Advanced finite element analysis for strain measurement in a threaded connection

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ADVANCED FINITE ELEMENT ANALYSIS FOR STRAIN MEASUREMENT IN A THREADED CONNECTION

By

András Bulkai. B.Eng.(Hons)

A Doctoral Thesis Submitted in Partial Fulfilment of the Requirements for the Award of Doctor of Philosophy of Loughborough University.


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Abstract

There is no established method of measuring load accurately in a threaded connection at working temperatures exceeding 500°C. At these conditions conventional methods cannot be used due to the sensitivity of the instruments and it is suggested that a non-contact method should be used. The laser strain gauge was developed by the Loughborough University Optical Research Group and it is a non-contact way of measuring surface strain. With the help of finite element analysis (FEA) a special nut was developed that can be used to measure the load on the connection by relating the surface strain of the nut to the load. Experimental work later revealed that due to the threads sticking in the connection there is hysteresis present between the load and surface strain relationship. To eliminate the hysteresis a new part was added to the connection which could be used to relate the surface strain on it to the load without any hysteresis. This new part was a specially designed washer with three grooves to allow easy access for the user to measure the surface strain using the laser strain gauge.

Part of the design specification was that the load has to be determined to an accuracy of 0.5%. Using sensitivity analysis the washer was analysed in terms of how manufacturing imperfections affect the accuracy of the load measuring device. The results revealed that to achieve the required 0.5% accuracy the washer would have to be manufactured to very tight tolerances. To achieve these tight tolerances the manufacturing process would not be cost effective so it was proposed that individual calibration is required for each load measuring washer. Tests showed that with sufficient calibration the specially designed washer and the laser strain gauge can be combined and used as an accurate non-contact load measuring device. As it is a non-contact method it can be used in extreme environments including high temperatures.

This thesis describes how background research, finite element analysis and experimental testing were used to develop the load measuring washer. Also it is shown, how in-depth sensitivity analysis was used to determine the accuracy of the prototype and that how manufacturing imperfections influence the working life of a threaded connection.
Keywords:

Threaded connections, manufacturing imperfections, sensitivity analysis, variation analysis, laser strain gauge, finite element analysis, fatigue life.
Acknowledgements

First of all I wish to thank Dr Andrew Nurse, my supervisor for all the help, direction and support he has given during the completion of the project. Without his enthusiasm and motivational skills I could not have succeeded.

I would like to thank Hydratight Sweeney and EPSRC for giving me the opportunity to work on this project and a special thanks to Dr Jeremy Coupland and Dr Feng Gao from optical engineering research group.

I would also like to thank members of the Loughborough University Engineering faculty for all their help and support with various aspect of the project. I would especially like to thank, Dr Jon Petzing, Prof Rachel Thomson, Dave Britton, Dr Alejandro Maranon, Clive Turner, Jo Mason, Gill Youngs, Charlie E Green, Andrew Scarborough, Brian Stapleton and Peter Thomas.

And finally I would like to thank my family, Kati, Dénes and Kristóf, who have supported me financially and emotionally during the past few years. They have always stood by me and given me encouragement, especially during my darkest hours.

I would like to dedicate this work to my grandfather, Pál Bulkai who passed away during the completion of this thesis. As an engineer himself he gave me a lot of motivation and encouragement and I know he would have been very proud to see my completed work.
<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Radius of drilled hole</td>
</tr>
<tr>
<td>A</td>
<td>Area</td>
</tr>
<tr>
<td>AC</td>
<td>Arbitrary Constant</td>
</tr>
<tr>
<td>BC</td>
<td>Body</td>
</tr>
<tr>
<td>c</td>
<td>Constant in equation for a line</td>
</tr>
<tr>
<td>C</td>
<td>Residual stress constant</td>
</tr>
<tr>
<td>CF</td>
<td>Forman constant</td>
</tr>
<tr>
<td>CP</td>
<td>Paris constant</td>
</tr>
<tr>
<td>dadt</td>
<td>Fatigue crack growth</td>
</tr>
<tr>
<td>D</td>
<td>Diameter</td>
</tr>
<tr>
<td>Db</td>
<td>Major diameter of nut thread</td>
</tr>
<tr>
<td>DS</td>
<td>Minor diameter of stud thread</td>
</tr>
<tr>
<td>Di</td>
<td>Inner diameter</td>
</tr>
<tr>
<td>Do</td>
<td>Outside diameter</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus</td>
</tr>
<tr>
<td>EE</td>
<td>Scalar strain</td>
</tr>
<tr>
<td>dE/E</td>
<td>Error</td>
</tr>
<tr>
<td>f</td>
<td>Function</td>
</tr>
<tr>
<td>F</td>
<td>Force</td>
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<tr>
<td>Fr</td>
<td>Friction</td>
</tr>
<tr>
<td>hg</td>
<td>Groove depth</td>
</tr>
<tr>
<td>k</td>
<td>Number of threads</td>
</tr>
<tr>
<td>K</td>
<td>Spring stiffness</td>
</tr>
<tr>
<td>ΔK</td>
<td>Stress intensity factor</td>
</tr>
<tr>
<td>KT</td>
<td>Thread stiffness</td>
</tr>
<tr>
<td>KBC</td>
<td>Body stiffness</td>
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<tr>
<td>KSC</td>
<td>Stud stiffness</td>
</tr>
<tr>
<td>L</td>
<td>Nut load</td>
</tr>
<tr>
<td>m</td>
<td>gradient</td>
</tr>
<tr>
<td>mv</td>
<td>Number of variables</td>
</tr>
</tbody>
</table>
N  Number of cycles
No  Normal component
p  Parameter
P  Total Load
P(r, \theta)  Load distribution on the washer
Pn  Load distribution on the nut
Pt  Thread load
Ph  Pitch
r  Radius
rd  Distance from centre of the hole
ri  Internal radius
ro  External radius
R  Stress ratio
s  Length / distance
S  Bolt load
SC  Stud
SF  Sensitivity Function
SF_A  Sensitivity Function of Area
SF_E  Sensitivity Function of Young’s modulus
SF_{hg}  Sensitivity Function of groove depth
SF_{p}  Sensitivity Function of a parameter
SF_{P}  Sensitivity Function of Load
SF_{ri}  Sensitivity Function for internal radius
SF_{ro}  Sensitivity Function for external radius
SF_{x’L}  Sensitivity Function of grating location in “x” (horizontal) direction
SF_{y’L}  Sensitivity Function of grating location in “y” (vertical) direction
SF_{\theta g}  Sensitivity Function of groove trajectory
SF_{\theta hg}  Sensitivity Function of groove geometry
SF_{\rho}  Sensitivity Function of Poisson’s ratio
T  Thread
u  Absolute deflection
v  Variables
w  Width across the flat faces of the nut
x’G  Groove centre location in “x” (horizontal) direction
\( x'_L \) Grating location in "x" (horizontal) direction

\( y'_G \) Groove centre location in "y" (vertical) direction

\( y'_L \) Grating location in "y" (vertical) direction

\( \alpha \) Angle of segment

\( \beta \) Thread angle

\( \beta_C \) Constant coefficient

\( \delta \) Displacement

\( \delta_T \) Thread displacement

\( \delta_{BC} \) Body displacement

\( \delta_{SC} \) Stud displacement

\( \epsilon \) Strain

\( \epsilon' \) Surface Strain within the groove

\( \epsilon_A \) Rosette Strain in direction A

\( \epsilon_B \) Rosette Strain in direction B

\( \epsilon_C \) Rosette Strain in direction C

\( \epsilon_1 \) Principle Axial Strain

\( \epsilon_1' \) Axial Strain within the groove

\( \epsilon_2 \) Principle Hoop Strain

\( \epsilon_2' \) Hoop Strain within the groove

\( \epsilon_m \) Measured Strain

\( \theta_G \) Groove trajectory

\( \theta_{hG} \) Groove geometry, trajectory and depth combined

\( \lambda \) root of characteristic equation

\( \mu \) Friction coefficient

\( \nu \) Poisson's ratio

\( \sigma \) Stress

\( \sigma^R \) Residual Stress

\( \sigma_a \) Stress amplitude

\( \sigma_m \) Mean stress

\( \sigma_{\text{max}} \) Maximum stress

\( \sigma_{\text{min}} \) Minimum stress

\( \sigma_u \) Tensile strength

\( \sigma_o \) Yield strength

\( \Phi \) Angle of force acting on the thread from normal
Abbreviations

<table>
<thead>
<tr>
<th>Abbr.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>API</td>
<td>American Petroleum Industry</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge Coupled Device</td>
</tr>
<tr>
<td>ECP</td>
<td>Electro Chemical Polishing</td>
</tr>
<tr>
<td>ERSG</td>
<td>Electric Resistance Strain Gauge</td>
</tr>
<tr>
<td>FEA</td>
<td>Finite Element Analysis</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Model</td>
</tr>
<tr>
<td>HP</td>
<td>Hewlett-Packard</td>
</tr>
<tr>
<td>LSG</td>
<td>Laser Strain Gauge</td>
</tr>
<tr>
<td>PSD</td>
<td>Position Sensitive Detectors</td>
</tr>
<tr>
<td>SF</td>
<td>Sensitivity Function</td>
</tr>
<tr>
<td>VSM</td>
<td>Variation Simulation Modelling</td>
</tr>
</tbody>
</table>
Content

Abstract................................................................................................. i
Certificate of Originality........................................................................ iii
Acknowledgments................................................................................ iv
Notation............................................................................................... v
Abbreviations........................................................................................ vi

1 Introduction.......................................................................................... 1

2 Background theory.............................................................................. 4
  2.1 Introduction................................................................................... 4
  2.2 Load measuring techniques......................................................... 5
  2.3 Strain measuring devices............................................................... 6
  2.4 Load distribution analysis in threaded connections....................... 9
     2.4.1 Experimental work.............................................................. 10
     2.4.2 Analytical work................................................................. 14
  2.5 Variation analysis.......................................................................... 20

3 The Assembly...................................................................................... 22
  3.1 Introduction................................................................................... 22
  3.2 Threaded connection – Structural integrity...................................... 24
  3.3 User manual for the laser strain gauge........................................ 27
  3.4 How to determine the load from measured strain........................... 30
  3.5 VA of assemblies due to manufacturing imperfections.................... 34

4 Sensitivity Analysis of Surface Strain Measurements......................... 35
  4.1 Introduction................................................................................... 35
  4.2 Sensitivity Analysis...................................................................... 36
  4.3 Fundamental equation for the nut sensitivity analysis.................... 40
  4.4 Nut sensitivity analysis................................................................. 45
5 Variation Analysis

5.1 Introduction

5.2 Variation in load distribution

5.2.1 Introduction to the 2D spring model

5.2.2 Spring stiffness

5.2.3 Variation in spring constants

5.2.4 Imperfections in the spring constants

5.2.5 Method to find variation affect on load distribution

5.2.6 Results of variation in load distribution

5.2.7 Discussion of variation in load distribution

5.3 Variation analysis of grating position

5.3.1 Modelling for grating position

5.3.2 Results for grating position

5.3.3 Discussion for grating position

5.4 Variation analysis on the groove geometry

5.4.1 Variation analysis of face angle

5.4.2 Modelling to find the variation due to the face angle

5.4.3 Results of variation in the face angle

5.4.4 Discussion of variation in the face angle

5.5 Summary

6 Development of the Prototypes

6.1 Introduction

6.2 Initial design

6.3 Results and experimental testing

6.4 Re-optimisation of prototype / Washer

6.4.1 Further analysis of the washer — Residual stress

6.4.2 Determination of bolt load from measured strain

6.4.3 Sensitivity analysis of the washer

6.5 Variation analysis of non-uniform loading on the washer

6.5.1 Modelling for non-uniform loading

6.5.2 Results for non-uniform loading

6.5.3 Discussion & Conclusions for non-uniform loading

6.6 Summary
Chapter 1 - Introduction

In engineering applications it is often required to join a number of elements together. There are different methods available like welding or riveting, but threaded fasteners have the advantage of being a temporary connection. By temporary connection it is meant that the joint can be disassembled and reassembled time after time. Therefore threaded connections have become widely used in all sorts of applications.

The dismountable quality of the threaded connection however means that the joint can become loose when not intended. Due to vibration and other external forces acting on a joint during employment can cause the threads to slip. This behaviour is called thread loosening. Thread loosening can occur without actual fracture taking part within the joint but it is regarded as a type of thread failure. If a joint becomes loose it is no longer capable of performing it prescribed task which can cause the connection to fall apart or lead to rupture of the components. Thread loosening is governed by a number of factors including load type and thread profile and with careful design the chances of it occurring can be reduced but not eliminated. For applications where thread loosening can have a disastrous effect the load in the threaded connections are monitored to ensure the tightness of the joint.

One of the most common ways of monitoring load within a threaded connection is by the means of ultrasounds. Special sensors are attached at the free end of the connection and the change in length on the bolt is measured. This change in length can be used to calculate the strain on the bolt which is then related to the load. By monitoring this load it can be ensured that thread loosening will not take place. Knowing the load on a threaded connection is also important to ensure that the joint is not over tightened which can cause fracture.

Most methods currently available to monitor the load in a threaded connection, including ultrasounds can only be used in certain environments. There is no method currently available to monitor the load on a threaded connection at elevated temperature. A non contact method to measure the load in a threaded connection would mean that the load can be monitored in most extreme environments.
The Loughborough University Optical Research Group has designed the Laser Strain Gauge (LSG). The laser strain gauge is a non contact method of measuring surface strain. A special grating is manufactured onto the surface of the specimen and with the use of lasers and a CCD camera the diffraction of the grating can be measured. The diffraction can then be related to surface strain.

The aim of the project is to develop a device which can be used to determine the load on a threaded connection at elevated temperatures in the region of 500°C. The measuring device needs to have an accuracy of 0.5% and should be easy to use and the load should be monitored at regular intervals.

The Loughborough University Structural Integrity Group has decided to use the laser strain gauge to try and develop a non contact load measuring design for a threaded connection. By relating the surface strain on a threaded connection to the load the LSG could be used to measure and monitor the load by non contact means. With the use of advanced finite element analysis a special nut with three grooves was designed.

Tests showed that the surface strain can be related to the load accurately assuming no manufacturing imperfections are present. To investigate the effects of manufacturing imperfections on the accuracy of the load measuring device sensitivity analysis was carried out. Sensitivity analysis takes the geometrical properties into consideration and the changes in these geometrical parameters are then related to the accuracy of the device. For each parameter a sensitivity function can be found so the accuracy of the device can be determined. The sensitivity functions can also be used to determine the required dimensional tolerances on the load measuring device for a required accuracy.

Experimental testing later revealed that the load cannot be determined accurately using the specially designed nut due to hysteresis caused by friction of the threads. Under test conditions when the load was reduced on the threaded connection the surface strain on the nut did not lessen due to the threads sticking. These findings meant a new component had to be designed which moves independently of the threads but experiences the same load as the joint.
A special washer was designed which experienced the full load on the joint but was not directly under the influence of the threads. This meant hysteresis was reduced so the washer combined with the laser strain gauge could be used as a non contact load measuring device. After carrying out further tests and sensitivity analysis on the washer it was found that the special washer can be used as a non contact way of measuring and monitoring the load on a threaded connection if calibrated.

In the same way that manufacturing imperfections affect the measured strain from an imposed load on the threaded joint the stresses and strains experienced in the threads at stress concentrations will also be affected. To complete this thesis a novel in-depth investigation carried out to analyse the effects of manufacturing imperfections on the fatigue life of threaded connections is reported.

The thesis will be presented the following way:

**Chapter 2 – Background Theory**
**Chapter 3 – The Assembly**
An introduction into how the laser strain gauge can be used to measure the load on a threaded assembly.

**Chapter 4 – Sensitivity Analysis of Surface Strain Measurements**
An introduction into sensitivity analysis and its importance highlighting how sensitivity analysis can be carried out on the modified nut.

**Chapter 5 – Variation Analysis**
An in depth analysis carried out on the modified nut showing how manufacturing imperfections affect the accuracy of the nut as a load measuring device.

**Chapter 6 – Development of the Prototypes**
A detailed description of how the modified nut was developed including experimental testing which lead to the development and sensitivity analysis of the washer.

**Chapter 7 – Effects of Manufacturing Imperfections on Thread Strength**
An in depth analysis of how threaded connections can fail and that how manufacturing imperfections influence the life of such a joint.

**Chapter 8 – Discussion and Conclusion**
**Chapter 9 – Recommendations for Further Work**
Chapter 2 – Background Theory

2.1 Introduction

The aim of this work is to find a way of determining the load in a threaded connection by the means of measuring the surface strain on the threaded connection. The surface strain is measured using a non contact method so that the temperature of the connection does not interfere with the strain measuring instrument.

There are a number of areas that are connected to this project which need to be researched further. In the following section these topics will be explored and discussed. The project as a whole is a completely new idea so all the areas that are researched here will only be related to the work partially. The aim is to get a general understanding of these areas so when certain aspects of each are combined a solution to the original problem can be found.

The concerned areas include load measurement techniques, strain measuring techniques and load distribution analysis within the threaded connection. By understanding these areas in depth should help find a way of measuring load in a threaded connection by the means of a non contact method.

Once a solution is found it is crucial to examine the accuracy and the reliability of it, especially as it will be used as a measuring device. To do this variation analysis will be used, which also have to be researched further.
2.2 Load measuring techniques

Threaded connections are used in many mechanical installations, as they have advantages over other joining methods. Unlike welding or riveting threaded connections can be disassembled and reassembled relatively easily. Due to this characteristic it is important to be able to determine the load in threaded connections to avoid loosening or failure.

Presently the most accurate way of determining load in a threaded connection is by the use of an ultrasonic extensometer (US3759090). By the use of ultrasound the change in the bolt length is measured and converted to load. Present ultrasonic instruments, which are used to measure the change in bolt length, have a resolution of 0.002 mm (www.norbar.com 2005). This method involves attaching sensitive ultrasonic sensors to the connection and it only works at room temperatures. For high temperature applications different load measuring methods have to be used.

From torque measurements it is possible to predict the load but due to internal friction this relies on assumptions causing an inaccuracy (Blake and Kurtz 1965). Due to this friction between the fastener and its mating hole, a torque wrench has an accuracy of ± 4%. Also torque measurements do not allow constant monitoring of the load. For a more exact measurement pre-tensioning using hydraulic means are widely used (US4659065). Aided by mechanical devices, such as a deformable washer, the loading on a bolt can be measured to an accuracy of ± 5% (US5226765). This setup was developed by Rotabolt (www.rotabolt.co.uk 2004) who specialise in different load measuring devices for threaded connections. This method can not be used for routine monitoring and due to creep it is not an ideal way of predicting loads. Rotabolt’s newest product is an inbuilt measuring device which can be used in a variety of applications. A standard bolt is machined out and a special indicator is inserted and then calibrated. This indicator is then used to determine the load within the joint to ± 5% accuracy.

Each of the methods discussed can be used to determine load in a threaded connection, but none of them is accurate enough or could be used at high temperatures.
2.3 Strain measuring devices

The general definition of strain is the change of length of a segment divided by the original length of that segment. There are a variety of methods which enable us to measure strain. The accurate monitoring of strain is very important in many engineering applications for safer and stronger designs. There are several methods available at the present to measure and monitor strain, these include mechanical, electrical and optical methods. Vernon (1992) expresses this as:

\[ \varepsilon = \frac{\Delta s}{s} \]

where \( \varepsilon \) is the strain

\( \Delta s \) is the change in length

\( s \) is the original length.

The simplest method to measure strain is to use an engineer's rule to determine original and change in length, or similar methods of measuring length. Extensometers can be employed for more accurate measurement of these distances.

![Extensometer](Figure 2-1 Extensometer (www.geokon.com 2004))

Figure 2-1 shows an example of an extensometer (Geokon Model 4450 VW Displacement Transducer). Such extensometers are capable of measuring to a resolution of 25 microstrains. It measures the change in distance between two points, assuming that the strain is uniform between the points. Some extensometers use electrical resistance strain gauges (ERSG) as the transducer which can give a
resolution in the order of 1 microstrain. ERSG rely on the discovery made by Lord Kelvin, that the electrical resistance of a material is changed when stressed. By applying a voltage through a strain gauge, the change in resistance can be measured usually using a Whetstone bridge and the reading is then related to the strain (Koch, Boiten et al. 1952).

Figure 2-2 Electric resistance strain gauge (www.dundee.ac.uk 2004)

Figure 2-2 shows an electrical stain gauge. Strain gauges are a very popular way of measuring strain as they are simple to use and inexpensive. The measurements are at a high accuracy and can be used even in dynamic applications. However, the positioning of the strain gauges is important to achieve good results and they only give a single reading, hence a slow method for building a full dataset. Recently much research has gone into optical engineering and as a result new techniques are being developed to measure strain. These methods include speckle photography (Archbold, Ennos et al. 1978), holographic interferometry (Charmet 1978) and photoelasticity (Khan and Wang 2000). These methods can collect large amounts of data in a short period of time. In some cases surface preparation is required (Dally and Riley 1991). The initial costs are very high due to the complex and sensitive equipment used in these measurement techniques and requires trained operators to use the equipment. In a comparative accuracy study between speckle photography and holographic interferometry it was shown that in speckle photography errors can arise due to focusing the camera incorrectly, whereas holography interferometry is not affected instrumentally (Ennos 1980). Also the sensitivity of the speckle method can approach that of holography interferometry, but without requiring the same interferometric stability of recording apparatus.
For approximately half a century diffraction gratings have been studied by various researchers. The main pioneer was Bell who invented the Diffraction Grating Strain Gauge (Bell 1956). Bell used a rod with a thread and used the diffraction of the thread and photodetectors to measure the strain. This method was in the region of 1 microstrain accurate. As technology improved lasers have became more widely used and the laser strain gauge was developed (Coupland, Creasey et al. 1994).

![Laser strain gauge diagram](image)

Figure 2-3 Laser strain gauge diagram (Coupland, Creasey et al. 1994)

Figure 2-3 shows how the laser strain gauge works. A very small diffraction grating is etched into the surface of the specimen (Wileman, Coupland et al. 1994). These gratings are very small, 1 mm² and have been successfully etched into a number of different surfaces such as glass and various steels. Two laser beams are then directed towards the grating, where one is used as a reference beam and any distortion can be recorded with help of a Charge Coupled Device (CCD) camera. This distortion in the grating can be related to accurately measure strain when calibrated correctly. It has been shown that this method is still accurate even after degradation of the gratings or when dirty.

It is proposed that this LSG method should be used to measure strain on threaded fasteners at temperatures exceeding 600 K. The strain then can be used to find the loading on the fastener.
2.4 Load distribution analysis in threaded connections

In order to understand threaded connections in greater depth, it is important to study the load distributions. The entire load is transferred from the bolt to the nut through the threads and the distribution of this load can influence the surface strain on the nut as well as the working life of the threaded connection.

Much work has been carried out on the load distribution in threaded connections. The research can be identified into two main categories, experimental and analytical techniques. These categories will now be discussed:
2.4.1 Experimental work

Before the use of computers and modelling methods, all results were empirical from testing and experimental methods. Experimental methods are useful in verifying analytical results where many assumptions are introduced. Experimental methods are usually destructive and hence cost a lot of money and are time consuming. For reliable results a dataset of many practical tests needs to be carried out, increasing the cost and time involved to gather data.

In the previous section, the uses of extensometers have been mentioned as a method of strain measurement. Extensometers have been used to determine the strain within a threaded connection (Goodier 1940). The radial and axial expansion of the nut was measured to try to determine the strain. By using special nuts with only a single thread it was possible to get a load distribution along the whole nut. The results showed that the method is not sufficiently accurate enough at lower loads, but at higher loads the results proved that the load distribution along the threads is non-uniform but exponential. In a compression scenario, the load on the first thread is approximately ten times as much as the load on the last thread.

Photoelasticity is an effective way of measuring stress distributions. By taking a single photoelastic image it can be used to analyse large areas of stress. This method usually requires surface preparation of the specimen for accurate results. By tensioning a bolt and nut assembly and annealing it to a high temperature it is possible to "stress freeze" the threaded connection (Hetenyi 1943). Photoelastic images were taken of the assembly and then sliced so images of the inside of the nut could be taken. It was shown that the load distribution on tapered threads are more uniform. Hetenyi also did experiments on determining nut and bolt failures using photoelastic methods (Hetenyi 1950). It was shown that 65 % of the failures occurred close to the first thread in the nut. University College London further investigated the strength of the whole joint photoelastically (Jessop, Snell et al. 1955). More recently Patterson attempted to verify the analytical methods used to calculate load distributions in threads by experimental procedures (Patterson 1990). In this work, Patterson used photoelastic methods, like Hetenyi (Hetenyi 1950), to stress freeze the experimental specimens. Patterson sliced an M12 ISO nut into 8 slices each 1.5 mm thick. A
polariscope was used to get the photoelastic fringes. The fringe order was determined using Tardy compensation techniques (Kenny and Patterson 1985). More current research employed three-dimensional photoelastic models (Cretu and Lazar 1997). These axially loaded nut bolt assemblies were examined in a monochromatic polarized light (Na). The Boundary Element method was used to verify the photoelastic results and it was found that the two results are in satisfactory agreement.

Another method of measuring strain in a threaded connection is by the means of sensitive pins (Sawa, Kumano et al. 1995). Five sensitive pins with strain gauges were placed inside the drilled body of the nut. The contact stress was then measured at three different points, upper, middle and lower. Sawa also used ultrasonic waves and sensitive films to attempt to measure strain in a nut and bolt assembly. The results were compared to numerical calculations which were performed using the axisymmetric theory of elasticity and good correlation of 1.5% was found. Inoue and Shimotsuma (1982) combined two experimental methods. An epoxy model was constructed at a four times scale, and by cutting flanks out it was possible to measure the deflection at the threads (Inoue and Shimotsuma 1982). These epoxy models were then subjected to three-dimensional photoelasticity to determine the stress distribution in the epoxy models. The results correlated well with previous numerical results.

The most recent experimental method involves X-ray diffraction and hole drilling techniques (Martin 1998). The difference with these techniques is that they are not used on a nut and bolt assembly but on a threaded plate. For the X-ray method, a monochromatic X-ray beam is employed to determine the strain within the atomic lattice. This method is limited to surface measurement only, but electro-chemical polishing (ECP) can be used to clean the surface and allow a deeper profile of strains. Hole drilling is a destructive method as it involves drilling a small hole into the specimen, then a special strain gauge rosette is fitted inside the hole to measure the localised strain. Martin then validated the experimental findings using Finite Element Evaluation (Martin 1999). Overall the trends and results for these tests showed fairly good correlation. There were differences in magnitudes of the final deformations, which were attributed by the compliance of the rolling mill and were not included in the finite element model.
Other researchers have performed finite element analysis as well as experimental studies. Using the Copper-electroplating method developed by Okubo (Okubo 1968), Maruyama analysed a bolt and nut joint (Maruyama 1973). From the results obtained by Maruyama it was concluded that the copper-electroplating method is a highly accurate measuring method. Also the accuracy of the finite element method carried out by Maruyama can be improved by dividing the elements more finely. In later research the same two methods were employed to analyse the stress at the root of a thread (Maruyama 1974). In these tests it was found that once again there is fairly good agreement between the experimental and the finite element method, but due to the difficult geometry of the thread the element division introduce inaccuracies.

Threaded connections are often used in pipe lines especially in the oil industry. These companies have their own American Petroleum Industry (API) recommended practises for testing, which are then verified using FEA (Hilbert and Kalil 1992). The API procedures consist of a number of different tests, material, pressure and failure loads. These experiments test the specimen to destruction, and special testing rigs are required to cope with high pressures and forces involved. Using FEA for testing helps cut costs and is less dangerous than experimental testing.

A nut and bolt assembly in most working environment is subjected to fatigue. Hommel (Hommel 1998) carried out fatigue tests until the specimen failed and compared these results to FEA predictions (Hommel 1998). The results showed that FEA is a rational approach to predict fatigue failures. Englund and Johnson (Englund and Johnson 1997) compared a number of previously published experimental results to FEA results (Englund and Johnson 1997). This research showed that FEA results correlate well with experimental data and that by introducing friction between the two surfaces the modelled results are more consistent with the experimental findings. Overall, it has been shown that analytical analysis is a good representation of experimental tests. As validated analytical results do not require further laboratory testing of a physical model, the scope in cost reduction is of much importance to industry, reducing time and manufacture process of items.

Most of the experimental work done by other people presented in this section was carried out on particular connections under specific conditions. Therefore the
accuracy of them cannot be examined precisely. In works where finite element analysis was used to validate the results good correlations were found, but errors were present due to the complicated geometry of the threaded connection. The complicated geometry meant that assumptions were made to reduce the complicity of the finite element analysis and the amount of computational power required. In the next section previous analytical works done on threaded connections will be discussed and the accuracy of them will be examined in more depth.
Analytical analysis has a number of advantages over experimental techniques. With analytical methods no laboratory or testing is required so there is no need to manufacture a prototype. It can all be done with a pen and paper or by the use of a computer. If any modifications are required in the design, there is no need to remanufacture the sample again but all the modification can be done numerically. The disadvantage of numerical methods is that for complicated geometries the solution might be difficult to find so assumptions are introduced to simplify the problem. The main analytical studies are finite element analysis and numerical methods.

**Finite Element Analysis**

Finite Element Analysis (FEA) is a numerical method for analysing forces within a specimen without carrying out any experimental testing. There has been a lot of research into the FE analysis of threaded connections, mainly two-dimensional modelling and some three-dimensional.

Before modelling a problem in 3-D it is good practise to draw it 2-D to get an idea of what is happening and to keep the variables to a minimum. As a nut and bolt assembly is practically symmetrical all the way around it is possible to just take a slice and model it in 2-D. The force transfer between the threads is not straight forward. The easiest way of modelling it is by introducing a layer of elements with orthotropic properties between the nut and the bolt (Bretl and Cook 1979). The properties of the orthotropic elements depend on the geometry and direction of the threads. In the work done by Bretl and Cook the finite element analysis results were compared to experimental results presented in other literature and the correlation was good both between tapered and conventional threads. In some cases the thread is analysed by using contact boundary conditions (Assanelli and Dvorkin 1993). For this method nonlinearities must be included in the model to achieve sufficient accuracies. A more popular method is by introducing interface contact elements (MacDonald and Deans 1995). The difficulty with these sorts of elements is that a non-linear analysis is required. Interface elements cannot merge and every time they come into contact they try and push each other apart until they come to equilibrium. For this an interpolation
method is used. MacDonald increased the mesh density at critical areas to overcome the problem of the non-linear analysis not converging. As the bolt is circular it is possible to model it using an axisymmetric model (O'Hara 1974). Axisymmetric model is a 2-D model which is fully rotated around an axis to make it seem like it is 3-D. The advantage of this model is that only one slice of it has to be drawn and meshed in 2-D. O'Hara correlated the results from the axisymmetric simulation to the Heywood equation which is based on photoelastic data. The finite element results data demonstrated useful information about thread design, but the accuracy was not perfect due to the lack of contact capability of the finite element software used (NASTRAN). In the case of the nut and bolt scenario an axisymmetric model is not ideal as it does not take the helical thread and the hexagonal nut into account, but can be very useful to calculate individual thread loads (Percy and Sato 1981). The helix angle in the nut introduces a bending moment which can be accounted for by introducing a bending moment. Hommel analysed two different axisymmetric models to see how different loadings affect them (Hommel 1999). Hommel found that there was only a 4% difference between the two methods and due to the extra time and complications it is better to use the simplified model.

Tanaka has done a lot of research into the modelling of threaded connectors. Tanaka used axisymmetric model with contact elements to determine the stress distribution (Tanaka and Hongo 1981). Later he extended the studies to look at flange coupling due to transverse loading on the thread (Tanaka, Hongo et al. 1982). Tanaka used finite element analysis with help of spring models to analyse the loading within a bolt-nut joint with a fastened plate (Tanaka, Miyazama et al. 1981). The FEA method was used to relate the tensile load within a bolt to the actual service load to assure a good tightening (Tanaka and Yamada 1986). Zhao also did a lot of work involving FEA and threaded connections. The virtual contact loading method was developed to study the load distributions (Zhao 1994). Zhao found that the smooth contact method corresponds to the numerical and analytical results but in a frictional sliding contact case its not so. By increasing computational efficiency the accuracy is higher. A similar method was used to compare the differences in thread distribution between tapered and straight threads (Zhao 1998). Zhao found that with a suitable taper the stress and load distribution in threaded connections can be improved efficiently. Zhao used the same method to find the load distribution in threaded pipes (Zhao 1996). As
mentioned earlier, threaded connections in pipes are very important due to the offshore applications. The traditional thread used is API (American Petroleum Industry) but with the use of FEA a premium threaded connection "FOX" was designed (Yamamoto, Kobayashi et al. 1990).

2-D analysis can represent the load distribution in a threaded connection but it was shown that the pitch effects on the solution are significant when compared to a 3-D model (Rhee 1990). This can be up to 20%, so even though the 3-D analysis is more expensive in some cases where accuracy is essential the 3-D model should be used.

3-D FEA requires very high computational power which usually involves high costs, and due to this reason only a handful of people have researched this topic further. Bahai, mentioned earlier is one of these people. Using PATRAN, the two dimensional model was rotated around using a transfer function to give a full three-dimensional model (Bahai, Esat et al. 1992). ABAQUS finite element program was used to analyse the model consisting of isoparametric and interface elements. The results were compared to a so called hybrid model which consists of bending elements with the use of spring constants. The stresses from the 3-D model showed slight variation around the circumference. This discrepancy however is minimal at the critical site where the a failure is likely to take place, hence the cost of the 3-D model is not justified.

In a pressure vessel the 3-D threaded connection was modelled using MSC/NASTRAN (Grewal and Sabbaghian 1997). On the contact surface adaptive frictional elements (GAP) were used in order to account for the various levels of friction. The model was divided up into 45 degree segments as in the full version only four fully engaged threads could be analysed due to lack of computational power. Tafreshi analysed drillstring threaded joints using the IDEAS package for modelling and ABAQUS for the finite element analysis (Tafreshi and Dover 1993). Different scenarios were analysed including bending and torsion. For the torsion scenario the contact elements were changed to a continuous mesh allowing linear analysis, for bending the 2-D model was rotated around correspondingly with Bahai. Tafreshi’s results of the finite element analysis have been compared with full-scale fatigue test results on similar joint sizes, and generally showed good agreement. Later Tafreshi
improved these models by introducing SAXA elements and Fourier interpolation, but the helix angle was still ignored (Tafreshi 1999). It was shown that using axisymmetric solid elements with non-linear, asymmetric deformation with Fourier interpolation reduces the computational time and modelling and increases accuracy in comparison to full 3-D analysis.

The above listed 3-D attempts of modelling threaded connections using finite element analysis have all used some sort of an assumption or simplification. Most recently Zadoks and Kokatam have managed to create a model, which is fully three-dimensional and helical and uses contact element (Zadoks and Kokatam 1999). The thread was modelled separate from the body of the bolt and nut, and were joined together using fixed contact. This allowed non consistent meshing between the thread and body. The contact scenario was modelled in FEM code, and using PROTON3D algorithms it was solved. The connection was then modelled in a dynamic situation. Both analyses require high computational powers are very expensive. They have found that helical threads can be modelled accurately using PROTON3D dynamic analysis and that the results correlate well with the empirical workings (Zadoks and Yu 1997). The results were also compared with hand calculations, and it was found that good agreement could be achieved using dimensions that were selected based on the FEM results.

Numerical methods

In terms of threaded connectors the first real numerical findings were done by Sopwith. Sopwith identified that the loading along the thread is not uniform and that the force on a single thread is concentrated at mid-depth (Sopwith 1948). Sopwith also realised that the maximum load which is at the first thread in a compression scenario is about 4 times as big as the mean loading on the threads. By changing the thread profile it is possible to make the load distribution more uniform. Using Sopwith and experimental work Miller and Marshek developed the two dimensional spring model theorem (Miller, Marshek et al. 1983). The spring model idea is based on the assumption that a threaded connector can be divided up into a series of springs, where all these springs interact with each other. Depending on the material properties
and the profile of the thread these spring constants may change (Wang and Marshek 1995).

The spring model was then adapted to different threaded applications. As it was mentioned before tapered threads are used to make the load distribution more uniform. It was shown using analytical methods that there is a significant improvement in the order of 2 to 1 by tapering the thread (Stoeckly and Macke 1952). Analytical studies have gone into threads used in pipes due to the high cost of fatigue testing (Liebster and Glinka 1994). Using numerical methods it was possible to determine the fatigue life of an off shore pipe. At Lulea University researchers looked at the load distribution of coupling sleeve type joints (Lundberg, Beccu et al. 1989). This type of joint consists of a cylindrical coupling sleeve with an internal thread which is used to connect two drill rods. The numerical model disregards the wave motion of the sleeve and only accounts for the axial movement in the sleeve. Assumptions like this simplify the numerical model but can introduce errors and make the model less real. The spring model can be modified in such way that it can be applied to a knuckle shape threaded joint (Daabin 1990). These joints are used to transfer high loads in power switchgears. In further studies sinusoidal and impulse loading was applied to look at the affect of thread loosening (Daabin and Chow 1991). In these dynamic models the finite difference method was used.

In the method mentioned above only axial displacement was taken into account. Due to the helix angle and the shape of the thread there are bending forces acting on a nut and bolt assembly. These tangential forces can be found using numerical methods (Yazawa and Hongo 1988). These forces contribute to thread loosening. When a nut is under loading it is subjected to radial deformation (Hosokawa, Sato et al. 1989). Hosokawa calculated the radial deformation of the nuts and verified it using experimental methods. In these results it was shown that the all the calculated radial displacements of the nuts almost agree with the measured values, except in the high nut whose distribution of load at the mating threads is considered to be distributed uniformly in the bolt-nut unit. The results were also compared to Goodier's report on load distribution (Goodier 1940). Goodier's measurements showed similar tendencies to that of the calculated values in Hosokawa's work.
Numerical studies are used to calculate the maximum strength of certain designs. Using the Alexander theory it is possible to find the maximum loadability of a bolt-nut assembly (Hagiwara, Itoh et al. 1995). Such calculations are useful when redesigning nuts and trying to keep them safe. The results show that the height of the nut has to be at least 40% of its diameter for it to be effective. The calculations were verified using experimental techniques carried out with accordance to the British Standards and it was found that the results are consistent on the whole, with some exceptions.

The spring model is an accurate way of modelling the loading in threads as it will be demonstrated later on. For the spring model to work the spring constants have to be specified accurately. If the material properties are known then the spring constant for the nut and the bolt is found by working out the volume and using Young’s modulus. To find the spring constant for the actual thread is more difficult as it involves a contact between two surfaces. To overcome this problem Finite Element can be used. By only modelling the thread no model is required for the rest of the assembly, keeping the number of elements to a minimum. Bahai converted the spring model into a matrix and used FEA to find the spring constant for the thread (Bahai and Esat 1991). The model used for the thread was an axisymmetric model with interface contact elements. The results between the spring model and the finite element analysis showed a very good comparison. Bahai then introduced bending elements to compensate for the helical thread, and make the model like if it would be three-dimensional (Bahai, Esat et al. 1992). Introducing bending elements meant a slightly more complicated analysis but improved the accuracy of the results.
2.5 Variation Analysis

Variation analysis has become an important research area in the late 1980s. It is used to help predict how components which are manufactured to be within a certain tolerance will behave under working conditions and most importantly when assembled. This is important in terms of safety but performance is also a main concern especially when designing measuring devices.

The tolerance represents the permissible variation of a dimension in an engineering drawing. The tighter the tolerances the higher the costs so it is important to optimise between cost of an assembly and the tolerances. In order to design, manufacture and assemble products most effectively it is necessary to evaluate the effects of dimensional and material variation. There are a number of different methods that can be used.

Monte Carlo Simulation (Early and Thompson 1989) is one of the most popular methods although it expensive and time consuming. Worst Case (Greenwood and Chase 1988) and Root Sum Square (Greenwood and Chase 1990) are commonly used as well but advanced statistical analysis is required to interpret the findings properly. One of the most straight forward analysis is the Stack-up method (Lee and Woo 1990) which sums the individual tolerances to find the overall variation.

The type of variation simulation modelling (VSM) tool used depends on the specific problem and the accuracy of the required analysis. It is important to introduce VSM in the early design phase to avoid problems in later stages of the development. VSM is a reliable prediction technique for built dimensions of a product, provided representative input for individual components is accurate (Doepker and Nies 1989).

In some cases where deformation occurs in the specimen more specialised variation simulation modelling needs to be used. Liu and Hu used finite element methods (FEM) in developing mechanistic variation simulation models for deformable sheet metal parts (Liu and Hu 1997). Similar methods can be applied to look at the affects of errors in threaded connections. Maruyama investigated the influence of pitch and flank angle error in threaded connections using FEA and copper-electroplating
methods (Maruyama 1976). Maruyama found that the pitch error on the first thread causes major variation in the load distribution along every thread. This may increase the stress concentration factor, therefore it is necessary to equalise the stress distribution along the root of the bolt thread which is screwed in the thread. The flank angle variation has minimal affect on the load distribution.

Most of the work done on variation analysis involved assemblies and apart from Maruyama there has been very little work done on the variation analysis of threaded connections. In later sections it will be shown how variation analysis can be used to help find the sensitivity of parameters errors within the threaded joint.
Chapter 3 - The Assembly

3.1 Introduction

Threaded connections are used to fasten two or more items together, with only the end of the bolt and the fastening nut visible (Figure 3-1). The inside of the connection is hidden from the naked eye and there is no simply way of analysing what goes on exactly within the joint. In this section the threaded connection will be analysed using finite element analysis as an assembly looking at the factors that influence the strength and the uniformity of them.

![Figure 3-1 Threaded connection assembly with modified nut](image)

The threads are a very important part of the connection as all the force is transferred through them. All threads are manufactured within a certain tolerance according to the British Standards (BS 1768:1963) and it cannot be guaranteed that two threads will be identical. When looking at the engaged threads it is possible to say that no two threads are joined the same way, hence it is very unlikely that two connections are the same.
In a threaded joint not all parts have threads, for example a washer. When a joint is assembled with unthreaded parts, it is difficult to assure that all parts are perfectly aligned axially. If parts are off centre the whole load distribution within the joint can be affected. This would mean that the laser strain gauge might give misleading results, but more importantly high stress areas might be created which would be more likely to cause failure.
3.2 Threaded Connection – Structural integrity

Like all engineering components every threaded connection has a tensile strength specification in order to prevent failure. The critical part of the threaded joint is the first engaged thread as it is under the most amount of load. The tensile strength of any threaded connection can be found in the equivalent British Standard Specification.

The tensile test on a bolt or screw shall be carried out as follows:

An ordinary nut or its equivalent in the form of an adaptor shall be screwed on to the bolt or screw so as to be clear of the run-out of the thread towards the head and also clear of any imperfect threads at the point, the load shall then be applied to the head and to the nut or adaptor. (BS 1768: 1963)

The strength of the nut and bolt will depend on its nominal diameter, its material and its grade. The tensile strength is only guidance; a safety margin should be taken into account when using threaded connections. So the maximum load a connection can take is governed by the tensile strength (with a suitable safety factor) assuming that all the other components in the joint have a higher structural integrity.

Whatever the purpose of the fastener is, it will be required to be under a minimal load to hold the joint together. Therefore for a threaded connection to function properly a maximum and a minimum load has to be specified. In some cases the specified load has to be even more accurate to allow efficiency and longer working life for the components. When assembling a joint a torque drive can be used to apply the required load, but due to thread loosening constant load monitoring is required. Thread loosening will be explained in detail in Chapter 7.

In most cases ultrasonic means are used to monitor the load in a connection by measuring the change in length of the bolt. Ultrasonic measurement works by applying an electrical pulse to a piezoelectric element inside a transducer which creates a longitudinal shock wave. This wave travels through the length of the bolt, until it encounters a change in density (i.e.: the end of the bolt) where most of the waves are reflected. The wave travels back along the bolt into the transducer and it
produces a small electric signal when it hits the piezoelectric element. Using this electric signal the time it takes for the shock wave to travel through the bolt is measured. Under loading the length of the bolt changes and this change in length can be calculated from the time it takes for the shock wave to travel. For accurate results a number of samples are taken and averaged. Sufficient time has to be left between readings to allow the ultrasound to diminish. Current ultrasonic methods can measure to a resolution of 0.002 mm. As using ultrasound is a contact way of measuring the load, extreme temperatures can interfere with the readings, also the equipment could be damaged. At present where the temperatures are extreme due to working conditions the only way of determining the load accurately is by stopping the machinery and cooling it down to more user-friendly temperatures. This is time consuming and not cost effective in most cases, hence a non contact method of determining and monitoring the load is required. In heavy duty machinery where the working temperatures are high the level of vibration the joints are subjected to increase as well. This requires more frequent load monitoring so an easy and quick method would be essential.

Finding the load on a threaded connection is useful to help prevent thread failure but also to help determine the residual life of the joint. By understanding the stress distribution within the joint the maximum stress area can be found. It is most likely for a joint to break at this point but by using finite element analysis the maximum stress points can be found with respect to the load and the residual life can be predicted. The FEA assumes perfect geometry but in real life parts are manufactured to within a certain tolerance. Due to this dimensional variation in the joint the relationship between the load and the maximum stress points is subject to vary. Variation analysis can be used to see how the maximum stress point is affected due to dimensional discrepancy; this will be described later in Chapter 7.

In the previous chapter (Background Theory) a few load measuring methods were discussed but due to the high temperatures only non contact methods can be taken into serious consideration if accurate load monitoring is required. Non contact methods limit the types of operations that can be carried out, especially at high temperatures dealing with metal objects. The laser strain gauge was developed in order to measure surface strain using lasers. If the surface strain on any visible part of the connection
can be related to the loading, then the laser strain gauge can be adopted as a load measuring device.
3.3 User manual for the laser strain gauge

The laser strain gauge was developed at Loughborough University Optical Engineering Research Group as a non contact strain measuring device (Coupland, Creasey et al. 1994). Using a hand held device a laser beam is reflected off a grating engraved onto the surface of the specimen and by measuring the change in the diffraction the surface strain can be determined.

In order for the laser strain gauge to work, a very fine reflective grating has to be manufactured onto the specimen surface. The grating manufacture is carried out using an Excimer laser at high frequency (1000 lp/mm). The Excimer laser operates in the ultraviolet light range and uses a combination of inert and reactive gases. Under the appropriate conditions of electrical stimulation, these gases give rise to a laser light which can etch the surface of the test material. The grating can be manufactured into a number of different surfaces, and once it has been etched it can be used to measure surface strain for the rest of the specimen's working life (Wileman, Coupland et al. 1994). The area of the grating is about 1 mm² (Figure 3-2).

![Etched Grating](image)

Figure 3-2  Etched Grating
The schematic diagram of the laser strain gauge can be seen in Figure 3-3. The reflective grating is illuminated using a 1mW HeNe laser with an expanded and collimated beam. Using Position Sensitive Detectors (PSDs) the angular position of the diffracted orders is measured. The signals from the PSDs are processed using and analogue electronic circuit to obtain the position of the laser spots on each of the detectors. The signal is then digitised to aid the data collection. The angle measured is used to determine the spatial frequency of the gratings, which is proportional to the surface strain (Coupland, Creasey et al. 1994).

![Figure 3-3 Schematic diagram of the laser strain gauge](image)

The laser strain gauge itself is built into a small handheld device (Figure 3-4). Two laser beams are used, one is to measure the diffraction and the other is a reference beam. The LSG works in a similar manner to a handheld barcode reader. The laser is directed at the grating and the reflected image is recorded using a CCD camera. By
measuring the distortion in the grating the surface strain can be determined. It was shown in laboratory conditions that that strains as low as $10^{-6}$ can be measured with the use of modern laser sources (Wileman, Coupland et al. 1994).

The laser strain gauge can be adapted to work as a load measuring device on a threaded connection. By carrying out a number of experimental and analytical tests the correlation between the surface strains and the load on a threaded connection can be found. This correlation can be programmed into a data logger so as soon as the surface strain is measured for a certain connection the load can be calculated instantaneously. The user then will know whether to tighten or loosen the joint.
3.4 How to determine the load from measured strain

There are two ways of finding the load distribution within a threaded connection, analytical and experimental. In the initial design stages, analytical methods will be used. All the previous work which was done on threaded connections looked at the load distribution in the inside of the thread. For the laser strain gauge method the strain has to be measured on the outside. With the help of finite element analysis initial testing was done on different shapes. Once the initial tests were finished more complicated modelling was performed using non-linear analysis to try and create a prototype design which could be manufactured and tested using strain gauges. Numerical methods were used to validate and aid the finite element results.

With the use of finite element analysis it was possible to investigate the strain distributions on threaded connections. It was found that the surface strain on the nut, which is the only visible part of the joint, is very small and it would be inaccurate to try and relate the load to it. Different ideas were investigated and it was decided that by cutting grooves into the nut the surface strain can be increased while maintaining sufficient strength in the joint. All the initial ideas and a detailed description on how the final prototype was designed are shown in Chapter 6.

Figure 3-5 shows the final prototype designed using finite element analysis. The grooves are at 45 degrees to the flat face and they are positioned on alternate sides to
give the user more options when taking readings. The grooves exact location was found by optimising the amount of material that got cut away with the magnitude of the surface strain.

The surface strain on the modified nut can be seen on Figure 3-6. The surface strain within the grooves is much higher than on the surface of the rest of the nut. This allows the load to be determined more accurately as a larger margin of error can be used.

Presently using ultrasonic methods are used to find the elongation in the bolt which is then used to work out the strain. By knowing the material properties of the bolt the load can be determined directly from the bolt strain. With the laser strain gauge the surface strain within the grooves can be related directly to the load using results obtained from FE models and experimental results. When a nut is modified by cutting out the grooves and adding the grating, it can be supplied with a correlation table to show how the surface strain corresponds to the overall load.

For the initial prototype this relationship was found using 2-D axisymmetric finite element modelling and was later verified using experimental testing. (Note: During the verification it was found that hysteresis is present on the surface strain due to the threads sticking. A different solution had to be found, this will be explained in depth in Chapter 6.)
An axisymmetric model is a 2-dimensional model representing a 3-dimensional object. The 2-D model is drawn so when it is rotated around the Y-axis it becomes the required 3-D model. It is a very useful simplification as it requires much less computational power. It is especially useful when working with non-linear analyses such as this one involving contact elements. The disadvantage of using this technique is that the model assumes that geometry is circular. In this case the helical effect of the threads and the hexagonal shape of the nut were neglected. This will be explained in more detail when the development of the prototype is described in Chapter 6.

Figure 3-7 2D axisymmetric model

Figure 3-7 shows the modified nut and bolt joint in an axisymmetric view. The strain plotted on this figure is the scalar strain (EE), it is used to find high strain concentrations on a FE model. This type of contour plot was used as a quick way of identifying where the surface strain is high. EE is found using:

\[ EE = \sqrt{\frac{1}{2} \left[ (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 \right]} \]

Equation 3-1
The elements within the model were lined up so a direct strain reading could be taken at the cut. In LUSAS the coordinate system is orientated in such a way that the horizontal direction is “x”, the vertical direction is “y” and depth is “z”. Therefore the axial direction is in the “y” direction and the hoop direction is “z”. The following preliminary results were found.

<table>
<thead>
<tr>
<th>Strain reading</th>
<th>Bolt Strain</th>
<th>Load</th>
</tr>
</thead>
<tbody>
<tr>
<td>Microstrain</td>
<td>Microstrain</td>
<td>kN</td>
</tr>
<tr>
<td>Axial Direction</td>
<td>-132</td>
<td>1215</td>
</tr>
<tr>
<td>Hoop Direction</td>
<td>397</td>
<td>1215</td>
</tr>
</tbody>
</table>

Figure 3-8 Preliminary results table

Preliminary tests showed that the strain along the bolt is about ten times the strain along the cut, and about three times as much as the hoop strain along the cut. The hoop strain is the strain in the z direction, going around the nut.

Strain is proportional to load and Young’s modulus. As Young’s modulus is constant within the elastic limits of a material the relationship between load and surface strain will be linear. These results are preliminary and not accurate. They are used as a representation how the load can be found by measuring the surface strain on a modified nut. Further tests will be carried out.
3.5 Variation analysis of assemblies due to manufacturing imperfections

In this section it was shown how the loading on a threaded connection can be determined by measuring the surface strain on a specially modified nut. Using finite element analysis a relationship was found between the surface strain in both the axial and the hoop direction and the load. In theory using this relationship the load can be calculated accurately.

It was mentioned before that the relationship was found using finite element analysis. Finite element analysis assumes conditions where the geometry, the assembly and the boundary conditions are all perfect. In real life this is rarely the case and it is very unlikely that an actual prototype will behave in exactly the same manner as predicted from previous models. It is even more unlikely that every sample ever manufactured will be identical.

Parts are usually manufactured within a certain tolerance so some variation will occur between the same parts. When assembling these parts, further variation will occur. Variation can also occur when assembling completely identical parts due to fitting tolerances. For example if a washer is added to a threaded joint it can move slightly allowing it to be off centre.

Variations can have a harmful effect on the accuracy of the laser strain gauge. The surface strain will be affected by differences in the tolerances, in the assembly and in the materials. The severity of this variation has to be investigated further to insure that the laser strain gauge can be used as an accurate load measuring device.

The next chapter will look at the main factors that can affect the accuracy of the load measurements.
Chapter 4 – Sensitivity Analysis of Surface Strain Measurements

4.1 Introduction

It was shown before that surface strain measured on a specially modified nut can be related to the load on the joint. However it is not known how manufacturing imperfections and other variations within the assembly might introduce inaccuracies into the load measuring mechanism. In this part a number of possible sources of variations will be investigated and analysed.

Variation can occur due to material defects, manufacturing imperfections and disparity in assembly. These variations can affect the surface strain measured on the modified nut which would then introduce errors into the load and surface strain relationship. For example, if a nut is manufactured so it's cross sectional area is smaller; it would mean that under the same load the surface strain would be higher. Hence, the surface strain reading would suggest a higher load than the actual one. During production all imperfections are kept within a tolerance level to maintain a certain level of quality and safety, but some variation will come about. The smaller the tolerance levels are the higher the production costs, so it is important to understand how variation affects the suitability of the product. To see how the accuracy of the load measuring nut changes with variation the errors due to variation needs to be quantified.

In variation analysis this is called the sensitivity factor. The sensitivity factor is a multiplier which is used to predict the error within a certain tolerance. With respect to the nut, the error is the difference between the actual and the measured load. By quantifying these errors, the impact of the variation is found. This will help decide whether:

a. The nut can be used as a load measuring device using the standard tolerances
b. The nut needs to be manufactured to a tighter tolerance
c. Each individual nut will have to be calibrated to achieve the required accuracy.
4.2 Sensitivity Analysis

The variation in geometric dimensions due to manufacturing tolerance will have an effect on the surface strain under the same loads. To find the significance of this impact the error needs to be quantified. This is done by introducing a sensitivity factor.

In variation analysis the sensitivity factor is a function that governs the relationship between the variations due to imperfections and the error. In numerical terms:

\[ \frac{dE}{E} = SF_p \times \frac{dp}{p} \quad \text{Equation 4-1} \]

Where \( SF \) is the sensitivity factor with respect to parameter \( p \), \( \frac{dp}{p} \) is the error in parameter \( p \) and \( \frac{dE}{E} \) is the error caused by parameter \( p \). The idea of the sensitivity factor is that if the maximum accepted error is known then the tolerance levels can be found. For example: If a component has a sensitivity factor of 5% per 1 mm and the maximum error allowed is 1 percent then the component has to be manufactured within a tolerance of 0.2 mm.

The sensitivity factor is unique for each parameter varied. It can be related to a dimension, and angle or even a material property. The object is to try and find the sensitivity factor for every parameter that can interfere with the surface strain reading. The total error will then depend on the sum of all the factors.

This project (sponsored by Hydratight Sweeney) has the requirement to develop a threaded connection load measuring device. The required accuracy by the company is to determine the load to be within 0.5% of the actual value. This means that even if the surface strain and the load can be related without any variation, then for an accurate load reading the measured surface strain has to be within 0.5% of the actual strain. So for the project to be successful \( \delta e/e < \pm 0.5 \% \).

To understand how the sensitivity factor works its worth looking at a simple example. The example chosen is to look at the parameters that can affect the surface strain on a solid shaft. The solid shaft was chosen as an example because the surface strains in
both the axial and the hoop direction can be found simply using standard formulas. Also the number of parameters that can affect the strain readings are relatively small so it can be used as a simple example. The example can be seen in Appendix A.

A simple example like this shows, that there are a number of parameters that can affect the strain on a simple shape. Each parameter has its own sensitivity factor that governs how the change in that parameter affects the change in the measurement. Once all the sensitivity factors are found they can be summed together to find the total change.

This example is very simple, but the same principles can be applied to more complicated problems like finding the surface strain on the nut. The geometry is more complicated so the errors cannot be found in a mathematical way but by the use of finite element analysis and variation analysis. In the simple example the sensitivity functions were simple but when more complicated geometries are analysed using FE the sensitivity function will be more complicated.

The example showed mathematically how a sensitivity function can be found for simple geometries. It was mentioned before that the nut has a complicated geometry and the sensitivity functions cannot be found mathematically but FE has to be used. With FE the sensitivity function can be found directly by analysing how the results vary with respect to a certain change in the parameter. In some cases, even though the sensitivity function can not be found mathematically accurately, the mathematical relationship can be used to help predict the type of sensitivity function (Katz, Khilyuk et al. 1996). Using the solid shaft as an example, if the measured property depends on the area, then it is possible to predict that the sensitivity with regards to radius will be at a factor of 2 due to area being proportional to $r^2$. Using similar techniques it is possible to predict how a parameter affects the sensitivity function.

With the use of variation analysis the parameters that may affect surface strain measurement on the nut are varied. These changes can be modelled using finite element analysis and the affects can be recorded from the analysed model. The change in parameter is then related to the change in the measured strain. This allows the sensitivity factor to be found for each parameter directly.
The special nut was designed with grooves to measure surface strain which then can be related to the load in the threaded connection. There are a number of parameters that can interfere with the surface strain which may give an incorrect load so it is important to determine the sensitivity factor for these parameters.

The method of determining the sensitivity functions for a complicated design like the special nut will be discussed in the following section.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Proposed solution</th>
<th>Assumptions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Material</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young’s Modulus</td>
<td>$E$</td>
<td>Find sensitivity function mathematically</td>
<td></td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>$\nu$</td>
<td>Find sensitivity function mathematically</td>
<td>Homogeneous, variations in bulk properties only</td>
</tr>
<tr>
<td>Residual strain</td>
<td>-</td>
<td>Future work</td>
<td>Residual strains are removed by heat treatment or other</td>
</tr>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Inner radius</td>
<td>$r_s(\theta)$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Bulk geometry not affected by groove, geometry variation changes the spring constant $K_{bc}$</td>
</tr>
<tr>
<td>Distance between flat faces</td>
<td>$F_N$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>No variation in z-direction</td>
</tr>
<tr>
<td>Distance between corners</td>
<td>$C_N$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>No variation in z-direction</td>
</tr>
<tr>
<td>Angle between flat faces</td>
<td>$\beta$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>No variation in z-direction</td>
</tr>
<tr>
<td>Length of flat faces</td>
<td>$L_N$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>No variation in z-direction</td>
</tr>
<tr>
<td>Chamfer angle</td>
<td>$\alpha$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td></td>
</tr>
<tr>
<td>Concentricity</td>
<td>$\Delta h$, $\Delta y$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td></td>
</tr>
<tr>
<td>Height</td>
<td>$h(\theta)$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>All walls are vertical in z</td>
</tr>
<tr>
<td>Diameter of washer face</td>
<td>$W_d$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Not threaded hence does not effect load distribution</td>
</tr>
<tr>
<td>Height of washer face</td>
<td>$W_h$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Not threaded hence does not effect load distribution</td>
</tr>
<tr>
<td>Flatness of faces</td>
<td>-</td>
<td>N/A</td>
<td>Surfaces are smooth</td>
</tr>
<tr>
<td><strong>Threads</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minor diameter</td>
<td>$D_m$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Geometry variation changes the spring constant $K_T$</td>
</tr>
<tr>
<td>Major diameter</td>
<td>$D$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Thread stiffness is constant for a single thread rotation</td>
</tr>
<tr>
<td>Pitch</td>
<td>$\rho_T$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Thread stiffness is constant for a single thread rotation</td>
</tr>
<tr>
<td>Thread angle</td>
<td>$\gamma$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Thread stiffness is constant for a single thread rotation</td>
</tr>
<tr>
<td><strong>Loading</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Load</td>
<td>$P$</td>
<td>Parameter to be measured</td>
<td>Strains vary linearly with load</td>
</tr>
<tr>
<td>Distribution</td>
<td>$P = \Sigma P_s$</td>
<td>Find sensitivity function using the spring model and FE</td>
<td>Force distribution is governed by the spring model</td>
</tr>
<tr>
<td><strong>Grooves (1,2, or 3)</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selection of groove</td>
<td>-</td>
<td>N/A</td>
<td>Grooves are equally spaced and of same geometry including any variations</td>
</tr>
<tr>
<td>Centre Location</td>
<td>$x'<em>{C}$, $y'</em>{C}$</td>
<td>Reference point</td>
<td>Positioned on washer outer wall for orthotropic model</td>
</tr>
<tr>
<td>Depth (height)</td>
<td>$h_g$</td>
<td>Find sensitivity function numerically</td>
<td></td>
</tr>
<tr>
<td>Trajectory</td>
<td>$\theta_g$</td>
<td>Find sensitivity function numerically</td>
<td></td>
</tr>
<tr>
<td>Radius</td>
<td>$r_g(\theta)$</td>
<td>N/A</td>
<td></td>
</tr>
<tr>
<td>Location of Grating</td>
<td>$x'<em>{G}$, $y'</em>{G}$</td>
<td>Find sensitivity function numerically</td>
<td>Bulk geometry not affected by groove</td>
</tr>
<tr>
<td><strong>Laser</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trajectory</td>
<td>-</td>
<td>LSG calibrated</td>
<td></td>
</tr>
<tr>
<td>Operating distance</td>
<td>-</td>
<td>LSG calibrated</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4-1**  Nut parameters
4.3 Fundamental Equation for the Nut Sensitivity Analysis

The first step when trying to find the fundamental sensitivity equation for a specimen is to identify all the parameters that may vary and affect the performance. The parameters are grouped and listed for the modified nut in Figure 4-1, and can be visualised in Appendix B.

Before varying each and every parameter it is important to have a reference point which is kept constant so all the other parameters can be varied with regards to it. This point was chosen to be the groove centre location \((x'_G, y'_G)\), which is the centre point on the wall of the nut from where the groove is manufactured from where “x” is the horizontal and “y” is the vertical direction (Appendix B). The other parameter in the table which is not varied is the total load as this is the parameter which is to be determined.

The nut has a very complicated geometry with a number of parameters that can change with regards to manufacturing imperfections. Manufacturing imperfections are present in everything that is manufactured to be within a certain tolerance. Due to this no two items can be assured to be perfectly identical. With regards to the nut these imperfections can occur at a number of places. The hexagonal nut is manufactured in such a way that the distance between the flat sides is within a certain tolerance. This governs the effective diameter of the nut, and with it the cross-sectional area of it. The minor diameter of the nut is also manufactured to be within a certain tolerance and this governs the size of the threads. The major diameter (which is the size of the hole in the nut without the threads) is kept to zero tolerance in the nut. In the bolt however both the width across the flats, the minor diameter, and the major diameter is varied for the unified threads. For ISO metric threads the major diameter on the bolts is kept constant, but all the other dimensions are varied the same way as for the unified threads. The tolerances tables for selected threads can be seen in Appendix R. When the nut is modified and the grooves are added, the geometry becomes even more complex introducing more sources of error. In order to simplify the problem the sensitivity analysis for the nut is broken down into a number of steps.
Initially to keep the geometry relatively simple, the grooves are replaced by an orthotropic material and the surface strain on the nut is analysed without the grooves. The orthotropic material will allow the model to behave similarly to the original geometry with the grooves, but with less parameter to vary. The method of finding the orthotropic material and how it works is explained in the next chapter when the sensitivity analysis is carried out. In these analyses the surface strain is not measured within the grooves (as they replaced by an orthotropic material), but the strain is measured at the groove centre location which is the reference point. This simplification is valid as it is assumed that the manufacturing imperfections affect the strain at the reference point in the same manner to the strain within the groove. The magnitudes of these strains will be different, but for the sensitivity analysis only the change in strain with regards to imperfections matter.

The second step is to include the grooves in the model and see how the changes in the groove itself affect the surface strain readings. This time the strain is measured within the grooves at the location of the grating point \((x'_L, y'_L)\). By carrying out the analysis in two separate steps simplifies the problem as we can first vary the geometry of the nut and then vary the geometry of the groove. To understand why these steps can be carried out independently we can show it in a mathematical way.

In the example in Appendix A it is shown that the surface strain on a solid object is a function of material properties, cross sectional area and the applied load. For the example the relationship between these and the surface strain was straight forward so it could be modelled easily mathematically. For the nut this relationship is rather more complicated, but as the sensitivity functions can be found using finite element analysis there is no need to know the exact formula. It is enough to say that at the reference point the axial and the hoop strain will be:

\[
\begin{align*}
\epsilon_1 &= f(P, E, A) \\
\epsilon_2 &= f(P, E, A, \nu)
\end{align*}
\]

So the axial strain \((\epsilon_1)\) is a function of load, Young's modulus and area, whereas the hoop strain \((\epsilon_2)\) is the function of all of the above and Poisson's ratio.
The exact relationship between these parameters and the strain is not known, but each of these parameters will have a sensitivity function (SFp) which will govern how the strain changes with regards to variation in that parameter. The sensitivity function will be found using finite element analysis. For some parameters it is possible to predict the sensitivity function in a mathematical way. For example, Young’s modulus will be directly related to surface strain as shown in Appendix A, so the sensitivity function for E will be $SFE = -1$.

The variation of the axial ($\varepsilon_1$) and the hoop strain ($\varepsilon_2$) at the reference point will be given as follows.

$$\frac{\delta \varepsilon_1}{\varepsilon_1} = SF_p \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_A \frac{\delta A}{A}$$  \hspace{1cm} \text{Equation 4-4}$$

$$\frac{\delta \varepsilon_2}{\varepsilon_2} = SF_p \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_A \frac{\delta A}{A} + SF_v \frac{\delta v}{v}$$  \hspace{1cm} \text{Equation 4-5}$$

The equations above show the relationships between the error in strain and the imperfection in each parameter if the sensitivity functions are known for the surface strain at the reference point ($x'G, y'G$).

The second step is to find the surface strain ($\varepsilon'$) within the groove ($x'L, y'L$). The surface strain within the groove ($\varepsilon'$) is a function of the strain at the reference point ($\varepsilon$) with respect to the groove geometry ($hG, \theta_G$) and the grating position ($x'L, y'L$). Therefore the expression for axial strain ($\varepsilon_1'$) and hoop strain ($\varepsilon_2'$) within the groove are as follows:

$$\varepsilon_1' = f \{ \varepsilon_1[h_G, \theta_G, (x'L, y'L)], \varepsilon_2[h_G, \theta_G, (x'L, y'L)] \}$$  \hspace{1cm} \text{Equation 4-6}$$

$$\varepsilon_2' = f \{ \varepsilon_1[h_G, \theta_G, (x'L, y'L)], \varepsilon_2[h_G, \theta_G, (x'L, y'L)] \}$$  \hspace{1cm} \text{Equation 4-7}$$

Using the same method as before the variation in the surface strain within the groove due to manufacturing imperfections can be found with the following expressions:

$$\frac{\delta \varepsilon_1'}{\varepsilon_1'} = SF_p \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_A \frac{\delta A}{A} + SF_v \frac{\delta v}{v} + SF_{hG} \frac{\delta h_G}{h_G} + SF_{\theta G} \frac{\delta \theta_G}{\theta_G} + SF_{(x'L)} \frac{\delta(x'L)}{(x'L)} + SF_{(y'L)} \frac{\delta(y'L)}{(y'L)}$$  \hspace{1cm} \text{Equation 4-8}$$
\[
\frac{\delta \varepsilon_2'}{\varepsilon_2'} = SF_P \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_A \frac{\delta A}{A} + SF_v \frac{\delta v}{v} + SF_{hG} \frac{\delta h_G}{h_G} + SF_{\theta g} \frac{\delta \theta_g}{\theta_g} + SF_{(x'L)} \frac{\delta (x'L)}{(x'L)} + SF_{(y'L)} \frac{\delta (y'L)}{(y'L)}
\]

*Equation 4-9*

So the surface strain within the groove will vary with respect to the sensitivity of the parameters listed above. This surface strain is measured using the laser strain gauge which is then related to the load on the joint. Depending on the accuracy of the laser strain gauge there might be a variation between the measured and the actual strain. This is the third step in the sensitivity analysis, where the measured strain \((\varepsilon_m)\) is a function of the actual strain.

\[
\varepsilon_m = f(\varepsilon_1', \varepsilon_2')
\]

*Equation 4-10*

If the laser is calibrated it is assumed that the measured strain and the actual strain will be identical.

The total variation of the measured surface strain within the groove can be given with the following fundamental equation:

\[
\frac{\delta \varepsilon_{m_r}}{\varepsilon_{m_r}} = \sum_{p=1}^{m_r} SF_p \frac{\delta v_p}{v_p}
\]

*Equation 4-11*

Where "SF" is the sensitivity function, "v" is the variables, "m_r" is the number of variables considered and \(\varepsilon_m\) is the measured strain.

The laser strain gauge works in such a way that both the hoop and the axial strain can be determined. Once all the analyses are carried out it will be possible to establish which strain can be measured more accurately so the strain in that direction can be used to be related to the load.

The accuracy required for this project is 0.5%, so for the project to be successful \(\frac{\delta \varepsilon_m}{\varepsilon_m} < \pm 0.5\%\). If this is not the case calibration is necessary. If the manufactured
modified nuts each behave slightly differently under loading due to manufacturing imperfections, then they have to be calibrated individually so they can be used as accurate load measuring devices. Individual calibration can be achieved in a test rig where the known load can be related to the measured surface strain.
4.4 Nut Sensitivity Analysis

In the previous section it was described how the sensitivity analysis of the nut will be broken down into two parts to simplify the problem. Firstly the original nut geometry will be investigated finding the affects of manufacturing imperfections on the surface strain at the reference point. In the second part the affects of manufacturing imperfections appearing during the modification of the nut will be investigated within the groove. Due to the complexity of the geometry finite element analysis will be used to find the sensitivity functions.

In Figure 4-1 all the parameters that may vary due to material or manufacturing imperfections are listed and grouped. The nut has a very complex geometry that means that there are a large number of dimensional parameters that may vary. Also there is no simple numerical way of finding the sensitivity factors for each so finite element analysis has to be used. This would be very time consuming so a simplification is suggested. Instead of varying the geometry of the nut the load distribution on the threads should be varied.

The dimensional variations have an affect on the way the load \( (\Sigma P_n) \) is distributed over the threads (this is explained in the spring constant section, Chapter 5). Initial test have shown that the variation in the load distribution due to the change in geometry can have a significant affect on the surface strain. The tests have shown that the difference in load distribution between the maximum and the minimum tolerance levels on the nut can cause a variation of 15% in the surface strain. This is much higher than the variation caused simply by the changes in the geometry if the load distribution would be unaffected. For this reason it is suggested to determine how the load distribution varies with manufacturing imperfections and find the sensitivity function with respect to the load distribution.

The load distribution within a threaded connection can be found by the use of Marshek's spring model. The spring model uses a set of spring constants which is determined by mathematical ways and by finite element analysis. A threaded connection is made up of three spring constants; these are the stiffness of the body, the stud and the thread. The spring constants will change according to the variation in
the geometry. Changes in the spring constants affect the load distribution on the threads which affect the surface strain. The sensitivity factor between each spring constant and the surface strain can be found using variation analysis. The advantage of relating the spring constant directly to the surface strain is that the number of separate parameters that need to be investigated is reduced. As all dimensional changes affect the spring constants it is easier to relate every parameter to the stiffness rather than finding individual sensitivity factors for each.

Equations 4-4 & 4-5 give an expression to help find the change in surface strain with regards to certain parameters. These parameters are material properties, force and cross sectional area. Figure 4-1 shows all of these parameters grouped, bearing in mind that the cross sectional area depends on the geometry as well as on the threads. From the table it is evident that to find the individual sensitivity function for each would be time consuming. Instead it is suggested that one single sensitivity function for the load distribution ($\Sigma P_n$) should be sufficient to represent all the sensitivity functions for the nut.

\[
SF_{\Sigma P_n} \frac{\delta \Sigma P_n}{\Sigma P_n} = SF_P \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_A \frac{\delta A}{A} + SF_v \frac{\delta v}{v}
\]

*Equation 4-12*

*Equation 4-12* shows this relationship in a numerical form. If we substitute this into *Equation 4-4 & 4.5* a much simpler equation can be expressed by:

\[
\frac{\delta \epsilon_1}{\epsilon_1} = SF_{\Sigma P_n} \frac{\delta \Sigma P_n}{\Sigma P_n}
\]

*Equation 4-13*

\[
\frac{\delta \epsilon_2}{\epsilon_2} = SF_{\Sigma P_n} \frac{\delta \Sigma P_n}{\Sigma P_n}
\]

*Equation 4-14*

The load distribution ($\Sigma P_n$) depends on the three spring constants ($K_T$, $K_{BC}$ and $K_{SC}$, where $T$ is the thread, $BC$ is the body and $SC$ is the stud) which can be calculated from the geometry. Once the sensitivity function with respect to load distribution is found it can then be further related to the spring constants and to the tolerance levels. This however can be carried out numerically and finite element analysis will not be necessary.
After determining how the surface strain at the reference point vary with respect to manufacturing imperfections the surface strain variation within the groove has to be found. The strain variation within the groove is governed by Equation 4-8 & 4-9. The strain within the groove will behave similarly to the strain at the reference point but the geometry of the groove will introduce more parameters. Combining Equations 4-8 & 4-12 the surface strain variation within the groove ($\varepsilon'$) will be given as:

$$\frac{\delta \varepsilon'}{\varepsilon'} = SF_{Eg} \frac{\delta \Sigma p_n}{\Sigma p_n} + SF_{h_g} \frac{\delta h_g}{h_g} + SF_{\theta_g} \frac{\delta \theta_g}{\theta_g} + SF_{(x'_{L})} \frac{\delta (x'_{L})}{(x'_{L})} + SF_{(y'_{L})} \frac{\delta (y'_{L})}{(y'_{L})}$$

Equation 4-15

Assuming that the laser strain gauge is calibrated so that the measured strain and the actual strain are the same then Equation 4-15 now becomes the fundamental equation governing the changes in the surface strain with respect to manufacturing and material imperfections.

Using variation analysis and finite element analysis the sensitivity functions for the above parameters will be found in the next chapter. After all the sensitivity functions are found, it will be possible to determine if the accuracy is within the given limits ($\delta \varepsilon_m/\varepsilon_m < \pm 0.5 \%$). If the error is outside the specified limits calibration will be required.
Chapter 5 – Variation Analysis

5.1 Introduction

It was shown earlier how a specially designed nut and the laser strain gauge can be used together to determine the load within a threaded connection. The surface strain can be measured using the LSG by measuring the change in the diffraction grating on the surface of the nut. Preliminary results from finite element analysis showed a good correlation between measured surface strain on the nut and the load, which means that the measured surface strain can be related to the load. Finite element analysis however assumes perfect conditions with perfect geometry. In real life this is not the case and imperfections will occur. The question is how these imperfections affect the accuracy of the load measuring design and the predicted strength of the joint.

In the previous chapter sensitivity analysis was introduced. It was shown how by finding the sensitivity function for each parameter that may vary during manufacture the accuracy of the load measuring device can be measured. The aim of this section is to find these sensitivity functions for the nut with respect to each parameter and determine how the variations influence the accuracy and the performance. The parameters can lead to inaccuracies independently so each parameter can be analysed individually to find the sensitivity function respectively. If the overall variation in the strain readings is more than 0.5% then the specially modified nut needs to be calibrated.

Figure 5-1 Prototype nut
The geometry of the nut (Figure 5-1) is complicated and will require complex model analyses to find the sensitivity functions.

St Venant’s Principle states: The stresses and strains in a body at points that are sufficiently remote from points of application of load depend only on the static resultant of the loads and not on the distribution of the loads (Barré de Saint-Venant 1797-1886). (Ugural 1991)

Based on St Venant’s principle the model is broken down into two steps to simplify the problem and to reduce the analysis time. The first step does not take account of the groove geometry but treats the nut as a solid model while the second step will only look at how the groove geometry affects the accuracy.

The nut has a number of dimensional parameters that may vary. These parameters are listed in Figure 4-1. The dimensional change in the geometry and the threads has a significant affect on the load distribution as well. Instead of investigating the effects of all these parameters with regards to surface strain individually it was suggested that the parameters should be related to the load distribution. Afterwards the variation in the load distribution can be analysed to find the sensitivity function relating load distribution and surface strain.

In aid of finding the load distribution it is important to understand how the spring model works. In the next section the basics of the spring model will be explained together with the methodology of how each spring constant is found from the geometry. The variation in the geometry will cause the spring constants to change which will affect the load distribution. Using this relationship the sensitivity function can be found.

Once the sensitivity function is determined for the load distribution the variation in the groove will be investigated. The laser strain gauge works by measuring the diffraction of a grating etched onto the surface. For this reason the surface strain can only be measured at the location of the grating. One of the possible sources of error during manufacture is the positioning of this grating. The sensitivity function for the grating location is one of the parameters that will be investigated. The geometry of the
The groove itself is given by two parameters; these are the depth and the angle of the groove face. These will be discussed in more detail later on in this chapter.

The governing equation will be the sum of all the errors with respect to load distribution, grating location and groove geometry.

The aim of this chapter is to find all the sensitivity functions in the governing equation (Equation 4-14) with respect to the parameter it describes. Once the sensitivity functions are found the accuracy of the prototype can be determined as a strain measuring device. The accuracy has to be within 0.5% or calibration for each nut will be necessary.
5.2 Variation in load distribution

The geometry of the nut is governed by a number of parameters as it is shown in Figure 4-1. To analyse each parameter individually and to find the sensitivity function for each would be time consuming and would result in a very complicated governing equation (Equation 4-14).

The changes in all these parameters which govern the geometry have a significant influence on how the load distribution changes. So instead of looking at each parameter individually it was decided to find how the geometry relates to the load distribution and find the sensitivity function with respect to the load distribution.

The load distribution within the threaded connection is very important for the maximum stresses in the threads and it is reasonable to assume it may affect the surface strain reading in the groove. In this section the factors that affect the load distribution will be looked at and its overall affect on the surface strain reading will be analysed.

To understand the load distribution within a threaded connection it is important to know how the two dimensional spring model works and that how the individual spring constants are calculated. Once the load distributions are found and varied the affects on the surface strain will be found using finite element analysis.
5.2.1 Introduction to the 2D spring model

A threaded connection consists of two main parts, the nut and the bolt. All the forces are transferred from one to the other through a set of helical threads. The force distribution on the threads within the nut is not uniform; it is an exponential relationship (Miller, Marshek *et al.* 1983) with the first thread taking most of the overall force and the last thread only carrying a fraction of the load.

![Thread Load/Mean Thread Load](image)

**Figure 5-2** An example of load distribution on a threaded connection over 9 threads

As Figure 5-2 shows, the load distribution is a smooth graph, and the exact shape of the curve is governed by the geometry and the material properties of the connection. In theory the threaded connection is the same throughout, apart from the points where thread run off takes place. This uniformity assumes identical material and geometrical properties at each thread which produces a smooth load distribution. Faults in the connection (i.e.: a broken thread) can introduce irregularities to the load distribution even though the total load transferred remains constant.
The special nut designed to calculate the load by measuring the surface strain on it, is designed in such a way that a regular load distribution is assumed. It is important to investigate how material and manufacturing imperfections might influence the structural properties of the thread slices. The variation in the thread slices will be used to find any discrepancy in the load distribution along the engaged threads. The new load distributions will be used to investigate how the surface strain is influenced. To achieve this finite element and numerical methods will be used.

The most efficient way of finding the load distribution is by the use of spring models. The spring model is a numerical way of calculating the forces within a threaded connection. The calculations can be done using a spreadsheet and once the initial model is set up it is very easy to use and determine the load distributions. The spreadsheet makes it simple to vary different parameters, which makes the spring model a very useful tool. To use the spring model the structural properties of the thread slices (spring constants) need to be found. In this section it will be explained how the spring model works and how the spring constants are found. The variation of the spring constants and their effect on the load distribution and on the surface strain will be investigated as well.

A threaded connection is a relatively complicated problem and it is difficult to analyse numerically, and practical testing is not always suitable. A nut and bolt connection consists of two parts in which are brought into contact via a helical thread. To model such a problem the 2-D spring model was developed. The 2-D spring model is a numerical method used to calculate the force distribution on a threaded connection. The model assumes that the threads are not helical, and that each thread revolution is an independent ring. This assumption is needed to simplify the otherwise complicated scenario. The thread therefore is no longer treated as a continuous helical line, but as separate slices within the connection. The threaded connection is then built up of all these slices. Each slice consists of three parts, the bolt, the thread and the stud. The spring model is the mathematical way of connecting all these parts and slices so they give a realistic representation of the force distribution.
There are two possible scenarios, compression and tension. To demonstrate how the spring model works, a compression case will be analysed as the prototype nut is designed to work in a compression scenario as well. The model consists of a stud and a body, with a force $F$, acting on the stud compressing the joint.

When the force is applied it is transmitted through the threads in the following form:

$$ P_T = S_{i-1} - S_i \quad 1 \leq i \leq n $$  \hspace{1cm} \textit{Equation 5-1}  

Where $P_T$ is the force on the thread and $S$ is the force on the stud, and the subscript $i$, shows the numbering of each thread revolution. The force acting on the body is represented with $L$. The forces can be represented as a function of displacement and stiffness. The stiffness in affect is a spring constant, which represents flexibility of each component, body ($K_{BC}$), stud ($K_{SC}$) and thread ($K_T$). The method of calculating these constants will be discussed later. Similarly to the spring constants, the displacements for each section of body, stud and thread can be shown as $\delta_{BC}$, $\delta_{SC}$ and $\delta_T$ respectively. The forces now can be represented as follows:

$$ S = K_{SC} \delta_{SC} \quad \textit{Equation 5-2} $$
The displacement of the stud at each thread has an absolute deflection \( u_S \) which can be represented as follows at the \( k \)-th thread:

\[
 u_S^k = \delta_T^k + \sum_{j=k}^{k-1} \delta_{BC}^j \quad 1 \leq k \leq n \tag{Equation 5-5}
\]

and at two consecutive threads

\[
 u_S^i - u_S^{i+1} = \delta_{SC}^i \tag{Equation 5-6}
\]

so summing the two equations gives

\[
 \delta_T^i - \delta_T^{i+1} + \sum_{j=i}^{i-1} \delta_{BC}^j - \sum_{i=1}^{i+1} \delta_{BC}^j = \delta_{SC}^i \tag{Equation 5-7}
\]

but the \( i+1 \) summation term can be written as

\[
 \sum_{j=1}^{i} \delta_{BC}^j - \delta_{BC}^{i+1} \tag{Equation 5-8}
\]

by substitution

\[
 \delta_T^i - \delta_T^{i+1} - \delta_{BC}^{i+1} = \delta_{SC}^i \tag{Equation 5-9}
\]

By replacing the displacements using Equations 5-2, 5-3 and 5-4

\[
 \frac{P_T}{K_T} - \frac{P_{T+1}}{K_T} - \frac{L_{i+1}}{K_{BC}} = \frac{S_i}{K_{SC}} \tag{Equation 5-10}
\]

The force on a body segment is the sum of the thread loads between that thread and the last thread. So the load \( L_k \) can be written in the following form:
\[ L_k = \sum_{j=k}^{n} P_{Tj} \quad \text{Equation 5-11} \]

From \textit{Equation 5-1}

\[ L_k = \sum_{j=k}^{n} [S_{j-1} - S_j] \quad \text{Equation 5-12} \]

\[ L_k = S_{k-1} - S_n \quad \text{Equation 5-13} \]

\( S_n \) is the last load on the stud. For compatibility reasons it must equal zero. Substituting into \textit{Equation 5-10}:

\[ [(S_{i-1} - 2S_i + S_{i+1})/K_T] - S_i/K_{BC} = S_i/K_{SC} \quad \text{Equation 5-14} \]

\textit{Equation 5-14} can be rearranged

\[ S_{i+1} - S_i(2 + [K_T/K_{BC}] + [K_T/K_{SC}]) + S_{i-1} = 0 \quad \text{Equation 5-15} \]

The constant coefficient in the equation can be replaced by \( \beta_C \) to yield a second order differential equation.

\[ S_{i+2} - \beta_C S_{i+1} + S_i = 0 \quad \text{Equation 5-16} \]

Where

\[ \beta_C = 2 + [K_T/K_{BC}] + [K_T/K_{SC}] \quad \text{Equation 5-17} \]

\textit{Equation 5-16} can be rewritten in the following form (Wylie 1975)

\[ (\lambda^2 - \lambda \beta_C + 1) S = 0 \quad \text{Equation 5-18} \]

Where \( \lambda \) is the root(s) of the characteristic equation. The roots of the second order differential equation can be found using:
\[ \lambda_{1,2} = \frac{\beta_c \pm \sqrt{\beta_c^2 - 4}}{2} \]  \hspace{1cm} \textit{Equation 5-19}

The root \( \lambda \) has two solutions due to the quadratic form, hence the subscripts \( \lambda_1 \) and \( \lambda_2 \).

The solution to the characteristic equation is discreet, which means it can be separated into parts. Therefore the complete solution to \textit{Equation 5-16} is given by:

\[ S_i = A_C \lambda_i^j + B_C \lambda_i^j \]  \hspace{1cm} \textit{Equation 5-20}

Where \( A_C \) and \( B_C \) are arbitrary constants and the subscript of the components are exponents. The values of \( A_C \) and \( B_C \) can be found using the boundary conditions, \( S_0 = F \) and \( S_n = 0 \).

An example verifying the spring model can be seen in Appendix C.
5.2.2 Spring Stiffness

When a solid is subjected to a force, it is compressed, elongated or sheared. This relationship is linear within the elastic region of that material and its property relating load to deflection is called its stiffness. The stiffness depends on the geometry and the linearly elastic material constants of a given solid. In the spring model the threaded connection is divided up into identical slices, with each slice consisting of three parts. All these parts have their own stiffness, which is used to relate them together. As the slices are identical the same stiffness can be used for each part.

The stiffness of each part depends on its geometry and its material properties. For a solid geometry the stiffness will be dependent on its cross sectional area, its depth and Young’s modulus. To find the stiffness of the body and the stud is relatively straightforward as both of them have simple geometries. Simple geometry means that it has a constant cross-sectional area and the force transferred through all of this area. The stiffness therefore can be found by numerical means. The depth depends on the pitch, Young’s modulus depends on the material and the cross sectional area can be calculated using a simple formula (Wang and Marshek 1995).

Stiffness = Force / displacement
Stiffness = Young’s Modulus * Cross-sectional Area / width

Stud:

\[ K_{SC} = \frac{E\pi D_s^2}{4 \, P_H} \]  \hspace{1cm} \text{Equation 5-21}

Body:

\[ K_{BC} = \frac{E\pi (D_o^2 - D_b^2)}{4 \, P_H} \]  \hspace{1cm} \text{Equation 5-22}

Where

\[ E = \text{Young’s modulus} \]
\[ P_H = \text{Pitch} \]
\( D_b = \) Major diameter of nut thread
\( D_s = \) Minor diameter of stud thread
\( D_o = \) Equivalent outside body diameter

\( D_o \) can be found from determining the area of the hexagonal nut. If the width \( (w) \) across the flat faces is known then the area of a hexagon can be calculated as follows (See Appendix D for verification):

\[
\text{Area} = 1.5 * w^2 * \tan 30^\circ
\]

\textit{Equation 5-23}

To find the stiffness of the thread is more complicated. The stud and the body are being compressed or elongated, with the force acting uniformly over the area. So by knowing the material properties the stiffness could be calculated. However the thread acts more like a beam that is being deflected. If the deflection of the thread would be known, then the stiffness would be easily calculated using the above expression.

It would be preferable to find the stiffness of the beam mathematically so any variation can be calculated quickly and accurately. The deflection in a beam can be calculated using mathematical methods, if the cross sectional area, the material properties and the boundary conditions are known, then the deflection can be calculated using the stress functions. This method can only be used for simple problems where the cross section area is constant and the beam is assumed to be long and thin. If the thread is treated like a beam then it needs to be taken into consideration that it is short and that the cross sectional area is not uniform. There are methods available such as the finite difference method (Ugural 1991) which is used to find the deflection in non uniform beams. Hetenyi (1946) found solutions for special short beams (Hetenyi 1946) but in his calculations the modulus of the foundation was taken into account. With respect to a threaded connection the modulus of the foundation could not be found easily in a mathematical way, also many assumptions would have to be introduced that would affect the accuracy of the calculations. Due to these complications it was decided that finite element analysis is a more reliable way of finding the displacement of the thread.
The disadvantage of using finite element analysis over a mathematical model is that every time a dimension is changed in the thread the model would have to be redrawn and reanalysed. For complicated models this can be difficult and time consuming. It was decided that a simple axisymmetric model will be used to represent a whole slice of the threaded connection, and by measuring the relative displacements of the threads the stiffness can be found.

The finite element model needs to be kept simple so it can be modified easily for different geometries and that variation analysis can be carried out. As it was mentioned before an axisymmetric model will be used but this simplification will ignore the helical effect on the thread, but allow the geometry to be modified simply, by moving nodes. Also the 2-D geometry allows the model to be meshed easier and less computational power is required for the calculations. It was also assumed that the friction between the threads is negligible and they were not taken into account when modelling.

![Figure 5-4 Finite element model to find $K_T$](image)

The figure above shows the axisymmetric finite element model which was used to find $K_T$. The vertical axis acts as the centre of symmetry where the 2-D axisymmetric model is rotated 360° to give a full 3-D model. Due to this rotation the nut is assumed to be round and not hexagonal, but the model is designed in such a way that the area of the nut is the same. This assures that the spring constant for the nut $K_{BC}$ is not affected.
There are two ways of modelling a contact problem using LUSAS. In the first scenario the two geometries are drawn with a finite distance between them. In this case two load cases are used, the first loadcase is a simple displacement which brings the surfaces into contact, and the second loadcase is used to apply the required force. This method is advantageous as the two geometries are completely separate and it makes it easier to edit the geometries. The second method only uses one loadcase, as the geometries are already in contact before the analysis. In this model the second method was used for two reasons. Having only one loadcase means a more simple analysis also, it is easier to apply displacement constraints when the geometries are already in contact.

The model represents one single thread so the spring constant for the thread can be found easily. A full model which shows the whole connection would be similar to this but with the threads stacked up on top of each other. To model this correctly the boundary conditions have to account for the threads not shown. The top and bottom surfaces of this single thread (both nut and bolt) are constrained so they move together. This ensures that the single thread behaves just like if another thread would be joined on.

A uniform force is applied along the top surface of the bolt and the bottom surface of the nut is fixed in the vertical direction. The uniform force represents the loading transferred through each layer of the shaft and it is also used to find the spring constant. No other supports are required as the nut is an axisymmetric model and it is assumed to be a ring so it cannot move in the horizontal direction freely.

Slidelines were added to the two contact surfaces and merging of the two surfaces was restricted. As mentioned before friction was not taken into account, therefore the slidelines used were non-friction.

Once the model is analysed it is possible to use the Value command in LUSAS, to find the displacement any node. The spring constant \(K_T\) represents the stiffness of the two threads acting together. At each thread the mid point is taken where the thread is joined onto the nut or the bolt respectively. The relative displacement of these
points gives the overall displacement of the threads. The stiffness is then calculated from the total load and displacement.

To test this method of finding $K_T$ an example was taken from Wang’s and Marshek’s work (Wang and Marshek 1995). The same geometry was used as described in their paper, although exact material properties were not given.

The geometry given in Wang’s and Marshek’s paper is a 1 inch diameter steel bolt and nut with 8UN thread. Their value for spring constant $K_T$ was $0.113 \times 10^8$ lb/in (1.97 $\times 10^6$ N/mm).

The overall displacement was $5.18 \times 10^{-4}$ mm under a 1000 N load, giving a value of $1.93 \times 10^6$ N/mm for $K_T$. The results were within 2% of each other proving that the above described method is a valid way of calculating the stiffness of two threads in contact.
5.2.3 Variation in Spring Constants

There are three main types of variation analysis; these are Worst Case, Root Sum Square (RSS) and Monte Carlo. All these methods are based on the stack-up method which is the adding up of the variation in the parts (Liu and Hu 1997). Worst case was the first one, and it works on using extreme examples. The worst case scenarios are taken as an example and analysed. The RSS method is based on a statistical distribution for the tolerance variation. This gives a more realistic view, but it is difficult to work out the distribution. Both of these methods are difficult to apply to complicated 2-D and 3-D assemblies. The Monte Carlo simulation can be applied to more complex assemblies and it is carried out by taking tolerances randomly from a known distribution.

For the variation analysis on the spring constants the worst case method was used due to the large number of different variations. The extreme (maximum and minimum) values are found for each spring constant and are then used to find the variation of the load distribution. Even though the nut is a 3-D problem the load distribution is found in a single plane so the worst case method is a suitable variation tool.

Marshek and Wang have looked at how the load distribution changes if the ratio between the nut and the bolt spring constant is varied (Wang and Marshek 1995). In their studies they kept the same spring constant ratio throughout the threads and they found that in a compression case there is a minimal change in the load distribution even at large variations. They also found that in the tension case the effects of changing the ratio between the spring constants have a much greater effect, in some extreme cases the load distribution is reversed so most of the load is carried on the last thread not on the first one. In this report all the three spring constants will be varied and the variation will be randomised between each thread looking at the compression case. This will introduce much larger changes in the load distribution especially as the thread spring constant \( K_T \) is varied as well. The effects of this on the surface strain will be measured using finite element analysis.

The 2-D spring model is a relatively simple model, so once the spring stiffness is calculated the force distribution can be found easily and quickly. This is an advantage
when large numbers of threads are needed to be analysed. One of these cases is when carrying out variation analysis on a set of nuts and bolts.

Nuts and bolts are very widely used in all sorts of applications. Each nut and bolt manufactured comes with its given tolerance, but the exact effect of this variation in dimension is unknown. These changes in the dimension will influence the magnitude of the stiffness, which will affect the force on each thread. The aim of the variation analysis is to find the effect of changes in the dimension with respect to the force distribution.
5.2.4 Imperfections in the Spring Constants

The stiffness is governed by two main factors, the material properties and the geometry. Any variation in one of these properties and the spring constant will change, which will affect the load distribution.

Nuts and bolts are made to standard sizes with in a given tolerance. Using tolerance tables from the British Standards (BS916:1953), the variations in geometry can be determined. As the stud and the body stiffness ($K_{SC}$ & $K_{BC}$) are directly proportional to the surface area the maximum and the minimum stiffness can be found directly from the tables. To find the effect of the geometry variation on the stiffness of the thread finite element analysis has to be used.

**Stud Stiffness (Bolt) - $K_{SC}$**

The spring stiffness for the stud (bolt) is affected by the minor diameter of the bolt. From the minor diameter the cross sectional area of the bolt can be found which can then be related to the stiffness using the pitch and the material properties. The minor diameter is manufactured to be within a certain tolerance and using the two extreme dimensions the maximum and the minimum values for $K_{SC}$ can be found.

The prototype nut was designed on the basis of a 2 ½ inch UNF8 nut. Therefore all the finite element modelling and variation analysis will be done using the dimensions for this nut from the British Standards.

### Properties:

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value 1</th>
<th>Unit 1</th>
<th>Value 2</th>
<th>Unit 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's Modulus</td>
<td>195000</td>
<td>N/mm$^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch</td>
<td>0.125</td>
<td>inch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pitch</td>
<td>3.175</td>
<td>mm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-5 Property table for the connection

The formula to find the spring constant for the stud was shown previously.
Table 5-6

<table>
<thead>
<tr>
<th></th>
<th>Major diameter</th>
<th>Minor diameter</th>
<th>Surface area</th>
<th>Ksc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>inch</td>
<td>inch</td>
<td>mm²</td>
<td>N/mm</td>
</tr>
<tr>
<td>Maximum value</td>
<td>2.4976</td>
<td>2.3442</td>
<td>2784.50</td>
<td>1.71E+08</td>
</tr>
<tr>
<td>Minimum value</td>
<td>2.4826</td>
<td>2.3270</td>
<td>2743.79</td>
<td>1.69E+08</td>
</tr>
<tr>
<td>Variation in Ksc</td>
<td>1.46%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-6  Variation in stud spring constant (Bolt)

Figure 5-6 shows the two extreme values for $K_{SC}$. The variation between the two values is relatively small (1.5%) but this is expected as the tolerances are very tight. The variation is different for different sized connections, and it cannot be assumed that all joints have such tight tolerances. In general it is possible to say that the smaller the nominal diameter, the bigger the variation due to the relative magnitude of the tolerances. For very small joints (nominal diameter < 5 mm) the variation can be as high as 8%. The variation in the spring constant has to be found individually for each size.

Body Stiffness (Nut) – $K_{BC}$

The spring stiffness for the body (nut) is found in a similar manner to the stud stiffness. The formula to find $K_{BC}$ is shown in the previous part.

Table 5-7

<table>
<thead>
<tr>
<th></th>
<th>Minor diameter</th>
<th>Major diameter</th>
<th>Width across flats</th>
<th>Surface area</th>
<th>Kbc</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Inch</td>
<td>inch</td>
<td>inch</td>
<td>mm²</td>
<td>N/mm</td>
</tr>
<tr>
<td>Maximum value</td>
<td>2.3897</td>
<td>2.5</td>
<td>3.75</td>
<td>4690.15</td>
<td>2.88E+08</td>
</tr>
<tr>
<td>Minimum value</td>
<td>2.3647</td>
<td>2.5</td>
<td>3.69</td>
<td>4440.73</td>
<td>2.73E+08</td>
</tr>
<tr>
<td>Variation</td>
<td>5.32%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-7  Variation in body spring constant (Nut)

The spring constant for the body is governed by two dimensions and due to this the variation is slightly higher than for the stud spring constant. Once again this variation is unique to this size and for smaller nominal diameters the variation can be as high as 10%. As the diameter increases the variation in the spring constant is reduced.
It was mentioned before that the variation in the spring constants vary with the nominal diameter and to calculate the level of variation each size has to be investigated individually. Alternatively, if the level of accuracy is not such a high importance data tables can be used to predict the variation. Using the British Standards a selection of nominal diameters can be investigated and the variation is found for each size. If a large enough sample is collected a trend can be found to relate nominal diameter and variation in their respected constants. This relationship is exponential and very inaccurate for small diameters. However it is a useful tool when the variation in spring constants has to be found quickly for any diameter.

**Thread Stiffness – Kₜ**

The stiffness of the thread cannot be found mathematically in a convenient manner due to a number of reasons as explained previously. Therefore finite element (numerical) analysis is used. This means that the variation in the thread spring constant has to be found using FE as well.

The threads are manufactured to within a certain tolerance just like the rest of the dimensions in the joint. These tolerances allow the geometry of the thread to vary which affects the stiffness of the thread. To find the maximum and the minimum value for the thread stiffness two extreme geometries are modelled and analysed. It is assumed that the minimum value will be when the thread bends the easiest and this will occur when the thread is the smallest. Small thread means less material which allows the thread to bend more. It is also assumed that the maximum thread stiffness is when the thread bends the least and this occurs when the thread is as big as possible. Both these geometries can be modelled using finite element analysis and the thread stiffness can be found for each. These will be the maximum and minimum values for Kₜ.

There is another factor that may change and affect the thread stiffness. This is the thread angle. However the tolerance for this is not specified in any British Standards. In a later chapter looking at residual life an investigation is carried out to examine the affect of thread angle on the maximum stress areas in the thread. It was found that the variation in the angle has a less significant effect on the thread than the dimensional
variation. For this reason it was decided to assume that effects due to the thread angle are negligible and only vary the geometry with respect to the dimensional changes when calculating the two extreme values for $K_T$.

The results can be seen in the table below. Delta 1 & 2 represent the two nodes where the displacement is measured from.

<table>
<thead>
<tr>
<th></th>
<th>Maximum</th>
<th>Minimum</th>
<th>units</th>
</tr>
</thead>
<tbody>
<tr>
<td>minor diameter of bolt</td>
<td>2.327</td>
<td>2.344</td>
<td>inch</td>
</tr>
<tr>
<td>major diameter of bolt</td>
<td>2.498</td>
<td>2.483</td>
<td>inch</td>
</tr>
<tr>
<td>major diameter of nut</td>
<td>2.5</td>
<td>2.5</td>
<td>inch</td>
</tr>
<tr>
<td>minor radius of bolt</td>
<td>29.553</td>
<td>29.771</td>
<td>mm</td>
</tr>
<tr>
<td>major radius of bolt</td>
<td>31.720</td>
<td>31.529</td>
<td>mm</td>
</tr>
<tr>
<td>major radius of nut</td>
<td>31.75</td>
<td>31.75</td>
<td>mm</td>
</tr>
<tr>
<td>Force applied</td>
<td>1000</td>
<td>1000</td>
<td>N</td>
</tr>
<tr>
<td>delta 1</td>
<td>-4.41E-04</td>
<td>-5.57E-04</td>
<td>mm</td>
</tr>
<tr>
<td>delta 2</td>
<td>-1.85E-05</td>
<td>-1.92E-05</td>
<td>mm</td>
</tr>
<tr>
<td>Displacement</td>
<td>-4.23E-04</td>
<td>-5.38E-04</td>
<td>mm</td>
</tr>
<tr>
<td>$K_T$ - N/mm</td>
<td>2.37E+06</td>
<td>1.86E+06</td>
<td>N/mm</td>
</tr>
<tr>
<td>Variation</td>
<td></td>
<td>21.44%</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5-8  Variation in thread spring constant

The thread behaves like a beam as it is only supported from one end. This suggests that the amount it is displaced by when a load is applied is more sensitive to changes in the geometry. This explains the high discrepancy between the maximum and the minimum value.

To find the effect of the variation in the spring constants on the surface strain an in-depth analysis needs to be carried out. First the variation in the load distribution needs to be found and then it can be used to find the variation on the strain. This will be presented in the following section.
5.2.5 Method to find variation affect on load distribution

The variation in geometry due to the dimensional tolerances will affect the individual spring constants which will cause the load distribution to vary. The load distribution can be found if the spring constants are known using the spring model described earlier. The best way to use the spring constant is by putting all the information in a spread sheet so each thread can be related to the previous one. This way the load on each thread can be found by the use of a table.

Using Excel and Visual Basic programming the load distribution table can be generated automatically if the user specifies the spring constants. The Macro for this can be seen in Appendix E. The advantage of using a Macro like this to generate the load distribution tables is that each time a parameter is changed the table automatically updates itself with the new distribution. This is especially useful when carrying out variation analysis where a considerable number of parameters have to be investigated.

For each spring constant there is a range of values between a maximum and a minimum. The load distribution changes with respect to these values and one of the ways to find the effect of dimensional tolerances is by carrying out variation analysis on the load distribution. For this the values of the spring constants need to be varied and the load distribution needs to be found in each instance.

The load distribution along the threads is governed by the values of the three spring constants \((K_{SC}, K_{BC} \text{ and } K_T)\). To find the most extreme load distributions each spring constant was allowed to vary between its maximum, minimum and normal values, with the normal value being perfect value which occurs at ideal geometries. This gives \(27 \ (3^3)\) different combinations for each thread. The aim of the variation analysis using the worst case method is, to find the most extreme distributions, so the same extreme values were used on each thread. In theory the spring constants can vary between individual threads but in real life because of the way the connections are manufactured it is highly unlikely. The spring constants depend on the geometry of the connection (cross sectional area) and for a single joint it is unlikely that the geometry will change between threads. Also when manufacturing a threaded fastener
the same methods and tools are used so it is unlikely that there will be any variation between individual threads, hence it is valid to assume that the spring constants will not vary between individual threads. Keeping the same spring constants between threads give enough variations with the most extreme combinations for the variation analysis to be carried out accurately. Varying the spring constants between threads would mean a lot more possible combinations which would require longer analysis time and more computational power.

The method of carrying out the variation analysis of the spring constants with respect to measured strain will be discussed in the results section.

Once the load distributions are determined its affect on the surface strain need to be found. For this finite element analysis is used. Even if the spring constants are not varied between threads the number of analyses required is quite high. To carry out all the required analyses manually would be time consuming and tedious. Once again it was decided that a special macro should be written which would carry out the analyses on behalf of the user.

So far all the finite element analysis was carried out using LUSAS. LUSAS has some advantages over other finite element software when carrying out contact analysis or using axisymmetric models. By using two-dimensional axisymmetric models contact analysis in LUSAS can be carried out by two methods. One of these methods is to use two load cases where the first load case brings the two surfaces into contact and the other is the actual applied force. With the other method the surfaces are already in contact and then the load is applied. With both methods the contact analysis can be done automatically or manually, giving many advantages over other FEA software. However, the disadvantage of LUSAS is that it’s very difficult to add macros to it, so for this problem it was decided that Ansys would be more suitable.

As described in Chapter 4 the sensitivity analysis is carried out in two steps. In the first step the sensitivity function is found with regards to the nut geometry only and in the second one the groove is investigated. In the nut geometry analysis the surface strain is measured at the groove centre location \((x'_G, y'_G)\) which is the reference point. For these analyses the grooves were simplified which meant that finite element
analysis was kept simpler. If the grooves would be present during the FE analysis the
geometry is no longer a regular shape and to model it would require an irregular mesh
and more elements. This would require a lot of computational power and time.

The irregular shape of the groove can be simplified if it is assumed that it can be
modelled using a regular shape with orthotropic material properties. The orthotropic
material properties allow the material to have different Young’s modulus in each
direction, so under loading it behaves like an irregular shape. The original geometry
of the nut with the grooves was modelled and it was subjected to a vertical load acting
on the top face. The displacement of nodes around the groove an on the top surface
were noted for reference.

The properties of the orthotropic material were found in two steps, firstly in the
vertical “y” direction, secondly in the radial “x” and “z” directions. Please note that in
LUSAS finite element software the coordinate system is orientated in such a way that
“y” is vertical, “x” is horizontal and “z” is the depth. Due to this, the same coordinate
system was kept when referring to the orthotropic material properties.

The orthotropic material property in the vertical “y” direction was relatively simple to
find. The original nut was divided into three layers, the top layer without the grooves,
the middle groove layer and the bottom layer without the grooves. The vertical
displacement of each layer was noted by measuring the displacement of the nodes at
each layer under a given load. Using these values the vertical displacement of the
grove layer was calculated and from this, its stiffness. The orthotropic model did not
include the grooves; hence it had a uniform cross sectional area. Using this
information the new calculated stiffness was used to calculate the Young’s modulus
for the groove layer by knowing the cross sectional area. As the grooves are only
present on three sides (not all six) it had to be taken into consideration. Therefore on
the solid sides the material properties were not modified, but on the three sides where
the grooves were located the new orthotropic material accounted for this. Therefore
the groove layer with its six segments (three orthotropic and three normal) had the
same stiffness as the original model with the grooves, hence when the same load was
applied the vertical displacement was the same. This was verified in LUSAS by
comparing the displacements in the two models.
Finding the radial ("x" and "z" directions) orthotropic material properties was slightly more complicated. The same method as before was used, by dividing the nut into three layers. This time the displacements in the radial direction were noted around the groove layer. Using the radial displacement values and Poisson's ratio the radial stiffness of the groove layer was found. Once again, taking into consideration that only three of the six sides have grooves on it, the new calculated stiffness was used to find the radial ("x" and "z" direction) Young's modulus for the constant cross sectional area of the orthotropic model. The new material properties were assigned to the orthotropic model and the same force as before was applied measuring the displacement at given nodes. It was found that due to the irregular shape of the grooves the radial displacement in the orthotropic material differs slightly from that of the original model with the grooves. The results were very close, and the material properties were adjusted by trial and error until the radial displacements agreed.

The orthotropic model was then validated by applying the same loads as before to the model with the grooves and the displacements were compared. The values for the orthotropic material can be seen in Figure 5-9.

<table>
<thead>
<tr>
<th></th>
<th>Steel</th>
<th>Orthotropic Material</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's modulus x (N/mm²)</td>
<td>197000</td>
<td>195677</td>
</tr>
<tr>
<td>Young's modulus y (N/mm²)</td>
<td>197000</td>
<td>188609</td>
</tr>
<tr>
<td>Young's modulus z (N/mm²)</td>
<td>197000</td>
<td>195677</td>
</tr>
<tr>
<td>Shear Modulus xy (N/mm²)</td>
<td>75770</td>
<td>72542</td>
</tr>
<tr>
<td>Shear Modulus yz (N/mm²)</td>
<td>75770</td>
<td>72542</td>
</tr>
<tr>
<td>Shear Modulus xz (N/mm²)</td>
<td>75770</td>
<td>75260</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>0.3</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Figure 5-9 Material Properties

The nut was then modelled as an unmodified nut but orthotropic material properties were used at the groove areas allowing the nut to behave in a similar manner as it would with grooves. The simplified model was subjected to the same force as the original nut earlier and the behaviour was noted and compared for correlation. Figure 5-10 shows the nut with the orthotropic material represented with a different colour.
It was mentioned before that it is very difficult to add macros to LUSAS and therefore Ansys was used to carry out the analysis. Figure 5-10 shows the Ansys model of the nut with the orthotropic material. Ansys uses a different coordinate system to LUSAS, as it can be visualised on the figure. With Ansys the vertical direction is "z", the horizontal direction is "x" and the depth is represented by "y".

Instead of modelling the helical threads, the nut was modelled by layers. The layers were modelled so that the top of each layer represented the centre of that thread. This made it straightforward to apply the individual loads to each thread from the load distribution tables. The force distribution calculated using the spring model represents the vertical forces, but due to friction and the shape of the thread there is a horizontal component as well.
Figure 5-11 shows that the vertical force $F$ has a horizontal component as well due to the thread angle and the friction (Sopwith 1948). When applying the load to the finite element model this needs to be taken into consideration. The horizontal component of the force can be calculated by knowing the friction and the thread angle. The vertical forces are given in the force distribution tables, and extra column can be added for the horizontal component as well. These forces can then be assigned to the equivalent threads.

The model is fixed in the vertical direction at the bottom face and the friction is assumed to be negligible. The strain can not be measured on the groove face at the groove centre location $(x'_{G}, y'_{G})$ as the grooves are replaced by an orthotropic material. It is not necessary to take the exact strain readings as the variation analysis is carried out to see the affect of different load distributions so the results only have to be relative to each other. By investigating how the strain on the surface of the orthotropic material changes with the different load distributions a conclusion can be drawn to how it would behave within the actual groove. The surface strain therefore is recorded on the centre of the orthotropic material surface in both the hoop and the axial direction.

A special macro was written and then adopted by Dr Alejandro Maranon of Loughborough University (Appendix F) to operate with Ansys. This macro connects the finite element model with the VB macro that finds the different load distributions
due to the variation in the spring constants. The macro assigns the loads to the
equivalent threads in the FE model in both the vertical and the horizontal direction
and the analysis is ran. The strains are measured in both the hoop and the axial
direction at a given point \((x'_G, y'_G)\) and recorded in a table. Once an analysis is
complete the macro resets the model and the next load distribution is assigned. With
the help of this macro large number of analyses can be carried out automatically.
5.2.6 Results of variation in load distribution

Variation analysis is carried out to find how variations in the spring constant affect the surface strain. By relating the change in the spring constants to the error in the surface strain the sensitivity function ($S_{F\gamma\gamma}$) can be found.

The variation analysis will be carried out by varying the amount the spring constants can change by. With each analysis the spring constants will be restrained to be within certain limits of its original value. The possible load distributions for these values will be found using a specially written macro (Appendix F). For each analysis there are 27 spring constant combinations which give a range of different extreme load distributions. The surface strain readings for these load distributions are found using Ansys and the error in the surface strains is recorded. The error is calculated as a percentage between the surface strain recorded using a perfect geometry and the surface strain measured using the extreme load distributions. From the results the maximum errors can be found which show the most amount of variation that is possible within the surface strain for a given amount of spring constant variation.

The maximum the spring constants can vary is 1.46 % for $K_{sc}$, 5.32 % for $K_{bc}$ and 21.44 % for $K_T$ (from section 5.2.3). These numbers show the variation between the maximum and the minimum values. For these constants the variation in the strain is as follows.

![Figure 5-12 Variation in strain with maximum (100%) varying spring constants](image)
Figure 5-12 shows how the surface strain varies from the normal surface strain where perfect conditions are assumed. As the graph shows most of the 27 cases vary by more than ± 2.5 %. The range between the maximum and the minimum possible surface strain reading is 18.93 % for the axial strain and 4.22 % for the hoop strain.

The results also show that 9 out of the 27 scenarios give a small amount of variation in the strain. These scenarios are when KT is not at an extreme value. When KT is either maximum or minimum the errors become significant.

In terms of the sensitivity function the area of interest is the range. For maximum spring constant variation (K_SC = 1.46 %, K_BC = 5.32 % and K_T = 21.44 %) the axial strain can vary by 18.93 % (± 9.47 % from the normal) and the hoop strain can vary by 4.22 % (± 2.11 % from the normal). At this point the spring constants are varied by the maximum which is 100 % variation. This amount of variation is reduced by 5 % increments and the same analysis is run and the results are recorded to see how the surface strain varies with respect to the variation in the spring constants. So for example, 80 % variation in the spring constants means that K_SC varies by 1.17 % (80 % of 1.46), K_BC varies by 4.26 % (80 % of 5.32) and K_T varies by 17.15 % (80 % of 21.44). The required accuracy for the load measuring device is 0.5 % so the maximum variation in surface strain has to be within 0.5 %. A selection of the results can be seen below to demonstrate how reducing the spring constant variation reduces the surface strain variation. The full set of results for all the analyses can be seen in Appendix G.
Figure 5-14  Variation in strain with 65% varying spring constants

Figure 5-15  Variation in strain with 50% varying spring constants

Figure 5-16  Variation in strain with 35% varying spring constants
The charts above show how the variation in the surface strain is reduced if the variation of the spring constants are restricted. The results for all the analyses can be seen in Appendix G. As Figure 5-18 shows, when the variation of the spring constant is restricted to 5% of its original variation is the only scenario when both the hoop and the axial strain readings are within the required limits.

To determine the relationship between spring constant variation and surface strain variation, the two variations are plotted against each other as percentage errors. For each analysis the maximum error of the surface strain is measured and then plotted against the spring constant variation (Figure 5-19 & 5-20).
The graphs above show how the errors in the spring constants affect the strain. Both the spring constant and the surface strain errors are measured as the variation from the normal strain reading, which is half the range.

The sensitivity function tells us how the surface strain can vary with the load distribution which is affected by the spring constants. Therefore the sensitivity function is given as the change in spring constant value ($\delta K/K$) and the change in strain ($\delta \varepsilon/\varepsilon$).
The change in spring constant value is governed by 3 spring constants. As it is a combination of the spring constants the change is treated together. The maximum the spring constants can change was specified earlier, and that is when $\delta K/K = 100\%$. The sensitivity function then becomes the gradient of the graphs shown in Figure 5-19 and 5-20.

In the governing equation the sensitivity function which relates the load distribution and the strain was given as:

$$\frac{\delta \varepsilon'}{\varepsilon'} = SF_k \frac{\delta \Sigma P_n}{\Sigma P_n}$$  \hspace{1cm} \text{Equation 5-24}

As the load distribution is governed by the change in the spring constant the strain can be related to the spring constant directly. So \textit{Equation 5-24} becomes:

$$\frac{\delta \varepsilon'}{\varepsilon'} = SF_k \frac{\delta K}{K}$$  \hspace{1cm} \text{Equation 5-25}

$SF_k$ is the sensitivity function that shows the relationship between the spring constant change and the surface strain. The strain is measured in both the axial and the hoop direction; therefore the sensitivity function has two components. In the axial direction $SF_{K1}$, and in the hoop direction $SF_{K2}$. The values for these sensitivity functions are found from the graphs above (Figure 5-19 & 5-20).

<table>
<thead>
<tr>
<th>Axial Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SF_{K1}$</td>
</tr>
<tr>
<td>0.095</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SF_{K2}$</td>
</tr>
<tr>
<td>0.0211</td>
</tr>
</tbody>
</table>

\textit{Figure 5-21}  \hspace{0.5cm} Sensitivity functions with respect to spring constants

The equation then becomes for axial strain:
\[ \frac{\delta e_1'}{e_1'} = 0.095 \frac{\delta K}{K} \]  
*Equation 5-26*

For hoop strain:

\[ \frac{\delta e_2'}{e_2'} = 0.0211 \frac{\delta K}{K} \]  
*Equation 5-27*

Note that \( \delta K/K \) is not directly the change in a spring constant but the combination of all spring constants. When \( \delta K/K \) is 100% then each spring constant is at its maximum variation, which is governed by the maximum dimensional tolerances specified in the British Standards.
5.2.7 Discussion of variation in load distribution

It was shown in the results section that the dimensional variation in the threaded connection influences the load distribution which affects the surface strain. The change in the spring constants is linearly related to the change in the strain which means that the sensitivity function is constant.

In the analyses the strain was measured for both the axial and the hoop direction. The sensitivity function was found for both the axial and the hoop strain. The results showed that the axial strain is significantly more sensitive to dimensional error than the hoop strain. At maximum variation the axial strain varied by nearly 10 % while the hoop strain only varied by 2 %.

The results also show that the most influential spring constant is $K_T$. There are two reasons for this. The thread is the connection in the joint and it is influenced both by the bolt and the nut. When the load is transferred from the bolt to the thread, it is a function of the thread stiffness and the bolt stiffness. Similarly when the load is transferred from the thread to the nut, it is a function of the thread stiffness and the nut stiffness. The thread stiffness therefore influences both load transfers making it the most influential parameter when finding the load distributions.

The other reason is due to the amount it varies by. The thread is under bending, and this bending gives its stiffness. Due to this it is more sensitive to dimensional variation and the smallest of change in its geometry can have an influential effect on the stiffness. When the variation in the constants was worked out $K_T$ varied by over 20 % while the other two constants were around 5 % and 1.5 %.

This amount of variation in $K_T$ caused significant change in the load distribution which introduced large errors when measuring the surface strain. The required accuracy for the load measuring nut is 0.5 % which would require very tight tolerances when manufacturing the nut. The required tolerances can be calculated using the sensitivity functions assuming that each spring constant changes by the same relative amount. Relative value means that the value it changes by is relative to
its maximum possible variation found from the given dimensional tolerances. This assumption is made to make it possible to quantify the change in the spring constants.

For axial strain $SF_{k1} = 0.095$, so to achieve a 0.5 % accuracy $\delta K/K$ has to be less than 5.26 %. This means that each individual spring constant can vary 5.26 % of its maximum variation.

For hoop strain $SF_{k2} = 0.021$, so to achieve a 0.5 % accuracy $\delta K/K$ has to be less than 23.7 %. This means that each individual spring constant can vary 23.7 % of its maximum variation. Figure 5-22 shows the summary of the results.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Maximum variation to maintain 0.5% accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thread stiffness</td>
<td>$K_T$</td>
<td>%</td>
<td>Axial 1.13 Hoop 5.08</td>
</tr>
<tr>
<td>Bolt stiffness</td>
<td>$K_{SC}$</td>
<td>%</td>
<td>0.08 0.35</td>
</tr>
<tr>
<td>Nut stiffness</td>
<td>$K_{NC}$</td>
<td>%</td>
<td>0.28 1.26</td>
</tr>
</tbody>
</table>

Figure 5-22 Maximum allowed spring constant variation for required accuracy

These tolerances are very tight and it would be very difficult to manufacture. Also it is not just the nut that would have to be manufactured to these tight tolerances but also the shaft.
5.3 Variation Analysis of Grating Position

The laser strain gauge works by etching a grating on the surface of the specimen and measuring the diffraction of that grating when under loading. In terms of the specially modified nut these gratings are positioned in the centre of the grooves.

The nut was designed using finite element analysis where all the conditions are assumed to be perfect. The strain on the FE model is measured by taking the strain readings at nodes. As the model is perfect the strain is always measured at the required location. However in real life this is not always the case.

The gratings are manufactured onto the surface of the groove using lasers (Wileman, Coupland et al. 1994). Unlike the finite element model the strain is measured over a small area not just a single node. This may introduce errors if the strain on the surface of the groove is not uniform. The other concern with the grating, which is probably the more important one, is that due to the non uniform surface strain on the groove major errors might be introduced if the grating is not in the intended position precisely. These factors need to be considered, and the effects of grating position need to be investigated in greater depth. To carry out these investigations 3-D finite element analysis was used.
5.3.1 Modelling for Grating Position

The nut with the groove has a relatively complicated geometry when modelled using finite element analysis. Once the model is drawn to the right geometry a mesh has to be assigned to the model so the analysis can be carried out. In most cases the mesh is assigned automatically, but for a complicated 3-D geometry like the nut, the mesh has to be assigned individually to each volume. The reason for this is that around the groove area, the model consists of irregular volumes which can not be meshed using the usual hexahedral mesh, but pentahedral mesh has to be used. Using two different meshes in a model can introduce a number of problems. Two adjacent volumes with different types of mesh can only be joined by the common surface if the mesh is lined up, so the nodes can be shared by both volumes. To avoid problems with the mesh it is important to orientate the volumes right the first time, so that triangular surfaces always have similar surfaces opposite to allow a regular mesh to be added.

The critical area within the model is the groove face as that is the area where the surface strain is measured on. The groove face therefore needs to have a larger mesh density so more surface strain readings can be taken. It is not possible to increase the number of elements in individual volumes, as every volume is connected, so if the number of mesh divisions is increased in one, the knock-on effect is that others have to be increased as well to allow the nodes to line up. A higher mesh density was achieved on the groove surface by dividing it up into 4 surfaces, allowing 4 times as many nodes as on a single surface (Figure 5-23).
In practise the force is transferred to the nut through the threads. The modelling of the helical thread would complicate the nut model very considerably. It would be very difficult to align the mesh within the helical thread with the perfectly lined up mesh on the existing model. The force on the thread can be modelled instead as a shear force on the inside wall of the nut. In reality the force is exponentially distributed between the threads, this was discussed in the Spring Model (Section 2.2, Figure 5-3).

For this model, however, it was assumed that a uniform shear stress on the inside wall of the nut would be appropriate to investigate the surface strain variation on the groove surface.

The bottom surface of the nut was fixed in the vertical direction and a shear force was assigned to the inner wall to simulate a contraction scenario within the nut. The shear force applied was given in the force/unit area format to ensure uniform loading. The total force applied was 703000 kN, which is equivalent to 200 bar on the test rig.
5.3.2 Results for Grating Position

The aim of this experiment was to look at the surface strain on the whole groove face. LUSAS, the finite element package used can show the result required numerically at each node, or show the bigger picture using contour plots. The advantages of contour plots are that large areas of surface strains can be analysed in an instant, therefore it was decided that the results will be presented as contour plots.

The two strains which are of interest to the laser strain gauge are the hoop and the axial strain along the surface of the groove face. The most accurate way of getting these results from the finite element model is by looking at the strain in the axial and hoop direction with respect to the local coordinate system. The local coordinate system depends on the orientation of the mesh elements, and to get accurate results using this method, all the elements have to be lined up in the same way within the analysed area. Due to the combination of meshes used, hexagonal and pentahedral, the meshes cannot be lined up. The triangular pentahedral elements will always be off angle with respect to the hexahedral ones. Therefore the hoop and the axial strain were found by looking at the principal strains. There are 3 principal strains in a 3 dimensional model, E1, E2 and E3. Following the LUSAS coordinate system, in the nut model E1 is the hoop strain, E3 is the axial strain and E2 would be the strain perpendicular to the groove face, but it is of no interest to us and to the laser strain gauge.

Figure 5-24 show the two principal strains on the whole nut.
Figure 5-24  Principle strains (axial & hoop), mesh shown is the geometrical configuration, the FE mesh is shown in Figure 5-23

These plots show that the surface strain within the groove is not uniform, but for more detail the contour plots of the grooves alone can be seen on Figures 5-25 & 5-26.

Figure 5-25  Hoop strain on the groove face
As the outline shows the groove face is circular. When it is meshed the nodes are connected with straight lines which give the impression that the groove face octagonal. Due to each element having a straight line the finite element mesh can never be fully circular. A finer mesh can be used to increase the number of sides, but this would mean the mesh density would have to be increased on the whole model. Due to the complicity of the model this would mean a significant increase in the number of elements increasing the analysis time and required computational power extensively.

As the initial findings suggested, the hoop strain is very close to uniform on the groove face. There is a small discrepancy of a couple of percentages, but the largest difference is on the upper part of the face which is too close to the groove wall, hence it is out of range for the laser strain gauge anyway.

Looking at the axial strain, there is a substantial difference between the strain on the upper and the lower part of the face. The strain along the top is much higher due to nut having a smaller cross-sectional area where the groove is cut the deepest.
To quantify these discrepancies graphs were plotted in the vertical (Y) and the horizontal (X) direction to see how the hoop (E1) and the axial (E3) strain changes (Figure 5-27). Once again, the “y” and “x” directions to refer to the coordinate system used in LUSAS.

The graphs show how the surface strain changes along the groove face. To find the sensitivity functions the results have to be converted. The sensitivity function is the relationship between the change in strain ($\delta \epsilon / \epsilon$) and the change in distance ($\delta x'/x'$, $\delta y'/y'$) with regards to the ideal reading.

The ideal reading means that perfect conditions are assumed. In this case the ideal strain is measured at the centre of the groove where $x = y = 6.25$ mm. As the value of $x$ and $y$ change so does the strain reading. By plotting the change in distance against the change in strain, the trend line of the graph will be the sensitivity function. The converted results can be seen in Appendix H.
The analysis was carried out over the whole groove face so the readings were taken near the groove wall as well. Near the sides the strain changes significantly due to the boundary conditions so the results at these points can not be used. When the trend lines were found for the graphs in Appendix H the reading at the edge were not taken into consideration.

There are four sensitivity factors. There are two strain directions, axial and hoop, and for both the displacement of the grating can be specified by two coordinates \((x_L', y_L')\). Hence the sensitivity factors will be:

- For axial strain in the x direction: \(SF_{x'L}\)
- For axial strain in the y direction: \(SF_{y'L}\)
- For hoop strain in the x direction: \(SF_{x'2L}\)
- For hoop strain in the y direction: \(SF_{y'2L}\)

With each unit change of distance the strain will change by the sensitivity function (Figure 5-28).

<table>
<thead>
<tr>
<th>Axial Strain</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(SF_{x'L})</td>
<td>-0.26</td>
</tr>
<tr>
<td>(SF_{y'L})</td>
<td>-1.24</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hoop Strain</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(SF_{x'2L})</td>
<td>0.019</td>
</tr>
<tr>
<td>(SF_{y'2L})</td>
<td>0.091</td>
</tr>
</tbody>
</table>

Figure 5-28  Sensitivity of grating position

This means that the total error in the surface strain with regards to the grating position can be written as:

\[
\frac{\delta \varepsilon'}{\varepsilon'} = SF_{x'L} \frac{\delta x_L'}{x_L'} + SF_{y'L} \frac{\delta y_L'}{y_L'}
\]

*Equation 5-28*
Where \( \delta x_L'/x_L' \) and \( \delta y_L'/y_L' \) are the errors in position in their specific directions. This equation is the same for axial and hoop strain.

Axial Strain:

\[
\frac{\delta \varepsilon_1'}{\varepsilon_1'} = -1.24 \frac{\delta x_L'}{x_L'} - 0.26 \frac{\delta y_L'}{y_L'}
\]

Equation 5-29

Hoop Strain:

\[
\frac{\delta \varepsilon_2'}{\varepsilon_2'} = 0.019 \frac{\delta x_L'}{x_L'} + 0.091 \frac{\delta y_L'}{y_L'}
\]

Equation 5-30

The surface strain is used to find the overall load on the threaded connection. If there is a variation in the surface strain reading, it will cause an error in the load calculations.

In previous results it was shown that at a load of 200 bar, the surface strain is in the magnitude of about -0.5 millistrain in the axial and 0.6 millistrain in the hoop direction. This means that a 1mm variation in either direction will introduce around 0.5% error in the load if the hoop strain readings are used and over 5% error in the load if the axial strain readings are used.

In terms of magnitude, both sets of results correlate well with the experimental test results which were carried out using 200 bar load.
5.3.3 Discussion for Grating Position

There are two areas of interest. The first one is whether measuring the strain at a single point in FE corresponds to the strain reading over an area when using the laser strain gauge. The second one is looking at how the surface strain reading changes if the grating is out of position.

The results suggested that there is some variation present near the centre of the groove, but as the grating area is very small, 1 mm², the difference in the strain reading between a single point and the area is negligible. Even though the strain is shown at a single point on the FE model, it is calculated from the surface strains from neighbouring nodes. The grating measures the strain over a very small area as well so results are equivalent.

The position of the grating is more sensitive to error when measuring the surface strain. In the hoop direction to achieve 0.5 % accuracy the maximum the grating can be out of position is 1.59 mm in the “x” horizontal direction and 0.34 mm in the “y” vertical direction.

The measurements in the axial direction are more susceptible to the position. This sensitive relationship can be due to the groove face being at an angle. To maintain the 0.5 % accuracy the grating has to be within 0.12 mm in the x direction and 0.025 mm in the y direction of the centre location.
5.4 Variation analysis on the groove geometry

The geometry of the groove has been broken down into two parts, the angle of the face and the depth of the groove. However these two parameters are related as if the face angle is changed the depth changes as well. It was decided that depth of the groove will not be investigated individually for the following two reasons:

- The depth of the groove can only decrease. If the depth would increase a lip would appear by the edge changing the geometry significantly (Figure 5-29).
- The user licence for the finite element software (LUSAS) used to find the sensitivity functions for the groove geometry has expired. The variation in change angle in effect changes the depth as well so it was decided that it is sufficient to only vary the angle and not the depth.

Figure 5-29  Varying groove depth on nut (side view)

For these reasons it was assumed that the two parameters (face angle and groove depth) can be treated as one. Hence only one sensitivity function will describe the change in surface strain due to the variation in face angle and in the depth of the groove. The parameters \( h_G \) and \( \theta_G \) can be combined to become \( \theta_{hg} \) and Equation 4-14 (Section 5.1) then becomes:

\[
\frac{\delta e'}{e'} = SF_{\Sigma P_n} \frac{\delta \Sigma P_n}{\Sigma P_n} + SF_{\theta G} \frac{\delta \theta_G}{\theta_G} + SF_{(x'_{L})} \frac{\delta (x'_{L})}{(x'_{L})} + SF_{(y'_{L})} \frac{\delta (y'_{L})}{(y'_{L})}
\]

Equation 5-31

Where "SF_{\theta_{hg}}" is the sensitivity function for the groove angle and depth variation.
5.4.1 Variation Analysis of Face Angle

The geometry of the modified nut is based on the specifications given in the equivalent British Standards (BS916:1953). Both the dimensions and the tolerances are given in these specifications so every standard nut produced will be manufactured to these given standards. The difference between a standard nut and the modified nut is the three grooves that are manufactured to the three alternate hexagonal sides.

The grooves are manufactured by drilling in to the flat side at a 45 degree angle in such a way that the bottom of the groove is a flat circular surface (Figure 5-30).

The grating which is used to help determine the surface strain is manufactured onto this flat surface. The importance of the location of the grating was discussed in the previous part. In this part the importance of the groove manufacture accuracy will be looked at in greater detail. The groove is manufactured using a drill with the given diameter to cut away the excess material. As the drilling is carried out with standard drill bits, its diameter will not vary by a significant amount so the bulk geometry of the nut will not be affected.

The other areas where manufacturing imperfections can occur are trajectory (face angle) and the depth of the groove. It was discussed earlier in this chapter even though...
these two parameters are not the same, they are closely related. Also variation in the depth of the groove doesn't just introduce dimensional variation but changes the geometry significantly. Figure 5-22 shows that if the groove is too deep a lip is produced at the edge of the groove. If this is the case the manufacturer can spot error with the naked eye the specimen can be discarded as a reject. If the groove is too shallow then the groove face will not be a full circle changing the geometry significantly once again.

Also due to the finite element software licence expiring it was decided that not to investigate the depth parameter individually. Instead it was assumed that the variation in the face angle can represent the change in depth as well.

The effects of variation in the face angle are investigated here in aid to find the sensitivity function $SF_{\theta_h}$. 
5.4.2 Modelling to find the variation due to the face angle

In the previous section it was shown how the strain varies on the groove face so the exact location of the grating is important. To find the strain distribution on the groove face a single analysis was enough once the right geometry was modelled. The modelling of the nut using FE was described earlier. To investigate the effect of variation in the face angle a number of analyses have to be carried out with a slightly modified model each time. To start with, the previous model with a perfect geometry can be used. This model can then be modified by moving the nodes of the flat face in such a way that the angle changes.

The original "perfect" model had an angle of 45 degrees between the side of the nut and the flat face. Using the bottom of the groove as a pivot point, the groove was pivoted around this point to allow a number of different angles varying from 40 to 50 degrees, in one degree increments.

![Figure 5-31 Varied face angles](image)

Figure 5-31 shows two side views of the nut with the two extreme face angles. The only difference between the two models is the angle of the specified face. Each time the angle of the face was modified an analysis was carried out to find the surface strain at the centre of the groove face. For each analysis the same boundary conditions were used as for the previous test looking at the grating position. Once the surface strains were recorded the face angle was pivoted again and the analysis was repeated.
5.4.3 Results of variation in the face angle

As shown in Figure 5-31, the angle was measured from the vertical face to the groove face. At each analysis the strains were measured in both the axial and the hoop direction.

<table>
<thead>
<tr>
<th>Face Angle</th>
<th>Hoop Strain</th>
<th>Axial Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>degrees</td>
<td>millistrain</td>
<td>millistrain</td>
</tr>
<tr>
<td>40</td>
<td>0.5297</td>
<td>-0.4753</td>
</tr>
<tr>
<td>41</td>
<td>0.5322</td>
<td>-0.4650</td>
</tr>
<tr>
<td>42</td>
<td>0.5347</td>
<td>-0.4540</td>
</tr>
<tr>
<td>43</td>
<td>0.5373</td>
<td>-0.4424</td>
</tr>
<tr>
<td>44</td>
<td>0.5400</td>
<td>-0.4301</td>
</tr>
<tr>
<td>45</td>
<td>0.5429</td>
<td>-0.4173</td>
</tr>
<tr>
<td>46</td>
<td>0.5458</td>
<td>-0.4039</td>
</tr>
<tr>
<td>47</td>
<td>0.5489</td>
<td>-0.3902</td>
</tr>
<tr>
<td>48</td>
<td>0.5521</td>
<td>-0.3761</td>
</tr>
<tr>
<td>49</td>
<td>0.5554</td>
<td>-0.3617</td>
</tr>
<tr>
<td>50</td>
<td>0.5590</td>
<td>-0.3473</td>
</tr>
</tbody>
</table>

Figure 5-32 Face angle results (table)

From Figure 5-33 it can be seen that the relationship between the strains and the face angle is linear. To find the sensitivity function the results have to be converted so the
change in strain ($\delta \varepsilon / \varepsilon$) can be plotted against the change in angle ($\delta \theta / \theta$). The trend line will then give the sensitivity function required. The converted results can be seen in Appendix I.

<table>
<thead>
<tr>
<th>Sensitivity Function for Face Angle</th>
<th>$SF_{\theta_h \theta_g}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in axial strain / Change in Face angle</td>
<td>-1.39</td>
</tr>
<tr>
<td>Change in hoop strain / Change in Face angle</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 5-34  Sensitivity functions for variation in the face angle

$SF_{\theta_h \theta_g}$ represents the sensitivity function for the face angle with respect to surface strain. Under perfect conditions $\theta$ is 45 degrees, any variation in this and an error will be introduced to the strain readings. This error is governed by the following sensitivity equations with respect to face angle variation:

$$\frac{\delta \varepsilon'}{\varepsilon'} = SF_{\theta_h \theta_g} \frac{\delta \theta_g}{\theta_g}$$  \hspace{1cm} Equation 5-32

In each direction:

Axial Strain

$$\frac{\delta \varepsilon'_1}{\varepsilon'_1} = -1.39 \cdot \frac{\delta \theta_g}{\theta_g}$$  \hspace{1cm} Equation 5-33

Hoop Strain

$$\frac{\delta \varepsilon'_2}{\varepsilon'_2} = 0.24 \cdot \frac{\delta \theta_g}{\theta_g}$$  \hspace{1cm} Equation 5-34

Where $\delta \theta_g / \theta_g$ is the variation of the face angle from 45 degrees. $\theta$ is measured in degrees and $\varepsilon$ is measured in strain.
5.4.4 Discussion of variation in the face angle

During manufacturing some variation in dimensions will occur. This is accepted in engineering as it is impossible to manufacture everything exactly the same. The important thing is to understand how these variations affect the performance of that certain component. In this section the effect of variation in the manufacture of the groove was investigated.

As expected the face angle of the groove has an influence on the measured surface strain. A variation in the face angle will introduce errors in the surface strain readings. This relationship is linear and from the results shown previously it is not very influential, never the less it needs to be taken into consideration. A one degree change in the angle would introduce an error of 3 microstrains in the hoop and 12 microstrains in the axial direction. When converted to load this is equivalent to 0.5% error due to the hoop reading and 2.5% error due to the axial reading with each degree variation.

In terms of manufacturing a one degree variation is substantial so it is unlikely that the groove is manufactured to a tolerance bigger than 0.5 degrees. But as all possible sources of error this needs to be taken into consideration and accounted for.
5.5 Summary

The aim of this chapter was to find the sensitivity functions which relate the variation in dimensional parameters to surface strain on the nut. These terms were specified in the governing equation, Equation 4-14. There have been two changes to the governing equation. The expression $\Sigma P_n$ was used to symbolize the load distribution. As the load distribution is dependent on the spring constants the sensitivity function was found in terms of the change within the spring constants, so the term $\Sigma P_n$ was replaced by $K$. Secondly the geometry of the groove (depth and face angle) was combined as a single expression. Therefore the governing equation is:

$$\frac{\delta e'}{e'} = SF_K \frac{\delta K}{K} + SF_{\theta h_G} \frac{\delta \theta h_G}{\theta h_G} + SF(x'_L) \frac{\delta (x'_L)}{(x'_L)} + SF(y'_L) \frac{\delta (y'_L)}{(y'_L)}$$

Equation 5-35

For each term the sensitivity function has been found in both the axial and the hoop direction. These can be seen the figure below.

<table>
<thead>
<tr>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SF_{K1}$</td>
<td>0.095</td>
</tr>
<tr>
<td>$SF_{x'1L}$</td>
<td>-0.26</td>
</tr>
<tr>
<td>$SF_{y'1L}$</td>
<td>-1.24</td>
</tr>
<tr>
<td>$SF_{\theta h_G}$</td>
<td>-1.39</td>
</tr>
<tr>
<td>$SF_{K2}$</td>
<td>0.021</td>
</tr>
<tr>
<td>$SF_{x'2L}$</td>
<td>0.019</td>
</tr>
<tr>
<td>$SF_{y'2L}$</td>
<td>0.09</td>
</tr>
<tr>
<td>$SF_{\theta h_2G}$</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 5-35  Summaries of sensitivities

Substituting into the general equations gives the following expressions.

For axial strain:

$$\frac{\delta e'_1}{e'_1} = 0.095 \frac{\delta K}{K} -1.3896 \frac{\delta \theta h_G}{\theta h_G} -0.26 \frac{\delta x'_L}{x'_L} -1.24 \frac{\delta y'_L}{y'_L}$$

Equation 5-36
For hoop strain:

\[
\frac{\delta \varepsilon_2'}{\varepsilon_2'} = 0.0211 \frac{\delta K}{K} + 0.2415 \frac{\delta \theta h_g}{\theta h_g} + 0.0196 \frac{\delta x'L}{x'L} + 0.0908 \frac{\delta y'L}{y'L}
\]

Equation 5-37

The results show in general that the axial strain measurements are more sensitive to error than the hoop strain measurements. To maintain the 0.5 % accuracy (even only in the hoop direction) would require very tight tolerances which would make the nut very difficult to manufacture.

Assuming that only one parameter can change in the governing equation, these are the limits to maintain 0.5 % accuracy:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Maximum variation to maintain 0.5% accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Axial</td>
</tr>
<tr>
<td>Thread stiffness</td>
<td>$K_T$</td>
<td>%</td>
<td>1.13</td>
</tr>
<tr>
<td>Bolt stiffness</td>
<td>$K_{SC}$</td>
<td>%</td>
<td>0.08</td>
</tr>
<tr>
<td>Nut stiffness</td>
<td>$K_{BC}$</td>
<td>%</td>
<td>0.28</td>
</tr>
<tr>
<td>Groove geometry</td>
<td>$\theta h_G$</td>
<td>degree</td>
<td>0.16</td>
</tr>
<tr>
<td>Groove location x</td>
<td>$x'_G$</td>
<td>mm</td>
<td>0.12</td>
</tr>
<tr>
<td>Groove location y</td>
<td>$y'_G$</td>
<td>mm</td>
<td>0.025</td>
</tr>
</tbody>
</table>

Figure 5-36  Maximum dimensional variations

In the next chapter the development of the nut will be discussed along with the experimental testing.
Chapter 6 – Development of the Prototypes

6.1 Introduction

It was shown in the previous chapters how the specially modified nut combined with the laser strain gauge can be used to measure the load on a threaded connection. The nut was developed by combining background knowledge and finite element analysis. The development process will be discussed in this chapter.

Up to this point all the testing was performed virtually using finite element analysis. Finite element analysis is a useful tool when designing new components as testing can be performed on the specimens without manufacturing taking place. This makes the process less time consuming and more economical. All the initial testing was carried out using finite element analysis which meant ideal conditions were assumed. Sensitivity analysis was performed to investigate how dimensional variation might affect the accuracy of the load measuring device. Finite element models are assumed to be perfect so sensitivity or variation analysis is used to ensure that the model is tested in a more realistic manner. However variation analysis still uses computer modelling and to achieve a real life scenario experimental testing on the final prototype is necessary.

This chapter explains how the specially designed nut was developed from first principles, how the final design was chosen and how the finite element findings compared to the experimental test results. The design stage has been broken down into a number of stages. Finite element modelling demands significant computational power, therefore simplifications are made at the initial design stage. The initial design was carried out using a course 3 dimensional finite element model where the threads were not taken into consideration. Modelling the threads would mean that contact surfaces have to be introduced, which would result in a non-linear analysis requiring considerable computational power and analysis time. After the initial design stage when more accurate modelling was required, two-dimensional axisymmetric modelling was used with contact surfaces. These models were used to determine the final prototype, which was then manufactured and tested experimentally on a
specially designed test rig. Later 3 dimensional finite element models were also developed in aid to carry out the variation analysis described in the previous chapter.

The experimental testing on the specially modified nut revealed that there is hysteresis present due to the friction between the threads. During experimental testing the load was cycled, and even though the load was reduced on the joint the surface strain did not decrease at the required rate. This meant that the strain reading on the nut could not be related to the load accurately; hence the nut cannot be used as a load measuring device.

To remove the hysteresis an independent part had to be introduced into the joint which was under the same load as the joint but the load was not transferred through the threads. The solution was found using a specially modified washer. Both experimental and 3 dimensional finite element analysis was carried out on the washer and results showed that hysteresis is not present when measuring strain and that the load can be determined more accurately. Further testing was done on the washer to find the reliability and the accuracy.

This chapter will show the development and the analysis of the specially modified washer. Sensitivity analysis with respect to dimensional variation has also been carried out and these are explained here as well. The aim of this chapter is to show how the final prototype for the load measuring device was developed, tested and verified to be within the required accuracy.
6.2 Initial design

The laser strain gauge is a non-contact method for measuring surface strain. By relating the surface strain at any visible part of a threaded connection to the load on the connection, the LSG can be used as a load measuring device. To understand how the surface strain is related to the load finite element analysis was used.

The finite element modelling of the threaded fastener was divided into parts. The initial modelling was performed using 3-D solid modelling followed by 2-D axisymmetric modelling and experimental testing.

For all the modelling LUSAS (FEA Ltd, Kingston upon Thames) was used, which meant that the coordinate system used was “x” vertical, “y” horizontal and “z” depth. The initial modelling was performed using completely solid parts. It was assumed that the nut and bolt joint are not connected via threads but are welded together for simplicity and to aid the learning process. Using an M80 bolt and nut combination, the solid model was drawn following the British Standards (BS4190:2001). A volume mesh using quadratic hexahedral elements was adopted assuming that the material is ungraded mild steel.

![Solid FE model](image)

Figure 6-1 Solid FE model
Figure 6-1 shows one of the initial models. The nut was fixed along the axis of the bolt and all the loading was applied end the end of the bolt to allow uniform loading. Using the slice function in LUSAS it was possible to look at the strains within the specimen.

Figure 6-2 is an example of a solid model sliced along the middle. Models like this were used to get an idea about how altering the shape of the nut affects the stain.

<table>
<thead>
<tr>
<th>Model name</th>
<th>Cross sectional of FEA model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid model of flat head bolt</td>
<td><img src="image1" alt="Cross section of FEA model" /></td>
</tr>
<tr>
<td>Solid model of unmodified nut</td>
<td><img src="image2" alt="Cross section of FEA model" /></td>
</tr>
<tr>
<td>Solid model of chamfered nut (30 degrees)</td>
<td><img src="image3" alt="Cross section of FEA model" /></td>
</tr>
<tr>
<td>Solid model of chamfered nut (50 degrees)</td>
<td><img src="image4" alt="Cross section of FEA model" /></td>
</tr>
</tbody>
</table>
Figure 6-3 shows some of the initial ideas. The nut was chamfered to see if it is possible to get a valid reading at the top of the nut. Nearly all the stress is located where the nut and the bolt are joined, where the first thread would be located, and hardly any stress can be seen on the other end of the nut. If the chamfered angle is increased, it is possible to get a reading on the outside top edge but would mean reducing the strength of the joint. This was a very coarse model which did not include the threads, but helped to see where the high stress concentration areas are. However in real life this is not totally realistic and two-dimensional modelling was used to optimise the geometry.

The bolt modelled was the same as before, M80 with an ISO nut. Ignoring the helical effect of the thread and the hexagonal shape of the nut an axisymmetric model was drawn and analysed. The results were initially plotted using EE strain to help identify the high stress areas. EE is the equivalent strain, by definition “The scalar strain state obtained by combining the individual component strains at a point according to the classical von Mises failure criterion”.

$$EE = \sqrt{\frac{1}{2}[(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_1 - \varepsilon_3)^2 + (\varepsilon_2 - \varepsilon_3)^2]} \quad \text{Equation 6-1}$$
Figure 6-4 shows the original nut. The model assumed to be rotated around the y axis 360 degrees to get the full model. The high strain areas are located near the first thread and there is reduced strain at the other end of the nut. Even after chamfering there is no measurable strain at the end as shown in Figure 6-5.

When using the 3 dimensional solid models the nut and the bolt were “welded” together instead of being connected by threads. The way the load is transferred through threads is only approximate for simplification to the load transferred in threaded connections. In Chapter 5 the spring model was introduced which is a mathematical way of modelling the load distribution in a threaded connection. As the spring model showed the load distribution on a threaded connection is in such a way that the first thread takes up 10 times as much force as the last thread. This explains why the high strain areas are located near the first thread.

The nut has to be modified in such way that the reading can be taken nearer the first thread, where there are high strain concentrations. There were three main ideas, and each was investigated more closely.

It was thought that by adding a deformable washer it would distort and a strain measurement could be taken on its surface. However a deformable washer would mean more parts are added to the joint which would introduce more friction surfaces...
allowing a larger margin of error. Therefore a special nut is therefore introduced, which has an inbuilt washer.

This idea would work as a clip ring. As the nut is tensioned the cut part would compress causing tension on the chamfered side. This solution though would introduce a large area of high stress concentrations and would be subject to failure. Also due to the cut being on the inside of the nut it would be very difficult to manufacture.

Figure 6-7 shows a simplified version of the previous idea. The difference is that the chamfered side is in compression. It is easier to manufacture but there are still areas of
high stress. A high percentage of the load is on the first thread so by cutting away the material at the base of the nut weakens the assembly and introduces high stress concentrations.

The problem is to get the strain out to the outside surface of the nut without weakening it too much. Studying the original nut the contour lines go diagonally from the base inwards. The outside contours are blue, indicating a low strain area. By cutting the low strain areas away it is possible to get to areas with more strain without weakening the structure by a significant amount.

Figure 6-8 shows the final idea. By simply cutting grooves into the side at 45 degrees higher strain areas can be accessed. It is possible to line up the mesh elements so the actual strain can be plotted along the edges. Using this method different size and trajectory grooves were analysed for optimum strength and strain measurement. The above picture shows the model with the optimised groove size. The grooves are also helpful for the user to take the strain measurement with the LSG as the gratings are at an angle. Preliminary tests showed that the strain along the bolt is about ten times the strain along the cut, and about three times as much as the hoop strain along the cut. The hoop strain is the strain in the z direction, going around the nut.
The initial 3 dimensional solid models and the two-dimensional axisymmetric models with non-linear analysis helped to find a design which is suitable to relate the surface strain to the load. Both types of models used in the development stage employ assumptions, therefore to get accurate results experimental testing is required. 3 dimensional finite element models were not used at this stage of the design. To carry out non-linear contact analysis with 3-D elements would have required considerable analysis time and computational power. 3 dimensional models were used when variation analysis was carried out on the prototype (Chapter 5) but even then contact analysis was avoided.

Using the results obtained from finite element analysis a full 3 dimensional model was drawn in CAD to aid in the manufacture of the prototype. The grooves were manufactured on alternate sides, so altogether there were 3 grooves. Having three grooves meant that the structural integrity of the nut is kept while allowing appropriate number of test points to measure the surface strain.

The engineering drawings with dimensions can be seen in Appendix J.
6.3 Results and Experimental testing

To validate the results obtained by analytical results experimental testing is required. Figure 6-9 shows the final idea. This is basically a standard nut with a small cut (groove) on the side at 45 degrees. Standard nuts were manufactured and the cuts were made by hand later. The surface strain within the grooves was measured using electrical resistance strain gauges and the axial strain of the bolt was measured using lasers.

In the initial testing all the modelling was done based on an M80 thread. The test rig provided by Hydratight Sweeney (www.hydration.com 2001) uses a 2 ½ UNF 8 thread, so when the prototype was manufactured this was taken into consideration. Hydratight Sweeney are a leading company who design and manufacture bolt tightening products. They are involved in this project to help develop a non contact bolt load measuring device.

The previous finite element models were modified to a 2 ½ inch UNF8 thread to keep the continuity between the test rig and the models. The test rig was set up for the 2 ½ inch bolt using a nut on both end with some spacers and a hydraulic pump in between (Figure 6-11). The schematic diagram of the test rig is shown on Figure 6-10. The modified finite element model was based on this test piece, so all the dimensions and the material properties were equivalent.

![Figure 6-10 Schematic diagram of the test rig setup](image)
The maximum loading was calculated using the material properties shown in Appendix K. Using a suitable safety factor it was decided that the maximum pressure that should be applied to the hydraulic pump is 200 bar which is equivalent to a 703 kN force applied to the threaded connection.

The schematic diagram of the test rig shows how the equipment was setup in order to carry out the experimental tests. The experiments were carried out in a temperature and humidity controlled laboratory in order to keep all the conditions the same throughout the tests. Also under these controlled conditions it was ensured that the errors introduced due to outside factors and the test environment were minimised. The test rig itself was set up on a solid stone table and attached to it. This was to ensure that vibrations and shaking does not interfere with the readings.

The test rig was designed and built to accommodate the 2 \( \frac{1}{2} \)" UNF8 threaded shaft (bolt) which has a length of 700 mm of which the active length is 617.62 mm. Active length means the length which the load is acting over, hence the distance between the two nuts at either end. A loading disk was positioned in the middle of the shaft which was connected to a hydraulic pump. The hydraulic pump was used to drive oil between the loading disks, hence forcing them apart. Two sleeve barrels were placed on either side of the loading disk which were used as spacers and were forced apart by

![Test Rig](image)

Figure 6-11 Test Rig
the discs. The sleeve barrels were locked in place by two nuts on either end of the shaft.

At the start of the experiment the nuts were loosely tightened to hold the sleeve barrels in place. The hydraulic pump was used to drive the oil in between the loading disks and push the barrels against the nuts. As the pressure in the pump increased, the sleeves were pushed apart with a higher force. This imitated a threaded joint which is being tightened, but instead of tightening the nut by turning it, it was achieved by pushing the two nuts apart. The oil pressure was monitored throughout the experiment and was related to the load by knowing the hydraulic pressure area of the loading disk (Appendix K).

The strain measurements were taken using two different methods. The first method involved the use of lasers. The two lasers used were a Renishaw ML10 and a HP 5529A Laser interferometers. Two reflective prisons were stuck to the end of the threaded shaft to ensure they move together. The laser beams were focused at the reflective prisons at either end as shown in Figure 10, and the reflected beams were directed back to the laser interferometers. The interferometers were connected to a laptop, and the length of the laser beams were measured and recorded. When force was applied to the sleeve barrels by the loading disk, the nuts were pushed apart which increased the length of the shaft. The change in length of the shaft was measured accurately with the laser interferometers, and by summing the displacement on either end the total change in length could be found. By knowing the original length of the shaft and the change in length, the axial strain in the bolt was calculated. The loading was increased in steps and the laser readings were taken at each stage before reaching the maximum load (200 bars, 703 kN) and reducing it back to zero again in steps.

The second method involved the use of electrical resistance strain gauges. The electric resistance strain gauges were used to imitate the LSG. Instead of etching the gratings into the surface of the nut and using the LSG to measure the surface strain, electrical resistance strain gauges were used for simplicity. The strain gauges were positioned on the six sides of the nut (three with grooves and three with flat unmodified surfaces) and were connected to a Wheatstone bridge (Figure 6-12).
On the three sides where the grooves have been manufactured the strain gauges were positioned on the centre of the flat face of the grooves. On the unmodified faces the strain gauges were positioned at the groove centre location point which was referred to earlier as the reference point (Chapter 4). Instead of single strain gauges, rosettes were used for more accuracy. Altogether 18 separate strain readings were taken, three on each of the six sides (Figure 6-14).

The strain gauges were attached to the nut surface using a special adhesive, and connected to the Wheatstone Bridge using colour coded wires. The Wheatstone Bridge measures how the resistance in the strain gauge change, and using a suitable gauge factor it converts a change of using rosette in that the principal strains in the gauge material. The hoop and axial strains were calculated using:

\[
\begin{align*}
\varepsilon_t &= \frac{1}{2} (\varepsilon_x + \varepsilon_y) + \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + 4 \varepsilon_z^2} \\
\varepsilon_z &= \frac{1}{2} (\varepsilon_x - \varepsilon_y) - \frac{1}{2} \sqrt{(\varepsilon_x - \varepsilon_y)^2 + 4 \varepsilon_z^2}
\end{align*}
\]

Where \( \varepsilon_x, \varepsilon_y, \) and \( \varepsilon_z \) are the axial strains in the gauge material.
Bridge measures how the resistance in the strain gauges change and using a suitable gauge factor it converts the resistances to strain. The advantages of using rosettes is that the principal strains can be determined regardless of the rosette orientation. The two principal strains in this scenario are the hoop and axial strains. The hoop and axial strains were calculated the following way (Dally and Riley 1991).

\[ \varepsilon_1 = \frac{1}{2}(\varepsilon_A + \varepsilon_C) + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2} \]

\[ \varepsilon_2 = \frac{1}{2}(\varepsilon_A + \varepsilon_C) - \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2} \]

Equation 6-2

Where \( \varepsilon_A, \varepsilon_B \) are \( \varepsilon_C \) are the rosette readings.

The strain readings were taken at the beginning of the experiment (at zero loading) and recorded. This was used as the reference strain gauge readings. The strain readings were taken at each load step (increasing and decreasing the load) and recorded in a table. The tests were repeated a number of times for continuity and the results were recorded. The results can be seen in the results section.

The same geometry as the prototype was modelled using two-dimensional axisymmetric contact analysis in LUSAS. By using the same boundary conditions for the finite element model as for the test rig the analytical and the experimental findings could be compared.

**Analytical findings**

As it was mentioned before the finite element results plotted were EE (equivalent) strain. EE is not an accurate way of telling what the exact strain is at each point but it is used to give a general overall idea. To find the strain readings more accurately graphs are plotted along certain edges. The meshed elements have to be lined up in the same direction for the graph reading to be taken. This is done by cycling the elements around until each are orientated the same way.
There are four types of usable strain plots in LUSAS finite element software. There is the overall strain which is a combination of all the strains from all directions, and there is the directional strain in the “x”, “y” and “z” directions. To get accurate strain results all the elements in the model have to be orientated in the same direction. Figure 6-14 shows the model with correctly aligned elements.

In Figure 6-14 the axes are orientated in such a way, that the “x” axis is in the vertical direction, the “y” axis is in the horizontal direction and the “z” axis is perpendicular to the page. When all the elements are pointing in the same direction a graph can be plotted at any cross section to show the strain at each node. Figures 6-15a, 6-15b and 6-15c show the strain at the required places on the assembly.
These results show the strain distribution under a load of 703 kN applied at 90° to the bolt at an angle of 15° to the bolt axis. The normal stress along the groove was measured along the cut and is maximum after the first 10 mm and then decreases along the cut way with distance. It is important to position the gauges in each way to get the maximum reading. Generally strain increases, while the strain along the cut reduces. Therefore, the best position for the gauges is at the centre of the groove, both in terms of the measurement magnitude and ease of manufacture.

Figure 6-15a Axial Strain along the bolt

Figure 6-15b Axial strain along the cut

Figure 6-15c Hoop strain along the cut
These results show the strain distribution under a load of 703 kN equivalent to 200 bar pressure on the test rig (Appendix K). The strain is measured along the bolt as well as along the groove. The axial strain on the bolt is measured as it can be used to find the loading if the material property and the geometry of the bolt is known. By relating the surface strain within the groove to the axial strain along the bolt the nut can be calibrated as a load measuring device.

The results show that the strain on the shaft increases away from the nut and it reaches its maximum after the first thread. Both the hoop strain and the axial strain along the cut vary with distance. It is important to position the grating in such way to get the maximum reading. Going away from the tip of the cut the hoop strain increases, while the strain along the cut reduces. Therefore the best position for the grating is at the centre of the groove, both in terms of the measurement magnitude and ease of manufacture.

<table>
<thead>
<tr>
<th>Load</th>
<th>703 kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial Strain on the Shaft</td>
<td>1.022 millistrain</td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0.17 millistrain</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>-0.15 millistrain</td>
</tr>
</tbody>
</table>

Figure 6-16 Tabulated results

The Finite element results show that roughly a 1/6th of the bolt strain can be seen on the hoop strain and about a -1/7th can be seen on along the cut. These numbers are indications only, as the two-dimensional axisymmetric model uses assumptions that can introduce major errors. For more accurate results experimental testing was carried out using a special test rig.

Experimental Results:

Similarly to the analytical test, the strain was measured in two places. The axial strain on the bolt was measured using laser interferometers, while the strain within the grooves was measured using electric resistance strain gauges. Using electric resistance
strain gauges meant that the strain can only be measured at one single point within the
groove, this point was the centre of the groove. (On the flat surfaces the strain was
measured at the groove location point)

The lasers were used to measure the extension in the shaft. Strain is calculated by
dividing the extension by the original length. What is the original length? The original
length when calculating the strain is taken between the points where the force is
acting. In this case it is between the nuts, but the nut acts over a length rather than a
point. It is a valid assumption that the original length can be worked out by taking the
distance between the mid points of the nuts. The length of the shaft between the nuts
was measured using a vernier under no load. The original length of the shaft was
measured to be 617.62 mm.

The experiment was carried out in a temperature and humidity controlled room. The
test rig was positioned on a solid stone table to reduce the chances of shakes and
vibrations. The pressure in the hydraulic assembly was adjusted by a hand pump
(Figure 6-17). The pressure was monitored by a digital pressure dial.

The experimental tests were carried out under varied loading. The pressure was
increased slowly by an equal amount each time, and the laser readings were taken.
Once 200 bar was reached, the pressure was steadily reduced taking readings at the
same pressures as before. The tests were repeated 10 times to eliminate any bad
readings ensure the accuracy of the results. The results were collected in a table which
can be seen in Appendix L. The results were averaged and tabulated in Figure 6-18,
the plotted averaged axial strain results can be visualized in Figure 6-19.
<table>
<thead>
<tr>
<th>Pressure Bar</th>
<th>HP reading mm</th>
<th>Renishaw reading mm</th>
<th>Combined reading mm</th>
<th>Normalised reading mm</th>
<th>Axial Strain microstrain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-0.001</td>
<td>0.000</td>
<td>-0.001</td>
<td>0.000</td>
<td>0</td>
</tr>
<tr>
<td>20</td>
<td>-0.072</td>
<td>0.000</td>
<td>19.928</td>
<td>0.000</td>
<td>115</td>
</tr>
<tr>
<td>40</td>
<td>-0.142</td>
<td>-0.004</td>
<td>39.858</td>
<td>0.004</td>
<td>235</td>
</tr>
<tr>
<td>60</td>
<td>-0.220</td>
<td>-0.009</td>
<td>59.780</td>
<td>0.009</td>
<td>369</td>
</tr>
<tr>
<td>80</td>
<td>-0.291</td>
<td>-0.014</td>
<td>79.709</td>
<td>0.014</td>
<td>492</td>
</tr>
<tr>
<td>100</td>
<td>-0.367</td>
<td>-0.019</td>
<td>99.633</td>
<td>0.019</td>
<td>623</td>
</tr>
<tr>
<td>120</td>
<td>-0.447</td>
<td>-0.030</td>
<td>119.553</td>
<td>0.030</td>
<td>770</td>
</tr>
<tr>
<td>140</td>
<td>-0.513</td>
<td>-0.031</td>
<td>139.487</td>
<td>0.031</td>
<td>880</td>
</tr>
<tr>
<td>160</td>
<td>-0.586</td>
<td>-0.034</td>
<td>159.414</td>
<td>0.034</td>
<td>1002</td>
</tr>
<tr>
<td>180</td>
<td>-0.661</td>
<td>-0.038</td>
<td>179.339</td>
<td>0.038</td>
<td>1130</td>
</tr>
<tr>
<td>200</td>
<td>-0.731</td>
<td>-0.042</td>
<td>199.269</td>
<td>0.042</td>
<td>1251</td>
</tr>
<tr>
<td>220</td>
<td>-0.802</td>
<td>-0.045</td>
<td>219.194</td>
<td>0.045</td>
<td>1371</td>
</tr>
<tr>
<td>240</td>
<td>-0.873</td>
<td>-0.048</td>
<td>239.121</td>
<td>0.048</td>
<td>1491</td>
</tr>
<tr>
<td>260</td>
<td>-0.944</td>
<td>-0.051</td>
<td>259.047</td>
<td>0.051</td>
<td>1611</td>
</tr>
<tr>
<td>280</td>
<td>-1.015</td>
<td>-0.054</td>
<td>279.017</td>
<td>0.054</td>
<td>1731</td>
</tr>
<tr>
<td>300</td>
<td>-0.086</td>
<td>-0.057</td>
<td>299.194</td>
<td>0.057</td>
<td>1851</td>
</tr>
<tr>
<td>320</td>
<td>-0.057</td>
<td>-0.060</td>
<td>319.361</td>
<td>0.060</td>
<td>1971</td>
</tr>
</tbody>
</table>

Figure 6-18  Averaged axial bolt strain measured by laser interferometers (Table)

![Graph showing the relationship between pressure and axial strain](image)

Figure 6-19  Averaged axial bolt strain measured by laser interferometers (Graph)
The results show that the strain in the bolt is 1.2 millistrains at 200 bar pressure which is equivalent to 703 kN force.

It was described before how strain gauge rosettes were placed on each side of the nut (Figure 6-20). The grooves were placed on alternate sides. The rosettes were used to measure the strain on all sides, so the flat face readings and the readings in the grooves could be compared.

The tests were repeated 10 times, and the results were tabulated in Appendix L. In Figure L-6 the raw test results for a single experiment can be seen along with the normalised results. The raw results are the direct readings from the Wheatstone bridge for each strain gauge (18 in total) which are normalised by adjusting the initial reading to zero strain at no load. The three rosette readings were used to calculate the hoop and the axial strain as it was shown earlier. The hoop and the axial strains are the two principal strains on the nut, $\epsilon_1$ and $\epsilon_2$ respectively. Figures L-7 to L-16 in Appendix L, show the converted results for the raw test and the converted results for the repeated experiments. The averaged tabulated results can be seen in Figure 6-21.
### Principle strain

<table>
<thead>
<tr>
<th></th>
<th>Pressure/Load</th>
<th>0 bar</th>
<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Groove</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>214</td>
<td>389</td>
<td>531</td>
<td>665</td>
<td>639</td>
<td>530</td>
<td>286</td>
<td>-3</td>
<td></td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>-178</td>
<td>-295</td>
<td>-407</td>
<td>-531</td>
<td>-475</td>
<td>-429</td>
<td>-255</td>
<td>-10</td>
<td></td>
</tr>
<tr>
<td><strong>Flat face</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>144</td>
<td>243</td>
<td>316</td>
<td>375</td>
<td>380</td>
<td>351</td>
<td>206</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>-9</td>
<td>-81</td>
<td>-168</td>
<td>-255</td>
<td>-197</td>
<td>-125</td>
<td>-42</td>
<td>-12</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6-21  Averaged strain gauge results calculated from rosette readings over 10 experiments (all units are in microstrain)

Figures 6-22a and 6-22b show the axial strain, $\varepsilon_2$ in the groove and on the flat face.

**Figure 6-22a**  Axial strain in groove

**Figure 6-22b**  Axial strain on flat face
At a pressure of 200 bars, the axial strain in the groove is -0.531 millistrains while on the flat face it is -0.245 millistrains. Figures 6-23a and 6-23b show the hoop strain. The hoop strain is positive due to the circumference increasing while the nut is being compressed.

The hoop strain is 0.665 millistrains in the groove and 0.375 millistrains on the flat face at 200 bar pressure. There is a significant difference between the surface strain...
measured within the groove and the surface strain measured on the flat unmodified face. The summarised results can be seen in Figure 6-24.

<table>
<thead>
<tr>
<th>Groove</th>
<th>Flat face</th>
<th>Factor between strain in the groove and strain on the flat face</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>millistrain</td>
<td>millistrain</td>
</tr>
<tr>
<td>Axial strain</td>
<td>-0.531</td>
<td>-0.245</td>
</tr>
<tr>
<td>Hoop strain</td>
<td>0.665</td>
<td>0.345</td>
</tr>
</tbody>
</table>

Figure 6-24 Summarised experimental results

From the summarised results it is possible to see the importance of the groove. By introducing the groove a higher magnitude of strain can be measured which helps improve accuracy. Also the grooves make it easier for the user to take the strain measurement as it is oriented at a 45 degree angle.

When finite element analysis is used a number of assumptions are introduced. In this case the assembly geometry was simplified so a two-dimensional axisymmetric model could be used, but also it was assumed that both the material and geometry are correct. Friction at the threads and at other contact surfaces was neglected.

The experimental results show a large variation from the finite element results. The results for the axial strain measurements have a slight error, while there is an enormous discrepancy in the surface strain measurements. The summary of the results are listed in Figure 6-25.

<table>
<thead>
<tr>
<th>Analytical results</th>
<th>Experimental results</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>millistrain</td>
<td>millistrain</td>
</tr>
<tr>
<td>Axial bolt strain</td>
<td>1.022</td>
<td>1.244</td>
</tr>
<tr>
<td>Axial strain in groove</td>
<td>-0.150</td>
<td>-0.531</td>
</tr>
<tr>
<td>Hoop strain in groove</td>
<td>0.170</td>
<td>0.665</td>
</tr>
</tbody>
</table>

Figure 6-25 Summary of results and error in terms of experimental results

(Load is 200 bars or 703 kN)
The discrepancy is due to the two-dimensional axisymmetric model using assumptions to simplify the model. Assuming that the nut is circular and not hexagonal meant that the results would be inaccurate; however they were useful to help determine the high stress areas. More significantly the experimental results show a large amount of hysteresis. The hysteresis means that a single strain reading corresponds to a range of loads, therefore the load can not be determined accurately (Figure 6-22 & 6-23). At a given strain reading there is a variation of nearly a 100 Bar in pressure. This amount of pressure is equivalent to nearly 350 kN in loading.

When the finite element testing was carried out there was no opportunity to investigate the hysteresis. There are two reasons for the hysteresis in the experimental testing. The most significant one is due to the friction in the threads. When the load is increased in the assembly the nut tightens, and the strain on the nut surface increases as expected. However, when the load is slowly reduced there is friction between the threads, which make the nut "stick", so the surface strain is not reduced. Due to this behaviour there will always be a certain amount of hysteresis on the nut. The other, less significant cause for the hysteresis is because of the hydraulic pump. As a result of the viscosity of the fluid within the pump, when the pressure is being reduced the pump responds slower. This effect can be seen in the laser testing, when there is a small amount of hysteresis on the shaft.

The results show that the redesigned nut is not a suitable solution to the given problem. The hysteresis means that the surface strain measurement cannot be related to the load accurately. Due to the threads on the nut there will always be hysteresis, therefore the strain measurements have to be taken from another location. The other option is the shaft, but only the free end of the shaft is visible and it is not under any loading. An alternative solution has to be found which experiences the same load as the joint and visible for the laser strain gauge to measure the surface strain on it.
6.4 Re-optimisation of Prototype / Washer

The hysteresis on the nut means that the strain reading can not be related to the load accurately. The hysteresis needs to be eliminated for the laser strain gauge to work. The main reason for the hysteresis is thought to be the threads locking through friction. The laser strain gauge is a non contact method and the article which is measured needs to be visible. As it was mentioned before the only visible part of the threaded connection is the end of the shaft and the nut. The end of the shaft does not see enough of the force for the laser strain gauge to take a reading, and due to the threads there is hysteresis on the nut. Another component needs to be introduced which experiences enough force, but is not affected by the thread.

A washer was suggested as a solution. If a washer is introduced all the force going through the shaft has to go through it. Also the washer is not connected to the system through threads but through two flat surfaces. This means that the magnitude of the friction is reduced significantly, but more importantly the direction of the frictional forces are now horizontal and will not affect the vertical load. The suggested prototype is shown below.

![Figure 6-26 Redesigned washer](image)

The washer shown in Figure 6-26 was designed based on a standard washer with some modification. Once again three grooves were manufactured at a 45 degree angle with each groove being 120 degrees apart. The grooves had two main functions. Firstly, as some material was removed it meant that the surface strain within the
grooves would be at a higher magnitude. Measuring higher strains mean there is more accuracy as if there is a larger margin of error. Secondly with the 45 degree angle it is easier for the laser strain gauge user to get readings. If the gratings would be placed straight onto the side of the washer it would be awkward for the user to get the right angle for the measurements. So it was decided that similar grooves should be added to the washer like to the ones on the modified nut. The rest of the washer was kept as standard to ensure ease of manufacture and keep the structural integrity of the joint. Full dimensions of the washer can be seen in Appendix M.

The initial tests were performed by finite element analysis, but this time full 3 dimensional modelling was used. As there were no threads involved in the analysis of the washer it was not necessary to use a model of the threads. This meant that contact analysis was not required to analyse the behaviour of the washer under loading.

The 3 dimensional finite element model was prepared similarly to the 3 dimensional nut model described in Chapter 5. Due to the complicated groove geometry a combination of hexahedral and pentahedral mesh was used. The washer was fixed on the bottom surface in the vertical direction and a distributed force was signed to the top surface to model a real life scenario.

The analysis was run and the strain was recorded at the centre of the groove in both the axial and the hoop direction. The analytical results showed that at maximum load (703 kN) the axial strain is -625 microstrain and the hoop strain is +301 microstrain.
To validate these results experimental testing was carried out. A prototype washer was manufactured and was included in the test rig described previously. The experimental tests were carried out using strain gauges in a similar manner to the experimental nut analysis. The only difference between the nut and the washer experimental tests was that on the washer the rosette strain gauges were placed in all three grooves, but only one rosette strain gauge was placed on the flat part. Therefore only 4 rosettes were used altogether to reduce the testing time. The strain on the flat surface was only measured as a comparison and it was assumed that it is the same throughout so one rosette is enough. The tests were repeated 10 times and the results are presented in Appendix N. The raw and normalised results for these tests can be seen in Figure N-1, with the calculated principle strain results are shown in Figure N-2 to N-11. The averaged test results are presented below in table and graphs formats.

<table>
<thead>
<tr>
<th>Pressure/Load</th>
<th>0 bar</th>
<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groove</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>-180</td>
<td>-345</td>
<td>-507</td>
<td>-669</td>
<td>-822</td>
<td>-113</td>
<td>-180</td>
<td>0</td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>91</td>
<td>176</td>
<td>266</td>
<td>356</td>
<td>440</td>
<td>528</td>
<td>616</td>
<td>704</td>
</tr>
<tr>
<td>Flat face</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>-345</td>
<td>-676</td>
<td>-970</td>
<td>-1247</td>
<td>-1520</td>
<td>-1803</td>
<td>-1806</td>
<td>0</td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>77</td>
<td>169</td>
<td>270</td>
<td>365</td>
<td>470</td>
<td>575</td>
<td>680</td>
<td>785</td>
</tr>
</tbody>
</table>

Figure 6-28  Averaged experimental results for the washer – Table

(all units are in microstrain)

Figure 6-29  Averaged experimental results of the strain within the groove of the washer (Graph)
The results are more satisfactory compared to the redesigned nut results. The hysteresis is considerably reduced on the hoop strain, and there is no hysteresis in the axial direction. The washer is in contact with the nut on one surface, and the friction on this surface causes some hysteresis in the hoop strain.

Using the same loading scenario on the finite element model and the test rig the analytical and the experimental analyses could be compared. The results show a very close correlation, even more so than with the axisymmetric model. This proves how the axisymmetric model uses assumptions, which introduce inaccuracies. The results can be seen in Figure 6-30.

<table>
<thead>
<tr>
<th></th>
<th>Analytical results</th>
<th>Experimental results</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>millistrain</td>
<td>millistrain</td>
<td>%</td>
</tr>
<tr>
<td>Axial strain in groove</td>
<td>-0.625</td>
<td>-0.672</td>
<td>6.99</td>
</tr>
<tr>
<td>Hoop strain in groove</td>
<td>0.301</td>
<td>0.329</td>
<td>8.51</td>
</tr>
</tbody>
</table>

Figure 6-30  Summary of washer results and error in terms of experimental results

(Load is 200 bars or 703 kN)

The new design using the washer had a number of advantages over the original nut proposal.

- The hysteresis was reduced.
- Introducing the washer meant that the nut does not have to be re-designed, which meant that the original nuts could be used keeping the bolt assembly’s strength.
- The washer was designed in such way that it has the same strength as the bolt so the assembly was not weakened.
- The strain readings on the washer were at a higher magnitude than on the redesigned nut and thereby improving the accuracy.
In order to use the redesigned washer as a load measuring device it is necessary to carry out further analyses. In the following sections the washer will be discussed in more detail.
6.4.1 Further analysis of the washer - Residual Stress

An important factor that can affect the accuracy of the laser strain gauge has not been mentioned before. This is the effect of residual stress left in the washer after the groove manufacture. Residual stress is the stress that remains in a body when all external forces have been removed. To be more specific these stresses within the body have to be net zero, as all the compressive residual stresses are cancelled out by the tensile stress regions.

Residual stresses are frequently formed during fabrication operations, such as casting, rolling or forging (Rowlands 1993). Residual stresses can be a real problem as they can cause fracture. Also they are very difficult to measure and they add to stresses due to applied loads. It is important to investigate the washer for residual stresses, so the surface strain readings are not affected by them.

Measuring residual stresses in a non-destructive way is extremely troublesome, so most methods are destructive. Methods include hole-drilling, ultrasonic techniques, X-ray, photomechanical techniques, and numerical analysis. The most common way is hole drilling technique (Boiten and Ten Cate 1952). There are a number ways of measuring the residual stress when using the hole drilling method, but as electric resistance strain gauges have been used in the previous experiments it was decided that special residual stress rosettes should be used.

The rosettes were cemented into the grooves of the washer the same way it was done in the previous experiments. The same experiment as before was carried out, increasing and decreasing the load in steps taking readings at certain pressures. Once the full cycle was carried out a hole was drilled in the centre of the flat face between the strain gauges. The diameter of the hole was 2 mm, so it was decided that the depth of the hole should be at least 8 mm to relieve all the stress. The change in strain was recorded before and after the drilling (Figure 6-31).

The initial test before the drilling showed a strong correlation with previous results as expected. However when the strain readings were taken after the drilling some unexpected results were found. When the 8mm deep hole was drilled on the flat face
one of the strain gauges in the rosette started to peel off, which would explain the unexpected results. The net change in the strain on the peeled off strain gauge was -2055 microstrain while the other two were only -312 and -253 respectively according to the rosette. Assuming that if the first gauge would have been unharmed the reading would have been in the region of -300 microstrains.

<table>
<thead>
<tr>
<th></th>
<th>0 bar</th>
<th>20 bar</th>
<th>40 bar</th>
<th>60 bar</th>
<th>80 bar</th>
<th>100 bar</th>
<th>120 bar</th>
<th>140 bar</th>
<th>160 bar</th>
<th>180 bar</th>
<th>200 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gauge 1</td>
<td>0</td>
<td>65</td>
<td>82</td>
<td>97</td>
<td>119</td>
<td>147</td>
<td>178</td>
<td>211</td>
<td>244</td>
<td>277</td>
<td>312</td>
</tr>
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<td>-104</td>
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<td>187</td>
<td>165</td>
<td>149</td>
<td>122</td>
<td>-4</td>
</tr>
</tbody>
</table>

Gauge 1 was damaged, after drilling value is estimated to be -300 microstrain

**Principal Stresses**

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<td>97.82</td>
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<td>-544.7</td>
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<th>After drilling</th>
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<td>302</td>
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<td>131.2</td>
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<td>-860</td>
<td>-769.1</td>
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<td>-568.5</td>
<td>-470.1</td>
<td>-367.4</td>
<td>-280.1</td>
<td>-193.2</td>
<td>-20.9</td>
<td>-320.0</td>
</tr>
</tbody>
</table>

All values are in microstrain

**Figure 6-31** Residual stress results without annealing

The strain gauge readings need to be converted into residual stress. The residual stress is biaxial and therefore has two principal values and directions. These stresses and their directions can be calculated using the following equations (Dally and Riley 1991).

\[
\sigma_1^R = \frac{\varepsilon_A + \varepsilon_C}{4C_1} + \frac{\sqrt{2}}{4C_2}\sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2}
\]

\[
\sigma_2^R = \frac{\varepsilon_A + \varepsilon_C}{4C_1} - \frac{\sqrt{2}}{4C_2}\sqrt{(\varepsilon_A - \varepsilon_B)^2 + (\varepsilon_B - \varepsilon_C)^2}
\]

\[
\tan 2\theta = \frac{\varepsilon_A - 2\varepsilon_B + \varepsilon_C}{\varepsilon_C - \varepsilon_A}
\]

*Equation 6-3*
Heat treatment methods can be used to remove the ‘history’ of stress accumulated from past operations upon the item of interest. The most thorough of these is annealing. Annealing occurs at or near the quenching temperature, however in this case the metal is slowly cooled to room temperature. The result is a microstructure that is totally free from stress, consisting of the crystal structure that is stable at room temperature, ferrite in the case of steel. Partial annealing, as the name implies, does less of a removing stress and provides a less uniform microstructure, due to the fact that lower temperatures and shorter times are used for the process. Stress relieving is conducted at even lower temperatures, and has the more limited goal of only dealing with residual stresses. The benefit of partial annealing and stress relieving lies in the amount of allowable distortion; if an item is already near net shape, the risk of distortion may be too great for complete annealing to be conducted.

It was decided to carry out stress relieving on one of the samples to investigate if there is any difference in the results. The material used for the prototypes is EN24 (817M40) alloy steel. The annealing temperature for this steel is around 800 Celsius degrees. The phase diagram for such steel shows that above 650 Celsius degrees the ferrite starts turning into austenite, changing the crystal structure of the material (Figure 6-32). Therefore stress relieving was used which was carried out at 600 Celsius degrees. The sample was heated to 600 Celsius degrees and it was soaked for 90 minutes then cooled slowly in the surface allowing the crystals structure to settle and for the internal stresses to disappear.

Figure 6-32 was generated for EN24 steel using MTDATA for Windows by knowing the chemical composition.
Experimental testing was carried out using the annealed washer. The results can be seen in Figure 6-33. The results for the annealed washer showed that the residual stress can be eliminated by heat treatment. Annealing is one of the most prevalent methods of relieving residual stresses but it is very expensive and can introduce changes in the dimensions of the specimen.

The results also show that the strain readings for the annealed and the non annealed washer correlate well. Hence even though there are residual stresses present in the washer the surface strain readings are not affected. So the residual stresses do not affect the accuracy of the laser strain gauge.
<table>
<thead>
<tr>
<th></th>
<th>0 bar</th>
<th>20 bar</th>
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<th>160 bar</th>
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<tr>
<td>Gauge 1</td>
<td>61</td>
<td>53</td>
<td>47</td>
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<td>38</td>
<td>36</td>
<td>34</td>
<td>32</td>
<td>45</td>
<td>59</td>
</tr>
<tr>
<td>Gauge 3</td>
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<td>34</td>
<td>43</td>
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<th>20 bar</th>
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<tbody>
<tr>
<td>Gauge 2</td>
<td>66</td>
<td>70</td>
<td>75</td>
<td>77</td>
<td>73</td>
<td>81</td>
<td>82</td>
<td>-7</td>
<td>-6</td>
<td>-6</td>
<td>-4</td>
</tr>
<tr>
<td>Gauge 3</td>
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<td>46</td>
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<td>14</td>
<td>6</td>
<td>-2</td>
<td>-25</td>
<td>-25</td>
<td>-22</td>
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</tbody>
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Principal Stresses

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<th>140 bar</th>
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<th>200 bar</th>
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</thead>
<tbody>
<tr>
<td>Hoop Strain</td>
<td>95.34</td>
<td>95.34</td>
<td>97.41</td>
<td>105.06</td>
<td>115.30</td>
<td>127.02</td>
<td>141.06</td>
<td>155.32</td>
<td>169.74</td>
<td>185.35</td>
<td>196.05</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>-34.64</td>
<td>-36.34</td>
<td>-39.41</td>
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<td>-55.02</td>
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<td>-78.74</td>
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<tr>
<th></th>
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<th>160 bar</th>
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<th>40 bar</th>
<th>20 bar</th>
<th>0 bar</th>
<th>After Drilling</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hoop Strain</td>
<td>183.23</td>
<td>183.23</td>
<td>173.93</td>
<td>160.05</td>
<td>135.19</td>
<td>138.40</td>
<td>128.97</td>
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<tr>
<td>Axial Strain</td>
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<td>-47.05</td>
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<td>-43.40</td>
<td>-40.97</td>
<td>-8.80</td>
<td>-36.30</td>
<td>-36.30</td>
<td>-29.64</td>
</tr>
</tbody>
</table>

All values are in microstrain

Figure 6-33  Residual stress results after annealing
6.4.2 Further analysis of the washer - Determination of Bolt Load from Measured Strains

The aim of this project is to show how the laser strain gauge can be used to measure the loading in a threaded connection. The laser strain gauge is used to find the surface strain on the washer which is then related to the strain in the threaded shaft. Finite element analysis and experimental methods were used to help find this relationship. The results indicated that half the strain on the shaft can be seen on the washer in the axial direction, and the hoop strain on the washer is a quarter of the shaft strain. The load can be worked out from the shaft, or directly from the washer. As the material and the cross-sectional area of the washer is known the load can be calculated in a theoretical way. As the shape of the washer is irregular due to the grooves these theoretical calculations are only estimations and assumptions are used. It is assumed that the washer has a constant cross sectional area and the grooves are neglected. It is also assumed that all conditions are perfect, there is no friction present, the surfaces are smooth and the loading is uniformly spread over the top surface. An example of the theoretical load calculations can be seen in Appendix 0. This theoretical load can be verified using the results from the finite element analysis and the experimental tests.

<table>
<thead>
<tr>
<th>Pressure (bar)</th>
<th>Applied Load (Experimental)</th>
<th>Equivalent Finite Element Load</th>
<th>Equivalent Estimated Mechanics of Material Load</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Force (kN)</td>
<td>Axial Strain (microstrain)</td>
<td>Hoop Strain (microstrain)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>175.8</td>
<td>-172.7</td>
<td>65.7</td>
</tr>
<tr>
<td>100</td>
<td>351.5</td>
<td>-345.5</td>
<td>148.5</td>
</tr>
<tr>
<td>150</td>
<td>527.3</td>
<td>-512.6</td>
<td>235.6</td>
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<td>200</td>
<td>703</td>
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<tr>
<td>100</td>
<td>351.5</td>
<td>-351.4</td>
<td>207.4</td>
</tr>
<tr>
<td>50</td>
<td>175.8</td>
<td>-173.1</td>
<td>123.1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>-5</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 6-34  Comparisons of experimental, analytical and theoretical loads
The comparisons of the theoretical, finite element and experimental results can be seen in Figure 6-34. The experimental and the finite element results are in close correlation, however, the theoretical results are different. This can be explained by the amount of assumptions introduced to simplify the theoretical calculations. From the results it is possible to say that with suitable calibration the load can be determined accurately using the laser strain gauge if perfect conditions are applied and the material inaccuracies are not taken into consideration.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Proposed solution</th>
<th>Assumptions</th>
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<tr>
<td>Material</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>$E$</td>
<td>Find sensitivity function mathematically</td>
<td>Homogeneous, variations in bulk properties only</td>
</tr>
<tr>
<td>Poisson's ratio</td>
<td>$\nu$</td>
<td>Find sensitivity function mathematically</td>
<td>Residual strains are removed by heat treatment or other</td>
</tr>
<tr>
<td>Residual strain</td>
<td></td>
<td>Future work</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Internal radius</td>
<td>$r_i(\theta)$</td>
<td>Find sensitivity function mathematically</td>
<td>Bulk geometry not affected by groove</td>
</tr>
<tr>
<td>External radius</td>
<td>$r_e(\theta)$</td>
<td>Find sensitivity function mathematically</td>
<td></td>
</tr>
<tr>
<td>Concentricity</td>
<td>$\Delta x_p, \Delta y_p$</td>
<td>Consider with load distribution $P(r, \theta)$</td>
<td>No variation in $z$-direction</td>
</tr>
<tr>
<td>Height</td>
<td>$h(\theta)$</td>
<td>Find sensitivity function mathematically</td>
<td>All walls are vertical in $z$</td>
</tr>
<tr>
<td>Flatness of faces</td>
<td></td>
<td>Consider with load distribution $P(r, \theta)$</td>
<td>Surfaces are smooth</td>
</tr>
<tr>
<td>Loading</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total Load</td>
<td>$P$</td>
<td>Parameter to be measured</td>
<td>No non-linear strain effects</td>
</tr>
<tr>
<td>Distribution</td>
<td>$P = P(r, \theta)$</td>
<td>Find sensitivity function mathematically</td>
<td>Strains vary linearly with load</td>
</tr>
<tr>
<td>Centre of loading</td>
<td>$x_p, y_p$</td>
<td>Consider with load distribution $P(r, \theta)$</td>
<td>Varies linearly with radius over upper surface, varies as Fourier series with angle</td>
</tr>
<tr>
<td>Grooves (1,2, or 3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selection of groove</td>
<td></td>
<td>N/A</td>
<td>Grooves are equally spaced and of same geometry including any variations</td>
</tr>
<tr>
<td>Centre Location</td>
<td>$x_G, y_G$</td>
<td>Reference point</td>
<td>Positioned on washer outer wall for orthotropic model</td>
</tr>
<tr>
<td>Depth (height)</td>
<td>$h_G$</td>
<td>Find sensitivity function numerically</td>
<td>Consider with trajectory</td>
</tr>
<tr>
<td>Trajectory</td>
<td>$\theta_G$</td>
<td>Find sensitivity function numerically</td>
<td>Measured angle is between face and outside washer wall</td>
</tr>
<tr>
<td>Radius</td>
<td>$r_G(\theta)$</td>
<td>N/A</td>
<td>Bulk geometry not affected by groove</td>
</tr>
<tr>
<td>Location of Grating</td>
<td>$x_L, y_L$</td>
<td>Find sensitivity function numerically</td>
<td>Not close to groove edges</td>
</tr>
<tr>
<td>Laser</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trajectory</td>
<td></td>
<td>LSG calibrated</td>
<td></td>
</tr>
<tr>
<td>Operating distance</td>
<td></td>
<td>LSG calibrated</td>
<td></td>
</tr>
</tbody>
</table>

Figure 6-35  Washer parameters
6.4.3 Further analysis of the washer - Sensitivity Analysis of the Washer

The importance of sensitivity analysis was introduced in Chapter 4. Similarly to the nut the washer was designed using finite element analysis. When finite element analysis is used perfect conditions and geometries are assumed which is not always the case in a real life scenario. In real life the product is manufactured to certain tolerances, and even though these tolerances are very tight some variation in the dimensions are permitted. Sensitivity analysis is used to investigate the relationship between the variation in dimensions and the desired surface strain reading. The sensitivity function is found with respect to each dimension so the overall relationship is found as the sum of all the sensitivity functions.

The washer was designed in such a way that by measuring the surface strain within the grooves the load on the joint can be determined. The strain and load relationship depends on the geometry of the washer. Any dimensional variation in the washer would introduce unwanted errors when calculating load. For this reason sensitivity analysis needs to be carried out for the washer to determine if it can be used to determine the load with 0.5% accuracy. If the errors introduced due to dimensional variation are significant calibration will be necessary.

The sensitivity analysis of the washer is carried out similarly to the nut. All the possible parameters that might vary are listed in Figure 6-35 and visualised in Appendix P. Once again a reference point is chosen which is kept constant so all the other parameters can be varied respect to it. This point is the same as it was for the nut, the groove centre location \((x'G, y'G)\). The total load \((P)\) is not varied either as this is the required parameter which needs to be measured.

The washer has a much simpler geometry than the nut but to investigate each parameter individually to find the sensitivity function would be time consuming so simplifications are made once again. The problem is split into two steps. The first step is to investigate the washer without any grooves and see how the surface strain at the groove centre location \((x'G, y'G)\) is affected with respect to dimensional variation. The
second step is to look at the effect of the groove changes on the surface strain within
the groove at the grating location \((x'_L, y'_L)\).

Similarly to the nut the surface strains in the axial \((\epsilon_1)\) and the hoop \((\epsilon_2)\) can be given
as a function of the following parameters.

\[
\begin{align*}
\epsilon_1 &= f(P, E, A) \quad \text{Equation 4-2} \\
\epsilon_2 &= f(P, E, A, \nu) \quad \text{Equation 4-3}
\end{align*}
\]

Due to the complicated geometry of the nut the sensitivity function was found by
investigating how the geometry changes affect the load distribution on the threads.
Therefore the sensitivity function found was relating the load distribution to the
measured strain. This assumption was a valid way to simplify the otherwise very
complicated problem. With the washer the geometry is less complicated therefore no
finite element analysis is required and the sensitivity functions can be found
numerically for each individual parameter. Assuming a uniform load on the washer
the surface strain can be determined by the following expressions:

\[
\begin{align*}
\epsilon_1 &= \frac{P}{E\pi(r_0^2 - \eta^2)} \quad \text{Equation 6-5} \\
\epsilon_2 &= \frac{P\nu}{E\pi(r_0^2 - \eta^2)} \quad \text{Equation 6-6}
\end{align*}
\]

Using these straightforward expressions the sensitivity functions for each can be
found in a similar manner to the example used in Chapter 4. The sensitivity equation
then becomes:

\[
\begin{align*}
\frac{\delta \epsilon_1}{\epsilon_1} &= \frac{\delta P}{P} + \frac{\delta E}{E} \frac{2\delta r_0}{r_0} + \frac{2\delta \eta}{\eta} \quad \text{Equation 6-7} \\
\frac{\delta \epsilon_2}{\epsilon_2} &= \frac{\delta \nu}{\nu} + \frac{\delta P}{P} + \frac{\delta E}{E} \frac{2\delta r_0}{r_0} + \frac{2\delta \eta}{\eta} \quad \text{Equation 6-8}
\end{align*}
\]
The sensitivity function for each of the parameters can be tabulated in Figure 6-36.

<table>
<thead>
<tr>
<th>Sensitivity Function</th>
<th>Axial Strain ($\varepsilon_1$)</th>
<th>Hoop Strain ($\varepsilon_2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$SF_p$</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$SF_E$</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>$SF_{ro}$</td>
<td>-2</td>
<td>-2</td>
</tr>
<tr>
<td>$SF_{rl}$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$SF_r$</td>
<td>NA</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 6-36  Sensitivity functions for washer geometry

These sensitivity functions were found assuming that the load is uniform over the washer. With this assumption two of the geometry parameters are not included when finding the sensitivity functions. The sensitivity functions for concentricity and flatness of the washer face will be investigated with respect to load distribution. The alteration of the load distribution on the washer can account for these changes in geometry. The sensitivity function with respect to the load distribution will be found later.

The second part of the sensitivity analysis investigates the effect of dimensional variation in the groove with respect to surface strain. Just as for the nut it is possible to say that the surface strain ($\varepsilon'$) at the grating location ($x'_L$, $y'_L$) is governed by the following function (assuming uniform loading):

$$\varepsilon_1' = f \{\varepsilon_1[h_G, \theta_G, (x'_L, y'_L)], \varepsilon_2[h_G, \theta_G, (x'_L, y'_L)]\}$$  \hspace{1cm} \text{Equation 6-9}

$$\varepsilon_2' = f \{\varepsilon_1[h_G, \theta_G, (x'_L, y'_L)], \varepsilon_2[h_G, \theta_G, (x'_L, y'_L)]\}$$  \hspace{1cm} \text{Equation 6-10}

The sensitivity functions governing the surface strain at the reference point ($\varepsilon_1$, $\varepsilon_2$) were shown earlier numerically. The groove geometry is much more complicated but
it is possible to find the sensitivity functions numerically using finite element analysis.

The geometries of the grooves on the washer are identical to the ones on the nut. Using this similarity it is possible to assume that the sensitivity functions found for the groove geometry when analysing the nut can be used for the washer as well. Instead of repeating the analyses on the washer grooves, the results from the nut grooves can be used. Using the same assumptions with regards to the groove depth and face angle the fundamental equation for the washer sensitivity will be as follows:

\[
\frac{\delta e_1}{e_1} = SF_p \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_{\eta} \frac{\delta \eta}{\eta} + SF_{v} \frac{\delta v}{v} + SF_{\theta h g} \frac{\delta \theta_g}{\theta_g} + SF(x_L) \frac{\delta(x'_L)}{(x'_L)} + SF(y_L) \frac{\delta(y'_L)}{(y'_L)}
\]

**Equation 6-11**

\[
\frac{\delta e_2}{e_2} = SF_p \frac{\delta P}{P} + SF_E \frac{\delta E}{E} + SF_{\eta} \frac{\delta \eta}{\eta} + SF_{v} \frac{\delta v}{v} + SF_{\theta h g} \frac{\delta \theta_g}{\theta_g} + SF(x_L) \frac{\delta(x'_L)}{(x'_L)} + SF(y_L) \frac{\delta(y'_L)}{(y'_L)}
\]

**Equation 6-12**

Under uniform loading the accuracy of the strain measuring device is governed by the equations above. Using the results from the groove analyses and the previous numerical results the sensitivity functions can be summarised (Figure 6-37).

<table>
<thead>
<tr>
<th>Sensitivity Function for Axial Strain</th>
<th>Sensitivity Function for Hoop Strain</th>
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</thead>
<tbody>
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<tr>
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<tr>
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<td>0.09</td>
</tr>
<tr>
<td>$SF_{\theta h 2}$</td>
<td>0.24</td>
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</tbody>
</table>

144
As it was mentioned before all these sensitivity functions were found assuming uniform loading. Variation in concentricity and a rough washer face can both cause non uniform loading. Also the load distribution can change during assembly even if exactly the same parts are reassembled. This is due to assembly tolerances and impurities getting into the joint. For these reasons it is important to carry out an analysis to investigate how different load distributions affect the measured strain.

The following section will show how finite element analysis was used to find the sensitivity function with respect to load distribution.
6.5 Variation Analysis of Non-uniform loading on the washer

The laser strain gauge was adopted so it can be used as a load measuring device in a threaded connection. A special washer is introduced to the joint with grooves, and by measuring the change in surface strain within the groove the load in the joint can be found. The relationship between the surface strain and the overall load has been found using experimental and finite element analyses; however there are a number of factors that can affect this relationship.

Just like with the nut manufacturing imperfections can cause inaccuracies when trying to relate the surface strain to the load. The grooves on the washer and the nut are identical and it is assumed that imperfections in the grooves will have the same affect on the washer as they did on the nut. So these factors will not be reinvestigated as the results would be identical. But there is one major difference between the washer and the nut. The force is transferred through the threads in the nut while the force is transferred through the ends of the washer. This means that there is a spread out force acting on the top surface of the washer which can vary due to a number of factors and can have a severe effect on the surface strain readings.

Using a simplified finite element model a number of different loading scenarios were analysed, and it was shown that even under the same overall load large errors are introduced when the load is not uniform. It was also shown that by taking a number of readings along the circumference of the washer, the mean strain will be the same as for a uniformly loaded washer. Hence by averaging the strain readings the errors introduced by the uneven loading can be cancelled out.

All of the load acting on the connection is transferred through the washer causing it to compress in the axial direction and expand in the radial direction (Figure 6-38). The surface strain in the groove is measured in both the axial and the hoop direction using the laser strain gauge. The strain readings are then used to calculate the load on the joint using data from FE and experimental analyses.
Using finite element analysis a correlation was found between the surface strain and the overall load, which would allow the load to be determined from any strain reading. However in the analytical FE model used to calculate total load from measured strain perfect conditions are assumed where the load is transferred through the washer is uniformly spread out over the cross sectional area.

In real life there are a number of factors that can interfere with the surface strain readings to give inaccurate results. In this section, the effect of non uniform loading on the washer will be investigated.

The load from the nut is transferred to the washer through the top face, and under perfect conditions it is assumed to be uniform. However dirt, impurities in the joint and imperfections in manufacture or the material can all cause uneven loading. With the use of finite element analysis a number of different loading scenarios were analysed to help understand the surface strain behaviour at uneven loads. As a number of analyses were carried out a simplified model was used where the grooves were replaced by an orthotropic material. The plain geometry with the orthotropic material ensured that the washer behaved the same way as it did with the grooves, but less time
and computational power was required. These analyses were used to look at the effect of different loadings on the surface strain at the reference point \((x'_G, y'_G)\).
6.5.1 Modelling for non-uniform loading

Geometry

A joint is only as strong as the weakest member, so the washer was designed in such a way that it has the same cross sectional area as the bolt. This meant that adding the extra part did not weaken the connection.

All the finite element analysis was carried out using LUSAS (www.lusas.com 2001). Unlike some finite element packages like Ansys where the modelling is done using Boolean operations, modelling in LUSAS is done with the use of nodes and lines. The nodes and the lines are then used to make volumes which are meshed. When modelling complicated curved geometries, like the grooves, the volumes become irregular and irregular meshing is required. This means large number of elements and longer analyses. To reduce the computational power required and the analysis time the model was simplified.

St Venant’s principle was stated in Chapter 5. In effect the principle says that if the points where the strain measurements are taken (the grooves) are far enough from where the uneven load is applied then the surface strain will be the same as if it would be under uniform loading. The tests carried out will show if Saints Venant’s principle can be used for this scenario.

In Chapter 4 when the effect of different load distributions were investigated on the nut a simplified model was used where the grooves replaced with an orthotropic material. The simplification was used again due to the large number of analyses that had to be carried out.
The original geometry of the washer (Figure 6-39) with the grooves was modelled using LUSAS and using the same method as before the material properties were found (Section 5.2.5). As the geometry of the grooves was the same as before the material properties were identical to the results found with the nut. The material properties can be seen in Figure 6-40.

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</tr>
<tr>
<td>Youngs modulus y (N/mm²)</td>
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<tr>
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<td>72542</td>
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</table>

The coordinate system used is “x” vertical, “y” horizontal and “z” depth.
Using the material properties shown in Figure 6-40 the washer could be modelled as a solid ring with two different materials. Once again the simplified model was compared to the original geometry to investigate the correlation in behaviour.

**Boundary Conditions**

The bottom surface of the washer was fixed in the vertical direction but it was allowed to move freely in the radial direction. It was assumed that the friction between the washer and the rest of the joint is negligible. In real life there is some friction present, but introducing friction in the finite element analysis would have meant that a non-linear contact analysis is required which requires a lot more computational power and time. The analyses are carried out as a comparison of different loading scenarios, so as long as the boundary conditions (i.e.: friction) are the same in each analysis, the required results will be achieved.

There are a number of loading methods that can be used from distributed loads to pressures. However, all these forces are uniform and the only way it is possible to achieve non uniform loading is by editing the force on certain nodes manually. This is time consuming and difficult to quantify. LUSAS finite element software is capable of...
using projected loads over a specified "search area". The search area can be any surface or surfaces within the model selected by the user.

A single point is specified in space above the washer. Using this point as a reference point a rectangular force field can be given using a Cartesian coordinate system allowing the magnitude of the force to be specified at each corner. By specifying the number of divisions within the force field and the magnitude at each corner a non uniform load distribution is created (Figure 6-42).

Figure 6-42 Non uniform loading visualised by definition in 3 and 2 dimensions

This force field is then projected onto the selected surface called the search area (Figure 6-43).

Figure 6-43 Non uniform loading visualised by effect on mesh in 3 and 2 dimensions
The advantage of this method is that the non uniform load can be projected to any surface, including the cylinder of the washer. As the search area is smaller than the force field, some of the force is lost and it is not taken into account. The overall force will be equal to the average magnitude multiplied by the search area. The average magnitude can be found from the four corners.

Analysis

The laser strain gauge is designed to measure the surface strain on the flat face of the groove. In the simplified model the grooves have been replaced by an orthotropic material that allows the washer to behave in a similar way as the original geometry however the surface strain within the groove can not be measured. It is assumed that the surface strain within the groove will be affected by different non uniform loading scenarios in the same manner as the outside surface of the washer. So if the surface strain of the washer is investigated under different non uniform loads, it will be a good representation of what happens inside the groove.

As there are three grooves, with each groove represented by an orthotropic material, it was decided that the surface strain is measured at the centre of the symbolised groove half way down the washer from its top. These three points are at 120 degrees apart and both the hoop and the axial strains are measured.

To allow a fast and an efficient way of finding the surface strain readings the elements are need to be orientated correctly. The elements are lined up so that the local element directions coincide with the hoop and the axial directions. This allows an easy way of recording the strains at the required nodes.

Loading variation

There are a number of factors that can influence the uniformity of the applied load. Imperfections in the material and the manufacture can result in an uneven surface. Impurities and dirt can get in between the washer and the nut surface introducing
points which are under higher loading. It would be impossible to model all the
different scenarios but by looking at a number of examples it is possible to reason out
the effects.

To investigate the effects of non uniform loading the projected load method was used.
By varying the magnitude of the force at each corner of the force field an uneven load
can be applied. The force field was varied by reducing the load on one side and
increased on the other (Figure 6-44). This means that the overall load is kept the same
but a constant change in the force is introduced from the left to the right side of the
washer.

Figure 6-44  Loading variations along the washer from LHS to RHS

The search area for the washer is the same as its cross sectional area which is 3044
mm². The load scenarios are given so that the average load is always one, which
means the total load will always be equal to 3044 N. At each analysis the total load
was checked by summing up the reaction forces on the fixed surface.

There are three test points where the three grooves are and they are each 120 degrees
apart. The load is given as a gradient between the left and the right hand side, but the
position of the test points can still vary as the washer can rotate. To be specific two
different orientations were tested.
In the first orientation the washer was orientated so one of the test points was always in the middle, so it was always experiencing the same force. For the second orientation the washer was turned by 30 degrees so one of the test points was always experiencing either the maximum or the minimum force. This was done so that the severest cases could be tested.

Figure 6-45  Top view of the washer (orientation 1)

The load scenarios for the first orientation are shown in Figure 6-45. The different loadings were analysed using LUSAS and the strain readings were recorded at each groove as shown on Figure 6-46. All together 21 scenarios were analysed but the last ten analyses were the same as the first ten reversed.
6.5.2 Results for non-uniform loading

When non-uniform load is applied to the washer, the strain distribution varies as expected. At the heavily loaded sides the strains are higher whereas at the lightly loaded sides the surface strain remains lower. An example of non-uniformly loaded finite element model is shown below.

Figure 6-46 Top view of the washer (orientation 2)

The second orientation is shown in Figure 6-46. For both orientations the strains were recorded at the three test points and tabulated. The results can be seen in the results section.

Figure 6-47 Contour plot of axial strain on washer (LHS load ratio W)

Figure 6-47 shows the strain distribution on the surface under an uneven load. The load ratio is 0.9 on the left side and 1.1 on the right side. From the contour plot it can be seen that the strains are much higher (blue is high) on the right loaded side, and also in the groove area. This is due to the orthotropic material having a different stiffness.

The surface strain was recorded at each groove in both the hoop and the axial direction. These results are shown in Figure 6-48 for orientation 1 and Figure 6-49 for orientation 2. The variation in individual grooves is significant, but the average strain on the three grooves remains constant. This means that regardless of the uniformity of
6.5.2 Results for non-uniform loading

When non-uniform load is applied onto the washer the strain distribution varies as expected. At the heavily loaded sides the surface strain is higher whereas at the lightly loaded sides the surface strain remains lower. An example of a non-uniformly loaded finite element model can be seen below.

Figure 6-47 Contour plot of axial strain on washer (LHS load ratio: 0.9)

Figure 6-47 shows the strain distribution on the surface under an uneven load. The load ratio is 0.9 on the left side and 1.1 on the right side. From the contour plot it can be seen that the strains are much higher (blue is high) on the right hand side, and also in the groove areas. This is due to the orthotropic material having a different stiffness.

The surface strain was recorded at each groove in both the hoop and the axial direction. These results are shown in Figure 6-48 for orientation 1 and Figure 6-49 for orientation 2. The variation in individual grooves is significant, but the average strain on the three grooves remains constant. This means that regardless of the uniformity of
the load the average of the three readings will only depend on the total loading. This will be discussed further in the next section.
<table>
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<th>RHS Loading ratio</th>
<th>Total Load</th>
<th>Test Point 1</th>
<th>Test Point 2</th>
<th>Test Point 3</th>
<th>Average Strain</th>
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Figure 6-45  Non-uniform loading results (orientation 1)
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<td>1.61161</td>
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</table>

Figure 6-49  Non-uniform loading results (orientation 2)
6.5.3 Discussion & Conclusions for non-uniform loading

Following the results the non uniform loading can be discussed with regards to two factors. The first one is the factor of the loading and the second is the position of the test point with regards to the direction of the non uniform load.

The first discussion looks at how the surface strain changes at a particular test point with regards to the load factor.

Figure 6-50  Axial strain results for non uniform loading (orientation 1)

Figure 6-51  Hoop strain results for non uniform loading (orientation 1)
The strain changes linearly with respect to the load factor however the gradient of this relationship depends on the position of the test point. If the test point is close to the left or right hand side of the washer it experiences more extreme loadings hence the relationship is at a steep gradient. Close to the middle where the washer experiences an average force factor of one, the gradient is close to zero. This leads onto the second discussion.

The second discussion is how the surface strain changes under the same load factor but at different positions throughout the washer. Figure 6-52 shows how the measurements were taken from the washer and Figure 6-53 shows the results.

![Washer outline](image)

**Figure 6-52** Washer outline

**Axial strain**

![Graph showing varying axial strain along the grooves](image)

**Figure 6-53** Varying axial strain along the grooves
Using the two different orientations it is possible to take measurements at 30 degrees apart. These angles can be converted to distances from the edge using trigonometry where the given load factor is applied. Recording these results and plotting them on a graph shows that surface strain changes linearly with respect to distance from the edge at a given load factor. Once again the gradient of the linear relationship depends on the severity of the load factor.

It is possible to find the sensitivity factor for each individual test point with respect to location and with respect to the load factor. This would mean a separate sensitivity factor for each location and for each load factor. Also if the test points are not treated individually but the average is found, then one sensitivity factor can be used for any type of loading. This however means that the load can only be determined if the strain is measured at each groove.

From the results figure it can be seen that both the axial and the hoop strains remain constant regardless of the load factor and position if the average is used. This means that the sensitivity factor for average strain is zero.

So the following conclusions can be drawn from the non uniform load analysis:

- Uneven loading does effect the individual surface strain readings around the groove area
- Even at uneven loadings the total load can be calculated by averaging the strain over the three readings
- The sensitivity factor for uneven loading is zero (SF_{LP} = 0)
- If the strain readings in the three grooves are different then uneven loading is present
- It was shown that the simplified model with orthotropic material is a fair way of modelling the otherwise complex shape
- Computational power and time is saved by using the simplified model in the analyses
• The results were conclusive so no more in depth analysis is required using complex or non-linear loads

• The surface strain readings were different at each groove when uneven loading was applied, hence St Venant's principle cannot be used as the test points are too close to the source of loading
6.6 Summary

It was shown in this chapter that the nut is an unsuitable solution due to the hysteresis present in the joint. By introducing an independent part the hysteresis effect can be removed. A modified washer was introduced to the joint which can be used to measure loading by measuring the surface strain in the grooves on the washer.

Using finite element analysis it was shown that the surface strain can be related to the load accurately if imperfections are not considered. Sensitivity analysis was then carried out to investigate how the accuracy of the washer as a load measuring device is affected by dimensional variation. Due to the relatively simple geometry of the washer it was possible to find some of the sensitivity functions numerically. As the groove geometry was unchanged from the nut the same sensitivity functions could be assumed for the washer. It was also found that the uniformity of the load does not affect the readings if the average surface strain is found on the grooves.

Using sensitivity analysis the effects of imperfections were investigated and the following results were found:

Axial Strain

\[
\frac{\delta \varepsilon_1'}{\varepsilon_1'} = \frac{\delta P}{P} - \frac{\delta E}{E} - 2 \frac{\delta r_0}{r_0} + 2 \frac{\delta \eta}{\eta} + \frac{\delta v}{v} - 1.39 \frac{\delta \theta_g}{\theta_g} - 0.26 \left( \frac{\delta (x'_L)}{x'_L} \right) - 1.24 \left( \frac{\delta (y'_L)}{y'_L} \right)
\]

\[\text{Equation 6-13}\]

Hoop Strain

\[
\frac{\delta \varepsilon_2'}{\varepsilon_2'} = \frac{\delta P}{P} - \frac{\delta E}{E} - 2 \frac{\delta r_0}{r_0} + 2 \frac{\delta \eta}{\eta} + \frac{\delta v}{v} + 0.24 \frac{\delta \theta_g}{\theta_g} + 0.02 \frac{\delta x'_L}{x'_L} + 0.09 \frac{\delta y'_L}{y'_L}
\]

\[\text{Equation 6-14}\]

The above equations show how the surface strain is affected if either of the parameters vary due to imperfections. Each parameter has its own sensitivity function. The amount the surface strain changes by is the product of the sensitivity function and
the change in the parameter. The change in the parameter is a ratio between the error and the magnitude of the parameter. Some parameters like grating location have a small sensitivity function but even a small change will have a big impact. This is because \( x'_{L} \) and \( y'_{L} \) are both very small. The change in Young's modulus for example has a less significant effect as the Young’s modulus is high. The summary of the sensitivity values can be seen in Figure 6-54.

<table>
<thead>
<tr>
<th>Sensitivity Function for Axial Strain</th>
<th>Sensitivity Function for Hoop Strain</th>
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</thead>
<tbody>
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<td>SF_{P2}</td>
</tr>
<tr>
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<td>1.00</td>
</tr>
<tr>
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<td>SF_{E2}</td>
</tr>
<tr>
<td>-1.00</td>
<td>-1.00</td>
</tr>
<tr>
<td>SF_{E1'}</td>
<td>SF_{E2'}</td>
</tr>
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<td>-2.00</td>
<td>-2.00</td>
</tr>
<tr>
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<td>SF_{y'22}</td>
</tr>
<tr>
<td>-1.24</td>
<td>0.09</td>
</tr>
<tr>
<td>SF_{\theta h1\theta}</td>
<td>SF_{\theta h2\theta}</td>
</tr>
<tr>
<td>-1.39</td>
<td>0.24</td>
</tr>
</tbody>
</table>

Figure 6-54  Summary of sensitivity functions for the washer

The required accuracy is 0.5%, so the change in strain can not exceed this number. To achieve this accuracy the washer would have to be manufactured to very tight tolerances and it would not be cost effective. The required tolerances to achieve the required accuracy are listed in Figure 6-55.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Maximum variation to maintain 0.5% accuracy</th>
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<td></td>
<td></td>
<td>Axial</td>
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<tr>
<td>Young’s modulus</td>
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<tr>
<td>Outer radius</td>
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<td>%</td>
<td>0.25</td>
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<tr>
<td>Inner radius</td>
<td>r_i</td>
<td>%</td>
<td>0.25</td>
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<tr>
<td>Poisson’s ratio</td>
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<td>%</td>
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<tr>
<td>Groove location y</td>
<td>( y'_{G} )</td>
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</table>

Figure 6-55  Maximum dimensional variations

The results suggest that the hoop strain measurements are a more accurate way of relating the surface strain to the load than the axial strain. From the analyses shown previously it is possible to conclude that the washer is a suitable way of measuring load although individual calibration is required.
Chapter 7 – Effects of Manufacturing Imperfections on Thread Strength

7.1 Introduction

A component or a structure is deemed to have failed when it is no longer capable of fulfilling its original design function. The residual life is defined to be the remaining period when the component can be considered to be fully functional. The purpose of a threaded connection is to fasten two or more components together securely. When the threaded connection can no longer fulfil this purpose it has failed or exceeded its residual life.

There are two main ways a threaded connection can fail, these are thread loosening and thread rupture. Failure can also take place due to corrosion, erosion and other time and environment related factors. Thread loosening can occur without fracture taking place, but it is still regarded as failure as it can no longer perform its specified function. Thread loosening is the main reason for needing to monitor the load in a threaded connection, hence the development of the laser strain gauge load measuring device. Thread loosening is affected by 4 major factors (loading, friction, thread angle and pre-load) and by finding the optimum of each factor, loosening can be reduced or even eliminated.

Thread rupture occurs when fracture and/or crack growth (either partial or complete) takes place in the threaded connection and the component is no longer capable of performing its function. There are two ways failure can take place due to loading. Fracture can occur when the maximum stress in any one stress cycle exceeds the fracture stress of the material. In other words when the load exceeds the tensile strength of the material the component will fracture. A component can also experience fracture without exceeding the tensile strength of the material. Repeated or cyclic stressing (loading) can cause fatigue failure. Failure in fatigue is the result of processes of crack nucleation and growth bought about by the application of cyclic stresses (Vernon 1992). Most steels have a definite fatigue limit or endurance strength which is usually about one-half of the value of the tensile strength, although many
non-ferrous materials have no fatigue limit. Thread failure will depend on the tensile strength and the fatigue limit of the material.

The objective of this section is to investigate the residual life of a threaded connection with respect to manufacturing imperfections. Manufacturing imperfections will cause changes in the geometry of the connection which can affect the residual life of the joint resulting from disadvantageous changes to the local stresses. The residual life of any component depends on a number of factors. These include loading magnitude, loading type, temperature and usage. These can be affected by manufacturing imperfections where the geometry is altered.

The strength of a threaded joint depends on the strength of the material and the local stress state. The most likely place where a threaded connection might fracture is at the first thread. This is due to \(1/3\)rd of the overall force being transferred through the first thread. The highest tensile stress occurs by the base of the thread which is the critical area. Using the tensile and the fatigue properties of the materials within the connection the change in residual life of the connection can be determined with respect to manufacturing imperfections.

Some of the factors that effect thread loosening will be studied and possible solutions to reduce thread loosening will be suggested. To help predict thread failures finite element modelling will be used to find high stress concentration areas. By modifying the finite element model different scenarios will be studied to help predict how the residual life of the threaded connection is affected by variation in the geometry due to manufacturing imperfections. The variation in geometry can have a direct and an indirect effect on the maximum stress concentration points. If the thread is slightly shorter or off angle, then the stress concentration will change. Also these changes in the thread can have an effect on the load distribution which again will cause the stress distribution to vary. Some extreme cases will be presented and analysed to represent these changes and to help predict how the residual life of a threaded component is affected.
7.2 Thread Loosening

There are a number of different joining methods available including soldering, welding or riveting. The disadvantage of these methods are that once implemented they are not easily removed. This is why threaded connections are so popular, as they are easily disassembled and reassembled again. This quality is however can be a disadvantage as the connection could become loose when not intended. This is called thread loosening, and there are a number of factors that can affect it. Due to thread loosening it is important to monitor the load on a threaded connection and that is why the laser strain gauge is so important.

Thread loosening takes place in a number of steps. Shock loading and vibration reduces friction between the threads and permits rotation. The rotation causes a reduction in pre-load which can result in thread loosening. This procedure depends on a number of factors (Daabin and Chow 1991).

![Figure 7-1 Thread loosening](image)

Figure 7-1 shows how the load and the friction behave on a single thread. Friction is a very significant factor in terms of thread loosening. Frictional forces oppose the direction of the thread slipping hence high frictional forces help reduce thread loosening. Friction is governed by three main factors as the expression above shows. These are thread angle, coefficient of friction and the magnitude of load.
The normal component of the applied force on the thread depends on the thread angle $\beta$. As the thread angle is reduced the normal force becomes larger due to $\cos \beta$ increasing. Higher normal forces mean higher frictional forces reducing the chances of thread loosening. In general a coarse thread has a larger thread angle so to avoid thread loosening fine threads are recommended. The coefficient of friction ($\mu$) depends on the surface of the thread. Rough thread surface means high coefficient of friction which also increases the frictional forces reducing the chances of thread slipping. Thread loosening can also be reduced by increasing the pre-load on the joint. By increasing the pre-load the normal forces between the thread surfaces are increased.

All these factors mentioned above help reduce thread loosening by increasing the frictional forces between the threads directly. The type of loading can also have a significant affect on the loosening of the threads. The loosening rate is inversely proportional to the duration of the applied load. The shorter the duration the higher the loosening rate, hence under shock loading the connection is more likely to come loose. Any types of shock loading or vibration allow the threads to slip past each other increasing the possibility of thread loosening.

Combinations of these factors have to be taken into consideration when designing a threaded joint. In order to help avoid thread loosening constant loading, high pre-load, high friction and fine thread profile is recommended.
7.3 Fatigue Life

In the previous section thread loosening was described as one of the possible ways a threaded connection can fail, so it is no longer capable of performing its specified function. Thread loosening takes place due to the threads slipping and it does not involve actual fracture or breakage of the component. When a threaded connection fails due to fracture, it is called thread failure.

As with all engineering parts a threaded connection has a structural integrity and if that limit is exceeded fracture becomes inevitable. There are two ways how the connection can fracture due to external loading. Each material has a tensile strength that can be determined using a short-term static test. A short-term static test is a single test to measure the maximum stress a material can withstand. The maximum stress a material can take is the tensile strength of that material. A material can also experience fracture without ever exceeding the tensile strength of the material due to cyclic loading. This type of failure is termed as fatigue failure.

Failure by fatigue is the result of processes of crack nucleation and growth. Crack propagation takes place in two distinctive steps. In the first step the crack propagates very slowly along the crystallographic planes of high shear stress. In the second step the crack growth rate increases and changes direction, moving perpendicular to the applied stress. Figure 7-2 shows how an example of crack propagation.

![Figure 7-2 Optical micrograph showing an example of crack propagation](www.materials.unsw.edu.au 2005)
There have been many attempts to describe the crack growth rate by applying crack growth laws. The two most common crack growth laws are given in the following equations (Ewalds and Wanhill 1984).

The Paris equation:

$$\frac{da}{dn} = C_p (\Delta K)^m$$  \hspace{1cm} \textit{Equation 7-1}

The Forman equation:

$$\frac{da}{dn} = \frac{C_F (\Delta K)^m}{(1-R)K_c - \Delta K}$$  \hspace{1cm} \textit{Equation 7-2}

Where $\frac{da}{dn}$ is the fatigue crack growth rate, $\Delta K$ is the stress intensity factor and $R$ is the stress ratio. The significance of these equations is limited, but they can be used to provide a first estimate in crack growth behaviour.

There are two types of fatigue failure, low-cycle fatigue and high-cycle fatigue. During low-cycle fatigue the tensile strength is not exceeded by the maximum stress at any cycle but it does exceed the yield stress. The number of stress cycles to failure is low, usually less than 1000. The yield stress is not exceeded during high-cycle fatigue and therefore large number of cycles can be achieved in the region of $10^5$-$10^6$ repetitions.

When a material is tested for fatigue characteristics, the stress conditions involve the application of an alternating stress cycle with a mean stress of zero (BS3518-3:1963). The results are plotted on an S-N curve (Figure 7-3), where $S$ is the maximum stress in a single cycle and is the number of stress cycles to failure.
When analysing different metals with the help of an S-N curve it becomes clear that the test material can behave either one of two ways as the figure above shows. Ferrous metals show an S-N curve with a definite fatigue limit which is sometimes called endurance strength. Below a certain maximum stress the number of cycles becomes irrelevant as below fatigue limit the metal should never break. The fatigue limit is usually about one-half of the value of the tensile strength as measured using a short-term static test. Non-ferrous materials like aluminium for example, show a different S-N curve. There is no definite fatigue limit for these materials so it is only possible to design for a limited life usually about $10^6$ cycles.

There are different types of S-N curves used for different applications. Instead of plotting the number of cycles with respect to the maximum stress, the range of the alternating stress can be used. In this case the average stress is kept constant by adjusting the minimum and the maximum stress. The experiment can be repeated with different average stresses to get a wider range of results for a specific material. The results show that for each value of mean stress there is a different value of limiting range of stress ($\sigma_{\text{max}} - \sigma_{\text{min}}$), which can be withstood without failure.

The Goodman diagram is one of the methods used to help show the dependence of limiting range of stress on mean stress (Goodman 1926). The Goodman diagram can
be constructed from fatigue data from an S-N curve showing the relationship between number of cycles to failure and alternating stress. An example of a Goodman diagram is shown in Figure 7-4.

![Goodman diagram](image)

Figure 7-4  Goodman diagram (Dieter 1981)

The diagram shows that as the mean stress ($\sigma_m$) becomes more tensile the allowable range of stress ($\sigma_r$) is reduced, until at the tensile strength ($\sigma_u$) the stress becomes zero. For practical purposes the testing is usually stopped when the yield stress ($\sigma_y$) is exceeded. The alternating stress ($\sigma_a$) is the difference between the mean and the maximum ($\sigma_{max}$) or the minimum ($\sigma_{min}$) stresses. In reality the test data shows that these lines are curves, but a conservative approximation of the Goodman diagram allows straight lines to be used. The Goodman diagram is a useful visual tool that shows the maximum alternating stress that can be applied to a material. The diagram can be modified to show the fatigue strength at any given number of cycles as well.
There is an alternative way of presenting the mean-stress data which is sometimes referred to as the Haig-Soderberg diagram. In these diagrams the alternating stress is plotted against the mean stress. Following Goodman’s suggestion, this relationship is linear but Gerber proposed a parabolic curve to show the relationship (Dieter 1981). The test results show a closer correlation for ductile metals to Gerber’s curve. However due to the scatter of the results and the fact that notched specimen results fall closer to the Goodman line; the straight line is preferred in engineering design. The relationship can be shown with the following formula (Dieter 1981):

\[ \sigma_a = \sigma_e \left[ 1 - \left( \frac{\sigma_m}{\sigma_u} \right)^x \right] \]

\[ \text{Equation 7-3} \]

where \( x = 1 \) for the Goodman line, \( x = 2 \) for the Gerber parabola and \( \sigma_e \) is the fatigue limit.

![Alternative Goodman diagram “Haig-Soderberg diagram”](Dieter 1981)
The dashed Soderberg line is used when the design is based on the yield strength instead of the tensile strength. In this case the yield strength ($\sigma_y$) is substituted for the tensile strength ($\sigma_t$) in Equation 7-3.

Obtaining fatigue data for specific materials is not straightforward as fatigue testing is time consuming. Fatigue testing is usually carried out for simple geometries which give an idea of the behaviour of that specific material but in most cases the fatigue life will depend on the geometry of the specimen and the type of loading. When testing is carried out on a simple geometry a notch is usually added to weaken the structure. When the specimen is loaded it is most likely to break at the notch so the experiment can be repeatable. Without the notch the specimen would fail at different locations where there is a high concentration of cracks. Microscopic cracks exist in all materials and sometimes bigger cracks can weaken the structure. The more complicated the geometry, the most likely for cracks to be present which can introduce errors when testing for fatigue strength.

The material used for the threaded connection prototype is EN24 stainless steel. After extensive research it was proved to be very difficult to obtain fatigue data for this material. With the help of a private communication from Corus it was possible to obtain some Japanese data (CORUS) on 0.4% C-Ni-Cr-Mo (SNCM439) steels with compositions not far away from EN24. The test data can be seen in Appendix Q.

The data shown in Appendix Q is the raw test data for a number of tests. The results need to be averaged so the S-N curve can be determined. The test was carried out for a number of specimens with different heat treatments and experiencing different types of loadings. Using the test data for SNCM439 steel tempered at 630 °C under rotating bending the S-N curve was found.
Further study of the test data revealed that the results are incomplete. As it was mentioned before these tests were carried out on a simple geometry with a "v" notch. Even though the threaded connection's geometry is more complicated it would be possible to use the material properties obtained from "v" notch specimen. However the tests were carried out under a mean stress ($\sigma_m$) of 0 N/mm$^2$, and a stress ratio ($\sigma_{\min}/\sigma_{\max}$) of -1. When a threaded connection is under loading it experiences a different stress conditions.

During use a threaded connection is tightened to the required stress and due to vibration on the joint the stress alters slightly. This change in stress is never reversed so the stress ratio cannot be negative and due to the connection being tightened the mean stress is relatively high. Under these conditions the material has a different fatigue life characteristics to the data obtained at mean stress = 0 N/mm$^2$. For this reason it is not possible to use the results obtained above.

Obtaining fatigue data for steel at a high mean stress proved to be very difficult so it was decided to look at previous works on fatigue life. Patterson carried out comparative study of methods to estimate bolt fatigue limit and it in his work fatigue...
data was published involving threaded fasteners (Patterson 1990). He compared theoretical and experimental methods to look at the fatigue behaviour of an ISO M12 thread with pitch of 1.75 mm.

Patterson used bolts of grade 8.8 steel (BS3692) made out of EN43 carbon steel (080M50) oil quenched at 840 °C and tempered at 600 °C. He carried out the testing for different lengths to see how the fatigue behaviour of a threaded connection is affected by length. The British Standard recommended nut length for an M12 connection is 10 mm, therefore the length tested were 4.8 mm, 7 mm, 10 mm and 12 mm.

Before the tests were carried out the components were lubricated with oil and were then placed under cyclic tensile loading of frequency 20 Hz. As it was mentioned before a threaded connection when tightened experiences a high mean load which increases under vibration and usage. To account for this a mean stress of 238 N/mm² was applied to the connection whilst the maximum stress amplitude was 200 N/mm². It was assumed that if the specimen survived more than 5 * 10⁶ cycles it has infinite life.

![Fatigue curves for the M12 connection for different lengths](image)

Figure 7-7 Fatigue curves for the M12 connection for different lengths ($\sigma_m = 238 \text{ N/mm}^2$) - Logarithmic scale on 'x' axis
By using a logarithmic scale on both axes the fatigue data can be visualised as a series of straight lines.

![Fatigue curves for the M12 connection for different lengths](image)

Figure 7-8 Fatigue curves for the M12 connection for different lengths ($\sigma_m = 238 \text{ N/mm}^2$) – Logarithmic scale on both axes

This fatigue data will be used to help predict how the residual life of a threaded connection is affected due to manufacturing imperfections.
7.4 Thread Failure

To predict the fatigue life of a thread involves a number of difficulties, complications and uncertainties. It is possible with the use of modern techniques of fracture mechanics to calculate the bolt life to crack initiation (Glinka, Dover et al. 1986) but to calculate the life to failure of a threaded connection from crack propagation remains unsolved. As thread failure cannot be predicted using theoretical methods alone it is necessary to resort to different methods like finite element analysis.

In a nut and bolt connection most of the total load is experienced by the first thread as it was shown in the Spring Model section. What this means is that the highest load concentration will occur on the first threads. When designing a threaded connection the first thread is the critical factor, so this has to be taken into consideration.

The stress concentration in a threaded connection was shown previously using 2D axisymmetric finite element models. In these models it was clear that the highest stress concentrations are around the edges of the threads. All the force exerted on the threaded connection is transferred through the threads and because the threads are in effect acting like short wide beams they experience large stress concentrations.

The load distribution along the threads is in such a way that the load on the first thread is a $\frac{1}{3}$rd of the total load. Due to this the first thread experiences much larger forces and cause of this it is the most likely place for a threaded connection to fail. When looking at the stress concentration in a threaded connection the first engaged thread is investigated as it is under the most stress. If the first thread is strong enough, the rest of the connection is predicted to hold as well.
Figure 7-9  2-D Axisymmetric model of thread (stress contour)

Figure 7-9 shows a finite element picture of the force being transferred between two threads on a nut and bolt connection. The maximum stress contour is plotted to help identify the highest stress areas (units: N/mm²). The threads were modelled using a 2-D axisymmetric mesh with contact surfaces where the two threads come into contact. It can be clearly seen that the area with the highest stress concentrations is near the thread base. To understand the stress distribution on the thread more accurately Figure 7-10 shows the close up of a slightly modified model.

Figure 7-10 2-D Axisymmetric model of thread close-up (stress contour)
Instead of using contact elements and non-linear analysis the previous model was modified by replacing the nut thread with a concentrated force acting on the middle of the bolt thread shown by the arrow on Figure 7-10. As two threads come into contact the adjacent surfaces bend away from each other in such a way that the point of contact is only at the mid point (Sopwith 1948). Comparing the two models it can be seen that there is a slight change in the maximum stress (about 7 %) when the second thread is replaced by a concentrated force. For this report the behaviour of the stress concentration is studied and not the exact magnitude of it. Therefore it is suitable to use Sopwith's assumption.

The loading for the models above was calculated using Marshek's spring model (described in Chapter 5). Marshek's spring model is a numerical way of calculating the load on each thread in a threaded connection (Miller, Marshek et al. 1983). The relationship between the loads on the threads is governed by the spring constants. The value for the spring constants depends on the geometry of each individual part. For the model above it was assumed that the geometry is perfect all the way through the threaded connection, hence the spring constants will have the same value through out the joint.

The model was based on a 1 inch 12UNF nut and bolt with 10 engaged threads with only the first thread modelled. If a total load of 1000 N is applied to the joint, then 213 N will act on the first thread alone under the conditions specified above. Under this load the maximum stress is 10.17 N/mm² and it is located at the base of the loaded side of the thread. This high stress concentration area is due to the thread bending upwards. The first thread is under the most load in a threaded connection (Wang and Marshek 1995), therefore the maximum stress point on the first thread will be the highest stress concentration within the whole connection.

The thread is most likely to fail at the areas with the highest stress concentrations. It was shown that this point is at the base of the loaded side of the thread. The maximum strength of the connection will depend on the first thread. The stress concentration on the thread shown previously is for one type of connection only where the geometry is assumed to be perfect. In real life this is rarely the case as all engineering parts are
manufactured to a certain tolerance so all connections will be slightly different introducing a variation into the maximum stress areas.

Using a number of case studies, threaded connections with different sizes and profiles will be analysed. With the use of finite element analysis manufacturing imperfections will be introduced and the behaviour of the high stress concentration areas will be studied in greater depth and the affects of it will be analysed in terms of strength. These case studies will help understand how different types of connections behave due to manufacturing imperfections.

Further to the case studies an in depth analysis will be carried out on the M12 pitch 1.75 mm connection. Using the fatigue data from Patterson's work the M12 connection will be studied in terms of how the working life of a joint may be affected due to manufacturing imperfections.
7.5 Case study analyses

During engineering design it is vital to identify the critical area where the highest stress concentrations occur. In terms of a threaded connection this area is at the base of the first engaged thread as it was shown previously. The magnitude of the stress depends on a number of factors including size, profile and loading which are known but also there are factors that are not and have to be accounted for. These are dimensional variations due to manufacturing imperfections and tolerances.

In order to get a better understanding on how these dimensional and geometrical variation affect the high stress areas a number of analyses (case studies) were suggested.

In previous chapters the accuracy of the laser strain gauge as a load measuring device was analysed with respect to dimensional variation. The variation in geometry was found using tolerance tables from British Standards and the same method will be used to study the effects on residual life. The dimensions of the threads can have a direct and an indirect affect on the stress concentrations and both of these have to be studied closer.

The varying dimensions have a direct effect on the geometry of the threads. Depending on the accuracy of the manufacture, the same thread can vary from being shorter and thicker to longer and thinner. Using data from British Standards both extreme geometries will be modelled and compared to the optimum geometry.

Changing the geometry of the threads will have an indirect effect on the stress concentrations as well. It was shown in Chapter 5 how the load distribution on a threaded connection depends on the stiffness of each thread slice called spring constants. The spring constants can be calculated numerically for the nut and the bolt and analytically for the thread. By changing the geometry the value of the spring constants may vary changing the load distribution. The change in load distribution can add to the amount of force acting on the first thread increasing the stress in the critical area.
The stress concentration areas are also influenced by the profile of the threads. There are two common types of profile used in engineering. For empirical units the Whitworth thread is used which has a thread angle of 55° and both the tip and the base are curved.

![Whitworth thread profile](image)

Figure 7-11  Whitworth thread profile (BS82:1956)

For metric units a slightly different thread is used where the thread angle is 60° and the edges are not rounded.

![ISO metric thread profile](image)

Figure 7-12  ISO metric thread profile (BS 3643-1:1981)
For both types of thread profile the threads on the nut and on the bolt are identical. The thread on the bolt (shaft) is referred to as the internal thread and the thread on the nut (body) is referred to as the external thread. Threaded connections are defined in terms of bolt thickness and pitch. The bolt thickness is in effect the size of the bolt (nominal diameter) and the pitch defines the size of the thread for that thickness. Usually there are at least two pitch sizes available for each diameter going from fine thread to coarse thread.

The case studies will look at how dimensional variation affects the two different thread profiles. The same analyses will be carried for fine and coarse threads to see how each pitch reacts to manufacturing imperfections. The tests will be repeated for different nominal diameters as well.

Up this point the material of the threaded connections was assumed to be EN24 stainless steel for both the nut and the bolt, which is a common type of steel used in thread manufacture. In some engineering applications the nut and the bolt are made from different materials, for example a steel bolt joined with an aluminium cast engine block (acting as a nut). Having dissimilar materials affect the load distributions which has an influential affect on the maximum stress.

All case studies will be carried out using finite element analysis. Using the method described in Chapter 5, the spring constants for all the geometries will be found using mathematical and analytical methods. The spring constant are used to find the load distribution on each connection. A summary of all spring constant values can be found in Appendix O.

The models will be drawn using the dimensions obtained from the British Standards. Assuming that bolt and the nut are subjected to the same force and they have similar geometries only the bolt thread will be modelled. Like in Figure 7-7 the force will be modelled as a concentrated force acting on the middle of the thread so linear analyses can be used. The top surface of the thread will be fixed in the vertical direction and the bottom surface will be free, but restrained to move together. These boundary conditions are there to try and simulate the connection as realistically as possible.
7.5.1 Case Study I - Stress concentration study on different profiles and pitches

The first case study will look at how changing the dimensions of the thread according to the tolerance tables from British Standards influence the stress concentration areas and the maximum stress. The analysis will be carried out for both the Whitworth thread profile and the ISO metric thread profile. For each profile two different pitches will be looked at.

The connections chosen for this case study are the 1 inch 12 UNF, the 1 inch 8 UNC, the M22 pitch 1 mm and the M22 pitch 2 mm. The first number always refers to the nominal diameter (1 inch, 22 mm) and the second number refers to the pitch size. For empirical units the number means the amount of threads per inch, hence 12 UNF means 12 threads per inch (pitch = 1/12 inch). UNF and UNC indicate whether it is a fine or a coarse thread. These connections were chosen as they are all similar in nominal diameter so the strength of the Whitworth and the metric thread could be compared.

Each connection is put under the same overall force of 1000 N, and using the spring constants and the spring model the equivalent loading for the first threads will be calculated individually. Using the tolerance tables the two extreme geometries for each thread will be modelled and analysed. One extreme is when the thread at its biggest, this will be referred to as the upper tolerance limit, and the other extreme is when the thread is the shortest; this will be referred to as the lower tolerance limit.

After each analysis the maximum stress will be recorded and compared with the other results to see how the change in geometry affects the stresses. The location of the maximum stress area will also be identified. The tests will be repeated for different profiles and pitches to compare how different shape threads behave.

The first thread to be analysed is the 1 inch 12 UNF which was shown as an example previously in Figure 7-10. The calculated load on the first thread for this joint is 213 N, and the maximum stress on the “perfect” thread was 10.17 N/mm² under this load.
The second connection tested is the 1 inch 8 UNC. This connection has the same nominal diameter as the previous one, but the thread is coarser. Therefore there are fewer number of engaged threads within the joint so the load will be slightly higher on the first thread. The load on the first thread was worked out to be 293 N on the first thread.

For the M22 threads the force was worked out to be 127 N for the 1 mm pitch and 245 N for the 2 mm pitch. The results are shown in Figure 7-15 and 7-16.
Table of Maximum Stress and Location of $\sigma_{max}$

<table>
<thead>
<tr>
<th></th>
<th>Maximum Stress ($\sigma_{max}$)</th>
<th>Location of $\sigma_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Tolerance Limit</td>
<td>12.75 N/mm$^2$</td>
<td>Node 460</td>
</tr>
<tr>
<td>Lower Tolerance Limit</td>
<td>17.20 N/mm$^2$</td>
<td>Node 460</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>25.9 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-15  Maximum stress variation for a M22 thread with pitch 1 mm

Table of Maximum Stress and Location of $\sigma_{max}$

<table>
<thead>
<tr>
<th></th>
<th>Maximum Stress ($\sigma_{max}$)</th>
<th>Location of $\sigma_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper Tolerance Limit</td>
<td>12.93 N/mm$^2$</td>
<td>Node 460</td>
</tr>
<tr>
<td>Lower Tolerance Limit</td>
<td>16.78 N/mm$^2$</td>
<td>Node 460</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>22.9 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-16  Maximum stress variation for a M22 thread with pitch 2 mm

There is quite a significant amount of variation in the maximum stress as the results show. The amount of variation depends on the thread profile and the pitch. For the Whitworth thread the variation is smaller than for the metric profiles, and it is further reduced when the pitch is increased. As the tolerance changes between the upper and the lower limit the location of the maximum stress point is different. The upper tolerance limit allows the thread to be longer so when the force is applied there are larger bending forces acting at the base of the thread. When the lower tolerance limit
is used the thread is much shorter so the stress distribution is different and the maximum stress point is near to where the load is applied.

The variation significantly increases for the metric threads, but for these the location of the maximum stress remains the same. Unlike the Whitworth thread the metric threads are not rounded but sharp corners are used. These sharp corners can account for the larger stress concentrations, as well as for the significant variation.

For both types of thread the variation is reduced when the pitch is increased. Smaller pitches are more sensitive to manufacturing imperfections and dimensional variation, as a small change can have a significant affect on the small pitch. For larger pitches more significant changes in dimensions are required to reach the same affect.

Regardless of the pitch, the strength of joint is not affected as the results show. For each threaded connection analysed the overall force was kept constant at 1000 N. For the different pitches the spring constants are altered as well as the number of engaged threads. This means the load distribution is changed. Even though the smaller pitch thread is weaker, due to more threads being engaged in the joint the force applied to the first thread is significantly smaller as well. Overall the maximum stress concentration does not change significantly with the pitch if the nominal diameter is kept the same.
7.5.2 Case Study II – Stress concentration study on different sizes

The second case study is carried out in a similar manner to the first one but this time connections with different nominal diameters will be analysed. Using the British Standards tolerance tables the two extreme geometries will be modelled and compared.

The geometries studied are ¼ inch 28 UNF, 1 Inch 12 UNF, 2.5 inch 8 UNF and 5 inch 6 UN. The 1 inch 12 UNF thread was analysed in case study one so only the 3 new nominal diameters will be modelled here.

The ¼ inch 28 UNF thread is the first one to be analysed. The overall load is kept at 1000 N and the load on the first thread is calculated using the spring constants and spring model. The first thread load for the ¼ inch connection is 319 N. This load is much larger than the previous loads, but this is due to the small size of the connection.

<table>
<thead>
<tr>
<th>Upper Tolerance Limit</th>
<th>147.7 N/mm²</th>
<th>Node 246</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lower Tolerance Limit</td>
<td>185.5 N/mm²</td>
<td>Node 500</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>20.4 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-17 Maximum stress variation for a ¼ inch 28 UNF thread

The second thread analysed would be the 1 inch 12 UNF thread which is shown in Figure 7-13. The variation for the maximum stress was 14.3 % for this thread under a load of 213 N on the first thread. For the 2.5 inch 8 UNF thread the load on the first thread was worked out to be 134 N.
Maximum Stress ($\sigma_{\text{max}}$) | Location of $\sigma_{\text{max}}$  
---|---  
Upper Tolerance Limit | 1.808 N/mm$^2$ | Node 246  
Lower Tolerance Limit | 1.802 N/mm$^2$ | Node 606  
Variation in Maximum Stress | 0.33 % |

**Figure 7-18** Maximum stress variation for a 2.5 inch 8 UNF thread

The final geometry to be analysed as part of the second case study is the 5 inch 6 UN thread. For large nominal diameters like 5 inch, there is no fine or coarse thread, so the connection is referred to as UN and not UNF or UNC. The load on the first thread for the 5 inch nominal diameter was worked out to be 89.7 N.

| Maximum Stress ($\sigma_{\text{max}}$) | Location of $\sigma_{\text{max}}$  
---|---  
Upper Tolerance Limit | 0.381 N/mm$^2$ | Node 246  
Lower Tolerance Limit | 0.444 N/mm$^2$ | Node 606  
Variation in Maximum Stress | 14.1 % |

**Figure 7-19** Maximum stress variation for a 5 inch 6 UN thread

In the first case study the results showed that for a Whitworth thread the location of the maximum stress point is different for the upper and the lower tolerance limit. As the results show in the second case study this behaviour is true for all dimensions of Whitworth threads.
The results of the 2.5 inch 8 UNF thread show a variation of only 0.33%. The other results suggest that this can not be right so there must have been an error introduced into the analysis. The other results show that as the nominal diameter increases the variation of the maximum stress is reduced up to a certain level. The variation for the 1 inch nominal diameter and the 5 inch nominal diameter were both around 14% so it can be assumed that variation can not be reduced further even with larger diameters.

For small nominal diameters like the 1/4 inch connection the smallest of changes in the dimensions can have an influential affect on the maximum stress. As the diameter gets bigger the thread is less sensitive to dimensional variation so the variation in maximum stress is reduced.

As expected the magnitude of the maximum stress is reduced when the size is increased due to larger connection being stronger.
7.5.3 Case Study III - Stress concentration study on varying load distributions and dissimilar materials

In the third case study the affect of different load distributions will be looked at. Even though the overall load on the joint is kept at 1000 N, the load distribution on the threads may change if the stiffness of the bolt or the nut is varied due to manufacturing imperfections. The threaded connection is treated as three separate parts. These are the bolt, the nut and the thread. Each of these components has a maximum and a minimum stiffness according to its geometry given by the tolerance tables. The two extreme cases were found where the loading on the first thread is at its maximum and at its minimum.

From British Standards the tolerances for the geometry of a threaded connection can be found. From the geometry the spring stiffness of the nut and bolt can be found numerically using the dimensions and the material properties. For a 1 inch 12 UNF connection the variation in the bolt stiffness (KSC) is 3.3 % and for the nut (KBC) it is 2.7 %. The amount of variation between the spring constants change depending on the nominal diameter of the bolt as all dimensions have different tolerances. In general the variation in the bolt and the nut spring constants for a threaded connection is below 5-6 %. Finding the spring stiffness off the thread (KT) is very difficult numerically so finite element analysis was used. The two extreme geometries were modelled in LUSAS finite element package and the two stiffness were found. The variation in KT was 33.6 % for the 1 inch connection.

The maximum loading on the first thread occurs when KBD and Ksc are at their minimum value and KT is at its maximum. Under these conditions at a total load of 1000 N, the load on the first thread would be 240 N. The minimum loading on the first thread occurs when Kac and Ksc are at their maximum and KT is at its minimum. The loading on the first thread then becomes 194 N under a total load of 1000 N.

The variation between the maximum and the minimum loads is 19.16 %, which would suggest the same amount of variation in the maximum stress as the stress is directly related to the load. However these loading scenarios occur at different thread geometries. The maximum load occurs when the thread is at its upper tolerance limit.
and the minimum loading occurs when the thread is at its lower tolerance level. The results can be seen in Figure 7-20.

<table>
<thead>
<tr>
<th>Maximum Load</th>
<th>11.44 N/mm²</th>
<th>Node 391</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum Load</td>
<td>10.79 N/mm²</td>
<td>Node 608</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>5.68 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-20  The affects of load distribution variation on a 1 inch 12 UNF thread

Even though the variation between the minimum and the maximum load scenarios is nearly 20 % the actual variation of the maximum stress is under 6 %. This is due to the geometry of the thread changing when the load distribution changes.

The load distribution is affected by the spring constants which are affected by the geometry of the threads. The spring constants can also be affected by the material properties. In the following examples the joints analysed will have dissimilar materials to see what affect it has on the maximum stress.

Figure 7-21 shows a 1 inch 12 UNF joint with an aluminium nut and a steel bolt. Using the same methods as previously the spring constants were found for such a connection and the load was calculated for the first thread. The two extreme tolerance levels were modelled and analysed. The load on the first thread was worked out to be 420 N.
Maximum Stress ($\sigma_{\text{max}}$) | Location of $\sigma_{\text{max}}$
---|---
Upper Tolerance Limit | 20.04 N/mm$^2$ | Node 391
Lower Tolerance Limit | 23.36 N/mm$^2$ | Node 608
Variation in Maximum Stress | 14.3 % |

Figure 7-21 Maximum stress variation for a 1 inch 12 UNF thread (aluminium nut & steel bolt)

The results shown in Figure 7-21 is in fact the same as the one in Figure 7-13 but due to a different load distribution on the joints the forces are higher. The variation of the maximum stress remains the same as the aluminium nut only affects the load distribution and not the behaviour of the steel bolt.

The analysis was repeated, but this time an aluminium bolt was used and a steel nut. Once again the load distribution was found using the spring constants. The load on the first thread was worked out to be 396 N.

Maximum Stress ($\sigma_{\text{max}}$) | Location of $\sigma_{\text{max}}$
---|---
Upper Tolerance Limit | 18.90 N/mm$^2$ | Node 391
Lower Tolerance Limit | 21.92 N/mm$^2$ | Node 608
Variation in Maximum Stress | 13.8 % |
The results show that the variation in the maximum stress is reduced when an aluminium bolt is used instead of a steel one. Due to the different material properties the same geometry is less sensitive to material variation when it is made out of aluminium.

In the final analysis for this case study the same force and the same geometry is used, the only property changed is the material. The model used is the lower tolerance level for the 1 inch 12 UNF connection with a load of 420 N on the first thread.

<table>
<thead>
<tr>
<th>Material</th>
<th>Maximum Stress ($\sigma_{\text{max}}$)</th>
<th>Location of $\sigma_{\text{max}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>23.36 N/mm²</td>
<td>Node 608</td>
</tr>
<tr>
<td>Aluminium</td>
<td>23.24 N/mm²</td>
<td>Node 608</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>0.51 %</td>
<td></td>
</tr>
</tbody>
</table>

Figure 7-23  Maximum stress variation for a 1 inch 12 UNF thread for different materials

The results show that the aluminium thread experiences slightly less stress under the same conditions as the steel thread. This is due to the two materials having different Young’s Modulus so the stress is distributed slightly differently. The steel thread is much stiffer so whereas the aluminium thread bends more it experiences less stress. Even though the aluminium thread experiences less stress under the same loading, the steel thread has a much higher strength.
7.5.4 Case Study IV – Stress concentration study on varying thread angle

For a Whitworth thread the specification is that the thread angle is 22.5 degrees on either face of the thread. As the tolerance for the thread angle is not specified in British Standards it was decided that two extreme cases will be investigated to see the effect of different thread angles on the maximum stress. In the first case, the thread will be distorted downwards so the angles on both faces are changed by 3 degrees. The second case will be the reverse of the first one with the thread distorted upwards.

Using the perfect geometry specified in the British Standards, the first thread of the 2 ½ inch bolt was modelled. Using trigonometry the thread angles were distorted in such way that bottom face thread angle was reduced by 3 degrees and the top face was increased by the same amount. The load applied on the first thread was 134 N which is equivalent to a total load of 1000 N.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Maximum Stress ($\sigma_{\text{max}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle down</td>
<td>1.815 N/mm$^2$</td>
</tr>
<tr>
<td>Angle up</td>
<td>1.744 N/mm$^2$</td>
</tr>
<tr>
<td>Variation in Maximum Stress</td>
<td>3.9 %</td>
</tr>
</tbody>
</table>

Figure 7-24 Maximum stress variation for a 2.5 inch 8 UNF thread for different thread angles

The variation between +3 degree angle and -3 degree angle is 3.9 %. This is roughly 0.6 % variation per each degree of change. In British Standards it is assumed that the thread angle remains constant so it is unknown how much the thread angle may vary. As the results show the thread is more sensitive to dimensional variation than to thread angle change.
7.6 Fatigue analysis

In the previous case studies the stress concentration on different threads were analysed and the affects of manufacturing imperfections were recorded. The affects of manufacturing imperfections were measured in terms of change in the maximum stress value. It is known that a component is likely to break where the highest stress occurs, and if due to changes in the geometry or other factors the maximum stress increases the working life of that component will be reduced. To find out how much the life of the component is affected fatigue data is required. There was no fatigue test data available for the range of threads analysed in the case studies. The case studies therefore could not be used to predict the fatigue life of the threads instead the stress behaviour within different threads with altered imperfections was analysed.

In this section with the use of Patterson’s work the fatigue life of the M12 connection will be looked at in greater depth. Using the information provided by Patterson a single M12 thread with pitch of 1.75 mm was modelled and $K_T$ was found. Using the tolerance tables from British Standards the two extreme geometries were analysed and the variation in $K_T$ was determined. The results for the spring constants can be seen in Appendix O. The variation for $K_T$ is small compared with other thread constants whereas the variation for the bolt and nut constants is large. This can be explained by the small nominal diameter and the relatively large pitch. As the results for the M22 thread suggests, small pitches are more sensitive to manufacturing imperfections. The variation between the maximum and the minimum values of $K_T$ was 7% ($\pm$ 3.5 % from the mean).

It was decided to use the spring model to help determine the variation of the first thread load due to the spring constants varying. It was shown previously that the thread constant $K_T$ is the most influential factor when determining the load on the first thread. A special macro was written which varied the value of $K_T$ between the ranges of $-4\%$ and $+4\%$ at each of the first three threads and the change in first thread load was recorded. Even though the change in $K_T$ is $3.5\%$ the values of $-4\%$ and $+4\%$ were chosen to allow a 1 % increment change between analyses. The affects of $K_T$ being varied on the load distribution is shown in the following graphs. The load
distribution was found for four different length 4.8 mm (n=3), 7 mm (n=4), 10 mm (n=6) and 12 mm (n=7). “n” is the number of threads.

Figure 7-25  Length 4.8 mm, $K_T$ variation on the first thread

Figure 7-26  Length 4.8 mm, $K_T$ variation on the second thread

Figure 7-27  Length 4.8 mm, $K_T$ variation on the third thread
Figure 7-28  Length 7 mm, $K_T$ variation on the first thread

Figure 7-29  Length 7 mm, $K_T$ variation on the second thread

Figure 7-30  Length 7 mm, $K_T$ variation on the third thread
Figure 7-31  Length 10 mm, $K_T$ variation on the first thread

Figure 7-32  Length 10 mm, $K_T$ variation on the second thread

Figure 7-33  Length 10 mm, $K_T$ variation on the third thread
Figure 7-34    Length 12 mm, $K_T$ variation on the first thread

Figure 7-35    Length 12 mm, $K_T$ variation on the second thread

Figure 7-36    Length 12 mm, $K_T$ variation on the third thread
The macro was programmed to record the value of the load on the first thread and variation of it was tabulated.

<table>
<thead>
<tr>
<th>Affected Thread</th>
<th>First Thread</th>
<th>Second Thread</th>
<th>Third Thread</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variation in Kt</td>
<td>Load on first thread</td>
<td>Variation</td>
<td>Load on first thread</td>
</tr>
<tr>
<td>%</td>
<td>N</td>
<td>%</td>
<td>N</td>
</tr>
<tr>
<td>-4</td>
<td>411.44</td>
<td>-0.68</td>
<td>414.26</td>
</tr>
<tr>
<td>-3</td>
<td>412.15</td>
<td>-0.51</td>
<td>414.26</td>
</tr>
<tr>
<td>-2</td>
<td>412.86</td>
<td>-0.34</td>
<td>414.26</td>
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<tr>
<td>-1</td>
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<td>-0.17</td>
<td>414.26</td>
</tr>
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<td>0.17</td>
<td>414.26</td>
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<td>0.34</td>
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<th>Third Thread</th>
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<td>Variation</td>
<td>Load on first thread</td>
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<td>Variation</td>
<td>Load on first thread</td>
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<td>342.55</td>
<td>1.49</td>
<td>337.52</td>
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Figure 7-37 First thread load variation summary
The results clearly show that only variation on the first thread has any affect on the first thread load. Using finite element analysis the change in maximum stress was found for each calculated load. Even though the loads were calculated using a different value of $K_T$ each time the geometry of the finite element model was assumed to be perfect at all times. This assumption was made to reduce the number of required analyses.

Figure 7-38  Maximum contour plot on M12 pitch 1.57 mm thread with total load of 1000 N on the connection ($n = 3$)

Figure 7-38 shows the stress contour of a thread under perfect conditions with no variation in $K_T$. Assuming that the thread geometry does not change with a varying $K_T$, the change in stress will be directly proportional to the change in load on the first thread. Therefore it is valid to assume that variation for load on the first thread shown in Figure 7-37 is the same as the variation on the maximum stress.

Using this variation in the maximum stress and the fatigue data from Patterson’s work it is possible to predict how the life of the thread will change with respect to variation in $K_T$. Changing $K_T$ affects the maximum stress even though the applied stress is kept constant. This variation in maximum stress changes the S-N characteristics, as the same amount of applied stress gives different maximum stress concentrations. To compensate for this the applied stress can be changed to keep the maximum stress the
same. Using the S-N data from Paterson’s work the change in applied stress can be related to the change in number of cycles ($\Delta N$) to predict how $K_T$ affects the life of a thread.

The S-N data obtained from Patterson’s paper was given as a set of graphs on an exponential scale (Figures 7-7 & 7-8). To find the change in the number of cycles with respect to change in applied stress the S-N data has to be digitised and reversed so the number of cycles is plotted with respect to applied stress. Using the curve fit function in Excel the formula for the S-N curve could be found.

![Figure 7-39 Example of change in S-N curve when maximum stress is increased](image)

Figure 7-39 shows an example of a reversed S-N curve. The curve is made up of two straight lines, one vertical and one diagonal. The vertical part of the curve tends towards infinity at a constant stress so that part of the curve can not be used to find $\Delta N$. The logarithmic scale is used so the data can be presented as a straight line instead of a curve. Logarithmic scale means that the data is presented as powers of ten. Because of the logarithmic scale the equation of the straight line is presented in
the power form, which is \( N = c \sigma_a^m \). Using the Power Law it is possible to convert this equation into a more familiar format by taking log of both sides.

\[
N = c\sigma_a^m
\]

\[
\log(N) = m\log(\sigma_a) + \log(c)
\]

*Equation 7-4*

*Equation 7-5*

*Equation 7-5* is in the same format as the equation of a straight line \( y = mx + c \) where “m” is the gradient and “c” is a constant.

The formula for the diagonal part can be found using the curve fit function in Excel, bearing in mind that the fitted curve has to be in the power format. Once the equation of the line is found in the \( N = c \sigma_a^m \) format it is possible to calculate the number of cycles (N) with respect to the stress amplitude (\( \sigma_a \)). By finding the change in maximum stress the applied stress can be adjusted accordingly. Using the new applied stress values and the formulas obtained using Excel the new N values can be found.

Figure 7-39 shows an example of how the reversed S-N curve changes with the maximum stress. With the curve fit formula the two different N values can be found for the different applied stress values. By subtracting the two S-N curves from each other it is possible to calculate the change in number of cycles (\( \Delta N \)).

Using this method of subtracting S-N curves from each other to determine the change in the number of cycles the different \( K_T \) variations were plotted for each length of connections.
Figure 7-40 Change in fatigue life due to $K_T$ variation, length = 4.8 mm

Figure 7-41 Change in fatigue life due to $K_T$ variation, length = 7.0 mm

Figure 7-42 Change in fatigue life due to $K_T$ variation, length = 10.0 mm
The fatigue data used to obtain these results were carried out at a mean stress of 238 N/mm$^2$ and with varying stress amplitude. Realistically if the maximum stress is changed by a certain amount it should affect the mean stress as well as the amplitude stress. Changing the mean stress would change the S-N characteristics so in theory a different set of S-N curves would be required. The change is minimal in the mean stress and the affect of the mean stress on the fatigue life is negligible.

Assuming that the amount the mean stress changes due to the increase in maximum stress is does not affect the S-N characteristics significantly the results become as follows.

Figure 7-43  Change in fatigue life due to $K_T$ variation, length = 12.0 mm

Figure 7-44  Change in fatigue life due to $K_T$ variation, length = 4.8 mm
Varying the mean and the amplitude stress together increases the discrepancy between the original S-N curve and the modified one reducing its effect. The results show that as $K_T$ increases the mean stress to the number of cycles are reduced.

For high values of $\Delta N$ it is relatively low but as the stress amplitude is reduced $\Delta N$ increases. This set of data is unique to the current test geometry. However the above example shows that increase of 1.49% in maximum stress caused by a 4% variation in $K_T$ reduces the number of cycles to failure by 40000 cycles.

Deterioration in fatigue life can be noticed due to $K_T$ variation.

Figure 7-45  Change in fatigue life due to $K_T$ variation, length = 7.0 mm

Figure 7-46  Change in fatigue life due to $K_T$ variation, length = 10.0 mm

Figure 7-47  Change in fatigue life due to $K_T$ variation, length = 12.0 mm
Varying the mean and the amplitude stress together increases the discrepancy between the original S-N curve and the modified one enhancing its affect. The results show that as $K_T$ increases so does the maximum stress so the number of cycles are reduced.

For high values of amplitude stress $\Delta N$ is relatively low but as the stress amplitude is reduced $\Delta N$ increases rapidly. This is because at small stress amplitudes on the S-N curve $n$ is very large in the region of $10^6$. With such high number of cycles even a small change in stress has a significant affect on $\Delta N$. As the amplitude stress is increased the life of the specimen is shortened, so $\Delta N$ is relative to this shorter life. Below the fatigue limit the life of the connection is not affected.

This set of data is unique to the M12 pitch 1.75 mm geometry. However the above example shows that even a small increase of 1.49 % in maximum stress caused by a 4% variation in $K_T$ can reduce the number of cycles to failure by 400000 cycles.
7.7 Discussion

There are two ways the thread can fail while in operation. These are thread loosening and thread failure. Thread loosening can be reduced or avoided by applying a high preload, increasing the friction between threads, reducing the thread angle and applying a constant load instead of shock loading which would introduce vibrations. Thread loosening is one of the main reasons why it is important to monitor the load in threaded connections. By introducing the laser strain gauge load measuring device, the thread loosening can be monitored to avoid malfunction.

The second way a threaded connection can fail is thread failure which involves fracture of the thread due the maximum stress concentration exceeding the strength of the material. The highest stress concentration is found at the base of the first thread. The strength of a threaded connection is usually known, but in all these cases perfect conditions are applied. Manufacturing imperfections can introduce variations to the maximum stress which can have an affect on the overall strength of the joint.

Using case studies some extreme scenarios were studied and the variation of the maximum stress was calculated. It is important to look at the variation of the maximum stress because if the maximum stress is increased by 10 % it is like decreasing the strength of the material by the same amount. From the results the following conclusions can be drawn:

- For connections with the same nominal diameter the variation of the maximum stress can be reduced by increasing the pitch
- Dimensional variation in metric threads has a much greater effect on the variation of the maximum stress
- As the nominal diameter is increased the variation of the maximum stress is reduced up to a point. Even at large diameters the stress variation is 14 %
- The different tolerance levels mean the spring constants change as well. This affects the load distribution which changes the maximum stress variation
- Dissimilar materials change the load distribution, but do not affect the amount the maximum stress varies by
- The change in thread angle has a minimal affect on the maximum stress
The summary of the results are shown in Figure 7-48.

<table>
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<tr>
<th>Case Study I - Dimensional variation of different profiles</th>
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</tr>
<tr>
<td>1 inch 8 UNC</td>
<td>3.84</td>
</tr>
<tr>
<td>M22 pitch 1 mm</td>
<td>25.9</td>
</tr>
<tr>
<td>M22 pitch 2 mm</td>
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<table>
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</tr>
<tr>
<td>1 inch 12 UNF</td>
<td>14.3</td>
</tr>
<tr>
<td>2.5 inch 8 UNF</td>
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<tr>
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<table>
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</thead>
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<td>Steel nut / Aluminium bolt</td>
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</tr>
<tr>
<td>+3 degrees / -3 degrees</td>
<td>3.9</td>
</tr>
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</table>

Figure 7-48 Results summary

There are a number of factors that can affect the maximum stress on the first thread. The most significant is changing the geometry according to the tolerance tables. However as the geometry changes so does the load distribution. In Case Study I it was found that there is a 14.3 % variation in the maximum stress between the lower and the upper tolerance levels. If the load distribution is taken into consideration as well this variation is reduced to 5.68 % as it was shown in Case Study III. This variation is likely to increase if the nominal diameter or the pitch is reduced.

The fatigue analysis showed that even a small change in strength can affect the life of a threaded connection by the region of $10^5$ cycles if the specimen is used at its fatigue
limit. This however can only take place when the expected number of cycles to failure is in the region of $10^6$ cycles. For such large values of $N$ reducing the number of cycles even by a considerable amount is insignificant especially because components are rarely operated close to their fatigue limit.

In conclusion it is possible to say that manufacturing imperfections will have a significant effect on the maximum stress concentration of a threaded connection and by that influencing the residual life of the connection. The case studies showed that it is only possible to determine the maximum stress to be within a certain percentage. This variation needs to be taken into consideration as it can affect the maximum strength of the joint.
Chapter 8 – Discussion and Conclusion

8.1 Discussion

The aim of the project is to gain understanding with the use of finite element analysis and experimental testing of how the surface strain on a visible part of a threaded connection can be used to find the loading in order to help develop a measuring device that can be used to determine and monitor the load on a threaded connection to an accuracy of 0.5% at elevated temperatures. Current methods use sensitive equipment such as ultrasound to help determine the load on threaded connections and therefore can not be used in extreme environments. It was decided to design a non contact device that can be used to measure the load on a threaded connection.

The Laser Strain Gauge is a non contact surface strain measuring device which works by measuring the distortion of a grating on the surface of the specimen. By relating the surface strain on a visible part of a threaded connection to the load the LSG can be used to determine the load on the joint. As it is a non contact method it can be used in high temperatures and in other extreme environments.

Using finite element analysis and previous works a modified nut was developed which could be used to relate the surface strain on it to the load on the joint. Initial findings using FEA showed that the surface strain within the grooves of the modified nut can be related to the load accurately assuming a perfect geometry. In reality all parts are manufactured to a certain tolerance which means that each part will be slightly different. Using FEA, sensitivity analysis was carried out to help determine how manufacturing variation influences the surface strain measurements and the accuracy of the device. The sensitivity analysis revealed that to keep within the required accuracy of 0.5% the parts have to be manufactured to very tight tolerances. To verify the FEA findings a prototype was manufactured and tested on a test rig using a hydraulic tensioner and electric resistance strain gauges to measure the surface strain. The experimental tests revealed that due to friction between the threads there is hysteresis present in the surface strain and load relationship. This meant the modified
nut cannot be used as an accurate load measuring device and an alternative solution is required.

To reduce the hysteresis it was necessary to measure the surface strain on a part which moves independently from the threads but still experiences the entire load. A washer was introduced in the assembly which was modified the same way as the nut by the addition of three grooves. The three grooves ensured that there are three places where the readings can be taken from as well as giving better accessibility for the user to take the measurement with the LSG.

Initially the washer was tested using finite element analysis and the results showed good correlation between the surface strain and the applied load. The measured surface strain was at a higher magnitude compared to the surface strain on the nut which improved the accuracy. A prototype washer was manufactured and experimental tests were carried out to verify the FEA model and to see if hysteresis was present. The tests showed an insignificant amount of hysteresis between the measured surface strain on the washer and the load on the joint. As both the experimental and the FEA results were consistent and there was no hysteresis present it was decided that the modified washer can be used as a load measuring device.

The surface strain on the washer can be measured using the laser strain gauge. Previous tests showed that the surface strain is linearly related to the applied load, so by knowing this relationship the load can be determined instantly from the surface strain measurement. The relationship between the surface strain and the load was determined using finite element analysis. This relationship however is based on a perfect geometry so sensitivity analysis was carried out to investigate the affects of manufacturing imperfections. Unlike for the nut where the load is transferred though the threads for the washer the load is transferred via the top and bottom surfaces. This meant fewer parameters affect the accuracy reducing the number of required sensitivity functions.

The sensitivity analysis revealed that manufacturing imperfections introduce errors into the surface strain measurements. These errors were especially significant in the axial direction and for some parameters in the hoop direction as well. Using the
sensitivity functions the maximum allowed dimensional variation for each parameter was calculated. It was revealed that to achieve the required 0.5% accuracy very tight tolerances are required. To manufacture every component to be within these specified tolerances would not be cost effective. It has been suggested that by individually calibrating each specially designed washer the accuracy can be maintained without tightening the manufacturing tolerances.

There are a number of advantages to calibration; the most obvious of these is to ensure accuracy. Without knowing the manufacturing and material imperfections of an individual part, calibration enables the part to be used as an accurate strain measuring device. Each part can be tested over a range of loads and the relationship between the load and the surface strain can be established. From previous tests it is possible to say that within the elastic limit of the material this relationship will be linear. So when a part is calibrated it is enough to know the gradient of this linear relationship. When a part is calibrated it should be labelled accordingly so it can be used over and over again as a load measuring device, even if it is used in different connections.

When a calibrated washer is placed in different connections the only varying factors are the boundary conditions. The effects of different boundary conditions were investigated in Chapter 6 when non-uniform loading was examined. It was found that even though varying boundary conditions have an effect on the individual strain readings in the grooves the mean strain over the three grooves is not affected. For this reason the washer only has to be calibrated once and then it can be used in different assemblies.

The other advantages of calibration are quality control and safety factor. Each part is calibrated individually which allows each part to be examined for strength and other properties. During calibration it is possible to cast-off parts that do not meet the required standard in terms of relating surface strain and load. Also the structural integrity of the washer can be tested in a safe environment during calibration to ensure the required safety under working conditions.
The disadvantage of calibration is the extra time and cost it requires. For this reason a simple calibration method is required which allows the washer to be tested over a range of loads including the maximum load it has to withstand. This is important both for accuracy and for safety reasons. There are a number of ways the washer can be calibrated but probably the easiest one is by using a similar set up to the test rig which was described earlier in this thesis. To increase accuracy the test rig would have to be modified.

Presently the test rig is operated using a hydraulic pump to increase and decrease the load. The hydraulic pump has two major disadvantages. Firstly the load is measured using a digital pressure gauge with accuracy to the nearest bar. Secondly the viscosity of the hydraulic setup means that there is some hysteresis in the loading mechanism. Both these factors reduce the accuracy hence unsuitable for calibration.

To increase accuracy the hydraulic clamp should be replaced by a gearbox and a motor so the viscosity is no longer a problem. The load can then be determined directly from the motor or for more accuracy the load can be measured from the connection itself.

Even though calibration can be used to eliminate the inaccuracies manufacturing imperfections within the joint can affect the structural integrity and the working life of the connection. The critical part of a threaded connection is the first engaged thread as it experiences the highest stress concentration. With the help of variation analysis and finite element analysis the affects of manufacturing imperfections were studied. The results showed that the life of the connection may be reduced by a large number of cycles due to imperfections but this is only the case if the component is used close to its fatigue limit. If a reasonable safety factor is taken into consideration, the affect of manufacturing imperfections on the joint is insignificant.

In summary the modified washer and the laser strain gauge can be used to measure the load to 0.5% accuracy on a threaded connection via a non contact if calibrated. The device is easy to use and can be used to monitor the load at regular intervals.
8.2 Conclusions

- There are no methods currently available to measure the load on a threaded connection in an extreme environment accurately. Methods that are currently used consist of sensitive equipment which cannot operate at high temperatures.
- To help measure the load on a threaded connection in an extreme environment a non-contact method is required.
- The laser strain gauge (LSG) can be adapted to be used as a non-contact load measuring device. By relating the surface strain on a threaded connection to the load on the connection the LSG can be used to measure load.
- The laser strain gauge can only determine the surface strain on a visible part of the connection. The only existing visible parts of a threaded connection are the end of the bolt and the nut. The end of the bolt does not experience any load so it was decided to use a modified nut to see if the surface strain on it can be related to load.
- Initial tests using finite element analysis showed that the surface strain on the modified nut can be related to the load and that the relationship is close to linear. At a load of 703 kN the axial strain was -150 microstrain and the hoop strain was 170 microstrain.
- Experimental tests revealed that the surface strain readings obtained from the two-dimensional axisymmetric finite element models were inaccurate due to the number of assumptions used to simplify the contact analysis for the complicated geometry.
- For the modified nut the hoop strain was at higher magnitude than the axial strain. For a load of 350 kN the axial strain was -350 microstrain and the hoop strain was 450 microstrain.
- Sensitivity analysis revealed that the modified nut is very responsive to manufacturing imperfections. The number of parameters affecting the accuracy of the nut as a load measuring device is too great for the load to be determined accurately.
- Manufacturing imperfections affect the load distribution on a threaded connection which can introduce even more inaccuracies when trying to relate surface strain to the applied load.
The summary of how manufacturing imperfections affect the accuracy of the modified nut are shown below:

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<th>Symbol</th>
<th>Units</th>
<th>Maximum variation to maintain 0.5% accuracy</th>
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<td>Maximum variation</td>
<td></td>
<td></td>
<td>Axial</td>
</tr>
<tr>
<td>to maintain 0.5%</td>
<td></td>
<td></td>
<td>accuracy</td>
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<tr>
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</tr>
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<td>%</td>
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<td>%</td>
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<tr>
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</table>

Figure 8-1  Maximum dimensional variations on the modified nut

- Experimental testing revealed that the modified nut cannot be used as an accurate load measuring device due to hysteresis in the threads. The threads sticking meant the relationship between surface strain and load cannot be related accurately. At a load of 350 kN the axial surface strain varies between -290 and -430 microstrain (over 30 %) and the hoop strain varies between 390 and 520 microstrain (25%) due to hysteresis.
- The hysteresis was present due to the threads sticking so an independent part with no threads needs to be introduced. A washer was chosen as it has no threads, it is visible, and experiences the entire load that goes through the connection.
- The modified washer is not affected by hysteresis significantly. As the force is not transferred through the threads but through the flat surfaces of the washer the friction and the hysteresis are reduced significantly.
- The hoop strain on the washer experiences some hysteresis due to the washer face sticking, but the axial strain does not experience any hysteresis and demonstrates a perfectly linear relationship between surface strain and load. At a load of 350 kN the axial strain varies between -374 and -377 microstrain (under 1%) and the hoop strain varies between 175 and 200 microstrain (12.5%) due to hysteresis.
- The measured axial surface strain on the washer is at a higher magnitude than the axial surface strain on the nut improving the accuracy of the measuring device. At higher magnitude of strains errors have less of an influence on the calculated load. At a load of 703 kN the axial surface strain on the nut is -660 microstrain and -550 on the modified nut.

- The measured axial strain is at a higher magnitude compared to the hoop strain; therefore it is recommended that the strain in the axial direction should be used to calculate the load instead of the hoop strain. At a load of 703 kN the hoop strain on the washer is 315 microstrain and the axial strain is -660 microstrain.

- The sensitivity analysis revealed that the manufacturing imperfections have a significant affect on the accuracy of washer as a load measuring device. To achieve the required accuracy of 0.5 % the washer would either have to be manufactured to very tight tolerances or calibrated individually.

- When measuring the surface strain on the washer, the strain in the axial direction is more sensitive to manufacturing imperfection. Even a slight change in one of the dimensional parameters can introduce significant errors to the axial strain reading. If calibrated properly the inaccuracies can be eliminated.

- The summary of the affects of manufacturing imperfections on the modified washer are shown below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Axial</th>
<th>Hoop</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young’s modulus</td>
<td>E</td>
<td>%</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Outer radius</td>
<td>r₀</td>
<td>%</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Inner radius</td>
<td>r₁</td>
<td>%</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>v</td>
<td>%</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>Groove geometry</td>
<td>θ₀</td>
<td>degree</td>
<td>0.16</td>
<td>0.93</td>
</tr>
<tr>
<td>Groove location x</td>
<td>x₀</td>
<td>mm</td>
<td>0.12</td>
<td>1.59</td>
</tr>
<tr>
<td>Groove location y</td>
<td>y₀</td>
<td>mm</td>
<td>0.025</td>
<td>0.39</td>
</tr>
</tbody>
</table>

Figure 8.2 Maximum dimensional variations on the modified washer
Manufacturing imperfections can affect the life of a threaded connection. Manufacturing imperfections influence the load distribution on a threaded connection. The first thread on a connection experiences the most load so it is most likely to break. If the load distribution changes so will the first thread load which can have an effect on the life significantly. For example, for an M12 pitch 1.75 mm threaded connection with 4% variation in the thread spring constant the maximum stress is increased by 1.49%. This can reduce the life of the connection by 400000 cycles.

The summary of the effects of manufacturing imperfections on the maximum stress concentration is shown below:

<table>
<thead>
<tr>
<th>Case Study I - Dimensional variation of different profiles</th>
<th>Model</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 inch 12 UNF</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>1 inch 8 UNC</td>
<td>3.84</td>
<td></td>
</tr>
<tr>
<td>M22 pitch 1 mm</td>
<td>25.9</td>
<td></td>
</tr>
<tr>
<td>M22 pitch 2 mm</td>
<td>22.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case Study II - Dimensional variation of different sizes</th>
<th>Model</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/4 inch 28 UNF</td>
<td>20.4</td>
<td></td>
</tr>
<tr>
<td>1 inch 12 UNF</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>2.5 inch 8 UNF</td>
<td>0.33</td>
<td></td>
</tr>
<tr>
<td>5 inch 6 UN</td>
<td>14.1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case Study III – Load variations on 1 Inch 12 UNF</th>
<th>Model</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum and minimum load</td>
<td>5.68</td>
<td></td>
</tr>
<tr>
<td>Aluminium nut / Steel bolt</td>
<td>14.3</td>
<td></td>
</tr>
<tr>
<td>Steel nut / Aluminium bolt</td>
<td>13.8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case study IV – Angle variation</th>
<th>Model</th>
<th>Variation %</th>
</tr>
</thead>
<tbody>
<tr>
<td>+3 degrees / -3 degrees</td>
<td>3.9</td>
<td></td>
</tr>
</tbody>
</table>

Figure 8-3 Results summary of manufacturing imperfections on maximum stress concentration
In conclusion the modified washer and the laser strain gauge can be used as an accurate non contact load measuring device if calibrated.
Chapter 9 – Recommendations for Further Work

In the previous chapters it was shown that the laser strain gauge can be used to measure the surface strain on a specially designed washer which can be related to the load on a threaded connection. The relationship between the surface strain and the load was found using finite element analysis and later verified using experimental techniques and the results showed good correlation. These results were based on perfect geometry and material but if the washer is to be used commercially manufacturing imperfections need to be taken into account.

Using variation analysis the sensitivity factor for the measured strain was found for each individual parameter which may affect the readings. The required accuracy for the load measuring device is ± 0.5% but to achieve this, the washer would have to be manufactured to extremely close tolerances. Instead it was suggested that the each washer should be calibrated individually to ensure the accuracy of the measuring device.

In order to complete this project a number of different areas have been studied and investigated. Even though a wide range of fields have been looked at due to time constraints some less relevant areas have been left out. In this section these areas will be discussed to show how this project may be expended in the future.

An broad sensitivity analysis has been carried out both on the nut and the washer, which helped predict the accuracy of each component with respect to manufacturing imperfections. The analysis was limited to a single size and geometry. To gain a better understanding on the behaviour of the proposed measuring device the sensitivity analysis could be repeated for different geometries.

When the sensitivity analysis was carried out, all parameters were investigated individually. It was assumed that the parameters are independent from each other and that the error caused by the parameters can be summed up. An extensive sensitivity analysis is proposed that investigates the combined effect of the sensitivity functions.
The project was originally proposed to help determine the load on threaded fasteners within a turbine. Turbines operate at extremely high temperatures and that is why it was necessary to design a non contact device to measure the load. The idea can be modified and used in different harsh environments or in areas where there are simply no means of getting to the threaded connection so a non contact method has to be used.

The aim of the project was to determine the load within a threaded connection. This was achieved by relating the surface strain to the load. As a modification of this project it would be possible to relate the surface strain directly to the maximum stress within the connection. The device could be used to ensure the maximum stress never exceeds the fatigue limit of the material ensuring an infinite life for the joint.

When the effects of manufacturing imperfections on the life of the threaded connection were investigated all the testing was done using finite element analysis. FEA was used as all the modelling and testing could be done using computers and there was no need for manufacture. This ensured lower costs and shorter analysis times. FEA however uses assumptions which introduce errors. For a more accurate way to predict how the life of the connection is affected with respect to manufacturing imperfections experimental testing could be carried out. The findings could then be compared to the results presented in this work for verification.
References


BS82:1956 Parallel screw thread of Whitworth form. British Standard


Hetenyi, M. (1946). Beams on elastic foundation : theory with applications in the field of civil and mechanical engineering, Michigan UP.


Okobu, H., (1968) "Memoirs of the Faculty of Engineering" Nagoya University, Vol 20 No.1


www.rotabolt.co.uk (2004) http://www.rotabolt.co.uk/


Appendices
Appendix A

There are three types of parameters that can vary in a solid shaft; these are geometry, material properties and boundary conditions. To be more specific the geometrical parameters are radius (r) and height (h). The material properties are Young’s modulus (E) and Poisson’s ratio (ν), and the boundary condition is force (P).

![Solid shaft parameters](image)

The axial strain can be calculated in the solid shaft using the following equation:

\[
\varepsilon_a = \frac{P}{E \pi r^2}
\]

*Equation A-1*

The strain is governed by three parameters, so a change in either of these would introduce an error. Assuming that all parts can vary *Equation A-1* is differentiated:

\[
\delta \varepsilon_a = \frac{\delta P}{E \pi r^2} - \frac{P \delta E}{E^2 \pi r^2} - \frac{2P \delta r}{E \pi r^3}
\]

*Equation A-2*

Divide left side by ε and right side by \( \frac{P}{E \pi r^2} \):

\[
\frac{\delta \varepsilon_a}{\varepsilon_a} = \frac{\delta P}{P} - \frac{\delta E}{E} - \frac{2 \delta r}{r}
\]

*Equation A-3*
The equation above shows how the changes in different parameters affect the calculated axial strain. So the change in axial strain depends on the change in load, the change in Young's modulus and twice the change in radius.

\[
\delta \varepsilon_a = \frac{\delta P}{P} \varepsilon_a - \frac{\delta E}{E} \varepsilon_a - \frac{2 \delta r}{r} \varepsilon_a
\]

Equation A-4

Equation 4-4 can be rearranged to get an expression for the change in axial strain. In this format it can be seen that for \( P \) the sensitivity factor is +1, for \( E \) it is -1 and for \( r \) it is -2.

The hoop strain can be calculated similarly to the axial strain but by introducing Poisson’s ratio. The hoop strain can be calculated using the following equation:

\[
\varepsilon_h = \frac{\nu P}{E \pi r^2}
\]

Equation A-5

Similarly to before assuming that all parts may vary the above equation is differentiated.

\[
\delta \varepsilon_h = \frac{\nu \delta P}{E \pi r^2} + \frac{P \delta \nu}{E \pi r^2} - \frac{\nu P \delta E}{E^2 \pi r^2} - \frac{2 \nu P \delta r}{E \pi r^3}
\]

Equation A-6

Divide left side by \( \varepsilon \) and right side by \( \frac{\nu P}{E \pi r^2} : \)

\[
\frac{\delta \varepsilon_h}{\varepsilon_h} = \frac{\delta \nu}{\nu} + \frac{\delta P}{P} - \frac{\delta E}{E} - \frac{2 \delta r}{r}
\]

Equation A-7

So the change in hoop strain is affected the same way as the axial strain plus the change in Poisson’s ratio.
Figure B-1
Nut parameters

Thread Profile

DETAIL A

DRW-AB011
LOUGHBOROUGH UNIVERSITY
Nut with varying parameters

DRAWN A BULKAI
MATERIAL: STEEL
SCALE: 1:2 UNITS: INCH DATE
Appendix C

To verify the spring model an example was chosen. Using the same geometry as previously used by Kenny and Patterson for experimental testing, it was possible to compare the experimental and the spring model results. The spring constants in the bolt and the nut were calculated using Young's modulus, and the thread spring constant was found using an axisymmetric finite element model. The spring model was then found using the method in Section 5.2.

The figures below show good correlation between previous experimental findings and the spring model proving that the spring model is a suitable tool to find the load distribution in a threaded fastener.

Figure C-1 Experimental findings from previous works for a 1 inch 8UN connection (Kenny and Patterson)
Figure C-2  Load distribution on a 1 inch 8UN connection found using the spring model shown in Section 5.2
Appendix D

Figure D-1 Hexagonal nut area calculation

- $s$ – Length of flat face
- $w$ – Width across flat faces
- $\alpha$ – Angle of segment = 30°

$A_S$ – Area of segment
$A_T$ – Total area

\[
A_s = \frac{1}{2} s \times \frac{1}{2} w = \frac{1}{8} sw
\]

Equation D-1

But:

\[
\frac{1}{2} s = TAN\alpha
\]

Equation D-2

\[
\frac{1}{2} w
\]

Therefore:

\[
s = \frac{w}{TAN\alpha}
\]

Equation D-3

Substituting into Equation 1:
\[ A_s = \frac{1}{8} w^2 \tan \alpha \]  

Equation D-4

The total area can be found:

\[ A_T = 12 \times A_s \]  

Equation D-5

Therefore:

\[ A_T = 1.5 \times w^2 \times \tan \alpha \]  

Equation D-6
Appendix E

Sub Macro1()
    Msg = "Is the material EN24?"
    Ans = MsgBox(Msg, vbYesNo)
    If Ans = vbNo Then GoTo opt2
        Young's = 200000000000#
        Poisson = 0.3
        Range("A3") = "Young's Modulus"
        Range("A4") = "Poisson's ratio"
        Range("B3").Value = Young's
        Range("B4").Value = Poisson
    Exit Sub
opt2:
    Young's = InputBox("What Is the Young's Modulus of the material?")
    Poisson = InputBox("What is the poisson ratio of the material?")
    Range("A3") = "Young's Modulus"
    Range("A4") = "Poisson's ratio"
    Range("B3").Value = Young's
    Range("B4").Value = Poisson
    pitch = InputBox("What Is the Pitch?")
    Range("A6") = "Pitch"
    Range("B6").Value = pitch
    Thread = InputBox("Number of threads")
    Range("A7") = "Number of Threads"
    Range("B7").Value = Thread
    d1 = InputBox("What is the nut diameter?")
    Range("E6") = "Nut Diameter"
    Range("F6").Value = d1
    d2 = InputBox("What is the bolt diameter?")
    Range("E7") = "Bolt Diameter"
    Range("F7").Value = d2
    load1 = InputBox("What is the load?")
    Range("A8") = "Load1"
    Range("B8").Value = load1
    dis = InputBox("What is the thread displacement")
    Range("A9") = "Thread Displacement"
    Range("B9").Value = dis
    Range("A11") = "KBC"
    Range("B11").Formula = "=(B3 * 3.14 * (F6 ^ 2 - F7 ^ 2)) / 4 * B6"
    Range("A12") = "KSC"
    Range("B12").Formula = "=(B3 * 3.14 * F7 ^ 2) / 4 * B6"
    KT = load1 / dis
    Range("A13") = "KT"
    Range("B13").Value = KT
    Range("A15") = "Beta"
    Range("A16") = "gamma 1"
    Range("A17") = "gamma 2"
    Range("A18") = "A"
    Range("A19") = "B"
    Range("B15").Formula = "=2 + (B13/B11) + (B13/B12)"
    Range("B16").Formula = "=(B15^2)+(SQRT((B15^2)-4*B2)+4)/2"
    Range("B17").Formula = "=(B15^2)-(SQRT((B15^2)-4*B2))"
    Range("B18").Formula = "=(B8*(1+(B16*B7)+(B17*B7-B16*B7))"
    Range("B19").Formula = "=-B8*((B16*B7)+(B17*B7-B16*B7))"
    Range("B22:C22").Select
      With Selection
        .MergeCells = True
      End With
    Range("D22:F22").Select
      With Selection
        .MergeCells = True
      End With
    Range("C23:C24").Select
      With Selection
        .MergeCells = True
        .WrapText = True
      End With
    Range("E23:E24").Select
      With Selection
        .MergeCells = True
  End Sub
Sub ThreadLoad

.Range("F23:F24").Select

With Selection

.MergeCells = True
.WrapText = True

End With

.Range("B22") = "Bolt Loads"
.Range("D22") = "Thread Loads"
.Range("A23") = "Thread Number"
.Range("B23") = "Load S"
.Range("C23") = "Percentage of Load"
.Range("D23") = "Load P"
.Range("E23") = "Thread Load/Mean Thread Load"
.Range("F23") = "Percentage of Load"
.Range("B25") = "N"
.Range("C25") = "%"
.Range("D25") = "N"
.Range("F25") = "%"

n = Range("B7").Value
A = Range("B18").Value
B = Range("B19").Value
Gamma1 = Range("B16").Value
Gamma2 = Range("B17").Value
Load1 = Range("B8").Value

For Count = 1 To n

.Range("A26").Offset(Count - 1, 0) = Count
.Range("B26").Offset(Count - 1, 0).Value = A * (Gamma1 ^ Count) + B * (Gamma2 ^ Count)
.Range("C26").Offset(Count - 1, 0).Value = (100 * (A * (Gamma1 ^ Count) + B * (Gamma2 ^ Count))) / Load1
.Range("D26").Value = 100 - (A * (Gamma1 ^ 1) + B * (Gamma2 ^ 1))

Next Count

For Count1 = 1 To n - 1

.Range("D27").Offset(Count1 - 1, 0).Value = (A * (Gamma1 ^ Count1) + B * (Gamma2 ^ Count1)) - (A * (Gamma1 ^ (Count1 + 1)) + B * (Gamma2 ^ (Count1 + 1)))

Next Count1

.Range("E19").Formula = "=SUM(D26:D39) / B8"
.Range("D19").Value = "Mean Load"
.Range("E26").Value = (100 - (A * (Gamma1 ^ 1) + B * (Gamma2 ^ 1))) / MeanLoad
.Range("E27").Offset(Count2 - 1, 0).Value = ((A * (Gamma1 ^ Count2) + B * (Gamma2 ^ Count2)) - (A * (Gamma1 ^ (Count2 + 1)) + B * (Gamma2 ^ (Count2 + 1)))) / MeanLoad

Next Count2

.Range("F25").Value = (100 - (A * (Gamma1 ^ 1) + B * (Gamma2 ^ 1))) * 100 / load1
.Range("F27").Offset(Count3 - 1, 0).Value = ((A * (Gamma1 ^ Count3) + B * (Gamma2 ^ Count3)) - (A * (Gamma1 ^ (Count3 + 1)) + B * (Gamma2 ^ (Count3 + 1)))) / 100 / load1

Next Count3

Charts.Add

ActiveChart.ChartType = xlXYScatterSmoothNoMarkers
ActiveChart.SetSourceData Source:=Sheets("Sheet1").Range("E13")
ActiveChart.SeriesCollection.NewSeries
ActiveChart.SeriesCollection(1).XValues = "+=Sheet1!R26C1:R34C1"
ActiveChart.SeriesCollection(1).Values = "+=Sheet1!R26C5:R34C5"
ActiveChart.Location Where:=xlLocationAsNewSheet

With ActiveChart

.HasTitle = True
.ChartTitle.Characters.Text = "Thread Load/Mean Thread Load"
.Axes(xlCategory, xlPrimary).HasTitle = True
.Axes(xlValue, xlPrimary).HasTitle = False
.Axes(xlCategory, xlPrimary).AxisTitle.Characters.Text = "Thread Number"
.Axes(xlValue, xlPrimary).AxisTitle.Characters.Text = "Thread Load/Mean Thread Load"
.EndWith

ActiveChart.HasLegend = False

End Sub
Appendix F

This Macro has been copyrighted by Dr Alejandro Maranon and András Bulkai of Loughborough University.

Option Explicit
Option Base 1

Private Declare Function OpenProcess Lib "kemel32" (ByVal dwDesiredAccess As Long, ByVal bInheritHandle As Long, ByVal dwProcessId As Long) As Long
Private Declare Function WaitForSingleObject Lib "kemel32" (ByVal hObject As Long, ByVal dwMilliseconds As Long) As Long
Private Declare Function CloseHandle Lib "kemel32" (ByVal hObject As Long) As Long

Private Const SYNCHRONIZE = &H100000
Private Const INFINITE = -1
Private Const INFINITE_ = &HFFFF

Public pitch As Double
Public n As Integer
Public dial1 As Double
Public dial2 As Double
Public load1 As Double
Public dis As Double
Public dial3 As Double
Public Young's As Double
Public Poisson As Double
Public KBC1 As Double
Public KBC2 As Double
Public KSC1 As Double
Public KSC2 As Double
Public KT1 As Double
Public KT2 As Double
Public h As Integer

Public ThreadLoads() As Double 'Array to store the loads calculated
Public AnsysResults() As Double 'Stores results from Ansys

* Start the indicated program and wait for it
* to finish, hiding while we wait.
Private Sub ShellAndWait(ByVal program-name As String, ByVal window style As VbAppWinStyle)
Dim process_id As Long
Dim process_handle As Long

* Start the program.
On Error GoTo ShellError
process_id = Shell(program_name, window_style)
On Error GoTo 0

* Hide.
Workbooks.Applicatlon.Visible = False
DoEvents

* Wait for the program to finish.
* Get the process handle.
process_handle = OpenProcess(SYNCHRONIZE, 0, process_id)
If process_handle <> 0 Then
WaitForSingleObject process_handle, INFINITE
CloseHandle process_handle
End If

* Reappear.
Workbooks.Applicatlon.Visible = True
Exit Sub

ShellError:
MsgBox "Error starting task " & Err. Description, vbOKOnly Or vbExdamation, "Error"
End Sub

Private Sub ReadVariables()

* This procedure define the variables numerically
pitch = Sheets("Input").Range("B6").Value
n = Sheets("Input").Range("B7").Value
dial = Sheets("Input").Range("B8").Value
dia2 = Sheets("Input").Range("B9").Value
load1 = Sheets("Input").Range("B10").Value
dis = Sheets("Input").Range("B11").Value
Young's = Sheets("Input").Range("B13").Value
Poisson = Sheets("Input").Range("B14").Value
KBC1 = Sheets("Input").Range("B15").Value
KBC2 = Sheets("Input").Range("B16").Value
KSC1 = Sheets("Input").Range("B17").Value
KSC2 = Sheets("Input").Range("B18").Value
KT1 = Sheets("Input").Range("B19").Value
KT2 = Sheets("Input").Range("B20").Value
h = Sheets("Input").Range("B21").Value

End Sub

Sub CalculateVariations()
  'Declare Internal variables
  Dim steve As Integer
  Dim count As Integer
  Dim c: ountt As Integer
  Dim KBC As Double
  Dim KSC As Double
  Dim KT As Double
  Dim Meanload As Double
  Dim CountArray As Long 'Iterator on sheets names
  Dim NamesArray() As String
  Dim IteratorInfinity As Long
  Dim Beta As Double
  Dim Gammal As Double
  Dim Gamma2 As Double
  Dim A As Double
  Dim B As Double
  Dim S As Double
  Dim Percentage As Double

  'Call the definition of the variables
  Call ReadVariables

  'Call: Delete Worksheets if they exists
  Call DeleteVariations

  Application.DisplayAlerts = False
  Application.EnableEvents = False

  'Create array with names of sheets
  ReDim NamesArray(h + 1)

  For CountArray = 1 To (h + 1)
    NamesArray(CountArray) = "Input"
  Next CountArray

  'Do Andras’ stuff
  For steve = 1 To h
    Worksheets.Add count:=1, After:=Sheets(NamesArray(steve))
    Sheets(steve + 1).Name = NamesArray(steve + 1)
    Sheets(NamesArray(steve)).Activate
    If steve = 1 Then KBC = (KBC1 + KBC2) / 2: KSC = (KSC1 + KSC2) / 2: KT = (KT1 + KT2) / 2:
      Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Norm":
      Sheets(NamesArray(steve + 1)).Range("A1") = NamesArray(steve + 1)
    If steve = 2 Then KBC = (KBC1 + KBC2) / 2: KSC = (KSC1 + KSC2) / 2: KT = KT1:
      Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Norm":
      Sheets(NamesArray(steve + 1)).Range("G1") = "Norm"
  Next CountArray
If steve = 3 Then KBC = (KBC1 + KBC2) / 2: KSC = (KSC1 + KSC2) / 2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 4 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC1: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 5 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC2: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 6 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 7 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC1: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 8 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC2: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 9 Then KBC = (KBC1 + KBC2) / 2: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Norm": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Min"

If steve = 10 Then KBC = KBC1: KSC = (KSC1 + KSC2) / 2: KT = (KT1 + KT2) / 2: Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Norm"

If steve = 11 Then KBC = KBC1: KSC = KSC1: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Norm"

If steve = 12 Then KBC = KBC1: KSC = KSC2: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Norm"

If steve = 13 Then KBC = KBC1: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 14 Then KBC = KBC1: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 15 Then KBC = KBC1: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 16 Then KBC = KBC1: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 17 Then KBC = KBC1: KSC = KSC2: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 18 Then KBC = KBC1: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 19 Then KBC = KBC2: KSC = (KSC1 + KSC2) / 2: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 20 Then KBC = KBC2: KSC = KSC1: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 21 Then KBC = KBC2: KSC = KSC2: KT = (KT1 + KT2) / 2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 22 Then KBC = KBC2: KSC = (KSC1 + KSC2) / 2: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 23 Then KBC = KBC2: KSC = (KSC1 + KSC2) / 2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 24 Then KBC = KBC2: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 25 Then KBC = KBC2: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 26 Then KBC = KBC2: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 27 Then KBC = KBC2: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 28 Then KBC = KBC2: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 29 Then KBC = KBC2: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 30 Then KBC = KBC2: KSC = KSC1: KT = KT1: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

If steve = 31 Then KBC = KBC2: KSC = KSC2: KT = KT2: Sheets(NamesArray(steve + 1)).Range("G9") = "Max": Sheets(NamesArray(steve + 1)).Range("G10") = "Max": Sheets(NamesArray(steve + 1)).Range("G11") = "Max"

Sheets(NamesArray(steve + 1)).Range("A3") = "Young's Modulus"
Sheets(NamesArray(steve + 1)).Range("A4") = "Poisson's ratio"
Sheets(NamesArray(steve + 1)).Range("B3").Value = Young's Modulus
Sheets(NamesArray(steve + 1)).Range("B4").Value = Poisson Modulus
Sheets(NamesArray(steve + 1)).Range("A6").Value = "Pitch"
For IteratorInfinity = 1 To n
ActiveSheet.Range("A15").Offset(IteratorInfinity - 1, 0).Value = IteratorInfinity
ActiveSheet.Range("B15").Offset(IteratorInfinity - 1, 0).Value = KBC
ActiveSheet.Range("C15").Offset(IteratorInfinity - 1, 0).Value = KSC
ActiveSheet.Range("D15").Offset(IteratorInfinity - 1, 0).Value = KT
Beta = 2 + (KT / KBC) + (KT / KSC)
Gamma1 = (Beta / 2) + (Sqr((Beta ^ 2) - 4) / 2)
Gamma2 = (Beta / 2) - (Sqr((Beta ^ 2) - 4) / 2)
A = 703000 * (1 + (Gamma1 ^ 17) / (Gamma2 ^ 17 - Gamma1 ^ 17))
B = -703000 * ((Gamma1 ^ 17) / (Gamma2 ^ 17 - Gamma1 ^ 17))
S = A * (Gamma1 ^ IteratorInfinity) + B * (Gamma2 ^ IteratorInfinity)
Percentage = 100 * S / load1
ActiveSheet.Range("E15").Offset(IteratorInfinity - 1, 0).Value = Beta
ActiveSheet.Range("F15").Offset(IteratorInfinity - 1, 0).Value = Gamma1
ActiveSheet.Range("G15").Offset(IteratorInfinity - 1, 0).Value = Gamma2
ActiveSheet.Range("H15").Offset(IteratorInfinity - 1, 0).Value = A
ActiveSheet.Range("I15").Offset(IteratorInfinity - 1, 0).Value = B
ActiveSheet.Range("J15").Offset(IteratorInfinity - 1, 0).Value = S
ActiveSheet.Range("K15").Offset(IteratorInfinity - 1, 0).Value = Percentage

Next IteratorInfinity
ActiveSheet.Range("L15").Formula = "=703000 - J15"
ActiveSheet.Range("L16").Formula = "=J15 - J16"
ActiveSheet.Range("L16: L31").FillDown
ActiveSheet.Range("B34").Formula = "=SUM(L15:L32) / 17"
ActiveSheet.Range("A34").Value = Meanload = ActiveSheet.Range("B34").Value
ActiveSheet.Range("M15").Formula = "=L15/M34"
ActiveSheet.Range("M15:M31").FillDown
ActiveSheet.Range("N15").Formula = "+L15*100/703000"
ActiveSheet.Range("N15:N31").FillDown

' Gadgets
Cells.Select
With Selection.Font
    .Name = "Microsoft Sans Serif"
    .Size = 8
    .Strikethrough = False
    .Superscript = False
    .Subscript = False
    .OutlineFont = False
    .Shadow = False
    .Underline = xlUnderlineStyleNone
    .ColorIndex = xlAutomatic
End With
Selection.Interior.ColorIndex = xlNone
With Selection.Interior
    .ColorIndex = 2
    .Pattern = xlSolid
End With
Cells.EntireColumn.AutoFit
Range("A1").Select

Next steve
Sheets("Input").Activate

Application.EnableEvents = True

End Sub

Sub DeleteVariations()
    ' Delete previous sheets if they exist
    ' deletes sheets in the active workbook
    Application.DisplayAlerts = False

    Dim CountSheet As Integer ' Iterator on sheets
    Dim NumSheet As Integer ' Number of sheets on xls file
    Dim NameSheet() As String ' Array of names

    NumSheet = Worksheets.Count

    If NumSheet > 1 Then
        ' Redimension NameSheet array
        ReDim NameSheet(NumSheet)

        ' Store sheet names in an array
        For CountSheet = 1 To NumSheet
            NameSheet(CountSheet) = Sheets(CountSheet).Name
        Next CountSheet

        ' Delete sheets
        For CountSheet = 1 To NumSheet
            If NameSheet(CountSheet) <> "Input" Then
                If NameSheet(CountSheet) <> "ANSYS RESULTS" Then
                    Sheets(NameSheet(CountSheet)).Delete
                End If
            End If
        Next CountSheet

    End If
    Application.DisplayAlerts = True
End Sub

Sub DeleteFeaResults()
Delete FEA RESULTS
' deletes sheets in the active workbook
If IsSheetExists("ANSYS RESULTS") Then
    Application.DisplayAlerts = False
    Sheets("ANSYS RESULTS' ).Delete
    Application.DisplayAlerts = True
End If
End Sub

Private Sub ExtractLoads()
' Extracts the effective loads for every analysis
Dim I As Integer
Dim J As Integer

ReDim Thread Loads(n, h)

For J = 1 To h
    ' Select Variation Number
    Sheets("Variation & J' ).Activate
    ' Extracts Thread Loads of Variation(NumAnalysis)
    For I = 1 To n
    Next I
    Next J

Sheets("Input") . Activate
End Sub

Sub FEA()
    Call ReadVariables
    Call ExtractLoads

    ' CALL THE WAITING FORM
    Call UserForm_Initialize

    ' WRITE TO HD THE ANALYSIS FILE
    Call AnsysFile

    ' WRITE TO HD THE STARTER FILE
    Call ExeFile

    ' EXECUTE ANSYS PROGRAM
    ' Shell FilePath & "master.bat"
    Call ShellAndWait(Excel. ActiveWorkbook. Path & "master.bat", vbMinimizedFocus)

    ' READ RESULTS FROM FILE
    ' Call ReadResults

    ' CLOSE THE WAITING FORM
    ' Call UserForm_Hide

    ' Clean rubbish
    ' Call Clean

End Sub

Private Sub ExeFile()

Dim FileName As String
Dim FilePath As String

FilePath = Excel. ActiveWorkbook. Path

' WRITE MASTER.BAT FILE FOR ANALYSIS
FileName = FilePath & "master.bat"

Open FileName For Output As #1
    Print #1, "@echo off"
    Print #1, "set ANS_CONSEC=YES"
End Sub
Private Sub AnsysFile()

Dim FileName As String
Dim I As Long
Dim J As Long

FileName = Excel.ActiveWorkbook.Path & \"FEA.MAC\" 

Open FileName For Output As #1
Print #1, "/TITLE, NUT FEA"
Print #1, "/FILNAME, NUT, 1"
Print #1, "/PREP7"
Print #1, "/DEFINE ARRAY [FEALOADS) WITH VARIATION LOADS"
Print #1, "I"
Print #1, "/DIM,FEALOADS,ARRAY, &n &", &h
Print #1, "I"
Print #1, "/DEFINE ARRAY [FEARES) TO STORE RESULTANT STRAINS"
Print #1, "/DIM,FEARES,ARRAY, &h &", 6"
Print #1, "/FILLING FEALOADS ARRAY"

For j = 1 To h
  For I = 1 To n
    Print #1, "; FEALOADS( &I &", &")= &Thread Loads(i, j) / 24 \There are 24 nodes in every thread\n    Next I
  Next j

  ' Here starts the nasty bit
  '-> Global cycle of h variations
  Print #1, "/DO, N, 1 &h
  Print #1, "ICOM SAVING PARAMETERS TO RESTART THE PROBLEM"
  Print #1, "PARSAV,ALL,PARAM,DAT"
  Print #1, "FINISH"
  Print #1, "/COPY, ORIG,db, ' &Excel. ActiveWorkbook. Path & \"NUT,db\"
  Print #1, "ICOM RETRIEVING PARAMETERS FROM, DAT FILE"
  Print #1, "PARRES,NEW,PARAM,DAT"

  '-> Enter solution processor
  Print #1, "/SOLU"
  Print #1, "/DO, M, 1 ;/ &n

  '-> Select thread M
  Print #1, "CMSEL,S,TN%M%,NODE"
  Print #1, "F,ALL,FZ,FEALOADS(M,N)"
  Print #1, "F,ALL,FX,0.236,FEALOADS(M,N)"

  '-> Re-select everything

Close #1
End Sub
Print #1, "ALLSEL, ALL"

' --> End cycle to load every thread
Print #1, "ENDDO"

' --> Starts a solution
Print #1, "SOLVE"

' --> After solution enters postprocessor
Print #1, "POST1"

' --> Change results csys to 1
Print #1, "RSYS, 1"

' --> Get ey and ez from nodes 1929, 1957, 1991
Print #1, "GET, FEARES(N, 1), NODE, 1929, EPEL, Y"
Print #1, "GET, FEARES(N, 2), NODE, 1929, EPEL, Z"
Print #1, "GET, FEARES(N, 3), NODE, 1957, EPEL, Y"
Print #1, "GET, FEARES(N, 4), NODE, 1957, EPEL, Z"
Print #1, "GET, FEARES(N, 5), NODE, 1991, EPEL, Y"
Print #1, "GET, FEARES(N, 6), NODE, 1991, EPEL, Z"

' --> Exit postprocessor
Print #1, "FINISH"

' --> Delete files
Print #1, "ICOM DELETING FILES"
Print #1, "DELETE, NUT, esav"
Print #1, "DELETE, NUT, emat"
Print #1, "DELETE, NUT, mntr"
Print #1, "DELETE, NUT, rst"
Print #1, "DELETE, NUT, br"

' --> Save parameters
Print #1, "ICOM SAVING PARAMETERS TO RESTART THE PROBLEM"
Print #1, "PARSAV, ALL, PARAM, DAT"

' --> Close FE MODEL WITHOUT SAVING
Print #1, "EXIT, NOSAVE"

' --> Clears database
Print #1, "ICOM CLEARING THE DATABASE"
Print #1, "CLEAR"

Print #1, "ICOM RETRIEVING PARAMETERS FROM .DAT FILE"
Print #1, "PARRES, NEW, PARAM, DAT"

Print #1, "FINISH"
Print #1, "DELETE, NUT, db," & Excel.ActiveWorkbook.Path & ""

' --> End cycle of h variations
Print #1, "ENDDO"

' --> Write final results to a file
Print #1, "CFOPEN, RESULTS, csv"
Print #1, "VWRITE, FEARES(1,1), FEARES(1,2), FEARES(1,3), FEARES(1,4), FEARES(1,5), FEARES(1,6)"
Print #1, ";E11.5, ,E11.5, ,E11.5, ,E11.5, ,E11.5, ,E11.5"
Print #1, "CFCLOSE"

Close #1
End Sub

Public Sub ReadResults()
    Dim I As Integer
    Dim J As Integer

' Read Variables
    Call ReadVariables

    ReDim AnsysResults(h, 6)

' Creates a new spreadsheet "ANSYS RESULTS"
    If Not IsSheetExists("ANSYS RESULTS") Then
Worksheets.Add After:=Sheets("Input")
ActiveSheet.Name = "ANSYS RESULTS"
Else

Application.DisplayAlerts = False

Sheets("ANSYS RESULTS").Delete

Application.DisplayAlerts = True

Worksheets.Add After:=Sheets("Input")
ActiveSheet.Name = "ANSYS RESULTS"
End If

' Gadgets
Cells.Select
With Selection.Font
 .Name = "Microsoft Sans Serif"
 .Size = 10
 .Strikethrough = False
 .Superscript = False
 .Subscript = False
 .OutlineFont = False
 .Shadow = False
 .Underline = xlUnderlineStyleNone
 .ColorIndex = xlAutomatic
End With

With Selection.Interior
 .ColorIndex = 2
 .Pattern = xlSolid
End With

' Fills the sheet
ActiveSheet.Range("A1") = "ANSYS RESULTS"
ActiveSheet.Range("A3") = "Variation"
ActiveSheet.Range("B3") = "Hoop Strain (node: 1929)"
ActiveSheet.Range("C3") = "Axial Strain (node: 1929)"
ActiveSheet.Range("D3") = "Hoop Strain (node: 1957)"
ActiveSheet.Range("E3") = "Axial Strain (node: 1957)"
ActiveSheet.Range("F3") = "Hoop Strain (node: 1991)"
ActiveSheet.Range("G3") = "Axial Strain (node: 1991)"

Range("A1").Select
Selection.Font.Bold = True
With Selection.Font
 .Name = "Microsoft Sans Serif"
 .Size = 14
 .Strikethrough = False
 .Superscript = False
 .Subscript = False
 .OutlineFont = False
 .Shadow = False
 .Underline = xlUnderlineStyleNone
 .ColorIndex = xlAutomatic
End With

Rows("3:3").Select
Selection.Font.Bold = True
With Selection
 .HorizontalAlignment = xlCenter
 .VerticalAlignment = xlBottom
 .WrapText = False
 .Orientation = 0
 .AddIndent = False
 .IndentLevel = 0
 .ShrinkToFit = False
 .ReadingOrder = xlContext
 .MergeCells = False
End With

Cells.EntireColumn.AutoFit

Range("A1").Select

*Open the results and stored in array RESULTS
 Open Excel. ActiveWorkbook.Path & "\RESULTS.csv" For Input As #1

250
For I = 1 To h
    For J = 1 To 6
        Input #1, AnsysResults(i, j)
        ActiveSheet.Range("A4").Offset(i - 1, 0).Value = i
        ActiveSheet.Range("B4").Offset(i - 1, j - 1).Value = AnsysResults(i, j)
    Next J
    Next I
Close #1

'Clean Files
Call Clean
End Sub

Public Function IsSheetExists(sname) As Boolean
    Dim x As Object
    On Error Resume Next
    Set x = ActiveWorkbook.Sheets(sname)
    If Err = 0 Then IsSheetExists = True _
        Else IsSheetExists = False
    End Function

Private Sub Clean()
    Dim Path As String
    Path = Excel.ActiveWorkbook.Path
    Call DeleteFile(Path & ".master.bat")
    Call DeleteFile(Path & ".FEA.MAC")
    Call DeleteFile(Path & ".file.bat")
    Call DeleteFile(Path & ".file.err")
    Call DeleteFile(Path & ".file.log")
    Call DeleteFile(Path & ".NUT.BCS")
    Call DeleteFile(Path & ".NUT.err")
    Call DeleteFile(Path & ".NUT.full")
    Call DeleteFile(Path & ".NUT.log")
    Call DeleteFile(Path & ".NUT.PVTS")
    Call DeleteFile(Path & ".out.bat")
    Call DeleteFile(Path & ".PARAM.DAT")
    Call DeleteFile(Path & ".RESULTS.csv")
    Call DeleteFile(Path & ".NUT.DO2")
    Call DeleteFile(Path & ".NUT.DO3")
    Call DeleteFile(Path & ".NUT.db")
End Sub

Public Sub DeleteFile(Killfile As String)
    If Len(Dir$(Killfile)) > 0 Then
        SetAttr Killfile, vbNormal
        Kill Killfile
    End If
End Sub

Private Sub UserForm_Initialize()
    Load Form
    Form.Show
End Sub

Private Sub UserForm_Hide()
    Unload Form
End Sub
Appendix G

Surface strain variation with respect to different range of spring constant variations –

All units are in percentage (%)

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>18.93</td>
<td>4.22</td>
</tr>
<tr>
<td>Mean</td>
<td>0.25</td>
<td>0.06</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>7.04</td>
<td>1.57</td>
</tr>
</tbody>
</table>

Figure G-1  Maximum spring constant variation

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>17.88</td>
<td>3.99</td>
</tr>
<tr>
<td>Mean</td>
<td>0.22</td>
<td>0.05</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.65</td>
<td>1.48</td>
</tr>
</tbody>
</table>

Figure G-2  95 % spring constant variation
Spring Constant Variation - 90% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
<td>8.00</td>
</tr>
<tr>
<td>-1.5</td>
<td>6.00</td>
<td>4.00</td>
</tr>
<tr>
<td>-0.5</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0</td>
<td>2.5</td>
<td>-1.5</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
<td>2.5</td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure G-3 90% spring constant variation

Spring Constant Variation - 85% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>16.85</td>
<td>3.76</td>
</tr>
<tr>
<td>-1.5</td>
<td>15.82</td>
<td>3.53</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
<td>4.00</td>
</tr>
<tr>
<td>0</td>
<td>2.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.5</td>
<td>-1.5</td>
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<tr>
<td>1.5</td>
<td>0.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Figure G-4 85% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>16.85</td>
<td>3.76</td>
</tr>
<tr>
<td>Mean</td>
<td>0.19</td>
<td>0.04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>6.26</td>
<td>1.40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>15.82</td>
<td>3.53</td>
</tr>
<tr>
<td>Mean</td>
<td>0.17</td>
<td>0.04</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.88</td>
<td>1.31</td>
</tr>
</tbody>
</table>
**Spring Constant Variation - 80% error**

![Graph showing frequency distribution for Spring Constant Variation - 80% error](image)

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure G-5**  80 % spring constant variation

<table>
<thead>
<tr>
<th>Range</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>14.81</td>
<td>3.30</td>
</tr>
<tr>
<td>Mean</td>
<td>0.15</td>
<td>0.03</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.50</td>
<td>1.23</td>
</tr>
</tbody>
</table>

**Spring Constant Variation - 75% error**

![Graph showing frequency distribution for Spring Constant Variation - 75% error](image)

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure G-6**  75 % spring constant variation

<table>
<thead>
<tr>
<th>Range</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>13.81</td>
<td>3.08</td>
</tr>
<tr>
<td>Mean</td>
<td>0.13</td>
<td>0.03</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>5.12</td>
<td>1.14</td>
</tr>
</tbody>
</table>
Spring Constant Variation - 70% error

![Graph of spring constant variation with 70% error showing frequency distribution for Percentage Error ranging from -2.5 to 2.5.](image)

- **Axial Strain**
- **Hoop Strain**

### Table G-7: 70% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>12.82</td>
<td>2.86</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.11</td>
<td>0.03</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>4.75</td>
<td>1.06</td>
</tr>
</tbody>
</table>

**Figure G-7** 70% spring constant variation

---

Spring Constant Variation - 65% error

![Graph of spring constant variation with 65% error showing frequency distribution for Percentage Error ranging from -2.5 to 2.5.](image)

- **Axial Strain**
- **Hoop Strain**

### Table G-8: 65% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Range</strong></td>
<td>11.84</td>
<td>2.64</td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>0.09</td>
<td>0.02</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>4.39</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**Figure G-8** 65% spring constant variation
Spring Constant Variation - 60% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- Blue bars represent Axial Strain.
- Red bars represent Hoop Strain.

Figure G-9  60% spring constant variation

<table>
<thead>
<tr>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>10.87</td>
</tr>
<tr>
<td>Mean</td>
<td>0.08</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>4.03</td>
</tr>
</tbody>
</table>

Spring Constant Variation - 55% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

- Blue bars represent Axial Strain.
- Red bars represent Hoop Strain.

Figure G-10  55% spring constant variation

<table>
<thead>
<tr>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>9.92</td>
</tr>
<tr>
<td>Mean</td>
<td>0.07</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>3.67</td>
</tr>
</tbody>
</table>
Spring Constant Variation - 50% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Axial Strain: Blue | Hoop Strain: Red

Range: 8.97 | Mean: 0.05 | Standard Deviation: 3.32

Figure G-11 50 % spring constant variation

Spring Constant Variation - 45% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
</tr>
<tr>
<td>1.5</td>
<td>0.00</td>
</tr>
<tr>
<td>2.5</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Axial Strain: Blue | Hoop Strain: Red

Range: 8.03 | Mean: 0.04 | Standard Deviation: 2.97

Figure G-12 45 % spring constant variation
### Spring Constant Variation - 40% error

![Bar chart showing frequency distribution of Percentage Error for Axial Strain and Hoop Strain with 40% spring constant variation.](image)

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
<td>-2.5</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
<td>-1.5</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
<td>-0.5</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>-2.00</td>
<td>1.5</td>
</tr>
<tr>
<td>2.5</td>
<td>-4.00</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Figure G-13** 40% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>7.10</td>
<td>1.58</td>
</tr>
<tr>
<td>Mean</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.62</td>
<td>0.58</td>
</tr>
</tbody>
</table>

### Spring Constant Variation - 35% error

![Bar chart showing frequency distribution of Percentage Error for Axial Strain and Hoop Strain with 35% spring constant variation.](image)

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>10.00</td>
<td>-2.5</td>
</tr>
<tr>
<td>-1.5</td>
<td>8.00</td>
<td>-1.5</td>
</tr>
<tr>
<td>-0.5</td>
<td>6.00</td>
<td>-0.5</td>
</tr>
<tr>
<td>0</td>
<td>4.00</td>
<td>0.00</td>
</tr>
<tr>
<td>0.5</td>
<td>2.00</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>-2.00</td>
<td>1.5</td>
</tr>
<tr>
<td>2.5</td>
<td>-4.00</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**Figure G-14** 35% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>6.18</td>
<td>1.38</td>
</tr>
<tr>
<td>Mean</td>
<td>0.03</td>
<td>0.01</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>2.28</td>
<td>0.51</td>
</tr>
</tbody>
</table>
Figure G-15  30 % spring constant variation

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>-2.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.5</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Figure G-16  25 % spring constant variation

<table>
<thead>
<tr>
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<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>-2.5</td>
<td>-2.5</td>
</tr>
<tr>
<td>-1.5</td>
<td>-1.5</td>
<td>-1.5</td>
</tr>
<tr>
<td>-0.5</td>
<td>-0.5</td>
<td>-0.5</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Spring Constant Variation - 20% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>0</td>
</tr>
<tr>
<td>-1.5</td>
<td>5</td>
</tr>
<tr>
<td>-0.5</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>5</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
</tr>
</tbody>
</table>

[Diagram showing frequency distribution with bars for Axial Strain and Hoop Strain.]

Figure G-17 20% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>3.48</td>
<td>0.77</td>
</tr>
<tr>
<td>Mean</td>
<td>0.01</td>
<td>0.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.28</td>
<td>0.29</td>
</tr>
</tbody>
</table>

Spring Constant Variation - 15% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>0</td>
</tr>
<tr>
<td>-1.5</td>
<td>5</td>
</tr>
<tr>
<td>-0.5</td>
<td>10</td>
</tr>
<tr>
<td>0</td>
<td>15</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
</tr>
<tr>
<td>1.5</td>
<td>5</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
</tr>
</tbody>
</table>

[Diagram showing frequency distribution with bars for Axial Strain and Hoop Strain.]

Figure G-18 15% spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>2.60</td>
<td>0.58</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.96</td>
<td>0.21</td>
</tr>
</tbody>
</table>
Spring Constant Variation - 10% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>0</td>
</tr>
<tr>
<td>-1.5</td>
<td>0</td>
</tr>
<tr>
<td>-0.5</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
</tr>
</tbody>
</table>

- Axial Strain
- Hoop Strain

Figure G-19 10 % spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>1.72</td>
<td>0.38</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.63</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Spring Constant Variation - 5% error

<table>
<thead>
<tr>
<th>Percentage Error</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2.5</td>
<td>0</td>
</tr>
<tr>
<td>-1.5</td>
<td>0</td>
</tr>
<tr>
<td>-0.5</td>
<td>5</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
</tr>
<tr>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>0</td>
</tr>
</tbody>
</table>

- Axial Strain
- Hoop Strain

Figure G-20 5 % spring constant variation

<table>
<thead>
<tr>
<th></th>
<th>Axial Strain</th>
<th>Hoop Strain</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>0.86</td>
<td>0.19</td>
</tr>
<tr>
<td>Mean</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.32</td>
<td>0.07</td>
</tr>
</tbody>
</table>
Appendix H

The results in section 5.3 show how the surface strain changes along the groove. In aid to find the sensitivity function the results have to be modified. The change in strain with respect to normal strain and the change in location with respect to ideal location are plotted below and the gradient of the graphs are the individual sensitivity functions.

![Summary table for hoop strain variation](Image)

![Variation in hoop strain with respect to the grating location (y' direction)](Image)

---

**Figure H-1**

Summary table for hoop strain variation

<table>
<thead>
<tr>
<th>Distance</th>
<th>Strain</th>
<th>$\delta y'$</th>
<th>$\delta y'/\gamma'$</th>
<th>$\delta \epsilon'$</th>
<th>$\delta \epsilon'/\epsilon'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.00E-04</td>
<td>-</td>
<td>-1</td>
<td>-3.82E-05</td>
<td>-7.10E-02</td>
</tr>
<tr>
<td>2.902</td>
<td>5.08E-04</td>
<td>3.421</td>
<td>-0.541</td>
<td>-3.02E-05</td>
<td>-5.61E-02</td>
</tr>
<tr>
<td>6.281</td>
<td>5.37E-04</td>
<td>0.041</td>
<td>-0.007</td>
<td>-6.25E-07</td>
<td>-1.16E-03</td>
</tr>
<tr>
<td>6.309</td>
<td>5.38E-04</td>
<td>0.014</td>
<td>-0.002</td>
<td>-2.33E-08</td>
<td>-4.34E-03</td>
</tr>
<tr>
<td>6.323</td>
<td>5.38E-04</td>
<td>0</td>
<td>0</td>
<td>0.00E+00</td>
<td>0.00E+00</td>
</tr>
<tr>
<td>6.363</td>
<td>5.38E-04</td>
<td>0.04</td>
<td>0.006</td>
<td>2.84E-07</td>
<td>5.27E-04</td>
</tr>
<tr>
<td>9.496</td>
<td>5.61E-04</td>
<td>3.173</td>
<td>0.502</td>
<td>2.33E-05</td>
<td>4.33E-02</td>
</tr>
<tr>
<td>10.816</td>
<td>5.69E-04</td>
<td>4.494</td>
<td>0.711</td>
<td>3.16E-05</td>
<td>5.87E-02</td>
</tr>
<tr>
<td>12.686</td>
<td>5.74E-04</td>
<td>6.363</td>
<td>1.006</td>
<td>3.62E-05</td>
<td>6.73E-02</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Distance</th>
<th>Strain</th>
<th>$\delta x'$</th>
<th>$\delta x'/\gamma'$</th>
<th>$\delta \epsilon'$</th>
<th>$\delta \epsilon'/\epsilon'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5.04E-04</td>
<td>-</td>
<td>-1</td>
<td>-3.42E-05</td>
<td>-6.36E-02</td>
</tr>
<tr>
<td>3.095</td>
<td>5.41E-04</td>
<td>-3.11</td>
<td>0.501</td>
<td>2.76E-06</td>
<td>5.13E-03</td>
</tr>
<tr>
<td>6.191</td>
<td>5.38E-04</td>
<td>0.015</td>
<td>0.002</td>
<td>-3.02E-08</td>
<td>-5.62E-05</td>
</tr>
<tr>
<td>6.201</td>
<td>5.38E-04</td>
<td>0.005</td>
<td>0.001</td>
<td>-1.31E-08</td>
<td>-2.44E-05</td>
</tr>
</tbody>
</table>

**Figure H-2**

Variation in hoop strain with respect to the grating location (y' direction)
Figure H-3  Variation in hoop strain with respect to the grating location (x' direction)

![Hoop Strain - x' direction](image)

Figure H-4  Summary table for axial strain variation

<table>
<thead>
<tr>
<th>Distance</th>
<th>Strain</th>
<th>$\delta y'_{y'}$</th>
<th>$\delta y'/x'$</th>
<th>$\delta e'$</th>
<th>$\delta e'/e'$</th>
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<td>0.00E+00</td>
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</tbody>
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<table>
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<th>$\delta x'$</th>
<th>$\delta x'/x'$</th>
<th>$\delta e'$</th>
<th>$\delta e'/e'$</th>
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Figure H-5  Variation in axial strain with respect to the grating location (y' direction)

Figure H-6  Variation in axial strain with respect to the grating location (x' direction)
## Appendix I

<table>
<thead>
<tr>
<th>Face Angle</th>
<th>$\delta \theta$</th>
<th>$\delta \theta / \theta$</th>
<th>Hoop Strain</th>
<th>$\delta e_2$</th>
<th>$\delta e_2 / \theta$</th>
<th>Axial Strain</th>
<th>$\delta e_1$</th>
<th>$\delta e_1 / \theta$</th>
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</table>

Figure I-1: Variation in hoop and axial strain with respect to the groove trajectory
Figure J-1    Modified nut design
The bolt and nut were made out of EN24 steel with the following properties.

- Tensile Strength: 980 N/mm²
- Yield Stress: 870 N/mm²
- Elongation: 16.8%

So for 2 ¼ UNF8 nut and bolt connection the maximum strength can be worked out the following way.

\[
\text{Area} = \frac{\pi}{4} (D - 0.93820P_H)^2
\]

\(D\) = Diameter
\(P_H\) = Pitch

Stress Area (BS 1580 Parts 1 & 2:1962) = 4.46 inch²
Therefore:
Maximum Strength = 2503349 N = 2.50 MN

It was decided that the maximum load on the bolt should not exceed 200 bars, which is equivalent to 703 kN load. This gives a safety factor of 3.5. This was calculated the following way.

Hydraulic Pressure Area = 35151 mm²
Pressure = 200 bar = 20000 kN/m²

Pressure = Force / Area
Force = Pressure * Area
Force = 20000 kN/m² * 35151 * 10⁻⁶ m²
Force = 703 kN
Appendix L

The results of the experimental tests performed on the nut are presented here. Firstly, the axial strain measurements on the bolt by the laser interferometers will be shown. Ten sets of results are presented here; the averaged results can be seen plotted in Chapter 6.

<table>
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<tr>
<th>Pressure (Bar)</th>
<th>First test</th>
<th></th>
<th></th>
<th></th>
<th>Second test</th>
<th></th>
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<th></th>
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<td>mm</td>
<td>Renishaw reading</td>
<td>Combined reading</td>
<td>Normalised reading</td>
<td>Axial Strain</td>
<td>HP reading</td>
<td>Renishaw reading</td>
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<td>-0.001</td>
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<td>-0.001</td>
<td>0.000</td>
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Figure L-1  Axial strain measured by laser interferometers - First & Second test results
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Figure L-2 Axial strain measured by laser interferometers - Third & Fourth test results

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Figure L-3 Axial strain measured by laser interferometers - Fifth & Sixth test results
Seventh test

Eights test

HP
Renlshaw Combined NormalisedAxial Strain HP Renishaw Combined Normalised
Axial Strain
reading
reading reading reading
reading reading reading
reading

ressure
Bar

mm
-0.001
2

4(

10

mm
0.00
0.00

-0.07

mm
-0.001

mm
microstrain mm
mm
0.00d
0. -1E-021

19.93

0.00

11

0.004

23

mm
-0.001

0.000

18.93

0.0

10

-0.13

-0.004

37.86

0.004

22

36

-0.20

-0.00

56.791

0.00

351

49

-0.27

-0.01

75.72

0.01

-0.01

94.651

0.01

464
59

0.02

73

-0.0

-0.14

-0.004

39.862

-0.22
-0.291

-0.00
-0.01

59.786
79.71

0.00
0.014

-0.01

99.64

0.01

62

0.03

77

-0.424

-0.02

113.57

-0.36

mm
microstrain
0.00N
c

-0.34

12(

-0.44

-0.03

119.56

14(
16(
18
20

-0.51

-0.031

139.501

0.031

88

-0.488

-0.02

132.51

0.02

83

-0.58

-0.034

159.43

0.034

100

-0.557

-0.03

151.44

0.03

95

-0.661
-0.731

0.03
0.04

107
1184

0.05

0.05

115

1

-0.62

-0.05
-0.054

179.32

-0.03
-0.040

0.03
0.04

-0.69

113 -0.62
1251 -0.69

170.37
189.30

18

-0.03
-0.04

179.357
199.28
159.39

0.054

104

14

-0.551

-0.051

139.463

12

-0.48

-0.04

10

-0.41

-0.04
-0.03

-0.26

-0.02

59.741

-0.1
-0.111

-0.02
-0.01

39.81

-0.031

-0.001

-0.341
4
2

-0.05

109

-0.6
-0.59

170.34

-0.051

151.40

0.051

97

-0.523

-0.04

132.47

0.051
0.04

119.530

0.04

856 -0.458

0.04

99.597

0.04

73

-0.04
-0.04

113.54
94.60

0.04

81
69

79.66

0.03
0.02

61
474

0.03

58

-0.251

-0.03
-0.02

75.67
56.74

0.02

450

34
201

-0.184

-0.021

331

-0.012

0.01

191

-0.031

0.001

51

-0.107
-0.03

37.81
18.89

0.021

19.88

0.02
0.013

-0.001

-0.03(

0.001

121

-0.392
-0.324

Tenth test

Ninth test
HP
reading

ar

mm
2

-0.001
-0.07

4

-0.13
-0.21
-0.28

Renishaw Combined Normalised
HP
Renishaw Combined Normalised
Axial Strain
Axial Strain
reading
reading
reading
reading reading
reading
reading

mm
0.00

-0.00

58.40

0.00

361

-0.01

77.87
97.341
116.804

0.01
0.01
0.02

481

136.27

0.030
0.03

85
97

-0.50

-0.03
-0.03

12

10

4
2

11

60
75

mm
0.000
0.00

mm

-0.13

-0.004

-0.001
19.290
38.58

-0.21

-0.00

-0.07

-0.28
-0.35
-0.43

mm
microstrain
0.000
0.
0.00
0.004

111
22

57.86

0.00

35

-0.01

77.1

0.01

47

-0.01
-0.02
-0.030

96.44

0.01

115.72
135.02

0.02
0.03

60
74

-0.03

154.31
173.600

0.03
0.03

1094

192.89

0.041

1211

-0.053

173.56

0.0

117

0.05

10601

85
97

-0.037

155.747
175.214

-0.041

194.68

0.037
0.041

-0.054

175.181

0.054

-0.497
-0.56
1104 -0.64
122
-0.70
118
-0.673

-0.60

-0.05

155.711

0.053

107

-0.603

-0.05

154.27

-0.53

-0.04

136.242

0.04

94

-0.53

-0.04

134.98

0.04

941

-0.04

116.76

0.04

83

-0.46

-0.046

115.69

0.04

82

-0.041
-0.03

97.297
77.82

0.041
0.036

-0.041
-0.03

96.40
77.11

0.041
0.03

711
5

-0.02

0.02
0.02

57.82
38.53

0.02
0.02

-0.01

19.43

0.01

197

-0.012

19.251

0.01

19

-0.001

-0.031

0.001 1

-0.18
-0.10

-0.02
-0.02

45

-0.02

58.361
38.890

71
-0.4
596 -0.33
463 -0.256

50

-0.0

-0.001

-0.03

0.001

4

-0.57
-0.64
-0.71

14

-1 E-03

0.000
23

14
1

1

microstrain mm

-0.0

-0.01
-0.02

18

mm
0.00
0.004

0.00

-0.35
-0.43

2

mm
-0.001
19.470
38.941

10
12

18

4

Axial strain measuredby laser interferometers- Seventh& Ninth test
results

Figure L-4

Pressure

92

-0.67

-0.471

-0.40
-0.33
-0.25
-0.1
-0.11
-0.031

Figure L-5

341

-0.03
-0.041

Axial strain measuredby laser interferometers- Ninth & Tenth test
results

270

33


The next set of results show the strain gauge results for the modified nut as collected by the Whetstone Bridge.

**RAW RESULTS**

<table>
<thead>
<tr>
<th>Gauge</th>
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<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
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</table>

All values are in microstrain

**NORMALISED RESULTS**

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<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
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<td>133</td>
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<td>-10</td>
</tr>
</tbody>
</table>

All values are in microstrain

Figure L-6 Electric resistance strain gauge results in Raw and Normalised formats on all six sides of the nut.
The results presented in Figure L-6 show the readings collected for each strain gauge. To calculate the principle strains (the axial and the hoop strains) these results need to be converted using Equation 6-2. The converted results for the ten tests carried out can be seen below. (All results are in microstrain)

**First test**

<table>
<thead>
<tr>
<th>Pressure/Load</th>
<th>Groove</th>
<th>Flat face</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 bar</td>
<td>50 bar</td>
<td>100 bar</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>214</td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>-178</td>
</tr>
</tbody>
</table>

**Second test**

<table>
<thead>
<tr>
<th>Pressure/Load</th>
<th>Groove</th>
<th>Flat face</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 bar</td>
<td>50 bar</td>
<td>100 bar</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>217</td>
</tr>
<tr>
<td>Hoop Strain</td>
<td>0</td>
<td>-176</td>
</tr>
</tbody>
</table>

**Third test**

<table>
<thead>
<tr>
<th>Pressure/Load</th>
<th>Groove</th>
<th>Flat face</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 bar</td>
<td>50 bar</td>
<td>100 bar</td>
</tr>
<tr>
<td>Axial Strain</td>
<td>0</td>
<td>213</td>
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<td>Hoop Strain</td>
<td>0</td>
<td>-175</td>
</tr>
</tbody>
</table>

Figure L-7 Electric resistance strain gauge converted results – First test

Figure L-8 Electric resistance strain gauge converted results – Second test

Figure L-9 Electric resistance strain gauge converted results – Third test
### Fourth test

<table>
<thead>
<tr>
<th>Principle strain</th>
<th>Pressure/Load</th>
<th>0 bar</th>
<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
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</thead>
<tbody>
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<td>212</td>
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<td>529</td>
<td>663</td>
<td>637</td>
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<tr>
<td>Flat face</td>
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<td>-180</td>
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<tr>
<td>Flat face</td>
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<td>0</td>
<td>143</td>
<td>242</td>
<td>315</td>
<td>374</td>
<td>379</td>
<td>350</td>
<td>205</td>
<td>8</td>
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Figure L-10  Electric resistance strain gauge converted results – Fourth test

### Fifth test

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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<td>-532</td>
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<td>-256</td>
<td>-11</td>
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<td>0</td>
<td>143</td>
<td>242</td>
<td>315</td>
<td>374</td>
<td>379</td>
<td>350</td>
<td>205</td>
<td>8</td>
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<tr>
<td>Flat face</td>
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<td>-81</td>
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<td>-255</td>
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Figure L-11  Electric resistance strain gauge converted results – Fifth test

### Sixth test

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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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</thead>
<tbody>
<tr>
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<td>Axial Strain</td>
<td>0</td>
<td>211</td>
<td>386</td>
<td>528</td>
<td>662</td>
<td>636</td>
<td>527</td>
<td>283</td>
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<td>-428</td>
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Figure L-12  Electric resistance strain gauge converted results – Sixth test

### Seventh test

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<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
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<th>0 bar</th>
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<tbody>
<tr>
<td>Groove</td>
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<td>0</td>
<td>211</td>
<td>386</td>
<td>528</td>
<td>662</td>
<td>636</td>
<td>527</td>
<td>283</td>
<td>-6</td>
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<tr>
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<td>Hoop Strain</td>
<td>0</td>
<td>-181</td>
<td>-298</td>
<td>-410</td>
<td>-534</td>
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<td>-256</td>
<td>-13</td>
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<tr>
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<td>0</td>
<td>142</td>
<td>241</td>
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<td>373</td>
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<td>349</td>
<td>204</td>
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<tr>
<td>Flat face</td>
<td>Hoop Strain</td>
<td>0</td>
<td>-11</td>
<td>-83</td>
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Figure L-13  Electric resistance strain gauge converted results – Seventh test
### Eighth test

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<th>150 bar</th>
<th>200 bar</th>
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<th>0 bar</th>
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<td>644</td>
<td>535</td>
<td>291</td>
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<td>-291</td>
<td>-403</td>
<td>-527</td>
<td>-471</td>
<td>-425</td>
<td>-251</td>
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<tr>
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<td>Axial Strain</td>
<td>0</td>
<td>148</td>
<td>247</td>
<td>320</td>
<td>379</td>
<td>384</td>
<td>355</td>
<td>210</td>
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<td></td>
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<td>0</td>
<td>-9</td>
<td>-81</td>
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Figure L-14 Electric resistance strain gauge converted results – Eighth test

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<th>150 bar</th>
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<td>-409</td>
<td>-533</td>
<td>-477</td>
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<td>-257</td>
<td>-12</td>
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<td>Flat face</td>
<td>Axial Strain</td>
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<td>139</td>
<td>238</td>
<td>311</td>
<td>370</td>
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<td>346</td>
<td>201</td>
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Figure L-15 Electric resistance strain gauge converted results – Ninth test

### Tenth test

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<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
</tr>
</thead>
<tbody>
<tr>
<td>Groove</td>
<td>Axial Strain</td>
<td>0</td>
<td>212</td>
<td>388</td>
<td>530</td>
<td>664</td>
<td>638</td>
<td>529</td>
<td>285</td>
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<td>0</td>
<td>-180</td>
<td>-297</td>
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<td>-431</td>
<td>-257</td>
<td>-12</td>
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<td>Flat face</td>
<td>Axial Strain</td>
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<td>238</td>
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<td>346</td>
<td>201</td>
<td>4</td>
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<td></td>
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<td>0</td>
<td>-13</td>
<td>-77</td>
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<td>-121</td>
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<td>-8</td>
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</table>

Figure L-16 Electric resistance strain gauge converted results – Tenth test
Figure M-1  Modified washer design
Appendix N

The experimental test results for the washer are presented here. Figure N-1 shows the raw and the normalised strain reading results from the Whetstone Bridge, followed by the calculated principle strain results for the 10 tests carried out. The principle strains were calculated according to Equation 6-2.

### RAW RESULTS

<table>
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<th>Gauge Location</th>
<th>Pressure</th>
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<th>50 bar</th>
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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<tbody>
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<td>-54</td>
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<td>-68</td>
</tr>
<tr>
<td>2 Groove 2</td>
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<td>-36</td>
<td>-76</td>
<td>-115</td>
<td>-154</td>
<td>-194</td>
<td>-155</td>
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<td>-121</td>
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<tr>
<td>4 Flat</td>
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</table>

All values are in microstrain

### NORMALISED RESULTS

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<th>Pressure</th>
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<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
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<td>74</td>
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<td>304</td>
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</table>

All values are in microstrain

Figure N-1 Electric resistance strain gauge results in Raw and Normalised formats for the three grooves and the flat side of the washer
### First test

<table>
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<tr>
<th>Principle strain</th>
<th>Pressure/Load</th>
<th>0 bar</th>
<th>50 bar</th>
<th>100 bar</th>
<th>150 bar</th>
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<th>50 bar</th>
<th>0 bar</th>
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<tr>
<td>Axial Strain</td>
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<td>-677</td>
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<td>-345</td>
<td>-177</td>
<td>-3</td>
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<tr>
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<td>226</td>
<td>148</td>
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Figure N-2  Electric resistance strain gauge converted results – First test

### Second test

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<th>150 bar</th>
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<td>224</td>
<td>146</td>
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<td><strong>Flat face</strong></td>
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Figure N-3  Electric resistance strain gauge converted results – Second test

### Third test

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<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<tr>
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<td>155</td>
<td>240</td>
<td>351</td>
<td>298</td>
<td>224</td>
<td>146</td>
<td>-2</td>
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<tr>
<td><strong>Flat face</strong></td>
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<td>259</td>
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Figure N-4  Electric resistance strain gauge converted results – Third test

### Fourth test

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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<td></td>
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</tr>
<tr>
<td>Axial Strain</td>
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<td>-518</td>
<td>-352</td>
<td>-174</td>
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<tr>
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<td>234</td>
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<tr>
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<td>-707</td>
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<td>-7</td>
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<td>272</td>
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Figure N-5  Electric resistance strain gauge converted results – Fourth test
### Fifth test

**Principle strain**

<table>
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<tr>
<th>Pressure/Load</th>
<th>0 bar</th>
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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
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<th>0 bar</th>
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<td></td>
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<td></td>
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</tr>
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<td>Axial Strain</td>
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<td>-345</td>
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<td>-672</td>
<td>-517</td>
<td>-351</td>
<td>-173</td>
<td>-5</td>
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<td>Hoop Strain</td>
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<td>236</td>
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<td>207</td>
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<td></td>
<td></td>
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</table>

**Figure N-6** Electric resistance strain gauge converted results – Fifth test

### Sixth test

**Principle strain**

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<th>0 bar</th>
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<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
</tr>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Axial Strain</td>
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<td>-180</td>
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<td>-507</td>
<td>-669</td>
<td>-515</td>
<td>-346</td>
<td>-176</td>
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<tr>
<td>Hoop Strain</td>
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<td>176</td>
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<td></td>
</tr>
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<td>-1247</td>
<td>-987</td>
<td>-708</td>
<td>-389</td>
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<tr>
<td>Hoop Strain</td>
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<td>365</td>
<td>288</td>
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**Figure N-7** Electric resistance strain gauge converted results – Sixth test

### Seventh test

**Principle strain**

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<th>0 bar</th>
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<th>100 bar</th>
<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<td><strong>Groove</strong></td>
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<td></td>
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<tr>
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<td>-670</td>
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<td>-347</td>
<td>-177</td>
<td>-1</td>
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<td>Hoop Strain</td>
<td>0</td>
<td>89</td>
<td>174</td>
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</tr>
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<tr>
<td>Hoop Strain</td>
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<td>173</td>
<td>274</td>
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<td>292</td>
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**Figure N-8** Electric resistance strain gauge converted results – Seventh test

### Eighth test

**Principle strain**

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<th>Pressure/Load</th>
<th>0 bar</th>
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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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</thead>
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<tr>
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<td>-340</td>
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<td>Hoop Strain</td>
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<td>259</td>
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<td>297</td>
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<td>-693</td>
<td>-380</td>
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<td>158</td>
<td>258</td>
<td>349</td>
<td>275</td>
<td>177</td>
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**Figure N-9** Electric resistance strain gauge converted results – Eighth test
### Ninth test

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<th>150 bar</th>
<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<tr>
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<td>-671</td>
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<td>-344</td>
<td>-174</td>
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<td>269</td>
<td>358</td>
<td>307</td>
<td>231</td>
<td>151</td>
<td>5</td>
</tr>
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<td><strong>Flat face</strong></td>
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<td></td>
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<td>-1240</td>
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<td>-698</td>
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Figure N-10  Electric resistance strain gauge converted results – Ninth test

### Tenth test

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<th>200 bar</th>
<th>150 bar</th>
<th>100 bar</th>
<th>50 bar</th>
<th>0 bar</th>
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<td>388</td>
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<td>664</td>
<td>638</td>
<td>529</td>
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<td>-4</td>
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<td>-431</td>
<td>-257</td>
<td>-12</td>
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<td>-77</td>
<td>-164</td>
<td>-251</td>
<td>-193</td>
<td>-121</td>
<td>-38</td>
<td>-8</td>
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</tbody>
</table>

Figure N-11  Electric resistance strain gauge converted results – Tenth test
Appendix O

The axial and the hoop strain on the washer can be determined theoretically by using the principles of mechanics material assuming that the washer has no grooves.

\[ \varepsilon_a = \frac{4P}{E\pi(D_o^2 - D_i^2)} \]

Equation O-1

\[ \varepsilon_h = \frac{4P \nu}{E\pi(D_o^2 - D_i^2)} \]

Equation O-2

Using these equations it is possible to find the strains at maximum load of 703 kN. At this load the theoretical surface strains for the washer are -1137 microstrains in the axial direction and 341 microstrains in the hoop direction. Assuming that the surface strain is linearly related to the load (and at zero load the strain is zero) it is possible to predict the load on the washer by working backwards from the measured surface strains.
For example at a load of 50 bars (175.8 kN) the experimentally measured strain in the axial direction is -172.7 microstrain and in the hoop direction it is 65.7 microstrain. Using these values the theoretical loading can be calculated as follows.

Axial direction:

\[ \text{Load}_{\text{estimated}} = \frac{-172.7 \times 703 \text{kN}}{-1137} = 106.8 \text{kN} \]  \hspace{1cm} \text{Equation O-3}

Hoop direction:

\[ \text{Load}_{\text{estimated}} = \frac{65.7 \times 703 \text{kN}}{341.1} = 135.4 \text{kN} \]  \hspace{1cm} \text{Equation O-4}
Figure P-1  Washer parameters
Appendix Q

Fatigue data for 0.4% C-Ni-Cr-Mo steel obtained from Corus through a private communication.

Figure Q-1  S-N data showing rotating bending fatigue properties of SNCM439 steel tempered at 580 °C

Figure Q-2  S-N data showing rotating bending fatigue properties of SNCM439 steel tempered at 630 °C
Figure Q-3  S-N data showing rotating bending fatigue properties of SNCM439 steel tempered at 680 °C

Figure Q-4  S-N data showing uniaxial stress fatigue properties of SNCM439 steel tempered at 630 °C
### Appendix R

<table>
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<th>Nominal diameter</th>
<th>Pitch</th>
<th>Thread type</th>
<th>External threads (bolts)</th>
<th>Internal threads (nuts)</th>
<th>Nut and Bolt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Major diameter</td>
<td>Minor diameter</td>
<td>Width across flats</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td>max</td>
<td>tol</td>
<td>min</td>
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<td>mm</td>
<td></td>
<td>mm</td>
<td>mm</td>
<td>mm</td>
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<th>Pitch</th>
<th>Thread type</th>
<th>External threads (bolts)</th>
<th>Internal threads (nuts)</th>
<th>Nut and Bolt</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Major diameter</td>
<td>Minor diameter</td>
<td>Width across flats</td>
</tr>
<tr>
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<td>in</td>
<td>in</td>
<td>in</td>
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<td>0.2052</td>
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**Figure R-1** Summary of thread tolerance tables
## Appendix S

### Empirical

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<th>Size</th>
<th>Pitch</th>
<th>Bolt (Stud - Ksc)</th>
<th>Nut (Body - Kbc)</th>
<th>Thread (Kt)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td>Normal N/mm</td>
<td>Minimum N/mm</td>
<td>Maximum N/mm</td>
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<td>inch</td>
<td></td>
<td>Normal %</td>
<td>Minimum %</td>
<td>Maximum %</td>
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<td>4.25E+06</td>
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<td>1/12</td>
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<td>3.62E+07</td>
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</tr>
<tr>
<td>1 - 8UNC</td>
<td>1/8</td>
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### Metric

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<th>Nut (Body - Kbc)</th>
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<td>Minimum N/mm</td>
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**Figure S-1** Summary of spring constants