA PARAMETER VALUES OF THREE SUBJECTS

The output files GILLR, JOHNr and CARLR containing the link lengths are listed in Table 4. The output files GILL, JOHN and CARL containing the segmental inertia parameters are listed in Table 5.

ACCURACY OF THE ESTIMATION OF TOTAL BODY MASS

The accuracy with which program ISEG calculates the total body mass is given in Table 6.

SUMMARY

A computer program has been developed in order to calculate segmental masses, mass centre locations and moments of inertia from a set of anthropometric measurements. A number of arbitrary choices have been made concerning the solids used to model the segments, the uniform segmental densities and joint centre locations. Such simplifying assumptions can only be properly justified on the basis of how well the model performs. The fact that total body mass is estimated to within 3% of values obtained by weighing is encouraging but cannot be used as a justification for using the program ISEG to estimate other quantities. It is important that the values produced by ISEG should give accurate results when used in conjunction with the film and simulation programs. The validation of the combined use of the three programs is given in Chapter 6 and provides the justification for the simplifying assumptions.
Table 3. Anthropometric data files

ANTHROPOMETRIC DATA FILE PER

DISTANCE IN CM.
VOLUME IN ML.

GIL

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<td></td>
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<td></td>
</tr>
</tbody>
</table>

JOH

<p>| | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>13.5</td>
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<tr>
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<td>75.3</td>
<td>87.5</td>
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<td>56.8</td>
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<td></td>
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<td>26.4</td>
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<td>25.3</td>
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<td></td>
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<td>377</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>52.1</td>
<td>47.8</td>
<td>36.0</td>
<td>33.6</td>
<td>20.2</td>
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<td>810</td>
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</tbody>
</table>

CAR

<p>| | | | | | | | | |</p>
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<td>56.0</td>
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<td></td>
</tr>
<tr>
<td>31.1</td>
<td>28.4</td>
<td>29.8</td>
<td>33.6</td>
<td>36.0</td>
<td>13.6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.2</td>
<td>28.2</td>
<td>34.2</td>
<td>53.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>28.5</td>
<td>25.1</td>
<td>27.0</td>
<td>16.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>364</td>
<td>18.2</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>8.1</td>
<td>23.1</td>
<td>37.9</td>
<td>51.5</td>
<td>80.7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>53.0</td>
<td>48.3</td>
<td>34.6</td>
<td>34.5</td>
<td>21.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>840</td>
<td>25.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 4. Output files containing link lengths

OUTPUT FILE PERFR FOR USE WITH PROGRAM FILL4

LINK LENGTHS IN METRES

GILLR

<table>
<thead>
<tr>
<th>LP</th>
<th>L2P</th>
<th>LT</th>
<th>LJ</th>
<th>LJ1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2420</td>
<td>0.0792</td>
<td>0.2180</td>
<td>0.8150</td>
<td>0.4100</td>
</tr>
</tbody>
</table>

JOHNR

<table>
<thead>
<tr>
<th>LP</th>
<th>L2P</th>
<th>LT</th>
<th>LJ</th>
<th>LJ1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1850</td>
<td>0.0777</td>
<td>0.3260</td>
<td>0.8260</td>
<td>0.4010</td>
</tr>
</tbody>
</table>

CARLR

<table>
<thead>
<tr>
<th>LP</th>
<th>L2P</th>
<th>LT</th>
<th>LJ</th>
<th>LJ1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2150</td>
<td>0.0777</td>
<td>0.3440</td>
<td>0.8070</td>
<td>0.3790</td>
</tr>
</tbody>
</table>
Table 5. Output files containing segmental inertia parameters

OUTPUT FILE PERF FOR USE WITH PROGRAM SIM

MASS IN KG.

MOMENT OF INERTIA IN KG*M**2

DISTANCE IN METRES

DATA SEQUENCE:

PERF

<table>
<thead>
<tr>
<th>MP</th>
<th>XIP</th>
<th>YIP</th>
<th>ZIP</th>
<th>ZP</th>
<th>LP</th>
<th>L2P</th>
</tr>
</thead>
<tbody>
<tr>
<td>MJ1</td>
<td>X1J1</td>
<td>Y1J1</td>
<td>Z1J1</td>
<td>J1</td>
<td>L1J1</td>
<td></td>
</tr>
<tr>
<td>MJ2</td>
<td>X1J2</td>
<td>Y1J2</td>
<td>Z1J2</td>
<td>J2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MT</td>
<td>XIT</td>
<td>YIT</td>
<td>ZIT</td>
<td>IT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC</td>
<td>XIC</td>
<td>YIC</td>
<td>ZIC</td>
<td>IC</td>
<td>C</td>
<td></td>
</tr>
<tr>
<td>MA1</td>
<td>XIA1</td>
<td>YIA1</td>
<td>ZIA1</td>
<td>A1</td>
<td>A1</td>
<td></td>
</tr>
<tr>
<td>MA2</td>
<td>XIA2</td>
<td>YIA2</td>
<td>ZIA2</td>
<td>A2</td>
<td>A2</td>
<td></td>
</tr>
</tbody>
</table>

GILL

| 10.6238 | 0.0767 | 0.1127 | 0.0839 | 0.1106 | 0.2420 | 0.0792 |
| 8.3716 | 0.1150 | 0.1150 | 0.0273 | 0.1812 | 0.4100 |
| 4.2192 | 0.0974 | 0.0974 | 0.0053 | 0.2205 |
| 3.5275 | 0.0118 | 0.0202 | 0.0281 | 0.0439 | 0.2180 |
| 13.3189 | 0.2122 | 0.2486 | 0.0990 | 0.0212 | 0.1650 |
| 1.7724 | 0.0097 | 0.0097 | 0.0020 | 0.1089 | 0.2460 |
| 1.3052 | 0.0130 | 0.0130 | 0.0008 | 0.1467 |

JOHN

| 9.1672 | 0.0535 | 0.0748 | 0.0756 | 0.0871 | 0.1850 | 0.0777 |
| 7.4813 | 0.1010 | 0.1010 | 0.0219 | 0.1814 | 0.4010 |
| 4.1400 | 0.1167 | 0.1167 | 0.0048 | 0.2413 |
| 8.2058 | 0.0483 | 0.0615 | 0.0697 | 0.0906 | 0.3260 |
| 14.3885 | 0.2419 | 0.2803 | 0.1059 | 0.0140 | 0.1645 |
| 1.9389 | 0.0126 | 0.0126 | 0.0022 | 0.1195 | 0.2680 |
| 1.5780 | 0.0191 | 0.0191 | 0.0010 | 0.1651 |

CARL

| 10.9412 | 0.0752 | 0.1072 | 0.0958 | 0.0994 | 0.2150 | 0.0777 |
| 7.1191 | 0.0857 | 0.0857 | 0.0212 | 0.1692 | 0.3790 |
| 4.3222 | 0.1208 | 0.1208 | 0.0051 | 0.2463 |
| 8.3995 | 0.0442 | 0.0743 | 0.0829 | 0.0841 | 0.3440 |
| 16.0864 | 0.2964 | 0.3667 | 0.1381 | -0.0074 | 0.1800 |
| 2.1130 | 0.0152 | 0.0152 | 0.0025 | 0.1238 | 0.2820 |
| 1.6312 | 0.0179 | 0.0179 | 0.0011 | 0.1566 |
Table 6. Accuracy of total body mass estimates

<table>
<thead>
<tr>
<th>SUBJECT</th>
<th>mass</th>
<th>calculated mass</th>
<th>error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GILL</td>
<td>60.0</td>
<td>58.8</td>
<td>-2.0</td>
</tr>
<tr>
<td>JOHN</td>
<td>60.9</td>
<td>62.0</td>
<td>1.8</td>
</tr>
<tr>
<td>CARL</td>
<td>64.3</td>
<td>65.8</td>
<td>2.3</td>
</tr>
</tbody>
</table>
CHAPTER 4

DEVELOPMENT OF THE FILM ANALYSIS PROGRAM

INTRODUCTION

This chapter details the development of the computer program FILM which calculates the orientation angles specifying body configuration from film data obtained by digitisation. FILM provides quintic spline coefficients which define the time histories of the orientation angles for use with the simulation program SIM.

The errors in displacement values arising from digitising the film and the corresponding errors in the calculated orientation angles are presented in Table 9 at the end of this chapter. The accuracy of the combined use of the three programs ISEG, FILM and SIM is evaluated in Chapter 6.

JOINT CENTRES AND BODY LANDMARKS

Corresponding to the 11 body segments described in the previous chapter (Figure 38) there are 10 joint centres at the junctions between segments (Figure 48). These 10 joint centres are listed in Table 7. Spinal flexion is assumed to occur at a single point X which is the junction of the longitudinal axes of the pelvis P and thorax T. The motion of the chest-head segment C relative to T is assumed to take place about the point N which is defined to be the midpoint of the shoulder centres and is assumed to lie on the longitudinal axis of T. The joint centre N is defined in this way because it is easily obtained from the locations of R and S. In addition N approximates to the centre of rotation corresponding to asymmetrical movements of the shoulders.

In addition to these 10 joint centres Figure 48 also shows the left and right wrist centres W and V, the left and right ankle
Figure 48. Joint centres and body landmarks.
Table 7. Joint centres connecting body segments

<table>
<thead>
<tr>
<th>Joint Centre</th>
<th>Location</th>
<th>Adjacent Segments</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td>left shoulder</td>
<td>C-A1</td>
</tr>
<tr>
<td>R</td>
<td>right shoulder</td>
<td>C-B1</td>
</tr>
<tr>
<td>Oa</td>
<td>left elbow</td>
<td>A1-A2</td>
</tr>
<tr>
<td>Ob</td>
<td>right elbow</td>
<td>B1-B2</td>
</tr>
<tr>
<td>H</td>
<td>left hip</td>
<td>P-J1</td>
</tr>
<tr>
<td>I</td>
<td>right hip</td>
<td>P-K1</td>
</tr>
<tr>
<td>Oj</td>
<td>left knee</td>
<td>J1-J2</td>
</tr>
<tr>
<td>Ok</td>
<td>right knee</td>
<td>K1-K2</td>
</tr>
<tr>
<td>X</td>
<td>junction P,T</td>
<td>P-T</td>
</tr>
<tr>
<td>N</td>
<td>midpoint R,S</td>
<td>T-C</td>
</tr>
</tbody>
</table>

Table 8. Landmarks used in film digitisation

<table>
<thead>
<tr>
<th>Landmark</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>left wrist</td>
</tr>
<tr>
<td>Oa</td>
<td>left elbow</td>
</tr>
<tr>
<td>S</td>
<td>left shoulder</td>
</tr>
<tr>
<td>H</td>
<td>left hip</td>
</tr>
<tr>
<td>Y</td>
<td>left ankle</td>
</tr>
<tr>
<td>V</td>
<td>right wrist</td>
</tr>
<tr>
<td>Ob</td>
<td>right elbow</td>
</tr>
<tr>
<td>R</td>
<td>right shoulder</td>
</tr>
<tr>
<td>I</td>
<td>right hip</td>
</tr>
<tr>
<td>Z</td>
<td>right ankle</td>
</tr>
</tbody>
</table>
centres Y and Z, and the point G which is defined to be the midpoint of the hip centres H and I.

The body landmarks used when digitising the film are \( W, V, O_a, O_b, S, R, H, I, Y, Z \) and are listed in Table B. The joint centres \( N, X, O_j \) and \( O_k \) are calculated from the locations of these 10 landmarks using the data on link lengths provided by file PERFR.

**LINK LENGTH DATA IN THE FILE PERFR**

As mentioned in the previous chapter the program ISEG outputs the link lengths \( LP, L2P, LT, LJ, LJ1 \) to the file PERFR. Using the prefix \( d \) to denote the distance between two points and referring to Figure 48 these link lengths are defined as follows:

\[
\begin{align*}
LP &= d_{GX} \\
L2P &= d_{GH} \\
LT &= d_{XN} \\
LJ &= d_{HY} \\
LJ1 &= d_{HO_j}
\end{align*}
\]

The ways in which these values are used will be described in the section on the calculation of orientation angles.

**LAYOUT OF THE CHAPTER**

The remainder of this chapter comprises two main sections. In the first section the calculations used in the program FILM are detailed sequentially whilst the procedures and equipment used in the filming sessions are described in the second section.

**SEQUENCE OF THE CALCULATIONS USED IN THE PROGRAM FILM**

The displacement data used as input to the program FILM is obtained by digitising the filmed views of two cameras whose optical axes intersect at right angles at the origin of a reference frame. Prior to the performance of a twisting somersault, film is taken of
the reference frame so that horizontal and vertical scale factors may be established by digitising points of the reference frame. In addition to obtaining film coordinates for each of the ten body landmarks for each frame, the frame time is read from the image of a timing device placed in view of both cameras.

The orientation angles are determined from the film data using the following three stages:

1. Derivation of synchronous data sets from the film data
2. Determination of the spatial coordinates of the body landmarks
3. Determination of the orientation angles from these spatial locations.

DERIVATION OF SYNCHRONOUS DATA SETS FROM THE FILM DATA

The film taken by each camera is digitised twice so that there are two estimates of each film coordinate. This facilitates the detection of digitising errors and enables estimates to be made of the standard deviations of the film coordinates for use with spline fitting routines.

SPLINE FITTING THE FRAME TIMES

If frame times are read to the nearest millisecond and the camera framing rate is constant more accurate time estimates may be obtained by fitting a straight line to the time values using the frame number as independent variable. For spring driven cameras the framing rate may not remain constant and a better procedure is to fit the cubic spline of Reinsch (1967). This spline fitting algorithm requires values for the standard deviation of each time value.

If the true time \( t \) is rounded to the nearest millisecond to give the estimate \( T \) the error \( (t-T) \) has variance \( v \) where:

\[
v = \int_{T-\frac{1}{2}}^{T+\frac{1}{2}} (t-T)^2 \, dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} t^2 \, dt = \frac{[t^3/3]}{\frac{1}{2}} = \frac{1}{12}
\]
Thus the standard deviation is $1/\sqrt{12} = \sqrt{3}/6 \approx 0.3$ milliseconds.

The Reinsch algorithm chooses the smoothest function satisfying the sums of squares constraint imposed by the standard deviation estimates so that when these estimates are sufficiently large a linear function is chosen. Thus in the case of a constant framing rate the Reinsch spline reduces to a least squares linear fit.

When the time values cannot be read accurately, so that the two readings $T_1$, $T_2$ of the same frame time are different, the variance of their average is estimated as $(T_1-T_2)^2/4$ as described in the next section.

Subroutine TIME calculates the new time estimates from the two sets of time data and calls subroutine REINSH which is a translation of the ALGOL 60 program of Reinsch (1967) into FORTRAN 77.

UNBIASED ESTIMATES OF VARIANCE

If $x_i$ ($i=1,n$) are normally distributed then an unbiased estimate of the variance $\text{var}(x)$ is given by:

$$\text{var}(x) = \frac{\sum_{i=1}^{n} (x_i - a)^2}{n-1}$$

where $a = \frac{\sum_{i=1}^{n} x_i}{n}$ is the average.

The variance of the average is $\text{var}(a) = \text{var}(x)/n$ so that an estimate of $\text{var}(a)$ is given by:

$$\text{var}(a) = \frac{\sum_{i=1}^{n} (x_i - a)^2}{n(n-1)}$$

The derivations of these estimates are given in Shchigolev (1965, pp.215-219).

Thus for $n=2$: $\text{var}(x) = (x_1 - a)^2 + (x_2 - a)^2 = (x_1 - x_2)^2/2$

and $\text{var}(a) = \text{var}(x)/2 = (x_1 - x_2)^2/4$

When $n=4$: $\text{var}(a) = \frac{\sum_{i=1}^{4} (x_i - a)^2}{12}$
The second of these estimates has been used in the previous section for the spline fitting of frame times. The remaining two estimates of variance will be used when fitting quintic splines to the film data and orientation angles.

**DETERMINING FRAME TIMES FROM THE DISPLACEMENT DATA**

If frame times are not available for the film taken by one camera then the vertical displacement data may be used to synchronise the two films.

Let the two cameras be named the side camera and the front camera and suppose that the frame times of the film taken by the side camera are known whilst those corresponding to the front camera are unknown. Let TF(J) be the unknown frame times and let an alternative set of time values T(J) be defined by T(J)=J where J is the frame number. If the framing rate of the front camera is constant there will be a linear relationship between the time sets T(J) and TF(J). In order to determine this relationship it is sufficient to obtain time value pairs T1,T1F and T2,T2F which give the times of events E1 and E2 in each timing system. The unknown frame times TF(J) may then be obtained from the frame numbers T(J) using the equation:

\[
\frac{TF(J) - T_{1F}}{T_{2F} - T_{1F}} = \frac{T(J) - T_1}{T_2 - T_1}
\]

The events E1 and E2 are defined to be the two crossings of the horizontal plane containing the cameras and reference frame origin by the left ankle centre. Let ZF(J) be the digitised coordinate for the front camera film corresponding to the vertical displacement of the left ankle at time T(J) and let ZF₀ be the vertical coordinate of the reference frame origin. If the pairs (T(J), ZF(J)) are fitted by a suitable function Z=Z(T) the times T1 and T2 are solutions of the equation Z(T)=ZF₀. The same procedure is followed for the side camera data using the known frame times to give time values T1S, T2S corresponding to events E1 and E2.

If the values T1F, T2F are defined by T1F=T1S, T2F=T2S the
frame times TF(J) may be calculated as described above.

If a straight line is fitted to 5 or 6 points of a data set (T(J),Z(J)) in each of the two neighbourhoods of Z=Z₀ then the times at which Z=Z₀ may be obtained analytically. The derivation of the least squares linear fit is given in the next section. Although this simple technique does introduce a systematic error the following example shows that the linear fit can produce accurate synchronisation.

Consider the case in which 5 points are taken on the parabola z=ut-\frac{1}{2}gt² at times t=-2s,-s,0,s,2s. The linear least squares fit is obtained as \( z_l = ut - \frac{1}{2}gs² \) with a maximum deviation of gs². The time at which \( z_l = 0 \) is given by \( t = gs²/u \) which corresponds to a value \( z_0 = ut - \frac{1}{2}gt² \) on the parabola. Thus the use of a straight line fit determines the time \( t \) for which \( z = z_0 = gs² \) rather than \( z = 0 \). Besides being small this difference will be approximately the same for another set of 5 or 6 points spaced around \( t = 0 \) at intervals of \( s² \) providing \( s² \approx s² \).

In order to estimate the accuracy of the method take \( u = 7 \text{ms}^{-1} \), \( g = 9.8 \text{ms}^{-2} \) and \( s = 0.015 \text{sec} \) to obtain:

\[
\begin{align*}
z₀ &\approx gs² \approx 0.002 \text{metres} \\
t &\approx gs²/u \approx 0.0003 \text{seconds}.
\end{align*}
\]

These are the systematic errors introduced by the use of a straight line fit but as remarked above the error in synchronisation will be an order of magnitude smaller than this.

The accuracy of the synchronisation is dependent upon the error \( \Delta z \) in \( z_0 \) introduced by digitisation errors. If \( \Delta z = 0.01 \text{metres} \) and \( u = 7 \text{ms}^{-1} \) then the corresponding error in time is \( \Delta t = \Delta z / u \approx 0.001 \text{sec} \).

The calculations described in this section are made in the separate computer program Z2TIME which is listed in Appendix B.

When frame times are not known for either film, frame numbers may be used instead and synchronisation may again be effected using Z2TIME.
FITTING A STRAIGHT LINE TO DATA USING THE METHOD OF LEAST SQUARES

A linear fit to the n data points \((T_j, Z_j)\) is given by \(Z = aT + b\) where \(a\) and \(b\) are constants. The least squares fit is the one for which:

\[
S = \sum_{j=1}^{n} [(aT_j + b) - Z_j]^2 \text{ is a minimum.}
\]

\[
\frac{\partial S}{\partial a} = \sum_{j=1}^{n} 2(aT_j + b - Z_j)T_j = 0 \text{ if } S \text{ is a minimum}
\]

\[
\frac{\partial S}{\partial b} = \sum_{j=1}^{n} 2(aT_j + b - Z_j) = 0 \text{ if } S \text{ is a minimum}
\]

Thus:

\[
a \sum_{j=1}^{n} T_j^2 + b \sum_{j=1}^{n} T_j - \sum_{j=1}^{n} Z_j T_j = 0
\]

and

\[
a \sum_{j=1}^{n} T_j + b \sum_{j=1}^{n} 1 - \sum_{j=1}^{n} Z_j = 0 \text{ giving } a \text{ and } b.
\]

SPLINE FITTING THE DISPLACEMENT DATA

For \(n\) data points \((t_i, x_i)\) and estimates \(\delta x_i\) of the errors in the ordinates \(x_i\) Reinsch (1967) determined the natural cubic spline \(g(t)\) for which:

\[
\int_{t_1}^{t_n} [g''(t)]^2 dt \text{ is minimum and } \sum_{i=1}^{n} (g(t_i) - x_i)^2 / \delta x_i^2 \leq n
\]

The endpoint conditions \(g''(t_1) = g''(t_n) = 0\) imply that the acceleration values are zero at the endpoints and so this spline function is unsuitable for fitting displacement data.

If the smoothness condition is replaced by the requirement:

\[
\int_{t_1}^{t_n} [g'''(t)]^2 dt \text{ is a minimum}
\]

a quintic spline is obtained for which \(g''\) is not constrained. The routine used is supplied by Dr. L.S. Jennings of the University of Western Australia.

Prior to the fitting of quintic splines the raw digitised data of the body landmarks is transformed in the following way. Using the digitised values of points on the reference frame the coordinates
of the origin of the reference frame are determined and horizontal and vertical scale factors are calculated. Using these scale factors the digitised values of the displacements of the body landmarks are converted into metres and a translation of coordinate axes is made so that the reference frame origin has the new coordinates \((0,0)\). The new displacement values are the coordinates in metres of the projections of the landmarks on a vertical plane which is normal to the optical axis of the camera and which passes through the origin of the reference frame. Since each film is digitised twice the linear transformation produces two sets of displacement values in the same coordinate system.

Let \(X_1(i)\) and \(X_2(i)\) be the two estimates of one coordinate of a landmark in frame number \(i\). The quintic spline routine requires values \(\delta X_1, \delta X_2\) for the error in each estimate. Since there are only two estimates available it is only possible to obtain a single value \(\delta X_1\) based upon the difference between \(X_1(i)\) and \(X_2(i)\). In the earlier section on unbiased estimates of variance it was shown that the variance is given by \(V_i = \frac{1}{4} (X_2(i) - X_1(i))^2\) from which \(\delta X_1\) may be obtained as \(\delta X_1 = \sqrt{V_i}\).

Any difference in the scale factors obtained for the two digitisations will introduce a systematic error in \((X_2(i) - X_1(i))\) in addition to the error arising from the digitisation of the landmark. As a consequence the calculated value of \(\delta X_1\) will be artificially high. In order to remove the systematic error arising from the slightly different linear transformations, a straight line with equation \(X_2 = aX_1 + b\) is fitted to the pairs \((X_1(i), X_2(i))\) and \(X_1(i)\) is replaced by \((aX_1(i) + b)\) in the equation defining \(V_i\).

A further complication arises when the errors in \(X_1(i)\) and \(X_2(i)\) are nearly equal so that the calculated value of \(\delta X_1\) underestimates the actual error. If \(V\) is the average of the \(V_i\) then \(\delta = \sqrt{V}\) is an estimate of the average error and the setting of each \(\delta X_1\) equal to \(\delta\) ensures that \(\delta X_1\) does not approach zero but then every large error is grossly underestimated. A compromise solution is obtained by taking \(\delta X_1 = (kV_i + (1-k)V)^{\frac{1}{4}}\) with \(0 < k < 1\) so that \(\frac{1}{4} \delta X_1^2\) is unchanged from its value of \(nV\). A choice of \(k = \frac{1}{4}\) means that a large
error is underestimated by at most a factor of 2 and a lower bound of about 0.876 is placed on the $\delta X_i$.

Quintic splines are now fitted to the 2 coordinates of the 10 landmarks for the 2 digitisations of the film from the 2 cameras resulting in a total of 80 quintic splines. Subroutine SPLIN calculates the quintic spline coefficients for the two data sets $(T(i), X1(i))$ and $(T(i), X2(i))$ where $T(i)$ are the frame times.

**INTERPOLATION OF THE DISPLACEMENT DATA**

A start time $ST$ and end time $ENDT$ are chosen to lie within the range of frame time values of both the front and side camera films. The interval $[ST, ENDT]$ is divided into 100 equally spaced sub-intervals and the quintic splines are evaluated at each of the 101 subinterval endpoints. In this way the displacement values corresponding to front and side views are synchronised. Since there are two digitisations of each view there will be four possible combinations from which the three-dimensional coordinates of landmarks may be calculated. This will lead to four estimates of each orientation angle so that error parameters may be calculated prior to the fitting of quintic splines to the orientation angles.

**CALCULATION OF THE SPATIAL COORDINATES OF THE BODY LANDMARKS**

Figure 49 shows the reference frame origin O $(0,0,0)$, the front camera $F(O, -d_f, 0)$, the side camera $S(d_s, 0, 0)$, the point $A(x_f, 0, z_f)$ in the x-z plane and the point $B(0, y_s, z_s)$ in the y-z plane. The distances $d_f$ and $d_s$ are measured during the filming session whilst the displacement coordinates $(x_f, z_f)$ and $(y_s, z_s)$ are obtained by applying linear transformations to the digitised film data as described in the earlier section on spline fitting.

The lines $FA$ and $SB$ may be expected to pass through the landmark location $R$ but because of digitising errors $FA$ and $SB$ will not intersect in general. In this case the point $R$ is defined to be the point which is closest to the lines $FA$ and $SB$ in the following sense.
Figure 49. Reconstruction of spatial location of a landmark

Figure 50. Location of the point R relative to P and Q
Let \( P, Q \) and \( R \) be chosen so that \( P \) is on \( FA \), \( Q \) is on \( SB \) and \((dRP)^2 + (dRQ)^2\) is a minimum where \( dRP \) denotes the distance from \( R \) to \( P \).

\( R \) must lie on \( PQ \) since otherwise \((dRP)^2 + (dRQ)^2 > (dR_1P)^2 + (dR_1Q)^2\) where \( R_1 \) is the foot of the perpendicular from \( R \) onto \( PQ \) (Figure 50). Of all points on \( PQ \) the midpoint \( R \) gives the minimum value of \((dRP)^2 + (dRQ)^2 = \frac{1}{4}(dPQ)^2\). Of all choices of \( P \) and \( Q \) the distance \( dPQ \) is a minimum when \( PQ \) is the common perpendicular to \( FA \) and \( SB \). Thus the point \( R \) is the midpoint of the common perpendicular to the lines \( FA \) and \( SB \) (Figure 49).

The points \( P \) and \( Q \) are defined by the vectors \( \mathbf{OP} = \mathbf{OF} + p\mathbf{FA} \) and \( \mathbf{OQ} = \mathbf{OS} + q\mathbf{SB} \) where the scalars \( p \) and \( q \) are chosen so that the dot products \( \mathbf{PQ} \cdot \mathbf{FA} \) and \( \mathbf{PQ} \cdot \mathbf{SB} \) are zero.

Thus
\[
0 = \mathbf{PQ} \cdot \mathbf{FA} = (\mathbf{OQ} - \mathbf{OP}) \cdot \mathbf{FA} = (\mathbf{OS} + q\mathbf{SB} - p\mathbf{FA}) \cdot \mathbf{FA}
\]
and
\[
0 = \mathbf{PQ} \cdot \mathbf{SB} = (\mathbf{OQ} - \mathbf{OP}) \cdot \mathbf{SB} = (\mathbf{OS} + q\mathbf{SB} - p\mathbf{FA}) \cdot \mathbf{SB}
\]
so that \( p \) and \( q \) are given by the simultaneous linear equations:
\[
(FS \cdot FA) + q(SB \cdot FA) - p(FA \cdot FA) = 0
\]
\[
(FS \cdot SB) + q(SB \cdot SB) - p(FA \cdot SB) = 0
\]
Finally the midpoint \( R \) is defined by the position vector \( \mathbf{OR} = \frac{1}{2}(\mathbf{OP} + \mathbf{OQ}) \). The various calculations are made by the subroutine COORD.

**CORRECTIONS FOR CAMERA MISALIGNMENTS**

Cameras \( F \) and \( S \) may be positioned as shown in Figure 49 so that the plane \( FOS \) is horizontal, angle \( FOS \) is a right angle and the distances \( dFO, dSO \) can be measured accurately. It is not so easy to align the optical axes of the cameras along \( FO \) and \( SO \) with accuracy. In addition it may be desirable to incline the camera axes so that the field of view of each camera accurately frames the region in which movement occurs. This section describes a method of determining the camera inclinations and makes appropriate transformations of the image coordinates prior to the calculation of the spatial location of landmarks by subroutine COORD.

Figure 51 shows the four points \( C_1, C_2, C_3, C_4 \) of the reference
frame which are used to establish the horizontal and vertical scale factors for the transformation of the digitised coordinate data of the film taken by the side camera S. Each of the four points is one metre from the origin 0 of the reference frame with \( C_3 C_1 \) directed along the horizontal axis \( i_2 \) and \( C_4 C_2 \) directed along the vertical axis \( i_3 \). There are two additional points on axis \( i_1 \) so that the horizontal scale factor of the film taken by the front camera can be calculated. The spatial locations of the four points are \( C_1(0,0,1) \), \( C_2(0,0,-1) \), \( C_3(0,1,0) \), \( C_4(0,-1,0) \) where the suffix \( i \) denotes the coordinate frame comprising the orthogonal axes \( i_1, i_2, i_3 \) with origin 0.

Using the scale factors the digitised \( y-z \) values of the four reference points and body landmarks are transformed by linear equations to give normalised image coordinates. The image coordinates of the four reference points are \( C_1(y_1, z_1) \), \( C_2(y_2, z_2) \), \( C_3(y_3, z_3) \), \( C_4(y_4, z_4) \) where \( y_1 = 1 \), \( y_3 = -1 \), \( z_2 = 1 \), \( z_4 = -1 \). If the axis of camera S is aligned precisely along the line \( SO \) the image coordinates of the landmark R will be the coordinates in metres of the projection B of R on the plane \( i_2 i_3 \) (Figure 52).

If the camera axis is not parallel to axis \( i_1 \) define a coordinate frame \( S \) with origin 0 and orthogonal axes \( s_1, s_2, s_3 \) such that axis \( s_1 \) is parallel to the camera axis and axis \( s_3 \) lies in the vertical plane \( i_1 i_3 \). Let the landmark R have normalised image coordinates \( (y_n, z_n) \) and let \( SR \) meet the plane \( s_2 s_3 \) at the point \( B_S \) with coordinates \( (0, y_s, z_s) \) and \( (x_i, y_i, z_i) \) in the coordinate frames \( S \) and \( i \). Let \( SR \) meet the plane \( i_2 i_3 \) at the point \( B(0, y, z) \).

If the \( z \)-axis of the digitiser has been aligned along the image of the line \( C_4 C_2 \) and if the optical systems of the camera and digitiser produce undistorted images of the projections of points onto the plane \( s_2 s_3 \) the image coordinates \( y_n \) and \( z_n \) will be linearly related to the coordinates \( y_s \) and \( z_s \). Thus there exist constants \( k_y \), \( e_y \), \( k_z \), \( e_z \) such that:

\[
y_s = k_y y_n + e_y \quad (1)
\]

and

\[
z_s = k_z z_n + e_z \quad (2)
\]
Figure 51. Locations of the four reference points

Figure 52. The geometry for inclined camera axes
The orientation of the coordinate frame $s$ relative to frame $i$ may be specified by angles $\theta_s$ and $\psi_s$ where successive rotations of frame $i$ through $\theta_s$ about $i_2$ and $\psi_s$ about $i_3$ will bring frame $i$ into coincidence with frame $s$.

Once the 6 unknowns $k_x, e_y, k_z, e_z, \theta_s, \psi_s$ have been determined it will be possible to calculate the coordinates $(x_i, y_i, z_i)$ of the point $B_s$ from the image coordinates $(y_n, z_n)$ of the landmark $R$ so that the direction of the line $SB$ may be obtained for use in subroutine COORD.

Let $S_{si}$ be the matrix which transforms the coordinates of a point in frame $s$ into the coordinates in frame $i$. The frame $s$ will be brought into alignment with frame $i$ by successive rotations through angles $-\psi_s$ and $-\theta_s$ about axes $s_3$ and $s_2$. Let $R_3(-\psi_s)$ be the rotation matrix which transforms $s$-coordinates to the intermediate frame and let $R_2(-\theta_s)$ be the rotation matrix which transforms the intermediate coordinates into $i$-coordinates. The columns of these rotation matrices are merely the new coordinates of the previous directions of the unit vectors $s_1, s_2, s_3$.

Thus

$$R_2(-\theta_s) = \begin{bmatrix} \cos \theta_s & 0 & \sin \theta_s \\ -\sin \theta_s & 0 & \cos \theta_s \end{bmatrix}$$

and

$$R_3(-\psi_s) = \begin{bmatrix} \cos \psi_s & -\sin \psi_s & 0 \\ \sin \psi_s & \cos \psi_s & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

so that

$$S_{si} = R_2(-\theta_s).R_3(-\psi_s) = \begin{bmatrix} \cos \theta_s \cos \psi_s & -\cos \theta_s \sin \psi_s & \sin \theta_s \\ \sin \theta_s \cos \psi_s & \sin \theta_s \sin \psi_s & \cos \theta_s \\ -\sin \psi_s & \cos \psi_s & 0 \end{bmatrix}$$

The point $B_s$ has coordinates $(0, y_s, z_s)$ in frame $s$ and coordinates $(x_i, y_i, z_i)$ in frame $i$.

Thus

$$S_{si} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} = \begin{bmatrix} O \\ y_s \\ z_s \end{bmatrix} = \begin{bmatrix} -y_s \cos \theta_s \sin \psi_s + z_s \sin \theta_s \\ y_s \cos \psi_s \\ y_s \sin \theta_s \sin \psi_s + z_s \cos \theta_s \end{bmatrix}$$
The points \( S(d^i_s,0,0)_i, B_s(x^i_s,y^i_s,z^i_s)_i \) and \( B(0,y,z)_i \) are collinear so that the vectors \( S_B_s \) and \( S_B \) are parallel. The collinearity conditions may be written as \( (d-x^i_s)/d^i_s=y^i_s/y \) and \( y^i_s/z=Z^i_s/z \).

Substituting for \( x^i_s, y^i_s, z^i_s \) and separating \( y^i_s \) from \( z^i_s \) gives:

\[
y^i_s(d_s \cos \theta \cos \psi - y^i_s \sin \psi + z^i_s \sin \theta \cos \psi) = d_s y^s \cos \theta \\
z^i_s(d_s \cos \theta \cos \psi - y^i_s \sin \psi + z^i_s \sin \theta \cos \psi) = d_s z^s \cos \psi
\] (3) (4)

Equations (1), (2), (3) and (4) express the image coordinates \( (y^i_n, z^i_n) \) in terms of the spatial coordinates \( (0,y,z)_i \) of the point \( B \).

The reference points \( C_1(0,1,0)_i, C_2(0,0,1)_i, C_3(0,-1,0)_i, C_4(0,0,-1) \) have image coordinates \( (1,z^1), (y^2,1), (-1,z^3), (y^4,-1) \) and substitution of these values results in the following four equations:

\[
k_y = d_s^2 \cos^2 \theta \cos \psi / (d_s^2 \cos^2 \theta - \sin^2 \psi) \\
e_y = -d_s \cos \theta \sin \psi / (d_s^2 \cos^2 \theta - \sin^2 \psi) \\
k_z = d_s^2 \cos^2 \theta / (d_s^2 \cos^2 \theta - \sin^2 \theta) \\
e_z = d_s \sin \theta / (d_s^2 \cos^2 \theta - \sin^2 \theta)
\] (5) (6) (7) (8)

Before the six unknowns \( k_y, e_y, k_z, e_z, \theta, \psi \) can be determined a further two equations relating them must be obtained.

The image coordinates \( (y^m_m, z^m_m) \) of the centre of the frame image may be obtained by averaging the image coordinates of the four corners of the film frame. Let \( M_s(0,y_s,z_s)_s \) be the corresponding point in the plane \( s_2s_3 \) so that the line \( SM_s \) will be parallel to the camera axis \( s_1 \).

Equations (1) and (2) give \( y^s = k_y y^m + e^y \) and \( z^s = k_z z^m + e^z \) so that:

\[
C_{M_s} = \begin{bmatrix} y^s \\ z^s \end{bmatrix} = \begin{bmatrix} 0 \\ \begin{bmatrix} y^m \\ z^m \end{bmatrix} + \begin{bmatrix} e^y \\ e^z \end{bmatrix} \end{bmatrix}
\]
If \( S_{is} \) is the matrix which transforms coordinates in frame \( i \) to coordinates in frame \( s \) then \( S_{is} = R_3(\psi_s) \cdot R_2(\theta_s) \) where:

\[
R_2(\theta_s) = \begin{bmatrix}
\cos \theta_s & 0 & -\sin \theta_s \\
0 & 1 & 0 \\
\sin \theta_s & 0 & \cos \theta_s
\end{bmatrix}
\]

and

\[
R_3(\psi_s) = \begin{bmatrix}
\cos \psi_s & \sin \psi_s & 0 \\
-\sin \psi_s & \cos \psi_s & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Thus \( S_0 = S_{is} \begin{bmatrix}
-d_s \\
0 \\
0
\end{bmatrix} = \begin{bmatrix}
-d_s \cos \theta_s \cos \psi_s \\
d_s \cos \theta_s \sin \psi_s \\
-d_s \sin \theta_s
\end{bmatrix} \)

Now \( S_{M_s} = S_0 + O_{M_s} = \begin{bmatrix}
-d_s \cos \theta_s \cos \psi_s \\
d_s \cos \theta_s \sin \psi_s + (k_y \cdot y_m + e_y) \\
-d_s \sin \theta_s + (k_z \cdot z_m + e_z)
\end{bmatrix} \)

and \( S_{M_s} \) will be parallel to axis \( s_1 \) when the dot products \( S_{M_s} \cdot s_2 \) and \( S_{M_s} \cdot s_3 \) are zero. These two conditions produce the required additional two equations which may be written in the form:

\[
\sin \theta_s = (k_z \cdot z_m + e_z)/d_s \tag{9}
\]

and

\[
\sin \psi_s = -(k_y \cdot y_m + e_y)/d_s \cos \theta_s \tag{10}
\]

Equations (5) through (10) comprise 6 non-linear equations in the six unknowns \( k_y, e_y, k_z, e_z, \theta_s \) and \( \psi_s \). They may be solved by an iterative process in the following way:

Initially set \( k_y = 1, e_y = 0, k_z = 1, e_z = 0 \)

Equation (9) gives the first approximation to \( \sin \theta_s \) from which \( \cos \theta_s \) may be calculated. Equation (10) then gives \( \sin \psi_s \) from which \( \cos \psi_s \) is calculated.

Equations (5), (6), (7), (8) provide new values for \( k_y, e_y, k_z, e_z \) and the process is repeated until suitable accuracy is achieved.

Once values have been obtained for \( k_y, e_y, k_z, e_z, \theta_s \) and \( \psi_s \) the
image coordinates \((y_n, z_n)\) of a point \(R\) may be used to obtain the
\(i\)-coordinates of the vector \(SB_s\) for use in subroutine COORD.

\[
\begin{bmatrix}
0 \\
y_s \\
z_s
\end{bmatrix} = \begin{bmatrix}
x_i \\
y_i \\
z_i
\end{bmatrix}
\]

where \(y_s = k_y y_n + e_y\) and \(z_s = k_z z_n + e_z\)

and then \(SB_s = SO + OB_s\).

Exactly the same procedure is followed for the front camera
which is located at the point \(F(O, df, 0)_i\). If the coordinate frame
\(i\) is brought into coincidence with the camera frame \(f\) by successive
rotations through \(\phi_f\) about \(i_1\) and \(\psi_f\) about \(i_3\) equivalents to
equations (5) through (10) are obtained by changing \(\theta_s\) to \(\phi_f\), \(\psi_s\) to
\(\psi_f\), \(d_s\) to \(df\), \(k_y\) to \(k_x\) and \(e_y\) to \(e_x\).

Once values have been obtained for \(k_x, e_x, k_z, e_z, \phi_f\) and \(\psi_f\) the
\(i\)-coordinates of the point \(A_f(x_f, 0, z_f)_f\) may be obtained from the
image coordinates \((x_p, z_p)\) as:

\[
\begin{bmatrix}
x_f \\
0 \\
z_f
\end{bmatrix} = S_{fi} \begin{bmatrix}
x_i \\
0 \\
z_i
\end{bmatrix}
\]

in frame \(i\) where \(x_f = k_x x_p + e_x\) and \(z_f = k_z z_p + e_z\)

(It should be noted that the values of \(k_z\) and \(e_z\) will be different
from those obtained for the side camera analysis). The vector \(FA_f\)
is then evaluated in frame \(i\) as \(FA_f = FO + OA_f\) for use in subroutine
COORD.

CALCULATION OF THE ORIENTATION ANGLES

Once the spatial coordinates of the ten body landmarks have
been obtained the sine and cosine of each orientation angle are
determined as indicated in the following subsections. Each
orientation angle is then calculated from its sine and cosine using
an incremental technique which avoids the discontinuities inherent
in the simple application of inverse trigonometric functions. Since
the films taken by the front and side cameras are digitised twice
there are four combinations of the digitised data and this leads
to four estimates of each orientation angle. The standard error of
the average of the four estimates is calculated at each time value
and a quintic spline is fitted to the average values for each
orientation angle.

ORIENTATION OF THE WHOLE BODY

The whole body orientation in space is specified by the
orientation of a reference frame f in the body relative to the
frame of reference i which is fixed in space. The frame i comprises
a right-handed triad \( (i_1, i_2, i_3) \) of unit vectors (Figure 51) and is
the same as the filming reference frame i described in the previous
section. The frame f is not fixed relative to one particular body
segment but is defined so as to be representative of the orientation
of the whole system. Frame f comprises a right-handed triad \( (f_1, f_2, f_3) \) of mutually orthogonal unit vectors which are defined as
follows:

\[
\begin{align*}
    f_3 & \text{ is a unit vector parallel to }QN \text{ where } Q \text{ is the midpoint of} \\
    & \text{the knee centres and } N \text{ is the midpoint of the shoulder centres.} \\
    & \text{(The calculation of } Q \text{ is given in the subsection describing the} \\
    & \text{orientation of the legs).} \\
    \text{Let } p_1 & \text{ be a unit vector parallel to the line }IH \text{ joining hip} \\
    & \text{centres. (The calculation of } p_1 \text{ is given in the next subsection).} \\
    f_2 & \text{ is a unit vector parallel to the vector product } f_3 \times p_1 \\
    f_1 & = f_2 \times f_3.
\end{align*}
\]

The orientation of frame f relative to frame i is given by the
angles \( \phi, \theta, \psi \) where the coordinate frame i is brought into alignment
with frame f by successive rotations through \( \phi \) about \( i_1 \), \( \theta \) about \( i_2 \)
and \( \psi \) about \( i_3 \) (Figure 53). \( \phi \) will be called the somersault angle,
\( \theta \) the angle of tilt and \( \psi \) the twist angle (Figure 54).

Let \( S_{fi} \) be the matrix which transforms the f-coordinates of a
vector into i-coordinates. Frame f will be brought into alignment
with frame i by successive rotations through \( -\psi \) about \( f_3 \), \(-\theta \) about \( f_2 \).
Figure 53. Orientation of frame $f$ relative to frame $i$

Figure 54. Angles of somersault, tilt and twist
and -ϕ about f₁. Let R₃(-ψ), R₂(-θ), and R₁(-ψ) be the rotation matrices corresponding to the successive rotations. The columns of these rotation matrices are merely the new coordinates of the previous directions of the unit vectors f₁, f₂ and f₃.

Thus:

\[
R₁(-ψ) = \begin{bmatrix}
1 & 0 & 0 \\
0 & cψ & -sψ \\
0 & sψ & cψ
\end{bmatrix}
\]

\[
R₂(-θ) = \begin{bmatrix}
cθ & 0 & sθ \\
0 & 1 & 0 \\
-sθ & 0 & cθ
\end{bmatrix}
\]

\[
R₃(-ψ) = \begin{bmatrix}
cψ & -sψ & 0 \\
sψ & cψ & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

where c denotes cosine and s denotes sine.

Similarly:

\[
S_{fi} = F₁(1) F₂(1) F₃(1) \quad F₁(2) F₂(2) F₃(2) \quad F₁(3) F₂(3) F₃(3)
\]

where Fk(j) denotes the jth component of f_k in frame i.

\[
S_{fi} = R₁(-ψ).R₂(-θ).R₃(-ψ) \quad giving:
\]

\[
S_{fi} = \begin{bmatrix}
cθcψ & -cθsψ & sθ \\
cψsψ+sψsθcψ & cθcψ-sψsθsψ & -sψcθ \\
sψsψ-cψsθcψ & sψcψ+cψsθsψ & cψcθ
\end{bmatrix}
\]

Comparing corresponding elements in the two expressions for S_{fi} permits the calculation of the sine and cosine of each orientation angle as:
\[ \tan \theta = F_3(1) \]
\[ \cos \theta = (1 - \sin^2 \theta)^{1/2} \] so that \( \theta \) lies in the range \(-\pi/2 < \theta < \pi/2\)
\[ \sin \phi = -F_3(2)/\cos \theta \]
\[ \cos \phi = F_3(3)/\cos \theta \]
\[ \sin \psi = -F_2(1)/\cos \theta \]
\[ \cos \psi = F_1(1)/\cos \theta \]

The components \( F_k(j) \) in frame \( i \) are calculated from the \( i \)-coordinates of the body landmarks.

**THE PIKE AND HULA ANGLES**

The orientations of the thorax \( T \) and legs \( L \) relative to the pelvis \( P \) are defined in terms of the pike angle \( \gamma \) and the hula angle \( \psi_p \). These angles are used because they are easily visualised and are the natural parameters to use for specifying hula movements.

Figure 55 shows the midpoint \( Q \) of the knee centres, the midpoint \( G \) of the hip centres \( H \) and \( I \), the junction \( X \) of the pelvis and thorax and the midpoint \( N \) of the shoulder centres. The point \( X \) was not used as a landmark since it is difficult to locate from a film image. In order to determine the location of \( X \) from the landmark locations it is assumed that the points \( Q, G, X, N \) are coplanar so that \( QG \) and \( NX \) intersect at a point \( O \). The pike angle \( \gamma \) is defined to be the angle \( NOQ \) between the longitudinal axis \( XN \) of the thorax and the midline \( QG \) of the upper legs.

The hula angle \( \psi_p \) is the angle between the pike plane \( QON \) and the plane through \( G \) which is normal to the hip-line \( IH \). When \( \psi_p = 0 \) the point \( Q \) will coincide with \( Q_0 \) and the configuration will be called a forward pike. When \( \psi_p = \frac{\pi}{2} \) the point \( Q \) will coincide with \( Q_1 \) and the body will be in a side pike with the right hip \( I \) interior to the pike angle \( NOQ \).

Let \( p_1, p_2, p_3 \) be orthogonal unit vectors with \( p_1 \) parallel to \( IH \) and \( p_3 \) parallel to \( GX \) (Figure 56). Let \( l_3 \) be a unit vector parallel to the midline \( QG \) of the upper legs and let \( \beta \) be the angle between \( l_3 \) and \( p_3 \). \( \psi_p \) is the angle between the pike plane \( p_3 l_3 \) and the
Figure 55. The pike angle $\gamma$ and the hula angle $\psi_p$

Figure 56. Unit vectors defining the hula angle
plane $p_2 p_3$ and so $l_3$ will have components $L3(1)$, $L3(2)$, $L3(3)$ in the directions $p_1, p_2, p_3$ where:

$$L3(1) = \sin \beta \sin \psi_p$$

$$L3(2) = \sin \beta \cos \psi_p$$

$$L3(3) = \cos \beta$$

Thus the sine and cosine of $\psi_p$ are obtained as:

$$\sin \beta = (L3(1)^2 + L3(2)^2)^{\frac{1}{2}}$$

$$\sin \psi_p = \frac{L3(1)}{\sin \beta}$$

$$\cos \psi_p = \frac{L3(2)}{\sin \beta}$$

Before these calculations can be made it is necessary to make some assumptions about the amounts of flexion occurring at $X$ and $G$ (Figure 55). In a forward pike it will be assumed that flexion occurs only at the hip centre $G$ providing the pike angle $\gamma$ is greater than a right angle (Figure 57). For flexion beyond a right angle it is assumed that the additional flexion is shared equally at the points $X$ and $G$ so that $\alpha = \frac{1}{2} (\pi \gamma)$ and $\beta = \frac{1}{2} \pi + \frac{1}{2} (\pi \gamma)$ (Figure 58). In a side pike or back arch position it is assumed that the flexion is shared equally at the point $X$ and $G$ so that $\alpha = \beta = \frac{1}{2} (\pi \gamma)$ (Figure 59).

(Whilst these assumptions are not unreasonable it should be observed that spinal flexion occurs along the length of the spine and not at a single point $X$. Such assumptions may be said to be justified providing there is close agreement between the values of $\phi, \theta$ and $\psi$ as determined by film analysis and by simulation).

The angle $\alpha$ between the pelvis axis $GX$ and the thorax axis $XN$ is defined as a continuous function $f(\gamma, \psi_p)$ of $\gamma$ and $\psi_p$ as follows:

If $\cos \psi_p > 0$ and $\gamma > \frac{1}{2} \pi$ then $\alpha = \frac{1}{2} (\pi \gamma) \sin^2 \psi_p$

If $\cos \psi_p > 0$ and $\gamma < \frac{1}{2} \pi$ then $\alpha = \frac{1}{2} (\pi \gamma) \sin^2 \psi_p + \frac{1}{2} (\pi \gamma) \cos^2 \psi_p$

If $\cos \psi_p < 0$ then $\alpha = \frac{1}{2} (\pi \gamma)$

The angle $\beta$ between the midline $OG$ of the upper legs and the pelvis axis $GX$ is defined by $\beta = \pi - \alpha$ so that $\alpha + \beta + \gamma = \pi$. 
Figures 57, 58. Angles of flexion for forward pike

Figure 59. Angles of flexion for side pike or back arch
The pike and hula angles may now be calculated from the landmark locations. The points N and G are obtained as the midpoints of the shoulder centres and hip centres whilst the midpoint Q of the knee centres is obtained as described in the subsection on the orientation of the legs. Next the distances dGQ and dGN and the angle QGN=γ are evaluated. The link lengths LP=dGX and LT=dXN are provided by the file PERFR. Ψ_p and γ are calculated iteratively in the following manner:

An initial estimate of Ψ_p is obtained by supposing that GX is parallel to QN. The unit vectors p_1, p_2, p_3 are then obtained as:

\[ p_3 \text{ is parallel to } GX \]
\[ p_2 \text{ is parallel to the vector product } p_3 \times IH \]
\[ p_1 = p_2 \times p_3 \]

(It should be noted that even when p_3 is accurately directed the vector p_1 will not be exactly parallel to IH since the locations of I and H are subject to digitisation errors).

\[ \lambda_3 \text{ is a unit vector parallel to } QG. \]

\[ \sin \psi_p \text{ and } \cos \psi_p \text{ are then calculated from the components of } \lambda_3 \text{ in frame } p \text{ as described earlier.} \]

\[ g \text{ is used as an initial estimate of } \gamma \text{ (Figure 60)} \]
\[ \alpha \text{ is obtained as the function } f(\gamma, \psi_p) \text{ defined above} \]
\[ \beta = \pi - \gamma - \alpha \]
\[ \sin(e) = \frac{dXN}{dGN} \sin \alpha \text{ (applying the Sine Rule to triangle GXN)} \]
\[ e = \sin^{-1}(\sin(e)) \]
\[ \gamma g/\left(\pi - \beta - e\right) \text{ is used as a corrected estimate for } \gamma \text{ and the iteration is continued until the calculated value of } \gamma \text{ changes by less than } 0.01 \text{ radians.} \]

\[ dGO = dGX(\sin \psi / \sin \gamma) \text{ (applying the Sine Rule to triangle GOX)} \]
\[ O \text{ is obtained as the point dividing } GQ \text{ in the ratio } -dGO/dGQ \]
\[ dOX = dGX(\sin \beta / \sin \gamma) \text{ (applying the Sine Rule to triangle GOX)} \]
\[ X \text{ is obtained as the point dividing } ON \text{ in the ratio } dOX/dON. \]

Thus a new estimate is obtained for GX and the whole process
Figure 60. Calculation of pike and hula angles
is repeated until the calculated values of \( \sin \psi_p \) and \( \cos \psi_p \) change by less than 0.01.

**ORIENTATION OF THE LEGS**

The orientation of the midline QQ of the upper legs relative to the pelvis is given by the angles \( \psi_p \) and \( \beta \) defined above. A measure of the angle of abduction of each leg is obtained as follows:

Figure 61 shows the midpoint G of the hip centres H, I and the midpoint P of the ankle centres Y, Z. If \( \epsilon_{jk} \) is the angle between GP and HY and GP is perpendicular to both GH and PY the angle between GP and IZ will also equal \( \epsilon_{jk} \) where \( \sin \epsilon_{jk} = (dPY - dGH) / dHY \). The link length \( L_2P \) given by the file PERFR is used in preference to the distance \( dGH \) which is subject to digitisation errors and the average leg length \( \frac{1}{2}(dHY + dIZ) \) replaces \( dHY \) so that \( \epsilon_{jk} \) is given by:

\[
\begin{align*}
\sin \epsilon_{jk} &= (dPY - L_2P) / \left( \frac{1}{2}(dHY + dIZ) \right) \\
\cos \epsilon_{jk} &= (1 - \sin^2 \epsilon_{jk})^{\frac{1}{2}}
\end{align*}
\]

Since the filmed movements were to be performed with straight legs it was not thought necessary to digitise the knee centres. However since the total leg length \( L_J \) and the upper leg length \( L_J1 \) are provided by file PERFR it is possible to calculate the knee angle \( \gamma_k \) by applying the cosine rule to the triangle shown in Figure 62. Here \( L_J2 = L_J - L_J1 \) is the lower leg length and \( dJ = \frac{1}{2}(dHY + dIZ) \) is the average distance between hip and ankle. Unfortunately the calculated value of \( \gamma_k \) is particularly sensitive to errors in \( dJ \) so that the method should not be used if the knee centre locations are known.

Having found \( \gamma_k \) the left knee centre \( O_j \) is located at a distance \( h \) from HY where \( h = \frac{dHY}{L_J1} \). \( \sin \gamma_j = \text{twice area of triangle } H_0jY \) in Figure 63. The right knee centre \( O_k \) is located in the same way and the midpoint \( Q \) of \( O_j \) and \( O_k \) is obtained for use in the calculation of the orientation angles of the previous two subsections.
Figure 61. The angle of leg abduction

Figure 62. Calculation of knee angle

Figure 63. Location of knee centre
ORIENTATION OF THE CHEST

Figure 64 shows the midpoint N of the shoulder centres S, R and the junction X of the thorax with the pelvis. The coordinate reference frame t of the thorax comprises unit vectors \(t_1, t_2, t_3\) defined as:

- \(t_3\) is parallel to XN
- \(t_2\) is parallel to the vector product \(t_3 \times p_1\) where \(p_1\) is the unit vector of the pelvis defined in the subsection on the pike and hula angles.
  \[t_1 = t_2 \times t_3.\]

The chest reference frame c comprises the unit vectors \(c_1, c_2, c_3\) defined by:

- \(c_1\) is parallel to RS
- \(c_3\) is parallel to \(c_1 \times t_2\)
- \(c_2 = c_3 \times c_1\)

The orientation of frame c relative to frame t is given by the angles \(\theta_c, \psi_c\). If frame c is initially aligned with frame t then successive rotations through \(-\theta_c\) about \(c_2\) and \(\psi_c\) about \(c_3\) bring frame c into its final orientation (Figure 64).

Let \(S_{ct}\) be the matrix which transforms c-coordinates into t-coordinates. Frame c will be brought into alignment with frame t by successive rotations through \(-\theta_c\) about \(c_3\) and \(\psi_c\) about \(c_2\). If \(R_3(-\psi_c)\) and \(R_2(\theta_c)\) are the rotation matrices corresponding to these rotations then:

\[
R_2(\theta_c) = \begin{bmatrix}
\cos \theta_c & 0 & -\sin \theta_c \\
0 & 1 & 0 \\
\sin \theta_c & 0 & \cos \theta_c
\end{bmatrix},
\quad
R_3(-\psi_c) = \begin{bmatrix}
\sin \psi_c & \cos \psi_c & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

Since \(S_{ct} = R_2(\theta_c) \cdot R_3(-\psi_c)\) the first column of \(S_{ct}\) is:

\[
\begin{bmatrix}
\cos \theta_c \\
\sin \psi_c \\
\sin \theta_c \cos \psi_c
\end{bmatrix} = \begin{bmatrix}
Cl(1) \\
Cl(2) \\
Cl(3)
\end{bmatrix}
\]
Figure 64. Orientation of the chest

Figure 65. Orientation of the lower arm

Figure 66. Orientation of the upper arm
where $C_1(j)$ is the $j$th coordinate of $c_1$ in frame $t$.

Thus $\theta_c$ and $\psi_c$ are obtained as:

\[
\begin{align*}
\sin\psi_c &= C_1(2) \\
\cos\psi_c &= (1 - \sin^2\psi_c)^{\frac{1}{2}} \\
\cos\theta_c &= C_1(1)/\cos\psi_c \\
\sin\theta_c &= C_1(3)/\sin\psi_c
\end{align*}
\]

**ORIENTATION OF THE ARMS**

Figure 65 shows the left shoulder $S$, left elbow $0_a$ and left wrist $W$. The coordinate frame $a_1$ of the left upper arm comprises unit vectors $a_{11}, a_{12}, a_{13}$ and the direction of the left lower arm is given by the unit vector $a_{23}$ where:

- $a_{13}$ is parallel to $0_a S$
- $a_{23}$ is parallel to $W_0_a$
- $a_{12}$ is parallel to $a_{13} \times a_{23}$
- $a_{11} = a_{12} \times a_{13}$

If frame $a_1$ is initially aligned with frame $c$ then successive rotations through $-\delta_a$ about $a_{11}, -\epsilon_a$ about $a_{12}, \psi_a$ about $a_{13}$ bring frame $a_1$ into its final orientation (Figure 66).

Let $S_{alc}$ be the matrix which transforms $a_1$-coordinates into $c$-coordinates. Frame $a_1$ will be brought into alignment with frame $c$ by successive rotations through $-\psi_a$ about $a_{13}, \epsilon_a$ about $a_{12}$ and $\delta_a$ about $a_{11}$. If $R_3(-\psi_a), R_2(\epsilon_a)$ and $R_1(\delta_a)$ are the corresponding rotation matrices:

\[
\begin{align*}
R_1(\delta_a) &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\delta_a & s\delta_a \\ 0 & -s\delta_a & c\delta_a \end{bmatrix} \\
R_2(\epsilon_a) &= \begin{bmatrix} c\epsilon_a & 0 & -s\epsilon_a \\ 0 & 1 & 0 \\ s\epsilon_a & 0 & c\epsilon_a \end{bmatrix} \\
R_3(\psi_a) &= \begin{bmatrix} c\psi_a & -s\psi_a & 0 \\ s\psi_a & c\psi_a & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
S_{alc} &= \begin{bmatrix} A_{11}(1) & A_{12}(1) & A_{13}(1) \\ A_{11}(2) & A_{12}(2) & A_{13}(2) \\ A_{11}(3) & A_{12}(3) & A_{13}(3) \end{bmatrix}
\end{align*}
\]
where \( c \) denotes cosine, \( s \) denotes sine and \( \text{Alk}(j) \) is the \( j \)th coordinate of \( \text{Alk}_k \) in frame \( c \).

\[
S_{\text{alc}} = R_1(\delta_a) \cdot R_2(\varepsilon_a) \cdot R_3(-\psi_a)
\]

\[
\begin{bmatrix}
\cos\varepsilon\psi & -\cos\varepsilon\psi & -\sin\varepsilon \\
\sin\delta\cos\varepsilon\psi + \cos\delta\sin\varepsilon & -\sin\delta\cos\varepsilon\psi + \cos\delta\sin\varepsilon & \sin\delta \\
\cos\delta\cos\varepsilon\psi - \sin\delta\sin\varepsilon & -\cos\delta\cos\varepsilon\psi + \sin\delta\sin\varepsilon & \cos\delta
\end{bmatrix}
\]

Comparing corresponding elements in the two expressions for \( S_{\text{alc}} \) permits the calculation of the sine and cosine of each orientation angle as:

\[
\sin\varepsilon_a = -\text{Alk}_3(1) \\
\cos\varepsilon_a = (1 - \sin^2\varepsilon_a)^{\frac{1}{2}} \text{ so that } \varepsilon_a \text{ lies in the range } -\frac{\pi}{2} < \varepsilon_a < \frac{\pi}{2}
\]

\[
\sin\delta_a = \text{Alk}_3(2)/\cos\varepsilon_a \\
\cos\delta_a = \text{Alk}_3(3)/\cos\varepsilon_a \\
\sin\psi_a = -\text{Alk}_2(1)/\cos\varepsilon_a \\
\cos\psi_a = \text{Alk}_1(1)/\cos\varepsilon_a
\]

The orientation of the lower arm relative to the upper arm is given by the elbow angle \( \gamma_a \) (Figure 65) where \( -\cos\gamma_a \) equals the dot product \( \text{Alk}_3 \cdot \text{Alk}_2 \).

A similar procedure is followed for the right arm with unit vectors \( \text{bl}_1, \text{bl}_2, \text{bl}_3, \text{b2}_3 \) defined in terms of the right shoulder \( R \), elbow \( O_b \) and wrist \( V \) by:

\[
\begin{align*}
\text{bl}_3 & \text{ is parallel to } O_b R \\
\text{b2}_3 & \text{ is parallel to } V O_b \\
\text{bl}_2 & \text{ is parallel to } b2_3 \times b1_3 \\
\text{bl}_1 & = \text{bl}_2 \times \text{bl}_3
\end{align*}
\]

If frame \( \text{bl} \) of the right upper arm is initially aligned with frame \( c \) then successive rotations through \( -\delta_b \) about \( \text{bl}_1 \), \( \varepsilon_b \) about \( \text{bl}_2 \), \( -\psi_b \) about \( \text{bl}_3 \) bring \( \text{bl} \) into its final orientation.

These orientation angles have been defined in such a way that
the arms are symmetrical relative to the chest when $\delta_a = \delta_b$, $\epsilon_a = \epsilon_b$, $\psi_a = \psi_b$ and $\gamma_a = \gamma_b$ where $\gamma_b$ is the angle at the right elbow. Starting from positions aligned with frame c: increasing $\delta_a$, $\delta_b$ lifts the arms forwards, increasing $\epsilon_a$, $\epsilon_b$ abducts the arms and increasing $\psi_a$, $\psi_b$ rotates the lower arms outwards.

The calculation of the sine and cosine of each of the 17 orientation angles $\phi, \theta, \psi, \gamma_p, \epsilon_i, \gamma_c, \delta_c, \epsilon_a, \psi_a, \gamma_a, \delta_b, \epsilon_b, \psi_b, \gamma_b$ is performed by subroutine ORANG.

**CALCULATION OF AN ANGLE FROM ITS SINE AND COSINE**

The sine and cosine of each orientation angle $A$ are obtained at each of the 101 time values. The initial value $A_0$ is obtained in the range $-\pi/2 < A_0 < 3\pi/2$ from $\sin A_0$ and $\cos A_0$ using subroutine ANG0. Subsequent values are calculated inductively by subroutine CSANG as follows:

In order to calculate the value of angle $A$ from $\sin A$, $\cos A$ and the preceding value $A_1$ let $B$ be the angle in the range $-\pi < B < \pi$ defined by:

\[
\cos B = \cos A \cos A_1 + \sin A \sin A_1 = \cos (A - A_1)
\]

\[
\sin B = \sin A \cos A_1 - \cos A \sin A_1 = \sin (A - A_1)
\]

$A$ is then obtained as $A = A_1 + B$ so that $A$ differs from the preceding value $A_1$ by less than $\pi$ radians. This procedure avoids the discontinuities inherent in the calculation of an angle solely from its sine and cosine.

**SINGULARITIES OF THE ORIENTATION ANGLES**

In the calculation of the hula angle $\psi_p$ from film data there is a singularity when the pike angle $\gamma$ equals $\pi$ radians so that $\psi_p$ is indeterminate for this value of $\gamma$. In practice this indeterminacy is not a problem since $\gamma$ is never exactly equal to $\pi$. However if $\gamma$ is close to $\pi$ the calculated value of $\psi_p$ is very sensitive to
Since the films taken by the two cameras are digitised twice there are four combinations of the digitised data and this leads to four estimates of $\psi$. When $\gamma$ is near $\pi$ the four estimates may lie in two or more quadrants and since subsequent values are calculated inductively by subroutine CSANG the divergence of the four estimates can be cumulative. Because each estimate is calculated inductively any such divergence will still be present even when $\gamma$ is no longer close to $\pi$ and it is quite possible for the difference of two estimates to be close to a multiple of $2\pi$ radians. Although the four $\psi$ values will correspond to similar body configurations their average value is likely to correspond to quite a different configuration.

In order to prevent this divergence of the four estimates the following procedure is adopted. Whenever the $n$th estimate of $\psi$ differs from the fourth estimate by more than $\pi$ radians the $n$th estimate is replaced by the average of itself and the fourth estimate. This ensures that the four estimates are never far apart so that their average will be representative of the actual configuration.

The same procedure is used for the following angles:

- $\delta_a$ which has a singularity at $\epsilon_a = \frac{1}{2}\pi$
- $\delta_b$ which has a singularity at $\epsilon_b = \frac{1}{2}\pi$
- $\psi_a$ which has singularities at $\epsilon_a = \frac{1}{2}\pi$ and $\gamma_a = \pi$
- $\psi_b$ which has singularities at $\epsilon_b = \frac{1}{2}\pi$ and $\gamma_b = \pi$

SPLINE FITTING THE ORIENTATION ANGLES

In the simulation program SIM it will be necessary to evaluate the orientation angles and their derivatives at arbitrary times determined by the integration routine. This is accomplished by fitting a quintic spline to each orientation angle in the program FILM and writing the spline coefficients to a file for use with SIM.
If \( X_1, X_2, X_3, X_4 \) are the four estimates of an orientation angle at time \( t_i \) their average \( A_i \) will have variance \( V_i \) where:

\[
A_i = \frac{1}{4} \sum_{r=1}^{4} X_{r_i} \quad \text{and} \quad V_i = \sum_{r=1}^{4} \frac{(X_{r_i} - A_i)^2}{12}
\]

as shown in the subsection on unbiased estimates of variance. If \( V \) is the average of the \( V_i \) then error estimates \( \delta A_i \) may be defined by

\[
\delta A_i = (kV_i + (1-k)V)^{\frac{1}{2}} \quad \text{with} \quad 0 < k < 1 \quad \text{as done in the fitting of the displacement data. A choice of} \ k = \frac{3}{4} \ \text{places a lower limit of} \ \frac{1}{6} \ \text{on the} \ \delta A_i \ \text{where} \ \delta = \sqrt{V} \ \text{is the average error estimate.}
\]

The 101 data points \((t_i, A_i)\) are then fitted by the quintic spline of Jennings using the \( \delta A_i \) as error estimates of the \( A_i \). Subroutine SPLIN4 calculates \( A_i \) and \( \delta A_i \) and calls the spline fitting routine.

The processing time for program FILM is around 300 seconds on a PRIME 750 computer with most of this time being spent on the computation of the 80 quintic splines of the displacement data and the 17 splines of the orientation angles. A listing of program FILM is presented in Appendix C.

DETAILS OF THE FILMING SESSIONS

Three filming sessions were held in the Victory Hall at Loughborough University of Technology. All movements were performed on a Nissen Goliath trampoline which had a six millimetre bed and short steel springs.

PERFORMERS AND MOVEMENTS

At the first filming session on March 24, 1982 the trampoline movements were performed by Gill Statham who had made her first attempt of the front somersault with \( \frac{11}{2} \) twists only three weeks previously. The filmed movements comprised:
G02 piked jump with \( \frac{1}{4} \) twist
G08 piked front somersault with \( 1\frac{1}{2} \) twists
G12 piked front somersault with 1 twist

At the second filming session on September 28, 1982 the trampoline movements were performed by Carl Furrer who had won the World Trampoline Championship four months previously. The filmed movements comprised:

C11 full twisting back somersault
C39 piked back somersault with full twist
C41 double front somersault with \( 1\frac{1}{2} \) twists (\( 1\frac{1}{2} \) out piked)
C45 double back somersault with 1 twist (1 out piked)
C47 double back somersault with 2 twists (\( \frac{1}{2} \) in 1\( \frac{1}{2} \) out piked)
C43 double back somersault straight

At the third filming session on December 21, 1983 the trampoline movements were performed by John Cryer who was an accomplished springboard diver with one year's experience of trampolining. The filmed movements comprised:

J71 piked front somersault with \( 1\frac{1}{2} \) twists
J73 piked back somersault with 1 twist

REFERENCE FRAME

The reference frame was composed of six threaded rods which screwed into a machined steel cylinder (Figure 67). The lower rod was a 2.4 metre length of 15mm diameter steel rod whilst the remaining five rods were 1.03 metre lengths of 15mm diameter aluminium rod. The triangular base could be levelled using the adjustable feet in conjunction with the spirit levels attached to the base.

TIMING DEVICES

At the first filming session a conical timer based on the
Figure 67. The reference frame
design of Blievernicht (1967) was used. A reed switch on the rim of the rotating cone provided a pulse once per revolution so that the rotation rate could be determined using a Venner electronic counter. The rotation rate was set to one revolution per second but it was found that the rate varied from 0.9 to 1.2 rps. The rate was measured during the filming of each movement and frame times corrected on the assumption that the revolution rate remained constant during a particular movement.

For the second filming session a cylindrical timer was constructed using plywood and perspex (Figure 68). The timer was driven by a small 12 volt DC motor which provided a torque of up to 150 milli-Newton-metres and rotated at one revolution per second. A variable voltage power pack was used in conjunction with the Venner electronic counter to set the rotation rate to one revolution per second. During the filming session the rate varied from 0.996 to 1.002 rps. Unfortunately the timer could not be read for the movements C41, C43, C45 and C47 since the colour film used had insufficient resolution. The frame times of these movements were determined from the displacement data using program Z2TIME.

For the third filming session an electronic timer was constructed. A 100 kHz crystal oscillator was used in the oscillating circuit (Figure 69) to produce square wave pulses at 100 kHz. The circuit was tuned by varying capacitor C3 and using the BBC Radio 4 station as a standard 200 kHz signal. Two 4017 decade divider chips in series were used to reduce the pulse frequency to 1 kHz. The signal then went to two counter units which were connected in parallel (Figure 70). Each counter unit used three decade divider chips in series so that light emitting diodes displayed the time in tenths, hundredths and thousandths of a second. The decade circuit (Figure 71) used two 4049 hex inverting buffers to amplify the pulses to give 10 mA input to each light emitting diode. A 555 timer chip was used to reset all counter stages to zero (Figure 72). When the switch was thrown the chip emitted a pulse which held all counters on zero until capacitor C charged sufficiently for tag 6 to reach a certain voltage. All
Figure 68. The cylindrical timer
Figure 69. The oscillator circuit

Figure 70. Layout of the timer circuit
Figure 71. The decade divider circuit

Figure 72. The reset circuit
circuits were powered by one 6 volt lamp battery.

The counter units were housed in two wooden boxes connected by a six metre cable which carried the signal and power supply. One box also contained the battery, the remainder of the circuitry and the power switch. Holes were drilled in the boxes and the high intensity light emitting diodes mounted on three concentric circles on each box. The outer circles displayed the milli-second value which was taken to be the greatest value when more than one LED was visible. Figure 73 shows the two counter units displaying a time of 0.325 seconds.

PHOTOGRAPHIC EQUIPMENT

Two spring driven Bolex H16 reflex cine-cameras were used in each filming session. Lenses used ranged in focal length from 10mm to 25mm. Camera framing rates were set to 64 frames per second and were found to range from 50 to 70 frames per second. The speed of a given camera when fully wound varied by less than 2%. Variable shutters were set to 1/2 or 3/4 closed positions corresponding to exposure times of 1/320 and 1/640 seconds. Apertures ranged from f1.8 to f4.

Tungsten and quartz-halogen lights were used in each filming session. In the first session four lights with a total power rating of 6 kilowatts were used with daylight providing almost as much illumination. In the second and third sessions two 25 metre electrical extension leads were used to connect additional lights to the ring main of the first floor. The total power of 13 kilowatts was close to the maximum since each 30 ampere 240 volt ring main could provide 7.2 kilowatts. Daylight provided little additional contribution.

Kodax Tri-X black and white Reversal Film 7278 was used in the filming of the movements G92, G98, G12, C11 and C39. The film was rated at 400 ASA and processed to produce a negative. For the movements C41, C43, C45, C47, J71 and J73 Eastman Ektachrome high
Figure 73. The electronic timer
speed colour Video News Film 7250 (Tungsten) was used. The film was rated at 400 ASA and processed to produce a positive. For a contrast of 1000:1 the resolving power of the colour film is stated to be 80 lines per millimetre compared with 119 lines per millimetre for the Tri-X film.

**LAYOUT OF EQUIPMENT**

Figure 74 shows the layout of the equipment. The distance D in metres of a camera from the origin was calculated using the formula $D = \frac{fH}{h}$ where $f$ is the focal length of the lens in millimetres, $H$ is the required vertical field of view in metres and $h$ is the film frame height of 6.75mm. In the first and third filming sessions $H$ was 5 metres whilst in the second session $H$ was 8 metres. The origin $o$ and camera locations $s$ and $f$ were marked on the floor and the theorem of Pythagoras was used to ensure that angle $sof$ was a right angle. The trampoline was erected with its centre above point $o$ and the cameras were positioned on tripods on tables with the objective lenses 2.4 metres above the points $s$ and $f$. Ropes were hung from pipes on the walls to provide vertical reference lines and square cards were attached to the walls to serve as check points (Figure 75). The timing unit was positioned in view of both cameras and the lights were placed as high as possible and as close to the trampoline as possible without obscuring the field of movement. Thick crash mats were placed over the springs at each end of the trampoline as a safety measure.

**PROCEDURE**

The equipment was set out as described above, all extension leads were fully unwound and the lights turned on. Light readings were taken of the performer on the trampoline and the camera apertures determined. The cameras were loaded with film, the leader was run off, frame rates were set to 64 frames per second, variable shutters were set to the 1/2 or 3/4 closed positions, apertures were set to values between f1.8 and f4 and the springs fully wound.
Figure 74. Layout of equipment
Figure 75. Performer and equipment
Whilst the performer practised on the trampoline each camera was checked to ensure that the field of view was adequate for the vertical range of movement.

Anthropometric measurements of the performer were then taken as described in Chapter 3 and 15mm tape was used to mark the joint centres as shown in Figure 75. The reference frame was erected through the centre of the trampoline bed and was adjusted until the base was horizontal. Next the cameras were aligned so that the origin of the reference frame appeared in the centre of each viewfinder and the reference frame was filmed and then removed from the trampoline. The timing unit was switched on and in the case of the conical and cylindrical timers the rotation rate was adjusted to one revolution per second.

The count-in procedure for each movement was described to the performer and camera operators. Counts of 'one, two, three' were made by the investigator as the performer made successive contacts with the trampoline bed. On the count of 'three' the performer started the required movement. Cameras were started between the counts of 'two' and 'three' and were stopped after a call by the investigator. The number of each movement was displayed using score cards in view of each camera (Figure 75). Cameras were rewound after each movement.

DIGITISATION OF THE FILMS

A Vanguard projection head mounted on a PCD X-Y reader was connected to a MINC mini-computer which provided data logging facilities. A cursor could be positioned over any point of the image which was back-projected via a mirror onto the screen. Upon depressing a footpedal the MINC computer converted the X and Y output voltages of the PCD reader into digital form and stored the values on floppy disc. The vertical resolution of the system was one part per thousand so that adjacent digital values corresponded to a difference of 5mm when the vertical field of view was 5 metres. The accuracy of the system was affected by the PCD reader which did
not output constant voltages for a fixed position of the cursor. As a result the digital output of each channel ranged over three adjacent values corresponding to a standard deviation of 4mm.

Prior to digitisation the movement to be analysed was viewed a number of times frame by frame to familiarise the operator with the successive images. The film was then rewound to display the reference frame and the projection head was rotated until the image of the hanging rope was aligned with the Y-axis of the reader. The data logging program was then run so that the MINC computer was ready to accept data. The four points of the reference frame were each digitised 8 times and the two check points were each digitised 12 times. The film was then wound on to the first frame of the required movement. In each frame the two check points were digitised followed by the 10 landmarks after which the check points were digitised a second time. The frame time was entered via the MINC keyboard and the procedure repeated with the next frame.

The digitised coordinates of the check points were needed since the images of the check points moved on the screen from frame to frame. Landmarks were located by assuming that joint centres lay on the midlines of the limbs. When there was little flexion at a joint the tape on the performer was used as a guide to joint centre location. Care was taken to preserve the order of landmark digitisation since it was possible to confuse the left and right sides of the body when the performer was twisting in an inverted position.

The number of film frames of a movement varied from 60 to 105 and the digitisation time lasted from two to four hours. The data on the floppy disc was transferred via a Prime 400 computer to the Prime 750 computer filestore. Since each film was digitised twice it was possible for the program FILM to indicate where there were large differences in the data files. When an error was detected it was replaced by the average of the values in adjacent frames.
ERROR ESTIMATES OF THE FILM DATA AND ORIENTATION ANGLES

The error estimate of the film data is the average of the error estimates used in the quintic spline fitting of the displacement data. The error estimate of the orientation angles is the average of the error estimates used when fitting quintic splines to the orientation angles. Values of these errors are given in Table 9 for each of the 11 filmed movements. These values should be regarded as underestimates of the actual errors since they do not reflect any systematic errors inherent in the program FILM. The errors in the movements C11-C47 of the second filming session are larger than those of the other sessions since a larger field of view was used.

Table 9 also gives the flight time which is taken to be the difference between the start time ST and end time ENDT used in the interpolation of the displacement data. The actual times for which the performers were airborne are about 0.09 seconds greater than the flight times since the crash mat obscured foot contact for the front camera and the start time and end time were stepped in by two frames from the available range in order to avoid endpoint problems.

SUMMARY

A computer program has been developed in order to calculate 17 orientation angles from the digitised coordinates of 10 joint centres from two camera views. A number of arbitrary choices have been made concerning the definition and calculation of the orientation angles. Whilst the error estimates suggest that angles are calculated with some accuracy it is possible that the program FILM contains some systematic error. Even if the error estimates are reliable it remains to be seen whether the accuracy is sufficient to produce accurate simulations. The validation of the program FILM is undertaken in Chapter 6 which evaluates the combined use of the programs ISEG, FILM and SIM.
Table 9. Error estimates and flight times

<table>
<thead>
<tr>
<th>movement number</th>
<th>displacement error (metres)</th>
<th>angle error (degrees)</th>
<th>flight time (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02</td>
<td>0.009</td>
<td>0.8</td>
<td>1.14</td>
</tr>
<tr>
<td>G08</td>
<td>0.013</td>
<td>1.6</td>
<td>1.02</td>
</tr>
<tr>
<td>G12</td>
<td>0.010</td>
<td>1.8</td>
<td>0.93</td>
</tr>
<tr>
<td>C11</td>
<td>0.015</td>
<td>2.2</td>
<td>1.59</td>
</tr>
<tr>
<td>C39</td>
<td>0.014</td>
<td>1.5</td>
<td>1.24</td>
</tr>
<tr>
<td>C41</td>
<td>0.014</td>
<td>1.6</td>
<td>1.51</td>
</tr>
<tr>
<td>C43</td>
<td>0.012</td>
<td>1.9</td>
<td>1.61</td>
</tr>
<tr>
<td>C45</td>
<td>0.014</td>
<td>1.0</td>
<td>1.54</td>
</tr>
<tr>
<td>C47</td>
<td>0.014</td>
<td>1.6</td>
<td>1.55</td>
</tr>
<tr>
<td>J71</td>
<td>0.011</td>
<td>1.2</td>
<td>0.99</td>
</tr>
<tr>
<td>J73</td>
<td>0.008</td>
<td>1.5</td>
<td>1.23</td>
</tr>
</tbody>
</table>
CHAPTER 5
DEVELOPMENT OF THE SIMULATION MODEL

INTRODUCTION

This chapter details the development of the computer program SIM which simulates aerial movement. Using the initial conditions and the time histories of the internal orientation angles SIM calculates the values of the angles defining somersault, tilt and twist.

The validation of SIM is undertaken in Chapter 6 where the accuracy of the combined use of the three programs ISEG, FILM and SIM is evaluated.

OPTIONS AVAILABLE IN THE PROGRAM SIM

In Option 1 SIM calculates the angular momentum of a filmed movement from the spline coefficients of the orientation angles given by program FILM.

In Option 2 SIM simulates a filmed movement using the angular momentum given by Option 1 and the internal orientation angles obtained from film. Comparison of the simulation values of the somersault, tilt and twist angles with the film values provides a measure of the accuracy of the program SIM when used in conjunction with values provided by FILM and ISEG.

In Option 3 SIM simulates a modified filmed movement so that it is possible to establish the type of twist and to determine the contributions to the twist made by various techniques.

In Option 4 SIM simulates an arbitrary movement for which the initial conditions and internal orientation angles are specified by the user.
In addition to these four basic options SIM provides the opportunity to control the unstable layout somersault and can output data on principal axes, principal inertias and momenta terms.

**INPUT AND OUTPUT DATA**

Segmental inertia parameters are read from file PERF created by program ISEG. For filmed movements the spline coefficients in file NAMFIT created by the program FILM are used to provide the time histories of the 14 internal orientation angles and the initial values of the three system angles. For Option 4 angles are defined by specifying end point values for time intervals in which the angle is monotonic. For simulations the three components of angular momentum are provided by file NAMMOM created using Option 1 or are defined by the user in Option 4. For Option 1 the spline coefficients are used to provide the time histories of the somersault, tilt and twist angles so that the total angular momentum can be calculated.

The output of a simulation comprises the values of the somersault, tilt and twist angles during the movement. The units used are revolutions for somersault and twist, and degrees for the tilt. For Option 1 the three components of the total angular momentum are calculated at 101 equally spaced times and average values are obtained together with accuracy estimates. The unit of time is taken to be the flight time and angular momenta components are given in the equivalent number of straight somersaults per unit time.

**INPUT ANGLES DEFINED BY THE USER**

Consider the quintic \( q(x) = x^3(6x^2-15x+10) \) which has derivatives

\[
q'(x) = 30x^2(x-1)^2 \\
q''(x) = 120x(x-\frac{1}{2})(x-1)
\]

\( q(x) \) increases from 0 to 1 on the interval \([0,1]\) and has zero first and second derivatives at the endpoints (Figure 76). These conditions make \( q(x) \) a suitable function for modelling changes in angle values since angular velocities and accelerations change smoothly.
Figure 76. The quintic function $q(x)$ and its derivatives
A trigonometric function satisfying the same conditions is \( t(x) = \frac{27\pi x - \sin 27\pi x}{2\pi} \) which is similar to \( q(x) \) and has maximum slope \( t'(x) = 2 \) compared with \( q'(x) = 15/8 \). \( q(x) \) is used rather than \( t(x) \) in order to be consistent with the choice of quintic spline representation used for orientation angles calculated from film data.

If the user requires that an angle \( \alpha \) change from \( \alpha_0 \) to \( \alpha_1 \) on the time interval \( [t_0, t_1] \), angle \( \alpha \) and its derivative \( \alpha' \) are defined by:

\[
\begin{align*}
\alpha(t) &= \alpha_0 + (\alpha_1 - \alpha_0) q(x) \\
\alpha'(t) &= (\alpha_1 - \alpha_0) q'(x)/(t_1 - t_0)
\end{align*}
\]

where \( x = (t - t_0)/(t_1 - t_0) \).

By using a succession of non-overlapping intervals the angle is defined as a quintic spline with continuous second derivative.

**SEGMENTS AND JOINT CENTRES**

The body is modelled as 11 rigid segments as described in Chapter 3. Figure 77 shows these segments and uses the notation:

- C: chest-head
- T: thorax
- P: pelvis
- A1: left upper-arm
- A2: left forearm-hand
- B1: right upper arm
- B2: right forearm-hand
- J1: left thigh
- J2: left calf-foot
- K1: right thigh
- K2: right calf-foot

The 11 segments are connected at 10 joint centres as described in Chapter 4. Figure 77 shows these joint centres and uses the notation:

- S: left shoulder
- R: right shoulder
Figure 77. Segments and joint centres of the simulation model
O\textsubscript{a}: left elbow
O\textsubscript{b}: right elbow
H: left hip
I: right hip
O\textsubscript{j}: left knee
O\textsubscript{k}: right knee
X: junction of pelvis and thorax
N: midpoint of shoulders

Note that N is assumed to be the junction of segments T and C so that movement of C relative to T is given by rotations of C about N.

In addition to the eleven segments the following supra-segments will be used in the next section which expresses the angular momentum of the system in terms of the relative angular velocities.

F: the whole body comprising the 11 segments
U: the upper body comprising segments T, C, A\textsubscript{1}, A\textsubscript{2}, B\textsubscript{1}, B\textsubscript{2}
D: the chest and arms comprising C, A\textsubscript{1}, A\textsubscript{2}, B\textsubscript{1}, B\textsubscript{2}
A: the left arm comprising A\textsubscript{1} and A\textsubscript{2}
B: the right arm comprising B\textsubscript{1} and B\textsubscript{2}
J: the left leg comprising J\textsubscript{1} and J\textsubscript{2}
K: the right leg comprising K\textsubscript{1} and K\textsubscript{2}

DERIVATION OF THE ANGULAR MOMENTUM EQUATION

Greenwood (1965, p.375) shows that the angular momentum of a rigid segment S about a point F is given by:

\[ h_{si} = I_{ss} \omega_{si} + m_s \mathbf{s}_f \times \mathbf{s}_f \]  

(1)

where: 
\( I_{ss} \) = inertia tensor of S referred to its mass centre
\( \omega_{si} \) = angular velocity of S relative to the non-rotating frame i
\( m_s \) = mass of S
\( \mathbf{s}_f \) = position vector of mass centre of S relative to the point F

.i indicates a vector derivative in frame i
The angular momentum of the whole body about its mass centre $F$ will be $\sum h_{s_i}$ where the sum is taken over the $11$ body segments. Three relations referred to as (DR), (IT) and (PA) will now be developed so that $h_{s_i}$ may be expressed in terms of the relative angular velocities of body segments.

The following equation, which is adapted from Greenwood (1965, p.48), expresses the difference of the derivatives of a vector $s_f^i$, in the frames $i$ and $p$, in terms of the angular velocity $\omega_{pi}$ of frame $p$ relative to frame $i$

$$s_f^i = \omega_{pi} \times s_f^i + s_f^p$$  \hspace{1cm} (DR)

Greenwood (1965, p.284) shows that if $I_{mr_f}$ is the inertia tensor of a point mass $m_r$ with position vector $r_f$ then for an arbitrary vector $\omega$:

$$m_r r_f \times (\omega \times r_f) = I_{mr_f} \omega \hspace{1cm} (IT)$$

In coordinate form $I_{mr_f} = m_r \begin{bmatrix} r_2^2 + r_3^2 & -r_1 r_2 & -r_1 r_3 \\ -r_1 r_2 & r_1^2 + r_3^2 & -r_2 r_3 \\ -r_1 r_3 & -r_2 r_3 & r_1^2 + r_2^2 \end{bmatrix}$ where $r_f = [r_1, r_2, r_3]^T$

The equation (IT) may be used to develop a generalisation of the parallel axes theorem. Suppose that the mass $m_r$ with position vector $r_f$ is an element of the segment $S$ with mass $m_s$ and mass centre location given by the vector $s_f$. Let $r_s$ be the position vector of mass $m_r$ relative to the mass centre of $S$ so that $r_f = r_s + s_f$. Taking sums over all such elemental masses in $S$:

$$\sum m_r = m_s = \text{mass of segment } S$$

$$\sum m_r r_s = m_s s_s = 0 \text{ i.e. the first moment about the mass centre is zero}$$

$$\sum I_{mr_f} = I_{sf} = \text{inertia tensor of } S \text{ referred to point } F$$

$$\sum I_{mrs} = I_{ss} = \text{inertia tensor of } S \text{ referred to its mass centre.}$$

Consider:

$$m_r r_f \times (\omega \times r_f) = m_r (r_s + s_f) \times (\omega \times (r_s + s_f))$$

Expanding:

$$m_r r_f \times (\omega \times r_f) = m_r r_s \times (\omega \times r_s) + m_s s_f \times (\omega \times s_f) + m_r r_f \times (\omega \times s_f) + m_s s_f \times (\omega \times r_s)$$
Using (IT): \[ I_{\text{rft}} = I_{\text{rfs}} + (\omega \times s_f) + s_f \times (\omega \times r_s) \]

Summing over all elemental masses in S and using the above expressions for sums:

\[ I_{sf} \omega = I_{ss} \omega + m_s s_f \times (\omega \times s_f) \quad \text{since } \sum m_r r_s = 0 \]

using (IT): \[ I_{sf} \omega = I_{ss} \omega + I_{msf} \omega \]

Thus \[ I_{sf} = I_{ss} + I_{msf} \] (PA)

since \( \omega \) is arbitrary.

The total angular momentum \( h \) of the whole body about its centre of mass \( F \) will be conserved during flight since the external (gravitational) forces on the body will have no moment about \( F \).

The equation of motion is \( h = \text{constant} \).

\[ h = \sum h_{si} \] where the sum is taken over the 11 segments \( P, T, C, A_1, A_2, B_1, B_2, J_1, J_2, K_1, K_2 \) and equation (1) gives \( h_{si} \) as:

\[ h_{si} = I_{ss} \omega_{si} + m_s s_f \times s_i \]

Expanding:

\[ I_{ss} \omega_{si} = I_{ss} (\omega_{sf} \omega_{fi}) = I_{ss} \omega_{sf} + I_{ss} \omega_{fi} \]

where \( \omega_{fi} \) is the angular velocity of the system frame \( f \) relative to the inertial frame \( i \).

Also: \[ m_s s_f \times s_i = m_s (s_f \times (\omega_{fi} + \omega_{sf})) \]

Expanding:

\[ = I_{msf} \omega_{fi} + m_s s_f \times s_i \]

Adding these expressions gives:

\[ h_{si} = I_{ss} \omega_{fi} + I_{msf} \omega_{fi} + I_{ss} \omega_{sf} + m_s s_f \times s_i \]

\( = I_{sf} \omega_{fi} + h_{sf} \) (PA)

where

\[ h_{sf} = I_{ss} \omega_{sf} + m_s s_f \times s_i \]

(2)

Summing over all segments:

\[ h = \sum h_{si} = \sum I_{sf} \omega_{fi} + \sum h_{sf} \]

i.e.

\[ h = I_{ff} \omega_{fi} + \sum h_{sf} \]

(3)
where $I_{ff} = I_{sf}$ is the whole body inertia tensor referred to the mass centre $F$.

We may write $h = h_{\omega fi} + h_{rel}$ where $h_{\omega fi} = I_{ff}\omega_{fi}$ is the angular momentum due to the motion of the body as a whole and $h_{rel} = \sum h_{sf}$ is the angular momentum due to movements of segments relative to the system frame $f$.

The same process is repeated moving from frame $f$ to the pelvis frame $p$ and expressing $\sum h_{sf}$ in terms of $\omega_{pf}$ and $\sum h_{sp}$ which is the angular momentum due to movement relative to frame $p$.

From equation (2):

$$h_{sf} = I_{ss}\omega_{sf} + m_s s_f \times s_f^f$$

Now

$$I_{ss}\omega_{sf} = I_{ss}\omega_{sp} + I_{ss}\omega_{pf}$$

and

$$m_s s_f \times s_f^f = m_s s_f (s_f^p \times s_p^f) + m_s s_f \times s_p^f$$

$$= I_{msf}\omega_{pf} + m_s s_f \times s_f^f$$

$$= I_{msf}\omega_{pf} + m_s s_f \times s_p^f + m_s s_f \times s_p^f$$

Thus:

$$h_{sf} = (I_{ss} + I_{msf})\omega_{pf} + I_{ss}\omega_{sp} + m_s s_f \times s_p^f$$

Thus:

$$h_{sf} = I_{ss}\omega_{sp} + m_s s_f \times s_p^f$$

where

$$h_{sp} = I_{ss}\omega_{sp} + m_s s_f \times s_p^f$$

From equation (3):

$$h = I_{ff}\omega_{fi} + \sum h_{sf}$$

and

$$\sum h_{sf} = \sum I_{sf}\omega_{pf} + \sum h_{sp} + \sum m_s s_f \times p_f^f$$

Thus

$$h = I_{ff}\omega_{fi} + I_{ff}\omega_{pf} + \sum h_{sp}$$

since

$$\sum I_{sf} = I_{ff}$$ and $\sum m_s s_f = 0$.

We may write

$$h = h_{\omega fi} + h_{\omega pf} + \sum h_{sp}$$

where $h_{\omega fi} = I_{ff}\omega_{fi}$, $h_{\omega pf} = I_{ff}\omega_{pf}$ and the sum $\sum h_{sp}$ includes all segments except $P$. The pelvis $P$ connects to the segments $T, J_1, K_1$, via the joint centres $X, H$ and $I$. Let the upper body $U$ comprise segments $T, C, A_1, A_2, B_1, B_2$, let the left leg $J$ comprise $J_1$ and $J_2$ and let the right leg $K$ comprise $K_1$ and $K_2$. The sum $\sum h_{sp}$ may be separated into three parts:
\[ \sum_{sp} h_{sp} = \sum_{sp} h_{sp} + \sum_{sp} h_{sp} + \sum_{sp} h_{sp} \]

It will next be shown that:

\[ \sum_{sp} h_{sp} = h_{\omega tp} + \sum_{st} h_{st} \]

where \( h_{\omega tp} \) is the angular momentum due to the motion of \( t \) relative to \( p \).

The process will be continued outwards along the network of joint centres by applying the same procedure to \( \sum_{sp} h_{sp} \) and \( \sum_{sp} h_{sp} \).

From equation (4):

\[ h_{sp} = I_{ss} \omega_{sp} + m_{sf} \times \dot{\omega}_{sp} \]

If \( X \) is the junction of the pelvis and thorax the vector \( \dot{x}_{p} \) is fixed in \( P \) so that \( \dot{x}_{p} = 0 \). Thus \( \dot{s}_{p} = \dot{s}_{x} + \dot{x}_{p} = \dot{s}_{x} \).

Hence

\[ h_{sp} = I_{ss} \omega_{sp} + m_{sf} \times \dot{\omega}_{sp} \]

This is the form of \( h_{sp} \) that will be used for the segments of the upper body \( U \). For the segments in \( J \) and \( K \), \( s_{p} \) will be replaced by \( s_{h} \) and \( s_{i} \) using the joint centres \( H \) and \( I \).

Expanding:

\[ I_{ss} \omega_{sp} = I_{ssp} \omega_{sp} + I_{ss} \omega_{tp} \]

and

\[ m_{sf} \times \dot{s}_{p} = m_{sf} \times (\omega_{sp} \times s_{p}) + m_{sf} \times \dot{s}_{p} \] (DR)

Writing \( s_{x} = s_{u} + u_{x} \), where \( s_{u} \) is the vector connecting the mass centres of segments \( U \) and \( S \), gives:

\[ m_{sf} \times (\omega_{tp} \times s_{x}) = m_{sf} \times (\omega_{tp} \times s_{u}) + m_{sf} \times (\omega_{tp} \times u_{x}) \] (IT)

Then using \( s_{f} = s_{u} + u_{f} \) gives:

\[ m_{sf} \times (\omega_{tp} \times s_{u}) = m_{sf} \times (\omega_{tp} \times s_{u}) + m_{sf} \times (\omega_{tp} \times s_{u}) \]

Substituting in equation (6):

\[ h_{sp} = (I_{ss} + I_{su}) \omega_{sp} + m_{sf} \times (\omega_{sp} \times u_{x}) \]

\[ + I_{ss} \omega_{st} + m_{sf} \times (\omega_{sp} \times s_{f}) \times s_{f} \]

\[ = I_{su} \omega_{tp} + m_{sf} \times (\omega_{sp} \times u_{x}) + h_{st} + m_{uf} \times (\omega_{sp} \times s_{u}) \] (PA)
where \( h_{st} = I_{ss} \omega_{st} + m_s \times s \times \frac{\dot{s}}{n} \) (7)

since \( s = s + \frac{\dot{s}}{n} = \frac{\dot{s}}{n} \) as the points N and X are fixed in T.

Taking sums over the segments U and noting that:

\[
\sum I_{su} = I_{uu} = \text{the inertia tensor of U referred to its mass centre}
\]

\[
\sum m_s f = m_u \text{ where } m_u \text{ is the mass of U}
\]

\[
\sum m_s u = 0
\]

gives:

\[
\sum_{U} h_{sp} = I_{uu} \omega_{tp} + m_u \times \frac{\omega_{tp} \times u}{x} + \sum_{D} h_{st}
\]

where \( D \) comprises segments \( C, A_1, A_2, B_1, B_2 \).

Following precisely the same procedure for \( \sum_{J} h_{sp} \) and \( \sum_{K} h_{sp} \) gives:

\[
\sum_{J} h_{sp} = I_{jj} \omega_{jp} + m_j \times (\omega_{jp} \times j) + h_{j2j1}
\]

(9)

and

\[
\sum_{K} h_{sp} = I_{kk} \omega_{klp} + m_k \times (\omega_{klp} \times k) + h_{k2kl}
\]

(10)

with

\[
h_{j2j1} = I_{j2j1} \omega_{j2j1} + m_{j2j1} \times j_{2j1}
\]

and

\[
h_{k2kl} = I_{k2kl} \omega_{k2kl} + m_{k2kl} \times k_{2kl}
\]

corresponding to equation (7).

Note that \( j_{2j1}^{*} = \omega_{j2j1} \times j_{2j1} \) and \( k_{2kl}^{*} = \omega_{k2kl} \times k_{2kl} \)

Adding equations (8), (9) and (10):

\[
\sum_{U} h_{sp} = h_{\omega tp} + h_{\omega jl p} + h_{\omega kl p} + \sum_{D} h_{st} + h_{j2j1} + h_{k2kl}
\]

(11)

where \( h_{\omega tp} \) is the angular momentum due to the moment of U relative to P.

Again the same procedure is applied three times to expand
\[ \sum_{D} h_{st} = h_{wct} + h_{walc} + h_{wblc} + h_{a2al} + h_{b2bl} \]  

(12)

Equation (5) stated:  
\[ h = h_{wfi} + h_{wpf} + \sum h_{sp} \]

so that using equations (11) and (12) we obtain:

\[ h = h_{wfi} + h_{wpf} + h_{wtp} + h_{wct} \]

+ \[ h_{walc} + h_{wblc} + h_{wa2al} + h_{wb2bl} \]

+ \[ h_{wjp} + h_{wklp} + h_{wj2jl} + h_{wk2kl} \]  

(13)

where:

\[ h_{wfi} = I_{ff} \omega_{fi} \]  

(14)

\[ h_{wpf} = I_{ff} \omega_{pf} \]  

(15)

\[ h_{wtp} = I_{uu} \omega_{tp} + m_{u} x_{u} (\omega_{tp} x_{u}) \]  

(16)

\[ h_{wct} = I_{dd} \omega_{ct} + m_{d} x_{d} (\omega_{ct} x_{d}) \]  

(17)

\[ h_{walc} = I_{aa} \omega_{alc} + m_{a} x_{a} (\omega_{alc} x_{a}) \]  

(18)

\[ h_{wblc} = I_{bb} \omega_{blc} + m_{b} x_{b} (\omega_{blc} x_{b}) \]  

(19)

\[ h_{wa2al} = I_{a2a2} \omega_{a2al} + m_{a2} x_{a2} (\omega_{a2al} x_{a2}) \]  

(20)

\[ h_{wb2bl} = I_{b2b2} \omega_{b2bl} + m_{b2} x_{b2} (\omega_{b2bl} x_{b2}) \]  

(21)

\[ h_{wjp} = I_{jj} \omega_{jp} + m_{j} x_{j} (\omega_{jp} x_{j}) \]  

(22)

\[ h_{wklp} = I_{kk} \omega_{klp} + m_{k} x_{k} (\omega_{klp} x_{k}) \]  

(23)

\[ h_{wj2jl} = I_{jj} \omega_{j2jl} + m_{j} x_{j} (\omega_{j2jl} x_{j}) \]  

(24)

\[ h_{wk2kl} = I_{kk} \omega_{wk2kl} + m_{k} x_{k} (\omega_{wk2kl} x_{k}) \]  

(25)

The expressions (13)-(25) for the angular momenta may also be derived in the following way:

Suppose that the system comprises \( n \) rigid segments \( S_k \) (\( k=1,n \)) which are linked by \( (n-1) \) joint centres \( O_k \) (\( k=2,n \)). The link
system emanates from segment $S_1$ and continues out to the extremities in the following way:

Let each segment $S_k$ which shares a joint centre $O_k$ with $S_1$ be known as an immediate successor of $S_1$. The immediate successors of $S_k$ will be those segments which share a joint centre with $S_k$ other than $O_k$. Provided that each segment shares a joint centre with some other segment and that two segments share at most one joint centre and that there are no closed loops, the link system will enter $S_k$ at exactly one point $O_k$ ($k=2,n$).

The orientation of the system will be given by the three orientation angles $\alpha_1, \alpha_2, \alpha_3$ which specify the orientation of $S_1$ relative to the inertial frame $i$. The orientation of $S_k$ relative to its predecessor $S_{k-1}$, which shares the joint centre $O_{k-1}$, will be specified by the three orientation angles $\alpha_{3k-2}, \alpha_{3k-1}, \alpha_{3k}$ ($k=2,n$). The system has $3n$ degrees of freedom with the orientation and configuration specified by $3n$ independent orientation angles $\alpha_j$ ($j=1,m$ where $m=3n$). If a joint is required to have less than three degrees of freedom this may be accommodated at a later stage by considering motions for which certain of the $\dot{\alpha}_j$ are zero.

For the segment $S_k$ let:

- $h_k = \text{the angular momentum of } S_k \text{ about the mass centre } F$ of the system
- $I_k = \text{the inertia tensor of } S_k \text{ referred to its mass centre } G_k$
- $\omega_k = \text{the angular velocity of } S_k \text{ relative to the inertial frame } i$
- $m_k = \text{the mass of } S_k$
- $r_k = \text{the position vector of the mass centre } G_k \text{ relative to } F$
- $v_k = \text{the velocity of the mass centre } G_k \text{ relative to } F$

Equation (1) then becomes:

$$h_k = I_k \omega_k + m_k r_k \times v_k$$

where $r_k$ is a vector function of $\alpha_j$ ($j=1,m$) (Kane, 1968, p.15) so that:
\[ r_k = r_{k1}i_1 + r_{k2}i_2 + r_{k3}i_3 \]

where \( i_1, i_2, i_3 \) are orthogonal unit vectors of the inertial frame \( i \) and \( r_{k\ell} (\ell=1,3) \) are scalar functions of \( \alpha_j (j=1,m) \)

Kane (1968, p.10) shows that:

\[ \dot{v}_k = \dot{r}_k = \dot{r}_{k1}i_1 + \dot{r}_{k2}i_2 + \dot{r}_{k3}i_3 \]

where \( \dot{r}_{k\ell} = \sum_{j=1}^{m} \left( \frac{\partial r_{k\ell}}{\partial \alpha_j} \right) \dot{\alpha}_j \) (\( \ell=1,3 \))

Thus:

\[ v_k = \sum_{j=1}^{m} v_{kj} \dot{\alpha}_j \]

where \( v_{kj} = \left( \frac{\partial r_{k1}}{\partial \alpha_j} \right)i_1 + \left( \frac{\partial r_{k2}}{\partial \alpha_j} \right)i_2 + \left( \frac{\partial r_{k3}}{\partial \alpha_j} \right)i_3 \), (\( j=1,m \))

so that \( v_{kj} (j=1,m) \) are vector functions of \( \alpha_p (p=1,m) \) and are independent of \( \dot{\alpha}_p (p=1,m) \).

The angular velocity \( \omega_k \) may also be expressed in the form:

\[ \omega_k = \sum_{j=1}^{m} \omega_{kj} \dot{\alpha}_j \]

where \( \omega_{kj} \) are vector functions of \( \alpha_p (p=1,m) \)

(Kane, 1968, p.17)

Thus:

\[ h_k = I_k \omega_k + m_k \dot{r}_k \times v_k \]

\[ = I_k \left( \sum_{j=1}^{m} \omega_{kj} \dot{\alpha}_j \right) + m_k \dot{r}_k \times \left( \sum_{j=1}^{m} v_{kj} \dot{\alpha}_j \right) \]

\[ = \sum_{j=1}^{m} h_{kj} \dot{\alpha}_j \]

where \( h_{kj} = I_k \omega_{kj} + m_k \dot{r}_k \times v_{kj} (j=1,m) \)

so that each \( h_{kj} \) is a vector function of \( \alpha_p (p=1,m) \) and is not dependent upon \( \dot{\alpha}_p (p=1,m) \).

The total momentum may now be written as:

\[ h = \sum_{k=1}^{m} h_k = \sum_{k=1}^{n} \left( \sum_{j=1}^{m} h_{kj} \dot{\alpha}_j \right) \]

\[ = \sum_{j=1}^{n} \left( \sum_{k=1}^{m} h_{kj} \right) \dot{\alpha}_j \]
The three orientation angles which govern movement at the joint centre $O_k$ are $\alpha_{3k-2}$, $\alpha_{3k-1}$, $\alpha_{3k}$ so that $h$ may be written as:

$$h = \sum_{k=1}^{n} (h_{3k-2}^{i} \dot{\alpha}_{3k-2} + h_{3k-1}^{i} \dot{\alpha}_{3k-1} + h_{3k}^{i} \dot{\alpha}_{3k})$$

where $h_{k}^{i}$ is dependent on $\dot{\alpha}_{3k-2}$, $\dot{\alpha}_{3k-1}$, $\dot{\alpha}_{3k}$ but is independent of the remaining $\dot{\alpha}_{p}$.

Thus $h$ has been expressed as a sum of terms $h_{k}^{i}$ each of which is a function of the relative movement at the joint centre $O_k$ but is independent of the movement elsewhere. Note that $h_{1}^{i}$ is a function of the movement of $S_1$ relative to the inertial frame $i$.

When all the internal orientation angles are held fixed the total angular momentum will equal $h_{1}^{i}$ since $h_{k}^{i}=0$ ($k=2,n$). In this situation the system moves as a rigid body with angular velocity $\omega_1$ so that:

$$h_{1}^{i} = I_{ff} \omega_1$$

where $I_{ff}$ is the inertia tensor of the system referred to the mass centre $F$.

When movement occurs only at the joint centre $O_k$ the total angular momentum will equal $h_{k}^{i}$ since $h_{k}^{i}=0$ ($p \neq k$). In this situation the system comprises two supra-segments $U$ and $L$ which are linked by the joint centre $O=O_k$. $U$ comprises the segment $S_k$ and all its successors in the link system whilst $L$ comprises the remaining segments. $L$ will include the segment $S_1$ and so will maintain a fixed orientation relative to the inertial frame $i$.

The angular momentum of the system will be:

$$h_{k}^{i} = I_{uu} \omega_i + m_{uf} \times \dot{\ell}_f^i + L \ell_i \omega_i + m_{lf} \times \dot{\ell}_f^i$$
where $I_{uu}, I_{ll}$ are the inertia tensors of $U$ and $L$ referred to their mass centres

$\omega_{ui}', \omega_{li}'$ are the angular velocities of $U$ and $L$ relative to frame $i$

$m_i', m_l$ are the masses of $U$ and $L$

$u_i', l_f$ are the position vectors of the mass centres of $U$ and $L$ relative to the mass centre $F$ of the system.

Since $L$ maintains a fixed orientation relative to frame $i$ the angular velocity $\omega_{li}'$ will be zero and $\omega_{ui}'$ will equal the angular velocity $\omega_{ui}$ of $U$ relative to $L$.

Since $F$ is the mass centre of the system $m \ell_f = -m u_f$ so that:

$$ m u_f \times i_f + m \ell_f \times i_f = m u_f \times i_f $$

Now $i_f = u_f + \ell_f = i_f$ since $O=O_k$ is fixed in $L$

and $i_f = u_f + \omega \times u_f = \omega \times u_f$ since $O$ is fixed in $U$.

Thus $h_k$ takes the form:

$$ h_k = I_{uu} \omega u + m u_f \times (\omega u_f \times u_f) $$

and this is precisely the form taken by equations (16)-(25).

$$ h_k = I_{ff} \omega l $$ may be expanded by writing $\omega_l = \omega_{pf} + \omega_{fi}$ which gives:

$$ h_k = I_{ff} \omega_{pf} + I_{ff} \omega_{fi} $$

and these are precisely the terms given in equations (14) and (15).

Thus the equation $h = \sum h_k$ is equivalent to equation (13) and this alternative derivation of the angular momenta is complete.

Since equations (16)-(25) have the same form the expressions are evaluated using a subroutine HRL in the program SIM. However before the angular momenta can be evaluated the angular velocities, position vectors and inertia tensors must be expressed in terms of the orientation angles.
In Chapter 4 a frame of reference was defined for each segment and the orientation of one frame relative to an adjacent frame was specified in terms of orientation angles. The frames of reference are:

i: inertial frame  
f: system or whole body frame  
p: pelvis frame  
t: thorax frame  
c: chest frame  
a1: left upper arm frame  
a2: left forearm frame  
b1: right upper arm frame  
b2: right forearm frame  
c: combined leg frame  
j1: left thigh frame  
j2: left calf frame  
k1: right thigh frame  
k2: right calf frame

In the standard configuration (Figure 78) all reference frames are aligned with their axes parallel to \( i_1, i_2, i_3 \) where:

\( i_1 \) is directed from right to left  
\( i_2 \) is directed from front to back  
\( i_3 \) is directed from bottom to top

The origins of the frames of reference need not be specified since the vector quantities in the angular momentum equation are unaffected by movement of origin.

In the following subsections the relative angular velocities are evaluated in frame f. The notation \( (v)_f \) will be used to indicate that the components of vector v are given in frame f.
Figure 78. The standard configuration of body segments
ORIENTATION OF THE WHOLE BODY

As described in Chapter 4 the orientation of the system frame \( f \) relative to the inertial frame \( i \) is given by the somersault angle \( \phi \), the tilt angle \( \theta \) and the twist angle \( \psi \). If \( f \) is initially aligned with frame \( i \) then successive rotations through \( \phi \) about \( f_1 \), \( \theta \) about \( f_2 \) and \( \psi \) about \( f_3 \) bring frame \( f \) into its final orientation (Figure 79). The rotation matrices \( R_1(\phi) \), \( R_2(\theta) \), \( R_3(\psi) \) transform the coordinates of a vector in one frame to the next. The columns of these matrices use the new coordinates of the previous directions of the unit vectors \( f_1, f_2, f_3 \), so that:

\[
R_1(\phi) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \phi & \sin \phi \\
0 & -\sin \phi & \cos \phi
\end{bmatrix}
\]

\[
R_2(\theta) = \begin{bmatrix}
\cos \theta & 0 & -\sin \theta \\
0 & 1 & 0 \\
\sin \theta & 0 & \cos \theta
\end{bmatrix}
\]

and

\[
R_3(\psi) = \begin{bmatrix}
\cos \psi & \sin \psi & 0 \\
-\sin \psi & \cos \psi & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

The angular velocity of \( f \) relative to \( i \) is given by:

\[
\omega_{fi} = \dot{\phi} f_1 + \dot{\theta} f_2' + \dot{\psi} f_3
\]

where \( f_2' \) is a unit vector with the direction of \( f_2 \) after the first rotation through \( \phi \).

Thus in frame \( f \):

\[
(\omega_{fi})_f = R_3(\psi)R_2(\theta)R_1(\phi) \begin{bmatrix}
\dot{\phi} \\
0 \\
0
\end{bmatrix} + R_3(\psi)R_2(\theta) \begin{bmatrix}
\dot{\theta} \\
0 \\
0
\end{bmatrix} + R_3(\psi)R_2(\theta) \begin{bmatrix}
\dot{\psi}
\end{bmatrix}
\]

so that

\[
(\omega_{fi})_f = R_\omega \begin{bmatrix}
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi}
\end{bmatrix}
\]

where \( R_\omega = \begin{bmatrix}
\cos \theta \cos \psi & \sin \psi & 0 \\
-\cos \theta \sin \psi & \cos \psi & 0 \\
\sin \theta & 0 & 1
\end{bmatrix} \)

The matrix which transforms \( i \)-coordinates into \( f \)-coordinates is evaluated as:
Figure 79. Angles of somersault, tilt and twist
\[ S_{if} = R_3(\psi)R_2(\theta)R_1(\phi). \]

The inverse of \( S_{if} \) is \( S_{fi} = R_1(-\phi)R_2(-\theta)R_3(-\psi) \) is evaluated as the transpose \( S'_{if} \).

The matrices \( S_{if} \) and \( S_{fi} \) will be used to transform the total angular momentum from one frame to another.

**THE PIKE AND HULA ANGLES**

The unit vectors \( p_3, t_3, \hat{p}_3 \) of the pelvis, thorax and legs are parallel to \( GX, OX \) and \( GO \) where \( G \) is the midpoint of the hip centres and \( X \) is the junction of the pelvis with the thorax (Figure 80). The pike angle \( \gamma \) is the angle between \( GO \) and \( OX \). The hula angle \( \psi_p \) is the angle between the plane \( GOX \) and the plane which bisects the hip centres.

The angle \( \alpha \) between \( p_3 \) and \( t_3 \) is defined as a continuous function of \( \gamma \) and \( \psi_p \) as follows:

If \( \cos \psi_p > 0 \) and \( \gamma > \frac{\pi}{2} \) then \( \alpha = \frac{1}{2}(\pi - \gamma) \sin^2 \psi_p = \alpha_1 \)
so that \( \dot{\alpha} = -\frac{1}{2}\gamma \sin^2 \psi_p \cos \psi_p \cos \psi_p (\dot{\psi}_p) = \dot{\alpha}_1 \)

If \( \cos \psi_p > 0 \) and \( \gamma < \frac{\pi}{2} \) then \( \alpha = \alpha_1 + \frac{1}{2}(\pi - \gamma) \cos^2 \psi_p \)
so that \( \dot{\alpha} = \dot{\alpha}_1 + \frac{1}{2}\gamma \cos^2 \psi_p - (\pi - \gamma) \cos \psi_p \sin \psi_p \frac{\dot{\psi}_p}{\psi_p} \)

If \( \cos \psi_p < 0 \) then \( \alpha = \frac{1}{2}(\pi - \gamma) \) so that \( \dot{\alpha} = \frac{1}{2}\dot{\gamma} \).

The angle \( \beta \) between \( p_3 \) and \( \hat{p}_3 \) is given by \( \beta = \pi - \gamma - \alpha \) so that \( \dot{\beta} = -\dot{\gamma} - \dot{\alpha} \).

Note that although \( \alpha \) and \( \beta \) are continuous across the boundaries \( \gamma = \frac{\pi}{2} \) and \( \cos \psi_p = 0 \) their derivatives \( \dot{\alpha} \) and \( \dot{\beta} \) are discontinuous at \( \gamma = \frac{\pi}{2} \).

The orientation of \( t_3 \) relative to frame \( p \) may be expressed in terms of \( \alpha \) and \( \psi_p \). Let \( VT \) be the column vector \( (t_3) p \) with components \( VT_i \) (i=1,3). Referring to Figure 80:
Figure 80. The pike angle GOX and the hula angle } \psi_p \\n
Figure 81. Orientation of the pelvis
The orientation of frame $t$ relative to frame $p$ may be defined in terms of angles $\delta_t$ and $\epsilon_t$ as follows. If $t$ is initially aligned with $p$ then successive rotations through $\delta_t$ about $t_1$ and $\epsilon_t$ about $t_2$ bring $t$ into its final orientation. Since there is no rotation about $t_3$ we may say that $t$ has no twist (torsion) relative to $p$.

The transformation matrix $S_{tp}$ which converts $t$-coordinates to $p$-coordinates is evaluated as $S_{tp} = R_1(-\delta_t)R_2(-\epsilon_t)$ where $R_1(-\delta_t)$ and $R_2(-\epsilon_t)$ are rotation matrices.

Thus

$$
\begin{bmatrix}
VT_1 \\
VT_2 \\
VT_3
\end{bmatrix}
= (t_3)_p = S_{tp}
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
= \begin{bmatrix}
sin\epsilon_t \\
-sin\delta_t \cos \epsilon_t \\
\cos \delta_t \cos \epsilon_t
\end{bmatrix}
$$

The sine and cosine of $\delta_t$ and $\epsilon_t$ are then obtained as:

$$
sin\epsilon_t = VT_1 \\
cos\epsilon_t = (VT_2^2 + VT_3^2)^{\frac{1}{2}} \\
sin\delta_t = -VT_2/cos \epsilon_t \\
cos \delta_t = VT_3/cos \epsilon_t
$$

The derivatives $\dot{VT}_1, \dot{VT}_2, \dot{VT}_3$ are evaluated in terms of $\dot{a}$ and $\dot{\psi}_p$. The derivatives $\dot{VT}_1, \dot{VT}_2, \dot{VT}_3$ may also be expressed in terms of $\dot{\delta}_t$ and $\dot{\epsilon}_t$ so that $\dot{\delta}_t$ and $\dot{\epsilon}_t$ are obtained as:

$$
\dot{\delta}_t = (VT_2 \cdot \dot{VT}_3 - VT_3 \cdot \dot{VT}_2)/cos^2 \epsilon_t \\
\dot{\epsilon}_t = VT_1/cos \epsilon_t
$$

The angular velocity of $t$ relative to $p$ is given by:

$$
(\omega_{tp})_p = \begin{bmatrix}
\dot{\delta}_t \\
0 \\
0
\end{bmatrix} + R_1(-\delta_t)
\begin{bmatrix}
0 \\
\dot{\epsilon}_t \\
0
\end{bmatrix}
= \begin{bmatrix}
\dot{\delta}_t \\
\dot{\epsilon}_t \cos \delta_t \\
\dot{\epsilon}_t \sin \delta_t
\end{bmatrix}
$$

and in frame $f$: $(\omega_{tp})_f = S_{pf}(\omega_{tp})_p$ where the transformation matrix
is evaluated as described in the next subsection.

The same procedure is followed for the orientation of the leg frame \( l \). The column vector \( VL=(l)_p \) is evaluated (Figure 80) as:

\[
\begin{bmatrix}
VL_1 \\
VL_2 \\
VL_3 
\end{bmatrix} =
\begin{bmatrix}
\cos\beta \cos\psi_p \\
\cos\beta \sin\psi_p \\
\sin\beta
\end{bmatrix}
\]

The orientation of frame \( l \) relative to frame \( p \) is given by angles \( \delta_{l} \) and \( \varepsilon_{l} \) where successive rotations through \( \delta_{l} \) about \( l_1 \) and \( \varepsilon_{l} \) about \( l_2 \) bring frame \( l \) from an initial alignment with frame \( p \) to the final orientation. The sine and cosine of \( \delta_{l} \) and \( \varepsilon_{l} \) and the derivatives \( \dot{\delta}_{l}, \dot{\varepsilon}_{l} \) are then obtained as functions of \( VLI \) and \( VLI \).

The transformation matrix \( S_{lp} \) is given by \( S_{lp} = R_1(-\delta_{l})R_2(-\varepsilon_{l}) \).

**ORIENTATION OF THE PELVIS**

The angular velocity of the pelvis relative to frame \( f \) will be evaluated as \( \omega_{pf} = -\omega_{fp} \), so that it is sufficient to consider the orientation of frame \( f \) relative to frame \( p \).

The vector \( QN \) joining the midpoint \( Q \) of the knee centres and the midpoint \( N \) of the shoulder centres is obtained as (Figure 81):

\[
QN = QG + GX + XN = dQG.l_3 + dGX.p_3 + dXN.t_3
\]

where the distances \( dQG, dGX, dXN \) are obtained from the link lengths \( LJ_1, LP \) and \( LT \) as:

\[
dQG = LJ_1 \cos\varepsilon_{jk}, \quad dGX = LP, \quad dXN = LT
\]

where \( \varepsilon_{jk} \) is the angle of abduction of each thigh away from the midline \( QG \).

\( f_3 \) is defined to be a unit vector parallel to \( QN \).
Let $(QN)_P = VQ_1 = dQG + dGX + dXN = VT_1$

If $L_Q$ is the magnitude of $QN$ then $L_Q^2 = VQ_1^2 + VQ_2^2 + VQ_3^2$ and:

$$\begin{bmatrix}
VQ_1/L_Q \\
VQ_2/L_Q \\
VQ_3/L_Q
\end{bmatrix} = \begin{bmatrix}
VF_1 \\
VF_2 \\
VF_3
\end{bmatrix}$$

$$\dot{VQ}_i = L_Q \cos \epsilon_{jk} \dot{V}L_i - L_Q \sin \epsilon_{jk} (\epsilon_{jk}) V_L i + LT \dot{V}T_i$$

and $$L_Q = \left( \sum VQ_i \cdot VQ_i \right) / L_Q$$

Since $VQ_i = L_Q \cdot VF_i$: $\dot{V}Q_i = \dot{L}_Q \cdot VF_i + L_Q \cdot \dot{VF}_i$

so that $\dot{VF}_i = (\dot{V}Q_i - L_Q \cdot \dot{VF}_i) / L_Q$

The orientation of $f$ relative to $p$ is given by angles $\delta_f, \epsilon_f$

which are obtained from $VF_i$ and $\dot{VF}_i$ as in the previous subsection.

If $f$ is initially aligned with $p$ then successive rotations through $\delta_f$ about $f_1$ and $\epsilon_f$ about $f_2$ bring frame $f$ into its final alignment.

The transformation matrix $S_{fp} = R_1(-\delta_f)R_2(-\epsilon_f)$ and its inverse is obtained as $S_{fp}^T = S_{fp}$. The angular velocity is given by:

$$\begin{bmatrix}
\dot{\delta}_f \\
\dot{\epsilon}_f \cos \delta_f \\
\dot{\epsilon}_f \sin \delta_f
\end{bmatrix} \begin{bmatrix}
\delta_f \\
\epsilon_f \cos \delta_f \\
\epsilon_f \sin \delta_f
\end{bmatrix}$$

from which: $$(\omega_{fp})_f = S_{fp} (\omega_{fp})_p$$

and then: $$(\omega_{fp})_f = -(\omega_{fp})_f'$$

### ORIENTATION OF THE LEGS

The orientation of the combined thigh frame $l$ relative to frame $p$ has been defined by successive rotations through angles $\delta_l$ and $\epsilon_l$.

If $\epsilon_{jk}$ is the angle of abduction of each thigh away from the midline, the orientation of the left thigh relative to frame $p$ is given by
successive rotations through angles $\delta_j$ and $\epsilon_j$ and the orientation of the right thigh is given by successive rotations through angles $\delta_k$ and $\epsilon_k$ where:

$$\epsilon_j = \epsilon_l - \epsilon_{jk} \quad \text{and} \quad \epsilon_k = \epsilon_l + \epsilon_{jk}$$

The transformation matrices $S_{jlp}$ and $S_{klp}$ are evaluated as:

$$S_{jlp} = S_{lp} R_2(\epsilon_{jk}) \quad \text{and} \quad S_{klp} = S_{lp} R_2(-\epsilon_{jk})$$

The angular velocities are calculated as:

$$\omega_{jlp} = \begin{bmatrix} \dot{\delta}_j \\ \dot{\epsilon}_j \cos \delta_j \\ \dot{\epsilon}_j \sin \delta_j \end{bmatrix} \quad \text{and} \quad \omega_{klp} = \begin{bmatrix} \dot{\delta}_k \\ \dot{\epsilon}_k \cos \delta_k \\ \dot{\epsilon}_k \sin \delta_k \end{bmatrix}$$

so that $(\omega_{jlp})_f = S_{pf}(\omega_{jlp})_p$ and $(\omega_{klp})_f = S_{pf}(\omega_{klp})_p$.

The orientation of the left calf $J_2$ relative to the left thigh $J_1$ is defined by the angle $\phi_2$ where a rotation through $\phi_2$ about axis $j_{21}$ brings $J_2$ into its final orientation.

The orientation of the right calf $K_2$ relative to the right thigh $K_1$ is defined by the angle $\phi_2$ where a rotation through $\phi_2$ about axis $k_{21}$ brings $K_2$ into its final orientation.

$\phi_2$ is obtained from the knee angle $\gamma_2$ using $\phi_2 = \pi - \gamma_2$

$$S_{j2p} = S_{jlp} R_1(-\phi_2) \quad \text{and} \quad S_{k2p} = S_{klp} R_1(-\phi_2)$$

The angular velocities are:

$$(\omega_{j2j1})_{j1} = (\omega_{k2k1})_{k1} = \begin{bmatrix} \dot{\phi}_2 \\ 0 \\ 0 \end{bmatrix}$$

so that $(\omega_{j2j1})_f = S_{jlf}(\omega_{j2j1})_{j1}$ and $(\omega_{k2k1})_f = S_{klf}(\omega_{k2k1})_{k1}$

where $S_{jlf} = S_{pf} S_{jlp}$ and $S_{klf} = S_{pf} S_{klp}$.
The orientation of the chest-head frame \( c \) relative to the thorax frame \( t \) is given by the angles \( \theta_c \) and \( \psi_c \). If frame \( c \) is initially aligned with \( t \) then successive rotations through \(-\theta_c\) about \( c_2 \) and \( \psi_c \) about \( c_3 \) bring frame \( c \) into its final orientation.

Conversely frame \( c \) will be brought into alignment with frame \( t \) by successive rotations through \(-\psi_c\) about \( c_3 \) and \( \theta_c \) about \( c_2 \). Thus the transformation matrix \( S_{ct} = R_2(\theta_c) \cdot R_3(-\psi_c) \).

\[
(\omega_{ct})_t = \begin{bmatrix} 0 \\ -\theta_c \\ 0 \end{bmatrix} + R_2(\theta_c) \begin{bmatrix} 0 \\ 0 \\ \psi_c \end{bmatrix} = \begin{bmatrix} -\psi_c \sin \theta_c \\ -\theta_c \\ \psi_c \cos \theta_c \end{bmatrix}
\]

and \( (\omega_{ct})_f = S_{tf}(\omega_{ct})_t \) where \( S_{tf} = S_{pf} S_{tp} \).

**ORIENTATION OF THE ARMS**

The frame \( a_1 \) of the upper left arm is brought from initial alignment with frame \( c \) into its final orientation by successive rotations through \(-\delta_a\) about \( a_1 \), \(-\varepsilon_a\) about \( a_2 \) and \( \psi_a \) about \( a_3 \).

\[
S_{alc} = R_1(\delta_a) R_2(\varepsilon_a) R_3(-\psi_a)
\]

and \( (\omega_{alc})_c = \begin{bmatrix} -\delta_a \\ 0 \\ 0 \end{bmatrix} + R_1(\delta_a) \begin{bmatrix} 0 \\ -\varepsilon_a \\ 0 \end{bmatrix} + R_1(\delta_a) R_2(\varepsilon_a) \begin{bmatrix} 0 \\ 0 \\ \psi_a \end{bmatrix} \)

\[
\begin{bmatrix} -\delta_a -\psi_a \sin \varepsilon_a \\ -\varepsilon_a \cos \delta_a + \psi_a \sin \delta_a \cos \varepsilon_a \\ -\varepsilon_a \sin \delta_a + \psi_a \cos \delta_a \cos \varepsilon_a \end{bmatrix}
\]

so that \( (\omega_{alc})_f = S_{cf}(\omega_{alc})_c \) where \( S_{cf} = S_{tf} S_{ct} \).

The orientation of the left forearm \( a_2 \) relative to the left upper arm \( a_1 \) is defined by the angle \( \theta_a \) where a rotation through \( \theta_a \)
about axis $a_2$ brings $A_2$ into its final alignment. $\theta_a$ is obtained from the elbow angle $\gamma_a$ using $\theta_a = \pi - \gamma_a$.

\[
(\omega_{a_2a_1})_{a_1} = \begin{bmatrix} 0 \\ \theta_a \\ 0 \end{bmatrix} \quad \text{and} \quad (\omega_{a_2a_1})_{f} = S_{\alpha f} (\omega_{a_2a_1})_{a_1} \text{ where } S_{\alpha f} = S_{cf} S_{alc}.
\]

The transformation matrix $S_{a_2f} = S_{\alpha f} R_2(-\theta_a)$ and its inverse $S_{fa_2} = S_{a_2f}'$ will be used in the transformation of the inertia tensor $I_{a_2a_2}$ from frame $a_2$ to frame $f$.

The frame $b_1$ of the upper right arm is brought from an initial alignment with frame $c$ into its final orientation by successive rotations through $-\delta_b$ about $b_{11}, \epsilon_b$ about $b_{12}$ and $-\psi_b$ about $b_{13}$

\[
S_{b_1c} = R_1(\delta_b) R_2(-\epsilon_b) R_3(\psi_b)
\]

and

\[
(\omega_{b_1c})_c = \begin{bmatrix}
\dot{\delta}_b \\
\dot{\epsilon}_b \\
\dot{\psi}_b
\end{bmatrix}
\begin{bmatrix}
-\delta_b & \epsilon_b & \psi_b \\
-\epsilon_b & \psi_b & -\delta_b \\
-\psi_b & -\delta_b & \epsilon_b
\end{bmatrix}
\]

so that $(\omega_{b_1c})_f = S_{cf} (\omega_{b_1c})_c$.

The right forearm $B_2$ is brought into its final orientation by a rotation through $-\theta_b$ about $b_{22}$ where $\theta_b = \pi - \gamma_b$.

\[
(\omega_{b_2b_1})_{b_1} = \begin{bmatrix} 0 \\ -\theta_b \\ 0 \end{bmatrix} \quad \text{and} \quad (\omega_{b_2b_1})_{f} = S_{b_1f} (\omega_{b_2b_1})_{b_1} \text{ where } S_{b_1f} = S_{cf} S_{b_1c}.
\]

$S_{b_2f} = S_{b_1f} R_2(\theta_b)$ and $S_{fb_2} = S_{b_2f}'$.

**EULERIAN ANGLES IN THE SAMMIE GRAPHICS SYSTEM**

The SAMMIE man model developed at Nottingham University was used to obtain computer graphics displays of simulations. The orientation of one body segment $S$ relative to another is specified
by successive rotations through $\varepsilon_1$ about $s_3$, $\varepsilon_2$ about $s_2$ and $\varepsilon_3$ about $s_3$ in the SAMMIE system. This z-y-z choice of axes makes it difficult to visualise the orientation since there is a singularity at $\varepsilon_2=0$.

The transformation matrix $S_{ip}$ may be evaluated as $S_{ip} = S_{fp}S_{if}^\ast$. If the orientation of $p$ relative to the inertial frame $i$ is given by successive rotations through $\varepsilon_1, \varepsilon_2, \varepsilon_3$ about axes $p_3, p_2, p_3$ the transformation $S_{ip}$ may be expressed as $S_{ip} = R_3(\varepsilon_3)R_2(\varepsilon_2)R_1(\varepsilon_1)$ where:

$$R_2(\varepsilon_2) = \begin{bmatrix} \cos \varepsilon_2 & 0 & -\sin \varepsilon_2 \\ 0 & 1 & 0 \\ \sin \varepsilon_2 & 0 & \cos \varepsilon_2 \end{bmatrix} \quad \text{and} \quad R_3(\varepsilon_3) = \begin{bmatrix} \cos \varepsilon_3 & \sin \varepsilon_3 & 0 \\ -\sin \varepsilon_3 & \cos \varepsilon_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

giving:

$$S_{ip} = \begin{bmatrix} \cos \varepsilon_1 \cdot \cos \varepsilon_2 \cdot \cos \varepsilon_3 - \sin \varepsilon_1 \cdot \sin \varepsilon_3 & \sin \varepsilon_1 \cdot \cos \varepsilon_2 \cdot \cos \varepsilon_3 + \cos \varepsilon_1 \cdot \sin \varepsilon_3 & -\sin \varepsilon_1 \cdot \cos \varepsilon_3 \\ -\cos \varepsilon_1 \cdot \cos \varepsilon_2 \cdot \sin \varepsilon_3 - \sin \varepsilon_1 \cdot \sin \varepsilon_3 \cdot \cos \varepsilon_3 + \cos \varepsilon_1 \cdot \cos \varepsilon_3 & \cos \varepsilon_1 \cdot \cos \varepsilon_3 \cdot \cos \varepsilon_3 + \sin \varepsilon_1 \cdot \sin \varepsilon_3 & \cos \varepsilon_1 \cdot \sin \varepsilon_3 \\ \cos \varepsilon_1 \cdot \sin \varepsilon_3 & \sin \varepsilon_1 \cdot \cos \varepsilon_3 & \cos \varepsilon_1 \cdot \cos \varepsilon_2 \end{bmatrix}$$

so that the sine and cosine of the Eulerian angles may be evaluated as:

$$\cos \varepsilon_2 = S_{33}$$
$$\sin \varepsilon_2 = (S_{23}^2 + S_{33}^2)^{1/2}$$
$$\cos \varepsilon_3 = -S_{13}/\sin \varepsilon_2$$
$$\sin \varepsilon_3 = S_{23}/\sin \varepsilon_2$$
$$\cos \varepsilon_1 = S_{31}/\sin \varepsilon_2$$
$$\sin \varepsilon_1 = S_{32}/\sin \varepsilon_2$$

where $S_{ab} = S_{ip}(a,b)$ and the singularity at $\varepsilon_2=0$ is treated by setting $\varepsilon_1=0$ so that:

$$\cos \varepsilon_3 = S_{11}/\cos \varepsilon_2$$
$$\sin \varepsilon_3 = S_{12}$$

The same procedure is followed to obtain the Eulerian orientation angles of each segment using the transformation matrices:

$$S_{pt}', S_{tc}', S_{cal}', S_{cbl}', S_{alat}', S_{blb}'$$
The file PERF produced by the program ISEG from anthropometric measurements gives the mass centre distance $Z_S$ and the link length $L_S$ for each segment $S$. The pelvis $P$ has the two link lengths $L_P$ and $L_2P$. If $G$ is the midpoint of the hip centres $H$ and $I$, the position vector $GP$ of the mass centre of $P$ is obtained as:

$$(GP)_f = S_{pf}\begin{bmatrix}0, 0, Z_P\end{bmatrix}^T$$

where $'$ denotes the transpose.

The vectors $GX$, $GH$, $GI$ which link $P$ to the segments $T$, $J_1$, $K_1$ are given by:

$$(GX)_f = S_{pf}\begin{bmatrix}0, 0, L_P\end{bmatrix}^T$$

$$(GH)_f = S_{pf}\begin{bmatrix}L_2P, 0, 0\end{bmatrix}^T$$

$$(GI)_f = S_{pf}\begin{bmatrix}-L_2P, 0, 0\end{bmatrix}^T$$

The same procedure is followed along the link system using the mass centre distances $Z_{J_1}$, $Z_{J_2}$, $Z_T$, $Z_C$, $Z_{A_1}$, $Z_{A_2}$ and link lengths $L_{J_1}$, $L_T$, $L_C$, $L_{A_1}$ to obtain the mass centre and joint centre locations relative to the previous joint centre. All position vectors are evaluated in frame $f$ using the appropriate transformation matrices and are then given relative to the point $G$ by summation along the link system. Note that the mass centre distances and link lengths of the right limbs $K_1$, $K_2$, $B_1$, $B_2$ are assumed to be equal to those of their left counterparts $J_1$, $J_2$, $A_1$, $A_2$.

The mass and mass centre locations of the supra-segments $J, K, A, B, D, U, F$ are then obtained by summation over the relevant segments. By subtraction the following position vectors of the mass centres of the supra-segments are obtained:

$$u_f, d_f, a_f, b_f, a_2 f, b_2 f, j_f, k_f, j_2 f, k_2 f$$

$$u^a, d^a, e^a, b^a, a_2^oa, b_2^ob, j^h, k_i, j_2 o_j, k_2 ok$$
CALCULATION OF INERTIA TENSORS

The inertia tensor of segment S in frame s is obtained as:

\[
(I_{ss})_s = \begin{bmatrix}
  XIS & 0 & 0 \\
  0 & YIS & 0 \\
  0 & 0 & ZIS
\end{bmatrix}
\]

where the principal moments of inertia XIS, YIS, ZIS of the segment referred to its mass centre are provided by file PERF for each of the segments P, J1, J2, T, C, A1, A2. It is assumed that the right limbs K1, K2, B1, B2 have identical principal moments to their left counterparts J1, J2, A1, A2.

\[I_{ss}\] is then evaluated in frame f as: \[(I_{ss})_f = Ssf(I_{ss})_sSfs'\]

The inertia tensor of the whole body is obtained in frame f as:

\[I_{ff} = \sum I_{sf}\]

where \[I_{sf} = I_{ss} + I_{msf}\] using (PA) which is the generalised theorem of parallel axes obtained for the derivation of the angular momentum equation. \[I_{msf}\] is the inertia tensor of a point mass \(m_s\) situated at the mass centre of segment S and in frame f:

\[
(I_{msf})_f = m_s \begin{bmatrix}
  s_2^2 + s_3^2 & -s_1 s_2 & -s_1 s_3 \\
  -s_2 s_1 & s_1^2 + s_3^2 & -s_2 s_3 \\
  -s_3 s_1 & -s_3 s_2 & s_1^2 + s_2^2
\end{bmatrix}
\]

where \[(s_f)_f = (FS)_f\]

gives the location of the mass centre of S relative to the whole body mass centre.

The same procedure is used to evaluate \[I_{uu}', I_{dd}', I_{aa}', I_{bb}', I_{a2a2}', I_{b2b2}', I_{jj}', I_{kk}', I_{j2j2}', I_{k2k2}\] in frame f.

CALCULATION OF MOMENTA TERMS

Equation (13) states the angular momentum equation in the form:

\[
h = h_{wfi} + h_{wpf} + h_{wp} + h_{wct}
+ h_{walc} + h_{wblc} + h_{wa2al} + h_{wb2bl}
+ h_{wjlp} + h_{wklp} + h_{wjl2nl} + h_{wk2kl}
\]
where the twelve terms are given by equations (14) through (25). Each of these terms may now be evaluated in frame $f$ using the expressions for angular velocities, position vectors and inertia tensors in frame $f$. The total momentum in the inertial frame $i$ is then given by $(h)_i = S_{fi}(h)_f$.

**SOLUTION OF THE ANGULAR MOMENTUM EQUATION**

For a simulation the total momentum $(h)_i$ is given and the motion of the system has to be determined so that $h_{wfi}$ is not known.

The angular momentum equation may be written as:

$$ h = h_{wfi} + h_{rel} $$

where $h_{rel}$ is the angular momentum associated with the internal movements.

$h$ is evaluated in frame $f$ using $(h)_f = S_{if}(h)_i$ and $(h_{rel})_f$ is evaluated using the expressions (15) through (25).

$h_{wfi}$ is then given in frame $f$ by:

$$ h_{wfi} = h - h_{rel}. $$

From equation (14) $h_{wfi} = I_{ff} \omega_{fi}$ and the angular velocity $\omega_{fi}$ has been expressed in the form:

$$ (\omega_{fi})_f = R_\omega \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \text{ where } R_\omega = \begin{bmatrix} \cos\theta\cos\psi & \sin\psi & 0 \\ -\cos\theta\sin\psi & \cos\psi & 0 \\ \sin\theta & 0 & 1 \end{bmatrix} $$

so that:

$$ I_{ff} R_\omega [\dot{\phi}, \dot{\theta}, \dot{\psi}]' = h - h_{rel}. $$

This equation is solved for $\dot{\phi}, \dot{\theta}, \dot{\psi}$ using the NAG library subroutine F04ATF which uses Crout's method with partial pivoting to decompose the matrix $I_{ff} R_\omega$ into the triangular form $LU$ where $L$ is lower triangular and $U$ is upper triangular. An approximation to $[\dot{\phi}, \dot{\theta}, \dot{\psi}]'$ is found by forward and backward substitution in the equations:

$$ LB = h - h_{rel} \text{ and } U[\dot{\phi}, \dot{\theta}, \dot{\psi}]' = B. $$
The residual vector \( R \) is then calculated where:

\[
R = (h-h_{rel}) - \int \omega R_0 \dot{\phi}, \dot{\theta}, \dot{\psi}
\]

and a correction \( C \) to \([\dot{\phi}, \dot{\theta}, \dot{\psi}]'\) is found by the solution of \( LU.C=R \). \([\dot{\phi}, \dot{\theta}, \dot{\psi}]'\) is replaced by \([\dot{\phi}, \dot{\theta}, \dot{\psi}]' + C \) and the process is repeated until machine accuracy is obtained.

**INTEGRATION OF THE EQUATION OF MOTION**

In the simulation program SIM the values \( D(1), D(2), D(3) \) of \( \dot{\phi}, \dot{\theta}, \dot{\psi} \) are calculated in subroutine DERIV using \( \phi, \theta, \psi \) and the values and rates of change of the internal orientation angles.

In the main program segment the differential equations:

\[
\dot{Y}(1) = D(1), \quad \dot{Y}(2) = D(2), \quad \dot{Y}(3) = D(3) \quad \text{(with } Y(1) = \phi, \ Y(2) = \theta, \ Y(3) = \psi \text{)}
\]

are solved using the NAG library routine D02ABF.

D02ABF is used to advance the solution over a time interval \( INT \) using a number of steps of Merson's form of the Runge-Kutta method. The routine obtains an estimate of the local truncation error at each step and varies the step-length to keep this estimate below an error bound specified by the user.

The interval \( INT \) is set to one hundredth of the flight time to enable the collection of orientation angle values throughout the simulation.

The processing time used for a simulation on a PRIME 750 computer is about 90 seconds for user specified movements and about 150 seconds when the orientation angles are given by their quintic spline coefficients.

**OUTPUT FILES**

For the movement NAM the program SIM creates the following output files:
NAMSIM which contains 101 sets of values of time, somersault, tilt and twist to enable the drawing of graphs.

NAMSAM which contains the Eulerian orientation angles used by the SAMMIE system to enable the depiction of a simulation using computer graphics.

For Option 1 SIM outputs:

NAMMOM which contains 101 sets of the three components of angular momentum together with the mean values and standard error estimates.

NAMSMM which contains the SAMMIE orientation angles of a filmed movement.

SUMMARY

A computer program has been developed for the simulation of aerial movement. Orientation angles may be specified using the spline coefficients output by the program FILM or may be defined by the user. The simulation may be displayed using tables, graphs or computer graphics.

The validation of the program SIM is undertaken in Chapter 6 where the accuracy of the combined use of the three programs ISEG, FILM and SIM is evaluated.
CHAPTER 6

VALIDATION OF THE SIMULATION MODEL

INTRODUCTION

In this chapter the accuracy of the combined use of the programs ISEG, FILM and SIM is evaluated by comparing the simulation and film values of somersault, tilt and twist for ten filmed movements. The effects of anthropometric measurement errors and film digitisation errors are determined by comparing simulations and by evaluating the variance in the angular momentum estimates.

COMPARISON OF SIMULATION AND FILM VALUES

Table 10 lists the maximum absolute deviations of the simulation values from the film values for each of the ten filmed movements. For all ten movements the maximum deviations are:

- somersault deviation: 0.05 revolutions
- tilt deviation: 8 degrees
- twist deviation: 0.14 revolutions

If deviations are expressed as a percentage of the total rotation the maxima are:

- somersault deviation: 3%
- twist deviation: 9%

These levels of accuracy should be borne in mind when drawing conclusions from the results of simulations.

A more detailed comparison of simulation and film for movement C41 is presented in Table 11 and in Figures 82 and 83. Appendix E uses these three formats to compare simulation and film for each of the ten movements. Note that the unit of time is taken to be the flight time of the movement.
Table 10. Maximum differences between film and simulation values

<table>
<thead>
<tr>
<th>movement</th>
<th>somersalt (revolutions)</th>
<th>tilt (degrees)</th>
<th>twist (revolutions)</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02</td>
<td>0.01</td>
<td>2</td>
<td>0.02</td>
</tr>
<tr>
<td>G08</td>
<td>0.01</td>
<td>4</td>
<td>0.04</td>
</tr>
<tr>
<td>G12</td>
<td>0.01</td>
<td>2</td>
<td>0.05</td>
</tr>
<tr>
<td>J71</td>
<td>0.03</td>
<td>6</td>
<td>0.09</td>
</tr>
<tr>
<td>J73</td>
<td>0.03</td>
<td>3</td>
<td>0.06</td>
</tr>
<tr>
<td>C11</td>
<td>0.01</td>
<td>2</td>
<td>0.09</td>
</tr>
<tr>
<td>C39</td>
<td>0.01</td>
<td>2</td>
<td>0.08</td>
</tr>
<tr>
<td>C41</td>
<td>0.04</td>
<td>6</td>
<td>0.12</td>
</tr>
<tr>
<td>C45</td>
<td>0.03</td>
<td>7</td>
<td>0.07</td>
</tr>
<tr>
<td>C47</td>
<td>0.05</td>
<td>8</td>
<td>0.14</td>
</tr>
</tbody>
</table>
Table 11. Comparison of simulation and film for C41

<table>
<thead>
<tr>
<th>TIME (T)</th>
<th>SOMERSAULT SIM</th>
<th>SOMERSAULT FILM</th>
<th>TILT SIM</th>
<th>TILT FILM</th>
<th>TWIST SIM</th>
<th>TWIST FILM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>REVOLUTIONS</td>
<td>DEGREES</td>
<td>REVOLUTIONS</td>
<td>DEGREES</td>
<td>REVOLUTIONS</td>
<td>DEGREES</td>
</tr>
<tr>
<td>0.0</td>
<td>0.12</td>
<td>0.12</td>
<td>-2.</td>
<td>-2.</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.1</td>
<td>0.33</td>
<td>0.32</td>
<td>-3.</td>
<td>-3.</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
<tr>
<td>0.2</td>
<td>0.58</td>
<td>0.58</td>
<td>3.</td>
<td>1.</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.3</td>
<td>0.83</td>
<td>0.83</td>
<td>5.</td>
<td>5.</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.4</td>
<td>1.03</td>
<td>1.03</td>
<td>9.</td>
<td>11.</td>
<td>0.03</td>
<td>0.09</td>
</tr>
<tr>
<td>0.5</td>
<td>1.16</td>
<td>1.17</td>
<td>13.</td>
<td>16.</td>
<td>0.34</td>
<td>0.36</td>
</tr>
<tr>
<td>0.6</td>
<td>1.28</td>
<td>1.30</td>
<td>15.</td>
<td>18.</td>
<td>0.70</td>
<td>0.73</td>
</tr>
<tr>
<td>0.7</td>
<td>1.40</td>
<td>1.42</td>
<td>13.</td>
<td>16.</td>
<td>1.10</td>
<td>1.16</td>
</tr>
<tr>
<td>0.8</td>
<td>1.53</td>
<td>1.54</td>
<td>12.</td>
<td>14.</td>
<td>1.28</td>
<td>1.39</td>
</tr>
<tr>
<td>0.9</td>
<td>1.66</td>
<td>1.69</td>
<td>10.</td>
<td>8.</td>
<td>1.37</td>
<td>1.49</td>
</tr>
<tr>
<td>1.0</td>
<td>1.82</td>
<td>1.86</td>
<td>8.</td>
<td>2.</td>
<td>1.39</td>
<td>1.49</td>
</tr>
</tbody>
</table>
Figure 82. Graphical comparison of film and simulation for C41
(circles : film data ; lines : simulation)
Figure 83. Computer graphics of film and simulation of C41
(upper set: film; lower set: simulation)
The simulation of the non-twisting double layout somersault C43 will be presented in Chapter 7 when control of the unstable layout somersault is examined.

STANDARD DEVIATIONS OF THE ANGULAR MOMENTUM ESTIMATES

For Option 1 the program SIM calculates 101 estimates of each component of the total angular momentum. From these 101 estimates the mean value and standard error of the mean are calculated. Since the total angular momentum during flight is constant the standard errors of each component give a measure of the accuracy of the combined use of the programs ISEG, FILM and SIM.

The unit of time is taken to be the flight time of a movement and angular momenta are expressed in the equivalent number of straight somersaults per unit time. Table 12 lists the standard errors of the three components of momentum in the inertial frame i for each of the ten filmed movements.

These standard errors appear to be quite small. For movement C41 each component has a standard error of about 0.008 straight somersaults per unit time which is less than 0.6% of the total momentum. However it should be remembered that the standard deviation of the set of 101 estimates is greater than the standard error of the mean by the factor \( \sqrt{101} \).

For movement C41 the standard deviations for each component are 0.084, 0.076, 0.077 straight somersaults per unit time. The standard deviation of the magnitudes of the deviation from the mean of the angular momentum estimates may be used as a measure of the total error. This momentum error estimate may be calculated as the magnitude of the vector whose components are the standard deviations of the estimates of the angular momentum components.

For movement C41 the momentum error estimate is 0.137 which is about 10% of the total momentum. This measure of the error in the calculated angular momentum vector will be used together with the twist deviation to evaluate the effects of measurement errors.
Table 12. Standard errors of the momenta estimates

unit of momentum = straight somersault per flight time

<table>
<thead>
<tr>
<th>movement</th>
<th>standard error of each component</th>
<th>total momentum</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02</td>
<td>0.004 0.002 0.002</td>
<td>0.033</td>
</tr>
<tr>
<td>G08</td>
<td>0.006 0.006 0.005</td>
<td>0.813</td>
</tr>
<tr>
<td>G12</td>
<td>0.006 0.003 0.003</td>
<td>0.711</td>
</tr>
<tr>
<td>J71</td>
<td>0.009 0.008 0.006</td>
<td>0.857</td>
</tr>
<tr>
<td>J73</td>
<td>0.009 0.005 0.004</td>
<td>0.902</td>
</tr>
<tr>
<td>C11</td>
<td>0.006 0.005 0.006</td>
<td>0.904</td>
</tr>
<tr>
<td>C39</td>
<td>0.006 0.004 0.006</td>
<td>0.755</td>
</tr>
<tr>
<td>C41</td>
<td>0.008 0.008 0.008</td>
<td>1.390</td>
</tr>
<tr>
<td>C45</td>
<td>0.006 0.006 0.007</td>
<td>1.365</td>
</tr>
<tr>
<td>C47</td>
<td>0.012 0.008 0.007</td>
<td>1.487</td>
</tr>
</tbody>
</table>
ANTHROPOMETRIC MEASUREMENT ERROR AND SIMULATION ERROR

The effect of measurement error may be determined by using two sets of anthropometric measurements of the performer CARL to produce two sets of segmental inertia parameters. Option 1 of the program SIM can then be used to produce two sets of momenta estimates for a filmed movement.

For movement C41 the mean of the two momenta error estimates is 10% of the total momentum whilst the deviation between the two sets is 3%.

Comparisons may also be made using the two inertia sets to produce two simulations of a filmed movement.

For movement C41 the twist error was 0.12 revolutions whilst the maximum deviation between the two simulations was 0.01 revolutions.

Thus it appears that anthropometric measurement errors have only a small effect on the calculated momenta and simulations.

FILM DIGITISATION ERROR AND SIMULATION ERROR

The program FILM uses the four combinations of the film data to produce four estimates of the orientation angles. These four sets arise from two digitisations of each film and so give two pairings in which the film data sets do not intersect. Each pair may be used to produce values for mean momenta and twist errors and the deviations between the two momenta and twist sets. The values for each pair may then be averaged to produce a single set of values which describe the differences arising from separate digitisations.

For movement C41 the momenta error is 9% of the total momentum whilst the deviation between independent sets is 7%.

For movement C41 the twist error was 0.10 revolutions whilst the deviation between simulations was 0.09 revolutions.

These measures indicate that much of the error in the momenta
and simulation values arises from digitisation error. Indeed one of the four film data combinations for C41 produced a maximum twist error of only 0.03 revolutions. Figure 84 shows momenta estimates arising from inertia and angle values which have been independently determined. Whilst each graph shows some systematic deviation from a constant value, most of the deviation appears to be due to digitisation errors and measurement errors.

This means that the accuracy of the simulation model is likely to be better than that indicated by the comparison of simulation and film values.

Ways of reducing the digitisation error are:

1. use more than two cameras
2. use frame rates higher than 60 frames per second
3. use digitising hardware with resolution better than 1 part per thousand.
4. digitise each film more than twice.

The experience gained whilst digitising the films suggests that much of the error arises from digitising landmarks which are obscured by other parts of the body. The use of an additional camera situated above or possibly behind the performer would provide a clear view of landmarks which are obscured from the other camera views.

SYSTEMATIC ERRORS

The arbitrary choices of simple solids used in the inertia model introduce systematic errors in the calculated inertia parameters. It is theoretically possible to detect and correct such errors by using an optimisation technique to minimise the momentum error estimate. Such an attempt is unlikely to be successful unless the film digitisation error is reduced.

The procedures used in the calculation of orientation angles and the idealised system of linked rigid bodies in the simulation model will lead to systematic errors. In addition it is unlikely that the programs ISEG, FILM and SIM are completely free from
Figure 84. Momenta estimates for C41 using repeated measurements
programming errors. All that can be said about these errors is that they do not produce more than 9% twist error in simulations of the ten filmed movements. Any attempt to improve the agreement between simulation and film by introducing modifications to the model will be frustrated by the present level of digitisation error.

**SUMMARY**

The maximum deviations of the simulations of ten filmed movements are 3% for somersault and 9% for twist. Whilst there are indications that much of the error arises from errors in the film data the levels of accuracy of the simulation model should not be assumed to be better than these values.
CHAPTER 7

TWISTING TECHNIQUES

INTRODUCTION

In this chapter the simulation model is used to fulfil the purpose of the study as stated in Chapter 1, viz:

(a) To present quantifiable mechanical explanations of the operation of twisting techniques.

(b) To establish quantitatively the capacities of different techniques for producing and controlling twist.

(c) To establish the contributions of the various techniques to the twist produced in actual performances.

The mechanics of the various techniques are explained by using simple mathematical models which permit general analytical solutions and by using the simulation model to produce idealised simulations which illustrate the operation of the techniques.

The capacities of the various techniques for producing and controlling twist are established using simulations and whilst the range of simulations used is extensive it is not exhaustive so that the limits obtained should be regarded as indicative rather than absolute.

The contributions of the different techniques to the twist produced in the filmed movements are established by modifying the internal orientation angles and observing the effects on the corresponding simulations.

THE TORQUE-FREE MOTION OF A RIGID BODY

In direct and tilt twists a sustained twist is produced during which the body often maintains an apparently fixed internal
configuration. For such phases of twisting somersaults the system may be modelled as a single rigid body and this permits a number of general analytical results to be derived from the simplified equation of motion.

In Chapter 5 the equation of motion of the simulation model was obtained in the form:

\[ h = h_{\text{ref}} + h_{\text{rel}} \]

where \( h \) is the total angular momentum

\( h_{\text{ref}} \) is the angular momentum corresponding to the motion of the reference frame \( f \) of the system

and \( h_{\text{rel}} \) is the angular momentum arising from the internal movements.

When the system is modelled as a rigid body \( h_{\text{rel}} = 0 \) so that the equation of motion becomes:

\[ h = I_{ff} \omega_{fi} \]  

(1)

where \( I_{ff} \) is the whole body inertia tensor

and \( \omega_{fi} \) is the angular velocity of the body relative to the inertial reference frame \( i \).

If the reference frame \( f \) of the system is chosen to be oriented with the principal axes of the body then in frame \( f \) the inertia tensor \( I_{ff} \) has the diagonal form:

\[
(I_{ff})_f = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix}
\]

where \( A, B, C \) are the principal moments of inertia corresponding to the principal axes \( f_1, f_2, f_3 \).

In frame \( f \) the angular momentum is:

\[ (h)_f = S_{if} \cdot (h)_i = R_3(\psi) \cdot R_2(\theta) \cdot R_1(\phi) \cdot (h)_i \]

where \( R_1(\phi), R_2(\theta), R_3(\psi) \) are rotation matrices corresponding to the somersault angle \( \phi \), the tilt angle \( \theta \) and the twist angle \( \psi \).
Thus if $(h)_i = \begin{bmatrix} h \\ 0 \\ 0 \end{bmatrix}$ then $(h)_f = \begin{bmatrix} h\cos\theta \cos\psi \\ -h\cos\theta \sin\psi \\ h\sin\theta \end{bmatrix}$

where the symbol $h$ will now be used to denote a scalar quantity.

In Chapter 5 the angular velocity $\omega_{fi}$ was evaluated in frame $f$ as:

$$
(\omega_{fi})_f = \begin{bmatrix} 
\cos\theta \cos\psi & \sin\psi & 0 \\
-cos\theta \sin\psi & \cos\psi & 0 \\
\sin\theta & 0 & 1 
\end{bmatrix}
$$

and so equation (1) takes the form:

$$
\begin{bmatrix} 
h\cos\theta \cos\psi \\
-h\cos\theta \sin\psi \\
h\sin\theta 
\end{bmatrix} = \begin{bmatrix} 
A & 0 & 0 \\
0 & B & 0 \\
0 & 0 & C 
\end{bmatrix}
\begin{bmatrix} 
\dot{\phi} \\
\dot{\theta} \\
\psi 
\end{bmatrix} + \begin{bmatrix} 
h\sin^2\psi + B\cos^2\psi \\
-h(A-B)\cos\theta \sin\psi \cos\psi \\
(h/C - \dot{\psi}) \sin\theta 
\end{bmatrix}
$$

which gives rise to:

$$
\begin{align*}
\dot{\phi} &= h(A\sin^2\psi + B\cos^2\psi)/AB \\
\dot{\theta} &= -h(A-B)\cos\theta \sin\psi \cos\psi /AB \\
\dot{\psi} &= (h/C - \dot{\phi}) \sin\theta 
\end{align*}
$$

The conservation of rotational energy may be derived from equations (2), (3), (4) using a single integration or since the absence of an external torque implies that no work is done the energy equation may be written as:

$$
2T = A\omega_1^2 + B\omega_2^2 + C\omega_3^2
$$

where $T$ is the (constant) rotational energy and $\omega_1, \omega_2, \omega_3$ are the components of the angular velocity $(\omega_{fi})_f$.

Since the angular momentum components in frame $f$ are $h_1, h_2, h_3$ where:
The energy equation may be written as:

\[ 2T = \frac{h_1^2}{A} + \frac{h_2^2}{B} + \frac{h_3^2}{C} \]

which leads to:

\[ 2T = \frac{h^2 \cos^2 \theta \cos^2 \psi}{A} + \frac{h^2 \cos^2 \theta \sin^2 \psi}{B} + \frac{h^2 \sin^2 \theta}{C} \quad (5) \]

If \( C \) is the minimum principal moment of inertia positive constants \( c_\alpha^2 \) and \( c_\beta^2 \) may be defined as:

\[ c_\alpha^2 = \frac{(h^2/C-2T)/(h^2/C-h^2/A)} { (h^2/C-2T)/(h^2/C-h^2/B)} \quad (6) \]

\[ c_\beta^2 = \frac{(h^2/C-2T)/(h^2/C-h^2/B)} { (h^2/C-2T)/(h^2/C-h^2/A)} \quad (7) \]

so that substitution in equation (5) permits the energy equation to be written in the following two forms:

\[ \sin^2 \psi = \frac{(1-c_\alpha^2 \sec^2 \theta)B(A-C)/C(A-B)} { (1-c_\beta^2 \sec^2 \theta)A(B-C)/C(A-B)} \quad (8) \]

\[ \cos^2 \psi = \frac{(c_\beta^2 \sec^2 \theta-1)A(B-C)/C(A-B)} { (c_\alpha^2 \sec^2 \theta)B(A-C)/C(A-B)} \quad (9) \]

Equations (2) and (8) then give the somersault rate as:

\[ \dot{\psi} = h(1/C-1/A)c_\alpha^2 \sec^2 \theta \quad (10) \]

whilst equations (3) and (10) give the twist rate as:

\[ \dot{\psi} = h(1/C-1/A)c_\alpha^2 \sec^2 \theta \sin \theta \quad (11) \]

It should be noted that equations (2) through (11) hold for \( A>B>C \) and for \( B>A>C \).

Suppose that \( A>B>C \). Equation (5) shows that the maximum possible energy for a rigid body with angular momentum \( h \) is given by \( 2T=h^2/C \) corresponding to a pure twist about axis \( f_3 \) whilst the minimum energy is given by \( 2T=h^2/A \) which corresponds to a plain somersault about axis \( f_1 \).
From equation (6) we may deduce that $0 < c_{\alpha}^2 < 1$ and replace $c_{\alpha}^2$ by $\cos^2 \alpha$ in the above equations knowing that the equation $\cos^2 \alpha = c_{\alpha}^2$ produces a real value for the angle $\alpha$.

Following the procedure of Whittaker (1937, p. 146), equations (3) and (8) give:

$$\dot{y}^2 = d^2 (1-y^2) (y^2 + e)$$  \hspace{1cm} (12)

where $y = \sin \theta / \sin \alpha$

$$d^2 = h^2 \sin^2 \alpha (A-C) (B-C) / ABC^2$$

and

$$e = \csc^2 \alpha \cdot C(A-B)/A(B-C) - B(A-C)/A(B-C)$$

In order to integrate equation (12) two cases will be considered:

If $2T > h^2 / B$ then equation (7) shows that $0 < c_{\beta}^2 < 1$ so that there exists an angle $\beta$ for which $\cos^2 \beta = c_{\beta}^2$. In this case it will be shown that the twist angle $\psi$ is a monotonic function of time.

If $2T < h^2 / B$ then equation (7) gives $c_{\beta}^2 > 1$ and so equation (9) shows that $\cos \psi$ is never zero. In this case it will be shown that the twist angle $\psi$ is a periodic function of time.

**THE ROD MODE**

If a rod is defined to be an axially symmetric body with principal moments of inertia $A, B, C$ such that $A = B > C$, equation (5) shows that $2T > h^2 / B$ with equality only when $\theta = 0$. It is for this reason that a rigid body for which $2T > h^2 / B$ will be said to be in the rod mode.

Equations (6) and (7) show that the condition $2T > h^2 / B$ for a rigid body with principal moments $A > B > C$ is equivalent to:

$$\sin^2 \alpha > C(A-B) / B(A-C)$$

and to:

$$\sin^2 \beta > 0$$

Equation (12) may be written as:

$$\dot{y}^2 = p^2 (1-y^2) (y^2 - (1-k^2))$$  \hspace{1cm} (13)
where \( p^2 = h^2 \sin^2 \alpha (A-C) (B-C)/ABC^2 \)
and \( k^2 = \cot^2 \alpha . C(A-B)/A(B-C) < 1 \)

Equation (13) has solution \( y = dn(pt) \) where \( dn \) is a Jacobian elliptic function with modulus \( k \) (Whittaker and Watson, 1962, p.494).

Thus \( \sin \theta = \sin \alpha . dn(pt) \) (14)

The function \( dn(pt) \) oscillates between the values 1 and \( k_1 \)
(where \( k_1 \) is the complementary modulus defined by \( k_1^2 + k^2 = 1 \)) and has
time period \( 2K/p \) where \( K \) is the complete elliptic integral of the
first kind (Bowman, 1953, p.13).

Jahnke and Emde (1945) show that \( K \) may be evaluated as follows:

If \( M(a,b) \) is the arithmeto-geometric mean of the positive real
numbers \( a \) and \( b \) defined by \( M = \lim a_n = \lim b_n \) with:

\[
    a_1 = \frac{1}{2} (a+b), \quad b_1 = (ab)^{\frac{1}{2}}, \quad a_n = \frac{1}{2} (a_{n-1}+b_{n-1}), \quad b_n = (a_{n-1}b_{n-1})^{\frac{1}{2}}
\]

then \( K = \frac{\pi}{M(1,k_1)} \) (15)

From equation (14) \( \sin \theta \) varies between \( \sin \alpha \) and \( k_1 \sin \alpha = \sin \beta \)
where \( \beta \) is defined by \( \cos^2 \beta = \frac{c_\beta^2}{c_\alpha^2} \) in equation (7).

It may be concluded that \( \theta \) is periodic with time period \( 2K/p \) and
with bounding values \( \alpha \) and \( \beta \).

Equation (11) shows that the twist rate \( \dot{\psi} \) has the sign of \( h \sin \theta \)
and so \( \psi \) is monotonic.

Equations (8) and (9) show that \( \theta = \alpha \) corresponds to the zero and
half twist positions whilst \( \theta = \beta \) corresponds to the quarter and three-
quarter twist positions so that the time taken for a full twist is
twice the period of \( \theta \).

Thus the average twist rate is \( n = \frac{\pi p}{K} \) (16)

Equation (2) shows that the somersault rate \( \dot{\phi} \) varies between
\( h/A \) and \( h/B \) and equation (10) indicates that the time period of the
oscillations is the same as the period of $\theta$, namely $2K/p$.

From equation (10) the average somersault rate is:

$$\Omega = h(1/C-(1/C-1/A)V(a))$$

where $V(a)$ is the average value of $\cos^2a/\cos^2\theta$.

$V(a)$ may be expressed in terms of Jacobi's Zeta function and an elliptic integral of the first kind as follows:

From equation (14) $\sin^6 = \sin^6\alpha$. $dn\sqrt{pt}$ so that:

$$\cos^2 = 1-\sin^2\alpha. dn^2 pt$$

$$= 1-\sin^2\alpha (1-k^2 sn^2 pt)$$

$$= \cos^2 \alpha + k^2 \sin^2 \alpha. sn^2 u$$

where $u = pt$

Thus

$$V(a) = \frac{1}{K} \int_{0}^{K} \frac{du}{1+k^2 \tan^2 \alpha. sn^2 u}$$

which is Legendre's complete elliptic integral of the third kind.

Writing $\tan^2 \alpha = -sn^2 \alpha$ leads to:

$$V(a) = \frac{1}{K} \left[ K + \frac{sn a. cn a. dn a. \pi(K,a)}{sna. cna. dna. \pi(K,a)} \right]$$

where $\pi(K,a) = \int_{0}^{K} \frac{k^2 sn a. cn a. dna. sn^2 u. du}{1-k^2 sn^2 a. sn^2 u}$

is Jacobi's complete elliptic integral of the third kind and is equal to $K.Z(a)$ where $Z$ is Jacobi's Zeta function (Whittaker and Watson, 1962, pp.522-523) so that:

$$V(a) = 1 + \frac{sn a. cna. dna. \pi(K,a)}{sna. cna. dna} Z(a)$$

Using Jacobi's imaginary transformation $a = ib$ (Whittaker and Watson, 1962, p.519) leads to:

$$V(a) = 1 - sn^2 b + \frac{\frac{1}{dn b} + \frac{\pi b}{2K \sqrt{1}}}{dn b} [Z_1(b) + \frac{\pi b}{2K}]$$
where the function subscript 1 denotes that the modulus is $k_1$.

\[ \tan^2 \alpha = - \text{sn}^2 a = - \frac{\text{sn}^2 b}{\text{cn}^2 b} \]
and
\[ \sin^2 \alpha + \cos^2 \alpha = 1 = \frac{\text{sn}^2 b}{\text{cn}^2 b} \]

it may be concluded that $\sin^2 = \text{sn}^2 b$ so that $b = F_1(a)$ where $F_1$ is an elliptic integral of the first kind with modulus $k_1$ and $V(a)$ takes the form:

\[ V(a) = \cos^2 \alpha + \sin \alpha \cos \alpha (1 - k_1^2 \sin^2 \alpha) - \frac{1}{2} \left[ Z_1(F_1(a)) + \pi F_1(a)/2 \right] \] (18)

The general motion of a rigid body in the rod mode may now be described:

The somersault angle $\phi$ and the twist angle $\psi$ steadily increase whilst the tilt angle $\theta$ oscillates between $\alpha$ and $\beta$ (assuming that $h, \alpha, \beta > 0$).

At the half and full twist positions $\sin \psi = 0$ so that:

(3), (8): $\theta$ attains its maximum value of $\alpha$ and $\dot{\theta} = 0$
(2): $\dot{\phi}$ attains its minimum value of $h/A$
(4): $\dot{\psi}$ attains its maximum value of $(h/C - h/A) \sin \alpha$

At the quarter and three-quarter twist positions $\cos \psi = 0$ so that:

(3), (9): $\theta$ attains its minimum value of $\beta$ and $\dot{\theta} = 0$
(2): $\dot{\phi}$ attains its maximum value of $h/B$
(4): $\dot{\psi}$ attains its minimum value of $(h/C - h/B) \sin \beta$

During the motion the average somersault and twist rates are given by equations (17) and (16).

**THE MOTION OF A ROD**

A rod has principal moments $A, B, C$ such that $A = B > C$. As remarked earlier, equation (5) implies that $2T > h^2 / B$ providing $\theta \neq 0$ so that the rod must move in the rod mode.

From equation (2): $\dot{\psi} = h/A$
\[ (3): \dot{\theta} = 0 \]
\[ (4): \dot{\psi} = (h/C-h/A)\sin \theta \]

and so the somersault rate has the constant value \( \Omega \) where:
\[ \Omega = h/A \]  \hspace{1cm} (19)

the tilt angle \( \theta \) has the constant value \( \alpha \),
and the twist rate has the constant value \( p \) where:
\[ p = \Omega(A/C-1)\sin \alpha \]  \hspace{1cm} (20)

Equations (19) and (20) are identical to those given by Travis (1968). In the literature review it was noted that other equations were provided by Eaves (1969, 1971) and Frolich (1979). The apparent discrepancies between the various equations may be explained as follows:

Figure 85 shows that the angular velocity vector \( \omega \) may be
resolved into components \( \omega_1 \) and \( \omega_3 \) along the body axes \( f_1 \) and \( f_3 \) or
may be resolved into components \( \Omega \) and \( p \) along axes \( i_1 \) and \( f_3 \) where
\( i_1 \) is fixed in space. It has been shown above that \( \Omega \) and \( p \) are the
rates of change of the somersault and twist angles and so \( \Omega \) and \( p \)
may be referred to as the somersault and twist rates. On the other
hand \( \omega_1 \) and \( \omega_3 \) are components of the angular velocity along the
principal axes \( f_1 \) and \( f_3 \) and whilst it is permissible to refer to
them as the somersault and twist angular velocities, this can be
misleading since \( \Omega \) and \( p \) are also angular velocities associated with
somersault and twist.

Resolving along \( f_1 \) and \( f_3 \) gives:
\[ \omega_1 = \Omega \cos \theta \]
\[ \omega_3 = p + \Omega \sin \theta \]

In Figure 86 the angular momentum vector is parallel to \( i_1 \) and
has components \( A\omega_1 \) and \( C\omega_3 \) along the principal axes \( f_1 \) and \( f_3 \).
Because of the symmetry equation \( A = B \), the principal axis \( f_1 \) may be
chosen to lie in the plane defined by \( i_1 \) and \( f_3 \) and so the angular
velocity \( \omega_2 = \dot{\theta} \) must be zero since there is no component of angular
momentum perpendicular to this plane. Thus the tilt angle \( \theta \) has a
constant value.
Figure 85. Angular velocity components of a rod

Figure 86. Angular momentum components of a rod
Resolving parallel and perpendicular to \( i_1 \) gives:

\[
  h = \omega_1 \cos \theta + \omega_3 \sin \theta \\
  \omega_3 \cos \theta = \omega_1 \sin \theta
\]

The last equation may be written as:

\[
  \omega_3 = \omega_1 (A/C) \tan \theta \tag{21}
\]

and is equivalent to that used by Eaves (1969, p. 76).

Substituting for \( \omega_1 \) gives:

\[
  \omega_3 = \Omega (A/C) \sin \theta \tag{22}
\]

which is equivalent to that given by Frolich (1979).

Substituting for \( \omega_3 \) produces equation (20) again:

\[
  p = \Omega (A/C - 1) \sin \theta
\]

Whilst each of equations (20), (21) and (22) governs the motion only equation (20) gives the number of twists per somersault.

Eaves (1971) analysed the motion of a rod by considering the movement of the momental ellipsoid on the invariable plane (described in Synge and Griffith, 1959, pp. 375-377). Such an analysis leads once again to equation (20):

\[
  p = \Omega (A/C - 1) \sin \theta
\]

The momental ellipsoid has semi-axes inversely proportional to the square roots of the principal moments of inertia (Synge and Griffith, 1959, p. 284) so that the ellipsoid is cigar-shaped. Eaves, however, took the semi-axes to be directly proportional to the principal inertias and deduced that the ellipsoid was disc-shaped. This error accounts for the discrepancy between equation (20) and the statement of Eaves that:

\[
  p = \Omega \sin \theta.
\]
THE DISC MODE

If a disc is defined to be an axially symmetric body with principal moments of inertia $A,B,C$ such that $A>B=C$, equation (5) shows that $2Th^2/B$ with equality only when the disc rotates about a minor principal axis. It is for this reason that a rigid body for which $2Th^2/B$ will be said to be in the disc mode.

As noted earlier, equation (6) implies that $0<\alpha<\pi/2$ so that there exists an angle $\alpha$ for which $\cos^2\alpha=\alpha^2$. The condition $2Th^2/B$ for a rigid body with principal moments $A>B>C$ may then be written as:

$$\sin^2\alpha<\frac{C(A-B)}{B(A-C)}$$

or using equation (7) as:

$$\beta^2 > 1$$

which together with equation (9) implies that $\cos\psi$ is never zero and so the quarter twist position is not attained during the motion.

Equation (12) may be written as:

$$\gamma^2 = q^2(1-\gamma^2)(k^2 \gamma^2 + k_1^2)$$

where $q^2 = h^2 \cos^2\alpha(A-C)/(A-B)$ and $k^2 = \tan^2\alpha/(B-C)/(A-B)<1$

and $k_1^2 = 1-k^2$

Equation (23) has solution $y=cn(qt)$ where $cn$ is a Jacobian elliptic function with modulus $k$ (Whittaker and Watson, 1962, p.493).

Thus

$$\sin\theta = \sin\alpha \cdot cn(qt)$$  \hspace{1cm} (24)

The function $cn(qt)$ oscillates between 1 and -1 and has time period $4K/q$, where $K$ is the complete elliptic integral of the first kind (Bowman, 1953, p.13) and may be evaluated using equation (15).

As a consequence $\theta$ is periodic with time period $4K/q$ and with bounding values $\alpha$ and $-\alpha$.

Equation (11) shows that the twist rate $\dot{\psi}$ has the sign of $h\sin\theta$ and is zero only when $\theta=0$. Thus $\dot{\psi}$ oscillates between $\dot{\psi}_0$ and $-\dot{\psi}_0$. 


where the angle \( \psi \) corresponds to \( \theta = 0 \) and is given by equation (8) as:

\[
\sin^2 \psi = \sin^2 \alpha \cdot B(A-C)/(C(A-B)) > \sin^2 \alpha
\]

The time period of \( \psi \) is the same as the period of \( \theta \), namely \( 4K/q \) and since \( \psi \) is oscillatory the average twist rate is zero.

Equation (10) shows that the somersault rate \( \dot{\psi} \) varies between \( h/A \) and \( h\cos^2 \alpha/A + h\sin^2 \alpha/C \) and has time period one half that of \( \theta \), namely \( 2K/q \).

From equation (10) the average somersault rate is:

\[
\Omega = h(1/C - (1/C - 1/A)W(\alpha))
\]

where \( W(\alpha) \) is the average value of \( \cos^2 \alpha/\cos^2 \theta \).

From equation (24) \( \sin \theta = \sin \alpha \cdot \cn(\sqrt{q}t) \) so that:

\[
\cos^2 \theta = 1 - \sin^2 \alpha \cdot \cn^2 \sqrt{q}t
= 1 - \sin^2 \alpha (1 - \sn^2 \sqrt{q}t)
= \cos^2 \alpha + \sin^2 \alpha \cdot \sn^2 u \text{ where } u = \sqrt{q}t
\]

Thus

\[
W(\alpha) = \frac{1}{K} \int_{0}^{1} \frac{du}{1 + \tan^2 \alpha \cdot \sn^2 u}
= \frac{1}{K} \int_{0}^{\tilde{K} \alpha} \frac{du}{1 + \tilde{k} \tan^2 \alpha_1 \cdot \sn^2 u}, \text{ where } \tilde{k} \tan \alpha_1 = \tan \alpha
= V(\alpha_1) \text{ where } V \text{ is given by equation (18)}.
\]

The general motion of a rigid body in the disc mode may now be described:

The somersault angle \( \phi \) steadily increases (for \( h > 0 \)) whilst the tilt angle \( \theta \) oscillates between \( \pm \alpha \) and the twist angle \( \psi \) oscillates between \( \pm \psi \).

As the time \( t \) increases from 0 to the period \( 4K/q \) the oscillations occur in the sequence:
\[ t = 0 \quad \theta = \alpha \quad \psi = 0 \quad \phi \text{ is a minimum} \]
\[ t = \frac{K}{q} \quad \theta = 0 \quad \psi = \psi_0 \quad \phi \text{ is a maximum} \]
\[ t = 2\frac{K}{q} \quad \theta = -\alpha \quad \psi = 0 \quad \phi \text{ is a minimum} \]
\[ t = 3\frac{K}{q} \quad \theta = 0 \quad \psi = -\psi_0 \quad \phi \text{ is a maximum} \]

The average somersault rate is given by equation (25).

**THE MOTION OF A DISC**

A disc has principal moments \( A, B, C \) such that \( A > B = C \). As remarked earlier, equation (5) implies that \( 2T < h^2/B \) (providing \( \cos \theta \neq 0 \)) so that a disc must move in the disc mode.

With \( B = C \) equation (22) becomes:
\[ \dot{y}^2 = q^2 (1-y^2) \]

where \( \dot{y} = \sin \theta / \sin \alpha \) and \( q = h \cos \alpha (A-C)/AC \).

Thus \( \sin \theta = \sin \alpha \cdot \cos \theta t \) and \( \theta \) oscillates between \( \pm \alpha \) with time period \( 2\pi/q \).

Equation (8) then gives \( \tan \psi = \tan \alpha \cdot \sin \theta t \) so that \( \psi \) oscillates between \( \pm \alpha \) with period \( 2\pi/q \).

The average somersault rate \( \Omega \) is given by equation (25) where \( W(\alpha) \) is the average value of \( \cos^2 \alpha / \cos^2 \theta \) so that:
\[ W(\alpha) = \frac{2}{\pi} \int_0^{\pi} \frac{du}{1+\tan^2 \alpha \cdot \sin^2 u} \]

which may be evaluated as \( W(\alpha) = \cos \alpha \) using the substitution \( \tan u = \cos \alpha \cdot \tan v \).

Thus \( \Omega = h(1/C-(1/C-1/A) \cos \alpha) \) is the average somersault rate and the number of somersaults per oscillation is given by:
\[ \frac{\Omega}{q} = \sec \alpha (A/(A-C))-1 \]

(26)

and is greater than 1 since \( C > \frac{1}{2} A \).
For a rigid body with \( A > B > C \) the condition \( 2T = h^2/B \) is satisfied by steady rotations about the axis corresponding to the intermediate principal moment \( B \). In addition, there is a motion in which rotation about the intermediate axis in one direction leads to a half twist followed by rotation in the opposite direction:

Equation (6) shows that the condition \( 2T = h^2/B \) is equivalent to:

\[
\sin^2 \alpha_0 = \frac{C(A-B)}{B(A-C)}
\]

where \( \alpha_0 \) is the angle of tilt when the twist angle \( \psi \) is zero.

Equation (7) shows that \( 2T = h^2/B \) is equivalent to:

\[
\beta_0 = 0
\]

where \( \beta_0 \) is the angle of tilt at the quarter twist position.

Equation (12) becomes:

\[
\dot{y}^2 = d^2 y^2 (1-y^2)
\]

where \( d^2 = h^2 (A-B)(B-C)/AB^2C \).

Equation (27) has solution \( y = \text{sech} (dt) = 2/(\exp(dt) + \exp(-dt)) \) and so:

\[
\sin \theta = \sin \alpha_0 \text{sech}(dt)
\]

The function \( \text{sech}(dt) \) has a maximum value of 1 at \( t=0 \) and approaches zero for large values of \( |t| \) and so \( \theta = \beta_0 \) at \( t=0 \) with \( \theta \) decaying to zero as \( |t| \) approaches infinity.

Equations (9) and (28) lead to:

\[
\tan^2 \psi = \sec^2 \alpha_0 \sinh^2 dt
\]

and since equation (11) implies that \( \psi \) has the sign of \( h \sin \theta \) the twist angle \( \psi \) is given by:

\[
\tan \psi = \sec \alpha_0 \sinh (dt)
\]

in the case \( h \alpha_0 > 0 \).
Thus the twist angle $\psi$ increases from \(-\frac{1}{2}\pi\) radians at $t=-\infty$, through zero at $t=0$, to $+\frac{1}{2}\pi$ radians at $t=+\infty$.

With $\alpha=\alpha_0$ equation (10) may be rewritten as:

$$\dot{\psi} = h(1/B - (1/C - 1/B)\tan^2\theta)$$

which gives the somersault rate as a function of time using equation (28).

The resulting expression for $\dot{\psi}$ may be integrated to give:

$$\dot{\psi} = (h/B)t - \arctan(\tan\alpha_0 \tanh(dt))$$  \hspace{1cm} \text{(30)}$$

where $\tanh(dt) = (\exp(dt) - \exp(-dt))/ (\exp(dt) + \exp(-dt))$.

The somersault rate $\dot{\psi}$ decreases from $h/B$ at $t=-\infty$ to $h/A$ at $t=0$ and then increases to $h/B$ as $t$ approaches $+\infty$. Since the time period of the oscillation is infinite and for large values of $|t|$ equation (30) gives the somersault angle as approximately $(h/B)t - \alpha_0 \cdot \text{sign}(t)$, the average somersault rate is $h/B$ although at any given time the actual somersault rate is less than $h/B$.

Equation (11) shows that the twist rate has a maximum value of $h(1/C - 1/A)\sin\alpha_0$ at time $t=0$ and approaches zero for large values of $|t|$. Over the infinite time period of the half twist the average twist rate is zero although at any instant the actual twist rate is greater than zero.

At time $t=0$ the ratio $\dot{\psi}/\dot{\phi}$ has the maximum value:

$$\max(\dot{\psi}/\dot{\phi}) = (A/C - 1)\sin\alpha_0 = (A/C - 1)^{1/2} (A/B - 1)^{1/2}$$  \hspace{1cm} \text{(31)}$$

In this idealised situation in which $2T=h^2/B$ the motion of a rigid body comprises an infinite number of forward somersaults (for $h>0$), followed by a finite number of somersaults in which the half twist becomes evident, followed by an infinite number of backward somersaults. In any real situation $2T$ will not be exactly equal to $h^2/B$ and the motion will be either in the disc mode or the rod mode and the oscillation period of $\theta$ will be finite. Such motions for which $2T$ is approximately equal to $h^2/B$ will have the following
characteristics which are not dependent upon the mode of motion:

During the oscillation of the tilt angle $\theta$, the motion will exhibit the following three phases:

(a) somersault in one direction with no apparent twist
(b) somersault during which a half twist occurs
(c) somersault in the opposite direction with no apparent twist.

This cycle of movement will be repeated throughout the motion and in the rod mode the twist is monotonic so that consecutive half twists will be in the same direction, whilst in the disc mode the twist is oscillatory so that consecutive twists will be in opposing directions. In the disc mode the amount of twist in a cycle will be slightly less than a half twist.

If a rigid body is initially rotating about an axis close to the intermediate principal axis, $2T$ will approximately equal $h^2/B$ and the subsequent motion will consist of a sequence of half twists in which the twist rate varies from a value close to zero to a maximum at the quarter twist position. In other words rotations about the intermediate principal axis are unstable.

The above discussion serves three purposes:

(a) it shows how the rod and disc modes approach each other
(b) it explains the instability of rotations about the intermediate principal axis
(c) it provides a description of the motion in such cases.

THE PRINCIPAL AXES OF THE SIMULATION MODEL

In Chapters 4 and 5 the twist axis $f_3$ was defined to be parallel to the line $QN$ where $Q$ is the midpoint of the knee centres and $N$ is the midpoint of the shoulder centres (Figure 87). This procedure is used so that the somersault angle is not unduly affected by
Q is a fixed point on the spine XN
Q is the midpoint of the knee centres

Figure 87. Redefining the twist axis of the model
configurational changes as is the case when the trunk line \( XN \) is used as a reference axis. For a particular individual it is possible to find a point \( Q_1 \) on the line \( XN \), where \( X \) is the junction of the pelvis and thorax segments, such that the line \( QQ_1 \) remains very close to the vertical throughout a piked jump. Let the point \( Q_1 \) be defined for the performer CARL by \( d(XQ_1) = 0.6d(XN) \) where \( d \) is the distance function and redefine axis \( f_3 \) to be parallel to \( QQ_1 \).

During a piked jump (Figure 88) the angle between \( f_3 \) and the vertical is the somersault angle \( \phi \) (PHI) and is tabulated along with values of the pike angle \( \gamma \) (GAMMA) in Table 13. It can be seen that throughout the movement the angle \( \phi \) remains within 2 degrees of zero.

Table 13 also gives the angle \( \phi_p \) (PHP) between the principal axis corresponding to minimum moment of inertia and axis \( f_3 \). For pike angles greater than 60 degrees, the angle \( \phi_p \) remains within 2 degrees of zero so that axis \( f_3 \) is very close to the minimum principal axis. For configurations which are symmetrical about the sagittal \( f_2f_3 \) plane, the lateral axis \( f_1 \) will be principal and so the remaining axis \( f_2 \) will lie close to the remaining principal axis for pike angles greater than 60 degrees. Thus with this modified system frame \( f \) for the performer CARL, the simulation model may be used to produce simulations in which a fixed configuration is maintained so that the output angles are governed by equations (1) through (31).

For pike angles less than 60 degrees, axis \( f_3 \) will no longer lie close to the principal axis of minimum moment of inertia and it is debatable which axis should be referred to as the twist axis. This problem will be avoided by placing a lower bound of 60 degrees on the pike angle \( \gamma \) in all simulations used in this chapter.

Table 13 shows that, in the range \( 120^\circ < \gamma < 180^\circ \), the maximum and intermediate principal moments of inertia lie within 5% of each other so that equation (20) may be expected to give a reasonable approximation to the number of twists per somersault for motions in the rod mode. The maximum and intermediate principal moments of inertia are equal when the pike angle is 137 degrees.
pike angle in degrees

180  140  100  60  20

Figure 88. Graphics sequence of the piked jump simulation PK1
Table 13. Principal axes and principal moments of inertia of the simulation model during a piked jump

**Simulation : PK1**  
**100 integration steps**  
**Performer : Carl**

**Components of momentum :**  
$HX = 0.00$  
$HY = 0.00$  
$HZ = 0.00$

**Angular**  
GAMMA: angle of pike  
DA: angle through which the arms are raised  
PHI: angle between axis $f_3$ and the vertical  
PHP: angle between minimum principal axis and $f_3$  
IF1: principal moment of inertia about axis $f_1$  
II1: minimum principal moment of inertia  
II2: remaining principal moment of inertia

<table>
<thead>
<tr>
<th>ANGLES</th>
<th>PRINCIPAL INERTIAS</th>
</tr>
</thead>
<tbody>
<tr>
<td>GAMMA</td>
<td>DA</td>
</tr>
<tr>
<td>180.</td>
<td>0.</td>
</tr>
<tr>
<td>160.</td>
<td>10.</td>
</tr>
<tr>
<td>140.</td>
<td>20.</td>
</tr>
<tr>
<td>120.</td>
<td>30.</td>
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<tr>
<td>100.</td>
<td>40.</td>
</tr>
<tr>
<td>80.</td>
<td>50.</td>
</tr>
<tr>
<td>60.</td>
<td>60.</td>
</tr>
<tr>
<td>40.</td>
<td>100.</td>
</tr>
<tr>
<td>20.</td>
<td>140.</td>
</tr>
<tr>
<td>0.</td>
<td>180.</td>
</tr>
</tbody>
</table>

**Internal orientation angles in degrees**

**Time**  
PIKE  
JHLA  
LEGARD  
KNEE  
SHOULDERS  

<table>
<thead>
<tr>
<th>T</th>
<th>GA</th>
<th>PSP</th>
<th>EJK</th>
<th>GAL</th>
<th>THC</th>
<th>PSC</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
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<td>0.</td>
<td>-2.</td>
<td>180.</td>
<td>0.</td>
<td>0.</td>
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</table>

**Left Arm**

<table>
<thead>
<tr>
<th>Time</th>
<th>Raise</th>
<th>Abduct</th>
<th>Rotate</th>
<th>Elbow</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>DA</td>
<td>EA</td>
<td>PSA</td>
<td>GAA</td>
</tr>
<tr>
<td>0.0</td>
<td>0.</td>
<td>-2.</td>
<td>0.</td>
<td>180.</td>
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</tbody>
</table>

**Right Arm**

<table>
<thead>
<tr>
<th>Time</th>
<th>Raise</th>
<th>Abduct</th>
<th>Rotate</th>
<th>Elbow</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>DA</td>
<td>EA</td>
<td>PSA</td>
<td>GAA</td>
</tr>
<tr>
<td>0.0</td>
<td>0.</td>
<td>-2.</td>
<td>0.</td>
<td>180.</td>
</tr>
</tbody>
</table>
In the range \(60^\circ < \gamma < 180^\circ\) the intermediate principal moment of inertia is near the minimum principal moment of inertia only when \(\gamma\) is close to 60 degrees and even then the difference is some 23%. Because of this the disc model will not be a good approximation and the number of somersaults per oscillation for motions in the disc mode should be calculated from the disc mode equations rather than equation (26).

**SIMULATIONS SHOWING THE DIFFERENT MODES OF MOTION**

In each of the following three simulations a fixed configuration is maintained and the modified reference frame for CARL is used so that the body axes lie close to the principal axes. The characteristics of each simulation may be derived independently using equations (1) through (3) and these calculations are described for each simulation.

Each configuration is symmetrical about the sagittal plane so that the lateral axis \(f_1\) will be principal with principal moment of inertia \(I_1\). If \(A, B, C\) are the principal moments of inertia with \(A > B > C\) then:

(a) \(I_1 = A\) when the pike angle \(\gamma\) is less than 137 degrees

(b) \(I_1 = B\) when the pike angle \(\gamma\) is greater than 137 degrees

In case (a) the reference frame \(f\) used in the development of the rigid body equations is the same as that of the simulation model whereas in case (b) there is a 90 degree phase difference between the two twist angles. Thus in case (b) the rigid body equations should be modified by changing \(\psi\) to \(\psi - \pi\) radians.

For convenience the unit of time is taken to be the duration of the simulation and the simulation program SIM uses normalised angular momentum values as input data with a value of 1 corresponding to the momentum of a straight somersault.
THE ROD MODE SIMULATION RD1

During the simulation RD1 the body maintains a straight position and the principal moments of inertia A, B, C are given in Table 13 as:

\[ A = 11.01 \quad B = 10.56 \quad C = 0.70 \quad (\text{kgm}^2) \]

The initial tilt angle is \( \beta = 10^\circ \) and equations (6) and (7) give:

\[ (1/C-1/A)\cos^2\alpha = (1/C-1/B)\cos^2\beta \]

from which \( \alpha = 10.5^\circ \) and the tilt angle \( \theta \) oscillates between these two values during the simulation (Table 14).

From equation (2) the somersault rate varies between \( h/B \) and \( h/A \) and in this simulation the angular momentum has a normalized value of 1 so that \( h/B \) corresponds to a somersault rate of 1 revolution per unit time. Thus the somersault rate varies between 1.00 and 0.96 revolutions per unit time (Figure 89).

The twist rate is given by equation (4) as:

\[ \dot{\psi} = (h/C-h)\sin\theta \]

which varies between \( (h/C-h/B)\sin\beta \) and \( (h/C-h/A)\sin\alpha \). Thus the twist rate varies between 2.46 and 2.58 revolutions per unit time (Figure 89).

Equation (20) may be used to obtain the average number of twists per somersault in the form:

\[ p/\Omega = (I/C-1)\sin\theta \]

and taking \( I \) to be the average of \( A \) and \( B \), and \( \theta \) to be the average of \( \alpha \) and \( \beta \) gives \( p/\Omega = 2.57 \) twists per somersault which is in agreement with the value obtained from the simulation.

The motion may be described as a forward somersault with 2\( \frac{1}{2} \) twists (Figure 90).
Table 14. Simulation RD1 showing the rod mode of motion

<table>
<thead>
<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
<th>MODE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.00</td>
<td>10.0</td>
<td>0.00</td>
<td>ROD</td>
</tr>
<tr>
<td>0.1</td>
<td>0.10</td>
<td>10.5</td>
<td>0.25</td>
<td>ROD</td>
</tr>
<tr>
<td>0.2</td>
<td>0.20</td>
<td>10.0</td>
<td>0.50</td>
<td>ROD</td>
</tr>
<tr>
<td>0.3</td>
<td>0.29</td>
<td>10.5</td>
<td>0.76</td>
<td>ROD</td>
</tr>
<tr>
<td>0.4</td>
<td>0.39</td>
<td>10.0</td>
<td>1.01</td>
<td>ROD</td>
</tr>
<tr>
<td>0.5</td>
<td>0.49</td>
<td>10.5</td>
<td>1.26</td>
<td>ROD</td>
</tr>
<tr>
<td>0.6</td>
<td>0.59</td>
<td>10.0</td>
<td>1.51</td>
<td>ROD</td>
</tr>
<tr>
<td>0.7</td>
<td>0.69</td>
<td>10.5</td>
<td>1.76</td>
<td>ROD</td>
</tr>
<tr>
<td>0.8</td>
<td>0.78</td>
<td>10.0</td>
<td>2.02</td>
<td>ROD</td>
</tr>
<tr>
<td>0.9</td>
<td>0.88</td>
<td>10.5</td>
<td>2.27</td>
<td>ROD</td>
</tr>
<tr>
<td>1.0</td>
<td>0.98</td>
<td>10.0</td>
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<td>ROD</td>
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INTERNAL ORIENTATION ANGLES IN DEGREES

<table>
<thead>
<tr>
<th>TIME</th>
<th>PIKE</th>
<th>HULA</th>
<th>LEGABD</th>
<th>KNEE</th>
<th>SHOULDERS</th>
</tr>
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<tr>
<td></td>
<td>T</td>
<td>GA</td>
<td>PSP</td>
<td>EJK</td>
<td>GAL</td>
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<tr>
<td>0.0</td>
<td>180.</td>
<td>0.</td>
<td>-2.</td>
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LEFT ARM

<table>
<thead>
<tr>
<th>TIME</th>
<th>RAISE</th>
<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
<th>RAISE</th>
<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
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<tbody>
<tr>
<td></td>
<td>T</td>
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<td>GAA</td>
<td>DB</td>
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<td>PSB</td>
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<td>0.</td>
<td>180.</td>
<td>0.</td>
<td>-2.</td>
<td>0.</td>
<td>180.</td>
</tr>
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</table>
Figure 89. Oscillation during the rod mode simulation RDL
Figure 90. Graphics sequence of the rod mode simulation RDL
THE DISC MODE SIMULATION DS1

During the simulation DS1 the body maintains a 60 degree angle of pike and the principal moments of inertia A, B, C are given in Table 13 as:

\[ A = 5.78 \quad B = 3.83 \quad C = 3.10 \text{ (kgm}^2\text{)} \]

Initially \( \psi = 0 \) and the angle of tilt is \( \alpha = 10^\circ \) and so the tilt angle oscillates between \(-10^\circ\) and \(+10^\circ\) (Table 15).

At \( \theta = 0 \) equation (8) gives the twist angle \( \psi_0 \) as:

\[ \sin^2 \psi_0 = \sin^2 \alpha \frac{B(A-C)}{C(A-B)} \]

so that \( \psi_0 = 13.1^\circ \) and the twist angle oscillates between \(-13.1^\circ\) and \(+13.1^\circ\) (Figure 91).

Equation (10) shows that the somersault rate varies from \( h/A \) to \( h(1/C-(1/C-1/A)\cos^2 \alpha) \) which are in the ratio 1:1.026 and the disc equation:

\[ \Omega = h(1/C-(1/C-1/A)\cos \alpha) \]

may be used to give the average somersault rate as \((1.013)h/A\).

In equation (23) the modulus \( k \) is given by \( k^2 = \tan^2 \alpha \frac{A(B-C)}{C(A-B)} \)

so that \( k = 0.147 \) and the complementary modulus \( k_0 = 0.989 \) and from equation (15) \( K = \frac{1}{2} \pi \) where \( M = 0.995 \).

The time period of an oscillation is \( 4K/q \) where \( q \) is obtained from:

\[ q^2 = h^2 \cos^2 \alpha \frac{A(C)}{A^2 BC} \] as \((0.653)h/A\).

The number of somersaults per oscillation can then be calculated as:

\[ \Omega/Mq = 1.56 \] which is in close agreement with the simulation value.

Figure 92 shows the orientation of the body at quarter cycle intervals. The motion may be described as somersault with wobble.
Table 15. Simulation DS1 showing the disc mode of motion

<table>
<thead>
<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
<th>MODE</th>
</tr>
</thead>
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<tr>
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<td>10.0</td>
<td>0.00</td>
<td>DISC</td>
</tr>
<tr>
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<td>0.02</td>
<td>DISC</td>
</tr>
<tr>
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<td>0.31</td>
<td>3.2</td>
<td>0.03</td>
<td>DISC</td>
</tr>
<tr>
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<td>0.47</td>
<td>-2.9</td>
<td>0.03</td>
<td>DISC</td>
</tr>
<tr>
<td>0.4</td>
<td>0.63</td>
<td>-8.6</td>
<td>0.02</td>
<td>DISC</td>
</tr>
<tr>
<td>0.5</td>
<td>0.78</td>
<td>-10.0</td>
<td>0.00</td>
<td>DISC</td>
</tr>
<tr>
<td>0.6</td>
<td>0.93</td>
<td>-8.2</td>
<td>-0.02</td>
<td>DISC</td>
</tr>
<tr>
<td>0.7</td>
<td>1.09</td>
<td>-3.2</td>
<td>-0.03</td>
<td>DISC</td>
</tr>
<tr>
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<td>1.25</td>
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<td>10.0</td>
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<td>DISC</td>
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INTERNAL ORIENTATION ANGLES IN DEGREES

<table>
<thead>
<tr>
<th>TIME</th>
<th>PIKE</th>
<th>HULA</th>
<th>LEG</th>
<th>ABD</th>
<th>KNEE</th>
<th>SHOULDERS</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>T</td>
<td>GA</td>
<td>PSP</td>
<td>EJK</td>
<td>GAL</td>
<td>THC</td>
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LEFT ARM

<table>
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<tr>
<th>TIME</th>
<th>RAISE</th>
<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
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RIGHT ARM

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<tbody>
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<td>0.0</td>
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<td>-2.</td>
<td>0.</td>
<td>180.</td>
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</tbody>
</table>
Figure 91. Oscillation during the disc mode simulation DS1
Figure 92. Graphics sequence of the disc mode simulation DS1
THE SINGULAR SOLUTION

In the simulation SN1 the body maintains a straight position and the principal moments of inertia $A, B, C$ are given in Table 13 as:

$$A = 11.01 \quad B = 10.56 \quad C = 0.70 \quad (kgm^2)$$

In the singular solution the tilt angle at the quarter twist position is $\alpha_0$ where:

$$\sin^2 \alpha_0 = C(A-B)/B(A-C)$$

so that $\alpha_0 = 3.1^\circ$. During the motion the tilt angle increases from 0 to $3.1^\circ$ and then decreases to zero again (Table 16).

The somersault rate decreases from $h/B$ to $h/A$ and then increases to $h/B$ again (Figure 93).

Equation (31) gives the maximum of the ratio $\dot{\psi}/\dot{\varphi}$ as:

$$\max(\dot{\psi}/\dot{\varphi}) = (A/C-1)^{\frac{1}{4}}(A/B-1)^{\frac{1}{4}}$$

which is about 0.8 twists per somersault.

In the singular solution the time period of the oscillation is infinite whereas the simulation SN1 was created with initial conditions $\theta=0, \psi_0=0.172$ degrees which resulted in 3 somersaults for the oscillation. The motion SN1 is in fact in the disc mode with the twist angle increasing from $\psi_0$ to $(180^\circ-\psi_0)$ during the 3 somersaults. During the middle somersault the twist angle increases by 0.44 revolutions so that 88% of the twist occurs during this somersault. As a result of this, the motion appears to a forward somersault with no twist, followed by a forward somersault with a half twist, followed by a backward somersault (Figure 94).

DIRECT TWIST

Direct twist is initiated whilst the feet are in contact with the take-off surface. During this contact phase body segments are set in motion so that, at the instant of take off, there is angular
Table 16. Simulation SN1 showing the singular solution

<table>
<thead>
<tr>
<th>TIME (REVOLUTIONS)</th>
<th>SOMERSAULT DEGREES</th>
<th>TILT DEGREES</th>
<th>TWIST DEGREES</th>
<th>MODE</th>
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INTERNAL ORIENTATION ANGLES IN DEGREES

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<th>TIME (T)</th>
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<th>HULA</th>
<th>LEG</th>
<th>ABD</th>
<th>KNEE</th>
<th>SHOULDERS</th>
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<table>
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<th>TIME (T)</th>
<th>RAISE</th>
<th>LEFT ARM</th>
<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
<th>RAISE</th>
<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
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</table>
Figure 93. Oscillation during the singular simulation SN1
Figure 94. Graphics sequence of the singular solution SN1
momentum about the longitudinal axis of the body. Subsequently the relative segmental motions cease and the whole body assumes an angular velocity which is determined by the conservation of angular momentum. This process is known as transfer of momentum since initially only certain segments possess angular momentum whereas once the relative segmental motions cease all body segments possess angular momentum.

TRANSFER OF MOMENTUM

Expressions will be obtained for the angular momenta associated with relative segmental movements. Three cases will be considered:

(a) motions of the arms A and B relative to the chest C
(b) torsion of the chest C relative to the thorax T
(c) twisting of the whole body F relative to the feet

Case (c) will be used as an approximation to the situation in which torsion occurs at the ankles, knees and hips.

ANGULAR MOMENTUM DUE TO MOTIONS OF THE ARMS

Figure 95 shows an aerial view of the left arm A, right arm B, left shoulder centre S and right shoulder centre R.

The angular momentum of the left arm is given by:

\[ h_a = I_{aa} \omega_a + m_a v_a d(AN) \]

where \( I_{aa} \) is the moment of inertia of the arm about an axis through its mass centre
\( \omega_a \) is the angular velocity of the arm
\( m_a \) is the mass of the arm
\( v_a \) is the velocity of the mass centre of the arm
\( d(AN) \) is the distance from the mass centre of A to the midpoint N of the shoulder centres.

Since the shoulder centre S remains at rest during the motion,
Figure 95. Aerial view of arm rotation

Figure 96. Aerial view of chest torsion
\[ v = \omega \cdot d(AS) \] so that:
\[ h_a = (I_{aa} + m \cdot d(AS) \cdot d(AN)) \omega_a \]

The right arm will possess an equal amount of angular momentum if it also has angular velocity \( \omega_a \) so that the total momentum may be written as:
\[ h_{ab} = I_{ab} \omega_a \]

where \( I_{ab} = 2(I_{aa} + m \cdot d(AS) \cdot d(AN)) \)

**ANGULAR MOMENTUM DUE TO MOTION OF THE CHEST**

Figure 96 shows an aerial view of the left arm A, the right arm B and the chest C. During the motion the supra-segment D, which comprises A, B and C, moves as a rigid unit so that the total angular momentum is given by:
\[ h_D = I_{dd} \omega_c \]

where \( I_{dd} \) is the moment of inertia of D about a vertical axis through N and \( \omega_c \) is the angular velocity of the chest.

\( I_{dd} \) may be calculated as \( I_{dd} = I_{an} + I_{bn} + I_{vv} \) where \( I_{an} \) is the moment of inertia of the left arm A about a vertical axis through N and may be evaluated using the theorem of parallel axes.

**ANGULAR MOMENTUM OF THE WHOLE BODY**

If the whole body rotates about the twist axis \( f_3 \) the angular momentum will be:
\[ h_f = I_{ff} \omega_f \]

where \( I_{ff} \) is the moment of inertia of the body about \( f_3 \) and \( \omega_f \) is the angular velocity.
COMPARISON OF THE ANGULAR MOMENTA

The angular momenta of the three motions are $h_{ab}$, $h_{d}$ and $h_{f}$ where:

$$h_{ab} = I_{ab} \omega_{a}$$
$$h_{d} = I_{dd} \omega_{c}$$
$$h_{f} = I_{ff} \omega_{f}$$

For the segmental inertia parameters of CARL $I_{ab}$, $I_{dd}$ and $I_{ff}$ take the values:

$$I_{ab} = 1.11 \quad I_{dd} = 1.84 \quad I_{ff} = 2.17 \quad (\text{kgm}^2)$$

where the arms are abducted at right angles to the body in each case.

Thus for equal angular velocities $\omega_{a}, \omega_{c}, \omega_{f}$ the angular momenta $h_{ab}, h_{d}, h_{f}$ will be proportional to 1.11, 1.84, 2.17 and so using the arms will be about half as effective as using the whole body to initiate the twist.

Once the arms are adducted, the moment of inertia of the body about the long axis will be $I = 0.70 \text{ kgm}^2$ and, assuming that there is no somersault, the twist rate may be calculated as $h/I$ so that for the three cases the final twist rates will be:

$$I_{ab} \omega_{a}/I = 1.59 \omega_{a}$$
$$I_{dd} \omega_{c}/I = 2.63 \omega_{c}$$
$$I_{ff} \omega_{f}/I = 3.10 \omega_{f}$$

In order to calculate the number of twists that can be produced by using the arms an estimate of the maximum value of $\omega_{a}$ is needed. During a filmed movement in which Carl Furrer adducted his arms through 180 degrees the maximum angular velocity of the arms was 2.4 revolutions per second. For arm swings parallel to the sagittal plane Fenn, Brody and Petrilli (1931) obtained a maximum angular velocity of 1.9 revolutions per second. In the proposed movement of the arms in a horizontal plane, the arm position shown in Figure
95 will maximise $I_{ab}$ but will reduce the maximum $\omega_a$ value since the arms are near the limit of their range of movement.

If $\omega_a$ is taken to be 1 revolution per second the resulting twist rate will be initially $I_{ab}\omega_a/I_{ff} = 0.51$ revolutions per second, increasing to $I_{ab}\omega_a/I = 1.59$ revolutions per second once the arms are adducted. Even if the flight time is only one second and if half of this time is allowed for arm adduction and abduction, the total twist will be greater than one revolution.

These calculations indicate that it is possible to produce a full twist solely by use of the arms.

**COMBINING TWIST WITH SOMERSAULT**

In the preceding analysis the twist rates were calculated on the assumption that there was no somersault. If twist and somersault occur together it is of interest to see whether the twist rate is affected by the presence of somersault and whether the somersault rate is affected by the presence of twist. As a first approximation the body will be modelled as a rigid rod with principal moments of inertia $A=B>C$.

**THE MOTION OF A ROD**

The earlier description of rod motion showed that the body spins at a constant rate $p$ about the body axis $f_3$ which makes a constant angle $\alpha$ with the plane normal to the angular momentum vector $h$ and precesses about $h$ at the constant rate $\Omega$ (Figure 97). $\Omega, p, \alpha$ and $h$ are related by the equations:

\begin{align*}
(19) \quad &\Omega = h/A \\
(20) \quad &p = \Omega(A/C-1)\sin\alpha
\end{align*}

so that equation (20) may be rewritten in the form:

\begin{equation}
 p = hs\sin\alpha(1/C-1/A)
\end{equation} (20a)
Figure 97. Precession of a rod about the angular momentum vector $\mathbf{h}$

Figure 98. Precession of a rod when angle $\alpha$ is less than $45^0$
Suppose that the initial values of the somersault, tilt and twist angles are zero so that the body axes \( f_1', f_2', f_3' \) are coincident with the inertial axes \( i_1, i_2, i_3 \) and let the angular momentum vector \( h \) lie in the plane \( i_1 i_3 \) (Figure 97) so that the components of \( h \) in the inertial frame \( i \) will be:

\[
\begin{align*}
  h_1 &= h \cos \alpha \\
  h_2 &= 0 \\
  h_3 &= h \sin \alpha
\end{align*}
\]

The somersault angle \( \phi \) of the simulation model is the angle between the planes \( i_1 f_3 \) and \( i_1 i_3 \) and so the somersault rate \( \dot{\phi} \) is the rate of precession of the body axis \( f_3 \) about the horizontal axis \( i_1 \). If \( \alpha < 45^\circ \) then after half a somersault there will have been half a revolution of precession of \( f_3 \) about both \( i_1 \) and \( h \) (Figure 98). As a consequence the average somersault rate will be equal to the precession rate \( \Omega \).

The tilt angle \( \theta \) of the simulation model is the angle between \( f_3 \) and the vertical plane \( i_2 i_3 \) which is normal to \( i_1 \). Since \( f_3 \) makes a constant angle \( \alpha \) with the plane normal to \( h \), the tilt angle after half a somersault will be \( 2\alpha \) (Figure 98).

Thus half a somersault about \( i_1 \) followed by tilt through an angle \( 2\alpha \) about \( f_2 \) is equivalent to half a revolution of precession about \( h \). In order for the final orientations to be equivalent, the twist angle \( \psi \) must equal the angle of spin about \( f_3 \) and so the average twist rate will be equal to the spin rate \( p \).

If \( \alpha = 45^\circ \) then after half a revolution of precession about \( h \) the body will be horizontal as shown in Figure 99. This position is equivalent to half a somersault and 90 degrees of tilt although a complete cycle of precession is barely recognisable as a twisting somersault.

If \( \alpha > 45^\circ \) the body will never reach a horizontal position and the somersault angle will vary between approximately \( -\alpha_1 \) and \( \alpha_1 \) whilst the tilt angle will oscillate between 0 and \( 2\alpha_1 \) where \( \alpha_1 = 90^\circ - \alpha \) (Figure 100). One revolution of precession about \( h \) will be perceived as a full twist and so the average twist rate will be \( n \) where:

\[
n = p + \Omega = h \sin \alpha (1/C - 1/A) + h/A
\]
Figure 99. Precession of a rod when angle \( \alpha \) is equal to \( 45^0 \)

Figure 100. Precession of a rod when angle \( \alpha \) is greater than \( 45^0 \)
and when $\alpha=90^\circ$ the motion will be a pure twist with twist rate:

$$n = \frac{h}{C}.$$ 

Three different motions may now be compared:

(a) $\alpha=0$ for which the motion comprises somersault without twist with the constant somersault rate $\frac{h_1}{A}$.

(b) $0<\alpha<45^\circ$ for which the motion is a twisting somersault with average somersault rate $\Omega$ and average twist rate $p$ where:

$$\begin{align*}
\Omega &= \frac{h}{A} \\
p &= \frac{h_3}{C(1/C-1/A)}
\end{align*}$$

(c) $\alpha=90^\circ$ for which the motion comprises twist without somersault with the constant twist rate $\frac{h_3}{C}$.

The introduction of the angular momentum component $h_3$ changes the plain somersault (a) into the twisting somersault (b) and the somersault rate increases from $\frac{h_1}{A}$ to $\frac{h}{A}$ where $h^2 = h_1^2 + h_3^2$. Thus the introduction of twist increases the somersault rate.

The introduction of the angular momentum component $h_1$ changes the plain twist (c) into the twisting somersault (b) and the twist rate decreases from $\frac{h_3}{C}$ to $\frac{h_3}{C(1/C-1/A)}$. Thus the introduction of somersault decreases the twist rate.

To see whether the same results occur when $A$ and $B$ are only approximately equal, consider a simulation DRI in which the motion is identical to the rod simulation RD1 but has an initial tilt angle of zero. Comparing Tables 14 and 17 it can be seen that the average somersault and twist rates differ by less than 1%. In simulation RD1 the tilt angle oscillates between $\beta=10^\circ$ and $\alpha=10.5^\circ$ whilst in simulation DRI the boundary values of the tilt angle are $0^\circ$ and $20.4^\circ$ which are close to the theoretical limits $(\beta-\alpha)$ and $(\beta+\alpha)$.

For simulation DRI the average somersault rate is 0.98 revolutions per unit time and the average twist rate is 2.51 revolutions per unit time. These values will be compared with the
**Table 17. The direct twist simulation DR1**

**SIMULATION : DR1**  
**100 INTEGRATION STEPS**  
**PERFORMER : CARL**

**COMPONENTS OF MOMENTUM :**  
HX = 0.98  
HY = 0.00  
HZ = 0.17

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<thead>
<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
<th>MODE</th>
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**INTERNAL ORIENTATION ANGLES IN DEGREES**

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**LEFT ARM**

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<th>ABDUCT</th>
<th>ROTATE</th>
<th>ELBOW</th>
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**RIGHT ARM**

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</table>
somersault rate of a plain somersault and the twist rate of a plain twist.

The principal moments of inertia have values $A=11.01$, $B=10.56$, $C=0.70$ (kgm$^2$) and the angular momentum $h$ has a normalised value of 1 which means that $h/B$ is equivalent to 1 revolution per unit time.

The somersault rate of a plain somersault is $h_1/B = h \cos \beta / B$ which is equivalent to 0.985 revolutions per unit time.

The twist rate of a plain twist is $h_3/C = h \sin \beta / C$ which is equivalent to 2.62 revolutions per unit time.

Thus the introduction of somersault into a plain twist decreases the twist rate from 2.62 to 2.51 revolutions per unit time and the introduction of twist into a plain somersault decreases the somersault rate from 0.985 to 0.98 revolutions per unit time.

The reason for the decrease in somersault rate is that in simulation DR1 the somersault rate is approximately $h/I$ where $I = \frac{1}{2} (A + B)$ so that although the angular momentum increases by 1.5% from $h_1 = 0.985h$ to $0.1h$ this is more than offset by an increase of 2.1% in the inertia term from $B = 10.56$ to $I = 10.785$ kgm$^2$.

THE NUTATION EFFECT

In the rod mode the angle between the twist axis $f_3$ and the plane normal to the angular momentum vector increases from $\beta$ to $\alpha$ as the twist angle increases by a quarter twist. This variation is known as nutation and although the increase in angle is only about half a degree in simulations RD1 and DR1 the effect will be more pronounced when a wide arm position is employed. Let the movement DR2 be defined as follows:

Initially the arms are extended laterally and the angle $\beta$ is 10 degrees. In this position CARL has principal moments of inertia:

\[
A = 13.45 \quad B = 11.52 \quad C = 2.17 \quad \text{(kgm}^2)\]
The angle between \( f_3 \) and the plane normal to the angular momentum vector will increase from \( \beta \) to \( \alpha \) where equations (6) and (7) give the relation:

\[
\frac{1}{C} - \frac{1}{A} \cos^2 \alpha = \frac{1}{C} - \frac{1}{B} \cos^2 \beta
\]

from which \( \alpha = 14.4^\circ \).

If the arms are now rapidly adducted the principal moments of inertia will become:

\[
A_1 = 11.01 \quad B_1 = 10.56 \quad C_1 = 0.70
\]

and the angle between \( f_3 \) and the plane normal to the angular momentum vector will vary between \( \alpha \) and \( \beta_1 \) where:

\[
\frac{1}{C_1} - \frac{1}{A_1} \cos^2 \alpha = \frac{1}{C_1} - \frac{1}{B_1} \cos^2 \beta_1
\]

from which \( \beta_1 = 14.0^\circ \).

If the angular momentum components for the movement DR2 are the same as for simulation DR1 and the initial tilt and twist angles are zero the subsequent motion will be as shown in Table 18.

In simulation DR2 the tilt angle increases from zero to a maximum value of 24.2° which lies between the theoretical limits of \( (\beta + \beta_1) = 24.0^\circ \) and \( (\beta + \alpha) = 24.4^\circ \). It should be noted that this maximum is reached after approximately half a somersault and the precise value is dependent upon the twist position at that time.

The twist rate is initially slow because of the arm abduction but once the arms are adducted at the quarter twist position the body twists at an average rate of 3.45 revolutions per unit time which is 1.38 times the twist rate of simulation DR1.

The somersault rate has an average value of 0.84 revolutions per unit time during the first quarter twist and becomes 0.99 revolutions per unit time once the arms are adducted so that the final somersault rate is within 1% of the somersault rate in simulation DR1.
Table 18. The direct twist simulation DR2

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INTERNAL ORIENTATION ANGLES IN DEGREES

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LEFT ARM

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In simulation DR1 the arms may be considered to be initially extended laterally and to be instantaneously adducted at the start of the motion whereas in simulation DR2 the instantaneous arm adduction occurs at the quarter twist position. This delaying of the arm adduction produces an increase of about 4 degrees in the angle between axis $f_3$ and the plane normal to the angular momentum vector and so the final twist rate is correspondingly greater for simulation DR2. For a rod equation (20) shows that the twist rate is proportional to $\sin \alpha$ so that the ratio of the twist rates in simulations DR2 and DR1 will be approximately $(\sin 14.4^\circ / \sin 10.4^\circ) = 1.38$.

In practice the arms cannot be adducted instantaneously and so the final twist rate will be less than the theoretical value of 3.45 revolutions per unit time obtained in simulation DR2. Suppose that the arms are adducted between times $t=0.1T$ and $t=0.4T$ where $T$ is the flight time. If $T$ is near the 1.6 second flight time of the filmed movement C11, this allows an ample half second for the arm adduction. The resulting motion is given by the simulation DR3 (Table 19). After the arm adduction, the twist rate has an average value of 3.35 revolutions per unit time which is 1.33 times the twist rate of DR1 and 0.97 of the theoretical maximum attained in DR2. The simulations DR1, DR2 and DR3 are compared in Figure 101.

This technique of delaying the arm adduction in order to increase the nutation and subsequent twist rate will be referred to as the nutation technique.

The nutation effect will be greatest when the arms are extended laterally, as in simulation DR2, and the initial tilt angle $\beta$ is small.

The equation: 
\[
\frac{1}{C} - \frac{1}{A} \cos^2 \alpha = \frac{1}{C} - \frac{1}{B} \cos^2 \beta
\]

with $A = 13.45$, $B = 11.52$, $C = 2.17$ (kgm$^2$) and $\cos \beta = 1$ gives: $\alpha = 10.4^\circ$.

In practice the increase in tilt angle will be less than $10.4^\circ$ since very small values of $\beta$ will result in an excessive time for the first quarter twist. When $\beta = 3^\circ$ the above equation gives $\alpha = 10.6^\circ$. 

Table 19. The direct twist simulation DR3

<table>
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<tr>
<th>TIME</th>
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<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
<th>MODE</th>
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INTERNAL ORIENTATION ANGLES IN DEGREES

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LEFT ARM

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Figure 101. Comparison of simulations DR1, DR2, DR3
which corresponds to a nutation effect of 7.6°. Thus the maximum nutation effect will be about 7°.

STOPPING THE TWIST

Equations (19) and (20) show that for motion in the rod mode the twist rate is approximately \( p = h \sin \alpha (1/C - 1/A) \). When the body pikes \( C \) will increase and \( A \) will decrease (Table 13) so that the twist rate will decrease.

This is shown in simulation DR4 which is similar to DR3 but somersaults in the opposite direction (Table 20). Initially angle \( \beta = 10° \) but this increases to 12.6° as the arms are adducted between \( t = 0.1 \) and \( t = 0.4 \) and the nutation effect is exploited. From \( t = 0.7 \) to \( t = 1.0 \) the arms are abducted and the body pikes so that the twist rate is reduced from about 3 revolutions per unit time to 0.75 revolutions per unit time. At \( t = 1.0 \) the residual tilt angle is less than 2° and the twist angle is within 6° of two twists so that the movement is perceived as a double twisting backward somersault with no apparent tilt upon landing (Figure 102). The rod mode of motion is maintained throughout this movement and so the twist rate is always strictly positive.

In order to stop the twist it is necessary to change from the rod mode to the disc mode. The critical value of \( \alpha \) corresponding to the singular solution for which \( 2T = h^2/B \) is given by equation (6) as \( \alpha_0 \) where:

\[
\sin^2 \alpha_0 = \frac{C(A-B)}{B(A-C)}
\]

If \( \alpha > \alpha_0 \) the motion will be in the rod mode and if \( \alpha < \alpha_0 \) the motion will be in the disc mode. The values of \( \alpha_0 \) may be calculated for different pike angles by using the inertia values of Table 13 in the above equation (Table 21). In these piked positions the arms are close to the legs (Figure 88) and axis \( f_1 \) corresponds to the maximum principal moment of inertia providing the pike angle is less than 137°. If the arms were to be extended laterally the axis \( f_1 \) would correspond to the maximum moment of inertia only when the
### Table 20. The direct twist simulation DR4

**Simulation:** DR4  
**Integration Steps:** 100  
**Performer:** CARL  
**Components of Momentum:**  
HX = -0.98  
HY = 0.00  
HZ = 0.17

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**Internal Orientation Angles in Degrees**

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**Left Arm**

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**Right Arm**

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Figure 102. Graphics sequence of the simulation DR4
Table 21. Values of the angle $\text{ALPHA} \theta$ for different pike angles

For $\text{ALPHA} > \text{ALPHA} \theta$ motion is in the rod mode
For $\text{ALPHA} < \text{ALPHA} \theta$ motion is in the disc mode

Arm positions are close to the body as used in Table 13

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pike angle is between about 40° and 80° so that this position is not particularly suitable for changing the motion from the rod mode into the disc mode.

This is demonstrated in simulation DR5 since the motion remains in the rod mode even when the pike angle becomes 80° (Table 22). Initially angle $\beta=10^\circ$ and this rises to 11.2° which is less than the corresponding value for simulation DR4 as the arm adduction time is shorter and the nutation effect is not as great. In the straight position the twist rate is about 3 revolutions per unit time but this falls to about 0.3 revolutions per unit time once the arms are abducted and the 80° pike is reached. This dramatic reduction in the twist rate can give the impression that the twist stops when the body is piked (Figure 103). When the body is extended and the arms are raised above the head the twist rate increases to about 1 revolution per unit time. The simulation resembles a back or reverse 1½ somersault dive with 1½ twists. The final twist value of 1.53 is acceptable but the movement is under-somersaulted. This could have been avoided by having more somersault momentum at take-off but this would increase the problem of the tilt angle. Figures 97 and 98 show that, for direct twist, the tilt angle of a rod is zero for a complete somersault and is 2$\alpha$ at the half somersault position. For the double twisting somersault DR4 this does not present any landing problems but for simulation DR5 it may be expected that the angle of tilt after 1½ somersaults will be 2$\alpha$ or about 24°. The effect could be reduced by using a smaller $\beta$ value but so long as the motion remains in the rod mode there will be a large angle of tilt at the half somersault position.

Simulation DR6 shows how the disc mode may be used to obtain a tilt angle close to zero at the 1½ somersault position (Table 23). The nutation effect is used to boost the angle $\beta$ from an initial value of 10° to 12.6° which corresponds to $\alpha=13^\circ$. As the body pikes the value of $\alpha$ undergoes little change since the action of piking produces little change in the directions of the principal axes (Table 13). Once the pike angle reaches 137°, the angles $\alpha$ and $\beta$ become equal and, although they diverge as the pike deepens, angle $\alpha$ is largely unaffected since the piking occurs near to the 1½ twist
Table 22. The direct twist simulation DR5

SIMULATION : DR5  100 INTEGRATION STEPS  PERFORMER : CARL

COMPONENTS OF MOMENTUM :  HX = -1.21  HY = 0.00  HZ = 0.21

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<th>TWIST REVOLUTIONS</th>
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INTERNAL ORIENTATION ANGLES IN DEGREES

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LEFT ARM

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Figure 103. Graphics sequences of the simulation DR5
Table 23. The direct twist simulation DR6

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INTERNAL ORIENTATION ANGLES IN DEGREES

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LEFT ARM

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</table>
position. As the pike angle decreases, the critical value \( a_0 \) increases (Table 21) so that when the pike angle is about 90° the condition \( a < a_0 \) applies and the motion changes from the rod mode to the disc mode. In the disc mode the angle between the twist axis \( f_3 \) and the plane normal to the angular momentum vector oscillates between \(+a\) and \(-a\). In simulation DR6 the disc mode is maintained until this angle reaches about -10° so that with the body extended at the 1\( \frac{1}{2} \) somersault position the observable tilt is close to zero (Figure 104).

At the start of the simulation, angle \( \beta = 10° \) and the body twists to the left whilst, at the end, angle \( \beta = -10° \) and the body is twisting to the right. It is possible to arrange for the final twist rate to be zero by extending from the pike earlier so that the final value of \( \beta \) is zero but this will result in a tilt angle of -10°. Thus there is no ideal solution for 1\( \frac{1}{2} \) somersault dives with direct twist and a compromise has to be made between a zero angle of tilt with a non-zero twist rate and a zero twist rate with a non-zero angle of tilt.

**OSCILLATIONS IN THE DISC MODE**

Let the angle between the twist axis \( f_3 \) and the plane normal to the angular momentum vector (known as the invariable plane) be referred to as \( \theta_1 \). Figure 105a shows the effect that piking has on angle \( \theta_1 \). Initially motion is in the rod mode with \( \theta_1 \) oscillating between \( \beta = 10° \) and \( \alpha = 10.5° \) as the body twists in the straight position. At time \( t = 0.1 \), when the half twist position is reached, the body pikes rapidly to an angle of 90° and the motion changes to the disc mode. The angle \( \theta_1 \) then oscillates between \( \alpha_1 \) and \(-\alpha_1\) where \( \alpha_1 = 10° \) whilst the twist angle \( \psi \) oscillates about the half twist position with amplitude 33°.

If the body extends from the pike after one quarter of an oscillation, the angle \( \theta_1 \) will be zero and the twist angle will be 213°. The subsequent motion will approximate to the singular solution
Figure 104. Graphics sequences of the simulation DR6
(a) piking at 0.5 twists

\[ \theta_1 \] 

\[ \begin{array}{c}
15^\circ \\
\theta_1 \\
-15^\circ
\end{array} \]

time

(b) piking at 0.4 twists

\[ \theta_1 \] 

\[ \begin{array}{c}
15^\circ \\
\theta_1 \\
-15^\circ
\end{array} \]

time

(c) piking at 0.6 twists

\[ \theta_1 \] 

\[ \begin{array}{c}
15^\circ \\
\theta_1 \\
-15^\circ
\end{array} \]

time

\[ \theta_1 \] : angle between axis \( f_3 \) and the invariable plane

Figure 105. Disc mode oscillations for different pike times
with the initial twist rate being zero.

If the body extends from the pike after half an oscillation, the angle $\theta_1$ will be $-10^\circ$ and the subsequent rod motion will be similar to the original rod motion but with the twist in the opposite direction. This was the technique used in simulation DR6.

Figure 105b shows the effect of piking earlier at $t=0.08$ when the twist has reached only 0.4 revolutions. In the subsequent motion in the disc mode, $\theta_1$ initially increases (as may be deduced from equation (3) with $h>0$) and reaches a maximum value of $\alpha_1=14^\circ$ governed by equation (8).

Figure 105c shows the effect of piking later at $t=0.12$ when the twist is 0.6 revolutions. In the subsequent motion $\theta_1$ oscillates with amplitude $\alpha_1=14^\circ$ and the twist angle $\psi$ oscillates with amplitude $49^\circ$.

It should be noted that although the piking movements were made rapidly, the same effects can be produced using longer times for piking since it is a matter of being early or late relative to the half twist position that produces (b) or (c).

This variation in the response of $\theta_1$ to the timing of the pike provides a means of control. By piking early as in (b), extension may be made when $\theta_1$ is near the maximum so that the disc mode is used to boost $\theta_1$ and the twist rate. By piking late as in (c), $\theta_1$ may be reduced to zero more rapidly than in (a).

Another effect of piking early or late is to increase the oscillation period. In (a) one oscillation of $\theta_1$ occurs in 1.8 somersaults whilst in (b) and (c) this increases to 2.2 somersaults. In theory the oscillation period may be extended indefinitely by rapidly piking at the quarter twist position for which equation (8) gives the value of $\alpha_1$ equal to the critical value $\alpha_0$ which corresponds to the singular solution. The oscillation period is also a function of the pike angle as shown in Table 24 for $\alpha_1=10^\circ$. For a pike angle of about $108^\circ$ the critical value $\alpha_0$ is near $10^\circ$ so that the motion approaches the singular solution and the oscillation period becomes large.
Table 24. Oscillation periods for different pike angles

The amplitude of the tilt oscillations is 10 degrees

<table>
<thead>
<tr>
<th>Pike Angle (degrees)</th>
<th>Oscillation Period (somersaults)</th>
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</thead>
<tbody>
<tr>
<td>60</td>
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<tr>
<td>70</td>
<td>1.57</td>
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<td>2.13</td>
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Table 25. Comparison of C47 with the modified simulation P47

**Simulation:** A47  
**100 Integration Steps**  
**Performer:** CARL  
**Components of Momentum:**  
HX = -1.46  
HY = -0.27  
HZ = 0.00

<table>
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<tr>
<th>Time (T)</th>
<th>Somersault SIM</th>
<th>Somersault FILM</th>
<th>Tilt SIM</th>
<th>Tilt FILM</th>
<th>Twist SIM</th>
<th>Twist FILM</th>
<th>Mode SIM</th>
<th>Mode FILM</th>
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<th>Mode DEGREES</th>
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<td>-11.</td>
<td>-11.</td>
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<td>ROD</td>
<td>ROD</td>
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<td>DISC</td>
<td>DISC</td>
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<td>DISC</td>
</tr>
<tr>
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<td>ROD</td>
<td>DISC</td>
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<td>ROD</td>
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<td>4.</td>
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<td>ROD</td>
<td>ROD</td>
<td>-0.56</td>
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Thus the timing and depth of the pike will affect the phase angle, amplitude and time period of $\theta_1$ and enable diverse movements to be produced. Any technique capable of producing a wide range of response also introduces problems of control. If small differences in technique produce large differences in the subsequent motion then it will become necessary to modify technique during flight in order to produce a given movement.

In the filmed movement C47 the body pikes during the first half twist and then extends for the subsequent $1\frac{1}{2}$ twists. The piked position is held for a short time so that the motion is in the disc mode only briefly and on extension the twist continues in the original direction. In the modified simulation P47 the piked position is held for longer and this produces dramatic changes in the tilt and twist angles (Table 25, Figure 106).

The oscillation effect can increase the angle $\theta_1$ to values approaching $\alpha_0$ which will be large when the pike angle is small (Table 21). For a pike angle of 90° the critical value $\alpha_0=19°$ and so it is possible to boost the angle $\theta_1$ to more than 20° providing a pike angle of less than 90° is adopted.

**INERTIA PARAMETER VALUES**

The segmental inertia parameter values of CARL have been used in the calculations and simulations pertaining to direct twists. Whilst the values of the inertia parameters do not affect the occurrence of momentum transfer, nutation in the rod mode and oscillation in the disc mode, the magnitudes of these effects are dependent upon the individual's inertia values. Some idea of individual variation may be obtained by comparing the values of JOHN and GILL with those of CARL.

The largest variation occurs in the values of $I_{ab}/I$ which gives the ratio of the twist rate after momentum transfer to the initial angular velocity of the arms. The value for the girl GILL is some 20% lower than that for CARL and this difference is due in part to
Figure 106. Comparison of C47 with the modified simulation P47
a smaller relative arm mass (2%) but arises primarily from her shorter arm length (9%), the value of $I_{ab}$ being approximately proportional to the square of arm length.

The ratios of whole body principal moments of inertia of JOHN and GILL, for various pike and arm positions, all lie within about 10% of the corresponding ratios of CARL. As a consequence JOHN and GILL have inertia characteristics very similar to those of CARL:

(a) When piking, the directions of the whole body principal axes change by less than 4° (for pike angles greater than 80°).

(b) Nutation effects lie within 1° of those of CARL.

(c) The angles of pike required to change a given motion from the rod mode to the disc mode lie within 10° of those of CARL.

(d) The angles of pike corresponding to a given period of the disc mode oscillation lie within 10° of those of CARL.

**SPEED AND TIMING OF INTERNAL MOVEMENTS**

The nutation effect in the rod mode and the effects which can be produced using the disc mode all require a delay before changing the arm or pike angles. Although the nutation effect is greatest when the arms are adducted rapidly at the quarter twist position, simulation DR3 has shown that a relatively slow arm movement can produce a twist rate within 3% of the theoretical maximum. Similarly the duration of the piking movement has little effect on the disc mode oscillations. Thus it is not the duration but the timing of the internal movement that is of importance. As a consequence the ability of an individual to make rapid configurational changes is of marginal advantage whereas the ability to make internal movements at the appropriate time is crucial.
COUNTER-ROTATION TWIST

In Chapter 5 it was shown that the angular momentum \( h \) of a system of linked rigid segments may be expressed in the form:

\[
h = h_f + h_{rel}\]

where \( h_f \) is the angular momentum due to the motion of the system frame \( f \)
and \( h_{rel} \) is the angular momentum due to movements of the segments relative to frame \( f \).

If the whole body twists through an angle \( \psi \) as a result of the counter-rotation of body segments through an angle \( -\psi_s \) then:

\[
h_f = I_f \dot{\psi} \quad \text{and} \quad h_{rel} = -I_s \dot{\psi}_s \]

where \( I_f \) is the moment of inertia of the whole body about the twist axis
and \( I_s \) is an inertia term associated with the internal movement.

If the total angular momentum is zero the equation of motion takes the form:

\[
I_f \ddot{\psi} - I_s \ddot{\psi}_s = 0 \tag{32}
\]

The inertia terms \( I_f \) and \( I_s \) will vary as the configuration changes but the whole body inertia \( I_f \) will always be greater than the segmental term \( I_s \) and so the twist angle \( \psi \) will be less than the counter-rotation angle \( \psi_s \). This result places a restriction on all counter-rotation techniques and shows that in order to produce multiple twists it is necessary to use a number of cycles of counter-rotation.

ROTATION OF THE CHEST

Suppose that the arms are abducted at right angles to the twist axis and the chest segment is twisted through an angle \( -\psi_c \) relative to the thorax. As a result the whole system will twist through an angle \( \psi \). The angular velocity of the supra-segment D
which comprises the chest and arms will be $\omega_d = \dot{\psi} - \dot{\psi}_c$ and the angular velocity of the supra-segment $R$ comprising the remainder of the body will be $\omega_r = \dot{\psi}$. If the moments of inertia of $D$ and $R$ about the twist axis are $I_d$ and $I_r$ then the total momentum is given by:

$$h = I_d \omega_d + I_r \omega_r$$

$$= I_d (\dot{\psi} - \dot{\psi}_c) + I_r \dot{\psi}$$

$$= I_f \dot{\psi} - I_d \dot{\psi}_c$$

where $I_f = I_d + I_r$ is the moment of inertia of the whole body about the twist axis.

If $h=0$ the equation of motion becomes:

$$I_f \dot{\psi} - I_d \dot{\psi}_c = 0$$

which has precisely the same form as equation (32).

In this example $I_f$ and $I_d$ remain constant throughout the motion and so the final twist angle is given by:

$$\psi = \frac{I_d \dot{\psi}_c}{I_f}$$

For CARL $I_d = 1.84 \text{ kgm}^2$, $I_f = 2.17 \text{ kgm}^2$ and taking the maximum value of $\dot{\psi}_c$ to be $90^\circ$ gives a twist angle of $\psi = 76^\circ$.

If the arms are adducted the inertia terms become $I_d = 0.37 \text{ kgm}^2$, $I_f = 0.70 \text{ kgm}^2$ so that when the torsion of the chest is removed the twist angle is reduced by $48^\circ$ giving only $29^\circ$ of twist for the cycle. This reduction of $48^\circ$ could be decreased by abducting the legs during the removal of torsion but even if this is done a number of cycles are required to produce even a half twist.

**ROTATIONS OF THE ARMS**

In simulation CR1 the arms are moved in a horizontal plane, the upper arms moving through $90^\circ$ and the forearms moving through $180^\circ$ (Figure 107). The resulting twist is only 0.17 revolutions and there remains the problem of returning the arms to their original positions before the cycle can be repeated. If it were
Figure 107. Graphics sequence of the simulation CR1
possible to have twice the range of arm movement shown in this simulation it would still require more than one cycle of counter-rotation of the arms to produce a half twist.

THE HULA MOVEMENT

The simplest model of hula movement consists of two identical cylinders U and L which represent the upper body and lower body and are connected at the point O (Figure 108). During a cycle of hula movement, U and L rotate once about their axes of symmetry relative to the pike plane UOL which executes a revolution in the opposite direction about axis $f_3$ relative to the whole body reference frame $f$. These motions are in opposing directions since the body is always facing forwards in frame $f$.

If the hula angle $\psi$ is the angle through which the point O moves about axis $f_3$, each cylinder rotates about its own axis through an angle $\psi_p$ whilst the pike plane rotates through an angle $-\psi_p$ about axis $f_3$ (Figure 108). As a consequence the system will rotate through an angle $\psi$ about axis $f_3$ which remains aligned with the fixed axis $i_3$ (Figure 109).

During the motion each cylinder has three components of angular velocity:

(a) an angular velocity of magnitude $\dot{\psi}_p$ along its own axis
(b) an angular velocity of magnitude $\Omega = \dot{\psi}_p$ along $-f_3$
(c) an angular velocity of magnitude $\psi$ along $f_3$

These components are shown in Figure 110. The angular momentum arising from the rotations of the cylinders about their own axes is $2Cn \cos \alpha$ along $i_3$ where $C$ is the moment of inertia of each cylinder about its own axis. The axis $i_3$ passes through the mass centre of each cylinder and is a principal axis of the whole system and so the angular momenta due to the rotations of the whole system are $-I_f \Omega$ and $I_f \dot{\psi}$ along $i_3$ where $I_f$ is the principal moment of inertia of the system about axis $i_3$. 
Figure 108. Hula movement relative to frame $f$

Figure 109. Twist arising from hula movement
Figure 110. Angular velocity components of the two cylinders
Thus the total angular momentum is:

\[ h = I_f \dot{\psi} - I_f \Omega + 2C \cos \alpha \]

\[ = I_f \dot{\psi} - (I_f - 2C \cos \alpha) \dot{\psi}_p \]

Setting \( h = 0 \) the equation of motion may be written as:

\[ I_f \dot{\psi} - I_p \dot{\psi}_p = 0 \]

which has the same form as equation (32) with \( I_p = I_f - 2C \cos \alpha < I_f \).

If the angle \( \alpha \) between each cylinder axis and \( i_3 \) remains constant, the twist angle \( \psi \) is given by:

\[ \psi = \left( \frac{I_p}{I_f} \right) \dot{\psi}_p \]

If each cylinder has principal moments \( A \) and \( C \):

\[ I_f = 2C \cos \alpha + 2A \sin \alpha \]

so that:

\[ \psi = \left( \frac{1 - C \cos \alpha}{C \cos \alpha + A \sin \alpha} \right) \dot{\psi}_p \]  \( \text{(33)} \)

Equation (33) shows that the twist angle \( \psi \) is proportional to the hula angle \( \dot{\psi}_p \). When the angle \( \alpha \) is zero the body is straight and the hula motion is purely theoretical and does not produce any configurational change so that the resulting twist is zero. \( \psi \) increases with \( \alpha \) and has a theoretical maximum value of \( \psi_p \) when \( \alpha = 90^\circ \) which corresponds to complete flexion throughout the hula movement and is anatomically impossible. \( \psi \) also increases with the ratio \( A/C \) and this means that the twist will be large when the cylinders are long and thin.

In simulation CR2 the body pikes to an angle of 130°, makes one cycle of hula motion and then extends (Table 26). For CARL this results in 0.47 twists (Figure 111). In simulation CR3 the same movement is performed with the arms overhead and for CARL this results in 0.52 twists (Figure 112).

Using the inertia parameters of JOHN in simulations CR2 and CR3 increases the resulting twist by 6% whilst the parameters of GILL produce 3% less twist than for CARL.

A pike angle of \( \gamma = 130^\circ \) corresponds to \( \alpha = 25^\circ \) since \( \gamma + 2\alpha = 180^\circ \).
<table>
<thead>
<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
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<td>0.0</td>
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<td>***</td>
</tr>
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**INTERNAL ORIENTATION ANGLES IN DEGREES**

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<th>LEGABD</th>
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**LEFT ARM**

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**RIGHT ARM**

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<th>ROTATE</th>
<th>ELBOW</th>
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</table>
Figure 111. Graphics sequence of the simulation CR2

Figure 112. Graphics sequence of the simulation CR3
The values of A and C may be estimated by dividing the body into upper and lower units separated by the hip level and averaging the principal moments of inertia of the two units. This gives a value of 5.4 for A/C. Equation (33) then gives a twist value of 0.49 revolutions which compares favourably with the 0.47 revolutions given by simulation CR2. The twist values given by the simulation model will be less than those given by equation (33) since the simulation model incorporates the pelvis P as a third unit which has a smaller inclination to the twist axis $f_3$.

However the main purpose of the simple two cylinder model is to provide an understanding of the mechanics of the movement by deriving the equation of motion and to obtain an analytical solution which expresses the twist angle as a function of hula angle, pike angle and inertia values.

The maximum twist that can be produced by one cycle of hula movement depends on the limits of side pike and back arch for a particular subject. If $\alpha=45^\circ$ the angle between trunk and legs is $90^\circ$ throughout the movement and this results in about $3/4$ twists for one cycle. Since $90^\circ$ of side pike and back arch are beyond the anatomical limits of most performers the value of $3/4$ twists may be regarded as an upper bound.

Although the analysis of the hula movement has been made assuming that the total momentum is zero it is to be expected that the counter-rotation effect will still occur when somersault is present. If the internal movements of simulation CR2 are incorporated into a backward somersault, the result is simulation CR4 in which 0.50 twists occur (Figure 113). However when the same movements are made in a forward somersault, as in simulation CR5, the resulting twist is 0.95 revolutions (Figure 114). This shows that there is an effect additional to counter-rotation when somersault is present and this will be considered in the section on tilt twist.

**Tilt Twist**

Suppose that the body is performing a plain somersault with the lateral axis $f_1$ parallel to the horizontal angular momentum vector
Figure 113. Graphics sequence of the simulation CR4

Figure 114. Graphics sequence of the simulation CR5
If the motion is transformed into a somersault with a sustained twist, the twist rate $\dot{\omega}$ will be given by equation (4) as:

$$\dot{\omega} = \left(\frac{h}{C-\phi}\right) \sin \theta$$

where $\theta$ is the angle between the twist axis and the vertical plane normal to $h$, $\phi$ is the somersault rate, $C$ is the principal moment of inertia about the twist axis.

For a sustained twist the motion must be in the rod mode and the tilt angle $\theta$ will vary between the values $\alpha$ and $\beta$ defined by equations (6) and (7) for rigid body motion. Since the motion will change from a plain somersault to a somersault with sustained twist only when the tilt angle is changed from zero, the term tilt twist is appropriate.

Since the angular momentum vector is horizontal, the angle between the twist axis and the vertical somersault plane is identical to the angle between the twist axis and the plane normal to the angular momentum vector whereas for direct twist a distinction had to be made between the two angles.

For configurations that are symmetrical about the sagittal plane $f_2f_3$, the twist axis of the simulation model is a principal axis and so is identical to the twist axis of the rigid body model. For asymmetrical configurations there will be a difference between the two tilt angles and the angle $\beta$ will be used as a tilt parameter of the rigid body motion on the rod mode.

Any configurational change in which symmetry about the sagittal plane is not maintained will produce tilt. The ability of arm, chest and hip movements to produce tilt will be considered.

**ROTATIONS OF THE ARMS**

If the total momentum is zero then abduction of the left arm will produce a tilting of the whole body in the opposite direction as shown in simulation TLL (Figure 115). Since the tilt is produced
by counter-rotation of the arm the equation of motion will have the same form as equation (32):

$$I_f \ddot{\theta} - I_a \ddot{\epsilon}_a = 0$$

where $\theta$ is the angle of tilt of the mid-line $f_3$ of the body
$\epsilon_a$ is the abduction angle of the left arm
$I_f$ is the whole body moment of inertia about axis $f_2$
$I_a$ is the inertia term associated with the arm movement.

Table 27 shows how the tilt angle $\theta$ increases with the arm abduction angle $\epsilon_a$. As a result of the arm asymmetry, the principal axis corresponding to minimum moment of inertia makes an angle $\theta_p$ with axis $f_3$. Thus the tilt of the principal axis is given by $\beta = \theta + \theta_p$. Using the inertia parameters of the three subjects gives the following values of angle $\beta$:

CARL: $\beta = 8.6^\circ$, JOHN: $\beta = 8.5^\circ$, GILL: $\beta = 7.1^\circ$

If the same arm movement is made instantaneously during a somersault then the same angle of tilt will result. For a slower arm movement the nutation effect will occur and this will increase the tilt angle during the first quarter twist. However as the twist angle increases, the plane of arm movement moves away from the vertical somersault plane and less tilt is produced. In the following example maximum use is made of the nutation effect but the second half of the arm movement is ineffective.

Suppose that during a somersault the left arm is rapidly abducted through $90^\circ$ so that the total tilt is $\beta = 5.1^\circ$ (Table 27). The nutation effect will increase the tilt from $\beta$ to $\alpha$ where equations (6) and (7) give:

$$(1/C - 1/A) \cos^2 \alpha = (1/C - 1/B) \cos^2 \beta$$

Using the values for the principal moments $A, B, C$ given in Table 27 results in $\alpha = 8.4^\circ$ at the quarter twist position. If the left arm is abducted rapidly through a further $90^\circ$ at the quarter twist position the tilt angle will not change but will fall to $\beta_1 = 7.9^\circ$ at the half twist position as a negative nutation effect occurs. Although the nutation effect has increased the tilt from $5.1^\circ$ to
Figure 115. Graphics sequence of the simulation TL1

Figure 116. Graphics sequence of the simulation TL2
Table 27. Tilt produced by asymmetrical arm movement \( TL_1 \)

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<tr>
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Table 28. Twist rates for different tilt angles

For the asymmetrical arm position the abduction angles of the arms are 0 degrees and 180 degrees

For the symmetrical arm position the abduction angles of the arms are 0 degrees

<table>
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<tr>
<th>tilt angle</th>
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7.9° the final tilt angle is less than the value \( \beta = 8.6° \) produced by a rapid 180° arm abduction. However the fact that the gain in tilt from the nutation effect is offset by the decreased effectiveness of the arm movement means that the tilt angle is not particularly sensitive to the duration of the arm movement.

In simulation TL2 the arm movement takes place throughout the first half somersault (Figure 116) and results in a tilt angle of \( \beta = 9.3° \) which is greater than the 8.6° produced by a rapid arm movement and the 7.9° produced by a delayed arm movement. This indicates that maximum tilt will be produced by an arm movement that is of intermediate duration rather than by a rapid or slow arm movement. The maximum possible tilt will lie between \( \beta = 9.3° \) and \( \beta = 8.6° + 2.8° = 11.4° \) where the last figure has been obtained by adding the gain in tilt during the delayed movement to the tilt produced by a rapid arm movement.

Since the response of tilt to the duration of arm movement is relatively flat the particular arm movement which produces maximum tilt is of minor interest whereas it is of importance that an arm movement of almost any duration will produce about 9° of tilt. This lack of sensitivity will tend to make the technique easy to learn and execute.

Since different tilt techniques are being compared using the values of the tilt angle \( \beta \) for CARL, it will be helpful to have a corresponding measure of the twist rate. The number of twists per somersault may be calculated approximately using the rod equation (20) in the form:

\[
p/\Omega = (I/C-1)\sin\theta
\]

where \( p/\Omega \) is the number of twists per somersault

- \( I \) is the average of the principal inertias \( A \) and \( B \)
- \( C \) is the principal moment of inertia about the twist axis
- \( \theta \) is the average angle of tilt.

The average tilt angle \( \theta \) may be taken as the average of the extreme values \( \alpha \) and \( \beta \). If the body is straight either with both
arms by the sides or with one arm overhead θ may be approximated as \( \theta = \beta + 0.3^\circ \) with less than 1% error in the range \( 7^\circ < \beta < 20^\circ \).

For the asymmetrical arm position used in simulations TL1 and TL2 the ratio \( I/C = 17.6 \) whereas for the position with both arms adducted \( I/C = 15.0 \). Using these values in equation (20) gives Table 28.

In simulation TL3 the left arm is rapidly abducted through \( 90^\circ \) and this produces a tilt angle of \( \beta = 5.1^\circ \) (Table 27). At the half twist position the left arm is rapidly adducted to its original position and the tilt angle increases by a further \( 5.1^\circ \) to \( \beta = 10.2^\circ \) (Figure 117). If slower arm movements are used during the first and second quarter twists the resulting tilt angle will be close to \( \beta = 10^\circ \). There will be a positive nutation effect during the first quarter twist and a negative nutation effect during the second quarter twist and although these effects can be controlled by varying the timing of the arm movements any gain made will be offset by a reduced contribution from the arm movement. Since the second arm movement cannot be made before the quarter twist position is reached in this technique, there is an inherent delay in the timing of the arm movements and as a result the body will have completed more than half a somersault when the arm movement ends.

A similar technique is used in simulation TL4 in which the left arm is adducted through \( 170^\circ \) during the first quarter twist and the right arm is adducted through \( 170^\circ \) during the second quarter twist (Figure 118). Since the initial direction of somersault is backwards, the left arm is adducted to produce a twist to the left whereas in simulations TL2 and TL3 the initial direction of somersault is forwards and the left arm is abducted to produce a twist to the left. The tilt produced by the sequential adduction of the arms is \( \beta = 15.9^\circ \) which is close to the value \( \beta = 17.2^\circ \) corresponding to instantaneous arm movements at the zero and half twist positions. The maximum angular velocity of rapid arm adduction obtained from a filmed movement of Carl Furrer was 2.4 revolutions per second. If the maximum angular velocities of the arm movements in simulation TL4 are taken to be 2.4 revolutions per second the duration of each arm movement will be about 0.37 seconds.
Figure 117. Graphics sequence of the simulation TL3

Figure 118. Graphics sequence of the simulation TL4
Since the arm movements are made sequentially, it will require about 3/4 seconds to produce the tilt and a similar period to remove the tilt. This total time of about 1½ seconds leaves little or no time to make use of the 16° of tilt. The problem may be overcome in some measure by moving the right arm in the sagittal plane near the quarter twist position. This will permit an overlap of the periods of arm movement without reducing the resulting tilt angle.

In simulation TL5 asymmetrical arm movements are used to produce a full twist in a straight forward dive (Figure 119). From a symmetrical arm position the right arm is adducted and the left arm is raised overhead during the first quarter twist, producing a tilt angle of \( \beta = 8.5° \). The asymmetrical position is held until 0.8 twists have been completed and then the arms are spread to slow the twist and remove the tilt so that at the full twist position the tilt angle and twist rate are near zero (Table 29). The arms are then moved to an overhead position prior to entry in the hypothetical water. In an actual dive there may be less than the 0.46 somersaults used in this simulation so that the arm movements will have to be made somewhat faster. If the time of flight is around 1.5 seconds the angular velocities of the arms will be near to the maximum of 2.4 revolutions per second produced by Carl Furrer.

If the body is piked rather than straight when an asymmetrical arm movement is made, the resulting tilt will be correspondingly greater. In simulation TL6 the hands are initially near the knees and the body is in a 90° pike (Figure 120). The left arm is abducted through 90° at which time extension from the pike commences. At the completion of the movements the left arm is overhead, the right arm is by the side and the final angle of tilt is \( \beta = 12.3° \) which is significantly better than the value \( \beta = 8.6° \) for the simulation TL1 in which the body is straight during the arm movement. If the same movements are made rapidly during a somersault then 12° of tilt will be produced. Whilst the body is in a 90° pike the motion will be in the disc mode and the tilt angle will start to decrease due to the oscillation effect in the disc mode. Thus whilst it is advantageous to be piked so that the arm movement has greater effect it is disadvantageous to be piked for long. If the arm movements are made more
Figure 119. Graphics sequence of the simulation TL5
Table 29. The tilt twist simulation TL5

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<th>TIME</th>
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<th>TWIST REVOLUTIONS</th>
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INTERNAL ORIENTATION ANGLES IN DEGREES

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Figure 120. Graphics sequence of the simulation TL6

Figure 121. Graphics sequence of the simulation TL7
slowly then extension from the pike must be started earlier to reduce the oscillation effect. As a consequence the ability of this technique to produce tilt is reduced to about 11° which is not much greater than the 9.3° obtained in simulation TL2.

However the oscillation effect of the disc mode may be used to increase the tilt angle as discussed in the earlier section on direct twist. This technique is used in simulation TL7 in which the arm moves in the plane of the trunk (Figure 121). When the body is piked and the right arm is abducted in this plane not only is tilt produced but there is also a small counter-rotation twist to the right. Since in a backward somersault the tilt produced by this arm action is to the left, it is clear that the tilt angle will rise until the twist angle reaches the zero position again. (The situation is similar to that shown in Figure 105b where the tilt angle initially rises to its maximum before falling). The extension is made from the pike whilst the tilt angle is near its maximum and a value of β = 14.0° is obtained. Since the arm asymmetry introduces 1.7° of tilt the values of the tilt angles in Table 30 oscillate about the value 14.0°.

As the body extends from the pike the motion changes from the disc mode to the rod mode. Until the zero twist position is reached the tilt angle will continue to rise due to the oscillation effect in the disc mode and the nutation effect in the rod mode. These effects are essentially the same in the neighbourhood of ψ=0 and are governed by equation (8) which may be written in the form:

\[ \cos^2 \alpha = \cos^2 \theta (1 - \sin^2 \psi \sin^2 \alpha_0) \]

where \( \alpha \) is the final angle of tilt at \( \psi=0 \)
\( \theta \) is the tilt produced by the arm movement
\( \psi \) is the counter-rotation twist produced by the arm movement
\( \alpha_0 \) is the critical \( \alpha \)-value and is a function of the pike position.

In order for the increase in tilt to be large, it is necessary to have a large angle \( \psi \) of counter-rotation twist. The arm movement of simulation TL7 produces a counter-rotation twist of only 15° and,
Table 30. The tilt twist simulation TL7

SIMULATION : TL7  100 INTEGRATION STEPS  PERFORMER : CARL

COMPONENTS OF MOMENTUM :  HX = -1.00  HY = 0.00  HZ = 0.00

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<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST REVOLUTIONS</th>
<th>MODE</th>
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INTERNAL ORIENTATION ANGLES IN DEGREES

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although this is increased since the asymmetric arm position re- orients the directions of the principal axes, the gain in tilt is only about 2°. If the time taken for the arm movement and extension from the pike is increased from that of simulation TL7 the resulting tilt remains close to $\beta = 14.0^\circ$ even when the duration is increased by a factor of three.

If the same arm movement is used in a forward somersault, the direction of twist produced by counter-rotation of the arm is the same as the direction of twist resulting from the tilt. In this situation the oscillation of the tilt angle is past the peak and has started to fall so that time spent in the piked position has a detrimental effect on the tilt angle. If the timing of the internal movement is identical to that of simulation TL7 the resulting tilt is $\beta = 8.7^\circ$ compared with $14.0^\circ$ in a backward somersault. If the duration of internal movement is increased by a factor of three the tilt angle produced in a forward somersault falls to zero.

In order to exploit the nutation and oscillation effects in a piked forward somersault it will be necessary to use an arm movement which produces a counter-rotation twist in the direction opposite to the tilt twist. Thus the optimum plane of arm movement is dependent upon the direction of the somersault.

**ROTATION OF THE CHEST**

In order to obtain maximum tilt using the disc mode oscillation effect, the counter-rotation twist angle $\psi$ should approach $90^\circ$. If the chest is twisted through $60^\circ$ relative to the thorax with arms abducted at $90^\circ$ the counter-rotation twist is about $30^\circ$ but since the asymmetric position of the chest and arms causes a reorientation of the principal axes the effective counter-rotation twist is more than $60^\circ$. The tilt angles produced for the three subjects by such a movement are:

- CARL: $\beta = 5.5^\circ$
- JOHN: $\beta = 5.4^\circ$
- GILL: $\beta = 5.0^\circ$

If the same movement is made during a forward somersault, the
counter-rotation twist will be in the opposite direction to the twist resulting from the tilt so that the oscillation effect will increase the tilt angle. This is shown in simulation TL8 where the tilt angle has been increased from 5.5° to β = 14.0° (Figure 122). If the duration of the internal movement given in Table 31 is increased by a factor of two the resulting tilt angle is 20°. The maximum possible tilt angle α may be calculated from equation (8):

\[ \cos^2 \alpha = \cos^2 \theta (1 - \sin^2 \psi \sin^2 \alpha_0) \]

The maximum value of α_0 during the movement is 21° and the maximum possible value of sin ψ is 1. Taking \( \theta = 5.5° \) produces \( \alpha = 21.7° \).

In simulation TL9 the internal movement of simulation TL8 is used in a backward somersault. Since the direction of counter-rotation twist is the same as the direction of tilt twist, the tilt angle has passed its peak value in the oscillation cycle and is falling rapidly. As a consequence the tilt produced is near zero and very little twist occurs (Figure 123).

**THE HULA MOVEMENT**

As shown in the section on counter-rotation twist, the hula movement may be modelled using two equal cylinders. Whilst such a model is adequate for the description of counter-rotation twist, the assumed symmetry of the model implies that no tilt is produced during the movement. To overcome this deficiency a model comprising two unequal cylinders will be used.

Let \( u_3 \) and \( l_3 \) be the axes of symmetry of the upper cylinder U and lower cylinder L and let \( e_3 \) be the whole body principal axis corresponding to minimum moment of inertia (Figure 124). Let \( e_2 \) be the whole body principal axis lying in the pike plane \( u_3 \perp 3 \) so that the reference frame \( e \) moves with the pike plane. If each cylinder rotates about its own axis at the rate \( n = \dot{\psi} \) relative to frame \( e \), the angular momentum associated with this movement will be (in frame \( e \)): 
Figure 122. Graphics sequence of the simulation TL8

Figure 123. Graphics sequence of the simulation TL9
Table 31. The tilt twist simulation TLS

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INTERNAL ORIENTATION ANGLES IN DEGREES

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\[ h_{\text{rel}} = \begin{bmatrix} 0 \\ -C_1 \sin \alpha_1 + C_2 \sin \alpha_2 \\ C_1 \cos \alpha_1 + C_2 \cos \alpha_2 \end{bmatrix} \]

where \( C_1 \) and \( C_2 \) are the principal inertias of \( U \) and \( L \) about their axes of symmetry
\( \alpha_1 \) and \( \alpha_2 \) are the angles \( \alpha_3 \) and \( \Theta_3 \) make with \( e_3 \)

If the system counter-rotates at rate \( \omega \) about the inertial axis \( i_3 \), which lies in the pike plane (Figure 125), the associated angular momentum in frame \( e \) will be:

\[ h_e = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \begin{bmatrix} 0 \\ \omega \sin \delta \\ -\omega \cos \delta \end{bmatrix} \]

where \( A, B, C \) are the whole body moments of inertia about \( e_1, e_2, e_3 \) and \( \delta \) is the angle between \( i_3 \) and \( e_3 \).

If the total momentum \( h + h_{\text{rel}} \) is zero then:

\[ B \omega \sin \delta = (C_1 \sin \alpha_1 - C_2 \sin \alpha_2) n \]
\[ C \omega \cos \delta = (C_1 \cos \alpha_1 + C_2 \cos \alpha_2) n \]

so that:

\[ \tan \delta = \frac{C_1 \sin \alpha_1 - C_2 \sin \alpha_2}{B (C_1 \cos \alpha_1 + C_2 \cos \alpha_2)} \] (34)

As a result of the internal movement the system rotates about \( i_3 \) at rate \( \omega = \dot{\psi}_e \) and the angle of tilt between axis \( e_3 \) and the initial orientation of the pike plane will reach its maximum value of \( \delta \) when \( \psi_e = 90^\circ \). The amount of twist of the body frame \( f \) may then be obtained as \( \psi = \dot{\psi}_p - \dot{\psi}_e = \psi_p - 90^\circ \).

The tilt angle \( \delta \) will be large when the pike is deep so that \( C/B \) and \( \alpha_1 \) are large and when the arms are abducted so that \( C_1 \) is large.

In simulation TLLO a quarter hula movement is made with an arm abduction angle of \( 60^\circ \) and a pike angle of \( 130^\circ \) (Figures 126a-126d). This produces a tilt angle of \( \theta = 9.8^\circ \) and a twist angle of \( \psi = 22^\circ \).
Figure 124. The geometry of the axes $u_3$, $l_3$ and $e_3$

Figure 125. Angular velocities during a hula movement
In the second phase of the movement the arms are adducted and the body is extended from the piked position (Figures 126d-126g). This extension from the pike reduces the tilt angle by 7.8° to 2.0°.

If the hula movement is made during a forward somersault, the direction of twist due to the tilt will be the same as the direction of the counter-rotation twist. However when the body is in the side pike position (Figure 126d), the lateral body axis \( f_1 \) is aligned with the principal axis corresponding to the intermediate principal moment of inertia. Thus the tilt angle has yet to reach its maximum value due to the nutation effect and the twist angle of the principal axis reference frame is effectively \( \psi_1 = \psi - 90° = -68° \).

Using equation (8) in the form:

\[
\cos^2 \alpha = \cos^2 \theta (1 - \sin^2 \psi_1 \sin^2 \alpha_0)
\]

with \( \theta = 9.8°, \psi_1 = -68° \) and \( \alpha_0 = 10.7° \) gives a maximum tilt angle of \( \alpha = 14.9° \) at the quarter twist position when \( \psi = 90° \). If extension from the pike is made at this time, the tilt angle will not change and in the subsequent motion in the rod mode the tilt angle will vary between \( \alpha = 14.9° \) and \( \beta = 14.5° \).

If the extension from the pike is made between the quarter and half twist positions, the tilt angle will have fallen due to the nutation effect but this will be offset by an increase in the tilt angle due to the extension from the pike. If extension is made when \( \psi = 180° - 22° = 158° \) the tilt angle will have fallen again to 9.8° due to the nutation effect and will increase by 7.8° upon extension to \( \theta = 17.6° \). The cancelling of the two effects means that the timing of the extension is not critical and so long as extension is made between 1/4 and 3/4 twists the resulting tilt angle will be around \( \beta = 16° \).

In simulation TL11 the hula movement is made whilst the body is extending from a 90° pike during a forward somersault so that, after a quarter of a hula cycle, the body is in a side pike position with pike angle \( \gamma = 130° \) (Table 32). The extension from the side pike is made between the twist values 0.21 and 0.69 revolutions and the resulting tilt angle is \( \beta = 17.5° \) (Figure 127).
Figure 126. Graphics sequence of the simulation TL10
Table 32. The tilt twist simulation TL11

SIMULATION : TL11  100 INTEGRATION STEPS  PERFORMER : KARL

COMPONENTS OF MOMENTUM :  IX = 1.00  HY = 0.00  HZ = 0.00

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INTERNAL ORIENTATION ANGLES IN DEGREES

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RIGHT ARM

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Figure 127. Graphics sequence of the simulation TLL1

Figure 128. Graphics sequence of the simulation TLL2
If the hula movement is made during a backward somersault, the twist due to the tilt will be in the opposite direction and so the tilt angle will be falling rather than rising. In simulation TL12 the internal movements of TL11 are made during a backward somersault. During the hula movement the counter-rotation twist is to the right whilst the twist arising from the tilt is to the left so that, when the side pike position is reached, the twist angle is close to zero (Figure 128). Although the tilt angle is about 8° at this time, the subsequent extension reduces the angle of tilt to near zero since the twist angle is near zero. As a consequence less than half a twist is produced during the somersault.

If the side pike position were to be held until the quarter twist position is reached the subsequent extension would result in a tilt angle of about 16°. In a backward somersault such a delay becomes excessive since the body will have rotated through a complete somersault by the time the extension has been made.

Thus the hula movement is much more effective in producing tilt during a piked forward somersault than during a piked backward somersault.

If the body is somersaulting backwards in a back arched position then a partial hula movement to a side pike position will produce tilt twist in the same direction as the twist resulting from the counter-rotation. As a consequence the quarter twist position is reached without undue delay and effective use can be made of the nutation effect.

STOPPING THE TWIST

A plain somersault may be transformed into a sustained twisting somersault by producing tilt using rotations of the arms or chest or using a hula movement of the hips. Each of these techniques is also capable of removing the tilt and examples may be obtained by considering simulations which produce tilt to be run backwards in time. Since the total angular momentum remains constant, the
reversed movements are as valid as the original simulations and show that a twisting somersault may be transformed into a plain somersault by using any of the tilt producing techniques to remove the tilt.

If asymmetrical arm movements are used to produce and remove tilt then for an even number of half twists the final arm movement will be the reverse of the original arm movement. Thus in the forward somersault dives with 1, 2, 3 or 4 twists, it is advantageous to leave one arm above the head during the twist, in preparation for the final arm movement. In the case of an odd number of half twists the arm action required to remove the tilt is the same as that required to produce the tilt and so it is a disadvantage to maintain an asymmetrical arm configuration.

Since torsion of the chest, whilst extending from a pike, is effective in producing tilt only in a forward somersault the reverse movement will be effective in the removal of tilt only when the somersault rotation is backwards prior to landing.

In the same way the hula movement is effective in producing tilt when the initial somersault direction is forwards and is effective in removing tilt when the final somersault direction is backwards. It should be noted that, during a twist in the straight position, the value of the hula angle is notional and the configuration is symmetrical. If the hula movement is used to produce and remove tilt in a movement, which initially somersaults forwards and comprises an odd number of half twists, the direction of hula movement will be the same at the beginning and end of the movement. In each case the counter-rotation twist produced by the hula movement will be in the same direction as the body is twisting and so the direction of hula movement will be the natural one.

Tilt may also be removed by piking sufficiently so as to make use of the oscillation effect in the disc mode. As the tilt angle approaches zero, extension may be made from the pike so that the rod mode of motion applies and the tilt angle will remain small.

The techniques for the removal of tilt may also be used in
direct twists. However, for an even number of half somersaults, the tilt away from the vertical somersault plane will automatically disappear and so the role of asymmetrical arm movements will be to make fine adjustments in the tilt angle prior to landing. It should be remembered that in such a situation the twist is not stopped since the twist axis is not perpendicular to the angular momentum vector. Although it is possible to stop the twist by tilting the twist axis to be perpendicular to the angular momentum vector, this results in an undesirable tilt out of the vertical somersault plane since the angular momentum vector is not horizontal for a direct twist.

For an odd number of half somersaults, tilt removal techniques can reduce the tilt angle and twist rate in direct twists but some compromise has to be made since the angle of tilt from the vertical plane and the twist rate cannot both be zero.

THE SOMERSAULT RATE

Equation (2) gives the somersault rate $\dot{\theta}$ as:

$$\dot{\theta} = h(\cos^2\psi/I_1 + \sin^2\psi/I_2)$$

where $\psi$ is the angle of twist
h is the angular momentum
$I_1$ and $I_2$ are the moments of inertia about the principal axes $f_1$ and $f_2$.

Prior to the production of tilt the somersault rate will be $h/I_1$ since the twist angle will be zero. After the production of tilt the somersault rate will vary between $h/I_1$ and $h/I_2$ with an average value of approximately $h/I$ where $I$ is the average of $I_1$ and $I_2$.

If the body is in the straight position with arms adducted, $I_2$ will be greater than $I_1$ so that the somersault rate will be smaller during the twist. For CARL $I_1=10.56$ kgm$^2$ and $I_2=11.01$ kgm$^2$ so that the somersault rate will decrease by about 2%. It should be noted that this decrease is practically independent of the angle of tilt.
The rotational energy $T$ of a rigid body is given by equation (5) as:

$$2T = h^2 \cos^2 \theta \cos^2 \psi / I_1 + h^2 \cos^2 \theta \sin^2 \psi / I_2 + h^2 \sin^2 \theta / I_3$$

where $I_1, I_2, I_3$ are the principal moments of inertia about the body axes $f_1, f_2, f_3$.

If the body is in the straight position then $I_1 < I_2$ and we may write:

$$I_1 = B, \quad I_2 = A, \quad I_3 = C \quad \text{where } A > B > C.$$  

For motion in the rod mode the twist angle $\psi$ will be zero at some point and the energy $T$ is given by:

$$2T = h^2 \cos^2 \beta / B + h^2 \sin^2 \beta / C$$

If a plain somersault is transformed into a twisting somersault using a tilt twist technique the energy will increase from $T_0$ to $T$ where:

$$2T_0 = h^2 / B \quad \text{and} \quad 2T = h^2 \cos^2 \beta / B + h^2 \sin^2 \beta / C$$

Thus:

$$T/T_0 = 1 + (B/C-1)\sin^2 \beta$$

For CARL $B=10.56 \text{ kgm}^2$ and $C=0.70 \text{ kgm}^2$ and if $\beta=16^\circ$ then $T/T_0 = 2.07$ so that in such a movement the rotational energy is more than doubled. This increase in energy is equal to the work done by the internal movements against the centrifugal and coriolis forces.

Let the mode parameter $m$ be defined by:

$$m = (B-h^2/2T)/(A-C)$$

$m$ is an increasing function of $T$. When $T$ equals the minimum value $h^2/A$, $m$ will be a minimum and when $T$ takes the maximum value $h^2/C$ $m$ will be a maximum. Thus:

$$-1 \leq -(A-B)/(A-C) \leq m \leq (B-C)/(A-C) \leq 1$$

For the rod mode $2T>h^2/B$ and $0<m<(B-C)/(A-C)$

For the disc mode $2T<h^2/B$ and $-(A-B)/(A-C) \leq m < 0$. 

The sign of \( m \) determines the mode of motion whilst the magnitude of \( m \) is a measure of how much the motion differs from the bounding singular solution which corresponds to \( m=0 \).

In the simulation program SIM the term \( 2T \) is evaluated as the dot product \( \omega f_1 h_w f_1 \) where \( \omega f_1 \) is the angular velocity of the body frame \( f \) relative to the inertial frame \( i \) and \( h w f_1 \) is the corresponding momentum term. This procedure permits the determination of the mode of motion at times for which the configuration is changing, although the modes obtained at such times should be regarded as indicators rather than absolute descriptors.

### THE STABILITY OF NON-TWISTING SOMERSAULTS

Suppose that a somersault is performed in a piked position so that the lateral axis \( f_1 \) corresponds to maximum moment of inertia \( A \). For CARL this condition will be met providing the pike angle \( \gamma \) is less than \( 137^\circ \). The motion will be governed by equation (8):

\[
\sin^2 \psi = (1-\cos^2 \alpha \sec^2 \theta) B(A-C)/C(A-B)
\]

which may be written in the form:

\[
\sin^2 \alpha = \sin^2 \theta + \sin^2 \psi \cos^2 \theta \sin^2 \alpha_0
\]

where \( \sin^2 \alpha_0 = C(A-B)/B(A-C) \).

If the initial values of \( \theta \) and \( \psi \) are sufficiently small then \( \alpha \) will be less than \( \alpha_0 \) and motion will be in the disc mode. During the motion the tilt angle \( \theta \) will oscillate between \( -\alpha \) and \( \alpha \), and the twist angle \( \psi \) will oscillate between \( -\psi_0 \) and \( \psi_0 \). When \( \theta=0 \) the twist angle \( \psi \) will equal \( \psi_0 \) and so equation (35) gives:

\[
\sin \psi_0 = \sin \alpha / \sin \alpha_0
\]

If the initial values of \( \theta \) and \( \psi \) are small enough to make \( \alpha/\alpha_0 \) small then \( \psi_0 \) will be small and the tilt and twist angles will remain small throughout the motion. The usual criterion for stability of a somersault is that arbitrarily small initial values of \( \theta \) and \( \psi \)
lead to only small changes in \( \theta \) and \( \psi \) in the subsequent motion. This criterion will be met providing that the pike angle is small enough to make axis \( f_1 \) the principal axis corresponding to maximum moment of inertia.

However from a practical standpoint, although the initial values of \( \theta \) and \( \psi \) can be made small, they cannot be made arbitrarily small. In this case the angle of pike must be small enough to ensure that \( a_0 \) is an order of magnitude greater than the initial value of \( \theta \) so that \( a/a_0 \) and hence \( \psi_0 \) will be small.

Suppose now that a somersault is performed in an open pike position so that the lateral axis \( f_1 \) is the principal axis corresponding to the intermediate principal moment of inertia \( B \). For CARL this condition will be met when the pike angle \( \gamma \) is greater than 137°. The twist angle \( \psi_1 \) of the body frame \( f \) will be 90° out of phase with the twist angle \( \psi \) used in the rigid body equations so that equation (35) will become:

\[
\sin^2 \alpha = \sin^2 \theta + \cos^2 \psi_1 \cos^2 \theta \sin^2 a_0
\]

If the initial values of \( \theta \) and \( \psi_1 \) are small, \( \alpha \) will be close to \( a_0 \) and the motion will approximate to the singular solution in which a succession of half twists occur. Thus somersaults in an open pike or straight position are unstable.

In simulation ST1 the abduction angle of the left arm is one degree more than that of the right arm so that the principal axes are tilted through a small angle. Stability is maintained during the first 1½ somersaults by adopting a back arch with angle \( \gamma = 145^\circ \) and a knee bend with knee angle \( \gamma_k = 160^\circ \) (Figure 129). If there were to be less arch than this the movement would be unstable and the twist angle would not remain small. In the last half somersault the body moves through the unstable straight position into a stable pike with \( \gamma = 130^\circ \). Since the time spent in unstable configurations is short the twist angle remains small. Thus it is possible to perform an open double back somersault and maintain stability providing the arch is not less than that shown in Figure 129.
Figure 129. Graphics sequences of the simulation ST1
CONTROL OF THE UNSTABLE LAYOUT SOMERSAULT

In a twisting somersault asymmetrical arm movements may be used to alter the tilt angle so that the twist is stopped or even reversed. It should be possible to ensure that the twist angle remains small during an unstable straight somersault by making corrective arm movements. In a backward somersault abduction of the left arm will produce twist to the right whilst abduction of the right arm will produce twist to the left.

Suppose that abduction of one arm is accompanied by adduction of the other so that the sum $\varepsilon_a + \varepsilon_b$ of the abduction angles of the left and right arms remains constant. Let $\dot{\varepsilon} = \varepsilon_a = -\varepsilon_b$ be the rate of change of the arm abduction angles. The momentum associated with $\dot{\varepsilon}$ will be $-I_\varepsilon \dot{\varepsilon}$ in the direction of axis $f_2$ which is normal to the plane of the arm movements. The inertia term $I_\varepsilon$ will be a function of the angles $\varepsilon_a$ and $\varepsilon_b$.

As shown in Chapter 5 the equation of motion will be:

$$h = I_{ff} \omega_{fi} + h_{rel}$$

where $h$ is the total angular momentum

$\omega_{fi}$ is the whole body inertia tensor

$h_{rel}$ is the angular momentum corresponding to internal movements.

In frame $f$ the inertia tensor $I_{ff}$ will have diagonal form if there is symmetry about the sagittal plane. For asymmetrical arm positions the axes of the simulation model will not coincide exactly with the whole body principal axes but it will be assumed that $I_{ff}$ may be approximated in frame $f$ by the diagonal matrix obtained from $I_{ff}$ by setting off-diagonal elements equal to zero. The momentum term $I_{ff} \omega_{fi}$ may then be evaluated as for the motion of a rigid body and the equation of motion takes the form:

$$\begin{bmatrix}
h \cos \theta \cos \psi \\
-h \cos \theta \sin \psi \\
h \sin \theta
\end{bmatrix} = \begin{bmatrix} I_1 & 0 & 0 \\
0 & I_2 & 0 \\
0 & 0 & I_3
\end{bmatrix} \begin{bmatrix} \dot{\phi} \cos \theta \cos \psi + \phi \sin \psi \\
\dot{\phi} \cos \theta \sin \psi + \phi \cos \psi \\
\dot{\phi} \sin \theta + \phi \end{bmatrix} + \begin{bmatrix} 0 \\
-I_\varepsilon \dot{\varepsilon} \\
0
\end{bmatrix}$$
If the tilt and twist angles $\theta$ and $\psi$ are small the approximations $\sin \theta = \theta$, $\cos \theta = 1$, $\sin \psi = \psi$, $\cos \psi = 1$ may be used to yield:

\[ h = I_1 (\dot{\phi} + \dot{\psi}) \]  
(36)

\[ -h \dot{\psi} = I_2 (-\dot{\psi} + \dot{\phi}) - I_1 \epsilon \dot{\epsilon} \]  
(37)

\[ h \theta = I_3 (\dot{\phi} + \dot{\psi}) \]  
(38)

If $\psi^2$ is neglected (36) and (37) give:

\[ (I_2 - I_1) h \dot{\psi} = I_1 I_2 \dot{\theta} - I_1 \epsilon \dot{\epsilon} \]  
(39)

\[ \dot{h} I_2 = I_1 I_2 \dot{\phi} + I_1 \epsilon \dot{\psi} \]  
(40)

If $\theta \psi$ is neglected (38) and (40) give:

\[ \dot{\psi} = h \dot{\theta} (I_1 - I_3) / I_1 I_3 \]  
(41)

Differentiating (41) and using (39) gives:

\[ \ddot{\psi} = k^2 \psi - m \dot{\epsilon} \]  
(42)

where $k^2 = h^2 (I_1 - I_3) (I_2 - I_1) / I_1^2 I_2 I_3$

and $m = -h (I_\epsilon / I_2) (I_1 - I_3) / I_1 I_3$

A value for $I_\epsilon / I_2$ may be obtained using a simulation under conditions of zero angular momentum. If the arms are perpendicular throughout the movement and $\epsilon_a$ increases from $45^\circ$ to $90^\circ$ then the tilt angle $\theta$ of the principal twist axis increases from zero to $5.2^\circ$. Thus the average value of $\dot{\theta}/\dot{\epsilon}$ is about $0.12$. When $h=0$ equation (39) gives $I_\epsilon / I_2 = \dot{\theta} / \dot{\epsilon}$ and so the value $0.12$ may be used as an approximation $I_\epsilon / I_2$.

Let the control variable $\dot{\epsilon}$ be of the form:

\[ \dot{\epsilon} = p \psi + d \dot{\psi} \]

where $p$ and $d$ are constants.

Equation (42) becomes:

\[ \ddot{\psi} + 2a \dot{\psi} + \omega^2 \psi = 0 \]  
(43)

where $2a = md$ and $\omega^2 = mp - k^2$. 
The general solutions of equation (43) are:

For $\alpha < \omega$: $\psi = A e^{\alpha t} (B + \cos \beta t)$ where $\beta^2 = \omega^2 - \alpha^2$

so that the twist angle $\psi$ performs damped oscillations.

For $\alpha > \omega$: $\psi = A e^{\alpha t} + B e^{\alpha t} (\alpha - \beta)$ where $\beta^2 = \alpha^2 - \omega^2$

known as overdamped response (Elgerd, 1967, p.23).

If there is a time delay $T$ so that the value of $\dot{\epsilon}$ at time $t$ is based upon the values of $\psi$ and $\dot{\psi}$ at time $(t-T)$ then:

$\dot{\epsilon}(t) = \dot{p}\psi(t-T) + d\dot{\psi}(t-T)$

If $T$ is small then Taylor series approximations are:

$\psi(t-T) = \psi(t) - T\dot{\psi}(t)$

$\dot{\psi}(t-T) = \dot{\psi}(t) - T\ddot{\psi}(t)$

and equation (42) again takes the form of equation (43) but with:

$2\alpha = (md-m\dot{p}T)/(1-mdT)$

$\omega^2 = (m\dot{p}-k^2)/(1-mdT)$

These two equations may be solved for $p$ and $d$ if $T$, $\alpha$ and $\omega$ are specified.

This proportional plus derivative control can stabilise the somersault even when there is a time delay. If $d=0$ and only proportional control is used then $\alpha<0$ and $\psi$ does not remain small. Similarly if only derivative control is used $p=0$ and $\omega^2<0$ so that again unstable response results.

Proportional plus derivative control is incorporated into the simulation model by making the arm abduction angles change from $\epsilon_a$ to $(\epsilon_a + \Delta \epsilon)$ and from $\epsilon_b$ to $(\epsilon_b - \Delta \epsilon)$ over a time interval $0 \leq t \leq T_0$, where $T_0$ is the flight time and $\Delta \epsilon = \dot{\psi} + d\dot{\psi}$. A time delay is introduced by basing the correction $\Delta \epsilon$ upon earlier values of $\psi$ and $\dot{\psi}$.

If control is not used then small asymmetries will lead to large twist angles. In simulation ST2 a difference of only 0.1°
in arm abduction angles produces little twist after one somersault but almost half a twist after two somersaults (Figure 130). Since the effect is similar when the arm abduction angles differ by as much as 10°, this indicates that instability has little effect on a single straight somersault but is of great importance in double straight somersaults.

In simulation ST3 there is initially a difference of 10° in arm abduction angles and control is employed with a delay equivalent to 0.02 somersaults. The arms move to approximately symmetrical positions which change little during the movement so that the response is stable (Figure 131).

In simulation ST4 the delay is equivalent to 0.12 somersaults and although the twist is controlled the amplitude of the arm oscillations does not decrease (Figure 132).

In simulation ST5 the delay is equivalent to 0.24 somersaults and although the twist is controlled the difference in arm abduction angles becomes as much as 100° (Figure 133).

If the original difference in arm abduction angles is reduced from 10° to 1° then a delay of 0.24 somersaults produces response similar to Figure 132 whilst a difference of 0.1° in arm abduction angles produces response similar to Figure 131. If the delay is increased to 0.36 somersaults and the initial difference in arm abduction angles is 10° then there do not appear to be any values of p and d which result in control. If the difference is 1° control is possible with response similar to Figure 133. For a double somersault with a flight time of 1.7 seconds a delay of 0.36 somersaults is equivalent to about 300 milliseconds.

In the filmed movement C43 Carl Furrer performed a double back somersault in the straight position (Figure 134a). For almost the entire movement the lateral axis was close to the principal axis corresponding to the intermediate principal moment of inertia. This implies that control must have been employed during flight in order to prevent the build up of twist. When the movement is simulated using the film values of the internal orientation angles,
Figure 130. Graphics sequence of the simulation ST2

Figure 131. Graphics sequence of the simulation ST3
Figure 132. Graphics sequence of the simulation ST4

Figure 133. Graphics sequence of the simulation ST5
the agreement between simulation and film is good during the first somersault (Figure 134b). During the second somersault however the effects due to instability become pronounced and 0.38 twists occur in the simulated movement. This discrepancy is to be expected since the accuracy of the internal orientation angle values is about 1° and this is sufficient to produce large twist values in the second somersault.

If control is introduced into the simulation by allowing the arm abduction angles to vary from the film values by up to 5° then the twist can be controlled and agreement with the film is good throughout the simulation (Figure 134c). If more than two of the 14 internal orientation angles are used to control the twist, the constraint imposed on the difference between film and control angles can be reduced from the 5° used here.

The instability effect may be used to advantage in movements such as the back-in full-out straight where the twist occurs during the second somersault. In simulation ST6 the arm abduction angles are initially 44° and 46° and the nutation effect results in a 7° angle of tilt at the quarter twist position. By adducting the arms at this time the twist rate is increased and a full twist can be completed with time available for making adjustments prior to landing (Figure 135).

In simulation ST7 the initial difference in arm abduction angles is 10° and this leads to a more rapid build up of twist enabling two twists to be completed (Figure 136).

The control of twist using asymmetrical arm movements may be applied to twisting somersaults so that the tilt is removed and the twist is stopped. To do this requires a modification of the control algorithm so as to permit large values of the twist angle $\psi$. Near the half twist position the corrective arm movement is opposite to that near the zero twist position. At the quarter twist position arm movements are ineffective and should not be made.

The ultimate test of a method of twist control is to convert a pure twist into a plain somersault. In simulation ST8 the initial
Figure 134. Graphics sequences of the movement C43
Figure 135. Graphics sequence of the simulation ST6

Figure 136. Graphics sequence of the simulation ST7
Figure 137. Graphics sequence of the simulation ST8
angle of tilt is 89° and this is reduced to zero after three twists and one somersault using three complete cycles of arm movements (Figure 137). Although such a movement is theoretically possible the time required is probably in excess of the two seconds obtainable from a trampoline. The sequence may also be viewed in reverse to see how a somersault may be converted into a twist.

**TECHNIQUES USED IN THE FILMED MOVEMENTS**

The simulation program SIM provides the option of modifying the time histories of the internal orientation angles obtained from film data so that the effects of changing some aspect of the body configuration can be determined. The techniques used in counter-rotation twist and tilt twist primarily comprise internal movements for which symmetry is not maintained about the sagittal plane. The contribution of such aerial twisting techniques may be determined by modifying the filmed configurations so that symmetry is maintained about the sagittal plane and by noting the effect of this change on the simulated movement.

For arm movements, symmetry may be achieved by setting the orientation angles of the left arm equal to those of the right arm or vice-versa. Such a procedure is preferable to using averages of the arm orientation angles as this can result in arm positions which bear little resemblance to the original position of either arm. Torsion of the chest relative to the thorax may be removed by setting the angle \( \psi_c \) equal to zero. Hula movement of the hips may be removed by setting the hula angle \( \psi_p \) equal to zero when \( \cos \psi_p \) is positive, so that the body assumes a forward piked position, and by setting \( \psi_p \) equal to 180° when \( \cos \psi_p \) is negative, so that a back arch position is assumed.

In the movement G02 a half twist is produced from a piked jump (Figure 138). The final twist value of 0.46 twists in the simulated movement is in close agreement with the 0.48 twists obtained from the film data (see Appendix E). If the internal orientation angles are modified so that symmetry is maintained about
Figure 138. Graphics sequence of the filmed movement G02
the sagittal plane, the corresponding simulation produces only 0.06 twists which may be taken to be the amount of direct twist present in the filmed movement. Since there are two ways of producing symmetrical arm positions the final twist values of 0.05 and 0.07 of the two simulations are averaged to give the value 0.06 twists. This procedure of using the average will be used whenever simulations involve symmetrical arm positions.

When the symmetrical position is modified by allowing the arm angles to take the original film values, 0.10 twists are produced, indicating that counter-rotation of the arms produces the additional 0.04 twists. When the symmetrical position is modified by allowing the torsion angle of the chest to assume the film values, an additional 0.01 twists are obtained above the twist value of the symmetrical simulation. Using the same procedure the contribution of the hula movement of the hips is obtained as 0.34 twists.

The twist values corresponding to the various internal movements have been obtained as:

- symmetrical movements: 0.06 twists
- asymmetrical arm movements: 0.04 twists
- torsion of the chest: 0.01 twists
- hula movement of the hips: 0.34 twists

The sum of these twist values is 0.45 twists which is close to the 0.46 twists obtained in the simulation of the filmed movement. Since the twist values obtained are approximately additive the effects of arm chest and hip movements are independent of each other and the above twist values may be considered to be the contributions made to the twist in the filmed movement.

In the remaining nine filmed movements the twisting mechanics are more complicated since somersault is present and tilt twist can make a substantial contribution. In addition tilt techniques may be used to remove the tilt and stop the twist during the last phase of a movement. If a modified simulation deviates significantly from the filmed movement, such techniques of tilt removal can produce the opposite effect and increase the tilt. If this occurs
the final twist value will be a poor measure of the modified performance. In order to avoid this problem attention will be restricted to the production of twist.

If the total momentum is set to zero in each of the nine movements then the resulting simulations each produce less than a quarter twist. This indicates that only a small amount of twist can be accounted for by counter-rotation and that most of the twist is a combination of direct twist and tilt twist. For both direct twist and tilt twist the angle $\theta_m$ between the twist axis $f_3$ and the plane normal to the angular momentum vector provides a measure of the twist rate for a given configuration. Let $T_m$ be the time at which the angle $\theta_m$ reaches its maximum value. The time $T_m$ will mark the end of the phase which will be analysed. In modified simulations the angle $\theta_m$ will be evaluated at time $T_m$ and this value of $\theta_m$ will be used as a measure of the twist.

When there is no direct twist the angular momentum vector will be horizontal and $\theta_m$ will be identical to the angle of tilt $\theta$ of the axis $f_3$ away from the vertical somersault plane. For convenience $\theta_m$ will be referred to as the tilt angle although the distinction between $\theta_m$ and $\theta$ should be borne in mind.

The filmed movement G38 is a forward somersault with 1½ twists and is detailed in Appendix E. In the simulation of this movement the angle $\theta_m$ attains its maximum value of 18° at time $T_m=0.5T$ where $T$ is the flight time. Figure 139 depicts the movement up to time $T_m$. In the modified simulation for which symmetry is maintained about the sagittal plane, the value of $\theta_m$ at time $T_m$ is 3°. If the arm angles are permitted to assume the original film values, the resulting simulation produces an additional 12° in the value of $\theta_m$ at time $T_m$. If the torsion angle of the chest assumes the film values and symmetry is maintained elsewhere, the angle $\theta_m$ rises to 4° above the value of the symmetrical simulation. In the same way the tilt value corresponding to the hula movement of the hips is obtained as 3°.

Thus the tilt values corresponding to the internal movements are:
Figure 139. Graphics sequence of the filmed movement G08

Figure 140. Graphics sequence of the filmed movement G12
symmetrical movements: 3°
asymmetrical arm movements: 12°
torsion of the chest: 4°
hula movement of the hips: 3°

The sum of these tilt values is 4° more than the value of 18° produced in the simulation of the filmed movement. This difference may arise because the effects of the internal movements are not completely independent. On the other hand it may be because the axis $f_3$ is not exactly coincident with the principal axis of minimum moment of inertia. Since an error of 1° in the base value of 3° for the symmetrical simulation results in a 2° change in the sum of the tilt values, the difference of 4° should not be regarded as large. Because the tilt values are approximately additive they may be taken to be the approximate contributions made to the tilt angle $\theta_m$ of the filmed movement.

The movement G12 comprises a front somersault with one twist. The angle $\theta_m$ attains its maximum value of 8° at time $T'_m = 0.5T$ and Figure 140 shows the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:

symmetrical movements: 1°
asymmetrical arm movements: 2°
torsion of the chest: 3°
hula movement of the hips: 2°

These tilt values are representative of the contributions made to the tilt angle $\theta_m$ of the filmed movement since their sum is equal to the value of $\theta_m$.

The movement J71 comprises a forward somersault with 1½ twists. The angle $\theta_m$ attains its maximum value of 18° at time $T'_m = 0.5T$ and Figure 141 depicts the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:
Figure 141. Graphics sequence of the filmed movement J71
symmetrical movements: 1°
asymmetrical arm movements: 2°
torsion of the chest: 7°
hula movement of the hips: 2°

Whilst these values indicate that torsion of the chest is the major contributor to the tilt the sum of the values is only 12° compared with $\theta_m = 18°$. This suggests that the effects of arm, chest and hip movements are not independent so that the combined effect of two or more internal movements is greater than the sum of the individual effects.

If the internal movements are considered in pairs rather than singly the following tilt values are obtained:

symmetrical movements: 1°
arm and chest movements: 13°
arm and hip movements: 8°
chest and hip movements: 13°

Assuming that an internal movement makes equal contributions to its two pairings the individual contributions may be evaluated as:

symmetrical movements: 1°
asymmetrical arm movements: 4°
torsion of the chest: 9°
hula movement of the hips: 4°

The sum of these tilt values is equal to the angle $\theta_m$ and so they may be taken to be the contributions to the tilt angle of the filmed movement.

The movement C41 comprises a double forward somersault with 11 twists in the second somersault. The angle $\theta_m$ attains its maximum value of 16° at time $T_m = 0.6T$ and Figure 142 shows the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:

symmetrical movements: 0°
asymmetrical arm movements: 11°
Figure 142. Graphics sequence of the filmed movement C41
torsion of the chest: $1^\circ$

hula movement of the hips: $5^\circ$

These values may be taken to be the contributions to the tilt angle of the filmed movement since the sum of $17^\circ$ is close to $\theta_m = 16^\circ$.

If the movement is modified so that symmetry is maintained during phase (b) of Figure 142 the tilt contribution becomes $6^\circ$. This arises chiefly from arm asymmetry during phase (a) of Figure 142 and this is surprising since these asymmetries appear to be small. The tilt is clearly evident in the first position shown in Figure 142b.

In the four forward somersaulting movements G90, G12, J71 and C41 the tilt contributions for symmetrical movement are small and this indicates that little direct twist is used. The three late twisting backward somersaulting movements J73, C39 and C45 will be considered next. Since the twist occurs late in each movement, it may be expected that again little direct twist is used.

Movement J73 comprises a piked backward somersault with full twist. The angle $\theta_m$ attains its maximum value of $17^\circ$ at time $T_m = 0.6T$ and Figure 143 depicts the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:

- symmetrical movements: $2^\circ$
- asymmetrical arm movements: $12^\circ$
- torsion of the chest: $1^\circ$
- hula movement of the hips: $1^\circ$

The sum of these values is close to the $17^\circ$ value of $\theta_m$ in the simulation of the filmed movement.

Movement C39 is also a piked back somersault with full twist. The angle $\theta_m$ attains its maximum value of $18^\circ$ at time $T_m = 0.7T$ and Figure 144 shows the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:
Figure 143. Graphics sequence of the filmed movement J73

Figure 144. Graphics sequence of the filmed movement C39
symmetrical movements: 2°
asymmetrical arm movements: 14°
torsion of the chest: 3°
hula movement of the hips: 1°

The sum of these values is close to the 18° value of $\theta_m$ in the simulation of the filmed movement.

In both J73 and C39 the arms make the main contribution to the tilt. In J73 the left arm is abducted, causing a twist to the right (Figure 143), whilst in C39 the right arm is abducted, producing a twist to the left (Figure 144).

Movement C45 comprises a double backward somersault with a full twist which becomes noticeable after 3/4 somersaults. The maximum value of the tilt angle $\theta_m$ is 21° and is attained at time $T_m=0.6T$. Figure 145 shows the movement up to this time. The procedure of considering the effects of the internal movements singly gives the following tilt values:

symmetrical movements: -2°
asymmetrical arm movements: 17°
torsion of the chest: 1°
hula movement of the hips: -3°

Since the sum of these tilt values is 8° less than the value $\theta_m=21°$ the internal movements will also be considered in pairs. The tilt values obtained are:

symmetrical movements: -2°
arm and chest movements: 21°
arm and hip movements: 22°
chest and hip movements: 1°

Assuming that an internal movement makes equal contributions to its two pairings the individual contributions may be evaluated as:

symmetrical movements: -2°
asymmetrical arm movements: 21°
torsion of the chest: 0°
hula movement of the hips: 1°
Figure 145. Graphics sequence of the filmed movement C45
The sum of these tilt angles is now close to the value $\theta_m = 21^\circ$ of the simulation of the filmed movement.

In the three movements J73, C39 and C45 the symmetrical simulations produce little tilt and so direct twist makes only small contributions. In each case the aerial twist is chiefly a result of the asymmetrical arm movements.

In the remaining two movements C11 and C47 the twist is noticeable early in each movement and so it may be expected that direct twist makes a greater contribution than in the previous movements.

Movement C11 is a backward somersault with full twist (Figure 146). The maximum value of $\theta_m$ is $12^\circ$ and occurs at time $0.4T$.

Considering the internal movements singly gives the following tilt values:

- symmetrical movements: $9^\circ$
- asymmetrical arm movements: $2^\circ$
- torsion of the chest: $0^\circ$
- hula movement of the hips: $1^\circ$

These values are additive and indicate that asymmetrical movements about the sagittal plane contribute only $3^\circ$ to the tilt. At the moment of take-off the angle $\theta_m$ is $3^\circ$ but this rises to $9^\circ$ at time $T_m$ in the symmetrical simulation. From Figure 146 it can be seen that the arms move parallel to the sagittal plane at around the quarter twist position. If the total momentum is set to zero the corresponding simulation shows that this arm movement produces $3^\circ$ of tilt. If the arm angles are modified so that the arms move in the arm abduction plane, the tilt angle falls by $3^\circ$. Thus the symmetrical arm movement in Figure 146 can account for $3^\circ$ of the $6^\circ$ increase in the tilt angle. The remaining $3^\circ$ increase in the tilt angle occurs as a result of nutation. The contributions to the tilt angle may now be listed as:

- direct twist: $3^\circ$
- nutation: $3^\circ$
- symmetrical arm movements: $3^\circ$
- asymmetrical movements: $3^\circ$
Figure 146. Graphics sequence of the filmed movement C11
If the nutation contribution is considered to be a consequence of direct twist then direct twist and tilt techniques make equal contributions.

Movement C47 comprises a double backward somersault with a half twist in the first somersault and 1½ twists in the second somersault. The angle $\theta_m$ attains its maximum at time $T_m = 0.6T$ and Figure 147 shows the movement up to this time. The tilt values of the internal movements are not additive when they are determined singly or in pairs:

<table>
<thead>
<tr>
<th>Movement</th>
<th>Value determined from pairs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrical movements</td>
<td>$18^\circ$</td>
</tr>
<tr>
<td>Asymmetrical arm movements</td>
<td>$3^\circ$</td>
</tr>
<tr>
<td>Torsion of the chest</td>
<td>$9^\circ$</td>
</tr>
<tr>
<td>Hula movement of the hips</td>
<td>$-6^\circ$</td>
</tr>
</tbody>
</table>

The variation between the calculated values and their non-additivity indicates that there is considerable interaction between the internal movements. In phase (a) of Figure 147 nutation in the rod mode is followed by oscillation in the disc mode. The oscillation in the piked position is sensitive to the twist position at which piking starts and it is probably this that produces the interaction between the internal movements.

If modifications are confined to phase (b) in Figure 147 the tilt values of the internal movements are determined singly as:

<table>
<thead>
<tr>
<th>Movement</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrical movements</td>
<td>$12^\circ$</td>
</tr>
<tr>
<td>Asymmetrical arm movements</td>
<td>$4^\circ$</td>
</tr>
<tr>
<td>Torsion of the chest</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>Hula movement of the hips</td>
<td>$1^\circ$</td>
</tr>
</tbody>
</table>

Since the sum of these tilt values is close to the value $18^\circ$ taken by $\theta_m$, it may be concluded that asymmetries during the extension from the pike position account for $5^\circ$ of the tilt.

Although the simulations which are symmetrical throughout phases (a) and (b) give an average tilt value of $18^\circ$, the initial value of $\theta_m$ is only $4^\circ$. The $14^\circ$ increase must be a result of tilt.
Figure 147. Graphics sequence of the filmed movement C47
Table 33. Contributions to the twist of filmed movements

Note: For movement G02 the contributions to the final twist angle are given in revolutions. For the remaining 9 movements the contributions to the maximum tilt angle THM are given. THM is the angle between the twist axis f3 and the plane normal to the angular momentum vector.

<table>
<thead>
<tr>
<th>movement</th>
<th>symmetry</th>
<th>arms</th>
<th>chest</th>
<th>hips</th>
</tr>
</thead>
<tbody>
<tr>
<td>G02</td>
<td>0.06</td>
<td>0.04</td>
<td>0.01</td>
<td>0.34</td>
</tr>
<tr>
<td>G0B</td>
<td>3</td>
<td>12</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>G12</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>J71</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>C41</td>
<td>0</td>
<td>11</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>J73</td>
<td>2</td>
<td>12</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>C39</td>
<td>2</td>
<td>14</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>C45</td>
<td>-2</td>
<td>21</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C11</td>
<td>9</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>C47</td>
<td>18</td>
<td>*</td>
<td>*</td>
<td>*</td>
</tr>
</tbody>
</table>

*: These contributions are highly interactive and are not additive
produced by symmetrical arm movements, nutation in the rod mode and oscillation in the disc mode.

Thus in both of the movements C11 and C47, the twist is evident at take off but the value of the tilt angle $\theta_m$ at take off is only about one quarter of the maximum value of $\theta_m$. This somewhat surprising result indicates that movements which involve an early twist should not be classified simply as direct twists since aerial twisting techniques may make major contributions to the twist.

Table 33 lists the contributions of the different segmental movements to the twist for each of the ten filmed movements.

The results obtained in this chapter will be summarised and discussed in Chapter 8.
CHAPTER 8

SUMMARY AND DISCUSSION

In this study three related purposes have been pursued, viz:

(a) To present quantifiable mechanical explanations of the operation of twisting techniques.
(b) To establish quantitatively the capacities of different techniques for producing and controlling twist.
(c) To establish the contributions of the various techniques to the twist produced in actual performances.

SUMMARY OF FINDINGS

The extent to which these purposes have been fulfilled will now be considered.

RIGID BODY MOTIONS

(1) The possible motions of a rigid body fall into two classes which have been named as the rod mode and the disc mode. In the rod mode the twist steadily increases whereas in the disc mode the twist is oscillatory.

The two modes of motion are evident in the analytical solutions of Synge and Griffith (1959, p.378) and Whittaker (1937, p.146) and in the simulations of Grantham (1961). Until now only Nazarov (1978) has discussed twisting somersaults in terms of these two modes.

(2) The general analytical solutions of motions in these modes provide a description of all rigid body motions. The consideration of motions near to the singular solution,
which separates the two modes, shows how the modes approach each other and also explains why rotations about the intermediate principal axis are unstable.

Whilst Marion (1965, pp.407-410), Nigg (1974) and Hinrichs (1978) have established the instability of such rotations in an isolated manner, the present treatment includes these motions as extreme cases of the rod and disc modes and shows that the motions comprise sequences of half twists.

(3) In the rod mode the tilt angle $\theta_m$, between the twist axis and the angular momentum vector, nutates between two values which have the same sign. This phenomenon, named the nutation effect, may be controlled by varying the whole body principal moments of inertia.

Likins (1973, p.481) used a Poinsot construction to show that nutation will occur in the general motion of a rigid body.

(4) The nutation effect is capable of boosting the tilt angle $\theta_m$ by up to 7° in direct and tilt twists when the body is straight. The effect is greatest when a wide arm position is used during the first quarter twist, the arms being adducted as the quarter twist position is reached.

(5) In the disc mode the tilt angle $\theta_m$ oscillates about a mean value of zero. This phenomenon, named the oscillation effect, may be controlled by varying the whole body principal moments of inertia.

Although this oscillation of the tilt angle may be described as nutation, the term oscillation effect has been used for motions in the disc mode since the characteristics are somewhat different from those of the rod mode nutation effect.

(6) The oscillation effect is capable of boosting the tilt angle $\theta_m$ to more than 20° in direct and tilt twists when a pike angle of 80° is used. The effect is greatest when a deep pike position is adopted.
(7) In the rod mode the tilt angle $\theta_m$ provides a measure of the twist rate. If $\theta_m$ is in degrees and a straight body position is maintained during the twist, the number of twists per somersault will be approximately $\theta_m/4$.

This finding is in agreement with that of Travis (1968) and is applicable to both direct and tilt twists.

(8) For a given amount of momentum about the horizontal somersault axis $i_1$ and for a straight body position with arms adducted, the somersault rate of a twisting somersault will be within 2% of that of a non-twisting somersault.

This finding holds for both direct and tilt twists and shows that the conflicting opinions of Barrow (1959c), Aaron (1977, p. 29), Bunn (1972, p. 220) and Rackham (1970) are of minor importance.

DIRECT TWIST

(9) A quantifiable explanation of momentum transfer is obtained by deriving expressions for the angular momenta associated with arm, chest and whole body movements.

(10) Torsion of the chest is almost as effective as the use of the whole body in producing direct twist, whereas arm movements are about half as effective.

It should be remembered that this finding is based solely upon inertia values and that the abilities of the muscles to produce segmental angular velocities are of equal importance. However it does appear that, contrary to the opinion of Valliere (1976), the arms alone are capable of producing sufficient twist for a full twisting somersault, even if the nutation effect is not exploited.

(11) For direct twist movements in the rod mode, the twist axis will become tilted away from the vertical somersault plane after an odd number of half somersaults. This
effect is a consequence of the precession of the twist axis about the angular momentum vector which is not horizontal when direct twist is employed.

Eaves (1960b, 1960c) has noted that this effect causes problems during the entry phase of twisting dives.

(12) After an odd number of half somersaults the amount of tilt away from the vertical somersault plane will be $2\theta_m$ where $\theta_m$ is the angle between the twist axis and the plane normal to the angular momentum vector.

This is in agreement with Eaves (1969, p. 76).

(13) Even in movements for which the twist is evident at take off it may be that direct twist makes only a small contribution to the tilt angle $\theta_m$.

It is not possible to state this finding more strongly since it is based upon the analysis of only two filmed movements, namely C11 and C47. However it does show that it is not possible to categorise a twist as direct solely upon the basis of a cinemato-graphic study, as was done by Winter (1966) and Mood (1968).

COUNTER-ROTATION TWIST

(14) A quantitative explanation of counter-rotation twist is obtained by expressing the total momentum as the momentum arising from the motion of the system as a whole, together with the momentum associated with internal movement.

For hula movement of the hips, this treatment provides both the quantitative counterpart of the qualitative description of Van Gheluwe (1981b) and the analytical equivalent of the analysis of Frolich (1979).

(15) A cycle of movement of both arms is capable of producing $60^\circ$ of twist.
This finding is in close agreement with the experimental results of Beauchamp (1968) and Stepantsov et al (1966). In addition it may be stated that, contrary to the opinion of Batterman (1974, p.75), a cycle of arm movement is incapable of producing a counter-rotation twist of 180°, although it should be noted that the arm movement described by Batterman will produce tilt twist, in the expected direction, when used in a layout back dive.

(16) A cycle of chest torsion accompanied by arm abduction and adduction is capable of producing up to 30° of twist.

The Cat Reflex movement of Kulwicki et al (1962) produced approximately twice this amount of twist since leg adduction and abduction movements were also used.

(17) For a pike angle of 130°, one cycle of hula movement of the hips will produce a half twist.

Kosa (1968) and McDonald (1961) obtained similar results. Whilst the maximum amount of twist obtainable from one cycle of hula movement will be dependent upon the flexibility of the spine, it can be stated that the maximum value will be less than 3/4 twists. This contradicts the opinions of Gluck (1979,1982) and Biesterfeldt (1974b) who thought that a hula cycle could produce a full twist or more.

(18) Counter-rotation accounted for less than a quarter of a twist in each of the nine twisting somersaults which were filmed.

Thus the chest torsion and hula movement observed by Van Gheluwe and Duquet (1977) in full twisting backward somersaults does not indicate that the twist was produced by counter-rotation.

TILT TWIST

(19) A quantitative explanation of the counter-rotation tilt, produced by asymmetrical arm movements, is obtained by expressing the total momentum as the momentum arising from
the motion of the system as a whole, together with the momentum associated with internal movement. The same principle accounts for the production of tilt using torsion of the chest.

(20) A model consisting of two unequal cylinders shows that, in addition to producing counter-rotation twist, the hula movement results in precession of the twist axis so that tilt is obtained.

(21) In a straight somersault, asymmetrical arm movement can produce up to $9^\circ$ of tilt.

This finding is in close agreement with the tilt obtained in the simulations of Pike (1980). In addition, simulation TL5 shows that asymmetrical arm movement can produce a full twisting dive with a final tilt angle of zero. This is in agreement with the conclusion of Pike, although it should be noted that the speeds of the required arm movements are probably close to the physiological limits.

(22) Asymmetrical arm movement can produce up to $11^\circ$ of tilt in a piked forward somersault and up to $14^\circ$ of tilt in a piked backward somersault.

(23) The plane of arm movement which will produce maximum tilt in a piked somersault is dependent upon the direction of somersault.

(24) Torsion of the chest, prior to extension from a piked position, can produce up to $20^\circ$ of tilt in a forward somersault. Maximum tilt is produced when the arms are abducted, the angle of pike is small, the torsion angle is large and when the extension from the pike is delayed to exploit the oscillation effect. The technique is much less effective when used in a piked backward somersault.

In a backward somersault, torsion of the chest is not effective in producing tilt from a piked position, but is effective from a back arch. The opposite is true for forward somersaults. Since the
natural take off position is a pike in forward somersaults and a back arch in backward somersaults, the body is in an efficient position for starting the twist soon after take off. Rackham (1970) advocated the use of chest torsion in piked forward somersaults and arched backward somersaults.

(25) A quarter hula cycle of the hips, as the body extends from a piked position, can produce up to 18° of tilt in a forward somersault. Maximum tilt is produced when the arms are abducted, the angle of pike is small and when the extension from the pike is slow enough to exploit the oscillation and nutation effects. The technique is much less effective when used in a piked backward somersault.

As with the chest torsion technique, the natural take off positions in forward and backward somersaults are efficient for the production of tilt using this method. Aaron (1977, p.26) advocated the use of a pike-extension hip rotation technique for producing twist during a somersault.

(26) The three techniques for producing tilt in a forward somersault are more effective if the body is initially piked rather than straight.

This finding is in agreement with the experimental results of Bartee (1977) who found that greater twisting speeds occurred when the body was initially piked than when the body was initially straight.

(27) The three techniques for producing tilt have all made substantial contributions to the twist in piked forward somersaulting movements.

This statement, which is based on the analyses of movements G08, G12, J71 and C41, is in line with the capacities described in (22), (24) and (25).

(28) Of the three tilt techniques, only asymmetrical arm movement has made a substantial contribution to the twist in piked backward somersaulting movements.
This statement, which is based on the analyses of movements J73, C39 and C45, is in line with the capacities described in (22), (24) and (25) and is in agreement with the study of Van Gheluwe (1981b).

CONTROLLING THE TWIST AND TILT

(29) If the body is straight and motion is in the rod mode, abduction of the arms will reduce the twist rate by a factor of three. If a deep pike position is then adopted the twist rate will decrease by a further factor of three.

Rackham (1958) observed that abducting the arms when the body is straight increases the moment of inertia about the longitudinal axis by a factor of three.

(30) If the body pikes sufficiently so as to change the motion from the rod mode into the disc mode, the twist will have, in effect, been stopped since the twist angle will oscillate about the nearest half twist value.

This is in agreement with Nazarov (1978) but contradicts Rackham (1960) who stated that piking would slow but could not stop the twist.

(31) The adoption of a pike angle of 80° with the arms adducted will transform the motion from the rod mode to the disc mode, providing the tilt angle $\theta_m$ is less than 26° prior to piking.

(32) Once motion is in the disc mode, the tilt angle $\theta_m$ will oscillate around zero with a period equivalent to about $\frac{3}{4}$ somersaults. Thus piking can be used to control the angle of tilt.

This is in agreement with Barrow (1959a) who stated that piking into the entry of the $\frac{1}{2}$ twisting back $\frac{1}{2}$ somersault dive helps to avoid any tendency for the legs to drop sideways on finishing the dive. If extension is made after a half cycle of oscillation, twist
will again occur but in the direction opposite to the original twist. This contradicts the statement of Kosa et al. (1973) that reversing the twist is impossible but, in fairness, it should be noted that Kosa probably did not envisage the presence of somersault.

(33) Tilt may also be removed using asymmetrical arm movements. Torsion of the chest and hula movement of the hips may be used to remove the tilt if the final somersault direction is backwards.

If the tilt angle is controlled so as to obtain a vertical entry in a direct twist dive, the body will start to twist in the opposite direction. This problem will be reduced if, as in movements C11 and C47, direct twist accounts for only a fraction of the tilt angle \( \theta_m \).

(34) Forward and backward somersaults are stable providing the pike angle (or back arch) is less than 130°.

(35) The build up of twist in the unstable layout somersault will become noticeable after about 1 somersault and the quarter twist position will be reached after about \( \frac{1}{2} \) somersaults.

This shows that instability does not present a problem in the single layout somersault, whereas the double layout somersault is not possible if a fixed configuration is to be maintained.

(36) In backwards somersaults, abduction of the left arm will produce a twist to the right, whilst abduction of the right arm will produce a twist to the left. The build up of twist in the backward double layout somersault may be controlled by means of such arm movements, providing the response time is not greater than about 200 milliseconds.
DISCUSSION

The answers to the eleven questions raised in the introduction are contained in the findings (1)-(36).

Differences in the segmental inertia parameters of individual performers can affect the tilt angle and twist rate values by up to 20%. In addition, the simulation model may introduce errors of up to 10%. Where hypothetical movements have been simulated, the parameters governing body configurations have been chosen to lie within or near physiological limits but are nevertheless somewhat arbitrary. Although the delays required for the exploitation of the nutation and oscillation effects ensure that internal movement need not be rapid in order to produce maximum tilt, it is possible that not every eventuality has been foreseen. Thus, where findings have been quantified, the stated values should be taken as indications of the capacities of the techniques rather than as accurate estimates.

Despite these limitations, the findings do provide useful information which may be applied to the coaching of twisting somersaults. Although simulations have been restricted to movements without knee flexion, the tilt twist techniques may be used when twists are made from a tucked position. In particular, the hula tilt technique is still effective in forward somersaults when the body is initially tucked, although it does appear that more tilt can be produced when the movement is made from a piked position.

In June 1984 this method was used to teach Chris Maile, who was a student at Loughborough University of Technology, a full-in half-out tucked fliffus on trampoline. Since no twist was taken from the bed, the movement could be performed without sideways cast whereas, in the half-in half-out, which used direct twist, there was a distinct tendency for the movement to cast in the same direction as the twist.

Attempts to explain twisting techniques to coaches by means of written articles (Yeadon, 1973, 1974, 1978) have been rather limited in their success, whereas slide presentations and practical
demonstrations, based upon this study, have been more successful. It is to be hoped that, as information is disseminated and as further work is done in the area, the coaching of twisting somersaults will become more based upon established findings. For reference purposes it should be noted that the main elements of this study are described in Yeadon (1982, 1985a, 1985b).

Whilst a number of questions have been answered, others may now be posed. For example:

(1) Is it easier to produce tilt in a forward somersault when the body is piked rather than tucked and, if so, why?

(2) Although it has been found that the chest torsion and hula techniques are less effective in producing tilt in piked backward somersaults than in piked forward somersaults, what are the capacities of these techniques in piked backward somersaults?

(3) Whilst it has been shown that corrections, throughout the flight phase, are capable of preventing the build-up of twist in a double layout somersault, is it possible to control the twist by making a single corrective movement in mid-flight?

It can be seen that much work remains to be done in the field of twisting somersaults and there are undoubtedly a number of effects yet to be discovered.
APPENDIX A

LISTING OF THE PROGRAM ISEG
PROGRAM ISEG

ISEG CALCULATES INERTIA PARAMETERS FROM ANTHROPOMETRIC MEASUREMENTS

ANTHROPOMETRIC MEASUREMENTS

LEVELS L1 (I=0,8) ARE MEASURED FROM THE HIP CENTRE LEVEL L9

AT LEVEL L1: PI = PERIMETER
              WI = WIDTH
              DI = DEPTH

THE FOLLOWING MEASUREMENTS COMPRISE THE FIRST 3 LINES OF INPUT FILE PER

LEVELS L1 (I=10,14) ARE MEASURED FROM THE SHOULDER CENTRE LEVEL L10

THE HAND IS DEFINED BY LENGTH L15 = 2*WRIST-KNUCKLE DISTANCE
LEVELS LI (I=20, 25) ARE MEASURED FROM HIP CENTRE LEVEL L20

L21: CROTCH
L22: MID-THIGH
L23: KNEE
L24: MAX CALF
L25: ANKLE

THE FOOT IS DEFINED BY I26 = DISTANCE FROM HEEL TO TOE2

AVERAGES OF LEFT AND RIGHT LEGS ARE ENTERED ON LINES 7, 8, 9 OF INPUT FILE PER

L21 L22 L23 L24 L25
P21 P22 P23 P24 P25
V26 I26

*******************************************************************************

ISEG MODELS EACH BODY SEGMENT USING A NUMBER OF SUBSEGMENTS

SEGMENT NAME SUBSEGMENTS

PELVIS P S1 S2
THORAX T S3
CHEST-HEAD C S4-S8
UPPER-ARM A1 S11 S12
FOREARM-HAND A2 S13-S15
THIGH J1 S21-S23
CALF-FOOT J2 S24-S26

*******************************************************************************

ISEG READS DATA FROM INPUT FILE PER

WRITES DATA TO OUTPUT FILE PERF FOR USE IN PROGRAM SIM
WRITES DATA TO OUTPUT FILE PERFR FOR USE IN PROGRAM FILM

SAMPLE OUTPUT IS GIVEN AT END OF THIS LISTING

*******************************************************************************

DOUBLE PRECISION DI, DS, DT, DP, DA1, DA2, DH, DJ1, DJ2, DF, PI,
L1, L2, L3, L4, L5, L6, L7, L8, L11, L12, L13, L14,
L21, L22, L23, L24, L25,
P0, P1, P2, P3, P5, P6, P7, P10, P11, P12, P13, P14,
P20, P21, P22, P23, P24, P25,
E1, E2, E3, E4, E5, E6, E7, E8, E9, E10, E11, E12, E13, E14, E15,
E121, E122, E123, E124, E125, E126,
I1, I2, I3, I4, I5, I6, I7, I8, I9, I10, I11, I12, I13, I14, I15,
I121, I122, I123, I124, I125, I126,
I1, I2, I3, I4, I5, I6, I7, I8, I9, I10, I11, I12, I13, I14, I15,
I121, I122, I123, I124, I125, I126,
R0, R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, R11, R12, R13, R14, R15,
R121, R122, R123, R124, R125, R126,
R1, R2, R3, R4, R5, R6, R7, R8, R9, R10, R11, R12, R13, R14, R15,
R121, R122, R123, R124, R125, R126,
DOUBLE PRECISION

*Z0, Z1, Z2, Z3, Z4, Z5, Z6, Z7, Z8, Z11, Z12, Z13, Z14, Z15,

*Z21, Z22, Z23, Z24, Z25, Z26,

*ZT, ZP, ZC, ZTC, ZQI, ZHD, ZA1, ZA2, ZA, ZJ1, ZJ2, ZJ,

DOUBLE PRECISION

*VO, V1, V2, V3, V4, V5, V6, V7, V8, V11, V12, V13, V14, V15,

*V21, V22, V23, V24, V25, V26,

*VT, VP, VC, VTC, VCH, VA1, VA2, VA, VJ1, VJ2, VJ,

DOUBLE PRECISION

*X10, X11, X12, X13, X14, X15, X16, X17, X18,

*X11, X12, X13, X14, X15,

*X121, X122, X123, X124, X125, X126,

*XIT, XIP, XIC, XITC, XICI, XIHD, XI1A, XI2A, XI3A, XI4, XI5,

*Y10, Y11, Y12, Y13, Y14, Y15, Y16, Y17, Y18,

*Y11, Y12, Y13, Y14, Y15,

*Y121, Y122, Y123, Y124, Y125, Y126,

*YIT, YIP, YIC, YITC, YICI, YIHD, YI1A, YI2A, YI3A, YI4, YI5,

*ZI0, ZI1, ZI2, ZI3, ZI4, ZI5, ZI6, ZI7, ZI8,

*ZI1, ZI2, ZI3, ZI4, ZI5, ZI6, ZI7, ZI8,

*ZI21, ZI22, ZI23, ZI24, ZI25, ZI26,

*ZIP, ZIC, ZITC, ZICI, ZIHD, ZI1A, ZI2A, ZI3A, ZI4, ZI5,

*LP, L2P, LJ, LJ1, LT, LC, LC1, L2C1, L1,

*R2P, R2J, R2J1, R2J2, R2TC, R2A1, R2A2, R2J1,

*M, V, D,

*R1HD, R1CH, R1C, R1T, R1P, R1J, R1J1, R1J2, R1A, R1C1, R1TC, R1A1, R1A2

INTEGER K

CHARACTER PER*3, PERF*4, PERFR*5, DENS*4

EXTERNAL PDD, CC, FF, FUN4, FUN5, HH, MKS, RR, SS

INTRINSIC SQRT

C*******************************************************************

C OPE24 READ FILES D121S MM PER

C OPEN WRITE FILES PERF NJD PERFR

C*******************************************************************

PRINT *, 'It ivr PERFOi i£RS NNiE' 160
READ(*, 15)PFR

FOR-V T (A)

PERFR=PFR//'R'

D 7Sa' DI24P'
OPEN(5,FILE=DENS, FORM='FORMATTED')
OPEN(6,FILE=PER, FORM='FORMATTED')
OPEN(7,FILE=PERF, FORM='FORMATTED')
OPEN(8,FILE=PERFR, FORM='FORMATTED')

READ DENSITY VALUES FOR ALL SEGMENTS

READ(5,*)DN, DS, DT, DP, DA1, DA2, DH, DJ1, DJ2, DF

READ(6,*)L1, L2, L3, L4, L5, L6, L7, L8
READ(6,*)P0, P1, P2, P3, P5, P6, P7
READ(6,*)W0, W1, W2, W3, W4, D4

PELVIC SEGMENT P COMPRISSES SEGMENTS S1, S2
THORAX T COMPRISSES SEGMENT S3

RR CALCULATES STADIUM PARAMETERS R AND T FROM PERIMETER P AND WIDTH W

CALL RR(R0, T0, P0, W0)
CALL RR(R1, T1, P1, W1)
CALL RR(R2, T2, P2, W2)
CALL RR(R3, T3, P3, W3)

H1=L1
H2=L2-L1
H3=L3-L2
H4=L4-L3
H5=L5-L4
H6=L6-L5
H7=L7-L6
H8=L8-L7
* SS CALCULATES INERTIA PARAMETERS OF A SOLID BOUNDED BY TWO STADIA

CALL SS(Z1, V1, M1, XI1, YI1, ZI1, R0, T0, R1, T1, H1, DP)
CALL SS(Z2, V2, M2, XI2, YI2, ZI2, R1, T1, R2, T2, H2, DP)
CALL SS(Z3, V3, M3, XI3, YI3, ZI3, R2, T2, R3, T3, H3, DT)
CALL SS(ZT, VT, MT, XIT, YIT, ZIT, R2, T2, R3, T3, H3, DT)

Hr=H3

* ADD EVALUATES INERTIA PARAMETERS FOR THE SUM OF TWO ADJACENT SUBSEGMENTS
   THE FIRST SEVEN PARAMETERS GIVE THE OUTPUT

CALL ADD(HP, ZP, VP, MP, XIP, YIP, ZIP, H1, Z1, V1, M1, XI1, YI1, ZI1, H2, Z2, V2, M2, XI2, YI2, ZI2)

* CAST CH COMPRISSES S4 AND S5

R4=D4/2.0
PI=3.1415926536
R5=P5/PI/2.0
T4=W4/2.0-R4
T5=W4/2.0-R5

CALL SS(Z4, V4, M4, XI4, YI4, ZI4, R3, T3, R4, T4, H4, DT)
CALL SS(Z5, V5, M5, XI5, YI5, ZI5, R4, T4, R5, T5, H5, DS)

CALL ADD(HCH, ZCH, VCH, MCH, XICH, YICH, ZICH, H4, Z4, V4, M4,
   *XI4, YI4, ZI4, H5, Z5, V5, M5, XI5, YI5, ZI5)

* HEAD HD COMPRISSES S6, S7, S8

CC CALCULATES INERTIA PARAMETERS FOR TRUNCATED CONE

* CC CALCULATES INERTIA PARAMETERS FOR TRUNCATED CONE
CALL CC(Z6,V6,M6,XI6,YI6,ZI6,P5,P6,H6,DN)
CALL CC(Z7,V7,M7,XI7,YI7,ZI7,P6,P7,H7,DN)
C*******************************************************************
C HH CALCULATES PARAMETERS FOR SEMI-ELLIPSOID
C*******************************************************************
CALL HH(Z8,V8,M8,XI8,YI8,ZI8,P7,H8,DN)
CALL ADD(H0,Z0,V0,M0,XI0,YI0,ZI0,H6,Z6,V6,M6,XI6,YI6,ZI6,
*H7,Z7,V7,M7,XI7,YI7,ZI7)
CALL ADD(HHD,ZHD,VHD,MHD,XIHD,YIHD,ZIHD,H0,Z0,V0,M0,
*XI0,YI0,ZI0,H8,Z8,V8,M8,XI8,YI8,ZI8)
C*******************************************************************
C SEGMENT TC COMPRIS ES CHEST CH AND HEAD HD
C*******************************************************************
CALL ADD(HC,ZC,VC,MC,XIC,YIC,ZIC,HCH,ZCH,VCH,MCH,
*XICH,YICH,ZICH,HHD,ZHD,VHD,MHD,XIHD,YIHD,ZIHD)
C*******************************************************************
C SEGMENT TC COMPRIS ES SEGMENTS T AND C
C*******************************************************************
CALL ADD(HTC,ZTC,VT,MC,XITC,YITC,ZITC,
*HT,ZT,MT,XIT,ZIT,HC,ZC,VC,MC,XIC,YIC,ZIC)
C************************************************************x*******************
C READ MEASUREMENTS FOR SEGMENTS A1: UPPER ARM
C AND A2: FOREARM-HAND
C*******************************************************************
READ(6,*)L11,L12,L13,L14
READ(6,*)P10,P11,P12,P13,P14
READ(6,*)V15,H15
C*******************************************************************
C CALCULATE SEGMENT LENGTHS H11,H12,H13,H14
C*******************************************************************
CALL CC(Z11, V11, M11, XI11, YI11, ZI11, P10, P11, H11, DA1)
CALL CC(Z12, V12, M12, XI12, YI12, ZI12, P11, P12, H12, DA1)
CALL ADD(HA1, ZA1, VA1, MA1, XIA1, YIA1, ZIA1, H11, Z11, V11, M11, XI11, YI11, ZI11, H12, Z12, V12, M12, XI12, YI12, ZI12)

CALL CC(Z13, V13, M13, XI13, YI13, ZI13, P12, P13, H13, DA2)
CALL CC(Z14, V14, M14, XI14, YI14, ZI14, P13, P14, H14, DA2)

CALL FF(Z15, M15, XI15, YI15, ZI15, V15, H15, DH)
CALL ADD(H0, Z0, V0, M0, XI0, YI0, ZI0, H13, Z13, V13, M13, XI13, YI13, ZI13, H14, Z14, V14, M14, XI14, YI14, ZI14)
CALL ADD(HA2, ZA2, VA2, MA2, XIA2, YIA2, ZIA2, H1, Z0, V0, M0, XI0, YI0, ZI0, H15, Z15, V15, M15, XI15, YI15, ZI15)

CALL ADD(HA1, ZA1, VA1, MA1, XIA1, YIA1, ZIA1, HA1, ZA1, VA1, MA1, XIA1, YIA1, ZIA1, HA2, ZA2, VA2, MA2, XIA2, YIA2, ZIA2)
READ MEASUREMENTS OF SEGMENTS J1: THIGH AND J2: CALF-FOOT

READ(6, *)L21, L22, L23, L24, L25
READ(6, *)P21, P22, P23, P24, P25
READ(6, *)V26, H26

READ LENGTHS H21, H22, H23, H24, H25

H21 = L21
H22 = L22 - L21
H23 = L23 - L22
H24 = L24 - L23
H25 = L25 - L24

THIGH J1 COMPRISSE S S21, S22, S23

P20 IS CALCULATED FROM WIDTH W0

P20 = PI*W0/2.0

CALCULATE LEG J2 COMPRISSE S S24, S25, S26

THE FOOT IS MODELLED AS A CONE
CALL CC(Z24, V24, M24, X124, YI24, ZI24, P23, P24, H24, DJ2)
CALL CC(Z25, V25, M25, X125, YI25, ZI25, P24, P25, H25, DJ2)
CALL FF(Z26, M2.6, XI26, YI26, ZI26, V26, H26, DF)
CALL ADD(H0, Z0, V0, M0, XI0, YI0, ZI0, H24, Z24, V24, M24, X124, YI24, ZI24, H25, Z25, V25, M25, XI25, YI25, ZI25)
CALL ADD(HJ2, Z32, VJ2, MJ2, XIJ2, YIJ2, ZIJ2, H0, Z0, V0, MJ, XI0, YI0, ZI0, H26, Z26, V26, M26, XI26, YI26, ZI26)

CALL ADD(H3, ZJ, VJ, MJ, XIJ, YIJ, ZIJ, HJ1, ZJ1, VJ1, M71, XIJ1, YIJ1, ZIJ1, HJ2, ZJ2, VJ2, MJ2, XIJ2, YIJ2, ZIJ2)

CALL MKS (HT, ZT, VT, MT, XIT, YIT, ZIT)
CALL MKS (HP, ZP, VP, MP, XIP, YIP, ZIP)
CALL MKS (HC, ZC, VC, MC, XIC, YIC, ZIC)
CALL MKS (HTC, ZTC, VTC, MTC, XITC, YITC, ZITC)
CALL MKS (HCH, ZCH, VCH, MCH, XICH, YICH, ZICH)
CALL MKS (HHD, ZHD, VHD, MEID, XIHD, YIH, ZIHD)
CALL MKS (HA1, ZA1, VA1, MA1, XIA1, YIA1, ZIA1)
CALL MKS (HA2, ZA2, VA2, MA2, XIA2, YIA2, ZIA2)
CALL MKS (HA, ZA, VA, MA, XIA, YIA, ZIA)
CALL MKS (HJ1, ZJ1, VJ1, MJ1, XIJ1, YIJ1, ZIJ1)
CALL MKS (HJ2, ZJ2, VJ2, MJ2, XIJ2, YIJ2, ZIJ2)
CALL MKS (HJ, ZJ, VJ, MJ, XIJ, YIJ, ZIJ)

LP=HP
L2P=0.01*W0/4.0
LJ=0.01*L25
LJ1=HJ1
LT=HT+0.01*H4
**CALCULATE RATIO CENTROID DISTANCE/SEGMENT LENGTH FOR EACH SEGMENT**

\[ R_{ZP} = \frac{ZP}{LP} \]

\[ R_{ZJ} = \frac{ZJ}{LJ} \]

\[ R_{ZJ1} = \frac{ZJ1}{LJ1} \]

\[ R_{ZJ2} = \frac{ZJ2}{(L25 - L23)/0.01} \]

\[ R_{LJ1} = \frac{LJ1}{LJ} \]

\[ R_{ZTC} = \frac{ZTC}{LT} \]

\[ R_{ZAI} = \frac{ZA1}{LA1} \]

\[ R_{ZA2} = \frac{ZA2}{(L14 - L12)/0.01} \]

**CALCULATE WHOLE BODY MASS \( M \), VOLUME \( V \), DENSITY \( D \)**

\[ M = MP + MI' + r.1C + M3 + MA + MA + MA \]

\[ V = VP + VT + VC + VJ + VJ + VA + VA \]

\[ D = \frac{M}{V} \]

**CALCULATE RATIO SEGMENT MASS/TOTAL MASS**

\[ R_{MHD} = \frac{MHD}{M} \]

\[ R_{MCH} = \frac{MCH}{M} \]

\[ R_{MC} = \frac{MC}{M} \]

\[ R_{MT} = \frac{MT}{M} \]

\[ R_{MT} = \frac{MT}{M} \]

\[ R_{MT} = \frac{MT}{M} \]

\[ R_{M} = \frac{M}{M} \]

\[ R_{M} = \frac{M}{M} \]

\[ R_{M} = \frac{M}{M} \]

\[ R_{M} = \frac{M}{M} \]
WRITE VALUES FOR INPUT TO SIMULATION PROGRAM

WRITE(7,10) MP, XIP, YIP, ZIP, ZP, LP, L2P
WRITE(7,10) MJ1, XIJ1, YIJ1, ZIJ1, Z11, LJ1
WRITE(7,10) MJ2, XIJ2, YIJ2, ZIJ2, Z12
WRITE(7,10) MT, XIT, YIT, ZIT, LT
WRITE(7,10) MCC, XIC, YIC, ZIC, ZC, LC
WRITE(7,10) MA1, XIA1, YIA1, ZIA1, ZA1, LA1
WRITE(7,10) MA2, XIA2, YIA2, ZIA2, ZA2

WRITE VALUES FOR INPUT TO PROGRAM FILM WHICH CALCULATES ORIENTATION ANGLES FROM FILM DATA

WRITE (8,10) LP, L2P, LT, LJ, LJ1
WRITE(8,10) RZP, R7. JI, RZJ2, RZTC, RZA1, RZA2
WRITE (8,10) MP, MJ1, MJ2, MTC, MA1, MA2

WRITE OTHER VALUES TO AID CHECK FOR PROGRAM ERRORS

PRINT *, 'SEGMENTAL INERTIA PARAMETER VALUES'
PRINT *, 'UNITS: MASS IN KG'
PRINT *, 'DISTANCE IN METRES'
PRINT *, 'MOMENT OF INERTIA IN KG*M**2'
PRINT *, 'FORMAT AND SEQUENCE OF DATA PRESENTATION'
PRINT *, 'SEGMENT NAME'
PRINT *, 'MASS, DISTANCES FROM PROXIMAL JOINT OF MASS CENTRE'
PRINT *, 'AND DISTAL JOINT CENTRE(S)'
PRINT *, 'PRINCIPAL MOMENTS OF INERTIA'
PRINT *, 'HEAD HD'
PRINT 12, MHD, ZHD
PRINT 12, XIHD, YIHD, ZIHD
PRINT *, 'CHEST CH'
PRINT 12, MCH, ZCH, LCH, L2CH
PRINT 12, XICH, YICH, ZICH
PRINT *, 'CHEST-HEAD C'
PRINT 12, MC, ZC, LC
PRINT 12, XIC, YIC, ZIC
PRINT *, 'THORAX T'
PRINT 12, MT, ZT, LT
PRINT 12, XIT, YIT, ZIT
PRINT *, 'TRUNK TC'
PRINT 12, MTC, ZTC
PRINT 12, XITC, YITC, ZITC
PRINT *, 'PELVIS P'
PRINT 12, MP, ZP, LP, L2P
PRINT 12, XIP, YIP, ZIP
PRINT *, 'UPPER ARM A1'
PRINT 12, MA1, ZA1, LA1
PRINT 12, XIA1, YIA1, ZIA1
PRINT *, 'FOREARM-HAND A2'
PRINT 12, MA2, ZA2
PRINT 12, XIA2, YIA2, ZIA2
PRINT *, 'WHOLE ARM A'
PRINT 12, MA, ZA
PRINT 12, XIA, YIA, ZIA
PRINT *, 'THIGH J1'
PRINT 12, MJ1, ZJ1, LJ1
PRINT 12, XIJ1, YIJ1, ZIJ1
PRINT *, 'CALF-FOOT J2'
PRINT 12, MJ2, ZJ2
PRINT 12, XIJ2, YIJ2, ZIJ2
PRINT *, 'WHOLE LEG J'
PRINT 12, MJ, ZJ
PRINT 12, XIJ, YIJ, ZIJ
PRINT *, 'WHOLE BODY'
PRINT 13, M, D
10 FORMAT(7F9.4)
12 FORMAT(4F12.3)
13 FORMAT(' MASS = ', F6.2, ' DENSITY = ', F6.3)
14 FORMAT(' SUBJECT: ', A4)
C*******************************************************************
C CLOSE FILE UNITS 5-8
C*******************************************************************
DO 100, K=5,8
CLOSE(K)
100 CONTINUE
STOP
END
RR CALCULATES STADIUM PARAMETERS R AND T FROM PERIMETER P AND WIDTH W

SUBROUTINE RR(R,T,P,W)
DOUBLE PRECISION R,T,P,W,PI
PI=3.14159
R=(P-2.0*W)/(2.0*PI-4.0)
T=(PI*W-P)/(2.0*PI-4.0)
RETURN
END

SS CALCULATES INERTIA PARAMETERS OF A SOLID BOUNDED BY TWO STADIA

SUBROUTINE SS(Z,V,M,XI,YI,ZI,R0,T0,R1,T1,H,D)
DOUBLE PRECISION Z,V,M,XI,YI,ZI,R0,T0,R1,T1,H,D,PI,*A,B,F1,F2,F3,F1A,F2A,F3A,G1,G2,G3,G4,E1,E2,E3,E4,E5,XI0,YI0
EXTERNAL FUN4,FUN5
PI=3.14159
A=(R1-R0)/R0
B=(T1-T0)/T0
F1=1.0+(A+B)/2.0+A*B/3.0
F2=0.5+(A+B)/3.0+A*B/4.0
F3=1.0/3.0+(A+B)/4.0+A*B/5.0
F1A=1.0+A+A**2/3.0
F2A=0.5+2.0*A/3.0+A**2/4.0
F3A=1.0/3.0+A/2.0+A**2/5.0
V=H*R0*(4.0*T0*F1+PI*R0*R0*F1A)
M=D*V
Z=D*H**2*(4.0*R0*T0*F2+PI*R0*R0*F2A)/M
CALL FUN4(G1,A,B)
CALL FUN5(G2,A,B)
CALL FUN4(G3,B,A)
CALL FUN4(G4,A,A)
E1=4.0*R0*T0**3*G1/3.0
E2=PI*R0**2*T0**2*G2
E3=4.0*R0**3*T0*G3
E4=PI*R0**4*G4/2.0
E5=4.0*R0*T0*F3+PI*R0**2*F3A

C

ZI=D*H*(E1+E2+E3+E4)
XI0=H*D*(E3/3.0+E4/2.0)+H**3*D*E5
YI0=H*D*(E1+E2+2.0*E3/3.0+E4/2.0)+H**3*D*E5

C

RETURN
END

C*******************************************************************
SUBROUTINE FUN4(G, A, B)
DOUBLE PRECISION G, A, B
G=1.0+(A+3.0*B)/2.0+(A+B)*B+(3.0*A+B)*B**2/4.0+A*B**3/5.0
RETURN
END

C*******************************************************************
SUBROUTINE FUNS(G, A, B)
DOUBLE PRECISION G, A, B
G=1.0+(A+B)+(A**2+4.0*A*B+B**2)/3.0+(A+B)*A*B/2.0+(A*B)**2/5.0
RETURN
END

C*******************************************************************
SUBROUTINE ADD(H, Z, V, M, XI, YI, ZI, H1, Z1, V1, M1, XI1, YI1, ZI1,
H2, Z2, V2, M2, XI2, YI2, ZI2)
DOUBLE PRECISION H, Z, V, M, XI, YI, ZI, H1, Z1, V1, M1, XI1, YI1, ZI1,
H2, Z2, V2, M2, XI2, YI2, ZI2)
H=H1+H2
V=V1+V2
M=M1+M2
J1=Z1
J2=H1+Z2
Z=(M1*J1+M2*J2)/M
XI0=XI1+XI2+M1*J1**2+M2*J2**2
YI0=YI1+YI2+M1*J1**2+M2*J2**2
C *******************************************************************
C
C CC CALCULATES INERTIA PARAMETERS FOR TRUNCATED CONE
C LIMB SEGMENTS ARE MODELED AS TRUNCATED CONES
C
C NOTE THAT T0 AND T1 ARE SET EQUAL TO 0.001 CM. SINCE
C SUBROUTINE SS HAS SINGULARITY AT T0=0
C
C *******************************************************************

SUBROUTINE OC(Z, V, M, XI, YI, ZI, P0, P1, H, D)
DOUBLE PRECISION Z, V, M, XI, YI, ZI, P0, P1, H, D
C EXTERNAL SS
C PI=3.14159
RO=P0/PI/2.0
R1=P1/PI/2.0
T0=0.001
T1=0.001
CALL SS(Z, V, M, XI, YI, ZI, RO, T0, R1, T1, H, D)
RETURN
END

C *******************************************************************
C
C HH CALCULATES INERTIA PARAMETERS OF A SEMI-ELLIPSIOD
C CRANIUM IS MODELLED BY A SEMI-ELLIPSIOD
C
C *******************************************************************

SUBROUTINE HH(Z, V, M, XI, YI, ZI, P, H, D)
DOUBLE PRECISION Z, V, M, XI, YI, ZI, P, H, D, PI, XI0
C PI=3.14159
R=P/PI/2.0
V=2.0*PI*H*R**2/3.0
M=V*D
Z=0.375*H
XI0=0.2*H**2+H**2
XI=XI0-M**2+2
YI=XI
ZI=0.4*M**2
RETURN
END

C
SUBROUTINE FF(Z, M, XI, YI, ZI, V, H, D)
DOUBLE PRECISION Z, M, XI, YI, ZI, V, H, D, PI, P0, P1
EXTERNAL CC
INTRINSIC SQRT
PI=3.14159
P0=2.0*PI*SQRT(3.0*V/PI/H)
P1=0.0
CALL CC(Z, V, M, XI, YI, ZI, P0, P1, H, D)
RETURN
END

SUBROUTINE MKS(H, Z, V, M, XI, YI, ZI)
DOUBLE PRECISION H, Z, V, M, XI, YI, ZI
H=.01*H
Z=.01*Z
V=.001*V
M=.001*M
XI=1.0E-7*XI
YI=1.0E-7*YI
ZI=1.0E-7*ZI
RETURN
END

INPUT FILE OF ANTHROPOMETRIC MEASUREMENTS OF CARL FURRER
FILENAME = CAR

<table>
<thead>
<tr>
<th>L1</th>
<th>L8</th>
<th>L9</th>
<th>L14</th>
<th>L21</th>
<th>L25</th>
</tr>
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<tbody>
<tr>
<td>16.6</td>
<td>21.5</td>
<td>37.5</td>
<td>55.9</td>
<td>60.3</td>
<td>68.7</td>
</tr>
<tr>
<td>90.9</td>
<td>75.1</td>
<td>81.1</td>
<td>93.5</td>
<td>37.8</td>
<td>46.8</td>
</tr>
<tr>
<td>31.1</td>
<td>28.4</td>
<td>29.8</td>
<td>33.6</td>
<td>36.0</td>
<td>13.6</td>
</tr>
<tr>
<td>14.2</td>
<td>28.2</td>
<td>34.2</td>
<td>53.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.8</td>
<td>28.5</td>
<td>25.1</td>
<td>27.0</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>364</td>
<td>18.2</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.1</td>
<td>23.1</td>
<td>37.9</td>
<td>51.5</td>
<td>80.7</td>
<td></td>
</tr>
<tr>
<td>53.0</td>
<td>48.3</td>
<td>34.6</td>
<td>34.5</td>
<td>21.0</td>
<td></td>
</tr>
<tr>
<td>840</td>
<td>25.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
INPUT FILE CONTAINING SEGMENTAL DENSITIES OF DEMPSTER (1955)

FILENAME = DENIP

1.11 1.04 0.92 1.01 1.07 1.13 1.16 1.05 1.09 1.10
DN DS DT DP DA1 DA2 DH DJ1 DJ2 DF

OUTPUT FILE CARL

10.9412 0.0752 0.1072 0.0958 0.0994 0.2150 0.0777
7.1191 0.0857 0.0857 0.0212 0.1692 0.3790
4.3180 0.1206 0.1206 0.0051 0.2461
8.3995 0.0442 0.0743 0.0829 0.0841 0.3440
16.0864 0.2964 0.3667 0.1381 -0.0074 0.1800
2.1130 0.0152 0.0152 0.0025 0.1238 0.2820
*.6241 0.0178 0.0178 0.0011 0.1566

OUTPUT FILE CARLR

0.2150 0.0777 0.3440 0.8070 0.3790
0.4625 0.4464 0.5749 0.7267 0.4392 0.6238
10.9412 7.1191 4.3180 24.4859 2.1130 1.6241

OUTPUT OF ISEG ON PRIME 750

OK, FORTRAN77 ISEG

[FTN77 VER 154]

NO ERRORS [<.MAIN. >FTN77-VER 154]
NO ERRORS [<RR >FTN77-VER 154]
NO ERRORS [<SS >FTN77-VER 154]
NO ERRORS [<FUN4 >FTN77-VER 154]
NO ERRORS [<FUN5 >FTN77-VER 154]
NO ERRORS [<ADD >FTN77-VER 154]
NO ERRORS [<CC >FTN77-VER 154]
NO ERRORS [<HH >FTN77-VER 154]
NO ERRORS [<FF >FTN77-VER 154]
NO ERRORS [<MKS >FTN77-VER 154]
INPUT PERFORMERS NAME
CARL
SEGMENTAL INERTIA PARAMETER VALUES

UNITS: MASS IN KG
DISTANCE IN METRES
MOMENT OF INERTIA IN KG*M**2

FORMAT AND SEQUENCE OF DATA PRESENTATION
SEGMENT NAME
MASS, DISTANCES FROM PROXIMAL JOINT OF MASS CENTRE
AND DISTAL JOINT CENTRE(S)
PRINCIPAL MOMENTS OF INERTIA

SUBJECT: CARL

HEAD HD
4.623 0.122
0.024 0.024 0.014
CHEST CH
*1.463 -0.077 0.044 0.180
0.077 0.147 0.124
CHEST-HEAD C
*6.086 -0.007 0.180
0.296 0.367 0.138
THORAX T
8.399 0.084 0.344
0.044 0.074 0.183
TRUNK TC
24.486 0.250
0.692 0.793 0.221
PELVIS P
*0.941 0.099 0.215 0.078
0.075 0.107 0.096
UPPER ARM A1
2.113 0.124 0.282
0.015 0.015 0.002
FOREARM-HAND A2
*.631 0.157
0.018 0.018 0.001
WHOLE ARM A
3.744 0.261
0.124 0.124 0.004
THIGH J1
7.119 0.169 0.379
0.086 0.086 0.021
CALF-FOOT J2

4.322 0.246

0.121 0.121 0.005

WHOLE LEG J

*1.441 0.341

0.766 0.766 0.026

WHOLE BODY

MASS = 65.80  DENSITY = 1.018

**** STOP
APPENDIX B

LISTING OF THE PROGRAM Z2TIME
PROGRAM Z2TIME SYNCHRONISES THE FRAMES OF THE FRONT CAMERA
WITH THOSE OF THE SIDE CAMERA AT THE TIMES WHEN THE LEFT
ANKLE LIES IN THE PLANE Z = 0

Z2TIME REDEFINES THE FRAME TIMES OF THE FRONT CAMERA USING
THE FRAME TIMES OF THE SIDE CAMERA CORRESPONDING TO TWO
DIGITISATIONS OF THE SAME FILM
WHEN FRAME TIMES FOR THE SIDE CAMERA ARE NOT AVAILABLE FRAME
NUMBERS ARE USED

NN IS THE DIMENSION OF THE ARRAYS CONTAINING THE FILM DATA
NN MUST NOT BE LESS THAN THE NUMBER OF FRAMES NF, NS

PARAMETER (NN=120)

DOUBLE PRECISION
*T1F0,T2F0,T1S0,T2S0,
*ST,ET,
*SFZ1, SFZS1, SFZS2,
*XF1A(5,20),ZF1A(5,20),
*YS1A(5,20),ZS1A(5,20),YS2A(5,20),ZS2A(5,20),
*ZF1C0,ZS1C0,ZS2C0,
*ZF1R, ZS1R, ZS2R,
*TF1A(4),TS1A(4),TS2A(4),TSA(4),
*ZF1C(NN),ZS1C(NN),ZS2C(NN)

DOUBLE PRECISION
*AF1(NN,10),AS1(NN,10),AS2(NN,10),
*XF1(NN,14),ZF1(NN,14),YS1(NN,14),ZS1(NN,14),
*ZF1U(NN,14),YS2(NN,14),ZS2(NN,14),
*TF1(NN),TS1(NN),TS2(NN),
*TS(NN),ZS(NN,14)

INTEGER I,J,MI,F,M1S,M2S,NF,NS,N1F,N2F,N1S,N2S

CHARACTER*5 NAMF1,NAMS1,NAMS2,NWF1

COMMON/B1/XF1,ZF1,YS1,ZS1,YS2,ZS2

EXTERNAL EXIT,LINT
PRINT *, 'INPUT DATA FILE NAME NAMF1'
READ(*,15)NAMF1

PRINT *, 'INPUT DATA FILE NAME NAMS1'
READ(*,15)NAMS1

PRINT *, 'INPUT DATA FILE NAME NAMS2'
READ(*,15)NAMS2

PRINT *, 'INPUT NEW FILE NAME NEWF1'
READ(*,15)NEWF1

OPEN(6, FILE=NAMF1, F0RM='FORMATTED')
OPEN(8, FILE=NAMS1, FORM='FORMATTED')
OPEN(9, FILE=NAMS2, FORM='FORMATTED')
OPEN(10, FILE=NEWF1, FORM='FORMATTED')

READ (6, *) NF, ST, ET

DO 10 J=1,4
  DO 20 I=1,14
    READ(6, *) XF1A(J, I), ZF1A(J, I)
  20 CONTINUE
  READ(6, *) TF1A(J)
  10 CONTINUE

DO 30 J=1,4
  ZF1A(J,15)=(ZF1A(J,1)+ZF1A(J,7)+ZF1A(J,13))/3.0
  ZF1A(J,16)=(ZF1A(J,2)+ZF1A(J,8)+ZF1A(J,14))/3.0
  30 CONTINUE
ZF1A(J,17)=(ZF1A(J,3)+ZF1A(J,9))/2.0
ZF1A(J,18)=(ZF1A(J,4)+ZF1A(J,10))/2.0
ZF1A(J,19)=(ZF1A(J,5)+ZF1A(J,11))/2.0
ZF1A(J,20)=(ZF1A(J,6)+ZF1A(J,12))/2.0

30 CONTINUE

DO 40 I=15,20
ZF1A(5,I)=(ZF1A(1,I)+ZF1A(2,I)+ZF1A(3,I)+ZF1A(4,I))/4.0
40 CONTINUE

SFZF1=(ZF1A(5,18)-ZF1A(5,20))/2.0
ZF10R=(ZF1A(5,18)+ZF1A(5,20))/2.0
ZF1C0=(ZF1A(5,15)+ZF1A(5,16))/2.0
ZF1CQ=ZF1C0-ZF10R

C*******************************************************************
C READ MOM= DATA F1
C*******************************************************************
DO 50 J=1,NF
READ(6,*)XF1(J,11),ZF1(J,11)
READ(6,*)XF1(J,12),ZF1(J,12)
DO 60 I=1,10
READ(6,*)XF1(J,I),ZF1(J,I),AF1(J,I)
60 CONTINUE
READ(6,*)XF1(J,13),ZF1(J,13)
READ(6,*)XF1(J,14),ZF1(J,14)
ZF1C(J)=(ZF1(J,11)+ZF1(J,12)+ZF1(J,13)+ZF1(J,14))/4.0
READ(6,*)TF1(J)
50 CONTINUE

C*******************************************************************
C PERFORM THE CALCULATIONS FOR EACH FRAME:-
C CORRECT (XFI,ZFI) VALUES USING (XFIC,ZFIC)
C*******************************************************************

PERFORM THE CALCULATIONS FOR EACH FRAME:-
CORRECT (XFI,ZFI) VALUES USING (XFIC,ZFIC)
DO 210 J=1,NF

DO 220 I=1,10

ZZF1(J,I)=(ZF1(J,I)-ZF1C(J)+ZF1CO)/SFZF1

TF1(J)=J

220 CONTINUE

210 CONTINUE

C**************************************************************

READ REFERENCE DATA S1

READ(8,*)NS,ST,ET

DO 310 J=1,4

DO 320 I=1,14

READ(8,*)YS1A(J, I), zs1A(J, I)

320 CONTINUE

READ(8,*)TS1A(J)

310 CONTINUE

DO 330 J=1,4

ZS1A(J,15)=(ZS1A(J,1)+ZS1A(J,7)+ZS1A(J,13))/3.0

ZS1A(J,16)=(ZS1A(J,2)+ZS1A(J,8)+ZS1A(J,14))/3.0

ZS1A(J,17)=(ZS1A(J,3)+ZS1A(J,9))/2.0

ZS1A(J,18)=(ZS1A(J,4)+ZS1A(J,10))/2.0

ZS1A(J,19)=(ZS1A(J,5)+ZS1A(J,11))/2.0

ZS1A(J,20)=(ZS1A(J,6)+ZS1A(J,12))/2.0

330 CONTINUE

DO 340 I=15,20

ZS1A(5, I)=(ZS1A(1, I)+ZS1A(2, I)+ZS1A(3, I)+ZS1A(4, I))/4.0

340 CONTINUE

SFZS1=(ZS1A(5,18)-ZS1A(5,20))/2.0

ZS1OR=(ZS1A(5,18)+ZS1A(5,20))/2.0

ZS1CO=(ZS1A(5,15)+ZS1A(5,16))/2.0

ZS1CO=ZS1CO-ZS1OR
DO 350 J=1, NS
READ(8,*)YS1(J,11),ZS1(J,11)
READ(8,*)YS1(J,12),ZS1(J,12)
DO 360 I=1,10
READ(8,*)YS1(J,I),ZS1(J,I),AS1(J,I)
360 CONTINUE
ZS1(J)=(ZS1(J,11)+ZS1(J,12)+ZS1(J,13)+ZS1(J,14))/4.0
READ(8,*)TS1(J)
350 CONTINUE

READ(9,*)NS,ST,ET
DO 410 J=1,4
DO 420 I=1,14
READ(9,*)YS2A(J,I),ZS2A(J,I)
420 CONTINUE
READ(9,*)TS2A(J)
410 CONTINUE

ZS2A(J,15)=(ZS2A(J,1)+ZS2A(J,7)+ZS2A(J,13))/3.0
ZS2A(J,16)=(ZS2A(J,2)+ZS2A(J,8)+ZS2A(J,14))/3.0
ZS2A(J,17)=(ZS2A(J,3)+ZS2A(J,9))/2.0
ZS2A(J,18)=(ZS2A(J,4)+ZS2A(J,10))/2.0
ZS2A(J,19)=(ZS2A(J,5)+ZS2A(J,11))/2.0
YS2A(J,20)=(YS2A(J,6)+YS2A(J,12))/2.0
ZS2A(J,20)=(ZS2A(J,6)+ZS2A(J,12))/2.0
430 CONTINUE
379

262 C
263 DO 440 I=15,20
264 ZS2A(5,I)=(ZS2A(1,I)+ZS2A(2,I)+ZS2A(3,I)+ZS2A(4,I))/4.0
265 CONTINUE
266 C
267 SFZS2=(ZS2A(5,18)-ZS2A(5,20))/2.0
268 C
269 ZS2OR=(ZS2A(5,18)+ZS2A(5,20))/2.0
270 C
271 ZS2CO=(ZS2A(5,15)+ZS2A(5,16))/2.0
272 ZS2C0=ZS2CO-ZS2OR
273 C
274*******************************************************************
275 C
276 C READ MOVEMENT DATA S2
277 C*******************************************************************
278 C
279 C
280 DO 450 J=1,NS
281 READ(9,*)YS2(J,11),ZS2(J,11)
282 READ(9,*)YS2(J,12),ZS2(J,12)
283 C
284 DO 460 I=1,10
285 READ(9,*)YS2(J,I),ZS2(J,I),AS2(J,I)
286 CONTINUE
287 C
288 C
289 DO 450 J=1,NS
290 TS(J)=(TS1(J)+TS2(J))/2.0
291 C
292 DO 510 J=1,NS
293 TS(J)=(TS1(J)+TS2(J))/2.0
294 CONTINUE
295 C
296 DO 510 J=1,NS
297 TS(J)=(TS1(J)+TS2(J))/2.0
298 CONTINUE
299 C
300 TS(J)=(TS1(J)+TS2(J))/2.0
301 C
302 DO 520 I=1,10
303 C
304 ZS1(J,I)=(ZS1(J,I)-ZS1C(J)+ZS1C0)/SFZS1
305 ZS2(J,I)=(ZS2(J,I)-ZS2C(J)+ZS2C0)/SFZS2
306 ZS(J,I)=(ZS1(J,I)+ZS2(J,I))/2.0
307 C
308 C
309 CONTINUE
310 C
311 C
312 CONTINUE
PRINT T AND Z VALUES AROUND Z=0.0 FOR FRONT CAMERA

PRINT *
PRINT *, 'FRAME TIME Z'
DO 610, J=1, NF
PRINT 605, J, TF1(J), ZZF1(J, 5)
605 FORMAT(I8,6F10.3)
IF(N1F.GT.0) GO TO 606
IF(ZZF1(J, 5).GT.0.0D0) N1F=J+2-1
606 IF(J.EQ. N1F) GO TO 611
610 CONTINUE
M2F=0
DO 620, J=NF, 1, -1
PRINT 605, J, TF1(J), ZZF1(J, 5)
IF(M2F.GT.0) GO TO 616
IF(ZZF1(J, 5).GT.0.0D0) M2F=J+2-NF
616 IF(J.EQ.M2F) GO TO 621
620 CONTINUE

C USER SELECTS 5 OR 6 POINTS AROUND Z=0.0 TO WHICH A STRAIGHT LINE IS FITTED SO THAT THE TIMES T1FO, T2FO ARE DETERMINED FOR WHICH Z=0.0

THIS IS DONE FOR THE LEFT ANKLE BOTH RISING AND FALLING

INPUT M1F, N1F, M2F, N2F
READ *, M1F, N1F, M2F, N2F
CALL LINT(T1FO, TF1, ZZF1(1, 5), M1F, N1F)
CALL LINT(T2FO, TF1, ZZF1(1, 5), M2F, N2F)
PRINT 625, T1FO, T2FO
625 FORMAT(' Z = 0 AT TIMES', 2F10.3)

PRINT T AND Z VALUES AROUND Z=0.0 FOR SIDE CAMERA
365 C
366 PRINT *, J, TS, ZS
367 PRINT *, N1S=0
368 DO 630, J=1, NS
369 PRINT 605, J, TS(J), ZS(J, 5), TS1(J), ZS1(J, 5), TS2(J), ZS2(J, 5)
370 IF(N1S.GT.0) GO TO 626
371 IF(ZS(J, 5).GT.0.0D0) M1S=J*2-1
372 626 IF(J.EQ.M1S) GO TO 631
373 630 CONTINUE
374 C
375 631 M2S=0
376 DO 640, J=NS, 1, -1
377 PRINT 605, J, TS(J), ZS(J, 5), TS1(J), ZS1(J, 5), TS2(J), ZS2(J, 5)
378 IF(M2S.GT.0) GO TO 636
379 IF(ZS(J, 5).GT.0.0D0) M2S=J*2-NS
380 636 IF(J.EQ.M2S) GO TO 641
381 640 CONTINUE
382 C
383 PRINT *, 'INPUT M1S, N1S, M2S, N2S'
384 READ *, M1S, N1S, M2S, N2S
385 C
386 C*******************************************************************
387 C DETERMINE TIMES T1SO AND T2S0 AT WHICH Z=0.0
388 C*******************************************************************
389 CALL LINT(T1SO, TS, ZS(1,5), M1S, N1S)
390 CALL LINT(T2SO, TS, ZS(1,5), M2S, N2S)
391 C*******************************************************************
392 C RESET FRAME TIMES OF FRONT CAMERA TO SYNCHRONISE WITH
393 C SIDE CAMERA
394 C*******************************************************************
395 PRINT 625, T1SO, T2SO
396 C*******************************************************************
397 PRINT *, 'NEW FRAME TIMES'
398 PRINT *, J, T, Z
399 DO 650, J=1, NF
400 TF1(J)=(TF1(J)-T1FO)*(T2SO-T1SO)/(T2FO-T1FO)Pr1S0
401 PRINT 655, J, TF1(J), ZF1(J, 5)
402 FORMAT(I8,2F10.3)
403 650 CONTINUE
WRITE THE TRANSFORMED FRONT CAMERA VALUES TO NEW FILE

WRITE(10,705)NF,ST,ET
705 FORMAT(I8,2F10.3)

DO 710, J=1,4
    DO 720, I=1,14
       WRITE(10,706)XF1A(J, I),ZF1A(J, I)
    CONTINUE

SET INITIAL TIME VALUES THE SAME AS FOR SIDE CAMERA

TSA(J)=(TS1A(J)+TS2A(J))/2.0
WRITE(10,707)TSA(J)
706 FORMAT(3F6.0)
707 FORMAT(F8.3)

CLOSE ALL FILES

CALL EXIT
END
END OF MAIN SEGMENT

LIN FITS REGRESSION LINE

SUBROUTINE LIN(A, B, T, Z, M, N)
PARAMETER (NN=120)
DOUBLE PRECISION A, B, S1, ST, ST2, SZ, STZ, T(NN), Z(NN)
INTEGER J, M, N
S1=0.0
ST=0.0
ST2=0.0
SZ=0.0
STZ=0.0
DO 10, J=M, N
S1=S1+1.0
ST=ST+T(J)
ST2=ST2+T(J)**2
SZ=SZ+Z(J)
STZ=STZ+T(J)*Z(J)
10 CONTINUE

A=(S1*STZ-ST*SZ)/(S1*ST2-ST*ST)
B=(ST*STZ-ST2*SZ)/(ST*ST-ST2*S1)
RETURN
END
LINT determines the time T0 at which Z=0

SUBROUTINE LINT(T0, T, Z, M, N)
PARAMETER (NN=120)
DOUBLE PRECISION A, B, T0, T(NN), Z(NN)
INTEGER M, N
EXTERNAL LIN
CALL LIN(A, B, T, Z, M, N)
T0 = -B/A
RETURN
END
APPENDIX C

LISTING OF THE PROGRAM FILM
PROGRAM FILM

FILM READS DIGITISED FILM DATA FROM FILES NAMFI AND NAMSI
CALCULATES 3-D COORDS
AND FITS QUINTIC SPLINES TO THE ORIENTATION ANGLES

NN IS THE DIMENSION OF THE ARRAYS CONTAINING THE FILM DATA
NN MUST NOT BE LESS THAN THE NUMBER OF FRAMES NF, NS

PARAMETER (NN=120)

DOUBLE PRECISION T,
*R2P,RZJ1,RZJ2,RZTC,RZAI,RZA2,
*MP,MJ1,MJ2,MTC,MA1,MA2,
*DGP,L2P,DXN,LJ,LJ1,
*DF,DS,ST,ET,FT,
*PI,RTD,
*DG(4,10,4),R(3,10,4),SA(17),CA(17),
*DGL(4,10),DG2(4,10)

DOUBLE PRECISION
*PH(4,NN),PH0(4),CCPH(6,NN),
*TH(4,NN),TH0(4),CCTH(6,NN),
*PS(4,NN),PS0(4),CCPS(6,NN),
*GA(4,NN),GAL(4),CCGA(6,NN),
*PS(4,NN),PS0(4),CCPS(6,NN),
*EJK(4,NN),EJK0(4),CCEJK(6,NN),
*GAL(4,NN),GAL0(4),CCGAL(6,NN),
*THC(4,NN),THC0(4),CCTHC(6,NN),
*PSC(4,NN),PSCO(4),CCPSC(6,NN),
*DA(4,NN),DA0(4),CCDA(6,NN),
*EA(4,NN),EA0(4),CCEA(6,NN),
*PSA(4,NN),PSA0(4),CCPSA(6,NN),
*GAA(4,NN),GAA0(4),CCGAA(6,NN),
*DB(4,NN),DB0(4),CCDB(6,NN),
*EB(4,NN),EB0(4),CCEB(6,NN),
*PSB(4,NN),PSB0(4),CCPSB(6,NN),
*GAB(4,NN),GAB0(4),CCGAB(6,NN),
*TV(NN)
DOUBLE PRECISION

*SDPH, SDTH, SDPS,
*SDEJK, SDGAL, SDLTC, SDPSC,
*SDDA, SDEA, SDPSA, SDGAL,
*SDDA, SDEB, SDPSB, SDGAL,
*SDANG

DOUBLE PRECISION

*SFXF1, SFXF2, SFZF1, SFZF2, SFYS1, SFYS2, SFZS1, SFZS2,
*CP(2), XF1C0, XF2C0, ZF1C0, ZF2C0, YS1C0, YS2C0, ZS1C0, ZS2C0,
*SF(2), SFXF1, SFXF2, SFZF1, SFZF2, SFYS1, SFYS2, SFZS1, SFZS2,
*CP(2), XF1C0, XF2C0, ZF1C0, ZF2C0, YS1C0, YS2C0, ZS1C0, ZS2C0,
*OR(2), XF1OR, XF2OR, ZF1OR, ZF2OR, YS1OR, YS2OR, ZS1OR, ZS2OR,
*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,

*SFHF, SPFHF, CHF, CPF, RNFH(3, 3), RNPSF(3, 3), SFI(3, 3),
*STHF, SPSS, CTNS, CPSS, RNFH(3, 3), RNPSF(3, 3), SSI(3, 3),

*XFM, ZFM, YSM, ZSM,

*IXF, KZF, EXP, EZF, KYS, KZS, EYS, ERS, EE,

*PHF, PSF, THR, PSS,
COMMON/B11/CXF1  
COMMON/B12/CXF2  
COMMON/B13/CZF1  
COMMON/B14/CZF2  
COMMON/B15/CYS1  
COMMON/B16/CYS2  
COMMON/B17/CZS1  
COMMON/B18/CZS2  
COMMON/B21/XF1, XF2, ZF1, ZF2  
COMMON/B22/YS1, YS2, ZS1, ZS2  
COMMON/B23/TF1, TF2, TF, TS1, TS2, TS  
COMMON/B25/CCHP, CCTH, CCP  
COMMON/B26/CCHSC, CCHR, CCDA, CCEA, CCDB, CCEB  
COMMON/B27/CCGPA, CCGAB, CCEJK, CCPSP, CCAS  
EXTERNAL ANXG, CSANG, COORD, EXIT, ORANG, RM1, RM2, RM3,  
*SPLIN, SPLIN4, TIME, VALQ, V4, WRITE  
INTRINSIC ABS, ASIN, MOD, SQRT  
PRINT *, 'STATE OPTION NUMBER 1 TO 5'
READ(*, *)OPT
PRINT *, 'STATE DATA FILE NAME'
READ(*, 15)NAM
PRINT *, 'STATE PERFORMER NAME'
READ(*, 15)PERF
PRINT *, 'STATE OUTPUT FILE NAME'
READ(*, 15)OUT
15 FORMAT(A)
PRINT *, 'STATE OPTION NUMBER 1 TO 5'
READ(*, *)OPT
PRINT *, 'STATE DATA FILE NAME'
READ(*, 15)NAM
PRINT *, 'STATE PERFORMER NAME'
READ(*, 15)PERF
PRINT *, 'STATE OUTPUT FILE NAME'
READ(*, 15)OUT
15 FORMAT(A)
OPEN READ FILES PERFR, NAMFI, NAMS2, I=1,2
OPEN(5, FILE=PERFR, FORM='FORMATTED')
OPEN(6, FILE=NAMFI, FORM='FORMATTED')
OPEN(7, FILE=NAMF2, FORM='FORMATTED')
OPEN(8, FILE=NAMS2, FORM='FORMATTED')
OPEN(9, FILE=NAMS2, FORM='FORMATTED')
OPEN WRITE FILES

OUTFIT=OUT//'FIT'
OUTANG=OUT//'ANG'
OUTFLM=OUT//'FLM'

OPEN(10,FILE=OUTFIT,FORM='FORMATTED')
OPEN(11,FILE=OUTANG,FORM='FORMATTED')
OPEN(12,FILE=OUTFLM,FORM='FORMATTED')

READ ANTHROPOMETRIC DATA:
DISTANCES: DGX,L2P,DXN,LJ,LJ1
CENTROID LENGTH RATIOS: RZP,RZJ1,RZJ2,RZTC,RZA1,RZA2
SEGMENT MASSES: MP,MJ,MPC,MA1,MA2

THIS DATA WILL BE USED IN SUBROUTINE ORANG

READ(5, *)DGX, L2P, DXN, LJ, LJ1
READ(5, *)RZP, RZJ1, RZJ2, RZTC, RZA1, RZA2
READ(5, *)MP, MJ1, MJ2, MTC, MA1, MA2

READ THE DIGITISED X, Z CO-ORDS OBTAINED FROM FRONT CAMERA.
DISTANCES ARE CONVERTED INTO METRES AND TIME VALUES ARE NORMALISED SO THAT START TIME = 0.0 AND END TIME = 1.0

READ REFERENCE DATA FROM FRONT CAMERA
DETERMINE: SCALE FACTORS SFX, SFZ
ORIGIN (XOR, ZOR) OF REFERENCE FRAME
CONTROL POINT (XC0, ZC0)

CALL REF(SF, OR, CP, TOO, NF, 6)
SFXF1 = SF(1)
SFZF1 = SF(2)
XF1OR = OR(1)
ZF1OR = OR(2)
XF1C0 = CP(1)
ZF1C0 = CP(2)
TF10 = T00

READ MOVEMENT DATA F1

DO 50 J = 1, NF
READ(6, *) XF1(J, 11), ZF1(J, 11)
READ(6, *) XF1(J, 12), ZF1(J, 12)
DO 60 I = 1, 10
READ(6, *) XF1(J, I), ZF1(J, I)
CONTINUE
READ(6, *) XF1(J, 13), ZF1(J, 13)
READ(6, *) XF1(J, 14), ZF1(J, 14)
XF1C(J) = (XF1(J, 11) + XF1(J, 12) + XF1(J, 13) + XF1(J, 14)) / 4.0
ZF1C(J) = (ZF1(J, 11) + ZF1(J, 12) + ZF1(J, 13) + ZF1(J, 14)) / 4.0
READ(6, *) TF1(J)
CONTINUE

READ REFERENCE DATA F2 FROM FRONT CAMERA

RF. AD(7, *) NF
CALL REF(SF, OR, CP, T00, NF, 7)
SFXF2 = SF(1)
SFZF2 = SF(2)
XF2OR = OR(1)
ZF2OR = OR(2)
XF2C0 = CP(1)
ZF2C0 = CP(2)
TF20 = T00

READ REFERENCE DATA F1 FROM FRONT CAMERA

READ(7, *) NF
CALL REF(SF, OR, CP, T00, NF, 7)
DO 150 J=1,NF
 READ(7,*),XF2(J,11),ZF2(J,11)
 READ(7,*),XF2(J,12),ZF2(J,12)
 DO 160 I=1,10
 READ(7,*),XF2(J,I),ZF2(J,I)
 160 CONTINUE
 READ(7,*),XF2(J,13),ZF2(J,13)
 READ(7,*),XF2(J,14),ZF2(J,14)
 XF2C(J)=(XF2(J,11)+XF2(J,12)+XF2(J,13)+XF2(J,14))/4.0
 ZF2C(J)=(ZF2(J,11)+ZF2(J,12)+ZF2(J,13)+ZF2(J,14))/4.0
 150 CONTINUE

PERFORM THE CALCULATIONS FOR EACH FRAME:-

DO 210 J=1,NF
 DO 220 I=1,10
 XF1(J,I)=(XF1(J,I)-XF1C(J)+XF1C0)/SFXF1
 ZF1(J,I)=(ZF1(J,I)-ZF1C(J)+ZFlcO)/SFZF1
 220 CONTINUE

IF(ABS(TF1(J)-TF2(J)).LT.3.0D0) GO TO 226
 PRINT 225, TF1(J), TF2(J)
 225 FORMAT(' TF TIMES DIFFERENT', 2F8.0)
 TF1(J)=(TF1(J)-TFO-ST)/(ET-ST)
 TF2(J)=(TF2(J)-TFO-ST)/(ET-ST)
 210 CONTINUE
READ THE DIGITISED Y,Z CO-ORDS OBTAINED FROM SIDE CAMERA.
DISTANCES ARE CONVERTED INTO METRES AND TIME VALUES ARE NORMALISED SO THAT START TIME = 0.0 AND END TIME = 1.0

READ REFERENCE DATA S1 FROM SIDE CAMERA
DETERMINE: SCALE FACTORS SFY, SFZ ORIGIN (YOR, ZOR) OF REFERENCE FRAME
CONTROL POINT (YCO, ZCO)

READ(8, *)NS
CALL REF(SF, OR, CP, T00, NS, 8)
SFYS1=SF(1)
SFZS1=SF(2)
YS1OR=OR(1)
ZS1OR=OR(2)
YS1C0=CP(1)
ZS1C0=CP(2)
TS10=T00

READ MOVEMENT DATA S1

DO 350 J=1,NS
READ(8,*)YS1(J,11), ZS1(J,11)
READ(8,*)YS1(J,12), ZS1(J,12)
DO 360 I=1,10
READ(8,*)YS1(J,I), ZS1(J,I)
360 CONTINUE
350 CONTINUE

YS1C(J)=(YS1(J,11)+YS1(J,12)+YS1(J,13)+YS1(J,14))/4.0
ZS1C(J)=(ZS1(J,11)+ZS1(J,12)+ZS1(J,13)+ZS1(J,14))/4.0
READ(8,*)TS1(J)
350 CONTINUE
READ REFERENCE DATA S2 FROM SIDE CAMERA

READ(9,*)NS
CALL REF(SF, OR, CP, T00, NS, 9)
SFYS2=SF(1)
SFZS2=SF(2)
YS2OR=OR(1)
ZS2OR=OR(2)
YS2CO=CP(1)
ZS2CO=CP(2)
TS20=T00

READ MOVEMENT DATA S2

DO 450 J=1, NS
READ(9,*)YS2(J, 11), ZS2(J, 11)
READ(9,*)YS2(J, 12), ZS2(J, 12)
DO 460 I=1, 10
READ(9,*)YS2(J, I), ZS2(J, I)
460 CONTINUE
READ(9,*)YS2(J, 13), ZS2(J, 13)
READ(9,*)YS2(J, 14), ZS2(J, 14)
YS2C(J)=(YS2(J, 11)+YS2(J, 12)+YS2(J, 13)+YS2(J, 14))/4.0
ZS2C(J)=(ZS2(J, 11)+ZS2(J, 12)+ZS2(J, 13)+ZS2(J, 14))/4.0
TS0=(TS10+TS20)/2.0

PERFORM THE CALCULATIONS FOR EACH FRAME:-
CORRECT (YSI, ZSI) VALUES USING (YSIC, ZSIC)
CONVERT YSI TO METRES USING SCALE FACTOR SFYSI
CONVERT ZSI TO METRES USING SCALE FACTOR SFZSI
NORMALISE TIME VALUES TSI

TS0=(TS10+TS20)/2.0
DO 510 J=1,NS
DO 520 I=1,10

YS1(J,I)=(YS1(J,I)-YS1C(J)+YS1CO)/SFYS1
ZS1(J,I)=(ZS1(J,I)-ZS1C(J)+ZS1CO)/SFZS1

YS2(J,I)=(YS2(J,I)-YS2C(J)+YS2CO)/SFYS2
ZS2(J,I)=(ZS2(J,I)-ZS2C(J)+ZS2CO)/SFZS2

520 CONTINUE

IF(ABS(TS1(J)-TS2(J)).LT.3.0D0) GO TO 526
PRINT 525, 'TS TIMES DIFFERENT', TS1(J), TS2(J)
525 FORMAT (2F8.0)
526 TS1(J)=(TS1(J)-TSO-ST)/(ET-ST)
TS2(J)=(TS2(J)-TSO-ST)/(ET-ST)

510 CONTINUE

READ(6, *)DF, XFM, ZFM
READ(8, *)DS, YSM, ZSM

XF1CO=XF1CO+XF1OR
XF2CO=XF2CO+XF2OR
ZF1CO=ZF1CO+ZF1OR
ZF2CO=ZF2CO+ZF2OR

XF1CV=0.0
XF2CV=0.0
ZF1CV=0.0
ZF2CV=0.0

DO 550 J=1,NF

550 CONTINUE
0489 XF1CV=ZF1CV/NF
0490 XF2CV=ZF2CV/NF
0491 ZF1CV=ZF1CV/NF
0492 ZF2CV=ZF2CV/NF
0493 C
0494 XF1CV=(XF1CV-XF1CV)/SFxF1
0495 XF2CV=(XF2CV-XF2CV)/SFxF2
0496 ZF1CV=(ZF1CV-ZF1CV)/SFZF1
0497 ZF2CV=(ZF2CV-ZF2CV)/SFZF2
0498 C
0499 C
0500 XFM=XFM-(XF1CV+XF2CV)/2.0
0501 ZFM=ZFM-(ZF1CV+ZF2CV)/2.0
0502 C
0503*******************************************************************
0504 C
0505 C CALCULATE INCLINATION ANGLES PHF, PSF AND ROTATION MATRIX
0506 C
0507 C
0508*******************************************************************
0509 C
0510 '1 IOcF=1.0
0511 KZF=1.0
0512 EXF=0.0
0513 EZF=0.0
0514 C
0515 DO 560, I=1,3
0516 C
0517 SPHF=(KZF*ZFM+EZF)/DF
0518 CPHF=SQRT(1.0-SPHF**2)
0519 C
0520 SPSF=-(KXF*XFM+EXF)/DF/CPHF
0521 CPSF=SQRT(1.0-SPSF**2)
0522 C
0523 EE=(DF*CPHF)**2-SPHF**2
0524 C
0525 KZF=(DF*CPHF)**2/EE
0526 EZF=DF*SPHF/EE
0527 C
0528 EE=(DF*CPHF*CPSF)**2-SPSF**2
0529 C
0530 KXF=CPSF*(DF*CPHF)**2/EE
0531 EXF=-DF*CPHF*SPSF/EE
0532 C
0533 PI=3.1415926536
0534 RTD=180.0/PI
0535 PHF=RTD*ASIN(SPHF)
0536 PSF=RTD*ASIN(SPSF)
0537 C
0538 560 CONTINUE
CALL R1(RNPHF,CPHF,-SPHF)
CALL R13(RNPSF,CPSF,-SPSF)
CALL PM33(SFI,RNPHF,RNPSF)

CORRECT POSITION (YSM,ZSM) OF IMAGE CENTRE USING CONTROL POINTS

Y1C0 = YS1C0 + YS1OR
Y2C0 = YS2C0 + YS2OR
Z1C0 = ZS1C0 + ZS1OR
Z2C0 = ZS2C0 + ZS2OR

YS1CV = 0.0
YS2CV = 0.0
ZSICV = 0.0
ZS2CV = 0.0

DO 570 J = 1, NS
  YS1CV = YS1CV + YS1C(J)
  YS2CV = YS2CV + YS2C(J)
  ZSICV = ZSICV + ZSIC(J)
  ZS2CV = ZS2CV + ZS2C(J)
570 CONTINUE

YS1CV = YS1CV/NS
YS2CV = YS2CV/NS
ZSICV = ZSICV/NS
ZS2CV = ZS2CV/NS

YS1CV = (YS1CV - YS1C0)/SFYS1
YS2CV = (YS2CV - YS2C0)/SFYS2
ZS1CV = (ZS1CV - ZS1C0)/SFZS1
ZS2CV = (ZS2CV - ZS2C0)/SFZS2

YSM = YSM - (YS1CV + YS2CV)/2.0
ZSM = ZSM - (ZS1CV + ZS2CV)/2.0

CALCULATE INCLINATION ANGLES THS,PSS AND ROTATION MATRIX SSI FOR SIDE CAMERA

C.................................
KYS = 1.0
KZS = 1.0
EYS = 0.0
EZS = 0.0

DO 580, I = 1, 3

STNS = (KZS * ZSNI + EZS) / DS
CTHS = SQRT(1.0 - STHS**2)

SPSS = -(KYS * YSti1 + EYS) / DS / CTHS
CPSS = SQRT(1.0 - SPSS**2)

EE = (DS * CTHS)**2 - STHS**2

KYS = CPSS * (DS * CTHS)**2 / EE
EYS = -DS * CTHS * SPSS / EE

THS = RTD * ASIN(STHS)
PSS = RTD * ASIN(SPSS)

580 CONTINUE

CALL R42(RNTHS, CTHS, -STNS)
CALL R43(RNPSS, CPSS, -SPSS)
CALL PM33(SSI, RNTHS, RNPSS)

PRINT *
PRINT *, 'INCLINATION ANGLES OF CAMFRAS IN DEGREES'
PRINT *, PHF, PSF
22 FORMAT (' PHF = ', F3.0, 5X, 'PSF = ', F3.0)
23 FORMAT (' THS = ', F3.0, 5X, 'PSS = ', F3.0)

TIME SETS TF AND TS ARE REPLACED BY CUBIC SPLINE FIT

CALL TIHE(TF, TF1, TF2, NF)
CALL TIME(TS, TS1, TS2, NS)

C*******************************************************************
THE X, Y, Z VALUES ARE SPLINED SO THAT INTERPOLATED VALUES MAY BE OBTAINED.

C*******************************************************************
C
PRINT *
PRINT *, 'DIGITISING ERRORS IN METRES'
PRINT *
PRINT *, 'TIME FRAME XYZ1 XYZ2'

DO 610 I=1,10

CALL SPLIN(CXF1(1,1,I), CXF2(1,1,I), SDXF(I), NF, TF, XF1(1,I),
* XF2(1,I))

CALL SPLIN(CZF1(1,1,I), CZF2(1,1,I), SDZF(I), NF, TF, ZF1(1,I),
* ZF2(1,I))

CALL SPLIN(CYS1(1,1,I), CYS2(1,1,I), SDYS(I), NS, TS, YS1(1,I),
* YS2(1,I))

CALL SPLIN(CZS1(1,1,I), CZS2(1,1,I), SDZS(I), NS, TS, ZS1(1,I),
* ZS2(1,I))

PRINT 625,1

610 CONTINUE

625 FORMAT(I3)

C*******************************************************************
C
PRINT STANDARD DEVIATIONS OF FILM COORDINATES

C*******************************************************************

PRINT *
PRINT *, 'STANDARD DEVIATIONS OF FILM COORDINATES'
PRINT *
PRINT *, ' I XF ZF YS ZS AV'
PRINT *

SD1=0.0
SD2=0.0
SD3=0.0
SD4=0.0
SD5=0.0

DO 620, I=1,10

SDAV(I)=(SDXF(I)+SDZF(I)+SDYS(I)+SDZS(I))/4.0
SD1=SD1+SDXF(I)
SD2=SD2+SDZF(I)
SD3=SD3+SDYS(I)
SD4=SD4+SDZS(I)
SD5=SD5+SDAV(I)

PRINT 635,I,SDXF(I),SDZF(I),SDYS(I),SDZS(I),SDAV(I)

620 CONTINUE
DO 700, N=1,101
T=(N-1)/100.0

C*******************************************************************
C
C IN THIS LOOP THE 3-D COORDS OF EACH POINT ARE DETERMINED AT
C TIME T AND THE ORIENTATION ANGLES ARE EVALUATED USING
C SUBROUTINE ORANG
C
C*******************************************************************

DO 710 I=1,10

CALL VALQ(DG1(1, I), T, CXF1(1,1, I), NF, TF)
CALL VALQ(DG1(2, I), T, CZF1(1,1, I), NF, TF)
CALL VALQ(DG1(3, I), T, CYS1(1,1, I), NS, TS)
CALL VALQ(DG1(4, I), T, CZS1(1,1, I), NS, TS)
CALL VALQ(DG2(1, I), T, CXF2(1,1, I), NF, TF)
CALL VALQ(DG2(2, I), T, CZF2(1,1, I), NF, TF)
CALL VALQ(DG2(3, I), T, CYS2(1,1, I), NS, TS)
CALL VALQ(DG2(4, I), T, CZS2(1,1, I), NS, TS)

DG CONTAINS THE 4 PAIRINGS OF THE SPLINED DIGITISED DATA
DG(J, I, L): J=1,4 COMPONENTS XF,ZF,XS,ZS
I=1,10 JOINT CENTRES
L=1,4 PAIRINGS F1S1,F1S2,F2S1,F2S2
CALL V4(DG(1,1,1),DG1(1,1),DG1(2,1),DG1(3,1),DG1(4,1))
CALL V4(DG(1,1,2),DG1(1,1),DG1(2,1),DG2(3,1),DG2(4,1))
CALL V4(DG(1,1,3),DG2(1,1),DG1(2,1),DG1(3,1),DG1(4,1))
CALL V4(DG(1,1,4),DG2(1,1),DG2(2,1),DG2(3,1),DG2(4,1))

************************************************************************
SUBROUTINE COORD OBTAINS 4 ESTIMATES OF THE POSITION VECTOR
R(J,I,L) J=1,3 COMPONENTS X,Y,Z
I=1,10 JOINT CENTRES
L=1,4 ESTIMATES
************************************************************************
DO 720 L=1,4
IF(OPT.LT.5) CALL COORD(R(1,I,L),DG(1,I,O))
IF(OPT.EQ.5) CALL COORD(R(1,I,L),DG(1,I,L))
720 CONTINUE
710 CONTINUE
************************************************************************
EVALUATE 4 ESTIMATES OF THE ORIENTATION ANGLES AT TIME T
ORANG RETURNS THE SINES SA AND COSINES CA
CSANG CALCULATES THE ANGLES IN RADIANS
ANG0 CALCULATES THE INITIAL VALUES OF THE ANGLES
************************************************************************
DO 750 L=1,4
CALL ORANG(SA,CA,R(1,1,L))
-- IF(N.GT.1) GO TO 760
CALL ANG0(PH(L,N),PH0(L),SA(1),CA(1))
CALL ANG0(TH(L,N),TH0(L),SA(2),CA(2))
CALL ANG0(PS(L,N),PS0(L),SA(3),CA(3))
CALL ANG0(GA(L,N),GA0(L),SA(4),CA(4))
CALL ANG0(PSP(L,N),PSP0(L),SA(5),CA(5))
CALL ANG0(EK(L,N),EK0(L),SA(6),CA(6))
CALL ANG0(GAL(L,N),GAL0(L),SA(7),CA(7))
CALL ANG0(THC(L,N),THC0(L),SA(8),CA(8))
CALL ANG0(PSCL(N),PSCL0(L),SA(9),CA(9))
CALL ANG0(DA(L,N),DA0(L),SA(10),CA(10))
CALL ANG0(EA(L,N),EA0(L),SA(11),CA(11))
CALL ANG0(PSA(L,N),PSA0(L),SA(12),CA(12))
CALL ANG0(GAA(L,N),GAA0(L),SA(13),CA(13))
CALL ANGO(DB(L,N), DBO(L), SA(14), CA(14))
CALL ANGO(EB(L,N), EBO(L), SA(15), CA(15))
CALL ANGO(PSB(L,N), PSB0(L), SA(16), CA(16))
CALL ANGO(GAB(L,N), GAB0(L), SA(17), CA(17))
C
GO TO 770
C
CALL ANGO(EB(L,N), EBO(L), SA(15), CA(15))
CALL ANGO(PSB(L,N), PSB0(L), SA(16), CA(16))
CALL ANGO(GAB(L,N), GAB0(L), SA(17), CA(17))
C
GO TO 770
C
760 CALL CSANG(PH(L,N), PHO(L), SA(1), CA(1))
CALL CSANG(TH(L,N), THO(L), SA(2), CA(2))
CALL CSANG(PS(L,N), PSO(L), SA(3), CA(3))
CALL CSANG(GA(L,N), GAO(L), SA(4), CA(4))
CALL CSANG(PSP(L,N), PSP0(L), SA(5), CA(5))
CALL CSANG(EJK(L,N), EJK0(L), SA(6), CA(6))
CALL CSANG(GAL(L,N), GAL0(L), SA(7), CA(7))
CALL CSANG(THC(L,N), THC0(L), SA(8), CA(8))
CALL CSANG(PSC(L,N), PSC0(L), SA(9), CA(9))
CALL CSANG(DA(L,N), DAO(L), SA(10), CA(10))
CALL CSANG(EB(L,N), EBO(L), SA(15), CA(15))
CALL CSANG(PSB(L,N), PSB0(L), SA(16), CA(16))
CALL CSANG(GAB(L,N), GAB0(L), SA(17), CA(17))
C
770 CONTINUE
C
WRITE(11,75)T, PH(L,N)/2.0/PI, TH(L,N)*RTD, PS(L,N)/2.0/PI
75 FORMAT(4F8.3)
C
750 CONTINUE
C
C*******************************************************************
C
DA HAS A SINGULARITY AT EA=PI/2
REPLACE EACH ESTIMATE OF DA BY THE 4TH ESTIMATE WHENEVER
THE DIFFERENCE EXCEEDS PI RADIANS
C
DO 772 L=1,3
IF(ABS(DA(L,N)-DA(4,N)).LT.PI) GO TO 772
PRINT *, 'DA SINGULARITY AT T=', T, ' AND L=', L
DA(L,N)=(DA(L,N)+DA(4,N))/2.0
DA0(L)=DA(L,N)
IF(ABS(DA(L,N)-DA(4,N)).GT.PI) GO TO 771
772 CONTINUE
402

0862 C
0863 C*******************************************************************
0864 C
0865 C DB HAS A SINGULARITY AT EB=PI/2
0866 C REPLACE EACH ESTIMATE OF DB BY THE 4TH ESTIMATE WHENEVER
0867 C THE DIFFERENCE EXCEEDS PI RADIANS
0868 C
0869 C*******************************************************************
0870 C
0871 DO 774 L = 1, 3
0872 C
0873 IF(ABS(DB(L, N) - DB(4, N)).LT.PI) GO TO 774
0874 PRINT *, 'DB SINGULARITY AT T = ', T, ' AND L = ', L
0875 773 DB(L, N) = (DB(L, N) + DB(4, N))/2.0
0876 DB0(L) = DB(L, N)
0877 IF(ABS(DB(L, N) - DB(4, N)).GT.PI) GO TO 773
0878 774 CONTINUE
0879 C
0880 C*******************************************************************
0881 C
0882 C PSP HAS A SINGULARITY AT GA=PI
0883 C REPLACE EACH ESTIMATE OF PSP BY THE 4TH ESTIMATE WHENEVER
0884 C THE DIFFERENCE EXCEEDS PI RADIANS
0885 C
0886 C*******************************************************************
0887 C
0888 DO 776 L = 1, 3
0889 C
0890 IF(ABS(PSP(L, N) - PSP(4, N)).LT.PI) GO TO 776
0891 PRINT *, 'PSP SINGULARITY AT T = ', T, ' AND L = ', L
0892 775 PSP(L, N) = (PSP(L, N) + PSP(4, N))/2.0
0893 PSP0(L) = PSP(L, N)
0894 IF(ABS(PSP(L, N) - PSP(4, N)).GT.PI) GO TO 775
0895 776 CONTINUE
0896 C
0897 C*******************************************************************
0898 C
0899 C PSA HAS A SINGULARITY AT GAA=PI
0900 C REPLACE EACH ESTIMATE OF PSA BY THE 4TH ESTIMATE WHENEVER
0901 C THE DIFFERENCE EXCEEDS PI RADIANS
0902 C
0903 C*******************************************************************
0904 C
0905 DO 778 L = 1, 3
0906 C
0907 IF(ABS(PSA(L, N) - PSA(4, N)).LT.PI) GO TO 778
0908 PRINT *, 'PSA SINGULARITY AT T = ', T, ' AND L = ', L
0909 777 PSA(L, N) = (PSA(L, N) + PSA(4, N))/2.0
0910 PSA0(L) = PSA(L, N)
0911 IF(ABS(PSA(L, N) - PSA(4, N)).GT.PI) GO TO 777
0912 778 CONTINUE
PSB HAS A SINGULARITY AT GAB=PI
REPLACE EACH ESTIMATE OF PSB BY THE 4TH ESTIMATE WHENEVER
THE DIFFERENCE EXCEEDS PI RADIANS

DO 780 L=1,3
    IF(ABS(PSB(L,N)-PSB(4,N)).LT.PI) GO TO 780
    PRINT *, 'PSB SINGULARITY AT T=', T, ' AND L=', L
    779 PSB(L,N)=(PSB(L,N)+PSB(4,N))/2.0
    PSBO(L)=PSB(L, N)
    IF(ABS(PSB(L,N)-PSB(4,N)).GT.PI) GO TO 779
  780 CONTINUE

Tv(N)--T
700 CONTINUE

PRINT ORIENTATION ANGLES

DO 810, N=1,101,10
  PRINT 804, TV(N), PH(1, N)/2.0/Pi, TH(1, N)*RTD, PS(1, N)/2.0/Pi
810 CONTINUE

PRINT

PRINT *, 'INTERNAL ORIENTATION ANGLES'

DO 820, N=1,101,10
  PRINT 805, TV(N), GA(1, N)*RTD, PSP(1, N)*RTD, FJK(1, N)*RTD,
          GAL (1, N)*RTD, THC(1, N)*RTD, PSC(1, N)*RTD
820 CONTINUE
0963 C
0964 PRINT *,
0965 PRINT *,
0966 PRINT *, ' T DA EA PSA GAA DB EB',
0967 ' PSB GAB'
0968 PRINT *,
0969 C
0970 DO 830, N=1,101,10
0971 PRINT 805, TV(N),
0972 *DA(1,N)*RTD, EA(1,N)*RTD, PSA(1,N)*RTD, GAA(1,N)*RTD,
0973 *DB(1,N)*RTD, EB(1,N)*RTD, PSB(1,N)*RTD, GAB(1,N)*RTD
0974 830 CONTINUE
0975 C
0976 804 FORMAT(2F7.2,F7.0,F7.2)
0977 805 FORMAT(F7.2,F7.0)
0978 C
0979 C*******************************************************************
0980 C
0981 C SPLIN4 FITS A QUINTIC SPLINE TO THE AVERAGE OF THE 4
0982 C ESTIMATES
0983 C
0984 C*******************************************************************
0985 C
0986 NV=101
0987 C
0988 CALL SPLIN4(CCPH, SDPH, NV, TV, PH)
0989 CALL SPLIN4(CCTH, SDTH, NV, TV, TH)
0990 CALL SPLIN4(CCPS, SDPS, NV, TV, PS)
0991 C
0992 CALL SPLIN4(CCPS, SDPS, NV, TV, PS)
0993 CALL SPLIN4(CCPSP, SDPSP, NV, TV, PSP)
0994 CALL SPLIN4(CCEJK, SDEJK, NV, TV, EJK)
0995 CALL SPLIN4(CCGAL, SDGAL, NV, TV, GAL)
0996 C
0997 CALL SPLIN4(CGTH, STHC, NV, TV, THC)
0998 CALL SPLIN4(CPPSC, SDPSC, NV, TV, PSC)
0999 C
1000 CALL SPLIN4(CCDA, SDDA, NV, TV, DA)
1001 CALL SPLIN4(CCEA, SDEA, NV, TV, EA)
1002 CALL SPLIN4(CCPSA, SDPSA, NV, TV, PSA)
1003 CALL SPLIN4(CCGAA, SDGAA, NV, TV, GAA)
1004 C
1005 CALL SPLIN4(CCD, SDD, NV, TV, Db)
1006 CALL SPLIN4(CCEB, SDEB, NV, TV, EB)
1007 CALL SPLIN4(CCPSB, SDPSB, NV, TV, PSB)
1008 CALL SPLIN4(CCGAB, SDGAB, NV, TV, GAB)
1009 C
1010 C*******************************************************************
1011 C
1012 C PRINT STANDARD ERRORS OF ORIENTATION ANGLES
1013 C
1014 C*******************************************************************
1015 C
PRINT *, 'STANDARD ERRORS OF ORIENTATION ANGLES IN DEGREES'
PRINT *, ' PH TH PS'
PRINT 815, SDPH, SDTH, SDPS
PRINT *, ' GA PSP EJK GAL THC PSC'
PRINT 815, SDGA, SDPSP, SDEJK, SDGAL, SDTHC, SDPSC
PRINT *, ' DA EA PSA GAA DB EB PSR'
SDANG = SDPFH + SDTH + SDPS
SDANG = SDANG + SOGA + SDPSP + SDEJK
SDANG = SDANG + SDGAL + SDTHC + SDPSC
SDANG = SDANG + SDDA + SDEA + SDPSA + SDGAA
SDANG = SDANG + SDDB + SDEB + SDPSB + SDGAB
SDANG = SDANG / 17.0
PRINT 816, SDANG
PRINT 816 FORMAT('AVERAGE STANDARD ERROR =', F3.1, ' DEGREES')
C
C**** ***************************************************************
C WRITE THE SPLINE COEFFICIENTS OF THE ORIENTATION ANGLES TO
C THE OUTPUT FILE
C**** ***************************************************************
CALL WRITE(CCPH, 10)
CALL WRITE(CCM, 10)
CALL WRITE(CCPS, 10)
CALL WRITE(CCGA, 10)
CALL WRITE(CCPSP, 10)
CALL WRITE(CCFJ, 10)
CALL WRITE(CCGAL, 10)
CALL WRITE(CCTHC, 10)
CALL WRITE(CCPSC, 10)
CALL WRITE(CCDA, 10)
CALL WRITE(CCPSP, 10)
CALL WRITE(CCEJK, 10)
CALL WRITE(CCGAL, 10)
CALL WRITE(CCTHC, 10)
CALL WRITE(CCPS, 10)
CALL WRITE(CCM, 10)
CALL WRITE(CCPH, 10)
CALL WRITE(CCGA, 10)
CLOSE ALL FILES

DO 900 J=5,12
CLOSE(J)
900 CONTINUE
CALL EXIT
END

END OF MAIN SEGMENT

SUBROUTINE WRITE(C,U) WRITES THE SPLINE COEFFICIENTS ON FILE UNIT U

SUBROUTINE WRITE(C,U)
PARAMETER (NN=120)
DOUBLE PRECISION C(6,NN)
INTEGER U,I,J
WRITE(U,11)((C(I, J), I=1,6), J=1,NN)
11 FORMAT(4D20.7)
RETURN
END

SUBROUTINE REF(SF, OR, CP, TOO, N, U) READS REFERENCE DATA FROM FILE UNIT U AND CALCULATES SCALE FACTORS SF(1), SF(2) REFERENCE ORIGIN (OR(1),OR(2)) CONTROL POINT (CP(1),CP(2)) REFERENCE TIME TOO

SUBROUTINE REF(SF, OR, CP, TOO, N, U)
DOUBLE PRECISION SF(2), OR, CP, TOO, N, U
INTEGER I, J, N, U
SUBROUTINE REF(SF, OR, CP, TOO, N, U)
DO 10 J=1,4
DO 20 I=1,14
READ(U,*)XA(J,I),ZA(J,I)
20 CONTINUE
READ(U,*)TA(J)
10 CONTINUE
DO 30 J=1,4
XA(J,15)=(XA(J,1)+XA(J,7)+XA(J,13))/3.0
ZA(J,15)=(ZA(J,1)+ZA(J,7)+ZA(J,13))/3.0
XA(J,16)=(XA(J,2)+XA(J,8)+XA(J,14))/3.0
ZA(J,16)=(ZA(J,2)+ZA(J,8)+ZA(J,14))/3.0
30 CONTINUE
DO 40 I=15,20
XA(5,I)=(XA(1,I)+XA(2,I)+XA(3,I)+XA(4,I))/4.0
ZA(5,I)=(ZA(1,I)+ZA(2,I)+ZA(3,I)+ZA(4,I))/4.0
40 CONTINUE
TA0=(TA(1)+TA(2)+TA(3)+TA(4))/4.0
SFX=(XA(5,17)-XA(5,19))/2.0
SFZ=(ZA(5,18)-ZA(5,20))/2.0
XOR=(XA(5,17)+XA(5,19))/2.0
ZOR=(ZA(5,18)+ZA(5,20))/2.0
XCO=(XA(5,15)+XA(5,16))/2.0
ZCO=(ZA(5,15)+ZA(5,16))/2.0
XCO=XCO-XOR
ZCO=ZCO-ZOR
SF(1)=SFX
SF(2)=SFZ
OR(1)=XOR
OR(2)=ZOR
CP(1)=XCO
CP(2)=ZCO
RETURN
END
COORD(R,D) OBTAINS AN ESTIMATE OF THE POSITION VECTOR R USING THE DIGITISED DATA D

R(3) HAS COMPONENTS X,Y,Z

D(4) HAS COMPONENTS XF,ZF,YS,ZS

SUBROUTINE COORD(R,D)
DOUBLE PRECISION D(4), OF(3), OS(3), OA(3), OB(3), FA(3), SB(3),
* FS(3), FP(3), SQ(3), OP(3), OQ(3), R(3),
* XF, YS, ZS, ZF, C1,C2, C3, D1, D2, D3, P, Q, DF, DS,
* KXF, KZF, EXP, EZF, SFI(3,3), KYS, KZS, EYS, EZS, SSI(3,3)

EXTERNAL DIF, DOT, MID, PM331, SCAVEC, SUM, V3

COMMON/B6/DF, KXF, KZF, EXP, EZF, SFI

COMMON/B7/DS, KYS, KZS, EYS, EZS, SSI

XF=D(1)
ZF=D(2)
YS=D(3)
ZS=D(4)

CORRECT COORDINATES FOR CAMERA TILT

XF=XF*KXF+EXP
ZF=ZF*KZF+EZF
CALL V3(OA, XF, 0.0D0, ZF)
CALL PM331(OA, SFI, OA)

YS=YS*KYS+EYS
ZS=ZS*KZS+EZS
CALL V3(OB, 0.0D0, YS, ZS)
CALL PM331(OB, SSI, OB)

DETERMINE MIDPOINT R OF THE COMMON PERPENDICULAR PQ TO THE RAYS FA AND SB FROM FRONT CAMERA F AND SIDE CAMERA S
CALL V3(OF, θ, OD0, -DF, θ, OD0)
CALL V3(OS, DS, θ, OD0, θ, OD0)

CALL DIF(FA, OF, OA)
CALL DIF(SB, OS, OB)
CALL DIF(FS, OF, OS)

CALL DOT(C1, FA, FA)
CALL DOT(C2, FA, SB)
CALL DOT(C3, FA, FS)

CALL DOT(D1, SB, FA)
CALL DOT(D2, SB, SB)
CALL DDT(D3, SB, FS)

P=(C3*D2-C2*D3)/(C1*D2-C2*D1)
Q=(C3*D1-C1*D3)/(C1*D2-C2*D1)

CALL SCAVEC(FP, P, FA)
CALL SCAVEC(SQ, Q, SB)

CALL SUM(OP, OF, FP)
CALL SUM(OQ, OS, SQ)
CALL MID(R, OP, OQ)

RETURN
END

C*******************************************************************

SUBROUTINE ORANG USES THE POSITION VECTORS W, EA, S, H, Y,
V, OB, R, I, Z OF THE TEN LANDMARKS TO EVALUATE THE SINES
AND COSINES OF THE ORIENTATION ANGLES

LANDMARKS:
W, V: LEFT AND RIGHT WRISTS
OA, OB: ELBOWS
S, R: SHOULDERS
H, I: HIPS
Y, Z: ANKLES

ORIENTATION ANGLES:
PH, TH, PS
GA, PSP, EJK, GAL
THC, PSC
DA, EA, PSA, GAA
DB, EB, PSB, GAB

C*******************************************************************
SUBROUTINE ORANG(SA, CA, PP)

DOUBLE PRECISION

*SA(17), CA(17),


*RZP, RZJ1, RZT2, RZTC, RZA1, RZA2,

*MP, MJ1, MJ2, MTC, MA1, MA2,

*DGX, LP2, DXN, LJ, LJ1, LJ2, DGN, RLJ1, RLK1, RQ1, RQ2,

*RZK1, RZK2, RZF1, RZF2,

*MPC, MJ, MK, MK1, MK2, ML, MB1, MB2, MA, MB, MAB, MU, MF,


*P(3), TC(3), PC(3), J(3), K(3), L(3), U(3), AB(3), F(3),

*J1(3), J2(3), K1(3), K2(3),

*X(3), G(3), GN(3), QG(3), G(3), N3(3), Q1(3), Q2(3),


*DYX, DIZ, DJI, DJJ, EJH,

*DYZ, DGX, DGQ, DXN, DOX, RGO, ROX, ON(3),

*F1(3), F2(3), F3(3),

*P1(3), P2(3), P3(3),

*L2(3), L3(3),

*T1(3), T2(3), T3(3),

*C1(3), C2(3), C3(3),

*A11(3), A12(3), A13(3), A23(3),

*B11(3), B12(3), B13(3), B23(3),

*PL(3), VC(3),

*VA1(3), VA2(3), VA3(3),

*VB1(3), VB2(3), VB3(3),

*PI, PI2,

*STH, CTH, SPH, CPH, SPS, CPS,

*SGA, GGA, SPSP, CPSP, SEJK, CFJK, SGAL, CGAL,

*GA, GA0, GG, AL, ET, EE,

*CGG, SAL, SBT, SEE, S2PSP, C2PSP,

*ATV, SPSP0, CPSP0,

*STHC, CTHC, SPSC, CPS,

*SEA, CEA, SDA, CDA, CPSA, CPSSA, SGAA, CGAA,

*SEB, CEB, SDB, CDB, CPSB, CPSSB, SGAB, CGAB

INTEGER JJ

EXTERNAL CEN, DIF, DOT, MAG, MID, RAT, SCAVEC, SUM, UNIV, VECF, VP

INTRINSIC ABS, ACOS, ASIN, COS, MAX, MIN, SIN, SQRT

COMMON/B3/DGX, L2P, DXN, LJ, LJ1

COMMON/B4/RZP, RZJ1, RZJ2, RZTC, RZA1, RZA2

COMMON/B5/MP, MJ1, MJ2, MTC, MA1, MA2

COMMON/B31/F
DO 10 JJ=1,3
W(JJ)=PP(JJ,1)
Q(JJ)=PP(JJ,2)
S(JJ)=PP(JJ,3)
H(JJ)=PP(JJ,4)
Y(JJ)=PP(JJ,5)
V(JJ)=PP(JJ,6)
O(JJ)=PP(JJ,7)
R(JJ)=PP(JJ,8)
I(JJ)=PP(JJ,9)
Z(JJ)=PP(JJ,10)

10 CONTINUE

RZK1=RZJ1
RZK2=RZJ2
RZB1=RZA1
RZB2=RZA2
MK1=MJ1
MK2=MJ2
MB1=MA1
MB2=MA2

G AND N ARE THE MIDPOINTS OF HIP CENTRES H, I AND SHOULDER CENTRES S, R

CALL MID(G,H,I)
CALL MID(N,S,R)

RECONSTRUCT POSITIONS OF KNEE CENTRES AND THEIR MID-POINT Q USING AN ESTIMATE OF THE KNEE ANGLE GAL CALCULATED FROM THE HIP-ANKLE DISTANCE
CALL DIF(HY, H, Y)
CALL MAG(DHY, HY)
CALL DIF(IZ, I, Z)
CALL MAG(DIZ, IZ)
DDJ=(DHY+DIZ)/2.0
DDJ=DDJ+0.05
LJ2=LJ-LJ1
CGAL=(LJ1**2+LJ2**2-DDJ**2)/(2.0*LJ1*LJ2)
CGAL=MAX(CGAL,-1.0D0)
SGAL=SQRT(1.0-CGAL**2)
EEJ=LJ1*LJ2*SGAL/DDJ
RLJ1=LJ1/LJ
RLK1=RLJ1
CALL RAT(QJ, H, Y, RLJ1)
CALL RAT(OK, I, Z, RLK1)
CALL MID(OL, OJ, OK)
CALL DIF(L3, OL, G)
CALL DIF(IH, I, H)
CALL VP(L2, L3, IH)
CALL UNIV(L2, L2)
CALL DIF(P3, Q, N)
CALL UNIV(P3, P3)
CALL DIF(IH, I, H)
CALL VP(P2, P3, IH)
CALL UNIV(P2, P2)
CALL VP(P1, P2, P3)

C*******************************************************************
C
C DIRECTION VECTOR L3 GIVES THE ORIENTATION OF THE MID-LINE OF
C THE UPPER LEGS
C
C*******************************************************************
C
CALL DIF(L3, Q, G)
CALL UNIV(L3, L3)
CALL DIF(P3, Q, N)
CALL UNIV(P3, P3)
CALL DIF(IH, I, H)
CALL VP(P2, P3, IH)
CALL UNIV(P2, P2)
CALL VP(P1, P2, P3)
**C**

**EVALUATE DIRECTION VECTOR VL IN REFERENCE FRAME P**

**C**

**PSP = HULA ANGLE**

**C**

**ANGLE BETWEEN NORMAL TO PIKE PLANE AND AXIS P1 WHICH IS**

**APPROXIMATELY PARALLEL TO LINE OF HIP CENTRES**

**C**

CALL VECF(VL, L3, P1, P2, P3)

**C**

\( sBT0 = \sqrt{VL(1)^2 + VL(2)^2} \)

\( SPS0 = VL(1)/sBT0 \)

\( CPS0 = VL(2)/sBT0 \)

**C**

**C**

**CALCULATE PIKE ANGLE GA ITERATIVELY USING THE ABOVE**

**APPROXIMATIONS OF SPSP AND CPSP**

**C**

CALL DIF(N3, N, G)

CALL UNIV(N3, N3)

CALL DOT(C3G, L3, N3)

\( G3 = \arccos(C3G) \)

\( G0 = G3 \)

**C**

\( GP = GP0 \)

\( SPSP = SPSP0 \)

**C**

**C**

\( PI = 3.1415926536 \)

\( PI2 = PI/2.0 \)

**C**

**C**

\( GA = GA0 \)

\( S2PSP = SPSP^2 \)

\( C2PSP = CPSP^2 \)

**C**

\( AL = 0.50 \times (PI-\Delta) \times S2PSP \)

**IF**(GA.LT.PI2) AL = AL + 0.50*(PI2-\Delta) \times C2PSP

**IF** (CPSP.LT.0.01D0) AL = 0.50*(PI-\Delta)

**C**

**HT = PI-\Delta-AL**

**C**

**CALL DIF(GN, G, N)**

**CALL MAG(DGN, GN)**

\( SEE = \sin(\Delta) \times DXN/DGN \)

\( EE = \arcsin(SEE) \)

**C**

\( GN = GA*GG/(PI-\Delta-EE) \)

**C**

**IF** (ABS(GA-GA0).GT.0.01D0) GO TO 50
LOCATE POINT O AT INTERSECTION OF THORAX AND LEG AXES
LOCATE POINT X AT INTERSECTION OF PELVIS AND THORAX

SGA=SIN(GA)
CGA=COS(GA)
DGO=DGX*SAL/SGA
CALL DIF(GQ,G,Q)
CALL MAG(DGQ,GQ)
RGO=DGO/DGQ
CALL RAT(O,G,Q,-RGO)
SBT=SIN(BT)
DOX=DGX*SBT/SGA
CALL DIF(ON,O,N)
CALL MAG(DON,ON)
ROX=DOX/DON
CALL RAT(X,O,N,ROX)

RE-EVALUATE P1,P2,P3 AND ANGLE PSP

CALL DIF(GX,G,X)
CALL UNIV(P3,GX)
CALL VP(P2,P3,PH)
CALL UNIV(P2,P2)
CALL VP(P1,P2,P3)
CALL VECF(VL,L3,P1,P2,P3)
SBT0=SQRT(VL(1)**2+VL(2)**2)
SPSP0=VL(1)/SBT0
CPSP0=VL(2)/SBT0
IF(ABS(SPSP-SPSP0).GT.0.01D0) GO TO 40
IF(ABS(CPSP-CPSP0).GT.0.01D0) GO TO 40

EVALUATE DIRECTION VECTORS F1,F2,F3 FOR FRAME F

POINTS Q1 ON XN AND Q2 ON GL DEFINE F3
RQ1=1.0
RQ2=RQ1
CALL RAT(Q1,X,N,RQ1)
CALL RAT(Q2,G,Q,RQ2)
CALL DIF(F3,Q2,Q1)
CALL UNIV(F3,F3)
CALL VP(F2,F3,P1)
CALL UNIV(F2,F2)
CALL VP(F1,F2,F3)

CALL DIF(T3,X,N)
CALL UNIV(T3,T3)
CALL VP(T2,T3,P1)
CALL VP(T1,T2,T3)

CALL DIF(C1,R,S)
CALL UNIV(C1,C1)
CALL VP(C3,C1,T2)
CALL UNIV(C3,C3)
CALL VP(C2,C3,C1)

CALL DIF(A13,OA,S)
CALL UNIV(A13,A13)
CALL DIF(A23,W,OA)
CALL UNIV(A23,A23)
CALL VP(A12,A13,A23)
CALL UNIV(A12,A12)
CALL VP(A11,A12,A13)
EVALUATE DIRECTION VECTORS B11, B12, B13, B23 FOR RIGHT ARM B

CALL DIF(B13, OB, R)
CALL UNIV(B13, B13)
CALL DIF(B23, V, OB)
CALL UNIV(B23, B23)

CALL VP(B12, B23, B13)
CALL UNIV(B12, B12)
CALL VP(B11, B12, B13)

USING THE DIRECTION VECTORS CALCULATE THE SINE AND COSINE OF EACH ORIENTATION ANGLE

THE ORIENTATION OF THE SYSTEM IS DEFINED BY ANGLES PHI, THETA AND PSI WHICH ARE THE AMOUNTS OF SOMERSAULT, TILT AND TWIST

ST8=F3(1)
CTH=SQR(1.0-SIH**2)
SPH=F3(2)/CTH
CPH=F3(3)/CTH
SPS=F2(1)/CTH
CPS=F1(1)/CTH

THE ORIENTATIONS OF THE LEG FRAME L AND THE TORSO FRAME T RELATIVE TO THE PELVIC FRAME P ARE DEFINED BY THE PIKE ANGLE GA AND THE HULA ANGLE PSP. THESE ANGLES HAVE BEEN EVALUATED

THE ORIENTATION OF THE PELVIC FRAME P RELATIVE TO THE SYSTEM FRAME P WILL BE CALCULATED IN THE SIMULATION PROGRAM SIM USING ANGLES GA AND PSP
THE ORIENTATIONS OF THE LEGS J AND K RELATIVE TO FRAME L ARE DEFINED BY THE ABDUCTION ANGLE EJK.


THE ORIENTATION OF THE LEFT LOWER-ARM A2 IS DEFINED BY THE ELBOW ANGLE GAA.

CALL DIF(YZ,Y,Z)
CALL MAG(DYZ,YZ)
DYZ=MAX(DYZ,0.10DO)
SEJK=(0.5*DYZ-L2P)/0.5/(DHY+DIZ)
CEJK=SQR(1.0-SEJK**2)

CALL VECF(VC,C1,T1,T2,T3)
SPSC=VC(2)
CPSC=SQR(1.0-SPSC**2)
STHC=VC(3)/CPSC
CTHC=VC(1)/CPSC

CALL DIF(VA1,A11,C1,C2,C3)
CALL VECF(VA2,A12,C1,C2,C3)
CALL VECF(VA3,A13,C1,C2,C3)
SEA=VA3(1)
CEA=SQR(VA3(2)**2+VA3(3)**2)
SDA=VA3(2)/CEA
CDA=VA3(3)/CEA

SPSA=VA2(1)/CEA
CPSA=VA1(1)/CEA

CALL DOT(CGAA,A13,A23)
CGAA=CGAA
SGAA=SQR(1.0-CGAA**2)
IF(CGAA.LT.-0.7D0) SPSA=ABS(SPSA)
THE ORIENTATION OF THE RIGHT UPPER-ARM B1 RELATIVE TO THE CHEST C IS DEFINED BY SUCCESSIVE ROTATIONS THROUGH ANGLES -DB, EB, -PSB ABOUT AXES B11, B12, B13

THE ORIENTATION OF THE RIGHT LOWER-ARM B2 IS DEFINED BY THE ELBOW ANGLE GAB

CALL VECF(VB1, B11, C1, C2, C3)
CALL VECF(VB2, B12, C1, C2, C3)
CALL VECF(VB3, B13, C1, C2, C3)

SDB=VB3(2)/CEB
CDB=VB3(3)/CEB

SPSB=VB2(1)/CEB
CPSB=VB1(1)/CEB

CALL DOT(CGAB, B13, B23)
CGAB=-CGAB
SGAB=SQRT(1.0-CGAB**2)
IF(CGAB.LT.-0.7D0) SPSB=ABS(SPSB)

SA CONTAINS THE SINES OF THE ORIENTATION ANGLES

SA(1)=SPH
SA(2)=STH
SA(3)=SPS
SA(4)=SGA
SA(5)=SPSP
SA(6)=SEJK
SA(7)=SGAL
SA(8)=STHC
SA(9)=SPSC
SA(10)=SDA
SA(11)=SEA
SA(12)=SPSA
SA(13)=SGAA
**CA CONTAINS THE COSINES OF THE ORIENTATION ANGLES**

CA CONTAINS THE COSINES OF THE ORIENTATION ANGLES

### CA(1) - CA(17)

- CA(1) = CPH
- CA(2) = CTH
- CA(3) = CPS
- CA(4) = CGA
- CA(5) = CPS
- CA(6) = CEJK
- CA(7) = CGAL
- CA(8) = CTHC
- CA(9) = CPSC
- CA(10) = CDA
- CA(11) = CEA
- CA(12) = CPS
- CA(13) = CGAA
- CA(14) = CDB
- CA(15) = CEB
- CA(16) = CPS
- CA(17) = CGAB

**DETERMINE LOCATIONS OF CENTROIDS J1, J2, K1, K2, P, TC A1, A2, B1, B2**

- CALL RAT(J1, H, OJ, RZJ1)
- CALL RAT(J2, OJ, Y, RZJ2)
- CALL RAT(K1, I, OK, RZK1)
- CALL RAT(K2, OK, Z, RZK2)
- CALL RAT(P, G, X, RZP)
- CALL RAT(TC, X, N, RZTC)
- CALL RAT(A1, S, OA, RZA1)
- CALL RAT(A2, OA, W, RZA2)
- CALL RAT(B1, R, OB, RZB1)
- CALL RAT(B2, OB, V, RZB2)
DETERMINE LOCATIONS OF CENTROIDS L, PC, A, B, U, F

CALL CEN(J, MJ, J1, MJ1, J2, MJ2)
CALL CEN(K, MK, K1, MK1, K2, MK2)
CALL CEN(L, ML, J, MJ, K, MK)
CALL CEN(PC, MPC, P, MP, TC, MTC)
CALL CEN(A, MA, A1, MA1, A2, MA2)
CALL CEN(B, MB, B1, MB1, B2, MB2)
CALL CEN(AB, MAB, A, MA, B, MB)
CALL CEN(U, MJ, AB, MAB, PC, MPC)
CALL CEN(F, MF, U, MJ, L, ML)

RETURN
END

RM1 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE X-AXIS WHERE C=COS(A) AND S=SIN(A)

SUBROUTINE RM1(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1)=1.0
R(1,2)=0.0
R(1,3)=0.0
R(2,1)=0.0
R(2,2)=C
R(2,3)=S
R(3,1)=S
R(3,2)=C
R(3,3)=C
RETURN
END
RM2 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE Y-AXIS WHERE C = COS(A) AND S = SIN(A)

SUBROUTINE RM2(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = C
R(1,2) = 0.0
R(1,3) = -S
R(2,1) = 0.0
R(2,2) = 1.0
R(2,3) = 0.0
R(3,1) = S
R(3,2) = 0.0
R(3,3) = C
RETURN
END

RM3 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE Z-AXIS WHERE C = COS(A) AND S = SIN(A)

SUBROUTINE RM3(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = C
R(1,2) = S
R(1,3) = 0.0
R(2,1) = -S
R(2,2) = C
R(2,3) = 0.0
R(3,1) = 0.0
R(3,2) = 0.0
R(3,3) = 1.0
RETURN
END
C *******************************************************************

PM33 CALCULATES THE PRODUCT C OF THE 3 X 3 MATRICES A AND B

C*******************************************************************

SUBROUTINE PM33(C, A, B)
DOUBLE PRECISION A(3,3), B(3,3), C(3,3), D(3,3), SUM
INTEGER I, J, K
DO 10 I=1, 3
DO 20 J=1, 3
SUM=0.0
DO 30 K=1, 3
30 SUM=SUM+A(I, K)*B(K, J)
20 D(I, J)=SUM
10 CONTINUE
DO 40 I=1, 3
40 C(I, J)=D(I, J)
RETURN
END

C*******************************************************************

PM331 CALCULATES THE PRODUCT C OF THE 3 X 3 MATRIX A AND
THE 3 X 1 MATRIX B

C********************************************************************************

SUBROUTINE PM331(C, A, B)
DOUBLE PRECISION A(3,3), B(3), C(3), D(3), SUM
INTEGER I, K
DO 10 I=1, 3
SUM=0.0
DO 30 K=1, 3
30 SUM=SUM+A(I, K)*B(K)
10 CONTINUE
DO 40 I=1, 3
40 C(I)=D(I)
RETURN
END

C********************************************************************************

DOT(D, A, B) CALCULATES THE SCALAR PRODUCT OF TWO VECTORS A, B

C********************************************************************************

SUBROUTINE DOT(D, A, B)
DOUBLE PRECISION A(3), B(3), D
D=A(1)*B(1)+A(2)*B(2)+A(3)*B(3)
RETURN
END
VP calculates the vector product $C$ of vectors $A$ and $B$

```
SUBROUTINE VP(C, A, B)
  DOUBLE PRECISION A(3), B(3), C(3), D(3)
  INTEGER J
  D(1) = A(2) * B(3) - A(3) * B(2)
  D(2) = A(3) * B(1) - A(1) * B(3)
  D(3) = A(1) * B(2) - A(2) * B(1)
  DO 10 J = 1, 3
  C(J) = D(J)
10  RETURN
END
```

V3 calculates the vector $V$ with components $A, B, C$

```
SUBROUTINE V3(V, A, B, C)
  DOUBLE PRECISION V(3), A, B, C
  V(1) = A
  V(2) = B
  V(3) = C
  RETURN
END
```

SCAVEC($VS, S, V$) determines the product $VS$ of a scalar $S$ and a vector $V$

```
SUBROUTINE SCAVEC(VS, S, V)
  DOUBLE PRECISION VS(3), V(3), S
  VS(1) = S * V(1)
  VS(2) = S * V(2)
  VS(3) = S * V(3)
  RETURN
END
```
C******************************************************************************
C CEDT(AB, MAB, A, rA, B, r'B) CALCULATES THE CENTROID AB AND MASS
C MB OF TWO SEGMENTS WITH CENTROIDS A, B AND MASSES MA, MB
C******************************************************************************

SUBROUTINE CES (AB, M'AB, A, MA, B, MB)
DOUBLE PRECISION AB(3), A(3), B(3), A*11(3), BM2(3), MAB, hMA, r'B, M1, M2
EXTERNAL SCAVEC, SUM

ZA+MB
M1=MA/MAB
M2=MB/MAB

CALL SCAVEC(AM1, M1, A)
CALL SCAVEC(BM2, M2, B)
CALL SUM(AB, AM1, BM2)
RETURN
END

C******************************************************************************
C SANG CALCULATES ANGLE A FROM ITS COSINE CA USING THE
C PREVIOUS VALUE AO AND THE SIGN OF SIN(A)
C******************************************************************************

SUBROUTINE CSANG(A, A0, SA, CA)
DOUBLE PRECISION A, SA, CA, A@, SAO, CAO, B, SB, CB
INTRINSIC ACOS, COS, SIN, SIGN, SQRT

IF(1.0-CA**2.GE.0.0D0) SA=SIGN(SQRT(1.0-CA**2), SA)
CAO=COS (AO )
SAO=SIN(AO)
CB= A*CAO+SA*, 13AO
SB=SA*CA0-CA*SAO
IF(CB. GT. 1.0D0) CB=1.0D0
B=ACOS(CB)
B=SIGN (B, SB)
A=AO+B
A6 =A
RETURN
END
ANGØ CALCULATES ANGLE A FROM ITS SINE AND COSINE AND SETS THE INITIAL VALUE A FOR USE IN CSANG

```
SUBROUTINE ANGO(A, A0, SA, CA)
  DOUBLE PRECISION A, A0, TA, SA, CA, PI
  INTRINSIC ATAN
  PI=3.1415926536
  TA=SA/CA
  IF(CA.LT.0.0D0) GO TO 10
  A=ATAN(TA)
  GO TO 20
  A=ATAN(TA)+PI
  A0=A
  RETURN
END
```

V4 CALCULATES THE 'VECTOR' V WITH COMPONENTS A, B, C, D

```
SUBROUTINE V4(V, A, B, C, D)
  DOUBLE PRECISION V(4), A, B, C, D
  V(1)=A
  V(2)=B
  V(3)=C
  V(4)=D
  RETURN
END
```

DIF (AB, A, B) DETERMINES THE VECTOR AB=B-A FROM THE VECTORS A, B

```
SUBROUTINE DIF(AB, A, B)
  DOUBLE PRECISION AB(3), A(3), B(3)
  AB(1)=B(1)-A(1)
  AB(2)=B(2)-A(2)
  AB(3)=B(3)-A(3)
  RETURN
END
```
SUM(C,A,B) determines the sum C of vectors A and B

SUBROUTINE SUM(C,A,B)
DOUBLE PRECISION A(3), B(3), C(3)

C(1) = A(1) + B(1)
C(2) = A(2) + B(2)
C(3) = A(3) + B(3)
RETURN
END

MAG(M,V) calculates the magnitude M of vector V

SUBROUTINE MAG(M,V)
DOUBLE PRECISION M, V(3)

EXTERNAL SQRT
M = SQRT(V(1)**2 + V(2)**2 + V(3)**2)
RETURN
END

UNIV(U,V) defines a unit vector U parallel to vector V

SUBROUTINE UNIV(U,V)
DOUBLE PRECISION U(3), V(3), M

EXTERNAL MAG
CALL MAG(M,V)
U(1) = V(1)/M
U(2) = V(2)/M
U(3) = V(3)/M
RETURN
END
MID(M, A, B) DEFINES THE POSITION VECTOR OF THE MIDPOINT M OF POINTS A AND B

SUBROUTINE MID(M, A, B)
DOUBLE PRECISION M(3), A(3), B(3)
M(1) = (A(1) + B(1)) / 2.0
M(2) = (A(2) + B(2)) / 2.0
M(3) = (A(3) + B(3)) / 2.0
RETURN
END

RAT(C, A, B, Z) DETERMINES THE CENTROID C OF A SEGMENT WITH ENDPOINTS A AND B WHERE Z IS THE RATIO:
CENTROID DISTANCE (FROM A) / SEGMENT LENGTH AB

SUBROUTINE RAT(C, A, B, Z)
DOUBLE PRECISION A(3), B(3), C(3), AB(3), ABZ(3), Z
EXTERNAL DIF, SCAVEC, SUM
CALL DIF(AB, A, B)
CALL SCAVEC(ABZ, Z, AB)
CALL SUM(C, A, ABZ)
RETURN
END

VECF(VF, V, F1, F2, F3) DETERMINES THE COMPONENTS VF IN FRAME F OF VECTOR V

SUBROUTINE VECF(VF, V, F1, F2, F3)
DOUBLE PRECISION VF(3), V(3), F1(3), F2(3), F3(3)
EXTERNAL DOT
CALL DOT(VF(1), V, F1)
CALL DOT(VF(2), V, F2)
CALL DOT(VF(3), V, F3)
RETURN
END
TIME USES ESTIMATES $T_1, T_2$ OF FRAME TIME TO FIT CUBIC SPLINE TO THEIR AVERAGE

SUBROUTINE TIME($T, T_1, T_2, N$)
PARAMETER ($NN=120$)
DOUBLE PRECISION $C(NN, 4), AV(NN), R(NN), T_1(NN), T_2(NN), T(NN)$,
$V(NN), DD(NN), SD(NN), SDO, VAR, S$
INTEGER $J, N$
EXTERNAL VALC, REINSH
INTRINSIC ABS, MAX, SQRT
VAR=0.0
DO 10, J=1, N
R(J)=J
AV(J)=($T_1(J)+T_2(J)) / 2.0$
V(J)=($T_1(J)-T_2(J))^{2}/4.0$
SD(J)=SQRT($V(J)$)
VAR=VAR+$V(J)$
10 CONTINUE
VAR=VAR/N
SD0=SQRT(VAR)
DO 20, J=1, N
DD(J)=MAX(SD(J), 0.4*SD0)
DD(J)=MAX(DD(J), 0.0003D0)
20 CONTINUE
S=N
CALL REINSH($C, N, R, AV, DD, S$)
DO 30, J=1, N
CALL VALC($T(J), R(J), C, N, R$)
IF($ABS(AV(J)-T(J)).GT.0.001D0$) PRINT 5, R(J), AV(J), T(J)
5 FORMAT('TIMES', 3F10.3)
30 CONTINUE
RETURN
END
REINSH FITS SMOOHEST CURIC SPLINE

THIS SUBROUTINE IS A TRANSLATION OF THE ALGOL PROGRAM GIVFN
BY CARL REINSCH 1967, NUMERISCHE MATHEMATIK, PP. 177-183.

PARAMETERS:

CC =SPLINE COEFFICIENTS
N2 = NUMBER OF DATA POINTS
K = ARRAY OF INDEPENDENT VARIABLE VALUES
   = FRAME NUMBER WHEN SPLINING TIME VALUES
Y = ARRAY OF DEPENDENT VARIABLE
DY = ARRAY OF ESTIMATE OF ERROR IN Y
S = CONSTANT DETERMINING CLOSENESS OF FIT

SUBROUTINE REINSH(CC,N2,K,Y,DY,S)
PARAMETER (NN=120)
DOUBLE PRECISION A(NN),B(NN),C(NN),D(NN),CC(NN,4),
*E,F,F2,G,H,P,
*R(0:NN),R1(0:NN),R2(0:NN),T(0:NN),T1(0:NN),U(0:NN),V(0:NN)

INTEGER N1,N2,1,M1,M2,I1,I2,IN1,IN2

INTRINSIC SQRT

N1=1
M1=N1-1
M2=N2+1

R(M1)=0.0
R(N1)=0.0
R1(N2)=0.0
R2(N2)=0.0
R2(M2)=0.0
U(M1)=0.0
U(N1)=0.0
U(N2)=0.0
U(M2)=0.0
P=0.0

M1=N1+1
M2=N2-1
H=K(M1)-K(N1)
F=(Y(M1)-Y(N1))/H
DO 10, I=M1, M2
   I1=I+1
   IN1=I-1
   G=H
   H=K(I1)-K(I)
   E=F
   F=(Y(I1)-Y(I))/H
   A(I)=F-E
   T(I)=2.0*(G+H)/3.0
   T1(I)=H/3.0
10 CONTINUE

DO 20, I=M1, M2
   I1=I+1
   I2=I+2
20 CONTINUE

F2=-S
25 CONTINUE

DO 30, I=M1, M2
   IN1=I-1
   IN2=I-2
30 CONTINUE

R1(IN1)=F*R(IN1)
R2(IN2)=G*R(IN2)
R(I)=1.0/(P*B(I)+T(I)-F*R1(IN1)-G*R2(IN2))
U(I)=A(I)-R1(IN1)*U(IN1)-R2(IN2)*U(IN2)
37 CONTINUE

F=P*C(I)+T1(I)-H*R1(IN1)
G=H
H=D(I)*P
30 CONTINUE

DO 40, I=M2, M1-1
   I1=I+1
   I2=I+2
40 CONTINUE

U(I)=R(I)*U(I)-R1(I)*U(I1)-R2(I)*U(I2)
40 CONTINUE

E=0.0
H=0.0
DO 50, I=N1, M2
I1=I+1
G=H
H=(U(I1)-U(I))/(K(I1)-K(I))
V(I)=(H-G)*DY(I)**2
E=E+V(I)*(H-G)
50 CONTINUE

C
V(N2)=H*DY(N2)**2
G=G(V(N2)
E=E-G*H
G=F2
F2=E*P**2
C
IF(P2.GE.S.OR.F2.LE.G) GO TO 65
C
F=0.0
H=(V(M1)-V(N1))/(K(M1)-K(N1))
C
DO 60, I=M1, M2
I1=I+1
IN1=I-1
IN2=I-2
C
G=H
G=(V(I1)-V(I))/(K(Il)-K(I))
G=G-H-G-R1(IN1)*R(IN1)-R2(IN2)*R(IN2)
F=F+G*R(I)*G
60 CONTINUE
C
H=E-P*F
IF(H.LE.0.0) GO TO 65
C
P=P+(S-F2)/((SORT(S/E)+P)*H)
GO TO 70 25
C
65 CONTINUE
C
DO 70, I=N1, N2
A(I)=Y(I)-P*V(I)
C(I)=U(I)
CC(I,1)=A(I)
CC(I,3)=C(I)
70 CONTINUE
C
DO 80, I=N1, M2
I1=I+1
H=K(I1)-K(I)
D(I)=(C(I1)-C(I))/H/3.0
B(I)=(A(I1)-A(I))/H-(H*D(I)+C(I))*H
CC(I,4)=D(I)
CC(I,2)=B(I)
80 CONTINUE

RETURN
END
VALC3 EVALUATES A CUBIC SPLINE AND ITS FIRST TWO DERIVATIVES

SUBROUTINE VALC3(SP, T, CC, N, K)
PARAMETER (NN=120)
DOUBLE PRECISION A, B, C, D, CC(NN, 4),
*H, K(NN), T, SP(3)
INTEGER I, I1, N, N1

N1=N-1
DO 10, I=1, N1
I1=I+1
IF(T.LT.K(1)) GO TO 18
IF(T.GT.K(N)) GO TO 14
IF(T.GE.K(I).AND.T.LT.K(I1)) GO TO 20
IF(T.EQ.K(N)) GO TO 16
10 CONTINUE
14 PRINT *, 'T > K(N) T= ', T, 'K(N)= ', K(N)
16 I=N-1
GO TO 10
18 PRINT *, 'T < K(1) T= ', T, 'K(1)= ', K(1)
20 A=CC(I, 1)
B=CC(I, 2)
C=CC(I, 3)
D=CC(I, 4)
H=I-K(I)
SP(1)=A+B*H+C*H**2+D*H**3
SP(2)=B+2.0*C*H+3.0*D*H**2
SP(3)=2.0*C+6.0*D*H
RETURN
END

VALC EVALUATES A CUBIC SPLINE
SUBROUTINE VALC(SP1, T, C, N, K)
DOUBLE PRECISION C(*), K(*), SP(3), SP1, T
INTEGER N
EXTERNAL VALC3
CALL VALC3(SP, T, C, N, K)
SP1=SP(1)
RETURN
END

C*******************************************************************
C SPLIN FITS SPLINES TO ESTIMATES X1, X2 OF THE DIGITISED
COORDINATE OF A JOINT CENTRE
C PARAMETERS:
C C1, C2 = SPLINE COEFFICIENTS
N = NUMBER OF DATA POINTS
T = ARRAY OF TIME VALUES
X1, X2 = ARRAYS OF DIGITISED DATA
C*******************************************************************
SUBROUTINE SPLIN(C1, C2, SDO, N, T, X1, X2)
PARAMETER (NN=120)
DOUBLE PRECISION C1(6, NN), C2(6, NN), T(NN), X1(NN), X2(NN), ACC, D(NN), SDO, V(NN), VAR,
S, A, B, DD(NN)
INTEGER J, N
EXTERNAL LIN, LSQOSP
INTRINSIC MAX, SQRT
VAR=0.0
CALL LIN(A, B, X1, X2, 1, N)
IF(A.LT.0.98D0) A=0.98
IF(A.GT.1.02D0) A=1.02
DO 10, J=1, N
D(J)=(X2(J)-(A*X1(J)+B))/2.0
V(J)=2*D(J)**2
IF(V(J).GT.0.01D0) V(J)=1.01
IF(V(J).GT.0.05D0) PRINT 5, T(J), J, X1(J), X2(J)
5 FORMAT(F7.3, I7, 2X, 2F7.3)
VAR=VAR+V(J)
10 CONTINUE
434

C
2511 VAR=VAR/N
2512 SD0=SQR(T(VAR)

2515 C*******************************************************************************
2516 C
2517 C CALCULATE ERROR ESTIMATES DD FOR EACH POINT
2518 C
2519 C*******************************************************************************
2520 C
2521 DO 20,J=1,N
2522 ACC=0.003
2523 DD(J)=SQR(0.25*V(J)+0.75*VAR)
2524 IF(DD(J).LT.1.0D-5) DD(J)=0.01
2525 DD(J)=MAX(DD(J),ACC)
2526 20 CONTINUE

C
2527 S=N
2528 PRINT 
2529 C
2530 CALL LSQQSP(T,X1,DD,S,N,C1,FLAG)
2531 CALL LSQQSP(T,X2,DD,S,N,C2,FLAG)
2532 C
2533 C
2534 RETURN
2535 C
2537 C*******************************************************************************
2538 C
2539 C LIN FITS REGRESSION LINE
2540 C
2541 C*******************************************************************************
2542 C
2543 SUBROUTINE LIN(A,B,T,Z,M,N)
2544 PARAMETER (NN=120)
2545 C
2546 DOUBLE PRECISION A,B,S1,ST,ST2,SZ,STZ,T(NN),Z(NN)
2547 C
2548 INTEGER J,M,N
2549 C
2550 S1=0.0
2551 ST=0.0
2552 ST2=0.0
2553 SZ=0.0
2554 STZ=0.0
2555 C
2556 DO 10,J=M,N
2557 S1=S1+1.0
2558 ST=ST+T(J)
2559 ST2=ST2+T(J)**2
2560 SZ=SZ+Z(J)
2561 STZ=STZ+T(J)*Z(J)
2562 10 CONTINUE
REGRESSION LINE Z=A*T+B

A=(S1*STZ-ST*SZ)/(S1*ST2-ST*ST)
B=(ST*STZ-ST2*SZ)/(ST*ST-ST2*S1)
RETURN
END

VALQ EVALUATES A QUINTIC SPLINE

SUBROUTINE VALQ(SP,T,C,N,K)
DOUBLE PRECISION C(6,N),K(N),SP,T,QSPLIN
INTEGER N
SP=QSPLIN(N,K,C,T)
RETURN
END

SPLIN4 FITS A QUINTIC SPLINE TO THE AVERAGE OF 4 ESTIMATES OF ANGLE A
SPLIN4 DETERMINES THE WEIGHT OF EACH POINT, THE CLOSENESS OF FIT AND CALLS SUBROUTINE LSQOSP WHICH CALCULATES THE SPLINE COEFFICIENTS.
LSQOSP IS A SUBROUTINE IN THE SPLINE FITTING LIBRARY OF LES JENNINGS, UNIV. WESTERN AUSTRALIA.
SUBROUTINE LSQOSP IS HELD IN LIBRARY SUBJEN.
PARAMETERS:
C=ARRAY OF SPLINE COEFFICIENTS
N=NUMBER OF DATA POINTS
T=ARRAY OF TIME VALUES
A=ARRAY OF 4 ESTIMATES OF ANGLE A
V=SD**2=LOCAL ESTIMATE OF VARIANCE
VAR=SDG**2=GLOBAL ESTIMATE OF VARIANCE
SUBROUTINE SPLIN4(C,SDO,N,T,A)
PARAMETER (NN=120)
DOUBLE PRECISION C(6,NN),T(NN),A(4,NN),VAR,AV(NN),
*DB(NN),V(NN),DD(NN),SDO,S,PI,RTD
INTEGER J,L,N
LOGICAL FLAG
EXTERNAL LSQOSP
INTRINSIC SQRT

VAR=0.0
DO 10,J=1,N
AV(J)=(A(1,J)+A(2,J)+A(3,J)+A(4,J))/4.0
DB(J)=0.0
DO 20,L=1,4
DB(J)=DB(J)+(A(L,J)-AV(J))**2
20 CONTINUE
V(J)=DB(J)/12.0
VAR=VAR+V(J)
10 CONTINUE
VAR=VAR/N
SDO=SQRT(VAR)
PI=3.1415926536
RTD=180.0/PI
SDO=RTD*SDO
DO 30,J=1,N
DD(J)=SQRT(0.75*V(J)+0.25*VAR)
IF(DJ(J).LT.1.0D-5) DD(J)=0.01
30 CONTINUE
S=N
CALL LSQOSP(T,AV,DD,S,N,C,FLAG)
RETURN
END

JENNINGS QUINTIC SPLINE FITTING LIBRARY
THE SPLINE FITTING LIBRARY WILL FIT A QUINTIC
SPLINE TO DATA IN THE SENSE DESCRIBED BY REINSCH 1967,
NUMERISCHE MATHEMATIK 10,177-183.
THE LIBRARY CONSISTS OF THE FOLLOWING SUBROUTINES AND FUNCTIONS:

QUINTIC SPLINE
LSQSP - USER ENTRY TO QUINTIC SPLINE CALCULATION.
QFUNCP - INTERNAL ROUTINE.
QSPLIN - VALUE OF QUINTIC SPLINE.
QDSPLIN - DERIVATIVES OF QUINTIC SPLINE.
QINTSP - INTEGRAL OF QUINTIC SPLINE.

INTERNAL ROUTINES
ZEROSP
SBFAC
SBSOL
SBFSUB
SBBSUB

ANY QUERIES SHOULD BE DIRECTED TO DR. L.S. JENNINGS AT
MATHEMATICS DEPARTMENT,
UNIVERSITY OF WESTERN AUSTRALIA.
NEDLANDS, W.A.

SUBROUTINE LSQSP(T, Y, DY, S, N, CC, FLAG) CALCULATES THE QUINTIC SPLINE COEFFICIENTS CC FOR A SEQUENCE OF DATA POINTS (T, Y)

PARAMETERS:
N: INTEGER
   N = NUMBER OF DATA POINTS
T: DP ARRAY DIMENSION N
   T CONTAINS THE VALUES OF THE INDEPENDENT VARIABLE
   T COMPRISSE THE KNOT SET
Y: DP ARRAY DIMENSION N
   Y CONTAINS THE VALUES OF THE DEPENDENT VARIABLE
DY: DP ARRAY DIMENSION N
   DY CONTAINS THE VALUES OF THE STANDARD ERRORS OF Y
S: DP
   S IS THE PARAMETER OF REINSCH
   ON INPUT S IS USUALLY SET TO N FOR THE SMOOTHEST
   FIT CONSISTENT WITH THE BOUNDED SUM OF SQUARES
   S IS SET TO ZERO TO OBTAIN THE INTERPOLATING SPLINE
CC: DP ARRAY DIMENSION (6, N)
   ON EXIT CC CONTAINS THE QUINTIC SPLINE COEFFICIENTS
FLAG: LOGICAL
   FLAG INDICATES ERROR AS .FALSE.
FUNCTION QSPLIN(N, T, CC, T1) evaluates the quintic spline of N points with knot set T(N) and coefficients CC(6,N) at time T1.

FUNCTION QDSPLN(N, T, CC, T1, J) evaluates the Jth derivative of the quintic spline with coefficients CC(6,N) and knot set T(N) at time T1.

COMMON BLOCKS

These are used to predefine certain numerical constants of the computer, working storage and to control error messages.

PARAMETERS:

PRINT: LOGICAL VARIABLE
   CONTROLS ERROR MESSAGES

EPS: DP VARIABLE TO TEST FOR CONVERGENCE OF ITERATION
   EPS SHOULD BE SET > 100*EPMACH

EPMACH: DP CONSTANT
   SHOULD BE SET TO MACHINE PRECISION
   E.G. CDC6000 1.0E-14, IBM360 1.0E-6

RESTQ: DP ARRAY
   WORK STORAGE > 2*N+6

END OF PROGRAM FILM
APPENDIX D

LISTING OF THE PROGRAM SIM
SIMULATION PROGRAM SIM SIMULATES THE FREE-FALL MOTION OF A HUMAN BODY

7 OPTIONS ARE AVAILABLE:

OPT=1 MOMENTUM OF A FILMED MOVEMENT IS CALCULATED
OPT=2 A FILMED MOVEMENT IS SIMULATED (VALIDATION)
OPT=3 MODIFIED FILMED MOVEMENT IS SIMULATED
OPT=4 USER SPECIFIED MOVEMENT IS SIMULATED
OPT=5 MOMENTUM DUE TO INTERNAL MOVEMENT IS FOUND
OPT=6,7 WHOLE BODY PRINCIPAL INERTIAS ARE FOUND

THE FOLLOWING DATA MUST BE INPUT BY THE USER:

INERTIA PARAMETERS
INTERNAL ANGLES AS QUINTIC SPLINE FUNCTIONS OF THE TIME

ADDITIONAL INPUT DATA:

FOR OPT=1 : TIME HISTORIES OF THE 3 SYSTEM ANGLES
FOR OPT>1 : ANGULAR MOMENTUM
: INITIAL VALUES OF THE 3 SYSTEM ANGLES

FOR OPT=6,7 LINEAR SPLINE FUNCTIONS ARE USED

SIM OUTPUTS THE ANGLES PHI, THETA AND PSI WHICH DEFINE THE AMOUNTS OF SOMERSAULT, TILT AND TWIST OF THE SYSTEM

LIBRARY 'NAGP'

N-1=NUMBER OF INTERVALS USED IN THE INTEGRATION OF THE DIFFERENTIAL EQUATIONS OF MOTION

PARAMETER (N=101)

ALL VARIABLES ARE DECLARED TO FACILITATE DETECTION OF PROGRAM ERRORS USING COMPILER OPTION -DCLVAR

DOUBLE PRECISION IMPLEMENTATION OF NAG SUBROUTINES IS USED

ON PRIME COMPUTERS
DOUBLE PRECISION T, Y(3), G(3), INT, ESL, W1(3), W2(3), W3(3), W4(3),
*V(3), W(3), P(3), P12, H(3), H0(3), D(3), MAXTH, TRATE, RTD,
*H(3), HREL(3), HFI(3), HHIP(3), HARM(3), HCT(3), HREM(3),
*NM(3), VH(3), SH(3),
*MP, XIP, YIP, ZIP, ZP, LP, L2P,
*MJ1, XIJ1, YIJ1, ZIJ1, ZJ1, LJ1,
*MJ2, XIJ2, YIJ2, ZIJ2, ZJ2,
*MT, XIJ3, YIJ3, ZIJ3, ZJ3,
*MC, XIJ4, YIJ4, ZIJ4, ZJ4,
*MA1, XIJ5, YIJ5, ZIJ5, ZA1, IA1,
*MA2, XIJ6, YIJ6, ZIJ6, ZA2,
*SHPH, CPIH, PHP, PHP0, AAA, BIB, CCC, STM1,
*STHP, CTIP, PHP, PHP0,
*PHPH, THF, PSF, PH0, TH0, PS0, PH, TH, PS,
*EAF, EBF,
*GA0, PSP0, BJK0, GAL0, THC0, PSC0,
*DA0, EAO, PAS0, GA0, DB0, ES0, PSS0, GAB0,
*GA, PSP, BJK, GAL, THC, PSC,
*DA, EA, PAS, GA, DB, ES, PSS, GAB,
*SP(3,3), SPT(3,3), SPJ1(3,3), SPK1(3,3),
*STC(3,3), SCA1(3,3), SAI(3,3), SAI(3,3), SB1B2(3,3),
*SJJ1J2(3,3), SK1K2(3,3),
*CCF(6,101,17), CCF(4,14,3),
*ST, ENDI,
*IFF(3,3),
*BB, RBB, MODE,
*YY(3), KY, KPP, KKH, KA, KB, CKN,
*KE, KW, KK, KKH, KT
DOUBLE PRECISION
*DT(N), D3T(N), Y3T(N),
*TT(N), PHT(N), TH(N), PST(N), GAT(N), PSPT(N), EKTN(N), GALT(N),
*PSCT(N), THCT(N), DAT(N), DBT(N), EAT(N), EBT(N), GAAT(N), GABT(N),
*PSAT(N), PSBT(N),
*PHF(T), THF(T), PSFT(N),
*HIT(N), H2N(N), H3N(N),
*HRIT(N), HR2T(N), HR3T(N),
*IFIT(N), IF2T(N), IF3T(N), IF4T(N),
*ALOT(N), THMT(N), PHPT(N), THPT(N), MODET(N), AAT(N), BRT(N), OCT(N),
*HULT(3,19,N)
INTEGER I, J, JJ, J1, J2, L, NA, NCH, NH, NCL, NINT, OPT
CHARACTER NAM*3, OUT*3, NAMFIT*6, NAM*6, OUTSIM*6, OUTSAM*6,
*PERF*4, INIT*6, VAR*5, SAM*1, AUTO*1, CMODE(N)*4
SUBROUTINE DERIV DEFINES THE ANGULAR MOMENTUM EQUATION

FOR OPT=1 DERIV CALCULATES THE MOMENTUM H

FOR OPT=2,3,4 DERIV CALCULATES THE DERIVATIVES D(I) I=1,3

OF THE ORIENTATION ANGLES Y(1)=PHI, Y(2)=THETA, Y(3)=PSI.

FOR OPT=5 DERIV CALCULATES THE MOMENTUM OF INTERNAL MOVEMENT

FOR OPT=6,7 DERIV CALCULATES WHOLE BODY PRINCIPAL INERTIAS

THE DATA HELD IN COMMON BLOCKS B1-B9 IS READ AT THE START

OF THE MAIN SEGMENT AND IS USED IN SUBROUTINE DERIV

EXTERNAL DERIV,D02ABF,ANG0,CSANG,ANGLANG,EXIT,SCAVEC,V3

INTRINSIC ASIN,ATAN,ABS,COS,MAX,MOD,SIGN,SQRT

COMMON/B1/MP, XIIP, YIP, ZIP, LP, L2P

COMMON/B2/MJ1, XIJ1, YIJ1, ZIJ1, ZJ1, LJ1

COMMON/B3/MJ2, XIJ2, YIJ2, ZIJ2, ZJ2

COMMON/B4/MT, XIT, YIT, ZT, LT

COMMON/B5/MC, XIC, YIC, ZIC, ZC, LC

COMMON/B6/MA1, XIA1, YIA1, ZIA1, ZA1, LA1

COMMON/B7/MA2, XIA2, YIA2, ZIA2, ZA2

COMMON/B8/H0

COMMON/B9/CCF, CCU

COMMON/B10/GA, PSP, EJK, GAL, THC, PSC

COMMON/B11/DA, EA, PSA, GAA, DB, EB, PSB, GAB

COMMON/B12/HPF, THF, PSF

COMMON/B13/NA, NCH, NH, ST, ENDT

COMMON/B14/OPT

COMMON/B15/H, HREL, HFI, HHIP, HARM, HCT, HREM

COMMON/B16/HA2A1, HB2B1, HJ2J1, HK2K1

COMMON/B20/NNINT

COMMON/B21/H0, TH0, PS0

COMMON/B22/GA0, PSP0, EJK0, GAL0, THC0, PSC0

COMMON/B23/DA0, EA0, PSA0, GAA0, DB0, EB0, PSB0, GAB0

COMMON/B24/STHM, IFF, MODE

COMMON/B25/SPHP, CHPH, STHP, CTHPH, AAA, BBB, CCC

COMMON/B30/SIP, SPT, STC, SCA1, SAI2, SBC1, SBI2

COMMON/B31/SPJ1, SJ1J2, SPK1, SK1K2

PRINT *, 'STATE OPTION NUMBER 1-7'

READ(*,*)OPT
READ ICIERTIA PARAMETERS WHICH HAVE BEEN CALCULATED FROM
ANTHROPOMETRIC MEASUREMENTS BY PROGRAM ISEG

PRINT *, 'INPUT PERFORMERS NAME'
READ(*,15)PERF
OPEN(5,FILE=PERF,FORM='FORMATTED',STATUS='READONLY')

READ(5,*)MP,XIP,YIP,ZIP,LP,L2P
READ(5,*)MJ1,XIJ1,YIJ1,ZIJ1,ZJ1,LJ1
READ(5,*)MJ2,XIJ2,YIJ2,ZIJ2,ZJ2
READ(5,*)MT,XIT,YIT,ZIT,LT
READ(5,*)MC,XIC,YIC,ZIC,ZC,LC
READ(5,*)MA1,XIA1,YIA1,ZIA1,ZA1,LA1
READ(5,*)MA2,XIA2,YIA2,ZIA2,ZA2

IF(OPT.EQ.5) GO TO 20
IF(OPT.GE.4) GO TO 10

PRINT *, 'INPUT DATA FILE NAME'
READ(*,15)NAM

OPEN(6,FILE=NAMMOM,FORM='FORMATTED')
IF(OPT.GT.1) READ(6,*)HN(1),HN(2),HN(3)

OPEN(7,FILE=NAMFIT,FORM='FORMATTED')
READ(7,*)((CCF(I,J,L), I=1,6), 1=1,101), L=1,17)
OPEN FILES OUTSIM AND OUTSAM TO WHICH THE ORIENTATION ANGLES WILL BE WRITTEN

STATE OUTPUT FILE NAME
READ(*,15)OUT
OUTSIM=OUT//'SIM'
IF(OPT.EQ.1) OUTSIM=OUT//'ANG'
OPEN(10,FILE=OUTSIM,FORM='FORMATTED')

PRINT *, 'DO YOU REQUIRE FILE FOR SAMMIE'
READ(*,15)SAM
IF(SAM.NE.'Y') GO TO 12
OUTSAM=OUT//'SAM'
IF(OPT.EQ.1) OUTSAM=OUT//'SAM'
OPEN(11,FILE=OUTSAM,FORM='FORMATTED')

IF(OPT.EQ.1) GO TO 50
IF(OPT.GE.4) GO TO 20

CHOOSE OPTIONS FOR MODIFYING INTERNAL ANGLES

PRINT **'STATE OPTION NUMBERS NA,NCH,NH'
READ(*,*)NA,NCH,NH
PRINT *, 'STATE START TIME AND END TIME'
READ(*,*)ST,ENDT

IF(OPT.LT.4) GO TO 30

READ: NORMALISED ANGULAR MOMENTUM HN
: INITIAL VALUES OF OUTPUT ANGLES PHI, THETA AND PSI
: INITIAL VALUES OF THE INTERNAL ANGLES

PRINT *, 'INPUT INITIAL VALUES FILE NAME'
READ(*,15)INIT
OPEN(8,FILE=INIT,FORM='FORMATTED')

READ(8,*)HN(1),HN(2),HN(3)
READ(8,*)PH0,TH0,PS0
READ(8,*)GA0,PSP0,JEK0,GAL0
READ(8,*)THC0,PS0C
READ(8,*)DA0,EA0,PSA0,GA0
READ(8,*)DB0,EB0,PSB0,GAB0
30 IF(OPT.LT.3) GO TO 50

IN ORDER TO DEFINE THE INTERNAL ANGLES THE USER SPECIFIES

CCU(I) I=1,4 (OPT > 2) WHERE:

CCU(1)=AS=INITIAL VALUE OF ANGLE A
CCU(2)=AE=FINAL VALUE OF A
CCU(3)=AST=START TIME
CCU(4)=AET=END TIME

SUBROUTINE VARANG (CALLED IN DERIV) DEFINES ANGLE A AS A
MONOTONIC FUNCTION OVER THE TIME INTERVAL [AST,AET]
FOR TIME T<AST VARANG LEAVES ANGLE A UNCHANGED
FOR TIME T>AET ANGLE A TAKES THE END VALUE AE

THUS ANGLE A MAY BE DEFINED BY A NUMBER OF CALLS TO VARANG

PRINT *,'INPUT VAR FILE NAME'
READ(*,15)VAR
IF(OPT.GT.4) OUT=VAR
OPEN(9,FILE=VAR,FORM='FORMATTED')
DO 40,L=1,3
DO 40,J=1,14
READ(9,*) (CCU(I,J,L),I=1,4)
40 CONTINUE

PRINT *,'DO YOU REQUIRE AUTO-TWIST CONTROL'
READ(*,15)AUTO
IF(AUTO.NE.'Y') GO TO 50
PRINT *,'STATE KE,KW,KMK,NCL'
READ(*,*)KE,KW,KMK,NCL
IF(OPT.NE.4) GO TO 50
PRINT *,'STATE START TIME AND END TIME'
READ(*,*)ST,ENDT

SET STANDARD CONFIGURATION AND CALCULATE MOMENT OF INERTIA BB ABOUT AXIS F1

50 T=-1.0
Y(1)=0.0
Y(2)=0.0
Y(3)=0.0
0323 C
0324 PH=0.0
0325 TH=0.0
0326 FS=0.0
0327 C
0328 PI=3.1415926536
0329 GA=PI
0330 PSP=0.0
0331 EJK=0.0
0332 GAL=PI
0333 C
0334 THC=0.0
0335 PSC=0.0
0336 C
0337 DA=0.0
0338 EA=0.0
0339 PSA=0.0
0340 GAA=PI
0341 C
0342 DB=0.0
0343 EB=0.0
0344 PSB=0.0
0345 GAB=PI
0346 C
0347 H0(1)=0.0
0348 H0(2)=0.0
0349 H0(3)=0.0
0350 C
0351 CALL DERIV(D,Y,T)
0352 C
0353 BB=IFF(1,1)
0354 C
0355 IF(OPT.GT.1) GO 70 150
0356 C
0357 FOR OPT=1 CALCULATE MOMENTUM FROM FILM DATA
0358 C
0359 C
0361 CALL DERIV(D,Y,T)
0362 C
0363 MOM(1)=0.0
0364 MOM(2)=0.0
0365 MOM(3)=0.0
0366 C
0367 C
0368 INT=0.01
0369 C
0370 DO 60 J=1,101
0371 C
0372 T=(J-1)*INT
0373 C
0374 CALL DERIV(D,Y,T)
SET TABULAR VALUES OF ORIENTATION ANGLES

- Tr(J)=T
RTD=180.0/PI
PHT(J)=Y(1)/2.0/PI
THT(J)=Y(2)*RTD
PST(J)=Y(3)/2.0/PI
GAT(J)=RTD*GA
PSPT(J)=RTD*PS
DAT(J)=RTD*DA
PI2=PI/2.0
IF(SAM. NE. 'Y') GO TO 62

CALL EJLANG(EULT(1,1, J), SIP)
CALL V3(EULT(1,2, J),0.0D0,0.0D0,0.0D0)
CALL EULANG(EULT(1,3, J),SPT)
CALL HJLANG(EULT(1,4, J), STC)
CALL EULANG(EULT(1,5, J), SCA1)
CALL EULANG(EULT(1,6, J), SAI2)
CALL V3(EULT(1,7, J),0.0D0,0.0D0,0.0D0)
CALL EULANG(EULT(1,8, J), SLC)
CALL EULANG(EULT(1,9, J), SCB1)
CALL EULANG(EULT(1,10, J), SB1B2)
CALL V3(EULT(1,11, J),0.0D0,0.0D0,0.0D0)
CALL EULANG(EULT(1,12, J), SPJ1)
CALL EULANG(EULT(1,13, J), SJ1J2)
CALL V3(EULT(1,14, J),-PI2,1.0D0,PI2)
CALL EULANG(EULT(1,15, J), SPK1)
CALL EULANG(EULT(1,16, J), SK1K2)
CALL V3(EULT(1,17, J),-PI2,1.0D0,PI2)
CALL V3(EULT(1,18, J),0.0D0,0.0D0,0.0D0)
CALL V3(EULT(1,19, J),0.0D0,0.0D0,0.0D0)

NORMALISE MOMENTA USING TRANSVERSE MOMENT OF INERTIA BB

62 RBB=1.0/(2.0*PI*RBB)
CALL SCAVEC(HN,RBB,H)
CALL SCAVEC(HFI,RBB,HFI)
CALL SCAVEC(HREL,RBB,HREL)
CALL SCAVEC(HHIP,RBB,HHIP)
CALL SCAVEC(HARM, RBB, HARM)
CALL SCAVEC(HCT, RBB, HCT)
CALL SCAVEC(HREM, RBB, HREM)
CALL SCAVEC(HA2A1, RBB, HA2A1)
CALL SCAVEC(HB2B1, RBB, HB2B1)
CALL SCAVEC(HJ2J1, RBB, HJ2J1)
CALL SCAVEC(HK2K1, RBB, HK2K1)

PRINT CONTRIBUTIONS TO TOTAL MOMENTUM

PRINT 42, T, HN(1), HFI(1), HREL(1), HHIP(1), HARM(1), HCT(1), HREM(1)
PRINT 44, T, HA2A1(1), HB2B1(1), HJ2J1(1), HK2K1(1)

PRINT 42, T, HN(2), HFI(2), HREL(2), HHIP(2), HARM(2), HCT(2), HREM(2)
PRINT 44, T, HA2A1(2), HB2B1(2), HJ2J1(2), HK2K1(2)

PRINT 42, T, HN(3), HFI(3), HREL(3), HHIP(3), HARM(3), HCT(3), HREM(3)

42 FORMAT(4F8.2,6X, 4F8.2)
44 FORMAT(F8.2,30X, 4F8.2)

SET TABLE VALUES

IT(J)=T
H1T(J)=HN(1)
H2T(J)=HN(2)
H3T(J)=HN(3)

CALCULATE: AVERAGE OF MOMENTA ESTIMATES
STANDARD ERROR OF THIS AVERAGE

MOM(1)=MOM(1)+HN(1)
MOM(2)=MOM(2)+HN(2)
MOM(3)=MOM(3)+HN(3)

60 CONTINUE

MOM(1)=MOM(1)/101.0
MOM(2)=MOM(2)/101.0
MOM(3)=MOM(3)/101.0
DO 70, J=1, 101
VH(1) = VH(1) + (H1T(J) - MOM(1))**2
VH(2) = VH(2) + (H2T(J) - MOM(2))**2
VH(3) = VH(3) + (H3T(J) - MOM(3))**2
70 CONTINUE

C
VH(1) = VH(1) / 101.0 / 100.0
VH(2) = VH(2) / 101.0 / 100.0
VH(3) = VH(3) / 101.0 / 100.0

C
SH(1) = SQRT(VH(1))
SH(2) = SQRT(VH(2))
SH(3) = SQRT(VH(3))

C*******************************************************************
C- FOR OPT=1 WRITE TABLE VALUES OF COMPONENTS OF MOMENTUM
C*******************************************************************

PRINT **'COMPONENTS OF MJPENTUM IN STRAIGHT SOMERSAULT UNITS'
PRINT *, ' XY Z'
PRINT 105, Ma4(1), MOP4(2), MOM(3)
WRITE(6, 105) M(1), MOM(2), MMOM(3)
C
PRINT *, 'ERROR ESTIMATES OF COMPONENTS'
PRINT *, ' XY Z'
PRINT 105, SH(1), SH(2), SH(3)
WRITE(6, 106) SH(1), SH(2), SH(3)
105 FORMAT(3F12.3)
106 FORMAT(3F12.4)
C
DO 110, J=1, 101, 10
PRINT 115, TT(J), H1T(J), H2T(J), H3T(J)
110 CONTINUE

C
DO 120, J=1, 101
WRITE(6, 125) TT(J), H1T(J), H2T(J), H3T(J)
120 CONTINUE

C

DO 130, J=1,101
WRITE(10,135) TT(J), PHT(J), THT(J), PST(J), GAT(J), PSPT(J), DAT(J)
135 FORMAT(4F10.3,3F10.1)
130 CONTINUE

IF(SAM. NE. 'Y') GO TO 998

C-- DO 140, J=2,100,14
C DO 140, J=3,99,8
C DO 140, J=1,101,5
C DO 140, J=1,101,10
WRITE(11,145) ((EULT(I,L,J), I=1,3), L=1,19)
145 FORMAT(16X, 3F7.4)
140 CONTINUE

IF(OPT. EQ. 1) GO TO 999

C*******************************************************************
C FOR OPT>1 CONVERT NORMALISED MOMENTA VALUES
C*******************************************************************
150 HO(1) = HN(1)*BB*2.0*PI
HO(2) = HN(2)*BB*2.0*PI
HO(3) = HN(3)*BB*2.0*PI

C*******************************************************************
C EVALUATE INITIAL VALUES OF ORIENTATION ANGLES (IN RADIANS)
NOTE THAT VALUES OF THE ANGLES ARE HELD IN COMMON BLOCKS
B10, B11 SO THAT THEY MAY BE ACCESSED BY THE MAIN SEGMENT
C*******************************************************************

, T=0.0
CALL DERIV(D,Y,T)

PHO=PHF
THO=THF
PSO=PSF

CALL ANG0(PHP,PHP0,SPHP,CPHP)
CALL ANG0(THP,THP0,SHP,CTHP)

C*******************************************************************
C SET PARAMETERS FOR SUBROUTINE D02ABF
C*******************************************************************
D02ABF(T,Y,G,J1,J2,JJ,INT,ESL,DERIV,W1,W2,W3,W4,W5,W6)
SOLVES THE DIFFERENTIAL EQUATIONS D/DT(Y(1))=D(1)
D/DT(Y(2))=D(2)
D/DT(Y(3))=D(3)
FOR Y(1),Y(2),Y(3)
D(1),D(2),D(3) ARE SPECIFIED IN SUBROUTINE DERIV(D,Y,T)
**C**

**C** SET INITIAL TIME T=0

**C** SET INITIAL VALUES OF Y(1), Y(2), Y(3)

**C** G(I) SETS THE ERROR BOUNDS FOR Y(I) I=1,2,3

**C** J1=1 INDICATES THAT A MIXED ERROR TEST IS USED

**C** J2=NUMBER OF DIFFERENTIAL EQUATIONS =3

**C** JJ=0 INITIALLY AND IS UNCHANGED UNLESS THE ROUTINE DETECTS

**C** AN ERROR, E.G. STEP LENGTH BECOMES TOO SMALL

**C** INT=TIME INTERVAL OVER WHICH INTEGRATION OCCURS

**C** ESL=ESTIMATED STEP LENGTH NEEDED FOR INTEGRATION

**C** W1-W6 ARE ARRAYS USED AS WORKING SPACE

**C** MAXTH=0.0

**C** TRATE=0.0

**C** T=0.0

**C** Y(1)=PHF

**C** Y(2)=THF

**C** Y(3)=PSF

**C** G(1)=1.0D-3

**C** G(2)=G(1)

**C** G(3)=G(1)

**C** J1=1

**C** J2=3

**C** JJ=0

**C** INT=1.0/(N-1)

**C** ESL=INT

**C** NNINT=0

**C** FURTHER DETAILS OF D02ABF AND OTHER NAG SUBROUTINES ARE

**C** GIVEN AT THE END OF THIS PROGRAM

**C** INTEGRATE OVER N-1 INTERVALS EACH OF LENGTH 1.0/(N-1)

**C** DO 160, J=1, N

**C** IF(J.EQ.1) GO TO 163

**C** CALL D02ABF(T,Y,G,J1,J2,JJ,INT,ESL,DERIV,W1,W2,W3,W4,W5,W6)
C*******************************************************************
C- AUTOMATIC TWIST CONTROL ATTEMPTS TO ENSURE THAT TWIST DOES
C NOT OCCUR IN THE UNSTABLE LAYOUT SOMERSAULT BY USING
C ASYMMETRIC ARM MOVEMENTS TO ADJUST THE TILT ANGLE
C
C THIS OPTION MAY ALSO BE USED TO STOP THE TWIST IN TWISTING
C SOMERSAULTS
C*******************************************************************

CALL DERIV(D,Y,T)
YY(3)=Y(3)
IF(OPT.EQ.4) GO TO 161

YY(1)=Y(1)+0.01*D(1)
YY(2)=Y(2)+0.01*D(2)
YY(3)=Y(3)+0.01*D(3)
OPT=2
T=T+INT
CALL DERIV(D,YY,T)

IF(OPT.EQ.4. AND. NCL. GT. 0) YY(3)=Y3T(J-NCL)
IF(OPT.EQ.4. AND. NCL. GT. 0) D(3)=D3T(J-NCL)

EAF=EA
EBF=EB

KK=H0(1)**2*(IFF(1,1)-IFF(3,3))*(IFF(2,2)-IFF(1,1))
KK=KK/IFF(1,1)/IFF(3,3)
KM=-0.12*H0(1)*(IFF(1,1)-IFF(3,3))/IFF(1,1)/IFF(3,3)
SGN=SIGN(1.0D0,COS(YY(3)))
KM=SGN*KM

IF(J.EQ.51) PRINT *, 'KK = ',KK, ' KM = ',KM

KT=0.01*NCL
KPP=(KW**2+KK)/KM
KDR=2.0*KP*KW/KM
IF(J.EQ.51) PRINT *, 'KPP = ',KPP, ' KDR = ',KDR
KPP=KPP*ABS(COS(YY(3)))

KDR=KDR*COS(YY(3))**2
KYY=0.5*SIN(2.0*YY(3))

KYY=MAX(KW,1.0D-5)
IF(J.EQ.51) PRINT *, 'DAMPING FACTOR = ',KM*KDR/2.0/KW
RTD=180.0/PI

KA=0.01*RTD*COS(DA)*(KDR*D(3)+KPP*KYY)

KB=0.01*RTD*COS(DB)*(KDR*D(3)+KPP*KYY)

IF(KA. LT. -KP'X) KA = rt

IF(KB . LT . -IQ'a) KB=-IQ(a)

IF(KA. GT. INX) KA=KMX

IF(KB. GT. KMX) KB=KMX

CALL DERIV(D,Y,T)

IF(EAF. LT. 0.0D0) KA=P1AX(KA, 0.0D0)

IF(EAF. GT. PI) KA=MIN(KA, 0.0D0)

IF(EBF. LT. 0.0D0) KB=MIN(KB, 0.0D0)

IF(EBF. GT. PI) KB=MAX(KB, 0.0D0)

CCU(1,8,2)=EA*RTD

CCU(2,8,2)=EAF*RTD+KA

CCU(3,8,2)=T

CCU(4,8,2)=T+INT

CCU(1,12,2)=EB*RTD

CCU(2,12,2)=EBF*RTD-KB

CCU(3,12,2)=T

CCU(4,12,2)=T+INT

162 CALL DERIV(D,Y,T)

TT(J)=T

Y3T(J)=Y(3)

D1T(J)=D(1)/2.0/PI

D3T(J)=D(3)/2.0/PI

RTD=180.0/PI

PHF(J)=Y(1)/2.0/PI

THF(J)=Y(2)*RTD

PST(J)=Y(3)/2.0/PI

PHFT(J)=PHF/2.0/PI

THFT(J)=THF*RTD

PSFT(J)=PST/2.0/PI

GAT(J)=RTD*GA

PSPT(J)=RTD*PSP

EJKT(J)=RTD*EJK

GALT(J)=RTD*GAL

PSCT(J)=RTD*PSC

THCT(J)=RTD*THC
0754 C
0755  DAT(J)=RTD*DA
0756  DMT(J)=RTD*DB
0757  EAT(J)=RTD*EA
0758  EB(T(J)=RTD*EB
0759  PSAT(J)=RTD*PSA
0760  PSBT(J)=RTD*PSB
0761  GAAT(J)=RTD*GAA
0762  GABT(J)=RTD*GAB
0763 C
0764  TMT(J)=RTD*ASIN(STM)
0765  MODE(J)=NDE
0766  CMODE(J)='***'
0767  IF(MODE.GT.-1.0D0.AND.MODE.LT.-0.001D0) CMODE(J)='DISC'
0768  IF(MODE.GT.0.001D0) CMODE(J)='ROD'
0769  IF(OPT.EQ.6) CALL CSANG(PHP,PHP0,SPHP,CPHP)
0770  PIPT(J)=PHP*RTD/2.0
0771  IF(OPT.EQ.7) CALL CSANG(THP,THPO,STHP,CTHP)
0772  THIP(T(J)=TIP*RTD/2.0
0773 C
0774  PI2=PI/2.0
0775  IF(SAM.'NE.'Y') GO TO 164
0776 C
0777 C*******************************************************************
0778 C
0779 C SET EULERIAN ANGLE VALUES FOR SAMMIE GRAPHICS
0780 C
0781 C*******************************************************************
0782 C
0783  CALL EULANG(EULT(1,1,J),SIP)
0784  CALL V3(EULT(1,2,J),0.0D0,0.0D0,0.0D3)
0785  CALL EULANG(EULT(1,3,J),SPT)
0786 C
0787  CALL EULANG(EULT(1,4,J),STC)
0788  CALL EULANG(EULT(1,5,J),SCA1)
0789  CALL EULANG(EULT(1,6,J),SAIL2)
0790  CALL V3(EULT(1,7,J),0.0D0,0.0D0,0.0D0)
0791 C
0792  CALL EULANG(EULT(1,8,J),STC)
0793  CALL EULANG(EULT(1,9,J),SCB1)
0794  CALL EULANG(EULT(1,10,J),SB1B2)
0795  CALL V3(EULT(1,11,J),0.0D0,0.0D0,0.0D0)
0796 C
0797  CALL EULANG(EULT(1,12,J),SP1)
0798  CALL EULANG(EULT(1,13,J),SJ1J2)
0799  CALL V3(EULT(1,14,J),-PI2,1.0D0,PI2)
0800 C
0801  CALL EULANG(EULT(1,15,J),SK1)
0802  CALL EULANG(EULT(1,16,J),SKK2)
0803  CALL V3(EULT(1,17,J),PI2,1.0D0,PI2)
0804  CALL V3(EULT(1,18,J),0.0D0,0.0D0,0.0D0)
0805  CALL V3(EULT(1,19,J),0.0D0,0.0D0,0.0D0)
ALDT(J)=SQRT(CCC*(AAA-BBB)/BBB/(AAA-C))
ALDT(J)=RTD*ASIN(ALDT(J))
IF1T(J)=IF1(1,1)
IF2T(J)=IF2(2,2)
IF3T(J)=IF3(3,3)
HR1T(J)=IREL(1)/BB/2.0/PI
HR2T(J)=IREL(2)/BB/2.0/PI
HR3T(J)=IREL(3)/BB/2.0/PI
IF(T.LT.0.0D0) GO TO 166
MAXT=MAX(MAXT,ABS(Y(2)*RTD))
TRATE=MATRAT,ABS(D(3))/MAX(ABS(D(1)),1.0D-5))
CONTINUE
C*******************************************************************
C FOR OPT>1 WRITE TABLE VALUES OF SIMULATION
C*******************************************************************
IF(OPT.EQ.2.AND.NA*NCI*N1.EQ.1) PRINT 201,NAM
201 FORMAT(/8X,'Table 1. Comparison of simulation and film for'
*,1X,A)
IF(OPT.GT.2) PRINT 202,OUT
202 FORMAT(/22X,'Table 1. Simulation',A)
PRINT *
PRINT *
WRITE(1,203)OUT,NINT,PERF
203 FORMAT(1X,'SIMULATION : ',A,9,' INTEGRATION STEPS',6X,
*PERFORMER : ',A)
PRINT *
PRINT *
PRINT 204,IN(1),IN(2),IN(3)
PRINT *
204 FORMAT(1X,'COMPONENTS OF MOMENTUM : HX = ',F5.2,3X,'HY = ',
*F5.2,3X,'HZ = ',F5.2)
C
IF(OPT.EQ.5) GO TO 500
IF(OPT.EQ.6) GO TO 600
IF(OPT.EQ.7) GO TO 700
C
WRITE(1,215)
215 FORMAT(1X,'TIME',6X,'SOMERSAULT',10X,'TILT',12X,'TWIST',10X,'MODE')
C
IF(OPT.EQ.4) GO TO 410
456

0858 C
0859 WRITE(1,225)
0860 225 FORMAT(10X,
0861 *'SIM',5X,'FILM',6X,'SIM',4X,'FILM',5X,'SIM',5X,'FILM'/
0862 *2X,'T',8X,'PH',5X,'PHF',8X,'TH',4X,'THF',7X,'PS',5X,'PSF')
0863 C
0864 410 WRITE(1,235)
0865 235 FORMAT(10X,
0866 *'REVOLUTIONS',8X,'DEGREES',8X,'REVOLUTIONS'/)
0867 C
0868 IF(OPT.EQ.4) GO TO 420
0869 C
0870 DO 250,J=1,101,10
0871 C
0872 WRITE(1,245)TT(J),PH(J),PHF(J),TH(J),THF(J),PS(J),PSF(J)
0873 *,CM3DE(J)
0874 245 FORMAT(1X,F3.1,5X,F5.2,3X,F5.2,5X,F4.0,3X,F4.0,5X,F5.2,3X,
0875 245 FORMAT(1X,F3.1,5X,F5.2,3X,F5.2,5X,F3.1,2X,F5.1,2X,F5.2,3X,
0876 *F5.2,6X,A)
0877 C
0878 250 CONTINUE
0879 C
0880 GO TO 280
0881 C
0882 420 DO 430,J=1,101,10
0883 C
0884 WRITE(1,425)TT(J),PH(J),TH(J),PS(J),CH(J)
0885 425 FORMAT(1X,F3.1,9X,F5.2,12X,F5.1,12X,F5.2,10X,A)
0886 430 CONTINUE
0887 C
0888 DO 270,J=1,101
0889 C
0890 WRITE(10,265)TT(J),PH(J),TH(J),PS(J),GA(J),PSPT(J),DAT(J)
0891 265 FORMAT(16X,3F7.4)
0892 CONTINUE
0893 C
0894 280 IF(SM.EQ.'Y') GO TO 300
0895 C
0896 DO 290,J=1,101,5
0897 C
0898 DO 290,J=2,101,14
0899 C
0900 DO 290,J=3,99,16
0901 C
0902 WRITE(11,285) (EULT(I,L,J),I=1,3),L=1,19)
0903 285 FORMAT(16X,3F7.4)
0904 290 CONTINUE
0905 C
0906 300 CONTINUE
0907 C
0908 PRINT *,
0909 PRINT *,'INTERNAL ORIENTATION ANGLES IN DEGREES'
0910 C
0911 PRINT *,'TIME PIKE HULA LEGARD KNEE SHOULDERS'
0912 C
0913 PRINT *,
0914 C
0915 PRINT *,"GAL TIC PSC"
DO 311, J=1,101,10
PRINT 312, TT(J), GAT(J), PSPT(J), EJKT(J),
*GALT(J), TICT(J), PSCT(J)
311 CONTINUE

C
PRINT *
PRINT *, 'LEFT ARM',
*RIGHT ARM'
PRINT *, 'TIME RAISE ABDUCT ROTATE ELBOW',
*RAISE ABDUCT ROTATE ELBOW'
PRINT *, 'TIME RAISE ABDUCT I YTATE FLW+l',
PRINT *, 'T DA EA PSA GAA ',
PRINT *, 'DB EB PSB GAB '
PRINT *

DO 313, J-1,101,10
PRINT 312, TT(J),
*DAT(J), EAT(J), PSAT(J), GAAT(J),
*DIT(J), EIT(J), PSIT(J), GAIT(J)
313 CONTINUE

312 FORMAT(1X, F3.1, 8F7.0)

C**************************************************
C IF(OPT. CT.4) GO TO 998
C WRITE(1,325)MAX'TT, TRATE
325 FORMAT(/1X, 'MAX TILT = ', F4.0, ' DEGREES', 5X, 'MAX TWIST = ',
*F4.2, ' REVS PER SALTO')
WRITE(1,326) PIT(101)*360, TIM(101), PST(101)*360
326 FORMAT(/1X, 'FINAL ORIENTATION ANGLES IN DEGREES',
*/1X, 'SALM _', F6.0, ' TILT =', F6.1, ' TWIST =', F6.0)

C
PRINT *
PRINT *, 'TIME TES! '
PRINT *, 'TIME TIM'

DO 330, J=1,101,10
WRITE(1,335)TT(J), TMIT(J)
335 FORMAT(1X, F3.1, F9.1)
330 CONTINUE

C
IF(OPT. CT.3) GO TO 998

DO 340, J=1,101
WRITE(10,345)TT(J), TIM(T), PST(J), DIT(J), D3T(J)
345 FORMAT(F6.3,5F16.4)
340 CONTINUE
C 600 PRINT *, ' PRINCIPAL MOMENTS OF INERTIA'
0978    PRINT *, GA  DA  PH  PHP  ALØ  IF1  ',
0979    PRINT *, IF2  IF3  ',
0980    PRINT *,
0981 C  DO 610, J=1,101,5
0982    IF4T(J)=AT(J)+BT(J)-IF1T(J)
0983    WRITE(1,615)GAT(J),DAT(J),PHT(J)*360,PHPT(J),ALAT(J),IF1T(J),
0984    *IF4T(J),CCT(J)
0985 610 CONTINUE
0986 C 615 FORMAT(2FB.6,3F8.1,3X,3F8.2)
0987 C  GO TO 280
0988 C 700 PRINT *, ' PRINCIPAL MOMENTS OF INERTIA'
0989    PRINT *
0990    PRINT *, GA  PSP  EA  TH  THP  ALØ  ',
0991    PRINT *, IF1  IF2  IF3  ',
0992 PRINT *,
0993 C  DO 710, J=1,101,5
0994    IF4T(J)=AT(J)+BT(J)-IF2T(J)
1000    WRITE(1,715)GAT(J),PSPT(J),EAT(J),HT(J),HTPT(J),ALAT(J),
1001    *IF4T(J),IF2T(J),CCT(J)
1002 710 CONTINUE
1003 C 715 FORMAT(3F8.0,3F7.1,3F8.2)
1004 C  GO TO 280
1005 C
1006 C 998 PRINT *,
1007 PRINT *
1008 PRINT *
1009 PRINT *
1010 PRINT *
1011 C
1012 C******************************************************************
1013 C
1014 C CLOSE FILE UNITS
1015 C
1016 C******************************************************************
1017 C
1018 C 999 DO 990 J=5,11
1019 C  CLOSE(J)
1020 C 990 CONTINUE
1021 C
SUBROUTINE DERIV DEFINES THE ANGULAR MOMENTUM EQUATION
FOR OPT=1 DERIV CALCULATES THE MOMENTUM H
FOR OPT=2,3,4 DERIV CALCULATES THE DERIVATIVES D(I) I=1,3
OF THE ORIENTATION ANGLES Y(1)=PHI, Y(2)=THETA, Y(3)=PSI.
FOR OPT=5 DERIV CALCULATES THE MOMENTUM DUE TO INTERNAL
MOVEMENT
FOR OPT=6,7 DERIV CALCULATES THE WHOLE BODY PRINCIPAL AXES
THE DATA HELD IN COMMON BLOCKS B1-R10 IS READ AT THE START
OF THE MAIN SEGMENT AND IS USED IN SUBROUTINE DERIV
SUBROUTINE DERIV(D,Y,T)
DOUBLE PRECISION DD(3),D(3),Y(3),T,IW(3,3),ANGD(3),
*W1(3,3),W2(3),W3(3),
*DTR,
*MP,XIP,YIP,ZIP,ZP,LP,L2P,
*M11,XIJ1,YIJ1,ZIJ1,ZJ1,LJ1,
*M22,XIJ2,YIJ2,ZIJ2,ZJ2,
*MT,XIT,YIT,ZIT,LT,
*MC,XIC,YIC,ZIC,LC,
*MA1,XIA1,YIA1,ZIA1,ZA1,LA1,
*MA2,XIA2,YIA2,ZIA2,ZA2,
*MK1,MK2,LK1,ZK1,ZK2,
*M1,B1,MB1,L1,B1,B2,
*MJ,MR,MJ,MB,MAB,
*MD,MJ,MUP,MP
DOUBLE PRECISION
*PHF,THF,PSF,PHO,THO,PSO,
*STHM,SPHP,CPHP,BYC,STHP,CTHP,AHC,
*CCU(4,14,3),CCF(6,101,17),K(101),SP(2,17),
*ST,ENDT
1077 C  DOUBLE PRECISION
1079  *FF, FFD,
1080  *PI, PI2,
1081  *GA0, PSM0, EJK0, GAL0, THC0, PSC0,
1082  *DA0, EA0, PSA0, GA0, DB0, EB0, PSB0, GAB0,
1083 C
1084  *PH, TH, PS,
1085  *GA, PSP, EJK, GAL, THC, PSC,
1086  *DA, EA, FSA, GAA, DB, EB, PSB, GAB,
1087  *AL, BT,
1088 C
1089  *PHD, THD, PSD,
1090  *GAD, PSPD, EJKD, GALD, THCD, PSCD,
1091  *DAD, FAD, PSAD, GAAD, DBD, EBD, PSBD, GABD,
1092  *ALD, BTD, DFD, EFD, DLD, ELD, LD, ETD,
1093  *EJ, EKD,
1094  *PHLD, THAD, THBD,
1095 C
1096  *CPH, CTH, CPS,
1097  *CGA, CPSP, CEJK, CGAL, CTHC, CPSC,
1098  *CDA, CEA, CPSA, CGA, CDB, CEB, CPBS, CGAB,
1099  *CAL, CBT, CDF, CEF, CDL, CEL, CDT, CET,
1100  *CPHIL, CTHA, CTHB,
1101 C
1102  *SPH, STH, SPS,
1103  *SGA, SPSP, SELK, SGAL, STHC, SPSC,
1104  *SQA, SEA, SPSA, SGAA, SDB, SEB, SPSS, SGAB,
1105  *SAL, SET, SDF, SEF, SDL, SEL, SET,
1106  *SPHL, STHA, STHB,
1107  *S2PSP, C2PSP
1108 C
1109  DOUBLE PRECISION
1110  *RLQ1, RLQ2, RLQ, LQ1, LQ2, LQD,
1111  *QXQ(3), Q6Q(3), Q2X(3), VEC1(3), VEC2(3), DUMMY,
1112  *VO(3), VR(3), VL(3), VT(3), VOD(3), VFD(3), VLD(3), VTD(3),
1113 C
1114  *RPH(3,3), RTH(3,3), RPS(3,3), RNPS(3,3),
1115  *RNDF(3,3), RNDFE(3,3), RNDL(3,3), RNEL(3,3), RNDT(3,3), RNET(3,3),
1116  *RNJ(3,3), RNJK(3,3), RNPHL(3,3),
1117  *RTHC(3,3), RNPSC(3,3),
1118  *RDA(3,3), REA(3,3), RNPSA(3,3), RNTBA(3,3),
1119  *RDB(3,3), RNEB(3,3), RPSB(3,3), RTHB(3,3),
1120 C
1121  *SEF(3,3), SLP(3,3), STP(3,3),
1122  *SJ1P(3,3), SK1P(3,3), SJ2P(3,3), SK2P(3,3),
1123  *STC(3,3),
1124  *SA1C(3,3), SB1C(3,3), SA2C(3,3), SB2C(3,3),
1125 C
1126  *SIF(3,3), SPF(3,3), SLF(3,3),
1127  *SJ1F(3,3), SK1F(3,3), SJ2F(3,3), SK2F(3,3),
1128  *STF(3,3), SCF(3,3),
1129  *SA1F(3,3), SB1F(3,3), SA2F(3,3), SB2F(3,3),
DOUBLE PRECISION.

*GP(3), GX(3), GH(3), GI(3),

*HJ1(3), GJ1(3), HQJ(3), GQJ(3), QJ2(3), GJ2(3),

*IK1(3), GKL(3), IOK(3), GOK(3), OK2(3), GKL2(3),

*IT(3), GT(3), XN(3), GN(3),

*NC(3), GC(3), NS(3), GS(3), NR(3), GR(3),

*SAL(3), GAL(3), SQA(3), GQA(3), OA2(3), GA2(3),

*RB1(3), GBL(3), ROB(3), COB(3), OB2(3), GB2(3),

*HJ(3), IK(3), XU(3), ND(3), SA(3), RB(3),

*GJ(3), GK(3), GL(3), GGA(3), GB(3), GGAB(3),

*GD(3), GU(3), SUG(3), GP(3),

*JJ1(3), JJ2(3), KK1(3), KK2(3),

*AAL(3), AA2(3), BB1(3), BB2(3),

*LJ(3), IK(3),

*DDA(3), DDB(3), DC(3),

*UD(3), UT(3),

*FU(3), FJ(3), FK2(3), FK(3),

*FA2(3), FA(3), FB1(3), FB(3),

*FP(3), FL(3), FD(3), Fur(3),
THE DATA HELD IN COMMON BLOCKS B1–B10 IS READ AT THE START
OF THE MAIN SEGMENT AND IS USED IN SUBROUTINE DERIV FOR THE
CALCULATION OF ORIENTATION ANGLES AND INERTIA TENSORS
THE INTERNAL ORIENTATION ANGLES ARE:

GA, PSP: WHICH DEFINE THE RELATIVE ORIENTATIONS OF THE UPPER AND LOWER SEGMENTS

DT, ET: WHICH DEFINE THE ORIENTATION OF THE THORAX T RELATIVE TO FRAME P

DL, EL: WHICH DEFINE THE ORIENTATION OF THE LEGS L RELATIVE TO FRAME P

DF, EF: WHICH DEFINE THE ORIENTATION OF THE PELVIS P RELATIVE TO THE SYSTEM FRAME P

EJK: WHICH GIVES THE ANGLE OF ABDUCTION OF EACH LEG RELATIVE TO FRAME L

THC = TILT ANGLE OF CHEST RELATIVE TO PELVIS P

PSC = TORSION ANGLE OF CHEST C RELATIVE TO PELVIS P

DA, EA, PSA: WHICH DEFINE THE ORIENTATION OF THE UPPER LEFT ARM A RELATIVE TO THE CHEST

DB, EB, PSB: WHICH DEFINE THE ORIENTATION OF THE UPPER RIGHT ARM B RELATIVE TO THE CHEST

GAA=LEFT ELBOW ANGLE

GAB=RIGHT ELBOW ANGLE

DO 10, J=1,17

10 CONTINUE

IF(T.LT.0.0) GO TO 55

IF(OPT.LT.4) GO TO 20

DTR=PI/180.0

SP(1,1)=PHI*DTR

SP(1,2)=THETA*DTR

SP(1,3)=PSI*DTR
SP(1,4) = GA0*DTR
SP(1,5) = PSPO*DTR
SP(1,6) = FJKO*DTR
SP(1,7) = GALPJ*DTR
SP(1,8) = THCO*DTR
SP(1,9) = PSCO*DTR
SP(1,10) = DAO*DTR
SP(1,11) = EAQ1*DTR
SP(1,12) = PSA0*DTR
SP(1,13) = GAA0*DTR
SP(1,14) = DBO*EIrR
SP(1,15) = EB0*D'I'R
SP(1,16) = PSBO*DTR
SP(1,17) = GAB0*DTR

IF(OPT. GT. 3) GO TO 50

C*******************************************************************
C EVALUATE THE ORIENTATION ANGLES AT TIME T USING SUBROUTINE
C VALQ2 WHICH EVALUATES A QUINTIC SPLINE AND ITS FIRST
C DERIVATIVE FROM THE KNOT SET AND SPLINE COEFFICIENTS
C*******************************************************************

K(I) = (I-1)/100.0
DO 40 I = 1, 17
CALL VALQ2(SPI(1, I), T, CCF(1,1, I), 101, K)

DO 50 IF(OPT. EQ. 1) GO TO 55
IF(OPT. LT. 3) GO TO 60

C*******************************************************************
C IN ORDER TO DEFINE THE INTERNAL ANGLES THE USER SPECIFIES
C CCU(I) I = 1, 4 (OPT > 2) WHERE:
C CCU(1) = AS = INITIAL VALUE OF ANGLE A
C CCU(2) = AE = FINAL VALUE OF A
C CCU(3) = AST = START TIME
C CCU(4) = AET = END TIME

SUBROUTINE VARANG DEFINES ANGLE A AS A MONOTONIC FUNCTION OVER THE TIME INTERVAL [AST,AET] FOR TIME T<AST VARANG LEAVES ANGLE A UNCHANGED FOR TIME T>AET ANGLE A TAKES THE END VALUE AE

THUS ANGLE A MAY BE DEFINED BY A NUMBER OF CALLS TO VARANG

VDANG DEFINES THE DERIVATIVE AD OF ANGLE A OVER THE TIME INTERVAL [AST,AET]

FOR OPT = 3 FILM ANGLES ARE MODIFIED FOR ST < T < ENDT

***************************************************************************************

IF(OPT.GT.3) GO TO 69
IF(T.LT.ST) GO TO 60
IF(T.GT.ENDT) GO TO 60

DO 70, L=1,3
DO 70, J=1,14
J3=J+3

CALL VARANG(SP(1, J3), CQJ(1, J, L), T)
CALL VDANG(SP(2, J3), CCU(1, J, L), T)

70 CONTINUE

IF(OPT.GT.1) GO TO 60

PH=SP(1,1)
TH=SP(1,2)
PS=SP(1,3)

PHD=SP(2,1)
THD=SP(2,2)
PSD=SP(2,3)

Y(1)=PH
Y(2)=TH
Y(3)=PS
D(1)=PHD
D(2)=THD
D(3)=PSD

GA=SP(1,4)
PSP=SP(1,5)
EJK=SP(1,6)
GAL=SP(1,7)
1392 C
1393 THC=SP(1,8)
1394 FSC=SP(1,9)
1395 DA=SP(1,10)
1396 EA=SP(1,11)
1397 PSA=SP(1,12)
1398 GAA=SP(1,13)
1399 DB=SP(1,14)
1400 EB=SP(1,15)
1401 PSB=SP(1,16)
1402 GAB=SP(1,17)
1403 C
1404 65 GAD=SP(2,4)
1405 PSPD=SP(2,5)
1406 EJKD=SP(2,6)
1407 GALD=SP(2,7)
1408 C
1409 THCD=SP(2,8)
1410 PSCD=SP(2,9)
1411 DAD=SP(2,10)
1412 EAD=SP(2,11)
1413 PSAD=SP(2,12)
1414 GAAD=SP(2,13)
1415 DBD=SP(2,14)
1416 EB=SP(2,15)
1417 PSBD=SP(2,16)
1418 GABD=SP(2,17)
1419 C
1420 IF(OPT.LT.2.OR.OPT.GT.3) GO TO 200
1421 C
1422 C******************************************************************
1423 C
1424 FOR OPT = 2,3 MODIFY INTERNAL ORIENTATION ANGLES OBTAINED
1425 FROM FILM BETWEEN TIMES ST AND ENDT
1426 C
1427 C******************************************************************
1428 C
1429 IF(T.LT.ST) GO TO 200
1430 IF(T.GT.ENDT) GO TO 200
1431 C
1432 C******************************************************************
1433 C
1434 FOR NA = 1 ARM ANGLES ARE UNCHANGED
1435 C
1436 FOR NA = 0 DERIVATIVES ARE SET TO ZERO
1437 C
1438 FOR NA = 2 RIGHT ARM ANGLES ARE SET EQUAL TO LEFT
1439 C
1440 FOR NA = 3 LEFT ARM ANGLES ARE SET EQUAL TO RIGHT
1441 C
1442 FOR NA = -1 LEFT AND RIGHT ARM ANGLES ARE INTERCHANGED
1443 C
1444 IF(NA.NE.0) GO TO 110
1445 C  
1446   DAD=0.0  
1447   FAD=0.0  
1448   PSAD=0.0  
1449   GAAD=0.0  
1450 C  
1451   DBD=0.0  
1452   EBD=0.0  
1453   PSBD=0.0  
1454   GABD=0.0  
1455 C  
1456   THCD=0.0  
1457 C  
1458   110 IF(NA.NE.2) GO TO 120  
1459 C  
1460   DB=DB  
1461   DBD=DA  
1462   EB=EA  
1463   EBD=FAD  
1464   PSB=PSA  
1465   PSBD=PSAD  
1466   GAB=GAA  
1467   GABD=GAAD  
1468 C  
1469   THC=0.0  
1470   THCD=0.0  
1471 C  
1472   120 IF(NA.NE.3) GO TO 130  
1473 C  
1474   DA=DA  
1475   DAD=DBD  
1476   EA=EB  
1477   EAD=EAD  
1478   PSA=PSB  
1479   PSAD=PSBD  
1480   GAA=GAB  
1481   GAAD=GABD  
1482 C  
1483   THC=0.0  
1484   THCD=0.0  
1485 C  
1486   130 IF(NA.NE.-1) GO TO 140  
1487 C  
1488   FP=DA  
1489   DA=DB  
1490   DB=FF  
1491   FFD=DAD  
1492   DAD=DBD  
1493   DBD=FFD  
1494 C  
1495   FP=EA  
1496   EA=EB  
1497   EB=FF  
1498   FFD=EAD  
1499   EAD=EBD  
1500   EBD=FFD
C FOR NCH = 1 CHEST TORSION IS UNCHANGED
C FOR NCH = 0 DERIVATIVES ARE SET TO ZERO
C FOR NCH = 2 PSC IS SET TO 0
C FOR NCH = -1 CHEST TORSION IS REVERSED
C
C******************************************************************
C FOR NH = 1 HIP MOVEMENT IS UNCHANGED
C FOR NH = 0 DERIVATIVES ARE SET TO ZERO
C FOR NH = 2 PSP IS SET TO 0 OR PI
C FOR NH = -1 HULA MOVEMENT IS REVERSED
C
C******************************************************************
C FOR NH = 0 HIP MOVEMENT IS UNCHANGED
C FOR NH = 0 DERIVATIVES ARE SET TO ZERO
C FOR NH = 2 PSP IS SET TO 0 OR PI
C FOR NH = -1 HULA MOVEMENT IS REVERSED
C
C******************************************************************
1556 C
1557 180 IF(NH.NE.2) GO TO 190
1558 C
1559  CPSP=COS(PSP)
1560  IF(CPSP.GE.0.0D0) PSP=0.0
1561  IF(CPSP.LT.0.0D0) PSP=PI
1562  PSPD=0.0
1563 C
1564 190 IF(NH.NE.-1) GO TO 200
1565 C
1566  PSP=-PSP
1567  PSPD=-PSPD
1568 C
1569 200 CONTINUE
1570 C
1571  IF(OPT.EQ.1) GO TO 210
1572 C
1573 C*******************************************************************
1574 C
1575 C DEFINE VALUES OF OUTPUT ANGLES
1576 C
1577 C*******************************************************************
1578 C
1579  PH=Y(1)
1580  TH=Y(2)
1581  PS=Y(3)
1582 C
1583  210 CONTINUE
1584 C
1585 C*******************************************************************
1586 C
1587 C CALCULATE SINE AND COSINE OF EACH ANGLE
1588 C
1589 C*******************************************************************
1590 C
1591  CPH=COS(PH)
1592  SPH=SIN(PH)
1593  CTH=COS(TH)
1594  STH=SIN(TH)
1595  CPS=COS(PS)
1596  SPS=SIN(PS)
1597 C
1598  CGA=COS(GA)
1599  SG=Sin(GA)
1600  CPSP=COS(PSP)
1601  SPSP=SIN(PSP)
1602  CEJK=COS(EJK)
1603  SEJK=SIN(EJK)
1604  CGAL=COS(GAL)
1605  SGAL=SIN(GAL)
1606 C
1607  CTHC=COS(THC)
1608  STHC=SIN(THC)
1609  CPSC=COS(PSC)
1610  SPSC=SIN(PSC)
C**CALCULATE ANGLES AL AND BT AS FUNCTIONS OF THE PIKE ANGLE GA**

C AND THE HULA ANGLE PSP

C

C CALCULATE ANGLES AL AND BT AS FUNCTIONS OF THE PIKE ANGLE GA

C AND THE HULA ANGLE PSP

C

C AL=ANGLE BETWEEN T3 AND P3

C BT=ANGLE BETWEEN L3 AND P3: SO THAT AL+BT=PI-GA

C

C*******************************************************************

C

C

C

C

C

C

C

C

C

C

C

C*******************************************************************
DEFINE DIRECTION VECTORS VT AND VL WHICH DETERMINE THE ORIENTATIONS OF THE THORAX T IN FRAME P AND THE LEGS L IN FRAME P

VT(1) = SPSP*SAL
VT(2) = -CPSP*SAL
VT(3) = CPL

VL(1) = SPSP*SBT
VL(2) = -CPSP*SBT
VL(3) = CBT

DEFINE DERIVATIVES OF DIRECTION VECTORS VT, VL

VTD(1) = -CPSP*SAL*PSPD - SPSP*CAL*ALD
VTD(2) = SPSP*SAL*PSPD - CPSP*CAL*ALD
VTD(3) = -SAL*ALD

VLD(1) = CPSP*SBT*PSPD + SPSP*CBT*BTD
VLD(2) = SPSP*SBT*PSPD - CPSP*CBT*BTD
VLD(3) = -SBT*BTD

DETERMINE DIRECTION VECTOR VF WHICH GIVES THE ORIENTATION OF THE SYSTEM FRAME F RELATIVE TO THE PELVIS FRAME P

LOCATE POINTS Q1 AND Q2 WHICH DEFINE AXIS F3

CALL SCAVEC(XQ1, LQ1, VT)
CALL V3(GX, 0.9D0, 0.9D0, LP)
CALL SCAVEC(Q2G, LQ2, VL)
CALL SM31(Q2X, Q2G, GX)
CALL SM31(VQ, Q2X, XQ1)
CALL UNIV(VF, VQ)
CALL MAG(LQ, VQ)
RLQ=1.0/IQ
CALL SCAVEC(VECl, LQ1, VTD)
CALL SCAVEC(VEC2, LQ2, VLD)
CALL M31(VQD, VEC1, VEC2)
DUM'ý, MY=LJ1*SEJK*EJM
CALL SCAVEC(VEC1, DUM9Y, VL)
CALL SCAVEC(VEC1,1, QD, VF)
CALL DM31(VEC2, VQD, VBC1)
CALL SCAVEC(VFD, RLO, VEC2)

C*******************************************************************
C THE ORIENTATION OF F WITH RESPECT TO P MAY BE SPECIFIED BY
SUCCESSIVE ROTATIONS THROUGH ANGLES DF AND EF ABOUT AXES
FL AND F2
DF AND EF ARE USED TO CALCULATE WPF=-WFP
C*******************************************************************
SEF=VI(1)
CEF=SQR(T(VF(2)**2+VF(3)**2)
SDF=-VF(2)/CEF
CDF=VF(3)/CEF
DDF=(VF(2)*VFD(3)-VF(3)*VFD(2))/CEF**2
ETD=VFD(1)/CEF

C*******************************************************************
C THE ORIENTATION OF T WITH RESPECT TO P MAY BE SPECIFIED
EITHER IN TERMS OF PSP AND AL
OR BY SUCCESSIVE ROTATIONS THROUGH ANGLES DT AND ET ABOUT
AXES T1 AND T2
DT AND ET ARE USED TO FACILITATE THE CALCULATION OF WTP
C*******************************************************************
SET=VT(1)
CET=SQR(T(VT(2)**2+VT(3)**2)
SDT=-VT(2)/CET
CDT=VT(3)/CET
DTD=(VT(2)*VTD(3)-VT(3)*VTD(2))/CET**2
ETD=VTD(1)/CET
THE ORIENTATION OF \( L \) WITH RESPECT TO \( P \) MAY BE SPECIFIED EITHER IN TERMS OF \( \Psi SP \) AND \( \Theta T \) OR BY SUCCESSIVE ROTATIONS THROUGH ANGLES \( DL \) AND \( EL \) ABOUT AXES \( L1 \) AND \( L2 \). \( DL \) AND \( EL \) ARE USED TO CALCULATE \( WJP \) AND \( WKP \).

\[
\begin{align*}
SEL & = vL(1) \\
CEL & = \sqrt{vL(2)^2 + vL(3)^2} \\
SDL & = -vL(2)/CEL \\
CDL & = vL(3)/CEL \\
DLD & = (vL(2)vLD(3) - vL(3)vLD(2))/CEL^2 \\
ELD & = \frac{(-\Phi D)}{CEL} \\

\end{align*}
\]

THE ORIENTATION OF \( J1 \) RELATIVE TO \( L \) IS GIVEN BY A ROTATION THROUGH ANGLE \( -\Phi JK \) ABOUT AXIS \( J12 \).

THE ORIENTATION OF \( K1 \) RELATIVE TO \( L \) IS GIVEN BY A ROTATION THROUGH ANGLE \( \Phi JK \) ABOUT AXIS \( K12 \).

\[
\begin{align*}
FJD & = ELD - EJKD \\
II = & ELD + EJKD \\

\end{align*}
\]

THE ORIENTATION OF \( J2 \) RELATIVE TO \( J1 \) IS GIVEN BY A ROTATION THROUGH ANGLE \( Pll \) ABOUT AXIS \( J21 \).

THE ORIENTATION OF \( K2 \) RELATIVE TO \( K1 \) IS GIVEN BY A ROTATION THROUGH ANGLE \( PLl \) ABOUT AXIS \( K21 \).

\[
\begin{align*}
SPHL & = SGAL \\
CPHl & = -CGAL \\
PHLD & = -GALD \\

\end{align*}
\]

THE ORIENTATION OF \( C \) RELATIVE TO \( T \) IS GIVEN BY SUCCESSIVE ROTATIONS THROUGH ANGLES \( -\Theta HC \) AND \( \Psi SC \) ABOUT AXES \( C2 \) AND \( C3 \).

THE ORIENTATION OF \( A1 \) RELATIVE TO \( C \) IS GIVEN BY SUCCESSIVE ROTATIONS THROUGH \( -DA, -EA, \) AND \( \Psi AC \) ABOUT AXES \( A11, A12, A13 \).

THE ORIENTATION OF \( B1 \) RELATIVE TO \( C \) IS GIVEN BY SUCCESSIVE ROTATIONS THROUGH \( -DB, EB, -PSB \) ABOUT AXES \( B11, B12, B13 \).
**THE ORIENTATION OF A2 RELATIVE TO A1 IS GIVEN BY A ROTATION THROUGH ANGLE \( \theta_a \) ABOUT AXIS A2**

**THE ORIENTATION OF B2 RELATIVE TO B1 IS GIVEN BY A ROTATION THROUGH ANGLE \( \theta_b \) ABOUT AXIS B2**

\( \theta_a \) IS RELATED TO THE ELBOW ANGLE GAA BY: \( \theta_a = \pi - GAA \)

\( \theta_b \) IS RELATED TO THE ELBOW ANGLE GAB BY: \( \theta_b = \pi - GAB \)

**EVALUATE THE 3 X 3 ROTATION MATRICES**

CALL RM1(RPH, CPH, SPH)
CALL RM2(RTH, CTH, STH)
CALL RM3(RPS, CPS, SPS)
CALL RM3(RNPS, CPS, -SPS)
CALL RM2(KNEE, CEL, -SEL)
CALL RM1(RNDT, CDL, -SDL)
CALL BM3(RNPS, CPS, -SPS)
CALL R'12 (RNEF, CEF, -SEF)
CALL R41(RNDL, CDL, -SDL)
CALL R11(RNPHL, CPHL, -SPHL)
CALL RTHC, CTHC, STHC)
CALL R42 (RNTHA, CTHA, -STHA)
CALL R12(RNEB, CEB, -SEB)
CALL PM3(RPSI3, CPSB, SPSB)
CALL RM2 (RrHB, CTEIB, STHB)
EVALUATE DIRECTION COSINE MATRICES
NOTATION: PREMULTIPLICATION BY 3X3 MATRIX SIF CONVERTS
COORDS IN FRAME I TO COORDS IN FRAME F

CALL PM33(SIF, RTH, RPH)
CALL PM33(SIF, RPS, SIF)

CALL PM33(SIF, RNDF, RN6F)
CALL PM33(SLP, RNDL, RNEL)
CALL PM33(STP, RNDT, RNET)

CALL PM33(SJ1P, SLP, REJK)
CALL PM33(SK1P, SLP, RNEJK)
CALL PM33(SJ2P, S1JP, RNPHL)
CALL PM33(SK2P, SK1P, RNPHL)

CALL PM33(SCT, RrHC, RNPSC)

CALL PM33(SA1C, REA, RNPSA)
CALL PM33(SA1C, RDA, SAlC)
CALL PM33(SA2C, SAIC, RNTHA)

CALL PM33(SB1C, RNEB, RPSB)
CALL PM33(SB1C, RDB, SB1C)
CALL PM33(SB2C, SB1C, RTHB)

EVALUATE TRANSFORMATION MATRIX SGF FOR EACH SEGMENT G

CALL TR(SPF, SPF)

CALL PM33(SLF, SPF, SLP)
CALL PM33(SJ1F, SPF, SJ1P)
CALL PM33(SKLF, SPF, SK1P)
CALL PM33(SJ2F, SPF, SJ2P)
CALL PM33(SK2F, SPF, SK2P)

CALL PM33(STF, SPF, STP)

CALL PM33(SCF, STF, SCT)
CALL PM33(SA1F, SCF, SA1C)
CALL PM33(SB1F, SCF, SB1C)
CALL PM33(SA2F, SCF, SA2C)
CALL PM33(SB2F, SCF, SB2C)
EVALUATE TRANSPOSE MATRICES

CALL TR(SFI, SIF)
CALL TR(SFL, SLF)
CALL TR(SFJ1, SJ1F)
CALL TR(SFK1, SK1F)
CALL TR(SFJ2, SJ2F)
CALL TR(SFK2, SK2F)

CALL TR(SFT, STF)
CALL TR(SFC, SCF)

CALL TR(SFA1, SA1F)
CALL TR(SFB1, SB1F)
CALL TR(SFA2, SA2F)
CALL TR(SFB2, SB2F)

EVALUATE TRANSFORMATION MATRICES NEEDED TO CALCULATE THE EUCLERIAN ANGLES USED BY THE SAMMIE GRAPHICS SYSTEM

CALL PM33(SIP, SFP, SIF)
CALL TR(SPT, STP)
CALL TR(STC, SCT)

CALL TR(SCA1, SA1C)
CALL TR(SAL2, RNIHA)
CALL TR(SCB1, SB1C)
CALL TR(SB1B2, RTHB)

CALL TR(SPJ1, SJ1P)
CALL TR(SPI, SK1P)
CALL TR(SJJ2, RNPHL)
CALL TR(SK1K2, RNPHL)

CALCULATE POSITION VECTORS NEEDED TO EVALUATE THE INERTIA TENSORS AND MOMENTUM TERMS

ALL VECTORS ARE EVALUATED IN FRAME F
**C**

**THORAX T**

**C**

CALL V3(XT,0.0D0,0.0D0,ZT)

CALL PM331(XT,STF,XT)

CALL SM31(GT,GX,XT)

CALL V3(XN,0.0D0,0.0D0,LT)

CALL PM331(XN,STF,XN)

CALL SM31(GN,GX,XN)

**C**

**CHEST C**

**C**

CALL V3(NC,0.0D0,0.0D0,ZC)

CALL PM331(NC,SCF,NC)

CALL SM31(GC,GN,NC)

CALL V3(NS,LC,0.0D0,0.0D0)

CALL PM331(NS,SCF,NS)

CALL SM31(GS,GN,NS)

CALL V3(NR,-LC,0.0D0,0.0D0)

CALL PM331(NR,SCF,NR)

CALL SM31(GR,GN,NR)

**C**

**LEFT ARM A**

**C**

CALL V3(SA1,0.0D0,0.0D0,-ZA1)

CALL PM331(SA1,SA1F,SA1)

CALL SM31(GA1,GS,SA1)

CALL V3(SOA,0.0D0,0.0D0,-LA1)

CALL PM331(SOA,SA1F,SOA)

CALL SM31(SOA,GS,SOA)

CALL V3(OA2,0.0D0,0.0D0,-ZA2)

CALL PM331(OA2,SA2F,OA2)

CALL SM31(GA2,GOA,OA2)
RIGHT AR1 B

CALL V3(RB1, 0.0D0, 0.0D0, -ZB1)
CALL PM331(RB1, SB1F, RB1)
CALL SM31(GB1, GR, RB1)

CALL V3(ROB, 0.0D0, 0.0D0, -LB1)
CALL PM331(ROB, SB1F, ROB)
CALL SM31(GB2, GR, ROB)

CALL V3(OB2, 0.0D0, 0.0D0, -ZB2)
CALL PM331(OB2, SB2F, OB2)
CALL SM31(GB2, GB, OB2)

CALL V3(GJ, M3, GJ1, M71, GJ2, M72)
CALL CEN(GK, MK, GK1, MK1, GK2, MK2)
CALL CEN(GL, ML, GJ, G14, GK, MK)

CALL CEN(GA, MA, GA1, MA1, GA2, MA2)
CALL CEN(GL, ML, GJ, G14, GK, MK)

CALL CEN(GD, MD, GC, MC, GGAB, MAB)
CALL CEN(GU, M, GT, MT, GD, MD)

CALL CEN(GUP, MUP, GU, NIU, GP, MP)
CALL CEN(GF, NF, GUP, MUP, GL, ML)
DETERMINE VECTORS CONNECTING MASS CENTRES

CALL DM31(JJ1, GJ1, GJ)
CALL DM31(JJ2, GJ2, GJ)
CALL DM31(KK1, GK1, GK)
CALL DM31(KK2, GK2, GK)
CALL DM31(AA1, GA1, GGA)
CALL TX131(AA2, GA2, GGA)
CALL DM31(BB1, GB1, GB)
CALL DM31(BB2, GB2, GB)
CALL DM31(LJ, GJ, GL)
CALL DP931(LK, GK, GL)
CALL DM31(DDA, GGA, GD)
CALL DM31(DDB, GB, GD)
CALL TX131(DC, GC, GD)
CALL DM31(UD, GD, GU)
CALL DM31(UT, GT, GU)

DETERMINE POSITION VECTORS OF MASS CENTRES RELATIVE TO THE MASS CENTRE F OF THE SYSTEM

CALL DM31(FP, GP, GF)
CALL DM31(FJ2, GJ2, GF)
CALL DM31(FJ, GJ, GF)
CALL DM31(FK2, GK2, GF)
CALL DM31(FK, GK, GF)
CALL DM31(FL, GL, GF)
CALL DM31(FD, GD, GF)
CALL DM31(FU, GU, GF)
CALL DM31(FA2, GA2, GF)
CALL DM31(FA, GGA, GF)
CALL DM31(FB2, GB2, GF)
CALL DM31(FB, GB, GF)
DETERMINE POSITION VECTORS OF MASS CENTRES RELATIVE TO THE JOINT CENTRES

CALL DM31(HJ, GJ, GH)
CALL DM31(IK, GJ, GI)
CALL DM31(XU, GU, GX)
CALL DM31(ND, GD, GN)
CALL DM31(SA, GGA, GS)
CALL DM31(RB, GB, GR)

EVALUATE INERTIA TENSORS OF EACH SEGMENT IN FRAME F

CALL IM(IPP, XIP, YIP, ZIP)
CALL PM33(IPP, SPF, IPP)
CALL PM33(IPP, IPP, SFP)
CALL IM(IJ1J1', XIJI, YIJl, ZIJ1)
CALL PM33(IJ1J1, SJIF, IJIJ1)
CALL PM33(IJ1J1, IJ1JI, SFJ1)
CALL IM(IJ2J2, XIJ2, YIJ2, ZIJ2)
CALL PM33(IJ2J2, SJ2F, IJ2J2)
CALL PM33(IJ2J2, IJ2J2, SFJ2)
CALL IM(IK1K1, XIJ1, YIJ1, ZIJ1)
CALL PM33(IK1K1, SK1F, IK1K1)
CALL PM33(IK1K1, IK1I1, SFK1)
CALL IM(IK2K2, XIJ2, YIJ2, ZIJ2)
CALL PM33(IK2K2, SK2F, IK2K2)
CALL PM33(IK2K2, IK2K2, SFK2)
CALL IM(ITr, XIT, YIT, ZIT)
CALL PM33(ITT, =, ITT)
CALL PM33(ITT, ITT, SFT)
CALL IM(IAlA1, XIA1, YIA1, ZIA1)
CALL PM33(IAlA1, SA1F, IAlA1)
CALL PM33(IAlA1, IAlA1, SFA1)
CALL PM33(IA2A2, SA2F, TA2A2)
CALL PM33(IA2A2, IA2A2, SFA2)
CALL PM33(IB1B1, XIA1, YIA1, ZIA1)
CALL PM33(IB1B1, IB1B1, SFB1)
CALL PM33(IB2B2, XIA2, YIA2, ZIA2)
CALL PM33(IB2B2, IB2B2, SFB2)

IMSF IS THE INERTIA TENSOR OF MASS MS LOCATED AT MASS CENTRE
S RELATIVE TO ORIGIN F
ISF IS THE INERTIA TENSOR OF SEGMENT S RELATIVE TO ORIGIN F
ISF = ISS + IMSF
CALL PA(IMAD, DDA, HA)
CALL SM33(IAD, IAA, IMAD)
CALL PA(IMBD, DDB, MB)
CALL SM33(IAD, IBB, IMBD)
CALL PA(IMCD, DC, MC)
CALL SM33(IAD, IBB, IMBD)

CALL PA(IMDU, UD, MD)
CALL SM33(IPF, IFP, IMD)

CALL PA(IMLF, FL, ML)
CALL SM33(ILF, ILL, IMLF)

CALL PA(IMUF, FU, MU)
CALL SM33(IFP, IFP, IMUF)

CALL V3(wFP, DFD, EFD*CDF, EFD*SDF)
CALL SCAVEC(wPF, -1.0D0, w9FP)
CALL PM331(WPF, SPF, W9FP)

CALL V3(IMP, DLD, EJD*CDL, EJD*SDL)
CALL PM331(W1P, SPF, W71P)
CALL V3(WK1P, DLD, EKD*CDL, EKD*SDL)
CALL PM331(WKIP, SPF, W7K1P)

CALL V3(WJ2J1, PHLD, 0.0D0, 0.0D0)
CALL PM331(W2J1, SJ1F, W2J1)

CALL V3(WK2K1, PHLD, 0.0D0, 0.0D0)
CALL PM331(W2K1, SK1F, W2K1)

CALL V3(WTP, DTD, ETD*CDT, ETD*SDT)
CALL PM331(WTP, SPF, WTP)

CALL V3(WCT, -STHC*PSCD, -THCD, CTHC*PSCD)
CALL PM331(WCT, STF, WCT)

WA1C(1)=-DAD-PSAD*PSA
WA1C(2)=-EAD*CDA+PSAD*SDA*CDA
WA1C(3)=EAD*SDA+PSAD*CDA*CPE
CALL PM331(WA1C, SCF, WA1C)
C
2349 C
2350 WBIC(1) = EBD*PSBD*SEB
2351 WBIC(2) = EBD*CD*PSBD*SB*CEB
2352 WBIC(3) = EBD*SB*PSBD*CD*CEB
2353 CALL PM331(WBIC, SCF, WBIC)
2354 C
2355 CALL V3(WA2A1, 0.0D0, THAD, 0.0D0)
2356 CALL PM331(WA2A1, SAIF, WA2A1)
2357 C
2358 CALL V3(WB2B1, 0.0D0, -THBD, 0.0D0)
2359 CALL PM331(WB2B1, SB1F, WB2B1)
2360 C
2361 C*******************************************************************
2362 C
2363 C EVALUATE MOMENTA TERMS ARISING FROM RELATIVE MOTIONS
2364 C HFP IS THE MOMENTUM OF THE SYSTEM ARISING FROM THE ANGULAR
2365 VELOCITY WPF
2366 C
2367 C*******************************************************************
2368 C
2369 C
2370 C
2371 CALL HRL(HJ1P, IJJ, MJ, FJ, HJ, WJ1P)
2372 CALL HRL(HK1P, IKK, MK, FK, IK, WK1P)
2373 CALL HRL(HJ2J1, IJ2J2, MJ2, FJ2, OJ2, WJ2J1)
2374 CALL HRL(HK2K1, IK2K2, MK2, FK2, OK2, WK2K1)
2375 C
2376 CALL HRL(HTP, IUU, MJ, FU, XX, WTP)
2377 CALL HRL(HCT, IDD, MD, FD, ND, WCT)
2378 C
2379 CALL HRL(HA1C, IAA, MA, FA, SA, WA1C)
2380 CALL HRL(HB1C, IBB, MB, FB, RB, WB1C)
2381 CALL HRL(HA2A1, IAA2, MA2, FA2, OA2, WA2A1)
2382 CALL HRL(HB2B1, IB2B2, MB2, FB2, OB2, WB2B1)
2383 C
2384 CALL PM331(HFP, IFF, WPF)
2385 C
2386 C*******************************************************************
2387 C
2388 C HHIP = MOMENTUM DUE TO HIP MOVEMENT
2389 C HARM = MOMENTUM DUE TO ARM MOVEMENT
2390 C HCT = MOMENTUM DUE TO CHEST/SHOULDER MOVEMENT
2391 C HREM = MOMENTUM DUE TO REMAINING RELATIVE MOTIONS
2392 C
2393 C HREL = MOMENTUM DUE TO INTERNAL MOTIONS
2394 C = HHIP + HARM + HCT + HREM
2395 C
2396 C*******************************************************************
2397 C
2398 CALL SM31(HHIP, HJ1P, HK1P)
2399 CALL SM31(HHIP, HHIH, HTP)
2400 CALL SM31(HHIP, HHIP, HFP)
2401 C
2402 CALL SM31(HARM, HA1C, HB1C)
2403 C

CALL SM31(HREM, HJ2J1, HK2K1)
CALL SM31(HREM, HREM, HA2A1)
CALL SM31(HREM, HREM, HB2B1)
CALL SM31(HREM, HREM, HREM)
CALL SM31(HREL, HREL, HREL)
CALL SM31(HREL, HREL, HCT)
CALL SM31(HREL, HREL, HREM)
CALL SM31(HREL, HREL, HREL)
CALL SM31(HREL, HREL, HRF31)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HRIIN, RNPS, HRIIN)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HREM, RNPS, HREM)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HREM, RNPS, HREM)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HREM, RNPS, HREM)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HREM, RNPS, HREM)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(HHIP, RNPS, HHIP)
CALL PM331(HARM, RNPS, HARM)
CALL PM331(HCT, RNPS, HCT)
CALL PM331(HREM, RNPS, HREM)
CALL PM331(HA2A1, RNPS, HA2A1)
CALL PM331(HB2B1, RNPS, HB2B1)
CALL PM331(HJ2J1, RNPS, HJ2J1)
CALL PM331(HK2K1, RNPS, HK2K1)
CALL PM331(WFI, PHD, THD, PSD) = WFI (IN FRAME F) WHERE PHD,THD,PSD ARE THE DERIVATIVES OF PH,TH AND PS AND WHERE WFI IS THE ANGULAR VELOCITY OF FRAME F RELATIVE TO THE INERTIAL FRAME I
IF(OPT.GT.1) GO TO 220
CALL V3(WFI, PHD, THD, PSD)
CALL PM331(WFI, RW, WFI)
CALL PM331(HFI, IFF, WFI)
CALL SM31(H, HREL, HFI)
CALL PM331(H, SFI, H)
CALL PM331(HFI, RNPS, HFI)
CALL PM331(HREL, RNPS, HREL)
IF(OPT.EQ.1) GO TO 250

CALCULATE TOTAL MOMENTA IN FRAME F

CALL PM331(H, SIF, HΩ)

CALL PM33(IW, IFF, RW)

THE EQUATION OF MOTION IS: IFF, RW.(PHD, THD, PSD) = HFI
SUBROUTINE F04ATF SOLVES FOR ANGD = (PHD, THD, PSD)

CALL PM33(IW, IFF, RW)

F04ATF SOLVES IW, ANGD=HFI FOR ANGD

SET PARAMETERS: J1, J2, J3=ORDER OF MATRIX IW=3
JJ IS SET TO ZERO AND REMAINS UNCHANGED
UNLESS THE ROUTINE DETECTS AN ERROR
W1-W3 ARE ARRAYS USED AS WORKING SPACE

CALL F04ATF(IW, J1, HFI, J2, ANGD, W1, J3, W2, W3, JJ)

DO 230 J=1,3

D(J)=ANGD(J)
THM IS THE ANGLE BETWEEN AXIS F3 AND THE PLANE NORMAL TO THE
ANGULAR MOMENTUM VECTOR

CALL MAG(MAGH,H)
MAG=MAX(MAGH,1.0D-5)
MAG=SIGN(MAGH,H0(1))
STHM=H(3)/MAGH

TT IS TWICE THE ROTATIONAL ENERGY
HH IS THE SQUARE OF THE ANGULAR MOMENTUM
BBB IS THE INTERMEDIATE PRINCIPAL INERTIA
MODE DEFINES THE TYPE OF (RIGID BODY) MOTION:
MODE > 0 : ROD MODE (SOMERSAULT WITH TWIST)
MODE < 0 : DISC MODE (SOMERSAULT WITH WOBBLE)

CALL PM331(WFI,IV,ANGD)
CALL DOT(TT,WFI,HFI)
CALL DOT(HH,HFI,HFI)

F01AGF REDUCES THE SYMMETRIC INERTIA MATRIX IFF TO
TRIDIAGONAL FORM WITH LEADING DIAGONAL IN ABC AND OFF-
DIAGONAL ELEMENTS IN EE
F02AVF DETERMINES THE EIGENVALUES OF THE TRIDIAGONAL MATRIX
ON EXIT ABC CONTAINS THE EIGENVALUES IN ASCENDING ORDER
PARAMETERS: EPS=MACHINE ACCURACY
TOL=RMIN/EPS WHERE RMIN IS THE SMALLEST POSITIVE
NUMBER EXACTLY REPRESENTABLE ON THE COMPUTER
JJ=ERROR PARAMETER

J=0
N=3
TOL=1.2D-9736
EPS=5.7D-14

CALL TR(FFF,IFF)
CALL F01AGF(N,TOL,FFF,N,ABC,EE,EE)
CALL F02AVF(N,EPS,ABC,EE,JJ)
C        CCC=ABC(1)
2564     BBB=ABC(2)
2565     AAA=ABC(3)
2566 C
2567     IF(HH.LT.5.0D0) GO TO 240
2568     CALL DOT(HH0,H0,H0)
2569     HH0=SORT(HH0)
2570     IF(HH0.LT.5.0D0) GO TO 240
2571 C
2572     MODE=(BBB*TT-HH)/(AAA-CCC)/TT
2573     GO TO 250
2574 240  MODE=-3.0
2575 C
2576 *******************************************************************
2577 C
2578 FOR OPT=6 DETERMINE THE ANGLE PHP WHICH IS TWICE THE ANGLE
2579 BETWEEN F3 AND THE PRINCIPAL AXIS CORRESPONDING TO THE
2580 MINIMUM MOMENT OF INERTIA CCC
2581 NOTE THAT THE USER SPECIFIED BODY CONFIGURATIONS SHOULD BE
2582 SYMMETRICAL ABOUT THE SAGITTAL PLANE
2583 C
2584 *******************************************************************
2585 C
2586 250 BMC=IFF(2,2)+IFF(3,3)-2.0*CCC
2587     SPHP=(IFF(2,3)+IFF(3,2))/BMC
2588     CHP=(IFF(2,2)-IFF(3,3))/BMC
2589 C
2590 *******************************************************************
2591 C
2592 FOR OPT=7 DETERMINE THE ANGLE THP WHICH IS TWICE THE ANGLE
2593 BETWEEN F3 AND THE PRINCIPAL AXIS CORRESPONDING TO THE
2594 MINIMUM MOMENT OF INERTIA CCC
2595 NOTE THAT THE USER SPECIFIED BODY CONFIGURATIONS SHOULD BE
2596 SYMMETRICAL ABOUT THE FRONTAL PLANE
2597 C
2598 *******************************************************************
2599 C
2600 AMC=IFF(1,1)+IFF(3,3)-2.0*CCC
2601 STHP=(IFF(1,3)+IFF(3,1))/AMC
2602 CHP=(IFF(1,1)-IFF(3,3))/AMC
2603 C
2604 RETURN
2605 END
2606 C
2607 *******************************************************************
2608 C
2609 END OF SUBROUTINE DERIV
2610 C
2611 *******************************************************************
2612 C
RM1 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE X-AXIS WHERE C=COS(A) AND S=SIN(A)

```plaintext
SUBROUTINE B41 (R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = 1.0
R(1,2) = 0.0
R(1,3) = 0.0
R(2,1) = 0.0
R(2,2) = C
R(2,3) = S
R(3,1) = 0.0
R(3,2) = S
R(3,3) = C
RETURN
END
```

RM2 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE Y-AXIS WHERE C=COS(A) AND S=SIN(A)

```plaintext
SUBROUTINE RM2(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = 0.0
R(1,2) = 0.0
R(1,3) = -S
R(2,1) = 0.0
R(2,2) = 1.0
R(2,3) = 0.0
R(3,1) = S
R(3,2) = 0.0
R(3,3) = C
RETURN
END
```

RM3 IS THE MATRIX CORRESPONDING TO A ROTATION THROUGH AN ANGLE A ABOUT THE Z-AXIS WHERE C=COS(A) AND S=SIN(A)

```plaintext
SUBROUTINE RM3(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = C
R(1,2) = 0.0
R(1,3) = 0.0
R(2,1) = 0.0
R(2,2) = C
R(2,3) = S
R(3,1) = 0.0
R(3,2) = 0.0
R(3,3) = C
RETURN
END
```
SUBROUTINE R13(R, C, S)
DOUBLE PRECISION R(3,3), C, S
R(1,1) = C
R(1,2) = S
R(1,3) = 0.0
R(2,1) = -S
R(2,2) = C
R(2,3) = 0.0
R(3,1) = 0.0
R(3,2) = 0.0
R(3,3) = 1.0
RETURN
END

TR CALCULATES THE TRANSPOSE B OF A 3 X 3 MATRIX A

SUBROUTINE TR(B, A)
DOUBLE PRECISION A(3,3), B(3,3)
INTEGER J, K
DO 10 J = 1, 3
DO 10 K = 1, 3
10 B(J, K) = A(K, J)
RETURN
END

PM33 CALCULATES THE PRODUCT C OF THE 3 X 3 MATRICES A AND B

SUBROUTINE PM33(C, A, B)
DOUBLE PRECISION A(3,3), B(3,3), C(3,3), D(3,3), SUM
INTEGER I, J, K
DO 10 I = 1, 3
DO 20 J = 1, 3
SUM = 0.0
DO 30 K = 1, 3
30 SUM = SUM + A(I, K) * B(K, J)
DO 40 I = 1, 3
DO 40 J = 1, 3
40 C(I, J) = D(I, J)
RETURN
END
**SM33 Calculates the Sum C of the 3x3 matrices A and B**

**PM331 Calculates the Product C of the 3x3 matrix A and the 3x1 matrix B**

**SM31 Calculates the Sum C of the 3x1 matrices A and B**

```fortran
SUBROUTINE SM33(C, A, B)
  DOUBLE PRECISION A(3,3), B(3,3), C(3,3)
  INTEGER I, J
  DO 10 I=1,3
    DO 20 J=1,3
      20 C(I, J)=A(I, J)+B(I, J)
  10 CONTINUE
  RETURN
END

SUBROUTINE PM331(C, A, B)
  DOUBLE PRECISION A(3,3), B(3), C(3), D(3), SUM
  INTEGER I, K
  DO 10 I=1,3
    SUM=0.0
    DO 30 K=1,3
      30 SUM=SUM+A(I, K)*B(K)
    D(I)=SUM
  10 CONTINUE
  DO 40 I=1,3
    40 C(I)=D(I)
  RETURN
END

SUBROUTINE SM31(C, A, B)
  DOUBLE PRECISION A(3), B(3), C(3)
  INTEGER I
  DO 10 I=1,3
    C(I)=A(I)+B(I)
  10 CONTINUE
  RETURN
END
```
DM31 calculates the difference of the 3x1 matrices A and B.

SUBROUTINE DM31(C, A, B)
  DOUBLE PRECISION A(3), B(3), C(3)
  INTEGER J
  DO 10 J = 1, 3
    C(J) = A(J) - B(J)
  10 CONTINUE
RETURN
END

DOT(D, A, B) calculates the scalar product of two vectors A, B.

SUBROUTINE DOT(D, A, B)
  DOUBLE PRECISION A(3), B(3), D
  D = A(1) * B(1) + A(2) * B(2) + A(3) * B(3)
RETURN
END

VP calculates the vector product of vectors A and B.

SUBROUTINE VP(C, A, B)
  DOUBLE PRECISION A(3), B(3), C(3), D(3)
  INTEGER J
  D(1) = A(2) * B(3) - A(3) * B(2)
  D(2) = A(3) * B(1) - A(1) * B(3)
  D(3) = A(1) * B(2) - A(2) * B(1)
  DO 10 J = 1, 3
    C(J) = D(J)
  10 CONTINUE
RETURN
END
V3 calculates the vector V with components A, B, C

SUBROUTINE V3(V, A, B, C)
  DOUBLE PRECISION V(3), A, B, C
  V(1) = A
  V(2) = B
  V(3) = C
  RETURN
END

SCAVEC(VS, S, V) determines the product VS of a scalar S and a vector V

SUBROUTINE SCAVEC(VS, S, V)
  DOUBLE PRECISION VS(3), V(3), S
  VS(1) = S * V(1)
  VS(2) = S * V(2)
  VS(3) = S * V(3)
  RETURN
END

CEN(AB, MAB, A, MA, B, MB) calculates the centroid AB and mass MAB of two segments with centroids A, B and masses MA, MB

SUBROUTINE CEN(AB, MAB, A, MA, B, MB)
  DOUBLE PRECISION AB(3), A(3), B(3), AM1(3), BM2(3), MAB, MA, MB, M1, M2
  EXTERNAL SCAVEC, SM31
  MAB = MA + MB
  M1 = MA / MAB
  M2 = MB / MAB
  CALL SCAVEC(AM1, M1, A)
  CALL SCAVEC(BM2, M2, B)
  CALL SM31(AB, AM1, BM2)
  RETURN
END
CSANG calculates angle $A$ from its cosine $CA$ using the previous value $A_0$ and the sign of $\sin(A)$.

SUBROUTINE CSANG($A, A_0, SA, CA$)
DOUBLE PRECISION $A, SA, CA, A_0, SM, CM, B, SB, CB$
INTRINSIC ACOS, COS, SIN, SIGN, SQRT
IF (1.0 - CA**2.0 .GE. 0.0D0) $SA = \text{SIGN}(\text{SQRT}(1.0 - CA**2.0), SA)$
$CA_0 = \text{COS}(A_0)$
$SA_0 = \text{SIN}(A_0)$
$CB = A * CA_0 + SA * SA_0$
$SB = SA * CA_0 - CA * SA_0$
$B = \text{ACOS}(CB)$
$B = \text{SIGN}(B, SB)$
$A = A_0 + B$
$A_0 = A$
RETURN
END

ANGO calculates angle $A$ from its sine and cosine and sets the initial value $A_0$ for use in CSANG.

SUBROUTINE ANGO($A, A_0, SA, CA$)
DOUBLE PRECISION $A, A_0, TA, SA, CA, PI$
INTRINSIC ATAN
$PI = 3.1415926536$
$TA = SA / CA$
IF (CA .LT. 0.0D0) GO TO 10
$A = \text{ATAN}(TA)$
GO TO 20
10 $A = \text{ATAN}(TA) + PI$
20 $A_0 = A$
RETURN
END

...
C MAG(M, V) calculates the magnitude M of vector V

-- Subroutine MN, (M, V)
DOUBLE PRECISION M, V(3)
INTRINSIC SQRT

M = SQRT(V(1)**2 + V(2)**2 + V(3)**2)
RETURN
END

C UNIV(U, V) defines a unit vector U parallel to vector V

C SUBROUTINE UNIV(U, V)
DOUBLE PRECISION U(3), V(3), M
EXTERNAL MAG

CALL MAG(M, V)
U(1) = V(1)/M
U(2) = V(2)/M
U(3) = V(3)/M
RETURN
END

C IM calculates the inertia matrix I with principal moments equal to A, B, C

C SUBROUTINE IM(I, A, B, C)
DOUBLE PRECISION I(3, 3), A, B, C

I(1, 1) = A
I(1, 2) = 0.0
I(1, 3) = 0.0
I(2, 1) = 0.0
I(2, 2) = B
I(2, 3) = 0.0
I(3, 1) = 0.0
I(3, 2) = 0.0
I(3, 3) = C
RETURN
END
PA calculates the inertia matrix I of a point mass m with the position vector P.

I is the transfer term occurring in the theorem of parallel axes.

Subroutine PA(I,P,M)

DOUBLE PRECISION I(3,3),P(3),M

I(1,1)=(P(2)**2+P(3)**2)*M
I(1,2)=-(P(1)*P(2)*M)
I(2,1)=I(1,2)
I(1,3)=P(1)*P(3)*M
I(3,1)=I(1,3)
I(2,2)=(P(1)**2+P(3)**2)*M
I(2,3)=-(P(2)*P(3)*M)
I(3,2)=I(2,3)
I(3,3)=(P(1)**2+P(2)**2)*M

RETURN
END

Function ANG is a monotonic function defined on the interval [T0,T1] with first and second derivatives equal to zero at the end-points.

DOUBLE PRECISION FUNCTION ANG(F0,F1,T0,T1,T)

DOUBLE PRECISION F0,F1,T0,T1,T,Z,PI

INTRINSIC SIN

PI=3.1415926536
Z=(T-T0)/(T1-T0)
ANG=F0+(F1-F0)*(Z-SIN(2.0*PI*Z)/2.0/PI)
ANG=ANG*PI/180.0
RETURN
END
ANGC IS THE MONOTONIC CUBIC ON THE INTERVAL \([T_0, T_1]\) WHICH TAKES END-POINT VALUES \(F_0\) AND \(F_1\) AND WHICH HAS ZERO FIRST DERIVATIVES AT THE END-POINTS.

DOUBLE PRECISION FUNCTION ANGC(F0, F1, T0, T1, T)

DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI

PI = 3.1415926536

Z = (T - T0) / (T1 - T0)

ANGC = F0 + (F1 - F0) * (3.0 * Z**2 - 2.0 * Z**3)

RETURN

END

ANGQ IS THE MONOTONIC QUINTIC ON THE INTERVAL \([T_0, T_1]\) WHICH TAKES END-POINT VALUES \(F_0\) AND \(F_1\) AND WHICH HAS ZERO FIRST AND SECOND DERIVATIVES AT THE END-POINTS.

DOUBLE PRECISION FUNCTION ANGQ(F0, F1, T0, T1, T)

DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI

PI = 3.1415926536

Z = (T - T0) / (T1 - T0)

ANGQ = F0 + (F1 - F0) * (6.0 * Z**5 - 15.0 * Z**4 + 10.0 * Z**3)

ANGQ = ANGQ * PI / 180.0

RETURN

END

ANGL IS THE LINEAR FUNCTION ON THE INTERVAL \([T_0, T_1]\) WHICH HAS ENDPOINT VALUES \(F_0\) AND \(F_1\).

DOUBLE PRECISION FUNCTION ANGL(F0, F1, T0, T1, T)

DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI

PI = 3.1415926536

Z = (T - T0) / (T1 - T0)

ANGL = F0 + (F1 - F0) * Z

ANGL = ANGL * PI / 180.0

RETURN

END
SUBROUTINE VARANG(F, CC, T)
DOUBLE PRECISION CC(4), F, F0, F1, T0, T1, T, ANGQ, ANGL, PI
INTEGER OPT

C FOR T<T0 VARANG LEAVES THE VALUE OF F UNCHANGED
C FOR T0<T<T1 F=ANGC
C FOR T>T1 F=F1

SUBROUTINE VARANG(F, CC, T)

DOUBLE PRECISION CC(4), F, F0, F1, T0, T1, T, ANGQ, ANGL, PI
INTEGER OPT

COMMON/B14/OPT

F0=CC(1)
F1=CC(2)
T0=CC(3)
T1=CC(4)
PI=3.1415926536

IF (T .LT. T0) GO TO 500
IF (T .LT. T1) GO TO 200
F=F1*PI/180.0
GO TO 500

200 F=ANGQ(F0, F1, T0, T1, T)
IF (OPT .EQ. 6 .OR. OPT .EQ. 7) F=ANGL(F0, F1, T0, T1, T)
CONTINUE
RETURN
END

C
C*********************************************************************************
C DANG IS THE DERIVATIVE OF THE TRIGONOMETRIC FUNCTION ANG
C*********************************************************************************

DOUBLE PRECISION FUNCTION DANG(F0, F1, T0, T1, T)

INTRINSIC COS

PI=3.1415926536
Z=(T-T0)/(T1-T0)
DANG=(PI/180.0)*(F1-F0)*(1.0*COS(2.0*PI*Z))/(T1-T0)
RETURN
END
DANGC IS THE DERIVATIVE OF THE CUBIC ANGC

DOUBLE PRECISION FUNCTION DANGC(F0,F1,T0,T1,T)

DOUBLE PRECISION F0,F1,T0,T1,T,Z,PI

PI=3.1415926536
Z=(T-T0)/(T1-T0)
DANGC=(PI/180.0)*(F1-F0)*6.0*(Z-Z**2)/(T1-T0)
RETURN
END

DANQ IS THE DERIVATIVE OF THE QUINTIC ANGQ

DOUBLE PRECISION FUNCTION DANQ(F0,F1,T0,T1,T)

DOUBLE PRECISION F0,F1,T0,T1,T,Z,PI

PI=3.1415926536
Z=(T-T0)/(T1-T0)
DANQ=(PI/180.0)*(F1-F0)*30.0*(Z**2-Z)**2/(T1-T0)
RETURN
END

DANGL IS THE (CONSTANT) DERIVATIVE OF THE LINEAR FUNCTION ANGL

DOUBLE PRECISION FUNCTION DANGL(F0,F1,T0,T1,T)

DOUBLE PRECISION F0,F1,T0,T1,T,PI

PI=3.1415926536
DANGL=(PI/180.0)*(F1-F0)/(T1-T0)
RETURN
END
**Vdang** defines derivatives in the same way that varang defines functions.

```fortran
SUBROUTINE VDANG(F, CC, T)
DOUBLE PRECISION CC(4), F, F0, F1, T0, T1, T, DANGQ, DANGL
INTEGER OPT
COMMON/B14/OPT

FO=CC(1)
F1=CC(2)
T0=CC(3)
T1=CC(4)
IF (T.LT.T0) GO TO 500
IF (T.LT.T1) GO TO 200
F=0.0
GO TO 500
200 F=DANGQ(F0, F1, T0, T1, T)
IF(OPT.EQ.6.OR.OPT.EQ.7) F=DANGL(F0, F1, T0, T1, T)
CONTINUE
RETURN
END
```

**D2ang** is the second derivative of the function ang.

```fortran
DOUBLE PRECISION FUNCTION D2ANG(FQ1, F1, T0, T1, T)
DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI
INTRINSIC SIN

PI=3.1415926536
Z=(T-T0)/(T1-T0)
D2ANG=(PI/180.0)*(F1-F0)*2.0*PI*SIN(2.0*PI*Z)/(T1-T0)**2
RETURN
END
```
D2ANGC IS THE SECOND DERIVATIVE OF THE CUBIC ANGC

DOUBLE PRECISION FUNCTION D2ANGC(F0, F1, T0, T1, T)

DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI

PI = 3.1415926536

Z = (T - T0) / (T1 - T0)

D2ANGC = (PI / 180.0) * (F1 - F0) * 6.0 * (1.0 - 2.0 * Z) / (T1 - T0)**2

RETURN

END

D2ANGQ IS THE SECOND DERIVATIVE OF THE QUINTIC ANGQ

DOUBLE PRECISION FUNCTION D2ANGQ(F0, F1, T0, T1, T)

DOUBLE PRECISION F0, F1, T0, T1, T, Z, PI

PI = 3.1415926536

Z = (T - T0) / (T1 - T0)

D2ANGQ = (F1 - F0) * 60.0 * (2.0 * Z**3 - 3.0 * Z**2 + 2.0 * Z) / (T1 - T0)**2

RETURN

END

EULPNG CALCULATES THE THREE EULERIAN ANGLES EUL(I) FROM

THE TRANSFORMATION MATRIX S

ANGLES EUL(I) ARE SUCCESSIVE ROTATIONS ABOUT THE Z, Y, Z AXES

AND ARE USED BY THE SAMMIE GRAPHICS SYSTEM

EULANG CALCULATES THE THREE EULERIAN ANGLES EUL(I) FROM
SUBROUTINE EULANG(EUL, S)
DOUBLE PRECISION EUL(3), S(3, 3), CE1, SE1, CE2, SE2, CE3, SE3
EXTERNAL ANG1
INTRINSIC SQRT
CE2=S(3, 3)
SE2=SQRT(S(1, 3)**2+S(2, 3)**2)
IF(SE2.LT.0.01D0) GO TO 10
CE3=-S(1, 3)/SE2
SE3=S(2, 3)/SE2
CE1=S(3, 1)/SE2
SE1=S(3, 2)/SE2
GO TO 20
CE3=S(1, 1)
SE3=S(1, 2)
IF(CE2.LT.0.0D0) CE3=-CE3
CE1=1.0
SE1=0.0
CALL ANG1(EUL(1), SE1, CE1)
CALL ANG1(EUL(2), SE2, CE2)
CALL ANG1(EUL(3), SE3, CE3)
RETURN
END

C*******************************************************************
C ANG1 CALCULATES ANGLE A (-PI < A < PI) FROM ITS SINE AND
COSINE
C**************************************************r*****************
SUBROUTINE ANG1(A, SA, CA)
DOUBLE PRECISION A, SA, CA, PI
INTRINSIC ABS, ACOS, SIGN
PI=3.1415926536
IF(ABS(CA).GT.1.0D0) CA=SIGN(1.0D0, CA)
A=ACOS(CA)
IF(SA.LT.0.0D0) A=-A
RETURN
END
**HRL Calculates the Angular Momentum Associated with the Angular Velocity W of One Segment Relative to Another**

**SUBROUTINE HRL(H, I, M, A, B, W)**

- DOUBLE PRECISION H(3), I(3,3), A(3), B(3), W(3), H1(3), H2(3)
- EXTERNAL PM331, SCAVEC, SM31, VP

- CALL PM331(H1, I, W)
- CALL VP(H2, W, B)
- CALL VP(H2, A, H2)
- CALL SCAVEC(H2, M, H2)
- CALL SM31(H, H1, H2)

**RETURN**
**END**

**VALQ2 Evaluates a Quintic Spline and Its First Derivative**

**SUBROUTINE VALQ2(SP, T, CCF, N, K)**

- DOUBLE PRECISION CCF(6, N), K(N), T, SP(2), QSPLIN, QDSPLN
- INTEGER N

- SP(1) = QSPLIN(N, K, CCF, T)
- SP(2) = QDSPLN(N, K, CCF, T, 1)

**RETURN**
**END**

**JENNINGS QUINTIC SPLINE FITTING LIBRARY**

**THE SPLINE FITTING LIBRARY WILL FIT A QUINTIC SPLINE TO DATA IN THE SENSE DESCRIBED BY REINSCH 1967, NUMERISCHE MATHEMATIK 10, PP177-183.**

**THE LIBRARY CONSISTS OF THE FOLLOWING SUBROUTINES AND FUNCTIONS:-**
LSQSP - USER ENTRY TO QUINTIC SPLINE CALCULATION.
QFUNCP - INTERNAL ROUTINE.
QSPLIN - VALUE OF QUINTIC SPLINE.
QDSPLN - DERIVATIVES OF QUINTIC SPLINE.
QINTSP - INTEGRAL OF QUINTIC SPLINE.

INTERNAL ROUTINES
QFUNCP
ZEROSP
SBFAC
SBSOL
SBFSUB
SBSSUB

THERE IS A SEPARATE WRITEUP ON HOW TO USE THE LIBRARY.
ANY QUERIES SHOULD BE DIRECTED TO DR. L.S. JENNINGS AT
MATHEMATICS DEPARTMENT,
UNIVERSITY OF WESTERN AUSTRALIA.

******************************************************************
SUBROUTINE LSQSP(T,Y,DY,S,N,CC,FLAG) CALCULATES THE QUINTIC
SPLINE COEFFICIENTS CC FOR A SEQUENCE OF DATA POINTS (T,Y)
PARAMETERS:
N: INTEGER
N=NUMBER OF DATA POINTS
T:DP ARRAY DIMENSION N
T CONTAINS THE VALUES OF THE INDEPENDENT VARIABLE
T COMPRISES THE KNOT SET
Y:DP ARRAY DIMENSION N
Y CONTAINS THE VALUES OF THE DEPENDENT VARIABLE
DY:DP ARRAY DIMENSION N
DY CONTAINS VALUES OF THE STANDARD ERRORS OF Y
S:DP
S IS THE PARAMETER OF REINSCH
ON INPUT S IS USUALLY SET TO N FOR THE SMOOTHEST
FIT CONSISTENT WITH THE BOUNDED SUM OF SQUARES
S IS SET TO ZERO TO OBTAIN THE INTERPOLATING
SPLINE
CC:DP ARRAY DIMENSION (6,N)
ON EXIT CC CONTAINS QUINTIC SPLINE COEFFICIENTS
FLAG:LOGICAL
FLAG INDICATES ERROR AS .FALSE.
FUNCTION QSPLIN(N,K,CC,T1) EVALUATES THE QUINTIC SPLINE
OF N POINTS WITH KNOT SET K(N) AND COEFFICIENTS CC(6,N)
AT TIME T1

FUNCTION QDSPLN(N,K,CC,T1,J) EVALUATES THE JTH
DERIVATIVE OF THE QUINTIC SPLINE WITH COEFFICIENTS CC(6,N)
AND KNOT SET K(N) AT TIME T1

NAG LIBRARY SUBROUTINE

D02ABF(T,Y,G,J1,J2,JJ,INT,ESL,DERIV,W1,W2,W3,W4,W5,W6)
SOLVES THE DIFFERENTIAL EQUATIONS
D/DT(Y(1))=D(1)
D/DT(Y(2))=D(2)
D/DT(Y(3))=D(3)
FOR Y(1),Y(2),Y(3)

D(1),D(2),D(3) ARE SPECIFIED IN SUBROUTINE DERIV(D,Y,T)

THE ROUTINE ADVANCES THE SOLUTION OF THE DIFFERENTIAL
EQUATIONS FROM T TO TINT USING A NUMBER OF STEPS OF
MERSON'S FORM OF THE RUNGE-KUTTA METHOD.

THE SYSTEM IS DEFINED BY A SUBROUTINE DERIV (SUPPLIED BY
THE USER) WHICH EVALUATES THE DERIVATIVES D(1),D(2),D(3)
IN TERMS OF T AND Y(1),Y(2),Y(3).

THE ROUTINE OBTAINS AN ESTIMATE OF THE LOCAL TRUNCATION
ERROR AT EACH STEP, AND VARIES THE STEP-LENGTH TO KEEP
THIS ESTIMATE BELOW AN ERROR BOUND SPECIFIED BY THE USER.
PARAMETERS:

T: DOUBLE PRECISION (DP)
BEFORE ENTRY T MUST BE SET TO THE INITIAL VALUE
OF TIME T AND ON EXIT IT WILL CONTAIN T+INT

Y: DP ARRAY DIMENSION J2
BEFORE ENTRY Y MUST BE SET TO THE INITIAL VALUES
OF Y(1), Y(2), Y(3) AND ON EXIT IT WILL CONTAIN
THE COMPUTED VALUES AT T+INT

G: DP ARRAY DIMENSION J2
BEFORE ENTRY G MUST BE SET TO ERROR BOUNDS
SPECIFIED BY THE USER FOR Y(I), I=1, J2

J1: INTEGER
J1 DEFINES THE TYPE OF ERROR TEST
J1=1 GIVES A MIXED TEST:
IF THE LOCAL ERROR IN Y(I) IS ESTIMATED AS E(I)
ABS(E(I)) < G(I)*(1+ABS(Y(I)))

J2: INTEGER
ON ENTRY J2 MUST CONTAIN THE NUMBER OF
DIFFERENTIAL EQUATIONS

JJ: INTEGER
BEFORE ENTRY JJ SHOULD BE SET TO ZERO.
UNLESS AN ERROR IS DETECTED JJ=0 ON EXIT

INT: DP
ON ENTRY INT MUST CONTAIN THE INTERVAL OVER
WHICH INTEGRATION IS REQUIRED

ESL: DP
ON ENTRY ESL MUST BE SET TO AN ESTIMATE OF THE
STEP-LENGTH NEEDED FOR INTEGRATION

DERIV: SUBROUTINE (SUPPLIED BY THE USER)
DERIV EVALUATES THE DERIVATIVES OF Y(1), Y(2), Y(3)
AND PLACES THEM IN D(1), D(2), D(3).
DERIV MUST BE DECLARED AS EXTERNAL IN THE PROGRAM
FROM WHICH DO2ABF IS CALLED

W1-W6: ARRAYS OF DIMENSION J2 USED AS WORKING SPACE
SUBROUTINE M4ATF(I, J1, H, J2, A, W1, J3, W2, W3, JJ) SOLVES THE MATRIX EQUATION \( I \cdot A = H \) FOR ANGD.

THE ROUTINE USES CROUT'S METHOD WITH PARTIAL PIVOTING TO DECOMPOSE THE MATRIX I INTO TRIANGULAR FORM \( I = L \cdot U \) WHERE L IS LOWER TRIANGULAR AND U IS UPPER TRIANGULAR.

AN APPROXIMATION TO A IS FOUND BY FORWARD AND BACKWARD SUBSTITUTION IN L. B = H AND U. A = B.

THE RESIDUAL VECTOR \( R = H - I \cdot A \) IS THEN CALCULATED AND A CORRECTION (C) TO A IS FOUND BY THE SOLUTION OF \( L \cdot U \cdot C = R \).

A IS REPLACED BY \( (A + C) \) AND THE PROCESS REPEATED UNTIL FULL MACHINE ACCURACY IS OBTAINED.

ADDITIONAL PRECISION ACCUMULATION OF INNER-PRODUCTS IS USED THROUGHOUT THE CALCULATION.

PARAMETERS:

- **I**: REAL ARRAY OF DIMENSION \((J1, P)\), \(P > J2\).
  - BEFORE ENTRY I SHOULD CONTAIN THE ELEMENTS OF THE MATRIX.

- **J1**: INTEGER
  - ON ENTRY J1 SPECIFIES THE FIRST DIMENSION OF ARRAY I AS DECLARED IN THE CALLING SUB-PROGRAM (DERIV).

- **H**: DP ARRAY OF DIMENSION AT LEAST J2
  - BEFORE ENTRY H SHOULD CONTAIN THE ELEMENTS OF THE RIGHT HAND SIDE.

- **J2**: INTEGER
  - ON ENTRY J2 SPECIFIES THE ORDER OF MATRIX I.

- **A**: DP ARRAY OF DIMENSION AT LEAST J2
  - ON SUCCESSFUL EXIT A WILL CONTAIN THE SOLUTION VECTOR.

- **W1**: DP ARRAY OF DIMENSION \((J3, Q)\) WHERE \(Q > J2\)
  - USED AS WORKING SPACE.

- **J3**: INTEGER
  - J3 SPECIFIES THE FIRST DIMENSION OF ARRAY W1 AS DECLARED IN THE CALLING SUB-PROGRAM.

- **W2**: DP ARRAYS OF DIMENSION AT LEAST J2
  - USED AS WORKING SPACE.

- **JJ**: INTEGER
  - BEFORE ENTRY JJ SHOULD BE SET TO ZERO UNLESS AN ERROR IS DETECTED JJ = 0 ON EXIT.
NAG LIBRARY SUBROUTINE

SUBROUTINE F01AGF(N, TOL, A, IA, ABC, EE, EE)
GIVES THE HOUSEHOLDER REDUCTION OF A REAL SYMMETRIC MATRIX TO TRIDIAGONAL FORM FOR USE IN F02AVF

PARAMETERS:
N: INTEGER
   N SPECIFIES THE ORDER OF MATRIX A
TOL: DP
   TOL = RMIN/EPS WHERE:
   RMIN IS THE SMALLEST POSITIVE NUMBER EXACTLY
   REPRESENTABLE ON THE COMPUTER
   EPS IS THE SMALLEST POSITIVE NUMBER SUCH THAT:
     0 + EPS > 1.0 ON THE COMPUTER
A: DP ARRAY OF DIMENSION (IA,Q) WHERE Q.GE.N
   BEFORE ENTRY A MUST CONTAIN THE ELEMENTS OF THE
   REAL SYMMETRIC MATRIX
   ON EXIT A IS OVERWRITTEN
IA: INTEGER
   IA SPECIFIES THE FIRST DIMENSION OF ARRAY A
ABC: DP ARRAY OF DIMENSION AT LEAST N
   ON EXIT ABC CONTAINS THE DIAGONAL ELEMENTS
   OF THE TRIDIAGONAL MATRIX
EE: DP ARRAY OF DIMENSION AT LEAST N
   ON EXIT EE CONTAINS THE N-1 OFF DIAGONAL ELEMENTS
IlC NAG LIBRARY SUBROUTINE

SUBROUTINE F02AVF(N, EPS, ABC, EE, JJ) CALCULATES THE
EIGENVALUES OF A REAL SYMMETRIC TRIDIAGONAL MATRIX

PARAMETERS:

N : INTEGER
   N SPECIFIES THE ORDER OF THE MATRIX

EPS : REAL
   EPS IS THE SMALLEST POSITIVE NUMBER SUCH THAT:
      EPS > 1.0 ON THE COMPUTER

ABC : REAL ARRAY OF DIMENSION N
   ON ENTRY ABC MUST CONTAIN THE DIAGONAL ELEMENTS
   OF THE TRIDIAGONAL MATRIX
   ON EXIT ABC CONTAINS THE EIGENVALUES IN ASCENDING
   ORDER

EE : REAL ARRAY OF DIMENSION N
   ON ENTRY EE(2) TO EE(N) CONTAIN THE SUB-DIAGONAL
   ELEMENTS OF THE TRIDIAGONAL MATRIX

END OF PROGRAM SIM
APPENDIX E

Comparison of film and simulation for ten movements using tables, graphs and computer graphics

Contents:

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<thead>
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<th>movement</th>
<th>table</th>
<th>graph</th>
<th>graphics</th>
</tr>
</thead>
<tbody>
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<td>G02</td>
<td>E1</td>
<td>E1a</td>
<td>E1b</td>
</tr>
<tr>
<td>G08</td>
<td>E2</td>
<td>E2a</td>
<td>E2b</td>
</tr>
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<td>E3b</td>
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<td>E5</td>
<td>E4a</td>
<td>E4b</td>
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<td>E6b</td>
</tr>
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<td>E7</td>
<td>E7a</td>
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<td>E8</td>
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<td>E8b</td>
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<tr>
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<td>E10a</td>
<td>E10b</td>
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</table>
Table El. Comparison of simulation and film for G02

SIMULATION : G02  100 INTEGRATION STEPS  PERFORMER : GILL

COMPONENTS OF MOMENTUM :  \( H_x = -0.03 \)  \( H_y = -0.01 \)  \( H_z = -0.01 \)

<table>
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<tr>
<th>TIME</th>
<th>SOMERSAULT REVOLUTIONS</th>
<th>TILT DEGREES</th>
<th>TWIST DEGREES</th>
<th>MODE</th>
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<td>T</td>
<td>PH SIM</td>
<td>PHF FILM</td>
<td>TH SIM</td>
<td>THF FILM</td>
</tr>
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<td>0.0</td>
<td>0.00 0.00</td>
<td>0.00 0.00</td>
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<td>0.00 0.00</td>
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<td>0.00 0.00</td>
<td>0. 0.</td>
<td>-0.01 -0.01</td>
<td>***</td>
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<tr>
<td>0.2</td>
<td>0.00 -0.01</td>
<td>-1. -1.</td>
<td>0.01 -0.01</td>
<td>***</td>
</tr>
<tr>
<td>0.3</td>
<td>-0.01 -0.02</td>
<td>-3. -1.</td>
<td>-0.01 -0.01</td>
<td>***</td>
</tr>
<tr>
<td>0.4</td>
<td>-0.02 -0.03</td>
<td>-5. -3.</td>
<td>-0.02 -0.01</td>
<td>***</td>
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time

twist in revolutions

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time

Figure Ela. Graphical comparison of film and simulation for G02
(ooooo : film data ; —— : simulation)
Figure Elb. Computer graphics of film and simulation for G02
Table E2. Comparison of simulation and film for G08

SIMULATION : G08  100 INTEGRATION STEPS  PERFORMER : GILL

COMPONENTS OF MOMENTUM :  
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time

tilt in degrees

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time

twist in revolutions

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time

Figure E2a. Graphical comparison of film and simulation for G08
(ooooo : film data ; — : simulation)
Figure E2b. Computer graphics of film and simulation for G08
Table E3. Comparison of simulation and film for G12

**SIMULATION : G12**  
100 INTEGRATION STEPS  
**PERFORMER : GILL**

**COMPONENTS OF MOMENTUM:**  
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Figure E3a. Graphical comparison of film and simulation for G12
(ooooo : film data ; --- : simulation)
Figure E3b. Computer graphics of film and simulation for G12
Table E4. Comparison of simulation and film for J71

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COMPONENTS OF MOMENTUM :  \( h_x = 0.86 \)  \( h_y = 0.01 \)  \( h_z = -0.01 \)

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Figure E4a. Graphical comparison of film and simulation for J71
(ooooo : film data ; — : simulation)
Figure E4b. Computer graphics of film and simulation for J71
Table E5. Comparison of simulation and film for J73

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**PERFORMER : JOHN**

**COMPONENTS OF MOMENTUM :**  
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Figure E5a. Graphical comparison of film and simulation for J73
(ooooo : film data ; ——— : simulation)
Figure E5b. Computer graphics of film and simulation for J73
Table E6. Comparison of simulation and film for C11

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**PERFORMER : CARL**

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Figure E6a. Graphical comparison of film and simulation for C11

(ooooo : film data ; —— : simulation)
Figure E6b. Computer graphics of film and simulation for C11
Table E7. Comparison of simulation and film for C39

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Figure E7a. Graphical comparison of film and simulation for C39

(ooooo : film data ; ——— : simulation)
Figure E7b. Computer graphics of film and simulation for C39
Table E8. Comparison of simulation and film for C41

SIMULATION: C41 100 INTEGRATION STEPS  PERFORMER: CARL

COMPONENTS OF MOMENTUM:  \( H_X = 1.39 \)  \( H_Y = -0.04 \)  \( H_Z = 0.02 \)

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Figure E8a. Graphical comparison of film and simulation for C41
(ooooo : film data ; ——— : simulation)
Figure E8b. Computer graphics of film and simulation for C41
Table E9. Comparison of simulation and film for C45

SIMULATION : C45  100 INTEGRATION STEPS  PERFORMER : CARL

COMPONENTS OF MOMENTUM :  HX = -1.37  HY = 0.00  Hz = 0.02

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Figure E9a. Graphical comparison of film and simulation for C45
(ooooo : film data ; --- : simulation)
Figure E9b. Computer graphics of film and simulation for C45
Table E10. Comparison of simulation and film for C47

**SIMULATION : C47** 100 INTEGRATION STEPS  **PERFORMER : CARL**

**COMPONENTS OF MOMENTUM :**  
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**RIGHT ARM**

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Figure El0a. Graphical comparison of film and simulation for C47

(ooooo : film data ; —— : simulation)

somersault in revolutions

tilt in degrees

twist in revolutions

(time)

(time)

(time)
Figure El0b. Computer graphics of film and simulation for C47
LIST OF REFERENCES


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McDonald, D. 1960. How Does a Cat Fall on its Feet? New Scientist 7,(189),1647-1649.


Orner, W. 1959. I Was Doubly Dismayed... Swimming Times 36,9,258.


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