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Engineering Aerodynamics of Horizontal Axis Wind Turbines (HAWTs)

by

James R. Shawler

Doctoral Thesis

Submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of Loughborough University

31.03.2004

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Abstract

This thesis comprises two main original contributions. The first concerns the aeroelastic modelling of a large-scale prototype wind turbine undertaken specifically to explain experimentally observed mechanical instabilities. The second explores the aerodynamic aspect of turbine modelling in greater detail since this is the main identified technical challenge, this process makes use of detailed large-scale wind tunnel test data from NREL for model validation purposes.

The MS4 prototype wind turbine was modelled using ADAMS/WT software, the aerodynamic model was provided by the NREL AERODYN subroutines. The drivetrain instability of the machine of 0.75Hz was reproduced by the computer simulation. The causes of the instability were found to be negative aerodynamic damping, complex blade bending modes caused by the blade design and rapid yawing and tilting inducing Coriolis forces in the rotor structure.

Accurate analysis of the aerodynamic forces acting on the MS4 was not possible because of the lack of detailed data available and the complicated aeroelastic response of its flexible structure.

Theoretical comparisons with the results from the NREL wind tunnel tests were made using several different engineering aerodynamic models (including those used with AERODYN). It was found that blade element aerofoil data had a controlling influence on the blade forces predicted through theory. The effect of inflow models was found to be marginal at lower tip speed ratios and to decrease with decreasing
tip speed ratio.

Experimental blade forces at low tip speed ratios were found to be defined by gross 3 dimensional effects and the use of 2 dimensional aerofoil data led to inaccurate prediction of blade forces. The use of a stall delay model improved results but was not convincing.

Yawed flow predictions were again controlled by the blade element aerofoil data used, use of a stall delay model again improved results in a steady state fashion. A dynamic stall model also improved results but the phasing of results towards the blade root was questionable and may be caused by unsuitable time constants or the influence of the delayed stall effect.
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List of significant symbols used: General.

\( a \): Axial induction factor.
\( a_{av} \): Axial induction factor, averaged over the rotor.
\( a_{t} \): Tangential induction factor.
\( A \): Rotor disc swept area. \((m^2)\)
\( c \): Blade section chord length. \((m)\)
\( C, D \): Arbitrary constants associated with associated Legendre polynomials of the first and second kinds.
\( C_D \): Aerofoil drag force coefficient.
\( C_L \): Aerofoil lift force coefficient.
\( C_{Me}, C_{Ms} \): Co-sinusoidal and sinusoidal rotor moment coefficients respectively.
\( C_n \): Aerofoil normal force coefficient.
\( C_t \): Aerofoil tangential force coefficient.
\( C_T \): Rotor thrust force coefficient.
\( C_P \): Rotor power coefficient.
\( \delta Nf \): Element blade force, normal to rotor plane.
\( dQ \): Elemental (annular) rotor torque. \((N/m)\)
\( \delta r \): Section of blade span, length of blade element. \((m)\)
\( dT \): Elemental (annular) rotor thrust force. \((N)\)
\( E \): Young's modulus of elasticity.
\( F \): Prandtl's tip loss factor.
\( G \): Torsional modulus of elasticity.
\( I \): Second moment of area. \((m^4)\)
\( j \) (subscript): Polynomial number.
\( J \): Polar moment of area. \((m^4)\)
\( L \): Lift force. \((N)\), Length \((m)\).
\( [L] \): L matrix. \((\text{Pitt and Peters theory})\)
\( [L^c], [L^s] \): Co-sinusoidal and sinusoidal L matrices respectively. \((\text{GDW theory})\)
\( m \) (superscript): Harmonic number.
\( M \): Moment \((N/m)\)
\( [M] \): Added mass matrix
\( n \) (subscript): Polynomial number.
\( N \): Number of rotor blades.
\( p \): Air pressure. \((N/m)\)
\( P_n^m \): associated Legendre polynomial of the first kind.
$P_n^m$: Normalised associated Legendre polynomial of the first kind. $(-1)^m \frac{P_n^m(\omega)}{p_n}$
$Q_n^m$: Associated Legendre polynomial of the second kind.

$\tau$: Local blade radius. (m)

$\tau^*$: Harmonic number.

$\tau^*$: Non-dimensionalised radius, $\left(\frac{\tau}{R}\right)$

$R$: Rotor tip radius. (m)

$t$: Time (seconds)

$i$: Non-dimensionalised time, $(\Omega t)$, (radians)

$u$: Axial induced velocity. (m/s)

$u_{av}$: Axial induced velocity, averaged over the rotor. (m/s)

$u_z$, $u_x$, $u_y$: Axial induced velocity components. (m/s)

$u_i$, $u_j$, $u_k$: Induced velocities in the x,y,z cartesian directions respectively. (m/s)

$U^*$: Friction velocity.

$V_D$: Air velocity at the rotor disc (axial flow in momentum theory). (m/s)

$V_T$: Average resultant velocity at the rotor disc. (m/s)

$V_W$: Air velocity in the far rotor wake (axial flow in momentum theory). (m/s)

$V$: Free stream air velocity. (m/s)

$W$: Resultant flow velocity at a blade element. (m/s)

$\alpha$: Blade element angle of attack in blade element theory, (degrees/radians)

or rotor disc angle of attack in inflow theory. (90 degrees - $\gamma$)

$\alpha_j^r, \beta_j^r$: Non-dimensionalised axial induced velocity component coefficients.

$\gamma$: Rotor yaw angle. (90 degrees - $\alpha$)

$\theta$: Local blade twist angle (degrees/radians)

$\lambda$: Tip speed ratio, $(\frac{\Omega R}{V})$, or axial non-dimensionalised induced velocity, (nominally $\frac{u}{\Omega R}$)

$\lambda_{av}$: Axial non-dimensionalised induced velocity averaged over the rotor, $u_{av}$ calculated from momentum theory.

$\lambda_0, \lambda_c, \lambda_s$: Nondimensionalised axial induced velocity components. $(\frac{u_0}{\Omega R}, \frac{u_c}{\Omega R}, \frac{u_s}{\Omega R})$

$\mu$: Rotor advance ratio $(\frac{v_{\cos(\alpha)}}{\Omega R})$

$\nu', \nu'^r, \psi$: Elliptical co-ordinate system.

$\eta$: Rotor inflow ratio $(\frac{v_{\sin(\alpha)}}{\Omega R})$

$\psi$: Blade azimuth angle. (degrees)

$\psi_{wind}$: Wind based rotor blade azimuth angle. (degrees)

$\rho$: Air density. (Kg/m$^3$)
\[(\rho_m^n)^2 : \text{integral (0 to 1) of } (P_m^n(v))^2\]

\[\tau_{nc}^n, \tau_{ns}^n : \text{Co-sinusoidal and sinusoidal non-dimensionalised pressure coefficients respectively.} \]

\[\phi_j^n : \text{Radial shape function.} \]

\[\varphi : \text{Resultant flow angle at a blade element. (degrees/radians)} \]

\[\kappa : \text{Wake skew angle. (degrees)} \]

\[\Gamma : \text{Bound circulation around an aerofoil.} \]

\[\Phi : \text{Acceleration potential function, non-dimensionalised pressure.} \]

\[\Omega : \text{Rotor rotational speed (rads/s)} \]

**Beddoes/Leishman Dynamic Stall model.**

\[\alpha_0 : \text{Zero lift angle of attack.} \]

\[\alpha_E : \text{Effective angle of attack.} \]

\[\alpha_E^{press} : \text{Effective angle of attack relating to leading edge pressure time lag.} \]

\[C_{Na} : \text{Aerofoil normal force curve slope.} \]

\[C_{Nv} : \text{Aerofoil normal force coefficient equating to critical leading edge pressure for flow separation.} \]

\[C_{N}^C : \text{Aerofoil normal force coefficient, circulatory component.} \]

\[C_{N}^{IL} : \text{Aerofoil normal force coefficient, impulse loading component.} \]

\[C_{N}^{pot} : \text{Aerofoil normal force coefficient, total from potential flow. } (C_N^C + C_N^{IL}) \]

\[C_{T}^{press} : \text{Aerofoil tangential force coefficient, total from potential flow.} \]

\[C_{N}^{press} : \text{Aerofoil normal force coefficient, relating to leading edge pressure time lag.} \]

\[C_{N}^{K} : \text{Aerofoil normal force coefficient, Kirchoff flow theory.} \]

\[C_{T}^{K} : \text{Aerofoil tangential force coefficient, Kirchoff flow theory.} \]

\[C_{N}^{sep} : \text{Aerofoil normal force coefficient, total from separated flow.} \]

\[C_{T}^{sep} : \text{Aerofoil tangential force coefficient, total from separated flow.} \]

\[C_v : \text{vortex component.} \]

\[C_{N}^{K} : \text{Aerofoil normal force coefficient, vortex lift component.} \]

\[C_{N} : \text{Total aerofoil normal force coefficient.} \]

\[C_{T} : \text{Total aerofoil tangential force coefficient.} \]

\[D, D_I, D^{press}, X, Y : \text{Exponential lift deficiency functions.} \]

\[f, f', f'' : \text{Aerofoil flow effective separation points.} \]
Chapter 1

Introduction

In the last decade the wind turbine industry has developed into a global market worth hundreds of millions of dollars/euros a year. It is strange then that the basic flow physics of these machines in all their operating conditions is still relatively poorly understood.

Currently, commercial turbine design and development consists of a series of small improvements to existing designs resulting in a slow evolution in terms of size, efficiency and robustness. The mainstay of this process is empirical field test experience. The theoretical computer (aeroelastic) codes used in the design process being to a large extent tuned to field measurements (by adjusting blade element aerofoil data used as input to the codes (Hansen, Ref 60)). This approach is taken because the engineering aerodynamic theories as currently formulated for wind turbine rotors have major shortcomings in certain operating conditions, especially when rotors are stalled, quoting A.D. Garrad of GarradHassan from 1999 (Ref 56) "Up to the present it does seem that the aerodynamicists have failed to provide an acceptable prediction of stall for wind turbines and, in failing to do so are severely hindering the technology development."

In this thesis an engineering aerodynamic theory is assumed to mean a theory based on classical rotor theories such as momentum theory, circular wing theory,
actuator disc theory or fixed vortex wake theory. Models based on Computational Fluid Dynamics and on Free Vortex and Prescribed Vortex wake models are currently not used for wind turbine certification work and although they will become useful design tools they are currently considered by the present author to be too computationally demanding for practical design work.

Typically, for engineering purposes a wind turbine aeroelastic code will have three main components; a dynamic structural model, a model for the dynamic effects of the flow through the rotor (dynamic inflow) and an unsteady aerodynamic model (including dynamic stall) at the blade element level. These separate components are typically coupled using Blade Element theory for the calculation of blade loads. These codes are necessarily simple and fast to be of use in design/certification work. However they still need to be reasonably accurate when applied to new or innovative designs for which there is little or no operational experience. This is open to question.

An attempt to build a structurally advanced wind turbine (complete with a new blade design) was carried out by the British company WEG (Wind Energy Group), this prototype (the MS4, built between 1997-98, Ref 52) proved to have an instability resulting in large amplitude oscillations of the machines drive train. An investigation into the performance of the machine using ADAMS mechanical dynamics software was carried out to try to identify the source of this instability. The instability was found to be largely structural. However, because of the limited nature of the field measurements collected from the machine the aerodynamic conclusions that can be drawn from this study are limited.

This highlights a major problem with improving the engineering aerodynamic theories used for the design of wind turbine rotors, which is a lack of accurate experimental data with which to compare theoretical results. Until recently all data sets collected on turbines of commercial size have been obtained in the field
where the turbine is subjected to the effects of atmospheric winds ie: turbulence, gusts, direction changes, wind shear etc. To a large extent these natural phenomena obscure the flow physics of the energy extracting process of the wind turbine rotor.

During 2000, the US National Renewable Energy Laboratory carried out wind tunnel testing on a full scale wind turbine, (Rotor diameter of 10m). The data acquisition on the turbine for the tests was highly detailed and resulted in Gigabytes of accurate aerodynamic information of a full scale turbine operating in a variety of carefully controlled flow conditions (Ref 65). NREL then issued a worldwide invitation to wind turbine research groups to model the experimental turbine in a variety of selected flow conditions and to have their theoretical results compared to the experimental results with no prior knowledge of the experimental results available for the simulation work. The present author took part in this comparison using NREL's YAWDYN wind turbine code.

The results of this "blind" comparison showed that almost all the engineering aerodynamic models produced serious inaccuracies when predicting blade loads and power output of the rotor. These discrepancies are very probably due to the unavailability of empirical data for model tuning.

A thorough investigation of the current engineering aerodynamic models is required to assess why there are such large disparities between the results from the differing theories and the experimental results. Only after an assessment is made of the assumptions and limitations of the current models when compared to the experimental results/flow physics is it possible to find a way to try and resolve some of the problems.

In short, a better understanding of the flow physics of rotating wind turbine blades will provide more reliable aeroelastic codes.

The number of engineering aerodynamic models and their different components
which can be considered in research of this kind is necessarily limited by the time available, however it is hoped that the results from those models which are considered will provide conclusions which are more generally applicable. The principal aim is to give perspective on the influence of each type of model component rather than a definitive judgement of the accuracy of any particular engineering model. The engineering models which are used throughout this thesis (including the ADAMS modelling of the MS4) are primarily those which have been and are used with the US National Renewable Energy Laboratories (NREL) wind turbine design codes. These include the Blade Element/Momentum (BEM) theory, the Pitt and Peters dynamic inflow theory, the Generalised Dynamic Wake theory (GDW) and the Beddoes/Leishman dynamic stall theory.

In addition to these theories certain other classical rotor theories are considered, these include a fixed skewed vortex wake model for yawed rotors based on the work of Coleman et al (Ref 10) and Glauert’s circular wing theory for yawed rotors, (Ref 5). Also considered is a theory for delayed stall on wind turbine blades by Snel, (Ref 40).

However the reader should be aware of other engineering aerodynamic models which have not been considered here due to lack of time and direct relevance to the NREL codes. These include dynamic inflow models based on the non-linear acceleration potential work of Van Holten (helicopters) (Ref 20) and Van Bussel (wind turbines) (Ref 46) and various dynamic inflow models including models based on differential forms of classical theories developed in a European project (Ref 43). There are also other dynamic stall models which haven’t been considered here such as the Gormont (Ref 18) and ONERA (Ref 25) models.
Chapter 2

Steady State HAWT

Aerodynamics

2.1 Theories for axial-symmetric flow

2.1.1 Simple Momentum Theory

One of the first interpretations of simple momentum theory for wind turbines was presented by Hoff (Ref 2). It is based on the assumption that the turbine rotor is extracting energy from the air by slowing it down and causing a pressure drop in the air as it flows through the rotor, i.e.: the air slows down as it approaches the rotor due to a rising pressure gradient (this also causes the flow to expand radially), as the flow passes through the rotor there is an instantaneous pressure drop and then there is another rising pressure gradient as the pressure begins to rise back to its original level, this of course continues to slow the air down and the flow continues to expand radially until the original (atmospheric) pressure upstream is again obtained. The manner of energy extraction for this simple theory is unimportant and all velocity changes and associated forces considered are purely axial and normal to the plane of the rotor.
Figure 2.1: Side view of flow expansion through a wind turbine rotor according to simple momentum theory.

The derivation of simple momentum theory has been reproduced many times since and can be found in many text books on wind turbines (Refs 28, 42, 62, 64). Referring to figure 2.1, if \( V \) is the free-stream velocity, \( V_D \) is the velocity at the rotor and \( V_W \) is the velocity in the far wake (all three defined as being positive in the downwind direction), \( A \) is the rotor area and \( \rho \) is the fluid density. Also, given that the axial induction factor, \( a \), by definition has the value of \( 1 - \frac{V}{V_D} \), then the following important results can be presented:

\[
V_D = \frac{1}{2}(V + V_W) \quad (2.1)
\]

\[
Power = 2\rho V^3 a(1 - a)^2 A \quad (2.2)
\]

\[
Thrust = 2\rho V^2 a(1 - a) A \quad (2.3)
\]

Power and Thrust can be non-dimensionalised to produce the following coefficients:
\[ CP = \frac{Power}{\frac{1}{2} \rho V^2 A} = 4a(1 - a)^2 \quad (2.4) \]
\[ CT = \frac{Thrust}{\frac{1}{2} \rho V^2 A} = 4a(1 - a) \quad (2.5) \]

By differentiation of (2.4) it can be shown that a maximum value of \( CP \) exists for \( a = \frac{1}{3} \), equal to \( \frac{16}{27} \), called the Betz limit. \( CT \) also has a maximum value of 1 corresponding to \( a = \frac{1}{2} \); however by experimentation it can be shown that the assumptions of this theory break down somewhere between \( a = \frac{1}{3} \) and \( a = \frac{1}{2} \); this is caused by a transition of the flow into a turbulent wake state. By definition, when the value of \( a \) reaches one half, the far wake velocity drops to 0, (or the area of the assumed wake becomes infinite). This is physically nonsense and instead large eddies are entrained into the rotor wake from the surrounding flow. For a wind turbine this occurs at low wind speeds (or high tip speed ratios, where the tip speed ratio is given by, \( \lambda = \frac{\Omega R}{V} \)). An empirical correction for the thrust coefficient will be presented in a later section for values of \( a \) greater than \( \frac{1}{3} \).

2.1.2 Blade Element Theory

Blade Element theory is two-dimensional and is concerned with calculating the forces on a spanwise section of blade or blade element (\( dr \)) from a knowledge of the lift and drag coefficients of the aerofoil concerned and the relative flow angle and velocity of the air approaching the blade element. The forces on the whole blade can then be calculated from the contributions of each blade element. Figure 2.2 shows an axial (downwind or upwind) view of a 2 bladed rotor with the definitions of variables used in blade element theory, in addition the flow through the rotor \( V \), needs to be considered along with the twist of the blade \( \theta \), which is also described for each blade element in a similar manner to the radius.
Figure 2.2: Axial view of 2 bladed rotor showing definitions of variables used in blade element theory.

If the influence of the energy extracting process on the fluid velocities is initially ignored for the sake of simplicity, then the airflow vector relative to the blade element \( W \), can be found from the free-stream velocity \( V \) and the rotational speed of the element \( \Omega r \), equation (2.6). Also the angle that this vector makes with the plane of rotation of the rotor is given by trigonometry, equation (2.7). If the pitch of the blade element relative to the plane of rotation is \( \theta \), then the angle of attack \( \alpha \), seen by the blade element is given by equation (2.8).

\[
W = \sqrt{V^2 + \Omega^2 r^2} \quad (2.6)
\]
\[
\tan \varphi = \frac{V}{\Omega r} \quad (2.7)
\]
\[
\alpha = \varphi - \theta \quad (2.8)
\]

The lift and drag forces on an aerofoil are dependent on \( \alpha \) and are expressed in two components, the lift component is normal to the relative airflow seen by the blade element and the drag component is parallel. So by resolving these components
relative to the plane of rotation expressions can be obtained for the torque and thrust of a given blade element. Note the use of force coefficients for determining both the lift and drag force on the blade element, these coefficients have to be determined empirically for the aerofoil section in question.

\[
\frac{dT}{dr} = \frac{1}{2} \rho N W^2 c (C_L \cos \varphi + C_D \sin \varphi) \tag{2.9}
\]

\[
\frac{dQ}{dr} = \frac{1}{2} \rho N W^2 c r (C_L \sin \varphi - C_D \cos \varphi) \tag{2.10}
\]

\(N\) is the number of blades in the rotor, \(c\) is the element chord length, \(r\) is the radius of the element and \(C_L\) and \(C_D\) are the lift and drag coefficients respectively. These two expressions enable the forces on the blades and hence on the entire rotor to be determined, however there remains the problem of how to accurately determine the inflow velocity \(W\), to each blade element. In the above analysis \(W\) is defined by the freestream velocity and the rotational velocity of the blade element. However the momentum theory states that the velocity through the disc is less than the freestream velocity, also the wake will contain rotating air with an angular velocity opposite to the rotational speed of the blades due to the acquisition of angular momentum by the rotor. So the definition of \(W\) above is clearly inaccurate. Lifting line theory is required to define the inflow velocity \(W\) for each blade element.

### 2.1.3 Lifting Line Theory

It was not until Prandtl's lifting line theory provided a means of expressing the lift of an aerofoil as a circulation of fluid around the aerofoil that the correct expressions to define the relative velocity \(W\) could be obtained. This theory defines the lift force on a blade element as:
Figure 2.3: Shed helical vortices of uniform strength from one blade of a two bladed rotor.

\[ L = \rho V \Gamma \]  

(2.11)

Where \( V \) is the fluid velocity approaching the aerofoil, in this case \( W \). \( \Gamma \) is the bound circulation around the aerofoil operating in the Kutta condition. An assumption is made that the circulation is constant along the blade and that the only circulation (or vorticity) which can be shed into the wake comes from either the tip or root of the blade. Figure 2.3 shows the shed vorticity from one rotor blade. Since a vortex has to form a closed loop the circulation shed from each tip and root must be of value \( \Gamma \) and forms a free vortex in the wake of the rotor. Since the freestream air is moving through the rotor, the resulting shed vorticity from the blade is carried downstream in the form of helices. The shed vorticity from the roots becomes bound into an axial vortex of strength \( N\Gamma \), where \( N \) is
the number of blades. The majority of rotation in the flow caused by this axial vortex is opposite to the direction of rotation of the blades and it is this which makes up the rotating wake of a wind turbine.

If the rotor contains a large enough number of blades then the shed helical vortices from the blade tips can be assumed to approximate a cylindrical vortex sheet, this assumption also requires that the expansion of the wake is ignored. This vortex sheet can then be resolved into vortices that have axes that are parallel to the plane of rotation of the rotor (circumferential about the wake), and axes which are normal to the plane of rotation (axial and lying on the streamtube boundary). The axial vortices on the streamtube boundary contribute to the rotation of the wake in a direction opposing the direction of the blades, this is in addition to the contribution from the axial vortex in the centre of the wake. The vortices circumferential about the wake are responsible for the slowing down of the axial velocity of the wake, ie: from \( V \) to \( V_D \) to \( V_w \).

The cylindrical sheet representation of the outer part of the wake vorticity makes possible the intuitive observation that the induced axial velocity at the rotor plane has half the value of the induced axial velocity in the far wake. The reason for this is the assumption that the induced axial velocities are caused only by the vortex cylinder of the wake. This has twice the strength in the far wake than at the rotor plane because in the far wake the vortex cylinder extends indefinitely upstream and downstream, whereas at the rotor plane it only extends downstream. Hence the resulting relationship is the same as the one from the simple momentum theory.

\[
V_D = \frac{1}{2}(V + V_w) \tag{2.12}
\]
Figure 2.4: Induced rotational velocities in the rotor plane.

For axial velocities the effect of the bound vorticity on the blades can be considered negligible in the far wake and cancelled out at the rotor plane by the fact that these bound vortices are centred at the rotor plane. For rotational flow in the wake the bound vorticity on the blades ($\Gamma$) is very important. From figure 2.4 it can be seen that due to this bound vorticity there must exist a rotational component of the flow (in the direction of blade rotation) immediately upstream of the blade ($-\kappa$), and an equal and opposite rotational component immediately downstream of the blade ($+\kappa$). However it is assumed that there is no fluid rotation upstream of the rotor, so there must be an induced rotational velocity at the rotor plane which cancels the rotation immediately upstream of the rotor, ie: $\kappa = \omega'$. If this rotation is $\omega'$ then it follows that the resulting rotational flow immediately behind the rotor disc is $2\omega'$.

Given:

$$2\omega' = 2\alpha' \Omega \quad (2.13)$$

and at the plane of rotation:
\[ W = \frac{1}{2}(W_1 + W_2) \]  

(2.14)

then the new definition of the relative velocity approaching a blade element becomes:

\[ W = \sqrt{V^2(1 - a)^2 + \Omega^2r^2(1 + a')^2} \]  

(2.15)

This theory also means that the appropriate non-dimensionalised lift and drag data to be used in the blade element equations should be that calculated for aerofoil sections of infinite aspect ratio ie: 2D data, since the effect of the aspect ratio of the blades is taken into account in the shedding of vorticity and hence the induced velocities mentioned above.

Another question relating to this method is the assumption of treating the blade elements separately for the application of theory, this was verified experimentally by Lock (Ref 3) by a series of tests carried out on propellers with varying twist distributions to see if the thrust produced by a certain blade element with a constant angle of pitch and speed remained constant while the pitch of neighbouring elements changed. It has also recently been checked numerically as will be mentioned in a later section.

2.1.4 Iterative BEM method

Glauert (Ref 8) presents blade element/momentum (BEM) theory for a propeller and outlines its application for performance prediction of wind turbines. However it wasn't until 1974 that Wilson and Lissaman (Ref 19) presented the classical iterative method for the calculation of flow conditions at a blade element
and the determining of blade loads from this result using suitable aerofoil data (lift/drag/pitching moment coefficients). This has been the favoured method for the performance prediction of wind turbines due to its relative computational simplicity.

The method is based on equating expressions for torque and thrust from the blade element theory with the equivalent expressions from simple momentum theory. For the thrust produced on a rotor, equation (2.3) can be rewritten as an elemental (annular) expression and equated with equation (2.9). Also taking into account the revised definition of $W$:

Rewriting (2.3):

$$
\frac{dT}{dr} = 2\rho V^2 a(1 - a)(2\pi r) \quad (2.16)
$$

This can then be equated to (2.9):

$$
a(1 - a) = \frac{NW^2 c(C_L \cos \varphi + C_D \sin \varphi)}{8V^2 \pi r} \quad (2.17)
$$

The same technique can be applied to calculate the torque produced on a blade element. From lifting line theory it can be shown that the angular momentum supplied to the wake (in one annulus) from the action of the blade elements (in that annulus) is equal to:

$$
\frac{dQ}{dr} = 2\rho \Omega r (1 - a) V(2\pi r) r \quad (2.18)
$$

This can then be equated to (2.10):
\[ a'(1 - a) = \frac{NW^2 c (C_L \sin \phi - C_D \cos \phi)}{8 \sqrt[3]{\Omega}} \] (2.19)

For steady state axial flow conditions, equations (2.17) and (2.19) can be used to find the thrust and torque on a wind turbine rotor subject to the assumptions inherent in the lifting line theory used to formulate the equations.

The solution of the equations is iterative, by guessing a value for \( a \) and \( a' \), \( W \) and \( \alpha \) can be found and hence \( C_L \) and \( C_D \) can be defined for the element concerned, then new values of \( a \) and \( a' \) can be calculated and the process repeated until convergence.

2.1.5 Glauert's Empirical Expression for large values of \( C_T \)

Wind turbines do have to operate at tip speed ratios appreciably above their design value, usually during very low wind speeds when the rotational speed of the rotor becomes much greater than the wind speed. In such conditions the airflow around the blade tips may develop into the "turbulent wake" state. This phenomena is a result of small angles of attack at the outer blade elements causing induced velocities, (generated by the shed vorticity at the blade tips), which are similar in magnitude to the freestream velocity of the air flow, ie values of \( a \) above \( \frac{1}{3} \). This means the wake shed by the outer part of the rotor no longer consists of annular streamtubes but rather a turbulent flow with vortices entrained in the slipstream. In extreme cases the flow can be reversed through the rotor, ie a greater than \( \frac{1}{2} \). This means the momentum/vortex theories become inapplicable.

Glauert (Ref 4) presents an empirical solution for this problem. The solution was derived from testing carried out on propellers operating in turbulent wake and vortex ring states, (Lock et al (Ref 6)). Glauert never presented a formulation for
Figure 2.5: Plots of Power and Thrust Coefficients from simple momentum theory and Hansen's curve fit for high values of 'a'.

this curve, however it given by Hansen (Ref 62) as:

\[ C_T = 4a(1 - \frac{1}{4}(5 - 3a)a) \]  

(2.20)

The formula for the thrust on a blade element \((dT)\), would be redefined for the iteration process based on this new definition of \(C_T\), ie: re-evaluation of (2.16). Figure 2.5 shows the 3 different curves for the power and thrust coefficients from simple momentum theory and Hansen's curve fit.

The results of a BEM analysis where the axial induction factor rises above \(\frac{1}{3}\) must be suspicious as the theory used to calculate the torque and power is no longer valid, however since this is usually under conditions of low wind speeds and hence low torque and power its empiricism is normally tolerated.
2.1.6 Prandtl’s Tip Loss Model

Besides the empirical correction of the axial (directly) and tangential (indirectly) induction factors for high thrust coefficients there is also a need in an applied BEM method to account for the fact that a rotor has a finite number of blades. Since the theories presented above are all based on the assumption of a very high or infinite number of blades a correction for this needs to be applied.

There exist two main approaches to this problem, one is a detailed mathematical analysis of the problem given by Goldstein (Ref 7) using an infinite set of Bessel functions. The alternative method is an approximation given by Prandtl in an appendix of a paper by Betz (Ref 1). The sheer complexity of Goldstein’s method and the reasonable accuracy and simplicity of Prandtl’s approximation is the reason why the latter is normally adopted.

A comparison between Goldstein’s method and Prandtl’s approximation shows that, of the two, Prandtl’s approximation becomes less accurate at low tip speed ratios or with a low number of blades. This should be noted for wind turbine applications, since 2 bladed rotors are sometimes still used. Both theories also neglect the expansion of the wake and so are only truly applicable to lightly loaded rotors.

Prandtl’s approximation is based on the consideration that the helical vortex sheets shed into the wake can be considered to be a rigid helical screw moving downwind at a velocity $V_W$, the velocity of the air outside the screw is $V$ (both velocities defined as before). The result of this situation is that the air inside the screw will be constrained to move at $V_W$ while the faster moving air outside will move past the edges of the helical sheets and will be drawn in between the sheets and then out again around the next sheet. This represents two phenomena of the wake, the first is the shedding of circulation at the tips of the blades resulting in significant radial velocities in front of and behind the blade tip and the second
being the fact that the velocity in the wake at the streamtube boundary does not slow down entirely to $V_W$ but to a velocity between $V_W$ and $V$ which is radially dependent.

By constructing a velocity field around the edges of a series of flat plates moving with a velocity $V_W$ in a flow of velocity $V$, (Figure 2.6), an analogy to the helical sheets is achieved. From analysis of this velocity field by the use of conformal mapping an expression is obtained for the tip loss factor:

$$F = \frac{2}{\pi} \cos^{-1} \exp\left(-\frac{ss}{x}\right)$$  \hspace{1cm} (2.21)

where $x$ is the distance from the edge of the plate inwards away from the freestream and $s$ is the distance between successive plates. For application to the wind turbine rotor these distances have to be defined, (see figure 2.7). Since the expansion of the streamtube is being neglected the expressions for $\varphi$, $a$ and $s$ become:
Figure 2.7: Geometry of shed helical vortex sheets

\[
\sin \varphi = \frac{(1 - a)V}{W_2} \tag{2.22}
\]

\[
x = R - r \tag{2.23}
\]

\[
s = \frac{2\pi R}{N} \sin \varphi \tag{2.24}
\]

Hence:

\[
\frac{\pi x}{s} = \frac{(R - r)NW_2}{2R(1 - a)V} \tag{2.25}
\]

(2.25) can be substituted into (2.21) for use in a BEM method. The technique would be to modify the axial and tangential induction factors by multiplying them by the tip loss factor for each blade element. Since the helical sheets are shed at an angle \(\varphi\) the tip loss applies as much to the tangential induction factor as it does to the axial induction factor.

So equations (2.17) and (2.19) become:
During the iteration process it should be noted that the use of $a$ and $a'$ in the right hand side of the above equations to determine the flow angle $\phi$ should not be factored by $F$.

### 2.1.7 The General Momentum Theory

The general momentum theory as presented by Glauert (Ref 8) for a propeller extends the simple momentum theory from a 1 dimensional to a 2 dimensional theory by including wake rotation. The need to include wake rotation comes from defining the energy extraction device to be that of a rotor which generates a torque and hence imparts angular momentum to the rotor wake.

The assumption of conservation of axial flow in each annular element of the wake is retained along with a conservation of angular momentum in each annular element as the wake expands, these are required to keep the problem mathematically tractable. The results of the analysis (following Sharpe, Ref 64), show that the rotation of the wake of the wind turbine adds to the axial pressure drop across the rotor due to a reduction in static pressure, replaced by a velocity (wake rotation/dynamic pressure), in a direction opposite to that of the blades.

This result has consequences for the application of momentum theory with blade element theory. Immediately behind the disc there is an additional pressure discontinuity (in comparison to the free stream), which comes from the additional drop in static pressure caused by the rotation of the wake. However due to this rotation being maintained throughout the wake it doesn't effect the extraction of...
axial momentum but it does increase the pressure loading on the disc and therefore needs to be included in the equation for thrust force on the rotor while the equation for the torque on the rotor remains unchanged. The rotation of the wake is maintained by a radial pressure gradient which balances the centrifugal force of the rotating air.

As the wake slows down and expands the additional pressure drop from the rotating wake must also be taken into account in the rise to ambient pressure at the edge of the wake. Since the edge of the far wake must be at atmospheric pressure and also contain rotating fluid it must follow that its axial velocity is lower than for the case where wake rotation is not considered.

If the rotation of the wake is not radially uniform (rigid body rotation, a function of constant blade circulation) then the radial pressure gradient will also be non-uniform and this will effect the expansion of the wake and hence the energy extraction of the disc.

This implies that the axial momentum change can now only really be equated between the far wake and the flow upstream. Depending upon flow expansion, a part of the static pressure drop from the rotation in the wake immediately behind the disc will now contribute to the extraction of axial momentum, only the radial pressure gradient which develops in the far wake will be maintained throughout the flow and will contribute to the thrust on the disc but not the axial momentum change. In order to employ the general momentum theory accurately then it is required to treat the expanded wake as the case to be solved in order to determine induced velocities at the rotor.

The rotation of the wake is often seen as a loss of energy for the wind turbine, it is however merely a reaction to the generation of torque by the rotor which increases the thrust loading on the rotor. Conversely the rotating wake does represent a loss for a propeller since the thrust force is reduced by the drop in static pressure
caused by the rotation of air in its wake, the angular momentum of which is equal to the torque supplied to the propeller.

As far as practical application of the momentum theory is concerned Sorenson (Ref 59) has shown that the major assumptions involved in the simple momentum theory (including the neglecting of the pressure drop due to wake rotation) have a negligible effect on the results of the momentum theory when applied to a rotor. The general momentum theory does however provide an insight into the energy extraction process of an idealised wind turbine rotor.

2.2 Theories for Yawed Flow

The BEM method as outlined in the preceding section will provide accurate blade loads in axial flows and under steady conditions (the conditions which mostly satisfy the assumptions made in formulating the method).

However when the flow through a rotor is no longer purely axial, ie: the freestream velocity is no longer normal to the plane of rotation, then this can be defined as non-axisymmetric flow. The term yawed flow is commonly applied to any non-axisymmetric flow, however non-axisymmetric flow may occur in either the horizontal or vertical planes, ie: yaw and tilt or a combination of both.

The BEM theory as formulated for axial flow does not consider the flow patterns which occur through yawing of the rotor, indeed application of BEM theory to yawed flow produces results which are significantly different to those obtained through experiment. Figure 2.8 shows a 2 bladed rotor viewed from above in 'yawed' flow, where the 'yaw error' is given by $\gamma$, experiment shows that there exists a significant moment $M$ which tries to restore the rotor to a position in which its plane of rotation is normal to the oncoming flow, $V$. (The same diagram could represent the same rotor side on, tilted and with its blades in a vertical plane.
The physical explanation for the restoring moment on a yawed rotor lies in a change in the overall flow field through the rotor. Simple methods have been presented to try to account for this effect, based more on trigonometry than fluid dynamics. (Refs 21, 38). When the rotor is operating in a yawed condition the thrust force on it is no longer parallel to the freestream flow, this is because the thrust force must be normal to the rotor and the rotor is no longer normal to the oncoming flow. This results in a deflection of the flow through the rotor. In fact the only moment predicted by the BEM theory outlined in the preceding chapter is due to another phenomenon, the advancing and retreating blade effect. This is caused by the relative velocity of flow approaching a blade element changing sinusoidally as the blade rotates. This is 90 degrees out of phase with the restoring moment shown in figure 2.8.

Figure 2.9 again shows the yawed two bladed rotor seen from above, this time the blades are vertical, (6 and 12 o’clock positions). The advancing blade sees a higher flow velocity at a shallower angle than the retreating blade, which sees
a slower velocity at a larger angle of attack. Hence there is an force imbalance between the two blade elements caused by the differences in magnitude of $W_1$ and $W_2$ and the relative angles of attack.

The total moment on a yawed rotor will be a combination of the restoring moment shown in figure 2.8 and the moment due to the advancing and retreating blade effect.

### 2.2.1 Glauert's Approximation for Yawed flow

One of the first attempts to explain the fluid flow through a yawed rotor was made by Glauert (Ref 5), in a report on autogyros, an autogyro is basically a highly yawed wind turbine where the rotor is nearly horizontal and the thrust force it develops due to a forward velocity is used to keep the craft airborne. In this case the flow parallel to the rotor plane is very much higher than the flow normal to the rotor plane, Glauert therefore considered the induced velocity normal to the rotor plane to be analogous to that of the induced velocity (downwash) of a
circular wing with semi-span equal to the radius $R$, of the rotor.

\[ u_{av} = \frac{T}{2\pi R^2 \rho V_T} \]  \hspace{1cm} (2.28)

Where $u_{av}$ is the average induced velocity due to the shed tip vortices (downwash), $T$ is the thrust on the rotor, $V_T$ is the average resultant velocity at the rotor and $\pi R^2$ is the area required to produce an induced drag by the rotor consistent with an elliptic lift distribution (Ref 17), this area is always normal to the resultant velocity at the rotor. Due to the assumed small angle of incidence of the rotor plane the thrust force $T$ is approximately normal to the rotor plane as is the average induced velocity $u_{av}$.

If an angle of incidence of 90 degrees is now considered (0 degrees yaw) it can be seen that equation (2.28) will default to the result for simple momentum theory, equation(2.29). Since $V_T = V - u_{av}$ and $u_{av} = a_{av}V$.

\[ T = 2\pi R^2 \rho (V - u_{av})u_{av} \]  \hspace{1cm} (2.29)

The theory's applicability at anything other than 0 or 90 degrees of yaw however must involve some uncertainty as it effectively assumes that the average induced velocity (in the rotor plane) produced by the shed wake of the rotor is always normal to the rotor plane, which for a yawed wind turbine extracting a significant amount of energy must be open to question. This also represents an inconsistency in the analogy between wing and rotor and the direction of the lift and thrust forces respectively.

However the solid wing analogy also allowed Glauert to present an expression for the variation of induced velocity across the rotor disc for an autogyro, viewing
the yawed rotor from above, figure 2.10, it could be viewed as a series of discrete vortices generating an overall flow around the rotor from upwind edge to downwind edge. If these vortices are of equal strength then the variation in induced velocity they cause is linear with reference to the upwind/downwind diameter of the rotor plane:

\[ u = u_o + ku_s \frac{r}{R} \sin \psi \]  

(2.30)

Where \( u_o \) is a constant component of induced velocity, \( u_s \) is a varying component, \( k \) is a scaling factor and \( \psi \) is the azimuth angle of the rotor blade which is zero when the blade is retreating, (12 o'clock position in Figure 2.9). The induced velocity is only considered normal to the plane of rotation since this is the direction of the thrust on the rotor. Figure 2.10 shows the case \( u_o = ku_s \) which is the assumption that Glauert made.
An equation for the relative velocity at the rotor $V_T$, can be obtained from the freestream velocity components and the average induced velocity, $u_{av}$, after Sharpe (Ref 64), equation (2.31). A differential form of the thrust equation can then be deduced for coupling with blade element theory, equation (2.32).

$$V_T = \sqrt{V^2(1 - 2a_{av}\cos\gamma + a_{av}^2)}$$ (2.31)

$$\frac{dT}{dr} = 2\rho V^2 a \sqrt{1 - a(2\cos\gamma - a)(2\pi)}$$ (2.32)

### 2.2.2 Vortex Wake Theory

Coleman et al (Ref 10), further developed the idea of varying induced velocities normal to the rotor plane of a yawed rotor.

The model used assumes a lightly loaded rotor with an infinite number of blades, the rotor being at an angle ($\gamma$) to the oncoming flow. The circulation on the rotor blades is assumed to be uniform and to not vary with azimuth angle, this results in a uniform, continuous, helical vortex sheet being shed from the edge of the rotor in the form of an elliptical vortex tube which is assumed to remain unexpanded. The vortex tube wake is skewed relative to the rotor plane and the oncoming flow by an angle that depends upon the free-stream velocity and the rotor thrust loading, Sharpe (Ref 64) gives an approximate relationship as:

$$\kappa = (0.6a + 1)\gamma$$ (2.33)

The wake skew angle $\kappa$ is defined as the angle between the skewed wake axis and a plane normal to the rotor plane such that $\kappa$ is a maximum. The direction of the induced velocity at the centre of the rotor is given by the angle $\kappa/2$, this bisects the angle between the rotor axis and the wake axis, the induced velocity has a
Figure 2.11: Coleman et al's skewed wake geometry

The magnitude of \( aV \sec(\frac{\chi}{2}) \).

If the induced velocity component parallel to the wake axis at the centre of the rotor is given by \( u = aV \), then from trigonometry the magnitude of the component normal to the wake axis can be obtained, \( aV \tan(\frac{\chi}{2}) \) this is the component responsible for the skewing of the wake. The induced velocity can also be resolved normal and parallel to the rotor plane as in figure 2.11. As a result of the elliptical wake a geometry of induced velocity components is produced which is identical at the rotor plane and in the far wake, the only difference being the magnitude of the induced velocities is doubled in the far wake compared to the rotor plane.
Coleman et al (using the Biot-Savart law) produced a formula for the distribution of induced velocities normal to the rotor plane across the upstream/downstream diameter of the rotor. It is presented in a linearised form similar to Glauert's theory and it only considers induced velocities normal to the rotor plane but with the advantage that the magnitude and distribution of induced velocities are directly related to the flow velocity and the geometry of the wake. Sharpe (Ref 64) has extended the analysis using numerical evaluation of the wake structure to arrive at formulas for the flow expansion that occurs due to the presence of the vortex wake.

In order to arrive at a theory that can be coupled with blade element theory it is necessary to evaluate the flow field upwind and downwind of the rotor using Bernoulli's equation taking account of the change of momentum which is normal to the rotor axis, this can then applied in a differential form to a annular element to be coupled with blade element theory:

\[
\frac{dT}{dr} = 2\rho V^2 a \left( \cos(\gamma) + \tan\left(\frac{\psi}{2}\right) \sin(\gamma) - a \sec\left(\frac{\psi}{2}\right)^2 \right) (2\pi r) \quad (2.34)
\]

In order to couple the torque produced by the rotor with the angular momentum represented by the wake structure it is necessary to transpose the rotation of the fluid around the wake axis into the rotor plane through the wake skew angle, \( \psi \), this results in the following expression for the tangential induction factor:

\[
d' = \frac{NW^2c (\cos \psi (C_L \sin \phi - C_D \cos \phi) - \cos \theta \sin(\psi)(C_L \cos \phi + C_D \sin \phi))}{8V \pi r^2 \Omega (\cos \gamma - a)(\cos \psi^2 + \cos \psi^2 \sin \psi^2)}
\]

(2.35)

Where \( \psi \) is the blade azimuth angle defined in the same way as for the Glauert theory. The Coleman theory can only give average values of induced velocities
therefore it necessary to analyse a complete revolution using the above equations and then average the induced velocities before applying the flow expansion equations of Sharpe over a repeat revolution of the rotor in order to determine the varying blade loadings over that revolution.

Meijer Drees (Ref 12) completed a similar analysis to Coleman et al, using the Biot and Savart law applied to a skewed elliptical tube wake of the same form. However, he also considered the changing circulation on the blades due to the advancing and retreating blade effect. As shown earlier this effect is 90 degrees out of phase to the induced velocities which occur on the upwind/downwind diameter. Meijer Drees considered both the effect of the changing bound vorticity on the blades and the changing shed vorticity in the wake. A result of this is that the induced velocities across the rotor plane are no longer symmetrical about the upwind/downwind diameter.

In principle this effect of changing bound vorticity could also be included in a Biot and Savart analysis of the flow expansion caused by the skewed vortex wake.

2.2.3 Application ofYawed Momentum Theory

The Glauert and Coleman et al yawed flow theories presented above are only capable of determining average induced velocities for the whole rotor. They are developed from what is assumed to be an essentially steady state condition. Since the circulation along the infinite number of blades is assumed uniform and the shed vorticity from the blade tips and roots is assumed unvarying with azimuth it follows that the pressure drop across the rotor is also uniform across the rotor and has a discontinuity at the rotor's edge.

These assumptions can also be shared for blade element theory in axial flow since this is also a steady state condition at a blade element level, however in yawed flow the forces on a blade element are no longer constant, they change constantly due
to the advancing and retreating blade effect and the varying induced velocities over the rotor. If these effects become significant then clearly the use of static 2D aerofoil data for the blade element side of the equations becomes untenable. Also the momentum theories have to be put into differential form in order to be coupled with blade element theory, again the assumptions which were made with the simple momentum theory in axial flow cannot be significantly violated, especially the assumption that each annular element can be treated separately for the purposes of momentum balance, if there is significant differences in the thrust forces between annular elements then it is likely that a radial change of momentum will take place due to the component of induced velocity which is normal to the rotor axis in the yawed case.

2.3 Chapter Summary

This chapter presents various aerodynamic models in order to illustrate their origins and the assumptions made in their formulation. The blade element / momentum theory (BEM) is the most basic, complete, aerodynamic model for a rotor. It is based on several crucial assumptions, it is assumed that the rotor is operating in axial flow, it is assumed that there are an infinite number of rotor blades (effectively an actuator disc model) and it is assumed that the bound circulation on the rotor blades can be described based upon radius and that the annular flowstreams so formed can be treated independently, also circulation is assumed to be shed only at the absolute tip and root of a rotor blade. Across the rotor plane this results in uniform axially induced velocities in separate concentric flowstreams with a discontinuity at the rotor's edge. Lifting line theory implies that 2D aerofoil force coefficients are to be used for calculation of blade forces when applying the BEM theory.
Prandtl's tip loss model is an ingenious attempt to correct for a finite number of rotor blades, however the assumption that circulation is shed only at the absolute root or tip of the blade still applies.

The BEM theory with and without Prandtl's model will be used in later chapters to generate results for comparison with experimental wind tunnel data. An extension to the BEM theory will also be made to approximate the axial pressure effects of wake rotation, this model will also be used for comparison with wind tunnel data.

Modified blade element / momentum theories based upon Glauert's wing analogy and the skewed vortex wake theory of Coleman et al will also be used to provide theoretical results for comparison to wind tunnel data. However they are steady state theories and applying them using steady state aerofoil data to yawed flow cases where the blade element conditions are dynamic and unsteady is not really appropriate. It will however provide a comparison to dynamic inflow based yawed flow aerodynamic models (presented in chapter 3) as well as wind tunnel data.
Chapter 3

Dynamic HAWT Aerodynamics

3.1 The demands of Aeroelastic Analysis

Of most interest to the wind turbine analyst is the response of a turbine design to given wind conditions. Specified wind conditions such as those laid out in the IEC international standards for turbine safety (Ref 55) are by necessity time based and unsteady in nature. Therefore the aeroelastic code used by the analyst has to be capable of dealing with unsteady rotor aerodynamics and the coupled structural response of the turbine design in an accurate and computationally efficient manner.

The previous chapter dealt with some of the classical theories of rotor aerodynamics. Although they give an insight into the flow physics of a rotor they principally deal with steady flow conditions and are not capable of dealing with the dynamic flow conditions imposed by the wind turbines operating environment.

The dynamic analysis of rotor aerodynamics is usually split into an inner and outer loop mirroring blade element theory and momentum theory respectively. The outer loop involves the determination of induced velocities in the rotor plane, this has to deal with the unsteady nature of the wind environment and the resulting unsteady rotor wake. It is usually referred to as a dynamic inflow model.
The inner loop of the analysis is concerned with determining the forces on the blades (from these forces the turbine's structural response is determined). The blade forces across the whole rotor for a given time step are also used in the dynamic inflow analysis and hence this calculation takes place in an inner loop of the dynamic inflow analysis. This loop has to incorporate unsteady aerofoil aerodynamics and separated flow (stalling), it is usually called a dynamic stall model.

3.2 Dynamic Inflow

3.2.1 Acceleration Potential Theory

In the 1930's Kinner (Ref 9) developed a mathematical model of a pressure discontinuity across a circular disc in a steady flow field. The pressure discontinuity is made up of Legendre polynomials and was arrived at through a linearisation of Euler's momentum equation. The theory allows much more generalised pressure distributions to be formulated across the disc than the uniform pressure distribution of the momentum theory, it also removes the singularity at the edge of the disc which is a result of a uniform pressure distribution.

Kinner assumed that the disc was lightly loaded and that the induced velocities in the flow field caused by the presence of the disc are very much smaller than the free-stream velocity, \( V \). Meaning in effect that \( V \) is constant throughout the flow field (even at the disc). Starting with the Euler equation in Cartesian coordinates (centred on the disc) such that \( x \) is normal to the disc, and \( u', v', w' \) are the induced velocities in the \( x, y, z \) directions respectively. The rate of change of momentum of the flow in the \( x \) direction will be in relation to the pressure gradient in that direction:
\[
\rho \left[ (V + u') \frac{\delta (V + u')}{\delta x} + v \frac{\delta (V + u')}{\delta y} + w \frac{\delta (V + u')}{\delta z} \right] = -\frac{\delta p}{\delta x} \quad (3.1)
\]

The assumption that \( u', v', w' \) are very small compared to \( V \) allows second order terms to be dropped from the momentum equations for all 3 dimensions:

\[
\rho V \frac{\delta u'}{\delta x} = -\frac{\delta p}{\delta x} \quad (3.2)
\]

\[
\rho V \frac{\delta v'}{\delta x} = -\frac{\delta p}{\delta y} \quad (3.3)
\]

\[
\rho V \frac{\delta w'}{\delta x} = -\frac{\delta p}{\delta z} \quad (3.4)
\]

By differentiating each momentum equation with respect to its particular direction and then summing the results and imposing the equation for the continuity of the flow (3.5), it is possible to arrive at the Laplace equation governing the pressure field on and surrounding the disc, equation (3.6):

\[
\frac{\delta u'}{\delta x} + \frac{\delta v'}{\delta y} + \frac{\delta w'}{\delta z} = 0 \quad (3.5)
\]

\[
\frac{\delta^2 p}{\delta x^2} + \frac{\delta^2 p}{\delta y^2} + \frac{\delta^2 p}{\delta z^2} = 0 \quad (3.6)
\]

Considering the special case where the disc is normal to the flow field (zero yaw) and integrating in the downwind direction \( x \), also remembering that an assumption of the theory is that \( V \) is uniform throughout the flow field:
\[ \rho V' u' = -p \quad (3.7) \]
\[ \rho \Phi = -p \quad \text{where:} \quad \Phi = V u' \quad (3.8) \]

Equation (3.8) implies that when the disc is normal to the flow field then the distribution of induced velocities normal to the disc will be identical in shape to the pressure distribution across the disc. In the more general case a formula linking the acceleration potential with the pressure drop across the disc is of the form: (\( u \) denotes upwind, \( D \) denotes downwind)

\[ \rho (\Phi_D - \Phi_U) = p_U - p_D \quad (3.9) \]
\[ 2\rho \Phi = p_U - p_D \quad \text{since} \quad \Phi_U = -\Phi_D \quad (3.10) \]

Kinner (Ref 9) found that using a co-ordinate transformation to elliptical co-ordinates (centred on the disc) and through separation of the variables of the resulting form of Laplace's equation that the potential functions describing the pressure field can be separated into 3 (one for each co-ordinate) ordinary differential equations. If the elliptical co-ordinates are given by \( \nu' , \eta' \) and \( \psi' \). Then the 3 differential equations are:

\[ \frac{d}{d\nu'} \left[ (1 - \nu'^2) \frac{d}{d\nu'} \Phi_1(\nu') \right] + \left[ n(n+1) - \frac{m^2}{1 - \nu'^2} \right] \Phi_1(\nu') = 0 \quad (3.11) \]
\[ \frac{d}{d\eta'} \left[ (1 + \eta'^2) \frac{d}{d\eta'} \Phi_2(\eta') \right] + \left[ \frac{m^2}{1 + \eta'^2} - n(n+1) \right] \Phi_2(\eta') = 0 \quad (3.12) \]
\[ \frac{d^2}{d\psi'^2} \Phi_3(\psi') + m^2 \Phi_3(\psi') = 0 \quad (3.13) \]
The first two equations are in the form of Legendre’s associated differential equations, since they are second order they have two sets of possible solutions, Legendre’s associated polynomials of the first $P_n^m(x)$ and second kinds $Q_n^m(x)$. Also they are linear and any combination of solutions, including arbitrary constants, will satisfy the equations. The third equation (for the azimuth co-ordinate) has trigonometric solutions. Taking into account practical considerations only polynomials of the first kind are used as solutions to the first equation since those of the second kind become infinite at the centre of the disc (although they could be used to represent the pressure drop due to the rotation of the wake). For the second equation only polynomials of the second kind (with imaginary arguments) are used as those of the first kind become infinite in the far field. The combined solution satisfying the above equations and hence Laplace’s equation at the disc becomes:

$$\Phi(\nu', \eta', \psi_{\text{wind}}) = \sum_{m=0}^{M} \sum_{n=0}^{N} P_n^m(\nu')Q_n^m(i\eta')(C_n^m \cos(m\psi_{\text{wind}}) + D_n^m \sin(m\psi_{\text{wind}}))$$

(3.14)

It is simple to relate this solution to a position on the disc in 2 dimensional polar co-ordinates. At the disc $\eta' = 0$. $\psi'$ is the azimuth angle around the disc and $\psi_{\text{wind}}$ is the azimuth angle around the disc measured with reference to the flow field, the sinusoidal variations are lateral with respect to the flow field and the cosinusoidal are longitudinal, therefore the zero azimuth position has to be consistent and it lies (when there is a yaw angle) directly downwind on the surface of the disc. $\nu'$ is related to the non-dimensionalised radius $r'$, through the relationship $\nu' = \sqrt{1 - r'^2}$. Also for a valid solution $m+n$ has to be an odd number, necessary for continuously varying pressure away from the disc.
Only the first distribution \((m = 0\) and \(n = 1\)) gives a net thrust force on the disc, all the remaining pressure distributions when integrated over the disc give zero thrust. Hence a thrust and a thrust coefficient for the disc can be defined from the first pressure distribution.

\[
p_U - p_D = 2\rho \Phi = 2\rho C_1^0 \sqrt{1 - r^2} \tag{3.15}
\]

Where \(C_1^0\) is an arbitrary constant coming from the Legendre polynomial solutions. Integrating across the disc to get the thrust force and a thrust coefficient yields:

\[
\text{Thrust} = \frac{4}{3} \rho C_1^0 \pi R^2 \quad \text{and} \quad C_T = \frac{8 C_1^0}{3 V^2} \tag{3.16}
\]

In the special case where the disc is normal to the fluid flow:

\[
\Phi = V u' = C_1^0 \sqrt{1 - r^2} \quad \text{and} \quad \frac{u'}{V} = \frac{3}{8} C_T \sqrt{1 - r^2} \tag{3.17}
\]

So the average induced velocity (normal to the disc) and the hence the average value of \(a\) over the disc is found by integration over the disc:

\[
a_{av} = \frac{u_{av}}{V} = \frac{\frac{3}{8} \pi R^2 C_T}{\pi R^2} = \frac{C_T}{4} \tag{3.18}
\]

Comparison with the simple momentum theory for axial flow shows that the non-linear term \((1 - a)\) is missing from the relationship of thrust to induced velocity in acceleration potential theory, it is a direct consequence of the linearisation of the momentum equations. Acceleration potential theory deviates from simple
momentum theory because of this linearisation and at a value of $a = \frac{1}{3}$ there is a difference of $\frac{1}{3}$ in the thrust value predicted by the two theories.

Work has been done by Van Holten (Ref 20) and Van Bussel (Ref 46) to develop non-linear versions of acceleration potential theory for rotor analysis. Van Holten was interested in the aeroelastic analysis of helicopter rotors, later Van Bussel extended this approach to wind turbine rotors.

As was stated in the introduction, the reader should be aware of these engineering aerodynamic models even though they have not been considered here due to the constraints of time and their lack of direct relevance to the NREL codes used by the present author for modelling work.

3.2.2 The Pitt and Peters Inflow model

Mangler (Ref 13) extended Kinner’s work by considering cases where the disc is at angles to the flow other than 0 or 90 degrees. Finding the induced velocity field from the pressure fields for these cases is mathematically much more complicated than for the two cases treated by Kinner. However Mangler succeeds in producing a general formula for an induced velocity distribution across the disc (normal to the disc) arising from a given solution to equation (3.14), (albeit only where $m = 0$), at a given angle of the disc to the flow field.

Mangler was researching the velocities induced in the rotor plane of a helicopter during flight, for this he required a suitable circular pressure field to represent a helicopter rotor, he selected a linear combination of two solutions to equation (3.14), ($m = 0, n = 1$, equation (3.15), with $C_1^0 = 1$ and $m = 0, n = 3$, equation (3.19), with $C_3^0 = \frac{3}{2}$). The two distributions and the solution resulting from their combination are shown in figure 3.1. It can be seen that the combined distribution falls to zero at the blade tip and at the centre of the disc. Also the rate of change of pressure falls to zero at the centre of the disc. If this combined distribution is
Kinner distributions, a) m=0, n=1 b) m=0, n=3, c) combined solution

Figure 3.1: Kinner distributions, a) m=0, n=1, Coeff=1, b) m=0, n=3, Coeff=3/2 c) distribution a) - distribution b)

equated to the thrust loading on the disc then the result is equation (3.20).

\[
\frac{P_U - P_D}{2\rho} = C_3^0 \frac{1}{3} (2 - 5r^2) \sqrt{1 - r^2} \quad (3.19)
\]
\[
\frac{P_U - P_D}{\frac{1}{2}\rho V^2} = \frac{15}{4} C_T r^2 \sqrt{1 - r^2} \quad (3.20)
\]

Mangler went on to represent the normal induced velocity distribution across the disc for equation (3.20) by a Fourier series, it is valid at any angle of incidence to the flow field (Refs 14, 15). This representation of a solution to two combined solutions to Kinner’s acceleration potential functions forms the basis of the dynamic inflow model of Pitt and Peters (Ref 24). This model considers the normal induced velocities across an actuator disc to be made up of three individual linear components of the form:
\[ \lambda(r', \psi_{\text{wind}}) = \lambda_0 + \lambda_s r' \sin(\psi_{\text{wind}}) + \lambda_c r' \cos(\psi_{\text{wind}}) \] (3.21)

Where \( \lambda \) represents the non-dimensionalised induced velocity normal to the disc at a point on the disc given by polar co-ordinates. It is non-dimensionalised by the tip speed of the rotor \( \Omega R \) instead of by the wind speed for two reasons, firstly the tip speed is likely to be more constant than the wind speed, given the large rotating inertia of the rotor. Also for the purposes of unsteady flow it allows the non-dimensionalised time to be given by the change in azimuth of the rotor per time step.

The steady state part of the governing equation of the Pitt and Peters model takes the form:

\[
\frac{1}{\Omega R} \begin{bmatrix} u_o \\ u_s \\ u_c \end{bmatrix} = [V] [L] \left( \frac{1}{\rho A \Omega^2 R^3} \right) \begin{bmatrix} \text{Thrust} \times R \\ \text{Moment}_s \\ \text{Moment}_c \end{bmatrix}
\] (3.22)

The \( L \) matrix takes the form: (after Sharpe Ref 64):

\[
\begin{bmatrix}
\frac{1}{2} & 0 & \frac{15\pi}{64} \tan \left( \frac{\pi}{2} \right) \\
0 & -2 \sec^2 \left( \frac{\pi}{2} \right) & 0 \\
\frac{15\pi}{64} \tan \left( \frac{\pi}{2} \right) & 0 & -2 - 2 \tan^2 \left( \frac{\pi}{2} \right)
\end{bmatrix}
\] (3.23)

where:
\[ x = \tan^{-1}\left( \frac{\mu}{\lambda_{mv} + \eta} \right) \]  

(3.24)

The V matrix takes the form:

\[
\begin{bmatrix}
\sqrt{(\lambda_{mv} + \eta)^2 + \mu^2} & 0 & 0 \\
0 & \frac{\mu^2 + (2\lambda_{mv} + \eta)(\lambda_{mv} + \eta)}{\sqrt{(\lambda_{mv} + \eta)^2 + \mu^2}} & 0 \\
0 & 0 & \frac{\mu^2 + (2\lambda_{mv} + \eta)(\lambda_{mv} + \eta)}{\sqrt{(\lambda_{mv} + \eta)^2 + \mu^2}}
\end{bmatrix}
\]  

(3.25)

where:

\[ \lambda_{mv} = \frac{1}{2} \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} [L]^{-1} \begin{bmatrix} \lambda_o \\ \lambda_s \\ \lambda_c \end{bmatrix} \]  

(3.26)

\( \lambda_{mv} \) is the momentum theory value of the non-dimensionalised induced velocity, \( \eta \) is the non-dimensionalised wind component normal to the disc and \( \mu \) is non-dimensionalised wind component in the plane of the disc.

The elements of the L matrix are obtained by imposing the linear distributions of equation (3.21) and equating them to Mangler’s Fourier series representation of the induced velocity distribution caused by the pressure field of equation (3.20) at a general angle of the disc to the flow field. \( L_{11} \) comes directly from the thrust on the rotor (the first term in the Fourier series) and is unaffected by the yaw angle. \( L_{31} \) comes from equating the first moment about the lateral axis of the disc (subscript \( \omega \)) of equation (3.21) with the first moment of Mangler’s Fourier series about the same axis. To determine \( L_{13} \) and \( L_{33} \) it is necessary to consider a
pressure distribution which is antisymmetric, the first solution to equation (3.14) to do this is with \( m = 1, n = 2 \), equation (3.27).

\[
\frac{p_U - p_D}{2\rho} = C_2^1 r' \sqrt{1 - r'^2} \cos(\psi_{wind}) \tag{3.27}
\]

Mangler never considered the induced velocity field from this pressure distribution but numerical evaluation against the linear velocity distribution imposed by Pitt and Peters yields the two terms.

\( L_{22} \) is obtained by rotating the antisymmetric pressure distribution by 90 degrees on the disc and considering the first moment about the longitudinal axis of the disc. (see Sharpe, Ref 64).

The unsteady part of the Pitt and Peters model comes from the differentiation of a chosen pressure field, (Pitt and Peters chose equation 3.20 again) by redefining the linear momentum equations of equations 3.2 to 3.4 to include unsteady terms and remembering that we only require the acceleration normal to the disc then the equation to solve becomes:

\[
\frac{\delta u'}{\delta t} = -\frac{\delta p}{\delta x} \tag{3.28}
\]

A similar assumption is made regarding the unsteady term as was made for the potential function in that the unsteadiness in the flow is assumed to be negligible in comparison to the magnitude of the flow itself. By solving equation (3.28) for a differential of a given pressure field it is possible to find the unsteady term \( \frac{\delta u'}{\delta t} \), this requires a co-ordinate transformation from the Kinner elliptical co-ordinate system to a Cartesian system, the resulting expressions are then equated to the thrust force or moment on the disc to find the elements of the \( M \) matrix, the 'added mass' terms.
The full governing equation of the Pitt and Peters model then becomes:

\[
\begin{bmatrix}
\frac{128}{75\pi} & 0 & 0 \\
0 & \frac{16}{45\pi} & 0 \\
0 & 0 & \frac{16}{45\pi}
\end{bmatrix}
\]  

(3.29)

The full governing equation of the Pitt and Peters model then becomes:

\[
[M] \begin{bmatrix}
\lambda_o \\
\lambda_s \\
\lambda_c
\end{bmatrix} + [V] [L]^{-1} \begin{bmatrix}
\lambda_o \\
\lambda_s \\
\lambda_c
\end{bmatrix} = \begin{bmatrix}
C_T \\
C_M \theta \\
C_M \phi
\end{bmatrix}
\]

(3.30)

This is in the form of a first order differential equation and the usual method for finding a solution for a given time step is to solve the equation for the differentials of the induced velocities determining the disc loading coefficients from the blade element forces. Then using the previous values of induced velocities and the previous values of the differentials it is possible using a predictor/corrector regime to determine the induced velocity coefficients for the current time step which can then be applied using equation (3.21). It should be noted that the non-dimensionalised time is the angular rotation per unit time step in radians. Peters and HaQuang (Ref 30) present a co-ordinate transformation from a wind based system to a rotor based system which makes the application of the Pitt and Peters model with a structural model much simpler. A tip loss model like Prandtl's still has to be employed with the Pitt and Peters model as it only employs 3 linear induced velocity distributions with a discontinuity at the disc edge which is a feature of actuator disc theory.

The Pitt and Peters model was first applied to the analysis of wind turbine rotors by Swift (Ref 23). It has since been used in many different wind turbine
codes including the National Renewable Energy Laboratories (NREL) AERO-DYN subroutines (up to version 11). AERODYN is used in conjunction with both YAWDYN and ADAMS software for aero-elastic modelling of wind turbines and both codes have been used by the author in this thesis.

3.2.3 The Generalised Dynamic Wake model

This method can be viewed as a direct extension of the Pitt and Peters model, instead of just 3 linear induced velocity distributions the GDW model has an infinite series to match the infinite series of pressure distributions represented by equation (3.14). The induced velocity distributions being expressed azimuthally by a Fourier series and radially by Legendre functions.

He et al (Refs 34, 35) have managed to develop the GDW model in an entirely closed form which makes it very attractive for aeroelastic analysis. The governing equations of this model are as follows:

\[
[M] \begin{bmatrix} \alpha_j^r \\ \vdots \end{bmatrix}^{*} + [V] \begin{bmatrix} \hat{L}^r \end{bmatrix}^{-1} \begin{bmatrix} \alpha_j^r \\ \vdots \end{bmatrix} = \frac{1}{2} \tau_{mc}^n \\
[M] \begin{bmatrix} \beta_j^r \\ \vdots \end{bmatrix}^{*} + [V] \begin{bmatrix} \hat{L}^r \end{bmatrix}^{-1} \begin{bmatrix} \beta_j^r \\ \vdots \end{bmatrix} = \frac{1}{2} \tau_{ms}^n
\]

The induced velocity series takes the form:
\[
\lambda(r', \psi_{\text{wind}}, \ell) = \sum_{r=0}^{\infty} \sum_{j=r+1, r+3, \ldots} 0 \phi_j^r(r') \left[ \alpha_j^r(\ell) \cos(\ell \psi_{\text{wind}}) + \beta_j^r(\ell) \sin(\ell \psi_{\text{wind}}) \right]
\]

(3.33)

Where the radial shape functions are based on Legendre polynomials of the first kind and the elliptical co-ordinate \( \nu \):

\[
\phi_j^r(r') = \frac{1}{\nu} P_j^{rr}(\nu)
\]

(3.34)

\( P_j^{rr}(\nu) \) is termed the normalised Legendre polynomial of the first kind since if it is squared and integrated radially it always has a value of 1. The closed form solution is:

\[
\phi_j^r(r') = \sqrt{(2j + 1)H_j^{r}} \sum_{q=r,r+2,\ldots}^{j-1} r^q (-1)^{(q-r)/2} (j+q)!! \frac{(j+q+1)!! (j-r-1)!!}{(q-r)!! (q+r)!! (j-q-1)!!}
\]

(3.35)

\[
H_j^{r} = \frac{(j+r-1)!!(j-r-1)!!}{(j+r)!!(j-r)!!}
\]

(3.36)

The added mass matrix \( M \) takes the form:

\[
[M] = \frac{2}{\pi} \begin{bmatrix}
\vdots & \vdots & \vdots \\
& H_n^m & \\
& & \ddots
\end{bmatrix}
\]

(3.37)

The pressure field series is represented in the following way:
\[
\Phi(r', \psi_{\text{wind}}, \nu) = \sum_{m=0}^{M} \sum_{n=0}^{N} P_n^m(\nu') \left[ \tau_n^{mc} \cos(m\psi_{\text{wind}}) + \tau_n^{ms} \sin(m\psi_{\text{wind}}) \right]
\]  

(3.38)

where the forcing functions are:

\[
\tau_n^{0c} = \frac{1}{2\pi \rho \Omega^2 R^4} \sum_{q=1}^{Q} \sum_{i=0}^{I} \delta N f_{q,i} \phi_j^{0}(r'_i)
\]  

(3.39)

\[
\tau_n^{mc} = \frac{1}{\pi \rho \Omega^2 R^4} \sum_{q=1}^{Q} \left( \sum_{i=0}^{I} \delta N f_{q,i} \phi_j^{c}(r'_i) \right) \cos(m\psi_{\text{wind}})
\]  

(3.40)

\[
\tau_n^{ms} = \frac{1}{\pi \rho \Omega^2 R^4} \sum_{q=1}^{Q} \left( \sum_{i=0}^{I} \delta N f_{q,i} \phi_j^{s}(r'_i) \right) \sin(m\psi_{\text{wind}})
\]  

(3.41)

Where \(\delta N f\) is the normal force on a blade element and \(q\) and \(i\) are the blade and element indices respectively. The elements of the \(\hat{L}\) matrices consist of the cross coupling elements between the pressure and induced velocity distributions. The analytical forms are:

\[
[L_{jn}^{0m}]^c = \tan^m \left| \frac{\nu}{2} \right| [\Gamma_{jn}^{0m}]
\]  

(3.42)

\[
[L_{jn}^{rm}]^c = \left[ \tan^{m-r} \left| \frac{\nu}{2} \right| + (-1)^{\min(r,m)} \tan^{m+r} \left| \frac{\nu}{2} \right| \right] \left[ \Gamma_{jn}^{rm} \right]
\]  

(3.43)

\[
[L_{jn}^{rm}]^s = \left[ \tan^{m-r} \left| \frac{\nu}{2} \right| - (-1)^{\min(r,m)} \tan^{m+r} \left| \frac{\nu}{2} \right| \right] \left[ \Gamma_{jn}^{rm} \right]
\]  

(3.44)

where equation (3.45) is used where \((r+m)\) is even, equation (3.46) is used where \((r+m)\) is odd and \(j = n \pm 1\), equation (3.47) is used where \((r+m)\) is odd and \(j \neq n \pm 1\).
The method for solution is exactly the same as for the Pitt and Peters model although the momentum theory value of induced velocity is now calculated at each time step by:

\[
\lambda_{mv} = \frac{2}{\sqrt{3}} \left\{ 1 \ 0 \ 0 \ \ldots \right\} \left[ \tilde{L}^c \right]^{-1} \begin{bmatrix} \alpha^m_n \\ \vdots \end{bmatrix} \tag{3.48}
\]

Suzuki replaced the Pitt and Peters theory in NREL’s AERODYN subroutines
Figure 3.3: The 10 GDW radial shape functions used by the present author for comparison work with the NREL test results.

with a version of the GDW theory (AERODYN version 12, Refs 48, 53, 61), figure 3.2 shows the 6 radial shape functions used by Suzuki, however an error in the implementation of the theory may cast doubts over his results for yawed cases. Suzuki used data from the Tjaereborg turbine and only had access to blade root flap bending moments, not actual blade forces. Therefore it is impossible to tell if the blade force spanwise distributions predicted by his model are accurate, most probably his results are defined (as are the present authors) by the aerofoil data, either steady or unsteady, which is used as input to the aeroelastic model employed.

A corrected formulation with the same 6 radial shape functions has been produced by the present author for the modelling of the MS4 prototype and for comparison work with the NREL wind tunnel test results. The corrected formulation has subsequently been included in AERODYN by NREL.

Figure 3.3 shows the 10 different distributions chosen by the present author to
best represent the rotating pressure spikes of a two bladed rotor. The higher values of the $r$ superscript are all multiples of two, this superscript controls the azimuthal trigonometric variations of induced velocities, see equation 3.33. This was also formulated for comparison work with the NREL wind tunnel test results, the NREL rotor having two blades. An induced velocity flow field produced by the GDW theory will be made up of a combination of the radial shape functions included in its formulation.

A tip loss model is not required with the GDW model as the azimuthal Fourier series representation of the induced velocity distributions means that as more distributions are included in the model the closer it should become to representing the individual rotating pressure spikes of a rotor with a finite number of blades.

### 3.3 Unsteady Aerodynamics and Dynamic Stall

In order to apply a dynamic inflow model in an aeroelastic model of a wind turbine rotor it is necessary to determine the right hand side of the governing equation, ie the forces on the rotor blades. These forces are related to the blade design, the wind conditions, the structural response of the turbine including the blades and tower and the inflow into and hence the wake shed by the rotor. Therefore the determination of the forces at a given time step has to take place in an inner loop to the rest of the analysis.

All the theories for rotor inflow presented so far are suitable for use with blade element theory, this requires the use of lift and drag coefficients for the determination of blade element forces. Several dynamic stall models have been developed that are intended to give dynamic values of these coefficients in response to a given time history of flow conditions at the blade element level.

The ONERA dynamic stall model (Ref 25) is based on a set of non-linear differ-
The model requires a significant number of empirical coefficients to be determined from experimental results on oscillating aerofoils. These coefficients have to be determined for individual aerofoils but once obtained the model seems to give good results. It was decided not to use this model because of the necessity of obtaining these coefficients from experiment and the fact that this model is not implemented in the codes used by the present author in both the MS4 modelling or the NREL code comparison.

The Gormont model (Ref 18) has been used in wind turbine aeroelastic codes in the past, this model uses Theodorsen's theory to calculate unsteady (linear) airloads together with an empirically determined "gamma" function which is used to determine dynamic airloads during separated (non-linear and stalled) flow. However it has been found by Hansen and Pierce (Refs 44, 47) that key coefficients in the model had to be tuned to different aerofoil sections and that the reliance of the "gamma" function on the rate of change of angle of attack proved to cause numerical instabilities. These factors led to the replacement of the Gormont model by the Beddoes/Leishman dynamic stall model in the NREL AERODYN subroutines.

It was decided to use the Beddoes/Leishman dynamic stall model in all the work covered in this thesis, the model is already implemented in the NREL codes used in modelling the MS4 and the code comparison. Also, the majority of coefficients used in the model can be obtained directly from the static force coefficients rather than from unsteady experimental data. This is a real advantage when considering aerofoils designed specifically for use on wind turbines, with these aerofoils experimentally determined unsteady airloads can be hard to obtain.
3.3.1 Beddoes/Leishman Dynamic Stall Model

The Beddoes/Leishman dynamic stall model is detailed in (Refs 22, 27, 32). It is formulated in 3 distinct stages, the first involves the modelling of attached flow unsteady aerodynamics, the second involves the extension of the first stage response into the non-linear regime of the aerofoil as the flow begins to separate and the third stage involves the modelling of a sudden loss of lift including vortex shedding.

The model includes formulations for dynamic pitching moment coefficients, however it was decided to disregard this part of the model due to the typically high torsional stiffness of wind turbine blades and hence the negligible effect of the pitching moments on the blades structural response.

Attached Flow

The Beddoes/Leishman dynamic stall model uses indicial response functions to model the dynamic attached flow behaviour. Since the model is intended for use in a time-based aeroelastic code the indicial functions are formulated into exponential functions in the time domain, also, in the context of the model each time step represents a new indicial "disturbance", so each of the components of the indicial response can be found for each time step. Then the total response to some arbitrary forcing function can be found using Duhamel's superposition theory.

\[
C_{N}^c = C_{N \alpha}(\alpha_n - \alpha_0 - X_n - Y_n) = C_{N \alpha}(\alpha_{E_n} - \alpha_0) \quad (3.49)
\]
\[
C_{N \alpha}^l = \frac{4K_{\alpha}T_1}{M} \left( \frac{\Delta \alpha_n}{\Delta t} - D_n \right) \quad (3.50)
\]
\[
C_{N \alpha}^p = C_{N \alpha}^c + C_{N \alpha}^l \quad (3.51)
\]
\[
C_{T \alpha}^p = C_{N \alpha}^c \tan(\alpha_{E_n}) \simeq C_{N \alpha}(\alpha_{E_n} - \alpha_0) \tan(\alpha_{E_n}) \quad (3.52)
\]
where $K_\alpha$ and $T_1$ are Mach number dependent variables, $X$, $Y$ and $D$ are exponential lift deficiency functions, $C_N^C$ is the circulatory part of the normal force coefficient, $C_N^I$ is the impulse loading part of the normal force coefficient, $C_T^P$ is the chordwise/tangential force coefficient which is determined from the circulatory part of the normal force. $C_N^P$ is the total normal force coefficient from potential flow.

**Non-linear Regime**

The next part of the model considers progressive trailing edge separation which introduces non-linear force behaviour due to a loss of circulation. This modifies the attached flow response by using an effective flow separation point on the low pressure side of the aerofoil. This approach is based upon Kirchoff inviscid flow theory, the effective flow separation point is calculated from static aerofoil data using the following equations:

\[
C_N = C_{Na}(\alpha - \alpha_0) \left( \frac{1 + \sqrt{f}}{2} \right)^2 \tag{3.53}
\]

\[
C_G = C_{Na}(\alpha - \alpha_0) \tan(\alpha) \sqrt{f} \tag{3.54}
\]

where $\alpha$ is the angle of attack, $\alpha_0$ is the zero-lift angle of attack and $f$ is the effective separation point given in the ratio $\frac{x}{c}$, where $x$ is the distance from the leading edge and $c$ is the chord length. Equations (3.53) and (3.54) are rearranged to solve for $f$ over a range of $\alpha$. This results in a separation point/angle of attack curve.
\[
\frac{C_N^P - D^{\text{press}}}{C_{\text{Na}}} = \frac{C_N^{\text{press}}}{C_{\text{Na}}} = \alpha_E^{\text{press}} \tag{3.55}
\]

\[f'' = f' - D' \tag{3.56}\]

\[C_{N_n}^{\text{sep}} = C_{N\alpha} \left(1 + \frac{\sqrt{f''}}{2}\right)^2 (\alpha_{E_n} - \alpha_0) + C_{N_n}^I \tag{3.57}\]

\[C_{T_n}^{\text{sep}} = C_{N\alpha} (\alpha_{E_n} - \alpha_0) \tan(\alpha_{E_n}) \sqrt{f''} \tag{3.58}\]

\(D^{\text{press}}\) is an exponential deficiency function related to a lag in the leading edge pressure of the aerofoil, this gives an effective angle of attack \(\alpha_E^{\text{press}}\), which is used in the determination of \(f'\) (\(\alpha_E^{\text{press}}\) is also used in the vortex shedding model). \(D'\) is a second exponential deficiency function which relates to a lag in the boundary layer response of the aerofoil. This leads to an overall effective separation point, \(f''\) for a given time step and the complete normal and chordwise force coefficients for the non-linear regime are given by equations (3.57) and (3.58).

**Vortex shedding**

To extend the Beddoes/Leishman model into the deep stall regime it is necessary to include functions which represent the physical phenomena which occur during gross separation (deep stall). This includes leading edge separation, vortex shedding and flow reattachment.

An empirical normal force coefficient, \(C_{N_n}^{\text{crit}}\), is defined as being representative of the critical leading edge pressure required for sudden leading edge separation, this can be obtained from static aerofoil data using the appropriate Mach number. \(C_N^{\text{press}}\) is monitored with respect to \(C_{N_n}^{\text{crit}}\) and if it exceeds \(C_{N_n}^{\text{crit}}\) then sudden separation and vortex shedding are initiated.

Under attached flow and gradual trailing edge separation the build up of vorticity on the low pressure side of the aerofoil is given by a vortex lift coefficient \(C_v\):
\[ C_v = C_N^p (1 - K_N) \quad (3.59) \]
\[ K_N = \left( \frac{1 + \sqrt{f''}}{2} \right)^2 \quad (3.60) \]

\( C_v \) represents the difference between the potential flow circulatory lift, \( (C_N^p) \), and the corresponding (lower) non-linear circulatory lift including a separation point, \( (C_N^{sep} - C_N^I) \). \( C_v \) is then used in the exponential deficiency function \( (C_N^\gamma) \) which defines the total accumulated vortex lift. The deficiency function means that for low rates of change of angle of attack the vortex lift decays as fast as it accumulates, allowing a smooth return to static values when the rate of change slows to zero.

If \( C_N^{press} \) exceeds \( C_N^{crit} \) then the vorticity on the low pressure side is assumed to detach and convect along the chord of the aerofoil before being shed into the wake from the trailing edge. The strength of the vortex lift coefficient \( (C_v) \), is still determined in the manner described above until it leaves the trailing edge. The speed of convection of the vortex is given by an empirical time constant \( \tau_v \), the vortex being shed when \( \tau_v = T_{el} \), this empirical variable being the time taken to reach the trailing edge.

An empirical time constant is used to control the build up of successive vortices on the aerofoil to generate a model which will reproduce the effects of multiple vortex shedding. Simple logic is outlined by Beddoes and Leishman which controls the interaction of the different parts of the model during different flow conditions, in this way vortex shedding can be either initiated or stopped and allowed to decay or time constants can be modified during certain conditions such as during flow reattachment.

Finally the total normal and tangential force coefficients during unsteady condi-
Unsteady Lift Coefficient prediction by different components of the Beddoes/Leishman dynamic stall model

Figure 3.4: Lift coefficient prediction for the S809 aerofoil undergoing sinusoidal pitching; mean angle of attack 8 degrees +/- 10 degrees, Reynolds number of 0.99 million, Reduced frequency of 0.052.

Equations are given by:

\[ C_n = C_{n}^{sep} + C_{n}^{v} \]  \hspace{1cm} (3.61)
\[ C_t = C_{t}^{sep} \]  \hspace{1cm} (3.62)

Figure 3.4 shows plots produced using the 3 different parts of the Beddoes/Leishman model. The static 2D lift coefficient curve is for the S809 aerofoil. The other plots are for the same aerofoil undergoing sinusoidal oscillation with a Reynolds number of 0.99 million, and a reduced frequency of 0.052.

The "attached flow" plot is obtained using the first part of the model which comes from unsteady (linear) airloads based on Duhamel superposition. The "non-linear" plot results from the addition of the second part of the model (based on flow separation) to the attached flow equations. The final "vortex shedding" plot
is produced by the full dynamic stall model where the vortex shedding equations are added on top of the non-linear and attached flow equations along with suitable logic to control their application during dynamic events.

This model is used in AERODYN by Hansen and Pierce (Refs 44, 47). They used this model to replace the Gormont model (Ref 18) which was used in the early versions of AERODYN.

The implementation of the Beddoes/Leishman model into AERODYN by Hansen and Pierce was made with a few minor modifications. First, in order to make the model applicable over a range of angles of attack up to 360 degrees the ±90 degree range was mirrored into the ±180 degree range. Second, to improve the prediction of pressure drag, two different lookup tables for the separation point/angle of attack value were used, one for the normal force coefficient and one for the chordwise force coefficient.

A major advantage of the Beddoes/Leishman model is that necessary parameters for a given aerofoil can be deduced directly from static aerofoil data, hence the model needs no tuning to work for different aerofoil sections, although its accuracy with aerofoils specifically developed for wind turbine applications hasn't been extensively tested.

### 3.4 Chapter Summary

The dynamic analysis of wind turbine rotors for engineering purposes requires time dependent inflow models which are relatively simple and fast to compute.

The development of the acceleration potential method in its various forms has proved very popular for this application because it results in solutions which are complete closed form dynamic inflow models suitable for use with blade element / dynamic stall models. The theory also allows (in the case of the GDW theory)
the simulation of complex pressure distributions across the rotor, including the rotating pressure spikes of rotor blades, if this is the case then no tip loss model is required.

The major question which arises with the use of acceleration potential based methods is the assumption of a lightly loaded rotor / disc. The whole theory is built on a paradox. First it is assumed that all induced velocities in a flow field which are caused by a pressure drop across a rotor / disc are negligible. It is then possible for the pressure drop across that disc to be described by first order differential equations. This in turn allows the calculation of the "negligible" induced velocities in the flow field which are caused by the pressure drop across the rotor / disc.

Even when the linear models are adjusted so that the overall average induced velocity normal to the disc is calculated according to the non-linear momentum theory, the induced velocity distributions are still determined by the solution to the linearised Euler momentum equation.

The work of Van Holten and Van Bussel has been directed to overcome the limitations of linear acceleration potential methods but for the reasons stated they are outside the remit of this thesis.

The YAWDYN model used by the present author in NREL's code 'blind' comparison contained the Pitt and Peters inflow theory. This theory was subsequently coded (using MATHCAD) with and without Prandtl's tip loss model for further comparison to wind tunnel data and also to the theoretical results obtained from the YAWDYN model during the code comparison.

The GDW theory, (as part of AERODYN, after being corrected), was used for the modelling of the MS4 with ADAMS/WT. The theory was also coded into MATHCAD in two different forms for further comparison work against wind tunnel data. One version was the same as that used in AERODYN with 6 pressure
induced velocity distributions. The second version was extended and used 10
pressure induced velocity distributions, these distributions were carefully chosen
to capture the rotating pressure spikes of a 2 bladed rotor. Both versions of the
GDW theory were coded with and without the Beddoes/Leishman dynamic stall
model.

The Beddoes/Leishman dynamic stall model forms part of AERODYN and is also
used in GarradHassan’s BLADED code. This dynamic stall model was used in
the ADAMS/WT modelling of the MS4 and also with both the Pitt and Peters
and GDW dynamic inflow models during their comparison with wind tunnel data.
Chapter 4

Aeroelastic Modelling of the Wind Energy Group’s MS4 Prototype Turbine.

4.1 MS4 Turbine Description

The MS4 is a highly flexible design of horizontal axis wind turbine. It is a downwind, free yaw machine with 3 blades, rated at 600kW. It is essentially stall regulated and has a nodding hinge at the tower top to which the nacelle and rotor are attached. When combined with the yaw bearing this effectively results in a "gimbled" 3 bladed rotor.

Details of the turbine’s design and operation are given in the DTI report (Ref 52). The turbine displays a serious instability consisting of a low speed shaft torsional oscillation which occurs in high winds when the blades are operating in stall, this is of a large magnitude and a frequency of approximately 0.75Hz which matches the natural frequency of the drive train.
4.2 Software Description

The code used for the aeroelastic investigation into the behaviour of the MS4 is the ADAMS/WT software package. It is an extension of the commercially available mechanical dynamics software, ADAMS (Automatic Dynamic Analysis of Mechanical Systems) produced by Mechanical Dynamics Inc, (details are available in Refs 49, 50, 51). ADAMS/WT was developed under contract to the National Renewable Energy Laboratory, part of the US Department of Energy.

ADAMS/WT includes a wind turbine aerodynamic model (AERODYN) developed at the university of Utah. This model generates the forces on the blades of the wind turbine model constructed using ADAMS. Version 12 of AERODYN was used with ADAMS/WT to model the MS4. This version contains the GDW dynamic inflow model as coded by Suzuki and subsequently corrected by the present author.

The interface to ADAMS/WT has been developed to simplify HAWT model development, making use of dialog boxes and macros for building structural elements. However the user is still left with the task of establishing appropriate parameters for the components in a basic turbine model. Additionally the user must implement modifications through the standard ADAMS interface where the turbine design varies from the options available in ADAMS/WT, this is especially true for a design like the MS4 prototype.

This is not the first ADAMS/WT model constructed by the author. Models of a small (5kW) stall-regulated turbine and a medium sized (20kW) 2 bladed, teetered turbine (Ref 54) had been completed in the past. The model of the MS4 is without doubt the most complicated attempted to date however, owing to its novel and flexible design.
4.3 MS4 Model Construction

4.3.1 Tower Model

Both the tower and blades are modelled in ADAMS as tapered beams using a linear lumped stiffness approach. The standard ADAMS/WT model assumes that the tower is a cantilever beam which is axisymmetric along its vertical axis; this is adequate for most tubular steel wind turbine towers.

The parameters required by ADAMS/WT to construct a flexible tower model are listed as a function of height, (The X-axis is vertically upwards, the Z-axis is downwind), they are: Station height \( (m) \), Mass/unit length \( (Kg/m) \), Running Mass moment around the y-axis and around the z-axis \( (Kgm) \), Torsional Stiffness \( (Nm^2) \), Extensional Stiffness \( (N) \), Bending Stiffness around the y-axis and around the z-axis \( (Nm^2) \).

These parameters are then assembled by an ADAMS/WT macro into rigid bodies (number specified by the user), connected by stiffness elements defined by the matrix in Figure 4.1 \( (EI_z = EI_y \) for the tower, due to assumed symmetry). These stiffness elements define the application of a translational and a rotational action-reaction force between two rigid bodies based upon the structural information supplied per unit length for the beam. The transverse elements are derived from cubic shape functions and the axial and torsional elements are derived from linear shape functions. The 'l' and 'o' subscripts represent structural values at the points of force application on two adjacent rigid bodies (Ref 51). This gives 6 degrees of freedom between two adjacent rigid bodies. Hence the user defines the number of degrees of freedom of the tower by the number of rigid bodies specified. The damping of the degrees of freedom of the beam are given as a ratio of the stiffness values, in this case 1%.
\[
\begin{bmatrix}
\frac{1}{2L}(EA_1+EA_o) & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{6}{L^3}(El_{Zl}+El_{Zo}) & 0 & 0 & 0 & \frac{-2}{L^2}(2El_{Zl}+El_{Zo}) \\
0 & 0 & \frac{6}{L^3}(El_{Y1}+El_{Yo}) & 0 & \frac{2}{L^2}(2El_{Y1}+El_{Yo}) & 0 \\
0 & 0 & 0 & \frac{1}{2L}(GJ_{Y1}+GJ_o) & 0 & 0 \\
0 & 0 & \frac{2}{L^2}(2El_{Y1}+El_{Yo}) & 0 & \frac{1}{L}(3El_{Yl}+El_{Yo}) & 0 \\
0 & \frac{-2}{L^2}(2El_{Zl}+El_{Zo}) & 0 & 0 & 0 & \frac{1}{L}(3El_{Zl}+El_{Zo})
\end{bmatrix}
\]

Figure 4.1: Stiffness matrix used in lumped stiffness modelling.

4.3.2 Spar Model

Each blade shell of the MS4 turbine is attached to the turbines hub via a flexible glass fibre spar, each spar is around 5.2 metres in length and is mounted on the hub via a cast bracket. This hub bracket has two points of contact with the spar, one at the inboard end of the spar and one at 1.2 metres out from the hub. The spar itself resembles a long thin plank, orientated so as to make the blade root flexible in the flapwise direction but still extremely stiff in the chordwise direction. The blade shell is then mounted on the hub/spar via two bearings, one is a spherical bearing at the very end of the spar (5.2 metres out from the hub). The second bearing (further in towards the hub) joins the blade shell to the hub and not the spar. The bearings are shown as blue spheres in figures 4.3, 4.4 and 4.5. Since the spars are coned by 6.2 degrees and the blade shells are coned by 8 degrees (to the rotor plane) the inner bearing is able to be mounted on the upwind side of the spar but in line with the outer bearing and parallel with the blade shell reference axis, this means that when the blade pitches around these two bearings there is little translational movement of the blade shell, this inner bearing is also spherical but has an additional degree of freedom in that it can
slide in a direction parallel with the blade reference axis and this it does when the blade shell/spar bends.

The pitch of the blades is controlled by pitch actuators for start up and low wind speeds. These are attached between the hub and the trailing edges of the blade shells, during the modelling this is shown to introduce some pitch/flap coupling independently to each blade. This is suspected of contributing to the drive train instability although it is of a small magnitude (less than a degree with large flap movement). The pitch actuator is shown as a light blue 'rod' in figures 4.3, 4.4 and 4.5.

It was decided to model the spars in a similar manner to the tower, however this had to be done "by hand" as the spars are unique to the MS4 design. The spar was divided into 5 rigid bodies for the purpose of modelling. It was known that the spars were made from unidirectional fibreglass composite with a high fibre content (the fibres running along the length of the spar ie: radially outwards), so mass and inertial properties could be calculated for each part of the spar, similarly stiffness properties could be calculated in a similar manner as for the tower in the previous section. However these had to be calculated and input into the stiffness matrices (Fig 4.1) between each rigid body using the standard ADAMS interface. Also the various constraints (bearings) joining the hub/spar/blade shell structure together had to be created using the standard ADAMS interface.

4.3.3 Blade Model

The blade input file is more complicated than the tower input file as the assumptions made in the tower structure no longer apply for the blade structure. The elastic axis of the beam and centre of gravity of each rigid body created no longer have to lie on the reference axis of the beam. The elastic axis can be twisted to capture the blade's structural twist, also the twist of the elastic axis doesn't have
Flapwise Blade Stiffness

Figure 4.2: Blade Shell Mounting Spar Flapwise Stiffness distributions (Inboard blade shell stiffnesses continue to increase up to $2 \times 10^{-8}$ Nm$^{-2}$, left out for clarity).

To follow the aerodynamic twist of the blade, the two can be quite separate.

The parameters required by ADAMS/WT to construct a flexible blade model are listed by radial distance as: (The X-axis is spanwise, the Y-axis is towards the pressure side of the blade, the Z-axis towards the leading edge) Radius of Station (m), Mass/unit length (Kg/m), Running Mass Moment around the y-axis and around the Z-axis (Kgm), CG offset along Y-axis (m), CG offset along the Z-axis (m), Elastic axis offset along the Y-axis (m), Elastic axis offset along the Z-axis (m), Structural Twist (degrees), Torsional Stiffness (Nm$^2$), Extensional Stiffness (N), Bending Stiffness around the Y-axis and around the Z-axis (Nm$^2$), Chord Length (m), Chordwise aerodynamic centre offset (m), Aerodynamic Twist (degrees).

The geometric values such as chord length and aerodynamic twist were obtained from a manufacturing drawing of the blade shell. The mass distribution was obtained from a mass balance spreadsheet used by WEG to obtain the balance of the nacelle and rotor relative to the support hinge. Also obtained from this
Figure 4.3: 1st Flapwise bending mode of the ADAMS blade/spar/hub assembly (frequency = 0.55Hz)

Figure 4.4: 2nd Flapwise/1st Chordwise coupled mode (frequency = 1.49Hz).
spreadsheet were the offsets of the centre of gravity of each radial station, this was found to be negligible in the chordwise direction, but significant in the flapwise direction.

The running mass moment distributions were estimated by using the mass distribution as above coupled with the cross sectional area of the blade which was estimated using ellipses of major and minor axes matching the chord length and the thickness of the aerofoil section at the given radial station. Then, by calculating the second moment of area around each axis and by producing a material density obtained by assuming that the blade was made from a homogeneous material the estimated running mass moments were produced. In the absence of better information the elastic axis was assumed to be coincident with the blade reference axis in the chordwise direction and to lie on the chordline in the flapwise direction. This means that at the root of the blade shell the elastic axis follows the 'knuckle' in the blade shell that accommodates the spar assembly.

Figure 4.5: 2nd Flapwise/1st Chordwise coupled mode (frequency = 1.58Hz).
The initial flapwise stiffness used was the one provided by Aerolaminates Ltd, this was used to estimate a chordwise stiffness distribution by estimating a radius of gyration in the chordwise direction based on the flapwise stiffness and the cross sectional area of the blade at a given radial station.

The extensional stiffness of the blade shell was obtained from a value for the Youngs modulus of the blade material (assuming an homogenous material) estimated from the flapwise stiffness. This was multiplied by the cross sectional area of the blade at each radial station. This could well be in significant error due to the assumption of an homogenous material.

The torsional stiffness of the blade was obtained from the experimental result of a tip deflection for a given applied torque, the distribution of this stiffness along the span of the blade was estimated from the magnitude of the polar second moment of area at each radial station; the stiffness at each radial station being adjusted based on these values, until the total for the blade matched the measured experimental result.

This approach could well create significant error, not only due to the unknown internal composite structure, but also because the torsional stiffness of a beam moves away from being directly related to its polar moment of area as its cross section moves away from being circular. Hence the thinner blade sections towards the tip of the blade will have a lower torsional stiffness than estimated by the above method. However, the low torsional loads experienced by wind turbine blades compared to their stiffness makes this property less important.

The structural twist of the blade was assumed to match the aerodynamic twist except at the root where it was angled to match the orientation of the blade shell internal spar.

Stiffnesses and running mass moment distributions were all estimated based on ellipses matching the chord/thickness of the aero section at each radial station.
A flexible blade shell model was then created based upon this input file data. It was relocated into position relative to the hub and constraints created to attach the blade shell to the spar/hub/pitch actuator assembly in a manner which reproduced the degrees of freedom of the real assembly.

Access to details of the internal composite structure of the blade shell would have provided information which could have been used in conjunction with a software package like 3D Beam from Stanford University to build a more accurate blade shell model.

Once this assembly was created, the hub was rigidly fixed and an Eigen analysis performed on the whole assembly. The natural frequencies of the first flapwise and first chordwise modes were noted. Experimental frequencies for the first two modes of the hub/spar/blade shell assembly were kindly provided by NEG Micon UK along with a comment that the first chordwise mode had proven hard to ascertain it being highly coupled. In order to match the natural frequencies of the real assembly, the flapwise and chordwise stiffness distributions of the blade shell were altered in a trial and error manner by adjusting the location of the radius of gyration.

The resulting natural frequencies are given in table 4.1, the matching mode shapes are shown in figures 4.3, 4.4 and 4.5. As can be seen, the two mode shapes around 1.5 Hz are highly coupled confirming the experimental observation at this frequency.
4.3.4 Generator Model

The standard generator model used in ADAMS/WT is based upon a steady-state Thévenin equivalent circuit model that due to the structural flexibility of the ADAMS/WT model, where the 3-phase rotating axis system is represented in a global rotating reference system. Fortunately the implementation of the induction machine model had already been accomplished and hence the ADAMS/WT wind turbine model had a certain amount of user-friendliness.

4.3.5 Low Speed Shaft

The low speed shaft model was created and the gear model was incorporated into this model. The model includes the flexible blades and the torsion of the tower. The tower material and the gear model, the shaft model was also modified using the ADAMS/WT software.

Figure 4.6: WEGs MS4 600kW prototype wind turbine

Figure 4.7: Graphic representation of the ADAMS model of the MS4 turbine.
4.3.4 Generator Model

The standard generator model available in ADAMS/WT is based upon a steady state Thevenin equivalent circuit model. It was felt that due to the structural flexibility of the MS4 rotor, a model which captured some of the magnetising dynamics of the generator would be more appropriate.

For this Park's differential equations (Ref 26) were utilised, where the 3-phase rotating axis system is replaced by a stationary direct-quadrature reference system. Fortunately the implementation of this type of dynamic induction machine model had already been accomplished by the author in another ADAMS/WT wind turbine model. (Ref 58)

4.3.5 Low Speed Shaft Model

The low speed shaft of the MS4 is over 7 metres long due to the need for the flexible blades to clear the tower and also for the rotor to balance the generator/gearbox which are upwind of the tower. It consists of a steel tube of average internal radius of 0.156m and average external radius of 0.2132m. Based on these figures a flexible low speed shaft model was created consisting of 4 rigid bodies each connected to each other with 6*6 stiffness elements (4.1) in exactly the same way as the tower and the spar models, this shaft model was also constructed using the standard ADAMS interface in the same manner as the spar models.

The values of the elements of the stiffness matrices were determined in the same manner as for the tower, using the material properties of steel and the geometry described above. The mass and inertial properties were also calculated using this information.

The low speed shaft model was attached to the nacelle in the same manner as the real low speed shaft, with a single bearing at each end. The upwind (gearbox) end of the shaft was constrained by a single rotational degree of freedom and the
rotor end was constrained to 3 rotational degrees of freedom, this allows the shaft to flex under the loads imposed on it from the dynamics of the rotor, which was considered important because one of the instabilities being investigated involved the drive train of which the low speed shaft is the major component.

4.3.6 Other Turbine Components

Other values which had to be determined for accurate modelling of the aerodynamic performance of the MS4 were the yaw damping and the nodding stiffness and damping. Calculations for these values came from two sources, the yaw damping had been determined by WEG and is based on the rotational velocity ($\omega$ in degs/sec) squared at the yaw bearing which comes from consideration of the drag of the electrical yaw drives which when inactive in free yaw operation nevertheless provide damping by remaining attached to the yaw drive ring gear between the tower and nacelle, see equation 4.1.

$$\text{Torque}(Nm) = 1000 \times \omega \times \text{ABS}(\omega)$$  \hspace{1cm} (4.1)

The nodding stiffness and damping were also determined by WEG and the torque applied upon the nacelle is based upon the nacelle's tilt angle ($\delta$ in degs) and rotational velocity ($\omega$ in degs/sec), see equation 4.2, the constant 5250Nm is used to make up the difference between application of the torque (level Nacelle) and the point where zero torque is applied (3 degs nose down).

$$\text{Torque}(Nm) = 2800 \times \omega \times \text{ABS}(\omega) + 1750 \times \delta - 5250$$  \hspace{1cm} (4.2)

For the drivetrain stiffnesses, the Wind Energy Group determined the rotational
Various Components | Rotational Stiffness (referred to the Low Speed Shaft)
--- | ---
High Speed Shaft | 240 $MNm/\text{rad}$
Gearbox | 15 $MNm/\text{rad}$
Gearbox mountings | 55 $MNm/\text{rad}$
Nodding Hinge | 50 $MNm/\text{rad}$
Low Speed Shaft | 24.9 $MNm/\text{rad}$

Table 4.2: Rotational stiffnesses of major drive train components.

stiffnesses of the major drivetrain components. These are given in Table 4.2. These figures gave a total drive train stiffness of 6.704 $MNm/\text{rad}$. With the generator locked and with a total aerodynamic rotor inertia of 289375 $Kgm^2$, a drive train natural frequency of 0.76 $Hz$ was determined.

The nacelle of the MS4 had to be accurately modelled in terms of mass and inertia because of the way it is mounted on a nodding hinge which itself is mounted on a tower top casting which is free to yaw, (with damping and within certain wind speed limits). Fortunately the mass balance spreadsheet provided by WEG contained the positions of all of the major nacelle components relative to the hinge position.

It was found best to lump components together where possible in order to limit the number of rigid bodies that had to be created in ADAMS, the list of rigid bodies in the MS4 ADAMS model are given in Table 4.3.

4.4 Aerodynamic Model

4.4.1 AERODYN

Details of AERODYN as used with ADAMS are available in (Refs 50, 51)

The aerodynamic code used to generate blade forces with the completed ADAMS structural model of the MS4 are the AERODYN subroutines written by Hansen et al (Ref 50). The version used was 12, this version of AERODYN contains 2
<table>
<thead>
<tr>
<th>Rigid Bodies created in ADAMS</th>
<th>Turbine Parts Represented</th>
<th>Rigid body Masses (Total) Kg</th>
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</thead>
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<td>Spar Parts (5)</td>
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<td>Spar Brackets</td>
<td>Spar Mounting Brackets</td>
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<td>Actuators + Accumulators</td>
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<td>Tower Top Casting</td>
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<td>Yaw Bearing + drives</td>
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<td>Nacelle Hinge Pin</td>
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<td>Generator (non rotating)</td>
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</tr>
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<td>HSS (non rotating)</td>
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<td>Support Tube Bell Housing</td>
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<td>LSS support tube</td>
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<td>LSS Rotor end bearing</td>
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<td>Nacelle Ballast (nodding)</td>
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<td>Generator Space Frame</td>
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<td>Remainder</td>
<td>Nacelle Sundries</td>
<td>3029</td>
</tr>
</tbody>
</table>

Table 4.3: Rigid body list of the ADAMS MS4 model
major components, the "Generalised Dynamic Wake" model of dynamic inflow to
determine induction/inflow to the rotor, making use of the Beddoes/Leishman un-
steady aerodynamics/dynamic stall model to determine the dynamic blade forces
used by the inflow model.

The Generalised dynamic wake model as implemented in AERODYN by Suzuki
(Ref 61) was found to have a serious error in its application, Suzuki had applied
the inflow model in terms of a rotor based co-ordinate system with a zero azimuth
datum fixed in a horizontal position. The GDW as formulated by He is in a
wind based co-ordinate system where the zero azimuth datum is fixed by the
direction of the inplane wind component, obviously this datum will change as
the yaw/tilt/wind direction change, this is not the case in Suzuki's application.
The GDW needs to be calculated entirely in a co-ordinate system based on wind
direction or it needs a matrix transpose from the co-ordinate system in which it
is formulated to one fixed in the rotor frame of reference as is done by Peters and
Haquang (Ref 30) for the Pitt and Peters model. Suzuki has done two things
to try to solve the problem neither of which are satisfactory. He has extended
the range of definition for the wake skew angle to +/- 90 degrees so that the
azimuth definition is correct for either pure negative or pure positive yaw (given
his azimuth datum is horizontal). Also Suzuki has split the definition of the wake
skew angle into horizontal and vertical components again in the rotor co-ordinate
system. This is wrong and misleading, it means that even if the rotor is subject to
pure yaw the Sine L matrix will be zero thereby eliminating the sine components
of the inflow distribution in pure yaw, this means that Suzuki's model will never
produce the correct theoretical inflow distribution as formulated by He.

These problems were solved by the present author before using AERODYN v12.
First the zero azimuth position was derived for a given time step by consideration
of the inplane wind components already available in the code, the entire dynamic
inflow calculation was then carried out based on this azimuth datum (in the wind co-ordinate system). The split wake skew angle components were removed in favour of a single wake skew angle based on the inplane wind component direction and measured relative to the rotor plane. This resulted in Suzuki's extension of the range of wake skew angles into the negative being unnecessary.

The resulting corrected code was verified by running simplified test cases on a simple rotor design in YAWDYN (simple structural WT code using AERODYN aerodynamic subroutines).

The verification consisted of trying various fixed yaw and tilt angles in a constant, steady flow field to see how the inflow distributions varied in each case. Clearly if the resultant angle of incidence of the rotor plane to the oncoming flow remains the same then changing the amount of yaw and tilt will simply rotate the inflow distribution according to the amount of yaw and tilt present because the yaw and tilt angles will determine the azimuth position of the resultant inplane wind component. This proved to be the case with the corrected code but not with Suzuki's version where differences in the shape as well as the orientation of the inflow distribution were seen to take place.

Subsequently these corrections have been included in the AERODYN code available from NREL.

**Tower shadow model**

AERODYN contains a tower shadow model to model the effects of the wake of the tower and the effect that this has on the flow of air through the wind turbine rotor when the rotor is downwind of tower as is the case with the MS4.

In AERODYN the model is basically a deficit applied to the free stream wind horizontal wind vector of the form:
\[ \text{Velocity Deficit} = u_1 \cos^2 \left( \frac{\pi \cdot d}{2 \cdot b} \right) \quad \text{where: } d \leq b \]

\[ \text{Velocity Deficit} = 0 \quad \text{where: } d > b \]

Where \( u_1 \) is the centreline velocity deficit of the tower shadow, \( b \) is the half width of the tower shadow and \( d \) is the perpendicular from the wake centreline of the point in question.

For the model the user has to supply 3 values, \( b_{\text{ref}}, u_{1\text{ref}} \) and \( l_{\text{ref}} \). These represent the tower shadow half width and centreline velocity deficit at a reference distance \( l_{\text{ref}} \) downwind of the tower centreline. The values used for the ADAMS model were \( b_{\text{ref}} = 1.5m \) (tower radius = 0.5m), \( u_{1\text{ref}} = 0.2 \) and \( l_{\text{ref}} = 4.6m \), these were determined using guidance from the AERODYN reference manual (Ref 50).

The magnitude of the tower shadow velocity deficit is then calculated at a given point behind the tower (\( l \)) using the above and:

\[
\begin{align*}
    b &= b_{\text{ref}} \sqrt{\frac{l}{l_{\text{ref}}}} \\
    u_1 &= u_{1\text{ref}} \sqrt{\frac{l_{\text{ref}}}{l}}
\end{align*}
\]

The velocity deficit is then subtracted from the horizontal free stream wind component before the aerodynamic calculations take place.

### 4.4.2 Blade Element Data

In order to generate blade forces, the subroutines of AERODYN require lift and drag data for the aerofoil profiles used on the blades of the MS4. These are NACA 6 series laminar flow aerofoils and data for them was obtained from Abbott and
<table>
<thead>
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<th>Radial Distance</th>
<th>Span ($\delta R$)</th>
<th>Chord</th>
<th>Twist</th>
<th>Aerofoil</th>
<th>T/C</th>
</tr>
</thead>
<tbody>
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<td>1.812</td>
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<td>-</td>
</tr>
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<td>0.526</td>
<td>0.20</td>
<td>63-2</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 4.4: MS4 turbine blade element data

Von Doenhoff (Ref 11). The MS4 blade shell had been divided into 12 rigid bodies for the purposes of structural modelling, for aerodynamic modelling each rigid body was treated as an individual blade element. The aerodynamic properties of the MS4 blade as used for the ADAMS simulations are shown in table 4.4.

The aerofoil sections ranged from 30% thick near the root to 14% at the tip. By taking into account rotational speed, blade element radius and wind speed it was found that the blades of the MS4 experience Reynolds numbers of between 2 and 4 million. The lowest Reynolds number for which data was available from Abbott and Von Doenhoff was for 3 million. However the lower Reynolds number operation of the MS4 blades corresponds to operation in lower wind speeds and is therefore not so important as higher wind speed operation where the machines instability is seen to be excited. Aerofoil data for 4 million could have been prepared and used in the modelling process utilising a function in the AERODYN code to interpolate between force coefficient data sets based on operating Reynolds number during a simulation. This wasn't done but with hindsight this might have improved the accuracy of the modelling work especially with subsequent realisation of the dominating importance of the force coefficient data in calculating the blade loads on a wind turbine operating in high wind speeds.
A 3D correction such as Snel's model for delayed stall (Ref 40) was not applied to the force coefficient data. Such a model could have improved the blade force prediction at higher wind speeds when the blades are beginning to stall. However, Snel's model requires an empirical coefficient to be used in its application. This is a problem with a unique blade design such as that used on the MS4 as there is no easy guide as to the value of this coefficient.

5 different sets of lift and drag data were produced for different aerofoil thicknesses at a Reynolds number of 3 million and applied along the blade according to aerofoil section. Each set of data was extrapolated over 360 degrees range angle of attack by the Foilcheck utility program from NREL. Again in retrospect this utility program may not have been the best method of extrapolating the force coefficient data. Referring to section 7.2.4 the equations used in this utility seem to produce extrapolated force coefficients that are questionable, see figures 7.32 and 7.33. This utility program does however generate the Beddoes/Leishman dynamic stall parameters for each aerofoil data set with is a useful feature. Although an engineer using the Beddoes/Leishman model should really become familiar with the different aspects of this model and the origin of the model parameters.

The input subroutines of AERODYN allow interpolation between different sets of aerofoil data to allow for variations in blade chord to thickness ratios and aerofoil profile, this interpolation function was used for differing aerofoil sections and chord to thickness ratios but not for different operating Reynolds numbers.

The performance of the NACA 6 series aerofoils used on the MS4 blade is very sensitive to manufactured blade profile and surface roughness, there was no way of checking either of these factors on the blades fitted to the operating prototype. However they are the primary reasons why such aerofoils are now seldom used on commercial wind turbines, blade chord to thickness ratios also are now usually 18% or higher. Experience has shown that dependability and predictability in
different operating environments is more important than the improvements in performance obtained by using aerofoils like the NACA 6 series.

4.4.3 SNWind3D

This code is available from NREL and is used to generate the atmospheric "virtual wind" required by the AERODYN routines. It represents an expansion of the stochastic wind simulator, SNLWIND, developed by Paul Veers (Ref 31) of the Sandia National Laboratory. SNLWIND only simulated the longitudinal component of the wind under neutral flow conditions in rotationally sampled space. Using Veer's original computational kernel, Kelley (Ref 41) expanded SNLWIND to map the three components of the full wind vector in Cartesian coordinates. In the current code, now referred to as SNLWIND-3D, the turbulence is now scaled by the boundary layer scaling parameters of friction velocity, $u^*$ and the Richardson number stability parameter rather than the turbulence intensity. The incorporation of these parameters means that a wider degree of inflow conditions can be simulated.

It also provides simulations based on either the Kaimal or von Karman neutral-flow spectral models as specified in various drafts of the IEC-61400 Document, "Safety of Wind Turbine Generator Systems". (Ref 55)
Chapter 5

MS4 Modelling Results

5.1 Available Data: Field Test Results

5.1.1 Prototype Data Acquisition.

The MS4 turbine was installed and commissioned by January 1998. After commissioning several data sets were collected from the machine, these were either of 10 or 5 minute duration and recorded in varying wind conditions, the channels of data which were recorded from the machine were detailed in the testing documentation as being recorded from the following sets of transducers.

- Blade 1 in-plane load (kNm) measured on the blade mounting spar centreline 1.134m from the blade root (2.134m radius).
- Blade 1 out of plane load (kNm) measured on the blade mounting spar centreline 1.134m from the blade root (2.134m radius).
- Azimuthal position of Blade 1 (degrees).
- Rotational Speed (RPM).
- Nacelle Tilt angle (degrees).
• Nacelle Tilt Nodding Moment (kNm).

• Low Speed Shaft Bending Moment (kNm, X-axis), measured 0.717m downwind of the tower centreline.

• Low Speed Shaft Bending Moment (kNm, Y-axis), measured 0.717m downwind of the tower centreline.

• Met Mast wind speed (m/s).

• Met Mast wind direction (degrees N).

• Blade 1 Pitch position (degrees).

• Blade 2 Pitch position (degrees).

• Blade 3 Pitch position (degrees).

• Electrical Power (kW).

• Nacelle orientation (degrees N).

In order to verify the ADAMS model against the constructed prototype it was necessary to try and reproduce the channels of data given above. Most of the signals could be reproduced by tracking the positions of the relative rigid bodies in the model during a simulation. However for the low speed shaft and blade bending moments this was not adequate. It was necessary to identify the rigid body (in the lumped stiffness model of the relevant component) where the bending moments needed to be assessed and then this rigid body was split into two separate rigid bodies divided at the point of interest. The two rigid bodies were then rejoined with no degrees of freedom and the bending moment could then be requested at the fixed joint in the lumped stiffness model of the component.
5.1.2 Prototype Measured Campaigns.

There were 13 measured campaigns made available in digital form covering a range of different operating conditions, these conditions are listed as follows:

- 2 * 10 minute campaigns, with a high speed start up, 5 to 10 m/s wind speed.
- 5 * 10 minute campaigns, high speed generation, above 5 m/s wind speed.
- 2 * 5 minute campaigns, high speed generation, where there were large nodding forces.
- 2 * 5 minute campaigns, high speed generation, where the electrical power exceeded 900kW.
- 2 * 10 minute campaigns, high speed generation, where the average wind speed exceeded 15m/s.

To enable direct comparison between the field data collected from the MS4 prototype and the simulated forces from the ADAMS model of the turbine several constraints needed to be met. First, the data set should be from a period of 10 minutes constant operation, this is in order to match the spectral gap in the wind characteristics and also to match the length of wind file produced by the SNLWIND3D software. The algorithm of which is written to generate 10 minute turbulent wind files with consistent, stationary statistics.

This leaves 7 suitable campaigns for comparison, of these campaigns only 2 had an average power output over the 10 minutes which was greater than 125kW. The 5 "low power" campaigns contained periods of powered yawing and blade pitching which would have required control algorithms to be added to the ADAMS model whereas the 2 high power campaigns were conducted with free yaw and no blade pitching, the blades being at their stall-regulation setting. One of these campaigns,
8fc, where the average wind speed exceeded 15 m/s, was chosen for a comparative simulation using the ADAMS model.

5.1.3 Campaign 8fc, Wind Statistics.

A suitable wind file needed to be generated as an input file for the AERODYN subroutines in order to generate the aerodynamic forces on the ADAMS structural model of the MS4 prototype.

A 10 minute turbulent wind file was required with a statistical nature which matched as closely as possible the measured wind statistics. To do this the met mast channels of wind speed and direction from campaign 8fc were analysed.

From consideration of the wind speed and the difference between the instantaneous wind direction and the mean wind direction at each time step the mean longitudinal component of the wind speed was found to be 14.99 m/s. The standard deviation of the longitudinal wind component was then calculated and found to be 1.39. The turbulence intensity was then calculated by dividing the standard deviation by the mean and found to be 0.0927. Finally, assuming neutral stability and a flat terrain the friction velocity was estimated from 0.4 * the standard deviation, (Ref 27), giving $U^*$ as 0.556.

These statistics were then input into the SNLWIND-3D software which, using an algorithm with a random seed, produced a 10 minute file with the following statistics:

The power law exponent was 0.143, the mean horizontal and vertical wind speeds were zero and the friction velocity at hub height was 0.654, other statistics are given in table 5.1.
### Table 5.1: Wind file statistics for campaign 8fc simulation.

<table>
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<tr>
<th>Parameter</th>
<th>mean</th>
<th>peak</th>
<th>min</th>
<th>( \sigma )</th>
<th>Turb Intensity</th>
</tr>
</thead>
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<tr>
<td>Longitudinal wind comp</td>
<td>14.99</td>
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<td>10.87</td>
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<td>9.010%</td>
</tr>
<tr>
<td>Horizontal wind speed</td>
<td>15.02</td>
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<td>8.983%</td>
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<td>19.70</td>
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<td>1.348</td>
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</tr>
<tr>
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<td>-3.67</td>
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<td>6.487%</td>
</tr>
<tr>
<td>Vertical wind comp</td>
<td>0.00</td>
<td>2.79</td>
<td>-2.65</td>
<td>0.763</td>
<td>5.073%</td>
</tr>
</tbody>
</table>

#### 5.2 Simulation Results.

##### 5.2.1 Verification, Campaign 8fc

The comparison of modelling results with field test campaign 8fc provides an insight into the behaviour of the ADAMS model of the turbine, however the conclusions of the modelling are difficult to draw for several reasons. First, the complexity of the structural model makes any inaccuracies contained therein difficult to identify. Second, the aerodynamic model is untested with this turbine/rotor design as is the aerofoil data used as input into the aerodynamic subroutines. The MS4 rotor blades are a unique design and therefore it behaves in a manner which is difficult to predict. Thirdly the flow field used with the aerodynamic model is by necessity statistical and based upon neutral stability and flat terrain assumptions which are not strictly applicable to the top of a Welsh hill where the MS4 is installed.

Figure 5.1 shows the wind speed comparison between the field test and the wind file used as input to the ADAMS model of the MS4 for case 8fc. The windspeed histogram has a small bin size of 0.05m/s, it shows a reasonable correlation between the windspeed measured at the met mast (100 yards or so from the turbine) and that used as input to the model. The 1 dimensional wind speed recorded during the field test cannot be directly converted into a full 3 dimensional wind file, instead the 3D turbulent wind file used for modelling was created based on statistics from the measured wind file, this results in the slight differences between the
Figure 5.1: Campaign 8fc, 10 minute verification case, wind speed comparison.

two histograms in figure 5.1.

Figure 5.2 shows a comparison of power curves, both for case 8fc, one is the measured curve and the other is the theoretical curve from the ADAMS model. This plot has a bin size of 0.1m/s, it shows an increasing discrepancy as the wind speed increases with an almost constant power output from the turbine of a little over 600kW whereas the model shows an increasing power level rising to over 800kW at 18m/s. The model was recording rotor mechanical power whereas the signal from the field test was of generator active power, so maybe there were significant drive train or electrical generator losses which were unrepresented in the ADAMS model. However, these losses would need to vary with wind speed so it is more probable that the difference is caused by aerodynamics, either the aerodynamic model itself or the aerofoil data used as input. Without aerodynamic information from the turbine blades themselves it is impossible to tell.

The azimuthally averaged blade bending moments (Figure 5.3, with a bin size of 2 degrees), show a close correlation for the out of plane bending moments of
Figure 5.2: Campaign 8fc, 10 minute verification case, binned power curve comparison.

Figure 5.3: Campaign 8fc, 10 minute verification case, azimuthally averaged bending moment comparison.
the turbine and those of the model, however the inplane bending moments show quite a difference, the mean values appear similar but the field test results show a much larger amplitude over one revolution. The main causes of inplane bending moment are going to be the aerodynamic forces driving the turbine and the forces due to gravity. The aerodynamic forces will be fairly consistent with azimuth, excepting tower shadow effects, whereas the gravity induced ones will be wholly dependent upon azimuth.

Since the mean of both is similar the cause of the difference would seem to be gravitational. This is interesting, since it would seem to suggest that the real turbine blade is heavier than the figures supplied by WEG would suggest. If this is not the case then it could be due to the method of attaching the blade shell to the hub. It must be remembered that the strain gauge is attached to the intermediate spar between various bearings which will induce complex stresses in the spar.

The comparisons of yaw error and tilt angle (histograms in figures 5.4 and 5.5,
Figure 5.5: Campaign 8fc, 10 minute verification case, nacelle tilt angle comparison.

bin sizes of 0.25 degs), illustrate a reasonable correlation between field test and simulation. The difference in yaw error of about 10 degrees is probably due the fact that the wind direction for the turbine is measured on the met mast 100 yards from the turbine, with possible differences in wind shear and direction over the 100 yards. The tilt angle plots are similar although the modelled model nacelle made excursions to 4 or 5 degrees nose up which the real turbine did not, the reason almost certainly lies in the complex gimbled mounting arrangement of the nacelle and the differences in the real and simulated aerodynamics of the rotor, but further investigation is limited by the few available field test data signals.

The instability displayed by the MS4 is a drivetrain oscillation and figures 5.6 and 5.7 (both having a bin size of 0.25 degs) are interesting as they show lift coefficient and blade forces at blade element 9, (approximately 0.7R). Once a wind speed of 18m/s is reached the binned quantities become quite variable, indicating large unsteady aerodynamic loads produced by the Beddoes/Leishman dynamic stall
model. It is probable that such loads play an important part in the turbine's mechanical instability.

5.2.2 Unstable Operation

Field test results: Campaign 91d

None of the electronic results supplied by GarrahHassan on CD contained examples of the MS4 undergoing the low speed shaft oscillation which is the machines major instability. However, hardcopies of campaign 91d were available from the Centre for Alternative Technology (CAT) which owned and operated the MS4 at the time of the investigation.

Plots of the MS4 becoming unstable and shutting down are shown in figures 5.8 to 5.12. Plots include the wind speed at the met mast upwind of the MS4, the yaw error, the yaw angle, the nacelle tilt angle and the low speed shaft torque. It can
be seen that the low speed shaft torque begins to oscillate at around 145 seconds and finally the MS4 shuts itself down at around 165 seconds after 20 seconds of severe torsional drivetrain oscillations of approximately 0.75Hz.

The low speed shaft resonance of figure 5.12 captures the drivetrain instability of the machine, the frequency of this oscillation is approximately 0.75Hz which is very close to the calculated drive train natural frequency (generator locked) stated earlier as being 0.76Hz. The excitation of the natural frequency of the drivetrain appears to be a major component in the instability of the MS4. Insights into how the drivetrain oscillation is sustained and overcomes the inherent damping of the induction generator to which it is attatched can be provided by looking at results produced by the MS4 ADAMS model.
Figure 5.8: Met mast wind speed during unsteady operation field test, campaign 91d.

Figure 5.9: Yaw error during unsteady operation field test, campaign 91d.

Figure 5.10: Yaw angle during unsteady operation field test, campaign 91d.
Figure 5.11: Tilt angle during unsteady operation field test, campaign 91d.

Figure 5.12: Low Speed Shaft torque during unsteady operation field test, campaign 91d.
ADAMS model simulation

The wind speed plot of figure 5.8 shows that the wind speed during the unsteady event of campaign 91d has a mean value of approximately 17.5m/s. In order to investigate whether the ADAMS model also shares this instability, an SNWind3D wind file was created with a mean wind speed of 17.5m/s and an IEC Kaimal "B" turbulence model from the IEC 61400 part 2 standard on wind turbine safety simply to provide a wind profile with quite high turbulence to try and excite the structure of the MS4.

An 10 minute ADAMS model simulation was then run with this new wind file and it could be seen clearly from the results that the ADAMS model has a similar low speed shaft instability to the MS4. In fact the instability was exhibited in the first 40 seconds of the simulation. Figures 5.13 to 5.16 show selected outputs from the ADAMS model over 20 seconds of the simulation, the wind speed (averaged from the 3D wind field file), the yaw error, the yaw and nacelle tilt angles and the low speed shaft torque.

Low Speed Shaft oscillation, comments:

If the ADAMS simulation results and the field test data are compared some possible causes for the MS4s instability can be developed. The wind speed during the ADAMS plots is slightly higher than for the field test plots resulting in higher LSS torque values (plus the uncertain effects of using 2D aerofoil data untuned to the blade design to model stalled aerodynamics). In both cases it is noticeable that the rotor yaws and tilts at quite a fast rate before and during the low speed shaft oscillations. It appears that rapid changes in wind direction result in rapid yawing taking place (in the ADAMS simulation first in one direction then the other), because the nacelle is hinged on top of the yaw casting this results in significant gyroscopic loads being developed by the rotating blades which in turn
Figure 5.13: Wind speed during a period of unstable operation of the ADAMS model simulation.

Figure 5.14: Yaw error during unsteady operation of the ADAMS model simulation.
Figure 5.15: Yaw and Tilt angles during unsteady operation of the ADAMS model simulation.

Figure 5.16: Low Speed Shaft torque during unsteady operation of the ADAMS model simulation.
causes rapid tilting of the nacelle, it appears that this rapid change in orientation of the rotor plane is caused by a rapid and quite large change in wind direction. Another factor which plays a part in this process is negative aerodynamic damping, because the MS4 is designed as a stall regulated turbine the blades are operating well into stall in wind speeds between 15 and 20 m/s. Coupled with the fairly low (by modern standards) thickness to chord ratio of the NACA 6-series aerofoils used (which themselves have quite severe stalling properties) all adds up to the blades being subject to significant negative aerodynamic damping.

Referring to the mode shapes which were derived for the complex blade/spar/hub arrangements, the effect of the negative aerodynamic damping coupled with the gyroscopic loads producing Coriolis accelerations of the blades around the low speed shaft together with highly coupled flapwise/chordwise modes means that the blades flex in both the flapwise and chordwise directions, it is impossible to note this effect from the data collected from just one blade as is the case with the prototype where only one blade is instrumented, however with the ADAMS model it is possible to produce outputs for various structural and aerodynamic variables. This has been done in figures 5.17 to 5.20.

Before the low speed shaft oscillation begins, the variations in both the chordwise and flapwise moments of the 3 blades are pretty much 120 degrees apart resulting in reasonably constant thrust and torque loadings on the rotor as a whole. However, as can be seen in figures 5.17 and 5.18, as the oscillation begins the bending moments on the blades begin to exhibit higher frequency oscillations due to the highly coupled chordwise/flapwise modes illustrated in figures 4.4 and 4.5. Figure 5.19 shows the combined blade moments for the rotor and this clearly shows that the driving force of the torsional oscillation comes from the combined response of the 3 blades and their coupled modes. The chordwise bending moments are transmitted directly to the low speed shaft but the flapwise moments also contribute.
through Coriolis accelerations around the low speed shaft because the rotor is also yawing and nodding relative to the ground.

Figure 5.20 shows the relationship between angle of attack and lift coefficient at the 9th blade element of blade 1 (approximately 70% span there being 12 equi-distance blade elements on each blade in the ADAMS model). The angle of attack varies between 0.2 and 0.45 radians (12 to 25 degrees), which in terms of 2D aerofoil data is a range from light separation through to almost total separation and this is reflected in the lift coefficient plot for the element which illustrates the negative aerodynamic of the blade element since the peaks in the angle of attack plot correspond to troughs in the lift coefficient plot and hence the blade element force.
Figure 5.18: Individual Blade Flapwise moments during a period of unstable operation.

Figure 5.19: Total Flapwise and chordwise Blade moments during a period of unstable operation.
Figure 5.20: Lift Coefficient and angle of attack at the 9th blade element during unstable operation of the ADAMS model simulation.

5.3 MS4 Modelling Conclusions

It is apparent from the results of the modelling that the versatility of the ADAMS software in building mathematical models of complex structures and then simulating these models dynamically with a complicated range of input forces has allowed the instability of the MS4 prototype to be modelled successfully.

However, even with an ADAMS model constructed using manufacturing drawings and detailed specifications, there still exist substantial discrepancies between the results from the ADAMS model simulation and from the measured loads on the MS4 over a 10 minute simulation (campaign 8fc). It is hard to try and determine the causes of these discrepancies from further analysis of the MS4. It is a highly flexible design with quite large structural responses to aeroelastic forces which cloud the investigation of the rotor aerodynamics. Also the MS4 is operated (like all turbines of any size) in an atmospheric environment where the fluid which
provides its driving forces is subject to turbulence, shear, rapid changes of direction and a continual variation in velocity across the swept area of the rotor, this can only be modelled statistically and again clouds any clear perception of a rotor's aerodynamic performance. In the particular case of the MS4 there are only a limited number of data acquisition channels available, only blade 1 is instrumented for blade bending moments and there is certainly no direct aerodynamic information available from the prototype.

With hindsight it would have been interesting to see the effects of varying the aerodynamics applied to the MS4 model, for instance a stall delay model such as Snel's could have been applied to the blade element data used to drive the model to see the effect this had on the response of model. Different data sets representing different aerofoils with gentler stalling characteristics could even have been tried out on the blades to try and improve the behaviour of the machine.

Analysing the sensitivity of the model to varying model components could have been carried out. This would have been time consuming but would have given insight into which of the identified shortcomings of the MS4 was most responsible for the machines instability. For example the drivetrain stiffness could have been varied to see if the drivetrain oscillation is eliminated or changes frequency.

Various other parameters could also have been varied to see their effect on the models response, such as the blade shell/spar stiffnesses, the rotational speed of the rotor and the yawing and nodding stiffnesses and damping. By varying these components and maybe even the machines design itself it may have been possible to identify an individual MS4 design flaw rather than a combination of factors.

However, since the entire range of software used to model the MS4 is already available for commercial use (albeit with the documented correction by the present author) the further investigation of the MS4 prototype with the ADAMS model becomes an exercise in engineering design analysis rather than a valid piece of
academic research.

One conclusion which can be drawn from the MS4 is that light flexible structures and stalled rotors are a combination where successful design is very difficult and given commercial considerations is one which may even be best avoided.

5.4 Influence on Future Work

It is clear that the area of greatest uncertainty in the MS4 modelling work is the aerodynamic model implemented in AERODYN. It is not possible with the MS4 model to take the validation of this aerodynamic model much further. What is required is highly detailed aerodynamic data from a large scale wind turbine rotor operating under controlled conditions. This will allow the AERODYN code and other aerodynamic models which are used for engineering purposes to be compared directly to experimental results.

Chapter 6 details experimental research carried out by the National Renewable Energy Laboratory (NREL) of the US. The aim of their work was to provide experimental results on a wind turbine of reasonable size which is free from most of the limitations usually encountered in trying to carry out academic research into wind turbine rotor aerodynamics. Following on from the wind tunnel testing NREL issued an invitation for a code 'blind' comparison to the experimental results which had been obtained. This invitation was taken up by many researchers around the world including the present author. The results and conclusions from the code comparison and the accompanying meeting at NREL provide the basis for the rest of the work presented in this thesis, namely the assessment of the effectiveness of different engineering aerodynamic model components.
Chapter 6

NREL - NASA Ames Wind Tunnel Test

Since 1987 the National Renewable Energy Laboratory of the US (NREL) has maintained an ongoing project called the Unsteady Aerodynamics Experiment (UAE). The aim of this project has been the development of systems for highly accurate data acquisition from a full scale HAWT in the field. The collected data being used for theoretical comparisons and to provide insight into HAWT performance.

The latest phase of the UAE (since 1998) saw the turbine completely rebuilt with state of the art data acquisition equipment in preparation for wind tunnel testing. The redesigned turbine was extensively tested in the NASA Ames full scale wind tunnel facility in California during May 2000. A small part of the data acquired from these tests forming the basis for an aeroelastic code 'blind' comparison in December 2000. 17 different wind turbine aerodynamics research groups from Europe and the US, both from academia and industry took part in this landmark code comparison. A total of 20 different models of the UAE turbine were employed representing BEM models, Acceleration Potential models, Vortex Wake models
and CFD models. Details of the participants and their chosen methods are given in Figure 6.1.

As the NREL website states 'The aim of the wind tunnel testing was to acquire accurate aerodynamic and structural measurements on a wind turbine that is geometrically and dynamically representative of full scale machines in an environment free from pronounced inflow anomalies.' Meaning that the controlled flow conditions in the wind tunnel will not have the complex structure of the meteorological free wind as was the case for earlier phases of the experiment.

6.1 NASA Ames Wind Tunnel

The UAE turbine wind tunnel tests were undertaken in NASAs 24.4 by 36.6 meter (80 foot by 120 foot) wind tunnel. This tunnel is part of the National Full-Scale Aerodynamics Complex (NFAC) which is located at the NASA Ames Research Center in Moffett Field, (Silicon Valley) California. The tunnel is primarily used for determining low- and medium-speed aerodynamic characteristics of full-scale aircraft and rotorcraft. The tunnel is powered by six 18,000-hp fans that produce test section wind velocities up to 50 m/s (115 mph).

Figure 6.3 shows how the NASA Ames wind tunnel was used for the testing of the UAE turbine, the test section of the tunnel having the air intake upwind and the fans driving the air flow downwind, the length of the open jet wind tunnel being approximately 500m long from inlet to outlet. Schreck (Ref 65) states that across the test section the flow speed varies by less than 0.25% and the turbulence intensity is typically less than 0.5%. Schreck also estimates that due to the large cross sectional area of the test section the boundary effects and the blockage effect of the presence of the turbine were substantially less than 1%.
<table>
<thead>
<tr>
<th>Legend Identifier*</th>
<th>Participants (Attendees in Bold)</th>
<th>Organizations</th>
<th>Codes</th>
<th>Incomplete Output Files</th>
<th>Code Type</th>
</tr>
</thead>
</table>
| UIUC/Enron-C*     | Michael Selig  
|                   | Philippe Ciglere | University of Illinois/Enron | PROPID-C  
|                   |                    |                             | PROPID-UI  
|                   |                    |                             |                | Performance model with BEM and Corrigan stall delay model  
|                   |                    |                             |                | Performance model with BEM and UIUC stall delay model  
| ROTABEM-DTU*      | Martin Hansen  
|                   | Takis Chaviaropoulos | Tech Univ of Denmark/CRES Greece | ROTABEM  
|                   |                    |                             |                | Performance model with BEM using 3-D corrected airfoil data based on a quasi-3D Navier-Stokes solver  
| Loughborough University | James Shawler | Loughborough University CREST UK | YAWDYN | No dynamic pressure. | Aerodynamic model with rigid blade flapping hinge blade using Aerodynl aerodynamics (BEM/Leishman-Beddoes)  
| Windward (1)      | Craig Hansen, Dave Laino | Windward Engineering | ADAMS |                | Multi-body aeroelastic model using Aerodynl aerodynamics (equilibrium wake, BEM/Leishman-Beddoes)  
| Windward (2)      |                    |                             |                |                | Multi-body aeroelastic model using Aerodynl aerodynamics (dynamic inflow, BEM/Leishman-Beddoes)  
| Windward (3)      |                    |                             |                |                | Multi-body aeroelastic model using Aerodynl aerodynamics (dynamic inflow and delayed stall, BEM/Leishman-Beddoes)  
| Garrad Hassan     | Robert Rawlinson-Smith | Garrad-Hassan | BLADED | No pitch moment, root bending, torque, yaw moment. | Assumed modes aeroelastic model with BEM/Beddoes-Leishman aerodynamics  
| NASA Ames         | Wayne Johansen | NASA Ames | Campd II |                | Aero-mechanical rotorcraft analysis tool  
| RISOE & HAWC      | Hege Madsen | Riso | HawC | No yaw, | Aeroelastic model with BEM theory  
| RISOE & HAWC/3D   |                    | HawC-3D | Yaw only. |                | Aeroelastic model with 3D CFD actuator disc model  
| ECN               | Herman Snel, Koert Lindenburg | ECN | PHATAS |                | Aeroelastic model with BEM theory  
| Teknikgruppen AB  | Bjorn Montgomery, Anders Bjorck, Hans Canander | FFA/Nordic Windpower/Teknikgruppen | VIDYN |                | Aeroelastic code with IAERFORCE aerodynamics and IDynStall dynamic stall (BEM/Leishman-Beddoes)  
| Riso NNS          | Niels Sorensen | Riso | EllipSys3D | No yaw, no downwind | 3D incompressible Navier-Stokes model  
| DTU1              | Robert Mikkelson | Tech.Univ. of Denmark | ADDWANS | No pitch moment, no yaw, no downwind | Combined Navier-Stokes BEM approach  
| Georgia Tech      | Lakshade Sankar  
|                   | Guanpeng Xu | Georgia Tech | Hybrid CFD | No yaw, no downwind, | 3D multi-domain analysis unsteady Navier-Stokes model  
| CRES-NTUA         | Michael Beleas  
|                   | Spyros Voutsinas | CRES-NTUA | GENUVP | No pitch moment, root bending, torque, yaw moment, downwind. | Vortex element, free wake model  
| Glasgow University | Frank Coton, Tongguang Wang, Roddy Gharb | University of Glasgow | HawtDawg |                | Prescribed wake model (Horizontal Axis Wind Turbine Directly Allocated Wake Geometry)  
| TU Delft*         | Gerard Van Ransel | Tech. Univ. of Delft | PREDICDYN | No S1500600, no pitch moments. | Asymptotic acceleration potential model  
| CRES-NTUA NS3D    | Michael Beleas  
|                   | Spyros Voutsinas | CRES-NTUA | NS3D | S1000000 only, Ca, Ct only. | 3D steady RANS finite volume formulation, Chimera mesh only.  

* Did not provide documentation as specified in Blind Comparison Overview
Figure 6.2: UAE turbine under test in the NASA Ames Wind tunnel during a smoke test, demonstrating the minimal turbulence in the test section.

80x120 MODE OF OPERATION

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Figure 6.3: NASA Ames wind Tunnel Schematic showing the open jet 80' x 120' test section
6.2 NREL Wind Turbine Configuration

6.2.1 Turbine Specification

For the wind tunnel tests the turbine was completely rebuilt, among the components redesigned were the tower, the rotor hub, the rotor blades and the yaw bearing/brake. Only the generator, drive train and nacelle being retained from the earlier field test machine.

The structure of the turbine was designed to be as rigid as possible in order to minimise aeroelastic response during the wind tunnel tests. The turbine blades themselves being of carbon fibre construction with a first static-flapwise natural frequency of 7.3Hz.

The rebuilt turbine had a hub height of 12.2m (placing the rotor in the centre of the test section), the rotor could be run upwind or downwind of the tower in either a rigid or teetering configuration combined with a variable cone angle. The blades had fully variable pitch angles, the pitch angles being controlled by electric servo motors. There was no rotor tilt option. The machine was rated at 20kW and with its induction generator was operated at a fixed synchronous speed of 71.6rpm, although there was a variable speed option this was not employed in most test cases. NREL tested 160 different steady state flow conditions in the wind tunnel (20 of which were used in the code comparison), as well as many dynamic cases including various yaw rates and blade pitch rates and blade pitch step changes.

6.2.2 Rotor Configuration

The rotor for the wind tunnel tests consisted of a 2 bladed design with both twisted and tapered blades the design of which is documented in (Ref 57). The twist distribution of the blade is shown in figure 6.4, the pitch of the blade is
defined at the 0.75R spanwise position. Included in the diagram is the twist of the blade tip extensions out to the 1.1R spanwise position, these extensions were not used in collecting the test data used in the code comparison. The chord length has a linear taper from 0.737m at 0.25R to 0.356m at the tip (1R).

The aerofoil used along the length of the blades is one of the SERI designs (S809) which was designed specifically for use on wind turbines. The aerofoil is consistent along the length of the blade with no changes in the chord/thickness ratio of 21%. The tip radius of the blades is 5.029m giving a rotor diameter of 10.06m, the hub radius to the first true aerofoil section is 1.257m (0.25R), however there is a transition from a cylindrical shape at a radius of 0.66m to the previously defined S809 aerofoil at 1.257m.
6.3 NREL Aerodynamics Code Blind Comparison

The author accepted the invitation from NREL to take part in the code comparison of theoretical results to experimental results. This work had to be carried out by November 2000, in preparation for the meeting of research groups in Boulder, Colorado in December.

For comparison purposes 20 different steady state flow cases (cases 1-14 with the rotor upwind of the tower and cases 15-20 with the rotor downwind of the tower) with the turbine operating in the wind tunnel were analysed by NREL who produced results averaged over one revolution and given per degree of azimuth. The results from the modelling work carried out by the different research groups on the 20 different flow cases were compared with this averaged experimental data by NREL in preparation for the meeting in December. None of the research groups had any access to any of the experimental results during their modelling work or before the meeting in Boulder, Colorado.

6.3.1 Model Description

It was decided by the author to use the YAWDYN structural code for the wind tunnel comparison work, it is a relatively simple structural code with limited degrees of freedom allowing a model to be quickly constructed. However it shares with ADAMS the same AERODYN aerodynamic subroutines. This work was carried out before the release of AERODYN v12 however so v11 of AERODYN was used with the YAWDYN model of the NREL machine. Version 11 includes the Pitt and Peters inflow model with Prandtl’s tip loss model and the Beddoes/Leishman dynamic stall model.

The YAWDYN model of the UAE turbine was constructed with a rigid hub/flapping
<table>
<thead>
<tr>
<th>Test Case</th>
<th>Blade Flap Angle (degs)</th>
<th>Blade Pitch Angle at 0.75R (degs)</th>
<th>Yaw Angle (degs)</th>
<th>RPM</th>
<th>Wind Speed (m/s)</th>
<th>Air Density (Kg/m³)</th>
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<td>72.1</td>
<td>17.2</td>
<td>1.234</td>
</tr>
</tbody>
</table>

Table 6.1: 20 test cases used for the NREL code 'blind' comparison
hinges for the blades, the rotor itself had a fixed rotational speed and a fixed yaw angle. The mass distribution of the UAE blade in conjunction with the first measured flapwise frequency was used to fix the stiffness value of the flapping hinge. The model only had 2 structural degrees of freedom (flapping hinges).

For the purposes of modelling the rotor the blade had to be divided up into discrete blade elements, these blade elements are detailed in table 6.2.

Blade Elements 2,5,8,11 and 14 have a specific span (6R) of 0.2m and were carefully selected to match the spanwise positions at which the instrumented blade had extensive pressure tappings, see figure 6.5. Chord and Twist are averaged over the blade element and the radial distance for an element is from the centre of rotation to the centre of the blade element.

Various sets of measured aerofoil data for the 21% thick S809 were provided by NREL in a preparatory document produced for participants in the code comparison, (Ref 63). These data sets came from several different wind tunnels and covered Reynolds numbers from 0.3 to 1 million. In looking at the operation of the NREL turbine at wind speeds between 7 and 25m/s it was determined that

<table>
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<th>Element No</th>
<th>Radial Distance</th>
<th>Span (δR)</th>
<th>Chord</th>
<th>Twist</th>
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</table>

Table 6.2: UAE turbine blade element data
the blades experienced Reynolds numbers from 0.6 to 1.3 million. Only the blade root experienced Reynolds numbers lower than 0.8 million at wind speeds less than 10m/s.

Measured aerofoil data from wind tunnel tests at the Delft University of Technology was chosen to be used as input to the YAWDYN model of the NREL turbine. This data was measured at a Reynolds number of 1 million and covered an angle of attack range between -1.04 to 17.21 degrees.

To be of use in modelling wind turbine aerodynamics measured aerofoil data frequently needs to be extrapolated to negative and post stall angles of attack. Thefoilcheck software (Appendix D of Ref 50) is a utility program written for this purpose, it extends the measured data over a complete 360 degrees angle of attack range. It also adjusts the data for to the aspect ratio of the blade design in question, this has been shown in earlier chapters to be theoretically incorrect (2D
<table>
<thead>
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<th>Value</th>
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<td>Stall angle of attack (deg)</td>
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</tr>
<tr>
<td>Zero lift angle of attack (deg)</td>
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<tr>
<td>$C_n$ at stall angle</td>
<td>1.89</td>
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<tr>
<td>Angle of attack for minimum drag (deg)</td>
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</tr>
<tr>
<td>Minimum drag ($C_d$)</td>
<td>0.0094</td>
</tr>
</tbody>
</table>

Table 6.3: Dynamic stall model parameters

data should be used) and raises questions about the methods used in Foilcheck to extrapolate measured data. However, the Foilcheck software does calculate the necessary parameters for the Beddoes/Leishman dynamic stall model used in the AERODYN subroutines (used in conjunction with the YAWDYN code). The dynamic stall parameters used in the code comparison are given in table 6.3.

At the time of the code comparison the Foilcheck extrapolation of the Delft university measured aerofoil data was used with YAWDYN in the modelling of the NREL turbine. Subsequently doubts arose about the methods of extrapolation used in the foilcheck software especially the aspect ratio adjustment of the data according to the blade design. After the NREL code comparison this data (labelled “aspect adjusted” in figure 6.6) was no longer used and for the results achieved in subsequent modelling of the NREL turbine (chapters 7 and 8) a different approach to extrapolating the data was taken, this involved extrapolating the Delft measured data with measured post stall data from Critzos et al (Ref 16). Even though this 2D post stall data is produced for the NACA 0012 aerofoil it was felt that it would still provide a better set of high angle of attack aerofoil data than the Foilcheck program.

Figure 6.6 shows the two sets of extrapolated data for the S809 aerofoil, one is the data set produced by Foilcheck (used to obtain the results in this chapter only). The second set labelled '2D' comes from the data produced by Critzos et al and is used in obtaining the results in chapters 7 and 8. The dynamic stall parameters
are not affected by this change in the extrapolated range of aerofoil data but the post stall unsteady force coefficients produced by the Beddoes/Leishman model will be affected due to the change in steady data at high angles of attack.

6.3.2 Code Comparison Results

Fig 6.7 shows a plot of rotor torque against wind speed from the unyawed test cases 1 to 6. It is clear that there is a wide spread of results from the different models employed by the different research groups. The YAWDYN model employed by the author gives a good estimate of the rotor torque up to a wind speed of 10m/s. The design tip speed ratio of the blades is around 5 which corresponds to a wind speed of approximately 8m/s. This means that at 10m/s and above the fixed speed turbine is operating at progressively lower tip speed ratios (lower than 5) with the blades becoming more and more stalled.

As can be seen in fig 6.7, most of the models in the code comparison perform reasonably below 10m/s but the spread of results diverges dramatically once sig-
Figure 6.7: Experimental and theoretical low speed shaft torque developed by the UAE turbine during fixed wind speed tests.

Significant flow separation and 3 dimensional effects begin to occur at the higher wind speeds.

The author's YAWDYN model appears to perform quite well at most of the wind speeds, however further investigation reveals that the force coefficients measured experimentally along the blade have values which differ significantly from those predicted by the model.

Fig 6.8 shows the total blade bending moment for the instrumented blade during the same 6 test cases. The spread of results for this plot doesn't have such a dramatic divergence of values at 10m/s but the results from the various models do diverge as the wind speed increases.

The author's YAWDYN model again performs reasonably well, however it does display an over-prediction of bending moment at lower wind speeds with an under-prediction at higher wind speeds, this is obviously not a good result for a model
Figure 6.8: Experimental and theoretical blade root bending moments on the UAE turbine during steady wind speed tests.

intended for use in engineering design work.

Fig 6.9 reveals the main reason for the under-prediction of blade bending moments by the YAWDYN model at higher wind speeds, it shows the normal force coefficient at the 0.3R spanwise position.

The YAWDYN model has no delayed stall model and so it is limited to the static coefficient values when modelling a steady state condition. As can be seen the experimental results reveal significant delayed stall effects resulting in much higher normal forces on the blade near the root at higher wind speeds.

Fig 6.10 shows the normal force coefficient values at the 0.8R spanwise position, there is little delayed stall effect this far out along the blade and the YAWDYN model prediction of the coefficient values is reasonably accurate. However by studying the normal force coefficient values at the other spanwise positions at the different wind speeds it seems the delayed stall effect is detected as far out from
Figure 6.9: Experimental and theoretical normal force coefficient at the 0.3R spanwise position on the UAE turbine during steady wind speed tests.

the blade root as 0.5R, resulting in under-prediction of the normal force values out to this point. At around the 0.75R position the normal forces are generally well predicted, but from there out to the tip a real tailing off of the experimental normal force coefficient values can be seen resulting from the loss of circulation around the blade near the blade tip which is not accounted for theoretically.

The combined effect of delayed stall at the blade root and the loss of circulation at the blade tip is to quite dramatically change the flow physics around the blade from that assumed in formulating the theoretical model. The fact that the blade bending moment predictions are quantitatively reasonable is coincidental and is rather like adding 1 and 3 and arriving at 4 instead of the 2 plus 2 that you had assumed in the theory.

Figs 6.11 and 6.12 show some of the results from test case 12 (upwind configuration, 15m/s, 30 degree yaw), normal force coefficients over one revolution at 0.3R
Figure 6.10: Experimental and theoretical normal force coefficient at the 0.8R spanwise position on the UAE turbine during steady wind speed tests.

and 0.8R respectively. The YAWDYN model again under-estimates the value of the 0.3R force coefficient owing to the lack of a delayed stall model. Qualitatively at both the 0.3R and 0.8R positions the initial dynamic stall event is captured but there appears to be extended dynamic stall effects throughout the time the blade element is above the theoretical static stall angle, the dynamic stall model seems to miss these effects.

Figs 6.13 and 6.14 show some of the results from test case 14 (upwind configuration, 15m/s, 60 degree yaw), normal force coefficients over one revolution at 0.3R and 0.8R respectively. The 60 degree yaw seems to be dominated by the advancing and retreating blade effect. The YAWDYN model achieves reasonable prediction of the force coefficients with the exception of a quite severe over-estimate of the 0.3R normal force coefficient while the blade element is above the theoretical static stall angle.
Figure 6.11: Experimental and theoretical normal force coefficients at the 0.3R spanwise position with the turbine yawed by 30 degrees, 15m/s steady wind speed.

Figure 6.12: Experimental and theoretical normal force coefficients at the 0.8R spanwise position with the turbine yawed by 30 degrees, 15m/s steady wind speed.
Figure 6.13: Experimental and theoretical normal force coefficients at the 0.3R spanwise position with the turbine yawed by 60 degrees, 15m/s steady wind speed.

Figure 6.14: Experimental and theoretical normal force coefficients at the 0.8R spanwise position with the turbine yawed by 60 degrees, 15m/s steady wind speed.
Figure 6.15: Experimental and theoretical normal force coefficients at the 0.47R spanwise position showing the effect of tower shadow, 7 m/s steady wind speed.

Figs 6.15 and 6.16 show the 0.47R normal force coefficients for test cases 15 and 16 concentrating on the tower shadow effect. For case 15 where the rotor is operating near its design tip speed ratio it appears that the velocity deficit caused by the presence of the tower is under-estimated both in width and in strength by the YAWDYN model.

At the higher wind speed (test case 16) there is a slight delayed stall effect raising the normal force coefficient anyway but the tower shadow effect appears to be offset by 10 degrees or so and be of a lower magnitude than at the lower wind speed, perhaps due to the delayed stall effects.

6.4 Chapter Summary

The results from the code comparison presented in this chapter make it clear that there are several areas of wind turbine rotor aerodynamic performance which are
Figure 6.16: Experimental and theoretical normal force coefficients at the 0.47R spanwise position showing the effect of tower shadow, 17m/s steady wind speed. 

not fully understood or modelled accurately by the current engineering models. In particular, in the unyawed cases, there seems to be large differences between the predicted and the experimental force coefficients especially in terms of spanwise distribution. Surprisingly, the predicted rotor torque and blade bending moments seem to be reasonably accurate. However, if the spanwise distribution of blade forces are not accurate then the result of their integration can not be relied upon and the accuracy of the predicted torque and bending moment results must be purely coincidental.

If the simplest flow cases (unyawed, with a steady wind) are not being predicted with any consistent accuracy by the engineering models available then there is no hope of the models producing consistently accurate results with more complicated flow cases. It is necessary to understand the limitations of the different components of the engineering models in order to try to determine their effect on results.
obtained through their use. Only by doing this against accurate experimental data is it possible to draw definite conclusions and improve their accuracy. An attempt at this process is presented in chapters 7 and 8.
Chapter 7

Theoretical comparisons to the NREL wind tunnel data;

Unyawed Test Cases

In order to carry out an assessment of the aerodynamic theories, (both steady state and dynamic), which are presented chapters 2 and 3 it was decided to use the turbine data already obtained through the code comparison and to use this data together with programs of the various theories written using MATHCAD to obtain theoretical/experimental comparisons. This approach has the benefit that the theories/programs can be easily modified to include or exclude various components of the different aerodynamic theories.

It should be noted that the S809 aerofoil data used throughout the modelling in chapters 7 and 8 differs from that used in the code comparison modelling of the previous chapter, see section 6.3.1.
7.1 Steady State Theories

Many of the engineering aeroelastic models in use today use Blade Element Momentum, or Blade Element/Pitt and Peters or GDW theories as their basis. This includes such engineering codes as YAWDYN, BLADED, ADAMS/WT, FLEX, FAST AD, WTperf etc. Therefore the first step in this investigation is to look at the performance of simple BEM theory (and its usual extensions) and how it compares to the first 6 cases from the code comparison which represent 6 different steady state, unyawed wind cases.

7.1.1 BEM Theory

The unyawed test cases were first modelled using BEM theory as formulated in equations (2.17) and (2.19), using the classical iterative procedure to arrive at a solution for each test case.

The first case in the code comparison provides a representation of a fixed speed wind turbine operating close to its design tip speed ratio (7m/s), the next 5 cases progressively lower the tip speed ratio pushing the turbine blades further into stall with each increase in wind speed. See figure 7.9 to see the angles of attack along the blade span predicted by BEM theory.

Figure 7.1 shows the axial (a) and tangential (a/) induction factors predicted by BEM theory. Only the first test case has a predicted level of axial induction which is above 0.1, as the wind speed increases the predicted level of axial induction falls meaning that the rotor is extracting relatively less energy from the air compared to that which is available. Also, as the wind speed rises the relative importance of the predicted induced velocities falls, simply because they are relatively lower compared to the overall flow velocity.

Figure 7.2 shows the normal force coefficients (relative to the localised blade chord)
Figure 7.1: Test cases 1-6, Induction factors calculated using BEM theory.

Figure 7.2: Test cases 1-6, Normal force coefficients calculated using BEM theory.
Figure 7.3: Test cases 1-6, Normal force per unit span calculated using BEM theory.

derived from experiment compared to those from BEM theory, the only reasonable prediction is obtained for the first test case (7 m/s). As the wind speed rises the measured data shows a distinct change in shape, remaining low towards the tip, having a levelling off through the mid span before rising significantly at the root. The theoretically predicted force coefficients however have a much flatter and uniform shape as the wind speed rises having a poor correlation with the measured results. Significantly the force coefficients obtained from BEM theory represent a direct relationship between the angle of attack and the predicted blade element forces arrived at through the process of iteration. Hence figures 7.2, 7.3 and 7.9 are all related by equations (2.17) and (2.19) and the data in figure 6.6. Figure 7.3 shows the difference between the distribution of theoretical and experimental normal forces along the blade span. There is a reasonable correlation for the first test case (to be expected from the force coefficients plot) but as the wind speed rises the distribution of forces becomes increasingly inaccurate. Again the
Figure 7.4: Test cases 1-6, Tangential force per unit span calculated using BEM theory.

Theoretical results will be dominated by the 2D aerofoil data used as input in the iteration process.

Figure 7.4 shows the distribution of tangential forces (again relative to the localised blade chord) along the blade span. There appears to be a qualitatively good agreement between experiment and theory. Although there do exist significant differences they are not as dramatic as those seen with the normal forces. Again the theoretical results will be dominated by the 2D aerofoil data used as input to the iteration process.

The phenomena which causes the dramatic effects on the spanwise distribution of the normal forces appears to have a significantly lesser effect on the spanwise distribution of tangential forces.
7.1.2 BEM + Prandtl’s Tip Loss Model

Prandtl’s tip loss model was used to modify BEM theory according to equations (2.26) and (2.27). Theoretically this adjusts the spanwise distribution of induced velocities to account for a finite number of blades (in this case 2), resulting in the level of induction rising as the blade tip is approached as in figure 7.5. The rise of the induction factors at the blade tip has the effect of lowering the angle of attack in this area. This effect can be seen in figure 7.9, however it can also be seen that in the post stall area this may not necessarily lead to a reduction in the force coefficients predicted for the outermost blade elements. This can be seen by comparing the angles of attack predicted in figure 7.9 with the 2D aerofoil data of figure 6.6.

Figure 7.6 shows the spanwise normal force coefficients predicted by the BEM/Prandtl theory. At 7m/s the value of the force coefficient is lowered at the blade tip and this slightly improves the match with the measured data. However at the higher
Figure 7.6: Test cases 1-6, Normal force coefficients calculated using BEM theory with Prandtl’s tip loss model.

At the higher wind speeds it is clear that the addition of Prandtl’s tip loss model has little effect in improving the theoretical results. The theory is still bound to the use of the 2D aerofoil data and this defines the results.

In figure 7.7 it can be seen that the theoretical normal forces now fall away slightly at the blade tip in comparison to the forces predicted in figure 7.3 this is due to the increased induction at the blade tip from the Prandtl tip loss model. It improves the already reasonable prediction of the forces at 7 m/s but does little to improve the prediction of normal forces at higher wind speeds.

The prediction of tangential forces in figure 7.8 appears to be very good at 7m/s, definitely improved by the addition of the tip loss model. At the other wind speeds the results are quantitatively reasonable, but it is interesting to note that in the 15m/s wind case the spanwise distribution of both normal and tangential forces has been increased towards the blade tip by the inclusion of the tip loss model. This results (by referring to figures 7.9 and 6.6) from the tip loss model lowering...
Figure 7.7: Test cases 1-6, Normal force per unit span calculated using BEM theory with Prandtl's tiploss model.

Figure 7.8: Test cases 1-6, Tangential force per unit span calculated using BEM theory with Prandtl's tiploss model.
Figure 7.9: Test cases 1-6, Angles of attack calculated using BEM theory with and without Prandtl's tiploss model.

the angle of attack towards the blade tip (18 to 14 degrees) but raising the force coefficients due to the area of 2D aerofoil data concerned. Again emphasizing the controlling influence of the 2D aerofoil data.

7.1.3 BEM + Prandtl + Snel’s Stall Delay Model

Snel et al (Ref 40) have produced an empirical modification for 2D aerofoil data to account for the effects of stall delay on rotating wind turbine blades, equation (7.1). It is an approximation based on results obtained using a three dimensional computational fluid dynamics code and aerofoil coefficients measured by Ronsten (Ref 37).

The fluid dynamics code was based on a simplified form of the 3D boundary layer equations which were used to calculate the effect of Coriolis forces which arise in a rotating aerofoils boundary layer. The assumption is that these Coriolis forces are responsible for the modified pressure distributions which have been
measured on rotating aerofoils during delayed stall. It is the modified pressure
distribution (a lowered suction peak and a reduction in adverse pressure gradient)
which results in delayed flow separation and so raises the value of the normal
force coefficients attained at high angles of attack. Snel's modification was used
to attempt to correct for this delayed stall effect which is so evident in the normal
force coefficients derived from the NREL experiment.

\[ C_{nSNEL} = C_n + t \left( \frac{c}{r} \right)^2 \Delta C_n \]  

Equation (7.1) is applied to the 2D aerofoil data for a particular blade element to
adjust the normal force coefficient based on blade element parameters. \( c \) and \( r \)
are the local chord length and radius respectively, \( \Delta C_n \) is the difference between
the value of the linear, potential flow, normal force coefficient \( (C_{N\alpha} \alpha - \alpha_\alpha) \)
and the measured 2D normal force coefficient \( (C_n) \) at the angle of attack in question.
The ratio of chord length to local radius was identified by Snel as being of prime
importance in determining how 2D aerofoil data should be modified for application
to a given blade design. The square of this ratio forms part of the multiplier to
\( \Delta C_n \) in equation (7.1).

Snel gives the value of the factor \( t \) (in the \( \Delta C_n \) multiplier) as 3 but comments
that the multiplier of \( \Delta C_n \) should not be more than 1, since at this point the
inviscid limit is reached. He suggests that \( \tanh(\xi) \) would be a good alternative as
a multiplier but notes that the multiplier of \( \Delta C_n \) may also be aerofoil profile and
Reynolds number dependent which highlights the empirical nature of equation
(7.1). Given this inherent empiricism it was decided to lower the value of the
factor \( t \) to 1.4 (determined by a tuning process) in order to achieve reasonable
results with the NREL data. The effect of lowering the value of \( t \) is to reduce the

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value of the multiplier of $\Delta C_n$ and hence lower the relative value of the modified normal force coefficient for a given blade element. Raising or lowering the value of $t$ effectively raises or lowers the influence of Snel’s delayed stall model on the magnitude of blade forces which are obtained by the aerodynamic code as a whole. In effect blade forces as a whole will be raised or lowered coupled with a gradual change in radial distribution which is determined by the square of the $\frac{c}{r}$ ratio.

A last point is that the results obtained here from using Snel’s model show a different radial distribution than the measured NREL results, two possible causes of these differences are either the effect of the particular blade planform of the NREL blade (twist and taper) on the boundary layer radial flow or the influence of 3D induced flow external of the boundary layer as argued by Wood (Ref 36) or Tangler (Ref 66).

Figure 7.10 shows the effect of Snel’s function (with $t=1.4$) on the static normal force coefficients at each blade element of the NREL rotor. The curves move from close to the (linear) potential flow solution near the blade root back to the original 2D curve near the blade tip due to the ratio of $c$ to $r$ decreasing with radius.

As can be seen in figures 7.11 and 7.12 the delayed stall model has a dramatic effect, improving the normal force coefficients and normal forces predicted using BEM/Prandtl theory. It fails however to reproduce the curves of the measured data in the mid span region and the drop in the derived coefficients and measured forces towards the blade tip.

### 7.1.4 BEM + Wake Rotation

An approximation of the effect of the axial pressure drop caused by wake rotation as described in section 2.1.7 can be made by modifying equation (2.17) to include the rotational pressure drop in the BEM iteration calculation, the equation used takes the form:
Figure 7.10: Adjustment of 2D normal force coefficient using Snel's stall delay model, plots for the NREL blade's 15 elements are shown.

Figure 7.11: Test cases 1-6, Normal force coefficients calculated using BEM theory with Prandtl's tiploss model and Snel's stall delay model.
Normal Forces (BEM + Prandtl + Snell)

![Graph showing normal forces for different wind speeds and blade spans.]

Figure 7.12: Test cases 1-6, Normal force per unit span calculated using BEM theory with Prandtl’s tip loss model and Snell’s stall delay model.

\[ a(1 - a) = \left( \frac{NW^2c(C_l \cos \phi + C_d \sin \phi)}{8\pi rV^2} \right) - \left( \frac{a'\Omega r}{V} \right)^2 \]  
(7.2)

As can be seen in figure 7.13 the effect of including the wake rotation pressure drop in the calculation is negligible. The inclusion of the wake rotation pressure term is physically correct but like other simplifications made to formulate the simple momentum theory (as compared to the general momentum theory) it has little practical effect and can be omitted from an engineering code.

7.1.5 Steady State Theories - Blade Bending Moment and Rotor Torque Comparison

The measured blade root bending moment in figure 7.14 looks to be fairly well predicted by all the models. The calculation for this bending moment involves
the integration of blade forces distributed along the blade span (local normal and tangential forces resolved into the out of plane direction). The integration process results in similar bending moments being calculated from very different theoretical and experimental spanwise force distributions. Indeed it is interesting to see that the addition of stall delay slightly degrades the prediction of blade root bending moment.

The overestimation of all the theories is due to the loss of circulation at the experimental blade tip, this is not properly accounted for in any of the theories. Figure 7.15 shows theoretical and experimental low speed shaft torque results. There is a slight improvement in the predictions as the different extensions to BEM theory are added to the model, but there remains a large underprediction of torque centred on the results at 15m/s. The rotor torque calculation under these steady state conditions is also simply an integration of resolved blade forces, this time dominated by the tangential blade forces due to the inplane direction.
of the rotor torque. Examining the results for BEM theory at 15m/s (the largest discrepancy) it can be seen that (figure 7.4) the theoretical tangential forces are negative until rising above zero at 4.5m span and then rising steeply until the blade tip. In contrast the experimental tangential forces are closer to zero until rising sharply at 4m span until the blade tip. The differences between these two distributions (especially near the blade tip) are sufficient to cause the large difference in theoretical and experimental rotor torques at this wind speed.

### 7.2 Dynamic Theories

Few engineering aeroelastic codes use BEM theory for inflow calculations since it is not a dynamic theory. Of the dynamic inflow theories that are available one of the most popular is the Pitt and Peters theory, it being used in GarradHassan’s BLADED, NREL’s AERODYN and the Danish FLEX codes.

Another dynamic inflow model is the more sophisticated generalised dynamic
Figure 7.15: Rotor torque calculated using BEM theory with and without Prandtl's model and Snell's/Prandtl's model.

wake (GDW) theory which was introduced into the AERODYN code in version 12. These theories were programmed using MATHCAD by the present author to compare their performance in analysing the data from the NREL wind tunnel tests.

A dynamic stall model is not required with these theories for test cases 1 to 6. The constant wind, unyawed test case is a steady state condition for the wind turbine rotor and once the dynamic inflow theory has arrived at a final flow pattern across the rotor (usually 2 revolutions for these tests) the dynamic stall model will default to 2D aerofoil data. Hence the final flow pattern will be unaffected by the dynamic stall model. This allows 2D aerofoil data to be used directly with these inflow models in the analysis of test cases 1 to 6.
Figure 7.16: Test cases 1-6, Normal force coefficients calculated using the Pitt and Peters theory.

7.2.1 The Pitt and Peters Theory

The unyawed test cases were modelled with the Pitt and Peters theory as formulated in equations (3.20) to (3.27), using an Adams Moulton Bashforth predictor/corrector subroutine to solve the time based differential governing equation. Over several revolutions the rotor induced velocities were allowed to build up until a steady state solution was reached for comparison to the experimental data.

The theoretical normal force coefficients and their spanwise distribution as predicted by the Pitt and Peters theory are shown in figure 7.16, they are very similar to those obtained using BEM theory. This is unsurprising as the Pitt and Peters model as formulated (with the average induced velocity through the rotor used in the calculation of the velocity matrix \([V]\)) effectively makes the model a linear small perturbation theory around a disc level BEM theory (non-linear) with blade forces integrated to derive the forcing functions which the model needs.

The normal and tangential blade forces predicted by the Pitt and Peters theory
Figure 7.17: Test cases 1-6, Normal force per unit span calculated using the Pitt and Peters theory.

are again very similar to those found using BEM theory. The same observations made about the BEM theory results can be made about the Pitt and Peters theory results especially the controlling influence of the 2D aerofoil data used as input to the modelling process.

7.2.2 The Pitt & Peters theory + Prandtl's Tip Loss model

Like the BEM theory the Pitt and Peters theory takes no account of the finite number of blades used in a physical rotor (there is no azimuthal variation in pressure distribution around the rotor). Therefore a tip loss model has to be included with the theory and this is usually the Prandtl model due to its relative simplicity.

When used with the Pitt and Peters theory it is found that Prandtl's tip loss model can only be applied to axial induced velocities \( (\lambda(r', \psi_{wind})) \) after they are calculated from the induced velocity coefficients \( (\lambda_o, \lambda_s, \lambda_c) \), see equation (3.20).
This is because the Prandtl model is radially dependent and cannot be applied directly to the coefficients of a disc averaged induced velocity distribution.

It is the unmodified coefficients, \((\lambda_o, \lambda_s, \lambda_c)\), which are used to calculate the average axial induced velocity \((\lambda_{mv})\) in the Pitt and Peters theory using equation (3.25). The result of this appears to be that the Prandtl tip loss model has a greater effect on the induced velocities towards the blade tip when used with the Pitt and Peters theory than it does when used with the BEM theory.

Figure 7.19 shows the levels of axial induction produced by the Pitt and Peters theory both with and without Prandtl’s tip loss model. The larger influence of the tip loss model in this case as opposed to BEM theory can be clearly seen. In fact the tip loss model causes the average level of induced velocity to be greater as the wind speed increases.

In figure 7.23 there is a large reduction in angles of attack at the blade tip caused by the tip loss model, this is approximately the same at each wind speed. However
Figure 7.19: Test cases 1-6, Induction factors calculated using the Pitt and Peters theory with and without Prandtl’s tiploss model.

Figure 7.20: Test cases 1-6, Normal force coefficients calculated using the Pitt and Peters theory with Prandtl’s tip loss model.
Normal Forces (P&P + Prandtl)

![Plot of Normal Forces (P&P + Prandtl)](image)

Figure 7.21: Test cases 1-6, Normal force per unit span calculated using the Pitt and Peters theory with Prandtl's tiploss model.

it is only in the 7m/s test case (figure 7.20) that this causes a significant reduction of the normal force coefficients. This again due to the controlling influence of the 2D aerofoil data which determines the force coefficients for a given angle of attack.

The spanwise force distributions shown in figures 7.21 and 7.22 are not that different to those achieved using BEM with Prandtl's tip loss model. Again, the influence of the 2D aerofoil data is the predominant factor with the prediction of tangential forces being generally better than the prediction of normal forces. The larger influence of the tip loss model when combined with the Pitt and Peters theory can also be seen.

### 7.2.3 The Generalised Dynamic Wake Theory

The unyawed test cases were modelled with the Generalised Dynamic Wake theory in the same way as the Pitt and Peters theory. The theory was coded as formulated in equations (3.29) to (3.46) with an Adams Moulton Bashforth pre-
Tangential Forces (P&P + Prandtl)

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Calculated Angles of Attack (P&P/P+Prandtl)

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Figure 7.22: Test cases 1-6, Tangential force per unit span calculated using the Pitt and Peters theory with Prandtl's tiploss model.

Figure 7.23: Test cases 1-6, Angles of attack calculated using the Pitt and Peters theory with and without Prandtl's tiploss model.
dictor/corrector subroutine was used to solve the time based differential governing equations. Over several revolutions the rotor induced velocities were allowed to build up until a steady state solution was reached for comparison to the experimental data.

Two different forms of the theory were programmed in MATHCAD, the first form follows the theory as implemented by Suzuki in AERODYN v12 and as such is used in all of NREL's aeroelastic analysis codes. This form uses 6 different pressure/induced velocity distributions, they are (as defined by the cumulative solutions in equations (3.31) and (3.36)), \( m=0 \ n=1, m=0 \ n=3, m=1 \ n=2, m=1 \ n=4, m=2 \ n=3, m=3 \ n=4 \). These distributions were chosen by Suzuki to give at least one distribution which covers the azimuthal pressure spikes of 1, 2 and 3 bladed rotors (the value of \( m \) defines the azimuthal variation of pressure/induced velocity for a given distribution).

Since the NREL rotor has 2 blades it was decided to remove the case with \( m=3 \) and add in 5 more distributions with higher order \( m \) values which are multiples of 2 to try and get more definition in the induced velocity distributions predicted by the theory. Therefore the second form of the theory has 10 distributions included of the form, \( m=0 \ n=1, m=0 \ n=3, m=1 \ n=2, m=1 \ n=4, m=2 \ n=3, m=2 \ n=5, m=4 \ n=5, m=4 \ n=7, m=6 \ n=7, m=6 \ n=9 \).

The levels of axial induction predicted by the 2 different forms of the GDW theory are shown in figure 7.27. The effect of including the extra terms in the theory has a marked effect on the induced velocities. The spanwise shape of the induced velocity distributions is governed by the Legendre polynomials which make up the solution to the GDW theory, the form with the larger number of terms shows more variation in spanwise shape as the wind speed increases as well as a greater average level in axial induction. It would be interesting to see the effect of including even more terms in the solution of the theory.
Figure 7.24: Test cases 1-6, Normal force coefficients calculated using the GDW theory with 6 pressure/induced velocity distributions.

As the GDW theory includes azimuthal variation of pressure there is no need to include a tip loss model with this theory, however it is interesting to note that the theory with more terms (10), shows a spanwise distribution of axial induction which falls towards the blade tip at the higher wind speeds. Theoretically the level of axial induction should rise sharply towards the blade tip due to the presence of the tip vortex which is shed into the wake at this point.

Despite the added complexity of the GDW theory over the BEM or Pitt and Peters theories, the results that are achieved in predicting normal force coefficients and normal forces are still very similar and are still dependent on the 2D aerofoil data despite the different axial induction distributions achieved by the different theories. It appears that the theories are only really applicable when the turbine is operating such that the blades are experiencing angles of attack in the linear attached flow region (close or near to their design tip speed ratio). When the blades are pushed significantly into stall then there appear to be significant 3D
Figure 7.25: Test cases 1-6, Normal force per unit span calculated using the GDW theory with 6 pressure/induced velocity distributions.

effects occurring predominately at the blade root and tip and these effects have a much larger effect on the normal forces than the tangential forces.

The prediction of tangential forces by both forms of the GDW model appear to be reasonably good except at the highest two wind speeds. The measured results do show (at most wind speeds) an increase in tangential force at the blade root which is not reproduced by any of the theories, this could well be related to the significant effect upon the normal forces by whatever 3D effect is occurring on the blades.

The 10 pressure distribution GDW theory produces normal force coefficients and normal forces which increase towards the blade tip, again this is due to the falling axial induction wrongly predicted by this theory towards the blade tip. The spanwise distribution of axial induction achieved by this theory must be directly related to the shape and magnitude of the Legendre polynomials which are combined to give the final pressure/induced velocity distribution and also the 2D aerofoil data
Figure 7.26: Test cases 1-6, Tangential force per unit span calculated using the GDW theory with 6 pressure/induced velocity distributions.

Figure 7.27: Test cases 1-6, Axial induction factors calculated using the GDW theory with either 6 or 10 pressure/induced velocity distributions.
Figure 7.28: Test cases 1-6, Normal force coefficients calculated using the GDW theory with 10 pressure/induced velocity distributions.

which will define the forcing functions of the theory. So again a question must arise about the suitability of using what is essentially a linear actuator disc theory combined with 2D data to model what is in reality a gross 3D flow pattern occurring on the finite number of blades of a wind turbine rotor.

7.2.4 Pitt and Peters Theory + Prandtl's tip loss model using aspect ratio adjusted Lift and Drag data

By looking at the results obtained in this chapter, specifically the plots of rotor torque and of blade root bending moments produced by the various theories (figures 7.14, 7.15, 7.34 and 7.35) and comparing these with the results obtained in the NREL code comparison (chapter 6, figures 6.7 and 6.8) which were obtained using a YAWDYN model it can be seen that there are some dramatic differences between the torque and bending moment predictions made by the YAWDYN model and the results presented in this chapter. The rotor torque predicted by
Figure 7.29: Test cases 1-6, Normal force per unit span calculated using the GDW theory with 10 pressure/induced velocity distributions.

Figure 7.30: Test cases 1-6, Tangential force per unit span calculated using the GDW theory with 10 pressure/induced velocity distributions.
Calculated Angles of Attack (GDW 6 and 10 pressure distributions)

Figure 7.31: Test cases 1-6, Angles of attack calculated using the GDW theory with either 6 or 10 pressure/induced velocity distributions.

the YAWDYN model is consistently higher than the torque predicted by any of the theories in this chapter. At low wind speeds the bending moments predicted by the YAWDYN model are of a similar value to the ones predicted in this chapter but they do not increase at the same rate and consequently they diverge from the results in this chapter as the wind speed increases.

The YAWDYN model was coupled with AERODYN version 11 subroutines for the NREL code comparison. AERODYN version 11 used a Pitt and Peters inflow model with Prandtl’s tip loss model and the Beddoes/Leishman dynamic stall model. The only major difference between this aerodynamic code and the MATHCAD Pitt and Peters + Prandtl’s tip loss model used in this chapter is the aerofoil data used as input to the codes. The dynamic stall model is irrelevant in steady unyawed flow cases.

It was decided to use the MATHCAD program (containing the Pitt and Peters theory combined with the Prandtl tip loss model) to try to reproduce the quite
different results achieved by the author in the code blind comparison using the YAWDYN model. In the MATHCAD program the 2D extrapolated aerofoil data created using data from Critzos et al (Ref 16) (used in this chapter and chapter 8) was replaced with the Foilcheck aspect ratio extrapolated aerofoil data used in the code comparison (chapter 6) see section 6.3.1 and figure 6.6.

As can be seen from figures 7.32 and 7.33 the prediction of spanwise forces has been dramatically altered by the selection of aerofoil data used as input into the model. The prediction of forces at 7m/s is still very good as this lies in the linear range of the aerofoil data which is the same for both data sets. As the wind speed increases the prediction of both normal and tangential forces becomes increasingly inaccurate, especially the tangential forces which become almost linear at the higher wind speeds, a direct result of the equations used in Foilcheck for extrapolation of the aerofoil data.

This must cast serious doubt over the equations used in Foilcheck for extrapolating measured data into the post stall range of angles of attack and confirms the controlling influence of aerofoil data in the post stall modelling of wind turbine rotor aerodynamics.

7.2.5 Dynamic Theories - Blade Bending Moment and Rotor Torque Comparison

The blade root bending moment and rotor torque predictions in figures 7.34 and 7.35 again show the equalising effect of spanwise integration of the blade forces, the results of all the theories with their many different inflow distributions are all similar to each other and also similar to the results from the steady state theories. The rotor torque does show a significant difference between the torque predicted by the different theories at 15m/s however it is no coincidence that this is also where large changes in force coefficients occur (post stall 15 to 25 degrees angle
Figure 7.32: Test cases 1-6, Normal force per unit span calculated using the Pitt and Peters theory with Prandtl’s tip loss model and Foilcheck extrapolated aerofoil data.

Figure 7.33: Test cases 1-6, Tangential force per unit span calculated using the Pitt and Peters theory with Prandtl’s tip loss model and Foilcheck extrapolated aerofoil data.
Figure 7.34: Out of plane blade root bending moments calculated using the various dynamic inflow theories.

of attack) for small changes in angle of attack.

The biggest change is not effected by a change in theory but by a change in the aerofoil data used in the modelling process, the results achieved by the present author in the code blind comparison using the YAWDYN code in chapter 6 are reproduced here by swapping the theoretically correct 2D aerofoil data for the Foilcheck extrapolated, aspect ratio adjusted data originally used with YAWDYN, (results labelled "Old L&D data" in figures 7.34 and 7.35).
Figure 7.35: Rotor torque calculated using the various dynamic inflow theories.
Chapter 8

Theoretical comparisons to the NREL wind tunnel data; Yawed Test Case 10

This chapter contains analysis of only one test case, case 10 (table 6.1). During this test the NREL rotor was operated in a fixed yaw position upwind of the turbine tower with a yaw angle of 30 degrees and a wind speed of 10 m/s. Since yawed flow is no longer a steady state condition at the blade element level it is necessary to plot individual variables over one revolution. This means that for a comparison of different inflow theories (both with and without a dynamic stall model) there is simply not space for the comparison of several different test cases.

Case 10 was chosen because it has a large enough yaw angle to induce significant unsteady aerodynamics and at a wind speed which minimises the 3D effects occurring on the blades at higher wind speeds. These effects have already been shown to be inadequately modelled by the theories under consideration. 7 m/s would have been a preferred wind speed for yawed tests but this was unavailable
from the code blind comparison results, (table 6.1).

8.1 Steady State Theories

MATHCAD was again used to programme theories for comparison to the NREL experimental results. For wind turbines in yaw the two steady state theories used here are the Glauert circular wing analogy and Coleman et al's cylindrical vortex wake theory. Both of these theories default to basic BEM theory in the unyawed case and are applied on an annular ring/blade element basis in the same manner as BEM. By yawing the rotor two main effects are added to the theoretical problem, one is the advancing and retreating blade effect which is purely geometric and the other is the non-symmetric distribution of induced velocities caused by the skewing of the rotor wake. The level of average axial induction caused by the energy extraction of the rotor will also change and each theory handles this slightly differently. Glauert's theory is less specific being based on results at 0 and 90 degrees of yaw which are assumed to apply to all cases in between. Coleman's theory uses an assumed vortex wake structure to determine the level of average axial induced velocity and also how this is then distributed across the rotor. Prandtl's tip loss model was included in both theories programmed into MATHCAD.

8.1.1 Glauert and Coleman Theories

Both theories are based upon an actuator disc analogy, a circular wing for Glauert and a disc with a fixed elliptical wake structure for Coleman. In order to apply these theories and find the theoretical distribution of induced velocities across the NREL rotor it is first necessary to find the average induced velocities for the rotor as a whole. To do this each theory has to be applied over one entire
revolution of the rotor, in the MATHCAD programmes this was done every 10 degrees of azimuth. Once completed the induced velocities are then averaged out before being redistributed across the rotor according to whichever theory is being applied. A simple linear variation in the case of Glauert’s theory (equation 2.30) and for the Coleman theory by using flow expansion equations determined by Sharpe (Ref 64) by use of the Biot Savart law.

Both approaches rely on the use of static 2D aerofoil data for the initial revolution when determining the average induced velocities. However since the rotor is yawed the blade elements will be affected by both varying wake conditions around the rotor and by the advancing and retreating blade effect resulting in unsteady aerodynamics at the blade element level. Hence a rotor in steady yaw with a steady wind is not really in a steady state, the blade loads are unsteady and cyclical and really have to be determined by a suitable unsteady aerodynamics model at the blade element level. This is really incompatible with the Glauert and Coleman theories which don’t allow for interaction between the cyclically varying blade loads and rotor inflow.

A further problem to consider relating specifically to the wind tunnel case under consideration is that the yaw angle is sufficient to cause large changes in angle of attack along the blade span. This means that the blade elements will be varying through angles of attack in the range of light and possibly deep stall. Also, it was shown in the previous chapter that the delayed stall phenomena was beginning to occur at this wind speed and interactions between delayed stall and dynamic stall phenomena could well occur. These points should be borne in mind when considering the results of these theories.

For yawed cases the aerodynamic variations around one revolution require azimuthal as well as spanwise positions to be defined for all quantities plotted. The azimuthal definition used for the NREL tests are that zero azimuth occurs when
Figure 8.1: Case 10; Axial induction factors calculated using Glauert’s circular wing analogy.

Figure 8.2: Case 10; Angles of attack calculated using Glauert’s wing analogy and the Coleman et al vortex wake theory.
the blade being considered is in the twelve o'clock position and for positive yaw
the blade is retreating when at zero azimuth.

Once the average levels of induction over one revolution have been calculated
from the Coleman theory, the azimuthally varying tangential and axial induced
velocities are calculated from the flow expansion equations derived by Sharpe. The
Glauert theory is much simpler and only axially induced velocities are allowed for
in the theory, these vary azimuthally according to the value of a factor $k$, see
equation (2.30). The distribution of the axial induction factors for case 10 (10.1
m/s and 30.2 degrees of yaw) as predicted by Glauert's theory (using $k = \sin(\gamma)$)
is shown in figure 8.1. The variation is sinusoidal with azimuth and dependent on
momentum balance in the annulus concerned for its radial variation. The theory
has been applied with Prandtl's tip loss model and so the highest levels of axial
induction are to be found at the blade tip. Maximum axial induction occurs at 90
degrees (blade horizontal and pointing downwind), minimum induction occurs at
270 degrees (blade horizontal and pointing upwind). These results are consistent
with the theoretical linear variation of induction over a circular wing as proposed
by Glauert.

Figure 8.2 shows the calculated angles of attack from the Glauert and Coleman
theories for case 10, (10.1 m/s and 30.2 degrees of yaw), this case combines a large
yaw angle with the largest levels of axial induction present in the experimental
yawed test cases. Even in this case there appears to be little to choose between the
results of the two theories. The advancing and retreating effect is most apparent
at the blade root (0.3R) between 0 and 180 degrees azimuth. The effect of the
skewed wake and resulting redistribution of the induced velocities is seen at the
blade tip (0.95R) where it is combined with the advancing and retreating blade
effect to give a minimum angle of attack at 135 degrees of azimuth.

The Glauert and Coleman theories have both been applied on an annular ring
Figure 8.3: Case 10; Wake skew angles calculated for the 15 annular sections of the NREL rotor using the Coleman et al vortex wake theory.

basis in the same manner as BEM was for the unyawed cases. This highlights an inconsistency in applying discretised aerodynamic theories to yawed flow, namely the assumption that each blade element or annular ring is independent or isolated from its neighbour, as is done in the unyawed case.

Part of the application of Coleman’s theory is the calculation of a wake skew angle. In reality the theory is only capable of producing an average wake skew angle for the whole rotor / disc along with average values of induced velocity at the centre of the disc for the whole disc and associated skewed wake structure. However applying this theory on an annular ring basis is, in effect, treating each annular ring as a separate actuator disc with its own averaged values of induced velocity and its own wake skew angle. Azimuthally averaged wake skew angles for the 15 blade elements of the NREL rotor for case 10 are shown in figure 8.3. Each annular ring is assumed to have its own elliptical vortex wake structure at its own wake skew angle, clearly this can not exist in reality since all the ellipses will commence
Figure 8.4: Case 10; Normal force coefficients calculated using the Coleman et al vortex wake theory.

...as concentric circles in the rotor plane. There will have to be interaction between the wake structures from each annular ring and this interaction will vary with azimuth. Calculating the induced velocities for a given blade element from the momentum balance or wake structure for that blade element alone is therefore incorrect and will contain errors. How big those errors are is unclear, they will however increase in direct relation to the differences in wake skew angle between neighbouring blade elements.

The Glauert and Coleman theories produce very similar results when analysing case 10, therefore only the results from the Coleman theory will be presented here. It was identified in the previous chapter that the aerofoil data chosen as input for the analysis has a dominating effect on predicted blade loads. For the yawed cases it is reasonable to expect a similar influence over the results.

Theoretical and measured normal force coefficients for case 10 are shown in figure 8.4. The predicted force coefficients are centred around 1 and are obtained from
theory in conjunction with the predicted angles of attack in figure 8.2. As in the unyawed case very little change in force coefficient can be expected with significant changes in angle of attack due to the use of steady state 2D aerofoil data, this aerofoil data defines the results of whatever aerodynamic theory is applied. The measured values are higher at the blade root and lower at the blade tip. Unlike the unyawed case there is a pronounced variation of force coefficient towards the blade root caused by the angle of yaw. Clearly the steady state 2D aerofoil data is not capable of capturing this large variation in force coefficient.

The normal forces shown in figure 8.5 give reasonable agreement where the flow at the blade is not influenced significantly by either the loss of circulation at the tip or the unsteady effects at the blade root, namely at the 0.63R and 0.8R positions. The normal force at 0.95R is over estimated by theory. The loss of circulation at the blade tip (related to the tip design) which is responsible for this is not part of Coleman's theory or Prandtl's tip loss model.

There is an interesting effect at 180 degrees azimuth at the 0.3R position, here the higher velocities of the advancing blade coupled with the relatively constant force coefficient predicted by steady state theory results in a peak in the predicted normal force. However the measured normal force actually shows a minimum at this position, this is due to the previously mentioned unsteady effects which are probably a result of a combination of delayed and dynamic stall phenomena.

The predicted restoring yaw moment caused by a redistribution of the axially induced velocity is most clearly seen at 0.95R which has a peak at around 220 degrees caused by a combination of the advancing blade effect and the lower induced velocities and hence higher wind speed through the rotor on its upwind side.

There appears to be no clear conclusions to be drawn from figure 8.6, the measured tangential force at 0.95R has a reduced and flatter response than that predicted
Figure 8.5: Case 10; Normal force per unit span calculated using the Coleman et al vortex wake theory.

Figure 8.6: Case 10; Tangential force per unit span calculated using the Coleman et al vortex wake theory.
by theory. The measured results at 0.47R and 0.63R show a large azimuthal variation between upwind and downwind blade positions. This effect seems to have disappeared from the 0.3R station to be replaced by a variation around the tilt rather than the yaw axis. A curious feature of the figure is that while blade is retreating the measured tangential forces at 0.3R and to a lesser extent at 0.47R do not fall to negative values as predicted by the theory.

8.1.2 Coleman Theory + Snel’s Stall Delay model

In the unyawed cases of the previous chapter there was a significant improvement in prediction of blade forces when the Snel stall delay model was used to adjust the 2D aerofoil data for high angles of attack based upon the ratio of chord to radius squared (equation 7.1 and figure 7.10).

In the yawed case shown here the stall delay model is used with the Coleman theory to see if the azimuthal changes in the blade forces towards the blade root are reproduced by the addition of the stall delay model. Figure 8.7 shows the normal force coefficients for case 10, the prediction at the 0.3R station is improved but with the azimuthal variation underestimated. At the 0.47R station the azimuthal variation is still mostly missed by the theoretical model.

Figure 8.8 also shows the improvement in the prediction of the normal force distribution at 0.3R.

It is interesting to note that the results presented here still come from steady state aerofoil data (albeit modified by Snel’s equation), meaning that the azimuthal variations predicted are in phase with the large sinusoidal changes in angle of attack at the blade root and that underestimation of these azimuthal variations is probably due to the steady state nature of Snel’s model which will not reproduce the unsteady effects which will be present.

Figure 8.9 is presented here to show the minimal difference between the Glauert
Figure 8.7: Case 10; Normal force coefficients calculated using the Coleman et al vortex wake theory with Snel’s stall delay model.

Figure 8.8: Case 10; Normal force per unit span calculated using the Coleman et al vortex wake theory with Snel’s stall delay model.
and Coleman theories. The main difference between these two theories lies in the induced velocities which they calculate. The Glauert theory only calculates induced velocities normal to the rotor plane whereas the Coleman theory considers the rotating air in the rotor wake and hence allows the calculation of tangential induced velocities in the rotor plane.

As discussed in chapter 2, the results of studying an idealised vortex wake of a wind turbine rotor leads to the conclusion that the rotation of the air in the rotor wake comes largely from the axial vortex of the wake which is formed from the root vortices shed from the inboard ends of the rotor blades.

In the Coleman theory the rotation of the wake is calculated from the momentum balance of inplane blade forces (in the same manner as for simple BEM theory), however because of the skewed wake the wake axis is not normal to the rotor plane and hence the rotation of the wake has to be transposed into the rotor plane in order to be equated with blade forces, see equation (2.35).

In axial flow cases the induced velocities caused by the rotation of the wake are azimuthally constant, in the yawed case however they are not and they have components which affect not only the tangential but also the normal induced velocities. These effects are included in the Coleman theory but not in the Glauert theory.

In reality only a slight improvement in normal force prediction can be seen in figure 8.8 as opposed to figure 8.9 leading to the conclusion that the inclusion of wake rotation (caused primarily by shed root vortices) has little effect on the results obtained. However this conclusion is one resulting from the assumptions made in formulating the Coleman theory, the same assumptions that qualify 2D aerofoil data as being that which should be used with the theory and which limits the effect of the shed vorticity from the blade roots as being consistent with that obtained from attached linear flow on the rotor blade.
Figure 8.9: Case 10; Normal force per unit span calculated using the Glauert wing analogy with Snel’s stall delay model.

The actual root vortices shed by the NREL rotor blades during case 10 are going to be produced by a combination of complicated flow phenomena including delayed stall effects possibly caused by boundary layer radial flow and/or 3D external induced velocities and sinusoidal angle of attack changes seen in figure 8.2 with the inevitable unsteady aerodynamics this will induce. The blade forces measured at the root during these flow conditions need to be determined by a model that considers the interaction of these flow phenomena.

8.2 Dynamic Theories

The dynamic inflow theories considered in this thesis inherently overcome the major problems of the steady state theories due to their basis in acceleration potential theory. The result of this type of analysis is a first order differential governing equation. This equation in two different forms appears in both the Pitt and Peters and the GDW inflow theories. The derivatives of the induced velocity
distributions provide a complete dynamic theory by incorporating "added mass" terms into the governing equation.

This type of time-based aerodynamic theory allows the calculation of unsteady aerodynamics at a blade element level which is not possible with steady state theories. This enables an analysis of two dimensional unsteady aerodynamics on the modelling of the NREL wind tunnel results, it has already been shown that the static 2D aerofoil data has a controlling influence on the theoretical predictions obtained using various inflow theories. No change can be expected in the results for the unyawed test cases but there should be some effect on the yawed test cases with the resulting ever changing angles of attack along the blade span.

### 8.2.1 Dynamic Stall - Beddoes/Leishman model

The Beddoes/Leishman dynamic stall model is a time-based aerodynamics model which produces unsteady two dimensional force coefficients for a given aerofoil section using two dimensional steady state aerofoil data as input. The model is divided into three main parts, linear unsteady aerodynamics, non-linear unsteady aerodynamics (light and/or slow stall) and vortex shedding (deep and fast stall - leading edge separation). A full description of the model is given in section 3.3.1. A version of this dynamic stall model was programmed into MATHCAD with the intention of incorporating it into the dynamic inflow models already programmed. First however the model had to be validated against experimental results to prove it was functioning correctly. Steady state 2D aerofoil data measured in the Delft university wind tunnel had already been selected for the S809 aerofoil, see section 6.3.1 and figure 6.6. The dynamic stall model parameters for the S809 aerofoil were obtained from the static data (the set extrapolated with data from Critzos Ref 16) and then the various parts of the dynamic stall model were implemented to arrive at the full model, see figure 3.3.
Wind tunnel results for the unsteady forces on an oscillating S809 aerofoil are available from an Ohio state university study (Ref 45). Results from twelve different oscillating tests are presented in the Ohio study, these tests were modelled for comparison using the dynamic stall model. Typical sample plots from this comparison process are shown in figures 8.10 to 8.13. Figures 8.10 and 8.12 relate to lift and pressure drag coefficients measured in three of the Ohio wind tunnel tests, the mean angle of attack was varied in the three tests between 8, 14 and 20 degrees, everything else remained constant, the sinusoidal oscillation was of +/-10 degrees, the Reynolds number was 0.99 million and the reduced frequency was 0.078. Figures 8.11 and 8.13 are the results from modelling the same three tests with the dynamic stall model. The results are plotted against angle of attack and so the unsteady force coefficients appear as hysteresis loops around the static data.

There is a general agreement between the experimental and the theoretical results, the main differences are similar to those highlighted by Pierce (Ref 47). The theoretical model has a tendency to achieve significant negative pressure drag for decreasing angles of attack just above zero. Also, the model underestimates the high levels of pressure drag achieved at high angles of attack and the model produces slightly smaller hysteresis loops in the plots for the lift coefficient by not dropping in value as far as the experimental results during the decreasing angle of attack part of the aerofoil oscillation (just prior to upper surface flow reattachment).

The main reason for these discrepancies probably has a lot to do with the model's original formulation and its intended use with relatively thin helicopter aerofoils, the model is semi-empirical and the time constants in the model were tuned largely to experimental results with thin symmetrical aerofoils. Discrepancies should not be unexpected then when it is applied to an aerofoil designed specifically for wind...
Figure 8.10: Unsteady lift coefficients for the S809 aerofoil measured by Ohio state university, Reynolds number of 0.99 million and reduced frequency of 0.078.

turbines with a quite large chord to thickness ratio.

A possible source of discrepancy in the validation process could lie in the 2D aerofoil data used as the input for the dynamic stall model, as stated this data came from the Delft university wind tunnel and was used to derive model parameters such as normal force slope, normal force at the stall angle, zero lift angle of attack, etc. However the oscillating aerofoil tests were carried out in the Ohio state university wind tunnel which produced slightly different static data for the same aerofoil under the same conditions.

A comparison of steady state 2D aerofoil coefficients from the Delft and Ohio state university wind tunnels is shown in figure 8.14. This figure highlights experimental differences between the two wind tunnels and these differences will add to any discrepancies in figures 8.10 to 8.13.

The effect of unsteady flow on the forces exerted on an aerofoil depend upon the
Figure 8.11: Predicted unsteady lift coefficients for the S809 aerofoil from the Beddoes/Leishman dynamic stall model, Reynolds number of 0.99 million, reduced frequency 0.078.

Figure 8.12: Unsteady pressure drag coefficients for the S809 aerofoil measured by Ohio state university, Reynolds number of 0.99 million, reduced frequency of 0.078.
Figure 8.13: Predicted unsteady pressure drag coefficients for the S809 aerofoil from the Beddoes/Leishman dynamic stall model, Reynolds number 0.99 million, reduced frequency of 0.078.

Figure 8.14: Lift and drag force coefficients for the S809 aerofoil from Delft university and Ohio state university.
Figure 8.15: Case 10; Unsteady lift force coefficients over one revolution calculated using the Beddoes/Leishman dynamic stall model.

angle of attack, reduced frequency and Reynolds number of the aerofoil/fluid flow interaction. These quantities can all vary with time and this is reflected in the formulation of the dynamic stall model. By making use of the angle of attack and relative velocity at a blade element level over one revolution determined earlier using the Coleman theory and steady state aerofoil data it is possible to get an idea of the unsteady force coefficients for a blade element over one revolution for a given yawed test case.

Figure 8.15 shows such a study for test case 10. By using the information obtained from the steady state analysis, (the angle of attack variation for case 10 is shown in figure 8.2), the dramatic effect of the unsteady aerodynamics becomes apparent. The largest effect is at the blade root where there is the largest variation in angle of attack, but all blade elements show a dramatic departure from the steady state aerofoil data that is used with the steady state inflow theories.
8.2.2 Pitt and Peters model + Dynamic Stall

The mode of operation of this fully dynamic model is as follows. Initially at $t=0$ there are no induced velocities and the induced velocity differentials are all zero. However it is possible to calculate blade forces from the flow conditions by resolving free stream and rotational velocities. These blade forces then allow the calculation of the non-dimensionalised thrust force and non-dimensionalised yaw and tilt moments on the rotor (the $C$ vector in the governing equation, 2.28). The governing equation can now be rearranged to solve for the derivatives of the induced velocities, since all other quantities are known. By making use of a predictor-corrector scheme it is possible to find the induced velocity coefficients for the next time step, this requires the stacking and updating of previous values of induced velocity coefficients and their derivatives. Once the induced velocity coefficients are known the velocity matrix and wake skew angle can be found and new blade loadings calculated. In this way the rotor inflow is built up over successive time steps, solving for the blade forces and the induced velocity derivatives at each time step. Note, $\lambda_{mv}$, the average induced velocity is recalculated at each time step and is not averaged over one revolution as is the case with the steady state theories.

The Pitt and Peters model has been used with steady state 2D aerofoil data for analysing the unyawed test cases. This is valid since these situations involve a steady state flow condition at the blade element level.

In the yawed case however unsteady aerofoil data needs to be used in the calculation of rotor inflow and blade forces. The Beddoes/Leishman dynamic stall model is written specifically to provide such unsteady aerofoil data by deriving unsteady force coefficients using exponential functions which respond well to discrete time steps and the corresponding discrete changes in flow conditions at the blade element level.
Figure 8.16: Case 10; Axial induction factors calculated using the Pitt and Peters theory with and without the Beddoes/Leishman dynamic stall model.

By considering test case 10 again, the axial induced velocity and angle of attack variation over one revolution for the Pitt and Peters model calculated with both steady state aerofoil data and with the Beddoes/Leishman dynamic stall model can be compared to the results obtained by the Glauert and Coleman theories, figure 8.16.

The variation in induced velocities has shifted in azimuth, the maximum and minimum induced velocities are no longer at 90 degrees and 270 degrees (the upwind/downwind axis), as was the case with the Glauert theory. They are now at approximately 140 degrees and 320 degrees, this is due to the sinusoidal terms in the Pitt and Peters theory that respond to the varying blade forces produced by the advancing and retreating blade effect. This produces a vertical as well as a horizontal variation in induced velocities.

The inclusion of the dynamic stall model causes the azimuthal variation of induced velocity to be much reduced. Since the blade forces drive the response of the Pitt
Figure 8.17: Case 10; Angles of attack calculated using the Pitt and Peters theory with and without the Beddoes/Leishman dynamic stall model.

and Peters theory it is not surprising that modification of these blade forces by the dynamic stall model should result in a redistribution of axially induced velocities around the rotor. Indeed in comparing figures 8.20 and 8.21 a definite azimuthal shift in blade forces can be seen, especially at the blade root.

The angle of attack plots in figure 8.17 show the influence of the sinusoidal as well as the co-sinusoidal variation in induced velocity possible with the Pitt and Peters theory, the resulting azimuthal shift is reflected in the shift of the minimum angle of attack at 0.95R from 135 degrees to 160 degrees of azimuth.

The inclusion of the Beddoes/Leishman dynamic stall model has a dramatic effect on the normal force coefficients and the normal forces predicted for case 10.

Figures 8.18 and 8.19 show the coefficient predictions with and without the dynamic stall model. Figure 8.18 is very similar to the Coleman theory results of figure 8.4 except with a very slight azimuthal shift in the plot for 0.95R.

Figure 8.19 in contrast shows a much improved prediction of force coefficients. It
Figure 8.18: Case 10; Normal force coefficients calculated using the Pitt and Peters theory with steady state aerofoil data.

Figure 8.19: Case 10; Normal force coefficients calculated using the Pitt and Peters theory with the Beddoes/Leishman dynamic stall model.
is interesting that the dynamic stall model and Snel's delayed stall model both provide significant improvement to the results.

The onset of the delayed stall phenomenon can be seen in the results of the unyawed test case for 10m/s (case 2). Including Snel's model improved the results due to the high angles of attack reached at the blade root. Significantly these improvements were in phase with the measured results, mirroring the angle of attack change as would be expected with a simple steady state normal force coefficient modification.

The results from the dynamic stall model however show a phase shift occurring with the value of the predicted force coefficients preceding the measured results by 30 to 45 degrees, this effect occurs at the blade root (0.3R and 0.47R), the same area of the blade affected by the delayed stall phenomenon.

It could be that the empirically derived time constants used with the Beddoes/Leishman model which are derived from experimental results using thin helicopter aerofoils are inapplicable for use with wind turbine aerofoils/operating conditions. It is also possible that the delayed stall phenomenon is affecting the dynamic stall process at the blade root, it is reasonable to expect the delayed stall effect to become azimuthally varying with a yawed rotor and as such to have an effect on the dynamic stall and unsteady aerodynamics at the blade root.

The normal forces of figure 8.20 are again very similar to the normal forces predicted using the Coleman theory in figure 8.5. Slight differences can be attributed to the slightly different inflow distributions but both results are generally defined by the 2D aerofoil data used with both theories as was the case with the unyawed test cases.

Figure 8.21 shows that the inclusion of the dynamic stall model in the modelling process generally improves the predictions, but that there still exists the azimuthal shift in the forces which was apparent in the normal force coefficient plots. This
Figure 8.20: Case 10; Normal force per unit span calculated using the Pitt and Peters theory with steady state aerofoil data.

Figure 8.21: Case 10; Normal force per unit span calculated using the Pitt and Peters theory with the Beddoes/Leishman dynamic stall model.
Figure 8.22: Case 10; Tangential force per unit span calculated using the Pitt and Peters theory with steady state aerofoil data.

shift is centred on the blade root and is absent from the normal force predictions of figure 8.8 which uses Snel’s delayed stall model with the Coleman et al vortex wake theory. Away from the blade root the predictions of normal forces is generally good and in phase. Still present is the over-estimation of the force at 0.95R which is almost certainly due to the loss of circulation towards the blade tip caused by the poor tip design.

The prediction of tangential forces in figures 8.22 and 8.23 is fairly poor. Figure 8.22 shows little difference to figure 8.6, both being largely defined by the 2D aerofoil data used with the models and the angle of attack variation of the yawed test case. The inclusion of the dynamic stall model shows little improvement in figure 8.23.

The blade root seems particularly bad, there may be a connection with the out of phase normal force predictions of figure 8.21, it is possible that the cause of
Figure 8.23: Case 10; Tangential force per unit span calculated using the Pitt and Peters theory with the Beddoes/Leishman dynamic stall model.

delayed stall in unyawed cases has an effect on both the normal and tangential forces at the blade root in the yawed case.

### 8.2.3 Generalised Dynamic Wake model + Dynamic Stall

The plots in figures 8.24 and 8.28 show the inflow distributions predicted by the two forms of the GDW theory programmed with and without the Beddoes/Leishman dynamic stall model. The resulting angle of attack variations both with and without the dynamic stall model are shown in the figures 8.25 and 8.29.

The GDW theory is much more responsive to blade forces than is the Pitt and Peters theory owing to the more flexible pressure/induced velocity distributions which are possible with this theory. This is reflected in figures 8.24 and 8.28 where the azimuthal variation of the inflow is more complex than that associated with the Pitt and Peters theory.

Despite the more sophisticated nature of the GDW theory it is still clear that the
Figure 8.24: Case 10; Axial induction factors calculated using the GDW theory with 6 pressure/induced velocity distributions with and without the Beddoes/Leishman dynamic stall model.

Figure 8.25: Case 10; Angles of attack calculated using the GDW theory with 6 pressure/induced velocity distributions with and without the Beddoes/Leishman dynamic stall model.
controlling influence on the prediction of blade forces is the aerofoil data, either steady state or from a dynamic stall model.

Figures 8.26 to 8.31 show normal force predictions from the two forms of the GDW theory which have been programmed, both with and without the Beddoes/Leishman dynamic stall model. These predictions show a remarkable similarity to the predictions made using the Pitt and Peters theory.

This shows that whichever inflow model is used, whether it be Pitt and Peters, GDW6 or GDW10, it is the source of the aerofoil data (steady state 2D, 3D adjusted or 2D unsteady) which is used as input to the inflow model which is really the defining part of the modelling process.

The dynamic inflow models presented here could be applied with a combination of both dynamic and delayed stall models. The first step would be to adjust initial sets of aerofoil data using Snel’s delayed stall model to produce individual aerofoil data sets for each blade element based on radius and chord length (as in
Figure 8.27: Case 10; Normal force per unit span calculated using the GDW theory with 6 pressure/induced velocity distributions with the Beddoes/Leishman dynamic stall model.

Figure 8.28: Case 10; Axial induction factors calculated using the GDW theory with 10 pressure/induced velocity distributions with and without the Beddoes/Leishman dynamic stall model.
Figure 8.29: Case 10; Angles of attack calculated using the GDW theory with 10 pressure/induced velocity distributions with and without the Beddoes/Leishman dynamic stall model.

Figure 8.30: Case 10; Normal force per unit span calculated using the GDW theory with 10 pressure/induced velocity distributions with steady state aerofoil data.
Figure 8.31: Case 10; Normal force per unit span calculated using the GDW theory with 10 pressure/induced velocity distributions with the Beddoes/Leishman dynamic stall model.

The dynamic stall parameters for each blade element / aerofoil data set could then be determined, these parameters would then be used in to determine an individual blade element’s unsteady response when running an aeroelastic simulation.

The delayed stall adjustment of the aerofoil data would colour the dynamic stall results, particularly towards the blade root where the aerofoil data would be adjusted to the greatest extent. The hysteresis and phase shift of the dynamic stall model would still occur but with normal force coefficients generally raised by the 3D adjustment. Towards the blade tip the delayed stall model would have little influence on the performance of the dynamic stall model due to the radius / chord length relationship in Snel’s model.
Chapter 9

Summaries and Conclusion

9.1 MS4 ADAMS/WT Modelling

The MS4 is a unique 3 bladed wind turbine of industrial size. Its design is highly flexible with several unusual design features. The nacelle is effectively gimbled at the top of the turbine tower, it being mounted on yaw and tilt bearings. The 3 blades of the downwind mounted rotor are attached to the machines hub with flexible spars which greatly reduce their out of plane stiffness. Also, the need to balance the nacelle on the tilt bearing with the generator upwind and the blades downwind results in a very long drive train because of the necessary blade/tower clearance.

ADAMS was chosen to model this turbine because of the ease with which complicated mechanically-dynamic systems can be modelled using this software. ADAMS/WT adds the basic components of the wind turbine to the software using the stiffness method to model structural components. It also adds an aerodynamic model in the form of AERODYN.

The MS4/ADAMS model was constructed from manufacturing drawings and design stiffnesses supplied by the turbine’s owners NEG-Micon UK. The finished model had over 320 degrees of freedom. 80 in each blade alone. The aerody-
damic model used originally was AERODYN v11 which used the Pitt and Peters dynamic inflow model and the Beddoes/Leishman dynamic stall model. The 2D aerofoil data required as input for the model came from Abbott and Von Doenhoff (Ref 11) and was extrapolated using the Foilcheck software, no stall delay model was used. The selected aerofoil data was for a Reynolds number of 3 million, calculations having revealed that the MS4 blades operated at between 2 and 4 million, the higher Reynolds numbers equating to higher wind speeds where the drivetrain instability of the MS4 was known to occur.

Obtaining 2D aerofoil data for a Reynolds number of 4 million and interpolating between the sets of aerofoil data based on operating Reynolds number may in hindsight have improved the results obtained for high wind speed conditions. However it was when extrapolating the aerofoil data into the post stall region where the greatest inaccuracies were probably introduced into the modelling process. As was shown in the later wind tunnel comparison work the use of measured post stall aerofoil data (instead of Foilcheck extrapolation) showed a great improvement in the results produced by an aeroelastic code for a wind turbine blade operating in post stall conditions. This does however still neglect the 3D effects which are also occurring in these conditions.

Later AERODYN v12 became available for use with the MS4/ADAMS model, this version of AERODYN replaced the Pitt and Peters inflow model with the more sophisticated Generalised Dynamic Wake inflow model. Errors were found in the way this model had been implemented but once they had been corrected the new version of AERODYN was used to drive the MS4 model together with the aerofoil data used with v11 (Re number of 3 million and extrapolated by Foilcheck).

Unfortunately there was limited data available to verify the MS4 model, only 13 measured campaigns of the prototype operating were available for analysis, of these only 2 contained data of the turbine operating in free yaw and tilt for 10
minutes. One of these campaigns was chosen and the wind characteristics were used to generate a statistically similar wind file to drive the ADAMS model.

The results from the ADAMS model were compared to the limited range of outputs from the prototype machine, the comparisons showed no results which were unreasonable but neither did it fully verify the ADAMS model against the prototype. Further verification would have been difficult given the limited data channels and the lack of measured campaigns for comparison.

No digital copies of the machine undergoing its drivetrain instability were available, but there were hardcopies of the prototype’s outputs during such an instability which were provided by the Centre for Alternative Technology who owned the turbine at the time of the modelling work.

A reasonably turbulent wind file was created to try and excite the ADAMS model into simulating the prototypes instability. This was achieved and a theory was developed to explain the machines instability, this was possible because of the range of outputs it was possible to extract from the ADAMS model.

The main cause of the turbines operating instability (of 0.75Hz) proved to be the very similar natural frequency of the long drivetrain. The complicated nature of the blade/spar/hub assemblies was also found to be a contributing factor, the design of these assemblies results in structures which have highly coupled flap and chordwise modes.

Several dynamic and aerodynamic factors were then found to occur before and during the period of instability. There was a fairly rapid change in wind direction which led to quite a high yaw rate and since the nacelle is free to tilt as well as yaw the high yaw rate induced the nacelle to tilt through gyroscopic loading. Due to the MS4 being a stall regulated machine the blades are frequently stalled and the accompanying aerodynamic negative damping helps to drive the complex mode shapes of the blade assemblies during the rapid yaw and tilt events.
This process is only clear when the chordwise and flapwise bending moments of all 3 blades are analysed together (the flapwise vibrations could be transferred to the drivetrain through the gyroscopic loading). This analysis could not be done with the field test results from the prototype because it only has one instrumented blade.

It may have been possible by careful varying of the MS4 model parameters to shed more light on the sensitivity of the MS4 design to the various factors contributing to its instability. This would have been time consuming as the MS4 model was not quick to run. Therefore it was felt that further analysis of the MS4 would produce diminishing returns. The design of the MS4 is unique and any solutions to the machines instability would probably have involved changing the blade mounting arrangements, the nacelle yawing and tilting arrangements and the aerofoil sections used in the blade design. All the software used in the ADAMS modelling work is commercially available and so the problem becomes one of engineering analysis rather than academic research.

While the MS4 modelling was taking place the author was invited to take place in a 'blind' comparison of aeroelastic codes organised by the National Renewable Energy Laboratory (NREL) in the US. It was felt that this code comparison work would enable a much greater insight into wind turbine aerodynamics than was possible through further analysis of the MS4. If there were serious inaccuracies in the ADAMS/WT model of the MS4 then it was felt that these would lie in the aerodynamic subroutines.

One conclusion to be drawn from the MS4 work which relates to wind turbine design in general is that the development of stall regulated flexible wind turbine designs is inherently problematic and that it may not even be possible to produce a stable design given the combination of infinitely varying turbulent air flow, flexible rotating structures and separated airflow on aerofoils. It may be possible
to design a stable wind turbine which satisfies two out of three of these conditions but maybe not all three.

### 9.2 NREL wind tunnel data comparisons.

In May of 2000 NREL undertook extensive wind tunnel testing of a 20kW wind turbine in the NASA/AMES wind tunnel. The turbine was 10m in diameter and was heavily instrumented to obtain accurate aerodynamic information. The design of the turbine was very stiff in order to minimise any aeroelastic response. Data was collected for 160 different test cases investigating various turbine configurations, operation of the rotor upwind or downwind of the tower, various yaw angles and yaw rates, various blade pitch angles including varying pitch of different forms, and of course different fixed and varying wind speeds. The tests were repeated on different days to prove the consistency of the results during varying climatic conditions.

20 of these tests were chosen, they comprised fixed rotational speeds with steady wind speeds and included unyawed, yawed, upwind and downwind rotor configurations. The results from these 20 cases were each averaged over one revolution, these processed results were then used as the basis for a code 'blind' comparison. NREL issued a turbine description (Ref 63) together with a selection of relevant aerofoil data from different sources. Operating conditions for the 20 tests were released but without any of the test results. Several different research groups from around the world took up the challenge of modelling the 20 cases.

At the time the author had not programmed any aerodynamic codes and so chose to use YAWDYN together with AERODYN v11 to model the NREL test cases. YAWDYN provided a simple structural model with 2 degrees of freedom to which was coupled the AERODYN subroutines. These were the same aerodynamic sub-
routines as had been used with the ADAMS/WT model of the MS4. They included
the Pitt and Peters dynamic inflow model and the Beddoes/Leishman dynamic
stall model. Various sets of 2D aerofoil data for the S809 aerofoil used for the test
turbine blades were provided by NREL, a set produced in the Delft University
wind tunnel at a Reynolds number of 1 million was selected to be used as input to
the YAWDYN/AERODYN aeroelastic model. This data was extrapolated using
the Foilcheck software and no stall delay model was used.

A meeting in Boulder, Colorado during December 2000 took place at which the
results from the modelling work carried out by the different research groups was
compared for the first time to the experimental results. This showed that most
of the aerodynamic codes used produced quite good results when the turbine was
operating at or close to its design tip speed ratio. However at lower tip speed ratios
(higher wind speeds) the results from the different codes quite rapidly diverged
from the experimental results and from each other.

Most of the codes used at the meeting were engineering codes and used relatively
simple aerodynamic models. These models included Blade Element/Momentum
theory, Pitt and Peters theory, the Generalised Dynamic Wake theory and the
Beddoes/Leishman dynamic stall model. One important conclusion from the
meeting centred on the fact that codes for engineering design of wind turbines
need to be most accurate when the forces on the turbine structure are at their
greatest and it was clear from the wind tunnel results that these conditions of high
loading were exactly when the greatest inaccuracies in load prediction occurred.

In order to understand why these inaccuracies occur it is first necessary to evaluate
the effect that each component of an aerodynamic model has on the eventual
prediction of blade forces at various tip speed ratios.

MATHCAD was used to program a selection of engineering aerodynamic models
for comparison to the experimental data obtained from the code comparison work.
For the unyawed test cases these included the BEM theory, the BEM theory with a wake rotation approximation, the Pitt and Peters dynamic inflow theory and 2 versions of the GDW dynamic inflow theory. The effect of Prandtl's tip loss model and Snel's stall delay model were investigated when applied with these various theories. A change was also made in the extrapolation of 2D aerofoil data for the S809 aerofoil, the post stall data generated by the algorithms in the Foilcheck software (used for the code comparison) was discarded in favour of post stall 2D data derived from measured experimental results.

From the analysis of the unyawed test cases it was found that the normal and tangential blade force predictions from the various inflow models were all very similar to each other. 2D aerofoil data was found to have a controlling influence on all the theoretical results. At the lowest wind speed (near to the rotors design tip speed ratio) the prediction of the blade forces by all the inflow theories was reasonably accurate, only towards the blade tip was there an over-estimation which wouldn't have been helped by the poor blade tip design. The inclusion of Prandtl's tip loss model improved the blade forces prediction in the tip region but not completely as this is a correction for a finite number of rotor blades and is not a correction for poor blade geometry.

The inclusion of an axial pressure drop in the momentum balance of the BEM theory consistent with wake rotation was investigated and was found to have a negligible effect on the predicted results.

As the wind speeds were increased and the rotor blades were pushed further into stall the theoretical predictions proved to be increasingly inaccurate. General prediction of normal force spanwise distribution in particular does not match the experimental results, the shape of these experimental normal force distributions shows large delayed stall effects taking place towards the blade root with the associated high static force coefficients, also there is a definite reduction in the
relative blade forces measured at the blade tip.
The addition of Snel's stall delay model improved the theoretical predictions but it still did not capture the characteristic spanwise shape of the stalled blades normal force spanwise distribution. The performance of Snel's model was unconvincing in this respect.
The overall controlling influence of the 2D aerodynamic data used as input to the inflow models was confirmed by reproducing the quite different theoretical results which had been obtained while using YAWDYN for the NREL code comparison. This change in extrapolated post stall data for the S809 aerofoil had a profound effect on the results obtained. The effect was much greater than any of the changes in results caused by using different inflow models.
When blade forces predicted by the different theories were integrated radially to produce blade bending moments and rotor torque values it was found that the plotted results were actually quite close to the measured values, this is deceptive however since the predicted spanwise distributions could be very different to the measured spanwise distributions and still result in similar bending moments and rotor torque values due to the averaging effect of the integration process.
There was only space to present the results of comparisons to one yawed test case. The aerodynamic models used for comparison included the two steady state yawed flow theories of Glauert and Coleman et al. The results of these two theories were produced using steady state aerofoil data, however the Beddoes/Leishman dynamic stall model was programmed and applied to the time history of results from the Coleman theory over one revolution to see the impact of unsteady aerodynamics. Both the Glauert and Coleman theories were also applied with Snel's stall delay model.
The Pitt and Peters theory and 2 versions of the GDW theory were also used to model the yawed test case, the use of the dynamic stall model with these theories
was investigated.

As with the unyawed cases it was found that the controlling influence on the theoretical predictions came from the 2D force coefficient data used as input into the inflow model. In the yawed case this data came from either static 2D aerofoil data, aerofoil data modified with Snel's stall delay model or from the 2D dynamic stall model. The choice of inflow model had much less effect on the results produced.

An unsteady aerodynamic model has to be used with yawed flow conditions, this was proved by the poor results achieved using static 2D data, and the dramatic effect of using the dynamic stall model on the results from the Coleman theory. The dynamic stall model also dramatically improved the results obtained from the other, dynamic, inflow models.

Snel's stall delay model also caused a dramatic improvement in predicted results over the results obtained using 2D static aerofoil data. This improvement was in phase with the measured results and also the azimuthal variation of angle of attack, this is clear because Snel's model is a static adjustment to aerofoil data and contains no dynamic element. The dynamic stall model improved results but also introduced a large phase shift in predicted normal blade forces, particularly towards the blade root.

9.3 Conclusion

There are engineering models and theories which are not covered by the work in this thesis. The reader should be aware of their existence, they include some of the dynamic inflow models developed in the EU JOULE project of the mid 90's, the acceleration potential work of Van Holten and Van Bussel, the ONERA dynamic stall model and the Gormont (Boeing-Vertol) dynamic stall model among others. However all of these theories share one thing in common with the models which
are covered here, until recently there has been a lack of detailed experimental wind tunnel test data from a large scale wind turbine for model validation and verification work. Data of this kind has now been produced through the efforts of NREL.

Continuously developing new models is not really productive unless there exists appropriate detailed experimental data to prove the theoretical models validity. The comparison work carried out in this thesis represents a preliminary attempt to try and do this for the NREL test results. Modelling results are compared and contrasted with experimental results to try and identify the effects and influences of different model components and how the assumptions used to formulate the models may differ from the actual flow physics.

The results of the modelling work carried out in this thesis has serious implications for the engineering design of wind turbines. Although complex structural models can be coded and simulated with commercially available software packages like ADAMS there is a lack of confidence in the results due to the previously mentioned difficulty in verifying the aerodynamic model that must be used to drive the structural model during an aeroelastic simulation.

The modelling of the MS4 was largely successful because the instability of the prototype appears to be a function of its design, detailed structure and mode of operation. If the problem had been more aerodynamic in nature, and driven more by aeroelastic response of the turbine blades then it is quite possible that even with an accurate structural model the instability might not have been reproduced.

As commercial wind turbines continue to grow in size the aeroelastic response of the resulting blade designs will become more important. Larger, heavier and inherently more flexible blades will require very careful design. Failure to accurately verify the aerodynamic models used in their design, may lead to design faults or misunderstanding of their aeroelastic behaviour, this may result in structural fail-
ures or aerodynamic underperformance.
The current conservative design procedures are therefore understandable. The tuning of aerofoil data with field test results for input to aeroelastic models provides a method of gradually evolving blade design without undue risks. This method does have limitations where unsteady and stalled aerodynamics are involved however, if the flow physics are not understood and/or are not modelled very well then even the tuned aerofoil data cannot produce the correct dynamic results.

As stated the problem with verification of engineering aeroelastic codes has always been the scale of commercial wind turbines, their size preventing their testing in the controlled flow conditions of a wind tunnel. The wind tunnel work carried out by NREL and outlined in chapter 6 has provided data that will be analysed for years to come and will hopefully lead to a better understanding of the flow physics of wind turbine blades. This understanding is necessary to develop better engineering models which will capture the detail necessary to accurately model the aeroelastic response of new large scale blade designs straight off the drawing board.

As has been mentioned, the NREL code comparison produced comparisons of engineering aerodynamic models to wind tunnel data which showed that the current models are inadequate in accurately predicting the blade forces that occur at tip speed ratios much lower than the design tip speed ratio. This equates to high wind speeds and stalled blades for a fixed speed wind turbine. The present author agrees with Garrad (Ref 56) that the stalling of wind turbine blades in particular is an area where there is a crucial lack of understanding. Wind turbines experience their greatest structural and operational loads during high winds, meaning that inaccurate model simulations of these conditions can be very dangerous.

The simplest cases to be considered in looking at the results of the NREL wind
tunnel testing are the unyawed test cases at fixed wind speeds, these consist of wind speeds corresponding to the rotors design tip speed ratio and lower. An important assumption can be made about these unyawed cases, because it is a steady state condition for the wind turbine rotor there is no need for a dynamic stall model since if one is used it will only default to steady state aerofoil data after initial transients.

When the NREL rotor was operating close to its design tip speed ratio it was found that nearly all the engineering models in the code comparison made good predictions of blade forces. This is because the rotor blades had low angles of attack along the blade span and the entire blade was experiencing linear (attached flow) aerodynamics. Since most of the inflow models used in the code comparison are principally based upon an axial pressure drop across an actuator disc it is useful to note that this approach requires the rotor blades to be operating as efficient converters of axial fluid momentum into mechanical torque. To be an accurate assumption this requires the Kutta condition of blade circulation to be present along the blade span. This conclusion can be seen in the assumptions made in formulating the inflow models particularly the blade element / momentum / lifting line theory. The acceleration potential theory also relies on directly equating blade forces normal to the rotor plane (axial forces) with induced axial velocities. If there is significant flow separation on the wind turbine blade this assumption must be in doubt and it may explain the shape of the induced velocity distributions produced by the GDW theory for high wind speed cases of the wind tunnel tests. In these cases the GDW theory will be equating axial blade forces directly to produce axial induced velocity distributions despite the 3D separated flows which might be occurring on the turbine blades during these tests.

Another facet of separated flows at lower tip speed ratios is that the relative energy extracted by the rotor falls, this means that the ratio of induced axial velocity to
freestream velocity falls. This has the effect of reducing the importance of the inflow model in predicting the blade forces. The violated assumptions of the inflow models as given above means that at higher wind speeds the increasingly inaccurate inflow models become less important in defining the results of a aeroelastic code. As the inflow models become increasingly irrelevant the aerofoil data used as input to the code assumes much more importance and here the use of static 2D aerofoil data for modelling separated flows on rotating wind turbine blades must be highly questionable.

However, in the present models, aerofoil data does become almost the only controlling factor in predicting the blade forces, these predictions are seen to become increasingly inaccurate as the tip speed ratio falls. The influence of the aerofoil data is confirmed by the fact that the blade forces predicted by the different inflow models are all very similar and it is only by changing the aerofoil data that a significant change in predicted results can be achieved.

This has important implications for the ADAMS model of the MS4. The prototype only had one blade instrumented for blade bending moments and no direct blade force measurement, there is no way of proving it, but, in all likelihood the spanwise blade force distribution used to drive the ADAMS model in simulating high wind speed operation was probably entirely incorrect despite producing a reasonably accurate blade bending moment. The similar bending moments (predicted/measured) being a result of the averaging effect of spanwise integration.

The obvious result of tuning of this aerofoil data with field test results in order to achieve correlation with measured results is to render the aeroelastic code almost completely empirical, using such models to predict the performance of new blade designs is almost bound to produce inaccurate results.

The measured results from the unyawed test cases reveal a gradual spanwise redistribution of normal blade forces as the tip speed ratio falls, the forces at tip
do not increase, at the root they increase dramatically, along the blade there is a curving distribution which gradually changes shape. It is clear that as the blades begin to stall there are 3D effects occurring which are controlling the flow physics of the turbine blade.

One attempt at correcting for these 3D effects is to use Snel’s stall delay model, this is based on the assumption that the increased normal forces at the blade root are caused by centrifugal forces in the boundary layer on the low pressure side of the blade, the model simply adjusts normal force coefficients towards the linear potential flow lift force curve based on the ratio of local chord to local radius. However even though this adjustment does improve the prediction of spanwise blade forces it doesn’t entirely convince. The curving shape of the force distributions is not reproduced and the relative reduction of forces at the blade tip is not reproduced.

Concentrating on the blade tip, the forces at 0.95R are consistently overestimated by the models, some loss of circulation at the blade tip is to be expected but the poor design of the NREL blade tip (which is square ended) does not help the efficiency of the test rotor in this respect. This loss of circulation results in a lowering of the blade forces measured at the tip (around 20% at 0.95R). As the tip speed ratio drops the forces at 0.95R do not significantly increase, is it the case that the increasing angle of attack increases the loss of circulation at this point? Does the tip vortex play a part in this process? If the tip vortex plays a part in lowering the relative blade forces at the tip could it be that the circulation shed at the blade root plays a part in raising the blade forces towards the blade root?

There are a few opinions being expressed about possible causes for the delayed stall effect as it is known, from measurements it is clear that the increased force coefficients which are measured at the blade root during high angles of attack are caused by the adverse pressure gradient on the suction side of the blade being
reduced and so delaying flow separation. The cause of the pressure gradient modification is subject to two theories, Snel et al (Ref 40) propose Coriolis forces in the boundary layer as being responsible whereas Wood (Ref 36) maintains that it is three dimensional effects in the external flow which are responsible for the changes in the pressure gradient.

The results from the wind tunnel testing undertaken by NREL may be crucial in understanding the flow physics of delayed stall, it already appears that opinion is moving in favour of 3 dimensional effects in the external flow as put forward by Wood. Tangler (Ref 66) for instance suggests that the effect may be caused by a standing vortex on the suction side of the blade root. It could well be that the formation of such a vortex is closely associated with the way circulation is shed at the blade root.

If the delayed stall phenomena is indeed caused by 3D induced velocities in the overall flow around stalled wind turbine blades then it becomes clear that modelling stalled wind turbine rotors requires the individual blades to be modelled as rotating wings with full consideration given to blade planform (aerofoil geometry, twist distribution, chord distribution, root and tip design) in the formulation of a 3 dimensional flow model. The axial pressure drop of an actuator disc model as used to formulate many engineering models is simply not adequate without resorting to significant empirical modification. This conclusion applies to even the simplest constant wind speed unyawed test cases, it would explain why Snel's delayed stall model doesn’t entirely convince. This empirical approximation is based on boundary layer equations and takes no account of the way circulation is shed at the blade root and tip or of blade twist, although interestingly it does use the local chord and radius in its formulation, variables which would also have to be included in a 3D flow model.

The examination of a yawed test case using the engineering models reveals more
interesting points. As in the unyawed cases, the selection of dynamic inflow model has very little effect on the prediction of blade forces. Again it is the aerofoil data used as input to the model which proves the controlling factor over the results obtained. When using 2D static aerofoil data the large unsteady azimuthal variations in blade forces were almost entirely missing from the predicted results. Interestingly Snel's stall delay model reproduces most of the azimuthal variation in blade force caused by the varying angle of attack along the blade. However this improvement comes from an empirical steady state correction to the input aerofoil data and doesn't reflect any unsteady aerodynamics. This means that the azimuthal variation in blade forces at the blade root is almost entirely in phase with the geometric changes in angle of attack which occur as the blade rotates. Indeed when the same case is modelled with the Beddoes/Leishman dynamic stall model it can be seen that the amplitude of force variation is improved over the results obtained using Snel's model but that the response of the dynamic stall model is out of phase at the blade root, gradually coming into phase along the blade span towards the blade tip. This points to one of two things, either the time constants used in the dynamic stall model which are empirical and come from helicopter research are unsuitable for use with wind turbine rotors or the delayed stall effect is colouring the dynamic stall flow physics towards the blade root.

The Beddoes/Leishman dynamic stall model is based on a 2D aerofoil assumption and the Snel delayed stall model is an approximation based on 3D boundary layer equations, the effects of combining them have not been explored in this thesis but the results obtained using them in their current forms do not inspire confidence if they are used to analyse a new blade design.

When applying existing engineering aerodynamic models to the design of new wind turbine blades there are several useful insights provided by the work presented in
This thesis. If the simplest case of a fixed speed rotor operating in unyawed flow conditions is considered then conclusions can be drawn about the different components of an engineering model and how their influence changes as the wind speed through the rotor changes.

Starting at a low wind speed, which equates to a high tip speed ratio for a fixed speed machine, high axially induced velocities can be expected at the rotor in comparison to the free-stream velocity and the empirical correction formulated by Glauert for high values of thrust coefficient is usually invoked. However, the structural loads experienced are usually so low that this modification is sometimes omitted from an engineering model.

As the wind speed rises we approach the wind speed that the turbine was designed for, the design tip speed ratio. At this wind speed the turbine blades will be operating efficiently, with aerofoils along the blade span close to their optimum lift to drag ratio. At this wind speed the dynamic inflow model will be important together with the 2D aerofoil data used as an input, a tip loss model (if required) will also help improve the performance prediction towards the blade tip. It should be remembered that the engineering model will not make any allowance for circulation lost towards the blade root or tip, since this will be dependent on blade geometry.

As the wind speed continues to increase the inflow model will become less important as the relative level of axial induction falls. Also the accuracy of the inflow model will decrease as the flow over the turbine blades begins to be affected by 3D effects and separated flow.

The predicted blade forces will be defined more and more by the aerofoil data used as input to the model, therefore dynamic stall (in yawed and unsteady flow) and delayed stall models will become increasingly important. In particular the amount of 3D "adjustment" applied using the stall delay model and the relevance
of the time constants used in the dynamic stall model are critical. The post stall 2D aerofoil data used as input to the dynamic stall model is also crucial and must be carefully assessed. Basically, when a wind turbine is operating in high wind speeds and is experiencing stalled blades and high structural loading, the only way to have real confidence in load predictions from the current crop of engineering models is to tune the aerofoil data used as input to the model with field test results from an operating machine of similar design.

The main areas for new research work would be to develop an engineering model which concentrates on modelling individual rotor blades, maybe a vortex panel model or a lifting line model, it should remain relatively simple in order to be of use for engineering design or certification work but it should also capture the points made above about 3 dimensional flow and the effects of differing blade geometries. Dynamic stall and delayed stall modelling are a crucial part of developing such a model and efforts should also focus on improving existing dynamic stall models to include delayed stall effects and the unsteady force response of aerofoils designed specifically for use with wind turbines. Efforts at improving dynamic stall models have already taken place in the helicopter field, (Refs 33, 39). When completed the new engineering model could be added to the ADAMS software in the same way as AERODYN and would hopefully enable the aerodynamic model to approach the accuracy of the structural/mechanical model in the modelling of new and innovative wind turbine designs.
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