Measuring performance in higher education

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MEASURING PERFORMANCE IN HIGHER EDUCATION

By

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A Doctoral thesis submitted in partial fulfilment of the requirements for the award of Doctor of Philosophy of the Loughborough University of Technology

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## CONTENTS

**A RESUME**

**ACKNOWLEDGMENTS**

<table>
<thead>
<tr>
<th>CHAPTER 1</th>
<th>OBJECTIVES AND ACTIVITIES OF AN EDUCATIONAL INSTITUTION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1.</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.2.</td>
<td>Objectives of Education</td>
<td>2</td>
</tr>
<tr>
<td>1.3.</td>
<td>Objectives of an Educational Institution</td>
<td>13</td>
</tr>
<tr>
<td>1.4.</td>
<td>Activities of an Educational Institution</td>
<td>15</td>
</tr>
<tr>
<td>1.5.</td>
<td>Summary of Chapter 1</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>References for Chapter 1</td>
<td>21</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 2</th>
<th>RESOURCE ALLOCATION BY FORMULA</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1.</td>
<td>Introduction</td>
<td>24</td>
</tr>
<tr>
<td>2.2.</td>
<td>Formulae Based on Staff Workload</td>
<td>26</td>
</tr>
<tr>
<td>2.3.</td>
<td>Student Load Schemes</td>
<td>34</td>
</tr>
<tr>
<td>2.4.</td>
<td>Allocation of Support Staff, Space and Materials</td>
<td>37</td>
</tr>
<tr>
<td>2.5.</td>
<td>Summary of Chapter 2</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>References for Chapter 2</td>
<td>39</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 3</th>
<th>THE MEASUREMENT OF INPUTS AND OUTPUTS FOR MANAGEMENT INFORMATION</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1.</td>
<td>Introduction</td>
<td>41</td>
</tr>
<tr>
<td>3.2.</td>
<td>Teaching Activity</td>
<td>42</td>
</tr>
<tr>
<td>3.3.</td>
<td>Research Activity</td>
<td>51</td>
</tr>
<tr>
<td>3.4.</td>
<td>Administration and Other Central Services</td>
<td>53</td>
</tr>
<tr>
<td>3.5.</td>
<td>Management Information Systems and their Impact on Input and Output Measurement</td>
<td>54</td>
</tr>
<tr>
<td>3.6.</td>
<td>Summary of Chapter 3</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>References for Chapter 3</td>
<td>57</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 4</th>
<th>THE RELATIONSHIPS BETWEEN INPUTS AND OUTPUTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1.</td>
<td>Introduction</td>
<td>60</td>
</tr>
<tr>
<td>4.2.</td>
<td>The Analytical Approach</td>
<td>61</td>
</tr>
<tr>
<td>4.3.</td>
<td>The Statistical Approach</td>
<td>72</td>
</tr>
<tr>
<td>4.4.</td>
<td>Limitations of Production Function Analysis</td>
<td>83</td>
</tr>
<tr>
<td>4.5.</td>
<td>Summary of Chapter 4</td>
<td>84</td>
</tr>
<tr>
<td></td>
<td>References for Chapter 4</td>
<td>85</td>
</tr>
</tbody>
</table>
## CONTENTS (continued)

### REFERENCES (in alphabetical order)  
Page 158

### APPENDICES

1. **MATHEMATICAL NOTES**
   1.1. The Legg Formula - Theory and Practice  
       Page 167
   1.2. The Effects on Cost per Meeting of an Allocation of  
        Resources to Research  
       Page 168
   1.3. Jean Benard's Ideal University  
       Page 175
   1.4. Layard and Verry's Value-Added  
       Page 179
   1.5. The Categorisation Approach - An Example  
       Page 186
   1.6. Measures of Teaching Activity  
       Page 188
   1.7. The "Delany" Ratios  
       Page 191
   1.8. A Measure of Research Performance  
       Page 195

2. **PUBLISHED PAPERS (Bound as Volume II)**
   Statement of Author's Contribution  
   Page (i)

   2.1. "How Profitable is Teaching?"  
       Page (ii)
   2.2. "Academic Staffing Formulae: With Particular Reference to  
        Advanced Further Education"  
       Page 12
   2.3. "A Note on Academic Staffing Formulae"  
       Page 32
       plus
       "Academic Staffing Schemes Reconsidered - A Comment"
   2.4. "A Proposal for a Management Information System for  
        Further Education"  
       Page 43
   2.5. "Identification of Performance Indices for Teaching Activities"  
       Page 82
   2.6. "A Study of Some Performance Indicators in Higher Education  
        With Particular Reference to Loughborough University and  
        Lanchester Polytechnic"  
       Page 120
   2.7. "A Comparative Timetable Analysis for Undergraduate Programmes  
        in a Polytechnic and a University"  
       Page 184
   2.8. "A Case Study of Some Performance Indicators in Higher  
        Education in the UK"  
       Page 195
   2.9. "A Note on Costing the Teaching Activity in Higher Education"  
       Page 207
   2.10. "Performance Indicators in Higher Education: A Comparative  
        Study"  
       Page 215
   2.11. "Tracing the Efficient Frontier in British Universities"  
       Page 228
       Page 261
<table>
<thead>
<tr>
<th>FIGURES</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A Stage in a Path Analysis Approach</td>
</tr>
<tr>
<td>2</td>
<td>A Stage in a Student Flow Model</td>
</tr>
<tr>
<td>3</td>
<td>Possible Production Functions</td>
</tr>
<tr>
<td>4</td>
<td>The CES Model and its types</td>
</tr>
<tr>
<td>5</td>
<td>Subgroups of the Sample</td>
</tr>
<tr>
<td>6</td>
<td>Standardised Course Parameters</td>
</tr>
<tr>
<td>7</td>
<td>College X's Teaching Activity</td>
</tr>
<tr>
<td>8</td>
<td>College X - Departmental Unit Costs</td>
</tr>
<tr>
<td>9</td>
<td>College X - Intermediate Changes in Teaching Activity</td>
</tr>
<tr>
<td>10</td>
<td>College X - Changes in Departmental Unit Costs</td>
</tr>
<tr>
<td>11</td>
<td>College X - Logic of Cost Allocation on a Meetings Basis</td>
</tr>
<tr>
<td>12</td>
<td>College X - Analysis of Changes for Subject Element M</td>
</tr>
<tr>
<td>13</td>
<td>College X - Analysis of Changes for Subject Element N</td>
</tr>
<tr>
<td>14</td>
<td>College X - Analysis of Changes for Subject Element O</td>
</tr>
<tr>
<td>15</td>
<td>College X - Analysis of Changes for Subject Element P</td>
</tr>
<tr>
<td>16</td>
<td>College X - Allocation of Costs to Courses</td>
</tr>
<tr>
<td>17</td>
<td>College X - Analysis of Changes for Course A</td>
</tr>
<tr>
<td>18</td>
<td>College X - Analysis of Changes for Course B</td>
</tr>
<tr>
<td>19</td>
<td>The More Efficient Institutions</td>
</tr>
<tr>
<td>20</td>
<td>The Frontier for Inputs Per Unit Output</td>
</tr>
<tr>
<td>21</td>
<td>The Frontier for Outputs Per Unit Input</td>
</tr>
<tr>
<td>22</td>
<td>Extra Points for One Input One Output</td>
</tr>
<tr>
<td>23</td>
<td>Extra Points for Two Inputs and One Output</td>
</tr>
<tr>
<td>24</td>
<td>Extra Points for One Input and Two Outputs</td>
</tr>
<tr>
<td>25</td>
<td>Adding the Origin</td>
</tr>
<tr>
<td>26</td>
<td>Definition of the Frontier</td>
</tr>
<tr>
<td>27</td>
<td>A Definition of Efficiency</td>
</tr>
<tr>
<td>28</td>
<td>Carlson's Search Procedure</td>
</tr>
<tr>
<td>29</td>
<td>No Changes in Slope Sign</td>
</tr>
<tr>
<td>30</td>
<td>Nearness to the Frontier</td>
</tr>
<tr>
<td>31</td>
<td>The Effects on Cost Per Meeting of a Resource Allocation</td>
</tr>
<tr>
<td>32</td>
<td>Two Dimensional Performance</td>
</tr>
</tbody>
</table>
Increasingly Institutions in Higher Education are being required to justify themselves. In the absence of agreed objectives, this is difficult. The Author proceeds by representing the objectives of an institution by the mix of activities it chooses to involve itself in. Each activity is examined in turn and the problems of measurement of inputs and outputs for that activity are identified. The point is made that measures of performance implicitly relate to some concept of the process that turns inputs into outputs. The Author, therefore, discusses the various suggestions that have been made. There are few acceptable measures of overall performance and so the Author suggests the use of a profile of performance or an input-output list instead. The thesis draws on the research carried out by the Author (with others) in the area and discusses in detail two approaches: variance analysis and the efficiency frontier, which it is suggested are fruitful areas for further research.
ACKNOWLEDGMENTS

I must take this opportunity to thank Derek Birch for his contributions to our joint efforts. It is not by accident that his name appears regularly within these pages. For five years now we have worked together on a range of projects in Educational Management and I look forward to continuing collaboration.

I must also thank my Head of Department, John Sizer, "without whom all this would not have been possible". His never-ending encouragement and involvement have been invaluable.

Sympathy is offered to Linda Harland who has womanfully struggled through my manuscript and transformed it into the neat, tidy and almost unrecognisable thesis presented here.
CHAPTER 1  OBJECTIVES AND ACTIVITIES OF AN EDUCATIONAL INSTITUTION

1.1. INTRODUCTION

The question "what are the objectives of an educational institution?" assumes that an educational institution is a single entity with clear objectives. Indeed Clark Kerr (1963) coined the term "Multiversity" to emphasise the federal nature of a University as a whole series of communities and activities held together only by a common name, a common governing body and related purposes. However, John Millett (1962) advocates that it is precisely the goals and objectives which bind the University community together. Since educational institutions are usually judged as if single entities it seems appropriate when considering performance to treat them as such and to treat the effects of internal differences over objectives as inefficiencies of those entities, and they will be judged, for as Gerald Fowler stated recently (1978) all public institutions are being asked to account for themselves and higher education cannot expect to be an exception to this.
1.2. OBJECTIVES OF EDUCATION

The purpose of education itself is central to any consideration of institutional objectives. Angus Maddison (1974) suggests that there are five possible purposes of education, namely: a means of personal fulfilment, a mechanism for social continuity and cohesion, an aid to social mobility, the promotion of social equality, and, finally, an economic investment for individuals and society. The last of these leads to the consideration of manpower planning itself and the process of certification.

1.2.1. A Means of Personal Fulfilment

Self-fulfilment often seems a less important aim of education policies than other aims such as examination success. Indeed the present controversy over the examination success or otherwise of certain comprehensive schools reflects this thinking. In Higher Education itself there are few schools of Independent Study within institutions where students can follow a less stereotyped form of education. Indeed, Gideon Fishelson (1972) in considering students' choice of University department concludes that one of the most important factors is the choice of specialisation in school. Thus the student gets onto a narrow escalator early on in his or her education. In the past the extension of compulsory formal schooling has been introduced as an aid to personal fulfilment but there is now more support for educational voucher schemes which allow everyone the possibility of taking time off during working life for further study. Indeed the Minister for Higher Education (Oakes 1978) in a recent speech commented that, of the alternative strategies for Higher Education into the 1990's put up for discussion by the Department of Education and Science (DES), he personally preferred the model which
involved more mature students returning to study in mid-career.

Mark Blaug, in his book (1970), discusses comprehensively the operation of educational voucher schemes but he does make the point that if parents are allowed complete choice in their offspring's education there may be a conflict with any social aims of education. Be that as it may, if personal fulfilment is to be reflected in the objectives of an educational institution then the institution must plan for greater flexibility for increased numbers of mature students and be guided much more by student views than at present.

1.2.2. The Impact on Society

The first report of the Carnegie Commission on Higher Education (USA, 1968) starts with the sentence: "From the beginning of the Republic education at various levels has played a vital role in the building of a strong democratic society". So this important body assumed from the start that the principal role of education was to serve society and meet its needs. If education is to have an impact on the structure of society it can do it in three main ways: It can promote social continuity and cohesion; in other words, perpetuate the status quo. It can help to promote social mobility in that people born into 'lower' socio-economic groups can with its aid move 'up' into other groups. It can also help to achieve social equality by levelling down the benefits of initial socio-economic standing.

In many countries education is financed and provided by governments because they consider it important in promoting or establishing social continuity and cohesion. Indeed, Durkheim (1911) suggested that "Society can survive only if there exists a sufficient degree
of homogeneity; education perpetuates and reinforces this homogeneity ...". However, it seems less clear today what are the agreed ethical standards and accepted truths and, as a result, education does not seem a force for social cohesion and continuity but rather a source of dynamic and unpredictable social change.

In many countries recently there have been cases of troops occupying the universities, situations of student-driven national disturbances as well as the emergence of the highly-educated but anarchic urban guerilla. Obviously these are isolated examples, but they do raise doubts about the efficacy of aiming for social continuity and cohesion as an explicit goal.

If social continuity and cohesion is no longer a wholly realistic aim, how about social mobility or social equality? Although, theoretically, Higher Education is available to able students from all types of background, there is still an uneven distribution of social class amongst those that take it up. The Robbins Report (1963) estimated that only 25% of the University population came from working class backgrounds. This point was emphasised by R. N. Morris (1969) who combined statistical information prepared for the Crowther and Robbins reports and was able to suggest that whereas children of manual workers made up 81% of the total population of maintained secondary schools, they were 65% of the total in grammar schools, 44% of the total in sixth forms, and only 26% of the total in universities. The Crowther Report (1960) itself showed that very able children were leaving school at 15 or 16 years of age and that these were predominantly children from working class backgrounds. There is similar evidence for other European countries. Thus in a situation where about 14% of the
age group in the UK participate in Higher Education (DES, 1978) and where that 14% is biased in its mix of social groups, there is little chance that Higher Education could promote either social mobility or social equality and, in fact, it may do the reverse in reinforcing the relationship between socio-economic standing and education. Paul Taubman (1975) suggests from a survey of 5100 US males that inequalities in earnings can be explained in the main by socio-economic standing and that education simply exaggerates the skewness and kurtosis of the earnings data.

When education is provided for pupils from all types of background there is little evidence of an increase in social mobility or the approach of social equality. The Coleman Report (1966), in its examination of all US schools, found that the educational facilities provided for the various ethnic groups were fairly similar but that the lower performance of the non-white groups was due mainly to family background influences and that the differences increased with the length of the period of schooling. George Mayeske (1969) in a further analysis of the same data attempted to partition the variability in the achievement data between the various suggested factors. He stated that the main factor in promoting achievement was either family background or school quality or both since he was unable to separate the two effects. Alex Mood (1969) in a similar analysis of the data found that school quality was dominated by teacher quality. These analyses led to the controversial 'bussing' policy in the United States where pupils were transported daily outside their neighbourhoods to schools where they mixed with other ethnic groups and thus the school quality effects were balanced out. However, this laudable attempt to promote social mobility and
equality has been under increasing criticism including some from James Coleman himself (1978 THES). He argues now that the 'bussing' policy has not worked because the family background effects have swamped any improvements in schooling and that the answer may lie in better housing and social welfare policies. As a final comment on the objective of affecting society in some way, consider the observation made by Rodmell (1974):

"A broad objective such as preparation for life is really a composite of innumerable sub-objectives each one of which has to be tackled separately if a meaningful answer is to be obtained. To tackle them simultaneously would require a combination of data and analytical resources unlikely to be available in the foreseeable future. What was earlier characterised as a plain man's view of educational output - increased knowledge and understanding - thus has the great merit of being more amenable to measurement and hence capable of reducing uncertainty about attainment of educational objectives; even though it is open to the theoretical objection of being really an intermediate rather than a final output".

1.2.3. An Economic Investment for Individuals and Society

Education adds to the productivity and earning power of the individual and can raise a nation's output. The concept of treating education as an investment in "human capital" was first brought to a wider audience as recently as 1960 by Walter Shultz (in his presidential address to the American Economics
Shultz, in that address and subsequently (1963), argues that much of what, in the past, economists called consumption is really an investment in human capital, although he does point out that most relevant activities involve an element of consumption and investment. He also comments that whereas public investment in physical capital is not transferred to particular individuals, the concept of human capital implies that public investment in education produces benefits to the individuals concerned. However, he argues that earnings represent productivity and so higher earnings for educated individuals imply higher productivity for the economy as a whole.

One of his major contributions to the economics of education has been to establish that the earnings foregone by a student are a major element in the cost of education.

About the same time, Gary Becker (1960) posed the rhetorical question "Is there under-investment in college education?". In his answer and subsequently (1964), he treats education for an individual as a stream of earnings, negative during the course to show earnings foregone and positive after the course to show the higher earnings due to that education. He then calculates the rate of return of this earnings stream treating it as an investment. This is nowadays called the 'private rate of return'. He also calculates the return to society by including public subsidies and by measuring returns using pre-tax increments. This is nowadays called the 'social rate of return'.

Jacob Mincer (1958) also about the same time, produced a pioneering work on personal income distribution in which he drew attention to
the importance of training both in school and on the job as a major explanation of income inequality. In his book (1974), he derives an earnings function to try and explain differences in earnings patterns in terms of education. He argues that the logic of the private rate of return implies that students choose the options which lead to the largest amounts of lifetime earnings discounted to the time of the decision. Comay et al (1973) extend the idea to suggest that students might weigh up the alternative earnings stream at every decision point not just at the time of choice of Higher Education course. Mincer in his empirical work (1974), by considering the 1960 census data, suggests that differences in schooling explain about one-third of the inequality in annual earnings after eight years. If experience (i.e. weeks worked) is taken into account, the explanatory power rises to 50%. He demonstrates that schooling has more explanatory power for groups of the same experience than of the same age, the peak being at seven to nine years after the education ceases, which fits an investment approach to education.

Becker, Mincer and others have produced many empirical studies for the USA but until recently British studies have been retarded through a lack of data. However, the inclusion of an educational qualification in the sample census of 1966 allowed the Author and Derek Birch to apply, in 1973, the ideas of an investment in education and the private rate of return on that investment to the choice of a teaching certificate course following a similar approach to that of Morris and Ziderman (1971) and Khanna and Bottomley (1960). This study (Birch and Calvert, 1973) is appended in Appendix 2.1 of this thesis. It illustrates very well the problems of applying rate of return studies in a British context. Ideally
the age-earnings profiles to be enjoyed by teachers qualifying in 1970 should be adjusted for economic activity and survival and then compared with similarly adjusted age-earnings profiles for similar people who did not choose to go into teaching. However, in reality, the published age-earnings profiles for teachers are cross-sectional not longitudinal, the economic activity rates for teachers and other groups come from the 1966 sample census and the survival rates for teachers and other groups are published only every ten years. It is also difficult to establish comparable age-earnings profiles for those who choose not to teach, hence the profiles for the total population are used instead. Fortunately the authors were able to confirm that virtually all those qualified to teach do, in fact, take up a teaching career. This is a reasonable assumption but one which becomes very dubious when translated into an assumption about the consequences of choosing an engineering degree, for example, as in the Khanna and Bottomley study (1970). Morris and Ziderman in their study (1971) use age-earnings profiles obtained by plotting the available data points and simply joining the points to get a jagged line. The author with Derek Birch (1973), however, believing the age-earnings profile to be more like a curve, produced their profiles by drawing smooth curves through the points available. The same approach is used for economic activity and survival data. This illustrates the basic paucity of the data available upon which these relatively sophisticated techniques are usually applied.

Education as an investment is a concept which has come a long way. Shultz's original comment that public investment in human capital leads to benefits to the particular individuals involved has led to the situation where it is suggested that attempts are made to recoup
those benefits by means of a graduate tax (Glennerster et al, 1968) or by replacing student grants by loans (Blaug, 1970). However, the author has calculated that such a tax for male teachers would be exorbitant given the low returns to investment in a teaching certificate for a man (Birch and Calvert, 1973).

1.2.4. Education for Manpower Planning

One of the aims for Higher Education set out in the DES Education Planning Paper No.1 (1970) is that of meeting the requirements of society for qualified manpower. However, manpower planning always sounds simpler than it is in practice, Mark Blaug (1970) suggests four approaches to manpower planning. Firstly, the consumers of the qualified manpower can be consulted. This is an inexact exercise, even in the short term, since employers are not at present geared up to provide realistic manpower forecasts which take into account expected industrial growth rates, forecast production level, market share and so on. Indeed, Sir Solly Zuckermann in his evidence to the Robbins Committee (1963) admitted that "we have discovered in our successive inquiries that one of the least reliable ways of finding out what industry wants is to go and ask industry!". Secondly, a relationship can be established between industrial output and the demand for qualified manpower. This relationship then enables a forecast of qualified manpower to be produced from a forecast of industrial output. Thirdly, the proportion of the total work-force qualified in a particular way can be established and then manpower forecasts can be derived from demographic forecasts of the future work-force. Finally, the most widely used is the Parnes MRP method which starts with a future target GNP which is then broken down into major sectors for which forecasts of
labour-force requirements can be made. From these forecasts estimates of qualified manpower can be made, and hence educational requirements can be deduced taking into account mortality, migration and retraining possibilities. In all these approaches the major problem is the lack of data and the uncertainty involved given the long lead times between a decision to encourage particular educational programmes and the impact on the labour market which may be entirely different from that postulated at the time of allocation of funds. In any case, even if national targets for certain educational programmes can be established, it is still very difficult to translate this into meaningful objectives for individual institutions unless the overall target is split down, as for teacher training in the UK, into targets for every institution involved.

1.2.5. Education as a Filter

The final concept of an objective for education which is considered here is the credentials approach or the screening hypothesis. Arrow (1973) argues that there is no very close connection between the content of people's education and the content of their jobs, their economically significant skills being learnt on the job. If this is the case then the main significance of education is to provide a screening device for employers to identify people of higher ability. Viewed in this way Higher Education is very expensive and could be replaced by a battery of aptitude tests. However, Layard and Psacharopoulos (1974) show that the rates of return for uncompleted courses are as high as those for completed courses. Also they show that standardised educational differentials rise with age, although employers by then have increased their knowledge about their employee's abilities, and finally aptitude tests, although cheaper, have not
replaced Higher Education in practice. Mincer (1974), as stated earlier, found that the earnings differential due to education existed and peaked about seven or eight years after graduation, which also reinforces the investment concept of education rather than a screening mechanism.
1.3. **OBJECTIVES OF AN EDUCATIONAL INSTITUTION**

The Department of Education and Science, in its Education Planning Paper No.1 (1970), list three main objectives in Higher Education:

(a) to provide higher education for all those who could benefit from it;

(b) to meet the requirements of society for qualified manpower; and

(c) to meet the requirements of society for postgraduates with research experience.

These objectives encompass most of the possible objectives of education discussed earlier and as such do not easily translate into institutional objectives. Jean Benard (1967) notes that the educational sector operates in at least four directions when viewed as an industry providing qualified manpower. These are:

(i) It provides pupils with knowledge essential for the general or occupational skills they will later possess as members of the work-force;

(ii) It raises their cultural level and so influences the choices they will make and their ability to absorb fresh knowledge within their working lives;

(iii) It develops scientific knowledge within the institutions themselves; and

(iv) It helps to disseminate cultural, scientific and technical knowledge within the population as a whole through books and reviews, broadcasts and the extramural activities of teachers.

In other words, he suggests that the output of educational institutions should be regarded for practical purposes as consisting entirely of its intermediate products.
Similarly, Gross (1973) postulates that the following set of institutional objectives would quickly be agreed for a University:

1. Stay in existence;
2. Provide undergraduate education;
3. Provide opportunities for postgraduate education;
4. Provide continuing education;
5. Advance knowledge through research and publication;
6. Organise the vast amount of knowledge into manageable form; and
7. Enable the cultural, economic and political advancement of society by increasing the accessibility of learned men to society, government and industry.

Most of these objectives are simply descriptions of what universities do already. Perhaps this is because, as James McNamara (1973) points out, the problem is one of multi-dimensional outputs and, as Jean Benard (1967) suggests, there are multi-dimensional objectives hence the easiest way forward is to follow Lars Thulin's advice (1974) and forget about formal objectives and concentrate on identifying the mix of activities carried on in the institution since the chosen mix of activities reflects the underlying mixture of objectives.
1.4. ACTIVITIES OF AN EDUCATIONAL INSTITUTION

If the objectives of an educational institution are to be taken as that of providing a certain mix of activities in a particular academic year, then the next question is that of what are the activities involved. In 1972 the Committee of Vice-Chancellors and Principals of the Universities of the United Kingdom carried out a survey on the use of academic staff time. Now, admittedly, the results were obtained by a questionnaire diary filled in on a voluntary basis by the academic staff themselves but they do give some indication of the time an academic professes to spend on different activities. The returns suggest that academic staff on average spend 24% of their working year on personal research, 11% on external professional work, 6% on graduate research, 42% on undergraduate and postgraduate work, with an unallocatable proportion of 18%. The average working week for all these activities came out at 50 hours a week for 47 weeks. However, these averages hide a wide range of institutional variations. For example, personal research was quoted as being as low as 14% of the staff's time to as high as 34%, and external professional time ranged from 7% to 14%. This does confirm, however, that the major activities are undergraduate/postgraduate teaching and personal research. Included in some of these categories, of course, is the associated administrative activity carried out by academics.

1.4.1. Teaching Activity

The transmission of knowledge from the teacher's point of view involves more than formal class contact. It necessitates desk research and preparation and it produces a marking and examining load. Simpson et al (1971), in the course of their wider study of University development planning, attempted to identify the amount
of time spent on these non-class contact components with a view to identifying spare teaching capacity. They asked lecturers to specifically identify the time spent on preparation, class contact and on post-mortem time. This proved very difficult, particularly in the split between personal research and lecture preparation. For example, the time taken in preparing and giving lectures varied from 250 hours to 450 hours p.a. for members of staff in the sample. In the end the study used values which were "generally recognised as reasonable in the departments" in building up their predictions of teaching load.

The Author and Derek Birch (1977) have suggested that preparation time is a function of the experience and method of working of the individual member of staff and that post-mortem time is a function of the number of students involved. Since most academics see a mix of different levels and sizes of course, it seems more realistic to look at teaching as an activity consisting of class contact involving a number of students. If it is necessary to estimate teaching capacity, then preparation time and post-mortem time can be catered for by a reduction in the figure used for the maximum teaching load of an individual. Indeed, it may be erroneous to assume that preparation time is a function of the level of teaching or the size of the group as is often done (e.g. Simpson et al, 1971).

1.4.2. Research Activity

Much of research is involved with the organisation of current knowledge into a more manageable form and as such involves a steady investment of time and produces a steady stream of conference papers, journal articles, extra-mural lectures and
even books. Some research involves the extension of current knowledge and as such is less predictable in its consumption of time and less productive in its pieces of paper. In the technological field research evidences itself by attracting research grants from industry and government. This activity sometimes strays into industrial research and development and, as such, is sometimes hard to distinguish from consultancy for individual profit. Research activity, however, must be catered for in any discussion of the activities of an institution since by the figures in the CVCP study (1972) graduate research, personal research and external professional work, which includes consultancy and extramural lectures, accounts for half of the allocatable time. This makes it all the more inexplicable that the DES, in its discussion paper "Higher Education into the 1990's" (1978), should attempt to discuss the future of Higher Education without mentioning research except to say that a steady state staff situation would make it difficult to recruit the young staff necessary to the vitality of the Higher Education system's research function. The unit costs quoted of course assume all the staff salaries should be 'charged' to teaching, i.e. to the students enrolled.

1.4.3. Consultancy

No academic will admit that any consultancy he or she is involved in is undertaken purely for private profit. Always it is said to feed into teaching or research. Indeed, several institutions have organisations which co-ordinate and, in some cases, initiate consultancy work for the institution's academic staff. However, it is a brave administrator who will plan the development of his institution on the basis that everybody should have time for private
consultancy. In fact, many institutions require academics to ask for permission before undertaking outside work. It seems safe, therefore, to treat consultancy as a spare time activity.

1.4.4. Administration

Jean Benard (1973), in his theoretical model of a University, suggests that such a model is incomplete without adding administration as an activity to those of teaching and research. However, he sees administration as an intermediate activity whose outputs are all inputs to teaching or research. He suggests for simplicity that these output/inputs relating to administration should be used as the measures of the level of activity. However, Duggan and O'Donoghue (1977) made attempts to measure central administrative activity by counting the numbers of committees and so on, but the results were inconclusive. Similarly Rivett et al (1974) looked at the effectiveness of alternative administrative structures from both a behavioural and a systems approach and perhaps as a consequence also produced inconclusive results (Thulin, 1975).

Departmental administration carried out by academics is very difficult to disentangle from the teaching and research it is concerned with. However, in Polytechnics and Colleges of Further Education the actual conditions of service are based on the dubious premise that senior academics do most of the administration and so require lower teaching loads. Gerald Stockdale (1974), in his examination of course mix in two educational institutions, attempted to measure the administrative load generated by running a course once. He then suggested that the amount of administrative, clerical and secretarial support available should be used as an additional constraint on the choice of courses
to mount. This assumes that there is a linear relationship between administrative load and number of times a course is put on. However, for simplicity's sake, when he applied his model in practice he took care to make this constraint inoperative! The Author and Derek Birch (1977) have pointed out that estimates of such non-class contact activities carried out by academics are subjective and hence suspect and since most academics teach a range of levels and types of course it seems more sensible to cater for administrative academic activity carried out by academics by adjusting their maximum teaching load, especially since it is not obvious how to relate it systematically to the teaching and research it obviously stems from.
1.5. SUMMARY OF CHAPTER 1

Institutions at present make little attempt to aid in personal fulfilment, nor are they particularly successful in promoting social mobility or achieving social equality. They do seem to offer an economic investment for individuals and they aid the economy by doing more than simply screening students for ability. The most useful set of institutional objectives is one which basically specifies the mix of activities to be carried out, those activities being teaching and research with administration as an intermediate activity and consultancy as a spare time activity.
REFERENCES FOR CHAPTER 1 (in order of mention)

Section


REFERENCES FOR CHAPTER 1 (continued)

Section


REFERENCES FOR CHAPTER 1 (continued)

Section


CHAPTER 2 RESOURCE ALLOCATION BY FORMULA

2.1. INTRODUCTION

Ideally institutions in Higher Education should allocate resources internally on the basis of performance in the various activities of the institution. However, most simply use formulae, incorporating norms, which are applied to the level of activity not its quality. Indeed Cooke (1976) makes a valiant attempt to prove that the University Grants Committee, for all its deliberations, actually allocates recurrent grants on the basis of student numbers alone. His analysis has been discredited though, firstly by the UGC itself in the form of Sir Frederick Dainton himself (1977), and secondly by the production of counter examples which produce the same results with widely different assumptions (Barnard, 1977), and finally by closer examination of the linear regression itself (Green and Chatfield, 1977). It is surprising, therefore, that the first thing any University Head calculates on hearing of his allocation seems to be the allocation per student!

In most institutions the largest element in the budget is academic remuneration and so most of them concentrate on the allocation of staff and leave the rest (materials, equipment, space, etc.) to be allocated pro rata. There are basically two ways of allocating staff. Firstly, the allocation can be made on the basis of staff's teaching workload and secondly, it can be made on the basis of the numbers of students involved. Research workload is not usually taken into account except that related to research students.

Many recent studies have concentrated on the staff workload based approach despite the fact that, as Fielden and Lockwood (1973) point
out, most universities use student numbers as the basic unit of measurement, and despite the situation on the other side of the binary line where staff-student ratios reign supreme. It must be pointed out, however, that as soon as the concept of the full-time equivalent student rears its ugly head, the distinction between the two approaches becomes blurred since class contact hours are then involved in both sets of calculations.
2.2. FORMULAE BASED ON STAFF WORKLOAD

The Robbins Committee (1963) identified the following parameters of the requirements for staff:

(i) Full time equivalent staff (denote by T)
(ii) Full time equivalent students (denote by S)
(iii) Average teaching-load - formal class contact (denote by t)
(iv) Average group or class size (denote by g)
(v) Average tuition load - formal teacher contact by one of these groups (denote by h)

These lead to a simple formula for the specification of staff, namely:

\[ T = \frac{S}{g} \times \frac{h}{t} \quad \text{(1)} \]

where the student-staff ratio (SSR) is defined as:

\[ SSR = \frac{g \times t}{h} \quad \text{(2)} \]

This relationship is the one postulated by John Delany (1971) and is the basis for the Pooling Committee's recommendations in the "Assessment of Curricular Activity and Utilisation of Staff Resources" (1972). Equation (1), however, is a ratio of averages and as such is a crude approximation to reality. It can be improved, however, if account is taken of the varying sizes of groups involved. For example, it is often said that there is no maximum size for a lecture but there is a maximum size for a tutorial. If the tuition load of a student (h) is split into lecture time (t) and tutorial time (m) and the lecture is made explicitly open-ended in size formula (1) becomes the following:
where $S$ is, as before, the number of students but $g$ is now the tutorial class size.

Keith Legg (1971) suggests that the level of the group is important since there are differences in the teaching pattern, for example between undergraduates and postgraduates. He, therefore, constructs for each programme of study the following type of relationship:

$$T = \frac{S \times m}{g} \quad \text{(3)}$$

Thus an undergraduate course would consist of two years at level 1 and one year at level 2, and a postgraduate course would be 1 year at level 3. This type of formula for a course is then used to distribute teaching load amongst departments to produce a formula for a department which takes into account short courses and industrial visits. This is all very reasonable but then he weights the different levels of work and starts to move away from reality. A full discussion of his work is contained in Appendix 1. His unique contribution to the development of staffing formula, however, is his use of an international survey to establish norms for various parts of his formulae. Thus he produces, via some rather tortuous logic, norms for the ratios $(k_i/t)$ and $(m_i/t \times g_i)$ for each level.
and each discipline area. This enabled Loughborough University to implement a simple version of his formula and use it until quite recently. However, as is shown in Appendix 1.1, the use of norms for teaching hours rather than the actual class contact turns a work load based formula into one that counts students and courses i.e. into a student load based formula! The main criticism that can be made of this approach is that there is little evidence that the chosen ratios are meaningful in their own right and have a distribution that is sensible to treat by averaging. It is important, however, to follow Legg's lead and treat discipline areas separately. Birger Fredriksen (1971); following an analysis of the same international survey, showed that discipline areas behaved (statistically) significantly differently in their provision of teaching as far as lectures and tutorials are concerned, both for undergraduate and postgraduate work.

Simpson et al (1971) at Lancaster, suggest a similar approach but they extend formula (3) by including preparation time and post-mortem time. They also follow Legg (1971) in treating lectures as open-ended in size and thus not repeated, and seminar/tutorials as limited in size and hence of necessity repeated several times. It is worth pointing out that Legg, when he examined service teaching, adapted his formulae to cater for repeated lectures thus negating the basic idea!

The Lancaster formula is:

\[ T = \frac{k(1 + p) + Sm (1 + \frac{q}{r}) + Su}{t} \quad \ldots \ldots \quad (5) \]
where $h_{ij}$ are as before and

\[ p = \text{lecture preparation time} \]
\[ q = \text{seminar preparation time} \]
\[ r = \text{average number of seminar repeats} \]
\[ u = \text{post-mortem time} \]

In practice, academics found it very difficult to identify $p$, $q$, $u$ as has been already mentioned in Chapter 1.

Bottomley et al (1971) started from formula (1) and extended it in a different way. They have put forward a more generalised version similar to formula (6) below which emphasises the whole range of meeting types - lectures, seminars, tutorials, examples classes, laboratories, etc.

\[
T = \sum_{i} \sum_{j} \frac{h_{ij} \times S_{j}}{g_{ij}} \quad \ldots \ldots \quad (6)
\]

where $h_{ij}$ = average number of formal tuition hours received by each type of teaching meeting $i$ in the $j$th year of the course.

$S_{j}$ = number of students enrolled on the course.

$g_{ij}$ = maximum group size for each type of meeting $i$ in the $j$th year of the course.

This is very commendable but, as the Author and Derek Birch (1975a) have pointed out, once a formula asks for numbers of lecture hours, numbers of tutorial hours and so on, what is to stop a department timetabling all its meetings as tutorials? Why, in practice,
should a 'lecture' to 20 students require different treatment to a tutorial to 20 students? On many timetables there are 'lectures' to 5 students and 'tutorials' to 20. From the point of view of the teaching techniques to be employed, the critical factor should be the number of students in the group not its timetabled description. A similar criticism can be made of schemes which weight the hours in some way. Terence Burlin (1976) outlines a system at the Polytechnic of Central London in which staff-student contact is said to be the vital statistic. However, the system does not measure actual hours, it utilises weighted hours. For example, a one hour meeting given to evening students is weighted at 1.3 whereas an hour given to a short course is weighted at 3.0 and an hour given to a postgraduate is weighted at 2.0 and so on. It is very debateable what meaning the final result of such calculations can have. There is little evidence that postgraduate teaching per se is twice as difficult or onerous as undergraduate teaching. In particular subjects the reverse may be true! If the weights are there only as incentives then they should not be built into the data collection but added at the end each year for flexibility.

The formulae mentioned above were all derived in the way they were because of their various authors' institutional experience, e.g. Legg with his experience of sandwich degrees involving large lectures with some tutorial back-up, and Simpson et al at Lancaster with their experience of a pattern involving many seminars. However, of all the ones mentioned above, only Legg in his work directly approaches the two problems of service teaching and joint groups, but his approach to joint groups is very crude. The Author and Derek Birch (1976) have followed the Robbins Committee approach of identifying the parameters necessary to allocate staff but more parameters are used in order to cope with joint meetings and service teaching. The argument is as follows:
A study programme is a set of meetings where a meeting is defined as a timetabled hour of contact between academic staff and students. This set can be broken down into subsets on the basis of: the department providing the tuition; the type of space utilised; the student group size; whether the meeting is compulsory or optional; and most important, whether it is taught to a single study programme or involves a number of study programmes, i.e. is "joint". Consequently to analyse a set of meetings the following information is required:

(i) total enrolment to a study programme (E);
(ii) the enrolment from a study programme to a particular subset of meetings (S where S ≤ E);
(iii) total enrolment from all study programmes attending a particular subset of meetings (E*);
(iv) the department providing the tuition;
(v) the type of space utilised;
(vi) the number of student groups (each assigned to one teacher) formed in a particular subset of meetings (g); and
(vii) the total number of hours attended by a student in a particular subset of meetings of a particular group size (h).

Given the above information, it is possible to establish for each year of a study programme, for a department's programmes, for discipline areas and for the institution as a whole, the meetings needed and hence the staff required using:
\[ T = \sum \left( \frac{h \times g \times S}{E^*} \right) \] \hspace{1cm} \ldots \ldots \hspace{1cm} (7)

(where the summation is over the appropriate set of subject elements).

When \( S \) is not equal to \( E^* \) it means that students from one course are being taught for that subject together with students from other courses. The formula means that the hours of class contact required for these joint meetings are shared out between the courses involved using the proportion \( S : E^* \), i.e. staff time is shared out pro rata to student numbers. Since this formula does not contain norms, averages or involve double counting, it is an explanatory formula rather than an allocative formula: it simply allocates staff on the basis of the work done measured in terms of meetings staffed.

As the Author and Derek Birch (1974, 1975b) have commented, the staffing formulae described above suggests that the basis for determining and allocating teaching staff should be the timetabled commitment. This means that an increase in enrolment to a course should not mean a proportionate increase in staff hours and hence staff numbers.

Bottomley et al (1971) were, in fact, able to show for the University of Bradford that if enrolments doubled there were potential economies in staff hours of between 52% and 82%. Since they assumed no change in teaching pattern, this result reflected the large proportion of open-ended (in size) lectures. Unfortunately, Bottomley has been overtaken by events and the problem is now one of holding enrolment steady and changing the teaching pattern! One of the disadvantages of workload based schemes is that they encourage departments to fill
up the timetable but since there is only a finite amount of staff resources to share out this simply means in the long run that departments work harder hoping to be rewarded with more staff in the future. Any published allocation formulae will have similar drawbacks. This is why the Author in all his work prefers to work with students and hours and to leave any weightings to be applied by the decision-makers at the moment of decision. In this way the decisions can be taken using the actual data but encouragement can be given to various activities or departments as required each year.

There can be no doubt that workload based formulae highlight some of the consequences of particular teaching patterns in a way that head-counting will never do, but they cannot indicate the effects of larger classes or shorter courses or larger teaching loads for individual teachers on the standard of educational provision, e.g. attrition rates, examination success, or the ultimate employability of the graduates. The Author with others (Birch, Calvert and Sizer, 1977) suggests, following a study of Loughborough University and Lanchester Polytechnic which will be discussed in more detail in Chapter 3, that it is possible to make some savings in staff costs by utilising joint meetings and larger group sizes in general without seemingly affecting wastage rates, examination performance or salary shortly after graduation. However, it is not obvious what the long term effects would be of such changes in teaching patterns and this awaits some longitudinal research.
2.3. STUDENT LOAD SCHEMES

Student load orientated schemes operate by converting student numbers into full-time equivalents and then by applying an appropriate student-staff ratio obtaining a requirement for academic staff. Thus an increase in students should be followed by a proportionate increase in staff. However, as Alan Crispin (1975) points out, many institutions retain more control over staffing by using different student-staff ratios in different discipline areas or for different levels of study. Thus, at his own institution (Institute of Education, London University), full-time student numbers receive a 10-1 ratio, part-time undergraduate numbers receive an 11-1 ratio whereas part-time postgraduate numbers receive a 20-1 ratio. Sheffield University carry this concept considerably further by applying one student-staff ratio for small departments and another, less beneficial, to large departments, i.e. they assume economics of scale exist. At the heart of most schemes is the calculation of the full-time equivalent student numbers. Recently the Department of Education and Science has set up a working party to re-examine student-staff ratios and costs in polytechnics and colleges (THES, 13 October 1978) and the main task is seen as the derivation of a new formula for counting students since the absence of a uniform formula for full-time equivalents has frustrated previous attempts to compare unit costs and measure the efficiency of individual institutions.

There are several approaches available: there are the UGC weightings for sandwich students etc.; there are the Burnham points weightings; and finally there are the weighting factors that can be obtained from the ratios of the contact hours of a particular course to the contact hours of some suitable full-time course. The latter approach was
recommended by the Pooling Committee in 1972 and is reasonable for
the institution as a whole, but it leads to difficulties when the
allocations are assessed within the institution particularly when
faced with joint meetings and service teaching. For example, the
Author and others (1976) have shown that, for Loughborough University
in the academic year 1972-73, the tuition load would have totalled
129,980 hours without joint meetings whereas the actual total using
joint meetings between courses was only 71,251 hours.

Aston University attempts to cater for service teaching by allocating
all students to their departments then credits are given to each
department that teaches a course outside and taken from the enrolling
department. Thus, each department ends up with a notional student
load to which an assortment of student-staff ratios is applied.
However, the calculation of credits for service teaching stretches
the bounds of credulity. For example, an hour taught by a service
department to a sandwich course merits 1/32 of a student, whereas a
similar hour to a social science course merits 1/16 of a student,
with other 3 year courses meriting 1/25 of a student per hour. The
Author and Derek Birch (1975a) recommend instead a straight head-count
under the appropriate headings - full-time sandwich, part-time day,
and so on, and these can be distributed between departments on the
basis of class contact. This avoids the artificiality of the FTE
and ensures that the figures available are not measured in "funny
money". It also means that weights can be applied to different
activities at the time of decision and not built into the formula as
so many institutions try to do. This is an important consideration
since, as Fowler (1973) states, in other contexts the allocation is
"by heads with some heads deemed to weigh more than others". If
institutions wish to promote, say, postgraduate study, then let them do so, but explicitly rather than implicitly via hidden weights in the data collection process.

There are some who believe, however, that the student-staff ratio, rather than simply a ratio of weighted averages, norms or estimates, has a life of its own! Grossack (1969) has estimated student-faculty ratios for all the post-secondary schools in Indiana for the year 1985. He does this by explaining the national figures in terms of size of institution, improving technology, course proliferation, and the graduate-undergraduate mix. He then predicts how these factors will vary during the period up to 1985. The resulting forecast for the national US student-faculty ratio is then compared with 1967 and the resulting multiplier used to extrapolate one year's data for Indiana for 1967 to produce values up to 1985 on a straight line basis. This shows touching faith in the existence of the student-staff ratio as an entity in itself rather than simply a ratio of other variables! However, when these figures were utilised by Perkins and Paschke (1973) in a simulation model of the Higher Education System in Indiana, their results were fairly robust to changes in the particular assumptions made. Furthermore, it must be said that many of the sophisticated management information systems devised for Higher Education Institutions have at their heart staff-student ratios input as norms (see Chapter 3).
2.4. ALLOCATION OF SUPPORT STAFF, SPACE AND MATERIALS

The elements of the institutional budget that remain after academic remuneration has been dealt with are usually allocated pro rata to staff or students. Again, there is little attempt at measuring performance. Bottomley et al (1972) outline a planning model for the University of Bradford which uses norms, supplied by the Academic Planning Committee, for space per student, technicians per student, and so on. However, in the course by course analysis they deduce that technician numbers, for example, are dependent on laboratory space not students, and that teaching space requirements are related to numbers of meetings not numbers of students. Dunworth (1974), in an analysis of laboratories at the University of Bradford, suggests that the UGC norms of space per student are too global for effective use and should be modified to take account of occupancy rates, area per working place, and the proportion of a student's classtime allocated to laboratory work. In other words, he is suggesting that the time spent in the laboratories is the important factor not the number of students on the courses using them. Keith Legg (1971), faced with the dilemma of choosing between norms derived via student numbers and norms derived via staff numbers, solved his problem in typical fashion by simply averaging the two! In another study (1973), he suggests that administrative staff in particular should be allocated pro rata to academic and teaching support staff rather than students. Thus, it is not yet clear what is the true relationship between space, support staff, materials, etc., and academic staff, students, or a combination of the two. This makes it all the more important to avoid global ratios that have no logical justification other than they happened to have a particular value at some point in the past.
2.5. SUMMARY OF CHAPTER 2

Although, ideally, resources should be allocated within an educational institution on the basis of performance, most institutions allocate academic staff resources on the basis of student-staff contact or, more likely, on the basis of student numbers. Most workload based schemes suffer from a surfeit of hidden weights which may have been appropriate at some point in the past but are most likely inappropriate now. It is suggested that the actual workload should be measured and used for staff allocation. Student number based schemes suffer from a range of sometimes inexplicable student-staff ratios and cannot always cope with service teaching or joint meetings between courses or levels. It is suggested that, if allocation is to be based on student numbers, the various types should be amalgamated separately using unweighted data. The decision-makers can then put in appropriate weights for the year in question at the time of decision. It is not yet clear how non-academic staff resources relate to students; academic staff or a combination of the two and so it is recommended that global ratios be treated with suspicion when used for allocating these items.
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Section


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2.2. Burlin, Terence (1976). "Staff-Student Contact is the Vital Statistic". THEES, 5.3.76.

REFERENCES FOR CHAPTER 2 (continued)

Section


CHAPTER 3  THE MEASUREMENT OF INPUTS AND OUTPUTS FOR MANAGEMENT INFORMATION

3.1. INTRODUCTION

The first stage in the measurement of performance must be the identification and measurement of the inputs and outputs of the various activities of an educational institution. The measurement may be direct or may have to be made via proxy measures. If performance is to be measured at course or department level then there must also be an allocation of those inputs and outputs to each course or department. Ideally the measurement of inputs and outputs should be a continuing systematic exercise, i.e. should be part of a management information system. Unfortunately, once complex systems are introduced, the system often chooses the input and output measures, not the institution. This can only happen because, as Mehar Arora (1972) states, there is no agreement amongst researchers and theoreticians on the inclusion and exclusion of particular inputs or outputs, and there are no agreed standardised units of measurements. This is yet another justification for measuring inputs and outputs in a commonsense way avoiding, as far as possible, full-time equivalents, weightings, conversion factors and the rest so that the decision-makers understand the meaning of the figures they are presented with and can apply their own weightings at the time of decision.
3.2. TEACHING ACTIVITY

As part of the OECD/CERI/IMHE Programme*, the Author with others carried out a study of Loughborough University and Lanchester Polytechnic (hereafter called the Loughborough-Lanchester study) with the aim of developing a methodology for constructing performance measures for the teaching function in Higher Education. The study involved the identification of the inputs to and the outputs from the teaching activity and their measurement either directly or via proxies. Parts of this study have been published (Birch, Calvert et al, 1975, 1976a, 1976b, 1977a, 1977b, 1977c) and are reproduced in Appendix 2 of this thesis which is bound separately as Volume 2.

Teaching has a lifelong impact and so the designation of the various factors as inputs or outputs depends on which part of the system is examined. The factors themselves can be loosely identified as students, academic staff, support staff, materials, equipment and space. Alex Mood (1969) emphasises the need for quantitative and qualitative measures of these factors but Hector Correa suggests (1967) that due to their heterogeneous nature they should be converted to costs and thus be measured in a common unit.

3.2.1. Students - Quantity and Quality

Students are one of the important factors in the teaching activity and they can simply be counted, although when different programmes of study are involved the problem of defining a full-time equivalent looms large, as pointed out in Chapter 2. Their quality before,

during and after their studies can be measured using tests and examinations although, as Attiyeh and Lumsden (1971, 1972) point out, the comparison of educational skills requires standardised pre-course and post-course tests covering common syllabi. Alex Mood (1969) suggests that family background affects the results of such tests and another influence is the time and effort each student expends on his studies. Clift and Thomas (1973) describe studies undertaken at Monash University, Australia, which indicate that their students spend at least 36 hours a week studying. However, as Mehr Arora (1972) reports, there is no agreement between researchers and theoreticians on whether student time should be included in the set of inputs. The Author has commented elsewhere (1978a) that the knowledge and skills developed by the student reflect more his learning curve than the performance of the institution. Indeed, its performance is related more to the provision of educational opportunities.

The Loughborough-Lanchester study measured not only enrolments to the different types of course but also measured examination performance before, during and after the courses, both as scores and as Pass/Fail rates. The quality of the graduates can be measured in terms of their subsequent salaries, as in the rate of return studies described in Chapter 1, but the Loughborough-Lanchester study showed that the discipline area was the over-riding determinant of salary. Thus salary as a quality measure reflects the initial choice of programme of study and virtually ignores any effects of the programme of study itself. The study also considered modifying influences on student quality and quantity, namely age, sex, whether married or single, and whether home or overseas based. Socio-economic background was not easily available for both sets of students so this was not
included. The report to the Paris Conference of the OECD/CERI/IMHE Programme in 1976 (copied in Appendix 2) contains several tables showing the results of such measurements of students. The problem remains though of how to measure the quality of those students that do not complete the course; as stated in Chapter 1 when discussing the screening hypothesis, the evidence is that employers take more notice of the subjects studied than of the final qualification. Some studies treat such "drop-outs" as of zero value, others treat them as proportionate in value between enrolled students and graduates.

3.2.2. Academic Staff - Quantity or Quality

Academic staff are obviously the other major factor in the teaching activity and they can simply be counted if an acceptable definition of full-time equivalent can be found. This is more difficult for Universities where an agreed staff workload is not enshrined in their conditions of service. The Committee of Vice-Chancellors and Principals (1972), in their study, identified time spent on undergraduate and postgraduate teaching but, as stated earlier, 18% of time was unallocated to any activity.

The quality of academic staff can be measured by their status (Professor, Associate Professor, Senior Lecturer, and so on), by their academic qualifications, by their salary, or by their performance in teaching or research. The Coleman Report (1966) referred to in Chapter 1, examined not only the pupils but also the teachers. An extensive questionnaire-survey involved questions not only on academic qualifications and teaching experience but also questions relating to their learning experience during their teaching training course! Most studies leave staff quality alone, although
student feedback can be used as a guide to teaching ability. Another approach is to look at the time spent on teaching and the quality of that time as measured by the teaching environment provided.

The Loughborough-Lanchester study measured the quality of teachers by using their salary scale mid-points and the quality of their teaching by reference to the size of the group, the nature of the space utilised, and the nature of the teaching in the sense of whether it was service or own department teaching.

3.2.3 Support Staff, Materials, Equipment and Space

Support staff can all be counted or measured by their salaries. However, they are involved in different activities and hence the time they spend on each would be a more appropriate measure. Thus Gerald Stockdale (1974) attempted, with mixed success, to measure the administrative time needed to set up one year courses. Most other studies treat support staff as an adjunct to academic staff and allocate their time accordingly, although, as pointed out in Chapter 2, there is little evidence regarding the relationship of support staff to academic staff, students, space or any other factors. In the Loughborough-Lanchester study the support staff were split into department administrative/clerical and technical staff since some departments had large laboratories and other had none.

The use of materials is clearly best measured on a consumption basis whereas space and equipment are fixed assets which are influences on the teaching activity. In the Loughborough-Lanchester study the nature of the space utilised is noted as specialistic or non-
specialist. Specialist space being space with equipment available. It is possible to measure the quantity of space utilised, as Dunworth (1974) did in his survey of laboratories at the University of Bradford, but there is an argument, relevant to all types of space, that its alternative use is virtually non-existent and so changes in space utilisation are simply changes in occupancy rates and not changes in cost or activity.

3.2.4. Inputs or Outputs?

Students can be an input or an output for, as the Author has written elsewhere (1978b), if the system is perceived as the provision of educational opportunities then the enrolments to study programmes are outputs whereas if the system is seen as the provision of graduates then the enrolments are inputs and the output includes the quantity and quality of those graduates. Alex Mood (1969) states that the educational state of the student at the beginning of the year is an input and that at the end is an output, and this is the approach used in the Loughborough-Lanchester study, although in Chapter 7 of this thesis, for purposes of illustration, student numbers are used as a proxy for the teaching environment provided.

Academic staff are certainly an input but the precise measure to use is less certain. Staff numbers and salaries are used as a measure of input in Chapter 7 whereas the Loughborough-Lanchester study utilises the staff time and its cost as a measure of their input. Student-Hours (students multiplied by their hours) is more a measure of the output from a teaching pattern and as such is an output.

Support staff, materials, space and equipment are obviously all inputs
but whatever measures are used they have to be allocated to the various activities undertaken by a department.

3.2.5. Problems of Allocation of Inputs or Outputs

If the institution as a whole is considered, then the total staff numbers, salaries, total recurrent expenditure and money resources generally can be used as crude measures of input as in Chapter 7. If a within-institutional examination is carried out, the various inputs and outputs must be allocated to the various parts of a department's or course's activities. For example, students have different types of courses and are rarely taught wholly by their enrolling department, hence students, if used as an input or output, have to be allocated between their teaching departments perhaps on the basis of student-hours involved, i.e. in full-time equivalents.

Similarly a department's staff do not teach only one course, hence their activity has to be split between the courses that are involved. If joint meetings between courses exist then the staff member's time must be shared between the participating students, if it is not, double counting occurs. In the Loughborough-Lanchester study staff time in joint meetings is shared out pro rata to students. Keith Legg (1971), in his analysis, split it in proportion to the number of courses involved but this is misleading if the courses are of widely different enrolments. At Loughborough, for example, one class of 127 contained 1 student from a particular course and 5 courses altogether. Surely the staff member's time should be allocated mostly to the other courses? Administrative and technical staff are usually given to departments and so they also need to be allocated. In the Loughborough-Lanchester study this is done simply on the same basis as that used for the academic staff, although the data base
set up would allow a separate allocation of administrative and technical staff on some other basis.

One major problem of allocation so far ignored is that of allocating these inputs between teaching and research. In Chapter 7 the inputs and outputs are measured at the institutional level and so research activity is simply another output. But when resources have to be allocated between departments or courses some assumption has to be made about the resources 'consumed' by research. Now in the Loughborough-Lanchester study the computer programs were written by the Author in such a way that each department could have a different split of resources between teaching and research but, in the event, there was no evidence of any agreed differences between departments' research activity and hence they were all treated the same and the departmental resources were initially allocated wholly to the teaching activity as measured by staff workload. The argument used is in essence that research is a "free good". The results were modified to show an assumption of 20% of time spent on research (the VCPC figure for Loughborough) but this was a symbolic rather than rational reduction in teaching resources allocation. In Appendix 1.2 the Author shows the effects on the allocation of cost to the teaching activity of an allocation of cost to research.

3.2.6. Costing the Teaching Activity

Measuring some of these inputs and outputs in monetary terms requires an allocation of cost between activities. The Author with others (1977b) has described one way of costing the teaching activity. It is an absorption costing approach involving the assumption that departmental financial resources are provided for teaching activity
only. Thus, for each department, its teaching activity is measured by class contact hours and hence a cost per meeting is calculated. This is then used to calculate the cost of a course or group of courses. Since it is an absorption costing approach, the total cost of all the courses in the institution equals the cost of the resources shared out in the first place. The cost per course is more useful than a cost per student because the former reflects the demand for tuition and space so that one extra student may necessitate no increase in the cost per course at all.

The cost per course can be used to establish a cost per student if necessary but, as Babbeau, Cossu and Cuenin (1975) point out, this is exclusively an accounting cost and hardly lends itself to comparisons because it has to cope with anomalies, economies and diseconomies of scale, and the arbitrary nature of the allocation of indirect costs. John Sizer (1978) makes a similar point when he comments that an absorption costing approach must contain a process of allocating fixed costs between activities and hence the results will be affected by the choice of allocation. Jean Benard (1973), on the other hand, distinguishes between fixed non-separable costs which should not be split in any way, and variable non-separable costs which should be treated as costs of indivisible collective assets and hence shared out amongst the activities involved. Thus, it seems feasible to allocate some, if not all, of a department's resources to its activities. Indeed, the alternative approach of incremental costing is not appealing. With this approach an extra student may, depending on room capacity, incur no extra teaching, and if one extra hour of teaching is required the result may simply be one hour less of administration or research, the effect of which may not be measurable.
The cost per course, cost per student and cost per student-hour are all unit costs that are affected by changes in enrolment and changes in teaching patterns. Hence, in Chapter 6, there is an examination of how these influences can be identified using an approach similar to the accountant's variance analysis. This examination also indicates the usefulness of the cost per student-hour as a unit-cost unbedevilled by concepts of full-time equivalents.
3.3. **RESEARCH ACTIVITY**

As stated in Chapter 1, research consists of the organisation of knowledge and the extension of knowledge. Institutions try to encourage the latter but usually measure the former. Research is an activity that consumes inputs, produces output and proceeds at a level of activity. It obviously consumes staff time and this can be measured via diary exercises and the results used to establish a cost of research time. This staff time could be used for teaching and so it implies a loss of potential student-hours received. Terence Burlin (1976) describes a procedure at the Polytechnic of Central London in which this opportunity cost of research time is estimated. For example, a full paper in a technical journal is treated as if it replaces 3000 student-hours (e.g. 14 students for 6 hours a week for 36 weeks), a major book review or case note is assumed to replace 1500 student-hours, and a book itself is assessed on an individual basis. Of course, these various pieces of paper can simply be counted up to measure the output from research treating, for example, a book as equivalent to eight papers. As regards the level of activity of research, David Katz (1973), in his model for staff promotion, points out that supervision of research students requires similar abilities to those required to carry out research itself. He further reflects that the number of dissertations supervised is a good predictor of academic salary for his sample. Research grants received also indicate that research is going on and in Chapter 7 the amount of research grant spent in a particular academic year is used as a crude proxy for research output. This is really a measure of input and is being used as a measure of the level of research activity and hence as a measure of research output. Moulitt and Line (1975), in their study of the costing and management of research grants and contracts in universities, make the
point that these should allow institutions to recover overheads. By this phrase they mean that research activity paid for by outside bodies should be charged with the cost of the non-academic staff, materials, etc., that may be utilised in the research supported by the grant. However, in practice most institutions simply require "x%" to be included in all applications for research finance. The percentage chosen is fairly arbitrary and it is not the case that teaching and other activities are charged with "(100-\(x\))%" so it is a symbolic rather than a rational allocation of overheads between activities.
3.4. **ADMINISTRATION AND OTHER CENTRAL SERVICES**

It has already been said several times in this thesis that administration is an intermediate activity whose outputs are inputs to teaching or research. Some institutions have central administration or other services whose activities are more difficult to allocate to teaching and research. The companion study to the Loughborough-Lanchester study, namely the Leeds-Huddersfield project (Norris et al, 1975) examined the computer, the library, committee work, administration and student welfare, but it was only in the computing area where real progress was made and that was due to the existence of a measured departmental and student use. Michael Pickford (1974) carried out a statistical analysis of University Administration Expenditure and he shows that there is no simple relationship between it and student numbers as a measure of university size. In the Loughborough-Lanchester study, since it was impossible to calculate the input by the Local Authority into the administrative activity of Lanchester Polytechnic, this and the central administration at Loughborough University were both ignored in the cost calculations. Barreau, Cossu and Cuenin (1975), in their extensive study of costing methods, discuss fairly detailed measures of central services such as pieces of paper copied by a copying service and so on. In Chapter 7 of this thesis non-academic staff salaries in total are used as an input for the institution as a whole.
3.5. MANAGEMENT INFORMATION SYSTEMS AND THEIR IMPACT ON INPUT AND OUTPUT MEASUREMENT

A Management Information System (hereafter MIS) can be implemented in two ways. Either an MIS can be bought "off the peg" and adapted for the institution, or the information needs of the institution can be identified and used to specify a "custom-built" system. The first approach gives access to a whole range of sophisticated planning, allocating and costing systems but requires input and output to be measured in particular ways. The second approach enables the right information to be collected but means that often only data collection and preliminary analysis is possible. The Author and Derek Birch (1976c), in a proposal for a MIS for a Further Education college, discuss the various computerised MIS packages available and this paper is in Appendix 2. It is worth repeating here, however, that they are all comparatively recent, only one being operational before 1970. The most widely implemented system is the Research Requirements Prediction Model (RRPM) which was first released in 1971. However, this system requires input of student credit hours which are converted to actual contact hours via a conversion factor. Similarly it calculates staff required by dividing contact hours required by an average class size and an average for faculty contact hours. It also works at a high level of aggregation and thus has less flexibility in its input. However, David Hopkins (1971), in an analysis of RRPM, concludes that these types of system are suitable mainly for making cost per student calculations under current operating conditions and it is questionable whether the expense of building in a large amount of detail for this purpose is justified. One final point on computerised MIS packages is that most of them when allocating resources have to make assumptions about student flow and they invariably use a Markov Model which requires only transition probabilities. Another more realistic
approach is to use the cohort method based on longitudinal data but this also has its limitations due to a lack of data and, as a result, analyses have to be based on one particular pattern of student flow only.

The Author and Derek Birch (1976c) have looked at the Management Information needs of a College of Further Education and tried to build up a system, initially manual, which on the one hand provides the basic aggregated data but on the other allows the calculation of some measures of performance at the course of department level. This necessitates, however, the collection of data on all the parameters mentioned in Chapter 3, namely enrolment to a subset of meetings, total enrolment to that subset, number of groups formed and so on.

One major advantage of such "custom-built" systems is that customer resistance is low whereas Gary Rice (1977) points out the very real dangers an administrator is faced with if he tries to implement a sophisticated MIS. The problems include combating fear of exposing weaknesses, coping with deliberate distortion of the input data, and the establishment of the integrity of the system as a source of information in allocations of staff, resources, and even in promotions.
3.6. SUMMARY OF CHAPT. 3

The inputs and outputs of teaching and research include students and their time, academic staff and their time, non-academic staff and their time, materials, space and equipment. All these can be measured quantitatively, qualitatively or in monetary equivalents with varying degrees of difficulty. The designation of a factor as an input or output depends on the boundary of the system considered. The analysis of inputs and outputs within an institution requires allocation of those inputs and outputs as does the analysis of costs within an institution. There is no agreed method of allocation, however. Management Information Systems by their very presence influence the way in which these inputs and outputs are measured and also the way in which allocations are made between activities, and there is usually a trade-off between detail of data collected and sophistication of the analyses on that data.
REFERENCES FOR CHAPTER 3  (in order of mention)

Section


3.2. Loughborough-Lanchester Study. This resulted in the following publications: (copied in Appendix 2)


REFERENCES FOR CHAPTER 3 (continued)

Section


REFERENCES FOR CHAPTER 3 (continued)

Section


CHAPTER 4  THE RELATIONSHIPS BETWEEN INPUTS AND OUTPUTS

4.1. INTRODUCTION

Once inputs and outputs have been identified and measured, either directly or via proxy measures, the next step must be an assumption about the relationships between those inputs and outputs. The usefulness of a measure of performance depends on the type of model that is used to represent the educational process. For example, the effects of various inputs may be additive or they may be multiplicative and the measures of performance should take this into account.

There are two main ways of modelling the educational process. Firstly, an explanatory model can be created which indicates how particular inputs affect particular outputs. Secondly, the process can be treated as a "black box" and examined statistically for mathematical relationships between the inputs and outputs. The first approach is difficult to implement, whereas the second is difficult to justify.
4.2. **THE ANALYTICAL APPROACH**

Ideally an analytical approach consists of the creation of a model which can be solved mathematically to provide measures of performance. However, this usually means that the model has to be a deterministic one with no random variations in it. If a stochastic model is developed then it usually has to be analysed by simulation techniques, although sometimes the nature of the random variations is such that a mathematical solution is still possible. Most models assume a static or a steady state situation but simulation does allow the analysis of dynamic models.

Most analytical models can be categorised as student flow models, student achievement models or consumption models where there are several main outputs each of which consumes some of the available inputs.

4.2.1. **Student Flow Models**

Student flow models are based on the concept of the educational process as a series of stages through which most, but not all, students move. The differences between the various types of model centre on the mechanism for determining transitions from one stage to another.

Comay *et al.* (1973) suggest that if education is viewed as an investment then the educational system is a series of paths for each student to choose from. The student chooses his route by reference at each decision point to the alternative discounted lifetime benefits, taking into account the possibility of failure at some future stage. Each stage can then be represented by a diagram similar to Figure 1.
This model is easily solved by dynamic programming techniques but suffers from the lack of data on future benefits resulting from particular career decisions.

As Roger Schroeder (1973) points out in his survey of Management Science in University operations, the most popular form of student flow model is that based on a Markov Chain. In other words, the transition from one particular stage to another occurs according to a laid-down probability which is dependent only on the two stages involved and not on how particular students reached one of those stages. This is a simplifying assumption but it enables the considerable array of Markov Chain techniques to be used to solve the model. Thus, most of the computerised resource allocation packages in use today contain Markov type models of student flow. The concept can be represented by a diagram similar to Figure 2 in which all the routes are determined by the laid down transition probabilities. The resulting student flow can be used to produce requirements for staff space and other requirements.
However, the Markov type model has a number of disadvantages. Firstly, in contrast to Comay et al (1973), it assumes that the students behave as a group of which a certain proportion drop out, a certain proportion fail and the rest 'pass'. However, since the previous stage is the only consideration, it ignores the fact that a weak student in one stage will also be a weak student in the next, so that the actual students that drop out or fail may well not be a random selection of the students in a particular stage. Indeed, the Loughborough–Lanchester study showed that the most important factor in examination performance was previous examination performance. Secondly, the Markov Model is relatively insensitive to changes in the transition probabilities. Johnstone and Philp (1973) show this by comparing the results of using an absurd transition probability matrix with the actual outcomes and producing nearly as good a fit as the suggested matrix. They point out that for slight fluctuations in the transition probabilities the model is even more insensitive. From the complex package designer's point of view this may be a good thing in that the model produces answers without being too affected by the initial estimates of the
probabilities involved. The third weakness of the Markovian Model is that it cannot handle a trend in the probabilities easily without upsetting the mathematical benefits of the approach, in other words it is a steady state static model. A fourth weakness identified by Sinha and Singh (1973) is that the transition probabilities are deterministic whereas the institution to be modelled is very stochastic in behaviour. They suggest the introduction of some stochastic behaviour by adding random variations or "noise" into the transition probabilities and using a simulation approach to see what happens.

Thus, the transition probabilities are related by the following equation:

\[ f_{ij}(k+1, k) = a_{ij} f_{ij}(k, k-1) + n_{ij}(k) \quad j < i \]

where \( f_{ij}(k+1, k) \) is the transition probability for moving from stage \( i \) to stage \( j \) in time \( (k, k+1) \)

\( f_{ij}(k, k-1) \) is the same probability for the previous time period

and \( n_{ij}(k) \) is a normal deviate with constant mean and covariance.

Unfortunately, this model when applied to real data by Sinha and Singh produces very poor predictions and hence their introduction of random variations turns out to be a symbolic rather than a practical move towards a stochastic approach.

Many of these disadvantages can be overcome by using actual past student flows as input to the model in a simulation approach. This necessitates the analysis of cohorts of past students. Oliver and Hopkins in 1972 split the student body into eight types of cohort including students who complete an undergraduate degree, students
who complete a postgraduate degree, first year drop outs, and so on.

Each of these cohorts is examined and an average lifetime in the educational system is calculated. Also relationships of a nature more complex than those of the Markov Model are introduced between these cohorts. Oliver and Hopkins solve their model by a network analysis approach and provide enrolment predictions that differ individually by no more than 3.3 per cent and in total by no more than 1.6 per cent from actual figures. However, the Author contends that all this means is that their model created out of longitudinal data fits that data very well. Thus, the disadvantage of the cohort method is that it assumes past patterns can be used as representatives of present and future patterns.

A different approach altogether to transition probabilities is utilised in an earlier paper by Oliver and Marshall (1970) in which they suggest that the educational process requires a certain constant amount of work by the student and he will not graduate until he has completed all the necessary units. Thus, instead of looking at the transition probabilities for the whole course, they suggest the examination of transition probabilities for individuals taking into account their time in the system. Now since those who complete all their units on time will have graduated after four years, those still there after four years will be weaker students and so the transition probabilities for individuals should show a marked change after four years' study (8 semesters). This is borne out by a study of students at the University of California using two cohorts entering in 1955 and 1960. For example, the percentages of the 1955 entry that completed so many consecutive semesters were as follows: (0-100, 1-97, 2-90, 3-73, 4-65, 5-52, 6-48, 7-41, 8-38, 9-10, 10-5, 11-1). Thus, the
proportions staying on from one semester to the next were, in the same order, (.97, .93, .81, .89, .89, .92, .85, .85, .93, .26, .5, .20) i.e. a break in the pattern after eight semesters. They use these proportions to predict the enrolments per semester for a cohort of 1961-1966 and are able to achieve reasonable predictions.

4.2.2. Student Achievement Models

Student flow models all input the transition probabilities either explicitly or implicitly. Student achievement models relate examination or test performance to the resource utilised. The model is then analysed using, perhaps, control theory. Sinha, Gupta and Sisson (1969) suggest that student performance is best measured in the aggregate so that a class pass rate is replaced by a class average mark. They assume that student performance depends on space per student, materials per student and teachers per student, and that these effects all interact multiplicatively. Hence they produce an equation similar to the following one:

\[ D = \sum A_i n_i U_i T_i V_i S_i M_i n_i \]

where \( D = \) student achievement

\( A_i = \) constant for course group \( i \)

\( n_i = \) number of students in course group \( i \)

\( T_i = \) teachers per student for course group \( i \)

\( S_i = \) space per student for course group \( i \)

\( M_i = \) materials per student for course group \( i \)

\( U_i, V_i, n_i = \) parameters for group \( i \) which reflect the returns to scale of each input ratio
This model can be solved using past data to find the parameters $U_i, V_i, n_i$ to examine the effects these input ratios have, although as the Author has emphasised in Chapter 2, these ratios may not be very meaningful being the ratio of two variables which may not be related in a simple linear way or even directly related at all. Roger Sisson himself (1969) carried this approach to the extreme when he produced the following equation at a symposium.

$$\text{Achievement} = \left\{ f_1(\text{SSR}) + k_1 \times \frac{M}{(1+S)} \right\} (l-e^{-A/A_o}) + f_2 \left( \frac{C}{1+p} \right) f_3(p)$$

(Test Scores)

where $f_1, f_2$ are functions defined by

$$f_i(x) = \frac{k_i}{1+\exp(k_i+1+k_i+2x)}$$

$A_o, k_i$ are constants

$M =$ materials per student

$A =$ space per student

$S =$ support staff per student

$SSR =$ student staff ratio

$C =$ community effort/student

$p =$ parental education

This has everything an equation could want for except justification!

The form is governed to some extent by the need to be able to solve the equation although it is still too complex for intelligibility.

4.2.3. Consumption Models

If students are not the major output but are only one of a series of outputs, the educational process can be viewed as a consumption process where each of the outputs "consumes" part of each input. Then an educational institution can be represented by a series of
mathematical relationships and if these are linear or can be approximated to by linear equations then the technique of linear programming can be used to solve the model. Alex Mood (1969) suggests that in practice an assumption of linearity is good enough for institutional analysis.

Jean Benard (1973) produces almost the last word in consumption models when he outlines his mathematical model of a university. His approach is described in detail in Appendix 1.3 but it is worth noting several points here. Firstly, he treats students as an input and an output. Enrolments are an input but numbers of successful students are an output. Secondly, he measures the input into the teaching activity by staff time but he measures the output of the teaching activity by student numbers. This inevitably means that since departments provide teaching, but students enrol on courses which may require teaching from several departments, he has to distinguish between a set of meetings attended by students from several courses (total student numbers $E$ in the notation of section 2.2) and a programme of study involving combinations of such meetings (total student numbers $E$ in the notation of section 2.2). Thirdly, he measures time as the main input into research activity and publications as the main output. Administration, he visualises as an intermediate activity which can be incorporated into the model if necessary. Thus, his final outputs for the university are graduates and research publications. He postulates linear relationships between the inputs and outputs and so he is able to formulate a linear programming model. The constraints involved are described in Appendix 1.3, but it should be noted here that of the eight inequalities, four are capacity limits, three are consistency equations, and the other states that research output
will not exceed that expected based on the time put in. The objective on the other hand is the discounted social utility of graduates and research publications. Thus, Jean Benard achieves a set of constraints that Paul Alper (1970) would describe as internal consistency rules and an objective that is not amenable to practical implementation. Thus, there is no equation which relates educational performance to educational environment. There are only book-keeping relationships which ensure that "something" this year equals "something" last year plus movements in minus movements out. Thus, the model is tautological arithmetic and this is one of the problems of linear programming type approaches to education. As Peter Armitage (1973) points out, such models have no inherent educational character so that if the labels of the variables were removed the equations could be representing any physical system where particles move from state to state.

Goal Programming is a technique which attempts to remove one disadvantage of linear programming, namely the need for one unique objective. Now this can be found by weighting the outputs, but this boils down to Benard's approach (1973) of using utilities which are never easy to identify. The technique instead involves a set of targets for each output. The outputs are represented by linear equations in the inputs but the discrepancies between the actual output from the model and each target is input as a variable itself and it is these discrepancies that are minimised. To do this in one iteration requires the attachment of weights to each discrepancy which comes back to the problem of utilities. However, Lee and Clayton (1972) and Geoffrion et al (1972) get around this problem by asking the decision maker to attach weights to the various discrepancies from his targets. The linear program is
then run and the results are presented to the decision maker. Probably not all the targets are reached so if he does not like the answer he is invited to revise his weightings and the linear program is run again. Eventually an acceptable solution is found in a situation where not all targets can be achieved simultaneously. Unfortunately, in both of these studies the equations are really only consistency equations. Also it is a debatable point whether the approach leads to a final solution that the decision maker had not thought of but finds acceptable or whether it is really an estimation of the weightings which explain his choice of output mix at the time of the computer runs. The latter is only useful if the weightings can be used again but this assumes they are stable over time. In many situations even if the decision maker is entirely rational and consistent, it is reasonable to use different weightings at different times so that differential encouragement can be given to the various activities of the institution.

An entirely different approach to the concept of a consumption model is that based on Industrial Dynamics. Thus the system is modelled via a set of levels, flows and valves in which students, staff and space provide the levels, the educational process provides the flows, and the valves are pass rates, enrolment rates, drop out rates, staff recruitment rates, staff retirement rates, staff promotion rates, and so on. Such a model has been created by Robert Thompson (1969) and, although it requires a lot of initial values for rates to be input, it does give a good idea of how changes in enrolments or teaching patterns work through the system causing ripples to spread and then die away as the system settles down again, perhaps in a slightly different state which is something that the LP models cannot do. However, although it is a dynamic model, it is deterministic in
the sense that given the initial values the simulation will produce the same outcomes and the relationships are again all of the consistency type.

Thus, it is clear that consumption models are simple to solve and if linear programming is used there are a range of sensitivity analyses available. However, in return for ease of solution, much of the educational flavour is missing from the relationships involved.
4.3. THE STATISTICAL APPROACH

If the educational process is treated as a "black box" which consumes inputs and produces outputs, then these inputs and outputs can be examined and a relationship found statistically. However, Weathersby and Truehart (1977) state that one of the key concepts in a "production function" analysis is that the production function describes the most efficient use of resources actually observed in practice, not the average, median or any other measure of resource use. Hence, the first stage in any statistical analysis should be a decision regarding whether the whole sample of departments, courses, or institutions should be considered or only some more efficient subset. Daryl Carlson (1976), however, points out that previous studies can still be split into those that identify the more efficient or frontier institutions and use those and the others who use the whole sample. If the whole sample is used, the relationship may well be found by regression techniques and thus represents a truly average performance. Indeed, the situation can be illustrated symbolically as in Figure 3.

![Figure 3: Possible Production Functions](image-url)
Chapter 7 discusses the identification of this subset of the sample observations - "the efficient frontier". Given that an appropriate set of data has been chosen then there are three main ways that it can be examined statistically. Firstly, certain easy to solve (difficult to justify) non-linear relationships can be fitted to the data as in an econometric analysis. Secondly, linearity can be assumed throughout and multiple regression techniques applied. Finally, the relationships can be specified only in terms of the inputs used and the outputs produced, without the establishment of any formal mathematical relationship.

4.3.1. Curve Fitting

Weathersby and Truehart (1977) offer a good example of the curve fitting approach to identifying the production function in Higher Education. Starting with a set of "more efficient" colleges, they attempt to fit three relationships to a model of a college system consisting of one output and two inputs. The output is the student enrolment as a proxy for the standard of educational provision made available. The two inputs are the full-time equivalent numbers of administrative and academic staff and an estimate of the cost of the capital utilised by a college each year based on a proportion of the value of land, buildings and equipment. The relationships are fitted by ordinary least squares linear regression, hence the chosen three relationships reflect this. They consist of a linear model, a multiplicative model (Cobb-Douglas), and a hybrid model which has constant elasticity of substitution (CES model).

Let $Q_i$ = output of college $i$

$L_i$ = labour input to college $i$
\( K_i \) = capital input to college \( i \)

\( U_i \) = random error term for college \( i \)

and let \( a, \beta, \varepsilon, \rho, \nu, \delta \) be constant parameters.

Then we have the three types of relationship

(a) \( Q_i = a_0 + a_1L_i + a_2K_i + U_i \) (Additive-Linear)

(b) \( Q_i = AL_i^\alpha K_i^\beta U_i \) (Multiplicative-Cobb Douglas)

(c) \( Q_i = \left( \gamma \left[ L_i^{-\rho} + (1-\delta)K_i^{-\rho} \right]^{1/\rho} \right)^\nu (U_i) \) (CES)

The linear model is very simple but, of course, every extra unit of input produces the same response in the output, i.e. there is no decline in the marginal productivity of inputs. However, it is useful as a means to explore the data and it does provide a reference point to determine the degree of non-linearity.

The multiplicative Cobb-Douglas model seems more realistic in that there will be an interaction between the inputs. This model will show positive returns to scale if \( (\alpha + \beta) > 1 \); constant returns to scale if \( \alpha + \beta = 1 \), and negative returns to scale if \( (\alpha + \beta) < 1 \). It has, however, constant elasticity of substitution between the two inputs (equal to 1) so a 5\% change in the ratio of labour to capital produces a 5\% change in the marginal rate of substitution of labour for capital.

The CES model has constant elasticity of substitution but the constant is equal to \( 1/(1+\rho) \), hence if \( \rho = -1 \) the model becomes a linear model, if \( \rho \) lies in \( (0, -1) \) the constant is greater than 1 and approaches
the Cobb-Douglas form and if $\rho$ is positive it has a different form again, as shown in Figure 4.

![Diagram showing the Cobb-Douglas Model and its types](image)

Figure 4: The Cobb-Douglas Model and its types as shown by their unit in-scans.

These models are obviously chosen because they are well-known, simple to solve, models from economic theory rather than because they are particularly apt for an educational system.

C. P. Timmer (1971) tackles the problem of choosing between fitting a function to the whole sample of observations and to a more efficient subset by trying both out admittedly on agricultural data. He finds an average production surface by least squares regression and he finds a frontier production surface by linear programming. However, he has made an important contribution to the area by pointing out that a surface found by picking out the "best" observations may be very sensitive to data errors. Some of these "more efficient" observations may be spurious. Thus, he suggests that they be withdrawn from the sample set one at a time to see how stable the calculated production function is, and he notes that for his agricultural data it only needs 27 to be disregarded for the frontier function to become very similar but parallel to the average function. He uses a Cobb-Douglas function in which logarithms are taken so that it becomes linear and
so amenable to regression or linear programming but it is still a valid point.

Layard and Very (1973) try a more econometric approach by attempting to find cost functions for departments using additive, multiplicative and polynomial type models. They suggest that the cost of a group of departments (e.g. a discipline area) is a function of the number of student-years, the number of departments in the group, and the research activity as measured by publications.

Hence, \( C = \text{function} (D, U, P, R) \)

where

- \( C \) = cost of discipline area
- \( D \) = number of departments in that discipline area
- \( U \) = undergraduate-years involved
- \( P \) = postgraduate-years involved
- \( R \) = weighted research papers
- \( S \) = student numbers

If analysis is confined to fairly disjoint discipline areas then these measures of output can be easily estimated. However, in practice, whatever the aggregation service teaching and, in particular, joint meetings between courses make such measures difficult to calculate unless the data is collected in the detail of section 2.2. Layard and Very try a function linear in all outputs except student numbers (teaching output is represented by student-years), a function cubic or quadratic in student numbers only, and a function involving a multiplicative relationship between the variables.

\[ C = a_0 D + a_1 U + a_2 P + a_3 R \]
This is a linear relationship and as such allows no encouragement for small departments and makes the allocation of students relatively unimportant.

\[ C = a_0 D + a_1 S + a_2 S^2 + \ldots \]

A polynomial in student numbers gives a size effect. If it is quadratic this suggests one optimal size. If it is cubic this implies that expansion should occur between two extremes of large and small departments.

\[ \frac{C}{D} = a_0 \left( \frac{U + wP}{D} \right)^{a_0} x \left( \frac{R}{D} \right)^{a_2} \]

where \( w \) is the UG/PG relative weighting.

A multiplicative model theoretically caters for interaction between the variables but this relationship has a higher ratio of standard error of estimate to its mean value than the linear model. Hence, Layard and Verry suggest that interactions should be built into the linear model instead of using a multiplicative model at all (see next section).

Their original contribution, however, is the introduction into the equations of a measure of learning gain. They measure the quality of the student before and after each student-year by relating A-level scores and degree classes to salary data. This is discussed in detail in Appendix 1.4. Unfortunately, they found that the use of their measure of learning gain led to less explanatory power rather than more so they left it out again! They also emphasise that the data for educational institutions exhibits a degree of heteroscedasticity.
(i.e. different variability about the mean at different levels of activity). Hence they divided all their variables by $\sqrt{r}$ of the number of students before carrying out the regressions. They suggest that without this the absolute errors are positively correlated with student numbers and, furthermore, that if student numbers are the divisor the absolute errors are negatively correlated with student numbers. Hence, the square root is the more appropriate. This is one of the dangers of using multiple regression techniques, namely that the data may have to be transformed to fit the assumptions underlying a regression analysis. Alex Mood (1969) points out strongly that analyses should be carried out with the major determinants as independent variables rather than transformed variables or orthogonal variables created by a factor analysis. If a model is to be applied by administrators and laymen they have to understand it so the terms used must be wholly meaningful to laymen. The usefulness of a model could be ruined by the use of esoteric jargon.

4.3.2. Linearity Throughout

If each output is assumed to be a linear combination of all the inputs then the appropriate coefficients can easily be found via multiple regression. The problem, however, is that often the various inputs are highly correlated, e.g. input of technical staff, input of equipment and specialist space, and so several of the inputs may remove nearly as much of the variability of the data as all of them so, in other words, there may be interaction between them. Alex Mood (1969) suggests a way of partitioning this variability into unique and common parts. In essence his argument is that if one variable $P$ removes $R_p^2$ of the variability when used alone and $Q$ removes $R_Q^2$ when used alone but $P$ and $Q$ together remove $R_{pQ}^2$ of the
variability then the unique contribution of $P$ is $(R_{PQ}^2 - R_Q^2)$, the unique contribution of $Q$ is $(R_{PQ}^2 - R_P^2)$ and the remainder, which is $(R_Q^2 + R_P^2 - R_{PQ}^2)$, is the proportion which may be associated with either $P$ or $Q$. Indeed, this is the approach which George Mayeske (1969) applied in his follow-up to the Coleman Report to produce the finding referred to in section 1.2.2 of Chapter 1 of this thesis, that the main factor in promoting achievement was either family background or school quality or both. Layard and Verry (1973), in their study referred to in the previous section, also attempt to cater for interaction effects by using an equation of the form

$$C = a_0 + a_1 U + a_2 P + a_3 R + a_4 \sqrt{UR} + a_5 \sqrt{PR} + a_6 \sqrt{UP}$$

where $C$, $D$, $U$, $P$, $R$ are as defined in that section.

They found, however, that in the regressions they carried out the interaction terms rarely received significant coefficients and so they suggest there is no reason to depart from a simple linear model.

The Author, in the course of the Loughborough-Lanchester study, carried out a series of regression studies to try and identify the important determinants of student performance and student salary. The most important influence on student performance was past student performance but the explanatory power was not very high. Student descriptor variables were used as well, e.g. sex, age, discipline area, but the only consistent variable of this type was the percentage of the course that was married, which the Author postulates is a reflection of discipline differences. The salaries were determined very much by discipline area and very little by examination performance. But the major reason why these regressions were not very useful to the
study was that the variation in the output measure (student examination marks) was extremely low between courses. This implies that examination marking is relative not absolute.

4.3.3. Direct Analysis of Input-Output Vectors

There are obvious problems discussed in the previous sections when an attempt is made to determine mathematically a function which explains the consumption of inputs and production of outputs. A more direct method is to treat the educational process as a "black box" and represent it by a list of the inputs and outputs, in other words by an input-output vector. These input-output vectors can then be examined as a set of vectors without any other assumptions about input-output relationships.

There are several ways of doing this. Firstly, the input-output vectors can be ranked in some way. Clarke and Rivett (1978) show an example where a set of 24 five-dimensional input-output vectors is examined for groups that a decision-maker finds equally important. This leads to an indifference table. Then the vectors are transformed using multi-dimensional scaling so that they are represented on a two-dimensional graph by 24 points whose separation distances correspond to the actual differences between the original vectors. The most preferred and the most disliked turn up as extremes on the two-dimensional map. Unfortunately, this has not yet been applied in practice although, in theory, it should illustrate how far particular institutions are from the most favoured ones. However, it obviously assumes the existence of one source for the indifference table and hence is really another version of the utility approach in that instead of attaching weights to outputs the decision-maker
simply notes which input-output vectors are equally acceptable; although it must be said that treating all inputs and outputs as equally important is a form of weighting too!

Secondly, the input-output vectors can be categorised and the subgroups can be more easily examined. Fritschi et al (1978) split a sample of institutes (within institutions) into various groups, each of which is then examined for size effects. They measure total expenditure as the input and have three outputs, teaching hours, research publications, and extra-mural courses and lectures. They divide all the outputs by total expenditure to give a three-dimensional problem which can be examined with the aid of diagrams similar to those in Figure 5. They find their groups by looking at high, medium and low values for each of the output (per unit expenditure) values. Thus one group is low on teaching hours, low on publications, but high on extra-mural activity per unit total expenditure. Each group is examined to see if the size pattern reflects the overall pattern (see Appendix 1.5 for an example of this approach).

![Diagram](image)

**Figure 5:** Subgroups of the sample

L - Low
M - Medium
H - High
Thirdly, the input-output vectors can be grouped using a technique such as Cluster Analysis. Strigl and Traumüller (1975), at the Johannes Kepler University in Linz in Austria, apply a cluster analysis to student achievement data to try and identify the factors which determine student success. They produce nine clusters which can be grouped into three groups of generally high marks, generally medium marks and generally low marks. The failed students are clearly differentiated from the others by the clustering method but the successful and the slower students tend to merge inside the groups.

Finally, the input-output vectors can be used to define a set of points in a multi-dimensional space. Intuitively the more efficient institutions, departments or courses will be represented by points on the boundary of this set of points and, if necessary, a production surface can be produced by using the hyperplanes through these more efficient points. The Author suggests, however, that it may be useful simply to identify more efficient institutions and use them as reference points for the others. Weathersby and Truhart (1977), in their curve-fitting approach discussed earlier, only examined the more efficient institutions whereas the Author (1978) postulates that it is important to measure the inefficiency of the non-frontier institutions as well. Two papers on the subject are presented in Appendix 2 in Volume 2 of this thesis and the topic is discussed further in Chapter 7.
4.4. LIMITATIONS OF PRODUCTION FUNCTION ANALYSIS

Truchhart and Weatherby (1978) point out that this approach of inferring a production function from observed resource usage contains several important assumptions.

Firstly, it is implicitly assumed that the decision-makers in the institution know what the production function is. Henry Levin (1974) argues that in reality the technology of the educational system is largely unknown by educational decision-makers at all levels and H. Liebenstein (1976) argues that, even with perfect information, managers have differential ability and enthusiasm.

Secondly, it is assumed that decision-makers have complete control over the inputs and outputs. Paul Alper (1972) states that some elements of the system are controllable, others are simply observable.

Thirdly, the value of inputs and outputs is assumed to be agreed and known. But in practice different funding bodies put different weightings on research, postgraduate students, sandwich students, and so on.

Fourthly, it is assumed that the system has no external constraints on the use of resources and this is countered by the "guidance" the polytechnics in the UK receive on staff-student ratios, for example.

Finally, as has been indicated in Chapter 3, there is still a lot of research to be carried out on the measurement of inputs and outputs as the choice of input or output measure may critically affect the production function identified.
4.5. SUMMARY OF CHAPTER 4

Measures of performance of an educational institution relate to the performance of a system which consumes inputs and produces outputs. Hence, their selection depends on the assumptions made about the relationship between those inputs and outputs. Relationships can be postulated and then fitted to the data available but the more theoretically justifiable relationships are always the ones that are more difficult to solve and, in practice, the assumption of a linear relationship can provide enough information about performance.

Relationships can also be found by statistical analysis of the available data and this is easiest to do by fitting standard economic relationships, but these have little educational justification.

Finally, the relationship can be represented by a set of input-output vectors rather than a formal mathematical relationship and this allows the identification of more efficient institutions by ranking, clustering or frontier finding methods.
REFERENCES FOR CHAPTER 4 (in order of mention)

Section


REFERENCES FOR CHAPTER 4 (Continued)

Section


5.1. INTRODUCTION

There are several ways of looking at performance, the most obvious being the concepts of efficiency and effectiveness. Efficiency involves the comparison of all the inputs with all the outputs whereas effectiveness involves the comparison of the outputs with goals or objectives. It is difficult in practice to identify an overall measure of performance so attention is usually focussed on partial measures of performance which only involve consideration of a few inputs and outputs. The most obvious example of this approach is the widespread calculation of the cost per student. Sorenson and Grove (1977), when discussing non-profit making organisations generally, propose several concepts of partial performance measurements, namely:

1. Availability - what facilities are available?
2. Awareness - who knows of them?
3. Accessibility - who uses them?
4. Extensiveness - what should be available?
5. Appropriateness - what could be available?
6. Acceptability - are the users satisfied?

At the institutional level, some of these measures require a degree of institutional self-evaluation that is unknown outside of the United States where, due to the accreditation system, many institutions have to do this at regular intervals. Thus, in the UK the data or the mechanisms for collecting it for such purposes are very limited.

John Sizer (1978) argues that measures of performance must not only be relevant, they must also be verifiable, unbiased, quantifiable and
relatively cheap to obtain. The last two in particular rule out a lot of possible measures. P. F. Gross (1973) adds another restriction when he states that some measures of output show very little variability compared with the measures of input and so relationships are difficult to justify statistically. In the Loughborough-Lanchester study, the examination marks were fairly uniform across departments, discipline areas and even institutions, despite differences in teaching patterns and use of resources. As a result it was not possible to produce acceptable relationships between the inputs and examination marks by regression analysis. The Author would add another restriction on the choice of measures of performance by emphasising that the level of analysis is important. For example, the infamous student-staff ratio is useful at institutional level but is virtually meaningless at departmental or course level given the amount of service teaching and combined meetings between courses unless its definition is altered drastically.

The Author suggests that partial measures of performance should be considered as a set of indicators rather than be used as candidates for one overall measure. A measure of efficiency relates all inputs to all outputs and so must incorporate performance in all the activities of the institution not just one.
5.2. **TEACHING ACTIVITY MEASURES**

In Chapter 3 it was suggested that teaching activity is best considered as an activity involving class contact between staff and students leaving preparation time and marking time to be taken into account when setting a maximum workload for academic staff. Even so the possible measures of performance are legion, varying from input-output ratios to direct measures of educational provision.

5.2.1. **Input-Output Ratios for the Teaching Activity**

The most widely used and hence the most often misused input-output ratio is the cost per student. Most of the statistics published by the DES treat students as the only output and costs as the only input. However, the Author suggests that the process is not as simple as that. Resources are allocated to institutions and within institutions to departments who provide teaching and carry out research. Students, on the other hand, enrol on courses and are taught by several departments. Hence, before a cost per student can be found either the departmental resources must be allocated to its teaching and research activity or the students must be allocated to departments. In Chapter 3 the allocation of inputs and outputs is discussed and in Chapter 6 the breakdown of the cost per student to show the effects of changes in enrolment and/or changes in teaching pattern is discussed further. In fact, the Author suggests that the cost per course is more useful than the cost per student since extra students may well not affect the cost of the course but will obviously decrease the notional cost per student.

The output from the teaching activity can also be measured by the number of meetings given or by the student hours supplied (student
numbers multiplied by their hours), so if a department's resources
can be allocated wholly or partly to teaching the cost per meeting or
the cost per student hour is a useful measure of departmental teaching
activity. These two measures are also discussed further in Chapter 6.

The student-staff ratio is not relevant at course level unless it is
redefined as the ratio of the student enrolment to the relevant staff
which can only be calculated by measuring the contact hours of staff
that can be fairly allocated to the course in question and converting
these hours to full-time equivalent staff numbers. At departmental
level the same procedure should be carried out to measure staff input
into teaching and the student numbers have to be found by allocating
them between the departments involved in their teaching. Thus, at
departmental level, a measure of teaching activity is the ratio of
relevant students to relevant staff. If the actual staff establishment
is used, this assumes that all the staff time is spent on teaching and
none is spent on research or administration. At the institutional
level, the relevant students are simply the total full-time equivalent
students but the relevant staff should again be calculated from
teaching hours.

The Author and Derek Birch (1974) argue that the student-staff ratio
is good enough for allocation of resources to institutions since
students are the major output and staff are the major input, but that
allocations within the institution should take account of teaching
hours rather than students involved. Appendix 1.6 shows the mathematical
definition of these input-output ratios and an example of their
application.
5.2.2. Direct Measures of Educational Provision

Decisions about educational provision relate to courses in the main and so measures of performance relate to courses and are aggregated and averaged to get equivalent measures at higher levels.

Student taught hours for a course measures the hours each student receives and as such is one measure of the learning environment provided. If optional subjects are treated as examples of small split group working, then the student's taught hours is simply the total of the student taught hours for each subject. It does not, however, indicate the sizes of the classes involved or the type of teaching activity.

Average class size for a course means nothing unless it is the average class size as perceived by the students on the course. It is not the Delany "ACS" which relates to the class sizes perceived by the teachers involved with the course who utilise split group working and joint meetings with other courses. See Appendix 1.7 for a discussion of the "Delany" ratios. It must be calculated by averaging the various class sizes experienced by a student on the course weighted by student taught hours. The measure is then another indication of the teaching environment provided.

There are negative measures of educational provision. The amount of time spent in joint meetings with other students or being taught by staff from other departments are arguably measures of the inappropriate-ness of the teaching to the course in question, although in theory it should not be so. Appendix 1.6 shows the mathematical definitions of these measures of performance at the course level.
Theoretically the level of marks or variability of marks should reflect the teaching patterns but in the case of the Loughborough-Lanchester study it did not and the Author is inclined to suggest that examination marking is inherently relative, not absolute, and so examination marks are not a useful measure of teaching performance at the course level.

These measures can be averaged across a department's courses or across a discipline area, but double counting must be avoided by some method of allocation between departments. If different modes of attendance are combined by using full-time equivalents then these measures lose a lot of their usefulness. Indeed, the average student taught hours, if calculated as in Appendix 1.7 'according to Delany', becomes simply the norm - the full-time load of a typical student - not an average of any kind.

At the departmental level the viewpoint must be that of the teacher since resources are provided to carry out teaching and research. But it is important to distinguish between students enrolled in a department and measures associated with the teaching activity of that department, since they do not match unless a department gives or receives no service teaching at all. Hence the departmental measures have to be calculated from data aggregated from all the courses in the institution. But since all courses are considered, the problems of allocating students disappear and the "Delany" type measures come into their own, namely: the average class size provided by a teacher and the student-staff ratio for that teaching activity.

The average class size provided is a measure of the effort being supplied by a member of staff and it will affect marking, examining,
and so on. It is calculated as the ratio of the total student hours supplied to the total class contact hours needed. Hence, it can be easily calculated from an institutional aggregation for each mode of attendance. The student-staff ratio is also a measure of the teaching activity of a department if it is calculated as outlined above and in Appendix 1.6.

It then indicates how many staff are involved in the teaching activity. However, it does not indicate how hard the staff in a department are working — for that the class contact per member of staff is an appropriate measure.

The final measure of educational provision is the acceptability by the user which usually has to be obtained by student feedback methods. There is wide experience of this in the United States and one disadvantage is that it inevitably is taken into account in promotion and allocation decisions, although not always collected for that purpose. It's mere existence means that it will be used. Another more important disadvantage is that student groups differ in their reactions to particular teachers so that student feedback can vary dramatically for the same teacher in successive years and for different teachers in the same year. The Author has already commented that individual student benefits are more a function of the student's learning gain than the level of educational provision and as such are more useful to the individual teacher than to the allocator of resources.
5.3. RESEARCH ACTIVITY MEASURES

If part of a department's resources can be allocated to its research activity then a cost per unit output can be calculated if a measure of research output can be widely accepted within the institution. Thus, a cost per paper or research grants awarded per pound invested in research activity can be produced. In Holland there is a tradition that academic staff spend 30% of their time on research and so the cost per paper can easily be calculated, although the measure of research output may not be widely accepted in the institution. In the UK there is no standard figure for research input and diary exercises have indicated a wide variation between institutions let alone departments in the time allocated to research. In Appendix 1.8 the Author outlines one method of deriving a unit cost for research activity based on the VCCP Survey of Academic Staff Time. Terence Burlin (1976), in an article mentioned in Chapter 3, introduces the concept of the opportunity cost of research. He suggests that the production of a full paper in a technical journal is at the expense of 6 hours a week tuition for 36 weeks to an average size group of 14 students. So the opportunity cost of a full paper is 3000 student-hours. This is an interesting idea but it is dependent on an estimate of the time needed to produce a particular type of research output. Indeed, if this is available, the pieces of paper can be converted into staff time and the actual cost established! In Appendix 1.2 the Author shows the effects of assuming research output consumes specific amounts of resources, in particular that research activity is measured only by total resources allocated to it. Whereas in Appendix 1.8 the Author shows that a comparison of research output with a fixed proportion of total departmental expenditure does allow a more useful measure of research activity, namely the unit cost.
In fact, most studies of institutional performance ignore research activity. Peter Mertens (1978), for example, in a study of six German Universities, utilises eight measures of expenditure, nine measures of capacity, ten measures of student study time and throughput, but no measures of research activity!
5.4. OTHER ACTIVITIES

Administration, teaching support and central services are intermediate activities but they can lead to measures of performance such as books per student, computing time per student, technicians per student, and so on. Performance can also be estimated by carrying out opinion surveys of the staff and students of the institution. However, there would not be much support for the idea of promoting administrative performance over teaching or research performance.

At departmental or course level, the only inputs to be taken into account of this type are the departmental administrative, clerical, technical and secretarial staff, plus expenditure on materials and equipment. Space is often allocated from a central pool so its opportunity cost is virtually zero as far as a particular course or department is concerned. Central Services should be taken into account when measuring institutional performance, but are not really relevant to analyses at course or departmental level.
5.5. PERFORMANCE PROFILES

Educational Activity is a multi-dimensional process and hence the use of only one of the measures discussed above, e.g. cost per student, is to be deplored. Either a measure of performance should incorporate all the inputs and all the outputs or all the partial performance measures should be presented in a profile for comparison, evaluation and decision. Thus, in the presentation of the Loughborough-Lanchester study to the Third General Conference of the OECD/CERT/EMHE Programme in 1976, the Author produced some of the results in the form of profiles showing student quality measures, teaching quality measures, plus cost measures. Since some measures are considerably larger in absolute terms than others, the Author standardised the profiles by converting each set for a course to deviations from the mean for each indicator measured in units of the standard deviation of that indicator. Figure 6 shows an example of this approach. Such profiles can also be compared with those for the discipline areas. This approach leads to the identification of courses or discipline areas with a lot of extreme values for measures of performance. Douglas Porter (1978) modifies this idea to suggest that the profiles be compared simply with targets or averages on a histogram to show where the profile is out of step.
**Figure 6: Standardised Course Parameters**

<table>
<thead>
<tr>
<th>Student Profile</th>
<th>Response</th>
<th>Resource Utilization</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average A-level Score</td>
<td>Enrolment</td>
<td>Student's Tuition Load</td>
</tr>
<tr>
<td>% Female</td>
<td>Pass Rate</td>
<td>Student's Group Size:</td>
</tr>
<tr>
<td>% Married</td>
<td>Fail Rate</td>
<td>Mean</td>
</tr>
<tr>
<td>% Overseas</td>
<td>Dropout Rate</td>
<td>Standard Deviation</td>
</tr>
<tr>
<td>% Without A-levels</td>
<td>Average Exam. Mark</td>
<td>% Meetings Saved</td>
</tr>
<tr>
<td></td>
<td>Coefficient of Variation of Exam. Marks</td>
<td>% Meetings Serviced</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total Direct Cost per Student</td>
</tr>
</tbody>
</table>
5.6. OVERALL MEASURES

Some of the partial measures of performance discussed in the previous section can be used as crude measures of overall performance, although inevitably the absence in the measures of consideration of some of the outputs restricts their usefulness. For example, cost per student, if it is calculated as in section 3.2.6. by allocating departmental resources to teaching and that teaching to students and courses, includes some measure of teaching activity as well as general use of resources, but it, of course, ignores research output.

Efficiency is the relationship between all inputs and all outputs and in a multi-dimensional situation is difficult to define mathematically. All that can be done is to identify the more efficient institutions or departments by comparisons rather than to calculate an absolute measure of efficiency. The latter can only be done if the inputs are measured in a common unit, i.e. money, and the outputs are measured in a common unit, i.e. utility. In Chapter 7 the Author examines in detail the identification of the more efficient institutions and measures of relative efficiency by representing performance by a list of the inputs consumed and the outputs produced.
5.7. SUMMARY OF CHAPTER 5

Educational institutions are involved in a range of activities and, as a result, many of the common measures of performance are partial performance measures. These are either input-output ratios or direct measures of performance. The Author believes that an important consideration is the level of analysis. Some measures are more relevant at course or departmental level and others only make sense at institutional level. It is further suggested that an institution's performance is best represented by a profile involving several measures of performance or by a list of its inputs and outputs. These profiles or input-output vectors can be compared with each other or with the institutional average to establish relative measures of performance rather than absolute ones. After all, when deciding allocations of resources, it is relative performance that is important.
REFERENCES FOR CHAPTER 5  (in order of mention)

Section


6.1. INTRODUCTION

Unit costs are often used as overall measures of performance, although they do not take account of all the outputs of an institution or departments. However, they do reflect changes in enrolments and changes in teaching patterns and so are crude measures of teaching activity. John Delany (1976) suggests that unit costs at the institutional level can be broken down using the accountant's concept of variance analysis to show the effects of changes in student numbers and costs of resources on the differences between the actual and the planned situation. The Author with others (1977) has suggested that since resources are, in the main, allocated to departments and they take the decisions about the teaching environment to be provided for each course, it is more important to examine unit costs at the department or course level. This necessitates an allocation of departmental resources to its activities and from its teaching activity to the students involved. The examination of the effects of changes in enrolment and teaching activity demand the detailed data collection carried out by the Author and others in the Loughborough-Lanchester study to cope with joint meetings and split groups.

Variance analysis consists usually of comparing 'actual' with 'planned', but the Author suggests that the changes in unit costs from "Year One" to "Year Two" can be usefully analysed in a similar way. For purposes of illustration, consider a mythical institution (College X) with only two departments (X and Y) each of which recruits for one course only (A, B respectively). In Year One there are 40 enrolments to course A
and 10 to course B. This is not enough information to calculate unit costs since following a course involves attending a number of subject elements, some of which may be compulsory, some optional, some may be put on for one course alone, and others may be taught jointly with other courses. Thus, the analysis of unit costs requires not only information about the cost of the resources available for the teaching activity, but also information about the pattern of teaching provided. Figure 7 shows the student taught hours, the number of groups formed, and the enrolments from courses A and B respectively for each subject element. The latter is necessary since Department Y provides two subject elements, one for half of course A and all of course B, and the other only for course B in Year One, since course A students all attend subject element N but they have a choice for the other subject element between M and O which, incidentally, have different hours and group sizes.
### YEAR ONE (19X1)

<table>
<thead>
<tr>
<th>Department</th>
<th>Resources allocated to teaching</th>
<th>Subject elements provided</th>
<th>Groups formed</th>
<th>Student contact hours (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>£2400</td>
<td>M N 0 P</td>
<td>2 1 1 1</td>
<td>15 30 30 20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Course</th>
<th>Enrolment (E)</th>
<th>Enrolments to subject elements (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>40</td>
<td>20 40 20</td>
</tr>
<tr>
<td>B</td>
<td>10</td>
<td>10 10 10</td>
</tr>
</tbody>
</table>

Enrolment from all courses (E*) 20 50 30 10

Meetings provided (g) x (h) 30 30 30 20

---

### YEAR TWO (19X2)

<table>
<thead>
<tr>
<th>Department</th>
<th>Resources allocated to teaching</th>
<th>Subject elements provided</th>
<th>Groups formed</th>
<th>Student contact hours (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>£2500</td>
<td>M N 0 P</td>
<td>1 1 2 1</td>
<td>20 30 30 20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Course</th>
<th>Enrolment (E)</th>
<th>Enrolments to subject elements (S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>30</td>
<td>10 30 20</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>20 20 20</td>
</tr>
</tbody>
</table>

Enrolment from all courses (E*) 10 50 40 20

Meetings provided (g) x (h) 20 30 60 20

---

*Figure 7: College X's Teaching Activity*
6.2. COST ANALYSIS AT THE DEPARTMENTAL LEVEL

As has already been stated, one measure of the teaching activity of a department is the number of meetings it provides. Another measure of its commitment to teaching is the total student-hours it makes possible (students multiplied by their hours). Therefore, for College X described in Figure 7 it is possible to calculate a cost per meeting and a cost per student-hour for each of the two years as shown in Figure 8. However, these unit costs will vary with enrolments, patterns of teaching, and the cost of the resources used. It is important, therefore, to break these unit costs down to show changes due to these factors.

<table>
<thead>
<tr>
<th>Department</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td>19X1</td>
<td>19X2</td>
</tr>
<tr>
<td>Resources allocated to teaching (£)</td>
<td>2400</td>
<td>2500</td>
</tr>
<tr>
<td>Meetings provided (£)</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td>Student-hours total (£hE*)</td>
<td>1800</td>
<td>1700</td>
</tr>
<tr>
<td>Cost per meeting (£)</td>
<td>40</td>
<td>50</td>
</tr>
<tr>
<td>Cost per student-hour (£)</td>
<td>1.33</td>
<td>1.47</td>
</tr>
</tbody>
</table>

**Figure 8: Departmental Unit Costs**

For example, does the cost per meeting for department X change from 40 to 50 because of increases in resource cost or decreases in teaching activity or disproportionate changes in both factors? What is needed is a way of bringing out the effects of such changes.

The Author suggests an approach based on variance analysis in which the situation of Year One is changed to that of Year Two in three steps by first changing the teaching pattern and producing one intermediate situation, then also changing the number of enrolments to that...
of Year Two, thus producing a second intermediate situation, and finally by changing the cost of resources used to that of Year Two producing the Year Two situation. Each of the four situations can be analysed and unit costs calculated. The changes in these unit costs from one situation to the next, where only one factor has been changed, indicate the effect of that factor on the overall change from Year One to Year Two. Figure 9 shows the changes in teaching activity that this approach produces. The change in cost of resources obviously leaves the teaching pattern unaltered so this is not shown in this figure.

If Figure 9 is examined, Subject element M is seen to involve a decrease in teaching activity which on its own actually increases the student taught hours, and hence the student-hours, but when accompanied by a drop in enrolment produces a drop in student-hours. This illustrates how the method splits up the two effects. As will be seen later, the order in which the factors are changed does not affect the results of the analysis. The results shown in Figure 9 can be aggregated to show the changes in teaching activity of each department, both as measured by meetings given and student hours provided. Then if the resources allocated to teaching are known in money terms then a cost per meeting and a cost per student-hour can be calculated for the two intermediate situations and compared with those for Year One and Year Two. The results of such an analysis are shown in Figure 10, which shows the variances or changes from one situation to the next in the total cost and the unit costs.
<table>
<thead>
<tr>
<th>Subject Element M</th>
<th>Student Taught Hours (h)</th>
<th>Meetings Provided (hg)</th>
<th>Students Involved (E*)</th>
<th>Student-Hours (hE*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern of 19X1</td>
<td>15</td>
<td>30</td>
<td>20</td>
<td>300</td>
</tr>
<tr>
<td>Only meetings changed</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>400</td>
</tr>
<tr>
<td>Pattern of 19X2</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject Element N</th>
<th>Student Taught Hours (h)</th>
<th>Meetings Provided (hg)</th>
<th>Students Involved (E*)</th>
<th>Student-Hours (hE*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern of 19X1</td>
<td>30</td>
<td>30</td>
<td>50</td>
<td>1500</td>
</tr>
<tr>
<td>Only meetings changed</td>
<td>30</td>
<td>30</td>
<td>50</td>
<td>1500</td>
</tr>
<tr>
<td>Pattern of 19X2</td>
<td>30</td>
<td>30</td>
<td>50</td>
<td>1500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject Element O</th>
<th>Student Taught Hours (h)</th>
<th>Meetings Provided (hg)</th>
<th>Students Involved (E*)</th>
<th>Student-Hours (hE*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern of 19X1</td>
<td>30</td>
<td>30</td>
<td>30</td>
<td>900</td>
</tr>
<tr>
<td>Only meetings changed</td>
<td>30</td>
<td>60</td>
<td>30</td>
<td>900</td>
</tr>
<tr>
<td>Pattern of 19X2</td>
<td>30</td>
<td>60</td>
<td>40</td>
<td>1200</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subject Element P</th>
<th>Student Taught Hours (h)</th>
<th>Meetings Provided (hg)</th>
<th>Students Involved (E*)</th>
<th>Student-Hours (hE*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern of 19X1</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>Only meetings changed</td>
<td>20</td>
<td>20</td>
<td>10</td>
<td>200</td>
</tr>
<tr>
<td>Pattern of 19X2</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>400</td>
</tr>
</tbody>
</table>

Figure 9: Intermediate Changes in Teaching Activity
### DEPARTMENT X

<table>
<thead>
<tr>
<th>Year</th>
<th>Meetings Provided ($\sum hg$)</th>
<th>Student-Hours ($\sum hE*$)</th>
<th>Resources</th>
<th>Cost per Meeting</th>
<th>Cost per Student-Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>(a) 60</td>
<td>1800</td>
<td>2400</td>
<td>40</td>
<td>1.33</td>
</tr>
<tr>
<td></td>
<td>Teaching Changed (b) 50</td>
<td>1900</td>
<td>2400</td>
<td>48</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>Enrolment also (c) 50</td>
<td>1700</td>
<td>2400</td>
<td>48</td>
<td>1.41</td>
</tr>
<tr>
<td>19X2</td>
<td>(d) 50</td>
<td>1700</td>
<td>2500</td>
<td>50</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a) n/a +8 -(.07)
Effect of Enrolment Change (c) - (b) n/a n/a +.15
Effect of Cost Change (d) - (c) +100 +2 +.06

<table>
<thead>
<tr>
<th>Year</th>
<th>Meetings Provided ($\sum hg$)</th>
<th>Student-Hours ($\sum hE*$)</th>
<th>Resources</th>
<th>Cost per Meeting</th>
<th>Cost per Student-Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>(a) 50</td>
<td>1100</td>
<td>2000</td>
<td>40</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>Teaching Changed (b) 80</td>
<td>1100</td>
<td>2000</td>
<td>25</td>
<td>1.82</td>
</tr>
<tr>
<td></td>
<td>Enrolment also (c) 80</td>
<td>1600</td>
<td>2000</td>
<td>25</td>
<td>1.25</td>
</tr>
<tr>
<td>19X2</td>
<td>(d) 80</td>
<td>1600</td>
<td>2800</td>
<td>35</td>
<td>1.75</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a) n/a - (15) 0
Effect of Enrolment Change (c) - (b) n/a n/a -(.-57)
Effect of Cost Change (d) - (c) +800 +10 +.50

### DEPARTMENT Y

<table>
<thead>
<tr>
<th>Year</th>
<th>Meetings Provided ($\sum hg$)</th>
<th>Student-Hours ($\sum hE*$)</th>
<th>Resources</th>
<th>Cost per Meeting</th>
<th>Cost per Student-Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>(a) 50</td>
<td>1100</td>
<td>2000</td>
<td>40</td>
<td>1.82</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a) n/a -(15) 0
Effect of Enrolment Change (c) - (b) n/a n/a -(.-57)
Effect of Cost Change (d) - (c) +800 +10 +.50

Figure 10: Changes in Departmental Unit Costs
From Figure 10, Department X can be seen to have adverse cost changes (i.e. increases) from Year One to Year Two and, in fact, all the intermediate ones are adverse except for the slight benefit due to the teaching pattern change which does increase the cost per meeting but actually decreases the cost per student hour. The latter effect, however, is wiped out by the effects of the reduced enrolment.

Department Y, on the other hand, despite an adverse change in the cost of resources, achieves favourable changes in unit costs from Year One to Year Two because of a favourable effect due to changes in teaching patterns and a favourable effect due to increased enrolments.

In this example cost per meeting and cost per student hour behave in a similar manner, with the occasional exception, but it must be remembered that they measure different things. Cost per meeting measures how much instruction is being provided by the teachers whereas cost per student-hour reflects how much tuition is received by each student. Clearly, the amounts that these unit costs change by in the analysis will depend on the order in which the factors are altered. However, the direction of the variances (adverse or favourable) will remain the same. For example, if the order is reversed for Department X the analysis of changes would be:

<table>
<thead>
<tr>
<th>Department X</th>
<th>Cost per Meeting</th>
<th>Cost per Student-Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resource Change</td>
<td>+1.7</td>
<td>+.06</td>
</tr>
<tr>
<td>Enrolment Change</td>
<td>n/a</td>
<td>+.13</td>
</tr>
<tr>
<td>Teaching Change</td>
<td>+8.3</td>
<td>-(+.05)</td>
</tr>
<tr>
<td></td>
<td>+10.0</td>
<td>+.14</td>
</tr>
</tbody>
</table>
Thus, whichever order is chosen, the directions of the changes in unit costs remain the same. Therefore, as long as the analysis is consistent across departments a decision on the order is not important.

The departmental analysis has not explicitly looked at the cost per student. However, since the only way to do this is to calculate the full-time equivalent student numbers which itself requires the aggregation of student-hours (students times hours), the cost per student-hour is actually directly proportional to the cost per full-time equivalent student and so can be used in its place for measuring changes in departmental unit costs.
6.3. COST ANALYSIS AT THE SUBJECT ELEMENT LEVEL

At the subject element level students enrolled, as well as student-hours, can be identified and an analysis of changes in these parameters can be carried out. But this necessitates the establishment of the cost of providing a subject element. The Author has already suggested several times that the number of meetings provided by a department is the most appropriate measure of teaching activity. Thus, the cost per meeting calculated in the previous section can be used to calculate the total cost of a subject element. This, of course, means that it is no longer possible to examine changes in the cost per meeting, but it is now possible to examine changes in cost per student as well as cost per student-hour. The latter could be used as a means of allocating costs to subject elements but, as has been said in the last section, it measures tuition received rather than instruction provided. The logic of an allocation based on cost per meeting is set out in Figure 11.

Direct Teaching Resources $\rightarrow$ Meetings Provided $\rightarrow$ Cost per Meeting of Department X by Department X (say £2400) (say 60) $\frac{2400}{60} = 40$

Subject Element M takes 30 Meetings

Cost of Subject Element M is $30 \times 40 = 1200$

Figure 11: Logic of Cost Allocation on a Meetings Basis
This is an allocation of cost and so the results, to some extent, reflect the method of allocation. For example, if subject element M still takes 30 meetings but Department X as a whole provides less meetings, M will be allocated a larger share of the direct teaching costs. Figures 12, 13, 14, 15 show the results of such an allocation of cost and the effects of changes in enrolment, teaching patterns, or resource cost on the unit costs for each subject element. It should be noted in passing that an increase in enrolment to a subject element will have a favourable effect on the cost per student hour.
<table>
<thead>
<tr>
<th>Subject Element M</th>
<th>Meetings Provided (hg)</th>
<th>Students Enrolled (E*)</th>
<th>Student-Hours (hE*)</th>
<th>Allocated Cost of Subject Element</th>
<th>Cost per Student-Hour</th>
<th>Cost per Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>30</td>
<td>20</td>
<td>300</td>
<td>1200</td>
<td>4.0</td>
<td>60</td>
</tr>
<tr>
<td>Teaching Changed</td>
<td>20</td>
<td>20</td>
<td>400</td>
<td>960</td>
<td>2.4</td>
<td>48</td>
</tr>
<tr>
<td>Enrolment also</td>
<td>20</td>
<td>10</td>
<td>200</td>
<td>960</td>
<td>4.8</td>
<td>96</td>
</tr>
<tr>
<td>19X2</td>
<td>20</td>
<td>10</td>
<td>200</td>
<td>1000</td>
<td>5.0</td>
<td>100</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a)  
Effect of Enrolment Change (c) - (b)  
Effect of Resources Change (d) - (c)  
For subject element M there is a beneficial change in teaching activity (negative entry) which shows up in all three variances - cost, cost per student, and cost per student-hour. However, the adverse effect of a drop in enrolment is enough to turn both unit costs changes to adverse changes without considering the adverse effect of changes in resource cost.

Figure 12: Analysis of Changes for Subject Element M
<table>
<thead>
<tr>
<th>SUBJECT ELEMENT N</th>
<th>Meetings Provided (h)</th>
<th>Students Enrolled (E*)</th>
<th>Student-Allocated Hours (hE*)</th>
<th>Cost of Subject Element</th>
<th>Cost per Student-Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>(a)</td>
<td>30</td>
<td>50</td>
<td>1500</td>
<td>1200</td>
</tr>
<tr>
<td></td>
<td>Teaching Changed</td>
<td>(b)</td>
<td>30</td>
<td>50</td>
<td>1500</td>
</tr>
<tr>
<td></td>
<td>Enrolment also</td>
<td>(c)</td>
<td>30</td>
<td>50</td>
<td>1500</td>
</tr>
<tr>
<td>19X2</td>
<td>(d)</td>
<td>30</td>
<td>50</td>
<td>1500</td>
<td>1500</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a) = +240, Effect of Enrolment Change (c) - (b) = n/a, Effect of Resources Change (d) - (c) = +60

| Effect of Teaching Change | +240 | +.16 | +4.8 |
| Effect of Enrolment Change | n/a  | 0    | 0    |
| Effect of Resources Change   | +60  | +.04 | +1.2 |

Enrolments to Subject Element N remain the same throughout and so the enrolment effect shows up as a zero change. The meetings also remain the same. Unfortunately, they become a larger proportion of the meetings given by Department X and so produce adverse teaching changes. This effect is increased by the adverse change in resource cost.

**Figure 13: Analysis of Changes for Subject Element N**
## Figure 14: Analysis of Changes for Subject Element 0

<table>
<thead>
<tr>
<th>SUBJET ELEMENT 0</th>
<th>Meetings</th>
<th>Students</th>
<th>Student-Allocated</th>
<th>Cost per</th>
<th>Cost per</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Provided</td>
<td>Enrolled</td>
<td>Subject Element</td>
<td>Hours</td>
<td>Student-</td>
</tr>
<tr>
<td></td>
<td>(hg)</td>
<td>(E*)</td>
<td>(hE*)</td>
<td>(E*)</td>
<td>Hour</td>
</tr>
<tr>
<td>19X1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>30</td>
<td>30</td>
<td>900</td>
<td>1200</td>
<td>1.33</td>
</tr>
<tr>
<td>(b) Teaching Changed</td>
<td>60</td>
<td>30</td>
<td>900</td>
<td>1500</td>
<td>1.67</td>
</tr>
<tr>
<td>(c) Enrolment also</td>
<td>60</td>
<td>40</td>
<td>1200</td>
<td>1500</td>
<td>1.25</td>
</tr>
<tr>
<td>19X2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>60</td>
<td>40</td>
<td>1200</td>
<td>2100</td>
<td>1.75</td>
</tr>
</tbody>
</table>

**Effect of Teaching Change (b) \(\rightarrow\) (a)**

| +300 | .34 | +10 |

**Effect of Enrolment Change (c) \(\rightarrow\) (b)**

| n/a | -.42 | -(12.5) |

**Effect of Resources Change (d) \(\rightarrow\) (c)**

| +600 | .50 | +15.0 |

| +900 | .42 | +12.5 |

Enrolments to Subject Element 0 increase and so produce favourable changes in cost per student-hour and cost per student, but these are completely reversed by the combination of the change in meetings which increase both absolutely and as a proportion of total meetings by Department Y and the change in resource cost.
### Subject Element P

<table>
<thead>
<tr>
<th></th>
<th>Meetings Provided (hg)</th>
<th>Students Enrolled (E*)</th>
<th>Subject Allocated Cost (hE*)</th>
<th>Cost per Student-Hour</th>
<th>19X1</th>
<th>19X2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Changed (a)</td>
<td>20</td>
<td>10</td>
<td>200</td>
<td>800</td>
<td>4.00</td>
<td>80</td>
</tr>
<tr>
<td>Enrolment also (b)</td>
<td>20</td>
<td>20</td>
<td>400</td>
<td>500</td>
<td>2.50</td>
<td>50</td>
</tr>
<tr>
<td>19X2 (c)</td>
<td>20</td>
<td>20</td>
<td>400</td>
<td>700</td>
<td>1.75</td>
<td>35</td>
</tr>
</tbody>
</table>

Effect of Teaching Change (b) - (a)  

Effect of Enrolment Change (c) - (b)  

Effect of Resources Change (d) - (c)  

| Effect of Teaching Change (b) - (a) | -(300) | -(1.5) | -(30)   |
| Effect of Enrolment Change (c) - (b) | n/a    | -(1.25) | -(25)   |
| Effect of Resources Change (d) - (c) | +200   | +.50   | +10     |
| Total Effect | -(100) | -(2.25) | -(45)   |

The meetings provided for Subject Element P remain the same throughout but since the total meetings provided by Department Y increases, their cost decreases and produces favourable changes and also the enrolment increase shown by the favourable changes. These two favourable effects swamp the increased cost of resources.

**Figure 15: Analysis of Changes for Subject Element P**
6.4. COST ANALYSIS AT THE COURSE LEVEL

Resources are usually allocated to departments to allow them to carry out teaching and research, but the approach followed in the previous sections can usefully be used to examine the costs associated with a course. This necessitates taking the costs derived for the subject elements and sharing them out between the courses involved. The Author has suggested several times already that this should be done pro rata to students involved. For example, in Year One Subject Element 0 has an enrolment of 30 of which 20 students are from course A and the rest are from course B. Hence, it is suggested that the cost of subject element 0 be split between the two courses in the ratio 20 : 10. The cost could be split 50 : 50 between the two courses but would then make the analysis of effects of changes in enrolment impossible.

Figure 16 shows the results of sharing out the cost of each subject element between the two courses involved. This is again an allocation of cost and so the results may be affected by the method chosen in that the allocation to one course may change only because another course has changed its enrolment. So in contrast to the situation at the subject element level, a unilateral increase in enrolment to a course will increase the cost of the course and may, therefore, actually increase the cost per student.

Figure 16 shows, for example, that for course A the cost per course decreased from Year One to Year Two but the cost per student went up and for course B the reverse happened. Now in each case the enrolments altered so the question arises of whether this was the sole cause of the changes. The cost per course, the cost per student and student-hour can be calculated for the intermediate situations identified in the previous sections and this enables these effects to be identified.
### 19X1

<table>
<thead>
<tr>
<th>SUBJECT ELEMENTS</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation to Course A</td>
<td>1200</td>
<td>960</td>
<td>800</td>
<td>-</td>
<td>2960</td>
</tr>
<tr>
<td>Allocation to Course B</td>
<td>-</td>
<td>240</td>
<td>400</td>
<td>800</td>
<td>1440</td>
</tr>
<tr>
<td><strong>Total Subject Cost</strong></td>
<td>1200</td>
<td>1200</td>
<td>1200</td>
<td>800</td>
<td>4400</td>
</tr>
<tr>
<td><strong>Total Department Cost</strong></td>
<td>2400</td>
<td></td>
<td></td>
<td>2000</td>
<td></td>
</tr>
</tbody>
</table>

### 19X2

<table>
<thead>
<tr>
<th>SUBJECT ELEMENTS</th>
<th>M</th>
<th>N</th>
<th>O</th>
<th>P</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>Allocation to Course A</td>
<td>1000</td>
<td>900</td>
<td>1050</td>
<td>-</td>
<td>2950</td>
</tr>
<tr>
<td>Allocation to Course B</td>
<td>-</td>
<td>600</td>
<td>1050</td>
<td>700</td>
<td>2350</td>
</tr>
<tr>
<td><strong>Total Subject Cost</strong></td>
<td>1000</td>
<td>1500</td>
<td>2100</td>
<td>700</td>
<td>5300</td>
</tr>
<tr>
<td><strong>Total Department Cost</strong></td>
<td>2500</td>
<td></td>
<td></td>
<td>2800</td>
<td></td>
</tr>
</tbody>
</table>

So the total changes are as follows:

<table>
<thead>
<tr>
<th>Course</th>
<th>19X1</th>
<th>19X2</th>
<th>Cost per Course</th>
<th>Enrolment</th>
<th>Cost per Student</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course A</td>
<td></td>
<td></td>
<td>2960</td>
<td>40</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>2950</td>
<td></td>
<td></td>
<td>30</td>
<td>98.3</td>
</tr>
<tr>
<td></td>
<td>-10</td>
<td></td>
<td></td>
<td></td>
<td>+24.3</td>
</tr>
<tr>
<td>Course B</td>
<td>1440</td>
<td></td>
<td></td>
<td>10</td>
<td>144</td>
</tr>
<tr>
<td></td>
<td>2350</td>
<td></td>
<td></td>
<td>20</td>
<td>117.5</td>
</tr>
<tr>
<td></td>
<td>+910</td>
<td></td>
<td></td>
<td></td>
<td>-26.5</td>
</tr>
</tbody>
</table>

**Figure 16: Allocation of Costs to Courses**
Figure 17 shows the results of such an approach for course A and
Figure 18 shows the results for course B. The assumption throughout
this Chapter is that all teaching activity changes from Year One to
Year Two simultaneously and likewise for enrolments. This means that
since costs are distributed to courses on the basis of student numbers,
a favourable change in cost per course will result if a course provides
a smaller share of the enrolment to its subject elements. An adverse
change in cost per student will result if the enrolment to the course
drops in absolute terms but the cost per student-hour will change
accordingly to the effects of the changes in teaching patterns as well.
For example, if a course receives 30 hours, the cost per student is
determined but this 30 hours may be 30 hours to all students or 15 hours
to each half of the course. The cost per student will be the same in
both cases, but the cost per student-hour will be doubled by a change
from the former to the latter pattern. Thus, the method of allocation
means that at course level as distinct from departmental level, the
cost per student reflects the meetings provided but the cost per
student-hour is not directly proportional to the cost per student
anymore since it reflects the student's taught hours as well as the
enrolment situation. Thus, both unit costs are needed at the course
level. Incidentally, Figure 17 also shows that, in contrast to the
situation at departmental level, an increase in teaching activity
provided for a course increases the cost and, hence, the unit costs.
Thus, it is best to talk of enrolment effects or teaching activity
effects rather than increases or decreases.
<table>
<thead>
<tr>
<th></th>
<th>Cost per Course</th>
<th>Enrolment (E)</th>
<th>Cost per Student</th>
<th>Student Hours ($)</th>
<th>Cost per Student Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(a)</td>
<td>2960</td>
<td>40</td>
<td>74</td>
<td>2100</td>
<td>1.41</td>
</tr>
<tr>
<td>Teaching Change (b)</td>
<td>3112</td>
<td>40</td>
<td>77.8</td>
<td>2200</td>
<td>1.42</td>
</tr>
<tr>
<td>Enrolment also (c)</td>
<td>2574</td>
<td>30</td>
<td>85.8</td>
<td>1700</td>
<td>1.51</td>
</tr>
<tr>
<td>19X2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d)</td>
<td>2950</td>
<td>30</td>
<td>98.3</td>
<td>1700</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Effect of Teaching: +152  
Effect of Enrolment: -(538)  
Effect of Resources: +376

The favourable variance in cost per course due to the effect of enrolment is almost wiped out by the more costly resources and their increased use by changes in teaching activity by the departments offering tuition to course A. These changes in teaching increase the cost per student though so the total increase in cost per student is made up of unfavourable changes in all three effects as is the cost per student hour.

Figure 17: Analysis of Changes for Course A
<table>
<thead>
<tr>
<th>COURSE B</th>
<th>Cost per Course</th>
<th>Enrolment (E)</th>
<th>Cost per Student</th>
<th>Student Hours (h)</th>
<th>Cost per Student Hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>19X1</td>
<td>1440</td>
<td>10</td>
<td>144</td>
<td>800</td>
<td>1.80</td>
</tr>
<tr>
<td>Teaching Change</td>
<td>1288</td>
<td>10</td>
<td>128.8</td>
<td>800</td>
<td>1.61</td>
</tr>
<tr>
<td>Enrolment also</td>
<td>1826</td>
<td>20</td>
<td>91.3</td>
<td>1600</td>
<td>1.14</td>
</tr>
<tr>
<td>19X2</td>
<td>2350</td>
<td>20</td>
<td>117.5</td>
<td>1600</td>
<td>1.47</td>
</tr>
</tbody>
</table>

Effect of Teaching: -(152)  
Effect of Enrolment: +538  
Effect of Resources: +524

\[\text{Total: +910} - (26.5) - (.33)\]

Since enrolments are the basis of cost allocation between courses, the doubling in enrolment increases the cost per course and decreases the cost per student. The cost per course is further increased by the resource changes despite favourable changes in teaching activity. These changes in teaching decrease the cost per student and so the total cost per student decreases despite the effects of the large increase in resource costs.

Figure 18: Analysis of Changes for Course B
6.5. **DRAWBACKS OF THE APPROACH**

The method works because it is feasible to suggest that the teaching pattern is independent of actual enrolment so that the effect of changes in these two factors can be split between them. However, the analysis at departmental level requires information on meetings given and student-hours provided and the analysis at course level requires information for each subject element involving the student taught hours (h), the number of groups (g), the number of students from each course (s), and the department providing the teaching. The approach, therefore, requires a detailed timetable analysis each year. However, most institutions these days collect such data albeit not in a systematic way.

Another drawback of the method is that it requires an allocation of departmental resources to its teaching, a procedure which awaits widespread approval. Throughout the Chapter the changes in resource cost have implicitly been assumed to be changes in quantity of resources rather than simply changes in price levels. If this is deemed to be important then the salary costs can be converted to "Year One" salary scales and the recurrent expenditure can be adjusted using an appropriate index.

The approach is similar to that of the accountant's concept of variance analysis but the analysis has been of changes over time rather than changes from standards or budgeted values. It is difficult to see how standards could be introduced into this situation since the use of norms for average class-size, average student taught hours, and average teacher-class contact would prohibit any analysis of the effects on cost of changes in teaching patterns. The use of
budgeted values at the course or departmental level would involve an exercise as extensive as doing the analysis itself.

The major drawback of the approach is that it is based on a hierarchy of allocation methods, some of which are, of necessity, arbitrary but if resource usage is to be examined at the course or departmental level, attempts such as this have to be made so that attention can be focussed on "unfavourable" trends. Of course, there will always be teaching experiments which may call for extra resources but the decisions which support these initiatives should be made explicit and an analysis such as the one outlined above would help to ensure that this happens. Such an analysis enables a department that is labelled 'expensive' to show whether it is so or whether it has not yet adapted its teaching activity to a lower than planned enrolment.
Given a breakdown of the timetable of an institution at the level of the subject element, it is possible to allocate meetings and student-hours to departments and hence obtain unit costs. The change from one year to the next in these unit costs can be broken down into effects due to changes in enrolment, effects due to changes in teaching patterns, and changes in the cost of resources used. If the department's resources are wholly or partly allocated to its teaching activity, then the cost of a subject element and, hence, the cost of a course, can be identified and unit costs calculated. The changes in these unit costs can be broken down in a similar way to show the various effects of changes in the factors involved. The analysis illustrates that cost per meeting and cost per student-hour are appropriate unit costs at the departmental level whereas cost per student and cost per student-hour are more appropriate at the subject element or course level. The analysis has all the faults of an absorption costing approach, but it is a first step to establishing the reasons behind changes in unit costs within institutions.

CHAPTER 7  THE EFFICIENT FRONTIER

7.1. INTRODUCTION

At various points in the Loughborough-Lanchester study the Steering Committee, which included several polytechnic and university administrators as well as a representative of the DES, raised the question of the construction of an overall measure of efficiency for an institution or department within an institution. This was not possible in the time with the resources available. However, as pointed out in Chapters 4 and 5, it is possible to produce an overall measure of relative efficiency by identifying some institutions or departments as "more efficient" and using them as benchmarks for the rest. Intuitively if there is a true production function which efficiently produces outputs from inputs then these more efficient institutions or departments will provide the best estimate of that function. However, care must be taken to test the estimated frontier since, as C. P. Timmer (1971) points out, the estimated function is based only on some extreme values and, hence, is highly subject to errors in the data. He suggests the random deletion of some of the "good" observations until the estimated function stabilises. He also points out, and this is confirmed for student achievement data by Henry Levin (1974), that a function based on the extreme values will often have a different shape to one based on the whole set. However, to show this both have to assume a linear production function and apply it to a one output many input example. Indeed, the use of the whole set to "fit to the average" necessitates the specification of the form of the production function whereas if the extreme values are used the function can be specified by those values. The Author has considered (1978a, 1978b) multi-output multi-input situations and so has no production function to fit. The production function in this case can only be specified by the extreme
points it is assumed to involve.

If the inputs and outputs of each department or institution are listed in the form of an input-output vector then these vectors define points in a multi-dimensional space and allow the use of geometrical arguments to justify the selection of the more efficient institutions or departments.
7.2. **THE "MORE EFFICIENT" POINTS**

To illustrate the approach, first consider the situation where several institutions have two inputs (staff cost and capital cost) and one output (students). In this simplistic situation, if there are constant returns to scale, each institution can be represented by a point on a graph similar to Figure 19.

![Figure 19: The More Efficient Institutions](image)

Now this set of points has a boundary and intuitively the more efficient institutions are represented by some of the boundary points. The ones ringed in red have two useful properties. Firstly, there is no other point with the same input ratio as a ringed point that uses less inputs. Secondly, if two adjacent ringed points are examined, there is no point with an input ratio between those of the two ringed points that uses less inputs than a linear combination of the two adjacent points. Thus, if a convex surface is drawn through the selected points all the others lie on the wrong side of it. This surface is hereafter described as the efficient frontier and the selected points are denoted as more efficient.
This concept of efficiency is akin to that of technical efficiency in the sense that it does not consider the value price or utility of the inputs. However, M. J. Farrell (1957) points out that if an institution is always compared only with another, perhaps hypothetical, institution that has the same mix of inputs, then the distinction is unnecessary since the ratio of their technical efficiencies equals the ratio of their overall efficiencies. This point is discussed in greater detail in the two papers by the Author in Appendix 2 (2.11, 2.12).
7.3. **THE EFFICIENT FRONTIER**

The production frontier is usually assumed to be convex in a geometrical sense. This means that if two points represent situations that are attainable in practice then a linear combination (with weights that add to 1) of them is also attainable in practice. In two dimensions this means any point on the line joining two frontier points is attainable in practice. If the shape of the production function is unknown and it is borne in mind that some of the frontier points might be "too good" then the simplest assumption is that the production surface is established by joining the adjacent points by straight lines in two dimensions, planes in three and hyperplanes in four or more dimensions. Thus, every point on the frontier is a linear combination (with weights that add to 1) of the frontier defining points, and the surface consists of a set of linked line segments in two dimensions, a set of linked faces in three dimensions and a set of linked facets in four or more dimensions.

However, the points that are used to estimate the frontier are not simply on the boundary, they are also "more efficient" than the others. So, for example, in the situation of Figure 20, a line through two adjacent points on the frontier must be nearer the origin than all the other points otherwise the frontier will not be convex.

![Figure 20: The Frontier for Inputs Per Unit Output](image)

---

**Notes:**

- The above explanation is a simplified version of the actual content, focusing on the core concepts of the efficient frontier in production economics.
- The diagram illustrates the concept with a graph showing the trade-off between staff cost per student and capital cost per student.
- The points on the frontier are labeled as "more efficient", indicating their superior performance in terms of cost optimization relative to the others.
In such a two input one output case with constant returns this can be done visually. Similarly in the two output-one input case with constant returns to scale, the Frontier can be found visually as in Figure 21.

![Figure 21: The Frontier for Outputs Per Unit Input](image)

Figure 21: The Frontier for Outputs Per Unit Input

However, in the multi-input multi-output case, this geometrical concept of a facet joining adjacent points being nearer or further from the origin than any others in the set is harder to pin down.

There are several ways of finding the Frontier. M. J. Farrell (1957), in his study of the Efficient Frontier in agriculture, uses complete enumeration of all possible facets to find the Frontier. However, he is able to find the efficient points by inspection since he considers a one output-four input situation. He eliminates some points as obviously inefficient but still has to examine the rest of the 48 points and, hence, consider 1820 facets to end up with 39 efficient facets involving 9 efficient points and four extra points which will be referred to in the next section.
Robert Cray (1977) tried a similar but more sophisticated approach by using an iterative procedure. He first chooses a set of points from the sample observations to form an arbitrary boundary. He then considers each other point one at a time. At each such stage if the new point is inside the boundary no action is taken, but if it is outside the boundary is extended to include it and some facets no longer on the boundary are discarded. Thus he ends up with the convex hull of the sample observations, in other words, the whole boundary. He then has the problem of deciding which bit is the efficient frontier and he is unable to offer anything other than inspection which, in a multi-input multi-output situation, is not good enough. M. J. Farrell himself suggested in 1957 that a Linear Programming approach could usefully be used if there were the facilities available. Since then the Linear Programming approach has been utilised by several authors but the major contribution is that of Daryl Carlson (1972, 1975) who deploys it in the measurement of educational provision and it is his approach that the Author has investigated (1978b). However, before discussing this approach, there are one or two conceptual problems to sort out, namely, should the Frontier have any specific properties in addition to lying on the "right side" of the observations?
7.4. PROPERTIES THE FRONTIER MIGHT HAVE?

M. J. Farrell (1957) visualises the frontier as a surface which continues to infinity but which touches the set of observations along the efficient part of the boundary. He wishes this surface to have certain properties namely: convexity, consistent slope, and involve constant returns to scale. These conditions could be introduced by adding mathematical constraints to the identification of the efficient facets or, as Farrell suggests, extra points could be added to the set in such a way that these properties are achieved automatically. The Frontier, if produced by linear combinations of adjacent points, is automatically convex along the boundary of the sample observations so as long as the Frontier does not meet the set again it cannot be labelled as non-convex. The concept of a consistent slope comes from economic theory. Where one input and one output is concerned, it means that extra input always produces some extra output, i.e. the slope is always positive, as in Figure 22.

\[\begin{array}{c}
\text{Output} \\
\text{Input}
\end{array}\]

\textbf{Figure 22: Extra Points for One Input One Output}

This can be achieved by a mathematical condition on each acceptable facet or by adding extra points to make it so. Similarly, when two inputs are considered, the slope should always be negative so that a drop in one input always requires an increase in the other to produce a particular level of output. This can also be achieved by adding
extra points such as points at infinity [(0,∞) and (∞, 0)] as in Figure 23.

These extra points will automatically be on the Frontier and so are involved in the definition of several facets. Points at infinity, as M. J. Farrell found out in his example, are difficult to cope with mathematically, although ideal for geometrical needs. A similar problem arises in the two output one input case. If all facets are examined mathematically then the slope can only remain consistent if extra restrictions are applied or if extra points are added. In this case they are [max, 0] and [0, max] as shown in Figure 24.
This approach of adding extra points to force certain properties on the frontier is also difficult to generalise to the many input many output situation, whereas Linear Programming ensures that the Frontier is convex and only selects Frontier Points that produce a consistent slope (see Figure 28 in section 7.6).

The concept of constant returns to scale is not so easily introduced into the analysis. Farrell considers a one input and many input problem and by assuming constant returns to scale makes his problem easier. He, therefore, suggests that the multi-input multi-output case should be treated as if it has this property. Daryl Carlson (1972,1975) and others have suggested that this is too restrictive and, of course, in educational circles there is a persistent search for economies of scale! Farrell introduces constant returns to scale by adding the origin to the frontier points and insisting that the origin belongs to every facet (analogue of face for multi-dimensional situations) if necessary with a negative weight in any linear combination of the defining points. Figure 25 shows the result of this in two dimensions.

Figure 25: Adding the Origin
Any point on the dotted red line is a linear combination of the origin and the other point with a negative weight for the origin.
7.5. FARRELL'S DERIVATION OF THE FRONTIER

If extra points are added to the sample observations to ensure consistent slope and constant returns to scale, then the Frontier spans the set of points in the sense that every point of the set has a segment of the Frontier between it and the origin when considering inputs, and every point lies between some line segment and the origin when considering outputs. So first consider the case previously discussed of two inputs and one output with constant returns to scale. The set of observations can be represented on a graph of inputs per unit output such as Figure 26.

![Figure 26: Definition of the Frontier](image)

The line segments making up the Frontier in this case can be identified mathematically as follows:

Consider two Frontier Points, $P_i$ and $P_j$.

Let $P_i$ and $P_j$ be the respective vectors of inputs per unit output.

Let $\lambda_{ijk}$ and $U_{ijk}$ be solutions of

$$\lambda P_i + U P_j = P_k$$

where $P_k$ is another point in the set.
Since any point on the line through $P_i$ and $P_j$ must have $\lambda + U = 1$. The line segment joining $P_i$ to $P_j$ can only lie between all the other points and the origin if and only if $\lambda_{ijk} + U_{ijk} > 1$ for all the other points $P_k$ in the set including any added points.

This idea can be easily extended to the case of $n$ inputs and one output with constant returns. Instead of lines and line segments there are hyperplanes and facets. A facet that is part of the Frontier can be represented as a linear combination of the $n$ defining points with weights that add up to 1.

Thus if $\lambda$ is the vector solution of

$$\left[ P_i, P_{i+1}, \ldots P_{i+n-1} \right] \lambda = P_k$$

The facet defined by $P_i, \ldots P_{i+n-1}$ is part of the Frontier Surface if and only if $\sum \lambda_i > 1$ for all other points $P_k$ in the set.

It is more difficult to generalise further to the $n$ inputs and $m$ outputs situation since each institution can now only be represented by a point in $m + n$ dimensional space, i.e. by an input output vector. M. J. Farrell wants to retain constant returns to scale so the origin is made part of every facet, if necessary with a negative weight, and it is assumed that other points have been added to ensure a consistent slope.

Let institution $i$ be represented by an input vector $X_i$ and an output vector $Y_i$. Then instead of dividing through by the output find a linear combination that matches the inputs whilst exceeding the outputs of any point $P_k$ by a constant multiple.
Thus if $\bar{\delta}$ is the vector solution of

$$\begin{bmatrix} Y_i, Y_{i+1}, \ldots, Y_{i+m+n-2}, 0 \end{bmatrix} = (\sum \delta_j) Y_k$$

$$\begin{bmatrix} X_i, X_{i+1}, \ldots, X_{i+m+n-2}, 0 \end{bmatrix} = X_k$$

then the facet defined by $P_i, P_{i+1}, \ldots, P_{i+m+n-2}, 0$ is part of the Frontier if and only if $\sum \delta_j \geq 1$ for all points $P_k$ in the set.

(This is a generalisation of the previous formulation and this can be checked by defining $\lambda_i$ by $Y_i+j-1 \delta_j = Y_k \lambda_j$; the first matrix equation for $m = 1$ gives $\sum \lambda_j = \sum \delta_j$ and the second is the linear combination).

It is important to note that a similar generalisation to $m$ outputs and $n$ inputs can be carried out from the one input $m$ output case but leads to a different formulation as shown below.

Thus if $\bar{\delta}^*$ is the vector solution of

$$\begin{bmatrix} Y_i, Y_{i+1}, \ldots, Y_{i+m+n-2}, 0 \end{bmatrix} \bar{\delta}^* = Y_k$$

$$\begin{bmatrix} X_i, X_{i+1}, \ldots, X_{i+m+n-2}, 0 \end{bmatrix} \bar{\delta}^* = (\sum \delta_j^*) X_k$$

then the facet defined by $P_i, P_{i+1}, \ldots, P_{i+m+n-2}, 0$ is part of the Frontier if and only if $\sum \delta_j^* \leq 1$ for all points $P_k$ in the sample set.

M. J. Farrell argues that if the data is forced to have constant returns to scale then it allows only one of these approaches to be used since they would both give the same frontier albeit with different weights in the conditions for an efficient facet. It is also the case that the property of constant returns to scale as
reflected in the conditions for a facet make the solution easier. For example, consider the simple case of two inputs and one output. The facet condition is then as follows:

\[
\delta_1 y_i + \delta_2 y_j = (\delta_1 + \delta_2 + \delta_3) y_k \\
\delta_1 x_{1i} + \delta_2 x_{1j} = x_{1k} \\
\delta_1 x_{2i} + \delta_2 x_{2j} = x_{2k} 
\]

Since \( P_i \) and \( P_j \) are boundary points the last two equations always provide values for \( \delta_1 \) and \( \delta_2 \) and then the first gives \( \delta_3 \) thus \( \delta_j \) can be checked.

If constant returns are dropped then one of two things happens. If there are diseconomies of scale then the estimated frontier, without including the origin, will have the property that points on a line or facet joining adjacent frontier points will be attainable but inefficient as in Figure 27. If there are economies of scale then the estimated frontier, without including the origin, will have the property that a line or facet joining adjacent frontier points will contain points that are measured as over-efficient and are not attainable in practice. The only way to avoid the latter problem is to break the sample set down into similar subsets where the estimated Frontier should be nearer the true Frontier. In each case the two formulations mentioned earlier would lead to different Frontiers and different measures of relative efficiency.
7.6. FARRELL'S DEFINITION OF RELATIVE EFFICIENCY

M. J. Farrell (1957) bases his concept of relative efficiency on a generalisation from the two inputs one input case with constant returns to scale. Consider Figure 27 which shows the inputs per unit output for a set of institutions.

\[
\begin{array}{c|c|c}
\text{Input 2} & \text{Per Unit Output} & \\
\hline
0 & 0 & \\
\hline
Input 1 & \text{Per Unit Output} & \\
\hline
\end{array}
\]

Consider two frontier points \( P_i \) and \( P_j \) that lie between \( P_k \) and the origin. Since they do this then there exist \( \lambda_{ijk} \) and \( U_{ijk} \) such that

\[
\lambda_{ijk} P_i + U_{ijk} P_j = P_k
\]

However, the line joining \( P_k \) with the origin cuts the Frontier between \( P_i \) and \( P_j \) so this point must be able to be expressed in the form

\[
a P_i + (1 - a) P_j
\]

However, this intersection point has the same ratio of inputs per unit output as \( P_k \) but uses less. If the Frontier were the true efficient production surface then this point could be represented by \( \beta P_k \) where \( \beta \) is the relative efficiency of \( P_k \).
Thus \( \beta_P = \alpha_P + (1 - \alpha)P_j \)

and \( \beta_k = \lambda_{ijk}P_i + U_{ijk}P_j \)

so \( \beta = 1 / (\lambda_{ijk} + U_{ijk}) \)

Hence if \( OP_k \) cuts the Frontier between \( P_i \) and \( P_j \) the measure of efficiency for \( P_k \) is the reciprocal of the sum of the weights in the facet condition for \( P_i, P_j \) and \( P_k \).

Generalising this to \( n \) inputs and one output suggests that the relative efficiency of a point is similarly defined as the reciprocal of the sum of the weights in the facet condition. In actual fact, due to convexity, it can also be defined as the maximum value of the reciprocal of the sum of weights for \( P_k \) in the facet condition of every frontier facet which is perhaps an easier approach which does not require identification of any particular facet before the efficiency of a point \( P_k \) can be calculated.

Generalising this to \( n \) inputs and \( m \) outputs leads to the concept of relative efficiency defined as that constant multiple of outputs where all inputs are matched.

So if the facet condition is

\[
\begin{bmatrix}
Y_{i1}, Y_{i1+1}, \ldots, Y_{i+m+n-1}
\end{bmatrix}
\frac{\sum Y_j}{\delta} = Y_k
\]

\[
\begin{bmatrix}
X_{i1}, X_{i1+1}, \ldots, X_{i+m+n-1}
\end{bmatrix}
\frac{\sum X_j}{\delta} = X_k
\]

then the relative efficiency of a point \( P_k \) is given by the maximum value of the reciprocal of the sum of the weights for every facet condition.
Similarly if the generalisation to m outputs and n inputs comes from m outputs and input the measure of relative efficiency is the constant multiple of inputs where outputs are matched.

So if the facet condition is

\[ \left[ \frac{Y_i, Y_{i+1}, \ldots, Y_{i+m+n-1}}{X_i, X_{i+1}, \ldots, X_{i+m+n-1}} \right] \delta^* = \frac{Y_k}{X_k} \]

then the relative efficiency of a point \( P_k \) is given by the maximum value of the sum of the weights for every facet condition.

M. J. Farrell's approach, therefore, consists of identifying frontier facets and using them to provide a measure of relative efficiency based on matching inputs and considering the frontier point with a constant multiple of outputs or vice versa.
7.7. CARLSON'S LINEAR PROGRAMMING APPROACH

Rather than locate a Frontier segment that contains a point with the same input mix but a constant multiple of outputs, Daryl Carlson (1972, 1975) uses a geometrical approach by searching for the Frontier nearest to a point $P_k$ in each output direction. Thus, he finds a Frontier segment containing a point with the same or less inputs and the same or more outputs than $P_k$. If the base point is on the Frontier then no other point will be found. If it is not on the Frontier then such a point will be found and will be a linear combination of some of the points in the set which are thus identified as Frontier points. If this process is carried out for every output and every point in the set then the Frontier points will be identified. Unfortunately, the specification of the facets connecting them requires further calculations. Thus, Carlson finds the Frontier points not the facets as Farrell does.

The procedure can be represented by Figure 28 which shows a cross-section of the multi-dimensional space which involves two of the outputs. If $P_k$ lies in this cross-section, then the Frontier must lie in the direction of increasing output 1 and increasing output 2.

![Figure 28: Carlson's Search Procedure](image-url)
The points on the Frontier that have the maximum of output 1 or the maximum of output 2 (redspots) are linear combinations of some of the Frontier points, and so the location of this point also locates some Frontier points. If this process is carried out for all points in the set and for all outputs, then the whole set of Frontier points will be found. It should be noted that this approach ensures convexity for the Frontier, and without the use of added points only chooses Frontier points that give a consistent slope as shown by Figure 29.

The best linear combination of all other points in the set that has the same or less inputs and the same or more of the other outputs will, therefore, never include as a defining point a point that would lead to a change in sign of the slope of the Frontier.

The simplest linear programming formulation of this procedure is as follows:

Consider T institutions with m outputs and n inputs. Then if $X_{it}$ is the quantity of the $i^{th}$ resource used by institution $t$ and $Y_{jt}$ is the quantity of the $j^{th}$ output produced by institution $t$, the point
on the Frontier in the direction of one output \( r \) starting from the
point representing institution \( s \) is given by the optimal solution
to the following Linear Program (L.P.).

Maximise \[ \sum_{t=1}^{T} z_t Y_r t \]

where \[ \sum_{t=1}^{T} z_t X_i t \leq X_i s \] for \( i = 1, \ldots, n \)

and \[ \sum_{t=1}^{T} z_t Y_j t \geq Y_j s \] for \( j = 1, \ldots, r-1, r+1, \ldots, m \)

with \( z_t \geq 0 \) for \( t = 1, \ldots, T \)

The problem is solved for the weights \( z_t \) which will be positive or
zero in the optimal solution. Since there are \((m+n-1)\) constraints
the final L.P. solution will contain only \((m+n-1)\) or less non-zero
\( z_t \)'s and each non-zero \( z_t \) means that institution \( t \) is a Frontier
Point. Hence, all the Frontier points can be identified if the L.P.
is solved for each institution \( s \) and every output \( r \). Since an L.P.
always finishes up at a vertex, the situation of Figure 28 results,
in that only points that maintain the sign of the slope of the
Frontier are identified as Frontier points. The estimated Frontier,
of course, does not exhibit constant returns to scale. Carlson is
concerned only to find the Frontier points and is not so interested
in the facets or in a measure of efficiency for the non-Frontier
institutions.

Faced with this problem the Author's first attempt (1978a, 1978b)
involved measurement of how near to the Frontier a point is in each
output direction. The L.P. solution provides the maximum output in the direction of increasing output r starting from institution s and the optimal value of the objective, namely $\sum z_t Y_{rt}$ can be compared with output r by institution s. Thus a measure of partial efficiency is $Y_{rs} / (\sum z_t Y_{rt})$ and these were calculated for all five outputs of the illustration used in the two papers in Appendix 2. It then seemed sensible to rank the institutions in order of their maximum partial efficiency rather than average partial efficiency, the logic of which is illustrated by Figure 30.

![Figure 30: Nearness to the Frontier](image)

There are several other possibilities which the Author intends to investigate in the future. Firstly, there is Farrell’s measure of efficiency which requires further analysis of the Frontier points, possibly via another L.P. Secondly, there is the geometrical idea of the shortest distance to the Frontier, but this, of course, involves combining different inputs and outputs and this needs a decision on weights, values or utilities, for simply to calculate the distance means that all inputs and outputs have been weighted equally.
In the two papers in Appendix 2, the Author introduces for purposes of illustration an analysis of British Universities for the academic years 1972/73 and 1973/74 and it is worth adding some comments here. The analysis uses crude measures of input and output. Inputs are measured by staff numbers, staff salaries and other institutional expenditure. Outputs are measured by student numbers at undergraduate and postgraduate level plus the expenditure from research grants as a proxy for research involvement. The analysis shows that for British Universities the number of full-time students is as good a measure as the number of full-time equivalent students. It also shows that staff numbers is a more constraining variable than staff salaries. In other words, most non-frontier institutions do not have enough staff for the salary bill involved. In fact, some of the frontier institutions were new universities with young staff and hence lower salary bills.

The introduction of a measure of research effort, albeit crude, produced noticeable effects on the choice of Frontier institutions. In particular, it would inevitably be biased towards technological universities where research can attract large amounts of funding. The measures themselves were fairly consistent over the two academic years and, as a result, the ranking of institutions on the basis of maximum partial efficiency produced a fairly stable situation. The Author intends to apply this method to within-institutional analysis using the Loughborough-Lanchester study data plus data on research and extra-mural activity.
7.8. QUALITY AS WELL AS QUANTITY

Up to this point the implicit assumption has been that different outputs are measured in different ways but that each output is measured in one way, e.g. student numbers or student achievement, numbers of graduates or salaries of graduates. Daryl Carlson (1972, 1975) makes an important contribution, however, when he introduces quality constraints into his formulation so that a particular output is maximised in quantity terms subject to conditions relating to quality. Now he calls these intervening variables characteristic variables and the examples he suggests include institutional ratings, growth rates, research emphasis and, most important of all, scale of operation.

He extends his formulation by adding constraints of the following forms on the one output being maximised \( r \)

\[
- \sum_{t} y_{rt} (c_{kt} - c_{ks}) \leq 0 \quad \text{for } k = 1, \ldots, p
\]

\[
- \sum_{t} y_{rt} (c_{kt} - c_{ks}) \geq 0 \quad \text{for } k = p + 1, \ldots, q
\]

For example, high enrolments may lead to high levels of wastage so if \( w_t \) is the wastage rate for institution \( t \) and \( Y_{rt} \) is the enrolment to institution \( t \) then wastage can be taken into account by adding a constraint such as

\[
\sum_{i} (w_{i}Y_{rt}) \leq (\sum_{i} w_{i}Y_{rs})
\]

or in L.P. form

\[
\sum_{i} (w_{i}Y_{rt} - w_{i}Y_{rs}) \leq 0
\]

Similarly, high enrolments might imply low entry grades and so if \( a_t \) is the A-level score of institution \( t \) then the total A-level score of
an institution can be safeguarded by adding a constraint of the form

\[ \sum_{i} z_i a_t Y_{rt} \geq \left( \sum_{i} z_i \right) a_Y_{rs} \]

or in L.P. form \[ \sum_{i} z_i a_t Y_{rt} - a_Y_{rs} \leq 0 \]

The introduction of the concept of scale into the problem is an important step also. It allows the Frontier to exhibit constant returns to scale over a small range of size but have different levels of efficiency for different sizes of operation. However, it assumes that scale can be measured by one measure. Should it be based on inputs consumed, e.g. total staff or academic staff, or should it be based on outputs, e.g. students or student-hours? The choice of measure of scale will affect the surface produced and the measure of efficiency calculated. For example, if student-hours are the measure of scale and research output is being maximised, then each institution requires a research output per student hour ratio, say \( R_t \), then scale can be taken into account by adding a constraint of the form

\[ \sum_{t} z_t (R_t Y_{rt}) \geq \left( \sum_{t} z_t \right) R_Y_{rs} \]

or in L.P. form \[ \sum_{t} z_t (R_t Y_{rt} - R_Y_{rs}) \geq 0 \]

Thus, when research output only is maximised, the choice is restricted by the need to produce the appropriate research output for an institution of the resulting scale.
7.9. TESTING AND USING THE FRONTIER

A Frontier estimated from sample observations should be tested for stability and homogeneity before use. A satisfactory Frontier should be used in either of two ways. Institutions that are identified as being on the Frontier should, if they wish to change, move along the Frontier and institutions that are not on the Frontier should move towards it.

7.9.1. Stability

As already stated, C. P. Timmer (1971) shows that a Frontier based on extreme observations only is vulnerable to errors in the data. The Frontier, therefore, should be checked for stability by recalculating it with some of the best points missing. Thus, if 5% of the "good" points are left out and the Frontier stabilises then it could be described as 95% stable. The Author (1978b) checked for stability by carrying out the approach in two successive years and looking for differences in the Frontier and the ranking of non-Frontier institutions. He has yet to apply Timmer's approach. Another approach to stability is that of the resistance to changes in the choice of measures of input and output and this is another approach that the Author intends to explore.

7.9.2. Homogeneity

From the illustration in Appendix 2 (2.11), it is evident that the approach leads to a Frontier which includes some very dissimilar institutions and indicates that the Frontier should be calculated for subsets of the sample, e.g. ex CAT's, to check this out. The natural processes of growth and decay of departments within an institution may make this check not so important for a within-
institutional analysis.

7.9.3. Moving Along the Frontier

The illustration in Appendix 2 also shows that an institution may be on the Frontier because it is very economical in its use of one or two inputs, not because it is economical "on average". It is on the Frontier because no other is "near" the true Frontier. This means that a Frontier institution may wish to move along the Frontier to get a different mix of inputs and outputs. If relative weights are applied to the inputs or outputs, this can easily be done (Goal Programming, for example). Otherwise, some method has to be found to locate adjacent institutions and compare differences. There is a need for further research to produce such a method.

7.9.4. Moving Towards the Frontier

Institutions that are not on the Frontier if they wish can move towards it in several ways. They could become more efficient in the Farrell sense by producing the same output mix but with less inputs, or use the same input mix to produce more outputs. Alternatively, they could move towards a point representing different input and output mix. Again, if relative weights are attached to the inputs and outputs, this is easily done. There is a need, therefore, for further research to produce a method of heading towards the Frontier in a non-Farrell sense.

7.9.5. Using the Frontier

It is obvious from the illustration in Appendix 2 (2.11) that some of the proxy measures are inadequate but the introduction of measures of teaching and research does indicate the right direction, and if
Contradictory results occur they indicate after examination what may be wrong with the proxy measures. For example, research expenditure is biased towards technological universities whereas research publications may be biased in some other way.
7.10. SUMMARY OF CHAPTER 7

Using a geometrical approach it is possible to identify some departments or institutions as "more efficient" and use them as benchmarks for the rest. So far the approach has been applied to either simple one output situations or to multi-input multi-output situations involving crude measures of input and output. Daryl Carlson (1972, 1975) has produced an approach which easily identifies the "more efficient" points but leaves the problem of measures of efficiency for non-Frontier points. M. J. Farrell (1957), on the other hand, has produced a theoretically more complete approach, but one that is slightly restrictive and is difficult to solve in practice. The concepts of scale or quality can be introduced into the analysis so that problems of wastage, student quality, even staff quality, could be straightforwardly taken into consideration.

Thus, the Efficient Frontier and, in particular, the use of L.P. to find it, seems to be a concept with a lot of potential but is a concept that has yet to be fully developed as a practical aid to decision-making within educational institutions.
REFERENCES FOR CHAPTER 7 (in order of mention)

Section


The Author suggests that the examination of the "State of the Art" in performance measurement in Higher Education contained in the previous chapters clearly illustrates a need for further research in specific areas.

The most obvious area for further research is the choice of input and output measures. What should be the measure of research activity? Should teaching be measured by staff input or by student-hours received? The Author suggests, however, that it would be dangerous to concentrate on improving individual measures of input and output at the expense of the development of mechanisms to analyse those measures. The two strands should be followed simultaneously.

Another area for further research is the allocation of costs. Absorption costing implies the sharing out of fixed costs and, if applied to research activity, implies that departments which produce a lot of research publications or attract a lot of research grants provide cheaper teaching as a result. Unit costs are too all-embracing to be used globally but there is potential in the variance analysis approach for showing which factors are influencing the unit costs.

The Author hopes that he has also made out a case for the continued research into the concept of the efficiency frontier. If treated as a mathematical optimisation technique, it is rather limited, but if used as a way of broadly classifying institutions and suggesting comparisons that should be made with similar institutions, it also has potential.
The Author, at present, is involved in informal discussions with the DES about research in the area of variance analysis as outlined in Chapter 6. The contents of Chapter 7 form the basis of a successful application by the Author to the SSRC for a grant to carry out further work on the concept of the efficiency frontier. This study will commence 1 December 1978 and it is hoped it will both check the methodology of the approach and lead to better measures of input and output. In particular, the research will use the Loughborough-Lanchester study data-base to examine the use of the efficiency frontier concept to compare departments.
REFERENCES (In Alphabetical Order)


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### REFERENCES (In Alphabetical Order - Continued)

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APPENDIX 1 - MATHEMATICAL NOTES

1.1. The Legg Formula - Theory and Practice.
1.2. The Effects on Cost Per Meeting of an Allocation of Resources to Research.
1.3. Jean Benard's Ideal University.
1.4. Layard and Verry's Value-Added.
1.5. The Categorisation Approach - An Example.
1.6. Measures of Teaching Activity.
1.7. The "Delany" Ratios.
APPENDIX 1.1. THE LEGG FORMULA – THEORY AND PRACTICE

Keith Legg (1971), as indicated in Chapter 2, develops a staff allocation system based on staff workload. However, he makes three assumptions before he starts. Firstly, he differentiates between lectures which are theoretically open-ended in size and seminars which are limited in size. Secondly, he assumes there are three levels of work corresponding to first/second year work in a UK undergraduate programme of study, final year in the same, and postgraduate work, each of which should be weighted separately because of the "extra effort" involved. Thirdly, he assumes that an academic member of staff can participate in more hours of seminars in a week than hours of lectures.

Thus, he defines the following:

\[ x_i \] = student's lecture hours at level i

\[ m_i \] = student's seminar hours at level i

\[ g_i \] = seminar group size at level i

\[ s_i \] = student numbers at level i

\[ p_i \] = project student at level i

\[ b_i \] = project 1-1 time at level i

\[ w_i \] = weeks at level i

\[ w \] = total weeks available

\[ k_i \] = weighting factor for level i work

with \( t \) = staff load if all lectures

and \( t_s \) = staff load if all seminars.
APPENDIX I. I. continued

Then academic staff required for particular activities are as follows:

Lectures at level i \[ \to \left( k_i \right) \frac{w_i}{t} \]

Seminars at level i \[ \to \left( k_i \right) \frac{s_i m_i}{g_i t_s} \frac{w_i}{w} \]

Projects at level i \[ \to \left( k_i \right) \frac{p_i b_i}{t_s} \frac{w_i}{w} \]

Short courses of a few weeks need more concentrated effort. Therefore, Keith Legg adds a concentration factor "f".

So Short courses at level i \[ \to (f)(k_i) \left( \frac{w_i}{w} \right) \left( \frac{\ell_i}{t} + \frac{s_i m_i}{g_i t_s} \right) \]

This would be sufficient to calculate the staff required to teach the courses involved. However, a particular department will, in general, not teach all of any course and so the staff required by a department has to be built up from its share of each course.

Let \( a_i \) = a department's share of a course's lectures

and \( \beta_i \) = a department's share of a course's seminars

Then the total academic staff required by a department to provide its teaching can be calculated from the formula below.
APPENDIX 1.1. continued

Teaching staff = \[ \sum_{\text{full-time courses}} \left\{ \sum_{i} \left( k_i \frac{w_i}{w} \right) \left( \frac{a_i l_i}{t} + \frac{b_i s_{i,m_i}}{g_i t_s} \right) \right\} \]  

\[ + \sum_{\text{its own full-time courses}} \left\{ \sum_{i} \left( k_i \frac{w_i}{w} \right) \left( \frac{p_i b_i}{t_s} \right) \right\} \]  

\[ + \sum_{\text{short courses}} f \left\{ \sum_{i} \left( k_i \frac{w_i}{w} \right) \left( \frac{a_i l_i}{t} + \frac{b_i s_{i,m_i}}{g_i t_s} \right) \right\} \]  

(The notation has been altered slightly from that of Keith Legg).

As a workload based formula this is perfectly acceptable apart from the weighting factors \( k_i \) and the distinction between \( t \) and \( t_s \). The important point is that the \( a_i \) and the \( b_i \) should take account of joint meetings between courses to avoid double counting and Keith Legg does, in fact, suggest this.

This formula can be applied to a variety of programmes of study in a variety of countries which, in fact, was one of the aims of Keith Legg's work. However, its application to a technological university in the UK allows some simplification to be carried out.
APPENDIX 1.1. continued

Keith Legg suggests the following changes:

1. Undergraduate courses can be treated as two years at level 1 and one year at level 2 with Postgraduate and Short Courses treated as level 3.

2. Seminars are not provided at level 1 and Projects are only provided in the final year of undergraduate courses.

3. The weighting factors $k_1, k_2, k_3$ should reflect the UGC relative weights for undergraduates and postgraduates.

4. Seminars require less effort than lectures so put $t_s = 1.25 t$ (this is another weighting being slipped in).

5. Short courses require double the concentration of postgraduate courses, i.e. $f = 2.0$.

These basically restrict the formula to undergraduate, postgraduate and short courses in a technological university and so are fairly acceptable. However, Keith Legg at this point introduces norms to replace workload measures, as follows:

1. Staff required per week for lectures, ignoring weighting factors, is given by $\frac{t_i}{t}$. It is assumed that the staff required at each level is a constant multiple $u_i$ of the staff required at level 1.

   Thus $\frac{t_2}{t} = u_2 \left( \frac{t_1}{t} \right)$ and $\frac{t_3}{t} = u_3 \left( \frac{t_1}{t} \right)$.

2. Staff required per week for seminars per student, ignoring weighting factors, is given by $\frac{m_i}{g_i t_s}$. It is assumed that the ratio $\frac{m_i}{g_i t_s}$ is a constant multiple of $\frac{m_i}{g_i t_s}$ so that $\frac{m_i}{g_i t_s} = v_i \left( \frac{m_i}{g_i t_s} \right)$. 
APPENDIX 1.1. continued

3. The distribution factors $a_i$ and $b_i$ can be calculated with reference to all levels of a course.

Hence the general formula can be transformed to the following:

$$ Teaching\ staff = \sum a\left(\frac{t}{t}\right) (k_1U_1 + k_1U_1 + k_2U_2) $$

all full-time u.g. courses

$$ + \sum \beta\left(\frac{m_1}{g_1t(1.25)}\right) (0 + 0 + k_2V_2s_2) $$

all full-time u.g. courses

$$ + \sum k_3p_3b_3 $$

own u.g. t(1.25) courses

$$ + \sum \left\{ \alpha\left(\frac{U_3}{t}\right) (k_3U_3) + \beta\left(\frac{m_1}{g_1t(1.25)}\right) (k_3V_3s_3) \right\} $$

all p.g. courses

$$ + \sum \left\{ \alpha\left(\frac{U_3}{t}\right) (k_3U_3) + \beta\left(\frac{m_1}{g_1t(1.25)}\right) (k_3V_3s_3) \right\} $$

short courses

Now $k_1$, $k_2$, $k_3$ are chosen to reflect UGC weightings and Keith Legg shows that they should be 1, 1.25, 1.5.

The various norms $U_1$, $U_2$, $U_3$, $V_1$, $V_2$, $V_3$ and $\frac{g_1}{t}$ and $\frac{m_1}{g_1t}$ are selected from his international survey to reflect the country and the discipline areas involved.
Hence the formula as implemented at a particular technological university is as follows:

\[
\text{Staff} = \sum \{3.26a + 0.0175 s (2.84\beta + 1)\} \\
\text{own u.g. courses}
\]

\[
+ \sum \{3.26a + 0.0497 s \beta\} \\
\text{u.g. servicing}
\]

\[
+ \sum \{2.18a + 0.199 s \beta\} \\
\text{p.g. courses}
\]

\[
+ \sum \{0.0604 w (1 + 0.0916 s)\} \\
\text{short courses}
\]

\[
+ 0.2 \times \text{(number of research students)}
\]

Thus, there is a fixed credit for each course plus a credit per student for seminar work. The first result of the introduction of this formula was a proliferation of overlapping courses instead of the development of courses with a number of options. The second was a tendency to label meetings as seminars rather than small lectures. The third, and perhaps most important, effect was the drift towards large joint lectures between courses backed up by single course tutorials as in the Loughborough-Lanchester study. This is because the number of lectures given is important rather than the number of students involved so the credit is established by providing as large a lecture as possible. However, the seminar credit is based on student numbers and so the credit is established by single course
seminars to give the maximum share of a course's credits. This is an example of how a formula starts off as a workload based formula that is widely applicable but ends up as a narrow norm-ridden formula that is virtually student number based instead and the tragedy is that the calculation of the distribution factors $a_i$ and $b_i$ requires the collection of data on actual hours, actual group sizes, enrolments from a particular course to a particular subject element, and so on, which would allow a true workload based formula to be applied. In fact, in the university in question, the Legg Formula has been replaced by a workload based formula involving some but few norms and requiring virtually the same information from departments each year as the Legg Formula.
APPENDIX 1.2.  
THE EFFECTS ON COST PER MEETING OF AN ALLOCATION OF RESOURCES TO RESEARCH

In Chapter 3 (section 3.2.5.) the problems of allocating resources to research are discussed in general terms. In this Appendix the Author considers the effect on the cost per meeting measure of teaching activity of an allocation of resources to research.

The first assumption is that research input can be measured by time spent and research output by numbers of publications produced.

One approach is to assume that, since the Vice Chancellors and Principals' Committee's Survey (1962) revealed that on average academic staff spent 24% of their time on personal research, each department should charge its research activity with 24% of its total expenditure and charge its teaching activity with the rest. This will simply reduce all unit costs by 24% and lead to course costs being similarly reduced.

Another is to assume that the institution as a whole should charge 24% of its expenditure to the research activity but that individual departments can charge a percentage based on the level of research activity as measured by publications produced. The VCPC Survey showed that the percentage of academic staff time spent on research varied from 14% to 34% for each institution as a whole. Hence, it is suggested that each department should charge its research activity with between 14% and 34% of its expenditure whilst assuming that the institution as a whole charges 24% as follows:

Let $c_i$ be the total recurrent expenditure allocated to department $i$ and let $P_i$ be the publications produced by department $i$ in the
Then the institution as a whole provides teaching and \( \sum P_i \) publications for an "investment" of \( \sum c_i \). If 24% of this is charged to the research activity, the average cost per publication is given by

\[
C = \frac{\sum c_i}{\sum P_i} \times 0.24
\]

Department \( j \) has \( c_j \) expenditure and produces \( P_j \) publications, so let it charge its research activity with the following amount:

\[
(0.24) \frac{\sum c_i}{\sum P_i} \times P_j
\]

This amount should lie within the range 14% to 34% of \( c_j \). If it does not then the appropriate limiting figure should be used instead. The rest of \( c_j \) can then be charged to the teaching activity and used to calculate a cost per meeting and hence a cost per course.

This means, of course, that a department producing more than its fair share of publications will be deemed to be providing cheaper teaching as a result. On the other hand, if a department is producing less than its fair share of publications, its expenditure should rightly be charged mainly against its teaching and hence it should be deemed to be providing more expensive teaching. This method of allocation though removes the possibility of producing a unit cost measure of research performance since cost per publication is fixed by the method as the overall average. Although it does allow the use of the percentage to be allocated to research to be used as a measure of research activity.
Thus for the Loughborough-Lanchester study data for 1972-73, the effect of such a procedure is shown below.

<table>
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<th>Department</th>
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<th>Publications Produced</th>
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<th>Research</th>
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</tr>
<tr>
<td>Tech.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chemistry</td>
<td>191868</td>
<td>16105</td>
<td>73</td>
<td>11.91</td>
<td>*34</td>
<td>7.86</td>
</tr>
<tr>
<td>Ergonomics</td>
<td>118028</td>
<td>7904</td>
<td>9</td>
<td>14.93</td>
<td>*14</td>
<td>12.84</td>
</tr>
<tr>
<td>Maths</td>
<td>112178</td>
<td>8317</td>
<td>22</td>
<td>13.49</td>
<td>30</td>
<td>9.43</td>
</tr>
<tr>
<td>Physics</td>
<td>85681</td>
<td>4497</td>
<td>6</td>
<td>19.05</td>
<td>*14</td>
<td>16.38</td>
</tr>
<tr>
<td>Economics</td>
<td>47330</td>
<td>4263</td>
<td>10</td>
<td>11.10</td>
<td>32</td>
<td>7.50</td>
</tr>
<tr>
<td>Management</td>
<td>113303</td>
<td>5922</td>
<td>26</td>
<td>19.13</td>
<td>*34</td>
<td>12.63</td>
</tr>
<tr>
<td>Soc.Studies</td>
<td>41446</td>
<td>3059</td>
<td>7</td>
<td>13.55</td>
<td>26</td>
<td>10.04</td>
</tr>
<tr>
<td>Library</td>
<td>54395</td>
<td>2726</td>
<td>4</td>
<td>19.95</td>
<td>*14</td>
<td>17.16</td>
</tr>
<tr>
<td>Euro.Studies</td>
<td>33140</td>
<td>4283</td>
<td>7</td>
<td>7.74</td>
<td>32</td>
<td>5.23</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1881372</td>
<td>133170</td>
<td>294</td>
<td>14.13</td>
<td>24</td>
<td>10.74</td>
</tr>
</tbody>
</table>

However, the costs per meeting vary so much that the variations in research output do not change the relative positions of the departments to any noticeable degree as shown by the graph below of cost per meeting treating research as free against cost per meeting modified to take account of research.
Figure 31: Effects on Cost Per Meeting of a Research Allocation
APPENDIX 1.3  JEAN BENARD'S IDEAL UNIVERSITY

In Chapter 4 (section 4.2.3.) Jean Benard's consumption model of a university (1973) is outlined briefly. In this Appendix the Author discusses the approach in more detail albeit with modified notation.

It is a consumption model in that outputs are assumed to "consume" parts of each input so that the inputs are shared out between the outputs using linear relationships. The activities involved are teaching, research and administration, although the latter is termed an intermediate activity and incorporated into the model at a later stage.

When considering the teaching activity Jean Benard distinguishes between a "pedagogically defined unit of value occurring at a particular time and place" (UVTP) which the Author hereafter refers to as a set of meetings, and a "permissible combination of such meetings" (PUVC) which the Author hereafter refers to as a programme of study. Thus, he has a vector of student enrolments to each set of meetings in each year \( Z_1(t) \), a vector of enrolments to each programme of study in each year \( Y_1(t) \), and a vector of successful students from each programme of study in each year \( X_1(t) \).

When considering research activity he suggests that different types of research produce different quantities of research output. Thus, he has a vector of research time input into each type of research \( Z_2(t) \) in each year and a vector of research publications from each type of research in each year \( X_2(t) \).

Teaching and research consume other inputs besides time, however, and they will be equally limited by the capacity of these other inputs.
APPENDIX 1.3. continued

Hence, Jean Benard defines four such limits.

There is a limit on recurrent expenditures in year (t) + Limit Vector \( A(t) \)

There is a limit on fixed equipment and space in year (t) + Limit Vector \( B(t) \)

There is a limit on staff time in each speciality in year (t) + Limit Vector \( C(t) \)

There is a limit on new enrolments to the University in year (t) in each speciality + Limit Vector \( S(t) \)

Each activity "consumes" some of these inputs and so there are a set of matrix operators which define how much input is needed to produce particular levels of output.

\[
[A_1] = \text{matrix of recurrent expenditure coefficients relating to each set of meetings.}
\]

\[
[B_1] = \text{matrix of fixed capital coefficients relating to each set of meetings.}
\]

\[
[C_1] = \text{matrix of staff time coefficients relating to each set of meetings.}
\]

\[
[E_1] = \text{matrix showing maximum pass rates for each programme of study.}
\]
APPENDIX 1.3. continued

\[ A_2 \] = matrix of recurrent expenditure coefficients relating to types of research.

\[ B_2 \] = matrix of fixed capital coefficients relating to types of research.

\[ C_2 \] = matrix of staff time coefficients relating to types of research.

\[ D_2 \] = matrix showing research publications produced by each type of research.

In addition, the different classification involved need to be balanced.

\[ D_1 \] = matrix showing the permissible combinations of sets of meetings, i.e. the study programmes.

\[ F_1 \] = matrix showing the proportions from each speciality attending the programmes of study.

Thus, eight constraints can be written down in linear form. Four of these are the capacity constraints corresponding to recurrent expenditure, fixed equipment/capital expenditure, staff time and student enrolments. Three are consistency style constraints which ensure firstly that the total study body when aggregated via programmes of study equals the aggregation by sets of meetings; secondly, that the aggregation of study programmes within a speciality should not exceed the total enrolments to that speciality; and finally that the number of successful students should not exceed the maximum permitted for each programme of study. The eighth constraint ensures that research output never exceeds that expected from the time put in. It must be remembered that the variables refer to several years so that it is necessary to have consistency equations to allow departments to build up or slow down
APPENDIX 1.3. continued

particular activities over the time period of the model.

The "final" outputs of the model are successful students and research publications and so the objective is the discounted utility of those two outputs over the time period of the model

i.e. Objective is \( \sum_t \alpha^t \left[ U_1(t)X_1(t) + U_2(t)X_2(t) \right] \)

where \( U_i(t) \) are utility coefficients.

So the linear programming formulation of Jean Benard is as below (with revised notation).

Maximise \( \sum_t \alpha^t \left[ U_1(t)X_1(t) + U_2(t)X_2(t) \right] \)

Subject to

\[
\begin{align*}
[A_1]Z_1(t) + [A_2]Z_2(t) & \leq A(t) \\
[B_1]Z_1(t) + [B_2]Z_2(t) & \leq B(t) \\
[C_1]Z_1(t) + [C_2]Z_2(t) & \leq C(t) \\
[D_1]X_1(t) - Z_1(t) & = 0 \\
-E_1]X_1(t) + X_1(t) & \leq 0 \\
[F_1]Y_1(t) - S_1(t) & \leq 0 \\
S_1(t) - X_1(t-1) & = S(t) \\
-D_2]Z_2(t) + X_2(t) & \leq 0
\end{align*}
\]

It should be noted that each line is a matrix equation with a time variation as well so that each line represents a whole set of constraints. This is a linear programming formulation and as such has a dual problem.
APPENDIX 1.3. continued

Jean Benard suggests that it is useful to consider the constraints that appear in the dual problem. The variables of the dual problem will consist of the inputed value or opportunity cost of the various inputs and outputs.

Let:

- Recurrent Expenditures in year t \( \rightarrow M(t) \)
- Fixed equipment and space in year t \( \rightarrow N(t) \)
- Staff time in year t \( \rightarrow P(t) \)
- Teaching activity of each programme of study in year t \( \rightarrow Q(t) \)
- Successful students in year t \( \rightarrow R(t) \)
- Students in a particular speciality in year t \( \rightarrow U(t) \)
- Students that succeed and continue into the next year in year t \( \rightarrow V(t) \)
- Research publications \( \rightarrow W(t) \)

All these dual variables will be positive or zero except for \( Q(t) \) and \( V(t) \) which by complementary slackness will be able to be positive or negative since the corresponding inequalities in the primal are, in fact, equalities.
APPENDIX 1.3. continued

Thus, the dual problem is as follows:

Minimise
\[
\sum_t \left[ A(t)M(t) + B(t)N(t) + C(t)P(t) + S(t)V(t) \right]
\]

Subject to
\[
\begin{align*}
[D_1]Q(t) - [E_1]R(t) + [F_1]U(t) & \geq 0 \\
[A_1]M(t) + [B_1]N(t) + [C_1]P(t) - Q(t) & \geq 0 \\
-U(t) + V(t) & \geq 0 \\
R(t) + V(t+1) & \geq \bar{U}_1(t) \\
[A_2]M(t) + [B_2]N(t) + [C_2]P(t) - [D_2]W(t) & \geq 0 \\
W(t) & \geq \bar{U}_2(t)
\end{align*}
\]

(where \( \bar{U}_1(t) = \sum_t \alpha^t U_i(t) \))

All these dual constraints can be interpreted as statements about the imputed value of the inputs and outputs. They also can be used to indicate which programmes of study or specialisms in teaching should be encouraged and extended using the idea of complementary slackness. For example, the first constraint insists that the imputed value of teaching activity should exceed the difference between the imputed value of registered students and the imputed value of successful students. In other words, the cost of the teaching activity involved must exceed the value added to the students involved. All the others lead to similar statements. If the first constraint set came out in the optimal solution as including an equality, this would imply that the corresponding variable in the primal should have a value, i.e. a corresponding programme of study should run. And if a particular variable in the dual has a value, say \( R(t) > 0 \) then this implies that the corresponding equation in the primal is an equality. In
other words, some programme of study is passing as many as it can and it should be allowed to pass more.

Since administration is deemed an intermediate activity it's introduction leaves the objective unchanged and simply adds some more variables and some more consistency style constraints.

The model is a theoretical one but it does bring out a lot of the consequences of assuming particular relationships and it does allow consideration of the concept of opportunity cost as measured by the imputed values of the dual problem.
APPENDIX 1.4. LAYARD AND VERRY'S "VALUE ADDED"

In Chapter 4 (section 4.3.1.) Layard and Verry's econometric approach to cost functions in Higher Education is discussed and their concept of "value-added to the student" is briefly mentioned. It is, however, a concept which merits further discussion.

Layard and Verry (1975) suggest that the value-added to a student passing through an educational institution should be related to the other more measurable inputs and outputs of a department. Thus they suggest that the quality of the students entering a discipline area should be aggregated using weights based on salary data and the quality of the students leaving the institution should be similarly aggregated. This, of course, ignores the fact that students enrol on courses and are taught by a number of departments. However, ignoring this, if U student years are involved (a similar concept to man-hours) then the value added to the students as a group is given by (g-a)U where gU is the aggregated quality of the graduates and drop-outs and aU is the aggregated quality of the enrolling students. This is a cross-sectional approach not a cohort based study.

Output quality is based on degree classes. If the classifications correspond to a linear scale then a first class degree is (1+3x), an upper second is (1+2x), a lower second is (1+x) where other types of degrees are (1). Teachers on average with a first or second class degree earn 10% more than other graduate teachers so, since the UGC returns for 1968-69 show that 8% of graduates obtained first class degrees, 20% upper seconds, 29% lower seconds and 43% other types of degrees, the scale must be as follows: a first = 1.18, an upper second = 1.12, a lower second = 1.06, and other types = 1.00.
APPENDIX 1.4. continued

This establishes the relative quality in salary terms of the different classifications.

Input quality is based on A-levels. If a linear scale is assumed with 3A's equivalent to \((1+9x)\) and 3D's equivalent to 1.00, then a similar approach based on much less general data leads to a scale consisting of 3A's = 1.18 down to 3D's = 1.00.

Drop-outs are assumed to be .94 on the output scale, i.e. one step below "other types of degree".

The remaining task is to find the relation between the two scales. A survey of electrical engineers aged 35-39 shows that the differential between "other honours degrees" (output scale 1.00) and no degrees but two A-levels or more (input scale 1.06 approx.) was £2145 to £1578. Thus, earnings for a graduate with a weight of 1 are 1.50 times those of A-level holders with a weight of 1. This 50% increase in earnings comes from, in the main, a three year course. Hence each course-year completed produces a 15% increase in earnings. Thus, if \(GU\) = aggregated degree classifications using the output scale and \(AU\) = aggregated A-level qualifications using the input scale

\[
\text{Value Added} = U(G - \frac{A}{1.15})
\]
APPENDIX 1.5. THE CATEGORISATION APPROACH - AN EXAMPLE

As indicated in Chapter 4 (section 4.3.3.) Fritschi et al (1978) have suggested a categorisation approach to multi-dimensional problems. They suggest that performance on each output per unit total expenditure be graded as high, medium and low so that the sample set can be split into distinct groups each of which can be examined for size effects. If this approach is tried on the Loughborough-Lanchester Study data for departments, the grading has to be cruder since there are only 18 departments altogether. Consider two outputs - teaching meetings and research publications, and one input - total departmental expenditure. If each output is divided by the input the results can be graded as above average (H) and below average (L), then the 18 departments can be split into four groups HH, HL, LH and LL.

Thus for the Loughborough-Lanchester Study data for 1972-73 the total expenditure and the two outputs for departments were as follows:
APPENDIX 1.5, continued

<table>
<thead>
<tr>
<th>Department</th>
<th>Cost</th>
<th>Meetings Provided</th>
<th>Publications Provided</th>
<th>Performance Teaching</th>
<th>Performance Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>29638</td>
<td>1060</td>
<td>3</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Chem.Eng.</td>
<td>139413</td>
<td>10359</td>
<td>24</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Civil Eng.</td>
<td>128494</td>
<td>6991</td>
<td>10</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Elec.Eng.</td>
<td>163109</td>
<td>11304</td>
<td>32</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Eng. Prod.</td>
<td>175028</td>
<td>18378</td>
<td>11</td>
<td>H</td>
<td>L</td>
</tr>
<tr>
<td>Mat. Tech.</td>
<td>41705</td>
<td>3557</td>
<td>11</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Mech. Eng.</td>
<td>151587</td>
<td>8973</td>
<td>7</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Trans. Tech.</td>
<td>183472</td>
<td>12012</td>
<td>22</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Inst. Poly. Tech.</td>
<td>71557</td>
<td>3460</td>
<td>10</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Chemistry</td>
<td>191868</td>
<td>16105</td>
<td>73</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Ergonomics</td>
<td>118028</td>
<td>7904</td>
<td>9</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Maths.</td>
<td>112178</td>
<td>8317</td>
<td>22</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Physics</td>
<td>85681</td>
<td>4497</td>
<td>6</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Economics</td>
<td>47330</td>
<td>4263</td>
<td>10</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Management</td>
<td>113303</td>
<td>5922</td>
<td>26</td>
<td>L</td>
<td>H</td>
</tr>
<tr>
<td>Soc. Studies</td>
<td>41446</td>
<td>3059</td>
<td>7</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>Library</td>
<td>54395</td>
<td>2726</td>
<td>4</td>
<td>L</td>
<td>L</td>
</tr>
<tr>
<td>Euro. Studies</td>
<td>33140</td>
<td>4283</td>
<td>7</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>TOTAL</td>
<td>1881372</td>
<td>133170</td>
<td>294</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The resulting four groups are made up as follows:

<table>
<thead>
<tr>
<th>LL</th>
<th>HH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>Chemical Engineering</td>
</tr>
<tr>
<td>Civil Engineering</td>
<td>Materials Technology</td>
</tr>
<tr>
<td>Mechanical Engineering</td>
<td>Chemistry</td>
</tr>
<tr>
<td>Transport Technology</td>
<td>Mathematics</td>
</tr>
<tr>
<td>Institute of Polymer Technology</td>
<td>Economics</td>
</tr>
<tr>
<td>Ergonomics</td>
<td>Social Studies</td>
</tr>
<tr>
<td>Physics</td>
<td>European Studies</td>
</tr>
<tr>
<td>Library Studies</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LH</th>
<th>HL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical Engineering</td>
<td>Engineering Production</td>
</tr>
<tr>
<td>Management</td>
<td></td>
</tr>
</tbody>
</table>
The resulting categories are not obviously biased in any way. For example, the discipline areas are evenly distributed in HH and slightly less so in LL. The total expenditure as a measure of size is evenly spread through each so the effect of dividing through by the input does not, in this example, affect the choice of categories.

HH has a mean expenditure of 102,857 with an S.D. of 48,406.

LL has a mean expenditure of 86,726 with an S.D. of 57,295.

The two sample means are not significantly different either.

This approach has the advantages of being simple but in return the types of analysis that can be carried out are limited. Thus category HH can be treated as efficient and category LL can be treated as inefficient, but what can be said about LH and HL? The method is at its best when applied to the solution of a particular question such as "Is size an important factor in performance?" rather than in the establishment of performance measures themselves.
APPENDIX 1.6. MEASURES OF TEACHING ACTIVITY

In Chapter 5 (section 5.2.1. and section 5.2.2.) when discussing input-output ratios and direct measures of provision for the teaching activity it is emphasised that there is a need for careful definition of the various measures, particularly where averages are concerned. The Author suggests in Chapter 2 (section 2.2) that the following information is necessary if the teaching activity is to be analysed properly.

For a set of meetings;

(i) the total enrolment to a study programme (E)
(ii) the enrolment from a study programme to a particular subset of meetings (s where s < E)
(iii) total enrolment from all courses to a particular subset of meetings (E*)
(iv) the department providing the tuition
(v) the type of space utilised
(vi) the number of student groups (each assigned to one teacher) formed in a particular subset of meetings (g)
(vii) the total number of hours attended by a student in a particular subset of meetings of a particular group size (h).

The various input-output ratios and direct measures of educational provision can then be defined precisely. Most of them require the calculation of class contact hours relating to a course or department. The calculation for a course must take joint meetings into account and the Author suggests that the time of a staff member should be shared out pro rata to the students involved.
APPENDIX 1.6. continued

Thus, for a particular subset of meetings that a particular course attends

Student taught hours involved = h

Student hours involved = hs

Meetings allocated to those s students from the course = (hg) x $\frac{s}{E^*}$

Group size = $\left(\frac{E^*}{g}\right)$.

Then

Student-Staff Ratio for a course = $\frac{\text{Relevant Students}}{\text{Relevant Staff}}$

= $\frac{E \times \text{Average Staff Load}}{\text{Allocated Staff Hours}}$

= $\frac{E \times \text{Average Staff Load}}{\sum_{\text{subjects of course}} (hg) \times \frac{s}{E^*}}$

Student-Staff Ratio for a department

= $\frac{\text{Relevant Students}}{\text{Relevant Staff}}$

= $\frac{\sum_{\text{departments}} \sum_{\text{subjects}} h \times E^*}{\sum_{\text{departments}} \sum_{\text{courses}} (hg)} \times \frac{\text{Average Staff Load}}{\text{Full-time Student's Load}}$
APPENDIX 1.6. continued

A Student's Taught Hours for a course including options

$$\text{Average Class Size as perceived by a student on a course including options}$$

$$\text{Proportion of time saved by use of Joint Meetings}$$

$$\text{Average Class Size provided by a Department}$$

Note that at institutional level when summation via courses equals summation by departments

For example, if the first term of the Painting and Decorating Course at St. Albans College of Building is examined the pattern of meetings is as follows:
Enrolment = 6

| Lectures | 6 | 13$\frac{1}{2}$ | 5 | 1 | 6 | Home |
| Labs. | 6 | 14$\frac{1}{2}$ | 5 | 1 | 6 | Home |
| Options | 6 | 2$\frac{1}{2}$ | 5 | 6 | 69 | Service |
| Ind. Studies | 6 | 1 | 5 | 1 | 6 | Service |

This information can be summarised as follows:

<table>
<thead>
<tr>
<th>Own Student-Hours (shw)</th>
<th>Group Size $E^*$</th>
<th>Meetings Given (hgw)</th>
<th>Meetings Allocated $s(E^*)$</th>
<th>Service</th>
<th>Special Space</th>
</tr>
</thead>
<tbody>
<tr>
<td>405</td>
<td>6</td>
<td>67$\frac{1}{2}$</td>
<td>67$\frac{1}{2}$</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>435</td>
<td>6</td>
<td>72$\frac{1}{2}$</td>
<td>72$\frac{1}{2}$</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>67$\frac{1}{2}$</td>
<td>11$\frac{1}{2}$</td>
<td>67$\frac{1}{2}$</td>
<td>5.87</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>30</td>
<td>6</td>
<td>5</td>
<td>5</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>937$\frac{1}{2}$</td>
<td>212$\frac{1}{2}$</td>
<td>150.87</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

A Student's Average Taught Hours = $\frac{937\frac{1}{2}}{6} = 156\frac{1}{4}$ per term

A Student's Average Group Size = $\frac{5996\frac{1}{2}}{937\frac{1}{2}} = 6.4$

Student Staff Ratio for the course = $\frac{6 \times \text{Staff Load}}{150.87}$

= 9.31 for the term but 3.6 for the 5 weeks the block runs for.

So 3.6 reflects the actual environment provided during 5 weeks whereas 9.3 reflects the actual resources used for the whole term.
APPENDIX 1.7.  THE "DELANY" RATIOS

In 1972 the Pooling Committee published its proposals for the calculation of student-staff ratios for advanced work together with targets bands for the different discipline areas, and these became known as the "Delany" ratios after the Civil Servant who carried out the survey on which they were based. The method has to cope with differing modes of attendance, for example, block release, and part-time day and evening, and so is bedevilled by the problem of defining full-time equivalents. So much so that, as mentioned in Chapter 2, a working party has been set up by the DES under the same Civil Servant to try and devise an acceptable method of calculating full-time equivalents.

The ratios are teacher based in that they involve calculations of student-staff ratios for departments and groups of departments rather than for courses and groups of courses, and so care should be taken to treat them accordingly. A measure of teaching time provided by a department is not necessarily any indication of the teaching environment experienced by particular students on particular courses.

Full-time Equivalent Staff are defined as

\[
\frac{\sum_{\text{grades of staff}} \text{Teaching Hours by each grade of staff}}{\text{Average Hours for that grade}}
\]

This is to enable the existence of different maximum teaching loads for different grades of staff to be taken into account but it immediately moves into an area of credits and norms rather than genuine full-time equivalents.
Full-time Equivalent Students are defined as

\[
\frac{\sum_{\text{all courses}} \text{student-hours}}{\text{Average Taught Hours for a full-time student}}
\]

This is to enable the aggregation of different modes of attendance. However, the denominator of the ratio can be chosen in several ways (all acceptable to the DES), namely as a genuine average, as an average for particular discipline areas, or as a norm.

Then the Student Staff Ratio for a Department

\[
\frac{\text{F.T.E. Students}}{\text{F.T.E. Staff}}
\]

This is acceptable provided the appropriate student hours and staff hours are allocated to the Department in question to cater for the effect of servicing.

Average Student Taught Hours (ASH)

\[
\frac{\sum_{\text{Full-time courses}} \sum_{\text{subjects}} (sh)}{\sum_{\text{Full-time subjects}} (E)}
\]

Average Lecture's Load (ALH)

\[
\frac{\text{Total Teaching Hours}}{\text{Total F.T.E. Staff}}
\]

The denominator of this ratio involves the different workloads for different members of staff and so this is not a true average.
APPENDIX 1.7. continued

Average Class Size = \( \frac{\text{Total Student Hours}}{\text{Total Teaching Hours}} \)

This is the average class size provided which is not the same as that perceived by the student since if the one is an arithmetic mean, the other is the harmonic mean as shown in Appendix 1.6.

However, given the last three ratios, the student-staff ratio can be written as

\[
SSR = \frac{\text{ACS} \times \text{ALH}}{\text{ASH}}
\]

At first sight this formula could be applied to a course but it must not be because ACS is the average class size provided to all sorts of students by a department not the average class size perceived by a student on the course, as pointed out in Appendix 1.6.

It should also be noted that if "ASH" is calculated as total curricular hours divided by the full-time equivalent students, then ASH becomes by the definition of full-time equivalent students equal to the tuition load of a typical full-time student, i.e. equal to a norm, and so is not a true average any more.
APPENDIX 1.8. A MEASURE OF RESEARCH PERFORMANCE

In Chapter 5 (section 5.3), when discussing measures of research performance, it is suggested that if the time spent on research is known then a cost per publication produced can be calculated and used as a measure of performance. The Vice Chancellors' and Principals' Committee's Survey (1962) indicated that on average an academic would allocate 24% of his time as research time. Hence, 24% of a department's expenditure could be allocated to its research activity and compared with the research output. This means, of course, that 76% of a department's expenditure will be charged to its other activities, mostly teaching. If it is assumed for purposes of illustration that there are two activities — teaching and research — then a cost per meeting and a cost per publication can be established in this way.

It should be noted that since a fixed proportion is used the relative performance on the teaching measure will be unaffected by relative performance on the research measure unlike the situation in Appendix 1.2.

Thus, for the data of the Loughborough-Lanchester Study for 1972-73 produces the following Teaching/Research Performance Measures for Departments.
<table>
<thead>
<tr>
<th>Department</th>
<th>Cost per Meeting (£)</th>
<th>Cost per Publication (£00's)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education</td>
<td>21.25</td>
<td>23.71</td>
</tr>
<tr>
<td>Chem. Eng.</td>
<td>10.24</td>
<td>13.94</td>
</tr>
<tr>
<td>Civil Eng.</td>
<td>13.96</td>
<td>30.84</td>
</tr>
<tr>
<td>Elec. Eng.</td>
<td>10.97</td>
<td>12.23</td>
</tr>
<tr>
<td>Eng. Prod.</td>
<td>7.24</td>
<td>38.19</td>
</tr>
<tr>
<td>Mat. Tech.</td>
<td>8.91</td>
<td>9.10</td>
</tr>
<tr>
<td>Mech. Eng.</td>
<td>12.84</td>
<td>51.97</td>
</tr>
<tr>
<td>Trans. Tech.</td>
<td>11.61</td>
<td>20.02</td>
</tr>
<tr>
<td>Inst. Poly. Tech.</td>
<td>15.72</td>
<td>17.17</td>
</tr>
<tr>
<td>Chemistry</td>
<td>9.05</td>
<td>6.31</td>
</tr>
<tr>
<td>Ergonomics</td>
<td>11.35</td>
<td>31.47</td>
</tr>
<tr>
<td>Maths.</td>
<td>10.25</td>
<td>12.24</td>
</tr>
<tr>
<td>Physics</td>
<td>14.48</td>
<td>34.27</td>
</tr>
<tr>
<td>Economics</td>
<td>8.44</td>
<td>11.36</td>
</tr>
<tr>
<td>Management</td>
<td>14.54</td>
<td>10.46</td>
</tr>
<tr>
<td>Social Studies</td>
<td>10.30</td>
<td>14.21</td>
</tr>
<tr>
<td>Library</td>
<td>15.16</td>
<td>32.64</td>
</tr>
<tr>
<td>European Studies</td>
<td>5.88</td>
<td>11.36</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>10.74</strong></td>
<td><strong>15.36</strong></td>
</tr>
</tbody>
</table>

This pair of unit costs can be plotted on a graph to show the spread of departmental values.
Arguably Chemistry (C) and European Studies (ES) are better than the rest. However, the spread is fairly wide and indicates little relationship between teaching and research performance, as measured in this way. So departments performing well on research vary just as much in cost per meeting as departments who do not perform so well as measured in this way.