Performance assessment of fighter aircraft incorporating advanced technologies

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PERFORMANCE ASSESSMENT OF FIGHTER AIRCRAFT INCORPORATING ADVANCED TECHNOLOGIES

by

Antony Kutschera

A Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

September 2000
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Thanks to you all!
ABSTRACT

The performance assessment of modern combat aircraft has been the subject of considerable research in recent years. This thesis considers the performance assessment at the preliminary design stage in an aircraft’s life, when the amount of data available for calculating metrics will be limited. Conventional metrics do not completely assess agile aircraft. In particular the advantages of future aircraft implementing Thrust Vectoring Control (TVC) and Post Stall Maneuvrability (PSM) are not shown by conventional metrics. This thesis reviews the suitability of both conventional and proposed metrics for assessing TVC and PSM. It suggests an extension to existing point performance metrics and then considers a new metric which can be evaluated early in the design cycle. The new metric is analysed via a sensitivity study and a validation study to show that the new metric is suitable. Results and design studies are also included which quantify the changes which are the result of TVC/PSM. Further extensions to the new metric are also briefly considered. Several important results which disagree with traditional thinking are highlighted.
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# Nomenclature

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<th>Acronym</th>
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<tr>
<td>AA</td>
<td>Axial Agility</td>
</tr>
<tr>
<td>AFFTC</td>
<td>Air Force Flight Test Center</td>
</tr>
<tr>
<td>AoA</td>
<td>Angle of Attack</td>
</tr>
<tr>
<td>CCT</td>
<td>Combat Cycle Time</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>Mean aerodynamic chord</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Drag Coefficient</td>
</tr>
<tr>
<td>$C_{D_0}$</td>
<td>Profile Drag Coefficient</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Rolling Moment Coefficient</td>
</tr>
<tr>
<td>$C_L$</td>
<td>Lift Coefficient</td>
</tr>
<tr>
<td>CLFLA</td>
<td>Continuously Load Factor Limited Area</td>
</tr>
<tr>
<td>$C_M$</td>
<td>Pitching Moment Coefficient</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Yawing Moment Coefficient</td>
</tr>
<tr>
<td>$C_Y$</td>
<td>Side Force Coefficient</td>
</tr>
<tr>
<td>D</td>
<td>Drag</td>
</tr>
<tr>
<td>DASA</td>
<td>Deutsche Aerospace</td>
</tr>
<tr>
<td>DERA</td>
<td>Defence Evaluation &amp; Research Agency</td>
</tr>
<tr>
<td>DoF</td>
<td>Degree of Freedom</td>
</tr>
<tr>
<td>DST</td>
<td>Dynamic Speed Turn</td>
</tr>
<tr>
<td>$e$</td>
<td>Error function</td>
</tr>
<tr>
<td>EFM</td>
<td>Enhanced Fighter Manoeuvrability</td>
</tr>
<tr>
<td>FCS</td>
<td>Flight Control System</td>
</tr>
<tr>
<td>$g$</td>
<td>Acceleration due to gravity</td>
</tr>
<tr>
<td>GS</td>
<td>Gain Scheduling</td>
</tr>
<tr>
<td>$h_e$</td>
<td>Energy Height</td>
</tr>
<tr>
<td>HARV</td>
<td>High Alpha Research Vehicle</td>
</tr>
<tr>
<td>HQ</td>
<td>Handling Qualities</td>
</tr>
<tr>
<td>HUD</td>
<td>Head Up Display</td>
</tr>
<tr>
<td>ITR</td>
<td>Instantaneous Turn Rate</td>
</tr>
<tr>
<td>$I_y$</td>
<td>Moment of inertia about the body pitch axis</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Generic gain</td>
</tr>
<tr>
<td>L</td>
<td>Lift</td>
</tr>
<tr>
<td>m</td>
<td>Mass</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>$\bar{M}$</td>
<td>Mach number</td>
</tr>
<tr>
<td>M</td>
<td>Pitching Moment</td>
</tr>
<tr>
<td>MBB</td>
<td>Messerschmitt, Boelkow, Blohm.</td>
</tr>
<tr>
<td>MSB</td>
<td>Manoeuvre Stall Boundary</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration</td>
</tr>
<tr>
<td>$n_x$</td>
<td>Axial load factor</td>
</tr>
<tr>
<td>$n_z$</td>
<td>Normal load factor</td>
</tr>
<tr>
<td>p</td>
<td>Body roll rate</td>
</tr>
<tr>
<td>$p_e$</td>
<td>Roll rate about velocity vector</td>
</tr>
<tr>
<td>P+D</td>
<td>Proportional plus Derivative</td>
</tr>
<tr>
<td>P+I+D</td>
<td>Proportional plus Integral plus Derivative</td>
</tr>
<tr>
<td>PLP</td>
<td>Power Loss Parameter</td>
</tr>
<tr>
<td>POP</td>
<td>Power On Parameter</td>
</tr>
<tr>
<td>$P_S$</td>
<td>Specific Excess Power</td>
</tr>
<tr>
<td>PSM</td>
<td>Post Stall Manoeuvre</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>( q )</td>
<td>Body pitch rate</td>
</tr>
<tr>
<td>( \bar{q} )</td>
<td>Dynamic pressure</td>
</tr>
<tr>
<td>( r )</td>
<td>Body yaw rate</td>
</tr>
<tr>
<td>RES</td>
<td>Relative Energy State</td>
</tr>
<tr>
<td>RNDI</td>
<td>Robust Non-linear Dynamic Inversion</td>
</tr>
<tr>
<td>( \text{stsl} )</td>
<td>Static Thrust, Sea Level</td>
</tr>
<tr>
<td>( S )</td>
<td>Reference (wing) area</td>
</tr>
<tr>
<td>SEP</td>
<td>Specific Excess Power</td>
</tr>
<tr>
<td>STR</td>
<td>Sustained Turn Rate</td>
</tr>
<tr>
<td>( t )</td>
<td>time</td>
</tr>
<tr>
<td>( T )</td>
<td>Thrust</td>
</tr>
<tr>
<td>TVC</td>
<td>Thrust Vector Control</td>
</tr>
<tr>
<td>( V )</td>
<td>Velocity</td>
</tr>
<tr>
<td>( V_C )</td>
<td>Corner Velocity</td>
</tr>
<tr>
<td>( W )</td>
<td>Weight</td>
</tr>
<tr>
<td>( X )</td>
<td>Down range distance</td>
</tr>
<tr>
<td>( Y )</td>
<td>Cross range distance</td>
</tr>
<tr>
<td>( Z )</td>
<td>Altitude</td>
</tr>
</tbody>
</table>

\( \alpha \) Angle of Attack
\( \beta \) Angle of Side Slip
\( \bar{c} \) mean aerodynamic chord
\( \gamma \) Flight path angle, measured upwards between horizon and velocity vector
\( \delta \) Control Deflection
\( \mu \) Roll angle, measured about the velocity vector
\( \chi \) Heading angle, measured between the initial heading (taken as zero) and the velocity vector
\( \rho \) Air density
\( \phi \) Roll angle
\( \psi \) Horizontal Turn Rate

**Superscripts**
- \( \cdot \) Infers a commanded value
- \( \cdot \) Derivative with respect to time

**Subscripts**
- \( a \) aileron
- \( e \) elevator
- \( \text{err} \) error value
- \( l \) left
- \( r \) rudder, right
- TVC thrust vector control
CHAPTER 1

INTRODUCTION

1.1 Background

One of the aircraft to fly at the 1995 Paris Air Show was the X-31a, an aircraft stemming from a project to investigate Enhanced Fighter Manoeuvrability (EFM), funded mainly by NASA and Deutsche Aerospace (DASA). This aircraft enhanced manoeuvrability with the use of thrust vectoring, and thrilled spectators by performing manoeuvres like Herbst manoeuvres, J turns, the Mongoose, etc \(^{[1]}\).

The X-31a project incorporated simulated combat between the X-31a and an F/A-18. The X-31a was first flown with the thrust vectoring switched off, and according to Smith\(^{[2]}\), tended to kill only once in every 2.5 sorties flown. The thrust vectoring was then switched on, and the kill ratio went as high as 32:1 in favour of the X-31a\(^{[1]}\).

Up to 1996 there were no combat ready aircraft that incorporated thrust vectoring. However at the Farnborough Air Show in September 1996, the Russians displayed the Sukhoi Su-37 which also implemented thrust vectoring. This demonstrated that thrust vectoring as implemented on these two aircraft was now ready to be used in combat.

The question that arose from the above information was a simple one. If thrust vectoring had been proven to enhance fighter manoeuvrability using combat simulation, and it was already being implemented on combat ready aircraft, can the improvements be quantified on paper, without combat having to take place first? If two aircraft were to meet in combat, one with thrust vectoring (referred to as an advanced aircraft), the other without (referred to as a conventional aircraft) then which one would perform better? These questions are not simple ones to answer, but are very important for conceptual designers and threat analysts alike to be able to answer.
1.2 What is Thrust Vectoring?

Thrust Vector Control (TVC) is the ability to change the direction of the vector of thrust acting on an aircraft. The method of doing this is traditionally by moving the exhaust nozzles at the rear of the engine, however engines are now being developed that use fluidic injection into the outlet exhaust to get the same result. The effect of vectoring the thrust is to change the total force and moment acting on the aircraft, with the aim to enhance the manoeuvrability of the aircraft. On the BAe Harrier, the thrust vectoring can be used to allow the aircraft to hover in mid-air. On systems where the engine exhaust nozzles are at the rear of the aircraft, vectoring the thrust can provide a control force and moment which is independent of Angle of Attack (AoA), and can hence be used where traditional aerodynamic forces fail, namely in the post stall domain. It is TVC implemented at the rear of the aircraft which was concentrated upon during this project.

The uses of thrust vectoring can be summarised in no particular order, as follows:

1. Departure control. Smith\(^2\) suggested that the number of fighter aircraft lost over the past ten years due to departure (about one fifth of peace time accidents according to Ashley\(^1\)) would have been drastically reduced if thrust vectoring had been in use on those aircraft.

2. Post stall control and nose pointing. TVC provides a control moment in the post stall domain where conventional aerodynamic controls fail.

3. Turn rate enhancement at speeds below corner speed. If the thrust is vectored towards the centre of the turn, then the turning force will be increased, leading to a higher turn rate. Above the corner velocity the aircraft will be load factor limited, and no advantage from TVC will be gained.

4. Improved field performance. Thrust vectoring can benefit field performance by supplementing the aircraft’s control power which often limits the aircraft’s minimum take off/landing speed.

5. Trim drag alleviation. Thrust vectoring can be used to trim an aircraft in pitch, so that the control surface drag is minimised, leading to trim drag reduction.

Herbst\textsuperscript{[4]} was beginning to look into the need for thrust vectoring, around the 1970’s. It seems to have come from the requirement to increase aircraft performance. This was traditionally done via increasing the thrust to weight ratio. However, what he identified was that increasing the thrust to weight ratio was becoming increasingly expensive. From statistical work, he found that there was eventually a point where it was cheaper to have more fighters with a lower thrust to weight ratio, than a few with large thrust to weight ratios. Thus, a requirement arose to find a way of enhancing fighter aircraft performance cheaply and with little weight increase. Herbst\textsuperscript{[5]} concluded that turn rates of 25°/s (high in the early 1980’s) could only be surpassed using new technology like thrust vector control. His conclusions were that to achieve a significant advantage over the enemy, a turn rate excess of 2-3°/s is required.

Of the above uses of TVC, this project primarily aimed at determining the change in aircraft performance due to using TVC as a means of auxiliary control power and post stall control. However the use of TVC without use of Post Stall Manoeuvrability (PSM) to increase the turning force as well as its use to decrease trim drag were also considered.

1.3 Accuracy

This project was aimed at providing performance analysis tools to conceptual designers and tacticians, meaning that the amount and accuracy of input data was limited. In fact, because of the lack of accurate input data, it was required that an output accuracy of only 10% compared to the real aircraft be achieved. This margin of accuracy was chosen since it is often used as a goal by conceptual designers and threat analysts. For the study, input data were limited to information that could be extracted from a three view general assembly drawing of an aircraft. In addition, basic design data that could be derived from this information were also allowed, for example, lift and drag data for the aircraft, centre of gravity position, mass, control surface size, etc. The use of dynamic stability derivatives and like information was not considered except for lift, drag and pitching moment elevator-deflection-derivatives. Engine data were based upon a generic engine model and sea level wet and dry thrusts.
1.4 References


CHAPTER 2

EXISTING AND PROPOSED PERFORMANCE METRICS

This chapter aims to review existing and proposed metrics. This was done to gain an understanding of what was available, after all, it was possible that a solution to the problems highlighted in chapter 1 already existed.

The individual metrics will now be discussed. To provide a structure for logical analysis of the metrics, they are split into the axis about which they act: longitudinal, lateral, axial, or a combination of these. The discussion is based upon the opinions of the author. During the early stages of the research, meetings were held with experts in the field (some of whom were pilots), to help the author gain an understanding of what was lacking in existing metrics and what the experts common requirements for a metric were. These meeting were held with the Defence Evaluation & Research Agency (DERA), BAE Systems, and the UK MoD. Note, some of the metrics found from this study were not directly relevant to this thesis, and so they are placed in Appendix A for reference.

2.1 Longitudinal Metrics

2.1.1 Minimum Nose Down Pitching Acceleration\textsuperscript{[1,2]}

This metric is defined as the minimum nose down pitching acceleration that can be created due to a full pitch down control input. The metric is important to post stall manoeuvres since aircraft are often very slow to recover from such manoeuvres. A larger negative pitching moment will assist in a quicker recovery, meaning that the aircraft will be less vulnerable after a nose-pointing manoeuvre. The quicker the aircraft can recover from a high AoA position, the less energy will be lost. The stability of the aircraft in question plays an important role here, as an unstable aircraft will need a higher nose down pitching acceleration as it will naturally tend to pitch up and away from low AoA. A stable aircraft will naturally recover from a pitch up
attitude, and hence will require less of a pitching down acceleration to recover as quickly. With the engine rotating, its engine gyro effect will increase the aircraft's overall pitch inertia, which will also affect this metric, as it will oppose motion about the pitch axis. Hence engine mass, inertia and rotation speed may become important. This metric will be a function of the AoA at which it is measured.

2.1.2 Maximum Angle of Attack
This metric quantifies the maximum attainable AoA of the aircraft. It is unclear if the aircraft must be controllable at this AoA, or whether it is simply the maximum attainable. In the latter case, the Su-37 would have a maximum AoA of 180° which it has shown to achieve during the Kulbit manoeuvre, where the aircraft completes a 360° pitch rotation about its own centre of gravity. The stability of the aircraft will again affect this metric. Since unstable aircraft can pitch away from the trimmed initial position more quickly, they will have more kinetic energy (rotational) and hence be able to travel to a higher angle of attack limit.

2.1.3 Speed of Control Response at High Angle of Attack
This is a measure of how quickly the control surfaces can react. Lags in this variable will affect how quickly the aircraft can recover when pitching to post stall angles of attack which are generally related to very high pitch accelerations. This metric is closely linked to handling qualities (HQ).

2.1.4 Pitch and Yaw Rate at High AoA
This metric will help to define how agile and controllable the aircraft is during a post stall manoeuvre. It is important to have an aircraft that can be controlled in the post stall regime, and according to Francis (interviewed by Ashley) improved HQ very much rely on pitch and yaw control. However, pitch rate is felt to be more dominant because it defines how quickly the aircraft can get to post stall AoAs.

The main idea behind these four metrics is to describe the ability of the aircraft to attain or recover from post stall flight (although pre stall flight should be considered equally). However, none of these metrics do this completely, since all are only a part of the answer. A combination of the metrics would give a better solution, since the recovery of the aircraft for example, depends upon the maximum AoA, the nose down
EXISTING AND PROPOSED PERFORMANCE METRICS

pitching acceleration, the speed of the control responses, and the pitch rate. Things like time to get to the post stall regime, the energy to get to the post stall regime, the amount of controllability once there, and the amount of post stall envelope available will also matter. It has been suggested by M°Kay\textsuperscript{5} that defining a task and carrying it out is a better way to measure performance than to simply measure for example the time to get to maximum angle of attack, since this value will change for different aircraft in different parts of the flight envelope.

These four metrics are closely related to the HQ of the aircraft. To assess the HQ of an aircraft requires a high fidelity aircraft model. It was however a requirement of this project that the kind of data that would be available would not support the accuracy required to develop metrics that looked purely at a pitching model. It was however acceptable to reduce the dominance of the pitch model by incorporating pitch into manoeuvres where the pitch response was only a small segment of the entire manoeuvre.

2.1.5 Pointing Margin, Point and Shoot Parameter

Proposed by Tamrat\textsuperscript{6}, this metric is defined as the angle between the nose of the adversary, and the line of sight of the friendly fighter at the time when the adversary is aligned with the line of sight. This is depicted in Figure 2.1. This metric implies that both aircraft are constrained to a single manoeuvre plane. It also requires a standard adversary turn - a function of load factor, speed loss, altitude change, etc.

When considering one aircraft versus another, the timing of the nose pointing manoeuvre becomes very important. The diagram in Figure 2.2 gives a good idea
about this sensitivity to timing. From Position A', the aircraft will fail to gain a pointing advantage. However, from point A there will be a successful kill from the post stall fighter (labelled PST Fighter).

Since pitch acceleration rate would give an idea of how much time is required to get to the post stall AoA, it would perhaps be useful here. However it would need to be combined with the spatial position for it to have any combat relevant meaning. The problem with this kind of analysis is that it becomes more complex as more degrees of freedom are added to the analysis.

Also suggested by Tamrat[6] and Kalviste[7] is the point and shoot parameter (sometimes known as the D-t parameter). This is the product of vertical distance from the start of the manoeuvre to the point where the right hand aircraft (in the Pointing Margin Figure) shoots, and the time taken to get to that position.

This set of metrics was generally felt as quite suitable to the requirements of this project, and so was analysed in further depth, as discussed in chapter 4.

2.1.6 Instantaneous and Sustained Turn Rates, Corner Speed and Turn Radius

Figure 2.3 shows the conventional lines for instantaneous turn rate (ITR) and sustained turn rate (STR). Also shown are load factor limit and stall limit lines, and lines of constant turn radius. Construction of such a plot is elementary, and discussed by Raymer[8]. Speed bleed rate lines, that give accelerations for level turns can also be added to such a plot.
Figure 2.3: Turn Rate Plot – ITR Plot at Constant Altitude (Sea Level), showing STR.
The turn rate plot can be plotted on axes of either turn rate against velocity, for a constant altitude (as shown in Figure 2.3), or on axes of velocity against the altitude, with contours for turn rate. The former presentation allows a better understanding of the difference between the load factor limited portion of the ITR and the stall limited section of the ITR. These occur to the right and the left respectively of the corner velocity (at 147m/s in Figure 2.3), which is the point where the aircraft is at maximum load factor and also on the stall boundary, hence giving the maximum turn rate.

The significance of the STR is that it is the turn rate at which the aircraft will not lose speed or altitude, and is capable of maintaining the energy until fuel is burnt, which will change the weight which will affect the SEP of the aircraft, which in turn will change the sustainability of the turn.

The corner velocity is labelled in Figure 2-4.

\[ L(\alpha, M) + T(M, Z) \cdot \sin(\alpha) = n_1 \cdot W \]

Equation 2-1

Where, \( L \) is lift (a function of AoA and Mach Number), \( T \) is thrust (a function of Mach number, altitude, and AoA – although not modelled here, intake pressure recovery is poor at very high angle of attack), \( n_1 \) is maximum load factor, and \( W \) is weight. Since the corner velocity is the minimum velocity at which the aircraft can
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attain the load factor limit, the turn rate is greatest, as shown in Figure 2-4. Furthermore, the line of constant radius that emanates from the origin shows the minimum radius. The line of constant radius that would pass through the corner velocity (if it were drawn) would have almost the same gradient, and hence almost the same turn radius, from,

\[ R = \frac{V}{\psi} \]

Equation 2-2

This indicates that the aircraft not only has its highest turn rate, but also nearly its minimum turn radius at the corner velocity. Hence the corner velocity is often cited as being where the aircraft is most manoeuvrable. Due to the dependency of Equation 2-1 on altitude, the corner velocity changes with altitude.

These metrics in the form of the dog house (turn rate) plot (as shown in Figure 2.3) are traditionally a major source of aircraft analysis. These metrics are worth further analysis because existing research such as that done by Raymer\cite{81} suggests that the turn rate plot already demonstrates the effects that post stall manoeuvring and thrust vector control have on an aircraft.

2.2 Lateral Metrics

Due to the limitations of available input data, it was not possible to analyse lateral metrics in this project. Although data can be derived for the aircraft at low AoA, it is currently not possible to easily derive input data suitable for analysis of lateral metrics for post stall capable aircraft. For the interested reader, the lateral metrics found during this study are given in Appendix A.

2.3 Axial Metrics

Proposed by Eidetics\cite{9}, this metric is the difference between minimum and maximum available Specific Excess Power (SEP or \( P_s \), see 2.4.2) at a given flight condition, divided by the time to transition between the two levels. It is defined by the following equation,

\[
\frac{\Delta(P_{s\min \ power, \ max \ drag} \Rightarrow P_{s\max \ power, \ min \ drag})}{\Delta t_{\text{transition}}}
\]

Equation 2-3
which has been referred to as the Power On Parameter (POP), which measures the performance of the aircraft accelerating from a minimum power and maximum drag flight condition to a maximum power and minimum drag flight condition. The opposite manoeuvre to this is the Power Loss Parameter (PLP) and is defined as,

\[
\Delta \left( \frac{P}{m \text{ax power, min drag}} \xrightarrow{\Delta t_{\text{transition}}} P_{m \text{ax power, max drag}} \right)
\]

Equation 2-4

Also, axial agility has been defined as simply the ability to change SEP level,

\[
AA = \frac{\partial^2 h_s}{\partial t^2} = \frac{\partial P_t}{\partial t}
\]

Equation 2-5

The axial agility is driven by the transient performance of the engine and drag devices. Since this manoeuvre is dominated by transient terms, and since the drag and thrust reverser deployment times will be low relative to the engine spool time, it is likely that this time will be the over riding term. Reference 10 concludes that axial agility does not quantify the acceleration of the aircraft, but is dominated by engine transients and the effectiveness of drag producing devices. In general (within reference 10) the aircraft with the faster spooling engine and the more effective speed brakes tended to have greater axial agility than the aircraft that was superior in acceleration.

The Power Onset Parameter would be of use in considering the energy management of a post stall recovery manoeuvre. Many of the suggested metrics consider the time taken to return to the original energy state. If energy management were not a primary concern, then the metrics could simply be the time to perform a given manoeuvre, regardless of energy state. Then for energy management considerations, the POP could be examined.

In general, the axial agility of an aircraft was considered to be a good metric by many interviewees. Although it is dominated by transients, it should not be totally ignored since the metrics give a good indication of how quickly an aircraft can slow down or accelerate. Also, many manoeuvres are performed at constant throttle (combat power or maximum afterburner) and so are not affected by this problem.
2.4 General/Combination Metrics

These metrics could fall either into several of the above categories, or none of them.

2.4.1 CCT, DST and Bleed Rates, RES

This set of functional metrics appears to have been developed by Tamrat[6], Kalviste[7], and McAtee[11]. Combat Cycle Time (CCT) is the time to perform a 180° heading change and return to the original energy state. It is not clear whether the end condition is for original energy level, or for original mach number and altitude. The CCT can be found from a turn rate plot by integrating around the boundary as shown above, by adding \( t_1, t_2, t_3, t_4 \). Liefer[31] suggests that times \( t_1 \) and \( t_3 \) are probably negligible relative to the others and so the metric is dominated by turn rate and SEP. Thus, it is more a measure of performance as opposed to agility.

From the conventional turn rate plot, the DST or Dynamic Speed Turn plots can be developed. The diagram helps to explain. Two new plots are produced (A&B). The first plot (A) gives the speed bleed rate for a maximum turn rate manoeuvre. The second plot (B) shows level acceleration for a 1g straight and level flight.

Figure 2.5: Concept of CCT

Figure 2.6: DST Plots (taken from Liefer)
Case A describes how the aircraft decelerates while in segments $t_{21}$ and $t_{22}$ of the CCT. Case B describes how quickly the aircraft can accelerate while in segment $t_4$ of the CCT. DST can be plotted onto turn rate plots in the form of contours, similar to the STR contours. This is useful because it provides additional information on plots that are already widely used and readily understood by pilots/tacticians and designers.

To go with the DST, are Energy Loss During Manoeuvre, and the Maximum Speed Bleed Rate. Energy Loss During Manoeuvre is a measure of change of energy height during a manoeuvre. For a high amount of energy loss, the aircraft will be left vulnerable after the manoeuvre. The Maximum Speed Bleed Rate shows how quickly the aircraft can lose speed. This leads on to the RES, or Relative Energy State. RES is simply a plot of the ratio of velocity over corner speed versus the aircraft’s heading angle, which changes with time as the aircraft turns. The smaller the gradient and the higher the initial velocity, the better the metric, and the more times the aircraft can complete turns. Once the velocity ratio drops below unity, the aircraft will be moving along its stall limit at the end of the manoeuvre. Note that the RES alone is not good enough. It must be analysed in conjunction with values like CCT, since it does not give any idea of the time taken to complete the turn. This metric is driven by SEP and hence thrust and drag.

Ryan and Downing\cite{12} did some interesting work on scheduling the AoA during the manoeuvre such that CCT was optimised. The conclusions were that the CCT could be optimised with correct scheduling of the AoA, but that it was of little use to the pilot since the scheduling was complex with no natural cues for the pilot to follow.
They also suggested that a metric to incorporate energy efficiency, time and turn radius should be developed, suggesting something along the lines of,

\[
\text{Metric} = \eta_{\text{ENERGY}}^k \cdot CCT^l \cdot R^m
\]

Equation 2-6

where the constants \(k, l, m\) are used to weight the individual metrics: CCT, R (radius) and \(\eta_{\text{ENERGY}}\) (energy consumption). It is suggested that the individual metrics, energy efficiency, CCT and turn radius be examined individually, so that there is no danger of losing information. When using weightings, as in Equation 2-6, the relationship between the individual metrics needs to be considered to determine which is the most important, and how much more important it is than the other metrics. This is why it is often valuable to consider the metrics individually and not combine them.

These metrics appear to be very powerful. They tell the analyst a large amount of information about turning flight. It was noted that the plots should be produced for turning in the vertical plane as well as the horizontal plane, as this may point out a tactical advantage, not noticed before (during the X-31 vs. F/A-18 combat simulations it was found that the X-31 did not perform as successfully while performing in the vertical plane\(^{[13]}\). It was resolved that creating a metric to incorporate the energy efficiency, CCT, and turn radius would be considered. This metric was chosen for further analysis.

### 2.4.2 Energy Height (\(h_e\)) and Specific Excess Power (SEP or \(P_s\))

These traditional metrics\(^{[18]}\) measure the energy and rate of change of energy of the aircraft, but in terms of equivalent altitude and rate of change of altitude, as opposed to Joules and Watts. First consider the most basic assumptions. These are that the aircraft configuration will be fixed, the weight constant, the load factor constant, the thrust level fixed and that potential and kinetic energy can be exchanged instantly without any losses. For the energy performance of an aircraft, specific energy height is given by,

\[
h_e = \frac{E}{W} = h + \frac{1}{2g}V^2 \quad [\text{m}]
\]

Equation 2-7
Also, specific excess power (Ps, or commonly referred to outside equations as SEP), defined as the rate at which energy height can be changed,

\[
P_s = \frac{\partial h_e}{\partial t} = \frac{\partial h}{\partial t} + \frac{V}{g} \frac{\partial V}{\partial t}
\]

or, more conventionally,

\[
P_s = \frac{\partial h_e}{\partial t} = \frac{V(T \cdot \cos[\alpha + \delta] - D)}{W}
\]

Equation 2-8

where, \( T \) is thrust, and \( W \) is weight, \( V \) is velocity, \( D \) is drag, \( \alpha \) is AoA and \( \delta \) is TVC pitch deflection. Now, as load factor increases, available specific excess power decreases, because the drag increases. At a zero excess power, \( T = D \). If \( P_s = 0 \), then the aircraft is flying straight and level without acceleration, or it is climbing and decelerating or it is descending and accelerating.

Now, plots of \( P_s \) vs. Mach Number can be plotted at constant altitude, showing lines of varying load factor.

![Figure 2.8: SEP Curve for Fixed Altitude, Varying Load Factor (from Raymer\textsuperscript{(8)})](image-url)
In addition, Level Turn Rate can be plotted against $P_s$ for given load factor, altitude and Mach number,

![Diagram showing Level Turn Rate against $P_s$](image)

Figure 2.9: $P_s$ Curve for Turn Rate (from Raymer[8])

The above plot is at a fixed $h_c$ level. This results in a plot, which allows two aircraft to be matched against each other. Finally, zero $P_s$ plots can be produced, which are of great use because they allow two aircraft to be compared at all Mach numbers, altitudes and load factors on one chart. The superior aircraft will be the one with contours that envelope the other aircraft.

![Diagram showing Zero $P_s$ Curves](image)

Figure 2.10: Zero $P_s$ Curves (from Raymer[8])

The zero SEP plot gives lines of constant load factor. These can be shown as turn rate instead, in which case the plot shows the Sustainable Turn Rate (STR).

These metrics are very well understood for conventional aircraft. Studies have not been conducted that consider the effects of TVC/PSM, although since energy height is
simply a static measure of the sum of the aircraft’s potential and kinetic energies, it will not be affected by TVC or PSM. However, it was decided to analyse SEP in further depth to understand the effects of TVC on it.

2.4.3 Herbst Agility\textsuperscript{[14]} and Specific and Tactical Agility derived from Frenet’s Formulae\textsuperscript{[15]}

Three metrics have been suggested by Herbst. They are called longitudinal, curvature and torsional agility, and come from examining manoeuvres such as those shown in Figure 2.10, but can be applied to any manoeuvre.

The metrics actually give values of the second derivatives of the axial velocity, the turn angle and the pitch angle (flight path angle). These are basically the rate at which the axial acceleration can be changed, and the rate at which the turn rate and flight path rate can be changed. The metrics are calculated using the following formulae.

Axial Agility,
\[
\ddot{v} = \left( \frac{g}{W_r} \right) \left[ T \cos \alpha - T \dot{\alpha} \sin \alpha - D \right] - \dot{W}_r \left[ \sin \gamma + \frac{\dot{V}}{g} \right] - \frac{[T \sin \alpha + L][\dot{\psi} \cos \phi + \dot{\psi} \sin \gamma \cos \phi]}{V [\dot{\psi} \cos \phi + \dot{\psi} \cos \gamma \sin \phi]^2 + (\dot{\psi} \cos \gamma \cos \phi - \dot{\gamma} \sin \phi)^2]}
\]
\text{Equation 2-9}

Turn Agility,
\[
\ddot{\psi} = \left( \frac{1}{V \cos \gamma} \right) \left( \frac{g}{W_r} \right) \left[ T \sin \alpha + L \right] \left( \phi - \dot{\psi} \sin \gamma \right) \cos \phi + \frac{[T \cos \alpha - D][\dot{\psi} \cos \gamma]}{V \left( \frac{V}{g} \right) \dot{\psi} \cos \gamma - 2 \dot{V} \psi \cos \gamma + 2 \dot{V} \dot{\psi} \sin \gamma]}
\]
\text{Equation 2-10}
Pitch Agility,
\[ \dot{\gamma} = \left( \frac{1}{V} \right) \left[ \frac{-G}{W_T} \right] \left[ T \sin \alpha + L \right] \left( \dot{\phi} - \psi \sin \gamma \right) \sin \phi \\
- \left[ T \cos \alpha - D \right] \dot{\gamma} - \left( \dot{L} + \dot{T} \sin \alpha + T \ddot{\alpha} \cos \alpha \right) \cos \phi \\
- 2\dot{V} \dot{\gamma} - V \psi^2 \sin \gamma \cos \gamma \]

Equation 2-11

The above equations were derived using the equations of motion, and differentiating them with respect to time. The equations of motion are discussed in the next chapter. Fox\textsuperscript{[14]} went on to plot these metrics against Mach number at constant weight and altitude with contours for constant load factor (and included stall boundaries). Fox’s results plots are shown in Figure 2.12.

These plots provide a snapshot of the aircraft’s instantaneous agility. On inspection of the plots, it appears that two optimal conditions exist, one for maximum positive agility and one for maximum negative agility. Furthermore, one of the manoeuvring conditions coincides with the more traditional corner velocity, already discussed. That is to say, at the corner velocity, the aircraft will also have the greatest turn agility.
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(the greatest potential to change the rate of turn), the greatest pitch agility (the greatest potential to change rate of pitch) and the greatest axial agility (the greatest potential to change acceleration). The axial agility is of use because it shows how the aircraft can accelerate for given Mach number. Notice the drop in performance due to transonic drag, indicated at point 'A' on the axial agility plot of Figure 2.12.

The Herbst equations become undefined at a vertical climb and a vertical dive due to the definition of the Euler angle. That is to say there is confusion between roll angle and yaw angle when pitch angle is 90°. Apart from these two conditions, the equations hold true for any constant manoeuvre condition.

These metrics appear to be very useful as they describe aircraft agility independently of the chosen manoeuvre. Because of this they could be applied to virtually all of the metrics discussed in this chapter. However, some of the inputs required for the calculations will be hard to determine accurately using numerical methods (dT/dt for example), since software differentiation is required.

Very similar to the Herbst Agility is a metric called the Specific and Tactical Agility, which is derived from Frenet's Formulae\cite{16,17}. According to AGARD Working Group 19\cite{15}, this metric can be calculated using three formulae, one for axial agility, one for curvature (pitch) agility and one for torsional (roll) agility. The formulae are used to calculate the components, which are then resolved to determine the total specific agility. Furthermore, the calculations are completed after each time step and so RMS values are taken.

\[
\begin{align*}
A_A &= \dot{Y} - V\omega^2 \\
A_C &= 2\dot{v}\omega + v\dot{\omega} \\
A_T &= v\omega[\ddot{\mu} - \dot{\chi}\sin\gamma] \\
\text{where, } \omega &= \sqrt{\dot{\chi}^2 + \dot{\chi}^2 \cos\gamma}
\end{align*}
\]

Equation 2-12

According to reference 15, Equation 2-12 can be derived alternatively so that it is not so reliant upon purely kinematic terms. Equation 2-13 give an alternative, also from reference 15.
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\[ A_A = \dot{\bar{v}} \]
\[ A_c = \bar{V} \omega + V \dot{\omega} \]
\[ A_r = \bar{m} - \dot{\chi} \sin \gamma - \dot{\chi} \dot{\gamma} \cos \gamma \]

where, \( \omega = \sqrt{\dot{\chi}^2 + \dot{\gamma}^2 \cos \gamma} \)

Equation 2-13

Since the Specific and Tactical Agility is less reliant upon derivative terms that are not readily available from low order simulation (for example dT/dt), it was decided that these metrics would be examined instead of Herbst Agility.

2.4.4 Energy Agility

This metric, suggested by Dorn\textsuperscript{181}, shows the aircraft’s efficiency at executing a prescribed manoeuvre and the ability to disengage and recover the original energy state. Energy Agility, \( EA \), is defined by Equation 2-14.

\[ EA = \int_{t=0}^{t_{final}} \Delta h_e dt \]

Equation 2-14

This metric can be further extended. By connecting this metric to others, the energy consumption during a manoeuvre can be studied. For the Point and Shoot Parameter discussed above, Dorn suggests a ‘large amplitude task agility’ parameter,

\[ A_{L.A.T.} = \frac{1}{Point \ and \ Shoot \ Parameter} \int \Delta h_e dt \]

Equation 2-15

This metric is interesting because it shows the ability to turn, point the nose, and the energy efficiency in doing so. This metric is thus similar to the DST and RES metrics discussed above. However, a metric like the CCT already incorporates the energy efficiency, and so then dividing by the EA means that the energy efficiency has been
accounted for twice. Furthermore, by producing a single value in the attempt to quantify the performance of the aircraft, there is great danger in losing the true meaning of the individual metrics. This has already been discussed above, and for the torsional agility metric, in Appendix A. It was felt that it is better to consider metrics individually, and not attempt to determine a single value to quantify the aircraft's performance. Beyond that, this metric is simply a measure of the energy consumed during a manoeuvre. Hence this specific metric was not considered further, however the need to quantify the energy consumption of the aircraft was considered extremely important for any metric.

2.5 Common Requirements

In addition to the requirements set out above, there are also common requirements that became apparent. These are as follows.

2.5.1 Closed Loop Tasks

It is important to consider closed loop tasks. In this text, closed loop is taken to mean including specific goals, as opposed to having the final conditions matching the initial conditions. For example, a manoeuvre that measures the time for an aircraft to get to a specific angle of attack is closed loop. However, a manoeuvre where the final and initial AoA are the same, but the maximum AoA is not explicitly specified is not considered closed loop. Furthermore, the measure should incorporate the rate, and the acceleration, as well as define explicitly the value of AoA (in this case) to be achieved. Bitten\(^{19}\) backs this suggestion up with the following statement, "defining a metric that uses the time to achieve a relative state change as a measure of performance incorporates the initial state conditions, the rates affected by manoeuvrability, and the accelerations affected by agility. This is termed functional agility by the AFFTC". The reason that all segments (initial, rate and acceleration) must be included is partly because the transient terms become evermore important in high performance situations, and partly because the true performance of an aircraft can be thought of as an average of all segments of a manoeuvre, not simply the start or finish condition of one segment.
2.5.2 The Order of the Metric

Aircraft performance is generally split into one of point performance, manoeuvrability or agility. These three terms have been the subject of discussion in many papers\cite{6,20-23, for example}, and they are generally described as follows. Performance assesses the steady state of the aircraft or its performance at a point in the flight envelope, and is often termed zero order. Manoeuvrability describes the time derivative of performance, and is often termed first order. Finally, agility is the second time derivative of performance, and hence often termed second order.

At this time, there is a lot of interest in agility metrics rather than in performance metrics. Comparing two aircraft will easily show that one is more agile than another (as defined in the above paragraph). However, there are two problems with agility metrics. The first is that these metrics do not always consider combat relevance. Knowing that an aircraft can pull a given load factor faster than the adversary, does not necessarily mean an advantage is conferred. According to Bitten\cite{19}, the U.S. Air Force Flight Test Center (AFFTC) emphasise, that it is tactically relevant to obtain a desired final state for the aircraft. Factors like the time to pitch to maximum load factor do not tell the designer about the final state of the aircraft, and are hence not functional. Also, many agility metrics use units that are unfamiliar to the designer or pilot. Terms such as time, turn diameter and energy state have much greater meaning than a metric with units of say $\omega^2/s^2$.

The second problem is that agility metrics do not always allow the conceptual designer to assess the aircraft. Accurate values of control derivatives cannot be known during conceptual and preliminary design\cite{8}. Many research projects have been undertaken that use complex models of aircraft to evaluate performance in the more general sense, references 24, 25, and 26 for example. None of these models are suitable at the initial design stage of an aircraft's life, where input data for mathematical analysis is limited, and certainly not suitable for threat analysis.

Determining the type (agility, manoeuvrability or performance) of a metric is a subjective matter. While it is not always obvious which type the metrics described in sections 0-2.4 are, it is clear that many are agility metrics. They tend to be the metrics
that are dominated by the transient terms. For example, velocity is a performance metric, SEP is a manoeuvrability metric, and hence rate of change of SEP (that is, POP/PLP) are agility metrics.

2.6 Chosen Metrics

Based upon the individual discussions above, the following metrics were chosen for further analysis.

<table>
<thead>
<tr>
<th>Axis</th>
<th>Suitable For Further Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal</td>
<td>A Post Stall Attainability And Recovery Metric</td>
</tr>
<tr>
<td></td>
<td>ITR / STR</td>
</tr>
<tr>
<td></td>
<td>Pointing Margin</td>
</tr>
<tr>
<td>Lateral</td>
<td>None due to input restraints, discussed in chapter 3.</td>
</tr>
<tr>
<td>Axial</td>
<td>Acceleration Metric</td>
</tr>
<tr>
<td></td>
<td>SEP and Energy</td>
</tr>
<tr>
<td>General/Combination</td>
<td>CCT, DST, RES</td>
</tr>
<tr>
<td></td>
<td>Specific and Tactical Agility</td>
</tr>
</tbody>
</table>

Table 2-1: Selected Metrics

From the discussions above, several paramount points came about, which it was decided would be used throughout the analysis of existing metrics as well as during the development of any new metrics.

Firstly, it was decided that it is important to consider the benefits of both energy conservation and maximum manoeuvrability. It was felt that while the majority of the time the pilot will want to keep energy levels high, there would be times when it is more beneficial to exchange large amounts of energy to gain a tactical advantage (to gain a shoot solution for example). For this reason, metrics were examined from both viewpoints. For example, when the turn rate is considered, the strategy used to fly around a 180° heading change and return to the original energy level can change the value of the metric greatly. Hence a strategy to optimise energy consumption (STR) as well as a strategy to optimise the aircraft’s spatial position (ITR) needed to be
examined. It has been noted that certain strategies can even further improve performance (for example those pointed out by Ryan and Downing\cite{12}) but if these strategies cannot be realistically flown due to lack of pilot cues then it is pointless to analyse them until technologies arrive that can supply the pilot with the required cues.

Secondly, many of the above metrics attempt to quantify the performance of the aircraft in terms of a single parameter. Often the parameter has odd units, which are unfamiliar to designers and pilots. Even SEP with units of m/s is often difficult to gauge without a comparison to some known situation! Hence it was decided that wherever possible, parameters would be considered separately in addition to the proposed combination. That way it was possible to determine if something was either being hidden by the composite parameter, or even if the composite parameter could offer something that the individual parameters could not.

2.7 References

5. Personal communication with Keith M'Kay, BAE Systems.


27. Personal communication with Lt. Commander Andy Sinclair, RN, of CDA, DERA.
CHAPTER 3

THE MODELS USED – AN F/A-18A

To analyse the performance of an aircraft, a model was required. Because of input limitations a model was required that consisted of two degrees of freedom (referred to as low order), namely acceleration along the velocity vector and pitch about the aircraft centre of gravity. To derive this model, a six degree of freedom model was used (referred to as high fidelity). This chapter discusses the depth of the six degree of freedom model, and then derives a low order model for analysing metrics in this work.

3.1 The High Fidelity Model

This model was created from data that were available on the Internet\(^1\). The data were put together from flight test data and provided a 6 DoF non-linear model of the F/A-18a. This aircraft is a naval fighter used predominantly by the US Navy, and has had a thrust vectoring retrofit used for research purposes under the NASA HARV (High Alpha Research Vehicle) project. The flight test data that were used to construct the model were actually from the HARV research project. The model was supplied as a Matlab Simulink\(^2\) model, by Flight Management Control, DERA, Bedford, who, in an internal report\(^3\) have stated that the model is sufficient and believed to represent the real aircraft. The inputs to the model were the commanded AoA, sideslip angle, roll rate (about the velocity vector), throttle setting, and a TVC on/off toggle. The equations of motion were solved within Matlab. The model was non-linear, in that the force coefficients varied with several parameters in a non-linear fashion, mostly Mach number and Angle of Attack (AoA), but depending upon the coefficient, sometimes other parameters (see equations below). To cater for these non-linearities, the equations used look-up tables with linear interpolation to find the non-linear coefficients from data files stored with the model. The equations could then be solved linearly over each time step, as long as the time step was small. The size of the time step was taken care of by Matlab, which used variable time step
integration. The forces on the aircraft were found in terms of the lift, drag, and side force; pitch, roll and yaw moments (all relative to the wind axes). These were then converted into body axes and used to solve the equations of motion in terms of body axis motion. To clarify which terms the force equations were dependent upon (to make them non-linear) they are given below. Showing them here also gives an idea of the depth that was used to model the ‘real’ aircraft, and which terms were considered negligible even for high accuracy. First, the longitudinal equations, using standard notation (see nomenclature).

\[
C_L = C_{L0} \left( \alpha \cdot k_{CL} (M) \right) + \Delta C_L (M) + \frac{\bar{c}}{2V} \cdot C_L \left( \alpha \right) \cdot q + k_{CL} \delta_e (M) \left( C_{\delta e, l} (\alpha) \cdot \delta_e, l + C_{\delta e, r} (\alpha) \cdot \delta_e, r \right)
\]

\[
C_D = C_{D0} \left( \alpha \cdot k_{CD} (M) \right) + \Delta C_D (M) + \frac{\bar{c}}{2V} \cdot C_D \left( \alpha \right) \cdot q + k_{CD} \delta_e (M) \left( C_{\delta e, l} (\alpha) \cdot \delta_e, l + C_{\delta e, r} (\alpha) \cdot \delta_e, r \right)
\]

\[
C_M = C_{M0} (\alpha) + \Delta C_M (M) + \Delta x_{NP} (M) \cdot C_{N_{wing}} (\alpha, M) + \frac{\bar{c}}{2V} \cdot C_M (\alpha) \cdot q + k_{C_M} \delta_e (M) \left( C_{\delta e, l} (\alpha) \cdot \delta_e, l + C_{\delta e, r} (\alpha) \cdot \delta_e, r \right) + \Delta C_M (\alpha, \beta, \bar{p}_w)
\]

Equations 3-1

where, subscript 0 refers to the aircraft body, \( k_{\alpha} \) is a factor to cope with the effective change in AoA at high speed, and is a function of \( M \), Mach number, the \( \Delta \) term refers to the change due to high speed flight, the q terms refer to the effect of pitch rotation, the \( k_{\delta_e} \) reflects a change in the elevator effectiveness at high speed, and the \( \delta_e \) terms refer to the elevator deflections (l being left elevator, r being right elevator). For the pitch moment equation, \( \Delta x_{NP} (M) \) is the neutral point displacement due to Mach number, and \( C_{N_{wing}} \) is the normal force coefficient due to the wing (i.e. without accounting for elevator deflection or damping terms), where,

\[
C_{N_{wing}} (\alpha, M) = C_L \cos \alpha + C_D \sin \alpha
\]

\( \Delta C_m (\alpha, \beta, \bar{p}_w) \) takes account of the non-steady state rotation of the model.

And now, the lateral equations.

\[
C_l = k_{C_{l\beta}} (M) \cdot C_{l\beta} (\alpha) \cdot \beta + \Delta C_{l\beta} (M) \cdot C_{l\alpha} (\alpha) \cdot \delta_a + k_{C_{l\delta_e}} (M) \cdot C_{l\delta_e} (\alpha) \cdot \delta_e, l + C_{l\delta_e, r} (\alpha) \cdot \delta_e, r
\]

\[
C_n = C_{n0} (\alpha) + k_{C_{n\beta}} (M, \alpha) \cdot C_{n\beta} (\alpha) \cdot \beta + \Delta C_{n\beta} (M) \cdot C_{n\alpha} (\alpha) \cdot \delta_a + k_{C_{n\delta_e}} (M) \cdot C_{n\delta_e} (\alpha) \cdot \delta_e, l + C_{n\delta_e, r} (\alpha) \cdot \delta_e, r
\]

\[
C_Y = k_{C_{Y\beta}} (M) \cdot C_{Y\beta} (\alpha) \cdot \beta + \Delta C_{Y\beta} (M) \cdot C_{Y\delta_a} (\alpha) \cdot \delta_a + k_{C_{Y\delta_e}} (M) \cdot C_{Y\delta_e} (\alpha) \cdot \delta_e, l + C_{Y\delta_e, r} (\alpha) \cdot \delta_e, r
\]

3-2
where, subscript 0 refers to the aircraft body, $k_{\alpha, \beta}$ accounts for the change in the derivative due to high speed, $k_{\alpha, \beta}$ factors the control derivatives according to high speed changes, $\delta_a$ is the aileron deflection, and $\delta_r$ is the rudder deflection. Also, the terms with the subscript rot account for the changes due to rotation, and are similar for all three equations. For the rolling moment coefficient, $C_l$, the algorithm to find this term is as follows.

1. At high Mach number, the moment increment due to rotation was computed as follows:
\[
\Delta C_{l_{\text{rot}}} = \frac{b}{2V} \cdot k_{C_l p} (M) \cdot C_l p (\alpha) \cdot p + \frac{b}{2V} \cdot k_{C_l r} (M) \cdot C_l r (\alpha) \cdot r
\]

2. At low speeds, both forced-oscillation and steady-rotation data were involved in the computation:
\[
\Delta C_{l_{\text{rot}}} = C_{l_{\text{rot}}} (\alpha) \cdot \bar{r}_w + C_{l_{\text{steady, rot}}} (\alpha, \beta, \bar{\rho}_w) - C_{l_{\text{steady, rot}}} (\alpha, \beta, 0)
\]

where,
\[
C_{l_{\text{rot}}} (\alpha) = C_{l_{\text{rot}}} (\alpha, \cos \alpha - C_{l_{\text{steady}}} (\alpha) \sin \alpha
\]

3. There was a smooth switch from one representation to the other, using a function that was equal to 1 at $M = 0.6$, and was equal to 0 at $M = 0.8$, with a smooth spline between these two points.

The aerodynamics were based on a low speed model, and a high speed model, the two of which were plugged together. This was represented by the equations above which are functions of $M$, the Mach number.

Yaw asymmetry was also included into the model, allowing it to show typical high AoA departure from the commanded flight path. Typically an aircraft performing a manoeuvre such as the Cobra manoeuvre will sideslip away from the commanded flight path, once at high AoA.

The control law synthesis procedure for this model was based upon the use of a methodology called Robust Non-linear Dynamic Inversion (RNDI). The model was controlled via commanded control inputs of AoA, sideslip and roll rate (about the velocity vector, $p_e = p \cos \alpha + r \sin \alpha$). For the controller to satisfy these inputs, a
reference model generator was used, which could incorporate handling quality design specifications. This basically allowed the gains of the model to be optimised by the model knowing the type of output that was expected (from the design specification). The gains were set so that these control criteria were matched. This type of analysis is beyond the scope of this project, and so different methods of control design were used for the low order model.

The controllers determined the control deflections (aerodynamic and TVC) that were required to produce the forces/moments that led to the commanded angle of attack, sideslip and roll rate.

The model was first initialised with values of velocity and altitude, for example (other values such as the initial displacements, rates and accelerations were also defined). The model was then given a constant input command, after which it was run for a given time. All of the state variables and control variables of the model were tracked and stored in the Matlab workspace, so that they were available for plotting and analysis.

3.2 The Low Order Model

To analyse the aircraft performing manoeuvres such as involved in the Combat Cycle Time (CCT) and Pointing Margin, discussed previously, the model had to be capable of at least acceleration and pitch motion. Hence a minimum of these 2 DoF were required. A third degree of freedom was required for the aircraft to manoeuvre out of the vertical plane, namely roll. To facilitate roll about the velocity vector, the information required to model rolling would not only be rolling moment coefficients, but also yaw moment coefficients. This is because aileron input induces roll about the body axis, which at high AoA translates the AoA to a sideslip angle. Hence to transfer the roll about the body axis to a roll about the velocity vector to ensure that the sideslip remains zero, requires a yawing motion. Furthermore, rolling and yawing coefficients at high alpha are currently impossible to predict accurately without extensive wind tunnel model testing. Hence, rather than dynamically modelling these motions, a simplistic approach was taken in assuming that the aircraft could fly in the horizontal plane if enough vertical load factor could be produced to overcome the weight of the aircraft.
The basis for the model was the assumption that it could be constructed from taking two 1 DoF models and combining them to create the 2 DoF model. The pitch degree of freedom is described by the following equation,

\[ M = I_y \dot{q} + rq(I_x - I_z) + I_{xz}(p^2 - r^2) \]

Equation 3-2

It was assumed that all body rotation rates except pitch would be zero, or small enough such that the cross coupling could be ignored, which reduced Equation 3-2 to,

\[ \dot{q} = \frac{M}{I_y} \]

Equation 3-3

where, \( \dot{q} \) is pitch acceleration, \( M \) is moment and \( I_y \) is pitch moment of inertia.

To determine the AoA, it was assumed that the pitch rate, \( q \), was equal to the rate of change of AoA. Hence, the following equation was derived.

\[ \alpha = \int \frac{M}{I_y} \, dt \]

Equation 3-4

It was assumed that the pitching moment of inertia was constant. This was a safe assumption, since it was also assumed that the weight of the aircraft did not change over a maximum time of one minute over which the aircraft manoeuvre was simulated.

The only unknown in Equation 3-4 was hence the pitching moment, which is produced by the aerodynamics of the aircraft. It was assumed that the pitching moment dependency on pitch rate was negligible. Hence, starting with the high fidelity model, and reducing its complexity by removing the pitch moment dependency on pitch rate we have that the pitch moment is,

\[ C_M = C_{M_0}(\bar{M}, \alpha) + 2 \frac{\partial C_M}{\partial e}(\bar{M}, \alpha) \cdot \delta_e \]

Equation 3-5

where, subscript 0 refers to the aircraft body, and pitch moment, \( M \) is found from,

\[ M = C_M \cdot \frac{1}{2} \rho V^2 S \bar{c} \]
where, $\rho$ is density, $V$ is velocity, $S$ is reference (wing) area, and $\bar{c}$ is the reference length (mean aerodynamic chord). Note that in Equation 3-5, the symbol $\overline{M}$ refers to the Mach number at which the aircraft is flying. Note also, another difference between the high fidelity model and this model is that this one assumes that the elevators (two all moving control surfaces, one on the port side, one on the starboard side) work together, hence the factor of two in front of the second term in Equation 3-5. So, in Equation 3-5, the pitching moment derivative with respect to elevator angle is per elevator, as opposed to the more usual case where it refers to entire elevator (both together). The data used in the low order model incorporated high and low speed effects, just as the high fidelity model did. Again, this was allowed for by the use of data tables in finding the coefficient data. Linear interpolation was used to find the coefficients at points between the tabulated speeds and angles of attack.

The second 1 DoF component of the model was the ability to accelerate along the velocity vector. Acceleration is given by the following equation, derived from force equilibrium on the aircraft (see Figure 3-1),

$$\sum F_x = T \cos(\alpha + \delta_{TVC_{pitch}}) - D - W \sin \gamma = m \dot{V}$$

$$\dot{V} = g(n_x - \sin[\gamma])$$

where, $n_x = \frac{T \cos(\alpha + \delta) - D}{W}$

Equation 3-7

where $\dot{V}$ is acceleration along the velocity vector, $g$ is gravity, $n_x$ is the axial load factor, $\gamma$ is the flight path angle (between the velocity vector and the horizon). Also, $T$ is thrust, and is a function of Mach number and altitude, $\delta$ is thrust vector deflection (measured as the angle between the thrust vector and the angle of attack). Finally, $D$ is drag, and found from,

$$C_D = C_{Do}(\overline{M}, \alpha) + 2 \cdot \frac{\partial C_D}{\partial \delta} (\overline{M}, \alpha) \cdot \delta,$$

$$D = \overline{q} \cdot S \cdot C_D$$

Equation 3-8
where, as before, the drag consists of a component due to the body (subscript 0), and a component due to the aerodynamic deflections. Also, $\bar{q}$ is dynamic pressure, and $S$ is wing reference area.

Equation 3-4 and Equation 3-7 provided the two degrees of freedom for the model. Now, the states of the aircraft could be calculated. To start with, vertical load factor was defined as,

$$n_x = \frac{T \sin(\alpha + \delta) + L}{W}$$

Equation 3-9

where lift, $L$, is found from

$$C_L = C_{L0} \left( \bar{M}, \alpha \right) + 2 \cdot \frac{\partial C_L}{\partial \delta_e} \left( \bar{M}, \alpha \right) \cdot \delta_e$$

$$L = \bar{q} \cdot S \cdot C_L$$

Equation 3-10

Miele\textsuperscript{61}, discusses the derivation of the following standard equations, describing the states of the aircraft.

$$\dot{X} = V \cos(\gamma) \cos(\chi)$$

$$\dot{Y} = V \cos(\gamma) \sin(\chi)$$

$$\dot{Z} = V \sin(\gamma)$$

$$\dot{\gamma} = \frac{g (n_z \cos(\mu) - \cos(\gamma))}{V}$$

$$\dot{\chi} = \frac{g \cdot n_e \sin(\mu)}{V \cos(\gamma)}$$

Equations 3-11

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Figure3-1.png}
\caption{Forces acting on the aircraft.}
\end{figure}
where X, Y, and Z are the global co-ordinates of the aircraft, \( \mu \) is the bank angle about the velocity vector (roll angle), \( \gamma \) is the flight path angle measured from the horizon up to the velocity vector, and \( \chi \) is the velocity vector heading angle with a value of zero being in the initial direction of flight. All of the above equations could then be used to create a simulation of the aircraft. The method employed for this simulation was a fixed time step, discrete and linear integration method. Hence, after each time step, the states of the aircraft were calculated and used to track the time history of the aircraft. However, there was still something missing, namely the control of the model. When Equation 3-4, Equation 3-7 and Equations 3-11 are combined, there are terms for the control deflections that were still undetermined. These control deflections dictate where the model flies during the simulation, and hence a controller was required to ensure that the control deflection caused the correct flight path to be flown, for a chosen manoeuvre.

A similar control strategy as used on the X-31a was implemented in the low order model. Namely, it had the following control inputs,

- AoA,
- Roll angle,
- Thrust setting.

These control inputs are the commanded values. Since the roll DoF was not modelled, the roll angle was set to the required input value after each time step. Since the AoA was modelled, the difference between the commanded AoA and the actual AoA could be used to determine the required elevator deflection to reduce the difference to zero. Finally, the thrust setting was chosen to be constant, at the maximum throttle position.

The commanded value of the AoA was controlled using simple logic conditions depending upon the manoeuvre that was to be flown. A manoeuvre was chosen, and the logic required to fly that manoeuvre was designed, and incorporated into the model. For example, if the manoeuvre were to have a post stall nose pointing segment at the end, then the commanded AoA would change from some pre-stall value to the required post stall value, once the time came for the aircraft to point. The commanded roll angle was determined depending upon whether the aircraft was to fly
in the vertical plane or not. For example, if flight in the horizontal plane was required, then the roll angle was set so that the aircraft flew in the horizontal plane.

The elevator controls the actual AoA. Hence a control system was required to determine the error between the commanded AoA and the actual AoA, and then set the elevator so that the commanded AoA was achieved. To keep the controller simple, a traditional proportional controller was chosen. The simplicity of such a controller by far outweighed the use of more modern controllers, such as those implementing fuzzy logic, neural networks, or RNDI, particularly as the proportional controller was more than capable of the task. From Equation 3-4, it can be seen that the 1 DoF pitch model was undamped since there are no exponential terms in the equation, and so artificial damping needed to be included. The artificial damping was added via the use of derivative feedback control.

The block diagram in Figure 3-2 represents this system, which is a classical P+D controller. Notice the derivative action in the P+D controller shown in Figure 3-2 (the $\frac{du}{dt}$ block). There was a problem in using this block, because the system was discrete – fixed length time steps were used to calculate the simulation. One solution of finding the derivative of a discrete system is to compare values in adjacent time steps. However, this is not always a very accurate method, and can lead to large errors in the controller as the time steps in the simulation are increased to increase the speed of the simulation. Using smaller time steps reduces the errors, but this is at the expense of the overall simulation speed.

![Figure 3-2: Implementation 1 - Classical P+D Controller.](image-url)
To overcome this problem, an identical controller was designed, that did not require the derivative block. Consider the block diagram shown in Figure 3-3.

![Block Diagram](image)

Figure 3-3: Implementation 2 – P+D Controller with simpler implementation.

This implementation provided an identical solution, but with no derivative block required. It was also simpler to implement in the software, with fewer calculations being required. It was possible to determine values for the gains of this system to provide identical output, as the previous implementation would give, which meant that a P+D controller without the problematic derivative action block could be used.

Implementation 1 can be described by the following equations.

\[
\delta_e^* = K_p \alpha_{err} + K_d \dot{\alpha}_{err} = K_p (\alpha^* - \alpha) + K_d \frac{d(\alpha^* - \alpha)}{dt}
\]

Equation 3-12

where, \( \delta_e^* \) is the commanded elevator deflection, \( K_p \) and \( K_d \) are the proportional and derivative gains respectively, and \( \alpha_{err} = \alpha^* - \alpha \). Implementation 2 can be described similarly.

\[
\delta_e^* = K_b K_o (\alpha^* - \alpha) - K_b \dot{\alpha} = K_b K_o (\alpha^* - \alpha) - K_b \dot{\alpha}
\]

Equation 3-13

If identical outputs are required from both implementations of the controllers shown in Figure 3-2 and Figure 3-3, then Equation 3-12 and Equation 3-13 must be equal. Since,

\[
\frac{d(\alpha^* - \alpha)}{dt} = -\dot{\alpha}
\]

\( \therefore \alpha^* \) is constant

Equation 3-14
then Equation 3-12 and Equation 3-13 are equal if,

\[ K_p = K_a K_b \]
\[ K_d = K_b \]

Equation 3-15

This allowed gains to be calculated for the second implementation such that its output was identical to that of the first classical P+D implementation. This is shown in Figure 3-4, since the two lines are identical (red \( \bullet \) versus green \( + \)), meaning the outputs are the same.

![Plot to show how two different p+d feedback strategies are equal](image)

Figure 3-4: Comparison of outputs of both control strategies.

There was no steady state error, and so no integrator was required.

Now that a control strategy was in place, the gains \( K_A \) and \( K_B \) could be designed. Their choice would affect the stability and robustness of the AoA controller. Traditionally, in a classical control theory design like this, the values would be chosen such that the aircraft conformed to handling qualities criteria. These may be such that the aircraft lies within a typical thumb print plot (of system natural frequency versus damping ratio) for its type class, or that it handled to a certain degree according to the Cooper-Harper rating. However, these methods would require knowledge of the aircraft a priori, and can be rather subjective. Values of the design criteria for the real aircraft are not always known – another input limitation for this project. To get around this, there are other methods for design, notably, those used for measuring the performance of controllers, specifically their step responses\[^7\]. Since the controller was to be used to control AoA step inputs, it was the most obvious design strategy to optimise for such inputs. The method used here was to choose a performance index
for which the controller gains could be designed such that the controller's performance was maximised. For this design, the performance index chosen was based upon Equation 3-16,

\[
Performance = \int_0^t e(t)^2 \, dt
\]

This performance index ensured that the error (\(e\), a function of time) was weighted with time, and so the response was as quick as possible with minimum overshoot. Optimising this performance index also implicitly reduced the steady state error of the model, due to the increasing weighting of the error with time, from the squared term. This contributed to the fact discussed earlier that there was no steady state error and so no integrator action was required within the controller (see Figure 3-4). Optimising Equation 3-16 led to a good step response in terms of amount of overshoot, number of oscillations, etc. This is shown in Figure 3-5.

![Figure 3-5: Typical Step Response, optimised to minimise performance index. Initial AoA is 3°.](image)

Now, because the controller had to control a non-linear system with respect to altitude and Mach number, gain scheduling (GS) was required to ensure that the response was always acceptable. Note that both TVC on and off cases required different gains.

Using Equation 3-16, gains could be chosen for any point in the flight envelope (Mach number and altitude ranges) such that the response shown in Figure 3-5 was typical, with one overshoot before settling. The amount of overshoot and the time to settle did, however, vary across the flight envelope. This design method did not require prior knowledge of the aircraft that was being modelled, and was in effect a perfect pilot/controller, obtaining the best out of the system. Hence the pilot and flight control system were not modelled.
Once the controller was designed, the entire implementation of the controller and model could be examined. Figure 3-6 shows a diagram of the entire system. The Plant block calculated the forces and moments on the aircraft and used these to calculate the states. These were fed back to the Flight Path Logic block, which decided upon the commanded angle of attack. Roll angle and throttle position were also fed straight to the plant, as they were not modelled, and it was assumed that the plant would respond immediately. There was also a load factor limiter to ensure that the aircraft did not encroach the aircraft structural limits. The FCS block and control deflection limiter simply allowed the signal to be converted to a control deflection with modelling of physical limits.

The load factor limiter was designed so that if the AoA became sufficient that the normal load factor went above the structural limit, then the commanded AoA was changed so that it was less than the current value of AoA. This then drove the control system to cause the aircraft to pitch down. The actual implementation used worked backward to derive the AoA for the exact structural limit, and set the commanded AoA value to this derived value (a form of dynamic inversion). A typical time response of this limiter in action is shown in Figure 3-7, which shows the initial spike of the load factor (in green) going through 15g (on an aircraft with a limit set at 9g). After that, the load factor hovers around 9g. There are small oscillations but these do not matter since it is the average value that affects the final spatial position of the aircraft. The oscillations occur due to small variations in the AoA (blue) which are too small to be seen (typically < 0.5°). The oscillations in the AoA occur due to the oscillations in the commanded value of the AoA (red), which can be seen clearly. These oscillations are present because of the load factor limiter. As discussed above, when the load factor becomes greater than the limit, the commanded AoA is set at the AoA to give the exact load factor limit. Then, if the load factor falls below the limit, the commanded AoA returns to the previous value. The humps in the commanded AoA (red) are where the load factor instantaneously falls below the limit, so the controller resets the commanded AoA to 20°, in this case. The humps appear large, but often they are only over 100th of a second (the simulation time step). They appear longer since the plotting data was captured at 10Hz. The actual AoA (in blue) sits at
Figure 3-6: Full Model and Controller Implementation.

Figure 3-7: Time histories of commanded AoA (AoA* [degrees], in red), AoA ([degrees] in blue), velocity vector bank angle (Mu [degrees], in cyan)(all using the left hand axis scale), and normal load factor (n_\text{zwing} [g], in green), using the right hand axis scale.
THE MODELS USED - AN F/A-18A

the value so that the load factor is right on the limit, throughout the simulation. This value of AoA varies from approximately 5-7°, changing as speed reduces.

The very brief spike in the load factor at the start of the simulation was due to the reaction of the controller, while the aircraft bled its pitch rate to a negative value so that it could pitch down to an AoA such that the load factor was within the limits.

One problem encountered with the load factor limiter is demonstrated in the following explanation. Consider the aircraft in straight and level flight. The aircraft is then required to pitch up to respond to the commanded AoA (20° in the case of Figure 3-7). Before the aircraft gets to this value of AoA, the load factor reaches the limiting value, and so the aircraft has to reduce the pitching moment. To do this, the controller moves the elevators positively (to produce negative moment) and in so doing, creates more positive lift at the tail. The effect of this is to increase the load factor, not to decrease it. This is shown on the force diagram in Figure 3-8.

![Figure 3-8: Force Diagram Showing Effect of Control Surface Force on Total Force.](image)

The left hand diagram of Figure 3-8 shows the aircraft in equilibrium. The right hand diagram shows the changes in red, when positive tail deflection is provided to pitch the aircraft down. Notice the increase in total lift. This will make the limiter cut in, and the commanded AoA will again drop, commanding a large AoA change. This is similar to having a large gain in the system, forcing it to go unstable.

---

3-15
To overcome this problem, the load factor limiter needed to be based upon the load factor produced by the aircraft alone, ignoring any load factor due to control deflections. Doing this meant that the load factor was not breached upon reduction of pitching moment.

At this point it is worth returning to the quasi degree of freedom, namely the roll angle. It was stated earlier that the roll angle was set equal to the required input angle after each time step. However, this required angle needs to be found. For the aircraft to be able to fly out of the vertical plane, it must be capable of roll. To maintain flight in the horizontal plane, the following relationship must be satisfied.

$$\mu_{\text{required}} = \cos^{-1}\left(\frac{1}{n_z}\right)$$

Equation 3-17

where, $n_z$ is the normal load factor, and $\mu$ is the bank angle. Diagrammatically, this is shown in Figure 3-9, where the aircraft velocity vector is pointing perpendicularly into the page.

Equation 3-17 means that the bank angle will be different depending upon which load factor is chosen – the load factor due to the wing/body alone, or that due to the entire aircraft including the control deflections. Physically, the real aircraft would base this bank angle upon the real load factor, that is, the one inclusive of the control deflection effects. However, because of the similar problems discussed with the load factor limiter, the flight path simulated was not horizontal as expected. One solution would
be to reset the flight path angle of the aircraft such that it was zero (that is, erasing the error due to the previous load factor). However doing so would certainly lead to errors since the aircraft could command a given bank angle and set the elevators for a given AoA and then have the small effect of the elevator instantly deleted. Overcoming this problem is the aim of the next section.

3.2.1 Design of a Flight Path Controller

After initial testing of the model, it was found that small errors would creep into the simulation and the flight path would deviate from the horizontal plane. By the end of the manoeuvre, there was a significant error in altitude, which also meant that further errors would have been introduced since the aircraft would have a different energy height, and the slight decent would have meant that the aircraft had accelerated during the manoeuvre.

Furthermore, it is possible that planes other than the vertical and horizontal could be examined. Initially one might take this as a requirement for the aircraft to fly along any given plane, as shown in Figure 3-10.

![Graphical depiction of the plane angle, $\phi$, the angle rotated about the global horizon $X$-axis between the global vertical and the flight path's plane.](image)

Figure 3-10: Graphical depiction of the plane angle, $\phi$, the angle rotated about the global horizon $X$-axis between the global vertical and the flight path's plane.

However, the cues that are available to the pilot do not easily allow such a manoeuvre to be flown, and it is much more likely that a pilot would fly a turn with a constant
flight path angle (the flight path angle being the vertical angle between the horizon and the velocity vector). All modern Head Up Displays (HUD) give constant flight path bars which make such a turn possible.

The control strategy used to fly such a manoeuvre would be to set the aircraft at a constant AoA, and once the flight path angle is achieved, bank the aircraft so that any excess load factor is used to induce a horizontal turn. The requirement to keep the AoA constant stems from the requirement to maximise load factor below the corner velocity and fly with maximum lift which occurs at a constant AoA.

Once the flight path angle is achieved, the aircraft may be producing too much load factor (lift), since it is at constant AoA. This excess of load factor would drive the flight path angle upwards, and so to get around this and keep the flight path angle constant, the aircraft is banked out of the vertical plane. Hence a 3D flight path is produced.

To control the flight path angle in a simulation, the first step is to say that the bank angle should drive a zero rate of change of the flight path angle, since the AoA will be constant, and sideslip is zero. From the equation of motion given below (taken from Equations 3-11), the bank angle to keep the flight path angular rate to zero can be found.

\[
\dot{\gamma} = \frac{g(n_z \cos \mu - \cos \gamma)}{V} = 0
\]

\[
\Rightarrow \mu = \cos^{-1}\left(\frac{\cos \gamma}{n_z}\right)
\]

Equation 3-18

When the bank angle is set to this value, the flight path angle will not change. However, there will initially be an error between the flight path angle set at zero, and the required flight path angle. This can be corrected by adding a term in the controller. This is shown in Figure 3-11.
The correction term is that shown in the dotted box. The gain, $K_{mu}$, that is chosen can be infinitely high, and this controller will not be unstable, neither will any overshoot exist. The reason is that the roll DoF is not modelled, and so what the controller demands, it gets - the system is ideal. The only problem that needs to be avoided is setting the gain too high which will cause the bank angle to go from $+180^\circ$ to $-180^\circ$ and back again, alternating every time step. This is because the required bank angle calculated is a function of the gain.

Using this controller now means that simulation can be performed in any plane, and that the aircraft will track the required flight path angle accurately. Most importantly, the problems highlighted above, for flight in the horizontal plane are eliminated.

Finally, the low order model described in this section was implemented using Microsoft Excel 97 using Visual Basic for Applications for the driving code, and the Excel computational engine for the calculations between each time step.

### 3.3 References


8. Microsoft ® Excel 97, The Microsoft Corporation, USA.
CHAPTER 4

ANALYSIS OF SELECTED EXISTING METRICS

An important part of this project was to measure the effect of Thrust Vector Control (TVC) on the performance of aircraft, using existing metrics. This had to be done to ensure that any new work would not be a repetition of existing work. Chapter 1 suggested that the effect of TVC be considered in terms of a) no post stall capability, where the TVC is used to enhance turning force via a deflection to the centre of the turn, or to increase the Specific Excess Power (SEP) capability of the aircraft via a deflection to maximise SEP; and b) post stall capability where nose pointing is the objective for the aircraft. This chapter is hence split into two parts – the first of which examines the no post stall capability, and the second, which examines the post stall capability. Furthermore, the non-post stall analysis can be further split using traditional methods, or an enhancement of these methods, derived in this chapter.

4.1 No Post Stall Capability – Traditional Methods

4.1.1 Specific Excess Power

Specific Excess Power can be found in terms of the aircraft forces or in terms of energy change. Both these forms are well documented in Raymer\textsuperscript{11}, refer to a system as drawn in Figure 4-1, and are given in Equation 4-1 and Equation 4-2, where \( F_Z \) is the wind axes normal force, \( F_X \) is the wind axes axial force, \( L \) is lift, \( T \) is thrust, \( D \) is drag, \( V_c \) is the climb velocity, \( \alpha \) is angle of attack, \( \gamma \) is flight path angle, and \( \delta_{TV} \) is the thrust deflection. Figure 4-1 shows the forces that act on an aircraft during steady flight. It is assumed that no control forces exist and that the forces all act through the centre of gravity.

\[
P_S = \frac{\partial h_c}{\partial t} = \frac{V}{g} \frac{\partial V}{\partial t} + \frac{\partial h}{\partial t}
\]

Equation 4-1
Depending upon the values that are substituted into Equation 4-2, there are three values of SEP that can be measured. These are the actual SEP of the aircraft at its current flight condition (Mach number and altitude), the maximum potential SEP, and the minimum potential SEP. The maximum and minimum values come when the throttle, AoA and thrust vector deflection are changed to respectively maximise or minimise SEP. During simulation or real flight, it is most common to calculate the actual SEP to determine the state of the aircraft. However in performance assessment, one might be interested in calculating the maximum possible SEP of the aircraft. This is an optimisation problem, which requires some thought. If the aircraft has no TVC, then the maximisation of Equation 4-2 comes from setting the thrust to a maximum and for each velocity, determining the AoA to maximise the difference of the thrust component and the drag (itself a function of AoA). To find the angle of attack to set SEP to a maximum will require some form of numerical optimisation for a non-linear case such as the models described in Chapter 3. Either a secant method[^2] or a numerical solver such as that provided within Microsoft Excel 97[^3] could be used for the problem.
For an aircraft with TVC, the optimisation problem becomes slightly more analytical. By examining Equation 4-2 it can be seen that for SEP to be a maximum (most positive), the thrust component must be largest (most positive). Hence the term $\cos(\alpha + \delta_{rv})$ needs to be unity. For this to be true, the thrust vector should be set equal and opposite to the AoA. Equation 4-2 can then be optimised in terms of AoA to find which AoA will give the most positive SEP. Conversely, if the minimum SEP is required, then the sum of the AoA and the thrust vector deflection would be set to 90 degrees, so that the thrust component became zero. These optimisation methods are well known, and Raymer[11] has shown this result previously.

There is however, a problem with these optimisations. If the thrust vector nozzle is at the rear of the aircraft, as assumed for this study, then an associated pitching moment is induced. For the aircraft to maintain the required AoA such that the SEP is optimum, the moment will require countering. This problem will be dealt with after a similar problem with turn rate is considered.

4.1.2 Turn Performance

Instantaneous Turn Rate (ITR) and Sustained Turn Rate (STR) are two metrics that are extremely well understood. Traditionally their results are displayed on so called Dog House plots, or as contours on flight envelope axes (Mach number versus altitude). The ITR is the maximum possible turn rate that the aircraft can achieve, and is regardless of energy. The STR is the maximum turn rate that the aircraft can sustain, and so requires the SEP to be zero. To determine the turn rate, the normal load factor must be known, which comes from Equation 4-3.

$$n_z = \frac{L(\alpha) + T \sin(\alpha + \delta_{rv})}{W}$$

Equation 4-3

where $n_z$ is normal load factor, $L$ is lift and a function of AoA, $T$ is thrust, and $W$ is weight. Once the load factor is known, the horizontal plane turn rate can be found from Equation 4-4.

$$\dot{\gamma} = \frac{g(n_z - \cos \gamma)}{V}$$

Equation 4-4
where \( g \) is acceleration due to gravity, \( V \) is velocity and \( \gamma \) is flight path angle.

Consider first the ITR. Turn rate is maximised by increasing the load factor to a maximum. To effect this, Equation 4-3 needs to be optimised. For an aircraft with no TVC, this is again a difficult problem (as for SEP and Equation 4-2) and requires a suitable numerical solver. Again, however, for the aircraft with TVC the optimisation is a little more analytical, in that optimisation can be determined by examination of the equations. For any AoA the load factor is greatest when the thrust component in Equation 4-3 is greatest, meaning that the sum of AoA and the thrust deflection should be equal to 90 degrees. With this in mind, Equation 4-3 can then be maximised using a numerical solver. Note that these calculations will be subject to a load factor limit due to structural constraints of the aircraft. In the case where the optimum load factor is greater than this limit, the AoA needs to be constrained, and the thrust deflection reduced to zero.

The maximisation of STR is more complex than this. The challenge that is being solved is to set SEP to zero, and to maximise the normal load factor for each velocity. This involves both Equation 4-2 and Equation 4-3. For a given speed, once the AoA has been found to set SEP to zero, the thrust vector deflection can be found to maximise normal load factor. However to determine the SEP in the first case, a value of thrust vector deflection must be known. Hence solving this problem requires a multivariable solver, or iteration solving first AoA, and then thrust vector deflection, and repeating until the difference between iterations is acceptably small.

A comparison between an aircraft with TVC and one without can now be made. The results (Figure 4-2) show a traditional turn rate plot, which provides the turn rate for a given velocity at one altitude. The figure shows that the ITR is higher for the TVC aircraft, but only below the corner velocity. Above the corner velocity, the load can be produced by lift alone, meaning that the TVC has no added use. Below the corner velocity, the thrust vectored aircraft can set the thrust deflection to the optimum angle, and so it can obtain a higher normal load factor and so the turn rate is higher. At the lowest speed in the case shown, the turn rate advantage is as much as 5 degrees per second, dropping to around a degree per second as the corner velocity is approached.
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35
Sea Level Altitude

25
20
15
10
5
0

0 0.2 0.4 0.6 0.8 1 1.2 1.4 1.6 1.8
Mach Number

ITR without TVC
STR without TVC
ITR with TVC Optimised
STR with TVC Optimised

Corner Velocity, \( V_e \)

Figure 4-2: Turn Rate Plots: Comparing a standard aircraft to one with TVC

Note that the maximum value is nearly double that quoted by Herbst & Krogull\(^{[4]}\) as significant. Note that there is also a problem with all of these results, in that the pitching moment generated by the TVC has not been considered. This problem will be addressed shortly, in section 4.2.

### 4.1.3 CCT and Pointing Margin

Using the equations derived in Appendix C, the use of TVC for the CCT manoeuvre was assessed. For this section (4.1) the aim was to assess the use of TVC without a PSM capability. Hence to optimise the turning part of CCT, which depends upon the ITR, the TVC was set to point to the centre of the turn. Hence, for a horizontal turn, the axial force would be due to drag alone, as shown in Equation 4-5,

\[
\Sigma F_z = T \cos(\alpha + \delta_{V\phi}) - D - W \sin \gamma = m \dot{V}
\]

Equation 4-5

This meant that the TVC aircraft would slow quicker than the aircraft without TVC, and so the time taken to accelerate back to the original energy level would be increased. This is shown in Figure 4-3, where the total time for the CCT is much greater for the TVC aircraft (67.8s for the TVC aircraft, versus 54.3s for the standard aircraft).
The tick marks on the lines indicate elapsed seconds. Thus it is possible to compare the positions of the aircraft after each second, to see if an advantage exists. Compare the thrust vectored aircraft to the standard aircraft. To start with, the TVC aircraft falls very slightly behind and inside the standard aircraft. The standard aircraft is always the furthest ahead, because the deceleration is the least. Once the aircraft are past the 90° total turn angle, the thrust vectored aircraft begins to move inside of the conventional aircraft because its radius is smaller. Basically, although a TVC aircraft will have the worse CCT, it will get the first shoot opportunity, meaning that CCT can be misleading. Alternatively, the argument could be given that TVC does not improve CCT at all, and that it should not be used in such a manoeuvre.

Next, consider the situation where two aircraft are passing each other for visual identification. This leads to a Pointing Margin type of scenario, where the two aircraft commence the maximum performance turn once they pass each other. This is also shown in Figure 4-3. This shows that neither of the aircraft will have a significant advantage when finishing the heading change. This is because both aircraft are limited to the same maximum AoA.
It is possible to optimise the CCT. It is important to look at this to consider a flaw in the logic of this metric. To do this, the dominating terms need to be found. The governing equation for the CCT, in Appendix C and repeated here as Equation 4-6 is,

\[
CCT = \int_{V_o}^{V_c} \frac{V}{P_S g} dV + \int_{V_c}^{V_{md}} \frac{V}{P_S g} dV + \int_{V_{md}}^{180° - \frac{d\chi}{d\tau}} \frac{V}{P_S g} dV + \int_{V_{md}}^{V_o} \frac{V}{P_S g} dV
\]

where, \( V \) is velocity, \( P_S \) is specific excess power, \( g \) is acceleration due to gravity, \( \chi \) is heading angle, subscript o refers to initial, subscript c refers to corner velocity, and subscript md refers to minimum drag speed. From Equation 4-6, it can be seen that \( P_S \) and \( V \) dominate. It also needs to be understood which segments of the CCT dominate. This is shown in Figure 4-4 for an altitude of 5km.

It can be seen from Figure 4-4 that the initial velocity will change which segments take the longest. This reflects the dominance of the initial velocity. The altitude will change the plot, but the trend will be the same. The kink in the lines occurs at an initial speed above which the manoeuvre contains no STR segment. Above this speed, the STR becomes insignificant, the ITR segment becomes roughly constant and the acceleration segment becomes increasingly dominant. Below the corner velocity, the ITR segment becomes less important, the acceleration segment becomes less important and the STR segment becomes increasingly important.
Most importantly about Figure 4-4 is the fact that the overall time taken is also shown and that the minimum time for the CCT is at the minimum drag speed (indicated by the vertical line labelled $V_{md}$ in Figure 4-4). This implies that the aircraft is most manoeuvrable (in terms of the CCT) at the minimum drag speed, however every pilot that was spoken to throughout the duration of this project confirmed that the aircraft generally performed best at least at the corner velocity, if not some speed higher. This kind of statement is a direct contradiction to the findings of the CCT, and hence doubt is cast over the validity of the CCT to determine the performance of any aircraft, let alone an aircraft with TVC/PSM capability.

Next, consider the Relative Energy State. Relative Energy State is simply a plot of the velocity ratio (current velocity divided by corner velocity) against the heading angle (which changes with time as the aircraft turns). This plot will help the analyst to understand the energy levels during the heading change, and help to explain why the standard aircraft in Figure 4-3 has a lower CCT.

As before, the tick marks indicate elapsed seconds. The gradient of the RES plot shows how efficient the aircraft is. If the gradient is zero, then the aircraft can perform back to back turns and its efficiency is 100%. The steeper the gradient gets, the less efficient the aircraft is. This confirms that the standard aircraft has a superior CCT and that it can complete the CCT quicker than the TVC aircraft.
It should be realised that the Relative Energy State and the Dynamic Speed Turn are simply tools for analysing the CCT and so if the CCT cannot show the effects of TVC, then neither will its tools.

4.2 No Post Stall Capability – Enhanced Methods

This section considers the pitching moment induced by TVC.

4.2.1 Turn Rate and SEP

The YF-22 was a technology demonstrator and prototype of the F-22 Raptor aircraft. The aircraft had the capability to switch its TVC on and off from within the cockpit. An experiment was undertaken to measure the STR at a supersonic flight condition, without TVC. The STR was then flown at the same flight condition, but this time with TVC switched on. It was found that the YF-22 had an increased supersonic STR when using TVC. At 38,000 feet and Mach 1.2, the turn rate advantage was 31 m/s in terms of SEP, meaning for the same turn rate, the YF-22 had an excess of power when the TVC was used. When considering this fact, and considering the STR as shown in Figure 4-2, there is a discrepancy. Figure 4-2 shows that there is no change to the high speed or supersonic STR. Further investigation reveals that the reason that the YF-22 had an increased turn rate was that the pitching moment required to balance (trim) the aircraft at this flight condition was the same regardless of whether the TVC was on or not. With the TVC, the required moment could be achieved with less elevator, since the TVC compensated for the loss of moment due to a smaller elevator deflection. The lower elevator deflection meant less drag, and so an excess of power was present.

Traditional metrics such as SEP and STR fail to account for this phenomenon. To account for it, a pitching moment balance is required, which determines the amount of elevator to use. With TVC this value is less, and so such a calculation should account for the increased sustainable performance as seen on the YF-22. To obtain suitable results, the angle of elevator and thrust deflection (if present) are set to balance the pitching moment to zero. Using this enhanced method of calculation will lead to more accurate results. Then the required AoA is found from Equation 4-2 or Equation 4-3, depending upon the metric being analysed. Iteration of this process is required, since the pitching moment is dependant upon the AoA, which depends upon the thrust
deflection, which in turn depends upon the pitching moment. By calculating the STR at combinations of velocity and altitude across the entire flight envelope, first for an aircraft without TVC, and then for the same aircraft with TVC, and subtracting one from the other, the change due to TVC can be found. This is simpler than calculating the excess power in each case, as was the case for the YF-22. It is also more meaningful, since the objective of the aircraft is to turn sustainably, as opposed to turning nearly sustainably with an excess of power. The change in STR due to TVC is shown in Figure 4-6.

Positive values in Figure 4-6 show that the advantage (increased turn rate) is with the TVC aircraft. It can be seen from Figure 4-6 that the largest change is only 0.2 degrees per second (7km, 375 m/s high speed or less than 8km, 100m/s low speed). At this flight condition, the turn rate is 4 degrees per second. This gives a 5% turn rate increase. In terms of SEP advantage this is approximately 5.5 metres per second, certainly nothing like the advantage discussed above for the YF-22 which has much higher thrust and is designed for super cruise. From Figure 4-6 it is also shown that there is an STR advantage at low speed (100m/s at all altitudes within the 1g zero SEP boundary). At low speed the amount of control deflection required to trim the aircraft is large, because dynamic pressure is low. The addition of TVC means again, that less elevator deflection is required.
This kind of analysis should also be done for the ITR. However, adding TVC to the ITR will not see any changes to turn rate. To balance the aircraft, the same pitching moment is required from the controls whether the aircraft is using TVC or not. The moment is produced by a force acting at the tail/nozzles of the aircraft. This force will also contribute to the overall normal force acting on the aircraft, which induces the turn. Whether this force is supplied from the elevator, or the TVC nozzles, the only difference between these two will be that the TVC force will mean that the aircraft has less drag since the elevators work less hard. This in turn will mean that the SEP will be higher, or rather less negative for an ITR manoeuvre. This will not affect the turn rate directly (Equation 4-3 and Equation 4-4), and so no change would be seen on the turn rate plot. The changes would be seen on a constant load factor SEP plot (for example a 9g SEP plot, as given by Raymer\(^1\)).

Figure 4-7 shows the changes in SEP for the F/A-18a at 1g when TVC is switched on. This is effectively the change in straight and level acceleration of the aircraft.

![Figure 4-7: Changes to 1g SEP due to TVC when pitching moment is balanced at zero.](image-url)
aircraft with TVC sees an increase in SEP of between 0 and 12 m/s. These values are not significant, especially since they are all outside of the 1g zero SEP contour. The significance of the 1g zero SEP contour is that the aircraft cannot get to any point on the right of it without first diving to gain speed. Even if this is the case, the aircraft will soon decelerate to return to the boundary. Because the contours all lie to the right of this boundary, they are therefore not significant.

4.2.2 CCT and Pointing Margin

Just as the turn rate and SEP calculations were enhanced to consider the trim of the aircraft, the same modification can be made to the CCT and pointing margin metrics. However, there will be no difference between the two aircraft in this case. It was discussed above in section 4.2.1 that the aircraft would not have an improved ITR due to TVC if the aircraft were trimmed in pitch. It was also stated that during the ITR segment, the SEP may be enhanced, however Figure 4-7 showed that this is not the case for this aircraft. The same figure also showed that there would be no significant acceleration available to the TVC aircraft. Figure 4-6 showed that the STR is also not significantly affected. Even if the STR were affected, the CCT only sometimes contains an STR segment (using the definition in Appendix C) which is likely to be very short. For these reasons, there is no significant difference made to the CCT when implementing TVC.

4.3 Post Stall Capability

4.3.1 Turn Rate Plot

By not constraining the aircraft to stay within the AoA for maximum lift, and allowing it to pitch into the post stall regime, the results of the turn rate plot are modified somewhat. This is shown in Figure 4-8, where faster than approximately Mach 0.25 the trends of the STR and ITR are similar to those shown in Figure 4-2 for non-post stall aircraft. The TVC allows a slightly greater STR at lower speeds, and a greater ITR also. The calculations used to produce the plots of Figure 4-8 ignored the pitching moment, as it was in section 4.1. Here it is assumed that TVC is used for performance enhancement via post stall manoeuvrability, and not for control alone. Below Mach 0.25 Post Stall Manoeuvrability (PSM) makes a marked difference. First, the ITR changes gradient and theoretically the aircraft gains an infinite turn rate.
at zero velocity (Equation 4-4). The STR also follows this path. Standard aircraft are limited to a minimum speed due to the fact that they will otherwise stall. It is in this region that traditionalists such as Raymer[^1] assume that all of the benefits of TVC and PSM take place.

Assuming that the thrust deflection is zero, makes the analysis independent of pitching moment. From Equation 4-3 it can be seen that load factor is a function of AoA. This implies that for the horizontal plane, the turn rate is a function of AoA. For zero TVC deflection, lines of constant AoA can be shown on a dog house plot. This creates a plot as shown in Figure 4-9, which shows that at any given turn rate and Mach number, there are two values of AoA. The two AoA represent a pre stall value and a post stall value, that is, beyond 35°. Also shown, in a dashed line, is the boundary for ITR. Furthermore, it can be shown that post stall AoA are only obtainable when the aircraft flies below the corner velocity. In this case the corner velocity is MO.48. Consider the aircraft flying at 5°AoA and MO.55 (above the corner velocity). The aircraft can pitch up to 15°AoA at the same speed, because this AoA is within the ITR boundary (and hence load factor boundary). However, to then pitch up to 30°AoA (still at MO.55) would mean that the aircraft would cross the load factor boundary. To get to a higher AoA is therefore impossible, and so the aircraft cannot
go post stall whilst maintaining a speed of M0.55. If the aircraft is below the corner velocity, it can go post stall without encroaching the load factor boundary. For example, at M0.40 the aircraft starts at 5° AoA and can pitch to 15° and 30°. To get to 45°, the aircraft would move up the M0.40 line on the turn rate plot and then intersect the ITR line. At this point it would then go back down the M0.40 line to 45°, 60° and so on.

A post stall boundary therefore exists at the corner velocity. In reality the aircraft will actually slow down as it is pitching up because it will have negative SEP at post stall AoA. To demonstrate this on the turn rate plot, the dynamic case should be considered.

The 2 DoF model developed in chapter 3 was used to determine the dynamic effects on the turn rate plot. The simulation was started at the corner velocity with an AoA
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corresponding to the ITR. The model was given a constant commanded AoA. Figure 4-10 and Figure 4-11 show traces of where the aircraft travels on the turn rate plot as time goes by. The simulation was started at point A, indicated in both figures.

Figure 4-10 represents post stall angles of attack. For a commanded AoA of 45°, the aircraft pitches up from point A to point B. Then as AoA is maintained, the aircraft loses speed and travels towards point E, until it intersects the STR line at M0.20 and 15°/s turn rate. If the commanded AoA is 75°, the aircraft pitches from A to C. It does not stop when it intersects the STR line because it is still pitching up. Once at point C, the aircraft maintains 75° AoA and moves along the 75° AoA line on to
around E, where it intersects the STR line and has zero SEP. Similarly, for commanded AoA of 90°, the aircraft travels from A to D and on to near E. Figure 4-11 shows the boundaries for pre stall flight. The aircraft starts at point A with AoA of around 35°. For a commanded AoA of 30°, the aircraft pitches down to F. It then has negative SEP and hence decelerates to point J. At this point, it intersects the STR line and has zero SEP. Because of this, it will neither accelerate nor decelerate, and continues a sustained 30° AoA turn. For a commanded AoA of 15°, the aircraft starts at point A and pitches down to point I. As it does so, it intercepts the STR line at 15° AoA and again, stays at that point. Finally, for a commanded AoA of 5°, the aircraft pitches down to point G, from A. Here, it has positive $P_s$ and so accelerates along the 5° AoA line past H, until it intercepts the STR line at a point not shown in the figure.

Figure 4-11: Pre Stall Dynamic Boundaries.

Figure 4-12 shows the post stall boundary. To obtain a post stall AoA, the aircraft has to be within this boundary. The high speed side of the boundary (A-B) is discussed above in Figure 4-10. It is now assumed that the aircraft is limited to 75° AoA and hence cannot fly a post stall AoA below B-C. Angles of attack below 30° do not constitute post stall flight (C-D). Below M0.10, the aircraft is still limited to 75°
which results in D-E-F-G. Finally, the aircraft cannot turn quicker than the ITR boundary (G-A).

Once a post stall boundary is drawn, it indicates where the aircraft can fly post stall. This is important because it shows the area in which the aircraft may gain an advantage. As altitude is increased, area EFG decreases as the value of the turn rate at F decreases. Above around 2000m altitude for this aircraft, point F intersects the x-axis (1g line).

4.3.2 SEP

For any given velocity, either of the pre-stall or post stall AoA for any given turn rate, can be substituted into Equation 4-2 to solve for the SEP of the aircraft at that point. This was done to produce the plot shown in Figure 4-13, which shows that lines of constant SEP cross each other. The thick black line shows STR. The maximum turn rate below M0.20 is the sustained turn rate. Upon closer examination, immediately to the left of this portion of the STR line (M0.07 and less), the SEP is slightly positive (shown as positive in the figure) in the order of <10m/s. This means that at such a
point, the aircraft would accelerate. Reducing thrust to stay at that point would not work. This is because as thrust is reduced, the minimum point (A) of the STR line gets closer to the x-axis. Eventually this point (A) intercepts the x-axis, and turn rate is reduced to zero. Keeping the thrust sufficient to ensure the aircraft turns will mean that it has positive SEP at all times. Since it will have positive SEP to the left of point A, it will not actually be able to get to this region. As soon as the aircraft gets slightly to the left of point A, it accelerates and returns to point A. Hence the portion to the left of point A is not actually useable in the horizontal plane, because the aircraft can never get there. Moreover, for T/W ratio less than unity, point A will intersect the x-axis and horizontal turn rate will reduce to zero, still making this portion unusable. In doing so, the normal load factor also reduces to less than unity, and the aircraft starts to fall out of the sky. Equation 4-3 shows that for very high AoA, when the lift is very small, if T/W < 1 then load factor will be less than unity.
4.3.3 Combat Cycle Time

At first glance, post stall application will mean that the aircraft will be able to slow down quickly once below the corner speed (by using high AoA) and attain the very high turn rates on the left hand side of the post stall turn rate plot (Figure 4-8). On application however, this does not occur. The aircraft cannot complete the CCT any faster than a conventional aircraft. This is because the CCT constrains the aircraft to the ITR, and as the aircraft slows along the ITR curve, it intercepts the STR line. Once this happens, the aircraft will stay at that point on the turn rate plot. This is shown as point A in Figure 4-14. The aircraft cannot go post stall above the corner velocity as stated earlier, but if the aircraft were to pitch up to high AoA below corner velocity to slow it faster and reduce the turn radius, then it would not be at the ITR. The turn rates would be less than the ITR in this case and so it would take longer to change the flight path heading angle. The acceleration segment would also be longer since the aircraft will have lost more energy while at post stall AoA. Hence the total time taken will be longer than for the conventional aircraft. Also, pitching to post stall AoA to get the nose around is of no use since the heading change (the angle between the nose of the aircraft and the target heading) will no longer be complete once the aircraft pitches back down to accelerate. This is known as Dynamic Turn and is discussed by Raymer\[6].

Considering the turn rate plot, more loci can be added to help determine where the aircraft would move to as time elapses. Figure 4-14 shows these in the form of arrows. At any point on the ITR curve, the aircraft would follow the arrows until it intercepts the STR curve. There is a small exception to this, if the aircraft were post stall (within the post stall limit of Figure 4-12), then it would head towards the minimum, indicated by point B in Figure 4-14. The above paragraphs show that although nose pointing can be achieved, the very left hand portion of the turn rate plot is not accessible when restricted to the horizontal plane. Raymer\[6] shows how gravity can be used to assist in turning the aircraft. Equation 4-4 contains a term for the flight path angle, $\gamma$. When the aircraft manoeuvres in the vertical plane, this term will change from negative unity to positive unity, as the aircraft goes from normal to inverted flight. This fact can be used to assist the aircraft in achieving the high turn
rates shown on the left hand side of the post stall turn rate plot. The diagrams in Figure 4-15 explain this, and are derived from Raymer.

Figure 4-14: Travel on the Turn Rate Plot as Time Elapses.

Figure 4-15: The effect of flight path angle on ITR, a) pull up, b) vertical, c) pull down.
Simulations of turns in the vertical plane can now be made. If the turn rate plot is examined with plots for pull up, vertical and pull down superimposed on the same axes, then the changes that the vertical plane has on turning motion can be illustrated. This is shown in Figure 4-16 with a profile (the contour taken by the aircraft on the turn rate plot) of an aircraft flying at a constant 70° AoA also shown.

The profile of the simulation in pink shows that the aircraft now crosses over to the very left hand side of the turn rate plot. In fact the aircraft simulated reaches turn rates just greater than 70°/s – more than twice the rate of a conventional F-16 for example. This suggests that an aircraft using the post stall regime should be able to outperform a conventional aircraft. However, this high turn rate only lasts while the aircraft goes through inverted and is not sustainable. Once the aircraft is again upright, it returns to a low turn rate at a lower speed (the aircraft will have less energy than when the manoeuvre was initiated). Such a manoeuvre is similar to the Herbst Turn[^7], where the aircraft pitches to high AoA, rotates 180° about the velocity vector, and in so doing, reverses the heading angle. It then returns to low AoA and accelerates away, having completed a small radius turn reversal. This manoeuvre has been shown to enhance performance during simulated combat in reference 7. The vertical turn shown here is a similar manoeuvre in that it initiates with high AoA. Then, once inverted, it is a pull down manoeuvre exactly like the latter stage of the Herbst Turn. The difference is that there is no reverse of heading angle with the
vertical turn, as there is with the Herbst manoeuvre, and so it is likely that the vertical turn is of less tactical use. Most importantly however, if static calculations are made, then it can be shown that the aircraft can attain turn rates as high as $70^\circ/s$, in this case. But if the dynamic case is considered, where the manoeuvre is simulated, it soon becomes clear that the actual turn rate cannot sustain this high value for more than a very short period of time.

4.3.4 Pointing Margin

The pointing margin metric is potentially very useful. However, unfortunately it relies on a simulation of an advanced fighter against an adversary. This then depends upon the turn flown by the enemy. Although this metric begins to test performance enhancements, it does not really go into enough depth. Before it becomes useful it will need to be extended to include such things as energy usage and remove the dependency on the enemy tactics.

The previous section showed that very high turn rates at low speed could be achieved for while manoeuvring in the vertical plane. Figure 4-17 is a plot of the vertical plane showing the profile of a conventional aircraft flying against a TVC PSM aircraft. The conventional aircraft flies at an AoA for ITR throughout the vertical loop manoeuvre. The advanced aircraft flies at a constant $70^\circ$ AoA to achieve the high turn rates discussed in the last section and use these for an overall advantage.

Figure 4-17: Plot of the Vertical Plane Showing Disadvantage of Flying Post Stall (Both aircraft start at 1km altitude and 150m/s velocity).
The tick marks represent elapsed seconds, and point along the direction of the gun vector of the aircraft. As the red dotted lines show, the conventional aircraft gets a shot after 9 seconds, where the advanced aircraft still cannot shoot. Furthermore, the conventional aircraft can continue to track by simply reducing angle of attack. The advanced aircraft cannot gain a shot until at least two seconds later. It will also be in a very low energy state and be bleeding much energy quickly. The reason that it has the disadvantage is that it decelerates so much during the first few seconds of the manoeuvre. Note that even though the turn radius is reduced, the advanced fighter still has the disadvantage.

So this shows that the advanced aircraft has no advantage in using post stall flight, but that it actually has great disadvantage. The question is of how to use post stall manoeuvrability to gain an advantage. The best advantage that can be obtained is via the use of nose pointing. The advanced aircraft should fly as a conventional aircraft until the point where pitching to high angle of attack means that the target shoot solution is obtained. This is shown graphically in Figure 4-18, where the same conventional strategy as before is being used, and this time the advanced TVC PSM aircraft is flown at an angle of attack for ITR until pitching leads to an advantage.

**Vertical Plane**

![Figure 4-18: Plot of the Vertical Plane Showing Advantage of Flying Post Stall.](image)

- TVC PSM A/C
- Conventional A/C

Figure 4-18: Plot of the Vertical Plane Showing Advantage of Flying Post Stall.
Here a clear advantage for the advanced aircraft is shown. After 4 seconds the advanced fighter pilot would realise that if the aircraft were pitched to 70° AoA then a clear shot could be taken. The aircraft is pitched after 4 seconds and after 6 seconds the advanced aircraft can now shoot at the conventional aircraft. To maintain the shooting advantage, this time the advanced aircraft need only reduce angle of attack (not shown on figure). It is more than another 2 seconds until the conventional fighter obtains a shoot solution.

This has shown that a clear advantage exists for the advanced aircraft as long as the correct strategy is flown. Although existing metrics such as the energy management plots and the turn rate plots show differences between aircraft, they do not necessarily show advantages. For example the turn rate plot shown above for the vertical plane shows that flying a constant 70° AoA gives extraordinarily high turn rates, but as the simulation showed, there was no advantage at all. The reality is that for an advantage to be gained using post stall flight, the aircraft needs to point its nose.

### 4.4 Other Metrics

Sections 4.2 and 4.3 considered the non-PSM and PSM capabilities respectively. However, of the seven metrics given in Table 2-2, only five have so far been examined (ITR/STR, SEP, acceleration, CCT/DST/RES, Pointing Margin). This leaves: a metric to examine post stall attainability/recovery, and Frenet Agility. Consider the former. If a metric was like those proposed in chapter 2, then it would measure for example load factor rate, or time to pitch to a given AoA and perhaps return. This kind of metric measures a highly dynamic performance of the aircraft over a very short time, and because of this and the fact that the available input data is not likely to be that accurate, it is not feasible to use such a metric in the analysis. This is discussed in more depth later with validation of the model in chapter 6.

As far as Frenet’s Agility is concerned, this metric requires a manoeuvre on which it can be calculated. Because of this, it is not yet considered due to the fact that no truly suitable metric has been found. Once such a metric is developed, Frenet’s agility will be considered.
4.5 Conclusions of Existing Metrics

- Traditional methods of calculating Specific Excess Power do not provide useful information when TVC is used as a means of reducing trim drag. The method needs to be extended to incorporate a pitching moment balance.
- It is possible to determine a pre-stall and a post stall SEP, and to show these static values on the turn rate plot.
- A post stall boundary can be applied to the turn rate plot to show where the aircraft is capable of post stall flight.
- Although the CCT metric shows that an optimum velocity can be found, its value does not agree with that shown by other traditional metrics, nor with values suggested by combat experienced pilots.
- Using TVC while ignoring pitching moment does not make it possible to gain an accurate understanding of the CCT metric. Only when both pitching moment and a post stall capability are considered does it become clear that implementing TVC will not improve the performance of the CCT. Hence the CCT does not add to the analysis of advanced aircraft.
- Although traditional metrics show extremely high values of turn rate at very low speed, it is not possible to gain a tactical advantage using these statically calculated values in a real combat scenario.
- Pointing Margin comes closest to providing more information about an advanced aircraft, but still lacks information that is essential to performance assessment.

It is clear that thrust vectoring and post stall flight together can give a powerful combat tool to the pilot for close in combat dog fighting. It is also clear now that although conventional metrics show differences between aircraft, these differences are not necessarily representative of what really happens, especially over a period of time during combat. Hence new methods of performance assessment do need to be found.

4.6 References

3. Microsoft ® Excel 97, The Microsoft Corporation, USA.


It is of interest to the pilot and hence designer, how quickly the aircraft state can be changed. Questions like, 'How long does it take to manoeuvre the aircraft to a given position with a given direction? ', 'How much energy will be expended? ' and 'What is the final rate of change of energy? ', need to be answered. In an attempt to answer these questions, Valasek & Downing\[1\] called for a metric that incorporates time, turn radius and energy efficiency of a manoeuvre. Such parameters will be considered by the new metric.

The objective of work by Costes\[2\] was to determine the effects of thrust vectoring and post stall flight on combat. This work used simple aircraft models without pitch or roll modelling to simulate combat. The equations were developed and simple laws were added to the Flight Control System (FCS) to fly the aircraft so that they were always trying to improve their positional relationship to the enemy. Using a probability function (called iso-threat curves), the probability of victory was calculated over the combat run. To test this, the aircraft were programmed to always try to point towards the enemy. Because the aircraft were free to roam the skies limited only by their control laws, the number of different flight paths was potentially infinite, not allowing straightforward analysis of the results.

By constraining this work, the results could be made more meaningful to designers and tacticians. The objective of any manoeuvre is to position the aircraft in such a way that the flight path is set up as the pilot requires. Hence to test an aircraft's ability to change the flight path from one state to another will test the aircraft's agility, manoeuvrability and performance (as defined earlier in 2.5.2). The proposed metric considers the aircraft flying from an origin to a point where the gun vector of the aircraft pointed towards a pre-defined target, meaning that nose pointing is allowed.
To reduce the order of complexity the target is assumed to be stationary. That is, possible manoeuvres by an enemy aircraft are ignored.

Figure 5-1, shows that to one side of an aircraft (assuming that it is symmetrical about the longitudinal axis) there are 17 points of interest. Each node tests different aspects of the functional agility of the aircraft. For example, node 2 tests the vertical plane performance, as does node 13. Node 10 shows the horizontal plane performance, whilst node 8 tests the axial performance of the aircraft in acceleration. The functional agility could be measured in terms of the time taken to get a first shot at the node, the amount of time that the aircraft can track the point thereafter, the time taken to fly through the node, or the energy lost during the manoeuvre.

At this stage however, the distance between the node and the origin was undetermined. For example, should the target point be 1km or 10km away from the aircraft? There is also an issue that if the target is too close and the aircraft’s speed is too high, the aircraft may not be able to manoeuvre to point at the target. This is illustrated in Figure 5-2.
A NEW METRIC

Figure 5-2: The Effect of Initial Mach Number on a Vertical Manoeuvre Aircraft Flight Paths.

From the left hand schematic, it can be seen that there is no problem with the target distance from the origin at low speed. As speed increases, so does the turn radius, and the aircraft performs less and less of a conventional combat realistic manoeuvre. Eventually there comes a point where the aircraft will never be able to get a shot on the target, as shown in the far right hand depiction.

To overcome problems in defining target position, the strategy used in flying the manoeuvres for the new metric was changed slightly. Instead of flying the aircraft towards a given target, the objective of the manoeuvre was to perform a heading change. This chapter considers 180° heading changes, although heading changes other than this may also be considered. Of the seventeen nodes shown in Figure 5-1, not all are actually necessary since there are too many to provide a concise analysis. However the four most important nodes are those used to measure 180° turning performance in the vertical pull up, vertical pull down, and horizontal planes, as well as acceleration in the straight and level sense.

Since the objective of the manoeuvres was to measure the performance of the aircraft using post stall manoeuvring, it was assumed that the pilot would be neglecting energy performance in favour of maximum turn performance. Since this was the case, the logic used to control the aircraft was that it would start at a 1g straight and level flight condition.
Figure 5-3 shows an arbitrary starting condition as point A. Also shown on Figure 5-3 is the ITR (both load factor and stall limited portions), lines of constant AoA in chain dashed, and the proposed path of the aircraft, in thick dashed black. The aircraft would then pitch up to an AoA that gives the instantaneous turn rate (ITR) which is its maximum turn rate for any given flight condition, shown as point B. In this case the aircraft is flying fast enough to be load factor limited, and so the AoA increases as the aircraft loses speed. The aircraft loses speed and decelerates through the corner velocity, point C, and flies at a constant 20° AoA. The aircraft then pitches to its maximum post stall AoA as soon as the required pointing angle becomes less than the maximum AoA. This is shown as point D, where the aircraft pitches from 20° to 70°. At point E, the manoeuvre is deemed to be complete, however, if the simulation continued, the aircraft would track the profile shown in Figure 5-3 to point F and beyond.

It was assumed that this logic is quite representative of what a real pilot may try. Above the corner velocity, the AoA for ITR is cued via the aircraft being at the load factor limit. Below the corner velocity, the AoA for ITR is cued via the aircraft being approximately on the stall limit, which is known to the pilot via any AoA sensors that
supply a reading inside the cockpit, as well as any form of buffet feedback that is transferred through the control stick. The point at which to pitch the aircraft to the post stall regime will be part of the pilot's experience in judging how far off the target the aircraft is. Although this pilot experience is not directly modelled in this work, it is included via the use of the logic which is based upon a perfect pilot's assumptions as to when pitching should commence.

In line with the thinking of Valasek, four parameters were chosen to quantify the aircraft's performance. These were, time taken to complete the manoeuvre, final SEP of the aircraft at the end of the manoeuvre, energy change over the manoeuvre and turn diameter. Each of these four parameters is extremely important. The time taken is required since it is a natural parameter for pilots to discuss. When pilots were interviewed, the parameter of time was frequently brought up and discussed as a desirable term to use in aircraft performance. The SEP and energy consumed are important since they show the change in the energy state of the aircraft. However, both are as important as each other. The former shows what the aircraft is capable of in the instant that the manoeuvre is complete, and is ever more important for a PSM capable aircraft because of the relationship between high AoA and large energy bleed rates. Final energy state is also important in the modern combat arena because a) a weapon's success is increased as its initial energy state is made larger, and b) it shows the potential for subsequent manoeuvres. The energy consumed shows what has happened to the aircraft's energy as the manoeuvre is flown and allows the effectiveness of manoeuvres to be assessed. Finally the turn diameter is given since it ties in with traditional metrics. It gives an idea of the geometry of the manoeuvre, and allows spatial analysis to be undertaken – analysing where two aircraft will be in relation to each other after a manoeuvre.

5.1 Example of Results Obtained from the New Metric

Results are obtained when the four result parameters (time, final SEP, energy change and turn diameter) are recorded for each manoeuvre flown where the initial conditions are varied such that the entire speed-altitude flight envelope is examined. The following grid over the flight envelope was used for all of the results shown.
Velocity (m/s): 10, 44, 78, 112, 146, 180, 214, 248, 282, 316, 350, 384, 418, 452, 486, 520, 554, 588. (These are Mach intervals of approximately 0.1 at sea level).

Altitude (m): 500, 1000, 1500, ..., 16000, 16500, 17000.

The aircraft started at 1g and followed the path shown in Figure 5-3. The control of the aircraft relied upon the controller knowing when to point the aircraft’s nose – this was effected by the use of the target solver, derived in Appendix B. The results can be viewed graphically using a contour plot for each of the four results. This is shown in Figure 5-4, where the results are for the aircraft turning through 180° in the vertical pull up plane. Figure 5-4 shows the results for a standard aircraft with no thrust vector control or post stall manoeuvre capability. The contours for the first plot show how many seconds are taken to complete the manoeuvre – to point at a heading change of 180° with the aircraft guns. For example, at an initial speed of 300m/s and an initial altitude of 5km, the manoeuvre is completed in 12 seconds. The final SEP of the aircraft is approximately -100m/s, the aircraft loses around 2000m of energy height during the manoeuvre, and the turn diameter is nearly 1600m.

5.1.1 The Manoeuvre Stall Boundary

There are two noteworthy points shown in Figure 5-4. The first is that there is the white area labelled MSB, which stands for Manoeuvre Stall Boundary. This area is present in all four of the plots in Figure 5-4. An aircraft starting with initial conditions in this area will not be capable of completing the manoeuvre because it will lose velocity to the point where it will stall at some point before completing the manoeuvre. In the horizontal plane the aircraft will simply begin to dive, but will still be able to complete the turn in one sweep. In the vertical plane the situation is totally undesirable to the pilot. The aircraft will get to a point on the flight path where it starts to stall. If the pilot were to pull back on the stick at that point, with the aim of pointing at the target, the aircraft would not be able to do so without pitching above the AoA maximum limit. This means that the aircraft will pitch down (even if AoA is held constant, the flight path will pitch down) and gain speed. This is clarified pictorially in Figure 5-5.
A NEW METRIC

Figure 5-5: A typical vertical flight path when the manoeuvre is initiated to the left and above of the MSB.

Until now the only boundaries on a flight envelope plot have been the 1g SEP boundary and the stall boundary. These are compared to the MSB in Figure 5-6.

Figure 5-6 shows how the MSB is very different to both the stall limit and the 1g zero SEP limit. In fact, at the low speed end (around 50m/s in Figure 5-6) the MSB is dominated by the traditional stall boundary since the aircraft cannot produce enough lift to create the 1g starting condition. At the high speed end of the MSB, it is actually altitude limited (above 350m/s in Figure 5-6). This is to say that the aircraft will go
above the model altitude limit, which in this case was 17km. In reality, the MSB always consists of a stall limited portion at the low speed end and, in the vertical plane, an altitude limit exists at the high speed end (horizontally the altitude does not change). Note that the MSB shown in Figure 5-6 is not smooth due to the grid density used in calculating the contours. If a finer data grid were used, then the MSB would be smooth.

Apart from low altitudes, the speed for the MSB is higher than both the 1g zero SEP limit and the stall limit. This shows that traditional belief in the stall boundary gives a false security, as in reality, the aircraft would not be capable of completing a maximum performance manoeuvre such as the one simulated here. Currently, only pilot experience would provide such information.

5.1.2 Corner Velocity and the Optimum

The second noteworthy point mentioned in section 5.1.1, is the corner velocity. Section 2.1.6 discussed how the corner velocity was often cited as the velocity at which the aircraft is most manoeuvrable, with maximum turn rate and near minimum radius. This velocity is hence of interest to the performance analyst, and is so added to Figure 5-7 as a thick black solid line.

![Figure 5-7: Time Taken to Complete Vertical Pull Up Heading Reversal.](image)
Figure 5-7 shows the plot of time taken to complete the manoeuvre, and is identical to that shown in Figure 5-4. In the same way that corner velocity shows the speed for maximum turn rate at any given altitude, the results obtained from the new metric were used to find the velocity for minimum turn time (and hence maximum average turn rate) at any given altitude. This line is shown as a thick black dashed line. The turn diameter plot in Figure 5-4 can be examined, and it is seen that the optimum line in Figure 5-7 is also near to the minimum diameter, but not actually the minimum diameter. The important aspect of this line is that it does not line up with the corner velocity even though they both aim to show the same condition.

This difference is especially noteworthy at lower altitude. This difference can be explained by looking at the path taken by an aircraft as it travels around the turn rate plot.

Figure 5-8 shows an example, with a dashed black line marking where the aircraft travels (B to C). In a linear sense, the average turn rate along B to the corner velocity (A) will be 20 degrees per second. The average turn rate will then also be around 20 degrees per second between point A and point C. If the aircraft spends equal amounts of time between B and the corner velocity, and the corner velocity and C, then the total average turn rate will be 20 degrees per second. Now consider what happens if the aircraft were to start at point A, the corner velocity, and finish at point D (a similar
speed loss). The average turn rate will be roughly 18 degrees per second. Therefore, to reduce the time to turn through 180°, it is better for the aircraft to start at point B, above the corner velocity.

Hence to minimise the time, the initial position of the aircraft on the turn rate plot needs to be found. It is shown in Appendix C that the time taken for the turn is,

$$t = \int_{v_c}^{v_{\text{SEP}}} \frac{V}{\text{SEP} \cdot g} dV + \int_{v_c}^{v_{\text{STR}}} \frac{V}{\text{SEP} \cdot g} dV + \frac{180° - \int_{\psi_{\text{STR}}}^{\psi_{\text{MAX STR}}} d\psi}{\psi_{\text{MAX STR}}}$$

Equation 5-1

where $t$ is time, $V$ is velocity, SEP is specific excess power, $g$ is gravity, $\gamma$ is the heading angle, and $\psi$ is turn rate. Equation 5-1 is heavily dependent upon the SEP of the aircraft. The first term in Equation 5-1 is the time taken for the aircraft to slow to the corner velocity. The second term is the time taken for the aircraft to slow from the corner velocity to the sustained turn rate, point D in Figure 5-8. The final term is the time taken to complete the heading change once at the sustained turn rate. Obviously, these terms will have limits. If, for example, the initial velocity of the aircraft is less than the corner velocity, then the first term will be zero. If the aircraft completes the 180° heading change before it slows to the STR, then the second term is adjusted so that the integral takes place between the corner velocity and the final velocity, and not between the corner velocity and the velocity for STR.

Optimising Equation 5-1 can be done by looking at the effect on time of varying the initial velocity. This is done using numerical integration on a PC, and the results are shown in Figure 5-9, for three altitudes. For each line, there is an optimum point, where the time taken is a minimum (designated with $\Delta$); a corner velocity (designated with $\triangleright$); and for reference, a velocity where the STR and ITR are identical (designated with $\otimes$). Notice that the corner velocity is always less than the optimum turn velocity, for the reasons given above.

It is worth considering the optimum points as far as the other three result parameters are concerned. Figure 5-10 is a repeat of Figure 5-4, but also shows the optimum velocity for any given altitude according to the individual parameters.
Figure 5-9: Time to Turn 180°.

Figure 5-10 shows that for a given altitude, the optimum SEP (zero or positive) can be achieved at speeds that coincide with the MSB. It shows that for a given altitude, the optimum energy change (zero or positive) can be achieved at speeds that coincide with the MSB. It also shows that for a given altitude, the optimum turn radius (smallest) can be achieved at speeds that coincide with the MSB, except at low altitude where a slightly higher velocity is required. If the velocity were kept slightly higher than the MSB, then the turn radius would not be that much greater, except at higher altitude where the increase becomes significant.

Shaw[1] speaks of angles and energy fighting. The former refers to allowing combat to be dominated by turn rate (time). The latter refers to allowing combat to be dominated by energy consumption. It is clear from Figure 5-10 that there are two distinct types of optimum combat, and that one certainly relates to reduced turn time, and that the other relates to increased energy efficiency. Using the results from Figure 5-10 it is possible to describe quantitatively where these two classifications are, and by how much they differ.
Figure 5-10: Optimum velocity for a given altitude, according to each of the four results parameters, shown in red.
5.2 Summary of New Metric

The new metric has so far only been used to introduce some very simple results, but has already shown that there are a couple of important conclusions. The first of these is that there is a new boundary called the Manoeuvre Stall Boundary, which is akin to the traditional stall boundary, except that it considers an aircraft manoeuvring over a time span. Because of this, it is superior to existing boundaries, in that it shows a result which, currently only pilot experience provides. The second conclusion is that the traditional corner velocity is not necessarily the most efficient speed at which to manoeuvre an aircraft, especially considering that the fact that it will decelerate while turning at its maximum rate. These conclusions provide simple yet very different results from any traditional or other existing metrics.

5.3 References

An important part of this work was to be able to validate the results. It was required that the low order model compared favourably with the high fidelity model. Making this comparison meant that the level of confidence in the low order model could be found, in particular, relative to the results from the new metric.

6.1 Comparison Between High Fidelity Model and Low Order Model

To validate the low order model, a standard manoeuvre was examined that modelled both acceleration and pitching. The manoeuvre represented the one used for the new metric developed in chapter 5, that is, a $180^\circ$ heading change in the vertical plane. Since the models could represent either an aircraft with TVC/PSM capability, or one without, two sets of validation were required. For the TVC/PSM validation, the manoeuvre chosen was one where the aircraft was initially trimmed at $1g$, then pitched to $20^\circ$ AoA. The roll angle was set at zero so that the manoeuvre was performed in the vertical plane. Then, when the aircraft got to a flight path angle of $110^\circ$, the commanded AoA was set at $70^\circ$. As soon as the aircraft pointed at the target, the manoeuvre was complete. For the non-TVC/PSM validation, the same manoeuvre is used, only instead of pitching to $70^\circ$, the aircraft only pitched to $30^\circ$.

To compare the two models across the entire flight envelope, 16 combinations of initial velocity and altitude were chosen. These were at combinations of the following points, spread evenly throughout the flight envelope.

<table>
<thead>
<tr>
<th>Initial Velocity:</th>
<th>100m/s</th>
<th>250m/s</th>
<th>400m/s</th>
<th>550m/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Altitude:</td>
<td>1km</td>
<td>5km</td>
<td>9km</td>
<td>12km</td>
</tr>
</tbody>
</table>
There were a few exceptions to this. For velocities of 100m/s at altitudes above 1km, and 250m/s at altitudes above 9km, the aircraft would start within the Manoeuvre Stall Boundary, so these points were not validated. Also, at speeds of 550m/s the upper altitude test point was lowered to 11km instead of 12 km. This was due to the fact that if the aircraft started at the higher altitude, it would then climb above the altitude limit of the model (17km) before the end of the manoeuvre.

The high fidelity model was not supplied with a load factor limiter, and so as opposed to adding one to this model, the limiter was removed from the low order model because this was the simpler solution. Furthermore, adding a load factor limiter to the high fidelity model would not have proven anything because it would have been identical to the low order model load factor limiter. The results shown in later chapters all include the load factor limiter because in real life such a limit must be imposed to obtain realistic and representative results.

The high fidelity model also incorporated modelling of the pilot’s physical ability to move the control stick. Although the value of commanded AoA was an input to the model, the value that was used as input to the controller was calculated from this input, by the model, and included the pilot modelling. Such modelling was too accurate to include in the low order model, because of its input data limitations. However, since the pilot model was embedded within the Robust Nonlinear Dynamic Inversion method\textsuperscript{[1,2]}, it was simpler to add the lag from the pilot model to the low order model. This was done by running a simulation using the high fidelity model and recording the commanded AoA (the stick deflection) that was input to the controller. This recording was then entered as the commanded input to the low order model. It did not matter that the low order model then included the pilot since it was a comparison between the two models that was desired, and subtracting one model from the other would remove this similar element.

The values used to make the comparisons between the two models were the four parameters used for recording the results of the new metric, that is, time, final SEP, energy consumed, and turn diameter. By using these parameters, a level of confidence in the low order model was given in relation to the metric. The exception to this was that the final AoA was used as opposed to the final SEP. The reason for
this was that the final SEP is dependent upon the accuracy of the AoA and the velocity. Since the velocity itself is dependent upon time, the SEP was sensitive. For this reason, and to aid in assessing the pitch modelling (where the biggest differences between the models existed), the final AoA was used. It is however important to use final SEP when using the metric, and not final AoA, for the reasons given in chapter 5.

The results of the validation for the TVC/PSM aircraft are summarised in Table 6-1. The results of the validation for the standard aircraft are summarised in Table 6-2.

Table 6-1 shows the results of the difference between the high fidelity and low order simulations at each of the test points for the TVC/PSM aircraft model. The four results parameters are shown for each test point as a sub-table, for example at 1000m and 100m/s:

<table>
<thead>
<tr>
<th>time</th>
<th>Energy consumption</th>
<th>Final AoA</th>
<th>Turn Diameter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.9s</td>
<td>6.7%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.7%</td>
<td>-11.3%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-47m</td>
<td>12m</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-11.0%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6-1: Sub-table taken from Table 6-1 to demonstrate which cell represents which values.

There are two numbers in each cell, the upper one being the numerical difference between the two simulations (positive values indicate that the low order model had the greater value), and the lower number being the percentage change of the low order simulation over the high fidelity simulation. Marked in red are those cells with percentages over 10%.

Table 6-2 shows the results of the difference between the high fidelity and low order simulations at each of the test points for the standard aircraft model. It can be seen from Table 6-1 and Table 6-2 that there are only a few cells with differences greater than 10%. Furthermore, the cases where the difference is more than 10% are not consistently the same points in the flight envelope, nor do they consistently affect the
<table>
<thead>
<tr>
<th>TVC/PSM</th>
<th>100</th>
<th>250</th>
<th>400</th>
<th>550</th>
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<tr>
<td></td>
<td>0.9s</td>
<td>0.2s</td>
<td>0.1s</td>
<td>0.0s</td>
</tr>
<tr>
<td></td>
<td>6.7%</td>
<td>3.3%</td>
<td>2.3%</td>
<td>0.0%</td>
</tr>
<tr>
<td>1000</td>
<td>-7.8°</td>
<td>-2.0°</td>
<td>-3.9°</td>
<td>-11.0°</td>
</tr>
<tr>
<td></td>
<td>-11.3%</td>
<td>-3.3%</td>
<td>-7.3%</td>
<td>-20.6%</td>
</tr>
<tr>
<td>Altitude [m]</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td>0.4s</td>
<td>0.1s</td>
<td>-0.2s</td>
<td>-1.8°</td>
</tr>
<tr>
<td></td>
<td>4.3%</td>
<td>1.5%</td>
<td>-2.6%</td>
<td>-3.0%</td>
</tr>
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<td></td>
<td>-45m</td>
<td>9m</td>
<td>34m</td>
<td>6m</td>
</tr>
<tr>
<td></td>
<td>2.8%</td>
<td>1.2%</td>
<td>4.1%</td>
<td>0.5%</td>
</tr>
<tr>
<td>9000</td>
<td>0.9s</td>
<td>0.5s</td>
<td>-0.5s</td>
<td>2.9°</td>
</tr>
<tr>
<td></td>
<td>5.1%</td>
<td>3.7%</td>
<td>-3.7%</td>
<td>4.8%</td>
</tr>
<tr>
<td></td>
<td>-53m</td>
<td>-37m</td>
<td>-1027m</td>
<td>-109m</td>
</tr>
<tr>
<td></td>
<td>3.9%</td>
<td>0.7%</td>
<td>10.7%</td>
<td>-5.3%</td>
</tr>
<tr>
<td>12000</td>
<td>0.5s</td>
<td>-0.5s</td>
<td>-4.2°</td>
<td>-6.9%</td>
</tr>
<tr>
<td></td>
<td>5.1%</td>
<td>-5.5%</td>
<td>0.9%</td>
<td>7.3%</td>
</tr>
<tr>
<td>(11000 @ 550m/s)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-274m</td>
<td>-46m</td>
<td>-607m</td>
<td>-258m</td>
</tr>
<tr>
<td></td>
<td>7.5%</td>
<td>-1.3%</td>
<td>7.3%</td>
<td>-4.7%</td>
</tr>
</tbody>
</table>

Table 6-1: TVC/PSM Aircraft Validation Summary
<table>
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<tr>
<th>STANDARD</th>
<th>Velocity [m/s]</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>100</td>
</tr>
<tr>
<td>1000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.5s</td>
</tr>
<tr>
<td></td>
<td>2.9%</td>
</tr>
<tr>
<td></td>
<td>3m</td>
</tr>
<tr>
<td></td>
<td>0.2%</td>
</tr>
<tr>
<td>5000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3s</td>
</tr>
<tr>
<td></td>
<td>2.3%</td>
</tr>
<tr>
<td></td>
<td>-30m</td>
</tr>
<tr>
<td></td>
<td>-0.4%</td>
</tr>
<tr>
<td>9000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.3s</td>
</tr>
<tr>
<td></td>
<td>1.2%</td>
</tr>
<tr>
<td></td>
<td>-106m</td>
</tr>
<tr>
<td></td>
<td>-0.9%</td>
</tr>
<tr>
<td>12000</td>
<td></td>
</tr>
<tr>
<td>(11000 @</td>
<td></td>
</tr>
<tr>
<td>550m/s)</td>
<td></td>
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<tr>
<td></td>
<td>-0.3s</td>
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<tr>
<td></td>
<td>-0.9%</td>
</tr>
<tr>
<td></td>
<td>-168m</td>
</tr>
<tr>
<td></td>
<td>-1.0%</td>
</tr>
</tbody>
</table>

Table 6-2: Standard Aircraft Validation Summary
same parameter. What is noticeable however, is that the errors are consistently greatest near the MSB and also at high speed. Furthermore, the magnitude of the errors is larger for the TVC/PSM aircraft. The reasons for these differences are that i) unlike the high fidelity model, the low order model does not include the term due to pitch rate, and ii) the controller gains (and methods) are different between the two models.

The former of these is seen by comparison of the model equations in chapter 3, and can be written as Equation 6-1, which shows the term that is missing in the low order equations.

\[ C_L = \ldots + \frac{c}{2V} C_{Lq}(\alpha) \cdot q + \ldots \]

\[ C_D = \ldots + \frac{c}{2V} C_{Dq}(\alpha) \cdot q + \ldots \]

\[ C_M = \ldots + \frac{c}{2V} C_{Mq}(\alpha) \cdot q + \ldots \]

Equation 6-1

There are two aspects about this term, which cause greater differences at high speed and near the MSB, as well as for TVC/PSM. The first is that at low speed, the velocity term, \( V \), in Equation 6-1 will mean that the change due to pitch rate will be up to 5½ times larger (compare 100m/s versus 550m/s at low speed). The second is that for PSM, \( q \), the pitch rate, will be larger (due to increased pitch acceleration from the TVC), and for longer (since the aircraft pitches to 70° AoA as opposed to 30° for the standard aircraft). These reasons mean that the difference, due to Equation 6-1, will be more significant for the TVC/PSM case.

The difference in control strategy (RNDI versus P+D for the low order model) also leads to discrepancies. At high speed, the body pitching moment coefficient, in both models becomes very large, and so the margin of excess control moment reduces. The body pitching moment increases due to the dynamic pressure, but also due to Mach number effects, and so is much greater than first expected. Although Mach number effects are included in the elevator moment data, the elevator moment only really increases due to dynamic pressure – the effects of Mach number on the elevator are less than on the body pitching moment. The result of this is that the elevator has less authority, and the pitch response contains more oscillations while the aircraft
pitches up from 1g to the AoA for ITR. Since the two controllers are different, their respective responses will also vary. Furthermore, because of the oscillations from the lack of control power, the difference between the model responses was larger than when there was no oscillation. The low order model controller actually oscillates with more overshoot, and so a difference between the two models was seen. This difference in pitch response at high speed (400m/s, 5km altitude) is shown in Figure 6-2.

![Figure 6-2: Typical High Speed Pitch Response Differences](image)

The difference in responses is due to the low order model effectively having a higher controller gain. It would however be incorrect to change the gains of the controller to match those of the high fidelity model since the actual AoA response would not normally be known.

Figure 6-2 also illustrates why metrics considering purely pitching motion should not be used. In the case illustrated, the high fidelity model takes an extra 2 seconds to capture the AoA. Since the low order model only takes 5 seconds this is a 40% discrepancy. However for the entire manoeuvre, according to Table 6-2 there is only a 0.1 second difference between the two models which is equivalent to 1.1%. Hence the effect of considering more than simply the pitch model is demonstrated. Furthermore, with the hindsight of knowing that the metric that was developed in chapter 5 can include a post stall AoA attainability component, but which is not the
dominant part of the metric (which would be the case if only pitching was examined), it is fair to applaud the AFFTC in their demand for a functional metric — one that examines all parts of a manoeuvre. For these reasons, it was argued that existing post stall attainability metrics would not be considered, and that this element would be considered as part of a metric.

It is now possible to conclude from all of the above discussion that the low order model that was used was suitable for providing results of the new metric within the required accuracy of 10%. This is true of both the high speed area and the area around the MSB, except that these results should be treated with a little more caution.

Finally, it is argued that since the confidence between the two models is known for the vertical plane, it is also known for the horizontal (or any other) plane. It is assumed that the aircraft starts any manoeuvre so that it is banked correctly, such that during the initial time step, flight will commence in the required plane. In the former case, the only difference between flying in the vertical plane and the horizontal plane is given by Equations 6-2.

\[
\begin{align*}
\dot{\gamma} &= \frac{g(n_z - \cos \gamma)}{V} \quad \text{(Vertical plane)} \\
\dot{\chi} &= \frac{g \cdot n_z \sin \mu}{V} \quad \text{(Horizontal plane)}
\end{align*}
\]

Equations 6-2

where \( \dot{\gamma} \) and \( \dot{\chi} \) are the respective turn rates. For the horizontal plane, the inputs that affect the flight path are AoA (via load factor, \( n_z \)), and bank angle, \( \mu \). Equations 6-2 show that the difference is only in the bank angle. Since the validation above has given a confidence in the pitch and acceleration modelling (vertical plane is only affected by these two degrees of freedom), it can be assumed that if bank angle is perfect, then the two planes will have similar accuracies. Since banking was not modelled, and was assumed to be perfect, the accuracy of the horizontal plane must be equal to that of the vertical plane.
6.2 References


CHAPTER 7

RESULTS AND DESIGN STUDIES

7.1 The Effect of TVC and PSM

7.1.1 Absolute Effect of TVC and PSM

The first step in assessing the effect of TVC is to consider two aircraft - identical apart from the fact that one is retrofitted with TVC. Neither have AoA limiters, and so the pilot could command a post stall AoA from either. However, the pilot is unlikely to do this while flying the standard aircraft (with no TVC retrofit) since recovery from the post stall envelope is likely to be slow and with no real control of the aircraft. On the other hand, the aircraft with TVC should have no problems recovering and staying in total control throughout any post stall manoeuvre. With this in mind, a comparison between the two aircraft can be made, using the new metric developed in this project.

First, a few notes. The manoeuvre used was a 180° turn carried out in the horizontal plane. It was assumed that the aircraft both weighed the same, and had the same rotational inertia. In the case of the F/A-18 HARV (High Alpha Research Vehicle, NASA) the weight increase was 952kg[1] however, this was a retrofit, included trim ballast in the nose, and was one of the first TVC installations developed. It is often cited[2] that engines of the future will have negligible weight increase. The TVC was assumed to have control deflection limits of ±15° and the aircraft was assumed to have a new maximum AoA of 70°. This aircraft is referred to as the advanced aircraft. It was assumed that both the aircraft would travel the ITR segment of the manoeuvre at an AoA of 20°. This value is below the AoA that would maximise the ITR. Theoretically, the AoA chosen for the ITR should maximise the load factor, and for the F/A-18a model this AoA is close to 35°. However, for the F/A-18a model that was used in this study, there were points in the flight envelope where the elevator could not produce enough pitching moment to hold the aircraft in a trimmed state.
while at 35° AoA. For these reasons, the AoA chosen for the ITR segment of the manoeuvre was less than that to maximise the ITR. Although the advanced aircraft had an increased pitch control power, initially it was assumed that the aircraft would also hold the AoA for ITR at 20°, so that the effect of nose pointing alone could be examined.

The maximum commanded AoA for the standard aircraft was set at 30°, whereas the advanced aircraft was allowed to be commanded to a maximum of 70°. It was also assumed that both aircraft had a load factor limit of 9g, and started with an initial load factor of 1g. For reference, the results for the standard aircraft are given first.

In the standard fashion for this report, the results are shown starting from top left: Time taken to complete the manoeuvre in seconds; top right, the SEP of the aircraft at the end of the manoeuvre, in metres per second; bottom left, the energy change of the aircraft during the manoeuvre in metres of equivalent height of energy; and bottom right, the maximum turn diameter in metres.

Figure 7-1 shows that it takes anything from just less than 8 seconds, near the optimum, to around 44 seconds for the standard aircraft to complete the turn, depending upon where it starts in the flight envelope. Similarly, it can be said that the SEP is always negative, but never hugely so, with the most energy draining portion of the flight envelope being at the high speed, at altitudes around 6 – 8 km. The energy bleed during the entire manoeuvre is dominated by the velocity. Velocity is said to dominate since the contours are vertical, and do not change much with altitude. This is true apart from the interesting bubble that occurs in the low altitude transonic region. This is likely to be an effect of transonic aerodynamics together with dynamic pressure effects. Finally, the turn diameter varies from 1km to ten times that. It is dominated by altitude, apart from the low to medium altitude supersonic region, when it is dominated by speed. This region is shortly shown to be where the aircraft is continuously load factor limited.
Overall, there is a clear Manoeuvre Stall Boundary (MSB). Even a little to the right of the MSB, the result will be a poor performance manoeuvre, since the time is greatly increased close to this boundary. Furthermore, the aircraft will finish the manoeuvre right on the stall limit for the aircraft, meaning that the control and handling qualities will be poor.

The results for the advanced aircraft are very similar, and are shown in Figure 7-2, to highlight a few points. The grey chain dashed line is the 1g, zero SEP contour, and will be discussed shortly. Figure 7-2 also has a red line drawn on it, which shows the MSB for the standard aircraft. What can be seen is that the advanced aircraft's MSB (the black boundary) totally encloses the MSB of the standard aircraft. This is interpreted as meaning that the advanced aircraft has a larger flight envelope than the standard aircraft. Apart from this, the time plot in this figure is not too dissimilar to the plot for the standard aircraft. The second plot (the SEP at the end of the manoeuvre) shows contours of up to -1750m/s of SEP. At this point, the aircraft is actually decelerating whilst at an AoA of around 60°. Because of this, the pilot would experience approximately an 8g longitudinal loading, with the axial loading approximately equal to zero. This condition is close to the maximum structural limits of the aircraft and pilot. Note that this condition is near the lg zero SEP high speed boundary. Since the aircraft cannot sustain flight to the right of this boundary, the high deceleration area shown in the SEP plot is actually smaller than it first appears. The contours that are within the boundary have slightly smaller SEP values than the maximum of -1750m/s.

Both the energy change plot and the turn diameter plot from Figure 7-2 are not that different from those in Figure 7-1. This is because the two aircraft are performing an almost identical manoeuvre for the majority of the time, until the advanced aircraft points its nose.
Figure 7-2: Results for the F/A-18 HARV.
7.1.2 Difference Due to TVC and PSM

For each of the four plots, the standard aircraft results are subtracted from the advanced aircraft results. These new results are shown in Figure 7-3, which has the 1g zero SEP contour from Figure 7-2 added, but this time in blue. The figure also has a red dashed contour, which is the "less than -1 second difference" contour from the time plot of Figure 7-3. It is significant since it shows where there is hardly any difference between the two aircraft. This is because both aircraft are load factor limited throughout the manoeuvre. When the heading change is great enough that the flight path controller wants to pitch the advanced aircraft to the post stall regime, it is not allowed to do so. The load factor limiter cuts in, and prevents the AoA going any higher (let alone to the post stall regime) since this would cause the aircraft to encroach the structural limit. Since this is the case throughout the manoeuvre, the advanced aircraft never gets to go post stall, and so the only differences between the aircraft are that the advanced one has greater pitch acceleration (from the greater pitch moment due to the TVC). This will cause small differences between the two aircraft since the advanced one can get to the commanded AoA quicker.

Again, due to the 1g zero SEP boundary on the right hand side, the actual area in which the two aircraft are virtually identical is reduced. The area where the biggest difference between the two aircraft exists, is near the MSB. Differences of between 10 and 15 seconds in favour of the advanced aircraft are seen, which is approximately a one third reduction in time taken to point. This length of time is also long enough to obtain a shoot solution from the weapons computer, fire the weapon and gain a kill before the enemy has the opportunity to return fire[3].

The final SEP differences from Figure 7-3 shows that the advanced aircraft is always at a disadvantage since it is always losing more energy at the end of the manoeuvre. This intuitively makes sense, since it is at a post stall AoA. Note however, the area inside the red dashed line - there is not really any difference between the two aircraft, again, since they are both load factor limited throughout the manoeuvre.
Figure 7-3: Difference plots - Advanced aircraft subtract Standard Aircraft.
As far as the energy change during the manoeuvre is concerned, the majority of the plot shows the advantage to the standard aircraft. This makes sense since the majority of the manoeuvre is very similar until the advanced aircraft points at the end of the manoeuvre, it loses more energy, giving the advantage to the standard aircraft. The exceptions to this are again the continuously load factor limited area, bound by the red dashed line, as well as the low speed area. At low speed, the advanced aircraft loses slightly less energy since it finishes the manoeuvre quicker, even though it is at a post stall AoA.

Finally, the turn radius difference is up to 2km. In fact, across the flight envelope, there is about a 20% reduction in turn diameter. The reduction is not actually a true reflection of what is happening though, since the advanced aircraft will continue to increase its turn diameter, after the manoeuvre is finished, as shown in Figure 7-4.

Although the advanced fighter is pointing at the target heading, there is a component of its velocity that is still in the upward direction of the diagram, which will serve to increase the diameter. The final diameter will however still be smaller for the advanced aircraft.

The results given above were for the horizontal plane. Similar results for the vertical (pull up) plane also exist, as shown in the last chapter in terms of the aircraft’s absolute performance. The difference that TVC/PSM makes in the vertical plane is
shown in Figure 7-5, which shows very similar trends to Figure 7-3. There are however two exceptions to this. First, the MSB is lower in the vertical plane. This is because there is an altitude limitation imposed on the model. For altitudes where the aircraft ended above this limitation, the results are flagged with a warning, and so the MSB was drawn. The second difference is shown best in the time plot, but also to some extent in the turn diameter plot. The contours double back on themselves close to the MSB. The meaning of this is that very close to the MSB, the advantage of the TVC/PSM is reduced in comparison to slightly to the left of the MSB, which is an effect not seen for the horizontal plane performance.

To summarise the results of these difference plots, there are two perspectives. The first is that of the aircraft with TVC/PSM capability added. The best advantage in this case will come from flying close to the MSB, but not actually on it. This optimum position is true for all four parameters, with the final SEP difference being small (less than 300m/s horizontally, or less than 100m/s vertically) and the horizontal turn diameter advantage being biggest for any given velocity when the aircraft is close to the MSB.

The other perspective is that of the aircraft without TVC or PSM. The best that can be achieved by the standard aircraft is to match the performance of the advanced aircraft, by flying low, inside the continuously load factor limited region (shown in red on Figure 7-3 & Figure 7-5). In this case the time to turn will be roughly the same, the final SEP will be similar, and the turn diameter will be similar. However, anywhere that the standard aircraft does fly, it will always use energy more efficiently.
Figure 7-5: Difference between Standard and Advanced aircraft in the Vertical (Pull Up) Plane.
These summaries can be shown on the flight envelope axes, in Figure 7-6, with red hatching showing the TVC/PSM disadvantage, or rather no advantage, and the green hatching showing the TVC/PSM advantage.

![Figure 7-6: Areas of advantage and disadvantage due to TVC/PSM](image)

### 7.1.3 Using TVC to enhance controllability

Consider next the difference between the standard aircraft, and the same aircraft with TVC, but without the PSM capability. That is, using the TVC to provide added control power, to improve the handling qualities. The same assumptions as above are used, and both aircraft have identical AoA demands and limits. Figure 7-7 shows the horizontal plane difference between the two aircraft, and effectively shows the change that adding TVC alone has made to the entire manoeuvre.

Figure 7-7 shows that there are few differences to the overall manoeuvre. The biggest area of difference is in the high altitude, high speed region. Here, there are turn diameter differences of up to 400m. This is not a very large proportional increase over the diameter in this region (approximately 10km), but it is significant. Precisely the same reasons that were discussed for Figure 6-2, cause this diameter difference. The aircraft basically does not have enough control power at this flight condition since the body pitching moment becomes very large due to Mach effects.
Figure 7-7: Difference that extra pitch power makes, to the standard F/A-18a.
Apart from this difference, no other significant differences are seen. From this it is clear that TVC alone does not affect the manoeuvre as a whole. It is far more likely that TVC alone will change the agility of the aircraft - agility being that property previously defined as the second differential of performance, and dominated by effects such as pitching moment/acceleration.

### 7.1.4 Changes due to Increased Control Power

It was stated above that the model of the F/A-18a used in this study suffered from a lack of control power at high speed, meaning that the AoA for ITR was chosen to be 20°, and not the ideal value of 35°. From Figure 7-8 it can be seen that according to the traditional turn rate plot, manoeuvring at the ideal value of 35° leads to a higher maximum turn rate. In fact an increase in turn rate of between 3°/s (near the corner velocity) and 8°/s (at low speed) is seen in this case. These values are at least as high as the turn rate excess quoted by Herbst\(^4\)(2-3°/s) as giving an advantage.

The increased control power from the TVC means that the higher AoA can be flown. This should result in an even greater turn time advantage for the advanced aircraft with higher AoA for ITR, referred to from here on as the super advanced aircraft.

![Figure 7-8: Schematic showing difference in ITR between AoA of 20°, and 35°, shown on a traditional Turn Rate plot.](image)
Figure 7-9 shows the absolute performance of the Super Advanced Aircraft. This can be directly compared to Figure 7-1 and Figure 7-2. Figure 7-9 also shows a green line which is the MSB for the advanced aircraft, a red dashed line which is the envelope for both aircraft in which the entire manoeuvre is load factor limited, and a blue chain dashed line which is the 1g zero SEP contour for the F/A-18.

The most striking change in Figure 7-9 is the fact that the MSB is much lower than in previous figures. Although the super advanced aircraft should on average have a higher turn rate, and hence improved performance, the MSB is not affected by this. With the higher turn rate, comes a higher energy bleed rate during the manoeuvre (SEP). This means that the manoeuvre is less sustainable, and so at a given altitude, the aircraft will need to start with a higher speed to complete the manoeuvre. The effect of this is to shift the MSB to the right - as seen in comparison to the green line in Figure 7-9.

Apart from this change, the plots are similar in shape and trend to those in Figure 7-2. The red dashed line (continuous load factor limit) is the same for all three aircraft. The optimum point in each plot is very similar for both the advanced and the super advanced aircraft. The optimum velocity for any given altitude in each plot is also very similar for both aircraft. To determine the true difference that the AoA for the ITR segment makes to the performance, the difference between the two aircraft is taken at each point in the flight envelope and plotted next, in Figure 7-10.

The values of the contours on the time plot of Figure 7-10 show that the advantage is with the advanced aircraft (since the values are positive, the advanced aircraft takes less time to complete the manoeuvre). This is explained by considering the time histories for the two aircraft, which are shown in Figure 7-11, for both aircraft starting at the same initial conditions (280m/s velocity and 9km altitude).
Figure 7-9: Absolute performance of Super Advanced Aircraft in the horizontal plane.
Figure 7-10: Difference in performance between Advanced Aircraft and Super Advanced Aircraft.
Figure 7-11: Velocity Variation - Advanced Aircraft (Red) versus Super Advanced Aircraft (Blue).

The heading angle (where the nose/gun points with respect to the initial condition) for the super advanced aircraft increases quicker than for the advanced aircraft, due to the higher pitch rate afforded by TVC and the higher commanded AoA. However after approximately 8 seconds, the advantage is lost to the advanced aircraft which, using the left hand axes, has always maintained a higher velocity. The reasons for this are as follows. Once the two aircraft slow to below the corner velocity, the super advanced aircraft is at a higher AoA (35°). The higher AoA will mean that the super advanced aircraft also has more drag, and so it decelerates quicker than the advanced aircraft. There then comes a point where although the super advanced aircraft is at a higher AoA and hence a higher lift coefficient, it is actually producing less lift force because the dynamic pressure is lower. The lower lift force leads to a lower load factor, which leads to a lower turn rate. Eventually, by the end of the manoeuvre, the advanced aircraft has the advantage overall. The above analysis relies on the fact that the aircraft will not be load factor limited for the entirety of the manoeuvre, in which case the two aircraft will perform identically.
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This result is a very good example of where traditional metrics such as turn rate plots fail. A turn rate plot at a given altitude, and start velocity would show the super advanced aircraft with a higher ITR, and hence an advantage, although an energy manoeuvrability diagram for the same initial conditions would show the super advanced aircraft losing energy quicker than the advanced aircraft. It only becomes clear how much of a disadvantage the super advanced aircraft has when a simulation of the two aircraft is run. This example is backed up by Hinchcliffe, who claims that although pilots are taught about the turn rate plot and instantaneous turn rate, from experience they come to know that the performance is improved by pulling less load factor, and hence lower AoA. Using the new metric allows this to be taught to pilots more easily than was previously possible by using traditional metrics.

There is a final noteworthy point. The SEP plot of Figure 7-10 shows an interesting result. Since the results come from the subtraction of the advanced aircraft from the super advanced aircraft, the positive values of SEP indicate that the super advanced aircraft is actually bleeding less energy per second at the end of the manoeuvre than the advanced aircraft. It could be expected that the higher AoA of the super advanced aircraft would mean that the final SEP would be more negative. However, the very large velocity bleed that occurs during the manoeuvre means that its SEP (which is a function of velocity) is actually more favourable.

7.2 Alternative Technologies

This section demonstrates the uses of the new metric, by qualifying TVC and PSM in terms of more traditional technologies, namely an increase in thrust and a decrease in weight. First an increase in thrust to the standard aircraft will be considered. Then a decrease in weight to the standard aircraft will be discussed, without changing the thrust. For reference, note that the studies in this section were completed arbitrarily for the vertical plane.

7.2.1 Increased Thrust

Figure 7-12 shows the effect of changing the engines to increase the overall thrust by 20%. The F-404-400 engines used on the F/A-18a have a thrust to engine weight ratio of about 5.0:1 (dry). Using advanced engines with a ratio of about 6.0:1 (dry) would increase the overall thrust by 20%. This example would be similar to
Figure 7-12: The effect of increasing thrust by 20% over the standard aircraft in the vertical plane.
exchanging the F-404-400 with the EJ-200, which according to Reference 6 have very similar dimensions.

Figure 7-12 has a thick black line drawn, which is the manoeuvre stall boundary for the standard F/A-18a, taken from Figure 7-1. Also shown on Figure 7-12, is a thin black line which is similar in shape and position to the thick black line. This limit is the manoeuvre stall boundary for the modified aircraft. It is above the thick line since the increased thrust allows the modified aircraft to turn at higher altitude without stalling.

The time plot in Figure 7-12 shows contours evaluating the difference in time taken between the standard aircraft and the modified aircraft. Contours with positive values show where the modified aircraft has an advantage in turn time (at any speed to the left of the zero second contour). It can be seen that the maximum amount of time advantage that the modified aircraft has, is about 2 seconds, at speeds slower than 175m/s. However, for the majority of the flight envelope, there is not more than about a second of advantage conferred by increasing the thrust of the engines. At higher speeds, the modified aircraft appears to have a small disadvantage. However, this result would tell the pilot that at higher speed, the throttle should be reduced in order to reduce the turn time. The turn time would be reduced since the aircraft would slow to the corner velocity quicker, and hence have a higher average turn rate. It should be noted that the simulations were run using full throttle. It was not the objective of this study to optimise the turn with throttle scheduling. However, this example shows that the new metric could be used for such a purpose.

The SEP plot in Figure 7-12 shows that there is virtually no difference between the two aircraft, except for the 200m/s bubble. At the speed and altitude where the 200m/s bubble exists, the standard aircraft is losing about 200 or more metres of energy height every second. Hence the modified aircraft is maintaining its energy at the end of the turn. This not only means that it is capable of sustaining a continued turn, but also that since less energy was used during the manoeuvre, there is a larger choice of follow on manoeuvres available.
The energy consumption plot in Figure 7-12 shows that for higher speeds, the modified aircraft loses slightly less energy height, with the difference between the two aircraft no greater than 800m. Since the modified aircraft has more thrust, it will lose less speed during the turn, leaving it with more energy at the end.

The turn diameter plot in Figure 7-12 shows that the modified aircraft has a small disadvantage (its turn diameter is typically less than 10% larger when compared to Figure 7-1 for any speed or altitude). However, for the time plot in Figure 7-12, it was stated that thrust could be reduced at higher speed. If this were done, then the turn diameter would reduce, hence equalising any disadvantage shown in the fourth plot.

The plots in Figure 7-12 show that increasing the thrust by as much as 20% only has a small effect on the performance of the aircraft for the vertical turn manoeuvre. The most advantage that comes is from the reduction in energy loss during the turn reversal. The increased thrust overcomes the drag, and leaves the aircraft with more energy at the end of the manoeuvre. Consider Equation 2-6, which gives SEP. This equation is integrated over time to give the energy used during the manoeuvre. At low angles of attack (at which the manoeuvre is performed at high speeds due to the load factor limit), it is dominated by the thrust term, and hence increasing the thrust will reduce the energy used during the manoeuvre.

The high speed, high altitude regime, which is a typical BVR scenario, shows that although there is no turn advantage (in terms of turn time or diameter), there are quite large energy savings made, by increasing the thrust. BVR is dominated by the ability to continue turning without losing energy, and increasing the thrust helps to allow the aircraft to do this. At low speed, low altitude (typically WVR), there are no energy advantages for the increase in thrust. There are however small turn time advantages, in this region.

### 7.2.2 Decreased Weight

Figure 7-13 shows the effect on performance of reducing the combat weight of the aircraft by 20%. The extensive use of composite/advanced materials in a total redesign of the airframe, but still keeping the same shape, outer mould lines and
Figure 7-13: The effect of decreasing weight by 20% over the standard aircraft in the vertical plane.
configuration. This could reduce the overall combat weight by as much as 20%, which Reference 7 would suggest to be quite reasonable for future aircraft.

The first plot in Figure 7-13 considers the time difference between the weight reduced aircraft and the standard aircraft. The thin black line above the thick black boundary in Figure 7-13 shows that a reduction in combat weight would mean that the modified aircraft could fly vertical turn reversals at higher altitude while at speeds less than 375 m/s. At any initial speed, the advantage of the modified aircraft increases with increase in altitude. The maximum advantage occurs near the manoeuvre stall boundary and lies between 5 and 10 seconds, for any initial speed. This is an advantage of around 30% for all speeds, compared to the standard aircraft. For WVR, an advantage of more than 5 seconds may be enough to obtain a shoot solution, before the enemy can return fire. The figure also shows that at low altitude and high speed, the standard aircraft gains the advantage. This is because the heavier aircraft will slow down quicker and get to its corner velocity sooner. The advantage is however very small, and unlikely to be significant.

The SEP plot in Figure 7-13 shows that the difference in final SEP is not that great. Since the modified aircraft will not gain a time advantage from flying low and fast (discussed in the previous paragraph), the difference in SEP is not significant, and is very similar to that for the first case study where thrust was increased.

In the areas where there is a time and diameter advantage for the modified aircraft (that is, near the manoeuvre stall boundary), the energy consumption plot in Figure 7-13 shows that there is also a small energy disadvantage, shown by the +200 m contours. However, at medium to low altitude and high speed (to the right and below the zero energy contour), the modified aircraft loses significantly less energy than the standard aircraft (up to 1400 m less).

The fourth plot in Figure 7-13 shows the change to the turn diameter. The higher the aircraft fly, the more of an advantage the modified aircraft gets. The maximum is as much as 1 km, which is about 15% less than that of the standard aircraft. At low altitude and high speed, there is an advantage for the standard aircraft. This is again because the heavier aircraft will slow quicker and hence reduce the turn diameter.
However, the modified aircraft could match this by reducing thrust as it enters the manoeuvre, although this would not give any time or diameter advantage. To maximise its superiority, the modified aircraft would do much better to fly as high as possible, while still being able to reverse the turn in the vertical plane.

7.2.3 Conclusions of Alternative Technologies

It is not intended that this discussion go any further than to show the capabilities of the new metric in this kind of parametric study. It is only intended to demonstrate that the results could be used in helping to determine appropriate levels of new technologies. Note that if the new metric is to be used to determine benefits/disadvantages of new technologies, or simply for assessing the performance of aircraft, then a much fuller analysis than that shown here is required. For a fuller assessment, there are many more combat realistic manoeuvres that should be considered, for example horizontal turn reversal, and axial acceleration (that is, SEP as enhanced in chapter 4). When considering technology such as TVC/PSM, it should be realised that this technology provides a capability to execute many new manoeuvres. It is possible that some of these may have tactical relevance, and so these manoeuvres should also be analysed in the full assessment, as long as the limitations of the models used are appreciated.

Once all of the relevant manoeuvres have been considered, and the full analysis has been completed, conclusions can be drawn about where the aircraft is most manoeuvrable. These conclusions can then be used to develop tactics. On the other hand, the designer can determine where the aircraft is under performing, and can use the new metric to execute a parametric study to see what can be modified in the design to give performance closer to that desired.

7.3 Conclusions of the New Metric

The metric was used in this chapter to quantify the differences that TVC and PSM make to an aircraft, throughout the flight envelope, in terms of a complete, combat relevant manoeuvre. The fact that TVC/PSM affects the time and turn diameter to give the advanced aircraft an advantage was highlighted. It was also highlighted that while these advantages exist, the energy performance was affected adversely. Most importantly, these effects can be quantified using the new metric.
It was then discussed that TVC alone does not nearly affect the entire manoeuvre as much as TVC when used to allow PSM. The effect of using TVC to produce a super advanced aircraft capable of performing the low speed ITR segment of the manoeuvre at an increased AoA (and hence static turn rate) was then considered. This result showed that traditional metrics such as the turn rate plot might lead to incorrect conclusions – since the super advanced aircraft is much less energy efficient than either the standard or the advanced aircraft. Although a traditional SEP plot could be used to pick up on this result, it is not guaranteed that the overall result of the super advanced aircraft gaining no advantage would be clear.

Finally, the metric was used to consider other changes to aircraft, other than TVC and PSM. This is extremely important since it highlights the point that this metric is not only suitable for assessing TVC/PSM, but rather that the lessons it can teach make it suitable for all aircraft and technologies.

7.4 References

2. Airfleet Magazine, Russian Airforce, Aircraft and Space Review, PO Box 77, Moscow, Russia, Published by RA Intervestnik Publishing House, July 1999.
CHAPTER 8

FURTHER RESULTS

8.1 Initial Conditions

It is important to consider what happens if the manoeuvre used in the assessment is changed. The original manoeuvres were initiated at one g load factor, straight and level. From the conversations that took place with pilots, it was determined that a more realistic manoeuvre would consist of the maximum turn performance manoeuvre used so far, but starting at the maximum load factor. This would simulate an aircraft breaking off a manoeuvre to initiate the maximum turn manoeuvre.

To realise this, the software was modified, so that the initial AoA was chosen so that the load factor was at the maximum, and not unity. The results were then obtained as before, and are shown in Figure 8-1, which shows all four results parameters for the vertical plane. It shows contours in grey and in black. The grey contours are those introduced for the standard F/A-18a in the previous chapter. The black contours are for the same aircraft, starting at 9g initial load factor. It can be seen from Figure 8-1 that there are only negligible changes in contours, except to the left of the black MSB. In fact, the only real change is that the MSB for the 9g aircraft has been shifted to the right. This intuitively makes sense because it shows that for the aircraft to be able to generate 9g of lift at the initial flight condition, it must be starting at least as fast as the 9g stall limit. This effect of shifting the MSB to the right is exactly the same as the shift to the right of the 1g stall limit compared to the 9g stall limit. It was discussed in chapter 5 that the MSB may consist of a segment that represents the 1g stall boundary, and a segment that represents stall during the manoeuvre. For an initial 9g load factor, the segment that represents the initial stall limit is simply adjusted to match the 9g stall limit.
The negligible differences between the contours that are on the right of the 9g MSB are where there is a difference in time taken to pitch from the initial load factor to the AoA commanded for ITR. In the original 1g case, the aircraft would need to pitch up to the commanded AoA. In the 9g case, the aircraft tended to need to pitch down. All differences in the contours are because of this initial difference in pitch.

8.2 **Turns Other than 180°**

For all of the results shown previous to this section, the target heading angle was set to 180°. This value was subjectively chosen since it was felt to be representative of an average turn during modern combat. However because no quantitative value can be chosen to describe what the target heading angle should be, it was required that this parameter be studied. To do this, two flight conditions were chosen, namely one to represent the Within Visual Range (WVR) and the other to represent the Beyond Visual Range (BVR) regimes. This provides an assessment in the two key areas of the flight envelope. The heading angle was then varied and simulations of the manoeuvre described in chapter 5 were run, in the horizontal plane. The target heading angle was varied from 30° to 360° in steps of 30°. The WVR flight condition was chosen at an initial velocity of 150m/s (below the corner velocity) and sea level altitude. The BVR flight condition was chosen at an initial velocity of 450m/s and 12km altitude. To see if any trends to do with TVC/PSM existed, the results were obtained for both the standard and the advanced aircraft.

8.2.1 **Within Visual Range**

Figure 8-2 shows each of the four results parameters for the WVR flight condition. The plots each have two lines, a blue and a pink line. The blue line represents the advanced aircraft, and the pink line represents the standard aircraft. The exception to this is the final SEP plot, which also has lines of final AoA (using the right hand axis). These lines are shown in dashed.
$V_o = 150 \text{ m/s}, \ h_o = 100\text{m}, \ n_{zo} = 1g$

Figure 8-2: WVR Comparison of Standard and Advanced aircraft performing turn reversal, showing Result Parameter for given target angle.
The time plot shows that there was always a constant time advantage of around 2 seconds for the advanced aircraft, except for low target heading angles. The reason that there was a smaller advantage at low target heading angles is explained by considering how the aircraft complete the manoeuvre. For a heading angle of 30° AoA, both aircraft pitch up from their initial AoA to 30°, and the manoeuvre is complete. Since the advanced aircraft can produce a higher pitch rate due to the TVC, it can complete this manoeuvre slightly quicker. For a heading angle of 60°, both aircraft start at their initial AoAs, then pitch up. The standard aircraft pitches up to 20° AoA, and then stays there. When the heading angle gets to 50°, it pitches up to 30° AoA, and as soon as it points at a heading angle of 60°, the manoeuvre is complete. In the mean time, the advanced aircraft will continue pitching right up to 60° AoA, at which point the manoeuvre is complete. Because of this, the difference between the two aircraft is much greater than for the 30° heading angle case. So, for target heading angles of less than the advanced aircraft’s 70° AoA, the advanced aircraft need only pitch up, whereas the standard aircraft will also need to turn. This is also why on the time plot the time taken to complete the manoeuvre for the advanced aircraft has a change in gradient at 70°.

Final SEP is dependent upon both the final velocity and the final AoA. The final SEP plot shows that above a target heading angle of 70°, the final SEP difference between the two aircraft reduces, and is smallest when the manoeuvre takes longest to complete (360° target heading change). This is because the longer the manoeuvre takes, the lower the final velocity. At low velocity the SEP becomes less significant and so the difference reduces. Also, the longer a manoeuvre takes, the closer to the sustainable speed the aircraft finish.

For a target heading angle of 30°, both aircraft pitch up immediately. However the advanced aircraft can pitch up quicker and so its final velocity, and hence SEP is larger in magnitude. For a target heading angle of 60°, the advanced aircraft pitches up to a post stall AoA, and so has a large magnitude SEP. This value drops as the target heading angle reduces since the final velocity of the aircraft drops. The final velocity is shown in Figure 8-3.
The fourth and final plot of Figure 8-2 shows the final cross range distance of the aircraft. The cross range distance is that shown as the “diameter” in Figure 8-4, which measures the distance in the horizontal plane from the original flight path (at the start of the manoeuvre) along the global Y axis. It gives more information than the maximum turn diameter. In effect, the maximum turn diameter is simply the maximum cross range distance achieved during the manoeuvre. Figure 8-2 shows that the advanced aircraft appears to be out of phase with the standard aircraft. The reason for this is simply that once the initial 70° heading angle is surpassed, the advanced aircraft always finishes the manoeuvre 2 seconds ahead, and so in effect always has the turn diameter of the standard aircraft 2 seconds later.

Figure 8-3: Final Velocity of both aircraft for WVR flight condition.

Figure 8-4: Effect of Heading Angle on Turn Diameter
8.2.2 Beyond Visual Range

Figure 8-5 shows very similar results for the BVR flight condition. The major differences between Figure 8-2 and Figure 8-5 are due to the fact that the BVR aircraft are initially load factor limited. Using the final AoA curves in the final SEP plot of Figure 8-5 (with the right hand axis), it can be seen that the first 90° of the heading change is load factor limited (this is because the standard aircraft does not point to its maximum). For target heading angles higher than this, the trends shown in Figure 8-5 are identical to those shown in Figure 8-2.

8.2.3 Entire Flight Envelope

As far as the entire flight envelope is concerned, there are two differences that the target heading angle makes. Figure 8-6 shows the entire flight envelope in terms of absolute performance of the standard aircraft. It shows contours in red for a target heading angle of 180°, and contours in black for a target heading angle of 360°. The time plot shows that the contours are flattened out, but that their trend is the same. The energy consumed and the final SEP are less for the longer manoeuvre, since it takes longer to complete and the final velocity is lower. The turn diameter is unchanged for the longer manoeuvre. This is because the final diameter is a measure of the maximum cross range distance. The cross range distance will only decrease after the aircraft has turned 180°, and so the maximum remains unchanged.

The second difference that the target heading angle makes is that the MSB is lowered (or shifted right) for the longer manoeuvre. This is because the final velocity is lower for the longer manoeuvre and so the aircraft will stall late on during the longer manoeuvre, which it will not do during the shorter manoeuvre since it never slows down enough.

Finally it is worth considering the optimum velocity for any given altitude, to reduce the turn time. This is shown in thick red and thick black for the 180° and 360° turns respectively on the time plot in Figure 8-6. The outcome is that the optimums are almost identical and so the argument over corner velocity in chapter 5 remains unchanged, and independent of heading angle.
\[ V_0 = 450 \text{ m/s}, \ h_0 = 12000\text{m}, \ n_{zo} = 1g \]

Figure 8-5: BVR Comparison of Standard and Advanced aircraft performing turn reversal, showing Result Parameter for given target angle.
Figure 8-6: Effect of Target Heading Angle on entire flight envelope.
8.3 Proposed Metrics – Dt Parameter, Compound Metrics, Frenet Agility (sections 2.1.5, 2.4.1, and 2.4.3 respectively)

Now that this manoeuvre and metric have been developed and are understood, it is time to return to these suggested metrics.

8.3.1 Dt Parameter

Using the results for the standard F/A-18a aircraft turning in the horizontal plane, the results of the Dt parameter shown in Figure 8-7 were derived.

![Figure 8-7: The Dt Parameter for the Standard F/A-18a turning horizontally.](image)

Figure 8-7 shows contours for products of time and diameter, with both parameters equally weighted. For each altitude, the speed to minimise the Dt parameter can be determined. This has been done and is shown as a red line on Figure 8-7. This optimum is very similar to that shown in chapter 5, where the optimum velocity for each altitude was determined to minimise turn time and diameter. It should be noted that the Dt parameter shown here assumes that time and diameter are weighted equally. Changing the weightings of each parameter may change the overall result shown here, but the choice of weighting would need to be determined by the designer/tactician.

8.3.2 Compound Metrics

Just as the Dt parameter shows the optimum time and diameter combination, a similar metric can be developed to determine the most energy efficient initial flight conditions. The final SEP is always negative. The energy consumption is nearly
always negative. It is desired to keep SEP as close to zero (a minimum) and it is also desired to keep energy consumption as close to zero (also a minimum). Hence the product of these two parameters will provide a measure of the energy efficiency. The optimum velocity for any given altitude can also be determined by locating the minimum valued contour in Figure 8-8. Figure 8-8 shows the energy efficiency parameter for the standard F/A-18a turning in the horizontal plane.

![Figure 8-8: The Energy Efficiency Parameter, the product of SEP and energy consumption for the standard F/A-18a turning horizontally.](image)

The red line in Figure 8-8 shows the optimum velocity for any given altitude so that the energy efficiency parameter is minimised giving the most energy efficient solution. The result is identical to that shown in chapter 5, where the energy consumption and the final SEP are examined independently. The result is that the aircraft should fly right on the MSB for the most efficient manoeuvre.

As well as the Dt parameter, it has been suggested by Valasek[41] that the Dt parameter be combined with the total energy consumed. The aim of this is to provide a metric that gives a single solution to the most efficient initial flight condition, as well as a single solution to the aircraft’s capability throughout the flight envelope. The results of this parameter for the standard F/A-18a turning horizontally are shown in Figure 8-9.
Figure 8-9 also shows a red line. Again, this is the optimum velocity for any given altitude, to minimise the DtE parameter, which minimises all three parameters. The expected result might be something in between the two red lines shown in Figure 8-7 and Figure 8-8. However, it is clear from Figure 8-9 that the optimum is the same as that shown in Figure 8-8. This indicates that the DtE parameter places too much emphasis upon energy performance, because its optimum is right on the MSB, and not somewhere between the two optimums shown in Figure 8-7 and Figure 8-8. This statement of course assumes that the energy performance and the turn performance are equally weighted. If the user were to consider energy performance far more important than turn performance, then the DtE parameter might actually show a promising result. This argument is however far too subjective for this work. The result shown here also indicates exactly why it is so important to consider the four results parameters individually and to understand what each one tells the user.

### 8.3.3 Discussion of Proposed Metrics

In this section it was shown that the Dt parameter and the Energy Efficiency parameters could be used to find similar optimum flight conditions as the new metric. The reason for this is that the turn time and turn diameter are both the dominant terms in turn performance\(^{[1]}\). Furthermore, the Energy Efficiency parameter contains both the terms that dominate energy performance, and so the conclusions that are drawn from it are the same as when considering the parameters separately. However, the DtE parameter is a good example of where contrasting parameters that contain
different agendas can be combined to become misleading. It is not only contrasting parameters (contrasting meaning that one dominates turn performance and the other dominates energy performance) that can be misleading, but also matching ones. Consider the Energy Efficiency parameter. Although it indicates good initial flight conditions to perform the turn efficiently, it does not give any idea about the final SEP of the aircraft. Since all four parameters are required before they can be post-processed into Dt and Energy Efficiency parameters, there is little point in considering the compound metrics because information is so easily lost, and the interpretation becomes very subjective, in that it can be determined that one aircraft might be more energy efficient, but in exactly which way is not determined.

8.3.4 Frenet Agility

The formulae to calculate this metric have been used to calculate the resultant RMS agility vector, whose magnitude is plotted in Figure 8-10. To produce this plot, values of the three agility components are calculated after each time step. These are then used to calculate an RMS value for each component over the entire simulation, and the resultant agility vector is then calculated from the three components. See the description of this metric in Chapter 2.

Figure 8-10: Absolute Frenet Agility

Figure 8-10 shows contours for the absolute agility of the standard F/A-18a turning in the horizontal plane. The trends of this figure do not match any of those for any of the four results parameters used with the new metric that was developed in chapter 5. In fact, the results appear to indicate that the larger the dynamic pressure becomes, the
better the agility. It is nearly impossible to grasp exactly what total agility means in this sense, although it comprises something akin to rate of change of acceleration of the aircraft, rate of change of pitch rate, and rate of change of roll rate. It is possible that the relationship that Figure 8-10 is showing is true, if agility is taken as these three terms. Then it intuitively makes sense that agility increases with dynamic pressure, since the control forces will also increase with dynamic pressure, meaning that the aircraft can accelerate and decelerate its pitch and roll quicker. Furthermore, by pitching quicker, the aircraft will decelerate axially at a quicker rate. Since the metric appears to be measuring terms that are not directly related to the manoeuvrability of the aircraft, as defined in section 2.5.2, it is not expected that the trends should be the same as those shown for the four results parameters of the new metric.

In the same fashion that the effect of TVC/PSM was examined in chapter 7, Figure 8-11 shows the difference between the two aircraft for Frenet Agility.

Figure 8-11 shows that near the MSB there is hardly any difference between the two aircraft. Further, it shows that the biggest differences are at high speed and low altitude (within the continuously load factor limited area). This is in direct contrast to the results obtained from the new metric, which means that Frenet Agility does not show the same effect of TVC and nose pointing. If Frenet Agility is indeed dominated by dynamic pressure then an explanation for this contrast has been determined.
Working Group 19 in their report\cite{5} and Booz\cite{6} in her report both comment that various forms of the equations exist to calculate Frenet Agility. The reason for this is that various authors have concluded that Frenet Agility is too dependent upon velocity. It is the conclusion of this study too, that Frenet Agility does not appear to determine the difference between the two aircraft because it is too reliant upon velocity (dynamic pressure).

8.4 References


2. Private communication with George Appleyard, Concept Engineering Group, BAE Systems.

3. Personal communication with Lt. Commander Andy Sinclair, RN, of CDA, DERA.


CHAPTER 9

SENSITIVITY ANALYSIS

9.1 Introduction

It is important to gain an understanding of which variables the new metric is sensitive to. This will aid in determining which variables need to be derived with high accuracy, and on which variables the accuracy tolerance can be relaxed. The sensitivity analysis carried out here was performed for the aircraft manoeuvring in the vertical plane. In chapter 6 it was discussed that no validation of the horizontal plane was required. A very similar argument can be given that no sensitivity study of the horizontal plane is required, since the only differences between the planes are due to flight mechanics. The sensitivity analysis was carried out by varying each of the inputs independently. This allowed the effect of a sensitivity in the input to be quantified. The sensitivity analysis was carried out for the advanced aircraft, with TVC/PSM capability, since the validation in chapter 6 showed this configuration to contain the largest differences.

The results were then post-processed into absolute form, relative form and percentage change. These values were then compared to the baseline aircraft, i.e. the advanced aircraft with the original inputs containing no changes. Also, mean and standard deviation in percentage change and relative form were calculated for the entire flight envelope.

The variables tested were as follows. The amount by which to change the variables was taken as the tolerance to which the project sponsor believed that it was possible to approximate the inputs.

- Baseline model
- Lift Coefficient, $C_L$ +10%, -10%
- Drag Coefficient, $C_D$ +10%, -10%
- Pitching Moment Coefficient, $C_M$ +10%, -10%
Sensitivity Analysis

- Elevator Lift Derivative, $dC_L/d\delta_e +10\%$, -10\%
- Elevator Drag Derivative $dC_D/d\delta_e +10\%$, -10\%
- Elevator Pitching Moment Derivative $dC_M/d\delta_e +10\%$, -10\%
- Weight, $W +10\%$, -10\%
- Pitching Moment of Inertia, $I_y +10\%$, -10\%
- Wing Reference Area, $S +10\%$, -10\%
- X Axis TVC Moment Arm, $l_x +10\%$, -10\%
- Thrust, $T +10\%$, -10\%
- Normal Load factor limit, $n_x +10\%$, -10\%
- Maximum Elevator Deflection, $\delta_e +5^\circ$, -5\%
- Minimum Elevator Deflection, $\delta_e_{\text{min}} +5^\circ$, -5\%
- Maximum Pitch TVC Deflection, $\delta_{\text{tvc}}$ (assume that maximum and minimum are of same magnitude) +5\%, -5\%
- Angle of Attack for Instantaneous Turn Rate Segment, $AoA_{\text{ITR}} +5^\circ$, -5\%
- Maximum Angle of Attack, $AoA_{\text{max}} +10^\circ$, -10\%
- Controller Gain, $K_c +10\%$, -10\%

9.2 Results

For each combination of velocity and altitude calculated for the baseline aircraft (default parameters are shown in Appendix E), the sensitivity case was calculated. The advanced, or rather baseline aircraft was then subtracted from the sensitivity case. The results shown are the mean values of this absolute difference, and their standard deviations. Also, the mean value of this absolute difference is calculated as a percentage change from the mean value of the parameter.

Plots of the parameter changes were made in terms of percentage change with absolute change overlaid on the plot. This was done so that critical values could be identified. Each of the plots (shown in Appendix E) contain grey contours that show the absolute value for the baseline aircraft. The blue contours show the absolute change between the baseline aircraft and the sensitivity case. The red contours show the percentage change. The final SEP plot and the energy change plot, often contain many closely grouped red contours. These seem to show that there are large sensitivities and changes. However, where the grey contours (absolute value) have
values of close to zero, it means that the percentage change is magnified since the denominator (baseline value) is close to zero. For the final SEP plot, all of the red contours that are close to the MSB can be ignored for this reason. On the energy change plot, all of the red contours at low speed can also be ignored. Figure 9-1 summarises in which areas the red contours can be ignored.

Figure 9-1: Summary of in which areas red percentage change contours can be ignored.

Table 9-1 summarises the findings in the sensitivity analysis. The left hand column shows the variables where an input change of 10% led to an output sensitivity of much less than 10%. The middle column shows the variables where an input change of 10% led to an output sensitivity of approximately 10%. The right hand column shows the variables where an input change of 10% led to an output sensitivity of much greater than 10%. It is only of interest to discuss the sensitive inputs here, as the fact that certain variables are insensitive is irrelevant.

<table>
<thead>
<tr>
<th>Insensitive Variables</th>
<th>Neutral Variables</th>
<th>Sensitive Variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pitching Moment</td>
<td>Drag</td>
<td>Lift</td>
</tr>
<tr>
<td>Lift Derivative</td>
<td>Pitching Derivative</td>
<td>Weight</td>
</tr>
<tr>
<td>Drag Derivative</td>
<td>Wing Area</td>
<td></td>
</tr>
<tr>
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<td>Thrust</td>
<td></td>
</tr>
<tr>
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<td>Load Factor Limit</td>
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<td>Elevator Deflections</td>
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<td>AoA ITR</td>
<td>Maximum AoA</td>
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<tr>
<td>Maximum AoA</td>
<td></td>
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</tbody>
</table>

Table 9-1: Summary of Insensitive, Neutral and Sensitive Variables.
9.3 Discussion

Lift Coefficient. With the subtraction of 10% of the entire lift coefficient throughout the flight envelope (shown in Figure 9-2), there comes a significant sensitivity. On average the time parameter change is 16.9%, with a spread of 1.71s. The time change is least at the high speed, low altitude region, and increases more than linearly as the manoeuvre stall boundary is approached. At initial conditions faster than 300m/s, and lower than 4km altitude, the sensitivity is less than 10%. However, nearer the manoeuvre stall boundary, this sensitivity is always greater then 20% and often greater then 25%, with a maximum sensitivity of almost 35%. The final SEP tells a similar story near the manoeuvre stall boundary, but with sensitivities as great as 100%. At the fast/low region, there are also very significant sensitivities of 100%. The energy change is rather acceptable (with the greatest sensitivity less than 10%), with exception to the very low speed region, between 100 and 200m/s. The reason for this is that the absolute performance is close to 0m of energy change. When calculating the percentage, the absolute value is used in the denominator, and hence the percentage becomes very sensitive and large percentages are seen. However, from a relative difference point of view, the change is much less than 100m difference, and hence insignificant. Finally the height change is also fairly insensitive, with the greatest sensitivities at around 10%.

On an addition of 10% in lift coefficient, very similar trends are seen (shown in Figure 9-3). The relative differences are almost identical, however the percentage differences appear less sensitive. Again, the SEP is very sensitive and erratic, and the energy change is very sensitive at the low speed region where the absolute values are close to zero. The height change has very similar trends to the above height changes.

From this analysis it can be stated that the outcome is rather sensitive to the lift coefficient. For this reason, it is recommended that to achieve a sensitivity of less than 10%, the allowable input sensitivity be tightened to less than 5%.
Figure 9-2: Lift Coefficient - 10%.
Figure 9-3: Lift Coefficient + 10%.
Weight. Increasing the weight by 10% (as shown in Figure 9-4) means that the time parameter has a sensitivity as great as 20%, right up near the manoeuvre stall boundary. However, the average sensitivity is 9.9% and at the low altitude and high speed region, the sensitivity is 0 or even -5%. The SEP parameter again has problems near the manoeuvre stall boundary where the absolute value is close to zero (hence this problem can be ignored). At the high speed and low altitude region, again, the SEP parameter appears sensitive with changes as great as + and - 500m/s. The energy change again appears sensitive at the low speed region, but as discussed previously it is not, and the greatest percentage change is about 15% (but only at one grid point) again in the low altitude/transonic region. The height change only gets sensitive near the manoeuvre stall boundary, with the greatest sensitivity about 10%.

Decreasing the weight (as shown in Figure 9-5) leads to smaller percentage changes as far as the time parameter is concerned (but, the difference values are similar). Generally, the changes are around 10%, with there being no change at the low altitude and high speed region. The SEP parameter has a very similar trend to the increase in weight. This time, the greatest sensitivities are a little smaller at about 400m/s, again in the high speed, low altitude region. The energy change again suffers in the low altitude transonic region with sensitivities as high as 20%, but the general sensitivity is much closer to 5%. The height change parameter shows sensitivities as high as 10% near the manoeuvre stall boundary, but closer to 0% in the high speed, low altitude region.

Based on this discussion, it is recommended that sensitivities in weight of less than 10% be allowed. The area around the manoeuvre stall boundary should be investigated to gain a better understanding of why the sensitivity increases in this area.
Figure 9-5: Weight - 10%.
9.4 Summary in Terms of Result Parameters

The four parameters have each been examined and tend to show similar trends, regardless of the variable tested. These can be summed up in the following schematics with recommendations for solving problems with each trend. Each schematic shows a picture of the flight envelope with the MSB drawn. The circled numbers indicate the area that the numbered notes discuss.

9.4.1 The Time Parameter.

![Figure 9-6: The Time Parameter Sensitivity Trends](image)

1. This region is particularly sensitive on parameters where the *average* time sensitivity is around 10%.

2. On insensitive parameters, it is common for a sensitivity of around 5% to occur in this region.

3. For the normal load factor, the region near the manoeuvre stall boundary exhibits no sensitivity, but the transonic and faster regions show large sensitivities around 20%.
9.4.2 The SEP Parameter.

1. All variables exhibit sensitivity here, but the absolute values are close to zero, meaning that percentages will be sensitive. The difference values are not significant.

2. There are typically large sensitivities exhibited in this region. However, it is outside of the 1g SEP = 0 m/s boundary, so it is not that important.

3. For the normal load factor, the region near the manoeuvre stall boundary exhibits low sensitivities, but the transonic and faster regions show very large sensitivities around 600m/s.

9.4.3 The Energy Change Parameter.

1. Many variables exhibit sensitivity here, but the absolute values are close to zero, meaning that percentages will be sensitive. The difference values are not significant.
2. There are typically large sensitivities around 20% exhibited in this region, when all other areas exhibit sensitivities less than 10% and often less than 5%.

3. For the normal load factor, this region exhibits sensitivities in the region of 20%, when all other areas show no sensitivity.

9.4.4 The Height Change Parameter.

1. Larger sensitivities are exhibited here when typical sensitivities of other results parameters are less than 5%.

2. This region is particularly sensitive on parameters where the average sensitivity is around 10%.

3. For the normal load factor, the region near the manoeuvre stall boundary exhibits low sensitivities, but the transonic and faster regions show large sensitivities around 15%.

In general terms it can be stated that when using current tolerances, the results are in general believable, but caution should be used when considering the confidence given to results in the following regions.

1. Near the manoeuvre stall boundary for the time parameter for the more sensitive variables.

2. In the high speed, low altitude region for the SEP.

3. The low altitude transonic region for energy change for the more sensitive variables.
10.1 Conclusions

It was not an aim of this project to conclude anything about the use of Thrust Vector Control (TVC) / Post Stall Manoeuvrability (PSM), but rather to purely provide methods for assessing these and other technologies, for aircraft where the engine exhaust nozzles are located at the rear of the aircraft. The conclusions that were drawn are as follows.

- Traditional point performance metrics such as Instantaneous Turn Rate (ITR), Sustained Turn Rate (STR), and Specific Excess Power (SEP) do not show the true changes that TVC and PSM bring, when TVC is applied as moveable nozzles at the rear of the aircraft.
- The addition of a pitching moment balance to the STR and SEP does show the changes made by TVC nozzles at the rear of the aircraft. Direct changes to ITR are not shown using such a pitching balance since the changes to ITR are in the form of implicit changes to SEP.
- The existing manoeuvrability metric called Combat Cycle Time (CCT) does not show the changes that PSM and TVC make in a combat realistic sense, partly due to the inclusion of energy recovery in the last segment of the CCT.
- The existing manoeuvrability metric called Pointing Margin does show the changes that PSM and TVC make, although with reference to another aircraft, and in terms that are not easily grasped, such as the angle between the two aircraft.
- No other existing metrics were found to provide suitable quantification of the incorporation of TVC/PSM for simple inputs.
• A new metric has been developed which can be used to assess and quantify the changes made to an aircraft’s manoeuvrability by the incorporation of TVC and PSM.
• The metric is suitable for use with low order models, and can therefore be used at the conceptual design stage of an aircraft.
• The traditional Corner Velocity can be misleading because it shows a lower velocity than the optimum velocity determined using the new metric which considers the more realistic case of a dynamic manoeuvre. This fact was backed by pilot opinion.
• The new metric shows the Manoeuvre Stall Boundary (MSB), above which an aircraft cannot complete the manoeuvre due to a lack of energy. This has been shown to be a more meaningful boundary than the traditional 1g stall limit.
• The above two points refer to traditional aircraft as well as those incorporating new technologies, and so it is concluded that the new metric is suitable for assessing traditional aircraft.
• The conclusions on Corner Velocity and the Manoeuvre Stall Boundary show that the new metric can provide a pilot with knowledge not previously available without combat experience.
• Traditional metrics such as the turn rate plot can show an advantage to an aircraft that can sustain a higher AoA. However the reality will not necessarily be a better performing aircraft, since the increased usage of energy might lead to a disadvantage. This example was clarified using the new metric, which will show the true result with much less doubt.
• The new metric was used to show that compound metrics like the Dt parameter do not provide additional information, but rather cloud the actual results by averaging turning and energy – two distinct types of combat, as discussed in previous chapters.
• The new metric was used to show similar conclusions about Frenet agility which other studies have also shown – the fact that it appears to be dominated by velocity.
• The metric can assess the aircraft starting a manoeuvre after breaking off from a previous manoeuvre, by considering different starting conditions.
• The ability to assess both the horizontal and the vertical plane provide a method not formerly available to conceptual designers, to design an aircraft to meet performance criteria for both planes.

### 10.2 Further Work

There are three areas of further work that are recommended, to improve the methods and metrics developed within this thesis.

- **Load factor limiter redesign.** The validation and sensitivity studies both showed that there were problems with the accuracy of the load factor limiter that was used here. To rectify these problems a new load factor limiter should be designed and incorporated into the models used. This will not directly affect the new metric, but should ensure greater accuracy from the results obtained with it.

- **Weapons system.** A large amount of interest was shown from industry about including a weapons system into this metric. It is suggested that initially this consist of simply including the ability of an off-bore sight missile. Further work could also include firing of a missile and a portion of its initial fly out path.

- **Input accuracy.** To aid in determining the inputs, especially the pitching moment modelling inputs, it is recommended that work be carried out looking into new methods of input determination. Furthermore, inputs to allow post stall roll modelling are currently unavailable to the conceptual designer. If such input data were available accurately at low cost (in terms of time as well as fiscal expense), then the new metric developed here could be extended to include analysis of manoeuvres such as the Herbst Turn and Helicopter Turn, over both of which there is doubt as to whether they are beneficial when used in combat.
APPENDIX A

MORE METRICS

This appendix is an extension to chapter 2 and contains descriptions of metrics that were found during the study, but which are not directly related to the work that took place. These metrics are, as in chapter 2, are listed according to their acting axis.

A.1 Longitudinal Metrics

A) Pitch Agility Criteria\[^{[1,2]}\],

This is defined as pitching moment coefficient due to a control surface deflection, scaled with wing area (S), mean aerodynamic chord (c), and pitch axis inertia (I\textsubscript{yy}). It is extracted from the pitching moment derivative,

\[
M_\delta = \frac{qS\varepsilon C_{M\delta}}{I_{yy}}
\]

Equation A.1

Hence, it can be calculated purely from aerodynamic and configuration data (although note that \(C_{M\delta}\), the Pitching Moment coefficient derivative with respect to deflection, will be AoA dependant). Thus it is a measure of the airframe's ability to generate pitch acceleration, and does not reflect the pilot or FCS ability. This metric can easily be extended to the Roll Agility Criteria, through,

\[
L_\delta = \frac{qSbC_{L\delta}}{I_{xx}}
\]

Equation A.2

These metrics give the steady state pitch and roll agility, and so do not incorporate the transient terms contained within the time for the aircraft to reach the steady state roll rate (say). As aircraft become more agile, and the time to bank to say 90° becomes less, the transients involved in such a manoeuvre become more important. Thus it may not be acceptable to ignore the transient lags involved, meaning that this metric becomes too simple.
B) Pitch Agility
Suggested by Eidetics\(^3\), this is the time to pitch to maximum load factor plus the time to pitch back to zero load factor. It was developed to be a measure of existing aircraft, and so come from flight test data. Other possibilities are, load factor rate (dn/dt), time to capture a given incidence, or time to capture a given load factor. For time to capture a given incidence, and time to capture a given load factor, the problem arises that not all aircraft have the same maximum incidence or load factor. The load factor rate is thus more useful, but apparently very hard to obtain from flight test data (using current instrumentation). This load factor rate is very similar to the curvature agility proposed by MBB (see section 2.4.3).

These metrics come from the fact that nose up and down pitch agility is important, especially for post stall recovery. The capture element of any such manoeuvre is very difficult for the pilot to fly. This means that the same manoeuvre could be flown many times and very different times for pitching to angle of attack or load factor could be obtained. To help alleviate this problem, it has been suggested that measuring time to fly through a certain angle of attack or load factor might be more repeatable and hence more useful. However, care must be taken when flying through a given load factor – the maximum allowable load factor must not be exceeded. Furthermore, if the aircraft has poor handling qualities such that the capture time dominates the manoeuvre, then this will not be reflected in the metric, meaning that the metric should indeed include capture time.

C) Pitch Rate. Both the maximum positive and negative values are measured. This rate is a function of AoA but also a function of the initial AoA and the amount of control deflection. If pitch rate is plotted against AoA, then there will be a maximum value of the pitch rate, and where this occurs will be the optimum maximum angle of attack. That is to say that below this angle of attack, the aircraft will be able to recover well. Above this angle of attack, the recovery time begins to degrade and so it may not be worth flying above this angle of attack. However, if nose pointing is the main objective of a manoeuvre, then this optimum angle of attack will be of little use. It is likely that this metric will show the differences between conventional and thrust vectored aircraft in pitch well.
Pitch agility and pitch rate are again aimed at determining the post stall attainability/recovery of an aircraft, and so were treated in the same manner in that it was suggested that a new metric be developed to incorporate all these needs.

**A.2 Lateral Metrics**

A) Maximum Lateral Acceleration (Side Slip)

This metric presented by Tamrat gives an insight into how much side slip occurs during say a loaded roll. It shows the body yaw and roll acceleration requirements for a given AoA and flight condition, as shown in Figure A.1 and Figure A.2. It is important to know this, as the thrust vectoring control system can then be designed to cope with this sideslip. This metric is affected by thrust vectoring when using the TVC to maintain the attitude of the aircraft relative to the horizon.

Although the loaded roll has been suggested as important, Tamrat suggests that the roll about the velocity vector is more important. This is because the loaded roll is continuously interchanging the angle of attack with the side slip angle and so the loaded roll is very hard to analyse. Also, because of the interchanging, the lift is continuously changing to become side force. To examine the roll about the velocity

---

1 A loaded roll is a roll about the aircraft body axis when at a specific AoA. This type of roll is the type normally discussed for lateral motion. During this manoeuvre the incidence is changed to become side slip angle, then returns to negative incidence and then negative side slip angle, followed by a return to positive incidence. The other type of roll is the roll about the velocity vector. During this type of roll, the incidence remains constant. A loaded roll is initiated with the use of the aileron. The control stick is simply pushed to the side and a roll commences. To control a roll about the velocity vector requires rudder input to maintain zero sideslip and constant AoA. To complete this type of roll, often the FCS is designed to control longitudinal stick as AoA and lateral stick as roll rate or acceleration. This was the case for the X-31, where foot pedal inputs would change the sideslip from zero.

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MORE METRICS

vector, the required aerodynamic, thrust vector and total control power are investigated for varying angle of attack.

These metrics are not of great use to the pilot or tactician (although they will be useful to the designer). However, pilots may find it useful to know that if the thrust vectoring power (or aerodynamic control power) is damaged that they should not try to fly above a certain angle of attack because sufficient control power will no longer exist.

This would be useful to have programmed into the FCS. The FCS could then impose an angle of attack limit is either system is damaged. Also useful to the pilot may be the knowledge of how much control power is available. This would be very similar to knowing how quickly the aircraft could manoeuvre at a given angle of attack (like pitch rate and roll rate for given angle of attack).

This metric is really a tool for designing the thrust vector control system. Its complexity means that its use in conceptual design is limited. It has been suggested that a loaded roll is less realistic of the practises used in high angle of attack combat, and that sideslip will never be very large because it is very uncomfortable for the pilot. Hence it was suggested that this metric not be studied at any further level.

B) \( T_{90} \)
Suggested by Eidetics\(^{3,5}\), this is the time to roll and capture (stop rolling and hold attitude) a 90° bank angle change, while maintaining angle of attack. It has been suggested by Rosa\(^{6}\) that this will be hard to measure using at most a simple model (as will most lateral metrics). Thus, perhaps a more realistic metric would be to
measure the time to roll through 90°. For the longitudinal metrics in chapter 2, it was discussed that it was not desirable to pitch through a given AoA, but rather it was desired to pitch and hold a given AoA. However due to the lack of input data available to roll modelling, it is acceptable to consider rolling through a given angle. This could be easily measured with a simple step integral over the bank angle change, with given roll capability for given bank angle. Also, Liefer[2] has suggested that an easier metric would be to measure the time to reduce angle of attack to zero, capture 90° bank angle, and then return to the original angle of attack. Thus, the original metric measures performance for a loaded roll. The unloaded roll technique is less complicated because it does not incorporate changing the heading angle – this is because the aircraft is at zero AoA for the unloaded roll. However, Eidetics and others feel that it is important to measure the agility of the loaded roll as it shows the agility in roll at high angle of attack. This metric is very pilot-dependant as it is a quick manoeuvre and depends on the pilots reactions. An alternative manoeuvre would be to roll to 180°. As indicated by Liefer[2], it is worth noting that US Military Specification (MIL-F-8785-C) does not require the aircraft to hold the bank angle, just to achieve it.

Steady roll rate would be a good measure of merit. This is simpler to calculate. It has been suggested that the capture (and the onset) elements of a roll should only be noted if they are out of the ordinary (that is particularly large or small), although this would require a definition of "out of the ordinary".

This metric, although it seems to be very useful, has some problems. Calculating the time to roll through 90° would not be possible using the data that was available in the specification of this project, as described in chapter 3, since there was no data other than for basic pitch modelling. In fact, data was not available for body roll, let alone body yaw and hence roll about the velocity vector. Because of this, the T90 metric was not considered any further.
C) Torsional Agility
Also proposed by Eidetics, this metric is defined as Turn Rate / T90 (see above). This metric gives an idea of how the aircraft can roll and turn during the T90 manoeuvre. The equation used to calculate the turn rate is,

\[
\dot{\psi} = \frac{g \sqrt{\frac{n^2 - 1}{V}}} {\text{[rad/s]}}
\]

Equation A.3

and is based on velocity and load factor. The discussion above explained that it would not be possible to analyse the T90 metric, and so it was also not possible to analyse Torsional Agility. However, there were still a few points about this metric that came out during discussions that were important to all metrics, as discussed next.

This metric suggests that the turn rate and T90 are equally important to torsional agility. That is, an aircraft with twice the load factor is equal to another with half the T90. Does this really mean the two aircraft are equal? Raising each term to a given power (weighting) need to be added – factoring each term by a weighting will not distinguish the terms since the weightings might as well be a single compound weighting. It has also been considered that the torsional agility metric needs to be examined alongside the turn rate and T90 metrics individually, as more can be read from this.

In discussion with Sinclair, an example was given where the A-4 was in trial dogfights with the F-14. As far as the theory was concerned, the F-14 should have won every fight because it has a far superior pitch rate, and thus could attain a higher turn rate. However, the two aircraft turned out to be rather evenly matched. This was said to be because the roll rate of the A-4 is three times that of the F-14. Hence a metric that considers the roll rate and the turn rate (as torsional agility does) would give a clearer picture for comparing the two aircraft.

The torsional agility metric seems to have developed by considering the agility along the three axes of an aircraft as Skow did (Skow worked for Eidetics). Along these axes, the following are considered: the acceleration along the flight path, the symmetrical turning perpendicular to the flight path, and rolling about the velocity vector. Plotting the above on three dimensional axes can be seen in Figure A-3.
If this plot is broken down and made more basic, elementary information is found. Setting roll rate to zero leads to a SEP vs. turn rate plot (typically shown in Raymer\textsuperscript{10}). Setting SEP equal to zero leads to the plot of turn rate vs. roll rate. It is known that AoA is directly proportional to turn rate (through load factor) and so gives a plot of roll rate vs. AoA, again a familiar plot. This type of three dimensional plot could be used to compare two aircraft by having different coloured surfaces for each fighter. To make the above plot easier to read, the turn rate and roll rate can be joined into one function, by dividing the turn rate by time to roll and capture 90°. Then, two plots can be examined, the torsional agility vs. SEP plot and the classical turn rate vs. SEP plot. Things like acceleration and turn rate advantages can then be found quite easily for two aircraft.

This metric was generally liked by those interviewed. It describes aircraft agility in the lateral plane and so helps to measure the turning performance (scissors, rolling, etc.). However, because of its relationship to the T\textsubscript{90} metric and the roll DoF, not enough data was in the data set available to this project, and the metric could not be
examined any further. The comments by Sinclair about the comparison between the F-14 and A-4 however were borne in mind during the project.

D) Roll Reversal Agility Parameter

Proposed by Kalviste[8], this metric is defined as the product of the time required to reverse a turn and the cross range displacement that occurs during the turn. The metric is derived from the scissors manoeuvre where the cross range displacement is the maximum distance in the horizontal direction between positive and negative rolls, as shown in Figure A-4. The parameter is based on reversing a level turn for a given load factor, or banking from +90° to -90°. The cross range distance is a function of the aircraft’s normal acceleration during the roll, and the smaller the metric, the more agile the aircraft. Similar to the T90 metric, this metric implies that the aircraft should hold a constant load factor, whilst reversing the turn. According to Liefer[12], if the aircraft were allowed to unload first, reverse the turn (roll 180°) then reload, the cross range distance would become very small, thus reducing the roll reversal agility parameter, and theoretically improving performance. However, the time taken to unload the aircraft, and then reload and capture the initial load factor may add up to make a longer total time for this new strategy than for the original strategy. That is to say that this metric is strongly affected by the strategy used and that it could give false indication.

Another measure of aircraft rolling agility is the Rearward Separation Distance[4], RSD. After an unsuccessful first shot, the scissors manoeuvre is likely to be utilised, where the aim is to get behind the enemy for another shot. As can be seen in Figure A-5, the better the

![Figure A-4: Cross Range Displacement](image)

![Figure A-5: Definition of RSD](image)
roll agility, the more the RSD will be.

Note that this metric examines the horizontal and vertical scissors only, and not the rolling scissors (a series of barrel rolls), which also exists as a common combat manoeuvre. A similar metric could be used for the rolling scissors.

Although extremely combat relevant, the modelling required for these two metrics is complex, and beyond the scope of the data allowed here. Hence this metric was not considered any further.

**A.3 General/Combination Metrics**

These metrics could fall either into several of the above categories, or none of them.

A) Elevation and Azimuth Gun Aiming Ranges

Suggested by Herbst\[^{10}\] in the early 1980's, this metric shows the potential of how far up and down and to each side the gun can aim. For example, if the sideslip limits are $\pm 30^\circ$, then the azimuth gun aiming range is $60^\circ$. When considering the dynamics of the aircraft and the weapons trajectories this value may change. The metric allows manoeuvres like nose pointing (in both pitch and yaw axes) and the helicopter gun attack, where the aircraft enters a flat spin and uses yaw thrust vector control to chase the aircraft circling around him, to be analysed. This metric could be hard for pilots to use, but it could be given as an envelope on the Head Up Display (HUD). To the authors knowledge this has never been attempted, partly due to the complexity of it. If the enemy aircraft is within the box, the pilot could point the nose at the aircraft for a shoot solution. This metric is the basis for on-line documentation which can be displayed real-time to the pilot, whereas the aim of this project to concentrate on off-line documentation producing information that is of primary use to the designer. Due to this and the complexity of involving multiple degrees of freedom and a weapons system, this metric was not considered any further.

B) T/W and W/S (wing loading) Ratios

Herbst\[^{10}\] (and others) have suggested that viewing data about these ratios gives a good idea about the performance of aircraft. Herbst summarises that,
1. to improve performance, T/W should be increased, W/S decreased,
2. for optimum combat capability, reduce W/S,
3. for a defensive capability, lower W/S,
4. for an offensive capability increase T/W.

Herbst also did a number of statistical studies on aircraft performance related to these ratios. It was in fact this work that lead Herbst to consider TVC and nose pointing. Herbst's work in this field gained him the unofficial title of "The Grandfather of Thrust Vectoring". Since TVC and PSM do not explicitly affect either of these metrics, they were not considered for further study.

C) Agility Potential
Suggested by Spearman\(^{[1]}\), it is defined as the ratio of maximum thrust loading to wing loading,
\[
\frac{T}{W}/\frac{W}{S} = \frac{TS}{W^2}
\]

Equation A.4

The two ratios relate the aircraft size and configuration to agility using traditional measures of merit. Since the metric is so simple, it does not connect the flight control characteristics, high angle of attack capability, nor body rate controllability. It is not a metric affected by transients because of this. However, from a conceptual design point of view, this is still a worthwhile metric, giving a very brief insight into the potential of an aircraft at a higher level. Since neither of its components are explicitly affected by TVC or PSM, it was decided not to consider this metric beyond this stage.

D) Herbst's \(^{[1]}\) Correlation Factor
Defined as,
\[
CF = 5\sqrt{AR + 100\frac{t}{c} + 10\cos\Lambda} + 10\sqrt{\frac{T}{W}} - 0.2\frac{W}{S} + 1.5MD
\]

Equation A.5

where,
- AR = aspect ratio,
- t/c = thickness to chord ratio of the wing,
- \(\Lambda\) = leading edge sweep angle,
- MD = control symbol for manouevring devices having the value of either
one or zero.

The differences of the correlation factors,

\[ \Delta CF_1 = CF_1 - CF_2 \]
\[ \Delta CF_2 = CF_2 - CF_1 \]

Equation A.6

show for each opponent the approximate chances of kill and survival against each other.

This metric seems to have been developed in studies done to justify the need for a new technology for enhancing turn performance. The trouble with such a metric is that it appears to have come from statistical analysis, meaning that the use of different configurations/technologies of aircraft outside the envelope of those used to create the statistical comparison cannot be used with any surety. For example, from Jane's\(^{[12]}\), an F/A-18a can be compared to a Boeing C-17a Transport Aircraft. Using the figures in Jane's, each aircraft has a correlation factor of 30.7 (for the F/A-18a) and 31.7 (for the C-17a). This appears to indicate that the two aircraft are similar in performance, however this claim is blatantly false! This is because we have effectively extrapolated the figures in the statistical study. In addition, the limits of the variables involved in the statistical data that were used are unknown, so it is impossible to understand whether using some new data stretches the limits of the metric beyond the bounds of its acceptable interpolative limits. For these reasons it was decided not to consider this metric any further.

**A.4 References**


6. Personal communication with Martin Rosa, DERA Pyestock, Farnborough, GS14 0LS, UK.

7. Personal communication with Lt. Commander Andy Sinclair, RN, of CDA, DERA, UK.


APPENDIX B

MODELS

This appendix includes the data used to model the high fidelity model and the low order model, as well as the derivation of the control inputs for simulation to satisfy any given target.

B.1 High Fidelity Model

The data given in this section provide all that is required to create a high fidelity model as discussed in chapter 3. They are taken from reference 1.

B.1.1 Tabulated Steady Rotation Aerodynamic Model

```matlab
function [C1,Cn,CY,Cm]=srmodel(a,b,w)

% Math model for F18 steady-rotation experiment data

b2=b*b;
b3=b2*b;
b4=b3*b;
b5=b4*b;
w2=w*w;
w3=w2*w;
w4=w3*w;
w5=w4*w;
b2w=b2*w;
bw2=b*w2;
ab=abs(b);
aw=abs(w);

% F18 Sh-5 symmetrized data/ cubical model for C_1

Aoa b w w^3 b^3 b^2 w w^2
D=[
  0  0.000498582 -0.358202 -0.763273 0.000000 5.98004e-05 -0.00279272
  5 -0.000509063 -0.367297 0.735233 1.40458e-06 6.73433e-05 0.0167081
 10 -0.00159156 -0.234052 -0.574399 2.55137e-06 -0.000142666 0.0167081
 15 -0.0021594 -0.250049 0.733023 4.27867e-06 -7.74524e-05 0.00000000
 20 -0.00168547 -0.146429 0.000000 3.42052e-06 0.000000000 0.00000000
 25 0.000543171 0.0568933 0.000000 2.33778e-06 -0.00493678 -0.0233069
 30 0.00341378 0.268717 -2.55252 -9.09886e-06 -0.00119224 -0.0374232
 35 0.00237138 0.24241 -2.49194 -9.15245e-06 -0.00102669 0.00000000
 40 0.000000000 0.144113 -2.46376 -5.22525e-06 -0.00018963 -0.0273333
 45 -0.00186309 -0.000000 -0.827176 0.000000000 -0.000228947 -0.0196925
 50 -0.00228643 -0.0790319 0.832763 8.51551e-07 0.000000000 -0.0119174
 55 -0.000225616 -0.065876 0.000000 6.26495e-07 0.000146375 -0.0142746
 60 -0.00250671 -0.0803257 0.000000 8.38482e-07 0.000146375 -0.0128996
];

kb =vtable(D(:,1),D(:,2),a);
kw =vtable(D(:,1),D(:,3),a);
kww=vtable(D(:,1),D(:,4),a);
kbbb=vtable(D(:,1),D(:,5),a);
```

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**MODELS**

\[
k_{bbw} = \text{vtable1}(D(:,1), D(:,6), a);
k_{bww} = \text{vtable1}(D(:,1), D(:,7), a);
\]

\[
C_Y = k_{b}b + k_{w}w + k_{www}w^3 + k_{bbb}b^3 + k_{bbw}b^2w + k_{bww}bw^2;
\]

```plaintext
% F18 Sh-5 symmetrized data/ cubical model for C_Y

%

\[
\% AoA  b    w    w^3    b^3    b^2w    w^2b
D=[
0 -0.0131281 0.0254259 -0.255744 2.43921e-05 -0.000387864 -0.0497845
5 -0.0139488 0.12394 -0.677275 7.0331e-06 -0.000496331 -0.0245781
10 -0.0145427 0.158114 -0.677275 7.0331e-06 -0.000496331 -0.0245781
15 -0.0154525 0.202288 -0.163407 2.92016e-05 -0.000702277 -0.024922
20 -0.0150963 0.264985 -0.966979 2.30117e-06 -0.002622752 -0.0349755
25 -0.00324409 0.497228 -7.64125 -1.27838e-05 -0.002882266 -0.34023
30 -0.00226162 0.0268914 -1.04482 -2.35942e-05 -0.00311219 -0.272466
35 -0.00581119 -0.278065 -3.00776 -1.5303e-05 -0.00212751 -0.212383
40 -0.00861118 -0.209607 -12.7722 6.33014e-06 -0.00701794 -0.200699
45 -0.01080022 -0.197596 -29.6414 -3.70298e-05 -0.00835454 -0.641569
50 -0.00466783 0.140934 -3.12297 -3.16408e-05 -0.00786426 -0.601256
55 -0.00785578 -0.141937 -22.9497 -4.65716e-05 -0.00664952 -0.524324
60 -0.00985193 -0.447273 -9.82908 -5.04017e-05 -0.00153534 -0.193794
];

% AoA  w^4b    w^3b2    w^2b3    wb4    b^5
DD=[
0 0.0 0.0 0.0 0.0 -6.11185e-08
5 0.0 0.0 0.0 0.0 -5.40612e-08
10 0.0 0.0 0.0 0.0 0.0
15 0.0 0.0283141 0.0 -3.5947e-06 -7.55082e-08
20 0.0 0.072587 0.0 -3.32184e-06 0.0
25 2.66206 0.0437627 0.0 0.0 0.0
30 0.0 0.0 0.0 0.0 0.0
35 0.0 0.0 0.0 0.0 0.0
40 0.0 0.101079 0.0 -1.27838e-05 -0.70282e-05
45 7.4129 0.17763 0.0 -1.09786e-05 -0.70282e-05
50 6.67415 0.184909 0.0 0.0 0.0
55 7.49658 0.171605 0.0 0.0 0.0
60 0.0 0.0791464 0.0 0.0 0.0
];

kb = \text{vtable1}(D(:,1), D(:,2), a);
kw = \text{vtable1}(D(:,1), D(:,3), a);
kwww = \text{vtable1}(D(:,1), D(:,4), a);
kbbb = \text{vtable1}(D(:,1), D(:,5), a);
kbbw = \text{vtable1}(D(:,1), D(:,6), a);
kbww = \text{vtable1}(D(:,1), D(:,7), a);

kw4b = \text{vtable1}(DD(:,1), DD(:,2), a);
kwb3b = \text{vtable1}(DD(:,1), DD(:,3), a);
kw2b3 = \text{vtable1}(DD(:,1), DD(:,4), a);
kwb4 = \text{vtable1}(DD(:,1), DD(:,5), a);
kbs = \text{vtable1}(DD(:,1), DD(:,6), a);

C_Y = kb*b + kw*w + kwww*w^3 + kbbb*b^3 + kbbw*b^2w + kbww*bw^2 +
kw4b*w^4b + kw3b2*w^3b2 + kw2b3*w^2b^3 + kw*b^4 + kw*b^5;

% F18 Sh-5 symmetrized data/ cubical model for C_n

%

\[
\% AoA  b    w    w^3    b^3    b^2w    w^2b
D=[
0 0.000365873 -0.13177 -0.0782539 -2.67445e-06 -5.17449e-05 -0.0163185
5 -0.000545806 -0.0835566 -0.849516 7.12662e-05 7.58311e-05 -0.0188721
10 -0.000147644 -0.0602936 2.24157 4.30158e-09 -0.0015147 -0.031463
15 0.000698089 -0.11495 -0.10159 -8.09757e-07 -5.15535e-05 -0.0429239
20 -0.00092932 -0.194418 -1.13711 8.54821e-06 5.99935e-05 -0.0208948
25 -0.00108676 -0.10555 2.46199 3.90079e-06 -0.00051582 0.0183845
30 -0.000959643 -0.0647284 -2.32754 -3.27195e-05 -0.000905023 -0.0211535
35 -0.00310525 -0.138846 -4.92282 1.79872e-05 -0.002620864 -0.0103511
40 0.00446765 -0.159977 -3.45946 -2.03175e-06 -0.000384115 0.00765704
45 0.00821303 0.353085 0.0933354 1.76155e-06 0.010946
50 0.0116601 0.542444 -16.8893 -1.13722e-06 -0.00104928 0.0295916
];
```

B-2
<table>
<thead>
<tr>
<th>AoA</th>
<th>C_Y</th>
<th>C_1</th>
<th>w5</th>
<th>w4b</th>
<th>w2b3</th>
<th>w3b2</th>
<th>w4b</th>
<th>b5</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.047797</td>
<td>-0.348476</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>60</td>
<td>0.00143003</td>
<td>0.0435067</td>
<td>-8.9461</td>
<td>-1.13319e-05</td>
<td>-0.00336062</td>
<td>-0.121998</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[(DD) = \begin{bmatrix}
0 & -0.047797 & -0.348476 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
5 & -0.10697 & -0.142482 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
10 & -0.0129708 & 1.12977 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
20 & 0.013689 & 0.315057 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 & 0.0 \\
30 & -0.00125369 & -0.155798 & 0.966645 & 0.0 & -0.0279424 & 0.0 & -0.0335125 |
\end{bmatrix} \]

\[\text{Cn} = \text{kb*b} + \text{kw*w} + \text{kwww*w} + \text{kbbb*b} + \text{kbw*b*w} + \text{kbww*b*w2} + ... \]
\[\text{kw5*w5 + kW4b*w4*b + kw3b2*w3*b2 + kw2b3*w2*b3 + kw4b*w4*b + kb5*b5 + ...} \]
\[\text{kcy*CY + kcl*C1;} \]

\[
<table>
<thead>
<tr>
<th>AoA</th>
<th>\text{Cm0}</th>
<th>\text{w1}</th>
<th>\text{w2}</th>
<th>\text{b2}</th>
<th>\text{b*w}</th>
<th>\text{w*b}</th>
<th>\text{Cm0}</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>-0.00125369 &amp; -0.155798 &amp; 0.966645 &amp; 0.0 &amp; -0.0279424 &amp; 0.0 &amp; -0.0335125</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>-0.00264721</td>
<td>0.12052</td>
<td>-1.7387</td>
<td>0.0 &amp; -0.0413794 &amp; 0.0 &amp; -0.0563169</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### B.1.2 Tabulated Model Coefficients (Low Speed)

% File AERODATA.M Origin: NASA
% this loads in the aerodata

```plaintext
global ALPHA_BREAK
ALPHA_BREAK = [-14.0000 -10.00000 -6.00000 -2.00000 2.00000 6.00000 ...
   10.0000 14.0000 18.0000 22.0000 26.0000 30.0000 34.0000 ...
   38.0000 42.0000 46.0000 50.0000 54.0000 58.0000 62.0000 ...
   66.0000 70.0000 74.0000 78.0000 82.0000 86.0000 90.0000 ];
```

% here is cy_b
```plaintext
cy_b = [-1.71957E-02 -1.71957E-02 -1.77407E-02 -1.82857E-02 -1.8419E-02 ...
   -1.84159E-02 -1.87763E-02 -1.77142E-02 -1.55057E-02 -1.40980E-02 ...
   -1.28732E-02 -1.26852E-02 -1.25160E-02 -1.35594E-02 -1.40314E-02 ...
   -1.33128E-02 -1.19496E-02 -1.30678E-02 -1.37112E-02 -1.37528E-02 ...
   -1.33426E-02 -1.29076E-02 -1.33076E-02 -1.38320E-02 -1.37811E-02 ...
   -1.31714E-02 -1.27360E-02 ];
```

% here is cyp
```plaintext
cyp = [-1.06465E-03 -1.06465E-03 -1.06465E-03 -8.63938E-04 -2.09440E-04...
   4.14516E-04 5.80322E-04 5.32325E-04 4.53786E-04 2.22529E-04 ...
   -1.74533E-04 -5.49779E-04 -9.42478E-04 -1.35699E-03 -2.60752E-03 ...
   -4.39823E-03 -5.23599E-03 -3.00197E-03 -1.65806E-03 -7.85399E-04 ...
   -9.77384E-04 -1.39626E-03 ];
```

% here is cy_r
```plaintext
cy_r = [2.69653E-03 2.69653E-03 2.69653E-03 2.89725E-03 3.36412E-03 ...
   3.94444E-03 4.59322E-03 3.82227E-03 1.22609E-03 -1.61443E-03 ...
   -3.46480E-03 -4.66876E-03 -5.36689E-03 -5.67232E-03 -3.89208E-03 ...
   -2.96706E-03 -4.36322E-03 -9.25024E-04 -9.94838E-04 -9.25024E-04 ...
   -9.77384E-04 -1.39626E-03 ];
```

% here is cy_da
```plaintext
cy_da = [-5.77397E-04 -5.77397E-04 -5.77397E-04 -5.46240E-04 -4.60488E-04...
   -4.06750E-04 -4.16183E-04 -3.70449E-04 -4.98886E-04 -2.88800E-04 ...
   -8.56568E-04 -1.00528E-03 -1.48179E-04 -7.14772E-04 -1.95743E-04 ...
   -2.37525E-04 -2.40677E-04 -2.78191E-04 -3.95206E-04 -3.28602E-04 ...
   -3.47581E-04 -3.65875E-04 ];
```

% here is cy_del
```plaintext
cy_del = [-1.21826E-03 -1.21826E-03 -1.21826E-03 -1.18543E-03 -1.08569E-03 ...
   -4.99497E-05 2.06273E-04 4.27499E-04 5.91445E-04 7.25571E-04 ...
   -8.56568E-04 -1.00528E-03 -1.13120E-03 -1.22332E-03 -1.30397E-03 ...
   -1.29672E-03 -1.07641E-03 5.04952E-04 -4.54204E-05 -5.69973E-04 ...
   -1.00132E-03 ];
```

% here is cy_der
```plaintext
cy_der = [1.21826E-03 1.21826E-03 1.21826E-03 1.18543E-03 1.08569E-03 ...
   9.69152E-04 8.62255E-04 7.02810E-04 4.98886E-04 2.88800E-04 ...
   4.99497E-05 -2.06273E-04 -4.27499E-04 -5.91445E-04 -7.25571E-04 ...
   -8.56568E-04 -1.00528E-03 -1.13120E-03 -1.22332E-03 -1.30397E-03 ...
   -1.29672E-03 -1.07641E-03 -5.04952E-04 4.54204E-05 5.69973E-04 ...
   1.00132E-03 ];
```

% here is cy_dr
```plaintext
cy_dr = [3.46480E-03 3.46480E-03 3.46480E-03 3.52987E-03 3.66813E-03 ...
   3.73320E-03 3.66813E-03 3.46480E-03 3.08253E-03 2.57827E-03 ...
   2.11467E-03 1.79747E-03 1.59413E-03 1.47213E-03 1.50595E-03 ...
   1.59771E-03 1.48189E-03 1.26506E-03 1.04479E-03 9.72747E-04 ...
   1.04790E-03 1.15331E-03 1.10125E-03 1.01504E-03 1.04757E-03 ...
   1.15266E-03 ];
```
```
% here is croll_b

croll_b = [-3.70196E-05 -3.70196E-05 -4.41520E-04 -8.46019E-04 -1.27065E-03...
          -1.70123E-03 -2.11290E-03 -2.82588E-03 -2.75756E-03 -2.70552E-03...
          -2.67292E-03 -2.66024E-03];

% here is croll_p

croll_p = [-7.05113E-03 -7.05113E-03 -7.05113E-03 -7.05113E-03 -7.05113E-03...
           -7.05113E-03 -6.24828E-03 -5.25344E-03 -4.62512E-03 -3.85718E-03...
           -5.38434E-03 -7.94125E-03 -8.68301E-03 -8.72665E-03 -4.88692E-03...
           -3.49064E-05 -3.31613E-03 -3.73500E-03 -4.04916E-03 -4.36332E-03...
           -4.67748E-03 -4.88692E-03 -5.02655E-03 -5.16617E-03 -5.30580E-03...
           -5.41052E-03 -5.41052E-03];

% here is croll_r

croll_r = [2.14676E-04 2.14676E-04 2.14676E-04 5.85558E-04 1.52804E-03...
           2.57087E-03 3.39177E-03 3.84496E-03 1.97600E-03 1.89664E-03...
           2.14676E-04 5.13071E-03 1.71042E-03];

% here is croll_da

croll_da = [1.16543E-03 1.16543E-03 1.16543E-03 1.16686E-03 1.17021E-03...
            1.14013E-03 1.04034E-03 8.40294E-04 6.31892E-04 5.23513E-04...
            5.13071E-03 9.61564E-03 5.02267E-05 1.77607E-05];

% here is croll_del

croll_del = [5.65136E-04 5.65136E-04 5.65136E-04 5.65136E-04 5.65136E-04...
             6.78982E-04 6.92797E-04 6.82421E-04 6.50348E-04 5.94802E-04...
             6.99129E-04 8.19937E-05 2.43878E-05 5.32750E-05 2.92725E-05...
             7.97000E-05];

% here is croll_der

croll_der = [-5.65136E-04 -5.80368E-04 -6.12176E-04 -5.65136E-04 -5.65136E-04...
              -6.78982E-04 -6.89279E-04 -6.82421E-04 -6.50348E-04 -5.94802E-04...
              -6.99129E-04 -8.19937E-05 -2.43878E-05 -5.32750E-05 -2.92725E-05...
              -7.97000E-05];

% here is croll_dr

croll_dr = [2.86293E-04 2.86293E-04 2.86293E-04 2.86293E-04 2.86293E-04...
             2.10653E-04 2.07400E-04 1.80560E-04 1.24440E-04 1.24440E-04...
             6.91336E-04 3.49733E-05 1.87076E-05 4.80000E-05 3.44428E-05...
             8.67339E-05 -1.08661E-03 -7.83403E-05 -2.63520E-05 -3.68277E-05...
             9.48021E-05 1.25253E-05 9.58432E-05 7.66811E-05 6.73115E-05...
             6.47088E-05 6.32773E-05];

% here is cn_b

cn_b = [ 1.42071E-03 1.42071E-03 1.52521E-03 1.62971E-03 1.65099E-03...
        1.67084E-03 1.80024E-03 1.46016E-03 9.91781E-04 6.82316E-04...
        1.75902E-03 -1.09086E-03 -1.11070E-03 -1.59429E-03 -1.89776E-03...
        1.76721E-03 -1.35917E-03 -1.52852E-03 -1.63273E-03 -1.59147E-03...
        -1.44456E-03 -1.44456E-03 -1.47796E-03 -1.60918E-03 -1.69618E-03...
        -1.72843E-03 -1.73807E-03];

% here is cn_p

cn_p = [-1.26406E-03 -1.26406E-03 -1.26406E-03 -1.23533E-03 -1.17787E-03...
        -1.12041E-03 -9.39611E-04 -6.36739E-04 -2.47236E-04 1.00356E-04...
        -3.70882E-04 4.58149E-04 3.97062E-04 2.48709E-04 2.33874E-04];
<table>
<thead>
<tr>
<th></th>
<th>4.18879E-04</th>
<th>6.9132E-04</th>
<th>9.77384E-04</th>
<th>-1.04720E-03</th>
<th>-1.60570E-03</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3.24631E-04</td>
<td>-2.26839E-04</td>
<td>-1.44164E-03</td>
<td>-6.99132E-04</td>
<td>-8.81388E-14</td>
<td>...</td>
<td></td>
</tr>
<tr>
<td>-9.39512E-14</td>
<td>-9.97960E-14</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

% here is cn_r

cn_r = [ -3.11978E-03 | -3.11978E-03 | -3.11978E-03 | -3.11973E-03 | -3.11105E-03 | ... |
| -3.10669E-03 | -3.12850E-03 | -3.17649E-03 | -3.27686E-03 | -3.46448E-03 | ... |
| -3.97499E-03 | -2.87796E-03 | -1.70606E-03 | -6.93768E-04 | -4.36324E-06 | ... |

% here is cn_da

| -3.93752E-05 | -4.53709E-04 | -5.23318E-04 | -6.15525E-05 | -7.74300E-05 | ... |
| -9.85781E-05 | -1.20466E-03 | -1.53810E-03 | -1.97276E-03 | -2.31246E-03 | ... |

% here is cn_del

cn_del = [ 4.45020E-04 | 4.45020E-04 | 4.45020E-04 | 4.36729E-04 | 4.17690E-04 | ... |
| 3.94303E-04 | 3.56959E-04 | 2.82299E-04 | 1.66782E-04 | 6.38125E-05 | ... |
| -4.76974E-04 | -4.53709E-04 | -5.23318E-04 | -6.14640E-04 | -7.97428E-04 | ... |
| -9.85781E-05 | -1.20466E-03 | -1.53810E-03 | -1.97276E-03 | -2.31246E-03 | ... |

% here is cn_dr

cn_dr = [ -0.227325 | 0.170673 | 7.47868E-02 | 3.2097E-02 | 2.79305E-02 | ... |
| 6.13288E-02 | 0.154023 | 0.261263 | 0.363863 | 0.466635 | 0.643989 | 0.863792 | 1.08558 | 1.27900 | 1.47638 | 1.61911 | 1.74604 | 1.84448 | 1.92402 | 1.98384 | 2.02834 | 2.06708 | 2.11362 | 2.15514 | 2.17363 | 2.16283 | 2.12622 | ... |

% here is cd0

cd0 = [ 0.227325 | 0.170673 | 7.47868E-02 | 3.2097E-02 | 2.79305E-02 | ... |
| 6.13288E-02 | 0.154023 | 0.261263 | 0.363863 | 0.466635 | 0.643989 | 0.863792 | 1.08558 | 1.27900 | 1.47638 | 1.61911 | 1.74604 | 1.84448 | 1.92402 | 1.98384 | 2.02834 | 2.06708 | 2.11362 | 2.15514 | 2.17363 | 2.16283 | 2.12622 | ... |

% here is cd_q

cd_q = [ 1.48740E-16 | -2.21604E-16 | -8.70931E-05 | -7.79330E-04 | -8.31959E-04 | ... |
| -6.60702E-05 | 1.33279E-03 | 1.67868E-04 | 5.84481E-06 | -9.59607E-17 | ... |
| -6.54823E-17 | -5.38470E-16 | 2.03264E-16 | 1.14528E-15 | -2.50755E-16 | ... |
| 8.43503E-16 | -8.45493E-16 | -2.54842E-15 | -7.66583E-16 | -3.79368E-16 | ... |

% here is cd_del

cd_del = [ -2.63851E-03 | -2.02960E-03 | -2.06960E-03 | -1.61668E-03 | -1.11176E-03 | ... |
| 4.33892E-04 | 4.67839E-04 | 4.93824E-03 | 5.59555E-03 | 5.88711E-03 | ... |
| 5.82388E-03 | 5.49440E-03 | 5.22574E-03 | 5.15071E-03 | 5.18088E-03 | ... |
| 5.20101E-03 | 5.06911E-03 | 3.65306E-03 | 2.50369E-03 | 1.69598E-03 | ... |
| 1.25007E-03 | 1.13131E-03 | ... |
```plaintext
 MODELS

% here is clift0

    clift0 = [-0.13860  -0.808970  -0.551318  -0.216206  0.154461  0.560857 ...  
              0.951575  1.24168  1.39786  1.88940  1.80975  1.68556  
              1.02151  0.858875  0.685334];

% here is clift_q

    clift_q = [-7.87143E-02  6.91587E-02  6.28753E-02  0.159698 ...  
                  0.131772  8.63938E-02 ... 8.49975E-02];

% here is clift_del

    clift_del = [-5.03735E-03  7.18902E-03  5.97515E-03  3.56060E-03  
                  1.18163E-03 -3.65314E-04 -1.10325E-03 -2.47676E-03];

% here is cmO

    cmO = [9.62520E-02  8.07290E-02  5.54066E-02  2.80686E-02  5.08530E-03 ...  
               -6.61034E-03 -1.64371E-02 -1.60245E-02 -5.86329E-02 -8.42348E-02 ...  
               -8.5266E-02 -0.1055 -0.1170 -0.1101 -0.1149 -0.1183 -1.018E-01 ...  
               -0.146057 -0.228243 -0.351592 -0.426319 -0.476897 -0.601749 -0.563499 ...  
               -0.547688 -0.570013 -0.580308];

% here is cm_q

    cm_q = [-8.68301E-02  -8.68301E-02  -8.65683E-02  -8.53902E-02 ...  
               -8.0524E-02  -7.57473E-02  -7.23875E-02  -7.56606E-02 ...  
               -8.02932E-02  -1.02364E-01  -0.147655  -0.205556  -0.132732  
               -1.8064E-02  -4.31969E-02  1.80642E-02  -1.96350E-02 ...  
               -9.8174E-02  -9.53822E-02  -9.25991E-02  -8.52593E-02 ...  
               -8.24666E-02];

% here is cm_del

    cm_del = [-6.46703E-03  -6.58967E-03  -6.83004E-03  -7.05535E-03 ...  
                 -7.82965E-03 -7.99808E-03 -7.99808E-03 -7.99808E-03 ...  
                 -5.05606E-03 -3.75061E-03 -3.75061E-03 -3.75061E-03 ...  
                 -1.37503E-03 -8.13315E-04 -7.80549E-04 -1.77291E-04 ...  
                 -2.35499E-03 -2.89985E-03];

% here is cm_der

    cm_der = [-6.46703E-03  -6.58967E-03  -6.83004E-03  -7.05535E-03 ...  
                 -7.82965E-03 -7.99808E-03 -7.99808E-03 -7.99808E-03 ...  
                 -5.05606E-03 -3.75061E-03 -3.75061E-03 -3.75061E-03 ...  
                 -1.37503E-03 -8.13315E-04 -7.80549E-04 -1.77291E-04 ...  
                 -2.35499E-03 -2.89985E-03];

cd_der = [-2.63851E-03  -2.42289E-03  -2.00960E-03  -1.61666E-03 ...  
               -4.31951E-04  1.15200E-03  1.97203E-03  2.82662E-03 ...  
               5.59955E-03  5.86711E-03  5.15071E-03  5.18088E-03 ...  
               3.59917E-03  4.29851E-03  5.82388E-03  5.49440E-03 ...  
               1.25007E-03  1.13131E-03];

% here is cmO

    cmO = [9.62520E-02  8.07290E-02  5.54066E-02  2.80686E-02  5.08530E-03 ...  
               -6.61034E-03 -1.64371E-02 -1.60245E-02 -5.86329E-02 -8.42348E-02 ...  
               -8.5266E-02 -0.1055 -0.1170 -0.1101 -0.1149 -0.1183 -1.018E-01 ...  
               -0.146057 -0.228243 -0.351592 -0.426319 -0.476897 -0.601749 -0.563499 ...  
               -0.547688 -0.570013 -0.580308];

% here is cm_q

    cm_q = [-8.68301E-02  -8.68301E-02  -8.65683E-02  -8.53902E-02 ...  
               -8.0524E-02  -7.57473E-02  -7.23875E-02  -7.56606E-02 ...  
               -8.81282E-02 -1.02364E-01 -0.147655 -0.205556 -0.132732  
               -1.8064E-02  -4.31969E-02  1.80642E-02  -1.96350E-02 ...  
               -9.8174E-02  -9.53822E-02  -9.25991E-02  -8.52593E-02 ...  
               -8.24666E-02];

% here is cm_del

    cm_del = [-6.46703E-03  -6.58967E-03  -6.83004E-03  -7.05535E-03 ...  
                 -7.82965E-03 -7.99808E-03 -7.99808E-03 -7.99808E-03 ...  
                 -5.05606E-03 -3.75061E-03 -3.75061E-03 -3.75061E-03 ...  
                 -1.37503E-03 -8.13315E-04 -7.80549E-04 -1.77291E-04 ...  
                 -2.35499E-03 -2.89985E-03];

% here is cm_der

    cm_der = [-6.46703E-03  -6.58967E-03  -6.83004E-03  -7.05535E-03 ...  
                 -7.82965E-03 -7.99808E-03 -7.99808E-03 -7.99808E-03 ...  
                 -5.05606E-03 -3.75061E-03 -3.75061E-03 -3.75061E-03 ...  
                 -1.37503E-03 -8.13315E-04 -7.80549E-04 -1.77291E-04 ...  
                 -2.35499E-03 -2.89985E-03];

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MODELS

AEROTABLE = [cy_b cy_p cy_r cy_da cy_del cy_der cy_dr ...
croll_b croll_p croll_r croll_da croll_del croll_der croll_dr ...
cn_b cn_p cn_r cn_da cn_del cn_der cn_dr ...
cd0 cd_q cd_del cd_der ...
clift0 clift_q clift_del clift_der ...
cm0 cm_q cm_del cm_der];

B.1.3 Tabulated Model Coefficients (High Speed)

% AERODAT2.M
% June 1998, by Andrew Khramtsovsky
% This file contains data for supersonic extension of F-18 aerodynamic model. Below M=0.7 combined model coincides with initial low-speed model.

% grid for Mach number

AM_BREAK=[.6 .7 .8 .9 .95 1.05 1.1 1.2 1.52 1.8 2.0];

LONGITUDINAL

% Longitudinal supersonic dependencies were extracted from TsAGI's wind tunnel data for prototype F-18 configuration.

%%%%%%%%%%%%%%%%%%%%%%%%
% C_L (alpha,M)  
%%%%%%%%%%%%%%%%%%%%%%%%

% C_L_basic (alpha,M) = C_L_basic (alpha * kacl(M)) + dclm(M) (low speed) ------- -------

kacl=[ 1.0000 1.0000 1.1373 1.1685 1.1799 1.0646 ...
1.0656 1.0522 1.0054 0.7432 0.6576 0.5789 ];

dclm=[ 0.0 0.0 -0.0299 -0.0306 -0.0393 -0.0414 ...
-0.0284 -0.0295 -0.0574 -0.0593 -0.0851 -0.1029 ];

%%%%%%%%%%%%%%%%%%%%%%%%
% C_D (alpha,M)  
%%%%%%%%%%%%%%%%%%%%%%%%

% C_D_basic (alpha,M) = C_D_basic (alpha * kacd(M)) + dcdm(M) (low speed) ------- -------

dcdm= [ 0.0 0.0 0.0014 0.0060 0.0157 0.0264 ...
0.0309 0.0329 0.0332 0.0334 0.0335 0.0334 ];

kacd=[ 1.0000 1.0000 1.0196 1.0336 1.0406 1.0476 ...
1.0713 1.0808 1.0508 0.8953 0.8661 0.8240 ];

%%%%%%%%%%%%%%%%%%%%%%%%
% C_m (alpha,M)  
%%%%%%%%%%%%%%%%%%%%%%%%


MODELS

\[
\begin{align*}
\text{C}_m_{\text{basic}} (\alpha, M) &= \text{C}_m_{\text{basic}} (\alpha) + \\
&\quad + \text{dcm}(M) + \text{dxn}(M) \cdot \text{C}_N_{\text{wing}} (\alpha, M) \\
\text{dcm} &= [0.0, 0.0, -0.003490, 0.013710, 0.013710, 0.000920, -0.025660, 0.002324, 0.025810, 0.025420, 0.019120, 0.022450]; \\
\text{dxn} &= [0.0, 0.0, 0.0101, 0.0254, -0.0595, -0.1449, ... -0.2002, -0.2610, -0.2996, -0.3073]; \\
\end{align*}
\]

Elevator effectiveness corrections

\[
\begin{align*}
\text{C}_D_{\text{De}} (\alpha, M) &= k_{\text{CDDE}} (M) \cdot \text{C}_D_{\text{De}} (\alpha) \\
k_{\text{CDDE}} &= [1.0000, 1.0000, 0.9876, 0.9207, 0.8838, 0.8799, ... 0.9072, 0.9112, 1.0219, 1.0557, 0.9147, 0.8222]; \\
\end{align*}
\]

\[
\begin{align*}
\text{C}_L_{\text{De}} (\alpha, M) &= k_{\text{CLDE}} (M) \cdot \text{C}_L_{\text{De}} (\alpha) \\
k_{\text{CLDE}} &= [1.0000, 1.0000, 1.0331, 1.0504, 1.0562, 1.0845, ... 1.1486, 0.9659, 0.9307, 0.6798, 0.4364]; \\
\end{align*}
\]

\[
\begin{align*}
\text{C}_m_{\text{De}} (\alpha, M) &= k_{\text{CMDE}} (M) \cdot \text{C}_m_{\text{De}} (\alpha) \\
k_{\text{CMDE}} &= [1.0000, 1.0000, 1.0413, 1.0999, 1.1500, 1.2185, ... 1.3176, 1.1312, 1.0929, 0.7863, 0.6330, 0.5380]; \\
\end{align*}
\]

---

LATERAL AND LATERAL-DIRECTIONAL

Corrections for lateral and lateral-directional characteristics were taken from prototype models, since no lateral supersonic data were available for F-18

\[
\begin{align*}
\text{C}_1 & \\
\end{align*}
\]

\[
\begin{align*}
\text{C}_1_{\text{beta}} (\alpha, M) &= k_{\text{clb}} (M) \cdot \text{C}_1_{\text{beta}} (\alpha) \\
k_{\text{clb}} &= [1.0000, 1.0000, 0.9712, 0.9392, 0.9232, 0.9072, ... 0.9002, 0.8931, 0.8791, 0.7739, 0.6295, 0.5263]; \\
\end{align*}
\]

\[
\begin{align*}
\text{C}_1_{\text{Da}} (\alpha, M) &= k_{\text{clda}} (M) \cdot \text{C}_1_{\text{Da}} (\alpha) \\
k_{\text{clda}} &= [1.0000, 1.0000, 1.0288, 0.9186, 0.8635, 0.8084, ... 0.7522, 0.6960, 0.5836, 0.4394, 0.3526, 0.2907]; \\
\end{align*}
\]

---
% C_l_Dr (alpha,M) = kcldr(M) * C_l_Dr (alpha)
%              -------- low speed

kcldr=[ 1.0000 1.0000 1.0370 1.0000 0.9815 0.9630 ...
       0.9074 0.8519 0.7407 0.5298 0.4800 0.4444 ];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % C_n % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % C_n_beta (alpha,M) = kcnb(M, alpha) * C_n_beta(alpha)
%              -------- low speed

ALPHA_CNB=[-10. -8. -4. 0. 4. 8. 12. 16. 18.];
% Aoa ->
% M
% kcnb=[
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000 1.0000
1.0433 1.0433 1.0433 1.0433 1.0256 0.9946 0.9423 0.8383 0.8383
1.1369 1.1369 1.1193 1.1007 1.0641 0.9728 0.7981 0.5649 0.5649
1.1837 1.1837 1.1559 1.1294 1.0833 0.9620 0.7260 0.4282 0.4282
1.2305 1.2305 1.1934 1.1581 1.1026 0.9511 0.6538 0.2916 0.2916
1.2124 1.2124 1.1808 1.1519 1.0859 0.9076 0.6058 0.2551 0.2551
1.1943 1.1943 1.1682 1.1456 1.0692 0.8641 0.5577 0.2187 0.2187
1.1581 1.1581 1.1430 1.1330 1.0359 0.7772 0.4615 0.1458 0.1458
0.8711 0.8711 0.8711 0.8711 0.7715 0.5374 0.3023 0.0383 0.0383
0.7301 0.7301 0.7301 0.7301 0.6595 0.4522 0.2269 -0.0638 -0.0638
0.6294 0.6294 0.6294 0.6294 0.5795 0.3913 0.1737 -0.1367 -0.1367
];

% C_n_Dr (alpha,M) = kcdnr(M) * C_n_Dr (alpha)
%              -------- low speed

kcdnr=[ 1.0000 1.0000 1.0221 0.9534 0.9190 0.8846 ...
       0.8427 0.8008 0.7170 0.4700 0.3761 0.3090 ];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % C_Y % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % C_Y_beta (alpha,M) = kcyb(M) * C_Y_beta(alpha)
%              -------- low speed

kcyb=[ 1.0000 1.0000 1.0197 1.0640 1.0899 1.1158 ...
       1.1305 1.1453 1.1379 1.0177 0.9532 0.9163 ];

% C_Y_Dr (alpha,M) = kcydr(M) * C_Y_Dr (alpha)
%              -------- low speed

kcydr=[ 1.0000 1.0000 1.0178 0.9636 0.9366 0.9095 ...
       0.8441 0.7787 0.6480 0.4236 0.3335 0.2691 ];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%% % Corrections of rotational derivatives % %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
MODELS

% \text{C}_1_p(\alpha, M) = kclp(M) \cdot \text{C}_1_p(\alpha)
% \quad \text{low speed}
%
% \text{C}_n_r(\alpha, M) = kclp(M) \cdot \text{C}_n_r(\alpha)
% \quad \text{low speed}

kclp = [ 1.0000 1.0000 1.0205 1.0557 1.0738 1.0964 ... 1.1113 1.1263 1.1439 1.0941 0.9859 0.8949 ];
%
% \text{C}_1_r(\alpha, M) = kclr(M) \cdot \text{C}_1_r(\alpha)
% \quad \text{low speed}
%
% \text{C}_n_p(\alpha, M) = kclr(M) \cdot \text{C}_n_p(\alpha)
% \quad \text{low speed}

kclr = [ 1.0000 1.0000 1.0977 1.1510 1.1667 1.1731 ... 1.1785 1.1743 1.1289 0.8761 0.6295 0.4534 ];
%
% end of file

B.1.4 Engine Model Listing

function \[\text{thaft}, \text{thmax}\] = engine(Mach, Height, \text{t}_{aft\_st}, \text{t}_{max\_st})
% Generic engine model
% Computes wet and dry thrust as function of Mach number
% and altitude (meters)
% \text{t}_{aft\_st} and \text{t}_{max\_st} - wet and dry static thrust at \text{M}=0, \text{H}=0
% \text{M}=1.0 .2 .4 .6 .8 1.0 1.2 1.4 1.6 1.8 2.2.21;
% \text{HH}=[0.5000 11000. 15000. 18000.];
% \text{TAFT}=[
% 1.000000 1.066886 1.130164 1.231009 1.348384 ... 1.437208 1.482456 1.512676 1.545903 1.582135 ... 1.600335 1.609521
% 0.617429 0.647653 0.695904 0.780221 0.887077 ... 1.010462 1.148870 1.224170 1.284445 1.332698 ... 1.326290 1.287132
% 0.255296 0.282513 0.312735 0.360989 0.412246 ... 0.487547 0.604919 0.740325 0.848684 0.932998 ... 0.984289 0.100849
% 0.130578 0.133753 0.148949 0.167149 0.209392 ... 0.263654 0.329939 0.394720 0.473026 0.549829 ... 0.595077 0.631309
% 0.074981 0.081161 0.087341 0.093521 0.117732 ... 0.141942 0.190194 0.232436 0.280688 0.331946 ... 0.366678 0.381874
% 0.046326 0.053994 0.063578 0.076997 0.094249 ... 0.117252 0.191054 0.233227 0.275399 0.315655 ... 0.338658 0.321406
% 0.046326 0.053994 0.063578 0.076997 0.094249 ... 0.117252 0.140256 0.159425 0.186262 0.209265 ... 0.209265 0.199681
% ];
% \text{TMAX}=[
% 1.000000 0.963578 0.942492 0.961661 1.000000 ... 1.024920 1.030671 1.032588 1.026837 1.021086 ... 1.005751 0.988498
% 0.536102 0.545687 0.566773 0.605112 0.662620 ... 0.729712 0.803514 0.839936 0.853355 0.850479 ... 0.833227 0.806390
% 0.215974 0.225559 0.240895 0.267732 0.307029 ... 0.362620 0.430671 0.507348 0.597859 0.662198 ... 0.584026 0.518850
% 0.099042 0.108626 0.120128 0.133546 0.162300 ... 0.191054 0.233227 0.275399 0.315655 0.338658 ... 0.338658 0.321406
% 0.046326 0.053994 0.063578 0.076997 0.094249 ... 0.117252 0.140256 0.159425 0.186262 0.209265 ... 0.209265 0.199681
% ];
% \text{TMAX}=\text{TMAX}';
% \text{TAFT}=\text{TAFT}';

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thaff = t_aft_st*vtable21(M, HH, TAFT, Mach, Height, 1, 1);
thmax = t_max_st*vtable21(M, HH, TMAX, Mach, Height, 1, 1);

% end of file

Other data that was used is listed in Table B-1, Figure B-1, and Figure B-2.

<table>
<thead>
<tr>
<th>Wing Area, S, ft²</th>
<th>400</th>
</tr>
</thead>
<tbody>
<tr>
<td>c, ft</td>
<td>11.52</td>
</tr>
<tr>
<td>b, ft</td>
<td>37.4</td>
</tr>
<tr>
<td>Mass, slug</td>
<td>1111.74</td>
</tr>
<tr>
<td>Gravitational Acceleration, ft/s²</td>
<td>32.17</td>
</tr>
<tr>
<td>Iₓₓ, slug ft²</td>
<td>22632.6</td>
</tr>
<tr>
<td>Iᵧᵧ, slug ft²</td>
<td>174246.3</td>
</tr>
<tr>
<td>Izz, slug ft²</td>
<td>189336.4</td>
</tr>
<tr>
<td>Iₓz, slug ft²</td>
<td>-2131.8</td>
</tr>
<tr>
<td>xref / c</td>
<td>0.017</td>
</tr>
<tr>
<td>xref, ft</td>
<td>-0.449</td>
</tr>
</tbody>
</table>

Table B-1: Constants used in the model.
Wing area = 400 ft$^2$
MAC = 11.52 ft
Aspect ratio = 3.5
Stabilator area = 88.26 ft$^2$
Weight = 31,980 lb
(Includes 60 percent fuel condition of 6,480 lb)

Figure B-1: Three View of the F-18 with major dimensions shown.
B.2 Low Order Model

Chapter 3 shows the equations used in modelling the low order model. The data to do this are as follows.

Other data used for the low order model were extracted from the high fidelity model as well as Janes\textsuperscript{[3]}, as necessary.
| Aircraft: | Cl | Alpha | Mach | -10 | -5 | 0 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 |
|---------|----|-------|------|-----|----|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 0.00    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.10    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.20    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.30    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.40    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.50    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.60    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.464 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.70    |    |       |      | -0.881 | -0.468 | -0.031 | 0.459 | 0.952 | 1.281 | 1.465 | 1.614 | 1.777 | 1.887 | 1.850 | 1.717 | 1.575 | 1.425 |
| 0.80    |    |       |      | -0.965 | -0.555 | -0.061 | 0.499 | 1.021 | 1.331 | 1.522 | 1.694 | 1.857 | 1.924 | 1.671 | 1.511 | 1.329 | 1.121 |
| 0.90    |    |       |      | -0.978 | -0.569 | -0.062 | 0.514 | 1.043 | 1.349 | 1.538 | 1.720 | 1.857 | 1.901 | 1.635 | 1.470 | 1.273 | 1.056 |
| 1.00    |    |       |      | -0.948 | -0.536 | -0.072 | 0.451 | 0.957 | 1.277 | 1.466 | 1.621 | 1.789 | 1.848 | 1.750 | 1.592 | 1.441 | 1.257 |
| 1.10    |    |       |      | -0.931 | -0.519 | -0.060 | 0.456 | 0.960 | 1.282 | 1.470 | 1.622 | 1.791 | 1.859 | 1.778 | 1.619 | 1.471 | 1.294 |
| 1.20    |    |       |      | -0.940 | -0.527 | -0.088 | 0.405 | 0.898 | 1.227 | 1.411 | 1.560 | 1.724 | 1.830 | 1.788 | 1.652 | 1.510 | 1.357 |
| 1.30    |    |       |      | -0.876 | -0.494 | -0.089 | 0.362 | 0.819 | 1.173 | 1.356 | 1.503 | 1.641 | 1.783 | 1.831 | 1.761 | 1.623 | 1.495 |
| 1.40    |    |       |      | -0.809 | -0.460 | -0.090 | 0.320 | 0.738 | 1.083 | 1.294 | 1.440 | 1.562 | 1.700 | 1.810 | 1.819 | 1.793 | 1.619 |
| 1.50    |    |       |      | -0.742 | -0.426 | -0.090 | 0.278 | 0.658 | 0.994 | 1.229 | 1.372 | 1.494 | 1.602 | 1.729 | 1.828 | 1.830 | 1.755 |
| 1.60    |    |       |      | -0.716 | -0.416 | -0.098 | 0.250 | 0.610 | 0.942 | 1.190 | 1.330 | 1.450 | 1.551 | 1.668 | 1.775 | 1.821 | 1.792 |
| 1.70    |    |       |      | -0.700 | -0.413 | -0.107 | 0.225 | 0.571 | 0.899 | 1.149 | 1.291 | 1.410 | 1.513 | 1.617 | 1.728 | 1.811 | 1.813 |
| 1.80    |    |       |      | -0.684 | -0.409 | -0.116 | 0.200 | 0.532 | 0.853 | 1.095 | 1.252 | 1.370 | 1.474 | 1.566 | 1.678 | 1.771 | 1.803 |
| 1.90    |    |       |      | -0.650 | -0.402 | -0.125 | 0.171 | 0.485 | 0.767 | 1.029 | 1.205 | 1.322 | 1.425 | 1.512 | 1.609 | 1.708 | 1.793 |
| 2.00    |    |       |      | -0.637 | -0.394 | -0.134 | 0.142 | 0.437 | 0.720 | 0.963 | 1.157 | 1.270 | 1.370 | 1.460 | 1.540 | 1.638 | 1.725 |

Figure B-3: Lift Coefficient Data for the Low Order Model.
<table>
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<tr>
<th>Aircraft:</th>
<th>Cd</th>
<th></th>
<th>-10</th>
<th>-5</th>
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<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
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<td>0.030</td>
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<td>0.605</td>
<td>0.864</td>
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<td>0.864</td>
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Figure B-4: Drag Coefficient Data for the Low Order Model.
Figure B-5: Pitching Moment Coefficient Data for the Low Order Model.
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Figure B-6: Lift Coefficient / Elevator Derivative Data for the Low Order Model.
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Figure B-8: Pitching Moment Coefficient / Elevator Derivative Data for the Low Order Model.
B.3 Determination of Values for $\alpha$ and $\mu$ to Satisfy a Target Solution

Depending upon the requirement of the manoeuvre that is being examined, the aircraft will need to turn about a given angle, and then point its nose so that it is aligned with the final target heading. For the pure vertical and pure horizontal cases, equations can be found from inspection of the co-ordinate plane that solve for $\alpha$ and $\mu$. For example in the pure vertical plane, at any time, the equation that describes the required $\alpha$ is,

$$\alpha_{\text{required}} + \gamma = \theta_{\text{target}}$$

Equation B-1

where $\alpha$ is the required AoA, $\gamma$ is the flight path angle, measured from the horizon to the velocity vector, and $\theta$ the angle measured between the horizon and the gun vector of the aircraft. This equation gives the required AoA. If the required AoA is more than the maximum allowable, then the aircraft will not point, but rather stay with an AoA which gives the ITR solution. As the flight path angle, $\gamma$, increases, there comes a point where the required AoA falls within the limit, and so the aircraft pitches up. A similar equation exists for the horizontal plane, however, simply pointing when the time is right will not actually give a correct solution. This is because the aircraft is canted over in a bank angle so that the horizontal turn is induced. The bank angle will not be $90^\circ$, and so if the aircraft simply pitches up to its maximum AoA without changing the bank angle to $90^\circ$, it will not truly point along the target heading.

When the problem is expanded to allow the aircraft to venture from the pure vertical/horizontal planes, finding when the aircraft is to point becomes a further puzzle. The correct way about solving what the values of $\alpha$ and $\mu$ should be, is to consider the translations from the body axes to the global (or horizon) axes. Miele[21] touches on this problem for a flat earth. In his text the following translations are provided,
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\]

Equation B-2

where, \( H \) refers to the horizon axes, and \( B \) refers to the body axes. Also, \( M_1 \) and \( M_2 \) are given as,

\[
M_1 = \begin{bmatrix}
  \cos \gamma \cos \chi & \sin \mu \sin \gamma \cos \chi & \cos \mu \sin \gamma \cos \chi \\
  \sin \mu \sin \gamma \cos \chi & \sin \mu \sin \mu \sin \chi & \cos \mu \sin \gamma \sin \chi \\
  \sin \mu \sin \chi & \sin \mu \cos \chi & \cos \mu \cos \chi \\
\end{bmatrix}
\]

Equation B-3

\[
M_2 = \begin{bmatrix}
  \cos \alpha \cos \beta & -\sin \beta & \sin \alpha \cos \beta \\
  \cos \alpha \sin \beta & \cos \beta & \sin \alpha \sin \beta \\
  -\sin \alpha & 0 & \cos \alpha \\
\end{bmatrix}
\]

Equation B-4

where the angles are Euler angles, as shown in Figure B-9.

These equations allow transformation of body co-ordinates into world co-ordinates. This is required, since the target solution for any manoeuvre will be to align the
aircraft gun vector with a given unit vector in the world axes system. For example, a 180° heading change can be represented as \([x \ y \ z]_T = [-1 \ 0 \ 0]^T\), called the heading vector. Also, since it is the gun vector that needs to be aligned with this heading vector, let the gun vector be described by the unit vector \([l \ 0 \ 0]^T\). Using Equation B-2 to Equation B-4, the solution that the gun vector equals the heading vector, \([-1 \ 0 \ 0]^T\), can be found in terms of the control variables, \(\alpha\) and \(\mu\), assuming that sideslip, \(\beta = 0\). Combining Equation B-2, Equation B-3, and Equation B-4, and setting equal to the heading vector gives Equation B-5.

\[
\begin{bmatrix}
-1 \\
-cos \gamma \cdot cos \chi \cdot cos \alpha - cos \mu \cdot sin \gamma \cdot cos \chi \cdot sin \alpha \\
-sin \mu \cdot sin \chi \cdot sin \alpha \\
0 \\
-cos \gamma \cdot sin \chi \cdot cos \alpha - cos \mu \cdot sin \gamma \cdot sin \chi \cdot sin \alpha \\
+ sin \mu \cdot cos \chi \cdot sin \alpha \\
0 \\
-sin \gamma \cdot cos \alpha - cos \mu \cdot cos \gamma \cdot sin \alpha
\end{bmatrix} = [1 \ 0 \ 0]^T
\]  
Equation B-5

This gives three equations for two unknowns, a solvable problem. Rearranging Equation B-5c leads to,

\[
\tan \alpha = \frac{-\tan \gamma}{\cos \mu}
\]  
Equation B-6

Rearranging Equation B-5b leads to,

\[
\tan \alpha = \frac{\cos \gamma \cdot sin \chi}{\cos \mu \cdot sin \gamma \cdot sin \chi - sin \mu \cdot cos \chi}
\]  
Equation B-7

Substitution of Equation B-7 into Equation B-6, leads to a solution for \(\mu\),

\[
\tan \mu = \frac{\tan \chi}{\sin \gamma}
\]  
Equation B-8
A value of bank angle can now be found from Equation B-8. This value can be back substituted into Equation B-6 to solve for $\alpha$. When Equation B-6 and Equation B-8 are solved numerically on a PC (watching for singularities and taking appropriate precautions), the inverse tangent functions will be limiting, in that they return answers in the domain of $0^\circ$-$180^\circ$. This will not always present a solution to point at the target. Consider the following case. $\gamma = 0^\circ$, $\chi = 270^\circ$. There is no solution for $\alpha$ and $\mu$ in the limits of $0^\circ$-$180^\circ$, since whatever the values of $\alpha$ and $\mu$, the aircraft will be pointing in the forward hemisphere (in the world co-ordinate system). In reality, the aircraft has limits on AoA of $-10^\circ < \alpha < 90^\circ$, but $\mu$ is unlimited. Hence, using the modulation function (mod), we can overcome these limitations set by the use of numerical analysis. The solutions for $\alpha$ and $\mu$ now becomes,

\[
\mu_{\text{required}} = \begin{cases} 
\tan^{-1}\left(\frac{\tan \chi}{\sin \gamma}\right) \mod 180 & \text{IF } (\chi \mod 360 > 180) \text{ THEN} \\
\tan^{-1}\left(\frac{\tan \chi}{\sin \gamma}\right) \mod 180 & \text{ELSE}
\end{cases}
\]

Equation B-9

\[
\alpha_{\text{required}} = \tan^{-1}\left(\frac{-\tan \gamma}{\cos \mu_{\text{required}}}\right) \mod 180
\]

Equation B-10

These equations can be tested to ensure that the target unit vector is always $[-1 0 0]^T_y$. This has been done using the above equations, and testing values in every quadrant in the domains of $-360^\circ < \gamma < 360^\circ$, $-360^\circ < \chi < 360^\circ$. Equation B-9 and Equation B-10 always return suitable values for a correct solution.

The flight path controller can now track a given AoA (the value to provide ITR), flight path angle, $\gamma$, and bank the aircraft at a given $\gamma$ to offload any excess load factor and induce an out-of-vertical-plane turn. It can also compute values of the required AoA and bank angles (from Equation B-9 and Equation B-10). When the required AoA falls within the limits of the maximum and minimum allowables for the AoA,
the controller can track the required AoA and the required bank angle. This will provide a solution, with the aircraft pointing at the target.

**B.4 References**


APPENDIX C

DERIVATION OF ANALYTICAL CCT EQUATIONS

C.1 Analysis

What is required is a mathematical analysis of the Combat Cycle Time (CCT). Once this analysis is done, analysis of CCT and Pointing Margin will become easier. Consider the definition of CCT, repeated here from chapter 2.

CCT is the time to perform a 180° heading change and return to the original energy state. The CCT can be found from a turn rate plot by integrating around the boundary as shown above, by adding t₁, t₂₁, t₂₂, t₃ and t₄.

There can be a slight anomaly with this definition. Consider an aircraft with thrust such that the STR and ITR have the relationship as shown in Figure C-2, where they meet at low speed. In this case, it is possible that the aircraft could continue turning on the ITR until it reaches the 1g turn rate line (the x-axis). The problem is that the aircraft will not turn at 1g, but rather continue in a straight flight path (or at least a turn with a very large radius), leaving the aircraft at a tactical disadvantage. It is also unrealistic to consider that the pilot would not take evasive action. According to Hinchcliffe[1], an experienced pilot would not fly the aircraft using full control deflections if possible, since doing so wastes energy – it is more beneficial to be more gentle on the stick. In a similar vein, it is assumed that a pilot flying an aircraft with characteristics as shown in Figure C-2 would consider the following strategy for performing this manoeuvre. The aircraft starts at a given speed and loads up to
maximum load factor. The aircraft will begin to decelerate. Once the aircraft decelerates to the corner speed, it has to be progressively unloaded to continue unstalled flight, along the stall portion of the turn rate plot. Once the aircraft reaches the minimum drag speed (the speed at which sustained turn rate, STR is a maximum), it then unloads to maintain the maximum STR. The aircraft then stays at this point on the turn rate plot until the full 180° heading change is achieved. The aircraft unloading to 1g load factor and then accelerating back to the initial velocity (energy level) completes the manoeuvre. No altitude change is assumed for this strategy, only axial velocity changes, and hence flight is in the horizontal plane. This strategy is shown on a turn rate plot in Figure C-3 with the thick blue line. The same strategy for a thrust vectored aircraft is shown in thick green.

Figure C-3: CCT Turn Rate Plot Profile

1 Note that depending upon the initial velocity into the manoeuvre, the 180° heading change may be completed during the ITR segment. If this is the case, the STR segment must be ignored.
CCT is simply a contour integral around the turn rate plot. The curve that it follows around the plot is however discontinuous and hence requires the calculation to be split into several discrete parts to allow mathematical analysis. The above contour can be split into several segments, which can be analysed individually. When CCT is redefined to include the maximum sustained turn rate segment to increase efficiency, and avoid the anomaly discussed in chapter 4, these segments are the ITR segment, STR segment, and the acceleration segment. Between these segments there are also pitching segments. However, these segments will be ignored for now as they introduce too much complexity into the problem, and can be considered negligible compared to the total time that the CCT takes[2].

So, CCT can be defined,

\[ \text{CCT} = \int_{\chi=\theta \rightarrow 180^\circ} dt \]

Equation C-1

where, \( \chi \) is the heading angle. At any point in time, velocity can be found based on the acceleration. This acceleration comes from Specific Excess Power, \( P_s \), assuming no altitude change,

\[ \dot{V} = \frac{P_s \cdot g}{V} \]

Equation C-2

Next, we define the limits of each segment of the CCT. These are as follows,

- **ITR:** \( V > V_{md} \) \( \chi \leq 180^\circ \)
- **STR:** \( V = V_{md} \) \( \chi \leq 180^\circ \)
- **Acceleration:** \( V \geq V_{md} \) \( \chi = 180^\circ \)

where the subscript \( \text{md} \) refers to minimum drag, the point where STR is maximum. This leads to the CCT being redefined as,

\[ \text{CCT} = \int_{\text{ITR}} dt + \int_{\text{STR}} dt + \int_{\text{Acceleration}} dt \]

Equation C-3
The analysis can now be done in parts, allowing the discontinuities to be handled. The STR segment is the easiest to analyse. Because the STR is a constant turn rate manoeuvre, the time to complete the STR segment comes from the linear relationship as follows,

\[ \int_{\text{STR}} \frac{dx}{dt} = \frac{180^\circ - \chi}{\dot{x}_{\text{MAX,STR}}} \]

Equation C-4

As can be seen, this does rely upon the ITR segment. The maximum turn rate occurs at the velocity for minimum drag, obtained from,

\[ V_{\text{min,drag}} = \sqrt{\frac{2W}{\rho S}} \sqrt{\frac{k}{C_{do}}} \]

Equation C-5

where \( k \) is the induced drag factor, and \( C_{do} \) is the profile drag coefficient. \( W \) is weight, \( S \) is wing reference area and \( \rho \) is air density. Now, velocity is constant (due to no acceleration or altitude change as STR requires), thrust is constant (since Mach number and altitude are constant) as are dynamic pressure, wing area, drag coefficients, and weight (assume that the change in weight over a period of the CCT is negligible compared to the overall weight of the aircraft).

Consider Equation C-4 and Equation C-5. These can be used to calculate the STR segment. The velocity can be determined, then the AoA for zero SEP can be found, which will lead to the load factor and sustainable turn rate. For the calculations presented here, no pitch model was considered, and so all control deflections were ignored in the calculations.

Next, the acceleration segment can be analysed. Assuming a 1g normal load factor manoeuvre without altitude loss, acceleration can be found Equation C-2.

Now, from \( \dot{V} = \frac{\partial V}{\partial t} \),
\[
\int_{\text{Acceleration}} dt = \int_{v_{\text{MD}}}^{v_0} \frac{1}{V} dV = \int_{v_{\text{MD}}}^{v_0} \frac{V}{P_s g} dV
\]

Equation C-6

where \( P_s \) itself is also a function of velocity.

Finally, the ITR segment needs to be calculated. The ITR segment in itself is discontinuous because of the change from constant load factor to constant lift coefficient (separated by the corner velocity). The limits for the two sub-segments are as follows,

- ITR 1: \( V > V_c \) \( n = n_{\text{limit}} \)
- ITR 2: \( V < V_c \) \( n = \frac{C_{L_{\text{max}}} \rho V^2 S}{2W} \)

Thus,

\[
\int_{\text{ITR}} dt = \int_{v_o}^{v_c} dt + \int_{v_c}^{v_{\text{MD}}} dt
\]

Equation C-7

Finally, expanding Equation C-7 using Equation C-6, the ITR segment is cast as,

\[
\int_{\text{ITR}} dt = \int_{v_o}^{v_c} \frac{V}{P_s g} dV + \int_{v_c}^{v_{\text{MD}}} \frac{V}{P_s g} dV
\]

Equation C-8

Also required is the turn angle of the ITR segment, so that the STR segment can be calculated. Noting that \( \partial \gamma = \dot{\chi} dt \),

\[
\int_{\text{ITR}} dy = \int_{v_o}^{v_c} \frac{\dot{\chi} V}{P_s g} dV + \int_{v_c}^{v_{\text{MD}}} \frac{\dot{\chi} V}{P_s g} dV
\]

Equation C-9

Hence a governing equation for CCT can be written. It is,

\[
\text{CCT} = \int_{v_o}^{v_c} \frac{V}{P_s g} dV + \int_{v_c}^{v_{\text{MD}}} \frac{V}{P_s g} dV + \frac{180^\circ - \int_{\text{ITR}} \dot{\chi} d\gamma}{\dot{\chi}_{\text{MAX STR}}} + \int_{v_{\text{MD}}}^{v_{\text{max}}} \frac{V}{P_s g} dV
\]

Equation C-10
Next, consider the horizontal plane that the CCT manoeuvre takes place in. It is possible to find equations that allow the analyst to calculate the position on this plane of the aircraft at any time, t. These equations are repeated and simplified from the equations of motion in chapter 3.

At any stage,

\[ dx = V \cos(\chi) \, dt \]
\[ dy = V \sin(\chi) \, dt \]

Equation C-11

where, \( \chi \) is the total turn angle at that point, and \( x \) and \( y \) are the down range and cross range distances respectively. By substituting Equation C-2 into Equation C-11 and recasting, functions for these distances can be found in terms of time.

\[ x(t) = \int_{v_0}^{v(t)} \frac{V^2 \cos[\chi(V)]}{P_s g} \, dV \]
\[ y(t) = \int_{v_0}^{v(t)} \frac{V^2 \sin[\chi(V)]}{P_s g} \, dV \]

Equation C-12

where the limits are a function of time and the total turn angle is a function of velocity. In real terms, this can be done by plotting time against velocity (using the CCT equations) and then obtaining a spline function for velocity as a function of time. To ensure that this spline is not multi-valued, it needs to be split into discrete parts, as discussed for the CCT calculation.

Now that all the equations are in place, numerical integration can take place. To allow the use of the equations as they are written, without having to translate them into the usual numerical equivalent that most programming languages require, Mathsoft's Mathcad 7131 Professional was used for all of the results.

C.2 References


APPENDIX D

DISCUSSION OF WVR COMBAT

Often, performance is considered in two distinct combat arenas, namely Within Visual Range (WVR) and Beyond Visual Range (BVR). Early in military aviation, combat was fought purely in the WVR arena. However, the advent of modern detection equipment, radar, infrared, and modern weapons (rocketry technology) have led to current combat tactics dictating combat in the BVR arena. Yet TVC and especially PSM tend to be cited as more useful in the WVR arena. It is hence important to understand that although modern tactics do tend to lead to BVR combat, this project will consider both arenas equally. The reasons for choosing to consider TVC in both arenas are discussed below.

In a multi-bogey environment (which is representative of modern combat arenas) the amount of energy lost in high performance manoeuvring done to kill the first bogey leaves the advanced fighter in a very low energy state ready for the enemy’s wingman to kill the advanced fighter. When questioned about this, Smith stated that he agreed, but would rather have the thrust vectoring capability than not. It has also been argued that one could use thrust vectoring/post stall flight to keep the enemy predictable. Then the advanced aircraft’s wingman could easily take a shot at the conventional predictable aircraft. Having thrust vectoring does not mean that it has to be used to its full effect (and hence leave the fighter in a low energy state), but it could be used to gain small advantages to kill the first bogey. For example, consider the Mongoose manoeuvre demonstrated by the X-31 as the maximum effect that TVC can provide. This manoeuvre is where the aircraft slows to zero speed and pitches to very high angle of attack, pointing skyward. This is called the Mongoose because of the similarity with the animal sitting up on its hind legs before pouncing. If an aircraft is capable of this, then it is capable of far less and using only 1% of this ability is likely to lead to an advantage.
Guetter, Friehmelt, and Haiplik[2] give the main reasons behind the implicit assumption that within visual range/close in combat will still develop in combat in the future, as opposed to a BVR only restriction. These are,

1. The dynamic merge during prolonged engagements will eventually bring the aircraft close together,
2. Measures to enhance low observability may conceal aircraft until they are detected visually in a WVR regime,
3. Various optical and electronic counter measures (ECM) can limit sensors in their ability to detect aircraft BVR,
4. Limits on number and types of stores carried as well as failed missiles may drive aircraft into a WVR arena,
5. Special rules of Rules of Engagement (RoE) especially concerning target identification, i.e. Identification Friend or Foe (IFF), requirements can make an approach into WVR necessary,
6. And last but not least fighting outnumbered, surprised, or having to defend fixed assets on the ground may require WVR engagement.

**D.1 References**

E.1 Default Values

Default values for the variables under test were as follows. For \textit{as supplied} values, refer to Appendix B.

\begin{itemize}
  \item $C_L$ As supplied
  \item $C_D$ As supplied
  \item $C_M$ As supplied
  \item $dC_L/d\delta_e$ As supplied
  \item $dC_D/d\delta_e$ As supplied
  \item $dC_M/d\delta_e$ As supplied
  \item $W$ 159174 N
  \item $I_y$ 256604 kg.m²
  \item $S$ 37.1612 m²
  \item $l_v$ 5.2 m
  \item $h_v$ 0 m
  \item $T$ As supplied, N
  \item $n_z$ limit 9g
  \item $\delta_e$ maximum limit 10.5°
  \item $\delta_e$ minimum limit -24.0°
  \item $\delta_{ve}$ limit ±15°
  \item $\alpha$ ITR 20°
  \item $\alpha$ max 70°
  \item $K_a$ As calculated to minimise cost function.
  \item TVC On
\end{itemize}

E.2 Results

The following pages show the figures that contain the results of the sensitivity study. Each figure contains the normal four results parameter. The plots show the baseline aircraft values in grey, the absolute change in blue, and the percentage change in red. Also shown with the title of each plot are absolute mean and standard deviation values, as well as percentage mean values.
**Lift Coefficient + 10%**

**Time**
- mean = -7.78, sd = 1.41, mean % = -12.6

**Final SEP**
- mean = -36.5, sd = 177.3, mean % = 7.3

**Energy Change**
- mean = 130.4, sd = 153.9, mean % = -2.9

**Height Change**
- mean = -132.2, sd = 86.9, mean % = -5.8

Figure E-1: Lift Coefficient + 10%.
Figure E-2: Lift Coefficient - 10%.
Drag Coefficient + 10%

**Time**
- mean = 0.23, sd = 0.72, mean % = 1.6

**Final SEP**
- mean = 7.7, sd = 133.7, mean % = 1.5

**Energy Change**
- mean = -29.18, sd = 199.4, mean % = 6.3

**Height Change**
- mean = -95.9, sd = 92.2, mean % = -4.3

Figure E-3: Drag Coefficient + 10%.
**Drag Coefficient - 10%**

**Time**
mean = -0.15, sd = 0.66, mean % = -1.1

**Final SEP**
mean = 16.2, sd = 123.3, mean % = -3.2

**Energy Change**
mean = 321.0, sd = 208.9, mean % = -7.0

**Height Change**
mean = 112.0, sd = 108.6, mean % = 5.0

Figure E-4: Drag Coefficient - 10%.
Pitch Moment Coefficient + 10%

Figure E-5: Pitching Moment Coefficient + 10%.
Figure E-6: Pitching Moment Coefficient - 10%.
Figure E-7: Elevator Lift Derivative + 10%.

dCG/δδe + 10%
Figure E-9: Elevator Drag Derivative + 10%.
Figure E-10: Elevator Drag Derivative - 10%.
Figure E.11: Elevator Pitching Moment Derivative + 10%.
**dCm/dde - 10%**

**Time**
mean = 0.32, sd = 0.41, mean % = 2.3

**Final SEP**
mean = 12.1, sd = 58.1, mean % = 2.4

**Energy Change**
mean = -6.5, sd = 76.9, mean % = 0.2

**Height Change**
mean = 34.1, sd = 66.1, mean % = 1.5

Figure E-12: Elevator Pitching Moment Derivative - 10%.
Figure E-13: Pitching Inertia + 10%.
Pitch Inertia - 10%

Figure E-14: Pitching Inertia - 10%.
Wing Reference Area - 10%.

Figure E-16: Wing Reference Area - 10%.
TVC Longitudinal Moment Arm - 10%

**Time**
mean = 0.05, sd = 0.07, mean % = 0.4

**Final SEP**
mean = 0.1, sd = 47.1, mean % = 0.0

**Energy Change**
mean = -3.4, sd = 26.0, mean % = 0.1

**Height Change**
mean = 4.2, sd = 1.7, mean % = 0.2

Figure E-18: TVC Moment Arm - 10%.
Installed Thrust + 10%

Time
mean = 0.24, sd = 0.82, mean % = -1.7

Final SEP
mean = 9.4, sd = 81.7, mean % = 1.9

Energy Change
mean = 133.1, sd = 95.3, mean % = -2.9

Height Change
mean = 41.4, sd = 53.5, mean % = 1.8

Figure E-19: Thrust + 10%.
Normal Load Factor Limit + 10%

**Time**
mean = -0.40, sd = 0.56, mean % = 3.4

**Final SEP**
mean = -41.0, sd = 142.0, mean % = 8.2

**Energy Change**
mean = -159.3, sd = 158.2, mean % = 3.4

**Height Change**
mean = -113.8, sd = 122.4, mean % = -5.0

Figure E-21: Normal Load Factor + 10%.
Normal Load Factor Limit - 10%

Figure E-22: Normal Load Factor - 10%.
Maximum Elevator Deflection + 5 Degrees

**Time**
mean = 0.30, sd = 0.81, mean % = 2.1

**Final SEP**
mean = -1.6, sd = 51.6, mean % = 0.3

**Energy Change**
mean = 150.1, sd = 165.6, mean % = -2.9

**Height Change**
mean = 99.8, sd = 160.9, mean % = 4.0

Figure E-23: Maximum Elevator Deflection (10.5° baseline) + 5°.
Maximum Elevator Deflection - 5 Degrees

- **Time**
  - mean = -0.10, sd = 0.23, mean % = -0.7

- **Final SEP**
  - mean = -6.3, sd = 55.3, mean % = 1.3

- **Energy Change**
  - mean = -68.8, sd = 71.7, mean % = 1.4

- **Height Change**
  - mean = -36.2, sd = 51.7, mean % = -1.7

Figure E-24: Maximum Elevator Deflection (10.5° baseline) - 5°.
Minimum Elevator Deflection - 5 Degrees

Final SEP
mean = 4.2, std = 0.8, mean % = -0.8

Height Change
mean = 4.3, std = 0.3, mean % = 0.2

Time
mean = 0.13, std = 0.06, mean % = 1.1

Elevon Change
mean = -25.3, std = 10.3, mean % = 0.5

Figure E-26: Minimum Elevator Deflection (24.0° baseline) - 5°.
Thrust Vector Deflection - 5 Degrees

Figure E-38: TVC Deflection - 5°.
**ITR AoA + 5 Degrees**

**Time**
- Mean = 0.48, sd = 0.66, mean % = 3.4

**Final SEP**
- Mean = 64.7, sd = 100.3, mean % = 18.5

**Energy Change**
- Mean = 421.8, sd = 298.9, mean % = -9.3

**Height Change**
- Mean = 221.5, sd = 215.4, mean % = 10.0

Figure E-29: AoA for ITR + 5°.
Maximum AoA + 10 degrees

Time
mean = -0.47, sd = 0.58, mean % = -3.6

Final SEP
mean = -43.9, sd = 77.3, mean % = 6.7

Energy Change
mean = -66.1, sd = 55.8, mean % = 1.4

Height Change
mean = -64.6, sd = 69.7, mean % = -3.6

Figure E-31: Maximum AoA + 10°.
Maximum AoA - 10 degrees

**Time**

- mean = 0.80, sd = 0.56, mean % = 5.6

**Final SEP**

- mean = 92.6, sd = 156.4, mean % = -12.5

**Energy Change**

- mean = 23.4, sd = 47.0, mean % = -0.5

**Height Change**

- mean = 88.3, sd = 73.9, mean % = 3.9

Figure E-32: Maximum AoA - 10°.
Figure E-33: $K_a + 10\%$.
Figure E-34: $K_a$ - 10\%.
E.3 Discussion of Non-Sensitive Plots

Drag Coefficient. Decreasing the drag coefficient by 10% will have hardly any effect on the time parameter. The SEP maximum will be as high as 400 m/s in the high speed, low altitude region, but close to zero in the other regions. At very low altitude and transonic speeds, the energy change error is greatest at around 25% at one grid point (about 600 m difference). However, everywhere else the error is very acceptable (except again near the low speed area where the absolute value is close to zero). The height change is also considered to be insensitive to drag errors.

Increasing the drag shows identical trends.

From this analysis, it is recommended that drag errors of 10% are very acceptable with the overall results being insensitive to the errors (bar the SEP, again see section E.4).

Pitch Moment Coefficient. Decreasing the pitch moment means that there is effectively no error on the time (apart from a small error at highest altitude and fastest speed). The SEP has a largest error of around 200 m/s in the region where the absolute SEP is close to -1600 m/s. There is absolutely no problem with the energy change (percentage errors much less that 5%), and the height change is also considered entirely insensitive, except for the region at maximum altitude/velocity where an error of 5% can be seen.

Increasing the pitching moment error shows exactly the same trends.

For these reasons it is recommended that a ±10% error is acceptable and that it is even possible that the methodology would be insensitive to an error as great as ±20%. For this to be recommended, further sensitivity analysis would need to be carried out.

Lift Coefficient/Elevator Derivative. Changing dC_L/dδ_e has no effect on the time parameter. The SEP parameter is mostly insensitive to the error, but at the high
speed, low altitude region differences as great as 300m/s occur. There is not effect on energy change or height change.

It is recommended that a ±10% error is acceptable and that it is even possible that the methodology would be insensitive to an error as great as ±20%. For this to be recommended, further sensitivity analysis would need to be carried out.

**Drag Coefficient/Elevator Derivative.** Changing \( \frac{dC_D}{d\delta} \) has no effect on the time parameter. The SEP parameter is mostly insensitive to the error, but at the high speed, low altitude region differences as great as 100m/s occur. There is not effect on energy change or height change.

It is recommended that a ±10% error is acceptable and that it is even possible that the methodology would be insensitive to an error as great as ±20%. For this to be recommended, further sensitivity analysis would need to be carried out.

**Pitching Moment Coefficient/Elevator Derivative.** Changing \( \frac{dC_M}{d\delta} \) has no effect on the time parameter, except at highest altitude and maximum speed where an error of 5% is seen. The SEP parameter is mostly insensitive to the error, but at the high speed, low altitude region differences as great as 300m/s occur. There is not effect on energy change or height change.

It is recommended that a ±10% error is acceptable.

**Wing Reference Area.** The effect of changing the wing reference area is that the time parameter has an error of 15%-20% near the manoeuvre stall boundary, but tapering down to zero error at the high speed, low altitude area. The average error is around 8%. The SEP parameter is most sensitive again in the high speed low altitude area, with errors around 500m/s. The energy change has an error of 5%-10% around the low speed transonic area, but else where, the error is insignificant. The height change has a maximum error of 10% near the manoeuvre stall boundary, dropping down to zero with higher speed and lower altitude. The average is around 5%.
It is recommended that the wing reference tolerance is left at ±10%, and that the areas around the manoeuvre stall boundary be investigated, to determine why they are more sensitive.

**Pitch Inertia.** Changing the pitch inertia has no significant effect on the time, energy change or height change parameters. The SEP parameter has a maximum error of 200m/s in the high speed, no altitude region.

It is recommended that the pitch inertia be calculated to within ±10%, and that this be increased to ±20%, once further sensitivity analysis is completed.

**TVC Longitudinal Moment Arm.** Changing this by ±10% has little effect on the time parameter, SEP, energy change or height change. The average errors are well below ±2%, and because of this it is recommended that the tolerance be increased to ±20%, after suitable sensitivity analysis is completed.

**Installed Thrust.** Changing the installed thrust through the flight envelope by 10% gives a maximum error of 5% near the manoeuvre stall boundary for the time parameter. The average however is much less, generally around zero. The SEP parameter again suffers in the high speed, low altitude region with errors as great as 300m/s. The energy change has an error at the low altitude transonic region of 10%-15%, but generally around zero. There is no significant effect on the height change.

It is recommended that the tolerance for the installed thrust be left at ±10%.

**Normal Load Factor Limit.** Changing the load factor limit will mean that there is little to no error near the manoeuvre stall boundary. However, the error creeps up to around 10% at the highest speed, low altitude region. The SEP parameter also shows no effect near the manoeuvre stall boundary, but in the low altitude, high speed region shows huge errors, as much as 600m/s. The energy change also shows no significant errors near the manoeuvre stall boundary, but increasing in the low altitude transonic region to 10%-15%. The height change also shows no error near the manoeuvre stall boundary, but as much as 20% in the low altitude transonic region.
It is recommended that the tolerance for the normal load factor limit be reduced to 5%. One explanation of why the load factor is sensitive in the low altitude transonic region is that the load factor limit will only affect the initial and early portions of a manoeuvre, and only when the initial conditions start it at above the corner velocity (so that the aircraft is load factor limited at the outset). For this reason, no effect at all is seen for initial conditions below the corner velocity, but the higher the initial speed above the corner velocity, the greater the effect. This trend is exhibited by the time, energy change and height change parameters. It is not shown by the SEP parameter, but this is not expected since the SEP is simply calculated at the end conditions and is only related to the load factor in the continuously load factor limited area.

Maximum Elevator Deflection (Induces Pitch Down). Reducing the maximum elevator deflection by 5° (so that the deflection is less than base line) has little effect on the time parameter, except that at the highest altitude and fastest speeds, there is an error of up to 15%. Everywhere else in the flight envelope, the error is insignificant. The SEP parameter again shows errors of up to 100m/s in the high speed, low altitude region. There is no significant error for the energy change, with a maximum error of 5%. The height change parameter shows a similar trend to the time parameter, with little or no error generally, except for the maximum altitude/speed region where errors are as high as 20%.

Increasing the maximum elevator deflection (that is allowing more control power) has no significant effect on the time parameter. The SEP shows errors as high as 200m/s in the usual region, but generally zero (as usual). The energy change is mostly zero, except in the low altitude transonic region where the error is as high as 10%. There is no effect on the height change where the average error is about 5%.

Based on this discussion it is recommended that the tolerance on the maximum elevator deflection should be kept at ±5°. However, it can also be stated that it is better to over estimate than to under estimate (more control power is desirable), except in the case where the real aircraft suffers from a lack of control power.
Minimum Elevator Deflection (Induces Pitch Up). Increasing the minimum elevator deflection by 5° has little effect on the time parameter. The SEP has a maximum error of 100m/s at high speed, and low altitude, with the average error being much smaller. There is no significant error in the energy change or in the height change.

Decreasing the minimum elevator deflection by 5° (reducing the amount of deflection) gives errors as high as 20% in the transonic region for all altitudes, but little effect elsewhere on the flight envelope. The SEP shows the usual trends with a maximum error of 200m/s. The energy change again shows no significant errors except in the low altitude, transonic region where the maximum error is around 20%. There is no significant effect on the height change.

It is recommended that the tolerances not be changed (±5°). However, it is also recommended that the errors can be reduced if the deflections are chosen to be larger so that more control power is available, unless the real aircraft is known to suffer from low control power, in which case doing so will make the modelling unrepresentative.

Thrust Vector Deflections. Changing the thrust vector deflections by ±5° has little or no effect on the time or height changes. The SEP has the usual trends with maximum errors around 200m/s. The energy change shows almost no errors, except at the low altitude transonic speeds with errors approaching 5%.

It is recommended that the tolerances be maintained, or increased to ±10°, once suitable sensitivity analysis is completed. It is also recommended that the deflection be over estimated rather than under, again to ensure that enough control power exists.

Manoeuvre Angle of Attack (AoA for ITR). If this angle is altered by ±5°, then no significant errors are shown in the time parameter. The SEP shows large errors throughout the flight envelope, with maximum errors of around 200m/s. The energy change will have errors around 20% near the manoeuvre stall boundary, reducing to zero as speed is increased and altitude is decreased. The same is true of the height change.
It can be said that these errors displayed are not too bad, since the AoA for ITR is so critical (it will change the amount of lift and drag for the entire manoeuvre!). This variable is also of more importance to the tacticians. It is for them to decide what a suitable AoA would be. This can be found automatically by setting the AoA as high as possible to still allow suitable controllability. For these reasons it is recommended that the AoA for ITR be set automatically using a rigid algorithm, independent of aircraft. More research is required in this area, however it does become a control issue — what is a suitable margin? After some consideration, it was decided that the value to use for this parameter was up to the user to decide since no standard form of calculating it could be developed.

Maximum Angle of Attack. Changing the maximum AoA by 10° will have very little effect on the time parameter, although a maximum error of 10% is seen at high altitude. The SEP shows the usual trends and maximum errors in the region of 300m/s. The energy change also shows the usual error in the low altitude transonic region of around 10%. There is little error in the height change, typically less than 5%.

It is recommended that the tolerance on this variable be maintained.

AoA Outer Loop Gain. Changing the gain of the controller by 10% has very little effect on the overall sensitivity of the manoeuvre. Studying the effect of this parameter has little meaning since it is calculated using a rigid method. The method gives an exact value of controller gain to use, and so there is no possibility of introducing an error. It is however still of interest to determine the overall sensitivity of the metric to this input.

**E.4 Discussion of the SEP Parameter**

In chapter 5 where the metric was developed it was discussed that the final SEP was extremely important for post stall aircraft, since it describes what is about to happen to the aircraft. For this reason, it was not possible to drop the final SEP as a result parameter, however some consideration is required because of its apparent sensitivity. The reason that the SEP parameter is so sensitive is that it is dominated by the final AoA of the aircraft. At the final segment of the manoeuvre, the aircraft is pitching up
very quickly. Since the simulation takes place over discrete time steps, the AoA at the point at which the simulation finishes may not be exactly as required to point at the target. The final AoA may be somewhere between the exact required final AoA or up to one time step above it. This small error in time, combined with the high pitch rate means that the final AoA becomes sensitive (can be up to several degrees out), leading to the problem in final SEP discussed above. Furthermore, when examining the plots, it becomes clear that the final SEP is even more sensitive in the high speed, low altitude region, where the manoeuvres are continuously load factor limited, throughout the simulation. In this case, because of the load factor limiter, the final AoA can also vary by several degrees from what it should be to maintain the aircraft within the load factor limit. This problem was discussed in chapter 5 and chapter 6.

To get around this problem, the SEP needs to be made less sensitive, or rather the AoA needs to be made less sensitive. To do this, it is could be proposed that the SEP be calculated at a fixed AoA using the final velocity and altitude of the aircraft. For non-load-factor limited final conditions, the AoA would be chosen to be some angle slightly less than the maximum, or indeed could be chosen to be the maximum. An angle slightly less than the maximum might be chosen to allow for the fact that no pilot is being modelled, and so the aircraft does not commence the nose pointing until the target AoA is exactly the maximum. By the time the aircraft pitches up to high AoA, the required target AoA will be slightly less than maximum since the aircraft will have turned by some amount as it travels around its flight path. It does not really become significant what angle is chosen as long as it is close to the maximum and so long as the same angle is used for all grid points. For the regions in which the aircraft is load factor limited at its final conditions, the exact AoA to give the maximum load factor should be found. In doing so, the SEP should become more uniform and less sensitive throughout the flight envelope, solving the problems highlighted by the sensitivity analysis. This is an ideal view however. In practice there are two problems. The first is that it is actually very difficult to calculate the required AoA such that the aircraft is exactly load factor limited, since the load factor is also dependant upon the elevator and thrust vector control deflections. Solving the AoA, and elevator/TVC deflections did prove to be a more difficult task. The second problem was for conditions where the aircraft was load factor limited until some short period before the end of the manoeuvre. In these cases, the aircraft would begin to
point the nose before the end of the manoeuvre, but would not have enough time to get to the maximum AoA. In this case, it is impossible to calculate the final AoA and so there was no way of finding the final SEP accurately.

Due to these problems it is was not possible to rectify the sensitivity problem encountered with the final SEP. It is however recommended that the load factor limiter be redesigned so that it is less sensitive, and that it tracks the required AoA better. Doing so would reduce the largest sensitivity encountered in the continuously load factor limited area.